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An antisymmetrized folding procedure for the interaction between composite particles

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DECLARATION

I declare that this dissertation is my own work performed under the supervision of Professor C. Toepffer and has not been previously submitted for a degree in any other university.

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Abstract

Reactions between composite fragments are described in the generator coordinate method (GCM). In this method the many-body wave function is expanded in terms of totally anti-symmetrized bound state wave functions, describing various given fragments' separation.

Pauli principle and absorption are treated consistently on a microscopic level with the help of the Feshbach projection techniques. The imaginary part of the scattering operator for bound nucleons is obtained in closed form by using internal harmonic oscillator wave functions and simple nucleon-nucleon interactions (like the Skyrme force).

CHAPTER 1.

INTRODUCTION

1.1 General Remarks

Recent years have seen a remarkable progress in the theoretical study of the interaction of composite nuclear particles towards a more genuine microscopic approach. This progress has been made possible because of a better description of the collective motion from first principles. Also computational procedures have been developed which have greatly reduced the technical barriers which are encountered in studying composite particles on a microscopic level. From this standpoint, we can be hopeful that a wide application to various problems can be expected. This work is an attempt to look into the recent work and progress on computational procedures in a unified form and, in particular, a method for the microscopic treatment of the absorption in composite-particle scattering is discussed. The development of the study of the composite-particle interaction provided the means to investigate concepts such as the cluster model in a unified way, where the bound state and scattering problems can be treated on the same footing.

A major contributing factor among those to stimulate the development of the microscopic theory of the composite-particle interaction is the progress being made in heavy-ion physics with vastly accumulating experimental data, as a result of the availability of accelerators producing high energy heavy ion beams. This led to an interest in theoretical and experimental investigations of both the elastic and inelastic

scattering of heavy ions which led to a better understanding of a large range of phenomena in the interaction of composite particles, viz. transfer, compound nucleus and fusion reactions. Therefore, to proceed further beyond the phenomenological and semi-phenomenological stages in theoretical research of heavy-ion physics, progress in the microscopic theory of composite-particle interaction is an indespensable route.

1.2 Survey of reaction models

A number of reaction models have been recently developed to serve as vehicles to carry the microscopic theory to its logical destination, namely, the many-body description of heavy-ion reactions without the always somewhat arbitrary presumptions of collective parameters. We may look, very briefly, into the following reaction models that have come up so far.

(a) The two-center shell model

As a starting point for a microscopic description of heavy-ion scattering, a basis of wave functions is needed which describes the nuclear shapes correctly for all distances from the asymptotic limit of two separated nuclei until complete overlap in a compound nucleus. For this purpose the two-center shell model shown in Fig.(1.1) is suitable.

This model was used in 1970 by Pruess and Greiner[1] in their calculations of the single particle levels. Two extreme situations can be envisaged. In the sudden limit of Fig. (1.1) and left side of Fig.(1.2), the oscillator constant characterising the average potential for single particle motion is

assumed to be frozen, which leads to a compression of nuclear matter. In the adiabatic limit it is supposed that the collision is sufficiently slow for nuclear matter to be in a relaxed state at every way that no compression takes place during the collision. The single particle levels in the adiabatic limit are shown on the right side of Fig. (1.2). It is the distinguishing feature of this model that the fragments are increasingly polarized with decreasing distance until the compound nucleus structure emerges for large overlap.

(b) The cluster model

While the two-center shell model serves as a good basis for microscopic treatment of the intrinsic motion of the nucleus in a heavy-ion collision, there arise difficulties in treating the relative motion (and other collective degrees of freedom) of the fragments. In 1971 Fliessbach[2] proposed to consider two harmonic oscillator model wave functions ϕ_A and ϕ_B for clusters localized at a mean separation distance $\langle \underset{\sim}{\rho} \rangle = \langle \underset{\sim}{\rho}_A - \underset{\sim}{\rho}_B \rangle = \underset{\sim}{R}$.

The total wave function being assumed to be:

$$1.2.1 \qquad \Psi(\underset{\sim}{K}, \underset{\sim}{R}) = N\hat{A}\{\exp i \underset{\sim}{K} \underset{\sim}{\rho}\} \phi_A(\underset{\sim}{r}_A, \dot{O}) \phi_B(\underset{\sim}{r}_B, \underset{\sim}{R})$$

where N is a normalization constant, $\hat{A}$ the anti-symmetrizing operator, $\underset{\sim}{r}_A$, $\underset{\sim}{r}_B$ represent the set of nucleon coordinate $\{\underset{\sim}{r}_i\}$ in cluster A and B respectively, $\underset{\sim}{\rho}_A = A^{-1} \sum_{i \varepsilon A} \underset{\sim}{r}_i$ and $\underset{\sim}{\rho}_B = A^{-1} \sum_{i \varepsilon B} \underset{\sim}{r}_i$ are the cluster C.M. coordinates and $\underset{\sim}{R}$ is the distance between the oscillator potentials.

The total wave function (1.2.1) is the solution of the Schrödinger equation:

1.2.2 $$\langle \Psi(\underset{\sim}{K}, \underset{\sim}{R}) \,|(H-E)|\, \Psi(\underset{\sim}{K},\underset{\sim}{R})\rangle = \frac{\hbar^2 K^2}{2\mu} + V(\underset{\sim}{K},\underset{\sim}{R}) - E = 0$$

where $\frac{\hbar^2 K^2}{2\mu}$ is the kinetic energy operator for the relative motion in the C.M. - system. The Schrödinger equation is solved after transforming the cluster wave functions ϕ_A and ϕ_B into an orthogonal set of basis functions. The microscopic Hamiltonian H consists of the usual kinetic and two-body interaction terms. The anti-symmetrization in Eq (1.2.2) leads to a repulsion between the clusters in the overlap region, which had already been noted by van der Spuy (1959)[3] who considered α-α scattering. It is a particularly attractive feature of this treatment that it allows a discussion of Pauli principle effects in both ordinary and momentum space.

(c) The folding model.

This is an approach in which one attempts to derive the heavy-ion potential by finding out how collisions between composite particles are determined by the "elementary" encounters between the constituents, making allowance for many-body effects.

(d) The GCM and RGM methods.

In Chapter 2 a discussion and a general review of the generator coordinate method (GCM)[4] and the resonating group method (RGM) which was proposed by Wheeler[5] is given. The GCM was initially proposed by Brink[6] in his microscopic α - particle model. The reactions between composite particles are described in the generator coordinate method (GCM), in which the many-body wave function is expanded in terms of totally anti-

symmetrized bound state wave functions, which describe various given fragment separations.

This generator coordinate method (GCM) approach has the merits that -

(i) it preserves the intuitive meaning of geometrical configurations of the clusters in the cluster-intrinsic states and the resulting collective features,

(ii) it enables one to do calculations with Slater-determinants and also

(iii) it contains the corresponding shell-model configurations by taking the appropriate limit.

At the time Brink proposed this approach, the applicability of the GCM was considered to be limited to the bound states only, but in the meantime the difficulties associated with the scattering problem have been overcome[7].

The resonating group method (RGM) was proposed by Wheeler about fourty years ago, primarily to describe the effects resulting from the exchange of particles between the clusters. Since then, this method has been applied to various problems. Recently Saito[8] has proposed an additional model to the two above known as the orthogonality condition model (OCM). It is a simplified model for the treatment of the effects of the Pauli principle. In this model, the inter-cluster wave function is required to be orthogonal to the forbidden states which give rise to the null states if the complete antisymmetrization is performed, so that the interaction takes place only in the space orthogonal to the forbidden states.

1.3 Absorption

Chapter 3 outlines the folding model of Toepffer. The consequences of the Pauli principle on the absorption is considered. The G - matrix concept of Brueckner is introduced here. The construction of the imaginary part of the effective nucleon interaction G between bound particles becomes the theme of Chapter 4. Pauli principle and absorption are here treated on a microscopic level with the help of the Feshbach projection technique. The imaginary part of the scattering operator for bound nucleons is obtained in closed form by using internal harmonic oscillator (HO) wave functions and simple nucleon - nucleon interactions (like the Skyrme force).

The shortcomings of the use of plane waves and the reasons for switching to the use of moving displaced harmonic oscillator (HO) functions are discussed. In chapter 5 the analytical techniques adopted in deriving an expression for calculating the matrix elements of the imaginary part of the scattering operator are discussed. The results and analysis appear in the concluding chapter 6.

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CHAPTER 2

GENERATOR COORDINATE METHOD AND THE RESONATING GROUP METHOD

2.1 Generator Coordinate Method (GCM).

During the past few years numerous attempts have been made to describe reactions of composite fragments in the generator coordinate method (GCM). In this method the many-body wave function Ψ is expanded in terms of totally antisymmetrized bound state wave functions, describing various given fragment separations. Solving a scattering problem then amounts to (i) calculating matrix elements in the basis spanned by the totally antisymmetrized bound state wave functions, and to (ii) solving a set of integral equations for the expansion coefficients of Ψ in the given basis of the bound state problem. Problem (i) can be handled by the well-known shell-model techniques for calculating matrix elements of one - and two - body operators in a non-orthogonal basis. We want to analyse the interaction of composite particles like nuclei with A nucleon coordinates $\underset{\sim}{r}_i$. The clusters A and B, separated by the distance $\underset{\sim}{\rho}$, are considered, for which the single particle oscillator potentials are separated by a distance $\underset{\sim}{R}$ Fig (2.1). This separation distance $\underset{\sim}{R}$ is a collective variable. It is superfluous and there arises the task to extract a wave function for the true coordinates of relative motion $\underset{\sim}{\rho}$. In the generator coordinate method (GCM) the total many-body wave function is assumed to be a superposition of generating functions $\Phi(\underset{\sim}{r}, \underset{\sim}{R})$

$$2.1.1 \qquad \Phi^{GCM}(\underset{\sim}{r}) = \int \Phi(\underset{\sim}{r},\underset{\sim}{R})\, f(\underset{\sim}{R})\, d^3R \,,$$

oscillator length b is taken for both fragments. If $b_A \neq b_B$, or if other models are used, the factorization property (2.1.6) breaks down, and a further GC-integration is required for projecting on state of good total momentum.

2.2 Resonating Group Method (RGM)

The resonating group method (RGM), a method proposed by Wheeler in 1937, was introduced primarily to describe microscopically the nuclei from a molecular viewpoint[5]. In this method it is assumed that the nucleons form groups or clusters with certain internal wave functions. There is a resonant exchange of nucleons between the clusters and the total wave function is antisymmetrized to account for the indistinguishability of the nucleons. An equation of motion for the inter-cluster wave function is derived through the use of a many-body Hamiltonian with the help of the variational principle. There occur non-local terms which represent exchanges of nucleons between one cluster and another resulting from the antisymmetrization. The microscopic study of cluster structures for light nuclei was initiated by Wildermuth and Kanellopoulos[10]. Many calculations appeared thereafter, e.g. ref. 3, which use the RGM to investigate both structure and scattering problems of light nucleon systems. Especially, the importance of the alpha-like four-body correlation in light nuclei was recognized extensively. This became the motivating factor in fostering investigations on molecule-like structure of nuclei on a wide scale. Also recent developments in heavy-ion reactions stimulated microscopic

calculations[11] including complete antisymmetrization. Actual calculations in the RGM, however, are very tedious for heavier systems because of the large number of exchange terms. Independently, the generator coordinate method (GCM) was developed to find the asymptotic conditions for the coefficients $f(\underset{\sim}{R})$ in the GCM.

2.3 Relationship between GCM and RGM

One notices that in the GCM, auxiliary parameters, like the distance $\underset{\sim}{R}$ were introduced; Slater determinant many-body wave functions were generated for each parameter set, and the total wave function was constructed by integrating the generating functions over these parameters. Using the variational principle for the energy expectation values, one obtains the Hill-Wheeler equation, the independent variables being the generator coordinates, and not the dynamical variables such as the particle coordinates. Here, all the dynamical variables are already integrated out when the integral kernels of the Hill-Wheeler integral equation are obtained. In the generating function the inter-cluster coordinate $\underset{\sim}{R}$ is fixed, and only the other independent coordinates are integrated over in deriving the non-local kernels of the GCM equation. The RGM will allow us to relate the expansion coefficients $f(\underset{\sim}{R})$ (in the GCM) to the wave function of relative motion $g(\underset{\sim}{\rho})$ of the fragments A and B. In this method, one substitutes Eq (2.1.6) into Eq(2.1.1) to obtain the form:

2.3.1 $$\Psi^{RGM}(\underset{\sim}{r}) = \phi_{cm} A\{\phi_A(\underset{\sim}{r}_A)\phi_B(\underset{\sim}{r}_B)g(\underset{\sim}{\rho})\}$$

Where $\hat{A}$ denotes the rest antisymmetrizer between fragments A and B, and g is the relative wave function of motion. Under certain conditions (to be discussed below) Ψ^{RGM} spans the same space as Ψ^{GCM}. The internal wave functions of the separated fragments are well-defined. In order to include any polarization effects, one can in principle, admix excited fragment configurations. Because of the antisymmetrization operator $\hat{A}$, Eq(2.3.1) does not mean that the densities of the separated fragments are simply added if the distance between the fragments A and B tends to zero. There is, naturally, a well-defined limiting configuration which, e.g. for the case α-α scattering, turns out to be ^{8}Be ground state, provided one neglects the difference between the oscillator lengths of the α-particle and the ^{8}Be nucleus. However, if one could calculate the effective Hamiltonian properly, the question of polarization would be irrelevant since the effective Hamiltonian accounts for polarization effects. To determine the relative motion wave function g, one projects the many-body Schrödinger equation onto the space spanned by Eq(2.3.1):

$$2.3.2 \qquad (\phi_A \phi_B | H_{eff} - E | \Psi) = 0$$

where the round brackets indicate integration over internal fragments only. The explicit form of the Schrödinger equation of relative motion is obtained by splitting the effective Hamiltonian into:

$$2.3.3 \qquad H_{eff} = T_{\underline{\rho}} - T_{cm} + H_A + H_B + V_{AB}$$

where $\underline{\rho}$ is the actual variable of relative motion, and by

separating direct and exchange contributions according to:

2.3.4 $$A = 1 + (A-1)$$

The resulting "resonating group equation" reads as follows:

2.3.5 $$\{T_{\underline{\rho}} + V(\underline{\rho}) - \varepsilon\}g(\underline{\rho}) = \int K(\underline{\rho},\underline{\rho}')g(\underline{\rho}')d^3\rho'$$

where

2.3.6 $$V(\underline{\rho}) = (\phi_A\phi_B|V_{AB}|\phi_A\phi_B)$$

is the direct part of the nucleus-nucleus potential, V is local if the underlying nucleon-nucleon interaction is local, as is assumed in Eq(2.3.6).

The energy ε of relative motion appears after subtracting the internal energies of A and B and the center-of-mass energy from the total energy E. The right hand side of Eq(2.3.5) contains exchange contributions (including those of overlap and kinetic energy),

2.3.7 $$\int K(\rho,\rho')g(\rho')d^3\rho' = (\phi_A\phi_B|(H_{eff}-E)(A-1)|\phi_A\phi_B g)$$

and is non-local like the Hartree-Fock exchange potential. The two types of potentials shown in Fig(2.2) are different local approximations of the non-local kernel V-K. If the fragments are spherical and if we restrict ourselves to central forces, then the angular momentum of relative motion ℓ will be a good quantum number. Expanding g, V and K in terms of spherical harmonics, one arrives at the radial equation:

2.3.8 $$\left\{\frac{d^2}{d\rho^2} - \frac{\ell(\ell+1)}{2\mu\rho^2} - V(\rho)+\varepsilon\right\}g_\ell(\rho) = \int K_\ell(\rho,\rho')g_\ell(\rho')d\rho'.$$

The non-local operator K depends on two vectors $\underline{\rho}$ and $\underline{\rho}'$. Under the above assumptions, (spherical fragments, central

forces) there is no coupling between different ℓ-values. Although both methods, GCM and RGM, are equivalent in principle, it is difficult in practice to construct generating functions $\Phi(\underset{\sim}{r},\underset{\sim}{R})$ such that, for given ϕ_A and ϕ_B the model spaces spanned by Eq(2.1.1) and Eq(2.3.1) are identical. As an example where the equivalence of GCM and RGM can be shown explicitly, let us assume that:

2.3.9 $$\Phi(\underset{\sim}{r},\underset{\sim}{R}) = \Phi_{cm}A\{\phi_A^{int.}(\underset{\sim}{r}_A-\underset{\sim}{\varrho}_A)\phi_B^{int.}(\underset{\sim}{r}_B-\underset{\sim}{\varrho}_B)\Gamma(\underset{\sim}{\varrho},\underset{\sim}{R})\}$$

where Γ is a wave packet describing fluctuations of the relative motion coordinate $\underset{\sim}{\varrho}$ around the mean distance $\underset{\sim}{R}$. It is now possible to actually construct Slater determinants with the above factorization property. Suppose that fragment A is described by a Slater determinant $\phi_A(\underset{\sim}{r}_A,\underset{\sim}{R}_A)$ centered around $\underset{\sim}{R}_A$, and fragment B by $\phi_B(\underset{\sim}{r}_B,\underset{\sim}{R}_B)$ respectively. Then,

2.3.10 $$\Phi(\underset{\sim}{r},\underset{\sim}{R}) = A\{\phi_A(\underset{\sim}{r}_A,\underset{\sim}{R}_A)\phi_B(\underset{\sim}{r}_B,\underset{\sim}{R}_B)\}$$

with $\underset{\sim}{R} = \underset{\sim}{R}_A-\underset{\sim}{R}_B$. Now, if the center-of-mass of the individual fragment is in the oscillator ground state, then Eq(2.2.3) holds with Γ being Gausian. The extraction of a gausian Γ in going from 3A-particle wave functions $\phi_A(\underset{\sim}{r}_A,\underset{\sim}{R}_A)$ to the 3A-3 intrinsic particle wave functions $\phi_A^{int}(\underset{\sim}{r}_A'-\underset{\sim}{\varrho}_A)$ is, however, only possible in the two-center harmonic oscillator cluster-model, provided the same oscillator length b is taken for both fragments. Combining Eqs(2.1.1) and (2.3.9) and comparing the results with Eq(2.3.1), one finds the desired connection between the GCM and the RGM as:

2.3.11 $$g(\underset{\sim}{\varrho}) = \int\Gamma(\underset{\sim}{\varrho},R)f(\underset{\sim}{R})d^3R$$

We are now in a position to discuss the scattering boundary condition to be imposed on f. As the asymptotic behaviour of the relative motion wave function g is known (Coulomb waves), one has to invert Eq(2.3.11) to obtain the asymptotic behaviour of f. Details of such an inversion have been discussed by Beck et al[7] and Weiguny et al[12,13]. The GCM matrix elements and the RGM kernels are connected accordingly:

$$2.3.12 \qquad \int \Gamma(\underset{\sim}{\rho},\underset{\sim}{R}) V(\underset{\sim}{\rho}) \Gamma(\underset{\sim}{\rho},\underset{\sim}{R}') d^3\rho = \langle \Phi(\underset{\sim}{R}) \sum_{\substack{i\in A \\ j\in B}} V_{ij} | \Phi(\underset{\sim}{R}') \rangle_D$$

$$\int\int \Gamma(\underset{\sim}{\rho},\underset{\sim}{R}) K(\underset{\sim}{\rho},\underset{\sim}{\rho}') \Gamma(\underset{\sim}{\rho}',\underset{\sim}{R}') d^3\rho d^3\rho'$$

$$= \langle \Phi(\underset{\sim}{R}) | H_{eff} - E | \Phi(\underset{\sim}{R}') \rangle - \langle \Phi(\underset{\sim}{R}) | H_{eff} - E | \Phi(R') \rangle_D$$

where the subscript D indicates that anti-symmetrization is only performed within the fragments A and B. By means of Eq(2.3.12) it is possible to calculate the RGM kernels from the GCM matrix elements by defolding Eq(2.3.12). This is important since direct calculation of the RGM kernels becomes impracticable with increasing particle number. It is, however, discomforting when one notices that, in deed, the inverse of the defolding operator Γ is singular and thus Eq(2.3.11) is not easy to handle. This causes numerical problems few of which can be briefly outlined below. In practice, the defolding equation (2.3.12) is carried out by discretization and subsequent matrix inversion[14]. Since Γ depends on the difference $(\underset{\sim}{\rho}-\underset{\sim}{R})$, the matrix $\Gamma(\underset{\sim}{\rho}_i,\underset{\sim}{R}_i)$ degenerates with decreasing step length of discretization, and the inverse matrix is made up of large numbers, alternating in sign. As a result,

the RGM kernels cannot be calculated to arbitrary accuracy (even if the GCM kernels are given exactly), and the method is limited to low energies where complete resolution of the fine structure of the RGM kernels is not required. Another aspect of the abovementioned problem is the irregular nature of f. A typical example is the problem of representing a Gaussian of width α by the convolution Eq(2.3.11) with Γ being a Gaussian of width β. The solution f is an ordinary function only if $\alpha>\beta$. In the limiting case $\alpha=\beta$ one contains $f(\underset{\sim}{R}) = \delta(\underset{\sim}{R})$ while f behaves even worse than a δ-distribution for $\alpha<\beta$. The irregular behaviour has led to the serious doubts concerning any solution of the Hill-Wheeler equations through discretization. The answer to this problem is given by a theorem of Wiener[15] concerning the closure of translated square-integrable functions. It guarantees that the short-range part of g can always be approximated by a finite sum of the form:

$$2.3.13 \qquad g_{\ell}^{S.R.}(\varrho) = \sum_{i=1}^{M} C_{\ell i} \Gamma_{\ell}(\varrho_i, \underset{\sim}{R}_i)$$

irrespective of the fact that coefficients $C_{\ell i}$ may cluster strongly on variation of the mesh points $\underset{\sim}{R}_i$. Details concerning this problem have been thoroughly treated by Friederich, Hüsken and Weiguny[13] and Giraud and Le Tourneux[16,17]. Conceptually the irregular behaviour of f does not cause any problem since $f(\underset{\sim}{R})$ has no direct physical meaning. The quantity $|f(\underset{\sim}{R})|^2$, e.g., cannot be interpreted as the probability for finding the fragments at mean distance $\underset{\sim}{R}$ since the basis functions $\Phi(\underset{\sim}{r},\underset{\sim}{R})$ are non-orthogonal.

2.4 Effects of anti-symmetrization

A perculiar feature of microscopic theories of composite-particles scattering is the appearance of square-integrable states of relative motion which lead to the solution of Eq(2.3.5) at arbitrary energy. Such "redundant states" are obtained whenever the eigenvalue problem

$$2.4.1 \qquad (\phi_A^{int}\phi_B^{int}|1-A\,|\phi_A^{int}\phi_B^{int}U) = \int N(\varrho,\varrho')U(\varrho')d^3\varrho' = \lambda U(\varrho)$$

possesses solutions U^R with eigenvalue $\lambda = 1$. After anti-symmetrization, the corresponding many-body wave function

$$2.4.2 \qquad \Psi^R = A\{\phi_A^{int}\phi_B^{int}U^R\}$$

vanishes identically, and provides a trivial solution of Eq(2.3.2) at any energy E. Redundant states are square integrable solutions of the RGM Eq(2.3.12) at arbitrary energy and have to be counted in the Levinson theorem like true bound states,

$$2.4.3 \qquad \delta_\ell(o) - \delta_\ell(\infty) = \pi(m_\ell+n_\ell)$$

where m_ℓ is the number of redundant states in the ℓth partial wave, n_ℓ the number of true bound states. Redundant states actually occur if the internal states $\phi_A^{int}, \phi_B^{int}$ are described in the oscillator shell-model, with equal b-values for both fragments as has been in most of the microscopic calculations done so far. In this model, the redundant states have harmonic wave functions with $N<N_R$ quanta excited, the redundancy limit N_R depending on the system under consideration. The redundant states, which are direct consequence of the Pauli principle, are important in two ways. Firstly, Eq(2.4.3) shows

that as far as the phase shifts are concerned the Pauli principle leading to the occurance of the redundant states can be simulated by an extra bound states. This explains the equivalence of repulsive core and deep attractive potentials, Fig.(2.2). Secondly the physical solutions g orthogonal to U^R are damped in the interaction region where absorption takes place. It is, perhaps not surprising that both refractive and diffractive effects show up as backward anomalies and are often hard to disantangle as shown in Fig.(2.3). In more realistic models where the internal wave function deviates from the pure oscillator shell-model by admixture of the other configurations or by introducing different size parameters for different fragments, there will be eigenfunctions of N with eigenvalues λ close to 1. There have been controversial views regarding the nature of such "almost redundant states", whether they are positive energy bound states[18] or resonances of finite width[19]. Careful studies of simple systems prove the latter version to be true. Consider, e.g., the fictitious problem of dineutron-dineutron scattering[20] with the internal dineutron wave function taken as a sum of two Gausian functions with different widths $b \neq b'$. The nucleon-nucleon interaction is switched off artificially in order to study the effect of anti-symmetrization separately. Fig.(2.4) shows the phase $\delta_o(K)$, obtained by solving the resonating group equation (2.3.5). In the oscillator shell-model limit, $b = b'$, no resonance occurs, and the Levinson theorem is established as $\delta_o(o) - \delta_o(\infty) = \pi$ due to the presence of one redundant state. Leaving the pure oscillator shell-model by choosing $b \neq b'$, one observes a resonance whose width

converges to a finite value if the step length d, used in the numerical treatment of Eq(2.3.5) is reduced. From the energy dependence one finds in this case $\delta_o(o)-\delta_o(\infty) = o$ which demonstrates that no positive energy bound state is present. As a more realistic example the α-^{16}O system has been studied[20], allowing for different size parameters, $b_\alpha \neq b_{^{16}O}$, and again, the nuclear and Coulomb forces were turned off. The phases, shown in Figs. (2.5) and (2.6) for various partial waves ℓ, display the same general behaviour as in the dineutron case. For low ℓ-values the resonance structures are more complicated since several redundant states are degenerate. On a formal basis[19,20] the finite width of the resonances and discontinuous transition from "redundant" to "almost redundant" states can be explained as follows:

One starts from the internal waves ϕ_A, ϕ_B in the harmonic oscillator (HO) model with $b_A = b_B+\eta$. The projection P onto the elastic channel can then be expanded in terms of the solutions U_i of Eq(2.4.1) as

$$2.4.4 \qquad P = \sum_i |\Psi_i><\Psi_i|$$

where

$$2.4.5. \qquad \Psi_i = \left(\frac{N_A!N_B!}{(N_A+N_B)!}\right)^{\frac{1}{2}} (1-\lambda_i)^{-\frac{1}{2}} A\{\phi_A\phi_B U_i\}; \quad \lambda_i \neq 1$$

Eq(2.4.5) is then equivalent to:

$$2.4.6 \qquad PH_{eff}P|\Psi> = EP|\Psi>$$

To compare with the case $b_A = b_B$, P is split into -

$$2.4.7 \qquad P = P_o+P_1$$

such that P_o comprises those states U_i for which $\lambda_i \neq 1$ if

$b_A \to b_B$.

The resonating group equation (2.3.2) then reads:

2.4.8 $$P_o H_{eff} P_o |\Psi\rangle = E P_o |\Psi\rangle$$

for $b_A = b_B$. Eqs(2.4.6) and (2.4.8) constitute two different problems because of the coupling term $P_o H_{eff} P_1$ appearing in Eq(2.4.6). This term connects the P_o-space with states Ψ_i from P_I-space which describe internally excited fragments, since

2.4.9 $$(\phi_A\ \phi_B|\Psi_i) = \left(\frac{N_A!\, N_B!}{(N_A+N_B)!}\right)^{\frac{1}{2}} (1-\lambda_i)^{\frac{1}{2}} U_i(\varrho) \to 0$$

for $\lambda \to 1$, and causes resonances in the spectrum of Eq(2.4.6) which do not occur in Eq(2.4.8).

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AN ANTISYMMETRIZED FOLDING PROCEDURE FOR THE INTERACTION BETWEEN COMPOSITE PARTICLES

BY

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concept of Brueckner. An approach such as the folding model was proposed. In it heavy-ion potentials are derived and by studying how far collisions between composite-particles are determined by the elementary encounters between the constituents making allowance for many-body effects. This can, at least presently, only be done in rather crude way, in particular one encounters again the familiar problem which arises when nuclei loose their identity in the overlap region. The procedure to calculate nucleon-nucleus scattering was first suggested by Greenlees et al.[28] in 1968 and in the same year by McManus[29]. An effective interaction $\tilde{G}$ compatible with nucleon-nucleon scattering is folded with density distribution of the target to give the real nucleon-nucleus optical potential $U_{N,2}(\underset{\sim}{R})$

$$3.2.1 \qquad U_{N,2}(\underset{\sim}{R}) = \int \rho_2(\underset{\sim}{r}_2)\tilde{G}(\underset{\sim}{R}-\underset{\sim}{r}_2)d^3r_2$$

There are two possibilities to extend this method in order to describe the scattering between two composite-particles.
One adopts the double folding

$$3.2.2 \qquad U(\underset{\sim}{R}) = \int\int \rho(\underset{\sim}{r}_1)\tilde{G}(\underset{\sim}{r}_1-\underset{\sim}{r}_2+\underset{\sim}{R})\rho(\underset{\sim}{r}_2)d^3r_1 d^3r_2$$

as has been employed, for example, in 1973 by Vary and Dover[30] and also two years later by Dover and Vary[31] or an empirical nucleon-nucleus potential $U_{N,2}(\underset{\sim}{r}_1,+\underset{\sim}{R})$ is folded with density distribution of the other nucleus to yield,

$$3.2.3 \qquad U(\underset{\sim}{R}) = \int \rho(\underset{\sim}{r}_1)U_{N12}(\underset{\sim}{r}_1+\underset{\sim}{R})d^3r_1$$

as was done by Kadmenskii et al.[32], or Broglia and Winther[33]. If necessary, Eq(3.2.3) must be symmetrized in order to ensure that the potential is independent of the arbitrary choice of the indices 1 and 2. A similar prescription has recently been used by Sinha[34] to describe the absorption in heavy-ion scattering. However, Fink and Toepffer[35] have shown that the absorption in heavy-ion reactions is mainly determined by the open phase space allowed by the Pauli principle which for scattering between bound nucleus is entirely different from the one for scattering between a free nucleon and bound target nucleus.

3.3 Absorption in the Folding model.

In 1976 Toepffer formulated a folding model in momentum space. Consider two Fermi fluids with densities $\frac{\rho_1}{\rho_1+\rho_2}$ and $\frac{\rho_2}{\rho_1+\rho_2}$ in ordinary space, which in momentum space are represented by Fermi spheres of radius k_F which centres are separated by the collective momentum $\underset{\sim}{K}$ of relative motion per nucleon in Fig.(3.1). Fig.(3.1) shows the Fermi spheres F_1 and F_2 separated by the local momentum K per nucleon. Scattering into $F_1 \cap F_2$ is forbidden by the Pauli principle. Because of momentum and energy conservation the final momenta for elementary collisions lie on the sphere R. The Pauli-forbidden region is marked as a heavy black line on the circumfrence of R.

Instead of Eq(3.2.2) one considers the interaction energy

$$3.3.1 \qquad U = \sum_{m,n} \{\langle \underset{\sim}{m},\underset{\sim}{n};\underset{\sim}{R},\underset{\sim}{K}|G|\underset{\sim}{m},\underset{\sim}{n};\underset{\sim}{R},\underset{\sim}{K}\rangle - \langle \underset{\sim}{m},\underset{\sim}{n};\underset{\sim}{R},\underset{\sim}{K}|G|\underset{\sim}{n},\underset{\sim}{m};\underset{\sim}{R},\underset{\sim}{K}\rangle\}$$

with the convention that $n \varepsilon F_1$ and $m \varepsilon F_2$
This interaction energy is between fragment 1 and 2 and use is made of the Thomas-Fermi model to write vectors $|\underset{\sim}{m},\underset{\sim}{n}>$ as:

3.3.2 $$|\underset{\sim}{m},\underset{\sim}{n}> = (d\Omega)^{-1}\exp\{i(\underset{\sim}{k}_m+\underset{\sim}{K})(\underset{\sim}{r}_1+\underset{\sim}{R})+\underset{\sim}{k}_n\underset{\sim}{r}_2\}$$

$$|\underset{\sim}{n}\ \underset{\sim}{m}> = (d\Omega)^{-1}\exp\{i(\underset{\sim}{k}_n(\underset{\sim}{r}_1+\underset{\sim}{\rho})+(\underset{\sim}{k}_m+\underset{\sim}{K})\underset{\sim}{r}_2\}$$

in a certain volume element $d\Omega$. Introducing a local total momentum $\underset{\sim}{P}_{mn} = \underset{\sim}{k}_m+\underset{\sim}{K}+\underset{\sim}{k}_n$ and a relative momentum $\underset{\sim}{k}_{mn} = \frac{1}{2}(\underset{\sim}{k}_m+\underset{\sim}{K}-\underset{\sim}{k}_n)$ (see Fig.(3.1)) in the local CM-frame, one obtains with $\underset{\sim}{\Upsilon} = \frac{1}{2}(\underset{\sim}{r}_1+\underset{\sim}{\rho}+\underset{\sim}{r}_2)$ and the relative coordinate of two nucleons $\underset{\sim}{r}' = \underset{\sim}{r}_1+\underset{\sim}{\rho}-\underset{\sim}{r}_2$ and interaction energy density

3.3.3 $$d^3u/d^3r_2 = (d\Omega)^{-2}\sum_{m,n}\int\exp(-i\,\underset{\sim}{P}_{mn}\underset{\sim}{R})\exp(-i\,\underset{\sim}{k}_{mn}\underset{\sim}{r}')$$

$$G\ \exp(i\underset{\sim}{P}\underset{\sim}{R})\{\exp(i\underset{\sim}{k}_{mn}\underset{\sim}{r}')-\exp(-i\underset{\sim}{k}_{mn}\underset{\sim}{r}')\}d^3r_1$$

Assuming local translational invariance, the total momentum $\underset{\sim}{P}_{mn}$ of two colliding nucleons is conserved. The summations in Eq(3.3.3) can be converted into integrations over the intrinsic momenta $\underset{\sim}{k}_1 = \underset{\sim}{k}_m$ and $\underset{\sim}{k}_2 = \underset{\sim}{k}_n$. Then

3.3.4 $$d^3u/d^3r_2 = (2\pi)^{-6}\int_0^{k_F}d^3k_1\int_0^{k_F}d^3k_2(<\underset{\sim}{k}|G|\underset{\sim}{k}>-<\underset{\sim}{k}|G|-\underset{\sim}{k}>$$

with $\underset{\sim}{k} = \underset{\sim}{k}_{mn}$ and the reaction matrix elements

3.3.5 $$<\underset{\sim}{k}|G|\pm\underset{\sim}{k}> = \int\exp(-i\ \underset{\sim}{k}\ \underset{\sim}{r}')G\ \exp(\pm i\ \underset{\sim}{k}\ \underset{\sim}{r}')d^3r'$$

for forward and backward scattering.

The Fermi momentum k_F in Eq(3.3.4) is calculated by inserting the compound density $\rho_{cs} = \rho_1+\rho_2$ at the position $\underset{\sim}{r}_2$ of the volume element $d\Omega$ into the following equations.

3.3.6 $$\underset{\sim}{k}_F(\underset{\sim}{r}) = (3\pi^2\rho(\underset{\sim}{r})/2)^{\frac{1}{3}}$$

In order not to count particles twice a factor $\rho_1\rho_2\rho_{cs}^{-2}$ is introduced and the optical potential becomes

3.3.7 $$U = (2\pi)^{-6}\int d^3r_2\rho_1\rho_2\rho_{cs}^{-2}\int_0^{k_F} d^3k_1\int_0^{k_F} d^3k_2\{\langle\underset{\sim}{k}|G|\underset{\sim}{k}\rangle-\langle\underset{\sim}{k}|G|-\underset{\sim}{k}\rangle\}$$

U is complex, since G obeys the Brueckner-Goldstone equation

3.3.8 $$G = \tilde{VQ}(H_o-E-i\eta)^{-1}G$$
$$= V\tilde{PQ}(H_o-E)^{-1} - i\pi\tilde{VQ}\,\delta(H_o-E)G$$

which is not Hermitian. Here H_o is the unpertubed Hamiltonian for the motion of the nucleons in their respective fragments at infinite separation $\underset{\sim}{\rho}$,$\tilde{Q}$ is the Pauli operator which ensures that no particles can be scattered into the overlap region of both Fermi spheres in Fig.(3.1) above, and $\tilde{P}$ denotes that the principal value should be taken upon integration over intermediate states. In contrast to the Brueckner theory for nuclear matter the imaginary part of G does not vanish here since real excitations can take place in which kinetic energy of relative motion is converted into internal excitation energy giving rise to an absorptive imaginary part. On a microscopic level probability is conserved and the imaginary part of G fulfils the optical theorem:

3.3.9 $$\mathrm{Im}\langle k|G|k\rangle = \pi(2\pi)^{-3}\int d^3k'\tilde{Q}|\langle\underset{\sim}{k}|G|\underset{\sim}{k}^1\rangle|^2\delta(E(k')-E(k))$$

where the exchange terms have been surpressed for simplicity. Although the Fermi momentum k_F is calculated here with the density of the compound system ρ_{cs}, the treatment is not fully anti-symmetrized and concepts like the distance ξ between the fragments and the momentum K per nucleon of relative motion are used which loose their meaning with increasing overlap, where the real parts of the folding potentials become too deep. This leads to problems if the absorption is weak in the surface region so that the interacting nuclei can probe each other deeply even in the case of elastic scattering.

Toepffer et al, calculated the real potentials and by using the phase shift approximation for the G-operator as:

3.3.10 $$G = 4\,\hbar^2 m^{-1} k^{-1} \sum_{\ell} (2\ell+1)\,\tan\delta_{\ell}$$

or alternatively the first order G_s+V_ℓ of the expansion

3.3.11 $$G = G_s^f + V_\ell - G_s^f \{\frac{\tilde{Q}}{H_o - E} - \frac{1}{K-E}\} G_s^f - 2G_s^f \frac{\tilde{Q}}{H_o - E} V_\ell - V_\ell \frac{\tilde{Q}}{H_o - E} V_\ell + \ldots$$

which was proposed by Moszkowski and Scott[36]. In this, d is a separation distance introduced in such a way that the short ranged repulsion between the nucleus is cancelled by a part of the long ranged attraction:

3.3.12 $$V = \begin{cases} V_s & r<d \\ V_\ell & r>d \end{cases}$$

so that the free reaction matrix G_s^f corresponding to V_s and the long range component V_ℓ are sufficiently small quantities to allow the pertubation expansion Eq(3.3.11). After the judicious parametrization of the phases or $G_s^f + V_\ell$ the momentum

integrals in Eq(3.3.7) can be evaluated in closed form. If one inserts $G_s^f+V_\ell$ of Eq(3.3.11) into the direct part of Eq(3.3.7) it turns out that the real potentials become too deep in order to fit the data for surface transparent systems like ^{16}O - ^{16}O or ^{12}C - ^{12}C.

In the Thomas-Fermi model the unperturbed Hamiltonian H_o for the motion of the nucleons is the local kinetic energy operator K. A simple way to estimate the imaginary part of G is to iterate Eq(3.3.8) once and to replace the G-matrix by the free two-nucleon scattering matrix T:

$$3.3.13 \qquad G = V+V\frac{\tilde{Q}}{E-K+i\eta}G$$

$$\approx V+V\frac{\tilde{Q}}{E-K+i\eta}T$$

The imaginary part of G(Eq(3.3.9)) can then be evaluated with the help of the optical theorem which yields the frivolous scattering model of Goldberger[37], in which the effective cross-section for nucleons is given by the free cross-section averaged over the Pauli-allowed region of momentum space.

3.4 The effective Hamiltonian

It remains to investigate whether it is justified to use the G-matrix of Eq(3.3.8) as the effective force in the elastic channel. Following Feshbach[9], the effective Hamiltonian (H_{eff}) for the elastic channel can be expressed as:

$$3.4.1 \qquad H_{eff} = P\{H+HQ(E-QHQ+i\eta)^{-1}QH\}P$$

where P projects onto the elastic channel defined in Eq(2.1.1) and Q is related with P by

3.4.2 $Q = 1-P$

Let us split the Hamiltonian H into

3.4.3 $H = H_o+V$

such that

3.4.4 $QH_o = H_oQ$

then Eq(3.4.1) can be written as

3.4.5 $H_{eff} = P\{H_o+G\}P$

where

3.4.6 $G = V+V\frac{Q}{E-H_o+i\eta}G$

is in general a many-body operator. In the independent pair approximation G reduces to the two-body operator G of Eq(3.3.13) if we choose $H_o = K$. Here Q projects onto the states which are unoccupied in the elastic channel and plays therefore formally the same role as the Pauli-operator $\tilde{Q}$ in Eq(3.3.8). The projection operator Q will therefore subsequently be referred to as the Pauli operator. To improve the convergence of this expansion we introduce a reference matrix:

3.4.7 $g = G(R\to\infty,$ momentum $K\to 0)$

describing the static problem and is related to G through:

3.4.8 $G = g+g(\frac{Q}{\omega-K+i\eta} - \frac{Q_o}{\omega-K})G.$

The Pauli operator Q_o of the reference problem as well as g are real. Their reality is due to the fact that they both

refer to a bound state problem, i.e. separated fragments at rest. One expects that the iteration of Eq(3.4.8) converges faster than for Eq(3.3.13). We, thus obtain for the second order (for the imaginary part in which we are presently interested),

3.4.9 $$\mathrm{Im}\ G = -\pi g Q \delta(\omega - K) g.$$

As reference matrix one may take a Volkov[25] or a Skyrme[38] force which are known to give a reasonable account of static nuclear properties like binding energies, radii etc. Relative motion is treated incorrectly when interpreting Eq(3.3.1) or optical potential since the parameter $\underset{\sim}{R}$ differs from the dynamical variable $\underset{\sim}{\rho}$ of relative motion. In other words, Eq(3.3.1) neglects non-diagonal elements $\langle\Phi(\underset{\sim}{R}',\underset{\sim}{K})|G|\Phi(\underset{\sim}{R},\underset{\sim}{K})\rangle$. They can be taken care of by using the above matrix elements as input in the symmetrized form:

3.4.10 $$\tfrac{1}{2}\langle\Phi(\underset{\sim}{R}',\underset{\sim}{K})|G(\underset{\sim}{R}',\underset{\sim}{K}) + G(\underset{\sim}{R},\underset{\sim}{K})|\Phi(\underset{\sim}{R},\underset{\sim}{K})\rangle$$

This procedure is not fully self-consistent since the above G is calculated with respect to a single configuration $\Phi(\underset{\sim}{R},\underset{\sim}{K})$ of the two-center shell model, which cover only part of the GCM-space of functions defined by

3.4.11 $$\Psi(\underset{\sim}{r}) = \int d^3\underset{\sim}{R}\,\Phi(\underset{\sim}{r},\underset{\sim}{R})\,f(\underset{\sim}{R})$$

* * * * * * * * * *

CHAPTER 4

MOVING OSCILLATOR BASIS AND PAULI OPERATOR.

The use of plane waves in the construction of Im G has a number of shortcomings, for example

(i) plane waves are not square-integrable and can therefore not be used to construct generator functions.

(ii) when using plane waves basis, surface and shell effects are neglected.

In the present work, we use moving displaced harmonic oscillator functions which are totally anti-symmetrized. The harmonic oscillator (HO) functions are given as:

$$4.1.1 \qquad (\underset{\sim}{r}|\underset{\sim}{\alpha};\underset{\sim}{R},\underset{\sim}{K}) = C_{\alpha} \exp\{i\underset{\sim}{K}(\underset{\sim}{r}-\underset{\sim}{R}_{\alpha})\cdot(\underset{\sim}{r}-\underset{\sim}{R}_{\alpha})^{2}/2\}H_{\underset{\sim}{\alpha}}(\underset{\sim}{r}-\underset{\sim}{R}_{\alpha})$$

with $\underset{\sim}{R}_{\alpha}$ and $\underset{\sim}{K}_{\alpha}$ given by

$$4.1.2 \qquad \underset{\sim}{R}_{\alpha} = \begin{Bmatrix} \underset{\sim}{R}_{A} \\ \underset{\sim}{R}_{B} \end{Bmatrix} = \pm\tfrac{1}{2}\underset{\sim}{R}, \text{ for } \alpha\varepsilon\begin{Bmatrix} A \\ B \end{Bmatrix}$$

$$\underset{\sim}{K}_{\alpha} = \begin{Bmatrix} \underset{\sim}{K}_{A} \\ \underset{\sim}{K}_{B} \end{Bmatrix} = \pm\ \underset{\sim}{K}, \quad \text{for } \alpha\varepsilon\begin{Bmatrix} A \\ B \end{Bmatrix}$$

and

$$C_{\alpha} = \pi^{-\frac{3}{4}}$$

$H_{\underset{\sim}{\alpha}}$ are the three -dimensional Hermite polynomials, in particular for the s-state, $\underset{\sim}{\alpha} = (0,0,0)$ becomes $H_{\underset{\sim}{\alpha}}(\underset{\sim}{r}-\underset{\sim}{R}_{\alpha}) = 1$. (This is the case treated in the actual calculations).

With the moving oscillator functions (4.1.1) we calculate the imaginary part of

4.1.3 $$\langle \Phi(\underset{\sim}{r},\underset{\sim}{R}) | H-E | \Psi(\underset{\sim}{r},\underset{\sim}{R}) \rangle$$

which contains the matrix elements

4.1.4 $$\langle \Phi(\underset{\sim}{R}',\underset{\sim}{K}) | \mathrm{Im}\ G | \Phi(\underset{\sim}{R}',\underset{\sim}{K}) \rangle$$

$$= \langle \Phi(\underset{\sim}{R}',\underset{\sim}{K}) | \Phi(\underset{\sim}{R},\underset{\sim}{K}) \rangle \sum_{\substack{\alpha,\beta,\gamma,\delta \\ \mathrm{occ.}}} (\underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{R}',\underset{\sim}{K} | \mathrm{Im}\ G | \underset{\sim}{\gamma},\underset{\sim}{\delta};\underset{\sim}{R},\underset{\sim}{K})$$

$$(B^{-1}_{\alpha\gamma} B^{-1}_{\beta\delta} - B^{-1}_{\alpha\delta} B^{-1}_{\beta\gamma})$$

The summation over α includes the spin and isospin summation. This, therefore, implies a factor $(\delta_{\tau_\alpha\tau_\gamma}\delta_{\tau_\beta\tau_\delta} - \delta_{\tau_\alpha\tau_\delta}\delta_{\tau_\beta\tau_\gamma})$ for isospin and similarly for spin. All the lengths in Eq(4.1.4) are measured in units of the oscillator length $b = (\frac{\hbar}{\mu\omega})^{\frac{1}{2}}$ and all momenta in units of b^{-1}.

The $B_{\alpha\gamma}$ are the oscillator overlap matrix $(\underset{\sim}{\alpha};\underset{\sim}{R}',\underset{\sim}{K} | \underset{\sim}{\gamma};\underset{\sim}{R},\underset{\sim}{K})$ in the non-orthogonal basis of the moving oscillator states, and they are given by

4.1.5 $$B_{\alpha\gamma}(\underset{\sim}{R}',\underset{\sim}{R},\underset{\sim}{K}) = C_{\alpha\gamma} \int d^3r \exp\{-(\underset{\sim}{r}-\underset{\sim}{R}'_\alpha)^2/2-(\underset{\sim}{r}-\underset{\sim}{R}_\gamma)^2/2\}$$

with $$C_{\alpha\gamma} = C_\alpha C_\gamma = \pi^{-3/2}$$

Toepffer et al. approximated the operator Q in the model of overlapping Fermi spheres such that the Pauli principle is fulfilled on average only. In the moving oscillator basis, and if the states were orthogonal, $B_{\alpha\gamma} = \delta_{\alpha\gamma}$, the Pauli

where $\underset{\sim}{r}$ stands for all coordinates of the nucleons and $\underset{\sim}{R}$ is the parameter which measures the mean distance between the fragments A and B. The generating functions Φ are square integrable and antisymmetrized with respect to all nucleons of the system. The characteristic feature of the GCM is that there are no superfluous degrees of freedom in the theory. This is due to the fact that the integration in Eq (2.1.1) extends over all $\underset{\sim}{R}$ values. The expansion coefficients $f(\underset{\sim}{R})$ are determined by solving the Schrödinger equation of the many-body system in the space spanned by:

$$2.1.2 \qquad \langle \Phi(\underset{\sim}{r}, \underset{\sim}{R}) \mid H - E \mid \Psi^{GCM}(\underset{\sim}{r}) \rangle = 0$$

where $\langle\ \rangle$ means integration over $\underset{\sim}{r}$, with appropriate scattering boundary conditions. In practice $\Phi(\underset{\sim}{r}, \underset{\sim}{R})$ will be represented by a Slater determinant in a two-center cluster model so that the GCM-Kernels

$$2.1.3 \qquad \begin{Bmatrix} H(\underset{\sim}{R},\underset{\sim}{R}') \\ N(\underset{\sim}{R},\underset{\sim}{R}') \end{Bmatrix} = \langle \Phi(\underset{\sim}{r},\underset{\sim}{R}) \mid \begin{Bmatrix} H \\ 1 \end{Bmatrix} \mid \Phi(\underset{\sim}{r},\underset{\sim}{R}') \rangle$$

can be calculated using well-established shell-model techniques. Here $H(\underset{\sim}{R},\underset{\sim}{R}')$ and $N(\underset{\sim}{R},\underset{\sim}{R}')$ are the energy and normalization kernels, and E the total energy. By taking only the ground-state Slater determinant into account in the integral equation 2.1.2 for the expansion coefficient $f(\underset{\sim}{R})$, one describes only elastic scattering and neglects the coupling to all other channels. This is clearly unrealistic for most cases of composite particle scattering where many channels are open in general. One could generalize Eq(2.1.2) to a system of coupled integral equations by replacing Eq(2.1.1)

by a linear combination:

2.1.4 $$\Psi^{GCM} = \sum_{\lambda} \int \Phi_{\lambda}(\underset{\sim}{r},\underset{\sim}{R}) f_{\lambda}(\underset{\sim}{R}) d^3R$$

where the subscript labels various channels, and then one can do a coupled channel calculation with a real Hamiltonian. Although this is possible in principle, it has a disadvantage in that it is impracticable since the number of channels which contribute to the absorptive part of the optical potential can be very large. Alternatively one may account for the non-elastic channels by introducing an effective microscopic Hamiltonian so as to account for the loss of flux in the inelastic channels. This microscopic Hamiltonian must be non-Hermitian. It can be calculated by using Feshbach's projection techniques. For scattering and decay problems, the separation parameter $\underset{\sim}{R}$ is related to the actual variable of relative motion $\underset{\sim}{\rho}$ of the two fragments, by:

2.1.5 $$\underset{\sim}{R} = \langle \Phi(\underset{\sim}{r},\underset{\sim}{R}) | \underset{\sim}{\rho} | \Phi(\underset{\sim}{r},\underset{\sim}{R}) \rangle$$

under certain circumstances,
One can further assume that:

2.1.6 $$\Phi(\underset{\sim}{r},\underset{\sim}{R}) = \phi_{cm} A\{\phi_A(\underset{\sim}{r}_A)\phi_B(\underset{\sim}{r}_B)\Gamma(|\underset{\sim}{\rho}-\underset{\sim}{R}|/b)\}$$

where the fragments wave functions ϕ_A, ϕ_B depend only on the internal fragments coordinates and ϕ_{cm} describes the center-of-mass motion. A takes care of complete antisymmetrization and Γ is a peaked function which describes fluctuations of $\underset{\sim}{\rho}$ about $\underset{\sim}{R}$ with root mean square deviation b. Assumption (2.1.6) is valid for the α-particle model of Brink[6] or in the harmonic-oscillator cluster model provided the same

From this, one notices that Eq. (5.1.2) implies that $\underset{\sim}{\mu}$ and $\underset{\sim}{\nu}$ also become:

5.1.6 $\quad \underset{\sim}{\mu} = (0,0,0) = \underset{\sim}{\nu}$

and therefore, the expansion coefficients reduce to:

5.1.7 $\quad \hat{M}(\underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{\mu},\underset{\sim}{\nu}) = 1,$

and that

5.1.8 $\quad c_{\alpha\gamma} = c_{\alpha}c_{\gamma} = \pi^{-\frac{3}{2}}$ become $c_{oo} = \pi^{-\frac{3}{2}}$

5.2 Matrix elements of Q

These are calculated with $Q = 1 - P$ and from Eq(4.1.9):

5.2.1 $\quad \langle r,s;\underset{\sim}{R},\underset{\sim}{K}|P|r,s;\underset{\sim}{R},\underset{\sim}{K}\rangle$

$$= \sum_{\substack{\alpha',\beta',\gamma',\delta' \\ \text{occ.}}} B^{-1}{}_{\alpha\gamma'}\langle r,s;\underset{\sim}{R}|\underset{\sim}{\alpha}'\underset{\sim}{\beta}';\underset{\sim}{R},\underset{\sim}{K}\rangle(\underset{\sim}{\alpha}',\underset{\sim}{\beta}';\underset{\sim}{R},\underset{\sim}{K}|r,s;\underset{\sim}{R},\underset{\sim}{K}\rangle B^{-1}{}_{\beta'\delta'}$$

$$= \sum_{\substack{\alpha',\beta',\gamma',\delta' \\ \text{occ.}}} B^{-1}{}_{\alpha'\gamma'}(\underset{\sim}{R},\underset{\sim}{K})B^{-1}{}_{\beta'\delta'}(\underset{\sim}{R},\underset{\sim}{K})(2\pi)^{6}i^{-\underset{\sim}{\alpha}'-\underset{\sim}{\beta}'+\underset{\sim}{\gamma}'+\underset{\sim}{\delta}}C_{\alpha'\beta'\gamma'\delta'}$$

$$\exp\{\tfrac{i}{2}(-\underset{\sim}{R}_{\alpha'},-\underset{\sim}{R}_{\beta'},+\underset{\sim}{R}_{\gamma'},+\underset{\sim}{R}_{\delta'})\underset{\sim}{K}_{rs}+i(-\underset{\sim}{R}_{\alpha'},-\underset{\sim}{R}_{\beta'},+\underset{\sim}{R}_{\gamma'},+\underset{\sim}{R}_{\delta'})\underset{\sim}{k}_{rs}\}$$

$$\exp\{-\tfrac{1}{2}K^{2}{}_{rs}-k^{2}{}_{rs}-\tfrac{1}{2}(K^{2}{}_{\alpha'}+K^{2}{}_{\beta'}+K^{2}{}_{\gamma'}+K^{2}{}_{\delta'})+\tfrac{1}{2}\underset{\sim}{K}_{rs}(\underset{\sim}{K}_{\alpha'}+\underset{\sim}{K}_{\beta'}+\underset{\sim}{K}_{\gamma'}+\underset{\sim}{K}_{\delta'})$$

$$+(K_{\alpha'}-K_{\beta'}+K_{\gamma'}-K_{\delta'})\}$$

$$\sum_{\underset{\sim}{\lambda},\underset{\sim}{\omega}=o}^{\underset{\sim}{\alpha}'+\underset{\sim}{\beta}'}\ \sum_{\underset{\sim}{\lambda}',\underset{\sim}{\omega}'=o}^{\underset{\sim}{\gamma}'+\underset{\sim}{\delta}'} \hat{M}(\underset{\sim}{\alpha}',\underset{\sim}{\beta}';\underset{\sim}{\lambda},\underset{\sim}{\omega})\hat{M}(\underset{\sim}{\gamma}'\underset{\sim}{\delta}';\underset{\sim}{\lambda}',\underset{\sim}{\omega}')H_{\underset{\sim}{\lambda}}(\underset{\sim}{K}_{rs})H_{\underset{\sim}{\omega}}$$

$$(\underset{\sim}{k}_{rs})H_{\underset{\sim}{\lambda}'}(\underset{\sim}{K}_{rs})H_{\underset{\sim}{\omega}'}(\underset{\sim}{k}_{rs})$$

This can, however, be further simplified with the convention

$$5.2.2 \qquad H_{\underset{\sim}{\lambda}}(\underset{\sim}{K}_{rs})H_{\underset{\sim}{\lambda}'}(\underset{\sim}{K}_{rs}) = \sum_{\varrho=o}^{\underset{\sim}{\lambda}+\underset{\sim}{\lambda}'} \hat{N}(\underset{\sim}{\lambda},\underset{\sim}{\lambda}';\varrho)H_{\varrho}(\underset{\sim}{K}_{rs})$$

where $\hat{N}(\underset{\sim}{\lambda},\underset{\sim}{\lambda}';\varrho)$ are expansion coefficients given by

$$5.2.3 \qquad \hat{N}(\underset{\sim}{\lambda},\underset{\sim}{\lambda}';\varrho) = \prod_{i=1} \frac{2^{(\lambda_i+\lambda'_i-\rho_i)/2}\times\lambda_i!\lambda!_i!}{(s_i-\lambda_i)!(s_i-\lambda'_i)!(s_i-\rho_i)!}$$

where $s_i = \lambda_i+\lambda'_i+\rho_i$ = even, otherwise $\hat{N}(\lambda_i,\lambda';\rho_i) = 0$.

5.3 Matrix elements of the reference interaction.

We use as an example of the reference interaction, the Skyrme force[38]. This effective interaction was designed for the Hartree-Fock calculations of nuclei. It is represented in momentum space by

$$5.3.1 \qquad g = t_o(1+x_oP_\sigma) + \tfrac{1}{2}t_1(k^2+k'^2) + t_2\underset{\sim}{k}\cdot\underset{\sim}{k}' + iW_o(\underset{\sim}{g}_1\times\underset{\sim}{g}_2)(\underset{\sim}{k}\times\underset{\sim}{k}')$$

where $\underset{\sim}{k}$ and $\underset{\sim}{k}'$ are relative wave vectors of two nucleons. The original Skyrme force also includes a three-body term which is equivalent to a density dependent force and does not conserve momentum. There exists, however, a Skyrme force in which this term is zero as in the treatment by Beiner et al.[38]. The spin-orbit term will also be neglected in this calculation.

The matrix elements of the Skyrme force are therefore given by

$$5.3.2 \qquad \langle i,\ell;\underset{\sim}{R},\underset{\sim}{K}|g|r,s;\underset{\sim}{R},\underset{\sim}{K}\rangle = \exp\{i(\underset{\sim}{k}_{i\ell}-\underset{\sim}{k}_{rs})\cdot\underset{\sim}{R}\}(2\pi)^3 \times$$

$$\delta(\underset{\sim}{K}_{rs}-\underset{\sim}{K}_{i\ell})[t_o\ (1+P_\sigma)+\tfrac{1}{2}t_1(k^2_{i\ell}+k^2{}_{rs})+t_2\underset{\sim}{k}_{i\ell}\ \underset{\sim}{k}_{rs}+W_o\text{-term}$$

In Eq(5.3.2), the conventions (4.1.13), have been used. In the actual calculation only the t_o and t_1 terms are employed for reasons of numerical simplicity. One notices that the t_2-term averages zero upon integration over the momenta.

5.4 The energy conserving δ-function.

This function is defined here with E_r given by

5.4.1 $\quad E_r = \{\hbar(\underset{\sim}{K}+\underset{\sim}{k}_r)b^{-1}\}^2=\tfrac{1}{2}(\underset{\sim}{K}+\underset{\sim}{k}_r)^2(\hbar\omega)$

The value of the oscillator length b being $\left(\frac{\hbar}{\mu\omega}\right)^{\frac{1}{2}}$. Similar relations are obtained for E_s, E_m and E_n. One can then express the δ-function in the form

5.4.2 $\quad \delta(E_r+E_s-E_m-E_n)=\frac{2}{\hbar\omega}\delta\{(\underset{\sim}{k}_r+\underset{\sim}{K})^2+(\underset{\sim}{k}_s-\underset{\sim}{K})^2-(\underset{\sim}{k}_m+\underset{\sim}{K})^2-(\underset{\sim}{k}_n-\underset{\sim}{K})^2\}$

with $\quad \hbar\omega = 41\ A^{-\frac{1}{3}}$ MeV.

Using Eq(4.1.14)the argument of the δ-function on the right hand side reduces to the form

5.4.3 $\qquad \delta(E_r+E_s-E_m-E_n) = \frac{2}{\hbar\omega}\ \delta(2k^2{}_{rs}-2k^2{}_{mn})$

For s-states all the Hermite polynomials are equal to 1. The expansion coefficients for the expression of the products of two Hermite polynomials as one polynomial also become 1. Large R has also been assumed, i.e. orthogonality, so that $B_{\alpha\gamma}=B^{-1}{}_{\alpha\gamma}=\delta_{\alpha\gamma}$.

For an analytical solution of the integrals occuring in Eq(4.1.11) one represents the δ-functions as Fourier-transforms of exponentials. For intrinsic s-states, as

considered here, it is simpler to evaluate the integrals in spherical coordinates, (see Appendix A.) as treated by Toepffer and Koller[39].

We are to calculate:

$$5.4.4 \qquad \sum_{\substack{\alpha,\beta,\gamma,\delta \\ occ.}} (\underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{R}',\underset{\sim}{K}|\,\mathrm{Im}\ G|\underset{\sim}{\gamma},\underset{\sim}{\delta};\underset{\sim}{R},\underset{\sim}{K})\,(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma})$$

and the Pauli operator (4.1.6) reduces to:

$$5.4.5 \qquad Q = 1 - \sum_{\alpha'\beta'} |\underset{\sim}{\alpha}',\underset{\sim}{\beta}';\underset{\sim}{R},\underset{\sim}{K})\,(\underset{\sim}{\alpha}',\underset{\sim}{\beta}';\underset{\sim}{R},\underset{\sim}{K}|$$

This form of the Pauli operator will be sufficient to demonstrate the important features of the model.

5.5 Matrix elements of Im G.

In the actual calculation the coordinate system has been orientated in such a manner that various vectors $\underset{\sim}{R}'$, $\underset{\sim}{R}$ and $\underset{\sim}{K}$ have the following cartesian form:

$$5.5.1 \qquad \underset{\sim}{R}' = (R'\sin\theta';\ 0;\ R\cos\theta'),$$

$$\underset{\sim}{R} = (R\sin\theta\cos\phi,\ R\sin\theta\sin\phi,\ R\cos\theta)$$

and

$$5.5.2 \qquad K = (0;\ 0;\ K)$$

The matrix elements in Eq.(4.1.11) for Im G for large R where $B_{\alpha'\beta'} = \delta_{\alpha'\gamma'}$ etc. are:

$$5.5.3 \qquad (\underset{\sim}{\alpha},\underset{\sim}{\beta};\ \underset{\sim}{R}'\underset{\sim}{K}|\,\mathrm{Im}\ G|\underset{\sim}{\gamma},\underset{\sim}{\delta};\underset{\sim}{R},\underset{\sim}{K}) = -\pi \sum_{\substack{i,\ell,r,s \\ r',s',m,n,}} (\underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{R}'\underset{\sim}{K}|i,\ell\rangle$$

$$\langle i,\ell|g|r,s\rangle\langle r,s|1-\sum_{\alpha'\beta'}|\underset{\sim}{\alpha}',\underset{\sim}{\beta}')(\underset{\sim}{\alpha}',\underset{\sim}{\beta}'|r',s'\rangle$$

$$\langle r's'|g|m,n\rangle\langle m,n|\chi,\xi;R,K\rangle\ \delta(E_r,+E_s,-E_m-E_n)$$

and similarly for the symmetrized term.

For convenience this can be split into two contributions originating from the "1" operator (NPT-terms) and the P-operator (PT-terms) in Q = 1-P:

5.5.4 $$(\underline{\alpha},\underline{\beta}|\mathrm{Im}\ G|\chi,\xi)_{NPT} = -\pi \sum_{i,\ell,r,s,m,n} (\underline{\alpha},\underline{\beta}|i,\ell\rangle\langle i,\ell|g|r,s\rangle$$

$$\langle r,s|g|m,n\rangle\langle m,n|\chi,\xi\rangle\,\delta(E_r+E_s-E_m-E_n)$$

and the Pauli part:

5.5.5 $$(\underline{\alpha},\underline{\beta}|\mathrm{Im}\ G|\chi\ \xi)_{PT} = \pi \sum_{\substack{i,\ell,r',s',r,s\\ m,n,\alpha',\beta'}} (\underline{\alpha},\underline{\beta}|i,\ell\rangle\langle i,\ell|g|r,s\rangle$$

$$\langle r,s|\underline{\alpha}',\underline{\beta}'\rangle(\underline{\alpha}',\underline{\beta}'|r',s'\rangle\langle r',s'|g|m,n\rangle\langle m,n|\chi,\xi\rangle\,\delta(E_r,+E_s,-E_m-E_n)$$

and similarly for the antisymmetrized form

$$\langle r,s|\underline{\alpha}',\underline{\beta}'\rangle(\underline{\alpha}',\underline{\beta}'|r',s'\rangle\langle r',s'|g|m,n\rangle\langle m,n|\chi,\xi\rangle\,\delta(E_r,+E_s,-E_m-E_n)$$

The NPT and PT terms (5.5.4) and (5.5.5), also subdivide further into three parts due to the reference force g, which gives rise to (t_o-t_o), (t_o-t_1) and (t_1-t_1) terms. These subdivisions are here refered to as NPT(t_o-t_o), NPT(t_o-t_1) etc. The t_2-term in the Skyrme force, whose angle average is, anyway zero, has been dropped for simplicity. The other terms are calculated in Appendix B. The results of the calculations are:

5.5.6 $$\mathrm{NPT}(t_o-t_o) = \frac{\sqrt{2}\,C^2{}_{oo}}{(2\hbar\omega)} \sum_{\alpha\beta\gamma\delta} \exp(-\tfrac{W^2}{4})\,\phi\,(-\tfrac{1}{2};\tfrac{3}{2};\tfrac{V^2}{4})\,F(\alpha,\beta,\gamma,\delta;t_o-t_o)$$

$$\times \exp\{-\tfrac{1}{2}(\xi'_{\alpha\beta}-\xi^*_{\gamma\delta})^2\}\exp(-\tfrac{V^2}{4})$$

5.5.7 $$NPT(t_o-t_1) = \frac{\sqrt{2}\,C^2_{oo}}{(2\hbar\omega)} \sum_{\alpha,\beta,\gamma,\delta} 3\exp(-\tfrac{w^2}{4})\exp(-\tfrac{v^2}{4})\exp\{-\tfrac{1}{2}(\xi'_{\alpha\beta}-\xi^*_{\gamma\delta})^2$$

$$\{\tfrac{1}{4}\phi(-1;\tfrac{3}{2};\tfrac{w^2}{4})\phi(-\tfrac{1}{2};\tfrac{3}{2};\tfrac{v^2}{4})+\phi(-\tfrac{3}{2};\tfrac{3}{2};\tfrac{v^2}{4})\}F(\alpha,\beta,\gamma,\delta;t_o-t_1)$$

5.5.8 $$NPT(t_1-t_1) = \frac{\sqrt{2}C^2_{oo}}{(2\hbar\omega)} \sum_{\alpha\beta\gamma\delta} 3\exp(-\tfrac{w^2}{4})\exp(-\tfrac{v^2}{4})\exp\{-\tfrac{1}{2}(\xi'_{\alpha\beta}-\xi^*_{\gamma\delta})^2\}$$

$$\{\phi(-1;\tfrac{3}{2};\tfrac{w^2}{4})\phi(-\tfrac{3}{2};\tfrac{3}{2};\tfrac{v^2}{4})+2\phi(-\tfrac{5}{2};\tfrac{3}{2};\tfrac{v^2}{4})\}F(\alpha,\beta,\gamma,\delta;t_1-t_1)$$

5.5.9 $$PT(t_o-t_o) = \frac{C^2_{oo}}{(2\hbar\omega)} \sum_{\alpha\ldots\beta'} \exp(-\tfrac{w^2}{4})\exp(\tfrac{u^2}{4})\exp\{-\tfrac{1}{2}(\xi'_{\alpha\beta}-\xi_{\alpha'\beta'})^2\}$$

$$\times \exp\{-\tfrac{1}{2}(\xi_{\alpha'\beta'}-\xi_{\gamma\delta})^2\}F(\alpha,\beta,\gamma,\delta;t_o-t_o)$$

$$\times\begin{cases} \frac{1}{uv}\{\phi(1;\frac{1}{2};-\frac{1}{8}(u-v)^2)-\phi(1;\frac{1}{2};-\frac{1}{8}(u+v)^2)\} & \text{if } u\neq o;v\neq o \\ \exp(-\frac{v^2}{8})\phi(-\frac{1}{2};\frac{3}{2};\frac{v^2}{8}) & \text{if } u=o \text{ but } v\neq o \\ \exp(-\frac{u^2}{8})\phi(-\frac{1}{2};\frac{3}{2};\frac{u^2}{8}) & \text{if } u\neq o \text{ but } v=o \end{cases}$$

5.5.10 $$PT(t_o-t_1) = \frac{C^2_{oo}}{2h\omega} \sum_{\alpha..\beta'} \exp(-\tfrac{w^2}{4})\ \exp\{-\tfrac{1}{2}(\xi'_{\alpha\beta}-\xi_{\alpha'\beta'})^2\}\exp(-\tfrac{u^2}{4})$$

$$\exp\{-\tfrac{1}{2}(\xi_{\alpha'\beta'}-\xi^*_{\gamma\delta})^2\}F(\alpha,\beta,\gamma,\delta;\ t_o-t_1)$$

$$\Big[\tfrac{3}{4}\{\phi(-1;\tfrac{3}{2};\tfrac{w^2}{4})+\phi(-1;\tfrac{3}{2};\tfrac{u^2}{4})\}\times\begin{cases} \frac{1}{uv}\{\phi(1;\frac{1}{2};-\frac{1}{8}(u-v)^2)-\phi(1;\frac{1}{2};-\frac{1}{8}(u+v)^2)\} \\ \exp(-\frac{v^2}{8})\phi(-\frac{1}{2};\frac{3}{2};\frac{v^2}{8}) \text{ If } u=o \\ \exp(-\frac{u^2}{8})\phi(-\frac{1}{2};\frac{3}{2};\frac{u^2}{8}) \text{ If } v=o \end{cases}$$

$$+\Gamma(\tfrac{2}{3})\begin{cases} \frac{1}{2uv}\{\phi(\frac{2}{3};\frac{1}{2};-\frac{1}{8}(u-v)^2)-\phi(\frac{2}{3};\frac{1}{2};-\frac{1}{8}(u+v)^2)\} \\ \frac{1}{3}\exp(-\frac{v^2}{8})\phi(-\frac{1}{6};\frac{3}{2};\frac{v^2}{8}) \quad \text{If } u=o \\ \frac{1}{3}\exp(-\frac{u^2}{8})\phi(-\frac{1}{6};\frac{3}{2};\frac{u^2}{8}) \quad \text{If } v=o \end{cases}\Big]$$

$$5.5.11 \quad PT(t_1-t_1) = \frac{C^2_{oo}}{2\hbar\omega} \exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta}-\underset{\sim}{\xi}_{\alpha'\beta'})^2\} \exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}_{\alpha'\beta'}-\underset{\sim}{\xi}^*_{r\delta})^2\}$$

$$F(\alpha,\beta,\gamma,\delta;\ t_1-t_1)\times \begin{cases} \frac{1}{2uv}\{\phi(\frac{2}{3};\tfrac{1}{2};-\frac{1}{8}(u-v)^2)-\phi(\frac{2}{3};\tfrac{1}{2};-\frac{1}{8}(u+v)^2)\} \\ \frac{1}{3}\exp(-\frac{v^2}{8})\,\phi(-\frac{1}{6};\frac{3}{2};\frac{v^2}{8}) \quad \text{If } u=\eta_{\alpha'\beta'}=o \\ \frac{1}{3}\exp(-\frac{u^2}{8})\,\phi(-\frac{1}{6};\frac{3}{2};\frac{u^2}{8}) \quad \text{If } v=\eta^*_{\gamma\delta}=o \end{cases}$$

where $\underset{\sim}{\eta}'_{\alpha\beta}$, $\underset{\sim}{\eta}^*_{\gamma\delta}$, $\underset{\sim}{\eta}_{\alpha'\beta'}$ are defined in Appendix A and for brevity $\underset{\sim}{u}=\underset{\sim}{\eta}_{\alpha'\beta'}$, $\underset{\sim}{v}=\underset{\sim}{\eta}^*_{\gamma\delta}$ and $\underset{\sim}{w}=\underset{\sim}{\eta}'_{\alpha\beta}$

The Skyrme force constants t_o, t_1 and x_o are taken as t_o=-1248,29 Mev.fm^3; t_1=970,56 Mev.fm^5 and x_o=-0,17 as in Beiner et al.[38].

The matrix elements of the imaginary part of the two-body operator G are then:

$$5.5.12 \quad \langle\Phi(\underset{\sim}{R}',\underset{\sim}{K})|\text{Im } G|\Phi(\underset{\sim}{R},\underset{\sim}{K})\rangle = \langle\Phi(\underset{\sim}{R}',\underset{\sim}{K})|\Phi(\underset{\sim}{R},\underset{\sim}{K})\rangle$$

$$\sum_{\substack{\alpha,\beta,\gamma,\delta \\ occ.}} (\underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{R}',\underset{\sim}{K}|\text{Im } G|\underset{\sim}{\chi},\underset{\sim}{\delta};\underset{\sim}{R},\underset{\sim}{K})\,(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma})$$

with the overlaps (ref.6)

$$5.5.13 \quad \langle\Phi(R',K)|\Phi(R,K)\rangle = \frac{\det B(\alpha,\underset{\sim}{R}',\underset{\sim}{K};\gamma,\underset{\sim}{R},\underset{\sim}{K})}{\sqrt{\det B(\alpha,\underset{\sim}{R}',\underset{\sim}{K};\gamma,\underset{\sim}{R}',\underset{\sim}{K})\ \det B(\alpha,\underset{\sim}{R},\underset{\sim}{K};\gamma,\underset{\sim}{R},\underset{\sim}{K})}}$$

and hence

5.5.14 $$\sum_{\substack{\alpha,\beta,\gamma,\delta \\ occ.}} (\underline{\alpha},\underline{\beta};\underline{R}',\underline{K}|\,\mathrm{Im}\ G|\underline{\gamma},\underline{\delta};\underline{R},\underline{K})$$

$$= \frac{\sqrt{2}\ C^2_{oo}}{2h\omega} \sum_{\substack{\alpha,\beta,\gamma,\delta \\ occ.}} [\ NPT(t_o-t_o)+NPT(t_o-t_1)+NPT(t_1-t_1)\}$$

$$+ \frac{1}{\sqrt{2}} \sum_{\alpha',\beta'} \{PT(t_o-t_o)+PT(t_o-t_1)+PT(t_1-t_1)\}]$$

So that:

5.5.15 $$<\Phi(R',K)\,|\,\mathrm{Im}\ G|\Phi(R,K)><\Phi(R',K|\Phi(R,K)>^{-1}$$

$$= \frac{\sqrt{2}\ C^2_{oo}}{2h\omega} \sum_{\alpha,\beta,\gamma,\delta} [\{NPT(t_o-t_o)+\ NPT(t_o-t_1)+NPT(t_1-t_1)\}$$

$$+ \frac{1}{\sqrt{2}} \sum_{\alpha',\beta'} \{PT(t_o-t_o)+PT(t_o-t_1)+PT(t_1-t_1)\}]$$

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CHAPTER 6

RESULTS AND DISCUSSION

The scattering of composite particles has been considered with primary emphasis on two aspects viz.-

(a) how to describe it microscopically and

(b) how to describe the absorption.

Conventionally one assumes that the clusters are intrinsically in their ground states, since otherwise the treatment turns out to be too complicated. In such a description one only describes the elastic channel in isolation, although many channels are open in general. It was therefore proposed to use the Feshbach projection technique with the aim of constructing an effective (non-Hermitian) Hamiltonian for the channel.

Some results of actual calculation of

$$W = \sum_{\substack{\alpha,\beta,\gamma,\delta \\ \text{occup.}}} \langle \chi | \mathrm{Im}\, G | \chi, \delta \rangle \, (\delta_{\alpha\gamma}\, \delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma})$$

(formula 5.4.4) for $B_{\alpha\gamma} = \delta_{\alpha\gamma}$ etc. and s-states (corresponding to α-α scattering) are presented in Figs. 6.1 - 6.3. In Fig.(6.1) W is plotted as a function of R the mean separation, for various values of R' and K=0, $\theta=0^0$. As anticipated in chapter 5, there arise inconsistencies because of our approximation for the Pauli operator, which is not a true Pauli operator because of the non-orthogonality of the basis functions as described by the overlap matrices $B_{\alpha\gamma}$ etc.

CHAPTER 3.

OUTLINE OF THE MOMENTUM SPACE FOLDING MODEL

3.1 Pauli principle: Its role in microscopic studies

In static problems the Pauli principle allows an independent particle motion in nuclear systems, although the nucleon-nucleon force is short ranged and in fact very repulsive at short distances (the healing property). In composite-particle scattering, the Pauli principle is responsible for the destruction of the structure of the fragments during overlap: Since every cell in phase space may only be singly occupied the overlapping system must either extend itself in ordinary or in momentum space (compression). An important role of the Pauli principle in composite-particle interaction was firstly recognised clearly in the study of the α-α interaction. The repulsive core of the phenomenological α-α potentials was shown to be originating from the Pauli principle. The calculations with the use of the resonating group method (RGM) showed the appearance of the almost energy-independent inner oscillations whose outermost node just corresponds to the phenomenological repulsive core radius[21]. The fundamental role played by the Pauli principle, becomes apparent in a number of puzzling features of phenomenological approaches. One can but just mention the following three of such phenomenological approaches.

(a) Consider the real part U(Re U) of the optical potential for elastic scattering. To the experimental data, reasonable fits have been obtained by both repulsive core and deep attractive potentials, as shown

in Fig(2.2). At first glance it seems unitelligible that such different types of potentials can be phase-equivalent. As mentioned above, the Pauli principle offers an explanation for this type of ambiguity. One would then expect this ambiguity to disappear at high energies where the Pauli principle looses its influence because the intrinsic states do not ovelap any more in momentum space. In fact, with increasing energy, the repulsive core is reduced[22] while purely attractive potentials become more and more shallow. Thus we notice then that the two types of potentials tend to converge to a unique high energy limit.

(b) Generally, distinguishing between refraction and diffractive effects[23] is often quite a difficult issue. An illustration in this regard can be noted in Fig.(2.3). Here the typical behaviour and quantitative features of elastic scattering cross section are indicated. The typical behaviour of angular distributions at forward angles can often be reproduced by both strong absorption and refractive models. A rise of the cross section at backward angles is generally assigned to particle exchange which can be described by a real non-local potential. On the other hand, when absorption sets in abruptly as soon as the nuclei touch, one will obtain reflection, as in the scattering of light from a polished metallic surface.

(c) Alpha reduced widths as calculated from the traditional shell-model approach are much too small when compared

to experiment. The absolute values may be wrong by factors 10 to 1000, varying from light to heavy nuclei. Alpha-cluster models were invoked to remedy the situation but it led to unreasonable large alpha-clustering. This discrepancy can be removed by proper anti-symmetrization as has recently been shown by Fliessbach[24]. A similar situation occurs for alpha-and two-nucleon transfer where the absolute values of spectroscopic factors cannot be explained. In contrast, there seems to be no problem in the case of one-nucleon transfer.

The three examples quoted above demonstrate that a development of practicable theories which incorporate both Pauli principle and absorption on a fundamental level in a consistent way is needed. For that purpose it seems worthwhile to investigate whether the GCM and the RGM can be combined with Feshbach's formalism of the effective Hamiltonian to account for the absorption [9].

3.2 The Folding Model

Most of the microscopic calculations done so far neglected absorption completely. Use is made of real effective interactions, like e.g., the Volkov force[25], adapted to the system at infinite fragment separation, and are at best useful approximations at low energies. Simple approaches[26] which introduce absorption phenomenologically in the Hill-Wheeler or resonating group Eqs(2.1.2) and (2.3.5) cannot solve the problem of energy dependence and radial shape of the optical potential. A semi-microscopic approach is due to Toepffer et al.[27] who base their work on the G-matrix

A.3 $\underline{\underset{\sim}{k}_{rs},\ \underset{\sim}{k}_{mn}\text{-integration}}$

$$\int d^3k_{rs}\ k_{rs}^n\ \exp(i\underset{\sim}{k}_{rs}\ \underset{\sim}{u} - bk_{rs}^2)\int d^3k_{mn}\ k_{mn}^\ell\ \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - ck_{mn}^2)$$

$$\delta(k_{rs}^2 - k_{mn}^2)$$

$$= 2\pi\int d(\cos\phi)\int dk_{rs}\ k_{rs}^{(n+2)}\ \exp(ik_{rs}\ u\ \cos\phi)\exp(-bk_{rs}^2)$$

$$\int d^3k_{mn}\ k_{mn}^\ell\ \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - ck_{mn}^2)\delta(k_{rs}^2 - k_{mn}^2)$$

$$= 2\pi\left(\frac{2i}{iu}\right)\int dk_{rs}\ k_{rs}^{(n+1)}\sin(k_{rs}\ u)\exp(-bk_{rs}^2)$$

$$\int d^3k_{mn}\ k_{mn}^\ell\ \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - ck_{mn}^2)\delta(k_{rs}^2 - k_{mn}^2)$$

$$= \frac{4\pi}{u}\int d(k_{rs}^2)\left(\frac{1}{2k_{rs}}\right)k_{rs}^{(n+1)}\sin(k_{rs}\ u)\exp(-bk_{rs}^2)$$

$$\int d^3k_{mn}\ k_{mn}^\ell\ \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - ck_{mn}^2)\delta(k_{rs}^2 - k_{mn}^2)$$

$$= \frac{2\pi}{u}\int d(k_{rs}^2)k_{rs}^{(n)}\sin(k_{rs}\ u)\exp(-bk_{rs}^2)$$

$$\int d^3k_{mn}\ k_{mn}^\ell\ \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - ck_{mn}^2)\delta(k_{rs}^2 - k_{mn}^2)$$

$$= \frac{2\pi}{u}\int d^3k_{mn}\ k_{mn}^{(\ell+n)}\sin(k_{mn}\ u)\exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v})$$

$$\exp\{-(b+c)k_{mn}^2\}$$

$$= \frac{4\pi^2}{u}\int d(\cos\phi)\int dk_{mn}\ k_{mn}^{(\ell+n+2)}\sin(k_{mn}\ u)$$

$$\exp(-ik_{mn}\ v\ \cos\phi)\exp\{-(b+c)k_{mn}^2\}$$

$$= \frac{8\pi^2}{uv}\int dk_{mn}\ k_{mn}^{(\ell+n+1)}\ \exp\{-(b+c)k_{mn}^2\}[\tfrac{1}{2}\{\cos(u-v)k_{mn} - \cos(u+v)k_{mn}\}]$$

$$= \frac{4\pi^2}{uv}\int dk_{mn}\{k_{mn}^{(\ell+n+1)}\ \exp\{-(b+c)k_{mn}^2\}[\cos(u-v)k_{mn} - \cos(u+v)k_{mn}]\ .$$

Each k_{mn}-integration in the sum above is similar to Erdélyi Eq. (1.4.14), p.15 with:

$a = b + c$; $\upsilon = \ell + n + 1$; $y \in (u-v;\ u+v)$, giving the result:

$$\tfrac{1}{2}(b+c)^{-\frac{1}{2}(\ell+n+2)}\,\Gamma\left(\tfrac{1}{2} + \frac{1}{2(\ell+n+1)}\right)\left[\phi\left(\tfrac{1}{2} + \frac{1}{2(\ell+n+1)};\ \tfrac{1}{2};\ -\frac{(u-v)^2}{4(b+c)}\right)\right.$$
$$\left. - \phi\left(\tfrac{1}{2} + \frac{1}{2(\ell+n+1)} - \tfrac{1}{2};\ -\frac{(u+v)^2}{4(b+c)}\right)\right]$$

Thus the final result is of the form:

A.3.1 $$\int d^3k_{rs}\, k_{rs}^n \exp(i\underset{\sim}{k}_{rs}\underset{\sim}{u} - bk_{rs}^2) \int d^3k_{mn}\, k_{mn}^\ell \exp(-i\underset{\sim}{k}_{mn}\underset{\sim}{v} - ck_{mn}^2)\ \delta(k_{rs}^2 - k_{mn}^2)$$

$$= \frac{2\pi^2}{uv}\,\frac{\Gamma\left(\frac{\ell+n+2}{2(\ell+n+1)}\right)}{(b+c)^{\frac{1}{2}(\ell+n+2)}}\left[\phi\left(\frac{\ell+n+2}{2(\ell+n+1)};\ \tfrac{1}{2};\ -\frac{(u-v)^2}{4(b+c)}\right) - \phi\left(\frac{\ell+n+2}{2(\ell+n+1)};\ \tfrac{1}{2};\right.\right.$$
$$\left.\left. - \frac{(u+v^2}{4(b+c)}\right)\right]$$

In the limit $u \to 0$ (or $v \to 0$), we use L'Hospital's rule:, which gives the result:

A.3.2 $$\frac{2\pi^2}{(b+c)}\,\frac{\Gamma\left(\frac{\ell+n+2}{2(\ell+n+1)}\right)}{(b+c)^{\frac{1}{2}(\ell+n+2)}} \times \frac{\ell+n+2}{\ell+n+1} \exp\left(-\frac{v^2}{4(b+c)}\right)\phi\left(\frac{3\ell+3n+4}{2\ell+2n+2} + \tfrac{3}{2};\ \tfrac{3}{2};\ \frac{v^2}{4(b+c)}\right).$$

Hence the complete result is:

$$\text{A.3.3}\quad 2\pi^2 \frac{\Gamma\left(\frac{\ell+n+2}{2(\ell+n+1)}\right)}{(b+c)^{\frac{1}{2}(\ell+n+2)}} \begin{cases} \frac{1}{uv}\left[\phi\left(\frac{\ell+n+2}{2\ell+2n+2};\ \frac{1}{2};\ -\frac{(u-v)^2}{4(b+c)}\right) - \phi\left(\frac{\ell+n+2}{2\ell+2n+2};\ \frac{1}{2};\ -\frac{(u+v)^2}{4(b+c)}\right)\right. \\ \qquad \text{if } u = 0 \\ \frac{\ell+n+2}{(b+c)(\ell+n+1)} \exp\left(-\frac{v^2}{4(b+c)}\right)\phi\left(\frac{3}{2} - \frac{3\ell+3n+4}{2\ell+2n+2};\ \frac{3}{2};\ \frac{v^2}{4(b+c)}\right) \\ \frac{\ell+n+2}{(b+c)(\ell+n+1)} \exp\left(-\frac{u^2}{4(b+c)}\right)\phi\left(\frac{3}{2} - \frac{3\ell+3n+4}{2\ell+2n+2};\ \frac{3}{2};\ \frac{u^2}{4(b+c)}\right); \\ \qquad \text{if } v = 0 \end{cases}$$

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APPENDIX B

SKYRME FORCE AS REFERENCE INTERACTION

B.1 Skyrme force direct non-Pauli term (DNPT)

is of the form:

B.1.1 $$[t_o(1-x_o) + \tfrac{1}{2}t_1(k_{i\ell}^2 + k_{rs}^2) + t_2 \underset{\sim}{k}_{i\ell}\, \underset{\sim}{k}_{rs}][t_o(1-x_o) + \tfrac{1}{2}t_1(k_{rs}^2 + k_{mn}^2) + t_2\, \underset{\sim}{k}_{rs}\, \underset{\sim}{k}_{mn}]$$

$$+ 3[t_o(1+x_o) + \tfrac{1}{2}t_1(k_{i\ell}^2\, k_{rs}^2) + t_2 \underset{\sim}{k}_{i\ell}\, \underset{\sim}{k}_{rs}][t_o(1+x_o) + \tfrac{1}{2}t_1(k_{rs}^2 + k_{mn}^2) + t_2 \underset{\sim}{k}_{rs}\, \underset{\sim}{k}_{mn}].$$

In this calculation, we have taken $t_2 = 0$, which thus reduces the Skyrme two-body interaction to:

B.1.2 $$[t_o(1-x_o) + \tfrac{1}{2}t_1(k_{i\ell}^2 + k_{rs}^2)][t_o(1-x_o) + \tfrac{1}{2}t_1(k_{rs}^2 + k_{mn}^2)]$$

$$+ 3\, t_o(1+x_o) + \tfrac{1}{2}t_1(k_{i\ell}^2 + k_{rs}^2)][t_o(1+z_o) + \tfrac{1}{2}t_1(k_{rs}^2 + k_{mn}^2)]$$

$$= 4t_o^2(1+x_o + x_o^2) + t_o t_1(2+x_o)(k_{i\ell}^2 + 2k_{rs}^2 + k_{mn}^2) + t_1^2(k_{rs}^2 + k_{i\ell}^2)(k_{rs}^2 + k_{mn}^2)$$

For simplicity, the three terms of (DNPT) are treated separately and represented by $(t_o\text{-}t_o)$, $(t_o\text{-}t_1)$ and $(t_1\text{-}t_1)$.

B.1.3
$$\mathrm{DNPT}(t_o\text{-}t_o) = 4t_o^2(1 + x_o + x_o^2)$$
$$\mathrm{DNPT}(t_o\text{-}t_1) = t_o t_1(2 + x_o)(k_{i\ell}^2 + 2k_{rs}^2 + k_{mn}^2)$$
$$\mathrm{DNPT}(t_1\text{-}t_1) = t_1^2(k_{i\ell}^2 + k_{rs}^2)(k_{rs}^2 + k_{mn}^2).$$

Each of these has to be multiplied by the product of the two overlap matrices: $B^{-1}_{\alpha\gamma}$ and $B^{-1}_{\beta\delta}$, which as explained previously, have been taken as $B^{-1}_{\alpha\gamma} = \delta_{\alpha\gamma}$ and $B^{-1}_{\beta\delta} = \delta_{\beta\delta}$.

Thus Eq. (B.1.3) becomes:

B.1.4 $$DNPT(t_o-t_o) = 4t_o^2(1+x_o+x_o^2)\delta_{\alpha\gamma}\ \delta_{\beta\delta}$$

$$DNPT(t_o-t_1) = t_o t_1(2+x_o)(k_{i\ell}^2 + 2k_{rs}^2 + k_{mn}^2)\delta_{\alpha\gamma}\ \delta_{\beta\delta}$$

$$DNPT(t_1-t_1) = t_1^2(k_{i\ell}^2 + k_{rs}^2)(k_{rs}^2 + k_{mn}^2)\delta_{\alpha\gamma}\ \delta_{\beta\delta}\ .$$

B.2 Skyrme force exchange non-Pauli term (ENPT)

This one is of the form:

B.2.1 $2[t_o(1+x_o) + \frac{1}{2}t_1(k_{i\ell}^2 + k_{rs}^2)][t_o(1+x_o) + \frac{1}{2}t_1(k_{rs}^2+k_{mn}^2)]$.

(t_2 has already been taken as $t_2 = 0$).

Multiply Eq. (B.2.1) out, we obtain:

B.2.2 $2t_o^2(1+x_o)^2 + t_o t_1(k_{i\ell}^2 + 2k_{rs}^2 + k_{mn}^2)+\frac{1}{2}t_1^2(k_{i\ell}^2+k_{rs}^2)(k_{rs}^2+k_{mn}^2)$.

Treating the three terms separately and multiplying each by $\delta_{\alpha\delta}\ \delta_{\beta\gamma}$ and -1:

B.2.3 $ENT(t_o-t_o) = -2t_o^2(1+x_o)^2\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

$ENT(t_o-t_1) = -t_o t_1(1+x_o)(k_{r\ell}^2 + 2k_{rs}^2 + k_{mn}^2)\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

$ENPT(t_1-t_1) = -\frac{1}{2}t_1^2(k_{i\ell}^2 + k_{rs}^2)(k_{rs}^2 + k_{mn}^2)\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

B.3. Spin and Isospin

The spin and isospin summation adds a factor of 4 in the direct term and a factor of 2 in the exchange term.

Combining the (DNPT) and ENPT) we get

B.3.1 $NPT(t_o-t_o) = 4\times 4t_o^2(1+x_o+x_o^2)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - 2\times 2t_o^2(1+x_o)^2\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

$= 4(4t_o^2(1+x_o+x_o^2)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - t_o^2(1+x_o^2)\delta_{\alpha\delta}\ \delta_{\beta\gamma})$

B.3.2 $NPT(t_o-t_1) = (4\times t_o t_1(2+x_o)\delta_{\alpha\gamma}\delta_{\beta\delta} - 2\times t_o t_1(1+x_o)\delta_{\alpha\delta}\delta_{\beta\gamma})$
$(k_{i\ell}^2 + 2k_{rs}^2 + k_{mn}^2)$

$=2\{2t_o t_1(2+x_o)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - t_o t_1(1+x_o)\delta_{\alpha\delta}\ \delta_{\beta\gamma}\}(k_{i\ell}^2 + 2k_{rs} + k_{mn}^2)$

B.3.3 $NPT(t_1-t_1) = (4\times t_1^2\,\delta_{\alpha\gamma}\,\delta_{\beta\delta} - 2\times\tfrac{1}{2}t_1^2\delta_{\alpha\delta}\,\delta_{\beta\gamma})(k_{i\ell}^2 + k_{rs}^2)(k_{rs}^2 + k_{mn}^2)$

$= 2(t_1^2\delta_{\alpha\gamma}\,\delta_{\beta\delta} - \tfrac{1}{2}t_1^2\delta_{\alpha\delta}\,\delta_{\beta\gamma})(k_{i\ell}^2 + k_{rs}^2)(k_{rs}^2 + k_{mn}^2)$

B.4 Non-Pauli Matrix elements of Im G

The matrix elements of Im G for (NPT) become

B.4.1 $(\underset{\sim}{\alpha},\underset{\sim}{\beta}|\text{Im } G|\underset{\sim}{\gamma},\underset{\sim}{\delta})_{NPT} = \sum_{i,\ell,r,s} (\underset{\sim}{\alpha},\underset{\sim}{\beta}|i,\ell\rangle\langle i,\ell|g|r,s\rangle$

$\langle r,s|g|m,n\rangle\langle m,n|\underset{\sim}{\gamma},\underset{\sim}{\delta})\,\delta(E_r + E_s - E_m - E_n)$

where

B.4.2 $(\underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{R}',\underset{\sim}{K}|i,\ell;\underset{\sim}{R},\underset{\sim}{K}\rangle = c_{oo}(2\pi)^3\exp\{\tfrac{i}{2}\underset{\sim}{K}_{i\ell}(\underset{\sim}{R}'_\alpha - \underset{\sim}{R}'_\beta) - \tfrac{1}{4}K_{i\ell}^2\}$

$\exp\{i\underset{\sim}{k}_{i\ell}(\underset{\sim}{R}'_\alpha - \underset{\sim}{R}'_\beta - \underset{\sim}{R}) - k_{i\ell}^2\}$

B.4.3 $g = g_o\delta(\underset{\sim}{r}_1 - \underset{\sim}{r}_2)$ (Skyrme)

B.4.4 $\langle i,\ell;\underset{\sim}{R},\underset{\sim}{K}|g|r,s;\underset{\sim}{R},\underset{\sim}{K}\rangle = g_o\exp\{\tfrac{i}{2}(\underset{\sim}{k}_{i\ell} - \underset{\sim}{k}_{rs})\underset{\sim}{R}\}(2\pi)^3\delta(\underset{\sim}{K}_{rs} - \underset{\sim}{K}_{i\ell})$

B.4.5 $g_o^2 = "(t_o - t_o)" + "(t_o - t_1)" + "(t_1 - t_1)"$

B.5 Constant value in (NP) calculation

From the above equations in B.5.4, we note the following constants: π, from the two-body operator Im G; $c_{oo}^2(2\pi)^6$, from the two overlaps $(\underset{\sim}{\alpha},\underset{\sim}{\beta}|i,\ell\rangle$ and $\langle m,n|\underset{\sim}{\gamma},\underset{\sim}{\delta})$; $(2\pi)^6$ from the matrix elements $\langle i,\ell|g|r,s\rangle$ and $\langle r,s|g|m,n\rangle$; $(2\pi)^{-18}$ from the six integrations over $\underset{\sim}{k}_{i\ell}, \underset{\sim}{K}_{i\ell}$; $\underset{\sim}{k}_{rs}, \underset{\sim}{K}_{rs}$; and $\underset{\sim}{k}_{mn}, \underset{\sim}{K}_{mn}$; and $\frac{1}{\hbar\omega}$ from the δ-function $\delta(E_r+E_s-E_m-E_n) = \frac{1}{\hbar\omega}\delta(k_{rs}^2 - k_{mn}^2)$.

Multiplying out all the constants mentioned above, we obtain:

B.5.1 NP-constant $= \frac{\pi}{\hbar\omega} \frac{c_{oo}^2}{(2\pi)^6}$.

B.6 Integrations carried out

The integration variables are $\underset{\sim}{K}_{i\ell}, \underset{\sim}{K}_{rs}$ and $\underset{\sim}{K}_{mn}$ on one hand, and $\underset{\sim}{k}_{rs}$, $\underset{\sim}{k}_{i\ell}$ and $\underset{\sim}{k}_{mn}$ on the other hand.

Substituting the equations of the type (B.4.2) and (B.4.4) which appear in (B.4.1), we can write down (NPT)-matrix elements of the two-body operator IM G as:

B.6.1 $(\underset{\sim}{\alpha},\underset{\sim}{\beta};\ \underset{\sim}{R}'\underset{\sim}{K}|\,\mathrm{Im}\ G|\underset{\sim}{\gamma},\underset{\sim}{\delta};\ \underset{\sim}{R},\underset{\sim}{K})_{NPT}$

$$= -\frac{\pi c_{oo}^2}{\hbar\omega(2\pi)^6} \int d^3K_{i\ell} \int d^3k_{i\ell} \int d^3K_{rs} \int d^3k_{rs} \int d^3K_{mn} \int d^3k_{mn}$$

$$\exp\{\tfrac{i}{2}\underset{\sim}{K}_{i\ell}(\underset{\sim}{R}'_\alpha+\underset{\sim}{R}'_\beta) + i\underset{\sim}{k}_{i\ell}(\underset{\sim}{R}'_\alpha-\underset{\sim}{R}'_\beta-\underset{\sim}{R}) - \tfrac{1}{4}K_{i\ell}^2-k_{i\ell}^2$$

$$+ i(\underset{\sim}{k}_{i\ell}-\underset{\sim}{k}_{rs})\underset{\sim}{R}\}\delta(\underset{\sim}{K}_{rs}-\underset{\sim}{K}_{i\ell})\exp\{-\tfrac{i}{2}\underset{\sim}{K}_{mn}(\underset{\sim}{R}_\gamma+\underset{\sim}{R}_\delta)$$

$$- i\underset{\sim}{k}_{mn}(\underset{\sim}{R}_\gamma-\underset{\sim}{R}_\delta-\underset{\sim}{R}) - \tfrac{1}{4}(K_{mn}^2-k_{mn}^2)\mathrm{expi}(\underset{\sim}{k}_{rs}-\underset{\sim}{k}_{mn})\underset{\sim}{R}$$

$$\delta(k_{rs}^2-k_{mn}^2)\{NPT(t_o-t_o) + NPT(t_o-t_1) + NPT(t_1-t_1)\}$$

$$= \frac{\pi c_{oo}^2}{\hbar\omega(2\pi)^6} \int d^3K_{i\ell}\ \exp\{i\underset{\sim}{K}_{i\ell}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}^*_{\gamma\delta}) \div \tfrac{1}{4}K_{i\ell}^2\}$$

$$\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w} - k_{i\ell}^2)\int d^3k_{rs} \int d^3k_m\ \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{y} - k_{mn}^2)$$

$$\delta(k_{rs}^2-k_{mn}^2)[NPT(t_o-t_o) + NPT(t_o-t_1) + NPT(t_1-t_1)]$$

where

operator

4.1.6 $$Q = 1 - \sum_{\alpha',\beta'} |\underset{\sim}{\alpha}',\underset{\sim}{\beta}';\underset{\sim}{R},\underset{\sim}{K})(\underset{\sim}{\alpha}',\underset{\sim}{\beta}';\underset{\sim}{R},\underset{\sim}{K}|$$

is a projection operator on the unoccupied states. This can be shown by taking matrix elements with $(\underset{\sim}{\alpha},\underset{\sim}{\beta}|$ and $|\underset{\sim}{\alpha},\underset{\sim}{\beta})$

4.1.7 $$Q|\underset{\sim}{\gamma},\underset{\sim}{\delta}) = |\underset{\sim}{\gamma},\underset{\sim}{\delta}) - \sum_{\substack{\alpha',\beta' \\ \text{occup.}}} |\underset{\sim}{\alpha}',\underset{\sim}{\beta}')(\underset{\sim}{\alpha}',\underset{\sim}{\beta}'|\underset{\sim}{\gamma},\underset{\sim}{\delta})$$

$$= |\underset{\sim}{\gamma},\underset{\sim}{\delta}) \sum_{\substack{\alpha'\beta' \\ \text{occup.}}} \delta_{\alpha'\gamma}\delta_{\beta'\delta}|\underset{\sim}{\alpha}',\underset{\sim}{\beta}')$$

$$= \begin{cases} 0 & \text{if } \gamma \text{ and } \delta \text{ are occupied} \\ |\underset{\sim}{\gamma},\underset{\sim}{\delta}) & \text{if } \gamma \text{ and } \delta \text{are unoccupied} \end{cases}$$

Since the states are not orthogonal in the overlap region:

4.1.8 $$(\underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{R}'\underset{\sim}{K}|\underset{\sim}{\gamma},\underset{\sim}{\delta};\underset{\sim}{R},\underset{\sim}{K}) = B_{\alpha\gamma}\, B_{\beta\delta}$$

one might attempt to construct a Pauli operator by:

4.1.9 $$Q = 1-P = 1-\sum_{\alpha'\beta'\gamma'\delta'} B^{-1}_{\alpha'\gamma'} |\underset{\sim}{\alpha}',\underset{\sim}{\beta}';\underset{\sim}{R},\underset{\sim}{K}><\underset{\sim}{\gamma}',\underset{\sim}{\delta}';\underset{\sim}{R},\underset{\sim}{K}| B^{-1}_{\beta'\delta'},$$

Here $B_{\alpha'\gamma'}$, as mentioned above, are the overlap matrix elements in the non-orthogonal basis of the moving oscillator states Eq(4.1.1). As expected Q tends to 1 for high energies since

4.1.10 $$<\underset{\sim}{k}_1,\underset{\sim}{k}_2;\underset{\sim}{R},\underset{\sim}{K}|Q|\underset{\sim}{k}'_1,\underset{\sim}{k}'_2;\underset{\sim}{R},\underset{\sim}{K}> \propto 1 - \exp(-K^2)$$

the influence of the Pauli principle on absorption disappears with increasing energy. However, it is easy to see that the operator Q in Eq(4.1.9) is not in general a proper Pauli operator because

i) if B^{-1} is the inverse of B in the full Hilbert space of all states $(\underline{\alpha}, \underline{R}, \underline{K}|$, then the sum over the occupied states does not give orthogonality

ii) if B^{-1} is the inverse of B in the space of occupied states, Q does not project on the occupied states, since the application on an unoccupied state will not give zero.

While the oscillator basis has the advantages cited above, their disadvantage is that one cannot easily construct a proper Pauli operator. The use of Eq(4.1.9) for Q will therefore lead to inconsistencies. Because of the appearance of the kinetic energy operator in Eq(3.4.9) for Im G, the matrix elements of Im G must be evaluated by inserting complete sets of plane waves as intermediate states. This is the source of another difficulty in connection with the Pauli operator, since it is the oscillator states (4.1.1) rather than plane wave states which are occupied or unoccupied. Inserting complete sets of plane waves into Eq(3.4.9) and taking the matrix elements required in Eq(4.1.4) one obtains after symmetrizing with respect to $i,\ell,r's' \longleftrightarrow m,n,r,s$

$$4.1.11 \quad (\underline{\alpha},\underline{\beta};\underline{R}',\underline{K}|\,\mathrm{Im}\,G\,|\underline{\chi},\underline{\delta};\underline{R},\underline{K}) = \tfrac{1}{2}\pi \sum_{\substack{i,\ell,r,s \\ m,n,r's'}} (\underline{\alpha},\underline{\beta};\underline{R},\underline{K}|$$

$$i,\ell;\underline{R},\underline{K}\rangle\langle i,\ell;\underline{R},\underline{K}|g|r's';\underline{R},\underline{K}\rangle\langle r',s';\underline{R},\underline{K}|Q|$$

$$r',s';\underline{R},\underline{K}\rangle\langle r',s';\underline{R},\underline{K}|g|m,n;\underline{R},\underline{K}\rangle\langle m,n;\underline{R},\underline{K}|$$

$$\underline{\chi},\underline{\delta};\underline{R},\underline{K})\{\delta(E_r+E_s-E_m-E_n) + \delta(E_r+E_s-E_i-E_\ell)\}$$

where the expression has been symmetrized against

$$\delta(E_r+E_s-E_m-E_n) \quad + \quad \tfrac{1}{2}\delta(E_r+E_s-E_m-E_n)+\tfrac{1}{2}\delta(E_r+E_s-E_i-E_\ell)$$

In the foregoing representation, one notices the appearance of various overlaps, like e.g. $\langle \underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{R},\underset{\sim}{K}|i,\ell;\underset{\sim}{R},\underset{\sim}{K}\rangle$. These have been fully calculated in the Mathematical Appendix. These overlaps are expressed in terms of momenta, with the convention

4.1.12 $\quad i,\ell \epsilon A$ and $m,n \epsilon B$

4.1.13 $$\underset{\sim}{K}_{i\ell} = \underset{\sim}{K}_i+\underset{\sim}{K}_\ell+\underset{\sim}{k}_i+\underset{\sim}{k}_\ell$$

$$2\underset{\sim}{k}_{i\ell} = \underset{\sim}{K}_i-\underset{\sim}{K}_\ell+\underset{\sim}{k}_i-\underset{\sim}{k}_\ell$$

We can however conversely give the above momenta as:

4.1.14 $$\underset{\sim}{k}_i+\underset{\sim}{K}_i = \tfrac{1}{2}\underset{\sim}{K}_{i\ell}+\underset{\sim}{k}_{i\ell}$$

$$\underset{\sim}{k}_\ell+\underset{\sim}{K}_\ell = \tfrac{1}{2}\underset{\sim}{K}_{i\ell}-\underset{\sim}{K}_{i\ell}$$

Therefore, in terms of momenta $\underset{\sim}{K}_{i\ell}$ and $\underset{\sim}{k}_{i\ell}$ we need the expressions for the following overlaps:

(i) matrix elements of the Pauli operator

(ii) the overlaps between localized states and plane waves

(iii) matrix elements of the reference operator

(iv) δ-function - Fourier transforms

From the full calculations of expressions (i) - (iv) mentioned above, which are carried out in the Appendix, we obtain the results as detailed in the next chapter.

* * * * * * * * * *

CHAPTER 5.

ANALYTICAL TECHNIQUES FOR THE CALCULATION OF Im G

5.1 The overlaps between localized states and plane waves.

These are given as

5.1.1 $$(\underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{R}',\underset{\sim}{K}|i,\ell;\underset{\sim}{R},\underset{\sim}{K}\rangle = C_{\alpha\beta}(2\pi)^3 i^{\underset{\sim}{\alpha}+\underset{\sim}{\beta}}\exp\{\tfrac{i}{2}(\underset{\sim}{R}'_\alpha+\underset{\sim}{R}'_\beta)\underset{\sim}{K}_{i\ell}$$

$$+i(\underset{\sim}{R}'_\alpha-\underset{\sim}{R}'_\beta-\underset{\sim}{R})\underset{\sim}{k}_{i\ell}\}\exp\{\tfrac{1}{4}K^2_{i\ell}-k^2_{i\ell}-\tfrac{1}{2}(K^2_\alpha+K^2_\beta)+\tfrac{1}{2}(\underset{\sim}{K}_{i\ell}(\underset{\sim}{K}_\alpha+\underset{\sim}{K}_\beta)$$

$$+(\underset{\sim}{K}_\alpha-\underset{\sim}{K}_\beta)\underset{\sim}{k}_{i\ell}\}\sum_{\underset{\sim}{\mu},\underset{\sim}{\nu}=0}^{\underset{\sim}{\alpha}+\underset{\sim}{\beta}} H_{\underset{\sim}{\mu}}(\tfrac{1}{2}\underset{\sim}{K}_{i\ell})H_{\underset{\sim}{\nu}}(\underset{\sim}{k}_{i\ell})\hat{M}(\underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{\mu},\underset{\sim}{\nu})$$

Here the conventions (4.1.13) and (4.1.14) and the following convention

5.1.2 $$H_{\underset{\sim}{\alpha}}(\tfrac{1}{2}\underset{\sim}{K}_{i\ell}+\underset{\sim}{k}_{i\ell}-\underset{\sim}{K}_\alpha)H_{\underset{\sim}{\beta}}(\tfrac{1}{2}\underset{\sim}{K}_{i\ell}-\underset{\sim}{k}_{i\ell}-\underset{\sim}{K}_\beta) = \sum_{\underset{\sim}{\mu},\underset{\sim}{\nu}=0}^{\underset{\sim}{\alpha}+\underset{\sim}{\beta}} H_{\underset{\sim}{\mu}}(\underset{\sim}{K}_{i\ell})H_{\underset{\sim}{\nu}}(\underset{\sim}{k}_{i\ell})\hat{M}(\underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{\mu},\underset{\sim}{\nu})$$

have been employed. A similar expression can be written down for the overlap $\langle m,n;\underset{\sim}{R},\underset{\sim}{K}|\underset{\sim}{\chi},\underset{\sim}{\delta};\underset{\sim}{R},\underset{\sim}{K})$.

The actual calculation was undertaken for the s-states $\underset{\sim}{\alpha} = (0,0,0) = \underset{\sim}{\beta} = \underset{\sim}{\chi} = \underset{\sim}{\delta}$; and thus the following expressions reduced to the simple forms given below:

5.1.3 $\underset{\sim}{\alpha} + \underset{\sim}{\beta} = (0,0,0)$; $i^{\underset{\sim}{\alpha}+\underset{\sim}{\beta}+\underset{\sim}{\chi}+\underset{\sim}{\delta}} = 1$ because we have defined the symbol $i^{\underset{\sim}{n}}$ as:

5.1.4 $$i^{\underset{\sim}{n}} = i^{n_x+n_y+n_z}$$

Also

5.1.5 $H_{\underset{\sim}{\mu}} = 1 = H_{\underset{\sim}{\nu}}$ etc.

B.7.2 $\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2_{i\ell}) = \pi^{3/2}\ \exp(-\frac{w^2}{4})\ ;\ \ w = \eta'_{\alpha\beta}$

B.7.3 $\int d^3k_{rs}\ \int dk_{mn}\ \exp\{-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - k^2_{mn}\}\delta(k^2_{rs}-k^2_{mn})$

$= 4\pi^2\ \exp(-\frac{v^2}{4})\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{4})$

and $v = \eta^*_{\gamma\delta}$; $\phi(a,b,z)$ is the function $\phi(a,b,z) = 1+\frac{a}{b}\frac{z}{1!}+\frac{a(a+1)}{b(b+1)}\frac{z^2}{2!}+\ldots$
The complete NPT(t_o-t_o) - term is obtained by using (B.7.1), (B.7.2) , (B.7.3) and (B·6:5), giving:

B.7.4. $NPT(t_o-t_o) = \frac{\sqrt{2}c^2_{oo}}{(2\hbar\omega)}\ \exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})$

$\phi(-\frac{1}{2};\ \frac{3}{4};\ \frac{v^2}{4})F(\alpha,\beta,\gamma,\delta;\ t_o-t_o)\times\exp\{(-\frac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta}-\underset{\sim}{\xi}^*_{\gamma\delta})^2\}$

where

B.7.4 $F(\alpha,\beta,\gamma,\delta;\ t_o-t_o) = 4t^2_o(1+x_o+x^2_o)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - t^2_o(1+x_o)^2\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

B.8 The NPT(t_o-t_1) - Term

This part has the form:

B.8.1 $NPT(t_o-t_1) = 2\{2t_ot_1(2+x_o)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - t_ot_1(1+x_o)\delta_{\alpha\delta}\ \delta_{\beta\gamma}\}$

$\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2_{i\ell})\int d^3k_{rs}\ \int d^3k_{mn}\ \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - \underset{\sim}{k}^2_{mn})$

$\delta(k^2_{rs}-k^2_{mn})\{k^2_{i\ell}+2k^2_{rs}+k^2_{mn}\}$

$NPT(t_o-t_1)$ because of $(k^2_{i\ell}+2k^2_{rs}+k_{mn})$ can be treated as the sum of the following:

B.8.2 $\int d^3k_{i\ell}\ k^2_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2_{i\ell})\int d^3k_{rs}\ \int d^3k_{mn}\ \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - k^2_{mn})$

$\delta(k^2_{rs}-k^2_{mn}) = \frac{3}{2}\pi^{3/2}\exp(-\frac{w^2}{4})\phi(-1;\frac{3}{2};\frac{w^2}{4})\times 4\pi^2\exp(-\frac{v^2}{4})\phi(-\frac{1}{2};\frac{3}{2};\frac{v^2}{4})$

$= 6\pi^3(\pi^{1/2})\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\phi(-1;\ \frac{3}{2};\ \frac{w^2}{4})\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{4})$

B.8.3 $2\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2_{i\ell})\int d^3k_{rs}k^2_{rs}\ d^3k_{mn}\exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v}\ -\ k^2_{mn})$

$\delta(k^2_{rs}-k^{2'}_{mn}) = 2\times\pi^{3/2}\exp(-\frac{w^2}{4})\times 8\pi^2\exp(-\frac{v^2}{4})\phi(-\frac{3}{2};\ \frac{3}{2};\ \frac{v^2}{4})$

$= 16(\pi^{3/2})(\pi^2)\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\phi(-\frac{3}{2};\ \frac{3}{2};\ \frac{v^2}{4})$.

B.8.4 $\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2_{i\ell})\int d^3k_{rs}\ \int d^3k_{mn}\ \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v}\ -\ k^2_{mn})$

$\delta(k^2_{rs}-k^2_{mn})$

$= (8\pi^2)\pi^{3/2}\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\phi(-\frac{3}{2};\ \frac{3}{2};\ \frac{v^2}{4})$.

Adding (B.8.2); (B.8.3) and (B.8.4) we get:

B.8.5 $6\pi^3(\pi^{1/2})\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\{\phi(-1;\frac{3}{2};\frac{w^2}{4})\phi(-\frac{1}{2};\frac{3}{2};\frac{v^2}{4})$

$+\ 4\phi(-\frac{3}{2};\ \frac{3}{2};\ \frac{v^2}{4})\}$.

Substituting (B.8.5) into (B.8.1) and multiplying by (B.6.3), we obtain:

B.8.6 $NPT(t_o-t_1) = \frac{\sqrt{2}c^2_{oo}}{(\hbar\omega 2)}\ 3\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\exp\{-\frac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta}-\underset{\sim}{\xi}^*_{\gamma\delta})^2\}$

$\{\frac{1}{4}\phi(-1;\frac{3}{2};\ \frac{w^2}{4})\phi(-\frac{1}{2};\frac{3}{2};\ \frac{v^2}{4})+\phi(-\frac{3}{2};\frac{3}{2};\ \frac{v^2}{4})\}F(\alpha,\beta,\gamma,\delta;\ t_o-t_1)$

where

B.8.7 $F(\alpha,\beta,\gamma,\delta;\ t_o-t_1) = 2t_ot_1(2+x_o)\delta_{\alpha\gamma}\delta_{\beta\delta}\ -\ t_ot_1(1+x_o)\delta_{\alpha\delta}\delta_{\beta\gamma}$

B.9 The $NPT(t_1-t_1)$ - Term

The integrations in this part are of the type:

B.9.1 $\int d^3k_{i\ell}\exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2_{i\ell})\int d^3k_{rs}\int d^3k_{mn}\exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v}\ -\ k^2_{mn})$

$\delta(k_{rs}{}^2-k^2_{mn})\{(k^2_{i\ell}+k^2_{rs})(\kappa^2_{rs}+k^2_{mn})\}$.

Because of $\delta(k_{rs}^2 - k_{mn}^2)$, (B.9.1) is equivalent to the sum of

B.9.2 $\int d\, k_{i\ell} \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w} - k_{i\ell}^2) \int d^3k_{rs} \int d^3k_{mn} \exp(-i\underset{\sim}{k}_{mn}\underset{\sim}{v} - k_{mn}^2)$

$\delta(k_{rs}^2 - k_{mn}^2) k_{i\ell}^2 (2k_{rs}^2)$

$= \frac{3}{2}\pi^{3/2} \exp(-\frac{w^2}{4})\, \phi(-1; \frac{3}{2}; \frac{w^2}{4}) \times \{2 \times 8\pi^2 \exp(-\frac{v^2}{4})\, \phi(-\frac{3}{2}; \frac{3}{2}; \frac{v^2}{4})\}$

$= 3(2\pi)^3 (\pi^{\frac{1}{2}}) \exp(-\frac{w^2}{4}) \exp(-\frac{v^2}{4})\, \phi(-1; \frac{3}{2}; \frac{w^2}{4}) \phi(-\frac{3}{2}; \frac{3}{2}; \frac{v^2}{4})$

and

B.9.3 $2\int d^3k_{i\ell} \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w} - k_{i\ell}^2) \int d^3k_{rs} \int d^3k_{mn} k_{mn}^4 \exp(-i\underset{\sim}{k}_{mn}\underset{\sim}{v} - k_{mn}^2)$

$\delta(k_{rs}^2 - k_{mn}^2)$

$= 2 \times \pi^{3/2} \exp(-\frac{w^2}{4}) \times 4\pi^2 \Gamma(\,') \exp(-\frac{v^2}{4}) \phi(-\frac{5}{2}; \frac{3}{2}; \frac{v^2}{4})$

$= 6(2\pi)^3 (\pi^{\frac{1}{2}}) \exp(-\frac{w^2}{4}) \exp(-\frac{v^2}{4}) \phi(-\frac{5}{2}; \frac{3}{2}; \frac{v^2}{4})$

Adding (B.9.2) and (B.9.3)

B.9.4 $3(2\pi)^3(\pi^{\frac{1}{2}}) \exp(-\frac{w^2}{4}) \exp(-\frac{v^2}{4}) \{\phi(-1; \frac{3}{2}; \frac{w^2}{4}) \phi(-\frac{3}{2}; \frac{3}{2}; \frac{v^2}{4}) + 2\phi(-\frac{5}{2}; \frac{3}{2}; \frac{v^2}{4})\}$

From (B.3.3); (B.6.2) and (B.9.4), we have that:

B.9.5 $NPT(t_1 - t_1) = \frac{\sqrt{2}\, \sigma_{OO}^2}{(2h\omega)} \times 3\exp(-\frac{w^2}{4}) \exp(-\frac{v^2}{4}) \exp\{-\frac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}^*_{\gamma\delta})^2\}$

$\{\phi(-1; \frac{3}{2}; \frac{w^2}{4}) \phi(-\frac{3}{2}; \frac{3}{2}; \frac{v^2}{4}) + 2\phi(-\frac{5}{2}; \frac{3}{2}; \frac{v^2}{4})\} F(\alpha,\beta,\gamma,\delta; t_1 - t_1)$

where

B.9.10 $F(\alpha,\beta,\gamma,\delta; t_1 - t_1) = 2t_1^2\, \delta_{\alpha\gamma}\, \delta_{\beta\delta} - \frac{1}{2}\, t_1^2\, \delta_{\alpha\delta}\, \delta_{\beta\gamma}$

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B.10 Skyrme for Pauli Term ($\tilde{PT}$)

Like the (NPT) one treats the (PT) in three stages, viz. $(t_o - t_o)$, $(t_o - t_1)$ and $(t_1 - t_1)$ - terms. In all the three cases the results of the integrations, over $\underset{\sim}{K}_{i\ell}$, $\underset{\sim}{K}_{rs}$, $\underset{\sim}{K}_{r's'}$ and $\underset{\sim}{K}_{mn}$ are the same. The results of such integrations, with the help of $\delta(\underset{\sim}{K}_{r's'}-\underset{\sim}{K}_{mn})$ and $(\underset{\sim}{K}_{i\ell}-\underset{\sim}{K}_{rs})$ are as follows:

B.10.1 $$\int d^3 K_{i\ell} \exp\{i \underset{\sim}{K}_{i\ell}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}_{\alpha'\beta'}) - \tfrac{1}{2}K^2_{i\ell}\}$$
$$= (2\pi)^{3/2} \exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}_{\alpha'\beta'})^2\}$$

B.10.2 $$\int d^3 K_{mn} \exp\{-i \underset{\sim}{K}_{mn}(\underset{\sim}{\xi}_{\alpha'\beta'} - \underset{\sim}{\xi}^*_{\gamma\delta}) - \tfrac{1}{2} K^2_{mn}\}$$
$$= (2\pi)^{3/2} \exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}_{\alpha'\beta'} - \underset{\sim}{\xi}^*_{\gamma\delta})^2\}$$

Also from the factors π, $(2\pi)^{-24}$, $(2\pi)^{+18}$ $(c^2_{oo})^2$, $\hbar\omega$ we obtain

B.10.3 $$\frac{\pi(c^2_{oo})^2}{\hbar\omega(2\pi)^6} = \frac{c^2_{oo}}{\pi^2\hbar\omega(2\pi)^6}$$

Multiplying (B.10.3) by (B.10.1) and (B.10.2), we obtian the common factor in the (PT)-Term calculation.

B.10.4 $$\frac{c^2_{oo}}{(2\hbar\omega)} \times \frac{2}{\pi^4(2\pi)^3} \exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}_{\alpha'\beta'})^2\} \exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}_{\alpha'\beta'} - \underset{\sim}{\xi}^*_{\gamma\delta})^2\}$$

B.11 PT$(t_o - t_o)$ - Term

In addition to the terms above we have integration over $\underset{\sim}{k}_{i\ell}$, $\underset{\sim}{k}_{rs}$, $\underset{\sim}{k}_{r's'}$ and $\underset{\sim}{k}_{mn}$.

In the case of $PT(t_o - t_o)$ we consider the following:

B.11.1 $$\int d^3k_{i\ell}\ \exp(i\,\underset{\sim}{k}_{i\ell}\,\underset{\sim}{w} - k^2_{i\ell}) = \pi^{3/2}\ \exp(-\frac{w^2}{4});\ w=\eta'_{\alpha\beta}$$

B.11.2 $$\int d^3k_{rs}\ \exp(-i\,\underset{\sim}{k}_{rs}\,\underset{\sim}{u} - k^2_{rs}) = \pi^{3/2}\ \exp(-\frac{u^2}{4});\ u=\eta_{\alpha'\beta'}$$

B.11.3 $$\int d^3k_{r's'}\ \exp(i\underset{\sim}{k}_{r's'}\,\underset{\sim}{u}-k^2_{r's'})\int d^3k_{mn}\ \exp(-i\underset{\sim}{k}_{mn}\,\underset{\sim}{v}-k^2_{mn})\,\delta(k^2_{r's'},-k^2_{mn})$$

$$= \pi^2\begin{cases} \frac{1}{uv}\{\phi(1;\ \tfrac{1}{2};\ -\tfrac{1}{8}(u-v)^2)-\phi(1;\ \tfrac{1}{2};\ -\tfrac{1}{8}(u+v)^2)\} & \text{if } u=\eta_{\alpha'\beta'}\neq 0 \text{ and } v=\eta^*_{\gamma\delta}\neq \\ \exp(-\frac{v^2}{8})\,\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{8}) & \text{if } u=0 \\ \exp(-\frac{u^2}{8})\,\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{u^2}{8}) & \text{if } v=0 \end{cases}$$

The full $PT(t_o - t_o)$ - term is obtained by multiplying the equations (B.10.4), (B.11.1) through (B.11.3) and (B.3.1).

B.11.4 $$PT(t_o - t_o) = \frac{c^2_{oo}}{(2\hbar\omega)}\ \exp(-\frac{w^2}{4})\exp(-\frac{u^2}{4})\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}_{\alpha'\beta'})^2\}$$

$$\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}_{\alpha'\beta'} - \underset{\sim}{\xi}^*_{\gamma\delta})^2\}F(\alpha,\beta,\gamma,\delta;\ t_o - t_o)$$

$$\begin{cases} \frac{1}{uv}\{\phi(1;\ \tfrac{1}{2};\ -\tfrac{1}{8}(u-v)^2)-\phi(1;\tfrac{1}{2};-\tfrac{1}{8}(u+v)^2)\} \\ \qquad \text{if } u=\eta_{\alpha'\beta'}\neq 0 \text{ and } v=\eta^*_{\gamma\delta}\neq 0 \\ \exp(-\frac{v^2}{8})\,\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{8}) \quad \text{if } u=0 \\ \exp(-\frac{u^2}{8})\,\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{u^2}{8}) \quad \text{if } v=0 \end{cases}$$

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B.12 PT$(t_o - t_1)$ - Term

In this case we have the integrations over $\underset{\sim}{k}_{i\ell}, \underset{\sim}{k}_{rs}$ etc. as follows:

B.12.1 $\int d^3k_{i\ell} \exp(i\underset{\sim}{k}_{i\ell} \underset{\sim}{w} - k^2_{i\ell}) \int d^3k_{rs} \exp(-i\underset{\sim}{k}_{rs} \underset{\sim}{u} - k^2_{rs})$
$\int d^3k_{r's'} \exp(i\underset{\sim}{k}_{r's'} \underset{\sim}{u} - k^2_{r's'}) \int d^3k_{mn} \exp(-i\underset{\sim}{k}_{mn} \underset{\sim}{v} - k^2_{mn})$
$\delta(k^2_{r's'} - k^2_{mn}) \{k^2_{i\ell} + k^2_{rs} + k^2_{r's'} + k^2_{mn}\}$

Treating above as a sum of three terms. viz.

B.12.2 $\int d^3k_{i\ell}\, k^2_{i\ell} \exp(i\underset{\sim}{k}_{i\ell} \underset{\sim}{w} - k^2_{i\ell}) \int d^3k_{rs} \exp(-i\underset{\sim}{k}_{rs} \underset{\sim}{u} - k^2_{rs})$
$\int d^3k_{r's'} \exp(i\underset{\sim}{k}_{r's'} \underset{\sim}{u} - k^2_{r's'}) \int d^3k_{mn} \exp(-i\underset{\sim}{k}_{mn} \underset{\sim}{v} - k^2_{mn})$
$\delta(k^2_{r's'} - k^2_{mn})$

$$= \frac{3}{2}\pi^5 \exp(-\frac{w^2}{4}) \exp(-\frac{u^2}{4}) \phi(-1;\ \frac{3}{2};\ \frac{w^2}{4})$$

$$\times \begin{cases} \frac{1}{uv} \{\phi(1;\ \frac{1}{2};\ -\frac{1}{8}(u-v)^2) - \phi(1;\ \frac{1}{2};\ -\frac{1}{8}(u+v)^2)\} & \text{If } u \neq 0 \text{ and } v \neq 0 \\ \exp(-\frac{v^2}{8}) \phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{8}) & \text{If } u = 0 \\ \exp(-\frac{u^2}{8}) \phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{u^2}{8}) & \text{If } v = 0 \end{cases}$$

B.12.3 $\int d^3k_{i\ell} \exp(i\underset{\sim}{w}\underset{\sim}{k}_{i\ell}) \int d^3k_{rs}\, k^2_{rs} \exp(-i\underset{\sim}{k}_{rs} \underset{\sim}{u} - k^2_{rs}) \exp(-k^2_{i\ell})$
$\int d^3k_{r's'} \exp(i\underset{\sim}{k}_{r's'} \underset{\sim}{u} - k^2_{r's'}) \int d^3k_{mn} \exp(-i\underset{\sim}{k}_{mn} \underset{\sim}{v} - k^2_{mn}) \delta(k^2_{r's'} - k^2_{mn})$

$$= \frac{3}{2}\pi^5 \exp(-\frac{w^2}{4}) \exp(-\frac{u^2}{4}) \phi(-1;\frac{3}{2};\frac{u^2}{4}) \begin{cases} \frac{1}{uv}\{\phi(1;\frac{1}{2};-\frac{1}{8}(u-v)^2) - \phi(1;\frac{1}{2};-\frac{1}{8}(u+v)^2)\} & \\ & \text{If } u \neq 0 \text{ and } v \neq 0 \\ \exp(-\frac{v^2}{8})\ \phi(-\frac{1}{2};\frac{3}{2};\frac{v^2}{8}) & \text{If } u = 0 \\ \exp(-\frac{u^2}{8})\ \phi(-\frac{1}{2};\frac{3}{2};\frac{u^2}{8}) & \text{If } v = 0 \end{cases}$$

B.12.4 $\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\ \underset{\sim}{w}-k^2_{i\ell})\int d^3k_{rs}\exp(-i\underset{\sim}{k}_{rs}\underset{\sim}{u}\ -\ k^2_{rs})$

$\int d^3k_{r's'}\exp(i\underset{\sim}{k}_{r's'}\ \underset{\sim}{u}-k^2{}_{r's'})\int d^3k_{mn}\exp(-\ \underset{\sim}{k}_{mn}\underset{\sim}{v}-k^2{}_{mn})\,.$

$k^2{}_{i\ell}(k^2{}_{r's'}+k^2{}_{mn})\,\delta(k^2{}_{r's'}-k^2{}_{mn})$

$$= 2\Gamma(\tfrac{2}{3})\pi^5\exp(-\tfrac{w^2}{4})\exp(-\tfrac{u^2}{4})\begin{cases}(\frac{1}{2uv})\{\phi(\frac{2}{3};\ \frac{1}{2};\ -\frac{1}{8}(u-v)^2)-\phi(\frac{2}{3};\ \frac{1}{2};\ -\frac{1}{8}(u+v)^2)\}\\ (\frac{1}{3})\exp(-\frac{v^2}{8})\,\phi(-\frac{1}{6};\ \frac{3}{2};\ \frac{v}{8}) \quad \text{If } u=0\\ (\frac{1}{3})\exp(-\frac{u^2}{8})\,\phi(-\frac{1}{6};\ \frac{3}{2};\ \frac{u^2}{8}) \quad \text{If } v=0\end{cases}$$

Adding (B.12.2), (B.12.3) and (B.12.4) and multiply the sum by (B.10.4) and (B.3.3):

B.12.5 $PT(t_o,\ t_1)$

$$= \frac{c_{oo}^2}{(2h\omega)}\exp(-\tfrac{w^2}{4})\exp(-\tfrac{u^2}{4})\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta}-\underset{\sim}{\xi}_{\alpha'\beta'})^2\}F(\alpha,\beta,\gamma,\delta;t_o-t_1)\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}_{\alpha'\beta'}-\underset{\sim}{\xi}^*_{\gamma\delta})^2\}$$

$$\times\Big[\ \tfrac{3}{4}\{\phi(-1;\tfrac{3}{2};\tfrac{w^2}{4})+\phi(-1;\tfrac{3}{2};\tfrac{u^2}{4})\}\begin{cases}\frac{1}{uv}\{\phi(1;\ \frac{1}{2};\ -\frac{1}{8}(u-v)^2)-\phi(1;\ \frac{1}{2};\ -\frac{1}{8}(u+v)^2)\}\\ \exp(-\frac{v^2}{8})\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{8}) \quad \text{If } u=0\\ \exp(-\frac{u^2}{8})\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{8}) \quad \text{If } v=0\end{cases}$$

$$+\ \Gamma(\tfrac{2}{3})\begin{cases}(\frac{1}{2uv})\{\phi(\frac{2}{3};\ \frac{1}{2};\ -\frac{1}{8}(u-v)^2)-\phi(\frac{2}{3};\ \frac{1}{2};\ -\frac{1}{8}(u+v)^2) & \text{If } u\neq 0;\ v\neq 0\\ \frac{1}{3}\exp(-\frac{v^2}{8})\phi(-\frac{1}{6};\ \frac{3}{2};\ \frac{v^2}{8}) & \text{If } u=0\\ \frac{1}{3}\exp(-\frac{u^2}{8})\phi(-\frac{1}{6};\ \frac{3}{2};\ \frac{u^2}{4}) & \text{If } v=0\end{cases}\Big]$$

where

B.12.6 $F(\alpha,\beta,\gamma,\delta;\ t_1-t_1) = 2t_ot_1(2+x_o)\delta_{\alpha\gamma}\ \delta_{\beta\delta}-t_ot_1(\cdot+x_o)\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

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B.13 $PT(t_1 - t_1)$ - Term

In this term we have the following Fourier transforms:

B.13.1 $$\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\ \underset{\sim}{w}-k^2_{i\ell}) \int d^3k_{rs}\ \exp(-i\underset{\sim}{k}_{rs}\ \underset{\sim}{u}-k^2_{rs})$$
$$\int d^3k_{r's'}\ \exp(i\underset{\sim}{k}_{r's'}\ \underset{\sim}{u}-k^2_{r's'}) \int d^3k_{mn}\ \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v}-k^2_{mn})$$
$$\delta(k^2_{r's'} - k^2_{mn})\{(k^2_{i\ell} + k^2_{r's'})(k^2_{r's'} + k^2_{mn})\}$$

This is treated as a sum of two terms - viz.

B.13.2 $$\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\ \underset{\sim}{w}-k^2_{i\ell}) \int d^3k_{rs}\ \exp(-1\underset{\sim}{k}_{rs}\ \underset{\sim}{u}-k^2_{rs})$$
$$\int d^3k_{r's'}\ \exp(i\underset{\sim}{k}_{r's'}\ \underset{\sim}{u}-k^2_{r's'}) \int d^3k_{mn}\ \exp(-1\underset{\sim}{k}_{mn}\ \underset{\sim}{v}-k^2_{mn})$$
$$\delta(k^2_{r's'} - k^2_{mn})\{k^2_{i\ell}(k^2_{r's'} + k^2_{mn})\}$$
$$= 3\ \pi^5\ \Gamma(\tfrac{2}{3})\ \exp(-\tfrac{w^2}{4})\exp(-\tfrac{u^2}{4})\,\phi(-1;\ \tfrac{3}{2};\ \tfrac{w^2}{4})$$
$$\begin{cases} (\frac{1}{2uv})\{\phi(\frac{2}{3};\ \frac{3}{2};\ -\frac{1}{8}(u-v)^2) - \phi(\frac{2}{3};\ \frac{3}{2};\ -\frac{1}{8}(u+v)^2)\} & \text{If } u \neq 0 \text{ and } v \neq 0 \\ (\frac{1}{3})\ \exp(-\frac{v^2}{8})\,\phi(-\frac{1}{6};\ \frac{3}{2};\ \frac{v^2}{8}) & \text{If } u = 0 \\ (\frac{1}{3})\ \exp(-\frac{u^2}{8})\,\phi(-\frac{1}{6};\ \frac{3}{2};\ \frac{u^2}{8}) & \text{If } v = 0 \end{cases}$$

and

B.13.3 $$\int d^3k_{i\ell}\ \exp(i\ \underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2{}_{i\ell})\ \exp(-i\underset{\sim}{k}_{rs}\underset{\sim}{u}-k^2{}_{rs}) \int d^3k_{mn}$$
$$\int d^3k_{r's'}\ \exp(i\underset{\sim}{k}_{r's'}\underset{\sim}{u}-k^2{}_{r's'})\ \exp\ (-i\underset{\sim}{k}_{mn}\underset{\sim}{v}-k^2{}_{mn})$$
$$\delta(k^2{}_{r's'}-k^2{}_{mn})\{k^2{}_{rs}(k^2{}_{r's'}+k^2{}_{mn})\}$$

$$= \quad 3\pi^5\,\Gamma(\tfrac{2}{3})\ \exp(-\tfrac{w^2}{4})\ \exp(-\tfrac{u^2}{4})\ \ (-1;\tfrac{3}{2};\tfrac{u^2}{4})$$

$$\times \begin{cases} (\tfrac{1}{2}uv)\ \ \phi(\tfrac{2}{3};\tfrac{3}{2};\ -\tfrac{1}{8}(u-v)^2)-\phi(\tfrac{2}{3};\tfrac{3}{2};-\tfrac{1}{8}(u+v)^2)\} & \text{If } u\neq o \text{ and } v\neq o \\ (\tfrac{1}{3})\ \exp(-\tfrac{v^2}{8})\ \ (-\tfrac{1}{6};\tfrac{3}{2};\tfrac{v^2}{8}) & \text{If } u=o \\ (\tfrac{1}{3})\ \exp(-\tfrac{u^2}{8})\,\phi(-\tfrac{1}{6};\tfrac{3}{2};\tfrac{u^2}{8}) & \text{If } v=o \end{cases}$$

Adding (B.13.2) and (B.13.3) and multiplying the sum bv(B.10.4) and (B.3.3) we obtain:

$$\text{B.13.4}\quad PT(t_1-t_1) = \frac{C^2\omega}{2\hbar\omega}\sum_{\alpha\ldots\beta'}\exp(-\frac{\eta'^2_{\alpha\beta}}{4}\exp(-\frac{\eta^2_{\alpha'\beta'}}{4})\exp\{-\tfrac{1}{2}(\xi'_{\alpha\ \beta}-\xi_{\alpha'\beta'})^2\}$$

$$\exp\ -\tfrac{1}{2}(\xi_{\alpha\ \beta}-\xi^*_{\gamma\delta})^2\ \}F(\alpha,\beta,\gamma,\delta;\ t_1-t_1)$$

$$\times \begin{cases} (\frac{1}{2uv})\{\phi(\frac{2}{3};\frac{1}{2};-\frac{1}{8}(u-v)^2)-\phi(\frac{2}{3};\frac{1}{2};-\frac{1}{8}(u+v)^2)\} & \text{If } u\neq o \text{ and } v\neq o \\ \frac{1}{3}\ \exp(-\frac{v^2}{8})\phi\ (-\frac{1}{6};\frac{3}{2};\frac{v^2}{8}) & \text{If } u\neq o \\ \frac{1}{3}\ \exp\ (-\frac{u^2}{8})\,\phi(-\frac{1}{6};\frac{2}{3};\frac{u^2}{8}) & \text{If } v\neq o \end{cases}$$

$\sum_{\alpha\ldots\beta'}$ means the summation over $\alpha,\beta,\gamma,\delta,\alpha',\beta'$,

where $F(\alpha,\beta,\gamma,\delta\ ;\ t_1-t_1)$ is as defined previously.

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forward angles may be due to absorption and/or refraction and the backward rise may be due to particle exchange and/or reflection.

Fig. (2.4) : s-wave phase shift caused by the Pauli principle in dineutron-dineutron scattering. The dashed curve refers to b=b'=1 fm and the solid line to b=2b'=2 fm; near resonance the phase is displayed for various step lengths d in the numerical solution of eq.(2.3.8).

Fig. (2.5) : $\alpha - {}^{16}O$ phase shifts for even partial waves. The upper part refers to $b_\alpha = b_{^{16}O}$, the lower one to $b_\alpha = b_{^{16}O}$, the lower one to $b_\alpha \neq b_{^{16}O}$.

Fig. (3.1) : Fermi spheres F_1 and F_2 separated by the local momentum K per nucleon.

Fig. (4.1) : Fragments A and B as two Fermi spheres approaching each other with relative initial momentum.

Fig. (4.2) : $\underset{\sim}{R}_A$ and $\underset{\sim}{R}_B$ describe the position of the oscillator potentials of the clusters, which is the mean position of the fragments, $\underset{\sim}{R}$ describes the mean separation between the fragments.

Fig. (6.1) : Solid line: The imaginary potential

$$W = \sum_{\substack{\alpha,\beta,\gamma,\delta \\ occ.}} (\alpha,\beta|\,\mathrm{Im}\ G|\gamma,\delta)(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\ \delta_{\beta\gamma})$$

as a function of R for various values of R'(fm) and K = O, $\theta=0^0$.

dashed line: W as a function of R for R = R' and $\theta = 0^0$.

Fig. (6.2) : W as a function of R for R = R' and $\theta = 0^0$ and 30^0.

Fig. (6.3): Same as Fig. (6.1) but with K = O, $\theta = 30^0$.

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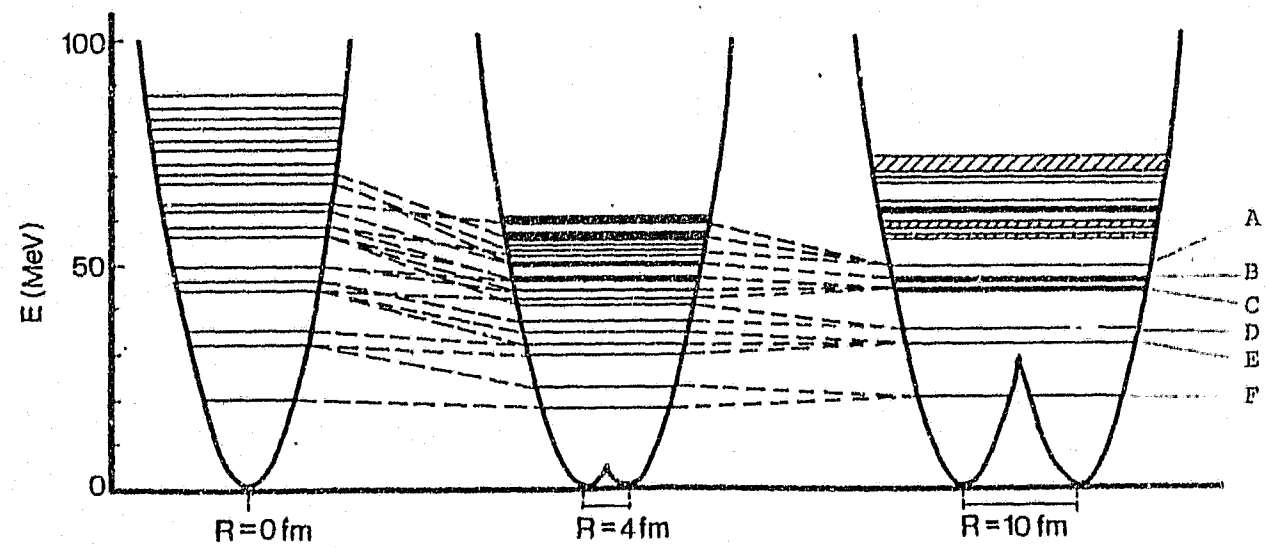


Fig. (1.1)

A = 1d $\frac{3}{2}$; $\frac{3}{2}\pm$; $\frac{1}{2}\pm$

B = 2s $\frac{1}{2}$; $\frac{1}{2}\pm$

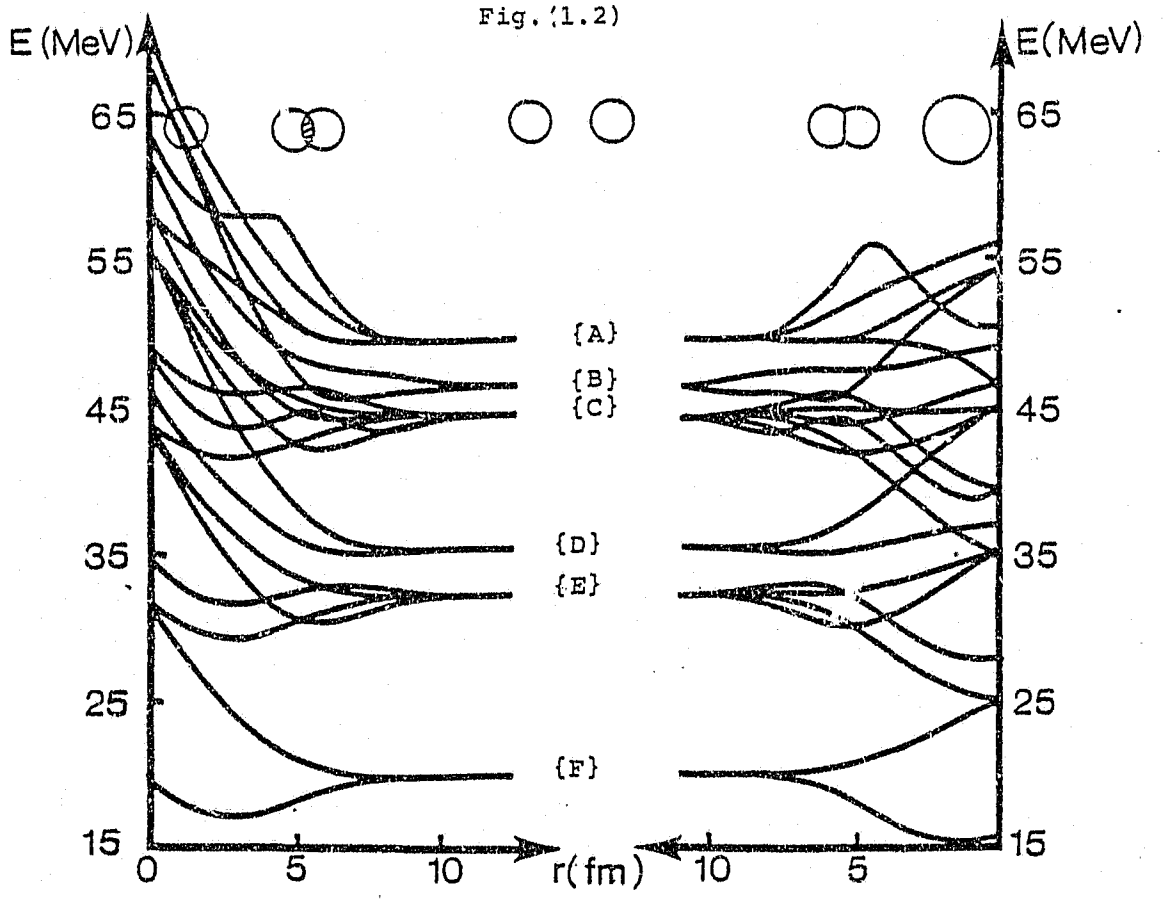
C = 1d $\frac{5}{2}$; $\frac{5}{2}\pm$; $\frac{3}{2}\pm$; $\frac{1}{2}\pm$

D = 1p $\frac{1}{2}$; $\frac{1}{2}\pm$

E = 1p $\frac{3}{2}$; $\frac{3}{2}\pm$; $\frac{1}{2}\pm$

F = 1s $\frac{1}{2}$; $\frac{1}{2}\pm$

Fig. (1.2)



$$A = \begin{vmatrix} 1d\ \frac{3}{2}\ \frac{1}{2}\ - \rangle \\ |1d\ \frac{3}{2}\ \frac{3}{2}\ + \rangle \\ |1d\ \frac{3}{2}\ \frac{3}{2}\ - \rangle \\ |1d\ \frac{3}{2}\ \frac{1}{2}\ + \rangle \end{vmatrix}$$

$$B = \begin{vmatrix} 2s\ \frac{1}{2}\ \frac{1}{2}\ - \rangle \\ |2s\ \frac{1}{2}\ \frac{1}{2}\ + \rangle \end{vmatrix}$$

$$C = \begin{vmatrix} 1d\ \frac{5}{2}\ \frac{1}{2}\ - \rangle \\ |1d\ \frac{5}{2}\ \frac{3}{2}\ + \rangle \\ |1d\ \frac{5}{2}\ \frac{3}{2}\ - \rangle \\ |1d\ \frac{5}{2}\ \frac{5}{2}\ + \rangle \\ |1d\ \frac{5}{2}\ \frac{5}{2}\ - \rangle \\ |1d\ \frac{5}{2}\ \frac{1}{2}\ + \rangle \end{vmatrix}$$

$$D = \begin{vmatrix} 1p\ \frac{1}{2}\ \frac{1}{2}\ - \rangle \\ |1p\ \frac{1}{2}\ \frac{1}{2}\ + \rangle \end{vmatrix}$$

$$E = \begin{vmatrix} 1p\ \frac{3}{2}\ \frac{1}{2}\ - \rangle \\ |1p\ \frac{3}{2}\ \frac{3}{2}\ + \rangle \\ |1p\ \frac{3}{2}\ \frac{3}{2}\ - \rangle \\ |1p\ \frac{3}{2}\ \frac{1}{2}\ + \rangle \end{vmatrix}$$

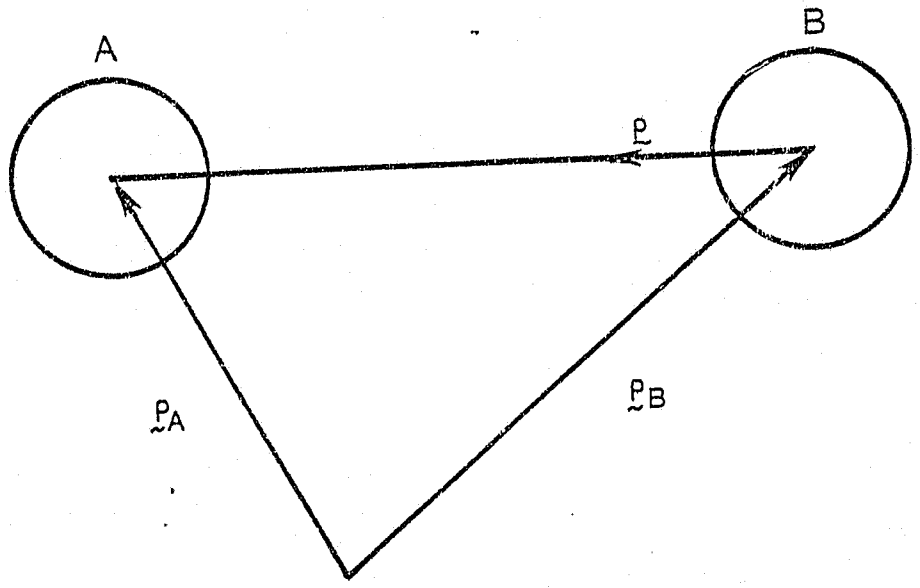


Fig. (2.1)

$$\underset{\sim}{\rho}_A = A^{-1} \sum_{i \in A} \underset{\sim}{r}_i \quad , \qquad \underset{\sim}{\rho}_\beta = \beta^{-1} \sum_{j \in \beta} \underset{\sim}{r}_j \quad , \quad \underset{\sim}{\rho} = \underset{\sim}{\rho}_A - \underset{\sim}{\rho}_B$$

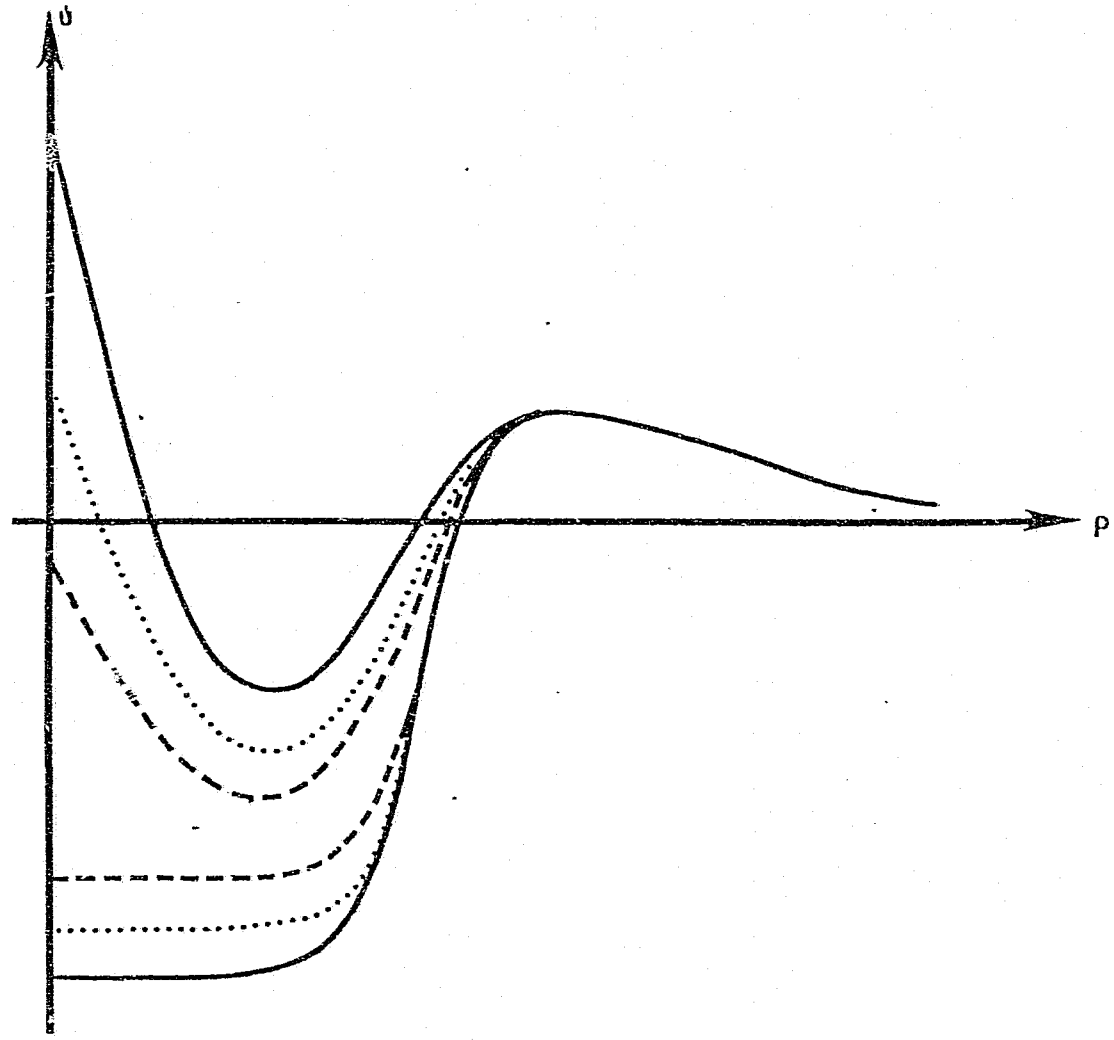


Fig. (2.2)

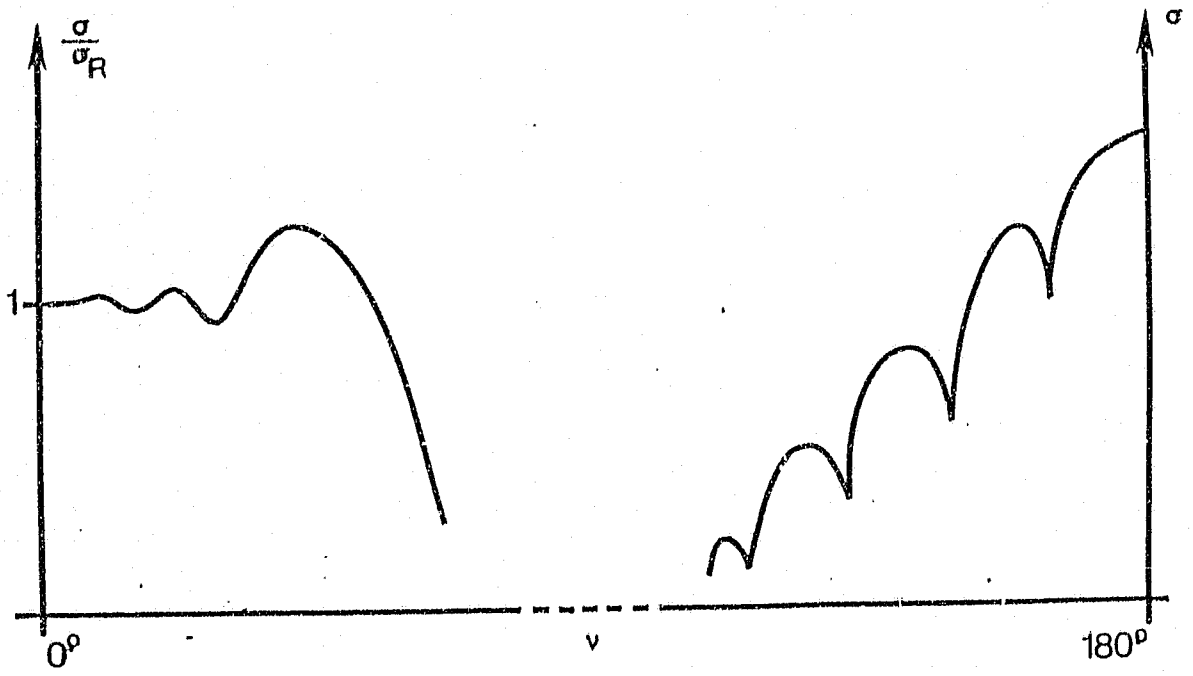


Fig. (2.3)

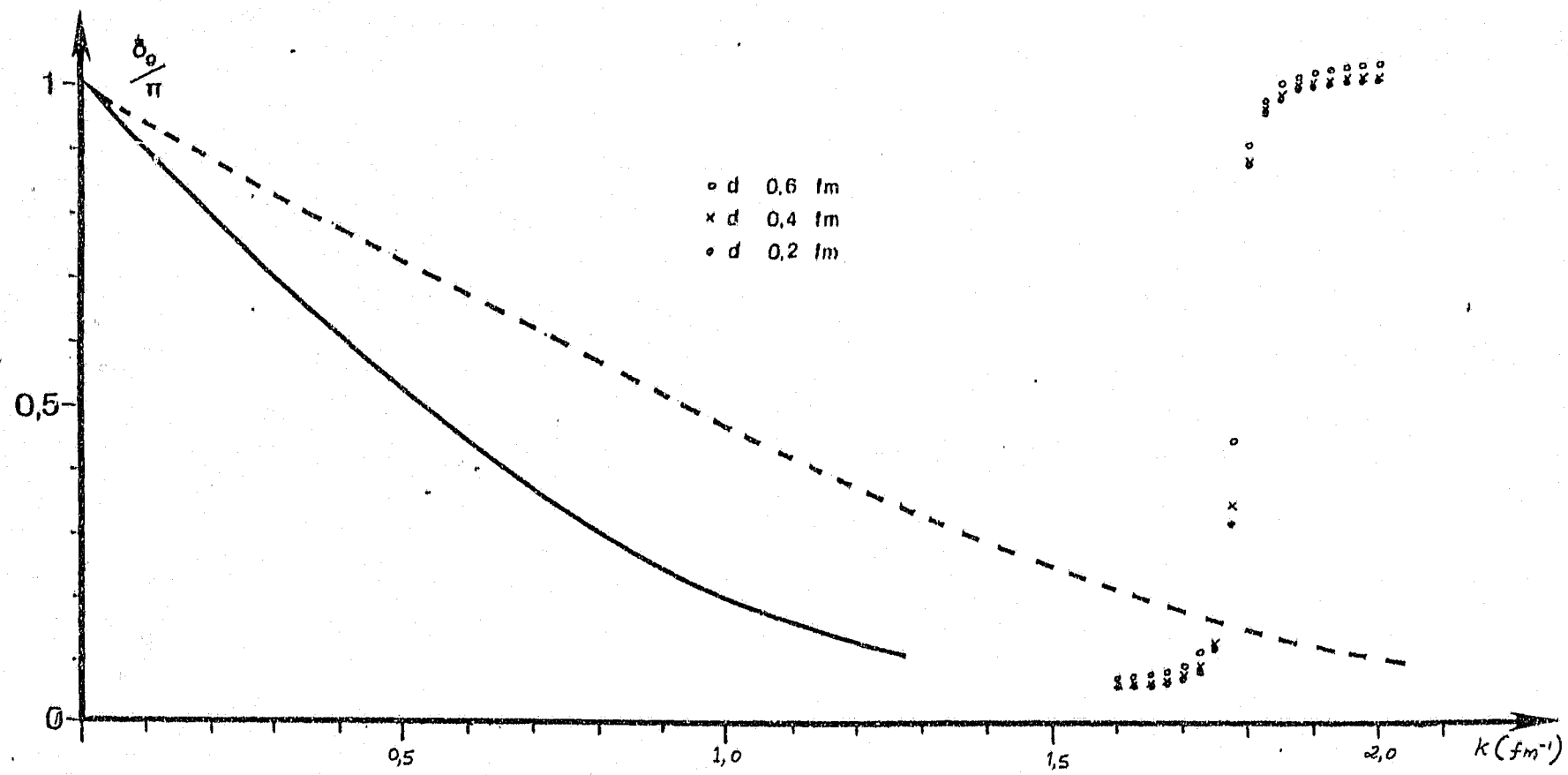
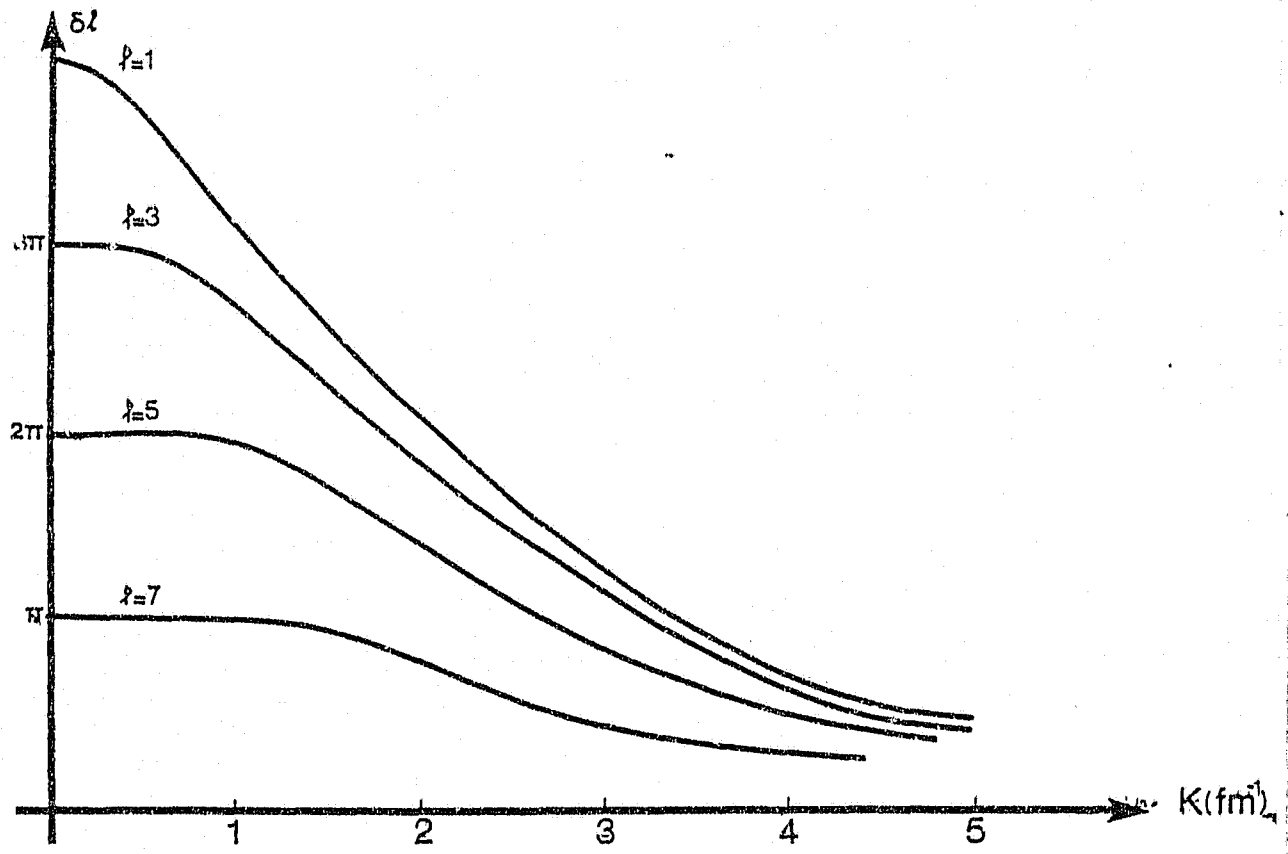


Fig. (2.4)


δl
l=1
l=3
l=5
l=7
3π
2π
π
1
2
3
4
5
K(fm⁻¹)

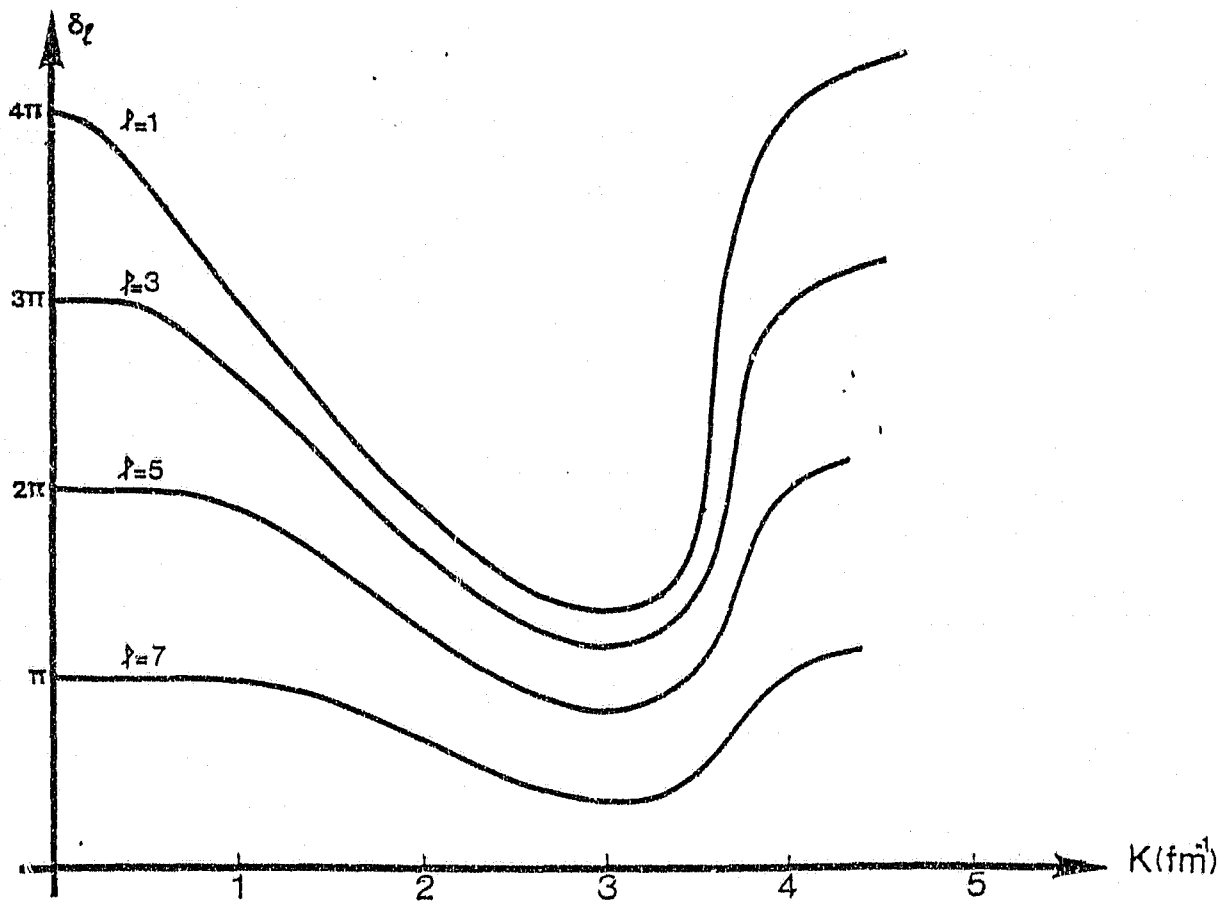
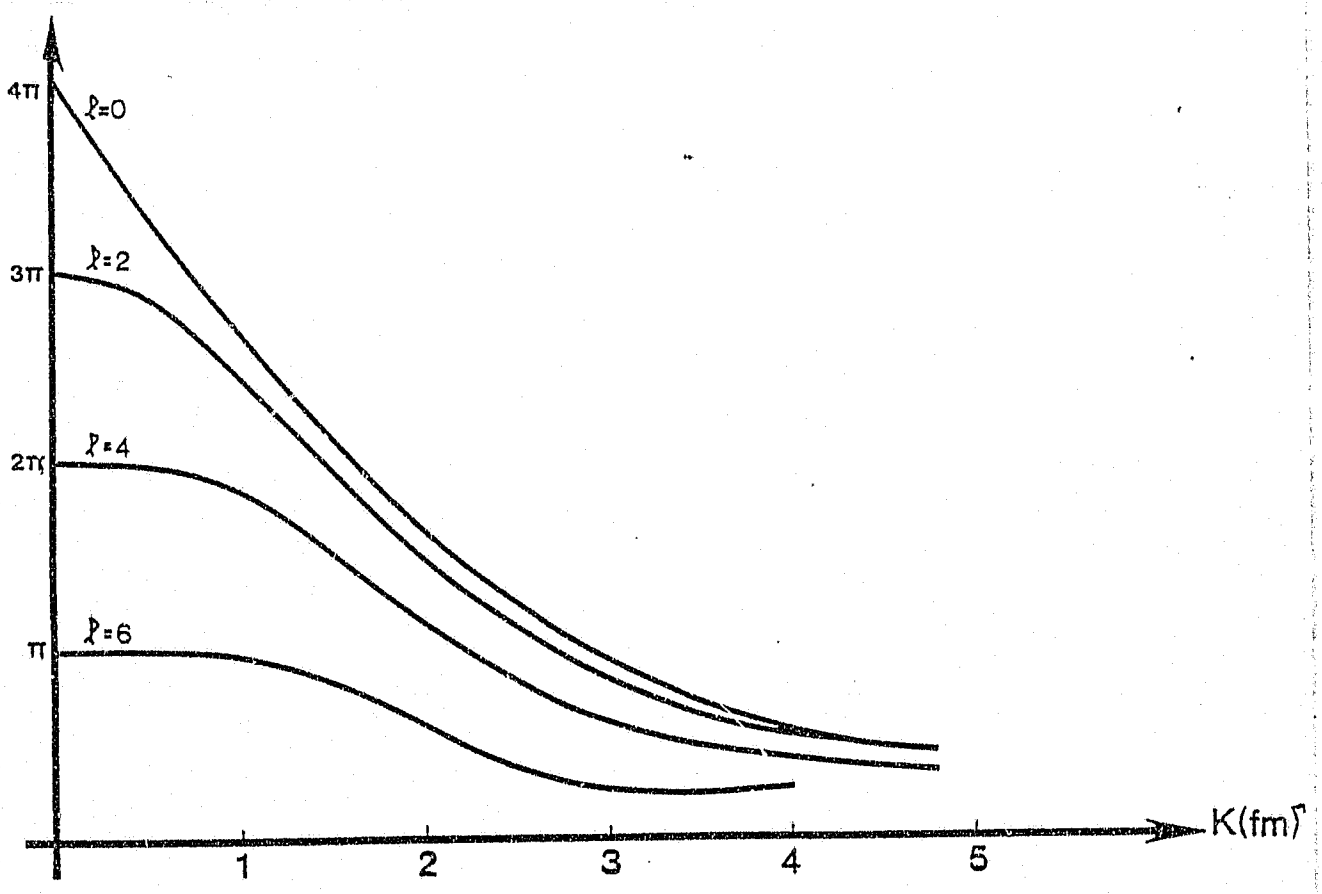

δl
4π
3π
2π
π
l=1
l=3
l=5
l=7
1
2
3
4
5
K(fm⁻¹)

Fig. (2.5)



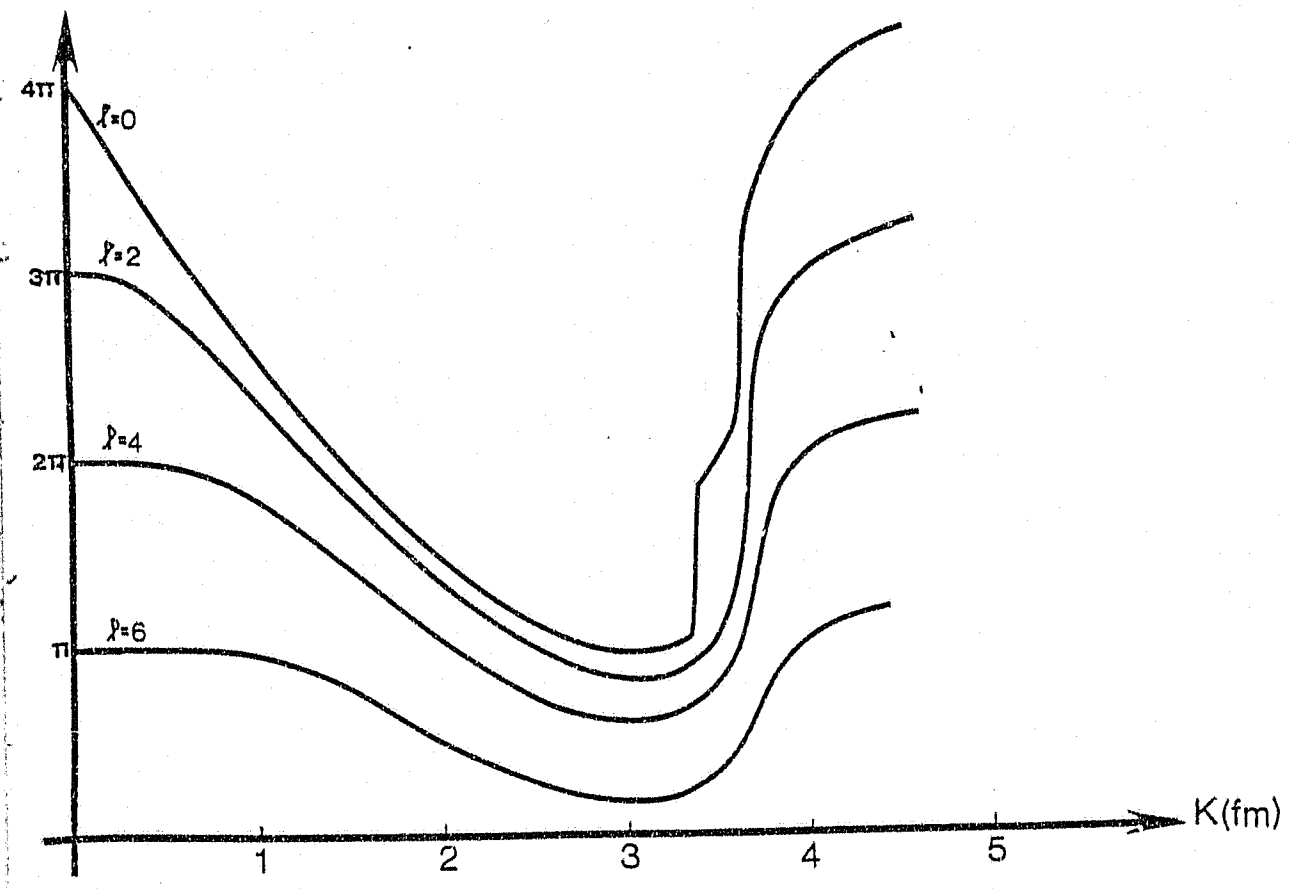


Fig. (2.6)

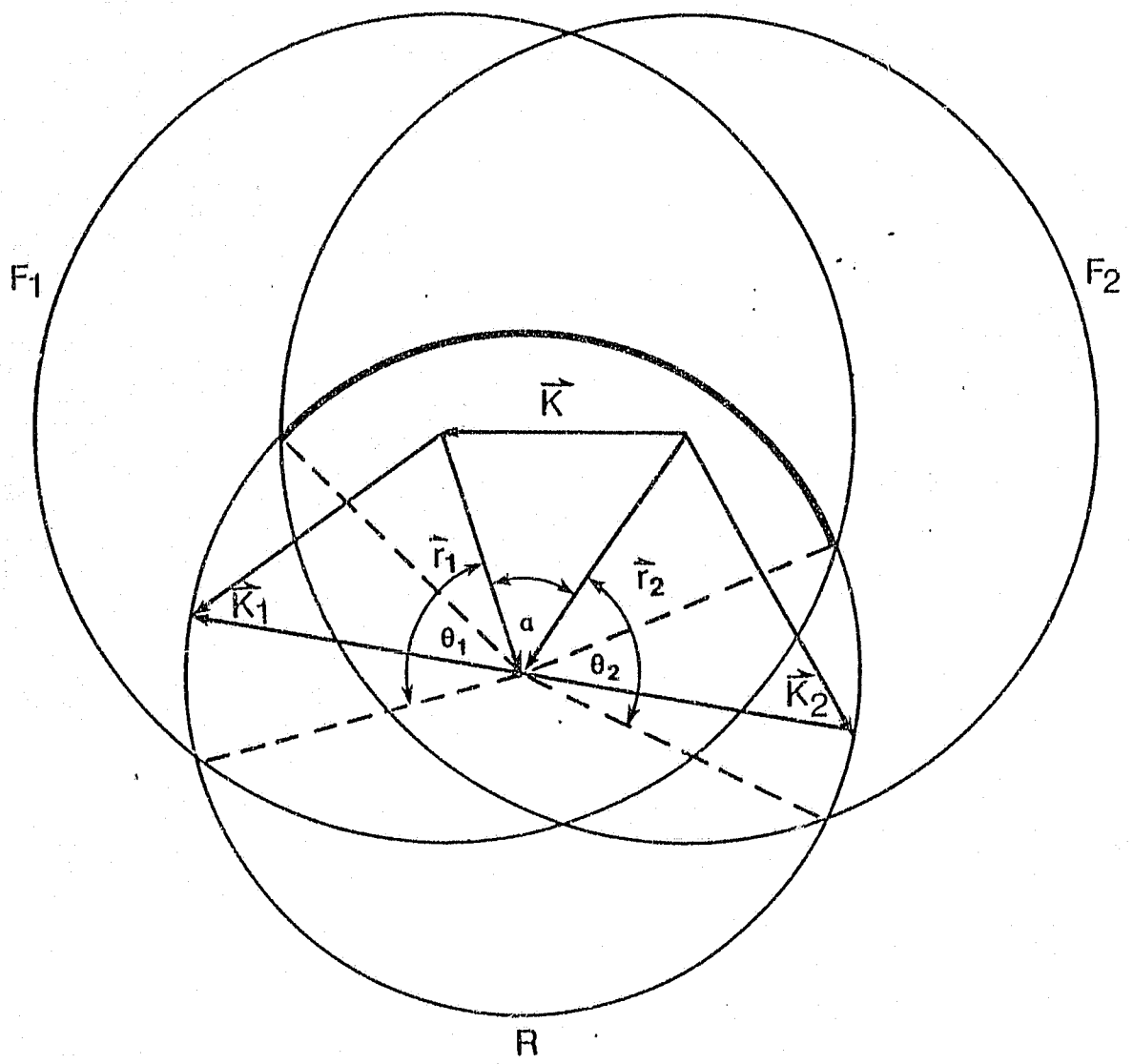


Fig. (3.1)

B.6.2 $\underset{\sim}{w} = \underset{\sim}{\eta}'_{\alpha\rho} = \underset{\sim}{R}'_\alpha - \underset{\sim}{R}'_\beta - i(\underset{\sim}{K}_\alpha - \underset{\sim}{K}_\beta)$; $\underset{\sim}{v} = \underset{\sim}{\eta}^*_{\gamma\delta} = \underset{\sim}{R}_\gamma - \underset{\sim}{R}_\delta + i(\underset{\sim}{K}_\gamma - \underset{\sim}{K}_\delta)$.

The $\underset{\sim}{K}_{i\ell}$-integration gives (See A.1)

B.6.3 $\int d^3K_{i\ell}\ \exp\{i\underset{\sim}{K}_{i\ell}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}^*_{\gamma\beta}) - \tfrac{1}{2}K^2_{i\ell}\} = (2\pi)^{3/2}\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}_{\gamma\delta})^2\}$

where

B.6.4 $\underset{\sim}{\xi}'_{\alpha\beta} = \tfrac{1}{2}(\underset{\sim}{R}'_\alpha + \underset{\sim}{R}'_\beta) - \frac{i}{2}(\underset{\sim}{K}_\alpha + \underset{\sim}{K}_\beta)$; $\underset{\sim}{\xi}^*_{\gamma\delta} = \frac{1}{2}(\underset{\sim}{R}_\gamma - \underset{\sim}{R}_\delta) + \frac{i}{2}(\underset{\sim}{K}_\gamma + \underset{\sim}{K}_\delta)$

(B.6.3) can also be factored out as a common factor in the NPT calculation and together with $\dfrac{c_{oo}^2\pi.}{\hbar\omega(2\pi)^6}$ it gives:

B.6.5 $\dfrac{(2\pi)^{3/2}\pi c_{oo}^2}{\hbar\omega(2\pi)^6}\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}^*_{\gamma\delta})^2\}$ and $c_{oo}^2 = \pi^{-3}$,

since $c_o = \pi^{-3/4}$ and $c_{oo} = c_o c_o = (\pi)^{-3/2}$.

We now have integrations over $\underset{\sim}{k}_{i\ell}$, $\underset{\sim}{k}_{rs}$ and $\underset{\sim}{k}_{mn}$ to carry out. We therefore treat the three (NPT)-terms separately.

B.7. NP$(t_o - t_o)$ - Term

It is of the form:

B.7.1 $4\{4t_o^2(1+x_o+x_o^2)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - t_o^2(1+x_o)^2\delta_{\alpha\delta}\ \delta_{\beta\gamma}\}$
$\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w} - k^2_{i\ell})\int d^3k_{rs}\int dk_{mn}\ \exp\{-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - k^2_{mn}\}$
$\delta(k^2_{rs} - k^2_{mn})$.

From (A.1), we notice that the individual integrations give the following results:

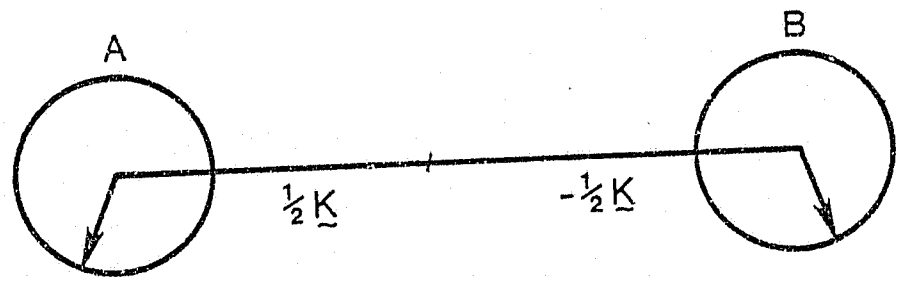


Fig. (4.1)

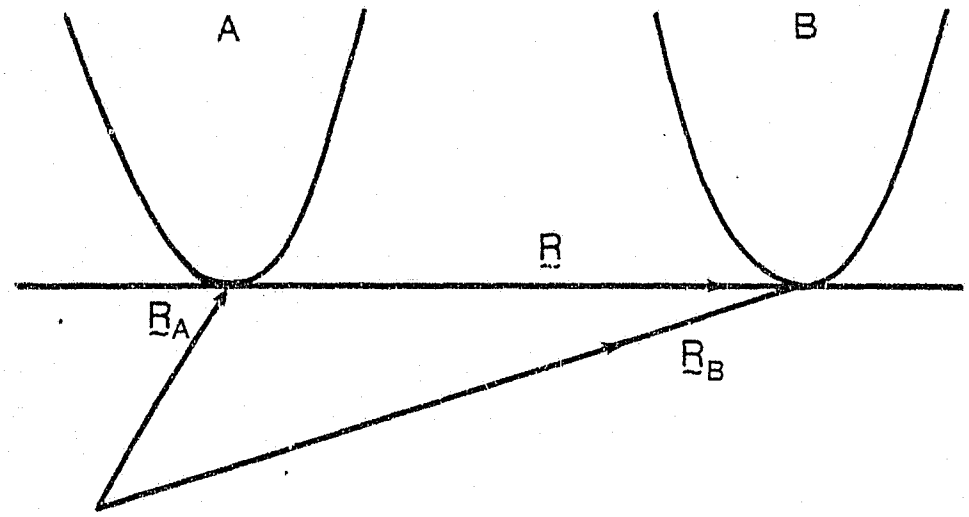


Fig. (4.2)

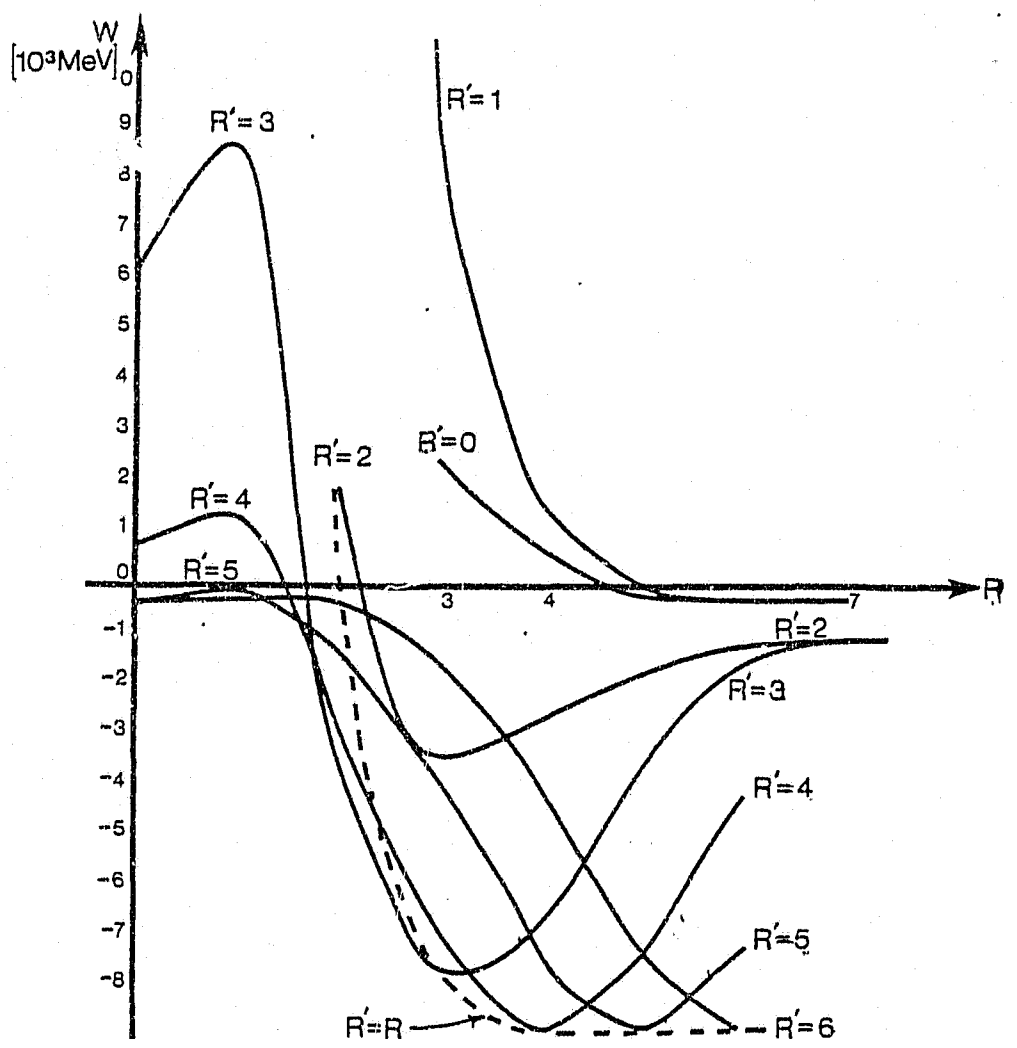


Fig. (6.1)

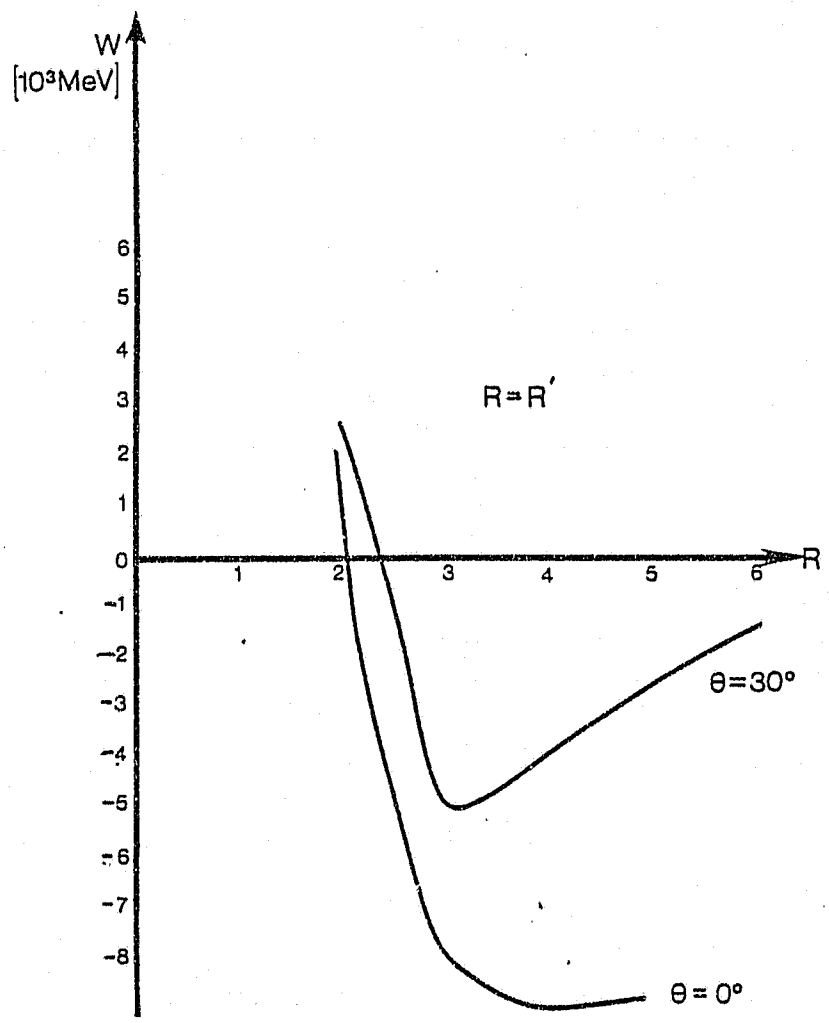


Fig. (6.2)

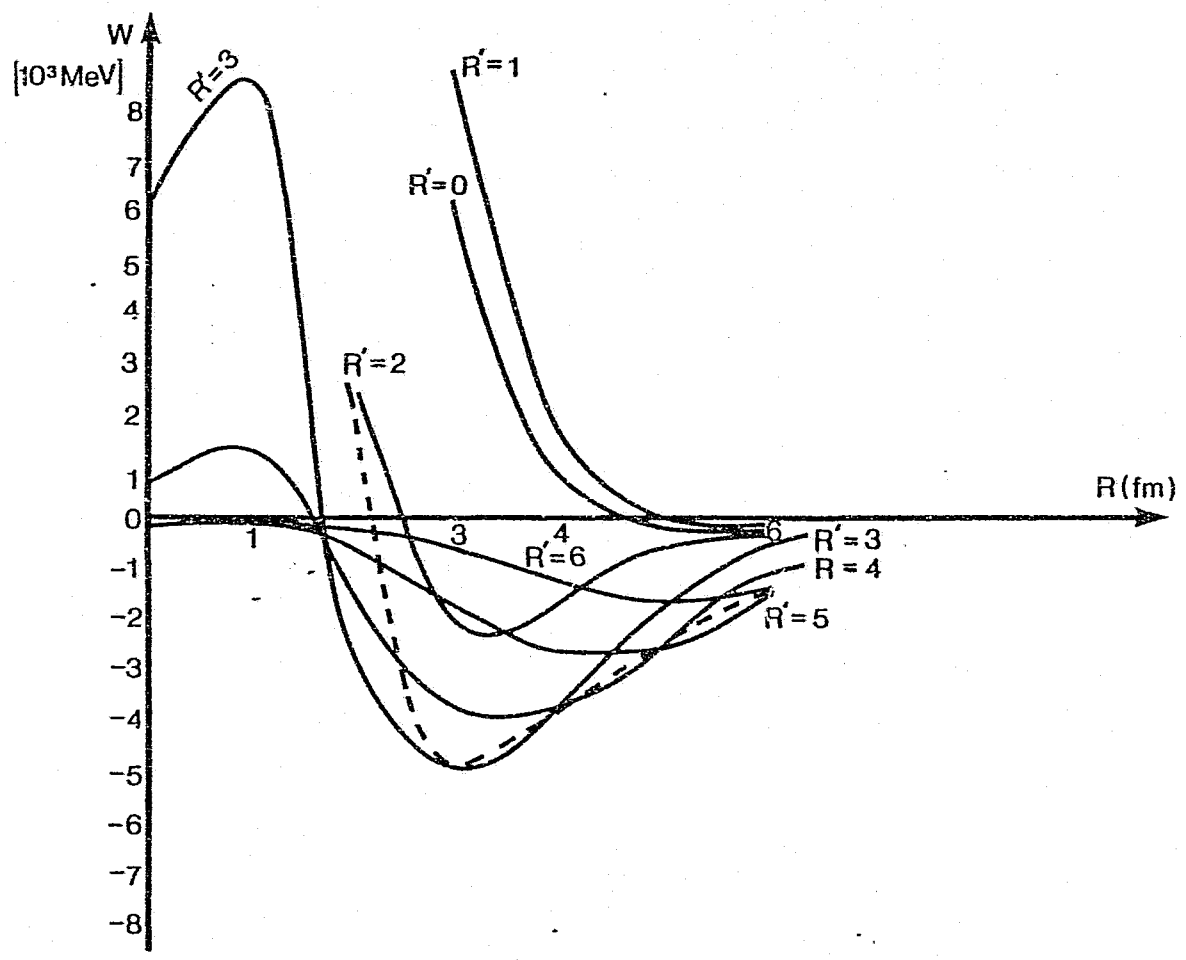


Fig. (6.3)

40. A. Erdelyi, W. Magnus, F. Oberhettinger and F.G. Tricomi, Tables of Integral Trandforms, Vol. 1 (McGraw-Hill, New York, 1954).

* * * * * * * * * * * *

FIGURE CAPTIONS

Fig. (1.1) : Instantaneous shell-model potentials for the nucleonic movement in the $^{16}O + ^{16}O$ collision ($\hbar\omega_\infty$=13,22 Mev). The single particle levels correspond to a spin-orbit force strength parameter κ=0,08 and are assigned with the quantum numbers N&j, m_π, $_\Lambda$ for the separated fragments. Regions of close lying levels are hatched.

Fig. (1.2) : Single particle levels for $^{16}O - ^{16}O$ with $\hbar\omega_\infty$=13,22 Mev and κ=0,08 as a function of relative distance for the "fast" and "slow" cases as (left and right, respectively). The levels are assigned with quantum numbers $N, \ell j, m_\pi$ for the separated fragments. The shape of the ion-ion system is shown on top of the figure.

Fig. (2.1) : Fragments A and B separated by the true seperation $\underline{\rho}$.

Fig. (2.2) : Qualitative behaviour of some heavy-ion potentials for various energies. With increasing energy (from solid through dotted to dashed lines) the deep attractive potentials become shallower while the repulsive core potentials are reduced.

Fig. (2.3) : Qualitative features of the elastic scattering cross section σ, divided by the Rutherford cross section σ_R as a function of the C.M. scattering angle. The drop of σ/σ_R below the value 1 at

A.2 $\underset{\sim}{k}_{rs}$, $\underset{\sim}{k}_{mn}$ integration with $\underset{\sim}{y} = (0,0,0)$

It is of the type:

$$\int d^3k_{rs}\ \underset{\sim}{k}_{rs}^{n}\ \exp(-bk_{rs}^2) \int d^3k_{mn}\ k_{mn}^2\ \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{y} - ck_{mn}^2)$$

$$\delta(k_{rs}^2 - k_{mn}^2)$$

$$= 2\pi \int d(k_{rs}^2) k_{rs}^{(n+1)} \exp(-bk_{rs}^2) \int d^3k_{mn}\ k_{mn}^{\ell}\ \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{y} - ck_{mn}^2)$$

$$\delta(k_{rs}^2 - k_{mn}^2)$$

$$= 2\pi \int d^3k_{mn}\ k_{rs}^{(\ell+n+1)} \exp\{-(b+c)k_{mn}^2\} \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{y})$$

$$= 4\pi^2 \int dk_{mn}\ k_{mn}^{(\ell+n+3)} \exp\{-(b+c)k_{mn}^2\} \int d(\cos\phi)$$

$$= 4\pi^2 \left(\frac{2i}{iv}\right) \int dk_{mn}\ k_{mn}^{(\ell+n+2)}\ \exp\{-(b+c)k_{mn}^2\} \sin(k_{mn}\ v)$$

$$= \frac{8\pi^2}{v} \int dk_{mn} \{k_{mn}^{(\ell+n+2)}\ \exp\{-(b+c)k_{mn}^2\} \sin(k_{mn}\ v).$$

The k_{mn}-integration is similar to Erdélyi[40] P.74 Eq. (2.2.24)) with:

$a = b + c$; $y = v$; $2\nu - 2 = \ell + n + 2 + \nu = \frac{\ell+n+4}{2}$;

and this therefore gives the result:

$$\tfrac{1}{2} v (b+c)^{-\frac{1}{2}(\ell+n+4)} \Gamma\left(\frac{\ell+n+4}{2}\right) \exp\left(-\frac{v^2}{4}\right) \phi\left(\frac{3}{2} - \frac{\ell+n+4}{2};\ \frac{3}{2};\ \frac{v^2}{4(b+c)}\right).$$

The final result is therefore:

A.2.1 $$\int d^3k_{rs}\ \underset{\sim}{k}_{rs}^{n}\ \exp(-bk_{rs}^2) \int d^3k_{mn}\ k_{mn}^{\ell}\ \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{y} - c\underset{\sim}{k}_{mn}^2)$$

$$\delta(k_{rs}^2 - k_{mn}^2)$$

$$= 4\pi^2 (b+c)^{-\frac{1}{2}(\ell+n+4)} \Gamma\left(\frac{\ell+n+4}{2}\right) \exp\left(-\frac{v^2}{4(b+c)}\right) \phi\left(-\frac{\ell+n+1}{2};\ \frac{3}{2}\ \ \frac{v^2}{4(b+c)}\right)$$

When in addition to b = 1 we have n = 0: The result is:

A.1.3 $$2\pi\Gamma'(\tfrac{3}{2})\exp(-\tfrac{w^2}{4}) = \pi^{\frac{3}{2}}\exp(-\tfrac{w^2}{4}) \qquad (\Gamma(\tfrac{3}{2}) = \tfrac{1}{2}\pi^{\frac{1}{2}})$$

For n = 2, b = 1; the result is:

A.1.4 $$2\pi\Gamma(\tfrac{5}{2})\exp(-\tfrac{w^2}{4})\phi(-1;\ \tfrac{3}{2};\ \tfrac{w^2}{4})$$
$$= \tfrac{3}{2}\pi^{\frac{3}{2}}\exp(-\tfrac{w^2}{4})\phi(-1;\ \tfrac{3}{2};\ \tfrac{w^2}{4}) \qquad (\Gamma(\tfrac{5}{2}) = \tfrac{3}{4}\pi^{\frac{1}{2}})$$

APPENDIX A

$\underset{\sim}{k}$ - INTEGRATIONS IN SPHERICAL COORDINATES

A.1 $\underset{\sim}{k}_{i\ell}$-integration

It is of the type:

$$\int d^3k_{i\ell}\ k_{i\ell}^n \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{W} - bk_{i\ell}^2)$$

$$= 2\pi\int d(\cos\theta)\int dk_{i\ell}\ k_{i\ell}^{(n+2)} \exp(ik_{i\ell}W\cos\theta)\exp(-bk_{i\ell}^2)$$

$$= 2\pi\left(\frac{2i}{iW}\right)\int dk_{i\ell}\left(k_{i\ell}^{(n+1)}\ \exp(-bk_{i\ell}^2)\right)\sin(k_{i\ell}W)$$

$$= \frac{4\pi}{W}\int dk_{i\ell}\{k_{i\ell}^{(n+1)}\ \exp(-bk_{i\ell}^2)\}\sin(k_{i\ell}W).$$

The δ-integration is now reduced to the Fourier sine Transform[40] (P.74 Eq. (2.4.24)) with a = b; y = w; and $2\nu - 2 = n + 1 \Rightarrow \nu = \frac{n+3}{2}$.

Hence $\int dk_{i\ell}\{k_{i\ell}^{(n+1)}\ \exp(-bk_{i\ell})\}\sin(k_{i\ell}W)$

$$= \tfrac{1}{2}b^{-\frac{1}{2}(n+3)}\Gamma\left(\frac{n+3}{2}\right)w\ \exp\left(-\frac{w^2}{4b}\right)\phi\left(\frac{3}{2} - \frac{n+3}{2};\ \frac{3}{2};\ \frac{w^2}{4b}\right).$$

A.1.1 Thus $\int d^3k_{i\ell}\ k_{i\ell}^n \exp(ik_{i\ell}\underset{\sim}{W} - bk_{i\ell}^2)$

$$= 2\pi b^{-\frac{1}{2}(n+3)}\Gamma\left(\frac{n+3}{2}\right)\exp\left(-\frac{w^2}{4b}\right)\phi\left(-\frac{n}{2};\ \frac{3}{2};\ \frac{w^2}{4b}\right),$$

when b = 1, (A.1.1) reduces to:

A.1.2 $$\int d^3k_{i\ell}\ k_{i\ell}^n \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{W} - b^2k_{i\ell}^2) = 2\pi\Gamma\left(\frac{n+3}{2}\right)\exp\left(-\frac{w^2}{4}\right)$$
$$\phi\left(-\frac{n}{2};\ \frac{3}{2},\ \frac{w^2}{4}\right).$$

In principle the absorption must vanish ($W \to 0$) in the limit $K \to 0$ with a proper Pauli operator (nuclear matter limit) for all values of R. This is not the case as one notices from the Figs.6.1 - 6.3. Even more, one does not even obtain $W \to 0$ in the $R=R' \to \infty$ case where the nuclei do not overlap. Even more and where the replacement $B_{\alpha\gamma} = \delta_{\alpha\gamma}$ is justified. This behaviour results from the use of plane waves as intermediate states in the evaluation of W which leads to unphysical excitations. In the overlap region one even obtains an unrealistic sign for the absorption. On the other hand, one obtains for a given R, maximal absorption for $R'=R$, i.e. for maximal overlap in respect to the fluctuations in the mean separation coordinate R. This is supported by the results of Figs. (6.2) and (6.3) for $\theta = 30^\circ$. These numerical results have been checked in the limits $R'=R \to \infty$ and $R'=R=0$ for selected values of the parameters x_o, t_o and t_1.

* * * * * * * * *

CHAPTER 6

RESULTS AND DISCUSSION

The scattering of composite particles has been considered with primary emphasis on two aspects viz.-

(a) how to describe it microscopically and

(b) how to describe the absorption.

Conventionally one assumes that the clusters are intrinsically in their ground states, since otherwise the treatment turns out to be too complicated. In such a description one only describes the elastic channel in isolation, although many channels are open in general. It was therefore proposed to use the Feshbach projection technique with the aim of constructing an effective (non-Hermitian) Hamiltonian for the elastic channel.

Some results of an actual calculation of

$$W = \sum_{\substack{\alpha,\beta,\gamma,\delta \\ \text{occup.}}} (\underline{\alpha},\underline{\beta}|\,\mathrm{Im}\ G\,|\underline{\gamma},\underline{\delta})\ (\delta_{\alpha\gamma}\,\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma})$$

(formula 5.4.4) for $B_{\alpha\gamma} = \delta_{\alpha\gamma}$ etc. and s-states (corresponding to α-α scattering) are presented in Figs. 6.1 - 6.3. In Fig. (6.1) W is plotted as a function of R the mean separation, for various values of R' and K=0, $\theta=0^0$. As anticipated in chapter 5, there arise inconsistencies because of our approximation for the Pauli operator, which is not a true Pauli operator because of the non-orthogonality of the basis functions as described by the overlap matrices $B_{\alpha\gamma}$ etc.

CHAPTER 2

GENERATOR COORDINATE METHOD AND THE RESONATING GROUP METHOD

2.1 Generator Coordinate Method (GCM).

During the past few years numerous attempts have been made to describe reactions of composite fragments in the generator coordinate method (GCM). In this method the many-body wave function Ψ is expanded in terms of totally antisymmetrized bound state wave functions, describing various given fragment separations. Solving a scattering problem then amounts to (i) calculating matrix elements in the basis spanned by the totally antisymmetrized bound state wave functions, and to (ii) solving a set of integral equations for the expansion coefficients of Ψ in the given basis of the bound state problem. Problem (i) can be handled by the well-known shell-model techniques for calculating matrix elements of one - and two - body operators in a non-orthogonal basis. We want to analyse the interaction of composite particles like nuclei with A nucleon coordinates $\underline{r}_i$. The clusters A and B, separated by the distance $\underline{s}$, are considered, for which the single particle oscillator potentials are separated by a distance $\underline{R}$ Fig (2.1). This separation distance $\underline{R}$ is a collective variable. It is superfluous and there arises the task to extract a wave function for the true coordinates of relative motion $\underline{\rho}$. In the generator coordinate method (GCM) the total many-body wave function is assumed to be a superposition of generating functions $\Phi(\underline{r}, \underline{R})$

$$2.1.1 \qquad \Phi^{GCM}(\underline{r}) = \int \Phi(\underline{r},\underline{R}) f(\underline{R}) d^3R \,,$$

1.3 Absorption

Chapter 3 outlines the folding model of Toepffer. The consequences of the Pauli principle on the absorption is considered. The G - matrix concept of Brueckner is introduced here. The construction of the imaginary part of the effective nucleon interaction G between bound particles becomes the theme of Chapter 4. Pauli principle and absorption are here treated on a microscopic level with the help of the Feshbach projection technique. The imaginary part of the scattering operator for bound nucleons is obtained in closed form by using internal harmonic oscillator (HO) wave functions and simple nucleon - nucleon interactions (like the Skyrme force).

The shortcomings of the use of plane waves and the reasons for switching to the use of moving displaced harmonic oscillator (HO) functions are discussed. In chapter 5 the analytical techniques adopted in deriving an expression for calculating the matrix elements of the imaginary part of the scattering operator are discussed. The results and analysis appear in the concluding chapter 6.

* * * * * * * * *

symmetrized bound state wave functions, which describe various given fragment separations.

This generator coordinate method (GCM) approach has the merits that -

(i) it preserves the intuitive meaning of geometrical configurations of the clusters in the cluster-intrinsic states and the resulting collective features,

(ii) it enables one to do calculations with Slater-determinants and also

(iii) it contains the corresponding shell-model configurations by taking the appropriate limit.

At the time Brink proposed this approach, the applicability of the GCM was considered to be limited to the bound states only, but in the meantime the difficulties associated with the scattering problem have been overcome[7].

The resonating group method (RGM) was proposed by Wheeler about fourty years ago, primarily to describe the effects resulting from the exchange of particles between the clusters. Since then, this method has been applied to various problems. Recently Saito[8] has proposed an additional model to the two above known as the orthogonality condition model (OCM). It is a simplified model for the treatment of the effects of the Pauli principle. In this model, the inter-cluster wave function is required to be orthogonal to the forbidden states which give rise to the null states if the complete antisymmetrization is performed, so that the interaction takes place only in the space orthogonal to the forbidden states.

The total wave function (1.2.1) is the solution of the Schrödinger equation:

1.2.2 $$\langle\Psi(\underline{K},\underline{R})\,|(H-E)|\,\Psi(\underline{K},\underline{R})\rangle = \frac{\hbar^2K^2}{2\mu} + V(\underline{K},\underline{R})-E = 0$$

where $\frac{\hbar^2K^2}{2\mu}$ is the kinetic energy operator for the relative motion in the C.M. - system. The Schrödinger equation is solved after transforming the cluster wave functions ϕ_A and ϕ_B into an orthogonal set of basis functions. The microscopic Hamiltonian H consists of the usual kinetic and two-body interaction terms. The anti-symmetrization in Eq (1.2.2) leads to a repulsion between the clusters in the overlap region, which had already been noted by van der Spuy (1959)[3] who considered α-α scattering. It is a particularly attractive feature of this treatment that it allows a discussion of Pauli principle effects in both ordinary and momentum space.

(c) <u>The folding model.</u>

This is an approach in which one attempts to derive the heavy-ion potential by finding out how collisions between composite particles are determined by the "elementary" encounters between the constituents, making allowance for many-body effects.

(d) <u>The GCM and RGM methods.</u>

In Chapter 2 a discussion and a general review of the generator coordinate method (GCM)[4] and the resonating group method (RGM) which was proposed by Wheeler[5] is given. The GCM was initially proposed by Brink[6] in his microscopic α - particle model. The reactions between composite particles are described in the generator coordinate method (GCM), in which the many-body wave function is expanded in terms of totally anti-

assumed to be frozen, which leads to a compression of nuclear matter. In the adiabatic limit it is supposed that the collision is sufficiently slow for nuclear matter to be in a relaxed state at every way that no compression takes place during the collision. The single particle levels in the adiabatic limit are shown on the right side of Fig. (1.2). It is the distinguishing feature of this model that the fragments are increasingly polarized with decreasing distance until the compound nucleus structure emerges for large overlap.

(b) The cluster model

While the two-center shell model serves as a good basis for microscopic treatment of the intrinsic motion of the nucleus in a heavy-ion collision, there arise difficulties in treating the relative motion (and other collective degrees of freedom) of the fragments. In 1971 Fliessbach[2] proposed to consider two harmonic oscillator model wave functions ϕ_A and ϕ_B for clusters localized at a mean separation distance $\langle \underline{\varrho} \rangle = \langle \underline{\varrho}_A - \underline{\varrho}_B \rangle = \underline{R}$.

The total wave function being assumed to be:

$$1.2.1 \qquad \Psi(\underline{K}, \underline{R}) = N\hat{A}\{\exp i \underline{K} \underline{\varrho}\} \phi_A(\underline{r}_A, \dot{O}) \phi_B(\underline{r}_B, \underline{R})$$

where N is a normalization constant, $\hat{A}$ the anti-symmetrizing operator, $\underline{r}_A$, $\underline{r}_B$ represent the set of nucleon coordinate $\{\underline{r}_i\}$ in cluster A and B respectively, $\underline{\varrho}_A = A^{-1} \sum_{i \in A} \underline{r}_i$ and $\underline{\varrho}_B = A^{-1} \sum_{i \in B} \underline{r}_i$ are the cluster C.M. coordinates and $\underline{R}$ is the distance between the oscillator potentials.

scattering of heavy ions which led to a better understanding of a large range of phenomena in the irteraction of composite particles, viz. transfer, compound nucleus and fusion reactions. Therefore, to proceed further beyond the phenomenological and semi-phenomenological stages in theoretical research of heavy-ion physics, progress in the microscopic theory of composite-particle interaction is an indespensable route.

1.2 Survey of reaction models

A number of reaction models have been recently developed to serve as vehicles to carry the microscopic theory to its logical destination, namely, the many-body description of heavy-ion reactions without the always somewhat arbitrary presumptions of collective parameters. We may look, very briefly, into the following reaction models that have come up so far.

(a) The two-center shell model

As a starting point for a microscopic description of heavy-ion scattering, a basis of wave functions is needed which describes the nuclear shapes correctly for all distances from the asymptotic limit of two separated nuclei until complete overlap in a compound nucleus. For this purpose the two-center shell model shown in Fig.(1.1) is suitable.

This model was used in 1970 by Pruess and Greiner[1] in their calculations of the single particle levels. Two extreme situations can be envisaged. In the sudden limit of Fig. (1.1) and left side of Fig.(1.2), the oscillator constant characterising the average potential for single particle motion is

CHAPTER 1-

INTRODUCTION

1.1 General Remarks

Recent years have seen a remarkable progress in the theoretical study of the interaction of composite nuclear particles towards a more genuine microscopic approach. This progress has been made possible because of a better description of the collective motion from first principles. Also computational procedures have been developed which have greatly reduced the technical barriers which are encountered in studying composite particles on a microscopic level. From this standpoint, we can be hopeful that a wide application to various problems can be expected. This work is an attempt to look into the recent work and progress on computational procedures in a unified form and, in particular, a method for the microscopic treatment of the absorption in composite-particle scattering is discussed. The development of the study of the composite-particle interaction provided the means to investigate concepts such as the cluster model in a unified way, where the bound state and scattering problems can be treated on the same footing.

A major contributing factor among those to stimulate the development of the microscopic theory of the composite-particle interaction is the progress being made in heavy-ion physics with vastly accumulating experimental data, as a result of the availability of accelerators producing high energy heavy ion beams. This led to an interest in theoretical and experimental investigations of both the elastic and inelastic

Abstract

Reactions between composite fragments are described in the generator coordinate method (GCM). In this method the many-body wave function is expanded in terms of totally anti-symmetrized bound state wave functions, describing various given fragments' separation.

Pauli principle and absorption are treated consistently on a microscopic level with the help of the Feshbach projection techniques. The imaginary part of the scattering operator for bound nucleons is obtained in closed form by using internal harmonic oscillator wave functions and simple nucleon-nucleon interactions (like the Skyrme force).

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DECLARATION

I declare that this dissertation is my own work performed under the supervision of Professor C. Toepffer and has not been previously submitted for a degree in any other university.

.....

The Anglo American Cooperation through its Chairman's Fund awarded me a generous scholarship during the tenure of this work. Words fail me to express how much I am indebted to them.

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* * * * * * * * * * * *

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AN ANTISYMMETRIZED FOLDING PROCEDURE FOR THE INTERACTION BETWEEN COMPOSITE PARTICLES

BY

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$b_A \to b_B$.

The resonating group equation (2.3.2) then reads:

$$P_o H_{eff} P_o |\Psi\rangle = E P_o |\Psi\rangle \qquad 2.4.8$$

for $b_A = b_B$. Eqs(2.4.6) and (2.4.8) constitute two different problems because of the coupling term $P_o H_{eff} P_1$ appearing in Eq(2.4.6). This term connects the P_o-space with states Ψ_i from P_1-space which describe internally excited fragments, since

$$(\phi_A \phi_B | \Psi_i) = \left(\frac{N_A! N_B!}{(N_A+N_B)!} \right)^{\frac{1}{2}} (1-\lambda_i)^{\frac{1}{2}} U_i(\varrho) \to 0 \qquad 2.4.9$$

for $\lambda \to 1$, and causes resonances in the spectrum of Eq(2.4.6) which do not occur in Eq(2.4.8).

* * * * * * * * * * * * *

converges to a finite value if the step length d, used in the numerical treatment of Eq(2.3.5) is reduced. From the energy dependence one finds in this case $\delta_0(o)-\delta_0(\infty) = o$ which demonstrates that no positive energy bound state is present. As a more realistic example the $\alpha-^{16}O$ system has been studied[20] allowing for different size parameters, $b_\alpha \neq b_{^{16}O}$, and again, the nuclear and Coulomb forces were turned off. The phases, shown in Figs. (2.5) and (2.6) for various partial waves ℓ, display the same general behaviour as in the dineutron case. For low ℓ-values the resonance structures are more complicated since several redundant states are degenerate. On a formal basis[19,20] the finite width of the resonances and discontinuous transition from "redundant" to "almost redundant" states can be explained as follows:

One starts from the internal waves ϕ_A, ϕ_B in the harmonic oscillator (HO) model with $b_A = b_B+\eta$. The projection P onto the elastic channel can then be expanded in terms of the solutions U_i of Eq(2.4.1) as

2.4.4 $$P = \sum_i |\Psi_i><\Psi_i|$$

where

2.4.5. $$\Psi_i = \left(\frac{N_A!N_B!}{(N_A+N_B)!}\right)^{\frac{1}{2}} (1-\lambda_i)^{-\frac{1}{2}} A\{\phi_A\phi_B U_i\};\ \lambda_i \neq 1$$

Eq(2.4.5) is then equivalent to:

2.4.6 $$PH_{eff}P|\Psi> = EP|\Psi>$$

To compare with the case $b_A = b_B$, P is split into -

2.4.7 $$P_o+P_1$$

such that P_o comprises those states U_i for which $\lambda_i \neq 1$ if

that as far as the phase shifts are concerned the Pauli principle leading to the occurance of the redundant states can be simulated by an extra bound states. This explains the equivalence of repulsive core and deep attractive potentials, Fig.(2.2). Secondly the physical solutions g orthogonal to U^R are damped in the interaction region where absorption takes place. It is, perhaps not surprising that both refractive and diffractive effects show up as backward anomalies and are often hard to disantangle as shown in Fig.(2.3). In more realistic models where the internal wave function deviates from the pure oscillator shell-model by admixture of the other configurations or by introducing different size parameters for different fragments, there will be eigenfunctions of N with eigenvalues λ close to 1. There have been controversial views regarding the nature of such "almost redundant states", whether they are positive energy bound states[18] or resonances of finite width[19]. Careful studies of simple systems prove the latter version to be true. Consider, e.g., the fictitious problem of dineutron-dineutron scattering[20] with the internal dineutron wave function taken as a sum of two Gausian functions with different widths $b \neq b'$. The nucleon-nucleon interaction is switched off artificially in order to study the effect of anti-symmetrization separately. Fig.(2.4) shows the phase $\delta_o(K)$, obtained by solving the resonating group equation (2.3.5). In the oscillator shell-model limit, $b = b'$, no resonance occurs, and the Levinson theorem is established as $\delta_o(o) - \delta_o(\infty) = \pi$ due to the presence of e redundant state. Leaving the pure oscillator shell-model by choosing $b \neq b'$, one observes a resonance whose width

2.4 Effects of anti-symmetrization

A perculiar feature of microscopic theories of composite-particles scattering is the appearance of square-integrable states of relative motion which lead to the solution of Eq(2.3.5) at arbitrary energy. Such "redundant states" are obtained whenever the eigenvalue problem

2.4.1 $$(\phi_A^{int}\phi_B^{int}|1-A|\phi_A^{int}\phi_B^{int}U) = \int N(\underline{r},\underline{r}')U(\underline{r}')d^3r' = \lambda U(\underline{r})$$

possesses solutions U^R with eigenvalue $\lambda = 1$. After anti-symmetrization, the corresponding many-body wave function

2.4.2 $$\Psi^R = A\{\phi_A^{int}\phi_B^{int}U^R\}$$

vanishes identically, and provides a trivial solution of Eq(2.3.2) at any energy E. Redundant states are square integrable solutions of the RGM Eq(2.3.12) at arbitrary energy and have to be counted in the Levinson theorem like true bound states,

2.4.3 $$\delta_\ell(o) - \delta_\ell(\infty) = \pi(m_\ell+n_\ell)$$

where m_ℓ is the number of redundant states in the ℓth partial wave, n_ℓ the number of true bound states. Redundant states actually occur if the internal states $\phi_A^{int}, \phi_B^{int}$ are described in the oscillator shell-model, with equal b-values for both fragments as has been in most of the microscopic calculations done so far. In this model, the redundant states have harmonic wave functions with $N \leq N_R$ quanta excited, the redundancy limit N_R depending on the system under consideration. The redundant states, which are direct consequence of the Pauli principle, are important in two ways. Firstly, Eq(2.4.3) shows

the RGM kernels cannot be calculated to arbitrary accuracy (even if the GCM kernels are given exactly), and the method is limited to low energies where complete resolution of the fine structure of the RGM kernels is not required. Another aspect of the abovementioned problem is the irregular nature of f. A typical example is the problem of representing a Gaussian of width α by the convolution Eq(2.3.11) with Γ being a Gaussian of width β. The solution f is an ordinary function only if $\alpha>\beta$. In the limiting case $\alpha=\beta$ one contains $f(\underline{R}) = \delta(\underline{R})$ while f behaves even worse than a δ-distribution for $\alpha<\beta$. The irregular behaviour has led to the serious doubts concerning any solution of the Hill-Wheeler equations through discretization. The answer to this problem is given by a theorem of Wiener[15] concerning the closure of translated square-integrable functions. It guarantees that the short-range part of g can always be approximated by a finite sum of the form:

$$2.3.13 \qquad g_\ell^{S.R.}(\underline{\rho}) = \sum_{i=1}^{M} C_{\ell i}\Gamma_\ell(\underline{\rho}_i,\underline{R}_i)$$

irrespective of the fact that coefficients $C_{\ell i}$ may cluster strongly on variation of the mesh points $\underline{R}_i$. Details concerning this problem have been thoroughly treated by Friederich, Hüsken and Weiguny[13] and Giraud and Le Tourneux[16,17]. Conceptually the irregular behaviour of f does not cause any problem since $f(\underline{R})$ has no direct physical meaning. The quantity $|f(\underline{R})|^2$, e.g., cannot be interpreted as the probability for finding the fragments at mean distance $\underline{R}$ since the basis functions $\Phi(\underline{r},\underline{R})$ are non-orthogonal.

We are now in a position to discuss the scattering boundary condition to be imposed on f. As the asymptotic behaviour of the relative motion wave function g is known (Coulomb waves), one has to invert Eq(2.3.11) to obtain the asymptotic behaviour of f. Details of such an inversion have been discussed by Beck et al[7] and Weiguny et al[12,13]. The GCM matrix elements and the RGM kernels are connected accordingly:

$$2.3.12 \qquad \int \Gamma(\underline{\rho},\underline{R})V(\underline{\rho})\Gamma(\underline{\rho},\underline{R}')d^3\rho = \langle\Phi(\underline{R})\sum_{\substack{i\in A\\ j\in B}} V_{ij}|\Phi(\underline{R}')\rangle_D$$

$$\int\int \Gamma(\underline{\rho},\underline{R})K(\underline{\rho},\underline{\rho}')\Gamma(\underline{\rho}',\underline{R}')d^3\rho d^3\rho'$$

$$= \langle\Phi(\underline{R})|H_{eff}-E|\Phi(\underline{R}')\rangle - \langle\Phi(\underline{R})|H_{eff}-E|\Phi(\underline{R}')\rangle_D$$

where the subscript D indicates that anti-symmetrization is only performed within the fragments A and B. By means of Eq(2.3.12) it is possible to calculate the RGM kernels from the GCM matrix elements by defolding Eq(2.3.12). This is important since direct calculation of the RGM kernels becomes impracticable with increasing particle number. It is, however, uncomforting when one notices that, in deed, the inverse of the defolding operator Γ is singular and thus Eq(2.3.11) is not easy to handle. This causes numerical problems few of which can be briefly outlined below. In practice, the defolding equation (2.3.12) is carried out by discretization and subsequent matrix inversion[14]. Since Γ depends on the difference $(\underline{\rho}-\underline{R})$, the matrix $\Gamma(\underline{\rho}_i,\underline{R}_j)$ degenerates with decreasing step length of discretization, and the inverse matrix is made up of large numbers, alternating in sign. As a result,

forces) there is no coupling between different ℓ-values. Although both methods, GCM and RGM, are equivalent in principle, it is difficult in practice to construct generating functions $\Phi(\underline{r},\underline{R})$ such that, for given α_A and α_B the model spaces spanned by Eq(2.1.1) and Eq(2.3.1) are identical. As an example where the equivalence of GCM and RGM can be shown explicitly, let us assume that:

2.3.9 $$\Phi(\underline{r},\underline{R}) = \Phi_{CM}\mathcal{A}\{\phi_A^{int.}(\underline{\xi}_A-\underline{\varrho}_A)\phi_B^{int.}(\underline{\xi}_B-\underline{\varrho}_B)\Gamma(\underline{\varrho},\underline{R})\}$$

where Γ is a wave packet describing fluctuations of the relative motion coordinate $\underline{\varrho}$ around the mean distance $\underline{R}$. It is now possible to actually construct Slater determinants with the above factorization property. Suppose that fragment A is described by a Slater determinant $\phi_A(\underline{r}_A,\underline{R}_A)$ centered around $\underline{R}_A$, and fragment B by $\phi_B(\underline{r}_B,\underline{R}_B)$ respectively. Then,

2.3.10 $$\Phi(\underline{r},\underline{R}) = \mathcal{A}\{\phi_A(\underline{r}_A,\underline{R}_A)\phi_B(\underline{r}_B,\underline{R}_B)\}$$

with $\underline{R} = \underline{R}_A-\underline{R}_B$. Now, if the center-of-mass of the individual fragment is in the oscillator ground state, then Eq(2.2.3) holds with Γ being Gausian. The extraction of a gausian Γ in going from 3A-particle wave functions $\phi_A(\underline{r}_A,\underline{R}_A)$ to the 3A-3 intrinsic particle wave functions $\phi_A^{int}(\underline{r}_A-\underline{\varrho}_A)$ is, however, only possible in the two-center harmonic oscillator cluster-model, provided the same oscillator length b is taken for both fragments. Combining Eqs(2.1.1) and (2.3.9) and comparing the results with Eq(2.3.1), one finds the desired connection between the GCM and the RGM as:

2.3.11 $$g(\underline{\varrho}) = \int\Gamma(\underline{\varrho},\underline{R})f(\underline{R})d^3R$$

separating direct and exchange contributions according to:

2.3.4 $$A = 1 + (A-1)$$

The resulting "resonating group equation" reads as follows:

2.3.5 $$\{T_{\underline{\rho}} + V(\underline{\rho}) - \varepsilon\}g(\underline{\rho}) = \int K(\underline{\rho},\underline{\rho}')g(\underline{\rho}')d^3\rho'$$

where

2.3.6 $$V(\underline{\rho}) = (\phi_A\phi_B|V_{AB}|\phi_A\phi_B)$$

is the direct part of the nucleus-nucleus potential, V is local if the underlying nucleon-nucleon interaction is local, as is assumed in Eq(2.3.6).

The energy ε of relative motion appears after subtracting the internal energies of A and B and the center-of-mass energy from the total energy E. The right hand side of Eq(2.3.5) contains exchange contributions (including thore of overlap and kinetic energy),

2.3.7 $$\int K(\rho,\rho')g(\rho')d^3\rho' = (\phi_A\phi_B|(H_{eff}-E)(A-1)|\phi_A\phi_B g)$$

and is non-local like the Hartree-Fock exchange potential. The two types of potentials shown in Fig(2.2) are different local approximations of the non-local kernel V-K. If the fragments are spherical and if we restrict ourselves to central forces, then the angular momentum of relative motion ℓ will be a good quantum number. Expanding g, V and K in terms of spherical harmonics, one arrives at the radial equation:

2.3.8 $$\left\{\frac{d^2}{d\rho^2} - \frac{\ell(\ell+1)}{2\mu\rho^2} - V(\rho)+\varepsilon\right\}g_\ell(\rho) = \int K_\ell(\rho,\rho')g_\ell(\rho')d\rho'.$$

The non-local operator K depends on two vectors $\underline{\rho}$ and $\underline{\rho}'$. Under the above assumptions, (spherical fragments, central

Where A denotes the rest antisymmetrizer between fragments A and B, and g is the relative wave function of motion. Under certain conditions (to be discussed below) Ψ^{RGM} spans the same space as Ψ^{GCM}. The internal wave functions of the separated fragments are well-defined. In order to include any polarization effects, one can in principle, admix excited fragment configurations. Because of the antisymmetrization operator A, Eq(2.3.1) does not mean that the densities of the separated fragments are simply added if the distance between the fragments A and B tends to zero. There is, naturally, a well-defined limiting configuration which, e.g. for the case α-α scattering, turns out to be ^{8}Be ground state, provided one neglects the difference between the oscillator lengths of the α-particle and the ^{8}Be nucleus. However, if one could calculate the effective Hamiltonian properly, the question of polarization would be irrelevant since the effective Hamiltonian accounts for polarization effects. To determine the relative motion wave function g, one projects the many-body Schrödinger equation onto the space spanned by Eq(2.3.1):

$$(\phi_A\phi_B|H_{eff} - E|\Psi) = 0 \qquad 2.3.2$$

where the round brackets indicate integration over internal fragments only. The explicit form of the Schrödinger equation of relative motion is obtained by splitting the effective Hamiltonian into:

$$H_{eff} = T_{\varrho} - T_{cm} + H_A + H_B + V_{AB} \qquad 2.3.3$$

where ϱ is the actual variable of relative motion, and by

calculations[11] including complete antisymmetrization. Actual calculations in the RGM, however, are very tedious for heavier systems because of the large number of exchange terms. Independently, the generator coordinate method (GCM) was developed to find the asymptotic conditions for the coefficients $f(\underline{R})$ in the GCM.

2.3 Relationship between GCM and RGM

One notices that in the GCM, auxiliary parameters, like the distance $\underline{R}$ were introduced; Slater determinant many-body wave functions were generated for each parameter set, and the total wave function was constructed by integrating the generating functions over these parameters. Using the variational principle for the energy expectation values, one obtains the Hill-Wheeler equation, the independent variables being the generator coordinates, and not the dynamical variables such as the particle coordinates. Here, all the dynamical variables are already integrated out when the integral kernels of the Hill-Wheeler integral equation are obtained. In the generating function the inter-cluster coordinate $\underline{R}$ is fixed, and only the other independent coordinates are integrated over in deriving the non-local kernels of the GCM equation. The RGM will allow us to relate the expansion coefficients $f(\underline{R})$ (in the GCM) to the wave function of relative motion $g(\underline{\rho})$ of the fragments A and B. In this method, one substitutes Eq (2.1.6) into Eq(2.1.1) to obtain the form:

2.3.1 $$\Psi^{RGM}(\underline{r}) = \phi_{cm}\mathcal{A}\{\phi_A(\underline{r}_A)\phi_B(\underline{r}_B)g(\underline{\rho})\}$$

oscillator length b is taken for both fragments. If $b_A \neq b_B$, or if other models are used, the factorization property (2.1.6) breaks down, and a further GC-integration is required for projecting on state of good total momentum.

2.2 Resonating Group Method (RGM)

The resonating group method (RGM), a method proposed by Wheeler in 1937, was introduced primarily to describe microscopically the nuclei from a molecular viewpoint [5]. In this method it is assumed that the nucleons form groups or clusters with certain internal wave functions. There is a resonant exchange of nucleons between the clusters and the total wave function is antisymmetrized to account for the indistinguishability of the nucleons. An equation of motion for the inter-cluster wave function is derived through the use of a many-body Hamiltonian with the help of the variational principle. There occur non-local terms which represent exchanges of nucleons between one cluster and another resulting from the antisymmetrization. The microscopic study of cluster structures for light nuclei was initiated by Wildermuth and Kanellopoulos [10]. Many calculations appeared thereafter, e.g. ref. 3, which use the RGM to investigate both structure and scattering problems of light nucleon systems. Especially, the importance of the alpha-like four-body correlation in light nuclei was recognized extensively. This became the motivating factor in fostering investigations on molecule-like structure of nuclei on a wide scale. Also recent developments in heavy-ion reactions stimulated microscopic

by a linear combination:

2.1.4 $$\Psi^{GCM} = \sum_{\lambda} \int \Phi_{\lambda}(\underset{\sim}{r},\underset{\sim}{R}) f_{\lambda}(\underset{\sim}{R}) d^3R$$

where the subscript labels various channels, and then one can do a coupled channel calculation with a real Hamiltonian. Although this is possible in principle, it has a disadvantage in that it is impracticable since the number of channels which contribute to the absorptive part of the optical potential can be very large. Alternatively one may account for the non-elastic channels by introducing an effective microscopic Hamiltonian so as to account for the loss of flux in the inelastic channels. This microscopic Hamiltonian must be non-Hermitian. It can be calculated by using Feshbach's projection techniques. For scattering and decay problems, the separation parameter $\underset{\sim}{R}$ is related to the actual variable of relative motion $\underset{\sim}{\rho}$ of the two fragments, by:

2.1.5 $$\underset{\sim}{R} = \langle \Phi(\underset{\sim}{r},\underset{\sim}{R}) | \underset{\sim}{\rho} | \Phi(\underset{\sim}{r},\underset{\sim}{R}) \rangle$$

under certain circumstances.

One can further assume that:

2.1.6 $$\Phi(\underset{\sim}{r},\underset{\sim}{R}) = \Phi_{cm}\, A\{\Phi_A(\underset{\sim}{r}_A)\Phi_B(\underset{\sim}{r}_B)\Gamma(|\underset{\sim}{\rho}-\underset{\sim}{R}|/b)\}$$

where the fragments wave functions Φ_A, Φ_B depend only on the internal fragments coordinates and Φ_{cm} describes the center-of-mass motion. A takes care of complete antisymmetrization and Γ is a peaked function which describes fluctuations of $\underset{\sim}{\rho}$ about $\underset{\sim}{R}$ with root mean square deviation b. Assumption (2.1.6) is valid for the α-particle model of Brink[6] or in the harmonic-oscillator cluster model provided the same

where $\underline{r}$ stands for all coordinates of the nucleons and $\underline{R}$ is the parameter which measures the mean distance between the fragments A and B. The generating functions Φ are square integrable and antisymmetrized with respect to all nucleons of the system. The characteristic feature of the GCM is that there are no superfluous degrees of freedom in the theory. This is due to the fact that the integration in Eq (2.1.1) extends over all $\underline{R}$ values. The expansion coefficients $f(\underline{R})$ are determined by solving the Schrödinger equation of the many-body system in the space spanned by:

$$2.1.2 \qquad \langle \Phi(\underline{r}, \underline{R}) \mid H - E \mid \Psi^{GCM}(\underline{r}) \rangle = 0$$

where $\langle\ \rangle$ means integration over $\underline{r}$, with appropriate scattering boundary conditions. In practice $\Phi(\underline{r}, \underline{R})$ will be represented by a Slater determinant in a two-center cluster model so that the GCM-Kernels

$$2.1.3 \qquad \begin{Bmatrix} H(\underline{R},\underline{R}') \\ N(\underline{R},\underline{R}') \end{Bmatrix} = \langle \Phi(\underline{r},\underline{R}) \mid \begin{Bmatrix} H \\ 1 \end{Bmatrix} \mid \Phi(\underline{r},\underline{R}') \rangle$$

can be calculated using well-established shell-model techniques. Here $H(\underline{R},\underline{R}')$ and $N(\underline{R},\underline{R}')$ are the energy and normalization kernels, and E the total energy. By taking only the ground-state Slater determinant into account in the integral equation 2.1.2 for the expansion coefficient $f(\underline{R})$, one describes only elastic scattering and neglects the coupling to all other channels. This is clearly unrealistic for most cases of composite particle scattering where many channels are open in general. One could generalize Eq(2.1.2) to a system of coupled integral equations by replacing Eq(2.1.1)

With the moving oscillator functions (4.1.1) we calculate the imaginary part of

4.1.3 $$\langle\phi(\underline{r},\underline{R})|H-E|\Psi(\underline{r},\underline{R})\rangle$$

which contains the matrix elements

4.1.4 $$\langle\phi(\underline{R}',\underline{K})|\text{Im } G|\phi(\underline{R}',\underline{K})\rangle$$

$$= \langle\phi(\underline{R}',\underline{K})|\phi(\underline{R},\underline{K})\rangle \sum_{\substack{\alpha,\beta,\gamma,\delta \\ \text{occ.}}} (\underline{\alpha},\underline{\beta};\underline{R}',\underline{K}|\text{Im } G|\underline{\gamma},\underline{\delta};\underline{R},\underline{K})$$

$$(B^{-1}_{\alpha\gamma} B^{-1}_{\beta\delta} \cdots B^{-1}_{\alpha\delta} B^{-1}_{\beta\gamma})$$

The summation over α includes the spin and isospin summation. This, therefore, implies a factor $(\delta_{\tau_\alpha\tau_\gamma}\delta_{\tau_\beta\tau_\delta} - \delta_{\tau_\alpha\tau_\delta}\delta_{\tau_\beta\tau_\gamma})$ for isospin and similarly for spin. All the lengths in Eq(4.1.4) are measured in units of the oscillator length $b = (\frac{\hbar}{\mu\omega})^{\frac{1}{2}}$

and all momenta in units of b^{-1}.

The $B_{\alpha\gamma}$ are the oscillator overlap matrix $(\underline{\alpha};\underline{R}'\cdot\underline{K}|\underline{\gamma};\underline{R},\underline{K})$ in the non-orthogonal basis of the moving oscillator states, and they are given by

4.1.5 $$B_{\alpha\gamma}(\underline{R}',\underline{R},\underline{K}) = C_{\alpha\gamma}\int d^3r \exp\{-(\underline{r}-\underline{R}'_\alpha)^2/2-(\underline{r}-\underline{R}_\gamma)^2/2\}$$

with $$C_{\alpha\gamma} = C_\alpha C_\gamma = \pi^{-3/2}$$

Toepffer et al. approximated the operator Q in the model of overlapping Fermi spheres such that the Pauli principle is fulfilled on average only. In the moving oscillator basis, and if the states were orthogonal, $B_{\alpha\gamma} = \delta_{\alpha\gamma}$, the Pauli

CHAPTER 4

MOVING OSCILLATOR BASIS AND PAULI OPERATOR.

The use of plane waves in the construction of Im G has a number of shortcomings, for example

(i) plane waves are not square-integrable and can therefore not be used to construct generator functions

(ii) when using plane waves basis, surface and shell effects are neglected.

In the present work, we use moving displaced harmonic oscillator functions which are totally anti-symmetrized. The harmonic oscillator (HO) functions are given as:

4.1.1 $$(\underset{\sim}{r}|\underset{\sim}{\alpha};\underset{\sim}{R},\underset{\sim}{K}) = C_{\alpha}\exp\{i\underset{\sim}{K}(\underset{\sim}{r}-\underset{\sim}{R}_{\alpha})\cdot(\underset{\sim}{r}-\underset{\sim}{R}_{\alpha})^{2}/2\}H_{\underset{\sim}{\alpha}}(\underset{\sim}{r}-\underset{\sim}{R}_{\alpha})$$

with $\underset{\sim}{R}_{\alpha}$ and $\underset{\sim}{K}_{\alpha}$ given by

4.1.2 $$\underset{\sim}{R}_{\alpha} = \begin{Bmatrix}\underset{\sim}{R}_{A}\\ \underset{\sim}{R}_{B}\end{Bmatrix} = \pm\tfrac{1}{2}\underset{\sim}{R}, \text{ for } \alpha\varepsilon\begin{Bmatrix}A\\B\end{Bmatrix}$$

$$\underset{\sim}{K}_{\alpha} = \begin{Bmatrix}\underset{\sim}{K}_{A}\\ \underset{\sim}{K}_{B}\end{Bmatrix} = \pm\underset{\sim}{K}, \text{ for } \alpha\varepsilon\begin{Bmatrix}A\\B\end{Bmatrix}$$

and

$$C_{\alpha} = \pi^{-\frac{3}{4}}$$

$H_{\underset{\sim}{\alpha}}$ are the three -dimensional Hermite polynomials, in particular for the s-state, $\underset{\sim}{\alpha} = (0,0,0)$ becomes $H_{\underset{\sim}{\alpha}}(\underset{\sim}{r}-\underset{\sim}{R}_{\alpha}) = 1$. (This is the case treated in the actual calculations).

refer to a bound state problem, i.e. separated fragments at rest. One expects that the iteration of Eq(3.4.8) converges faster than for Eq(3.3.13). We, thus obtain for the second order (for the imaginary part in which we are presently interested),

3.4.9 $\quad \text{Im } G = -\pi g Q \delta(\omega - K) g.$

As reference matrix one may take a Volkov[25] or a Skyrme[38] force which are known to give a reasonable account of static nuclear properties like binding energies, radii etc. Relative motion is treated incorrectly when interpreting Eq(3.3.1) or optical potential since the parameter $\underline{R}$ differs from the dynamical variable $\underline{\rho}$ of relative motion. In other words, Eq(3.3.1) neglects non-diagonal elements $\langle\Phi(\underline{R}',\underline{K}|G|\Phi(\underline{R},\underline{K})\rangle$. They can be taken care of by using the above matrix elements as input in the symmetrized form:

3.4.10 $\quad \frac{1}{2}\langle\Phi(\underline{R}',\underline{K})|G(\underline{R}',\underline{K}) + G(\underline{R},\underline{K})|\Phi(\underline{R},\underline{K})\rangle$

This procedure is not fully self-consistent since the above G is calculated with respect to a single configuration $\Phi(\underline{R},\underline{K})$ of the two-center shell model, which cover only part of the GCM-space of functions defined by

3.4.11 $\quad \Psi(\underline{r}) = \int d^3\underline{R}\,\Phi(\underline{r},\underline{R}) f(\underline{R})$

* * * * * * * * * * *

3.4.2 $Q = 1-P$

Let us split the Hamiltonian H into

3.4.3 $H = H_o+V$

such that

3.4.4 $QH_o = H_oQ$

then Eq(3.4.1) can be written as

3.4.5 $H_{eff} = P(H_o+G)P$

where

3.4.6 $G = V+V\frac{Q}{E-H_o+i\eta}G$

is in general a many-body operator. In the independent pair approximation G reduces to the two-body operator G of Eq(3.3.13) if we choose $H_o = K$. Here Q projects onto the states which are unoccupied in the elastic channel and plays therefore formally the same role as the Pauli-operator $\bar{Q}$ in Eq(3.3.8). The projection operator Q will therefore subsequently be referred to as Pauli operator. To improve the convergence of this expansion we introduce a reference matrix:

3.4.7 $g = G(R\to\infty,$ momentum $K\to 0)$

describing the static problem and is related to G through:

3.4.8 $G = g+g(\frac{Q}{\omega-K+i\eta} - \frac{Q_o}{\omega-K})G.$

The Pauli operator Q_o of the reference problem as well as g are real. Their reality is due to the fact that they both

integrals in Eq(3.3.7) can be evaluated in closed form. If one inserts $G_s^f+V_\ell$ of Eq(3.3.11) into the direct part of Eq(3.3.7) it turns out that the real potentials become too deep in order to fit the data for surface transparent systems like ^{16}O - ^{16}O or ^{12}C - ^{12}C.

In the Thomas-Fermi model the unperturbed Hamiltonian H_o for the motion of the nucleons is the local kinetic energy operator K. A simple way to estimate the imaginary part of G is to iterate Eq(3.3.8) once and to replace the G-matrix by the free two-nucleon scattering matrix T:

3.3.13 $$G = V+V\frac{\bar{Q}}{E-K+i\eta}G$$

$$\approx V+V\frac{\bar{Q}}{E-K+i\eta}T$$

The imaginary part of G(Eq(3.3.9)) can then be evaluated with the help of the optical theorem which yields the frivolous scattering model of Goldberger[37], in which the effective cross-section for nucleons is given by the free cross-section averaged over the Pauli-allowed region of momentum space.

3.4 The effective Hamiltonian

It remains to investigate whether it is justified to use the G-matrix of Eq(3.3.8) as the effective force in the elastic channel. Following Feshbach[9], the effective Hamiltonian (H_{eff}) for the elastic channel can be expressed as:

3.4.1 $$H_{eff} = P\{H+HQ(E-QHQ+i\eta)^{-1}QH\}P$$

where P projects onto the elastic channel defined in Eq(2.1.1) and Q is related with P by

where the exchange terms have been surpressed for simplicity. Although the Fermi momentum k_F is calculated here with the density of the compound system ρ_{CS}, the treatment is not fully anti-symmetrized and concepts like the distance R between the fragments and the momentum K per nucleon of relative motion are used which loose their meaning with increasing overlap, where the real parts of the folding potentials become too deep. This leads to problems if the absorption is weak in the surface region so that the interacting nuclei can probe each other deeply even in the case of elastic scattering.

Toepffer et al, calculated the real potentials and by using the phase shift approximation for the G-operator as:

3.3.10 $$G = 4\hbar^2 m^{-1} k^{-1} \sum_{\ell} (2\ell+1) \tan \delta_{\ell}$$

or alternatively the first order G_s+V_ℓ of the expansion

3.3.11 $$G = G_s^f + V_\ell - G_s^f \{\frac{\tilde{Q}}{H_o - E} - \frac{1}{K-E}\} G_s^f - 2G_s^f \frac{\tilde{Q}}{H_o - E} V_\ell - V_\ell \frac{\tilde{Q}}{H_o - E} V_\ell + \cdots$$

which was proposed by Moszkowski and Scott[36]. In this, d is a separation distance introduced in such a way that the short ranged repulsion between the nucleus is cancelled by a part of the long ranged attraction:

3.3.12 $$V = \begin{cases} V_s & r<d \\ V_\ell & r>d \end{cases}$$

so that the free reaction matrix G_s^f corresponding to V_s and the long range component V_ℓ are sufficiently small quantities to allow the pertubation expansion Eq(3.3.11). After the judicious parametrization of the phases or $G_s^f + V_\ell$ the momentum

The Fermi momentum k_F in Eq(3.3.4) is calculated by inserting the compound density $\rho_{cs} = \rho_1 + \rho_2$ at the position $\underline{r}_2$ of the volume element $d\Omega$ into the following equations.

3.3.6 $$\underline{k}_F(\underline{r}) = (3\pi^2\rho(\underline{r})/2)^{\frac{1}{3}}$$

In order not to count particles twice a factor $\rho_1\rho_2\rho_{cs}^{-2}$ is introduced and the optical potential becomes

3.3.7 $$U = (2\pi)^{-6}\int d^3r_2\rho_1\rho_2\rho_{cs}^{-2}\int_0^{k_F} d^3k_1\int_0^{k_F} d^3k_2\{\langle\underline{k}|G|\underline{k}\rangle - \langle\underline{k}|G|-\underline{k}\rangle\}$$

U is complex, since G obeys the Brueckner-Goldstone equation

3.3.8 $$G = \bar{V}\bar{Q}(H_o - E - i\eta)^{-1}G$$
$$= \bar{V}P\bar{Q}(H_o - E)^{-1} - i\pi\bar{V}\bar{Q}\,\delta(H_o - E)G$$

which is not Hermitian. Here H_o is the unpertubed Hamiltonian for the motion of the nucleons in their respective fragments at infinite separation $\underline{r}$, $\bar{Q}$ is the Pauli operator which ensures that no particles can be scattered into the overlap region of both Fermi spheres in Fig.(3.1) above, and P denotes that the principal value should be taken upon integration over intermediate states. In contrast to the Brueckner theory for nuclear matter the imaginary part of G does not vanish here since real excitations can take place in which kinetic energy of relative motion is converted into internal excitation energy giving rise to an absorptive imaginary part. On a microscopic level probability is conserved and the imaginary part of G fulfils the optical theorem:

3.3.9 $$\mathrm{Im}\langle k|G|k\rangle = \pi(2\pi)^{-3}\int d^3k'\bar{Q}|\langle\underline{k}|G|\underline{k}'\rangle|^2\delta(E(k') - E(k))$$

with the convention that $n \varepsilon F_1$ and $m \varepsilon F_2$
This interaction energy is between fragment 1 and 2 and use is made of the Thomas-Fermi model to write vectors $|\underset{\sim}{m},\underset{\sim}{n}>$ as:

3.3.2 $$|\underset{\sim}{m},\underset{\sim}{n}> = (d\Omega)^{-1}\exp\{i(\underset{\sim}{k}_m+\underset{\sim}{K})(\underset{\sim}{r}_1+\underset{\sim}{R})+\underset{\sim}{k}_n\underset{\sim}{r}_2\}$$

$$|\underset{\sim}{n}\ \underset{\sim}{m}> = (d\Omega)^{-1}\exp\{i(\underset{\sim}{k}_n(\underset{\sim}{r}_1+\underset{\sim}{R})+(\underset{\sim}{k}_m+\underset{\sim}{K})\underset{\sim}{r}_2\}$$

in a certain volume element $d\Omega$. Introducing a local total momentum $\underset{\sim}{P}_{mn} = \underset{\sim}{k}_m+\underset{\sim}{K}+\underset{\sim}{k}_n$ and a relative momentum $\underset{\sim}{k}_{mn} = \frac{1}{2}(\underset{\sim}{k}_m+\underset{\sim}{K}-\underset{\sim}{k}_n)$ (see Fig.(3.1)) in the local CM-frame, one obtains with $\underset{\sim}{r} = \frac{1}{2}(\underset{\sim}{r}_1+\underset{\sim}{R}+\underset{\sim}{r}_2)$ and the relative coordinate of two nucleons $\underset{\sim}{r}' = \underset{\sim}{r}_1+\underset{\sim}{R}-\underset{\sim}{r}_2$ and interaction energy density

3.3.3 $$d^3u/d^3r_2 = (d\Omega)^{-2}\sum_{m,n}\int\exp(-i\,\underset{\sim}{P}_{mn}\underset{\sim}{R})\exp(-i\,\underset{\sim}{k}_{mn}\underset{\sim}{r}')$$

$$G\ \exp(i\underset{\sim}{P}\underset{\sim}{R})\{\exp(i\underset{\sim}{k}_{mn}\underset{\sim}{r}')-\exp(-i\underset{\sim}{k}_{mn}\underset{\sim}{r}')\}d^3r_1$$

Assuming local translational invariance, the total momentum $\underset{\sim}{P}_{mn}$ of two colliding nucleons is conserved. The summations in Eq(3.3.3) can be converted into integrations over the intrinsic momenta $\underset{\sim}{k}_1 = \underset{\sim}{k}_m$ and $\underset{\sim}{k}_2 = \underset{\sim}{k}_n$. Then

3.3.4 $$d^3u/d^3r_2 = (2\pi)^{-6}\int_0^{k_F}d^3k_1\int_0^{k_F}d^3k_2(<\underset{\sim}{k}|G|\underset{\sim}{k}>-<\underset{\sim}{k}|G|-\underset{\sim}{k}>$$

with $\underset{\sim}{k} = \underset{\sim}{k}_{mn}$ and the reaction matrix elements

3.3.5 $$<\underset{\sim}{k}|G|\pm\underset{\sim}{k}> = \int\exp(-i\ \underset{\sim}{k}\ \underset{\sim}{r}')G\ \exp(\pm i\ \underset{\sim}{k}\ \underset{\sim}{r}')d^3r'$$

for forward and backward scattering.

as was done by Kadmenskii et al.[32], or Broglia and Winther[33]. If necessary, Eq(3.2.3) must be symmetrized in order to ensure that the potential is independent of the arbitrary choice of the indices 1 and 2. A similar prescription has recently been used by Sinha[34] to describe the absorption in heavy-ion scattering. However, Fink and Toepffer[35] have shown that the absorption in heavy-ion reactions is mainly determined by the open phase space allowed by the Pauli principle which for scattering between bound nucleus is entirely different from the one for scattering between a free nucleon and bound target nucleus.

3.3 Absorption in the Folding model.

In 1976 Toepffer formulated a folding model in momentum space. Consider two Fermi fluids with densities $\frac{\rho_1}{\rho_1+\rho_2}$ and $\frac{\rho_2}{\rho_1+\rho_2}$ in ordinary space, which in momentum space are represented by Fermi spheres of radius k_F which centres are separated by the collective momentum $\underline{K}$ of relative motion per nucleon in Fig.(3.1). Fig.(3.1) shows the Fermi spheres F_1 and F_2 separated by the local momentum K per nucleon. Scattering into $F_1 \cap F_2$ is forbidden by the Pauli principle. Because of momentum and energy conservation the final momenta for elementary collisions lie on the sphere R. The Pauli-forbidden region is marked as a heavy black line on the circumfrence of R.

Instead of Eq(3.2.2) one considers the interaction energy

$$3.3.1 \quad U = \sum_{m,n} \{\langle \underline{m},\underline{n};\underline{R},\underline{K}|G|\underline{m},\underline{n};\underline{R},\underline{K}\rangle - \langle \underline{m},\underline{n};\underline{R},\underline{K}|G|\underline{n},\underline{m};\underline{R},\underline{K}\rangle\}$$

concept of Brueckner. An approach such as the folding model was proposed. In it heavy-ion potentials are derived and by studying how far collisions between composite-particles are determined by the elementary encounters between the constituents making allowance for many-body effects. This can, at least presently, only be done in rather crude way, in particular one encounters again the familiar problem which arises when nuclei loose their identity in the overlap region. The procedure to calculate nucleon-nucleus scattering was first suggested by Greenlees et al.[28] in 1968 and in the same year by McManus[29]. An effective interaction $\tilde{G}$ compatible with nucleon-nucleon scattering is folded with density distribution of the target to give the real nucleon-nucleus optical potential $U_{N,2}(\underline{R})$

$$3.2.1 \qquad U_{N,2}(\underline{R}) = \int \rho_2(\underline{r}_2)\tilde{G}(\underline{R}-\underline{r}_2)d^3r_2$$

There are two possibilities to extend this method in order to describe the scattering between two composite-particles.
One adopts the double folding

$$3.2.2 \qquad U(\underline{R}) = \int\int \rho(\underline{r}_1)\tilde{G}(\underline{r}_1-\underline{r}_2+\underline{R})\rho(\underline{r}_2)d^3r_1d^3r_2$$

as has been employed, for example, in 1973 by Vary and Dover[30] and also two years later by Dover and Vary[31] or an empirical nucleon-nucleus potential $U_{N,2}(\underline{r}_1+\underline{R})$ is folded with density distribution of the other nucleus to yield,

$$3.2.3 \qquad U(\underline{R}) = \int \rho(\underline{r}_1)U_{N12}(\underline{r}_1+\underline{R})d^3r_1$$

to experiment. The absolute values may be wrong by factors 10 to 1000, varying from light to heavy nuclei. Alpha-cluster models were invoked to remedy the situation but it led to unreasonable large alpha-clustering. This discrepancy can be removed by proper anti-symmetrization as has recently been shown by Fliessbach[24]. A similar situation occurs for alpha-and two-nucleon transfer where the absolute values of spectroscopic factors cannot be explained. In contrast, there seems to be no problem in the case of one-nucleon transfer.

The three examples quoted above demonstrate that a development of practicable theories which incorporate both Pauli principle and absorption on a fundamental level in a consistent way is needed. For that purpose it seems worthwhile to investigate whether the GCM and the RGM can be combined with Feshbach's formalism of the effective Hamiltonian to account for the absorption [9].

3.2 The Folding Model

Most of the microscopic calculations done so far neglected absorption completely. Use is made of real effective interactions, like e.g., the Volkov force[25], adapted to the system at infinite fragment separation, and are at best useful approximations at low energies. Simple approaches[26] which introduce absorption phenomenologically in the Hill-Wheeler or resonating group Eqs(2.1.2) and (2.3.5) cannot solve the problem of energy dependence and radial shape of the optical potential. A semi-microscopic approach is due to Toepffer et al.[27] who base their work on the G-matrix

in Fig(2.2). At first glance it seems unitelligible that such different types of potentials can be phase-equivalent. As mentioned above, the Pauli principle offers an explanation for this type of ambiguity. One would then expect this ambiguity to disappear at high energies where the Pauli principle looses its influence because the intrinsic states do not ovelap any more in momentum space. In fact, with increasing energy, the repulsive core is reduced[22] while purely attractive potentials become more and more shallow. Thus we notice then that the two types of potentials tend to converge to a unique high energy limit.

(b) Generally, distinguishing between refraction and diffractive effects[23] is often quite a difficult issue. An illustration in this regard can be noted in Fig.(2.3). Here the typical behaviour and quantitative features of elastic scattering cross section are indicated. The typical behaviour of angular distributions at forward angles can often be reproduced by both strong absorption and refractive models. A rise of the cross section at backward angles is generally assigned to particle exchange which can be described by a real non-local potential. On the other hand, when absorption sets in abruptly as soon as the nuclei touch, one will obtain reflection, as in the scattering of light from a polished metallic surface.

(c) Alpha reduced widths as calculated from the traditional shell-model approach are much too small when compared

CHAPTER 3.

OUTLINE OF THE MOMENTUM SPACE FOLDING MODEL

3.1 Pauli principle: Its role in microscopic studies

In static problems the Pauli principle allows an independent particle motion in nuclear systems, although the nucleon-nucleon force is short ranged and in fact very repulsive at short distances (the healing property). In composite-particle scattering, the Pauli principle is responsible for the destruction of the structure of the fragments during overlap: Since every cell in phase space may only be singly occupied the overlapping system must either extend itself in ordinary or in momentum space (compression). An important role of the Pauli principle in composite-particle interaction was firstly recognised clearly in the study of the α-α interaction. The repulsive core of the phenomenological α-α potentials was shown to be originating from the Pauli principle. The calculations with the use of the resonating group method (RGM) showed the appearance of the almost energy-independent inner oscillations whose outermost node just corresponds to the phenomenological repulsive core radius[21]. The fundamental role played by the Pauli principle, becomes apparent in a number of puzzling features of phenomenological approaches. One can but just mention the following three of such phenomenological approaches.

(a) Consider the real part U(Re U) of the optical potential for elastic scattering. To the experimental data, reasonable fits have been obtained by both repulsive core and deep attractive potentials, as shown

CHAPTER 6

RESULTS AND DISCUSSION

The scattering of composite particles has been considered with primary emphasis on two aspects viz.-

(a) how to describe it microscopically and

(b) how to describe the absorption.

Conventionally one assumes that the clusters are intrinsically in their ground states, since otherwise the treatment turns out to be too complicated. In such a description one only describes the elastic channel in isolation, although many channels are open in general. It was therefore proposed to use the Feshbach projection technique with the aim of constructing an effective (non-Hermitian) Hamiltonian for the [illegible] channel.

Some resu[illegible] actual calculation of

$$W = \sum_{\substack{\alpha,\beta,\gamma,\delta \\ \text{occup.}}} \underline{\beta}|\mathrm{Im}\, G|\underline{\gamma},\underline{\delta})\ (\delta_{\alpha\gamma}\,\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma})$$

(formula 5.4.4) for $B_{\alpha\gamma} = \delta_{\alpha\gamma}$ etc. and s-states (corresponding to α-α scattering) are presented in Figs. 6.1 - 6.3. In Fig.(6.1) W is plotted as a function of R the mean separation, for various values of R' and K=0, $\theta=0^{\circ}$. As anticipated in chapter 5, there arise inconsistencies because of our approximation for the Pauli operator, which is not a true Pauli operator because of the non-orthogonality of the basis functions as described by the overlap matrices $B_{\alpha\gamma}$ etc.

5.5.14 $$\sum_{\substack{\alpha,\beta,\gamma,\delta \\ occ.}} (\underline{\alpha},\underline{\beta};\underline{R}',\underline{K}|\,\mathrm{Im}\ G|\underline{\gamma},\underline{\delta};\underline{R},\underline{K})$$

$$= \frac{\sqrt{2}\ C^2_{oo}}{2\hbar\omega} \sum_{\substack{\alpha,\beta,\gamma,\delta \\ occ.}} [\ NPT(t_o-t_o)+NPT(t_o-t_1)+NPT(t_1-t_1)$$

$$+ \frac{1}{\sqrt{2}} \sum_{\alpha',\beta'} \{PT(t_o-t_o)+PT(t_o-t_1)+PT(t_1-t_1)\}]$$

So that:

5.5.15 $$\langle\Phi(R',K)|\,\mathrm{Im}\ G|\Phi(R,K)\rangle\langle\Phi(R',K|\Phi(R,K)\rangle^{-1}$$

$$= \frac{\sqrt{2}\ C^2_{oo}}{2\hbar\omega} \sum_{\alpha,\beta,\gamma,\delta} [\{NPT(t_o-t_o)+ NPT(t_o-t_1)+NPT(t_1-t_1)\}$$

$$+ \frac{1}{\sqrt{2}} \sum_{\alpha',\beta'} \{PT(t_o-t_o)+PT(t_o-t_1)+PT(t_1-t_1)\}]$$

. . . . . .

$$5.5.11\quad PT(t_1-t_1) = \frac{C^2\,\alpha\alpha}{2\hbar\omega}\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta}-\underset{\sim}{\xi}_{\alpha'\beta'})^2\}\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}_{\alpha'\beta'}-\underset{\sim}{\xi}^*_{\gamma\delta})^2\}$$

$$F(\alpha,\beta,\gamma,\delta;\ t_1-t_1)\times\begin{cases}\frac{1}{2uv}\{\phi(\frac{2}{3};\tfrac{1}{2};-\frac{1}{8}(u-v)^2)-\phi(\frac{2}{3};\tfrac{1}{2};-\frac{1}{8}(u+v)^2)\}\\ \frac{1}{3}\exp(\frac{v^2}{8})\,\phi(-\frac{1}{6};\frac{3}{2};\frac{v^2}{8})\quad \text{If } u=\eta_{\alpha'\beta'}=0\\ \frac{1}{3}\exp(-\frac{u^2}{8})\,\phi(-\frac{1}{6};\frac{3}{2};\frac{u^2}{8})\quad \text{If } v=\eta^*_{\gamma\delta}=0\end{cases}$$

where $\underset{\sim}{\eta}'_{\alpha\beta}$, $\underset{\sim}{\eta}^*_{\gamma\delta}$, $\underset{\sim}{\xi}_{\alpha'\beta'}$ are defined in Appendix A and for brevity $\underset{\sim}{u}=\underset{\sim}{\eta}_{\alpha'\beta'}$, $\underset{\sim}{v}=\underset{\sim}{\eta}^*_{\gamma\delta}$ and $\underset{\sim}{w}=\underset{\sim}{\eta}'_{\alpha\beta}$

The Skyrme force constants t_o, t_1 and x_o are taken as t_o=-1248,29 Mev.fm^3; t_1=970,56 Mev.fm^5 and x_o=-0,17 as in Beiner et al.[38].

The matrix elements of the imaginary part of the two-body operator G are then:

$$5.5.12\quad \langle\Phi(\underset{\sim}{R}',\underset{\sim}{K})|\text{Im } G|\Phi(\underset{\sim}{R},\underset{\sim}{K})\rangle = \langle\Phi(\underset{\sim}{R}',\underset{\sim}{K})|\Phi(\underset{\sim}{R},\underset{\sim}{K})\rangle\cdot$$

$$\sum_{\substack{\alpha,\beta,\gamma,\delta\\ occ.}}(\underset{\sim}{\alpha},\underset{\sim}{\beta};\underset{\sim}{R}',\underset{\sim}{K}|\text{Im } G|\underset{\sim}{\gamma},\underset{\sim}{\delta};\underset{\sim}{R},\underset{\sim}{K})(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma})$$

with the overlaps (ref.6)

$$5.5.13\quad \langle\Phi(R',K)|\Phi(R,K)\rangle = \frac{\det B(\alpha,\underset{\sim}{R}',\underset{\sim}{K};\gamma,\underset{\sim}{R},\underset{\sim}{K})}{\sqrt{\det B(\alpha,\underset{\sim}{R}',\underset{\sim}{K};\gamma,\underset{\sim}{R}',\underset{\sim}{K})\ \det B(\alpha,\underset{\sim}{R},\underset{\sim}{K};\gamma,\underset{\sim}{R},\underset{\sim}{K})}}$$

and hence

5.5.7 $$NPT(t_o-t_1) = \frac{\sqrt{2}\,C^2_{oo}}{(2\hbar\omega)} \sum_{\alpha,\beta,\gamma,\delta} 3\exp(-\tfrac{W^2}{4})\exp(-\tfrac{v^2}{4})\exp\{-\tfrac{1}{2}(\xi'_{\alpha\beta}-\xi^*_{\gamma\delta})^2$$

$$\{\tfrac{1}{4}\phi(-1;\tfrac{3}{2};\tfrac{W^2}{4})\phi(-\tfrac{1}{2};\tfrac{3}{2};\tfrac{v^2}{4})+\phi(-\tfrac{3}{2};\tfrac{3}{2};\tfrac{v^2}{4})\}F(\alpha,\beta,\gamma,\delta;t_o-t_1)$$

5.5.8 $$NPT(t_1-t_1) = \frac{\sqrt{2}C^2_{oo}}{(2\hbar\omega)} \sum_{\alpha\beta\gamma\delta} 3\exp(-\tfrac{W^2}{4})\exp(-\tfrac{v^2}{4})\exp\{-\tfrac{1}{2}(\xi'_{\alpha\beta}-\xi^*_{\gamma\delta})^2\}$$

$$\{\phi(-1;\tfrac{3}{2};\tfrac{W^2}{4})\phi(-\tfrac{3}{2};\tfrac{3}{2};\tfrac{v^2}{4})+2\phi(-\tfrac{5}{2};\tfrac{3}{2};\tfrac{v^2}{4})\}F(\alpha,\beta,\gamma,\delta;t_1-t_1)$$

5.5.9 $$PT(t_o-t_o) = \frac{C^2_{oo}}{(2\hbar\omega)} \sum_{\alpha\ldots\delta'} \exp(-\tfrac{W^2}{4})\exp(\tfrac{u^2}{4})\exp\{-\tfrac{1}{2}(\xi'_{\alpha\beta}-\xi_{\alpha'\beta'})^2\}$$

$$\times \exp\{-\tfrac{1}{2}(\xi_{\alpha'\beta'}-\xi_{\gamma\delta})^2\}F(\alpha,\beta,\gamma,\delta;t_o-t_o)$$

$$\times \begin{cases} \frac{1}{uv}\{\phi(1;\frac{1}{2};-\frac{1}{8}(u-v)^2)-\phi(1;\frac{1}{2};-\frac{1}{8}(u+v)^2)\} & \text{if } u\neq o;\ v\neq o \\ \exp(-\frac{v^2}{8})\phi(-\frac{1}{2};\frac{3}{2};\frac{v^2}{8}) & \text{if } u=o \text{ but } v\neq o \\ \exp(\frac{u^2}{8})\phi(-\frac{1}{2};\frac{3}{2};\frac{u^2}{8}) & \text{if } u\neq o \text{ but } v=o \end{cases}$$

5.5.10 $$PT(t_o-t_1) = \frac{C^2_{oo}}{2\hbar\omega} \sum_{\alpha..\beta'} \exp(-\tfrac{W^2}{4})\ \exp\{-\tfrac{1}{2}(\xi'_{\alpha\beta}-\xi_{\alpha'\beta'})^2\}\exp(-\tfrac{u^2}{4})$$

$$\exp\{-\tfrac{1}{2}(\xi_{\alpha'\beta'}-\xi^*_{\gamma\delta})^2\}F(\alpha,\beta,\gamma,\delta;\ t_o-t_1)$$

$$\Big[\tfrac{3}{4}\{\phi(-1;\tfrac{3}{2};\tfrac{W^2}{4})+\phi(-1;\tfrac{3}{2};\tfrac{u^2}{4})\}\times \begin{cases} \frac{1}{uv}\{\phi(1;\frac{1}{2};-\frac{1}{8}(u-v)^2)-\phi(1;\frac{1}{2};-\frac{1}{8}(u+v)^2)\} \\ \exp(-\frac{v^2}{8})\phi(-\frac{1}{2};\frac{3}{2};\frac{v^2}{8}) \text{ If } u=o \\ \exp(-\frac{u^2}{8})\phi(-\frac{1}{2};\frac{3}{2};\frac{u^2}{8}) \text{ If } v=o \end{cases}$$

$$+\Gamma(\tfrac{2}{3})\begin{cases} \frac{1}{2uv}\{\phi(\frac{2}{3};\frac{1}{2};-\frac{1}{8}(u-v)^2)-\phi(\frac{2}{3};\frac{1}{2};-\frac{1}{8}(u+v)^2)\} \\ \frac{1}{3}\exp(-\frac{v^2}{8})\phi(-\frac{1}{6};\frac{3}{2};\frac{v^2}{8}) \quad \text{If } u=o \\ \frac{1}{3}\exp(-\frac{u^2}{8})\phi(-\frac{1}{6};\frac{3}{2};\frac{u^2}{8}) \quad \text{If } v=o \end{cases}\Big]$$

$$\langle r's'|g|m,n\rangle\langle m,n|\chi,\xi;\underline{R},\underline{K})\ \delta(E_r,+E_s,-E_m-E_n)$$

and similarly for the symmetrized term.

For convenience this can be split into two contributions originating from the "1" operator (NPT-terms) and the P-operator (PT-terms) in Q = 1-P:

5.5.4 $$(\underline{\alpha},\underline{\beta}|\mathrm{Im}\ G|\chi,\xi)_{NPT} = -\pi \sum_{i,\ell,r,s,m,n} (\underline{\alpha},\underline{\beta}|i,\ell\rangle\langle i,\ell|g|r,s\rangle$$

$$\langle r,s|g|m,n\rangle\langle m,n|\chi,\xi)\delta(E_r+E_s-E_m-E_n)$$

and the Pauli part:

5.5.5 $$(\underline{\alpha},\underline{\beta}|\mathrm{Im}\ G|\chi\ \xi)_{PT} = \pi \sum_{\substack{i,\ell,r',s',r,s\\ m,n,\alpha',\beta'}} (\underline{\alpha},\underline{\beta}|i,\ell\rangle\langle i,\ell|g|r,s\rangle$$

$$\langle r,s|\underline{\alpha}',\underline{\beta}')(\underline{\alpha}',\underline{\beta}'|r',s'\rangle\langle r',s'|g|m,n\rangle\langle m,n|\chi,\xi)\delta(E_r,+E_s,-E_m-E_n)$$

and similarly for the antisymmetrized form.

The NPT anc PT terms (5.5.4) and (5.5.5), also subdivide further into three parts due to the reference force g, which gives rise to (t_o-t_o), (t_o-t_1) and (t_1-t_1) terms. These subdivisions are here refered to as NPT(t_o-t_o), NPT(t_o-t_1) etc. The t_2-term in the Skyrme force, whose angle average is, anyway zero, has been dropped for simplicity.' The other terms are calculated in Appendix B. The results of the calculations are:

5.5.6 $$\mathrm{NPT}(t_o-t_o) = \frac{\sqrt{2}\ C^2_{oo}}{(2\hbar\omega)} \sum_{\alpha\beta\gamma\delta} \exp(-\tfrac{W^2}{4})\phi(-\tfrac{1}{2};\tfrac{3}{2};\tfrac{V^2}{4})F(\alpha,\beta,\gamma,\delta;t_o-t_o)$$

$$\times \exp\{-\tfrac{1}{2}(\xi'_{\alpha\beta}-\xi^*_{\gamma\delta})^2\}\exp(-\tfrac{V^2}{4})$$

considered here, it is simpler to evaluate the integrals in spherical coordinates, (see Appendix A.) as treated by Toepffer and Koller[39].

We are to calculate:

$$5.4.4 \quad \sum_{\substack{\alpha,\beta,\gamma,\delta \\ \text{occ.}}} (\underline{\alpha},\underline{\beta};\underline{R}',\underline{K}|\text{Im } G|\underline{\gamma},\underline{\delta};\underline{R},\underline{K})(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma})$$

and the Pauli operator (4.1.6) reduces to:

$$5.4.5 \quad Q = 1- \sum_{\alpha'\beta'} |\underline{\alpha}',\underline{\beta}';\underline{R},\underline{K})(\underline{\alpha}',\underline{\beta}';\underline{R},\underline{K}|$$

This form of the Pauli operator will be sufficient to demonstrate the important features of the model.

5.5 Matrix elements of Im G.

In the actual calculation the coordinate system has been orientated in such a manner that various vectors $\underline{R}'$, $\underline{R}$ and $\underline{K}$ have the following cartesian form:

$$5.5.1 \quad \underline{R}' = (R'\sin\theta';\ 0;\ R\cos\theta'),$$

$$\underline{R} = (R\sin\theta\cos\phi,\ R\sin\theta\sin\phi,\ R\cos\theta)$$

and

$$5.5.2 \quad K = (0;\ 0;\ K)$$

The matrix elements in Eq.(4.1.11) for Im G for large R where $B_{\alpha'\beta'} = \delta_{\alpha'\gamma'}$ etc. are:·

$$5.5.3 \quad (\underline{\alpha},\underline{\beta};\ \underline{R}'\underline{K}|\text{Im } G|\underline{\gamma},\underline{\delta};\underline{R},\underline{K}) = -\pi \sum_{\substack{i,\ell,r,s \\ r',s',m,n,}} (\alpha,\beta;R'K|i,\ell\rangle$$

$$\langle i,\ell|g|r,s\rangle\langle r,s|1-\sum_{\alpha'\beta'}|\underline{\alpha}',\underline{\beta}')(\underline{\alpha}',\underline{\beta}'|r',s'\rangle$$

$$\delta(\underset{\sim}{K}_{rs}-\underset{\sim}{K}_{i\ell})[t_o\ (1+P_\sigma)+\tfrac{1}{2}t_1(k^2_{i\ell}+k^2_{rs})+t_2\underset{\sim}{k}_{i\ell}\cdot\underset{\sim}{k}_{rs}+W_o\text{-term}$$

In Eq(5.3.2), the conventions (4.1.13), have been used. In the actual calculation only the t_o and t_1 terms are employed for reasons of numerical simplicity. One notices that the t_2-term averages zero upon integration over the momenta.

5.4 The energy conserving δ-function.

This function is defined here with E_r given by

5.4.1 $\quad E_r = \{\hbar(\underset{\sim}{K}+\underset{\sim}{k}_r)b^{-1}\}^2=\tfrac{1}{2}(\underset{\sim}{K}+\underset{\sim}{k}_r)^2(\hbar\omega)$

The value of the oscillator length b being $\left(\frac{\hbar}{\mu\omega}\right)^{\frac{1}{2}}$. Similar relations are obtained for E_s, E_m and E_n. One can then express the δ-function in the form

5.4.2 $\quad \delta(E_r+E_s-E_m-E_n)=\frac{2}{\hbar\omega}\delta\{(\underset{\sim}{k}_r+\underset{\sim}{K})^2+(\underset{\sim}{k}_s-\underset{\sim}{K})^2-(\underset{\sim}{k}_m+\underset{\sim}{K})^2-(\underset{\sim}{k}_n-\underset{\sim}{K})^2\}$

with $\quad \hbar\omega = 41\ A^{-\frac{1}{3}}$ MeV.

Using Eq(4.1.14)the argument of the δ-function on the right hand side reduces to the form

5.4.3 $\quad \delta(E_r+E_s-E_m-E_n) = \frac{2}{\hbar\omega}\ \delta(2k^2_{rs}-2k^2_{mn})$

For s-states all the Hermite polynomials are equal to 1. The expansion coefficients for the expression of the products of two Hermite polynomials as one polynomial also become 1. Large R has also been assumed, i.e. orthogonality, so that $B_{\alpha\gamma}=B^{-1}{}_{\alpha\gamma}=\delta_{\alpha\gamma}$.

For an analytical solution of the integrals occuring in Eq(4.1.11) one represents the δ-functions as Fourier-transforms of exponentials. For intrinsic s-states, as

This can, however, be further simplified with the convention

$$5.2.2 \qquad H_{\underset{\sim}{\lambda}}(\underset{\sim}{K}_{rs})H_{\underset{\sim}{\lambda}'}(\underset{\sim}{K}_{rs}) = \sum_{\varrho=0}^{\underset{\sim}{\lambda}+\underset{\sim}{\lambda}'} \hat{N}(\underset{\sim}{\lambda},\underset{\sim}{\lambda}';\varrho)H_{\varrho}(\underset{\sim}{K}_{rs})$$

where $\hat{N}(\underset{\sim}{\lambda},\underset{\sim}{\lambda}';\varrho)$ are expansion coefficients given by

$$5.2.3 \qquad \hat{N}(\underset{\sim}{\lambda},\underset{\sim}{\lambda}';\varrho) = \prod_{i=1} \frac{2^{(\lambda_i+\lambda'_i-\rho_i)/2}\times\lambda_i!\lambda'_i!}{(s_i-\lambda_i)!(s_i-\lambda'_i)!(s_i-\rho_i)!}$$

where $s_i = \lambda_i+\lambda'_i+\rho_i$ = even, otherwise $\hat{N}(\lambda_i,\lambda';\rho_i) = 0$.

5.3 Matrix elements of the reference interaction.

We use as an example of the reference interaction, the Skyrme force[38]. This effective interaction was designed for the Hartree-Fock calculations of nuclei. It is represented in momentum space by

$$5.3.1 \qquad g = t_o(1+x_oP_\sigma) + \tfrac{1}{2}t_1(k^2+k'^2) + t_2\underset{\sim}{k}\,\underset{\sim}{k}' + iW_o(\underset{\sim}{\sigma}_1\times\underset{\sim}{\sigma}_2)(\underset{\sim}{k}\times\underset{\sim}{k}')$$

where $\underset{\sim}{k}$ and $\underset{\sim}{k}'$ are relative wave vectors of two nucleons. The original Skyrme force also includes a three-body term which is equivalent to a density dependent force and does not conserve momentum. There exists, however, a Skyrme force in which this term is zero as in the treatment by Beiner et al.[38]. The spin-orbit term will also be neglected in this calculation.

The matrix elements of the Skyrme force are therefore given by

$$5.3.2 \qquad \langle i,\ell;\underset{\sim}{R},\underset{\sim}{K}|g|r,s;\underset{\sim}{R},\underset{\sim}{K}\rangle = \exp\{i(\underset{\sim}{k}_{i\ell}-\underset{\sim}{k}_{rf}')\underset{\sim}{R}\}(2\pi)^3 \times$$

From this, one notices that Eq. (5.1.2) implies that $\underline{\mu}$ and $\underline{\nu}$ also become:

5.1.6 $\qquad \underline{\mu} = (0,0,0) = \underline{\nu}$

and therefore, the expansion coefficients reduce to:

5.1.7 $\qquad \hat{M}(\underline{\alpha},\underline{\beta};\underline{\mu},\underline{\nu}) = 1,$

and that

5.1.8 $\qquad c_{\alpha\gamma} = c_{\alpha}c_{\gamma} = \pi^{-\frac{3}{2}}$ become $c_{oo} = \pi^{-\frac{3}{2}}$

5.2 Matrix elements of Q

These are calculated with $Q = 1 - P$ and from Eq(4.1.9):

5.2.1 $\qquad \langle r,s;\underline{R},\underline{K}|P|r,s;\underline{R},\underline{K}\rangle$

$$= \sum_{\substack{\alpha',\beta',\gamma',\delta' \\ occ.}} B^{-1}{}_{\alpha\gamma'}\langle r,s;\underline{R}|\underline{\alpha}'\underline{\beta}';\underline{R},\underline{K}\rangle(\underline{\alpha}',\underline{\beta}';\underline{R},\underline{K}|r,s;\underline{R},\underline{K}\rangle B^{-1}{}_{\beta'\delta'}$$

$$= \sum_{\substack{\alpha',\beta',\gamma',\delta' \\ occ.}} B^{-1}{}_{\alpha'\gamma'}(\underline{R},\underline{K})B^{-1}{}_{\beta'\delta'}(\underline{R},\underline{K})(2\pi)^{6}i^{-\underline{\alpha}'-\underline{\beta}'+\underline{\gamma}'+\underline{\delta}}C_{\alpha'\beta'\gamma'\delta'}$$

$$\exp\{\tfrac{i}{2}(-\underline{R}_{\alpha'},-\underline{R}_{\beta'},+\underline{R}_{\gamma'},+\underline{R}_{\delta'})\underline{K}_{rs}+i(-\underline{R}_{\alpha'},-\underline{R}_{\beta'},+\underline{R}_{\gamma'},+\underline{R}_{\delta'})k_{rs}\}$$

$$\exp\{-\tfrac{1}{8}K^{2}{}_{rs}-k^{2}{}_{rs}-\tfrac{1}{2}(K^{2}{}_{\alpha'},+K^{2}{}_{\beta'},+K^{2}{}_{\gamma'},+K^{2}{}_{\delta'},)+\tfrac{1}{2}\underline{K}_{rs}(\underline{K}_{\alpha'},+\underline{K}_{\beta'},+\underline{K}_{\gamma'},+\underline{K}_{\delta'},)$$

$$+(K_{\alpha'},-K_{\beta'},+K_{\gamma'},-K_{\delta'},)\}$$

$$\sum_{\substack{\underline{\lambda},\underline{\omega}=o}}^{\underline{\alpha}'+\underline{\beta}'}\sum_{\substack{\underline{\lambda}',\underline{\omega}'=o}}^{\underline{\gamma}'+\underline{\delta}'}\hat{M}(\underline{\alpha}',\underline{\beta}';\underline{\lambda},\underline{\omega})\hat{M}(\underline{\gamma}'\underline{\delta}';\underline{\lambda}',\underline{\omega}')H_{\underline{\Delta}}(\underline{K}_{rs})H_{\underline{\omega}}$$

$$(k_{rs})H_{\underline{\lambda}},(\underline{K}_{rs})H_{\underline{\omega}},(\underline{k}_{rs})$$

CHAPTER 5.

ANALYTICAL TECHNIQUES FOR THE CALCULATION OF Im G

5.1 The overlaps between localized states and plane waves.

These are given as

5.1.1 $$\langle \underline{\alpha},\underline{\beta};\underline{R}',\underline{K}|\underline{i},\underline{i};\underline{P},\underline{K}\rangle = C_{\alpha\beta}(2\pi)^{-1} i^{\underline{\alpha}+\underline{\beta}} \exp\{\tfrac{1}{2}(\underline{R}'_{\alpha}+\underline{R}'_{\beta})\underline{K}_{i\underline{i}}$$

$$+i(\underline{R}'_{\alpha}-\underline{R}'_{\beta}-\underline{R})\underline{k}_{i\underline{i}}\}\exp\{\tfrac{1}{4}K^2_{\perp\underline{i}}-k^2_{i\underline{i}}-\tfrac{1}{2}(K^2_{\alpha}+K^2_{\beta})+\tfrac{1}{2}(\underline{K}_{i\underline{i}}(\underline{K}_{\alpha}+\underline{K}_{\beta})$$

$$+(\underline{K}_{\alpha}-\underline{K}_{\beta})\underline{k}_{i\underline{i}}\}\sum_{\underline{\mu},\underline{\nu}=0}^{\underline{\alpha}+\underline{\beta}} H_{\underline{\mu}}(\tfrac{1}{2}\underline{K}_{i\underline{i}})H_{\underline{\nu}}(\underline{k}_{i\underline{i}})\hat{M}(\underline{\alpha},\underline{\beta};\underline{\mu},\underline{\nu})$$

Here the conventions (4.1.13) and (4.1.14) and the following convention

5.1.2 $$H_{\underline{\alpha}}(\tfrac{1}{2}\underline{K}_{i\underline{i}}+\underline{k}_{i\underline{i}}-\underline{K}_{\alpha})H_{\underline{\beta}}(\tfrac{1}{2}\underline{K}_{i\underline{i}}-\underline{k}_{i\underline{i}}-\underline{K}_{\beta}) = \sum_{\underline{\mu},\underline{\nu}=0}^{\underline{\alpha}+\underline{\beta}} H_{\underline{\mu}}(\underline{K}_{i\underline{i}})H_{\underline{\nu}}(\underline{k}_{i\underline{i}})\hat{M}(\underline{\alpha},\underline{\beta};\underline{\mu},\underline{\nu})$$

have been employed. A similar expression can be written down for the overlap $\langle m,n;\underline{R},\underline{K}|\underline{\gamma},\underline{\delta};\underline{R},\underline{K}\rangle$.

The actual calculation was undertaken for the s-states $\underline{\alpha} = (0,0,0) = \underline{\beta} = \underline{\gamma} = \underline{\delta}$; and thus the following expressions reduced to the simple forms given below:

5.1.3 $\underline{\alpha} + \underline{\beta} = (0,0,0)$; $i^{\underline{\alpha}+\underline{\beta}+\underline{\gamma}+\underline{\delta}} = 1$ because we have defined the symbol $i^{\underline{n}}$ as:

5.1.4 $$i^{\underline{n}} = i^{n_x+n_y+n_z}$$

Also

5.1.5 $$H_{\underline{\mu}} = 1 = H_{\underline{\nu}} \text{ etc.}$$

where the expression has been symmetrized against

$$\delta(E_r+E_s-E_m-E_n) \rightarrow \tfrac{1}{2}\delta(E_r+E_s-E_m-E_n)+\tfrac{1}{2}\delta(E_r+E_s-E_i-E_\ell)$$

In the foregoing representation, one notices the appearance of various overlaps, like e.g. $(\underline{\alpha},\underline{\beta};\underline{R},\underline{K}|i,\ell;\underline{R},\underline{K}>$. These have been fully calculated in the Mathematical Appendix. These overlaps are expressed in terms of momenta, with the convention

4.1.12 $\quad i,\ell\varepsilon A$ and $m,n\varepsilon B$

4.1.13 $$\underset{\sim}{K}_{i\ell} = \underset{\sim}{K}_i+\underset{\sim}{K}_\ell+\underset{\sim}{k}_i+\underset{\sim}{k}_\ell$$

$$2\underset{\sim}{k}_{i\ell} = \underset{\sim}{K}_i-\underset{\sim}{K}_\ell+\underset{\sim}{k}_i-\underset{\sim}{k}_\ell$$

We can however conversely give the above momenta as:

4.1.14 $$\underset{\sim}{k}_i+\underset{\sim}{K}_i = \tfrac{1}{2}\underset{\sim}{K}_{i\ell}+\underset{\sim}{k}_{i\ell}$$

$$\underset{\sim}{k}_\ell+\underset{\sim}{K}_\ell = \tfrac{1}{2}\underset{\sim}{K}_{i\ell}-\underset{\sim}{k}_{i\ell}$$

Therefore, in terms of momenta $\underset{\sim}{K}_{i\ell}$ and $\underset{\sim}{k}_{i\ell}$ we need the expressions for the following overlaps:

(i) matrix elements of the Pauli operator

(ii) the overlaps between localized states and plane waves

(iii) matrix elements of the reference operator

(iv) δ-function - Fourier transforms

From the full calculations of expressions (i) - (iv) mentioned above, which are carried out in the Appendix, we obtain the results as detailed in the next chapter.

* * * * * * * * * * *

i) if B^{-1} is the inverse of B in the full Hilbert space of all states (χ, $\underline{R}$, $\underline{K}$), then the sum over the occupied states does not give orthogonality

ii) if B^{-1} is the inverse of B in the space of occupied states, Q does not project on the occupied states, since the application on an unoccupied state will not give zero.

While the oscillator basis has the advantages cited above, their disadvantage is that one cannot easily construct a proper Pauli operator. The use of Eq(4.1.9) for Q will therefore lead to inconsistencies. Because of the appearance of the kinetic energy operator in Eq(3.4.9) for Im G, the matrix elements of Im G must be evaluated by inserting complete sets of plane waves as intermediate states. This is the source of another difficulty in connection with the Pauli operator, since it is the oscillator states (4.1.1) rather than plane wave states which are occupied or unoccupied. Inserting complete sets of plane waves into Eq(3.4.9) and taking the matrix elements required in Eq(4.1.4) one obtains after symmetrizing with respect to $i,\ell,r's' \longleftrightarrow m,n,r,s$

$$4.1.11 \quad (\alpha,\beta;\underline{R}',\underline{K}|\mathrm{Im}\ G|\chi,\delta;\underline{R},\underline{K}) = \tfrac{1}{2}\pi \sum_{\substack{i,\ell,r,s \\ m,n,r's'}} (\alpha,\beta;\underline{R},\underline{K}|$$

$$i,\ell;\underline{R},\underline{K}><i,\ell;\underline{R},\underline{K}|g|r's';\underline{R},\underline{K}><r',s';\underline{R},\underline{K}|Q|$$

$$r',s';\underline{R},\underline{K}><r',s';\underline{R},\underline{K}|g|m,n;\underline{R},\underline{K}><m,n;\underline{R},\underline{K}|$$

$$\chi,\delta;\underline{R},\underline{K})\{\delta(E_r+E_s-E_m-E_n) + \delta(E_r+E_s-E_i-E_\ell)\}$$

operator

4.1.6 $$Q = 1 - \sum_{\alpha',\beta'} |\underline{\alpha}',\underline{\beta}';\underline{R},\underline{K}\rangle\langle\underline{\alpha}',\underline{\beta}';\underline{R},\underline{K}|$$

is a projection operator on the unoccupied states. This can be shown by taking matrix elements with $(\underline{\alpha},\underline{\beta}|$ and $|\underline{\alpha},\underline{\beta})$

4.1.7 $$Q|\underline{\gamma},\underline{\delta}) = |\underline{\gamma},\underline{\delta}) - \sum_{\substack{\alpha',\beta' \\ \text{occup.}}} |\underline{\alpha}',\underline{\beta}')(\underline{\alpha}',\underline{\beta}'|\underline{\gamma},\underline{\delta})$$

$$= |\underline{\gamma},\underline{\delta}) \sum_{\substack{\alpha',\beta' \\ \text{occup.}}} \delta_{\alpha',\gamma}\delta_{\beta',\delta}|\underline{\alpha}',\underline{\beta}')$$

$$= \begin{cases} 0 & \text{if } \gamma \text{ and } \delta \text{ are occupied} \\ |\underline{\gamma},\underline{\delta}) & \text{if } \gamma \text{ and } \delta \text{are unoccupied} \end{cases}$$

Since the states are not orthogonal in the overlap region:

4.1.8 $$(\underline{\alpha},\underline{\beta};\underline{R}'\underline{K}|\underline{\gamma},\underline{\delta};\underline{R},\underline{K}) = B_{\alpha,}\, B_{\beta\delta}$$

one might attempt to construct a Pauli operator by:

4.1.9 $$Q = 1-P = 1-\sum_{\alpha'\beta',\gamma'\delta'} B^{-1}_{\alpha',\gamma'} |\underline{\alpha}',\underline{\beta}';\underline{R},\underline{K}\rangle\langle\underline{\gamma}',\underline{\delta}';\underline{R},\underline{K}| B^{-1}_{\beta',\delta'},$$

Here $B_{\alpha',\gamma'}$, as mentioned above, are the overlap matrix elements in the non-orthogonal basis of the moving oscillator states Eq(4.1.1). As expected Q tends to 1 for high energies since

4.1.10 $$\langle\underline{k}_1,\underline{k}_2;\underline{R},\underline{K}|Q|\underline{k}'_1,\underline{k}'_2;\underline{R},\underline{K}\rangle \propto 1 - \exp(-K^2)$$

the influence of the Pauli principle on absorption disappears with increasing energy. However, it is easy to see that the operator Q in Eq(4.1.9) is not in general a proper Pauli operator because

Multiplying out all the constants mentioned above, we obtain:

B.5.1 NP-constant = $\frac{\pi}{\hbar\omega}\,\frac{c_{oo}^2}{(2\pi)^6}$.

B.6 Integrations carried out

The integration variables are $\underset{\sim}{K}_{il}$, $\underset{\sim}{K}_{rs}$ and $\underset{\sim}{K}_{mn}$ on one hand, and $\underset{\sim}{k}_{rs}$, $\underset{\sim}{k}_{il}$ and $\underset{\sim}{k}_{mn}$ on the other hand.

Substituting the equations of the type (B.4.2) and (B.4.4) which appear in (B.1.1), we can write down (NPT)-matrix elements of the two-body operator IM G as:

B.6.1 $(\underset{\sim}{\alpha},\underset{\sim}{\beta};\ \underset{\sim}{R}'\underset{\sim}{K}|\,\mathrm{Im}\ G|\,\underset{\sim}{\gamma},\underset{\sim}{\delta};\ \underset{\sim}{R},\underset{\sim}{K})_{NPT}$

$$= -\frac{\pi c_{oo}^2}{\hbar\omega(2\pi)^6}\int d^3K_{il}\int d^3k_{il}\int d^3K_{rs}\int d^3k_{rs}\int d^3K_{mn}\int d^3k_{mn}$$

$$\exp i\tfrac{1}{2}\underset{\sim}{K}_{il}(\underset{\sim}{R}'_\alpha+\underset{\sim}{R}'_\beta) + i\underset{\sim}{k}_{il}(\underset{\sim}{R}'_\alpha-\underset{\sim}{R}'_\beta-\underset{\sim}{R}) - \tfrac{1}{4}\underset{\sim}{K}_{il}^2-\underset{\sim}{k}_{il}^2$$

$$+ i(\underset{\sim}{k}_{il}-\underset{\sim}{k}_{rs})\underset{\sim}{R})\delta(\underset{\sim}{K}_{rs}-\underset{\sim}{K}_{il})\exp\{-\tfrac{1}{2}\underset{\sim}{K}_{mn}(\underset{\sim}{R}_\gamma+\underset{\sim}{R}_\delta)$$

$$- i\underset{\sim}{k}_{mn}(\underset{\sim}{R}_\gamma-\underset{\sim}{R}_\delta-\underset{\sim}{R}) - \tfrac{1}{4}(K_{mn}^2-k_{mn}^2)\exp i(\underset{\sim}{k}_{rs}-\underset{\sim}{k}_{mn})\underset{\sim}{R}$$

$$\delta(k_{rs}^2-k_{mn}^2)\{NPT(t_o-t_o) + NPT(t_o-t_1) + NPT(t_1-t_1)\}$$

$$= \frac{\pi c_{oo}^2}{\hbar\omega(2\pi)^6}\int d^3K_{il}\ \exp\{i\underset{\sim}{K}_{il}(\underset{\sim}{\xi}'_{\alpha\beta}-\underset{\sim}{\xi}^*_{\gamma\delta}) - \tfrac{1}{4}K_{il}^2\}$$

$$\int d^3k_{il}\ \exp(i\underset{\sim}{k}_{il}\underset{\sim}{w} - k_{il}^2)\int d^3k_{rs}\int d^3k_m\ \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - k_{mn}^2)$$

$$\delta(k_{rs}^2-k_{mn}^2)[NPT(t_o-t_o) + NPT(t_o-t_1) + NPT(t_1-t_1)]$$

where

B.3.3 $NPT(t_1-t_1) = (4\times t_{1\ x\gamma}\ \delta_{\beta\delta} - 2\times\tfrac{1}{2}t_1^2\delta_{\alpha\delta}\ \delta_{\beta\gamma})(k_{i\ell}^2 + k_{rs}^2)(k_{rs}^2 + k_{mn}^2)$

$= 2(t_1^2\delta_{\alpha\gamma}\ \delta_{\beta\delta} - \tfrac{1}{2}t_1^2\delta_{\alpha\delta}\ \delta_{\beta\gamma})(k_{i\ell}^2 + k_{rs}^2)(k_{rs}^2 + k_{mn}^2)$

B.4 Non-Pauli Matrix elements of Im G

The matrix elements of Im G for (NPT) become

B.4.1 $(\underline{\alpha},\underline{\beta}|\text{Im } G|\chi,\underline{\delta})_{NPT} = \sum_{i,\ell,r,s} (\underline{\alpha},\underline{\beta}|i,\ell\rangle\langle i,\ell|g|r,s\rangle$

$\langle r,s|g|m,n\rangle\langle m,n|\chi,\underline{\delta})\delta(E_r + E_s - E_m - E_n)$

where

B.4.2 $(\underline{\alpha},\underline{\beta};\underline{R}',\underline{K}|i,\ell;\underline{R},\underline{K}\rangle = c_{oo}(2\pi)^3\exp\{\frac{i}{2}\underline{K}_{i\ell}(\underline{R}'_\alpha - \underline{R}'_\beta) - \frac{1}{4}K_{i\ell}^2\}$

$\exp\{i\underline{k}_{i\ell}(\underline{R}'_\alpha - \underline{R}'_\beta - \underline{R}) - k_{i\ell}^2\}$

B.4.3 $g = g_o\delta(\underline{r}_1 - \underline{r}_2)$ (Skyrme)

B.4.4 $\langle i,\ell;\underline{R},\underline{K}|g|r,s;\underline{R},\underline{K}\rangle = g_o\exp\{\frac{i}{2}(\underline{k}_{i\ell} - \underline{k}_{rs})\underline{R}\}(2\pi)^3\delta(\underline{K}_{rs} - \underline{K}_{i\ell})$

B.4.5 $g_o^2 = "(t_o - t_o)" + "(t_o - t_1)" + "(t_1 - t_1)"$

B.5 Constant value in (NP) calculation

From the above equations in B.5.4, we note the following constants: π, from the two-body operator Im G; $c_{oo}^2(2\pi)^6$, from the two overlaps $(\underline{\alpha},\underline{\beta}|i,\ell\rangle$ and $\langle m,n|\chi,\underline{\delta})$; $(2\pi)^6$ from the matrix elements $\langle i,\ell|g|r,s\rangle$ and $\langle r,s|g|m,n\rangle$; $(2\pi)^{-18}$ from the six integrations over $\underline{k}_{i\ell}, \underline{K}_{i\ell}$; $\underline{k}_{rs}, \underline{K}_{rs}$; and $\underline{k}_{mn}$, $\underline{K}_{mn}$; and $\frac{1}{\hbar\omega}$ from the δ-function $\delta(E_r+E_s-E_m-E_n) = \frac{1}{\hbar\omega}\delta(k_{rs}^2 - k_{mn}^2)$.

B.2 Skyrme force exchange non-Pauli term [illegible]

This one is of the form:

B.2.1 $2[t_o(1+x_o) + \tfrac{1}{2}t_1(k_{i\ell}^2 + k_{rs}^2)][t_o(1+x_o) + \tfrac{1}{2}t_1(k_{rs}^2+k_{mn}^2)]$

(t_2 has already been taken as $t_2 = 0$).

Multiply Eq. (B.2.1) out, we obtain:

B.2.2 $2t_o^2(1+x_o)^2 + t_o t_1(k_{i\ell}^2 + 2k_{rs}^2 + k_{mn}^2) + \tfrac{1}{4}t_1^2(k_{i\ell}^2+k_{rs}^2)(k_{rs}^2+k_{mn}^2)$.

Treating the three terms separately and multiplying each by $\delta_{\alpha\delta}\ \delta_{\beta\gamma}$ and -1:

B.2.3 $ENT(t_o\text{-}t_o) = -2t_o^2(1+x_o)^2\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

$ENT(t_o\text{-}t_1) = -t_o t_1(1+x_o)(k_{r\ell}^2 + 2k_{rs}^2 + k_{mn}^2)\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

$ENPT(t_1\text{-}t_1) = -\tfrac{1}{4}t_1^2(k_{i\ell}^2 + k_{rs}^2)(k_{rs}^2 + k_{mn}^2)\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

B.3. Spin and Isospin

The spin and isospin summation adds a factor of 4 in the direct term and a factor of 2 in the exchange term.

Combining the (DNPT) and ENPT) we get

B.3.1 $NPT(t_o\text{-}t_o) = 4\times 4t_o^2(1+x_o+x_o^2)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - 2\times 2t_o^2(1+x_o)^2\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

$= 4(4t_o^2(1+x_o+x_o^2)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - t_o^2(1+x_o^2)\delta_{\alpha\delta}\ \delta_{\beta\gamma})$

B.3.2 $NPT(t_o\text{-}t_1) = (4\times t_o t_1(2+x_o)\delta_{\alpha\gamma}\delta_{\beta\delta} - 2\times t_o t_1(1+x_o)\delta_{\alpha\delta}\delta_{\beta\gamma})$

$(k_{i\ell}^2 + 2k_{rs}^2 + k_{mn}^2)$

$= 2(2t_o t_1(2+x_o)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - t_o t_1(1+x_o)\delta_{\alpha\delta}\ \delta_{\beta\gamma})(k_{i\ell}^2 + 2k_{rs}^2 + k_{mn}^2)$

Thus Eq. (B.1.3) becomes:

B.1.4 $$DNPT(t_o-t_o) = 4t_o^2(1+x_o+x_o^2)\delta_{\alpha\gamma}\,\delta_{\beta\delta}$$

$$DNPT(t_o-t_1) = t_o t_1(2+x_o)(k_{i\ell}^2 + 2k_{rs}^2 + k_{mn}^2)\delta_{\alpha\gamma}\,\delta_{\beta\delta}$$

$$DNPT(t_1-t_1) = t_1^2(k_{i\ell}^2 + k_{rs}^2)(k_{rs}^2 + k_{mn}^2)\delta_{\alpha\gamma}\,\delta_{\beta\delta}$$

APPENDIX B

SKYRME FORCE AS REFERENCE INTERACTION

B.1 Skyrme force direct non-Pauli term (DNPT)

is of the form:

B.1.1 $$[t_o(1-x_o) + \tfrac{1}{2}t_1(k_{i\ell}^2 + k_{rs}^2) + t_2\underset{\sim}{k}_{i\ell}\,\underset{\sim}{k}_{rs}][t_o(1-x_o)$$

$$+ \tfrac{1}{2}t_1(k_{rs}^2 + k_{mn}^2) + t_2\,\underset{\sim}{k}_{rs}\,\underset{\sim}{k}_{mn}]$$

$$+ 3[t_o(1+x_o) + \tfrac{1}{2}t_1(k_{i\ell}^2\,k_{rs}^2) + t_2\underset{\sim}{k}_{i\ell}\,\underset{\sim}{k}_{rs}][t_o(1+x_o)+\tfrac{1}{2}t_1(k_{rs}^2+k_{mn}^2)$$

$$+ t_2\underset{\sim}{k}_{rs}\,\underset{\sim}{k}_{mn}].$$

In this calculation, we have taken $t_2 = 0$, which thus reduces the Skyrme two-body interaction to:

B.1.2 $$[t_o(1-x_o) + \tfrac{1}{2}t_1(k_{i\ell}^2 + k_{rs}^2)][t_o(1-x_o) + \tfrac{1}{2}t_1(k_{rs}^2+k_{mn}^2)]$$

$$+ 3\,t_o(1+x_o) + \tfrac{1}{2}t_1(k_{i\ell}^2 + k_{rs}^2)][t_o(1+z_o) + \tfrac{1}{2}t_1(k_{rs}^2 + k_{mn}^2)]$$

$$= 4t_o^2(1+x_o + x_o^2) + t_o t_1(2+x_o)(k_{i\ell}^2 + 2k_{rs}^2+k_{mn}^2)+t_1^2(k_{rs}^2+k_{i\ell}^2)(k_{rs}^2+k_{mn}^2)$$

For simplicity, the three terms of (DNPT) are treated separately and represented by $(t_o\text{-}t_o)$, $(t_o\text{-}t_1)$ and $(t_1\text{-}t_1)$.

L.1.3 $$\text{DNPT}(t_o\text{-}t_o) = 4t_c^2(1 + x_o + x_o^2)$$

$$\text{DNPT}(t_o\text{-}t_1) = t_o t_1(2 + x_o)(k_{i\ell}^2 + 2k_{rs}^2 + k_{mn}^2)$$

$$\text{DNPT}(t_1\text{-}t_1) = t_1^2(k_{i\ell}^2 + k_{rs}^2)(k_{rs}^2 + k_{mn}^2).$$

Each of these has to be multiplied by the product of the two overlap matrices: $B^{-1}_{\alpha\gamma}$ and $B^{-1}_{\beta\delta}$, which as explained previously, have been taken as $B^{-1}_{\alpha\gamma} = \delta_{\alpha\gamma}$ and $B^{-1}_{\beta\delta} = \delta_{\beta\delta}$.

Hence the complete result is:

$$\text{A.3.3}\quad 2\pi^2 \frac{\Gamma\left(\frac{\ell+n+2}{2(\ell+n+1)}\right)}{(b+c)^{\frac{1}{2}(\ell+n+2)}} \begin{cases} \frac{1}{uv}\left[\phi\left(\frac{\ell+n+2}{2\ell+2n+2};\ \frac{1}{2};\ -\frac{(u-v)^2}{4(b+c)}\right) - \phi\left(\frac{\ell+n+2}{2\ell+2n+2};\ \frac{1}{2};\ -\frac{(u+v)^2}{4(b+c)}\right)\right. \\ \qquad \text{if } u = 0 \\ \frac{\ell+n+2}{(b+c)(\ell+n+1)} \exp\left(-\frac{v^2}{4(b+c)}\right)\phi\left(\frac{3}{2} - \frac{3\ell+3n+4}{2\ell+2n+2};\ \frac{3}{2};\ \frac{v^2}{4(b+c)}\right) \\ \qquad \text{if } v = 0 \\ \frac{\ell+n+2}{(b+c)(\ell+n+1)} \exp\left(-\frac{u^2}{4(b+c)}\right)\phi\left(\frac{3}{2} - \frac{3\ell+3n+4}{2\ell+2n+2};\ \frac{3}{2};\ \frac{u^2}{4(b+c)}\right); \end{cases}$$

. . . . . .

Each k_{mn}-integration in the sum above is similar to Erdélyi Eq. (1.4.14), p.15 with:

$a = b + c$; $\upsilon = \ell + n + 1$; $y \in (u-v; u+v)$, giving the result:

$$\tfrac{1}{2}(b+c)^{-\frac{1}{2}(\ell+n+2)}\Gamma\left(\tfrac{1}{2} + \frac{1}{2(\ell+n+1)}\right)\left[\phi\left(\tfrac{1}{2} + \frac{1}{2(\ell+n+1)}; \tfrac{1}{2}; -\frac{(u-v)^2}{4(b+c)}\right)\right.$$
$$\left. - \phi\left(\tfrac{1}{2} + \frac{1}{2(\ell+n+1)} - \tfrac{1}{2}; -\frac{(u+v)^2}{4(b+c)}\right)\right]$$

Thus the final result is of the form:

A.3.1 $\int d^3k_{rs} k_{rs}^n \exp(i\underset{\sim}{k}_{rs}\underset{\sim}{u} - bk_{rs}^2) \int d^3k_{mn} k_{mn}^\ell \exp(-i\underset{\sim}{k}_{mn}\underset{\sim}{v} - ck_{mn}^2)$

$$\delta(k_{rs}^2 - k_{mn}^2)$$

$$= \frac{2\pi^2}{uv} \frac{\Gamma\left(\frac{\ell+n+2}{2(\ell+n+1)}\right)}{(b+c)^{\frac{1}{2}(\ell+n+2)}}\left[\phi\left(\frac{\ell+n+2}{2(\ell+n+1)}; \tfrac{1}{2}; -\frac{(u-v)^2}{4(b+c)}\right) - \phi\left(\frac{\ell+n+2}{2(\ell+n+1)}; \tfrac{1}{2}; \right.\right.$$
$$\left.\left. - \frac{(u+v^2}{4(b+c)}\right)\right]$$

In the limit $u \to 0$ (or $v \to 0$), we use L'Hospital's rule:, which gives the result:

A.3.2 $$\frac{2\pi^2}{(b+c)} \frac{\Gamma\left(\frac{\ell+n+2}{2(\ell+n+1)}\right)}{(b+c)^{\frac{1}{2}(\ell+n+2)}} \times \frac{\ell+n+2}{\ell+n+1} \exp\left(-\frac{v^2}{4(b+c)}\right)\phi\left(\frac{3\ell+3n+4}{2\ell+2n+2} + \frac{3}{2}; \frac{3}{2}; \frac{v^2}{4(b+c)}\right).$$

A.3 $\underline{k_{rs},\ k_{mn}}$-integration

$$\int d^3k_{rs}\ k_{rs}^n \exp(i\underset{\sim}{k}_{rs}\ \underset{\sim}{u} - bk_{rs}^2)\int d^3k_{mn}\ k_{mn}^\ell \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - ck_{mn}^2)$$

$$\delta(k_{rs}^2 - k_{mn}^2)$$

$$= 2\pi\int d(\cos\phi)\int dk_{rs}\ k_{rs}^{(n+2)} \exp(ik_{rs}\ u\cos\phi)\exp(-bk_{rs}^2)$$

$$\int d^3k_{mn}\ k_{mn}^\ell \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - ck_{mn}^2)\delta(k_{rs}^2 - k_{mn}^2)$$

$$= 2\pi\left(\frac{2i}{iu}\right)\int dk_{rs}\ k_{rs}^{(n+1)}\sin(k_{rs}\ u)\exp(-bk_{rs}^2)$$

$$\int d^3k_{mn}\ k_{mn}^\ell \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - ck_{mn}^2)\delta(k_{rs}^2 - k_{mn}^2)$$

$$= \frac{4\pi}{u}\int d(k_{rs}^2)\left(\frac{1}{2k_{rs}}\right)k_{rs}^{(n+1)}\sin(k_{rs}\ u)\exp(-bk_{rs}^2)$$

$$\int d^3k_{mn}\ k_{mn}^\ell \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - ck_{mn}^2)\delta(k_{rs}^2 - k_{mn}^2)$$

$$= \frac{2\pi}{u}\int d(k_{rs}^2)k_{rs}^{(n)}\sin(k_{rs}\ u)\exp(-bk_{rs}^2)$$

$$\int d^3k_{mn}\ k_{mn}^\ell \exp(i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - ck_{mn}^2)\delta(k_{rs}^2 - k_{mn}^2)$$

$$= \frac{2\pi}{u}\int d^3k_{mn}\ k_{mn}^{(\ell+n)}\sin(k_{mn}\ u)\exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v})$$

$$\exp\{-(b+c)k_{mn}^2\}$$

$$= \frac{4\pi^2}{u}\int d(\cos\phi)\int dk_{mn}\ k_{mn}^{(\ell+n+2)}\sin(k_{mn}\ u)$$

$$\exp(-ik_{mn}\ v\cos\phi)\exp\{-(b+c)k_{mn}^2\}$$

$$= \frac{8\pi^2}{uv}\int dk_{mn}\ k_{mn}^{(\ell+n+1)}\exp\{-(b+c)k_{mn}^2\}[\tfrac{1}{2}\{\cos(u-v)k_{mn} - \cos(u+v)k_{mn}\}]$$

$$= \frac{4\pi^2}{uv}\int dk_{mn}\{k_{mn}^{(\ell+n+1)}\exp\{-(b+c)k_{mn}^2\}[\cos(u-v)k_{mn} - \cos(u+v)k_{mn}]\ .$$

A.2 $\underline{k}_{rs}$, $\underline{k}_{mn}$ integration with $\underline{u} = (0,0,0)$

It is of the type:

$$\int d^3k_{rs}\ \underline{k}_{rs}^n\ \exp(-bk_{rs}^2) \int d^3k_{mn}\ k_{mn}^2\ \exp(i\underline{k}_{mn}\ \underline{v} - ck_{mn}^2)$$

$$\delta(k_{rs}^2 - k_{mn}^2)$$

$$= 2\pi \int d(k_{rs}^2) k_{rs}^{(n+1)} \exp(-bk_{rs}^2) \int d^3k_{mn}\ k_{mn}^{\ell}\ \exp(i\underline{k}_{mn}\ \underline{v} - ck_{mn}^2)$$

$$\delta(k_{rs}^2 - k_{mn}^2)$$

$$= 2\pi \int d^3k_{mn}\ k_{rs}^{(\ell+n+1)} \exp\{-(b+c)k_{mn}^2\} \exp(i\underline{k}_{mn}\ \underline{v})$$

$$= 4\pi^2 \int dk_{mn}\ k_{mn}^{(\ell+n+3)} \exp\{-(b+c)k_{mn}^2\} \int d(\cos\phi)$$

$$= 4\pi^2 \left(\frac{2i}{iv}\right) \int dk_{mn}\ k_{mn}^{(\ell+n+2)}\ \exp\{-(b+c)k_{mn}^2\} \sin(k_{mn}\ v)$$

$$= \frac{8\pi^2}{v} \int dk_{mn} \{k_{mn}^{(\ell+n+2)}\ \exp\{-(b+c)k_{mn}^2\} \sin(k_{mn}\ v).$$

The k_{mn}-integration is similar to Erdélyi[40] P.74 Eq. (4.2.24)) with:

$a = b + c$; $y = v$; $2\nu - 2 = \ell + n + 2 \rightarrow \nu = \frac{\ell+n+4}{2}$;

and this therefore gives the result:

$$\tfrac{1}{2}v(b+c)^{-\frac{1}{2}(\ell+n+4)}\, \Gamma\left(\frac{\ell+n+4}{2}\right) \exp\left(-\frac{v^2}{4}\right) \Phi\left(\frac{3}{2} - \frac{\ell+n+4}{2};\ \frac{3}{2};\ \frac{v^2}{4(b+c)}\right).$$

The final result is therefore:

A.2.1 $$\int d^3k_{rs}\ k_{rs}^n\ \exp(-bk_{rs}^2) \int d^3k_{mn}\ k_{mn}^{\ell}\ \exp(i\underline{k}_{mn}\ \underline{v} - ck_{mn}^2)$$

$$\delta(k_{rs}^2 - k_{mn}^2)$$

$$= 4\pi^2 (b+c)^{-\frac{1}{2}(\ell+n+4)}\, \Gamma\left(\frac{\ell+n+4}{2}\right) \exp\left(-\frac{v^2}{4(b+c)}\right) \Phi\left(-\frac{\ell+n+1}{2};\ \frac{3}{2};\ \frac{v^2}{4(b+c)}\right)$$

When in addition to b = 1 we have n = 0: The result is:

A.1.3 $2\pi\Gamma(\frac{3}{2})\exp(-\frac{w^2}{4}) = \pi^{\frac{3}{2}}\exp(-\frac{w^2}{4})$ $\quad(\Gamma(\frac{3}{2}) = \frac{1}{2}\pi^{\frac{1}{2}})$

For n = 2, b = 1; the result is:

A.1.4 $2\pi\Gamma(\frac{5}{2})\exp(-\frac{w^2}{4})\phi(-1;\ \frac{3}{2};\ \frac{w^2}{4})$

$= \frac{3}{2}\pi^{\frac{3}{2}}\exp(-\frac{w^2}{4})\phi(-1;\ \frac{3}{2};\ \frac{w^2}{4})$ $\quad(\Gamma(\frac{5}{2}) = \frac{3}{4}\pi^{\frac{1}{2}})$

APPENDIX A

$\underline{k}$ - INTEGRATIONS IN SPHERICAL COORDINATES

A.1 $\underline{k}_{i\ell}$-integration

It is of the type:

$$\int d^3k_{i\ell}\, k_{i\ell}^n \exp(i\underline{k}_{i\ell}\underline{w} - bk_{i\ell}^2)$$

$$= 2\pi \int d(\cos\theta) \int dk_{i\ell}\, k_{i\ell}^{(n+2)} \exp(ik_{i\ell} w \cos\theta) \exp(-bk_{i\ell}^2)$$

$$= 2\pi \left(\frac{2i}{iw}\right) \int dk_{i\ell} \left(k_{i\ell}^{(n+1)} \exp(-bk_{i\ell}^2)\right) \sin(k_{i\ell} w)$$

$$= \frac{4\pi}{w} \int dk_{i\ell} \{k_{i\ell}^{(n+1)} \exp(-bk_{i\ell}^2)\} \sin(k_{i\ell} w).$$

The δ-integration is now reduced to the Fourier Sine Transform[60] (P.74 Eq. (2.4.24)) with $a = b$; $y = w$; and $2\nu - 2 = n + 1 \Rightarrow \nu = \frac{n+3}{2}$.

Hence $\int dk_{i\ell} \{k_{i\ell}^{(n+1)} \exp(-bk_{i\ell})\} \sin(k_{i\ell} w)$

$$= \tfrac{1}{2} b^{-\frac{1}{2}(n+3)} \Gamma\left(\frac{n+3}{2}\right) w \exp\left(-\frac{w^2}{4b}\right) \phi\left(\frac{3}{2} - \frac{n+3}{2};\ \frac{3}{2};\ \frac{w^2}{4b}\right).$$

A.1.1 Thus $\int d^3k_{i\ell}\, k_{i\ell}^n \exp(i\underline{k}_{i\ell}\underline{w} - bk_{i\ell}^2)$

$$= 2\pi b^{-\frac{1}{2}(n+3)} \Gamma\left(\frac{n+3}{2}\right) \exp\left(-\frac{w^2}{4b}\right) \phi\left(-\frac{n}{2};\ \frac{3}{2};\ \frac{w^2}{4b}\right).$$

when $b = 1$, (A.1.1) reduces to:

A.1.2 $$\int d^3k_{i\ell}\, k_{i\ell}^n \exp(i\underline{k}_{i\ell}\underline{w} - b^2k_{i\ell}^2) = 2\pi\Gamma\left(\frac{n+3}{2}\right) \exp\left(-\frac{w^2}{4}\right) \phi\left(-\frac{n}{2};\ \frac{3}{2},\ \frac{w^2}{4}\right).$$

In principle the absorption must vanish ($W \to 0$) in the limit $K \to 0$ with a proper Pauli operator (nuclear matter limit) for all values of R. This is not the case as one notices from the Figs.6.1 - 6.3. Even more, one does not even obtain $W \to 0$ in the $R=R' \to \infty$ case where the nuclei do not overlap. Even more and where the replacement $B_{\chi\gamma} = \delta_{\chi\gamma}$ is justified. This behaviour results from the use of plane waves as intermediate states in the evaluation of W which leads to unphysical excitations. In the overlap region one even obtains an unrealistic sign for the absorption.
On the other hand, one obtains for a given R, maximal absorption for R'=R, i.e. for maximal overlap in respect to the fluctuations in the mean separation coordinate R. This is supported by the results of Figs. (6.2) and (6.3) for $\theta = 30^{\circ}$. These numerical results have been checked in the limits $R'=R \to \infty$ and $R'=R=0$ for selected values of the parameters x_o, t_o and t_1.

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CHAPTER 6

RESULTS AND DISCUSSION

The scattering of composite particles has been considered with primary emphasis on two aspects viz.-

(a) how to describe it microscopically and

(b) how to describe the absorption.

Conventionally one assumes that the clusters are intrinsically in their ground states, since otherwise the treatment turns out to be too complicated. In such a description one only describes the elastic channel in isolation, although many channels are open in general. It was therefore proposed to use the Feshbach projection technique with the aim of constructing an effective (non-Hermitian) Hamiltonian for the elastic channel.

Some results of an actual calculation of

$$W = \sum_{\substack{\alpha,\beta,\gamma,\delta \\ \text{occup.}}} (\underline{\alpha},\underline{\beta}|\,\mathrm{Im}\,G|\underline{\gamma},\underline{\delta})\ (\delta_{\alpha\gamma}\,\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma})$$

(formula 5.4.4) for $B_{\alpha\gamma} = \delta_{\alpha\gamma}$ etc. and s-states (corresponding to α-α scattering) are presented in Figs. 6.1 - 6.3. In Fig.(6.1) W is plotted as a function of R the mean separation, for various values of R' and K=O, $\theta=0^0$. As anticipated in chapter 5, there arise inconsistencies because of our approximation for the Pauli operator, which is not a true Pauli operator because of the non-orthogonality of the basis functions as described by the overlap matrices $B_{\alpha\gamma}$ etc.

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$$= 3^{-\frac{1}{2}}\,\Gamma(\tfrac{2}{3})\,\exp(-\tfrac{w^2}{4})\,\exp(-\tfrac{u^2}{4})\;\;(-1;\tfrac{3}{2};\tfrac{u^2}{4})$$

$$\times\begin{cases}(\tfrac{1}{2}uv)\;\;\phi(\tfrac{2}{3};\tfrac{3}{2};\;-\tfrac{1}{8}(u-v)^2)-\phi(\tfrac{2}{3};\tfrac{3}{2};-\tfrac{1}{8}(u+v)^2)\} & \text{If } u\neq o \text{ and } v\neq o\\ (\tfrac{1}{3})\;\exp(-\tfrac{v^2}{8})\;\;(-\tfrac{1}{6};\tfrac{3}{2};\tfrac{v^2}{8}) & \text{If } u=o\\ (\tfrac{1}{3})\;\exp(-\tfrac{u^2}{8})\,\phi(-\tfrac{1}{6};\tfrac{3}{2};\tfrac{u^2}{8}) & \text{If } v=o\end{cases}$$

Adding (B.13.2) and (B.13.3) and multiplying the sum bv(B.10.4) and (B.3.3) we obtain:

$$\text{B.13.4}\quad PT(t_1-t_1) = \frac{C^2}{2\hbar\omega}\sum_{\alpha\ldots\beta'}\exp(-\frac{\eta'^2_{\alpha\beta}}{4}\exp(-\frac{\eta^2_{\alpha'\beta'}}{4})\exp\{-\tfrac{1}{2}(\xi'_{\alpha\;\beta}-\xi_{\alpha'\beta'})^2\}$$

$$\exp\;-\tfrac{1}{2}(\xi_{\alpha\;\beta}-\xi^*_{\gamma\delta})^2\;\}F(\alpha,\beta,\gamma,\delta;\;t_1-t_1)$$

$$\times\begin{cases}(\tfrac{1}{2uv})\{\phi(\tfrac{2}{3};\tfrac{1}{2};-\tfrac{1}{8}(u-v)^2)-\phi(\tfrac{2}{3};\tfrac{1}{2};-\tfrac{1}{8}(u+v)^2)\} & \text{If } u\neq o \text{ and } v\neq o\\ \tfrac{1}{3}\;\exp(-\tfrac{v^2}{8})\phi\;(-\tfrac{1}{6};\tfrac{3}{2};\tfrac{v^2}{8}) & \text{If } u\neq o\\ \tfrac{1}{3}\;\exp\;(-\tfrac{u^2}{8})\,\phi\,(-\tfrac{1}{6};\tfrac{2}{1};\tfrac{u^2}{8}) & \text{If } v\neq o\end{cases}$$

$\sum_{\alpha\ldots\beta'}$ means the summation over $\alpha,\beta,\gamma,\delta,\alpha',\beta'$,

where $F(\alpha,\beta,\gamma,\delta\;;\;t_1-t_1')$ is as defined previously.

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B.13 $PT(t_1 - t_1)$ - Term

In this term we have the following Fourier transforms:

B.13.1 $$\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\ \underset{\sim}{w}-k^2_{i\ell})\int d^3k_{rs}\ \exp(-i\underset{\sim}{k}_{rs}\ \underset{\sim}{u}-k^2_{rs})$$
$$\int d^3k_{r's'}\ \exp(i\underset{\sim}{k}_{r's'}\ \underset{\sim}{u}-k^2_{r's'})\int d^3k_{mn}\ \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v}-k^2_{mn})$$
$$\delta(k^2_{r's'}-k^2_{mn})\{(k^2_{i\ell}+k^2_{r's'})(k^2_{r's'}+k^2_{mn})\}$$

This is treated as a sum of two terms - viz.

B.13.2 $$\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\ \underset{\sim}{w}-k^2_{i\ell})\int d^3k_{rs}\ \exp(-i\underset{\sim}{k}_{rs}\ \underset{\sim}{u}-k^2_{rs})$$
$$\int d^3k_{r's'}\ \exp(i\underset{\sim}{k}_{r's'}\ \underset{\sim}{u}-k^2_{r's'})\int d^3k_{mn}\ \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v}-k^2_{mn})$$
$$\delta(k^2_{r's'}-k^2_{mn})\{k^2_{i\ell}(k^2_{r's'}+k^2_{mn})\}$$
$$= 3\pi^5\,\Gamma(\tfrac{2}{3})\exp(-\tfrac{w^2}{4})\exp(-\tfrac{u^3}{4})\phi(-1;\tfrac{3}{2};\tfrac{w^2}{4})$$
$$\begin{cases}(\frac{1}{2uv})\{\phi(\frac{2}{3};\frac{3}{2};-\frac{1}{8}(u-v)^2)-\phi(\frac{2}{3};\frac{3}{2};-\frac{1}{8}(u+v)^2)\} & \text{If } u\neq 0 \text{ and } v\neq 0\\ (\frac{1}{3})\exp(-\frac{v^2}{8})\phi(-\frac{1}{6};\frac{3}{2};\frac{v^2}{8}) & \text{If } u=0\\ (\frac{1}{3})\exp(-\frac{u^2}{8})\phi(-\frac{1}{6};\frac{3}{2};\frac{u^2}{8}) & \text{If } v=0\end{cases}$$

and

B.13.3 $$\int d^3k_{i\ell}\ \exp(i\ \underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2{}_{i\ell})\ \exp(-i\underset{\sim}{k}_{rs}\underset{\sim}{u}-k^2{}_{rs})\int d^3k_{mn}$$
$$\int d^3k_{r's'}\ \exp(i\underset{\sim}{k}_{r's'}\underset{\sim}{u}-k^2{}_{r's'})\ \exp\ (-i\underset{\sim}{k}_{mn}\underset{\sim}{v}-k^2{}_{mn})$$
$$\delta(k^2{}_{r's'}-k^2{}_{mn})\{k^2{}_{rs}(k^2{}_{r's'}+k^2{}_{mn})\}$$

B.12.4 $\int d^3k_{il}\ \exp(i\underset{\sim}{k}_{il}\ \underset{\sim}{w}-k^2_{il})\int d^3k_{rs}\exp(-i\underset{\sim}{k}_{rs}\underset{\sim}{u}\ -\ k^2_{rs})$
$\int d^3k_{r's'}\exp(i\underset{\sim}{k}_{r's'}\ \underset{\sim}{u}-k^2_{r's'})\int d^3k_{mn}\exp(-\ \underset{\sim}{k}_{mn}\underset{\sim}{v}-k^2_{mn})\cdot$

$k^2_{il}(k^2_{r's'}+k^2_{mn})^{-1}(k^2_{r's'}-k^2_{mn})$

$$= 2\Gamma(\tfrac{2}{3})\,3^{-3}\exp(-\tfrac{w^2}{4})\exp(-\tfrac{u^2}{4})\begin{cases}(\frac{1}{2uv})\{\phi(\frac{2}{3};\ \frac{1}{2};\ -\frac{1}{8}(u-v)^2)-\phi(\frac{2}{3};\ \frac{1}{2};\ -\frac{1}{8}(u+v)^2)\}\\ (\frac{1}{3})\exp(-\frac{v^2}{8})\ \phi(-\ \frac{1}{6};\ \frac{3}{2};\ \frac{v}{8}) \quad \text{If}\quad u=0\\ (\frac{1}{3})\exp(-\frac{u^2}{8})\ \phi(-\ \frac{1}{6};\ \frac{3}{2};\ \frac{u^2}{8}) \quad \text{If}\quad v=0\end{cases}$$

Adding (B.12.2), (B.12.3) and (B.12.4) and multiply the sum by (B.10.4) and (B.3.3).

B.12.5 $PT(t_o,\ t_1)$

$$= \frac{c^2_{oo}}{(2\hbar\omega)}\exp(-\tfrac{w^2}{4})\exp(-\tfrac{u^2}{4})\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta}-\underset{\sim}{\xi}_{\alpha'\beta'})^2\}F(\alpha,\beta,\gamma,\delta;t_o-t_1)\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}_{\alpha'\beta'}-\underset{\sim}{\xi}^*_{\gamma\delta})^2\}$$

$$\cdot\Big[\ \tfrac{3}{4}\ \{\phi(-1;\tfrac{3}{2};\tfrac{w^2}{4})+\phi(-1;\tfrac{3}{2};\tfrac{u^2}{4})\}\begin{cases}\frac{1}{uv}\ \{\phi(1;\ \tfrac{1}{2};\ -\tfrac{1}{8}(u-v)^2)-\phi(1;\ \tfrac{1}{2};\ -\tfrac{1}{8}(u+v)^2)\}\\ \exp(-\frac{v^2}{8})\phi(-\ \frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{8}) \quad \text{If}\quad u=0\\ \exp(-\frac{u^2}{8})\phi(-\ \frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{8}) \quad \text{If}\quad v=0\end{cases}$$

$$+\ \Gamma(\tfrac{2}{3})\begin{cases}(\frac{1}{2uv})\{\phi(\frac{2}{3};\ \frac{1}{2};\ -\frac{1}{8}(u-v)^2)-\phi(\frac{2}{3};\ \frac{1}{2};\ -\frac{1}{8}(u+v)^2) & \text{If } u\neq 0;\ v\neq 0\\ \frac{1}{3}\exp(-\frac{v^2}{8})\phi(-\ \frac{1}{6};\ \frac{3}{2};\ \frac{v^2}{8}) & \text{If } u=0\\ \frac{1}{3}\exp(-\frac{u^2}{8})\phi(-\ \frac{1}{6};\ \frac{3}{2};\ \frac{u^2}{8}) & \text{If } v=0\end{cases}\Big]$$

where

B.12.6 $F(\alpha,\beta,\gamma,\delta;\ t_1-t_1) = 2t_ot_1(2+x_o)\delta_{\alpha\gamma}\ \delta_{\beta\delta}-t_ot_1(\cdot+x_o)\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

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B.12 $PT(t_0 - t_1/ -$ Term

In this case we have the integrations over $\underset{\sim}{k}_{i\ell}, \underset{\sim}{k}_{rs}$ etc. as follows:

B.12.1 $\int d^3k_{i\ell} \exp(i\underset{\sim}{k}_{i\ell}\ \underset{\sim}{w} - k^2_{i\ell}) \int d^3k_{rs} \exp(-i\underset{\sim}{k}_{rs}\ \underset{\sim}{u} - k^2_{rs})$
$\int d^3k_{r's'} \exp(i\underset{\sim}{k}_{r's'}\ \underset{\sim}{u} - k^2_{r's'}) \int d^3k_{mn} \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - k^2_{mn})$
$\delta(k^2_{r's'} - k^2_{mn})(k^2_{i\ell} + k^2_{rs} + k^2_{r's'} + k^2_{mn})$

Treating above as a sum of three terms. viz.

B.12.2 $\int d^3k_{i\ell}\ k^2_{i\ell} \exp(i\underset{\sim}{k}_{i\ell}\ \underset{\sim}{w} - k^2_{i\ell}) \int d^3k_{rs} \exp(-i\underset{\sim}{k}_{rs}\ \underset{\sim}{u} - k^2_{rs})$
$\int d^3k_{r's'} \exp(i\underset{\sim}{k}_{r's'}\ \underset{\sim}{u} - k^2_{r's'}) \int d^3k_{mn} \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - k^2_{mn})$
$\delta(k^2_{r's'} - k^2_{mn})$

$$= \frac{3}{4}\pi^5 \exp(-\tfrac{w^2}{4}) \exp(-\tfrac{u^2}{4}) \phi(-1;\ \tfrac{3}{2};\ \tfrac{w^2}{4})$$

$$\times \begin{cases} \frac{1}{uv}\{\phi(1;\ \frac{1}{2};\ -\frac{1}{4}(u-v)^2) - \phi(1;\ \frac{1}{2};\ -\frac{1}{4}(u+v)^2)\} & \text{If } u \neq 0 \text{ and } v \neq 0 \\ \exp(-\frac{v^2}{8})\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{8}) & \text{If } u = 0 \\ \exp(-\frac{u^2}{8})\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{u^2}{8}) & \text{If } v = 0 \end{cases}$$

B.12.3 $\int d^3k_{i\ell} \exp(i\ \underset{\sim}{w}\ \underset{\sim}{k}_{i\ell}) \int d^3k_{rs}\ k^2_{rs} \exp(-i\underset{\sim}{k}_{rs}\ \underset{\sim}{u} - k^2_{rs}) \exp(-k^2_{i\ell})$
$\int d^3k_{r's'} \exp(i\underset{\sim}{k}_{r's'}\ \underset{\sim}{u} - k^2_{r's'}) \int d^3k_{mn} \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - k^2_{mn}) \delta(k^2_{r's'} - k^2_{mn})$

$$= \frac{3}{2}\pi^5 \exp(-\tfrac{w^2}{4}) \exp(-\tfrac{u^2}{4}) \phi(-1;\tfrac{3}{2};\tfrac{u^2}{4}) \begin{cases} \frac{1}{uv}\{\phi(1;\frac{1}{2};-\frac{1}{4}(u-v)^2) - \phi(1;\frac{1}{2};-\frac{1}{4}(u+v)^2)\} & \text{If } u \neq 0 \text{ and } v \neq 0 \\ \exp(-\frac{v^2}{8})\ \phi(-\frac{1}{2};\frac{3}{2};\frac{v^2}{8}) & \text{If } u = 0 \\ \exp(-\frac{u^2}{8})\ \phi(-\frac{1}{2};\frac{3}{2};\frac{u^2}{8}) & \text{If } v = 0 \end{cases}$$

In the case of $PT(t_0-t_0)$ we consider the following:

B.11.1 $\int d^3k_{i\ell} \exp(i\, \underset{\sim}{k}_{i\ell}\, \underset{\sim}{w} - k^2_{i\ell}) = \pi^{3/2} \exp(-\frac{w^2}{4})$; $w = \eta'_{\alpha\beta}$

B.11.2 $\int d^3k_{rs} \exp(-i\, \underset{\sim}{k}_{rs}\, \underset{\sim}{u} - k^2_{rs}) = \pi^{3/2} \exp(-\frac{u^2}{4})$; $u = \eta_{\alpha'\beta'}$

B.11.3 $\int d^3k_{r's'} \exp(i\underset{\sim}{k}_{r's'}\, \underset{\sim}{u} - k^2_{r's'}) \int d^3k_{mn} \exp(-i\underset{\sim}{k}_{mn}\, \underset{\sim}{v} - k^2_{mn})\, \delta(k^2_{r's'} - k^2_{mn})$

$$= \pi^2 \begin{cases} \frac{1}{uv}\{\phi(1;\ \tfrac{1}{2};\ -\tfrac{1}{8}(u-v)^2) - \phi(1;\ \tfrac{1}{2};\ -\tfrac{1}{8}(u+v)^2)\} & \text{if } u = \eta_{\alpha'\beta'} \neq 0 \text{ and } v = \eta^*_{\gamma\delta} \neq \\ \exp(-\frac{v^2}{8})\, \phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{8}) & \text{if } u = 0 \\ \exp(-\frac{u^2}{8})\, \phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{u^2}{8}) & \text{if } v = 0 \end{cases}$$

The full $PT(t_0 - t_0)$ - term is obtained by multiplying the equations (B.10.4), (B.11.1) through (B.11.3) and (B.3.1).

B.11.4 $$PT(t_0 - t_0) = \frac{c^2_{00}}{(2\hbar\omega)} \exp(-\frac{w^2}{4}) \exp(-\frac{u^2}{4}) \exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}_{\alpha'\beta'})^2\}$$

$$\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}_{\alpha'\beta'} - \underset{\sim}{\xi}^*_{\gamma\delta})^2\} F(\alpha,\beta,\gamma,\delta;\ t_0 - t_0)$$

$$\begin{cases} \frac{1}{uv}\{\phi(1;\ \tfrac{1}{2};\ -\tfrac{1}{8}(u-v)^2) - \phi(1;\tfrac{1}{2};-\tfrac{1}{8}(u+v)^2)\} \\ \qquad \text{if } u = \eta_{\alpha'\beta'} \neq 0 \text{ and } v = \eta^*_{\gamma\delta} \neq 0 \\ \exp(-\frac{v^2}{8})\, \phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{8}) & \text{if } u = 0 \\ \exp(-\frac{u^2}{8})\, \phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{u^2}{8}) & \text{if } v = 0 \end{cases}$$

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B.10 Skyrme for Pauli Term (PT)

Like the (NPT) one treats the (PT) in three stages, viz. $(t_0 - t_c)$, $(t_0 - t_1)$ and $(t_1 - t_1)$ - terms. In all the three cases the results of the integrations, over $\underset{\sim}{K}_{il}$, $\underset{\sim}{K}_{rs}$, $\underset{\sim}{K}_{r's'}$ and $\underset{\sim}{K}_{mn}$ are the same. The results of such integrations, with the help of $\delta(\underset{\sim}{K}_{r's'}-\underset{\sim}{K}_{mn})$ and $(\underset{\sim}{K}_{il}-\underset{\sim}{K}_{rs})$ are as follows:

B.10.1 $$\int d^3 K_{il} \exp\{i \underset{\sim}{\xi}_{il}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}_{\alpha'\beta'}) - \tfrac{1}{2}K_{il}^2\}$$
$$= (2\pi)^{3/2} \exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}_{\alpha'\beta'})^2\}$$

B.10.2 $$\int d^3 K_{mn} \exp\{-i \underset{\sim}{K}_{mn}(\underset{\sim}{\xi}_{\alpha'\beta'} - \underset{\sim}{\xi}^*_{\gamma\delta}) - \tfrac{1}{2} K_{mn}^2\}$$
$$= (2\pi)^{3/2} \exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}_{\alpha'\beta'} - \underset{\sim}{\xi}^*_{\gamma\delta})^2\}$$

Also from the factors π, $(2\pi)^{-24}$, $(2\pi)^{+18}$ $(c_{00}^2)^2$, $\hbar\omega$ we obtain

B.10.3 $$\frac{\pi(c_{00}^2)^2}{\hbar\omega(2\pi)^6} = \frac{c_{00}^2}{\pi^2\hbar\omega(2\pi)^6}$$

Multiplying (B.10.3) by (B.10.1) and (B.10.2), we obtian the common factor in the (PT)-Term calculation.

B.10.4 $$\frac{c_{00}^2}{(2\hbar\omega)} \times \frac{2}{\pi^2(2\pi)^3} \exp(-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}_{\alpha'\beta'})^2) \exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}_{\alpha'\beta'} - \underset{\sim}{\xi}^*_{\gamma\delta})^2\}$$

B.11 PT$(t_0 - t_0)$ - Term

In addition to the terms above we have integration over $\underset{\sim}{k}_{il}$, $\underset{\sim}{k}_{rs}$, $\underset{\sim}{k}_{r's'}$ and $\underset{\sim}{k}_{mn}$.

Because of $\delta(k_{rs}^2 - k_{mn}^2)$, (B.9.1) is equivalent to the sum of

B.9.2 $\int d\,k_{i\ell} \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w} - k_{i\ell}^2) \int d^3k_{rs} \int d^3k_{mn} \exp(-i\underset{\sim}{k}_{mn}\underset{\sim}{v} - k_{mn}^2)$

$\delta(k_{rs}^2 - k_{mn}^2) k_{i\ell}^2 (2k_{rs}^2)$

$= \frac{3}{2}\pi^{3/2} \exp(-\frac{w^2}{4})\,\phi(-1;\frac{3}{2};\frac{w^2}{4}) \times \{2 \times 8\pi^2 \exp(-\frac{v^2}{4})\,\phi(-\frac{3}{2};\frac{3}{2};\frac{v^2}{4})\}$

$= 3(2\pi)^3(\pi^{\frac{1}{2}})\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\,\phi(-1;\frac{3}{2};\frac{w^2}{4})\,\phi(-\frac{3}{2};\frac{3}{2};\frac{v^2}{4})$

and

B.9.3 $2\int d^3k_{i\ell} \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w} - k_{i\ell}^2) \int d^3k_{rs} \int d^3k_{mn} k_{mn}^4 \exp(-i\underset{\sim}{k}_{mn}\underset{\sim}{v} - k_{mn}^2)$

$\delta(k_{rs}^2 - k_{mn}^2)$

$= 2 \times \pi^{3/2} \exp(-\frac{w^2}{4}) \times 4\pi^2 \Gamma(\,)\exp(-\frac{v^2}{4})\,\phi(-\frac{5}{2};\frac{3}{2};\frac{v^2}{4})$

$= 6(2\pi)^3(\pi^{\frac{1}{2}})\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\,\phi(-\frac{5}{2};\frac{3}{2};\frac{v^2}{4})$

Adding (B.9.2) and (B.9.3)

B.9.4 $3(2\pi)^3(\pi^{\frac{1}{2}})\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\{\phi(-1;\frac{3}{2};\frac{w^2}{4})\,\phi(-\frac{3}{2};\frac{3}{2};\frac{v^2}{4}) + 2\phi(-\frac{5}{2};\frac{3}{2};\frac{v^2}{4})\}$

From (B.3.3); (B.6.2) and (B.9.4), we have that:

B.9.5 $NPT(t_1 - t_1) = \frac{\sqrt{2}\,c_{00}^2}{(2h\omega)} \times 3\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\exp\{-\tfrac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta} - \underset{\sim}{\xi}^*_{\gamma\delta})^2\}$

$\{\phi(-1;\frac{3}{2};\frac{w^2}{4})\,\phi(-\frac{3}{2};\frac{3}{2};\frac{v^2}{4}) + 2\phi(-\frac{5}{2};\frac{3}{2};\frac{v^2}{4})\}F(\alpha,\beta,\gamma,\delta;t_1 - t_1)$

where

B.9.10 $F(\alpha,\beta,\gamma,\delta;t_1 - t_1) = 2t_1^2\,\delta_{\alpha\gamma}\,\delta_{\beta\delta} - \tfrac{1}{4}\,t_1^2\,\delta_{\alpha\delta}\,\delta_{\beta\gamma}$

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B.8.3 $2\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k_{i\ell}^2)\int d^3k_{rs}k_{rs}^2\ d^3k_{mn}\exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v}\ -\ k_{mn}^2)$

$\delta(k_{rs}^2-k_{mn}^2) = 2\times\pi^{3/2}\exp(-\frac{w^2}{4})\times 8\pi^2\exp(-\frac{v^2}{4})\phi(-\frac{3}{2};\frac{3}{2};\frac{v^2}{4})$

$= 16(\pi^{3/2})(\pi^2)\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\phi(-\frac{3}{2};\frac{3}{2};\frac{v^2}{4})$.

B.8.4 $\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k_{i\ell}^2)\int d^3k_{rs}\ \int d^3k_{mn}\ \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v}\ -\ k_{mn}^2)$

$\delta(k_{rs}^2-k_{mn}^2)$

$= (8\pi^2)\pi^{-3/2}\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\phi(-\frac{3}{2};\frac{3}{2};\frac{v^2}{4})$.

Adding (B.8.2); (B.8.3) and (B.8.4) we get:

B.8.5 $6\pi^3(\pi^{1/2})\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\{\phi(-1;\frac{3}{2};\frac{w^2}{4})\phi(-\frac{1}{2};\frac{3}{2};\frac{v^2}{4})$

$+ 4\phi(-\frac{3}{2};\frac{3}{2};\frac{v^2}{4})\}$.

Substituting (B.8.5) into (B.8.1) and multiplying by (B.6.3), we obtain:

B.8.6 $NPT(t_o-t_1) = \frac{\sqrt{2}c_{oo}^2}{(h\omega 2)}\ 3\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\exp[-\frac{1}{2}(\underset{\sim}{\xi}'_{\alpha\beta}-\underset{\sim}{\xi}^*_{\gamma\delta})^2)$

$(\frac{1}{4}\phi(-1;\frac{3}{2};\frac{w^2}{4})\phi(-\frac{1}{2};\frac{3}{2};\frac{v^2}{4})+\phi(-\frac{3}{2};\frac{3}{2};\frac{v^2}{4})\}F(\alpha,\beta,\gamma,\delta;\ t_o-t_1)$

where

B.8.7 $F(\alpha,\beta,\gamma,\delta;\ t_o-t_1) = 2t_ot_1(2+x_o)\delta_{\alpha\gamma}\delta_{\beta\delta} - t_ot_1(1+x_o)\delta_{\alpha\delta}\delta_{\beta\gamma}$

B.9 The $NPT(t_1-t_1)$ - Term

The integrations in this part are of the type:

B.9.1 $\int d^3k_{i\ell}\exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k_{i\ell}^2)\int d^3k_{rs}\int d^3k_{mn}\exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v}\ -\ k_{mn}^2)$

$\delta(k_{rs}^2-k_{mn}^2)((k_{i\ell}^2+k_{rs}^2)(\epsilon_{rs}^2+k_{mn}^2))$,

B.7.2 $\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2_{i\ell}) = \pi^{3/2}\ \exp(-\frac{w^2}{4});\ w = \eta'_{\alpha\delta}$

B.7.3 $\int d^3k_{rs} \int dk_{mn}\ \exp\{-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - k^2_{mn}\}\delta(k^2_{rs}-k^2_{mn})$

$= 4\pi^2\ \exp(-\frac{v^2}{4})\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{4})$

and $v = \eta^*_{\gamma\delta}$; $\phi(a,b,z)$ is the function $\phi(a,b,z) = 1+\frac{a}{b}\frac{z}{1}+\frac{a(a+1)}{b(b+1)}\frac{z^2}{2!}+\ldots$
The complete NPT(t_o-t_o) - term is obtained by using (B.7.1), (B.7.2) , (B.7.3) and (B·6.5), giving:

B.7.4. $NPT(t_o-t_o) = \frac{\sqrt{2}c^2_{oo}}{(2h\omega)}\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})$

$\phi(-\frac{1}{2};\ \frac{3}{4};\ \frac{v^2}{4})F(\alpha,\beta,\gamma,\delta;\ t_o-t_o)\times\exp\{-\frac{1}{2}(\xi'_{\alpha\beta}-\xi^*_{\gamma\delta})^2\}$

where

B.7.4 $F(\alpha,\beta,\gamma,\delta;\ t_o-t_o) = 4t^2_o(1+x_c+x^2_o)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - t^2_o(1+x_o)^2\delta_{\alpha\delta}\ \delta_{\beta\gamma}$

B.8 The NPT(t_o-t_1) - Term

This part has the form:

B.8.1 $NPT(t_o-t_1) = 2\{2t_ot_1(2+x_o)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - t_ot_1(1+x_o)\delta_{\alpha\delta}\ \delta_{\beta\gamma}\}$

$\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2_{i\ell})\int d^3k_{rs}\int d^3k_{mn}\ \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - \underset{\sim}{k}^2_{mn})$

$\delta(k^2_{rs}-k^2_{mn})(k^2_{i\ell}+2k^2_{rs}+k^2_{mn})$

NPT(t_o-t_1) because of $(k^2_{i\ell}+2k^2_{rs}+k_{mn})$ can be treated as the sum of the following:

B.8.2 $\int d^3k_{i\ell}\ k^2_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2_{i\ell})\int d^3k_{rs}\int d^3k_{mn}\ \exp(-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - k^2_{mn})$

$\delta(k^2_{rs}-k^2_{mn}) = \frac{3}{2}\pi^{3/2}\exp(-\frac{w^2}{4})\phi(-1;\frac{3}{2};\frac{w^2}{4})\times 4\pi^3\exp(-\frac{v^2}{4})\phi(-\frac{1}{2};\frac{3}{2};\frac{v^2}{4})$

$= 6\pi^3(\pi^{1/2})\exp(-\frac{w^2}{4})\exp(-\frac{v^2}{4})\phi(-1;\ \frac{3}{2};\ \frac{w^2}{4})\phi(-\frac{1}{2};\ \frac{3}{2};\ \frac{v^2}{4})$

B.6.2 $\underset{\sim}{W} = \underset{\sim}{D}'_{\alpha\beta} = \underset{\sim}{R}'_{\alpha} - \underset{\sim}{R}'_{\delta} - i(\underset{\sim}{K}_{\alpha}-\underset{\sim}{K}_{\beta})$; $\underset{\sim}{V} = \underset{\sim}{D}^{*}_{\gamma\delta} = \underset{\sim}{R}_{\gamma}-\underset{\sim}{R}_{\delta}+i(\underset{\sim}{K}_{\gamma}-\underset{\sim}{K}_{\delta})$.

The $\underset{\sim}{K}_{i\ell}$-integration gives (See A.1)

B.6.3 $\int d^3K_{i\ell}\ \exp\{i\underset{\sim}{K}_{i\ell}(\underset{\sim}{\xi}'_{\alpha\beta}-\underset{\sim}{\xi}^{*}_{\gamma\beta})- \tfrac{1}{4}K^2_{i\ell}\}=(2\pi)^{3/2}\exp\{-\tfrac{1}{4}(\underset{\sim}{\xi}'_{\alpha\beta}-\underset{\sim}{\xi}_{\gamma\delta})^2\}$

where

B.6.4 $\underset{\sim}{\xi}'_{\alpha\beta} = \tfrac{1}{2}(\underset{\sim}{R}'_{\alpha}+\underset{\sim}{R}'_{\beta}) - \frac{1}{2}(\underset{\sim}{K}_{\alpha}+\underset{\sim}{K}_{\beta})$; $\underset{\sim}{\xi}^{*}_{\gamma\delta} = \frac{1}{2}(\underset{\sim}{R}_{\gamma}-\underset{\sim}{R}_{\delta}) + \frac{1}{2}(\underset{\sim}{K}_{\gamma}+\underset{\sim}{K}_{\delta})$

(B.6.3) can also be factored out as a common factor in the NPT calculation and together with $\dfrac{c_{oo}^2\pi^{.}}{\hbar\omega(2\pi)^6}$ it gives:

B.6.5 $\dfrac{(2\pi)^{3/2}\pi c_{oo}^2}{\hbar\omega(2\pi)^6}\exp\{-\tfrac{1}{4}(\underset{\sim}{\xi}'_{\alpha\beta}-\underset{\sim}{\xi}^{*}_{\gamma\delta})^2\}$ and $c_{oo}^2 = \pi^{-3}$,

since $c_o = \pi^{-3/4}$ and $c_{oo} = c_o c_o = (\pi)^{-3/2}$.

We now have integrations over $\underset{\sim}{k}_{i\ell}$, $\underset{\sim}{k}_{rs}$ and $\underset{\sim}{k}_{mn}$ to carry out. We therefore treat the three (NPT)-terms separately.

B.7. NP(t_o-t_o) - Term

It is of the form:

B.7.1 $4\{4t_o^2(1+x_o+x_o^2)\delta_{\alpha\gamma}\ \delta_{\beta\delta} - t_o^2(1+x_o)^2\delta_{\alpha\delta}\ \delta_{\beta\gamma}\}$

$\int d^3k_{i\ell}\ \exp(i\underset{\sim}{k}_{i\ell}\underset{\sim}{w}-k^2_{i\ell})\int d^3k_{rs}\int dk_{mn}\ \exp\{-i\underset{\sim}{k}_{mn}\ \underset{\sim}{v} - k^2_{mn}\}$

$\delta(k^2_{rs}-k^2_{mn})$.

From (A.1), we notice that the individual integrations give the following results:

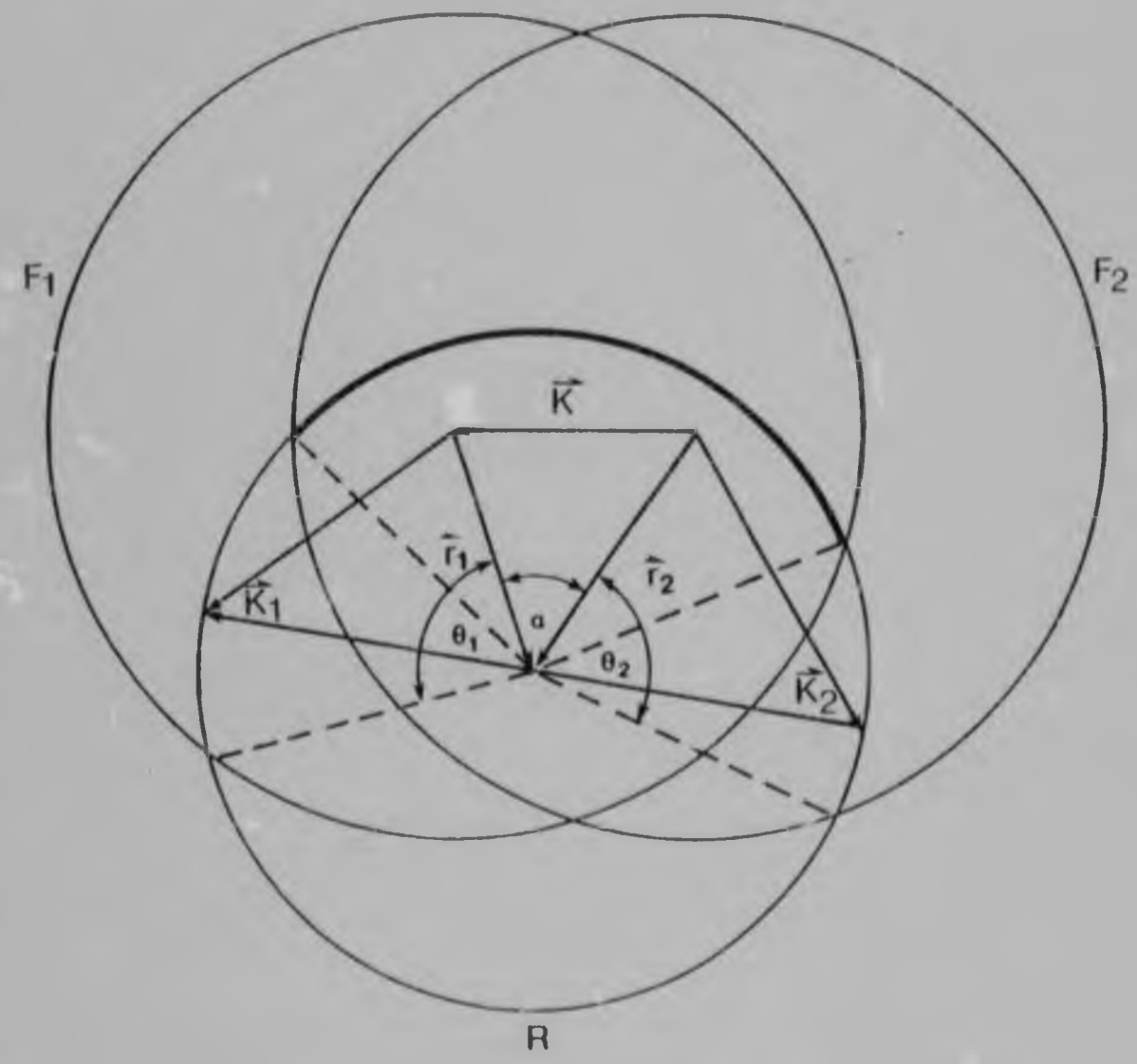


Fig. (3.1)

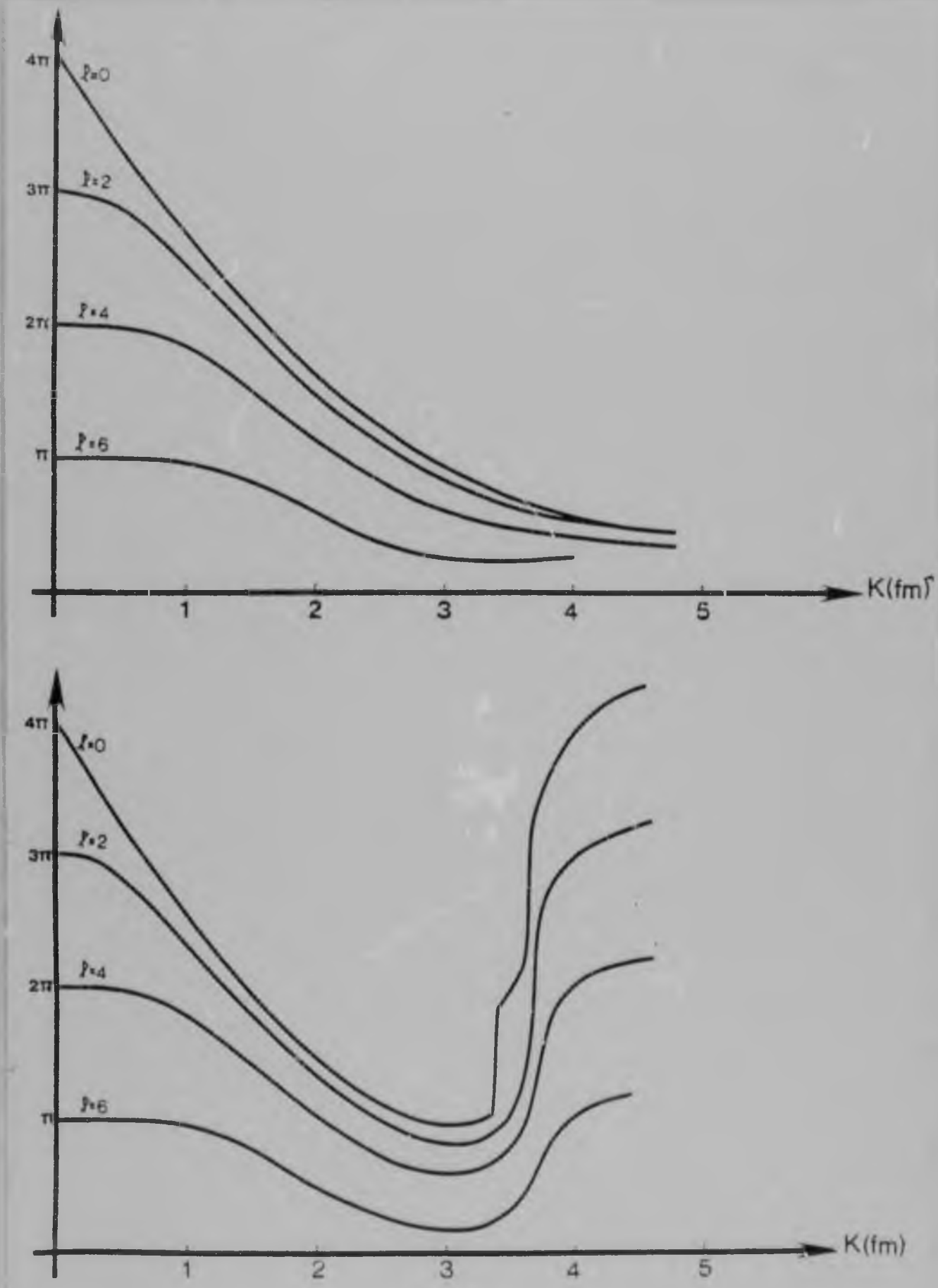
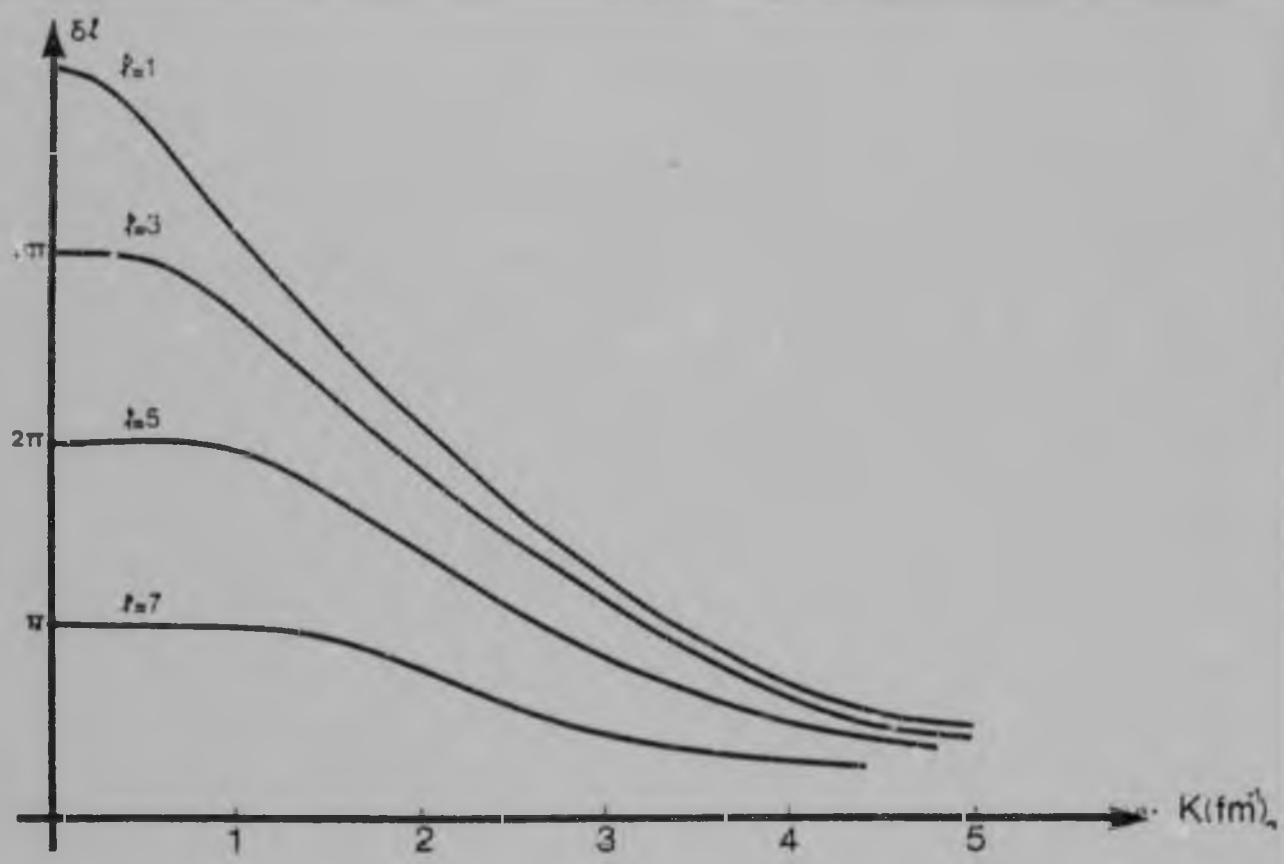


Fig. (2.6)



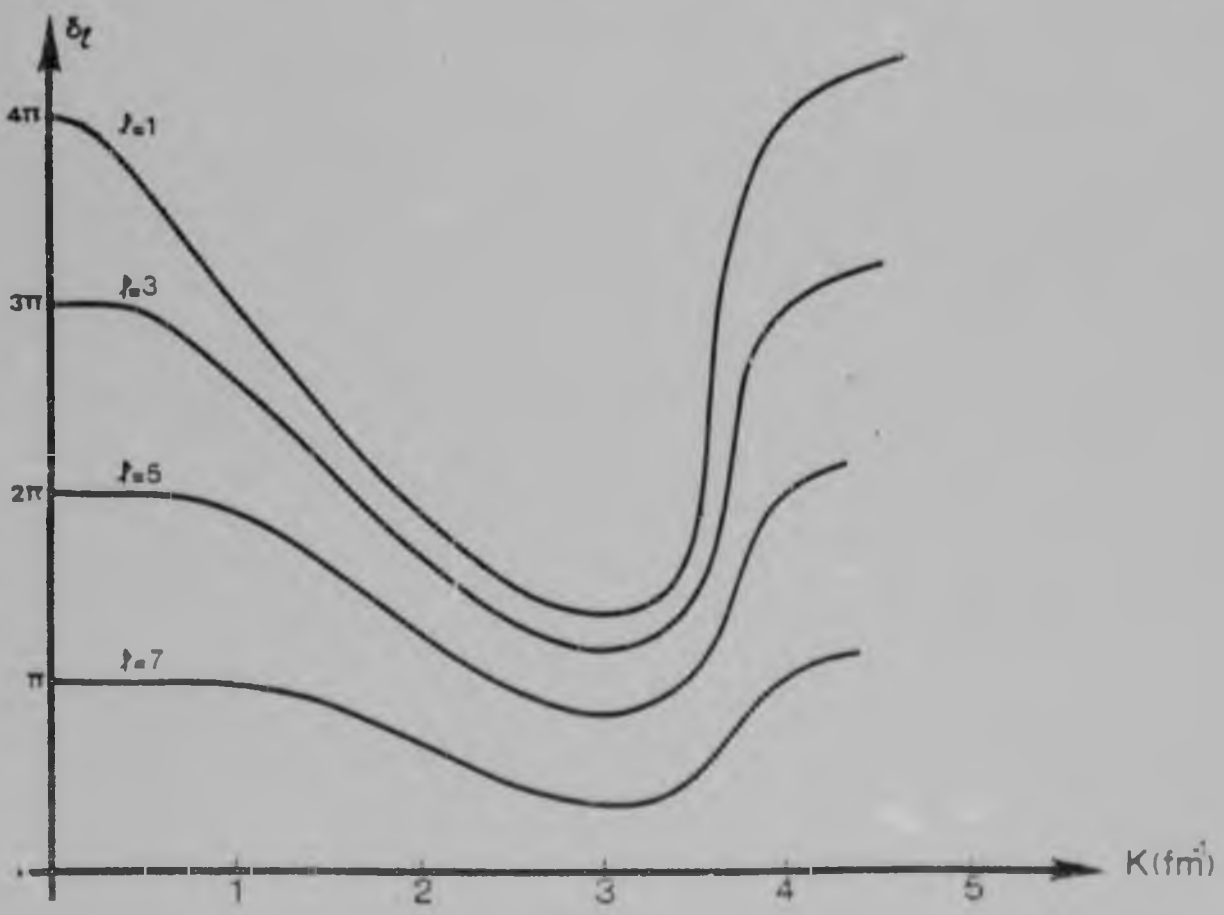


Fig. (2.5)

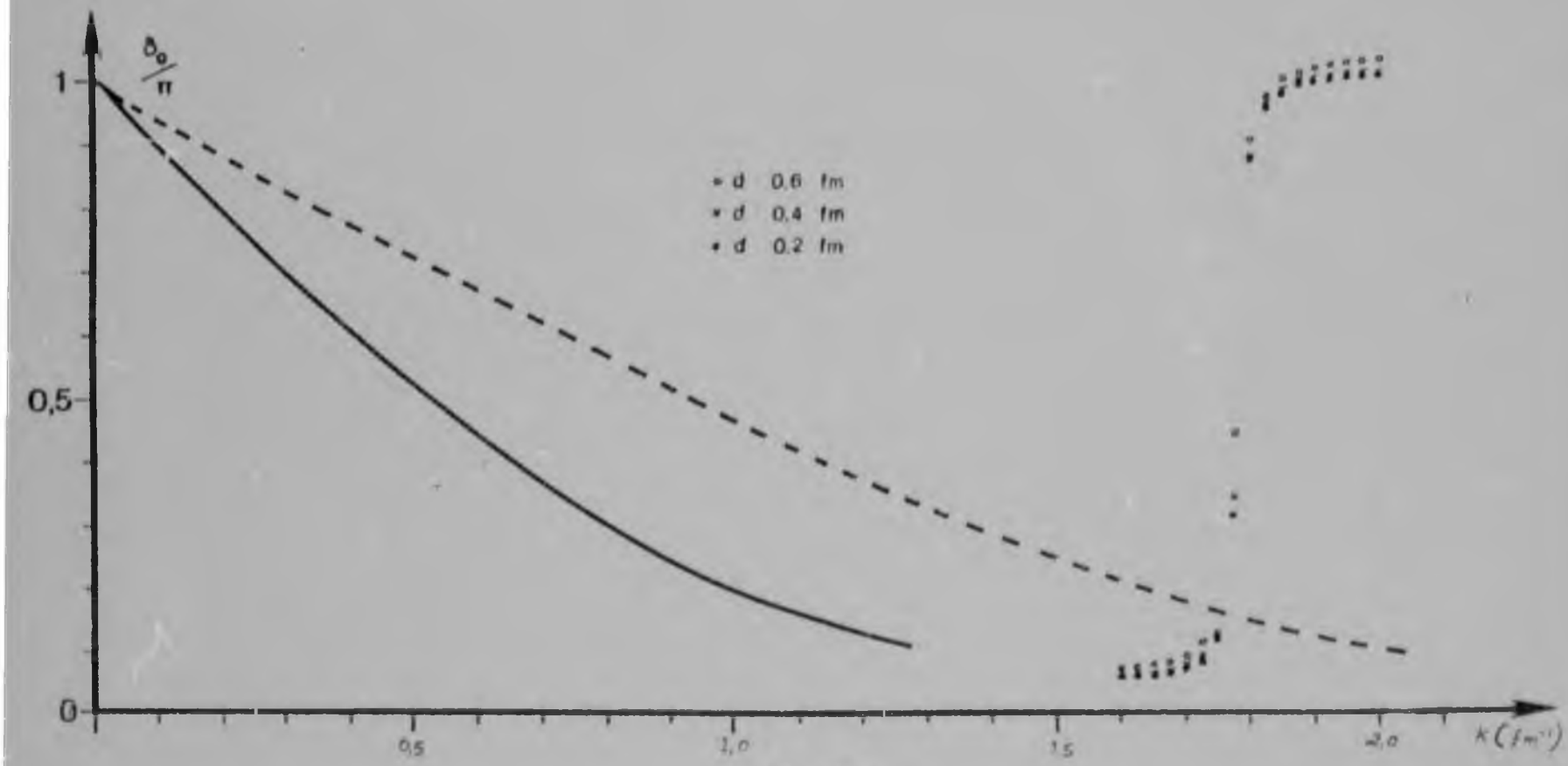
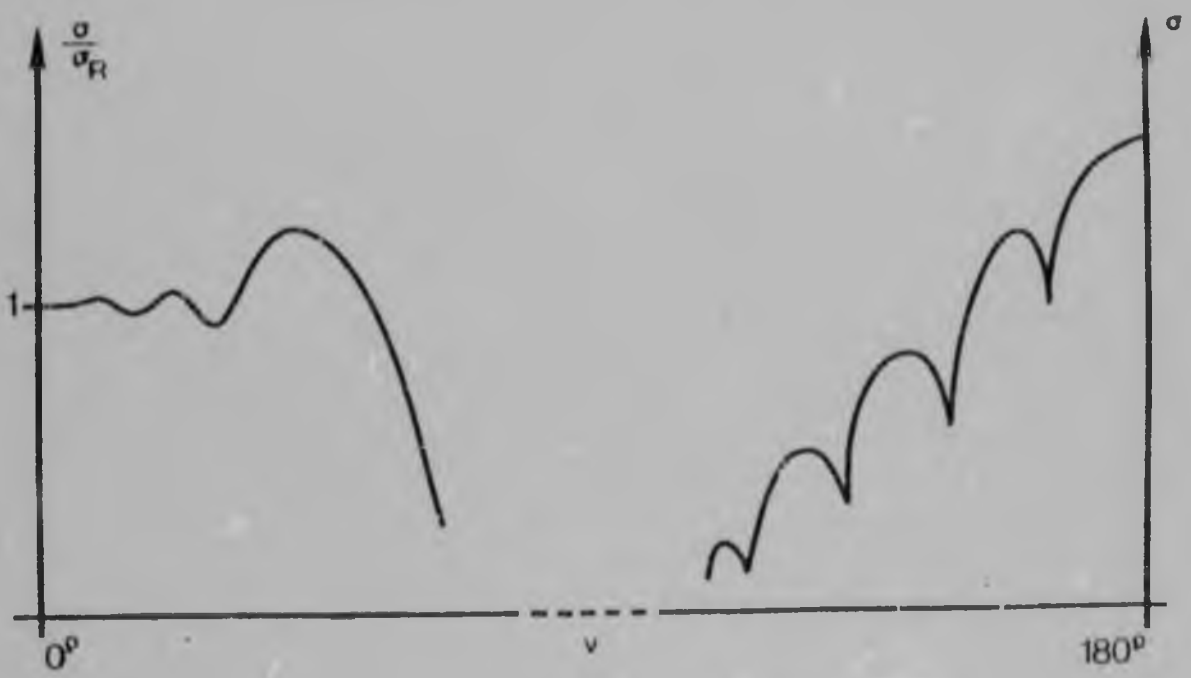


Fig. (2.4)



Flg. (2.3)

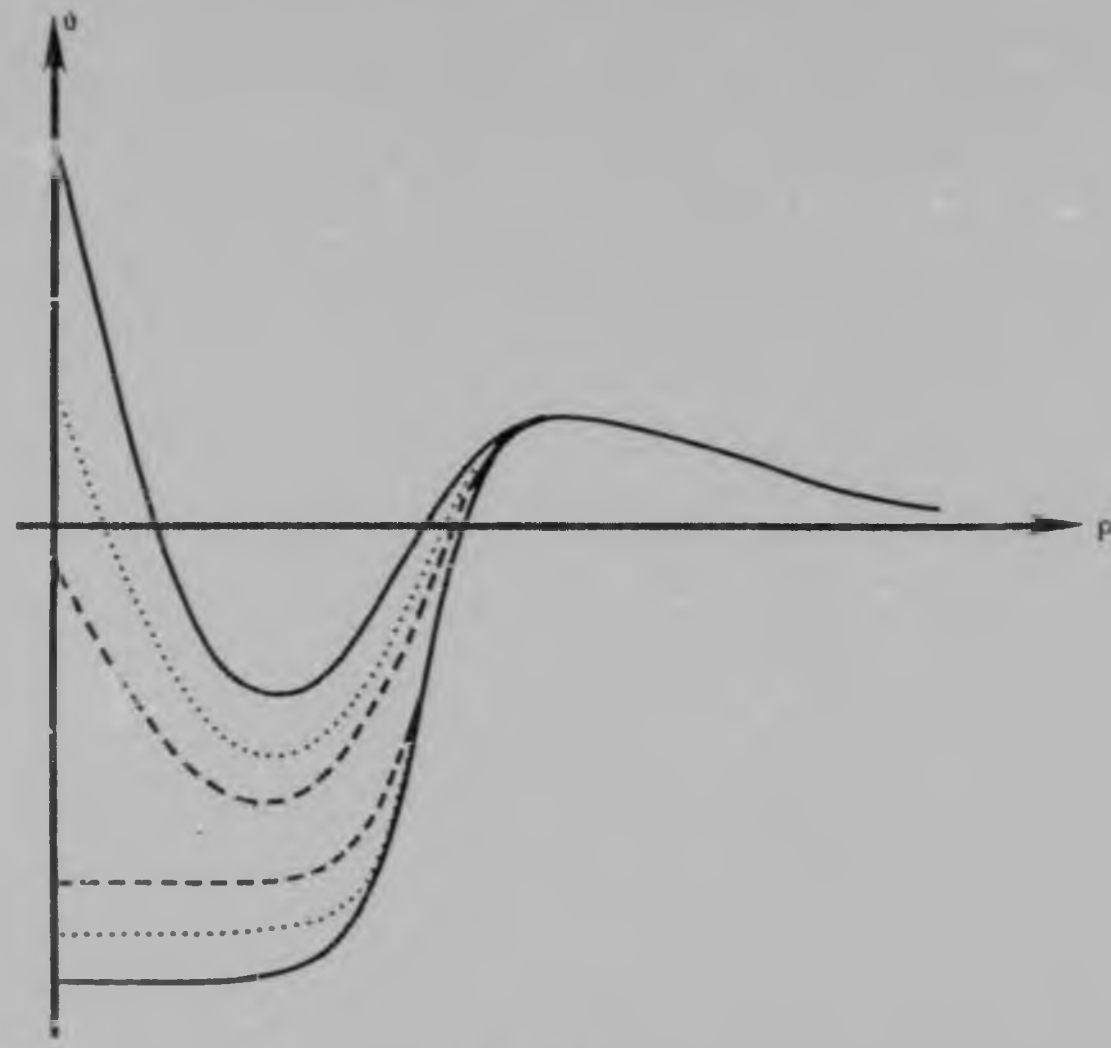


Fig. (2.2)

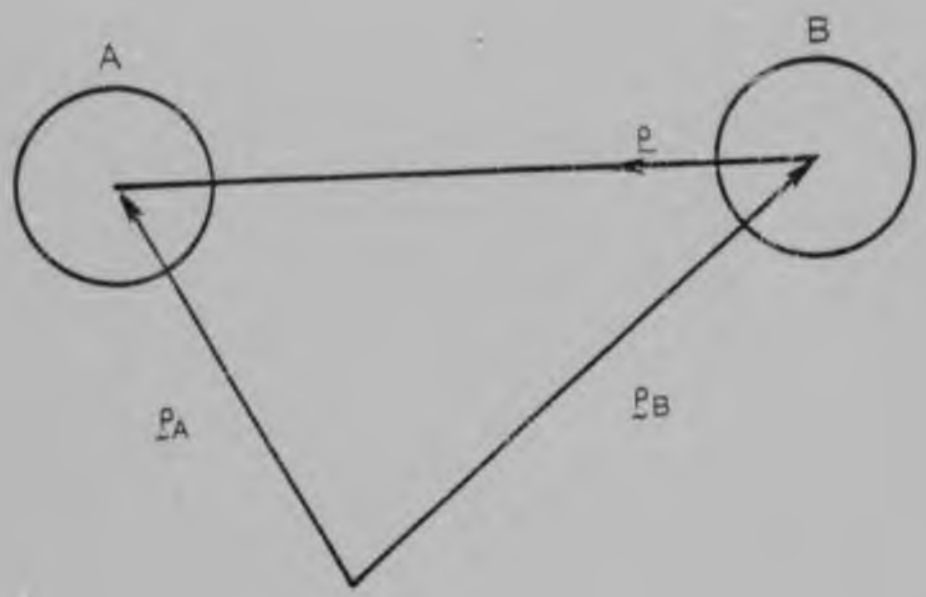
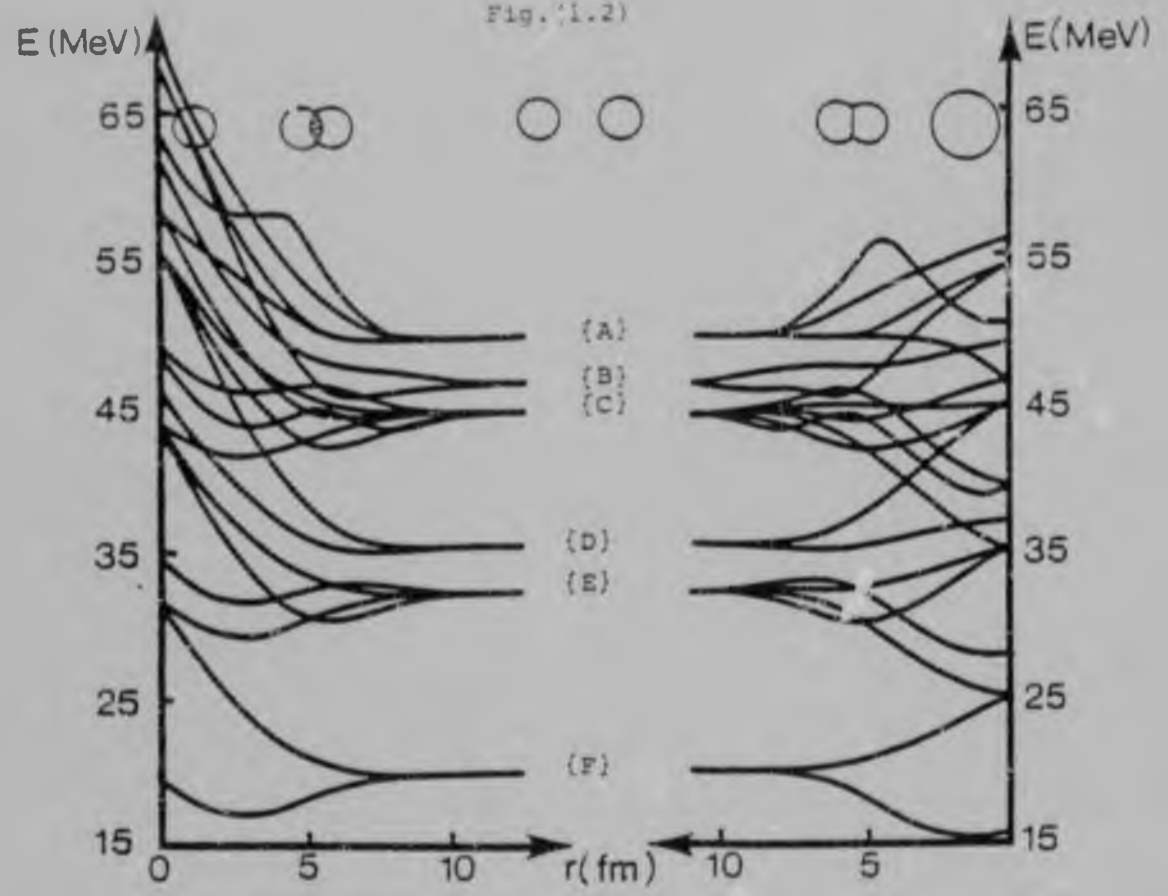


Fig. (2.1)

$$\underline{R}_A = A^{-1} \sum_{i \in A} \underline{r}_i \quad , \quad \underline{R}_B = B^{-1} \sum_{j \in B} \underline{r}_j \quad , \quad \underline{\rho} = \underline{R}_A - \underline{R}_B$$

Fig.(1.2)



$$A = \begin{vmatrix} |1d\ \frac{3}{2}\ \frac{1}{2}\ - > \\ |1d\ \frac{3}{2}\ \frac{3}{2}\ + > \\ |1d\ \frac{3}{2}\ \frac{3}{2}\ - > \\ |1d\ \frac{3}{2}\ \frac{1}{2}\ + > \end{vmatrix} \qquad B = \begin{vmatrix} |2s\ \frac{1}{2}\ \frac{1}{2}\ - > \\ |2s\ \frac{1}{2}\ \frac{1}{2}\ + > \end{vmatrix}$$

$$C = \begin{vmatrix} |1d\ \frac{5}{2}\ \frac{1}{2}\ - > \\ |1d\ \frac{5}{2}\ \frac{3}{2}\ + > \\ |1d\ \frac{5}{2}\ \frac{3}{2}\ - > \\ |1d\ \frac{5}{2}\ \frac{5}{2}\ + > \\ |1d\ \frac{5}{2}\ \frac{5}{2}\ - > \\ |1d\ \frac{5}{2}\ \frac{1}{2}\ + > \end{vmatrix} \qquad D = \begin{vmatrix} |1p\ \frac{1}{2}\ \frac{1}{2}\ - > \\ |1p\ \frac{1}{2}\ \frac{1}{2}\ + > \end{vmatrix}$$

$$E = \begin{vmatrix} |1p\ \frac{3}{2}\ \frac{1}{2}\ - > \\ |1p\ \frac{3}{2}\ \frac{3}{2}\ + > \\ |1p\ \frac{3}{2}\ \frac{3}{2}\ - > \\ |1p\ \frac{3}{2}\ \frac{1}{2}\ + > \end{vmatrix}$$

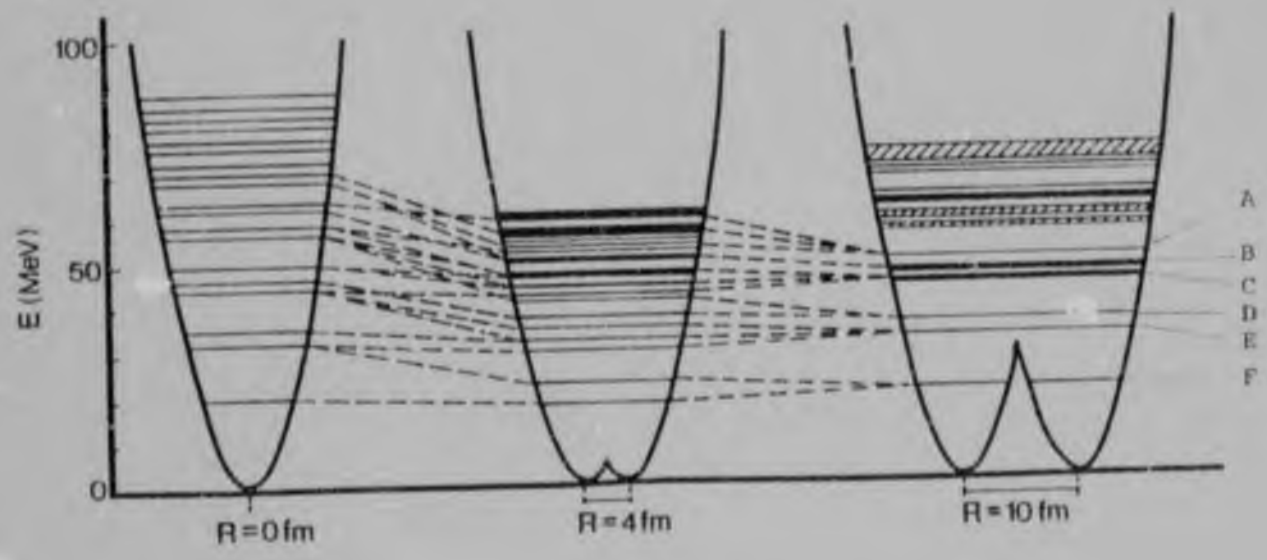


Fig. (1.1)

A = 1d $\frac{3}{2}$; $\frac{3}{2}$ ±; $\frac{1}{2}$ ±

B = 2s $\frac{1}{2}$; $\frac{1}{2}$ ±

C = 1d $\frac{5}{2}$; $\frac{5}{2}$ ±; $\frac{3}{2}$ ±; $\frac{1}{2}$ ±

D = 1p $\frac{1}{2}$; $\frac{1}{2}$ ±

E = 1p $\frac{3}{2}$; $\frac{3}{2}$ ±; $\frac{1}{2}$ ±

F = 1s $\frac{1}{2}$; $\frac{1}{2}$ ±

Fig. (6.2) : W as a function of R for R = R' and $\theta = 0^0$ and 30^0.

Fig. (6.3): Same as Fig. (6.1) but with K = 0, $\theta = 30^0$.

* * * * * * * * * * *

forward angles may be due to absorption and/or refraction and the backward rise may be due to particle exchange and/or reflection.

Fig. (2.4) : s-wave phase shift caused by the Pauli principle in dineutron-dineutron scattering. The dashed curve refers to b=b'=1 fm and the solid line to b=2b'=2 fm; near resonance the phase is displayed for various step lengths d in the numerical solution of eq.(2.3.8).

Fig. (2.5) : $\alpha - {}^{16}O$ phase shifts for even partial waves. The upper part refers to $b_\alpha = b_{^{16}O}$, the lower one to $b_\alpha = b_{^{16}O}$, the lower one to $b_\alpha = b_{^{16}O}$.

Fig. (3.1) : Fermi spheres F_1 and F_2 separated by the local momentum K per nucleon.

Fig. (4.1) : Fragments A and B as two Fermi spheres approaching each other with relative initial momentum.

Fig. (4.2) : $\underline{R}_A$ and $\underline{R}_B$ describe the position of the oscillator potentials of the clusters, which is the mean position of the fragments, $\underline{R}$ describes the mean separation between the fragments.

Fig. (6.1) : Solid line: The imaginary potential

$$W = \sum_{\substack{\alpha,\beta,\gamma,\delta \\ \text{occ.}}} (\alpha,\beta|\,\mathrm{Im}\,G|\gamma,\delta)(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\,\delta_{\beta\gamma})$$

as a function of R for various values of R'(fm) and K = 0, $\theta=0^0$.

dashed line: W as a function of R for K = R' and $\theta = 0^0$.

FIGURE CAPTIONS

Fig. (1.1) : Instantaneous shell-model potentials for the nucleonic movement in the $^{16}O + ^{16}O$ collision ($\hbar\omega_{\infty}$=13,22 Mev). The single particle levels correspond to a spin-orbit force strength parameter κ=0,08 and are assigned with the quantum numbers Nℓj, mπ, Λ for the separated fragments. Regions of close lying levels are hatched.

Fig. (1.2) : Single particle levels for $^{16}O - ^{16}O$ with $\hbar\omega_{\infty}$=13,22 Mev and κ=0,08 as a function of relative distance for the "fast" and "slow" cases as (left and right, respectively). The levels are assigned with quantum numbers Nℓj, mπ for the separated fragments. The shape of the ion-ion system is shown on top of the figure.

Fig. (2.1) : Fragments A and B separated by the true seperation $\underline{\rho}$.

Fig. (2.2) : Qualitative behaviour of some heavy-ion potentials for various energies. With increasing energy (from solid through dotted to dashed lines) the deep attractive potentials become shallower while the repulsive core potentials are reduced.

Fig. (2.3) : Qualitative features of the elastic scattering cross section σ, divided by the Rutherford cross section σ_R as a function of the C.M. scattering angle. The drop of σ/σ_R below the value 1 at

40. A. Erdelyi, W. Magnus, F. Oberhettinger and F.G. Tricomi, Tables of Integral Trandforms, Vol. 1 (McGraw-Hill, New York, 1954).

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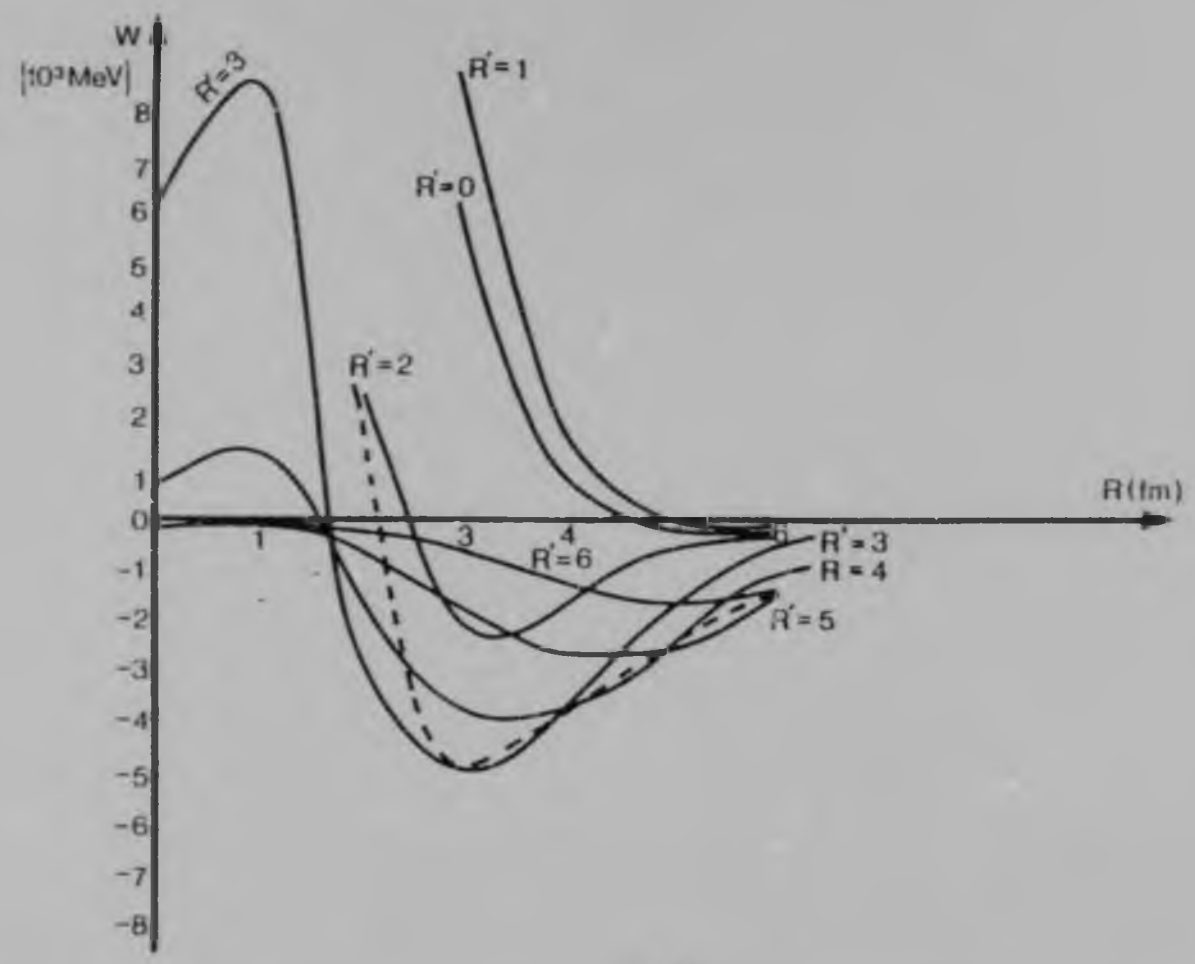

Fig. (6.3)

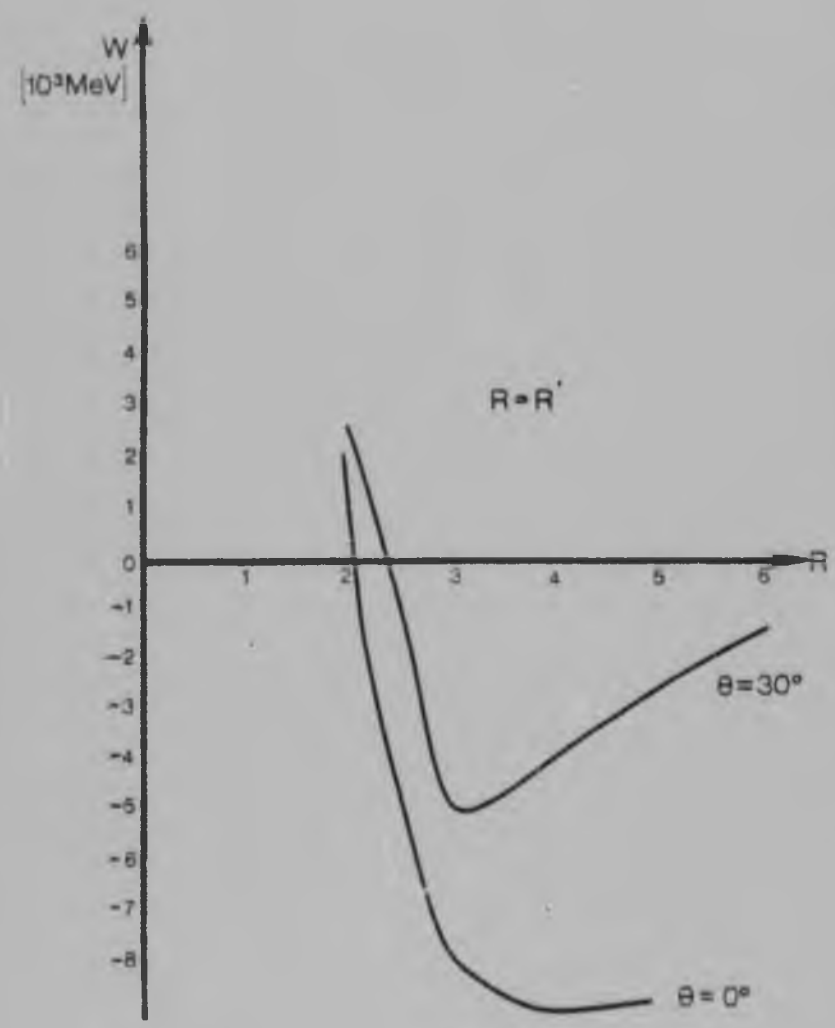

Fig. (6.2)

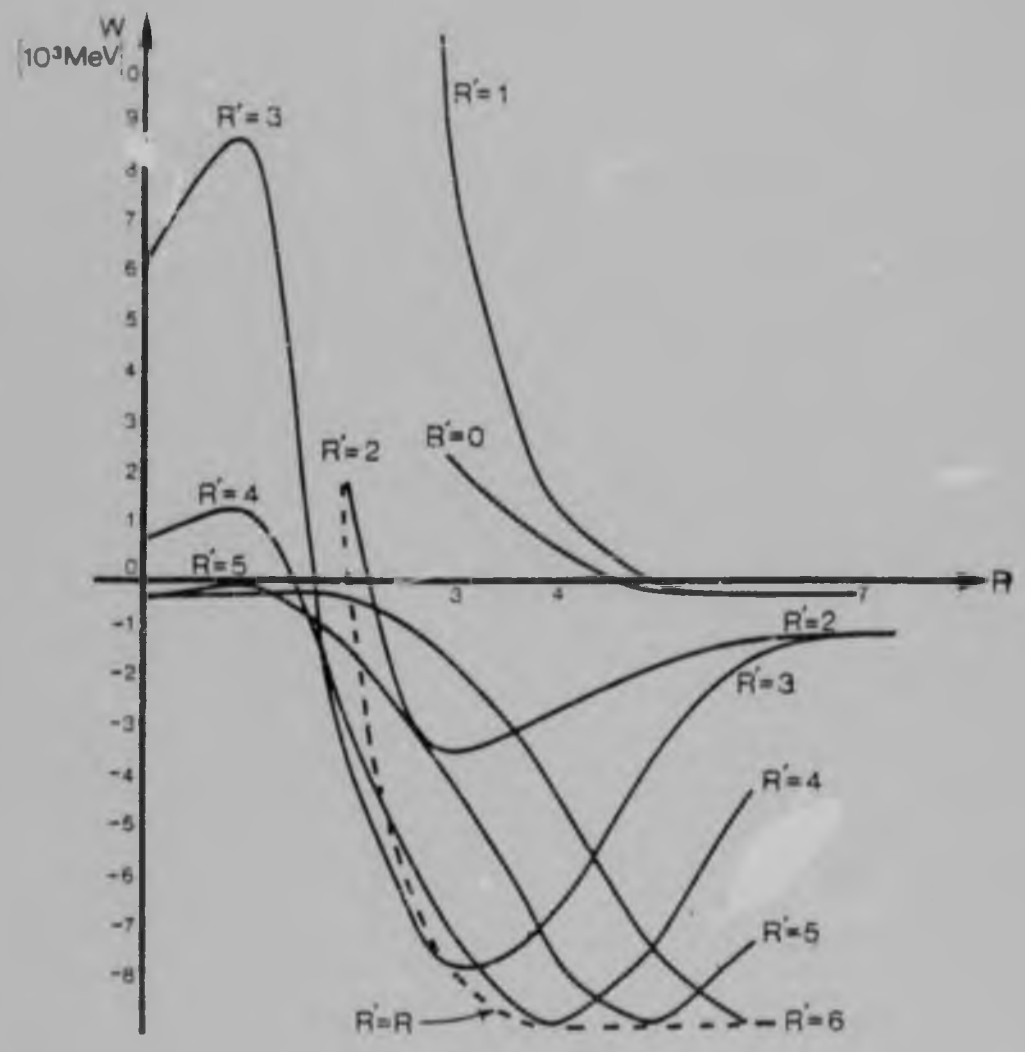
W
(10³MeV)
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
R'=3
R'=1
R'=2
R'=0
R'=4
R'=5
3
4
7
R
R'=2
R'=3
R'=4
R'=5
R'=R
R'=6

Fig. (6.1)

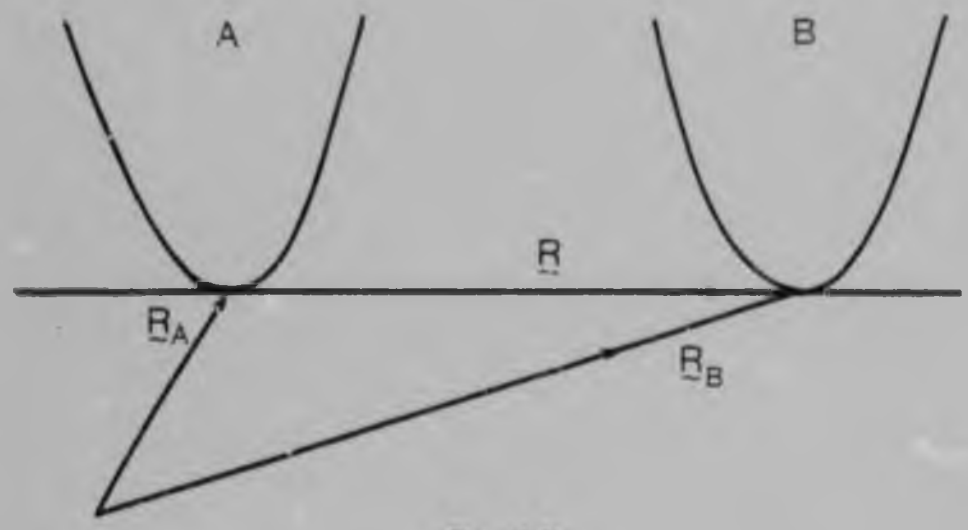

Fig. (4.2)

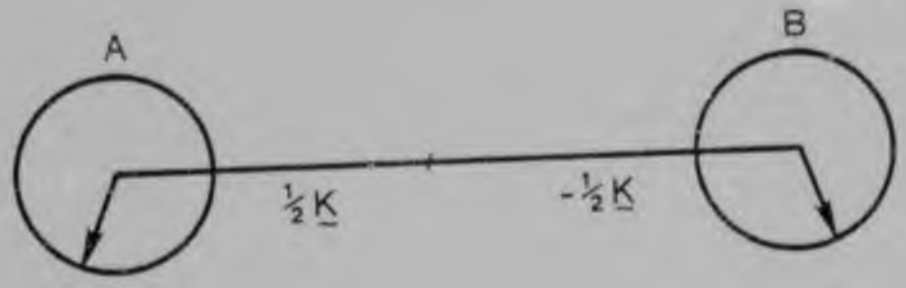

Fig. (4.1)

Author Gadinabokao W L
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