

in a 30-minute lesson the teacher may be working towards addition tasks with two collections, the first of which is covered with numbers in the range beyond 10, for example, $7 + 4$. The lesson could commence with saying short, FNWSs commencing at 5, then 6, 7, 8, and so on. This could be followed by the teacher saying one number and the child saying the next two, three or four number words. For example, Five ... 6, 7, 8; ... Nine 10, 11, 12; Thirteen ... 14, 15, 16; and so on. The activity could shade to saying the number word after a number or even saying two or three numbers after. Similarly activities could involve saying the number word before, two before, or even three before. A numeral track, or covered number line, would enable the same activities to be done in a different setting. Activities involving flashing spatial patterns using a filled Ten Frame and a partially filled Ten Frame with 2, 3, 4, or 5 dots could be used with the child being asked, 'How many altogether?' The Figurative Counting Key Topic involves a variety of settings such as seven animals in one barn and four animals in the field; seven bees in the hive and four flying around; seven letters in the post box and four more being posted, and so on. The key element of the activity is that the first collection is briefly displayed and then screened.

GUIDING PRINCIPLES IN MATHEMATICS RECOVERY TEACHING

The teaching sessions, whether for an individual, group or class, afford an opportunity to provide intensive, high-quality teaching. In order to do this the teacher must have a clear model of the children's current knowledge and strategies in early number, and a clear idea of the progress in learning that is a reasonable goal for them. The origins of Mathematics Recovery teaching lie in large part in research projects which involved longitudinal observation and study of children's developing strategies and learning as they occurred in interactive teaching sessions.

We conclude this chapter on how Judy, Denise and Michael were able to make such good progress by summarizing the teaching approach in the following set of nine guiding principles of Mathematics Recovery teaching.

1. The teaching approach is enquiry-based, that is, problem-based. Children are routinely engaged in thinking hard to solve numerical problems which for them, are quite challenging.
2. Teaching is informed by an initial, comprehensive assessment and ongoing assessment through teaching. The latter refers to the teacher's informed understanding of the child's current knowledge and problem-solving strategies, and continual revision of this understanding.
3. Teaching is focused just beyond the 'cutting-edge' of the child's current knowledge.
4. Teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations.
5. The teacher understands children's numerical strategies and deliberately engenders the development of more sophisticated strategies.
6. Teaching involves intensive, ongoing observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of her or his observation.
7. Teaching supports and builds on the child's intuitive, verbally based strategies and these are used as a basis for the development of written forms of arithmetic which accord with the child's verbally based strategies.
8. The teacher provides the child with sufficient time to solve a given problem. Consequently, the child is frequently engaged in episodes which involve sustained thinking, reflection on her or his thinking and reflecting on the results of her or his thinking.
9. Children gain intrinsic satisfaction from their problem-solving, their realization that they are making progress, and from the verification methods they develop.

Appendix 1

The Mathematics Recovery Programme

Assessment Interview Schedule 1.1 Early Arithmetical Strategies and Numerical Knowledge

Child's name:

DoB: Age: years months

Interviewer's name:

Date of interview:

Teacher/class:

	SEAL	FNWS	No. Id.	BNWS
Stage/level				

1 Forward Number Word Sequence

Start counting from ** and I'll tell you when to stop.

- | | |
|----------------|-----------------|
| (a) 1 (to 32) | (b) 48 (to 61) |
| (c) 76 (to 84) | (d) 93 (to 112) |

2 Number Word After

Say the word that comes straight after ***, Example: Say the word that comes straight after one.

(a) Entry task			
14	11	19	12
23	29	20	

(b) Less advanced task			
5	9	7	
3	6		

(c) More advanced task			
59	65	32	
70	99		

3 Numeral Identification

Show each card in turn, saying, *What number is this?*

(a) Entry task									
	10	15	47	13	21				
80	12	17	99	20	66				

(b) Less advanced task					(c) More advanced task				
8	3	5	7	9	100	123	206		
6	2	4	1		341	820			

4 Numeral Recognition

Arrange the cards from 1 to 10 randomly. Which number is ... ?

6 4 7 9 8

5 Backward Number Word Sequence

Example: Count backwards from 3. *Three, two, one.*

Now count backwards from ** and keep going until I say stop.

- | | |
|---------------------|---------------------|
| (a) 10 (down to 1) | (b) 15 (down to 10) |
| (c) 23 (down to 16) | (d) 34 (down to 27) |
| (e) 72 (down to 67) | |

6 Number Word Before

Say the number word that comes just before ***, Example: Say the number just before 2.

(a) Entry task			
24	17	20	11
13	21	14	30

(b) Less advanced task			
7	10	4	
8	3		

(c) More advanced task			
67	50	38	100
83	41	99	

7 Sequencing Numerals

Show the ten numbered cards face up in random order, asking the child to identify each number as you put it out. Then say, *Can you place the cards in order? Start by putting the smallest down here.*

(a) Entry task
Cards from 46 to 55

(b) More advanced task
Cards from 1 to 10

8 Additive Tasks (screened, use counters of two colours)

Introductory task:

There are three red counter under here, and one yellow counter under here. How many counters are there altogether?

3	+	1
---	---	---

(a) Entry tasks (both collections screened)

$$\boxed{5} + \boxed{4}$$

$$\boxed{9} + \boxed{6}$$

If one or more incorrect continue below to 8(b). If both correct go to 8(e).

(b) Less advanced task (first collection screened)

$$\boxed{5} + 2$$

$$\boxed{7} + 3$$

$$\boxed{9} + 4$$

If (b) is too difficult go to (c)

(c) Unscreened collections

$$5 + 2$$

$$7 + 3$$

$$9 + 4$$

(d) Perceptual counting

Would you count to see how many counters there are altogether in this group?

Place out 13 counters.

Place out 18 counters

(e) Supplementary additive tasks (screened, use counters of two colours)

Supplementary tasks to (a) if further clarification is needed. The tasks are presented totally screened.

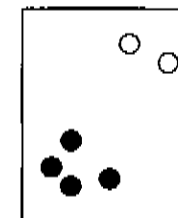
$$\boxed{8} + \boxed{5}$$

$$\boxed{9} + \boxed{3}$$

(f) Missing addend

Here are four red counters. Now look away. While you were looking away I put some more yellow counters under here. Now there are 6 counters altogether. How many yellow counters did I put under here?

Introductory task $4 + \boxed{} = 6$



Tasks $7 + \boxed{} = 10$

$12 + \boxed{} = 15$

9 Subtractive Tasks

(a) Subtraction sentences

Present the tasks as a written number sentence on card. Say to the child, *What does this say? Do you have a way to work out what the answer is?* [Note: using counters is not an option]

Entry task	Supplementary task
$16 - 12$	$17 - 14$

(b) Missing subtrahend

[Note: use counters of one colour only]

Here are five counters. (Ask the child to look away. Remove and screen two counters.) There were five counters. While you were looking away I took some away. Now there are only three. How many did I take away?

(i) Introductory task	5 to 3	$\{ 5 - \boxed{} = 3 \}$
(ii) Entry tasks	10 to 6	$\{ 10 - \boxed{} = 6 \}$
	12 to 9	$\{ 12 - \boxed{} = 9 \}$
(iii) More advanced task	15 to 11	$\{ 15 - \boxed{} = 11 \}$

(c) Removed items

Here are three counters. (briefly display, then screen). If I take away one, (remove one counter, display briefly, then re-screen) how many are left under here? (Indicate the first screen.)

(i) Introductory task $3 - 1$

(ii) Entry tasks $6 - 2$ $9 - 4$ $15 - 3$

(iii) More advanced task $27 - 4$

Mathematics Recovery Programme: 1.1 – pupil profile sheet

Pupil:	DoB:	Teacher:	Class:
Age: yrs months	Assessment date:	Assessor:	

Aspect/Task Item	Level	Comments
FNWS and NWA	1 2 3 4 5	
Numerical Id.	1 2 3 4	
BNWS and NWA	1 2 3 4 5	
Sequencing Nos		
Additive Tasks and Subtractive tasks. Use the information on each section to arrive at the Stage.		
Additive Tasks		
Missing Addends		
Subtractive Tasks		
Missing Subtrahends		
Removed Items		
STAGE on SEAL	1 2 3 4 5	

Appendix 2**The Mathematics Recovery Programme****Assessment Interview Schedule 1.2
Base-Ten and Advanced Arithmetical Strategies**

Child's name:
DoB: Age: years months
Interviewer's name:
Date of interview:
Teacher/class:

Tasks	Comments
Tens and Ones Level 0 1 2 3	
Non-Count-By-Ones strategies	

1 Tens and Ones Tasks

(a) Counting by tens with 'strips' - informal familiarization with material

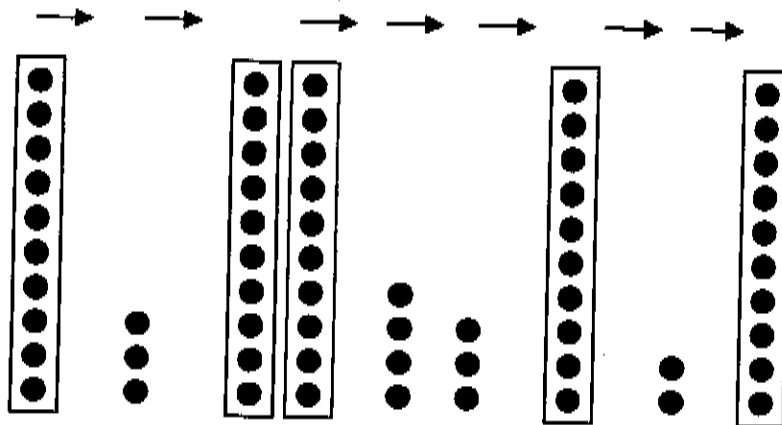
- (i) Put down a ten strip. *How many do we have?* If the child says 'one', ask, *How many dots are there?*
- (ii) '*How many altogether?*' Put down one ten strip at a time to 8 strips.
10 20 30 40 50 60 70 80
- (iii) Pick up all the strips.
How many dots do we have?
How many strips are there?

(b) Incrementing by ten

- (i) Place out the 'four dot' strip. *How many dots are there?*
- (ii) Place out a ten strip to the right of the four strip. *How many dots are there altogether?*
- (iii) Continue placing ten strips to the right of the four strip. *How many dots are there altogether?* 24 34 44 54 64 74
- (iv) If necessary, repeat the whole task with either the 3 strip or the 7 strip.

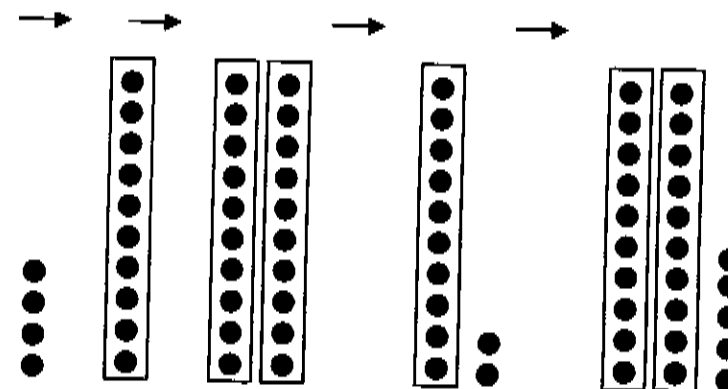
(c) Uncovering tasks: Board One

Upon each uncovering ask, *How many dots are there now?*



(d) Uncovering tasks: Board Two

Upon each uncovering ask, *How many are there now?*



Example of Uncovering tasks: Board One, fourth move



2 Horizontal Sentences

- (a) Do you have a way to figure out what is?

$$16 + 10 =$$

So what is $16 + 9$?

- (b) Do you have a way to figure out what is?

$$47 + 23$$

If correct ask, Do you have another way to work it out or check?

- (c) Do you have a way to figure out what is?

$$38 + 24$$

If correct ask, Do you have another way to work it out or check?

- (d) Repeat the above questions for:

$$39 + 53$$

- (e)

$$56 - 23$$

- (f)

$$43 - 15$$

- (g)

$$73 - 48$$

3 Tasks to Elicit Non-Count-By-Ones Strategies and Relational Thinking

(a)

$9 + 3$

*Can you use ...
to help you do?*

$9 + 4$

$9 + 5$

*9 + 6
and so on
if necessary*

(b)

$6 + 6$

*Can you use ...
you help you do?*

$7 + 5$

$8 + 4$

*9 + 3
and so on
if necessary*

(c)

$7 - 5$

*Can you use ...
to help you do?*

$27 - 5$

$47 - 5$

*What do you think
the next take-away
will be?*

(d)

$15 + 3$

*Can you use ...
to help you do ...?*

$18 - 3?$

*How do you use
... to help you?*

(e)

If I tell you

$21 - 16 = 5$

*Can you use ...
to help you do ...?*

$21 - 5?$

(f)

$14 + 4 = 18$

*There are three numbers here.
Can you use these to make
a take-away sum?**Can you make
another one?*

Appendix 3

The Mathematics Recovery
ProgrammeAssessment Interview Schedule 2.1
Early Grouping: Structuring Numbers 1 to 10

Child's name:

DoB: Age: years months

Interviewer's name:

Date of interview:

Teacher/class:

Subitizing and Spatial Patterns	Regular	Irregular	Domino
Finger Patterns	1 to 5		6 to 10
Five Frame Patterns	1 to 5		
Pair-wise Patterns on a Ten Frame	1 to 5		6 to 10
Five-wise Patterns on a Ten Frame	1 to 5		6 to 10
Combining to make Five			
Combining to make Ten			

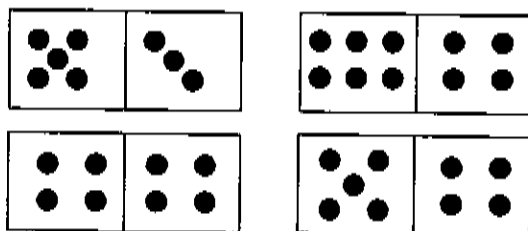
1 Subitizing and Spatial Patterns

Show each card briefly, for about half a second, saying, *I'm going to show you some cards very quickly. Tell me how many spots are on each one.*

- (a) Flashed Regular 4 3 2 5 6
 (b) Flashed Irregular 6 7 4 5 8
 (c) Domino Cards

Show each card briefly, for about half a second, saying, *I'm going to show you some cards very quickly.*

- (i) What did you see?
 (ii) Tell me how many spots are on one side.
 (iii) Tell me how many spots are on the other side.
 (iv) How many are there altogether?



2 Finger Patterns 1 to 5

- (a) Show me three on your fingers.
 3 2 5 1 4
 (b) Using two hands show me.
 3 2 5 4

3 Finger Patterns 6 to 10

- (a) Show me 6 on your fingers.
 (b) Show me 6 a different way.
 (c) Show me 9 on your fingers.
 (d) Show me 10 on your fingers.
 (e) Show me 8 on your fingers.
 (f) Show me 8 a different way.

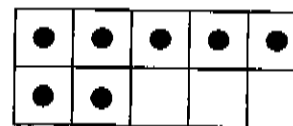
4 Five Frame Patterns



Flash the Five Frame.

- (a) What did you see?
 (b) How many spots did you see?
 3 2 5 1 4

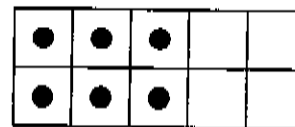
5 Five-wise Patterns on a Ten Frame



Flash five-wise cards 1-10

- (a) What did you see?
 (b) How many spots did you see?
 7 10 8 6 9

6 Pair-wise Patterns on a Ten Frame



Flash pair-wise cards 1-10

- (a) What did you see?
 (b) How many spots did you see?
 4 2 5 1 3
 7 10 8 6 9

7 Combining to make five

I will say a number and you say the number that goes with it to make five.

4 2 1 3 5

8 Combining to make ten

- (a) Tell me two numbers that add up to 10.
 (b) Tell me 2 other numbers that add up to 10.
 (c) Can you tell me another two?
 (d) I have 8 apples, how many more do I need to make 10?
 (e) I have 4, how many more to make 10?
 (f) I have 7, how many more to make 10?

Appendix 4

The Mathematics Recovery Programme

Assessment Interview Schedule 2.2 Advanced Grouping: Structuring Numbers 1 to 20

Child's name:

DoB: Age: years months

Interviewer's name:

Date of interview:

Teacher/class:

Tasks	Comments
Doubles	
Near Doubles	
Addition using five, ten or doubles, and so on	
Subtraction using five, ten or doubles, and so on	
Addition and subtraction with one addend/subtrahend > 10	
Relational Thinking	

Assessment Interview 2.2 Advanced Grouping: Structuring Numbers 1 to 20

1 Doubles

Show spot cards (two seconds)

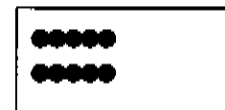
(a) $5 + 5$ (b) $4 + 4$ (c) $2 + 2$

(i) What do you see?

(ii) How many on the top row?

(iii) How many on the bottom row?

(iv) How many altogether?

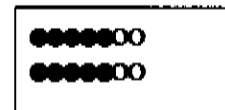
(d) $7 + 7$ (e) $9 + 9$ (f) $6 + 6$ (g) $8 + 8$

(v) What do you see?

(vi) How many on the top row?

(vii) How many on the bottom row?

(viii) How many altogether?



2 Near Doubles

Show the number problem.

Can you work this problem out?

How did you do that?

Do you have another way to work it out?

(a) $5 + 6$ (b) $4 + 3$ (c) $7 + 6$ (d) $9 + 8$ (e) $8 + 7$ (f) $3 + 2$

3 Addition Using Five, Ten or Doubles, and so on

Show the number problem. Do you have a way to work out this problem?

How did you do that?

(a) $9 + 3$ (b) $8 + 5$ (c) $9 + 6$ (d) $8 + 6$ (e) $5 + 7$ (f) $4 + 9$

4 Subtraction using five, ten or doubles, and so on

Show the number problem. Do you have a way to work out this problem?

How did you do that?

(a) $17 - 7$ (b) $11 - 4$ (c) $12 - 9$ (d) $17 - 8$ (e) $13 - 5$ (f) $14 - 8$

5 Addition and Subtraction with One Addend/Subtrahend Greater than Ten

Show the number sentence. Do you have a way to work out this problem?

How did you do that?

- (a) $13 + 5$ (b) $11 + 8$
(c) $19 - 7$ (d) $17 - 15$

6 Relational Thinking

- (a) Present the card
What does this say?
Can you work it out?

Can you use this number problem to help you work out ...?

- (b) Present the card
What does this say?

Can you use this ... to help you do ...

Can you use this ... to help you do ...

And so on ... if necessary.

- (c) Present the card
What does this say? Do you have a way to work this number problem out?
Can you use this number problem to help you work out this?

How did you do that?

- (d) Present the card

Can you use this number problem to help you work out this?

- (i) $8 + 6$

- (ii) $14 - 6$

- (iii) What other number facts would you know from this?

$$14 + 2$$

$$2 + 14$$

$$7 + 7$$

$$8 + 6$$

$$9 + 5$$

$$13 + 4$$

$$17 - 4$$

$$6 + 8 = 14$$

Appendix 5

The Mathematics Recovery Programme

Assessment Interview Schedule 3.1 Early Multiplication and Division

Child's name:

DoB: Age: years months

Interviewer's name:

Date of interview:

Teacher/class:

Forming Equal Groups

FNWS in multiples

Rows and Arrays

Equal groups of visible items

- (a) Multiplication
(b) Partition Division
(c) Quotition Division

Screened Items

- (a) Multiplication (equal groups)
(b) Partition Division (equal groups)
(c) Quotition Division (equal groups)
(d) Multiplication (array)
(e) Quotition Division (array)

Level 0 1 2 3

5 Addition and Subtraction with One Addend/Subtrahend Greater than Ten

Show the number sentence. Do you have a way to work out this problem?

How did you do that?

- (a) $13 + 5$ (b) $11 + 8$
(c) $19 - 7$ (d) $17 - 15$

6 Relational Thinking

- (a) Present the card
What does this say?
Can you work it out?

$14 + 2$
$2 + 14$

Can you use this number problem to help you work out ...?

- (b) Present the card
What does this say?

$7 + 7$
$8 + 6$
$9 + 5$

Can you use this ... to help you do ...

Can you use this ... to help you do ...

And so on ... if necessary.

- (c) Present the card
What does this say? Do you have a way to work this number problem out?
Can you use this number problem to help you work out this?

$13 + 4$
$17 - 4$

How did you do that?

- (d) Present the card

Can you use this number problem to help you work out this?

$6 + 8 = 14$

- (i)

$8 + 6$

 (ii)

$14 - 6$

(iii) What other number facts would you know from this?

Appendix 5

The Mathematics Recovery Programme

Assessment Interview Schedule 3.1 Early Multiplication and Division

Child's name:

DoB: Age: years months

Interviewer's name:

Date of interview:

Teacher/class:

Forming Equal Groups

FNWS in multiples

Rows and Arrays

Equal groups of visible items

- (a) Multiplication
(b) Partition Division
(c) Quotition Division

Screened items

- (a) Multiplication (equal groups)
(b) Partition Division (equal groups)
(c) Quotition Division (equal groups)
(d) Multiplication (array)
(e) Quotition Division (array)

Level 0 1 2 3

Assessment Interview 3.1

Early Multiplication and Division

TASK GROUP 1. FORMING EQUAL GROUPS

Present a pile of 15 counters to the child. (Put them out randomly spaced and not in a line.)
Using these counters, make three groups with four in each group.



How many counters did you use?

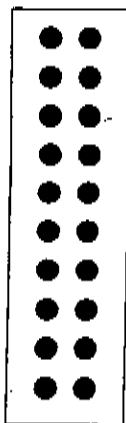
TASK GROUP 2. TASKS INVOLVING FNWS OF MULTIPLES

To discover how facile the child is in the FNWS of multiples and where they stop or have problems.

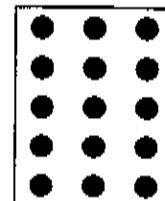
- (a) Count by twos. I'll tell you when to stop. (Stop at 20)
- (b) Count by tens. I'll tell you when to stop. (Stop at 120)
- (c) Count by fives. I'll tell you when to stop. (Stop at 55)
- (d) Count by threes. I'll tell you when to stop. (Stop at 15)

TASK GROUP 3. TASKS INVOLVING VISIBLE ITEMS ARRANGED IN ROWS OR ARRAYS

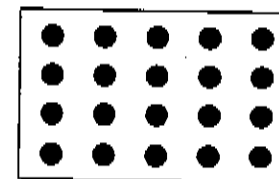
- (a) Display a 10×2 array of dots, that is ten rows and two columns.
Can you tell me how many dots there are altogether?



- (b) Display a 5×3 array of dots, that is five rows and three columns.
Can you tell me how many dots there are altogether?



- (c) Display a 4×5 array, that is four rows and five columns. Indicate rows in turn.
How many rows are there?

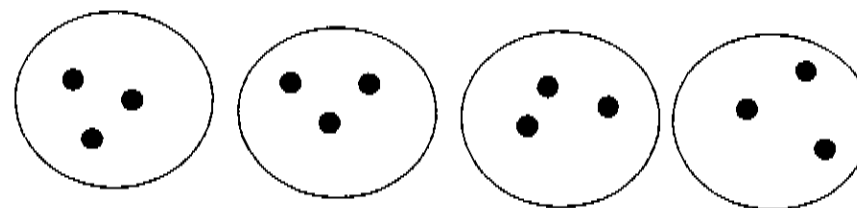


How many dots in each row?
How many dots altogether?

Turn the array through 90 degrees.
How many dots altogether now?

TASK GROUP 4. TASKS INVOLVING EQUAL GROUPS OF VISIBLE ITEMS

- (a) Multiplication
Place out four plates with three counters on each plate.



How many plates are there?
There are three counters on each plate, how many counters are there altogether?

- (b) Partition division
Place out a pile of 15 counters.
Here are 15 counters. If we shared them equally among three children, how many would each child get?



(c) Quotition division

Place out a pile of 12 counters.

Here are 12 counters. If we shared them equally among some children so they each got four, how many children would there be?



(d) Partition division with redistribution

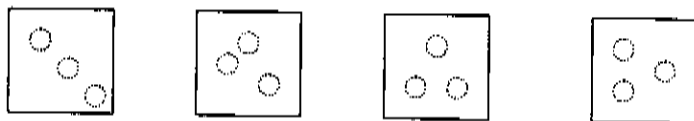
Place out a pile of 24 counters.

Here are 24 counters. If we shared them equally among three children, how many would each child get? If I now shared them equally among 4 children how many would each get?

TASK GROUP 5. TASKS INVOLVING SCREENED ITEMS

(a) Multiplication with equal groups

Ask the child to look away while you place out four screens with three counters under each screen.



Each screen has three counters under it. How many counters altogether?

(b) Partition division with equal groups

Place out a pile of 12 counters and three covered opaque containers.

Share these counters equally among the three containers and tell me how many counters there will be in each container?

Ensure that the child is not able to count the counters after having shared them.

(c) Quotition division with equal groups

Place out a pile of 30 counters and 7 covered containers.

Use 20 of these counters to make containers with five counters in each and tell me how many containers you will use?

(d) Multiplication with an array.

Using a 5×3 array, use one screen to screen the two upper rows and a second screen to screen the lower three rows. Unscreen the upper two rows for a few seconds. (Fig. 1)

How many rows do you see? Re-screen the two rows. (Fig. 2)

There are three more rows under this screen. How many counters are there altogether?

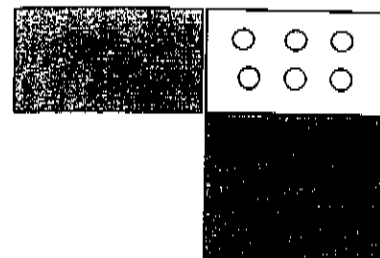


Fig. 1



Fig. 2

(e) Quotition division with an array

Using a 6×2 array, screen five rows and leave the uppermost row unscreened. (Fig. 3)

How many dots in this row? There are 12 dots altogether. How many rows altogether? Let the child check. (Fig. 4)

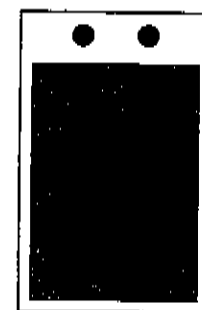


Fig. 3

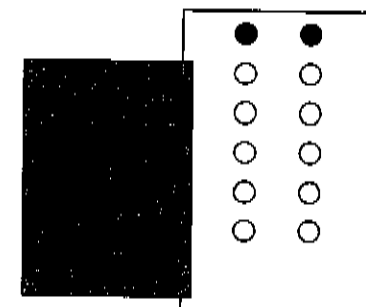


Fig. 4

Appendix 6

The Mathematics Recovery Programme

Assessment Interview Schedule 3.2 Advanced Multiplication and Division

Child's name:

DoB: Age: years months

Interviewer's name:

Date of interview:

Teacher/class:

Written sentences

- (a) Multiplication
- (b) Quotition division
- (c) Partition division
- (d) Quotition division with remainder
- (e) Partition division with remainder

Commutativity and Inverse relationship

Applied Multiplication

Level 3 4 5

Assessment Interview 3.2 Advanced Multiplication and Division

TASK GROUP 1: TASKS PRESENTED VERBALLY WITHOUT MATERIALS

(a) Multiplication

Six children have five marbles each. How many marbles altogether?

(b) Quotition division

There are 12 bananas and each child is given two bananas.
How many children are there?

(c) Partition division

If we shared eighteen apples among three children,
how many apples would each child get?

(d) Quotition division with remainder

There are 17 flowers and each person is given 5 flowers. How
many people are there and how many flowers left over?

(e) Partition division with remainder

If we shared 14 cookies equally among four children, how
many cookies would each child get and how many would be left over?

TASK GROUP 2: COMMUTATIVITY AND INVERSE RELATIONSHIP

- (a) What does this say? What does it make you think of?
or

What do you see in your mind when you read 9×7 ?

$$9 \times 7$$

How would you work this out?

Can you tell me another way to work it out?

- (b) What does this say?

$$3 \times 7$$

What does this say?

$$7 \times 3$$

What can you tell me about these two problems?

$$3 \times 7$$

$$7 \times 3$$

(c) What is the answer to this problem?

$$8 \times 4$$

Can you use that to help you do this problem?

$$32 \div 4$$

(d) If I tell you that eight times seven is 56 (show the card)

$$8 \times 7 = 56$$

Can you use these numbers and signs (symbols) to make a division (division sentence)?

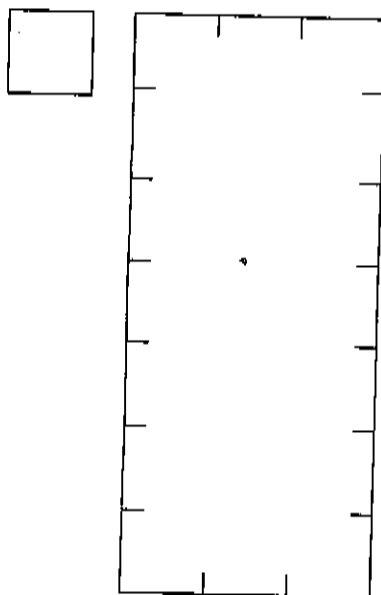
$$8 \quad 7 \quad 56 \quad = \quad \div$$

Can you make another division?

TASK GROUP 3: AREA MULTIPLICATION

Show the cardboard square unit and the 7×3 rectangle.

How many squares like this one would you need to cover the rectangle completely?



Provide the child with a copy of the grid and ask,
Can you draw what the squares would look like?

Levels in the Development of Multiplication and Division

Level 1: Forming Equal Groups or Initial Grouping

The child uses perceptual counting and sharing (that is, by ones) to form or make groups of specific sizes; to share items into groups of a given size (quotative sharing); to share items into a given number of groups (partitive sharing). The child does not see the groups as composite units and thus counts each item by ones instead of in multiples. (Similar to Stage 1 in SEAL)

Level 2: Perceptual Counting in Multiples

The child uses a multiplicative counting strategy to count visible items arranged in equal groups. The child cannot count screened groups. Strategies used can be rhythmic, skip counting and double counting. All are called perceptual because the child is reliant on seeing the items.

Level 3: Figurative Composite Grouping

The child uses a multiplicative counting strategy to count items arranged in equal groups where the individual items are not visible. The child is not dependent upon direct sensory experience where he or she relies on counting by ones. (Equivalent to Stage 3 - Counting-on, in SEAL)

Level 4: Repeated Abstract Composite Grouping

The child counts composite units in repeated addition or subtraction tasks, that is, he or she uses the composite unit a specified number of times. Double counting is a common strategy at this stage.

Level 5: Multiplication and Divisions as Operations

Can regard both the number in each group and the number of groups as a composite unit. Can immediately recall or quickly derive many of the basic facts for multiplication and division, for example, three sixes. A child at this level may possess the commutative principle of multiplication ($5 \times 3 = 3 \times 5$) and/or see the inverse relationship of multiplication and division.

Appendix 7: The Learning Framework in Number

Part A	Part B	Part C	Part D
Early Arithmetical Strategies Base-Ten Arithmetical Strategies	Forward Number Word Sequences and Number Word After Backward Number Word Sequences and Number Word Before Numerals	Structuring Numbers 1–20	Early Multiplication and Division
Stages: Early Arithmetical Strategies 0 – Emergent Counting 1 – Perceptual Counting 2 – Figurative Counting 3 – Initial Number Sequence 4 – Intermediate Number Sequence 5 – Facile Number Sequence Levels: Base-Ten Arithmetical Strategies 1 – Initial Concept of Ten 2 – Intermediate Concept of Ten 3 – Facile Concept of Ten	Levels: Forward Number Word Sequences (FNWSs) and Number Word After 0 – Emergent FNWS. 1 – Initial FNWS up to 'ten' 2 – Intermediate FNWS up to 'ten' 3 – Facile with FNWSs up to 'ten' 4 – Facile with FNWSs up to 'thirty' 5 – Facile with FNWSs up to 'one hundred' Levels: Backward Number Word Sequences (BNWSs) and Number Word Before 0 – Emergent BNWS. 1 – Initial BNWS up to 'ten' 2 – Intermediate BNWS up to 'ten' 3 – Facile with BNWSs up to 'ten' 4 – Facile with BNWSs up to 'thirty' 5 – Facile with BNWSs up to 'one hundred'. Levels: Numeral Identification 0 – Emergent Numeral Identification. 1 – Numeral to '10' 2 – Numerals to '20' 3 – Numerals to '100' 4 – Numerals to '1000'	Combining and Partitioning Spatial Patterns and Subitizing Temporal Sequences Finger Patterns Five-based (Quinary-based) Strategies	Levels: 1 Initial Grouping 2 Perceptual Counting in Multiples 3 Figuratives Composite 4 Repeated Composite Grouping 5 Multiplication and Division as Operations

Source: adapted from Wright et al., 2002, p. 10.

Appendix 8: Instructional Framework for Early Number

Instructional Phase	Number Words and Numerals	Counting	Grouping
Phase 1 Emergent	▶ FNWS 1 to 20 ▶ BNWS 1 to 10 ▶ Numerals 1 to 10	▶ Count involving visible items in collections and in rows ▶ Temporal sequences and temporal patterns	▶ Early spatial patterns ▶ Early finger patterns
Phase 2 Perceptual	▶ FNWS 1 to 30 ▶ BNWS 1 to 30 ▶ Numerals 1 to 20	▶ Counting involving screened items in collections and rows	▶ Developing spatial patterns ▶ Developing finger patterns ▶ Equal groups and sharing
Phase 3 Figurative	▶ FNWS 1 to 100 ▶ BNWS 1 to 100 ▶ Numerals 1 to 100	▶ Counting-on and counting-back to solve additive and subtractive tasks	▶ Combining and partitioning using 5 and 10 ▶ Combining and partitioning in range 1–10 ▶ Early multiplication and division
Phase 4 Counting-on and Counting-back	▶ NWSs by 2, 10s, 5s, 3s, and 4s in the range 1–100 ▶ Numerals 1 to 1000	▶ Incrementing by 10s and 1s	▶ Adding and subtracting to and from decade numbers ▶ Adding and subtracting to 20 using 5 and 10 ▶ Developing multiplication and division
Phase 5 Facile	▶ NWSs by 10s on and off the decade ▶ NWSs by 100s on and off the 100 and on and off the decade	▶ Two-digit addition and subtraction through counting	▶ Two-digit addition and subtraction involving collections ▶ Non-canonical forms of two- and three-digit numbers ▶ Higher decade addition and subtraction ▶ Advanced multiplication and division