2.2 Tracking Radars

The surveillance radar provides complete volume cover around the airfield or area. However, if the output of the radar is to be used to instruct a gun system to shoot at this target, some means of supplying the three-dimensional target position to the guns must be employed. If the accuracy of the position data is important then use is made of a tracking radar. The surveillance radar will hand over the range and azimuth position of the target to the tracking radar. The tracking radar has a pencil beam antenna pattern, so that a very limited elevation and azimuth area is covered. The tracking radar will execute an elevation search until it acquires the target following which it enters a tracking mode. In this mode the radar will accurately track the target in elevation, azimuth and range, the target position then continuously being supplied to the gun system. If an output device is needed use will be made of an A-scope and possibly also an R-scope display. Skolnik (1980) provides a discussion on the various types of displays. Fig. 2.2 illustrates the A-Scope display.

Here the received echo amplitudes are plotted against range.

2.3 The Track-While-Scan System

Once the tracking radar has acquired a target it essentially locks on to and tracks the target for as long as is necessary.

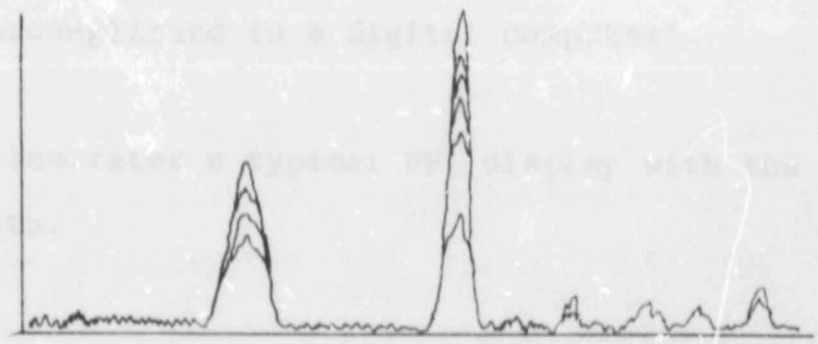


Figure 2.2 The Output of a Typical A-scope Display

This means that the radar will be used continously to track the single target for as long as it remains a threat. Other aircraft may be attacking from different directions or altitudes and if it is necessary to track these aircraft more than one tracking radar will be needed.

Similarly, if the surveillance radar is being used to monitor the air traffic around an airfield the operator's workload is decreased dramatically if target tracks are provided that will indicate to the operator which aircrat is being tracked and what their predicted positions will be.

Such a system is called a Track-While-Scan (TWS) system. It is a facility that will allow the normal surveillance operation of the radar as well as allowing target tracks to be implemented. Hovanession (1973) defines TWS as follows:

'The process of tracking targets based on discrete radar information obtained while the radar continues to scan the airspace is referred to as the Track-While-Scan process and may be accomplished in a digital computer'.

Figure 2.3 illustrates a typical PPI display with the inclusion of the TWS data.

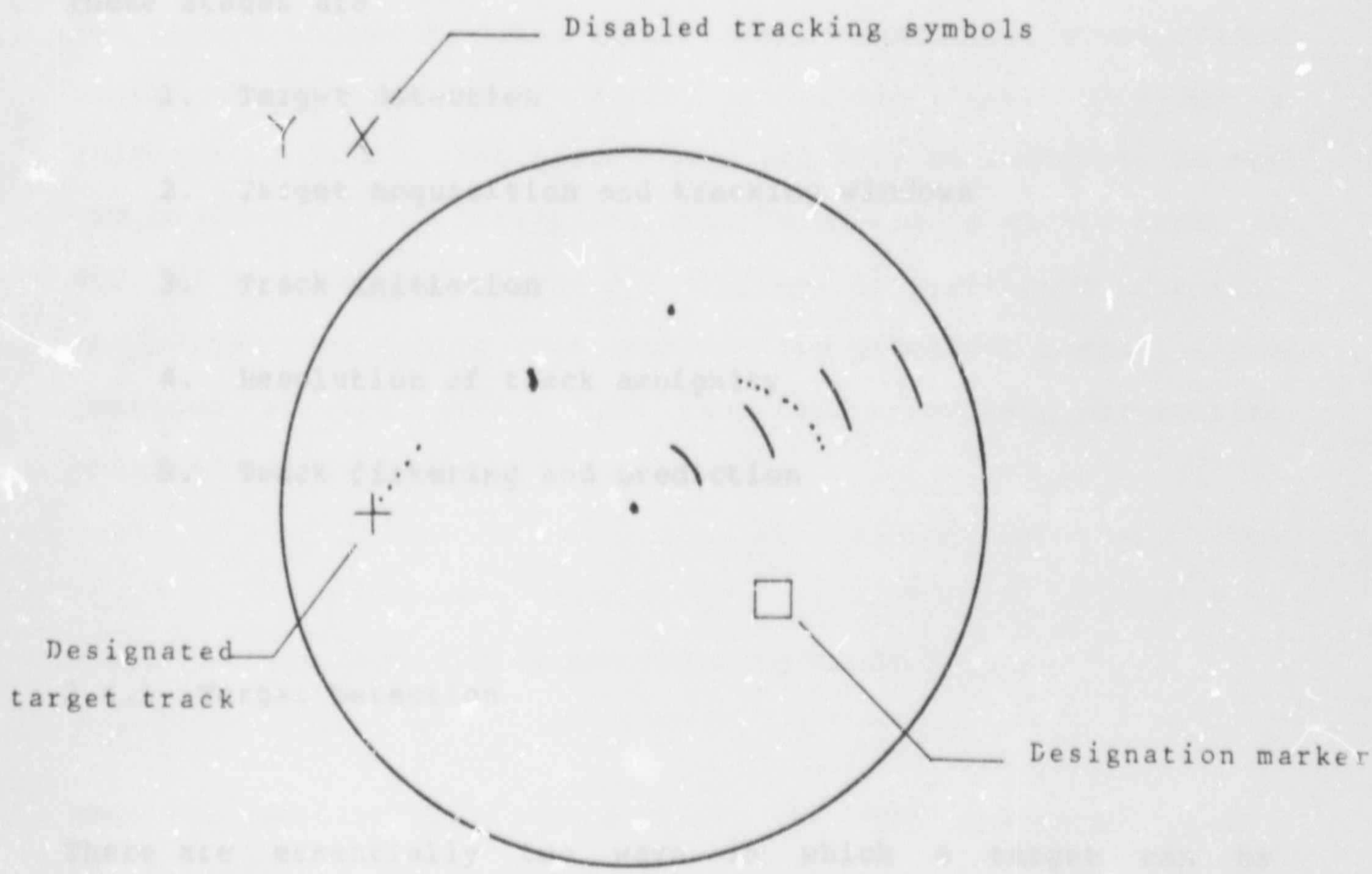


Figure 2.3 A PPI Display with TWS Symbols Included

2.4 TWS Process Stages

Various stages may be defined in the TWS process. Mayaitis (1979) identifies five stages, while Schleher (1980) identifies four stages. Essentially the two authors identify the same stages, with Schleher assuming the target to have been detected. These stages are

1. Target detection
2. Target acquisition and tracking windows
3. Track initiation
4. Resolution of track ambiguity
5. Track filtering and prediction

2.4.1 Target Detection

There are essentially two ways in which a target can be detected:
Automatic detection or manual detection.

In a system utilising automatic detection, the output from the radar is fed into a processor. An algorithm within the processor decides whether a target is present, what type of target it is and assigns this target a priority. A track is automatically initiated. If the operator decides that the track is unnecessary he may manually cancel the track.

In a system relying on manual detection, the operator plays the part of the detection algorithm. When a target is detected on the PPI display by the operator, he will ascertain the nature of the target and manually initiate a track. The target will then be tracked until the operator decides to cancel the track or, if the target is not detected by the TWS processor for a prescribed number of scans, the computer may cancel the track. In order to initiate a track the target position must be indicated to the TWS processor. The target designation marker, a cursor that is able to be moved across the PPI display, is positioned such that the target lies within this symbol. The processor reads in the position of the symbol and can then ascertain the initial position of the target.

2.4.2 Target Acquisition And Tracking Windows

When the operator initiates a track, the TWS processor inputs the position of the target designation marker and calculates the position at which the target is expected to lie. It then sets up a large volume in space called the acquisition window, the limits of this volume being calculated as follows:

Track designation symbol position (range, azimuth)

Range window (Range +/- ΔR)

Azimuth window (Azimuth +/- $\Delta \theta$)

where ΔR and $\Delta \theta$ may be set such that if the target executes a maximum expected manoeuvre at maximum velocity and acceleration the target will still fall within this volume on the following scan.

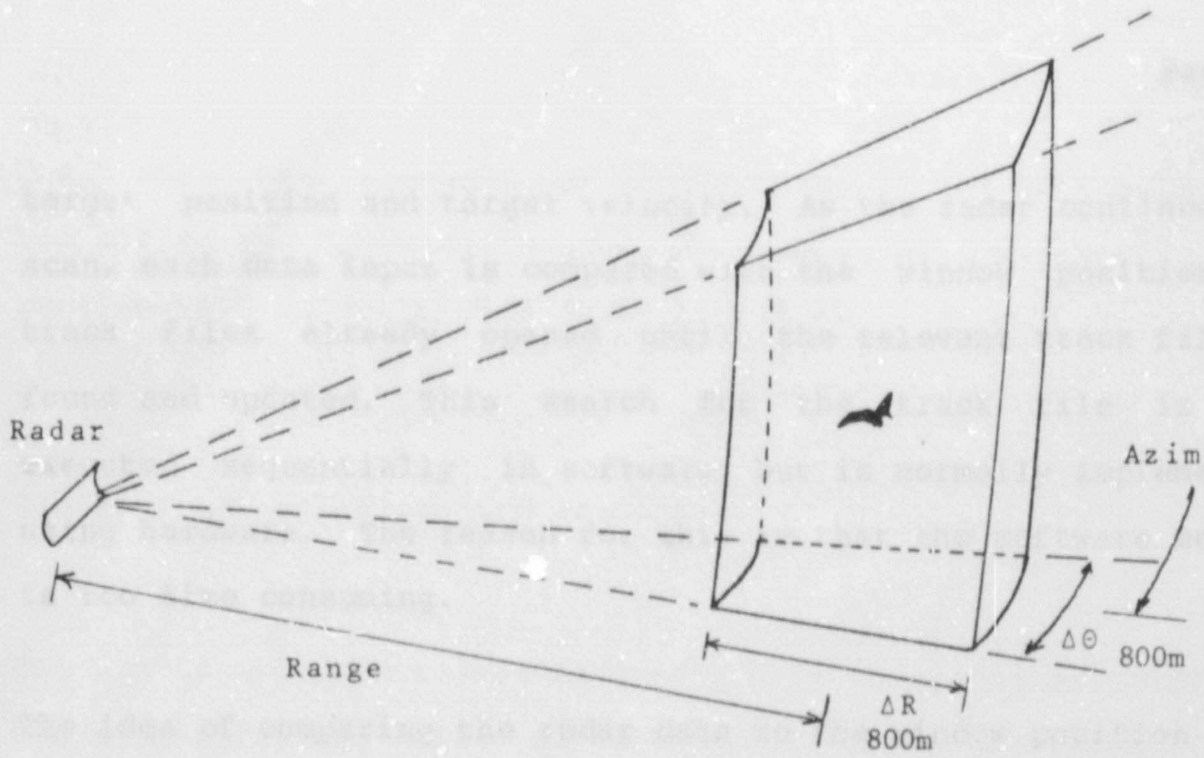
If the target does fall within the volume on the second scan, the large acquistion window may be reduced to form the smaller tracking window. The size of this window is not fixed and may vary. As the tracking filter reduces the error between the predicted and the actual measured target position it is desirable to reduce the size of this tracking window.

Figure 2.4 illustrates a view of the acquisition and tracking windows.

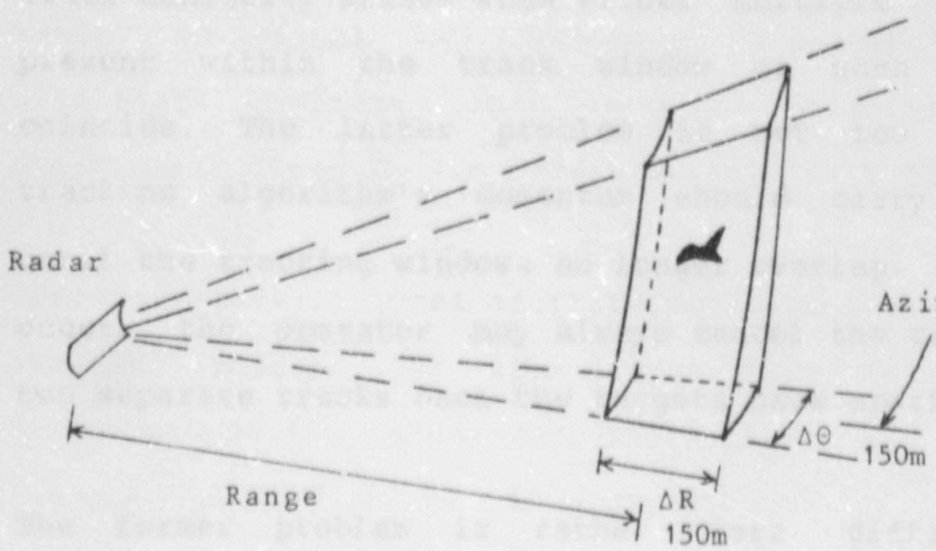
Note that the above windows are generated only for two-dimensional radars, where no indication of elevation is available. Typical physical sizes of the windows are also presented.

2.5 Track Initiation

When the operator initiates the track the acquisition window is immediately set up around the target. The processor simultaneously opens a track file in which all the relevant information associated with the track is stored: acquisition or track window position, window size, measured and predicted



(a) Acquisition Window



(b) Tracking Window

Figure 2.4 View of the Windows

target position and target velocity. As the radar continues to scan, each data input is compared with the window position of track files already opened until the relevant track file is found and updated. This search for the track file is not executed sequentially in software, but is normally implemented using hardware. The reason for this is that the software method is too time consuming.

The idea of comparing the radar data to the window position data leads to the problem of correlation of data input to the track files and how to resolve the problem of ambiguous correlation.

2.5.1 Resolution Of Track Ambiguity

Track ambiguity arises when either multiple radar echoes are present within the track window or when two track windows coincide. The latter problem is not too serious, as the tracking algorithm's momentum should carry the track through until the tracking windows no longer overlap. If this does not occur, the operator may always cancel the tracks and initiate two separate tracks once the targets have spatially separated.

The former problem is rather more difficult. Software algorithms need to be devised to extract the target position from the multiple echoes received. In the absence of intricate algorithms a simple method may be to find the 'centre of gravity' of the echoes and assume that to be the target position. The presence of false alarms is one of the reasons for making the tracking window as small as possible. This problem will be dealt with in section 4.4.

2.5.2 Track Filtering And Prediction

The position of the target as received by the radar is noisy and hence errors do exist in the position of the target. The tracking window set up in space has now replaced the antenna of the tracking radar and this tracking window must be mathematically positioned and smoothed by the TWS algorithm. This is accomplished by the use of smoothing equations which provide smoothed target position and velocity estimates. In addition these smoothed target estimates are used to provide a one-scan ahead prediction of target position. The window thus leads the target and smoothing is accomplished by comparing predicted parameters with observed parameters and making adjustments based on the errors derived from these comparisons.

There are several types of tracking filters suitable for use in the TWS system. The Alpha filter, Alpha-Beta filter, Adaptive Alpha-Beta filter, Alpha-Beta-Gamma filter and Kalman filter are examples to be discussed in section 3.

2.6 The Basic TWS System

Essentially a TWS system will form an additional unit to the radar. The processed video from the signal processor follows two paths: the first path directly to the PPI for display and the second path to the tracking system for target position extraction and prediction. The output of the tracking algorithm is fed into the PPI drive circuits for display of the TWS tracking symbol. Figure 2.5 details a block diagram of a typical TWS system.

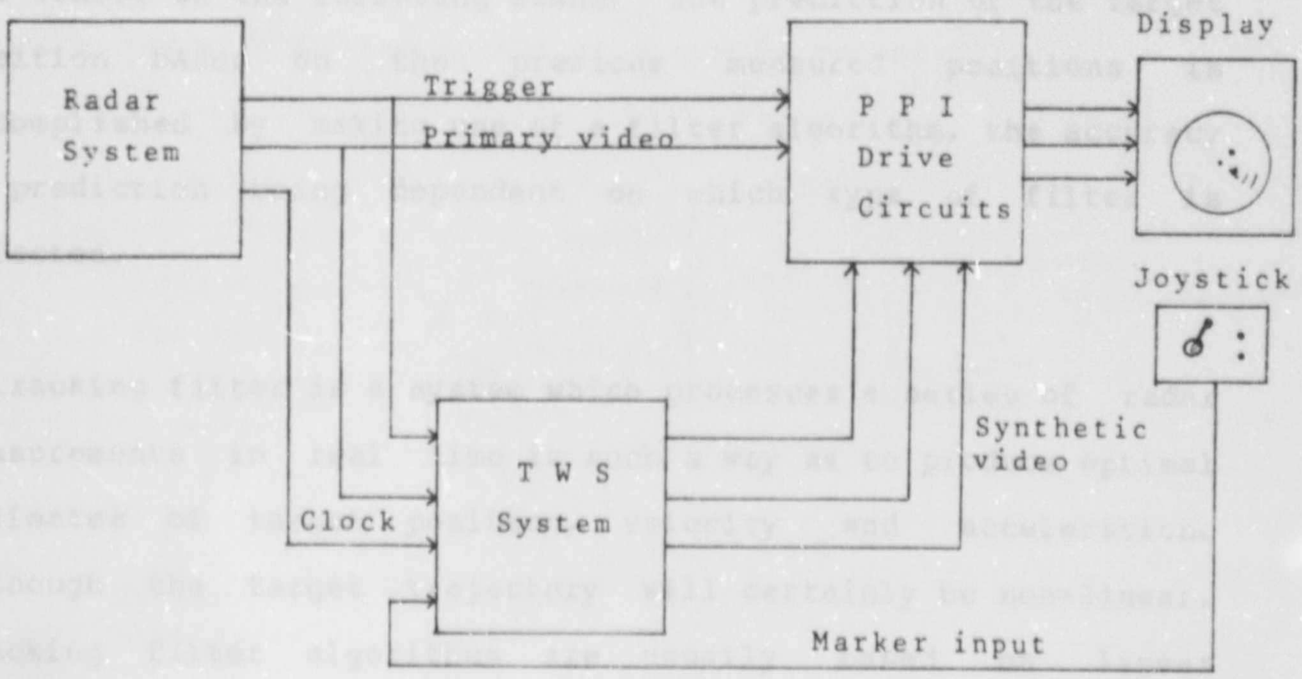


Figure 2.5 Block Diagram of the Basic TWS System

3.0 THE TRACKING FILTERS

3.1 How The Target Is Tracked

Once the track has been initiated the acquisition window is set up in space. For a two-dimensional surveillance radar the window will have a particular range, with a range spread about this value, and a particular azimuth, with an azimuth spread about this value. It is the function of the tracking algorithm to position the acquisition window and the subsequent tracking windows such that the target will coincide with the centre of the window on the following scans. The prediction of the target position based on the previous measured positions is accomplished by making use of a filter algorithm, the accuracy of prediction being dependent on which type of filter is selected.

A tracking filter is a system which processes a series of radar measurements in real time is such a way as to produce optimal estimates of target position, velocity and acceleration. Although the target trajectory will certainly be non-linear, tracking filter algorithms are usually based on linear mean-square estimation theory which provide very elegant and relatively easy solutions.

Because the radar measurements are corrupted with noise, the tracking filter is required to produce a noiseless or smoothed estimate of the current signal. In addition the filter can also predict the future values of the input signal. However, to do so requires complete knowledge of the target state vector. The measurements performed by the radar are usually not sufficient to completely determine the state of the target, the radar usually measuring only the target position whereas the state vector consists of the target position, velocity and acceleration in each of the two independent coordinates. The tracking filter is thus required to produce estimates of the unknown states of the target by using only the time series of the measured state. The problem is further complicated in that the measurements are corrupted by noise and the random disturbances of the target itself. A general schematic diagram of a tracking filter system is shown in figure 3.1.

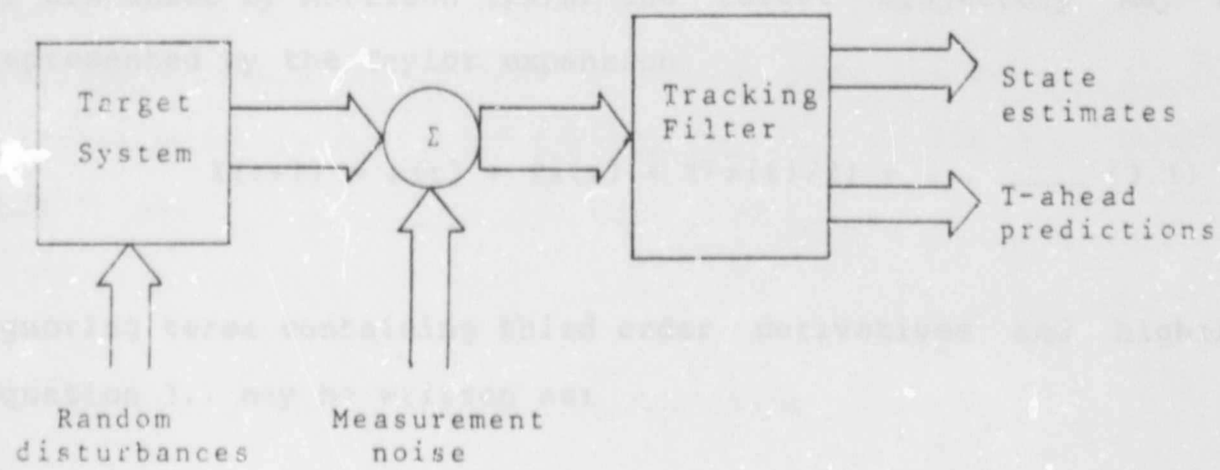


Figure 3.1 General Schematic of a Tracking Filter System

3.2 Basic Concepts Of Tracking Filters

The task of the tracking filter is to estimate current and future states of the target. Estimation of the state vector is complicated by measurement noise and random distribances of the target and hence some form of stochastic estimation must be used. Once one has an estimate of the system state X, together with the system model ϕ, one can predict the future states of the system.

3.2.1 The Tracking Model

As discussed by Morrison (1978) the target trajectory may be represented by the Taylor expansion

$$X(t+T) = x(t) + T\dot{x}(t) + T^2\ddot{x}(t)/2! + \dots \qquad (3.1)$$

ignoring terms containing third order derivatives and higher. Equation 3.1 may be written as:

$$\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 1 & T & T^2/2! \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} \qquad (3.2)$$

Equation 3.2 may be written in matrix form as

$$X(t+T) = \phi(T)X(t) \qquad (3.3)$$

$$\text{where} \quad \phi(T) = \begin{bmatrix} 1 & T & T^2/2! \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$$

$\phi(T)$ is the system model and is known as the transition matrix. Thus, given $X(t)$, the current system state, the transition matrix allows one to predict the system state $X(t+T)$ at the next measurement instant.

In the above equations the variable T is the radar scan rate. Surveillance radars are inherently sampled data systems. The target model is discussed further for each of the particular filter types.

3.2.2 Linearity And Coordinate Translations

The majority of tracking filters use some form of linear estimation algorithm and consequently linearity of the differential equations describing the target's motion become important. As explained by Morrison (1978), the trajectory of an aircraft flying past a radar with constant velocity in a straight line can be described by linear equations in Cartesian space but has a highly non-linear description in Polar coordinates as viewed by the radar. Barton (1964) refers to this problem as the "pass-course" problem.

Hence even if the target is in linear Cartesian motion with zero acceleration, it will perform acceleration when viewed in Polar coordinates. If the filtering is implemented in Polar coordinates this acceleration will have to be taken account of.

3.2.3 Filter Memory Types

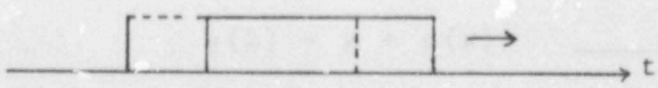
Morrison (1978) lists three basic families of filters.

The fixed memory filters take a flat window and move it in time, the distance between the back and the front of the window remaining fixed. Any observations that are on the same time axis and which land in this window are put into the algorithm to produce an estimate based on whatever is "seen", with all observations weighted equally.

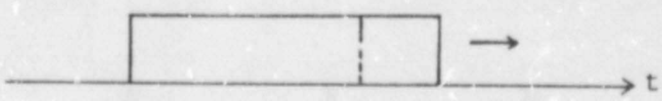
In the expanding memory filter, every observation adds to the total. The observation window, again flat, expands with time and, theoretically, any estimate gets increasingly better due to the expanded body of information.

Fading memory is based on the exponential function which is at the root of all linear systems. The observation window moves forward but is not flat. The emphasis has an exponential decay: theoretically, the tail goes to infinity but the weighting diminishes to zero. It is in this manner that observations are weighted - they are forgotten at an exponential rate.

Figure 3.2 illustrates the above filter types.



(a) Fixed memory



(b) Expanding memory

(c) Fading memory

Figure 3.2 Filter Memory Types

3.3 Review Of Tracking Filters

3.3.1 The Alpha Filter

Van Der Merwe (1981) gives a detailed account of the Alpha filter. A brief description of this filter follows.

The filter is a simple exponential smoothing filter and is used only when dealing with processes that can be modelled by a simple, constant state variable x.

$$y(k) = x + n(k) \qquad (3.4)$$

$$K=0,1,2...$$

The filter algorithms are given by:

$$\text{Smoothing:} \quad X^N_{smooth} = X^N_{pred} + \alpha(X^N_{meas} - X^N_{pred})$$

$$\text{Predicting:} \quad X^N_{pred} = X^N_{smooth} \qquad (3.5)$$

The difference between the actual measured value at interval N and the value forecast during the previous interval is the error, E.

$$E(N) = (X^N_{meas} - X^N_{pred}) \qquad (3.6)$$

Hence the estimate of X is simply the previous value of X plus the fraction of the last forecast error. (The factor is the smoothing constant and lies between 0 and 1). The prediction for the next interval is the estimate of the present interval.

Figure 3.3 illustrates the filter structure.

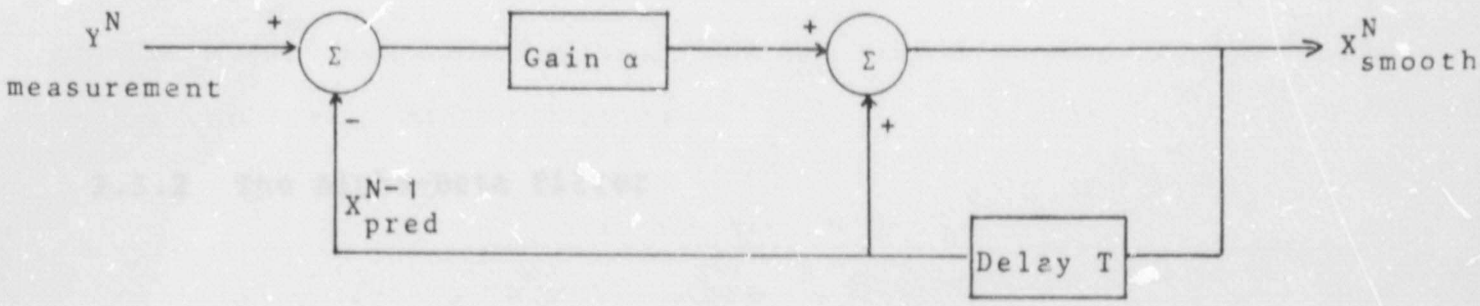


Figure 3.3 The Alpha Filter

As only one memory location is needed to store the previous estimate, the filter may be seen to have an exponentially fading memory. From Van der Merwe (198') it may be shown that the dynamic lag error due to uniform target velocity X is

$$\text{Lag error} = \dot{x}T(1-\alpha)/\alpha \qquad (3.7)$$

So for a constant velocity target the filter estimate will lag the target by the lag error.

It may also be shown that the average age of the data in the fading memory is (1-alpha)/alpha periods. Thus small values of alpha result in slow response and large dynamic lag, but good noise smoothing. Conversely if alpha is large one would obtain small lag errors but little noise smoothing.

The Alpha filter is not normally used as a tracking filter because it cannot account for uniform target velocities - i.e. it will only perform with zero error if the target has constant range and this is obviously unrealistic.

3.3.2 The Alpha-Beta Filter

This filter assumes a constant velocity target. Hence the transition matrix of equation 3.3 may be written as

$$\phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \qquad (3.8)$$

From Mayaitis (1979) the alpha-beta tracking equations may be written as

Smoothing:

$$X^N_{smooth} = X^N_{pred} + \alpha(X^N_{meas} - X^N_{pred})$$

$$V^N_{smooth} = V^{N-1}_{smooth} + \beta/T(X^N_{meas} - X^N_{pred})$$

Predicting:

$$X^N_{pred} = X^N_{smooth} + TV^N_{smooth}$$

$$V^N_{pred} = V^N_{smooth} \qquad (3.9)$$

where T = Sampling time

N = present measurement period

N+1 = following measurement period

X = position

V = velocity

The above equations imply a constant velocity target. They may be written in matrix form as

$$X_{smooth} = X_{pred} + G(X_{meas} - X_{pred})$$

$$X_{pred} = \phi X_{smooth} \qquad (3.10)$$

where $G = \begin{bmatrix} \alpha \\ \beta/T \end{bmatrix}$ and $\phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$

Figure 3.4 illustrates the general alpha-beta filter structure.

Brookner (1980) indicates that filter stability requirements dictate that

$$\alpha>0 \ , \ \beta>0 \ \text{ and } \ (4-2\alpha-\beta)>0$$

This region is graphically illustrated in figure 3.5.

It is evident from the schematic in figure 3.4 that the alpha-beta filter has the same structure as a second order servo system where alpha controls the bandwidth and beta controls the damping. Mayaitis (1979) conducts a stability analysis on the Alpha-Beta tracking filter and concludes that:

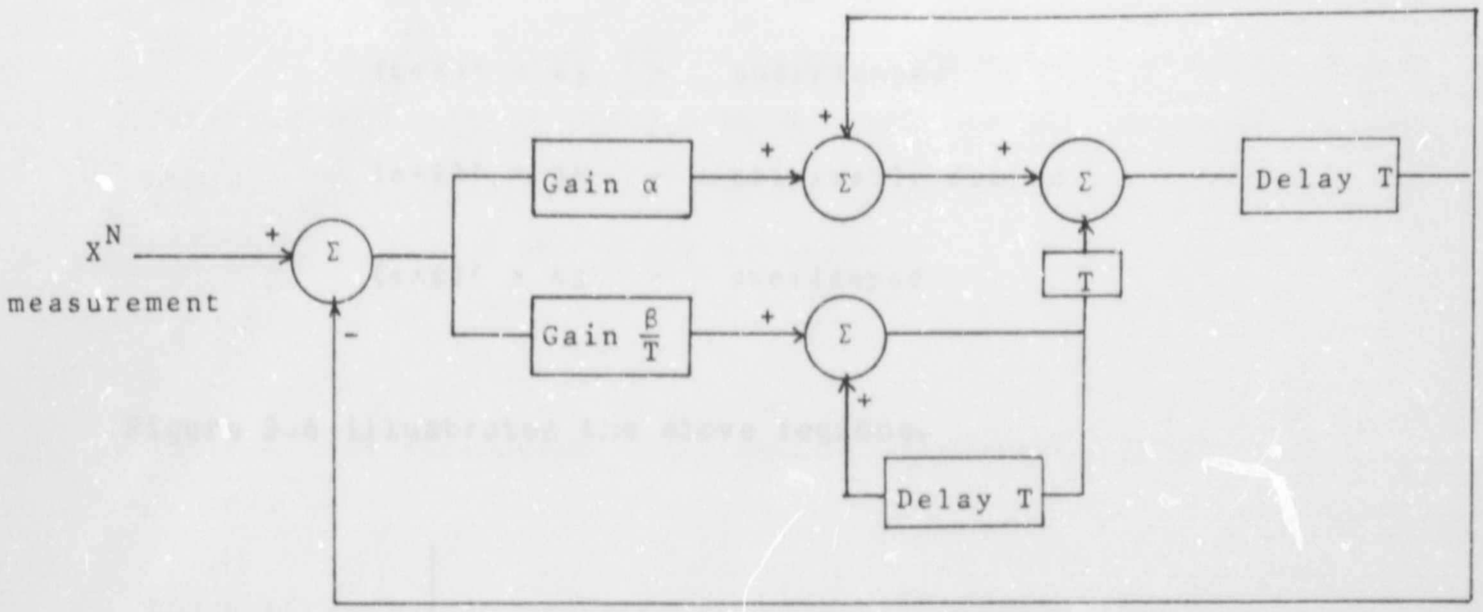


Figure 3.4 The Alpha-Beta Filter

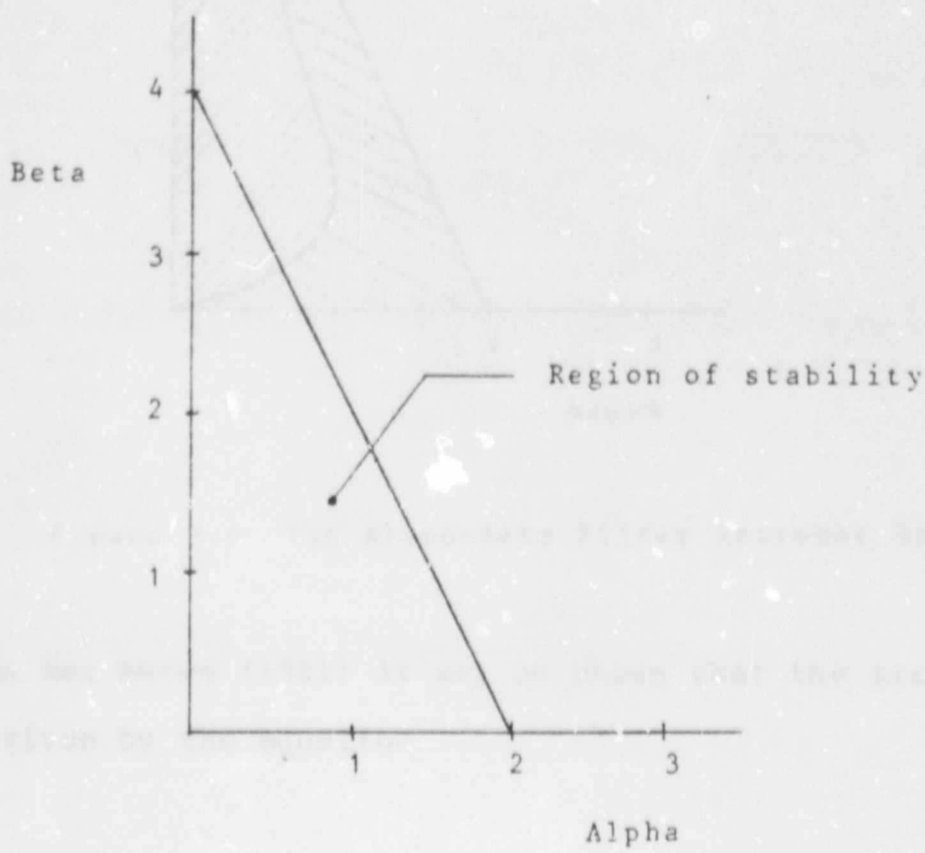


Figure 3.5 The Alpha-Beta Filter Stability Region