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COMPARISON OF PRICE-PREDICTION MODELS IN FORECASTING COMMODITY PRICES

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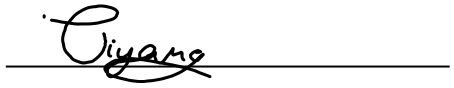
A research report submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

Johannesburg, 2020

DECLARATION

I declare that this research report is my own, unaided work. It is being submitted to the Degree of Master of Science to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.

Signed:



Thelma Thobile Jiyana

This 16 day of June year 2020

ABSTRACT

Commodity price is one of the vital inputs in mining projects valuations. If high incorrect price is used, the project will be overvalued. Subsequently, an uneconomic project may be commissioned and fail to yield expected targets. If a low incorrect price is used, the project will be undervalued. Consequently, an economic project may be shelved or abandoned due to an incorrect price being used. Mining companies are price takers; therefore, it is important to be able to apply an appropriate commodity price during valuation. However, it is difficult to predict commodity prices due to numerous uncertain factors that influence the price movements such as technology, supply, demand and macro-economics. The impact of global events further complicates the price prediction process.

There are various price prediction models that can be used to predict commodity prices with a certain degree of confidence such as mean reversion, autoregressive moving average, variants of conditional variance, dynamic model averaging and dynamic model selection. However, these models are based on different assumptions yielding different results. Copper and gold commodities were selected for this study in order to compare the forecast accuracy of the commonly used price-prediction models. Copper was selected because it is regarded as a relaible indicator for the strength of the market due to its widespread application in all sectors. A rising copper price suggests a strong economy and the converse is true. Gold was selected because it is affected by various factors such as business cycle, exchange rate, stock price and interest rate. Given that gold price movement is affected by numerous factors, it is important to investigate if there is a price prediction model that can be able to forecast the price of gold. The prices of these commodities were sourced from Market Index website.

This research study selected the most commonly used models to predict copper and gold prices. Python and MATLAB programming languages were used to apply these models because of availability and simplicity. The price-prediciton models used were Autoregressive Integrated Moving Average (ARIMA) and Glosten, Jagannathan, and Runkle Model (GJR). The selected model parameters were ARIMA (9, 1, 9) and ARIMA (7, 1, 7) for copper and gold price prediction. GJR (1, 1) model was used for

both copper and gold volatility forecasting. The measurement of forecast accuracy used was MAPE since it varies between 0 and 1, thus it is not influenced by the scale of the time series. Both ARIMA and GJR models considerably failed to forecast the commodity prices. For the purpose of comparison, the results showed that ARIMA (9, 1, 9) and ARIMA (7, 1, 7) models are only suitable to forecast copper and gold prices over a short-term, that is, periods less than three years. It was also found that GJR (1, 1) model yielded superior results when forecasting copper and gold prices conditional variances for periods over five years. Based on the findings of this study it is recommended that ARIMA (9, 1, 9) and ARIMA (7, 1, 7) models be used to forecast copper and gold prices over one-year and three-year periods. When forecasting price movements over three years, then GJR (1, 1) is recommended to forecast price volatility up to a seven-year period.

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CONTENTS **Page**

DECLARATION ii

ABSTRACT iii

ACKNOWLEDGEMENTS v

LIST OF FIGURES ix

LIST OF TABLES x

1. INTRODUCTION 1

1.1 Background 1

1.2 Problem Statement and Research Significance 2

1.3 Aim and Objectives of the Research Study 4

1.4 Outline of Chapters 4

2. LITERATURE REVIEW 6

2.1. Introduction 6

2.2. Lognormal Model 7

2.3. Mean Reversion Model 9

2.4. Time-varying Parameter Model 10

2.5. Bayesian Model Averaging 12

2.6. Dynamic Model Averaging and Dynamic Model Selection 13

2.7. Autoregressive Moving Average 15

2.8. Autoregressive Integrated Moving Average 16

2.9. Conditional Variance Models 17

2.9.1 Generalised Autoregressive Conditional Heteroscedasticity *17*

2.9.2 Exponential GARCH Model *18*

2.9.3 Glosten, Jagannathan, and Runkle Model *19*

2.10. Selection of Forecasting Models 19

2.11. Chapter Summary 19

3. DATA AND METHODOLOGY 20

3.1. Introduction 20
3.2. Data from Market Index Website 20
3.3. Generic Approach to Forecasting 21
3.3.1. Decomposing data 21
3.3.2. Removing non-stationarity 21
3.3.3. Model specification, diagnostic analysis, fitting and forecasting .. 22
3.3.4. Forecasting approach 23
3.4. Measures of Forecast Performance 24
3.4.1. Mean squared error 25
3.4.2. Root mean squared error 26
3.4.3. Mean absolute deviation 26
3.4.4. Mean absolute percentage error 26
3.4.5. Theil's U-statistics 27
3.5. Chapter Summary 27
4. ARIMA MODEL 28
4.1. Introduction 28
4.2. Copper Price Forecasting 28
4.2.1. Data stationarity test and differencing 28
4.2.2. Model goodness-fit test and forecasting copper prices 31
4.3. Gold Price Volatility Forecasting 33
4.3.1. Data stationarity test and differencing 33
4.3.2. Model goodness-fit test and forecasting copper prices 36
4.4. Summary for Forecasting Using ARIMA Models 38
5. GARCH VARIANT MODELS 39
5.1. Introduction 39
5.2. Model Selection and Forecasting for Copper Returns 39
5.3. Model Selection and Forecasting for Gold Returns 40

5.4. Summary for Forecasting Using GJR Model 42
6. COMPARISON OF ARIMA AND GARCH MODELS 43
6.1. Analysis of the Performance of the ARIMA and GJR Models 43
6.2. Chapter Summary 45
7. CONCLUSIONS AND RECOMMENDATIONS 46
7.1. Introduction 46
7.2. Findings 46
7.3. Recommendations 47
7.4. Limitations of the Research 47
7.5. Recommendations for Future Work 47
REFERENCES 48
APPENDIX A: INPUT DATA FOR COPPER PRICES IN US S/t 55
APPENDIX B: INPUT DATA FOR GOLD PRICES IN US $/Oz 56
APPENDIX C: ARIMA MODEL DIAGNOSTIC FOR COPPER USING AIC 57
APPENDIX D: ARIMA MODEL DIAGNOSTIC FOR GOLD USING AIC 60

LIST OF FIGURES

Figure 3.1: Data series with non-stationarity 22

Figure 3.2: Data series without non-stationarity 22

Figure 3.3: Procedure for data set analysis and forecasting 24

Figure 4.1: Autocorrelation in the copper price time series 28

Figure 4.2: Partial autocorrelation in the copper price time series..... 29

Figure 4.3: Differenced autocorrelation in the copper price time series 30

Figure 4.4: Differenced partial autocorrelation in the copper price time series..... 30

Figure 4.5: Copper price forecast results using ARIMA (9, 1, 9) 32

Figure 4.6: Autocorrelation in the gold price time series..... 33

Figure 4.7: Partial correlation in the gold price time series..... 34

Figure 4.8: Differenced autocorrelation in the gold price time series..... 35

Figure 4.9: Differenced partial autocorrelation in the gold price time series 35

Figure 4.10: Gold price forecast results using ARIMA (7, 1, 7) 37

Figure 5.1: GJR (1, 1): copper forecast conditional variance 40

Figure 5.2: GJR (1, 1): gold forecast conditional variance..... 42

LIST OF TABLES

Table 1.1: Valuation approaches and associated methods 3

Table 4.1: Non-stationary copper price time series using ADF test 29

Table 4.2: Stationary copper price time series using ADF test 31

Table 4.3: Various models: copper forecasting results over a seven-year period 32

Table 4.4: Non-stationary gold price time series using ADF test 34

Table 4.5: Stationary gold price time series using ADF test 36

Table 4.6: Various models: gold forecasting results over a seven-year period 36

Table 5.1: GARCH (1, 1) conditional variance model for copper price volatility 39

Table 5.2: EGARCH (1, 1) conditional variance model for copper price volatility 39

Table 5.3: GJR (1, 1) conditional variance model for copper price volatility 40

Table 5.4: GARCH (1, 1) conditional variance model for gold price returns 41

Table 5.5: EGARCH (1, 1) conditional variance model for gold price returns 41

Table 5.6: GJR (1, 1) conditional variance model for gold price returns 41

Table 6.1: Comparison of ARIMA and GJR models for a one-year period 43

Table 6.2: Comparison of ARIMA and GJR models for a three-year period 44

Table 6.3: Comparison of ARIMA and GJR models for a five-year period 44

Table 6.4: Comparison of ARIMA and GJR models for a seven-year period 45

1. INTRODUCTION

1.1 Background

Mining companies are concerned with improving performance such that the needs of all stakeholders are met. Stakeholders include, *inter alia,* shareholders, organised labour, communities, and government. These needs can be met only if the mining companies keep a close focus on protecting their profit margins and curtailing the rise in operational cost. The focus on operational cost reduction, consequently resulting in increased profits, is driven by market risks. Market risks include, amongst others, commodity price fluctuations, exchange rates and equity risks (Ernst and Young, 2013).

These market risks components need to be factored in during project valuation. Project valuation is conducted to determine if a project is feasible based on a number of assumptions. One of the assumptions is the commodity price. If a commodity price is underestimated, a good project may be abandoned because of the prospect of loss of capital. On the other hand, an uneconomic mining project may be commissioned at an overestimated commodity price, consequently failing to yield the expected returns. Consequently, the project will have to be shelved leading to loss of investment.

According to Mobtaker and Osanloo (2015), commodity price is one of the most important parameters of mine design and planning. However, it is difficult to predict given the numerous uncertain factors that affect the commodity price such as technology, supply and demand, macro-economics, hedging and futures contracts to name a few. Therefore, it is important that a proper analysis of commodity price is conducted in order to determine its true value and forecast future prices. However, forecasting of commodity prices is not straightforward, different assumptions and models result in different results. These models are based on certain assumptions, which may not always be true in the real world. The next chapter explores the differences in the application of these models.

1.2 Problem Statement and Research Significance

Mining projects require a large amount of upfront capital injection to establish a mine with long payback periods when compared to other sectors (Smith, *et al.*, 2007). Consequently, the commissioning of a project may occur under different market conditions than those experienced during early project development. This necessitates careful determination of all parameters required for the valuation of the feasibility of a mining project. There are four main factors and uses of mining project valuations, which are to (Lilford, 2011):

- Ascertain the economic feasibility and value as well as the intrinsic uncertainty of a mineral project;
- Indicate technical, economic and operational strategies for the optimal exploitation of the mineral reserves;
- Enable decision-making concerning project acquisitions, project financing, regulatory requirements and taxation considerations; and
- Provide management with adequate flexibility in enhancing operating standards and procedures and to control operating variances.

Numerous approaches and methodologies applicable in valuation of mining projects exist. However, application of these methods depends on the availability of information about the project and preference of the valuator in some instances. The majority of these approaches depends on the price of the commodity concerned. Furthermore, supporting the importance of applying an appropriate commodity price. The effect of using an incorrect price was discussed in Section 1.1. Table 1.1 shows the various valuation approaches and associated methods. Discussion of these approaches is beyond the scope of this research study. However, for illustration discounted cash flow of the income approach is briefly discussed. Discounted Cash Flow (DCF) analysis technique is the most commonly used technique in valuating mining project.

Table 1.1: Valuation approaches and associated methods

Approach	Methods	Summary of method
Cost	Appraised value	Application of past exploration costs and budgeted or projected future costs.
	Historical cost	Uses a principle of incurred costs less any outstanding obligations and/or depletion.
Income	Discounted cash flow (DFC) analysis	Net Present Value (NPV) performed on calculated free cash flows over a project's life.
	Gross contained metal value	Factors in decision flexibility, mitigates unknowns. Financial derivative.
	Gross contained metal value	Factors in decision flexibility, mitigates unknowns. Financial derivative.
	Gross contained on metal value	Tonnes x grade x commodity price (generally inappropriate)
	Statistical / probabilistic	Probability factor applied to NPV of a theoretical deposit.
	Decision tree analysis	Using yes or no analysis with probabilities.
Market	Comparable transactions	Similar properties reflect similar values.
	Market capitalisation per ounce	Market value divided by total resource ounces
	Market capitalisation per annual production in ounces	Market value divided by annual ounces produced
	Option agreement or joint venture terms	Required participation expenditure in proving up or developing a project by the incoming party.
	Geoscience factor	Kilburn method, using location, grade and geology factors.
	Rules of thumb	Back of the envelope analysis.

Source: Adapted from Lilford (2011)

The DCF technique is based on the calculation of free cash flows over the life of a project. These cash flows are discounted to present values using a discount rate. One of the main input parameters in calculating cash flow is the commodity price. Therefore, if the price is underestimated, the net value of the project will be understated leading to project performing above expectations. If the price is overestimated, the net value of the project will be overstated. Consequently, high expectations will be set, which the project will not meet.

The ability to apply an appropriate commodity price is fundamental to an optimum and robust assessment of the true value of any mineral project with all things being equal. Therefore, it is important to identify a model that can be used to estimate the appropriate commodity price with consideration of the market conditions.

This research study proposes the application of several commonly applied but different price prediction models to determine commodity price, such as the lognormal model and mean reversion models (Geman, 2007; Chinhamu and Chikobvu, 2010). This research study compares the differences in the fundamental theories between these models.

1.3 Aim and Objectives of the Research Study

The aim of this research study is to enhance the understanding of the appropriate price prediction model to be used under certain economic conditions for commodity price forecasting.The objectives of this research study are to:

- Determine if copper and gold time series are stationary for application of price-prediction models;
- Select and apply price prediction models to forecast commodity prices;
- Evaluate the accuracy of the selected prediction models; and
- Recommend a model to be used to predict commodity prices taking into account market conditions.

1.4 Outline of Chapters

Chapter 1 defined the problem statement, posed the research question and provided the significance of the research. In addition to the introductory chapter, the remainder of this report is structured as briefly discussed in the rest of this paragraph. Chapter 2

(Literature review) compares and contrast models (including literature incorporating all the major parameters that are present in the proposed study). Chapter 3 (Research methodology) outlines the tasks carried out in order to achieve the set objectives. The tasks include data used in the project; a method for sample selection is discussed in detail; and data analysis. Chapters 4 and 5 present the results of data analysis in light of the research question based on Autoregressive Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) variants models. Chapter 6 (Comparison of ARIMA and Glosten, Jagannathan, and Runkle Model (GJR) models) presents a comparison of the selected ARIMA and GJR models. Chapter 7 (Conclusion and recommendations) presents the implications of the results and discusses the recommendations for future research.

2. LITERATURE REVIEW

2.1. Introduction

It will be great if it was possible to accurately predict prices of commodities because the mine design will be based on true prices. This would enable investment decisions to be based on optimal designs with everything else being equal. However, it is difficult to accurately forecast prices. The difficulty is due to a number of uncertain factors such as economic conditions, political situations, and supply and demand patterns. Therefore, the commodity prices changes as time lapses, in other words, they are dynamic (Korn and Korn, n.d.; Taylor, 2007). Nonetheless, commodity prices can be predicted with some level of confidence through the application of models that take into consideration various points in time in the future. The models that have shown some level of success, owe their performance in the ability to (Drachal, 2018):

- Combine uncertainty of the model with the explanatory variables and time-varying parameters approach (TVP); and
- Handle various explanatory variables in the environment that are rich in data.

The common forecasting techniques are classified under qualitative and econometric methods. Qualitative forecasting models are subjective depending on the opinion and judgement of the user or the expert. They are appropriate when historical data set is not available. On other hand, econometric models are statistical models that specifies the assumed relationship between various economic parameters. The output of these statistical models is There are three main categories of econometric models, namely Giliola, *et al.* (2009):

- Time series models – predict future prices by exploiting the statistical characteristics for the historical data. These techniques are widely used due to their simplicity in application;
- Financial models – arise from different assumptions such as the arbitrage theory or from rational expectations hypothesis. These models are based on the premise that futures and forward prices are unbiased predictors of the future spot price values. However, it was found that it is impossible to identify casual direction of the relationship between future and spot prices.

Furthermore, these models may not be ideal for predict commodity prices given the lead time before commission of mining projects; and

- Structural models – define how specific economic factors and the behaviour of other economic drivers such at inventories influence the future price values. These models are not widely used in commodity price prediction.

Simulation models such as game theory can be used to forecast prices, however the major drawback of these models is that it requires detailed information of data and its computational cost is intensive (Pandey and Upadhyay, 2016). Time series models were selected because of the availability of the input data and their simplicity in application. The common time series models used to predict prices are Bayesian Model Averaging (BMA), Dynamic Model Averaging (DMA), Dynamic Model Selection (DMS), Lognormal Model, Time-Varying Parameters (TVP) Model, ARMA, and GARCH. These models are expected to manage the increasing complexity in commodity price estimation. The complexity is due to the nonlinearity of effects or behaviour that arise resulting in chaotic behaviour in the markets. Discussion of all of the prediction models is beyond the scope of this research study. Therefore, the next subsections provide a brief discussion of some of these models.

2.2. Lognormal Model

The lognormal model is also called the lognormal random walk model (Greene, n.d.). Greene (n.d.) applied the lognormal model to stock prices. This model predicts that the price will always take the form shown in Equation 1, in other words, the price will always be positive, consequently overcoming the challenge of the generic random walk model where negative prices can be realised. Therefore, this model remedies some of the deficiencies of the linear (normal) model (Greene, n.d.)

$$P_{t+1} = P_t e^r \quad (1)$$

Where:

P_{t+1} : price at the current period;

P_t : price at the previous period; and

r : rate of return for t = 1, 2, 3,…, n.

If the individual changes in prices (i.e. returns) are normally distributed with a mean (μ) and standard deviation (σ), then $P_n = N[n\mu, \sigma\sqrt{n}]$ where N represents a normal distribution (Greene, n.d.).

This model can be used to predict commodity prices and it implies that the best prediction price at period t+1 should be the price at t multiplied by the return from *t* to *t+1*. Thus, commodity price movements are independent. This model is associated with the efficient market as the more efficient the market, the more random the price changes. An efficient market means that market prices cannot be predicted by observing historical prices. Consequently, the prices might follow a random walk. Any deviations from the random walk will suggest some mean reversion (Chinhamu and Chikobvu, 2010). Chinhamu and Chikobvu (2010) applied the lognormal model to crude oil prices.

An example of the application of the lognormal random walk model is shown below. Determine the probability that the price after 25 days will be higher than R45 given that the current price is R40 with a mean of zero and standard deviation of 0.02 (Greene, n.d.).

Solution:

$$P[P_{25} > 45 = P[logP_{25} > log45] = P[logP_{25} > 3.8066]$$

$$P[logP_{25} > 3.8066] = P[\frac{logP_{25} - 3.6889}{0.1} > \frac{3.8066 - 3.6889}{0.1})]$$

$$P[Z > 1.177] = P[Z \leq -1.177] = 0.1196$$

Therefore, based on the prediction interval, there is a 95% confidence that $logP_{25}$ is in the interval:

$$logP_t + \mu_{25} - 1.96\sigma_{25} \quad to \quad logP_t + \mu_{25} + 1.96\sigma_{25}$$

Since $\mu = 0$ and $\sigma = 0.02$ then $\mu_{25} = 25(0) = 0$ and $\sigma_{25} = 0.02\left(\sqrt{25}\right) = 0.1$

Thus, the interval is $3.6889 - 1.96(0.1) to\ 3.6899 + 1.96(0.1)$ which translate into 3.4929 to 3.8849.

Therefore, there is a 95% probability that P_{25} is in the range of $e^{3.4929} = 32.88 \; and \; e^{3.8849} = 48.66$

The lognormal distribution is used when determining the value of assets because (Farooq, n.d.):

- The price at time $t + 1$ is dependent on the price at time, t and the rate of return (r) for the interval $t[t, t + 1]$;
- The rate of return follows a normal distribution; and
- For the interval $[t, t + 1]$ the return is continuously compounded represented by *e*.

It should be noted that returns follow a normal distribution is the only a model taken from Weiner process or Brownian motion. According to Geman (2007), the Geometric Brownian motion is a better approximation tool for forecasting commodity prices. The drawback of the lognormal pricing model is that even though the mean is zero it provides a value that might be greater than the previous period price. This translate as though there is free money, which is not true in reality (Farooq, n.d.; Greene, n.d.).

2.3. Mean Reversion Model

In the context of commodity prices, the mean reversion model is based on a principle that prices will tend to regress to the mean value. Therefore, a commodity price is mean-reverting if it tends to rise or fall after reaching a minimum or maximum fluctuation level, respectively. The mean reversion speed depends on the length of the burst period. However, if a rise or fall is likely to be followed by a rise or fall again, the prices are said to be mean averting (Geman, 2007; Exley, *et al.,* 2004). Geman applied the mean reversion model to oil and natural gas prices in 2007 and Exley, *et al.* (2004) applied the model United Kingdom (UK) and United State of America (USA) equity returns.

Chinhamu and Chikobvu (2010) stated that when a time series is weakly stationary, its mean (μ) and variance (σ^2) are constant and the covariance relies on the lag time. Thus, the time series will tend to return to its mean, hence mean reversion. The fluctuations around the mean, measured by variance, will have a relatively constant range. The classical approach in testing mean reversion in a series of prices is to test

whether coefficient p significantly differ from unit (1) or not, see Equation 2 (Geman, 2007.).

$$X_{t+1} = pX_t + \varepsilon_t \quad (2)$$

A null hypothesis (H_o) is created based on the existence of a unit, in other words, $p = 1$. If the $p - value$ is less than 0.05, it allows for the rejection of the null hypothesis with confidence levels higher than 95%. Consequently, one can conclude that the process is of a mean-reverting type. Otherwise, the process is of a random walk type. As the $p - value$ increases from 0.05, the more random walk existence is validated. There are two main types for testing mean reversion, namely (Geman, 2007):

- Augmented Dickey-Fuller (ADF) test: it estimates the regression coefficient $p(t+1)$ on $p(t)$. If the $p - value$ is significantly below a unit (1), it supports that the process is mean-reverting. On the contrary, if a $p - value$ is close to 1, the process is a random walk type; and
- Phillips-Perron test: it searches for a unit in the equation linking X_t and X_{t+1}. A high $p - value$ supports the hypothesis of unit value, subsequently, reinforcing the random walk type.

Ahmed, *et al.* (2018) applied mean reversion model to predict Deutscher Aktien Index (DAX), National Association of Securities Dealers Automated Quotations (NASDAQ) and Standard & Poor's 500 (S&P 500) indices. Ahmed, *et al.* (2018) stated that it is easy to predict prices of stocks if they follow a mean reversion process because past information can be used to forecast the future prices. This feature of mean reversion is important because mining engineers can be able to apply commodity prices with high confidence levels during pre-feasibility and bankable studies.

2.4. Time-varying Parameter Model

The time-varying parameter (TVP) model is based on the principle that the coefficients used to determine the forecasted objective may not be the same throughout all periods under different market conditions. These coefficients stochastically change over time due to how they react to different change in the underlying market conditions (Tucci, 1995).

The model allows for one model with same set of predictors to hold over the period under study (Buncic and Moretto, 2015). Buncic and Moretto (2015) applied this model to copper prices. Time-varying parameter models are estimated using methods such as Kalman filter (Koop and Korobilis, 2012). Koop and Korobilis (2012) used the model to forecast USA inflation. Suppose there is K number of models, which are characterised by having subsets, z_t as predictors. The subsets are denoted by z^k for $k = 1, \dots, K$.

$$y_t = z_t^k \beta_t^k + \varepsilon_t^k \quad (3)$$

$$\beta_t^k = \beta_t^k + \eta_t^k \quad (4)$$

Where:

y_t : is the dependent variable being forecast, such as return on commodity price;

β_t^k : time-varying coefficients;

ε_t^k : is a scalar assumed to be $N(0, H_t^k)$; and

η_t^k : is $N(0, Q_t^k)$.

McGough (2000) applied the time-varying parameter belief by adopting the principles of random walk with variable conditional variance. The reasons cited were:

- The random walk is a standard model for time-varying parameters;
- If the conditional variance is zero then the random walk reduces to a constant parameter model; and
- The simplicity of application allows for analytic tractability.

The main shortcoming of this model is that if the set of predictors is large, it tends to over-fit in-sample, consequently, having poor out-of-sample forecasting performance. The results do not improve even if the extension of this model is used such as time-varying parameter vector autoregression (TVP-VAR) models because they suffer from the same limitations. Hence, DMA has presented itself as an alternative model (Aye, *et al.,* (2015). Aye, *et al.* (2015) used the model to predict gold prices.

2.5. Bayesian Model Averaging

Bayesian Model Averaging is based on the principle that all uncertainties in the model are represented and measured by probabilities through the application of probability rules. These rules provide probabilistic inferences about the factor(s) measured, such as the impact on the commodity price volatility. This model requires that information about the probability distributions representing current uncertain information or data and beliefs be outlined. The major shortcoming of this model is that the beliefs are subjective to the forecaster (Hoeting, n.d.; Faust, *et al.,* 2011; Masih, *et al.,* 2010). Hoeting (n.d.) used this model to predict effect of "Mad-cow" disease, Faust, *et al.* (2011) applied the model to forecast credit spreads and Masih, *et al.* (2010) applied the model to predict Standard and Poor's 500 returns.

Equation 5 expresses the generic Bayesian model approach. The aim of the approach is to use available data to construct probability density distributions for parameters associated with hypothesis for parameter estimation. Consequently, it evaluates the parameters and identifies one that has a likelihood to affect the commodity price movement. Therefore, it supports the notion that the future can be predicted using historical data.

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B)} \quad (5)$$

Where:

$p(A|B)$: is the probability of A occurring given that B is correct;

$p(A)$: is the prior knowledge about the parameter being measured before the experiment;

$p(B)$: is the currently available information (observed data); and

$\frac{p(B|A)}{p(B)}$: is the support of B for A, that is the probability of A happening given that B has occurred.

The probability of the observed data, $p(O)$, occurring is 1 since it already exists. Thus, Equation 5 is rewritten as in Equation 6. Where prior is established based on acceptable belief using a hypothesis (H).

$$p(H|O) = p(O|H) \times p(H) \quad (6)$$

Where:

$p(H|O)$: represents posterior;

$p(O|H)$: represents the likelihood; and

$p(H)$: is prior.

According to Masih, *et al.* (2010), BMA has proved to yield superior results when used to forecast prices over a long horizon compared to the random walk model. However, its major downfall is the fact that there is subjectivity in the determination of beliefs regarding the parameter being measured. Furthermore, BMA requires application of discretisation method where numeric data have to organised categorical which further introduces some level of subjectivity depending on the experience of the forecaster. Baur, *et al.* (2014) stated that BMA suffers from computations burden if many regressors are included. Baur, *et al.* (2014) applied the model to forecast gold prices. This is due to its nature of always determining a transition matric during probability prediction process (Koop and Korobolis, 2012).

2.6. Dynamic Model Averaging and Dynamic Model Selection

Dynamic Model Averaging (DMA) and Dynamic Model Selection (DMS) were developed by Raftery, *et al.* (2010), and further fortified by Koop and Korobilis (2012). Raftery, *et al.* (2010) utilised the model to predict crime. These two models are relatively similar in application with a minor difference when choosing model(s) to use during forecasting. According to Aye, *et al.* (2015), this model is suitable for measuring variables of returns of volatile parameters over changing market conditions such as gold price. DMA and DMS allow the forecasting model to vary over time, hence, the name dynamic. Furthermore, it allows for predicting model and coefficient to change over a certain period, hence, it successes (Baur, *et al.,* 2014; Aye, *et al.,* 2015). The advantages of DMA are:

- The coefficients on the predictors can vary over time;
- As the number of predictors increases, the number of models increases as well; and

- That it can change over time as the predictors change or introduce some variables better capable of forecasting during market changes, such as a recession.

Bork and Moller (2015) stated that models that do not allow modifications to cater for market changes during forecasting tend to yield inferior results to those that do. DMA and DMS are built based on time-varying parameter (TVP) models as shown in Equations 3 and 4. Bork and Moller (2015) applied DMA and DMS models to predict house prices.

According to Bork and Moller (2015), DMA and DMS are implemented by calculating probabilities, $\Pr(L_t = k|Y^{t-1})$, for the model $k = 1, \dots, K$ based on $Y^{t-1} = \{y_1, y_2, \dots, y_{t-1}\}$ and averaging forecasts across the K models using principles of probabilities like in the Bayesian approach. This means determining the probability that model k is the best model to use in forecasting y_t given information up to time $t-1$. The determined probabilities are denoted by $\pi_{t|t-1,k} = \Pr(L_t = k|Y^{t-1})$.

DMA and DMS are used to calculate the recursive forecasts of y_t conditional on Y^{t-1} based on Equations 7 and 8.

$$\hat{y}^{DMA} = \sum_{k=1}^{K} \pi_{t|t-1,k}\, \chi_{t-1}^{k\prime} \hat{\beta}_{t-1}^{k} \quad (7)$$

$$\hat{y}_t = \chi_{t-1}^{(k^*)\prime} \hat{\beta}_{t-1}^{k^*} \quad (8)$$

Where:

$\hat{\beta}_{t-1}^{k}$: is the parameter prediction; and

k : refers to a model with maximum model probability at time $t-1$.

DMS is based on the principle of selecting a model with the highest b probability for $\Pr(L_t = k|y^{t-1})$, and applying this model during the forecasting process. The shortcomings of these models are that estimation may take long if (Aye, *et al.,* (2015):

- The model may have a large number of parameters; and
- The number of K models is large since the computational time will increase exponentially.

Buncic and Moretto (2015) stated that DMA and DMS bring the much-needed flexibility in forecasting as they allow the predictors to change at each point in time above the time-varying parameter function. Therefore, it prevents overfitting of data, which leads to poor-out-of-sample forecast performance. Naser (2016) found that DMA and DMS outperformed all other models when estimating oil prices. Naser (2016) and Aye, *et al.* (2015) stated that these models not only outperformed time-varying parameter model but even BMA. DMA and DMS are complementary to each other rather than exclusive as may have been believed before.

2.7. Autoregressive Moving Average

Autoregressive moving average (ARMA) model is a combination of autoregressive [AR(p)] and Moving Average [(q)] models. AR(p) model attempts to explain the momentum and mean reversion observed, in this case, in prices. It assumes that the current price is dependant on historical prices. Therefore, it can be concluded that there is s linear relationship between the current price and the prices observed in the past. MA(q) assumes that the current prices depend on the error terms between current and previously observed prices. These error terms are random, thus there is no linear relationship between the current price and error terms. MA(q) try to model the shock effects embedded in the data in the white noise terms such as unexpected earnings and war attacks (Adhikari and Agrawal, 2013).

The ARMA (p, q) offers both the benefits of AR(p) and MA(q) in the modelling process. The major drawback of this model is that it does not factor in volatility clustering. Volatility clustering is the volatility of an instrument that is dynamic in time. However, Adhikari and Agrawal (2013) stated that ARMA is suitable for univariate time series modelling. Univariate time series consists of single observations recorded over equal time increment such as stock price. Adhikari and Agrawal (2013) applied the model to various data set such as Canadian lynx, Wolf's sunspot, airline passenger, quarterly USA beer production and USA accidental deaths. Equation 9 shows the representation of ARMA model.

$$y_i = \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{j-1} + \varepsilon_t \quad (9)$$

Where:

y_i : is the actual value at time period t;

φ_i : are model parameters due to AR(p);

θ_j : are model parameters due to MA(q); and

ε_t : is a random error at time period t.

The ARMA model is manipulated using lag operator notation, defined as $Ly_t = y_{t-1}$. The lag polynomials are utilised to represent ARMA as shown in Equations 10 and 11.

$$AR(p)\ model: \varepsilon_t = \varphi(L)y_i \quad (10)$$

$$MA(q)\ model: y_t = \theta(L)\varepsilon_t \quad (11)$$

Where:

$$\varphi(L) = 1 - \sum_{i=1}^{p} \varphi_i L^i \quad (12)$$

$$\theta(L) = 1 + \sum_{j=1}^{q} \theta_j L_j \quad (13)$$

According to Hipel and McLeod (1994), one of the vital properties of AR(p) is that it can always be represented in terms of an $MA(\infty)$ process, thus being invertible. While MA(q) process can be invertible when all the roots of the equation $\theta(L) = 0$ lie outside the unit circle. ARMA (p,q) is only applicable to stationary time series only. However, commodity prices are known to be non-stationary, hence, the ARIMA model may better predict commodity prices.

2.8. Autoregressive Integrated Moving Average

Autoregressive Integrated Moving Average (ARIMA) takes into account the correlation in the observed data unlike other methods such as the exponential smoothing method. The main premise of the ARIMA model is that irregular component(s) of the time series can take a non-zero autocorrelation (Kasyoki, 2015). ARIMA works with stationary time series, however, if the observed data is non-stationary, differencing has to be performed. Differencing is done only once to convert non-stationary data to stationarity using Equation 15. According to Taylor (2005), the first differencing follows the stationary ARMA (p,q) model discussed in Section 2.7 expressed with Equation 9.

Hence, the model is written as ARIMA (p, d, q) with $d = 1$ being adequate in most cases. If $d = 0$, then the model becomes an ARMA(p, q) model (Adhikari and Agrawal, 2013).

$$X_t - X_{t-1} = \mu + \varepsilon_t \quad (14)$$

When considering a scenario where the first differences are returns, X_t is the current price and X_{t-1} is the price observed in the immediately preceding period. μ is a constant and ε_t is the white noise (Taylor, 2005).

2.9. Conditional Variance Models

Conditional variance models address two characteristics of financial time series viz. volatility clustering and leverage effects. Volatility is defined as the conditional square root of the variance of a time series. Leverage effect is an asymmetric clustering which occurs when a time series responds more to large decreases than to large increases in the values. Generalised Autoregressive conditional heteroskedasticity (GARCH), Exponential Generalised Autoregressive conditional heteroskedasticity (EGARCH) and Glosten, Jagannathan, and Runkle (GJR) models exhibit these qualities in part of wholly (MATLAB, 2019). These models are discussed in the following sub-sections.

2.9.1 Generalised Autoregressive Conditional Heteroscedasticity

Generalised Autoregressive conditional heteroskedasticity is the most frequently used model in financial applications (Engle, n.d.; Rossi, 2004). GARCH (*P, Q*) process with conditional normal distributions is the most popular autoregressive conditional heteroscedasticity (ARCH) specification. GARCH (*P, Q*) has four parameters that are relatively easy to estimate. These parameters are $\mu, \alpha, \beta \; and \; \omega$, which are constraints factors for a conditional variance. GARCH model is based on a premise that the distribution of the return for period, t is conditional on all preceding returns, see Equation 15 (Taylor, 2005).

$$r_t | r_{t-1},\ r_{t-2}, \dots ,\ \sim N(\mu, h_t) \quad (15)$$

$$h_t = \omega + \alpha(r_{t-1} - \mu)^2 + \beta h_{t-1} \quad (16)$$

r_t: return at period t;

h_t: conditional variance from recursive definition;

r_{t-1}: return at a period preceding t;

h_{t-1}: conditional variance preceding period t.

Equation 15 is subject to $\omega \geq 0,\ \alpha \geq 0\ and\ \beta \geq 0$ in order to guarantee that the conditional variance is never negative and GARCH (1, 1) is stationary provided Equation 17 holds and then the (Taylor, 2005):

- Unconditional variance is restricted;
- Unconditional kurtosis is greater than three and restricted;
- Correlation between the squared residual $s_t = (r_t - \mu)^2\ and\ s_{t+\tau}$ is positive for $\tau > 0$, and is equal to $C(\alpha + \beta)^\tau$ where C is determined by $\alpha + \beta$ and is positive; and
- Correlation between returns at t, r_t and successive period, $t + \tau$, $r_{t+\tau}$ is zero for $\tau > 0$.

$$\alpha + \beta < 1 \tag{17}$$

2.9.2 Exponential GARCH Model

Exponential GARCH (EGARCH) model is a variant of GARCH that models the logarithm of conditional standard deviation and it also captures the asymmetry in volatility clustering. EGARCH (*P, Q)* model has *P and Q* GARCH coefficients associated with lagged log variance terms and magnitude of lagged standardised innovations, respectively. Forecasts of conditional standard deviations (variances) from an EGARCH model are considered biased because of the Jensen's inequality shown in Equation 18 (MATLAB, 2019).

$$E(\sigma_t^2) \geq e^{\{E(log\sigma_t^2)\}} \tag{18}$$

Where σ^2 and t are the variance and time period, respectively.

GARCH and ARCH coefficients for EGARCH (1, 1) model are expected to be positive while the leverage effect coefficient is expected to be negative. Any unexpected downward shocks will cause an increase in the variance, thus making it difficult to infer volatility sequences and forecasting. These complexities make the EGARCH model a

less favourable model to model time series where the likelihood of unexpected outcome exists. EGARCH model is different to GARCH and GJR models in that it models the logarithm of the standard deviation (MATLAB, 2019).

2.9.3 Glosten, Jagannathan, and Runkle Model

Glosten, Jagannathan, and Runkle (GJR) model is a GARCH variant model which includes the leverage term for modelling asymmetric volatility clustering in the time series. In GJR large negative shocks in values are more likely to cluster than positive changes. GJR (*P, Q*) model has *P* GARCH and *Q* ARCH coefficients associated with lagged variances and, lagged squared innovations. When all the leverage coefficients are zero, the GJR model reduces to a GARCH model (MATLAB, 2019).

2.10. Selection of Forecasting Models

As discussed in Section 2.1, the models commonly applied for forecasting commodity prices are Bayesian Model Averaging (BMA), Dynamic Model Averaging (DMA), Dynamic Model Selection (DMS), Lognormal Model, Time-Varying Parameters (TVP) Model, Mean Reversion Model, Autoregressive Moving Average (ARMA), and Generalized Autoregressive Conditional Heteroskedasticity (GARCH). However, due to the simplicity and availability of resources ARIMA model and GARCH model variations were selected for analysis. These models can easily be applied in Python and MATLAB programming languages.

2.11. Chapter Summary

This chapter discussed the different models that can be used for price forecasting. These models have their advantages and disadvantages when forecasting due to the assumptions under which they were developed. For instance, some of the models such as ARMA and GARCH require that the time series must be stationary, however, it is uncontested that commodity prices are non-stationary. This further necessitates the need to determine which model best predict commodity prices. The next chapter discusses the research methodology adopted for this study and provides justifications.

3. DATA AND METHODOLOGY

3.1. Introduction

This chapter provides a description of the data and methodology utilised in comparing the price forecasting models. Section 3.2 describes the sources of data and the nature of the data required for this research study. The parameters used to compare and contrast the price prediction models are discussed in Section 3.5.

3.2. Data from Market Index Website

All commodities that can be economically exploited are important. However, it will be cumbersome to forecast the prices for all commodities known to man. Therefore, it is important to select specific commodity or commodities to study. In this research study, only copper and gold commodities were selected. Copper was selected because it is regarded as a reliable indicator for the strength of the market due to its widespread application in all sectors. A rising copper price suggests a strong economy and the converse is true (Investopedia, 2020; Southern African Development Community, 2018). South Africa consumes approximately 400 000 tonnes of copper per annum. If this figure is extrapolated to the African continent, it can be concluded that the demand for copper is extensive in nature. On the other side, according to Aye, *et al.* (2015), gold has several functions in the world and is linked to the macroeconomic variables. Gold is used by central banks as part of reserves, applied in modern finance as a hedge against inflation and a safe haven during market crises. Furthermore, gold has industrial uses such as manufacturing of electronics and can be transformed into jewellery. Gold price is affected by various factors such as business cycle, exchange rate, stock price and interest rate. Furthermore, predicting the price of gold is a difficult task (Baur, *et al.*, 2014). Given that gold price movement is affected by numerous factors, it is important to investigate if there is a price prediction model that can be able to forecast the price of gold. The model that best predict gold prices can be used to forecast the gold price to be used during project valuation phase. It should be noted that mining projects normally have a long lead-time from construction stage to extraction stage. Therefore, it is important to use an appropriate gold price during project valuation to ensure that projects are correctly appraised. Therefore, the ability

to accurately forecast its movement will be beneficial to mining companies and those who safeguard their investments against buying physical gold bullions.

Monthly gold and copper prices were sourced from Market Index website for a 30-year period from 1988 to 2018 due to availability of data, yielding 360 monthly observations (Appendices A and B). In addition, the period under study was chosen because it covers times of market booms and busts in the global economy thus it is representative of real-world events. Furthermore, this period provides substantial sample data.

3.3. Generic Approach to Forecasting

3.3.1. Decomposing data

The commodity price observations were graphically analysed for three components namely; trend, seasonal and cyclic parameters. A trend is observed when there is evidence of long-term increase or decrease in the observations. This effect does not have to be linear but can be an underlying pattern in the observation over time. Seasonality is experienced when the time series is influenced by the existence of seasonal factors, viz., quarter, monthly and days of a week. Cyclicity is observed when the time series show rise and fall patterns that are not of a fixed period (Chatterjee, 2018). A random feature can also be observed meaning certain part of data cannot be explained. If the data have been analysed and found to be non-stationary, the non-stationarity was removed through techniques such as differencing.

3.3.2. Removing non-stationarity

Most prediction models are based on assumptions that the test dataset is stationary. However, this is not true with commodity prices (Figure 3.1), thus, it was necessary to remove non-stationarity in the observations by using the process of differencing. Time series stationarity means that the statistical properties such as variance and autocorrelation are constant over the period concerned. Differencing is a process of computing the difference between consecutive observations. If differencing is done only once, the process is called first differencing. Differencing was performed until the data series exhibits stationarity (see Figure 3.2 for general illustration). Thereafter, non-constant variance is stabilised by taking a logarithm or square root of the data

series. If a trend is observed in the data, some type of a curve that fits the data and the residual from the fitted curve has to be modelled (Chatterjee, 2018).

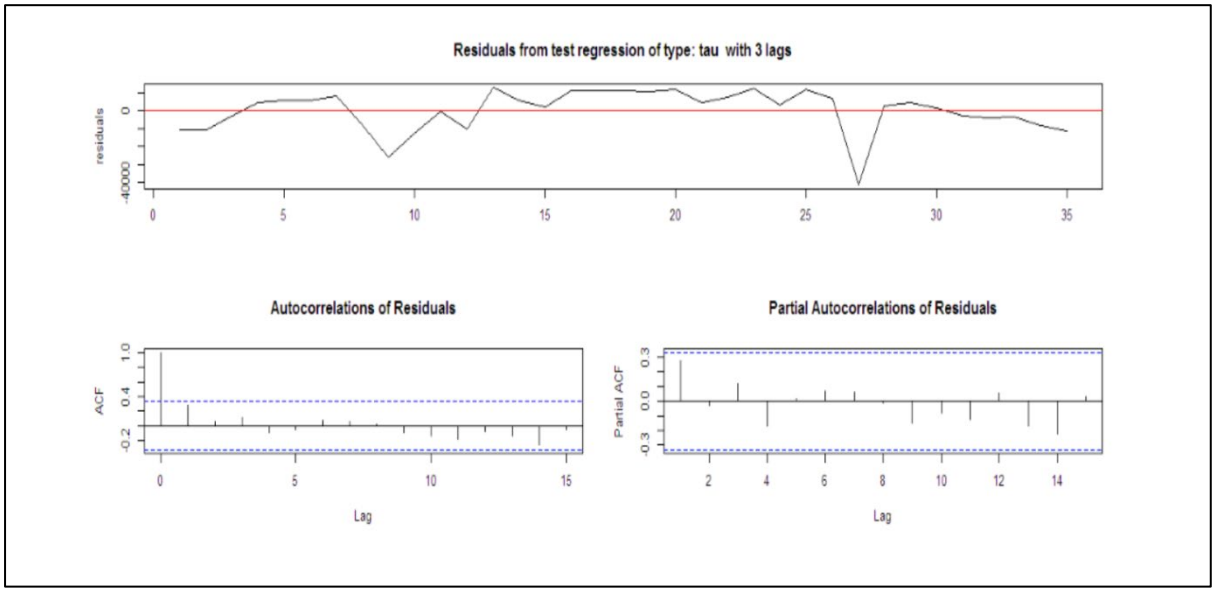

Figure 3.1: Data series with non-stationarity

Source: Chatterjee (2018)

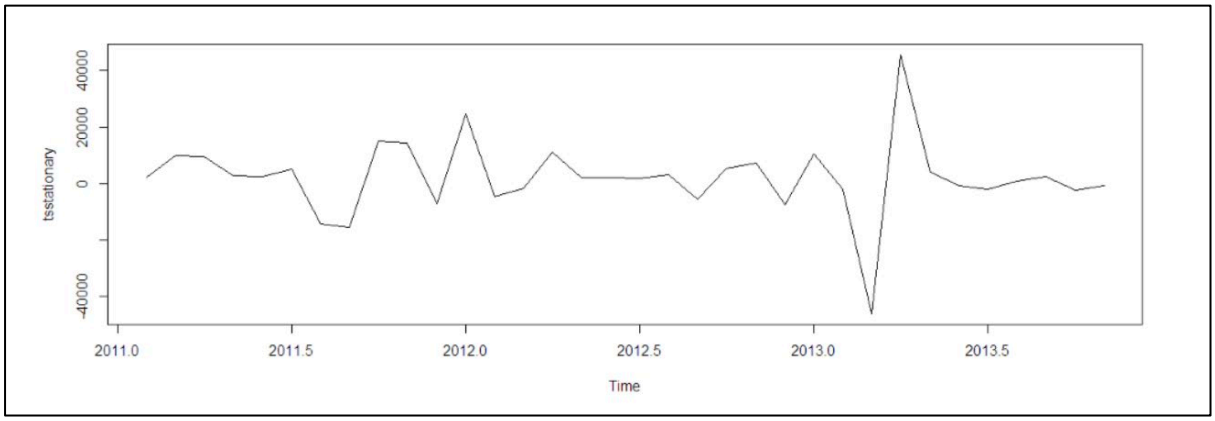

Figure 3.2: Data series without non-stationarity

Source: Chatterjee (2018)

3.3.3. Model specification, diagnostic analysis, fitting and forecasting

Fitting the model to the data refers to the selection of the appropriate model input parameters to be used. These input parameters are dependent on the type of data used in the analysis. The fitting criteria vary from model to model thus during

application of each model, the criteria used for selection of input parameters is discussed.

Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are some of the techniques used to select appropriate model parameters for price-prediction models. The AIC technique is the common technique used to determine the order of an autoregressive model. This technique considers how well the model fits the test data (observed series) and the number of parameters used in the fit. A model with the lowest AIC and/or BIC value(s) is considered to provide the best in-sample fit with number of parameters needed given the number of data points (Arowolo, 2013; Wennström, 2014).

The best ARIMA and GARCH models were used for forecasting the out of sample values to assess the goodness of fit. The assessment was conducted by comparing the forecasted prices or volatility against the actual commodity prices or volatility. Measures of forecast performance such as root mean squared error and mean absolute percentage error were used to select the best performing model. The selected measures of forecast performance used in this study are discussed in Section 3.5.The best ARIMA and GARCH models were selected to forecast copper prices and gold volatility for the testing period.

3.3.4. Forecasting approach

The data for each commodity were divided into two-parts namely; model training (in-sample) and testing (out of sample) data in order to generate appropriate prediction models and test their forecasting capabilities. Buncic and Moretto (2013) in the study of commodity price forecasting considered 217 monthly observations as adequate for the analysis. In their study, they used the first 70% price observations as training or model fitting period data, and the remaining 20% was used for out of sample evaluation. However, there is no standard approach to splitting data. This research study opted for using 75% of the observations for model training and 25% for model testing as recommended by Prabhakaran (2019). This split ensures that there are adequate in-sample observations to clean-up any noise or bias in model fitting, while there is adequate testing data set to evaluate the performance of a price prediction

model. The procedure for the analysis and forecasting for each commodity price is shown in Figure 3.3.

The available data set for testing each model is 25%, which is equivalent to 7.5 years given the 30-year period. Therefore, the price forecasting was divided into four parts. The models were used to predict over a 1-year, 3-year, 5-year and 7-year periods. Measures of performance were used to analyse the performance of the models for these periods. In addition, the entire 7.5 year period was graphically represented in forecasting in order to visualise how each model performed over the entire testing period.

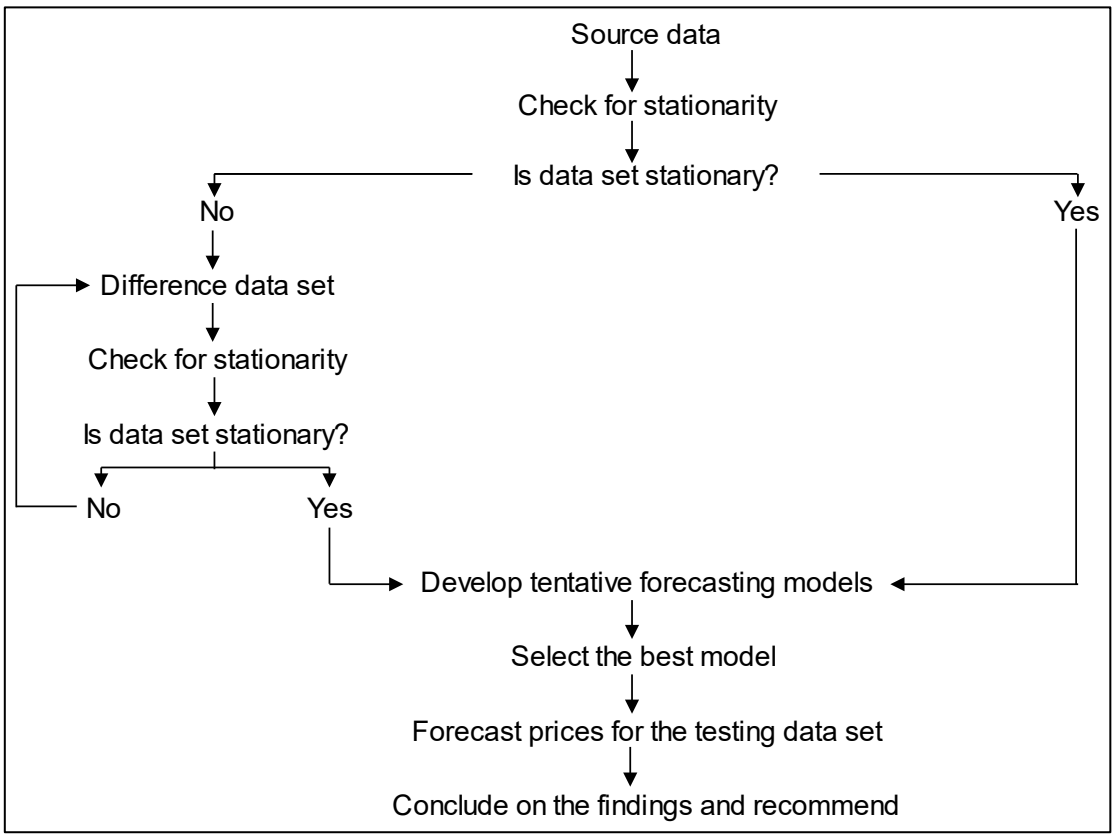

Figure 3.3: Procedure for data set analysis and forecasting

3.4. Measures of Forecast Performance

It is important to judge prediction models based on their forecasting quality when determining future commodity price movements. This will aid in selecting the appropriate forecasting technique. Failure to determine the forecasting error or accuracy may lead to the application of a wrong model in price prediction, consequently resulting in a biased forecast. There are numerous measures of

predictive accuracy that can be used such as mean forecast error, mean absolute error, mean absolute percentage error, mean percentage error, mean squared error, sum of squared error, signed mean squared error, root mean squared error, normalised mean squared error, Theil's U-statistics, mean absolute deviation, mean absolute scaled error, geometric mean absolute error, symmetric mean absolute percentage error (Adhikari and Agrawal, 2013; Armstrong and Collopy, 1992; Wallstrom, 2009; Woschnagg and Cipan, 2004; Schokry, 2010; Hyndman, 2006; Hyndman, 2014; Shcherbakov *et al.*, 2013). All of these measures of accuracy have drawbacks. It is beyond the scope of this research study to analyse each of these measures. However, to obtain reasonable knowledge of the forecast error and have high confidence in the analysis, it is advised to use more than one performance criteria (Adhikari and Agrawal, 2013). Therefore, the most commonly used measures of accuracy were adopted for analysing predictive power of the models and they are briefly discussed in the subsequent sections.

3.4.1. Mean squared error

Mean squared error (MSE) is related to standard deviation and it gives an overall idea about the errors present during forecasting. It is sensitive to outliers and error that are smaller than one (Adhikari and Agrawal, 2013; Wallstrom, 2009). MSE is calculated using Equation 19.

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(D_t - F_t)^2 \qquad (19)$$

Where t = 1, 2, …, n; D_t is the original data and F_t is the forecast data. The properties of MSE can be summarised as that it (Adhikari and Agrawal, 2013; Schokry, 2010; Wallstrom, 2009):

- Describes how well a forecasting model fit;
- Is dependent on the scale, thus cannot be used to make analysis across different time series;
- Recognises that large individual errors affect the overall forecast error;
- Penalises extreme forecast errors; and
- Is not easy to interpret as compared to other measures.

3.4.2. Root mean squared error

Root mean squared error (RMSE) is the square root of MSE, thus, all the properties of MSE hold for RMSE. It is calculated using Equation 20. Where t = 1, 2, …, n; D_t is the original data and F_t is the forecast data.

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(D_t - F_t)^2} \quad (20)$$

3.4.3. Mean absolute deviation

Mean absolute deviation (MAD) also called mean absolute error (MSE) functions like MSE as it measures variance, however, it lacks statistical robustness that exists in MSE (Wallstrom, 2009). Its major benefit is that it is easy to understand since the error has the same dimensions as the forecast. Unlike MSE, this measure is not sensitive to outliers because of the absolute value rather than quadratic value of each error. The MAD has to be as small as possible for a good forecast result. However, there is no empirical proof to know if the MAD error is small or large in relative to the actual data. In this measure, effects of negative and positive errors do not cancel out. It does not penalise extreme forecast errors, unlike MSE. It cannot be used for comparison across different time series because it is scale-dependent (Adhikari and Agrawal, 2013; Schokry, 2010; Wallstrom, 2009; Hyndman, 2006). MAD is calculated using Equation 21. Where t = 1, 2, …, n; D_t is the original data and F_t is the forecast data.

$$MAD = \frac{1}{n}\sum_{t=1}^{n}|D_t - F_t| \quad (21)$$

3.4.4. Mean absolute percentage error

Mean absolute percentage error (MAPE) is the same as MAD but deviations are measured as a percentage. The other difference is that it does not penalise extreme deviations. Moreover, it is independent of the scale of measurement, however, data transformation affects its capability to measure the forecast error. If there are periods in the series with zero values, measurements become infinite or undefined (Schokry, 2010; Hyndman, 2006). This is not a problem in this research study as there is no commodity that can be sold at no cost. Another drawback of MAPE is that it is biased towards low forecasts (Armstrong and Collopy, 1992). MAPE is calculated using Equation 22. Where t = 1, 2, …, n; D_t is the original data and F_t is the forecast data.

$$MAPE = \frac{100}{n}\sum_{t=1}^{n}\left|\frac{D_t - F_t}{D_t}\right| \quad (22)$$

3.4.5. Theil's U-statistics

Theil U-statistics is a borne out of the notion of standardised root mean squared error to aid assessment between different variables. The Theil coefficient (U) is invariant and lies between zero and a unit. Therefore, a coefficient close to zero is desirable (Adhikari and Agrawal, 2013; Woschnagg and Cipan, 2004). It is calculated using Equation 23.

$$U = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n} F_t^2}\sqrt{\frac{1}{n}\sum_{t=1}^{n} D_t^2}} \quad (23)$$

Where U and $e = D_t - F_t$ are Theil coefficient and error in the forecast. When the Theil coefficient is zero, it means a perfect fit exists.

3.5. Chapter Summary

This chapter discussed the research methodology adopted for this research study and presented justification for the selected commodities for prediction. ARIMA model and GARCH model variations were used for analysis based on their qualities. The price forecasting performance of these models was analysed using different measures of forecast accuracy. There are various measures of forecast accuracy that can be used, however, the commonly used measures are MAE, RMSE and MAPE (Ong and Chan, 2011; Botchkarev, 2018; Masoud, 2014). Consequently, these measures were adopted for this research study to analyse models performances. However, these measures have some drawbacks, hence, using more than performance criteria is recommended. The next chapter presents the forecasting of copper and gold prices using ARIMA models.

4. ARIMA MODEL

4.1. Introduction

This chapter presents the results of forecasting copper and gold prices using ARIMA models. Furthermore, the models are compared and a better prediction model(s) for forecasting copper and gold prices are identified (see Appendices C and D). The ARIMA model-related work was conducted in Python programming language.

4.2. Copper Price Forecasting

4.2.1. Data stationarity test and differencing

The stationarity properties of copper prices were analysed using ACF and PACF plots. Figure 4.1 depicts ACF plot with correlation values showing a steady decrease towards zero after more than 10 lags, thus, it can be concluded that the copper price time series is non-stationary. Figure 4.2 shows PACF plot where after the first two lags there is a drop of values below the zero line. Even so, the candles in the PACF plot are consistently out of the confidence limits (i.e. the shaded area), thus supporting the notion that the data set is non-stationary.

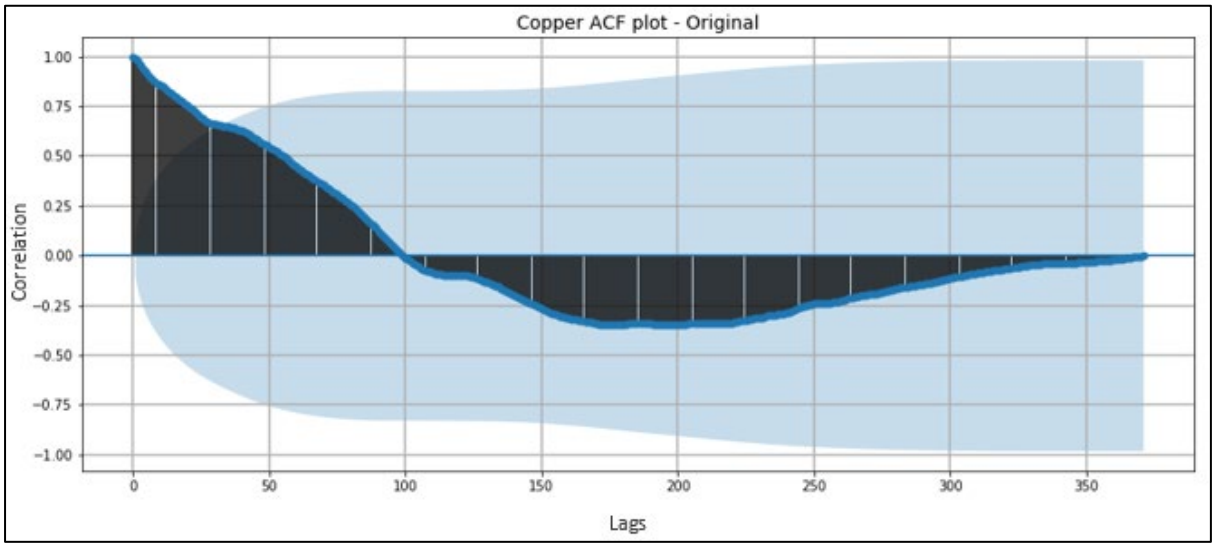

Figure 4.1: Autocorrelation in the copper price time series

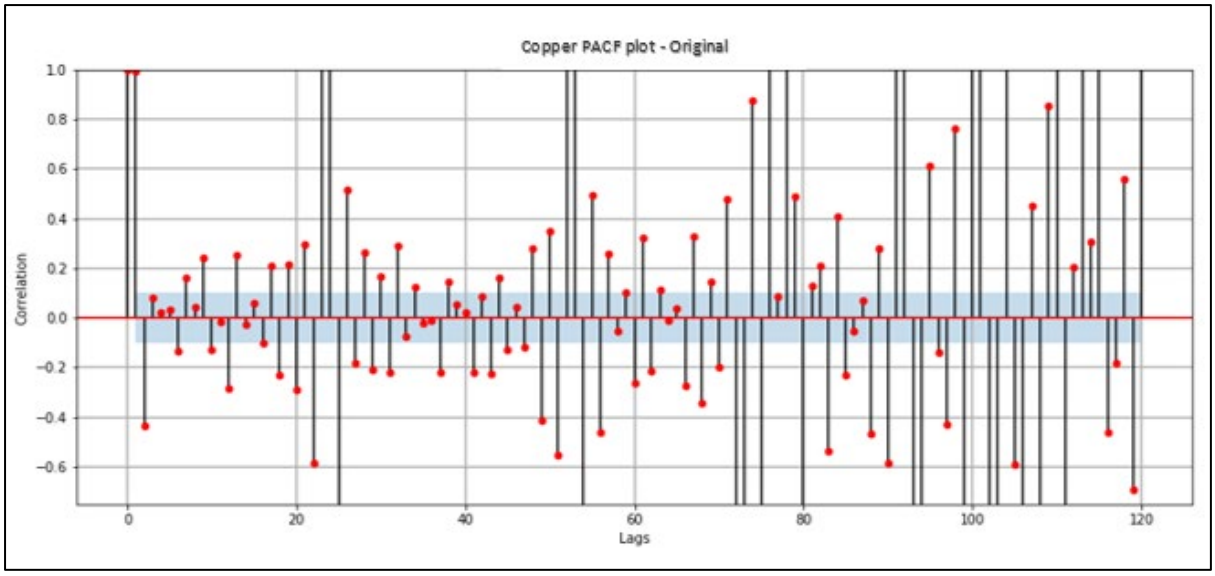

Figure 4.2: Partial autocorrelation in the copper price time series

ADF test was used to verify the results of ACF and PACF plots and it did not reject the null hypothesis that the time series contains a unit root as the p-value is 0.46. This value is greater than a p-value of 0.05, therefore, it is insignificant (see Table 4.1). Therefore, non-stationarity in the time series is confirmed. Consequently, the copper price time series was transformed into stationarity using the differencing technique.

Table 4.1: Non-stationary copper price time series using ADF test

Test parameters				
	Lags	Model	Test Statistic	Significance Level
1	1	AR	t1	0.05
Test results				
	Null Rejected	P-Value	Test Statistic	Critical Value
1	false	0.45958	-0.52367	-1.9415

Differencing was conducted in order to make the copper time series stationary using ACF and PACF techniques. The stationarity results were confirmed using the ADF test. The copper time series was differenced once (d = 1) and proved to be stationary as evident through the majority of candles being within the confidence limits as depicted in Figures 4.3 and 4.4 for ACF and PACF plots, respectively.

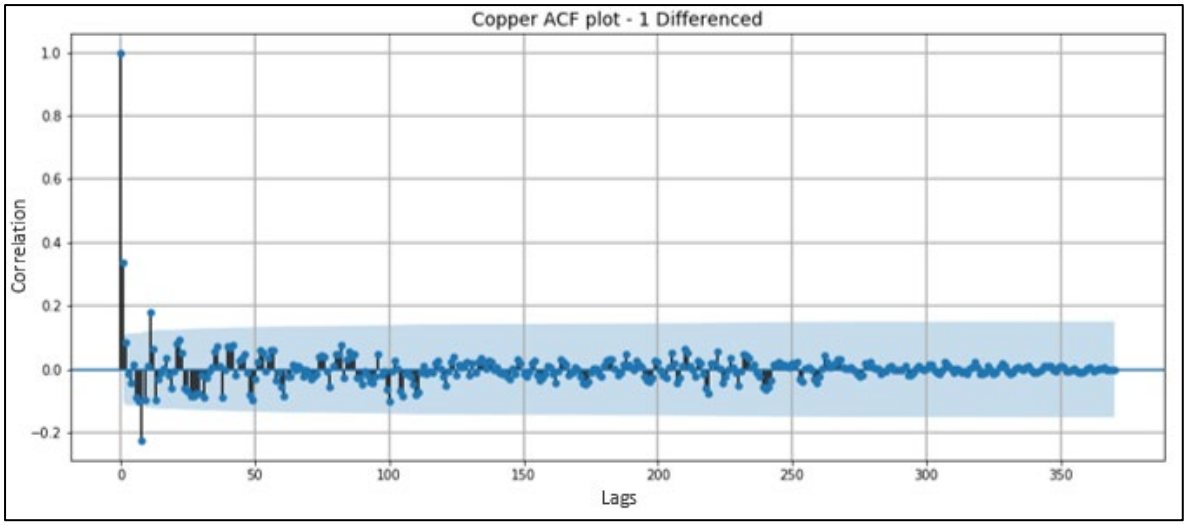

Figure 4.3: Differenced autocorrelation in the copper price time series

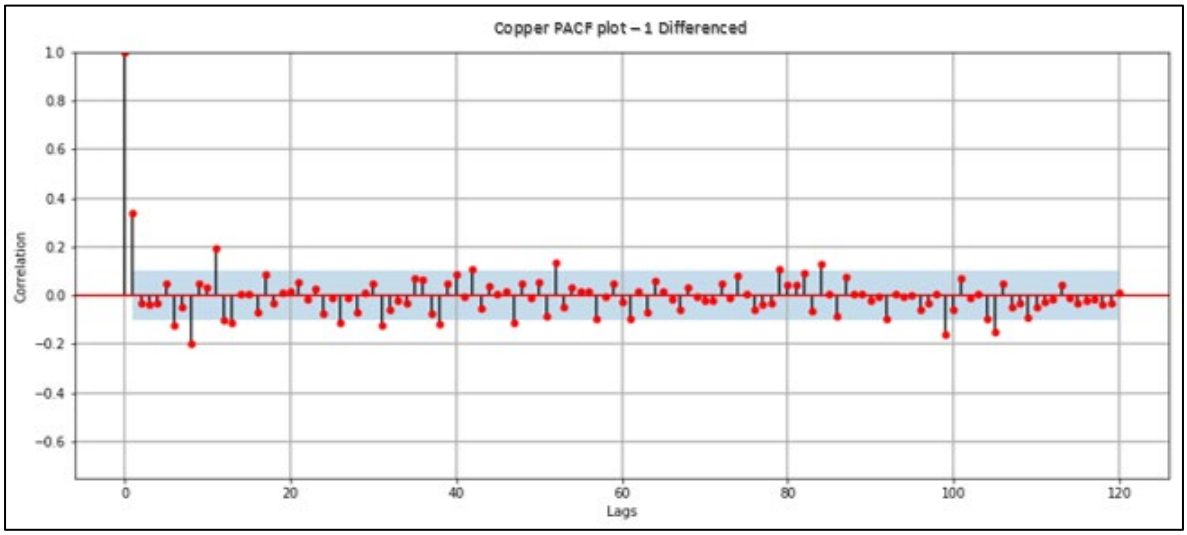

Figure 4.4: Differenced partial autocorrelation in the copper price time series

ADF test was conducted to verify the results of ACF and PACF plots and it confirmed that the time series was stationary because the p-value was significant at 0.001, which is less than a p-value of 0.05 (see Table 4.2). Therefore, the null hypothesis is rejected and the time series is stationary.

Table 4.2: Stationary copper price time series using ADF test

Test parameters				
	Lags	Model	Test Statistic	Significance Level
1	1	AR	t1	0.05
Test results				
	Null Rejected	P-Value	Test Statistic	Critical Value
1	true	0.001	-11.3777	-1.9415

Once the copper time series was stationary, tentatively appropriate models were selected, as discussed in the next section.

4.2.2. Model goodness-fit test and forecasting copper prices

The historical copper prices were split into model estimation period (in-sample data) and validation (out of sample data) periods as discussed in Section 3.2. AIC was used to select the tentatively best four ARIMA models for validation using measures of performance. The top four ARIMA models with the lowest AIC values were ARIMA (10, 1, 9), ARIMA (5, 2, 5), ARIMA (9, 2, 5) and ARIMA (9, 1, 9). Table 4.3 shows the performance of each model with regards to the goodness of fit with training data and forecasting the testing data over four different periods, i.e. one-, three-, five-, and seven-year periods. The measurement of performance indicators shows that ARIMA (9, 1, 9) is the best model to forecast copper prices because it has the lowest values for MAPE, MAE and RMSE at 3.413%, 113.709 and 132, 002, respectively. This model was used for subsequent prediction of copper prices in the next section.

Figure 4.5 shows the copper price forecast using the ARIMA (9, 1, 9) compared against the actual copper prices over a period of 7.5 years at a 95% confidence level. The 95% confidence level was selecte because it is widely used (Finch and Cumming, 2009). The forecast results show that the ARIMA (9, 1, 9) model managed to closely forecast prices in some instances from 2011 to 2014. Although the overall forecast trend was to the actual. After 2014, it forecasted a steady increase in prices while actual prices fluctuated with the lowest price being approximately US$4800/t and the highest price of about US$7000/t. The best results were realised when forecasting over the three-year period where the MAPE is about 0.7%.

Although the MAPE is about 35% for the 7-year period, overall, the forecasted prices are within the confidence higher and lower confidence limits, thus the model statistically managed to yield good results.

Table 4.3: Various models: copper forecasting results over a seven-year period

Model	Forecasting period (year)	MAE	RMSE	MAPE (%)
[10, 1, 9]	1	115.974	138.391	3.884
[5, 2, 5]		120.820	135.792	4.923
[9, 2, 5]		138.102	156.536	6.777
[9, 1, 9]		**113.709**	**132.002**	**3.413**
[10, 1, 9]	3	113.580	135.349	0.115
[5, 2, 5]		124.417	146.425	3.390
[9, 2, 5]		142.021	167.503	6.011
[9, 1, 9]		**109.268**	**130.225**	**0.688**
[10, 1, 9]	5	628.787	963.328	18.810
[5, 2, 5]		676.907	1028.126	22.407
[9, 2, 5]		719.159	1078.589	25.127
[9, 1, 9]		**614.923**	**946.682**	**18.043**
[10, 1, 9]	7	1880.148	2909.803	35.889
[5, 2, 5]		1958.109	3001.583	39.190
[9, 2, 5]		2022.245	3074.139	41.694
[9, 1, 9]		**1860.075**	**2887.920**	**35.172**

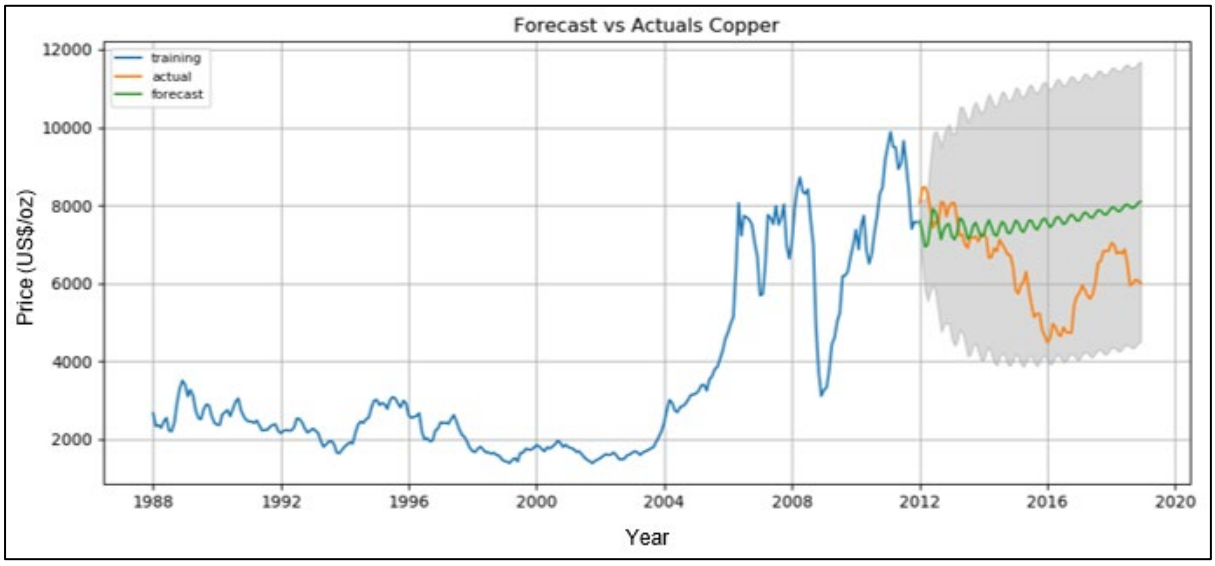

Figure 4.5: Copper price forecast results using ARIMA (9, 1, 9)

4.3. Gold Price Volatility Forecasting

4.3.1. Data stationarity test and differencing

The stationarity properties of the copper prices were observed by ACF and PACF plots. Figure 4.6 depicts ACF plot with correlation values showing a steady decrease towards zero lines after more than 10 lags, thus, it can be concluded that the copper price time series is non-stationary. Figure 4.7 shows PACF plot where after the first two lags there is a drop of values below the zero line. Nonetheless, throughout the lag period, the values are consistently outside the confidence limits (i.e. the shaded area). For that reason, the concept that non-stationarity exist in the data set is supported.

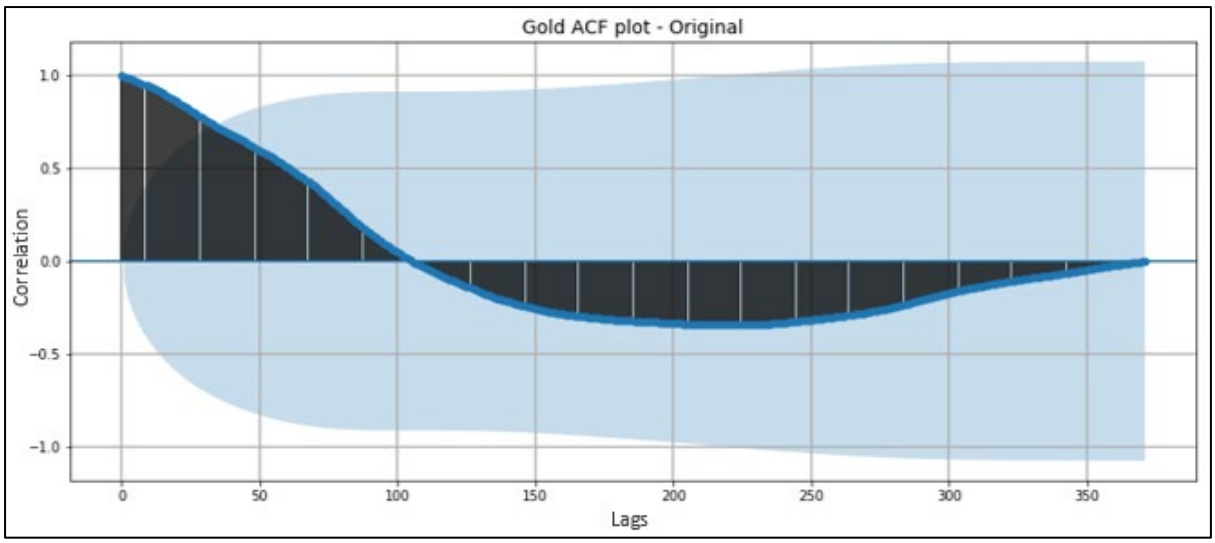

Figure 4.6: Autocorrelation in the gold price time series

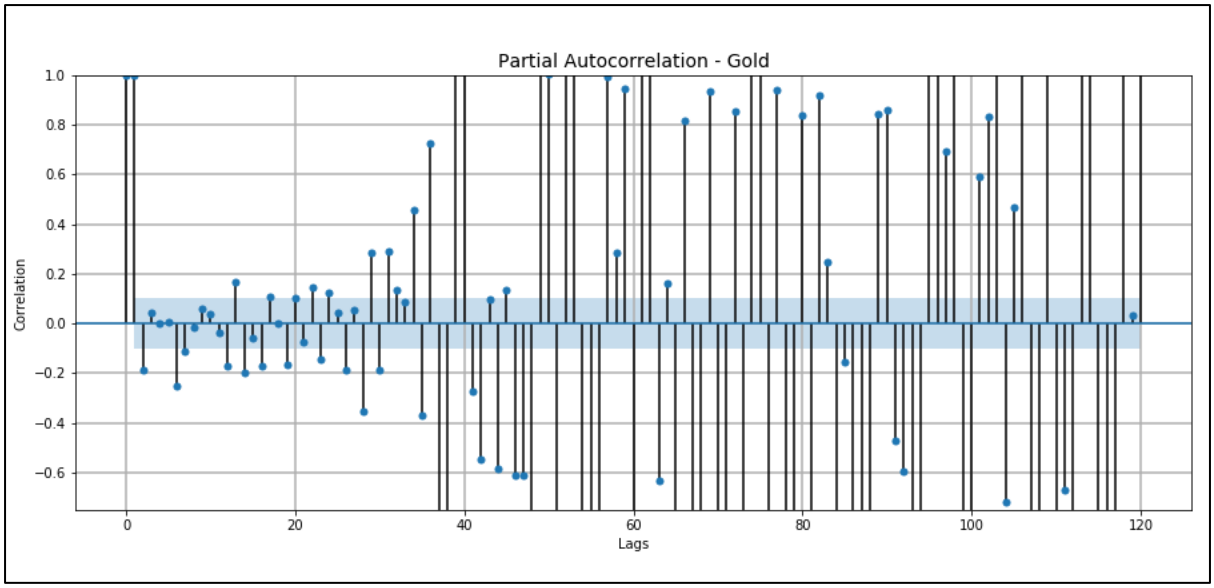

Figure 4.7: Partial correlation in the gold price time series

ADF test was used to validate the results of ACF and PACF plots and it does not reject the null hypothesis that the gold price time series contains a unit root, as the p-value is 0.88. This value is larger than a p-value of 0.05, therefore, it is insignificant (see Table 4.4). Hence, non-stationarity in the time series is definite. Accordingly, the gold price time series was transformed into stationarity using the differencing technique.

Table 4.4: Non-stationary gold price time series using ADF test

Test parameters				
	Lags	Model	Test Statistic	Significance Level
1	1	AR	t1	0.05
Test results				
	Null Rejected	P-Value	Test Statistic	Critical Value
1	false	0.87776	0.76211	-1.9415

Differencing was conducted in order to make the gold price time series stationary using ACF and PACF techniques. The gold price time series was differenced once (d = 1) and demonstrated to be stationary as apparent through the majority of candles being within the confidence limits as depicted in Figures 4.8 and 5.9 for ACF and PACF plots, respectively.

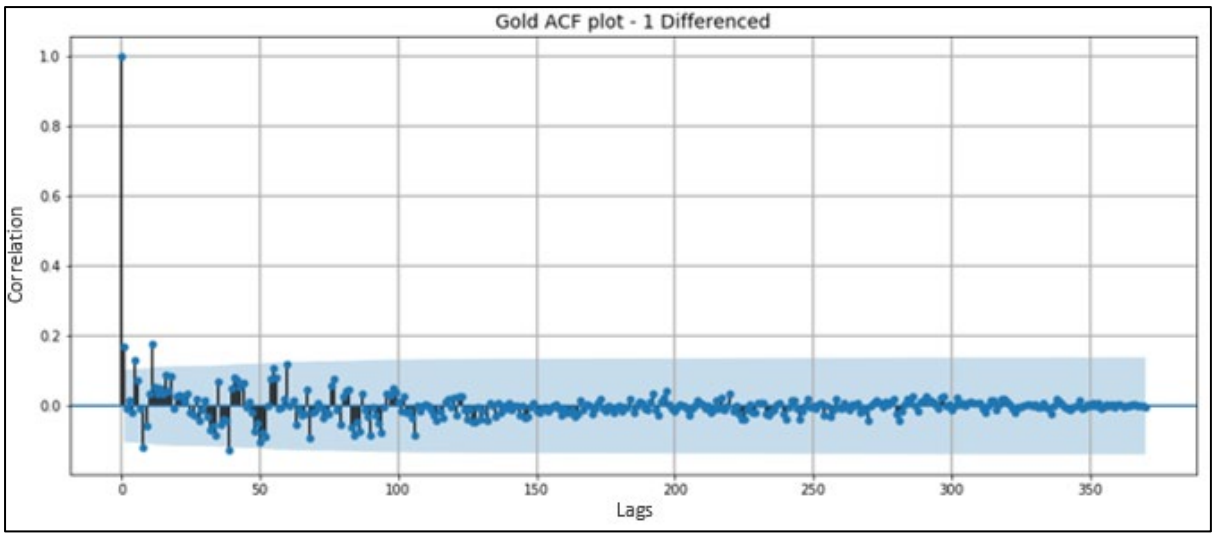

Figure 4.8: Differenced autocorrelation in the gold price time series

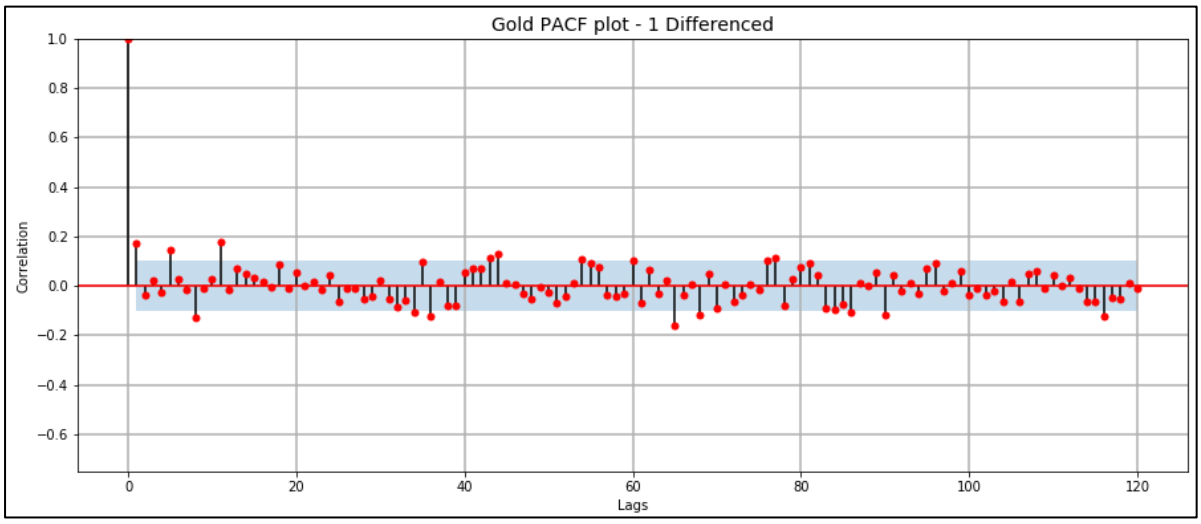

Figure 4.9: Differenced partial autocorrelation in the gold price time series

Furthermore, ADF test was performed to validate the results of ACF and PACF plots and it verified that the time series was stationary because the p-value was significant at 0.001, which is less than a p-value of 0.05 (see Table 4.5). Therefore, the null hypothesis is rejected and the time series is stationary.

Table 4.5: Stationary gold price time series using ADF test

Test parameters				
	Lags	Model	Test Statistic	Significance Level
1	1	AR	t1	0.05
Test results				
	Null Rejected	P-Value	Test Statistic	Critical Value
1	true	0.001	-13.6589	-1.9415

4.3.2. Model goodness-fit test and forecasting copper prices

The goodness-fit test was conducted to check if the residuals are 'white noise', highest-order coefficients are significant and forecasts are reasonable or not.

The historical gold prices were split into in-sample data and out of sample data periods. AIC and BIC techniques were used to select the tentatively best four (4) ARIMA models for validation using measures of performance. Table 4.6 shows the performance of each model with regards to the goodness of fit with the out of sample data and forecasting gold prices.

Table 4.6: Various models: gold forecasting results over a seven-year period

Model	Forecasting period (year)	MAE	RMSE	MAPE (%)
[5, 1, 4]	1	11.834	13.074	4.427
[7, 1, 7]		**9.292**	**10.396**	**3.437**
[8, 1, 6]		13.666	14.540	5.126
[11, 1, 6]		9.452	10.679	3.492
[5, 1, 4]	3	42.863	56.539	10.238
[7, 1, 7]		**41.212**	**55.083**	10.394
[8, 1, 6]		42.102	55.263	**9.562**
[11, 1, 6]		43.013	57.260	10.920
[5, 1, 4]	5	99.522	125.892	24.102
[7, 1, 7]		**95.426**	**120.866**	23.437
[8, 1, 6]		97.972	123.976	**23.425**
[11, 1, 6]		98.665	124.689	24.277
[5, 1, 4]	7	185.027	242.220	36.226
[7, 1, 7]		**178.369**	**234.017**	**35.123**
[8, 1, 6]		183.359	240.541	35.649
[11, 1, 6]		182.989	239.284	36.116

ARIMA (7, 1, 7) has the lowest MAPE values in one-year and seven-year periods. While ARIMA (8, 1, 6) has lowest MAPE values in three-year and five-year periods. The ARIMA (8, 1, 6) outperforms ARIMA (7, 1, 7) by 0.832% and 0.012% for the three-year and five-year periods, respectively. If the other measures of errors are considered, ARIMA (7, 1, 7) outperforms all the other ARIMA models. ARIMA (7, 1, 7) was selected for the forecasting of gold price returns.

Figure 4.10 shows the performance of ARIMA (7, 1, 7) model in forecasting gold prices against the actual gold prices over a period of 7.5 years at a 95% confidence level.

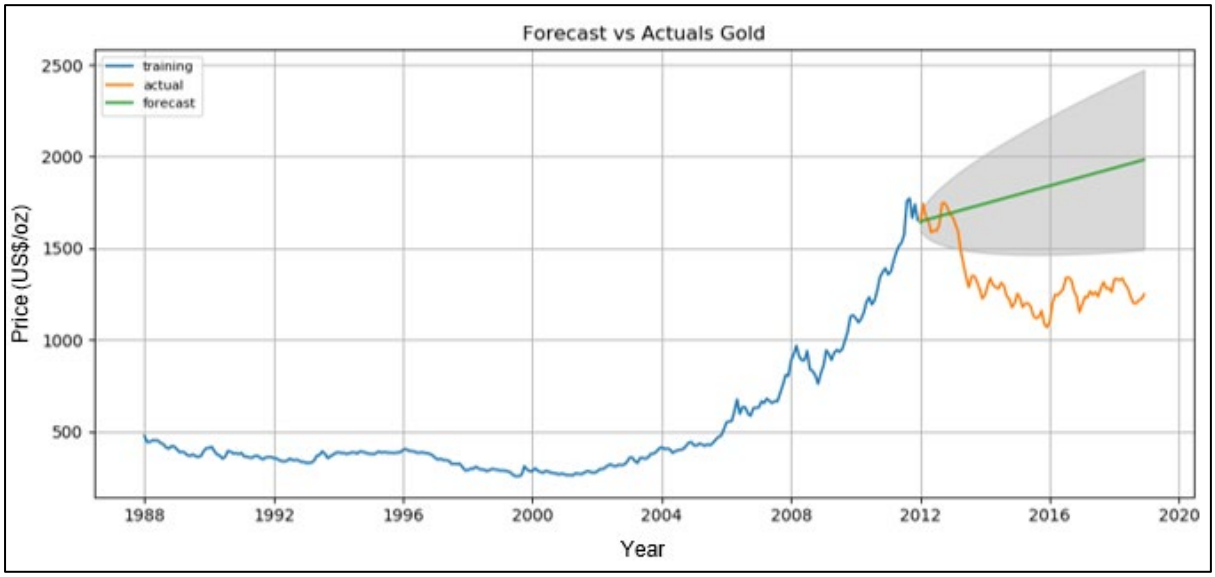

Figure 4.10: Gold price forecast results using ARIMA (7, 1, 7)

The forecast results show that ARIMA (7, 1, 7) model could not forecast prices. Only the forecast prices from the second half of 2010 to 2013 lie within the confidence limits even though they do not follow realistic gold price movements. The ability to forecast within a year is evidenced by the lowest MAPE value of about 3% in the one-year period. After 2013, it forecasted a steady increase in prices while actual prices fluctuated with the lowest price being about US$1100/oz and the highest price of about US$1400/t. This can allude to the drastic increase in gold price. Over a period of 16 years, the gold price averaged US$ 350.6/oz, then a sharp increase is observed from 2005 to 2012. This sharp appreciation in the price may have an influence on the prediction capabilities of the model. The gold price volatility is realistic thus the model failed to predict the gold prices.

4.4. Summary for Forecasting Using ARIMA Models

ARIMA (9, 1, 9) model produced as trend opposite to the actual copper prices from June 2010 to 2014. Thereafter, it continued to predict a steady increase while actual prices were fluctuating below the predicted prices. However, the forecast results remained within the 95% confidence limits. Thus, making the forecast results significant. ARIMA (7, 1, 7) only managed to forecast gold prices lying within the confidence limits from July 2010 to 2013. Thereafter, the model predicted a steady-state; however, the actual gold prices dropped outside the confidence limits of the model. Overall, the ARIMA models failed to accurately forecast the copper and gold prices. The next chapter presents model selection between the GARCH model variants and forecasting results.

5. GARCH VARIANT MODELS

5.1. Introduction

This chapter presents the GARCH variant models used to forecast copper and gold prices. The copper and gold prices were log-transformed into returns since the conditional variance models work with volatility, not the actual prices. Furthermore, better volatility prediction models for copper and gold were identified for forecasting. The original GARCH, EGARCH and GJR models were tested and analysed for the ability to forecast copper and gold volatility using MATLAB programming language.

5.2. Model Selection and Forecasting for Copper Returns

GARCH variant models were tested in order to analyse the goodness of fit for each model and selected the better performing model, which could be used for forecasting copper prices volatility. Tables 5.1 to 5.3 present the results for the goodness of fit for the GARCH, EGARCH and GJR models. From these tables, the GJR (1, 1) model has the lowest AIC and BIC values of -1040.64 and -1024.98, respectively. Therefore, the GJR (1, 1) model was selected as the best model to forecast copper price volatility.

Table 5.1: GARCH (1, 1) conditional variance model for copper price volatility

Model estimation results				
Parameter	Value	Standard Error	t Statistic	P-Value
Constant	0.00018356	8.2037e-05	2.2376	0.025249
GARCH{1}	0.84972	0.045004	18.8809	1.6374e-79
ARCH{1}	0.10277	0.032499	3.1621	0.0015665
Goodness of fit				
AIC	-1042.63			
BIC	-1030.89			

Table 5.2: EGARCH (1, 1) conditional variance model for copper price volatility

Model estimation results				
Parameter	Value	Standard Error	t Statistic	P-Value
Constant	-0.34595	0.18316	-1.8888	0.058917
GARCH{1}	0.9376	0.032613	28.7494	9.2126e-182
ARCH{1}	0.26527	0.055981	4.7387	2.1513e-06
Leverage{1}	-0.0063643	0.022034	-0.28884	0.77271
Goodness of fit				
AIC	-1040.70			
BIC	-1025.05			

Table 5.3: GJR (1, 1) conditional variance model for copper price volatility

Model estimation results				
Parameter	Value	Standard Error	t Statistic	P-Value
Constant	0.0001952	0.000103	1.8951	0.058077
GARCH{1}	0.84337	0.053667	15.7148	1.197e-55
ARCH{1}	0.10309	0.033455	3.0813	0.002061
Leverage{1}	0.0064774	0.036135	0.17925	0.85774
Goodness of fit				
AIC	-1040.64			
BIC	-1024.98			

Forecasting of copper price volatility was conducted using GJR (1, 1) model over the 7.5-year period from July 2010 to December 2018 and the results are graphically shown in Figure 5.1. The GJR (1, 1) model was not able to accurately predict the volatility changes in the copper prices. However, it can be concluded that the average volatility might approximately be the same as the actual averages if the spikes in periods 300 and about 340 are eliminated. However, in reality the spikes cannot be tempered with, thus model failed to predict the copper prices.

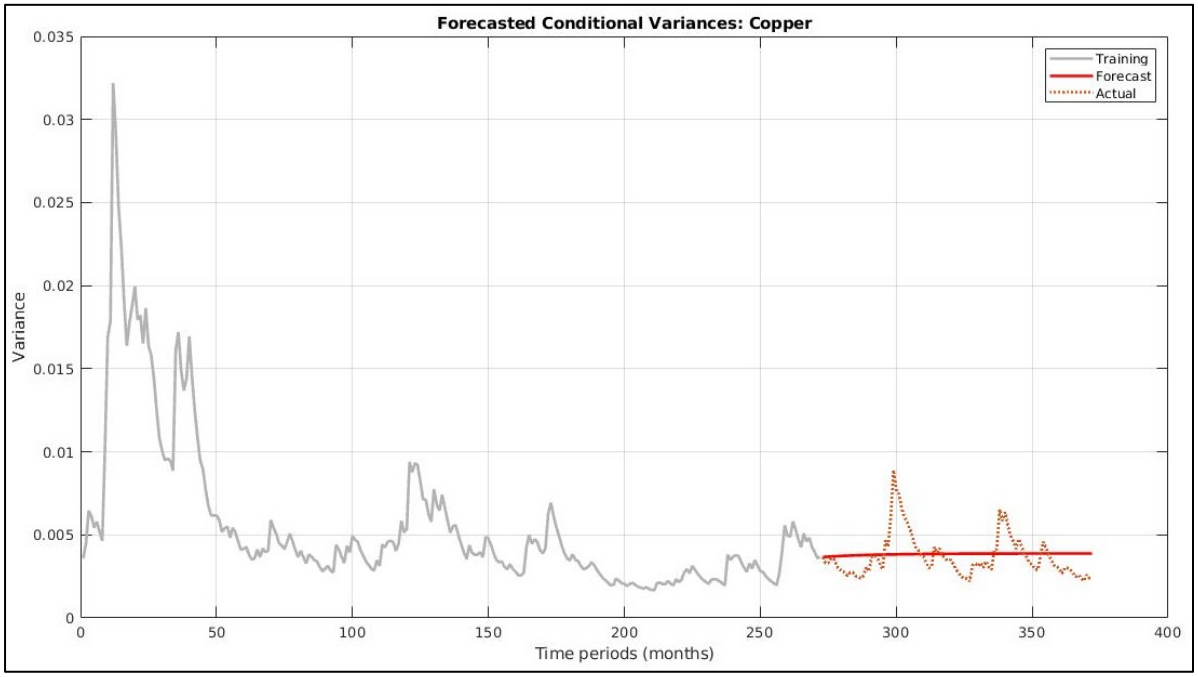

Figure 5.1: GJR (1, 1): copper forecast conditional variance

5.3. Model Selection and Forecasting for Gold Returns

Tables 5.4 to 5.6 show the models fitted to the gold price returns time series and their performances based on AIC and BIC values. The GJR (1, 1) model has the lowest

AIC and BIC values of -1451.80 and -1436.15, respectively. Therefore, the GJR (1, 1) model was used to forecast the gold price volatility.

Table 5.4: GARCH (1, 1) conditional variance model for gold price returns

Model estimation results				
Parameter	Value	Standard error	t Statistic	P-value
Constant	0.00015471	7.2359e-05	2.1381	0.032505
GARCH {1}	0.72965	0.099878	7.3054	2.7644e-13
ARCH {1}	0.14882	0.058297	2.5528	0.010686
Goodness of fit				
AIC	-1452.53			
BIC	-1440.79			

Table 5.5: EGARCH (1, 1) conditional variance model for gold price returns

Model estimation results				
Parameter	Value	Standard error	t statistic	P-value
Constant	-1.331	0.50298	-2.6463	0.0081386
GARCH {1}	0.80295	0.072324	11.1021	1.226e-28
ARCH {1}	0.29698	0.10329	2.8752	0.0040375
Leverage {1}	0.078272	0.058109	1.347	0.17798
Goodness of fit				
AIC	-1453.83			
BIC	-1438.18			

Table 5.6: GJR (1, 1) conditional variance model for gold price returns

Model estimation results				
Parameter	Value	Standard error	t statistic	P-value
Constant	0.0001975	8.5708e-05	2.3044	0.021203
GARCH {1}	0.68962	0.11429	6.0338	1.6013e-09
ARCH {1}	0.19142	0.0824	2.3231	0.020176
Leverage {1}	-0.094182	0.086973	-1.0829	0.27886
Goodness of fit				
AIC	-1451.80			
BIC	-1436.15			

Forecasting of gold price volatility was conducted using GJR (1, 1) model over the 7.5-year period from July 2010 to December 2018 and the results are graphically shown in Figure 5.2. Like for copper, the GJR (1, 1) model was not able to accurately predict the volatility changes in the gold prices.

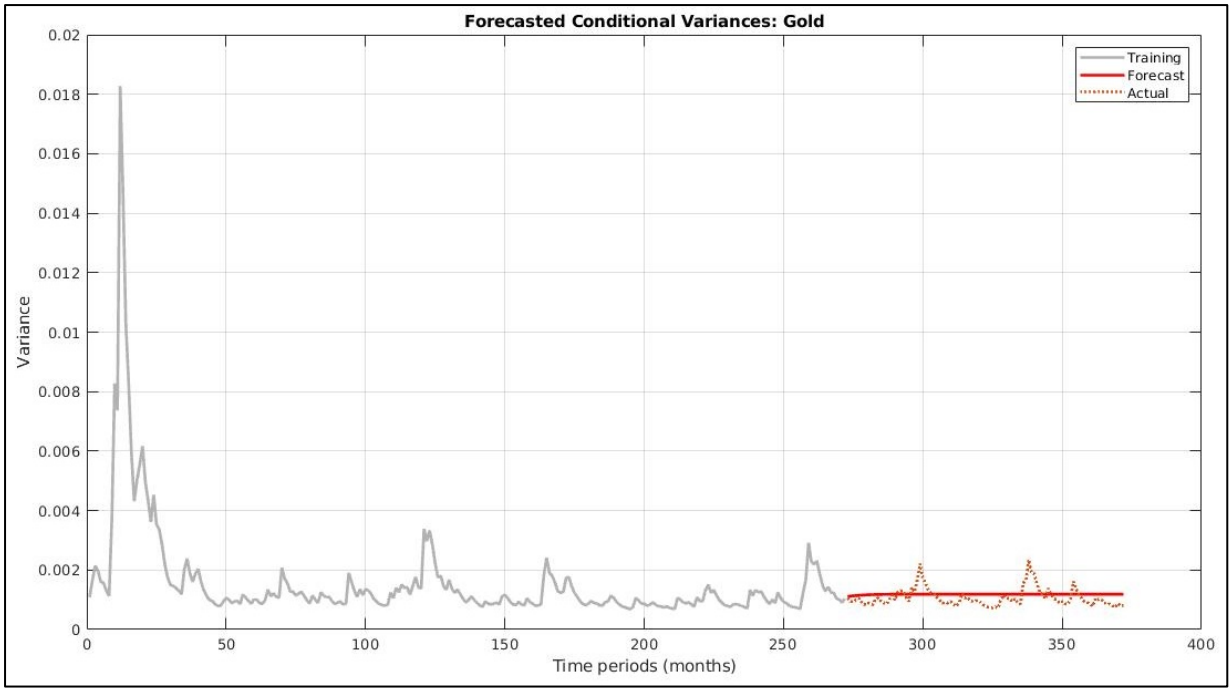

Figure 5.2: GJR (1, 1): gold forecast conditional variance

5.4. Summary for Forecasting Using GJR Model

The GJR (1, 1) model was not able to accurately predict the volatility changes in the copper price volatility over the period under study. The GJR (1, 1) model was not able to accurately predict the volatility changes in the gold prices. Nonetheless, over the period of forecasting which is 7.5 years, the model managed to capture the average volatility. The next chapter presents the comparison of ARIMA (p, d, q) model and GJR (1, 1) over the selected period of analysis and recommends the better model.

6. COMPARISON OF ARIMA AND GARCH MODELS

This chapter compares the performance of ARIMA models and GJR models using the measures of performance. The best measure of performance used are MAE, RMSE and MAPE. MAPE is the best performance measure since it varies between 0 and 1, thus it is not influenced by the scale of the time series. Whereas MAE and RMSE are quantities, thus are influenced by the magnitude of the values. For example, RMSE of 200 for a series with a mean of 1000 is better than RMSE of 5 for a series with a mean of 10 (Prabhakaran, 2019). Consequently, RMSE and MAE were not used to compare ARIMA models against GJR models since they are differently scaled time series. MAPE makes it simple to judge how good the forecast results are. The comparison was conducted over one-year, three-year, five-year and seven-year periods in Section 6.1.

6.1. Analysis of the Performance of the ARIMA and GJR Models

ARIMA (9, 1, 9) and ARIMA (7, 1, 7) models for copper and gold were compared with GJR (1, 1) models for copper and gold respectively for the different time periods. Table 6.1 shows the performance of these models over a one-year period. Using MAPE as the measure, it can be seen that ARIMA (9, 1, 9) and ARIMA (7, 1, 7) performed better than GJR (1, 1) models because they both have about 3.4% against 11% and 32% for GJR (1, 1) models for copper and gold, respectively. Thus, ARIMA (9, 1, 9) and ARIMA (7, 1, 7) models are suitable to forecast copper and gold prices over a one-year period.

Table 6.1: Comparison of ARIMA and GJR models for a one-year period

One-year forecast				
	Copper		**Gold**	
Criterion	**ARIMA (9, 1, 9)**	**GJR (1, 1)**	**ARIMA (7, 1, 7)**	**GJR (1, 1)**
MAE	113.709	0.076	9.292	0.023
RMSE	132.002	0.094	10.396	0.032
MAPE (%)	**3.413**	11.020	**3.437**	32.315

Table 6.2 shows the performance of the models over a three-year period. Using MAPE as the measure, it can be seen that ARIMA (9, 1, 9) and ARIMA (7, 1, 7) performed

better than GJR (1, 1) models because they have about 0.7% and 10% against 12% and 16% for GJR (1, 1) models for copper and gold, respectively. Thus, ARIMA (9, 1, 9) and ARIMA (7, 1, 7) models are suitable to forecast copper and gold prices over a three-year period.

Table 6.2: Comparison of ARIMA and GJR models for a three-year period

Three-year forecast				
	Copper		**Gold**	
Criterion	**ARIMA (9, 1, 9)**	**GJR (1, 1)**	**ARIMA (7, 1, 7)**	**GJR (1, 1)**
MAE	109.268	0.063	41.212	0.026
RMSE	130.225	0.077	55.083	0.034
MAPE (%)	**0.688**	12.083	**10.394**	15.967

Table 6.3 shows the performance of models over a three-year period. Using MAPE as the measure, it can be seen that GJR (1, 1) models for copper and gold performed better because they both have about 0.0% and 20% compared to approximately 18% and 20% for ARIMA (9, 1, 9) and ARIMA (7, 1, 7) models for copper and gold, respectively. Thus, GJR (1, 1) models are suitable to forecast copper and gold prices over a five-year period.

Table 6.3: Comparison of ARIMA and GJR models for a five-year period

Five-year forecast				
	Copper		**Gold**	
Criterion	**ARIMA (9, 1, 9)**	**GJR (1, 1)**	**ARIMA (7, 1, 7)**	**GJR (1, 1)**
MAE	614.923	0.050	95.426	0.022
RMSE	946.682	0.064	120.865	0.030
MAPE (%)	18.043	**Infinity**	23.437	**19.780**

Table 6.4 shows the performance of models over a seven-year period. Using MAPE as the measure, it can be seen that GJR (1, 1) models for copper and gold performed better because they both have about 0.0% and 24% compared to approximately 35% and 35% for ARIMA (9, 1, 9) and ARIMA (7, 1, 7) models for copper and gold, respectively. Thus, GJR (1, 1) models are suitable to forecast copper and gold prices over a seven-year period.

Table 6.4: Comparison of ARIMA and GJR models for a seven-year period

Seven-year forecast				
	Copper		Gold	
Criterion	**ARIMA (9, 1, 9)**	**GJR (1, 1)**	**ARIMA (7, 1, 7)**	**GJR (1, 1)**
MAE	1860.075	0.049	178.369	0.022
RMSE	2887.980	0.063	234.017	0.030
MAPE (%)	35.172	**Infinity**	35.123	**23.847**

6.2. Chapter Summary

This chapter compared the performance of ARIMA and GJR models in forecasting copper and gold movements based on prices and volatility of the prices over one-year, three-year, five-year and seven-year periods using MAPE as a measure of accuracy. ARIMA models yielded better results over one-year and three-year periods. Whereas GJR models showed superior results over five-year and seven-year periods. Both ARIMA and GJR models considerably failed to forecast the commodity prices. However, for the basis of comparison, ARIMA produces better results compared to the GJR model for periods under three years, while GJR model outperformed ARIMA models over the period of three years up to 7 years. This conclusion does not undermine the fact that there is evident divergence between actual and forecasted results. The next chapter presents the conclusion of the findings of this study.

7. CONCLUSIONS AND RECOMMENDATIONS

7.1. Introduction

This chapter summarises the findings, draw conclusions, and provides recommendations for future research. Sections 7.2 and 7.3 presents the summary of the findings and recommendations with reference to the research question, respectively. Limitations of the research study and proposals for future work are discussed.

7.2. Findings

Mining project valuation is conducted to determine the economic feasibility of the project based on a few assumptions such as commodity price. Mining companies are price takers thus it is vital to be able to apply an appropriate commodity price during valuation. However, it is difficult to predict commodity prices due to numerous uncertain factors that influence the price such as technology, supply, demand and macro-economics.

There are price prediction models that can be used to predict prices with a certain degree of confidence. However, these models are based on different assumptions and yield different results. This research study selected the most commonly used models to predict copper and gold prices. The selected models were ARIMA (9, 1, 9) and ARIMA (7, 1, 7) for copper and gold price prediction, while GJR (1, 1) model was used for both copper and gold volatility forecasting. The measurement of forecast accuracy used was MAPE since it varies between 0 and 1, thus it is not influenced by the scale of the time series.

All the commodity price models failed to accurately forecast the commodity prices for the period under study. The results showed that ARIMA (9, 1, 9) and ARIMA (7, 1, 7) models performed better than GJR (1, 1) when forecasting copper and gold prices over a short-term, that is, periods less than three years. The GJR (1, 1) model yielded superior results compared to the ARIMA models when forecasting copper and gold conditional variances for periods over five years.

7.3. Recommendations

Based on the findings of this study it is recommended that ARIMA (9, 1, 9) and ARIMA (7, 1, 7) models be used to forecast copper and gold prices over one-year and three-year periods. When forecasting price movements over three years, then GJR (1, 1) is recommended to forecast price volatility up to a seven-year period.

7.4. Limitations of the Research

There are various price prediction models that can be used to forecast commodity prices but this research study only focused on ARIMA and GRACH variants based models. The results of this study may change depending on the model(s) used, however, this study used commonly used models with different qualities. The period under study covers only 30 years, however, commodity prices may well be modelled over a century period depending on the availability of data and impact of market events. Large historical data set will results in adequate training data for the model which may improve the prediction capabilities of the models. This research study used few measures of performance, other researchers may employ other methods like machine learning and artificial neural methods for commodity prediction methods.

7.5. Recommendations for Future Work

Based on the findings of this study it is recommended that:

- Artificial neural network and machine learning techniques because they are capable of pattern recognition;
- Models used in this study be extended to other commodities like platinum, iron ore, etc.; and
- Measures of performance need to be expanded and evaluated for their accuracy.

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APPENDIX A: INPUT DATA FOR COPPER PRICES IN US S/t

Year	January	February	March	April	May	June	July	August	September	October	November	December
1988	2 660.98	2 328.08	2 358.94	2 283.99	2 442.72	2 539.72	2 213.44	2 200.21	2 433.90	2 936.55	3 302.52	3 496.53
1989	3394.06	3096.92	3264.14	3117.91	2738.98	2546.01	2505.00	2762.20	2884.40	2861.40	2591.30	2418.60
1990	2366.64	2360.16	2625.61	2686.15	2741.95	2584.56	2769.49	2957.03	3031.37	2743.56	2585.89	2485.38
1991	2448.52	2449.38	2409.63	2472.04	2305.88	2219.28	2236.28	2233.19	2324.57	2363.64	2379.84	2223.14
1992	2139.23	2205.97	2227.33	2215.33	2216.52	2299.20	2520.32	2521.64	2414.06	2249.15	2158.32	2206.82
1993	2256.85	2212.60	2152.59	1949.88	1794.50	1853.42	1927.16	1947.45	1861.86	1646.40	1630.02	1724.19
1994	1805.35	1866.40	1914.87	1881.82	2150.60	2364.20	2458.19	2406.23	2505.93	2547.67	2802.45	2985.30
1995	3008.93	2877.65	2924.04	2903.50	2773.31	2994.64	3075.67	3036.84	2915.52	2813.55	2977.36	2926.26
1996	2616.41	2537.71	2561.02	2595.78	2658.26	2173.40	1985.57	2008.57	1941.45	1961.17	2230.86	2268.08
1997	2434.93	2405.85	2421.29	2391.18	2514.33	2612.62	2450.46	2251.20	2107.30	2052.26	1917.45	1762.33
1998	1688.45	1664.80	1747.98	1800.90	1732.53	1660.52	1651.04	1620.93	1647.64	1586.39	1573.95	1473.57
1999	1431.18	1410.78	1378.35	1466.00	1511.16	1422.48	1640.00	1647.62	1750.34	1724.12	1727.55	1764.75
2000	1843.98	1800.83	1739.39	1678.75	1785.62	1753.18	1799.36	1855.86	1960.41	1898.59	1795.11	1850.55
2001	1787.50	1765.65	1738.77	1664.16	1682.21	1608.45	1525.21	1464.43	1426.33	1377.28	1427.73	1471.74
2002	1503.96	1561.90	1604.88	1590.33	1595.68	1647.53	1589.46	1479.55	1478.71	1483.76	1582.29	1595.68
2003	1647.66	1683.80	1658.98	1587.48	1648.28	1686.50	1710.00	1760.28	1789.52	1920.54	2055.43	2201.29
2004	2423.57	2759.53	3008.72	2948.73	2733.50	2686.71	2808.43	2846.10	2894.86	3012.24	3122.80	3145.45
2005	3170.00	3253.70	3379.49	3394.48	3249.10	3524.07	3614.21	3797.75	3857.84	4059.76	4269.34	4576.78
2006	4734.33	4982.40	5102.85	6387.78	8045.86	7197.61	7712.10	7695.66	7602.36	7500.39	7029.18	6675.11
2007	5669.66	5676.45	6452.48	7766.47	7681.42	7474.38	7972.57	7513.50	7648.98	8008.44	6966.71	6587.67
2008	7061.02	7887.69	8439.29	8684.93	8382.75	8260.60	8414.04	7634.70	6990.86	4925.70	3717.00	3071.98
2009	3220.69	3314.73	3749.75	4406.55	4568.63	5013.96	5215.54	6165.30	6196.43	6287.98	6675.60	6981.71
2010	7386.25	6848.18	7462.83	7745.08	6837.68	6499.30	6735.25	7283.95	7709.30	8292.41	8469.89	9147.26
2011	9555.70	9867.60	9503.36	9492.79	8959.90	9066.85	9650.46	9000.76	8300.14	7394.19	7581.02	7565.48
2012	8040.47	8441.49	8470.78	8289.48	7955.64	7423.02	7584.26	7515.53	8087.74	8062.03	7711.23	7966.49
2013	8047.36	8060.93	7645.58	7234.28	7249.41	7000.24	6906.64	7192.92	7159.27	7203.02	7070.65	7214.90
2014	7291.47	7149.21	6650.04	6673.56	6891.13	6821.14	7113.38	7001.84	6872.22	6737.48	6712.85	6446.45
2015	5830.54	5729.27	5939.67	6042.09	6294.78	5833.01	5456.75	5127.30	5217.25	5216.09	4799.90	4638.83
2016	4471.79	4598.62	4953.80	4872.74	4694.54	4641.97	4864.90	4751.67	4722.20	4731.26	5450.93	5660.35
2017	5754.56	5940.91	5824.63	5683.90	5599.56	5719.76	5985.12	6485.63	6577.17	6807.60	6826.55	6833.89
2018	7065.85	7006.52	6799.18	6851.51	6825.27	6965.86	6250.75	6051.05	6050.76	6219.59	6195.92	6075.32

APPENDIX B: INPUT DATA FOR GOLD PRICES IN US $/Oz

Year	January	February	March	April	May	June	July	August	September	October	November	December
1988	476.58	442.07	443.61	451.55	451.07	451.33	437.63	431.28	413.45	406.78	420.17	419.05
1989	404.01	387.51	390.15	384.40	371.05	367.60	375.04	365.14	361.75	366.88	394.26	409.39
1990	410.11	416.81	393.06	374.24	369.05	352.33	362.53	394.73	389.32	380.74	381.73	376.95
1991	383.64	363.83	363.34	358.38	356.95	366.72	367.69	356.31	348.74	358.69	360.17	361.73
1992	354.45	353.91	344.34	338.62	337.24	340.81	352.72	343.06	345.43	344.38	335.02	334.82
1993	329.01	329.31	330.08	342.15	367.18	371.89	392.19	378.84	355.28	364.18	373.83	383.30
1994	386.88	381.91	384.13	377.27	381.43	385.64	385.49	380.36	391.58	389.77	384.39	379.29
1995	378.55	376.64	382.12	391.03	385.22	387.56	386.23	383.67	383.06	383.14	385.31	387.44
1996	399.45	404.76	396.21	392.85	391.93	385.27	383.47	387.35	383.14	381.07	377.85	369.00
1997	355.11	346.58	351.81	344.47	343.84	340.76	324.10	324.01	322.82	324.87	306.04	288.74
1998	289.10	297.49	295.94	308.29	299.10	292.32	292.87	284.11	288.98	295.93	294.12	291.68
1999	287.08	287.33	285.96	282.62	276.44	261.31	256.08	256.69	264.74	310.72	293.18	283.07
2000	284.32	299.86	286.39	279.69	275.19	285.73	281.59	274.47	273.68	270.00	266.01	271.45
2001	265.49	261.87	263.03	260.48	272.36	270.23	267.53	272.39	283.42	283.06	276.16	275.85
2002	281.51	295.50	294.06	302.68	314.49	321.18	313.29	310.26	319.14	316.56	319.07	331.92
2003	356.86	358.97	340.55	328.18	355.68	356.35	351.02	359.77	378.95	378.92	389.91	406.95
2004	413.79	404.88	406.67	403.26	383.78	392.37	398.09	400.51	405.28	420.46	439.38	442.08
2005	424.03	423.35	433.85	429.23	421.87	430.66	424.48	437.93	456.05	469.90	476.67	510.10
2006	549.86	555.00	557.09	610.65	675.39	596.15	633.71	632.59	598.19	585.78	627.83	629.79
2007	631.17	664.75	654.90	679.37	667.31	655.66	665.38	665.41	712.65	754.60	806.25	803.20
2008	889.60	922.30	968.43	909.71	888.66	889.49	939.77	839.03	829.93	806.62	760.86	816.09
2009	858.69	943.00	924.27	890.20	928.65	945.67	934.23	949.38	996.59	1043.16	1127.04	1134.72
2010	1117.96	1095.41	1113.34	1148.69	1205.43	1232.92	1192.97	1215.81	1270.98	1342.02	1369.89	1390.55
2011	1360.46	1374.68	1423.26	1480.89	1512.58	1529.36	1572.75	1759.01	1772.14	1666.43	1739.00	1639.97
2012	1654.05	1744.82	1675.95	1649.20	1589.04	1598.76	1594.29	1630.31	1744.81	1746.58	1721.64	1684.76
2013	1671.85	1627.57	1593.09	1487.86	1414.03	1343.35	1285.52	1351.74	1348.60	1316.58	1275.86	1221.51
2014	1244.27	1299.58	1336.08	1298.45	1288.74	1279.10	1310.59	1295.13	1236.55	1222.49	1175.33	1200.62
2015	1250.75	1227.08	1178.63	1198.93	1198.63	1181.50	1128.31	1117.93	1124.77	1159.25	1086.44	1075.74
2016	1097.91	1199.50	1245.14	1242.26	1260.95	1276.40	1336.66	1340.17	1326.61	1266.55	1238.35	1157.36
2017	1192.10	1234.20	1231.42	1266.88	1246.04	1260.26	1236.84	1283.04	1314.07	1279.51	1281.90	1264.45
2018	1331.30	1330.73	1324.66	1334.76	1303.45	1281.57	1237.71	1201.71	1198.39	1215.39	1220.65	1250.40

APPENDIX C: ARIMA MODEL DIAGNOSTIC FOR COPPER USING AIC

```
Test MSE: 114.032Test MSE: 209.860Test MSE: 108.071                              ARIMA Model
==============================================================================
Dep. Variable:               D2.Price   No. Observations:                  148
Model:                 ARIMA(5, 2, 5)   Log Likelihood                -934.165
Method:                       css-mle   S.D. of innovations            127.867
Date:                Fri, 03 May 2019   AIC                           1892.329
Time:                        10:40:32   BIC                           1928.296
Sample:                  03-01-1988   HQIC                          1906.943
                       - 06-01-2000
==================================================================================
                     coef    std err          z      P>|z|      [0.025      0.975]
----------------------------------------------------------------------------------
const             -0.0178      0.352     -0.051      0.960      -0.707       0.671
ar.L1.D2.Price    -0.6888      0.088     -7.827      0.000      -0.861      -0.516
ar.L2.D2.Price    -0.4876      0.093     -5.253      0.000      -0.669      -0.306
ar.L3.D2.Price    -0.7906      0.080     -9.896      0.000      -0.947      -0.634
ar.L4.D2.Price    -0.6798      0.090     -7.524      0.000      -0.857      -0.503
ar.L5.D2.Price     0.2274      0.093      2.454      0.015       0.046       0.409
ma.L1.D2.Price     0.0631      0.047      1.332      0.185      -0.030       0.156
ma.L2.D2.Price    -0.2231      0.038     -5.889      0.000      -0.297      -0.149
ma.L3.D2.Price     0.2231      0.024      9.454      0.000       0.177       0.269
ma.L4.D2.Price    -0.0631      0.045     -1.397      0.165      -0.152       0.025
ma.L5.D2.Price    -1.0000      0.030    -33.178      0.000      -1.059      -0.941
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1           -0.8923           -0.5286j            1.0371           -0.4149
AR.2           -0.8923           +0.5286j            1.0371            0.4149
AR.3            0.3655           -0.9368j            1.0056           -0.1908
AR.4            0.3655           +0.9368j            1.0056            0.1908
AR.5            4.0427           -0.0000j            4.0427           -0.0000
MA.1            1.0000           -0.0000j            1.0000           -0.0000
MA.2            0.3348           -0.9423j            1.0000           -0.1957
MA.3            0.3348           +0.9423j            1.0000            0.1957
MA.4           -0.8663           -0.4995j            1.0000           -0.4168
MA.5           -0.8663           +0.4995j            1.0000            0.4168
-----------------------------------------------------------------------------
count  148.000000
mean     5.269087
std    137.255651
min   -505.134361
25%    -75.879493
50%      6.503951
75%     81.485438
max    395.503715Test MSE: 433135.647                        ARIMA Model Results
```

```
==============================================================================
Dep. Variable:                D2.Price   No. Observations:                  148
Model:                 ARIMA(9, 2, 5)   Log Likelihood                -931.819
Method:                       css-mle   S.D. of innovations            127.025
Date:                Fri, 03 May 2019   AIC                           1895.637
Time:                        10:40:42   BIC                           1943.593
Sample:                    03-01-1988   HQIC                          1915.121
                         - 06-01-2000
==============================================================================
                    coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const             -0.0411      0.340     -0.121      0.904      -0.707       0.624
ar.L1.D2.Price     0.2357      0.098      2.407      0.017       0.044       0.428
ar.L2.D2.Price     0.7102      0.119      5.977      0.000       0.477       0.943
ar.L3.D2.Price    -0.5872      0.118     -4.992      0.000      -0.818      -0.357
ar.L4.D2.Price    -0.8197      0.115     -7.131      0.000      -1.045      -0.594
ar.L5.D2.Price     0.5481      0.130      4.218      0.000       0.293       0.803
ar.L6.D2.Price    -0.0588      0.129     -0.457      0.648      -0.311       0.193
ar.L7.D2.Price    -0.2268      0.111     -2.040      0.043      -0.445      -0.009
ar.L8.D2.Price    -0.0722      0.097     -0.742      0.459      -0.263       0.118
ar.L9.D2.Price     0.1967      0.097      2.029      0.044       0.007       0.387
ma.L1.D2.Price    -0.8337      0.055    -15.177      0.000      -0.941      -0.726
ma.L2.D2.Price    -0.9801      0.094    -10.376      0.000      -1.165      -0.795
ma.L3.D2.Price     0.9801      0.052     18.743      0.000       0.878       1.083
ma.L4.D2.Price     0.8337      0.088      9.483      0.000       0.661       1.006
ma.L5.D2.Price    -1.0000        nan        nan        nan         nan         nan
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1           -0.9130           -0.4573j            1.0212           -0.4261
AR.2           -0.9130           +0.4573j            1.0212            0.4261
AR.3           -1.0350           -0.9890j            1.4315           -0.3786
AR.4           -1.0350           +0.9890j            1.4315            0.3786
AR.5            0.8111           -0.6013j            1.0097           -0.1015
AR.6            0.8111           +0.6013j            1.0097            0.1015
AR.7            0.5752           -1.1113j            1.2513           -0.1740
AR.8            0.5752           +1.1113j            1.2513            0.1740
AR.9            1.4903           -0.0000j            1.4903           -0.0000
MA.1           -0.8813           -0.4725j            1.0000           -0.4217
MA.2           -0.8813           +0.4725j            1.0000            0.4217
MA.3            0.7982           -0.6024j            1.0000           -0.1029
MA.4            0.7982           +0.6024j            1.0000            0.1029
MA.5            1.0000           -0.0000j            1.0000           -0.0000
-----------------------------------------------------------------------------
count  148.000000
mean     5.570463
std    134.439618
min   -450.422105
25%    -68.877029
50%      5.601292
75%     71.845478
max    373.558346Test MSE: 486271.802                          ARIMA Model Results
```

```
==============================================================================
Dep. Variable:                D.Price   No. Observations:                  149
Model:                ARIMA(10, 1, 9)   Log Likelihood                -929.504
Method:                       css-mle   S.D. of innovations            118.308
Date:                Fri, 03 May 2019   AIC                           1901.008
Time:                        10:41:44   BIC                           1964.091
Sample:                    02-01-1988   HQIC                          1926.637
                         - 06-01-2000
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          -7.4921      2.416     -3.100      0.002     -12.228      -2.756
ar.L1.D.Price  -0.0770        nan        nan        nan         nan         nan
ar.L2.D.Price  -0.2219        nan        nan        nan         nan         nan
ar.L3.D.Price   0.2666        nan        nan        nan         nan         nan
ar.L4.D.Price   0.1180        nan        nan        nan         nan         nan
ar.L5.D.Price  -0.7042        nan        nan        nan         nan         nan
ar.L6.D.Price  -0.1532        nan        nan        nan         nan         nan
ar.L7.D.Price   0.3212        nan        nan        nan         nan         nan
ar.L8.D.Price   0.1282        nan        nan        nan         nan         nan
ar.L9.D.Price   0.5527        nan        nan        nan         nan         nan
ar.L10.D.Price -0.1679        nan        nan        nan         nan         nan
ma.L1.D.Price   0.4331        nan        nan        nan         nan         nan
ma.L2.D.Price   0.3348        nan        nan        nan         nan         nan
ma.L3.D.Price  -0.3786        nan        nan        nan         nan         nan
ma.L4.D.Price  -0.5503      0.034    -15.973      0.000      -0.618      -0.483
ma.L5.D.Price   0.5507      0.010     55.854      0.000       0.531       0.570
ma.L6.D.Price   0.3785        nan        nan        nan         nan         nan
ma.L7.D.Price  -0.3354        nan        nan        nan         nan         nan
ma.L8.D.Price  -0.4331        nan        nan        nan         nan         nan
ma.L9.D.Price  -0.9996      0.051    -19.657      0.000      -1.099      -0.900
                                    Roots
==============================================================================
                  Real          Imaginary           Modulus         Frequency
------------------------------------------------------------------------------
AR.1           -0.9961           -0.3286j            1.0489           -0.4493
AR.2           -0.9961           +0.3286j            1.0489            0.4493
AR.3           -0.3457           -0.9384j            1.0001           -0.3062
AR.4           -0.3457           +0.9384j            1.0001            0.3062
AR.5           -0.1484           -1.1301j            1.1398           -0.2708
AR.6           -0.1484           +1.1301j            1.1398            0.2708
AR.7            0.7558           -0.6561j            1.0009           -0.1138
AR.8            0.7558           +0.6561j            1.0009            0.1138
AR.9            1.1528           -0.0000j            1.1528           -0.0000
AR.10           3.6078           -0.0000j            3.6078           -0.0000
MA.1            1.0000           -0.0000j            1.0000           -0.0000
MA.2            0.7567           -0.6538j            1.0000           -0.1134
MA.3            0.7567           +0.6538j            1.0000            0.1134
MA.4           -0.9518           -0.3066j            1.0000           -0.4504
MA.5           -0.9518           +0.3066j            1.0000            0.4504
MA.6           -0.3470           -0.9379j            1.0000           -0.3064
MA.7           -0.3470           +0.9379j            1.0000            0.3064
MA.8           -0.1745           -0.9848j            1.0002           -0.2779
MA.9           -0.1745           +0.9848j            1.0002            0.2779
------------------------------------------------------------------------------
```

APPENDIX D: ARIMA MODEL DIAGNOSTIC FOR GOLD USING AIC

ARIMA Model Results

Dep. Variable:	D.Price	No. Observations:	149
Model:	ARIMA(11, 1, 6)	Log Likelihood	-552.081
Method:	css-mle	S.D. of innovations	9.604
Date:	Fri, 03 May 2019	AIC	1142.161
Time:	10:40:00	BIC	1199.236
Sample:	02-01-1988 - 06-01-2000	HQIC	1165.350

	coef	std err	z	P>|z|	[0.025	0.975]
const	-1.1351	0.383	-2.966	0.004	-1.885	-0.385
ar.L1.D.Price	-0.7826	0.502	-1.560	0.121	-1.766	0.201
ar.L2.D.Price	-0.4574	0.211	-2.166	0.032	-0.871	-0.044
ar.L3.D.Price	-0.0941	0.184	-0.511	0.610	-0.455	0.267
ar.L4.D.Price	0.2726	0.119	2.295	0.023	0.040	0.505
ar.L5.D.Price	0.9046	0.173	5.219	0.000	0.565	1.244
ar.L6.D.Price	0.2301	0.423	0.544	0.587	-0.598	1.059
ar.L7.D.Price	0.1562	0.117	1.335	0.184	-0.073	0.386
ar.L8.D.Price	0.2323	0.152	1.533	0.128	-0.065	0.529
ar.L9.D.Price	0.1722	0.140	1.227	0.222	-0.103	0.447
ar.L10.D.Price	0.0502	0.117	0.429	0.669	-0.179	0.280
ar.L11.D.Price	0.0773	0.097	0.794	0.428	-0.113	0.268
ma.L1.D.Price	0.9256	0.498	1.857	0.066	-0.051	1.902
ma.L2.D.Price	0.3859	0.259	1.492	0.138	-0.121	0.893
ma.L3.D.Price	-0.0995	0.102	-0.971	0.333	-0.300	0.101
ma.L4.D.Price	-0.5738	0.104	-5.529	0.000	-0.777	-0.370
ma.L5.D.Price	-1.2128	0.251	-4.828	0.000	-1.705	-0.720
ma.L6.D.Price	-0.4254	0.496	-0.857	0.393	-1.398	0.547

Roots

	Real	Imaginary	Modulus	Frequency
AR.1	1.0205	-0.0000j	1.0205	-0.0000
AR.2	-0.8699	-0.5841j	1.0478	-0.4059
AR.3	-0.8699	+0.5841j	1.0478	0.4059
AR.4	-1.1602	-0.6288j	1.3196	-0.4210
AR.5	-1.1602	+0.6288j	1.3196	0.4210
AR.6	0.0791	-1.0159j	1.0189	-0.2376
AR.7	0.0791	+1.0159j	1.0189	0.2376
AR.8	1.0534	-1.0423j	1.4819	-0.1242
AR.9	1.0534	+1.0423j	1.4819	0.1242
AR.10	0.0623	-1.7045j	1.7056	-0.2442
AR.11	0.0623	+1.7045j	1.7056	0.2442
MA.1	1.0000	-0.0000j	1.0000	-0.0000
MA.2	0.0965	-0.9953j	1.0000	-0.2346
MA.3	0.0965	+0.9953j	1.0000	0.2346
MA.4	-0.8466	-0.5323j	1.0000	-0.4107
MA.5	-0.8466	+0.5323j	1.0000	0.4107
MA.6	-2.3505	-0.0000j	2.3505	-0.5000

count	149.000000
mean	-0.292102
std	9.908102
min	-33.364929
25%	-4.719153
50%	-0.187400
75%	4.959553
max	42.832735

ARIMA Model Results

```
==============================================================================
Dep. Variable:                 D.Price   No. Observations:                  149
Model:                 ARIMA(8, 1, 6)   Log Likelihood                -548.913
Method:                       css-mle   S.D. of innovations              9.281
Date:                Fri, 03 May 2019   AIC                           1129.825
Time:                        10:40:19   BIC                           1177.889
Sample:                    02-01-1988   HQIC                          1149.353
                         - 06-01-2000
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          -1.3217      0.902     -1.466      0.145      -3.089       0.446
ar.L1.D.Price   0.6465      0.264      2.446      0.016       0.129       1.165
ar.L2.D.Price  -0.3772      0.320     -1.181      0.240      -1.003       0.249
ar.L3.D.Price   0.7767      0.176      4.410      0.000       0.432       1.122
ar.L4.D.Price  -0.2861      0.175     -1.632      0.105      -0.630       0.057
ar.L5.D.Price   0.6050      0.183      3.298      0.001       0.246       0.965
ar.L6.D.Price  -1.0288      0.196     -5.253      0.000      -1.413      -0.645
ar.L7.D.Price   0.3688      0.309      1.194      0.235      -0.237       0.974
ar.L8.D.Price  -0.1637      0.249     -0.656      0.513      -0.653       0.325
ma.L1.D.Price  -0.4620        nan        nan        nan         nan         nan
ma.L2.D.Price   0.0778      0.072      1.081      0.282      -0.063       0.219
ma.L3.D.Price  -0.6846      0.082     -8.380      0.000      -0.845      -0.525
ma.L4.D.Price   0.0778      0.067      1.159      0.249      -0.054       0.209
ma.L5.D.Price  -0.4620      0.036    -12.688      0.000      -0.533      -0.391
ma.L6.D.Price   1.0000        nan        nan        nan         nan         nan
                                    Roots
==============================================================================
                  Real          Imaginary           Modulus         Frequency
------------------------------------------------------------------------------
AR.1           -0.7671           -0.6417j            1.0001           -0.3891
AR.2           -0.7671           +0.6417j            1.0001            0.3891
AR.3            0.0183           -1.0156j            1.0158           -0.2471
AR.4            0.0183           +1.0156j            1.0158            0.2471
AR.5            1.0132           -0.2938j            1.0549           -0.0449
AR.6            1.0132           +0.2938j            1.0549            0.0449
AR.7            0.8620           -2.1391j            2.3062           -0.1890
AR.8            0.8620           +2.1391j            2.3062            0.1890
MA.1           -0.7692           -0.6390j            1.0000           -0.3897
MA.2           -0.7692           +0.6390j            1.0000            0.3897
MA.3            0.0405           -0.9992j            1.0000           -0.2435
MA.4            0.0405           +0.9992j            1.0000            0.2435
MA.5            0.9597           -0.2810j            1.0000           -0.0453
MA.6            0.9597           +0.2810j            1.0000            0.0453
------------------------------------------------------------------------------
count  149.000000
mean     0.039452
std      9.717816
min    -33.178349
25%     -5.929367
50%     -0.214912
75%      4.754471
max     41.578426                          ARIMA Model Results
```

```
==============================================================================
Dep. Variable:                 D.Price   No. Observations:                  149
Model:                 ARIMA(7, 1, 7)   Log Likelihood                -552.161
Method:                       css-mle   S.D. of innovations              9.466
Date:                Fri, 03 May 2019   AIC                           1136.323
Time:                        10:40:27   BIC                           1184.386
Sample:                  02-01-1988   HQIC                          1155.850
                       - 06-01-2000
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          -1.0572      0.330     -3.200      0.002      -1.705      -0.410
ar.L1.D.Price   0.6856      0.075      9.127      0.000       0.538       0.833
ar.L2.D.Price  -0.6219      0.052    -11.847      0.000      -0.725      -0.519
ar.L3.D.Price   0.5320      0.059      9.061      0.000       0.417       0.647
ar.L4.D.Price  -0.5797      0.066     -8.813      0.000      -0.709      -0.451
ar.L5.D.Price   0.5852      0.066      8.881      0.000       0.456       0.714
ar.L6.D.Price  -0.6951      0.068    -10.154      0.000      -0.829      -0.561
ar.L7.D.Price   0.8969      0.060     14.843      0.000       0.778       1.015
ma.L1.D.Price  -0.6185      0.064     -9.607      0.000      -0.745      -0.492
ma.L2.D.Price   0.5321      0.072      7.372      0.000       0.391       0.674
ma.L3.D.Price  -0.4941      0.076     -6.518      0.000      -0.643      -0.346
ma.L4.D.Price   0.4941      0.075      6.577      0.000       0.347       0.641
ma.L5.D.Price  -0.5321      0.071     -7.510      0.000      -0.671      -0.393
ma.L6.D.Price   0.6185      0.073      8.469      0.000       0.475       0.762
ma.L7.D.Price  -1.0000      0.060    -16.681      0.000      -1.117      -0.883
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1           -0.7749           -0.6443j            1.0078           -0.3896
AR.2           -0.7749           +0.6443j            1.0078            0.3896
AR.3           -0.0530           -1.0132j            1.0146           -0.2583
AR.4           -0.0530           +1.0132j            1.0146            0.2583
AR.5            0.6927           -0.7350j            1.0100           -0.1297
AR.6            0.6927           +0.7350j            1.0100            0.1297
AR.7            1.0455           -0.0000j            1.0455           -0.0000
MA.1           -0.7853           -0.6191j            1.0000           -0.3938
MA.2           -0.7853           +0.6191j            1.0000            0.3938
MA.3           -0.0809           -0.9967j            1.0000           -0.2629
MA.4           -0.0809           +0.9967j            1.0000            0.2629
MA.5            1.0000           -0.0000j            1.0000           -0.0000
MA.6            0.6755           -0.7373j            1.0000           -0.1320
MA.7            0.6755           +0.7373j            1.0000            0.1320
-----------------------------------------------------------------------------
```