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Number pattern: developing a sense of structure with primary school teachers

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Chapter 4

Algebraic Reasoning in the Curriculum and Supporting Texts.

"What students learn is largely defined by the tasks they are given"

Hiebert and Wearne (1993, p395)

Introduction

The content that is covered in class, as well as the sequencing of content, is often determined by the texts that are used in teaching (Johannson, 2006). Frequently mathematics is, for both teachers and learners, simply what is written in the textbook (Lindqvist, Emanuelsson, Lindström, and Rönnberg, 2003). Textbooks are designed to provide an authoritative pedagogic version of an area of knowledge (Stray, 1994) and are regarded as artefacts that translate policy into pedagogy and are seen as the link between the intended and implemented curriculum. The nature of mathematical tasks, their richness and quality, mostly depends on the textbook that is used in classroom instruction (Chávez, 2003). The ways in which teachers use textbooks are related to their beliefs about mathematics and teaching, and there is evidence that a great number of teachers depend, to a large extent, on textbooks (Chávez, 2003; Love and Pimm, 1996). Textbooks do not always follow the guidelines set out in the curriculum (Johannson, 2006) and it is essential to monitor not only the use of texts in class but also the content of such texts, as they inform what is made available for learning in the classroom context.

A study conducted by Sun, Cullum and Capraro (2009) shows that the majority of teachers follow the pedagogy of textbooks, using textbooks as their primary, if not only, resource for instruction. This underscores the research findings that mathematics instruction is generally textbook-driven (American Association for the Advancement of Science, 2000; Chandler, 1995; McKnight et al., 1987; Reys et al, 2004). The findings of studies by Sun et al. (2009), AAAS (2000) and Sun and Kulm (2003) highlight the importance of high quality textbooks in supporting deep conceptual understanding in mathematics. Teachers who used high quality

textbooks were found to engage more learners in learning and asked more probing questions that promoted mathematical reasoning and cognition. The relationship between teachers and textbooks is largely dependent on the curriculum and the teacher's comfort with and interpretation of the curriculum. How the teacher interprets learners' feedback and reacts to learners in the context of the classroom setting also plays a significant role in shaping how teachers use textbooks in their classes (Remillard, 2005). Texts that are used in class are central to the learning experience, the teaching and enhancement of numeracy outcomes and the attainment of curricular goals.

In this chapter I focus on two sets of documents that, together, shape the teaching of mathematics in the FP classroom. The first set consists of two texts: The Curriculum and Assessment Policy Statement (CAPS) (2010) and the Numeracy Handbook for Foundation Phase Teachers (NHFPT) (2012). The second set of documents comprises the classroom resource materials contained in Foundation Phase Rainbow Workbooks (FPRW) and the Annual National Assessment (ANA) resource materials (guidelines, exemplar test items and assessment items). The CAPS (2010) and the NHFPT (DbE, 2012) are curriculum statements which inform and describe policy and curriculum content in a broader sense; and the FPRW series, together with the ANA resources, is what the teachers in the study used as classroom resource texts from which examples and exercises were chosen. I begin by discussing the relationship between the CAPS (2010) policy document and the NHPFT (2012) supporting documents, giving some background information related to each of the two documents. Thereafter I analyse the CAPS (2010) FP breakdown of time allotted to CA2, ('geometric' and number pattern from a general and specific content focus), and also the term overview of specifications for FP CA2.

This is followed by an analysis of the CAPS policy documents (2010) and NHFPT supporting documentation (2012) in relation to the triadic notion of structure, patterns and algebraic reasoning, and the visibility given to the triad in the policy and supporting documents. The key concepts that will be examined in this analysis are the types of patterns, the notions of repeat and growth, structural thinking, generalisation, objectification and algebraic reasoning. An analysis of the actual resource materials used by the participating teachers in class, the FPRW series and the ANA resource materials, will then be conducted.

In line with Valverde, Bianchi, Wolfe, Schmidt and Houang's (2002) framework, the CAPS (2010) and NHFPT (2012) are positioned as indicators of the *intended* curriculum, and the

FPRW (2012) are positioned as the *potentially implemented* curriculum. The *implemented* curriculum is represented by the pedagogical approaches taken by the teachers in class and the ANA achievement tests are an indicator of the *attained* curriculum.

Background to the Curriculum and Supporting Materials

The CAPS (2010) policy document has been developed in order to replace the RNCS (2002) policy documents and to provide teachers with a comprehensive, week-by-week plan which is to be implemented in order to help learners achieve expected levels of numeracy. The CAPS (2010) national curriculum statement contains the following definition for mathematics:

> Mathematics is a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem-solving that will contribute to decision-making.
>
> (CAPS, 2010, Foundation Phase, p.8)

This definition gives prominence to the notion of 'mathematics as... investigating patterns and relationships... between mathematical objects'. This is consistent with international trends, which posit that mathematics is the science of pattern and that pattern is integral to the development of mathematical cognition (Warren, 2005; Zazkis and Liljedahl, 2002; Burton, 1982; Liljedahl, 2004; Threlfall, 1999).

The CAPS (2010) policy document outlines five mathematics Content Areas (CA) which contribute to the acquisition of specific mathematical skills. To support the implementation and interpretation of the CAPS (2010) curriculum, the NHFPT Grades R-3 (DbE, 2009) was revised to be CAPS-compliant and re-issued in 2012 under the same name. Its essential purpose is to 'enhance the pedagogical and didactical capacity of FP teachers to teach mathematics more effectively in the early grades by providing numerous teaching ideas for mathematical concepts and skills that are set out in the CAPS (NHFPT, 2012, p.1). The NHFPT provides a 'discussion of important issues and recommendations for classroom practice based on current academic research in the field' (NHFPT, 2012, p.1). In the foreword by the Director-General, the NHFPT states that:

Teachers are encouraged to draw on this resource as much as they can in order to plan stimulating activities that:

- start with real problems that present children with mathematics processes that are embedded in meaningful contexts;
- encourage children to present their mathematical thinking/understanding verbally and graphically with symbols of their own during the emergent numeracy phase; and
- involve children in a variety of dialogues that encourage them to reflect on their mathematical thinking.

It continues to articulate that the handbook 'describes both what it means to do mathematics and provides guidance for the FP teacher on how to support children developing the required knowledge and skills' (NHFPT, 2012, p.1). While learners should be fluent at basic operations, number facts and mental arithmetic, they should 'understand the mathematics that they learn in flexible and meaningful ways so that they can apply it with confidence to make sense of the world' (Ibid, p.3-4). After establishing desired outcomes of a lesson or activity, teachers are encouraged to shape the situation/activity in such a way that it guides learners to see patterns and structures (Ibid, p.9). Pattern and structure appear to be positioned as central to all mathematical activity at FP. It is reasonable to expect that the FP teacher will find important pedagogical and didactical information related to their teaching of CA2 in this handbook. CAPS (2010) and the NHFPT (2012), used and read together, give teachers a clear indication of what is expected and what is intended by the curriculum policy documents regarding FP mathematics teaching in CA2.

Breakdown of Time Allotted to Mathematics and CA2 in FP

At FP there are four subjects that are studied (See Figure 4.1): a home language, a first additional language (studied from Grade 1 onwards), mathematics and life skills. Of a week

SUBJECT	Grade R (Hours)	Grades 1 and 2 (Hours)	Grade 3 (Hours)
Home Language	10	8/7	8/7
First Additional Language		2/3	3/4
Mathematics	7	7	7
Life Skills	6	6	7
• Beginning Knowledge	(1)	(1)	(2)
• Creative Arts	(2)	(2)	(2)
• Physical Education	(2)	(2)	(2)
• Personal & Social Well-being	(1)	(1)	(1)
TOTAL	23	23	25

Figure 4.1: Hours per week to be spent on each subject in Foundation Phase mathematics.

that consists of 23 school hours in Grades 1 and 2, mathematics is studied for seven hours. This constitutes an allocation of 30% of available time. At Grade 3 level, seven hours of the 25 available school hours (28%) are allocated to mathematics. In the CAPS (2012) curriculum statement, there are also specifications regarding the time to be spent on each subject and CA. The total of seven hours a week for Mathematics amounts to roughly 84 minutes per day in a five-day week. The entire time allocated to mathematics per year amounts to 160 hours. In the FP the curriculum assigns only 10% of the mathematics time to CA2 (See figure 4.2). The time (in hours) that is allocated to CA2 comes to 16 hours in total: 12 hours per year for number pattern (three hours per term) and then four hours per year for geometric pattern (one hour per term) (CAPS, 2010, Foundation Phase Grades R-3, pp.39 – 40). The time frames that are specified for CA2 are limited, despite the central positioning of pattern in mathematics as advocated by various researchers. The CAPS document indicates that the key focus for FP mathematics is operational fluency and number sense, which it advances through CA1 with 65% of mathematics contact time devoted to CA1.

PERCENTAGE WEIGHTING OF CONTENT AREAS				
	Content Area	**Grade 1**	**Grade 2**	**Grade 3**
CA1	Numbers, Operations and Relations	65	60	58
CA2	Patterns, Functions and Algebra	10	10	10
CA3	Space and Shape (Geometry)	11	13	13
CA4	Measurement	9	12	14
CA5	Data Handling	5	5	5
	TOTAL	100	100	100

Figure 4.2: Weighting per Content Area in Foundation Phase Grades 1-3.

Describing Geometric Pattern and Number Pattern at Foundation Phase

As the central concern of this study is pattern as a component of the triad pattern, structure and algebraic reasoning, Figure 4.3 (on the next page) is included to show the general focus as well as the specific focus of CA2 for the FP (CAPS, 2010, Foundation Phase Grades R-3, p.10). The overview of FP CA2 is given in Figure 4.4 (on the next page) which shows that there are two main types of pattern that learners focus on in FP, namely geometric and number patterns (CAPS, 2010, Foundation Phase Grades R-3, p.26). The CAPS document refers to number patterns as 'skip counting' and geometric patterns as 'pictures' (ibid, p.10). The progression the ranges of numbers that are dealt with at a particular grade signify the progression.

MATHEMATICS CONTENT KNOWLEDGE: CA2 (Patterns, Functions and Algebra)	
General Content Focus	**Foundation Phase Specific Content Focus**
Algebra is the language for investigating and communicating most of mathematics and can be extended to the study of functions and other relationships between variables. A central part of this content area is for the learner to achieve efficient manipulative skills in the use of algebra. It also focuses on the: • description of patterns and relationships through the use of symbolic expressions, graphs and tables; and • identification and analysis of regularities and change in patterns, and relationships that enable learners to make predictions and solve problems.	In this phase, learners work with both • number patterns (e.g. skip counting); and • geometric patterns (e.g. pictures). Learners should use physical objects, drawings and symbolic forms to copy, extend, describe and create patterns. Copying the pattern helps learners to see the logic of how the pattern is made. Extending the pattern helps learners to check that they have properly understood the logic of the pattern. Describing the pattern helps learners to develop their language skills. Focusing on the logic of patterns lays the basis for developing algebraic thinking skills. Number patterns support number concept development and operational sense built in Numbers, Operations and Relationships. Geometric patterns include sequences of lines, shapes and objects, but also patterns in the world. In geometric patterns learners apply their knowledge of space and shape.

Figure 4.3: The general and specific focus of the CA2 for the Foundation Phase.

FOUNDATION PHASE OVERVIEW: CA2

Progression in Patterns, Functions and Algebra

- *In Patterns, Functions and Algebra, learners get opportunities to (i) complete and extend patterns represented in different forms and (ii) identify and describe patterns.*
- *Describing patterns lays the basis for learners in the Intermediate Phase to describe rules for patterns. This in turn becomes more formalised in algebraic work in the Senior Phase.*

TOPICS	GRADE R	GRADE 1	GRADE 2	GRADE 3
Geometric Patterns	**Copy and extend** Copy and extend simple patterns using physical objects and drawings (e.g. using colours and shapes).	**Copy, extend and describe** Copy, extend and describe in words • simple patterns made with physical objects • simple patterns made with drawings of lines, shapes or objects **Create own patterns** Create own geometric patterns • with physical objects • by drawing lines, shapes or objects **Patterns around us** Identify, describe in words and copy geometric patterns • in nature • from modern everyday life • from our cultural heritage	**Copy, extend and describe** Copy, extend and describe in words • simple patterns made with physical objects • complex patterns made with drawings of lines, shapes or objects **Create and describe own patterns** • with physical objects • by drawing lines, shapes or objects **Patterns around us** Identify, describe in words and copy geometric patterns • in nature • from modern everyday life • from our cultural heritage	**Copy, extend and describe** Copy, extend and describe in words • simple patterns made with physical objects • more complex patterns made with drawings of lines, shapes or objects **Create and describe own patterns** • with physical objects • by drawing lines, shapes or objects **Patterns around us** Identify, describe in words and copy geometric patterns • in nature • from modern everyday life • from our cultural heritage
Number Patterns		**Copy, extend and describe** Copy, extend and describe simple number sequences to at least... **Create and describe own patterns**	**Copy, extend and describe** Copy, extend and describe simple number sequences to at least... **Create and describe own patterns**	**Copy, extend and describe** Copy, extend and describe simple number sequences to at least... **Create and describe own patterns**

Figure 4.4: Overview of Foundation Phase CA2 showing the progression over the Grades R – 3.

Most of the advancement in number range occurs in Grade 3 where learners progress in number range from 200 to 1000. The patterns are classified by a move from simple patterns, to complex and even more complex patterns[29].

This same progression in the number range also appears in CA1 (CAPS, 2010, p.18) which reflects the range over which counting activities are to be developed, as well as the skip counting magnitudes. The counting activities that take place when learners engage with CA1 (1.1 – Counting objects) vary in range. In Grade R, learners count objects up to at least ten, while Grade 1 learners must be able to estimate and count forwards and backwards in ones, twos, fives and tens between zero and 100. In Grade 2, learners are expected to estimate and count to (at least) 200 and forwards and backwards in ones, twos, threes, fours, fives and tens. In Grade 3, learners are required to count and estimate up to 1000. Here learners are expected to count forwards and backwards between zero and 1000 in ones, twos, threes, fours, fives, tens, twenties, twenty-fives, fifties and one hundreds.

Counting by grouping is encouraged in all three years of study. Thus, by the end of FP, learners should be able to count fluently in ones, twos, threes, fours, fives, tens, twenties, twenty-fives, fifties and one hundreds, forwards and backwards, over a number range from zero to 1000. CA1 and CA2 together develop the number ranges that teachers can use in number pattern activities. Counting and patterns are inter-related: moving forwards in a sequence of numbers is tantamount to adding the same quantity to numbers in the sequence to obtain the next number. In the same manner, counting backwards over a particular range of numbers equates to describing the regular amount by which numbers in the same sequence are diminished in order to obtain the next number in the sequence. Counting activities convey a strong pattern sense and teachers can harness these activities to develop the notion of sequencing in a relational way.

The general progression for CA2 across phases starts with identifying, completing and extending patterns that are represented in different forms at FP. These activities lay the basis for describing the structural extensions in the Intermediate Phase (Grades 4 to 6) which, in turn, becomes formalised in algebraic notation in the Senior Phase (Grades 7 to 9). The analysis of the documents in this chapter will focus only on CA2, although I acknowledge the

[29] The distinction drawn between the types of patterns will be discussed in detail later in this chapter when specific pedagogical issues related to pattern activities will be sequenced and classified using the three categories referred to in the CAPS (2010) documents. The classification of simple and complex patterns is also discussed in Chapter 1, pp.5-6 and Chapter 2, pp.36-37 in the context of structure and the repeating pattern.

need for pattern to underlie all mathematical activity in the various CAs across the curriculum.

Breakdown of CA2 for the Four Terms of the FP School Year

What follows is a term-by-term breakdown of what teachers are expected to teach in CA2 with regard to patterns. This breakdown informs teachers of the content areas that are to be covered with the learners. CAPS (2010) describes what learners must achieve in CA2 by listing the skills that should be developed when focusing on geometric (shape) patterns and number (growth) patterns.

Geometric Patterns

In Grade R (Figure 4.5), learners are expected to copy and extend simple geometric patterns

GRADE R OVERVIEW – CA2 – GEOMETRIC PATTERNS			
Copy and extend simple repeating patterns using physical objects and drawings. Creates own repeating patterns.			
TERM 1	**TERM 2**	**TERM 3**	**TERM 4**
• Identify patterns in clothes, objects and environment. • Copy and complete patterns. • Copy patterns using body percussions. • Copy, complete and create own patterns.	• Copy, extend and create own patterns. • Copy a given pattern using coins.	• Copy, extend and create own patterns using pictures.	• Copy, extend and create own auditory patterns. • Copy a noise pattern. • Play a game 'hop scotch' pattern.

Figure 4.5: Overview of what is expected in Grade R regarding Geometric Patterns in CA2 per term.

using colours and shapes. Patterns are to be studied using various objects in the environment, clothes, body percussion, noise patterns, play patterns (hop scotch), pictures and shapes. The patterns are classified as simple, which implies that the repeating component should not contain more than two core items. Learners are also required to create their own repeating patterns.

For Grades 1 to 3 (Figure 4.6 on the next page) the CAPS (2010) curriculum overview is written generically with many of the specifications per term overlapping in scope and skills development. Across all three grades and in all terms, learners are required to copy, extend and describe patterns, as well as create their own patterns. These patterns can consist of shapes, clothes, physical objects, drawings, lines and can be patterns in art or cultural

GRADES 1 to 3 OVERVIEW – CA2 – GEOMETRIC PATTERN							
DESCRIPTORS OF SPECIFICATIONS	**Copy and extend** Copy and extend simple patterns using • physical objects; • drawings (e.g. using colours and shapes)	**Suggested sequencing of work** Start copying and extending patterns using physical objects and once learners are comfortable with using a crayon or pencil, start copying and extending patterns by drawing them.	**Copy, extend and describe** Copy, extend and describe in words • simple patterns made with physical objects • simple patterns made by drawings lines, shapes or objects.	**Create and describe own patterns** • Create own geometric patterns - with physical objects - by drawing lines, shapes or objects • Describe own patterns.	**Patterns around us** Identify, describe in words and copy geometric patterns • in nature • from modern everyday life • from our cultural heritage.	**Range of patterns:** Simple patterns in which shapes, or groups of shapes are repeated in exactly the same way.	**Range of patterns:** Patterns in which the number or size of shapes in each stage changes in a predictable way i.e. regularly increasing patterns.
Gr 1	T1	T1	T2 / T3	T2 / T3 / T4	T4		
Gr 2			T1 / T2 / T3	T1 / T2 / T3	T4	T1 / T2	T2 / T3
Gr 3			T1 / T2 / T3	T1 / T2 / T3	T4	T1 / T2	T2 / T3

Figure 4.6: Overview of specifications for geometric patterns in Grades 1-3 in CA2 per term.

artefacts. Across all terms in all the grades, the overview specifies that learners are to work with simple patterns when copying, extending and describing patterns. The suggestion is made to start by sequencing physical objects and later to move to representing and extending these patterns by drawing them. In Grades 2 and 3 there is an additional description of the ranges of patterns. In both grades the patterns range from simple patterns, where shapes or groups of shapes are repeated in a congruent manner (Term 1 and Term 2), to patterns in which the number or size of shapes in each stage changes in predictable ways (Term 2 and Term 3). In both Grades 2 and 3, term 4 consists of the same content as in Grade 1 term 4: identifying, copying and describing in words patterns from nature, everyday life and cultural heritage.

Number Pattern

The overview of number pattern, similar to geometric pattern, is written very generally and across Grades 1 to 3 (See Figure 4.7 on the next page). In all four terms, learners are required to copy, extend and describe simple number sequences[30] that contain forward and backward counting. Across the grades and terms, the number ranges are specified as well as the dimensions for counting. In patterns for Grade 2 the specifications show progression to include number ranges for simple number sequences to at least 100 (term 1), 150 (term 2),

[30] In Grade R there are no number patterns to be studied, only repeating patterns.

180 (term 3) and 200 (term 4). Learners are now required to count forwards and backwards in all four terms, covering counting in ones, twos, fives and tens (terms 1 to 4) and including counting in threes and fours (terms 2 to 4) over the various number ranges specified per term. The counting in Grade 2 is also specified to start from zero and to progress in multiples of two, three, four, five and ten. In Grade 1 learners are expected to count only in ones, twos, fives and tens, backwards and forwards up to a range of zero to 100.

GRADE 1 to 3 OVERVIEW OF NUMBER PATTERN IN CA2		NUMBER RANGES SPECIFIED PER GRADE PER TERM									
		0-20	0-50	0-80	0-100	0-150	0-180	0-200	0-500	0-750	0-1000
Sequences should show counting forwards and backwards in ... from any multiple of ... over the specified number range.	1s	G1T1	G1T2	G1T3	G1T4 G2T1	G2T2	G2T3	G2T4 G3T1	G3T2	G3T3	G3T4
	2s		G1T2	G1T3	G1T4 G2T1	G2T2	G2T3	G2T4 G3T1	G3T2	G3T3	G3T4
	5s		G1T2	G1T3	G1T4 G2T1	G2T2	G2T3	G2T4 G3T1	G3T2	G3T3	G3T4
	10s		G1T2	G1T3	G1T4 G2T1	G2T2	G2T3	G2T4 G3T1	G3T2	G3T3	G3T4
	3s					G2T2	G2T3	G2T4 G3T1	G3T2	G3T3	G3T4
	4s					G2T2	G2T3	G2T4 G3T1	G3T2	G3T3	G3T4
	100s							G3T1	G3T1	G3T3	G3T4
	50s								G3T2	G3T3	G3T4
	20s									G3T3	G3T4
	25s									G3T3	G3T4

Figure 4.7: Overview of specifications for number patterns in Grades 1-3 in CA2 per term.

For Grade 3, the overview specifications include number ranges to at least 200 (term 1), 500 (term 2), 750 (term 3) and 1000 (term 4). Number sequences should show counting forwards and backwards in ones, twos, threes, fours, fives, tens and one-hundreds (terms 1 to 4) and later on include counting in 50s (terms 2 to 4) and counting in 20s and 25s (term 3 and 4).

The next five sections will provide an analysis of the CAPS (2010) document together with the supporting documents, in the form of NHFPT (2012), to ascertain the degree to which they foreground the development of key issues related to pattern, structure and algebraic reasoning in FP mathematics. The key ideas that will be examined in this analysis are:

1. types of patterns,
2. notion of repeat as a pedagogical resource,

3. notion of growth as a pedagogical resource,
4. structural thinking,
5. generalisation, objectification and algebraic reasoning

Types of pattern

The word 'pattern' is used in different ways: it describes an object (the 'thing' that is a collection of sequenced items); a process (the action of sequencing items); a visual aesthetic (artistic or cultural); a form (the shape or outline of a number or object); behaviour (such as trends and data interpretation); outcomes (doubling threes, bridging tens); formulating structural aspects ($a + b = b + a$); general trends (halving integers makes smaller);a template (copying a shape); an arrangement (arranging items in an array); games (hopscotch) and sound (rhythm, rhyme, repeated sounds, a bird's call). This is not an exhaustive list and many more meanings can be ascribed to the word 'pattern'. It is, therefore, necessary to clarify the meaning of the term 'pattern' as it is intended for use in CA2, and that these broad notions should not be conflated with the notions of pattern that are used in other CAs.

There are two main types of pattern described in the research literature: growth patterns and repeating patterns. In growth patterns there is a discernible element of growth evidenced by either sequenced numbers growing in a fixed manner (growth number patterns), or objects that are sequenced and growing in magnitude or in number of objects that are sequenced in a specific array (growth shape patterns). In repeating patterns, sequenced items can consist of objects, shapes, pictures, drawings, and numbers, and can also include patterns that are auditory or situated in some action like stomping feet or clapping hands to a repeating rhythm. Repeating patterns are recognised and described by referencing their cyclic structure and their complexity ranges from simple (two or three core items with single variability) to complex (two or three core items with multi variability or four items in the core). The core of the repeated pattern is never interrupted. Repeating patterns vary in complexity by the addition of variability in single or multiple layers. The complexity increases as more items are added to the core of the pattern and the variability may be based on colour, orientation or shape[31].

[31] This is illustrated in Chapter 2, Table 2.3.

The CAPS (2010) document refers to geometric, shape, auditory, game and number patterns, without providing a classification of these different patterns into the two broader categories of *repeat* and *growth*. CAPS (2010) and the NHFPT (2012) specify the use of *geometric patterns* and *numerical patterns*. Geometric patterns in the CAPS (2010) are regarded as pictures (p.10) and are formed by sequences of lines, shapes and objects (p.10) that can be made 'by repeating groups of objects' (p.115) or 'from identical repeating groups' (p.149; p.235). Geometric pattern is classified in Grade R as the use of concrete objects and words (p.131). Number patterns are not explicitly defined but are positioned as activities in counting. The NHFPT (2012) refers to repeating patterns as patterns where 'an element gets repeated' (p.103) and demonstrates this in the context of beading.

The NHFPT (2012) provides teachers with a discussion of pattern in general and pattern as a mathematical tool, where growth and repeat are foregrounded as key concepts that can be used as pedagogical resources in developing an understanding of the broader notion of patterning. It proposes (p.101) that pattern activities should be used to develop *advanced counting schemes* (skip counting and counting on), reveal the *properties of numbers* (completing and bridging tens and operating with multiples of ten) and *operations* (inverse, commutative, associative and distributive properties) *identify common properties* in shapes and objects, as well as *distinguish between* shapes and objects. The above description is provided in the discussion and analysis of CA2, provided in Part 2 of the handbook (under unit 1 of the NHFPT), which focuses on the specific CAs as specified in the CAPS (2010) curriculum document. This description of pattern in the NHFPT (2012) is a description that informs CA1 in which the notion of pattern is used as a pedagogical resource to explicate regularity in the outcomes of mathematical operations on number.

The NHFPT (2012) further identifies three *kinds* of patterns: numerical, geometrical and mathematical patterns (p.99). Definitions of each kind of pattern are not provided, but illustrations are used to represent visually the details of each type of pattern (see Figure 4.8). From the illustrations *numerical patterns* are those patterns with additive or multiplicative regularity or which consist of repeated patterns using numbers as core items, i.e. both growth and repeating patterns. The two growth patterns, illustrated under numerical patterns, consist of multiples of the growth item (adding and multiplying two). The repeated pattern is a complex pattern with four numbers in the core of the pattern.

Numerical patterns:

2; 4; 6; 8; .....; .....; .....; .....; .....; .....

2; 4; 8 ; 16; .....; .....; .....; .....; .....; .....

1; 2; 4; 2; 1; 2; 4; 2; .....; .....

Geometrical patterns:



Mathematical patterns:

$8 \times 2 = 2 \times 8$ $\quad 8 + 2 = 2 + 8$

BUT $\quad 8 \div 2 \neq 2 \div 8 \quad 8 - 2 \neq 2 - 8$

$(8 + 2) + 5 = 8 + (2 + 5)$

BUT $\quad (8 - 2) + 5 \neq 8 - (2 + 5)$

Figure 4.8: Different kinds of patterns as illustrated in NHFPT.

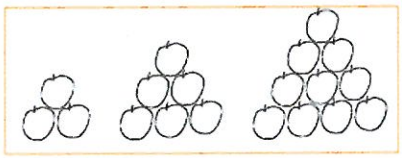
The first *geometrical pattern* in Figure 4.8 appears to consist of two shapes that repeat in a sequenced set and the second consists of the same shape orientated differently as one progresses through the sequenced array. In the latter pattern the shape changes orientation by rotating through a right angle in a clockwise direction until five orientations are shown. From the illustrations, it is not clear whether the cores of the first pattern consist of two (triangle, circle) or five (triangle, circle, triangle, circle, triangle) items, and the second pattern consisting of four (triangles rotated through three angles of ninety degrees each) or five elements (triangles rotated through four angles of ninety degrees each). If, for the first geometric pattern, the intention is to use a simple pattern[32] (two core elements) with single variability (shape), it is clear that the core of the pattern is interrupted in the way the illustration has been given since its form is then AB (two core elements) instead of ABABA (five core elements). If the core is not interrupted and the intention is to use a core of five items with the form ABABA, then this is an example of a complex pattern (five core elements) with single variability (shape). The second geometric pattern can be classified as a complex pattern (four/five core elements) of single variability (orientation is changing), albeit that the change illustrated is using a complex transformation. The same dilemma exists in terms of an interrupted core, as the form of the pattern is either ABCD (if the intention is to

[32] The complexity of repeated patterns is discussed in Chapter 2, pp.36-38.

represent four consecutive transformations collectively completing a revolution) or ABCDE in the event of five consecutive items being illustrated. This distinction is key in continuing the two patterns since it is the core and the form that determines how the pattern continues beyond the perceptual. The *mathematical patterns* are patterns that relate structural aspects of operations, in this case $a \times b = b \times a$ and $a + b = b + a$, etc. The NHFPT does not specify any of the details mentioned above to describe the nature of these patterns; that is, whether the focus is on core, form, complexity, variability, growth or repeat.

The NHFPT also provides the teacher with a guideline stating that there are two underlying didactical approaches, based on prediction, which can be used when working with pattern (see Figure 4.9). This particular illustration (p.190) is the only instance where the move towards objectification and generalisation is suggested in the CAPS (2010) and the NHFPT (2012) documents. It is also the only illustration that draws the teacher's attention to the need to extrapolate when describing the general trend in the pattern and where objects are represented with an embedded growth from one picture to the next. The illustration refers to the act of extrapolation as 'predicting', but the didactical approaches in developing this notion are not discussed and thus remain invisible.

Illustration of the two kinds of predictions that we can make:



For the pattern above, determine:

- How many apples will be needed to make the sixth picture in the pattern?
- Which picture in the pattern can be made with 45 apples?

Figure 4.9: Two kinds of predictions that can be made when working with pattern in NHFPT.

The NHFPT (2012) advocates that, from a patterning perspective, learners should experience both the recursive (adding/subtracting a quantity to/from one term in the pattern to get the next) and the functional relationships (seeing a relationship between a term and its position in the sequence) in a pattern. The NHFPT advances the use of tables as a pedagogical resource to enable and develop these two conceptions of pattern, but here too the lines between growth

and repeat are blurred as no guidance is given for the effective use of tables to advance covariant thinking[33].

The Notion of Repeat as a Pedagogical Resource

When teachers use the notion of repeat and make it explicit as a pedagogical instrument in FP mathematics, they can give learners access to cyclic structure and develop the tools of *core* and *form,* which are used in describing and working with repeating patterns. Conceptually, a repeating pattern is a pattern in which there is an apparent unit of repeat (the core). These patterns contain a cyclic structure that is generated due to the repeated application of a smaller portion of the pattern (Liljedahl, 2004). CAPS (2010) and the NHFPT (2012) suggest patterns that contain a discernible unit of repeat be incorporated into work programmes. The NHFPT (2012) describes repeating pattern as 'a recurrence or repetition of events or objects' (p.99). With regard to this point, the CAPS (2010) states that patterns can be made by repeating groups of objects. This characterises repeat as an action that is done to objects and shapes. Groups can be made up of several identical objects which are positioned in different ways (Ibid, p.115). This implies a variance in orientation in the sequencing of objects or shapes. In the policy document it is recommended that learners in Grades 1, 2 and 3 'work with patterns in which elements (shapes, lines or objects) are repeated in exactly the same way' (Ibid, p.136).

Learners are required to work with the repetition of the size or colour of objects and also with groups of objects that repeat in some way. The CAPS (2010) document depicts and describes various patterns, as illustrated in Figure 4.10 (following page), but fails to formalise the variants in such a way that they illuminate the notions of core and form, and fail to give explicit classifications of the complexity of the repeated patterns. The eight patterns in Figure 4.10 illustrate repeating patterns of changing complexity and variability. The core components of these patterns that repeat vary in complexity to include a core of two, three and four. Variability is illustrated through choice of object, colour, size and orientation, and includes combinations of any of these.

[33] This discussion is taken further in this chapter on p.104, and illustrated in Figure 4.14.

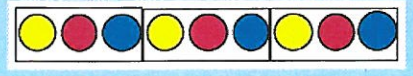
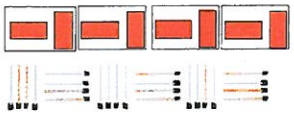
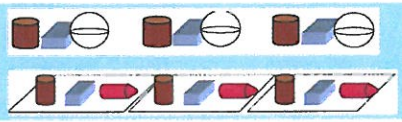
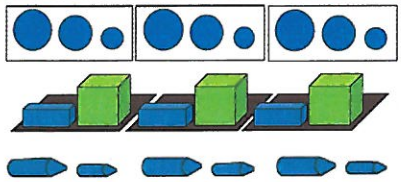
Illustration	Description
	Patterns can be made by using one object but having the colours of the object change in a regular way (p.235).
	Patterns can be made from identical repeating groups, where each group has only one kind of object but the position of the objects in a group change. Identical groups are repeated (p.236).
	In some patterns different objects are used to make up a group, but the groups of objects are repeated in exactly the same way (p.114; p.236).
	Patterns can be made in which the size of objects alternates in exactly the same way (p.114; p.236; p.269).

Figure 4.10: Illustration of the notion of repeat in the content clarification of the CAPS.

It is evident that both CAPS and the NHFPT do not make the notion of a core available as a pedagogical resource that gives access to the cyclic structure of repeating patterns. This omission can have a deleterious effect on learning about the structure (the logic) of repeating pattern, and it therefore precludes the possibility of generalising repeating patterns. It can result in repeating patterns remaining merely a study of pictures or shapes that are sequenced in a repeating manner, irrespective of the number of items that are repeated. The structural aspects of core and form of repeating patterns are not made available for learning and expressing the notion of repeat.

The Notion of Growth as a Pedagogical Resource

Growth patterns consist of sequenced items, either numbers or shapes, which have elements of growth embedded in their structure. This growth can be positive (increasing) or negative (decreasing) in magnitude or area (size) or number of congruent objects in the case of a collection of shapes. When working with growth as a concept in sequencing, this growth focuses on elements of additive, multiplicative or spatial reasoning. The growth in a number sequence can be described either by using a recursive approach, or by engaging a functional relationship to describe growth. Focusing on the notion of growth activates the embedded structural aspects (the growth element) related to the sequenced items.

In the FP, number patterns are linked with number operations and relationships (CAPS, 2010, Foundation Phase Mathematics, p.116). Learners have to deal with both the ordinal as well as

the cardinal (position of an item in a sequence) aspects of number. The CAPS (2010) and the NHFPT (2012) describe number pattern conception as the ability to copy, extend and describe simple number sequences and, to accomplish this, both documents acknowledge that being able to identify and describe the relationship between numbers is important. In order for a relational understanding to develop, teachers in the FP need to provide learners with a variety of tasks which incorporate an assortment of patterns that are constructed solely from numbers.

The CAPS (2010) document explains that number pattern activities will consolidate and develop learners' counting skills. Examples of such activities include learners:

- doing verbal counting,
- showing number sequences written in different ways,
- filling in missing numbers in any form of number sequence,
- pointing to numbers as they count,
- completing number patterns in different visual representations, such as
    - a number line,
    - a number grid,
    - a number chain or
    - a written sequence of numbers

(CAPS, 2010, pp. 137, 269, 394).

The NHFPT (2012) points out that, by recognising and reflecting on number patterns that are presented visually, learners will develop their patterning skills, their number sense and their conceptual knowledge (Ibid, p.104). The document maintains that involving physical objects or drawings in patterning activities provides a natural transition mechanism to introduce patterns involving numbers. The NHFPT (2012) document suggests that patterns are presented and introduced in the classroom using tables, diagrams, number chains and problems (Ibid, p.106), and advocates that learners be introduced to both increasing and decreasing patterns so that they understand both types of regularity in number pattern (Ibid, p.104). The items exemplified in Figure 4.11 (on the next page) are used as illustrations of growth, using objects and shapes. (Equivalent number sequences were added by author). These items of growth display spatial features which can be utilised by the teachers to work with elements of growth that are multiplicative and additive in nature. The NHFPT (2012) does not discuss the pedagogical tools that are used to work conceptually with growth items.

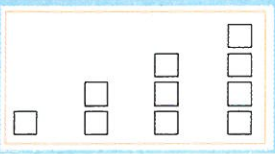
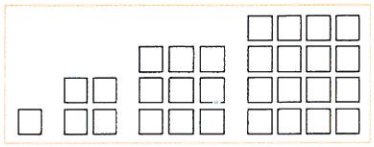
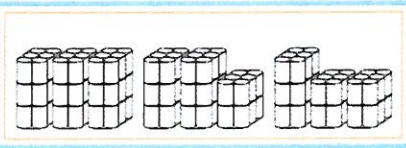
Illustration	Description
	Increasing pattern with a constant quantity being added from one element to the next. *The equivalent number sequence: 1; 2; 3; 4; ....*
	Increasing pattern with a changing quantity being added from one element to the next. *The equivalent number sequence: 1; 4; 9; 16; ....*
	Decreasing pattern with a constant quantity being subtracted from one element to the next. *The equivalent number sequence: 54; 48; 42; ....*
	Decreasing pattern with a changing quantity being subtracted from one element to the next. *The equivalent number sequence: 15; 10; 6; ....*

Figure 4.11: Illustration of the notion of growth in the NHFPT.

The concept of growth is also underscored in the CAPS (2010) document with illustrations (See Figure 4.12) that display the various types of growth that can be used in sequencing items. The growth varies from the continuous enlargement of the areas of sequenced shapes to include a collection of objects that grow or decline in number as the sequence progresses.

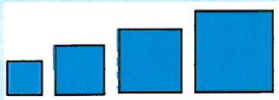
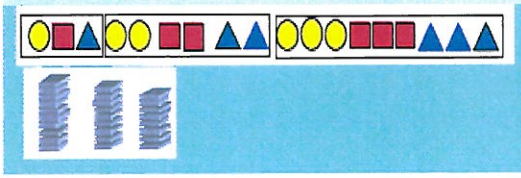
Illustration	Description
	Some patterns are made up of a single kind of shape, but each example of the shape increases or decreases in size (p.366).
	Some patterns are made up of groups in which the same shapes or objects occur, but the number of each kind of shape or object increases or decreases in a regular way (p.366).

Figure 4.12: Illustration of the notion of growth in the content clarification of the CAPS.

These illustrations provide a sample of sequenced items with embedded growth attributes, but the pedagogical approaches, which use this growth as a tool to foster the notion of relational thinking and structure, are not described or incorporated in extension activities.

Blurring repeating and growth pattern

Care must be taken not to conflate or blur avenues that develop the core activity of CA2. Examples of such conflations, blurring or omissions can be found in the NHFPT (p.105) in the activities illustrated in Figure 4.13. Using the number chart as a pedagogical resource when working with numbers carries with it the possibility of confusing growth number pattern with repeating patterns. As the illustration and discussion in Figure 4.13 claim, after placing the first few counters, learners will start 'observing a pattern' and this observation will help them 'anticipate the next block to be covered'. When learners notice the visual array formed by covering the numbers, they may ignore the numbers in the sequence and merely repeat the action of covering to extend the procedure in the same fashion as was suggested by the first few actions. The intended cognitive benefits – developing a sense of number and the relational properties of number – would consequently be negated. Activities like this blur the lines between growth pattern and repeated pattern (Liljedahl, 2004), as the relational attributes of the growth pattern is obscured by the action of covering. This then becomes merely an exercise in repeated pattern where the array of covering is extended to maintain the same form or direction suggested by the array.

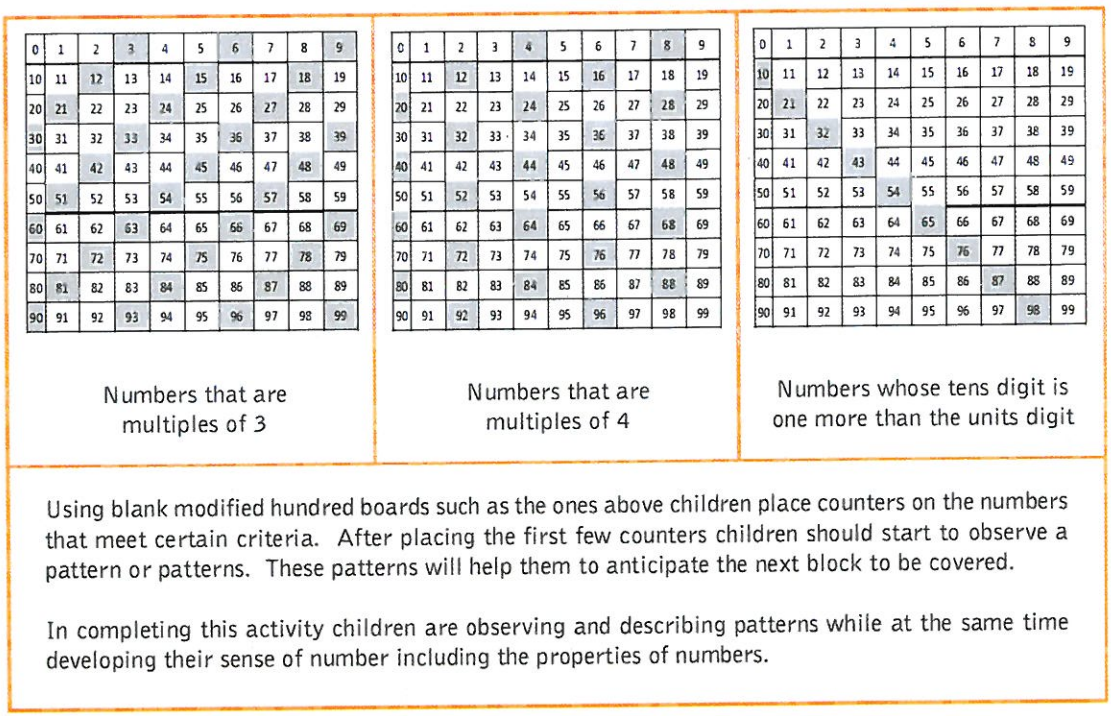


Figure 4.13: Illustrations and discussion from NHFPT of patterns formed by multiples and making use of a number chart to show the emerging pattern.

Blurring the line that separates repeating pattern and growth pattern, by using the number chart in the manner suggested by the NHFPT, is problematic since the two types of sequences develop distinct mathematical ideas. Repeating patterns deal with cyclic structure which is embodied in the repeated application of a 'unit of repeat' that can be used to generalise across items in a repeating sequence. Growth patterns deal with growth as a regularity which forms the surface structure of sequences, and can later be applied to give form to the structural aspects that are generalised through the use of both the systemic and surface structures of a sequence.

Conflating recursive and functional thinking

When choosing pedagogical resources for use in sequencing activities, teachers must be clear about the objective of the activity and the mathematical imprint it will leave in the mind of the learner. The choice of tables to represent the growth in a set of sequenced items is usually

Tables

Number of tables	1	2	3	4	5	6
Number of legs	4	8	12			

Number of tables	1	2	3	4	5	6
Number of legs	5	10	15			

Number of tables	1	2	3	4	5	6
Number of legs	6	12	18			

Tables are important in activities used to develop patterning skills because they allow children to explain how they get their answers in at least two different ways.

In the three tables above, the values in the second row can be established by:

- Adding 4, 5, and 6 respectively to the numbers in the second row. In that way the table is simply completed by extending the pattern in the second row.

  4; 8; 12; 16; .....; .....;
  5; 10; 15; 20; .....; .....;
  6; 12; 18; 24; .....; .....;

Alternatively:

- Children can also be supported to see that the number in the second row is 4×, 5× and 6× the number in the first row respectively. And that there is no co-incidence in the fact that in the pattern where we are adding 4 we are also multiplying by 4 and the same for 5 and 6.

Figure 4.14: An illustration using the completion of tables to demonstrate the use of recursive and functional thinking.

connected with the intention of developing and illustrating a co-variant relationship between two variants; the number and the position of the number in the sequence. Number sequences that are given in a sequenced, side-by-side consecutive way usually evoke a recursive approach to describe the surface structure embedded in the sequence (see tables illustrated in Figure 4.14). Learners will most likely describe the sequence 3; 7; 11; ... as a sequence of adding fours to get from one term to the next. If a table is arranged in the same way, as shown in the three tables in Figure 4.14 (as the NHFPT proposes on p.107 for the use of tables), learners will most probably not see the functional relationships, as the numbers in the first row of each table are all consecutive. The activities that make use of tables must require more than simply completing the table: they can include items that are outside the perceptual, and need at least a form of expressing the general relationship in a co-variant way. Thus if the table gives the first six terms, it is not sufficient to ask for item nine or ten as these are easily reached through recursion. When using a table to engage covariant relationships in a sequencing activity, it is essential to extend such activities to include items beyond the perceptual. Activities need to search for item 100 (a *big number* as Radford terms it) to engage learners in using structural extension in their functional reasoning. For this purpose, the table is a powerful pedagogical tool, if used in an appropriate manner. Neither the CAPS (2010) nor the NHFPT (2012) deals with the embedded structure of tables in order to develop the notion of co-variance. They simply provide examples of tables where each of the items is given consecutively, and there is no attempt to use the relational features to create a generalised object that represents all the terms in the sequence. These tables can thus be completed by recursively determining the value of each entry.

Structural Extension

CA2 has specific bearing on developing the notion of *sequencing*. It includes using the *function concept* (co-variance) to develop and support early algebra at FP. It is necessary that through carefully designed activities and purposeful instruction, the basic knowledge of number (counting and writing numbers) is already in place before CA2 activities, which involve sequencing numbers using repeat and growth regularity, are introduced. The idea of seeking patterns in other content areas is not the same as the pattern activity that is described in CA2. The NHFPT (2012) locates pattern and structure as core pedagogical activities in the instruction of mathematics in general, and encourages teachers to seek out opportunities to highlight patterns that occur and structures that shape mathematical activity. These

descriptions inform the use of *mathematical patterns*[34] as described in NHFPT (2012, p.99) and should ideally be described and applied in CA1.

Focusing on CA2, in the facilitation of the threefold understanding of arithmetic[35], the teacher can provide learners with the opportunity to understand and recognise the 'logic' of the pattern and use this logic to describe patterns. Learners can utilize an arithmetic procedure to notice the commonalities between the sequenced items (numbers or objects) in a manner which will allow them to extend the sequence beyond the perceptual. By enabling the learners to use conceptual arithmetic resources, like relational emphasis, the teacher can allow the explication of structure as a pedagogical resource to become the catalyst for structural extension and thus emergent algebraic understanding. For the FP classroom this means that in a set of sequenced numbers that possesses constant growth (process) between consecutive items, the next term in the sequence can be obtained by applying the growth element recursively (product). When using this growth element to find numbers beyond the perceptual, the sequence has to be decomposed into its structural components (relational) to extend towards the general.

The NHFPT (2012) refers to the notion of structure in its description of patterns by claiming that 'patterns in mathematics enable the individual to see structure in numbers, mathematical operations, data and geometry shapes and objects' (p.99). The NHFPT (2012) emphasises the importance of learners' exposure to a range of different patterns from an early age and proposes that they 'learn to look for *patterns within patterns*' (p.100). Pattern is used here to describe two separate notions: firstly, the notion of patterns as a meta-level regularity that is presented as structure (relational) and, secondly, to name the object that is being explored, a 'pattern', which can be described better by using the broader notion of a 'sequenced set of items' (product). Thus looking for a pattern within a pattern describes the process of looking for structure and relational attributes within a set of sequenced items. Using the notion of pattern within a pattern as a relational approach would imply that the focus is on the general structural aspects between sets of structures. The NHFPT (2012) continues to describe deeper embedded regularities by referring to patterns (the product) as possessing 'underlying patterns (common properties)' (thus relational) that are similar or different (p.100). The

[34] This is described in the NHFPT and is included in this chapter where types of patterns are discussed on p.96.
[35] This is referring to the threefold meaning of arithmetic proposed by Subramaniam and Bannerjee (2004) discussed in Chapter 2 pp.32-33.

properties that are referred to in the discussions that follow are based on structural aspects embedded in and across items through a focus on regularity and change.

Working with regularity and change

CA2 is designed to enhance the skill of searching for regularity and developing the tools that assist learners in identifying such regularities. Its focus is on developing learners' ability to use structure to work with relationships between items that are sequenced according to some (growth or repeat) regularity. The notions of structure that are proposed as pedagogical tools are cyclic, spatial and growth structure, working with single variability (recursions) or co-variance (functional thinking). The broad intention behind CA2 is to enhance algebraic reasoning skills among learners through a focus on the structural aspects of sequenced items. As the relationship between pattern, structure and algebraic reasoning has been described and researched internationally, more proof is surfacing that shows that CA2, with a focus on structure, enhances basic numeracy skills and supports the learning of more advanced algebra.

The NHFPT (2012) asserts that pattern recognition enables learners to predict what will happen under certain conditions and what must happen for a desired outcome to be achieved (p.99), but it does not draw on an understanding of structure or regularity as a pedagogical resource in CA2. The failure to establish clearly the significance of structure may impede the teachers' (and consequently the learners') acquisition of an analytical (micro-) understanding of pattern and the broader notion of sequencing. The CAPS (2010) and the NHFPT (2012) state that the teacher should provide learners with the opportunity to copy and complete or extend the pattern, and then to represent their own patterns (CAPS, 2010, p.24 and NHFPT, 2012, p.107). Teachers are encouraged to guide learners in the process of representing their extended pattern as an end *product* of the *process* of pattern recognition and generalisation. What is not made clear is the approach to establishing a route to relational thinking through a focus on variance and also what constitutes a meaningful end to a patterning activity.

Both the CAPS (2010) and NHFPT (2012) documents encourage the notion of describing relationships between different patterns and within a sequenced set of numbers or objects. Teachers are encouraged to use forms of structural thinking when teaching number pattern so that learners can engage with the 'logic of how a particular pattern is established and how it evolves' (CAPS, 2010, p.10). In identifying the logic behind a pattern, learners are working with forms of structural thinking in order to identify surface structure (NHFPT, 2012, p.48)

as discussed in Chapter 2. If this process is properly taught and negotiated, learners will be aided in developing a structural conception of number patterns, and sequencing in general, which in turn will create the habit of mind to think structurally in arithmetic and to search constantly for underlying regularities and change in mathematics. It can also enhance the use of structure to reach items that are beyond the perceptual in given sequences.

Making structure available as a pedagogical resource

FP patterns and patterning activities, across the curriculum, can play a critical role in revealing, and thus making available, the notion of structural thinking. In order to extend a number sequence, learners can be required to analyse the pattern, to observe the regularities within the pattern and finally use these regularities to extend the pattern. Through this process, learners will need to be guided by the teacher towards structural extension in order to complete and extend the number pattern, so that at the end of this process, learners' representations should contain elements of structural thinking. The necessity for using and expressing the notion of structure is not described in the policy documentation, nor are the pedagogical tools made available to teachers to enhance structural extension and understanding of sequenced sets of items.

The NHFPT (2012) describes part of the teacher's responsibility, when engaging learners with repeating patterns and growth patterns, as revealing the underlying structures, operations and mathematical relationships (p.8). 'Having established the desired outcome of the lesson/task the teacher then needs to shape the situation/problem/activity in a way that will invoke children to *"see"* the patterns and structures' (p.9). The document, however, does not provide a definition of what is meant by the notion of structure, particularly in the context of CA2. Structure is mentioned in the NHFPT as a characteristic of a particular domain or activity (pp. 17, 28, 57, 59, 60) and never as an extension of mathematical exploration of relational aspects embedded in sequenced items. Where structure is mentioned in a relational context, it is in CA1 where teachers are encouraged to use activities that expose learners to different structures in the context of addition and subtraction (pp. 67, 68).

In the CAPS (2010) and NHFPT (2012) policy documents, teachers are encouraged to enhance their pedagogical approach when teaching number patterns in such a way that learners develop an understanding of forms of structural thinking. However, no prominence is given to mathematical notation and symbolism in the FP or to the use of notation and symbolism to develop rules that will express the structural relationships in a sequenced set of

numbers or items. An example would be to relate the core and the form of the repeating pattern. All repeating patterns of the form ABC will have three components in their core that repeat. The core shows the shortest string of elements that repeat, and the form is thus a generalisation of the core repeating. Thus patterns with the form ABC can have core elements consisting of 1 2 3 or ▲♥● or ☺☼Ø. When working with growth sequences, teachers can foreground the surface structure as a pedagogical resource which helps to develop the generalised form of the sequence[36].

Generalisation, Objectification and Algebraic Reasoning

Radford (2006, 2007) argues that pattern generalisation is a powerful mathematical indicator of how learners reason algebraically. He maintains that observing how learners generalise ideas in patterning activities gives a good indication of the extent of algebraic reasoning. Pattern and structural extension are avenues to generalisation and thinking algebraically (Taylor-Cox, 2003). When a learner is capable of perceiving a commonality that applies to the terms in a number sequence (the surface structure), Radford (2006) argues that the learner demonstrates the ability to carry out *arithmetic generalisation*. Noticing and expressing this commonality also permits learners to form a *genus* that applies across the items in the pattern. The next level of generality, the *schema*, results from the evolution of the genus into a more sophisticated general rule that is *algebraic* in nature (Radford, 2006). The level of generality at which learners are working determines their ability to formulate schemas. Radford (2006) maintains that the learners' route to the general should progress through three levels: *factual, contextual* and *symbolic* generalisation, in the development of their algebraic generalisation skills. Underscoring all of this is the process of knowledge objectification which happens as learners use a variety of semiotic resources to express their thinking. As they progress through the levels of generality, they undergo a process of semiotic contraction to reach finally a semiotic mode where their reasoning becomes algebraic in nature. This process of knowledge objectification makes use of iconicity and contractions which are characterised by the semiotic means of objectification. Iconicity is utilised by the learner in order to draw a distinction between the same and different. Through this process, learners will rely on what they have previously experienced and use these experiences in order to deal with new situations (Radford, 2006). Then, through the process of contraction, an individual makes a

[36] In Chapter 2, Table 2.5, p.40 an example of such a decomposition is illustrated.

choice between what is relevant and what is irrelevant. Contraction is the process undertaken by the learner to focus attention on aspects that are perceived to be relevant, and ignore that which is superfluous. As fluency with and within a mathematical process is gained by the learner, contraction improves (Radford, 2007) and results in more sophisticated mathematical generalisations.

The concept of *generalisation* does not appear in either of the CAPS (2010) or the NHFPT (2012) documents under review, but it is implied that some form of generalising will be used in order to identify, copy, extend and describe a pattern. The general content focus of the CAPS (2010) in CA2 (Patterns, Functions and Algebra) proposes that teachers need to allow learners to *identify and analyse* patterns and enable them to *make predictions* when working with pattern problems. The NHFPT (2012) indicates that copying and extending patterns is 'more than a game of guess the next value' (p.109). Both of the documents underscore the need to allow learners to make predictions when analysing patterns. These prediction activities should ideally be based on mathematical analysis of the set of sequenced items, identification of the underlying structure in the sequence and application of this structure to extend the sequence, and the use of structural extension to move towards expressing a prediction which is more than merely guessing the next few numbers. In order to leave some mathematical imprint in the mind of the learner, this prediction should move beyond the perceptual. Radford (2006, 2007, 2010) calls these predictions finding 'big' figures in the set of sequenced items that work with a functional structure. When working with growth number patterns, learners are to focus on items in the perceptual field, that is the terms that are given, and move towards those that are not given and not so easy to reach – the big figures, the item in position 100 of a sequence. The process of prediction by learners is informed by some level of generalisation across the sequenced items. This sequence of cognitive events is not explicated in the NHFPT(2012) and can contribute to learners merely making naïve inductions when expressing and predicting items in a sequenced set, instead of making use of sound (arithmetic) generalisations which could enhance their algebraic reasoning skills.

The CAPS (2010) and the NHFPT (2012) encourage teachers to allow learners to copy, extend and describe the pattern in words. Extending the pattern 'helps learners to check that they have properly understood the logic of the pattern' (CAPS, 2010, p.136). I have argued that the CAPS document is advancing a recursive rather than a functional approach to the general. The CAPS document does not elaborate on what is meant by 'describing the pattern in words', nor does it elaborate on the notion of the 'logic' of the pattern. It is up to the reader

to imagine what is possible within these terms. The NHFPT (2012) provides a description under the heading *Guidelines for Practice* (p.103) which is theoretically framed within the concepts of *metacognitive* practices and the development of *conceptual knowledge*. It positions mathematical classroom practice firmly as enhancing the logico-mathematical development of understanding. Against this background, a description of both the teacher's role and the meaning of 'copy and extend' is provided.

> The teacher's role is to:
>
> - provide patterns that children must copy and extend
> - help children reflect on the activity – with particular focus on pattern recognition.
>
> To copy and extend patterns means to be able to:
>
> - reproduce an existing pattern made of shapes or objects, drawings and other representations, and numbers
> - add further elements to the pattern. (NHFPT, 2012, p.103)

These descriptions are general and do not provide sufficient depth as to the operationalization of the intended cognitive activity of learners in CA2 at FP. Foregrounded as central to the teacher's role is helping learners to recognise patterns, without providing and explaining the key pedagogical tools of regularity, structural extension and generalisation. These pedagogical tools are essential ingredients in developing a conceptual approach to sequencing at FP. The implication is that teachers who know how to elicit relational aspects in sequences will apply this knowledge in their teaching, and those who do not will not approach sequencing from this perspective. This approach is not in any way restricted by either the CAPS (2010) or NHFPT (2012) but is, at the same time, not included and elaborated upon when learners engage in activities of recognition, copying, extending and describing patterns. As expressed in NHFPT (2010, p.101), conceptual knowledge is described as no more than:

> - the recognition of patterns in and between situations; and
> - the realisation of the values of these patterns in explaining and/or predicting
>
> and is developed through the reflection on and recognition of patterns and the realisation of the value of these patterns in explaining and/or predicting.

This description of conceptual knowledge foregrounds recognition and application of regularity through a process of reflection, explanation and prediction.

The NHFPT (2012, p.103) and CAPS (2010, p.9) make reference to processes that can lead to arithmetic generalisation when positing that 'learners need to think about what changes and

what does not change as they progress from one element of a pattern to the next'. Here teachers are encouraged to 'provide enough of the pattern' in order for the 'extension to be obvious' (NHFPT, 2012, p.103). Mason (1996) places emphasis on the ability to study and discern regularity and change, presenting them as key mathematical reasoning skills for the conceptual development of generalisation skills and objectification of knowledge. The NHFPT (2012) describes the sameness and the difference not by a broader structural approach to arithmetic, but rather by an approach that advances surface features embedded in a sequenced set of items. As argued before, the notion of core and form are not mentioned in any way. It does not address the 'why' component of discerning regularity and change in sequenced items and leaves the reader pondering its value.

Learners ideally have to notice and use commonalities between terms in a sequenced set of items to be able to identify, continue and complete the sequenced set *beyond the perceptual*. Both the CAPS (2010) and the NHFPT (2012) do not propose the process of formally generalising across sequenced items where generalisation involves the use of an indeterminate (unknown) quantity to create a generalised term expressed either as an arithmetic generalisation[37] or an algebraic generalisation[38]. Generalisation in number pattern rests on the ability to grasp a commonality and be aware that this commonality extends to all the terms in a sequence (Radford, 2010). This is the essence of sequencing.

There are also instances in both the CAPS (2010) and the NHFPT (2012) documents in which problems require learners to work with a growth sequence, where there is a constant growth within the growth of the sequenced items (eg. 1; 2; 4; 7; 11; ...). These sequences are challenging and present an opportunity for learners to break from working with number sequences that grow constantly and replicate counting activities, to more compounded forms of growth. Learners are afforded the opportunity to experience growth that is different from the usual first order constant growth, and this is best mediated through pictures or objects that illustrate this growth using spatial structure. These activities, however, do not end in some formal generalised description or expression and they remain merely at the level of drawing the next item in the sequence, without exploring and extending structurally. The purpose of

[37] Radford proposes that an arithmetic generalisation is a generalisation that is based on a local commonality. It is informed and shaped by the perceptual and lies in the realm of arithmetic. A genus is formed which then serves as a warrant to build towards an expression of elements beyond the perceptual.

[38] An algebraic generalisation emerges from the genus evolving into a more sophisticated form, a schema, which represents all the terms in a sequence. This schema is shaped by a process of reasoning that moves between the three levels of factual, contextual and symbolic generalisation and takes shape through a process of semiotic contractions.

sequencing as a mathematical tool, which leads to expressing generality, is left impoverished by this approach.

In Summary

There is no clear differentiation between repeating patterns and growth patterns and often the NHFPT interrupts the core of the repeating pattern. The CAPS (2010) curriculum documents do not provide a definition for 'geometric pattern' as a repeated pattern, which foregrounds the notion of core and the complexity of repeating patterns. This type of omission, according to Liljedahl (2004), is problematic in the sense that it does not set the boundaries within which the concept of pattern is to be studied, and it also does not develop the mathematical landscape in which patterns with a cyclical structure are studied. At the entry level, this would mean that learners (and teachers) are not being made aware of the boundaries shaping the object of learning. Regularity and generality related to patterns are not introduced as pedagogical resources in any of the introductory documentation. This regularity expresses itself in the core of the repeated pattern and in the growth element of a sequence that consists of shapes or numbers.

At FP it would appear that generalisations remain at the level of the perceptual, and thus are contextually bound. Learners will identify the surface features in a set of sequenced items and use this knowledge to complete the missing intermediate items, or to reveal the next few items in the set, but will not use this regularity to move beyond the perceptual. Surface structures are easily accessible (Skemp, 1982). These surface structures allow learners to identify the 'rule' that is used and they then apply this rule (usually called 'a jump' when dealing with growth number sequences) in a recursive manner to get to either the next term in the sequence or to determine the terms that came before. The numbers that are used to sequence growth items are specified as multiples of the growth number, and can be easily accessed through the resemblance of these items to counting. The likelihood that learners would even use the surface structure to complete the sequence is negated by their having reached fluency in routine counting activities and working with the same numbers as counting.

Possibly the biggest omission in the documents under review is the lack of recommended pedagogical resources. Where recommendations are made, the intended didactical aims are not made explicit, so as to enable teachers to select activities that enhance the intended mathematical learning and reasoning skills. Teachers need to be assisted and supported in

choosing the correct pedagogical resources for learning particular mathematical concepts to develop a sound conceptual understanding of the mathematics with which they engage, and also to develop a more formal mathematical register (that is, one that constitutes algebraic notation). There is no reference to how activities can be used to develop learners' algebraic reasoning skills. This is an essential step in the progression towards seeing and working with structure, generalising from such explorations and building algebraic reasoning abilities. In addition, these pedagogical tools remain primarily invisible in those problems that are given as samples, even in those instances where there is the opportunity to enhance these thinking skills.

The NHFPT (2012) and the CAPS (2010) documents do not explicate the need for forming a genus by drawing attention to the structural aspects of the number sequence, and this remains part of the invisible didactical tools which are needed to complete some of the activities that are sampled in these documents. There are no individual exercises or examples that explain what the *logic* of the pattern is, and how this *logic* can be used to develop more generalised descriptions of the sequenced items. The CAPS (2010) document also explains that in Grade 2 and Grade 3 learners should begin to describe patterns without the aid of guiding questions (p.269), which implies that by the time they reach Grade 3 the language and syntax that is used to describe pattern should be fully developed. There is no elaboration on what is required to develop this ability to describe and express the structure (logic) of the sequence in a more generalised manner, other than merely invoking the rule and using surface features. The CAPS and the NHFPT also fall short in specifying indicators that signify learners' descriptions of patterns as using appropriate language and tools.

The rest of this chapter will analyse the activities that appear as sample items in the CAPS, the NHFPT, the FPRW as well as the ANA guidelines and test items, to investigate what is made available as sample activities as well as actual class resource material which is used in the teaching of CA2.

The key areas that will be examined in the analysis are:

1. the types of patterns and pedagogical resources
2. describing and classifying patterns
3. levels of cognitive demand of growth and repeating patterns in the resource texts
4. instructions that accompany activities
5. activities focusing on the structure of the pattern and moving towards generalisation

Activities in the Curriculum and in Classroom Resource Materials

When the first set of lesson data was recorded, it became evident that the teachers used two sets of classroom resources when teaching mathematics. The one resource was the Foundation Phase Rainbow Workbook (FPRW) series which is compiled, printed and distributed free of charge by the Department of Basic Education. The second set of resources was a collection of materials that together are known as the ANAs. These documents consist of a guidelines document, sets of exemplar question items and also actual assessment items that were used for the Annual National Assessment. As this study is focusing on the use of structure in the pattern classroom in FP, it was important to ascertain whether the learning materials that teachers use at FP provide activities and suggest pedagogical resources for foregrounding structure.

Types of Patterns and Pedagogical Resources

CAPS sample activities

In the CAPS curriculum documents, there are sample activities that illustrate aspects of the content that are discussed under CA2. These illustrations, and the discussions that accompany them, are for clarification and are intended to provide pedagogical guidelines for teachers. Table 4.1 illustrates what is foregrounded in the CAPS document breakdown, focusing on sample activities and guidelines included for teachers to assist with the interpretation of the

	Types of Patterns						Pedagogical Resources for Pattern										
CAPS	Geometric Patterns	Number Patterns	Symmetry Pattern	Auditory Patterns	Noise Pattern	Game Pattern	Shapes	Colour	Percussion	Repetition	Words	Pictures	Rhythm	Counters	Coins	Peg Board	Numbers
Gr R	12		1	3	1	1	5	6	3	12	1	2	2	1	1	1	
Gr 1	6	5	1				6	6		5		2		1			4
Gr 2	6	4					6	7		6		3					4
Gr 3	4	4	1				4	6		4		2					3
Totals	**28**	**13**	**3**	**3**	**1**	**1**	**21**	**25**	**3**	**27**	**1**	**9**	**2**	**2**	**1**	**1**	**11**

Table 4.1 – Types of patterns and pedagogical resources for pattern.

curriculum. There are no patterns with number in Grade R. Across all four grades, patterns are mainly repeating (geometric) patterns using shapes, colours and pictures that are repeated

in various ways. There are also auditory patterns that use percussion, rhythm and words to teach the notion of repetitiveness in pattern activity. The illustrations in the CAPS focus more on repeating patterns of varying complexity, with variability in shape and colour, than on growth patterns with shapes and numbers. Growth patterns are aligned with counting activity and none of these types of sequences are extended beyond counting backwards and forwards.

Symmetry is used in one example of pattern, yet the CAPS (2010) mentions that teachers should use patterns in art and beading to describe and develop the notion of pattern. These types of pattern show repetitive characteristics, but also develop the notion of symmetry across the pattern (line and rotational symmetry) in a visual manner. The notion of repeat due to a symmetrical attribute (blue circle, blue circle, red square; red square, blue circle, blue circle) is not explored across the four grades. Symmetry patterns are complex patterns, and they require a well-developed language of transformation to copy and describe the patterns[39]. Symmetry does not feature prominently as a developmental activity in the study of pattern across the three FP years.

From Grades 1 to 3, there are repeating patterns as well as growth patterns that are sampled. The repeating patterns are in the context of geometric shapes and colours that repeat in various ways. In all four grades, sequences that consist of identical geometric shapes that are iterated (circle, circle, circle), and sometimes with their interior colours varying (blue circle, red circle, blue circle, red circle), or with varying shapes and colours across a collection of shapes (blue circle, red triangle, blue circle, red triangle), are regularly used. From Grades R to 2, there are more repeating patterns than number patterns in the CAPS document. Where pictures (collections of leaves, matchsticks, crayons, stacked blocks) are used, these are usually not analysed for their structural attributes and pedagogical possibilities. Attributes that are present in the pictures of objects include object shape, size, colour, orientation and position in a sequenced collection. These attributes are more often than not combined in an activity, without drawing the necessary attention to the classroom discussion that could inform the heuristics necessary to notice the deeper structural components of the sequenced items.

Across the FP there is a strong focus on repeating patterns (68%). However, there is no development of the pedagogical tools to support the structural notion of these types of patterns, nor any development of the register to describe them. Furthermore, there is no

[39] The second pattern in Table 4.4, p.120 is an example of a complex pattern that uses symmetry.

reference to the core of the repeating pattern and the way in which the core and variability of attributes together determine the complexity of these patterns. The learners are merely engaged in focusing on the surface features of the geometric patterns and not on explicating the structural aspects of each pattern. The pictures are also already grouped together, indicating the unit that must be repeated and, therefore, the learner is not afforded the opportunity to group from within the continuous collection of objects to identify and explicate the specific core of a pattern. The CAPS suggests that units of repeat (when working with a collection of physical objects) should be placed as a combined unit on a separate sheet of paper, or in a block (when given as a drawing), in order for learners to see the unit that requires repetition (CAPS, 2010, p.115). The fact that the CAPS specifies and provides illustrations of 'making visible' the core of the pattern, negates the possibility of using repeat as a pedagogical resource to elicit cyclic structure. Identifying the core of the pattern, when it has been placed in a block enclosing the elements contained in the core, does not enhance the mathematical value and pedagogical purpose of working with repeating pattern.

The Foundation Phase Rainbow Workbooks and the Annual National Assessment materials

The two main resources that the teachers in my study used in class were the FPRWs and the ANA guidelines, exemplar test items and previous test materials, spanning the years 2010 to 2013. Each of the activities in the FPRWs consisted of a number of sub-questions and multiple instructions which collectively made up one activity. If, for example, Grade 1 had recorded seventeen activities, these were then fanned out to include the eighty-four individual sub-activities that were contained in the total of seventeen pattern activities. These sub-activities in the ANAs were all counted and numbered individually. The categories in the analysis were assigned to each sub-activity. Based on this selection, I decided to assign the activities to one of four categories that indicated the type of pattern. These categories were number pattern, repeating pattern, shape pattern with growth and a final category named 'different patterns'[40], as they were difficult to classify in the first three categories.

Teachers in my sample used the ANA resources as a check-list mechanism to measure whether the work in these assessment documents was adequately covered in class. They asked learners to use to their ANA booklets, and do the relevant problems in these books, in

[40] These activities will be discussed later in this section, and illustrated in Table 4.4 on p120.

class. Teachers used the FPRWs as the basis for classwork and then referred learners to the ANA booklets to finalise the day's mathematical activity.

Pedagogical Resources used in Sequencing

A variety of pedagogical resources was used when dealing with CA2, and these are illustrated in Table 4.2. For the FPRWs, the table shows that 75% (395/533) of the CA2 activities in the FPRW focused on growth number sequences when studying pattern. A further 16% (83/533) of the problems were based on repeating patterning activities. Of the 75% focusing on number sequences, a mere 1% (4/395) contained patterns that work across the grain (using co-variant thinking), by developing the function concept where position in the sequence and the value of the sequenced item were key.

		Pedagogical Resources													Types of Pattern (FPRW 533 activities) (ANAs 196 activities)						
	Number of activities	Number	Shape (pattern using shapes)	Alphabet	Drawing	Colours	Rhythmic Claps	Pictures of Objects	Shape Pattern (tessellation / transformation of a unit of repeat)	Money coins	Across the grain	Symmetry (reflective / Rotational)	Time	Different Pattern	Growth (number) pattern	Repeating pattern	Shape pattern with growth	Unit of repeat interrupted	Forwards (Positive growth)	Backwards (Negative growth)
ACTIVITIES IN THE FOUNDATION PHASE RAINBOW WORKBOOKS																					
Gr1	17	10	4	2	2	1								13	41	30		10	34	7
Gr2	26	17	7			13	1	5	2	2	2	2		25	101	47	7	23	76	25
Gr3	46	44	1			1			1		2		4	9	253	6	1		216	37
Tot	**89**	**71**	**12**	**2**	**2**	**15**	**1**	**5**	**3**	**2**	**4**	**2**	**4**	**47**	**395**	**83**	**8**	**33**	**326**	**69**
ACTIVITIES IN THE ANA GUIDELINES AND ASSESSMENT DOCUMENTS																					
Gr1	37	23	12		1			1							23	13	1	9	17	6
Gr2	58	47	11					1			1				46	8	4	8	36	10
Gr3	101	81	13		6					2	1			3	83	9	6	9	56	27
Tot	**196**	**151**	**36**		**7**			**2**		**2**	**2**			**3**	**152**	**30**	**11**	**26**	**109**	**43**

Table 4.2: Breakdown of the activities for CA2 and their pedagogical resources.

In the ANAs, 78% (152/196) of the total number of activities were growth number sequence items in CA2 content. A further 15% (30/196) of the questions worked with repeating patterns. Only 1% (2/152) of sequenced items engaged co-variant thinking, and this was comprised of two questions based on a table which recorded how coins grew in number as

one progressed. Since the ANAs are not printed in colour, they do not feature any problems where colour formed part of the variability in the sequenced set of shapes. Neither did the authors use shapes in different shades of grey. From the activities and sub-questions that were analysed, all of the 533 activities in the FPRW, and all of the 196 activities in the ANAs could be classified under four types of patterns shown in Table 4.2. The 'shape patterns' categorised as growth (8 in FPRWs and 11 in the ANAs) had two different types of growth enacted in the problem. The first growth type happened where the shapes in a sequenced set of items would be the same shape, but each consecutive shape would increase in size as one progressed through the sequence[41]. The second growth type was enacted by the number of shapes growing across consecutive items. Thus in a unit of repeat consisting of a square and a circle in the first item, the consecutive item would have two squares and two circles, and the next item three squares and three circles[42]. The growth could be indicated by arranging the shapes side by side, and also where shapes are placed in a particular array that grows either vertically or horizontally or in both directions. Both these growths, in number and in size, are complex to describe in words.

None of the documents (CAPS, NHFPT, FPRW and ANAs) deals with repeating pattern at a meta-level which focuses on the cyclic structure of the pattern or the mathematical register that is necessary to describe the object and the notion of its repeating in some way or another. Thus the notion of core and form are not made visible anywhere when dealing with the repeating patterns. The closest that descriptions come to the core of a pattern is to refer to the unit that repeats, but this concept is also not used correctly or effectively in a large number of activities, (FPRWs – 40% or 33/83 and ANAs – 97% or 26/30) as the cores in these activities are interrupted. It is worrisome to notice that there is no meta-level understanding that is developed or assessed in any of the resources with regards to the notions of repeat and cyclical structure which give shape to the core and form of a repeated pattern. The CAPS (2010) does not pose problems with a core that has been interrupted, but it also does not make this notion available at the meta-level to develop the necessary discourse and structural focus required for understanding the cyclicality of repeating patterns.

A further strong trend in the number pattern activities is the preference given to sequences with positive growth as opposed to those sequenced items that show negative growth. In the ANAs, 72% (109/152) of the number pattern has positive growth and 28% (43/152)

[41] This pattern is illustrated on p.101, Figure 4.12
[42] This pattern is illustrated on p.101, Figure 4.12.

possessing negative growth. Of the growth sequences given in the ANAs, 2% can move in any direction, as there is no specific instruction that suggests forwards or backwards[43]. This 2% will most likely be actioned as a forward moving sequence, and was thus recorded as positive growth sequences. The FPRW poses 83% (326/395) of its growth sequences with positive growth, and 17% (69/395) with negative growth. 4% of the number patterns can be developed in either direction (see Table 4.3 for details) and were included in the positive growth sequences. Interestingly the CAPS (2010) gives equal prominence to both sequences with positive growth and those with negative growth.

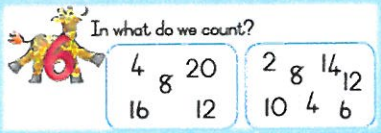
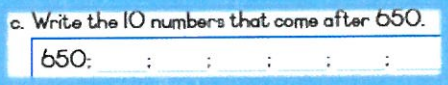
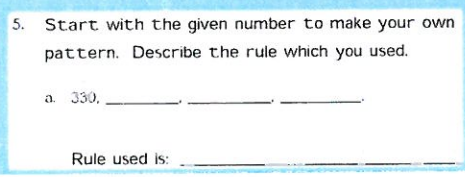
	Activity	Description
FPRW Grade 2 Book 2 p.112		Each of these activities can be written using either positive or negative growth.
FPRW Grade 3 Book 2 p. 70		The numbers can be based on a decreasing or an increasing set of sequenced numbers. Both sets will represent numbers that follow onto 650.
ANA Grade 3 Exemplar Set 3 p.15		Here the choice is left to the reader whether to increase or to decrease in magnitude based on the growth number that is used to form the sequence.

Table 4.3: Activities that could be answered by applying growth in an increasing or decreasing way.

Describing and Classifying Patterns

When learners are asked to describe the pattern in words, it is not clear what is required for such a description to be meaningful and contain important mathematical information about the object of learning (the broader notion of sequencing). The first four questions illustrated in Table 4.4 (on the next page) are examples of activities, listed under the heading of 'pattern', where learners are required to explain or describe the pattern. The activities use line and/or rotational symmetry, numbers, geometric shapes, highlighted number grids as well as coloured shapes that are layered in a collection of individual diagrams. These descriptions will most probably be at the visual level (surface features) of identifying the shape and the colour, and may not specifically cover the complexities inherent in the object based on some

[43] Illustrated in Table 4.4 on the next page.

sort of regularity (pattern) or relational feature. For instance, in question three, which is related to the shaded items in the number grid, will the description focus on implied vertical

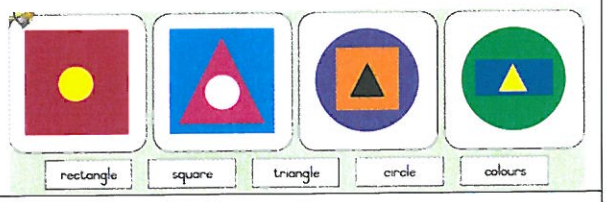
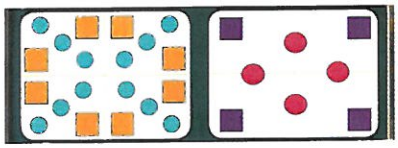
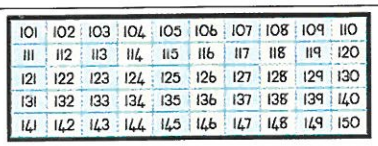
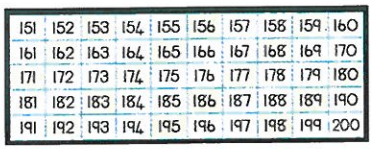
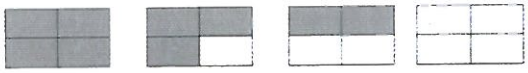
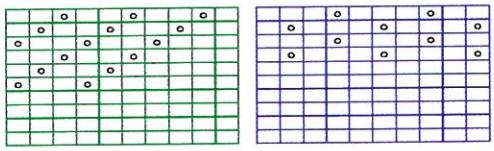
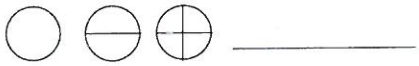
1) FPRW: Gr 2	*Describe each pattern in words. The words below might help you.*	
2) FPRW: Gr 2	*Describe the pattern.*	
3) FPRW: Gr 2	*Explain the pattern on each board.*	 
4) ANAs: Gr 3	*Colour in the diagram to complete the pattern. Describe the pattern in your own words.*	
5) FPRW: Gr 1	*Identify the shape pattern that is congruent to the one in the first block.*	
6) FPRW: Gr 1	*Completing each row by using the same pattern.*	
7) FPRW: Gr 3	*Identifying and naming the number pattern shown by the circles drawn on the grid, and continuing to draw circles in the grid to complete the pattern.*	
8) ANAs: Gr 3	*Draw the next diagram to continue the pattern by dividing the inside of the circle.*	
9) FPRW: Gr 3	*Queenie visits her Dad at the clinic at 3.45. She leaves at 5.15. How long did she visit for?*	

Table 4.4: Sample questions where the classification and description of pattern are complex.

or horizontal structure, both or neither? Where there is a definite symmetry in the pattern (numbers one and two), will the description require learners to name and describe such symmetries and also use the symmetry to describe the pattern? There are four items in the ANAs that relate to number pattern in which the requirement is to describe the pattern; two of these are patterns that the learners have to create themselves with only the starting number given. The other two questions give the instruction to fill in the missing numbers and then follow up by describing the pattern in words. The documents under review do not provide guidelines for the teacher to follow when focusing on the description of the patterns.

Activity eight, listed in Table 4.4, is an example of an open question which works with spatial features in a diagram. It is not certain how the division continues beyond the perceptual, but learners can choose the way forward by choosing one of three pathways. The most likely path would be to continue halving each shape, which will yield an equivalent number sequence 1; 2; 4; 8; ... by merely continuing to divide the area of the circle segments into half sizes. The continuation can also happen in the form 1; 2; 4; 6; ... where one line of division is added to the existing collection dividing the shape. The one relation is continually halving area, and the second relation focuses on adding another diameter to the circle.

Across both resources, the questions that reflect on the broader sequencing activity are limited, with the exception of the Grade 3 FPRW where 18% (96/533) of the activities require some form of reflection on relational features. Questions are asked about the rule (or the jump) that is used in the activity, noticing a pattern and describing the 'jump' as a counting activity. These questions are intended to draw on the relational aspects between the sequenced items or numbers by reflecting on the process and making connections between counting activity and sequencing of numbers.

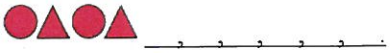
Levels of Cognitive Demand of Growth and Repeated Patterns Questions in the Resource Texts

Each of the sub-questions in the FPRW and the ANA resources was analysed and categorised for its level of cognitive challenge. The categories were labelled *Difficulty Level 1*, *2* and *3* respectively[44]. Table 4.5 (on the next page) shows typical examples of activities that were classified using the different levels based on cognitive demand. For a growth sequence problem to be classified as Difficulty Level 1 (DL 1), the growth sequence must show three

[44] These levels are again used later in Chapter 7, pp. 226-234 to indicate how these levels assigned to resource activities fit into the typology that is proposed for the growth sequences that teachers used in the lesson data.

<table>
<tr><th></th><th>NUMBER (Growth) PATTERNS</th><th>REPEATING (Shape) PATTERNS</th></tr>
<tr><td>Difficulty Level 1</td><td>
• Fill in the missing numbers:
<table>
<tr><td>20</td><td></td><td>40</td><td>50</td><td>60</td><td></td><td></td><td>90</td><td></td></tr>
</table>
• Count in 5s; Write the next 5 numbers counting in 5s.
400; 405; ____; ____; ____; ____; ____
• Count in threes starting at 6.
</td><td>
• Draw the next three diagrams in the pattern:

• Draw the next three diagrams in the repeating diagram pattern:

</td></tr>
<tr><td>Difficulty Level 2</td><td>
• Fill in the missing numbers:
7; __ ; __ ; 16 ; 19 ; __ ; __ ; __ ; __ .
15 ; __ ; 23
• Complete the table:
<table>
<tr><td>Number of hotdogs</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>Coins</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Cost in Rand</td><td>R5</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
</td><td>
• Complete the pattern:

• Draw the next two shapes or objects in each row:

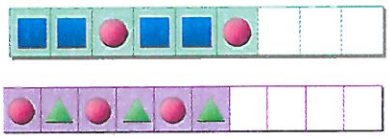
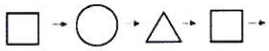
</td></tr>
<tr><td>Difficulty Level 3</td><td>
• Fill in the missing numbers:
37; __ ; __ ; __ ; __ ; 27
53; __ ; __ ; 41 ; __ ; __ ; __ .
• Working in the tables:
Sassy makes necklaces using beads. She uses 10 beads to make one necklace. Look at the table and fill in the missing numbers.
<table>
<tr><td>Number of necklaces</td><td>1</td><td>2</td><td>3</td><td>14</td><td></td></tr>
<tr><td>Number of beads</td><td>10</td><td>20</td><td>30</td><td></td><td>500</td></tr>
</table>
</td><td>
• Complete the Pattern

• Extend the pattern by drawing the shapes that come next:

</td></tr>
</table>

Table 4.5: Classification of the cognitive demand of sample problems from the ANAs and the FPRWs.

consecutive numbers (10; 20; 30; ...), or it must make the rule (adding in twos, threes etc.) available in the question. DL1 growth patterns include sequences that are made up of multiples of the growth number, and continue from a given multiple onwards (count in three starting at 6). These types of questions resemble counting activity and can easily be completed through recognition from counting. DL1 repeating patterns are those patterns where the core of the pattern is interrupted, since these activities simply require the drawing of the next item(s) in the collection of shapes. Here the focus is on order and not core.

A DL2 growth number pattern will have two consecutive terms given somewhere in the sequence, with other intermediate terms missing, and the rule is not given and must be found by accessing the surface structure of the sequence. Sequences that require the calculation of the mean number (eg. 5; ___ ;17) are also classified as DL2 questions. Tables where all the entries are consecutive, are classified as DL2 since these can be completed recursively. For DL2 repeating patterns, the core is not interrupted. DL2 repeated patterns are interpreted as *simple patterns* with a core of two or three elements with single variability (red circle, red triangle or green triangle, green square, green square). Here the colour is the same, and the shape varies, or the shape is the same and the colour varies. Even though DL2 patterns do not explicitly employ the notion of core to extend patterns, the given core is not interrupted as it is in DL1 repeating patterns where the focus is on continuing the order in which the shapes are placed, irrespective of the core of the pattern which is in any event interrupted.

A DL3 growth number sequence has no consecutiveness in the given terms and there are three, or more, spaces between the given items in a sequenced set of numbers. These questions mostly have a first and a last term given, with two or more intermediate terms missing. For DL3 growth number sequences, there is no rule given and the structural aspects embedded in the questions are used to establish the growth quantity. Growth shape pattern questions at DL3 contain growth either in the size of the sequenced objects, or in the number of sequenced items, growing either horizontally or vertically or both. Lastly, questions that work across the grain (functional thinking), were all classified as DL3 questions. DL3 repeating patterns are *complex patterns*, where the core of the pattern is fully shown and has two or more items with multi-variability.

There were a few items that were not classified into one of the three levels. This was mainly because the questions contained errors or merely asked learners to copy and colour a particular pattern with the shape outlines already in place, thus reducing the activity to 'trace and colour in'. Table 4.6 shows the

	Difficulty Level 1		Difficulty Level 2		Difficulty Level 3	
FPRW						
	GP	**RP**	**GP**	**RP**	**GP**	**RP**
Gr1	33	10	8	18	0	2
Gr2	76	23	21	16	4	8
Gr3	203	0	26	4	24	2
Totals	**312**	**33**	**55**	**38**	**28**	**12**
ANAs						
	GP	**RP**	**GP**	**RP**	**GP**	**RP**
Gr1	14	9	6	4	3	0
Gr2	20	8	16	0	10	0
Gr3	48	9	32	0	3	0
Totals	**82**	**26**	**54**	**4**	**16**	**0**

Table 4.6: Cognitive demand of CA2 items.

breakdown in detail (GP = Growth pattern; RP = repeating pattern). The total number of activities classified in the FPRWs amounted to 395 growth number patterns and 83 repeating patterns. For the ANAs 152 growth number patterns and 30 repeating pattern activities were classified. In the FPRWs 79% (312/395) of the growth patterns were DL1 questions, with 14% (55/395) at DL2 and 7% (28/395) at DL3. The ANAs classification of growth patterns were 54% (82/152) DL1 questions with and 36% (54/152) DL2 questions with 10% (16/152) DL3 questions.

With regard to repeating patterns, the FPRWs feature a total of 83 questions and the ANAs contain a total of 30 questions. For the FPRWs, 40% (33/83) were classified as DL1 questions, 46% (38/83) as DL2 questions and 14% (12/83) as DL3 questions. For the ANAs 87% (26/30) were classified as DL1 questions, and 13% (4/30) as DL2 questions, with no DL3 questions in the ANAs.

	DL 1				DL 2				DL 3			
	GP		RP		GP		RP		GP		RP	
	F	A	F	A	F	A	F	A	F	A	F	A
Grade 1	80%	61%	33%	69%	20%	26%	60%	31%	0%	13%	7%	0%
Grade 2	75%	43%	49%	100%	21%	35%	34%	0%	4%	22%	17%	0%
Grade 3	80%	58%	0%	100%	10%	38%	67%	0%	10%	4%	33%	0%

Table 4.7: Percentage of sample items per resource per grade, calculated to the nearest percentage.

Table 4.7 shows the percentage breakdown per grade, for growth patterns (GP) and repeating patterns (RP), per resource (F = FPRWs and A = ANAs), side by side, based on the figures in Table 4.6. This provides a clear sense of the preference in each resource with regards to the difficulty levels of questions. In the FPRW the authors tended to favour DL1 type sequences with 80% (Gr1), 75% (Gr2) and 80% (Gr3), showing that the DL1 category seems to predominate as one progress through the grades in the FPRW. This demonstrates that the majority of activities in the FPRWs require learners to work with sequenced items that resemble counting. This includes sequences where the growth element is revealed by either stating it (count in fives starting at 5) or by revealing three consecutive numbers in the sequence. These sequences are easily formed and recognised as they bear a distinct resemblance to counting activities. For this reason, the surface structure is obvious and very few, if any, relational aspects need to be accessed to determine the growth element. Except for Grade 1, the ANA activities exhibit a spread between DL1 and DL2. This suggests that

questions in the ANAs make use of consecutiveness together with either multiples or non-multiples of the growth number when working with growth sequences.

For Grade 1, DL1 and DL2 questions make up all the activities that are presented in the FPRWs. There are no activities in the FPRWs for Grade 1 that focus on co-variance and very few in Grade 2 (4%) and Grade 3 (10%), while for the Grade 2 ANAs 22% of questions fall into DL3 where there are no consecutive items given or where co-variance is foregrounded in some way. There is a significant discrepancy between the FPRWs and ANAs with regard to the levels of difficulty of growth pattern questions. The ANAs feature many more questions at DL2 and DL3 than the FPRWs. This shows that the level of assessment items is more advanced than the questions provided in the textbooks used in the classroom by the teachers in this study. This is most conspicuous in the area of growth pattern where the assessment items in the ANAs are significantly more demanding than those featured in the FPRWs.

Turning to repeating patterns across the grades for both resources, the trend appears different. From the data it is clear that the ANAs are problematic as far as the interrupted core is concerned, with all the questions at Grade 2 and Grade 3 working with cores that are interrupted, and Grade 1 having 69% of the questions working with an interrupted core. In the FPRWs 33% of activities in Grade 1 and 49% of activities in Grade 2 work feature repeating patterns where the core is interrupted.

Many activities in the FPRW and the ANA guidelines and workbooks deal with the growth number 2 and, in addition, the sequenced numbers that are chosen are usually multiples of two. The breakdown for growth numbers 2; 3; 4; 5 and 10 for these two documents are shown in Table 4.8, indicating the percentage of sequenced numbers that are either multiples (M) or non-multiples (N) of the growth number. The table shows that in both documents growths of two and five are used most frequently and the sequenced items are almost invariably also multiples of two or five.

	Growth 2		Growth 3		Growth 4		Growth 5		Growth 10		Total Counted
	M	N	M	N	M	N	M	N	M	N	
FPRW	23%	13%	9%	10%	7%	5%	14%	10%	6%	3%	**194/395**
ANA	28%	6%	12%	7%	8%	1%	24%	1%	11%	1%	**103/152**

Table 4.8: Percentage of activities in the FPRWs and the ANAs where sequenced items are multiples or non-multiples of the growth numbers 2,3,4,5 and 10.

Instructions that Accompany the Activities

The instructions that accompany activities are important, as they describe the action that needs to be taken in order to fulfil the requirements for learning embedded in the activity. Ideally these instructions can inform the process of learning by layering the activity in such a way that the learning agenda (sequencing, structure and algebraic reasoning) is enhanced for the particular section. The instructions that accompany each of the activities are summarised in Table 4.9 and are grouped together according to the intended action.

	What comes next	Fill in the missing numbers	Complete the patterns	Draw the next pattern	Circle the shapes that come next in the pattern	Draw the next diagram on the repeating pattern	Colour the pattern that comes next	Complete these number lines	Complete the tables	Extend the pattern	Colour the beads as you count in...	Count in...	Write the (5's, 10's) .... From ... To ...	Tick all the (fives, tens...) on number chart	Use number line to write a pattern	Copy the patterns into the spaces	Match the pattern	Trace the pattern and then colour it	Describe each pattern in words	Create your own pattern
Foundation Phase Rainbow Workbooks – Repeating Pattern (83 activities)																				
Gr 1	3		27																	
Gr 2			3	8			4			5	3					10	2	2	6	4
Gr 3	1		3							2										
Tot	**4**		**33**	**8**			**4**			**7**	**3**					**10**	**2**	**2**	**6**	**4**
ANA test items and ANA guidelines - Repeating Patterns (30 activities)																				
Gr 1				7		2				4										
Gr 2	2			2	2					1						1				
Gr 3				1		4				3										1
Tot	**2**			**10**	**2**	**6**				**8**						**1**				**1**
Foundation Phase Rainbow Workbooks – Growth Pattern (395 activities)																				
Gr 1		41																		
Gr 2		13	54					9		4		7			12				2	
Gr 3	32	71	51					23		28		39	6	3						
Tot	**32**	**125**	**105**					**32**		**32**		**46**	**6**	**3**	**12**				**2**	
ANA test items and ANA guidelines - Growth Pattern (152 activities)																				
Gr 1		15	3					3				1			1					
Gr 2	14	19	10					3												
Gr 3	14	24	9	4				5	12	2		4							5	4
Tot	**28**	**58**	**22**	**4**				**11**	**12**	**2**		**5**			**1**				**5**	**4**

Table 4.9: A breakdown of the instructions in activities in the FPRW and the ANAs.

The FPRW material appears to rely heavily on counting techniques when posing number pattern questions, and a great number of instructions start with 'count in' followed by a further instruction to do something ('point to the number on the chart' or 'fill in the spaces left blank') while counting. The bulk of the instructions require learners to 'fill in missing numbers' or continue an already given sequenced set of numbers by 'writing down the next few' (two to three) numbers. As seen in Tables 4.6 and 4.7, the majority of these questions start with the first three, or any three, consecutive items given and are thus level 1 questions. The same instruction accompanies repeated pattern questions where learners are asked to 'extend the pattern' by 'colouring and/or drawing' the item in the pattern that follows. The questions on repeated pattern often interrupt, or do not even work with, the core of the pattern. Instructions in repeated pattern questions require learners to draw the next shape in the repeated pattern, and do not work explicitly with the notions of the core and form of the pattern.

In the FPRW, 68% (56/83) of the instructions accompanying repeating patterns require a mere continuation of the given pattern, and 83% (326/395) of the number pattern instructions require learners to write what comes next, fill in the missing numbers or complete the patterns. The same level of mathematical activity is required in 93% (28/30) of the instructions for repeated pattern questions and 90% (137/152) of growth pattern questions contained in the ANA materials. The instructions require a completion of some sort, based only on what comes next, or what has been left out – usually three or four items in the sequence.

The FPRWs includes activities that are based on counting with 46 of activities instructing learners to count. These activities are a repetition of preceding activities, merely with different growth numbers, or repeat groups of beads that are then connected to the number in which one is counting. Twelve of the activities require learners to copy the same numbers given on a number line in an empty space below the given numbers. These activities do not work with the embedded structure in the number line and simply require learners to copy the number above the empty space provided. Fourteen activities in the FPRWs require learners to merely copy given shapes where the outline of the shape has been drawn, and then colour these in. In these activities the word 'pattern' is used to represent a geometric shape and not some sort of regularity that underlies the sequence. Learners are not required to understand and work with the notion of the core in repeated patterns, as it is not the core that repeats, but the order of the shapes, irrespective of the core or form of the pattern. A total of eight

activities (out of 478) in the FPRWs and five (out of 182) in the ANAs require learners to describe the pattern in their own words. Learners are rarely asked to make up their own patterns (four questions in the FPRW and five questions in the ANAs), and are not given an opportunity to express their understanding of the underlying relational concepts by applying these to their own patterns.

Activities Focusing on the Structure of the Pattern and Moving Towards Generalisation

In both the FPRW and the ANA booklets, there are questions based on number-lines, number grids and tables. All three of these pedagogical resources are potentially useful and can lead to questions that relate to the structure inherent in these manipulatives or pedagogical tools. Working with the inherent structure of physical objects, which learners use on a daily basis, can enhance the notions of structure and regularity and support generalisations about numbers that are sequenced through the use of some regularity.

The number line structure is used mainly in steps of one in the FPRW, particularly when it is linked to the number chart that is used for counting activities. There are very few instances where the unit on the number line is changed to a dimension of two, three, five or ten. Here the regularity of the objects goes unnoticed and learners can use 'count all' strategies to get to the desired next number on the number line or on the number charts. Table 4.10 shows that

	Number of Activities (Growth & Repeat)	Generalisation	Using visible structure	Describe each pattern in words	Use number line to write a pattern	Complete the tables	In what are we counting?	Colour in the pattern...what do you notice?	What is the jump?	What is the pattern?	What is the difference between the numbers?	Do you notice the pattern?	What rule did you see?
GRADE 1													
FPRW	71												
ANAs	36		4		1				1				
GRADE 2													
FPRW	148		30	8	12	4							
ANAs	54		3										
GRADE 3													
FPRW	259		23				14	2	1	14	2	6	2
ANAs	92		28	5		12		2					6

Table 4.10: Breakdown of questions that refer to structural aspects of pattern.

the FPRW contains no activity that focuses on structure in Grade 1, whereas in the ANAs 11% (4/36) of questions are based on the number line structure in Grade 1.

Learners are required to fill in the missing numbers on the unitised number line, and in a further question, they need to 'jump in twos' on the unitised number line. The latter shows regularity away from 'jumps of one', yet the unitised ticks on the number line are in jumps of one. In the Grade 2 FPRW, 20% (30/148) of activities are based on a structured object (number line, table, number chart), but this drops to 9% (23/259) in Grade 3. In the FPRW, across the three grades, tables are used in only 1% (4/395) of the questions that are posed in the workbooks. Tables lend themselves naturally to developing the notion of function (co-variance) and working across the grain in a number sequence. Using tables appropriately in pattern activities, represents a definite move away from a recursive perspective on number sequencing, and is critical in developing a sense of structure in and between numbers. The workbooks fail to engage with this notion of sequencing.

In the ANA question papers and guidelines 11% (Grade 1 – 4/36), 6% (Grade 2 – 3/54) and 30% (Grade 3 – 28/92) of the questions use pre-structured objects. The sample questions do not necessarily build on the inherent structure of the tables or number lines, and leave the potential to work structurally and generalise the relationships unexplored. The sequenced items that are given in activities do not provide the opportunity to work with the general through focusing on structural extension, and merely require the filling in of missing numbers based on recognisable surface features.

Summary

In my analysis of the documents, I was searching for how teachers are being supported by the documents, firstly, to emphasise a distinction between the different types of patterns and, secondly, to use the definitions to make the difference accessible to learners. I also searched for instances where the triadic notion of pattern, structure and algebraic reasoning is developed and supported in the policy documents and the classroom resource texts.

There appears to be a sufficient range of different patterns that are included in the documents under review. Each of these different patterns is accompanied by a short description and an example of the kind of pattern. The concept of sequencing is approached in a broad sense, where actions or grouped items that contain some form of regularity are foregrounded as objects of learning. The word 'pattern' is used in all the documents as representing one of

two notions: an object that is the product of some application of regularity, and the process of expressing regularity. In this sense, 'pattern' is used as a noun and a verb in FP mathematics. What differentiates or distinguishes between patterns is not made clear; for example, what makes a shape pattern and a geometric pattern different from one another? Is a numerical pattern not a mathematical pattern? The NHFPT (2012) states that:

> It is through copying and extending patterns using matchsticks, tiles, blocks and other apparatus that children develop a sense of behaviour of patterns (p.6).

The absence of clear definitions for repeated patterns and growth patterns in the documents under review has the consequence that pattern activities remain unclassified and this blurs the lines between repeat and growth in quite a few of the activities. Definitions set the parameters and provide the details that give a mathematical object shape and create the landscape and its boundaries for the study of a concept. None of the documents under review defines a growth pattern or a repeated pattern. Neither do they include activities that draw attention to the relational features embedded in these types of sequences.

Both policy documents under review refer to structure, but they fail to define the notion of structural thinking in the context of CA2. They also fail to show a relationship between growth pattern and structure, nor do they demonstrate how the latter can be used in class to enhance the notion of generalisation in the domain of number pattern at FP. Structure is referred to as 'the pattern within the pattern' (NHFPT, 2012, p.100), but no visibility is given to the notion of using structure to express the regularities and the variations that are observed in sequenced items. These remain undefined and unexplored and, because of this, the documents convey a largely operational view of sequencing, evidenced by the fact that the FP learners are expected to fill in the missing items or complete the next few items once the surface features are recognised. There is no deeper exploration of how surface features apply (and can be used) systemically.

The NHFPT (2012) makes it very clear that learners should be afforded the opportunity to look at pattern in both a recursive and a functional manner, both of which require a strong sense of structure and relational thinking. The CAPS (2010), however, do not refer to the structure of arithmetic expressions or number patterns. In this document, structure is used to signify an array of blocks placed to form a structure. Structured arithmetic does not feature as a pedagogical resource that teachers can use to decompose numbers according to the relational properties within the number or within a sequenced set of numbers in CA2. This is

a cause for concern, as it is the ability to identify and explicate the structural aspects of arithmetic that enables algebraic habits of mind to develop. Put differently, it is the exploration of structure that unlocks the opportunity to create generalised ideas about the relationships between numbers and within numbers themselves. Structure supports the natural development towards reasoning algebraically. The relationship between structure and algebraic reasoning is lacking in both of the policy documents. Learners are expected to identify, complete, extend and describe patterns. This process is to be supported by reflecting on the number sequence in order to identify what it is that is changing in the sequence, and what remains the same.

Although the structural aspects are spoken of in different ways, none of the documents provides the teacher with pedagogical resources that establish structure as central to the exploration of sequenced items. The policy documents and the resource materials encourage teachers to introduce growth patterns in class using spatial features. These spatial features are made available as pedagogical resources through the use of tables, flow diagrams and number chains. When recognising patterns within these visual representations, the learners are expected to develop patterning skills, number sense, relational and conceptual knowledge as well as a structural understanding of arithmetic. Although the documents underscore the importance of learners' being able to integrate the mathematical and spatial features when generalising patterns algebraically, they do not provide information and guidance related to the didactical and pedagogical tools that can be used to encourage learners to form a genus that later evolves into a schema by using the structure embedded within the pattern. There is also no suggestion as to how to use pedagogical resources effectively, such as tables and the number line, to enhance structural thinking.

The policy documents do not underline the necessity for learners to make use of algebraic means to generalise the patterns. Even when instructed to describe patterns 'in words', teachers are only encouraged to allow learners to recognise the arithmetic commonalities within a pattern, rather than to structurally extend patterns using algebraic generalisations (working with an indeterminate), which would be a more functional approach to pattern. It is also not clear how teachers are expected to use the description 'in words' as a pedagogical resource that can enhance the development of algebraic reasoning skills.

The resources that teachers use in class fall short of developing the notion of structure in the domain of pattern. They also fail to explore the structural extensions of patterning activities,

and do not pay sufficient attention to developing habits of mind that are specific to sequencing and algebraic reasoning. The resources merely extend the counting activities and do not approach sequenced items with generality in mind. The focus is predominantly recursive and, where there is an attempt at a functional approach to pattern, it is at a level of merely completing a table, which can also be done using simple recursion.

In the next chapter I discuss the series of intervention workshops which were designed to introduce the teachers to working structurally at FP and, as a result, enhance relational thinking and algebraic reasoning.