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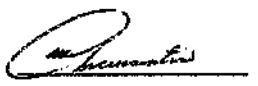
FORECASTING MODELS FOR THE DOLLAR/RAND SPOT RATES

Lungelo Gcilitshana

A research report submitted to the Faculty of Science, University of the Witwatersrand, Johannesburg, South Africa, in partial fulfillment of the requirements for the Degree of Masters of Science.

DECLARATION

I declare that this research report is my own work, submitted to the University of the Witwatersrand for the degree of Master of Science and has not been submitted before for any other qualification or examination at any other university.



Lungelo Gcilitshana

16 day of 04 199 8.

In memory of my late wife
Nomaxabiso Gcilitshana
1972-1997

ABSTRACT

Owing to the complexity of hedging against the unfavourable price movements, derivatives came into being to solve this problem if used in an effective and appropriate manner. Movements in share or stock prices, foreign exchange rates, interest rates, etc., make it difficult to anticipate or guess the next price or exchange rate or interest rates. Hence hedging ones'self against these movements becomes a hurdle that is difficult to overcome. Coming to the fore of the derivatives markets made a relief to many traders, but still then, no one could be certain about the move of the market which he is trading in. Forecasting appeared as an educated guess as to which direction and by how much the market will move.

This research report focusses on how to forecast the foreign exchange rates using the Dollar/Rand as an example. I have gathered the historical daily data for the Dollar/Rand spot rates which includes the mayhem period that happened in February 1996. The data was obtained from one of the biggest banks of South Africa; it was drawn from the Reuters historical data giving the open, high, low and close prices of the Dollar/Rand (USD/ZAR) spot rates. The data was then downloaded and copied to the spreadsheet for the calculation of the historical volatilities for different periods. To have a genuine comparison with the implied volatilities, a data of historical implied volatilities for approximately the same period was gathered from the SAIMB (South African International Money Brokers). The only snag with the data was that it only catered for

specific traded periods, like 1 month, 2 months, 3 months, 6 months, 9 months and 12 months only. Most financial institutions are using these implied volatilities for their pricing and end-of-day or -month or -year revaluation. By the same token the data was downloaded to the spreadsheet for further analysis and arrangement.

Chapter 1 gives the purpose and the meaning of forecasting, together with different methods that this process can be achieved. Views from Makridakis *et al.*, (1983) are used to beautify the world of forecasting and its importance. In Chapter 2 the concept of volatility and its causes, is discussed in detail. Besides the implied and historical volatility discussions, volatility '*smile*' concept is discussed and expanded. Volatility slope trading strategies and constraints on the slope of the volatility term structure are discussed in detail.

Chapter 3 discusses different models used to calculate both the historical and the implied volatility. This includes models by Kawaller *et al.*, (1994) and Figlewski *et al.*, (1990). The Newton-Raphson method is among of the methods that can be used to get a good estimate of the implied volatility. For a lot accurate estimates the Method of Bisection can be used in place of the Newton-Raphson method. Mayhew (1995) even suggest a method, which involves the use of more weighting with higher vegas (Latane and Rendleman 1976) or weighting not by vegas but elasticity (Chiras and Manaster 1978).

Chapter 4 dwells on different forecasting models for foreign exchange markets. This includes models by Engle (1993), who is one of the pioneers of the autoregression theory. He discusses the ARCH, GARCH and EGARCH models; Heynen *et al.*, (1994,1995) discusses the models for the term structure of volatility implied by foreign exchange. In the 1995 article he dwells on the specifications of the different autoregressive conditional heteroskedastic models. U.A. Muller *et al.*, (1990,1993) discusses some of the models for the changing time scale for short-term forecasting in financial markets. This includes discussion of some statistical properties of FX rates time-series. Xu and Taylor (1994) also discuss the term structure of volatility as implied, in particular, by FX options. Regression is used in computation of implied volatility

Chapter 5 dwells on the empirical evidence and the market practice. This includes the statistical analysis of the data; applying the scaling law; proprietary model which depicts the *edge* between the historical volatility and implied volatility; empirical tests and the volatility forecast evaluation applied to historical USD/ZAR daily data, using different models.
In the statistical analysis, using U.A. Muller *et al.*, (1993) theory, the scaling law, which involves the absolute price changes, which are directly related to the interval Δt, is discussed. Using my USD/ZAR data I managed to calculate the parameters described by the scaling law, using Δt as one day since my data is a daily data I could not calculate the activity model function, which calculates the intra-day and intra-hour trading using tick-by-tick data, because of the nature of my data. Had it not been the case, I would have been able to calculate the intra-day and intra-hour

volatilities. These statistics would have been able to depict the daily volatility, more especially on volatile days, like the day when the Rand took its first knock in February 1996.

In the second section of the chapter the proprietary model is discussed, where an edge between the actual volatility and implied volatility was identified. There is a positive correlation between the actual and implied volatility although the latter is always higher than the former; hence traders can play with this situation for arbitrage purposes. To get the estimates of historical volatility I used the well-known formula of using the log-relatives of the returns of any two consecutive days. Annualised standard deviation of these log-relatives resulted into the required historical volatility estimates. Moving averages were used to get estimates of different periods, as can be seen in the text.

The main theme of the research report is to expose forecasting models that can be used in foreign exchange currencies using Dollar/Rand as an example. Random walk model was used as benchmark to other models like stochastic volatility, ARCH, GARCH(1,1), and EGARCH (1,1). Due to the complexity of the specifications of these models, I used the SHAZAM 7.0 econometric program to generate the necessary parameters. Complex formulas of these models are given in the Appendices at the end of the report, together with the program itself.

The significance of the forecasted volatility estimates was checked using the p-value correlation statistic and the Akaike Information Criterion (AIC). The p-value gives us the significance of the parameters and the AIC gives us an indication of the *goodness-of-fit* of the model. The formulas used to calculate these statistics are given at the end of the report as part of the Appendices. An account of where and how these results can be of help in the practical situation is given under the

section of market practice. One of the areas worth mentioning is in risk management, where estimates of the historical volatility can be used together with correlation in risk-metrics to calculate VAR (value-at-risk). VAR is defined in simple terms as the 5th percentile (quantile) of the distribution of value changes. The beauty of working with the percentile rather than, say the variance of a distribution, is that a percentile corresponds to both a magnitude e.g., dollar amount at risk, and exact probability e.g., the probability that the magnitude will not be exceeded. This roughly the gist of the research report.

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Chapter 1

INTRODUCTION

1.1 Forecasting

Forecasting is one of the important techniques that most organizations use for their day-to-day activities as no one can predict the future with certainty. Frequently, there is a time lag between awareness of an impending event or need and occurrence of that event. This lead time is the main reason for planning and forecasting. If the lead time is zero or very small, there is no need for planning. If the lead time is long, and the outcome of the final event conditional on identifiable factors, planning can perform an important role.

Perspectives on forecasting, as suggested by Makridakis *et al.*, (1983), are probably as diverse as views on a set of scientific methods used by decision makers. The lay person may question the validity and efficacy of a discipline aimed at predicting an uncertain future. However, it should be recognized that substantial progress has been made in forecasting over the past several centuries. There is a large number of phenomena whose outcomes can now be predicted easily, for example, the sunrise can be predicted, as can the speed of a falling object, the onset of hunger, thirst or fatigue, rainy weather, and a myriad of other events. The ability to predict many types of events, seems as natural today as will the accurate forecasting of weather conditions in a few decades. The trend in being able to accurately predict more events, particularly those of an economic nature, will continue providing a better base from which to plan. Formal forecasting methods are the means by which this improvement will occur.

Forecasting situations vary widely in their time horizons, factors determining actual outcomes, types of data patterns and many other respects. To deal with such diverse applications, several techniques have been developed, namely *qualitative* and *quantitative* methods; of which the latter includes the

time series and casual or regressive methods. The following research will use the latter method. Makridakis *et al.*, claim that this method is generally applied when three conditions exist :-

- (i) information about the past is available
- (ii) this information can be quantified in the form of numeric data
- (iii) it can be assured that some aspects of the past pattern will continue into the future (*assumption of continuity*).

He claims that persons unfamiliar with quantitative forecasting methods often think that the past cannot describe the future accurately because everything is constantly changing, but after some familiarity with data and forecasting techniques, however, it becomes clear that although nothing remains the same, history does repeat itself in a sense.

Robert Engle (1993), the co-founder of the ARCH or GARCH models, argues that financial market volatility is predictable. This assertion has implications for asset pricing and portfolio management. Investors seeking to minimise risk, may choose to adjust their portfolios by reducing their commitments

to assets whose volatilities are predicted to rally high or by using more sophisticated dynamic diversification approaches to hedge predicted volatility increases. In a market in which such strategies operate, equilibrium asset prices should respond to forecasts of volatility, as well as to the risk aversion of investors. This is particularly true of the markets for derivative assets such as options and swaps, where the volatility of the underlying asset has a profound effect on the value of the derivative.

If large changes in financial markets tend to be followed by more large changes, in either direction, then volatility must be predictably high after large changes. This is one of the many ways traders typically predict volatility. Thus forecasts can be made over short horizons or long horizons, and forecasts can be for a single asset's volatility or for a whole set of asset variances and covariances.

The main theme of this research is to find the better model that can be used to forecast the volatility of the exchange rates, and hence the exchange rates, especially the Dollar/Rand exchange rate using the historical data.

1.2 Background

Generally, an option is a contract that gives the holder the right to buy or sell the underlying asset at a predetermined price and period. The underlying asset could be anything ranging from shares or stock, currency, bond, etc. These instruments can either be 'American' or 'European' style. In the latter type the option can only be exercised at expiration, while in the former, it can be exercised at any time during the life of the option. Pricing these options became a hurdle for most financial engineers until 1973, through a historical-breakthrough paper by two financial economist the late Fischer Black and Myron Scholes, who developed the pricing model, given below, for European options on non-paying dividend stock.

$$c = SN\left\{\frac{\ln\left(\frac{S}{X}\right) + \left[r + \frac{\sigma^2}{2}\right]T}{\sigma\sqrt{T}}\right\} - e^{-rT}XN\left\{\frac{\ln\left(\frac{S}{X}\right) + \left[r - \frac{\sigma^2}{2}\right]T}{\sigma\sqrt{T}}\right\} \quad (1.1)$$

-where c is the price of the European call option,

- S is the spot price of the underlying asset,

- X is the exercise or the strike price,

- $N(.)$ is the cumulative normal distribution function,

- r is the risk-free interest rate,

- σ is the volatility of the underlying asset,

- and T is time to maturity or expiration of the option.

Although the model is entrenched on many simplifications and assumptions of the real-world, it is the most used model in many financial institutions. In 1983 Garman and Kohlhagen modified the Black-Scholes pricing model to suit the valuation of foreign exchange options. The modification is a result of the difference between the two underlying instruments when we compare their equilibrium forward prices, i.e. non-dividend-paying stock and the foreign currency. When the interest rates are constant, as assumed by Black-Scholes, the forward price of the stock must, by arbitrage, command a forward premium equal to the interest rate. But in the foreign currency markets, forward prices can involve either forward premiums or discounts. This is because the forward value of the currency is related to the ratio of the prices of riskless bonds traded in each country. The familiar arbitrage relationship called the "*interest rate parity*" asserts that the forward exchange premium must equal the interest rate *differential*, which may be either posi-

tive or negative. Thus both domestic and foreign interest rates pl , a role in the valuation of these forward contracts, and it is therefore logical to expect such a role to extend to options as well.

This argument condones the formulation of the foreign exchange call options pricing model given below :-

$$c = e^{-r_F T} S^* N\left\{ \frac{\ln\left(\frac{S}{X}\right) + \left[r_D - r_F + \left(\frac{\sigma^2}{2}\right)\right] T}{\sigma\sqrt{T}} \right\} - e^{-r_D T} X^* N\left\{ \frac{\ln\left(\frac{S}{X}\right) + \left[r_D - r_F - \left(\frac{\sigma^2}{2}\right)\right] T}{\sigma\sqrt{T}} \right\} \tag{1.2}$$

where r_F and r_D are the interest rates for the foreign currency and the domestic currency respectively, and other parameters are as described in the preceding equation.

1.3 FX Options

Foreign exchange options are a recent market innovation, which provide a significant expansion in the available risk-control and speculative instruments for a vital source of risk, namely the foreign currency values [Garman and Kohlhagen 1983]. The deliverable instrument of an FX option is a fixed amount of underlying foreign currency. In the standard Black-Scholes (1973) option-pricing model, the underlying deliverable instrument is a non-dividend-paying stock. The difference between the two underlying instruments is readily seen when we compare their equilibrium forward prices, as discussed in the above section. The key to understand the FX options pricing is to properly appreciate the role of foreign and domestic interest rates, and this can be done by comparing the advantages of holding an FX option with those of holding its underlying currency. Like the basic and usual assumptions of the Black-Scholes model, Geometric Brownian motion governs the currency spot price : i.e., the differential representation of spot price movements is :

$$dS = \mu S dt + \sigma S dz \tag{1.3}$$

where z is the standard Wiener process,

- S is the spot price of the deliverable currency,

- μ the drift of the spot currency price,

- and σ is the volatility of the spot currency price.

The risk-adjusted expected excess returns of securities governed by our assumptions must be identical in an arbitrage-free continuous-time economy. This means that we must have :

$$\frac{\alpha_i - r_D}{\delta_i} = \lambda, \textit{ for all } i \tag{1.4}$$

where α_i is the expected return on security i,

δ_i is the standard deviation of the security i rate of return,

and λ does not depend on the security considered. Applying this fact to the ownership of foreign currency, we have :

$$\frac{(\mu + r_F) - r_D}{\sigma} = \lambda. \tag{1.5}$$

That is, the expected return from holding the foreign currency is μ, the '*drift*' of the exchange rate (domestic units per foreign unit), plus the riskless capital growth arising from holding the foreign currency in the form of an asset, like the foreign treasury notes and CD's (certificates of Deposit), paying interest at the rate of r_F. The denominator of the left-hand side of the above equation is σ, since this is the standard deviation of the rate of return on holding the currency. Now, let $C(S,T)$ be the price of a European call option with time T left to maturity, (1.4) implies :

$$\frac{\alpha_C - r_D}{\delta_C} = \lambda \tag{1.6}$$

where α_C, δ_C are the call option's expected rate of return and standard deviation of same, respectively. By Ito's Lemma, we have :

$$\alpha_C C = \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \mu S \frac{\partial C}{\partial S} - \frac{\partial C}{\partial T} \tag{1.7}$$

and

$$\delta_C S = \sigma S \frac{\partial C}{\partial S}. \tag{1.8}$$

Substituting (1.7), (1.8) into (1.6) yields :

$$\frac{\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \mu S \frac{\partial C}{\partial S} - \frac{\partial C}{\partial S} - r_D C}{\sigma S \frac{\partial C}{\partial S}} = \lambda.$$

Thus equating (1.4) and (1.5) we have :

$$\frac{\sigma^2}{2} S^2 \frac{\partial^2 C}{\partial S^2} - r_D C + (r_D S - r_F S) \frac{\partial C}{\partial S} = \frac{\partial C}{\partial T} \tag{1.9}$$

The foreign rate r_F can be considered as the '*dividend rate*' of the foreign currency. To convert to domestic terms, one would need to multiply it by the spot exchange rate S. The solution to equation (1.9) for a European FX call option must obey the further boundary condition that $C(S,0) = \max[0, S-K]$, yielding the valuation formula given in equation (1.2) above. The FX put option also satisfy the same differential equation, but with the boundary condition $P(S,0) = \max[0, K-S]$, where K is the strike price.

1.4 Conclusion

The appropriate valuation formulas for European FX options depends importantly on both foreign and domestic interest rates. The valuation formula for European put FX options can be developed in another way, using what is called the '*Put-Call Parity*', which states that, a long call and short put, at the same strikes and same maturity give the same payoff as the outright forward price for the same period. The comparative statistics are as might be expected, with two exceptions: the reaction of FX option prices to interest rate changes depends upon the nature of the concomitant changes required in either the spot or forward currency markets.

Chapter 2

VOLATILITY

2.1 What is volatility ?

There are various ways that one can define volatility. Roughly speaking volatility of the underlying asset is a measure of how uncertain we are about future underlying asset price movements. Robert Engle and Joseph Mezrich (1995: *Risk*) describe volatility as a fundamental characteristics of financial markets, hence measuring and forecasting volatility is always important. It is a measure of the intensity of random or unpredictable changes in an asset return. It is also associated with a visual plot of returns against time where the amplitude of the return fluctuates over time. The episode of high and

low volatility are often called clusters. As volatility increases, the chances that the underlying asset will do very well or very poorly increases. For the owner of the underlying asset, these two outcomes tend to offset each other. However, this is not so for the owner of a call or put. The owner of a call benefits from price increases but has limited downside risk in the event of price decreases since the most that he or she can lose is the price of the option. Similarly, the owner of a put benefits from price decreases but has a limited downside risk in the event of price increases. The values of both calls and puts therefore increase as volatility increases.

2.1.1 What causes volatility ?

Hull (1993) writes that proponents of the efficient markets hypothesis traditionally claimed that the volatility of a stock price, for example, is caused solely by the random arrival of new information about the future returns from the stock. Others have claimed that volatility is caused largely by trading. The latter statement is appropriate to the foreign exchange options market.

Generally, there is a perception, that price movements are largely affected by economic events. Kenneth Leong (*Risk :* 1992) argues that there is a very curious observation that the volatilities (implied) of exchange-traded options tend to fall, rather than to rise, after an important economic statistics release. Intuitively, one would expect the opposite to occur because the market needs to adjust to any new information that the economic number carries. Engle and Mezrich (1995) also substantiate this point by adding that historical data, for example, show that some volatility are short-lived, lasting only hours, while others may last a decade; and it is usual to think of these as driven by economic processes. The primary source of changes in market prices is the arrival of news about the asset's fundamental value. If the news arrives in rapid succession, the returns will exhibit a volatility cluster. Hence, we can conclude that economic news does influence the market to a large extent, hence the volatility of the underlying asset. Experience in our markets have shown that not only economic processes which can influence the market, but also political events and rumours.

Another interesting question is whether volatility is the same when the exchange is open as when it is closed. Fama (1965) and French (1980) have

tested this question empirically, by collecting data on the stock price at the close of each trading day over a long period of time, and calculated the following:-

- the variance of the stock price returns between the close of trading on one day and the close of trading on the next trading day when there are no intervening non-trading days.
- the variance of the stock price returns between the close of trading on Fridays and the close of trading on Mondays. They also found out that if trading and non-trading days are equivalent, the variance in the second situation should be three times as great as the variance in the first situation.

These results suggest that volatility is far larger when the exchange is open than when it is closed. The only reasonable conclusion seems to be that volatility is to some extent caused by trading itself. This implies that if daily data are used to measure volatility, the results suggest that days when the exchange is closed can be ignored, hence the volatility per annum is calculated from the volatility per trading day using the formula:-

$$volatility\ per\ annum = volatility\ per\ trading\ day \times \sqrt{number\ of\ trading\ days\ per\ annum.}$$

In FX options, volatility is one of the peculiar factors to options. It represents the anticipated volatility of the currency pair over the life of the option and is the only 'unknown' factor in the option price. It is for this reason that OTC (over-the-counter) markets quotes in volatility rather than the actual price, which is a rare case. The premium is easily calculated once volatility has been agreed between two counterparties. In FX option markets volatility is expressed as the annualized percentage rate of change of a currency pair. It is the key component to an option's " *time value*" and so the price of the option; hence high volatility equals high premium, low volatility gives low premium. Participants in the options market bid and offer around the perceived volatility level for any given period, with supply and demand dictating the final level. Like any liquid market, volatility rates can only rise to the point where sellers become evident and only fall to where buyers enter

the market. In the following section we will discuss different types of volatility as used in the derivatives market.

2.1.2 Historical and Implied volatility

Implied volatility

Alan Hicks (1995) claims that if the volatility is the key component and price of the option can be calculated by combining the other factors using Black-Scholes, it follows that volatility can be computed if the option price is available. This is called the *implied volatility* of the option and is frequently used in the case of exchange listed markets where options are priced in US cents or other currency.

Mayhew (1995) defines the implied volatility as 'the market's assessment of the underlying asset's volatility, as reflected in the option price or it is a theoretical volatility implied by an option price, using a particular option pricing model, like the Black-Scholes pricing model. This means that instead of inputting a volatility parameter into an option model like Black-Scholes,

to determine an option's fair value, the calculation can be turned around, where the actual current option price is the input and the volatility is the output. The implied volatility can be regarded as a measure of an option's "*expensiveness*" in the market, and is used by traders setting up combi strategies, where they have to identify relatively cheap and expensive options.

Traditionally, as stated above, implied volatility has been calculated using either Black-Scholes formula or The Cox-Ross-Rubinstein binomial model. Under strict assumption of the Black-Scholes model, implied volatility is interpreted as the market's estimate of the constant volatility parameter. If the underlying asset's volatility is allowed to vary deterministically over time, implied volatility is interpreted to be the market's assessment of the average volatility over the remaining life of the option. It is perhaps useful to note that implied volatility only has any meaning in the context of a particular option model, and it is not intrinsic to the option itself. Although options have existed for a long time, implied volatility has only had any meaning since the option pricing model of Fischer Black and Myron Scholes, devised in the early 1970's, stated that the value of an option was a function of the underlying share price.

Historical volatility

Another volatility measure which can cause confusion is the historical volatility. Generally, *historical volatility* is a measure of the past fluctuations of the share price or any underlying asset in question. Crudely, it is the indicator of the shares's up or downess. There is much discussion over the best method of calculating the historical volatility, but the most usual method is by taking th standard deviation of the log of price returns, which is a fairly standard method. While the calculation itself is straight-forward, it is accurate only within the parameters of each calculation e.g. the specific time period, 3 months, 3 years, etc. We will show different methods used in calculating both implied and historical volatility in the next chapter.

2.2 Volatility "*Smile*"

Volatility is of utmost importance for option pricing models, derivatives risk management, and option trading strategies. More often than not, volatility is used as an alternative way to quote option prices. As stated earlier on, Black and Scholes model assumes volatility *is* constant, but most option markets reveal there are systematic patterns in implied volatility versus option strike. These patterns are termed as the volatility "*strike structure*" or "*smile*" and the implied volatility versus maturity i.e., the volatility "*term structure*". In practice, especially in the USA after the 1987 Crash, out-of-the-money (OTM) puts typically exhibited higher implied volatilities than OTM calls. This led to a volatility "*skew*" or "*smile*".

The consensus opinion is that the Black-Scholes model performs reasonably well for at-the-money options with one or two months to expiration, and this experience has motivated the choice of such options for calculating implied volatility. For other options, however, discrepancies between market and Black-Scholes prices are large and systematic. If the market were to

price options according to the Black-Scholes model, all options would have exactly the same implied volatility, which of course, is not the case. Mayhew (1995) argues that even if market participants were to price options according to Black-Scholes, price discreteness, transactions costs, and nonsynchronous trading would cause observed implied volatilities to differ across options. In response to this problem, most early literature suggested calculating implied volatilities for each option and then using a weighted average of these implied volatilities as a point estimate of future volatility.

Non-trivial lower and higher bounds on the slope of the smile can be derived, and explained. A lower bound on the slope of the volatility term structure can be derived for options on non-dividend-paying assets. Hence, a methodology for translating the slope constraints on the smile into arbitrage bands on the volatility smile curve itself can be possible, and these constraints can be applied to trading strategies, and implications for volatility curve-fitting, of relevance to option pricing models.

2.2.1 Lower and Upper Bounds on the Volatility versus Strike Price curve

Arbitrage bounds on the slope of the implied volatility versus strike price curve restrict the slope of the smile and the level of the implied volatility skew, a measure that is closely related to the slope. As illustrated by Merton (1973) there exist some arbitrage constraints given below -:

$$\frac{dC^M}{dx} \leq 0, \tag{2.1}$$

$$\frac{dP^M}{dx} \geq 0 \tag{2.2}$$

where $x = \frac{X}{F}$, X is the strike price and F is the futures price of the underlying and C^M, P^M are respectively the market call and put option prices given by:

$$C^M = e^{-rT}F\left[N(d) - xN(d-v)\right]$$

and

$$P^M = e^{-rT}F\left[-N(-d) + xN(-d+v)\right]$$

respectively, where N is the cumulative normal distribution function, and

$$d = \frac{-\ln(x) + \frac{v^2}{2}}{v}$$

and τ is the time to maturity of the option expressed in years.

Intuitively, the first constraint states that, call options should not become more expensive with increasing strike price (all else being constant); and the second constraint states that, put options should not fall with increasing strike price. If either of these constraints were to be violated, a spread trade could be employed to capture a risk free arbitrage profit. For example, if equation (2.1) above, were found to be violated upon calculating the call option price slope between a specific strike price X and a slightly larger strike price $X+\Delta X$. Then it follows that a bull call spread trade, i.e., the purchase of a call at strike X and the sale of a call at a strike $X+\Delta X$, would generate an up-front premium to an investor. Since the full option position has no downside risk if it is held to expiration, a riskless profit is obtained.

Derivation of the Constraints on the Volatility Slope

The slope of the standard implied volatility versus strike price curve can be expressed mathematically as σ_x^M, where here, subscripts denote partial derivatives with respect to the subscript. Hence volatility slope is expressed as $v_x = \sigma_x^M F\sqrt{\tau}$ for $v = \sigma^M\sqrt{\tau}$. Constraints on the volatility slope can be

developed via a link to the slopes of call and put price versus strike price curves, obtained through the calculus chain rule :

$$\frac{dC^M[x, v(x)]}{dx} = C_x^M + C_v^M v_x \tag{2.3}$$

$$\frac{dP^M[x, v(x)]}{dx} = P_x^M + P_v^M v(x). \tag{2.4}$$

Substituting the above expressions for $\frac{dC^M}{dx}$ and $\frac{dP^M}{dx}$ in (2.1) and (2.2), the lower and the upper bounds on the scaled slope of the implied volatility versus strike price curve are obtained :

$$v_x^{U.B}(x) = -\frac{C_x^M}{C_v^M} \tag{2.5}$$

$$v_x^{L.B}(x) = -\frac{P_x^M}{P_v^M}. \tag{2.6}$$

Therefore, an arbitrage-free or scaled volatility slope must lie between these two bounds; because only a limited range of volatility slopes are arbitrage-free, the volatility versus strike curve $v(x)$ is restricted in shape, and not all smile or "skew" curves are possible. Equations (2.5) and (2.6) can be cast into a form that eliminates reference to option prices by evaluating the

partial derivatives using the Black-Scholes call and put equations given above:

$$v_x^{U.B} = +\sqrt{2\pi}\exp\left(\frac{d^2}{2}\right)N(d-v), \tag{2.7}$$

$$v_x^{L.B} = -\sqrt{2\pi}\exp\left(\frac{d^2}{2}\right)[1 - N(d-v)]. \tag{2.8}$$

The above expressions can also be expressed in the following way :

$$v_x^{L.B.}(d,v) = -v_x^{U.B.}(-d,-v),$$

$$v_x^{L.B.}(x,v) = -v_x^{U.B.}(x,-v).$$

Numerical values of the upper and lower boundaries of the scaled volatility slope can evaluated using the above equations.

Overall Restrictiveness of the Volatility Slope Constraints

The restrictiveness of the upper and lower slope constraints, taken together, can be studied by examining the range of allowed slopes, as measured by the difference in the upper and lower slope bounds (H.M. Hodges 1996) :

$$\text{Range Measure} \equiv \mathrm{R} = \left(\sigma_x^{U.B.} - \sigma_x^{L.B.}\right)$$
$$= \sqrt{\frac{2\pi}{\tau}} \exp\left(\frac{d^2}{2}\right). \tag{2.9}$$

Smaller values of R imply a tighter combination of lower and upper bounds. For options of a fixed maturity, the tightest combination of constraints can be found by minimizing R with respect to volatility and moneyness. A global minimum of R occurs if $d = 0$, in which case the exponential term in the above equation is unity, and corresponds to the curve

$$v = \sqrt{2\ln(x)}$$

for $x > 1$. Along this curve $R = \sqrt{2\pi/\tau}$.The measure is also minimized if

$$v = \sqrt{-2\ln(x)}$$

for $x < 1$. Along this curve, the unannualised market volatility slope is bounded as follows :-

$$|v| \leq \frac{\sqrt{\pi/2}}{x}.$$

and the range of annualized slopes is

$$R = \frac{\sqrt{2/\tau}}{x}.$$

Hence, in the limit of very long option maturity, it is possible to find combinations of annualized volatility and strike prices that essentially require a zero sloping volatility versus strike curve at those particular strikes.

2.2.2 Volatility Slope Trading Strategies

One advanced volatility trading strategy is based on the mean-reversion of the volatility slope or '*skew*'. For example, if the volatility spread is, say, 0.5 % between the out-of-the money calls and puts, and the trader believes this difference is essentially a volatility slope or skew, which deviates from the historical norms, the trader can buy or sell the skew to bet that the slope

will return to more normal levels. This trading strategy could be potentially be enhanced by applying the same mean-reversion strategy to the market slope expressed as a percentage of the slope boundaries. This implies that, instead of measuring and analyzing the historical market slope in a "vacuum", it can be measured relative to the bound, which can vary over time, depending on the volatility climate.

2.2.3 Constraints on the Slope of the Volatility Term Structure

In this section, constraints on the implied volatility versus option maturity are examined. At any given strike, call and put option prices must increase in value with increasing option maturity [Cox and Rubinstein 1985], simply because the more time left to maturity, the more chance of the option to be in-the-money and exercised. Hence this condition holds :

$$\frac{dC^M}{d\tau} \geq 0 \tag{2.10}$$

Instead of the strike price, the option time to maturity τ is the variable of interest, and analogous to the smile boundary derivations, it follows that :

$$\frac{dC^M[\tau, v(\tau)]}{d\tau} = C^M_\tau + C^M_v v_\tau \tag{2.11}$$

where the subscripts represent the partial derivatives. Now, substituting the above equation into (2.10), the slope of the term structure is constrained as follows :

$$v_\tau(\tau) \geq -\frac{C^M_\tau}{C^M_v}. \tag{2.12}$$

This constraint is intuitive, stating that, the volatility term structure cannot be too downward-sloping. Evaluating this constraint further by calculating the partial derivatives and using the forward price of, say, equities without dividends [Hodge 1996],

$$F = Se^{r\tau},$$

hence

$$v_\tau(\tau) \geq -\sqrt{2\pi r}\exp\left(-r\tau + \frac{d^2}{2}\right)N(d-v), \tag{2.13}$$

where

$$d = \frac{r\tau + \frac{v^2}{2}}{v}.$$

In this case the strike price has been equated to the underlying spot price, since at-the-money volatility is the conventional choice among volatility traders to measure volatility term structure. As long as the interest rates are not abnormally high, the right-hand side of (2.11) remains small. Hence

$$\lim_{v\to\infty}\left[-\sqrt{2\pi r}\exp\left(-r\tau + \frac{d^2}{2}\right)N(d-v)\right] \to 0,$$

and in the approximation the right-hand side is dropped altogether, it follows that :

$$v(\tau_2) \geq v(\tau_1) \text{ for } \tau_2 \geq \tau_1.$$

That is , the unannualised implied volatility must increase with increasing option maturity. If (2.11) is violated for some reason over a range of maturities $\tau_1 \leq \tau \leq \tau_2$, a simultaneous sale and purchase of at-the-money call options with maturities τ_1 and τ_2, respectively, will generate an up-front premium. Moreover, the options position can be managed to avoid a loss by the time of the final option expiration, and might even provide additional gains. The net result is a risk-free profit.

2.3 Conclusion

Constraints on the variation of option prices with respect to strike prices have been around since the development of option pricing theory; in practice, though, option pricing is typically viewed in terms of implied volatility. The volatility constraints can serve as reference points against which to measure unusual behaviour, and hence trading opportunities. This can be accom-

plished by tracking the ratio of market volatility slopes to slope boundaries over time. Alternatively, the slope can be placed on a volatility versus strike price curve through a numerical solution of non-linear first order differential equations [Hodge 1996].

Volatility versus strike data can thus be directly compared to volatility versus strike boundaries to gauge the attractiveness of potential trading opportunities. These volatility constraints are also relevant to modern option pricing models and applications, like generating probabilities of future underlying asset prices, which sometimes involve the fitting or "smoothing" of volatility versus strike price market data.

Chapter 3

DIFFERENT MODELS FOR CALCULATING HISTORICAL AND IMPLIED VOLATILITY

3.1 Models for Historical Volatility

More often than not, it is difficult to know the exact volatility for any underlying asset, otherwise risk-less arbitrage would prevail, because every dealer would know the future price of any traded security. For that matter, we wouldn't be having derivative products, because these products were created for protection against the adverse movements of any traded security. Hence to have an educated guess, sometimes we need to use the pass history of the asset in question. A good estimate of how volatile the asset was in the past, within a limited percentage of error, creates a better base for how the asset will probably be in the future.

The most commonly used formula, as I indicated in the last chapter, is given below (Banks , 1993) :-

$$s = \sqrt{\frac{1}{n-1}\sum_{T=1}^{n}(r_T - \bar{r})^2}, \tag{3.1}$$

- where n is the number of observations,

- S_T is the underlying price at period T,

- $r_T = \ln\left(\frac{S_T}{S_{T-1}}\right)$,

- $\overline{r}$ is the mean of the natural log of price relatives defined by r_T.

The same version of the formula is given by Hull (1993) in a different form :-

$$s = \sqrt{\frac{1}{n}\left(\sum_{i=1}^{n} u_i\right)^2 - \frac{1}{n(n-1)}\sum_{i=1}^{n} u_i^2}, \tag{3.2}$$

where $u_i = \ln\left(\frac{Si}{S_{i-1}}\right)$ and n is the number of observations.

Figlewski *et al.*, (1990) argues that the precision with which volatility is measured increases as more information is used to estimate it. One month of close-to-close data may be too short a period of time to provide useful estimates of volatility. An alternative hypothesis is that these data reflect changes in underlying volatility; standard statistical procedures can determine whether sampling variability is sufficient to explain observed changes in

volatility from month to month. Figlewski *et al.*, (1990) suggest the simplest of such a procedure, known as the *Barlett's test* given below :-

$$\textit{Barlett's test statistic} = \sum_{i=1}^{12}(n_i - 1)\ln\left(\frac{\hat{\sigma}^2}{\hat{\sigma}_i^2}\right), \tag{3.3}$$

- where $\hat{\sigma}_i^2$ is the variance estimated for month i,
- $\hat{\sigma}^2$ is the variance estimated for the entire year,
- n_i is the number of trading days in month i.

This test statistic is simply the difference between the logarithm of annual variance $\hat{\sigma}^2$ and the average value of the logarithm of monthly variance σ_i^2. It thus measure the extent to which monthly variances differ from annual variances.

On the other hand, instead of finding a simple average of changes in the logarithm of stock prices and of the squared deviations from the average, one might take weighted averages, where weights sum to one and decrease as one goes further back in time. This method is an example of exponential smoothing, which is a character found in the GARCH model, which is

the acronym for Generalized AutoRegressive Conditional Heteroskedasticity and EGARCH (exponential GARCH) models. If we assume the average of changes in the logarithm of prices is known, or for simplicity set equal to zero, then, for successive squared deviations, sd_t, the prior estimate of variance $\hat{\sigma}^2_{t-1}$ is updated by the formula :-

$$\hat{\sigma}^2_t = \alpha\, sd_t + (1-\alpha)\hat{\sigma}^2_{t-1}, \tag{3.4}$$

where the weighting constant α is determined by experience to be a number between zero and one. This is, in fact, a weighted average scheme, where the weights $\alpha, \alpha(1-\alpha), \alpha(1-\alpha)^2, \ldots$ sum to one and decrease as one goes further back in time. These measures of volatility depend crucially on an appropriate choice of weighting constants. Advanced statistical procedures have been recently developed to estimate appropriate values of these constants from the data. However, these methods assume the variance evolves in a fashion inconsistent with the standard option pricing formulae, which assume the variance will be constant over the remaining life of the option.

Figlewski *et al (1990);* again suggest another approach, which is to increase the frequency with which the data is measured. If data prior to last month is considered of limited use in determining volatility, an obvious approach is to use trade-to-trade data if available. If the assumptions of the model are correct, knowledge of the high's and low's of trading on a day-by-day basis can yield an estimate of volatility superior to that obtained by looking only at successive close-to-close data. Incorporating the open and close prices will lead to further improvements. The estimate of volatility is given by the following formula :-

$$\hat{\sigma} = \sqrt{\frac{0.361}{n} \sum_{i=1}^{n} [\ln(H_i) - \ln(L_i)]^2}, \tag{3.5}$$

where H_i is the trading high for day i,

L_i is the trading low for the day i,

n is the number of trading days under consideration.

In other words, simply take the average of the squared difference in logarithms between the high and low price for each trading day. We then multi-

ply this quantitv by the factor 0.361 to obtain the estimate of variance. The square root of this estimate is then the desired measure of volatility. Thus, an estimate of variance is given by the average squared range multiplied by the reciprocal of four times the natural logarithm of two, (ln 2) or 0.361. The superiority of this method using high's and low's has been hailed as the appropriate method by Heynen and Kat (1994) and J. Hull (1993). The use of opening and the closing rates or prices were ruled out on the grounds that, more often than not, the financial markets are dull in the morning; and during the closing time the markets tend to die down again, hence both the opening and the closing rates do not give a good reflection of the liquidity of the market for that particular day. Another point of concern is the time interval between successive day close-to-close rates or prices, i.e. length of the differing interval is not uniquely fixed because of weekends, holidays, and weekend-holidays; it varies from twenty-four hours for a single trading day to seventy-two hours for the weekend and even to ninety-six hours for the weekend-holiday.

In the later chapter, we will be able to test this formula and compare it to the conventional method that we mentioned earlier, using our historical

data. Finally, research suggests that, in many situations, measures of volatility based on historical data are unreliable where volatility changes through time. In many applications, only data for the recent past is considered; and that is to be kept in mind whenever using historical data.

3.2 Models for Implied Volatility

As highlighted in the very first chapter that volatility can be measured by physically inputting the market's option price and solve for the volatility parameter in any pricing model. We termed this type of volatility as the "implied volatility". As the name suggests, it is the volatility that is implied by market makers for a specific option on a specified underlying for specific period. Many financial economists substantiate this concept of implied volatility. Stewart Mayhew (1995) who is a doctoral student in Finance at the University of California at Berkerley, refers to implied volatility as the *markets's assessment* of the underlying asset's volatility as reflected in the option price.

Traditionally, implied volatility has been calculated using either the Black-Scholes formula or the Cox-Ross-Rubinstein binomial model. Under the strict assumptions of the Black-Scholes model, because of its assumptions, implied volatility is interpreted as the market's estimate of the constant volatility parameter. But if the underlying asset's volatility is allowed to vary deter-

ministically over time, implied volatility is interpreted to be the market's assessment of the average volatility over the remaining life of the option.

Option pricing formulas, more often than not, cannot be inverted analytically, so implied volatility must be calculated numerically. In general, this is accomplished by feeding the value-price difference :-

$$C(\sigma_0) - C_m$$

into a root-finding program, where $C(\ )$ is an option pricing formula, σ_0 is the volatility parameter, and C_m is the observed market price of the option. Various algorithms can be used to find the value of σ that makes the above expression equal to zero, e.g. the *Newton-Raphson method* as noted by Figlewski *et al* (1990). This method is highly efficient and accurate in the context of European call options. It also does not work well for American options on a dividend-paying securities. Newton-Raphson is one of the basic numerical methods of getting a solution through repeated iterations.

The computational problem is to find the volatility σ_0 such that the value of the option expressed as a function of volatility, $C(\sigma_0)$, is equal to the observed option price, C_m. This method starts out with the presumption that the option value is to a first approximation given by a linear function of volatility :-

$$[C_m - C(\sigma_0)] \approx \kappa * (\sigma_0 - \sigma), \tag{3.6}$$

where κ or kappa, is the derivative of the option value as a function of volatility. For any given value of σ, the implied volatility σ_0 may be approximated by :-

$$\hat{\sigma}_0 \approx \sigma + \frac{[C_m - C(\sigma)]}{\kappa}. \tag{3.7}$$

The method proceeds by first specifying a starting value for volatility, computing $C(\sigma)$ and κ for that value of σ, and approximating the implied volatility using the above formula. The precision of the approximation can be judged by the extent to which the option value, given that estimate, $C(\hat{\sigma}_0)$,

comes close to the observed price C_0. A closer approximation can be found by substituting the approximate measure of volatility $\hat{\sigma}_0$ into the above formula again through iterative methods.

In certain applications, kappa can be difficult to compute or not well defined, and in that case the Newton-Raphson procedure will not work well. In that case the Method of Bisection is an alternative iterative method which does not require any estimate of kappa and is not sensitive to choice of starting values, and is computationally efficient.

For the Method of Bisection, we first choose a '*low*' estimate of implied volatility σ_L, which would correspond to an option value of C_L, and a '*high*' estimate σ_H, corresponding to C_H, so that C_m lies between C_L and C_H. Then the estimate of implied volatility is given as the linear interpolation between those two points :-

$$\hat{\sigma}_0 = \sigma_L + \frac{(C_0 - C_L) \times (\sigma_H - \sigma_L)}{C_H - C_L}. \tag{3.8}$$

If the value of the option given this estimate of implied volatility is equal to

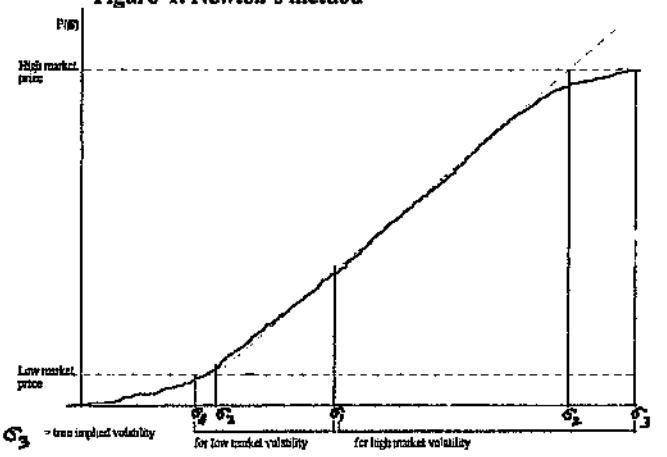
the traded option value, C_0, stop. Otherwise, if the value $C(\tilde{\sigma}_0)$ is less than C_0, replace σ_L with this value and repeat the exercise. If it is greater, use it to replace σ_H.

Ed Weinberger (1993, *Risk*) regard the Method of Bisection as the most obvious approach which brackets the true implied volatility between a series of successively tighter upper and lower bounds. replacing the upper or lower bound at each stage by the average of the bounds, depending on the option premium predicted by this average value. He claims that the Method of Bisection represents the safest possible "investment", in that it is guaranteed to find the right answer, but large portfolios can contain hundreds or thousands of options, each of which must be processed accurately to avoid cumulative errors in computing portfolio hedge ratios.

Weinberger suggests an alternative method, which he reckons to be faster and which doubles the number of correct digits in each volatility estimate after each iteration. The method, invented by Sir Isaac Newton is illustrated in figure I below. The solid curve is $P(\sigma)$, the Black-Scholes estimate of the option premium as a function of σ; the horizontal lines are actual market premia. For a given market premium, P_{market}, the implied volatility is the value

at which the $P(\sigma)$ curve intersects the horizontal line $P = P_{market}$. Newton's idea was that the diagonal dotted line, the tangent to $P(\sigma)$ at σ, intersects the market price line at a volatility σ_2 near the true implied volatility, and that a tangent drawn at σ_2 yields a still better estimate, σ_3.

Figure 1: Newton's method



Generally, given the estimate σ_i, the improved estimate, σ_{i+1}, is given by :-

$$\sigma_{i+1} = \sigma_1 - \frac{P(\sigma) - P_{market}}{P'(\sigma_i)}. \tag{3.9}$$

Newton's method is especially convenient for finding implied volatilities for

European options because $P'(\sigma_i)$, the slope of the tangent line at volatility σ_i, can then be computed explicitly via the formula :-

$$P'(\sigma_i) = S\sqrt{\frac{T}{2\pi}}\exp\left\{\frac{-\left[\ln\left(\frac{S}{X}\right)+\left(r-q+\left(\frac{\sigma_i^2}{2}\right)\right)T\right]^2}{2\sigma_i^2 T} - qT\right\} \qquad (3.10)$$

- where S is the underlying market variable,

- q is the payout rate, if any, of the underlying security (For non-dividend paying securities $q = 0$, for foreign currencies q is the foreign risk-free rate.),

- other parameters are as defined before.

We already know that $P'(\sigma_i)$ is also the vega of the option at that volatility, so that we get the option for free when σ_i converges to the true implied volatility. One exciting principle of this method is that it is guaranteed to converge from the starting point :-

$$\sigma_1 = \sqrt{\frac{2\left|\ln\left(\frac{S}{X}\right)+(r-q)T\right|}{T}}. \qquad (3.11)$$

At this sigma value, the corresponding vega value and thus the slope of the curve in figure I. is a maximum. If the estimated option premium using

(3.11) as the volatility, is larger than the actual market premium, the volatility iterates generated by Newton's method will be a decreasing sequence bounded below by the true volatility. Such a sequence must converge, and since (3.9) will continue to generate everdecreasing iterates until the true implied volatility is reached, (3.9) must converge to that value. As figure I suggests, a similar argument applies when the volatility estimate (3.11) is smaller than the market volatility.

Many options, which may vary in strike price and time to expiration, are written on the same underlying asset. If Black-Scholes model held exactly, these options would be priced so that they all have exactly the same implied volatility, which of course, is not the case. Systematic deviations from the predictions of the Black-Scholes model are often called the "*volatility smile*" (most texts discuss this phenomenon). Even if market participants were to price options according to Black-Scholes, price discreteness, transaction costs, and non-synchronous trading would cause observed implied volatilities to differ across options. In response to this problem, Stewart Mayhew (1995) argues that calculating implied volatilities for each option and then using a *weighted average* of these implied volatilities as a point estimate of future

volatility. The idea behind this approach is simple. If the model is correct, then deviations from the predicted prices represent noise, and noise can be reduced by using more observations. The simplest weighting scheme, used by Trippi (1977) and by Schumalense and Trippi (1978), places equal weights on all N implied volatilities :-

$$\widehat{\sigma} = \frac{1}{N}\sum_{i=1}^{N}\sigma_i. \tag{3.12}$$

The other shortcoming is that, the Black-Scholes model prices some options more accurately than others, and to place more weight on observations for which the model performs better is reasonable. Trippi and Schumalense simplified this problem by simply throwing out options that are near expiration or far from the money. Another problem with equal weighting is that some options are more sensitive to volatility than others (long-dated options more sensitive than short-dated options); hence estimation errors are likely to be higher for options whose prices are insensitive to volatility. Therefore, placing more weight on options with higher vegas appears to be preferable to equal weighting. Latane and Rendleman(1976) suggested this weighting scheme :-

$$\hat{\sigma} = \frac{1}{\sum_{i=1}^{N} w_i} \sqrt{\sum w_i^2 \sigma_i^2} \tag{3.13}$$

where the weights, w_i, are the Black-Scholes vegas of the options. This forecast has the advantage of weighing options according to their sensitivities, but it is subject to criticism that it is biased because the weights do not sum to 1. Chiras and Manaster (1978) suggested weighting not by vegas but by volatility elasticities :-

$$\hat{\sigma} = \frac{\sum_{i=1}^{N} \sigma_i \frac{\delta C_i}{\delta \sigma_i} \frac{\sigma_i}{C_i}}{\sum_{i=1}^{N} \frac{\delta C_i}{\delta \sigma_i} \frac{\sigma_i}{C_i}}. \tag{3.14}$$

Beckers (1981) and Whaley (1982) suggested choosing to minimize :-

$$\sum_{i=1}^{N} w_i \left[C_i - BS_i(\hat{\sigma})\right]^2$$

where C_i is the market price and BS_i the Black-Scholes price of option i. The weights, w_i, may be chosen in many ways, the most obvious choices being equal weights or Black-Scholes vegas.

Chapter 4

DIFFERENT FORECASTING MODELS FOR FX MARKETS

4.1 STATISTICAL MODELS FOR FINANCIAL VOLATILITY

In the previous chapter, we discussed different methods of calculating the implied volatility, of which one of them was to invert the options pricing model for a given option's market-related option price. The article discussed below is written by Robert Engle (1993), who is one of the pioneers of the

ARCH or conditional volatility clustering models. He discusses in detail the ARCH models, which can be used for forecasting in different scenarios. In the next subsection, the original autoregressive conditional heteroskedasticity or ARCH model is discussed as the statistical model.

4.1.1 The Statistical Model

In order to understand these conditional variance models, their similarities and differences, it is important to understand the difference between conditional and unconditional moments.

Let y_t be the return on an asset received in period t, and let E represent mathematical expectation. Then the mean of the return can be called μ, and

$$Ey_t = \mu. \tag{4.1}$$

This is the *unconditional* mean (Robert Engle 1993), which is not a random variable. The *conditional* mean, m_t uses information from the previous period and can generally forecast more accurately, and is given by :

$$m_t = E\left[y_t \mid F_{t-1}\right] \equiv E_{t-1}\left[y_t\right]. \tag{4.2}$$

This is in general a random variable depending on the information set F_{t-1}. Although $y - \mu$ can be forecast, $y_t - m_t = \varepsilon_t$ cannot, using the information in F_{t-1} alone. The *unconditional* and *conditional* variances can defined respectively, as :

$$\sigma^2 \equiv E\left[y_t - \mu\right]^2 = E\left[y_t - m_t\right]^2 + E\left[m_t - \mu\right]^2, \tag{4.3}$$

$$h_t \equiv E_{t-1}\left[y_t - m_t\right]^2. \tag{4.4}$$

The first part of the righ-hand side of (4.3) can be simplified as follws :-

$$\begin{aligned} E\left[y_t - \mu\right]^2 &= E\left[y_t - m_t + m_t - \mu\right]^2 \\ &= E\left[\left(y_t - m_t\right) + \left(m_t - \mu\right)\right]^2 \\ &= E\left[\left(y_t - m_t\right)^2 + \left(m_t - \mu\right)^2 + 2\left(y_t - m_t\right)\left(m_t - \mu\right)\right] \\ &= E\left(y_t - m_t\right)^2 + E\left(m_t - \mu\right)^2 + 2E\left(y_t - m_t\right).E\left(m_t - \mu\right) \\ &= E\left[y_t - m_t\right]^2 + E\left[m_t - \mu\right]^2 \text{ using } E\left[\varepsilon_t\right] = 0 \end{aligned} \tag{4.5}$$

The *conditional variance* potentially depends upon the information. For higher moments, the conditional skewness and kurtosis are defined respectively, as :

$$s_t = E_{t-1}\left[\frac{(y_t - m_t)}{\sqrt{h_t}}\right]^3 \tag{4.6}$$

$$k_t = E_{t-1}\left[\frac{(y_t - m_t)}{\sqrt{h_t}}\right]^4. \tag{4.7}$$

These potentially depend on past information.

4.1.2 Formulating Volatility Equations

The specification problem for analyzing a series y_t can be described by three steps:

- specify m_t
- specify h_t

- specify the conditional density of ε_t which is equal to $y_t - m_t$.

For financial markets, m_t is generally the risk premium, or the expected return. To show how conditional variances depend upon past information, Engle (1993) reckons that the best method to use in estimation is the maximum-likelihood estimation, which is generally recommended and used. When using this estimation method, the likelihood function typically assumes that the conditional density is Gaussian (random walk), so that the logarithmic likelihood of the sample is simply the sum of the individual normal conditional densities. Carol Alexander and Navtej Riyait (1992) give the simplest form of the log-likelihood function, up to a constant, for arbitrary parameters α, β, γ, and δ :

$$L_T(\alpha, \beta, \gamma, \delta) = T^{-1} \sum_{t=1}^{T} l_t(\alpha, \beta, \gamma, \delta),$$

where T is the sample size and

$$l_t(\alpha, \beta, \gamma, \delta) = -\frac{1}{2} \ln h_t^2 - \frac{1}{2} \varepsilon_t^2 / h_t^2$$

and estimates of the parameters are obtained by maximizing L_T. Normally, the likelihood function includes $-\frac{1}{2}\ln(2\pi)$ as the first term, but it is omitted because it is a constant. One of the reasons the likelihood function includes natural logarithm, is because the function e^x is a monotonic increasing function.

The simplest specification of the conditional variance equation is the ARCH(p) model, in which the conditional variance is simply a weighted average of past squared forecast errors :

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2. \tag{4.8}$$

If $\omega = 0$ and $\alpha = 1/p$, the above equation simply becomes the sample variance of the previous p returns, and this is widely used by market participants as the past volatility estimate, and the parameters can be estimated from historical data. This model can be used to forecast future patterns in volatility. Equation (4.8) can also be written as :

$$\varepsilon_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \left[\varepsilon_t^2 - h_t\right]. \tag{4.9}$$

Engle (1993) argues that the term in brackets is unforecastable and is therefore considered the innovation in the autoregression for ε^2; and this is the source of the name *autoregressive conditional heteroskedasticity.* Alexander and Riyait (1992) assert that the name autoregressive conditional heteroskedasticity refers to a particular type of *heteroskedastic* or *non-constant variance* error term in a regression model, the 'autoregressive conditional' means that a large past variance induces a large current variance for the error term. The following equation is the simple example of a standard linear regression model :

$$y_t = \alpha x_t + \varepsilon_t$$

where

- y_t is the value at time t of the variable we wish to model or is the dependent variable,

- x_t is a vector of explanatory or independent variables at time t,
- α is a vector of unknown parameters (to be estimated),

and ε_t is a normally distributed error term. This implies that any time series in which turbulent periods are interspersed with more tranquil spells may be suitable for this type of analysis.

A natural generalization is to allow past conditional variances, which result into a generalized ARCH model or GARCH(p,q), which is given by :

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i}. \tag{4.10}$$

By adding and subtracting

$$\sum \beta_i \varepsilon_{t-i}^2$$

equation (4.10) can be written as :

$$\varepsilon_t^2 = \omega + \sum_{i=1}^{p} (\alpha_i + \beta_i) \varepsilon_{t-i}^2 - \sum_{i=1}^{p} \beta_i \left[\varepsilon_{t-i}^2 - h_{t-i}\right] + \left[\varepsilon_t^2 - h_t\right] \tag{4.11}$$

where $p \geq q$ is assumed without the loss of generality. Like the ARCH model, GARCH parameters can be estimated from the historical data. Nelson's exponential form of GARCH(p,q), EGARCH is given by :

$$\log h_t = \omega + \sum_{i=1}^{p} \beta_i \log h_{t-1} + \sum_{i=1}^{p} \frac{\alpha_i |\varepsilon_{t-i}|}{\sqrt{h_{t-i}}} + \sum_{i=1}^{p} \frac{\gamma_i \varepsilon_{t-i}}{\sqrt{h_{t-i}}}. \tag{4.12}$$

If $p = q = 1$, the summation signs disappear, and you get a GARGH(1,1) model, which is regarded as the best volatility forecasting model, and is mentioned in detail in the next section.

There is a host of other ARCH models, which are not going to be considered in this research, because there is no special need for them, seeing that the GARCH(1,1), and EGARCH(1,1) are recognized as the best volatility forecasting models. This host consists of :

- ARCH-M or GARCH-M which incorporates the time-varying risk premium ($m_t = \delta h_t$) as the specification of the mean in equation (4.2), for δ interpreted as the coefficient of relative risk aversion. GARCH-M, is

useful for instruments such as equities or bonds where an increase in risk may be accompanied by increased return. In econometric terms this implies that the conditional variance should appear positively in the conditional mean equation.

- AARCH - augmented ARCH
- AARCH - Asymmetric ARCH
- MARCH -Modified ARCH
- MARCH - Multiplicative ARCH
- NARCH - Non-linear ARCH
- PNP ARCH - Partially non-parametric ARCH
- QTARCH - Qualitative threshold ARCH
- SP ARCH - Semi-parametric ARCH
- STARCH - Structural ARCH, and
- TARCH - Threshold ARCH

All these models are based on regression.

4.2 ANALYSIS OF THE TERM STRUCTURE OF IMPLIED VOLATILITIES

In this section Heynen *et al.*, (1994) article discusses the relation between short- and long-term implied volatilities based on three different assumptions of stock return volatility behaviour, i.e., mean-reverting, GARCH and EGARCH models. The last two models are analyzed simpler in the Heynen and Kat (1994) paper, and hence they will be discussed in the coming section.

4.2.1 Restrictions on the Average Expected Volatilities

This subsection explains how mean-reverting stock volatility models lead to restrictions on the average expected volatility.

Mean-Reverting Stock Return Volatility

A mean-reverting stock return volatility process can be modeled as :

$$\frac{dS_t}{S_t} = \mu dt + \sigma_t dW_1, \tag{4.13}$$

$$d\sigma_t^2 = \alpha_0 \left(\overline{\sigma}^2 - \sigma_t^2\right) dt + \alpha_1 \sigma_t dW_2 \tag{4.14}$$

- where S_t is the stock price at t,

- μ is the mean stock price return,

- dW_1, dW_2 are Wiener processes;

- in $dW_1.dW_2 = \omega dt$, ω is the instantaneous correlation between stochastic increments dW_1 and dW_2,

- σ_t is the stock return volatility,

- α_0 is the coefficient of mean reversion.

- α_1 is the instantaneous standard deviation of $\frac{d\sigma^2}{\sigma}$

- and $\overline{\sigma}^2$ is the mean-reversion level.

Heynen *et al.*, (1994) argues that for a stochastic volatility option model, it can be shown that the value of an at-the-money option is approximately equal to the Black-Scholes value with the volatility equal to the average expected volatility of the underlying stock over the remaining lifetime of the option. This can be seen as an extension of a result of Merton (1973), which states that Black-Scholes holds with the volatility replaced by its average volatility if the volatility is a deterministic function of time. Thus, Heynen *et al.*, defines the average expected volatility $\sigma_{Av}(t,T)$ as :

$$\sigma_{Av}^2(T,t) = \frac{1}{T}\int_t^{t+T} E_t\left[\sigma_s^2\right] ds \tag{4.15}$$

-where E_t is the conditional expectation operator at the current time t,

-and T is the time to expiration.

Assume that 'instantaneous' volatility σ_t evolves according to the following continuous-time mean-reverting AR1 process :-

$$d\sigma_t = -\alpha_0\left(\sigma_t - \overline{\sigma}\right) dt + \alpha_1 \sigma_t dz.$$

At the time t, the expectation of volatility as of $t+j$ will be given by :-

$$E_t\left(\sigma_{t+j}\right) = \overline{\sigma} + \rho^j\left(\sigma_t - \overline{\sigma}\right)$$

where $\rho = e^{-\alpha_0} < 1$. That is, volatility is expected to decay geometrically back towards its long-run mean level of $\overline{\sigma}$. Denote $\sigma(t,T)$ as the implied volatility at time t on an option with T remaining until expiration and this should be equal the average expected instantaneous volatility over the time span $[t, t+T]$. Using the above equation, this implies :-

$$\begin{aligned}\sigma(t,T) &= \frac{1}{T}\int_{j=0}^{T}\left[\overline{\sigma} + \rho^j(\sigma_t - \overline{\sigma})\right] dj \\ &= \overline{\sigma} + \frac{\rho^T - 1}{T\ln\rho}\left[\sigma_t - \overline{\sigma}\right]\end{aligned}$$

where ρ is as defined above. This last equation implies that, when instantaneous volatility is above its mean level, the implied volatility on an option should be decreasing in the time to expiration. Conversely when instantaneous volatility is below its mean, implied volatility should be increased in the time to expiration. Hence it can be deduced that :

$$\sigma^2_{Av}(t,T) = \overline{\sigma}^2 + \frac{1}{\alpha_0 T}\left[\sigma_t^2 - \overline{\sigma}^2\right]\left[1 - e^{\alpha_0 T}\right]. \qquad (4.16)$$

Equation (4.16) simply states that when the instantaneous variance σ_t^2 is above its mean level $\overline{\sigma}^2$, the average expected volatility should be decreasing

in time to maturity. When the instantaneous variance is below its mean, average expected volatility should be increasing in time to maturity. Although the instantaneous variance cannot be observed, a relation can be derived between two average expected volatilities, differing in time to maturity (say T_1 and T_2) thereby eliminating the instantaneous variance σ_t^2 :

$$\left[\sigma_{Av}^2(t,T_1)-\overline{\sigma}^2\right]=\frac{T_2}{T_1}\frac{\rho^{T_1}-1}{\rho^{T_2}-1}\left[\sigma_{Av}^2(t,T_2)-\overline{\sigma}^2\right] \tag{4.17}$$

where $T_1 > T_2$ and $\rho = e^{-\alpha_0}$.

The above equation is referred to as the term structure of average expected volatility (average expected volatility for different times to maturity) in the case of a mean-reverting stock return volatility model. For ρ smaller than 1, the first two fractions on the right-hand-side of equation (4.17) also become smaller than 1. This implies that, given a movement in the short-term average expected volatility $\sigma_{Av}^2(t,T_2)$, there should be a smaller movement in distant average expected volatility $\sigma_{Av}^2(t,T_1)$. The constant of proportionality depends on the mean-reversion parameter ρ, as well as on the remaining

time to maturities T_1 and T_2. Other models will be discussed in chapter 5.

This mean-reverting model will not be used in forecasting since all the other models have an intrinsic property of reversion, where conditional variance is reverting to the unconditional variance or vice versa.

4.3 THE TERM STRUCTURE OF VOLATILITY IMPLIED BY FX OPTIONS

In the previous section, we discussed the analysis of the term structure of implied volatility discussed by Heynen, Kemna and Vorst. We will test these models later using our historical data.

In this section we follow Xu and Taylor discussing the term structure of volatility implied by foreign exchange options. In the first section of the discussion, we get a model of both the term structure of expected volatility and the time series characteristics of the term structure. The second section discusses simple specifications for different term structures, namely short-term and long-term. The third section describes the estimation methods and how they were used in data from the Philadelphia Stock Exchange options for spot currency options on the British pound, German mark, Japanese yen, and Swiss franc quoted against U.S dollar for a five year period from Jan 1985-Nov 1989.

4.3.1 A Model for the Term Structure

Volatility is defined, for some time period, as the annualized standard deviation of the change in the price logarithm during the same period of time. Xu and Taylor argue that market agents will have expectations at time t about the volatility during future time periods. Suppose they form expectations of the quantities :

$$Var(\ln P_{t+\tau} - \ln P_{t+\tau-1}) \ , \tau = 1, 2, ..., \tag{4.18}$$

where P refers to the price of the asset upon which options are traded. These expectations can be annualized by multiplying them by n, where n is a smaller interval, which might either be calendar days or might be trading days. Let $\sigma_{t,t+\tau}$ denote the volatility expectation at time t for time interval $t+\tau$, so

$$\sigma^2_{t,t+\tau} = nVar\left(\ln(P_{t+\tau}/P_{t+\tau-1}) \mid M_t\right) \tag{4.19}$$

where M_t is the information set used by the options market.

The model supposes that the expectations $\sigma_{t,t+\tau}$ are functions of at most three parameters: the first is the short-term expectation α_t for the next time interval :

$$\alpha_t = \sigma_{t,t+1}. \tag{4.20}$$

The second parameter is the long-term expectation μ_t, given by assuming that the expectations converge for distant intervals,

$$\mu_t = \lim_{\tau \to \infty} \sigma_{t,t+\tau}. \tag{4.21}$$

Expectations are assumed to revert towards the time-dependent level μ_t as τ increases. The third parameter, ϕ, controls the rate of reversion towards μ_t and ϕ and is assumed to be same for all t. For practical purposes we suppose that reversion applies to variances rather than to standard deviations :

$$\sigma^2_{t,t+\tau} - \mu^2_t = \phi\left(\sigma^2_{t,t+\tau-1} - \mu^2_t\right), \tau > 1. \tag{4.22}$$

Hence the expectation for time interval $t+\tau$ depends upon α_t, μ_t, ϕ, and τ, thus :-

$$\sigma^2_{t,t+\tau} = \mu^2_t + \phi^{\tau-1}\left(\alpha^2_t - \mu^2_t\right), \tau > 0. \tag{4.23}$$

Market agents have mean-reverting expectations when $0 < \phi < 1$, whereas when $\phi = 0$ or $\phi = 1$, we get constant expectations as τ varies, consistent with the B-S (Black-Scholes) paradigm. This simple modeling, graph of $\sigma_{t,t+\tau}$ against τ, results in either monotonic increasing or decreasing expectation as τ increases or remains the same for τ. The expected volatility at time t for an interval of general length T, from time t to time $t+T$, is the square root of :-

$$\bar{\sigma}^2_T = \frac{1}{T}\sum_{\tau=}^{T}\sigma^2_{t,t+\tau} = \mu^2_t + \frac{1-\phi^T}{T(1-\quad)}(\alpha^2_t - \mu^2_t) \tag{4.24}$$

assuming that subsequent asset prices $\{P_{t,t+\tau}, \tau > 0\}$, follow a random walk. The numbers v_T, for $T = 1, 2, 3, ...$, time intervals, define the term structure of expected average volatility at time t. We can therefore estimate the time series $\{\alpha_t\}$ and $\{\mu_t\}$ and also the mean-reverting parameter ϕ, since (4.24) shows that v_T^2 is a linear function of α_t^2 and μ_t^2.

4.3.2 Estimation Method

The sought method seeks the best match between the model and a data set of implied volatilities. This method makes few assumptions about the time series properties of the $\{\alpha_t\}$ and $\{\mu_t\}$. The method seems to be very quick. The second method which Xu and Taylor discuss supposes that $\{\alpha_t\}$ and $\{\mu_t\}$ follow autoregressive processes and then uses the Kalman filter method to provide estimates of both the term structure and the parameters of the models assumed for $\{\alpha_t\}$ and $\{\mu_t\}$. We will not discuss this method in this research report.

4.3.3 A Regression method

In this method, the time t is supposed to count trading days. On day t, there will be implied volatility information for N_t expiry dates, supposed to be represented by a single number for each expiry date. Let $y_{j,t}$ denote the implied volatility for option expiry date j on day t and suppose the times to expiry are $T_{j,t}$, measured in calender days, with $T_{1,t} < T_{2,t} < ... < T_{N_t t}$. Forward implied variances $f_{j,t}$ can be calculated from the implied volatilities. At time t, the forward variance for the time interval from $t+T_{j-1,t}$ to $t+T_{j,t}$ is :-

$$f_{j,t} = \frac{T_{j,t} \cdot y_{j,t}^2 - T_{j-1,t} y_{j-1,t}^2}{T_{j,t} - T_{j-1,t}}. \tag{4.25}$$

This number is annualized and when $j = 1$, $T_{0,t} = 0$ in (4.25) above. Comparing (4.25) above with the expected value for the appropriate part of the term structure, the forward expected variance is given by :-

$$g_{j,t} = \frac{1}{T_{j,t} - T_{j-1,t}} \left(\sum_{\tau=T_{j-1,t}+1}^{T_{j,t}} \sigma^2_{C(t),C(t)+\tau} \right) \tag{4.26}$$

where $C(t)$ is the calender day count corresponding to the passage of t trading

days and τ is measured in calender days. From (4.23) in the above section, it can be seen that the forward expected variance is a linear combination of α_t^2 and μ_t^2 . The combination is :-

$$g_{j,t} = \mu_t^2 + x_{j,t}(\alpha_t^2 - \mu_t^2) \ , \tag{4.27}$$

with

$$x_{j,t} = \frac{\phi^{T_{j-1,t}} - \phi^{T_{j,t}}}{(1-\phi)(T_{j,t} - T_{j-1,t})} \tag{4.28}$$

assuming $\phi < 1$. If we let n denote the number of days for which there are implied volatilities, we need to find the estimates of :-

$$\phi, \alpha_1, \alpha_2, ..., \alpha_n, \mu_1, \mu_2, ..., \mu_n \ ,$$

resulting in small values for the differences,

$$e_{j,t} = f_{j,t} - g_{j,t}, \ 1 \leq j \leq N_t, \ 1 \leq t \leq n.$$

These estimates are given by minimizing sums of terms $e_{j,t}^2$ for various ϕ, followed by choosing ϕ to be the value giving the smallest sum across all times t. The estimation method can be summarized by three steps :-

- Step 1 : involves selecting a set of plausible values for ϕ, say $\phi_1, \phi_2, ..., \phi_m$.
- Step 2 : involves finding the best estimate $\alpha_{i,t}, \mu_{i,t}$, when $\phi = \phi_i, i = 1, 2, ..., m$. As $g_{j,t}$ is a linear function of $x_{j,t}$ (4.27), these estimates are given for period t by regressing $f_{j,s}$ on $x_{i,s}$, with $1 \leq j \leq N_s$ and $t - k \leq s \leq t + k$. From (4.27), the estimated intercept is $\mu_{i,t}^2$ and the sum of the estimated slope and the estimated intercept is $\alpha_{i,t}^2$. These estimates are obtained for $t = k + 1, ..., n - k$, and the sum of the squared regression errors calculated, summing over the three variables j, s, and t. Call the sum $S(\phi_i)$ when $\phi = \phi_i$.
- Step 3 : gives ϕ as the value that minimizes $S(\phi_i)$, and the time series of estimates $\{\alpha_t\}$ and $\{\mu_t\}$ as the regression estimates when $\phi = \widehat{\phi}$.

4.3.4 Data

In the data used by Xu and Taylor, several exclusion criteria were used to remove uninformative options records from the data base. Five of these criteria are listed and explained below :-

- i) options with time to expire less than 10 calendar days
- ii) options violating European boundary conditions,

$$c < Se^{-r_f T} - Xe^{-r_d T}, p < Xe^{-r_d T} - Se^{-r_f T},$$

- iii) options with premia less than or equal to 0.01 cents
- iv) options violating American boundary conditions,

$$C < S - X, P < X - S.$$

- v) options that are far in - or out -of- the money,

$$X < 0.8\ S \text{ or } X > 1.2\ S.$$

If one tries to reason out the above listed criteria one can conclude that they are justifiable. If we take these criteria one by one, you will find that :-

- i) this criterion was used in eliminating options with small times to maturity as the implied volatilities then behave erratically.

- ii) and iii) eliminate the options violating the boundary conditions for European and American options. Although this paper focuses on European style options, as our small OTC market *hardly* quotes American options, we know that American options could be exercised at any time up to expiration, and the holder entitled to the intrinsic value of the option at the point of exercising. Hence both boundary conditions must be satisfied, otherwise a riskless arbitrage could arise, that is : when an option price violates a rational pricing bound, there are good reasons for suspecting that trades could not be made at this price, resulting in a riskless arbitrage.

- iv) is used to exclude options for which the necessarily discrete market prices are particularly likely to distort calculations of implied volatility.

- v) is used to eliminate those options that are either deep in-the-money or deep out-of-the money as their implied volatilities are extremely sensitive to a small change in the options price, they could distort calculations of implied volatility, and the other factor is that they *hardly*

trade with much volume and thus are unrepresentative.

4.3.5 Computation of Implied Volatility and Results

Xu and Taylor used the American pricing model in calculating the implied volatilities. The calculations used an interval subdivision method, which always guarantees convergence to a unique solution. They decided to use closing prices of the nearest-the-money options; the nearest-the-money option on some day for a specific T is the option whose exercise price mininizes $/S - X/$. These nearest-the-money options where chosen for two reasons :-

- (1) Firstly, given the popular and widely reported "strike bias" or "smile effect" Alan Hicks (1995), etc., hence including out-of-the money and in-the-money options would introduce further noise into the term structure estimates. In theory, the smile effect can be a consequence of stochastic volatility, (Hull and White 1987).

- (2) Secondly, the approximation that the implied volatility of a rationally priced option will equal the mean expected volatility over the time to expiry is generally considered more satisfactory for an at-the-money

option than for all o her options (Stein, 1989).

Five conclusion• suggested by the results through the regression method. Firstly, the differenc.. oetween 15-day and long-term expectations is often several percent so the implied volatilities reveal a significant term structure. Secondly, the estimates of the 15-day and long-term expectations frequently crossover, so the slope of the term structure often changes. Crossovers occurs approximately at an average rate of once every two to three months. Thirdly, the long-term expected volatility varies significantly. Fourthly, as might be expected, the estimated 15-day volatility expectation is much more variable over time than the estimated long-term expectation. Finally, the implied volatility process may not have been stationary in the sense that the average level appears to have been higher in 1985 than in later years 1986 to 1989, although historic estimates of volatility are high also in 1985.

4.3.6 Conclusions

The regression method in particular, assume that expectations revert monotonically from a short-term value towards a long-term level as the horizon of the expectations increases. Further conclusions are that, first, there are significant term structure effects, because fifteen-day and long-term volatility expectations often differ by several percent, which causes implied volatilities to vary significantly across maturities. Secondly, the term structure sometimes slopes upwards, sometimes downwards, and its direction (up or down) frequently changes. The direction changes, on average approximately once every two or three months. Thirdly, there are significant variations in long-term volatility expectations, although these expectations change more slowly than both short term expectations and the spread between short and long-term expectations. Fourthly, the term structures of the pound, mark, Swiss franc and Yen at any moment in time have been very similar. Finally, there are non-stationary elements in the term structure in the sense that some of the parameters of the preferred autoregressive models changed during the period of five years investigated.

The volatility expectations provide insights into how the currency options market behaves. A constant volatility assumption is not made by the market. Volatility shocks are assumed to be transitory with an estimated half-life of approximately only one month. There is no evidence that the currency options market overreacts because this half-side is indistinguishable from the half-life for the mean reverting spread between short and long-term expectations.

The volatility term structure estimates summarize the market's beliefs about volatility for all future periods. These estimates are expected to be more informative than forecasts obtained from historic prices alone. The estimates can be used to enhance hedging strategies and to value options for all maturities T including those that are not traded at exchanges.

4.4 CHANGING TIME SCALE FOR SHORT-TERM FORECASTING IN FINANCIAL MARKETS

This paper was presented by A Mullër *et al.*, at the International Conference on "Financial Markets Dynamics and Forecasting" organized by 'Groupe Caisse des dēpôts' and held in Paris on September 2-4 1993. In the progression of the presentation, they used some of their previous papers published in 1990 and 1993.This paper is basically in tandem with a continuously updating and reoptimizing the forecasting model for financial markets which is presented for a time horizon going from a few hours to a few weeks. This model is based on the real-time collection and treatment of large amounts of FX quotes by market makers around the clock.

An analysis of the statistical behaviour of these time series leads to discussing the importance of choosing the appropriate time scale to optimize

forecasting models. In this paper, they introduce variable time scales in a general way and defined the new concept of intrinsic time. This particular time scale has an advantage of reflecting, more than the physical time scale does, the actual trading activity in terms of price variations. It is expanded during periods of high activity and contracted during periods of low activity and thus models the heteroskedasticity typical of these time series.

Using a different time scale means a forecast in two steps, first a forecast of the intrinsic time against physical time, then a forecast of the price against intrinsic time. The forecasting model consists, for both steps, of a linear combination of non-linear price-based indicators. The relative weights of these indicators are continuously re-evaluated through a modified linear regression technique on a moving sample of past prices. The sample size is related to the forecasting horizon. This algorithm has been running continuously on 56 different FX rates for three years and its performance, measured out-of-sample, is remarkably consistent. The results for a selected set of important FX rates and interest rates as well as some techniques to measure the forecasting performance, are presented and discussed in this paper. A Muller *et al.*, firstly look at some statistical properties of FX rates time series.

4.4.1 Some statistical properties of FX rates time series

A leptokurtic and non-stable price change distribution

The previous paper published by U.A. Müller *et al.*, (1990) reported a set of empirical results that had to do with the shape of the price change distribution, and how this distribution relate to the scaling law, discussed in the following section. Three distributions for the *USD/DEM* and *USD/CHF* rates for 30 minutes interval, 1 day interval and 1 week interval over a period of six years from 5 May 1986 and 5 May 1992 were plotted. The cumulative frequency of price changes on the scale of the cumulative Gaussian probability function, were plotted against units of the mean absolute value of each time interval. The overall analysis shows that the shorter the horizon, the more leptokurtic is the distribution , and only for weekly intervals does the distribution look almost a straight line, which is the form that the normal distribution would have on such a scale.

This implies that the kurtosis for the weekly price changes is approximately 0 and the distribution of half-hourly changes, has a general behaviour, i.e. a

decreasing leptokurticity with increasing intervals. McFarland *et al.*, (1982) and Boothe and Glassman (1987) suggest that these distributions are formed by reactions to different information flows; and other authors suggest that this instability can be explained by a heteroskedastic process, which will be dealt with in chapter 5.

A scaling law for absolute price changes

U. A. Müller *et al.*, (1990) have empirically found a law which relates the interval Δt directly to the average or mean absolute price changes. The mean absolute change of the logarithmic middle price over a time interval is related to the size of this interval, Δt :-

$$\overline{|x(t) - x(t - \Delta t)|} = \overline{|\Delta x|} = \left(\frac{\Delta t}{\Delta T}\right)^D \tag{4.29}$$

where the bar over $|\Delta x|$ indicates the average over a long sample interval t,

- ΔT is an empirical time constant depending on the FX rate,
- D is normally called the direction quality and

$$x(t) \equiv \frac{(\log p_{ask} + \log p_{bid})}{2}. \qquad (4.30)$$

Taking an average of the logarithms instead of the logarithm of the average has the advantage of behaving symmetrically when the price is inverted and unitless e.g. 1 ZAR expressed in USD instead of 1 USD expressed in ZAR. Using U.A. Müller *et al.*, (1990), the above equation can be expressed as :-

$$\overline{|\Delta x|} \equiv c\Delta t^{1/E}. \qquad (4.31)$$

The parameter $1/E$ is called the drift exponent, and c is a constant depending on the FX rate. IF Δt is expressed in hours, c is in the order of magnitude 10^{-3} for the main FX rates against the USD. The drift exponent $1/E$ is about 0.6, whereas the pure Gaussian random walk model would imply $1/E \equiv 0.5$. These distinct differences for the exponent can be only explained by varying distribution forms for the difference time intervals. Most authors use the term volatility for this quantity, the mean absolute price changes, which is our main scaling parameter. U.A. Müller also found that if one plots the

intervals Δt and the volatilities $\overline{|\Delta x|}$ on a logarithmic scale, a straight line is produced, with line fitting being done by linear regression. The $\overline{|\Delta x|}$ values for different intervals Δt are not totally independent, as the larger intervals are aggregates of smaller intervals.

Testing the scaling law using the USD/JPY and GBP/USD rates, U.A. Muller *et al.*, found that the correlation coefficients between the logarithms of ΔT and $\overline{|\Delta x|}$ exceed 0.999 and the standard errors of the exponents $1/E$ are less than 1.0%, reflecting that the volatility, $\overline{|\Delta x|}$ increased on average over the last fifteen years. The results indicate a very general scaling law that applies to different currencies as well as commodities such as gold and silver. This phenomenological law becomes more important in showing that the distributions of Δx are unstable and the scaling law cannot be explained as a trivial consequence of a stable random process. The evidence for unstable distributions is given by the scaling laws for $\left(\overline{|\Delta x|^2}\right)^{1/2}$ and the interquartile ranges of the distributions. U. A. Muller *et al* ., found lower exponents $1/E \approx 0.52$ for $\left(\overline{|\Delta x|^2}\right)^{1/2}$ and higher exponents $1/E \approx 0.7$ for the interquartile ranges and these can only be explained by varying distribution forms for different intervals.

Seasonal and Conditional Heteroskedasticity

The behaviour of a time series is called *seasonal* if it shows a periodic structure in addition to less regular movements. U.A. Muller *et al* (1990) demonstrated both daily and weekly seasonal heteroskedasticity, a seasonal behaviour of FX price volatility rather than of FX prices themselves. The first analysis of heteroskedasticity was an autocorrelation study of hourly changes i.e. intraday and intraweekly sampling, Δx of the logarithmic price, their absolute values, and their squares over the whole sample. Autocorrelation coefficients were found significantly higher for time lags that are integer multiples of the seasonal period than for other lags. The interval analysis shows that mean absolute price changes are much higher over working days than over Saturdays and Sundays when the market actors are hardly present. The corresponding intraday analysis shows that the mean absolute hourly price changes have distinct seasonal patterns, which are clearly correlated to the changing presence of main market places of the worldwide FX market.

The lowest market presence outside the weekend is during the lunch hour

in Japan i.e., noon break in Japan, night in America and Europe; it is at this time when the minimum of mean absolute hourly price changes is found. Conversely, the maximum is found during the overlap of business hours of the two main markets, Europe and America. Further evidence exist of a correlation between market presence and volatility, although it cannot be observed directly. There is also substantial evidence which has been highlighted in many empirical studies in favour of a positive correlation between price changes and volume in financial markets.

Another interesting property which has been discussed in asset p[illegible] conditional heteroskedasticity, which will be discussed and tested in chapter 5. This property asserts that the volatility of the price changes is clustered in periods of high volatility and periods of low volatility. The most popular models for the clustering of volatility are the ARCH, first proposed by Engle (1982) and GARCH, generalized version proposed by Bollerslev (1986). Many empirical works also find that conditional heteroskedastic effects tend to weaken with *less* frequently sampled data.

Optimal time scales for forecasting FX rates

As mentioned in the above introduction, forecasting models in economics that are based on time series analysis concentrate on discovering a process that generates equally spaced values, and little attention is given to the analysis of the underlying time scale. The general assumption is an equally-spaced and homogeneous time series with elements separated by constant intervals of physical time. This assumption is, however, wrong, for in most actual "daily" time series Saturdays and Sundays are skipped together with the business holidays. In this paper U.A. Müller *et al.*, call this underlying time scale as *business time scale*. This time scale can also be viewed as a way to introduce some of the "fundamentals" or some economic factors that are missing in usual time series analysis, which certainly have influence on price movements.

An intraday business time scale : the ϑ-scale

U.A. Müller *et al.*, attribute the observed seasonal heteroskedasticity to the changing presence of traders on the FX markets and they introduced a new

time scale, the ϑ-scale. In this scale, price changes have a non-seasonal volatility and they called its derivative against physical time the *activity*, $a(t)$. From the name itself this variable measures, for each time t, the active presence of traders on the FX market, geographically centered in East Asia, Europe and America, through the price changes they induce. The activity model used to construct the ϑ-scale as the integral of worldwide activity is :-

$$\vartheta(t) \equiv a_0(t - t_0) + \sum_{k=1}^{3} \int_{t_0}^{t} a_k(t')dt' \tag{4.32}$$

where a_0 is a basic activity and a_k is the activity of a particular market out of three major markets. The time series $\vartheta(t)$ is a strictly monotonic function of physical time t. The activity variable is normalized in such a way that ϑ - time can be measured in the same units as physical time e.g., hours, days, etc.

4.4.2 Discussion of the forecasting results

This section addresses the technicality in forecasting foreign exchange rates, in the sense that the type of data and the accuracy of forecasting are the

key items. Because of the statistical nature of FX rates, forecasting accuracy measured from the same data used for optimizing the model, has proved to have little significance. Dividing the data into portions for forecasting and reoptimizing, seems to be the better way to address this problem.

Appropriate measures of forecast accuracy

Some of the essentials for forecast accuracy were mentioned in the introduction of this paper by Makrikdakis *et al.*,(1983). Most standard measures rely on the Mean Square Error (MSE) and the Mean Absolute Error (MAE) for each time horizon. These errors are then compared to the similar ones produced by a forecasting model serving as a benchmark. These considerations led to U.A. Müller *et al.*, to formulate non-parametric methods of analyzing forecast accuracy. These are generally "distribution-free" measures in that they do not assume a normally distributed population, and so can be used when this assumption is not valid.

One measure which has this desirable property is the percentage of forecasts in the right direction, because forecast in direction of any trend is more important to a trader than its magnitude. U.A. Müller *et al.*, refer to this

measure the *direction quality*. Hence a D, direction quality, significantly higher than 50% means that the forecasting model is better than the random walk; significance level as the 95% confidence level of the random walk:

$$\sigma_D \approx \frac{1.96}{2\sqrt{n}}$$

where n is the number of tests. The factor 2 comes from the assumption of an equal probability of having positive and negative signals.

4.4.3 Conclusions

This section has shown the possibility and the feasibility of specifying forecasting models for every short-term horizons. It also summarizes the most important characteristics of a forecasting model. Here are some of these characteristics as noted by U.A. Müller *et al.*, : The model must be a :-

- univariate time analysis type of model but based on intraday non-homogeneous data

- variable time scales to capture both the seasonal heteroskedasticity ($\vartheta - scale$) and the autoregressive conditional heteroskedasticity ($\tau - scale$)
- multiple linear regression with two modifications to avoid inst bilities and to correct for the leptokurtic behaviour of the price changes
- continuous optimization of the model coefficients in a finite size, forecasting horizon-dependent sample.

The seasonal and conditional heteroskedasticity behaviour of price changes has been modeled using Engle and Bollerslev models, which will be tested using the historical data.

4.5 Volatility Prediction : Comparison of the Stochastic Volatility, GARCH(1,1), EGARCH (1,1) models

This article (Heynen and Kat 1994) investigates whether there are significant differences in the ability of random walk, GARCH(1,1) EGARCH(1,1), and stochastic volatility models to predict the realized volatility of seven different stock indexes and five different currencies over horizons ranging from two to hundred days, using close-to-close data over the period 1980-1992. The period 1988-1992 is used for out-of-sample evaluation purposes. During this period the models are (re)estimated over the previous eight years of data at the beginning of each month, starting in January 1988. As I stated before I will use these models as well to try and figure out the best model for the dollar/rand spot rates over a period of six years. In the following paragraphs, we will look at these models.

4.5.1 Asset return volatility specifications : Model specifications

Under the *random walk* assumption, as specified by the following equations, return volatility is constant:

$$R_t = \mu + \sigma_t \xi_t \tag{4.33}$$

$$\sigma_t = \overline{\sigma} \tag{4.34}$$

where R_t is the continuously compounded total return, i.e., including dividends, on the relevant asset over a period t, $\xi_t \overset{i.i.d}{\sim} N(0,1)$, and μ is the conditional mean, which for simplicity is assumed to be constant. The conditional variance σ_t^2 (the variance conditional on knowing all returns up to period t) is constant and equal to the unconditional variance $\overline{\sigma}^2$, which can be estimated using the classical estimator given by :-

$$\sigma^2 = \frac{1}{n-1} \sum_{t=1}^{n} (R_t - m)^2 \tag{4.35}$$

where the sample mean

$$m = 1/n \sum(R_t),$$

and n is the total number of observations.

He[illegible]nd Kat argue that observations are independent and identically di[illegible]ed, so predicted return variance over a period of length T can be calculated simply as $\sigma^2 T$, simply called the " *random walk predictor*".

In GARCH models, the variance of the conditional distribution of one-day returns, which is typically assumed to be normal, changes as a deterministic function of past *residual* returns. For a GARCH(1,1) specification, asset return is modeled by the equation (4.33) above, and return volatility is specified by :-

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \qquad (4.36)$$

where $\varepsilon_t = \sigma_t \xi_t$.

To ensure a well-defined process, all parameters are restricted to be non-

negative, and $\beta_1 + \beta_2$ must be smaller than 1. Although the unconditional variance of the process is constant, the conditional variance is time-dependent. Heynen and Kat argue that, more specifically, there is a tendency for extreme returns to be followed by other extreme returns, but of unpredictable sign, because only the squared values of past disturbances enter the model. As a result, squared returns and absolute returns are positively autocorrelated, even though returns themselves are uncorrelated. The conditional error distribution is normal, while the unconditional error distribution is somewhat leptokurtic ("fat-tailed"). Hence volatility forecasts one period ahead can be calculated directly from the explicit specification of the model's conditional variance. More-than-one-step-ahead forecasts can be generated by repeated substitution. The time t forecast of the variance of return over the next T days, expressed on a daily basis, is given by the following :

$$\sigma_{t,T}^2 = \frac{1}{T}\sum_{k=1}^{T} E_t\left[\sigma_{t+k}^2\right] = \overline{\sigma}^2 + \left(\sigma_{t+1}^2 - \overline{\sigma}^2\right)\frac{1-\gamma^T}{T(1-\gamma)} \qquad (4.37)$$

where

$$\gamma = \beta_1 + \beta_2,$$

and $\bar{\sigma}^2$ is the model's unconditional variance, which can be shown to be equal to

$$\frac{\beta_0}{1-\gamma},$$

γ can be interpreted as a measure of the speed by which shocks to volatility decay.

Heynen and Kat argue that, when γ approaches 1 from below, the effect of past volatility shocks increases, which may cause volatility to deviate from its long-term mean for a very large number of periods into the future. The maximum likelihood estimates of the GARCH model's parameters can be obtained by numerical maximization of the log-likelihood function. A thorough look at equation (4.37) above, clearly shows a mean-reversion in variance, with the variance forecast tending toward the model's unconditional variance as the length of the forecast horizon increases.

An alternative to GARCH (1,1) is EGARCH (1,1) model, where return is

also modeled by equation (4.33), and return volatility is specified by :

$$\ln \sigma_t^2 = \beta_0 + \beta_1 \ln \sigma_{t-1}^2 + \beta_2 \xi_{t-1} + \beta_3 \left(|\xi_{t-1}| - \sqrt{\frac{2}{\pi}} \right) \qquad (4.38)$$

where $\beta_0 ... \beta_3$ are time-independent parameters.

Unlike the GARCH model, there are no restrictions on the model's parameters necessary to ensure non-negativity of the conditional variance. Instead of making the conditional variance a positive linear combination of positive random variables, the EGARCH model ensures non-negativity by working with the logarithm of the conditional variance. Maximum likelihood estimates of the EGARCH (1,1) model's parameters can be obtained by the same way as in the GARCH (1,1) model.

As with the GARCH (1,1) model, volatility forecasts one day ahead can be calculated directly from the explicit specification of the model's conditional variance. Expressed on a variance per day basis, more-than-one-day volatility forecasts can be obtained from the following ("EGARCH predictor) :

$$\sigma_{t,T}^2 = \frac{\overline{\sigma}^2}{TC} \sum_{Tk=1} \sigma_{t+1}^{2\beta_1^{k-1}} \times \exp \left(\frac{-\left(\beta_0 - \beta_3 \sqrt{\frac{2}{\pi}}\right) \beta_1^{k-1}}{1 - \beta_1^2} \right) \qquad (4.39)$$

$$\times \exp\left(-\frac{\frac{1}{2}\left(\beta_2^2+\beta_3^2\right)\beta_1^{2(k-1)}}{1-\beta_1^2}\right) \times C_k\left(\beta_1,\beta_2,\beta_3\right)$$

where

$$\overline{\sigma}^2 = \exp\left(\frac{\left(\beta_0-\beta_3\sqrt{\frac{2}{\pi}}\right)}{1-\beta_1}+\frac{1}{2}\frac{\left(\beta_2^2+\beta_3^2\right)}{1-\beta_1^2}\right) \times C\left(\beta_1,\beta_2,\beta_3\right)$$

$$C\left(\beta_1,\beta_2,\beta_3\right) = \prod_{m=0}^{\infty} [F_m\left(\beta_1,\beta_2,\beta_3\right) + F_m\left(\beta_1,\beta_2,\beta_3\right)]$$

$$F_m\left(\beta_1,\beta_2,\beta_3\right) = N\left[\beta_1^m\left(\beta_3+\beta_2\right)\right] \times \exp\left[\beta_1^{2m}\beta_2\beta_3\right]$$

$C_1 = 1$ and

$$C_k = \prod_{m=0}^{k-2} [F_m\left(\beta_1,\beta_2,\beta_3\right) + F_m\left(\beta_1,-\beta_2,\beta_3\right)]$$

for k $\geq$ 2. C and F are the normal distribution functions and $N(x)$ denotes

the univariate cumulative standard normal distribution with upper limit of integration x.

As in GARCH (1,1) case, equation (4.39) exhibits mean-reversion, with forecasted variance tending toward the model's unconditional variance as the length of the forecast horizon increases and toward the next-day variance as the horizon shortens.

One of the popular volatility models in the option pricing literature is the continuous time stochastic volatility model of Hull and White (1987). In order to make paremeter estimates, discrete time observations have to be used, so it is necessary to specify a discrete time version of the stochastic volatility model. A natural candidate of this model is given by Autoregressive(1), AR(1) specification :

$$\ln \sigma_t^2 = \beta_0 + \beta_1 \ln \sigma_{t-1}^2 + \eta_t \qquad (4.40)$$

where η_t i.i.d. $N(0, \sigma_\eta^2)$.

Expressed on a daily basis, more-than-one-day volatility forecast from the

discrete time stochastic volatility model, is given by the following equation ("stochastic volatility predictor") :

$$\sigma^2_{t,T} = \frac{\overline{\sigma}^2}{T} \sum_{k=1}^{T} \left(\sigma_t^2\right)^{\beta_1^k} \exp\left(\frac{-\beta_0 \beta_1^k}{1-\beta_1}\right) \times \exp\left(-\frac{1}{2} \times \frac{\sigma_\eta^2 \beta_1^{2k}}{1-\beta_1^2}\right) \qquad (4.41)$$

where

$$\overline{\sigma}^2 = \exp\left(\frac{\beta_0}{1-\beta_1}\right) \exp\left(\frac{1}{2} \times \frac{\sigma_\eta^2}{1-\beta_1^2}\right).$$

Again as you can see above, the long-term volatility forecast tends toward the unconditional volatility $\overline{\sigma}$, while the short-term volatility prediction tends toward the short-term volatility σ_t. These models, together with their predictors will be used in forecasting

Chapter 5

EMPIRICAL EVIDENCE AND MARKET PRACTICE

5.1 Statistical Analysis

5.1.1

Scaling Law

As mentioned in the previous section, U. A Müller *et al.*, (1990) used the absolute price changes to define the scaling law which relates the interval Δt directly to these absolute price changes. Using my daily data of USD/ZAR

spot rates $|\Delta x|$ is approximately 0.47, using an approximate drift exponent of $c = 10^{-3}$, and unfortunately I could not determine the activity because of the type of my data. Some of the main reasons are the following :

- Both the scaling law and the activity function in U. A. Müller *et al.*, paper were tested using intra-hour and intra-day data, or *tick-by-tick* sample measurement of the spot rates for a period of three years. This data does not only affect one major market part, but it covers all the major markets in a 24-hour-a-day market basis.

- The daily data I managed to get only caters for a small, sometimes called an *emerging market* of South Africa, where 24-hour trading is impossible. Although it does overlap with major market centers, it is only for a limited number of hours, i.e., 08h00 to 17h00 South African time. This period is very limited since in two of the major markets the overlap is during mid-night or early morning. At least the European market opens when we have only completed two hours of trading.

If one looks and compares the major market centers, one will find that the intra-week analysis shows that absolute price changes are much higher over

the working days than over Saturdays and Sundays, when the market actors are hardly present. The empirical scaling law evidenced by U. A. Müller *et al.*, shows that when applied to the ith hourly subsample instead of the full sample, it mathematically transforms into :

$$\Delta\vartheta_i = \left(\frac{|\Delta x|}{c^*}\right)^E \tag{5.1}$$

where $\Delta t = 1$ is not constant and is replaced by $\Delta\vartheta_i$ on the time scale ϑ. The constant c^* is essentially the c of the scaling law but can slightly differ because of normalization condition. The activity of the ith hourly subsample is given by :

$$a_{eta,i} \equiv \frac{1}{\Delta t}\left(\frac{\Delta x_i}{c^*}\right)^E \tag{5.2}$$

where $\Delta t = 1$ hour.

This is the volatility-based activity definition used in the following analysis. The strong relation between price change-based activity and market presence leads to the explanation of the activity as the sum of geographical compo-

nents. Although the FX market is worldwide, the actual transactions are executed and entered in the bookkeeping of particular market centres, the main ones being London, New York and Tokyo. These centres contribute to the total market activity during different opening hours that sometimes overlap. The model activity of a particular geographical component k is called $a_k(t)$:

$$a(t) \equiv \sum_{k=1}^{3} a_k(t). \tag{5.3}$$

The total activity model should model the intra-weekly pattern of the statistical activity $a_{stat,i}$ as closely as possible. Unlike $a_{stat,i}$, which has a relatively complex behaviour, the components of $a_k(t)$ have a simple form, in line with knowing opening and closing hours and activity peaks of the market centres. The activity does not completely go to zero when the market is closed because it is defined in terms of price changes. The activity during the closing hours is modeled to stay on a small constant base level $a_{0,k}$. During the opening hours, a much stronger, varying, positive activity $a_{1,k}$ adds to the base level :-

$$a(t) \equiv \sum_{k=1}^{3} [a_{0,k} + a_{1,k}(t)] \equiv a_0 + \sum_{k=1}^{3} a_{1,k}(t) > 0. \tag{5.4}$$

U. A. Müller *et al.*, (1990) considered a statistical week starting from $t = 0$ on Monday 00:00 to $t = 168$ hours on Sunday 24:00 GMT. In order to define the opening and closing conditions of the markets in a convenient form, an *auxiliary* time scale, T_k was introduced; which essentially is GMT time; resulting into market-dependent shifts of plus or minus 24 hours :-

$$T_k = [(t + \Delta k) \bmod(24\ hours)] - \Delta t_k \tag{5.5}$$

where Δt_k has the value of 9 hours for East Asia; 0 for Europe (2 for South Africa); and 5 hours for America. The weekend condition also depends on the market :

$$t + \Delta k \bmod(168\ hours) \geq 120\ hours \tag{5.6}$$

The model for an individual market component can be formalized as :-

$$a_{1,k}(t) \equiv \left\{ \begin{array}{c} 0 \text{ if } T_k < 0_k \text{ or } T_k > c_k \text{ or weekend} \\ a_{open,k}(t) \text{ if } 0_k < T_k < c_k \text{ and not weekend} \end{array} \right\} \tag{5.7}$$

where 0_k and c_k are the opening and closing hours respectively.

5.2 Proprietory Model

As indicated in the beginning of the paper, I have gathered some historical USD/ZAR spot rates for a period of 4,5 years up to the end of March 1996. I have also organized some historical implied volatilities for at-the-money options, from the option's market OTC brokers, approximately for the same period. As mentioned earlier, I managed to get the raw data from one of the biggest banks of South Africa. I manually transferred the data to the spreadsheet, removed all non-trading days like weekends, holidays and I was left purely with the data of trading days. The reason of removing these days was that only a few trades went through in those days, as can be evidenced from the volume of trades for each particular day.

I initially calculated the historical volatilities using the first four years and after that I made my data to increment by two weeks. I then calculated the historical volatilities after each increment for the sought periods. I had also organised the historical implieds from the SAIMB, as mentioned earlier. SAIMB brokers use daily sheets to record the traded volatility levels, which

they file for later use. I was fortunate to negotiate a good deal with them, and I had the opportunity to play around with their big historical implied volatility files. Unfortunately I had to manually input them to the spreadsheet as well, leaving out non-trading days. I then calculated the moving average of the implied volatilities in line with the increments that I have made to my historical data. As can be witnessed from the graphs below, this method gave me a better way of comparing the two data.

The main intention was to show that there is an *edge* between the historical volatilities and the historical implied volatilities, hence one can be able to play the 'historicals' against the 'implieds' and generate some profit. This implies that knowing where the real volatility is, one can be able to shop around the market and buy at the volatility close to the 'historicals' and sell the same option back to the market at the volatility implied by the market. Below, there is a table showing the calculated historical volatilities over 2 week increments of the 4 year data. These volatility estimates were calculated using the general formula of calculating the historical volatility, mention by Banks (1993) and Hull (1993). Log relatives were calculated from successive rates and standard deviation of these log relatives was calculated to

get the required volatility estimates. For the specific periods, like 1 month, 2 months, etc., moving average was used and the estimates were annualised. For each 2 week increment in the data the whole exercise was repeated.

TABLE I : CALCULATED HIST. VOL. IN PERCENTA(.:

	4yr	4yr+1M	4yr+1.5M	4yr+2M	4yr+2.5M
0.5 M	3.38	3.34	3.33	3.34	3.33
1 M	3.81	3.77	3.75	3.77	3.80
2 M	4.40	4.36	4.33	4.33	4.33
3 M	5.00	4.96	4.94	4.92	4.92
4 M	5.23	5.34	5.32	5.29	5.28
5 M	5.21	5.36	5.47	5.45	5.43
6 M	5.38	5.37	5.54	5.51	5.49
7 M	5.60	5.60	5.59	5.56	5.52
8 M	6.11	6.08	6.07	6.04	6.00
9 M	6.51	6.50	6.48	6.44	6.40
10 M	6.90	6.87	6.86	6.82	6.78
11 M	7.13	7.16	7.15	7.10	7.05
12 M	7.32	7.42	7.44	7.40	7.35
18 M	7.22	7.61	7.72	7.69	7.64
24 M	7.18	7.48	7.78	7.78	7.75

The historical implied volatilities, from roughly the same period, are quoted in special periods in the market screens, as will be seen in the following table.

TABLE II HIST. IMPLIED VOLS IN PERCENTAGE

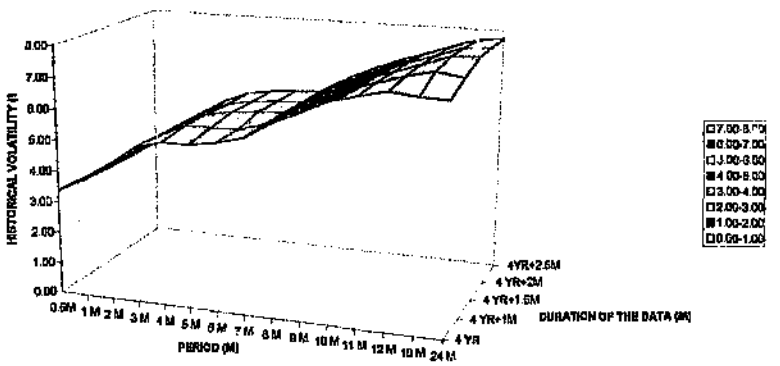
	1 M	2 M	3 M	6 M	9 M	12 M
1 YR	2.87	3.00	3.22	3.42	3.69	3.70
2 YR	4.88	5.20	6.34	6.25	6.71	6.82
2 YR +1 M	4.79	5.32	6.47	6.44	6.97	7.11
2 YR +2 M	5.14	5.49	6.65	6.67	7.25	7.45
2 YR +3 M	5.27	4.64	6.82	6.88	7.49	7.72
2YR +4 M	5.43	5.83	7.03	7.13	7.80	8.06
2YR +5 M	5.80	6.18	7.38	7.47	8.15	8.43

From the tables above, it is clearly seen that as the data is made to increase by two weeks, no significant change in the historical volatility estimates, in each of the respective periods. Although the 2 weeks periods used to increase the data include the period when the Rand against the Dollar exchange rate took a sliding dive, no significant effect shown in the historical volatility estimate. I believe if I was using an hourly data, this period would have made a bias in my estimations, in the sense that it would have resulted in higher

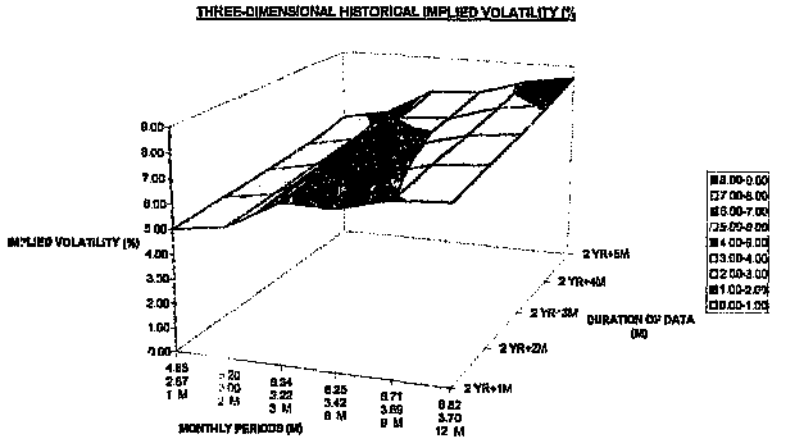
historical volatility estimates, in line with the way the Rand/Dollar exchange rate crashed.

Below there are charts representing the historical volatility estimates and the historical implied volatilities from the OTC option brokers. Although the underlying data does not perfectly match with the one that I used to calculate the historical actual volatilities, the two charts are perfectly correlated, and that they are a parallel replication of each other. This implies that the movement of the underlying spot rates in either direction, is evidenced in both the actual and implied volatilities, although the implied volatilities are approximately higher than the historical actual volatilities.

THREE-DIMENSIONAL HISTORICAL ACTUAL VOLATILITY GRAPH



The three-dimensional historical implied volatility graph is also given below :



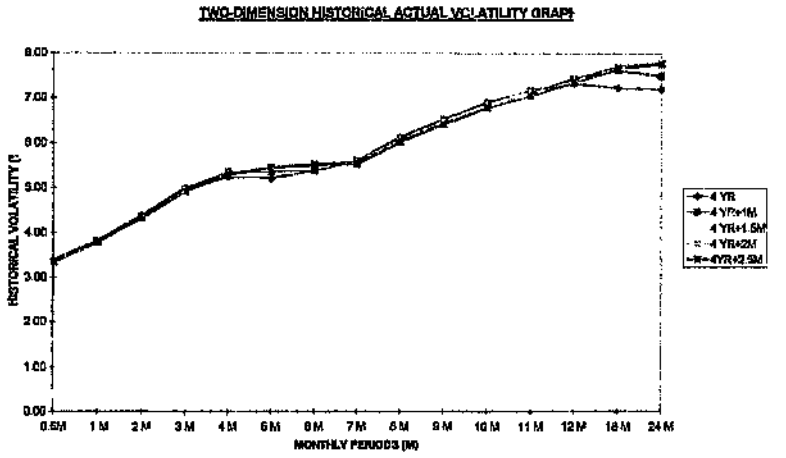
This discrepancy can be justified by the uncertainty the traders are having when making a volatility price, not knowing where the underlying spot rate will eventually settle. Hence they inflate their volatility prices by means of a margin, hence the inflation in the premium, so that they can have more cash to run the written option(s).

As highlighted above a dealer can sell at the 'implieds' and buy at the 'historicals', thereby making a profit in the difference of the two volatility rates or *spread.* Although buying low and selling high generates some profits, but

it is not the professional way of trading options, in fact that method is useful in the cash market, i.e., spot and the forward market, since there are many strategies that a trader can do to generate profit, without buying or selling back the same option. Selling or buying an option at a good volatility price, for a good options's trader, does not necessary imply buying or selling the same option; probably when he runs his position, he might find that there is no need or he can do another transaction totally different to the first one, as long as his position is immune to losses to a certain comfortable level.
The fore-gone discussion bears testimony to how important volatility is in FX option's market; infact, option traders themselves are sometimes called the *volatility traders*, since volatility is the key element in quoting and running the options book. Traders either *buy volatility* from or *sell volatility* to the market.

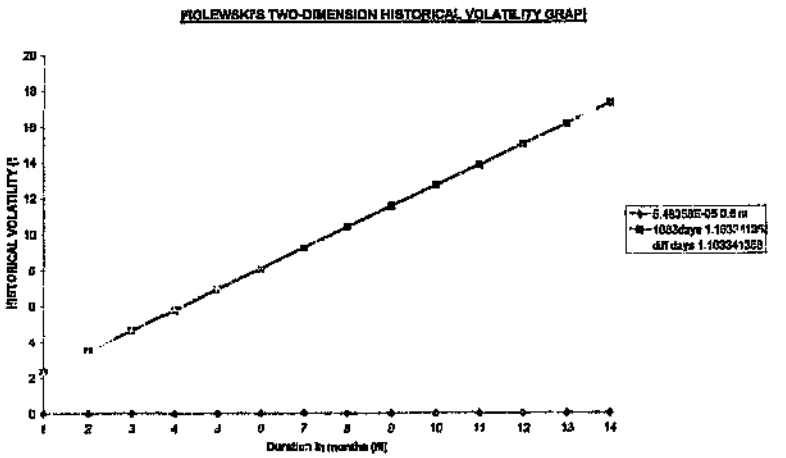
The two graphs can also be used to detect the *trend* and how closely the implieds follow the historicals. This implies that the movement of the spot rates in either direction has an impact in the movement of the implieds. Hence 'historicals' against the 'implieds' can be used as a forecasting measure of the movement of spot rates. This phenomenon is justified in the

following graphs :



The two-dimension historical implied volatility graph has the same pattern as the above, as can be seen in the three-dimensional cases. Some of the models discussed in the earlier chapters for calculating the implied volatility can also be used in the edge model, without having to get in the trouble of collecting the historical implieds from the brokers. The only problem is that since the FX option's market is 100% OTC unlike the stock or index and futures markets, it is difficult to find the option prices in the formal exchange, whereby one can just input the values in the formulas and compare them with

the historicals. For a serious trader, these models can even be more helpful when he tries to find out what volatility price can he get in the market for an option in the period of his interest. Figlewski's (1990) historical volatility model gave very strange results, implying that the volatility curve is linear, as it can be seen in the next graph. This is definitely not the case, as seen in the two different charts for both the historical implieds and the actual historicals discussed above.



5.3 Empirical tests

The main theme of the research is to investigate the better volatility forecasting model and the most important issue in volatility forecasting is whether the forecast should be based on historical price data, implied volatility or some combination of the two. For a broader investigation, I gathered both the historical spot rates, to calculate the historical volatility, and the historical implied volatilities. I needed this data for a good comparison between the volatility estimates from the models to be used and both the historical actual and implied volatilities.

Although there are many models that can be used, I decided to use random walk, stochastic volatility, ARCH, GARCH (1,1) and EGARCH (1,1) models. As indicated in the sections above, usability of all these models depends on calculating the respective important parameters. As I indicated in the text, I made use of the log-likelihood maximum estimation. Given the parameters one can be able to forecast to whatever period, by dividing the historicals into respective intervals. I have written a simple program (which is given in

the appendix) which calculates these parameters together with other statistical functions, using an econometric program known as SHAZAM, version 7.0 released in October 1993. Some of the other statistical parameters that it can generate are the following :

- standard error
- T- ratio
- partial p-value correlation
- standardized coefficients
- elasticity at means
- log of the likelihood function
- variance of the estimate-sigma (σ^2)
- standard error of the estimate-sigma (σ)
- sum of the squared errors (SSE)

and mean of dependent variable.

I copied the historical spot rates data from the spreadsheet and down-loaded

it to SHAZAM. I applied the simple program to the data and calculated the required parameters for all the respective models. I then down-loaded the output of the program back to the spreadsheet for analysis. Using the formulas of the different models, I then calculated the volatility estimates, as discussed in the above section (4.5).

From a practical point of view, the *Random walk model* can be considered a benchmark against which to compare the more sophisticated models, which do take account of changing return volatility. Some of the values of the respective parameters are given in the following table :

TABLE III : Parameter estimates for Different Models

	Random Walk	ARCH	GARCH	EGARCH	Stoch. Vol.
μ	3.3486				
	(0.01024)				
	(0.000)				
α		0.10076			
		(0.0002184)			
		(0.000)			
β_0			0.073239	-0.10462	-0.10645
			(0.0005985)	(0.03016)	(0.03020)
			(0.000)	(0.001)	(0.000)
β_1			0.034246	0.96713	0.96686
			(0.02205)	(0.00775)	(0.007763)
			(0.121)	(0.000)	(0.000)
β_2			0.3207	-0.042577	
			(0.03531)	(0.01690)	
			(0.000)	(0.012)	

β_3				0.00040649	
				(0.02831)	
				(0.089)	
L	-361.043	-361.0434	-410.675	-909.647	-906.471
AIC	724.086	724.0868	825.35	1818.942	1823.294

These parameters are estimated using the whole sample data. The AIC row indicates the Akaike Information Criterion, which gives an indication of the goodness-of-fit of the model estimated, with the lowest value indicating the best fitting model (Akaike 1973). The AIC is given by the formula.:

$$-2\ln(\text{max. likelihood}) + 2(\text{number of parameters}).$$

The L row represents the log-likelihood values. The values in the middle parentheses in each estimated parameter, represents the estimated standard errors; while the numbers in the bottom parentheses represents the p-values, which shows the significance of each parameter. β_1 in the GARCH model

and β_3 in the EGARCH model are insignificant in the models, hence the AIC of the EGARCH model is so enormous.

The similarity of the GARCH (1,1) and EGARCH (1,1) models obviously results from the absence of an asymmetrical relation between return volatility, which has been reported only in equity markets. The differences in ranking of goodness-of-fit of the various models are especially pronounced for the stochastic volatility model. This is clearly reflected in the size of the value for the parameter $\sigma^2\eta$ of the stochastic volatility model, indicating that the deterministic part of stochastic volatility model, which depends on the size of the parameters β_0 and β_1, contributes relatively more to the goodness-of-fit than its stochastic part, which depends only on the parameter $\sigma^2\eta$. The next table lists the value of the annualized unconditional volatility calculated for every model specification using the estimated parameters. From the table it can be seen that the unconditional volatility estimates vary across the different models.

TABLE IV : Annualised Percentage unconditional volatility for different model specifications, with Standard Errors

	Random walk	ARCH	GARCH	EGARCH	Stch. Vol
Vols	5.44	5.12	8.46	9.01	9.03
Errors	0.1139468	0.00000477	0.0275394	0.3124586	0.313712

This variation must be attributed to the fact that different return volatility models tend to take account of different characteristics of return volatility behaviour.

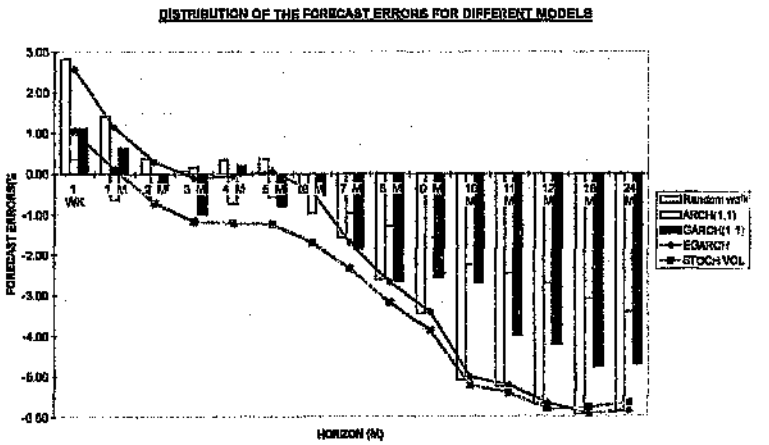
5.3.1 Volatility Forecast Evaluation

For a given forecast horizon of T days, forecasts are compared to the realized volatilities over the same subperiods, i.e., 1 week, 1month, 2months, etc.

TABLE V : Forecasted Volatility for Different Models

Horizon	Random Walk	ARCH	GARCH	EGARCH	Stoch. Vol.
1 wk	6.15	3.67	4.44	5.90	4.37
1 M	5.21	3.16	4.42	4.94	3.91
2 M	4.70	3.83	3.95	4.63	3.62
3 M	4.98	5.07	3.93	4.82	3.74
4 M	5.60	4.54	5.47	5.2	4.06
5 M	5.80	4.86	4.63	5.48	4.18
6 M	5.03	4.53	4.97	8.01	3.79
7 M	3.94	4.55	3.65	5.99	3.19
8 M	3.39	4.70	3.33	5.19	2.81
9 M	2.95	4.82	3.82	4.65	2.52
10 M	1.68	4.54	4.08	2.67	1.55
11 M	1.80	4.59	3.04	2.80	1.64
12 M	1.65	4.62	3.13	2.58	1.54
18 M	1.69	4.55	2.87	2.61	1.85
24 M	1.89	4.33	3.03	2.88	2.08

For every forecast and each predictor, the average forecast error, the median forecast error, and the squared forecast error, are calculated, where the forecast error is defined as the difference between the forecasted and the realized volatility over a specific time horizon. The distribution of the forecast errors is shown in the following chart, which really depicts the efficiency of the different models in forecasting.



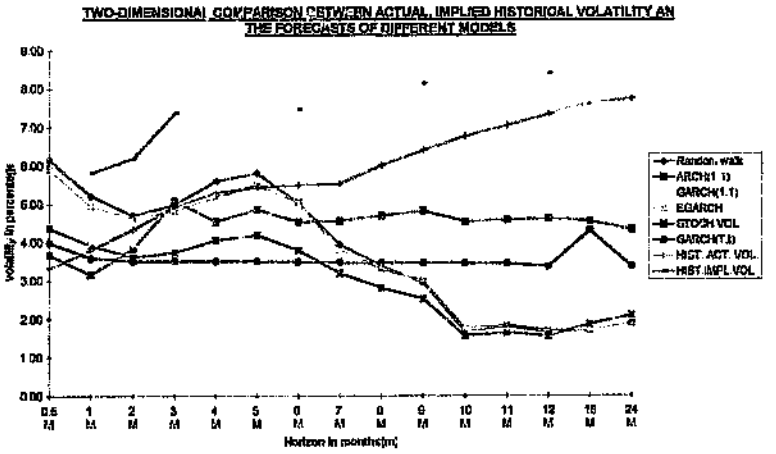
For comparing forecast performance across models. Heynen and Kat suggest that squared forecast error be used instead of the average, because the forecast error is not symmetric.

To compare the forecast performance for two different volatility predictors, pairwise comparisons of the squared error statistics are made for each time horizon. To decide which predictor performs better, it is simply, the number of horizons, say H, is counted for which a predictor has the lowest squared forecast error.

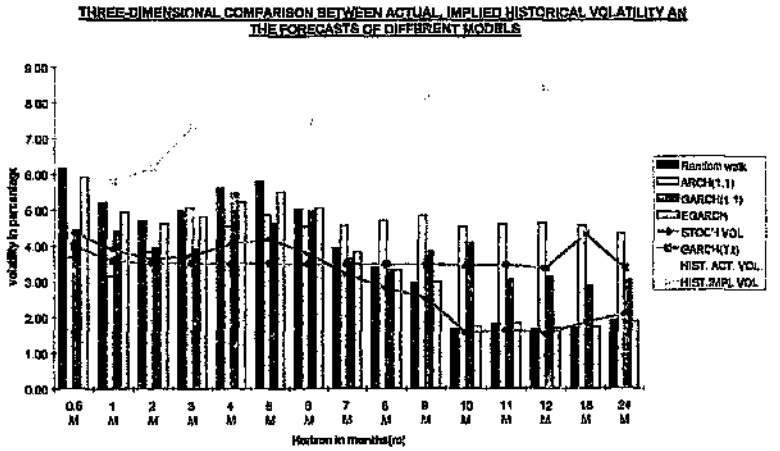
TABLE VI :Squared Forecast Errors for Different Horizons, and Predictors.

Horizon	Random Walk	ARCH	GARCH	EGARCH	Stoch.Vol.
1 wk	7.94	10.12	1.22	6.58	1.08
1 M	1.98	0.42	0.38	1.30	0.01
2 M	0.13	0.25	0.15	0.09	0.52
3 M	0.00	0.02	0.99	0.01	1.39
4 M	0.10	0.55	0.04	0.01	1.49
5 M	0.13	0.33	0.64	0.00	1.56
6 M	0.21	0.91	0.27	0.19	2.88
7 M	2.52	0.94	3.53	2.94	45.47
8 M	6.83	1.70	7.12	7.20	10.19
9 M	11.92	2.48	6.64	11.67	15.08
10 M	26.02	5.00	7.28	25.33	27.28
11 M	27.59	6.06	16.11	27.31	29.29
12 M	32.42	7.42	17.79	32.02	33.75
18 M	35.36	9.53	22.74	35.23	33.48
24 M	34.35	11.72	22.31	34.52	32.14

As can be seen in the above table, all the models seem to fail to track the actual historical volatility in the longer-term as opposed to the shorter-term, longer than the one week. This explains the fact that these models are good short-term volatility forecasters, although there is some forecasting error embedded in the forecasted values. Below is a graph which shows the distribution of the forecast errors, which clearly indicates the failure of the models in tracking longer terms.



Nice picture is seen in a three-dimensional representation of the comparison:



Nonetheless, EGARCH, Random Walk, GARCH, ARCH, and lastly Stochastic volatility, do give good forecasts from the two months to the seven months periods, respectively. The most interesting thing, is that in the EGARCH model, the third parameter was not contributing to the forecasts, because of its weird p-value. As can be seen in the table, the forecasts together with the actuals, in the attached graph, do reject the hypothesis of constant volatility in the Random walk model.

5.4 MARKET PRACTICE

5.4.1 Scaling Law

As discussed earlier, scaling law is a law that relates the interval Δt directly to the average price changes. The mean absolute price changes are much higher over working days than over Saturdays and Sundays, when the market actors are hardly present. This law basically helps in the intra-day analysis for the mean absolute hourly price changes which have distinct seasonal patterns. These patterns are correlated to the changing presence of main market places of world-wide FX market. Hence a relationship between the change in the prices and change of time intervals can be able to indicate the volatility and the difference, if any, between inter-day and intra-day behaviours.

This process requires a close observation of the market, tick-by-tick, and plotting the logarithm of change of prices against logarithm of time interval in seconds, hence can be able to give a healthy inter-day or intra-day analysis. This information can be of help to the risk managers since they can have an educated assumption of how trading can occur in a given time interval, hence advise the traders about the suitable and liquid intervals for venturing in certain markets.

5.4.2 Activity model

Activity model have close links with the scaling law except that the former goes further in considering the 'activity' in different markets of the world. There is further evidence of a strong correlation between market presence and volatility, and the former is even further related to another variable which cannot be observed directly, i.e., the worldwide transaction volume. The FX market has more or less 24-hours of trading, considering the fact that even if the local market is closed, the offshore markets are not, because of the difference in time horizons.

Knowing the activity of a certain pair of currencies can help even in a situation whereby, while a trader's local market is closed and there is a big move of the pair of currencies in 'still open' markets, and the trader is likely to incur a big loss because of the move, then he can contact the market where there is still some activity in that pair of currencies. So he can either close out the position or cut the position before the losses can become unbearable and accumulate to undesirable levels, i.e., 'out of limits'.

5.4.3 Proprietary model

As mentioned earlier, the model is trying to show that there is an *edge* between the actual volatility and the implied volatility. This edge can be profitable in the sense that one can play the actual volatility against the implied volatility. Although it is hard to know the actual volatility without using the past information, with the inception of new systems like Bloomberg and Telerate, one can be able to see the actual volatility for the previous few days. This volatility is purely the measurement of the liquidity of that particular currency on those particular days, whereas implied volatility is affected by market makers as well. Levels in implied volatility are sometimes effected by the supply and demand of that particular option in the market, dictating the final level. Just as any liquid market, implied volatility rates can only rise to the point where sellers become evident and only fall to where buyers enter the market.

The simply analysis of this edge is that a trader can be able to buy an option at the volatility closer to the actual volatility and sell the same option at the implied volatility and make the spread as a profit. This kind of trading is prevalent to speculators, who have no options position to keep and run,

except to make profits through the above transactions. Financial institutions can also act as speculators as well, more especially in currencies inwhich they are not active. For example, a local client who has an AUD/USD exposure can contact his financial institution for hedging his foreign exchange exposure. The most likely situation for a local bank is to contact the offshore banks for a price and quote back to the client, without running the position. This is exactly what most financial institutions do to keep their clientelé business.

Although most financial institutions use the implied volatility for their end-of-day book revaluations, in the event of poor systems failure, actual volatility can be used by adjusting the values according to the historical gapping between the historical actual volatility and the historical implied volatility. Similar process applies to less liquid currencies like the Rand or Zar crosses, where there is hardly a liquid market where implied volatility rates can be shown on screens for participants to see.

5.4.4 FORECAST. D RESULTS

I ifferent models have been used to investigate the better forecasting model for currencies, using USD/ZAR foreign exchange rates as an example. Over and above, these models can also be used in risk analysis for complex currency portfolios. A new r'-k management tool called VAR (value-at-risk) has been developed by international financial institutions like JP Morgan through their Risk-Metrics (1996). VAR calculations can be performed by modifying the models in question to calculate the 5th percentiles or quantiles of the return distribution from a given historical data. VAR calculations can alsc be achieved through other means without using standard deviation or correlation forecasts. The principal reason to work with standard deviation (volatility) is the strong evidence that the volatility of financial returns is predictable, and it makes sense to make forecasts of it to predict future values of the return distribution. J.P. Morgan have developed their RiskMetrics to calculate and get accurate estimates of VAR for their portfolios and their client's portfolios, including most of our local financial institutions.

According to J.P.Morgan, RiskMetrics generally measures change in value of a portfolio in terms of log-price changes also known as continuously-

compounded returns. RiskMetrics offers two methodologies, an analytical approximation and structured Monte-Carlo simulation, to compute the VAR of non-linear positions like options. The first method approximate the non-linear relationship via a mathematical expression that relates the return on the position to the return on the underlying rates. This is done by using Taylor series expansion. This approach no longer necessarily assumes that the change in value of the instrument is approximated by its delta alone, but also the second-order term using the option's gamma must be introduced to measure the curvature of changes in value around the current value. Other *'greeks'* like vega, rho and theta can also be used to improve accuracy of the approximation.

The second alternative, structured Monte Carlo simulation, involves creating a large number of possible rate scenarios and revaluing the instrument under each of these scenarios.. VAR is then defined, as stated above, as the 5th percentile of the distribution of value changes. The two methods differ not in terms of how market movements are forecast, since both use RiskMetrics volatility and correlation estimates, but in how the value of portfolio changes as a result of market movements. In essence forecasts of volatility and correlations play a central role in RiskMetrics framework and are required for

valuations in the derivatives and also are critical inputs for risk estimates. Market risk is often measured in terms of a percentile (quantile) of a portfolio's return distribution. The attractiveness of working with percentile rather than say, variance of a distribution, is that a percentile corresponds to both the magnitude, e.g., dollar amount at risk, and the exact probability e.g., the probability that the magnitude will not be exceeded.

5.5 CONCLUSIONS

In the early chapters of the paper, the term volatility was discussed and defined in precise terms, and the various models that are used to calculate volatility were highlighted. Also different types of models which are relevant to the FX markets were discussed of which many of them were used to achieve the sought goal of the research, i.e., to investigate to a certain degree of confidence which model better estimate or forecast the volatility on the Dollar/Rand exchange rate. The volatile period between mid-February and beginning of April 1996 did not have an impact in the calculation of the historical volatility, and even the forecasts were never affected. One of the reasons could be the fact that the data was long enough to swallow this

impact, and also the fact that the weakening of the Rand did not match the crash in the U.S markets on October 1989, where prices reached record levels.

One of the exciting characteristic of the ARCH models, is that, they do not only give the forecast for the next day, but can also forecast for more-than-one-day horizon. Although I managed to get the results, but I have a feeling that if I had used a more advanced and newer computer software, probably, I would have got more accurate results. As stated in the text, I did not get convincing results pertaining to the Scaling law and the activity model because of the limitations of my data. I reckon these two models are most important since there is a strong evidence of correlation between the market presence and volatility. Unfortunately these models need either intra-day or inter-day data as the daily data fails to expose their importance.

As indicated before, these forecasting models can be extended to other exchange rates as well, notwithstanding the difference in the respective markets, for example the Rand/Yen, Rand/Chf, etc; which strongly depends on the movement of the major exchange rates. The major exchange rates for Rand/Chf, for example, are Dollar/Chf and Dollar/Rand. Although the market is still thin in these crosses, with gradual removal of exchange con-

trols, thereby resulting into our country being among the major players in the world, this market can prove to be the most profitable ' if the game is played according to the rules '.

5.6

APPENDIX A

In this appendix, some statistical terms and functions which appear in the above tables are defined, using simply statistical principles which can be found in most statistical texts.

0.0.1 Standard Error of Estimate

After a regression line has been fitted to a set of points, it is usually possible to inspect its graph and observe how accurately it predicts say, y values. An arithmetic procedure for doing this is to calculate the sizes of all the errors, $y_i - y_i'$, by using the *regression line* of y on x :-

$$y' = \overline{y} + b\,(x - \overline{x})$$

where

$$b = \frac{\sum(x_i - \overline{x})y_i}{\sum(x_i - \overline{x})^2}$$

for a line equation

$$y' = a + b(x - \overline{x})$$

where

$$a = \overline{y_i}$$

A useful measure of the accuracy of prediction is obtained by calculating the mean of the squared errors of prediction. This mean is given by the expression :-

$$\frac{\sum_{i=1}^{n}(y_i - y_i')^2}{n}$$

dividing by $n-2$ instead of $n-1$, we get the variance of the errors of prediction when y_i values are those of a theoretical regression line than of the sample regression line. The square root of this expression, denoted s_e is called the *standard error of estimate* :-

$$s_e = \sqrt{\frac{\sum_{i=1}^{n} (y_i - y_i')^2}{n-2}}$$

0.0.2 Sum of the Squared Errors (SSE)

If

$$\hat{y}_i = \hat{\beta}_0 + \beta_1 x_i$$

is the predicted value of the ith y value when $x = x_i$, then the observed value of y from the $\hat{y}$ line (sometimes called the error) is :-

$$y_i = \hat{y}_i$$

and the sum of squares of deviations or sum of squares for error to be minimized is :-

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} \left[y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_i \right) \right]^2$$

0.0.3 p-Value Correlations

If we have any hypothesis to be tested, say, H_0, called the null hypothesis given $p = 0.5$, the alternative or research hypothesis, denoted by H_a would be $p < 0.5$. An error can be made in testing the hypothesis. A *type I* is made if H_0 is rejected when H_0 is true and the probability of a *type I* error is denoted by α. A type II error is denoted by β. The attained *significant level* or *p-value* is a statistic that represents the smallest value of α for which the observed data indicates that the null hypothesis should be rejected. If an experimenter's choice of α is greater than or equal to the *p-value*, the null hypothesis is rejected, otherwise, if α is less than the *p-value*, the null

hypothesis cannot be rejected.

0.0.4 Programme using SHAZAME 7.0

RANDOM WALK

OLS AVE / RESID=UH (This part generates the residuals using least squares)

*ARCH(1,1)

HET AVE (this generates the averaging of heteroskedasticity)

*GARCH(1,1)

GENR UHSQ = UH**2 (this generates the sigma estimates)

GENR LUHSQ = LOG (UHSQ) (this part generates the log of the residuals)

GENR E = NOR(0,1) (this generates the parameter ξ which is assumed to be normal distributed with mean zero and variance equal to 1)

GENR ESQ = E**2 (this part generates error terms squared ε_t^2)

GENR EPSQ = UHSQ* ESQ (this part generates the product $\sigma_t \varepsilon_t$)

*EGARCH

GENR LUH = LAG (UH, 1) (this part generates the residuals lagged by 1)

GENR UHSQ1 = LAG (UHSQ, 1) (this part generates σ_t^2 lagged by 1)

GENR LUHSQ1 = LAG (LUHSQ, 1)

GENR ESQ1 = LAG (ESQ, 1) (this part generates the ε_t^2 lagged by 1)

GENR EPSQ1 = LAG (EPSQ, 1)

OLS UHSQ EPSQ1 UHSQ1

*STOCHASTIC VOLATILITY

OLS LUHSQ LUHSQ1

GENR XI1 = LAG (E.1) (this part generates ξ_t lagged by 1)

GENR XI2 = ABS (XI1) -SQRT (2*7.0/22.0) (this part generates ($|\xi_{t-1}| - \sqrt{\frac{2}{\pi}}$)

OLS LUHSQ LUHSQ1 XI1 XI2

FORMAT (1x,6f12.8)

Write (output. data) luh uhsqi esq1 xi1 xi2 /format (this part generates a document that can be converted to a spreadsheet)

STOP.

0.1

APPENDIX B

The mathematics behind these models is basic, while the statistics is a lot tougher, which would be advanced for a novice person in this field. The random walk and the ARCH (1,1) models are easy to follow, unlike the GARCH(1,1) and the EGARCH (1,1) models. In the following paragraph, step by step analysis is given for each model.

0.0.1 Garch (1,1) derivation

For a GARCH (1,1) specification stock return an stock return volatility are modeled as follows :-

$$\ln(S_t/S_{t-1}) = \mu + \sigma_t \xi_t \tag{0.8}$$

$$\sigma_t^2 = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \tag{0.9}$$

where S_t is the stock price at t,

μ is the mean stock price return,

σ_t is th stock return volatility,

$\beta_0, \beta_1, \beta_2$ are time dependent parameters, $\beta_1 + \beta_2 < 1$,

ϵ_t are the innovation variables, $\epsilon_t = \sigma_t \xi_t$, ξ_t is Gaussian white noise, and $\xi_t \overset{i.i.d}{\sim} N(0,1)$. The average expected volatility is defined by :-

$$\sigma^2_{Av}(t,T) = \frac{1}{T}\sum_{k=1}^{T} E_t\left[\sigma^2_{t+k}\right] \qquad (0.10)$$

where T is the number of periods from t to the expiration date. To simplify the right-hand side of the above equation, one needs to express the σ^2_{t+k} into σ^2_t. So, using the second equation

$$\sigma^2_{t+k} = \beta_0 + \left(\beta_1 \xi^2_{t+k-1} + \beta^2\right)\sigma^2_{t+k-1} \qquad (0.11)$$

By induction (leaving out some steps in-between) from above :-

$$\sigma_{t+k}^2 = \beta_0 + \beta_0 \sum_{m=1}^{k-1} \prod_{n=1}^{m} \left(\beta_1 \xi_{t+k-n}^2 + \beta_2\right) + \sigma_t^2 \prod_{n=1}^{k} \left(\beta_1 \xi_{t+k-n}^2 + \beta_2\right) \qquad (0.12)$$

Using independence of ξ_k,

$$E_t\left[\sigma_{t+k}^2\right] = \beta_0 + \beta_0 \sum_{m=1}^{k-1} (\beta_1 + \beta_2)^m + (\beta_1 + \beta_2)^{k-1} \left(\beta_1 \xi_t^2 + \beta_2\right) \sigma_t^2 \qquad (0.13)$$

where E_t is the conditional expectation operator at t. Evaluating the summations,

$$E_t \sigma_{t+k}^2 = \beta_0 + \beta_0 \frac{\gamma - \gamma^k}{1 - \gamma} + \gamma^{k-1} \left(\beta_1 \xi_t^2 + \beta_2\right) \sigma_t^2 \qquad (0.14)$$

with $\gamma = \beta_1 + \beta_2$. Hence the right-hand side of the third equation can be simplified as :-

$$\sigma_{Av}^2(t, T) = \frac{1}{T} \sum_{k=1}^{T} E_t\left[\sigma_{t+k}^2\right] = \overline{\sigma}^2 + \left(\sigma_{t+1}^2 - \overline{\sigma}^2\right) \frac{1}{T} \frac{1 - \gamma^T}{1 - \gamma} \qquad (0.15)$$

where $\overline{\sigma}^2 = \beta_0/(1 - \beta_1 - \beta_2)$, and $\sigma_{t+1}^2 = \beta_0 + \beta_1 \epsilon_t^2 + \beta_2 \sigma_t^2$.

0.0.2 EGARCH (1,1) Derivation

For an EGARCH (1,1) process, stock return and stock return volatility are modeled as follows :-

$$\ln(S_t/S_{t-1}) = \mu + \sigma_t \xi_t$$

$$\ln \sigma_t^2 = \beta_0 + \beta_1 \ln \sigma_{t-1}^2 + \beta_2 \xi_{t-1} + \beta_3 \left(|\xi_{t-1}| - \sqrt{\frac{2}{\pi}} \right) \qquad (0.16)$$

where $\beta_0, ..., \beta_3$ are time independent parameters, and ξ_t is as before Gaussian white noise, $\xi_t \overset{i.i.d}{\sim} N(0,1)$. Average expected volatility is defined as follows :-

$$\sigma_{Av}^2(t,T) = \left[\prod_{k=1}^{T} E_t \left[{}_t\sigma_{t+k}^2 \right] \right]^{\frac{1}{T}} \qquad (0.17)$$

or by taking the natural logs on both sides :-

$$\ln \sigma_{Av}^2(t,T) = \frac{1}{T}\sum_{k=1}^{T} \ln E_t\left[\sigma_{t+k}^2\right] \tag{0.18}$$

where $\sigma_{Av}(t,T)$ is the geometric average expected volatility over the time span $[t, t+T]$,

E_t is the conditional expectation operator at t, and T is the number of periods from t to the expiration date. To calculate the average expected volatility, we need to express σ_{t+k}^2 into σ_t^2 as before. Define $y_t = \ln \sigma_t^2$ and $A_t = \beta_2 \xi_t + \beta_3 \left(|\xi_t| - \sqrt{\frac{2}{\pi}}\right)$, then using induction again, it follows that (eliminating some of the steps) :-

$$y_{t+k} = \beta_0 \sum_{m=0}^{k-1} \beta_1^m + \beta_1^k y_t + \sum_{m=0}^{k-1} \beta^m A_{t+k-m-1} \tag{0.19}$$

From this follows the expression for σ_{t+k}^2,

$$\sigma_{t+k}^2 = \sigma^{2\beta_1^k} \exp\left[\beta_0 \sum_{m=0}^{k-1} \beta_1^m + \sum_{m=0}^{k-1} \beta_1^m A_{t+k-m-1}\right] \tag{0.20}$$

Now the conditional expectation can be evaluated as,

$$E_t\left[\sigma_{t+k}^2\right] = \sigma^{2\beta_1^k} E_t\left[\exp\left[\beta_0 \sum_{m=0}^{k-1} \beta_1^m + \sum_{m=0}^{k-1} \beta_1^m A_{t+k-m-1}\right]\right] \tag{0.21}$$

Factoring out the expectation operator, for $k \geq 2$:-

$$E_t\left[\sigma_{t+k}^2\right] = \sigma^{2\beta_1^k} \exp\left[\beta_1^{k-1} A_t\right] \tag{0.22}$$

$$\times \exp\left[\beta_0 \sum_{m=0}^{k-1} \beta_1^m\right] \prod_{m=0}^{k-2} E_t\left[\exp\left[\beta_1^m A_{t+k-m-1}\right]\right] \tag{0.23}$$

Now for $k = 1$, $E_t[\sigma_{t+1}^2] = \sigma_{t+1}^2$, and the average expected volatility can be computed if one evaluate the factors $E_t[\exp[\beta_1^m A_{t+k-m-1}]]$. Because of the independence of the terms A_t as a result of the independence of ξ_t, we need to only evaluate expressions of the form

$$E\left[\exp\left[\beta_1^m\left(\beta_2\xi_t + \beta_3\left(|\xi_t| - \sqrt{\frac{2}{\pi}}\right)\right)\right]\right]$$

From calculus, the above equals :-

$$E\left[\exp\left[\beta_1^m\left(\beta_2\xi_t + \beta_3\left(|\xi_t| - \sqrt{\frac{2}{\pi}}\right)\right)\right]\right] = \tag{0.24}$$

$$\exp\left[-\beta_3\beta_1^m\sqrt{\frac{2}{\pi}}\right]$$
$$\left[\begin{array}{c} N\left[\beta_1^m(\beta_3+\beta_2)\right]\exp\left[\frac{1}{2}\beta_1^{2m}(\beta_3+\beta_2)^2\right] \\ +N\left[\beta_1^m(\beta_3-\beta_2)\right]\exp\left[\frac{1}{2}\beta_1^{2m}(\beta_3+\beta_2)^2\right] \end{array}\right]$$

with $N(a) = (1/\sqrt{2\pi})\int_{-\infty}^{a} e^{-z^2/2}dz$. Hence the right-hand side of the second last expression becomes :-

$$\begin{aligned} E_t\left[\sigma_{t+k}^2\right] &= \sigma^{2\theta_1^k}\exp\left[\beta_1^{k-1}A_t\right] \qquad (0.25) \\ &\exp\left[\begin{array}{c}\left(\beta_0-\beta_3\sqrt{\frac{2}{\pi}}\right)\frac{1-\beta_1^k}{1-\beta_1}+\beta_3\beta_1^{k-1}\sqrt{\frac{2}{\pi}} \\ +\frac{1}{2}(\beta_2^2+\beta_3^2)\frac{1-\beta_1^{2(k-1)}}{1-\beta_1^2}\end{array}\right] \\ &\times\prod_{m=0}^{k-2}\left[\begin{array}{c} N[\beta_1^m(\beta_3+\beta_2)\exp[\beta_1^{2m}\beta_2\beta_3] \\ +N[\beta_1^m(\beta_3-\beta_2]\exp[-\beta_1^{2m}\beta_2\beta_3] \end{array}\right] \end{aligned}$$

The expression for $\sigma_{Av}^2(t,T)$ follows from the above expression. For convenience, we can define

$$F_m(\beta_1,\beta_2,\beta_3) = N[\beta_1^m(\beta_3+\beta_2)\exp[\beta_1^m\beta_2\beta_3]$$

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