UNIVERSITY OF THE WITWATERSRAND

MASTERS RESEARCH PROJECT

Exploration of calculation strategies in doubling and halving with grade 3 learners.

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Research Report submitted to Wits School of Education,
Faculty of Humanities,
University of the Witwatersrand, Johannesburg
In partial fulfilment of the requirements
For the degree of Master of Education

Johannesburg, 2023

Declaration

I, **Thobile Mtsweni**, student number: **0419589p**, declare that the research project is my own work and has not been copied except for quotes where references have been used. All phrases, sentences and paragraphs taken directly from other works have been cited and the reference recorded in full in the reference list. This research project is being submitted as final preparation for Bachelor of Education Masters in Mathematics Education at the University of Witwatersrand, Johannesburg. This research project has not been submitted to any other university or likewise institution for consideration.

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Date: 15/03/2023

Abstract

A crisis reported is that the majority of learners do not achieve development in number sense. Unit counting is the preferred method of counting, and consequently, fluency and conceptual understanding of numbers are lost. This study addresses the need for early intervention that focuses on the progression of learners towards the use of more efficient strategies. Specifically, the study aimed to explore doubling and halving strategies with Grade 3 learners through an intervention to develop learners' calculation strategies using the adapted pre-test, intervention, and post-test from the Mental Starter Assessment Project (MSAP). The sample included 24 Grade 3 learners, which comprised a control group and an intervention group. The study employed Piaget's theory of cognitive development, which focuses on how learners process new knowledge. Findings indicate that before the intervention, the learners in the control and intervention groups relied on counting strategies to solve doubling and halving problems, and the alternative strategies that were used were not clear. The intervention group was exposed to the various doubling and halving representations of the strategy. However, the findings show that the intervention group performed only slightly better in the strategic calculating and strategic thinking categories than the control group. These findings indicate that a shift in learning can happen, albeit slower than expected. Further research is needed across contexts and learners to indicate ways in which the intervention could be improved.

Dedication

To my late parents, Mabuti and Thandiwe Mtsweni

Acknowledgement

I would like to first and foremost give praise and a big thank you to God who has allowed for a successful completion of my research project.

I would also like to express my gratitude to Dr. Zaheera Jina Asvat for her patience, guidance, insightful comments and motivation.

Lastly, I would like to thank my siblings for giving me time and space, I love you and thank you.

Contents

Chapter 1: Introduction

1.1 Background

The decline in mathematics results in the different grades reported by the Annual National Assessment (ANA) in 2011 and 2012, Trends in International Mathematics and Science Study (TIMSS) during the years 1995, 1999, 2002, 2011, and the Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ) in 2007 is an almost stark indication that learners are struggling with mathematics. Further results highlight the 37% of South African learners who have adequately acquired basic mathematical knowledge as compared to the 63% of learners who are lacking in basic mathematical knowledge (Reddy, Winnaar, Juan, Arends, Jaqueline, Hannan, Namome, and Ncamisile, 2019). Grade 3 learners in Quintiles 1–3 are already three years' worth of learning behind their Quintile 5 peers, and "this gap grows as they progress through school to the extent that by Grade 9, they are four years' worth of learning behind their Quintile 5 peers" (Spaull and Kotze, 2015, p. 23). These results indicate that a large percentage of learners are not likely to progress from the basic acquisition of mathematical concepts to more advanced concepts, which include manipulation, reasoning, and explanation of mathematical concepts. The vast majority of learners do not progress from the use of concrete to the use of abstract (Schollar, 2008; Weitz and Venkat, 2013). Below, I illustrate an example from the Annual National Assessment (ANA), which shows one learner working out for "double, 34" and "half, 34".

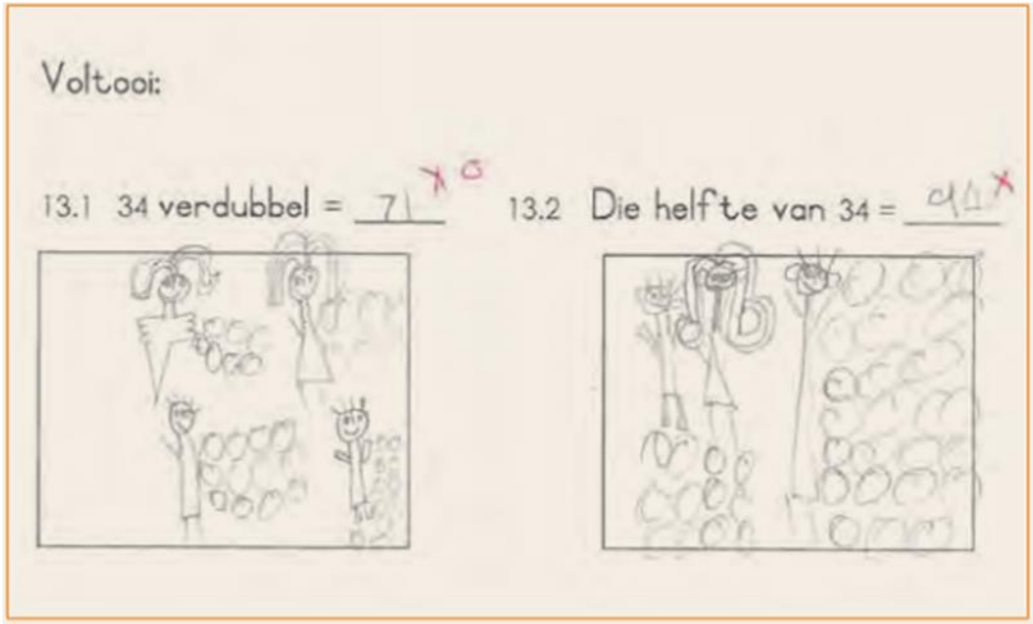

FIGURE 1: FIGURE 1: EXAMPLE OF COUNTING IN DOUBLING AND HALVING ANA SCRIPT

The learner used a form of tallying using circles, but s/he could not solve the problems. There are multiple reasons for this, and language could be one. However, it should also be highlighted here that doubling and halving are not recognised in curriculum documents as calculation strategies to be mastered on their own but are rather embedded within problem solving and mental math. In accordance with curriculum expectations, learners in grades one to three are expected to understand various meanings of doubling and halving, to solve problems, and to explain solutions to problems (DBE, 2011b). How do learners solve problems related to doubling and halving when they lack an understanding of this strategy?

The Mental Starters Assessment Project (MSAP) has developed tasks and assessments in response to them not being taught in schools (Graven and Venkat, 2021) to help learners develop structural understanding in strategies like doubling and halving. In this study, I have adopted the Mental Starters Assessment Project (MSAP) to address the concept of doubling and halving with grade 3 learners. I now give context to this project. Mental Starters Assessment Project (MSAP).

The project is aimed at helping grade 3 learners progress from counting in ones on their fingers or with tally marks on paper to strategies and skills that support number sense. Counting using concrete means is time-consuming and leads to errors, especially when the number range increases. The project includes six mental mathematics lesson starter units, and each unit focuses on a different calculation strategy that covers a particular group of connected skills. These skills include three types of calculations: fluency, strategic calculation, and strategic thinking. Strategies are paired with the application of a range of mental models that support a structural understanding of numbers. Emphasis is placed on the basic facts of the strategy as well as on using calculation strategies flexibly and efficiently. These strategies include bridging through tens, jump strategies, doubling and halving, rounding and adjusting, re-ordering, and linking addition and subtraction. Each unit is three weeks long and includes a pre-test, a set of intervention lessons, and a post-test. Information regarding the MSAP is readily available on the Department of Basic Education website (see: Mental Starters Assessment Project (education.gov.za). In this study, I worked through the lesson starters for the 'doubling and halving' unit with a group of grade 3 learners. In chapters 4 and 5, I report on the improvements in learners' performance from the pre- to post-tests. It should be noted that these improvements show progress in mental mathematics skills and number sense.

1.2 Research Questions

The purpose of this study was to explore the use of the doubling and halving strategy with Grade 3 learners with emphasis on understanding learners shift from counting to calculating strategies using an intervention. The following questions framed my study:

1. Is there any mean difference in performance between the intervention group and control group?
2. What strategies did learners use prior to the intervention in the pre-test?
3. What strategies did learners in the intervention group use after the intervention in the post-test?
4. Were there any shifts from counting strategies to calculating strategies after the intervention process in the intervention group

1.3 Rationale

Concrete counting is fundamental in the primary stages of developing number sense; however, learners need to develop towards more "flexible and efficient strategies as they work with larger numbers and move up the grades" (Graven and Venkat, 2021, p. 24). The reason for the poor performance as reported is that many learners do not achieve this development in number sense (Graven and Venkat, 2021; Schollar, 2008). Unit counting is the preferred method of counting, and consequently, fluency and conceptual understanding of numbers as being constituted as part-part-whole relations are lost. This study addresses the need for early intervention that focuses on the progression of learners towards the use of more efficient strategies. A similar study was conducted by Graven and Venkat (2021), which focused on the 'bridging through ten' strategy. The MSAP has made these strategies available to all; however, there has not been much research reported on their implementation in schools. I have therefore attempted to adapt Graven and Venkat's (2021) study on the implementation of the calculation strategies relating to doubling and halving as a context for my study.

1.3.1 A different setting and yet so similar

Graven and Venkat (2021) conducted a diagnostic assessment for strategic calculation focused on the 'bridging through ten' strategy from the MSAP with the aim of moving learners from the use of concrete counting methods to a conceptual understanding of number structure.

Data was collected from two government schools, one in Gauteng and the other in the Eastern Cape. A pre-assessment, lesson starter, and post-assessment cycle was administered over a period of 2-3 weeks. The authors employed the calculation strategies[1] of rapid recall, strategic calculation, and strategic thinking, which they adopted from Askew's (2012) study:

[1] These calculation strategies are common across all strategies addressed in the MSAP. The authors have addressed these strategies with the 'bridging through ten' strategy. I have employed them using the 'doubling and halving' strategy.

- Rapid recall: facts are automatically recalled, for example, $27 + 40 = 67$. The next ten is: 70; $63 + 7 = 70$; $5 = 3 + 2$; $70 + 2 = 72$.
- Strategic calculating: where the focus is on efficient working that pays attention to number relationships and/or the known facts at the level of fluent recall to derive results, for example, making up the multiple of ten- speedily and confidently being able to say $36 + 4$; $58 + 2$; $67 + 3$.
- Strategic thinking is focused on understanding and using the structure of numbers. For example, given $43 + 138 = 181$, what is $181 - 43 = \_$?

The results indicate an overall positive gain in the post-assessment as evidence of the intervention being effective in improving learners rapid recall, strategic calculation, and strategic thinking through the 'bridging through ten' strategy.

1.3.2 The 'Doubling and Halving' Strategy

An analysis of the doubling and halving strategy within the curriculum documents reveals that the doubling and halving strategy is embedded within the topics: solving problems in context, solving problems that are context-free, and performing mental calculations. This means that doubling and halving are not recognised within the foundation phase curriculum as concepts to master on their own. The CAPS document further fails to provide suggested ways of implementation but rather assumes that learners would be able to use doubling and halving as a strategy to solve problems.

The MSAP has designed intervention lessons to get grade 3 learners to work through the doubling and halving concept to improve their mental mathematics skills. The MSAP in comparison demonstrates how the doubling and halving strategy may be mastered using rapid recall, strategic calculation, and strategic thinking. Learning, according to the MSAP, is progressive in the sense that it gets grade 3 learners to move on from counting in ones on their fingers or with tally marks on paper to the mastery of strategies and skills to support number sense. The emphasis in the MSAP lies in the objective of mastering specific strategies to enable the solution of problems.

With reference to the research discussed above, I investigate whether learners can improve their doubling and halving strategies after progressing through a short intervention. I report on the findings of the implementation of the '"doubling and halving'" assessment and lesson starter cycle across two groups of grade 3 learners in a primary school in the Gauteng province. I wanted to see if there were differences that could be related to the starter lessons.

In this study, I draw on constructivist approaches, which provide insight into how learning and development take place within the mind of the learner. This study can be of benefit to teachers in the foundation phase in their approach to unit counting and number structure. It could provide teachers with knowledge on what learners need to know to acquire fluency, strategic thinking, and strategic calculating in their enactment of the doubling and halving strategy. Findings from this research could provide useful data that may serve as a basis for further investigation to explore learners' thinking when adopting the strategies learned. This study will also give insight into the role of different strategies that learners can use to progress from unit counting to a more conceptual understanding of number sense.

1.4 Structure of Research Report

Chapter 1 gives an introduction to the study and details the rationale and questions that framed the study.

Chapter 2 reviews the literature on doubling and halving and outlines the intervention lessons, the use of representations, and an explanation of the theoretical framework of the study.

Chapter 3 provides an explanation of the methodology and the administration of the pre-test, intervention, and post-test. Furthermore, I explain the sample group that made up the intervention and control groups and also explain how the data was analysed.

Chapter 4 analyses the data that was gathered and discusses the findings.

Chapter 5 is a presentation of the conclusion of the study, answering the research questions that framed the study and the limitations of the study with recommendations and consideration for future research.

Chapter 2: Theoretical Framework and Literature Review

2.1 Introduction

In this chapter, I will discuss the theoretical framework and related literature that has guided my research. The framework is based on Piaget's theory of cognitive development. The framework will assist me in trying to understand how the learners worked with the starter lessons and whether and how they shifted in their thinking from the pre-test to the post-test. I will also review related literature on doubling and halving strategies and calculating/thinking strategies in relation to doubling and halving which form part of the analytical framework used within this study. I then explore discursive demands in a second language classroom, which is pertinent for discussion within South African schooling.

2.2 Piaget's Theory of Cognitive Development

Piaget's theory of cognitive development focuses on building cognitive structures, which set the basis for how learners process new knowledge. Piaget termed these processes assimilation and accommodation and described them as pivotal notions that must be considered when learners go through the process of conceptual development in building cognitive structures. Naire (2014, p. 104) explains that "assimilation is the process through which we fit into or assimilate new experiences." "Young learners have experienced the concept of halving in their everyday lives, and therefore the first intervention lesson using concrete objects will allow the learner to assimilate this experience with previous knowledge. [Whereas] ""accommodation is the process through which we change or modify existing schemata to accommodate new experiences," "as learner's progress from the concrete to the abstract representation of doubling and halving, learner's cognitive structures ""accommodate" " the new knowledge, which systematically changes the learner's cognitive structure as they progress through the different lessons. Kazak, Wegerif, and Fujita (2015) note that a dialectical link exists between these processes. The interaction between the two concepts (assimilation and accommodation) contributes to learners' conceptual development by giving learners the opportunity to incorporate newly constructed knowledge through the process of accommodation, constructing new schemata in them (Piaget, 1964). Learners' exposure to the concrete representation of doubling and halving arranges their thoughts into what Piaget referred to as a schema. As learners are introduced to the doubling and halving strategy, learners tap into their already existing knowledge to incorporate new knowledge in the form of concrete, pictorial, or abstract representation. From the inception of the starter lessons, learners are required to ‘fit’ or assimilate the new information into what they already know about doubling and halving; consequently, they are accommodating more abstract ways of using the strategy. Continuation of teaching in this manner alters the learner’s knowledge base or schema about the use of doubling and halving, adding to the learner’s schema about the use of the doubling and halving strategy.

2.3 Counting Strategies

Counting strategies are used by learners with the aid of concrete objects such as fingers or physical objects (Betts, 2015). Baroody (1987) found that "“developmentally, the most basic strategy is *concrete counting all*... Fingers (or other countable objects such as blocks) are counted out one by one to represent an addend; the process is repeated for the other addend” “(p. 141). Clement and McMillen (1996) reiterate that learners who are learning to add must symbolise the addends with concrete objects. Carpenter and Moser (1984) identified addition and subtraction as early strategies that learners use when solving simple math problems. Therefore, it can be argued that learners as early as pre-school enter grade 1 with already existing counting strategies that can be further developed (Baroody, 1987). These strategies work well, and many learners continue to use them; however, it should be noted that counting in ones is time-consuming and can lead to errors and misconceptions in higher grades. The ideal would be for learners to progress on to calculation strategies.

2.4 Calculation Strategies

Thompson (1999) states that calculation strategies do not require learners to use concrete objects, but mental strategies applied to problem solving. What needs to be noted in this section is that calculating has been termed differently over the years in different literature. Mansson (2021) found that there are varying terms such as mental computation, mental calculation, mental math, and mental arithmetic that can be grouped as one. Mansson further found that the varying terms have the same underlying meaning, which is calculating without concrete objects. Threlfall (2002) found that ""in the school context [mental calculations] are translated into short-term curriculum objectives focused on skills, in particular being able to calculate accurately and reasonably quickly over a range of demands seen to be appropriate to the age of the pupil. By the late primary years, this usually includes mental calculations with two- and three-digit numbers" "(p. 30). According to Thompson (1999), ""mental strategies are more about the application of known or quickly calculated number facts in combination with specific properties of the number system to find the solution to a calculation whose answer is not known" "(p. 2). Mansson (2021) states that to be flexible in calculating, it's important to acquire a variety of strategies and be cognisant of the applied strategy to a problem. Strategies differ in complexity and are used differently in problems to ""help develop mental processes that enhance logical and critical thinking, accuracy, and problem-solving that will contribute to decision-making" "(DBE, 2011a, p. 8).

Murphy's (2004) study provides an analysis of children's use of a learned mental calculation strategy. Murphy conducted a study with three children ranging in age from 8 to 9 with """average ability" "in mathematics... using diagnostic interviews to determine the contrasting use of deductive mental calculation strategies (p. 5–6). Murphy's findings show that before the teaching of the compensation strategy with multiples of 10 to learners, "demonstrated contrasting spontaneous calculation approaches in the pre-teaching interviews" (p. 16). However, after the instructional teaching, the children showed some use of the compensation strategy with multiples of ten. This study supports lesson eight of the intervention, where learners are introduced to the use of compensation (double $39 = 40 + 40 - \_$) and are expected to add the addends and then subtract to maintain the balance.

2.5 Doubling and Halving

Wallace and Gurganus (2005) state that strategies that are used to solve multiplication problems are linked to repeated addition and doubling. This is seen in a study conducted by Beak (2005), who worked with 58 learners in a primary school with a focus on learners who developed their own strategies to solve multi-digit multiplication problems. The author's findings show an example (see Figure 3) of a learner who devised a doubling strategy and used the thinking that is associated with doubling to explain the following problem: “An elementary school has 24 classes. If there are 32 children in each class, how many children are there at the school?” The learner repeatedly doubled the previous number of children and classes until she reached 512 children and 16 classes. She stopped doubling at this point because she knew that if she doubled again, she would have 32 classes, which was more than the 24 classes in the problem. Instead, Lauren added 256 children for 8 classes that she had figured out in the process of doubling and reached her final answer. The author alludes to the fact that the use of the doubling strategy ‘significantly shortened the calculation procedure but also demonstrated a more sophisticated creation of units’ (p. 244).

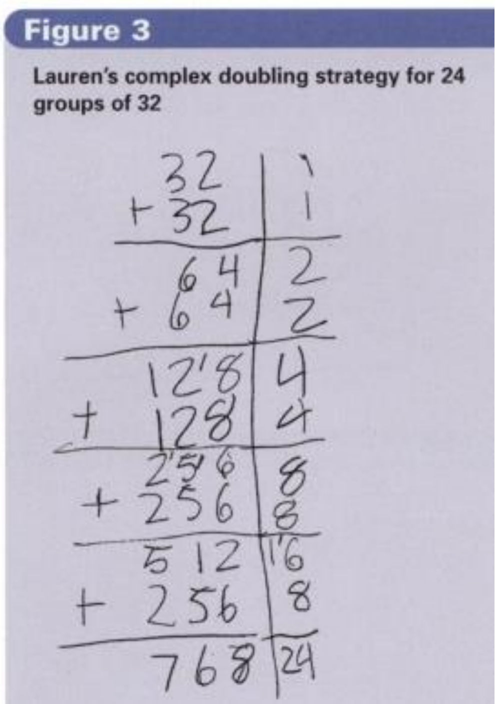

Figure 3: Learner’s response using a complex doubling strategy, Beak (2005, p.244)

Whereas, the halving strategy is rooted in partitioning, where the value of whole numbers is partitioned from the whole number into units. Downton (2008) states that in “partition division (commonly referred to as the sharing aspect) the number of subsets is known, and the size of the subsets is unknown” (p. 171). The halving strategy considers the whole number that is known, but the value of the partitioned value is not known. Pearson, Baker, and Smith (2013) described partitioning as the ability to think of a number as made up of two parts. The use of the halving strategy is seen in Anghileri's (2001) study, which provides learner approaches to solving division problems that stem from 'informal intuitive procedures to the application of standard algorithmic procedures' (p. 85). Anghileri (2001) conducted the study with primary school learners from 10 different schools using tests 1 and 2 with various division problems. Anghileri's findings show that learners display the use of what he refers to as 'low-level' strategies like tallying and direct addition of the divisor in problems that have large numbers. However, other learners used 'high-level' strategies, which among the categories of strategies listed in the findings was '[h]alving the divisor or the dividend. Working with small multiples of the divisor (low-level chunking) gained some efficiency but generally led to long calculations. This was also true for strategies involving halving and doubling, and 7, 8, and 9 were classed together as 3 (L). For some calculations (e.g., $64 \div 16$) halving and doubling were very efficient approaches, and it was not appropriate to classify these as low-level. In these cases, doubling and halving were included in 4 (H). Doubling and halving are intertwined notions that are stated in the curriculum as being inverse (DBE, 2011). French (2005) states that [d]oubling and halving, represent an inverse relationship between multiplication and division. This is echoed by Hope and Sherill's (1987), who illustrated the inverse relationship notion using the following example: "[o]ne [learner] reported that to calculate 12×16, "I did doubling and halving again. I did half of 12 is 6. Doubling 16 gives 32. And 6 times 30 is 180, so 192".

While the use of the doubling and halving strategies is elicited from learners solving various problems and showing their thinking, Flowers and Rubenstein (2010) developed a sequence of mental math problems to develop competency with the doubling strategy. They offer this sequence to shift learners thinking from using repeated addition to also "seeing" and using the doubling strategy as a "single replicating process" (p. 298). The author's sequence, as seen in Figure 1, presents a variety of problem sets, which are outlined from the simplest doubling problem to the most complex. The authors further suggest that in teaching the different problem sets, the teacher can follow the following suggestions:

1. Work on the sets in regular, brief sessions rather than too much at once.
2. Mix up the items within each problem set until you are sure that the student can complete the set easily.
3. Include a brief round of doubling at the outset of the lesson and another brief round at the end if you are teaching other topics.
4. Ensure that students are fluent with one problem set before moving on to the next.
5. Begin by doing the sets sequentially.
6. Ensure that students say aloud how they reasoned when completing the more challenging problems. Sharing one's thinking benefits the speaker, who clarifies what he has done; the listener, who is challenged to follow another's reasoning; and the teacher, who can learn the strategies that students are using.
7. Vary the language you use and connect the language to the equal group meaning of multiplication. For example, with Problem Set 2, you might say, “What are 2 eights?” “How much is in two groups of size 8?” “What is twice eight?” “What do you get if you double eight?” In particular, we minimize the simple use of “What is two times eight?” because it masks the mathematical process of doubling. (p.299)

The implementation suggestion gives “direction” on how the doubling strategy should be implemented, keeping track of the connection that enables learners to progress when working on different problems that range in complexity.

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Fig. 1 This sequence of mental-math problems allows students to develop competency with doubling.

Problem Set 1
Double digits 5 or less and 10: 1, 2, 3, 4, 5, 10

Problem Set 2
Double digits between 5 and 10: 6, 7, 8, 9

Problem Set 3
Double multiples of 10 to 50: 10, 20, 30, 40, 50

Problem Set 4:
Double small numbers in early decades: 11–15, 21–25, 31–35, 41–45

Problem Set 5:
Double multiples of 10 over 50: 60, 70, 80, 90, 100

Problem Set 6:
Double 5s in later decades: 55, 65, 75, 85, 95

Problem Set 7:
Double large numbers in early decades: 16–19, 26–29, 36–39, 46–49

Problem Set 8:
Double large numbers in later decades: 56–59, 66–69, 76–79, 86–89, 96–99

Figure 1.1: Sequence of mental maths problems, Flowers and Rubenstein (2010, p.298)

In reflection on the strategies employed above, doubling and halving can be employed to solve computations involving other computations. It is for this reason that the curriculum has also embedded doubling and halving as strategies to be used in the solution. My study, however, focuses on doubling and halving as concepts to be understood and worked within its own entirety. The Foundation Phase math curriculum states that "[doubling and halving] is quite sophisticated and requires a strong number sense. Learners who are able to choose this technique are quite flexible in the strategies they use. Knowing how to double will allow learners to use the strategy of near doubles" (DBE, 2011, p. 346). I will re-enforce here that the curriculum does not address doubling and halving as concepts to be mastered on their own but has included them as a technique to use when solving problems and performing mental operations. The MSAP has designed starter lessons to promote fluency, calculating and thinking strategies involving doubling and halving.

2.6 MSAP Doubling and Halving Starter Lesson Intervention

The MSAP offers eight starter lessons that should be presented to grade 3 learners. These include the progression from the concrete to abstract representation to build understanding of the use of the doubling and halving strategy. Prior to teaching the strategies, each intervention lesson begins with a 1-minute mental warm-up. Codding, Burns, and Lukito (2011) state that at the core of learning mathematics is the development of mathematical fluency. The authors focused on the mental analysis of mathematical basic-fact fluency intervention and found that the practice of modelling and drill is effective in developing fluency. The progression of the intervention lessons further develops learners' strategic calculating and conceptual thinking. In a study conducted by Johnson et al., (2001) on developing conceptual understanding and procedural skill in Mathematics: An Iterative process, they found four ways that procedural and conceptual knowledge are connected to advance each knowledge base. Firstly, they found that procedural knowledge may be used to recall conceptual knowledge. Secondly, learners' progression in the use of procedural and conceptual knowledge in solving problems gives them an opportunity to adapt newer processes to solve problems. Thirdly, the use of advanced procedural knowledge may lead learners to identify misconceptions in their previously used procedures, which will assist them in further developing their conceptual knowledge. Lastly, reflection on procedural and conceptual knowledge enables learners to obtain a deeper understanding of why a procedure worked when working out the problem. Askew (2012) identified fluency, reasoning, problem solving, and understanding as core mathematical proficiency. Graven and Venkat (2021) adapted Askew's categories and re-categorised them into fluency, strategic calculating and strategic thinking.

The first lesson in the MSAP on doubling and halving introduces the use of fingers and a concrete counting tool. Learners use their fingers to mirror match the number the teacher shows, for example, showing three fingers $3 + 3 = 6$. The lesson progresses to pictorial representation, which involves the use of a dot card to represent concrete objects. The second and third lessons continue with pictorial representations, where learners move from using the dot card to a dot strip and long strips, which also stand for 10. Both the 10-dot strip and the long strip introduce learners to working with multiples of 10. The fourth lesson continues with the use of the 10 dot and long strip to reinforce the use of multiples of 10, and then learners use the breaking down method using the doubling strategy. The use of the breaking-down method is demonstrated at an abstract level using numbers and mathematical symbols. In lesson five, learners continue using the breaking-down method with a focus on the halving strategy. Both lessons four and five use the doubling and halving strategy using two-digit numbers. In lesson six, learners are introduced to the different representations of doubling and halving, where they articulate different ways of representing, for example, double $26 = 52$. Lesson seven goes back to the breaking down method, where learners practice using the breaking down method using both doubles and halves in multiples of 10. Lesson eight is modelled using a web fact to build connected facts related to a given doubling and halving number fact, including near doubles. The eight lessons are designed to help learners move from concrete forms of counting and calculating to abstract forms that include fluency, strategic thinking, and strategic fluency. With the progression of the lesson, learners experienced processes of assimilation and accommodation of new knowledge. The pre-test and my interviews with them illustrate whether they made the necessary shifts.

2.7 Multiple Representation

Duval (2006) states the importance of learning mathematics with representations. Fennel (2001, p. 292) states that "[r]epresentation is [a] mathematical process… [that] clarifies mathematical ideas to students, to access students' mathematical thinking, and to help students translate mathematical ideas in a form that they can mentally or physically manipulate to gain understanding". Mcintosh, Reys, and Reys (1992) mention the importance of multiple representations as an aspect of number. The authors state that numbers can be depicted in a variety of symbolic and/or pictorial representations and appear in a range of contexts. Understanding numbers requires being aware that they may be manipulated in a variety of ways to meet a specific purpose. By way of illustration, MSAP (2020) exemplified the following: recognising that double 29 is the same as 29×2. Fennel (2001) states that it is easier for learners to understand a mathematical problem when they can engage with it in a way that makes sense to them, for example, the use of fingers to visually comprehend double 5 or the use of tally marks, which are symbolic. The use of different representations, whether they be drawings, mental images, concrete objects, or equations, enables learners to structure their thinking process in such a way that they are able to apply different solutions to the problem. Tripathi (2008) maintains that [c]onnecting multiple representations (such as manipulatives, spoken words, symbols, and actions) helps to build understanding and to connect the [learner]'s own concrete and symbolic mental representations, all while they are learning to use the tools to solve problems. The use of different representations shifts learners away from dependency on using the same strategy to solve problems that vary in complexity.

The intricacy of conceptual development requires the teacher to be aware that learners acquire the use of strategies such as doubling and halving in developing conceptual understanding differently. Dreher, Kuntze, and Lerman (2015) state that numerous research studies have found that giving learners access to different representations does not, in and of itself, promote learners learning as learners find it challenging to integrate and connect the various representations. A lack of coherence may result in learners using the same strategy or using the same representation to solve problems, as stated by Scott, Mortimer, and Ametller (2011), who highlight the importance of gaining a thorough understanding of mathematical ideas that require the ability to connect various modalities of representation. The reader is reminded that doubling and halving are embedded within other areas of the curriculum and not addressed as a concept of their own. The use of multiple representations to deliver this concept to grade three learners will assist learners in conceptually understanding doubling and halving. The level of complexity in each stage of the [doubling and halving strategy] from using visual objects to the most complex, which is abstract thinking to solve the problems, requires the teacher to maintain the connection between the different representations that progress from the use of concrete objects, words, numbers, and symbols. This means that mathematical language is foregrounded in mastering the concepts of doubling and halving.

2.8 Mathematics Language

At the core of teaching and learning mathematics is the use of mathematical language, which develops an understanding of mathematical concepts such as the doubling and halving strategy. Cody and Taylor (2009) state that there is an assumption that mathematics is a universal language and is understood across different cultures, despite the background knowledge learners bring to class, as mathematics is made up of numbers and symbols. Adams (2003, p. 786) argues that "[mathematics] is a language of words, numerals, and symbols that are… interrelated and interdependent and at times disjointed and autonomous". The Foundation Phase Math Curriculum states that "mathematics is a language that makes use of symbols and notations for describing numerical, geometric, and graphical relationships…[i]t helps to develop mental processes that enhance logical and critical thinking, accuracy, and problem solving that will contribute to decision making" (DBE, 2011, p. 8). Furthermore, the Foundation Phase Maths Curriculum has outlined specific skills that are needed by learners to develop essential mathematical skills, and learners should:

- Develop the correct use of the language of Mathematics.
- Develop number vocabulary, number concepts, calculation, and application skills.
- Learn to listen, communicate, think, reason logically and apply the mathematical knowledge gained.
- Learn to investigate, analyse, represent, and interpret information.
- Learn to pose and solve problems; … (p.8-9).

Embedded within each of these skills is mathematical language and symbolism, and although the language can be acquired, it's still complex for foundation phase learners. Symbolism is regarded as a main characteristic of mathematics (Rubenstein and Thompson, 2001). Adams (2003) highlights the importance of the construction of the meaning of symbols within a mathematical context. The author states that mathematical symbols in problems are used as a form of identification in the four mathematical operations (addition, subtraction, multiplication, and division). However, Brown, Cady, and Taylor (2009) argue that the commonality of symbols and numbers in different cultures is slightly different in how they are conveyed, which can result in inconsistency in how symbols and numbers are used when learning mathematics. For example, "[i]n English, the numeral 10,000 is read as "ten thousand" and 100,000 is read as "one hundred thousand. In Korea… "man", indicates 10, 000 and 100,000 is read as "ten-man", misrepresentation of numbers could negatively affect learners' conceptual development. The author further states that it is pivotal that learner background knowledge be known by the teacher, as learners whose mother tongue is not English may find difficulty identifying various ways mathematical symbols are utilized in that context. Rubenstein and Thompson (2001) stress that mathematical growth will eventually be impeded by students who are unable to communicate using conventional symbols.

Part of mathematical language is the use of mathematical vocabulary. Pierce and Fontaine (2009) state that "proficiency in mathematics has increasingly hinged upon a child's ability to understand and use two kinds of math vocabulary words: math-specific words and ambiguous, multiple-meaning words with math denotations. Barrow (2014) reiterates that vocabulary has multiple meanings of words, such as, for example, when doubling and halving are used in a mathematical context, their original meanings are altered. Adam (2003) identified that the meaning of words in the mathematical context and the everyday context differ. Similarly, Jones, Hopper, Knott, and Evits (2008) note that to understand a given [mathematical] problem, it is pivotal that learners recognise and comprehend the vocabulary and language structure. Goldstein and Randolph (2017) state that learners' mathematical vocabulary is acquired best by showing how words relate to other concepts, as exemplified by Overbeek and Sondergaard (2011), who first differentiate between "half" and "halve". The authors state that the former is a noun, and the latter is a verb, meaning that the use of the words functions differently. The authors state that the use of the word half in real-life examples requires learners to recognise that, for example, if they have a best friend and like to share things equally with him or her, then [they] will be splitting things in half to share. Another example states: If the learner has R10 and gives half to the sister or brother, they will divide by 2 to get the answer of 5 ($10 \div 2 = 5$). Overbeek and Sondergaard further state that if something is halved, it is divided into two equal parts. To halve is the action of dividing or splitting something into two equal parts. If a number is halved, you divide it by two. The authors do state [i]f [learners] don't know what half is, [they] could make mistakes in many areas of life" (p. 222). The use of math vocabulary shifts learners' use of everyday vocabulary to mathematical vocabulary, giving learners an understanding of the use of mathematical vocabulary in a mathematical context. The use of mathematical vocabulary contributes to the meaning and understanding of mathematical language, which gives the learner whose mother tongue is not the language of instruction an opportunity to differentiate vocabulary from their everyday context to mathematical context.

2.8 Multilingual Classroom

Multilingual classrooms are culturally diverse, and in them are learners with different languages. Mathematics as a second language for learners could cause hindrances in learning concepts such as doubling and halving and in implementing them when solving problems. Hoffert (2009) states that "[m]athematics is considered the universal language, but students who speak languages other than English have difficulty doing mathematics in English. For instance, because of a lack of familiarity with problems' context, many have trouble understanding exactly what operations to perform" (p. 130). South African classrooms are made up of multilingual learners, with many of the learners being non-English speakers. Barwell (2009) states that a mathematics classroom that uses more than two languages is multilingual; this is inclusive of bilingual. Taylor and Coetzee (2013) found that several schools whose learners are not native speakers of English use English as early as in Grade 1 as a language of instruction. According to Robertson and Graven (2020), this is due to the mistrust placed on the use of the mother tongue as the language of teaching and learning Mathematics.

Moschkovich (2007) identified that learning mathematics in multilingual classes encompasses the use of language switching and code switching, which will enable learners to work out arithmetic computation and, in the process, acquire the mathematical language. On the other hand, Moschkovich also recognised that it is possible for learners not to be considered proficient in simple arithmetic computation if they cannot use the preferred alternative language, as illustrated in Sesati's (2000) study, where she analysed the use of mathematical language in a multilingual classroom. The author collected the data through observation and interviews of teachers and learners' use of mathematical language in a multilingual classroom. The author found that in teaching mathematical language, some learners struggled to engage in calculation and conceptual discourses without using their main language, Tswana. A related study on teaching and learning mathematics is Essien's (2018) study, which provides a systematic review of 'on the role of Language in teaching and learning of Early Grade Mathematics'. The author's countries of interest were South Africa, Malawi, and Kenya, and he conducted a literature review using journal papers, monitoring and evaluation reports, and book chapters. The author found that in the reviewed literature, most South African schools use the language of instruction as the language of teaching and learning as early as in Grade 1. The same was found in Kenya, where learners in standards 1–3 received English from the start of primary school. In Malawi teaching learners in their home language points to difficulties in implementing indigenous languages as the language of teaching and learning.

The issues related to the language of mathematics and the language of learners are important to my study, especially because I employed this intervention with second-language learners. Consequently, language became a contributing factor in processes of assimilation and accommodation.

2.9 Conclusion

In this chapter, I have reviewed literature for understanding the shifts that learners make, if any, when taught the doubling and halving strategy, progressing from concrete to abstract ways. Piaget's notion of the development of cognitive structures forms the foundation for my analysis of the learner's use of fluency, strategic thinking, and strategic calculating when working with doubling and halving. In my discussion, I have argued that as learners progress from the concrete to the abstract representation of doubling and halving, their cognitive structures "accommodate" the new knowledge, which systematically changes their cognitive structure as they progress through the different lessons. I have looked at the importance of learners moving from counting to calculating strategies. I have also considered mathematics as a language, as learners with English as a second language may be disadvantaged in understanding instructions. In the next chapter, I discuss the research process that will be used to answer my research questions.

Chapter 3: Methodology

3.1 Introduction

Haralambos and Holborn (2007) state that knowledge production in an academic subject is achieved systematically through data collection and analysis. It is through the process of collection and analysis that “theories can be tested, accepted or rejected” (p. 787). This study focuses on testing whether an intervention to teach grade 3 learners to use doubling and halving strategies may result in better post-test results. In this chapter, I will detail the method that I used to undertake the research, the research method and design, the target population, and the sampling method. Furthermore, I have included the research instruments, which entail the pilot test, pre- and post-test, intervention lessons, and interviews, together with the data collection and analysis. Lastly, I have discussed the validity and reliability of the study.

3.2 Aim and Research Questions

The aim of this study is to explore the use of doubling and halving strategies with grade 3 learners. The aim of my study is guided by the following questions:

1. Is there any mean difference in performance between the intervention group and control group?
2. What strategies did the learners use prior to the intervention in the pre-test?
3. What strategies did learners in the intervention group use after they received the intervention lessons in the post-test?
4. Were there any shifts from counting strategies to calculating strategies after the intervention process?

3.3 Research Method

In my study, I used both qualitative and quantitative methods to determine the effectiveness of the intervention programme that comes from the Mental Starters Assessment Project (MSAP). This project is supported by the South African Research Chairs Initiative the

Department of Science and Technology, and the National Research Fund, together with the Wits Maths Connect project, which supports primary school math. The MSAP has designed tasks and assessments that focus on various strategies. I have chosen to focus on the doubling and halving strategy unit, which outlines fluency, strategic calculating and strategic thinking (MSAP, 2020) as areas of improvement. I was granted permission to use these instruments (see Appendix C for the permission letter).

I (the researcher) have used both qualitative and quantitative methods because of the differences each method brought to my study. Haralambos and Holborn (2007) state that the quantitative method entails numerical data, which can be measured, quantified, and statistically produced. In my study, learners wrote a pre-test and post-test, and the results they obtained were calculated into mean percentages and used to compare if they improved. In contrast, the qualitative method is underpinned by the interpretivist approach, where knowledge is constructed and interpreted to make meaning (Haralambos and Holborn, 2007). With focus given to the intervention group after the post-test, the researcher ascertained if learners understood the taught strategies. The use of both quantitative and qualitative methods in my study provided an in-depth explanation of the shift that resulted in the intervention group after the use of doubling and halving strategies.

3.4 Research Design

My study employed an experimental approach along with the tests and an intervention program involving eight lessons and interviews. Haralambos and Holborn (2007, p. 791) state that experimental research involves "intervening in the social world in such a way that hypotheses can be tested by isolating particular variables". In my study, learners in the control group did not receive intervention, and learners in the intervention group received intervention. The intervention group responded positively to the intervention lessons, yielding positive results.

3.5 Target population and Sample

The *target population* was grade 3 Foundation Phase learners with an age range that was between 9 and 10 years in a former Model C school. The school was identified by myself based on the closeness of the school to my residential home, which gave me easy access to the school, and the Head of Foundation Phase agreed for the grade 3 learners to be part of the study. The school has learners from different socio-economic backgrounds, and they are taught in English as the language of instruction, even though English is their second language.

The grade 3 learners formed two groups: a control group with 12 learners and an intervention group with 12 learners. The intervention group was given consent letters, which I explained to them, and then they gave consent to participating in the study. Initially, there was a dependency on *random sampling* in my study to make sure that all learners got an opportunity to be part of the research. Plooy (2009) states that “[a] simple random sample is drawn when a sampling frame is available and each unit of analysis in the target population has an equal chance of being selected” (p. 155). Twelve learners were chosen as part of the intervention, and the same was done for the control group.

3.5.1 Data Collection

Data Collection Method	**Frequency**	**Total Collections**
Pre-Test	Once	1
Intervention Lesson	Eight lessons	8
Post-Test	Once	1
Interviews	After the pre-and post-test	24 learners (Control group and Intervention group)

Table 3.5.1: Summary of data collection

The data was collected in the mornings during the learners scheduled time for math. The frequency of the pre- and post-test happened once; however, what needs to be noted is that the tests are divided into Part 1 and Part 2. In part 1, one of the pre-test learners had 2 minutes to complete it, and in part 2 of the pre-test learners had 3 minutes to complete it. The post-test was also divided into two. Part one of the post-test learners had 2 minutes to complete it, and part two of the post-test learners had 3 minutes to complete it. The interviews took place before the pre-test and after the intervention group completed the intervention starter lessons and the post-test. The control group also received the same test, and I interviewed them before and after the tests. The control group did not receive the intervention.

3.6 Research Instrument

The pre and post-test, intervention lessons and interviews were used as the instruments (see Appendix A). These instruments come from the MSAP project.

3.6.1 Pilot Test

Haralambos and Holborn (2007, p. 821) mention that a "pilot study is a small-scale preliminary study conducted before the main research in order to check the feasibility or to improve the design of the research". The pilot test that the learners wrote was accessed from MSAP. The pilot test had 20 questions in part 1, which focused on fluency. In addition, part 2 had 10 questions focused on strategic calculating and strategic thinking.

3.6.2 Pre-test and post-test

The diagnostic assessment that learners had been given formed part of the pre-test and post-test, which focused on mathematical questions that are accessible from the MSAP pack. What needs to be noted is that the use of the diagnostic assessment was to "measure specific knowledge structure and processing skills in students so as to provide information about their cognitive strengths and weaknesses" (Leighton and Gierl, 2007, p. 3). The diagnostic assessment gave an indication and exposed the strategies (in conjunction with the interview response) that the learners used in solving mathematical problems. Learners were given a pre-test to complete before the intervention. I then taught eight intervention lessons (which I elaborated on under intervention lessons). The learners in the intervention group were then given a post-test that determined whether the intervention had improved the learners' use of doubling and halving strategies when solving these mathematical problems. Plooy (2009, p. 184) states, "the post-test provides a comparison of the experimental group and the control group after the manipulation of the independent variable…". Both tests (see Appendix A) are comprised of doubling and halving-based problems taken from the MSAP project, which required that learners apply the different representations of doubling and halving strategies taught in the intervention lessons. Questions 1, 2, 5, 8, 11, and 18 in Part 1 of the pre-test are numerically different from the post-test. Whereas part 2 of the post-test question 5 is also numerically different in the post-test (see table 4). Both the control and intervention groups wrote the same test.

Part 1	Pre-test	Post-test
Questions		
1	$6+6$	$7+7$
2	Half of 12=_	Half of 14=_
5	$\_ \times 2 = 12$	$\_ \times 2 = 14$
8	Half of 14=_	Half of 12=_
11	$15+15 = \_$	$14+14 = \_$
18	$16 \div 2 = \_$	$18 \div 2 = \_$
Part 2		
Question		
5	Double 47	Double 99

Table 3.6.2: Numerically different pre and post test questions

3.6.3 Interviews

Interviews were conducted with learners to understand which strategies they chose to use when solving the questions in the pre- and post-tests. Mason (1996, p. 39) states that "interviewing as a method lays emphasis on depth, complexity, and roundedness in data". I asked the question(s) in the same order to avoid learners giving a different explanation and being confused. The interviews were used to understand which strategies the control and intervention groups used and which strategies the intervention group used after the intervention in solving the mathematical problems in the tests.

Below are examples of the type of questions that learners were asked:

- How did you work out double 10?
- Can you explain how you worked out 'half of_$= 52'$?
- What does the question ask you to do?

3.6.4 Intervention lessons

Eight intervention lessons were designed by the MSAP to introduce learners to doubling and halving strategies to improve their fluency, strategic calculating and strategic thinking (MSAP, 2020). Eight intervention lessons were taught to the grade 3 learners over a period of two weeks and each lesson lasted approximately 30–40 minutes. Each lesson consisted of a) a mental warm-up, b) a task sequence, and c) individual tasks. The structure of the intervention lessons progressed from the use of concrete material to an abstract representation, enabling learners to enact the various strategies when solving the mathematical problems with more sophisticated mental tasks that require that the learners apply more advanced representations of doubling and halving strategies in solving problems.

The intervention lessons consisted of eight lessons that differed in levels of complexity, and each lesson had a specific outcome that gave focus to the lesson. What the reader needs to note is that I (the researcher) had a dual role in my study. I refer to myself as a researcher and teacher, as seen in the explanation of the intervention lessons. Detailed intervention lessons can be found in the Appendix.

Lesson 1

Learning objective: Practise doubling and halving facts of 20

The teacher demonstrated the double numbers using fingers; for example, double four (the teacher holds up four fingers) is eight, double 5 is 10, double 3 is six, etc., and learners are asked to say doubles in a sentence. Learners were instructed to work in pairs, and each pair was asked to show a double of six. The teacher asked the pair, “How many fingers altogether?” Learners responded and said 12. The teacher showed the learners how the answer 12 is obtained by showing two hands $(5 + 5)$ and two fingers $(1 + 1)$, and when added together, it gives the answer 12. The teacher used the double-dot card with six dots on each side. The teacher asked the learners to number the dots they saw on one side. Learners: 6. The teacher asked the learners to open the dot card; now how many dots do you see altogether? Learners responded with 12. The teacher asked the learners, so what is double of six? The learners responded 12. I asked the learners how they knew a double of six is 12, and they responded, “Because 6 and 6 is 12. Task: Learners were given dot cards to write sentences using the dot cards.

Lesson 2

In lesson 2, the teacher recapped the use of the dot card and introduced an alternative doubles vocabulary. Instead of learners saying double 8 is 16, they could say 2 groups of 8 is 16. The teacher wrote double $8 = 16$ on the board and asked learners what is half of 16. They responded eight, and the teacher wrote the number sentence on the board. The teacher wrote double five, double 7, double 9 and half of 14 on the board. The learner wrote the doubles and halves of the number sentences.

Lesson 3

Learning Objectives: Practise doubles of multiples of ten.

The teacher used the 10-dot strips to help learners visualise a group of 10. I showed the learners how to double 30 using the 10-dot strips. The teacher reminded learners that $30 = 60$, so what is double 30? Learners responded 60; the teacher says if double 30 is 60, then what is half of 60? Learners responded and said 30. The teacher wrote double 2, double 5, and double 7 on the board and asked learners to write the single digit examples in 10s: double 20, double 50, and double 70. The teacher wrote more examples on the board for practise, e.g., Double 3, Double 8, Double 30, Double 80, half of 4, half of 8, half of 40, and half of 80.

Lesson 4

Learning Objective: Practise doubles of two-digit numbers

In lesson 4, the teacher asked a double 35 problem and used the 10 dot strips and five strips to show the learners the composition of two-digit numbers such as 35. The teacher asked the learners how we could work out the double of 35 (or $35 + 35$). Learners identified tens and units together. The teacher introduced the breaking down method to the learners and explained that 35 is partitioned into 30 and 5, and then 30 is doubled to 60 and 5 to 10. 60 and 10 are brought together to make 70. In the process of explaining the use of the breaking down method, the teacher reminded the class that multiplying by 2 is the same as doubling. The teacher should write examples on the board (double 41, double 36, double 47) for learners to try out.

Lesson 5

Learning Objective: Practise halving of two-digit numbers

In lesson 5, the teacher asked a halving problem ($62 \div 2$). Class to be reminded that dividing by 2 (or $\div$ 2) is the same as halving. The teacher asked learners how they would work out $62 \div 2$. Learner: I used the breaking down method the same way as I did for the doubles problem. The teacher explained to the learners that 62 is partitioned into 60 and 2, and then half of 60 is 30 and half of 2 is 1. The 30 and 1 are brought together to make 31. The teacher wrote the following examples on the board half of 42, $68 \div 2$, $34 \div 2$ for learners to try out.

Lesson 6

Learning Objective: Use doubles and halves in a range of ways

In lesson 6, the teacher wrote a problem of double 26 and asked learners for different ways they could use to represent double 26. Learners: double 26 can be the same as 26×2 because doubling is the same as multiplying, and 'you could share the 52 sweets with your brother ($52 \div 2$ or $52 - 26$). As learners were giving the different representations, the teacher wrote the learners ideas in a web of facts. The teacher wrote double $43 = 86$ problem and asked learners to draw web facts, the same one that was put together as a class.

Lesson 7

Learning Objective: Practise doubles and halves of multiples of 10

In lesson 7, the teacher wrote a double 34 problem on the board so that learners could work out the problem mentally. Learners who struggled the teacher used the breaking down method to work out double 34. The teacher asked the learners to use what they knew about double 34 to work out 340.

Lesson 8

Learning Objective: Building connected facts related to given doubling and halving number fact, including near doubles.

In lesson 8, the teacher introduced learners to building connected facts related to a given doubling and halving number fact, including near doubles. The teacher drew a web of facts on the board of the different ways of representing double $17 = 34$. Learners were asked to create their own web of facts to show how they would represent the given problem using doubling and halving. After the completion of the task, learners try to answer double 99. This problem was aimed at directing learners to use the near doubles fact.

Below are the intervention lesson objectives.

Intervention Lesson	**Objective of the lesson**
1	To use basic doubling and halving facts to 20
2	To practise basic doubling and halving facts to 20
3	To practise doubles of multiples of ten
4	To practise doubles of two-digit numbers
5	To practise halving two-digit numbers
6	To use doubles and halves in a range of ways
7	To practise doubles and halves of multiples of 10
8	To build connected facts related to a given doubling and halving number fact, including near doublings

Table 3.6.5: Intervention lesson objectives

3.7 Data Analysis

I analysed the pre-test and post-test data quantitatively and the interviews qualitatively. Plooy (2009) states that "[a] qualitative content analysis is conventionally not conducted or reported in numerical terms (frequencies or percentages) but guided by questions and reported as descriptions of attributes" (p. 219). Learner understanding of the doubling and halving strategies was dependent on their explanation and reasoning, which was an indicator for the researcher if learners understood the doubling and halving strategies and their use in solving mathematical problems.

Part 1 of the pre- and post-test had 20 questions that focused on fluency, and learners had 2 minutes to complete the test. Part 2 of the pre- and post-test focused on strategic calculating and strategic thinking, and learners had 3 minutes to complete the test. I went through the control and intervention groups strategies that they used to solve the problems and used Graven and Venkat's (2021) categories as units of analysis. Below is a summary and description of the calculation strategies.

Category	**Description**
Fluency	Facts are automatically recalled
Strategic calculation	Where the focus is on efficient working that pays attention to number relationships and/ or the known facts at the level of fluent recall to derive results
Strategic thinking	Is focused on using relationships to answer problems.

Table 3.7.1: Summary of calculation strategies

The strategies that the intervention group used after the intervention were presented using graphs (see Figures in 4.7). The intervention lessons were categorised into doubling and halving facts to 20, multiples of 10, doubles of two digits, halving of two-digit numbers, and doubling and halving number facts, including near doubles. Under each category, questions from parts 1 and 2 of the pre- and post-test were populated into the graph to see if there were any shifts with the intervention group with the use of the doubling and halving strategies from the pre-test to the post-test after the intervention, as per question three of the research question. For each category in the summarised graph (see Figure 4.7), a sub-graph was created to zoom in on individual questions to analyse how the learners in the intervention group performed after receiving the intervention lessons. Each individual question was assigned a specific category based on the outcome of the intervention lesson. Below is a summary table of how the doubling and halving strategies were classified.

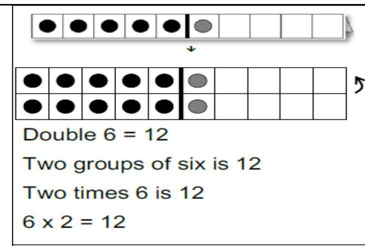
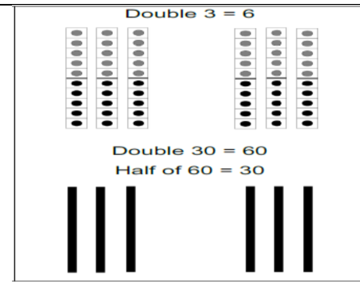
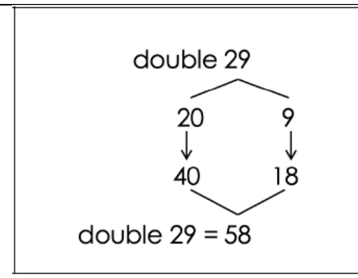
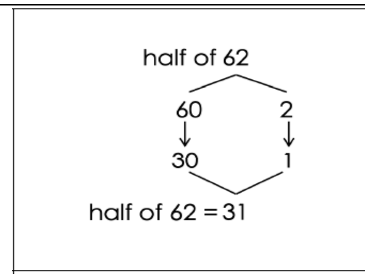
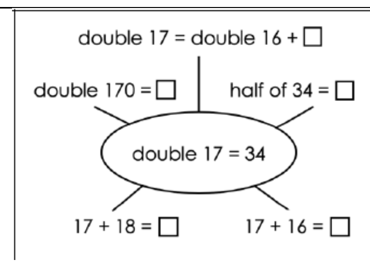
Category	Description	Example
Doubling and Halving facts of 20	Adding numbers to themselves numbers up to 20	
Multiples of 10	Solving problems using a value that ends with 0	
Doubles of two-digit numbers	To partition the value of each number by doubling	
Halving of two-digit numbers	To partition the value of each number by halving	
Doubling and halving number facts including near doubles	To use the doubles facts for example $5+5$ to solve questions that are near to the double's numbers for example $6+5=11,5+5+1=11,10+1=11$	

Table 3.7.2: Category and description of doubling and halving strategies used for the Post-test Part 1 and 2 by the Intervention group.

3.8.3 Analysis of Tests

The collected data from the pre- and post-test that the control and intervention groups wrote was marked to obtain the overall average performance from the 30 questions in each group. For each correct answer, a mark was allocated, and a mean percentage was worked out from the correct answers. The pre- and post-tests were divided into parts 1 and 2. Part 1 of the test had 20 questions, and part 2 had 10 questions, which were categorised into fluency, strategic calculating and strategic thinking and populated into bar graphs. After conducting the pre-test and post-test, a check of results was done using the t-test, a comparative data analysis instrument that entailed testing the hypotheses on the selected variables (Bakkabulindi, 2015). In my study, a comparison between the control and intervention groups had 12 learners in each group to establish if there was a difference in results after the intervention group was taught. The results that the control and intervention groups obtained were compared and showed that there was a difference in results in both groups, even though there was no statistical difference. Furthermore, Okeke and van Wyk (2015) state that in a study where there is a "two-sample t-test, a researcher is interested in comparing the mean values of the numerical dependent variable (DV) on the two categories of a binary categorical independent variable (IV)". To show the difference in results, the pre-test and post-test results were calculated to find the mean from the control and intervention groups. The results from the pre- and post-test in the different categories were compared to see if there were any shifts as per question 3 of the research questions.

3.8.4 Analysis of Interview

The data collection from the pre- and post-test guided the next phase, which was collecting data from the interview as learners were interviewed after the tests. The interview was used as the main measure of the effectiveness of the doubling and halving strategies as per questions one (What strategies did the learners use prior to the intervention in the pre-test?) and two (What strategies did learners in the intervention group use after receiving the intervention lessons in the post-test?) of the research question. The explanations that the learners gave from the questions in the tests were recorded and transcribed for further analysis.

3.9 Rigor

Research planning, data collection, analyses, and reporting required that each aspect undergo a rigorous process and ensure that the work that was conducted is of quality. To obtain quality in research, it is pivotal that validity and reliability are aligned with the stated instruments, such as the pre- and post-tests that were written by the learners.

3.9.1 Validity and Reliability

Plooy (2009, p. 28) maintains that validity is “to approximate reality as closely as possible”. The pre-tests, post-tests, and interviews were used to assess and measure learner acquisition of doubling and halving strategies from the intervention programme. The improvement in the post-test results in comparison to the pre-test means that the intervention lessons together with the tests were valid; they measured what the tests ought to measure (Sieborger, 2004).

Plooy (2009, p. 28) states that reliability entails “to consistently obtain the same answer when researched at different points in time”; the study must yield the same results every time it is conducted in a different context and time. Reliability of this study was maintained using the same pre-test and post-test with my supervisor and another master’s student.

Triangulation was used because of the pre- and post-tests and the interviews. Haralambos and Holborn (2007, p. 846) state that triangulation is “used to refer specifically to research where quantitative and qualitative research methods are used to cross-check the findings produced by other methods”. The use of qualitative and quantitative research methods improved the reliability and validity of the tests, interviews, findings, and analysis of the findings.

3.10 Ethical Consideration

I have abided by the University’s code of ethics for researchers on human subjects by applying for ethics clearance to enact this study and the Department of Education. Parents or legal guardians signed consent forms and learners signed assent forms. These forms were accompanied by a short description of the study. An information sheet detailing the study was given to the principal, head of foundation phase, class teacher and learners. I informed the participants that they could withdraw at any point in the research. Information letters, consent forms and assent forms can be found in Appendix of this study.

3.11 Conclusion

In this chapter, I described the proposed methodology, including the choice of methods and instruments to conduct the study. I have given a description of the context in which the study will take place. Issues of validity, reliability, and ethics were also discussed.

The doubling and halving strategies expose learners to improved fluency, strategic thinking, and strategic calculation, which are deemed important if learners are to shift from the use of counting strategies to calculation strategies. In my study, I investigated whether the intervention has mediated support and improved learners understanding of doubling and halving strategies in solving mathematical problems.

Chapter 4: Data Findings and Analysis

4.1 Findings and Analysis

In this chapter, I will present the findings and analyse the data that was collected in the pre- and post-test and interview to answer the following questions:

1. Is there any mean difference in performance between the intervention group and control group?
2. What strategies did the learners use prior to the intervention in the pre-test?
3. What strategies did learners in the intervention group use after they received the intervention lessons in the post-test?
4. Were there any shifts from counting strategies to calculating strategies after the intervention process?

4.2 Introduction

As initially stated, this study sought to explore learners use of the doubling and halving strategy to improve their fluency, strategic calculating and strategic thinking with grade 3 learners with an already developed intervention by the Mental Starters Assessment Programme (MSAP). The intervention lessons are composed of a series of doubling and halving strategies with multiple representations from lesson starter 1 to 8. Learners' use of the doubling and halving strategies was examined in the pre-test and post-test using interviews.

This chapter will present the findings of the study in relation to the strategies that were used for rapid recall, strategic calculating, and strategic thinking mathematical problems in conjunction with the pre-test that was written prior to the intervention and the post-test, as well as the interview that was conducted. Strategies presented by the learners in the control and intervention groups before the pre-test and after the post-test were transcribed and analysed to see the kinds of strategies that were used by both groups to work out part 1 of the test. Part 1 is composed of fluency mathematical problems, and Part 2 consists of strategic calculating and thinking mathematical problems.

The second part of the section will focus on the intervention group, which received a series of intervention lessons that exposed the learners to a range of representations of doubling and halving in solving mathematical problems.

The pre- and post-tests and interviews were used to collect data from 24 grade 3 learners, who were split into two groups of 12 to make up the intervention group and 12 to make up the control group. The pre- and post-test results were quantified and presented in the form of bar graphs, and interview responses were coded to check for strategies that learners in both groups used in the pre-test. I also analysed possible shifts. What the reader needs to note, as mentioned in Chapter 3, is that the sample (*n 12*) was not large enough to determine statistical improvement; however, the increase in results shows that there were improvements from the pre-test to the post-test in the intervention group.

4.3 Fluency

In this section, as the first part of the analysis, I focus on comparing the overall performance on different mathematical problems in the pre- and post-tests in fluency, strategic calculating and thinking between the control and intervention groups. Learners in the intervention and control groups were interviewed about the strategies they used to answer the fluency, strategic calculating and thinking mathematical questions. Lastly, the focus of the first part of the analysis was on learner gains in fluency, strategic calculating and thinking.

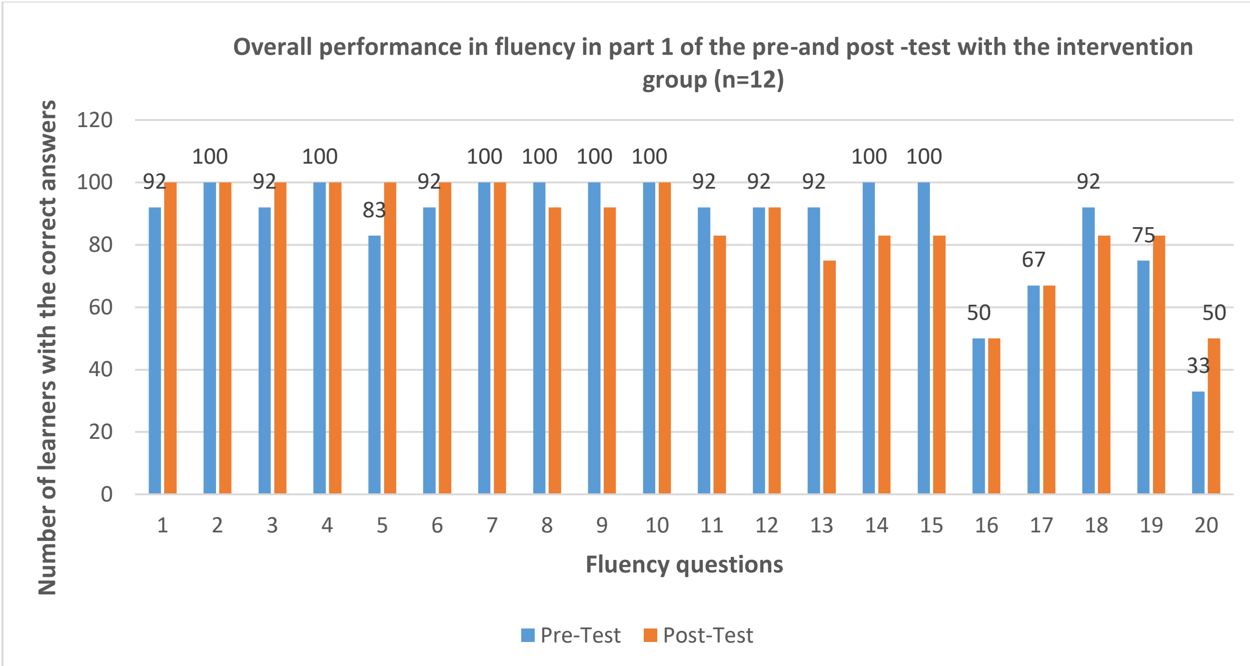


Figure 4.3.1: Overall performance in Fluency in part 1 of the pre-and post-test with the intervention group (*n=12*)

At the beginning of every lesson, the learners engaged in a 1-minute mental warm-up activity. Six items (questions 2, 4, 7, 10, 16, and 17) maintained the same performance in both the pre- and post-test. In six items (questions 1, 3, 5, 6, 19, and 20), there is improvement from the pre-test to the post-test, while there is a decline in seven items (questions 8, 9, 11, 13, 14, 15, and 18). In question 5 ($\_ \times 2 = 12$), in the pre-test, 10 learners answered correctly, whereas in the post-test, 12 learners got the correct answer. In question 19 (half of 30 = _), in the pre-test, nine learners got the correct answer, and in the post-test, 10 learners got the correct answer. In question 20 ($2 \times 60 = \_$), in the pre-test, 4 learners got the correct answer, whereas in the post-test, 6 learners achieved a correct answer. Learners improved from 33% to 50%.

A paired t-test was performed to determine whether a significant difference exists between the mean percentages of the pre-test (mean percentage = 87.60) and post-test results (mean percentage = 86.65) in fluency for the intervention group. The results of the test indicate that there is no statistically significant difference between the pre-test and post-test outcomes for fluency (t-statistic = 0.417, d.f. = 19, p-value = 0.681).

Although the learners result statistically suggest that the mental warm-ups had little to no impact on learners being able to quickly recall known facts within the intervention group in the post-test based on the decrease in learners results by 1%, what must be noted is that learners averaged above 85%, which suggests that learners' knowledge on recalling basic doubling and halving of known facts was already developed. The results of the post-test indicated a decrease of 1% from the overall results of the pre-test. When learners were writing the post-test, I observed that some learners rushed through the test so that they could complete it within the allocated time of 2 minutes. The decline in overall performance cannot be attributed to learners' lack of understanding of retrieving basic doubling and halving known facts but could be due to learners rushing through the test and not reading the questions properly, which led to learners making careless mistakes. Codding, Burns, and Lukito (2011), in their study of the mental analysis of mathematical basic-fact fluency intervention, found that practise with modelling and drill is effective in developing fluency. Even though there was a decline of 1% from the pre-test to the post-test, the use of mental warm-up in every lesson reinforced the use of doubling and halving, which resulted in learner improvement in some mathematical problems, as seen in the question.

4.3 Strategic Calculating and Strategic Thinking

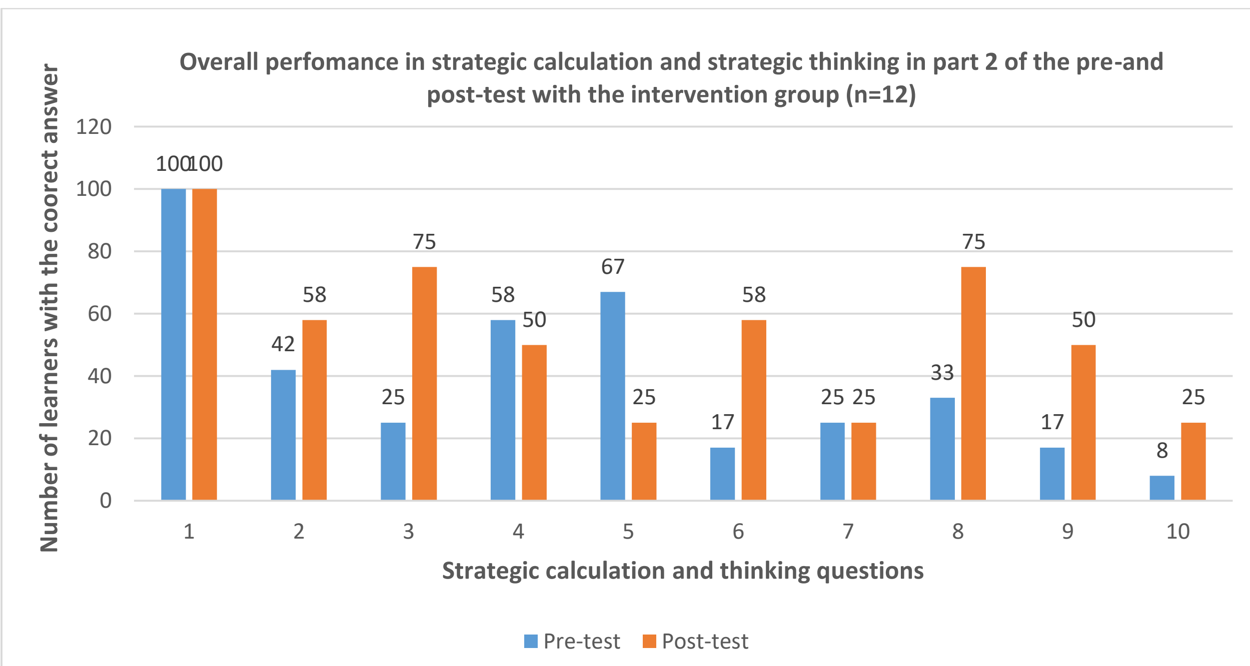


Figure 4.3.2: Overall performance in Strategic Calculation and strategic Thinking in part 2 of the pre-and post-test with the intervention group (*n=12*)

Figure 4.3.2 illustrates an overall performance in mathematical problems that required learners to shift from rapid recall to applying strategic calculating and strategic thinking to the mathematical problems. Part 2 of the pre- and post-test focused on mathematical problems that required learners to apply doubling and halving strategies that are procedural and progress to an abstract level, consequently shifting learners from the use of dot cards to mental calculations. Learners were given three minutes to complete the pre- and post-tests. Two items (questions 1 and 7) maintained the same performance in both the pre- and post-tests. On six items (questions 2, 3, 6, 8, 9, and 10), there is improvement from the pre-test to the post-test, while there is a decline in two items (questions 4 and 5). In question two (36×2) of the pre-test, five learners got the question correct, whereas in the post-test, seven learners got the answer correct. In question three ($64 \div 2 = \_$) of the pre-test, three learners got the answer correct, whereas in the post-test, nine learners got the question correct. In question six (half of 38 = _) of the pre-test, two learners got the question correct, whereas in the post-test, seven learners got the answer correct. In question nine ($39 + 38 = \_$) of the pre-test, two learners got the question correct, and in the post-test, six learners got the correct answer. Question 10 (double $= 40 + 40 - \_$): in the pre-test, one learner got the correct answer, whereas in the post-test, two learners got the question correct.

A paired t-test was performed to determine whether a significant difference exists between the mean percentages of the pre-test (mean percentage = 39.20) and post-test results (mean percentage = 53.30) in strategic calculation and thinking for the intervention group. The results of the test indicate that there is no statistically significant difference between the pre-test and post-test outcomes for strategic calculation and thinking (t-statistic = 1.575, d.f. = 9, p-value = 0.150).

As a result, the overall improvement in learners' performance suggests that strategic calculating and strategic thinking improved in the intervention group. Learners' performance in question 3, which focused on strategic calculating, showed the most improvement by 50%, as well as in questions 8, 9, and 10, averaging 50% with questions that focused on strategic thinking. In the pre-test, the learner's performance showed that the learner had some strategic calculating and strategic thinking knowledge from prior background to solve the problem. In the post-test, learners' strategic calculating and thinking had improved. Learner improvement in strategic calculating can be attributed to the strategic calculating skills that learners were taught during the intervention lessons. The repeated practice of the calculating skills in the individual tasks and take-home worksheets reinforced the strategic calculating and thinking. Learners' improvement in question 10 can be attributed to their understanding of the problem and their use of strategic thinking to solve it. The improvement suggests that learners were able to recognise that question 10 required more than strategic calculating but also recognised that the question had different parts; for example, double 39 was given as 40 + 40, which learners had to subtract from to get the final answer. By drawing on the different parts of the question, the learners were able to get to the answer. Johnson, Siegler, and Alibali's (2001) study found that procedural and conceptual knowledge are bidirectional, where procedural knowledge enhances conceptual knowledge. Procedural knowledge and conceptual knowledge are pivotal in improving the learners' conceptual understanding.

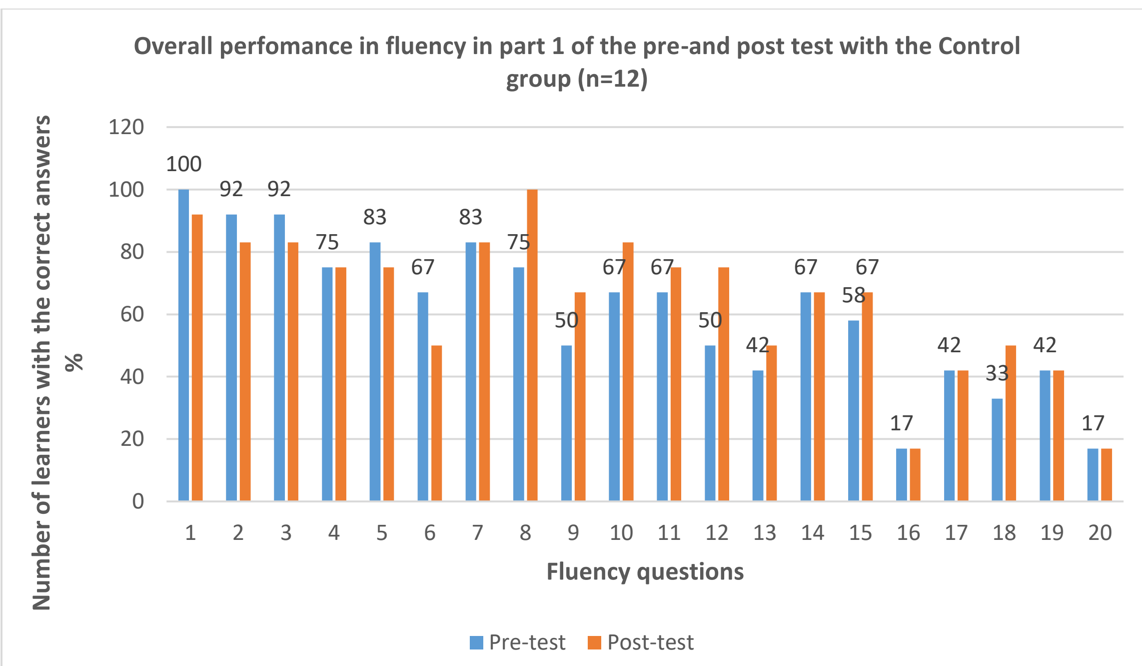


Figure 4.4.1: Overall performance in fluency in part 1 of the pre-and post-test with the Control group (*n=12*)

The reader is reminded that the control group did not receive an intervention. Seven items (questions 4, 7, 14, 16, 17, 19, and 20) maintained the same performance in both the pre- and post-test. In eight items (questions 8, 9, 10, 11, 12, 13, 15, and 18), there is improvement from the pre-test to the post-test, while there is a decline in five items (questions 1, 2, 3, 5, and 6). In the pre-test, 12 learners answered question 1 ($6 + 6 = \_$) correctly, and in the post-test, there was a drop in results where 11 learners got the answer correct. In questions 2 and 3, 11 learners got the answer correct, whereas in the post-test, there was a drop, and 9 learners got the answer correct. A further drop was also noted in questions 5 and 6, while no gains were noted in questions 7, 14, 16, 17, 19, and 20. However, questions 8, 9, 10, 11, 12, 13, 15, and 18 showed positive gains in the post-test.

A paired t-test was performed to determine whether a significant difference exists between the mean percentages of the pre-test (mean percentage = 61.00) and post-test results (mean percentage = 64.65) in rapid recall for the control group. The results of the test indicate that there is no statistically significant difference between the pre-test and post-test outcomes for fluency (t-statistic = 1.394, d.f. = 19, p-value = 0.180).

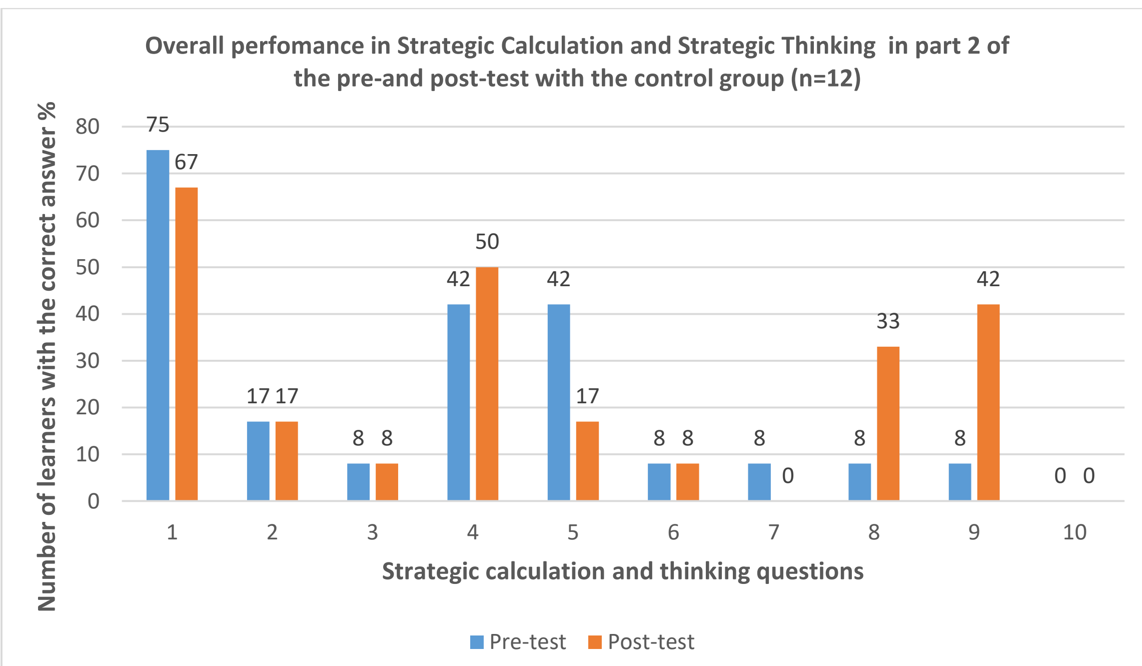


Figure 4.4.2: Overall performance in Strategic Calculation and strategic Thinking in part 2 of the pre-and post-test with the Control group (*n=12*)

The above figure illustrates the overall performance in strategic calculating and strategic thinking. In the pre-test, nine learners answered question 1 correctly, and in the post-test, eight learners got the answer correct. In question five, five learners got the answer correct. In questions 2, 3, and 6, no gains were noted; in question 7 of the pre-test, 1 learner got the answer correct, and in the post-test, 0 learners answered the question. In question 10 of the pre-test, zero learners answered the question. Questions 4, 8, and 9 showed an increase in results, averaging 42%. Learner gains in parts 1 and 2 of the pre- and post-test in the control group could be attributed to the continuation of teaching the foundation math content. The increase in some of the questions in the post-test can be attributed to learners engaging in the foundation phase math curriculum.

A paired t-test was performed to determine whether a significant difference exists between the mean percentages of the pre-test (mean percentage = 21.60) and post-test results (mean percentage = 24.20) in strategic calculation and thinking for the control group. The results of the test indicate that there is no statistically significant difference between the pre-test and post-test outcomes for strategic calculation and thinking. (t-statistic = 0.490, d.f. = nine, p-value = 0.636).

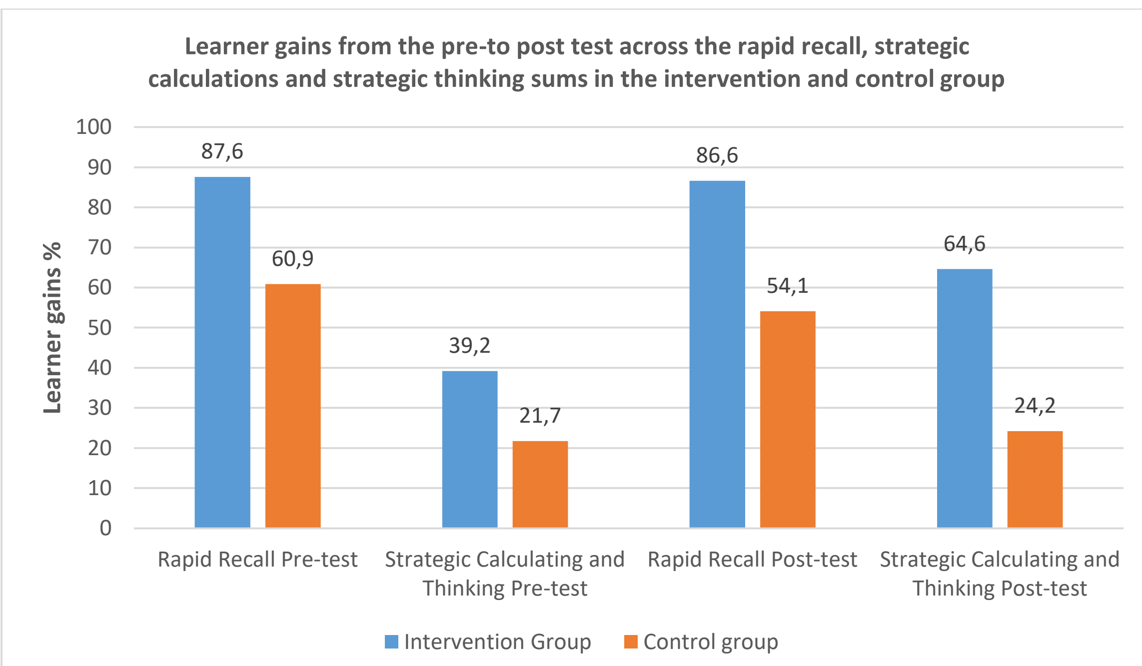


Figure 4.5.1: Learner gains from the pre-to post-test across the fluency, strategic thinking and strategic thinking questions in the intervention and control group

The above figure illustrates the overall gains of the pre-test parts 1 and 2 between the control and intervention groups. The control group in both parts 1 and 2 of the pre-test performed slightly poorly in comparison to the intervention group, whereas the intervention group showed positive gains. In part, in the pre-test, the control group scored 61% in comparison to the intervention group, which scored 87.6%. In addition, in Part 2 of the pre-test, both groups scored a mean of below 40%. The control group scored 21.7%, and the intervention group scored 39.2%. In Part 1 of the post-test, the control group scored 64.7% and the intervention group scored 86.7%, and in Part 2, the control group scored 24.2% and the intervention group scored higher with a score of 53.3%, showing a significant gain from the intervention group.

Evaluation	Group	Number of Questions per group (n)	PRE-TEST MEAN (%)	POST-TEST MEAN (%)	GAINS (%)	t-statistic	p-value
Rapid Recall	Intervention	20	87.6	86.7	-0.9	0.417	0.681
	Control	20	61.0	64.7	3.7	1.394	0.180
Strategic Calculating and Thinking	Intervention	10	39.2	53.3	14.1	1.575	0.150
	Control	10	21.6	24.2	2.6	0.490	0.636
Overall	Intervention	30	71.5	75.5	4.1	1.160	0.256
	Control	30	47.9	51.2	3.3	1.348	0.188

Table 4.5.2: Overall t-test results between intervention and control group in the pre-and post-test based on 30 mathematical problems.

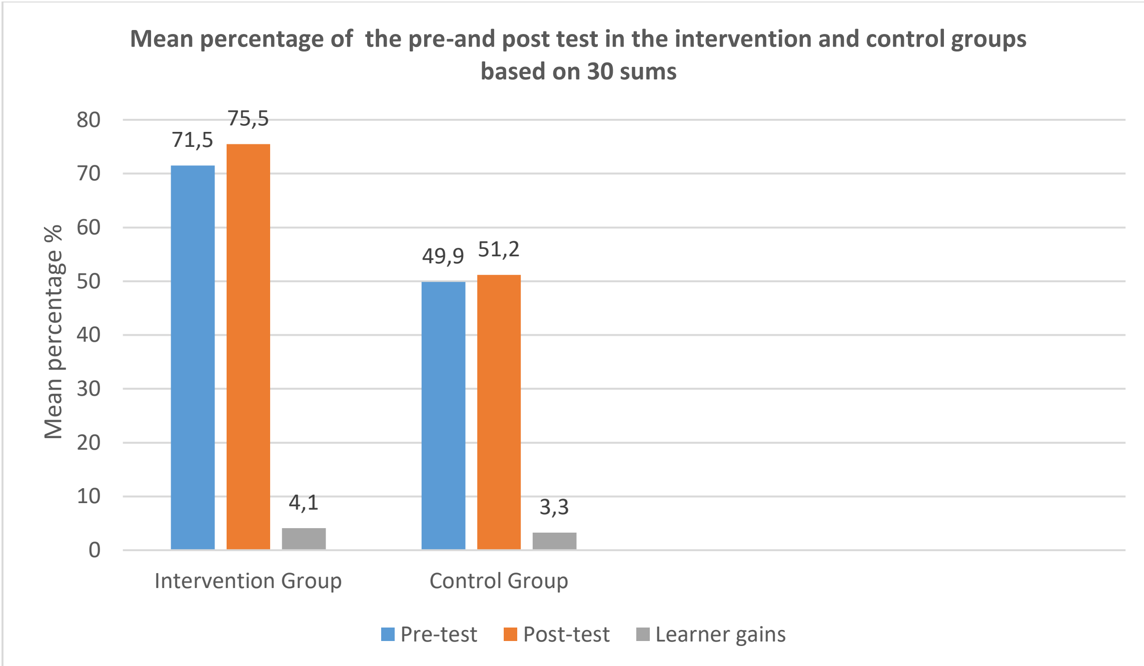


Figure 4.5.2: Mean percentage of the pre-and post-test based on 30 mathematical questions.

Table 4.5.2 and Figure 4.5.2 illustrate the overall mean percentage of the pre- and post-test based on 30 mathematical questions, which are divided into parts 1 and 2 to assess fluency, strategic calculating and thinking. The results of the overall performance in the pre- and post-tests show that there was a slight difference of -3.85% between the intervention and control groups. The intervention group showed a gain of +4.1%, whereas the control group showed a gain of +3.1%. Both the intervention and control groups received the same pre- and post-test, and both tests were timed, giving learners 2 minutes for part 1, which assessed fluency, and part 2, which assessed strategic calculating and thinking. The gain in the intervention group shows that the intervention starter lessons (lessons 1–8) that the learners received could have contributed to the shift in their overall performance. The reader is reminded that each intervention lesson employed a different doubling and halving strategy. The lessons aimed at getting learners to improve their fluency, strategic calculating and strategic thinking when solving problems. Learners in the intervention and control groups were interviewed about the strategies they used to answer the fluency, strategic calculating and strategic thinking mathematical questions. Below, I compare the overall performance of the learners in the intervention group on Part 1 and, consequently, Part 2 of the pre-test and post-test. Part 1 of both tests included short questions encouraging learners to employ fluency in their solutions. Whereas in Part 2, learners had to apply strategic calculating and strategic thinking when problem solving.

	Number of Questions per strategy(n)	PRE-TEST MEAN (%)	POST-TEST MEAN (%)	GAINS (%)	t-statistic	p-value
Doubling and Halving facts to 20	13	94.4	93.6	-0.8	0.296	0.772
Doubles of Multiples of 10	7	75.0	73.7	-1.3	0.275	0.793
Doubles of 2-digit numbers	2	71.0	79.0	8.0	1.000	0.500
Halving of 2-digit numbers	4	33.3	64.5	31.2	2.361	0.099
Doubling and Halving number facts including near doubles	4	0.8	29.3	28.5	0.000	1.000

Table 4.6: Overall t-test gains of doubling and halving strategies with intervention group in the pre and post-test across the mathematical questions in t-test.

Table 4.6 shows the overall view of the t-test gains in doubling and halving strategies and the learners' performance in the order in which the intervention lessons were presented. Analysis will be focused on doubling and halving facts to 20, multiples of 10, doubles of two digits, halving of two-digit numbers, and doubling and halving number facts, including near doubles used from the intervention lessons. The difference between the pre- and post-test results is evident; however, the largest difference in learners' results is in the use of doubles of multiples of 10, doubles of 2-digit numbers, halving of 2-digit numbers, and doubling and halving number facts strategy including near doubles strategies.

In the next section, I will focus on questions where learners showed positive gains. The reader is reminded that due to the small sample, I (the researcher) was limited in obtaining more than one interview for each question. I will analyse the strategies that were used in the pre-test and after the intervention by the intervention group to solve the problems. In my analysis of the strategies, I focused on the following doubling and halving strategies:

doubling and halving facts of 20, doubles of multiples of 10, doubles of two digits, halving of two digits, and doubling and halving number facts, including near doubles. Overall findings suggest that learners made positive gains in the above-mentioned doubling and halving strategies in the post-test.

4.7 Doubling and Halving Strategies

4.7.1 Doubling and halving facts of 20

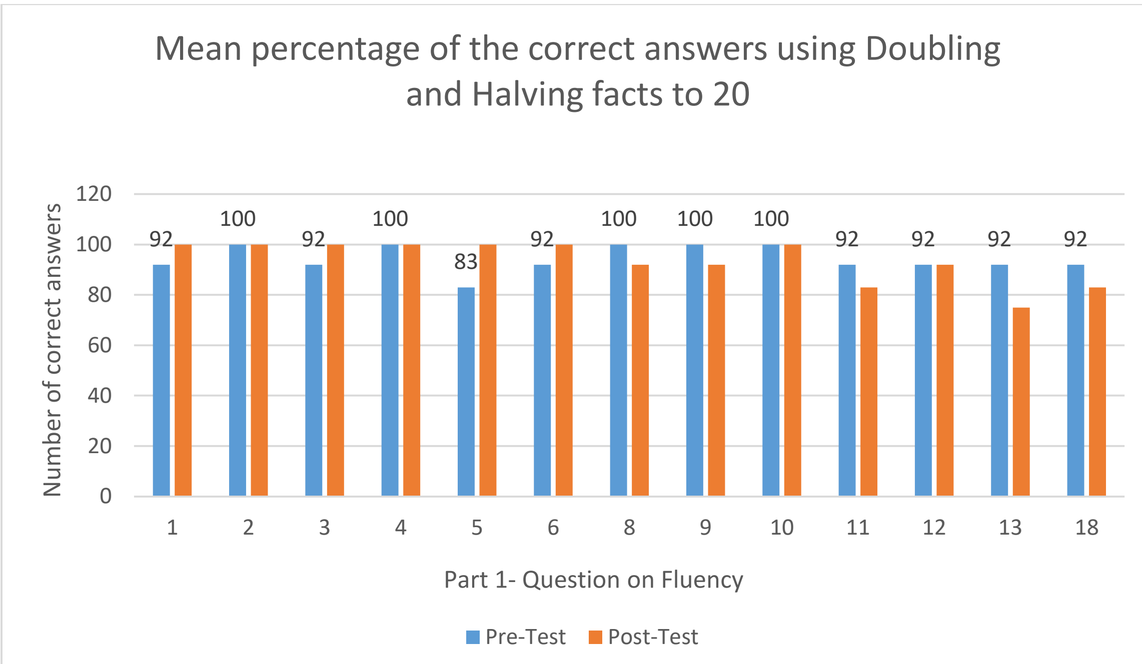


Figure 4.7.1: Overall results after the use of the Doubling and Halving facts to 20

Figure 4.7.1 illustrates the number of correct answers obtained by learners using the doubling and halving facts of 20. The results of learners' pre-tests using the strategy suggest that learner knowledge based on rapid recall might be developed from the everyday teaching of the curriculum or can be attributed to the mental warm-ups. In the pre-test, the learners obtained more than 80% of the correct answers. In question five ($\_ \times 2 = 12$) of the pre-test, 10 learners got the question correct, and in the post-test, 12 learners got the answer correct, showing a gain of +17%. After the pre-test, learners were interviewed about the strategy used to answer the question. The reader is reminded that due to the small sample size *n=12* and limited time that I was given to work with the intervention group, I could only interview one learner in the pre-test where learners scored the lowest and one learner in the post-test where learners showed the highest gain to compare the strategies, they used to solve the problems. When the learner was interviewed on the strategy used to work out $\_ \times 2 = 12$, the learner said: "I plussed $6 + 6$ and then I times it by my hand and then counted it, and then it equalled 12". The learner's response shows understanding of the use of doubling and halving facts in 20 strategies, where the learner added 6+6 and defaulted to using fingers to check his thinking.

In the post-test, the learner was asked how they worked out the mathematical problem. The learner said, "We know that 7 plus 7 is 14, so $7 \times 2 = 12$.". By mentioning "we know," it suggests that the learner had prior knowledge and understanding of retrieving the basic facts of 20 and had internalised the 1-minute mental warm-up class activities that were conducted before each intervention lesson. The learner was able to combine 7 and 7, which form part of the basic facts of 20. Isaacs and Caroll (1999) state that learners' understanding of basic facts strategy is a vital aspect of developing a basis for learners "conceptual understanding and procedural skills, helping students learn to explain their thinking and to understand others' explanations "(p. 509). In their study, the authors found that the "doubles" facts are helpful when learners' illicit unknown facts. The authors further emphasise that learners, as early as in their foundational years, should be taught double facts. In my study, teaching basic facts in the intervention lessons improved the learner's retrieval of 20 basic facts, which resulted in improved results.

4.7.2 Doubles of Multiples of 10

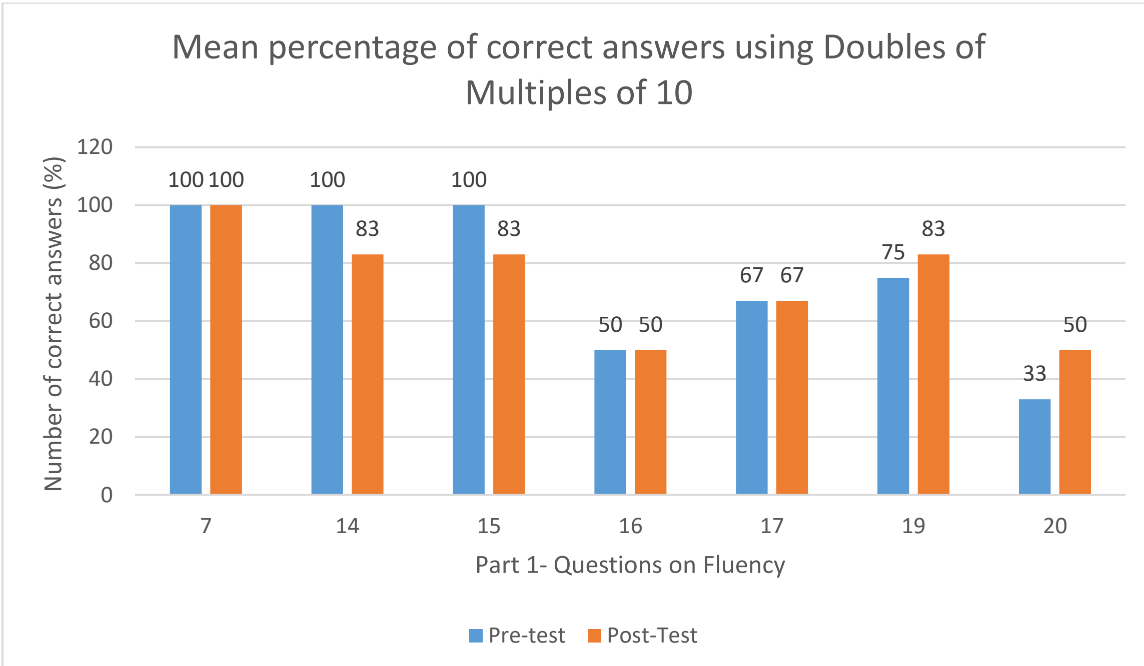


Figure 4.7.2: Overall results after the use of Doubles multiples of 10

Figure 4.5 illustrates learners' results with the use of the doubles of multiples of 10. In the pre-test and post-test, 12 learners got the correct answer for question 7 (double 10 = _). In question 19 (half of 30) of the pre-test, nine learners got the correct answer, and in the post-test, 10 learners got the correct answer. In question 20 ($2 \times 60 = \_$) of the pre-test, four learners got the answer correct, whereas in the post-test, six learners got the correct answer. Focusing on the lowest result, which was in question 20 of the pre-test, the learner was asked how she worked out question 20. The learner said, "I counted in 2s up to 60 with my fingers, and then when I stopped at 60, I had the number 10". The learner's use of fingers suggests that learners are dependent on the use of concrete material to work out problems and have not identified how to apply multiples of 10 to a problem. The learner's strategy suggests that the learner's prior knowledge shows that the learner was unable to recognise that the problem is based on the use of the number 10. After the intervention, the researcher asked the learner which strategy the learner used. The learner said, "I got $6 + 6$ is 12, and you just add a 0 and then you get 120. The learner applied the basic doubling facts of 20 by doubling six and was able to recognise that the zero had to be moved to the correct place value to get 120 as an answer. When I was teaching intervention lesson 3 to the learners, some of them were confused about the use of zero doubles problems; it seemed that learners were not cognizant of the zero when doubling or halving. I asked a learner to explain to their classmates why double 30 is 60. The learner said that because 10 is not a unit, it falls under tens, and in tens there is always one zero. The pictorial representation of base 10 in the intervention lesson presented how learners can make a connection between 3 and 30 or 6 and 60 when doubling and halving using multiples of ten.

4.7.3 Doubles of 2 Digit Numbers

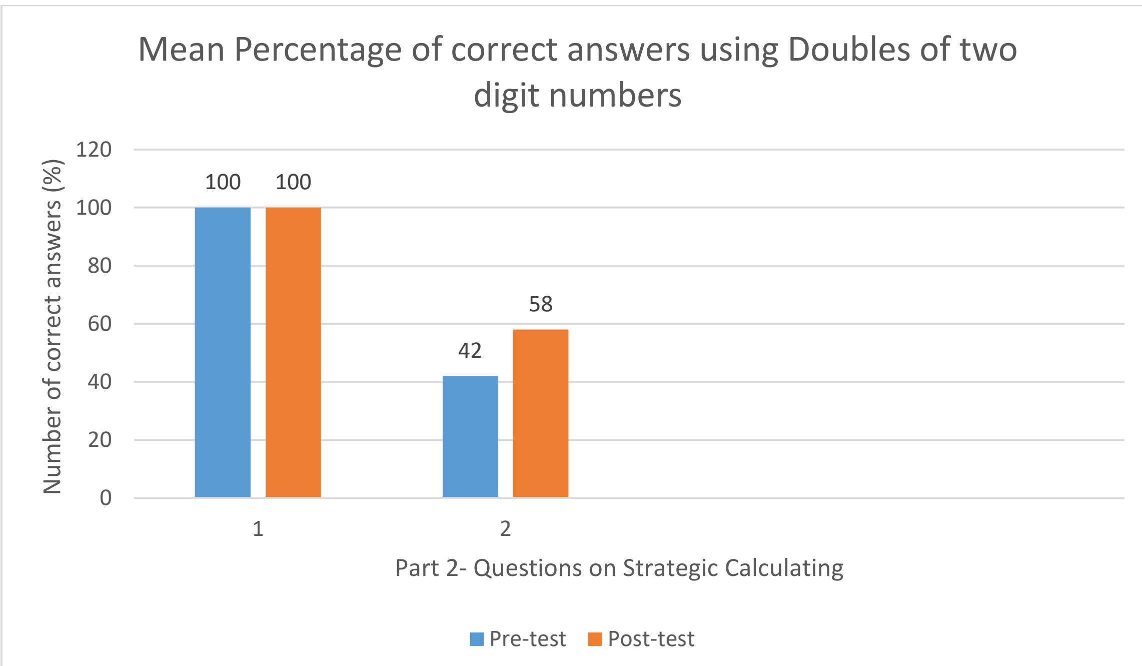


Figure 4.7.3: Overall results after the use of Doubles of two-digit numbers

Figure 4.7.3 illustrates the overall results of the use of doubles of two-digit numbers strategy. In question one (double 42 = _) of the pre-test and post-test, 12 learners got the answer correct. In question two (36×2) of the pre-test, five learners got the correct answer, whereas in the post-test, seven learners got the correct answer. In an interview that was conducted after the pre-test, the researcher asked the learner how he worked out question 2. The learner said, I counted in 2s." I first said, $36 + 36$ is $3 + 3$ is 60, and $6 + 6$ is 12. $60 + 12 = 72$." The learner's use of the strategy suggests that the learner had knowledge of doubling and was able to recognise that putting three tens together makes 30 and double of 6 makes 12.

In the post-test, the learners' results increased by 16%. The increment of the results in question two suggests that the learner used the correct strategy as taught in the intervention lesson, where emphasis was placed on the breaking down method. The learner explained using the breaking down method, which was explicitly taught in intervention lesson 4. The learner explained the strategy by saying: "Because I said $30 + 30 = 60$ and then I said $6 + 6 = 12$, then I took the ten from the tens in 12 and plussed it with the 60, and then I put the 2 to get 72". The learner's explanation using the breaking down method suggests that the learner was able to apply the method to the doubles of two-digit numbers strategy that was taught in the intervention lesson.

4.7.4 Halving of 2 Digit Numbers

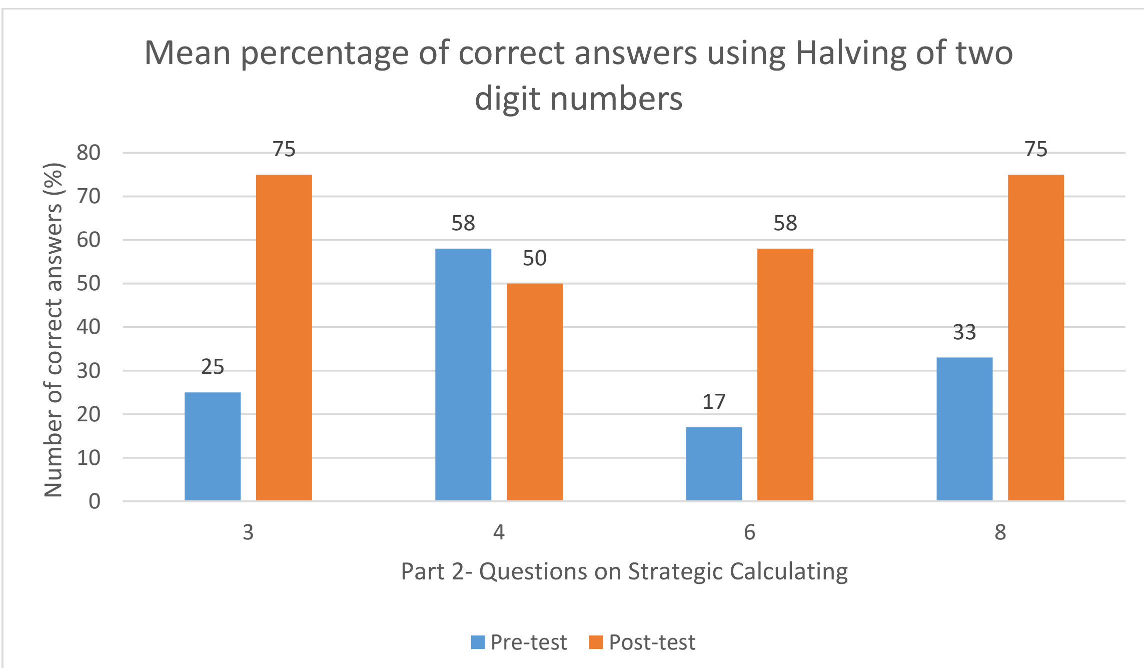


Figure 4.7.4: Overall results after the use of Halving of two-digit numbers

Figure 4.7.4 shows the results of the halving of using a two-digit number strategy. In question three $(64 \div 2)$ of the pre-test three learners got the correct answer whereas in the post-test nine learners got the correct answer. In question six (half of 38=_) two learners got the correct answer. In question eight (half of 78 is_) four learners got the correct answer and in the post-test nine learners got the correct answer. In explaining, the strategy used in the pre-test to work out half of 38 the learner said: "So I counted what half of 38 I counted is I counted I counted I counted then my answer was 19." The learner's response shows that the learner did not have a clear strategy but guessed the answer. However, after the intervention the post test results revealed an increment in questions 3 (50%), 6 (41%) and eight (42%). In solving the problem, the learner used the breaking down method: "I would put the lines here and say 30 and 8 and here I would say 15 and I would say 4 and I add 15 by 4, and I say 16,17,18,19." The learner recognised that halving numbers is the same as divide which was a reminder to learners when taught to half two-digit numbers in the intervention lesson. Learners' use of the halving of two-digits strategy suggests that the learner was able procedurally work out the mathematical problems using the breaking down method substituting doubling for halving but using the same procedure to work out problems hence the learner was able to get the correct answer.

4.7.5 Doubling and Halving Number Facts including Near Doubles

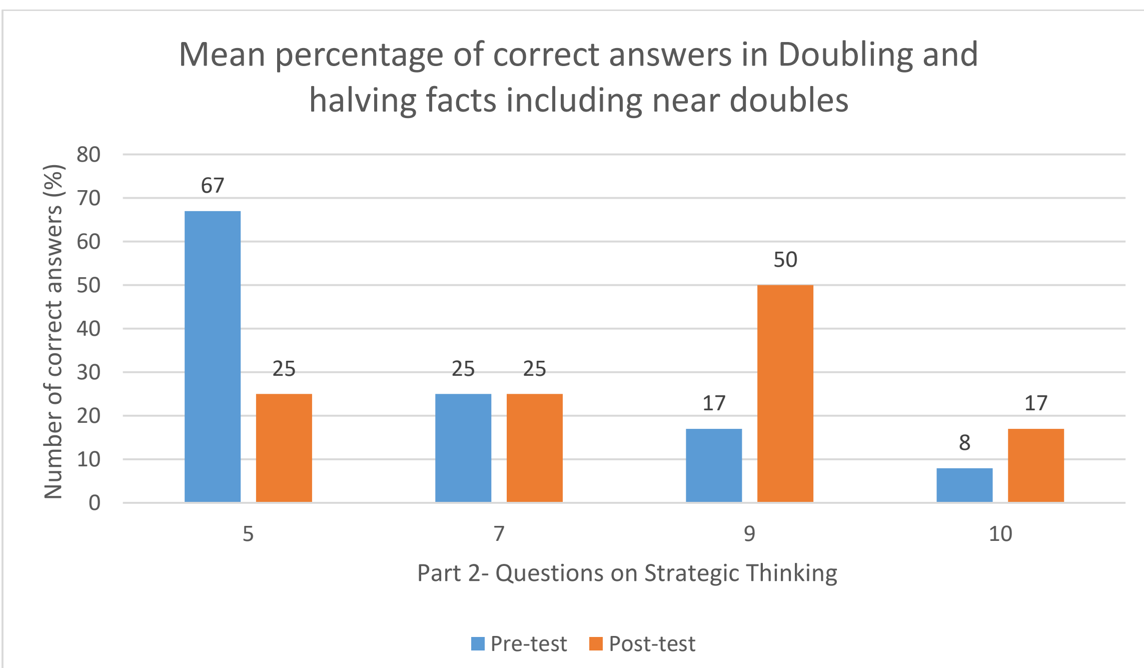


Figure 4.7.5: Overall results after the use of Doubling and halving facts including near doubles.

Figure 4.7.5 shows the results for doubling and halving facts, including near doubles. In question five (double 47 = _) of the pre-test, eight learners got the correct answer, whereas in the post-test, three learners got the correct answer. No difference in results was noted for question 7 in both the pre- and post-tests. In question nine $(39 + 38)$ in the pre-test, two learners got the correct answer. In question 10 (double $39 = 40 + 40 = \_$) of the pre-test, one learner got the answer correct, and in the post-test, two learners got the correct answer. With focus on the strategy used for question 10 in the pre-test and the overall results of 8%, it can be seen that learners did not understand that the problem was made up of different aspects to be drawn together to get to the answer. The learner did not have a clear strategy and explained that "double 39 is 78 plus $40 + 40$, so I said $4 + 4$ is 8, and then $0 + 0$, and then I wrote 8". The learner demonstrated no understanding of the complexity of the problem but applied doubling to show that the learner had some knowledge of how to double from prior knowledge.

In the post-test, the learners showed an understanding of the problem's complexity. When asked how the learner worked out question 10, the learner responded: I said double $39 = 40 + 40$, so they just added 1 to it to make 40 because $39 + 1$ is 40. Therefore, they write $40 + 40 - 2$. I got 2. If you don't understand how to do $39 + 39$ right, you just add 1 and another 40, $40 + 40 - 2$, because 39 is double right, so we want to double 39 right, so if it's very hard for you, you just add 1 and then you minus 2, and I get 78. The learner's use of the strategy shows a more sophisticated understanding of the strategy, where the learner was able to recognise that the problem moves beyond applying procedure but was able to show conceptual understanding of the strategy by recognising the use of near doubles. The learner's use of the doubling and halving facts strategy, which includes near doubles, suggests that the strategy is effective in solving complex problems.

4.8 Conclusion

Prior to the intervention, the learners in the control and intervention groups relied on unit counting strategies to solve problems, and the alternative strategies that were used were not clear. The strategies used in the pre-test were an indication that there was a need for an intervention to enhance learners' use of calculation strategies. After the pre-test, the intervention group was exposed to the various doubling and halving representations of the strategy. The findings show that the intervention group had performed slightly better in the strategic calculating and strategic thinking categories than the control group, which continued with the usual Foundation Phase Math Curriculum. These results help me draw conclusions that, doubling and halving calculation strategies are critical for learners to learn to improve their problem-solving skills, albeit minimal shifts

Chapter 5: Conclusion

5.1 Introduction

Following from the findings in the previous chapter, in this chapter I discuss the findings by re-looking at the research questions of the study and if the outcomes were met. I will then discuss the limitations, followed by the recommendations, and then close with consideration for future research.

5.2 Overview of Research Study

This research study was an intervention study that aimed at exploring the doubling and halving strategy with Grade 3 learners to shift learners from counting to calculation strategies. The acquisition of calculation strategies in the Foundation Phase is key to developing learners' conceptual understanding of the use of doubling and halving strategies in solving complex mathematical problems. This project study was adopted from the MSAP; whose aim is to shift learners' dependency on unit counting using starter intervention lessons to improve learners' use of counting strategies to calculation strategies.

My study was an experiment, and 24 learners were selected from a Grade 3 class and split into two groups of 12 to create a control and intervention group. The purpose of the control and intervention groups was to compare learner gains in results between the two groups to determine whether the intervention group's fluency, strategic calculating and strategic thinking had improved after the intervention. In another layer of analysis, with the use of strategies by the intervention group in the pre-test and post-test, learners use of doubling and halving strategies was compared to strategies used prior to the intervention after the implementation of the intervention to determine whether the doubling and halving strategies shifted learners' use of strategies in the post-test. The analysis of the strategies was to bring forth any noticeable change with the intervention group's pre-test and post-test results and if practice of the doubling and halving strategies resulted in learner improvement.

The theoretical framework that informed my study is the theory of cognitive development from Piaget, which focuses on building cognitive structures that set the basis for how learners process new knowledge. In my study, learners were taught strategies that differed in complexity, thus contributing to how these strategies are processed, assimilated, and accommodated. The intervention lessons that were presented to learners started with concrete representation where learners used their fingers, and then the lessons progressed to introducing pictorial representation and then abstract representation of the doubling and halving strategy. The progression of the lessons enabled learners to grasp the strategies as they systematically progressed through all eight starter intervention lessons. Learner progression on the use of strategies prior to the intervention and after can be seen in the following discussion, where I revisit the research questions that guided the study.

5.3 Key Findings

Is there any mean different in performance between the intervention group and control group?

Although in the t-test there was no significant difference between the control and intervention groups, the mean percentage showed improvement in strategic calculating and strategic thinking. Both groups averaged similar results of 80% in fluency; however, after the intervention and after the post-test, the intervention group’s performance slightly dropped in fluency and improved in strategic calculating and strategic thinking. The findings were aligned with Graven and Venkat’s (2021) findings, where they found learners who used the “bridging through tens” calculation strategy, a strategy from the MSAP, showed improvement in fluency, strategic calculating, and strategic thinking.

What strategies did learners use prior to the intervention in the pre-test?

In the pre-test, the results from the intervention group showed an overall score of above 80%, which meant that learners’ knowledge base was already established in the fluency questions, but there was a slight drop in comparison to the control group. Whereas the control group also obtained above 80% in the fluency questions and a drop in strategic calculation and strategic thinking. The intervention group and control group used low level strategies (use of fingers and counting) to work out fluency, strategic calculating and strategic thinking problems. The lack of sophistication in learner use of strategy in the pre-test shows learners inability to recognise the use of more sophisticated strategies that do not require the use of fingers or counting. At Grade 3, and according to the curriculum (DBE, 2011), learners should be using mental strategies when solving problems. Strategies used by learners in the pre-test have reflected that learners use of strategies is still less sophisticated.

What strategies did learners in the intervention group use after the intervention in the post-test?

In part one of the post-test, there was a slight decrease in results from the intervention group, as mentioned above. In Part 2, the focus was on strategic calculating and strategic thinking, which entailed pictorial and abstract representations of the doubling and halving strategy. The positive gain in strategic calculating and strategic thinking problems was achieved using the intervention strategies (doubling and halving facts to 20, doubles of multiples of 10, doubles of 2-digit numbers, halving of 2-digit numbers, and doubling and halving number facts, including near doubles). In the post-test, learners used a variety of strategies from the intervention lesson, where a positive percentage gain was achieved, but less so for the doubling and halving number facts, including near doubles, where only two learners were able to use this strategy. This is aligned with Murphy's (2004) study, which found that after teaching mental calculation strategies, learners showed some use of the compensation strategy with multiples of ten.

Were there any shifts from counting strategies to calculating strategies after the intervention process in the intervention group?

The analysis of the findings shows that there was a shift in strategies from counting to calculating using different representations of doubling and halving, even though the gain was not significant. In the pre-test, there was evidence that learners were dependent on the use of unit counting strategies with the use of fingers, and in other instances, learners displayed strategies that were not clear. After teaching the intervention lessons, learners in the intervention group used more of the doubling and halving strategies taught in the intervention. Learners preferred the breaking down method to check their answers as a form of manipulation, which was taught in lessons 4 and 5 in the intervention lessons, instead of guessing or defaulting back to using counting strategies or fingers, which is considered a “low-level” strategy (Anghileri, 2001). Although learners in the intervention group showed understanding of the strategies, they need to practice the doubling and halving number facts, including the near doubles strategy, to improve their understanding of how the procedure of the strategy is applied to multi-digit addition and subtraction problems. (Johnson et al., 2001). Thus, while we can conclude that shifts in strategies from the pre-test to the post-test can be attributed to the intervention, these shifts are not significant enough to argue that the intervention can produce definite results. Instead of banishing this intervention, I need to understand these results as a first step in shifting how learners assimilate and accommodate knowledge. These findings have implications for the teaching and learning of doubling and halving using the MSAP lesson starter intervention.

5.4 Implications

My research adds to other research that indicates that foundation phase learners continue to employ concrete counting strategies using fingers and tally marks. The MSAP intervention was able to get learners to move forward from using counting strategies to more abstract strategies using fluency, strategic calculating, and strategic thinking, but learners struggle to fully accommodate this knowledge into their cognitive structures. Accommodation, however, takes time, and within this process, a lot may be learned to improve the outcomes.

Graven and Venkat (2021), as discussed in the introduction of my study, focused on the ‘bridging through ten’ strategy from the MSAP with the aim of moving learners from the use of concrete counting methods to a conceptual understanding of number structure. The authors indicate that the results of their study present an overall positive gain in the post-assessment as evidence of the intervention being effective in improving learners rapid recall, strategic calculating and strategic thinking through the ‘bridging through ten’ strategy.

Although I have adapted Graven and Venkat’s (2021) study to contextualise my research, it needs to be noted that the authors employed a different strategy than the one employed in my study. This indicates that each strategy should be analysed in different contexts. We should be mindful of different findings. We cannot expect learners to master these strategies immediately. We should expect contestations on a range of levels and within limitations. These discussions should contribute to the revision of the starter lessons.

In the analysis of the pre-test and post-test in Chapter 4, I indicate that prior to the intervention, the learners in the control and intervention groups relied on counting strategies to solve problems, and the alternative strategies that were used were not clear. The strategies used in the pre-test were an indication that there was a need for an intervention to enhance learners' use of calculation strategies. The intervention group was exposed to the various doubling and halving representations of the strategy. However, the findings show that the intervention group performed slightly better in the strategic calculating and strategic thinking categories than the control group. These findings indicate that a shift in learning can happen, albeit slower than expected.

In relation to the above discussion, shifts through intervention studies need to be understood through further research. It appears learners are attempting to assimilate new knowledge when working with the MSAP strategies. In my study, the learners from the intervention group attempted to accommodate some of the strategies, which could be seen in how they responded to the post-test and in the interviews. However, they resorted to using unit counting. Further research is needed across contexts and learners to indicate ways in which the intervention could be improved. The challenge for learner education is to understand the small shifts that learners are making to develop ways to effectively facilitate the process. Doubling and halving are recognised as "sophisticated techniques". The curriculum states that "learners who choose doubling and halving are quite flexible in the strategies they use. However, the level of intricacy that is interrelated with the use of calculation strategies is not apparent in the curriculum. Doubling and halving need to be explained within the curriculum and its documents. Results from intervention studies can help move learners from there to where they need to be.

5.5 Limitations

This study is part of the MSAP, and like any small-scale study, it came with its limitations.

Piaget's theory of cognitive development, with a focus on learners' processes of acquiring new knowledge and developing their already existing schemas through doubling and halving strategies, had its limitations, which hindered the learners process of acquiring the more advanced strategies and applying them to problems. Block (1982) argues that "[s]chematizing (assimilation) and schemata modifying (accommodation) are very different adaptive modes" (p. 286). In the process of acquiring the doubling and halving strategies, the processes of assimilation and accommodation could have limited how the learners modified and understood the doubling and halving strategies in problems that were advanced.

The learner sample was small and not representative of the population. The learners in the intervention group were high attainers, and the learners in the control group were middle to low attainers academically. The small sample further limited the number of learners that could be interviewed to show their understanding and use of the strategies, which cannot be generalised.

The pilot study took place over ten days, with each lesson taught on different days as per the school's schedule. The time that I was allocated with the learners in the intervention group was not enough to teach the doubling and halving number facts near doubles, which were more challenging for learners to grasp. Thompson (1999) states that it is important to master single digit doubling when working with near doubles. I had to work with the time I was given, which sometimes did not give me enough time with the learners to check their understanding of the use of the strategies**.**

5.6 Conclusion

This study indicates that calculating strategies will take time. Counting strategies using concrete forms have been deeply engrained, and adjustment to new skills and strategies takes time. While the intention of the intervention was to get grade 3 learners to employ more sophisticated ways of calculation, this research shows that the results did not present significant shifts from the pre-test to the post-test.

From a cognitive stance, mathematical language as well as the language of instruction need to be considered when learners assimilate knowledge and accommodate cognitive structures. It is clear from the research that getting learners to progress from the use of concrete to the use of abstract is problematic (Schollar, 2008; Graven and Venkat, 2021). Research must identify doubling and halving strategies in order to help learners progress in number sense.

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Appendices

Appendix A

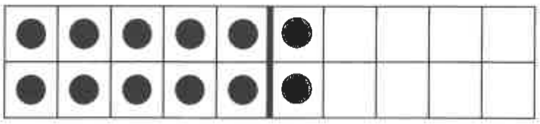
Tests and Intervention Lessons

Doubling & Halving

Name:

Doubling & Halving: Pre-Test

PART 1 — 2 minutes for this page

1. 

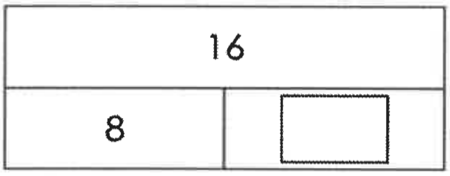
6 + 6 = ☐

2. half of 12 = ☐

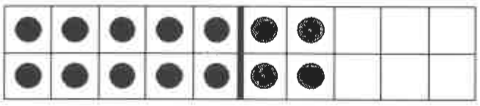
3. 9 + 9 = ☐

4. double 8 = ☐

5. ☐ x 2 = 12

6. 

7. double 10 = ☐

8. 

half of 14 = ☐

9. 10 ÷ 2 = ☐

10. half of 18 = ☐

11. 15 + 15 = ☐

12. 7 x 2 = ☐

13. half of ☐ = 7

14. double 100 = ☐

15. double 20 = ☐

16. half of ☐ = 40

17. half of 50 = ☐

18. 16 ÷ 2 = ☐

19. half of 30 = ☐

20. 2 x 60 = ☐

Total out of 20

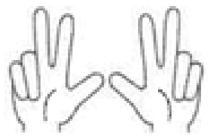
Doubling & Halving: Pre-Test PART 2	3 minutes for this page
1. double 42 = $\boxed{}$	
2. 36 x 2 = $\boxed{}$	
3. 64 ÷ 2 = $\boxed{}$	
4. half of 102 = $\boxed{}$	
5. double 47 = $\boxed{}$	
6. half of 38 = $\boxed{}$	
7. half of $\boxed{}$ = 52	
double 39 is 78 8. half of 78 is $\boxed{}$ 9. 39 + 38 = $\boxed{}$ 10. double 39 = 40 + 40 – $\boxed{}$	
Total out of 10	

DOUBLING & HALVING: LESSON STARTER 1

1-Minute Mental Warm-Up

'I show, you say' (whole class and then a learner paired activity)

a. The teacher shows a 'double' number using fingers on two hands, e.g.



Double 3 is 6. Now tell me the doubles sentence for the fingers I show.'

The teacher shows: Double 4 Double 1 Double 3 Double 5 Double 2

Learners say the appropriate sentence, e.g. "Double 4 is 8".

b. Learners working in pairs can extend this activity to show double 6 – double 10 using their fingers:

Teacher: Each pair, show me double 6.

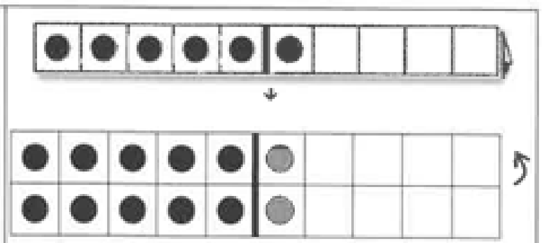
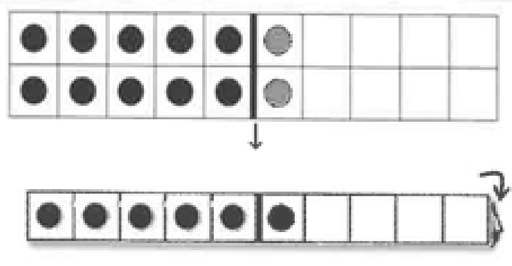
Learner pairs: 

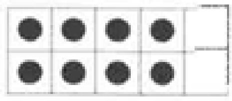
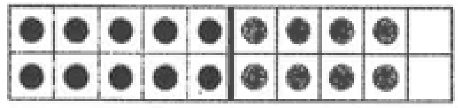
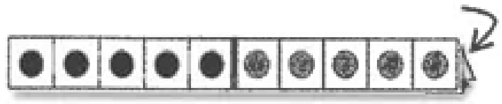
Teacher: How many fingers altogether in double 6?

Ask learners to mirror match their full hands and the hands with one finger open. Help learners to see that the answer 12 is made up of two hands with 5 fingers and two hands with 1 finger: 5 + 5 and 1 + 1.

Task Sequence

In this lesson we use basic doubling and halving facts to 20.

Problem: double 6 Show six dots on one half of the doubles dot card. Open the card. Teacher: Now I have double six. How many dots altogether? Learners: 12 Teacher: How do you know double 6 is 12? Listen for learners who say that the doubles card shows: '6 and 6', 'two groups of 6', 'two times 6', '6 x 2'.	 Double 6 = 12 Two groups of six is 12 Two times 6 is 12 6 x 2 = 12
Problem: half of 12 Show twelve dots on the doubles dot card. Fold the card in half lengthways. Teacher: Now I can see half of the 12 dots and you can see half. So what is half of 12? Learners: 6	 Half of 12 is 6

Teacher: How do you know the answer is 6? Listen for learners who give explanations like: 'half of 12 is 6' or 'twelve divided into two parts is 6' or 'twelve shared between two is six' or '12 ÷ 2 = 6'. If no similar offers are made, prompt the class to repeat these sentences after you. Write these different versions on the board.	Twelve divided into 2 equal parts Is 6. Twelve shared between 2 is 6. 12 ÷ 2 = 6 (These examples must remain on the board.)
Repeat with: Double 4 and double 9 dot cards Half of 8 and half of 20 dot cards Ask the class to say different sentences to match each of the double and half dot cards. Encourage learners to see the fives (dark dots) in the dot cards and to use these to find the number of dots quickly. So for double 9 we can see two 5s as 10 and two 4s as 8 so double 9 is 18.	 Double 4 and half of 8  Double 9  Half of 20

Individual Tasks

Learners should now try the individual task sheet provided for Lesson Starter 1. Learners should complete the sentences, and write sentences, below each image of the dot cards on the worksheet.

Support Video

Doubling & Halving 1

https://youtu.be/UMmzMVM-SS0

DOUBLING & HALVING: LESSON STARTER 2

1-Minute Mental Warm-Up

Pop-Fizz doubles and halves to ten

a. The teacher says 'pop', learners say 'fizz'; the teacher says a number, learners respond with **doubles** (or a multiple of 10 more):

Teacher: pop	→	Learners: fizz	
Teacher: 1	→	Learners: 2	
Teacher: pop	→	Learners: fizz	
Teacher: 5	→	Learners: 10	and so on…

Doubles to 10: 1 – 2; 3 – 6; 5 – 10; 4 – 8; 2 – 4.

b. The teacher says 'pop', learners say 'fizz'; the teacher says a number, learners respond with **halves** (or a multiple of 10 less):

Teacher: pop	→	Learners: fizz	
Teacher: 8	→	Learners: 4	
Teacher: pop	→	Learners: fizz	
Teacher: 6	→	Learners: 3	and so on…

Halves to 10: 10 – 5; 6 – 3; 4 – 2; 8 – 4; 2 – 1

Task Sequence

In this lesson we practise basic doubling and halving facts to 20.

Note: The double dot cards are available in the Print Master book.

Problem: Connect double 8 = ☐ ; half of 16 = ☐

Use the double 8 dot card. Fold the card to show one group of 8. Then open to show both groups of 8.

Teacher: So what is double 8?

Learners: 16

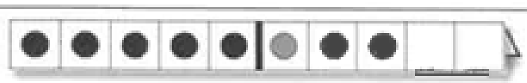
Write 'double 8 = 16' on the board

Teacher: So what is half of 16? (Fold the card as you say this to show how to halve by making two equal groups.)

Learners: 8

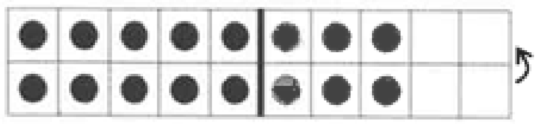
Write 'half of 16 = 8' below the double sentence on the board.

These examples must all remain on the board.

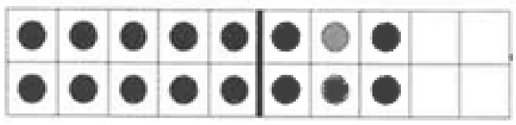


8 dots

↓

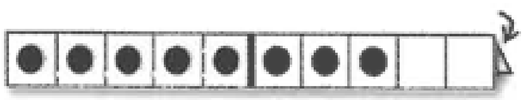


double 8 = 16



16 dots

↓



half of 16 = 8

Doubling & Halving

Individual Tasks

Put the double 5, double 7 and double 9 dot cards on the board. Learners should write the double and half number sentences for the cards.

Learners should be encouraged to explain their thinking.

Tell learners NOT to count in 1s.

If any learners finish these tasks quickly, ask them to write number sentences for other dot cards.

Support Video

Doubling & Halving 2

https://youtu.be/8g1unCfK1Lo

DOUBLING & HALVING: LESSON STARTER 3

1-Minute Mental Warm-Up

Pop-Fizz doubles and halves to twenty

Doubles to 20: 1 – 2; 3 – 6; 5 – 10; 4 – 8; 2 – 4; 6 – 12; 9 – 18; 7 – 14; 8 – 16; 10 – 20.

Halves to 20: 10 – 5; 6 – 3; 4 – 2; 8 – 4; 2 – 1; 12 – 6; 18 – 9; 14 – 7; 16 – 8; 20 – 10.

Task Sequence

In this lesson we practise doubles of multiples of ten.

Note: The dot strips are available in the Print Master book.

<table>
<tr>
<td>Problem: Connect double 3 = ☐; double 30 = ☐

Use six 10-dot strips and arrange them to show double 30.

Teacher: We know double 3 = 6, so what is double 30?

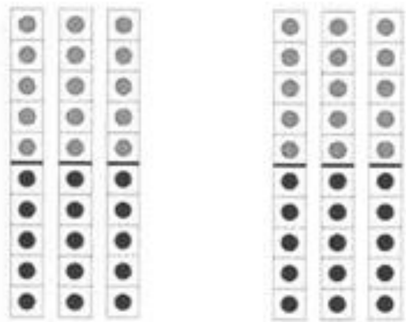
Learners: 60

Teacher: Double 30 is 60, so what is half of 60?

Learners: 30

Write the number sentences as shown, and point out that 6 and 60 are connected just like 3 and 30. Tell learners to remember that doubles and halves are connected.

Teacher: We can also use strips to stand for 10 instead of showing ten dots every time.

Show learners on the board how the tens can also be shown using long strips that you draw on the board.</td>
<td>Double 3 = 6

Double 30 = 60
Half of 60 = 30
</td>
</tr>
</table>

Individual Tasks

Learners should now try the following examples *mentally*:

Double 2	Double 5	Double 7
Double 20	Double 50	Double 70

Tell learners NOT to count in 1s. They should use the connection they have just learned to write the larger doubles quickly.

If any learners finish these tasks quickly, give them more to practice:

Double 3	Double 8	Half of 4	Half of 8
Double 30	Double 80	Half of 40	Half of 80

DOUBLING & HALVING: LESSON STARTER 4

1-Minute Mental Warm-Up

Doubles and halves of friendly numbers

Friendly numbers are numbers that are easy to work with. Often these are multiples of ten.

"What is…?"

Teacher: double 30	→	Learners: 60	
Teacher: double 10	→	Learners: 20	
Teacher: double 50	→	Learners: 100	
Teacher: half of 40	→	Learners: 20	
Teacher: half of 50	→	Learners: 25	
Teacher: half of 100	→	Learners: 50	and so on…

Task Sequence

In this lesson we practise doubles of two-digit numbers.

Note: The dot strips are available in the Print Master book.

Problem: Double 35

Use six 10-dot strips, and two 5-strips, and arrange them to show double 35.

Teacher: What double number sentence does the diagram show?

Remind the class that 1 strip (either dots or a solid line) shows 1 ten.

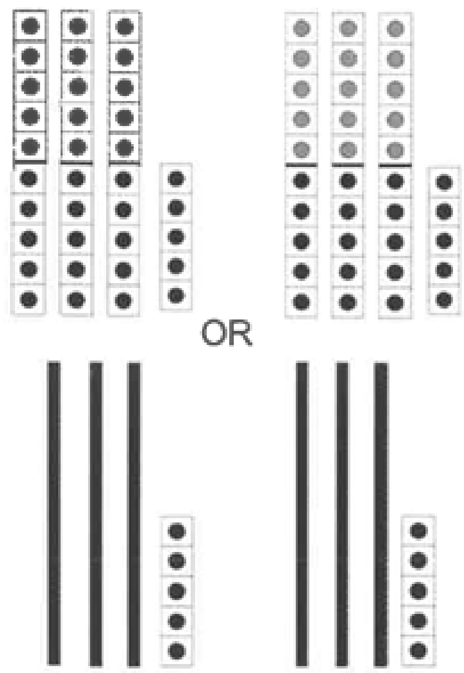
Learners: Double 35 (or 35 + 35)

Teacher: How can we work out the answer?

Listen for learners who talk about putting the tens together to get 6 tens or 60 and putting the two 5s together to give 10.

Write this 'breaking down' method as shown opposite on the board.

Double 35



double 35

30 5

↓ ↓

60 10

double 35 = 70

Problem: 29 x 2 Repeat the breaking down method for double 29 as shown. Remind the class that multiplying by 2 (or x2) is the same as doubling. Allow the learners to help you to fill in the doubles of the tens and the units. Some learners may say that 29 x 2 is 60 – 2 = 58. This approach should also be accepted.	double 29 20 9 ↓ ↓ 40 18 double 29 = 58

Individual Tasks

Learners should now try the following examples:

Double 41 Double 36 Double 47

Learners should write the breaking down and work out the doubles of the tens and units mentally. The idea is to work towards being able to answer these questions mentally.

Learners should explain their thinking, e.g. "double 47 is double 40 (that's 80) and double 7 (that's 14). 80 and 14 is 80, 90, 94."

Tell learners NOT to count in 1s.

Take-home task – Worksheet 1

At the end of today's session give learners Worksheet 1.

You do not need to time learners as they do this worksheet. The aim is to give learners some written practice of the work they have done mentally.

Support Video

Doubling & Halving 4

https://youtu.be/qnSniN-bIiU

DOUBLING & HALVING: LESSON STARTER 5

1-Minute Mental Warm-Up

Doubles and halves of friendly numbers

Task Sequence

In this lesson we practise halving two-digit numbers.

Problem: 62 ÷ 2 Remind the class that dividing by 2 (÷ 2) is the same as working out half. Teacher: How can we work out what half of 62 is? Listen for learners who talk about halving 60 to get 30 and halving 2 to get 1, to get the answer of 31. Write this 'breaking down' method as shown opposite on the board.	half of 62 60 2 ↓ ↓ 30 1 half of 62 = 31
Problem: 76 ÷ 2 Repeat the breaking down method for half of 76 as shown. Remind the class that dividing by 2 (or ÷2) is the same as halving. Allow the learners to help you to fill in the halves of the tens and the units.	half of 76 70 6 ↓ ↓ 35 3 half of 76 = 38

Individual Tasks

Learners should now try the following examples:

half of 42   68 ÷ 2   34 ÷ 2

Learners should write the breaking down and work out the halves of the tens and units mentally.

Learners should explain their thinking, e.g. "34 ÷ 2 is half of 30 (that's 15) and half of 4 (that's 2). 15 and 2 is 17."

Tell learners NOT to count in 1s.

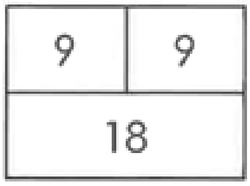
DOUBLING & HALVING: LESSON STARTER 6

1-Minute Mental Warm-Up

Say it another way:

Work with different representations of doubling and halving. These can include words like 'double 7' and 'half of 16', or alternatives like 'two groups of 7' or '7 and 7' or '7 + 7' or '16 ÷ 2' or 'sixteen shared between two'.

This can also include providing an image like the one below:



Learners should offer ways of saying or writing 'double 9 = 18' e.g.
9 + 9 = 18   18 – 9 = 9   two nines make 18   9 × 2 = 18

Task Sequence

In this lesson we use doubles and halves in a range of ways.

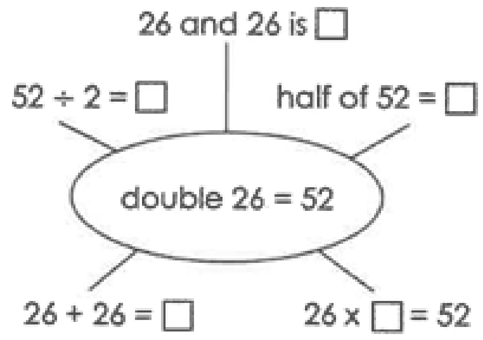
Problem: double 26

Ask learners for different ways of saying what they see in this diagram.

For example, double 26 is the same as 26 x 2 or 'two groups of 26'. Another example is that double 26 is 2 more than double 25 because each of the groups has 1 more in it.

Add any new ideas from the learners to the diagram.

Some learners may offer the subtraction calculation 52 – 26 = 26. Write this on the diagram if it is offered. Linking addition and subtraction is taught as a strategy in the final set of lesson starters.



Individual Tasks

Learners should now draw a web of facts, like the one above, linked to:

double 43 = 86

Learners should explain their thinking, e.g. "I know 43 and 43 is 86" or "double 43 is 86, so, I know double 430 is 860."

If any learners finish this task quickly, ask them to create another web of connected facts starting with any doubling or halving fact they choose.

DOUBLING & HALVING: LESSON STARTER 7

1-Minute Mental Warm-Up

Doubles and halves of multiples of 10, 100, 1000

"What is…?"

Teacher: double 10	→	Learners: 20
Teacher: double 100	→	Learners: 200
Teacher: double 1000	→	Learners: 2000
Teacher: half of 40	→	Learners: 20
Teacher: half of 400	→	Learners: 200
Teacher: half of 4000	→	Learners: 2000 and so on…

Task Sequence

In this lesson we practise doubles and halves of multiples of 10.

Problem: double 34 → double 340 → double 3400 Teacher: How can we work out what double 34 is? Listen for learners who talk about doubling 30 to get 60 and doubling 4 to get 8, to get 68. If learners struggle to calculate this mentally, write this 'breaking down' method as shown. Teacher: Can we use what we know about double 34 to work out what double 340 will be? Listen for learners who say that 340 is 10 times bigger than 34 so double 340 is ten times bigger than 68; that is 680.	double 34 30 ↓ 60 4 ↓ 8 double 34 = 68 double 34 = 68 double 340 = $\boxed{}$
Problem: half of 46 → half of 460 Teacher: How can we work out what half of 46 is? Listen for learners who talk about halving 40 to get 20 and halving 6 to get 3, to get 23. If learners struggle to calculate this mentally, write this 'breaking down' method as shown. Teacher: Can we use what we know about half of 46 to work out what half of 460 will be? Listen for learners who say that 460 is 10 times bigger than 46 so half of 460 is 10 times 23 = 230.	half of 46 40 ↓ 20 6 ↓ 3 half of 46 = 23 half of 46 = 23 half of 460 = $\boxed{}$

Individual Tasks

Learners should now draw a web of facts, like the one above, linked to:

double 45	double 27	half of 82	half of 76
double 450	double 270	half of 820	half of 760

Encourage learners to calculate the first double/half mentally if they can and to use the pattern to answer the subsequent doubles/halves as fast as they can.

Take-Home Task – Worksheet 2

At the end of today's session give learners Worksheet 2.

You do not need to time learners as they do this worksheet. The aim is to give learners some written practice of the work they have done mentally.

Support Video

Doubling & Halving 7

https://youtu.be/JJUPpmMdaAw

DOUBLING & HALVING: LESSON STARTER 8

1-Minute Mental Warm-Up

Doubles and halves of friendly numbers

Task Sequence

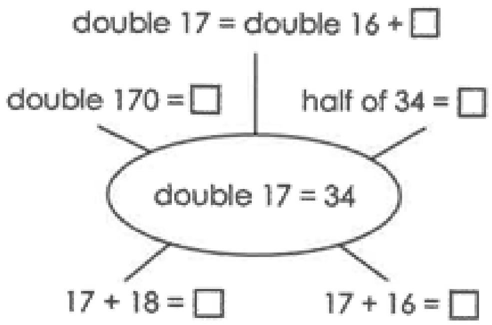
In this lesson we build connected facts related to a given doubling or halving number fact, including near doubles.

<table>
<tr>
<td>Problem: double 17

Ask learners for different ways of saying what they see in this diagram.

For example, 17 + 18 must be 1 more than 17 + 17. Another example is that double 17 is 2 more than double 16 because each of the groups has 1 more in it.

Add any new ideas from the learners to the diagram.

Some learners may offer the subtraction calculation 34 – 17 = 17. Write this on the diagram if it is offered. Linking addition and subtraction is taught as a strategy in the final set of lesson starters.</td>
<td>
</td>
</tr>
</table>

Individual Tasks

1. Learners should now draw a web of facts, like the one above, linked to:

   double 38 = 76

For example, '38 + 38 = 76' or 'half of 760 = 380' or 'half of a half of 76 = 19'

Learners should explain their thinking, e.g. "a quarter of 76 is 19 because I halved and then halved again".

Tell learners NOT to count in 1s.

2. Learners should try to answer these questions:

   What is double 99?

   Complete this sentence: Double 99 = double 100 – □

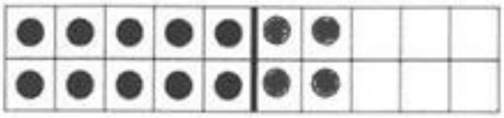
   What is double 49?

If any learners finish these tasks quickly, ask them to create another web of connected facts starting with any doubling or halving fact they choose.

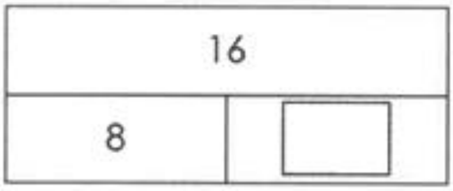
Name:

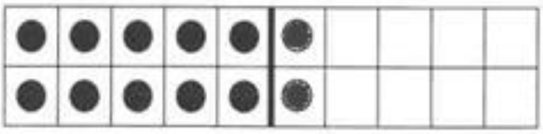
Doubling & Halving: Post-Test

PART 1 — 2 minutes for this page

1. 

   7 + 7 = ☐

2. half of 14 = ☐
3. 9 + 9 = ☐
4. double 8 = ☐
5. ☐ × 2 = 14
6. 

7. double 10 = ☐
8. 

   half of 12 = ☐

9. 10 ÷ 2 = ☐
10. half of 18 = ☐
11. 14 + 14 = ☐
12. 7 × 2 = ☐
13. half of ☐ = 7
14. double 100 = ☐
15. double 20 = ☐
16. half of ☐ = 40
17. half of 50 = ☐
18. 18 ÷ 2 = ☐
19. half of 30 = ☐
20. 2 × 60 = ☐

Total out of 20

Doubling & Halving: Post-Test

PART 2 — 3 minutes for this page

1. double 42 = ☐

2. 36 x 2 = ☐

3. 64 ÷ 2 = ☐

4. half of 102 = ☐

5. double 99 = ☐

6. half of 38 = ☐

7. half of ☐ = 52

double 39 is 78

8. half of 78 is ☐

9. 39 + 38 = ☐

10. double 39 = 40 + 40 – ☐

Total out of 10	

Appendix B.

Paired t-test results

Paired *t* test results

P value and statistical significance:

The two-tailed P value equals 0.6814

By conventional criteria, this difference is considered to be not statistically significant.

Confidence interval:

The mean of Pre-test minus Post-Test equals 0.95

95% confidence interval of this difference: From -3.82 to 5.72

Intermediate values used in calculations:

t = 0.4169

df = 19

standard error of difference = 2.279

Review your data:

Group	Pre-test	Post-Test
Mean	87.60	86.65
SD	18.30	15.87
SEM	4.09	3.55
N	20	20

Pre and Post Test Intervention Group Fluency

Paired *t* test results

P value and statistical significance:

The two-tailed P value equals 0.1497

By conventional criteria, this difference is considered to be not statistically significant.

Confidence interval:

The mean of Pre-test minus Post-Test equals -14.10

95% confidence interval of this difference: From -34.35 to 6.15

Intermediate values used in calculations:

t = 1.5752

df = 9

standard error of difference = 8.951

Review your data:

Group	Pre-test	Post-Test
Mean	39.20	53.30
SD	28.35	26.05
SEM	8.96	8.24
N	10	10

Pre and Post Test Intervention Group Strategic Calculating and Strategic Thinking

Paired *t* test results

P value and statistical significance:

The two-tailed P value equals 0.1795

By conventional criteria, this difference is considered to be not statistically significant.

Confidence interval:

The mean of Pre-test minus Post-Test equals -3.65

95% confidence interval of this difference: From -9.13 to 1.83

Intermediate values used in calculations:

t = 1.3936

df = 19

standard error of difference = 2.619

Review your data:

Group	Pre-test	Post-Test
Mean	61.00	64.65
SD	23.95	22.94
SEM	5.36	5.13
N	20	20

Pre and Post Test Control Group Fluency

Paired *t* test results

P value and statistical significance:

The two-tailed P value equals 0.6356

By conventional criteria, this difference is considered to be not statistically significant.

Confidence interval:

The mean of Pre-test minus Post-test equals -2.60

95% confidence interval of this difference: From -14.59 to 9.39

Intermediate values used in calculations:

t = 0.4903

df = 9

standard error of difference = 5.302

Review your data:

Group	Pre-test	Post-test
Mean	21.60	24.20
SD	23.80	22.83
SEM	7.53	7.22
N	10	10

Pre and Post Test Control Group Strategic Calculating and Strategic thinking

Paired *t* test results

P value and statistical significance:

The two-tailed P value equals 0.2555

By conventional criteria, this difference is considered to be not statistically significant.

Confidence interval:

The mean of Pre-test minus Post-Test equals -4.07

95% confidence interval of this difference: From -11.24 to 3.10

Intermediate values used in calculations:

t = 1.1599

df = 29

standard error of difference = 3.506

Review your data:

Group	Pre-test	Post-Test
Mean	71.47	75.53
SD	31.74	25.13
SEM	5.79	4.59
N	30	30

Overall Mean Pre and Post Test Intervention Group

Paired *t* test results

P value and statistical significance:

The two-tailed P value equals 0.1881

By conventional criteria, this difference is considered to be not statistically significant.

Confidence interval:

The mean of Pre-test minus Post-Test equals -3.270

95% confidence interval of this difference: From -8.231 to 1.691

Intermediate values used in calculations:

t = 1.3480

df = 29

standard error of difference = 2.426

Review your data:

Group	Pre-test	Post-Test
Mean	47.917	51.187
SD	30.072	29.681
SEM	5.490	5.419
N	30	30

Overall Mean Pre and Post Test Control Group

Paired *t* test results

P value and statistical significance:

The two-tailed P value equals 0.7722

By conventional criteria, this difference is considered to be not statistically significant.

Confidence interval:

The mean of Pre-test minus Post-Test equals 0.77

95% confidence interval of this difference: From -4.89 to 6.43

Intermediate values used in calculations:

t = 0.2962

df = 12

standard error of difference = 2.597

Review your data:

Group	Pre-test	Post-Test
Mean	94.38	93.62
SD	5.22	8.49
SEM	1.45	2.35
N	13	13

Doubling and Halving facts to 20

Paired *t* test results

P value and statistical significance:

The two-tailed P value equals 0.7927

By conventional criteria, this difference is considered to be not statistically significant.

Confidence interval:

The mean of Pre-test minus Post-test equals 1.29

95% confidence interval of this difference: From -10.16 to 12.73

Intermediate values used in calculations:

t = 0.2748

df = 6

standard error of difference = 4.679

Review your data:

Group	Pre-test	Post-test
Mean	75.00	73.71
SD	26.86	18.79
SEM	10.15	7.10
N	7	7

Doubles of Multiples of 10

Paired *t* test results

P value and statistical significance:

The two-tailed P value equals 0.5000

By conventional criteria, this difference is considered to be not statistically significant.

Confidence interval:

The mean of Pre-test minus Post-Test equals -8.00

95% confidence interval of this difference: From -109.65 to 93.65

Intermediate values used in calculations:

t = 1.0000

df = 1

standard error of difference = 8.000

Review your data:

Group	Pre-test	Post-Test
Mean	71.00	79.00
SD	41.01	29.70
SEM	29.00	21.00
N	2	2

Doubles of 2-digit numbers

Paired *t* test results

P value and statistical significance:

The two-tailed P value equals 0.0993

By conventional criteria, this difference is considered to be not quite statistically significant.

Confidence interval:

The mean of Pre-test minus Post-Test equals -31.25

95% confidence interval of this difference: From -73.38 to 10.88

Intermediate values used in calculations:

t = 2.3607

df = 3

standard error of difference = 13.237

Review your data:

Group	Pre-test	Post-Test
Mean	33.25	64.50
SD	17.75	12.56
SEM	8.87	6.28
N	4	4

Halving of 2-digit numbers

Paired *t* test results

P value and statistical significance:

The two-tailed P value equals 1.0000

By conventional criteria, this difference is considered to be not statistically significant.

Confidence interval:

The mean of Pre-test minus Post-Test equals 0.00

95% confidence interval of this difference: From -49.76 to 49.76

Intermediate values used in calculations:

t = 0.0000

df = 3

standard error of difference = 15.636

Review your data:

Group	Pre-test	Post-Test
Mean	29.25	29.25
SD	26.11	14.34
SEM	13.05	7.17
N	4	4

Doubling and Halving number facts including near doubles.

Appendix C.

Consent Letters

University of Witwatersrand, Johannesburg, South Africa

School of Education

Dear Parent/Guardian,

Information to participate in research project.

I write this letter to you as a Masters student at the University of the Witwatersrand. I am working with the Mental Starters Assessment Programme (MSAP) – Wits Maths Connect-Primary partnership project focused on Calculation Strategies. The project consists of a three-week intervention involving a written pre-test that your child will take, eight intervention lessons which will run thrice per week, followed by a post-test that is like the pre-test and an interview. I (the researcher) will mark the pre- and post-implement the eight intervention lessons and conduct the interview after the post-test.

I would like to collect and analyse the data on the class's pre- and post-test responses – your child's work on the tests would be part of this. This data will help me to understand whether the intervention lessons are helpful in improving learners' calculation strategies which entail fluency, strategic calculating and strategic thinking. In my research work, I will ensure that your child's name and school name are kept confidential and anonymized in all my collation and writing up of this data, so your child will not be identifiable in any way.

Learners' names will not be included on the tests and their names and identities won't be included in the research project. Any data collected will be stored in a password protected computer and will be destroyed after 5 years of completion of the project. If you are happy for your child's pre- and post-test responses to be used within my research work, I ask you to confirm this by signing and returning the tear-off slip below. If you do not wish for your child to be part of this project, then he/she will be allowed to join another Grade 3 class when her/his class either writes a test or does the lessons (three lessons per week for 3 weeks) linked to this project.

Please contact me if you have any questions regarding this research project using the following details 0419589P@students.wits.ac.za.

Yours sincerely,

Thobile Mtsweni

Researcher: Thobile Mtsweni, 0419589P@students.wits.ac.za

Supervisor: Dr. Zaheera Jina Asvat, zaheera.jina@wits.ac.za

Parent Consent Letter

TITLE OF THE STUDY: **Exploration of calculation strategies in doubling and halving with grade 3 learners**

I, ………………………………………………… (parent), agree for my child,
…………………………………………………………………

to participate in this research project. The research has been explained to me and I understand what.

my child's participation will involve. I agree to the following:

(Please circle the relevant options below).

I agree that my child's participation will remain anonymous
YES NO

I agree that the researcher may use anonymous quotes of my child in her academic articles
YES NO

I give consent to Zoom sessions being recorded with my child in them
YES NO

I give consent for part(s) of my child's working out notes to be used as part of the analysis
YES NO

I agree that the information my child provides may be used anonymously after this project has ended, for academic purposes by other researchers, subject to their own ethics clearance being obtained. YES NO

…………………………………… (name of participant)

…………………………………… (signature)

…………………………………… (date)

Thobile Mtsweni (name of person seeking consent)

…………………………………… (signature)

…………………………………… (date)

University of Witwatersrand, Johannesburg, South Africa

School of Education

LEARNER INFORMATION SHEET

Dear Learner,

I write this letter to you as a Masters student at the University of the Witwatersrand. I am working on a project which looks at calculation strategies and I will be working with you on this project. As part of the work, I will be giving you a short test that I will mark, and will then run eight lessons, thrice a week. At the end of this work, I will set you another short test to see if your work has improved. Your work on these tests does not count for marks, so you don't have to worry about how you do on them. I will just like you to try them and do as many of the questions as you can.

As part of my research project at Wits University, I would like to collect and look at your pre- and post-test answers and conduct an interview after the post-test. This will help me to understand whether these lessons are helpful for improving your mathematics learning.

In my research work, I won't put in your real name, so nobody reading my work will know that it is your answers we are talking about. All your answers will be kept on a computer with a password and will be deleted after 5 years. If you are happy for your work to be used in my research, please complete the assent form below. If you do not want to partake in this project, then you will visit another Grade 3 class when your class is doing a test or lessons linked to this project.

Yours sincerely,

Thobile Mtsweni

LEARNER ASSENT FORM

TITLE OF THE STUDY: **Exploration of calculation strategies in doubling and halving with grade 3 learners**

I, ……………………………………………………. agree to participate in this research project. The research has been explained to me and I understand what participation will involve. I agree to the following:

(Please circle the relevant options below).

I agree that my participation will remain anonymous
YES NO

I agree that the researcher may use anonymous quotes in her academic articles
YES NO

I give consent to Zoom sessions being recorded with me in them
YES NO

I give consent for part(s) of my working out notes to be used as part of the analysis
YES NO

I agree that the information I provide may be used anonymously after this project has ended, for academic purposes by other researchers, subject to their own ethics clearance being obtained. YES NO

…………………………………… (name of participant)

…………………………………… (signature)

…………………………………… (date)

Thobile Mtsweni (name of person seeking consent)

…………………………………… (signature)

…………………………………… (date)

University of Witwatersrand, Johannesburg, South Africa

School of Education

Dear Principal,

Request for permission into school as a research site

I write this letter to you as a Masters student at the University of the Witwatersrand. I request your permission for your school to participate in the Mental Starter Assessment Project (MSAP) – Wits Maths Connect-Primary Partnership Calculation strategies project. I am working with the Wits Maths Connect on this project where I (the researcher) will run the intervention lessons with focus on developing calculation strategies which entail fluency, strategic thinking and strategic calculation. The three-week intervention consists of a pre-test, eight intervention lessons run thrice per week, followed by a post-test and an interview. I (the researcher) will be marking the pre- and post-tests and conducting the interview after the post-test and implementing the eight intervention lessons. The proposed dates for the work in schools are attached on the next page.

On my side, for research purposes, I would like to collect and analyse the data on learners' pre- and post- test responses and explanations. This will help me to understand whether the calculation strategies will be helpful for improving learners use of calculation strategies. In all my collation and writing of this data, I will ensure that learner, teacher and school names are kept confidential and anonymized. Learners' names will not be included on the tests and their names and identities won't be included in the research project. Any data collected will be stored in a password protected computer and will be destroyed after 5 years of completion of the project.

At the end of the project, I will report on the findings detailing the project outcomes for learners in your school. The report will be sent out to you early in term 4, following the intervention's conclusion in term 3.

I ask you to confirm that you are happy for this project to proceed with me (the researcher) and the grade 3 learners in your school by signing and returning the tear-off slip below. Please contact me if you require any further details.

I am happy for the MSAP - Wits Maths Connect Primary Mental Mathematics project to be run in my school, and for the learner data to be collated for research purposes alongside the development purposes detailed above.

Principal name: ________________________School: ________________________ Date: _______

Intervention Timetable

	Week 1 Mon 2nd May	Week 2 Mon 9th May	Week 3 Mon 16th May

MSAP Intervention Lessons	Pre-test Lesson 1	Lesson 4	Lesson7
	Lesson 2	Lesson 5	Lesson8
	Lesson 3	Lesson 6	Post-test and Interview

Please contact me if you have any questions regarding this research project using the following details 0419589P@students.wits.ac.za.

Yours sincerely,

Thobile Mtsweni

Researcher: Thobile Mtsweni, 0419589P@students.wits.ac.za

Supervisor: Dr. Zaheera Jina Asvat, zaheera.jina@wits.ac.za

Appendix D.

Permission letter to use the MSAP resources.

Wits Maths Connect

University of the Witwatersrand, Johannesburg. Wits School of Education. Faculty of Humanities.
• Marang West Block, 27 St. Andrews Road, Parktown, Johannesburg, 2193
• Private Bag 3, Wits 2050 • Tel: +27 11 717 3412 • Fax:+27 11 717 3109 • www.wits.ac.za/WitsMathsConnect

Re: The Mental Starters Assessment
Programme (MSAP)

The two South African Numeracy Chairs: Prof Hamsa Venkat (Wits University) and Prof Mellony Graven (Rhodes University) have worked in partnership with the DBE on the national roll out of the Mental Starters Assessment Programme (MSAP) in Grade 3 in schools in 2022. This programme consists of assessment and teaching materials for six mental maths clusters that are linked to the mandated curriculum and to the research base in mathematics education. The rollout of this programme occurs in the 'mental starter' sectionof lessons; it is expected to support the broader teaching and learning of the whole mathematics curriculum in the main activity part of lessons by building strong foundations in mental calculation based on solid number sense.

The programme follows 6 years of iterative research and development in a partnership between the DBE andthe two South African Numeracy Chair teams. The iterations have consistently shown pre- to post-test gains for learners on different units. Provincial coordinators and district Subject Advisers are supporting teachers with the national rollout of the MSAP assessments and materials.

As the MSAP materials go into national rollout with the materials available open-source from the DBE website (https://www.education.gov.za/MSAP2022.aspx), these materials (teacher guides and print mastersin all the official South African languages) are available freely for access by undergraduate and postgraduatestudents for both research and professional development purposes. Videos and slides developed for virtual training of in-service teachers can also be made available where needed.

I look forward to hearing about student experiences of working with the materials and reading the outcomesof research studies looking at various aspects of the teaching and learning of mental mathematics based on these materials.

Prof Hamsa
Venkatakrishnan
SA Numeracy
Chair, Wits
Email: hamsa.venkatakrishnan@wits.ac.za

Appendix E.

Ethics Clearance Certificate

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: 2022ECE006M**
PROJECT TITLE	Exploration of calculation strategies in doubling and halving with grade 3 learners.
INVESTIGATOR	**THOBILE MTSWENI**
SCHOOL/DEPARTMENT OF INVESTIGATOR	WSoE
DATE CONSIDERED	18 March 2022
DECISION OF THE COMMITTEE	Approved unconditionally
RISK LEVEL	MINIMAL
EXPIRY DATE	Date of submission of the Research Report
ISSUE DATE OF CERTIFICATE	**CHAIRPERSON** Mmokane Dr. Batseba Mofolo-Mbokane

cc: Dr. Zaheera Jina Asvat

Appendix F.

Permission letter from Department of Education

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	16 May 2022
Validity of Research Approval:	08 February 2022– 30 September 2022 2022/56
Name of Researcher:	Mtsweni T
Address of Researcher:	1343 Diepkloof
	Thakadu Street
	Extension 3
Telephone Number:	082 585 0616
Email address:	0419589P@students.wits.ac.za
Research Topic:	Exploration of calculation strategies in doubling and halving with grade 3 learners.
Type of qualification	Master's degree in Education
Number and type of schools:	2 Primary Schools
District/s/HO	Johannesburg South

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

17/05/2022

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1. *The letter would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. ***Because of the relaxation of COVID 19 regulations researchers can collect data online, telephonically, physically access schools, or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate.***
4. ***The Researchers are advised to wear a mask at all times, Social distance at all times, Provide a vaccination certificate or negative COVID-19 test, not older than 72 hours, and Sanitise frequently.***
5. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s has been granted permission from the Gauteng Department of Education to conduct the research study.*
6. *A letter/document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs, and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
7. *The Researcher will make every effort to obtain the goodwill and cooperation of all the GDE officials, principals, and chairpersons of the SGBs, teachers, and learners involved. Persons who offer their cooperation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
8. *Research may only be conducted after school hours so that the normal school program is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
9. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
10. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
11. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
12. *The researcher is responsible for supplying and utilising his/her research resources, such as stationary, photocopies, transport, faxes, and telephones, and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
13. *The names of the GDE officials, schools, principals, parents, teachers, and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
14. *On completion of the study, the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
15. *The researcher may be expected to provide short presentations on the purpose, findings, and recommendations of his/her research to both GDE officials and the schools concerned.*
16. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a summary of the purpose, findings, and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Ndiwi: Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 17/05/2022..

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

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