University of the Witwatersrand
School of Education

INVESTIGATING THE NATURE OF CHALLENGES FACED BY GRADE 12 TEACHERS IN THE TEACHING AND LEARNING OF PROBABILITY AND COUNTING PRINCIPLES IN SOUTH AFRICA. A CASE OF THOKOZA, EAST RAND

By

OBADIAH DUBE

A research project submitted to the
Faculty of Education
University of the Witwatersrand, Johannesburg,
In partial fulfillment of the requirement
For the M.Ed. (Masters) degree.
Student number: **2290612**

Supervisor: Dr. George Ekol

Date of Submission: March 2022

Abstract

This study investigates the challenges faced by mathematics teachers in the teaching of probability counting principles (PCP) in Thokoza secondary schools in Johannesburg, South Africa. The motivation for doing this study came from two main observations. The first observation is learners' deficient performance in the topic of probability and counting principles (PCP) in the matric examinations over an extended period from 2014. Results from my school always indicated that PCP was the worst topic in terms of learner performance in different tasks. Despite all this, I realized that less attention has been paid to the development of teachers regarding probabilistic thinking. Department of Basic Education (DBE) diagnostic reports revealed that matric learners performed badly in PCP from the time the topic was introduced in 2014 to the present. Examiners' comments suggest that this topic is not properly taught in schools.

The second reason was that, when I did my Honours degree l investigated problems that teachers faced in teaching probability. In doing that study I found out that there was not much literature about teachers' challenges in teaching probability, yet the topic was poorly done by learners. This encouraged me to want to know more about probability and how its teaching could be improved.

This study focused on investigating the challenges faced by mathematics teachers in the teaching and learning of probability especially at grade 12 level. It was a qualitative study with 11 mathematics teachers participating from five different secondary schools. Data was collected through an open-ended questionnaire, short text messages (follow-up messages), classroom observations and document analysis. Questionnaire responses from teachers indicated that teachers found it difficult to explain probability terms. The majority of the teachers indicated that they faced challenges when teaching probability tree diagrams, dependent and independent events. Implications for this study have been made to address the specific challenges identified in this report.

DEDICATION

This research project is dedicated to my family.

ACKNOWLEDGEMENTS

I want to express my gratitude to everyone who gave any form of support, be it spiritual, material, or emotional, in the writing of this report. I would not have accomplished this alone.

- Special thanks to my Heavenly Father the source of all knowledge, strength, and encouragement for helping me throughout the course and for enabling me to undertake my research successfully.
- I want to express my sincere appreciation to my supervisor, Dr. George Ekol, for the guidance and advice provided throughout the planning and writing of this report. Thank you, sir.
- My family you are the best, especially my son Mbongeni.
- My school, for allowing me to do this research
- Finally, I would like to extend my heartfelt gratitude to all teachers who gave their time to participate in this study.

PLAGIARISM DECLARATION

I, Obadiah Dube (Student Number: 2290612) am a student registered for the M.Ed. Course EDUC 7031A (Research Report) in the year 2022. I hereby declare the following:

- I am aware that plagiarism is wrong.
- I confirm that the work submitted for assessment for the above course is my own unaided work except where I have explicitly indicated otherwise.
- I have followed the required conventions in referencing the thoughts and ideas of others.
- I understand that the University of the Witwatersrand may take disciplinary action against me if there is a belief that this is not my own unaided work or that I have failed to acknowledge the source of the ideas or words in my writing**.**

Signature: Dube

Date: 15 March 2022

TABLE OF CONTENTS

CHAPTER 3: METHODOLOGY

CHAPTER 4: FINDINGS AND ANALYSIS

CHAPTER 5 SUMMARY, DISCUSSION AND RECOMMENDATIONS AND CONCLUSIONS

Appendices

List of tables

List of Figures

CHAPTER 1 - GENERAL INTRODUCTION

1.1 Introduction

In South Africa, probability is taught and assessed at national examination at Grade 12 under the general topic of probability and counting principles (PCP). Throughout this report, however, and for simplicity's sake, I will only refer to probability to represent PCP at Grade 12. The teaching of probability is a challenge to many mathematics teachers all around the world to the extent that in some countries its introduction led to a decline in the mathematics pass rate. In Turkey for example, a qualitative study by Sezgin-Memnun (2019) involving 142 grade 11 learners showed that many of the students failed to solve open ended probability problems and had difficulty with almost every problem. In the United Kingdom, Pratt (2012) investigated students' reasoning about data and probability and acknowledged that there are serious problems associated with the teaching and learning of probability and that this problem is well documented in literature. In South Africa, Morkel (2014) reveals that the matric results of 2014 experienced a decline of 2,4% from 78,2% in 2013 to75,8% in 2014 and one of the reasons given for the decline was the introduction of probability and Euclidean geometry in that year in the matric examinations. The decline in percentage pass in South Africa caused by the introduction of the new topics was understandable given that this was the first time these topics were examined in the new syllabus, it meant that both the teachers and learners were still adapting to their inclusion. The worrying factor is that of performance in probability in matric examinations which has not improved in many years.

It is now more than seven years since probability became a compulsory section in the mathematics Paper 1 of South Africa's national matric examinations but the performance of learners in this topic has remained poor. For a long time, probability was not in the mainstream mathematics syllabus for the Further Education and Training (FET) band until 2012 when it was first introduced at grade 10 level under the new Curriculum Assessment Policy Statement (CAPS) (Mutara & Makonye, 2014). Under the new CAPS syllabus, probability has been examined in paper 1 grade 12 since 2014. It contributes about 18% of the total marks in paper 1 (Mutara and Makonye, 2014). However, since its introduction in 2014, learner performance in this topic has been low. The challenges in the teaching and learning of this topic need to be identified and discussed.

Considerable research in probability (DeKock, 2015; Kodisang, 2016; Makwakwa, 2012; Mutara and Makonye, 2014; Awuah, 2018) has been done but the focus has always been on learners rather than teachers. Problems faced by teachers in South Africa in teaching probability have not been given much attention. The present study aims to address that problem. It is profound that the problem of probability be investigated from both sides that is, learners and teachers. It is worth noting that probability is traditionally considered one of the most difficult areas of mathematics as probabilistic arguments often come with apparently counterintuitive results (Groth, Butler, & Nelson, 2016). This view is supported by Batanero et al. (2016) who argues that probability creates special challenges to teachers because of its characteristics. Counterintuitive concepts of probability make the teaching of this of this topic difficult as teachers are not only required to know that there are several ways to approaching a probability problem, but also mut be aware that there are many methods to solve probabilistic problems (Knudtzon, 2019). Also important is how the teacher communicates with learners and listen to their arguments to gain understanding of their probabilistic reasoning (Knudtzon, 2019). This requires great expertise from the teacher who should facilitate mathematical arguments and discussions that foster the skills of problem-solving in learners.

Promoting the acquisition of problem-solving skills is of great importance if the dream of the fourth revolution is going to be realised (Awuah, 2018). According to the Department of Education (DBE), (2011), "Mathematics is a discipline that aids the development of mental processes that improve logical and critical-thinking accuracy and problem-solving skills needed in the making of decisions." This means that mathematics is linked to most job opportunities and professions. Therefore, a strong foundation in mathematics concepts prepares young people in the acquisition of essential problem-solving skills. However, evidence abounds that mathematics in general has been denied many South Africans during the colonial and apartheid era, and currently some topics such as statistics and probability pose a challenge to most learners in South Africa (Makwakwa, 2012; Spaull & Kotze, 2015). Given that the overall mathematics results at matric are continuously being affected by the learner performance in probability, it is important for this study to explore teachers' experiences on how they teach the topic. The purpose of this study is to investigate the nature of challenges faced by Grade 12 teachers in the teaching and learning of probability and counting principles in Thokoza, East Rand, Gauteng.

1.2 **Background of the study**

Research done worldwide on probability indicates that teachers encounter enormous challenges in teaching probability concepts. Many researchers (e.g., Batanero, 2013, Groth, Butler &

Nelson, 2016) agree that teaching probability is difficult because learners are challenged by probabilistic reasoning. Groth, Butter, & Nelson (2016) contend that teachers face challenges in explaining the language used in probability especially probability terms which make it difficult for learners to comprehend classroom discussions. Ogbonnaya & Mogari (2014) affirm that challenges of learner understanding become the teacher's problem because the teacher should come up with teaching strategies to assist learners understand the concepts being taught. Studies carried out in South Africa indicate that problems of learning probability are escalated by teachers' inadequate knowledge of teaching probability, particularly in large classes in public schools (Kodisang, 2016; DeKock 2015). A South African study by DeKock (2015) investigating the relationship between teachers' content knowledge and learner performance in probability in grade 12 revealed that some mathematics teachers struggle with probability. A sample of six teachers from DeKock (2015)'s study who wrote probability tests equivalent to a grade 12 level obtained the following marks: 100%, 97%, 77%, 73%, 53% and 43%. From these results, it is the two topmost teachers who can be confident to teach probability effectively to grade 12 learners. The other teachers' scores reflect inadequate content knowledge especially those who scored 43% and 53% their scores are too low for a teacher to be able to teach probability at Grade 12 level.

1.2.1 Probability in the South African Curriculum

In the present South African Curriculum Assessment Policy Statements (CAPS) the topic of probability is one of the core topics in Paper 1 mathematics from grade 10, 11 and 12 (Mutara and Makonye, 2014). The content of probability covered in these grades includes theoretical and experimental probability, mutually exclusive events, dependent and independent events. At grade 12 the content is extended to include fundamental counting principles. The average weighting from grade 10, 11 and 12 is 18%. Students are expected solve probability problems using Venn Diagrams, contingency tables, and probability tree diagrams. According to the DBE (2011: p.5), learners are expected to, "identify and solve problems, and make decisions using critical thinking". The document also emphasises that students must be afforded the opportunity to communicate appropriately by using descriptions in words, graphs, symbols, tables, and diagrams and to develop problem-solving and cognitive skills (DBE, 2011). Moreover, teachers should guide learners in active and critical learning as opposed to rote and uncritical learning.

1.3 Low achievement by Grade 12 learners

The low achievement by grade 12 learners in the probability section in matric examinations indicates students' difficulties in answering probability questions. It may also be an indicator of misconceptions students have in understanding probability concepts

Table 1: South African learners' pass rate in probability (2014-2019)

Year	2014	2015	2016	2017	2018	2019	2020
Probability %	39%	28%	65%	41%	31%	21%	18%

Table 1 above shows learner performance in Grade 12 in probability from 2014 to 2020. Except the 65% performance achieved in 2016, learners have performed very poorly in probability over the period investigated. Using data presented in Table 1 above to calculate the average performance between 2014 and 2020 one gets 34, 7%. This is a very low average and is a cause for concern for mathematics national results. According to the percentages shown on Table 1, the 2020 percentage pass of the probability question was 18% which is the lowest since 2014 when the topic was first examined. The pattern does not show signs of improvement.

The main reason for the poor performance in probability suggested by the South Africa's Department of Education (DBE) (2016) was that the subject matter was still unfamiliar to some of the teachers then. However, this problem has persisted as reflected by low pass percentages up to 2020. The trend displayed in Table 1 above triggered the desire to want to find out the nature of the problems teachers faced in teaching probability in schools, especially teaching Grade 12 learners to improve their performance in probability.

Prolonged failure to achieve goals causes frustration and frustration might result in further decline in performance. Brodie (2017) although referring to the problems of teaching mathematics in South Africa maintains that prolonged failure to achieve something is frustrating. I argue that this could be the case with mathematics teachers regarding probability. Brodie (2017) argues that learners' failure often translates into teachers' failure. When learners fail, it reflects badly on the teachers. Frustration creeps in when teachers continue to put so much effort, trying very hard to change the situation and yet achieve nothing. For example, in some schools, there are catch-up lessons in the afternoon, during weekends, and during holidays that are organized by mathematics teachers all around South Africa to improve mathematics results, but the persistent poor performance remains the same particularly in rural and township public schools. This discourages both the teachers and the learners.

1.4 Problem Statement

There is widespread evidence from literature that mathematics is challenging for most of South African learners. In January 2020 when South Africa's Department of Basic Education released the 2019 matric results the country celebrated 81.3% overall pass in the 2019 examinations (see DBE, 2019). However, the story was different for the performance in mathematics which dropped from 58% in 2018 to 54% in 2019. The concern is that the 54% measured the number of learners that scored 30% or above, not 50% upwards. According to the DBE 2019 report, the decline was in two ways: first, the students who wrote mathematics in 2019 (222 034) were fewer than those who wrote in 2018 (270 516). Many learners over the years have been migrating from mathematics to mathematical literacy to avoid the more challenging mathematics. Schools also allow a limited number of learners to do mathematics to maintain good percentage pass rates in this subject. Spaull (2013) noted that within the period of four years (2008-2011), the number of learners doing mathematics as opposed to mathematical literacy has dropped from 56% to 45% as more learners view mathematical literacy as the easier option of the two.

A major concern about South Africa's poor learner performance in mathematics is the shortage of critical skills in some key areas of the country's economy. According to Siyepu (2013) South Africa is experiencing a shortage of critical skills in fields like engineering, architecture, medicine, and many other professions where mathematics is a prerequisite because of learners 'poor performance in the subject. This situation is worsened by the decline mentioned above in the number of learners who do pure mathematics as many opt for mathematical literacy. The problem of high failure rate in mathematics in South Africa has been linked to several factors. Olivier (2017) identified the following as contributory factors; models used to teach mathematics, lack of qualified mathematics teachers in schools, learners' attitude, feeling of discouragement and boredom in doing mathematics, lack of resources for example textbooks and disparities in schools created by the past political history. Literature also reveals that frequent changes in the school curriculum have contributed to poor mathematics results, (Schmidt, 2017; Tshiredo, 2013; Van der Horst & McDonald, 2014). Frequent curriculum changes in South Africa since 1994 have been found to be a major cause of teachers' struggles in successfully teaching some topics in the mathematics curriculum (Atagana, Mogari, Kriek, Ochonogor, Ogbonnaya, Dhlamini and Makwakwa, 2010, Ramnarain & Fortus, 2013).

The reason for the researcher's interest in carrying out a study in investigating challenges facing teachers in teaching probability is that probability is a relatively new concept in the South African mathematics syllabus. Prior to 2012, probability was not a compulsory section in the mathematics matric examinations (Mutara and Makonye, 2014). The topic was examined in an optional paper, mathematics Paper 3. With the introduction of CAPS, probability along with a few other topics became compulsory sections of the syllabus. The implication of this change was that not all mathematics teachers had the essential content knowledge, critical strategies required to teach probability, good understanding of language of probability and the expertise to manage interactions in the classroom to promote the teaching and learning of probability (Ogbonnaye and Awuah, 2019). Awauh (2018) in his study affirmed that the probability taught in South Africa's mathematics curriculum is problematic to both teachers and learners. A study by Kodisang (2016) which investigated teaching strategies used by Grade 6 teachers in teaching probability in a province in South Africa revealed that, first, teachers never gave their learners opportunities to take the lead in the activities and to interact with each other. Secondly class discussions whether between the teacher and the learners and amongst the learners were minimised in all the lessons observed in the study. Thirdly, teachers asked their learners closed questions which do not stimulate critical thinking in learners as they engage with probability. All the practices observed in the study above are contrary to Wells (2014)' findings that, learner reasoning increases through discussions and verbal arguments with the teacher and other learners. Wells (2014) argued that learners' cognitive processes are improved as learners engage in discussions with others and when explain and justify their claims. The points discussed above led to the research objective outlined below and the related research questions of the study.

1.5 Research Objectives

The main objective of the study was to investigate the challenges encountered by Grade 12 mathematics teachers in the teaching and learning of probability. The study considered the nature of the problems in relation to the following aspects: the language of probability, teachers' understanding of probability content, teachers' knowledge of specific subtopics of probability, use of technology in teaching probability, and facilitating classroom interaction when teaching probability. This is a qualitative study that involved eleven (11) Grade 12 teachers from Thokoza secondary schools, Johannesburg, South Africa.

1.6 Research questions

In conducting this research, the following research questions are addressed:

Main Question: What are the challenges faced by mathematics teachers when teaching probability in terms of: language of probability; teachers' content knowledge of probability; knowledge of subtopics of probability; use of technology in teaching probability; and classroom interaction.

To address the main research question above, the following sub-questions have been formulated:

(i) What are the challenges faced by teachers in terms of the language of probability?
(ii) What are the teachers' challenges with probability content knowledge?
(iii) What are teachers' challenges with some specific subtopics of probability?
(iv) What are teachers' challenges with the use of technology in teaching probability?
(v) What are the teachers' challenges in facilitating classroom interaction when teaching probability?

1.7 Rationale

There are two reasons for carrying out this kind of study. Firstly, the aim is to get the teacher's own perspectives about the problems on the ground that lead to poor performance in probability as revealed in the DBE annual diagnostic reports from 2014 up to 2020. I have also had many conversations and discussions with other mathematics teachers concerning the problems they face in teaching probability, but none of the problems have been documented. The second reason is that the researcher used google scholar to search for similar studies done in South Africa using characters such as, "barriers, challenges, problems, or obstacles faced by teachers in the teaching of probability in South Africa". The results from google scholar indicated that no similar studies have been carried out recently to investigate the difficulties faced by teachers in teaching probability Gauteng, South Africa. The studies that were found were broad studies that focused on obstacles, barriers, or problems faced by South African teachers in the teaching of mathematics. Studies on probability carried out in South Africa focused on learners' problem-solving skills rather than exploring problems related to the teaching of probability (e.g., Awuah, 2018; DeKock, 2015). Awuah (2018)'s focussed on learners' problem-solving skills while DeKock (2015) looked at the relationship between teachers' content knowledge and learner performance in probability.

1.8 Significance of the study

The study will inform practising teachers about the teaching challenges identified from the case study and the recommendations made. Moreover, all the information gathered from the questionnaire, lesson observations and document analysis about the challenges teachers are facing in teaching probability will be made available to all stakeholders including mathematics departmental heads, principals, district and provincial education officers, curriculum designers, mathematics researcher scholars and any other interested parties in mathematics education. Such information will be useful in designing future intervention programmes or professional development programmes.

1.9 Outline and organisation of the research report.

This research report consists of five chapters. Below is a brief outline of each chapter:

Chapter 1: General introduction and background of the study

Chapter 1 comprises seven sections including the introduction section. The next section 1.2 addresses the background of the study which discusses the general problems encountered in the teaching of probability including the situation in South Africa. Section 1.2.1 discusses probability as it is in the South African curriculum. This is followed by the statement of the problem in Section 1.3 and low achievement in probability by grade 12 learners in Section 1.4. In Section 1.5, I present research objectives, followed by research questions in Section 1.6. Section 1.7 explains the rationale of the study. Section 1.8 outlines the significance of the study, 1.9 gives an outline of the entire report or a summary of the five chapters and Section 1.10 concludes Chapter 1 by a summary.

Chapter 2: Literature review and theoretical framework

In Chapter 2, I discuss the literature review which dwells on the challenges teachers face in teaching probability. The literature reviewed on challenges teachers face when teaching probability included both international and South African studies. In the same chapter is a conceptual framework which the research study. This will involve the discussion of the concept of argumentation and its relevance to teaching probability.

Chapter 3: Research design and methodology

Chapter 3 provides a discussion of the qualitative methodological approach the researcher followed to conduct the study. This includes a description of the research paradigm and research design. The study population and sampling procedures are discussed leading to the data collection methods, data collection tools, trustworthiness of the study and ethical considerations. The chapter concludes with an analysis of data. A brief summative conclusion is provided to sum up the chapter.

Chapter 4: Data presentation and analysis

This chapter presents participants' responses and the analysis of the findings from these data. It provides a presentation and interpretation of participants' views on challenges teachers face in the teaching and learning of probability from the selected schools in Thokoza. The analysis revealed a multifaceted nature of challenges, some of which have been identified in the literature review and some others emerging from participants' responses.

Chapter 5: Summary and conclusion

Chapter 5 is the last chapter which presents the summary of the study, a discussion of the findings, a conclusion, and limitations of the study. The findings of the study are discussed in relation to the theoretical framework that guided the study. Implications of this are provided at the end.

Chapter summary.

Chapter 1 outlined the overall picture of what the study seeks to investigate. Apart from the introduction, the chapter gave a background of the study which provided the context the study and its relevance. The chapter also presented the rationale, the objectives and the research questions guiding the study. I have also explained the significance of the study in the field of mathematics education.

CHAPTER 2. LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.0 Introduction

This chapter focusses on the literature reviewed on the challenges of teaching probability around the world and in South Africa. It also provides a conceptual framework which is used as a 'map' to guide the writing of this research report. One sure way of addressing problems of teaching probability to grade 12 learners in Thokoza schools, Johannesburg is to review the existing literature of this topic in internationally and in South Africa. A literature review locates this study in probability research and helps the writer to find out what other researchers have already found or not found regarding the challenges teachers face in teaching probability. The review process will provide first, the introduction of all literature reviewed before discussing the findings from international and South African studies on probability. This will be followed by a discussion on the nature of probability. The next section focusses on teachers' probabilistic knowledge which encompasses discussions on specialised content knowledge (SCK), knowing content and students (KCS) and knowing content and teaching (KCT). Next, I will examine the professional training given to teachers to teach probability. The next subheading looks at curriculum documents and text books and show they impact on the teaching and learning of probability. From there I deliberate on language of probability and methodical practices in mathematics in South Africa. The last section outlines the conceptual framework.

2.1 Literature review.

2.1 Introduction

This section of the research explores some fundamental findings from existing literature on the challenges teachers face when teaching probability in schools. In addition, it discusses the essence and extent of these challenges especially in South Africa. As mentioned in Chapter 1, the main objective of this study was to investigate the nature of the challenges faced by mathematics teachers in selected schools in Thokoza, Johannersburg in teaching probability and counting principles. Although the problems of teaching probability both at primary and secondary school levels have been investigated, most research dwelt more on investigating problems related to learner understanding of probability concepts. The assumption made in this study is that learner understanding of any mathematical concepts taught is greatly influenced by how the content is presented by the teacher. In turn, teachers can only be effective in their teaching if they have adequate content knowledge of the topic they are teaching and they have the right pedagogical knowledge to do the work. Such sentiments are supported by Elbehary

(2020) who argues that teachers' knowledge is vital in determining the quality of teaching as it is through this knowledge that the correct activities are planned for learners. For example, it is the teacher who determines what learners must know, and how they can know it. Teachers are the ones that present probability ideas to learners, modify activities from textbooks and use their creativity to make learning both exciting and meaningful.

According to the literature examined in this research, even though teachers are not the only cause for learner poor performance in probability in South Africa, the studies highlight that teachers' s lack of adequate content knowledge of probability and lack of training to teach probability are among the causes of learner poor performance. For example, there is a consensus among researchers that many South African mathematics teachers find it difficult to teach probability to learners in a way that develops meaningful understanding of probability concepts (Kodisang, 2022, Kodisang, 2016; Awuah, 2018). Researchers affirm that the training of teachers to teach probability needs serious attention (Batanero et al., 2016; Batanero, 2013). In South Africa (DeKock, 2015; Brijlall and Maharaj, 2014; Bansilal et al., (2014); Awuah, 2018; Kodisang, 2015) all agree that many of the teachers teaching probability today did not do it either at school or during their teacher training. The findings above are a potential source of teachers' shortcomings and disappointing outcomes of teacher preparedness to teach probability (Bansilal et al., 2014).

According to Batanero (2012) teaching probability is difficult because it is not only about presenting different probability concepts and their applications to learners, but it involves deeper discussions that should aim at developing stochastic (statistical and probability) thinking. Stochastic reasoning is different from mathematical reasoning Scheaffer (2006) cited in Batanero and Diaz (2012) and hence requires a different approach to teaching (Batanero, 2012). Many high school teachers make a big mistake of treating probability as a subsidiary of mathematics and therefore teach it like any other mathematics topic. Teachers who view probability in this way tend to compromise the teaching of probability (Batanero, 2015). For teachers to have the correct perspective, they need to possess adequate background knowledge of the development of probability as a topic. This is important because familiarity with how probability emerged as a topic and understanding how it evolved over time to be where it is now gives them a holistic view which in turn helps them to master their students' conceptual difficulties (Batanero, 2015). I will discuss the nature of probability, followed by teachers' probabilistic knowledge, teachers' professional knowledge to teach probability and the language of probability.

2.1.1 Studies done internationally and in South Africa on probability

Many studies carried out around the world including South Africa show that the teaching and learning of probability present distinct challenges for both teachers and learners. Pratt (2011) investigated probability reasoning in the United Kingdom where he found out that both teachers and learners found the topic difficult. Elberhary (2019) found out that in Egypt learner achievement in statistics and probability remained the lowest among other mathematics areas. A study by Danisman and Tanisli (2016) in Turkey informed that secondary school mathematics teachers had inadequate pedagogical content knowledge about probability. Similarly, in South Africa authors including, (DeKock, 2015; Kodisang, 2016; Kodisang 2022; Awuah, 2018; Ogbonnaya & Awuah, 2019) concur that learner performance in probability needs to be improved. Borovcnik and Kapadia (2016 p. 42) argued in their study, 'Research and Developments in Probability Education Internationally' about the teaching of probability, that "the challenge is to teach probability in order to enable students to understand and apply it, by creating approaches that are both accessible and motivating". This means that teachers must possess a certain level of specialised content knowledge (SCK) for teaching probability that would enable them to present the probability content in a more simplified and comprehensible manner. Beside the challenges stated above, other challenges have been noted as follows, inadequate training of teachers to teach probability, teachers' lack of sufficient pedagogical knowledge related to probability education, and teachers' lack of requisite understanding of the probability concepts they teach to their learners (Batanero & Diaz, 2012; Batanero et al., 2016). With respect to South Africa, changing curriculum over time has also been suggested as another factor contributing to difficulties in teaching some mathematics topics such as probability (Kodisang, 2016; Awuah, 2018; Ogbonaye & Awuah, 2019). Because there were many curriculum reforms in South Africa since 1994, the last one being in 2012 which introduced topics like probability into the mainstream mathematics curriculum it is important to examine teachers' familiarity with the additional topics and explore the challenges encountered in the teaching and learning of such topics. Stohl (2005) highlighted that, the success of any probability curriculum for improving learners' probabilistic reasoning hangs on teachers' understanding of the topic. Thus, this study aimed at assessing the current scope and nature of challenges facing the teaching and learning of probability in selected schools in Johannesburg, particularly at Grade 12 level.

To address the main research question on challenges faced by teachers when teaching probability, the reports from studies below provide the seriousness of the problems experienced at different locations in South Africa. There is a consensus among researchers who did research on teaching and learning of probability at different grades in South Africa that many mathematics teachers are finding probability difficult to teach and that the performance of learners in this topic is poor. For example, (DeKock, 2015; Kodisang 2016; Kodisang, 2022), Awuah, 2018) investigated different problems in the teaching and learning of probability at both primary and secondary level in South Africa and came up with findings outlined below. A study by DeKock (2015) comprising of eight (8) teachers, 89 Grade 11 learners and 75 Grade 12 learners found some shocking results from tests written by teachers and learners. The analysis of test results from six teachers (two teachers did not write the test) revealed that some teachers struggled to answer correctly low cognitive level questions involving dependent, independent, and mutually exclusive events. The author adds that most teachers could not recall formulas and rules to apply in answering low-level knowledge questions. Another finding from the study was that, even though all the participating teachers possessed acceptable qualifications to teach mathematics and had attended professional development sessions, the analysis of their performance in the test items indicated that they lacked the requisite mathematical content knowledge (MCK) related to probability.

A more recent study by Kodisang (2022) got similar results where teachers were found to be struggling with basic definition and understanding of probability. Below is a summary of Kodisang (2022) study that aimed at finding ways of improving the teaching and learning of probability in South African schools. Although the study focused on Grade 7 teachers and learners, the findings are helpful to explain the misconceptions learners have at higher grades including Grade 12. The main finding of this study was that, participating teachers generally lacked creativity in teaching probability and that there is a need for a re-examination of the way teachers approach the teaching of probability. The researcher found out that teachers had problems in initiating discussions and in responding to students' questions (Kodisang, 2022). The study found out that participating teachers had a limited conceptual understanding of probability which reflected in teachers' difficulties in comprehending the concept of chance and failure to deal with complexities of probability language. Some of the useful findings from the study are listed below; teachers had problems defining probability, challenges in making sense of some probability terms, lack of required skills to develop situations that allow learners to explore probability in depth, and teachers not exposing learners to a variety of representations to develop learner understanding (Kodisang, 2022). It can be noted from these findings that teachers generally lack the required SCK to teach probability. SCK for teaching probability will be discussed below.

2.1.2 Nature of probability.

This section of the work is concerned with exploring the challenges faced by teachers in terms of the difficulty of probabilistic reasoning. Most of the difficulties of this topic are encapsulated in the nature and characteristics of probabilistic reasoning. For example, several authors agree that the counterintuitive characteristics of probability results make probability very challenging and difficult to teach and to learn (Koeller, Pittman & Brendefur, 2015; Knudtzon, 2010). Some authors argue that probabilistic reasoning is different from mathematical logical reasoning (Borovecnik & Kapadia, 2016; Borovecnik & Peard, 1996). In this argument the authors maintain that, while with other topics in mathematics there is always certainty about a proposition, this is not always the case with random events. In addition, while the other branches of mathematics, for example in arithmetic and geometry reversibility is possible and can be shown using concrete objects, this is not the same with random experiments where a different result is obtained every time the experiment is carried out and it is not possible to repeat an experiment and still get the same result (Batanero & Diaz, 2012). Probability operations are different from those of pure mathematics (Batanero & Diaz, 2012) and it is such characteristics of probability that lead to the existence of erroneous understanding of probability by both teachers and learners (Batanero, 2015). King & Park (2019) and Nabbot-Cheiban, (2017) posit that children and adults hold misconceptions about probability. The above observations about the nature of probability concepts are important for the present study because it seeks to establish challenges that exist in the case being studied. Teachers in the sample will be asked to identify the subtopics in probability that they find difficult to teach. Further, the open-ended questionnaire will be used to collect information about strategies teachers use when teaching problems with counterintuitive results.

2.1.3 Teachers' probabilistic knowledge

According to Batanero & Diaz (2012) everything the teacher is going to teach learners depends on the teacher's probabilistic knowledge. Batanero and Diaz (2012) assert that many teachers teaching probability have a weak understanding of the topic. In their study Batanero and Diaz (2012) also noted that many of the mathematics teachers teaching high school probability who majored in mathematics at tertiary only studied theoretical statistics and probability during their training. It is highly likely that such teachers may not be effective in teaching probability in higher classes such as grade 12. Mutodi & Ngirande (2014) corroborate this with respect to South Africa's situation when they argue that South Africa's teachers have little experience about probability. It was discussed in this study under background of the study that in DeKock (2015)'s study there are some grade 12 teachers who scored 43% and 53% in probability tests. This is evidence that some teachers struggle to understand probability, and this makes it highly impossible for such teachers to teach it effectively at grade 12. The expectation of this study is not to duplicate what other researchers have done, but it seeks to verify some of these claims or prove them otherwise using the qualitative data collected from participants.

Four different domains of knowledge articulated by Ball et al. (2008) which I found relevant for the successful teaching of probability to the grade 12 learners include: common content knowledge (CCK), specialised content knowledge (SCK), knowing content and students (KCS) and knowing content and teaching (KCT). According to Ball et al. (2008) common content knowledge (CCK) is the knowledge possessed by anyone who studied mathematics to a certain extent, for example engineers, accountants and other people who studied mathematics will have this content knowledge. Teachers need this basic content knowledge for their teaching too, but they need more than just CCK (Brijlall, 2014). They need specialised content knowledge (SCK), knowing content and students (KCS), and knowing content and teaching (KCT).

- **Specialised content knowledge (SCK)**

Specialised content knowledge (SCK) is exclusive to teaching and its use is solely for teaching (Brijlall, 2014). Teachers need SCK in their daily teaching to map up different strategies of solving the same problem. SCK gives teachers the ability to structure and represent mathematical concepts and identify the mathematics that supports an instructional task. With their SCK, teachers can anticipate different ways students might think about concepts including their misconceptions (Steele, 2013). The author further affirms that SCK also empowers teachers to assess and analyse unconventional solution methods of their students. This is the kind of knowledge teachers require to teach probability with counterintuitive concepts requiring different approaches to solutions.

- **Knowing content and students (KCS)**

A teacher must not know only the mathematics he is teaching, but a good teacher needs sound knowledge of the students he/she is teaching. This includes knowing and anticipating what the students will think and react to specific information from the teacher or class (Brijlall, 2014). The teacher should know the context and the kind of examples that will excite and motivate the students bringing eagerness to learn. Good choice and sequencing of examples make it easy for learners to understand what is taught. This content knowledge helps the teacher in designing tasks that will motivate and inspire learners to want to learn (Brijlall, 2014). Teachers who are rich in KCS can motivate their students to like mathematics and enjoy it.

- **Knowing content and teaching (KCT).**

KCT is very important for effective teaching of mathematics as acknowledged by Brijlall (2014) that this is the knowledge that helps the teacher to understand the curriculum and sequencing of mathematics topics and concepts to enhance and facilitate easier learning of new concepts. A teacher with high level of KCT will take advantage of concepts taught earlier to students to build on new knowledge. Ball et al. (2008) cited in Brijlall (2014) maintain that teachers should have a global picture of a concept taught. In illustrating this concept of global picture Brijlall (2014) explained that teachers when teaching a concept should know its application in higher grades and beyond. In other words, this knowledge relates to the vertical connections between mathematics topics and concepts. KCT is therefore critical in helping teachers sequence mathematics topics for easy teaching and promoting a good building up of concepts. Teachers also become aware of what the children they teach already know from lower grades which they use to facilitate the learning of new concepts (Brijlall, 2014). The four domains of knowledge discussed above are all important to teachers teaching grade 12 probability. Having been regarded as a difficult topic for a long time, probability must be taught using all the domains of knowledge discussed above.

2.1.4 Professional training of teachers to teach probability.

For teachers to be successful in their teaching of mathematics, proper and effective training is key. The success of learners in probability will depend on the effectiveness of teacher training to teach the topic (Batanero, 2013; Koparan, 2019). It is during training that teachers broaden their subject matter knowledge, be informed about the different philosophies of mathematics, acquire teaching skills, and learn about different teaching methodologies (Biehler, 1990, cited in Batanero & Diaz, 2012). Training teachers to teach probability is very critical because it increases their probabilistic reasoning (Batanero & Diaz, 2012).

Teachers can be unsuccessful in their teaching careers because they lack sufficient professional knowledge (Elberhary, 2020). The teachers' professional knowledge is critical for teaching probability because of the following reasons: teachers are able adapt their knowledge of probability to different teaching levels and reach learners' various levels of understanding

(Vasquez & Alsina, 2021). Teachers require professional knowledge to be able to plan for their lessons, anticipate their students' learning difficulties, errors, obstacles, and devise strategies to teach their students problem solving (Elberhary, 2020). Additionally, it is professional knowledge teachers require to develop and analyse curricular documents, assessment tasks and textbooks to maximise student learning (Batanero & Diaz, 2012). Consequently, teachers who lack professional knowledge of probability cannot teach it successfully. One wonders if the teachers teaching mathematics to grade 12 learners in South Africa received specific training in teaching statistics and probability as Batanero and Diaz (2012) observed that few teachers teaching probability in schools are trained this way. The authors argue that for teachers to be effective in their teaching of probability they need specific training in the pedagogical knowledge aligned to the teaching of probability, where general principles that apply to for example algebra and geometry and other topics in pure mathematics do not apply.

Most of South Africa's mathematics teachers may not have received the kind of learning and training in probability as outlined above. Makwakwa (2012) affirm that probability was not in the South Africa's Further Education and Training (FET) (Grade 10-12) mathematics syllabus prior to 2006 and was introduced after this period. However, even after its introduction in 2006 it was only assessed in an optional Mathematics Paper 3 at matric until 2014 when the topic became compulsory in Mathematics Paper 1. From my experience as a teacher then, only a few students registered to do Paper 3 because it was regarded as difficult. This means that the students who did matric before 2014 and did not do Paper 3 mathematics did not do probability at secondary school level. It is only the students who did matric mathematics from 2014 onwards who had the opportunity to learn probability at secondary school level. According to Makwakwa (2012) probability was initially offered as a component of statistics at university level in South Africa. Therefore, the same students who did not do probability at school were also denied the opportunity to do probability as an independent topic in their tertiary education. From the analysis of the literature provided by Makwakwa (2012), one can only expect teachers who completed their mathematics teaching degrees in South Africa's universities from 2018 onwards to have a high-level knowledge of probability because they had the opportunity to do probability at secondary school level as well as in their tertiary education.

In the South African case, professional development programmes have not been effective in addressing the needs of the teachers. Pournara, Hdgen, Adler and Pilly (2015) found out that most mathematics professional development programmes in South Africa can be described as taking either a repair approach or a conceptual approach in training teachers. The same authors claim that repair approaches focus on teachers re-doing school mathematics in the same ways as their learners would learn it. For Pournara et al., this is not effective as teachers simply rehearse the steps necessary to solve typical tasks from the school curriculum (Pournara et al., 2015). Teachers do not benefit significantly from such approaches as they are limited to just a narrow knowledge of the mathematics of the curriculum instead of providing a broader understanding of the content they need to teach. These practices simply position teachers as school learners which contrasts with the principles of professional development which should foster a systematic attempt to bring about change in classroom practices of teachers, change in their beliefs and attitudes, and change in the learning outcomes of students (Guskey (1986) cited in Keren and Patkin (2016)).

A study by DeKock (2015) agrees with Pournara et al., (2015) by revealing that professional development programmes initiated and hosted by the Gauteng Department of Education (GDE) in the form of training sessions to in-service teachers were found to be of little benefit to the teachers. In that study, when teachers were asked to comment about the training sessions, they expressed the opinion that the sessions they attended did not contribute to any gains in their mathematical content knowledge (MCK) (DeKock, 2015). The development programmes failed to make the intended impact to teachers because they did not take into consideration the teachers' needs when they were designed. According to Kleickmann et al., (2013) cited in DeKock (2015), for professional development programmes to be effective and beneficial to teachers, they need to be designed with individual teacher needs in mind and that teachers should attend such programmes over an extended period. Another proof that these professional development programmes may not be working is the persistence poor performance in probability by matric students since its inception in 2014 to the present as reflected in Table 1

Conceptual approaches on the other hand, work from the assumption that teachers' mathematical knowledge is procedural, and thus inadequate, and that interventions should provide them with a deep conceptual understanding to complement their procedural knowledge. Conceptual approaches, according to Klipatrick, Swafford and Findell (2001) tend to adopt an exclusively conceptual approach with little regard for the role of procedures or procedural fluency. This may be a problem in assessment as the CAPS document stipulates that, 35% of the questions in any maths test or examination should be based on routine procedural processes. It is therefore important that professional developmental programmes blend the two approaches and design the content of the programmes through consultation with the teachers they intend to service. Batanero and Diaz (2012) maintain that professional development programmes should be focused on designing, implementing, and assessing probabilistic content aligned with the country's curricular guidelines and in relation to the students' ages and individual needs. Programmes should promote sharing of both knowledge and strategies of teaching probability including the acquisition of the best resources to teach the topic. It can be argued that well trained teachers should be able to produce the desired learning outcomes and professional development programmes are a prerequisite for the introduction of new topics to enable teachers to extend and perfect their teaching skills of the new topics (DeKock, 2015).

2.1.5 Curriculum documents and textbooks

To explore further the challenges faced by teachers in teaching probability to grade 12 learners, it is tempting to look at curriculum documents and the teaching resources used to find out if they provide enough support to teachers to get the results required. Batanero and Diaz (2012) reported findings from statistics education which indicated that the curriculum documents and textbooks prepared for both primary and secondary teachers were not giving teachers adequate support. The authors argued that some of the textbooks present just a narrow a view of probability concepts, for example a textbook may focus on theoretical probability at the expense of experimental or classical approach (Batanero and Diaz, 2012). Some textbooks may use examples that are culturally not aligned to the students using them and hence cannot help students to understand the probability concepts taught.

2.1.6 Language of probability and probability terms.

Language can be a barrier in the teaching of mathematics concepts and more so in the teaching of probability because of the difficult terms used**.** The language and terminology used in probability is demanding and difficult to understand (Batanero et al., 2016). The problem of language may have a greater negative impact to second language learners, for example non-English speaking learners struggle with mastering the mathematics and the language at the same time (Ledibane et al., 2018). Language challenges contribute to some teachers' beliefs that probability and statistics are difficult to learn and impact their inclination to teach the subjects (Leavy et al., 2013). Awuah (2018) who carried a study to investigate Grade 12 learners' problem-solving skills in probability in South Africa found out that the language of probability was a challenge to the teaching and learning of probability. Paul and Hlanganipai (2014) noted that when teachers and learners find that the language is difficult, they reach a point where they develop negative attitudes towards the topic.

2.2 Methodological practices in mathematics in South Africa.

Boaler (2015) observed that there is a gap between what researchers have shown to work in teaching mathematics and what is practiced by teachers in schools. In South Africa, research shows that teachers are still grounded in traditional methods of teaching where the teacher must talk, and students must pay attention and learn. Vavrus et al (2011) and Vayrynen (2003) contend that, teacher centred approaches were dominant in South Africa before the mid-1990s. Back then, teachers were officially expected to use authoritarian approaches (Harber & Serf, 2006). These strategies had political implications, and were largely connected to apartheid policies (Naiker, 2006). The assumption was that, if teachers gave clear instructions to motivated students, learning would automatically occur. If learning did not happen, it was because students were not paying attention. As observed by Bray et al. (2010) teaching in schools still reflects these authoritarian practices even to present day. They may be modified in different schools, but they are still practised. In general, teachers tend to want to teach their students the way they were taught themselves.

The other reason that can be used to explain why teachers rely on autocratic methods of teaching is the problem of overcrowded classes that render the use of learner-centred approaches difficult to implement. DeKock (2015) posit that teacher-learner ratios are very high in non-fee-paying schools (Quantile 1-3), and this causes serious disciplinary problems for teachers who then resort to strict authoritarian strategies as a disciplinary measure. Teachers have also voiced the concern of long syllabi that are difficult to complete using learner-centred approaches. Mhlolo (2017) Julie & Gierdien (2020) stated in their studies that, South African teachers are usually caught between teaching for mathematical competency and meeting curriculum demands. Teacher classroom practices have been identified as one of the problems contributing to poor mathematics results in South Africa (Arends, Winnaar, & Mosimege, 2017).

2.3 Theoretical Frameworks.

It was revealed in the literature review that probabilistic reasoning differs significantly from the logical thinking found in other areas of mathematics and that this creates special challenges for both teachers and learners (Batanero et al., 2016). Batanero and Diaz (2012) highlighted in their study that some of the challenges teachers are facing are caused by lack of specific training in the pedagogical knowledge related to the teaching of probability. The authors added that, another challenge is that of lack of good textbooks and curriculum documents needed to support teachers when teaching probability. According to Batanero and Diaz (2012), probability is difficult to teach and learn because of the misconceptions both teachers and learners have about probability concepts. Developing the point above further, the authors affirmed that the evidence which is there suggests that it is difficult to eliminate the misconceptions through formal teaching (Batanero and Diaz, 2012). Teachers are therefore faced with the challenge of how to teach learners so that they can develop the correct intuitions in probability (Batanero and Diaz, 2015). Batanero et al., (2016) added that the language of probability is difficult for both teachers and learners. The main challenge faced by mathematics teachers teaching probability is the task of teaching this topic in such a way that learners are able to comprehend and apply it (Batanero et al., 2016) Summarising the challenges of teaching probability identified internationally by Batanero and Diaz (2012) and Batanero et al., (2016) the following items came up: teachers are challenged by probabilistic reasoning because it is different from the logic found in other areas of mathematics; teachers find the teaching of probability challenging because they are not given specific training in pedagogical knowledge related to teaching probability; teachers are not supported by good text books and curricular documents when teaching probability; teachers and learners hold misconceptions about probability which are difficult to eradicate through formal teaching; teachers are challenged by the language of probability; and teachers have the challenge of teaching probability to learners in a way that learners can comprehend and be able to apply it in life. In this study, the challenges outlined above shall be used as a framework of investigating challenges South African teachers are facing in the teaching and learning of probability.

To teach probability effectively, (Batanero et al., 2016) assert that teachers need to come up with approaches that are both accessible and motivating to learners. A study by Kodisang (2016) found out that teachers used teacher-centred approaches to Grade 6 learners. The findings of that study revealed that all teachers in the study were not keen to have learners involved in discussions but were more interested in correct answers given by learners. Learners were not given opportunities to discuss alternative strategies with other learners or the teacher. Asking learners to discuss their answers with others and the teacher, asking them to justify and defend their answers helps them to deepen their mathematical understanding. Justification is key in teaching probability because as learners are encouraged to justify their claims or thinking, their mathematical reasoning is improved (Burgin, 2020). It is such findings that influenced the researcher to explore pedagogical challenges faced by teachers when teaching probability through the theory of argumentation in mathematics. I found the argumentation theory relevant in this study to explore the nature of the pedagogical challenges teachers are facing in teaching probability. Mueller (2009) in Can and Isleyen (2020) determined teaching guidelines that foster the process of argumentation in teaching a mathematics topic such as probability. In investigating the nature of the pedagogical challenges the researcher will be guided by Mueller (2009)'s framework which considers the following questions: are teachers able to provide a conducive learning environment which allows cooperative learning; are students given open ended questions or tasks, allowed to research, discuss and report back to the teacher and others; are students encouraged to form their own representations; are learners encouraged to explain and defend their answers; are teachers' interventions carefully planned; are teachers able to encourage mathematical discourse in their lessons. The strategies outlined above will provide a framework with which to view the pedagogical strategies used by teachers in the study to teach probability content.

The department of education (DBE) can use the argumentation theory as a framework for their professional development programs to improve the teachers' content knowledge of probability. Once teachers are trained, they may use the theory themselves as methodology in teaching probability in schools. Figure 1 below shows how both teachers and learners may engage with probability concepts to foster deeper conceptual understanding of probability concepts.

Civil and Hunter (2015) define argumentation as a teaching strategy that simultaneously allows participants to extensively participate in class by explaining their reasoning, elaborating, and justifying mathematical thinking while also developing an understanding of opposing perspectives through classroom interactions with other learners. This is a methodology which represents models of teaching that contrast the traditional ways of teaching by telling, where teachers are viewed as transmitters of knowledge and learners as passive recipients. Current educational goals around the world are focused on getting students engaged in meaningful disciplinary discussions as opposed to where students are taught to simply apply procedures as they are given by their teachers. I support the perspective that learners attain better understanding when they are involved in mathematical arguments, explaining their answers, justifying their methods and solutions to others and to the teacher. In this process, and that of constructing arguments and responding to others' reasoning, students develop better understanding of fundamental mathematical ideas and practice critical thinking (Graham & Lesseig 2018). Uygun & Akyuz (2019) recommended argumentation as a useful strategy to foster the effective development of learners' knowledge of subject matter. Burgin (2020) agrees with Uygun & Akyuz (2019) when he affirms that as students seek to justify their claims, thinking, and methodologies they are required to dig deep in subject matter knowledge. Based on the Professional Standards for Teaching Mathematics (NCTM) (1991), mathematics teachers should foster in learners an attitude of questioning the teacher and other classmates and use a variety of mathematical tools to reason and make connections, solve problems, and communicate to others their own understanding. A model below is used to explain how the processes of argumentation may improve the level of learner reasoning leading to deeper mathematical understanding.

Figure 1.

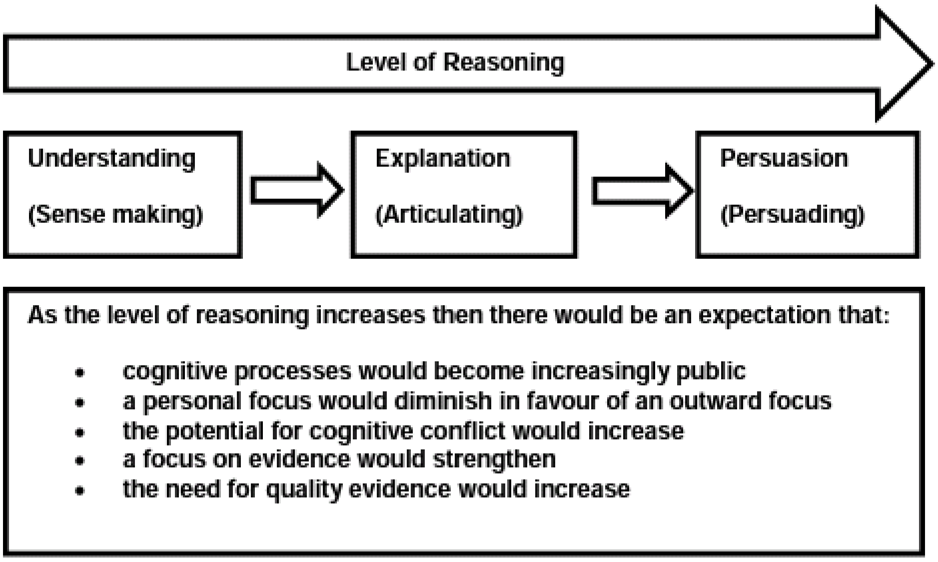


Figure 1. Model of potential interactions between reasoning, goals of argumentation and use of evidence. (Adapted from Wells, 2014)).

Explaining the interactions between reasoning, goals of argumentation and use of evidence Wells (2014) maintains that as the reasoning of learners increase through discussions and verbal arguments with other learners and the teacher there is a change in the cognitive processes of the learners. As learners try to explain and justify their claims to others their cognitive processes are exposed and identified by others who may challenge the exposed thinking and if this happens often it results in a dwindled focus on personal understandings, beliefs, and dependence on internalised sources of knowledge by learners concerned. Such processes help learners to shift from depending on their internalised sources of knowledge to relying more on externalised, objective, and defensible evidence (Wells, 2014).

This model encourages teachers to make use of class discussions, small group discussions and presentation of arguments as tools to foster deeper mathematical understanding in learners (Wells, 2014). The author further argues, based on Wells (2014)'s work that when learners are given an opportunity during discussions to propose their ideas and provide reasoning for the ideas, then, it enables the teacher to have insights into their thinking. That process might help teachers to identify and correct misconceptions that learners have. Initially the teacher plays the role of directing arguments through asking questions and act as a class resource, but this role diminishes as learners get used to argumentation. The teacher gets less involved, and learners become more independent, and the role of the teacher becomes that of posing few questions to provoke further thinking and alternate considerations to direct mathematical focus (Wells, 2014). In my opinion this strategy can work in teaching grade 12 probability. Grade 12 learners are grown up and need less of the teachers' involvement in their learning. Using this approach teachers can thoroughly prepare for their probability lessons including resources such as textbooks, teaching and learning charts and other useful teaching resources and then allow learners to engage with the task planned for that day. The teacher can be in class to manage time and facilitate the discussions while allowing learners to propose, explain and justify their ideas to others and the teacher.

While the benefits of argumentation are huge, its implementation in the classroom has always been problematic for mathematics teachers and learners (Civil and Hunter, 2015). The authors argue that the creation of constructive classroom mathematical arguments has always posed a challenge and calls for a variety of expertise. The teacher should create a permitting environment for argumentation where all learners feel comfortable to interact with each other during learning. Teachers remain responsible in ensuring that arguments remain academic, devoid of emotions, inferiority, and superiority complex from learners. Mistakes and errors should be regarded as part of the learning process. In other words, learners should view correction or criticism of their answers positively without being embarrassed. Civil & Hunter (2015) argue that the construction of mathematical arguments demands a diversity of expertise, and it also requires tactical strategies including code-switching so that learners are not restrained by language to develop their arguments. To effectively employ argumentation successfully in South African schools, in-service teachers may require some training and conscientisation because of the important role they play. Teachers have a role of directing and leading students' discussions by asking probing questions for students to be accountable to mathematics itself as a discipline, accountable to other learners and to the teacher. This requires a high level of competence from the teacher.

2.3.1 Linking argumentation to the study

Argumentation can create opportunities for teachers and learners alike to solve probability problems at Grade 12 level in South Africa. According to Can and Isleyen (2020) argumentation works more effectively in some topics than in others. Driver et al. (2000) cited in Can and Isleyen (2020) argue that the argumentation approach is more effective in teaching topics that provide the possibility of more than one opinion. Probability is one topic where a question can be approached from different ways and solutions can be formulated using different methods and representations. I suggest that argumentation might provide conducive teaching and learning environments for the effective teaching and learning of probability to South Africa's grade 12 learners.

- **Teacher professional development**

In the context of this study, the main question asked is: How could the propositions of the argumentation theory be used to enhance both teachers and student practices that will support the development and improvement of understanding probability concepts. Another question would be: How could teachers use the theory of argumentation to improve their own conceptual understanding of probability concepts and in turn use this knowledge to guide the process in developing classroom activities and other interventions to foster deeper understanding of probability concepts in their learners. Effective teaching and learning can only be realised if teachers know and understand what they teach. In addition, this theory can be used to provide a more supportive setting in which teachers could be assisted to try out different approaches to promote better outcomes for students. Argumentation is more engaging, and the live discussions encouraged through argumentation may be useful in stimulating thinking in learners through active participation. Reasons for difficulty in probability in many cases have been linked to use of teacher-centred approaches, lack of adequate knowledge of teachers, teachers' pedagogical deficiencies and learners' negative attitude and misconceptions caused by various reasons (Batanero & Diaz, 2012; Mutara & Makonye, 2014; Kodisang, 2016). Research has revealed that teachers who participate in effective continuous professional development programmes which are content-specific are more likely to achieve better mathematics results for their learners (Sithole et al., 2017). To improve teacher content knowledge, I suggest that this theory might be used by DBE to workshop teachers to improve their content knowledge to teach probability through peer live discussions, justification of answers and group presentations. If this is achieved, it might help teachers to improve their classroom practice and have an effect of increasing the quality of their teaching probability concepts.

The argumentation theory was used in developing both the questionnaire questions and the observation guide. These two instruments were used to collect most of the data for this study. The framework was the guide in answering sub research question (iii), on what could be the pedagogical practices that hinder effective teaching of probability to Grade 12 learners in South Africa. The online questionnaire contained specific questions on argumentation. The principles of argumentation defined by Akkus et al. (2007) were used to formulate questions on Section F of the questionnaire. Ekkus et al. (2007) defined argumentation as a teaching strategy where ideas are put forward, criticized, evaluated and arguments formed. The questions contained in the open-ended questionnaire about classroom interaction were prepared using these principles, for example, asking teachers if they used small group discussions or allowed learners to talk to each other. Teachers were asked through the open-ended questionnaire and follow-up messages if they gave their learners ample time to look for solutions and if they encouraged learners to challenge each other's solutions on probability and counting principles (PCP). In the same open-ended questionnaire, teachers were asked if they motivated leaners to challenge teachers' solutions and if they allowed learners to discuss probability problems freely with teachers taking the role of a facilitator.

In classroom observations, the researcher was guided by the principles of argumentation in observing teachers' methods of instruction. There was emphasis on learner-to-learner and teacher to learner interactions. The researcher observed if learners were given adequate opportunities to explain their answers and justify them. Teachers' questioning techniques were analysed. For example, the researcher observed the kind of questions teachers asked learners in class. Teachers applying the argumentation theory would ask questions such as, "why do you think like that", "how you can justify your thinking to other learners" and many other questions that encouraged learners to justify their claims. It is such questions that deepen learners' understanding which are needed in probability.

Application of argumentation elsewhere, for example a study done in Turkey have shown that this theory can be used effectively to improve the teaching and learning of probability (Can & Isleyen, 2020). Can and Isleyen (2020) did a quantitative study in Turkey with 21 control group and 23 experimental group of pre-service teachers to the application of argumentation in the teaching of probability. Their results from the two groups showed that the experimental group that used the argumentation approach outperformed the control group that used traditional teacher-centred approaches (Can & Islayen, 2020). For the control group, lowest scores of the pre-test (13) improved to post-test (25) after an intervention using traditional teacher-centred approaches as opposed to lowest score of a pre-test (9) and the post-test (39) in the experiment group after an intervention applying argumentation approaches. The highest scores improved from 49 to 89 for the control group compared to the experimental group where they increased from 21 to 108. The number of the control group was 21 and 23 for the experimental group (Can & Islayen, 2020).

2.4 Chapter summary

In this chapter I discussed the existing literature on the nature of probability and outlined the different types of knowledge and professional development required by teachers to be able to teach probability. The chapter also discussed the theoretical framework on which the study is grounded. The last section looked at the link between the present study and the argumentation approach where I explained how this approach was used to design questions on the questionnaire as well as giving guidelines during classroom observations.

CHAPTER 3: METHODOLOGY

3.1 Introduction

Having discussed in the literature review the different challenges faced by teachers in teaching probability both internationally and in South Africa, chapter 3 gives a synopsis of the research methodology, research design and the sampling techniques employed in the study. Moreover, the chapter explains data collection techniques and justifies them. Justification for the choice of the methodology and research design is also provided.

3.2 Research Paradigm.

Creswell (2009) defines a paradigm as a "worldview". It is defined by others as a "cluster of beliefs and dictates which for scientists in a particular discipline influence what should be studied, how research should be done and how results should be interpreted" (Bryman, 2012 cited in Du Plooy-Cilliers & Cronje, 2014, p. 19). In other words, a paradigm should be viewed as a way by which people interpret, relate to and gain understanding about their world.

This study adopted the interpretive paradigm which is explained by Cohen, Manion and Morrison (2011) as a paradigm that is concerned with individuals' personal judgements in relation to reality. As a result, it is concerned with humans' activities and the interpretations of their experiences. According to Check & Schutt (2012), the interpretive paradigm is a belief that reality is socially constructed. This paradigm is suitable for this study because it assisted the researcher to investigate and identify the challenges participants faced when teaching probability. This paradigm is best suited to semi structured interviews, but for this study, because of the Covid 19 restrictions, an open-ended questionnaire was used for all eleven participants. But, to cover up for lack of face-to-face interviews the researcher used follow-up or short text messages mainly through what's app to encourage more comments and more detailed explanations from participants. Participants were expected to provide interpretations of their daily experiences in the classrooms relating and explaining challenges they faced.

A case study was used to investigate the challenges faced by mathematics teachers in selected schools in Thokoza Johannesburg. The reasons for choosing a case study were based on Creswell (2012)' s argument that case studies allow the researcher to do an in-depth exploration of an activity, process, event, or individual based on extensive data collection. The event in this case is to investigate the challenges South African mathematics teachers face in teaching probability to grade 12 learners. Schwandt & Gates (2018) identified a case study as a strategy that is appropriate when the researcher wants to describe and explain how everyday practices in specific locations are connected to the large structure and processes. It is for this purpose that the case study was chosen, to assist the researcher to gain an understanding of the nature and extent of the challenges affecting the teaching of probability in selected schools particularly to grade 12 classes. Eleven Grade 12 mathematics teachers from five different secondary schools in Thokoza, Johannesburg participated in the study.

The second reason for a case study were the two factors of time and resources that made it impossible to think of large-scale research. Although it is often difficult to generalise findings from a case, it is assumed that results from this research may illuminate some of the contextual challenges faced by teachers with similar conditions. Marshall and Rossman (2016) reckon that case studies are now widely used in research because of their explicit focus on context and dynamic interactions often over time. This assertion aligns with the motivation of the study; to gather first-hand information from teachers about what happens daily in their classrooms. It aims to obtain teachers' views concerning challenges faced by teachers when teaching probability as mentioned in the research questions.

3.3 Sampling

Sampling is the process used to select a portion of the population for a study (Creswell, 2010). The author explains further that, a sample is studied to understand the population from which it was drawn. Therefore, studying a sample is not an end, but rather it is done to gain understanding of some facets of the population from which it was drawn (de Vos 2000 & Bryman, 2012). Lopez and Whitehead (2016) maintain that the critical purpose of sampling is the selection of suitable populations or units so that the focus of the study can be correctly researched. Moreover, the sampling should be conducted in an appropriate manner because if the selection of a sample is flawed the consequences can affect the findings and outcomes of a study (Lopez and Whitehead, 2016).

3.3.1 Types of sampling

There are two types of sampling namely a probability sample and a non-probability sample. For this qualitative study a non-probability sample was chosen in line with Lopez and Whitehead (2016)'s assertion that non-probability sampling is suitable for qualitative research because it covers a restricted size of community. Probability sampling on the other hand is used in quantitative research where researchers enlist the population with characteristics that represent a wider community (Lopez & Whitehead, 2016). Within non-random sampling, are different types of sampling procedures which include, convenience sampling, purposeful sampling, snowball sampling and theoretical sampling (Lopez and Whitehead, 2016). A purposeful sampling was chosen for this study because it allows the recruitment of participants based on a pre-selected criterion relevant to the research questions (Creswell, 2013). In other words, the inclusion of participants in the study depended on the judgement of the researcher. Mathematics teachers who taught in the selected schools were considered for sampling. The selection criterion followed a purposeful sampling procedure because participants were chosen based on their knowledge about the study (Merriam & Tisdell, 2016).

Purposeful sampling was used for selecting the schools also. Schools were chosen for the study if they were in the Thokoza circuit of secondary schools. Secondary schools only were chosen because the study was about Grade 12 mathematics teachers. There are five secondary schools in this circuit, and they were all considered. Data was collected from mathematics teachers in these five secondary schools. For the purposes of gaining insight into the problems of teaching probability to grade 12 learners, the researcher extended data collection to include all mathematics teachers from the selected schools who had Grade 12 teaching experience even though in 2021-2022 (period the study was done) the teachers were not involved with Grade 12 classes. In other words, the study considered mathematics teachers from selected schools with Grade 12 probability teaching experience. Those teachers who were not teaching Grade 12 during the period of data collection reported on the problems they faced in those grades. This was done to capture any challenges or barriers along the system. The teaching of mathematics is developmental. Probability concepts are introduced at lower levels and are developed up until grade 12. Students can develop misconceptions at lower grades which may not be corrected until they reach grade 12. So, the criterion for selecting participants was based on mainly Grade 12 mathematics teaching experience and location of school.

The reason for choosing Thokoza secondary schools was mainly due to proximity to the workplace and the fact that these were township secondary schools that offered conditions like in most public secondary schools around the country. Township schools more challenged in terms of access to resources, staffing and their mathematics results are always poor compared to private schools. All the five schools are Quantile 1 schools. The conditions in these schools are the same.

The schools in the sample show significant variation in the mathematics pass rate. One school in the sample had a pass rate of over 80% in mathematics over the last three years. The second and third schools had an average of 78% and 75% respectively and the fourth school had an average of 60% in mathematics for the same period. The last school had an average of 48% in mathematics also over the same period. Nothing separates these schools in terms of the kind of learners they enrol because they are all draw learners from the same catchment area and the distance between the five schools is within 3 kilometre or less.

3.4 Research Methodology.

For the current study, data was gathered through an online open-ended questionnaire for teachers, short text messages (follow-up messages or what's up messages), classroom observations, and document analysis. The researcher designed the open-ended questionnaire himself with the support of the supervisor and were then emailed to the different teachers to complete and send back to the researcher by email. Follow-up text messages or what's app messages on the open-ended questionnaire were used like semi-structured interviews to get more information on some of aspects data that the researcher required. For example, after receiving a questionnaire response from a participant, if the researcher thought more information or clarity was required then a follow-up text message (what's app) were sent to that participant depending on the information required. For the purposes of consistency in this work all text messages or what's app messages will be referred to as follow-up messages. Eleven (11) mathematics teachers completed and returned the questionnaires. The researcher also carried out lesson observations and document analysis with four of the eleven teachers who are teachers at the same school with the researcher. Due to the COVID-19 restrictions at the time of data gathering, the Gauteng Department of Education (GDE) only allowed the researcher permission to observe lessons in one school where the researcher is employed.

A qualitative research methodology was found to be in alignment with the case study's element of providing an in-depth understanding of participants' experiences, perspectives, and thoughts. This aspect of a qualitative research is emphasised by Darragh (2016) who asserts that a qualitative research approach is suitable to provide an in-depth and rich description of phenomenon. Qualitative methods were useful in helping the researcher to answer questions about the subjects' own experiences, meanings, and perspectives (Hammarberg et al., 2016). In addition, McMillan & Schumacher (2010) explain qualitative research as an analysis of people's personal and collective social actions, their beliefs, thoughts, and perceptions and is primarily concerned with understanding the social phenomena from the participants' perspective. An open-ended questionnaire supported by follow-up messages were utilised to collect detailed data about teachers' views regarding the challenge of poor learner performance in mathematics and probability.

3.5 Data Collection and Methods

Four data collection tools were used, and they are discussed below.

3.5.1 Online questionnaires.

Locally developed questionnaire (see Appendix I) was used to gather data from the teachers regarding their challenges with probability content, teaching strategies and the causes of the challenges identified that made the teaching of grade 12 probability difficult. Questionnaires are not commonly used to collect qualitative data because qualitative approaches acquire data in many cases through interpersonal contact with participants (Babbie, 2014). The original data collection tool for the current study was the semi-structured interview which was changed into an online questionnaire when the researcher was denied the opportunity of meeting the participants in person because of covid 19 restriction. The same questions that were prepared for the semi-structured interview were written down into an open-ended questionnaire. According to De Chesnay (2014) qualitative questionnaires can be used where a list of open-ended questions captures qualitative data. The open-ended questionnaire had the advantage of giving the participants more time to think about their responses. Follow-up messages were used to follow up on participants to respond to the questionnaire and give more detail where the researcher felt there was a need. Although it took others time to respond, all eleven teachers responded with detailed answers. The nature of the questions contained in the questionnaire required teachers to explain in detail their responses. Where possible participants were encouraged to comment on every question and explain themselves the way would do with semi-structured interviews.

3.5.2 Administering the questionnaire to teachers

The purpose of the questionnaire was to gather data on (1) teachers' personal and professional data (2) challenges encountered in the teaching of probability (3) the causes of the problems (4) the strategies teachers used to teach probability (5) teachers' suggestions to mitigate the problems. The questionnaire layout process was divided into eight sections. The sections were from Section A up to Section H. Each section dealt with a specific set of questions designed to collect data on different aspects in the teaching of probability.

The probability content was subdivided into different sections for teachers to respond to each subtopic. This was done to determine the sections that were problematic and the reasons for their difficulty. Responses from these questions would help address the main research question on the kind of challenges mathematics teachers face when teaching grade 12 probability. Answering the research questions was important as means of finding the source of poor performance in probability by grade 12 learners in matric examinations.

The questionnaire guide below was used as a source of broad themes to provide a set of questions that were included in the questionnaire. The guide was used to ensure that only those questions that addressed research objectives and research questions were covered (Lopez & Whitehead, 2013).

3.5.3 Questionnaire guide

- What kind of challenges do teachers face in schools in the teaching of probability?
- What aspects of probability do teachers regard as difficult in teaching probability to grade 12 learners?
- Which approaches do teachers use when teaching probability in grade 12?
- What support are teachers getting to improve the teaching of probability?
- Do teachers have sufficient resources to teach probability, for example textbooks, revision materials for learners?
- What are teachers' own suggestions on what should be done to improve the teaching of probability?

3.5.4 Classroom Observations

Initially, classroom observations were planned for all eleven teachers who participated in the study, but the plan was changed to include only four of the teachers because of the main reason stated below. The Gauteng department of education (GDE) barred researchers from physically collecting data at the schools except, in the case of researchers who are teachers, they were allowed to collect such data at the schools they were teaching at. This pronouncement meant that the researcher could do classroom observation with the teachers that taught at the same school with the researcher. This will be explained more under limitations of the study.

Creswell (2010) define observation as a process of capturing behavioural patterns of participants, objects, and phenomenon without necessarily relating with them. Classroom observations have become a fundamental part of qualitative research and have been used extensively in mathematics education (Bostic et al., 2019). The importance of classroom observations is found in their ability to give the researcher the opportunity to observe and describe the teacher's current instructional practices (Bostic et al., 2019). The researcher can then identify and delineate the instructional problems that confront the teacher and learners (Bostic et al., 2019). It was through this process that the researcher was able to examine the different levels of the quality of the teachers' instructional practices

in relation to fostering conceptual understanding of probability in their learners. In addition, the researcher was able to observe the teacher and students' actions and interactions during lessons.

Observations gave the additional advantage to the questionnaire in that the researcher captured even the information participants were unwilling or unable to provide (Creswell, 2010). In the present study, classroom observations focused on the following aspects of teaching, classroom organisation, classroom resources, teacher-learner interactions, learner to learner interaction, questioning techniques, teacher time management and any other aspects of the teaching process the researcher found useful in addressing research questions.

During the data collection process, I observed teachers in practice. I adopted a non-participant observation style (Marshall & Rossman, 2016), because I wanted to observe teachers during their habitual routines to understand how they interact daily with their learners and to avoid distracting lessons (Lopez & Whitehead, 2016). This is important in qualitative research because the idea was to capture data in more natural settings, capturing the whole setting and context of the environment in which participants function (MacMillan & Schumacher, 2010). However, this aspect was somehow compromised because the lessons were either taught in the morning during morning classes or in the afternoon during afternoon classes. What was consoling was that these lessons are conducted throughout the year with Grade 12 classes, they are formalised. The purpose of the morning and afternoon classes are for catching up with the syllabus at Grade 12.

The purpose of the observation was explained to the participants and an observation schedule (see Appendix J) was designed based on the teaching days and times agreed with the four teachers teaching at the researcher's school. All teaching periods were allocated one hour to match the normal one-hour mathematics periods teachers were used to. The researcher met with each teacher to discuss the subtopics the teacher would be observed teaching. This was done to avoid a situation where all the teachers would teach the same probability concepts. Using the grade 12 Annual Teaching Plan (ATP), the researcher used a pre-meeting with the teachers in the school three days before observing them to agree on the sections of probability each teacher was going to cover. The first teacher agreed to teach the use of probability tree diagrams, the second dealt with the concepts of mutually exclusive and independent events, the third Venn diagrams and the last one taught the concept of the contingency table.

The observation guide was extracted from the principles of teaching (Antony & Walshaw, 2009). The following principles of teaching were observed from the four teachers:

- **Arrangement of learners**
  The researcher focused on finding out if the arrangement of the class created opportunities for learners to work as individuals or in small groups.
- **Building on learners' thinking**
  The researcher focused on what and how the teacher introduced the probability concepts he or she was teaching. This included a focus on how the teacher introduced and explained probability concepts and the procedures that were explained to learners. Teachers' activities were put on a scale based on their level of involvement of learners' experiences and interests.
- **Making connections**
  Teachers were observed on their expertise in connecting probability to the learners' own real-life situations. This included the examples teachers used.
- **Classwork or tasks and assessment for learning**
  The researcher examined the teachers' tasks in terms of the opportunities they afforded learners to make sense of probability as a whole and how the tasks gave the learners a broad understanding of mathematics concepts. This involved observing the opportunities teachers created using the tasks for learners to explain and justify their answers.
- **Mathematical communication**
  In mathematical communication I was concerned with the time and learning space created for learners to engage more in using different ways of finding answers and being able to explain and justify their paths than just focussing on correct answers. Brodie et al. (2021) said this with reference to South African teachers, that teachers work mainly

to ensure that learners pass examinations thus they focus on drilling learners on procedures of mathematics at the expense of its full power. Brodie (2017) maintains that this is caused by prescriptiveness of the curriculum and excessive testing.

- **Mathematical language**

  This involved noting the use of probability language by the teacher and learners, including the use of correct probability vocabulary. Meaney et al. (2012) note that language can either be a support or a barrier to learners learning probability. It was expected of the teachers to explain probability terminology to the learners for better understanding of probability concepts.

- **Tools and representations**

  Teachers were expected to use different and appropriate resources, probability models, and technology to support learning.

- **Teacher knowledge**

  Substantial content knowledge and pedagogical knowledge of teachers is an indispensable requirement that enables them to develop in-depth understanding of mathematics concepts in their learners (Ogbonnaya & Mogari, 2014). The authors stated that lack of teacher content knowledge in any subject or curriculum is reflected in learners' performance. Teachers were examined in terms of their ability to respond to learners' mathematical needs using their content knowledge.

The other areas that were observed were those of teacher-learner interaction and the support teachers gave learners to develop their probabilistic reasoning. All learners need to be supported to persevere to succeed in mathematics. Success was viewed as the ability by learners to find new directions about a task, explaining ideas to others and engaging in mathematical conversations, improving thinking about a task rather than focussing on getting answers right (Lampen & Brodie, 2020). Teachers were examined on the support they gave learners to use and discuss different methods in solving problems. Lastly, I observed learner-learner interaction. Teachers are expected to support learners to have conversations that might enhance their understanding of new concepts (Lampen & Brodie, 2020).

3.5.5 Documents

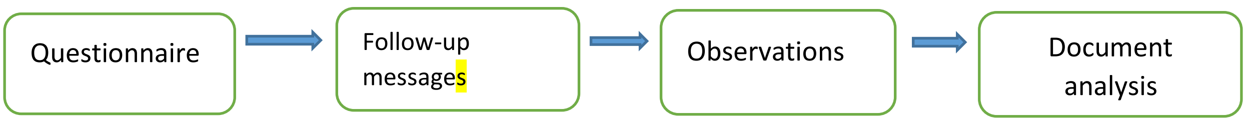
McMillan and Schumacher (2010) define documents as printed or written records of past plans and events. In this study documents included annual teaching plans (ATP) and lesson plans. The four teachers who were observed were asked to submit copies of these documents before they taught their lessons. This was done to elicit information pertaining to teacher planning and preparation and to ensure compliance to policy documents in the planning and teaching of probability (Batanero & Diaz, 2012).

3.6 Trustworthiness

Trustworthiness is not only crucial in qualitative research, but it is demanded to ensure the integrity and usefulness of the findings (Cope, 2014). According to Pilot and Beck (2014) trustworthiness of study refers to the measure of confidence in data, interpretation and methods used to ascertain the quality of a study. In other words, trustworthiness in qualitative research embraces everything to do with the aim, practices and procedures researchers do to ensure the credibility of a study. The credibility of this study was established by using multiple data sources, member checks and iterative questioning during data collection (Nieuwenhuis, 2010).

The first practice I used to establish credibility of this study was to engage four data collection strategies which promoted the use of multiple data collection techniques. This involved use of multiple sources of data mainly questionnaire, follow up messages, classroom observations and documentary analysis. Torrance (2012) argued that data use of multiple data collection sources is most popular because it provides deeper insight into research problems and mitigates shortfalls found in one-source of data. Hence, multiple sources of data were used to reduce the limitations of using one source which might be prone to errors and bias (Patton, 2015). They also provided opportunities for the researcher to cross examine and compare patterns that emerged from data with other findings obtained in the literature review. In addition, the multiple sources of data were used to enhance the credibility of the study because it allowed comparison of different data sets from the open-ended questionnaire, follow-up messages, classrooms observations and document analysis to arrive at richer findings and higher quality research (Creswell, 2013).

Figure 3.1. Multiple sources of data used in the study.



The second practice I used to ensure the credibility of this study was to engage participants in the verification process of all data I collected. Participants were cordially asked to read and verify all the researcher's interpretation of their responses and observations. This is in line with Whiting and Stines (2012)'s assertion that, the meaning-making process of the reported data must be checked and verified by participants to establish both trustworthiness and credibility of the study. Teachers were given time to cross check their questionnaire responses. Others sent some thoughts through what's app as additions (follow-up messages). Participants kept the questionnaire in their phones or laptops so that they can constantly reflect on the questions.

3.7 Ethical Considerations

As part of the research process, it is procedural to seek permission from the Gauteng Department of Education (GDE), school administration as well as the University of the Witwatersrand's Ethics Committee. All this was done, and formal permission was obtained from all the above listed departments to collect data from selected schools (see Appendix H). The second step involved a formal approach to the research participants who were eleven grade 12 teachers from five different secondary schools. The teachers were given both an oral explanation and a consent form that outlined the research project's objectives and data collection methods. This is in line with Henderson and Esposito (2017)'s argument that participants should be consulted on the focus and conduct of the study and that their views should be sought. Creswell (2010) adds that getting letters of consent, getting permission to send questionnaires to participants, and undertaking to destroy audiotapes and other recorded information should be part of the ethics principles. Each participant signed a consent form to confirm their willingness to participate in the study. Both oral and written communication between the researcher and participants were done and stressed the fact that participation was genuinely voluntary and that participants could withdraw from the study without bearing any consequences.

Assurance was given to participants regarding the confidentiality of all the information they contributed to the study. It was stressed that all their contributions will be kept safe in a password protected computer giving access only to the researcher. This process was done before the data collection process began. On issues of anonymity, all reporting from the study adhered to accepted ethical research standards including the use of pseudo names for all participating teachers and schools. For teachers, numbers were used to identify teachers, for example teacher 1, teacher 2 up to teacher 11. Alphabets were used to code schools for example there was school A, school B, school C, school D, and school E.

During data collection process, all Covid 19 protocols were always observed by both the researcher and the participants. This included registration at the entrance and screening, sanitising hands at the entry point, maintaining social distancing, wearing of face masks, sanitising before exchanging materials such as papers and other documents that were used. This was done to comply with GDE covid 19 protocols in their schools. Also, when granting permission, the five principals emphasised that covid 19 protocols were to be observed.

3.8 Analysis

In analysing the data, the frameworks by Batanero and Diaz (2012) and Batanero et al., (2016) about challenges faced by teachers when teaching probability internationally were used as a basis to understand problems teachers faced in South Africa. The authors identified teachers' challenges with respect to probabilistic reasoning; lack of specific training in pedagogical knowledge related to teaching probability; lack of good textbooks and curricular documents to support teachers when teaching probability; misconceptions about probability which are difficult to eradicate through formal teaching; language of probability; lack of effective teaching strategies to foster learner understanding. The theory of argumentation which was used in drafting questions of the open-ended questionnaire and the classroom observation schedule, was also used as a framework to guide data analysis. Muller (2009) described some of the application of the argumentation theory that were considered in the analysis of data in this research as:

- Teachers' knowledge in providing an environment that promoted cooperative learning,
- Teachers' knowledge in asking learners appropriate questions, for example open-ended questions or giving open-ended tasks
- Teachers' knowledge regarding classroom activities that give learners opportunities to explain and justify their answers
- teacher interventions that are carefully planned to foster learner's active involvement in constructing knowledge.
- Mathematical discourse that is carefully planned to promote live class discussions.

It was the above models that were used to guide the description of the different categories that emerged from the data. The interpretive paradigm also offered opportunities for the researcher to engage deeper with participants especially through follow up messages. The researcher was able to analyse the participants' knowledge, understanding, perceptions and experiences to approximate how they constructed phenomenon in their settings (Creswell, 2010 & McMillan, and Schumacher, 2010). This means that data analysis of this study is mainly interpretive.

Discussed in this section is how the data was gathered, categorised, and analysed. The section therefore explains how data from the open-ended questionnaire, follow-up messages, classroom observations, and document analysis were unpacked and analysed. The aim in analysing data was to reduce and synthesise information to make sense out of it and to note any emerging patterns and trends. The data was analysed manually by distinguishing and the describing patterns that allowed the researcher to determine themes. According to Maree (2007) qualitative data analysis is usually done based on an interpretive philosophy whose aim is to examine meaningful and symbolic content of qualitative data. Another dimension of a data analysis in qualitative research that it is an ongoing process (Merriam & Tisdell, 2016) and researchers take advantage of this process to refine or redefine some of their research questions as the focus moves from the particular to the general (Creswell & Creswell, 2018; Creswell & Poth, 2018; Merriam & Tisdell, 2016). For this study three steps were followed in analysing all data collected.

Step 1: Pre-analysis stage

In this first step all the data collected from the open-ended questionnaire, follow-up messages, classroom observations and document analysis were read by the researcher repeatedly to gain understanding of participants' views concerning their problems in teaching probability particularly in Grade 12. It is at this stage that follow-up messages were created and sent to some participants for further elaboration or clarification. Intense reading of the questionnaire responses and follow-up messages was done to capture the thoughts, views, and the experiences of participants.

Step 2: Actual analysis of data

The analysis process involved making sense and generating meaning of the data collected. This involved reading the response and the comments made about a particular question. This captured all expressions that indicated the magnitude and frequency of the problem and the implications of the participant's response. This was then followed by identification and coding of all data found to be aligned with research questions (Merriam & Tisdell, 2016). The process was done for the questionnaire category and follow-up messages, classroom observations and documents.

Step 3: Consolidating the codes

At this level all the patterns noted from the findings were related to the study's objectives and their implications noted. The different codes initially formed were further examined and compared to establish any recurring patterns or similar views. Noted patterns were grouped and further coded according to themes that emerged through analysis process. Further analysis from resulting themes were used to create either an interpretation or description of the case in relation to the research questions.

3.9 Chapter summary

This concluding chapter outlined, explained, and justified the research design, research methodology and the data collection techniques chosen for the study. The reasons for choosing a case study were discussed which included the advantages a case study offered. A case study isolates a case and study it in detail. To explain the persistent problems of poor learner performance in probability at grade 12 required an in-depth study of the teachers who taught probability; their experiences, and the specific challenges they faced with the different aspects of the teaching and learning of probability. The four data collection methods of questionnaire, follow-up messages, classroom observations and document analysis were employed to guarantee the trustworthiness of the data collected. The chapter also explored how ethical considerations were observed and how the data collected would be analysed.

CHAPTER 4: FINDINGS AND ANALYSIS

4.0 Introduction

The aim of this chapter was to present, analyse and interpret all the data collected through the online questionnaire, follow-up messages, classroom observations and document analysis. The chapter is organised into three sections. The first section deals with reports on findings from the teacher questionnaire and follow-up messages. The online questionnaire represented the widest data collection tool as it included all eleven teachers from the five schools. The second section focused on reporting and presenting data collected through classroom observations. Four out of the eleven teachers were observed teaching. It was not possible to observe all the teachers in the sample because of covid 19 restrictions. The third and last section paid attention to reporting on the findings from document analysis. The researcher was able to view teaching documents from four of the teachers because of covid 19 restrictions that did not allow physical contact with participants from other schools. To this study the documents from four teachers were sufficient as the researcher wanted just to have an idea if teachers were facing challenges with planning and preparation to teach probability. The two documents analysed were the annual teaching plan (ATP) and lesson plans because the researcher wanted to know if teachers were teaching according to the ATP and that they planned and prepared proper and adequate activities for their learners. Brahier (2013) asserts that, the effectiveness of a lesson hinges on how well the lesson plan was prepared.

The data analysis process was guided by the two frameworks of the study which are the problems of teaching probability as identified and discussed by Botanero and Diaz (2012) and Batanero et al., (2016) and the argumentation theory in mathematics. The responses of the participants from all the research tools were analysed based on the five problems identified in section 3.8 by Batanero and Diaz (2012) and Batanero et al., (2016) as well as on their relations with teachers' challenges in dealing with the creation of environments that foster discussions, asking open ended questions, giving opportunities to learners to discuss tasks and explore in depth the core content of probability. The questionnaire collected data on teachers' challenges on developing strategies to assist learners to conceptualise probability concepts. On classroom interaction teachers were asked to comment on the mathematical discourse they encouraged in the teaching of probability.

Data presentation began by providing first, the demographic and the professional information about the eleven teachers who participated in the study. This was followed by a presentation and analysis of data collected through online questionnaire and follow-up messages, classroom observations and document analysis. Data collected through follow-up messages were integrated with those from the questionnaire because they were used either to ask teachers to explain a point further or clarify their responses. The responses from the questionnaire were analysed qualitatively and themes from the narrative analysis were then classified into different themes that responded to research questions. Data from the four data collection instruments were analysed independently discussed except for the questionnaire and follow-up messages because these tools were used in conjunction.

4.1 Teachers' general and professional information.

Teachers' information gathered through the online questionnaire included the following:

- The name of the school the teachers were teaching
- The gender of the participating teachers
- The highest mathematics qualification of the teachers
- The years of probability teaching experience of the teachers

Teachers' general and professional information was included as the first part of the questionnaire questions for teachers. Information about the teachers' qualifications was important to ascertain that the teachers teaching probability at Grade 12 were rightly qualified for the task. For example, Fekeye (2012) affirms that teachers' qualifications have been found to contribute significantly to students' academic achievement. Amuche & Musa (2013) report a similar study carried out in Nigeria which suggested that teachers teaching a subject must be trained and certified in the subject they are teaching because they will have specific knowledge relevant to that subject. Teachers were asked to provide information on gender, age range, highest qualification obtained, experience in teaching mathematics, experience in teaching probability and the current grades teachers were teaching probability. They were also asked if they did probability in their years at school and/or during their training at university or college. A total of 11 teachers responded to the interview questions above and the responses are shown in Table 4.1 below. Teachers were numbered from teacher 1 (T1), teacher 2 (T2) up to (T11) without any order or meaning. Throughout the entire chapter the same number will be used for each teacher.

Table 4.1: General and professional information about teachers.

Teacher	Gender	Degree	Diploma	Certificate	
T1	F	B Ed Honours	NA	NA	1

		Mathematics			
T2	M	B Ed in Science & Mathematics	NA	NA	1
T3	M	B Ed Honours, Mathematics	NA	NA	1
T4	M	B Ed in Science & Mathematics	NA	NA	1
T5	F	B Sc Agric degree.	PGCE, Science & Mathematics	NA	1
T6	F	BSc, Science & Mathematics	NA	NA	1
T7	M	STD, ACE	NA	NA	1
T8	F	B Ed, Honours, Mathematics	NA	NA	1
T9	M	M Ed.	NA	Certificate in Education	1
T10	M		M in Operations Research	NA	1
T11	F	B Ed Honours, Mathematics	NA	NA	1

Key: NA represents Not Applicable.

Table 4.2: Summary of Teachers' Professional Information.

Degree Programs	Male	Female	Total
BSc- Science & Maths	0	1	1
BSc Agriculture	0	1	1
B Ed Honours Mathematics	1	3	4
Master's degree	2	0	2
ACE	1	0	1
B Ed Maths & Science	2	0	2
Total	6 (55%)	5 (45%)	11

A total of eleven (11) teachers completed the online questionnaire for teachers. It will be noted however that, the number of responses to some questions on the questionnaire might be less than eleven. This was caused by some teachers who skipped some questions in their responses. It will be observed on summaries that the number of responses varied from question to question because of this inconsistency from the teachers. Table 4.1 and 4.2 show that there were more male teachers 6 (55%) than female 5 (45%). The tables also reveal that the modal qualification was the B Ed honours in mathematics with 4 out of 11 (36%) teachers possessing this qualification. Two teachers had a bachelor's degree in education in science and mathematics (B Ed Maths & Science), 2 teachers had master's degree, one possessed a bachelor's degree in science and mathematics (B Sc-Maths & Science), one teacher had the Advanced Certificate in Education (ACE), and the last teacher had a B Sc degree in Agriculture plus Post Graduate Certificate in Education (PGCE).

Information on teachers' experiences in teaching probability was gathered and summarised in the table below.

Table 4.3: Teachers' experiences teaching probability.

Teacher	< 1 year	1-5yrs	6-10yrs	>10yrs
T1	NA	NA	X	NA
T2	NA	NA	X	NA
T3	NA	NA	X	NA
T4	NA	X	NA	NA
T5	NA	NA	X	NA
T6	NA	NA	X	NA
T7	NA	NA	NA	X
T8	NA	NA	X	NA
T9	NA	NA	X	NA
T10	NA	NA	NA	X
T11	NA	X	NA	NA

Regarding probability teaching experience, the data indicates that 7 out of 11 (64%) of the teachers had between six years and ten years teaching experience. 2 out of 11 (18%) had more than 10 years probability teaching experience. There were 2 (18%) teachers with probability teaching experience of five years or less.

The data concerning teachers' years of experience was collected to find out if there was a relationship between the kind of problems teachers faced and their level of experience. In other words, the researcher wanted to find out whether teachers with more experience in teaching probability had less challenges compared to the novice probability teachers. From the data collected, there was no clear relationship between the two. For example, a teacher who had two years teaching experience in both mathematics and probability answered that he was comfortable in teaching all subtopics of probability while teachers with more probability teaching experience answered that they had problems. The pattern that surfaced was that relating lack of probability experience during schooling years and at teacher training with problems of teaching probability. Many of the teachers who indicated that they did not do probability in their high school learning and teacher training also indicated that they were having problems in teaching probability. However, this relationship cannot be ascertained because of the size of the sample. In this study, there is only one teacher who did probability at high school and at university who answered that he was comfortable with teaching probability. This relationship needs further investigation.

4.1.1 Reporting on teachers' qualifications

An analysis of the data from Table 4.1 and 4.2 above, shows that all the teachers were qualified to teach High school mathematics with the lowest qualification being the Advanced Certificate in Education (ACE). ACE is a qualification that teachers needed to upgrade themselves from the REQV 13 to REQV 14 which was recommended by the Department of Education as the minimum basic qualification for all teachers in South Africa. REQV is equivalent to (M+4) that is a matric certificate plus four years post matric training. ACE also afforded teachers the opportunity to retrain themselves in a new subject area or to further specialise in a subject that they were already teaching (Welch, 2009). The data reflects that a high number of teachers in the study either had a bachelor's degree or an honours degree in mathematics.

Based on the qualification possessed by teachers in the study, there cannot be any justification to think that teachers were underqualified to teach mathematics or even probability. However, the analysis of the teachers' responses in relation to their qualification revealed similar patterns found in other studies carried out in South Africa before which indicated that, even though teachers met all the minimum qualification requirements to teach mathematics, many of them being holders of B Ed in mathematics and some with B Ed Hon degrees, their mathematics content knowledge (MCK) related to probability remained low (De Kock, 2015; Bansilal et al, 2014, Kodisang, 2016; Awuah ,2018).

4.1.2 Teachers' years of experience in teaching probability

The years of experience teaching probability at Grade 12 were relatively lower when compared with mathematics teaching experience from the same teachers (not shown in the table above). While mathematics teaching experience for the teachers averaged twenty and above with many teachers having over thirty years of mathematics teaching experience the number of years teaching probability averaged ten years or less. This accounts for a relatively limited probability teaching experience for the teachers because probability was introduced recently under the CAPS syllabus in 2012 in grade 10. The data collected suggests that experience could still be a challenge to some of the teachers. This view is derived from the teachers' responses who indicated that they were still learning the content and the methodology to teach probability. Teachers' remarks on teaching probability will be discussed under teachers' questionnaire, follow-up messages and classroom observation responses. Literature suggested that the main challenges teachers faced were lack of probability teaching experience and probability content (DeKock, 2015; Makwakwa, 2012). However, if teaching experience depends on the number of years teachers have been teaching a topic, then teachers in this study must have gained more experience since the two studies above were done. According to the data collected in this study, many of the teachers had more than six years probability teaching experience. However, despite the experience, teachers still wrote in their responses that they faced challenges when teaching probability.

4.1.3 Summary

The analysis of the data on teachers' qualifications and probability teaching experience indicated that all teachers were qualified to teach mathematics. The data also suggested that many teachers in the study had over six years of probability teaching experience. Teachers however when asked about their encounters in teaching probability, they answered that they had challenges ranging from explaining the probability terminology and teaching specific subtopics for example, dependent, independent, and mutually exclusive events.

4.2 Language in probability

Research question 1

The first research question was: What are the challenges faced by teachers in terms of language of probability?

To answer this research question, teachers' challenges were explored according to the literature reviewed and the responses from the open-ended questionnaire. Follow up messages brought up more discussions to collect more data about language related challenges. A thematic analysis was then done to identify the nature of the challenges teachers and learners faced in the teaching and learning of probability. The discussions are presented below according to the different terms identified and the challenges they pose.

The questionnaire was used to gather information from teachers about:

- Terms that teachers and learners were having problems to understand
- This question was designed to find out specific terms that teachers and learners found challenging and the comments on the nature of the challenges
- The strategies teachers used to help learners to understand the terms referred to.

This was done to establish the precise terms that made the teaching and the learning of probability to be problematic to the teachers. Literature revealed that probability was difficult both to teach and to learn because of the difficult terms used (Batanero and Diaz, 2012, Batanero et al., 2016; Groth et al., 2016)

Teachers were asked the question: *"Can you give some examples of terms in probability that your learners struggle with? What teaching strategies do you use to enable your learners understand the terms used in probability?"*

The extracts below are samples from the responses to the question above. The terms that were mentioned in the responses included mutually exclusive events, depended on and independent events, complementary events, and compound events. Ten out of the eleven teachers mentioned mutually exclusive, dependent, and independent events in their responses. Four teachers mentioned complementary and compound events. Below are samples of teachers' responses about the challenges teachers face in dealing with the different terms.

Section B. Language of/in probability

1. Can you give some examples of terms in probability that your learners struggle with? What teaching strategies do you use to enable your learners understand the terms used in probability?

(Please say more……. Random experiments, mutually exclusive, inclusive, dependent, and independent events. Is difficult to explain these terms to learners. I also confuse mutually exclusive events with independent events. I find it difficult to explain independent events. The textbook I use at my school Platinum explains independent events as events where the outcome of the first experiment does not affect the outcome of the second experiment. It does not give examples. As a teacher I must give examples myself and I find it difficult to find clear examples.

Considering the response above, it was clear that the teacher's challenges are related to her lack of clear understanding of the terms herself because she mentioned that she confused the terms. This response illustrated some of the difficulties teachers experienced in teaching probability terminology and this becomes an obstacle in the teaching and learning of probability. For effective teaching of probability to happen, teachers should know and use the correct probability language to teach learners. The teacher needs to know the correct terminology which he or she should also be able to explain it to the learners so that they are able to understand the meaning. The response revealed that the teacher lacked the mathematical content knowledge (MCK) for teaching probability. This is illustrated by the teacher's admission that she struggles to get clear examples to support the explanation of probability terms. Teachers with a high level of MCK can use practical tools and provide contextual examples that assist learners in problem solving and in presentation of mathematical materials to learners (Bill, Hill, & Bass, 2005; Baumert et al., 2010; Hawk et al., 1985; Kanyongo & Brown, 2013; Mogari et al., 2009; in DeKock, 2015).

(Please say more……. Mutually exclusive, Independent events, random events, experimental probability, equally likely. There are many terms that are difficult to explain but the ones I listed above are very problematic. I find it difficult to explain the difference between mutually exclusive and independent events. Some books say mutually exclusive means events that exclude each other while independent events are events that do not influence each other.

The response above shows that this teacher also had problems of confusing the terms used in probability. The teacher reported that he found it difficult to explain the difference between mutually exclusive and independent events. This confusion of the terms can be explained in terms of the teacher's limited understanding of both terms, because if the teacher clearly understood the two terms, he would understand that they were different. According to Bold, Filmer, Martin, Molina et al. (2017), for teachers to be effective in their teaching, it is needful that they possess a clear understanding of the subject they teach. Relating this assertion to the responses from the teachers, the conclusion that can be drawn is that the teachers who responded this way are not effective in teaching probability.

Learners struggle with, dependent and independent events, differentiating between independent events and mutually exclusive events, complementary events, contingency table, and probability tree diagram.

(Please say more....... Learners do not struggle with understanding the meanings of the above terms, but they confuse them. For example, learners confuse mutually exclusive events and independent events and this always lead to wrong use of the formula. Learners fail to identify from the question whether what they are dealing with in a question is dependent, independent, or mutually exclusive event. These terms are difficult to explain to learners so that learners can be able to identify them from the question. This requires a special skill from the teacher, in other words the teacher must be good in probability which many of us are not yet. A teacher must be able to use different examples to help learners understand the terms.

11. In your view what are the three major challenges faced by a teacher teaching probability and Counting principles at Grade 12? (If not Grade 12 specify the Grade. Feel welcome to say more...)
    (1) Language of probability it makes interpretation of questions difficult. In one workshop I attended when probability was first introduced, the facilitators also struggled with some probability questions
    (2) Teachers' content of probability is weak, especially for old teachers like me who did not do this topic at university, how are we supposed to all of a sudden supposed to know it and teach it especially in Grade 11 and 12. It is difficult.
    (3) Because this topic is still new, it must be given more time, they must not put it last in the ATP, it must come first or second.

The sample of responses above is part of the responses from other teachers that responded to the question on probability terms that are challenging teachers and learners. One teacher did not respond to the first question on the questionnaire. In analysing the data collected through this question from the 10 teachers, the themes that emerged were as follows; (a) I find it difficult to explain the terms mutually exclusive events, dependent and independent events as a teacher, (b) Learners confuse exclusive events with independent events, (c) Textbooks are not helping teachers and learners to understand the terms.

The responses confirmed that Grade 12 teachers have problems with some probability terms and that the challenges are twofold. The responses indicated that teachers had challenges of understanding and explaining mutually exclusive events and independent events to learners. The effect of this is that learners fail to understand the terms and hence confuse them when answering probability questions. The responses also show that learners are not able to differentiate between the different terms, mainly independent, dependent, and mutually exclusive events. This is what on teacher said in the extract above, *"Learners fail to identify from the question whether what they are dealing with in a question is a dependent, independent, or mutually exclusive".*

Other teachers reported that the difficult terminology made it difficult for them to interpret questions leading to use of wrong representations, diagrams, and formulae. One teacher said about the language of probability,

This is where the difficult is, understanding the language of probability. At times I read a probability question and fail to think of an answer. I think I must understand the language first before I can be able to explain it. I think language goes with notation also. There are questions that imply the use of a probability tree diagram, but if you have not mastered the language, you will not know that you must use the probability tree diagram

These sentiments were shared by another teacher who responded by saying,

I have not mastered the language and questions of Grade 12 probability. I am not able to structure an answer from the information given. I do not know whether to use the tree diagram or the formula for mutually exclusive or independent. I think the language in probability is difficult to understand

The teacher's response above explains how the teacher fails to make sense of probability questions because of failing to interpret the terminology. Brown and Adler (in Sharma, 2015) maintain that students can only be competent in mathematics if they understand the highly technical language used specifically in mathematics. In the literature review it was revealed that the language used in probability was more difficult understand than the usual mathematics language. Teachers and learners alike can excel in probability if they are proficient in the language of teaching and learning probability and from the responses coming from teachers in this study the challenges of language of probability are prevalent among teachers and learners.

6.2. Dependent and independent events. Score. _5__Grade_12__

(*Please, say more about your score.......)* I find it difficult to explain the difference between mutually exclusive and independent events. I have not found a good textbook that explains the difference clearly. But what I always emphasise are the formulae for "or" and "and" for independent events. Sometimes questions are not clear as to whether events are independent or mutually exclusive.

The comment above made by one of the teachers, reveals that this teacher experienced the same problems as others and then the teacher resorted to teaching learners the formulae without making them understand probability concepts. These practices are contrary to the conceptual framework of this study which emphasises active participation in learning. The teacher mentioned that the textbooks he used were not helping him to understand probability terminology.

In further analysing the themes, I found out that most teachers who had indicated that they did not do probability at school and at university had similar responses, that they found explaining probability terms to their learners very difficult. Out of the 11 teachers, 9 out 11 (81, 2%) claimed that they did not do probability at high school, during teacher training and at postgraduate. Out of these 9 teachers 8 wrote that they had problems explaining probability terms especially mutually exclusive and independent events. One teacher (T8) answered that she had no problems with explaining the terms or teaching them. Her response was.

When I explain mutually exclusive events, I bring a die in class and explain from sample space, I let them count the faces of the die and from there we choose prime numbers, even numbers, etc from a die that everyone sees…. I let my learners to be actively involved by encouraging them in the discussions

Many authors recommended the use of physical objects such as coins, dice, spinner, and simulation as a useful strategy that can help learners to have interest in probability and improve their understanding of the topic (Koparan, 2016; Batanero and Diaz, 2012). T8's teaching strategies are in line with Liu (2011)'suggestion that teachers need to use multimedia and provide rich real-life situations and games that facilitate learners' understanding of probability. In the response below T8 explained that when giving examples she used learners' names, and she said this made the lesson to be interesting. Use of real-life situations goes beyond generating interest from the learners to helping them to understand probability concepts better (Batanero et al., 2016). The idea of letting learners to be actively involved in class discussions as a teaching strategy is a positive step given that some authors assert that many teachers in South Africa use teacher-centred approaches when teaching mathematics (Kodisang, 2016; Machaba, 2013).

I explain the terminology first using explanations from different sources (Mind Action Series, Kevin Smith, Platinum textbooks). Integrating the African language into the lesson is the last resort to help them understand. When I give an example, I use their names to make the lesson interesting.

With respect to the review of the literature from studies done in South Africa, this raises concern because the assumption was that teachers were experiencing problems to teach probability because they lacked experience of probability during their schooling days and at teacher training. This then suggests that all teachers must be provided with strong in-service training to teach probability (Awuah, 2018). The summary is given in figure 4.4 below.

Table 4.4. Summary of teachers' responses on how they regard probability language/ terms.

Items	**Category B: Language/ Terminology used in Probability**	**Teachers' responses**				Comment
		Very difficult to explain	Difficult to Explain	Easy to Explain	Very easy to explain	
1.1	Explaining mutually exclusive events	T1, T2 T6,	T3, T5, T9, T11	T8, T7	T4,	Explaining term mutually exclusive- Difficult
1.2	Independent & dependent events	T1, T2, T6, T9	T3, T5, T7, T11	T4, T8	NA	Explaining terms- Dependent & Independent- Difficult
1.3	Complementary events	NA	T1, T9	T8, T7	T4	Not included by others. Easy

1.4	Compound Events	T1	T5, T6	T8	NA	Not included by others. Difficult.

Where the number of responses on the table is less than 11, the reader must know that other teachers did not respond to that question. The data collected in Category B above shows that 7 out of 10 (70%) teachers found the explanation of the term “mutually exclusive” events either difficult or very difficult. Three teachers indicated that they had no problems in explaining this concept to learners. One teacher did not attempt all the questions from this section. The three teachers who indicated that they had no problems in explaining this term were, the inexperienced teacher (2 years teaching experience both for mathematics and probability), the second is the teacher who did probability when he did ACE, and the last one was a well experienced teacher with 30 years mathematics teaching experience and 9 years probability teaching experience.

According to the table, the next question teachers responded to was to explain the term “independent” events to learners. Responses to this question were like those for mutually exclusive events with only one variation. One teacher moved from difficult to very difficult in explaining independent events. The last two questions in this category addressed the terms complementary events and compound events. Only five teachers included this term in their response of which four out of five regarded it either easy or very easy to explain. One teacher found it difficult. It was therefore classified as easy. Five teachers commented about explaining compound events with four out of five teachers regarding the term as either difficult or very difficult to explain.

Therefore, the results in table 4.4 indicate that teachers found it difficult to explain probability terms to their learners. The data included responses from Grade 12 teachers some of which were not necessarily teaching Grade 12 at the time of collecting data, but they have Grade 12 teaching experience. 3 out of 10 (30%) teachers who responded to the question of explaining the term “mutually exclusive” events answered that they found it very difficult, 4 out of 10 (40%) said it was difficult. 2 teachers said it was easy and only one teacher confessed that it was very easy. This suggests that more teachers had problems explaining exclusive events. Referring to independent events, 40% of the teachers viewed explaining the term “independent” events to learners as very difficult and another 40% regarded it as difficult. Only 2 teachers (20%) had no problems in explaining independent events to learners. Although fewer teachers responded concerning the terms compound and complementary events, the responses revealed that teachers found it generally difficult to explain compound events to learners. More teachers found it easy to explain complementary events.

4.3 Teacher Content Knowledge

Research question 2

The second research question was: What are the teachers' challenges with probability content knowledge?

To answer this research question content from the literature review plus the responses from the questionnaire were analysed. Data collected through classroom observation and document analysis from four of the teachers from the sample were used to strengthen and substantiate some of the claims made by teachers' responses to the questionnaire.

The question on the questionnaire on probability content knowledge addressed the following themes:

- Whether teachers did probability in their pre-service training
- Teachers' comments about the impact of professional development programmes organised by DBE for improving probability teacher content knowledge.
- How teachers personally rated their probability content knowledge.

Extracts below are samples of teachers' responses to the questions regarding their ratings of the probability content knowledge they possessed and what they thought about DBE workshops on probability.

> *(Please, say more.......* Inadequate. I am not confident in teaching probability like I am teaching other topics. I think I lack the foundation of this topic. This has made me to have a negative attitude in learning it and teaching. We are two teachers at my school teaching grade 12, so I always teach other topics and my colleague helps with probability.

I teach grade 10 and Grade 12, I tell learners to attempt only those questions they can answer easily in probability, if my learners can get any mark in probability that is a boost for a distinction. For the weak and the average learners, we do not encourage them to do probability. I find probability too abstract.

3. Since your graduation, have you had other teacher development training in the teaching of probability? How many and where? Were these programs organised by the DBE or b school?

1

(Please, say more……. I have attended many training workshops since I graduated. I cannot remember the number of times. But many of the workshops I attended back then probability was not covered. Probability came recently with the CAPS syllabus and I attended one or two workshops where probability was discussed. For the past 5 or 6 years I have not attended a workshop except road shows where they will be discussing areas where learners struggled answering questions. Probability is always one of the problematic questions in paper 1.

3. Since your graduation, have you had other teacher development training in the teaching of probability? How many and where? Were these programs organised by the DBE or by the school?

(Please, say more……. Yes, I attended a few workshops. I gained nothing from the workshops. I found it still very difficult to teach probability after the workshops. Facilitators did not accommodate teachers like me who were doing probability for the first time, we were not up grading what we knew but we were starting from zero. We needed background of probability. Another suggestion, I personally think the problem of probability must be addressed from grassroots. If possible, all Universities must pay attention to its teaching and address the content problem of probability at that level. In-service training will not help because not all teachers attend these trainings and workshops.

In analysing the data collected through the questions above, the themes that emerged were (a) teachers lack foundation in probability (b) teachers advise the weak learners not to answer questions on probability, (c) professional workshops organised by DBE were not effective, (d) teachers are not confident to teach Grade 12 probability. Based on the responses above, it meant that the teaching of probability in some schools is compromised, and learner performance is at risk. Teacher content knowledge is a fundamental element of what is to be taught and learned. Teachers who lack confidence in a subject content are less likely to be enthusiastic to teach that subject. Good and Lavigne (2007) argue that mathematics teachers who possess inadequate content knowledge of a topic may lose interest in what is to be taught and may end up teaching less or not teaching the topic at all.

Khaliliaqdam's (2014) argues that, for effective learning to take place, the knowledgeable other must be experienced in the specific field that is being taught. Relating this to the challenge of teaching probability content to learners, teachers should not only have the knowledge of the topic but should have the experience of solving problems of different levels from that topic. Literature supports the notion that challenges teachers have with the terms of probability and probability content in general are the results of the teachers' inexperience with probability both during their schooling days and during pre-service training (Ogbonnaye and Awuah, 2019).

There are teachers who mentioned in their responses that they lacked the background of probability content. According to one of the responses, professional development workshops failed to recognise that the teachers they were workshopping had not done probability at any level in their academic learning as well as in their training. This was problematic to many teachers who attended the workshops but did not benefit anything. In his response, the teacher stated that the teaching of probability should be developmental. By this the teacher meant that teachers were to be taught the basic concepts of probability from primary school level and be developed up to Grade 12. Many of the responses in this work indicated that what teachers lacked was a strong foundation of probability. For example, one teacher said, "*I am not confident in teaching probability like when I am teaching other topics. I think I lack the foundation of this topic*"

Another important point to be made about professional training programmes for teachers is the way the content is delivered to teachers. Early et al. (2016) affirms that, whatever programmes might be designed for teachers, they must be content focused and use active teaching methods where teachers are not passive listeners with very low participation. The programmes must involve teachers through practical activities rather than providing them with theory with no practical application (Parrot and Eu, 2018).

4.3.1 Teachers' experiences with probability

The data collected through the questionnaire (not shown in the tables above) indicated that only two teachers did probability both at high school and at university. Of the two teachers, one was a recent graduate from the Witwatersrand University. This teacher did probability at high school under the CAPS syllabus and did probability at university. The teacher explained that at university the emphasis was on methodology of teaching probability rather than content. This placed the teacher at an advantage to teach probability because he learnt probability content at high school and methods at teacher training at university. The second teacher (T10) who did probability at both high school and university did his education in Zimbabwe. This teacher indicated that probability was his favourite topic in the current syllabus.

Eight teachers did not do probability both at high school and at teacher training, meaning they missed on the content of probability which they must have gained at high school and the methodology of teaching it which they must have obtained from university. Such teachers were supposed to be supported by the department of education for both content and methodology by way of in-service training. One teacher who did ACE did not do probability at high school but did it when he studied ACE. Analysis of teachers' preservice knowledge of probability revealed that all the teachers who did not do probability at high school and teacher training indicated that explaining probability terms and teaching different subtopics was difficult for them despite the over six years probability teaching experience. Biyela et al. (2016) advanced that pre-service teacher training programmes represent the base for competent mathematics teaching, and because of this they should aim to produce competent and capable mathematics teachers who are prepared to produce positive results in the classroom. Craig, Evans, and Stokes (2017) add that, pre-service teacher preparation programmes should be transforming and relevant to give pre-service mathematics teachers adequate knowledge required for the tasks ahead.

The question on teacher professional development aimed at collecting data about the support given to mathematics teachers after the mathematics curriculum change. It was important to know whether the teachers were given enough support and training to meet the new challenges arising from the introduction of new topics, in this case probability. Questionnaire responses from the teachers showed that in-service training was provided but it was not enough in terms of providing adequate probability content knowledge for teaching up to grade 12 level. Many teachers felt that in-service training on probability was discontinued too soon before teachers had mastered both the content and the methodology. Their comments are supported by

Klieckmann et al. (2013) who argued that for in-service training to be effective, it needed to be designed to meet the teachers' individual needs and should engage teachers in the training sessions over an extended period.

Teachers thought that the training which was provided was in adequate because most teachers were doing this topic for the first time. A comment by one of the teachers is given below. This teacher argued that the workshops organised by the department when probability was introduced were not enough because most of the teachers never did this topic before. The teacher added that training in probability must be extended to the other teachers teaching probability in lower grades because probability is taught in those grades as well. This comment makes sense because if probability is not introduced properly in the lower grades, learners might develop misconceptions that may be difficult to eliminate at higher grades.

> *(Please, write more about ....)* With problem number 1, I think the department must organise more workshops. The one or two workshops they organised were not enough because many teachers never did this topic before and these workshops must include all maths teachers not only grade 12 teachers. This topic is also there in Grade 9. Grade 9 teachers must know it well to give learners a good foundation in probability.

> It is the same with content, teachers need to be taught probability first. Teachers are willing to learn probability, but the department is no longer organising workshops for teachers on probability. Their workshops were not enough, maybe they can organise one workshop every term until teachers understand the topic and this must target all maths teachers with problems in teaching probability

While most of the teachers reported that the professional development workshops organised by DBE did not benefit them, three teachers from the study had different views. These teachers reported that the in-service training they received from the DBE workshops and other workshops were very helpful in improving their content of probability. For example, one teacher wrote in his response; *"I attended some workshops organised by GDE in 2009, 2010, 2012,2013…I learnt a lot about probability and counting principles, and Euclidean geometry."* The reasons to justify why most of the teachers viewed these programmes as not beneficial to them might be found in Umugiraneza et al (2017)'s argument that for professional development programmes to be effective, they should be conducted continuously. Junqueira and Nolan

(2016) expanded this argument when they said that all mathematics teachers need to be continuously supported to progressively develop more in the mathematics they teach. Teachers need continuous support to possess rich mathematics knowledge, skills, and the best strategies of teaching mathematics in their schools. As the authors have explained, the programmes must be continued over a period until teachers and learners are proficient in the topic. This is what was supposed to happen with probability since it was a new topic. Moreover, literature has revealed that the concepts of probability are abstract and hence very difficult to teach and to learn. According to the teachers' responses the programmes were discontinued before teachers fully understood probability.

Below is summary of teachers' responses regarding teacher content knowledge of probability.

Table 4.5. Summary of teachers' responses regarding probability content.

Items	**Category C: Teacher content knowledge of probability**	**Teachers' responses**				Comment
		Inadequate	Lacking in some respect	Adequate	Excellent	
1.5	Knowledge gained during preservice training.	T1, T2, T5, T6, T3, T9,	T8 T11	T7, T4, T10	0	Knowledge gained through pre-service-Inadequate
1.6	Knowledge gained through in-service training.	T1, T2, T5, T6, T3, T9,	T4	T8, T7	0	Knowledge gained through in-service-Inadequate
1.7	How do you rate your current probability knowledge?	T1, T2, T5, T6	T3, T11, T9,	T8, T4, T7, T10	0	Teachers' knowledge of probability-Even
1.8	How do you rate your contribution to your school pass rate in	T1, T2, T5, T6	T3, T9	T4, T7, T10	T8	Contribution to school pass % pass-Inadequate.

	probability in the last 3 years?					

On the table above, question shows less than 11 responses it means that some participants did not respond to that question even after follow-up messages were sent to them. The data in category C of table 4.5 above informed that teachers regarded as very inadequate the probability knowledge they acquired in their pre-service training as well as in their in-service training. The data also revealed that teachers generally rated their content knowledge of probability as inadequate and that their own contribution to the school results in probability was very inadequate. These responses resonated findings by DeKock (2015) and Kodisang (2016) about teachers' low confidence in dealing with probability content. DeKock (2016) reported that some of the teachers in that study struggled to correctly answer questions on the cognitive level of knowledge. That study found out that most teachers could not recall the formulas and rules used to solve problems on dependent, independent, and mutually exclusive events and hence they could not apply the formulas and rules correctly to answer the questions. According to data collected through the questionnaire, these problems have not been solved as teachers answered in the questionnaire that their content was inadequate. Teachers indicated that the probability results for their schools in the interviews (not shown in this table), the results for most schools were all below 45%. The responses from this group of teachers indicated that they lacked a strong foundation of probability knowledge. To be effective in their work in teaching probability, teachers need to understand the probability they teach to their students (Batanero, 2016).

4.4 Exploring teachers challenges in teaching specific probability subtopics.

Research question 3

What are the teachers' challenges with some specific subtopics of probability?

In these set of questions, teachers rated the challenges of teaching their learners the probability subtopics on a scale of 1-10 and then elaborated on their experiences.

The questions sought information on teachers' challenges when teaching the following:

- Mutually exclusive events
- Dependent and independent events
- Explaining the language used in teaching probability

- The language used to teach mathematics
- Helping learners to understand and perform well in probability
- Help learners solve problems using Venn diagrams, contingency table, and probability tree diagrams.

In analysing responses from the above questions, themes which emerged were, (a) Difficult to explain mutually exclusive events and independent events (b) Lack of confidence in teaching probability. (c) Confusing language, (d) probability tree diagrams are most difficult to explain, (e) probability tree diagrams are difficult for learners to understand.

> *(Please, say more about your score.......)* I find it difficult to explain the difference between mutually exclusive and independent events. I have not found a good textbook that explains the difference clearly. But what I always emphasise are the formulae for "or" and "and" for independent events. Sometimes questions are not clear as to whether events are independent or mutually exclusive.

> *(Please, say more about your score.......)* It is difficult to distinguish between dependent, independent, and mutually exclusive events from a question. I understand independent events if the question involves replacement and no replacement. If those terms are not there and the questions requires proof of independence, I usually struggle. It goes to the point that to be good in probability one must understand the whole language of probability.

> *(Please, say more about your score.......)* I struggle with dependent and independent events. At times I am completely lost. I always prepare what I will be teaching in a lesson. I can work out all the questions before I go to class and am able to teach to learners. I can have problems when learners ask me some questions that I did not prepare myself for, but grade 10 normally do not ask questions except they are given by the teacher. I always refer my learners to ask from

Further analysing the different responses from the eleven teachers it was revealed to me that the same teachers who had difficulties with the other aspects of probability confessed the same challenges in this question.

The summary in Table 4.5 below also revealed that there were few teachers who crossed from one group of responses to another. For example, the teachers who answered the questionnaire having difficulties in explaining terms were the same teachers that said they did not gain anything from developmental workshops, they were the same teachers who confessed that they had inadequate knowledge to teach probability.

T9 said the following remarks with respect to dependent and independent events, *"There are questions that even after reading, and you cannot make sense of the question. The working for probability is not clearly implied in the question. There are terms you must understand to help you to answer that question."* T9 further commented that, for a teacher to be able to help learners understand a topic, the teacher must understand the topic more than the learners he/she is teaching. He added that the teacher must be able to use more than one approach to get to the answer and be able to give examples to explain the answers. T8 stated that without understanding the language the learners will not be able to understand probability. She added, *"to help my learners understand probability I expose them to all levels of questions in class from simple to complex, to build their confidence"* Earlier in the interview, she had stated that she works on helping her learners understand probability language before teaching the concepts. She said she uses different textbooks to build good understanding of probability language in her learners.

T4 did probability both at high school and university during his teacher training. This teacher shared the following remarks about his experiences in teaching probability,

"I find it easier to make learners understand the language used in Functions, Calculus, or Geometry. Probability requires more problem-solving skills than any other topic in the syllabus. I find probability more demanding in terms of analysing the text and formulating the answers and only to find that no procedures are the same in solving probability questions."

The same teacher responded to the question of helping learners to solve problems using probability tree diagrams like this, *"What is difficult is to help learners to figure out from the questions that they can apply the probability tree diagram. This is difficult to me as a teacher also. There is a lot of reasoning required in formulating probability answers especially in Grade 12"*

This observation by the teacher is in line with what many authors including Batanero and Diaz (2012) argued that stochastic (statistics and probability reasoning) is different from mathematical reasoning. Other teachers in the study stated that probability was unpredictable and difficult to map out answers from the word problems.

T6 narrated her own account that probability problems are tricky. She said,

".... the textbook is not helping because it does not explain the terms clearly. Problems in probability do not have clear steps to solve them. Every question comes with its way of solving depending on the wording of the question. Questions are very tricky...."

The teacher also mentioned that in her teaching of probability, she only focuses on what learners can easily understand. She continued,

"With probability we always concentrate on the questions learners can easily understand. Our learners fail to read and understand the questions." She spoke also about examination questions not resembling the questions they find in the textbook.

"Many learners struggle with probability because of the language and the unpredictability of the questions. As a teacher you may explain the questions in the textbook but when the exam comes the questions are different. May be the textbooks we are using are not aligned to the syllabus"

Table 4.6 Summary of teachers' responses about their challenges in teaching different subtopics of probability.

Items	**Category D: Exploring teachers' challenges in teaching specific probability subtopics**	**Teachers' responses**				Comment
		Not difficult	Less difficult	Difficult	Very Difficult	
1.9	Mutually Exclusive Events	T8, T7, T10	T3, T11, T4	T2, T5, T9, T6	T1	Mutually exclusive- Difficult
1.10	Dependent & independent events	T8, T7, T10	T3, T4	T2, T5	T1, T6, T9, T11	Dependent &

						independent-Difficult
1.11	Explaining language used in probability.	T8, T4, T10	T7, T3	T5, T11	T1, T6, T9, T2	Probability language-Very difficult
1.12	Explaining language used in mathematics.	T8, T4, T7, T3, T10	T11, T5, T2,	T6, T9, T1	0	Mathematics language-Not difficult
1.13	Help learners understand and answer probability questions.	T8, T10	T4, T7	T3,	T1, T6, T9, T5, T11	Probability questions-Very difficult
1.14	Helping learners to solve problems using Venn Diagrams.	T8, T4, T7, T10 T2, T11	T5, T9, T3	T6, T1	0	Venn diagrams-Not difficult
1.15	Helping learners solve probability problems using contingency tables.	T8, T4, T7, T2, T10,	T5, T9, T3, T11	T6	T1	Contingency table-Not Difficult
1.16	Help learners to solve problems using tree diagrams.	T8, T10	T4	T7, T3	T1, T2, T5, T6, T9, T11	Probability-tree diagrams-Very difficult

The summary of this section showed that teachers experienced problems in teaching most sub-topics of probability. The causes were revealed as lack of in-depth probability knowledge from the teachers and lack of understanding probabilistic reasoning. This was reflected in some of the teachers' responses who submitted that they selected those concepts which their learners could easily grasp and leave the rest. One teacher admitted that there are probability questions that she could not interpret herself. The data also showed that, teachers found it difficult to teach mutually exclusive, dependent, and independent event and found it even more difficult to explain probability language to the learners. The problem of language is central to the problems teachers were facing. The subtopic that was viewed as most difficult to teach was the probability tree diagram.

4.5 Use of Technology in Teaching Probability

Research question 4

Research question four was asked as follows: What are the teachers' challenges with the use of technology in teaching probability?

This of questions was asked because of the important role teacher support materials play in the teaching of probability. Some books are written in a language that is easier for both the teacher and learners to understand. Also important about the information in textbooks is the choice of examples and the sequencing of activities.

This section addressed issues about probability teaching and learning materials used by teachers and learners as sources of probability content. The question asked the teachers about "tools" they used to teach probability. The question clarified that the term tool was used in a broad sense to mean the sources that teachers and learners used in the teaching and learning of probability.

The fourth set of questions focused on collecting data on:

- The challenges teachers were facing about the tools they used when teaching probability.
- Whether teachers felt the school was doing enough to support the teaching and learning of probability.

In analysing data collected through these questions, the themes that emerged were (a) my school uses only one textbook, that is all, (b) I use a textbook and another source, for example online, (c) I use different textbooks for learners to understand probability language (d) I use the textbook plus past exam question papers from internet.

Section E. Use of technology in Teaching Probability

7.1 What tools do you use for teaching probability at different Grades that you teach? ('Tool' is used as a general term to include all resources, such as textbook, and online resources that you use for teaching). I use mainly the textbook, calculators, and past exam question papers.

> Do you think your school has adequate resources for the teaching of probability? Score 5 Grade 12. The school relies on one textbook and if learners fail to understand the textbook, they have no alternative.

The responses above from two different teachers indicated that one textbook was used as the main source of teaching probability by these teachers. While there is nothing wrong with using textbooks to teach mathematics, it becomes a problem when a teacher relies on one textbook. Teachers should have different sources of teaching a topic like probability. This is important for effective teaching and learning and for helping learners to have richer context of questions. The extract below gives a comment from a teacher who sounded frustrated by using one textbook.

> 6.5. Helping learners understand and perform well in answering questions in probability.
> Score. __5_Grade_10 and 11_ (Feel free to score other grades that you teach.)
>
> *(Please, say more about your score…….)* Many learners struggle with probability because of the language and the unpredictability of the questions. As a teacher you may explain the questions in the textbook but when the exams come the questions come different. May be the textbooks we are using are not aligned to the syllabus.

I suggest that learners may use one or two basic copies, but teachers need more than one source. Vidermanova & Vallo (2015) maintain that, when learners are taught according to what is written in the textbooks, they remain passive listeners and a single textbook may not meet the needs of all the learners.

Responding to the question, "Do you think your school has adequate resources for the teaching of probability?" The teacher's response was *"No"*.

Do you think your school has adequate resources for the teaching of probability? No, the school must provide internet facilities for learners especially Grade 12. These days there are a lot of online teaching and intervention programs. I believe our learners need more variety in their learning. I also think if the internet can be provided by the school and we educate our learners on how to use it, we may reduce the problems faced by maths teachers in teaching probability.

This teacher felt that the school was not doing enough in terms of supporting Grade 12 teachers and learners to understand probability. He believed that learners needed a variety of sources to understand probability. I believe in variety too and the variety that might bring some 'fun' in learning probability so that learners might enjoy doing the activities. The use of one textbook in every lesson might be boring for both the teacher and learners while using different textbooks might help the teacher to explore different methods of solving the same problem.

Table 4.7: Summary of teachers' responses about the tools they use to teach probability.

Item	**Category E: Available tools used to teach probability.**	Teachers' responses				**Comment**
		Always	Sometimes	In exceptional Circumstances	Never	
1.17	Use only one textbook	T6, T5	T9, T1 T11,	T4, T3, T2,	T8, T7	In exceptional Circumstances
1.18	Use a textbook, and online sources	T8, T4, T7	T2, T3	T9, T11, T1,	T6, T5	In exceptional circumstances
1.19	Use textbook & teaching Aids	T8, T11,	T4, T7, T9, T3	T6, T1, T6, T2	-	In exceptional circumstances
1.20	Use textbook, online sources & teaching Aids	-	T8, T11	T2, T3, T4, T5, T7, T9	T6, T1,	In exceptional circumstances

The summary above shows that many of the teachers did not use one textbook to teach probability. Only under exceptional cases do they use one textbook. However, there are two teachers who answered that they never use one textbook to teach probability. This is good because teachers can select the best probability content for their learners. As mentioned earlier on by T8, she said using different sources has helped her to reinforce better understanding of probability terms in her learners.

4.6 Classroom Interaction

Research question 5

Research question 5 was framed as: What are the teachers' challenges in facilitating classroom interaction when teaching probability?

The fifth set of questions on the questionnaire were designed to gather data about classroom interaction during teaching and learning of probability. The questions were asking teachers whether they were using whole class or small group discussions. Other questions looked for information pertaining to classroom interaction, including teacher-learner and/or learner-learner interaction. Teachers were asked if their learners were using correct probability terms during classroom discussions either with their peers or with their teacher. In addition, teachers were to respond to the question of giving their students ample time to work out probability questions on their own as well as with others in class. This is in line with the theoretical framework of this study which emphasises that teachers should create a teaching and learning environment which allows learners to share, evaluate and analyse the ideas of others (Can and Isleyen, 2020). Can and Isleyen contend that argumentation is a model based on the constructivist approach and hence its emphasis on allowing learners time to discuss probability problems with others and the teacher in class to defend and justify their thinking. It is against this background that the questionnaire asked teachers to comment on how they dealt with classroom interaction challenges. Burke et al. (2005) assert that argumentation theory in mathematics is based on inquiry-based activities, group work, group discussions, exchange of ideas, evaluation of ideas and making inferences. It was found in this study that most teachers had challenges with creating teaching and learning environments that allowed learners space to engage deeply with the content of probability. Most teachers in the study utilised whole class discussion strategy. Teachers were asked in the questionnaire to respond to the question of classroom interaction. Some of the responses from teachers are discussed below and it was found that in many cases although teachers were aware of various teaching strategies, they had challenges in implementing them.

(Please say more....... I tried to apply the methods I learnt at university, learner-centred, group work, guided discovery and many other and I realized that contextual factors were making their application difficult. Time and space limit the use of those learner-centred approaches. I am now using teacher-centred approaches as everybody else.

The remarks above indicated that this teacher was aware of learner-centred approaches which are recommended in the framework, for example groupwork, guided-discovery and many others as the teacher implied, but the teacher lacked the skill of implementing these strategies under the circumstances the teaching was conducted. While the challenges of space and time were observed during the classroom observations done with the four teachers, the researcher found out that teachers could still have encouraged more discussions in their classes by allowing learners to talk to each other in the way they were seated. What the researcher realised was that teachers failed to incorporate other teaching strategies and were content with using teacher-centred approaches that made work easier for them.

8.1. Whole class discussions to solve probability problems. Score. _9__Grade__9 and 10_

(Please, say more about your score.......) This is the main teaching method I use. A question is written on the board and it is discussed by the whole or one learner works it out on the board, if the answer is correct learners copy it in their books. If it is wrong, other learners are given chances to go and correct it. If all learners fail to find the answer, then the teacher shows them the solution.

8.1. Whole class discussions to solve probability problems. Score. _8__Grade__12_

(Please, say more about your score.......) **This is the only feasible method now. I know we were taught that teacher-centred are not good but in the school I am using it and it is difficult to use other methods because of big classes as well as limited time to teach probability. In recent years COVID-19 made it worse as I was seeing Grade 10 once a week.**

8.1. Whole class discussions to solve probability problems. Score. _10__Grade_12__

(Please, say more about your score.......) This is the method I use basically, number 1, it saves time, number 2 it helps me to deal with large classes. In 2022 this year my grade 12 class has 53 learners, and the classroom is small. There is no space to organise learners into smaller groups.

In analysing the data above and from other participants, the themes listed below emerged; (a) I use whole class discussions because it is the only feasible method available because of large classes (space) and time, (b) group discussions are discouraged because of covid 19 protocols (c), I encourage group discussions because they promote deeper understanding, (d) I encourage learners to use the correct terms of probability, it helps them to get used to the terms, (f) limited time is allocated on the annual teaching plan to teach probability.

4.6.1 Strategies used by teachers in teaching probability

As for the teaching strategy used to teach probability 6 out 11 teachers answered that they used whole class discussions often and the other three teachers said they used this method always. Together the teachers who used this strategy often and those who used it always accounted for 9 out 11 (81, 2%) of the teachers. This became the dominant strategy used to teach probability for this study. Every teacher had reasons for using whole class discussions. The scenarios that unfolded from these findings were that teachers faced challenges of moving away from the traditional teaching approaches to the facilitation of learning as guided by the research framework of this study because of various reasons. The reasons cited included teaching big classes, saving time and the advent of covid 19. The responses further revealed that teachers were challenged to provide learners opportunities to engage in constructive learning in their own as constructors of their own learning. Teachers were challenged in their teaching skills to incorporate students' ideas, questions, and responses into their lessons to inform teaching. Most teachers practiced strict teacher-centred approaches which denied learners opportunities to discuss and explore probability ideas.

Responding to the question, "How often in your probability class do you use whole class discussions to solve probability problems?" Some teachers argued that this was the only feasible method because of large classes, limited time and covid 19 restrictions of social distancing. This view was shared by other teachers who used whole class discussions who argued that small group strategy of teaching was not practicable because of limited space due to large classes. `

, As shown in one of the responses above that even those teachers who knew other methods that could be applied to improve learning of probability could not use them because of contextual factors. Although challenges of space and time were not frequently raised in the literature reviewed, in this research they pose serious challenges in the teaching of mathematics. Another teacher added that he was teaching a Grade 12 class with 53 learners in a small classroom. Facing such conditions, it would be difficult to apply the more learner-centred methodologies suggested by the framework.

Many other teachers in the study raised the same challenges of big classes and time as limiting the application of other teaching methods. Teachers argued that there was no space to organise learners into smaller groups. One teacher in the study responding to the question, how often he made use of small group discussions to solve probability problems in the classroom, the teacher answered this question by saying,

"Small group discussions are good to promote in-depth understanding of mathematical concepts, but they are difficult to use in big, congested classes.

This response shows that teachers are informed of the right strategies to use to teach both mathematics and probability, but contextual factors make it difficult to implement them. Teaching a class of 53 in a small classroom as mentioned by T2 might make it impracticable to engage learners in smaller groups. Ogbonnaye and Awuah (2019) found out that learners from the affluent Quantile 4 and 5 schools outperformed the learners from Quantile (1-3) because the affluent schools used their resources to employ additional teachers to lower the teacher-people ratio. The implications of this are that DBE must investigate the challenges of teacher-learner ratio in the Quantile (1-3) and build more classrooms and employ more teachers to reduce the number of learners per class and per teacher.

While the rest of the teachers resorted to whole class discussions because of the reasons they stated, T8 had a different view about whole class discussions. She stated in her questionnaire response, *"I do not prefer the whole class answering as the weak learners tend to take a back seat (not participating) and as a result they end up being lost in a big group".*

While I agree that there is a serious challenge of teaching large classes and that teachers have few options to use in teaching probability in the congested classrooms, I still hold a different view as T8 correctly noted. Using whole class discussions in such classes does not solve the problem. T8 argued that, when whole class discussions are used in big classes the weak learners are sacrificed in the process. When asked to elaborate her views through a follow-up message, she stated that, "*because of the big numbers it becomes easy for weak learners to take a back seat and are completely excluded from the teaching and learning process".* The implications for this are that the teacher will be teaching the fast learners in the class and the weak ones are excluded in the teaching and learning process. These could be some of the factors that contribute to the poor performance in mathematics and in probability. With respect to small group discussions, T8 answered that she uses this method often with grades 11 and 12S. he affirmed that, *"Peer to peer discussion works well as it allows me to spot the struggling learners when it's still early".* Moreover, research established that when learners can talk about and interact with each other as they solve mathematical problems, they become more confident in mathematics (Amit, Fried, & Abu-Naja, 2007; Diez-Palomar, Varley, & Simic, 2006). Generating interest and building confidence in learners is what is needed in the teaching and learning of probability, and this is in line with the theoretical framework of this study.

4.6.2 Classroom interaction

Teachers' views about language usage in the classroom

Teachers had different views about classroom interaction, whether learner to learner or teacher to learner. The question was about language usage. While some teachers believed that all communication during lessons should be done through use of correct probability terminology, others thought that would be a hindrance to the learners who struggled with the difficult probability terms. This was about teacher beliefs. Responding to the question whether learners should talk to each other in class using correct probability terms, T7 had this to say,

"I stress that learners should always use the correct terms. Practice makes perfect. I believe that children can only excel in those things they practise regularly."

The teacher added that his learners used correct probability terms every time they were talking to him, and he demanded that they did so. His argument was that using correct terms during all communication helped him to pick up misconceptions and misunderstandings during such conversations with learners. T7 continued, *"Learners use correct probability terms when* *talking in class and I demand it from all learners, where they use terms wrongly, I correct them during the discussion".*

T9 had different views about the language to be used during different interactions. His answer was, *"Learners use both English and their mother languages when answering questions in class. I allow them to use their languages to promote discussions".* The teacher added that he did not enforce learners to use correct probability terms, *"Not enforced because I have found out that learners understand maths problems better when they use their own languages."* These are different views that surfaced from the data collected. It was noticed that in almost all the questionnaire responses, the issue of probability language was raised many times by respondents because learners generally found probability terms difficult to understand. T9 viewed learners' own languages as helpful to them to understand probability problems which are written in English while on the other hand T7 saw learners' own languages as barriers to their understanding of probability problems. In my view I will suggest that, depending on the level probability is taught, teachers should be flexible with language. Personally, I do code switching between use of correct terms and learners' own languages more often with the lower classes, but for grades 11 and 12 I would encourage more of the use of correct probability terms to improve their understanding of the terms in their final examinations.

4.6.3 Teachers' views about giving learners time in class to solve probability problems

Teachers also responded to the question of giving learners ample time to solve probability problems during their lessons. Responding to the question; "How often are learners given ample time to look for solutions to probability problems"? Many teachers expressed their concern regarding limited time allocated to the teaching of probability on the annual teaching plan (ATP). According to the ATP in grade 10, 11 and 12, probability is allocated time just before the end of year final examinations. Around this time, both teachers and learners begin serious revision for the final examinations to the extent that the last topics are not given the attention given to other topics covered earlier during the year. According to many teachers in the study, learners are not given ample time to solve probability problems. T4's response was,

"As already cited above, probability is taught under pressure. In grade 12 I usually summarise the whole content in one week. Probability is for the 'Clever' learners. I always encourage weak learners to not waste time doing probability because they will waste time and still fail it".

The methodology teacher T4 outlined emphasises drilling learners for examinations. Broadie et al. (2021) observed the same pattern about the teaching of mathematics going on in schools. They contended that, teachers work mainly to ensure that learners pass examinations, thus emphasising the procedures of mathematics at the expense of its full power. Teachers and schools aim for the best results in examinations at the expense of giving their learners rich mathematical knowledge that might be of use to them in the future. This is contrary to the framework of this study which encourages teachers to create an environment that would allow students to share their ideas and to evaluate and analyse the ideas of other students (Can & Isleyen, 2020). Kovacs et al. (2018) supports this argument by saying that mathematics teaching should not only encourage memorisation of concepts but must create bridges between learners' experiences and mathematics curriculum.

Teachers also responded to the question whether teachers allowed the freedom to discuss questions while they acted as facilitators in the process. Different views were shared with many of the teachers saying they lacked time to do this. This is what T7 said,

"This is always encouraged because as learners discuss their solutions and respond to their peers' questions, their understanding of the topic is deepened. But because we teach probability to help learners pass it than understand it, this time is shortened. We want learners to memorise the most important things in probability to enable them to pass the exam"

Other teachers wrote that they teach their learners concepts that are easy to understand and leave the rest. T6 restated the problem of time,

"…I always teach probability last, and I look at a few simpler questions that my learners may be able to understand before they write the exam. The time is not there to teach and finish probability and normally there is even no time to assess it before the exam"

Table 4.8 below gives a summary of teachers' responses. The responses have been grouped, putting teachers who gave similar responses together. The indications are that teachers use mostly the whole class discussion method to teach probability. The table also shows that, it is on rare occasions that teachers allow learners to interact with each other and/ with the teacher using correct probability terms. It is also on rare occasions that those teachers gave their learners ample time to look at probability problems by themselves, or for teachers to allow their children to work while they take the role of facilitators.

Table 4.8: Summaries of teachers' responses regarding classroom interaction.

Items	Category F: Classroom interaction	Teachers' responses				Comment
		On very rare occasions	Sometimes but not often	Often	Always	
1.21	Whole class discussions	0	T8, T11	T7, T9, T4, T3, T2	T6, T1, T5	Often
1.22	Small group discussions	T5, T1, T3, T4	T7, T9, T2	T8, T11		On very rare occasions
1.23	Learners talk to each other in class using correct probability terms	T6, T1, T4, T9,	T2, T11	T8, T5, T3,	T7	On very rare occasions
1.24	Learners talk to the teacher using correct probability terms.	T6, T5, T1, T3, T4	T2, T11	T9	T8, T7	On very rare occasions
1.25	Learners talk to each other outside class using correct probability terms	T6, T5, T1, T3, T4, T2, T11, T7	T3, T8	0	0	On very rare occasions
1.26	Learners are given ample time to look for solutions to Problems	T6, T5, T1, T3, T2, T11,	T4, T9, T10	T8, T7	0	On very rare occasions
1.27	Learners challenge each other's solutions in probability	T6, T5, T1, T2, T11,	T9, T3	T4,	T7, T8	On very rare occasions
1.28	Learners challenge the teacher's solutions in probability.	T6, T5, T1, T2, T11, T3,	T4, T9	T7, T8	0	On very rare occasions
1.29	Learners freely interact with each other during probability class facilitated by the teacher.	T6, T5, T1, T2, T11,	T3, T4, T9	T7	T8	On very rare occasions
1.10	Learners freely interact with each other during probability lesson facilitated by the teacher.	T6, T5, T1, T2, T11,	T3, T4, T9	T7, T8	0	On very rare occasions

Based on the summary on Table 4.8 one can see that teachers use teacher-centred approaches to teach probability in their schools. Out of the 10 teachers who responded 8 out 10 (80%) confessed used whole class discussions often or always in their lessons. The same number (8 out of 10) indicated that they did not use small group discussions except on very rare occasions but not often. The summary also reveals that many of the teachers do not encourage learners to use appropriate probability language during lessons and that they do not give space for learners to interact freely in class discussions while the teacher assumes the role of a mere facilitator.

The observations above (Table 4.8) concur with Kodisang (2016)'s findings on teachers' practices in teaching probability to grade 6 learners. The study found out that teachers were not keen to see learners involved in learner-to-learner discussions, but they were only interested in learners getting correct answers. The same study found out that teachers were less willing to give learners opportunities to discuss alternative strategies amongst themselves or with their teachers, observe alternative ways of looking at probability problems (Kodisang, 2016). Teachers' responses in the questionnaire confirm similar practices. There is evidence from research that allowing learners to interact meaningfully during the learning process deepens their understanding and promotes construction of new knowledge (Graham & Lesseig, 2018; Uygun & Akyuz, 2019; Burgin, 2020). In their responses most teachers indicated that they did not allow their students ample time to think and make sense of probability problems. The findings from this questionnaire are consistent with what Kodisang (2016) observed in her study. Her findings were that teachers generally lacked essential knowledge of implementing various teaching and learning approaches.

4.7 Mathematics and probability subtopics

The sixth set of questionnaire questions asked teachers to choose any three topics they enjoyed teaching and any three topics they found difficult to teach. One teacher (T10) preferred teaching probability and he made probability his first choice in all the mathematics topics in grade 11. The other 10 teachers included probability as the topic they would not want to teach. Many of the teachers had already given reasons why they did not enjoy teaching probability, stating its unpredictability and difficult language. T10 who loved to teach probability did his education in Zimbabwe and he stated that he did the topic at school as well as at university.

The next question asked teachers to share their views on any three challenges faced by grade 12 teachers in teaching probability. The teachers not currently teaching grade 12 were free to give views in relation to the grades they were teaching at the time. The question was, In your view, what are the three major challenges faced by a teacher teaching probability and counting principles at grade 12?

The themes that emerged from analysing this question were, (a) teaching learners to use formulae in probability without explaining the formulae to them, (b) not attending professional development workshops, (c) teacher lack of passion to learn probability, (d) probability knowledge gaps created by incomplete coverage of probability in lower grades, (e) time, (f) probability language. T8 felt that learners developed negative attitudes towards probability because they were handed over probability formulae without proper explanation and hence, they are always frustrated when they fail to use the formula correctly to answer questions. She argued that the formula must be explained first, in other words learners must understand where the formula is coming from. This is true, my learners sometimes want to understand why and how the formula P(A or B) = P(A) +P(B) – P(A and B) for independent events is different from P(A or B) = P(A) + P(B) for mutually exclusive events. Explaining that in mutually exclusive events P(A and B) =0. The teacher might use Venn diagrams to explain (A and B) and (A or B) and then develop the formulae with learners. If learners are involved to construct their learning, they do not forget it easily, unlike something that was told to them. T7 thought some mathematics teachers had no passion of learning probability themselves before teaching it to the learners. The challenge of time and problematic language were once again raised by most teachers. T10 made this remark about time,

"Generally, the syllabus is too ambitious in terms of amount of work to be covered within a time frame. There is not enough time to teach for understanding. Teachers are now teaching for exams at the expense of understanding maths as it were".

This comment tallies with other comments from teachers who confessed that they taught the simpler probability concepts to prepare their learners for exams.

4.8 Specific Challenges and suggestions for solutions by the teachers

The last question collected data on solutions to the probability changes. In analysing responses from this question, the themes that emerged were, (a) teachers need to change their attitudes and be positive in their approach to teaching probability, (b) if teachers have a negative attitude towards a topic, learners become negative too, (c) re-organising the ATP, probability to be taught early, (d) DBE to take responsibility to re-train teachers in probability, not only grade 12 teachers, (e) HOD to supervise the teaching of probability to ensure that it is properly covered in all grades, (f) teachers must improve their probability content, teachers to educate themselves in probability. Teachers also felt that the textbooks used to teach probability must be reviewed and be more aligned to the current syllabus.

4.9 Classroom observations

Introduction

There were many reasons that compelled the researcher to request for permission from schools and teachers to conduct classroom observations. The first one was that classroom observations would enable the researcher to physically explore the approaches teachers used to teach probability to their grade 12 learners. The second, was for the researcher to see all the processes of teaching probability in a grade 12 class, from planning, preparation, presentation, and evaluation of the lesson. In this context therefore, classroom observation was used to examine the whole process of teaching to identify the areas of difficulty for the teachers.

Another reason for carrying out classroom observations was to examine how mathematics teachers dealt with the challenges they raised in the questionnaire and follow-up messages. The researcher wanted to see how teachers applied their knowledge of probability to teach different concepts of this topic to their learners, hence the researcher observed and captured data on how teachers introduced the concepts. This involved examining how teachers introduced their lessons, including the examples they used to capture learners' interest and attentiveness. In addition, the researcher explored how teachers defined and explained probability terms and the accompanying examples they gave to aid learners' understanding. The researcher also focused on strategies used to engage learners during the lessons. Teachers were observed regarding the time and the opportunities they gave to their learners to discuss probability problems with minimum teacher interference in line with the theoretical framework. Different interactions were also observed. These included learner to learner and teacher to learner. Language usage was examined closely in the interactions. This was done to compare teachers' responses on the questionnaire with what the researcher observed. Classroom observations were done and triangulated with the questionnaire, follow-up messages and document analysis to increase the validity and credibility of the findings.

4.9.1 Challenges in carrying out classroom observations

The execution of classroom observations was however not easy because of the circumstances leading to the data collection process. It was difficult to observe teachers teaching probability in the month of February when according to the Annual Teaching Plan (ATP) this topic is normally covered in term four for all grades. According to the researcher's original plan, data was supposed to be collected during the fourth term of 2021 when the teaching of probability was going on in schools. Delays in getting permission from the Gauteng Department of Education (GDE) and ethics clearance letter from the university meant that data could not be collected at that time.

Permission to collect data from schools was only obtained in mid-February by which time schools were not teaching probability as per the ATP. Moreover, covid 19 restrictions could not allow physical contact with participants from other schools. Classroom observation was done with four teachers teaching at the school where the researcher is teaching. Each of the four teachers was observed once. Teaching of different aspects of probability was observed from these four. T3 was observed teaching probability tree diagrams, T4 taught contingency table, T9 mutually exclusive events and independent, and T6 Venn diagrams in a grade 11 class.

4.9.2 Presentation and analysis of the data collected through classroom observations

As already has been highlighted, classroom observations were utilised to add another dimension in collecting data about teachers' challenges in teaching probability grade 12 learners and in some cases other grades as well. Four classes taught by four different teachers were observed from one school. The rest of the teachers could not be observed teaching because of the reasons already outlined. The four teachers included T3, T4, T6 and T9 were all from the same school with the researcher. The researcher was able to observe these teachers teaching because covid 19 physical restrictions did not apply to them because they were teaching at the same school with the researcher. This was a serious limitation because out of 11 teachers only 4 teachers were observed teaching. A classroom observation guide was prepared for this purpose (see Appendix J).

4.9.2.1 Classroom conditions

Before discussing what was observed from individual teachers the researcher made a comparison of teaching environments in the four classrooms observed. The environments were the same. Teachers made efforts to clean the classrooms and arranged the furniture orderly. Learners were seated in rows and all the four classes observed rooms were full. I had requested the teachers to use the classrooms they use daily for their mathematics lessons. This was done to keep the setting as natural as possible.

In all the classes there were no charts on the walls, nothing on the walls showed that this was a classroom used for mathematics lessons. When asked about charts on the walls T4 said he could not paste his charts on the walls because he did not have specific room allocated for mathematics. He explained that whenever he planned to use charts, he would paste them for that period and that at the end of the lesson he would remove them.

The classroom environments were observed to determine if they contributed to probability teaching problems. Apart from the generally overcrowded classrooms and lack of mathematics charts the classroom environments were conducive to teaching and learning. All classrooms had smart boards. What was observed with all the four teachers was that smart boards were used only as chalk boards for writing. Teachers did not utilise some important features of the smart board. T4 said he used the smart board more to teach graphs. Other teachers said they had not yet explored other uses of the smart board in their teaching. Table 4.9 gives a summary on how teachers performed based on the classroom observation guide. The guide measured the teacher's performance in each activity as either achieved or not achieved. Achieved measured anything above 50% of what the teacher was expected to do in that category. Not achieved meant that the teacher performed below expectation and the class did not benefit from the activity.

Table 4.9: showing summary of observation outcomes for the four teachers

Teacher's method of instruction	Achieved	Not achieved
Teacher provides well-designed teaching materials for the lesson and uses different sources and textbooks to present probability content.	T4	T3, T6, T9
Teacher allows learners to discuss problems in small groups and values learners' input.	NA	T4, T6, T3, T9
The teacher is loud and clear, and he/she pays attention to different categories of learners in the class, the fast, average and the slow	T3, T9	T4, T6
Teacher uses correct mathematical and probability language and encourages learners to use correct language in class discussions	T3	T4, T6, T9
Teacher gives learners ample time to discuss probability problems among themselves. Teacher asks questions in class and give learners tasks to foster creative and critical thinking.	NA	T4, T6, T3, T9

Teacher's content knowledge	**Achieved**	**Not achieved**
Teacher can explain probability concepts using clear and real-life examples to foster learner understanding. Different representations are used to illustrate the concepts.	T3	T4, T6, T3.
	T3,	T9, T4, T6.
The teacher demonstrates rich understanding of the topic and uses different approaches to explain probability problems	T4, T3.	T6, T9.

Teacher asked learners both open and closed questions and allows learners to ask questions, shows confidence when answering learners' questions.	T8,	T6, T4, T9
Teacher breaks down probability content into comprehensible units that can be easily understood by learners	T4	T3, T6, T9
Teacher presents the lesson according to curriculum expectations.	T4, T6, T3, T9	NA
Teacher summarises the content taught and highlights the key concepts taught during the lesson	T3, T9	T4, T6

4.9.2.2 Classroom observations in relation to teaching methodology

According to this summary of data collected from observations teachers had more problems with teacher's method of instruction than they had with content. The table indicates that one teacher achieved in the category of bringing to the lesson well-designed teaching materials including lesson plans, teaching aids such as cards, coins, dice, and many others. T4 had brought to counters and dice for the lesson. He asked learners to take out their coins which explained how they would use them in the lesson to learn probability. Everything the teacher brought and what learners could find was used as teaching aids to explain probability concepts. For example, the concept of an outcome was demonstrated using a coin and a die. Learners were able to find total number of outcomes when a coin is tossed and when a die is rolled. This discussion helped the teacher to explain the formula P(E) = number of desired outcome**s**/total number of outcomes. This concept was explored further using learners in class as resources. For example, teacher asked the question,

"A learner is chosen at random from 12B (their class) to represent you in the school quiz competition, what is the probability that probability that: (a) the learner chosen is a girl? (b) The learner chosen is a boy. (c) Karabo is chosen."

A whole class discussion method was used to find the answers until all questions were covered. I realised that when the teacher used these examples the whole class became involved. Learners were asked to think of other probabilities they could calculate. A learner raised a hand and asked the question of finding the probability of someone putting on a jersey in the class. The teacher praised the learner for the example. This encouraged other learners who all wanted to say something, but the teacher stopped the discussions because of time. A point taken from this observation was that the use of examples drawn from the learners' own environment triggered interest from the class and every learner had and wanted to say something.

The teacher explained at the end of the observation that the class he was teaching never did probability in grade 10. Probability was one of the topics in 2020 that were trimmed from the syllabus because of covid 19 disruptions. The teacher said the topic was brought back in 2021 but learners missed out on grade 10 probability content. He explained how probability is always compromised to the extent that teachers do not take it seriously. He said this, *"Teachers treat probability like an additional topic not like a core topic".* He added that this is always the last topic to be taught either at the end of the term or year.

Having developed the concept of outcomes and then possible outcomes with learners, T4 used these concepts to teach probability tree diagrams. He linked possible outcomes to the number of branches in a probability tree diagram. The teacher used three different textbooks. Examples from the three textbooks were used to strengthen what was discussed in class. What T4 did not achieve was organising learners to work in small groups and giving them ample time to solve problems by themselves. As shown in the table above, no teacher from the study permitted learners to work in groups or allowed them ample time to solve problems on their own. The data from classroom observations are consistent with the findings from Awuah (2018)'s study which attributed learner poor performance in complementary events to poor teaching strategies. In 2015 a study by Kodisang attributed learner poor performance in probability to lack of resources and poor teaching practices which she argued impaired students' performance in probability. Poor teaching practices were evident in this study as shown in the summary above. 75% of the teachers did not provide well-designed teaching materials and used one textbook. Batanero (2016) asserts that some school textbooks present a very narrow view of probabilistic concepts. This implies that by using one textbook the teachers give learners a limited breadth of probability. No wonder learners perform badly in probability questions in examinations. Table 4.9 also reveals that 100% of the teachers used the whole class discussion method, 50% gave loud and clear instructions to their learners. 75% of the teachers did not emphasise the use of correct probability and 100% did not give learners sufficient time to solve probability problems independently.

4.9.2.3 Classroom observations in relation to teacher content knowledge

Regarding teacher's content knowledge Table 4.9 shows that although many teachers still failed to achieve in many categories a fair balance between achieved and not achieved is displayed. Three out four teachers (75%) of the teachers failed to explain probability concepts using clear and real-life examples to help learners to understand the concepts taught. For example, these teachers did not use different representations such as diagrams, tables, symbols, pictures, technology, videos and so on in their lessons to help them to explain probability concepts. They taught only from the textbook. As far as language is concerned 50% achieved and 50% did not. The two teachers who did not achieve under language were reading the probability terms for their learners from the textbook.

For example, T9 introduced the lesson by trying to explain mutually exclusive events and independent events. The teacher emphasised that in the final matric paper there is always a question on mutually exclusive and independent events. The teacher asked learners if they were taught these terms before. The whole class was quiet. The teacher then referred the class to the textbook, Platinum Grade 12 page 256. Learners were asked to read through all the definitions and familiarise with all of them and two learners were chosen by the teacher, one to read the definition of mutually exclusive and the other independent events. In the textbook, mutually exclusive were defined as, *"Events are mutually exclusive if there is no overlap or intersection between the two events, that is, P (A and B) =0".* Independent events were defined from the textbook as, *"Two events are independent when one result does not affect the result of the other"*

After the reading was done, the teacher explained the terms using his own words and asked the learners to copy the definitions into their books. Straight from the definitions, the teacher introduced the formulae for both mutually exclusive and independent events. Learners were told to copy the formulae for both mutually exclusive and independent events. The teacher then emphasised that learners were supposed to learn them for use in the examinations. The teacher said, *"Learn these formulae by heart so that you may not confuse them in exams".* Learners were then given some sums to practice on mutually exclusive and independent events while the teacher moved around the class. This was the same approach used by T6. T6 had some few notes on a paper where terms were defined, and she wrote them on the board for learners to copy. The teacher emphasised that learners should read the definitions of terms until they understood them. No diagrams, tables or pictures or examples from learners' experiences were used. Regarding interaction learners never interacted among themselves and the interaction was centred on the teacher. He gave instructions and learners responded by lifting their hands or in many cases there was no response after which the teacher explained the concept further. T3 and T4 used diagrams, physical objects, drew tables and used learners in class to explain the terms and went further to explain the differences between the terms.

T3 and T4 demonstrated rich understanding of the topic and used different approaches to help their learners to understand probability problems. The same teachers entertained learners' questions and used such questions learners were asking to enrichen their knowledge. Concerning presenting the lesson according to curriculum expectations, all teachers that were observed followed the ATP and all that was supposed to be covered was done. T6 and T9 did not expose their learners to a variety of examples and contexts of probability. Finally, only T3 and T4 summarised the content for their learners through asking them closed questions to get feedback from learners about what they learnt in the lesson. Learners were given a few activities to write but these were not checked or marked and then homework was given. T6 and T9 closed their lessons by asking the learners to revise the definitions again and again.

It was noticed that teachers limited learner to learner interaction to whole class discussions where learners were lifting hands and were picked by the teacher to respond to others. Group work was completely missing. Teachers explained the reasons for not using small group discussions as overcrowding in classrooms and covid 19 restrictions. Information from classroom observations was useful in verifying some of the questionnaire responses.

4.10 Document Analysis

The teacher documents were requested to obtain data pertaining to teachers' expertise in analysing and breaking down national plans and adapting them to the needs of their learners. The following documents were requested from teachers for analysis, the annual teaching plan (ATP) and teacher's lesson plans. These two documents were required mainly in relation to classroom observation. It was important to verify that the content taught during classroom observation was in line with the ATP and the CAPS policy document. Lesson plans were requested to find out if teachers were able to plan and prepare for their lessons. Analysis of these two documents were to help to determine the depth with which the content was covered.

4.10.1 Annual teaching plan

The researcher had copies of the ATP for all grades but requested it from teachers first to find out if teachers had it and used it all the time. The second reason was, to compare teachers' plans with the ATP. There are teachers who prepare for their lessons using textbooks rather using ATPs. The challenge with such a scenario is that, not all content in the textbook is relevant or is contained in the ATP and it is also true to find content in the ATP that is not in the textbooks. It is important that teachers' planning and preparation be based on the ATP. It is through effective planning and preparation that effective teaching might be achieved. Thorough preparation saves time. A teacher who has prepared does not waste time on irrelevant content.

4.10.2 Lesson plans.

From my teaching experience I found out that the longer teachers teach the same subject or content the more they think it is no longer necessary to prepare lesson plans for that content because they can use experience. While this can be true with some teachers, I believe writing down a lesson plan capturing the main themes of the lesson will help to focus the teacher during the lesson and help the teacher to anticipate challenges learners might face during the lesson. Brahier (2013) maintains that the effectiveness of any lesson depends substantially on the seriousness with which the lesson is prepared. A thoroughly prepared lesson plan can help the teacher to anticipate learners' cognitive challenges (Brahier, 2013). Moreover, a lesson plan can help the teacher to look for the resources including the best textbooks to use for the lesson. The teacher already knows the activities including the questions that will be asked in the lesson.

Three of the teachers observed did not have detailed lesson plans and this was evident in their teaching. T6 had a piece of paper where she had written a few definitions of probability terms. She finished everything she had prepared in the first 30 minutes and the remainder of the time she gave learners to do some activities which she knew learners did not fully understand. T9 had the ATP and past examination question papers but no detailed lesson plan. He was not confident with what he wanted to do throughout the lesson. T3 although he did not have a formally written lesson plan, he had a detailed document on probability prepared by Sci-Bono. Sci-Bono is a science centre in Johannesburg that supports the teaching of mathematics, science, and technology in secondary schools.

4.10.3 Chapter summary

In this chapter four sets of data were presented and analysed. Data from the questionnaire, follow-up messages, classroom observations and document analysis were analysed. It was found that all teachers were qualified to teach mathematics at secondary school level. It was also found that teachers had inadequate knowledge to effectively teach mathematics, but they lacked probability content to teach this topic.

Teachers used whole class discussions in teaching probability which is a challenge when this is viewed the conceptual framework of the study. Teachers maintained that they used this method because it helped them to manage big classes and to save time which they needed to do revision.

CHAPTER 5: DISCUSSION, SUMMARY, CONCLUSIONS, IMPLICATIONS FOR THE STUDY, LIMITATIONS

5.1 Introduction

This chapter consist of the discussion, summary, implications of this study and the conclusions of the study. The aim of the study was to investigate the challenges faced by mathematics teachers in teaching grade 12 probability. The sample was therefore restricted to teachers that had Grade 12 teaching experience. To gain more insight into these challenges, some experienced Grade 12 teachers who were teaching other lower grades were asked to report on the challenges they were facing teaching probability in those grades. This was done because of the assumption that the challenges faced at 12 are not limited to this grade alone but might be inherited from the lower grades. Data from teachers teaching different grades were collected and analysed. When the data were qualitatively analysed, it was found that many teachers had challenges in teaching probability. This chapter, therefore, presents a summary of the study, discussion of the findings, conclusion, and recommendations. It also looks at the implications of these findings. The findings are discussed according to the research questions as outlined in chapter 1 section 1.7.

5.2 Discussion of the findings

Main research question

5.2.1 Main research question was as follows: What are the challenges faced by mathematics teachers when teaching probability?

5.2.1.1. Teacher qualifications and probability content knowledge

Based on the data collected through the open-ended questionnaire, all the teachers in the study were qualified to teach mathematics and all of them except one had been teaching probability for over 5 years. It was established that most of them (9 out of 11) did not have probability experience in their secondary education as well as in their training as teachers. Data analysis revealed that most of the teachers who did not experience probability in their secondary school as well as at university stated that their probability content was in adequate and that this was caused by lack of background knowledge of this topic. This group of teachers also reported that they found teaching probability to be more challenging than teaching all other topics in the current syllabus. Bietenbeck et al. (2017) affirm that teacher subject knowledge has a positive and significant impact on learner performance. The data from the questionnaire indicated that most of the teachers had gained five years or more probability teaching experience. This is important data because teacher qualifications and teaching experience play an important role in the teaching of mathematics (Khumalo et al., (2016).

5.2.1.2. **Research question 1:** What are the challenges faced by teachers in terms of the language of probability?

To respond to this research question, analysis of different data sets was done. The analysis of data collected through the questionnaire revealed that many teachers had challenges about the language of probability. Table 4.4 was completed to group and analyse teachers' responses about how they regarded probability terminology or probability language. Teachers expressed their problems and rated them in terms of their seriousness The researcher read each of the teachers' responses and grouped on a ranking scale of either difficult, very difficult, not difficult, or less difficult. On commenting, the researcher considered the number of teachers' comments that were ranked together. For example, if most teachers' responses were ranked as very difficult, then the comment for that question would be "very difficult". Using the above analysis, Table 4.4 shows that teachers found the task of explaining mutually exclusive, independent, and dependent events to the learners difficult. Note, that the comment "difficult" was given because the highest number of teachers found the task of explaining these terms to learners difficult. There are 3 teachers out of 10 who found these terms either easy or very easy to explain. 7 out of 10 found them either difficult or very difficult. One teacher did not respond to the question. From the few teachers who included complementary events, more teachers found it easy to explain complementary events. With regards to compound events, more teachers found the task of explaining compound events either difficult or very difficult.

Teachers' responses of viewing probability language as difficult are consistent with Batanero (2016)'s assertion that probability employs language and terminology that is challenging and is different from the language used in other mathematics topics. A study by Makwakwa (2012) found that learners had difficulties in understanding and interpreting probability terminology. There are teachers in the present study who indicated that they had challenges explaining probability terms and some admitted that they confused mutually exclusive and independent events. It was found in the literature review from studies done in South Africa that teachers found it difficult to understand probability terminology particularly mutually exclusive and independent events. In this study two teachers explained that the difficulties were caused by lack of background knowledge which is missing from the teachers who had no experience of probability in their secondary school learning and teacher training. The teacher argued that to address this problem, this topic was supposed to be addressed developmentally, that is the professional developmental programs offered to teachers were supposed to address the fundamental basics of probability. The programs were supposed to introduce probability to teachers starting from the basic concepts of probability and building up to Grade 12 rather than teaching teachers only the complex Grade 12 probability content which teachers must teach to Grade 12 learners. It is therefore not surprising if teachers have problems in interpreting probability problems themselves. The nature of the challenge was therefore identified as that of teachers lacking a good foundation of probabilistic reasoning or simply not having adequate background knowledge about probability.

5.2.1.3 Research question 2: What are the teachers' challenges with probability content knowledge?

Table 4.5 provides a summary and analysis of teachers' responses about their content knowledge of probability, including how they gained it and how they rated themselves in terms of probability knowledge. In this table, the rankings of adequate, very inadequate, lacking in some respects, adequate and excellent were used. Teachers indicated that the probability knowledge they had gained during pre-service training and in in-service training was inadequate. They also ascertained that the probability knowledge they had was inadequate in contributing to the school pass rate. Pertaining to rating their current knowledge of probability, 50% of the teachers believed that their current probability knowledge was adequate while the other 50% thought they had inadequate knowledge of probability. Teachers stated that they did not do probability during their secondary school learning and teacher training. Their responses resonate what Awuah (2018) said about South African learners performing poorly in probability. He argued that this poor performance was justified since many South African mathematics teachers lacked fundamental knowledge of teaching probability because they did not do the topic themselves either at school or at teacher training college (Awuah, 2018). Biyela et al. (2016) posit that pre-service training programmes constitute the foundation for mathematics teaching, and therefore such programmes are expected to produce competent and effective mathematics teachers who would produce results in schools. Drawing from teachers' responses it was clear that the teachers in this study were not trained adequately to teach probability. Several authors agree that current teacher education programmes training teachers do not yet train teachers adequately to teach probability and statistics (Batanero et al., 2016; Batanero, 2013; Franklin, Kader, Mewborn, Moreno, Peck, Perry, & Schaeffer, 2007). Most of the teachers in this study maintained that the training was in adequate given that many teachers had no background knowledge about probability.

5.2.1.4 **Research question 3**: What are the teachers' challenges with some specific subtopics of probability?

. Table 4.6 was completed to group and analyse teachers' responses about the challenges they faced in teaching specific probability subtopics. Results from the questionnaire analysis revealed that many teachers regarded the teaching of mutually exclusive, dependent, and independent events as difficult. The problem of language used in probability resurfaced in this question as teachers explained the challenges they faced teaching mutually exclusive events, dependent and independent events. The teachers reported having problems of making sense of probability statements and formulating solutions from probability statements. Batanero (2016) explained that probability language is different from the mathematics language used in other topics that and it does not share the same notation used in other mathematics topics. The language is unique to probability which makes it challenging to both teachers and learners.

5.2.1.5 **Research question 4:** What are the teachers' challenges with use of technology in teaching probability?

The responses for this question showed that there is lack of variety in the sources teachers use to teach probability. Most of the teachers used one textbook as a source. The data collected showed that teachers did not use online sources, did not use the internet, videos, games, and many other tools to give their learners a variety. One teacher responded to the question above by saying: *The school relies on one textbook and if learners fail to understand the textbook, they have no alternative.*

Use of different strategies and tools may motivate the learners and generate the much-needed interest in mathematics and probability.

Other possible reasons why teachers find the teaching of probability difficult, year after year is lack of preparation, to bring many other teaching and learning aids to provoke learners' curiosity. Classroom observations showed that teachers do not adequately prepare before they taught probability. Teachers prepared a few definitions and a few questions and went to class to teach. Thorough preparation should involve the use of more than one textbook. Teachers can also prepare together as a group in the department, discuss the examples they are going to use to explain terms and concepts.

5.2.1.6 **Research question 5:** What are the teachers' challenges in facilitating classroom interaction when teaching probability?

The questionnaire contained questions that asked teachers how often they used whole class discussions to solve probability problems, how often did teachers use small group discussions to solve probability. Other questions were focused on interactions in the classroom, for example, learner-learner, teacher-learner, and asked questions about the use of probability terms during and informal discussions. Teachers were asked if they gave learners enough time to solve probability problems in class on their own. The role of the teacher was to be clarified, where teachers were asked if they allowed their learners to discuss probability questions while they assumed the role of a facilitator. It was through the analysis of all the responses from the above questions that the researcher gained insight into teachers' pedagogical challenges in teaching probability. Classroom observations contributed more data to answer this question. More important, classroom observation helped the researcher to have a picture of what happens in a probability class. Some of things teachers wrote in their responses they used the traditional teacher-centred approaches where teachers are at the centre of the learning process.

According to the analysis of data in Table 4.7 eight out of ten (80%) teachers answered that they used whole class discussions often or always. Two out ten (20%) of teachers said that they used this method sometimes but not often. Small group discussions were used on very rare occasions. During classroom observations, small group discussions were not utilised, teachers used only whole class discussions. A study by Kodisang (2022) found similar results where Grade 7 teachers utilised teacher-centred approaches and had difficulty in incorporating resources into the lessons. This is contrary to one of the research frameworks of this study. Research shows that learners learn mathematics better if they are given opportunities to discuss problems in smaller groups. For example, when learners work together or alongside a partner, they are afforded the opportunity for interaction and support, enhancing their learning (Takeuchi, 2016; Kodisang, 2015). Freire (2018) argued that teacher-centred methods which involve whole class discussions perpetuate oppression of learners' thinking as the teacher dominates discussions and learners act as passive recipients. Instead, Freire (2018) advocated for learner-centred approaches where learners are given opportunities to express their voice through the creation of dialogue with other learners or the teacher. It is through such discussions and dialogue that learners actively construct their own knowledge.

The analysis done in Table 4.7 also revealed that teachers were not strict with the use of correct probability terms either when learners were discussing among themselves or with the teacher. It is further shown on the table that teachers did not give learners enough time to solve probability problems by themselves. Denying the learners quality time to engage deeply with probability problems is depriving them the opportunity to construct meaning for themselves in the learning process. This thought is supported by Warshauer (2015) who argued that teachers should allow learners to struggle productively with mathematics problems. He asserts that, students should be allowed time to struggle productively in their learning while teachers offer support through asking questions, providing encouragement, giving enough time, and acknowledging student contributions (Washauer, 2015). According to the data collected, teachers in this did not write about using such approaches, instead teachers reported on using the traditional teacher-centred approach. Reasons for using this approach were given by teachers although the researcher thinks that teachers could still do more in engaging learners in meaningful learning.

5.3 Summary of the study

Data on teachers' challenges in teaching probability in selected schools were collected through a questionnaire for teachers, short messages or follow-up messages, classroom observations and document analysis. The analysis of data revealed that the teachers' challenges were rooted in their lack of adequate content knowledge for teaching probability. This was reflected in their responses where many answered that they had difficulties in explaining probability terms such as mutually exclusive events, dependent and independent events, and compound events to their learners. Teachers also submitted that they had difficulties with probabilistic reasoning which led to challenges in understanding and interpreting probability statements or questions. It was revealed also that teachers had inadequate content knowledge to help learners to answer different probability questions at grade 12 level. The subtopics that gave teachers problems in their teaching were the use of probability tree diagrams and proofs involving mutually exclusive, dependent, and independent events.

Apart from challenges involving the probability content, teachers complained about overcrowded classes which they claimed were a major hindrance in the implementation of proper strategies to teach probability effectively. Teachers argued that it was not feasible to utilise small group class discussions because of overcrowding. The researcher was able to verify these claims during classroom observations. The classes observed were full and overcrowded. Teachers had problems with the period allocated to teach probability in the ATP. They argued that this topic was allocated time just before the writing of the final examination during which time teachers would be rushing through content to prepare learners for the examinations. Teachers suggested this as one of the reasons the topic is not given proper attention. Also, they argued that teachers were not able to teach the topic and assess learners in probability because of time constraints.

5.4 Conclusions

The study concludes by gathering the different parts of the proposal. A brief introduction and a general background were given to set the scene and highlight the importance of the research. This was followed up by the discussion of existing literature on problems affecting the teaching of probability. The problems associated with the teaching and learning of probability identified by Batanero and Diaz (2012) and Batanero et al., (2016) together with the argumentation teaching strategy were discussed as frameworks to guide the study. Next the study provided a brief discussion of the methodology adopted to carry out the study. Lastly, I provided a brief account of ethical considerations before a describing of the proposed analysis of the research.

Regarding the purpose of the study, the data collected has shown that there are problems that beset the teaching of probability to grade 12 learners. It was found also that these problems are not confined to teaching of grade 12 only but they are widespread including the teaching of probability in lower grades. Teachers, although qualified to teach mathematics and have 5 years probability teaching experience, the study found out that most teachers lacked adequate content knowledge to teach probability. The reasons for this were found in the literature and they were confirmed in this study. Literature already revealed that many mathematics teachers teaching probability in schools did not do this topic themselves either at high school or at university. In addition to this setback, it was found that the factors of overcrowding and the period allocated to teach probability in the ATP also contributed adversely to the successful teaching of probability. Teachers had concerns with the department’s planning which allocated the teaching of probability at the end of the year knowing that this topic was problematic to teach. Teachers argued that less attention was given to the topics that are taught just before the exams. Their argument was that, around that time teachers are focussing more on finishing the syllabus and engage learners in intensive revision for the exams. The study also revealed that overcrowding made it very difficult to utilise more learner-centred approaches.

5.5 Implications for the study

I ascertain that many of the findings from this study will be very valuable to teachers who are in similar situations as teachers involved in this study. While I agree that the department of education must take full responsibility in developing in-service teachers to meet the challenges of teaching every topic in the current curriculum, I want to challenge mathematics teachers to personally think of upgrading their content to teach probability. The current study has shown that many teachers lack adequate content knowledge to teach probability up to grade 12. I also recommend that the department of education should organise more in-service training for all teachers including primary school teachers to engage them with both content and pedagogical practices specifically designed for the teaching of probability.

Secondly, I recommend that the department of education reconsider the period it allocated to the teaching of probability in the ATP. If everyone admits that probability is very useful for the future of our children as was explained in the background of this study, it means that special attention must be paid to its teaching and assessment. Again, if it has been found that probability is difficult to teach and to learn then it must be given priority and be taught early in the year to allow teachers to have more time to teach and assess it. If learners fail the assessments remedial lessons may be organised while they still time. I therefore suggest that probability be taught in the first term as opposed to the current allocation of the fourth term. My final recommendation is that of utilising technology in the teaching and learning of probability. All the four lesson observations I carried out with the teachers, there were smart boards which teachers did not utilise. Teachers can download videos on the teaching of probability and any other videos on probability to make the topic interesting.

Teachers gave suggestions as to how the teaching and learning of probability could be improved. Some of the suggestion that came included self-improvement in probability by teachers, taking preparation for lessons seriously and heads of departments were called to supervise the teaching of mathematics to ensure that even the topics that are allocated time at the end of the year are taught. Teachers strongly felt that the department of education should organise more training works for probability and counting principles and that teachers who have challenges teaching the topic should be given preference to attend. A teacher who stated that probability was his favourite topic in the current grade 12 syllabus suggested that there are topics that support the teaching of probability that are not taught in the CAPS. He said, *"I believe probability is better understood together with set theory. The fact that set theory is not in the syllabus presents a huge content gap both to teachers and learners".* The same teacher argued that,

"…the syllabus is too ambitious in terms of amount of work to be covered within a certain time frame. There is not enough time to teach for understanding. Teachers are now teaching for exam at the expense of understanding mathematics as it were".

5.6 Limitations

This study was done during the covid 19 period, and this adversely affected the data collection process. Many times, data collection strategies had to be changed to comply with covid 19 restrictions. All data collection tools that involved physical contact between the researcher and participants were changed to online questionnaire. This affected the original structure of this research. Classroom observations were planned for all eleven teachers but in the end only four went through. Four of the teachers observed were teaching at the same school with the researcher, this allowed the process of observation to continue but not as originally planned. A major concern was that lessons were observed not during the normal school timetable, this greatly affected the natural setting under which normal classes were conducted at the school. The second limitation was that all the five schools were township public schools with almost the same enrolment and learners come from the same catchment area meaning they experience almost similar socioeconomic conditions. The three schools were all non-fee-paying schools which are Quantile 1 schools. It could have been ideal at least to have one private school to use to compare the resources and the teachers teaching in different schools in terms of qualifications and experience. The last limitation was that the study focused only on teachers and did not involve learners to get their views on how they are taught probability. Even though the researcher would have wanted to do this, time would have failed the study.

REFERENCES

Akkus, R., Gunel, M., & Hand, B (2007). Comparing an inquiry-based approach known as the science writing heuristic to traditional science teaching sciences: are there differences? International Journal of Sciences Education, 29(14): 1745-1766.

Amit, M., Fried, M. N., & Abu-Naja, M. (2007). The mathematics club for excellent students as common ground for Bedoium and other Israel youth. In B.Sriraman (Ed.), The Montana mathematics enthusiast, 2007 monograph1: International perspectives on social justice in mathematics education (pp. 75-90). Missoula, MT: Information Age Publishing and the Department of Mathematical Sciences-The University of Montana.

Arends, F., Wiinnaar, L., & Mosimege, M. (2017). Teacher classroom practices and Mathematics performance in South African schools: A reflection on TIMSS 2011. *South African journal of education,* 37(3), 1-11

Atagana, H.I., Mogari, L. D., Kriek, J., Ochonogor, E. C., Ogbonnaya, U.I., Dhlamin J. J., & Makwakwa, E. G. (2010). An intervention into the teachers' and learners' difficulties in some topics in mathematics, science, and technology: A report of the ISTE 2010 winter school. The Institute for Science and Technology Education, University of South Africa. Pretoria, South Africa.

Awuah, F. K (2018). Grade 12 learners' problem-solving skills in probability. (Unpublished Doctoral dissertation). University of South Africa, Pretoria. South Africa.

Babbie, E. (2014). The Basics of Social Research, ed 6, Wadsworth Publishing.

Bill, D. L., Hill, H.C., & Bass, H (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? American Educator, 29(1), pp.14-17. Retrieved from: http://hdl.handle.net/2027.42/65072.

Bansilal, S., Brijlal, D., & Mkhwananzi, T. (2014). An exploration of the common content knowledge of high school mathematics teachers. Perspectives in Education, 32(1), 34-50.

Ball, D. L., Themes, M. H., & Phleps, G. (2008). Content Knowledge for Teaching: What makes It Special? Journal of Teacher Education, 59(5), 59(5), 389-407.

Batanero, C., Chernoff, E., Engel, J., Lee, H. S., Sanchez, E. (2016). Research on teaching and learning probability. Ludwigsburg: Springer Nature.

Batanero, C., & Diaz, C. A. R. M. E. N. (2012). Training schoolteachers to teach probability: reflections and challenges. *Chilean Journal of Statistics,* 3(1), 3-13.

Batanero, C. (2015). Understanding randomness: Challenges for research and teaching. *CERME 9-Ninth Congress of the European Education Society for Research in Mathematics* (pp. 34-49)

Batanero, C. (2013). Teaching and learning probability. In S. Lerman (ED.), Encyclopaedia of Mathematics Education (pp. 491-496). Heidelberg: Springer.

Bietenbeck, J., Piopiunik, M., & Wiederhold, S. (2017). Africa's skill tradegy: Does teachers'lack of knowledge lead to low student performance? *Journal of Human Resources,* 0616-80022R0611.

Biyela, K., Sibaya, P. (2016). Comparative Analysis of Mathematics Syllabi for Secondary School and Teacher Training Programs. *International Journal of Educational Sciences, 13(3), 325-334.*

Boaler, J. (2016). Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching. Hoboken, NJ: Wiley.

Borovcnik, M., & Kapadia, R. (2018). Reasoning with Risk: Teaching probability and risk as twin concepts. In C. Batanero & E. J. Chernoff (Eds.), Teaching and learning stochastics.

Borovcnik, M., and Peard, R. (1996). Probability, In Bishop, et al (eds.), International handbook of mathematics education, Dordrecht: Kluwer, (pp. 239-288.)

Brahier, D. J. (2013). Teaching secondary and middle school mathematics (4th ed.). Boston, MA: Pearson.

Brijlall, D. (2014). Exploring the pedagogical content knowledge for teaching probability in middle school. A South African case study. *International Journal of Educational Sciences,* 7(3), 719-726

Brijlall, D., & Maharaj, A. (2014). Exploring support strategies for high school mathematics teachers from underachieving schools. International Journal of Educational Sciences, 7(1), 99-107.

Brodie, K. (2017). Mathematics pedagogy in South Africa: A human and academic project. In P. Webb & N. Roberts (Eds.), The pedagogy of mathematics in South Africa: is there a unifying logic (pp. 175-184). Johannesburg: Real African Publishers.

Brodie, K., Gopal, D., Moodliar, J., & Siala, T. (2021). Bridging powerful knowledge and lived experience: Challenges in teaching mathematics through COVID-19. Pythagoras, 42(1), a593. https://doi.org/10.4102/pythagoras.v42i1.593

Brown, S., and Wong, W. K. (2015). Probability and statistics with applications in finance and economics. The scientific World Journal, 2015, 1-2

Burgin, S. (2020). A three-dimensional conceptualization of authentic inquiry-based practices: a reflective tool for science educators. International Journal of Science Education, 42:9, 1465-1484.

Burke, K. A., Hand, P., Poack, J., and Greenbowe, T (2005). Using the science writing heuristic. Journal of College Science Teaching, 35(1), 36-41.

Campbell, P. F., Rust, A. H., Nishio, M., DePiper,J. N., Smith, T. M., Frank, T.,Choi, Y. (2014). The relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. Journal for Research in Mathematics Education, 45(4), 419-459. Doi:10,5951/jresematheduc.45.4.0419 (Crossref), (Web of Science R), (Google Scholar).

Can, O. S., & Isleyen, T. (2020). The effect of probability instruction through argumentation approach on the achievement of pre-service teachers and the permanence of their knowledge. Erzurum, Turkey.

Check, J., & Schutt, R. K. (2012). Survey research. Research methods in education, 159-185.

Civil, M. and Hunter, R. (2015). Participation of non-dominant students in argumentation in the mathematics classroom. Intercultural Education, 26(4), 296-312.

Cohen, I., Manion, L., & Morrison, K. (2011). Surveys, longitudinal, cross-sectional and trend studies. Research Methods in Education, 7th edition, Abingdon: Routledge, 261-264.

Cope, D. G. (2014). Methods and meanings: Credibility and trustworthiness of qualitative research. Oncology Nursing Forum, 41(1), 89-91. Doi: 10.1188/14.ONF.89-91

Craig, C., Evans, P., & Stokes, D. (2017). Developing STEM teachers through both informal and formal learning experiences: Ediciones Universidad de Salamnca.

Creswell, J. W. (2013). Qualitative inquiry and research design: choosing among five approaches. California: Thousand Oaks, Sage.

Creswell, J. W., & Creswell, J. D. (2018). Research design: Qualitative, quantitative, and mixed methods approach. Thousand Oaks, CA: Sage.

Creswell, J., & Poth, C. (2018). Qualitative inquiry & research design: Choosing among five approaches (4th ed.). Los Angeles: Sage.

Creswell, J. W., & Zhang, w. (2009). The application of mixed methods research designs to trauma research. Journal of Traumatic Stress: Official Publication of The International Society for Traumatic Stress Studies, 22(6), 612-621.

Danisman, S., & Tanisli, D. (2016). Examination of mathematics teachers' pedagogical content knowledge of probability. Malaysian Online Journal of Educational Sciences, 5(2), 16-34.

Darragh, L. (2016). Identity research in mathematics education. Educational Studies in Mathematics, 93(1), 19-33.

De Chesney, M. (2014). Nursing Research Using Ethnography, New York, Springer.

De Kock, S. M. (2015). Relating teachers' content knowledge to learner performance in probability, University of Pretoria, Pretoria, South Africa.

Du Plooy-Cilliers, F., & Cronje, J. (2014). Research paradigms and traditions. *Research matters*, 18-35

Early, D. M., Berg, J. K., Alicea, S., Si, Y., Aber, J. L., Rayan, R. M., & Deci, E. L. (2016). The impact of every classroom, every day on high school student achievement: Results from a school-randomised trial. *Journal of Research on Educational Effectiveness, 9(1), 3-29.*

Elberhary, S. G. A. (2020). Discussing the Conditional Probability from a Cognitive Psychological Perspective." American Journal of Educational Research, vol. 8, no 7 (2020): 491-501.

Elberhary, S. (2019). The necessity of revising primary school content of probability in Egypt to enhance students' Probabilistic reasoning. Decision making based on data. Proceedings of the Satellite conference of the International Association for Statistical Education (IASE).

Estrada, A., Batanero, C., & Diaz, C. (2018). Exploring teachers' attitudes towards probability and its teaching. In C. Batanero & E. Chernoff (Eds.), Teaching and learning stochastics (pp. 313-332). Springer, Cham. https://doi.org/10.1007/978-3-319-72871-1_18.

Fekeye, D. O. (2012). Teachers' qualification and subject mastery as predictors of achievement in English language in Ibarapapa division of Oyo State. Global Journal of Human Social Science, 12(3), 1-6.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Schaeffer, R. (2007). Guidelines for assessment and instruction in statistics education (GAISE) Report: A Pre-K-12 curriculum framework. Alexandria, VA: American Statistical Association.

Freire, P. (2018). Pedagogy of the oppressed (50th Anniversary Edition). New York, NY: Bloomsbury Academic.

Heyd-Metzuyanim, E., Tabach. M., & Nachlieli, T. (2015). Opportunities for learning given to prospective mathematics teachers: Between ritual and explorative instruction. Journal of mathematics Teacher Education, 16(9), 547-574.

Gardee, A., & Brodie, K. (2021). Relationships between teachers' interactions with learner errors and learners' mathematical identities. International journal of Science and Mathematics Education.

Graham, M., & Lesseig, K. (2018). Black-pocket strategies for argumentation. The Mathematics Teacher, 112(3), 172-178.

Groth, R. E., Butler, J., & Nelson, D. (2016). Overcoming challenges in learning probability vocabulary. Teaching Statistics, 38(3), 102-107.

Henderson, K. L. T., and Esposito, J. (2017). Using others in the nicest possible way. Qualitative Inquiry 00(0): 1-14DOI: 10.1177/1077800417743528.

Julie, C, & Gierdien, F. (2020). Reflections on Realistic Mathematics Education from a South African Perspective. In M. van den Heuvel-Panhuizen (Ed.), International Reflections on the Netherlands Didactics of Mathematics Education (pp. 71-82). Cham: Springer International Publishing.

Kabir,S. M. S. (2018). Methods of data collection. Zenodo. https://doi.org/10.5281/zenodo.5814115.

Kanyongo, G.Y., & Brown, L. I. (2013). Relationship between pedagogical strategies and teachers' content knowledge in mathematics. *African Journal of Research in MST Education*, 17, 106-112.

Khumalo, B., Molepo, G., & Mji, A. (2016). Attitude of Learners and Educators towards the Use of Digital Satellite Television (DSTV) in Teaching Mathematics in Senior Phase. *Empowering the 21st Century Learner*.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding It Up: Helping Children Learn Mathematics. Washington, DC: National Academies Press.

Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser, M., Krauss, S., & Baunert, J. (2013). Teachers' content knowledge and pedagogical content knowledge: The role of structural differences teacher education. Journal of Teacher Education, 64(1), 90-106.

Knudtzon, S. H. (2019). Pitfalls and surprises in probability: The battle against counterintuition. Eleventh Congress of the European Society for Research in Mathematics Education (CERME11), Utrecht University, Utrecht, Netherlands. (hal-02411606).

Kodisang, S. M. (2022). A framework towards improved instruction of probability Grade seven students: A case study of South African schools in Mpumalanga province. University of Pretoria, South Africa.

Kodisang, S. M. (2015). Teaching strategies used by mathematics teachers to teach Grade 6 probability in Nkangala District. University of Pretoria, Pretoria, South Africa.

Koparan , T. (2019). Teaching game and simulation-based probability. International Journal of Assessment Tools in Education, 6(2), 235-258. https://doi.org/10.1111/test.12062

Kovacs, Z., Recio, T., & Velez, M. P. (2018). Using automated reasoning tools in GeoGebra in the teaching and the learning of proving in geometry. *The International Journal for Technology in Mathematics Education, 13(1)*

Lampen, E., & Brodie, K., (2020). Becoming mathematical: Designing a curriculum for a mathematics club. Pythagoras, 41(1), a572. https://doi.org/10.4102/pythagorus.v41i1.572

Leavy, A. M., Hannigan, A., & Fitzmaurice, O. (2013). If you're doubting yourself then, what's the fun in that? An exploration of why prospective secondary mathematics teachers perceives statistics as difficult. Journal of statistics education, 21(3). https://doi.org/10.1007/BF00540060

Ledibane, M., Kaiser, K., & Van der Walt, M. (2018). Acquiring mathematics as a second language: A theoretical model to illustrate similarities in the acquisition of English as a second language and mathematics. Pythagoras, 39(1), a347. hhps//doi.org/10.4102/Pythagoras.v39i.

Liu, X. (2011). *Young children's social skills development and academic achievement: Longitudinal analysis of developmental trajectories and environmental influences.* State University of New York at Buffalo

Lo, C. K., & Hew, K. F. (2017). A critical review of flipped classroom challenges in K-12 education: Possible solutions and recommendations for future research. Research and Practice in Technology, 38(1), 3-16.

Lopez, V., & Whitehead, D. (2013). Sampling data and data collection in qualitative research. In nursing & Midwifery Research: Methods and Appraisal for Evidence-Based Practice (4th ed., pp. 123-140). Elsevier Mosby.

McMillan, J.H., & Schumacher, S. (2010). *Research in Education: Evidence-based Inquiry* (7th ed.). USA: Pearson Education, Inc.

Marshall, C., & Rossman, G. B. (2016). Designing qualitative research. Thousand Oaks, CA: Sage. Merriam, S. B., & Tisdell, E. J. (2016). Qualitative research: A guide to design and implementation. Hoboken, NJ: Wiley.

Meaney, T., Trinick, T., & Fairhall, U. (2012). Collaborating to meet language challenges in indigenous mathematics classrooms. New York, NY: Springer.

Merriam, S.B., & Tiisdell, E.J. (2016). Qualitative research: A guide to design and implementation. Hoboken, NJ: Wiley.

Mhlolo, M. K. (2017). Regular classroom teachers' recognition and support of the creative potential of mildly gifted mathematics learners. ZDM, 49(1), 81-94. https://doi.org/10.1007/s11858-016-0824-6

Mogari, D., Kriek, J., Stols, G., & Iheanachor, O. U. (2009). Lesotho's students' achievement in mathematics and their teachers' and professional development. Pythagoras (70), 3-15.

Morkel, G. (2014). Bloodstream infections and antimicrobial resistance patterns in a South African neonatal intensive care unit. Paediatrics and international child health, 34(2), 108-114.

Moschkovich, J. (2007). Using two languages when learning mathematics. Educational Studies in Mathematics, 64(2), 121-144.

Mukwakwa, E. G. (2012). Exploring problems experienced by Grade 11 mathematics teachers in the teaching and learning of statistics. University of South Africa, Pretoria South Africa.

Mutara, L., & Makonye, J. (2014). Learners' use of probability models in answering probability tasks in South Africa. University of the Witwatersrand, South Africa.

National Council of Teachers of Mathematics (1991), Professional standards for teaching mathematics, Reston: NCTM, [Google Scholar]

Nieuwenhuis, J. (2010). Qualitative research design and data gathering techniques in K. Maree (ed), First steps in research, rev. 4th impression, (pp. 67-97), Van Schaik, Pretoria.

Ogbonnaya, U. L., & Awuah, F. K. (2019). Quantile ranking of schools in South Africa and learners'achievement in probability, *Statistics Education Research Journal*, 18(1), 106-119

Ogbonnaya, U.I, & Mogari, D. (2014). The relationship between grade 11 students' achievement in trigonometric functions and their teachers' content knowledge. Mediterranean Journal of Social Sciences 5(4), 44-452.

Olivier, W. A. (2017). The Pedagogy of Mathematics-Is there a unifying logic? Chapter 12: Quest for a Modern Technology-assisted Model for Mathematics Teaching and Learning in South Africa. Published as a commissioned project of the DST & FirstRand Foundation by the Mapungubwe Institute for Strategic Reflection (MISTRA), ISBN 978-0-9870347-1-7.

Patton, M.Q. (2015). Qualitative research and evaluation methods (4th ed.). Thousand Oaks, CA: Sage.

Paul, M., & Hlanganani, N. (2014). Exploring mathematics students understanding of language of estimative probability in relation to probability scale. *International Journal Educational Sciences,* 7(3), 439-447

Patton, M. Q. (2015). Qualitative research and evaluation methods (4th ed.). Thousand Oaks, CA: Sage Publications.

Polit, D. F., Beck, C. T. (2014). Essentials of nursing research: Appraising evidence for nursing practice (8th ed.). Philadelphia: Wolters Kluwer. [Google Scholar]

Pratt, D. (2012). How do teachers foster students understanding of probability? In G. A Jones (Ed.). Exploring probability in school: Challenges for teaching and learning, (pp. (171-189).

Pratt, D. (2011). Re-connecting probability and reasoning about data in secondary school teaching. Proceedings from the 58th World Statistics Congress of the International Statistical Institute.

Pournara, C., Hodgen, J., Adler, J., and Pilly. (2015). Can improving teachers' knowledge of mathematics lead to gains in learners' attainment in mathematics? *South African Journal of teacher Education*, 35(3), Article no: 1083, 10pages.

Ramnarain, U., & Fortus, D. (2013). South African physical science teachers' perceptions of new content in a revised curriculum. South A frican Journal of Education, 33(1), 1-15.

Roberts, N. (2016). CAPS Mathematics curriculum map Grade 6. *African journal of Research in Mathematics, Science and Technology Education*, 20(20, 106-118.

Root, J. R., Browder, D. M., Saunders, A. F., & Lo, Y. Y. (2017). Schema-based instruction with concrete and virtual manipulatives to teach problem-solving to students with autism. Remedial and Special Education, 38(1), 42-52.

Schmidt, M.J. (2017). The Perils of Outcomes Based Education in fostering South African Educational Transformation. Open Journal of Political Science, 7, 368-379. https://doi.org/10.4236/ojps.2017.73030.

Schwandt, T. A., & Gates, E. F. (2018). Case study methodology. In *The Sage handbook of qualitative research* (pp.341-358).

Sithole, A., Chiyaka, E. T., McCarthy, P., Mupinga, D. M., Bucklein, B. K., & Kibirige, J. (2017). Student Attraction, Persistence and Retention in STEM Programs: Successes and Continuing Challenges. *Higher Education Studies*, 7(1), 46-59

Sharma, S. (2015). Teaching probability: A socio-constructivist perspective. Teaching Statistics, 37(3), 78-74. https://doi.org/10.1111/test.12075.

Siyepu, S. (2013). The zone of proximal development in the learning of mathematics. South African Journal of Education, 33(2). Art no 714, 13 pageshttp://www.sajournal of education.co.za.

Spaull, N. (2013) (Report Commissioned by Centre for Development and Enterorise). South Africa's Education Crisis: The quality of Education in South Africa 1994-2011. Available [Online]: http://www.section27.org.za/wp-content/uploads/2013/10/Spaull-2013-CDE-report-South Africas-Education-Crissis.pdf. Accessed on: 02/11/2016.

Spaull, N., & Kotze, J. (2015). Starting behind and staying behind in South Africa: The case of insurmountable learning deficits in mathematics. International Journal of Educational Development, 41, 13-24.

Steele, M. D. (2013). Exploring the mathematical knowledge for teaching geometry and measurement through the design and use of rich assessment tasks. Journal of Mathematics Teacher Education, 16, 245-268, DO1 10.1007/s10857-012-9230-3.

Stohl, H. (2005). Probability in teacher education and development. In G. Jones (Ed.). *Exploring probability in school: Challenges for teaching and learning* (pp. 345-366). New York: Springer.

Takeuchi, M. (2016). Friendships and group work in linguistically diverse mathematics classrooms: Opportunities to learn for English language learners. Journal of the Learning Sciences, 25(3), 411-437. https://doi.org/10.1080/10508406.2016.1169422

Torrance, H. (2012). Triangulation, respondent validation, and democratic participation in mixed methods research. Journal of mixed methods research, 6(2) 111-123.

Tshiredo, L. (2013). Impact of curriculum change in teaching and learning science: University of South Africa, Pretoria.

Umugiraneza, O., Bansilal, S., & North, D. (2017). Exploring teachers'practices in teachimg mathematics and statistics in KwaZulu-Natal schools. *South African Journal of Education, 37(2).*

Uygun, T., & Akyuz, D. (2019). Developing subject matter knowledge through argumentation. *International Journal of Research in Education and Science* (IJRES), 5(2), 532-547.

Van der Horst, H., & McDonald, R. (2014). Outcomes Based Education: theory and practice, Pretoria. University of South Africa.

Vasquez, Z, C., Alsina, A. (2021). Analysing Probability Teaching Practices in Primary Education: What Tasks Do Teachers Implement. Educ. Mathematics, 9(19), 2493.

Vidermanova, K., & Vallo, D. (2015). Practical geometry tasks as a method for teaching active learning in geometry. *Procedia-Social and Behavioural Sciences*, *191*, 1796-1800

Warshauer, H. K. (2015). Informing practice: Strategies to support productive struggle, Mathematics Teaching in the Middle School, 20(7), 390-393.

Welch, T. (2009). Teacher education qualifications: Contribution from Tessa Welch to working paper for teacher development summit. Unpublished document.

Wells, J. (2014). Developing argumentation in mathematics: The role of evidence and context. The University of Queensland.

Whiting, M., & Stines, D. (2012). Mind maps: Establishing ‘trustworthiness’ in qualitative research. Nurse Researcher, 20(1), 21-27.

Date: 6 August 2021

Appendix A– Information Sheet for Principal

INFORMATION SHEET FOR PRINCIPAL

Dear Principal

Request to participate in a research project.

My name is Obadiah Dube, and I am a mathematics teacher. I am a student in the School of Education at the University of the Witwatersrand. I am currently doing my master's degree in Mathematics Education under the supervision of George Ekol.

I am doing research on the topic of Probability.
The aim of this study is to investigate the challenges that teachers encounter in teaching probability at grade 12 level. The study tries to respond to the poor results in probability as reported in examiners' reports. The outcome of this study may lead/contribute to some solutions to the teaching and learning of probability in our schools. My research project involves examining teachers' preparation files, class observations, teacher interviews (30 minutes) The interview with teacher(s) aims to gather more information on the actual obstacles and contextual factors that hinder the effective teaching and learning of probability at grade 12.

I hereby request permission to carry out my research at your school. The information data collection process will take place after school hours so as not to cause any disruptions to the school. The research will require permission to:

- Examine teachers' preparation files and records.
- Do class observations once per teacher for 30 minutes.
- Interview Grade 12 Mathematics teacher(s) for 30 minutes, 15 before the lesson and 15 minutes after the lesson.

Research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating and no participants will be paid for this study. The identity of all participants and the school will be always kept confidential and in all academic writing about the study. Individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project. Please let me know if you require any further information.

Date ____________

Appendix B – Consent Form for Principal

CONSENT FORM FOR PRINCIPAL

Please fill in and return the reply slip below if you agree to allow the school to participate in my study.

I ______________________________________ the principal of ______________________________________ (schools name) agree that a research study conducted by -__________ (student number ________) from the University of the Witwatersrand may be undertaken in this school.

Informed consent

I understand that:

- All documents from teachers will be kept safe in the duration of use and will be handed back to teachers intact.
- Interview with e.g *Grade 12 Mathematics teacher(s) shall be conducted and audiotaped.*
- Classroom observations will not interfere with normal teaching and that teachers and learners will not be inconvenienced in any way.
  My name and information, as well as the names and information of teacher(s) that participate will be kept confidential and safe and that my name and the name of the school will not be revealed.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Signed: ____________________ Date: ________________

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

Appendix E – Information Sheet for Teachers

INFORMATION SHEET FOR TEACHERS

Date 06 August 2021

Dear Teacher

Request to participate in a research project.

My name is Obadiah Dube, and I am a mathematics teacher. I am a student in the School of Education at the University of the Witwatersrand. I am currently doing my master's degree in Mathematics Education under the supervision of George Ekol. I am doing a research on challenges faced by teachers in teaching probability at Gr 12. Mathematics P1 diagnostic reports show that grade 12 learners do not do well in probability at matric.
The aim of this study is to identify possible factors that may result in poor performance achieved by learners. The outcome of this study may lead to some solutions to the current performance of learners in mathematics.

My research involves doing interviews with teachers, observing lessons, analyzing teachers' documents, including preparation files and records. The reason why I have chosen your school is because it offers mathematics as a subject that is examinable, and the location of the school is in the community that I want to make a difference in. Teachers will be interviewed to obtain information on the challenges they experience when teaching probability. Classroom observations will be conducted to gather data about classroom environments, teaching methods and styles, teacher-learner interactions, and all other observable behavior.

Please note that the research will require the following:

- Teachers' preparation files and records of work.
- Classroom observations once per teacher for 30 minutes (15 minutes before lesson observation and 15 minutes after)
- Interviews with Grade 12 mathematics teacher(s) for 30 minutes.
- Interviews will be recorded (voice).

Your name and identity will be always kept confidential and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project. You will not be advantaged or disadvantaged in any way. Your participation is *voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.*

Please let me know if you require any further information.

Yours sincerely,
Obadiah Dube

Student _O.D__________(signature)	Supervisor __GL Ekol___________(signature)
Name___Obadiah Dube__________(student)	Name: Dr.George Ekol (Supervisor)
Cell-phone: __0783090112__________________	Tel__011_7170355______
e-mail: _dubeobadiah@gmail.com___________	e-mail: george.ekol@wits.ac.za______

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

Appendix F – Consent Form For Teachers

CONSENT FORM FOR TEACHERS

Date _______________

Please fill in and return the reply slip below indicating your willingness to be a participant in my research project.

I ____ volunteer to participate in this research by answering questions based on teaching probability and giving you access to learners scripts, my lesson plans and any other documents that may be of help to this study. I am aware that the findings of this research will be used to promote teaching and learning and will be published. I am aware that my identity will be protected and I will remain anonymous.

Permission to review/collect documents/artifacts **Circle one**

I agree that preliminary examination answer scripts can be used for this study only. YES/NO

Permission to be audiotaped **Circle one**

I agree to be audiotaped during the interview or observation lesson YES/NO

I know that the audiotapes will be used for this project only YES/NO

Informed Consent

I understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Signed: ________________________ Date: ___________________

Appendix G

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: 2021ECE086M**
PROJECT TITLE	Investigating the nature of challenges faced by Grade 12 teachers in the teaching and learning of Probability and Counting Principles in South Africa. A case of Thokoza, East Rand
INVESTIGATOR	**OBADIA DUBE**
SCHOOL/DEPARTMENT OF INVESTIGATOR	**WSoE**
DATE CONSIDERED	17 JANUARY 2022
DECISION OF THE COMMITTEE	Approved unconditionally
RISK LEVEL	MINIMAL
EXPIRY DATE	Date of submission of the Research Report
ISSUE DATE OF CERTIFICATE	**CHAIRPERSON** ____________ Dr. Paul Goldschagg

cc: Dr George Ekol

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am are authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

____

Signature

Date ______/______/______

27 St Andrews Road, Parktown, Johannesburg, 2193 | Pvt Bag 3, WITS, 2050 | T +27 11 717 3007 | www.wits.ac.za/education

IYUNIVESITHI YASEWITWATERSRAND | YUNIVESITHI YA WITWATERSRAND

Appendix H

GAUTENG PROVINCE

Department: Education

REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**12 August 2021**
Validity of Research Approval:	**08 February 2021 – 30 September 2021** **2021/222**
Name of Researcher:	**Dube Obadiah**
Address of Researcher:	**22 Carolina Street**
	Dawn Park
	1459
Telephone Number:	**078 3090 112**
Email address:	**2290612@students.wits.ac.za**
Research Topic:	**Investigating the nature of the challenges faced by Grade 12 South African teachers in the teaching and learning of Probability and Counting principles. A case of Thokoza cluster, Ekurhuleni South,**
Type of qualification	**M.Ed. Mathematics**
Number and type of schools:	**Secondary Schools**
District/s/HO	**01 District**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

16/08/2021

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

Making education a societal priority

1

Office of the Director: Education Research and Knowledge Management

7th Floor, 17 Simmonds Street, Johannesburg, 2001

Tel: (011) 355 0488

Email: Faith.Tshabalala@gauteng.gov.za

Website: www.education.gpg.gov.za

2. The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.
3. **Because of COVID 19 pandemic researchers can ONLY collect data online, telephonically or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate. The approval letter will then indicate the type of arrangements that have been made with the school.**
4. **The Researchers are advised to make arrangements with the schools via Fax, email or telephonically with the Principal.**
5. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.
6. A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
7. The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
8. Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
9. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.
10. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
11. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
12. The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
13. The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
14. On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.
15. The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.
16. Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: ....16|08|2021..............................

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix I

Online Questionnaire for Teachers.

Section A. Introduction:

Thank you for participating in the interview. My name is Obadiah Dube, a student at the University of Witwatersrand, School of Education. I am investigating challenges faced by probability teachers at high school in general, but at grade 12. Information you provide will be kept confidential including your personal details. Direct information will not be used in the report but only the aggregated data collected from all participants, to enable me to understand the challenges better. If you have any questions, please contact Mr. Obadiah Dube (078 309 0112 or 072 935 9571), or Dr George Ekol (email: george.ekol@wits.ac.za).

General Information.

Gender________M/F

Age (years). a. 20 - <25 b. 25-<30 c. 30-<35 d. 35-<40 e. 40-< 45 f. 45 - <50 g. 50+ ______

- School______________________________
- Highest academic qualification_______________________________
- Studied Mathematics at High school? ____Yes/No___and at university? ____ Yes/No________
- Studied probability at High school? ____Yes/No___and at university? ____ Yes/No________
- Years of teaching mathematics_______________________
- Years of teaching probability___________________
- Current grade(s) at which I teach mathematics topics___________
- Current grade(s) at which I teach probability___________

Please, say more about any of the questions above…

Section B. Language of/in probability

1. Can you give some examples of terms in probability that your learners struggle with? What teaching strategies do you use to enable your learners understand the terms used in probability?

(Please say more…….

Section C. Teacher Content Knowledge

2. Did you study probability in your pre-service teacher training program?

(Please, say more…….

3. Since your graduation, have you had other teacher development training in the teaching of probability? How many and where? Were these programs organised by the DBE or by the school?

(Please, say more…….

4. How do you rate your current content knowledge of probability (Adequate, Lacking in some topics, Excellent, …)? Use any descriptions that suits your case)

(Please, say more…….

5. What has been the performance in probability by your school in 2021, 2020, and 2019? Give percentage pass rate in probability?

(Please, say more…….

Section D. Exploring Teachers Challenges in Teaching Specific Topics

6. In a scale of 1-10 (please give only one specific number for each question).

How comfortable are you with teaching the following concepts in probability, and at which Grade, or Grades**?**

6.1. Mutually exclusive events. Score. ___Grade___

(Please, say more about your score…….)

6.2. Dependent and independent events. Score. ___Grade___

(*Please, say more about your score…….)*

6.3. Explaining the language used in probability_

Score__ Grade___ Score. __Grade___ Score. __Grade___

(Please, say more about your score…….)

6.4. Explaining the language used in mathematics topics

Score__ Grade___ Score. __Grade___ Score. __Grade___

(Please, say more about your score…….)

6.5. Helping learners understand and perform well in answering questions in probability. Score. ___Grade__ (Feel free to score other grades that you teach.)

(Please, say more about your score…….)

6.6. Helping learners solve probability problems using Venn Diagrams. Score____Grade_

(Please, say more about your score…….)

6.7. Helping leaners solve problems using contingency tables. Score. ___Grade___

(Please, say more about your score…….)

6.8. Helping learners solve problems using probability tree diagrams.

Score. _ __Grade___

(Please, say more about your score…….)

Please provide any additional comments that you may have about the above teaching topics in Question 10.

Section E. Use of technology in Teaching Probability

7.1 What tools do you use for teaching probability at different Grades that you teach? ('Tool' is used as a general term to include all resources, such as textbook, and online resources that you use for teaching).

7.2. Which of the above resources do you provide yourself, and which ones are provided by the school?

Do you think your school has adequate resources for the teaching of probability? Score__ Grade

Please provide any additional comments below that you may have about use of technology in teaching probability.

Section F. Classroom Interaction

8. In a scale of 1-10 (please give only one specific number for each question).

How often in your probability classes, and at which Grade levels do the following activities take place with your teaching arrangements.

8.1. Whole class discussions to solve probability problems. Score. ___Grade___

(Please, say more about your score…….)

8.2. Small group discussions to solve problems**.** Score. ___Grade___

(Please, say more about your score…….)

8.3. Learners talk to each other in class using correct probability terms. Score. ___Grade___

(Please, say more about your score…….)

8.4. Learners talk to the teacher using correct probability terms. Score. __ Grade___

(Please, say more about your score…….)

8.5. Learners talk to each other in class using correct probability terms. Score. __Grade___

(Please, say more about your score…….)

8.6. Learners are given ample time to look for solutions to problems. Score. __ Grade___

(Please, say more about your score…….)

8.7. Learners challenge each other's solutions in probability and counting principles. Score. __ Grade___

(Please, say more about your score…….)

8.8. Leaners challenge the teacher's solutions in probability and counting principles? Score. __ Grade___

(Please, say more about your score…….)

8.9. Learners freely interact with each other during probability class facilitated by the teacher. Score. __ Grade___

8.10. Learners freely interact with each other during probability lesson facilitated by the teacher. Score. __ Grade___

Please provide any additional comments below that you may have about the above teaching issues in Question 12.

Section G: Mathematics and probability topics

9. Which three topics in the current mathematics syllabus are your favourite, and at what Grade?
   i. Topic_1______________ Grade________________

   ii. Topic 2 ________________ Grade_______________

   iii. Topic 3_________________ Grade ______________

(Please, say more about …….)

10. Which three topics in the present mathematics are most difficult for you to teach and at what Grade? Order them from difficult to most difficult.
    iv. Topic_1______________ Grade________________

    v. Topic 2 ________________ Grade_______________

    vi. Topic 3_________________ Grade ______________

(Please, write more about …….)

Section H. Specific Challenges and Suggestions for Solutions

11. In your view what are the three major challenges faced by a teacher teaching probability and Counting principles at Grade 12? (If not Grade 12 specify the Grade. Feel welcome to say more…)

(Please, write more about …….)

12. What solutions do you suggest for each of the three challenges in 13?

(Please, write more about …….)

Appendix J : Classroom Observation guide

The teaching and learning environment was observed to determine if it contributed to probability teaching problems.

The following were observed:

1. Classroom resources
2. Classroom organisation
3. Teaching activities. These are important variables that influence successful teaching and learning.

They include:

- Teacher-learner interaction

Learner to learner interaction

- Learner participation and well planned teacher intervention

Methods used to teach probability are regarded as the major determinant of successful learning.

The methods used were observed for their effectiveness as regards children's successful learning

The framework of argumentation was used to observe the learning environment created by teachers.