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Exploring a teacher's selection and use of examples in Grade 11 probability multilingual classroom

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Exploring a Teacher's selection and use of examples in Grade 11 Probability multilingual classroom.

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A research report submitted to the WITS SCHOOL OF EDUCATION, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science (Science Education).

Johannesburg 2015

Declaration

I declare that this research report is my own work, except as indicated in the acknowledgements, the text and the references. It is being submitted for the Degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.



Signature

5 NOVEMBER 2015.

Dedication

To my wife Rosemary

To my son Vuyo Bradley

And daughter Leanne Mbali

Acknowledgements

I wish to thank the following:

My supervisor, Dr Anthony Anietie Essien for taking his time to engage with my work and offering advice that challenged me to clarify my thinking. Thank you for being patient and encouraging me to soldier on during the days when I felt like giving up. Without your unreserved support and great encouragement, this research project would never have been completed fruitfully.

The teacher who allowed me to observe and analyse his lessons.

My wife, Rosemary, for your unwavering support, encouragement, patience and love. I could have not done this without you- Thank you.

Abstract

Using qualitative methods, this study reports on the selection and use examples in Probability by a teacher in a multilingual mathematics classroom where learners learn in a language which is not their first or home language. The study involved one teacher together with his Grade 11 multilingual class in a township school in Ekurhuleni South Johannesburg. Data was collected through audio-visual recording of four lessons. In addition two one-on-one semi-structured interviews were conducted with the teacher. Data was analysed using Rowland's (2008) categories of exemplification alongside Staples' (2007) conceptual model of collaborative inquiry mathematics practices. In the study it emerged that it is important for teachers to select examples by considering the context, ability of the example to be generalised, consistency in the use of symbols, syllabus requirements and accessibility. It also emerged that the selection of examples together with the accompanying mathematical practices has the potential to support or impede the learning of mathematics. In particular the findings revealed that the practice of 'guiding the learners with the map' declines the cognitive level of examples and hence impedes learning. Code- switching and re-voicing were most frequently used practices seen in the findings with the use of code-switching encouraging full participation of the learners. The study recommends that methodology courses offered at tertiary institutions to pre-service teachers should include the selection, how to select or design and use examples in multilingual classrooms e.g. what constitutes a good example and how to maintain the cognitive level of an example. The study also recommends that more research needs to be done on effective mathematical practices that may be used to implement worked-out examples in multilingual classrooms.

Key words: example, exemplification, mathematical practices, multilingual classrooms.

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CHAPTER ONE

1.1General introduction to study

In my teaching experience, I have taught for eleven years in a school with one dominant home language. I have also taught in a multilingual school for seven years with home languages such as English, Afrikaans, IsiZulu, Sepedi and Sesotho dominating. In both set ups English as a language of instruction affected teaching and learning in one way or the other. One of the challenges was getting poor results after assessments especially on topics with highly specialised terms like Probability and Statistics. During the class discussions of the assessment items it would emerge that the poor results were largely due to language related problems like failing to understand the language used in the question. According to Hawkins & Kapadia (1984) there is a significant discrepancy between linguistic and technical interpretations in probability more than in any other branches of mathematics. Words like equal likely, random, mutually exclusive, certain, impossible, independent and dependent are often misinterpreted by learners (Bennie, 1998).

Probability was optional in the old syllabus of i.e. National Curriculum Statement (NCS) (2005) but is now compulsory for all learners in the amended NCS known as Curriculum Assessment Policy Statement (CAPS) (2012), with first examinations done in year 2013. It is interesting to find out the examples teachers use to teach this new component especially in schools that were not writing paper 3 which was previously an optional paper in matric under the old National Curriculum Statement (NCS).

The matric results of 2014 released by Umalusi showed a decline in Mathematics national pass rate by 2.4% from 78.2% in 2013 to 75.8 in 2014 (Morkel, 2014). One of the reasons given for the decline is the introduction of Euclidian Geometry and Probability in the new syllabus (Morkel, 2014). It is therefore worthwhile to find out how teachers are adjusting to the challenges posed by the change of syllabus through exploring the selection and use examples by teachers in the teaching and learning of Probability specifically and Mathematics in general and perhaps make some recommendations on what needs to be done. It was my intention to do my research on both Statistics and Probability as both have now been given some prominence than before. However, Statistics was done during the fourth term and the ethics clearance from Gauteng Department of Education did not allow me to do research at that time. In Statistics, students analyse and organise quantitative data so as to

evaluate and critique conclusions. While in Probability students use the collected data to make predictions. In both Statistics and Probability students use highly technical words, graphs, symbols (e.g. σ for standard deviation), tables and diagrams like tree diagrams.

It is also worth noting that the results of the Third International Mathematics and Science Study (TIMSS) of 1995 and 1999 summarised by Howie (2003) revealed that South Africa performed poorly in comparison to other countries in both studies. The research findings identified language as the main reason for the poor performance (Howie, 2003). What is interesting in the studies is that other countries like Indonesia, Morocco, Philippines and Singapore who use second language for teaching and learning performed better than South Africa. This is a worrying phenomenon without taking into cognisance other factors such as lack of resources, class sizes and so on, affecting the learning and teaching of mathematics in South Africa.

Furthermore it is a shared concern by all stakeholders i.e. teachers, students, parents and other education professionals that the mathematics matric results are far below acceptable level (Howie, 2003). There have been numerous suggestions offered by both academics and non-academics on improving mathematics teaching and learning. According to Adler (1995) there are three issues that affect the teaching and learning of mathematics i.e. the access: to the language of instruction, mathematical discourse and classroom discourse. In this research project, I focussed on the first issue of access to language of instruction. I investigated the role and nature of examples used by teachers to teach Probability at Grade 11 level.

It was my primary concern in this study to investigate the role played by teachers in selection and the use of examples because it is imperative that teachers possess both mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK) in order to be effective in the classroom *(*Rowland, 2008). In other words teaching mathematics involves both in-depth understanding of the content and how to impart the content. Mason & Spence (1999) argue that 'knowing- about' mathematics and mathematics teaching is achieved through 'knowing- to act' during the practice of teaching (p.135). In other words, content knowledge and methods of teaching mathematics should complement each other in order for effective learning and teaching to take place. It was therefore important to consider what the teacher made available for learning and how this knowledge was imparted in the classroom situation. It was possible to analyse what the teacher made available for learning by looking at his choice and use examples.

Bills, Dreyfus, Mason, Tsamir, Watson & Zaslavsky (2006) argue that examples provide

what is made available to learn in both theoretical and practical ways. Goldenberg & Mason (2008) point out that examples "…are a major means for 'making contact' with abstract ideas and a major means of mathematical communication, whether 'with oneself', or with others" (p.184). In other words examples are vehicles of bringing mathematical concepts to the learners. Viewing examples as communication tools implies the use of language.

Language is a very important component of learning as most of our thinking is done in a language (Setati, 2008). In South Africa the issue of language is compounded by the fact that there are 11 official languages. The Bill of Rights in South Africa places all languages at par (Setati, 2008). Theoretically this implies that all the eleven languages may be used as medium of instruction. However this has remained as just a policy lacking implementation. Setati (2008) argues that in South Africa language is connected to a large extent with political power. Many parents prefer their children to be taught in English as a Language of Learning and Teaching (LoLT) (Setati, 2008).

The use of examples in a multilingual classroom inevitably involves language related issues. While the selected examples may be applicable to all classrooms, whether multilingual or not the mathematical practices that accompany the examples involve the use of practices unique to multilingual classrooms. Adler & Venkat (2014) argue that the use of English in multilingual classrooms increases the '…demands on the teacher's mathematical discourse in instruction…' (p. 4). Adler & Venkat (2014) describe 'mathematical discourse in instruction' (MDI) as "…examples and the discourse that makes up the explanations associated with example…"(p.1). It is my contention that examples and the mathematical practices that accompany them are very critical in multilingual classrooms. Bills et al. (2006) further point out that more research needs to be done on selection and use of examples in the mathematics classroom.

The above concerns led me to explore the following research questions:

1. How does a teacher select examples when teaching Probability in Grade 11 multilingual mathematics classroom, and for what purposes?
2. What mathematical practices accompany these examples?
3. What are the obstacles or affordances offered by examples in Probability given by a mathematics teacher in Grade 11 multilingual classroom?

The first question allowed me to explore the criteria used by the teacher to select examples and the purpose for using any given example. The second question afforded me an opportunity to see the use of examples at implementation level i.e. in the classroom situation.

The third question was used to analyse the obstacles and affordances offered by the examples used by the teacher during the implementation stage. Stein, Grover, & Henningsen, (1996) point out that at implementation stage the teacher can either decline or maintain the cognitive level of a mathematical task. The question was critical in critiquing the potential learning opportunities or lack thereof offered by the examples used by the teacher in the classroom situation. It should be noted that my study was not aimed at making any judgemental analysis of the teacher.

1.2 Significance of the study

The study was aimed at improving my pedagogical content knowledge and subject content knowledge. The study will also inform practising teachers about the importance of carefully chosen examples more especially multilingual mathematics classrooms. I also think other stakeholders like curriculum designers; mathematics researcher scholars and other interested parties in education in general will find the study informative about the current exemplification practices of teachers in South African schools.

1.2Operational definitions

Example

The use of the term example in this study resonates with the use of the term by Stein, Grover, & Henningsen (1996) "…as a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea" (p.460). In other words examples are meant to illuminate, illustrate, demonstrate or explain concepts (Rowland, 2008). Examples include both worked examples and exercises. I will elaborate more in Chapter 2 on my use of examples since it is the crux of my study.

Exemplification

The term exemplification in the study refers to the use of a specific example to represent the general concepts to instruct learners (Bills et al., 2006).

Language of Learning and Teaching (LoLT)

This refers to the official language used for instruction.

Multilingualism

The term multilingualism in this study refers to the ability to use two or more languages. In

this case a classroom with learners proficient in two or more languages is considered as a multilingual classroom.

Mathematical Practices

Mathematical practices in this study is used to refer to what Godino, Batanero and Font (2007) define as "any action or manifestation (linguistic or otherwise) carried out by somebody to solve mathematical problems, to communicate the solution to other people, so as to validate and generalize that solution to other contexts and problems" (p.130). In other words these are actions that lead to the solution of the mathematical problem at hand. These include practices like defining terms, code-switching, re-voicing, scaffolding, using different types of representations, using the natural language to explain mathematical language, making connections between procedures, identifying misconceptions and so on.

1.4 Conclusion

This Chapter has given an overall picture of what the study seeks to investigate and briefly highlighted the language policy in South African education. In doing so it has presented the rationale and the purpose of the study. I have also outlined the significance of the study. The next chapter will focus on the theoretical framework that informs the study as well as some existing literature related to the study. In Chapter 3 methodology issues will be dealt with. These include research design, data collection methods, validity and reliability and ethical considerations. Chapter 4 looks at the analysis of findings. Chapter 5 then concludes the study by looking at the conclusions, recommendations, limitations of study and reflections.

CHAPTER TWO: Literature review and theoretical framework

2.1 Introduction

This chapter focuses on the theoretical framework that is used to underpin my study as well as the review of literature on multilingual classrooms. In this research study variation theory particularly propounded by Rowland (2008) is used as a lens for exploring the choice and use of examples by a high school teacher in a multilingual classroom for teaching Probability at Grade 11 level.

2.2 Literature Review

In Chapter 1, I defined the use of example in this study as referring to "… a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea" (Stein et al., 1996, p.460). Examples are therefore the critical link between the concepts taught by the teacher and the learner. In other words examples are meant to illuminate, illustrate, demonstrate or explain concepts (Rowland, 2008).

2.2.1 Types of examples

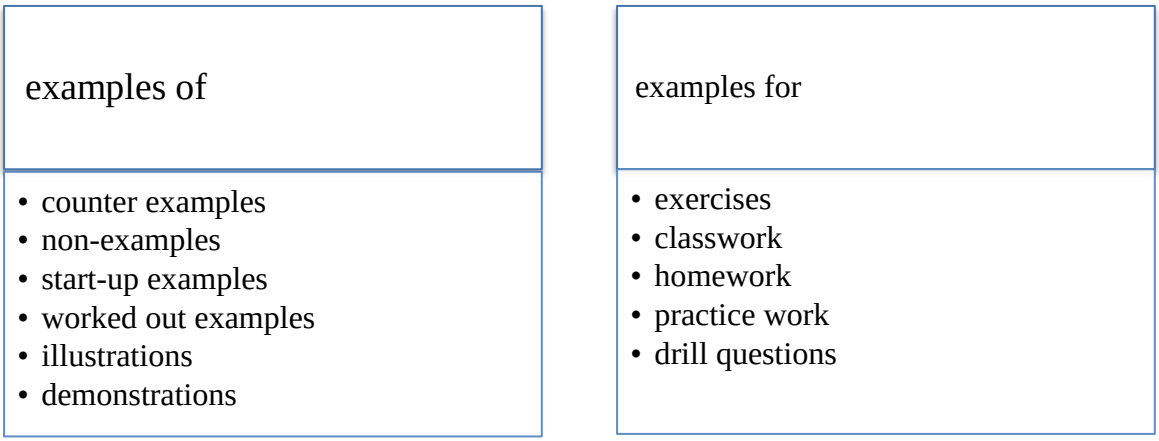
Examples of and examples for

According Bills et al. (2006) examples should be both transparent and offer opportunities for generalization. The examples are classified into two types by Rowland (2008) namely 'examples of' and 'examples for' (p.150). The first category refers to the examples used by the teacher to illustrate or demonstrate concepts and provide opportunities for generalization and abstraction (Bills et al., 2006; Rowland, 2008; Watson & Mason, 2006). Bills et al. (2006) call 'examples of', 'worked (out) examples' and looks at them as questions done by the 'teacher, textbook author or programmer, often with some sort of explanation or commentary' (p.2). In a classroom situation this could refer to examples done by the teacher on the board to demonstrate some concepts. Worked out examples are usually accompanied by explanations. It is therefore important for the teacher to come up with well-thought out examples that provoke generalisation and abstraction (Adler & Venkat, 2014). Start-up examples, model or generic examples, reference examples, counter examples and non-

examples used to explore new concepts in my view are part of 'examples of' (Goldenberg & Mason, 2008).

The second category referred to as 'exercises' are questions meant to provide learners with an opportunity to practice certain mathematical skills (Bills et al., 2006). The exercises are meant to consolidate the 'examples of'. It is therefore very important for the teacher to carefully select examples i.e. both worked examples and exercises that provide learners access to mathematical concepts. The following section will explore in detail the importance of examples. The research study will focus on both types of examples summarised in Figure 2.1 below:

Figure 2.1: Type of examples



2.2.2 The importance of examples in mathematics

According to Zaslavsky & Zodik (2007) examples are an important tool for generalisation, reasoning and communication. It becomes imperative in the South African context to deal with the issues of language as they affect communication. In Chapter 1, I mentioned that the results of the Third International Mathematics and Science Study (TIMSS) of 1995 and 1999 summarised by Howie (2003) revealed that South Africa performed poorly in comparison to other countries in both studies. The research findings identified language as the main reason for the poor performance. Research on the impact of language on examples is therefore essential to help mathematics teachers in multilingual classrooms.

On the other hand, viewing examples as a communication tool implies the use of language in the selection and use of examples. The language in the example itself communicates the intended mathematical concept. At the same time the accompanying explanation from the teacher uses language to communicate the mathematical concept in the example.

As mentioned before language issues affect the mathematics classrooms adversely in South Africa with 11 official languages and English language as the most dominant language (Howie, 2003). The examinations are written in English. The learners therefore, need to have a good understanding of the English language before understanding Mathematics language to achieve mathematical proficiency (Adler, 1995). It is very important that teachers choose examples that are well thought-out so as to cover key components of a topic and also help learners understand appropriate language used in assessments. In the next section I look at the language in the curriculum and its role. The exploration of language is in light of the argument that examples are communication tools which to a larger extent rely on a language. The main thrust of the research study was to explore the selection and use of examples. The selection part notwithstanding the language issues may be the same in all classrooms. However the mathematical practices in the use of examples in a multilingual classroom are not same with practices in any other classroom.

2.2.3 Language in the curriculum

The South African education policy states that there are 11 official languages. These languages are

> "…meant to facilitate communication across the barriers of colour, language and religion, while at the same time creating an environment in which respect for languages other than one's own would be encouraged" (Department of Basic Education, 1996, p. 2).

The policy theoretical puts all languages at the same level. According to Setati (2008) many schools in South Africa use English language as a medium of instruction because of political reasons.

The new curriculum in South Africa has an increased focus on the educator being the facilitator. This involves more interaction between the educator and learner and amongst the learners themselves. Students have to participate in mathematics classrooms in both verbal and written practices to prove conjectures, explain and present arguments among other activities meant to promote mathematical proficiency (Moschkovich, 1999). Jina and Brodie (2008) assert that language in the curriculum is used as a "…the tool through which meaning is constructed" (p.1). This means that learners interpret what they see and hear based on their prior knowledge. All this is done through a language. This suggests that teachers and learners must be able to communicate effectively in the classroom. Effective communication may

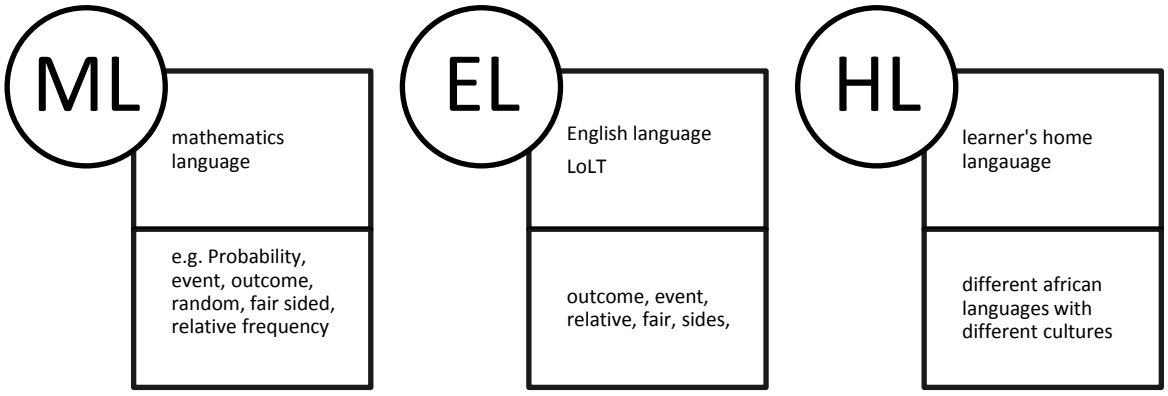
occur when learners are able to: speak mathematically, read mathematical texts, interpret and analyse mathematical questions (Pimm, 1981; Adams, 2003; Shuard, 1991).

2.3 Role of language in Mathematics classroom

2.3.1 Language in Mathematics

Language is a very important component of learning as most of the thinking is done in a language (Setati, 2008). Kilpatrick, Swafford & Findell (2001) denote that "students often understand before they can verbalise that understanding" (p.118). In other words learners think through a language. When learners talk in class they reveal their reasoning or understanding of the concepts under discussion in the classroom. Lovell (2002) asserts that people gain thought clarity by talking. Therefore talking about mathematics should aid in the understanding of mathematics. The pedagogic practices of teachers should therefore provide learners with opportunities to discuss and explain mathematics. The challenge faced by the teacher is largely on the appropriate language to use. Figure 2.2 shows the three main languages at play in any given multilingual classroom. ML is Mathematics language, EL-English language and HL is Home language.

Figure 2.2 Languages in the mathematics classroom



The main challenge in a multilingual classroom is for the teacher to understand the language problems that learners have, so as to develop pedagogical practices to overcome these problems. The language has an impact on the understanding of key concepts in mathematics. Pimm (1981) argues that mathematics is a language. Adams (2003) further elaborates that mathematics is language made up of words, numeracy and symbols that may be connected

and mutual reliant. Therefore doing mathematics in English language brings another dimension to the pedagogical challenges in multilingual classrooms. Adams (2003) asserts that English language is a channel through which the language of mathematics is accessed. The learners need to have a good understanding of the English language before understanding Mathematics language to achieve mathematical proficiency.

The question is how one navigates through the language of instruction and the subject matter. What mathematical practices promote the acquisition of mathematical proficiency without watering down the subject matter? In multilingual classrooms the biggest challenge is for educators to understand the difficulties that learners face in understanding and making sense of mathematical language (Boulet, 2007). In the next section I look at some of the mathematical practices used in multilingual classrooms documented in some studies.

Setati (2008) argues that Language of Learning and Teaching (LoLT) is a complex political issue in South Africa. Most learners and teachers prefer using English as a Language of Learning and Teaching (LoLT) despite research suggesting that the learners' home language should be used (Setati, 2008). The use of English to promote understanding by both the learners and teachers is politically motivated (Setati, 2008). English is seen as an international language used to access social goods as most textbooks are written in English and higher education uses it as well (Setati, 2008). The power of language cannot be under-estimated in the selection and use of examples.

2.3.2 Teaching and learning Mathematics in a multilingual context

According to Adler (1995) learners and teachers in a multilingual classroom are faced with three issues in pursuit of gaining conceptual understanding of mathematics i.e. the access: to the language of instruction, mathematical discourse and classroom discourse. Conceptual understanding involves doing mathematics with understanding (Ball, 2002). In this study conceptual understanding was measured only on how learners participate in class and more importantly how they explain and link probability concepts. Discourse may be described as "…both the way ideas are exchanged and what the ideas entail: who talks? About what? In what ways?" (National Council of Teachers of Mathematics, 1991: 34). The language of instruction which is also referred to as the Language of Learning and Teaching (LoLT) was dealt with in the previous section. Mathematical discourse relates to the mathematical strategies that may be employed in a multilingual classroom. The choice of different strategies will influence how mathematics is taught. According to Vygotsky (1978) learners

use tools that come from a culture like language, calculators and writing to mediate their social environments. Therefore it will be useful for the teacher to use these tools to promote conceptual understanding. This also implies that the teacher can use learners as resources in the classroom more especially in a multilingual classroom (Moschkovich, 1999, Essien, 2010). Classroom communication is therefore very important more especially in a multilingual classroom.

Effective communication may occur when learners are able to: speak mathematically, read mathematical texts, interpret and analyse mathematical questions (Pimm, 1987; Adams, 2003). Moschkovich (2002) describes communicating mathematically as what constitutes these features among others "…using multiple resources and participating in mathematical practices, such as abstracting, generalizing, being precise, achieving certainty, explicitly specifying the set of situations for which a claim holds and tying claims to representations" (p.196). In other words communicating mathematically means that the classroom talk or discussion focuses on conceptual understanding more than just computational fluency.

According to Pirie (1998) communicating mathematically involves use of different forms of resources to access mathematical concepts, namely 'ordinary language', 'mathematical verbal language', 'symbolic', 'visual representation', 'unspoken shared assumptions', and 'quasi-mathematical language' (p.8). The classroom discussion goes beyond the use of one form of communication for the sake of it but looks at what will enhance mathematical understanding. Put differently the classroom environment that promotes communicating mathematically allows learners to use their informal conceptual understanding to access the formal concepts (Moschkovich, 2003).

According to Moschkovich (2002) communicating mathematically is not limited to " acquiring vocabulary" and "constructing meanings" it is affording learners to participate fully in 'discourses' (p.191). This implies that acquisition of mathematical terms and understanding uses of words or terms in different contexts does not guarantee understanding of mathematical concepts. Allowing learners to participate in the classroom encourages them to communicate mathematically. As they participate in the classroom learners may acquire new words or terms and learn their multiple meanings. However acquisition of mathematical terms and understanding of word meanings in different contexts does not translate to mathematical proficiency. Pirie (1998) argues that teachers "…must not make superficial judgements" when trying to "…access the understanding being constructed by pupils in their language of whatever form, verbal or symbolic" (p.28). Learners, as argued by Pirie (1998) communicate mathematically by using a language that they best understand. It becomes the role of the teacher to use learners as resources in the classroom.

In the next section I will discuss some mathematical practices found in multilingual mathematics classrooms.

2.4 Mathematical practices in multilingual classrooms

According to Watson (2007) "…mathematical learning is hard to sustain without engaging in the mathematical practices by which such entities were originally created…" (p.118). In other words mathematical practices are a key element in analysing learning of mathematics. It is therefore essential for me to survey the mathematical practices in multilingual classrooms.

As mentioned in chapter 1 mathematical practices in this study is used to refer to what Godino, Batanero and Font (2007) define as "any action or manifestation (linguistic or otherwise) carried out by somebody to solve mathematical problems, to communicate the solution to other people, so as to validate and generalize that solution to other contexts and problems" (, p.130). This definition involves communication of the solution implying the use of language. There is interaction which is critical in promoting mathematical reasoning (Jina and Brodie, 2009).

Mathematical practices in multilingual classroom should address communication challenges faced by both teachers and learners. Vorster (2008) divides strategies that can be used in multilingual classrooms into two major groups namely English only and code switching. English only include strategies that develop proficiency in English and the English mathematics register while code switching relies on exploiting two languages (Vorster, 2008). The discussion below tries to capture some of the mathematical practices put forward by some researchers.

The mathematical practices as described by Ball (2002) are the actual actions that enable the development of mathematical thinking. The development of mathematical thinking in my view requires increased participation from learners. Thus in this research study mathematical thinking is seen as the ability to explain and justify claims and the capability of participating in meaningful mathematics discussions. For example as learners explain and justify answers they do it by actively participating as a group or individually. Kilpatrick et al (2001) argue that classroom norms need to encourage learners to justify their claims. In other words the classroom provides a platform for the learners to improve their mathematical thinking but it all depends on the prevailing situations as promoted by the teacher. The situations in the classroom either constrain or promote learning. Brown et al (1989) argue that "situations might be said to co-produce knowledge through activity" (p.32). In other words learning is

dependent on the set of circumstances in which it is developed and the examples used to produce it.

Boulet (2007) argues that it is "... important for teachers to use clear language that reveals the reasoning behind mathematical procedures" (p.4). The teacher should use simple language that goes beyond procedures. Many researchers are of the view that the mathematics teacher in a multilingual classroom should use mathematical language appropriately to assist learners understanding the mathematical concepts embedded in the subject content (Essien, 2010; Setati, 2008; Adler, 2003; Moschkovich, 1999). The teacher may use explicit mathematics language by making sure instructions and explanations are clear (Adler, 1995). Research done by Adler (1995) showed that explicit mathematics language benefits both the first language and second language speakers. It is therefore beneficial to the learners if the teacher does not only use appropriate language but also uses explicit mathematically language. Use of explicit mathematics language can be achieved through the mathematical practice of defining key or new terms.

Moschkovich (1999) argues that teaching strategies in the multilingual classroom should not be limited to exposing learners to content vocabulary. Teachers should move beyond terminology or grammatical errors to listen for the mathematical content (Moschkovich, 1999). This means the focus should be on acquisition of mathematical skills more than just mathematics terminology. The teacher should promote conceptual understanding. Learners should be allowed to use several expressions for the same concept (Boulet, 2007).

Code- switching is another useful mathematical practice Essien (2010) puts it this way: the teacher should "…recognise multilingualism as an asset rather than a liability in his/her class" (p.38). In other words the teacher should use multilingualism to his/her advantage... This implies that the teacher should be familiar with the learner's home language to use it effectively. By code–switching the teacher uses the learners' language as a resource and tool in the teaching and learning of mathematics (Adler, 2001; Setati, 2002; Vorster, 2008). On the other hand code-switching arguably promotes conceptual understanding (Setati, 2008). Learners are able to engage with concepts without any barrier. Setati (2008) argues that language is tool for thinking and communicating. Therefore allowing learners to code-switch empowers them with a tool to think and communicate ideas. Learners get an opportunity to reason out concepts and make sense and ultimately say what is in their minds without impediments which may be imposed by a foreign language and the examples.

However the context of examples brought by the teacher to the classroom also affects learning (Brown et al, 1989). Stein, Smith, Henningsen & Silver (2000) argue that selecting and using examples primarily based on carrying out procedures does not promote mathematical thinking. The role of the teacher is central in the development of mathematical thinking, firstly in choosing an appropriate task and later to lead the class discussion in a way that encourages full participation. The task or the example space should be well thought to stimulate learners into active participation. The quality of tasks should "…facilitate specific mathematical actions or observations" (Ozmantar & Monaghan, 2008). The task should lead learners into developing sustainable problem solving skills. In the next section I will present what the South African syllabus expects the examples in probability to look like.

2.5 Curriculum Expectations

Gal (2009) presents an analysis of how the South African curricula advocates the use of real life examples to teach probability at the same time places more emphasis on examples that neglect real- life situations . There is seemingly a gap between what the curriculum expects the learners to acquire and the examples used to achieve that goal. According to Lampen (2011) the curriculum requirement of incorporating real life contexts is spoiled by the examples given by teachers.

According to CAPS (2011) document

> Mathematical modelling is an important focal point of the curriculum. Real life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible .(p.8).

The CAPS (2011) document strongly emphasizes the use of realistic examples and discourages the use of artificial ones. However research shows many teachers teach statistics and probability using examples that lack real life context (Lampen, 2011;Garfield & Ben-Zvi, 2008). The result of neglecting real life context promotes fluency in getting the answer between 0 and 1 while in most real life situations probability requires one to interpret probability presented in words (Gal, 2009).

Here are some real life examples that contain some probability terms and symbols from various media:

1. Starting in 2007, less people have been dying on an annual basis. The total number of deaths fell by **6.2%** in 2010, as compared to 2009. **More males than females died** and the highest number of deaths was recorded amongst people aged 30 – 39 years. Tuberculosis (TB) was the leading cause of death in South Africa, accounting for about **12%** of deaths that occurred in 2010. During this year, the number of deaths caused by TB, influenza and pneumonia and intestinal infectious diseases decreased by at least **10%** per cause. The number of deaths due to diabetes mellitus and HIV disease increased by **3,8 and 3,0%** respectively. **Nearly 10%** of deaths were due to non-natural causes (e.g. transport accidents, assault, etc.). **3 out of every 8** deaths in the age group 15 – 19 were due to non-natural causes.
.(http://www.healthline.com/health/hiv-aids/chances-contracting#OtherSTIs6)[1]

2. People with untreated STIs are **more likely** to contract HIV than people without STIs. Why? First, some STIs cause ulcers, or sores, in the genital area. These sores create an opening in your skin, **making it easier** for HIV to get into your system. Second, when you have an infection, your immune system sends certain cells called CD4+ cells to help fight the infection. These CD4+ cells are what the HIV virus targets—therefore, having an STI **makes you more susceptible** to HIV.(http://www.healthline.com/health/hiv-aids/chances-contracting#OtherSTIs6)[2]

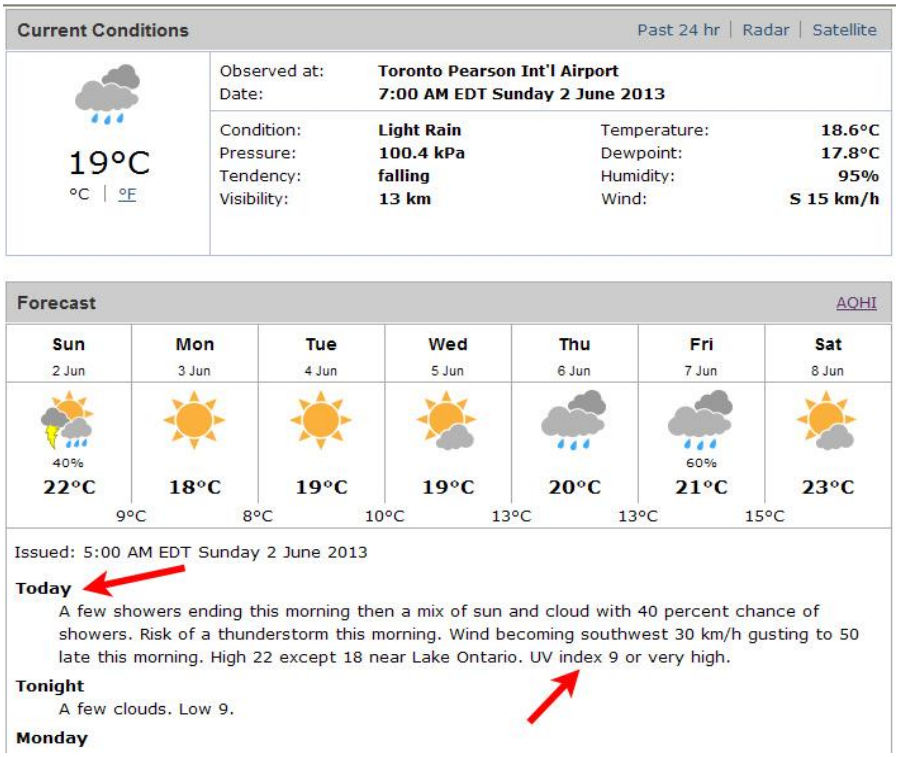
3. Mainly cloudy with a **little drizzle**. **Slight chance** of a rain shower. North eastern wind at 5 to 10 mph. Temperature **below normal** in the north and central regions. **Near normal** for southern regions. Maximum temperature 10°C.

(http://usefulenglish.ru/vocabulary/weather-and-temperature)[3]

[1] Accessed: 2015-07-28 21:18:43

[2] Accessed: 2015-07-28 21:18:43

[3] Accessed: 2015-07-28 21:18:43



(https://www.google.com/search?q=weather+forecasts+in+words&tbm=isch&imgil=Cd0p[4])

In the above examples I have highlighted terms that require probability knowledge for everyday use. In the above examples probability is represented in ratios, words/phrases and percentages. The words or phrases used are: nearly, more likely, making it easier makes you more susceptible, little drizzle, slight chance, below normal, near normal and risk. The above examples demonstrate that in real life probability is not always represented in numerical terms but in words or phrases as well (Gal, 2009). The use of words or phrases in probability poses linguist problems to learners in multilingual classroom who use English as a second language (Gal, 2009; Lampen, 2011). The teacher in a multilingual classroom needs to therefore select and use examples that will be effective in making learners understand probability in the real life context. It was therefore the focus of this study to analyse the selection and use of examples in probability using the theoretical framework laid out below.

2.6 Theoretical framework

[4] Accessed: 2015-07-28 21:18:43

Sun (2011) argues that the "…old-fashioned East Asian teaching practices have been able to produce students with high achievements in China" due to use of variation in examples given by teachers to learners (p.66). Goldenberg & Mason (2008) argue that the variation in well selected examples helps learners to differentiate between fundamental and subsidiary characteristics of mathematical concepts. It is useful to draw lessons from what Sun (2011) calls 'Chinese paradoxical phenomenon' (p.66). Sun (2011) describes 'Chinese paradoxical phenomenon' as characterised with teaching methods that,

> are content oriented and examination driven, while conducted with a large class size. Instruction is teacher dominated. Memorization of mathematical facts is stressed and students learn mainly by rote. Yet, the seemingly old-fashioned East Asian teaching practices have been able to produce students with high achievements in China (p.66)

The paradox in the above statement is that learners in the Chinese schools using what seem to be old methods of teaching and learning do very well when compared to other students from other countries. According to Sun (2011) most researchers generally attribute observed good practices in the Chinese mathematics to variation theory. In my view the conditions in East Asian are more similar to the South African context more especially with arguably a very prescriptive new curriculum (CAPS). However there are some notable differences like multilingualism which is a common feature in South African classrooms. Goldenberg & Mason (2008) maintain that variation in examples helps learners to see what stays the same and what changes as well as the boundaries for those changes.

2.6.1 Variation Theory

Variation is about what changes, what stays constant and the underlying rule that is discerned by learners in the process of learning (Leung, 2012). Variation theory was first propounded by Marton and Booth (1997) who argued that learning is largely affected by perception. Marton & Pang (2006) further assert that "perceptual learning is a process of discrimination and discernment" (p. 199). The assertion implies that perceptual learning involves recognition and understanding of the difference between one idea from another which underpins the theory of variation. It is therefore important to look at the views of variation theory about learning before looking at the theory itself.

According to Runesson (2005) learning is "…a change in the way something is seen, experienced and understood" (p.70). In other words seeing, experiencing and understanding are critical in learning. It is important for the teacher to provide the learners with examples that facilitate learning. Learners should be afforded a clear access to mathematical concepts.

On the other hand Marton & Pang (2006) describe learning as the "…sense of becoming able to discern and distinguish certain qualities …" (p.200). Discerning and distinguishing further strengthen the need of careful selection of examples in mathematics classrooms and more so in multilingual classrooms. Language problems might cloud the mathematical concepts which are the 'object of learning' in the mathematics classroom (Marton & Pang, 2006, p. 217).

According to Watson & Mason (2006) using variation theory by "…paying attention to variation can illuminate our understanding of learners' responses to sequences of questions" (p.95). In other words variation theory enables the teacher to engage meaningful with learners having a better understanding of the learners' perception of the mathematics concepts. On the other hand Rowland (2008) argues that choices and sequencing of examples are 'neither trivial nor arbitrary' in the mathematics classroom (p.150). One can argue that through variation learners are able to come to a deeper understanding of a concept by focussing on invariants within variation (Marton, Runesson, & Tsui, 2004).
In this study I would use variation theory based on Rowland's (2008) four categories of exemplification namely "variables, sequencing, representations and learning objectives" (p.153). I will elaborate each of the categories in some detail below.

Variables

The first category of exemplification deals with variables and is concerned with what changes or varies in what is being learnt (Rowland, 2008). Watson & Mason (2006) argue that 'dimensions of possible variation' and 'permissible range of change' should be part of the examples provided by the teacher in the classroom (p.98). The examples given by the teacher should expose learners to all possible variation of the concept being exemplified. According to Rowland (2008) all examples have something that varies or changes and 'what varies in our experience influences what we learn' (p. 153).

Sequencing

Rowland (2008) points out that teacher's examples in many instances come in a sequence. This means that the teacher or the exercise designer arranges the questions in a particular way. In textbooks the examples are often controlled or predetermined than in the classroom

situation with the teacher interacting with the learners (Rowland, 2008). The teacher may alter the order of his/her examples depending on the situation in the classroom. The choice of examples can be spontaneous or random (Rowland, 2008). The random selection of examples "may, among other things, confuse the role of variables" (Rowland, 2008, p. 158).

Representations

This category involves representing concepts in a flexible, connected and progressive manner (Heinze, Star, & Verschaffel, 2009; Rowland, 2008). Flexible representation involves representing examples in a multiple ways that clarify the concept using words, symbols, tree diagrams, pictures, contingency tables, graphs and so on (Heinze et al., 2009). There is need to connect all the representations whether spatial or symbolic (Rowland, 2008). Progressive representation is about moving from one form of representation to another for example using pictorial graphs to bar graphs (Nistal, Van Dooren, Clarebout, Ellen & Verschaffel, 2009). It is my contention that in a multilingual classroom flexible, connective and progressive representation of concepts is critical due to complications induced by the use of a second language.

Learning objectives

This category suggests that an example is chosen to attain a specific goal (Rowland, 2008). The learning objectives category is '….less coherent of all three…' as it is'…an attempt to gather several related issues under one banner' (Rowland, 2008, p. 160). The learning objective is described by Marton & Tsui (2004) as the 'object of learning' (p.22).In other words the learning objective of an example can be viewed as the intended purposes of giving the example or examples. Rowland (2008) identifies two objectives for examples namely 'examples of something' and 'examples for practice' (p.150). 'Examples of something' is used to mean examples chosen to promote abstraction and generalisations. While on the other hand 'examples for practice' refers to the exercises given by the teacher to help learners achieve procedural fluency (Rowland, 2008).

In this research study I wish to explore the extent to which variation practices used in South Africa provide or constrain opportunities for learning for learners in multilingual classrooms.

2.6.2 Implementation stage of examples

In order to analyse the use of examples I used Staples (2007) as my theoretical framework. The framework consists of three components namely "supporting students in making contributions, establishing and monitoring a common ground and guiding the mathematics" (Staples, 2007, p. 172). The three components of Staples' (2007) framework are interrelated and as such are not so easy to distinguish in practice.

Supporting students in making contributions

Supporting students in making contributions includes "eliciting students ideas", scaffolding the production of students ideas" and "creating contributions" by for example linking students' ideas (Staples, 2007:173). Scaffolding refers to the action of assisting learners to do tasks they would not be able to do on their own (Staples, 2007).

Establishment of a common ground

The second component of Staples' (2007) model is the establishment of a common ground in mathematics classroom. Common ground refers to "…suppositions, ideas, and objects that participants take as mutually held or recognized" (Staples, 2007, p. 180). In other words common ground ensures that all participants have a common understanding of the path the discussion is taking by keeping the discussion focussed. Common ground controls the flow of the discussion and even the concept development (Staples, 2007). In some sense common ground supports the learners in making contributions by linking their contributions to each other.

Guiding the mathematics

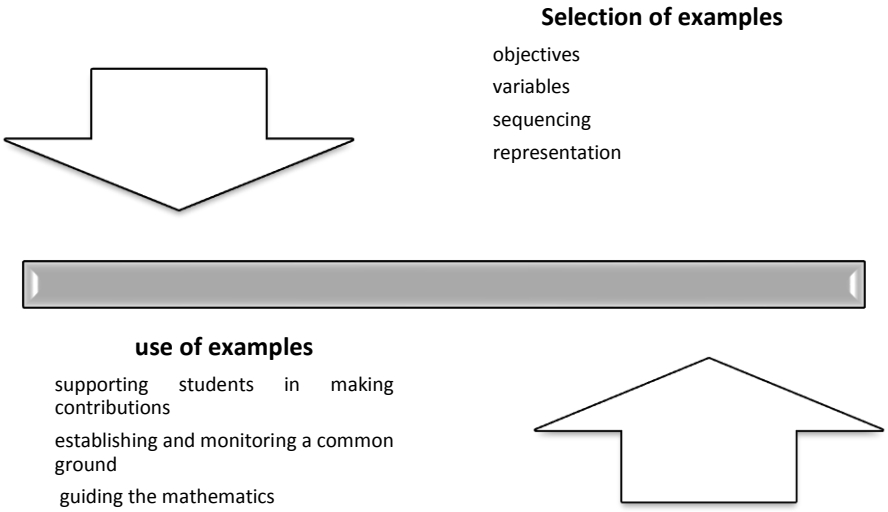
The third feature of Staples' (2007) is guiding the mathematics and is about "on-going assessing and diagnosing of students' understanding" (p.189). This has to do with maintaining the cognitive level of the task by listening without evaluating. Davies (1997) asserts that the quality of the articulations by learners is closely related to how educators listen to their learners. Coles (2000) argues that evaluative listening tends to constrain the teacher in establishing learners' understanding of the concepts. The teachers are encouraged

to use transformative form of listening which entails the willingness for the listener to alter ideas or entertain other points of view (Coles, 2000). This implies the teacher may "re-voice" learners' ideas, ask for "justifications" and "…is willing to divert from his teaching agenda somewhat to listen to and work with learners' questions" (Brodie, 2007:7). According to Setati & Adler (2001) re-voicing involves listening to learners talk and repeating and this leads to leading learners to correct and more formal mathematical discourse. Re-voicing has an effect of helping the rest of the class understand what the speaker said and re-enforcing correct terminology and conceptualisation. It also acts as motivation for the learners to participate in class discussions.

Guiding mathematics in my view is equivalent to what Engle and Conant (2002) call "productive disciplinary engagement" (p.607). The concept of productive disciplinary engagement includes encouraging learners to engage meaningfully in the class discussion, "holding learners accountable to others and to disciplinary norms, productive disciplinary engagement and providing relevant resources" like time, books and so on (Scott, Mortimer & Aguiar, 2006:607). According to Staples (2007) guiding mathematics includes among other things selecting and implementing tasks, on-going assessing and diagnosing of students' understanding and guiding by following. Task implementation involves maintaining the cognitive level of the task (Stein et al, 2000).

Summary of the two-fold theoretical framework

Figure 2.3



The use variation theory in the examples influences the interaction in the classroom. Using examples the teacher initiates the type of interaction in the classroom. In the diagram, the selection of examples and use of examples contribute to what I contend to be a balanced view of a mathematics lesson that promotes mathematical thinking. Examples provide "…what is accessible in response to a particular situation, to particular prompts and propensities" (Bills et al., 2006, p. 133). In other words examples depend on the situation and are actually provoked by external factors. These external factors are influential in the pedagogical use of examples (Bills et al., 2006). Sources of examples include textbooks and teacher's own experience. Selection or designing of tasks is followed by implementation stage which may involve the discussion of task at hand (Stein, 2000). Paying attention to variation in the example is equally important as the discourse that accompanies the example (Adler & Venkat, 2014). On the other hand classroom 'productive dialogue' depends largely on the task or activity made available by the teacher (Wells & Ball, 2008, p. 170). Variation theory is useful for analysing the set up rather than implementation stage of the lesson (Stein, 2000). Staples' (2007) framework was useful for analysing the implementation stage of the examples selected by the teacher as it relates to mathematics learning practices. It is at the implementation stage that language related mathematical practices sets apart the multilingual classroom from non-multilingual classrooms.

2.7 Conclusion

In this chapter I have described the theoretical framework underpinning my study. I have also looked at language in Education policy in South Africa. The literature review has outlined what other scholars say about the importance examples in mathematics classrooms and different methods that can be used by the educator to facilitate interaction in mathematics classroom. The Chapter also dealt with some mathematical practices found in multilingual classrooms. In the next chapter I engage with the methodology and research design of the study.

CHAPTER THREE: Research design and Methodology

3.1 Introduction

In this chapter I will outline the methodology used in the research. The population and sample is described. I will also look at the ethical issues that were taken into account before collecting data in this research and engage with issues of validity and reliability.

3.2 Research design

This is a qualitative study on the selection and use of examples by teachers in multilingual mathematics classrooms where learners learn in a language which is not their first or home language. According to Sulkind (2012) a qualitative study involves the use of techniques that are flexible and suitable for discovering underlying motivations, feelings, values, attitudes and perceptions. In other words qualitative studies use methods that dig deeper into the underlying reasons of certain practices. Qualitative research afforded me an opportunity to study the selection and use of examples in multilingual classroom in a more detailed form and learn more about the importance of careful selection of examples in mathematics instruction. The syllabus referred to in all the cases in the study is CAPS and the learners' textbook is Via Afrika Grade 11 by Abbot, Botsane, Bouman, Bruce, du Toit, Pillay, Schalekamp & Smith (2012).

The school where the research was done is a multilingual township school in Ekurhuleni South, Johannesburg. The medium of instruction in the school is English. Languages spoken by learners include isiZulu, Xhosa, Setswana, Sesotho and Xitsonga. English is a second if not a third language to most learners in the school.

Sample

The sample consisted of one teacher in one Grade 11 class with 40 learners between the ages of 17 to 19 years. The sample was a purposive one. Marshall (1996) argues that in purposive sampling the researcher '...actively selects the most productive sample to answer the research question(s)' (p.523).The teacher was selected after looking at his qualifications and experience. Themba has a B.Ed. (FET) in Mathematics and four years teaching experience in multilingual mathematics classrooms. He is fluent in IsiZulu and IsiXhosa and understands

Sesotho, Sepedi and Setswana very well.

3.3 Data collection methods

Observations and interviews were used as the two major means of data collection. Data was collected in three phases: first was the pre-observation interview, then lesson observations and then the post-observation interview. The interview done before the class observations was to understand how the teacher plans for his lessons. It was also meant to understand the teacher's beliefs and theoretical approach to the lesson. The second interview was done at the end of the lesson observations to reflect on what transpired during observation.

3.3.1 Classroom observation

I collected data by observing one teacher teaching one multilingual Grade 11 class. The classroom observations gave me some first-hand information of what is happening in multilingual classrooms. Dawson (2002) argues that observation has the advantage of revealing information that people are normally unwilling or unable to provide. In observations one gets to view certain practices or behaviours that people may be reluctant to give in interviews.

The focus of the observations was to record the obstacles and affordances of the examples chosen by teachers in multilingual classrooms. Eight lessons were observed but four were analysed. The lessons that were left out in the analysis were too short and did not develop fully to be considered for analysis. Observing more than one lesson ensured that the data collection captured a good portion of examples used by teachers in multilingual classroom. All the lessons were both visual and audio-recorded. The class observations informed my second interview questions. In other words what I observed in class helped me formulate my interview questions. The four lessons were transcribed and the transcripts given to the teacher to correct any errors made during the process of transcribing.

3.3.2 Interviews

According to Goddard and Melville (2001: p.49) an interview "…involves a one-on-one interaction between the researcher and a respondent". The interview provides a chance to get in-depth views of the respondent. Kothari (1985) asserts that interviews are a very flexible and quick way of obtaining data. The semi-structured interview is very flexible in

that issues are dealt with and probed as the situation arises (Kothari, 1985). Goddard and Melville (2001) further argue that interviews afford the researcher to ask the respondent to clarify unclear answers. The researcher has a chance to ask for more explanations. Responses from respondent determine the trajectory of the interview.

I conducted these interviews with the educator before and after the lesson observations. The discussion provided a deeper understanding of reasons behind certain mathematical practices observed during in the lessons. The teacher got the opportunity to explain the selection and use of some examples observed during the lesson. The interviews also afforded me a chance to record examples not observed in lessons but employed by the teacher from time to time. The teacher interviews were substantiated by class observations. Whatever the teacher told me during the first interview was checked against what transpired in class for truthfulness. This was done in order to improve trustworthiness of data using triangulation. Cohen, Morrison & Manion (2011) describe triangulation as using multiple methods of data collection and analysis to ensure validity and reliability.

The two interviews i.e. pre and post observation were transcribed and used in the data analysis. The transcripts were cross checked by the teacher for errors. Learners were not interviewed in this study because the research was more focused on the examples used by teachers. Lesson observations were meant to provide information about the selection and use of examples by teachers when teaching Probability.

Table 3.1: Sample of the first interview questions

Question	**Justification for inclusion**
What strategies do you use to teach probability?	To find out the mathematical practices the teacher uses
Where do get your examples?	To find where the teacher gets his examples
Do you alter or adapt your examples? What influences your choice of examples?	To find out the factors that the teacher considers when choosing examples.
How do you deal with language issues in your selection of examples?	To understand what the teacher considers as a good example in multilingual set up.
How do you deal with language issues in the classroom situation?	To find out the mathematical practices the teacher employs to deal with language issues

3.4 Analytical framework

The following analytical framework was developed a priori. The analytical framework was

informed by Rowland's (2008) theory of variation and Staples' (2007) conceptual model of collaborative inquiry mathematics practices. Rowland's (2008) four categories of exemplification consist of variables, sequencing, representation and lesson objectives. Combining Rowland's (2008) categories of exemplification and Staples' (2007) three components of teachers' role in supporting whole class collaborative inquiry afforded me an opportunity to analyse the selection and use of examples in multilingual set-up where language use is a critical issue. Staples (2007) was useful in analysing mathematical practices that played out in the multilingual classroom under study as the teacher used his chosen examples. Construction or selection of examples and the explanations or class discussions of examples complement each other. Well thought-out examples need to be accompanied by well thought explanations. The analytical framework was developed both a priori and a posteriori. The re-contextualised analytical framework in Section 3.5 was developed a posterior after data was collected.

Table 3.2 shows the possible ways the teacher could use variation theory to select examples in probability. In each category of exemplification, examples can be classified as varied and not varied. The last column of the table shows what is possibly made available to learn to the learners as the teacher uses a particular category of exemplification. For example the paying attention to sequencing by say moving from simple to complex examples allows the learners to compare, classify, contrast and associate ideas or concepts.

Table 3.3 shows the mathematical practices accompanying the selected examples. The contributory codes for each of Staples' (2007) three components of teachers' role in supporting whole class collaborative inquiry are listed in the second column.

Table 3.4 links the categories of exemplification and the mathematical practices. My contention is that examples used in a multilingual classroom are not necessarily different from any other classroom but the mathematical practices are different as argued in Chapter 1 and 2.

Table 3.2: Variation theory

Adapted from Rowland (2008)

	varied	Not varied	what is made available to learn to the learners
Variables - Use of the same or different symbols	Different symbols used Different numbers used Different situations used e.g. dice, cards, coins, marbles, balls	Restrictive use of symbols or situations	'dimensions of possible variation' and 'permissible range of change' of variables is visible to learners Creation objects with one feature Creation objects with multiple features the structural component can be discerned from the incidental
Sequencing - Simple to complex - Random - Moving from familiar context to non-familiar - Worked examples	Examples are controlled or predetermined by teacher to cater for the learners' special needs like language, understanding	Similar questions given for practice and pattern identification.	Seek pattern; Compare, classify, contrast and associate ideas; Generalisation; Re-description; Summarise development of ideas; Abstraction
Representations flexible, connected and progressive - Graphs - Symbols - Words - Tables	Different graphs, symbols, words, tables are used.	The questions are done using the same representation.	Flexible, connective and progressive representation of concepts. Connecting words, symbols, tree diagrams, pictures, contingency tables, graphs
Learning objectives	'combination of examples of something 'examples for practice' evident		Procedural fluency Conceptual understanding promoted

Table 3.3: Mathematical practices

Adapted from Staples (2007)

Mathematical practices	Contributory codes
supporting students in making contributions	**eliciting student ideas**- request and press, providing time, giving participation points **Scaffolding the production of students ideas**- representing, providing structure, extending **Creating contributions**- expanding what counts, demonstrating the logic, linking
establishing and monitoring a common ground	**Creating a Shared Context**- establishing prerequisite concepts, verbally marking, affording multiple opportunities to access ideas **Maintaining continuity over time**- using the board over time, pursuing discrepancies, grounding then building **Coordinating the Collective**- positioning students for collective work, controlling the flow.
guiding the mathematics	**Guiding High-level Task Implementation**- Modifying tasks, Providing 'food for thought', **On-going assessing and diagnosing** **Guiding with a Map of Students' Learning** **Attending to 'pressure points'** **Guiding by following, 'going with the students'**- flexibly following a student's thinking, keeping students positioned as the thinkers and decision-makers

Linking variation theory and mathematical practices

The link between variation theory and mathematical practices is not linear. A well thought-out example does not necessarily imply that the mathematical practices used at implementation stage of the example are successful in achieving the desired goal. Table 3.4 on the next page shows the potential that each component of Rowland's (2008) categorization has on influencing the mathematical practices employed in the classroom. Mathematical practices are meant to ensure that a particular example is successful in affording learners a chance to learn probability and mathematics in general. In a multilingual classroom this may involve practices meant to help learners overcome language problems. Table 3.4 attempts to show the link between Rowland's (2008) four categories of exemplification (variation theory) and Staples' (2007) three components of teachers' role in supporting whole class collaborative inquiry (mathematical practices). Figure 3.1 summarises the link between variation theory and mathematical practices.

Table 3.4 Variation and Mathematical practices

Category	Potential effect on Mathematical practices
Variables	**supporting students in making contributions** e.g. using instructive symbols like ∩ and ∪
	establishing and monitoring a common ground.e.g. Using symbols that all learners will understand
	guiding the mathematics. e.g. Using conventional variables
Sequencing	**supporting students in making contributions** e.g. moving from simple to complex
	establishing and monitoring a common ground e.g. constantly checking if the sequence of examples accommodates all learners
	guiding the mathematics e.g. following students' thinking flexibly(non-rigid sequencing)
Representation	**supporting students in making contributions** e.g. using examples that allow the use of a variety of representations e.g. Venn diagram, tree diagram, contingency tables
	establishing and monitoring a common ground e.g. preferring to use one form of representation over another
	guiding the mathematics e.g. allowing learners to use multiple representations
Learning objectives	**supporting students in making contributions** e.g. providing structure through stating learning outcomes expressly or otherwise
	establishing and monitoring a common ground e.g. spelling out the objectives of the lesson to monitor common ground
	guiding the mathematicse.g. Keeping learners focussed on the required skills

The use of variables in the examples may support students in making contributions, establish common ground and guide the mathematics (Staples, 2008). An example in probability is

quite instructive: P (A∩B) and P (A∪B) in my view guides the learners on what to do i.e. to use multiplication rule or addition rule in probability. P (A∩B) means the probability that A and B happen or probability of both A and B happening. The use of the symbol ∩ for intersection is meant to guide the learners in that the 'n' shape comes from the word intersection i.e. where both A and B meet. If there is no intersection between A and B it means A and B have nothing in common and hence are mutually exclusive events. While P (A∪B) means the probability of A or B happening. The use of the symbol 'U' for union has an effect of influencing learners to remember union as all elements in A or B. Therefore P (A∪B) evokes the need to add probabilities of A and B and subtracting the intersection to avoid double counting.

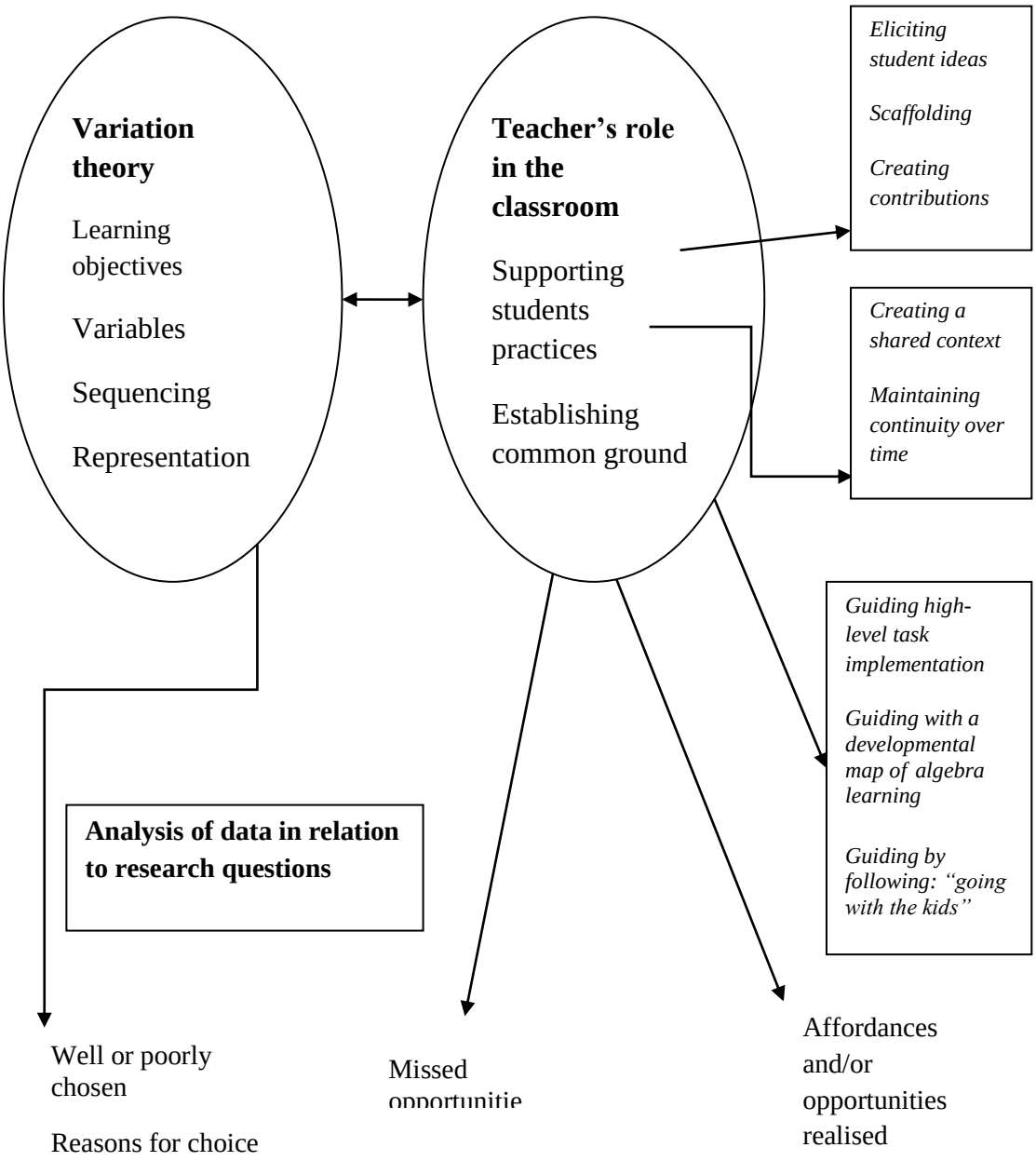
Sequencing of examples influences the learners in seeking a pattern, comparing, classifying, contrasting and associating concepts, and generalising, summarising, developing ideas and so on. A well sequenced set of examples will support learners in making contributions by among other ways affording learners to develop ideas in a controlled manner. More so the control of examples provides structure and in a way scaffolding the production of student's ideas. Sequencing also ensures that the common ground is established and monitored through starting with prerequisite concepts before moving to complex concepts. The sequencing of examples is primarily concerned with guiding the mathematics through modification of examples to suit the cognitive level of the learners. Paying attention to sequencing examples happens at both selection and implementation stages.

Representations are useful in supporting students make contributions, establishing and monitoring a common ground and guiding the mathematics. Using different ways to represent concepts affords learners multiple opportunities to access ideas and in the process establish and monitor common ground. Representations ensure that concepts are flexible, connected and progressive. Learners get grounded on the fundamentals of probability notwithstanding the use of myriad representations to solve a probability problem. The limitations of each form of representation become visible and therefore in way guide the mathematics by keeping students positioned as the thinkers and decision-makers.

Looking at learning objectives component it is worth noting that there are primarily for guiding the mathematics. Learning objectives can also be used to support learners in making contributions and this may be seen when the teacher reminds the learners the objectives of a certain concept, procedure, method or step. Learning objectives are important in ensuring that common ground is established by making sure learners stay focussed on the ultimate goal of

any mathematical practice.

Figure 3.1 links between variation and mathematical practices



Re-worked Analytical framework

The variation theory adapted from Rowland (2008) was used for analysing the selected examples and the obstacles or affordances they offer. Using the two analytical frameworks afforded me an opportunity to analyse both the selection and use of examples by the teacher. The selection of examples involved paying attention to the four components of Rowland's (2008) categories of exemplification, namely learning objectives, variables, sequencing and representation. It also involved finding out the purposes served by the examples selected by

the teacher. The use of examples was analysed using Staples' (2007) collaborative classroom practices model because it involved the implementation of the selected examples in the classroom situation in the form of mathematical practices. It should be noted that in some occasions the teacher had to think on his feet and alter some examples or use some unplanned examples. The codes in the analytical framework for analysing the mathematical practices of the teacher in Chapter 2 were developed a priori by using Staples' (2007) collaborative classroom practices model and other literature. Using the data collected the codes were reworked a posteriori as shown in Table 3.5 below. After analysing the first lesson I had to add and remove some details in the original Staples' (2007) collaborative classroom practices to make it effective in analysing my data. I added 'allowing choral answers' as part of establishing and monitoring a common ground practice (CGP) after realising that the teacher was using it to ensure that all learners remember what needs to be done in each worked-out example. I also added some guiding questions or phrases in each category like, what do you think? Why? What must we do? Tell us how you got that, what must we do first and so on. I removed 'Modifying tasks' under 'guiding the mathematics' as all examples that were used by the teacher taken from textbooks without any notable modifications.

Codes used in table 3.5

SCP-E- supporting students in making contributions practices through eliciting

SCP-SSCP-C- supporting students in making contributions practices through Scaffolding the production of students' ideas and creating contributions

CGP-ShC- Establishing and monitoring a Common ground Practice through creating a Shared Context

CGP-MCont- Establishing and monitoring a Common ground Practice through Maintaining Continuity over time

CGP- CC- Establishing and monitoring a Common ground Practice through Coordinating the Collective

GMP-HL- Guiding the Mathematics Practice through guiding High-Level task implementation

GMP-Ass- Guiding the Mathematics Practice through Coordinating the Collective

GMP-Map- Guiding the Mathematics Practice through Guiding with a Map of Students' learning

GMP-GWS- Guiding the Mathematics Practice through Guiding by following, 'going With the Students'

Table 3.5: Re-worked mathematics practices, Adapted from Staples (2007)

Mathematical practice	Contributory codes	Code
supporting students in making contributions practices (SCP)	**eliciting student ideas**- request and press, providing time, giving participation points E.g. what do think, why, what must we do first. **Scaffolding the production of students' ideas**- representing, providing structure, extending E.g. how else can you do this, how about **Creating contributions**- expanding what counts, demonstrating the logic, linking	SCP-E SCP-SSCP-C
establishing and monitoring a common ground practice (CGP)	**Creating a Shared Context**- establishing prerequisite concepts, verbally marking, affording multiple opportunities to access ideas, code- switching, re-voicing E.g. do we all agree, no you are wrong, if you do not get 1. **Maintaining continuity over time**- using the board over time, pursuing discrepancies, grounding then building **Coordinating the Collective**- positioning students for collective work, controlling the flow and allowing chorus answers. E.g. do you all agree, what do other people say, let's stop making noise and listen, now let's do this question.	CGP-ShC CGP-MCont CGP- CC
guiding the mathematics practice (GMP)	**Guiding High-level Task Implementation**- Providing 'food for thought' On-going assessing and diagnosing **Guiding with a Map of Students' Learning** **E.g.** - So we must stop, let's do this now, we need to, the question says…, we must answer the question, yes the answer should be half. **Guiding by following, 'going with the students'**- flexibly following a student's thinking, keeping students positioned as the thinkers and decision-makers E.g. tell us how you got that.	GMP-HL GMP-Ass GMP-Map GMP-GWS

3.5 Validity and Reliability

According to Maxwell (2005, p.106) validity refers "… to the correctness or credibility of a

description, conclusion, explanation, or other sort of account". In this study it refers to how credible are the results of the study. The results are directly influenced by among other things: teacher selection, class observations and teacher interviews. Validity in other words looks at the correctness of the instruments used to collect data. Maxwell (1992) argues that validity can be classified into five forms namely descriptive validity, interpretive validity, theoretical validity, generalizability and evaluative validity.

Descriptive validity

Maxwell (1992) says that descriptive validity is concerned with accuracy in the account as described by the researcher. In other words the report given by the researcher should reflect the factual account of what transpired. In order to achieve descriptive validity I video recorded all lessons and audio recorded the interviews. The data was transcribed and checked for accuracy using the recorded material.

Interpretive validity

According to Maxwell (1992) interpretive validity is about understanding the situation on the basis of the participants. Maxwell (1992) sheds more light by adding that the language used by the researcher must use as much as possible participants' own words and concepts. I used the semi-structured interviews to check on the accuracy of the participant's meanings of words and actions captured on the videos. The interview questions were piloted to clear out some of the language problems that were anticipated at the set-up stage of the questions. For example, the first question in the first interview was original on the mathematical practices used by the teacher but later altered because the use of the phrase 'mathematical practices' is not commonly used by teachers. I had to use the term strategies in place of 'mathematical practices'.

Theoretical Validity

Maxwell (1992) describes theoretical validity as that which is concerned with the legitimacy of the application of a given theory to establish facts. I ensured theoretical validity of the data collected in the research by giving a colleague my audio-recordings to analyse and give me feedback. I also developed some codes to help me interpret the results to ensure reliability and validity. The codes were developed using the theoretical framework.

Generalizability

Generalizability is the level at which one can use the account of a particular situation or

population to other persons, times or settings other than those directly studied (Maxwell, 1992).In this research study it was not my intention to make any claim about generalising.

Evaluative validity

Evaluative validity is about the use of an evaluative or judgemental framework or perspective on that which is being researched (Maxwell, 1992). The aim of this study was not to make any judgemental analysis of either the teacher in the study or the school where the research was done. The central concerns in this qualitative study were descriptive, interpretive, and theoretical validity.

Reliability

Hinds (2000) describes reliability as a term that "…refers to matters such as the consistency of a measure- for example the likelihood of the same results being obtained if the procedures were repeated" (p.45). I ensured that my study was reliable by developing codes using my theoretical framework discussed in Chapter 2.Whatever was observed was analysed using these codes. The codes were developed a posterior in consultation with my supervisor.

3.6 Ethical Considerations

Before carrying out the research project at the school, I had to seek permission from the Gauteng Department of Education, school administration as well as the Ethics Committee of the University of the Witwatersrand (*Protocol Number: 2014ECE042M).*

Before collecting data, the teacher and learners were given both an oral explanation and written outline information sheet of the research project's objectives and data collection methods. This was meant to stress out or make it clear that participation in the research project was voluntary and all reporting will keep participants' details anonymous. In the written outline was attached a separate tear-off section for educators / learners to sign giving their informed consent to participation. In the case of learners, the slip also contained a section seeking parents'/ guardian's informed consent.

All reporting from the study conformed to accepted ethical research practices on using pseudonyms for all names involved – of school, teacher and learners. Sharing and discussion of the 'raw' data within the research project group was according to conventions for use of pseudonyms.

3.7 Conclusion

In this chapter I have outlined the methodology used in the research project as well as research instruments used to collect data. A qualitative approach was adopted. The chapter also developed the analytical framework for the study. I have also mentioned the ethical considerations that needed to be taken into account before collecting any empirical data in this research. In the next chapter I will be discussing and analysing the results of my lesson observations and interviews.

CHAPTER FOUR- Analysis and Findings

4.1 Introduction

The main aim of this chapter is to focus on the analysis of the data collected from classroom observations and the interviews. The analytical framework used to analyse the data was informed by the Rowland's (2008) categories of exemplification and Staples' (2007) collaborative classroom practices model. The sections in this chapter are organised in major themes as obtained in class observations and teacher interviews. The emerging findings from the study were focussed on answering these questions:

- How does a teacher select examples when teaching Probability in Grade 11 multilingual mathematics classroom and for what purposes?
- What mathematical practices accompany these examples?
- What are the obstacles or affordances offered by examples in Probability given by a mathematics teacher in Grade 11 multilingual classroom?

4.2 Summary of the lessons

In order to analyse the examples used by the teacher I used the lesson transcripts and field notes. 'Examples of' are examples used by the teacher to 'provoke or facilitate abstraction' (Rowland, 2008, p. 150). As explained in Chapter 2 according to Bills et al. (2006) 'examples of '...are worked-out examples. On the other hand 'examples for' are exercises given by the teacher to help learners practise some procedure or concept (Watson & Mason, 2006, Bills et al. 2006). In the excerpts 'R' stands for Researcher, Tr- Teacher and Ln(s) - learner(s). **EOFS-** stands 'examples of' (worked-out examples) and **EFORS-** stand for 'Examples' for (exercises for practise).

Lesson 1

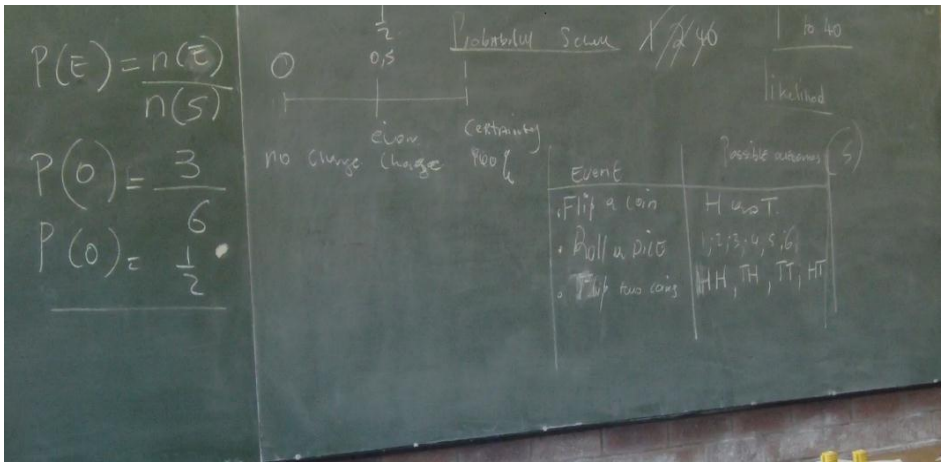
Lessons 1 was an introductory lesson meant to define probability and provide revision of Grade 10 work. Learners were given an exercise to help them revise Grade 10 work. The teacher defined some concepts like probability, sample space and complementary events to establish common understanding. Probability scale diagram was used with 0 % (no chance), 50% (even chance) and 100% (certainty).

Sample space was defined by the teacher as 'a collection of all possible outcomes in a statistical experiment'. To illustrate the concept the teacher drew a table similar to Table 4.1 below on the chalkboard:

Table 4.1: Lesson 1 examples

Event	Possible outcomes
Flip a coin Rolling a dice	H, T 1,2,3,4,5,6
Flipping two coins	HH, TH, TT, HT

Figure 4.1: Lesson 1 examples



Examples of **(EOFS 1)** are given below:

1.1. What is the probability of throwing a six when you toss a dice?

1.2. The numbers 1 to 40 are written on pieces of paper and then placed in a jar. What is the probability of picking a piece of paper with a multiple of 5 without looking at the numbers?

Examples for practice- exercises **(EFORS 1)** are shown in this extract on the next page:

1. If you roll a die, which of the following statements are false?
   a) The probability of getting a three is a little greater than that of getting a six.
   b) The probability of getting a prime number is $\frac{1}{2}$.
   c) The probability of getting a number greater than 2 is $\frac{2}{3}$.
   d) Throwing the die higher in the air will affect the probability of getting a one.
   e) The probability of getting a number less than seven is one.
2. You accidently drop a pack of shuffled cards and only one lands face up! What is the probability that the card is a (remember an ace counts as one):
   a) Jack? b) red card? c) prime number?
   d) number less than six e) diamond f) picture card?
3. A group of pupils comes to school on a particular bus. The bus is often late. The following table shows how often the bus was a certain number of minutes late.

Minutes late	0	5	10	15	20
No. of times	12	9	8	6	5

Assuming the same pattern occurs in the next term, what is the probability that the pupils will:

(a) On time (b) be 15 minutes late

(c) Less than 15 minutes late? (d) be more than 10 minutes late?

Lesson 2

The second lesson that I observed was on mutually exclusive and complementary events. The lesson started off with the definition of complementary events using the symbols: P(E') + P(E)= 1, and P(E')= 1- P(E). The teacher then went on to use the following examples to explain further the concept of complementary events:

Examples of **(EOFS) 2.1**

2.1.1. If you roll a die the probability of getting a 3 is $\frac{1}{6}$and therefore of not getting a 3 is equal to 1- 1/6 = 5/6.

2.1.2. The probability of drawing a diamond from a pack of 52 cards is 13/52 = ¼. What is the probability of not drawing a diamond?

The teacher then introduced mutually exclusive events as '...events that cannot happen at the same time '. The symbolical representation was given as: P (A or B) = P (A) + P (B). Three 'examples of ' were given as shown below:

Examples of (**EOFS**) 2.2

2.2.1 If a die is thrown what is the probability of getting of either even or odd number?

2.2.2 In a set of 24 balls 3 are black, 5 are green and 16 are neither black nor green. What is the probability that a ball is black or green?

2.2.3 The probabilities of three soccer teams (X, Y, Z) winning the World Cup are P(X) = ¼,

P(Y) = $\frac{1}{8}$ and P (Z) =$\frac{1}{10}$. Calculate the probability of the following

a) either X or Y will win
b) X or Y or Z will win
c) None of these teams will win

Examples for **(EFORS 2)**

1. A spinner, divided into 12 equal spaces and numbered 1 to 12, is spun in such a way that the pointer has an equal chance of landing on any one of the 12 numbers. We define events, A, B and C such that A is the set of prime numbers, B is the set of o dd numbers and C is the set of even numbers.
    a) Draw a Venn diagram to show events A, B and C.
    b) What is probability that the pointer will land on
        i) An odd number?
        ii) An even number?
        iii) A prime number?
        iv) A prime and an odd number?
        v) Either an even number or a prime number?
        vi) Either an odd number or an even number, but not a prime number?
2. On the throw of a die:
    a) What is the sample space?
    b) What is the probability of getting:
    i) An odd number? ii) an odd prime?
    iii) A number less than four ? iv) a nine?
    iv) An even prime? vi) a four or a five?

Lesson 3

This lesson was on Independent and Dependent events. The teacher started off the lesson by defining terms. The term Dependent event was defined as '...an event with outcomes that depend on other outcomes'. The term independent event was defined using a textbook definition. The teacher read the definition to the learners as ' we say two events are independent of each other when the outcome of either of the events is not affected by the occurrence or non-occurrence of the other event'.

Examples of (**EOFS**) 3

1. If a farmer plants x maize seeds, the number of plants that will grow depends on the quality of the seed, the amount of sunlight, the amount of fertilizer and the amount of rain that falls on the fields.

2. You can toss a coin to decide which team will kick off a soccer match.

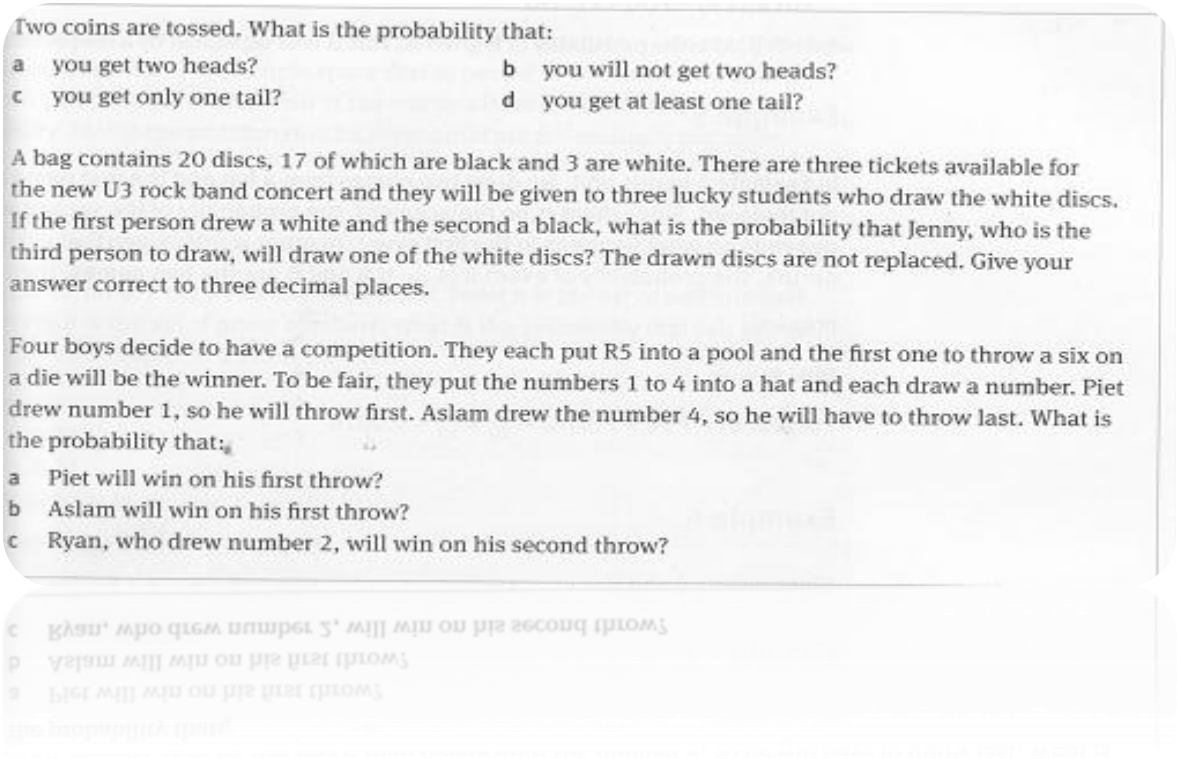
3. The probability that a leaner is left-handed is 0.15.

a) What is the probability that a learner is right-handed?
b) What is the probability that two learners chosen at random will both be left-handed?

4. Two coins are tossed. What is the probability that:

a) You get two heads?
b) You will not get two heads?
c) You will get only one tail?
d) You will get at least one tail?

Examples for **(EFORS 3)**



Two coins are tossed. What is the probability that:

a you get two heads?
b you will not get two heads?
c you get only one tail?
d you get at least one tail?

A bag contains 20 discs, 17 of which are black and 3 are white. There are three tickets available for the new U3 rock band concert and they will be given to three lucky students who draw the white discs. If the first person drew a white and the second a black, what is the probability that Jenny, who is the third person to draw, will draw one of the white discs? The drawn discs are not replaced. Give your answer correct to three decimal places.

Four boys decide to have a competition. They each put R5 into a pool and the first one to throw a six on a die will be the winner. To be fair, they put the numbers 1 to 4 into a hat and each draw a number. Piet drew number 1, so he will throw first. Aslam drew the number 4, so he will have to throw last. What is the probability that:

a Piet will win on his first throw?
b Aslam will win on his first throw?
c Ryan, who drew number 2, will win on his second throw?

Lesson 4

The lesson was a continuation of work on Dependent and Independent events but with more emphasis on using tree diagrams to solve problems on combined events.

EOFS 4 used

1. Two coins are tossed together. Draw a tree diagram to find the probability of tossing the following:

a) two heads
b) One head and one tail.

2. A school boarding house serves mince or chicken for supper on Tuesdays and

Wednesdays. Sometimes they serve the same meal on both nights. The students found that the probability of getting mince on Tuesday was 1/3 and on Wednesday was 5/6.

a) What is the probability they will serve chicken on Tuesday and mince on Wednesday?

b) What is the probability of mince on both days?

EFORS 4

1. The probability of snow in a Swiss village is P (snow) = 63% (or 63/100 or 0,63) during March. The probability of a person falling in dry weather (in other words, no snow) during March is 12% and three times more likely when it snows.

Draw a tree diagram illustrating the possible events and their probabilities.

What is the probability that:

a) a person will not fall?
b) a person will fall in dry weather?

2.

4 A school decided to serve hot meals in its canteen at lunchtime during winter. The school conducted a survey to determine students' preference for chicken, fish and vegetarian meals. The information below shows some of the results of the survey.

In all, 203 students were questioned. Two ate only chicken. Three ate only fish. Five ate only vegetables and two ate none of the choices. Then, 66 ate chicken and fish only and 75 ate fish and vegetables only. 40 ate chicken and vegetables only.

a Draw a Venn diagram showing all the relevant information.
b How many students like all three options?
c What is the probability that a student, selected at random, will only eat chicken and fish, but not eat vegetables? Leave the answer as a fraction.
d What is the probability that a student, selected at random, will eat any two of the three choices?

4.3 Findings

In the following sub-sections I discuss the main themes that emerged from the pre-lesson interview, classroom observations and post-lesson interview. In all the excerpts **R** stands for the Researcher and **Tr** for teacher. Parts of the excerpts were translated from isiZulu to English. The following themes emerged: learning objectives, time as a limiting factor, relevant features on examples, taking account of variables, taking account of sequencing, taking account of representation and most frequently used mathematical practices. Learning objectives, time as a limiting factor, relevant features on examples, taking account of variables, taking account of sequencing, taking account of representation were all connected to the first research question which sought to find out how the teacher selects examples and for what purposes. The mathematical practices that accompany the examples are discussed in

more detail under the theme 'frequently used mathematical practices'. The analysis of mathematical practices is a direct response to the second research question which sought to find out the mathematical practices that accompanied the chosen worked examples also referred to as examples of (EOFS) in this study.

4.3.1 Learning objectives

The pre-lesson interview revealed that the teacher considered syllabus coverage and syllabus objectives as influential in selecting examples, As mentioned in Chapter 2 learning objectives are a comprehensive category in that they influence variables, sequencing and representation (Rowland, 2008) . I therefore had to interview the teacher and use his lesson plans to find out the learning objectives in each lesson that was observed. Table 4.2 shows a summary of the learning objectives the teacher had for five out eight lessons observed.

Table 4.2 Summary of lesson objectives

Lesson	Objective(s)
1.Introduction	Revise work done in grade 10 so that all learners start on the 'same page'.
2. Mutually exclusive and complementary events Venn diagrams	Define terms mutually exclusive and complimentary events. To solve problems that deal with concepts such as mutual exclusive and complementary sets Represent or illustrate visually some relations between or among different sets of data
3. Dependent and independent events	Define the terms and solve questions involving such relationships
4. Tree diagram Venn diagrams with 3 sets	Use tree diagrams to solve probability problems with two or more outcomes. Use Venn diagrams with three sets, to solve probability problems.

In the following excerpt of the pre-lesson observation the teacher explains what influences his choice of examples.

Excerpt 1- Pre observation interview transcript

R: How do you decide that an example will suit your needs?

Tr: syllabus coverage. I consider if the particular example covers the syllabus or not. I also look at the easy access of the concept contained in the example.

R: what do you mean by easy access? Please unpack that for me?

Tr: Eh… it should be **easily understood by learners and in line with the syllabus. It should allow learners access to similar problems in future.**

R: In what way?

Tr: ok … **the language must be easy** for the multilingual learners.

R: how do you do that?

Tr: I alter the questions by using easy language.

In the above excerpt the teacher explained that he is syllabus driven. In other words he gives examples that are meant to satisfy the syllabus objectives. However seeing that his learners are linguistically challenged, the teacher claimed that he altered the language in the original example from the textbook. Altering the language in the question was therefore a way of scaffolding the concepts to the learners. The ultimate objective was for learners to understand probability. Excerpt 1 reveals that the syllabus has a profound effect on the selection of examples.

4.3.2 Time as a limiting factor

In addition to making sure the examples meet the syllabus requirements the teacher added time as a critical factor in the choice of examples. Time consuming examples were excluded to meet the demands of a fast paced syllabus (CAPS). In my view this had a drastic effect on the selection and use of examples. The teacher's selection of examples had to exclude some examples due to time constraints. The teacher pointed out that the syllabus does not prescribe the examples to be used but dictates the pace in which they must be done. Therefore the teacher had some freedom to choose examples. It was worthwhile to investigate how the teacher under these conditions selects and uses examples in a multilingual setup. In the excerpt below the teacher talks about how fast paced the CAPS syllabus is. This except is based on lesson 4 based on the context of food served at a boarding school.

Excerpt 2: Post observation interview

R: Let's look at the specific example that you used. The example of food served at a boarding school. Did you expect learners to ask you about why the probability of serving mince was $\frac{1}{3}$ on Tuesday and $\frac{5}{6}$ on Wednesday? This was example 2 in the lesson on independent and dependent events lesson.

Tr: Yes I did… eh I was disappointed that they did not ask. I expected them to ask. If you look at the question it says the probability for mince to be served on Tuesday is $\frac{1}{3}$ and

Wednesday is $\frac{5}{6}$ and this shows a change in two different days. I expected learners to ask why the probability for the same thing was different.

R: why did you not probe them to ask that question?

Tr: no I did not …eh because of **time constraint**. I did not get time to probe learners because one has to finish the syllabus on time.

R: so you were actually considering time in your examples. Explain further how it affected your choices.

Tr: CAPS has a lot of work that needs to be done more especially at Grade 11 level. The topics are too many and the schedule is more prescriptive. So **there is a lot of pressure to complete the syllabus**.

R: Sure, CAPS puts you under pressure…

Tr: Yes, it tells you what you must be teaching almost on a day to day basis. You need to be fast.

R: But it doesn't prescribe the examples that you must use.

Tr: Yes… but gives me pressure to push the syllabus.

What came out of the above excerpt was that the teacher was concerned about finishing the syllabus and ignored to look at his learners' understanding. The teacher acknowledged that the syllabus does not prescribe the examples he must use and the onus of selecting examples rests on him. In the classroom situation the example played out as shown in Except 3 below.

Except 3- lesson 4

Tr: Let's start with Tuesday. **We have two choices mince or chicken**. What is the probability of mince?

L2: One third.

Tr: Okay… **what is the probability of getting chicken on Tuesday?**

L3: two thirds.

Tr: **why is it two thirds?**

Ls: (inaudible)

Tr: okay guys please ngicela ninga jahi ukubhali ianswer (*please do not hurry)*… (points to a learner).

L4: **the probability of getting chicken is a complement**.

The teacher asked the learners for the probability of having chicken served given that of beef. The teacher further asked the learners to justify their answers. The practice of asking learners to justify their answers offers learners a chance to consolidate their understanding of mathematical concepts. It also helps them to clarify their thinking. The example coming after

the lesson on complementary events was well sequenced to allow learners to connect concepts. Asking learners 'Why' according to Staples (2007) is an example of supporting students in making contributions through eliciting their ideas. This affords the learner to share his thinking in public space.

However in my view the teacher possibly by trying to race against time denied learners an affordance inherent in the example by giving them a hint through mentioning that there were two choices on each day. In my view this had an effect of guiding the learners towards the correct answer (Staples, 2007). The level of cognitive demand in the question was lowered as the teacher took away the opportunity for learners to come up with that analysis on their own (Stein et al, 2000). The cognitive level in the task was going to be maintained by not giving away such details.

4.3.3 Relevant features on examples- accessibility of examples

In Excerpt 1 the teacher also mentions that it is important that an example allows learners 'access to similar problems in future'. In Excerpt 2 the teacher registers his concern that '…there is a lot of pressure to complete the syllabi' but '…it doesn't prescribe the examples that you must use'. The similar sentiments are echoed by Adler & Venkat (2014, p. 1) when they point out that there is a '…increasing specification of what to teach, and in what order or sequence' in South Africa. It is therefore imperative for the teacher to come up with well thought out examples that distinguish important features from coincidental feature (Goldenberg & Mason, 2008). According to Zodik & Zaslavsky (2008) teachers choose examples that draw learners' attention to relevant features of a concept and this affords the learners according to Watson & Mason (2006) to generalise or see structure in the concept.

In lesson 2 the teacher gave the example on the probability of getting an even or odd number from throwing die. The example does bring out the concepts of independent and dependent events. The two events are also both complementary or mutually exclusive and independent. This may be considered as an incidental feature of independent events. Giving this example first has the potential to impede learning as learners might be led to think that all complementary events are independent.

4.3.4 Taking account of Variables

According to Rowland (2008) all examples have something that varies or changes and to a

large extent influences what we learn. The variation in the examples is very important because by critically looking at "what changes and what stays the same?," and the nature of the changes offered, we can gain some understanding about what an example together with an established ways of working (collection of social practices) affords the learner and with what constraints (Greeno, 1994; Watson, 2003, Watson & Mason, 2006).

The above argument strengthens the justification of analysing the variation in worked examples or exercises given by the teacher because they influence the mathematical practices in the classroom. In my view the mathematical practices to a larger extent are influenced by the objects brought or not brought into the classroom by the teacher or other external factors like the curriculum. The focus of my study was examples and mathematical practices accompanying them and not the external factors like the curriculum. The examples were analysed on how variance and invariance offered opportunities of learning. Table 4.3 presents the different variables under the Examples of and Examples for practice columns and the representations that were observed in the lessons.

Table 4.3: Taking account of variables

Lesson topic	'Examples of'- Teacher illustration examples (EOFS)	'Examples for practice'- exercises (EFORS)	representations
Introduction- Probability	1. coin tossing 2.numbered papers	Mathematics g11 learner's textbook (no. 1-3) 1. dice 2. playing cards 3. table showing school bus's late coming	tables
Venn diagrams Mutually exclusive and complementary events	1. dice 2. playing cards 3.a bag containing some balls 4. sports- soccer teams winning world cup.	Mathematics g11 learner's textbook (Exercise 1:no. 1) Spinner	Venn diagrams Table
Dependent and independent events	1. Left-handed and right-handedness. 2. sports- hockey and basketball teams 3. throwing a die and a coin 4. tossing two coins	Mathematics G11 learner's textbook (Exercise 2:no. 1) Letters on words: STOMERS, SHARKS and BUILD.	Venn diagrams Tree diagrams
Tree diagrams and Venn diagrams	Food- mince and chicken	Mathematics G11 learner's textbook (Exercise 4:no. 2 & 4) 1. weather- snow or dry versus falling or not falling 2. food- chicken, fish and vegetarian meals.	Tree diagram Venn diagram

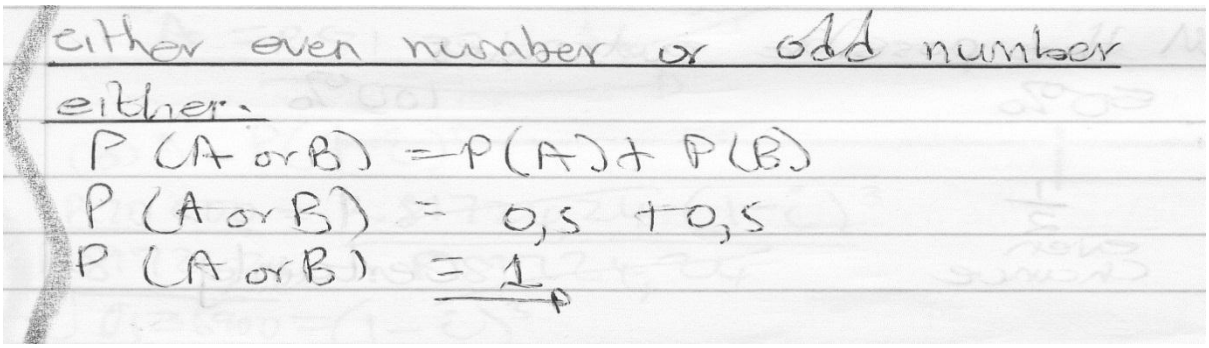
Using Rowland's (2008) categories of exemplification the variables across the four lessons were: coin tossing, die throwing, numbered papers, playing cards, left-handed and right-handedness, bag containing some balls, sports- hockey and basketball teams, throwing a die and a coin combination, weather, spinner, words and Food- mince and chicken. There is some variation that allows learners to make generalisations and hence opportunities to learn were enabled (Watson & Mason, 2006).The variation gave learners a chance to experience probability from different real life situations. The variation was helpful in showing that the application of probability is not confined to playing games but to other spheres of life as well

(Bennie, 1998). For example the inclusion of coin tossing and weather forecast. The weather forecast is an example of real life application of probability and in my view the most misunderstood concept.

The teacher in lesson 3 gave learners EFOR 3.1 as group work. The group discussions were all on drawing the tree diagram and finding the required probabilities according to the demands of the question. The question on what it means to have a probability of 63% of snowing was not part of group discussions. It should be noted that some of the implications of having a probability of 63% are contained in the question as it talks about dry and wet weather. It is my contention that there was need to talk about relative frequency so as to provide the practical use of relative frequency. If we say that the probability of an event is less than 1 or less than 100% we are giving room for the non-occurrence of that event. Relative frequency uses data that was collected in the past and is now used to make predictions. I also contend that this was an opportunity to connect Statistics and Probability, as agued by Bennie (1998) that "… pupils who have a sound understanding of probability…" are "… better equipped to make judgements and decisions regarding statistical data" (p. 3).

Going back to variables it was evident that tossing of a coin and throwing of a die were invariants as three out four lessons contained these examples. In lessons 2 and 3, four different examples were given. It is noteworthy to mention that in lesson 2 which was on independent and dependent events the teacher used an example (EOFS 2.1.1) which had some potential to hinder learners' understanding of the concept of independent events. The solution of that example is captured in the learners' exercise given in the Figure 4.2 below:

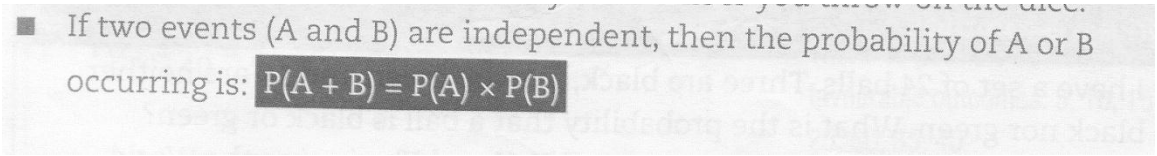
Figure 4.2: Complimentary events



The example shows that for any given independent events A and B, the probability of A or B happening is equal to probability of A plus probability of B. The use of odd and even

numbers on a dice in my view was a hindrance in the sense that the two events were both independent and complimentary. This may have an effect of making learners think that all independent events are complementary. According to Rowland (2008) this could be classified as an 'example of obscuring the role of variables' (p.155). Furthermore the teacher used a textbook which had the notation shown in Figure 4.3 below:

Figure 4.3: Independent events



The use + sign instead of 'and' or ∩ is a potential source of more problems. In the lesson discourse the teacher emphasised the importance of paying attention to language. The following excerpt demonstrates that point. The teacher says to the learners "if they say 'or' it means the two events are mutually exclusive". This means the use of the word 'or' implies that the learners will add the individual probabilities. Looking at the example from the textbook the sign for addition i.e. '+' is used for independent events. In this situation the variation in variables was somewhat confusing. In my view the use of '+', 'or' and 'and' in the same example had the potential to confuse the learners more especially after the teacher had emphasised that the use of 'or' always implies addition and the use of 'and' implies multiplication. Figure 3 starts by saying two events A and B and goes on to mention the probability of A or B to be represented by P(A + B)= P(A) × P(B). In Figure 2 the example used there says P (A or B) = P (A) + P (B). The variation of symbols here has a potential to distort the meaning of independent events. There was a need to use one variable consistently to avoid potential confusion. The symbol '+' is normally used for addition operation. In Excerpt 4 below the teacher emphasizes the use of 'or' to imply that the events are mutually exclusive. This is in sharp contrast to use of the same word in Figure 3 to explaining independent events. Bearing in mind that the example was done in multilingual classroom more clarity was required from the teacher.

Excerpt 4- lesson 4- EOFS 2

Tr: So it is food Tuesday or food Wednesday. There are two events. Kukhona i… (*there is*…)

Ls: mince and chicken.

> Tr: Ok… mince and chicken. Bathi (*they say….referring to the question*) mince or chicken. **If they say or it means the two events are mutually exclusive**. Mutually isho ukuthi enye ngeke yenzeke enye isenzeke (*mutually means one event cannot happen while the other is happening*). Ikhona i… (there is… inaudible response). If there no mince there is chicken available. It means there is food Tuesday and food Wednesday. (Reads part of the question). The probability of getting mince on Tuesday was one third and on Wednesday was five sixth.

4.3.5 Taking account of Sequencing

Rowland (2008) argues that all examples come in a sequence of some kind. Looking at Table 4.3 it is evident that the teacher decided to move from familiar context to non- familiar context. The teacher started with coin tossing example and moved to none familiar context of snow. The teacher controlled the examples by moving from simple to complex questions within and across the lessons.

The examples from lesson 1 to 4 labelled as EFORS are for illustrating concepts and provide learners with some practice (Rowland, 2008). In the examples used by the teacher to illustrate the concept of possible outcomes there is some evidence of moving from simple to complex questions. The teacher started with tossing of one coin then rolling a die and thirdly tossing two coins. The same sequencing strategy can be seen in the other examples given to illustrate theoretical probability. The examples move from the use of dice with six numbers to using papers numbered from 1 to 40. The possibilities are widened and therefore stretching learners' cognitively. In the example exercise (EFORS 1) from Lesson 1, sequencing was done by starting with multiple choice questions on possible outcomes on a die, playing cards and finally the table showing how often a given bus is late. There is some progression in the level of difficulty in the exercise. A case in point in that exercise would be when learners had to remember that being on time means one is zero minutes late. The question incorporated the concept of inequalities and thus challenging learners to think deeper. For example less than 15 minutes late demanded learners to decide on whether to include the number of times the bus was 15 minutes late or not. This question challenges learners' linguistically more especially second language speakers.

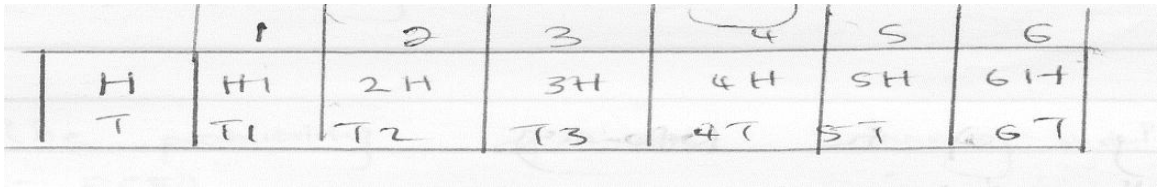
There was sequencing that was evident across all the lessons. The teacher started by revising Grade 10 work and slowly moved to Grade 11 work. Single events were dealt with first, followed by two and three events. Lesson one and two had examples on single events like throwing one die, tossing a coin, drawing a ball from a bag and so on. In the third lesson a combination of two events was introduced like Left-handedness and right-handedness, sports like hockey and basketball teams, throwing a die and a coin and tossing two coins. In the

fourth lesson a combination of two or more events was done. One of the questions taken from the textbook that was given in the 'example exercise' category captured in Lesson 4 as EFORS 2 demonstrates the effect of sequencing. The question has a well calculated sequence of its own as the sub-questions start with the drawing of a Venn diagram meant to make doing the other questions much simpler. It goes further to ask learners to find the missing number. The missing number unlocks everything else. The other two examples were on food choices at a boarding school and weather forecasts EOFS 4.2 and EFORS 4.1

4.3.6 Taking account of Representations

The visual representations used by the teacher included the following: tree diagrams, Venn diagrams and tables. Figure 4.4 and 4.5 capture some of the visual representations used by the teacher across the four lessons that were analysed in this research report.

Figure 4.4: Representations



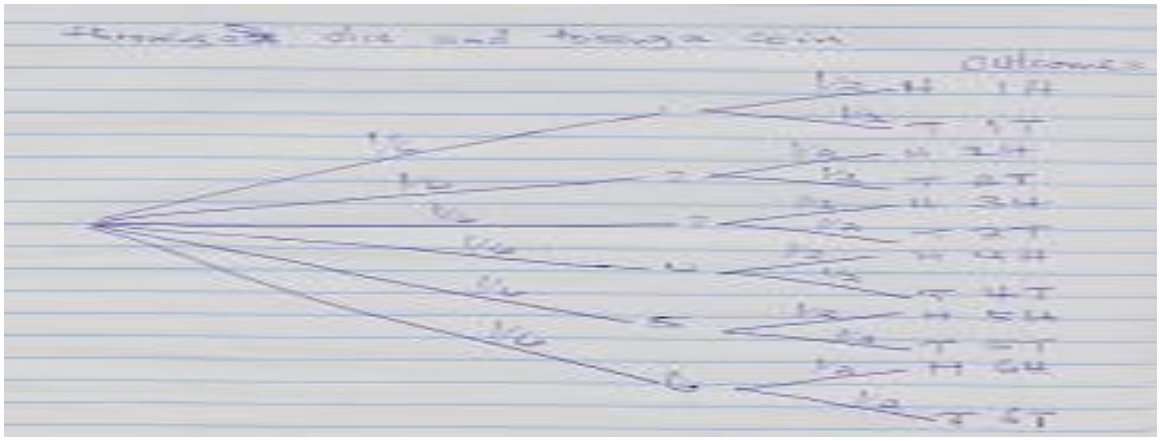
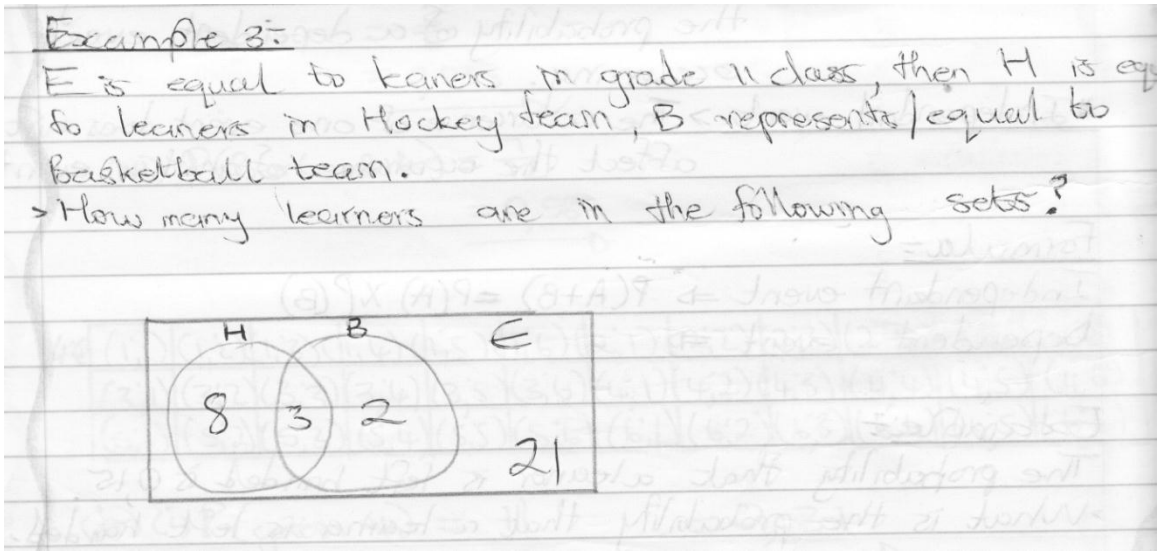


Figure 4.5: Venn diagram



In Figure 4.5 flexible representation can be seen in that the same problem is solved using two different representations, namely a table and tree diagram. It is also a good way of showing the connection between two visual representations. What remained to be explored in the classroom discussion were the advantages and limitations of each representation to show some progression in the use of each graph. For example the tree diagram could be used for more than two combined events while the same cannot be done using tables. This justifies the reason why these tables are commonly called two-way tables.

4.4 Most frequently used Mathematical practices

As mentioned in Chapter 3 the variation theory falls short as an analytical tool of language issues under this study. The Staples' (2007) collaborative classroom practices model is a useful tool for analysing the mathematical practices in the classroom. This is a direct attempt to answer question two which seeks to find out the mathematical practices that accompany the selected examples. Mathematical practices included any action used to communicate the solution of mathematical problems and this includes both the learners and teacher's explanations of examples. According to Bills et al. (2006*)* explanations are equally important as the examples. In other words examples and explanations are two sides of the same coin. It is not enough to develop good examples without accompanying meaningful explanations. The mathematical practices were divided into three main categories namely supporting students in making contributions practices **(SCP),** establishing and monitoring a common ground practice **(CGP)** and guiding the mathematics practice **(GMP)**. Table 4.4 captures the

dominant mathematical practices.

Table 4.4: Most frequently used mathematical practices

Mathematical practice	Number of times
Supporting students in making contributions practice	**[30]**
Eliciting	5
Scaffolding	22
Creating contributions	3
Establishing and monitoring common ground practice	**[110]**
Creating a Shared Context-	50
Maintaining continuity over time-	30
Coordinating the Collective-	30
Guiding the mathematics practice	**[31]**
Guiding High-level Task Implementation-	3
On-going assessing and diagnosing	8
Guiding with a Map of Students' Learning	20
Guiding by following, 'going with the students	0

In the above table it is evident that the most frequently used mathematical practice was establishing common ground. Establishing common ground practice included establishing prerequisite concepts, verbally marking, affording multiple opportunities to access ideas, code- switching, re-voicing. However the most frequently used common ground practices were code-switching, re-voicing and verbal marking.

Code-switching

In all the lessons the teacher used code-switching to establish common ground in the classroom. The teacher recognised students' language as legitimate language of mathematical communication (Moschkovich, 1999; Setati, 2008). Code-switching was used by teacher to create a shared context as well as a pedagogic tool to explain concepts. Excerpt 5 and 6 below are instances among many others where the teacher used code-switching practice to explain the probability concepts.

Excerpt 5

> T: So it is food Tuesday or food Wednesday. There are two events. Kukhona i… (*There is what?*)
>
> Ls: mince and chicken.
>
> Tr: Ok… mince and chicken. Bathi (*they say….referring to the question*) mince or chicken. If they say or it means the two events mutually exclusive. **Mutually isho ukuthi enye ngeke yenzeke enye isenzeke (mutually means one event cannot happen while the other is happening).** Ikhona i… (there is… inaudible response). If there no mince there is chicken available. It means there is food Tuesday and food Wednesday. (Reads part of the question). The probability of getting mince on Tuesday was one third and on Wednesday was five sixth.

Excerpt 6

> Tr: I **union** isho lokhu nalokhu (the union means this and that) (shading it on the Venn diagram). **Isho ukuthi yonke into ihlangene (it means everything combined)**. Yonke into i…. (everything….)
>
> Ls: ihlangene (combined).

The teacher uses the language learners can understand better to help them understand the concept of mutually exclusive events. The words in bold are the actual words used by the teacher which when translated read: 'Mutually exclusive means one event cannot happen while the other is happening'. The explanation is embedded with the idea that if two events cannot happen at the same time there are mutually exclusive and therefore the probability of both happening is zero. In my view there was need to consolidate the notation used in the second lesson which introduced the concept of mutually exclusive and independent events i.e. for mutually exclusive events. The teacher used the learners' home language as a resource for learning mathematics according to the argument put forward by most researchers (Adler, 2001; Essien, 2010; Moschkovich, 1999; Setati, 2008; Setati, Adler & Bapoo, 2002; Vorster, 2008). The teacher allowed the learners to code switch because he believed it afforded learners a chance to conceptualise the concepts in their main language. According to Vorster (2008) learners' main language is important in facilitating concept construction in two ways: firstly it forms part of prior knowledge and secondly it brings mastery of word sense.

It is worth noting here that code switching has the potential to pose serious challenges. Code- switching could impede on learning in cases where the language used by the teacher is not understood by some learners more especially considering the level of migration in South Africa. Also in some schools the mathematics teachers are not South Africans who may not know the local languages spoken in South Africa. In this research study the sampled teacher

was South African. The example on mutually exclusive events above demonstrates that code-switching fails to work for some specialised terms, to the extent that the teacher had to use the chalk board to visually demonstrate the concept of Union as shown in excerpt 6 above.

Re-voicing

The teacher used re-voicing practice to accept and build on the learner's response. The following excerpt captures an instance in which the teacher used re-voicing to establish common ground. The teacher according to Moschkovich (1999) accepted the learner's answer by re-voicing it. Re-voicing as a mathematical practice was used by the teacher to encourage learner participation in the classroom. The learners generally view the teacher as an authority and expert in learning of mathematics. Re-voicing learners' responses has an effect motivating learners to participate in the classroom**.** According to Goos, Galbraith and Renshaw (2002) in a teacher-student interaction the teacher should work with students' ideas and gradually transfer dominance in the exploration and discussion to the students.

Excerpt 7

> Tr: So it is food Tuesday or food Wednesday. There are two events. Kukhona i… (*there is*…)
>
> Ls: mince and chicken.
>
> Tr: **Ok… mince and chicken**. Bathi (*they say….referring to the question*) mince or chicken. If they say or it means the two events mutually exclusive. Mutually isho ukuthi enye ngeke yenzeke enye isenzeke (*mutually means one event cannot happen while the other is happening*). Ikhona i… (there is… inaudible response). If there no mince there is chicken available. It means there is food Tuesday and food Wednesday. (Reads part of the question). The probability of getting mince on Tuesday was one third and on Wednesday was five sixth.

The teacher repeated what the student had said and therefore adding weight to the student's words. In excerpt 7 the teacher re-voices the student words by saying '…so you are saying **the probability of the right handed is one minus the probability of left-handed'.** The teacher makes the students' thinking public and hence encourages other students in making contributions (Staples, 2007).

Verbally marking

The teacher through verbal marking of students' answers created a shared context and therefore established and monitored common ground. Throughout the four lessons the students were urged to check that they do not get probability of more than 1 in any situation.

The teacher in the excerpt 14 below verbally marks the learners' response to his question. This has an effect of motivating learners however it can also demoralise them depending on how the teacher does it and how the learners view the comments.

Excerpt 8

> Tr: okay guys please ngicela ninga jahi ukubhali ianswer (*please do not hurry to write the answer)*… (points to a learner).
>
> L4: the probability of getting chicken is a complement.
>
> Tr**: good**… kusho ukuthi uzothi (so it means) one minus one third. So the answer is two thirds. So we are sorted with Tuesday lets now go to Wednesday. We have two events on Wednesday. What are the events?

Supporting students in making contributions practices (SCP)

Under the practice of supporting students in making contributions practices (SCP) there were three categories that were observed namely eliciting students' ideas, scaffolding the production of students' ideas and creating contributions.

Eliciting students' ideas

The eliciting of students' ideas was in my view the first step in ensuring that students start making contributions that will bring their thinking to the fore and help further the class discussion.

Excerpt 9

> Tr: ok kasiqhubekeni *(lets proceed)*. Now here is the question: what is the probability that a learner is right-handed? Pule. If we say that the probability that your friend Xoli is left-handed is 0.15. What is the probability that uPule is right-handed.
>
> L: 0.85
>
> Tr: **why**
>
> L: Ngizothi one minus iprobability ye left-handed (I will say one minus the probability of the left-handed).
>
> Tr: **why u sithi 1 minus**? (**Why are you saying one minus**?)
>
> L1: because it is exclusive.
>
> L2: Because probability of mutually exclusive kumeli isinike u 1 (we *must get one for probability of mutually exclusive events*).
>
> Tr: So uthi (***you are saying)*** **the probability of the right handed is one minus the probability of left-handed.**
>
> L 1: yes

In Excerpt 9 above the teacher asked the learner to explain his reasoning and by so doing pressed the learner to put forward his thinking to the whole class. By putting forward his thinking to the whole class the teacher afforded the class to access the learner's ideas publicly (Staples, 2007).

Excerpt 10

R: Let's look at the specific example that you used. The example of food served at a boarding school. Did you expect learners to ask you about why the probability of serving mince was on Tuesday and on Wednesday? This was example 2 in the lesson on independent and dependent events lesson.

Tr: Yes I did… eh **I was disappointed that they did not ask. I expected them to .** If you look at the question it says the probability for mince to be served on Tuesday is and Wednesday is and this shows a change in two different days. I expected learners to ask why the probability for the same thing was different.

R: why did you not probe them to ask that question?

Tr: no I did not …eh because of time constraint. I did not get time to probe learners because one has to finish the syllabus on time.

In the above excerpt the teacher expressed that he is constrained by the time factor to probe learners to open up during classroom discussion. This concern was captured in Chapter 1 where it was noted that the current syllabus is fast paced.

Scaffolding the production of students' ideas

The use of representation was also one of the dominant mathematical practices used by the teacher to support students in sharing their ideas and making meaningful contributions. In Figure 4.1 from Lesson 1, the teacher used visual representations on the chalkboard to assist learners communicate their mathematical thinking.

Guiding with a map of students' learning

In all the four lessons analysed in this study the teacher did not change the course of the lesson as a result of students' request or question. All the lessons were fully controlled by the teacher. In Excerpt 11 below the teacher gives clear guidelines that 'if you do not get one you have a problem'. This statement was double barrelled in that it established common ground and also guided students' learning.

Excerpt 11

Tr: please add all the probabilities. What do you get?

Ls: one

Tr: okay ….eh if you do not get one you have a problem…

L9: how about zero comma nine nine…

L9: one comma zero zero zero zero zero one

Tr: you will wrong… **you must always get one because the question says the event will occur**. So what are chances of getting a student who will have two minces? Katlego.

The teacher in the above excerpt attends to a pressure point (Staples, 2007). The learners are drawn to the most basic rule in probability. The learners are guided and grounded in checking that their probabilities in each branch must always add up to one. Without common ground learners do not make meaningful contributions in the class discussion (Staples, 2007).

4.5 Conclusion

In this chapter, I have dealt with my data analysis and findings that emerged from my study. The findings were on the selection and use of examples. The mathematical practices accompanying the examples were also discussed. In the next chapter, I will deal with the conclusions, recommendations, limitations and reflections based.

CHAPTER FIVE- Conclusions and Recommendations

5.1 Introduction

The main aim of this chapter is to draw conclusions on the research findings. The implications, limitations and reflections of the study would also be discussed. The emerging findings from study were an attempt to answer these questions:

- How does a teacher select examples when teaching Probability in Grade 11 multilingual mathematics classroom and for what purposes?
- What mathematical practices accompany these examples?
- What are the affordances or obstacles offered by examples in Probability given by a mathematics teacher in Grade 11 multilingual classrooms?

Summary of findings

In this section I will give an overview of the research findings. In the study it emerged that it is important for teachers to select examples by considering the context, ability of the example to be generalised, consistency in the use of symbols, syllabus requirements and accessibility. It also emerged that the selection of examples together with the accompanying mathematical practices has the potential to support or impede the learning of mathematics. In particular the findings revealed that the practice of 'guiding the learners with the map' declines the cognitive level of examples and hence impedes learning. Code- switching and re-voicing were most frequently used practices seen in the findings with the use of code-switching encouraging full participation of the learners. The study recommends that methodology courses offered at tertiary institutions to pre-service teachers should include the selection, how to select or design and use examples in multilingual classrooms e.g. what constitutes a good example and how to maintain the cognitive level of an example. The study also recommends that more research needs to be done on effective mathematical practices that may be used to implement worked-out examples in multilingual classrooms.

5.2 Conclusions

The instruments that were used to collect data for this study were class observations and one-on-one teacher interviews. The study revealed that the context of activities brought by the teacher to the classroom affects learning (Brown et al, 1989). The context used in the example has the potential to support or impede leaning (Zaslavsky & Zodik, 2007). The example (EFOR 4.1) used in lesson four on weather challenged the learners to think about the context in the question and hence made the group discussions more lively. The group discussions were on trying to understand what happens during a snow more especially the probability of a person falling .Though the context was unfamiliar to learners the concepts of independent, dependent and complementary events were beautifully embedded in the question.

The context in the question influenced the mathematical practices that were employed by the teacher. Establishing common ground practices like creating a shared context became necessary as some learners struggled to understand the question (EFORS-4.1) (Staples, 2007). The teacher had to establish pre-requisite concepts through code switching and re-voicing. The teacher had to emphasise the basics like checking that in every branch of a tree diagram the total of the probabilities is one.

The example (EFOR 4.1) was able to facilitate mathematical reasoning. Stein et al (2000) argue that if a teacher selects and uses examples that promote the use of procedures mathematical reasoning is unlikely to be developed. The role of the teacher is central in the development of mathematical thinking; firstly in choosing an appropriate task and later to lead the class discussion in a way that encourages full participation as well as avoid reducing the cognitive level in the example. The task or the example space should be well thought to stimulate learners into active participation. The examples should lead learners into developing sustainable problem solving skills.

The other important finding on the selection of examples is the ability of examples to help learners see the general using the specific (Zaslavsky & Zodik, 2007). The main purpose of a particular example is to help learners have a global view about a concept. The example given in lesson 2 as a first EOFS 2.1 on the probability of getting an even or odd number if a die is thrown does not foster generalisation as it gives one as an answer making the two events exhaustive. The two events are also complementary/ mutually exclusive and independent.

This may have an effect of making learners think that all independent events are complementary. The example does not distinguish important features from incidental ones (Goldenberg & Mason, 2008). It is also worth noting that the example cannot be used to solve problems on complementary and independent events. Therefore the example can be viewed as an obstacle in the learning of probability.

It also came out in the research study that the use of symbols in the examples must be consistent with the use elsewhere in the topic. This became apparent in the use of '+' sign to denote intersection (∩) in the treatment of independent events. The '+' sign usually denotes the operation of addition. The use of '+' sign brought confusion when the teacher later introduced the concept of finding the probability of event A or B happening given that the events are either independent or mutually exclusive events.

In the post-observation interview the teacher claimed that he makes sure the language used in the examples is easy for multilingual learners. In keeping with this claim code-switching and re-voicing were present in all lessons. A case in point is Excerpt 5 in Chapter 4 where the teacher explains union in isiZulu as "yonke into ihlangene" meaning everything combined. However according to Adler (2001) the dilemma with code-switching is largely on whether to "develop learners' proficiency in English (the LoLT) vs. ensuring that learners understood the mathematics" (p.75). The teacher has to choose between encouraging participation which may lead to better understanding of mathematics and developing learners' proficiency in English. Adler (2001) writes about code-switching as a dilemma because reform mathematics encourages conceptual understanding. At the same time South African policies on language encourage code-switching without looking at the implications of it. Adler (2001) argues that "code-switching has become a taken-for-granted good thing" (p.100). In other words the benefits of code-switching have been overrated in general.

The use of flexible representations was also observed in one of the lessons. The teacher used a tree diagram and two-way table to solve the same question. However a comparative advantage of each representation was not done. In my view it would have been worthwhile to explore the benefits and limitations of different forms of representations.

The teacher also used analogical representations. In lesson 3 to explain the concept of dependence the teacher used the example of farming whereby the number of plants growing is dependent on the quality of the seed, the amount of sunlight, the amount of fertilizer and the amount of rain that falls on the fields. In this example there are no calculations but the concept of dependence is beautifully portrayed. The influence of one event on another is

clearly illustrated in the example. The example is useful as a start-up example that initiates learners into the concept(Goldenberg & Mason, 2008).

Another finding that came out of the study was the accessibility of examples to the learners (Goldenberg & Mason, 2008). The teacher in the pre- observation interview mentioned that examples should be easily accessible to learners. When asked to elaborate further he explained that examples must comply with the syllabus and use language understood by learners in multilingual classrooms. By making sure the examples are in line with curriculum the teacher ensured that the learners are able to do well in formal assessments. The accessibility of examples can be understood in different ways. In this case the teacher used accessibility to mean that the learners must be able to gain access to knowledge that is in accordance with curriculum requirements. The use of language that is easily accessible to learners in the examples was not evident during the observations as the teacher extracted his examples from the textbooks without making any changes. After the pre-interview I anticipated to see how an example in multilingual classroom looks like. In my view, the mathematical practices used to implement the examples make a world of difference between multilingual and non-multilingual.

5.3 Recommendations

Some of the findings mentioned in this study may be used in most classrooms as mentioned in Chapter 4. Explaining explicitly to learners using for instance good example space, defining mathematics register can be used in any mathematics classroom (Adler, 1995). It is my strong suggestion that in multilingual classroom these practices are not an option. The study showed that the teacher used a variety of strategies to in trying to promote mathematical proficiency. The findings revealed the need for teachers to choose examples carefully. I would further add that teachers need to choose examples carefully to ensure that there are generic and also help learners understand appropriate language used in assessments (Bills et al., 2006). In addition to coming up with well thought examples it is equally important for teachers to develop accompanying mathematical practices that promote mathematical proficiency.

In Excerpt 5 in Chapter 4 the teacher explains union in isiZulu as “yonke into ihlangene” meaning everything combined. The teacher in this case describes union in isiZulu. The

teacher did not use a single term. Describing union as everything combined lands one to confusing it with the universal set or sample space. However during the class observation the teacher pointed to the sets that he was referring to and avoided the confusion. This demonstrates that African languages are currently not well developed for academic use. There is a lot of research that needs to be done to develop the African Languages for academic use. Furthermore feasibility studies should be done considering the diversity and migration dynamics in South Africa. It is my view that the language policy of South Africa is good on paper but not actually implemented fully. Therefore developing the African languages for academic would be one of the ways towards practical implementation. Lessons can be drawn from the New Zealand's development of the Maori language for academic use (Barton, 2008). Drawing lessons from the Maori mathematics vocabulary, it is possible to develop a vocabulary for the African languages in South Africa but it is not easy. The development of this vocabulary requires a concerted effort from Department of Education, teachers, parents, learners and other interested parties like business community.

I also contend that the methodology courses offered at tertiary institutions to pre-service teachers should include how to select or design and use examples in multilingual classrooms e.g. what constitutes a good example and how to maintain the cognitive level of an example. More research needs to done on effective mathematical practices that may be used to implement worked-out examples in multilingual classrooms.

5.4 Limitations of the study

The research focused mainly on teachers' practices and did not look at what learners think is effective to them. Further study on the effectiveness of these mathematical practices needs to be done to consolidate the findings in this study. Interviewing the learners would be one of the methods of finding that out. I analysed only four lessons, which arguably may be viewed as too small to make widely applicable conclusions.

Another limitation of my study is that the data does not evaluate the effectiveness of examples by looking at the quantitative results it produces in assessments. If I were to re-do the research study I would give learners a post-test to ascertain the effectiveness of the examples.

5.5 Reflections

Notwithstanding that the study was qualitative in nature and the results cannot be duly generalised, valuable information was gained with regard to mathematical practices used by teachers and the reasons why those practices were employed. The research has afforded me a chance to critique my own teaching methods in a multilingual set up. In this study I came to understand some mathematical practices that are in line with the theoretical framework provided by Staples (2007). The categories of exemplification by Rowland (2008) summarise powerfully the key components that are important for selecting examples thoughtfully. The use of both theoretical frameworks provided me with both theoretical and practical handles to analyse the selection and use of examples in a multilingual classroom. Further study on examples generated by students needs to be explored to complement the teacher generated examples.

References

Abbot, T., Botsane, M.L., Bouman, J., Bruce, L., du Toit, G., Pillay, M., et al. (2012). *Via Afrika Mathematics: Grade 11 Learner's Book*. Cape Town: Paarl Media Paarl.

Adams, T.L. (2003). Reading Mathematics: More than words can say. *The Reading Teacher*, 53(8), 786-795.

Adler, J. (1995). Dilemmas and a paradox: secondary mathematics teachers' knowledge of their teaching in multilingual classrooms. *Teaching & Teacher Education*, 11(3), 263-274.

Adler, J. (2001). *Teaching Mathematics in Multilingual Classrooms*. Dordrecht: Kluwer Academic Publishers.

Adler, J., & Venkat, H. (2014). Teachers' mathematical discourse in instruction: Focus on examples and explanations. In M. Rollnick, H. Venkat, M. Askew & J. Loughran (Eds.), *Exploring content knowledge for teaching science and mathematics*. London: Routledge.

Ball, D.L. (Chair) (2002). *Mathematical proficiency for all students: Towards a strategic research and development program in mathematics education.* (DRU-2773-OER). Arlington, VA: RAND Education, Science and Technology Policy Institute.

Barton, B. (2008*). Language and Mathematics*. New York: Springer.

Bennie, K. (1998). The "slippery" concept of probability: Reflections on possible teaching approaches. *Proceedings of the Fourth Annual Congress of the Association for Mathematics Education of South Africa* (pp. 53-67). Pietersburg: University of the North.

Bills, L., Dreyfus, T., Mason, J., Tsamir, P., Watson, A., & Zaslavsky, O. (2006). *Exemplification in mathematics education.* Paper presented at the 30th Conference of the International Group for the Psychology of Mathematics Education, Prague, Czech Republic.

Boulet, G. (2007). How does language impact the learning of Mathematics? Let me count the ways. *Journal of Teaching and Learning*, 5(1), 1-12.

Brodie, K. (2007). *Dialogue in mathematics classrooms: beyond question and answer methods*. Pythagoras, 33, 3-13.

Brown, S.J., Collins, A. & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher*, 18(1), 32-41.

Cohen,L., Manion, L. & Morrison, K. (2011). *Research Methods in Education*, 7th Edition, Chapter 23. Routledge: London.

Coles, A. (2000). Teaching Strategies related to Listening and Hearing in two Secondary Classrooms. *Research in Mathematics Education*, 4, 21-34.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education*, 28(3).

Dawson, C. (2002). *Practical Research Methods*, New Delhi, UBS Publishers' Distributors.

Department of Education, South Africa (DBE). (2011). *Curriculum and Assessment Policy Statement Grades 10-12.*

Department of Education, South Africa (DBE). (2012). *Curriculum and Assessment Policy Statement*.

Department of Education, South Africa (DoE). (1996). *Language in Education Policy*.

Department of Education, South Africa (DoE). (2005). *National Curriculum Statement*.

Engle, R. A., & Conant, F. R. (2002). Guiding principles for fostering productive disciplinary engagement: Explaining an emergent argument in a community of learners classroom. *Cognition and Instruction*, 20(4), 399–483. Inc. Lawrence Erlbaum Associates.

Essien, A. (2010). Investigating proficiency in the language of instruction as means of improving mathematical proficiency in a multilingual classroom, *Education as change*, 14:2, 169-185.

Gal, I. (2009). South Africa's Mathematical Literacy and Mathematics curricula: is probability literacy given a fair chance? *African Journal of Research in Mathematics, Science and Technology Education*, 13(1), 50-61.

Garfield, J., & Ben-Zvi, D. (2008). *Developing Students' Statistical Reasoning: Connecting Research and Teaching Practice*. Springer.

Goddard, W. & Melville, S. (2001). *Research Methodology*: An Introduction, second edition, Lansdowne: Juta & Co. Ltd.

Godino, J. D., Batanero, C. Y. & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM. The International Journal on Mathematics Education,* Vol. 39 (1-2): 127-135.

Goldenberg, P., & Mason, J. (2008). Shedding light on and with example spaces. *Educational Studies in Mathematics, 69*, 183-194.

Goos, M., Galbraith, P., & Renshaw, P. (2002). *Socially mediated meta-cognition: Creating collaborative zones of proximal development in small group problem solving*. Educational Studies in Mathematics, 49, 193-223.

Hawkins, A.S., & Kapadia, R. (1984). Children's conceptions of probability- A psychological and pedagogical review. Educational studies in Mathematics, 15, 349-377.

Heize, A., Star, J. R., & Verschaffel, L. (2009). Flexible and adaptive use of strategies and representations in mathematics education. *ZDM Mathematics Education, 41*, 535-540.

Hinds, D. (2000), *Research Instruments.* In Wilkinson, D. (Ed.), The Researcher's Toolkit-The Complete Guide To Practitioner Research. RoutledgeFalmer, New York, pp. 41-54.

Howie, S. (2003).Language and other background factors affecting secondary pupils' performance in Mathematics in South Africa. *African Journal in SMT Education,* 7,120. Retrieved September 23, 2014, from http://www.docstoc.com/docs/19869494/Language-and-other-background-factors-affecting-secondary

Jina, Z., & Brodie, K. (2009). Teacher questions and interaction patterns in the new and old curriculum: a case-study. *Paper presented at the 16th annual conference of the Southern African Association for Research in Mathematics, Science and technology Education*. Maseru.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington DC: National Academic Press. Chapter 4, pp. 115-135.

Kothari, C.R. *(1985). Research Methodology-Methods and Techniques,* New Delhi, Wiley Eastern Limited.

Lampen, E. (2011). Structuring everyday contexts for statistical treatment: The struggle for shared focus. *African Journal for Research in MST Education, 15*(1), 51-66

Leung, A. (2012). Variation and mathematics pedagogy. In J. Dindyal, L.P. Cheng, & S.F. Ng (Eds.), *Proceedings of the 35th Annual Conference of the Mathematics Education Research Group of Australasia*(pp. 433–440). Singapore: MERGA. Available from http://www.merga.net.au/documents/Leung_2012_MERGA_35.pdf

Lovell, E. (2002). *Mathematical Discussion in the Classroom. Philosophy of Mathematics Education, 16, 1-12.* Jina, Z., & Brodie, K. (2008). Teacher questions and interaction patterns in the new and old curriculum: a case-study. Paper presented at the 16th annual conference of the Southern African Association for Research in Mathematics, Science and technology Education, Maseru, January, 2008.

Marshall, M. N. (1996). Sampling for qualitative research. *Family Practice,* 13(6), 522-525.

Marton, F., & Booth, S. (1997). *Learning and awareness*. Mahwah, NJ: Lawrence Erlbaum Associates.

Marton, F., & Pang, M. F. (2006). On some necessary conditions of learning. *Journal of Learning Sciences*, 15, 193–220.

Marton, F., Runesson, U., & Tsui, A. (2004). *Classroom discourse and the space of learning*. Mahwah, N.J: L. Erlbaum Associates.

Mason, J. & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing- to act in the moment. *Educational Studies in Mathematics,* 38(1-3), 135-161.

Maxwell, J. A. (1992). Understanding and validity in qualitative research. *Harvard Educational Review, 62*(3), 279-300.

Maxwell, J.A. (2005). *Qualitative Research Design: An interactive approach.* Thousand Oak, CA: Sage.

Morkel, D. (2014, December 30).Umalusi: maths and physics results. Retrieved from http://mobi.iol.co./#!/article/umalusi-maths-and-physics-results-worse-1.1800169.

Moschkovich, J. (1999). *Supporting the participation of English language learners in mathematical discussion*. For the Learning of Mathematics, 19(1), 11-19.

Moschkovich, J. (2003).What counts as mathematical discourse? In N. Pateman, B. Dougherty & J. Zilliox (Eds.) *Proceedings of the 2003 Joint Meeting of PME and PMENA.* Volume 3. University of Hawaii: USA, pp. 325-331.

National Council of Teachers of Mathematics. (1991). *Principles and Standards for school mathematics*. Reston, VA: NCTM.

Nistal, A. A., Van Dooren, W., Clarebout, G., Elen, J., & Verschaffel, L. (2009). Conceptualising, investigating, and stimulating representational flexibility in mathematical problem solving and learning: A critical review. *ZDM Mathematics Education, 41*, 627-636.

Ozmantar, M.F. and Monaghan, J. (2008). Are mathematical abstractions situated? In Watson, A. and Winbourne, P. (Eds.) *New directions for situated cognition in mathematics education*. Springer.

Pimm, D. (1981). Mathematics? I speak it fluently. In A. Floyd (Ed.), *Developing Mathematical thinking* (pp. 139-150). Addision Wesley: Wokingham.

Pirie, S. (1998). Crossing the gulf between thought and symbol: language as (slippery) stepping stones. In H. Steinbring, B. Bussi & A. Sierpinska (Eds.) *Language and Communication in the Mathematics Classroom.* NCTM: Reston, Virgin, pp. 7-29.

Rowland, T. (2008). The purpose, design and use of examples in the teaching of elementary mathematics. *Educational studies in mathematics, 69*, 149-163.

Runesson, U. (2005). Beyond discourse and interaction. Variation: a critical aspect for teaching and learning mathematics. *Cambridge Journal of Education*. London: Routledge.Vol. 35, No. 1, pp. 69–87.

Salkind, N.J. (2012). *Exploring research*. New Jersey: Pearson Education International.

Scott, P. H., Mortimer, E. F., & Aguiar, O. G. (2006). The tension between authoritative and dialogic discourse: A fundamental characteristic of meaning making interactions in high school science lessons. *Science Education, 90*(4), 605-631.

Setati, M. & Adler, J. (2001). Between languages and discourses: Language practices in primary multilingual classrooms in South Africa. *Educational Studies in Mathematics*, 43, 243-269.

Setati, M. (2008). Access to mathematics versus access to the language of power: the struggle in multilingual mathematics classrooms. *South African Journal of Education*, Vol. 28, pp. 103- 116.

Setati, M., Adler, J. & Bapoo, A. (2002). Incomplete journeys: Code-switching and other language practices in mathematics, science and English language classrooms in South Africa. *Language and Education*, 16(2), 128-149.

Shuard, H. (1991). *Calculators, children and mathematics*. London: Menthuen.

Staples, M. (2007). Supporting whole-class collaborative inquiry in a secondary mathematics classroom. *Cognition and instruction*, 25(2), 161-217.

Stein, M. K., Grover, B. W., & Henningsen, M. A. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, 33(2), 455–488. doi:10.3102/00028312033002455.

Stein, M.K., Smith, M.S., Henningsen, M.A., and Silver, E.A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers' College Press. Introduction and chapters 1-2.

Sun, X. (2011). "Variation problems" and their roles in the topic of fraction division in Chinese mathematics textbook examples. *Educational Studies in Mathematics*, 76, 65–85. http://doi.org/10.1007/s10649-010-9263-4.

Vorster, H. (2008). Investigating a Scaffold to Code-Switching as Strategy in Multilingual Classrooms. *Pythagoras*, 67, 31-41.

Vygotsky, L.S. (1978). *Mind in Society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press. Ch. 4, 6.

Watson, A. (2007). The nature of participation afforded by tasks, questions and prompts in mathematics classrooms, *Research in Mathematics Education*, 9:1, 111-126.

Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematics Thinking and Learning*, *8*(2), 91-111.

Wells, G. and Ball, T. (2008). *Promoting exploratory talk through dialogic inquiry*. In N. Mercer, & S. Hodgkinson Exploring Talk in School, (pp. 130-143). London: Sage.

Zaslavsky, O. & Zodik, I. (2007). Mathematics Teachers' choices of examples that potentially support or impede. *Research in Mathematics Education*, 9:1, 143-155.

Zodik, I. & Zaslavsky, O. (2008). Characteristics of teachers' choice of examples in and for the mathematics classroom. *Educational Studies in Mathematics*, 69, 165–182.

Appendices

Appendix A- Wits School Education Ethics clearance

Appendix B- GDE Research Approval Letter

Appendix C- Audio Recording Parents informed consent form

Appendix D- Audio Recording Teacher's informed consent form

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Appendix A

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

02 September 2014

Student Number: 584613

Protocol Number: 2014ECE042M

Dear Mlungisi Sibanda

Application for Ethics Clearance: Master of Science

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate, has considered your application for ethics clearance for your proposal entitled:

Teachers' selection and use of examples in Statistics and Probability in multilingual classrooms

The committee recently met and I am pleased to inform you that **clearance was granted.**

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

M Mabelo

Wits School of Education

Appendix B



GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

For administrative use:
Reference no: D2015 / 236

GDE RESEARCH APPROVAL LETTER

Date:	30 July 2014
Validity of Research Approval:	30 July 2014 to 3 October 2014
Name of Researcher:	Sibanda M.
Address of Researcher:	56 Van der Merwe Street
	Elsburg
	Germiston
	1401
Telephone Number:	082 049 8241
Email address:	mlusibanda@yahoo.com
Research Topic:	Teachers' selection and use of examples in Statistics and Probability in multilingual classrooms
Number and type of schools:	ONE Secondary School
District/s/HO	Ekurhuleni South

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

2014/07/31

1

Making education a societal priority

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

.....

Dr David Makhado
Director: Education Research and Knowledge Management

DATE: 2014/09/3

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix C

University of the Witwatersrand

Wits School of Education

Informed Consent Form for Audio Recording in mathematics classrooms

Research Topic: Teachers' selection and use of examples in Statistics and Probability in multilingual classrooms (Protocol no. 2014ECE042M).

Parents' Informed Consent for Audio Recording.

1. I accept that my child can be audio recorded during classroom interactions. Yes/No (please tick one).
2. I understand that the Researcher will lock away raw data for a period of up to 5 years. After this, the raw data will be destroyed
3. I have also received, read and understood the Information and Consent sheets regarding the educational study.
4. In view of the requirements of the research, I agree that the data collected during this audio recording can be processed in a computerized system by the researcher. Yes/No (please tick one).

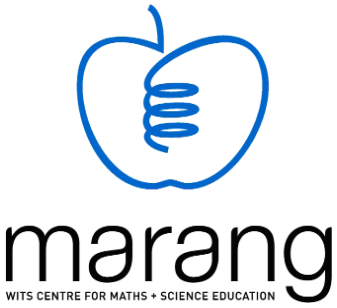
Name: _____

Signature: _____

Date: _____

Appendix D





University of the Witwatersrand

Wits School of Education

Informed Consent Form for Audio Recording in mathematics classrooms

Research Topic: Teachers' selection and use of examples in Statistics and Probability in multilingual classrooms (Protocol no. 2014ECE042M).

Teachers' Informed Consent for Audio Recording

1. I accept to be audio recorded by the researcher during classroom interactions. Yes/No (please tick one).

2. I accept to be audio recorded by the researcher during interview. Yes/No (please tick one).
3. I understand that the Researcher will keep all raw data under lock and key for a period of up to 5 years. After this, the raw data will be destroyed
4. I have also received, read and understood the Information and Consent sheets regarding the educational study.
5. In view of the requirements of the research, I agree that the data collected during this audio recording can be processed in a computerized system by the researcher. Yes/No (please tick one).

Name: _____

Signature: _____

Date: _____

Appendix E

University of the Witwatersrand

Wits School of Education

Information Sheet: Teachers

Dear Sir

My name is Mlungisi Sibanda and I am a master's student in the School of Education at the University of the Witwatersrand.

I am doing research on what examples are used by mathematics teachers when they teach statistics and probability. My research is looking at how probability and statistics are taught in schools. I would like to observe some of your lessons to see how learners are understanding statistics and probability so I can see what examples and techniques work in the understanding of the topics.

I would like you to please participate in my study. If you were to partake in the study you would mind if I may copy the examples that you use in class, video-record a few of your lessons and interview you to see why you chose those specific examples to see whether or not these examples work well in teaching statistics and probability.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

You will not be advantaged or disadvantaged in any way. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.

Please let me know if you require any further information.

Thank you very much for your help.

Name:

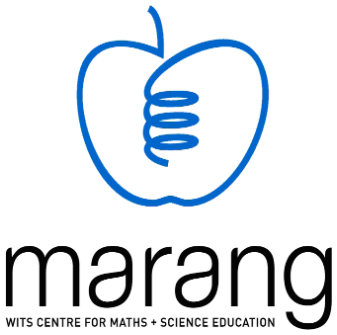
Signature: _____

Date: _____

(Protocol no. 2014ECE042M)

Appendix F





University of the Witwatersrand

Wits School of Education

Informed Consent Form for Conducting Research in a Mathematics classroom

Research Topic: Teachers' selection and use of examples in Statistics and Probability in multilingual classrooms (Protocol no. 2014ECE042M).

I, _____ agree to participate in this study to be conducted by **Mlungisi Sibanda** of the University of Witwatersrand for the research on **The selection and use of examples in multilingual classrooms** to Grade 11 class. I realize that no harm will result from my participation in this study, and that the study is being conducted for purposes of improving the teaching of Mathematics in our schools. I give permission for the material to be used for research or teaching only.

I am not forced to participate and understand that I may withdraw from the study at any time.

Interviews

I further agree to being interviewed as part of the study. I also understand I have the right to review the transcripts made of our conversations before these are used for analysis if I so choose. I can delete or amend any material or remove or revise any of my remarks. Everything I say will be kept confidential by the interviewer. My real name will not be used in the transcripts. In addition, any persons I refer to in the interview will be kept confidential.

Real words from me may be used in the research report as quotes, but they will be reported so that my identity is not known. Any specific individuals or courses I refer to will be given names that are different from their own. I understand that the results of the study may be published, but my name will remain unknown.

Name:

Signature:

Date: September 2014

Appendix G

University of the Witwatersrand

Wits School of Education

Informed Consent Form for video recording for Conducting Research in mathematics classrooms

Research Topic: **Teachers' selection and use of examples in Statistics and Probability in multilingual classrooms (Protocol no. 2014ECE042M).**

Parents' Informed Consent for video recording

1. I accept that my child can be video recorded during classroom interactions by the researcher, M.Sibanda. yes/no (tick one)
2. I hereby confirm that I have been informed by the researcher, M.Sibanda about the nature of the study. I have also received, read and understood the Information and Consent sheets regarding the educational study.
3. In view of the requirements of the research, I agree that the data collected during this study can be processed in a computerized system by the researcher. Yes/no (tick one)
4. I understand that the researcher will keep all raw data under lock and key for a period of up to 5 years. After this, the raw data will be destroyed. Yes/no (tick one)

M.Sibanda (082 049 8241)

mlusibanda@yahoo.com

Name: _____

Signature: _____

Date: _____

Appendix H

University of the Witwatersrand

Wits School of Education

Information Sheet: Parents

My name is Mlungisi Sibanda I am a researcher studying MSc in Mathematics Education in the School of Education at the University of the Witwatersrand.

I am carrying out a study on **Teaching and Communicating Effectively in Multilingual Mathematics classrooms** in schools in Johannesburg, mainly looking at **the selection and use of examples in multilingual classrooms**. My research should not only benefit the institutions where it is conducted, but also the South African educational system in improving the teaching and learning of Mathematics.

I will observe 10 lessons in their natural teaching and learning situations. To help me in my observation I will video tape the learners.
The length of time for each lesson will be determined by the duration of a mathematics lesson at the school.

It is important to note that participation in this study is voluntary, no harm is anticipated. All information collected in this study will be treated as confidential and names will not be disclosed. Participants can choose to accept or refuse to answer any questions, and can withdraw from the study at any given time. In order to maintain anonymity and confidentiality, all names I use will not be real.

I will provide participants with a summary of my research results on completion if they would like me to.

M.Sibanda (082 049 8241) mlusibanda@yahoo.com

Name:

Signature: _____

Date: _____

(Protocol no. 2014ECE042M)

Appendix I

University of the Witwatersrand

Wits School of Education

LETTER TO THE PRINCIPAL,

DATE

Dear Madam

My name is Mlungisi Sibanda and I am studying MSc in Mathematics Education in the School of Education at the University of the Witwatersrand.

I am doing research on what examples are used by mathematics teachers when they teach statistics and probability. My research involves looking through teachers' examples of Statistics and Probability used in their classes and interviewing the teachers so that I may understand how s/he explains the topics to the class. I would like to do this by observing the teachers teaching statistics and probability in their classrooms. I would then like to interview the teachers to see why they chose certain examples to use. I would like to look at some learners' books to see how the learners are interpreting the work. To help me in my observation I will video tape the learners.
The reason why I have chosen your school is because it is the top government school in Johannesburg and I would like to see how the teaching and understanding of mathematics at the beginning of learners' high school careers would impact their understanding in later years.

I am inviting your school to participate in this research so that I may investigate ways to enable understanding of mathematics to inform other mathematics teachers of possible ways to start teaching statistics and probability in grade 11 so that the teachers will enable their learners to excel in mathematics.

The research participants will not be advantaged or disadvantaged in any way. They can withdraw at any given time without any consequences against them. The participants will not be remunerated in any way. The names of the participants and identity of the school will be kept confidential at all times and in all academic writing about the study. All research data will be destroyed between 3-5 years after completion of the project.

Thank you.

M.Sibanda (082 049 8241) mlusibanda@yahoo.com **(Protocol no. 2014ECE042M)**

Appendix J

Interview schedule

Semi – structured interview questions for teachers

1. Do you agree that language helps us explain our thoughts and therefore enables us to talk about what we have learnt? Please elaborate your answer.
2. How do you view the level of interaction in your class? Does it in anyway improve your learner's conceptual understanding of mathematics?
3. Are there linguistic challenges your students experience while teaching them in the LoLT? How do you deal with these challenges why do you apply those strategies?
4. What do you consider as a carefully selected example for teaching mathematics in multilingual classroom?
5. How do you choose your examples?

6. Have you ever altered an example in class? What were your reasons for doing that?

Appendix K

University of the Witwatersrand

Wits School of Education

Information Sheet: Learners

My name is Mlungisi Sibanda I am a researcher studying MSc in Mathematics Education in the School of Education at the University of the Witwatersrand.

I am carrying out a study on **Teaching and Communicating Effectively in Multilingual Mathematics classrooms** in schools in Johannesburg, mainly looking at **the selection and use of examples in multilingual classrooms**. My research should not only benefit the institutions where it is done, but also the South African educational system in improving the teaching and learning of Mathematics.

I will observe 10 lessons in their natural teaching and learning situations. To help me in my observation I will video tape the lessons.
The length of time for each lesson will be determined by the duration of a mathematics lesson at the school.

Taking part in this study is a voluntary exercise, no harm is likely to happen, and all information will be treated with strictest confidentiality. Participants can choose to accept or refuse to answer any questions, and can withdraw from the study at any given time. I hope to publish part or all the results of this study in academic journals and non- real names shall be used.

A summary of my research results on completion will be available upon request.

M.Sibanda (082 049 8241)

mlusibanda@yahoo.com

(Protocol no. 2014ECE042M)