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The Prediction, Quantification and Evaluation of Advanced Process Control Benefits

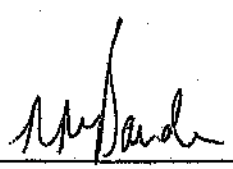
Neil Martin Bawden

A project report submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg in partial fulfillment of the requirements for the degree of Master of Science in Engineering.

Johannesburg 1992

DECLARATION

I declare that this project report is my own, unaided work. It is being submitted for the Degree of Master of Science in Engineering at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.



Neil Martin Bawden

The Second day of December 1992

Abstract

It has been suggested that the inability to quantify the financial benefits of advanced process control is a key barrier to its more widespread use and acceptance. This report discusses the financial analysis of advanced process control opportunities. Potential benefits of advanced control are predicted by considering the power spectral density of load disturbances likely to affect the control system, and the ideal characteristics of feedback systems as proposed by H.W. Bode. Financial ratio analysis and modern measures of profitabilty, such as Discounted Cash Flow Rate of Return (DCFRR) and Net Present Value (NPV) are used to evaluate the economic potential of control opportunities for an organisation. The results indicate that financial benefits of advanced process control strategies can be predicted, quantified and then evaluated, with the ultimate aim of improving the overall financial position of an organisation.

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I am grateful to Professor I. M. MacLeod for permission to publish the routines for determining the optimum loop gain shape for unity feedback systems.

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Chapter 1

Introduction

1.1 Background

Management in the Chemical Process Industry is frequently called upon to critically analyze the profitability of certain areas of plant and to consider methods to improve the operation of a plant. Experience has shown that the control upgrade project can offer significant improvements in the profitability of the operation of a plant [1]. A method is required, however, whereby the economic viability of such an upgrade project can be evaluated and compared with the existing mode of operation, before the upgrade is sanctioned. The application of advanced process control techniques can be considered to fall into the category of control upgrades, and hence this method is also applicable to the evaluation of advanced process control strategies.

The opportunities for applying advanced control in the Chemical Process Industry are vast and numerous. The need exists to be able estimate the financial returns that a particular control application will yield in order that management committment, investor confidence and finally, financial support can be gained. If the financial benefits of a control strategy can be predicted and quantified, comparisons can be made between various alternative strategies based on profit yield, as well as on technical merit. The end result is that the most profitable and suitable projects can be selected and implemented, thus improving the overall financial position of the organisation.

The aim of this project is to develop a set of analysis tools to assist with the prediction, quantification and evaluation of the benefits that advanced process control techniques have to offer. The financial decision-making process of selecting profitable investment opportunities for improving overall financial performance is a key theme throughout this work.

1.2 Related Work

It has been suggested that the inability to quantify the financial benefits of advanced process control is a key barrier to its more widespread use and acceptance (see Warren Centre Report [2]).The Warren Centre, at the University of Sydney, therefore initiated a 9.5 man-year investigation, known as the Advanced Process Control Project, to demonstrate a general benefits analysis methodology for the prediction and quantification of benefits of applying advanced process control technology.

The Warren Centre Advanced Process Control Project used a benefits analysis method developed by T.E. Marlin for large, continuous petrochemical plants. The method was extended during the project so that it could be applied to a wide range of plants in different process industries. The project culminated in a Project Report [2] and the detailed documentation of seven in-depth case studies in Australian industry. The extended methodology evaluated the potential of advanced process control projects for these case studies by comparing the proposed operation with the existing, so-called *base case*, mode of operation. The extended benefits analysis method developed during the project has been well documented in the literature by Marlin, Perkins, Barton and Brisk [3] and Brisk [4].

The extended methodology suggested that financial benefits of various control opportunities can be calculated using the following formula:

$$\begin{aligned} \textit{Benefit} \;=\; & \textit{Improvement} \times \textit{Incremental Value} \times \\ & \textit{Time} \times \textit{Throughput} \times \textit{Service Factor} \end{aligned} \tag{1.1}$$

Equation 1.1 requires that an incremental monetary value be associated with the improvement brought about through improved control. This incremental value is assumed to be the value of operating process variables closer to their optimum targets or set points. Improved control reduces the deviation of a process variable and allows the variable to be moved closer to the desired operating target or constraint. The incremental value can be derived from the profit function relating monetary value to the operating point. The Warren Centre work focussed on a linear profit function, up to a given constraint or set point. The benefits attributable to improved regulation were assumed only to be the contributions from moving the operating point of the variable closer to the constraint or target set point. The benefit of reducing the standard deviation of the variable about its operating point was not quantified, and was assumed only to enable the variable to be moved closer to the target.

Latour [5], in his work in the area of controlling the blending of fuel oils, concluded that improved regulation alone is also beneficial. Latour considered the consequences of violating the constraint or target beyond the linear region assumed in the Warren Centre Report [2]. This benefit of improved regulation was included in the development of his improved control profit function, which featured a non-linear penalty region around the constraint.

Stout and Cline [6] extended the analysis of profit functions to include quadratic and higher order functions. They proved mathematically that if the profit function is linear, reducing the standard deviation has no benefit; reducing variability however, allows the process variable to be operated at a mean value closer to a constraint. Furthermore, they showed that for quadratic profit functions, benefits are obtained from reducing the standard deviation and from moving the average operating point.

This study combines all of the above documented research in the area of profit and penalty functions by considering the formulation of an overall performance function. The performance function approach proposed in this report caters for linear and higher order functions as well as the handling of constraints.

Marlin, Perkins, Barton and Brisk [3] proposed four methods for the estimation of the improvement derived from advanced control:

1. Review of historical plant data to determine the best operation achieved in the plant for sustained periods. This approach assumes an advanced process control scheme can duplicate this best operation on a continuous basis.

2. Improvements calculated on experience gained with similar process units and the resultant benefits obtained with advanced process control strategies.

3. Plant tests conducted which emulate the proposed advanced process control strategy and from which the predicted improvements over existing *base case* operation can be calculated.

4. Predict the improvement based on knowledge of the typical disturbance experienced by the process and on estimates of the control strategy to be employed.

Brisk [4] states that in practice, plant test runs are seldom used and that simulation is substituted for the more rigorous approach of analyzing disturbances. Brisk suggests that advanced process control projects typically yield 2%–6% savings of total production costs, when implemented correctly. This suggestion is based on the seven documented case studies [2] and on subsequent experience gained in the ICI group of companies. This research project suggests that the rigorous approach of analyzing disturbances can be used to quantify the benefits or savings of each proposed control project in more specific terms. It is hoped that the quantification of real, specific control benefits will allow the more widespread use and acceptance of advanced control principles.

The costs for implementing control projects and for receiving their corresponding benefits, can be estimated once the implementation strategy is known. The Warren Centre methodology suggests that all generated control opportunitites be ranked according to their benefits to cost ratio. Projects for implementation can then be proposed from the list of ranked projects. The ranking of projects according to a benefits to cost ratio (a "traditional" measure of profitability) can, however, provide very misleading results. This method of ranking does not take into account *the time value of money*. Modern methods for determining profitability of projects have been developed which provide a more realistic overview of the returns a project will yield. Net Present Value methods and Discounted Cash Flow analysis are such modern methods and are presented in [7] and [8] respectively. The modern methods for profitability assessment will be used in this project to assist in the decision- making process of ranking control project opportunities.

1.3 Overview of the Project Report

The report is presented in the following format:

Chapter 2 discusses the financial analysis of an organisation using financial ratios. This analysis is used to assess the ability of an organisation to obtain financing for investment in projects and to set financial targets for control projects.

Chapter 3 discusses the benefits of feedback control by analyzing the power spectral density of load disturbances likely to affect the system. Ideal feedback systems, as proposed by Bode, are used to quantify the maximum achievable benefit feedback has to offer.

Chapter 4 presents the formulation of performance functions relating the operating point of a process variable to a corresponding financial value.

Chapter 5 examines various methods for ranking control project opportunities, based on different measures of profitability. This ranking is used in the final decision-making process of selecting the most suitable project opportunities for a particular organisation.

Chapter 6 presents a case study where the principles of the preceeding chapters are applied.

Chapter 7 concludes the report and presents the main results of the research study.

The financial and analysis tools are developed with the personal computer based software package Matlab. Matlab is widely used amongst control engineers and provides a suitable mathematical environment for the development of control and system identification tools. Application of the "toolbox" is demonstrated to identify and assess the economic viability of advanced process control techniques on a selected case study involving the classical two tank liquid level control problem. Economic evaluations are tested against financial targets set for an organisation, based on the financial performance of the organisation and guidelines are given for the decision-making process of selecting projects which improve the overall financial condition of an organisation.

Cost estimation of implementing advanced control projects is not within the scope of this research project.

Chapter 2

Financial Targets and Objectives

2.1 Introduction

The implementation of control upgrade and advanced process control projects requires the investment of capital resources. The ability of an organisation to obtain financing for the capital requirements of a project is dependent on the financial condition and performance of the organisation. The yardstick by which financial performance can be measured (and thus the ability to obtain financing for investments) is derived from financial ratios and interpretation of information recorded in the financial statements issued by the organisation.

This chapter discusses the financial analysis of an organisation using financial ratios. The AECI Group of companies is used as a case study for the period 1990–91. The insight gained from the financial performance of an organisation is then used to assess the ability to finance projects and to set targets which the control opportunities under consideration must comply with and meet. The targets set for control projects include: limitations on capital resources (forced by the ability to obtain finance) and minimum returns which the project must yield in order for the organisation to prosper. Control upgrades and advanced process control opportunities are presented as a possible means to achieve this prosperity.

2.2 Financial Statements and Ratios

Financial statements summarize the financial transactions of an organisation. The day-to-day transactions are recorded and then summarized into a format required by law.

The financial statements comprise of two parts: a balance sheet, which shows the financial position at a particular time and the income statement, a record of the loss or gain over a period of time. The financial information recorded in an organisation's financial statements can be analysed and interpreted using financial ratios. These financial ratios can be used to measure business performance, or be used as the basis for setting business objectives for an organisation.

Ratios allow comparisons to be made between the present performance of an organisation and the following measures:

1. predetermined expected standards or targets, expressed as financial ratios.
2. the historical performance of the same organisation.
3. the performances of similar organisations or organisations within the same industry.

One can, for example, compare the performance of two organisations operating in the same sector of industry by directly comparing ratios. Care must, however, be exercised when comparisons are made, as ratios can only be compared if the information used in the calculation has been standardized.

Business objectives can also be set for an organisation using financial ratios. The past business performance of an organisation can be measured from historical data and used to specify future targets and objectives for the organisation. Once again, care must be taken to ensure that historical information has been calculated according to the same standard as current data.

Financial ratios can be classified by type: liquidity, leverage, activity and profitability. A number of financial ratios are presented in table 2.1. Table 2.1 includes: the ratio type, method of derivation, typical average values for the Chemical Process Industry (reference Perry [9]) and ratio values for the AECI Group of companies for 1990. The AECI ratios were determined directly from information recorded in the AECI Limited Annual Report 1990 [10].

Type	Ratio	Equation	Industry	AECI
Liquidity	Current Ratio	$\frac{Current\ Assets}{Current\ Debt}$	2.60	1.37
	Acid Test	$\frac{Current\ Assets - Inventory}{Current\ Debt}$	1.00	0.64
Leverage	Debt Ratio(%)	$\frac{Total\ Debt}{Total\ Assets}$	33.0	57.0
	Times Interest	$\frac{Profit + Interest\ Charges}{Interest\ Charges}$	8.00	4.84
Activity	Inventory Turnover	$\frac{Sales}{Inventory}$	9.00	5.20
	Collection Period	$\frac{Account\ Receivables}{Sales(per\ day)}$	61.0	61.0
	Fixed Asset T/over	$\frac{Sales}{Fixed\ Assets}$	5.00	3.50
	Total Asset T/over	$\frac{Sales}{Total\ Assets}$	2.00	2.10
Profit	Profit Margin	$\frac{Net\ Profit}{Sales}$	5.00	4.90
	Return on Assets	$\frac{Net\ Profit}{Total\ Assets}$	10.0	7.60
	Return on Net Worth	$\frac{Net\ Profit}{Net\ Worth}$	15.0	18.0

Table 2.1: Types of Financial Ratios

2.2.1 Liquidity Ratios

Liquidity ratios are a measure of the availability of cash or the ability to meet short-term obligations from available funds. The most general and frequently used of these ratios, is the current ratio. The higher the current ratio for an organisation, the greater the ability an organisation has to pay its bills.

The quick or acid test ratio gives another perspective of liquidity by measuring the ability of an organisation to pay its bills without having to sell off inventory.

During 1990, AECI was below the Industry average for liquidity, indicating that the Group was vulnerable if forced to settle short-term obligations.

2.2.2 Leverage Ratios

Leverage can be considered a gauge of the level of debt or borrowings an organisation has entered into. In comparison to liquidity ratios, leverage ratios are based on long-term obligations. Organisations with high leverage ratios run the risk of large losses in an economic downturn, although they may be poised to earn large profits during an economic boom if their assets, acquired with borrowed capital, produce high returns.

The "times-interest-earned" ratio measures the extent to which an organisation can service annual interest charges. Failure to meet interest charges may result in bankruptcy.

The leverage ratios for AECI indicate a high level of borrowings to finance assets. This high level of leverage may prevent investors from making additional funds available to the Group for investment in projects.

2.2.3 Activity Ratios

Activity ratios measure the effectiveness of an organisation to utilize available resources, generated from sales.

AECI seems to be making good use of revenue generated from sales, although the fixed asset turnover is too low. Fixed assets may be under-performing.

2.2.4 Profitability Ratios

Profitability is a measure of the return from investments. Profitability ratios indicate the efficiency of operation of an organisation.

AECI is performing satisfactorily when profitability ratios are considered, indicating that operations management is sound.

2.2.5 Summary of Ratio Analysis

Financial interpretation of the results reported by the AECI Group of compa ies during 1990 [10], indicate that AECI must improve its liquidity and leverage position. In order to acquire additional resources for the financing of projects, AECI must be liquid to service any short-term obligations and correctly levered to avoid heavy losses. By improving upon its liquidity and leverage rating, an organisation can present projects as attractive investment opportunities to investors.

In 1991, AECI re-valued its assets, resulting in the debt ratio falling from 57% to 47%. By improving this ratio, it is hoped that investors will be attracted to invest in projects proposed by AECI in the future.

An alternative approach to improving the liquidity and leverage rating for AECI would be to invest in projects requiring a small capital investment but which provide high returns. Small investment, high yield projects provide a "quick fix" solution to poor performance in the short-term; long-term projects must be considered if financial prosperity is to be sustained. Control upgrades and advanced control can provide both short-term, quick fix solutions as well as longer-term, growth orientated opportunities.

Control projects are ideal candidates for improving benefits from existing fixed assets. Benefits can be achieved by: reducing maintenance and operating costs, improving process and equipment efficiencies and by increasing production capacity. Furthermore, these benefits can be achieved without large capital investment in a new plant, thus improving the financial ratios and reflecting an improved overall financial performance of the organisation.

2.3 Time Value of Money

The primary purpose of all normal business operations is to earn profit from investments made in the business. The profit made should be at least equal to, but preferably greater than, the cost of borrowing the capital to fund the investment. Even if the business funded its own capital requirements, the business would still want to earn at least as much as if it had been invested in a secure investment. A secure investment could be considered as one where a guaranteed return is offered, eg. a bank deposit or in stocks and shares.

The *time value of money* refers to the effect that interest rates or potential, alternative investment opportunities have on the value of money. The underlying maxim is that money received now is more valuable than money received later. This is because money received earlier can be re-invested and interest earned on this money. Conversely, money earned later is worth less because of the lost investment opportunities.

Consider an entrepreneur who wishes to make an investment. He has an opportunity to invest, in say, a fixed deposit which earns him a guaranteed return. His original investment capital remains in tact which he can reclaim at the end of the investment period. This opportunity is relatively free from risk. Alternatively, the entrepreneur also has an opportunity to invest his capital in an engineering project venture. The entrepreneur will be looking for the following returns from the venture:

1. The return of the original investment capital at the end of the project life.
2. Sums of money equivalent to the interest that could have been earned from a risk-free venture.
3. Extra sums of money as an inducement to undertake a potentially risky project, for example, in terms of: the technology used, political stability of the environment or the availability of markets.

The future worth, or future value, of money, relative to the present value, can be

determined using the compound interest formula:

$$FV = PV \times (1+i)^n \tag{2.1}$$

FV = future value of a sum of money at a future date
PV = present value or *principal*
i = interest rate (may be cost of borrowing capital)
n = number of periods specifying investment lifespan

From the above formula it can be clearly seen that the potential future value of capital invested at an interest rate, i, is greater by a factor:

$$Factor = (1+i)^n \tag{2.2}$$

Compound interest tables have been generated which list the factors for various interest rates and number of time periods. Tables, however, have limitations with respect to accuracy because of round-off errors. This accuracy limitation has been reduced with the advent of spreadsheet computer programs and the need for look-up tables of factors has diminished.

One can compound over different time periods to obtain annual, monthly, daily or even continuously compounded results. In this research project, compounding and interest rates quoted on an annual basis will be used throughout.

2.4 Formulation of Targets

Projects and investments must support the business objectives of an organisation. The justification of advanced control and control upgrade projects must therefore include an awareness of the business objectives set for an organisation and the economic climate the organisation operates in.

The financial performance of an organisation is reflected in financial ratios, derived from information recorded in the financial statements. Financial ratios, therefore, provide criteria against which financial performance can be measured. Ratios can be used to set objectives; performance can be judged against these objectives or targets. The financial performance of an organisation will influence the ability of that organisation to acquire financing from an institution or from investors. Ratio analysis therefore also allows one to determine the probability of success of an organisation acquiring financing. If this financial interpretation is available, control projects and upgrades can be tailored to meet the constraints exerted by business performance, preset targets or financing charges.

The time value that an organisation places on money can be used to set minimum rates of return that an organisation would expect from any investment—be it a control upgrade or any other investment opportunity. The minimum rate of return can be expressed as an interest rate i and is sometimes referred to as the "hurdle rate". The hurdle rate sets a minimum standard which projects must exceed before they can be considered to offer an acceptable rate of return. Different hurdle rates will be set by different organisations, but different hurdle rates may also apply to the same organisation during various stages of financial performance. The hurdle rate during poor financial performance will obviously be higher than during times of strong performance. The hurdle rate can be set using different criteria:

1. The cost of borrowing capital.

2. Interest rates that could be earned from risk-free ventures.

3. Interest rates returned from risk-free ventures plus an additional contribution for undertaking risk.

Inflation can also be included in the determination of a hurdle rate. The effects of inflation are beyond the scope of this project report and will therefore not be considered.

2.5 Summary

Financial performance of an organisation affects the ability of an organisation to acquire financing for projects. Ratio analysis can be used to estimate the probability of success of acquiring this financing from an institution or from other investors. Ratio analysis requires interpretation of information published in an organisation's financial statements—information which is usually freely available. AECI, the group of companies used as a case study, is currently suffering from a weak liquidity and leverage rating. Limitations will have to be placed on capital resources for any projects likely to be undertaken, until the financial performance of the group improves.

The time value of money, for an organisation, sets a minimum rate of return or "hurdle rate" which projects must show the potential to yield, before they can be considered to be economically feasible. Projects must offer returns which will attract investors and which will gather support for the proposed venture. AECI will be setting fairly high hurdle rates, hoping to attract the attention of investors and thereafter to reap the benefits of high yielding projects.

The scope of advanced control opportunities is vast; not all upgrades and opportunities need be capital-intensive. Upgrades which provide large returns for a relatively small capital outlay could well provide an organisation, suffering from poor financial results, with the potential for improving its short-term position. Upgrades requiring larger capital investments could yield larger, long-term benefits which organisations with a sound financial position could take advantage of to continue to prosper and grow. AECI would do well in the short-term to invest in a number of high yielding control upgrades requiring a small investment of capital resources. In the longer term, AECI would prosper if larger, growth orientated control upgrades were implemented, assuring that the improved short-term financial performance of the group continues into the future.

Control upgrades and advanced process control projects must support the business objectives of an organisation. An awareness of financial performance, preset targets and the economic climate an organisation operates in, provides valuable insight into possible financial constraints which control upgrade or advanced process control projects may face.

Chapter 3

The Benefits of Feedback Control

3.1 Introduction

The rejection of disturbances is one of the main functions of a feedback controller; without dis nces there would be no need for feedback control. The effect that disturbances have on a system can be quantified by investigating the characteristics of these disturbances and the overall system transfer functions.

The purpose of this chapter is to quantify the effects that deterministic and stochastic load disturbances have on a feedback control system. The description of disturbances and their effects is developed using continuous-time random signal theory, with special attention paid to the unity feedback system.

Ideal single loop feedback amplifier characteristics, as proposed by Bode, are then used to quantify the maximum achievable benefit feedback has to offer. The ability of the ideal single loop amplifier to reject disturbances and thus to minimize the variance of the output due to these disturbances, is investigated using the sensitivity function and the power spectral density of the disturbance.

3.2 Disturbance Rejection

The nature of disturbances affecting a system determines the quality of regulation that can be achieved by a control system. Disturbances, therefore, place limits on the performance of a controller design.

Consider the feedback system in figure 3.1. The main objectives of the controller $C(s)$ are:

1. To keep the change in output signal y caused by the introduction of a load disturbance d small.

2. To keep the tracking error e small for changes in set point r (sometimes referred to as the reference input).

Noise or measurement errors in the measurement signal can also be classified as disturbances. This class of disturbances will not be considered in detail in this project report, hence $n = 0$ for this investigation.

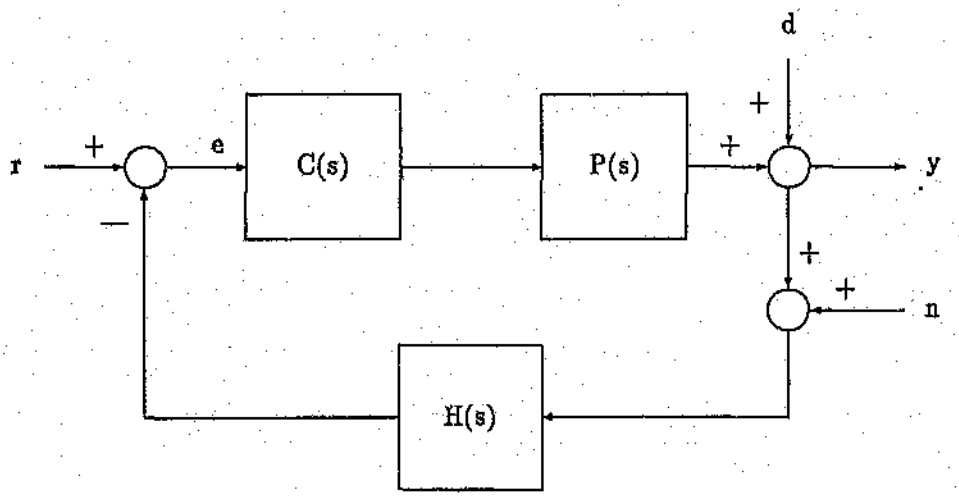

Figure 3.1: The Standard Feedback Control System

Referring to figure 3.1 and defining *G(s)* as the forward or open loop gain and *L(s)* as the loop gain:

$$\begin{aligned} G(s) &= C(s)P(s) \\ L(s) &= C(s)P(s)H(s) \\ &= G(s)H(s) \end{aligned}$$

The *sensitivity function S(s)* is defined as the transfer function from measured process variable y to disturbance d:

$$\begin{aligned} \frac{y}{d} &= \frac{e}{r-1} \\ &= \frac{1}{1+L(s)} \\ &= S(s) \end{aligned}$$

Therefore:

$$S(s) = \frac{1}{1+L(s)} \tag{3.1}$$

The *complementary sensitivity function T(s)*, or closed loop transfer function, is defined as the transfer function from measured variable y to set point r:

$$\begin{aligned} \frac{y}{r} &= \frac{G(s)}{1+L(s)} \\ &= T(s) \end{aligned}$$

The sum of the two sensitivity transfer functions is unity, $S(s) + T(s) = 1$. Ideally $S(s) = 0$ and $T(s) = 1$. This ideal situation is not physically realizable because unknown high frequency components in a system must be attenuated to avoid instability. Sensor noise and mechanical resonances are examples of undesirable high

frequency signals whose effect must be minimized. The result of this limitation is that for a physically realizable system:

$$\begin{aligned} \lim_{s\to\infty} S(s) &= 1 \\ \lim_{s\to\infty} T(s) &= 0 \end{aligned}$$

A trade-off must therefore be made between the requirements of disturbance rejection and the reduction of tracking error for a physically realizable system, where high frequency noise must be attenuated. Bode has developed ideal characteristics for single loop amplifiers. These optimum trade-off characteristics will be discussed later in this chapter.

3.3 Spectral Analysis of Load Disturbances

Load disturbances which influence the measured process variable can sometimes be described by so-called simple disturbance models. If this approximation can be made, four different types of models are typically used: impulse and pulse, step, ramp and sinusoid. Simple disturbance models fall into the broader category of *deterministic* signals, which are uniquely determined by mathematical expressions or rules. Deterministic signals (including simple disturbance models) can be directly transformed using the Fourier transform.

The effect of deterministic load disturbances on the magnitude of the output of the unity feedback controller, $H(s) = 1$, can be calculated from equation 3.1 and application of the Fourier transform:

$$\begin{aligned} |Y(j\omega)| &= \left|\frac{1}{1+L(j\omega)}\right| \times |D(j\omega)| \\ &= \left|\frac{1}{1+C(j\omega)P(j\omega)}\right| \times |D(j\omega)| \end{aligned}$$

In real-world measurement applications, disturbances are not always simple and deterministic. Disturbances may be very complex, making a precise description of the signal difficult, or they may be random in nature. By using a random or *stochastic* approach to disturbances, it is possible to describe a much wider range of disturbance characteristics. Stochastic signals do not directly have Fourier transforms, but many of these signals can be summarised by means of an autocorrelation function, for which a Fourier transform does exist. The effect of stochastic disturbances on the magnitude of the output of feedback controllers can therefore be investigated if the Fourier transform of the autocorrelation function is considered. *Power spectral density* is defined as the Fourier transform of the autocorrelation function.

To specify a random disturbance signal, certain statistical properties of the signal must be known. Amplitude behaviour can be specified in terms of means and standard deviations, whereas frequency behaviour can be specified in terms of the power spectral density.

Consider a stable, linear time-invariant system with impulse response h, subjected to a continuous-time stochastic input $u(t)$ as shown in figure 3.2. The relationship between output and input is:

$$y(t) = \int_{-\infty}^{t} h(t-s)u(s)\,ds \tag{3.2}$$

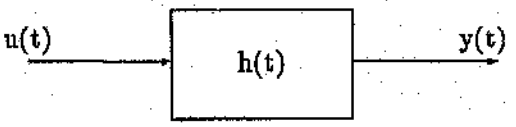

Figure 3.2: Linear Time-Invariant System

or

$$y(t) = \int_0^\infty h(s)u(t-s)\,ds \tag{3.3}$$

The mean of a process is defined as:

$$\mu_u = \varepsilon\{u(t)\}$$

where ε denotes expected value.

The autocorrelation function of a process is:

$$\phi_y(\beta) = \varepsilon\{y(t)y(t+\beta)\} \tag{3.4}$$

Substituting equation 3.3 into equation 3.4 and applying the Convolution theorem whilst taking Fourier transforms yields:

$$\Phi_{yy}(\omega) = |H(j\omega)|^2 \times \Phi_{uu}(\omega) \tag{3.5}$$

$\Phi_{yy}(\omega)$ and $\Phi_{uu}(\omega)$ are termed the *power spectral densities* of the output and input signals respectively.

A more detailed derivation of equation 3.5 is presented in Appendix A. Oppenheim and Schafer [11] and Åström and Wittenmark [12] can both be referred to if additional reference material is required. Both references provide detailed analysis of the discrete-time stochastic process yielding the analogous result:

$$\Phi_{yy}(e^{j\omega}) = \left|H(e^{j\omega})\right|^2 \times \Phi_{uu}(e^{j\omega})$$

From the properties of the correlation function for a stationary process with real input, and knowing that the variance of a signal is the square of the standard deviation, it can be shown that for the input signal u:

$$\begin{aligned} variance &= \sigma_u^2 \\ &= \varepsilon\{|u(t)|^2\} - |\mu_u|^2 \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_{uu}(\omega)\,d\omega - |\mu_u|^2 \end{aligned}$$

A more detailed derivation of $\sigma_u^2 = \varepsilon\{|u(t)|^2\} - |\mu_u|^2$ is provided in a later section, subsection 4.2.3, in the derivation of equation 4.2.

It can also be shown that Φ_{uu} is both real and even. Therefore, for a signal with a zero mean, $\mu_u = 0$, it follows that:

$$\sigma_u^2 = \frac{1}{\pi}\int_0^{\pi} \Phi_{uu}(\omega)\,d\omega \tag{3.6}$$

Thus the variance of a signal with zero mean is the area under the power spectral density curve.

Interpreting the results of equations 3.5 and 3.6 with reference to figure 3.1, the standard deviation of the output signal y can be determined by considering the power spectral density of the disturbance signal d, and the transfer function from the output signal to the disturbance—the sensitivity function.

From equations 3.1 and 3.5:

$$\begin{aligned}\Phi_{yy}(\omega) &= |S(j\omega)|^2 \times \Phi_{dd}(\omega) \\ &= \left|\frac{1}{1+L(j\omega)}\right|^2 \times \Phi_{dd}(\omega)\end{aligned}$$

In the unity feedback case, where $H(s) = 1$

$$\Phi_{yy}(\omega) = \left|\frac{1}{1+C(j\omega)P(j\omega)}\right|^2 \times \Phi_{dd}(\omega)$$

Applying equation 3.6:

$$\sigma_y^2 = \frac{1}{\pi}\int_0^\pi \left|\frac{1}{1+C(j\omega)P(j\omega)}\right|^2 \times \Phi_{dd}(\omega)\, d\omega \tag{3.7}$$

The variance of a controlled process variable y subject to a disturbance d can be calculated directly using equation 3.7. The variance of variable y is therefore the area under the curve of the power spectral density of d, modified by the sensitivity function. For a given disturbance signal, the performance of a controller can therefore be judged by the size of the variance of the process variable. The smaller the variance, the better the control performance.

3.3.1 Using Matlab for Spectral Analysis

Matlab provides an ideal environment for the spectral analysis of load disturbances. Signals are entered into Matlab as time series vectors and Matlab provides functions for the manipulation of this data. The area under a curve, represented as a time series vector, can be calculated directly by summing the individual components, between the upper and lower limits of interest. From equation 3.6 and using the Matlab function **sum**:

$$\sigma_u^2 = \frac{1}{\pi} \times \mathbf{sum}(\Phi_{uu})$$

Matlab offers a number of functions for the estimation of spectra and for spectral analysis. The signal processing toolbox [13] contains the function **spectrum**, which uses the Welch method of power spectrum estimation. The units of the power spectrum produced by this method are such that:

$$\sigma_u^2 = \mathbf{sum}(\Phi_{uu}) \tag{3.8}$$

The signal processing toolbox also offers the **detrend** function, which can be used to remove the mean value from an input time series. This feature can be used prior to the calculation of the variance σ_u^2 using equation 3.8, therefore ensuring that the mean value of the input, μ_u, is always equal to zero.

The system identification toolbox [14] has two functions for the estimation of spectra: **spa** and **etfe**. The first function **spa**, estimates the covariance function of

the input series, applies a Hamming window and then Fourier transforms the result. The second, **etfe**, computes the periodogram of the input series as the normalized absolute square of the Fourier transform of the series. Both these functions are normalized with respect to the sampling interval T, such that:

$$\sigma_u^2 = \frac{1}{\pi}\int_0^{\frac{\pi}{T}} \Phi_{uu}(\omega)\, d\omega$$

or using Matlab notation:

$$\sigma_u^2 = \frac{\mathbf{sum}(\Phi_{uu})}{NT}$$

where
T = sampling interval
N = length of the series generated by the estimation function, calculated using the Matlab function **length**(Φ_{uu})

The resultant power spectral density can be plotted for the **spectrum** and **spa** or **etfe** functions using the corresponding plot routines **specplot** and **bodeplot** respectively. It should be noted that the normalization differs between **spectrum** and the other two functions and hence care must be exercised when comparing plots.

Example

Consider a time-varying input signal of the form:

$$y(t) = A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t) + A_3 \sin(2\pi f_3 t)$$

For $A_1 = 2$, $A_2 = -0.5$, $A_3 = 1$, $f_1 = 0.0032$ Hz, $f_2 = 0.003$ Hz, $f_3 = 0.0025$ Hz and a one second sampling interval, the power spectral densities produced by each Matlab function are plotted in figure 3.3. The mean of the time-varying input signal was removed using the **detrend** function. The plot for the Welch method, **spectrum**, was generated using a 128-point FFT size and the plots for the spectral analysis and empirical methods, **spa** and **etfe**, were generated using function default values.

The sample variance is defined as:

$$\sigma_y^2 = \frac{\sum_{i=1}^{n}(y_i - \mu_y)^2}{n-1}$$

For a signal with its mean value removed, $\mu_y = 0$:

$$\sigma_y^2 = \frac{\sum_{i=1}^{n} y_i^2}{n-1}$$

For the above example, the variance is calculated as

$$\sigma_y^2 = 1.4337$$

Comparing the variance of the time-varying input signal with the variance derived from calculating the areas under the various power spectral density curves presented in table 3.3.1, one can see the errors introduced by the different smoothing, window functions.

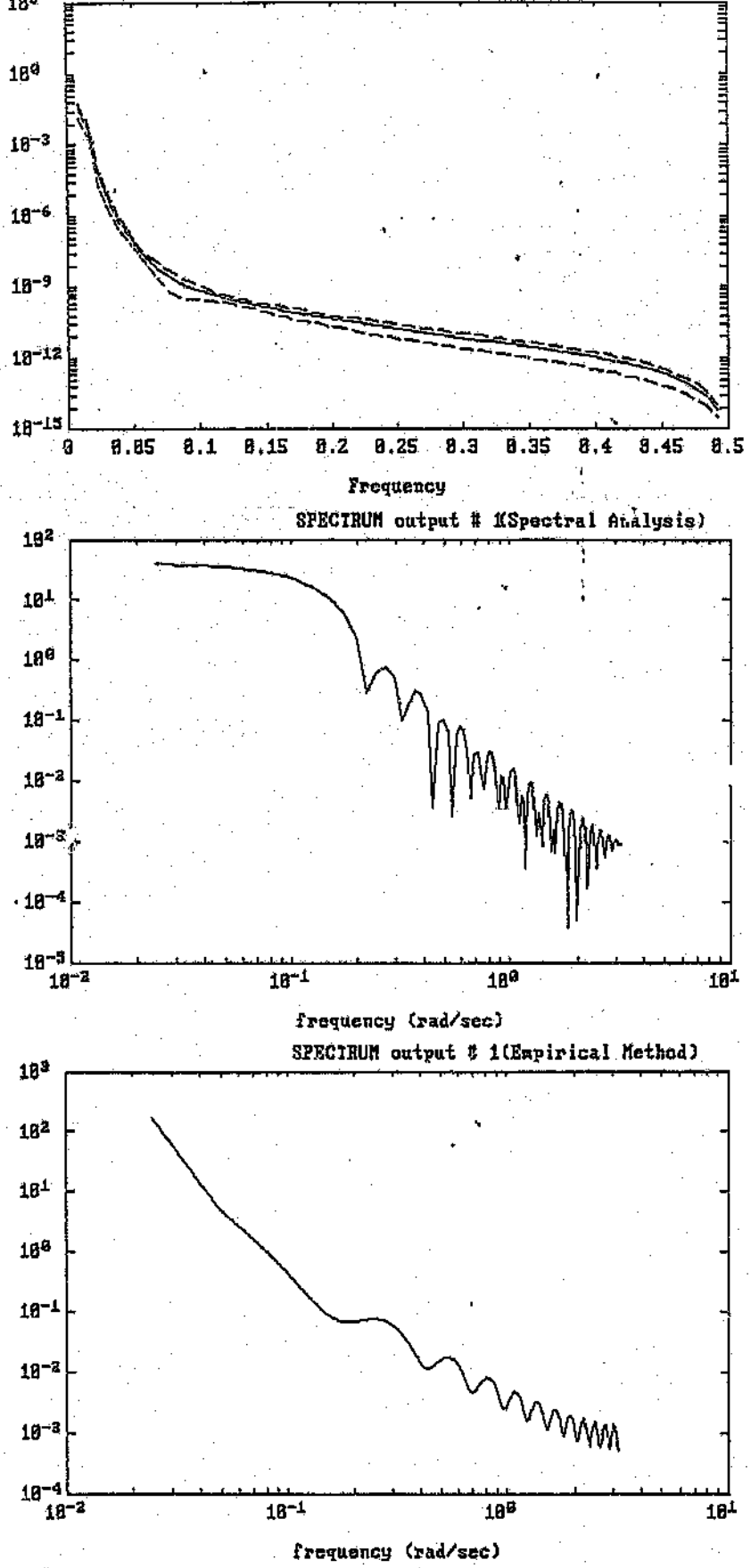

Figure 3.3: Comparison of Power Spectral Density Calculations

Method	Parameter	Variance	Error
spectrum	$m = 128$	0.0773	94.6 %
	$m = 256$	0.8138	43.2 %
	$m = 512$	3.9485	-175.4 %
spa	$M = 10$	1.4335	0.0 %
	$M = 30$	1.3130	8.4 %
	$M = 50$	1.1964	23.7 %
	$M = 100$	1.0200	28.9 %
etfe	$N = 64$	0.0954	93.3 %
	$N = 128$	0.0773	4.4 %
	$N = 256$	0.8138	-9.8 %
	$N = 512$	3.9485	0.1 %

m = number of points for FFT calculation; m must be a power of two
M = length of the lag window
N = number of frequency points; N must be a power of two

Smoothing windows are used to give graphical clarity—not necessarily numerical accuracy. Therefore, either the empirical method of calculating power spectral density, **etfe**, or the spectral analysis function, **spa**, with the correct lag window length specified, is recommended for the determination of the variance of a signal from disturbance spectra considerations.

3.4 Ideal Feedback Characteristics

Bode [15], in considering the design of feedback amplifiers, developed ideal characteristics for the loop gain of single loop amplifiers. A single loop amplifier is equivalent to a unity feedback system. Thus, ideal characteristics for the loop gain of a unity feedback system can be developed using the characteristics proposed by Bode.

Bode's work focussed on shaping the loop gain of a feedback amplifier in the following areas:

1. *Maximum obtainable feedback*—to specify the absolute level of gain in the useful band.

2. *Ideal cut-off characteristics*—to maximise the rate at which the gain outside the useful band is reduced, without producing a phase shift greater than a prescribed design margin.

3. *High frequency asymptotic characteristic*—the loop gain and phase must approach asymptotes determined by the pole-zero excess of the plant plus its controller, at high frequencies.

Macleod [16] has developed two Matlab m-files to assist the Matlab user to develop the ideal loop gain characteristics according to Bode's theorem. These m-files appear

in Appendix B as **bodeopt.m** and **bodeopt1.m**. Macleod has included the facility to have integrator(s) in the loop gain. This additional facility extends the ideal Bode loop gain characteristics to include a wide range of systems typical of the Chemical Process Industry. Integrating processes and the standard PI or PID controllers are found extensively in chemical processes, both of which introduce integrator(s) into the loop gain. It should be noted that the **bodeopt.m** function does not cater for systems with dead time. Dead time modifies the loop gain shape further, but the scope of this research project does not include the handling of systems with dead time.

The ideal loop gain characteristic features a phase margin which has been guaranteed for a large portion of the frequency spectrum. The crossing of the phase over the -180 degree stability point, is only permitted below a specified gain margin threshold. High frequency noise or resonant components which may affect the stability of the loop have therefore been attenuated by an amount at least equivalent to the gain margin.

Example

Consider a system requiring a loop gain of 200 in the useful operating band, which is required to extend up to 1 radian/second in frequency. If one specifies a gain margin of 12 dB and a phase margin of 40 degrees, **bodeopt.m** yields the bode plot for which the magnitude response appears in figure 3.4. For this example, the pole-zero excess is assumed to be 3.

A Matlab m-file has been written to calculate the sensitivity function of the ideal loop gain characteristic. This m-file is included in Appendix B. The magnitude response of the sensitivity function for the above example, as calculated using **sensit.m**, is plotted in figure 3.5.

Example

Including a single integrator at the origin changes the loop shape. If one considers the same system specifications for the above example and interprets the operating band as the region providing a minimum loop gain, up to a certain frequency point, **bodeopt.m** yields the bode plot in figure 3.6. The corresponding magnitude response of the sensitivity function for the above example, including the single integrator, is plotted in figure 3.7. The variance of a process variable, controlled by an optimum feedback controller, can therefore be calculated from equation 3.7 as:

$$\sigma_y^2 = \frac{1}{\pi}\int_0^\pi |S(j\omega)|^2 \times \Phi_{dd}(\omega)\, d\omega \tag{3.9}$$

The variance of a process variable calculated in this manner can be considered to be the best possible using feedback as compensation, and as such, indicates the maximum performance (benefit) feedback has to offer. Comparing this "best possible" variance value for a system under evaluation, with variance values obtained for existing or alternative control schemes, one obtains a quantitative measure of the performance of a control system relative to an optimum system.

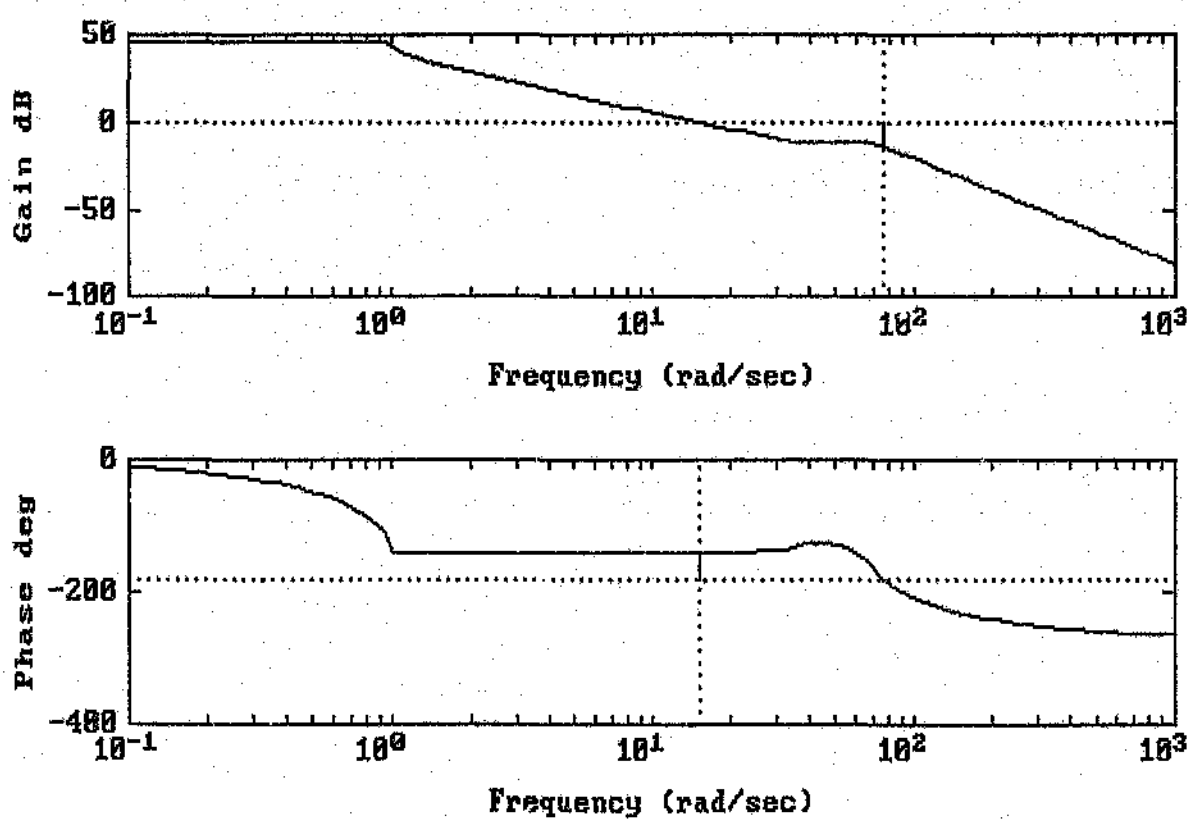

Figure 3.4: Optimum Loop Characteristic for a Unity Feedback System

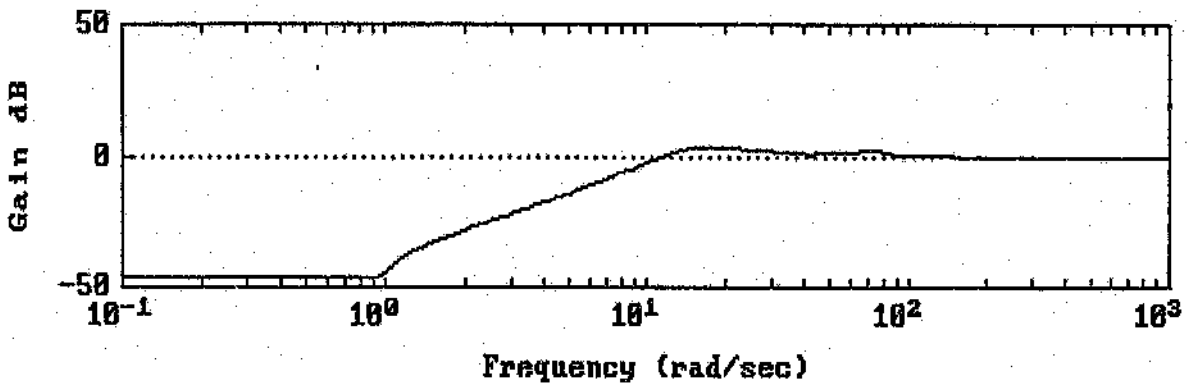

Figure 3.5: Sensitivity Function for Optimum Loop Characteristic

3.4.1 Controller Design

The design of a controller to meet specified performance criteria in an optimum manner, using the optimum loop characteristic proposed by Bode, can be achieved if the behaviour of the process is known. The disturbances likely to affect the system performance must also be characterized, so that the performance criteria the controller must meet can be specified.

The standard method of loop shaping can be used to design a controller for the known process, with the aim of producing a combined loop gain which approximates the optimum characteristic proposed by Bode.

Using the sensitivity function of the combined controller and known plant transfer functions, a measure of the performance of the control system design can be gained by calculating the variance of the process variable due to load disturbances. Care must be taken to ensure that the same frequency interval is used when multiplying the power spectral density with the square of the magnitude response of the sensitivity function.

3.5 Summary

Matlab routines and functions have been demonstrated as a means of quantifying the effects of deterministic and stochastic load disturbances. The quality of control of a system can be judged by the variation in the process variable, subject to such load disturbances.

For a signal with zero mean, the variation in process variable can be directly calculated from the area under the power spectral density of the disturbance and the sensitivity function.

It has also been shown that ideal characteristics for feedback amplifiers, developed by Bode, can be applied to unity feedback systems. Loop shaping can then be used to design a controller for a known process, with the combined loop gain

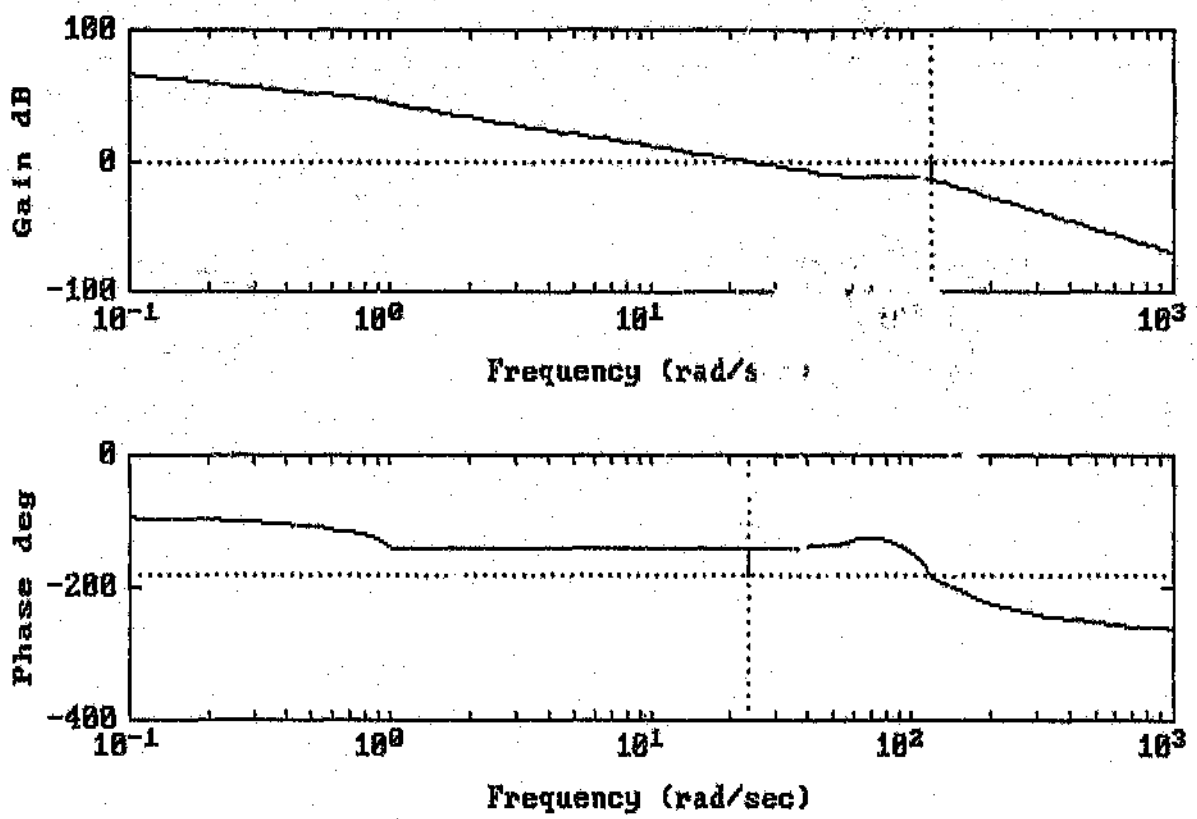

Figure 3.6: Optimum Loop Characteristics for Unity Feedback System with Integrator

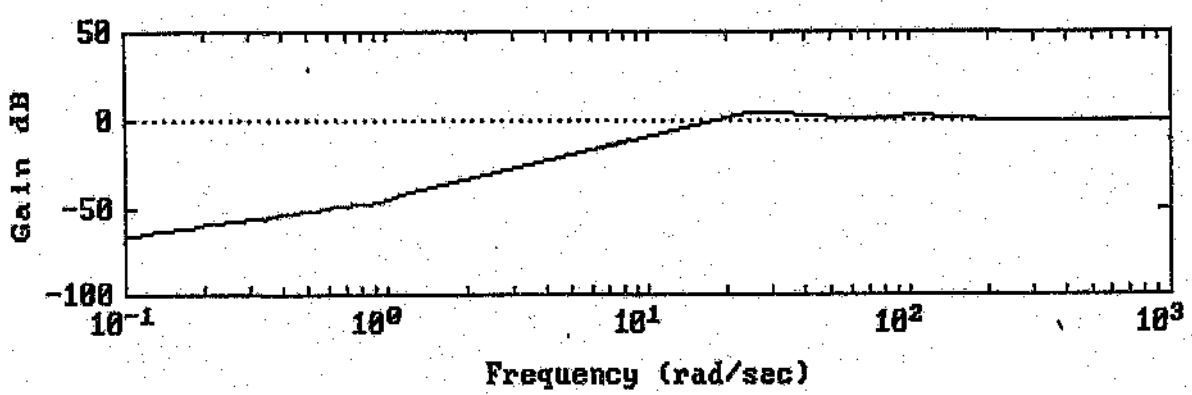

Figure 3.7: Sensitivity Function for Optimum Loop Characteristic with Integrator

approximating the optimum characteristic shape proposed by Bode.

Calculation of the process variable variance using the magnitude response of the sensitivity function for an optimum loop shape, gives an indication of the maximum achievable peformance feedback has to offer. The improvement in variance between an existing control scheme and the optimum feedback scheme can be used to quantify the potential benefits of applying advanced control strategies.

The financial value of these potential benefits will be quantified using performance functions in the next chapter.

Chapter 4

Quantifying the Benefits

4.1 Introduction

This chapter discusses the formulation of an overall performance function for a system. The performance function, relating the operating point of the process variable to financial value, is derived from knowledge of profit and penalty functions for the system. Linear, quadratic and higher order polynomial performance functions are investigated. The operation of the process variable close to an imposed system constraint is also considered. The benefits of improved regulation and optimization are then quantified using the derived performance function.

To conclude the chapter, a brief discussion of the estimation of the costs required to implement control strategies is presented. Once the financial benefits and the associated costs of implementation for a control system have been quantified, financial analysis can then be performed. This chapter presents the benefits of improved performance of the controlled system and the costs of implementation in a form suitable for financial evaluation.

4.2 Performance Functions

4.2.1 Regulation

Referring to Chapter 3, the main objectives of a control system are to keep changes in the measured process variable from a desired set point small, for changes introduced by load and set point disturbances. The performance of a control system can therefore be judged by the ability of the controller to maintain the process variable close to the desired value or set point—a process known as *regulation*. The variation of a process variable about the desired set point therefore indicates the quality of control (or lack of it!) and hence indicates the overall performance of the system.

If one considers the variation of a process variable about the set point to vary due to the influence of random disturbances, statistical parameters can be used to characterize this random behaviour. The gaussian, or normal, distribution fits many different types of data and statistical applications; it will be assumed for this project report that the variation of a process variable about the set point will approximate a normal distribution. The time-varying process variable signal can then be completely described by a mean value and a standard deviation. The statistical description of a signal is presented in figure 4.1

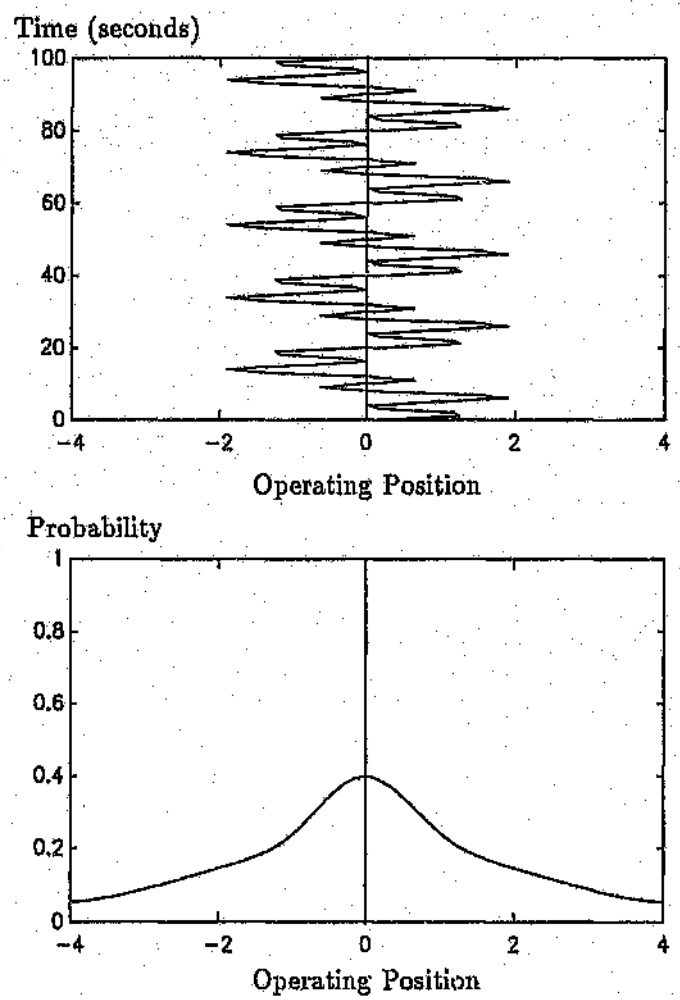

Figure 4.1: Statistical Representation of Time-Varying Process Signal

The aim of a control system is therefore to maintain the mean value of the variable at, or close to, a desired value and to minimize the variation of the signal about this value, despite the influence of disturbances. Minimizing the variation amounts to minimizing the standard deviation. An improved control system will reduce the standard deviation of the measured process variable and will result in financial benefit if a value is associated with tighter control. This potential for financial benefit is reflected in the performance function for the process.

4.2.2 Optimization

Optimization can be considered as the process of identifying and enforcing optimum set points for the controller. These optimum set points may correspond to conditions of maximum profit, minimum cost, maximum savings or any other economic objective set for the process. Optimization consists of moving the suboptimal operating point of a process variable to a new, optimum position which satisfies the economic objective.

Constraints and limitations may be imposed on the process of optimization by physical limitations of the process itself or by economic constraints affecting the financial performance of the process, if a limit or threshold value is exceeded. The result of these constraints is that the control set point may have to be moved away from the limit, in order to satisfy the overall economic objective. Figure 4.2 illustrates the control of a process variable subject to an operating constraint. Note the small, finite probability of exceeding the limit as represented by the area of the tail of the normal distribution beyond the constraint.

An improved control system will reduce the variation of the process variable and allow the operating point to be moved closer to the limit, without increasing the probability of exceeding the limit and without incurring the associated financial penalty. The control improvement has a corresponding financial benefit dependent on the value of the difference in operating postitions. The relationship between the operating position and financial performance is described by the performance function.

The handling of constraints will be discussed, in greater detail, later in this chapter.

4.2.3 Polynomial Performance Functions

If the performance of a system can be related to the economics of the process, an improvement in regulation and the benefits of optimization can be quantified in financial terms. If the assumption is made that performance is dependent on only one key process variable, for a particular control application, a relationship can be established which relates the operating postition of the key variable to economic performance. This relationship may be described in terms of a profit, cost (or penalty) or savings function, depending on the objective set for the process. These functions will be collectively referred to as performance functions.

A standard method for deriving performance and economic relationships cannot exist, because differences in processes and business objectives prevent any general observations from being made about the economic implications of improved control. However, if such a performance function can be derived for a specific application, the benefits of improved regulation and optimization can be quickly assessed.

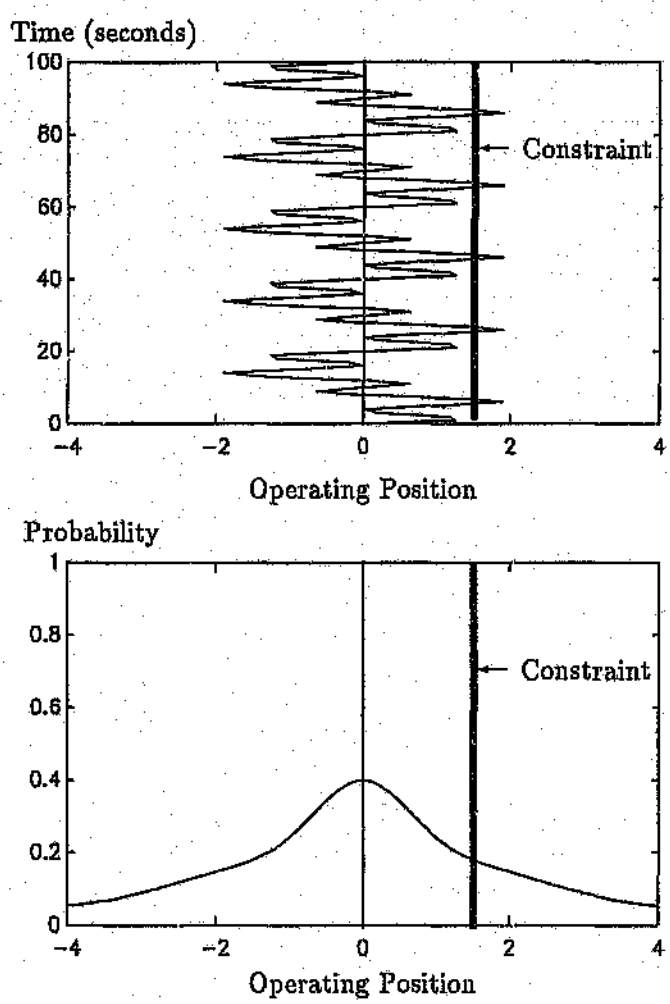

Figure 4.2: Time-Varying Process Signal subject to a Constraint

If the performance of a system can be expressed, or approximated, by a polynomial function of the key measured process variable y, then the benefits attributable to improved regulation and optimization can be calculated. Consider the mean value of the key process variable, y :

$$\mu_y = \varepsilon\{y(t)\} \tag{4.1}$$

and that y experiences positive and negative variations from this mean value μ_y, denoted by d.

The time-varying process variable signal, such as the example illustrated in figure 4.1, can then be written as:

$$y = \mu_y + d$$

The expected value ε, is defined as the mean value of a variable over a large number of observations. It follows that:

$$\begin{aligned} \varepsilon\{y\} &= \varepsilon\{\mu_y + d\} \\ &= \varepsilon\{\mu_y\} + \varepsilon\{d\} \end{aligned}$$

From the definition, $\varepsilon\{y\} = \mu_y$, it follows that:

$$\varepsilon\{d\} = 0$$

Also,

$$\begin{aligned} \varepsilon\{y^2\} &= \varepsilon\{\mu_y^2 + 2\mu_y d + d^2\} \\ &= \varepsilon\{\mu_y^2\} + 2\varepsilon\{\mu_y d\} + \varepsilon\{d^2\} \\ &= \mu_y^2 + 2\mu_y \varepsilon\{d\} + \varepsilon\{d^2\} \\ &= \mu_y^2 + \varepsilon\{d^2\} \end{aligned}$$

By definition, $\varepsilon\{d^2\} = \sigma^2$, where σ is the standard deviation.

Therefore,

$$\varepsilon\{y^2\} = \mu_y^2 + \sigma^2 \tag{4.2}$$

Linear Performance Function

A linear performance function, as discussed by the Warren Centre Report work, can be expressed as a polynomial in process variable y:

$$p(y) = my + C$$

The average performance value can be written as:

$$\begin{aligned} \mu_p &= \varepsilon\{p(y)\} \\ &= \varepsilon\{my + C\} \\ &= \varepsilon\{my\} + \varepsilon\{C\} \\ &= m\varepsilon\{y\} + C \\ &= m\mu_y + C \\ &= p(\mu_y) \end{aligned}$$

The average performance of a system, described by a linear performance function, is therefore the value of the performance function evaluated at the mean operating position, μ_y. It can be seen that the average performance is independent of any variation σ in y. Therefore, reducing the variation in y with improved control will have no effect on the performance of the system and will have no direct financial benefit, for a linear performance function. However, if constraints exist for the system, reducing variability will allow the operating position of y to be moved closer to the constraint.

Quadratic Performance Function

A quadratic performance function, typical of combustion efficiency versus excess air and the hydrogen-nitrogen ratio in the synthesis of ammonia, can be expressed as a polynomial in process variable y as:

$$p(y) = Ay^2 + By + C$$

The performance function has either a maximum or a minimum value where $\frac{dp}{dy} = 0$, ie. at:

$$y = -\frac{B}{2A}$$

The average performance value can be written as:

$$\begin{aligned} \mu_p &= \varepsilon\{p(y)\} \\ &= \varepsilon\{Ay^2 + By + C\} \\ &= A\varepsilon\{y^2\} + B\varepsilon\{y\} + C \end{aligned}$$

Applying equations 4.2 and 4.1:

$$\mu_p = A(\mu_y^2 + \sigma^2) + B\mu_y + C \tag{4.3}$$

$$= p(\mu_y) + A\sigma^2 \tag{4.4}$$

It can be seen that the average performance has two components. The first component is a contribution from the performance calculated at the average operating value of y; the second is a contribution from the variation in y. In order to maximize or minimize the value of the average performance, equation 4.4 should be equated with the performance function evaluated at $y = -\frac{B}{2A}$. It follows, therefore, that if the variance is non-zero, the average operating position must be moved away from the optimum operating point to achieve the best possible average performance:

$$p(\mu_y) = p(-\frac{B}{2A}) - A\sigma^2 \tag{4.5}$$

Higher Order Polynomials

Higher order performance functions can be treated in a similar fashion. It can be shown that for a cubic function of the form:

$$p(y) = Ay^3 + By^2 + Cy + D$$

that the average performance is:

$$\begin{aligned}\mu_p &= \varepsilon\{p(y)\} \\ &= p(\mu_y) + \sigma^2(3A\mu_y + B) + A\varepsilon\{d^3\}\end{aligned}$$

In practice, the performance functions of many applications can be approximated by linear and quadratic polynomials over the region of interest. The additional effort required to develop higher order polynomials may not always be justified.

In Chapter 3 it was shown that the variance in process variable can be calculated from the area under the power spectral density and the sensitivity function. This variance can be used to calculate the average performance of a system for quadratic and higher order polynomial performance functions. In the special case of a quadratic performance function, the variance and the performance function evaluated at the mean operating position completely describes the average performance for the system. However, for higher order polynomials, additional information is required before the average performance can be completely described. The cubic polynomial, for example, requires additional information regarding the $\varepsilon\{d^3\}$ term.

Based on this information, it was decided to concentrate on linear and quadratic performance functions during this research project. It is recommended that higher order polynomials be investigated in possible future research studies.

4.3 Constraint Handling

The process or economic objectives set for a process may fix maximum values for the operating positions of the process variable y. Maximum values may exist because of: product quality specifications, safety limits, physical limitations of equipment or environmental legislation. If this maximum limit cannot be exceeded under any circumstances, the operating position of y must be moved away from the limit, such that this limiting region is never entered. If the maximum value can be exceeded, an associated cost penalty is normally paid for doing so.

It has already been shown that for a linear performance function, reducing the variability of variable y alone has no benefit. If however, a constraint exists, reducing the variation will allow the operating position of y to be moved closer to the maximum limitation. This principle is clearly shown in the accompanying figure 4.3.

The improvement in control, resulting in a change in the operating position, has brought about a financial benefit which can be directly read off the performance function curve:

$$\Delta\mu_p = m\,\Delta y$$

where

$\Delta\mu_p$	=	change in average performance
Δy	=	change in operating position of y
m	=	slope of linear performance function curve

The calculation of variance, derived in Chapter 3 as being the area under the power spectral density curve and the sensitivity function, can thus be used to indicate the average performance possible for a system described by a linear performance function, subject to a constraint.

Latour [5], in his discussion of the control of the blending of fuel oils, starts off with a linear profit function with a specification limit as shown in figure 4.3 and

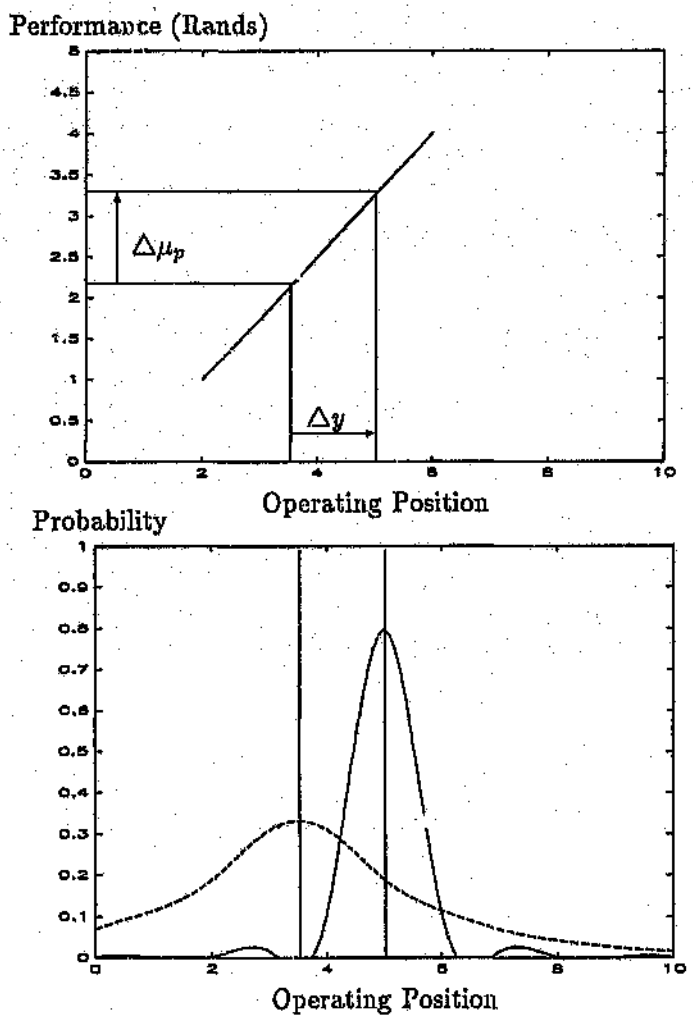

Figure 4.3: Constraint Pushing for a Linear Performance Function

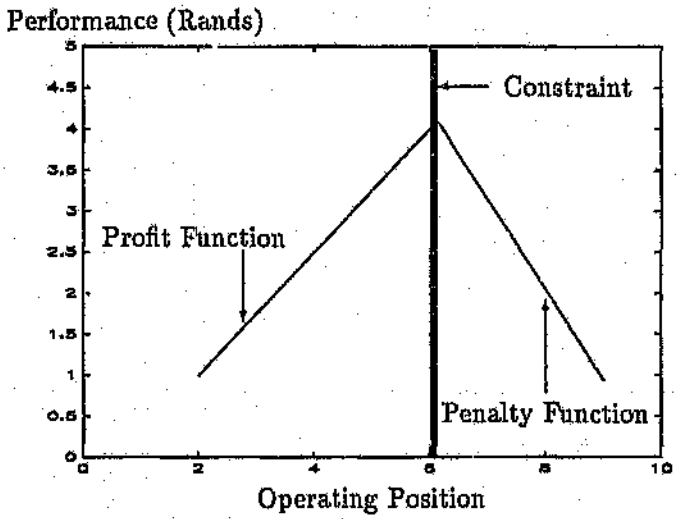

Figure 4.4: Penalty Function for Process Variable exceeding Specification

then adds a penalty function when the process variable exceeds the specification limit, see figure 4.4. The optimum operating point for the process variable is chosen which offers the best comprimise between the profit and penalty functions for a given variation. A signal with a large variation must be moved away from the constraint; a signal with a small variation can be moved closer to the limit.

If a polynomial is fitted to the combined profit and penalty functions, one has an overall performance function which includes the handling of constraints. A first approximation to the combined function could be a quadratic polynomial, or if greater accuracy is required, higher order polynomials can be fitted. In this project, a quadratic polynomial is fitted to the combined profit and penalty function. The variation of process variable together with the performance calculated at the mean operating position therefore completely describes the average performance of the system. The fitting of a quadratic polynomial to a linear profit and a linear penalty function is illustrated in figure 4.5

4.4 Cost Estimation

Cost estimation is a vital part of investment analysis; without costs, the profitability of projects yielding benefits cannot be assessed. The timing of these costs and when payments fall due must also be known before financial analysis can be performed. [17] outlines a number of methods for determining an estimate of the capital requirements for a project, depending on the level of accuracy required. Whichever method is selected, an estimate of the capital costs required to implement a control project must be obtained before the economic evaluation stage can be entered.

The focus of this research project is on predicting, quantifying and evaluating advanced control benefits. Although cost estimation is an important consideration, it is a vast area in itself and will not be investigated in this report.

Perry [9] can be referred to for estimating the capital cost of process equipment

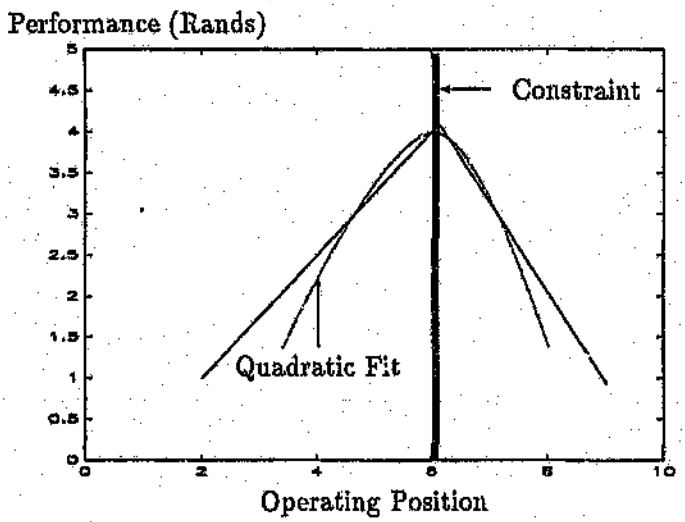

Figure 4.5: Quadratic Fit to Linear Function with a Penalty Constraint

and general instrumentation, but the estimation of advanced control application costs will require additional research work in the future.

4.5 Summary

The development of a performance function relating the operating point of a process variable to financial value and performance was investigated. It was concluded that linear and quadratic polynomials can be used to adequately describe the relationship between operating point and financial value in many applications. If a system is limited by a constraint, it is possible to develop an overall performance function which includes the financial impact of the constraint.

The variation and operating position of a process variable describes the average performance of a system if a normal distribution is assumed for the variable. Improved control offers the potential to reduce variations through better regulation and to move the operating point closer to the system constraint (optimization) and the financial benefit of these changes is reflected in the value of the improved average performance as described by the overall performance function.

In the following chapter, the relative value of different control projects will be evaluated and a methodology developed for the overall ranking of control opportunities.

Chapter 5

Ranking of Control Opportunities

5.1 Introduction

All projects, control upgrades and others, compete for the resources and capital made available by management investment opportunities. It is therefore the task of managers to allocate funds in such a manner that the profitability of the organization is maximized. A methodology for quantifying and presenting the benefits of projects and investments, specifically control upgrades, will assist in this decision-making process and in the correct allocation of funds.

The aim of this chapter is to present various measures of profitability and to select the most appropriate method for determining the profitability of control projects. The method selected must be flexible to cater for the wide range of different control project types and suitable for computer implementation. Both traditional and modern methods of profitability are investigated as a source of methods.

By correctly assessing the profitability of control investment opportunities under consideration, an overall ranking can be generated which will greatly assist with the task of allocating resources for investment opportunities.

5.2 Traditional Measures of Profitability

Rate of Return and the payback period methods have been widely used in the past for estimating the profitability of projects. Despite the advent of more modern methods for determining profitability, these techniques are still applied because of their simplicity and ease of use. Although the traditional methods have the advantage of being easy to use, they can provide very misleading results and should only be used if their limitations are known.

5.2.1 Rate of Return

The Rate of Return method, also commonly known as the Return on Investment (ROI) and the Return on First Cost, is a measure of the annual rate of return generated by an investment and the total original capital employed.

Rate of return is calculated, expressed as a percentage, as follows:

$$Rate\ of\ Return = \frac{Annual\ Profit}{Capital\ Investment} \times 100\% \tag{5.1}$$

The annual profit is normally calculated as an average once the project has reached maturity. The profit may either be calculated before or after tax has been deducted, and hence the method of calculation must be clearly indicated. Rate of return, is therefore the average annual profit, taken over the productive life of the plant, expressed as a percentage of the total initial investment.

Variations on the Rate of Return method include taking annual profit for a particular year (not on an average basis) and including depreciation of the initial investment as a charge against the annual profit. It is therefore important for the precise method of calculation to be known before comparisons can be made between the rates of return of different projects or investments.

Disadvantages

1. The Rate of Return method assumes that the income or profit generated by the project is reasonably constant over the project lifespan. Furthermore, the assumption is made that the capital outlay is made at one point in time. It is unlikely that a project will display such simple characteristics.

2. The method cannot reflect the advantages of receiving income earlier eg: early commissioning of projects.

3. The effects of replacing or expanding upon capital equipment after commissioning cannot be measured.

4. No account is taken of the residual value of the investment at the end of the project life.

Control upgrade projects do not display the simple characteristics required for the correct interpretation of Rate of Return methods.

5.2.2 Payback Period

Payback period is defined as the time taken for the cash generated by the project to recoup the original capital expenditure. Alternatively, the payback period can be thought of as the period of time during which the original capital is at risk, before the capital sum is regained by way of cash generated or savings realized.

Payback or payout period can be calculated as follows :

$$Payback\ Period = \frac{Original\ Capital}{Annual\ Income - Depreciation} \tag{5.2}$$

The annual income used to determine the payback period is normally calculated on an after tax basis. If a different definition of annual income is to be used , this must be clearly indicated in any comparative studies.

The payback period defines a breakeven point in time for the project. Prior to the breakeven point the project is still in overall debt; beyond the breakeven point the project shows an increasing profit. It follows, therefore, that the sooner the

breakeven point is reached (the shorter the payback period), the more attractive the project appears to be.

Payback period can also be calculated from cash flow considerations. Cash flow analysis and the determination of payback period from cash flows will be discussed in greater detail in section 5.3.

Disadvantages

1. The Payback Period method ignores the time period after the breakeven point has been reached! No account is taken of income generated or savings realised after the original capital has been regained. The underlying assumption for this method is that if a project exhibits an attractive payback period, the cash generated by the project after the breakeven point will continue at a favourable rate. This assumption is unlikely to be true for the control upgrade project.

2. Fluctuations in the income generated or the savings realised over time cannot be accounted for. Capital replacements or expansions also cannot be evaluated using this method.

3. No account is taken of residual value of the investment at the end of the project life.

4. Using the Payback Period as a single criterion of profitability places emphasis on short term income to the detriment of possible longer term, growth orientated projects. Payback period, if used exclusively, favours liquidity rather than profitability.

5.2.3 Annual Return on Capital Employed

This method is a measure, in each year, of the annual profit after tax expressed as a percentage of the capital outstanding in that year.

$$\textit{Annual Return on Capital Employed} = \frac{\textit{Annual Profit}}{\textit{Original Capital} - \textit{Depreciation}} \times 100\%$$

The original capital amount is depreciated by an annual depreciation charge. The capital employed, determined each year, is the decreasing (or declining) balance of the original capital minus the cumulative depreciation amount allowed up to the year of calculation.

Consider a 5 year investment requiring a capital investment of 100 units, returning an annual profit of 20 units. Assuming straight-line depreciation, the capital employed each year would be 100, 80, 60, 40 and 20 units respectively. The annual return on capital employed for each corresponding year would be 20%, 25%, 33%, 50% and 100% per annum.

This method is often used to present overall long term planning goals where the annual cash generated versus the capital requirements can be represented.

Disadvantages

1. The Annual Return on Capital Employed does not allow for easy comparison between investment opportunities because it yields varying profitability values

for each opportunity. No single index of profitability is given for a particular opportunity, thus making decision-making difficult.

2. An increasing profitability value is indicated despite a constant net income. It is preferable to have a method which indicates a constant profitability value for a constant net income.

3. During times of inflation, this method tends to indicate profitability results which are very optimistic. Hicks and Bridgwater [18] show graphically that the Return on Capital Employed method consistently indicates higher returns than other methods.

5.2.4 Summary of Traditional Methods

The traditional methods for measuring the profitability of projects do not take into account the time value of money. Due to this limitation, the traditional methods provide very misleading results when used in the selection of investment opportunities.

The Rate of Return method assumes a constant income over the life of the project and a capital outlay at one point in time; the Payback Period method ignores the time period after the capital investment has been repaid. Control upgrade projects do not display the simple characteristics required for the correct application and interpretation of the traditional methods.

5.3 Cash Flow Analysis

Projects cost money, the spending of which must usually be justified in terms of the expected benefits. These benefits may be direct returns in profit or indirect benefits such as increased public esteem, political advantage or enhanced social value.

Cash flow analysis is based on estimates of the monetary costs, the timing of the effort and the resources the project will consume, together with the monetary value and timing of the resultant benefits. Estimates are combined in an appropriate manner to find the pattern of cash flow throughout the anticipated life of the project.

The expenditures of money on capital and the returns of money from profit are termed *cash flows*. By convention, expenditure (cash outflow) is negative and income (inflow of cash) is positive. Cash flows are normally assumed to be received on an annual basis at the end of the financial year. Taxation charges and depreciation allowances can therefore all be brought to a common reference point in time. To allow for the initial capital investment signifying the start of the project, it is common practice to assume the initial cash flow occurs at the end of the financial year 0 (zero). Even though this initial cash flow is realized at the beginning of year 1, the notation of using year end results is consistent. In order to construct a cash flow statement for a project, the following information must be available:

1. Knowledge of the project investment capital (or principal), plus the timing thereof.

2. Knowledge of the project returns or savings, plus the timing thereof.

3. Knowledge of depreciation allowances and the relevant tax laws.

4. Knowledge of the intention to sell off assets at the end of the project life.

Example

Consider a project requiring a fixed capital investment of R50 000 payable over two years, with a working capital committment of R10 000. The estimated life of the project is 10 years and there will be no salvage value. If the estimated profit before tax in each of the ten years is known and straight line deprecication and a 50% tax rate is assumed, the cash flow statement can be constructed. Table 5.1 presents the cash flow statement.

Year	Capital	Depreciation	Profit	Tax	Cash Flow
0	-25 000				-25 000
1	-35 000	5 000	10 000	2 500	-27 500
2		5 000	20 000	7 500	12 500
3		5 000	30 000	12 500	17 500
4		5 000	20 000	7 500	12 500
5		5 000	25 000	10 000	15 000
6		5 000	30 000	12 500	17 500
7		5 000	30 000	12 500	17 500
8		5 000	30 000	12 500	17 500
9		5 000	25 000	10 000	15 000
10	+10 000	5 000	20 000	7 500	22 500

Table 5.1: Preparation of a Cash Flow Statement

The working capital includes raw materials and a spares holding required to start up the plant during commissioning. Working capital is recoverable at the end of the project as all raw materials and spares will have been converted into saleable products.

The characteristic pattern of cash flow for a project in time, can be identified by plotting the cumulative cash flow versus time, as illustrated in figure 5.1 for the above example. It should be noted that not all projects earn income or profit from sales. Projects can also be considered as generating income through savings in expenditure. Savings in expenditure are benefits which can be treated as positive, incoming cash flows for a project. The so-called control upgrade project can fall into both categories depending on the application. A cash flow statement can be prepared for both types of project including those which generate income through savings in expenditure.

Example

Consider a project which requires a capital investment of R50 000 payable over two years. The project yields maintenance savings of R15 000 per year for the full 10 year life span of the project. Assuming that there will be no salvage value at the end of the project life, the cash flow statement for the project can be constructed. Table 5.2 presents the cash flow information.

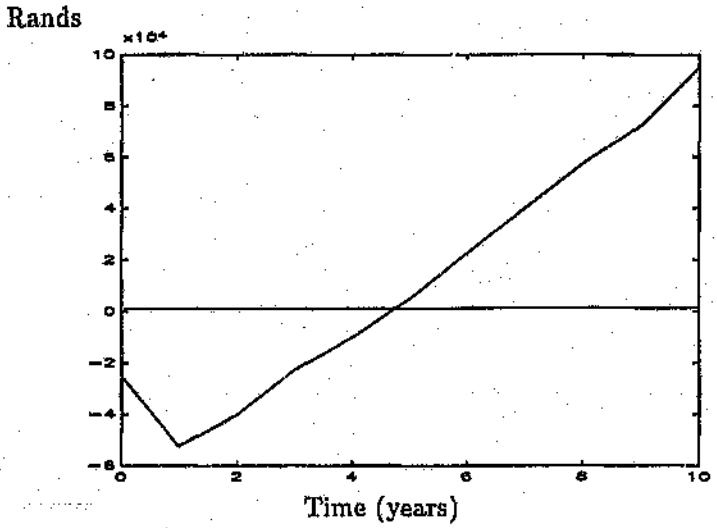

Figure 5.1: Cumulative Cash Flow Digram

A control upgrade is characterised by additions, replacements or improvements to existing plant equipment. Objectives for the control system may be to achieve savings in operating costs, to increase production or to meet other objectives. Examples of typical control projects might include:

1. The replacement of an existing plant control system to reduce operating and maintenance costs.
2. To uprate the existing facility to produce more product or to improve the product quality.
3. The introduction of new control measures to prevent the illegal discharge of waste to the environment.

Consider the pattern of cash flowing in the implementation of a control upgrade project. The e rly stages of the project consist of development and design work. These preliminary activities involve the expenditure of money and hence the cumulative cash flow diagram goes negative. This phase is followed by the purchase and installation of capital equipment. The rate of expenditure during this procurement phase is high and the cumulative curve drops steeply. A slowing down of the rate of expenditure is characteristic of the commissioning phase. The minimum point on the curve represents the maximum cumulative expenditure (or maximum debt incurred by the project). Beyond this minimum point, the income resulting from savings through, for example, improved operation or better quality product are realized. The positive incoming cash flows turn the cumulative curve trend and, hopefully, the incoming cash flows will exceed the maximum expenditure. As the cumulative cash flow curve changes from negative to positive, a breakeven point for the project is reached. The breakeven point can be thought of as the payback period for the project. At this point, the total income received from the project balances out the project expenditure. For a profitable project, the project continues to accumulate

Year	Capital	Savings	Cash Flow	Cumulative
0	-25 000		-25 000	-25 000
1	-35 000	15 000	-20 000	-45 000
2		15 000	15 000	-30 000
3		15 000	15 000	-15 000
4		15 000	15 000	0 000
5		15 000	15 000	15 000
6		15 000	15 000	30 000
7		15 000	15 000	45 000
8		15 000	15 000	60 000
9		15 000	15 000	75 000
10	+10 000	15 000	25 000	100 000

Table 5.2: C sh Flow Statement for Savings Project

net income beyond the payback period. Towards the end of the project life the rate of cash inflow may diminish due to increased maintenance costs as the project components show signs of wear. The characteristic pattern of cumulative cash flow for a control project is illustrated in figure 5.2. Note that the payback period can be read directly off the cumulative cash flow diagram, where the cumulative cash flow equals zero. The payback period is defined as the length of time required for the project to recoup the original capital expenditure—the cumulative cash flow diagram graphically depicts this value.

The calculation, preparation and presentation of cash flow information is well suited to the spreadsheet format offered by commercial Personal Computer programs such as Lotus 1–2–3 and Borland's Quattro–Pro. Matlab is however retained as the computational and presentation environment for this research project, to provide the user with an integrated environment for the analysis of control projects.

5.4 Modern Profitability Measures

The modern methods for determining profitability aim to take into consideration *the time value of money*. The traditional methods rely on the assumption that the initial investment is made at one point in time and that income is received continuously throughout the life of the project. Very few projects exhibit the characteristics required for this assumption. The modern methods overcome this limitation by considering the costs and benefits of a project as well as the timing of these cash flows.

A brief description of the principle of discounting is used to introduce the modern measures of profitability, namely the Discounted Cash Flow Rate of Return (DCFRR) and Net Present Value (NPV) methods.

5.4.1 The Principle of Discounting

In chapter 2 the concept of *the time value of money* was introduced; money earned now is worth more than money earned later. Using the compound interest formula,

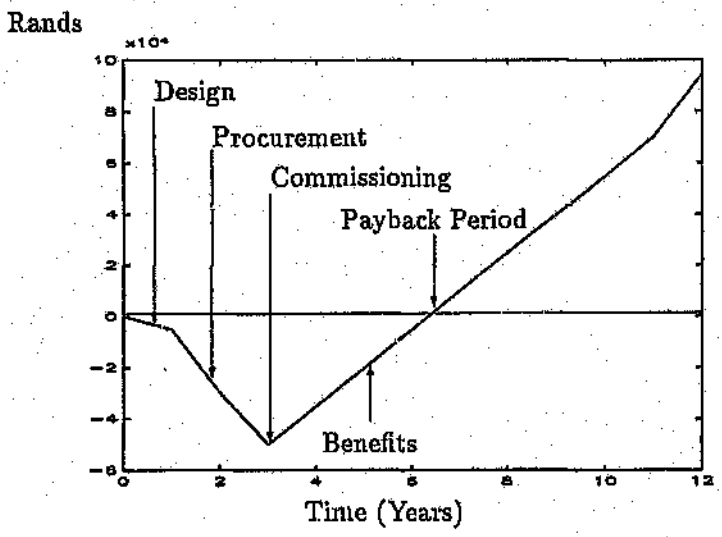

Figure 5.2: Characteristic Cumulative Cash Flow Pattern for Control Projects

it was found that the potential future value of capital invested at an interest i, was greater than the present value by a factor:

$$Factor = (1+i)^n$$

Discounting is the inverse operation to compounding. Money received in the future is discounted back to the present using an appropriate discount factor. Remembering that money received in the future is worth less than money received in the present because of lost investment opportunities, the appropriate discount factor is therefore:

$$Discount\ Factor = \frac{1}{(1+i)^n} \tag{5.3}$$

Discounting accounts for the time value of money in cash flow analysis by rewarding the project which receives money early and penalises the project where benefits are delayed. Applying the compound interest formula:

$$PV = \frac{FV}{(1+i)^n} \tag{5.4}$$

PV	=	present value of money received in the future
FV	=	future value of money as it is received
i	=	interest rate (interest of other opportunities)
n	=	number of periods specifying investment lifespan

Example

If a project earns R 100 a year for 10 years, the R 100 earned in the first year must be worth more than that received in the tenth year. Assuming that the interest rate of an opportunity lost to the project is 10%, then the value of the R 100 earnad in the first year is:

$$PV = \frac{100}{(1+0.1)^0} = 100$$

The value of the R 100 earned in the final year of the project is :

$$PV = \frac{100}{(1+0.1)^{10}} = 38.55$$

The procedure to apply discounting, is to draw up a series of cash flows for the lifespan of the project and then to discount each cash flow by the appropriate discount factor. The chosen interest rate for this factor could be the cost of capital (cost of borrowing money) or the rate of return an alternative investment may yield. The discounted cash flows for the Savings Project example in section 5.3 are presented in table 5.3 for a discount rate of 10%.

Year	Cash Flow	D.Factor	D.Cash Flow	D.Cumulative
0	-25 000	1.0000	-25 000	-25 000
1	-20 000	0.9091	-18 182	-43 182
2	15 000	0.8264	12 397	-30 785
3	15 000	0.7513	11 270	-19 515
4	15 000	0.6830	10 245	-9 270
5	15 000	0.6209	9 314	44
6	15 000	0.5645	8 467	8 511
7	15 000	0.5132	7 697	16 208
8	15 000	0.4665	6 998	23 206
9	15 000	0.4241	6 361	29 567
10	25 000	0.3855	9 639	39 206

Table 5.3: Discounted Cash Flow Statement for Savings Project

Plotting the discounted cumulative cash flows for a project versus time and comparing this plot to the undiscounted plot, one can see in figure 5.3 that discounting makes projects appear less attractive. The interpretation of this plot will be discussed in a later section.

Discounting supports the two principles of making money:

1. Receive or collect money early. Early collections will not be effected by discounting.

2. Pay later. Money paid in the future will be discounted by an appropriate factor.

Discount factors at various interest rates and period intervals are available in printed tables, but the advent of financial calculators and spreadsheet computer programs have superseded the use of these tables.

A disadvantage of the cash flows analysis method, is that the cash flows for the entire duration of the project or investment must be known. If the cash flows are not known exactly, they must be at least estimated within a fair degree of accuracy. Discounting, however, provides a mechanism whereby future cash flows have a smaller impact than present cash flows. Discounting therefore protects one from large inaccuracies and assumptions made when determining future returns. Conversely, one must have accurate estimates of present and not-too-distant costs and benefits, as discounting weights these costs and benefits heavily.

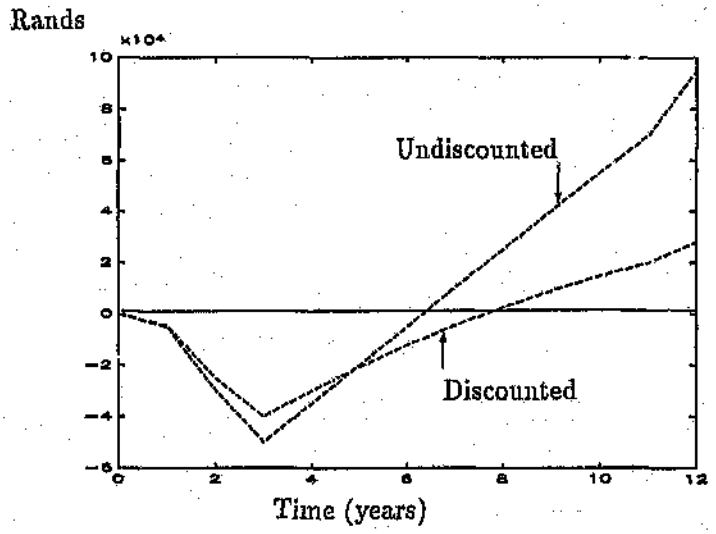

Figure 5.3: Comparison of Discounted and Undiscounted Cumulative Cash Flows

5.4.2 Net Present Value

In the discussion of cash flow analysis, it was shown that in economic terms a project can be regarded as a series of cash flows occurring throughout the lifespan of the project. To take account of the time value of money, these cash flows can be discounted back to the present time for evaluation purposes.

The Net Present Value (NPV) of a project is defined as the cumulative sum of the present values of the annual cash flows:

$$NPV = \sum_{j=0}^{n} PV_j = \sum_{j=0}^{n} \frac{C_j}{(1+i)^j} \tag{5.5}$$

PV_j = present value of investment in year j
C_j = cash flow in year j
n = number of periods in project lifespan
i = interest rate (minimum acceptable rate of return)

The objective of the NPV method is to assess the profitability of a project in terms of a lump sum of money at the present time, after allowances have been made for the time value of money. The NPV is therefore the sum of discounted cash flows for the duration of the project. If the NPV of a project is zero, the sum of all discounted cash outflows or costs is balanced by the sum of discounted cash inflows or benefits. The physical interpretation of this result is that for an NPV of zero, the project returns the initial capital investment plus interest on the investment equivalent to the discounting rate i.

The discounting interest rate i can be interpreted in different ways. If the discount rate i is chosen to be the cost of capital, ie. the interest charged for borrowing money to fund the project, an NPV of zero indicates that the project has no benefit other than returning the initial investment back to the investor. If however the discount rate is chosen to be the minimum acceptable rate of return or "hurdle rate"

for project acceptance, then an NPV of zero implies that the project has met the minimum acceptable requirements as laid down by the organisation.

A positive NPV indicates that the project yields returns in addition to the minimum acceptable rate, whereas a negative NPV indicates that the minimum rate has not been achieved. In cases where the NPV has a large negative value, all of the initial investment capital may not even be recovered.

Referring to table 5.3, the NPV for the Savings Project for a 10% discount rate can be calculated as the cumulative sum of the discounted cash flow column entries, or can be read directly from the table as the last entry of the cumulative column, ie. R39 206.

The interpretation of the NPV method ties up very closely with the aims of an investor mentioned previously, who looks for the following returns from an investment:

1. Return of the initial capital investment — NPV should be at least zero.

2. Sums of money equivalent to the interest rate, i, that could have been earned from a risk-free venture. If NPV is zero and above, this interest rate has been achieved.

3. Extra sums of money as an inducement to undertake risk. An NPV greater than zero indicates the value of money offered as inducement.

The greater the expected positive value of a project, the more economically attractive the project is. If a project is to be implemented to achieve objectives other than returns on investment, eg. to conform to environmental regulations, then the project with the greater (least negative) NPV, will be the more economically attractive for this type of project.

5.4.3 Discounted Cash Flow Rate of Return

The Discounted Cash Flow Rate of Return (DCFRR) or Internal Rate of Return (IRR) is defined and calculated as the discount interest rate which yields an NPV equal to zero. The DCFRR is therefore the value of i which satisfies the non-linear equation:

$$NPV = \sum_{j=0}^{n} \frac{C_j}{(1+i)^j} = 0 \tag{5.6}$$

The value of i can be obtained by solving the equation using a recursive technique such as Newton-Raphson. A solution can also be determined graphically by interpolating between points plotted on an NPV versus discount rate graph. The points are determined by selecting various values of i, and then solving for the corresponding NPV. The DCFRR is found where $NPV = 0$.

DCFRR can be thought of as the discount rate at which the present value of the benefits balances the present value of the costs. The DCFRR is analogous to the concept of calculating a rate to repay a mortgage bond. Annual payments on a mortgage consist of both interest payments on outstanding capital and payments to repay the initial loan amount. These annual payments are calculated such that the total payments over the period of the loan, meet the interest and capital repayment obligations, at a prescribed rate of interest.

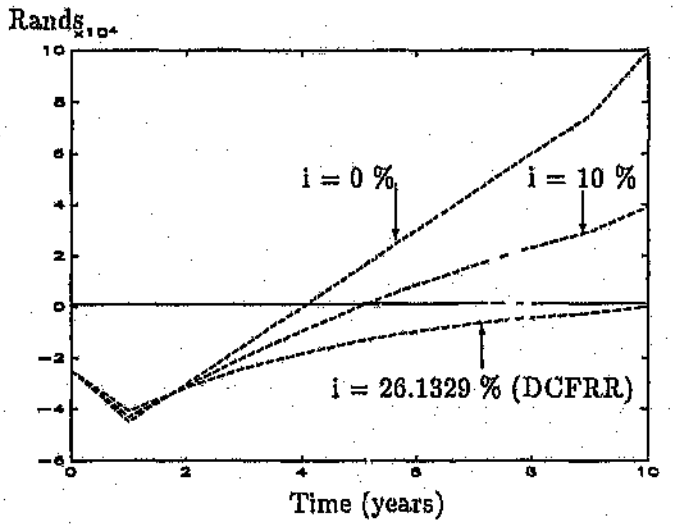

Figure 5.4: Cumulative cash flows discounted at different interest rates

Referring to the cash flow statement for the savings project in table 5.3, the DCFRR is 26.1329%. Figure 5.4 shows a graphical interpretation of the DCFRR when compared to cumulative cash flows discounted at different interest rates, i.

The DCFRR is used in practice as a single quantitative measure for comparing the profitability of projects. The higher the DCFRR, the more profitable the project.

5.4.4 Summary of Modern Methods

Two modern methods of profitability have been discussed, namely the discounted cash flow rate of return (DCFRR) and the net present value (NPV) method.

A risk-free project is economically viable if the NPV, discounted at the cost of capital, is positive, or if the DCFRR is greater than the cost of capital interest rate. Inducements to undertake a potentially risky project would have to be offered to an investor and these would be reflected in a greater positive NPV or a DCFRR significantly greater than the cost of capital.

Matlab routines to generate and plot cumulative undiscounted and discounted cash flow information, as well as the calculation of the DCFRR and NPV for projects, are provided in Appendix C.

5.5 Risk and Sensitivity Analysis

The information required to evaluate the economic potential of projects is based on estimates of costs and forecasts of benefits. Uncertainty will exist on the cost estimates, on the predicted benefits or savings and on external influences such as the inflation rate. Associated with uncertainty, is the economic risk that the project will not be as profitable as expected.

The object of investment decision-making is to select projects which have the highest probability of generating the highest returns, within limits dictated by the financial position of the organisation. Investment decisions made only on the basis

of maximum expected returns may not always be the best decisions. Risk and uncertainty must therefore be considered if correct decisions are to be made.

A simple approach to allow for uncertainty or risk when evaluating projects is to increase the "hurdle rate" for risky projects. However, to take risk and uncertainty into account in a more rigorous manner, the sources of uncertainty must be investigated. Two approaches to evaluate risk and uncertainty in economic evaluations are: sensitivity analysis and risk analysis (otherwise known as probability sensitivity analysis).

Sensitivity analysis is a means of evaluating the relative influence of parameter variations on the overall evaluation result. Individual parameters are changed one at a time and the effect of these changes on either the NPV or DCFRR calculation are observed. Sensitive parameters are identified as those parameters which have a large influence on the result. Sensitive parameters require the best possible estimates in order that the profitability calculation can accurately reflect the true economic potential of the project. [19] and [20] give details of how to apply sensitivity analysis to project evaluations.

Risk analysis involves assigning subjective probabilities to possible outcomes to quantify judgements about uncertain parameters. The assigned probabilities are combined to give an overall NPV or DCFRR result in terms of a probability distribution. One method to combine the parameter probability distributions is to use a computational technique known as Monte-Carlo simulation. The Monte-Carlo method involves performing numerous iterative calculations, solving for a DCFRR or NPV result using random parameter values selected from within user-defined, subjective probability distributions assigned to key parameters. In effect, all valid combinations of key variables are tested in order to simulate all possible outcomes. Monte-Carlo simulations can be performed on cash flow information in spreadsheet form using an add-in package for the Lotus 1-2-3 spreadsheet, known as @RISK. The @RISK package allows various probability distributions to be assigned to key variables and presents the simulated end result as a probability distribution of achieving a NPV or DCFRR value. The principles underlying Monte-Carlo simulation are developed in [21]. The @RISK user guide [22] is a useful reference, providing practical examples of risk analysis using simulation methods.

Analytical techniques can also be used to perform probability sensitivity analysis. These techniques require that probability distributions of uncertain variables be described mathematically and solved to yield an equation for the resultant distribution of possible outcomes. Analytical techniques are time-consuming and difficult to implement.

It should be noted that sensitivity and risk analysis do not provide definitive answers or decisions. A decision must still be made based on the information provided and targets and objectives set for an organisation. Risk analysis should never be substituted in place of sound judgement. A quantitative assessment of risk does, however, provide decision-makers with a better insight into the probability of achieving predicted benefits and results. Risk and sensitivity analysis is a vast area of research which has only been briefly discussed in this report. Further research into this field of study, specific to control project evaluations, is recommended.

5.6 Summary

The modern methods for assessing the profitability of projects, the Discounted Cash Flow Rate of Return (DCFRR) and the Net Present Value (NPV), provide a better insight into the financial prospects of projects than the traditional methods do, because they take into account the *time value of money*. These modern methods can be easily coded into a computational environment, such as Matlab, and lend themselves to graphical presentation and interpretation.

The cumulative cash flow diagram complements the modern methods of profitability by graphically presenting the information used in the DCFRR and NPV calculations. The combination of the DCFRR and NPV methods with the cumulative cash flow diagram provides a useful methodology for the assessment of control projects. The use of this methodology together with sensitivity and risk analysis greatly assists with the decision-making process of selecting the most profitable project opportunities for an organisation.

Chapter 6

Case Study

6.1 Introduction

The level control of a hypothetical two tank system in a chemical process plant within the AECI Group of companies is analyzed using the principles outlined in this research project.

A standard PID controller and a controller offering a loop shape approximating the ideal loop gain characteristic are designed. The benefits of both designs are identified and quantified by considering load disturbances likely to affect the system.

Financial analysis of both control systems is performed, based on knowledge of the financial position of the AECI Group. The control designs are then ranked according to their potential profitability and a design is proposed which will help to improve the overall financial position of the organisation.

6.2 Problem Statement

The two tank system under consideration is illustrated in figure 6.1. The side stream F_d, from the overflow of another process unit, can be considered to be a disturbance to the basic two tank system. The aim of the control scheme is to regulate the control valve such that the level in the second tank, h_2, is kept constant, despite the influence of the disturbance flow, F_d. F_{out} is constant, fixed by the delivery of the pump and $F = \alpha h_1$. The transfer function from output h_2 to input F_{in} for the level control of this two tank system is:

$$G(s) = \frac{1}{s(100s + 1)}$$

The derivation of the transfer function appears in Appendix D.

A block diagram of a unity feedback system to control the level in the two tank system is presented in figure 6.2. The known load disturbance is cyclical in nature and is modelled as a function of the form:

$$y(t) = 2\sin(2\pi 0.0032t) - 0.5\sin(2\pi 0.003t) + \sin(2\pi 0.0025t)$$

The power spectral density of the disturbance signal (with the mean value removed using **detrend**) appears in figure 6.3, as generated with the function **etfe**, using a 512 point FFT and no window. Calculating the area under the power spectral

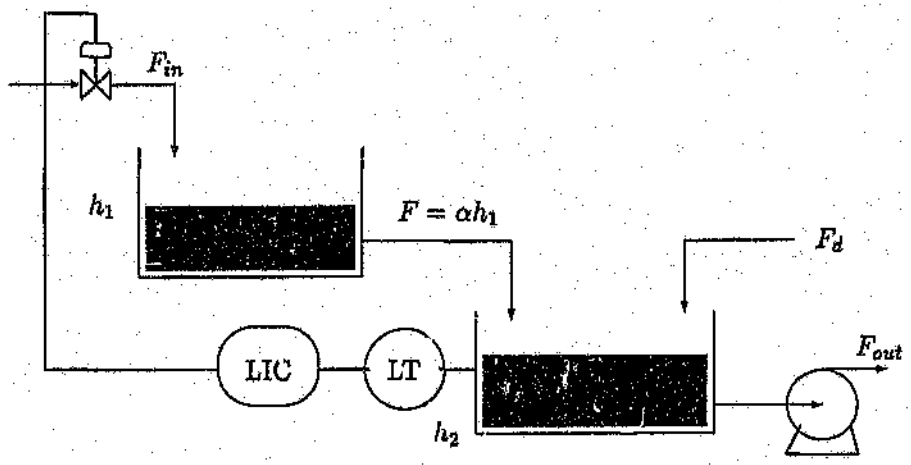

Figure 6.1: Case Study : Two Tank Level Control System

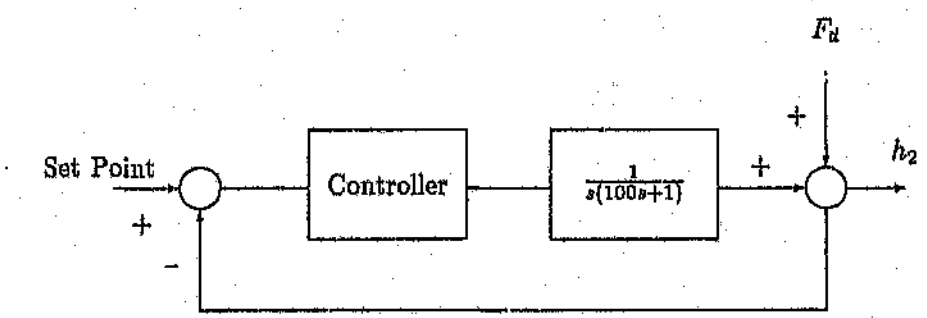

Figure 6.2: Unity Feedback System for Two Tank Level Control System

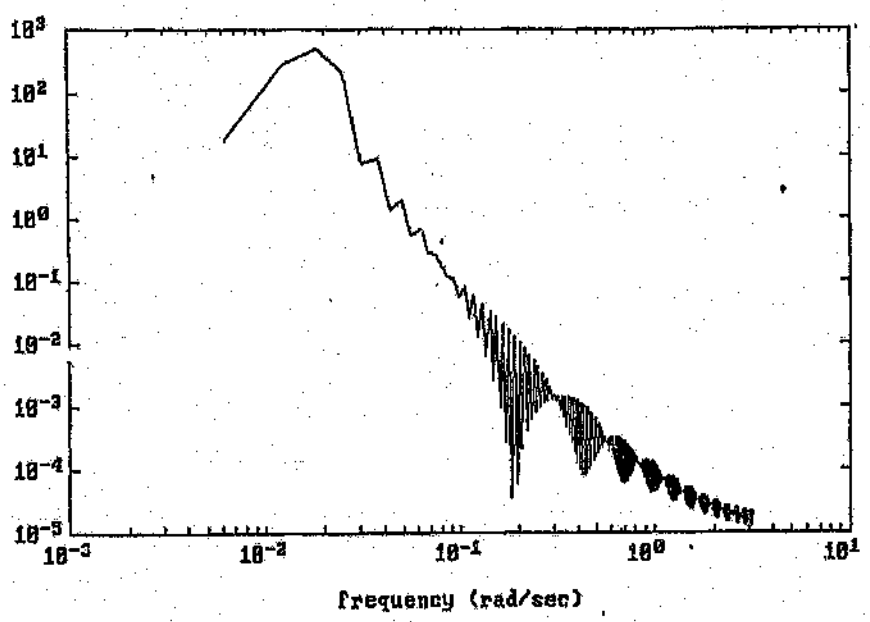

Figure 6.3: Power Spectral Density of Load Disturbance Model

density and comparing the result to the variance of the disturbance signal using statistical techniques, the numerical accuracy of the proposed technique can be validated. The results show that adequate accuracy can be achieved by careful selection of the sampling interval, FFT size and windowing technique. Using the Matlab statistical function **std**, the variance $\sigma^2 = 1.8863$, whereas calculating the area under the power spectral density, the variance $\sigma^2 = 1.8825$. The two tank system, without compensation, would pass this varying load disturbance through to the process variable without attenuation.

6.3 Controller Design

The bode plot of the uncompensated open loop system, as shown in figure 6.4, indicates a phase margin of 5.75 degrees, which would give a highly oscillatory step response if feedback compensation were applied. A controller must be designed to improve the system performance under closed loop conditions, as well as to reduce the effects of the varying load disturbance on the process variable.

The following values for a PID-type controller were chosen:

$$\begin{aligned} K &= 0.1 \\ T_i &= 250s \\ T_d &= 50s \end{aligned}$$

where

K	=	controller gain
T_i	=	integral time (seconds)
T_d	=	derivative time (seconds)

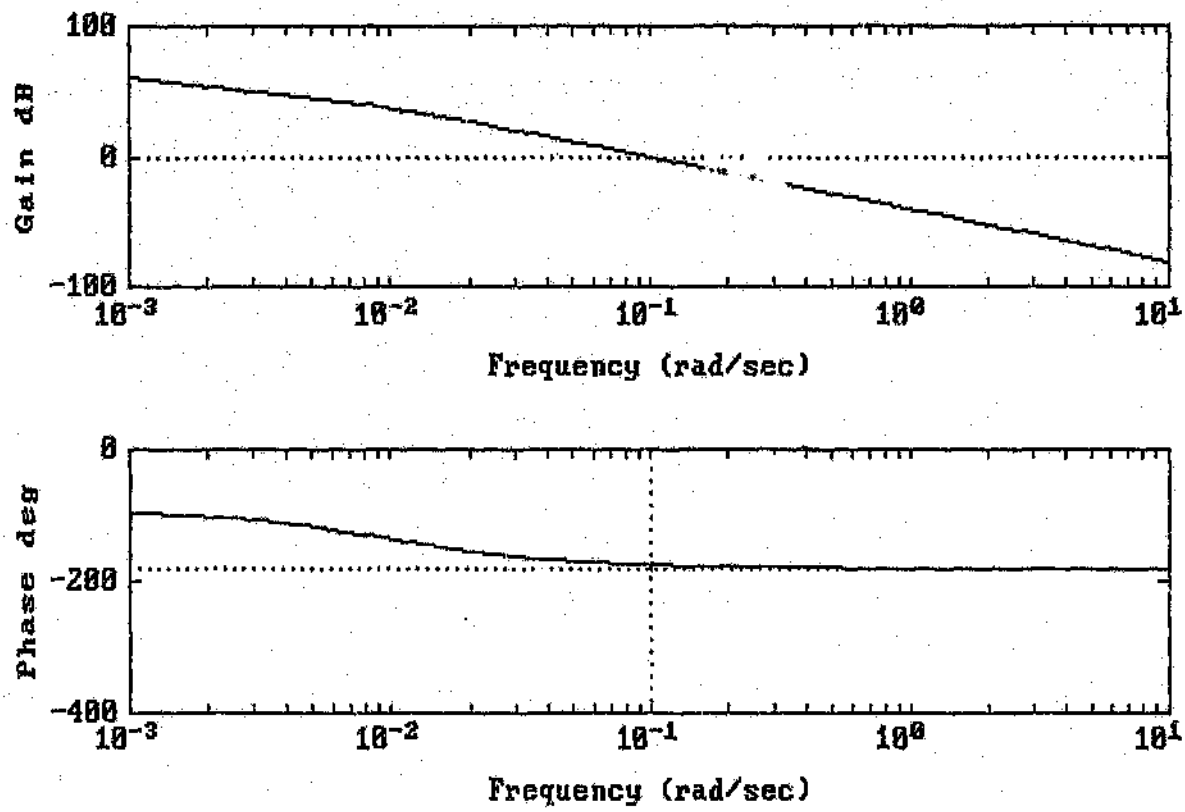

Figure 6.4: Bode Plot of Uncompensated Two Tank System

The phase margin is increased to an acceptable 48.65 degrees using PID compensation, whilst still maintaining an agreeable combination of rise time, overshoot and settling time. The bode plot for the plant with PID compensation is illustrated in figure 6.5. The sensitivity function for the plant plus PID-type compensation shows that disturbances containing frequency components below 0.04 radians/second are attenuated—see figure 6.6. The variance of the process variable, due to a varying load disturbance can be calculated from equation 3.3 as:

$$\sigma_y^2 = 0.2835$$

A check on the numerical accuracy of the result using the Matlab statistical function **std** yields:

$$\sigma_y^2 = 0.2751$$

A matlab m-file used to generate the above results appears in Appendix B as **petfesd.m**.

The introduction of a PID-type feedback controller reduces the variance of the process variable caused by the load disturbance. Further improvement in the rejection of disturbances, using feedback, can be achieved by considering the ideal loop characteristics of feedback systems as proposed by Bode. The penalty for this improvement is additional design effort and implementation complexity.

Choosing a loop gain of 200 at a 0.01 radian/second operating point and specifying gain and phase margins of 12 dB and 55 degrees respectively for the overall system, the optimum loop gain shape can be determined, as illustrated in figure 6.7. The parameters for the **bodeopt** routine are:

$$\begin{aligned} K &= 200 \\ w0 &= 0.01 \\ P &= 55 \\ G &= 4 \\ k2 &= 1 \\ n &= 3 \end{aligned}$$

where

K	=	gain at a frequency point in the operating band
$w0$	=	frequency point at which gain is specified
P	=	phase margin (degrees)
G	=	gain margin expressed as an arithmetic ratio
$k2$	=	number of integrators in system
n	=	pole-zero excess

The sensitivity function for the optimum loop gain characteristic is shown in figure 6.8 and illustrates that disturbance frequency components up to 0.3 radians/second are attenuated. Designing a controller which approximates this optimum loop gain shape therefore provides greater attenuation of load disturbances than the PID-type controller up to higher frequency values.

Using the standard method of loop shaping, the optimum loop gain characteristic can be closely approximated by introducing a controller to the two tank system of the form:

$$G_c(s) = 5000\frac{(s+1)(s+1)(s+0.03)}{(s+2)^2(s+3)^3}$$

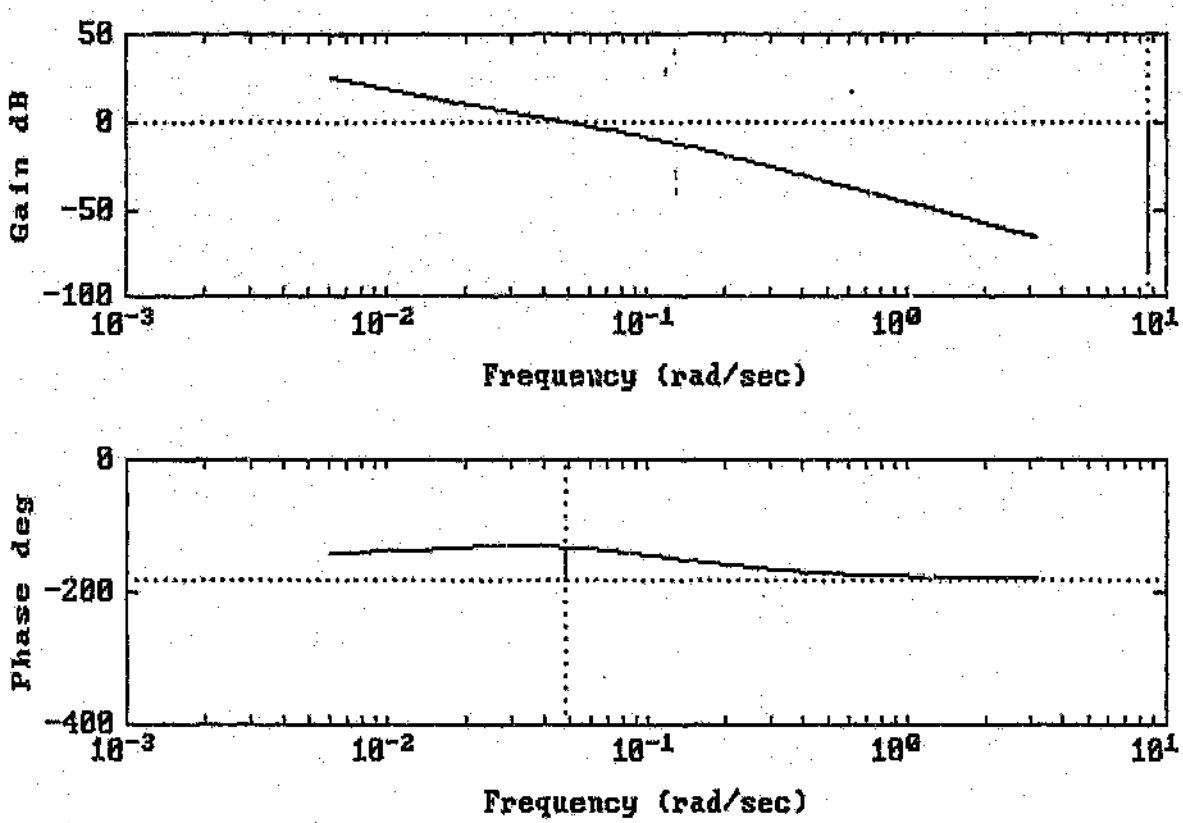

Figure 6.5: Bode Plot of Two Tank System with PID Compensation

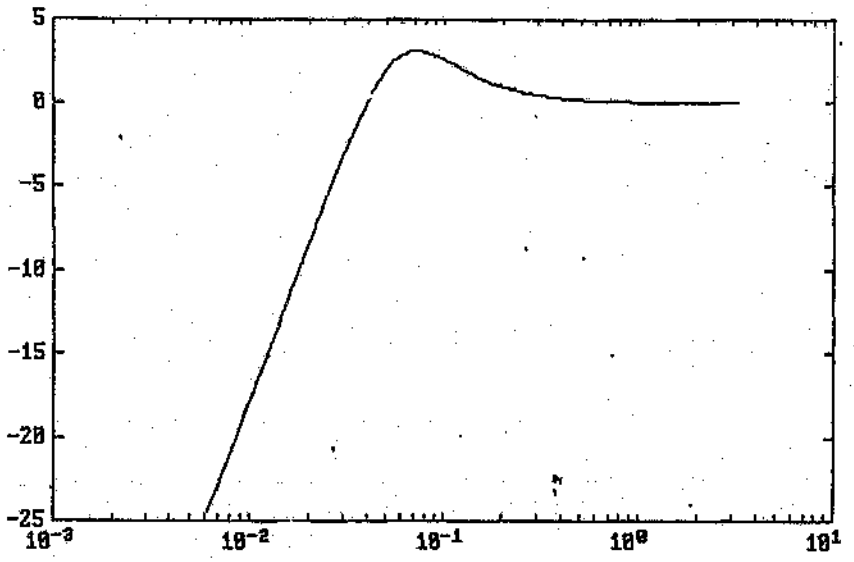

Figure 6.6: Sensitivity Function of Two Tank System with PID Compensation

where
$G_c(s)$ = transfer function of controller which approximates an optimum feedback controller

The approximation to the optimum loop gain shape is plotted in figure 6.9. The corresponding sensitivity function, figure 6.10, closely resembles the optimum shape designed for and provides attenuation of load disturbances up to approximately 0.6 radians/second. Calculating the variance of the process variable, due to the known varying load disturbance for this case study, using equation 3.9 yields:

$$\sigma_y^2 = 0.0016$$

A matlab m-file used to generate the above results appears in Appendix B as **optpsd.m**.

The time-domain performances of the two controller designs are compared using the step responses of the two compensated systems as presented in figure 6.11. The optimum controller offers improved disturbance rejection and superior time domain performance. Faster rise time, smaller overshoot and reduced settling time has been obtained in this case study with the correctly designed, optimum controller.

6.4 Performance Function

A hypothetical performance function for the two tank level control system has been generated using a combination of linear profit and penalty functions around a constrained operating position. Operating with a tank level below the optimum position increases the probability of draining the tank completely during a plant upset, resulting in pump damage. An operating level for the tank above the optimum position increases the probability of the tank overflowing, with an associated loss of profit due to loss of product. A quadratic polynomial has been fitted, over the operating region

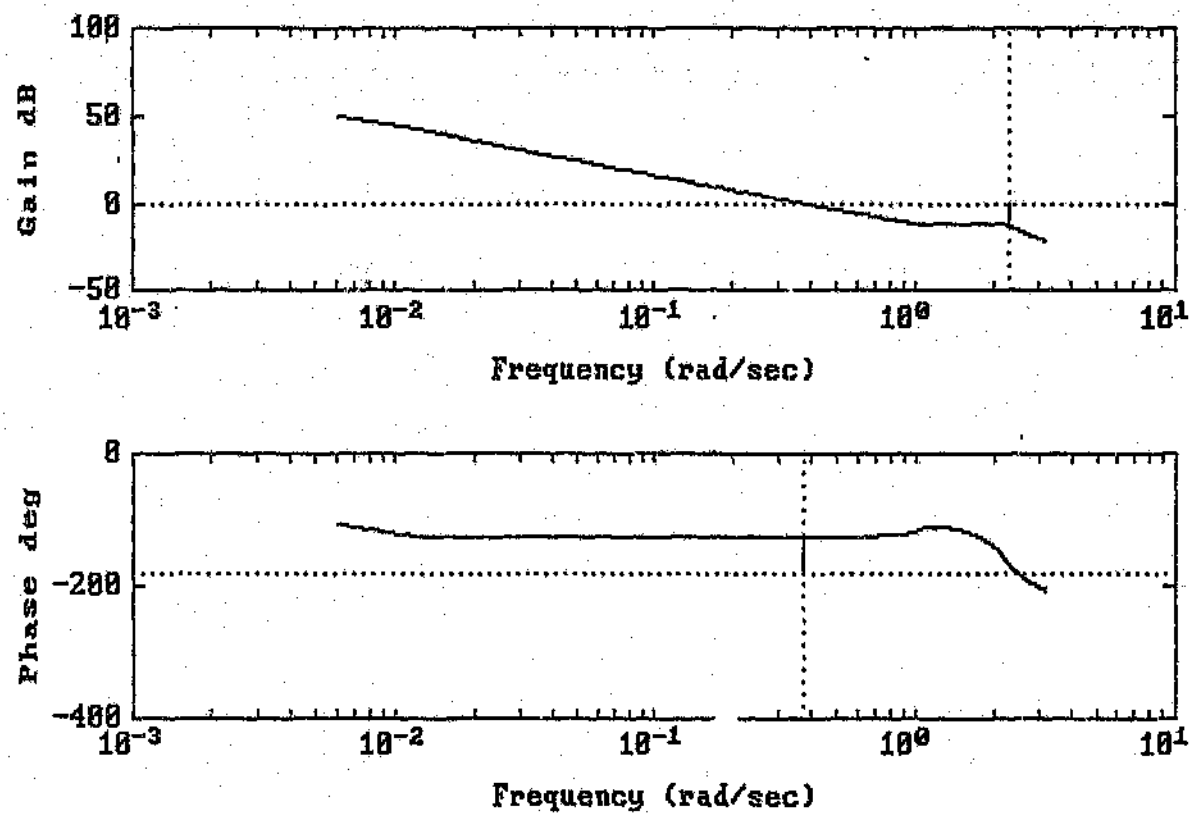

Figure 6.7: Optimum Loop Gain Shape for Two Tank System

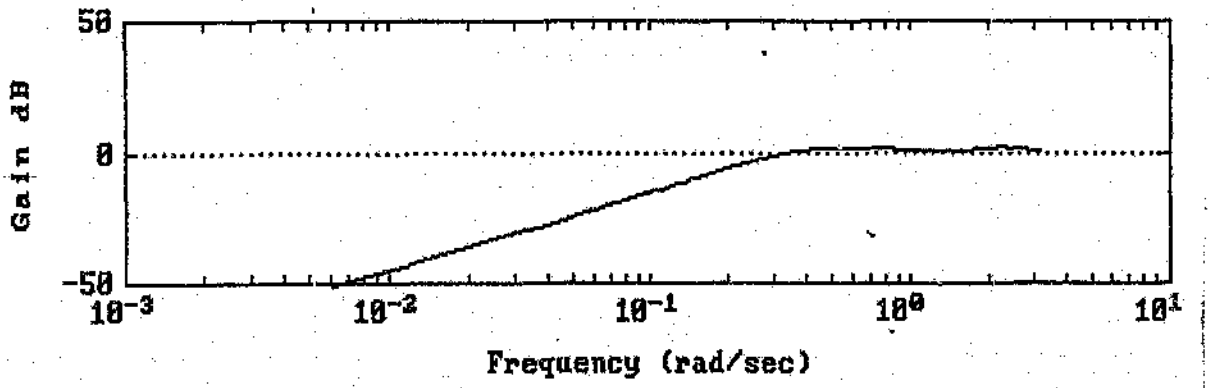

Figure 6.8: Sensitivity Function of Two Tank System with Optimum Feedback

of interest, to the overall performance function. The actual performance function and quadratic approximation appear in figure 6.12. The financial value associated with tank level operating position, is the annual contribution the tank level system offers to the overall plant performance. The quadratic was fitted to the performance function data using the Matlab **polyfit** routine and resulted in a polynomial:

$$p(y) = -321.4y^2 + 3857.1y - 7828.6$$

Assuming the intention is to operate the tank as close to the constraint as possible in order to maximize the financial potential of the system, the actual operating position will be governed by the expected variance of the process variable. A process variable subject to variance will have to be moved away from the constraint to avoid any cost penalty. Equation 4.5 can be used to calculate the average performance value for each control opportunity with its associated process variable variance. The results are presented in table 6.1.

Opportunity	Variance	Avg Performance
No Compensation	1.8825	R 3137.4
PID-type Control	0.2835	R 3652.5
Optimum Control	0.0016	R 3743.6

Table 6.1: Average Performances of Opportunities for Two Tank System

6.5 Financial Analysis

The uncompensated two tank system can be considered to be the base case of operation for financial evaluation. From table 6.1, if a PID-type control algorithm were

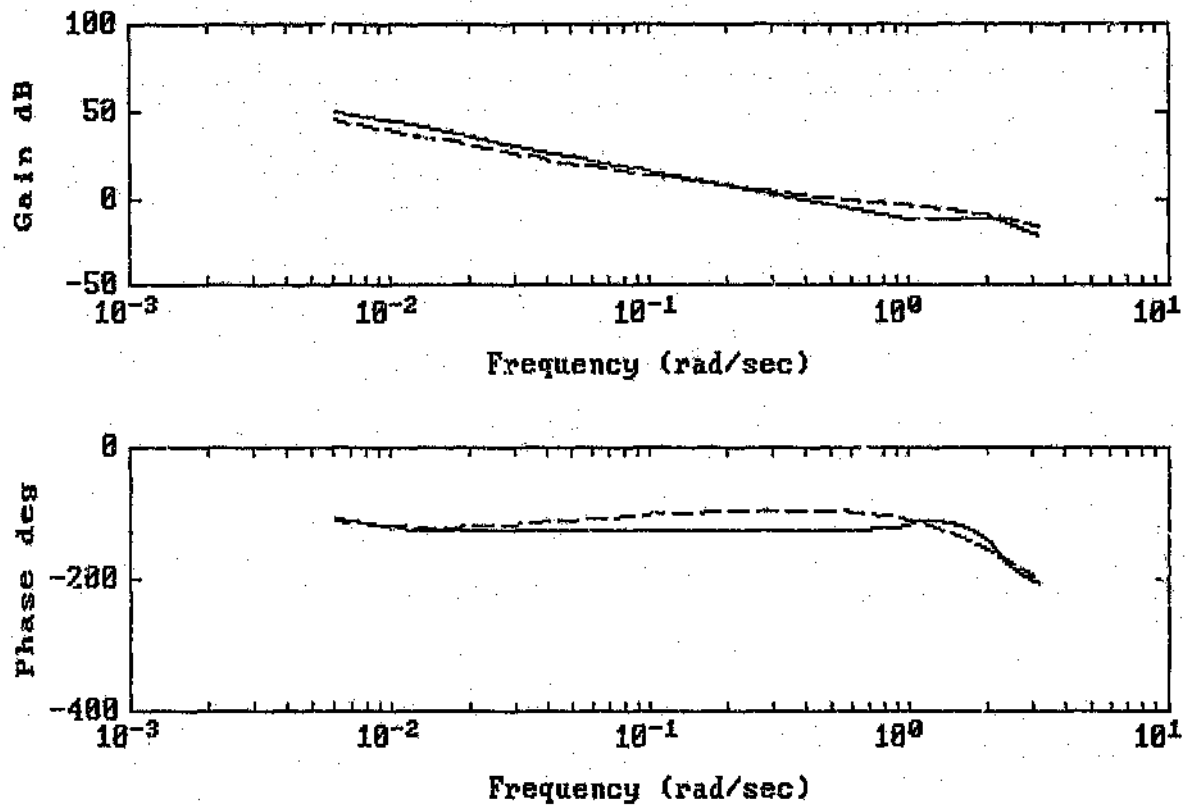

Figure 6.9: Comparison of Controller Design with the Optimum Loop Gain Shape for Two Tank System (Optimum Shape – bold; Controller – dash)

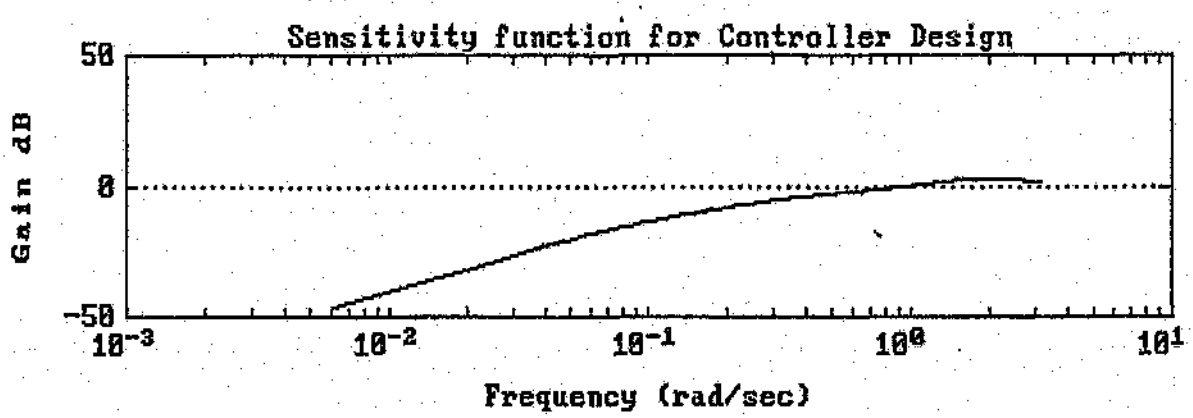

Figure 6.10: Sensitivity Function of Approximation to Optimum Feedback System

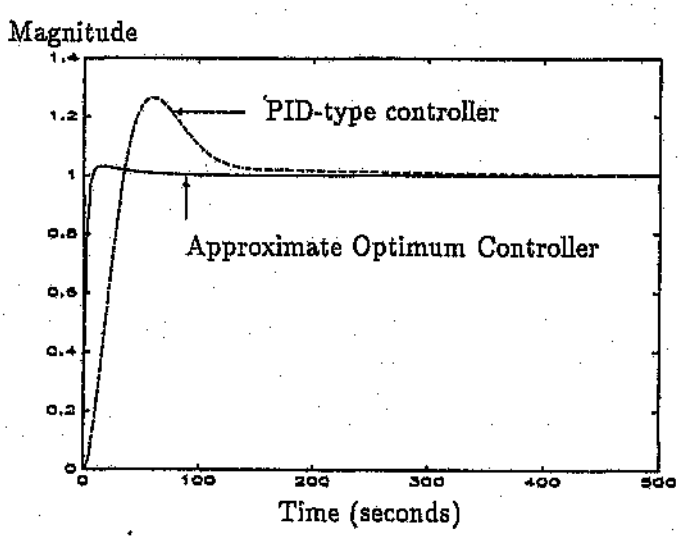

Figure 6.11: Comparison of Step Responses for Controller Designs

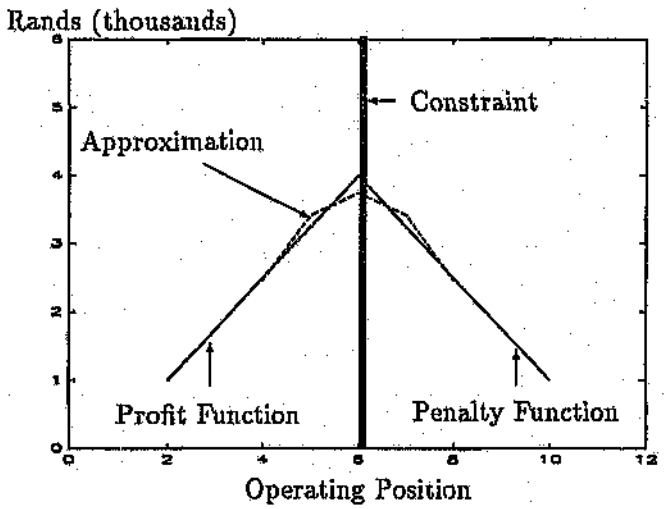

Figure 6.12: Performance Function and Quadratic Approximation for Two Tank System

to be implemented, the operation would benefit by an additional R 517 saved each year; if the optimum control scheme were implemented, an annual benefit of R 606 would be saved. These annual benefits can be used to perform cash flow analysis for each of the two control opportunities if the costs of implementation are known and if the life-span over which the benefits will be received can be predicted.

Assuming costs of implementation of R 2500 and R 3000 for the PID-type and optimum controllers respectively and a predicted life of 15 years, the cash flow statement can be prepared and the tools developed in Chapter 5 used to evaluate the potential of the two investment opportunities. The cash flow statements for the two control opportunities on the two tank system, PID-type and optimum feedback appear in tables 6.2 and 6.3 respectively.

Table 6.4 presents a summary of the results of the modern methods of profitability used to assess the financial potential of each opportunity.

As discussed in Chapter 2, AECI needs to improve its liquidity and leverage position. AECI should implement high yielding projects which require small capital investment.

The PID-type control opportunity offers a high yield from a relatively small capital investment. The DCFRR of 19.2% for the PID-type controller is higher than that of the optimum controller at 18.6%. The return is achieved with a smaller capital investment than that required for the optimum controller, which favours AECI's current financial position. It should be noted, however, that the optimum controller offers a greater return over the project life with an NPV discounted at 15% of R 543 versus R 523 for the PID solution.

It is recommended that AECI implement the PID-type control opportunity for the two tank system under consideration. Once the financial position of the organisation improves, AECI could prosper by investing in longer-term, larger return opportunities, such as the optimum controller, as the opportunities present themselves in the future.

Year	Capital	Savings	Cash Flow	Cumulative
0	-2500		-2500	-2500
1		517	517	-1983
2		517	517	-1466
3		517	517	-949
4		517	517	-432
5		517	517	85
6		517	517	602
7		517	517	1119
8		517	517	1636
9		517	517	2153
10		517	517	2670
11		517	517	3187
12		517	517	3704
13		517	517	4221
14		517	517	4738
15		517	517	5255

Table 6.2: Cash Flow Statement for PID Control of Two Tank System

Year	Capital	Savings	Cash Flow	Cumulative
0	-3000		-3000	-3000
1		606	606	-2394
2		606	606	-1788
3		606	606	-1182
4		606	606	-576
5		606	606	30
6		606	606	636
7		606	606	1242
8		606	606	1848
9		606	606	2454
10		606	606	3060
11		606	606	3666
12		606	606	4272
13		606	606	4878
14		606	606	5484
15		606	606	6090

Table 6.3: Cash Flow Statement for Optimum Feedback Control of Two Tank System

Opportunity	NPV (15 %)	DCFRR	Capital
No Compensation	R 0	N/A	R 0
PID-type Control	R 523	19.19 %	R 2500
Optimum Control	R 543	18.64 %	R 3000

Table 6.4: Profitability of Control Opportunites for Two Tank System

Chapter 7

Conclusion

7.1 Discussion of Results

A set of analysis tools to assist with the prediction, quantification and evaluation of the benefits of advanced process control techniques was developed. The analysis tools were developed using Matlab and presented as an aid to the financial decision-making process of selecting profitable control investment opportunities for improving the overall financial performance of an organisation.

The findings suggest that the rigorous approach of analyzing disturbances likely to affect a system can be used to quantify the benefits or savings of proposed control projects, using the power spectral density of the disturbance and the system sensitivity function.

The development of a performance function relating the operating point of a process variable to financial value was investigated. It was concluded that linear and quadratic polynomials can be used to adequately describe the perfomance function and if a system is limited by a constraint, it is possible to develop an overall performance function which includes the financial impact of the constraint.

The ranking of control project opportunities was performed by taking into account *the time value of money* using the modern methods for determining profitability of projects. These methods provide a more realistic view of the returns a project will yield than the traditional benefits-to-cost ratio method. The combination of the Net Present Value (NPV) and Discounted Cash Flow Rate of Return (DCFRR) methods of profitability assessment, together with graphical interpretation of the cumulative cash flow diagram, provides valuable insight into the financial potential of advanced control and control upgrade projects.

The significance of the study is that it has shown that the quantification of real, specific benefits for advanced control applications is feasible and will allow for the more widespread use and acceptance of advanced control principles.

7.2 Areas for Future Development

1. The estimation of costs to implement and install advanced control strategies was not included in the scope of this research project. Further research into the estimation of these costs, in the specialized field of advanced control and control upgrade projects, would assist management in their decision-making process.

2. The effects of inflation on the decision-making process have not been addressed in this project report. In these inflationary times, it would be worthwhile to investigate and quantify these effects on the ranking of opportunities.

3. The calculation of feedback benefits should be extended to include sensor noise or measurement errors in the measured process variable.

4. Only unity feedback systems were considered in this report. The effect of non-unity feedback in systems should be evaluated.

5. The effects of dead time, on the optimum loop gain characteristic of feedback systems (both unity and non-unity) as proposed by Bode, should be investigated.

6. Further development is required in the area of specifying performance functions using high order polynomials and other improved function fits.

7. Risk and sensitivity analysis is a vast area of research which has only been discussed briefly. Further research in this field could help to refine the art of decision-making.

8. The principles outlined in this report should be applied to more advanced control case studies in practice. The process of predicting, quantifying and evaluating benefits is an iterative one, with experience helping to continuously improve the methodology.

Appendix A

Derivation of Power Spectral Density

Consider a stable, linear time-invariant system with impulse response h. The system is subjected to a continuous-time stochastic input signal $u(t)$. The relationship between output and input is:

$$y(t) = \int_{-\infty}^{t} h(t-s)u(s)\,ds \tag{A.1}$$

$$= \int_{0}^{\infty} h(s)u(t-s)\,ds \tag{A.2}$$

The autocorrelation function of a process is:

$$\phi_y(\beta) = \varepsilon\{y(t)y(t+\beta)\} \tag{A.3}$$

Substituting equation A.2 into equation A.3

$$\begin{aligned}\phi_{yy}(\beta) &= \varepsilon\{\int_0^\infty h(k)u(t-k)\,dk \int_0^\infty h(r)u(t+\beta-r)\,dr\} \\ &= \varepsilon\{\int_0^\infty h(k)\int_0^\infty u(t-k)u(t+\beta-r)h(r)\,dk dr\} \\ &= \int_0^\infty h(k)\int_0^\infty h(r)\varepsilon\{u(t-k)u(t+\beta-r)\}\,dkdr\end{aligned}$$

If $u(t)$ is stationary, then $\varepsilon\{u(t-k)u(t+\beta-r)\}$ depends only on the the time difference $\beta - r + k$.

$$\phi_{uu}(\beta - r + k) = \varepsilon\{u(t-k)u(t+\beta-r)\}$$

$$\phi_{yy}(\beta) = \int_0^\infty h(k)\int_0^\infty h(r)\phi_{uu}(\beta - r + k)\,dkdr$$

By making the substitution $l = r - k$:

$$\phi_{yy}(\beta) = \int_0^\infty \phi_{uu}(\beta - l)\int_0^\infty h(k)h(l+k)\,dkdl$$

If $c(l)$ is defined as $\int_0^\infty h(k)h(l+k)\,dk$, noting that $\int_0^\infty h(k)h(l+k)\,dk$ is the convolution of $h(t)$ with itself, then

$$\phi_{yy}(\beta) = \int_0^\infty \phi_{uu}(\beta - l)c(l)\,dl$$

The autocorrelation of the output of a linear system $\phi_{yy}(\beta)$ is therefore the convolution of the input autocorrelation $\phi_{uu}(\beta)$ with a system impulse response.

Taking Fourier tranforms and applying the convolution theorem:

$$\begin{aligned} \Phi_{yy}(\omega) &= C(j\omega)\Phi_{uu}(\omega) \\ C(j\omega) &= H(j\omega)H^*(j\omega) \\ \Phi_{yy}(\omega) &= |H(j\omega)|^2 \times \Phi_{uu}(\omega) \end{aligned}$$

Appendix B

Matlab Routines for Feedback Analysis

```
function [A,B]=bodeopt(K,w0,P,G,k2,n,w);
%BODEOPT [A B] = bodeopt(K, w0, P, G, k2, n, w);
%  Generates the ideal Bode loop transfer function characteristic.
%  Parameters:
%     K  the gain in (at the edge of) the useful band
%     w0 the useful band extends from 0 to w0 rad/s
%     P  the phase margin in degrees
%     G  the gain margin as an arithmetic ratio
%     k2 the slope of an optional infinite (decreasing) charactristic
%     n  the required pole-zero excess for the loop transfer function
%     w  a vector of frequencies at which the magnitude A and
%         the phase angle B must be calculated
% Bode, H.W. (1940). "Relations between attenuation and phase
% in feedback amplifier design", Bell System Tech. J., 19, 421-454.
%  I. MacLeod 3-Feb-1992
K = log(K);           % convert gain at edge of useful band to nepers
hsflag = G>0;         % G<=0 means no horizontal segment
G = log(abs(G));      % gain margin or margin below unity gain
                      %  to maintain cutoff magnitude slope
kcutoff = 2*(1 - P/180);
k1 = kcutoff - k2;    % reduce if infinite magnitude char. is present
y = (K + G - k1*log(2))/kcutoff;
if hsflag,
   k4 = kcutoff;          % to bring magnitude slope to zero
   k3 = n;                % final slope
   wb = exp(y)*w0;
   wc = n/kcutoff*wb;
else
   k4 = 0;
   k3 = n - kcutoff;      % additional slope to reach final slope
   wb = 0;
   wc = exp(y)*w0;
end
```

```
w = w(:);
A = zeros(w);
B = zeros(w);
ilew0 = find(w <= w0);
igtw0 = find(w > w0);
r0 = w/w0;
s0 = r0(ilew0);
A(ilew0) = K - k2*log(s0);
B(ilew0) = -k1*asin(s0) - pi/2*k2;
t0 = r0(igtw0);
A(igtw0) = K -k1*log(sqrt(t0.*t0 - 1.0) + t0) - k2*log(t0);
B(igtw0) = pi/2*(-k1 - k2)*ones(t0);
if wc ~= 0,
   igtwc = find(w > wc);
   A(igtwc) = A(igtwc) - k3*log(w(igtwc)/wc);
   B = B - k3*bodeopt1(w/wc);
end
if wb ~= 0,
   igtwb = find(w > wb);
   A(igtwb) = A(igtwb) + k4*log(w(igtwb)/wb);
   B = B + k4*bodeopt1(w/wb);
end
A = exp(A);      % magnitude as arithmetic ratio
B = B*180/pi;

function phase=bodeopt1(xxc);
%BODEOPT1  phase = bodeopt1(xxc)
%  Subroutine for bodeopt.m
%  Calculates the approximate phase angle (in rads) corresponding
%  to an (increasing) semi-infinite magnitude characteristic
%  using the first two terms of the series derived in:
%  Bode, H.W. (1945). "Network Analysis and Feedback Amplifier
%  Design", van Nostrand, New York, p342.
%  Parameter xxc is a vector of frequency ratios w/wC, where
%  w0 is the "corner" frequency of the semi-infinite
%  characteristic. I. MacLeod 3-Feb-1992
phase = zeros(xxc);
ile1 = find(xxc <= 1.0);
igt1 = find(xxc > 1.0);
ilef = find(xxc(ile1) <= 0.414);
igtf = find(xxc(ile1) > 0.414);
xc = xxc(ilef);
phase(ilef) = 2/pi*xc.*(1 + xc.*xc/9);
xc = xxc(igtf);
yc = (1-xc) ./ (1+xc);
phase(igtf) = pi/4 - log(xc).*log(yc)/pi - 2/pi*yc.*(1 + yc.*yc/9);
% For xxc>1 use Ph(w/w0) = pi/2 - Ph(w0/w)
invxc = 1 ./ xxc;
```

```
ilef = find(invxc <= 0.414);
igtf = find(xxc > 1.0 & invxc > 0.414);
xc = invxc(ilef);
phase(ilef) = pi/2 - 2/pi*xc.*(1 + xc.*xc/9);
xc = invxc(igtf);
yc = (1-xc) ./ (1+xc);
phase(igtf) = pi/4 + log(xc).*log(yc)/pi + 2/pi*yc.*(1 + yc.*yc/9);

function [aS,bS] = sensit(a,B,w);
% Sensit generates the Bode Diagram of the Sensitivity function S.
% The M-file BODEOPT plots a Bode Diagram for the loop gain L.
% Sensit transforms L to S using the block diagram form S=1/(1+L).
% transfer function (H) = L = a.*exp(j*b) where b=B*pi/180.
% Parameters
%      a = magnitude response (in magnitude NOT dB's)
%      B = phase response     (in degrees NOT radians)
%      w = a vector of frequencies at which magnitude and phase
%          is calculated.
% Author N.Bawden Date 13/7/92
L=a.*exp(j*B*pi/180);
ontop=ones(length(w),1);
S=ontop./(1+L);
aS=abs(S);
BS=angle(S);
bS=BS*180/pi;              % convert radians back to degrees
%end sensit

% petfesd.m
% An m-file to test power spectral density calculations
% for determining standard deviations of PID controller
% disturbance rejection characteristics.NMB 16/8/92.
% modified NMB 7/11/92.
  t=0:1:500; % sampling interval = 1 second
  y=2*sin(2*pi*0.0032*t)-0.5*sin(2*pi*0.008*t)+sin(2*pi*0.0025*t);
% rand('normal')
% yn=y+0.5*rand(t);
  y=detrend(y);
% Calculate variance (deviation^2) of disturbance signal
  variance=std(y)^2,pause
  u = mean(y),pause
  varied2 =sum(y.^2)/(length(y)-1),pause
% Determine Power Spectral Density of the disturbance
  no = input('enter no of frequency points in spectrum ')
% P=spectrum(y,no); % no = no. of points in spectrum
% specplot(P,deltaT),pause
% Calculate standard deviation by integrating under PSD
% Pdev=sum(P),pause
%
```

```
% Calculate Power Spectral Density for preset frequencies
  M=input('Enter smoothing function window size ')
  g=etfe(y',M,no,1); % use default values if f1 & f2 > 0.02 Hz
  bodeplot(g),pause
  ffplot(g),pause
  [ws,amp]=getff(g);
  clear g ;
% Check consistency of calculations
  spavar=sum(amp)/length(amp),pause
% Set plant model and PID controller
  numP=1;
  denP=[100 1 0];
  [numC,denC]=cpid(0.1,250,50);
% Close the feedback loop around controller and plant
  [As,Bs,Cs,Ds]=sfb(numC,denC,numP,denP);
  pack
% Display Bode plots of closed loop system
  [dmag]=bode(As,Bs,Cs,Ds,2,ws);
  semilogx(ws,20*log10(dmag))
  title('Sensitivity Function for Compensated Plant'),pause
% Simulate response of system to disturbance at input2
  U=[zeros(1:length(y))' y']; % set point = input1
  clear y spavar;
  pack
  yout=lsim(As,Bs,Cs,Ds,U,t);
  yout=detrend(yout);
  plot(t,yout(:,1),'r',t,U(:,2),'g')
  clear t U As Bs Cs Ds;
  title('Disturbance effects on output signal'),pause
  newvariance=std(yout(:,1))^2,pause
  newu=mean(yout(:,1)),pause
  newvaried2 =sum(yout(:,1).^2)/(length(yout(:,1))-1),pause
  clear yout
% Compare simulated response with calculated effect on system
% using PSD and amplitude
  H=dmag(:,1).*dmag(:,1);
  semilogx(ws,20*log10(H))
  title('Sensitivity function squared'),pause
  output=H.*amp;
  semilogx(ws,20*log10(output))
  title('Output Power Spectral Density')
  finalvar=sum(output)/length(output)

% optpsd.m
% An m-file to test power spectral density calculations
% for determining standard deviations of Optimum controller
% disturbance rejection characteristics.NMB 8/11/92.
  t=0:1:500; % sampling interval = 1 second
```

```
  y=2*sin(2*pi*0.0032*t)-0.5*sin(2*pi*0.003*t)+sin(2*pi*0.0025*t);
% rand('normal')
% yn=y+0.5*rand(t);
  z=detrend(y);
% Calculate Power Spectral Density for preset frequencies
  no=input('Enter no. of frequency points in spectrum')
  M=input('Enter smoothing function window size ')
  g=etfe(y',M,no,1); % use default values if f1 & f2 > 0.02 Hz
  bodeplot(g),pause
  ffplot(g),pause
  [ws,amp]=getff(g);
% Set plant model and Optimum controller
  k=50;
  z=[-1 -1 -0.03]';
  p=[0 -0.01 -2 -2 -3 -3 -3]';
  [num,den]=zp2tf(z,p,k);
  [mag,phase]=bode(num,den,ws);
  [aS,bS]=sensit(mag,phase,ws);
  margin(aS,bS,ws),pause
  shg
% Calculate effect of disturbance on system using PSD and amplitude
  H=aS.*aS;
  semilogx(ws,20*log10(H))
  title('Sensitivity function squared'),pause
  output=H.*amp;
  semilogx(ws,20*log10(output))
  title('Output Power Spectral Density')
  finalvar=sum(output)/length(output)
```

Appendix C

Matlab Routines for Financial Analysis

```
function cashflow=cumcash(flows)
% A cash flow matrix is formed, including a cumulative cash flow.
% Author N.Bawden Date 7/7/92, modified 20/10/92.
% The cumulative cash flow is subsequently plotted versus time.
% Note: cash flows are entered in an n by 1 vector.
%
n=length(flows);
years=(0:1:n-1)';
cumflows=cumsum(flows);
cashflow=[years flows cumflows];
plot(years,cumflows),title('Cumulative cash flow');
xlabel('Time (years)');
ylabel('Cumulative Cash Flow');
grid
%end cumcash
```

```
function DCFRR = dcf(x)
% Calculation of the Discounted Cash Flow Rate of Return (DCFRR).
% Also commonly referred to as the Internal Rate of Return (IRR).
% Author N.Bawden ( Ref. p.187 D.H.Allen A3.2 - with modifications)
% DCFRR is defined as the discount rate required to yield of NPV = 0.
% The Newton-Raphson method is used to solve the non-linear equation.
% Iteration x[k+1] = x[k] - f{x[k]}/f'{x[k]}, where f{x[k]} = NPV.
% A guess for the DCFRR must be supplied to initialize function.
% A negative DCFRR implies no return and is therefore set to zero.
%
n=length(x);
guess=input('Enter initial guess (%) ');
DF=1/(1+guess/100);
p=1/n;
%
    while abs(p) > 0.00001  % limit sets degree of accuracy required
        PV=x(n);            % Calculation of NPV = f{x[j]}
```

```
   for j=1:n-1
          PV=PV*DF+x(n-j);
          end %for
        DPV=(n-1)*x(n);      % Calculation of d/dDF{NPV} = f'{x[j]}
          for j=1:n-2
              DPV=DPV*DF+(n-j-1)*x(n-j);
          end %for
        p=PV/DPV;            % Iteration x[j+1]=x[j]-NPV/f'{x[j]}
        DF=DF-p;
    end %while
%
I=1/DF-1;                    % Discount Factor = 1/(1+Interest)
if I<0                       % Negative DCFRR set to zero
   DCFRR=0;
else
   DCFRR=100*I;              % Convert to percentage result
end  %if
%end dcf

function discashflow=discount(flows)
% This function discounts a given vector of cash flows.
% Author N.Bawden Date 7/7/92, modified 20/10/92.
% User must supply appropriate discount factor as input to routine.
% Cumulative plots of discounted and undiscounted cash flows versus
% time are plotted.
%
n=length(flows);
years=(0:1:n-1)';
cumflow=cumsum(flows);
interest=input('Enter discount factor (%) ');
disfactor=ones(n,1)./((1+interest/100).^years);
discash=flows.*disfactor;
discumflow=cumsum(discash);
discashflow=[years discash discumflow];
plot(years,discumflow,'r',years,cumflow,'g');
title('Discounted versus Undiscounted Cumulative cash flows');
xlabel('Time (years)');
ylabel('Cumulative Cash flow');
grid
%end discount

function probdist=normdist(y,T)
% A normal probability distribution is calculated for a process
% variable, y, entered as a vector. The distribution is plotted
% versus the operating mean position for comparison with the
% plot of process variable versus time.
% Author N.Bawden Date 20/10/92.
% Normal Distribution is calculated according to the formula:
```

```
%
%                  -(x-mean)/(2*sigma^2)
%                 e
%        f(x) =  ----------------------
%
%                 sqrt(2*pi*sigma^2)
%
% Enter y = process variable in vector form
%       T = sampling period in seconds
%
   n      = length(y);
   points = (n-1)*T;
t = 0:T:points;
   sigma = std(y);
   mu    = mean(y);
   upperlim= mu+4*sigma;
   lowerlim= mu-4*sigma;
   x = lowerlim:sigma:upperlim;
probdist = exp(-abs(x-mu)/(2*sigma^2))/(sqrt(2*pi*sigma^2));
   xi    = lowerlim:sigma/25:upperlim;
   yi    = spline(x,probdist,xi);
   axis([lowerlim upperlim 0 points])
   subplot(211),plot(y,t)
   axis([lowerlim upperlim 0 1])
  subplot(212),plot(xi,yi);
%end normdist

function y = npv2(x)
% Calculation of the Net Present Value (NPV) of cash flows.
% Author N.Bawden ( Reference p.185 D.H.Allen A3.1 ) Date 6/7/92.
% The input x is an n by vector of undiscounted cash flows.
% User must supply an interest rate for discounting.
%
n=length(x);
disrate=input('Enter discount rate (%) ');
%
y=x(n);
    for j=1:n-1
        y=y/(1+disrate/100)+x(n-j);
    end %if
%npv2
```

Appendix D

Derivation of Two Tank Transfer Function

Consider the two tank system illustrated in figure D.1.

Performing a mass balance for each of the two tanks, assuming the density of the liquid is constant:

$$\begin{aligned} \frac{d}{dt}\{Volume\} &= Input\ Flow - Output\ Flow \\ A_1\frac{dh_1}{dt} &= F_{in} - F \\ A_2\frac{dh_2}{dt} &= F - F_{out} \end{aligned}$$

Eliminating h_1 and F and changing to incremental values yields:

$$\frac{d^2h_2}{dt^2} + \frac{\alpha\ dh_2}{A_1 dt} = \frac{\alpha\ F_{in}}{A_1 A_2}$$

If $A_1 = A_2 = 1m^2$ and $\alpha = 0.01$ then

$$\frac{d^2h_2}{dt^2} + 0.01\frac{dh_2}{dt} = 0.01F_{in}$$

The transfer function from output h_2 to input F_{in} is therefore

$$G(s) = \frac{1}{s(100s+1)}$$

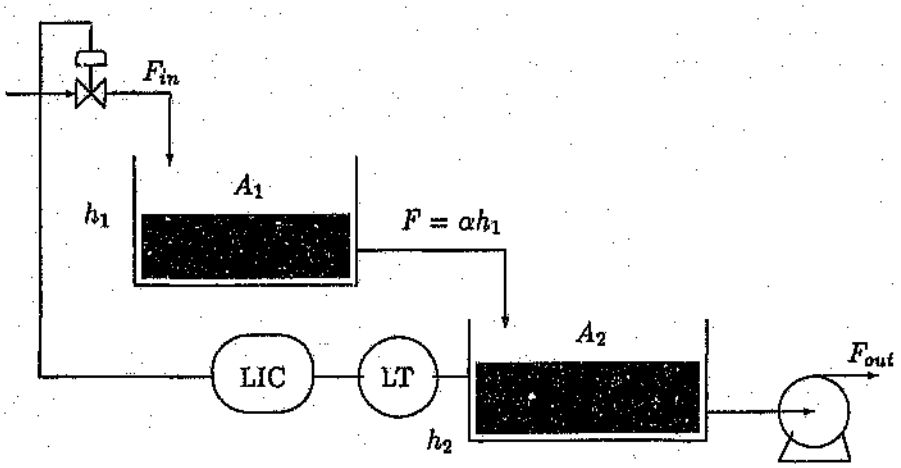

Figure D.1: The Two Tank Level Control System

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