

Optimisation of a Three Spring and Damper Suspension

Robert Berman

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Declaration

I declare that this dissertation is my own, unaided work, except where otherwise acknowledged. It is being submitted for the degree of Master of Science in Engineering in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other university.

Signed this 19th day of August 2016

A handwritten signature in black ink, appearing to read 'R Berman', with a long, sweeping horizontal stroke extending to the right.

Robert Berman

To
Miessie, whose love and support sustains me

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Abstract

This investigation considers the influence of a three spring and damper suspension system (SDS) on overall vehicle performance. Three SDS systems are used in high performance winged racing cars to manage the effects of the aerodynamic forces. The aim of the investigation was to quantify and compare the performance of a three SDS system to that of a conventional two SDS system. The investigation was carried out on the Bailey Cars LMP2 race car. Physical track testing was conducted on Zwartkops Raceway to measure the vehicle's performance, with further testing conducted on the vehicle's tyres. A software model of the vehicle and tyres was then created in ADAMS/Car, with models for the conventional two SDS system, as well as the three SDS system. The ADAMS/Car model was then validated against the test data. A Design of Experiments approach was used to investigate the influence of the parameters in both the suspension models. The optimal set of suspension parameters, that maximised vehicle performance on Zwartkops Raceway, was then identified. The performance of the optimal suspension systems was then compared to quantify the effect of the three SDS system. It was found that the optimised three SDS system travelled 4.38 m less than the optimal two SDS in a 60 second simulation on Zwartkops Raceway. However, the three SDS was effectively able to isolate the pitch and roll stiffness of the vehicle. The optimal three SDS had a greater pitch stiffness and less roll stiffness than the two SDS. This is significant for winged vehicles where aerodynamic forces are highly sensitive to vehicle pitch, such as the Bailey Cars LMP2 race car, allowing for a soft wheel rate without sacrificing the pitch stiffness of the vehicle.

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List of Symbols

The units of quantities defined by a symbol are indicated in square brackets following the description of the symbol. Quantities with no indicated units may be assumed to be dimensionless.

α slip angle [deg].

B magic tyre formula stiffness factor.

C magic tyre formula shape factor.

D magic tyre formula peak factor.

E magic tyre formula curvature factor.

κ longitudinal slip.

S_h magic tyre formula horizontal shift.

S_v magic tyre formula vertical shift.

σ Standard Deviation.

σ^2 Variance.

List of Acronyms

(SDS) spring and damper suspension.

ARB anti-roll bar.

C_D Coefficient of drag.

C_L Coefficient of lift.

CoG centre of gravity.

CoP Centre of pressure.

DOE design of experiment.

FRF frequency response function.

IRI International Roughness Index.

PDF probability density function.

SACAM The South African Conference on Computational and Applied Mechanics.

Chapter 1 Introduction

1.1 Background

Modern racing cars reach extremely high speeds with Formula One, Le Mans and Indy cars all reaching speeds of over 320 km/h. More significant than straight line speed are the speeds that these racing cars are able to sustain during cornering due to aerodynamic forces that are generated through the use of winged surfaces. The aerodynamic forces are reacted through the tyres which in turn generate greater accelerations than would otherwise be possible.

Managing the position and orientation of the winged surfaces on racing cars is important to prevent the wings from stalling inadvertently through excessive body pitch and roll. This leads to extremely stiff suspension settings, however an overly stiff set-up can cause a wheel to lose contact with the road surface leading to a loss in acceleration, which is undesirable in motor racing. In order to avoid this, race car designers have implemented a so-called three spring and damper suspension (SDS) system with a vertical anti-roll bar (ARB), illustrated in [1.1](#). A 3D CAD rendering of a 3 SDS is shown in [Figure 1.2](#)

The third spring and damper are activated when both the left and right wheels of the front suspension move in the same direction, such as under braking. The vertical ARB is activated as a torsion bar, when the left and right wheels of the front suspension move in opposite directions, such as when cornering.

The literature survey, presented below, shows that there is no published data on three SDS systems. The focus of this study, therefore, is to determine the exact effect that the third spring and damper, in combination with a vertical ARB, will have on overall vehicle performance.

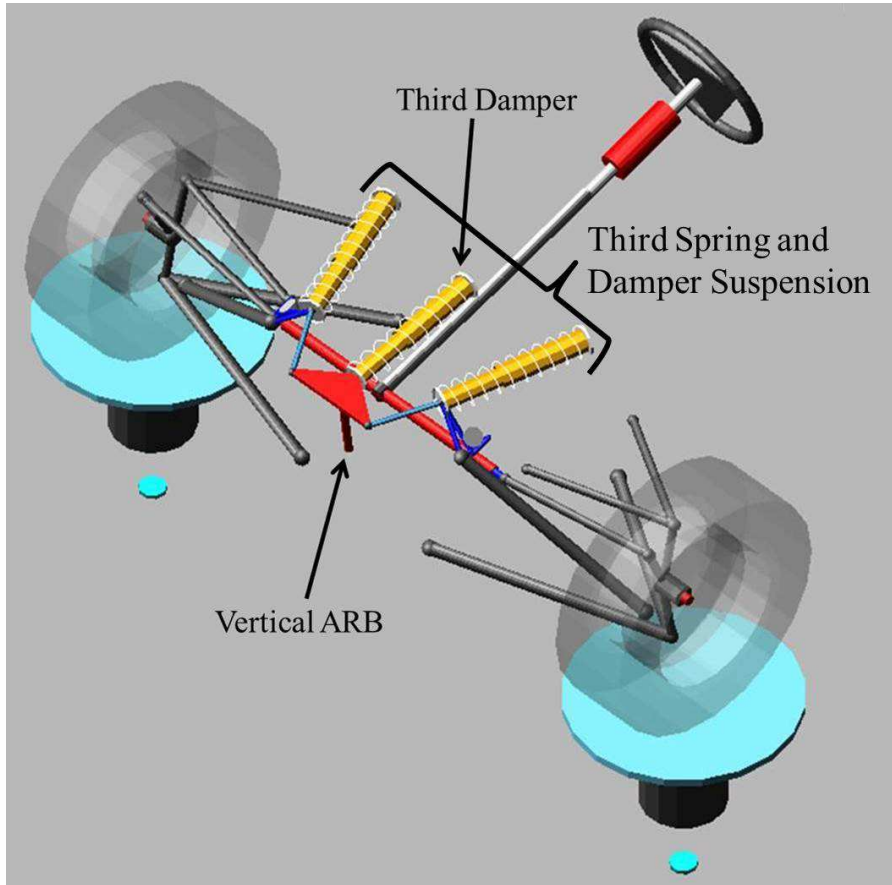


Figure 1.1: Three Spring and Damper Suspension (SDS)

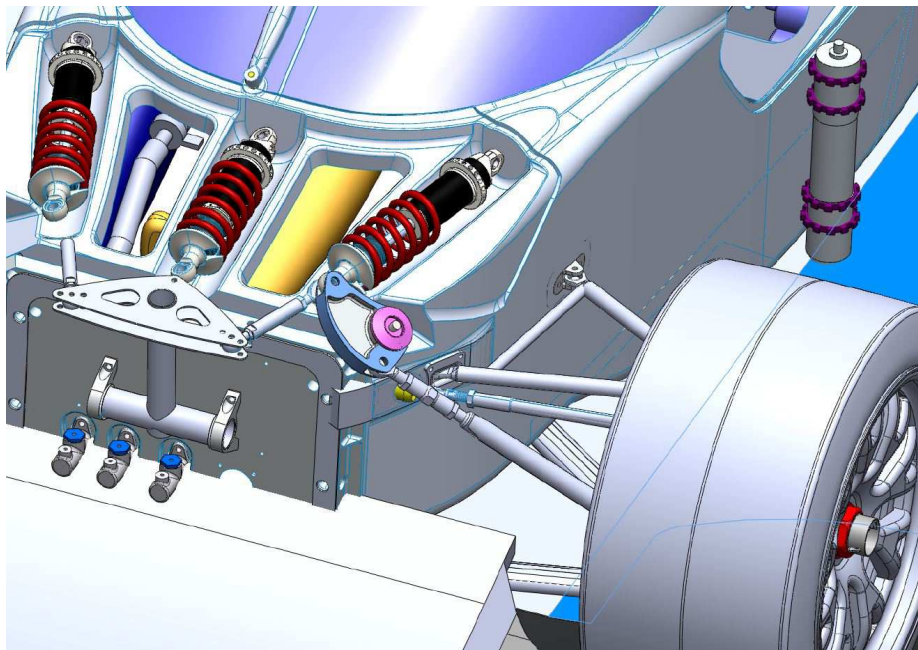


Figure 1.2: Three Spring and Damper Suspension 3D rendering

1.2 Literature Review

In this section, a number of models describing vehicle suspension and dynamics will be discussed. The first of these is the quarter car model, used throughout literature as a first pass calculation, before an expansion of the degrees of freedom of the system are explored. Suspension tuning and the use of various physical test methods will be reviewed. Track testing, data acquisition and tyre testing will then be discussed, before outlining the use of commercial software in suspension analysis.

The literature highlights that no results on three SDS systems, their development, or tuning have been published.

1.2.1 Suspension Modelling

A quarter car model is frequently used in suspension analysis due to its simplicity and closed form solution. The quarter car model illustrates body bounce and wheel hop, but it does not directly include the effects of suspension geometry. Geometric effects such as the motion ratio of the suspension need to be taken into account when considering the quarter car model. Kowalczyk [2] used a quarter car model in the analysis of a Champ Car suspension before progressing to higher order models and ultimately tuning the suspension on a test rig.

To account for pitch, heave and roll, as well as their interaction, a 7-DOF, or higher, model of the vehicle was required. Kowalczyk showed that using higher order models allowed one to assess the effects of the relative damping of the front and rear suspensions, as well as changes in vehicle mass properties, namely centre of gravity (CoG) position, pitch inertia and roll inertia.

Thoreson [4] presented an ADAMS/View model of an off-road vehicle that was used to determine optimum spring and damper rates for on and off-road ride and handling. The software model was used for a variety of test manoeuvres to aid in the optimisation process. ADAMS/View allows the user to include non-linear components, such as dampers and tyres, while still maintaining a reasonable level of simplicity [5].

Hegazy et al. [6] presented the development and testing of a complete ADAMS model of a generic road car, to conduct a handling analysis. The tests simulated in the analysis were carried out under the specifications of ISO and British Standards, but no actual test data were used to validate the results of the ADAMS model. The analysis highlighted the usefulness of ADAMS software especially in transient analyses as a result of steering inputs.

1.2.2 Suspension Testing

Ride simulators were introduced in 1997 and have become an integral part of vehicle development. Kelly et al. [7] used a four-post test rig to test for pitch, heave, roll and torsional frequencies of a vehicle suspension. A four-post rig, with a single actuator at each wheel contact patch is capable of simulating the inputs of the road surface at each tyre.

Kowalczyk [2] used Lagrange's equations of motion with a 7-DOF model, as mentioned above, to obtain analytical results for the behaviour of a single seater Champ Car suspension system. These results were then used in the optimisation of the dampers for the vehicle on a seven-post rig. The analysis was done in the frequency domain by considering the frequency response function (FRF) of the major modes of the vehicle.

The input used by Kowalczyk for the seven-post rig was sinusoidal in nature and tuned manually until the damper response of the vehicle on the test rig matched that of the vehicle on the track. This required a certain amount of operator skill and instinct to accurately predict. This approach to modelling allowed the frequency response of the vehicle to be calculated, but could not predict the performance of the vehicle on track, nor show the true extent of any set-up changes made to the vehicle. He found that the seven-post rig was an effective tool in the suspension optimisation process, as well as adding greater understanding to the dynamics of the vehicle.

Miller [8] tested a Formula SAE race car suspension on a seven-post shaker rig, which is an extension of the four-post rig, where three additional actuators simulate the effects of aerodynamic loadings and inertial forces on the vehicle, phenomena that cannot be included in a four-post rig analysis. The papers that describe the use of four-post test rigs usually describe the tuning of road vehicle ride comfort, whereas those that discuss the use of ride simulators to test race car suspensions discuss the use of seven-post rigs. Ride comfort is of little concern in a race car, with effects such as inertial and aerodynamic loading being

of critical importance. Miller used the seven-post shaker rig to determine the most suitable ARB set-up on, which was found in one session on the shaker rig as opposed to three track sessions.

1.2.3 Suspension Tuning

Day et al. [9] discussed the effects of CoG height, vehicle track, suspension stiffness and roll centre on vehicle performance. They noted that load transfer from the inside tyres to the outside tyres during cornering is a function of the CoG height and the vehicle track. Load transfer front to rear is a function of the front and rear roll stiffness and the roll centre location of the front and rear suspension. It was further noted that typically, the total roll stiffness is selected on the basis of the permissible amount of body roll.

1.2.4 Track Testing and Data Acquisition

To quantify a vehicle's performance on a racetrack, intuition and experience no longer suffice. In days gone by, the tools used to track race car performance were the stopwatch, tyre pressure gauge and the pyrometer [11]. Data acquisition has evolved much since then and will be briefly discussed in this section.

Segers [11] presented a detailed discussion on data acquisition and track testing of racing cars; the value of data logging systems and how detailed analysis of the data can provide insight into the set-up of the race car. A data logging system provides information about the car-driver combination performance at a particular instant on a racetrack.

Segers stated that data acquisition can be broken down into the following categories: vehicle performance analysis, driver performance analysis, vehicle development, reliability and safety, determining vehicle parameters and running logs. According to Segers, the basic data acquisition signals are: engine RPM, wheel speed, throttle position, steering angle and lateral and longitudinal acceleration.

Segers summarised that a race car's performance on a racetrack is ultimately measured by lap time and that cornering speed has the single largest effect on lap time.

1.2.5 Tyre Models

Stainforth [12] states that: "Every vehicle is connected to the ground through four small contact patches where the tyres meet the road. Through these four contact patches, all the cornering, acceleration and braking forces are transmitted from the road to the vehicle. The suspension of the vehicle is concerned with connecting the wheels to the vehicle, controlling how the vehicle manoeuvres and how the four small contact patches are utilised". Tyre data and models are thus of great importance to this handling analysis investigation and are discussed below.

Pacejka et al. [13] presented the now ubiquitous Magic Tyre Formula or Pacejka 89 tyre model, for vehicle handling, as follows:

$$y(x) = D \sin C \arctan[Bx - E(Bx - \arctan(Bx))] \quad (1.1)$$

$$Y(X) = y(x) + S_v \quad (1.2)$$

$$x = X + S_h \quad (1.3)$$

Where $Y(x)$ either represents lateral force, self-aligning torque or longitudinal force and x denotes tyre slip angle, (α) or longitudinal slip (κ). The general coefficients in equation 3.1 represent the following:

B = Stiffness factor

C = Shape factor

D = Peak factor

E = Curvature factor

S_v = Vertical shift

S_h = Horizontal shift

Lateral force is further characterised by 14 coefficients, a_0 to a_{13} , the longitudinal force by 11 coefficients, b_0 to b_{10} and the self-aligning torque by 18 coefficients, c_0 to c_{17} .

1.2.6 Tyre Testing and Equipment

Hugo et al. [14] presented an approach to characterising tyres in which they note that in order to adequately characterise a single tyre, the following set of tests should be carried out: 17 slip angles, 6 vertical loads and 5 camber angles. This process would not only be costly due to the number of tyres required, but also because of the time required to conduct the testing. Issues of repeatability due to test conditions and variations in the individual tyres also arise. Anomalies such as tread carcass asymmetries, tyre wear and test rig peculiarities, which create ply steer as well as conicity effects, all influence tyre test data. This leads to non-zero force data at zero slip angle, but it is also mentioned that for racing cars, small slip angle force data are of little interest. As such, small slip angle nuances can largely be ignored for racing car tyres.

Hugo et al. also stated that the road tyre friction coefficient can be estimated by taking the maximum ratio of lateral force to the vertical load on the tyre for each load. Day et al. [14] noted that aligning moments also have little effect on vehicle handling and are thus of far less concern than lateral tyre force data.

Kasprzak and Gentz [15] discussed that while drum type tyre testers are relatively simple, they have the drawback that a curved surface is used for the tyre contact patch. This leads to a different tyre contact patch pressure distribution to those recorded on flat roads. On-road tyre testing has the main drawback of low repeatability. The solution to this, according to Kasprzak and Gentz, is an indoor, flat surface testing machine. The Calspan Tire Research Facility tyre tester is one such machine, with the ability to vary normal load, normal load rate, roadway speed, slip angle, inclination angle, tyre inflation pressure and roadway surface.

1.2.7 Suspension Modelling using Commercial Software Packages

Simple mathematical models provide valuable insight into vehicle handling characteristics, but show only part of the picture. In order to more accurately simulate reality, the order and thus the complexity of the mathematical model must be increased to an appropriate level. In practice, this can only be done using computer simulation [5].

Dominy [16] presented an investigation into lap time between two cars from different Le Mans classes. For informative lap time simulation, just five basic aspects of the vehicle are required: a chassis model, a tyre model, the engine torque and power curves and a braking model. The last three submodels are determined almost entirely from physical testing. The commercial software package Laptime was used for the analysis.

ADAMS/Car, an addition to the ADAMS/View environment in the MSC.ADAMS package has been used extensively to model racing cars. ADAMS is a systems based analysis tool, developed as an application to solve non-linear numerical equations. ADAMS/Car [17] is a specialised vehicle analysis tool that allows all the mechanical components of a vehicle to be simulated. Each component is grouped into a subsystem in one of the following categories: suspension, chassis, steering, braking system and drivetrain.

ADAMS/Insight [18] is a design-of-experiments package that is part of the ADAMS software suite. ADAMS/Insight contains a selection of statistics tools that are used for parametric studies of ADAMS models. ADAMS/Insight allows the performance of a system to be explored and optimised.

Of the above mentioned papers that used shaker rigs in their analyses, Kowalczyk [2] and Miller [8] used linear mathematical suspension models. They did not develop their mathematical models to include non-linear elements such as dampers and tyres. They also made no use of computer simulation software to analyse and optimise the vehicle suspension.

1.2.8 Research Opportunities

Due to the competitive nature of motorsport racing, there exists very limited published research on suspension development and tuning in motorsport engineering. Carrol [1] notes that in the global context, the information that does exist is proprietary and thus closely guarded by individual race teams. The published data is usually outdated and irrelevant for modern innovations and applications.

As presented above, many of the papers focusing on suspension analysis and optimisation in motor racing give results of research carried out on vehicles with independent suspension systems. There are very few, if any, research papers that deal exclusively with anti-dive/squat features such as a third spring and damper.

In this study, a race car with a three SDS system that is validated with test track data is presented; with a validated tyre submodel. The analysis was conducted in collaboration with a commercial race car manufacturer. The vehicle parameters and model are taken from an actual vehicle and are representative of a high performance winged race car. The effect of using a 3 SDS on vehicle performance is evaluated.

To the best of the authors knowledge, this is the first publication of results from a vehicle using a 3 SDS.

Initial work done for this research was presented at The South African Conference on Computational and Applied Mechanics (SACAM) 2012. That work formed the foundation of the research for this thesis, investigating the suspension and ARB parameters [20].

1.3 Objectives

The specific objectives of this research are to:

1. Quantify the performance of a race car which uses a three spring and damper suspension system.
2. Compare the above vehicle performance with that of a conventional suspension system without the third spring and damper

1.4 Overview of the Testing and Simulation Methodology

The Bailey Cars Le Mans Prototype 2 (BC LMP2) race car, pictured in Figure 1.3, was selected as the basis of this investigation. The BC LMP2 was designed and built according to the technical regulations of the Le Mans 24 hour endurance race as specified by the Fédération Internationale de l'Automobile (FIA). The BC LMP2 features 3 SDS suspension both front and rear. A combination of physical testing and computer simulation were used in this investigation.

The physical testing was carried out in two phases: track testing of the entire vehicle and testing of the BC LMP2 tyres. Testing was conducted at Zwartkops Raceway to determine the overall performance of the vehicle, measuring lap time, GPS position, vehicle speed and accelerations. The vehicle was fitted with two data acquisition systems, a Motec system and a Racelogic VBOX.

Tyre testing was used to characterise the lateral tyre force and aligning moment versus slip angle curves of the vehicle's tyres. Testing was conducted using a trailer type tyre tester that requires the adjustment of both the left and right wheels for slip angle. Forces were measured by six load cells: 2 lateral, 3 vertical and 1 longitudinal; and slip angle was measured with a string potentiometer. Matlab[®] was used to process the data and to calculate the Pacejka 89 tyre model coefficients.

The computer simulation was carried out in MSC ADAMS/Car. Two versions of the BC LMP2 double wishbone suspension were created, one with a third spring and damper, and the other without. The model without the third spring and damper was used to validate the

ADAMS/Car model using the track test data. The model with the third spring and damper was then used to investigate the effects of the ARB and the third spring and damper.



Figure 1.3: Bailey Cars LMP 2 Race Car

Chapter 2 TRACK TESTING

2.1 Introduction

Track testing was conducted at the Zwartkops Raceway in Centurion, with the goal of recording the performance of the vehicle on its fastest lap around the circuit. The suspension set-up of the vehicle excluded the vertical ARBs and third springs and dampers as their design was not finalised at the time of testing.

The BC LMP2 race car was designed according to the regulations of the Le Mans 24 hour endurance race, and as such, the vehicle is well suited to long, high speed circuits. Zwartkops raceway is unfortunately a short circuit with tight turns and few high speed sweeping corners and as is thus not the ideal circuit to fully evaluate the performance of the BC LMP2. However, once again due to the prohibitive costs of race car testing, as well as availability of all parties required to conduct testing such as this, the Zwartkops raceway was the only option for track testing.

The vehicle was driven by South African racing driver, Hennie Groenewald. The testing consisted of a number of warm up and shakedown laps to ensure all systems were functioning optimally before the actual testing commenced. Hennie was instructed to explore the vehicle limits and to obtain the fastest possible lap time. This was done by running a series of two to three laps at a time, before pitting to analyse the data and fine tune the vehicle set-up.

2.2 Vehicle Description

Some of the relevant vehicle parameters of the BC LMP2 for the track testing are given as follows: Eibach Motorsport springs were fitted to the vehicle, with front and rear linear stiffness values of 115 N/mm and 145 N/mm respectively [21]. The damper curves of the Penske adjustable dampers that were fitted to the vehicle are given in Figure 2.1 [22].

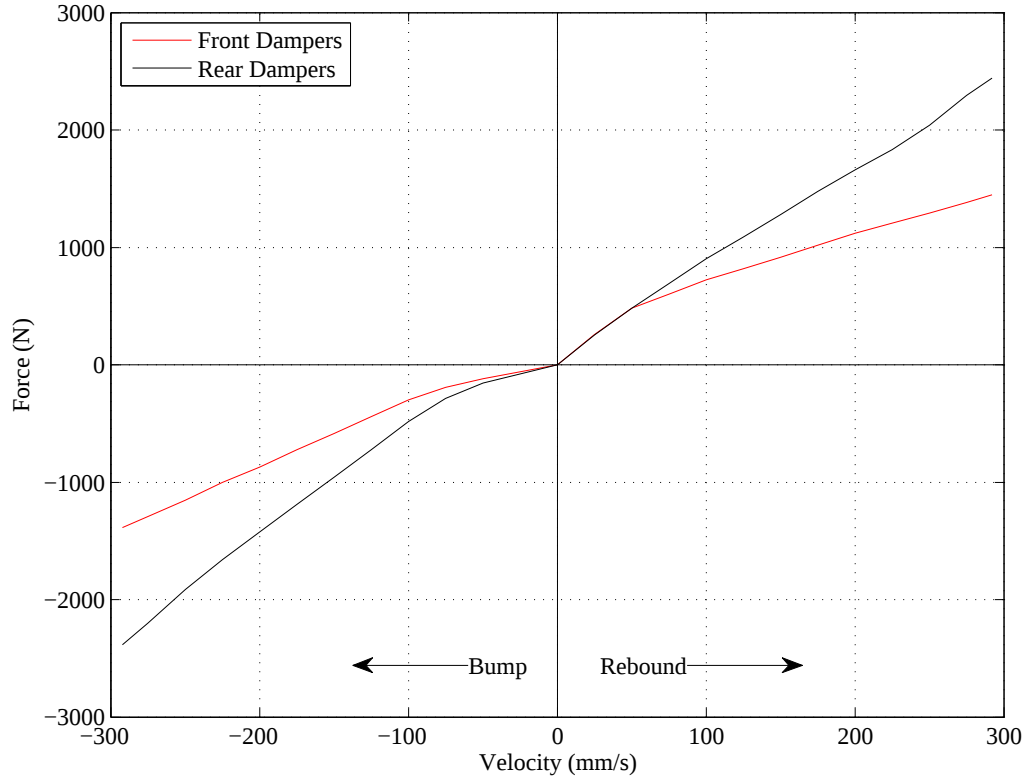


Figure 2.1: Damper Curves for the BC LMP2 Front and Rear Dampers [22]

The vehicle mass properties were obtained from physical measurements using a Longacre AccuSet vehicle scale and from the CAD model; these data are given in Table 2.1. The suspension coordinates, engine power and torque curves and gear ratios are given in Appendix A. The vehicle was fitted with a six speed Riccardo sequential gear box.

Table 2.1: Vehicle Mass Properties

Part	Mass (kg)	MOI (kg.mm ²)		
		Ixx	Iyy	Izz
Chassis	960	2.00E+08	5.00E+08	6.00E+08
Front wheel	12	3.50E+04	3.50E+04	8.00E+03
Rear wheel	15	4.38E+04	4.38E+04	1.00E+04

Aerodynamics play an important role in the overall performance of the BC LMP2 race car, with a maximum downforce of 1 456 kg ,at a speed of 320 kph. The maximum recorded speed of the vehicle during the Zwartkops testing was 211 kph which equates to a maximum possible downforce of 633 kg. Although the effect that downforce has on the vehicle's performance is not as significant as it would be on a faster circuit, it is still substantial compared to the total mass of the vehicle at 960 kg.

2.3 Data Acquisition

The vehicle was fitted with two separate data acquisition systems, the first a Motec M800 with fully integrated engine management and data acquisition system and the second a Racelogic VBOX. The Motec M800 system consisted of a CDL3 Track Kit, comprising: a GPS system, a series of engine management and monitoring devices and a 3-axis accelerometer placed near the vehicle's centre of gravity. A WPS-500-MK30-P10 Micro-Epsilon string potentiometer, $\pm 0.1\%$ full scale output with a resolution of 0.15 mm, was connected to the Motec system as a steering angle sensor.

The Racelogic VBOX, VB20 SL3 with an internal Kalman Filter, was used to measure vehicle: slip, pitch, roll and yaw using three GPS antennae. The VBOX system relies on connection to a large number of GPS satellites. However in practice, this was seldom possible and accurate roll, pitch and yaw data could not consistently be obtained from the VBOX.

2.4 Track Test Data

The parameters from the track testing used for the validation of the ADAMS/Car model were: distance travelled, vehicle speed, lateral acceleration, longitudinal acceleration, gear, engine RPM and steering angle. The fastest lap recorded in minutes and seconds was 1:01.99. The GPS position of the vehicle was used to create a road model of the track for ADAMS/Car. The GPS coordinates were converted to Cartesian coordinates to create the ADAMS/Car model of the Zwartkops Raceway. The method used to convert the coordinates is detailed in [Appendix B](#).

The GPS coordinates recorded by the VBox are shown overlaid on the Zwartkops Raceway in Google Earth in Figure 2.2. The GPS coordinates were then converted to Cartesian coordinates, as detailed in Appendix B. After conversion, the total track distance was calculated as 2408.2 m. The Motec system recorded distance travelled directly to be 2203.0 m. The conversion from GPS to Cartesian coordinates had therefore scaled the points, and as such, the Cartesian coordinates were scaled to rectify the lap distance. The rotated and scaled co-ordinates from the VBox are shown along with the rotated and unscaled co-ordinates in Figure 2.3.



Figure 2.2: VBox GPS Coordinates Overlaid in Google Earth

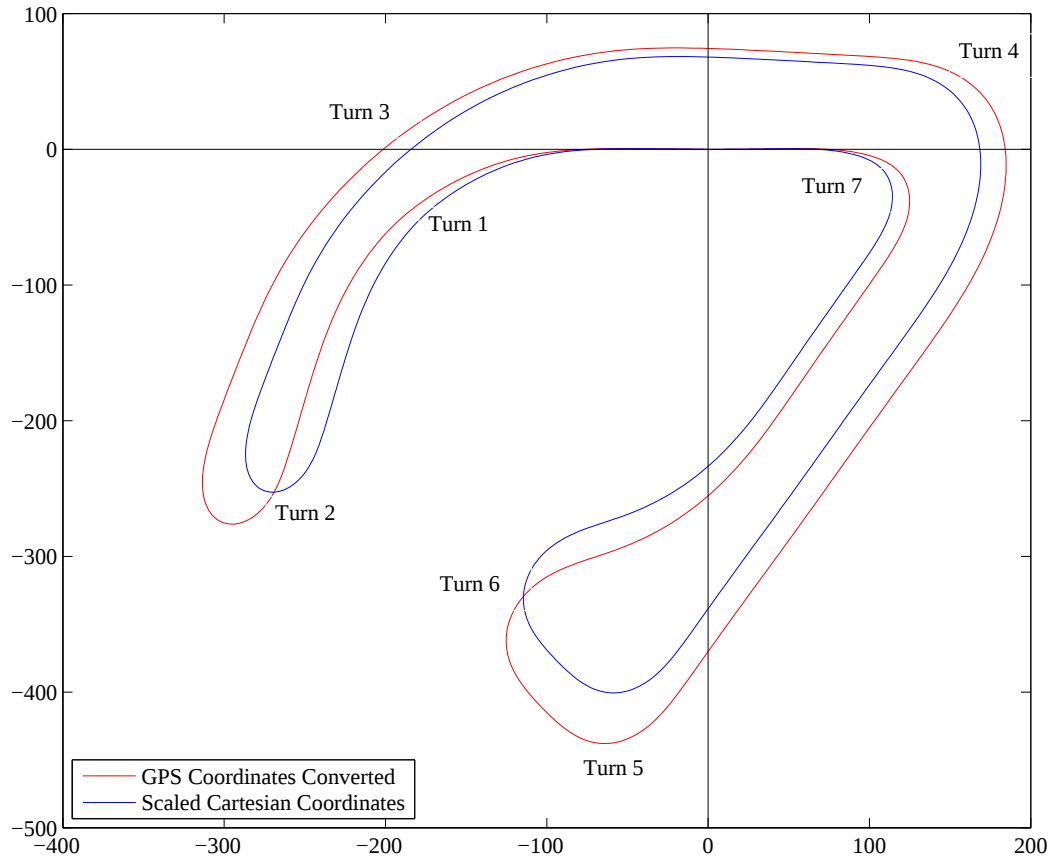


Figure 2.3: VBox: Unscaled versus Scaled GPS Co-ordinates

The vehicle velocity versus track position is shown in Figure 2.4. The vehicle reached a speed of 209.6 km/h on the straight between turns 3 and 4. The slowest vehicle speed is 43.3 km/h through the apex of the hairpin bend in turn 2.

The lateral acceleration versus track position is shown in Figure 2.5. The data is credible, with a peak lateral acceleration of just over 1.5 g recorded. The longitudinal acceleration versus track position is shown in Figure 2.6. The data is again credible, with a peak driving acceleration of 0.5 g and peak braking acceleration just over 1 g. According to the discussion of G-G diagrams in Milliken and Milliken, this is what is to be expected for a winged track car [25].

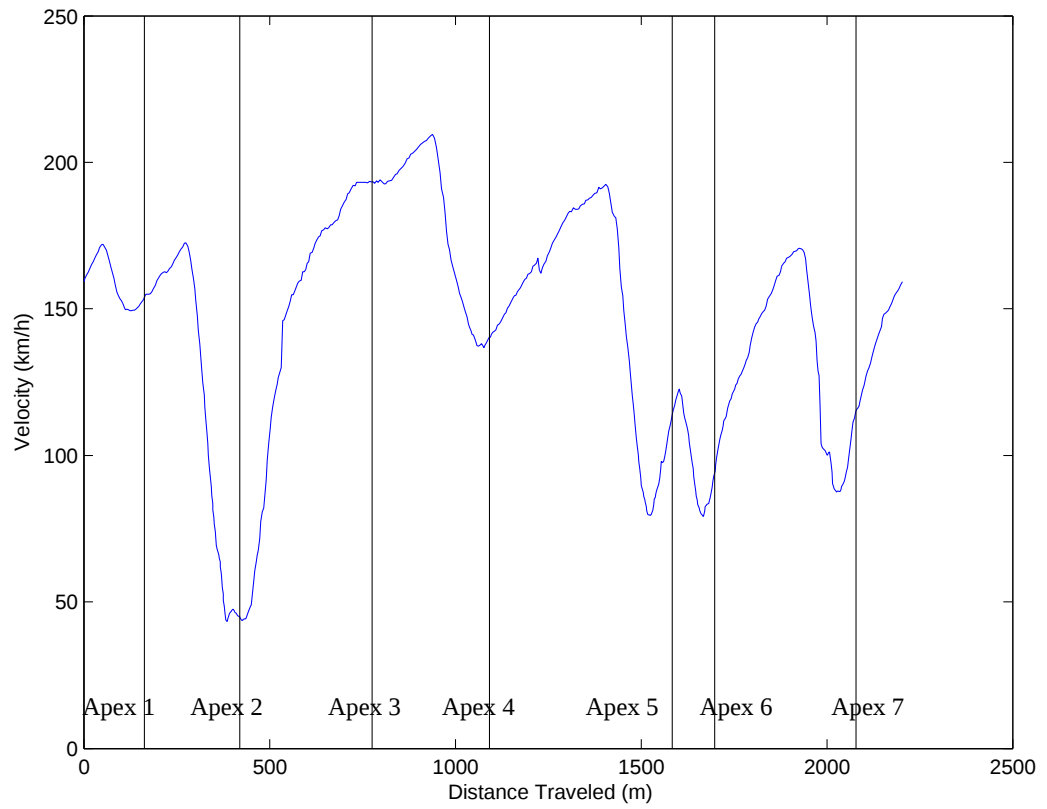


Figure 2.4: Zwartkops Track Test: Velocity vs Distance

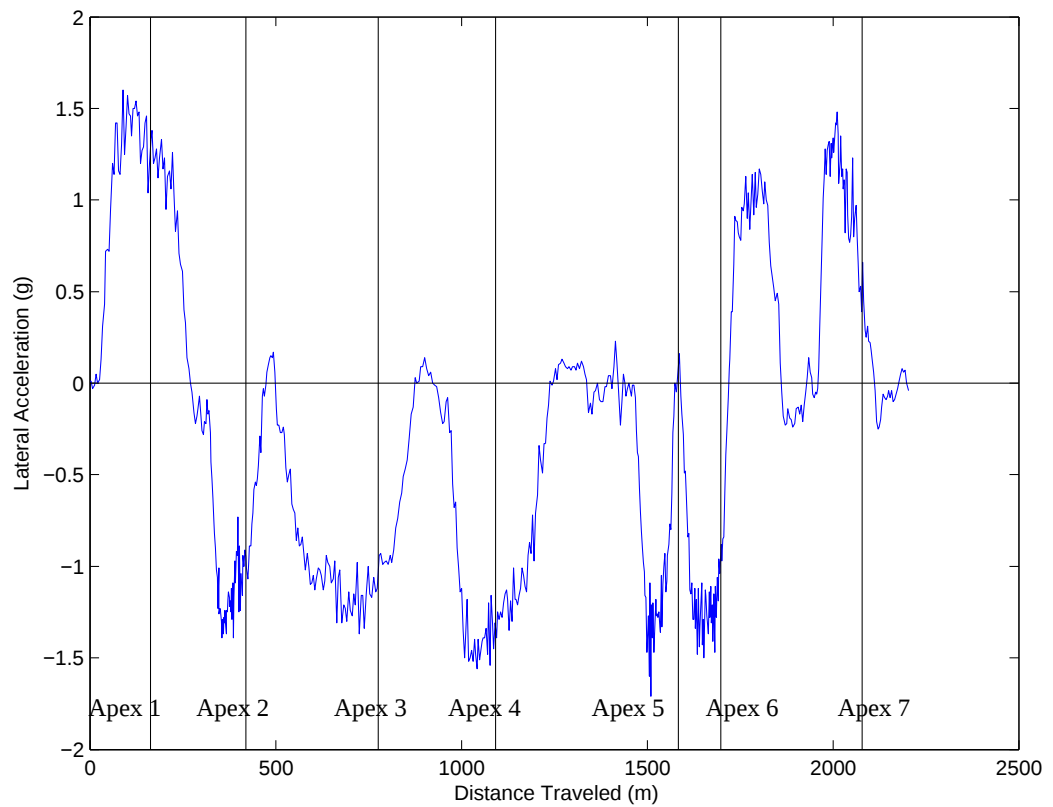


Figure 2.5: Zwartkops Track Test: Lateral Acceleration vs Distance

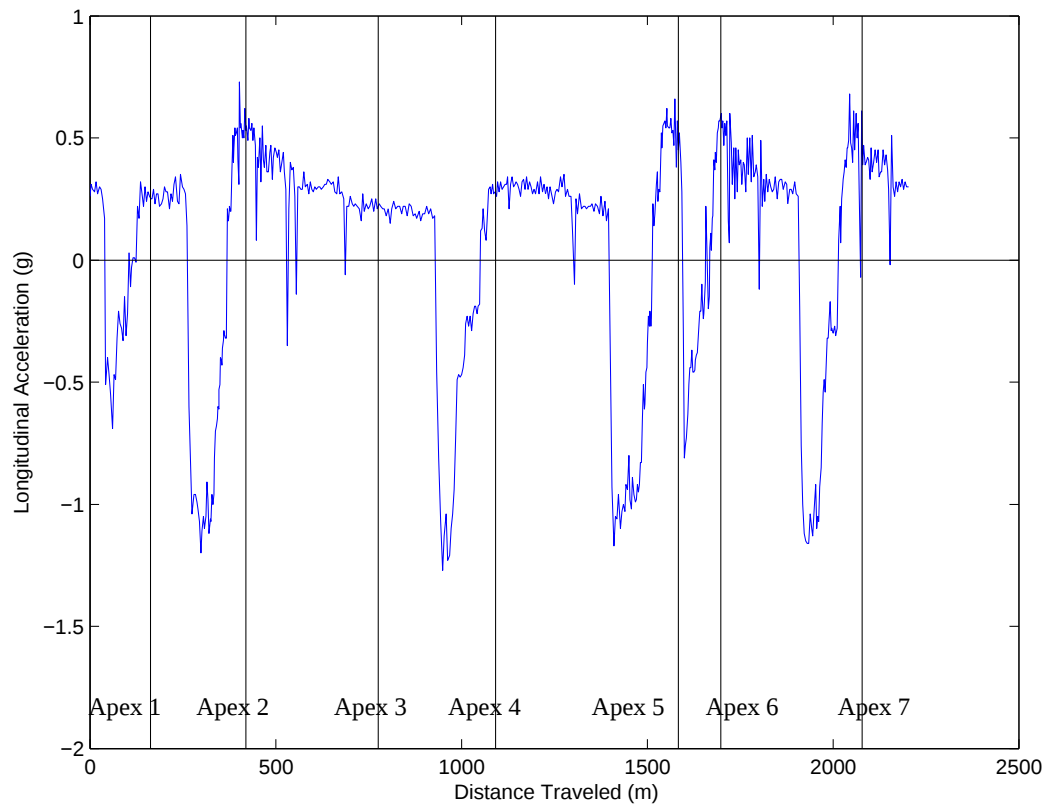


Figure 2.6: Zwartkops Track Test: Longitudinal Acceleration vs Distance

The gear ratios of the Riccardo six speed gear box were not well suited to the demands of the Zwartkops raceway. Figure 2.7 shows the gears versus track position and it can be seen that gears two to four were used for the majority of the lap. Closer gear ratios would have allowed the vehicle to accelerate more quickly, particularly out of the tight corners of Zwartkops Raceway. This mismatch between the vehicle set-up and the track would be challenging in a racing environment. However they are not significant in the context of this study as the focus is on the vehicle suspension arrangement, with all other factors, including aerodynamics, powertrain and gear ratios being held constant during analysis.

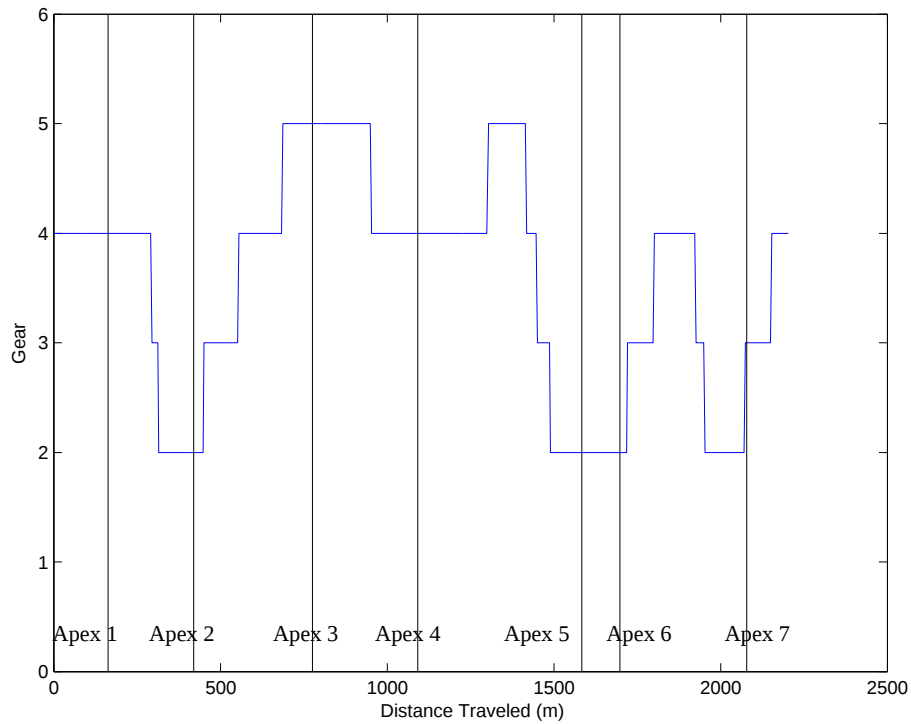


Figure 2.7: Zwartkops Track Test: Gear vs Distance

In addition to the data presented above, engine RPM and steering wheel angle were also recorded and will be used to validate the simulation model. The test data presented above are similar to what would be expected for a winged track car.

2.5 Summary

The performance of the BC LMP2 on the Zwartkops Raceway was recorded using two the integrated Motec system and a Racelogic VBOX data acquisition system. The GPS data recorded during the testing were used to create the ADAMS/Car model of the track. The vehicle velocity, lateral acceleration, longitudinal acceleration, gear, engine rpm and steering wheel angle were recorded and will later be used to validate the simulation model. Only the velocity, lateral acceleration and longitudinal acceleration data and gear were presented in this section. The test data look credible and are in line with what is expected for a winged track car.

Chapter 3 TYRE TESTING

3.1 Introduction

Tyres are the only connection between a vehicle and the road surface, with the wheels connected to the vehicle body through the suspension. To ensure that accurate forces are transmitted to the suspension, an accurate tyre model is required. Tyre characterisation testing was conducted on the rear tyres of the BC LMP2: Hankook Ventus F200 300/680R18 medium slick. Due to the cost of the tyres and the testing, only one set of rear tyres could be tested.

3.2 Data Acquisition and Test Equipment

The testing was carried out on the Long Straight at Gerotek Vehicle testing facility in Pretoria West. The ARMSCOR/CSIR Landward Sciences medium tyre tester, shown in Figure 3.1, was used for the tyre testing. This is a trailer type tyre tester with an automated slip adjustment system. The slip angles of each tyre were adjusted equally, but opposite amounts, to balance the lateral forces generated by the tyres.

The adjustable hub assembly is connected to a sub-frame, which in turn is connected to the main body via six load cells: five ULP 5 tonne load cells and one custom made 5 tonne load cell. The hub assembly allows camber angle to be adjusted for a run, while two linear actuators dynamically control the slip angle for each run.

The tyre data collected was processed using Matlab[®] to generate a Pacejka 89 tyre model. The tyre model was then imported into ADAMS/Car for the tyre model on the BC LMP2.

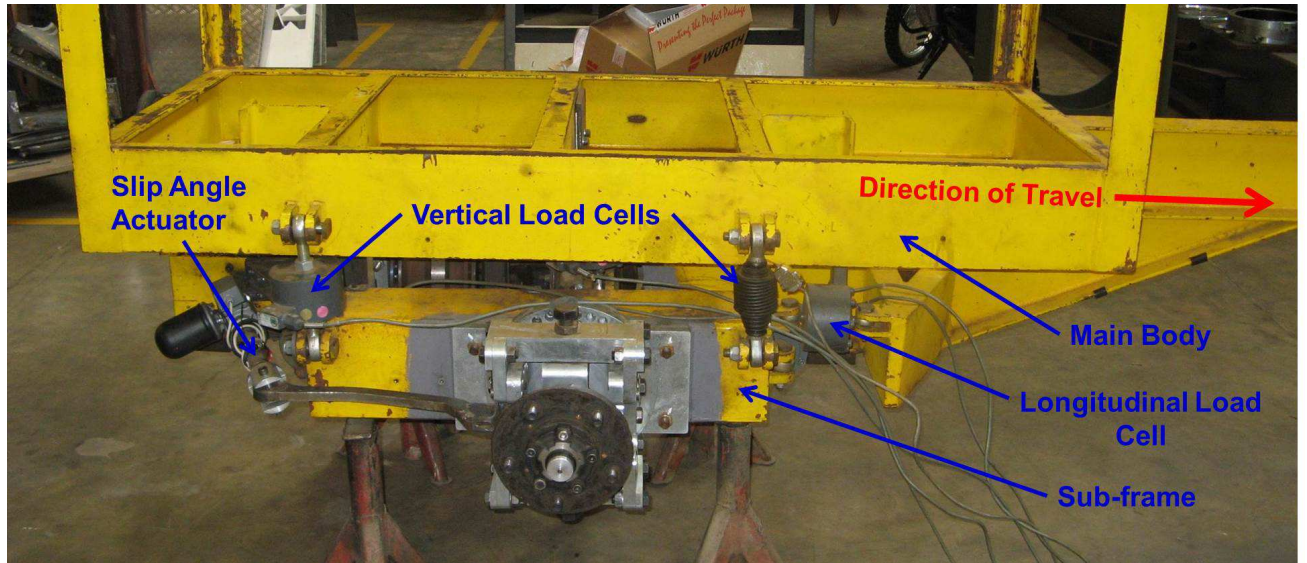


Figure 3.1: The ARMSCOR/CSIR Landward Sciences Medium Tyre Tester

The calibration of the load cells is detailed fully in Appendix C, with the calibration curves given in Table 3.1, where V is given in volts and F in newtons. The layout of the load cells is shown schematically in Figure 3.2.

Table 3.1: Load Cell Specification

Position	Serial No:	Calibration
Longitudinal	54336	$F_x = 2\,451\,664V - 90.66$
Lateral Front	41441	$F_{y1} = 2\,451\,664V - 90.66$
Lateral Rear	41436	$F_{y2} = 2\,451\,664V - 90.66$
Vertical Front	MTT Vert 1	$F_{z1} = -2\,427\,559V + 2759.10$
Vertical Mid	54340	$F_{z2} = 2\,451\,664V - 90.66$
Vertical Rear	54349	$F_{z3} = 2\,451\,664V - 90.66$

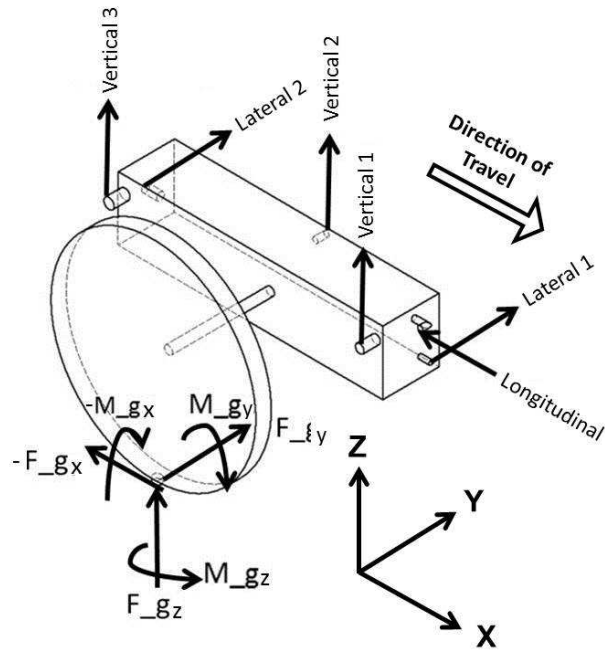


Figure 3.2: Load Cell Arrangement on the ARMSCOR/CSIR Landward Sciences Medium Tyre Tester

The BC LMP2 wheels are attached to the vehicle's racing hub via a single centre wheel nut. In order to attach the BC wheels to the tyre tester, a steel adapter was designed and manufactured to interface between the tyre tester hub and the BC wheel. This is discussed in further detail in Appendix C.

The slip angles of the wheels relative to the tyre tester (and thus the direction of travel) were controlled by a pair of Bircraft ALI - 3F linear actuators. The slip angle was measured using a pair of 250 mm ASM string potentiometers. The slip angle was calibrated from the string potentiometers output voltage and is given as: $\alpha = 11.11V - 10.54$ for the right hand side and $\alpha = 31.69V - 14.72$ for the left hand side of the tyre tester. The slip angle calibration process is described in Appendix C.

A SOMAT eDAQ Lite was used as a data acquisition system. The eDAQ was configured with four bridge boards and a total of 16 channels. The calibration curves were loaded to each channel on the eDAQ such that the outputs were force in newtons for the load cells and angle for the string potentiometers in degrees. The eDAQ was set to record at a sampling rate of 50 Hz.

The slip angle actuation was limited by the stroke of the Bircraft actuators as well as the geometry of the tyre tester hubs, resulting in a range from approximately 2° toe out (defined as negative) to 7° toe in (defined as positive). The testing was conducted for three different vertical loads at three camber angles with the exception of the maximum load at the highest camber angle. This resulted in eight tests sweeping through the range of slip angles as shown in Table 3.2.

Table 3.2: Tyre Testing Load Cases

	Vertical Load (N)	Camber (deg)
Run 1	5 101	0
Run 2	5 101	-2
Run 3	5 101	-4
Run 4	6 082	-4
Run 5	6 082	-2
Run 6	6 082	0
Run 7	7 063	0
Run 8	7 063	-2

The Bircraft slip angle actuators were controlled by a custom built motor controller that required that the upper and lower stroke limits be set. The controller would then divide the total range into 10 increments, pausing at each increment for 5 seconds during the test. The slip angle could thus not directly be set, but rather was swept dynamically from the minimum to the maximum angle.

For each test run, metal plates were loaded on to the tyre tester to increase the vertical load on the tyres and the camber angle was set on the adjustable hubs. The tyre tester was towed at a test speed of approximately 20 kph by a SAMIL 100. The Bircraft actuators were allowed to complete a full sweep of the slip angles before the run was completed. The set-up of the tyre tester was then adjusted by changing either the vertical load or the camber angle and the next test conducted. This was done for all eight test cases.

3.3 Tyre Testing Data

The data recorded on the eDAQ was processed in Matlab[®]. A Butterworth filter at 2 Hz was applied to the force and slip angle data. The raw and filtered data for Run 1 are shown in Figures 3.3 and 3.4. The data for the remaining tests are given in full in Appendix C.

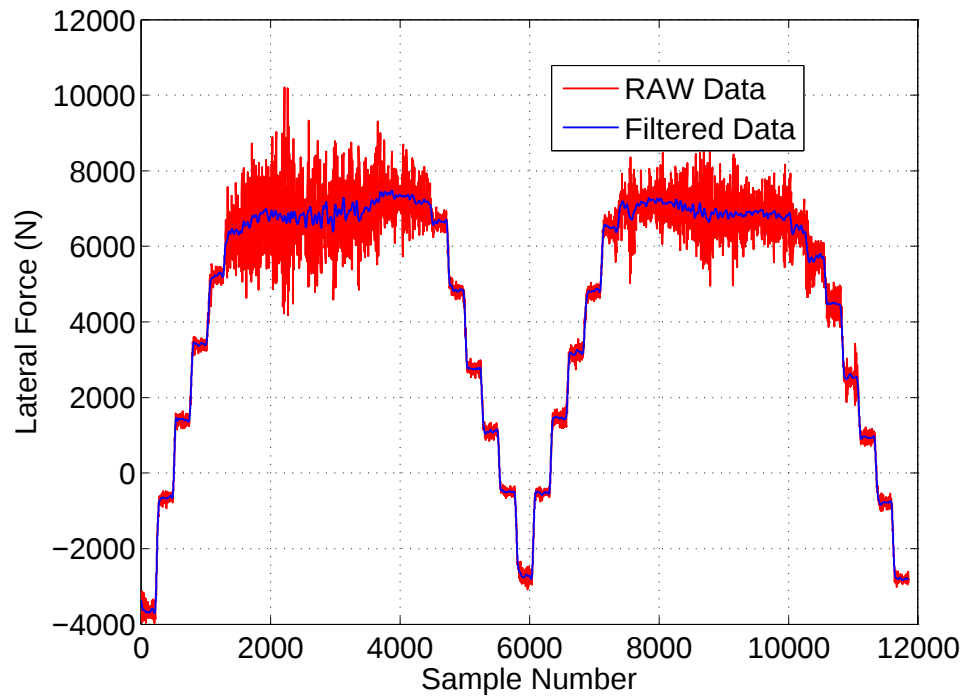


Figure 3.3: Recorded Lateral Force Data for Run 1

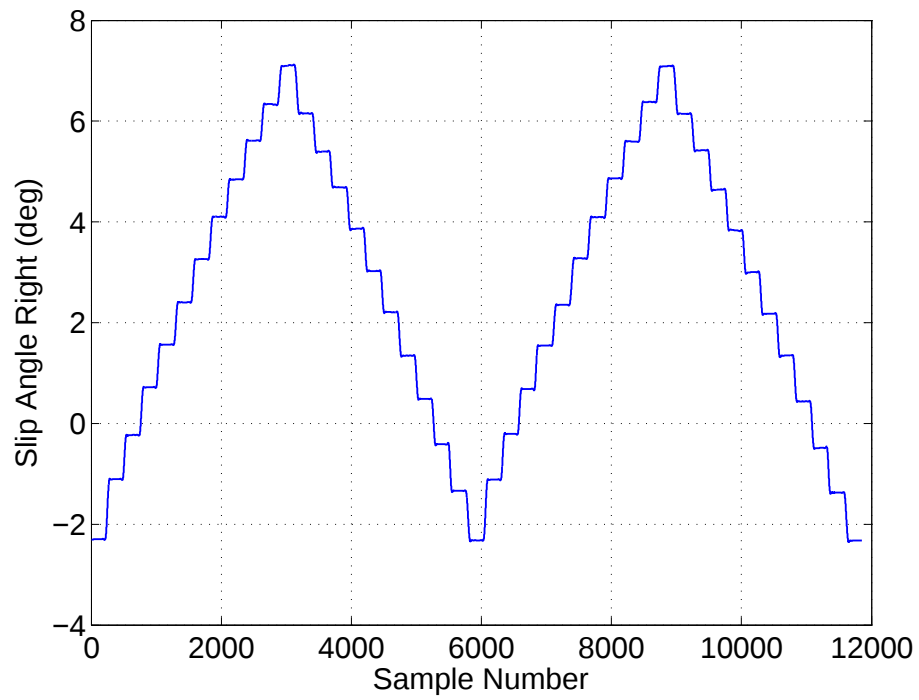


Figure 3.4: Recorded Lateral Force Data for Run 1

From this data, the average lateral force value for each slip angle was extracted giving the lateral force versus slip angle curves. The curves of each load for constant camber angle are given in Figures 3.5 to 3.7.

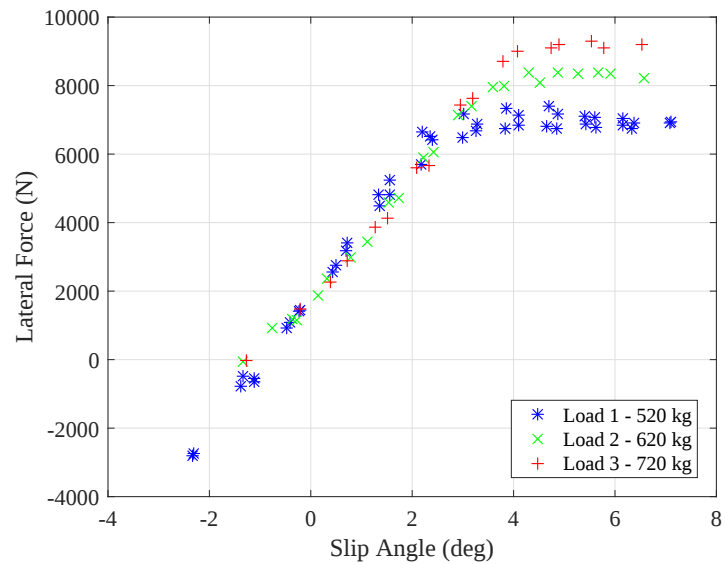


Figure 3.5: Lateral Force vs. Slip Angle for 0° Camber Angle

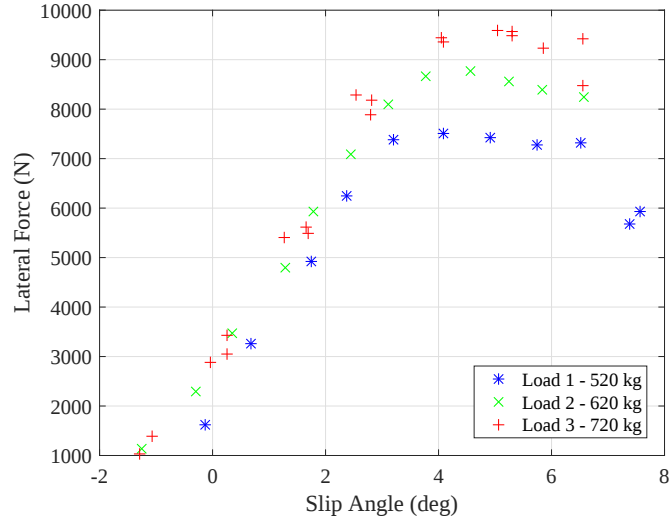


Figure 3.6: Lateral Force vs. Slip Angle for -2° Camber Angle

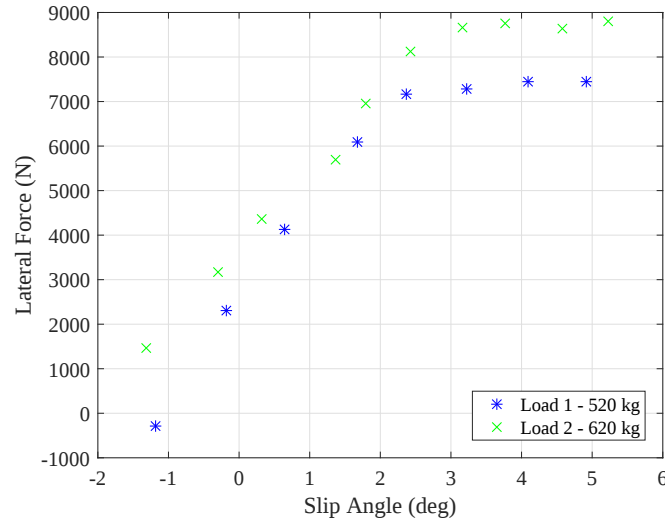


Figure 3.7: Lateral Force vs. Slip Angle for -4° Camber Angle

These curves were then used to regress the Pacejka 89 tyre model [13] that was then added to the ADAMS/Car model of the BC LMP2.

3.4 Regression of the Pacejka 89 Tyre Model

The ARMSCOR/CSIR Landward Sciences Medium Tyre Tester was used to measure lateral force and aligning moment tyre data, but longitudinal force data could not be collected due to the limitations of the tyre tester. According to UMTRI [23], for heavy vehicle tyres, the

longitudinal tyre forces can be approximated from the lateral tyre forces. They state that the peak lateral force of heavy vehicle tyres is reached at 90% of the longitudinal slip, measured in radians. This assumption was used to approximate the longitudinal force data for the BC LMP2 tyre.

Matlab[®] was used to regress the Pacejka 89 tyre model given by equations 1.1 to 1.3, repeated here for convenience:

$$y(x) = D \sin C \arctan[Bx - E(Bx - \arctan(Bx))] \quad (3.1)$$

$$Y(X) = y(x) + S_v \quad (3.2)$$

$$x = X + S_h \quad (3.3)$$

The Pacejka 89 Tyre model lateral force coefficients a_0 to a_{13} are defined by Pacejka [13], with slip angle in degrees, lateral force in newtons and vertical force in kilonewtons with coefficient of friction μ and camber angle γ as follows:

$$D = \mu F_z \quad (3.4)$$

$$\mu = a_1 F_z + a_2 \quad (3.5)$$

The gradient at 0° slip angle is given by BCD:

$$BCD = a_3 \sin[2 \arctan(F_z/a_4)][1 - a_5|\gamma|] \quad (3.6)$$

$$C = a_0 \quad (3.7)$$

$$E = a_6 F_z + a_7 \quad (3.8)$$

$$S_h = a_8 \gamma + a_9 F_z + a_{10} \quad (3.9)$$

$$S_v = a_{11} F_z \gamma + a_{12} F_z + a_{13} \quad (3.10)$$

Similarly, the aligning moment coefficients c_0 to a_{17} are defined as follows:

$$D = c_1 F_z^2 + c_2 F_z \quad (3.11)$$

$$BCD = (c_3 F_z^2 + c_4 F_z) \exp(-c_5 F_z F_z) \quad (3.12)$$

$$E = (c_7 F_z^2 + c_8 F_z + c_9)(1 - c_{10} |\gamma|) \quad (3.13)$$

$$S_h = c_{11} \gamma + c_{12} F_z + c_{13} \quad (3.14)$$

$$S_v = (c_{14} F_z^2 + c_{15} F_z) \gamma + c_{16} F_z + c_{17} \quad (3.15)$$

The Pacejka coefficients a_0 to a_{13} and c_0 to c_{17} in the above equations were required for the ADAMS/Car tyre input file. These coefficients were calculated in two phases. Firstly, the factors B to E, S_h and S_v were calculated for the test data of Figures 3.5 to 3.7 by means of a least squares regression. Secondly, the lateral force and aligning moment coefficients were calculated from equations 3.4 to 3.15 by means of a least squared non-linear optimisation. This was done in Matlab[®] using built-in optimisation functions. The Matlab script is given in the digital Appendix.

A number of obvious outliers were removed before the regression was conducted. The results of the fitted Pacejka 89 tyre model are shown in Figures 3.8 to 3.10.

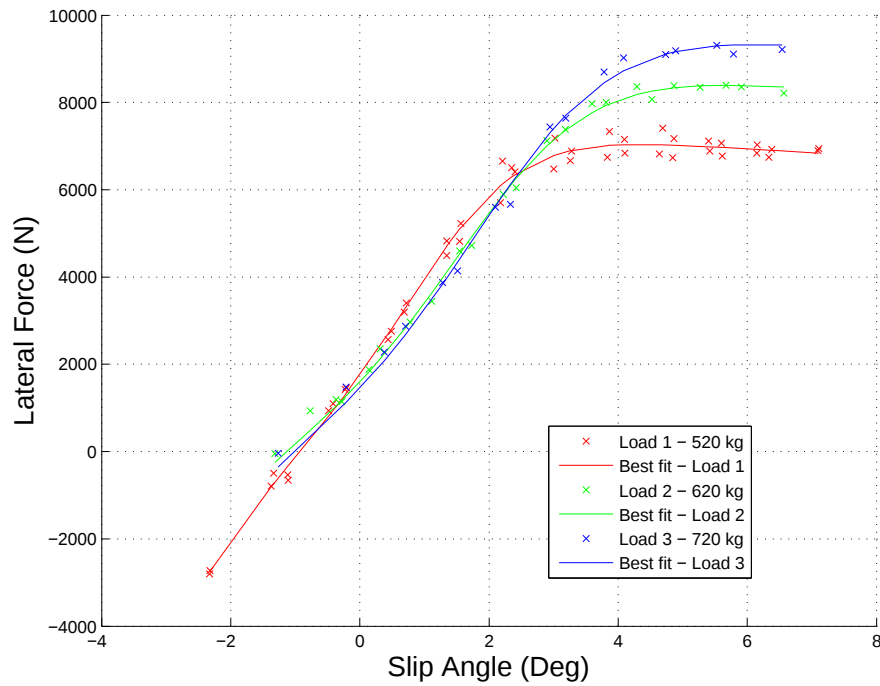


Figure 3.8: Regressed Lateral Force vs. Slip Angle for 0° Camber Angle

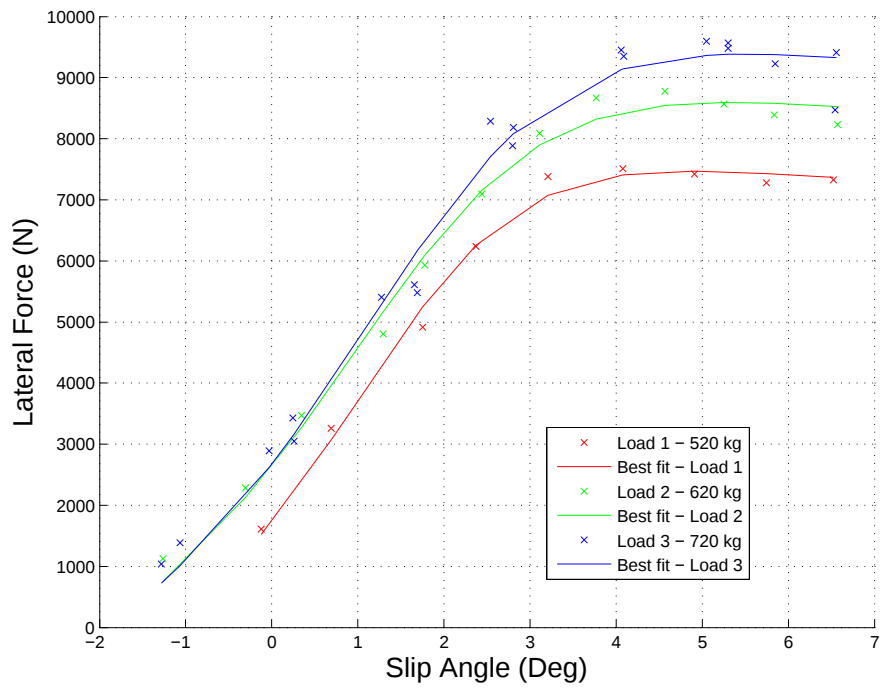


Figure 3.9: Regressed Lateral Force vs. Slip Angle for -2° Camber Angle

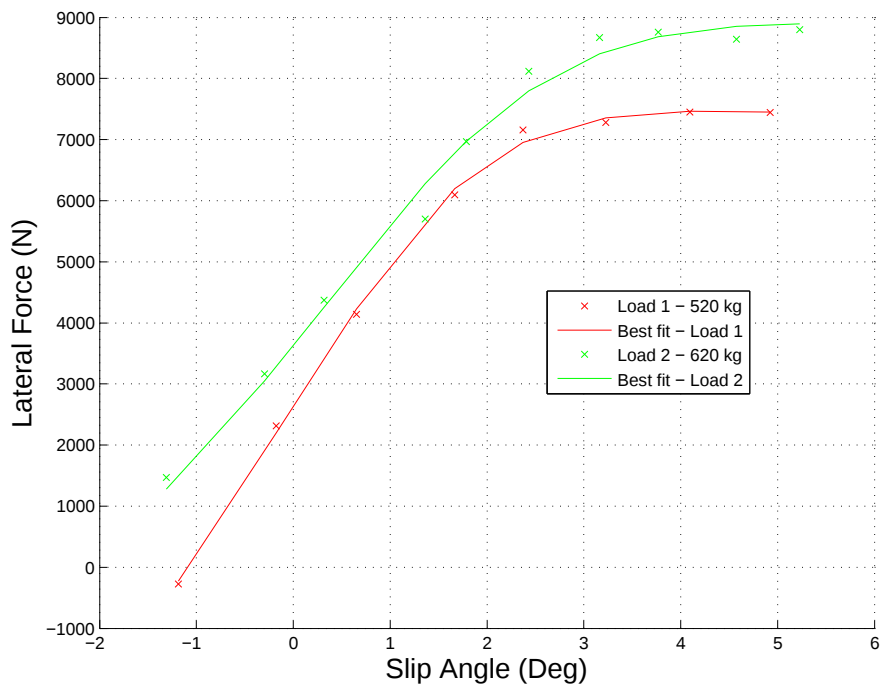


Figure 3.10: Regressed Lateral Force vs. Slip Angle for -4° Camber Angle

The regressed Pacejka 89 coefficients are given in Tables 3.3 and 3.4:

Table 3.3: Pacejka 89 Lateral Force Coefficients

a0	1.30
a1	-58.62
a2	1 751.55
a3	71 703.05
a4	0.05757
a5	-0.05075
a6	-5.13104
a7	19.12253
a8	-0.19725
a9	0.33588
a10	-0.95440
a11	0.00
a12	0.00
a13	0.00

Table 3.4: Pacejka 89 Aligning Moment Coefficients

c0	2.4
c1	-1.06005
c2	-11.7793
c3	0.000792
c4	-7.63492
c5	-0.06885
c6	0.020957
c7	-0.62946
c8	8.543519
c9	-30.6725
c10	-0.13362
c11	0.132583
c12	-0.03415
c13	0.020533
c14	0
c15	0
c16	0
c17	0

These data were fed into the ADAMS/Car model for use in the simulations. However it was found that this tyre model led to severe instability of the vehicle. This was determined to be due to the large offset of the tyre force at 0° lateral slip angle. The range of slip angles that could be tested was limited to approximately 2° toe out, defined as negative slip angle, to 7° toe in, defined as positive slip angle. It was not possible to determine the exact value of the shift due to this limitation, and the peak forces for negative slip angle could thus not be determined. As a result, the tyre forces were assumed to have the same peak magnitude for positive and negative slip angles.

The plot provided by Pacejka [24] is given for a slip angle range of 15° toe out to 15° toe in for a typical road car tyre. The peak forces are clearly evident for both positive and negative slip angles within that range. The plot also clearly shows the vertical and horizontal offset of forces to be almost zero at zero camber angle.

Milliken and Milliken [25] present lateral force versus slip angle data for an early 90's Formula 1 full slick front tyre at zero degree camber angle. The vertical and horizontal offset at zero slip angle can also be seen to be close to zero for this racing tyre. This data is the closest published data to that of the BC LMP2 Hankook racing slick tyres used here and will likely give a good indication of what can be expected of the Hankook tyres.

Further insight into the source of the large offset or measured force at zero slip angle is given by Pacejka [24]. He states camber causes a vertical offset (S_v) in the lateral tyre force for the entire curve. During the tyre testing, the tyres were predominantly aligned toe in due to limit in the range of slip angles. This resulted in the tyres wearing unevenly across the width of the contact patch, and due to the soft compound of the Hankook tyres, a camber was worn into the tyre.

The vertical and horizontal offset of the Hankook test data were removed, which in turn significantly reduced the instability of the ADAMS/Car model. The cornering stiffness and peak values were retained as far as possible. The resulting tyre curves are shown in Figures 3.11 to 3.14.

As mentioned, longitudinal tyre forces could not be measured and the approximation given by UMTRI [23] was used, with the result given in Figure 3.15.

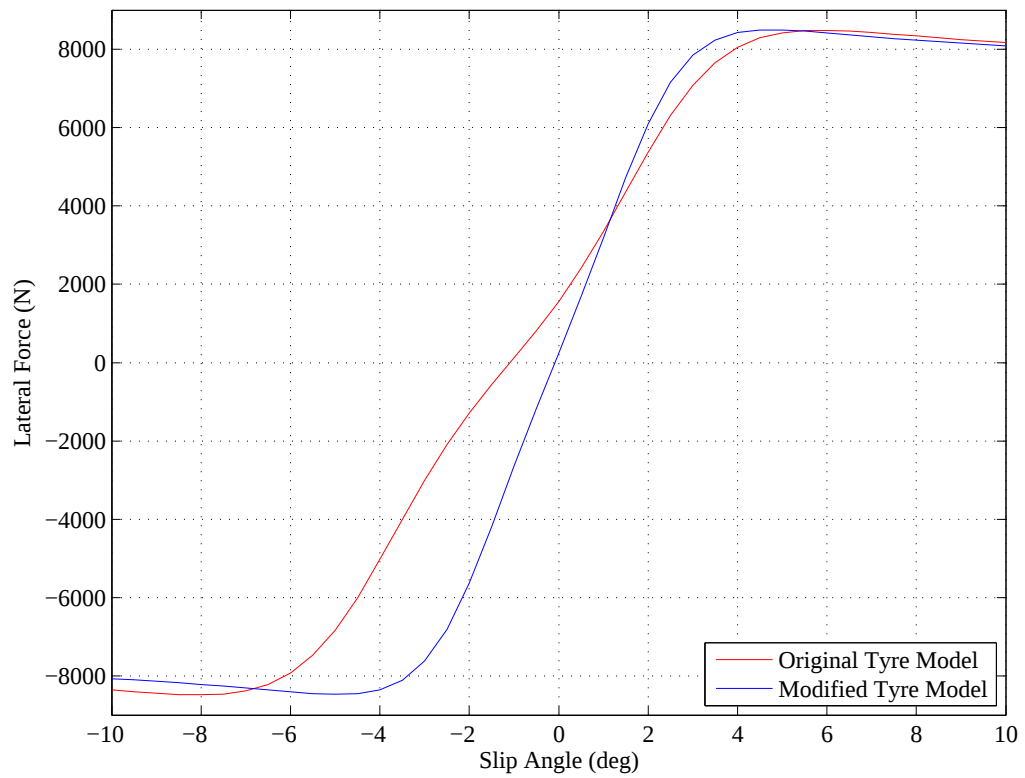


Figure 3.11: Modified Lateral Tyre Force Model, Load 2, 0° Camber

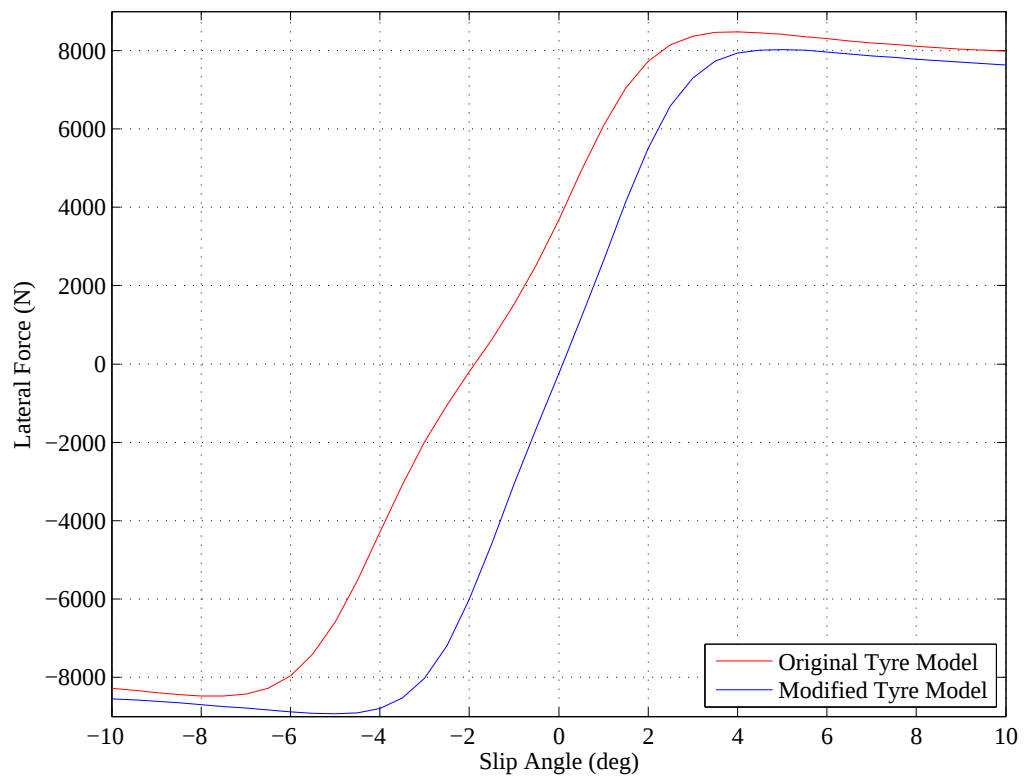


Figure 3.12: Modified Lateral Tyre Force Model, Load 2, -4° Camber

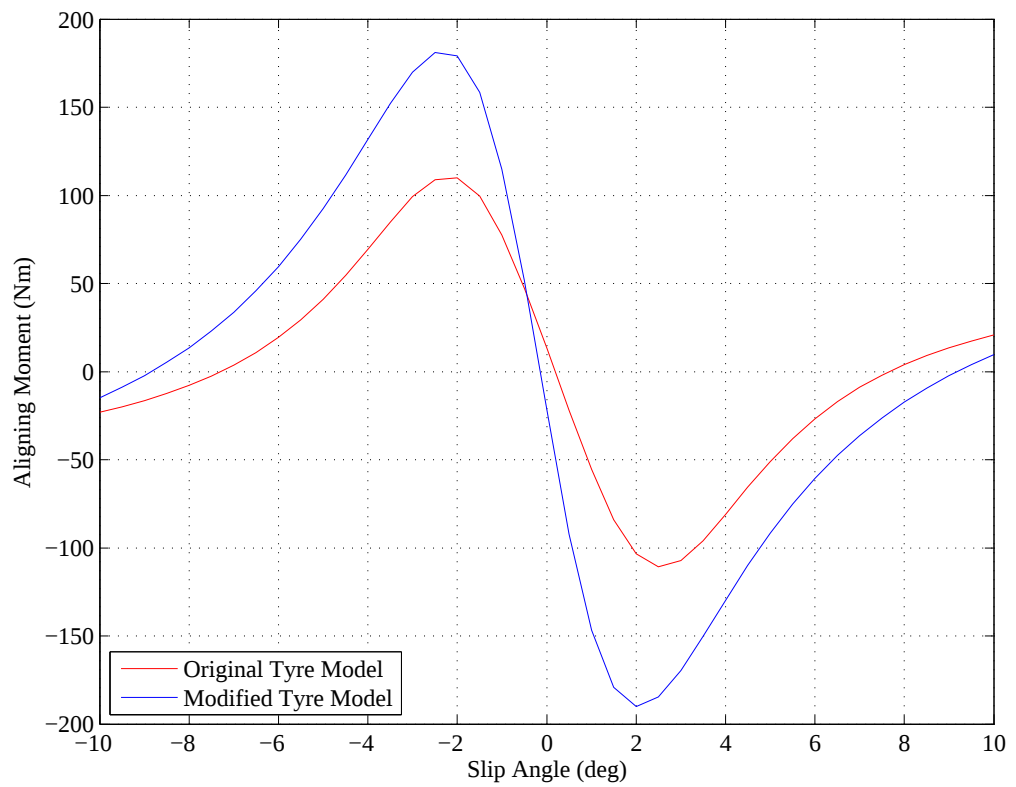


Figure 3.13: Modified Aligning Moment Model, Load 2, 0° Camber

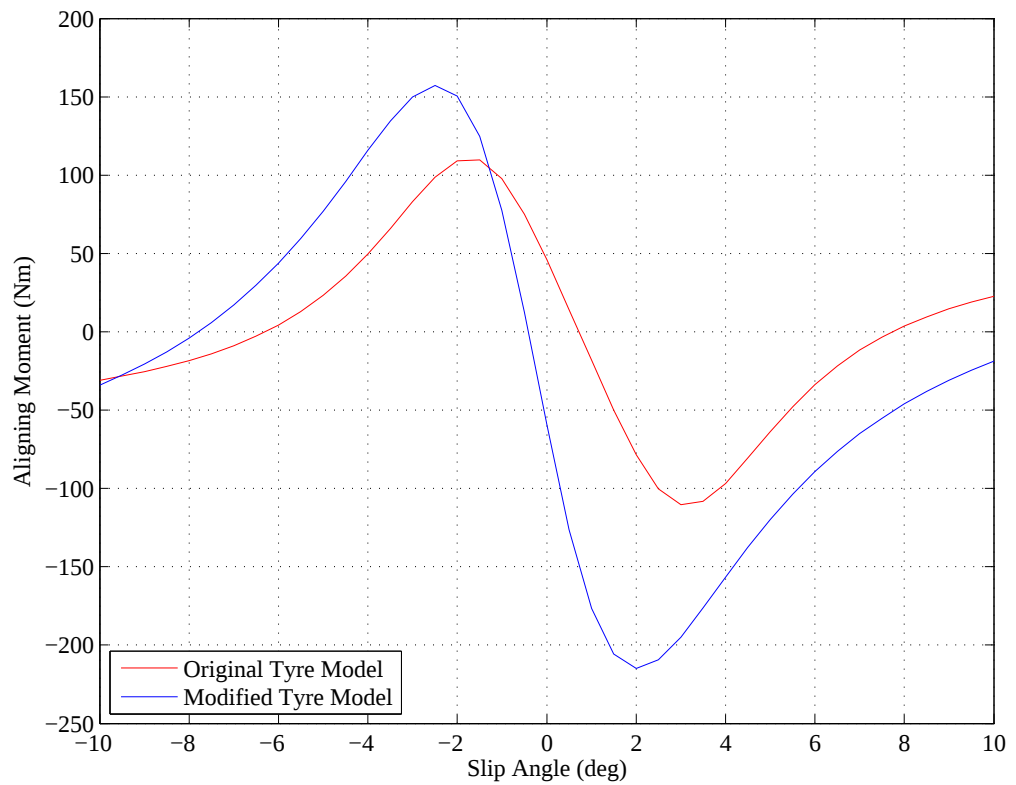


Figure 3.14: Modified Aligning Moment Model, Load 2, -4° Camber

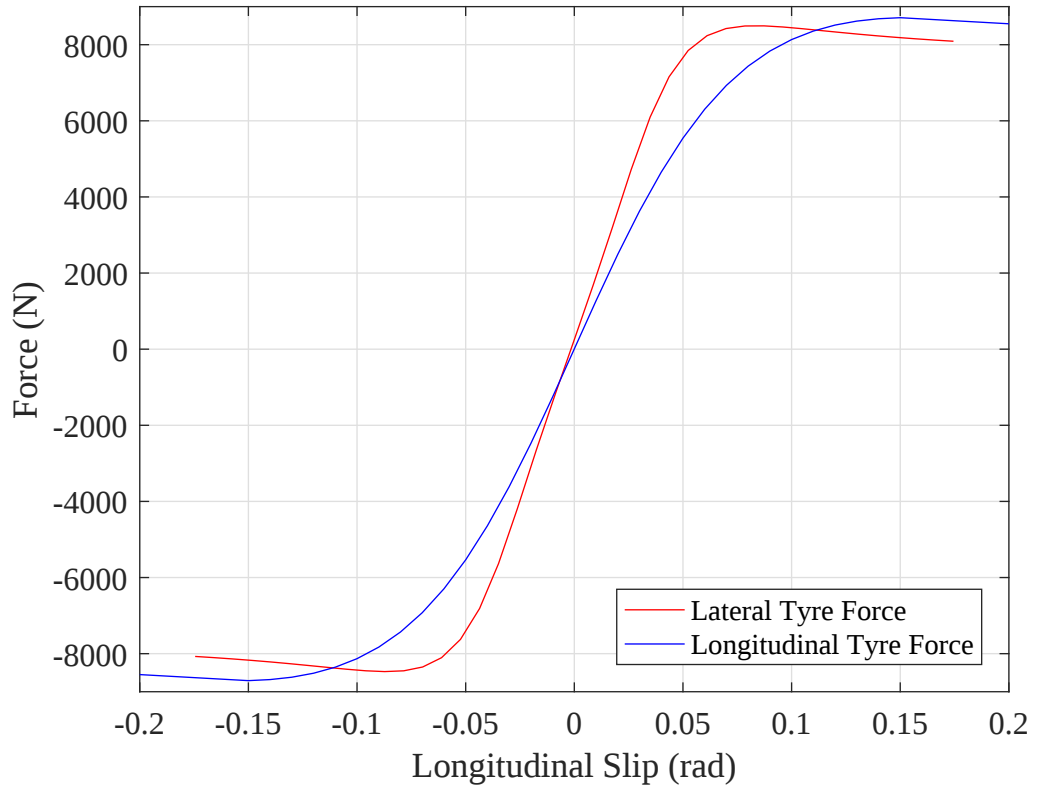


Figure 3.15: Longitudinal Force Curve Generated from Lateral Force Curve [23]

The data from these modified tyre curves were then read into the Matlab[®] Pacejka 89 regression model. The modified Pacejka 89 coefficients are given in Table 3.5.

The rubber compound used in the Hankook racing tyres has a specific operating temperature, of approximately 70° C where maximum traction is achieved. A surface temperature above or below this optimal operating temperature would result in suboptimal performance. During the testing, a surface temperature outside of the optimal operating range, as was most likely the case, would thus result in an underestimation of the tyre forces and cornering stiffness.

The Hankook tyres were designed preliminary for racing, where the tyres are subjected to one or two heat cycles before being changed. However, during testing the tyres were put through numerous heat cycles as they were heated during each run, for approximately 90 seconds, and would then cool as the test conditions were changed. This may have lead to a further underestimation of the tyre forces and cornering stiffness.

Table 3.5: Modified Pacejka 89 Coefficients

Lateral Force		Longitudinal Force		Aligning Moment	
a0	1.3	b0	1.65	c0	2.34
a1	-58.62	b1	-11.4077	c1	1.495
a2	1751.55	b2	1786.76	c2	6.416654
a3	3634.356	b3	-12.7882	c3	-3.57403
a4	12.76982	b4	232.174	c4	-0.08774
a5	0.004958	b5	-0.07811	c5	0.09841
a6	0.05	b6	0.00511	c6	0.00277
a7	-4.25	b7	-0.07735	c7	-0.00012
a8	0.001758	b8	0.10868	c8	0.1
a9	0.012895	b9	0.03058	c9	-1.33329
a10	0.005853	b10	-0.176	c10	0.025501
a11	19.1647			c11	-0.02357
a12	1.222729			c12	0.03027
a13	6.230245			c13	-0.0647
				c14	0.021133
				c15	0.89469
				c16	-0.09944
				c17	-3.33694

The front tyres fitted to the BC LMP2 during track testing were 285 mm wide compared to 300 mm at the rear. There is likely to be a slight difference in actual performance. However given the assumptions of eliminating the vertical offset of the lateral force as well as the approximation of longitudinal force. The same Pacejka 89 coefficients were used for both the front and rear tyres.

3.5 Summary

The ARMSCOR/CSIR Landward Sciences Medium Tyre Tester was used to collect lateral force and aligning moment curves for the BC LMP2 rear tyres with data processing conducted in Matlab®. The raw data was filtered with a Butterworth filter at 2 Hz to get the average value for the force and slip angle data. The Pacejka 89 coefficients were calculated by regression of the lateral force and aligning moment test data. Due to limited range of slip angles tested, -2° to 7° , the lateral and vertical offsets of the lateral force data could not accurately be determined, and were assumed to be zero. Pacejka 89 coefficients for the modified data were calculated once again using a regression model, with the longitudinal force being estimated from the lateral force data. The Pacejka 89 coefficients were then used to create the front and rear tyre models in ADAMS/Car.

Chapter 4 SOFTWARE SIMULATION

4.1 Introduction

The ADAMS/Car model of the BC LMP2 was created using data from the physical testing as well as the vehicle design drawings. Physical dimensions of the vehicle were measured directly from the car, or taken from CAD drawings of the vehicle and suspension as necessary.

The spring and ARB stiffness values and damper characteristics were taken from the manufacturers' specifications while the engine torque and power curves were measured by dynamometer test [26]. All of these parameters or curves are given in Appendix A along with the geometry and hardpoints of the suspension, steering and body. The tyre testing discussed above provided the inputs for the track and tyre subsystems.

4.2 Development of the ADAMS/Car Model

The ADAMS/Car model of the BC LMP2 was created by modifying the default vehicle in ADAMS/Car, a Ferrari Testarossa. The Testarossa model assembly comprises subsystems for the front and rear suspension, steering, chassis, brakes, engine and powertrain. Both front and rear suspension subsystems are double wishbone type suspension with vertical outboard springs connecting via a strut to the vehicle chassis. The BC LMP2 also employs double wishbone suspension, however it has inboard springs and dampers that are actuated via pushrods and rockers. The Testarossa suspension template was thus modified to include the pushrods, rockers and inboard springs and dampers as well their corresponding joints, communicators and mass properties.

Two versions of the BC LMP2 suspension were created, the first included only the pushrods and inboard springs and dampers as shown in Figure 4.1 below. The second also included the vertical ARB and third spring and damper as shown in Figure 4.2 below.

The pushrod suspension templates were created in the ADAMS/Car Template builder by modifying and creating hard points, joints and communicators. The steering, body, engine and brake subsystems of the Testarossa were also modified at a subsystem level. Only the physical and geometrical parameters were updated. The steering system was modified to reverse the direction of the output pinion as the track rod ends attaching to the hub in front of the wheel centre, as opposed to behind it on the Testarossa model.

Stevenson [27] presented an investigation into the aerodynamic properties of the BC LMP2. The combined aerodynamic effects of the wings and diffusers cannot be neglected due to the magnitude of forces developed, as well as the sensitivity of their magnitude to vehicle pitch. The aerodynamic drag and downforce functions, although complex, were reduced to single functions and incorporated into the ADAMS/Car body subsystem. Stevenson shows that the Coefficient of drag (C_D), Coefficient of lift (C_L) as well as the frontal area of the BC LMP2 are dependant on pitch angle. The aerodynamic properties with the pitch sensitivities are given in Appendix A, while the complete ADAMS/Car model of the BC LMP2, containing both the two and three spring and damper suspensions, is included in the Digital Appendix.

The ADAMS/Car simulation for this investigation consisted of three sub-sections: validation, parameter optimisation and comparison of results. The two spring and damper suspension subsystems were used in the ADAMS/Car model for the validation sub-section. These will be discussed in further detail below.

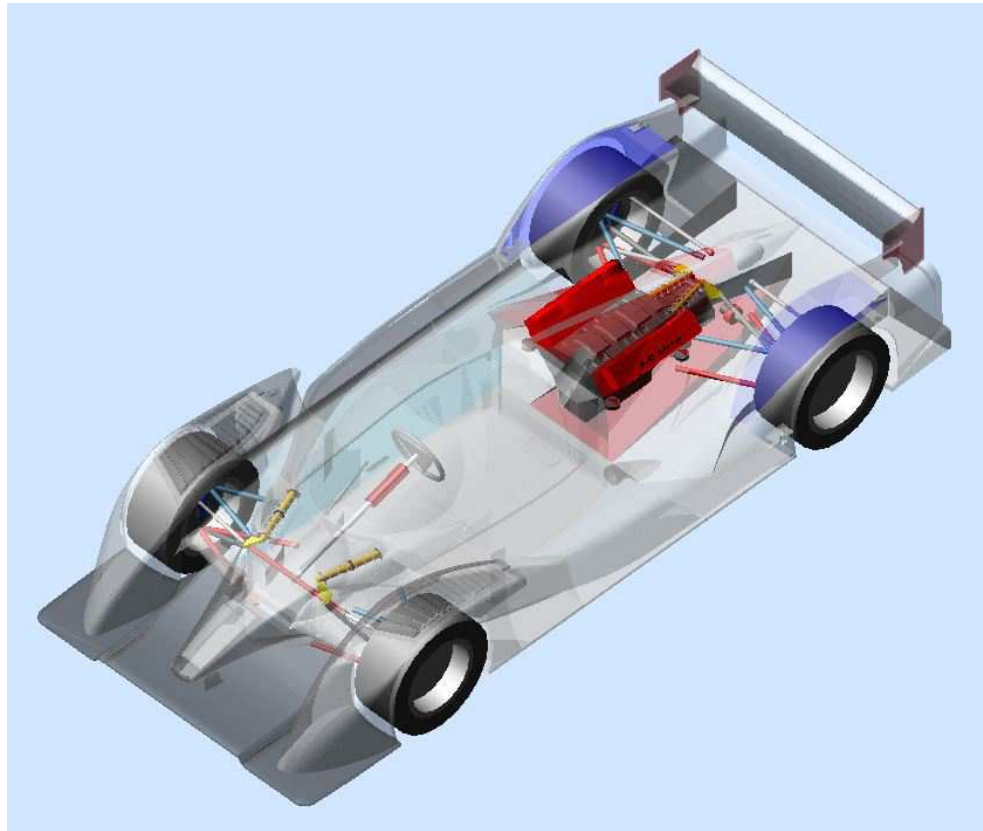


Figure 4.1: ADAMS/Car Model of the Two Spring and Damper Suspension

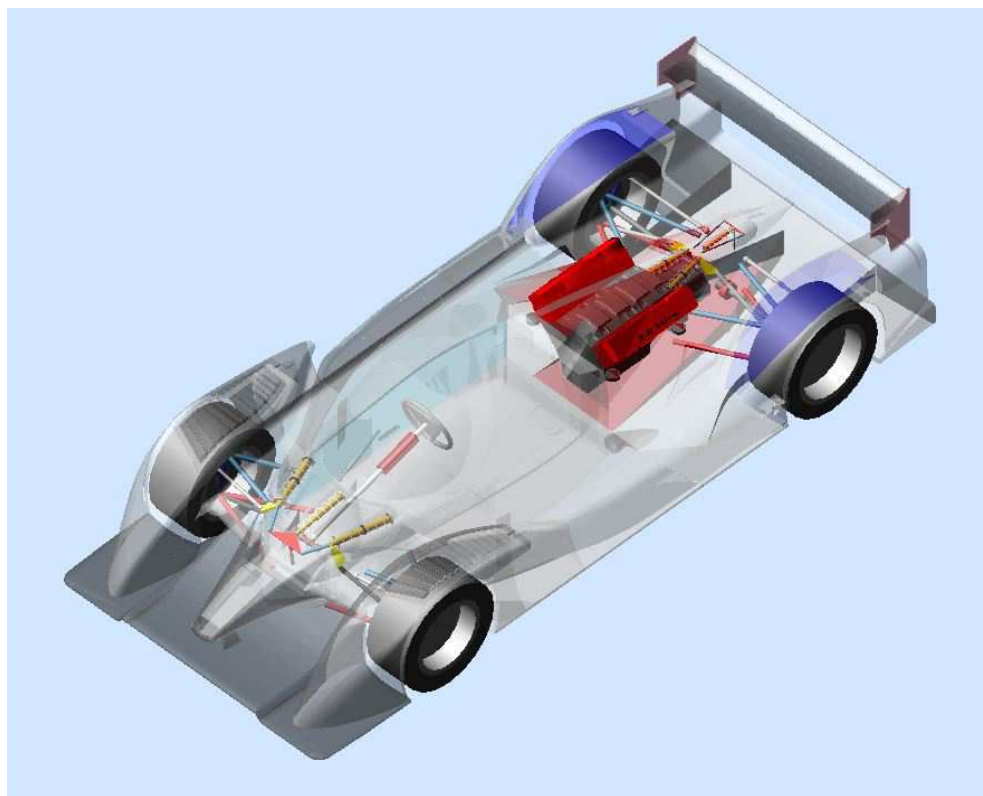


Figure 4.2: ADAMS/Car Model of the Three Spring and Damper Suspension

4.3 Development of the Zwartkops Raceway Model for ADAMS/Car

The model of Zwartkops Raceway was created using the ADAMS/Car road builder using GPS data recorded during the track testing. The vehicle path from the fastest lap was used as the centre line of the model as well as the path for the ADAMS/Car SmartDriver. Conversion of the GPS coordinates to Cartesian coordinates as discussed in Section 2.4, is outlined fully in Appendix B. The required inputs for the road builder are x, y and z coordinates of the complete path, the track width and road roughness.

Ascertaining the road roughness of Zwartkops Raceway is beyond the scope of this investigation, however Sayers and Karamihas [28] present the International Roughness Index (IRI) values for various road classes, shown in Figure 4.3. In discussion with racing driver Hennie Groenewald, it was determined that Zwartkops Raceway is a fairly smooth track [29] and an IRI value in the range of 2.0 - 2.5 m/km was chosen.

ProVal 3.4 is a software package that generates road roughness profiles for a given IRI value, as well as calculating the IRI value for a specific road profile [30]. A roughness profile with a known IRI value was first analysed in ProVal 3.4 to validate the software. Following this, a roughness profile with an IRI of 2.15 m/km was generated using the software.

The change in elevation of Zwartkops Raceway, although small, were found to negatively affect the ADAMS/Car SmartDriver, leading to instability and ultimately simulation failure. This was due to the inability of the SmartDriver to account for the elevation changes in the braking distance required for corner entry. The elevation was therefore removed from the track model. It was also found that the road roughness lead to instabilities in the simulations, and the SmartDriver was not able to match the performance of the test driver, Hennie Groenewald, during the track testing. Road roughness was thus omitted from the validation of the ADAMS/Car model, but was included for the detailed simulation that followed.

For the purposes of this study, the assumptions made in constructing the ADAMS/Car model of Zwartkops Raceway are considered to be acceptable, as it the relative performance of the vehicle with the two different suspension systems that is of greater interest than overall performance. These assumptions will however have an influence in the validation of the ADAMS/Car model.

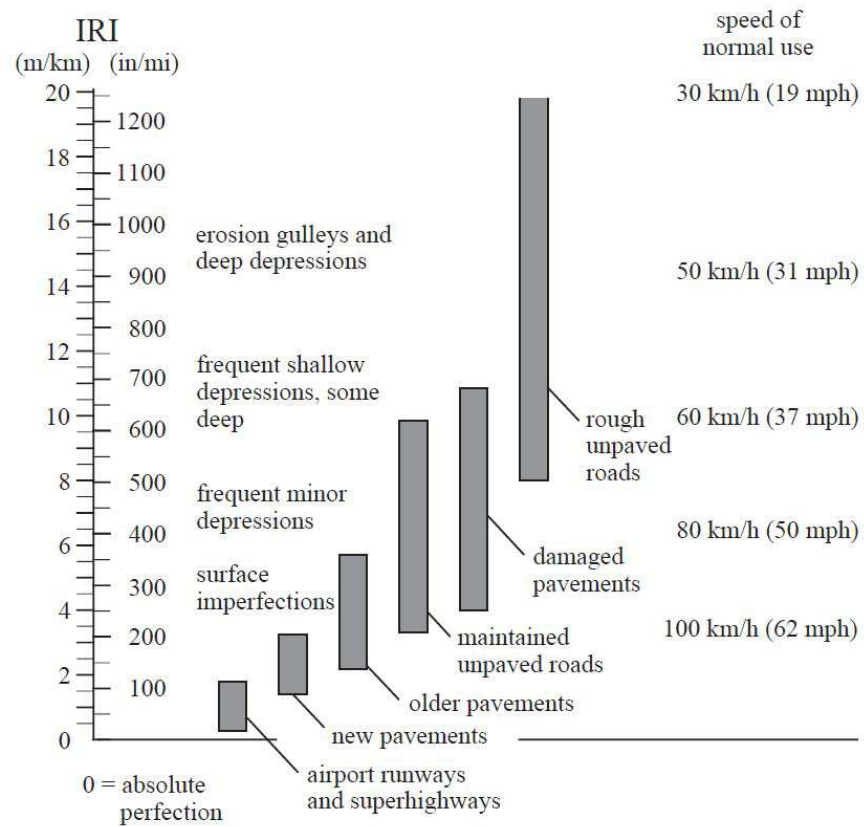


Figure 4.3: IRI Values for Various Road Classes [28]

4.4 Validation of the ADAMS/Car Model

The BC LMP2 race car was simulated in ADAMS/Car using the SmartDriver model, and the parameters calibrated using the track test data. The parameters that were adjusted to obtain a good correlation were: driving and braking acceleration, left and right cornering acceleration, brake balance and brake torque. The specifics of the braking system are beyond the scope of this investigation and as such, the parameters were tuned using the track test data as was done by Dominy [16]. The tuned SmartDriver parameters for the BC LMP2 model on the Zwartkops Raceway model, the SmartDriver conditions and the ADAMS/Car solver settings are given in Tables 4.1 to 4.3.

Table 4.1: Tuned SmartDriver Parameters for Validation

Driving	90%
Braking	85%
Cornering Left	97%
Cornering Right	97%
Front Brake Balance	0.7
Max Front Brake Torque	850 (Nm)
Max Rear Brake Torque	500 (Nm)

Table 4.2: SmartDriver Conditions

End Time	62 (s)
Time Steps	62000
Initial Velocity	44.44 (m/s)
Gear	4
Gear Shift Time	0.15 s

Table 4.3: ADAMS/Car Solver Settings

Dynamics		Equilibrium	
Integrator	GSTIFF	Error	1.00E-04
Formulation	SI2	Tlimit	10 (s)
Error	0.001	Alimit	5°
Corrector	Modified	Maxit	400
Integrator Order	6	Stability	1.00E-05
Maxit	10	Imbalance	1.00E-04

For the 62 second lap, the ADAMS/Car simulation is calculated to be 8.5 m short of the actual distance of 2 203 m measured by the Motec system, a 0.39% difference. The ADAMS/Car SmartDriver was not able to drive the vehicle as close to limits as the test driver during the track testing. The discrepancies will now be investigated and discussed.

Figure 4.4 shows a comparison of vehicle velocity versus distance. It can be seen that the ADAMS/Car SmartDriver was not able to achieve the recorded track test corner entry or apex velocity in turns 1, 3, 4, 5 and 7, while exceeding the recorded velocity on the straight between turns 4 and 5. The apex velocity for turns 2 and 6 are close.

The assumption of a zero elevation changes is most likely the source of the overestimation of velocity between turns 4 and 5 as in reality this is an incline. The overall variation of velocity between the track data and the model is similar, showing good correlation between the two.

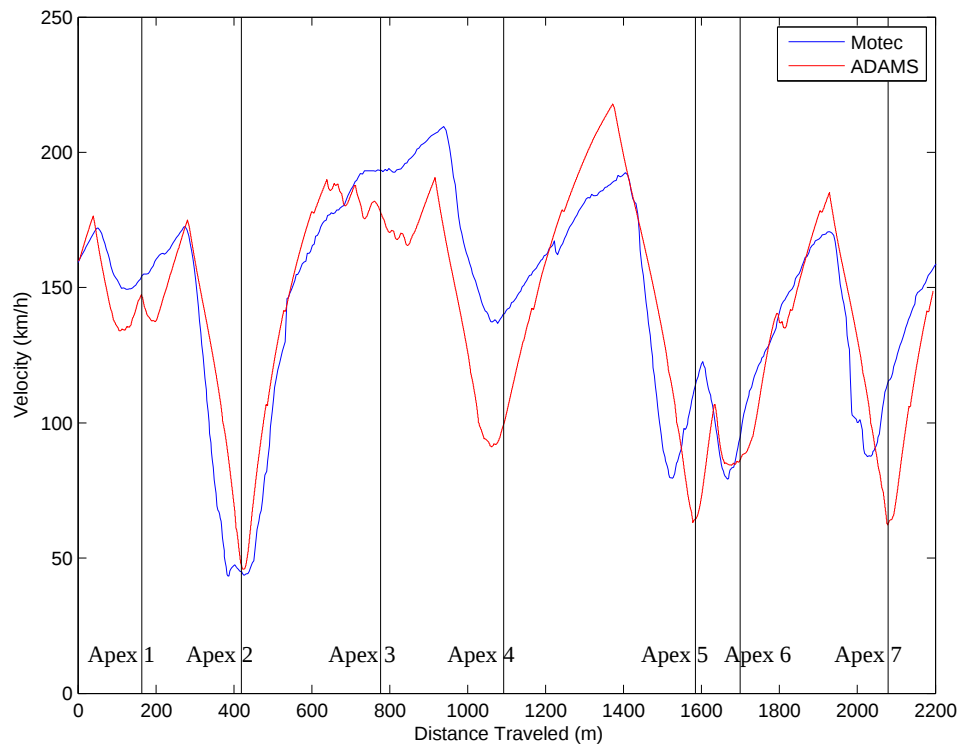


Figure 4.4: ADAMS/Car vs Track Test Data: Velocity vs Distance

The comparison of lateral acceleration shown in Figure 4.5 indicates good correlation between the actual and simulation values. The peak lateral acceleration values of the ADAMS/Car

model were however slightly less than the measured values. This is likely due to an under-estimation of lateral tyre force and cornering stiffness as discussed in Section 3.4.

The comparison of longitudinal acceleration in Figure 4.6 shows that the steady state longitudinal acceleration values of the ADAMS/Car model were higher than the measured values. This is likely due to the approximation of longitudinal tyre forces from the lateral force data as discussed in section 3.4. It can be seen that the ADAMS/Car model is far more erratic longitudinally than laterally.

The comparison of acceleration plots show that the ADAMS/Car model of the BC LMP2 track well with the measured test data, showing that the model is representative of the actual car in this respect.

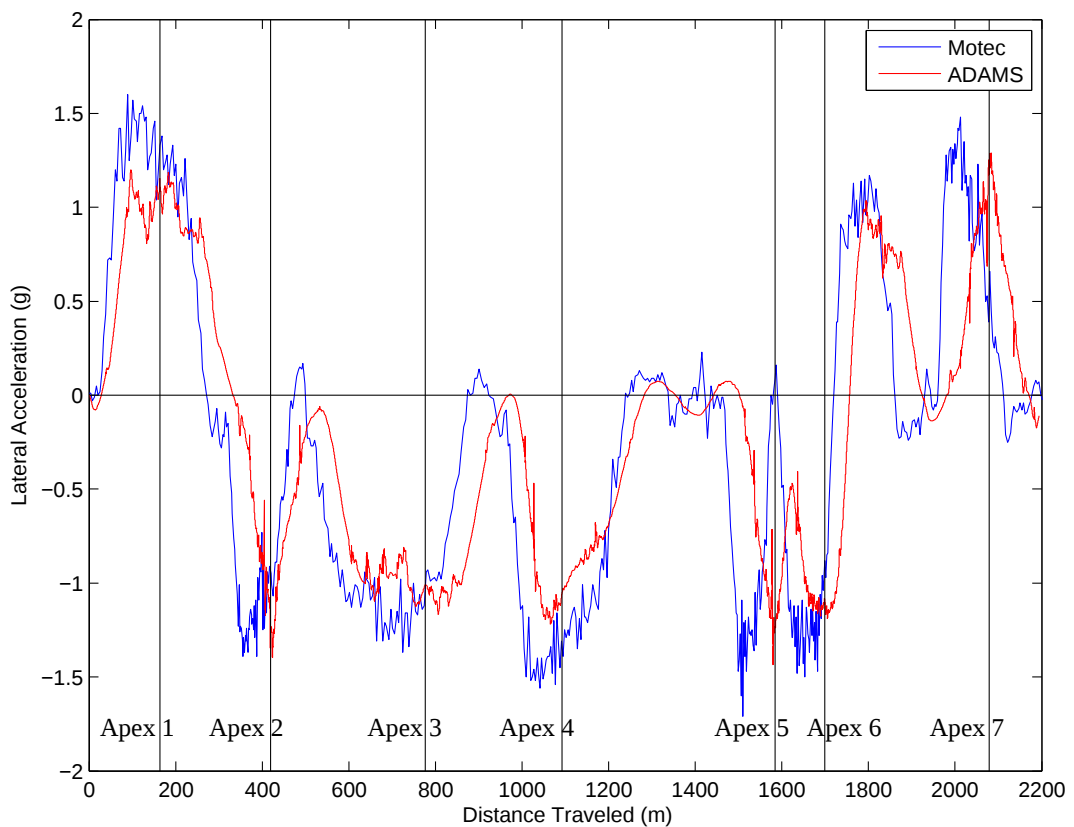


Figure 4.5: ADAMS/Car vs Track Test Data: Lateral Acceleration vs Distance

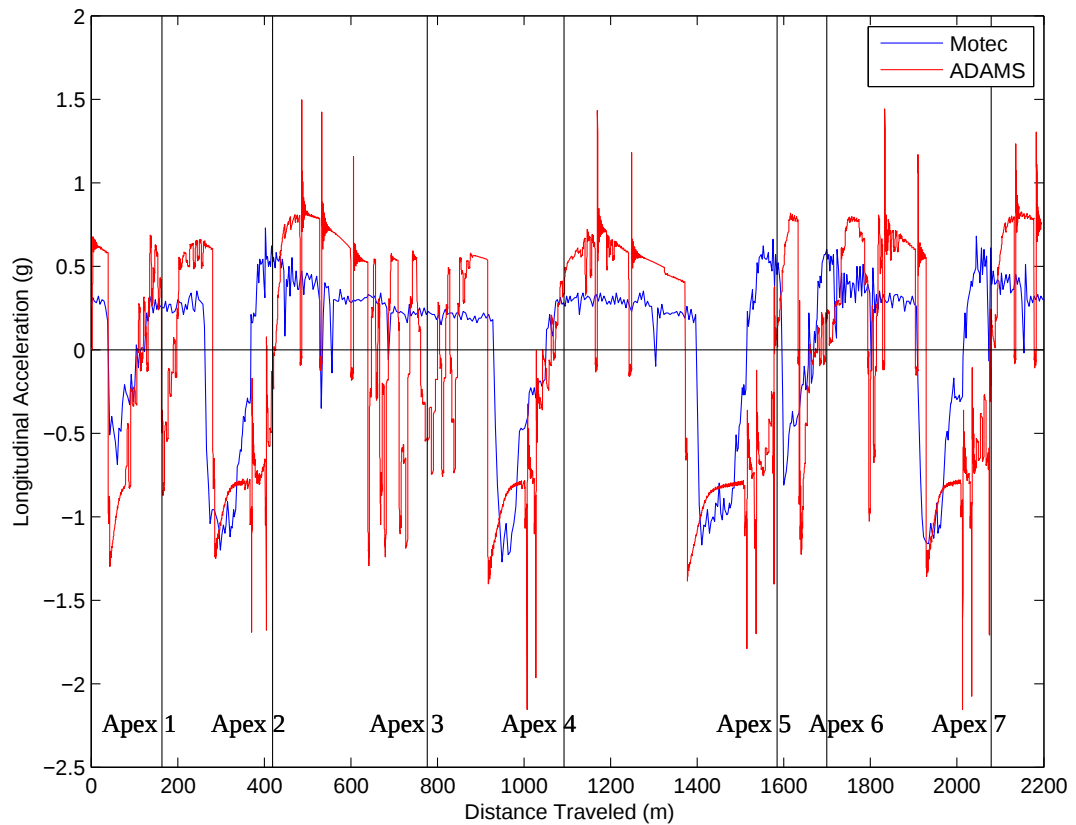


Figure 4.6: ADAMS/Car vs Track Test Data: Longitudinal Acceleration vs Distance

Figures 4.7 to 4.9 show a comparison of gear, engine RPM and steering angle all versus track position. These plots again show good correlation between the ADAMS/Car model and the test data.

Overall, the ADAMS/Car model correlates well with the track test data. The discrepancies between the test data and the ADAMS/Car model can largely be attributed to the assumptions made in creating the ADAMS/Car models of Zwartkops Raceway and the BC LMP2 tyres from test data. In addition to this, differences between the ability of the SmartDriver to that of Hennie Groenewald on Zwartkops Raceway will account for subtle differences. The overall differences between the ADAMS/Car simulation and track test data were deemed small enough to conduct a representative comparative study of the two and the three (SDS) systems.

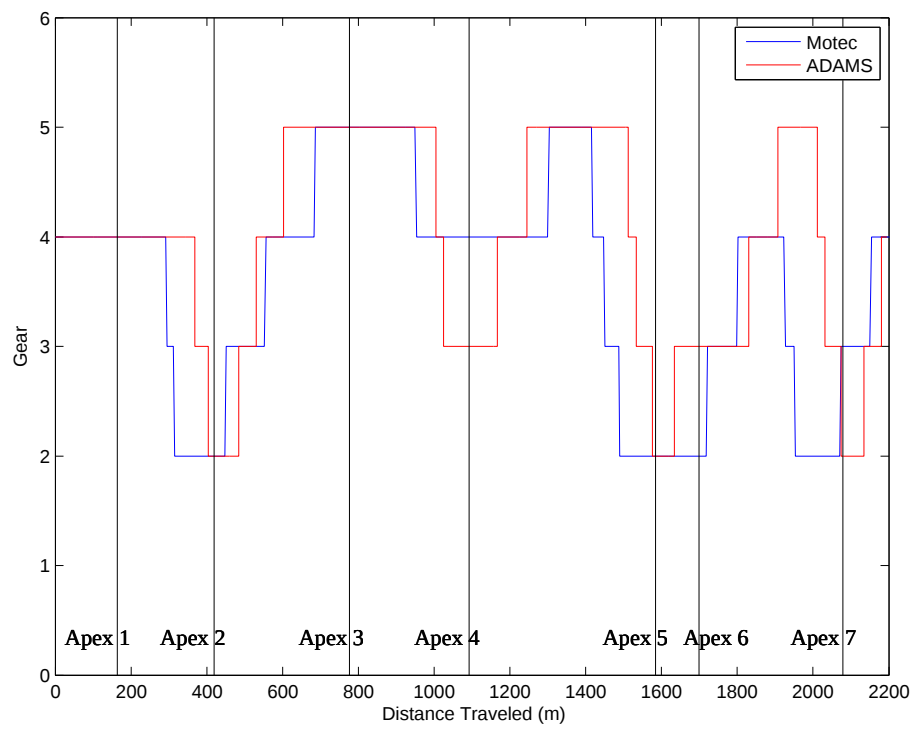


Figure 4.7: ADAMS/Car vs Track Test Data: Gear vs Distance

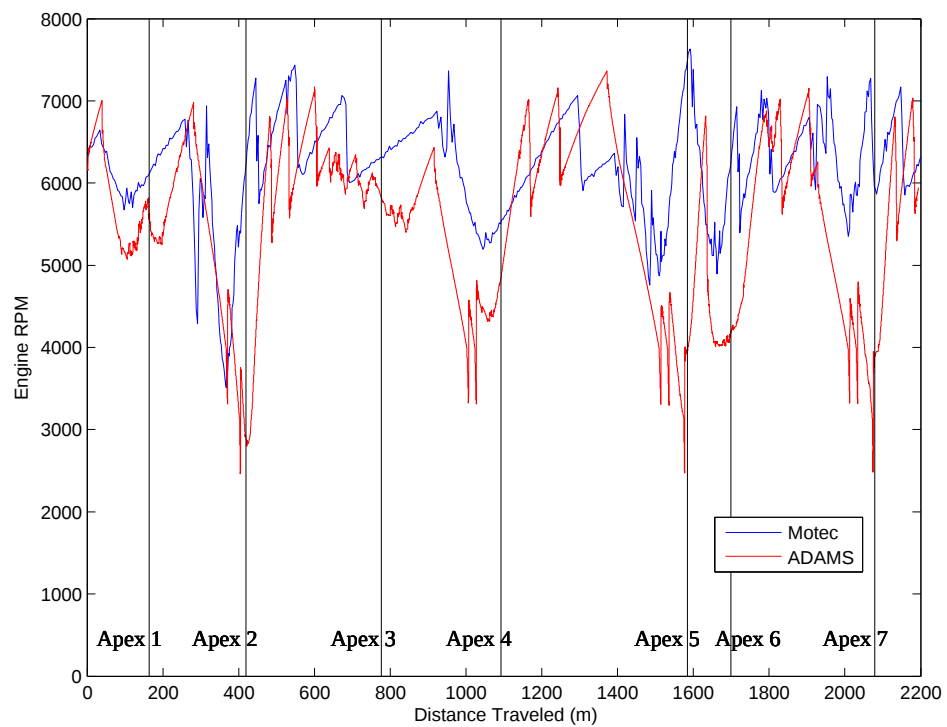


Figure 4.8: ADAMS/Car vs Track Test Data: Engine RPM vs Distance

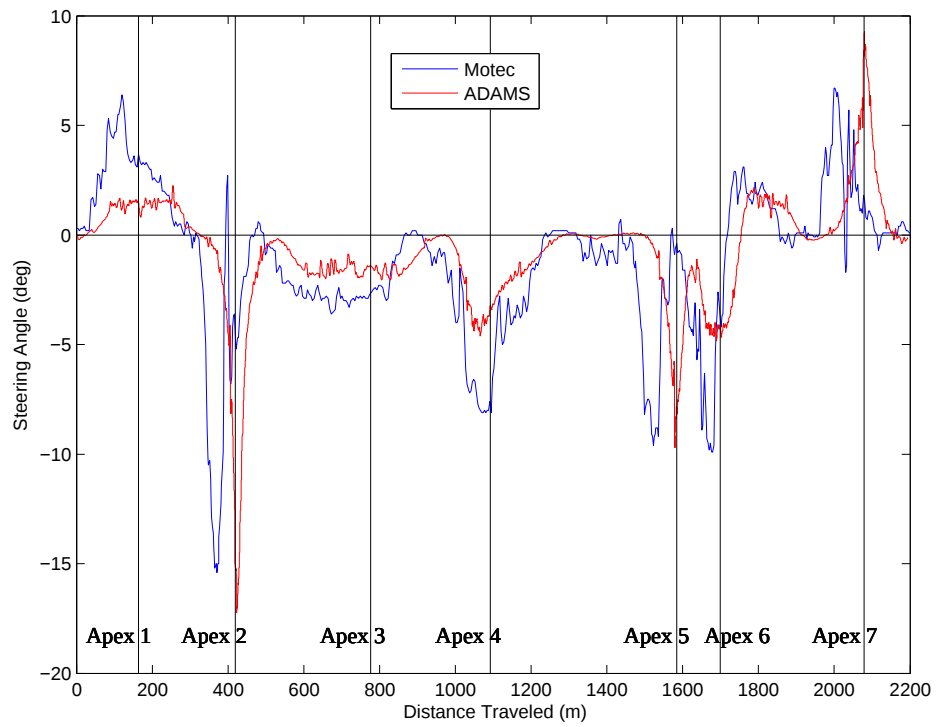


Figure 4.9: ADAMS/Car vs Track Test Data: Steering Angle vs Distance

4.5 Summary

An ADAMS/Car model of the BC LMP2 was created using inputs from both the track and tyre testing, with the model of Zwartkops Raceway was created using the recorded GPS data from the track testing. The BC LMP2 was then simulated on the model of Zwartkops Raceway for 62 seconds, the fastest lap time recorded in physical testing. The ADAMS/Car SmartDriver model parameters were tuned to ensure the model correlated well with the track test data and the model was validated against the test data.

Chapter 5 SUSPENSION OPTIMISATION

5.1 Introduction

Optimisation of the suspension parameters for the two and three spring and damper suspension (SDS) models was conducted using a design of experiment (DOE) approach in ADAMS/Insight. The DOE analyses were based on the baseline vehicle configuration with the original parameter values from the validation in Section 4.4.

Road roughness was introduced into the track model for the optimisation simulation runs, but as discussed in Sections 4.3 and 4.4, the SmartDriver parameters used for validation caused instability in the ADAMS/Car model. As such, the SmartDriver parameters were systematically scaled down until it was found that the original vehicle combination was consistently stable in a simulation on the Zwartkops Raceway. The set of SmartDriver parameters with maximum vehicle performance were found to be: Driving 55%, Braking 70%, Cornering left 70%, Cornering Right 70%. These parameters were used for all subsequent simulations.

5.2 ADAMS/Insight Optimisation of the Two Spring and Damper Suspension

For the DOE optimisation of the two (SDS) model, the design objective was selected as total distance travelled for 60 seconds, with the aim of maximising this objective.

The parameters that were selected as factors for the front and rear suspension were spring stiffness values and damping coefficients. These were controlled in ADAMS/Insight by

means of a scaling factor with a nominal value of 1. This would represent the stiffness and damping values of the suspension in track testing, given in Section 2.2 on page 13. The scaling factors were set to vary continuously between $\pm 50\%$ of the nominal values, with a uniform probability density function (PDF) used to select the value of each factor per run.

A Latin Hypercube investigation strategy was chosen for factor parameter selection for the DOE and a set of 350 simulations were run for the two (SDS) model resulting in 342 successfully completed runs. The spring and damper parameters were then optimised in ADAMS/Insight to maximise the design objective of total distance travelled. The ADAMS/Insight file with the full set of factors and distance travelled for each run is given in the Digital Appendix.

Once the optimal combination of spring and damper factors had been determined, the prediction was then confirmed by performing an ADAMS/Car simulation with that set of factors and a close correlation between the predicted optimal values and measured values was confirmed. The optimal values and vehicle performance will be compared to that of the three (SDS) model below.

The optimised factors for the two (SDS) model are given in Table 5.1, with the damper curves comparing the original and optimised values shown in Figure 5.1. The simulation results of the DOE optimisation for distance travelled for the two (SDS) are given in Table 5.2.

Table 5.1: Optimised Factors for the Two Spring and Damper System

	Factor	Optimised Value
Front Spring	1.105	127.098 N/mm
Rear Spring	0.825	119.628 N/mm
Front Damper	1.024	-
Rear Damper	1.005	-

Table 5.2: Two SDS DOE Results

	Distance Travelled (m)	Increase (m)	RMS Pitch (deg)	RMS Roll (deg)
Baseline	1992.32	-	0.71	4.46
ADAMS/Insight Optimised	1992.60	0.28	-	-
Optimised Actual	1992.34	0.02	0.90	4.44

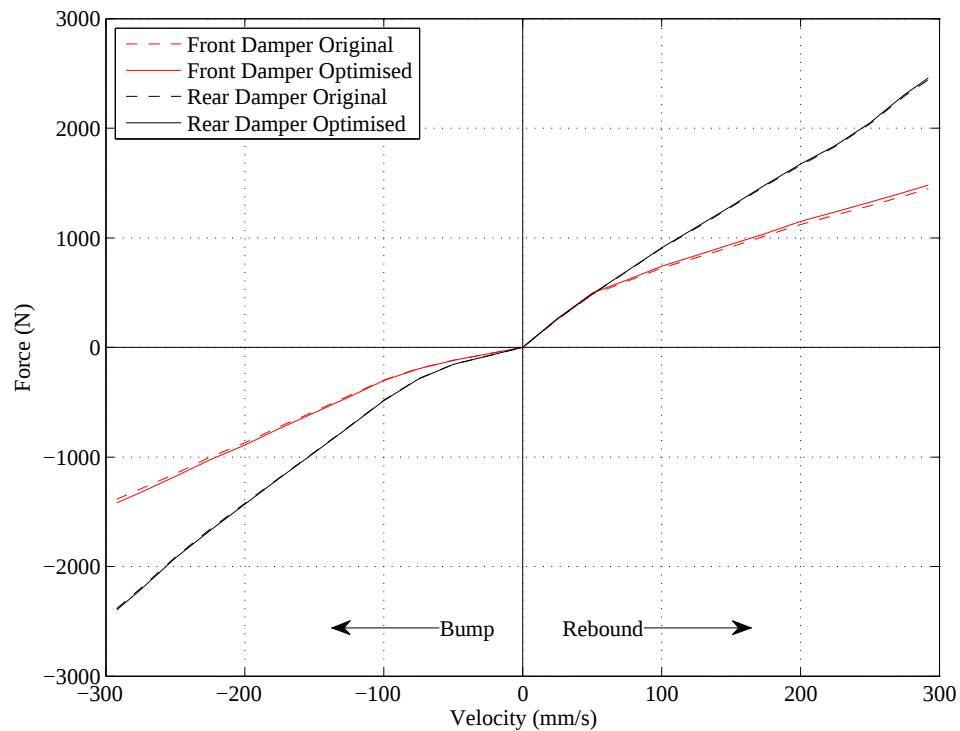


Figure 5.1: Optimised Damper Curves for the Two SDS

The optimised parameters resulted in a slight increase in distance travelled as well as the RMS pitch (forward), but a decrease in the RMS roll. In spite of the increase in pitch angle of the vehicle having an influence on the downforce, the combination of the softer rear spring stiffness and slight increase in the remaining parameters is marginally better for the vehicle's performance on Zwartkops Raceway. This suggests that the BC LMP2 is an inherently well balanced vehicle and that the baseline suspension parameters are close to optimal in terms of vehicle performance.

The reduction in rear spring stiffness suggests that the front tyres were not carrying enough load. The increased forward pitch also suggests that this was the case as an increase in pitch results in a forward shift of the centre of pressure of the downforce, as can be seen in Figure

[A.6](#) in Appendix A, resulting in more load on the front tyres.

Milliken and Milliken [25] note that increasing the front roll stiffness, in this case by a decrease of rear roll stiffness, will assist the vehicle in resisting the tendency to spin under braking at corner entry. This phenomenon was noted numerous times during the calibration and validation of the software model and thus, intuitively makes sense.

5.3 ADAMS/Insight Optimisation of the Three Spring and Damper Suspension

A similar DOE optimisation was conducted for the three (SDS) model, with the design objective once again being the total distance travelled for 60 seconds.

The front and rear suspension parameters selected as factors for the DOE were the coefficients for each of the three springs and dampers as well as the ARB stiffness values. The parameters were set to the values of the baseline two SDS model used for the validation, with the stiffness values of the third springs chosen as 145 N/mm and 115 N/mm for the front and rear respectively. The damping coefficients for both front and rear third dampers were chosen to be 10 Ns/mm. The ARB stiffness values were chosen to be 60 Nm/deg for both the front and rear as these were the original design stiffness values for the vehicle [19].

The scaling factors were set to $\pm 60\%$ of the nominal values, with the ARB stiffness values set to vary between 0 and 120 Nm/deg. A uniform PDF was again chosen for the selection of parameters. A Latin Hypercube investigation strategy was selected for a total of 800 simulations, resulting in 758 successfully completed runs.

The optimised factors for the three (SDS) model are given in Table 5.3, with the damper curves comparing the original and optimised values are shown in Figure 5.2. The ADAMS/Insight file with the full set of factors and results is given in the Digital Appendix.

Table 5.3: Optimised Factors for the Three (SDS)

	Factor	Optimised Value
Front Spring	1.001	115.150 N/mm
Front Damper	1.271	-
Front Third Spring	1.047	151.801 N/mm
Front Third Damper	1.001	10.006 Ns/mm
Front ARB	-	46.929 Nm/deg
Rear Spring	0.373	54.100 N/mm
Rear Damper	1.014	-
Rear Third Spring	1.408	161.897 N/mm
Rear Third Damper	1.072	10.721 Ns/mm
Rear ARB	-	59.219 Nm/deg

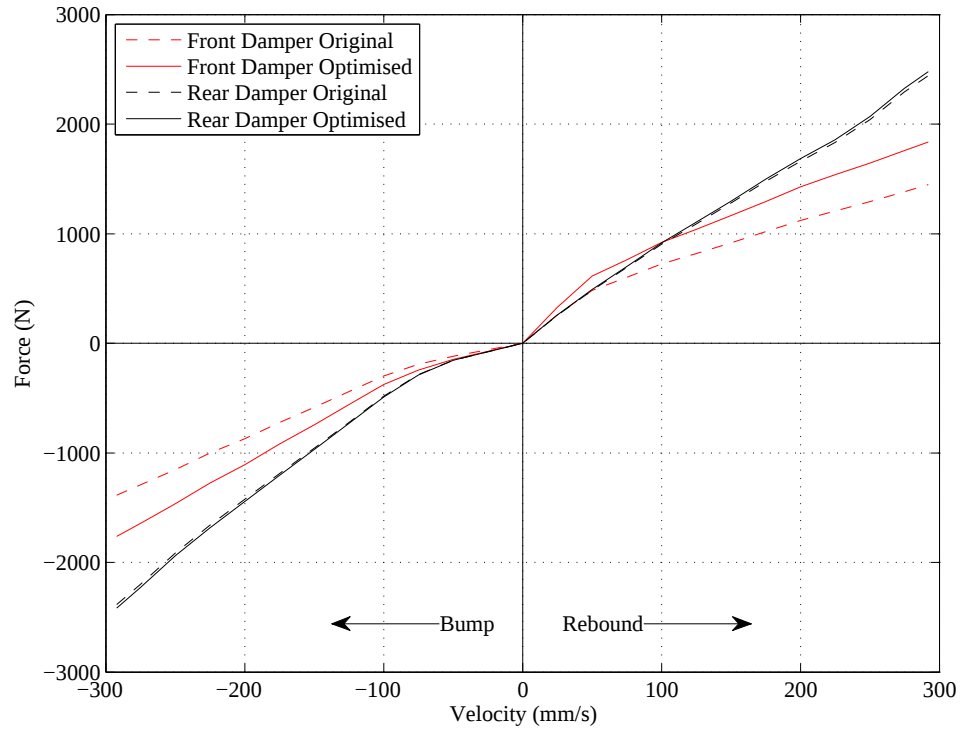


Figure 5.2: Optimised Damper Curves for the Three SDS

The simulation results of the DOE optimisation for distance travelled for the three (SDS) are given in Table 5.4.

Table 5.4: Three SDS DOE Results

	Distance Travelled (m)	Increase (m)	RMS Pitch (deg)	RMS Roll (deg)
Baseline	1987.14	-	1.22	4.40
ADAMS/Insight Optimised	1987.89	0.78	-	-
Optimised Actual	1987.96	0.82	0.94	4.17

The simulation results show an improvement in overall performance for the optimised suspension characteristics for both the two and three SDS systems. In the following section, the performance of the vehicle with optimised suspension parameters for two SDS will be compared to that of the three SDS.

5.4 Comparison of the Two and Three SDSs

Simulation data from the optimised suspension parameters for the two and three SDS models for the 60 second simulations above, are compared in this section. Table 5.5 shows a side by side comparison of the optimised factors and values for the two and three SDS models with Table 5.6 giving a summary of distance travelled, RMS pitch and RMS roll.

Table 5.5: Comparison of Two and Three SDS Factors

	3 SDS		2 SDS		
	Factor	Value	Factor	Value	
Front Spring	1.001	115.150	1.105	127.098	N/mm
Front Damper	1.271	-	1.024	-	Ns/mm
Front 3rd Spring	1.047	151.801	-	-	N/mm
Front 3rd Damper	1.001	10.006	-	-	Ns/mm
Front ARB	-	46.929	-	-	Nm/deg
Rear Spring	0.373	54.100	0.825	119.628	N/mm
Rear Damper	1.014	-	1.005	-	Ns/mm
Rear 3rd Spring	1.408	161.897	-	-	N/mm
Rear 3rd Damper	1.072	10.721	-	-	Ns/mm
Rear ARB	-	59.219	-	-	Nm/deg

Table 5.6: Two versus Three SDS DOE Results

	Distance Travelled (m)	RMS Pitch (deg)	RMS Roll (deg)
2 SDS Baseline	1992.32	0.71	4.46
3 SDS Baseline	1987.14	1.22	4.40
2 SDS Optimised	1992.60	0.90	4.44
3 SDS Optimised	1987.96	0.94	4.17

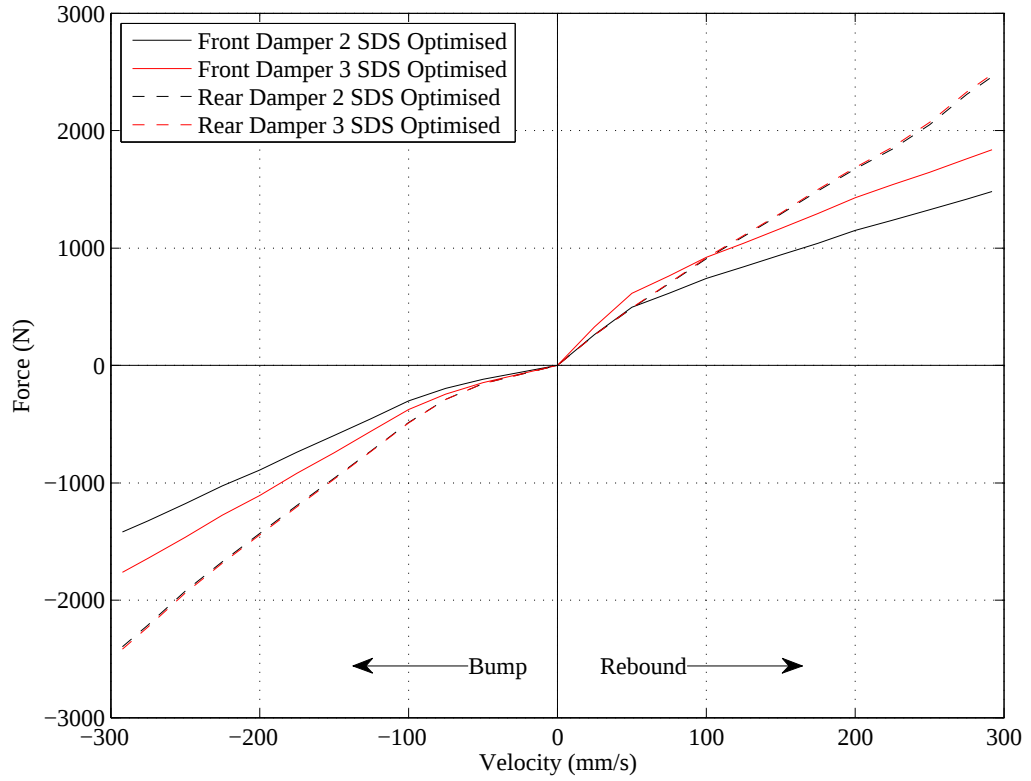


Figure 5.3: Comparison of Optimised Two and Three SDSs Dampers

The total distance travelled by the optimal three SDS model was 4.38 m less than the optimal two SDS model. Milliken and Milliken [25] state that the cornering capability of a vehicle is highly dependant on the dynamic vertical tyre loads, which are affected by either the CoG location or through mechanisms that affect the weight transfer distribution during cornering, such as spring and ARB stiffness as well aerodynamic forces. As already stated, the aerodynamic drag and downforce of the BC LMP2 are highly sensitive to pitch angle. Figure 5.4 shows a comparison of pitch versus track position for the 2 and 3 SDS systems.

A decrease in pitch angle (forwards) increases the magnitude of the C_L and thus also the magnitude of the downforce, however the frontal area and C_D are also increased. A lower variance in the pitch angle is therefore desirable, as is the case with the 3 SDS model, to reduce excessive fluctuations in the aerodynamic forces which in turn may upset the balance of the vehicle in dynamic manoeuvres.

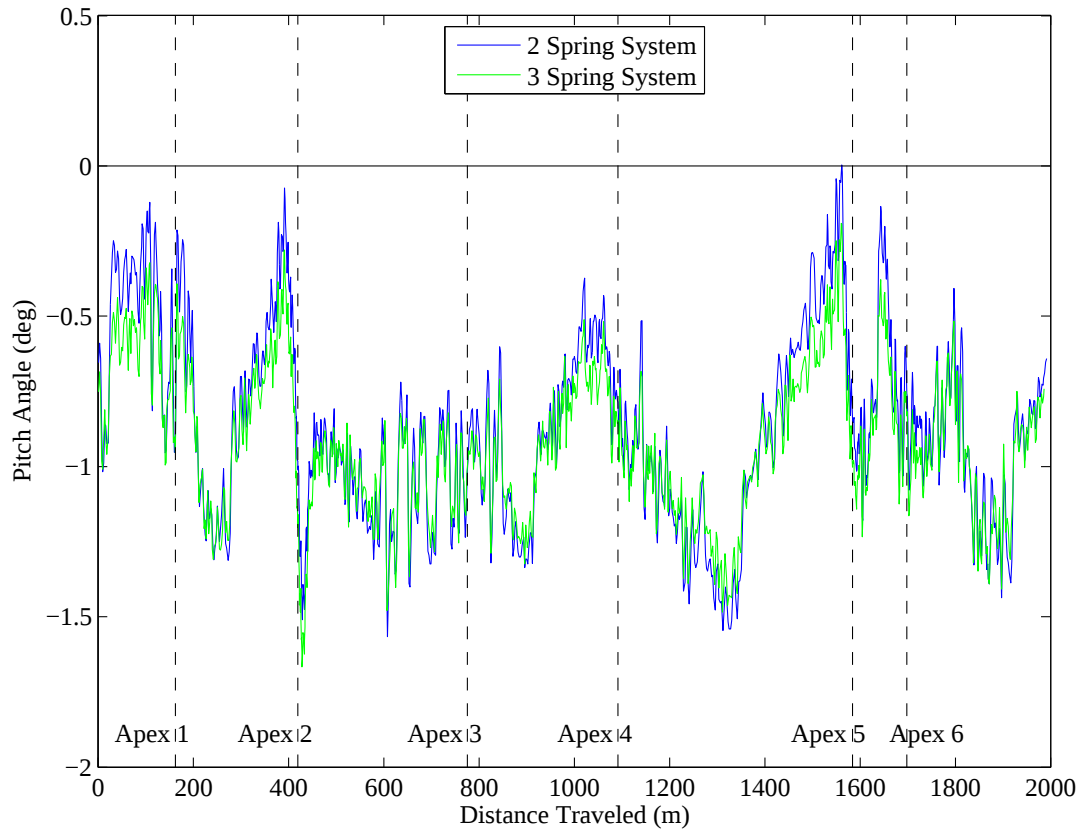


Figure 5.4: Pitch Angle vs Distance Travelled

Figure 5.5 shows a comparison of the pitching stiffness for the 2 and 3 SDS systems, with the average pitch stiffness values given in Table 5.7. It can be seen that the front Pitch stiffness of the optimised 3 SDS system is greater than the 2 SDS system while the 2 SDS system has a higher rear pitch stiffness. The 3 SDS model therefore has greater resistance to pitching under braking than the two SDS model, which reduces the variation in aerodynamic forces. The 2 SDS model however has a greater resistance to pitching rearward under acceleration.

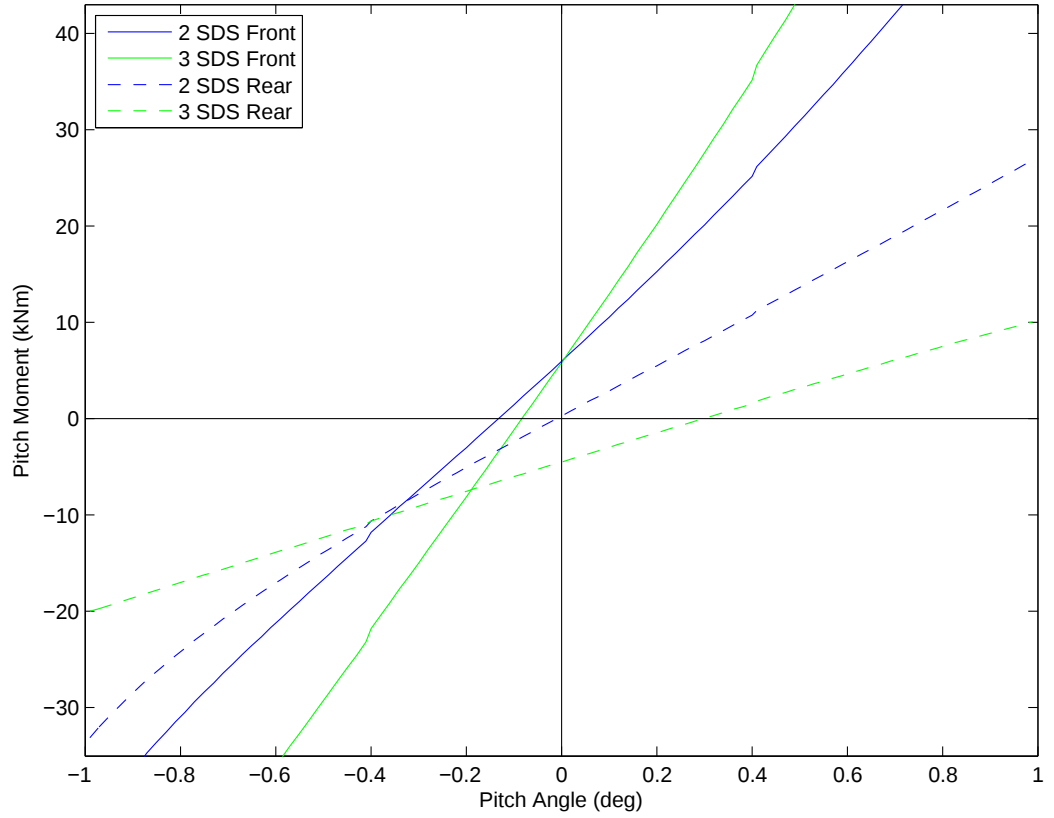


Figure 5.5: Pitch Stiffness of the Optimised SDS Systems

Table 5.7: Pitch Stiffness for the Two and Three SDS Systems

	Front (kNm/deg)	Rear (kNm/deg)
2 SDS Optimised	51.755	30.474
3 SDS Optimised	74.638	15.228

In spite of the additional pitch stiffness of the optimal 3 SDS model, the 2 SDS model overall shows slightly less pitch angle, but a greater standard deviation (σ) and variance (σ^2) in pitch, as shown in Table 5.8.

Table 5.8: Two versus Three SDS Pitch Angle

	RMS Pitch (deg)	Average Pitch (deg)	Std Deviation σ	Variance σ^2
2 SDS Optimised	0.895	-0.836	0.320	0.102
3 SDS Optimised	0.940	-0.904	0.260	0.068

Roll angle versus track position for both optimised models is shown in Figure 5.6.

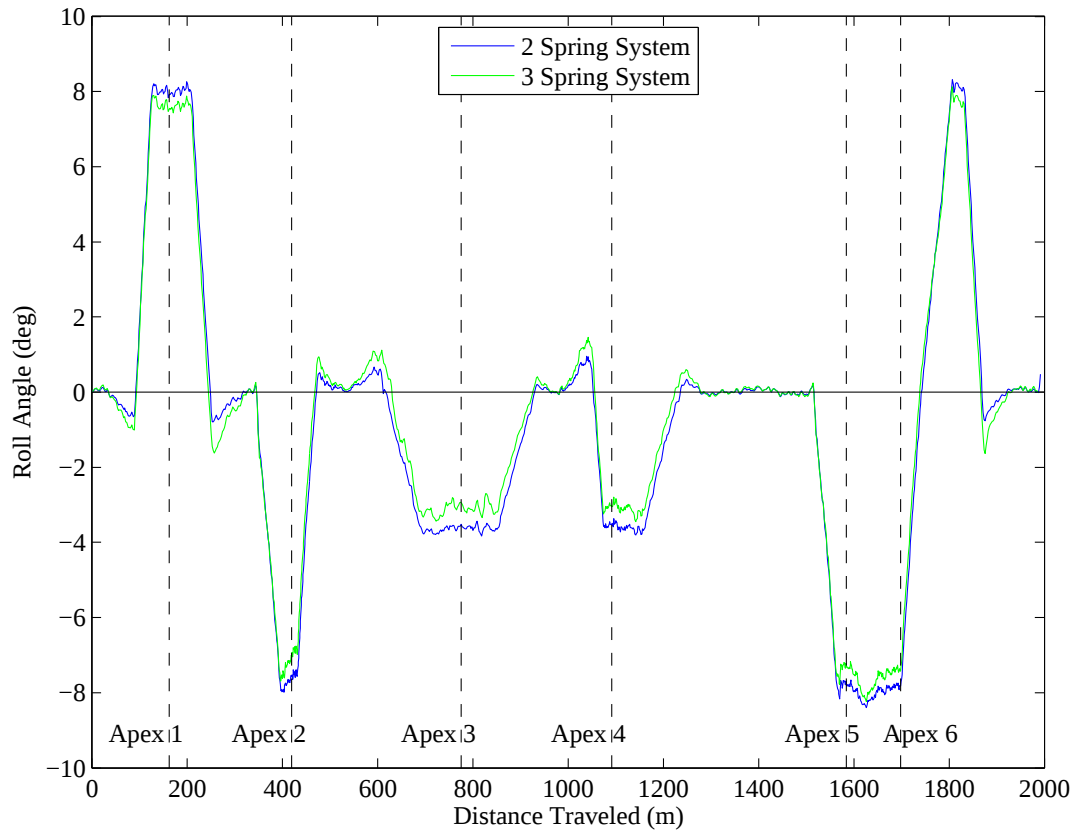


Figure 5.6: Roll Angle vs Distance Travelled

Figure 5.7 shows a comparison of the overturning moment for the 2 and 3 SDS systems, with the average roll stiffness values given in Table 5.9. It can be seen that the roll stiffness of the optimised 2 SDS system is greater than the 3 SDS system for the front, rear and overall stiffness values.

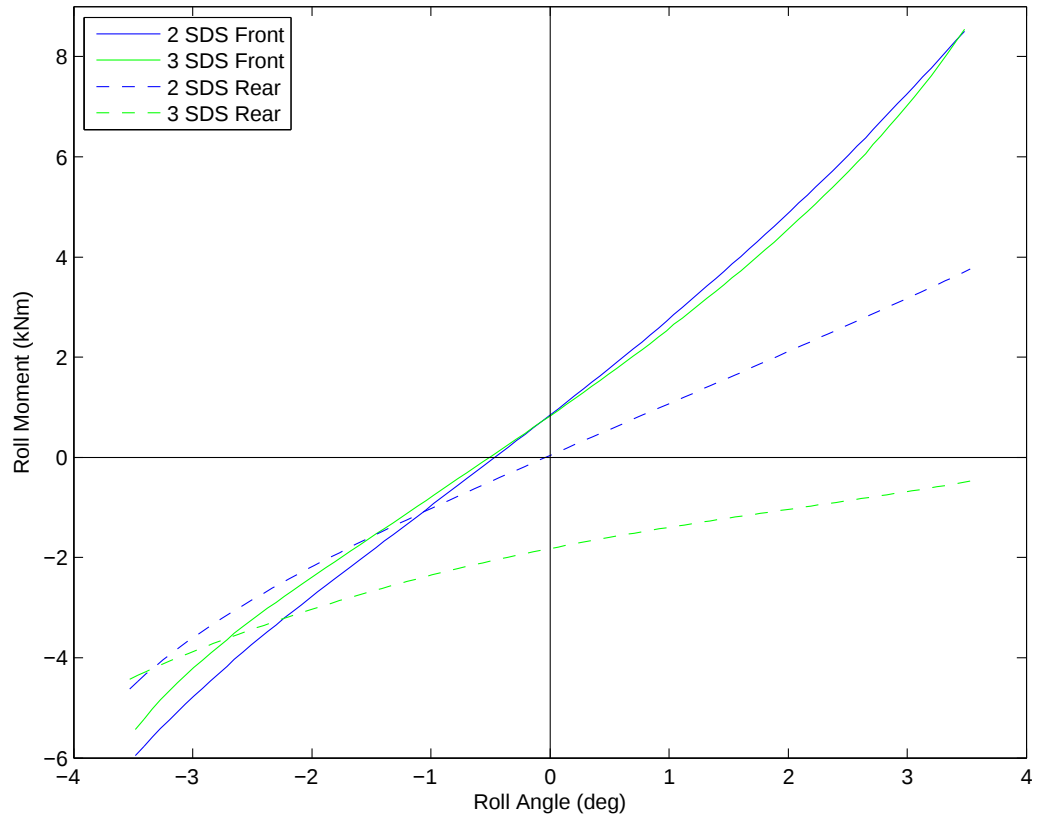


Figure 5.7: Roll Stiffness of the Optimised SDS Systems

Table 5.9: Roll Stiffness for the Two and Three SDS Systems

	Front (kNm/deg)	Rear (kNm/deg)	Combined (kNm/deg)
2 SDS Optimised	2.084	1.192	3.276
3 SDS Optimised	2.020	0.562	2.583

Surprisingly, in spite of the 3 SDS model's decreased roll stiffness, it shows less roll than the 2 SDS model on Zwartkops raceway. The 3 SDS model shows less average roll as well as a reduced standard deviation and variance compared to the 2 SDS model, as shown in Figure 5.10.

Figures 5.8 and 5.9 show the distance travelled versus RMS pitch and roll angles respectively for the 3 SDS DOE investigation. It can be seen that for all the combinations sampled in the DOE, those that resulted in the greatest distance travelled ($>1\,987.8$ m) consistently had an RMS pitch angle of less than 1.1° while the corresponding RMS roll angles had a far greater spread, ranging from 4° to 4.4° . This indicates that for the 3 SDS system, pitch angle

Table 5.10: Two versus Three SDS Roll Angle

	RMS Roll (deg)	Average Roll (deg)	Std Deviation σ	Variance σ^2
2 SDS Optimised	4.441	-1.558	4.159	17.298
3 SDS Optimised	4.168	-1.376	3.934	15.479

has a far greater influence on overall distance travelled than roll angle.

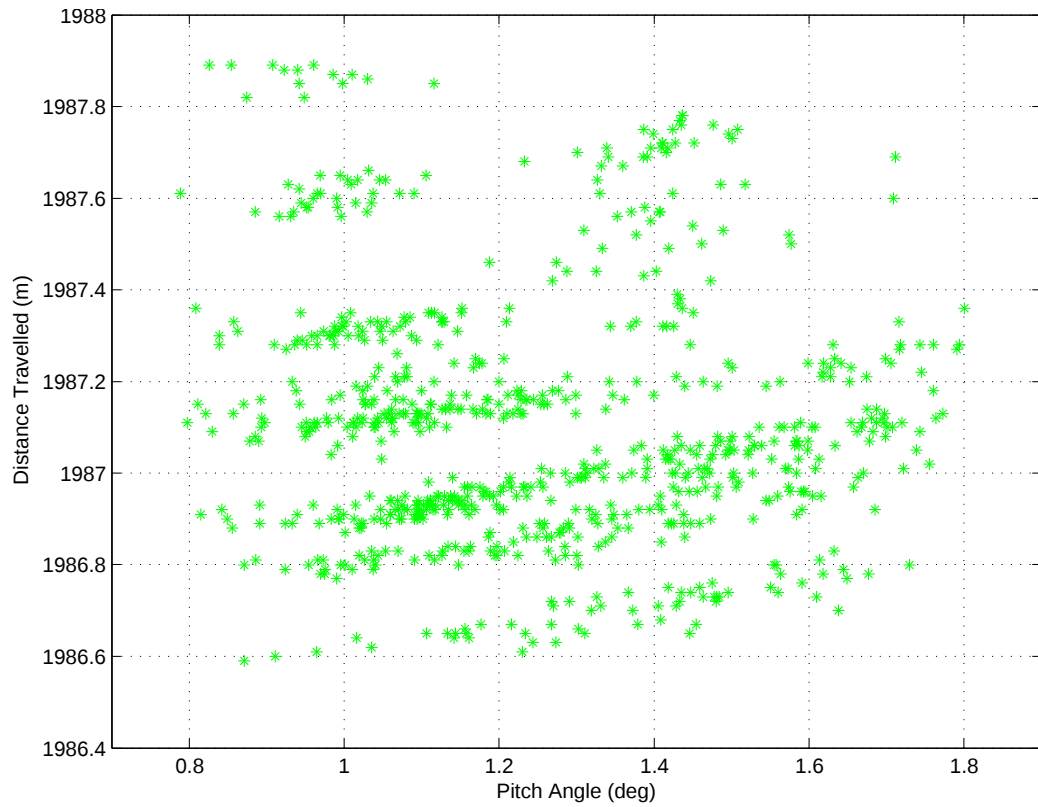


Figure 5.8: Distance Travelled vs RMS Pitch Angle for the 3 SDS

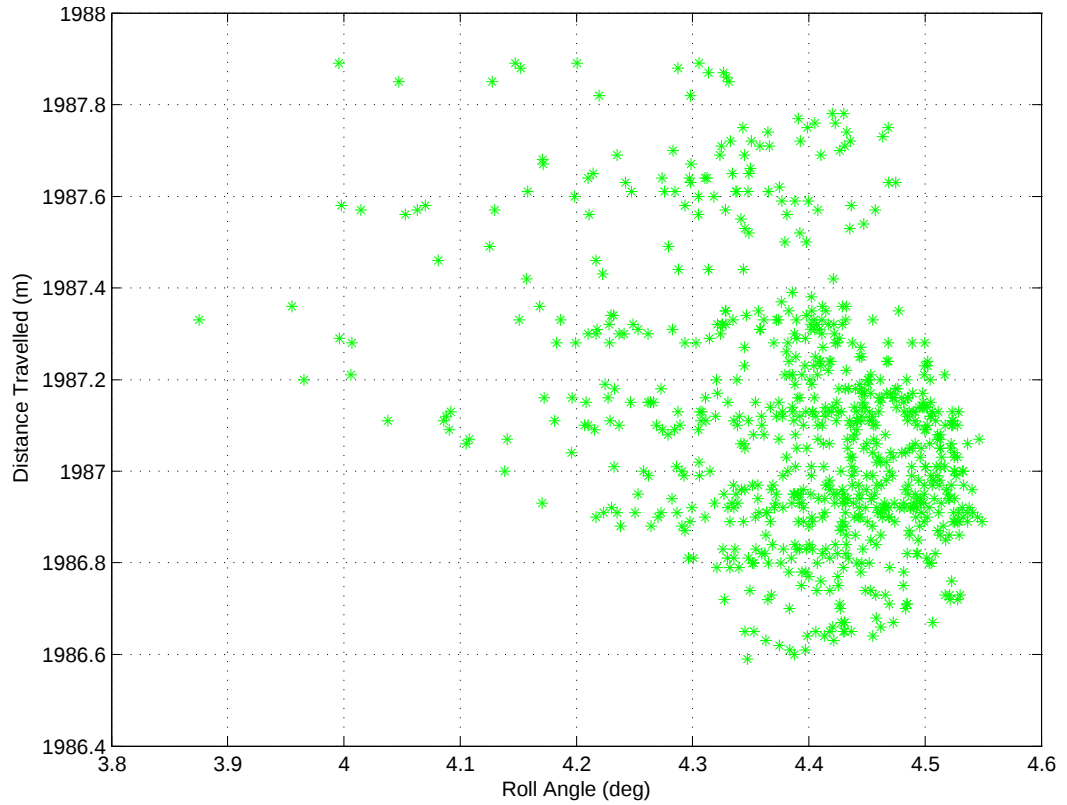


Figure 5.9: Distance Travelled vs RMS Roll Angle for the 3 SDS

Figures 5.10 and 5.11 show comparisons of the optimal 2 and 3 SDS models for lateral and longitudinal acceleration respectively. There is little distinguishable difference between the two models. However, as expected, the magnitudes of the peak lateral acceleration values for the 3 SDS system are marginally lower than those of the 2 SDS system. This is expected given the reduction in distance travelled by the 3 SDS model.

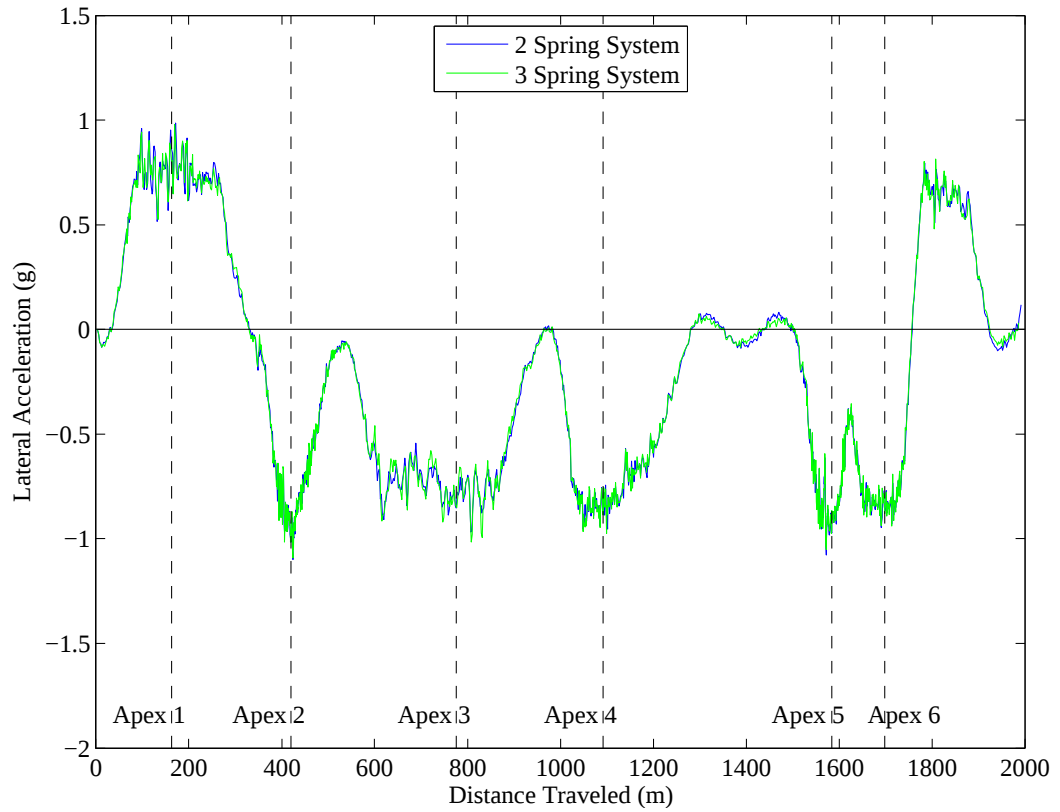


Figure 5.10: Lateral Acceleration vs Distance Travelled

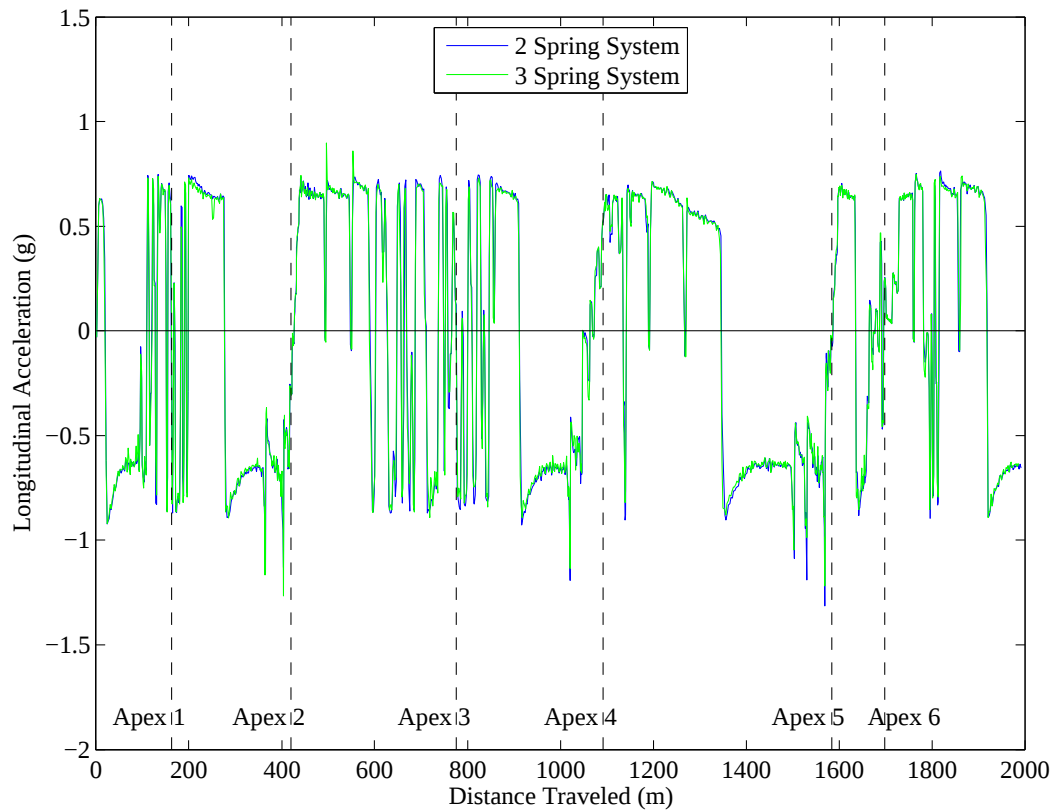


Figure 5.11: Longitudinal Acceleration vs Distance Travelled

A comparison of steering angle versus track position for the two optimal models is given in Figure 5.12, again showing that the two models produce similar results. The steering angle inputs are almost identical through the two low-speed corners, turns 2 and 5, while larger steering inputs are required by the 3 SDS model in the high-speed corners, turns 1 and 3.

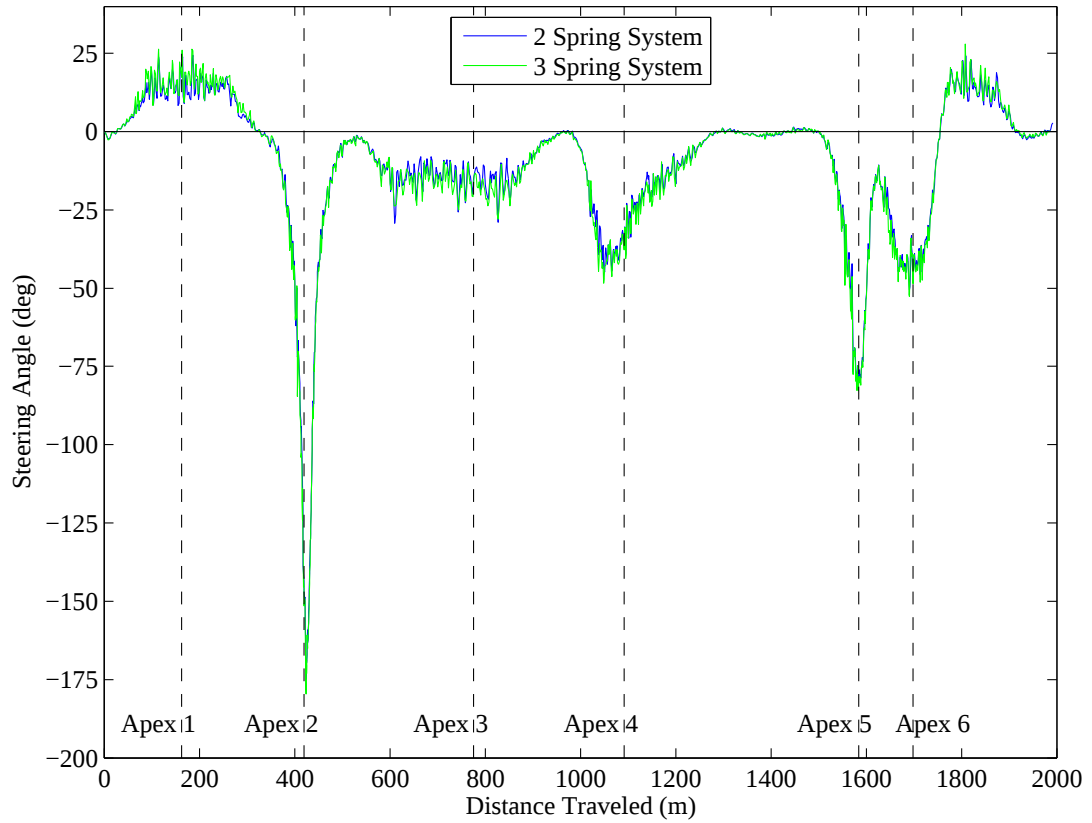


Figure 5.12: Steering Angle vs Distance Travelled

Milliken and Milliken [25] present the "g-g" diagram as a measure of vehicle performance on a circuit. Figure 5.13 shows a "g-g" diagram comparing the performance of the 2 and 3 SDS models. It can be seen that both suspension models allow the vehicle to consistently reach 1 g acceleration with peaks of 1.5 g under braking. The RMS of acceleration magnitude for the 2 SDS model is 0.870 g, compared to 0.864 g for the 3 SDS model. This is in agreement with the slight reduction in distance travelled by the 3 SDS model. Overall, both models are well balanced and produce similar results, indicating that the BC LMP2 has a fundamentally good design and is a stable platform that is not highly sensitive to changes in the suspension parameters.

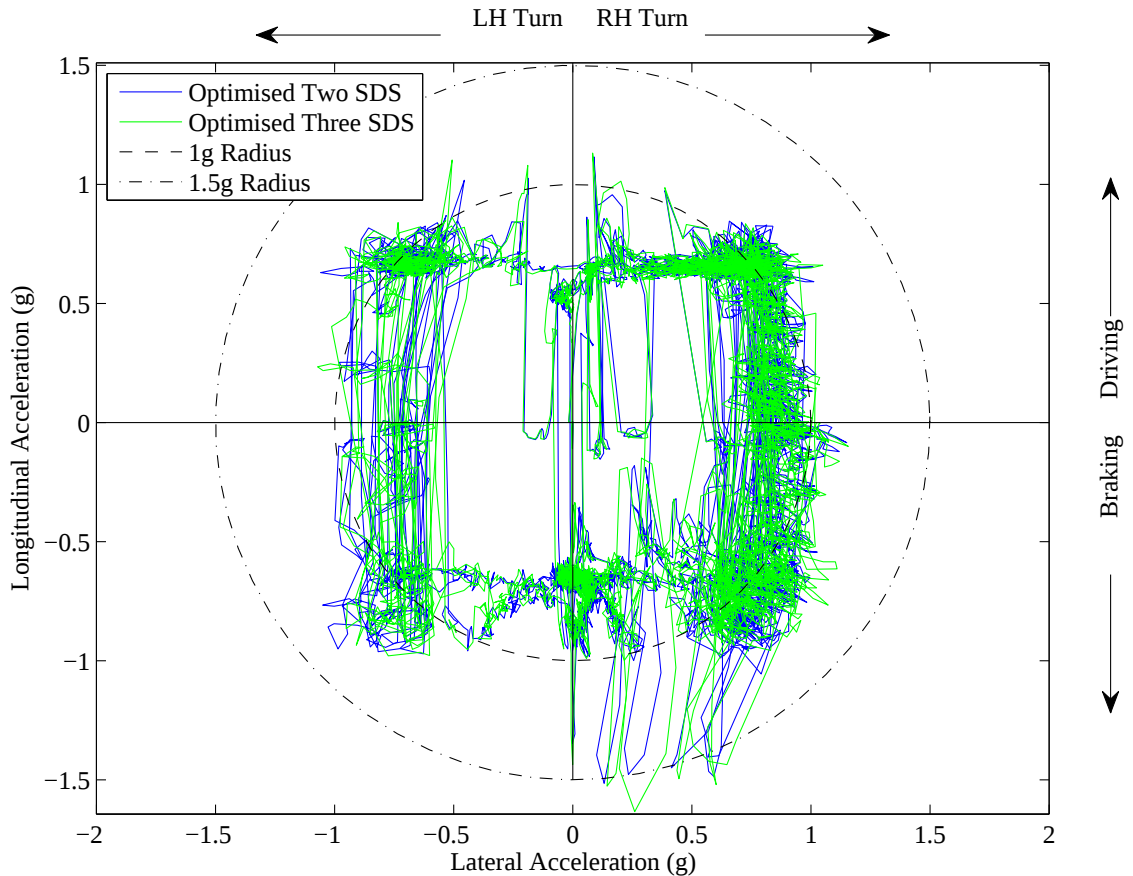


Figure 5.13: "g-g" Diagram

The results of the optimised models show that the 2 SDS model was able to achieve a greater distance travelled on Zwartkops Raceway than the 3 SDS system. However, it was seen that the 3 SDS system was effectively able to isolate pitch and roll stiffness. This is of particular interest to vehicles such as the BC LMP2 where the magnitude of aerodynamic forces is highly sensitive to pitch angle.

Due to the mechanics of a 2 SDS model, pitch and roll stiffness cannot be isolated. Reducing the stiffness of the 2 SDS springs to allow greater compliance for the wheels to better track road undulations, and thus better manage the vertical tyre loads, but sacrifices pitching stiffness, compromising the aerodynamics of the vehicle. Introducing the third spring and damper as well as the ARB allows pitch and roll stiffness to be controlled independently.

A 3 SDS system therefore gives that ability to allow for softer springs, which allow the wheels to closely follow the undulations of the road, without sacrificing pitch stiffness. The third spring and damper can be adjusted to allow for differing downforce set-ups according to the demands of a specific circuit. As mentioned in Section 2.1, Zwartkops Raceway is a

short circuit with tight turns and is not well suited to vehicles with large amounts of down-force. This limitation means that the full potential of the BC LMP2 3 SDS system could not be fully explored in this investigation.

The DOE analysis was selected to ensure the number of simulations runs required remained reasonable while still evaluating the effect of each parameter on vehicle performance. Although the performance of the 3 SDS system was not found to be equal or better than that of the 2 SDS system, the effects of a 3 SDS system were quantified.

5.5 Summary

An ADAMS/Car model of the BC LMP2 was created using inputs from both the track and tyre testing. A model of the Zwartkops raceway was created using the recorded GPS data from the track testing. The ADAMS/Car SmartDriver model parameters were tuned to ensure the model correlated well with the track test data. ADAMS/Insight was then used to conduct a set of 350 runs with unique suspension parameters for the 2 SDS model, varying the spring stiffness and damper coefficient parameters. The ADAMS/Car suspension sub-system models were modified to include the third springs and dampers as well as the ARBs. ADAMS/Insight was again used to complete 800 runs, again with unique suspension parameters, to optimise spring, damper and ARB stiffness values. The suspension parameters for the 2 and 3 SDS systems were optimised to maximise distance travelled around Zwartkops Raceway. The differences between the optimal models were then compared and discussed.

Chapter 6 Conclusions

The optimal suspension parameters for the BC LMP2 race car on Zwartkops Raceway were determined through a process of physical testing, software simulation and optimisation.

1. The performance of the BC LMP2 on Zwartkops Raceway is as follows:
 - (a) Total distance travelled was 1987.96 m
 - (b) RMS pitch angle was 0.94°
 - (c) RMS roll angle was 4.17°
2. The optimal suspension stiffness and damping coefficients for BC LMP2 with both SDS system and their performance on Zwartkops Raceway are as follows:
 - (a) The optimal two SDS front spring stiffness was found to be 127.098 N/mm (1.105 times the original value), and the rear spring stiffness was found to be 119.628 N/mm (0.825 times the original value). The front damping coefficient was found to be 1.024 times the original value, and the rear damping coefficient was found to be 1.005 times the original value.
 - (b) The optimal three SDS front spring stiffness was found to be 115.150 N/mm (1.001 times the original value), and the rear spring stiffness was found to be 54.100 N/mm (0.373 times the original value). The front damping coefficient was found to be 1.271 times the original value, and the rear damping coefficient was found to be 1.014 times the original value. The optimal values for ARB stiffness was found to be 46.929 Nm/deg and 59.219 Nm/deg for the front and rear respectively.
 - (c) The optimal 3 SDS model travelled 4.38 m less than optimal 2 SDS model, showing an increase in RMS pitch angle and a reduction in RMS roll angle. However, the variance of pitch angle for the 3 SDS model was less than that of the 2 SDS model.
 - (d) The front pitch stiffness values of the optimised models were found to be 74.638 kNm/deg and 51.755 kNm/deg for the 2 and 3 SDS models respectively. The rear

pitch stiffness values found to be 15.228 kNm/deg and 30.474 kNm/deg for the 2 and 3 SDS models respectively.

- (e) The combined roll stiffness values for the 2 and 3 SDS models were found to be 3.276 kNm/deg and 2.583 kNm/deg respectively.

The model of the BC LMP2 with the optimised 3 SDS system was shown to not be able to match the performance of the optimised 2 SDS model on Zwartkops Raceway. However as discussed in Section 2.1, Zwartkops Raceway is not an ideal circuit for high downforce vehicle due to it being a short circuit with tight corners. Additionally, as highlighted in Section 5.4, the computational requirements to conduct a comprehensive analysis of the combinations of suspension parameters is prohibitive.

It was shown that a 3 SDS model is capable of adjusting pitch and roll stiffness values in isolation, without sacrificing the other. This has particular relevance to high downforce vehicles, where the magnitude of the aerodynamic forces that are developed is highly sensitive to pitch angle.

The third spring and dampers in a 3 SDS system are able to control vehicle pitch, especially under braking, which in this study was found to be 1.5 times greater than the acceleration under driving. The third spring and damper can be adjusted such that they manage the aerodynamic forces, without also influencing the wheel rate of the suspension. This allows for a softer wheel rate, to ensure that the wheels remain in contact with the road surface to maximise traction, as well as the required pitch stiffness to manage the aerodynamic forces.

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Appendix A - VEHICLE PARAMETERS FOR ADAMS/CAR

The vehicle parameters used to create the BC LMP2 model in ADAMS/Car are given below. The ADAMS/Car model including both the 2 and 3 SDS systems are contained in the Digital Appendix which can be accessed through the following URL:

<https://drive.google.com/open?id=0B3dYunQGyC9wbGYwNzR0MF8tTjg>

For any further information regarding the ADAMS/Car model of the BC LMP2, the author can be contacted at the following email address: robjberman@gmail.com

The hard points for the BC LMP2 suspension are shown in Table [A.1](#) and Table [A.2](#) below for the Front and Rear suspensions, respectively. These hard points were imported into ADAMS/Car to create the software model of the vehicle.

The BC LMP2 engine was tested on a dynamometer, and the resulting Torque and Power curves are shown below in Figure [A.1](#) and in Figure [A.2](#), respectively. The dampers fitted on the BC LMP2 were also tested and the resulting damper curves obtained for the front and rear dampers are shown below in Figure [A.3](#).

Table A.1: Front Suspension Hard Points

	x	y	z
ARB blade pivot at 'T'	-238.12	0.00	558.72
ARB 'T' pivot	-214.70	0.00	365.13
ARB 'T' pivot axis	-439.10	0.00	227.00
Third damper at ARB	-189.86	0.00	568.85
Third damper at chassis	146.85	0.00	616.01
ARB link at blade	-271.66	-117.99	561.06
ARB link at rocker	-127.34	-201.52	592.11
Left damper at chassis	206.63	-306.25	620.99
Left damper at rocker	-127.34	-201.52	592.11
LCA front	-162.00	-197.93	161.55
LCA outer	0.50	-785.40	186.87
LCA rear	280.00	-340.00	169.60
Pushrod inner	-160.00	-295.00	510.00
Pushrod outer	-7.50	-712.50	219.20
Rocker pivot	-115.56	-284.32	533.18
Rocker pivot axis	-111.95	-274.77	521.57
Tierod inner	-88.00	-357.88	435.12
Tierod outer	-67.58	-783.01	486.13
UCA front	-33.51	-370.16	437.26
UCA outer	37.42	-755.51	486.11
UCA rear	235.06	-430.72	441.90
Wheel centre	0.00	-822.50	338.04

Table A.2: Rear Suspension Hard Points

	x	y	z
ARB blade pivot at 'T'	3119.42	0.00	538.69
ARB 'T' pivot	3119.42	0.00	377.54
Third damper at ARB	3110.00	0.00	538.69
Third damper at chassis	2900.00	0.00	539.12
ARB link at blade	3119.42	-75.00	538.69
ARB link at rocker	2845.77	-28.36	563.32
Left damper at chassis	2541.65	-84.00	539.12
Left damper at rocker	2808.02	-43.53	556.72
LCA front	2515.93	-138.07	208.06
LCA outer	2885.00	-805.21	194.00
LCA rear	3061.70	-130.96	205.49
Pushrod inner	2836.60	-148.38	511.13
Pushrod outer	2872.25	-678.11	233.28
Rocker pivot	2787.50	-134.93	516.98
Rocker pivot axis	2787.50	-129.48	504.46
Tierod inner	3059.91	-150.60	449.78
Tierod outer	3029.98	-756.54	480.59
UCA front	2549.50	-165.74	442.50
UCA outer	2840.00	-756.50	477.61
UCA rear	3010.00	-152.08	449.07
Wheel centre	2900.00	-810.32	343.78
Drive shaft inner	2900.00	-200.00	343.78

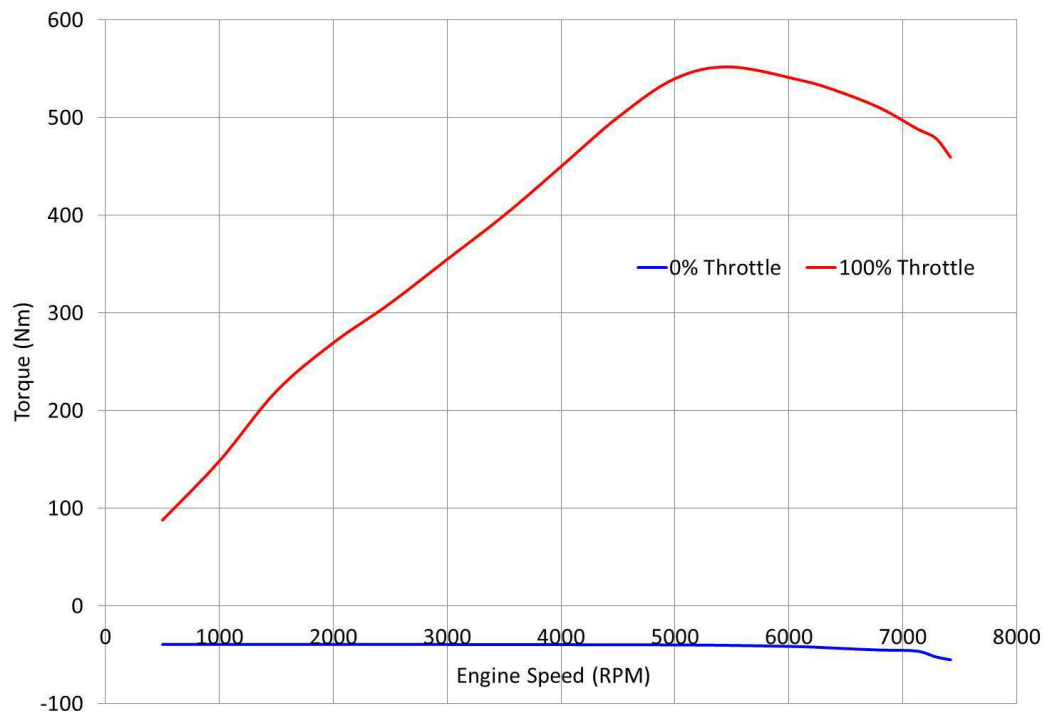


Figure A.1: Engine Torque for the BC LMP2 [26]

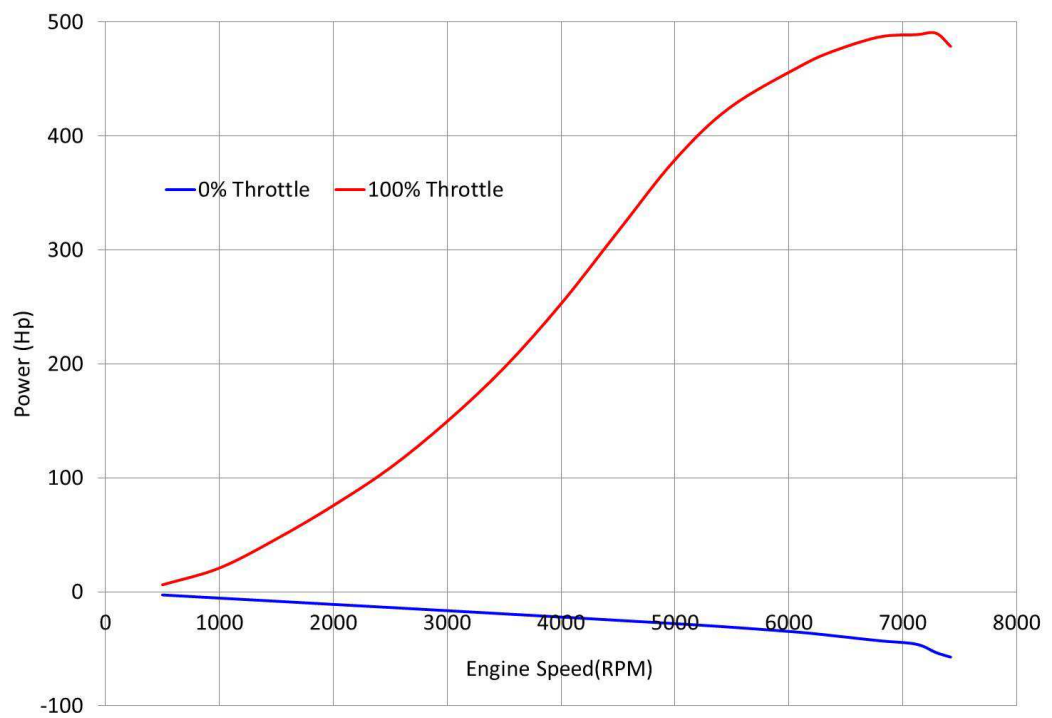


Figure A.2: Engine Power for the BC LMP2 [22]

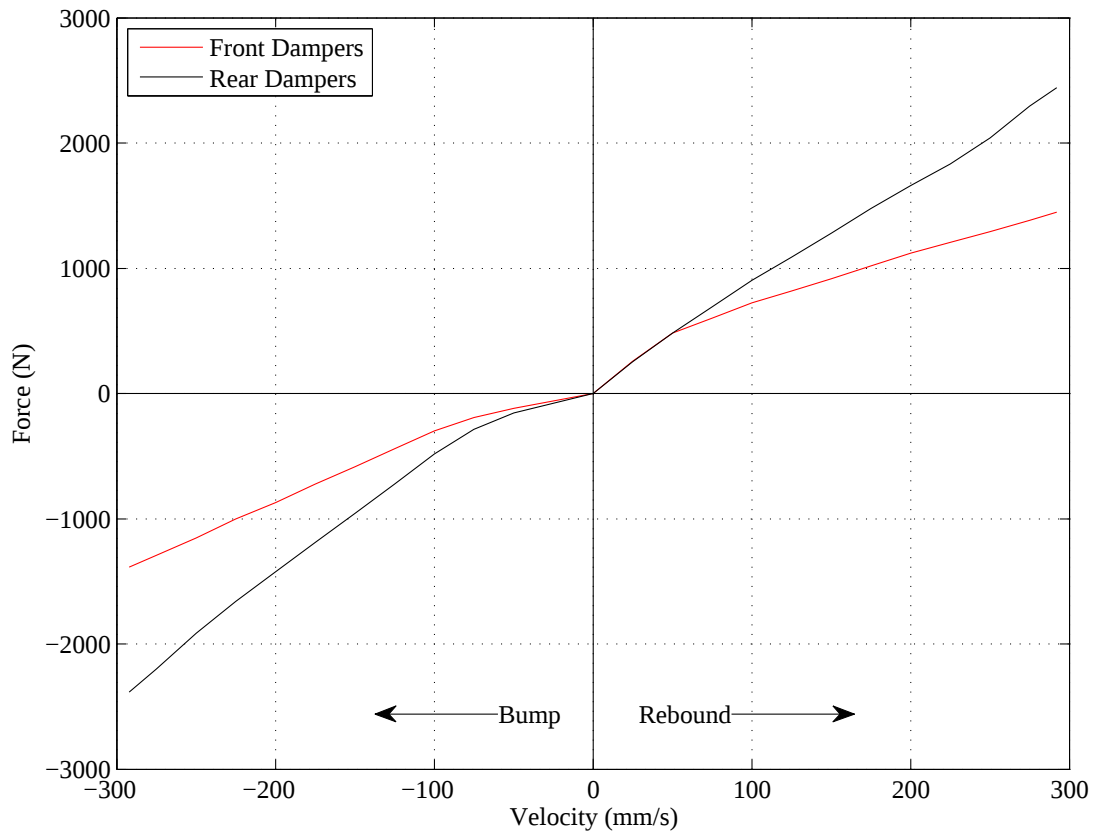


Figure A.3: Damper Curves for the BC LMP2 Front and Rear Dampers [22]

The gear ratios of the Riccardo 6-speed gearbox are given in A.3 [22]. The body, brake,

Table A.3: Gear Ratios

Gear	Ratio
1	0.34
2	0.45
3	0.59
4	0.72
5	0.85
6	0.96
Diff	3.50

powertrain and suspension subsystems were modified from the default subsystems in ADAMS/Car, including mass and moment of inertia values and where no data were available, all parameters were left as default. Table A.4 gives the data that were used for the BC LMP2 model.

Table A.4: Vehicle Mass Properties

Part	Mass (kg)	MOI (kg.mm ²)		
		I _{xx}	I _{yy}	I _{zz}
Chassis	960	2.00E+08	5.00E+08	6.00E+08
Front wheel	12	3.50E+04	3.50E+04	8.00E+03
Rear wheel	15	4.38E+04	4.38E+04	1.00E+04

The Centre of pressure (CoP) of the aerodynamic forces as well as the position of the vehicle's CoG are shown in Figure A.4 below. The functions for drag and downforce according to Stevenson [27] are given in equations A.1 and A.2.

$$F_D = \frac{1}{2}\rho v^2 C_D A_D \quad (\text{A.1})$$

$$F_L = \frac{1}{2}\rho v^2 C_L A_L \quad (\text{A.2})$$

Where:

F_D = Drag force (N)

F_L = Downforce (N)

ρ = Air density (1.225 kg/m³)

v = Velocity (m/s)

C_D = Coefficient of drag

C_L = Coefficient of lift

A_D = Vehicle frontal area

A_L = Effective wing area (2.219 m)

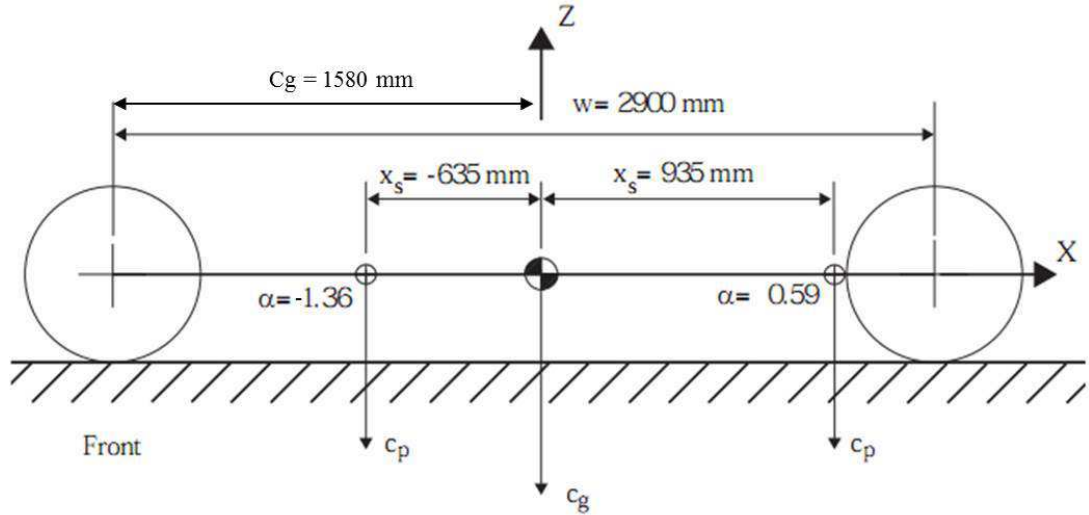


Figure A.4: Centre of Pressure for Downforce [27]

The frontal area, centre of pressure as well as the coefficients of lift and drag all vary according to pitch angle as shown in Figures A.5 to A.7. These parameters were assumed to vary linearly between their bounds stated by Stevenson [27], however limits were placed

on the upper and lower bounds so as to avoid instability in the ADAMS/Car model. The linear functions were approximated by sigmoid functions given by Equations and shown in Figures A.5 to A.8.

$$A_D = \frac{-0.4}{1+e^{\left(\frac{-2\alpha_p+0.7}{1.8}\right)}} + 1.925 \quad (\text{A.3})$$

$$C_p = 1.1 \left[0.16 + \tanh \left(\frac{0.5\alpha_p+0.2}{0.6} \right) \right] \quad (\text{A.4})$$

$$C_l = -0.8 \left[-1.6 + \tanh \left(\frac{0.5\alpha_p+0.2}{0.6} \right) \right] \quad (\text{A.5})$$

$$C_d = \frac{-0.15}{1+e^{\left(\frac{-2\alpha_p+1}{1.4}\right)}} + 0.468 \quad (\text{A.6})$$

Where:

α_p = pitch angle (deg) positive for nose up

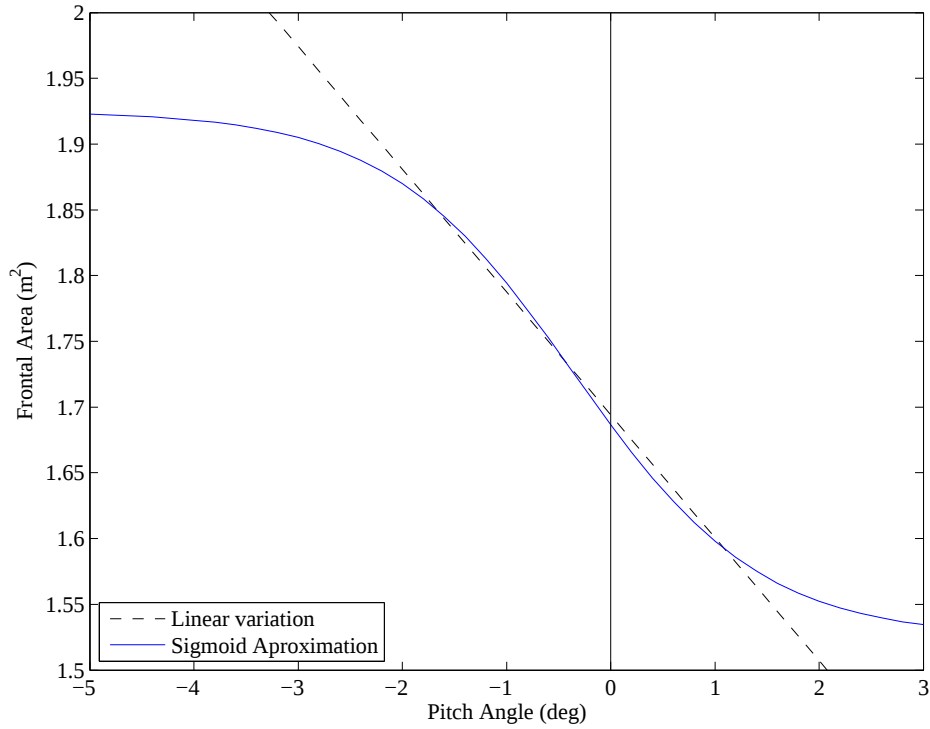


Figure A.5: Variation of Frontal Area with Pitch Angle

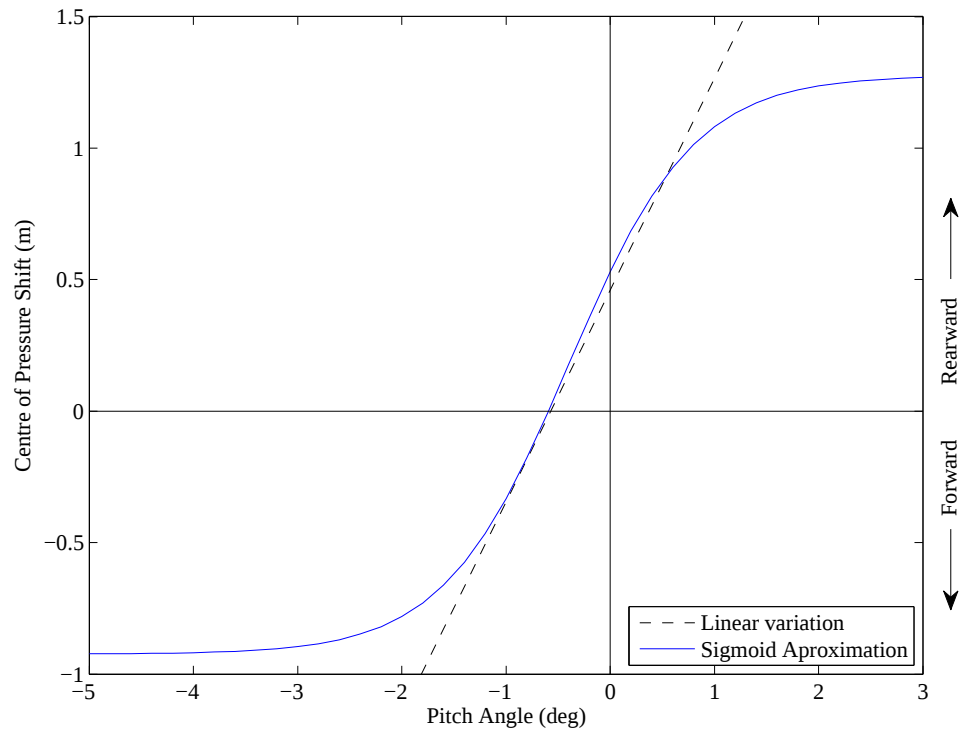


Figure A.6: Variation of Centre of Pressure with Pitch Angle

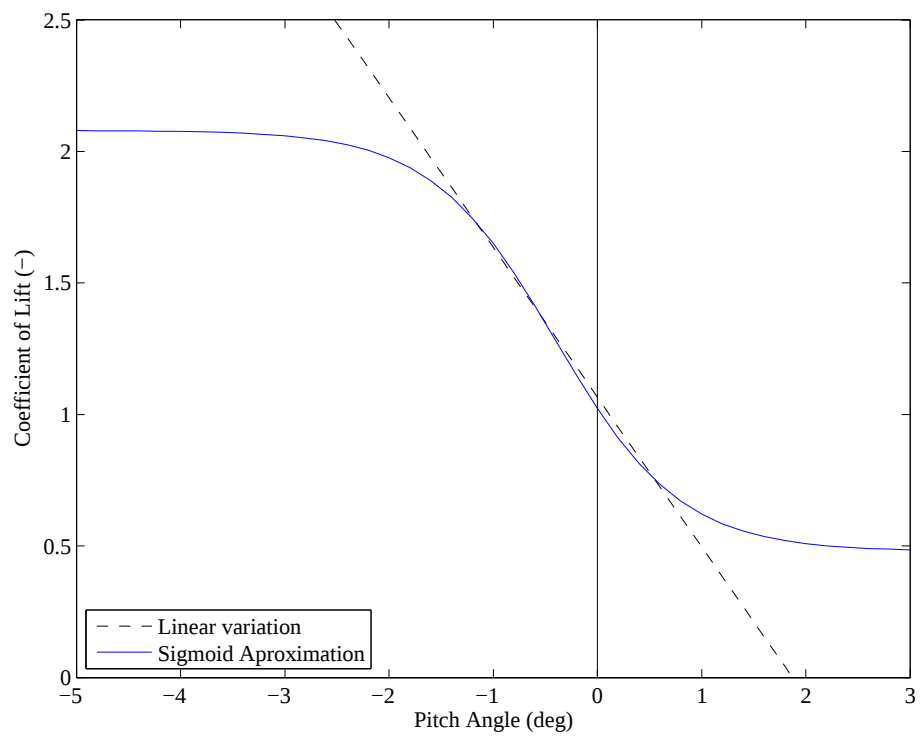


Figure A.7: Variation of Centre of Lift with Pitch Angle

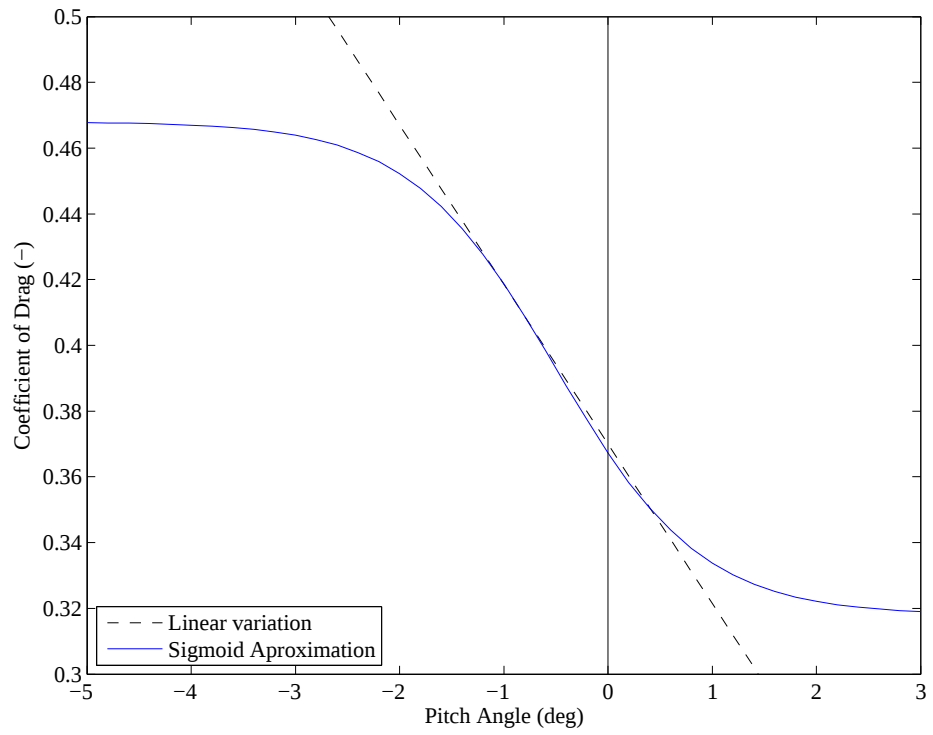


Figure A.8: Variation of Centre of Drag with Pitch Angle

Appendix B - DEVELOPMENT OF THE ZWARTKOPS TRACK FOR ADAMS/CAR

The process used to generate Cartesian coordinates for the centre line of the Zwartkops racetrack will be discussed here.

During the track testing of the vehicle at the Zwartkops raceway, GPS coordinates of the vehicles path were recorded using the Racelogic VBOX [31]. The VBOX captured the coordinates of the path that the vehicle followed around the track. This path was deemed to be the optimal racing line.

The VBOX recorded GPS coordinates in minutes format [31]. In order to import this data into ADAMS/Car, it was necessary to convert from Ellipsoidal coordinates, in minutes, to Cartesian coordinates, in meters. ϕ and λ represent the latitudinal and longitudinal coordinates respectively, and h represents the height. The relation of Spherical to Cartesian coordinates is illustrated in Figure B.1 below.

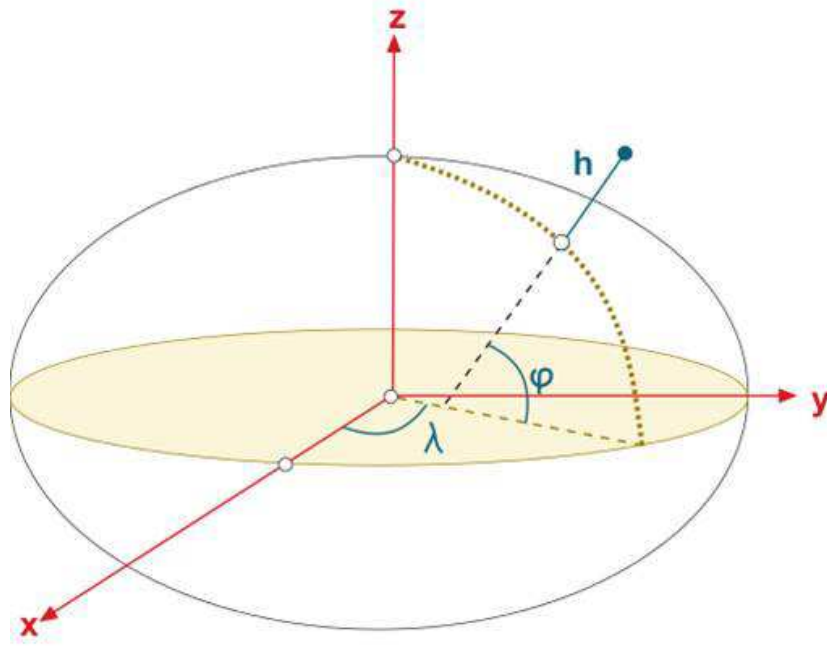


Figure B.1: Relation of Spherical to Cartesian Coordinates [32]

$$x = (N + h) \cos \phi \cos \lambda \quad (\text{B.1})$$

$$y = (N + h) \cos \phi \sin \lambda \quad (\text{B.2})$$

$$z = [(1 - e^2)N + h] \sin \phi \quad (\text{B.3})$$

Where N is the radius of curvature of the prime vertical:

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}} \quad (\text{B.4})$$

According to the World Geodetic System of 1984 [33], flattening is the difference in length between the major and minor axes. The flattening factor f is given by:

$$f = 1 - \frac{b}{a} \quad (\text{B.5})$$

Where: $a = 6,378,137.0$ meters and $b = 6,356,752.31$ meters

The eccentricity, e is given by [32]:

$$e = \frac{a^2 - b^2}{a^2} = 2f - f^2 \quad (\text{B.6})$$

The recorded Spherical coordinates were converted to Cartesian coordinates with the results plotted in Figure B.2 below.

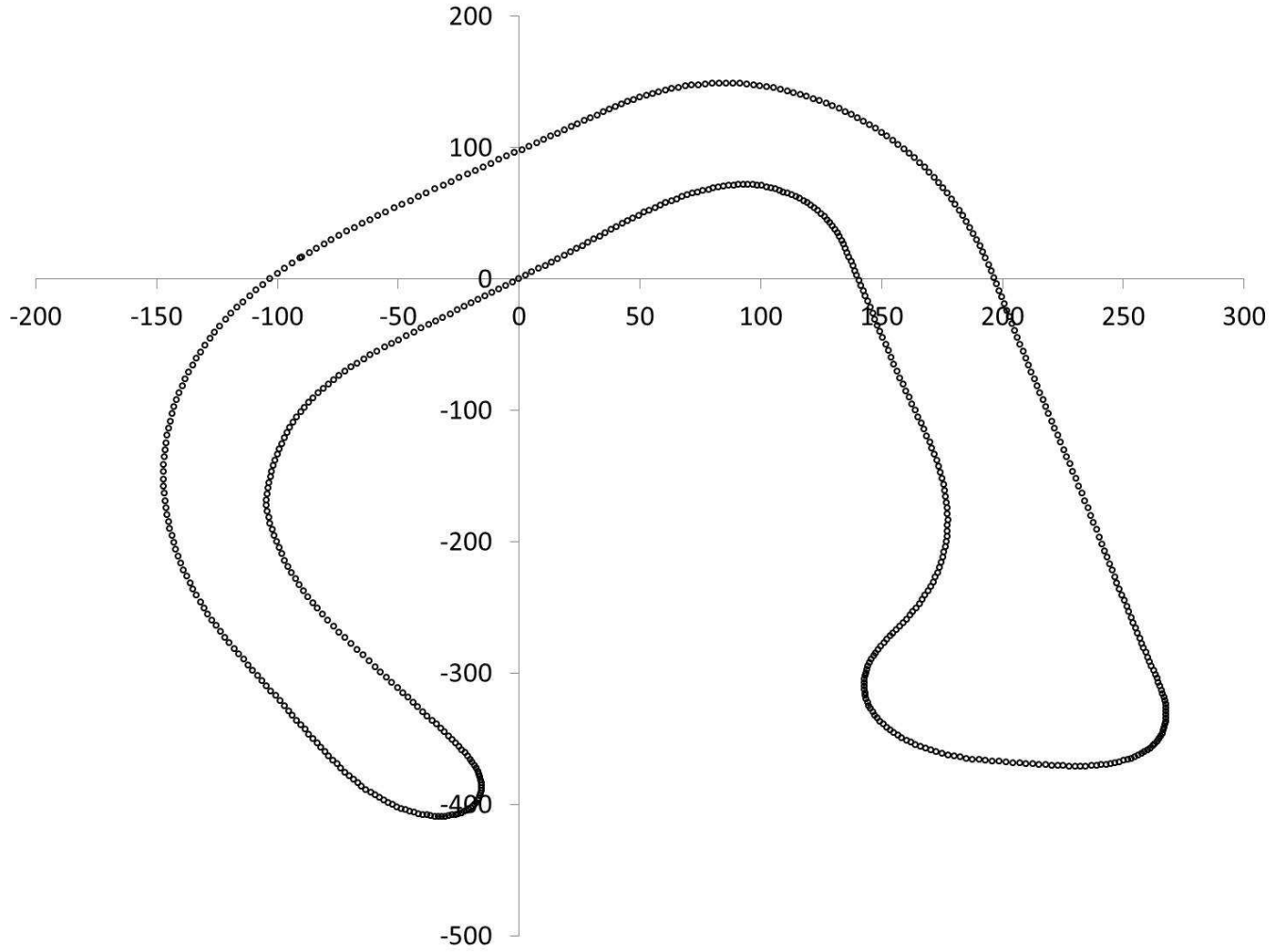


Figure B.2: Zwartkops GPS Data in Cartesian Coordinates

The start/finish line is represented by the origin and the direction of travel is clockwise. During the conversion from Spherical to Cartesian coordinates, the scale of the track was affected. The total length of the track as shown by the VBOX is 2 423 m, and the coordinates were scaled, such that the total distance was equal to this length.

The ADAMS/Car road builder requires that the initial direction of the track be along the X-axis. Thus, the coordinates were rotated using the following conversion:

$$X' = x \cos \theta + y \sin \theta \quad (\text{B.7})$$

$$Y' = -x \sin \theta + y \cos \theta \quad (\text{B.8})$$

The result is shown in Figure B.3 below.

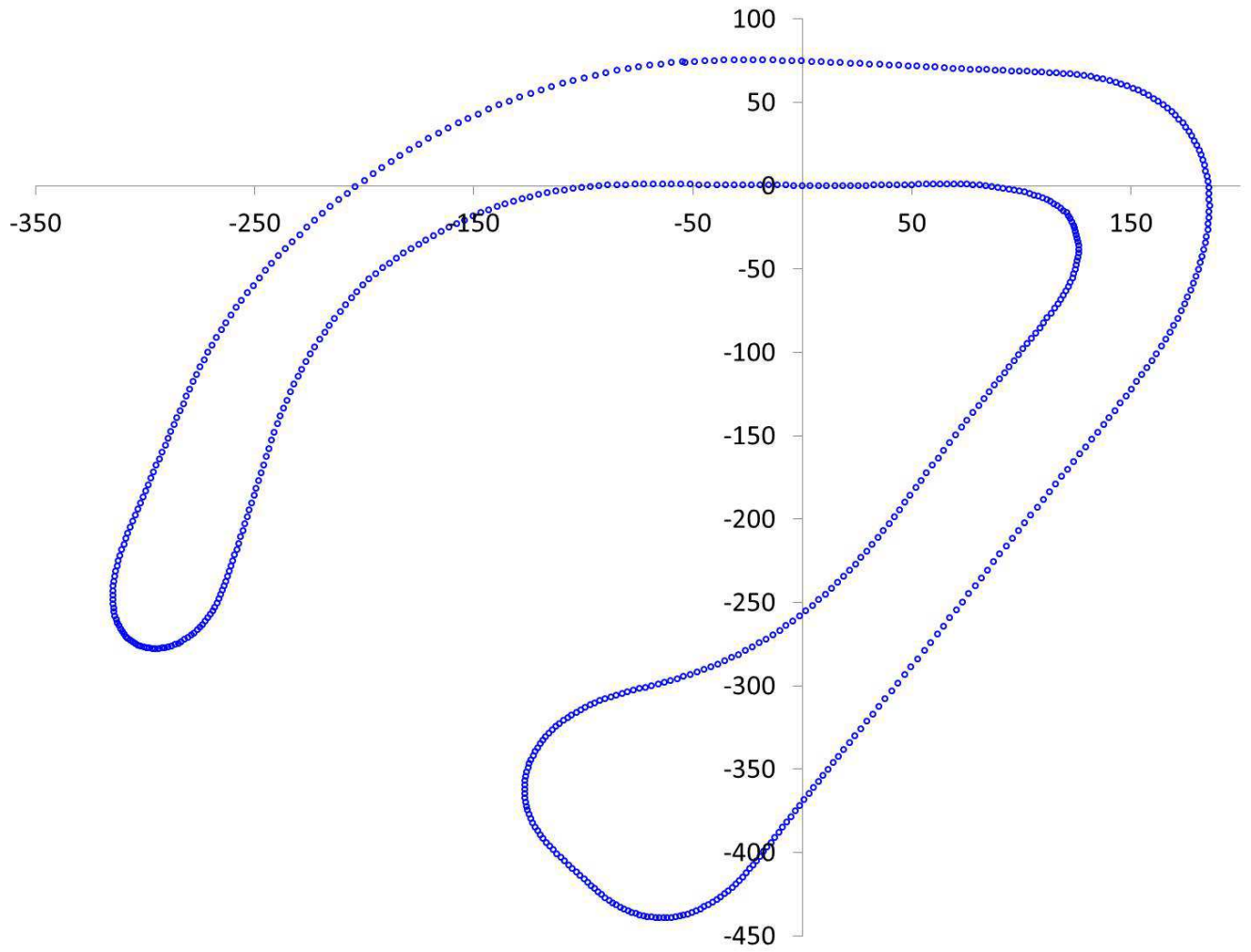


Figure B.3: Zwartkops GPS Coordinates Rotated for ADAMS/Car

The elevation was given by the VBOX in meters and is shown in Figure B.4 below

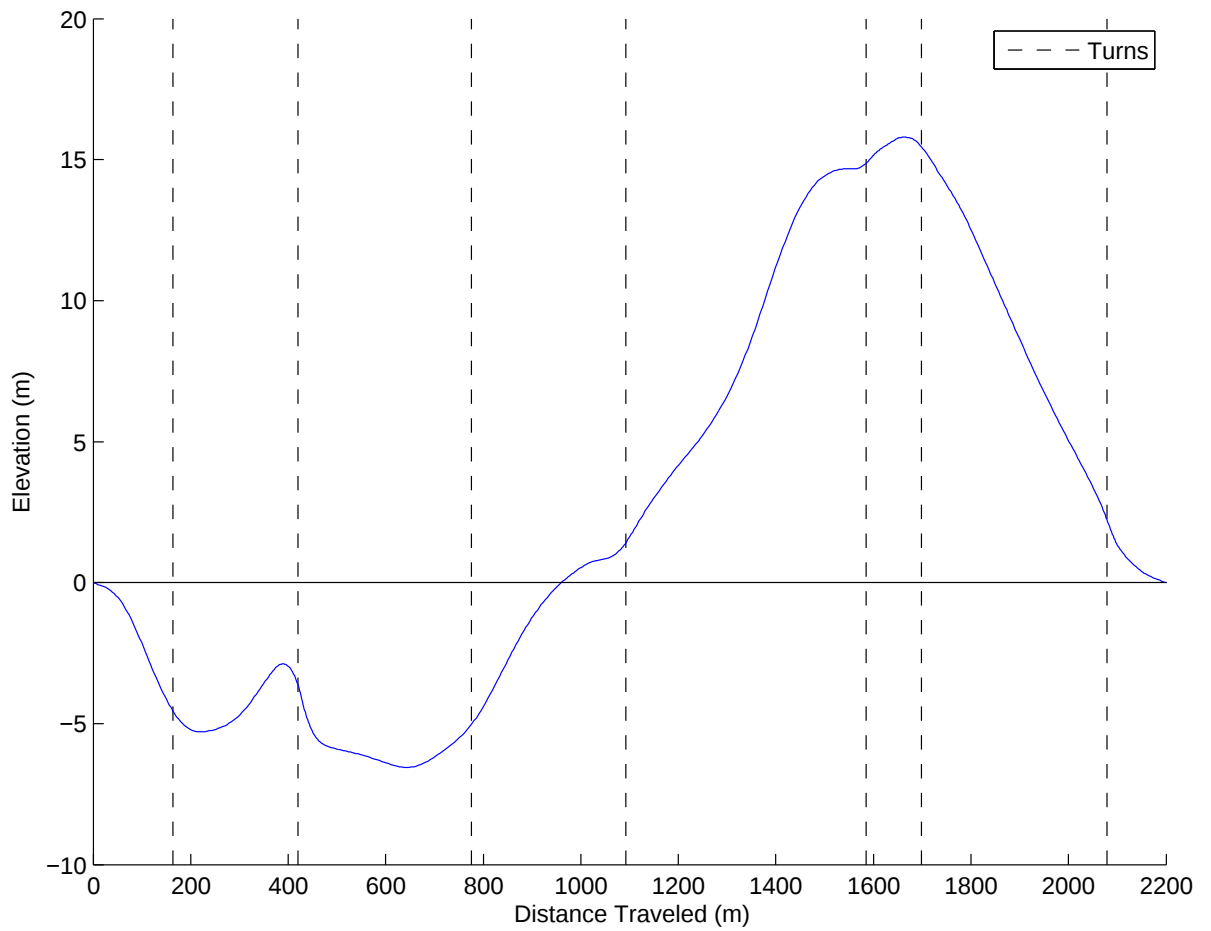


Figure B.4: Zwartkops Elevation: Distance vs Elevation

It was found that the changes in elevation along the lap adversely affected the SmartDriver's ability to control the vehicle. These elevation changes are relatively small, ranging from approximately -6 m between turns two and three, to a maximum of approximately 16 m at the entry to turn six. The most significant incline is situated after turn four in the run up to the tight right handed turn five. The SmartDriver consistently failed to allow a sufficient braking zone coming up to turn five and as a result would lose control of the vehicle at this point. Elevation changes were therefore excluded from the analysis.

Due to some irregularities in the data, a Jacobian smoothing operation was applied to both the track GPS coordinates, as well as the elevation [34]. The smoothing operation was performed through the dataset for a total of 10 times.

The results of the Jacobian smoothing operation on the elevation is shown overlaid to the original in Figure B.5 below. The effect is quite subtle, but has a significant effect in the formulation of the track in the ADAMS/Car road builder, as well as on the SmartDriver function. The smoothing also has the effect of reducing the radius of curvature of the corners, but it was deemed that this was acceptable in light of the overall effect that the smoothing operation had.

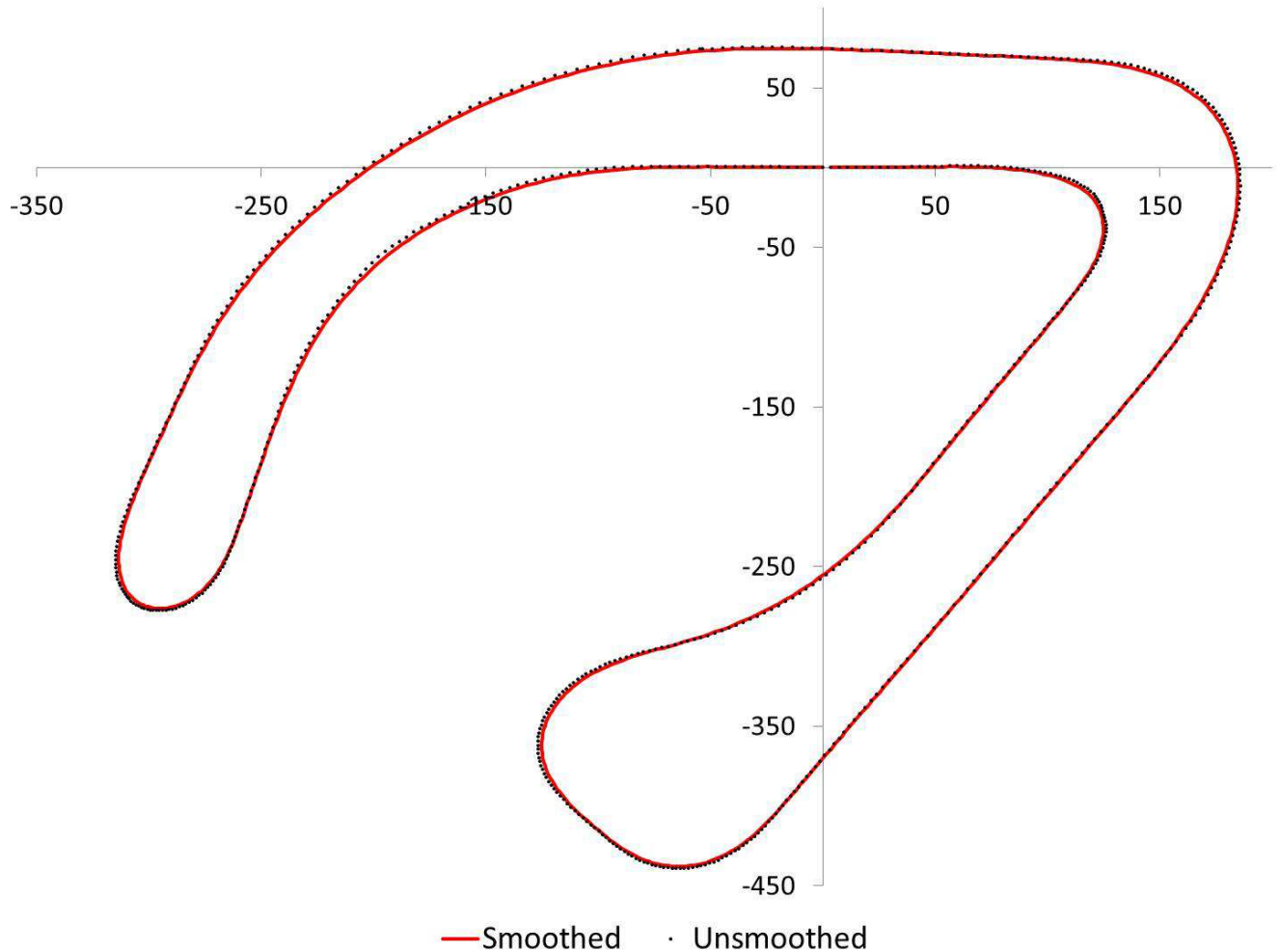


Figure B.5: Jacobian Smoothing for Zwartkops track

The path that the SmartDriver function seeks to follow can be generated automatically through an optimisation in ADAMS/Car. However in this case, the path that the driver followed around the Zwartkops track was deemed to be the optimal path around the circuit. This was deemed acceptable as the driver, Hennie Groenewald, is a highly accomplished racing driver and is well acquainted with the Zwartkops track and its nuances. The

coordinates used to create the Zwartkops track were imported into the path file and effectively represent the centre line of the ADAMS/Car track.

Appendix C - TYRE TESTING, EQUIPMENT AND DATA

The ARMSCOR/CSIR Landward Sciences Medium Tyre Tester is shown in Figure C.1 below. It consists of two main components, the body and a floating sub-frame. The sub frame is suspended from the main body by six load cells: three vertical, two lateral and a single longitudinal load cell. Two of the vertical and the longitudinal load cells can clearly be seen in the figure.

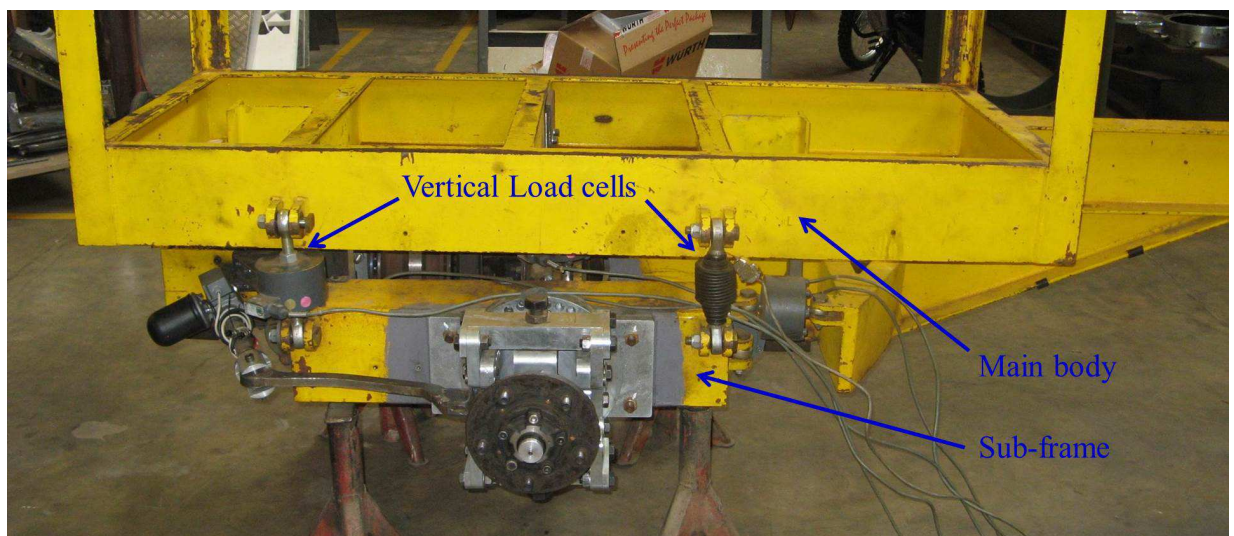
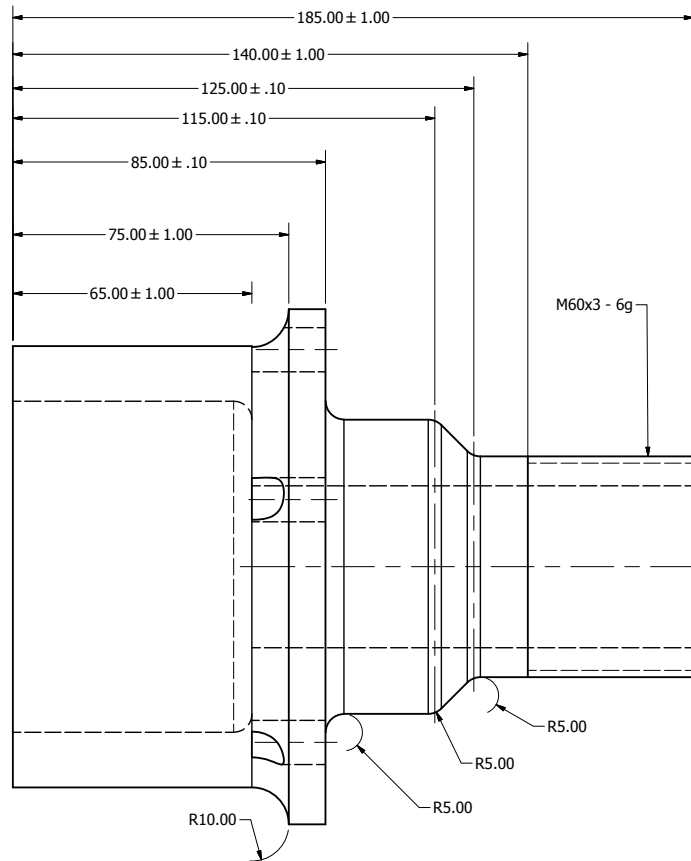


Figure C.1: The ARMSCOR/CSIR Landward Sciences Medium Tyre Tester

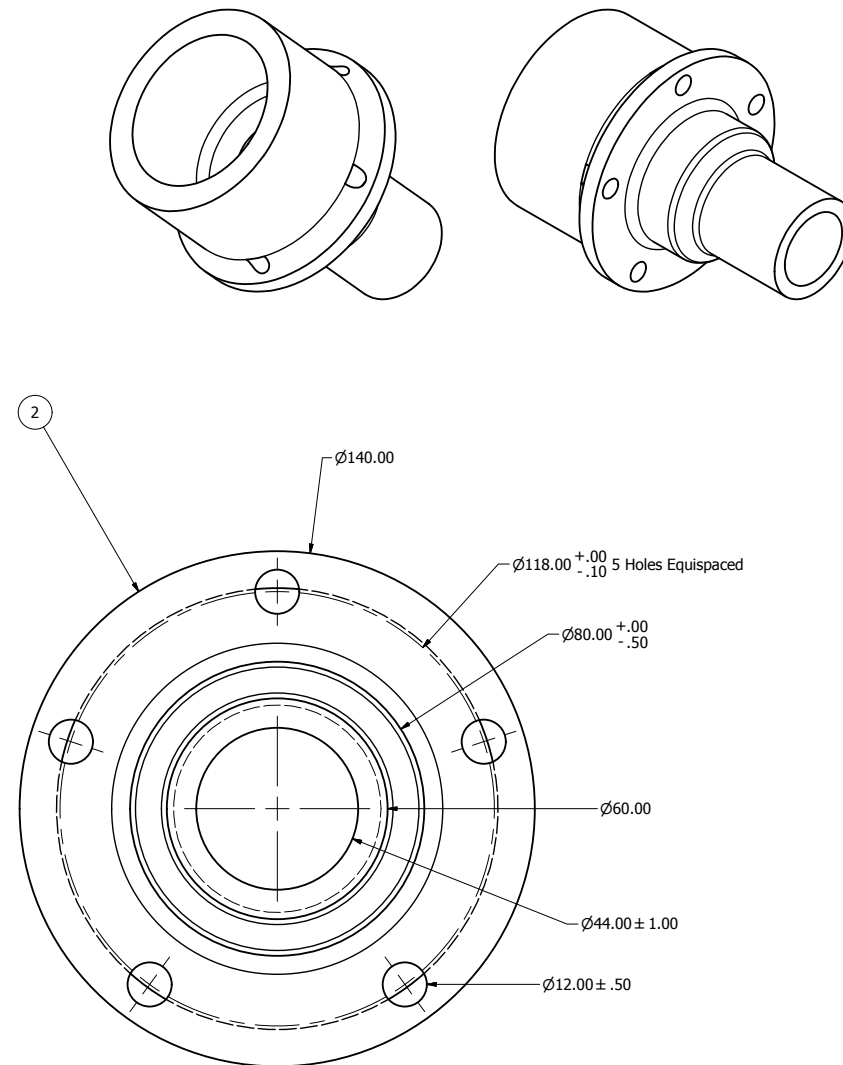
C.1 Wheel Adapters

The wheel adapters designed to attach the BC wheels to the tyre tester are shown on the following two pages.

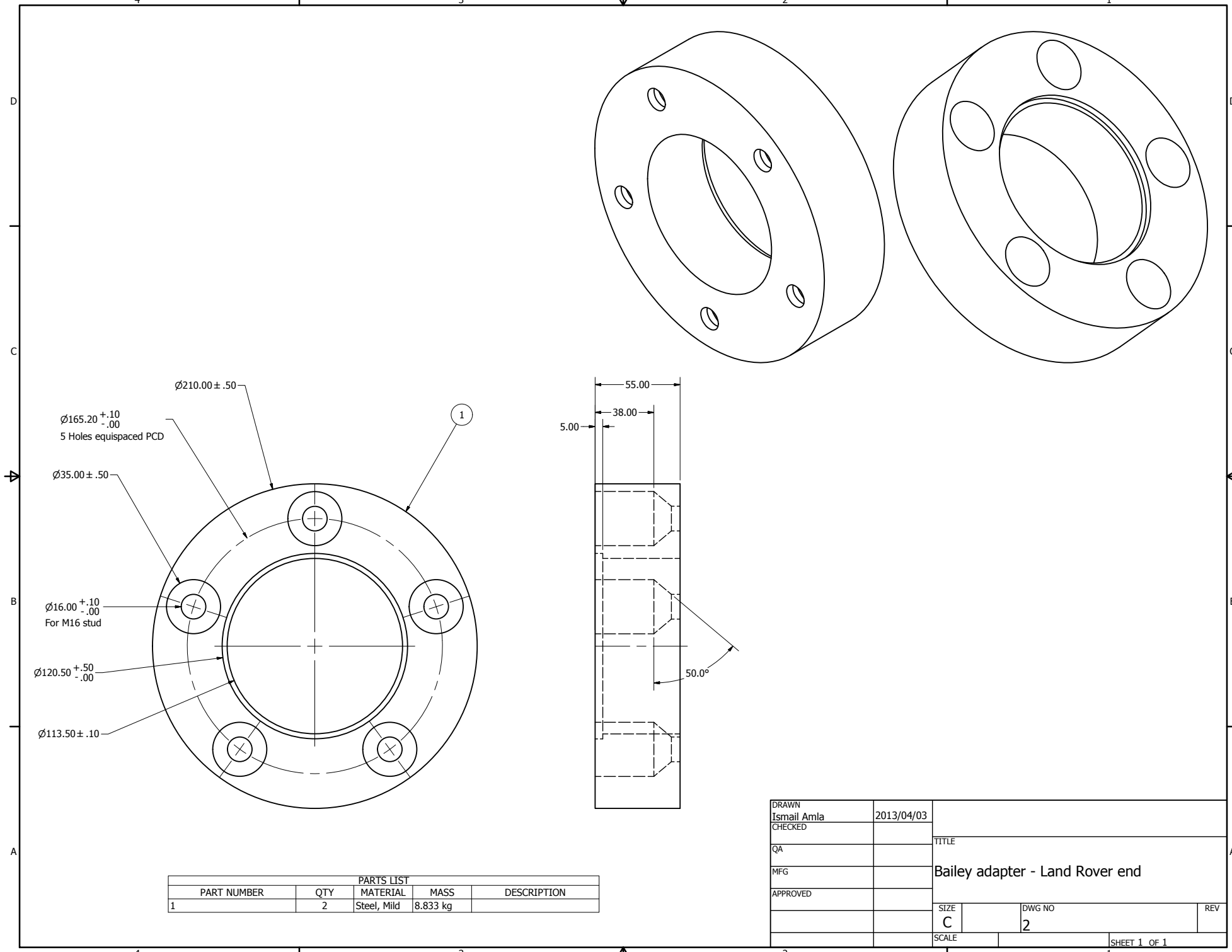


All dimensions are in mm
Tolerance of +/- 1mm if not specified
Material is EN 8

PARTS LIST				
PART NUMBER	QTY	MASS	MATERIAL	DESCRIPTION
2	2	6.027 kg	Steel, Mild -EN 8	EN 8



DRAWN	Ismail	2013/04/03	TITLE	
CHECKED				
QA			Bailey adapter front- Attachment to Bailey wheel	
MFG				
APPROVED			SIZE	DWG NO
			C	1
			SCALE	SHEET 1 OF 1



PARTS LIST				
PART NUMBER	QTY	MATERIAL	MASS	DESCRIPTION
1	2	Steel, Mild	8.833 kg	

DRAWN	Ismail Amla	2013/04/03	TITLE	
CHECKED				
QA				
MFG				
APPROVED			Bailey adapter - Land Rover end	
			SIZE	DWG NO
			C	2
			SCALE	REV
				1

Figure C.2 below shows the wheel adapter attached to the tyre tester.



Figure C.2: Hankook Wheel Adapter on the Tyre Tester

C.2 Load Cell Calibration

Each load cell was individually removed from the tyre tester and calibrated as follows. In each case, the tyre tester was used as the ballast, as shown in figure C.3. The load was determined using an AC USA scale, with the load reported in kg. The load cell was connected to the eDAQ, and using the eDAQ software, Test Control Environment v18 (TCE), the calibration was performed by inputting the load of the ballast measured by the scale. In this process, two known loads were applied to each load cell, whilst the eDAQ recorded the corresponding voltage, and the calibration automatically obtained.

The calibration was added to each channel in the eDAQ, with the outputs given as load in newtons.



Figure C.3: Calibration of the load cells

C.3 Slip Angle Calibration

The calibration of the slip angle was done by first fixing a bar of known length (L) to the hub, setting the slip angle to a minimum and measuring the perpendicular distance of each end of the bar to the sub frame. This process is illustrated in Figure C.4 below. This was repeated throughout the entire range of slip angles, whilst simultaneously recording the string potentiometer voltage.

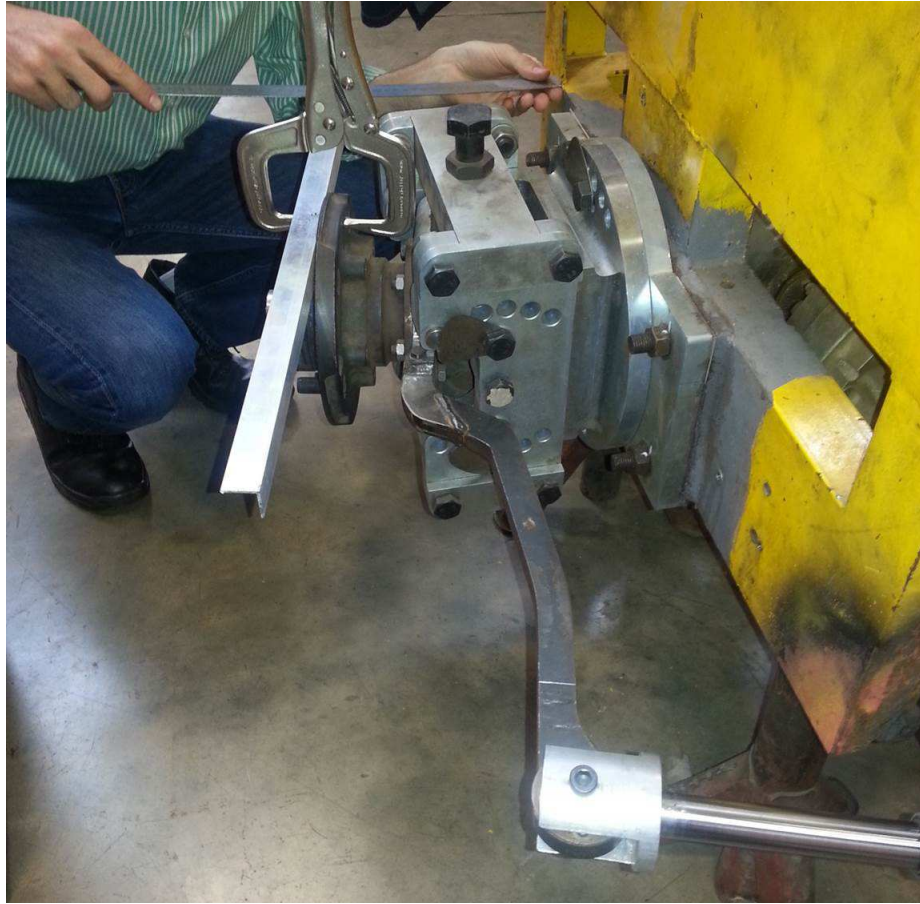


Figure C.4: Calibration of the Slip Angle

The calculation of slip angle is shown in Figure C.5 below. where:

$$\theta = \arctan \frac{(b - a)}{L} \quad (C.1)$$

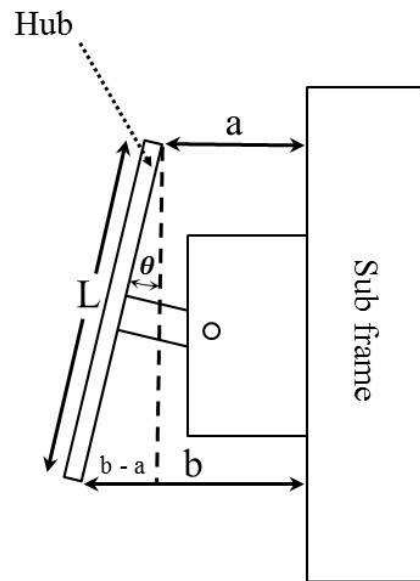


Figure C.5: Calculation of the Slip Angle

The calibration curve of slip angle vs. voltage for the test side (right) is given in Figure C.6 below.

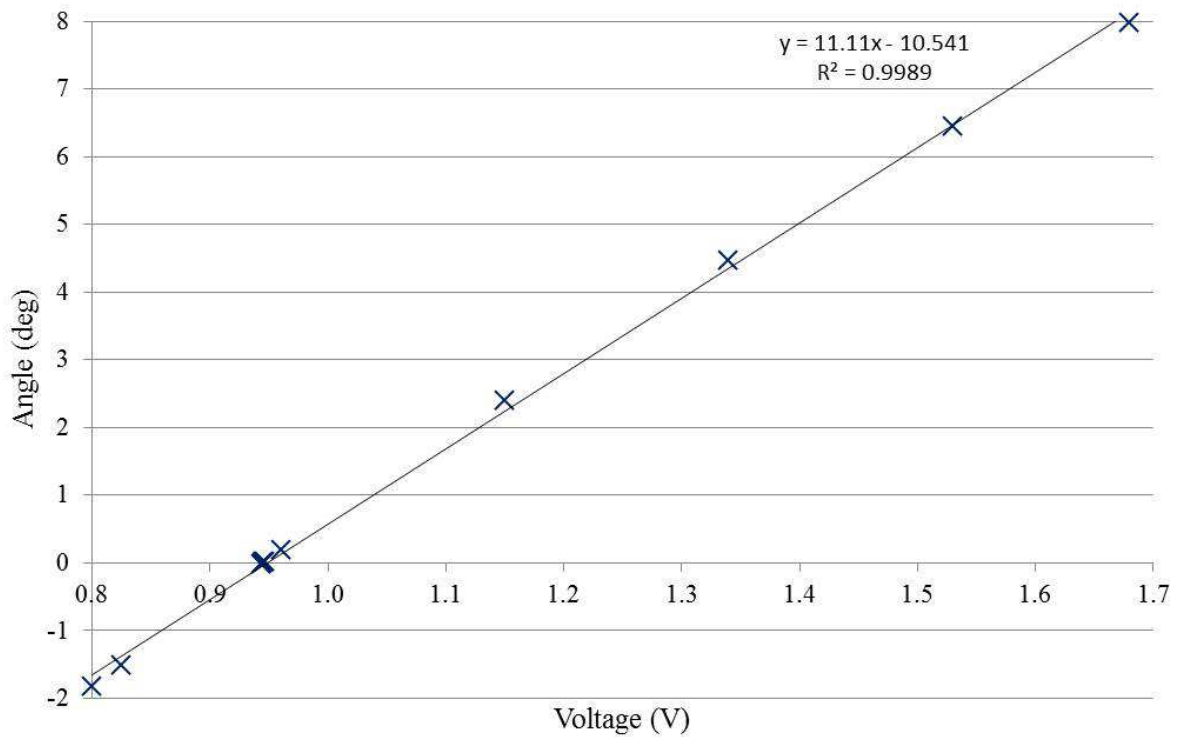


Figure C.6: Calibration Curve of Slip Angle vs. Voltage

C.4 Testing

Tyre testing was carried out at the Gerotek Vehicle testing facility in Pretoria West. The long straight was used as the test track. The tyre tester was towed by a SAMIL 100, as shown in Figure C.7 below. The test speed was approximately 20 kph.



Figure C.7: SAMIL 100 Towing the Tyre Tester

The recorded data for the all Runs are given in Figures C.8 to C.23 below.

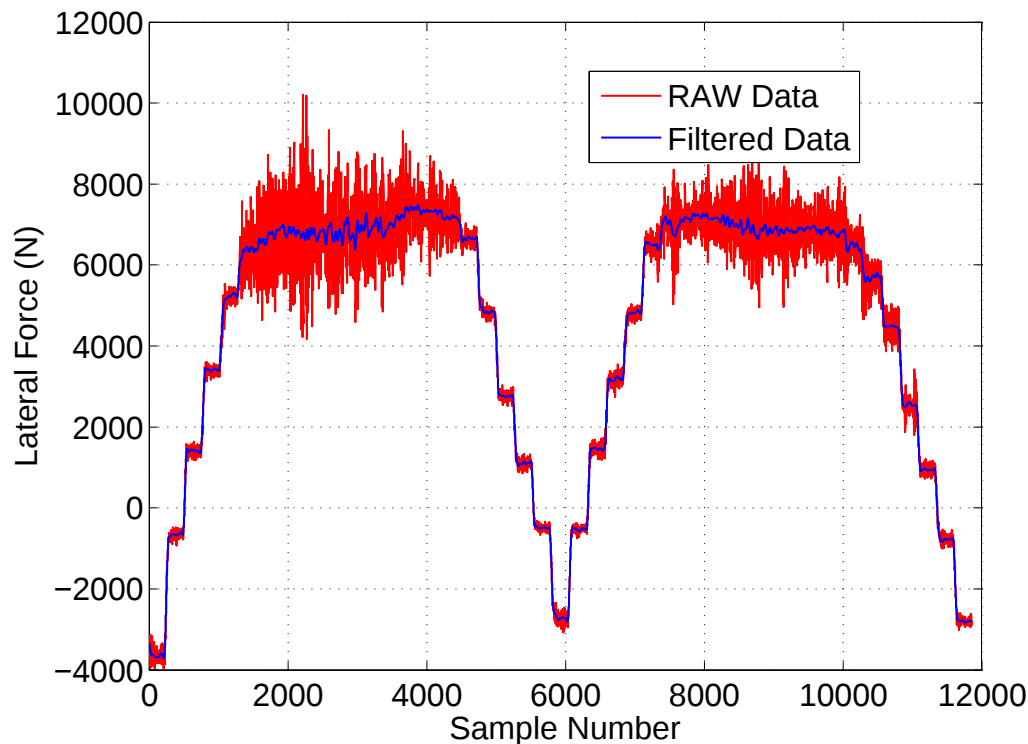


Figure C.8: Recorded Lateral Force Data for Run 1

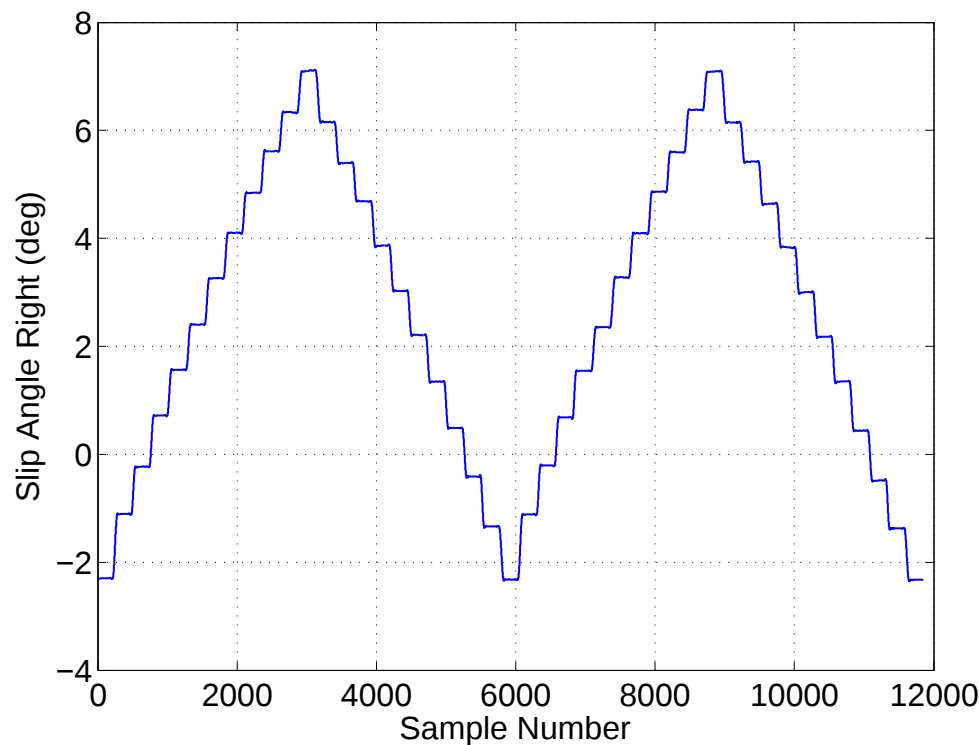


Figure C.9: Recorded Lateral Force Data for Run 1

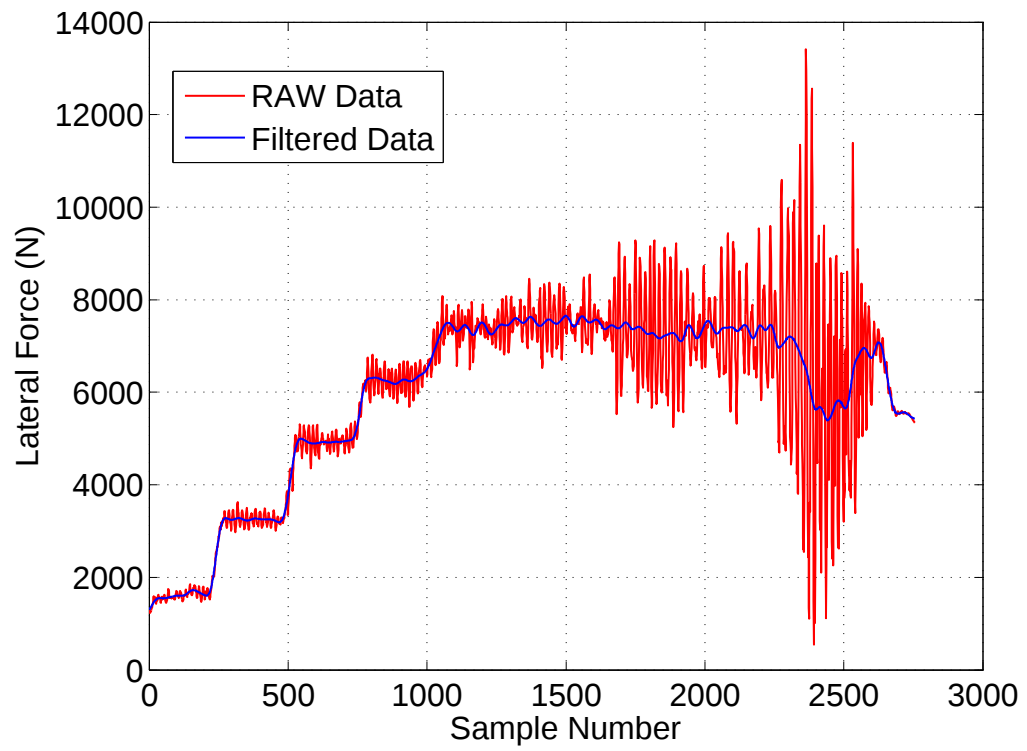


Figure C.10: Recorded Lateral Force Data for Run 2

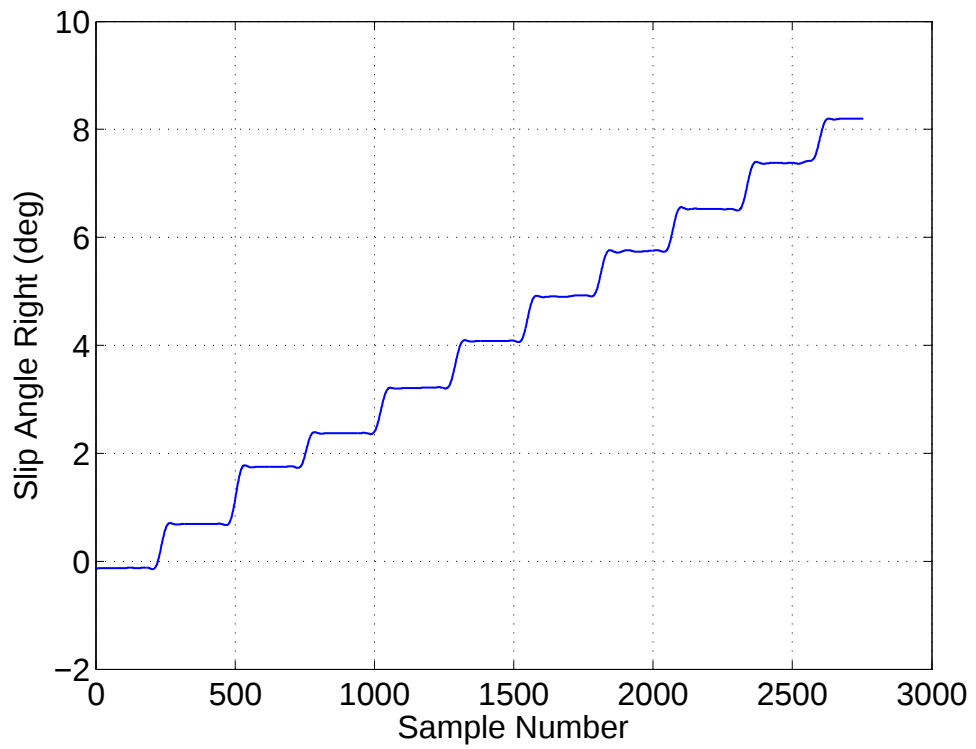


Figure C.11: Recorded Lateral Force Data for Run 2

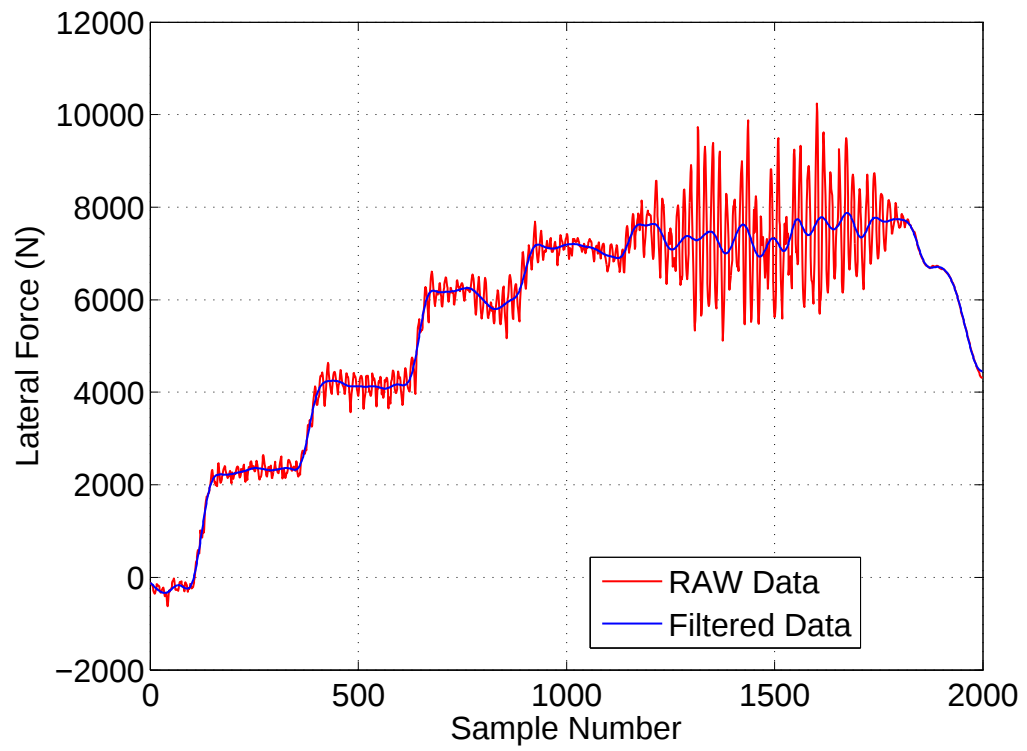


Figure C.12: Recorded Lateral Force Data for Run 3

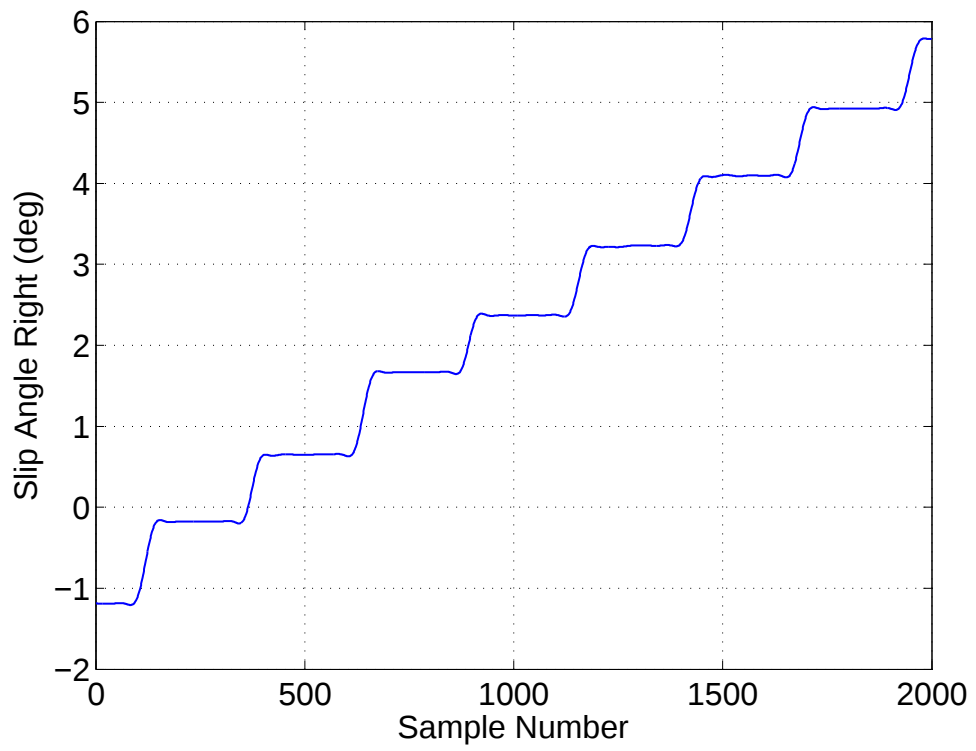


Figure C.13: Recorded Lateral Force Data for Run 3

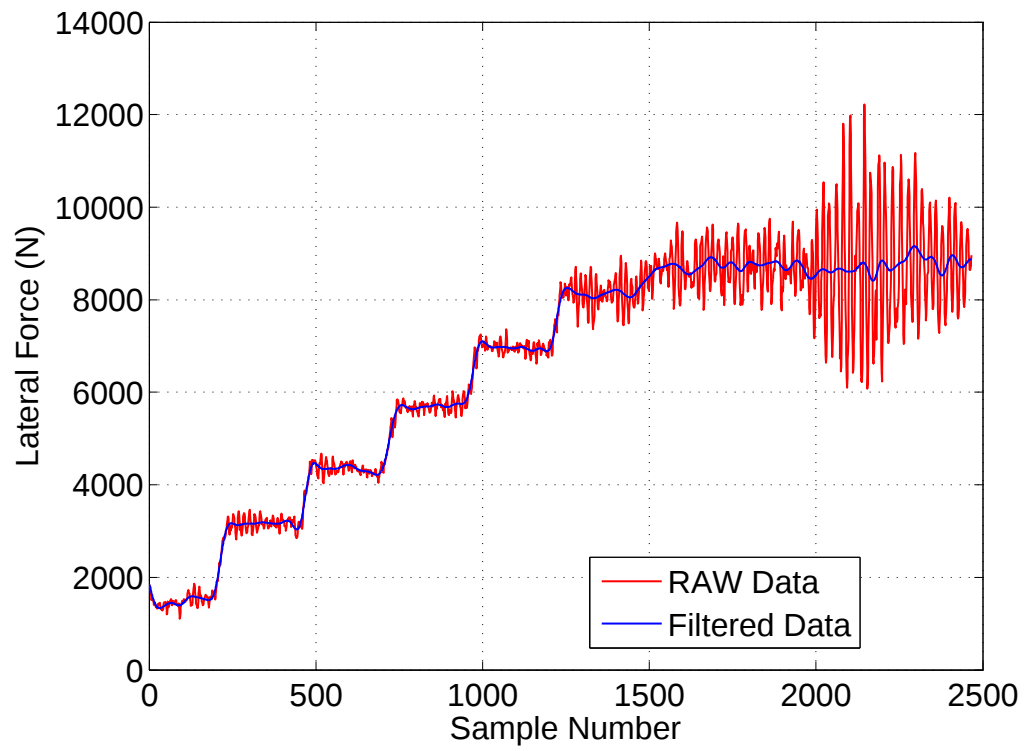


Figure C.14: Recorded Lateral Force Data for Run 4

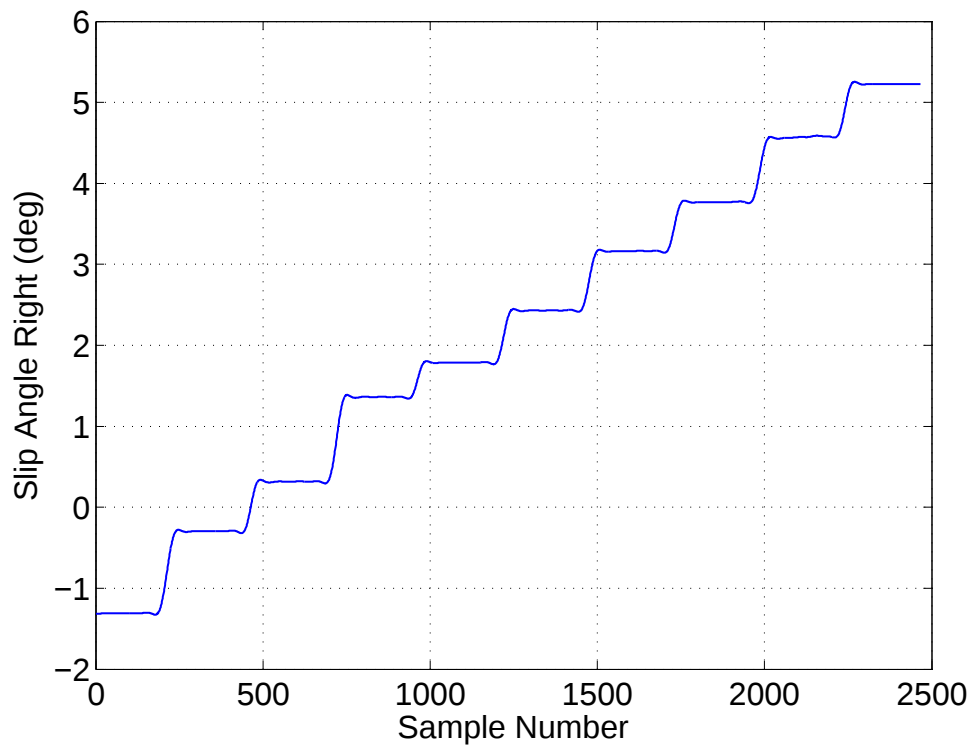


Figure C.15: Recorded Lateral Force Data for Run 4

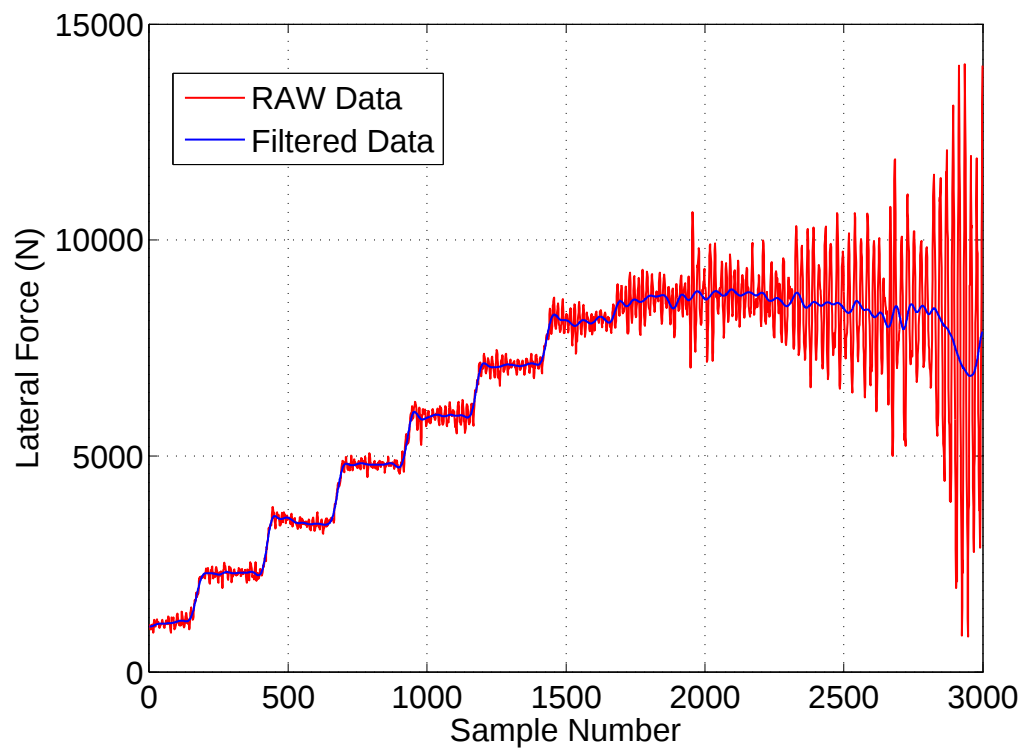


Figure C.16: Recorded Lateral Force Data for Run 5

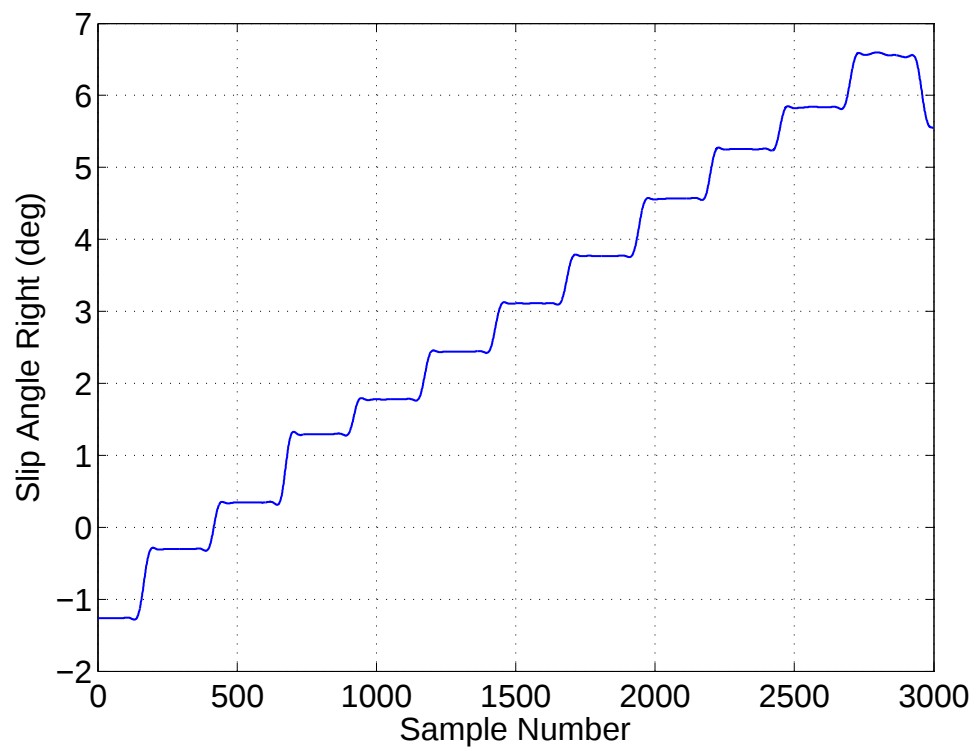


Figure C.17: Recorded Lateral Force Data for Run 5

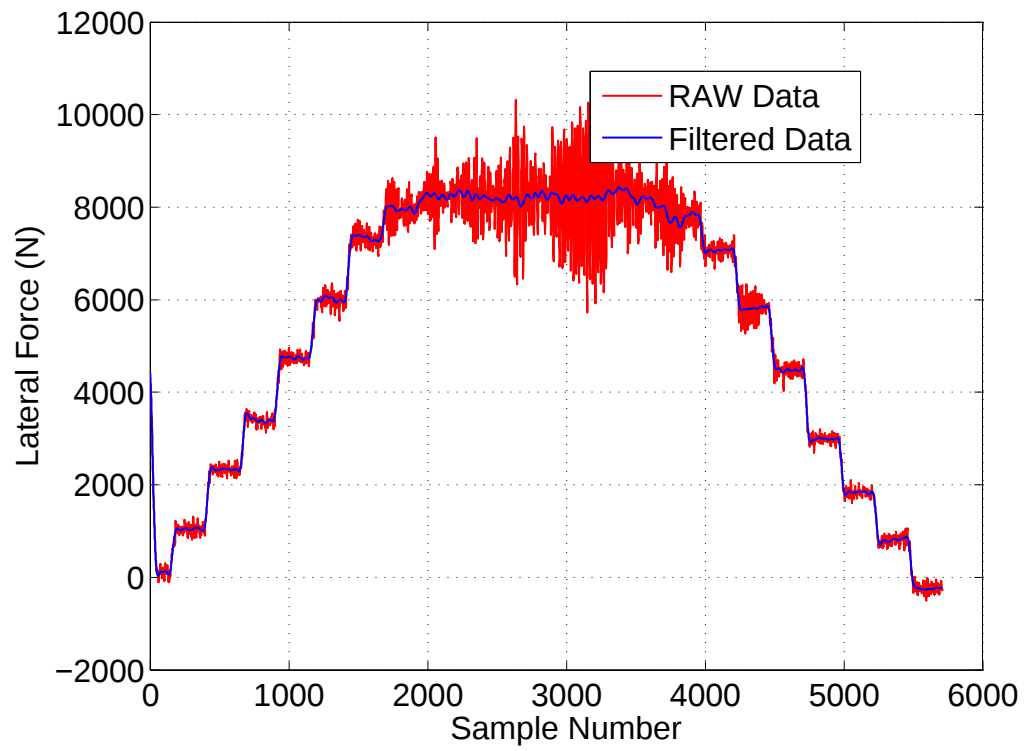


Figure C.18: Recorded Lateral Force Data for Run 6

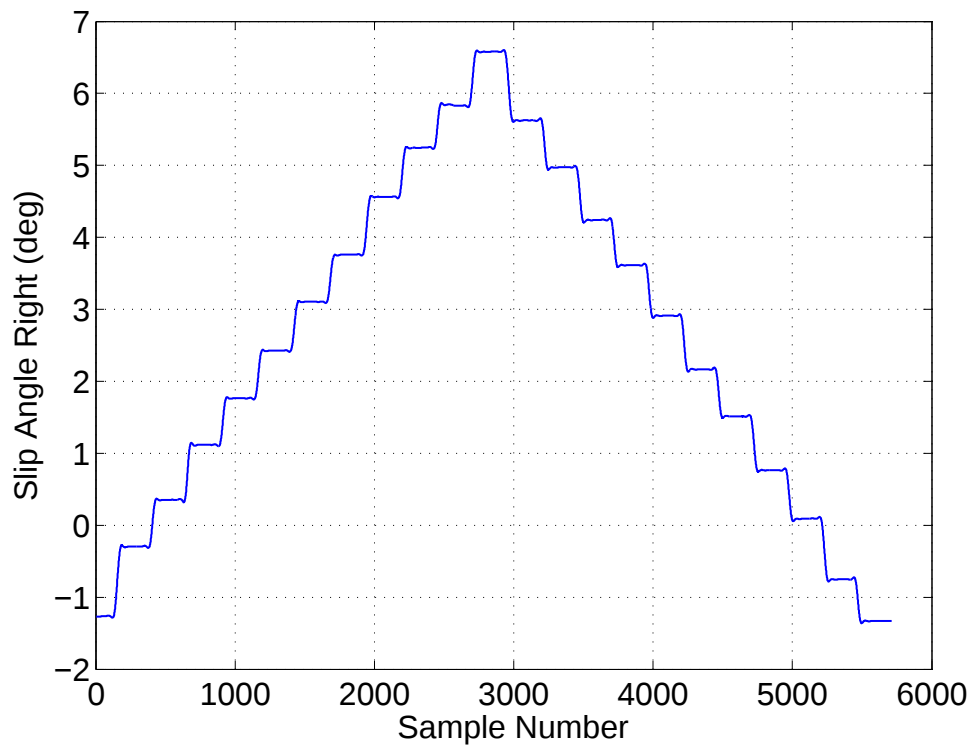


Figure C.19: Recorded Lateral Force Data for Run 6

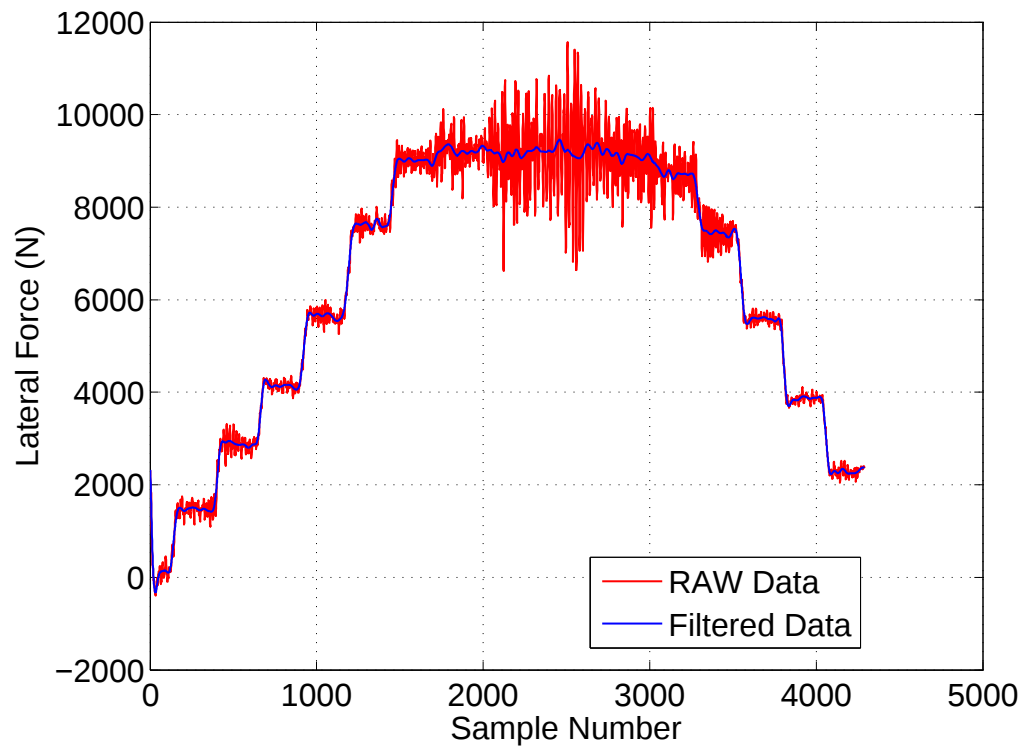


Figure C.20: Recorded Lateral Force Data for Run 7

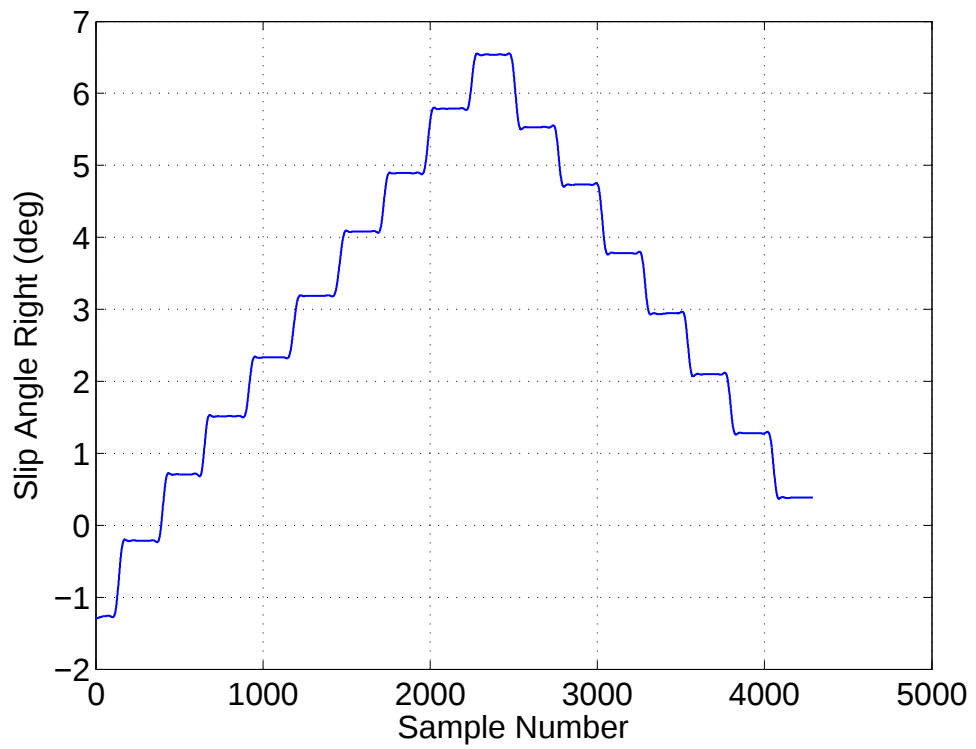


Figure C.21: Recorded Lateral Force Data for Run 7

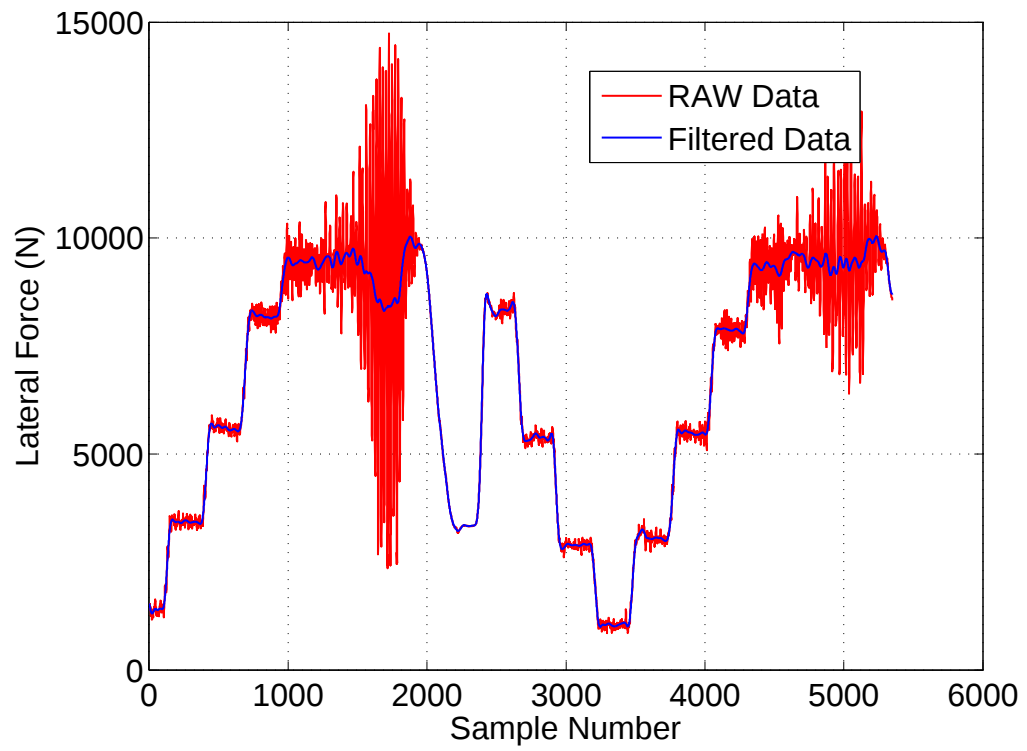


Figure C.22: Recorded Lateral Force Data for Run 8

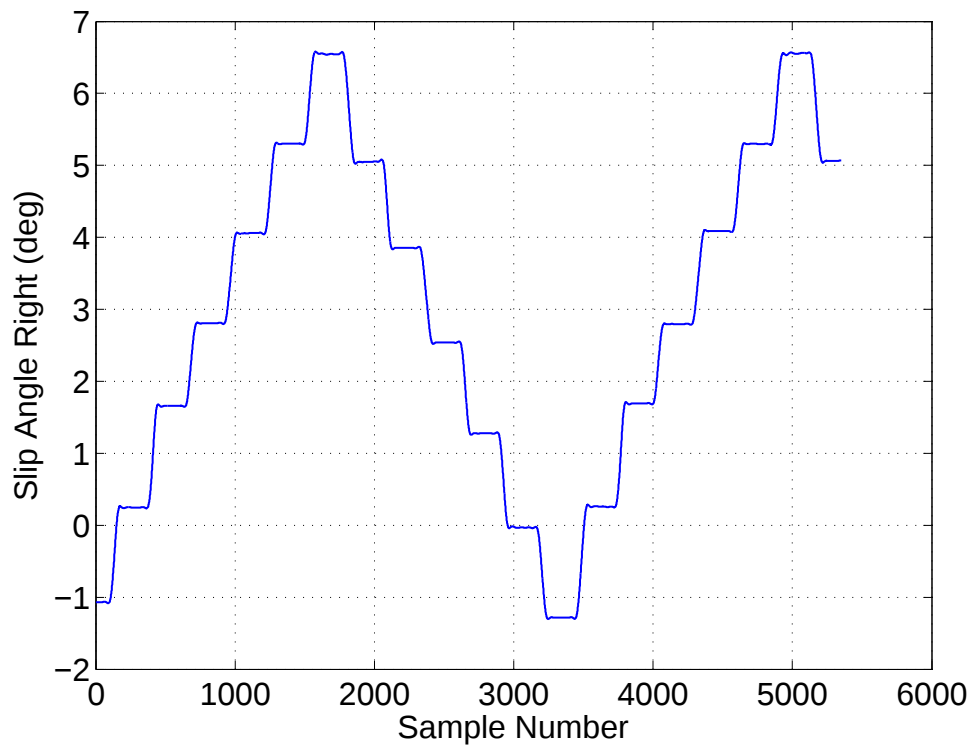


Figure C.23: Recorded Lateral Force Data for Run 8