Power Allocation and User Selection in Multi-cell Multi-user Massive MIMO systems

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POWER ALLOCATION AND USER SELECTION IN MULTI-CELL MULTI-USER MASSIVE MIMO SYSTEMS

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AUTHORIZATION

As the candidate's supervisor, I have approved this thesis for submission.

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DECLARATION

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ABSTRACT

The benefits of massive Multiple-Input Multiple-Output (MIMO) systems have made it a solution for future wireless networking demands. The increase in the number of base station antennas in massive MIMO systems results in an increase in capacity. The throughput increases linearly with an increase in number of antennas. To reap all the benefits of massive MIMO, resources should be allocated optimally amongst users. A lot of factors have to be taken into consideration in resource allocation in multi-cell massive MIMO systems (e.g. intra-cell, inter-cell interference, large scale fading etc.)

This dissertation investigates user selection and power allocation algorithms in multi-cell massive MIMO systems. The focus is on designing algorithms that maximizes a particular cell of interest's sum rate capacity taking into consideration the interference from other cells. To maximize the sum-rate capacity there is need to optimally allocate power and select the optimal number of users who should be scheduled. Global interference coordination has very high complexity and is infeasible in large networks. This dissertation extends previous work and proposes suboptimal per cell resource allocation models that are feasible in practice. The interference is introduced when non-orthogonal pilots are used for channel estimation, resulting in pilot contamination. Resource allocation values from interfering cells are unknown in per cell resource allocation models, hence the inter-cell interference has to be modelled. To tackle the problem sum-rate expressions are derived to enable power allocation and user selection algorithm analysis.

The dissertation proposes three different approaches for solving resource allocation problems in multi-cell multi-user massive MIMO systems for a particular cell of interest. The first approach proposes a branch and bound algorithm (BnB algorithm) which models the inter-cell interference in terms of the intra-cell interference by assuming that the statistical properties of the intra-cell interference in the cell of interest are the same as in the other interfering cells. The inter-cell interference is therefore expressed in terms of the intra-cell interference multiplied by a correction factor. The correction factor takes into consideration pilot sequences used in the interfering cells in relation to pilot sequences used in the cell of interest and large scale fading between the users in the interfering cells and the users in the cell of interest. The resource allocation problem is

modelled as a mixed integer programming problem. The problem is NP-hard and cannot be solved in polynomial time. To solve the problem it is converted into a convex optimization problem by relaxing the user selection constraint. Dual decomposition is used to solve the problem. In the second approach (two stage algorithm) a mathematical model is proposed for maximum user scheduling in each cell. The scheduled users are then optimally allocated power using the multi-level water filling approach. Finally a hybrid algorithm is proposed which combines the two approaches described above. Generally in the hybrid algorithm the cell of interest allocates resources in the interfering cells using the two stage algorithm to obtain near optimal resource allocation values. The cell of interest then uses these near optimal values to perform its own resource allocation using the BnB algorithm. The two stage algorithm is chosen for resource allocation in the interfering cells because it has a much lower complexity compared to the BnB algorithm. The BnB algorithm is chosen for resource allocation in the cell of interest because it gives higher sum rate in a sum rate maximization problem than the two stage algorithm. Performance analysis and evaluation of the developed algorithms have been presented mainly through extensive simulations. The designed algorithms have also been compared to existing solutions. In general the presented results demonstrate that the proposed algorithms perform better than the existing solutions.

Dedicated to my parents, Piniel Jealous Chiguvare and Evellin Chiguvare who relentlessly encouraged me to strive for excellence and who have been there for me through thick and thin.

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Finally, I would like to extend my heartfelt gratitude to my siblings, Addington, Pamela and Yolanda for their loyal support and continuous encouragement through the difficult times. I also wish to express my unreserved appreciation to Memory, Mufarowashe and Anotidaishe for whom I will always feel a special bond for the respect and the affection you have shown in these amazing years. You guys are great and I am blessed to have you in my life. I thank you!

CONTENTS

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NOMENCLATURE

Abbreviations and Acronyms

BS:	Base Station
UT:	User Terminal
MIMO:	Multiple-Input Multiple-Output
MRT:	Maximum Ratio Transmission
5G:	Fifth Generation of Cellular Wireless Standards
BD:	Block-Diagonalization
CSI:	Channel State Information
FDD:	Frequency Division Duplex
KKT:	Karush-Kuhn-Tucker
LMMSE:	Linear Minimum Mean Square Error
MMSE:	Minimum Mean Squared Error
MRC:	Maximum Ratio Combining
MSE:	Mean Squared Error
MU:	Multi user
NP-hard:	Non-Deterministic Polynomial-Time Hard
SINR:	Signal-to-Interference-and-Noise Ratio
SNR:	Signal-to-Noise Ratio
TDD:	Time Division Duplex
TDMA:	Time Division Multiple Access
ZF:	Zero-Forcing

ZFC:	Zero-Forcing with Receive Combining
BnB:	Branch and bound
UL:	Uplink
DL:	Downlink
MU-MIMO:	Multi user Multiple-Input Multiple-Output
MF:	Matched filter
EE:	Energy Efficiency
QoS :	Quality of service
EVD:	eigenvalue –decomposition
ILSP:	iterative least square with projection
CEO :	cross-entropy optimization framework
ZFBF:	zero forcing beamforming
DFT:	discrete Fourier transform

Mathematical Notation

Boldface upper case letters are used to denote matrices e.g. (**X,Y**) and bold face small letters are used to denote vectors e.g. (**x,y**)

$\mathbf{x}$	column vector
$\mathbf{X}$	matrices
$\mathbf{X}^T$	transpose
$\mathbf{X}^*$	conjugate
$\mathbf{X}^H$	conjugate transpose
$\mathbb{C}^{N\times M}$	The set of complex-valued N × M matrices.

$\lvert\cdot\rvert$	standard Euclidean norm
x_i	ith element of a vector **x**
x_{ij}	i,j th element of a matrix **X**
diag($x_i, \dots x_n, \dots x_N$)	diagonal matrix with $x_i, \dots x_n, \dots x_N$ at the diagonal
$log_a x$	logarithm of x using base a
O(·)	Big O Notation
tr{**X**}	The trace of a square matrix **X**.
vec(**X**)	The vector obtained by stacking the columns of **X**.
N(**x, R**)	multivariate Gaussian distribution with mean x and covariance matrix R.
CN(**x**, **R**)	The circular symmetric complex Gaussian
∀x	Means that a statement holds for all x
$x \in S$	*x* is a member of *S*
X ⊗ Y	The Kronecker product of two matrices X and Y
I_N	N × N identity matrix.
1_N	N × 1 vector of all ones.

Thesis Specific Notation

BnB technique	standard branch and bound technique
BnB algorithm	The proposed branch and bound algorithm in the thesis
$\mathrm{C}_{\mathrm{jk}}^{*}$	Optimal user capacity for user k in cell j
$\mathrm{p}_{\mathrm{jk}}^{*}$	Optimal user power allocated to user k in cell j
K^{*}	Optimal number of users selected in a cell

K_s	maximum number of users which can be selected in a cell
N	Number of base station antennas
K	Number of users in a cell
$\mathbf{h}_{jbm}$	channel vector from user m in cell b to a BS in cell j
β_{jbm}	large scale fading of user m in cell j to base station b
$\mathbf{v}_{jk}$	receive beamforming/precoding vector
x_{bm}	data sent by user m in cell b
$\hat{\mathbf{y}}_j$	data received signal after data transmission by BS j
$\mathbf{n}_j$	noise signal after UL pilot transmission in cell j during channel estimation
$\mathbf{Y}_j$	received signal after pilot transmission by BS j
q_{bm}	pilot sequence used by user m in cell b transmit power
p_{bm}	data power for user m in cell b
s_{bm}	pilot sequence for user m in cell b
$\mathbf{N}_j$	noise after data transmission in cell j
$\widehat{\mathbf{h}_{jjk}}$	estimated channel for user k in cell j to a BS in cell j
$\mathrm{SINR}_{jk}^{\mathrm{MRC}}$	Signal-to-interference-and-noise ratio after MRC precoding
z_{jk}	geometric position of user k in cell b
$d_b(z_{jk})$	variance of channel attenuation from BS b
I_{jk}	interference experienced by user k in cell j
$p_{jk,max}$	maximum power which can be allocated to a user k in cell j

CHAPTER 1

1 INTRODUCTION

This chapter gives a basic introduction to resource allocation in multi-cell multi-user massive Multiple-Input Multiple-Output (MIMO) systems. The chapter provides sufficient background that helps in understanding the research problem and how this research contributes to solving this research problem. A more detailed massive MIMO background is presented in Chapter 2.

The rest of the chapter is organized as follows: Section 1.1 gives a summary of the data transmission process in a Massive MIMO system. Section 1.2 discusses resource allocation in massive MIMO systems. Section 1.3 gives the problem statement as well as the research question. Section 1.4 discusses the contributions of the dissertation as well as the author publications.

1.1 Massive MIMO Systems

Fifth Generation of Cellular Wireless Standards (5G) technology is regarded as the future of mobile wireless communications due to their fast data transmission rates [1], [2], [3], [4]. A massive MIMO system (which is part of 5G technology) has an excess of base station (BS) antennas which are used to serve a large number of user terminals (UTs) [5]. Most of the work is transferred to the BS because the UTs are usually inexpensive single antenna devices [6] , [7]. The goal of a massive MIMO system is to improve the data rate, diversity gain and throughput for each user [8], [9]. A massive MIMO system has the advantage of reducing the transmit power whilst improving the data rate [10].

The massive MIMO gain is obtained through beamforming, which requires channel state information (CSI). To estimate the CSI, users send pilot signals to the BS which ideally should be orthogonal to reduce interference. However due to limited coherence time, it is not possible for orthogonal pilot signals to be used in all the cells. Pilot sequences are reused in neighboring cells leading to pilot contamination and subsequently inter-cell (pilots sequences reused in neighboring cells) and intra-cell (pilot sequences reused in the same cell) interference. Pilot contamination reduces the transmit power for each user by a factor inversely proportional to the number of BS antennas [11].

Global interference coordination becomes impractical in massive MIMO systems when the number of BSs and BS antennas increase. Sharing CSI amongst the BSs becomes almost impossible. The approach taken in multi cell massive MIMO literature is to assume constant power in the interfering cells. In this research we will adopt single cell processing and model the statistical properties of the inter cell interference in terms of the cell of interest’s intra cell interference.

Resource allocation problems in multi-cell multi-antenna systems are non-convex and non-deterministic polynomial-time hard (NP-hard), thus an optimal solution cannot be obtained in polynomial time. Heuristic methods can be applied when trying to optimize parameters such as performance or proportional fairness. These NP-hard problems can be solved using heuristic methods such as outer polyblock approximation and the branch and bound technique [12]. These algorithms have local convergence hence the choice of the starting point is of paramount importance.

This research focuses on designing resource allocation massive MIMO algorithms that maximizes a particular cell’s sum rate taking into consideration the interference from other cells. Due to the intra-cell and inter-cell interference the sum rate is significantly decreased. However resource allocation values from interfering cells are unknown hence the inter-cell interference has to be approximated and modelled.

These issues have been addressed to some extent in the literature. The primary problem is that an optimal solution would require optimization over all users and cells which becomes cumbersome. This research considers sub-optimal approaches that reduces multi cell optimization into single cell optimization

1.2 Resource Allocation Massive MIMO Systems

A multi cell massive MIMO system consists of a cellular network with a large number of cells and a large number of users in each cell. Each cell has at least one BS which serve the users which are closest to it. This research focuses on resource allocation per cell since it is cumbersome to globally optimize resources. However due to the effects of inter-cell interference the cells cannot exist as independent entities. The signal-to-interference-and-noise ratio (SINR) values of each of the users in each cell are influenced by the resource allocation of all the other users in all the cells. For example the SINR value of user k in cell j is affected by the resource allocation (e.g. power allocation, user selection etc.) of all the other users in all the cells except itself, this phenomenon is known as interference. The extreme case occurs when two users using the same pilot sequences are close to each other, their data signals will cause severe interference to each other.

The complexity of allocating resources in a large multi-cell system is very high. Due to the high computational complexity global optimal resource allocation cannot be applied in practical systems. To solve this multi-cell resource allocation problem a lot of authors have assumed constant interference from other cells. Some authors have also assumed an interference temperature, where the number of users chosen is limited by the interference temperature. Since it is nearly impossible to globally optimize resources in a multi cell massive MIMO system, we propose to reduce the multi cell structure into single cell system by assuming that the statistical properties of the interference in each of the other cells are the same as in the cell of interest. Hence the inter-cell interference is expressed in terms of the intra-cell interference multiplied by a correction factor, f_{jk}. The correction factor, f_{jk} takes into consideration pilot sequences used in the interfering cells in relation to pilot sequences used in the cell of interest and large scale fading between the users in the interfering cells and the users in the cell of interest.

This research focuses mainly on user selection and power allocation in a multi cell massive MIMO system for a particular cell of interest. The dissertation proposes three different approaches for solving resource allocation problem in multi-cell massive MIMO systems. In the first approach, branch and bound algorithm (BnB algorithm)[1], the inter-cell interference is modelled in terms of the intra-cell interference by assuming that the statistical properties of the intra-cell interference in the cell of interest are the same as in the other interfering cells. The resource allocation problem is

[1] In this dissertation BnB algorithm refers to the thesis proposed branch and bound algorithm algorithm. BnB technique refers to the standard branch and bound technique

modelled as a mixed integer programming problem. The problem is NP-hard and cannot be solved in polynomial time. To solve the problem it is converted into a convex optimization problem by relaxing the user selection constraint. Dual decomposition is used to solve the problem. In the second approach (two stage algorithm) a mathematical model is proposed for maximum user scheduling in each cell. The scheduled users are then optimally allocated power using the multi-level water filling approach. Finally a hybrid algorithm is proposed which combines the two approaches described above. Generally in the hybrid algorithm the cell of interest allocates resources in the interfering cells using the two stage algorithm to obtain sub optimal resource allocation values. The cell of interest then uses these sub optimal values to perform its own resource allocation using the BnB algorithm. The two stage algorithm is chosen for resource allocation in the interfering cells because it has a much lower complexity compared to the BnB algorithm. The BnB algorithm is chosen for resource allocation in the cell of interest because it gives higher sum rate in a sum rate maximization problem than the two stage algorithm. The differences in the approaches proposed in this research are that the BnB and two stage algorithm models the inter-cell interference in terms of the intra-cell interference by assuming that the statistical properties of the intra-cell interference in the cell of interest are the same as in the other interfering cells. In the hybrid algorithm, the cell of interest performs resource allocation in each interfering cell by assuming that the statistical properties of the intra-cell interference in the cell of interest are the same as in the other interfering cells. The cell of interest then uses the sub-optimal resource allocation values obtained in the interfering cells to perform its own resource allocation using the BnB algorithm.

The problem is solved iteratively for all the users in each cell in parallel. This is done by initially allocating zero power to all the users and increasing the powers iteratively until a maximum user sum rate is obtained considering both intra-cell and inter-cell interference. Consider multi-cell massive MIMO uplink (UL) transmission when all the users in all cells are allocated orthogonal pilot sequences, the power allocated to users is increased iteratively from zero the user sum rate is expected to increase as shown in Figure 1-1. In a sum rate maximization problem when interference is negligible, each user is allocated maximum power, p_{max}.

However when non orthogonal pilots are used, and the power allocated to users is increased iteratively from zero, the user capacity is expected to increase and then decrease as the interference becomes significant as shown in Figure 1-2. Only two users, *m* and *k* in cell *j* experiencing different interferences are shown in Figure 1-2. As the power allocated to a user is increased the user capacity and the interference from other users also increase (since resource are being allocated iteratively in parallel) until we get to an optimal user capacity, after which an increase in power doesn't result in an increase in capacity. Hence at higher power values the user capacity is limited by interference. The allocated powers to users when interference is significant is determined by the level of interference. The optimal sum rate for user k in cell j is denoted as C_{jk}^{*} and the optimal power for user k in cell j is denoted as p_{jk}^{*}.

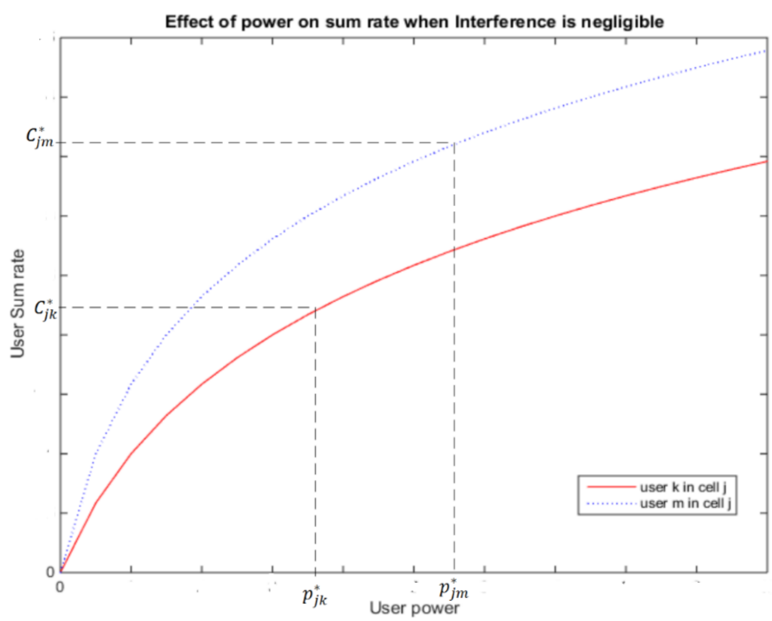

Figure 1-1: The user power-sum rate curve during power allocation when orthogonal pilots are used.

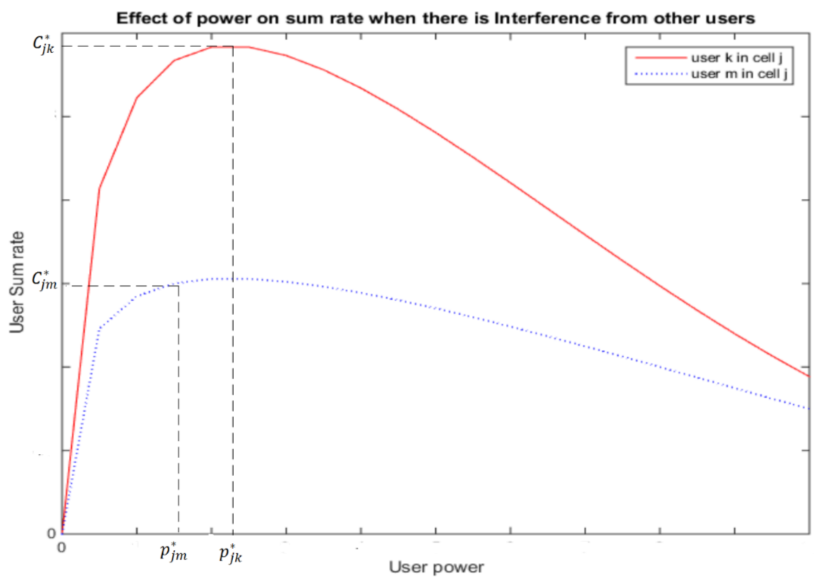

Figure 1-2: The user power-sum rate curve during power allocation when non-orthogonal pilots are used.

1.3 Problem Statement and Research Objectives

1.3.1 Problem Statement

A lot of work has been done on single cell multi user (MU) massive MIMO systems, not much has been done in multi cell MU massive MIMO systems. This has been largely due to the fact that data transmission rate of a user in a multi cell system is dependent on the interference values of users in the same cell and users in different cells. If users are transmitting in parallel, it is of paramount importance to take both the intra-cell and inter-cell interference into consideration in resource allocation. A global solution to the multi cell resource allocation problem gives an optimal solution, however the complexity becomes very high when the number of cells increases. Due to the high computational complexity global optimal resource allocation cannot be applied in practical systems.

However a lot of authors have researched on modelling the interference in the interfering cells and using the interference model to decouple the cells thereby reducing the multicellular system into a single cell massive MIMO system. In [13] the authors assume that interfering users in neighboring cells transmit at constant power. In [11] the authors introduce an interference temperature which gives a lower bound for the users' sum rate. The approaches taken in literature do not adequately model the interference in the other cells. In this research we are going to model the statistical properties of the interference in the interfering cells in terms of the statistical properties of the interference in the cell of interest.

1.3.2 Research Question

Based on the problem statement given in Section 1.3, the research question for the work can be stated as follows:

In a multi cell massive MIMO system which is subject to inter-cell interference, how should the optimal number of users and their respective transmit powers be determined?

1.3.2.1 Sub Questions

The sub questions of the research study are:

- What is the spectral efficiency for a Multi-user multi-cell massive MIMO system in UL for each user taking into consideration channel estimation and that each user can only be served by one BS?
- What are the appropriate power allocation algorithms for UL massive MIMO systems after MRC precoding, for a particular cell of interest?
- How can the unknown interference from the other cells be modelled?
- There are $\binom{K}{K^*}$ different ways of choosing K^* users from K. What are the appropriate user selection schemes that chooses K^* from K that maximize the sum rate in the cell of interest whilst minimizing interference?

1.4 Contributions and Thesis Organization

Authors in literature model the inter-cell interference as constant interference per iteration and optimize resources iteratively per cell until the solution converges to an optimal solution. The solutions in literature have a high computational complexity. The research aims at designing a low complexity optimal resource allocation algorithm that models the inter cell interference using optimal resource allocation parameters in the interfering cells. In the solution proposed by this research, the interfering cells' optimal resource allocation parameters are obtained by statistical approximations rather than iteratively which has a high computational complexity.

The main contributions of this work are derived from the research questions mentioned above, accordingly outlined as follows:

- Deriving a statistical model of the inter-cell interference in terms of the intra-cell interference model. The statistical model assumes that the statistical properties of the intra-cell interference in the cell of interest are the same as in the other interfering cells. The statistical model also takes into account the large scale fading as well as the distance the interfering users are from the user of interest.
- A dual decomposition user selection and power allocation algorithm is designed. This makes use of the Lagrangian decomposition method to allocate power to users and branch and bound technique for user selection.
- Designing a user selection and power allocation algorithm based on the maximum user scheduling expression derived in [14]. We then go on to derive an expression which determines how the users are to be selected in a cell. After user selection, multi-level water filling is then used to allocate power.
- A hybrid user selection and power allocation algorithm is designed. The cell of interest allocates resources in the interfering cells using the low complexity two stage algorithm. Resource allocation in the cell of interest is done using the BnB algorithm because it performs better than the two stage algorithm.

1.4.1 Author Publications

Part of the work has been presented by the author in the following publications:

- P. Chiguvare and F. Takawira, "Power Allocation and User Selection in Multi-cell Massive MIMO Systems*," IEEE Africon 2017 Proceedings*, pp. 376-381, Sept. 2017

- P. Chiguvare and F. Takawira, “Multi-Cell Multi user massive MIMO Systems optimal user selection and power allocation using the branch and bound Algorithm,” [prepared for submission to *International Journal of Communication Systems]*

The dissertation is organized as follows:

Chapter 2: This chapter presents a description of the massive MIMO principle. The chapter introduces massive MIMO basic theories and principles which includes channel estimation, signal detection in UL, precoding techniques in DL, pilot contamination and mitigation techniques.

Chapter 3: This chapter details the work that has been done by other researchers highlighting the recent works and accomplishments on resource allocation in multi cell massive MIMO systems. The previous research work in cell massive MIMO systems are therefore summarized with more emphasis on power allocation and user selection.

Chapter 4: This chapter models the inter-cell interference so that global resource allocation can be converted to a per cell resource optimization solution. The Lagrangian frame work and the Newton Raphson method are employed to determine optimal resource allocation in the cell of interest. The BnB technique is then used to select users that give optimal results. The chapter also presents simulation based performance evaluation.

Chapter 5: This chapter presents a low complexity two stage algorithm. The algorithm is divided into two parts; the user selection part and the power allocation part. A mathematical expression for selecting the maximum number of users who can be scheduled in a cell, K_s is derived for the user selection part. The K_s users that have high SINR value assuming equal power allocation are selected from K users. Finally the multi-level water filling technique is used to allocate powers to the selected users. Simulations are used to check the viability of the proposed algorithm.

Chapter 6: This chapter presents the hybrid algorithm. The cell of interest performs both user selection and power allocation in the interfering cells using the low complexity two stage algorithm. The cell of interest then uses these near optimal values to perform its own resource allocation. The chapter compares the proposed algorithms inter-cell interference modelling to the existing interference models in [13]. The BnB algorithm is chosen for resource allocation in the cell of interest because it performs better than the two stage algorithm. Finally the chapter presents simulation based performance evaluation.

Chapter 7: This chapter summarizes the main conclusions of the dissertation, which provides insights on the design of user selection and power allocation algorithms in multi cell massive MIMO systems.

CHAPTER 2

2 MASSIVE MIMO SYSTEMS BACKGROUND

2.1 Introduction

This chapter presents a description of the massive MIMO principle. A massive MIMO system is a multi-user MIMO (MU-MIMO) system with a large number of antennas. In massive MIMO hundreds or thousands of BS antennas serve a large number of users simultaneously [15], [16]. The number of antennas should be much greater than the number of users. Asymptotic analysis demonstrate that as the number of antennas approach infinity the effects of the uncorrelated noise and part of the interference which is not due to pilot contamination vanishes. This chapter will introduce massive MIMO basic theories and principles which includes channel estimation, signal detection in (UL), precoding techniques in downlink (DL), pilot contamination and mitigation techniques.

The rest of the chapter is organized as follows: Section 2.2 presents the system models and assumptions. Section 2.3 discusses system operation and channel estimation in massive MIMO system. Uplink transmission and signal detection is discussed in Section 2.4. Section 2.5 discusses downlink transmission and precoding. Section 2.6 discusses the detrimental effect pilot contamination has on massive MIMO systems and Section 2.6 presents different ways pilot contamination can be mitigated.

2.2 System Models and Assumptions

We consider a multi cell multi user massive MIMO system. The system consists of B cells with one base station (BS) in each cell. Each BS has large number of fixed transmit antennas N which serve K single antenna user terminals (UT) as shown in Figure 2-1 and Figure 2-2 for DL and UL transmission respectively. It is assumed that $N \gg K \gg 1$.

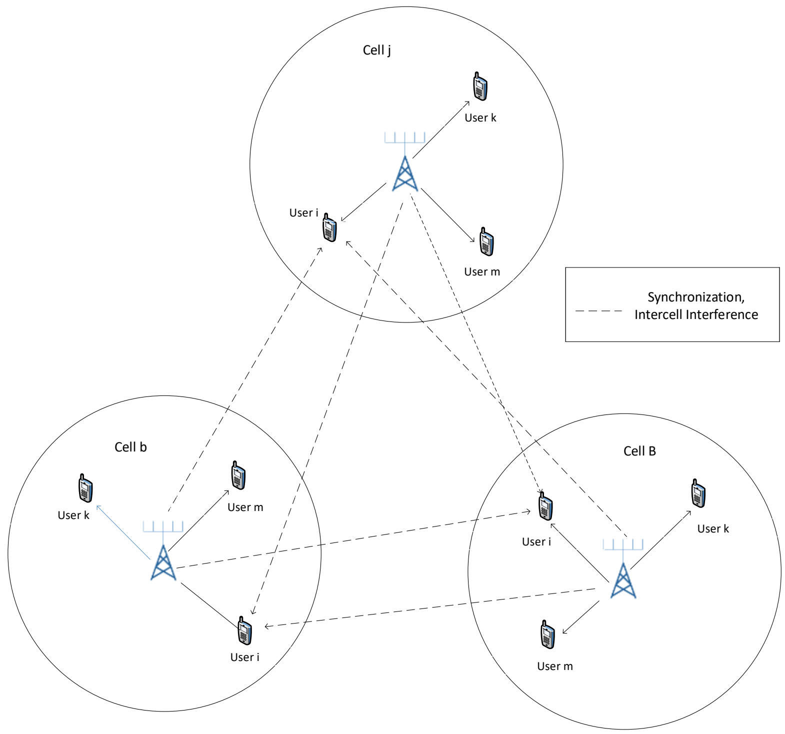

Figure 2-1: Multi cell multi user massive MIMO system model DL transmission

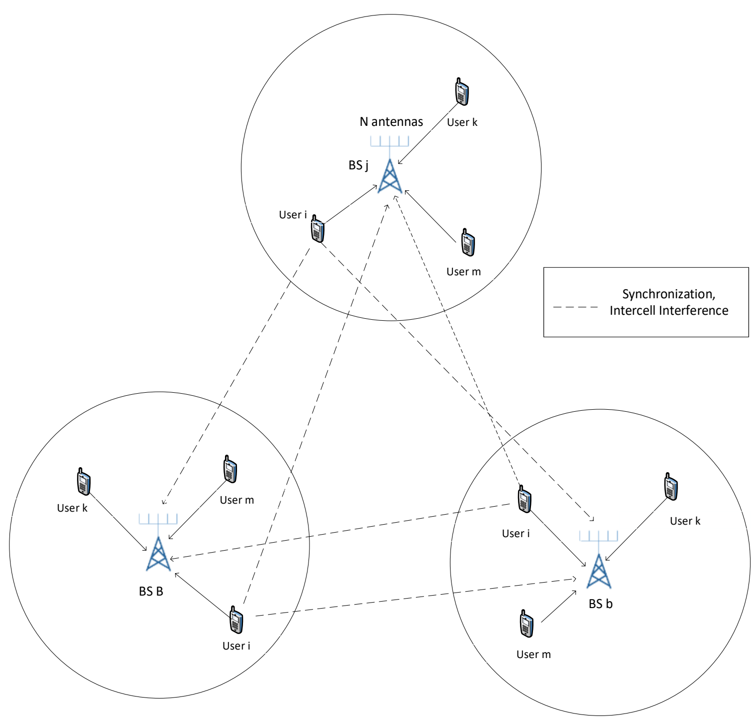

Figure 2-2: Multi cell multi user massive MIMO system model UL transmission

Ideally $N \gg K$ so to provide the system with N-K unused degrees of freedom [17]. The advantage of a large number of BS antennas is that the uncorrelated thermal noise and the small scale fading can be eliminated due to the law of large numbers [18].

The channel can be assumed to be narrowband and slow fading therefore the channel realization is constant for a coherent block duration of time T. The time frequency resources are divided into frames [19]. The number of transmitted symbols per frame is T*W, T is the time in seconds and W is the bandwidth in Hz. It is assumed that both T and W are smaller or equal to the coherence time and coherence bandwidth of all users respectively. Therefore all the channels are static within the frame. V symbols are transmitted during pilot transmission and the total number of symbols transmitted in time T (i.e. for both pilot and data transmission) is S.

Time -division duplex is considered in which the same frequency channel is used and divided into time slots for the UL pilots, UL data and DL data [20]. The channel matrix from user m in cell b to the nth antenna on a BS in cell j is $\mathrm{h_{jbmn}}$. Since the BS has N antennas the resultant channel matrix to a BS in cell j form user m in cell b is $\mathbf{h}_{\mathrm{jbm}} = [\mathrm{h_{jbm1}}, \dots, \mathrm{h_{jbmn}}, \ \dots \ , \mathrm{h_{jbmN}}]$. Each realization of the channel matrix is a circularly symmetric complex Gaussian distribution with zero mean and unit variance as summarized in Eq.(2-1). We denote β_{jbm} as the large scale propagation factor between user m in cell b and the BS in cell j. The large scale fading parameter captures the effects of path loss and shadowing that changes slowly and can be learnt over a long period of time [21], [14]. Hence the overall propagation factor between user m in cell b and a BS in cell j is $\sqrt{\beta_{\mathrm{jbm}}}\mathbf{h}_{\mathrm{jbm}}$.

$$\mathrm{h_{jbmn}} \sim \mathcal{CN}(0,1) \tag{2-1}$$

2.3 System Operation and Channel Estimation in FDD and TDD

Massive MIMO can be operated in both time division duplex (TDD) and frequency division duplex (FDD) [22]. In FDD different frequency resources are used for (UL) and (DL). The difference is that each sub-channel in TDD transmission toggles between UL and DL transmission and hence channels are reciprocal whilst in FDD pilot signals can only be sent in one direction and needs another channel to achieve CSI in the other direction [22]. For FDD the CSI has to be estimated for both the UL and the DL, this however becomes complex as the BS antennas increase [23]. Massive MIMO is usually operated in the time-division-duplex (TDD) mode [23]. Figure 2-3 summarizes the system operation for FDD and TDD.

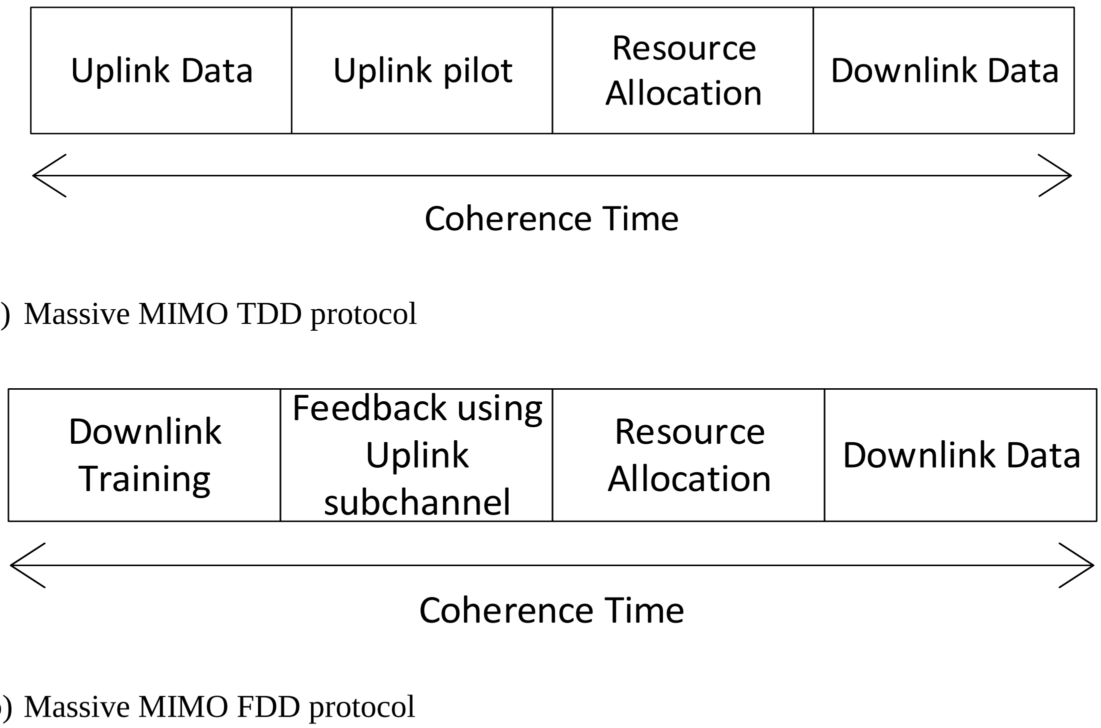

Figure 2-3: System operations for TDD and FDD

As can be seen from the Figure 2-3 the main components of both FDD and TDD are training signaling, data transmission and resource allocation. The above named can be described as follows:

1. **Training:** In FDD the BS sends pilot signals that enable users to estimate their channels. In TDD users send pilot signals in the UL that enables the BS to estimate the UL channel. A reciprocal of this channel is used for DL transmission.
2. **Data Transmission:** Data is transmitted throughout the coherence time
3. **Resource Allocation:** Resource allocation is performed by the BS in both FDD and TDD (user selection, power allocation, etc.)

4. **Feedback using UL sub-channel:** In FDD each user sends its channel estimate using an UL sub channel

The UL and DL corresponding CSI are different in FDD since different frequency bands are used in the UL and DL. In the UL users send pilot signals to the BS, and the BS performs channel estimation. The time required for pilot transmission in UL does not depend on the number antennas at the BS. To calculate CSI in FDD, the BS first sends pilot signals to the users, the users then send back the estimated CSI to the BS through an UL sub-channel. The time required for DL pilot transmission increases with the number of antennas [7]. In massive MIMO as the number of antennas increase FDD becomes infeasible.

Traditionally channel estimation is performed in the DL, the BS sends pilot signals to the users which in turn estimates the channels. However in massive MIMO, this is infeasible as the number of antennas increase since the channel estimation overhead is proportional to the number of antennas [17]. Therefore it is more logical to estimate the channels during UL transmission.

In massive MIMO, a BS uses its excess antennas for receive beamforming in the UL and transmit precoding in the DL which amplifies the desired signal and suppresses interfering signals [19]. The BS has to know the CSI for receive beamforming in the UL and precoding in the DL. The BS estimates the CSI using pilots sent by users. To mitigate the effects of pilot contamination orthogonal pilots should be used. This is not always possible when there is a large number of cells since $B \times K$ orthogonal pilots are needed.

In TDD operation, the UTs send pilot signals to the BS, the BS then estimates the channel matrix.

2.4 Uplink Transmission

Uplink transmission is the scenario where UT, k in cell j transmits a signal to the BS in cell j. Inter-cell interference is when part of the signal is received by a BS in another cell. Intra-cell interference is when the BS wrongly decodes the information sent by a user, k as coming from another user in the same cell. The $N \times 1$ received signal at the BS can be modelled as Eq.(2-4) [24], [25],[26]

2.5 Uplink Signal Detection

The BS will coherently detect the signals from the K users using the received signal together with the estimated CSI. Signal detection in the UL of a massive MIMO system can be achieved using linear and nonlinear signal detectors [7]. Compared with linear detectors, nonlinear detectors have better performance but higher computational complexity [7]. Linear precoders such as zero forcing (ZF), matched filter (MF) and minimum mean-square error (MMSE) are ideal for a massive MIMO system. A lot of researchers have studied the performance of a massive MIMO system under various linear precoders [26] ,[27]. The authors in [26] proved that when there is inter-cell interference in a multi cell scenario the MMSE receiver achieves the same computational complexity as the MF receiver with fewer antennas.

The N×1 received signal before receive beamforming at BS j, $\mathbf{y}_{\mathrm{j}}$ = $[\mathrm{y}_{\mathrm{j1}}, \dots, \mathrm{y}_{\mathrm{jn}} \dots, \mathrm{y}_{\mathrm{jN}}]$ can be modelled as

$$\mathbf{y}_{\mathrm{j}} = \sum_{\mathrm{b}=1}^{\mathrm{B}} \sum_{\mathrm{m}=1}^{\mathrm{K}} \sqrt{\mathrm{p}_{\mathrm{bm}} \beta_{\mathrm{jbm}}}\, \mathbf{h}_{\mathrm{jbm}} \mathbf{x}_{\mathrm{bm}} + \mathbf{n}_{\mathrm{j}} \tag{2-2}$$

where x_{bm} is the symbol transmitted by user m in cell b. The signal is normalized as $\mathbb{E}\{\mathbf{x}_{\mathrm{bm}}{}^{\mathrm{H}} \mathbf{x}_{\mathrm{bm}}\} = 1$, while p_{bm} is the corresponding UL transmit power The noise is modelled as Gaussian additive noise with each modelled as a complex Gaussian distribution, $\mathbf{n}_{\mathrm{j}} \sim CN\,(0, \sigma_w^2 I_N)$. σ_w^2 is the noise variance after data transmission

Since the BSs have multiple antennas, in the UL, the base station should apply a receive beamforming vector, $\mathbf{v}_{\mathbf{j}} = [\,\mathbf{v}_{\mathrm{j1}}, \dots, \mathbf{v}_{\mathrm{jk}} \dots .\, \mathbf{v}_{\mathrm{jK}}]$ so as to amplify the desired signal and reject interference thereby separating the signals into streams. This is achieved by multiplying the received signal by $\mathbf{v}_{\mathrm{jk}}{}^{\mathrm{H}}$ [19], [28].

$$\hat{\mathbf{y}}_{\mathrm{jk}} = \mathbf{v}_{\mathrm{jk}}{}^{\mathrm{H}}\, \mathbf{y}_{\mathrm{jk}} \tag{2-3}$$

Where $\hat{\mathbf{y}}_{\mathrm{j}}$ is the received signal after precoding.

The received vector, after linear beamforming is given by

$$\hat{\mathbf{y}}_{jk} = \sum_{b=1}^{B} \sum_{m=1}^{K} \sqrt{p_{bm}\beta_{jbm}} \mathbf{v}_{jk}{}^{H}\, \mathbf{h}_{jbm} \mathbf{x}_{bm} + \quad \mathbf{v}_{jk}{}^{H} \mathbf{n}_{j} \tag{2-4}$$

Each stream from each user is then decoded independently. The signal from the kth stream in BS j is given by

$$\hat{\mathbf{y}}_{jk} = \overbrace{\sqrt{\beta_{jjk} p_{jk}} \mathbf{v}_{jk}{}^{H} \mathbf{h}_{jjk} x_{jk}}^{\text{desired}} + \overbrace{\sum_{b=1}^{B} \sum_{m=1\,,(b,m)\neq(j,k)}^{K} \sqrt{\beta_{jbm} p_{bm}} \mathbf{v}_{jk}{}^{H}\, \mathbf{h}_{jbm} \mathbf{x}_{bm}}^{\text{interference}} + \overbrace{\mathbf{v}_{jk}{}^{H} \mathbf{n}_{j}}^{\text{noise}} \tag{2-5}$$

Some conventional massive MIMO receivers are reviewed below.

2.5.1 Maximum Ratio Combining (MRC)

Maximum Ratio Combining (MRC) is an example of passive rejection [19]. It maximizes the SINR by maximizing the effective channel gain, the interfering terms are rejected on the basis that the channels are nearly orthogonal if they do not use the same pilot sequence for channel estimation. With MRC, the BS maximizes each user's SINR whilst ignoring the effects of both the intra-cell and inter-cell interference.

From

$$\begin{aligned} \mathbf{v}_{MRC,jk} &= \underset{\mathbf{v}_{jk}}{\operatorname{argmax}} \frac{\text{desired signal power}}{\text{noise power}} \\ &= \underset{\mathbf{v}_{jk}}{\operatorname{argmax}} \frac{p_{jk} |\mathbf{v}_{jk}{}^{H} \mathbf{h}_{jjk}|^{2}}{||\mathbf{v}_{jk}||^{2}} \end{aligned} \tag{2-6}$$

Since

$$\frac{\mathrm{p_{jk}}|\mathbf{v}_{\mathrm{jk}}{}^{\mathrm{H}}\mathbf{h}_{\mathrm{jjk}}|^2}{||\mathbf{v}_{\mathrm{jk}}||^2} \leq \frac{\mathrm{p_{jk}}\left|\left|\mathbf{h}_{\mathrm{jjk}}\right|\right|^2\left|\left|\mathbf{v}_{\mathrm{jk}}\right|\right|^2}{\left|\left|\mathbf{v}_{\mathbf{jk}}\right|\right|^2} = \mathrm{p_{jk}}\left|\left|\mathbf{h}_{\mathrm{jjk}}\right|\right|^2 \tag{2-7}$$

The above equality holds when $\mathbf{v}_{\mathrm{jk}} = \mathbf{h}_{\mathrm{jjk}}$

2.5.2 Zero Forcing (ZF)

ZF is an example of active rejection. ZF precoding uses channel inversion to cancel out inter-user interference however effective channel gains are less than that of MRC precoding [19]. The ZF beamforming vector for the kth user satisfies

$$\begin{cases} \mathbf{v}_{\mathrm{jk}}{}^{H}\mathbf{h}_{\mathrm{jbm}} \neq 0\,, (b,m) = (j,k) \\ \mathbf{v}_{\mathrm{jk}}{}^{H}\mathbf{h}_{\mathrm{jbm}} = 0, (b,m) \neq (j,k) \end{cases} \tag{2-8}$$

The pseudo inverse of the channel matrix satisfies Eq. (2-8) for all users. The pseudoinverse is given by:

$$\mathbf{v}_{\mathrm{jk}} = \overline{\mathbf{h}_{\mathrm{jjk}}}^{\mathrm{H}}[\overline{\mathbf{h}_{\mathrm{jjk}}}\ \overline{\mathbf{h}_{\mathrm{jjk}}}^{\mathrm{H}}]^{-1} \tag{2-9}$$

ZF receivers have a higher computational complexity than MRC because of the pseudoinverse in Eq.(2-9).

2.5.3 Minimum Mean Square Error (MMSE) Precoding

MMSE finds a balance between effective channel gain and interference. It is therefore both passive and active [19]. MMSE minimizes the error between the estimate $\mathbf{v}_{\mathrm{jk}}{}^{H}\hat{\mathbf{y}}_{\mathrm{j}}$ and the transmitted signal $\mathrm{x_{jk}}$

$$\mathbf{v}_{\mathrm{MRC,jk}} = \underset{\mathbf{v}_{\mathrm{jk}}}{\operatorname{argmin}}\, \mathbb{E}\left\{\left|\mathbf{v}_{\mathrm{jk}}{}^{H}\hat{\mathbf{y}}_{\mathrm{j}} - \mathrm{x_{jk}}\right|^2\right\}$$

$$= cov\left(\hat{\mathbf{y}}_{\mathrm{j}}, \hat{\mathbf{y}}_{\mathrm{j}}\right)^{-1} cov(\mathrm{x_{jk}}, \hat{\mathbf{y}}_{\mathrm{j}})^{H}$$

$$=\sqrt{\mathrm{p_j}}\mathbf{h}_{\mathrm{jjk}}[\mathrm{p_j}\mathbf{h_j}\ \mathbf{h_j}^{\mathrm{H}}+\mathbf{I_K}]^{-1} \quad (2\text{-}10)$$

For two random column vectors $\mathbf{m}_1$ and $\mathbf{m}_2$ with zero-mean elements, $cov(\mathbf{m}_1, \mathbf{m}_2) \triangleq \mathbb{E}\{m_1 {m_2}^H\}$. MMSE functions by maximizing the user SINR, hence MMSE is better than ZF and MRC [19].

2.6 Downlink Transmission and Precoders

Downlink transmission is when the BS transmits signals to its users, however some of the signals may be received by users in the same cell (intra-cell interference) or by users in different cells (inter-cell interference). Let $\mathbf{x_j}$ be the transmitted signal from a BS in cell j, where $\mathbf{x_j} = [\mathbf{x}_{\mathrm{j1}}$, $\mathbf{x}_{\mathrm{j2}}, \dots, \mathbf{x}_{\mathrm{jk}}]$. The data symbol for each user has average power, $\mathbb{E}\{||\mathbf{x}_{\mathrm{jk}}||^2 = 1$.

$$\mathbf{y_{jk}} = \sum_{\mathrm{b=1}}^{\mathrm{B}} \sqrt{\beta_{\mathrm{jbk}}}\mathbf{h}_{\mathrm{jbk}}{}^{\mathrm{H}} \sum_{\mathrm{m=1}}^{\mathrm{K}} \sqrt{\mathrm{p_{bm}}}\,\mathbf{v_{bm}}\mathbf{x}_{\mathrm{bm}} + \mathbf{n}_{\mathrm{jk}} \quad (2\text{-}11)$$

It can be assumed that the data is precoded with an N×1 precoding vector $\mathbf{v}_{\mathrm{bm}}$ before transmission.

In massive MIMO both linear and nonlinear precoders may be used, however nonlinear precoders perform better than linear precoders but have a higher computational complexity. Nonlinear precoders include dirty-paper-coding (DPC), vector perturbation VP and lattice aided methods. The performance of linear precoders has proved to be near optimal as the number of antennas increase. Linear precoders include MMSE, ZF, and MRC. Due to an improved performance as number of antennas increase and their low computational complexity, linear precoders are mainly used in massive MIMO systems.

2.7 Interference and Pilot contamination

A typical massive MIMO system is made of many cells which share the same time-frequency resources. As the number of cells increases in a multi cell massive MIMO system, it becomes difficult to assign orthogonal pilots to all the users in all the cells, due to the limitation of the channel coherence interval [16]. Figure 2-4 and Figure 2-5 illustrate the pilot contamination concept in the UL and DL respectively. Two users are shown in neighboring cells using the same pilot sequence. The identical pilot sequences will cause pilot contamination and consequently inter-cell interference.

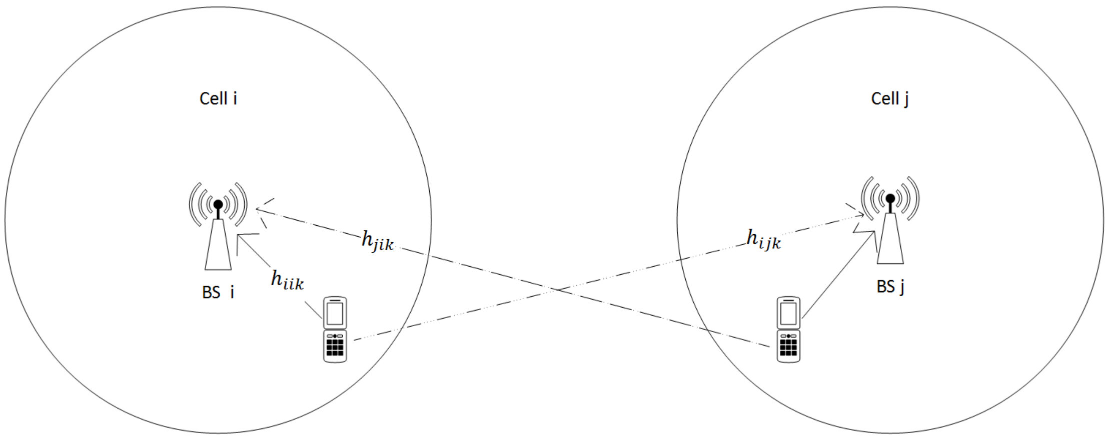

Figure 2-4: Illustration of pilot contamination concept in UL transmission. The two users shown are assigned the same pilot sequences

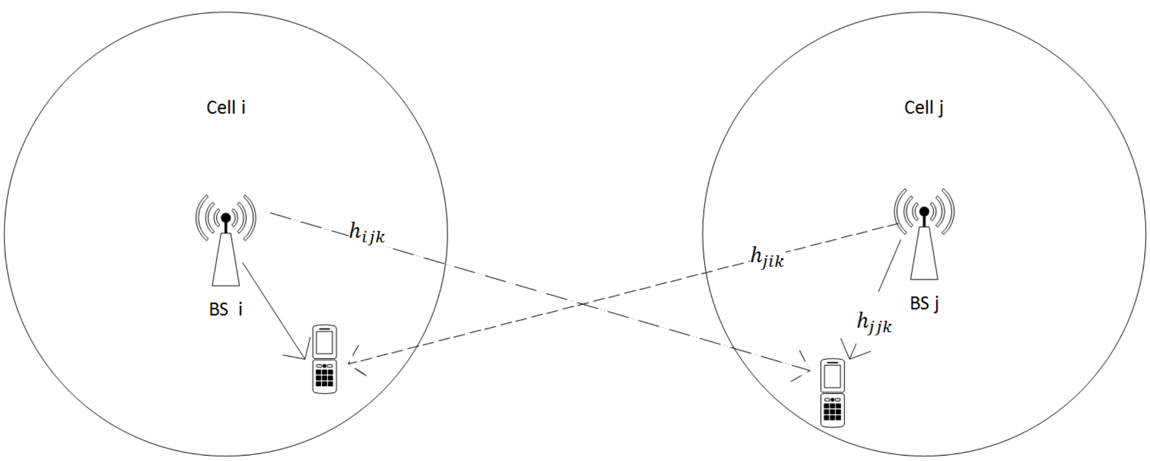

Figure 2-5: Illustration of pilot contamination concept in DL transmission. The two users shown are assigned the same pilot sequences

As discussed above TDD is most likely to be used for massive MIMO systems, pilot signals will be transmitted from the users during UL. To eliminate interference, the pilot signals used in the same cell and also neighboring cells should be orthogonal i.e.

$$\mathbf{x}_{\mathrm{bm}}\mathbf{x}_{\mathrm{jk}}{}' = \begin{cases} 1, & (\mathrm{b},\mathrm{m}) = (\mathrm{j},\mathrm{k}) \\ 0, & \text{otherwise} \end{cases} \tag{2-12}$$

If orthogonal pilot sequences are used, the channel vectors used to estimate the channels are uncorrelated and hence the BS obtains uncontaminated channel estimates. Due to the limitation of the channel coherence interval it becomes difficult to assign orthogonal pilots to all users hence

$$\mathbf{x}_{\mathrm{bm}}\mathbf{x}_{\mathrm{jk}}{}' \neq 0 \tag{2-13}$$

In UL training, the K users in cell j send their pilot sequences of length τ to BS j. If we consider the above system model the received signal at BS j, $\mathbf{Y}_{\mathrm{j}} \in \mathbb{C}^{N}$ during pilot transmission can be modelled as

$$\mathbf{Y}_{\mathrm{jk}} = \sum_{\mathrm{b}=1}^{\mathrm{B}} \sum_{\mathrm{m}=1}^{\mathrm{K}} \sqrt{\mathrm{q}_{\mathrm{bm}}\beta_{\mathrm{jbm}}}\,\mathbf{h}_{\mathrm{jbm}}\mathbf{S}_{\mathrm{bm}} + \mathbf{N}_{\mathrm{j}} \tag{2-14}$$

Where $\mathbf{S}_{\mathrm{bm}} = \mathbf{s}_{\mathrm{bm}} \otimes \mathbf{I}_{\mathrm{N}}$ is a τN × N matrix, $\mathbf{h}_{\mathrm{jbm}}$ is the UL channel vector from user m in cell b to a BS in cell j. The corresponding pilot transmit power is q_{bm} , $\mathbf{N}_{\mathrm{j}}$ is the noise due to the transmitted symbols. The noise is modelled as Gaussian additive noise with each modelled as a complex Gaussian distribution $\mathbf{N}_{\mathrm{j}} \sim \mathrm{CN}\,(0,\sigma^2 I_N)$. σ^2 is the noise variance after pilot transmission.

Using LS channel estimation, $\widehat{\mathbf{h}_{\mathrm{jjk}}}$ is given by

$$\widehat{\mathbf{h}_{\mathrm{jjk}}} = \mathbf{h}_{\mathrm{jjk}} + \sum_{\mathrm{b}=1}^{\mathrm{B}} \sum_{\mathrm{m}=1,(\mathrm{b},\mathrm{m})\neq(\mathrm{j},\mathrm{k})}^{\mathrm{K}} \mathbf{s}_{\mathrm{jk}}^{\mathrm{T}}\mathbf{s}_{\mathrm{bm}}\sqrt{\mathrm{q}_{\mathrm{bm}}\beta_{\mathrm{jbm}}}\,\mathbf{h}_{\mathrm{jbm}} + \mathbf{N}_{\mathrm{j}}\mathbf{S}_{\mathrm{jk}}^{\mathrm{T}} \tag{2-15}$$

Proof: See Appendix A1

The channel estimate in Eq. (2-15) shows how the desired channel information $\mathbf{h}_{jjk}$ is contaminated by other users' channel vectors. If $\mathbf{s}_{jk}^{T}\mathbf{s}_{bm}$=0, the channels are uncorrelated and the pilots used are mutually orthogonal. If $\mathbf{s}_{jk}^{T}\mathbf{s}_{bm} = 1$, there is maximum correlation and therefore maximum interference. If $\mathbf{s}_{jk}^{T}\mathbf{s}_{bm} \neq 0$ the estimated channel is contaminated by undesired $\mathbf{h}_{jbm}$ channel vectors. When there is no pilot contamination, channel estimation and precoding can be completely decoupled from each other [7].

It can be deduced that the SINR for the received signal given in Eq. (2-4) and Eq.(2-5) for user k in cell j is:

$$\mathrm{SINR}_{jk}^{MRC} = \frac{\beta_{jjk}p_{jk}}{\sum_{b=1}^{B}\sum_{m=1\,(b,m)\neq(j,k)}^{K} p_{bm}\beta_{jbm}(\psi_{bmjk}^{2}J_{jbm}^{2}) + \frac{\alpha_{jk}}{N}\overline{R_{jk}}} \tag{2-16}$$

Where $\psi_{bmjk}^{2} = s_{jk}^{T}s_{bm}$, $\overline{R_{jk}} = \sum_{b=1}^{B}\sum_{m=1}^{K} p_{bm}\beta_{jbm} + \sigma_{w}^{2}$, $\alpha_{jk}= \sum_{\substack{t=v\\u=z}}\psi_{tujk}^{2}J_{jtu}^{2} + \sigma^{2}$ and J_{jbm} $=\sqrt{q_{bm}\beta_{jbm}}$

Proof: see Appendix A2

As the number of antennas approaches infinity, a part of the interference term that is due to pilot contamination does not go to zero, the SINR tends to the limit

$$\mathrm{SINR}_{jk}^{MRC} = \frac{\beta_{jjk}p_{jk}}{\sum_{b=1}^{B}\sum_{m=1\,(b,m)\neq(j,k)}^{K} p_{bm}\beta_{jbm}((\psi_{bmjk}^{2})^{2}J_{jbm}^{2})} \tag{2-17}$$

Proof: Obtained from Eq.(2-16) taking $\lim_{N\to\infty} SINR_{jk}$

Therefore when non orthogonal pilots are used $s_{jk}^{T}s_{bm} \neq 0$, the BS cannot distinguish between users. Pilot contamination does not fully disappear when the number of antennas is increased without limit.

2.8 Pilot Contamination Mitigation

A lot of research work has been done on pilot contamination mitigation. We describe the developed pilot mitigation methods below

1. **Blind Methods:** Clever channel estimation techniques have been developed which eliminate the effect of pilot contamination. These include blind techniques that estimate the channels and payload data [29], [30]. The authors in [31] propose eigenvalue – decomposition (EVD) based channel estimation technique. The approximation technique exploits the assumption that the channels are orthogonal in a massive MIMO system. To improve the proposed technique, it is combined with the iterative least square with projection (ILSP) algorithm. In [32],[33] the authors designed an algorithm that mitigates pilot contamination on the assumption that as the number of antennas approaches infinity the transmitted signals become orthogonal. The algorithm separates the signal subspace from the interference subspace, thereby eliminating interference.
2. **Protocol-based Methods:** Pilot contamination can be eliminated by reducing the number of users that makes use of non-orthogonal pilot sequences. This can be achieved through frequency reuse [24], [34]. Pilot contamination can also be mitigated by using a scheme based on a time shifted protocol [7],[35], [36].The authors propose a time shifted protocol whereby users using non-orthogonal pilot sequences transmit at different non overlapping time slots (see Figure 2-6). In [37] the authors define a low complexity near optimal pilot scheduling algorithm that reduces the effects of pilot contamination in massive MIMO systems. The algorithm makes use of the cross-entropy optimization framework (CEO). CEO is an optimization algorithm that generates a population of candidate solutions from a solutions space [37]. It was proved that the designed algorithm provides excellent performance with reduced complexity.
3. **Precoding Methods:** Precoding techniques which are designed to take into consideration the network structure have been designed to mitigate pilot contamination. The precoding method described in [21] minimizes both the intra-cell and the inter-cell interference.

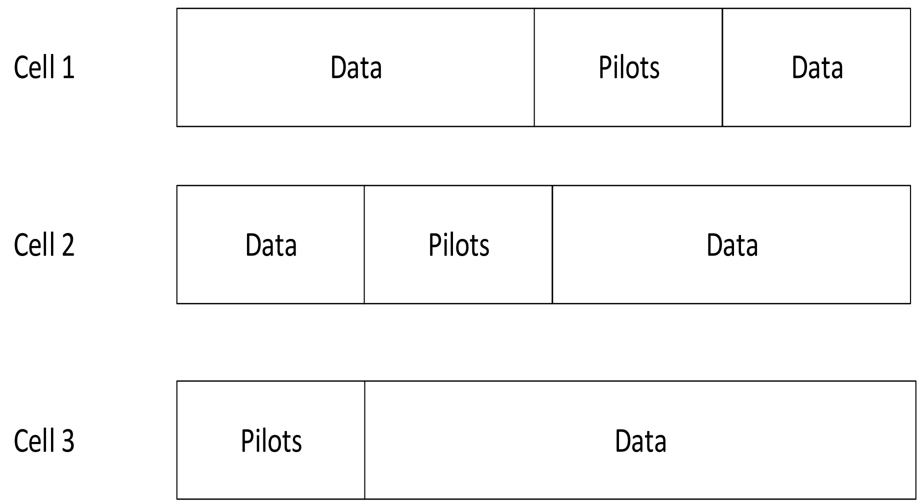

Figure 2-6: Shifted frame structure for eliminating pilot contamination

2.9 Chapter Summary

This chapter presents a description of the massive MIMO principle. The chapter introduces massive MIMO basic theories and principles which includes channel estimation, signal detection in UL, precoding techniques in downlink, pilot contamination and mitigation techniques. Massive MIMO system are adversely affected by pilot contamination, however a lot of research has been conducted to mitigate the effects of pilot contamination.

CHAPTER 3

3 MASSIVE MIMO RESOURCE ALLOCATION

3.1 Introduction

This section details the work that has been done by other researchers on resource allocation in multi cell massive MIMO systems. The previous research work in multi cell massive MIMO systems is therefore summarized with more emphasis on power allocation and user selection. A lot of factors affect resource allocation in multi cell massive MIMO systems. The major problem is the high complexity of obtaining a global resource allocation solution as the number of cells increase. However most authors model inter-cell interference and optimize resource allocation on a per cell basis.

The rest of the chapter is organized as follows: Section 3.2 presents the related work on power allocation in massive MIMO systems. Section 3.3 details the research accomplishments on user selection in massive MIMO systems. Section 3.4 presents the related work on both user selection and power allocation in massive MIMO systems. Finally Section 3.5 summarizes the chapter's main points

3.2 Power Allocation

A lot of work has been done on both multi cell and single cell massive MIMO systems power allocation. Single cell massive MIMO simplifies the problem because there is no inter cell interference. In multi cell massive MIMO, the inter-cell interference has to be modelled if a sub-optimal per cell optimization solution is to be obtained.

3.2.1 Power Allocation in Single Cell Uplink

The authors in [25] ,[38], [39] consider the UL of a single cell massive MIMO system, where K single antenna elements transmit to a BS with N antenna elements. Eq. (3-1) shows the received signal, $\hat{\mathbf{y}}_{\mathrm{jk}}$ by the user of interest, k in cell j after channel estimation. The expression for $\hat{\mathbf{y}}_{\mathrm{jk}}$ is similar to the one given in Eq.(2-5) , they differ in that Eq.(2-5) considers a multi cell system whilst Eq.(3-1) considers as single cell system.

$$\hat{\mathbf{y}}_{jk} = \overbrace{\sqrt{\beta_{jk} p_{jk}} \mathbf{v}_{jk}{}^{H} \mathbf{h}_{jjk} x_{jk}}^{\text{desired}} + \overbrace{\sum_{m=1, m\neq k}^{K} \sqrt{\beta_{jm} p_{jm}} \mathbf{v} j_{k}{}^{H} \mathbf{h}_{jm} x_{jm}}^{\text{interference}} + \overbrace{\mathbf{v}_{jk}{}^{H} \mathbf{n}_{j}}^{\text{noise}} \qquad (3\text{-}1)$$

The sum rate for user k, C_{jk} is given by

$$C_{jk} = \left(1 - \frac{V}{S}\right) \log_2 (1 + SINR_{jk}) \qquad (3\text{-}2)$$

In [38] , [39] the authors define the $SINR_{jk}$ lower bound as

$$SINR_{jk}^{MRC} = \frac{q_{jk} \beta_{jjk}{}^{2} p_{jk} (N-1)}{\left(1 + \sum_{m=1\ (m)\neq(k)}^{K} p_{jm} \beta_{jjm}\right)\left(1 + q_{jk} \beta_{jjk}\right) + p_{jk} \beta_{jjk}} \qquad (3\text{-}3)$$

where q_{jk} is the pilot power allocated to user k in cell j.

The difference between the SINR expressions in Eq.(3-3) above and Eq.(2-16) is that the Eq.(2-16) considers the lower bound of the SINR expression using the Jensen inequality. Eq.(2-16) considers UL power control such that $q_{jk}\beta_{jjk} = 1$. The authors in [38] maximize the sum rate for two cases; in the first case they assume the users have perfect CSI, in the second case they assume the users have to estimate their CSI. For the case when users have perfect CSI, the maximization problem is formulated as:

$$\max_{\{p_{jk}\}\in\mathbb{R}} \sum_{k=1}^{K} \{\log_2 (1 + SINR_{jk})\}$$

$$\text{subject to:}$$

$$A: \ p_{jk} \leq p_{jk,max}$$

$$B: p_{jk} \geq 0 \qquad (3\text{-}4)$$

It can be concluded that for receiver with perfect CSI, the users transmit at maximum power because there is no interference due to pilot contamination. However, the authors in [38] argue that for a case of imperfect CSI, the pilot powers become part of the optimization parameters. The problem formulation with imperfect CSI becomes

$$\max_{\{p_{jk},q_{kj}\}\in\mathbb{R}} \sum_{k=1}^{K} \{\log_2 (1 + \mathrm{SINR}_{jk})\}$$

$$\begin{aligned} &\text{subject to:} \\ &A:\ 0 \le p_{jk} \le p_{jk,max} \\ &B: 0 \le q_{jk} \le q_{jk,max} \end{aligned} \tag{3-5}$$

Where $q_{jk,max}$ is the maximum power that can be allocated to the pilot symbol k.

The problem is solved by convex optimization techniques using the dual Lagragian optimization technique which states that the Lagrangian multipliers are optimum if they satisfy Karush-Kuhn-Tucker (KKT) conditions. The results show that for UL transmission if the interference is negligible compared to the noise level i.e. $(1 + \sum_{m=1\,(m)\neq(k)}^{K} p_{jm}\beta_{jjm})(1 + q_{jk}\beta_{jjk}) \le p_{jk}\beta_{jjk}$ for Eq. (3-3), then the user optimal power allocation corresponds to the maximum power. Hence if interference is negligible UL power allocation allocates maximum power.

The authors in [39] optimize power allocation for both pilots and data. They formulate the power allocation problem by adding an energy budget constraint

$$Vq_{jk} + (S\text{-}V)\ p_{jk} = E_{max} \tag{3-6}$$

where E_{max} is the total energy budget for a coherence block, V is the number of symbols transmitted during pilot transmission and S is the total number of symbols transmitted in a frame.

The problem is formulated as follows:

$$\max_{\{p_{jk}, q_{kj}\} \in \mathbb{R}} \sum_{k=1}^{K} \{\log_2 (1 + \mathrm{SINR}_{jk})\}$$

subject to:

$$A: \; 0 \leq p_{jk}$$

$$B: 0 \leq q_{jk}$$

$$C: Vq_{jk} + (S\text{-}V)\, p_{jk} = E_{max} \tag{3-7}$$

The problem is reformulated using the epigraph form

$$\max_{\{p_{jk}, q_{kj}, \gamma_{jk}\} \in \mathbb{R}} \prod \gamma_{jk}$$

subject to:

$$A: \; (1 + \mathrm{SINR}_{jk}) \geq \gamma_{jk}$$

$$B: 0 \leq q_{jk}, 0 \leq p_{jk}$$

$$C: Vq_{jk} + (S\text{-}V)\, p_{jk} = E_{max} \tag{3-8}$$

The authors argue that the problem in Eq. (3-7) is NP-hard, hence a successive convex optimization approach is adopted to converge to a KKT point.

3.2.2 Power Allocation in Single Cell Downlink

The authors in [40], [41], [42], [43], [44], [45] consider a single cell DL massive MIMO system. The system model presented in this section is different from the system model presented in Section 3.2.1 in that the model for the latter is for UL, whilst in this section we are focusing on DL. However both sections assume a single cell system with one BS cell and K users.

The received signal model for user k is given by

$$\mathbf{y}_{jk} = \sqrt{\beta_{jjk}}\mathbf{h}_{jjk}{}^{H} \sum_{m=1}^{K} \sqrt{p_{jm}}\, \mathbf{v}_{jm} x_{jm} + \mathbf{n}_{jk} \tag{3-9}$$

The capacity for user k, $\mathrm{C_{jk}}$ is given by

$$\mathrm{C_{jk}} = \log_2(1 + \mathrm{SINR_{jk}}) \tag{3-10}$$

The $\mathrm{SINR_{jk}}$ lower bound is defined as

$$\mathrm{SINR_{jk}^{MRC}} = \frac{\mathrm{q_{jk}\beta_{jjk}}^2 \mathrm{p_{jk}}(N-1)}{\left(\sum_{\mathrm{m=1\ (m)\neq(k)}}^{\mathrm{K}} \mathrm{p_{jm}\beta_{jjm}}\right)(1 + \mathrm{q_{jk}\beta_{jjk}}) + \sigma_j} \tag{3-11}$$

The differences between Eq.(3-3) and Eq.(3-11) is that Eq.(3-3) looks at the UL and Eq.(3-11) looks at the DL. The power allocation problem in [43], [40] is formulated to maximize the energy efficiency (EE) with the constraint that the power per UT should be greater than zero. The system EE is given by

$$EE = \frac{\sum_{\mathrm{k=1}}^{\mathrm{K}} \mathrm{C_{jk}}}{\sum_{\mathrm{k=1}}^{\mathrm{K}} \mathrm{pj_k} + \sum_{\mathrm{n=1}}^{\mathrm{N}} \mathrm{p_{c,n}}} \tag{3-12}$$

$\mathrm{p_{c,n}}$ is the circuit power consumption per antenna.

The optimization problem is formulated as:

$$\max_{\{\mathrm{pj_k, q_{jkj}}\}\in\mathbb{R}} EE$$

subject to:

$$\mathrm{A}: \sum_{\mathrm{k=1}}^{\mathrm{K}} \mathrm{p_{jk}} \leq \mathrm{p_{jk,max}}$$

$$\mathrm{B}: \mathrm{C_{jk}} \geq \mathrm{C_{jk,min}} \tag{3-13}$$

However the resultant problem is a non-convex optimization problem which has an exponential complexity. To transform the problem into a convex problem, the authors in [43], [40] transformed the objective function of the EE optimization problem into subtractive form which can be solved by convex optimization techniques as shown in Eq.(3-14).

$$EE = \frac{\sum_{k=1}^{K} C_{jk}}{\sum_{k=1}^{K} p_{jk} + \sum_{n=1}^{N} p_{c,n}} = \left\{\sum_{k=1}^{K} C_{jk} - q^{*}\left(\sum_{k=1}^{K} p_{jk} + \sum_{n=1}^{N} p_{c,n}\right)\right\} \qquad (3\text{-}14)$$

Where q^{*} denote the maximum EE.

If q is the EE, then using Eq. (3-14) the above problem formulation can be simplified to:

$$\max_{\{pj_k, q_{kj}\} \in \mathbb{R}} \left\{\sum_{k=1}^{K} C_{jk} - q\left(\sum_{k=1}^{K} p_{jk} + \sum_{n=1}^{N} p_{c,n}\right)\right\}$$

$$\text{subject to:}$$

$$A: \sum_{k=1}^{K} p_{jk} \le p_{jk,max}$$

$$B: C_{jk} \ \geq \ C_{jk,min} \qquad (3\text{-}15)$$

The authors in [43],[40] utilize the Lagrangian dual function to transform the problem into an unconstrained form. They argue that due to the coupling problem caused by intra cell interference it is impossible to obtain the kth user transmit power explicitly. However it can be obtained implicitly by assuming the intra-cell powers are constant per iteration.

The authors in [45] address the EE and SE tradeoff in downlink single cell massive MIMO systems. They define EE as defined in Eq. (3-14). The total power consumption P_{tot} is defined as:

$$P_{tot} = \sum_{k=1}^{K} p_{jk} + \sum_{n=1}^{N} p_{c,n} \qquad (3\text{-}16)$$

To maximize both SE and EE the problem is formulated as follows:

$$\max_{\{p_{jk}\}\in\mathbb{R}} \sum_{k=1}^{K}\left\{\log_2\left(1+\ \mathrm{SINR_{jk}}\right)\right\}$$

$$\min_{\{p_{jk}\}\in\mathbb{R}} P_{tot}$$

subject to:

$$\mathrm{A}:\ \sum_{k=1}^{K} p_{jk} \le p_{j,max}$$

$$\mathrm{B}: p_{jk} \ge 0 \qquad (3\text{-}17)$$

It can be observed that objective functions in Eq.(3-17) conflict with each other. Therefore to obtain a solution, the weighted sum method is used, i.e.

$$\max_{\{p_{jk}\}\in\mathbb{R}} \quad w\sum_{k=1}^{K}\left\{\log_2\left(1+\ \mathrm{SINR_{jk}}\right)\right\} -(1-\mathrm{w})P_{tot}$$

subject to:

$$\mathrm{A}:\ \sum_{k=1}^{K} p_{jk} \le p_{j,max}$$

$$\mathrm{B}: p_{jk} \ge 0 \qquad (3\text{-}18)$$

where $w \in [0, 1]$ is the weighting parameter

However the problem is non convex due to intra user interference coupling. In order to decouple the interference an interference constraint I_{jk} is used.

$$(1+q_{jk}\beta_{jjk})\sum_{m=1\ (m)\neq(k)}^{K} p_{jm}\beta_{jjm} \le\ I_{jk} \qquad (3\text{-}19)$$

The sum rate optimization problem is converted into a convex optimization problem by substituting the interference term in Eq. (3-11) by I_{jk}, the $\mathrm{SINR_{jk}}$ expression becomes

$$\mathrm{SINR}_{\mathrm{jk}}^{\mathrm{MRC}} = \frac{\mathrm{q_{jk}}\beta_{\mathrm{jjk}}{}^{2}\mathrm{p_{jk}}(N-1)}{I_{jk} + \sigma_j} \tag{3-20}$$

The solution is obtained by the Lagrangian technique. Based on the KKT conditions the optimal solution is obtained by taking the derivative of the dual problem and equating it to zero [45].

The approaches presented by [38] , [39], [41], [42] are similar. The problem formulation in [41], [38] can be summarized as

$$\max_{\{\mathrm{p_{jk}}\}\in\mathbb{R}} \sum_{\mathrm{k}=1}^{\mathrm{K}} \left\{\log_2\left(1+\ \mathrm{SINR_{jk}}\right)\right\}$$

$$\begin{aligned} &\text{subject to:} \\ &\mathrm{A}:\ \mathrm{p_{jk}} \le \mathrm{P_{jk,max}} \\ &\mathrm{B}:\mathrm{p_k} \ge 0 \end{aligned} \tag{3-21}$$

The above problem is not convex in general and hence cannot be solved by convex optimization techniques. The problem in [38] is solved using the CVX package (Matlab Software for Disciplined Convex Programming) and the problem in [42] is solved using the alternating optimization technique.

The other problem encountered by many of the authors cited above is the concavity of the sum rate expression $\sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{C_{jk}}$ in the objective function of the problem formulation. The convexity of the problem depends largely on the inter-user interference in the denominator of the SINR expression. The authors in [40] go on to make an assumption to simplify the capacity of the channel summarized in the equation below

$$\mathrm{C_{jk}} = \left\{\log_2\left(1+\ \mathrm{SINR_{jk}}\right)\right\} \approx \left\{\log_2\left(\ \mathrm{SINR_{jk}}\right)\right\} \tag{3-22}$$

They argue that in high SINR region the SINR is much larger than 1.They go on to prove that the simplified expression for the sum rate is concave regardless of the coupling caused by the inter-user interference. In [45], [11] an interference temperature constraint is added to limit the interference, the interference temperature then replaces the interference expression so as to ensure concavity of the objective function. Replacing the interference term by the interference temperature lower bounds the data rate of the UT and decouples the interference from the objective function.

Generally Section 3.2.1 and Section 3.2.2 discuss the different resource allocation solutions in a single cell massive MIMO system. As noted above in both Section 3.2.1 and Section 3.2.2, the problem in single cell resource allocation is the intra- cell interference. The authors propose two approaches, in the first approach they decouple the intra-cell interference using an interference temperature. They then replace the intra-cell interference term in the denominator of the SINR expression by the interference temperature, thereby lower bounding the SINR. In the second approach they allocate resources iteratively per user by assuming the resource allocation values of the other users are constant per iteration. The second approach has a higher complexity than the first one since resource values are updated iteratively until convergence. Authors for both approaches argue that the approaches perform better than assuming equal power allocation.

3.2.3 Global Power Allocation in Multi Cell Massive MIMO

A lot of literature has looked at resource allocation in multi cell massive MIMO both at a global and cellular level. In [11], [19], [46], ,[47], [48], the authors adopt the same system model as the one proposed in Section 2.2. The difference with the system model proposed in this section and the model proposed in Sections 3.2.1 and 3.2.2 is that this section considers a multi cell system. However Sections 3.2.1, 3.2.2 and 3.2.3 propose a constant K users in each cell. The authors in [48], [49],[50] propose a global multi cell massive MIMO resource allocation solution. The authors in [50] consider UL transmission whist the authors in [48], [49] consider DL transmission. Since the authors in [48], [49] consider DL transmission the received signal is modelled as Eq.(2-11) and is repeated below

$$\mathbf{y}_{jk} = \sum_{b=1}^{B} \sqrt{\beta_{jbk}} \mathbf{h}_{jbk}{}^{H} \sum_{m=1}^{K} \sqrt{p_{bm}}\, \mathbf{v}_{bm} x_{bm} + \mathbf{n}_{jk} \tag{3-23}$$

They optimize the Energy Efficiency (EE) subject to a quality of service constraint and maximum power constraint. The EE is defined as the ratio of the sum total rate to the total power consumption. The EE for the whole multi cell massive MIMO system η is given by:

$$\eta = \frac{\sum_{b=1}^{B} \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{C}_{\mathrm{jk}}}{\sum_{b=1}^{B} \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{p}_{\mathrm{bk}} + \sum_{b=1}^{B} \sum_{\mathrm{n}=1}^{\mathrm{N}} \mathrm{p}_{\mathrm{c,jn}}} \tag{3-24}$$

The problem is formulated as follows

$$\max_{\{\mathrm{pj}_{\mathrm{k}}\}\in\mathbb{R}} \quad \eta$$

$$\text{subject to:}$$

$$\begin{aligned} &\mathrm{A}: \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{p}_{\mathrm{jk}} \leq \mathrm{p}_{\mathrm{j,max}} \\ &\mathrm{B}: \mathrm{p}_{\mathrm{jk}} \geq 0 \end{aligned} \tag{3-25}$$

To tackle the problem the EE is converted into a subtractive form. Optimal power allocation is obtained iteratively using the successive convex approximation technique which approximates the objective function based on the following result

$$\log\left(1 + \mathrm{SINR}_{\mathrm{jk}}\right) \geq \mathrm{a}_{\mathrm{jk}}\mathrm{SINR}_{\mathrm{jk}} + \mathrm{b}_{\mathrm{jk}} \tag{3-26}$$

a_{jk} and b_{jk} are adaptively calculated as shown below to get the tightest lower bound

$$\mathrm{a}_{\mathrm{jk}} = \frac{\mathrm{SINR}_{\mathrm{jk}}}{1 + \mathrm{SINR}_{\mathrm{jk}}}, \quad \mathrm{b}_{\mathrm{jk}} = \log\left(1 + \mathrm{SINR}_{\mathrm{jk}}\right) - \frac{\mathrm{SINR}_{\mathrm{jk}}\log(\mathrm{SINR}_{\mathrm{jk}})}{1 + \mathrm{SINR}_{\mathrm{jk}}} \tag{3-27}$$

In [47] and [19] statistically aware power control is considered where each UT k in cell b is allocated

$$p_{bk} = \frac{\pi}{d_b(\phi_{jk})} \qquad (3\text{-}28)$$

where ϕ_{jk} is the geometric position of user k in cell b and $d_b(\phi_{jk})$ gives the variance of channel attenuation from BS b. π is a design parameter such that $\mathbb{E}\{p_{jk}||h_{jjk}||^2\} = K\pi$. Hence the power allocation is not constant but depends on the geometric position of the UTs, and has the advantage of making the effective channel gain for all users the same. However the power allocation schemes proposed in Section 3.2.1 and Section 3.2.2 perform better than the statistically aware power control because they incorporate large scale fading parameters and pilot sequences in their design whilst the statistically aware power control only considers the geometric positions of the users.

3.2.4 Per cell Multi Cell Massive MIMO Power Allocation

The authors in [46], [19],[47], [11], [13],[51], [52] propose a per cell massive MIMO power allocation scheme. The same system model used for Section 3.2.3 is also used for this section. One of the main problems in per cell sub-optimal resource allocation in a multi cellular system is determining the inter cell interference. In [46] they divide the cells into groups that do not interfere with each other. They make an assumption that adjacent cells produce interference whilst other cells are neglected. However cells transmit with maximum sum rate for one time slot, after which there will be rate loss as a result of interference from the neighboring cells. To minimize the interference contribution of each cell the cell power is upper bounded. They propose a sub-optimal per cell optimization approach. The power allocation in the interfering cells is not known since there is no cooperation between cells. To solve the resource allocation problem BS-wise power allocation is used, where the resource allocation is solved per cell iteratively assuming that the inter cell interference is constant until the optimal solution is obtained. This solution however is not the best solution for a large number cells because of its high complexity.

In [46] they divide the optimization method into two parts, a scheduling part and an optimal power allocation part per cell. The scheduling problem is solved by frequency reuse patterns. They design a per cell optimization approach for the central cell in the UL by solving the following optimization problem.

$$\max_{\{p_{jk}\}\in\mathbb{R}} \sum_{k=1}^{K} \{\log_2\left(1 + \mathrm{SINR}_{jk}\right)\}$$

$$\text{Subject to:} \tag{3-29}$$

$$A: \; p_{jk} \leq P_{jk,max}$$

However in the solution the inter-cell interfering power is kept constant for each iteration. The power allocation problem is solved using a water-filling approach. The authors conclude that with an asymptotically large number of BS antennas the power allocation approaches equal power assignment. The approach to the resource allocation problem is similar to the approach in Section 3.2.2 where the interference for a single cell system is kept constant for each iteration. Hence since the solution described for this section is for a multi cell system it is equivalent to repeating the iterative solution in Section 3.2.2 for all the BS

In [11], [52],[53] pilots are reused for channel estimation in neighboring cells hence there is pilot contamination. The authors design a joint pilot and data power allocation scheme. The sum rate of the whole system is optimized while fulfilling the per user rate and power constraints. Hence the power allocated to a user should be smaller than the power used by the pilot plus the power used for data transmission.

The authors in [11] formulate their optimization problem to maximize the system EE in a massive MIMO multi cell UL. The EE, η_{jk} for user k in cell j is given by:

$$\eta_{jk} = \frac{\frac{S-V}{V} C_{jk}}{\frac{1}{S}\left(V q_{jk} + (S-V) p_{jk}\right)} \tag{3-30}$$

The differences between the EE expressions in Eq.(3-24) and Eq.(3-30) is that Eq.(3-24) looks at the DL and Eq.(3-30) looks at the UL. The authors optimized both pilot power allocation and data power allocation. The maximization problem is formulated as:

$$\max_{\{p_{jk}, q_{kj}\} \in \mathbb{R}} \sum_{b=1}^{B} \sum_{k=1}^{K} \eta_{jk}$$

$$\begin{aligned}
&\text{subject to:} \\
&\text{A}: \; q_{bk} + (T - V) p_{bk} \leq p_{bk,max} \\
&\text{B}: \mathrm{C_{bk}} \geq \mathrm{C_{bk,min}} \\
&\text{C}: q_{bk} \geq 0, p_{bk} \geq 0
\end{aligned} \tag{3-31}$$

They assume a non-cooperative approach in game theory where each cell optimizes its SE whilst there is fixed resource allocation in neighboring cells. The interference is separated into inter-cell interference and intra-cell interference. The inter-cell interference is assumed to be constant and each user maximizes its EE independently. The authors go on to discuss that since the users in each cell are coupled by intra cell interference, optimizing per user cannot be parallelized i.e.

$$\max \sum_{\mathrm{k}=1}^{\mathrm{K}} \left\{\log_2\left(1 + \mathrm{SINR_{jk}}\right)\right\} \neq \sum_{\mathrm{k}=1}^{\mathrm{K}} \max\left\{\log_2\left(1 + \mathrm{SINR_{jk}}\right)\right\} \tag{3-32}$$

An additional constraint is added which acts as an interference temperature.

$$\sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{p_{jk}} \beta_{\mathrm{jjk}} \leq I_{jk} \tag{3-33}$$

The SINR for each UT is lower bounded by replacing the intra-cell interference term with interference temperature which modifies the maximization problem. Replacing the intra-cell interference term with interference temperature decouples the users in each cell such that optimization per user can be parallelized. The intra-cell interference is modelled similarly to the single cell system in Section 3.2.2 where an interference temperature is introduced to decouple the intra-cell interference.

In [13] the intra-user interference is eliminated by making the assumption the BS has perfect CSI and zero forcing beamforming (ZFBF) is used. Hence only the inter cell interference remains and can be obtained as

$$I_{jk} = \sum_{b=1, b \neq j}^{B} p_{bm} \mathbb{E} |\mathbf{v}_{jk}{}^{H} \mathbf{h}_{jbm}|^2 \tag{3-34}$$

The problem that maximizes the EE is formulated subject to some power constraints. The authors argue that maximizing the EE is equivalent to minimizing the total energy consumption, since EE is defined as the ratio of the system sum rate to the energy consumption. Therefore the problem minimizes the transmit power and is formulated as follows.

$$\begin{aligned} &\min_{\{p_{bk}\} \in \mathbb{R}} \sum_{b=1}^{B} \sum_{k=1}^{K} p_{bk} \\ &\qquad \text{subject to:} \\ &\qquad A: \sum_{k=1}^{K} p_{bk} \leq p_{bk,max} \\ &\qquad B: C_{bk} \geq C_{bk,min} \end{aligned} \tag{3-35}$$

However since the optimization is sub-optimal per cell basis, there is no cooperation among the BSs, the BS has no knowledge of the power allocation results of the interfering cells. Most papers in literature consider global solution, which solves the problem by allocating power iteratively however the complexity is high when the cell number is large. In [13] the interference is modelled in two ways: first, they assume all interfering cells transmit with p_{max} regardless of the data rate requirements of the user. They also model the interference by allocating power depending on the instantaneous CSI of all the users. The problem is then converted into a linear programming problem. Assuming constant inter cell interfering power does not give a true reflection of the resource allocation in the neighboring cells, hence in this research we define an approximation for the inter-cell interference taking into consideration pilot contamination, large scale fading parameters etc.

3.3 User Selection

Not much work has been done on user selection in massive MIMO however there is a lot of work that has been done on user selection in multi user MIMO systems. The multi user MIMO user selection algorithms can generally be divided into capacity based user selection and Frobenius norm based user selection [54], [55], [56], [8], [57]. In all the multi cell MIMO systems listed above a single cell with one BS and K UTs is considered. Each BS is equipped with N antennas.

The authors in [54] propose two user selection algorithms, they use same system model described in Section 3.2.2. The received signal is modelled as in Eq.(3-9). They propose a capacity based suboptimal user selection algorithm which greedily maximizes both capacity and throughput. The algorithm selects the user with the highest capacity, then finds the user that provides the highest throughput with the selected users. The algorithm terminates if the total throughput decreases when more users are selected. This algorithm works if the assumption that there is no inter user interference is made. In a case where the interference is not neglected, the user's interference contribution should also be taken into account.

The second algorithm proposed by authors in [54] is the Frobenius norm user selection algorithm. This algorithm is based on channel Frobenius norm. The authors argue that the capacity of the channel is related to the eigenvalues of the effective channels after precoding [54]. The Frobenius norm gives an indication of the overall energy of the channel. The Frobenius norm can be calculated as

$$||\mathbf{h}_{\mathrm{jk}}||_{\mathrm{F}}^{2} = \mathbf{h}_{\mathrm{jk}}\mathbf{h}_{\mathrm{jk}}{}^{\mathrm{H}} \tag{3-36}$$

The algorithm selects the users that gives the highest Frobenius norm (effective channel energy) together with the already selected users. Some of these ideas have been adopted for a massive MIMO system. Based on some simulation results the authors conclude that although the Frobenius norm based algorithm has better computational complexity, the capacity based algorithm performs better. The capacity based user selection algorithm incorporates optimal power allocation, the same algorithm can be extended to a multi cell massive MIMO system however for global solution the computational complexity will be very high. The authors also assumed that the CSI is known and interference is negligible thereby simplifying the problem. The problem becomes complicated when there is interference, the solution for the capacity based algorithm has to incorporate the interference contribution from other uses. The Frobenius norm algorithm solution is independent of the power of the UT, hence the solution doesn't incorporate power allocation.

The authors in [8] propose a user selection algorithm for downlink multiuser MIMO systems. The authors use same system model described in Section 3.2.2. The received signal is modelled as in Eq.(3-9). They also assume that the BS has full CSI for all the UTs and the precoder is designed in a way that eliminates interference.

$$\mathbf{h}_{bk}\mathbf{v}_{bm} = 0, \text{for all } k \neq m \tag{3-37}$$

The solution is based on the fact that the capacity of MU-MIMO is directly related to the gain of the channel, g_{jk} (see Eq. (3-38)). However equal power allocation is assumed to all selected UTs.

$$C_{jk} = \left\{\log_2\left(1 + \frac{p_{jk}g_{jk}}{\sigma^2}\right\}\right. \tag{3-38}$$

The user with the highest gain, therefore highest SINR is chosen. The algorithm would perform better if power allocation is in cooperated in the solution because the capacity is also directly proportional to the power allocated to user k.

The authors in [58], [59] focus on the downlink user selection of a single cell massive MIMO system. The system model is similar to the system model in Section 3.2.2. There is no inter user interference because the system is a single cell system. This simplifies the problem because there is no dependency on other cells and hence the inter-cell interference doesn't need to be approximated.

The authors in [58] adopt a code word precoding scheme, where each user chooses a code word s_{jk} from the code book. The code book used in [58] is based on discrete Fourier transform (DFT). To maximize the system sum rate, the interference from other users should be minimized. To achieve this the authors proposed a solution that minimizes the correlation between precoding vectors. Hence the precoding vectors should be orthogonal. This however is very difficult to achieve if the system is large. A user selection variable, r_{jk} is defined such that

$$r_{jk} = \begin{cases} 1, & \text{if UT k is selected} \\ 0, & \text{otherwise} \end{cases} \tag{3-39}$$

r_j is then defined as a diagonal matrix, which has r_{jk} as its diagonal elements. The problem is formulated as a convex optimization problem and can be summarized as

$$\begin{gathered} \min_{r_{jk}} ||\mathbf{S}_j(\mathbf{r}_j)^H \mathbf{S}_j \mathbf{r}_j|| \\ \text{Subject to} \\ \text{A: } r_{jk} \in \{0,1\} \\ \text{B: } \sum_{k=1}^{K} r_{jk} = K^* \end{gathered} \tag{3-40}$$

The problem is solved using a linear programming relaxation method. However the solution in [58] does not explicitly state how the total number of optimal users is calculated. The solution only describes how to choose the individual users that give the optimal solution given K^*, the optimal number of users. For optimal user selection, optimal power allocation should be designed. The dissertation however aims to add on to the work that was done in [58] by designing a power allocation scheme and also designing a user selection scheme that determines the total number of selected users for optimal conditions.

The authors in [47] and [19] define an algorithm that estimates the total number of users that can be scheduled per cell in a massive MIMO system. The paper proposes the same multi cell massive

MIMO model used in Section 2.2, Section 3.2.1 and Section 3.2.2, where each cell has one BS with N antennas and K users. CSI is obtained by pilot signaling, each cell uses only a subset of the pilots and hence the pilots are bound to be reused leading to pilot contamination. Both [19] and [47] makes use of the MMSE channel estimation technique to estimate the channel. To simplify the ergodic capacity expression they look at the asymptotic limit i.e. when $N \rightarrow \infty$. The authors in [19] concluded that the optimum number of users scheduled per cell is K^* given in Eq.(3-41). The authors derived the expression for K^* as $M \rightarrow \infty$.

$$K^* = \frac{KS}{2V} \tag{3-41}$$

Where S is the total number of symbols sent during UL, and V is the number of symbols sent during pilot signaling.

The proposed solution however is not practical because the number of BS antennas cannot be infinity. This research proposes a more practical solution where the number of antennas is finite.

In [60], [61], [62], [63], the problem of BS-user association in massive MIMO hetnets is considered. They consider a system with B BSs and K users. The system model proposed by the authors in [63], [62] is similar to the one described in Section 3.2.3. The received signal model is as Eq.(3-23). The authors in [63], [62] model the average rate to a user as in Eq.(3-52). The system consists of a macro cell BS with a massive MIMO and Pico cell BSs with regular antennas. To simplify the resource allocation problem inter-cell interference is ignored. The received signal is the same as Eq.(3-23). In [62], [60], [63] all the users are allocated constant power. In [62],[60],[63] the problem is formulated as

$$\min_{\{p_{bl} \geq 0\} \in \mathbb{R}} \sum_{l=1}^{K} U\left(\sum_{b=1}^{B} r_{bl} C_{bl}\right)$$

subject to:

$$A : r_{bl} \leq 1 \tag{3-42}$$

$$B : r_{bl} \geq 0$$

$$C : \sum_{k=1}^{K} r_{bl} \leq K$$

The authors in [61] formulate their problem slightly different, rather than maximizing the sum rate, they maximize the energy efficiency. To preserve some degree of fairness the authors in [60], [61], [62], [63] adopt the energy efficiency proportional fairness criterion where U(·) =log(·). Convex optimization techniques are used to solve the problem.

3.4 Joint User Selection and Power Allocation

Several work have considered the problem of BS-user association in massive MIMO systems. The authors in [28] and [64] propose a multicellular system with B cells and each BS has N antennas. The total number of UTs is K, the K users do not have predefined cell indices and can be served by any BS. Their received signal is a superposition of the desired signal from all the BS, which differs from our received signal since in our system model each user is served by the BS in its cell.

$$\mathbf{y}_{jk} = \sum_{b=1}^{B} \overbrace{r_{bk}\sqrt{p_{bk}}\mathbf{v}_{bk}{}^{H}\mathbf{h}_{bk}x_{bk}}^{\text{desired}} + \overbrace{\sum_{b=1}^{B}\sum_{m=1\,,(b,m)\neq(j,k)}^{K} \sqrt{p_{bm}}r_{bm}\mathbf{v}_{jk}{}^{H}\,\mathbf{h}_{bm}x_{bm}}^{\text{interference}} + \overbrace{\mathbf{v}_{jk}{}^{H}\mathbf{N}_{j}}^{\text{noise}} \tag{3-43}$$

To solve the BS-user association problem, they formulate the problem as a power minimization problem, minimizing the BS transmit power, p_b.

$$\min_{\{p_{bk,} r_{bk,} k\geq 0\}\in\mathbb{R}} \sum_{b=1}^{B} \sum_{k=1}^{K} p_{bk,} r_{bk}$$

subject to:

$$A: \sum_{k=1}^{K} p_{bk} \leq p_{b,max},$$

$$B: C_{bk} \geq C_{k,min} \tag{3-44}$$

The similarities between our work and the work done in [28] and [64] is that the we consider quality of service (QoS) long term constraints that do not dependent on instantaneous fading realizations such as proposed in [65]. The authors in [28] and [64] conclude that BS-user association is affected by many factors such as interference, power allocation, large scale fading and QoS constraints. The work done in [64] and [28] differs from our work in they give a global solution and for a very large system the computational complexity is very high, where as our solution is a suboptimal solution which optimizes sum rate per cell.

In [51] resource allocation in the DL of a single cell massive MIMO system is considered. The authors propose the same system model described in Section 3.2.2. The received signal model is the same as in Eq.(3-9). The authors propose an algorithm that performs power allocation, user selection and antenna selection. The system model comprises of an N- antenna BS and K single-antenna UTs. The authors design a sub-optimal large scale fading (LSF) based resource allocation scheme. The resource allocation algorithm is transformed into two sub problems, they first optimize the users selected, K_c and the number of antennas N_c under constant power. They then perform optimal power allocation. The authors formulate their optimization problem as follows

$$\max \frac{\sum_{b=1}^{B} \sum_{k=1}^{K} C_{jk}}{\sum_{b=1}^{B} \sum_{k=1}^{K} p_{bk} + \sum_{b=1}^{B} \sum_{n=1}^{N} p_{c,jn}}$$

subject to:

$$A: N_j \in N,$$

$$B: K_j \in K \tag{3-45}$$

$$C : C_{jk} \geq C_{k,min}$$

The authors first order the UTs in descending order with respect to the separation distance between the UT and the BS (d_k). They start off with $N_j = 0$ and add UTs one at a time until $\frac{R_j}{P_j}$ starts decreasing under constant power. After obtaining the optimal number of users, they go on to perform power allocation using the Dinkelbach's method described in [66]. The above resource allocation solution only looks at a single cell massive MIMO scenario, but does not address multi cell massive MIMO systems.

The authors in [14] and [67] design a user scheduling and power allocation algorithm that ensure that the network user capacity is achieved for DL massive MIMO transmission. An SINR from the N antennas at BS j to the UT k is derived as:

$$\mathrm{SINR}_{jk}^{MRC} = \frac{\beta_{jjk} p_{jk}}{\sum_{b=1}^{B} \sum_{m=1\ (b,m)\neq(j,k)}^{K} p_{bm} \beta_{jbm} ((s_{jk}^{T} s_{bm})^2 J_{jbm}^2) + \frac{\alpha_{jk}}{N} \overline{R_{jk}}} \tag{3-46}$$

Where $\overline{R_{jk}} = \sum_{b=1}^{B} \sum_{m=1}^{K_b} r_{bm} p_{bm} \beta_{jbm} + \sigma_w^2$, $\alpha_{jk} = \sum_{\substack{t=v \\ u=z}} \psi_{tujk}^2 J_{jtu}^2 + \sigma^2$ and $J_{jbm} = \sqrt{q_{bm}\beta_{jbm}}$

Proof: see Appendix A2

The authors then derive an asymptotic expression for the SINR when the number of antennas approach infinity. The asymptotic SINR is derived to be:

$$\mathrm{SINR}_{jk}^{MRC} = \frac{\beta_{jjk} p_{jk}}{\sum_{b=1}^{B} \sum_{m=1}^{K} p_{bm} \beta_{jbm} \left(\left(s_{jk}^{T} s_{bm} \right)^2 J_{jbm}^2 \right) - \beta_{jjk} p_{jk}} \tag{3-47}$$

The authors use Eq.(3-47) to prove that in multi cell massive MIMO system the total number of users, K_s that can be served simultaneously through the DL transmission is upper bounded by:

$$K_s \leq \sqrt{V \sum_{b=1}^{B} \sum_{k=1}^{K} \frac{1 + SINR_{min,bk}}{SINR_{min,bk}}} \tag{3-48}$$

The work presented in [14] and [67] forms the basis of the two stage algorithm derived in Chapter 5 of this dissertation. The differences are that the authors in [14] consider the optimal number of users to be scheduled for the whole multi cell massive MIMO system whilst this research derives an expression only for the cell of interest. To allocate power amongst users the authors derive an expression for the upper limit of the maximum permitted SINR in cell j as

$$SINR_j \leq \frac{1}{B - 1} \tag{3-49}$$

The authors argue that since the effective bandwidth can be defined as

$$W = \frac{SINR_{max,j}}{1 + SINR_{max,j}} \tag{3-50}$$

The power allocated to user k in cell b is chosen to be

$$p_{jk} = \frac{SINR_{max,j}}{1 + SINR_{max,j}} \tag{3-51}$$

The authors in [63], [62] design a user association and power allocation algorithm in massive MIMO systems where $N \gg K$. The system model proposed by the authors in [63], [62] is similar to the one described in Section 3.2.3. The received signal model is as Eq.(3-23). The authors in [63], [62] model the average rate to a user as:

$$C_{jk} = log_2\left(1 + \frac{N - K + 1}{N} \frac{p_{jk}\beta_{jjk}}{1 + \sum_{b=1}^{B}\sum_{m=1}^{K} p_{jm}\beta_{jbm}}\right) \tag{3-52}$$

The resource allocation problem defined in [63] amounts to finding an association of each user to a BS. Ideally the algorithm they define tries to find the BS-user association with sufficiently high SINR values. The authors define an activity fraction r_{jk}, the fraction of BSs over which user k is served. Hence in their design a user can be served by multiple BSs. The BS which can potentially serve user k are denoted as J_k. Hence the total sum rate user k in cell j can achieve can be summarized as:

$$C_{jk,tot} = \sum_{j \epsilon J_k} r_{jk} C_{jk} \tag{3-53}$$

The resource allocation problem is then formulated as follows:

$$\max_{\{p_{bk} \geq 0, r_{bk}\} \in \mathbb{R}} \sum_{k=1}^{K} \sum_{j \epsilon J_k} r_{jk} C_{jk}$$

$$\text{subject to:}$$

$$\text{A: } p_{bk} \geq 0$$

$$\text{B: } \sum_{j \epsilon J_k} r_{jk} \leq 1$$

$$\text{C: } \sum_{k=1}^{K} r_{bk} \leq K \tag{3-54}$$

The above problem is non convex because of the user scheduling variable r_{bk}. The problem is then converted to a convex problem by relaxing the user scheduling variable. The problem is solved using the dual sub gradient method. Dual variables are introduced for each constraint which are then solved iteratively using the sub gradient method.

The authors adopted a global solution whose complexity increases with the number of cells. Hence the solution cannot be adopted for multi cell systems with a large number of BS

3.5 Chapter Summary

As noted above multi cellular massive MIMO systems are largely affected by both inter-cell and intra-cell interference. The authors propose several approaches for modelling the intra cell interference; in the first approach they decouple the intra-cell interference using an interference temperature. They then replace the intra-cell interference term in the denominator of the SINR expression by the interference temperature, thereby lower bounding the SINR. In the second approach they allocate resources iteratively per user by assuming the resource allocation values of the other users are constant per iteration. The second approach has a higher complexity than the first one since resource values are updated iteratively until convergence. Authors for both approaches argue that the approaches perform better than assuming equal power allocation.

Several papers in literature have modeled inter-cell interference as a constant interference by allocating equal power to all the inter-cell interfering users. This however is not the best model for the inter-cell interference because there is optimal resource allocation in the interfering cells. To simplify the multi cell resource allocation problem other authors considered global resource allocation, however global solutions become infeasible as the number of cells increase. The research looks into designing an inter-cell interference model based on the intra-cell interference.

CHAPTER 4

4 USER SELECTION AND POWER ALLOCATION BASED ON INTERCELL INTERFERENCE MODELLING

4.1 Introduction

This chapter derives a new algorithm for optimal user selection and power allocation in multi cell massive MIMO systems for a particular cell of interest. The main objective is to model inter-cell interference so that global resource allocation problem can be converted to a per cell resource allocation problem. To model the inter cell interference it is assumed that the statistical properties of the intra-cell interference in the cell of interest are the same as in the other interfering cells. Therefore the inter-cell interference of the cell of interest can be modelled in terms of the intra-cell interference multiplied by a correction factor. The correction factor takes into consideration pilot transmission, large scale fading and also the distance the users causing inter-cell interference are from the user of interest k in cell j. Hence the inter-cell interference is not evaluated but approximated using the intra-cell interference multiplied by a correction factor.

The problem is modelled as a non-convex optimization problem which is nondeterministic in polynomial time because the user selection variable can only take binary integer values. To convert the non-convex optimization into a convex problem, the user selection variable is relaxed. The Lagrangian frame work and the Newton Raphson method are employed to determine the optimal solution. The BnB technique is then used for optimal user selection. In general simulation results show that for optimal resource allocation in multi cell massive MIMO system, it is preferable to assign orthogonal pilots to users. It was also observed that the maximum power allocated to a user should be chosen carefully so as to minimize interference and maximize cell sum rate.

The rest of the chapter is organized as follows: Section 4.2 presents the system model, channel estimation and uplink transmission. Section 4.3 presents inter cell interference modelling. Section 4.4 describes how the sum rate maximization problem is formulated. Section 4.5 details how the non-convex problem is converted into a convex optimization problem. Section 4.6 presents the

BnB algorithm for optimal resource allocation in massive MIMO system. Simulation based performance and evaluation is presented in Section 4.7. Section 4.8 summarizes the chapter.

4.2 System Model

The system is modelled as described in Chapter 2.2 for UL transmission. The system model is similar to the system model proposed in previous research work discussed in Section 3.2.3 and Section 3.2.4. The authors described in Section 3.2.3 considered global resource allocation which becomes infeasible as the number of cells increases, in this research we are going to adopt the approach taken in Section 3.2.4 where resource allocation is performed per cell. However rather than allocating resources for all cells we are going to allocate resources for a particular cell of interest j taking into consideration the inter-cell interference from other cells. In this research the inter-cell interference is modelled in terms of the statistical properties of the intra-cell interference rather than modelling it as a constant interference or iteratively updating it as described in Section 3.2.4. Section 3.2.2 gives two methods of decoupling the intra-cell interference; in the first approach intra-cell interference is updated iteratively until the problem converges to an optimal solution. The second approach replaces the intra-cell interference by an interference temperature thereby lower bounding the sum rate. The first approach performs better than the second approach although it has a higher computational complexity, however, the authors in [45] argue that assuming an intra-cell interference temperature gives a performance lower bound of the original sum rate maximization problem and does not give an accurate representation of the intra-cell interference. In this research we are going to update the intra-cell interference iteratively until convergence.

In addition to the system model described in Section 2.2 for a multi cell massive MIMO UL system, a user selection variable $\mathrm{r_{bm}} \epsilon \{1,0\}$ is added such that if a user m in cell b is selected for transmission $\mathrm{r_{bm}} = 1$ and if the user is not selected $\mathrm{r_{bm}} = 0$. The above argument is summarized in Eq. (4-1) below

$$r_{bm} = \begin{cases} 1, & \text{if UT m in cell b is selected} \\ 0, & \text{otherwise} \end{cases} \tag{4-1}$$

4.2.1 Channel Estimation

During the UL training phase each user sends pilot sequences to enable channel estimation. The pilot sequence assigned to user m in cell b is denoted by a $\tau \times 1$ matrix, $\boldsymbol{s}_{bm}$ where τ is the pilot length.

The received signal at BS j, $\mathbf{Y}_j \in \mathbb{C}^N$ during pilot transmission can be modelled as Eq. (4-2) below. The signal in Eq. (4-2) is modelled in the same way as the received signal in Eq. (2-14).

$$\mathbf{Y}_j = \sum_{b=1}^{B} \sum_{m=1}^{K} \sqrt{q_{bm}\beta_{jbm}}\, \mathbf{h}_{jbm} \mathbf{S}_{bm} + \mathbf{N}_j \tag{4-2}$$

Where $\mathbf{S}_{bm} = \mathbf{s}_{bm} \otimes \mathbf{I}_N$ is a $\tau N \times N$ matrix, $\mathbf{h}_{jbm}$ is the UL channel vector from user m in cell b to a BS in cell j. The corresponding pilot transmit power is q_{bm} , $\mathbf{N}_j$ is the noise due to the transmitted pilot symbols. The noise is modelled as Gaussian additive noise with each modelled as a complex Gaussian distribution CN $(0,\sigma^2 I_N)$.

The correlation between different pilot sequences is modelled as $\psi_{bmjk} = \mathbf{s}_{jk}^T \mathbf{s}_{bm}$. The correlation factor ψ_{bmjk} ranges from -1 to +1. Where 0 indicates orthogonal sequences, -1 and +1 indicates negative and positive correlation respectively. Hence ψ_{bmjk} cannot be zero for some $(b,m)\neq(j,k)$ giving rise to pilot contamination. From the above argument it can therefore be deduced that $\mathbf{S}_{bm}{}^T \mathbf{S}_{bm} = \mathbf{I}_N$.

The channel estimate is obtained using the low complexity least square (LS) channel estimation method. The LS mean error has been proved to remain constant as the number of antennas increase, which make it ideal for massive MIMO networks [68].

Lemma 1. The expression for the LS channel estimate, $\widehat{\mathbf{h}_{jjk}}$ for a user k in cell j to a BS in cell j is

$$\widehat{\mathbf{h}_{jjk}} = \mathbf{h}_{jjk} + \sum_{b=1}^{B} \sum_{m=1,(b,m)\neq(j,k)}^{K} \psi_{bmjk} \sqrt{q_{bm}\beta_{jbm}}\, \mathbf{h}_{jbm} + \mathbf{N}_j \mathbf{S}_{jk}^{T} \quad (4\text{-}3)$$

Proof: The proof is given in the Appendix A1

We assume that power control is enabled, such that $q_{jk}\beta_{jjk} = 1$, so that the powers received by the BS from all the UTs are the same [14],[69].

It can be easily seen from the equation above that the estimated channel, $\widehat{\mathbf{h}_{jjk}}$ is a combination of the actual channel, $\mathbf{h}_{jjk}$ and some error arising from pilot contamination when non orthogonal pilots are used, i.e., $\psi_{bmjk} \neq 0$ (correlation between pilot sequences assigned to different users is not equal to zero).

4.2.2 Uplink Transmission

The received signal, $\mathbf{y}_j \in \mathbb{C}^N$ during UL transmission before applying the receive beamforming vector can be modelled in the same way as Eq. (2-2), the difference is in the addition of a user selection variable r_{bm} in Eq. (4-4) below.

$$\mathbf{y}_j = \sum_{b=1}^{B} \sum_{m=1}^{K} r_{bm} \sqrt{p_{bm}\beta_{jbm}}\, \mathbf{h}_{jbm} x_{bm} + \mathbf{n}_j \quad (4\text{-}4)$$

where x_{bm} is the symbol transmitted by user m in cell b. The signal is normalized as $\mathbb{E}\{{x_{bm}}^{H} x_{bm}\} = 1$, while p_{bm} is the corresponding UL transmit power The noise is modelled as Gaussian additive noise with each modelled as a complex Gaussian distribution CN (0, σ_w^2).

Since the BSs have multiple antennas, in the UL, the base station should apply a receive beamforming vector, $\mathbf{v}_{jk}$ so as to amplify the desired signal and reject interference. This is achieved by multiplying the received signal by ${\mathbf{v}_{jk}}^{H}$ [19],[28].

$$\hat{\mathbf{y}}_{jk} = \mathbf{v}_{jk}{}^{H}\, \mathbf{y}_{jk} \tag{4-5}$$

Where $\hat{\mathbf{y}}_{jk}$ is the received signal after precoding

The received vector, after linear beamforming is given by

$$\hat{\mathbf{y}}_{jk} = \sum_{b=1}^{B} \sum_{m=1}^{K} \sqrt{p_{bm}\beta_{jbm}}\, r_{bm} \mathbf{v}_{jk}{}^{H}\, \mathbf{h}_{jbm} x_{bm} + \mathbf{v}_{jk}{}^{H} \mathbf{n}_{j} \tag{4-6}$$

This can further be simplified to Eq. (4-7) below if the desired signal is separated from the noise-plus-interference signal.

$$\begin{aligned}\hat{\mathbf{y}}_{jk} = {} & \overbrace{r_{jk}\sqrt{\beta_{jjk} p_{jk}}\, \mathbf{v}_{jk}{}^{H} \mathbf{h}_{jjk} x_{jk}}^{\text{desired}} \\ & + \overbrace{\sum_{b=1}^{B} \sum_{m=1\,,(b,m)\neq(j,k)}^{K} \sqrt{\beta_{jbm} p_{bm}}\, r_{bm} \mathbf{v}_{jk}{}^{H}\, \mathbf{h}_{jbm} x_{bm}}^{\text{interference}} \\ & + \overbrace{\mathbf{v}_{jk}{}^{H} \mathbf{N}_{j}}^{\text{noise}}\end{aligned} \tag{4-7}$$

Lemma 2: The ergodic capacity for user k in cell j is given by

$$C_{jk} = \left(1 - \frac{V}{S}\right) \log_2(\, 1 + \mathrm{SINR}_{jk}\,) \tag{4-8}$$

The UL effective signal-to-interference-and-noise ratio (SINR) for a user k in cell j is given by

$$\mathrm{SINR}_{jk} = \frac{r_{jk}\beta_{jjk}p_{jk} |\mathbb{E}\{\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jjk}\}|^2}{r_{jk}\beta_{jjk}p_{jk}\mathrm{Var}|\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jjk}| + \sum_{b=1,b\neq j}^{B} \sum_{m=1,m\neq k}^{K} p_{bm}\beta_{jbm}r_{bm} \mathbb{E}|\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jbm}\,|^2 + \sigma_w^2 \mathbb{E}|\mathbf{v}_{jk}|^2} \tag{4-9}$$

where $\mathbb{E}\{\cdot\}$ is the expectation with respect to channel realizations and $\left(1 - \frac{V}{S}\right)$ is data transmission fraction.

Proof: The Lemma follows from making an assumption that only the desired signal is received over the effective channel mean $\mathbb{E}\{\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jjk}\}$ whilst the signal received over the uncorrelated channel $\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jjk} - \mathbb{E}\{\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jjk}\}$ and the interference are treated as noise. The noise is taken as worst Gaussian noise. Eq. (4-7) can be rewritten as

$$\hat{\mathbf{y}}_{jk} = r_{jk}\sqrt{\beta_{jjk}p_{jk}}\mathbb{E}\{\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jjk}\}x_{jk} + a_{jk} \quad (4\text{-}10)$$

Where a_{jk} is the noise that is uncorrelated with $r_{jk}\sqrt{\beta_{jjk}p_{jk}}\mathbb{E}\{\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jjk}\}$ and can be expressed as

$$\begin{aligned} a_{jk} = {} & r_{jk}\sqrt{\beta_{jjk}p_{jk}}\left(\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jjk} - \mathbb{E}\{\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jjk}\}\right)x_{jk} \\ & + \sum_{b=1}^{B}\sum_{m=1,(b,m)\neq(j,k)}^{K}\sqrt{\beta_{jbm}p_{bm}}r_{bm}\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jbm}x_{bm} \\ & + \mathbf{v}_{jk}{}^{H}\mathbf{N}_{j} \end{aligned} \quad (4\text{-}11)$$

Based on Eq. (4-11), the effective SINR expression can be expressed as Eq. (4-9).

This research makes use of the MRC receive beamforming vector to amplify the desired signal and reject interference. This is achieved by replacing is $\mathbf{v}_{jk}$ by $\mathbf{h}_{jjk}$ in Eq.(4-9) above (as explained in Section 2.5.1). For clarity the expectation expressions in Eq. (4-9) are computed one at a time

Theorem 1: The simplified SINR after MRC receive beam-forming becomes

$$\mathrm{SINR}_{jk}^{MRC} = \frac{r_{jk}\beta_{jjk}p_{jk}}{\sum_{b=1}^{B}\sum_{m=1\,(b,m)\neq(j,k)}^{K} r_{bm}p_{bm}\beta_{jbm}(\psi_{bmjk}^{2}J_{jbm}^{2}) + \frac{\alpha_{jk}}{N}\overline{R_{jk}}} \quad (4\text{-}12)$$

Where $\overline{R_{jk}} = \sum_{b=1}^{B}\sum_{m=1}^{K} r_{bm}p_{bm}\beta_{jbm} + \sigma_{w}^{2}$ $\alpha_{jk} = \sum_{\substack{t=v\\u=z}} \psi_{tujk}^{2}J_{jtu}^{2} + \sigma^{2}$ and $J_{jbm} = \sqrt{q_{bm}\beta_{jbm}}$

Proof: see Appendix A2

The following conclusions can be drawn from the simplified SINR_{jk} expression in Theorem 1; the SINR_{jk} expression in Eq. (4-13) is easy to compute it is independent of channel realizations.

Theorem 1 also shows that the $\mathrm{SINR_{jk}}$ increases with the number of BS antennas, N. It can also deduced that if $\psi_{\mathrm{bmjk}} \neq 0$ (pilot sequences are not orthogonal) $\mathrm{SINR_{jk}}$ is reduced.

4.3 Interference Modelling

In order to optimally allocate resources for a particular cell of interest in multi cell massive MIMO systems, an expression should be derived to model the inter-cell interference. Let the inter-cell and intra-cell interference experienced by user k in the cell of interest j from other users, $\mathrm{I_{jk}}$ be

$$\mathrm{I_{jk}} = \sum_{b=1}^{B} \sum_{m=1\ (b,m)\neq(j,k)}^{K} p_{bm}\beta_{jbm}r_{bm}\left(\psi_{bmjk}^{2}J_{jbm}^{2}\right) + \frac{\alpha_{jk}}{N}\left(\sum_{b=1}^{B}\sum_{m=1}^{K} p_{bm}\beta_{jbm}r_{bm}\right) \tag{4-14}$$

Which can then be separated into the inter-cell and intra-cell interferences.

$$\mathrm{I_{jk}} = \overbrace{\sum_{m=1,\ m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm}\left(\psi_{jmjk}^{2}J_{jjm}^{2}\right) + \frac{\alpha_{jk}}{N}\sum_{m=1}^{K} p_{jm}\beta_{jjm}r_{jm}}^{\text{intra-cell interference}} + \overbrace{\sum_{b=1,\ b\neq j}^{B}\sum_{m=1}^{K} p_{bm}\beta_{jbm}r_{bm}\left(\psi_{bmjk}^{2}J_{jbm}^{2}\right) + \frac{\alpha_{jk}}{N}\sum_{b=1,\ b\neq j}^{B}\sum_{m=1}^{K} p_{bm}\beta_{jbm}r_{bm}}^{\text{inter-cell interference}} \tag{4-15}$$

The goal of this research is to allocate resources in the cell of interest j, taking into consideration the interference from other cells. The inter-cell interference is unknown and has to be approximated. To model the inter-cell interference we adopt the approach taken in [70], [71] which assumes that the statistical properties of the intra-cell interference in the cell of interest are the same as in the other interfering cells. Using the same argument as above the above expression can be simplified as

$$I_{jk} = \overbrace{\sum_{m=1,\ \ m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm}\left(\psi_{jmjk}^2 J_{jjm}^2\right) + \frac{\alpha_{jk}}{N}\sum_{m=1}^{K} p_{jm}\beta_{jjm}r_{jm}}^{\text{intra}-\text{cell interference}}$$
$$+ \overbrace{f_{jk}\sum_{m=1}^{K} p_{jm}\beta_{jjm}r_{jm}\left(\psi_{jmjk}^2 J_{jjm}^2\right) + f_{jk}\frac{\alpha_{jk}}{N}\sum_{m=1}^{K} p_{jm}\beta_{jjm}r_{jm}}^{\text{inter}-\text{cell interference}} \quad \text{(4-16)}$$

In Eq. (4-16) the inter-cell interference is written in terms of the intra-cell interference. The correction factor, f_{jk} takes into consideration pilot sequences used in the interfering cells in relation to pilot sequences used in the cell of interest and large scale fading between the users in the interfering cells and the users in the cell of interest.

Proposition 1. In multi cell massive MIMO the inter-cell interference can be described in terms of the intra-cell interference multiplied by a correction factor f_{jk} defined as

$$f_{jk} = \frac{\sum_{b=1,\ \ b\neq j}^{B}\sum_{m=1}^{K}\beta_{jbm}\left(\psi_{bmjk}^2 J_{jbm}^2\right) + \frac{\alpha_{jk}}{N}\sum_{b=1,\ \ b\neq j}^{B}\sum_{m=1}^{K}\beta_{jbm}}{\sum_{m=1}^{K}\beta_{jjm}\left(\psi_{jmjk}^2 J_{jjm}^2\right) + \frac{\alpha_{jk}}{N}\sum_{m=1}^{K}\beta_{jjm}} \quad \text{(4-17)}$$

Proof: Refer to Appendix A3

The expression for I_{jk} in Eq. (4-16) can further be simplified to

$$\begin{aligned} I_{jk} =& \\ &\sum_{m=1,\ \ m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm}\left(\psi_{jmjk}^2 J_{jjm}^2\right) + \frac{\alpha_{jk}}{N}\sum_{m=1,m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm} \\ &\quad + \frac{\alpha_{jk}}{N}\left(\, p_{jk}\beta_{jjk}r_{jk}\right) \\ &\quad + f_{jk}\Big(\sum_{m=1,m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm}\left(\psi_{jmjk}^2 J_{jjm}^2\right) \\ &\quad + f_{jk}\frac{\alpha_{jk}}{N}\sum_{m=1,m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm} + f_{jk}p_{jk}\beta_{jjk}r_{jk} \\ &\quad + f_{jk}\frac{\alpha_{jk}}{N} p_{jk}\beta_{jjk}r_{jk} \end{aligned} \quad \text{(4-18)}$$

The expression can be further simplified to

$$I_{jk} = \left(f_{jk}\beta_{jjk} + f_k\frac{\alpha_{jk}}{N}\beta_{jjk} + \frac{\alpha_{jk}}{N}\beta_{jjk}\right)p_{jk}r_{jk} +$$

$$(f_{jk} + 1) \sum_{m=1,\ m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm}\left(\psi_{jmjk}^2 J_{jjm}^2\right) + f_{jk}\frac{\alpha_{jk}}{N} \sum_{m=1,m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm} \quad (4\text{-}19)$$

We can express Eq.

(4-19) in terms of I_1 and I_2 as shown below

$$I_{jk} = I_1 p_{jk} r_{jk} + I_2 \quad (4\text{-}20)$$

where $I_1 = f_{jk}\beta_{jjk} + f_{jk}\frac{\alpha_{jk}}{N}\beta_{jjk} + \frac{\alpha_{jk}}{N}\beta_{jjk}$ and

$$I_2 = (f_{jk} + 1) \ \sum_{m=1,\ m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm}\left(\psi_{jmjk}^2 J_{jjm}^2\right) + \ f_{jk}\frac{\alpha_{jk}}{N} \ \sum_{m=1,m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm}$$

4.4 Problem Formulation

Resource allocation is performed by the BS for users during uplink transmission under the following constraints

- Power constraint:

$$r_{jk}p_{jk} \leq p_{jk,max}, \forall\, j, k \quad (4\text{-}21)$$

Where $p_{jk,max}$ is the maximum transmit power of each user

- To maintain performance a minimum capacity should be set

$$r_{jk}\left(1-\frac{V}{S}\right)\{\log_2(\,1+\mathrm{SINR}_{jk}^{MRC}\}\;\geq\;C_{jk,min}\,,\forall\, j,k \tag{4-22}$$

- User selection constraint: The user selection variable can either be 1 or 0

$$r_{jk}\;\in\{0,1\}\,,\forall\, j,k \tag{4-23}$$

The power allocation and user selection problem can be formulated as a non-convex mixed integer programming problem [72]. Hence the optimization problem can be stated as follows

$$\max_{\{p_{jk},r_{jk}\}\in\mathbb{R}}\sum\nolimits_{k=1}^{K} r_{jk}\left(1-\frac{V}{S}\right)\left\{\log_2\left(1+\frac{p_{jk}\beta_{jjk}}{I_1 r_{jk}p_{jk}\,+\,I_2\,+\,z_{jk}}\right)\right\}$$

$$\begin{aligned}
&\text{subject to:}\\
&A:\; r_{jk}p_{jk}\;\leq\;p_{jk,max},\,\forall\, j,k\\
&B: p_{jk}\geq 0,\,\forall\, j,k\\
&C: r_{jk}\left(1-\frac{V}{S}\right)\left\{\log_2\left(1+\frac{p_{jk}\beta_{jjk}}{I_1 r_{jk}p_{jk}+I_2+z_{jk}}\right)\right\}\geq\;C_{jk,min}\,,\forall\, j,k\\
&D: r_{jk}\;\in\{0,1\}\,,\forall\, j,k
\end{aligned} \tag{4-24}$$

Where z_{jk}, the noise term is $\frac{\alpha_{jk}}{N}\sigma_w^2$

Constraint A limits the transmit power for each UT to be below $p_{jk,max}$, $p_{jk,max}$ is the power budget for the kth user in cell j. The power for each user, p_{jk} should be greater than zero. Constraint C imposes the minimum rate, $C_{jk,min}$ required for each. In D, r_{jk} the user selection variable can either be a 1 or a 0.

4.5 Transformation of Optimization Problem

The above maximization problem is a non-convex integer problem because of the user selection constraint above. The problem can be converted into a convex optimization problem by relaxing r_{jk} to be a continuous variable [73]. The user selection variable, r_{jk} is relaxed to be continuous real variable in the range [0,1]. Therefore r_{jk} can be interpreted as the likelihood/probability that a user is chosen. The power for user k in cell j can there be written as $s_{jk} = r_{jk}p_{jk}$

The problem in Eq.(4-24) can now be converted into:

$$\max_{\{p_{jk},r_{jk}\}\in\mathbb{R}} \sum_{k=1}^{K} r_{jk}\left(1-\frac{V}{S}\right)\left\{\log_2\left(1+\frac{s_{jk}\beta_{jjk}}{r_{jk}\left(I_1 s_{jk} + I_2 + z_{jk}\right)}\right)\right\}$$

subject to:

$$A: s_{jk} \le p_{jk,max}, \forall\, j,k$$

$$B: s_{jk} \ge 0, \forall\, j,k$$

$$C: r_{jk}\left(1-\frac{V}{S}\right)\left\{\log_2\left(1+\frac{s_{jk}\beta_{jjk}}{r_{jk}\left(I_1 s_{jk} + I_2 + z_{jk}\right)}\right)\right\} \ge C_{jk,min}, \forall\, j,k$$

$$D: r_{jk} \le 1, \forall\, j,k \tag{4-25}$$

$$E: r_{jk} \ge 0, \forall\, j,k$$

The objective function in Eq. (4-25) is concave because the Hessian matrix of every element of the objective function with respect to s_{jk} and r_{jk} is negative semi-definite [72]. Since the problem was transformed into a convex problem, it now has a unique optimal solution.

The Lagrangian function from (44) is given by

$$L(\{s_{jk}\},\{r_{jk}\},\nu,\eta,\lambda,\theta) =$$

$$\sum_{k=1}^{K} r_{jk} H\left\{\log_2\left(1+\frac{s_{jk}\beta_{jjk}}{r_{jk}\left(I_1 s_{jk} + I_2 + z_{jk}\right)}\right)\right\} + \sum_{k=1}^{K} \nu_{jk}\left(P_{max} - s_{jk}\right) +$$

$$\sum_{k=1}^{K}\lambda_{jk}(r_{jk}H\left\{\log_2\left(1+\frac{s_{jk}\beta_{jjk}}{r_{jk}(I_1 s_{jk}+I_2+z_{jk})}\right)\right\}-C_{k,min})+\sum_{k=1}^{K}\eta_{jk}\left(1-r_{jk}\right)+\sum_{k=1}^{K}\theta_{jk}\left(r_{jk}\right) \quad (4\text{-}26)$$

Where H is $\left(1-\frac{V}{S}\right)$ and $\nu_{jk}, \lambda_{jk}, \eta_j, \theta_j$ are the Lagrangian multipliers for the constraints A, C,D and C respectively. The dual function can be formulated as [72]

$$g(\nu,\eta,\lambda,\theta)=\inf_{\{s_{jk},r_{jk}\}\in\mathbb{R}} L(\{p_{jk}\},\{r_{jk}\},\nu,\eta,\lambda,\theta) \quad (4\text{-}27)$$

The subsequent dual problem can be expressed as

$$\min_{\nu,\eta,\lambda,\theta} L(\nu,\eta,\lambda,\theta) \quad (4\text{-}28)$$

$$\text{Subject to: } \nu,\eta,\lambda,\theta \geq 0.$$

Constraints B in Eq. (4-25) is absorbed in the Karush-Kuhn-Tucker (KKT) conditions [72]. The optimal solution of the sub problems can be obtained as

$$\frac{\partial L\{\dots\}}{\partial s_{jk}}\begin{cases}=0, p_{jk}>0\\ <0, p_{jk}=0\end{cases} \forall j,k \quad (4\text{-}29)$$

$$\frac{\partial L\{\dots\}}{\partial r_{jk}}\begin{cases}<0, r_{jk}=0\\ =0, 0<r_{jk}<1\ \forall j,k\\ >0, r_{jk}=1\end{cases} \quad (4\text{-}30)$$

By taking the partial derivatives of the Lagrangian with respect to the primal and dual variables, the KKT conditions can be expressed as

$$\frac{\partial L\{\dots\}}{\partial s_{jk}}=\frac{r_{jk}s_{jk}\beta_{jjk}(1+\lambda_{jk})(I_2+z_{jk})}{\ln 2\ (I_2+z_{jk}+I_1 s_{jk})(s_{jk}\beta_{jjk}+I_2 r_{jk}+r_{jk}z_{jk}+I_1 s_{jk}r_{jk})}-v_{jk} \quad (4\text{-}31)$$

$$\frac{\partial L_{jk}\{\dots\}}{\partial r_{jk}} = H\left(1+\lambda_{jk}\right)\log_2\left(1+\frac{s_{jk}\beta_{jjk}}{r_{jk}(I_2+z_{jk}+I_1 s_{jk})}\right) - \frac{H(1+\lambda_{jk})s_{jk}\beta_{jjk}}{\ln 2\times\left(s_{jk}\beta_{jjk}+r_{jk}(I_2+z_{jk}+I_1 s_{jk})\right)} - \eta_j - p_{jk}\nu_{jk} + \theta_{jk} \tag{4-32}$$

$$\nu_{jk}\left(P_{max} - r_{jk}p_{jk}\right) = 0 \tag{4-33}$$

$$\lambda_{jk}\ (r_{jk}H\left\{\log_2(1+\frac{p_{jk}\beta_{jjk}}{I_{jk}+z_{jk}})\right\} = 0 \tag{4-34}$$

$$\eta_{jk}\left(1 - r_{jk}\right) = 0 \tag{4-35}$$

$$\theta_j\left(r_{jk}\right) = 0 \tag{4-36}$$

The optimal power, p_{jk} allocated to user k, in cell j is obtained by setting Eq. (4-31) to zero.

$$s_{jk} = \left(\frac{\sqrt{b^2-4ac}}{2a} - \frac{b}{2a}\right)^+ \tag{4-37}$$

where $a = \left({I_1}^2 r_{jk} + \beta_{jjk}I_1\right)$

$$b = \beta_{jjk}I_2 + 2I_2I_1r_{jk} + z_{jk}\beta_{jjk} + 2I_1r_{jk}z_{jk}$$

$$c = \left({I_2}^2 r_{jk} + 2I_2z_{jk}r_{jk} + {z_{jk}}^2 r_{jk} - \frac{r_{jk}\beta_{jjk}(1+\lambda_{jk})(I_2+z_{jk})}{\nu_{jk}\ln 2}\right)$$

The optimal power allocation above follows a water-filling technique through which power is distributed to the users with a larger gain.

Newton-Raphson's method is applied to determine the optimal user selection variable, r_{jk}

$$r_{jk}{}^{i+1} = r_{jk}{}^{i} - \frac{f(r_{jk}{}^{i})}{f'(r_{jk}{}^{i})} \tag{4-38}$$

The function $f(r_{jk}{}^{i})$ is

$$f(r_{jk}{}^{i}) = (1+\lambda_{jk}) \log_2\left(1 + \frac{s_{jk}\beta_{jjk}}{r_{jk}(I_2 + z_{jk} + I_1 s_{jk})}\right) - \frac{(1+\lambda_{jk}) s_{jk} G_{jjk}}{\ln 2 \times (s_{jk}\beta_{jjk} + r_{jk}(I_2 + z_{jk} + I_1 s_{jk}))} \tag{4-39}$$
$$- \eta_j - p_{jk}\nu_{jk} + \theta_{jk}$$

$$f'(r_{jk}{}^{i}) = -\frac{(1+\lambda_{jk}) s_{jk}{}^{2} \beta_{jjk}{}^{2}}{r_{jk} \ln 2 \times \left(s_{jk}\beta_{jjk} + r_{jk}(I_2 + z_{jk} + I_1 s_{jk})\right)^2} \tag{4-40}$$

The dual variables are obtained by using the sub gradient method, the Lagrangian variables are updated according to:

$$\nu_{jk}^{i+1} = [\nu_{jk}^{i} - \theta_1^{i}\,(p_{jk,max} - r_{jk} p_{jk})]^{+} \tag{4-41}$$

$$\lambda_{jk}^{i+1} = [\lambda_{jk}^{i} - \theta_2^{i}(r_{jk} H \left\{\log_2(1 + \frac{s_{jk} D}{r_{jk} I}\right\} - C_{k,min})]^{+} \tag{4-42}$$

$$\eta_j^{i+1} = \left[\eta_j^{i} - \theta_3^{i}(1 - r_{jk})\right]^{+} \tag{4-43}$$

$$\theta_j^{i+1} = \left[\theta_j^{i} - \theta_3^{i}\left(\theta_{jk}(r_{jk})\right)\right]^{+} \tag{4-44}$$

The choice of the Newton steps, θ_1^i, θ_2^i, θ_3^i and θ_4^i is of paramount importance since it determines the rate of convergence of the dual decomposition problem. Initially both the primal and dual variables are initialized to feasible values and then iteratively updated towards an optimal solution.

The dual decomposition algorithm is simplified in the table below

Dual Decomposition algorithm for cell j

1. Input: I, p_{max}, c_{min}, β_j, N
2. Initialize $r_j = [r_{j1}, \ldots r_{jk}, \ldots r_{jK}] \leftarrow 1_k$, $p_j = [p_{j1}, \ldots p_{jk}, \ldots p_{jK}] \leftarrow 1_k$
3. Initialize $\eta_j = [\eta_{j1}, \ldots \eta_{jk}, \ldots \eta_{jK}] \leftarrow 0_k$, $\lambda_j = [\lambda_{j1}, \ldots \lambda_{jk}, \ldots \lambda_{jK}] \leftarrow 0_k$,

4. Initialize $\nu_j = [\nu_{j1}, \ldots \nu_{jk}, \ldots \nu_{jK}] \leftarrow 0_k$

5. Repeat

6. **for** k = 1 to K **do**

7. sum_rate_max⟵0

8. Calculate f_{jk} according to Eq. (4-17)

9. Calculate I_1 and I_2 according to Eq. (4-20)

10. Update p_{jk}^{i+1} according to Eq. (4-37)

11. Update ${r_{jk}}^{i+1}$ according to ${r_{jk}}^{i+1} = {r_{jk}}^{i} - \frac{f({r_{jk}}^{i})}{f\prime({r_{jk}}^{i})}$ in Eq. (4-39) and Eq. (4-40)

12. Calculate sumrate

13. if sumrate is greater or equal to sum_rate_max

14. sum_rate_max= sumrate

15. end

16. Update λ_{jk}, η_j, ν_{jk}, θ_j according to Eq.(4-42), Eq. (4-43), Eq. (4-41) and Eq. (4-44) respectively

17. End for

18. Until convergence

19. **return** (r_j, p_j)

4.6 Branch and Bound Algorithm

The goal of the BnB algorithm is to obtain the primal optimal solution from the relaxed solution. The relaxed r_{jk} (user selection) values represents the likelihood of a user being chosen. If computation complexity is not an issue the best approach would be to use the Brute force method by computing the solution for all possible combinations of the binary r_{jk} variables and chose the set of r_{jk} values that give maximum sum rate. However as the number of users in a cell increases the computational burden will become overwhelming. The BnB technique significantly reduces the complexity of the solution by pruning “less promising nodes” [74]. The solution obtained after integer relaxation is greater or equal to the solution obtained after BnB technique. Hence the UB of the BnB technique is obtained from the relaxed solution [12].

The BnB algorithm is performed by first selecting the user, k from the relaxed solution with a highest $r_{jk} \in r_j^*$. We then create two branches; the first branch, r_{j_one} by fixing $r_{jk} \leftarrow 1$, the second branch r_{j_zero} by fixing $r_{jk} \leftarrow 0$. The Lagrangian dual decomposition method is used to solve for optimal resource allocation values ($r_{j\_one}^*$,$p_{j\_one}^*$) and ($r_{j\_zero}^*$, $p_{j\_zero}^*$) for the cases when r_{jk} is set to one and zero respectively. The UB is set to the vector that gives the maximum sum rate. The branch that gives a lower sum rate is pruned. The resultant vectors, r_j^* may be comprised of fixed and relaxed values. The above described algorithm is repeated until r_j^* is comprised of only fixed r values. The algorithm is summarized below.

Branch and Bound Algorithm

Input: I, p_{max}, c_{min} , β, N
For the cell of interest, j
    Repeat
        Compute r_j^* and p_j^*, Using the dual decomposition algorithm
        If $r_j^* \in \{0,1\}\ \forall$ k
            Exit
        end
        Select maximum fraction, r_{jk} from relaxed r_j^*
        Branch: $\widehat{r_{jk}} \leftarrow 1$
            Compute $r_{j\_one}^*$ and $p_{j\_one}^*$, Using the dual decomposition algorithm
            Calculate sumrate_one when ($r_{j\_one}^*$, $p_{j\_one}^*$)
        Branch: $\widehat{r_{jk}} \leftarrow 0$
            Compute $r_{j\_zero}^*$ and $p_{j\_zero}^*$, Using the dual decomposition algorithm
            Calculate sumrate_zero when ($r_{j\_zero}^*$, $p_{j\_zero}^*$)
        If sumrate_one >= sum_rate_zero
            (r_j^* , p_j^*)=($r_{j\_one}^*$, $p_{j\_one}^*$)
        else
            (r_j^* , p_j^*)=($r_{j\_zero}^*$, $p_{j\_zero}^*$)

19. **end**

20. **until** $r_j^* \in \{0,1\}\ \forall k$

21. **end for**

4.7 Complexity Anlysis of BnB Algorithm

The brute force approach that gives an optimal solution has exponential complexity dependent on the number of users. Since the whole system has BK users and B BSs the overall complexity for the brute-force method is .$O(B^3K^2 2^{BK})$ [75] The dual decomposition method described above reduces the exponential complexity into linear unconstrained optimization problems [73], [22]. The complexity of the proposed BnB depends on the number of visited and nodes. The proposed BnB algorithm solves 2K(K-1) convex sub problems per cell with a complexity of $O(K^2)$ [75]. Obtaining the primal solution from the relaxed solution requires $O(K)$ computations which has linear complexity.

4.8 Simulations

4.8.1 Simulation Model

In this section the performance of the proposed BnB algorithm is analyzed and evaluated. The simulator used is custom built in MATLAB. In the simulations the BSs and UEs are deployed as shown in Figure 2-2 above. Each cell consists of one central BS and several UTs. To simplify the problem the number of UTs in each cell is fixed to K (all the cells have equal number of UTs). The cell can be divided into four equal quadrants, the UTs are randomly put around the cell making sure that there is at least one UT in each quadrant. The UTs are randomly put at a minimum distance of 50 m and a maximum distance of 250m from the BS. The BSs have a minimum distance of 500m between them. Each BS is equipped with N antennas. Pilot sequences are reused in all cells. The pilot correlation in each cell ranges from zero (orthogonal pilots) to one (non-orthogonal pilots). Since the project focuses on UL transmission, the UL data transmission fraction is 0.5. The large scale fading are dependent on the distance of the UTs from the BS and are modelled as [1].

$$\beta_{jbm} = 10^{-(128.1+37.6\log(d_{jbm})+\sigma_m g_{jbm}+n)/10} \tag{4-45}$$

Where d_{jbm} is the distance of the the mth user in cell b from the jth BS, σ_m is the shadow fading standard deviation which is set to 5dB. g_{jbm} is the shadow fading realization. The simulation parameters are set as shown in Table 4-1. To simplify the resource allocation problem we assume that the maximum power that can be allocated to a user is the same for all the users in the massive MIMO network hence $p_{jk,max}$ = pmax ∀ j,k.

Table 4-1: Simulation parameters for all proposed algorithms

Parameter	**Value**
Number of users per cell	10
Number of Base Stations	9
Number of frames	10000
Number of Cells	9
Number of antennas	10000

4.8.2 Average Cell Correlation

The average cell cross relation is a measure of the degree to which on average the individual users pilot sequences in the cell are correlated.

The cross correlation between two users k and m in cell j can be defined as $\psi_{bmjk} = \mathbf{s}_{jk}^{T}\mathbf{s}_{bm}$. The values of ψ_{bmjk} ranges from -1 to +1, Where 0 indicates orthogonal sequences, -1 and +1 indicates negative and positive correlation respectively.

The average cross correlation, R_{jk} of user k in cell j with every other user in the cell can be calculated as:

$$R_{jk} = \frac{1}{K-1}\sum_{m=1,m\neq k}^{K} |\,\psi_{bmjk}| \qquad (4\text{-}46)$$

Where $|\cdot|$ is the absolute value of the cross correlation matrix. We have taken the absolute value because the sign of the correlation value shows that the correlation is either negative or positive but does not have an effect on the user sum rate.

The average cell cross relation, $\mathrm{R_j}$ is obtained by summing and averaging the individual $\mathrm{R_{jk}}$ $\forall$ k.

$$\mathrm{R_j} = \frac{1}{\mathrm{K}}\sum_{k=1}^{K} \mathrm{R_{jk}} \tag{4-47}$$

A cell cross correlation value, $\mathrm{R_j}$ of zero shows that the pilot sequences used in cell are uncorrelated and hence the pilot sequences are orthogonal. This means that the intra cell interference is zero. However since the pilot sequences are reused in the other cells the inter cell interference is not zero.

Each pilot sequence is reused once in each cell hence minimum inter-cell interference occurs with a zero cell cross correlation. Both inter-cell and intra-cell interference increase as the cell cross correlation increases from zero to one. Since $\mathrm{R_j}$ ranges from zero to one, the best performance is expected when $\mathrm{R_j}$ is zero and the worst performance when $\mathrm{R_j}$ is one.

4.8.3 Results and Discussion

Multi cell massive MIMO systems are largely affected by both intra-cell and inter-cell interference. If non-orthogonal pilots are used in massive MIMO cells, the BS contains contaminated channel estimates resulting in interference. This research's proposed system model assumes that the same set of pilot sequences are re used in each cell, hence the intra-cell interference can be eliminated by using orthogonal pilots in each cell but the inter-cell interference cannot be eliminated since the pilot sequences are reused in each cell.

Figure 4-1 shows how the pilot correlation varies with the cell sum rate. The pilot sequence correlation varies from zero (when all users in a cell use orthogonal pilot sequences) to one (when all users in a cell use the same pilot sequence). A pilot sequence correlation of zero corresponds to negligible interference, whilst a pilot sequence correlation of one corresponds to maximum interference. It can be observed that total cell sum rate is maximum when the correlation between pilot sequences is zero, it is a minimum when the pilot sequence correlation is one. It can therefore be deduced from Figure 4-1 that the designed BnB algorithm maximizes capacity best when the users are assigned orthogonal pilots. The total cell sum rate decreases gradually as the pilot sequence correlation increases from zero to one. It is therefore of paramount importance to consider pilot sequence correlation in multi cell massive MIMO resource allocation. For the parameters in Table 4-1 the sum rate decreases by about 75% when the pilot sequence increases from zero to one.

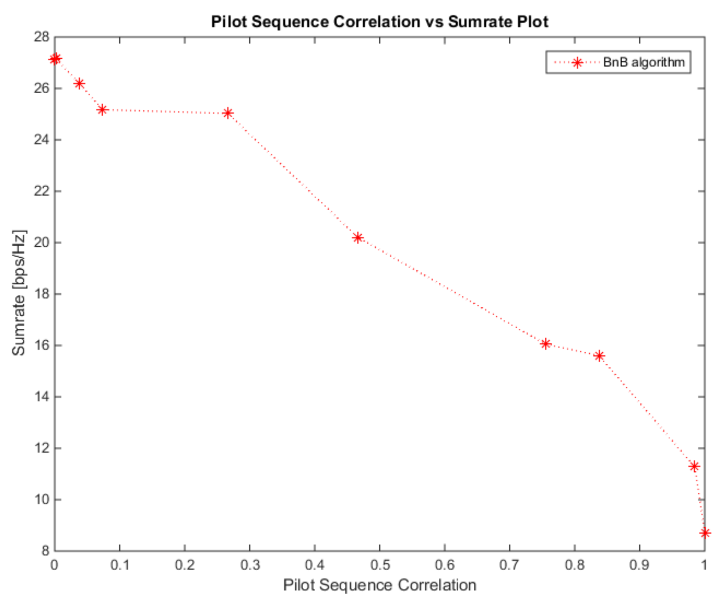

Figure 4-1: Effect of pilot sequence correlation in the cell of interest on cell sum rate

Considering UL transmission, if all users in a multi cell system are assigned orthogonal pilots, the resource allocation algorithm will assign maximum power to all users because interference will be negligible. Hence as shown in Figure 4-2, the total cell sum rate decreases as the pilot sequences correlation values increase from zero to one for the same minimum rate constraint. As can be seen in Figure 4-2, for smaller correlation values, sum rate increases with an increase in maximum power per user (pmax) for smaller pmax values. As the maximum allowable allocated power to a user is increased the total cell capacity and the interference also increases until we get to an optimal cell capacity, after which an increase in pmax does not result in an increase in capacity. Although both the user power and the interference from other users increase as pmax is increased, the rate of increase of the interference is greater than that of the power of the desired signal. Figure 4-2 below shows that there is an optimal pmax per user for maximizing cell sum rate.

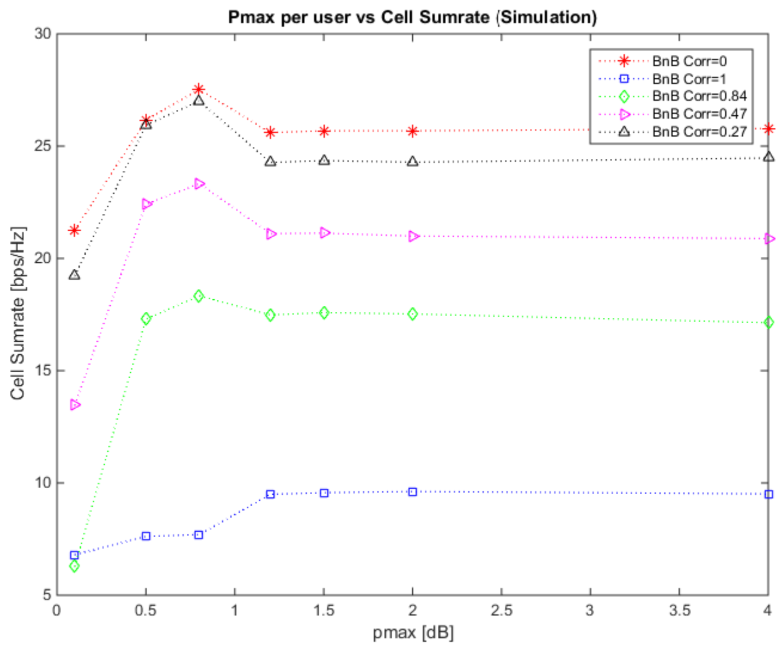

Figure 4-2: Effect of varying pmax per user on cell sum rate for the BnB algorithm for different pilot sequence correlation values

It can be seen in Figure 4-2 that the sum rate decreases to a constant value after the optimum point, this is because when interference is high the power allocated to a UT is limited by interference and thereby also limiting the cell capacity. It can be seen that the sum rate becomes constant as the pmax becomes greater than 1.5dB showing that at high interference levels an increase in pmax does not affect both the power allocated to a user and the interference being experienced by the user. When the interference is very high due to pilot contamination (pilot sequence correlation is one) increasing pmax does not affect sum rate.

A QoS constraint was put so as to make sure that the quality of the output signal is not compromised. A minimum rate constraint makes sure that users with low quality contaminated signals are not selected. It can be easily seen from Figure 4-3 that for the same minimum rate constraint the sum rate decreases with an increase in pilot sequence correlation. This is mainly because as the minimum rate requirement is increased fewer users meet the rate requirement and hence fewer users are selected for transmission.

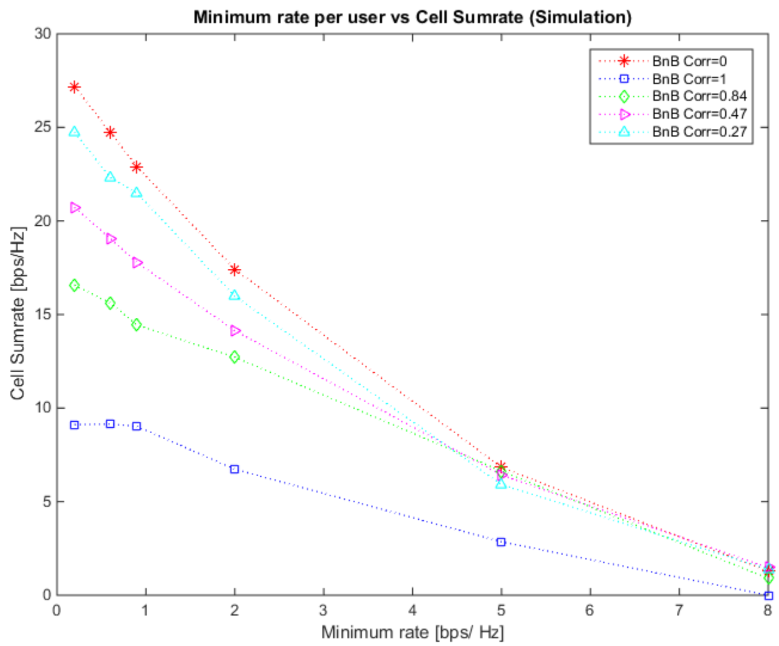

Figure 4-3: Effect of the Qos constraint on sum rate for BnB algorithm

Figure 4-4 shows the effect of increasing the number of BS antennas on the BnB resource allocation algorithm. It can be observed that generally for a small number of antennas there is not much difference in the cell sum rate for different correlation values. As the number of antennas increase, there is an increase in the cell sum rate. This is mainly because as the number of antennas increase the Gaussian noise and part of the interference which is not due to pilot contamination decreases. For large number of antennas (approx. 10^6 antennas) there is a significant difference in the cell sum rate for different correlation factors. This shows that multi cell MU massive MIMO system performance is greatly affected by pilot contamination. As noted in Chapter 2.5 a part of the interference due to pilot contamination does not decrease as the number of antennas increases, hence the difference in total cell sum rate for different correlation values in Figure 4-4.

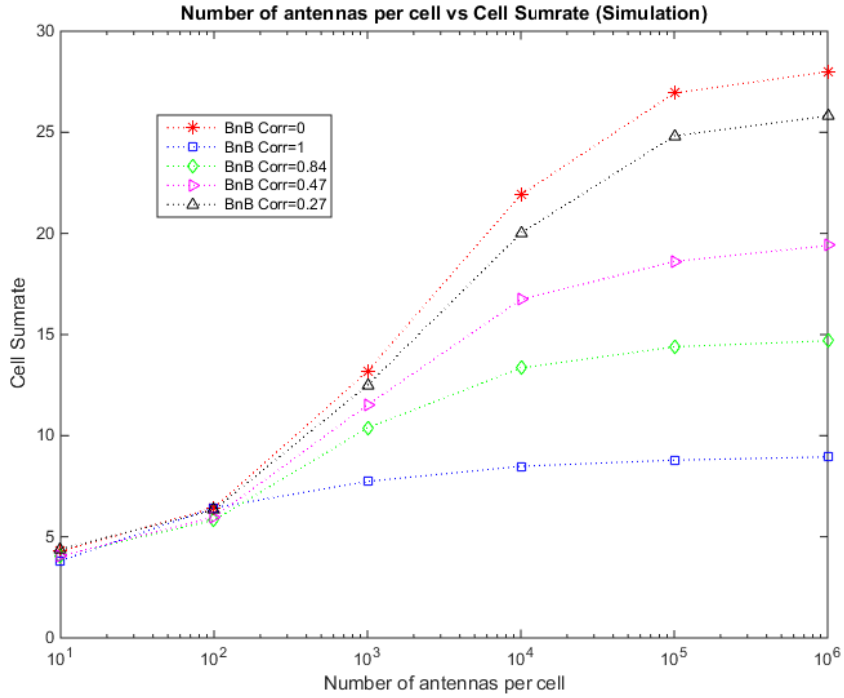

Figure 4-4: Effect of the number of BS antennas on cell sum rate on the BnB algorithm

4.9 Chapter Conclusion

The chapter presented a BnB algorithm for optimal resource allocation in a multi cell massive MIMO system. Optimal resource allocation in a multi cell system can be done at a global level however the complexity is very high. This chapter proposed a suboptimal per cell optimization method which solves the resource allocation problem by modelling the inter-cell interference. To model the inter cell interference it is assumed that the statistical properties of the intra-cell interference in the cell of interest are the same as in the other interfering cells. Therefore the inter-cell interference for each cell can be modelled in terms of the intra-cell interference multiplied by a correction factor. The correction factor takes into consideration pilot sequences used in the interfering cells in relation to pilot sequences used in the cell of interest and large scale fading between the users in the interfering cells and the users in the cell of interest.

The problem is modelled as a non-convex optimization problem which is nondeterministic in polynomial time because the user selection variable can only take binary integer values. To convert the non-convex optimization into a convex problem, the user selection variable is relaxed. The Lagrangian frame work and the Newton Raphson method is employed to determine the optimal solution.

In general the results show that for optimum recourse allocation in multi cell massive MIMO system, it is preferable to assign orthogonal pilots to users, since the sum rate decreases by about 75% when the pilot sequence correlation increases from zero to one for the designed BnB algorithm. The results also show that sum rate increases with an increase in maximum power per user (pmax) for smaller pmax values. As the maximum allowable allocated power to a user is increased the total cell capacity and the interference also increases until we get to an optimum cell capacity, after which an increase in pmax does not result in an increase in capacity. Hence the maximum power allocated to a user should be chosen carefully so as to minimize interference and maximize cell sum rate. The results proved that the performance of a multi cell multi user massive MIMO system improves significantly with an increase in the number of antennas.

CHAPTER 5

5 TWO STAGE RESOURCE ALLOCATION BASED ON MAXIMUM USER SCHEDULING

5.1 Introduction

This chapter proposes a two stage resource allocation algorithm for a particular cell of interest in a multi cell massive MIMO system. The proposed algorithm can be divided into two parts; the user selection part and the power allocation part. The user selection part derives an expression for the maximum number of users which can be served simultaneously through the uplink transmission per cell, $\mathrm{K_s}$. The $\mathrm{K_s}$ users that have a high SINR value assuming equal power allocation are selected from the K users in the cell of interest. Finally the multi-level water filling technique is used to allocate power to the selected users. The inter-cell interference is modelled the same way the inter-cell interference for BnB algorithm is modelled by assuming that the statistical properties of the intra-cell interference in the cell of interest are the same as in the other interfering cells. Hence the inter cell interference is approximated using the intra-cell interference multiplied by a correction factor which takes into consideration pilot sequences used in the interfering cells in relation to pilot sequences used in the cell of interest and large scale fading between the users in the interfering cells and the users in the cell of interest.

Simulation results demonstrate that although the two stage algorithm has a lower computational complexity than the BnB algorithm, the BnB algorithm gives a higher sum rate for the sum rate maximization problem under similar conditions. However, in general, the two stage algorithm selects more users than the BnB algorithm. It can therefore be deduced that the two stage algorithm selects users that interfere more with each other.

The rest of the chapter is organized as follows: Section 5.2 presents the system model. An expression for the SINR when the number of antennas approaches infinity is presented in Section 5.3. Section 5.4 derives an expression for the maximum number of users which can be served simultaneously through the uplink transmission per cell. Section 5.5 discusses the problem formulation. Section 5.6 presents the two stage algorithm. Simulation based performance and evaluation is presented in Section 5.7. Section 5.8 summarizes the chapter.

5.2 System Model

The system model follows the model described in Chapter 4.2.

5.3 Maximum Number of Users to be Scheduled per Cell

In this section we derive an expression for the maximum number of users who can be selected per cell in a multi cell massive MIMO network. To simplify the derivation we derive an expression for the SINR when the number of antennas grows very large i.e., N→∞. When the antenna size goes to infinity the effects of noise are ignored, and the uplink transmission operates in the interference limited region. Under these conditions the asymptotic achievable SINR from Eq. (4-12) becomes,

$$\mathrm{SINR}_{jk}^{MRC} = \frac{\beta_{jjk} p_{jk} r_{jk}}{f_k \beta_{jjk} p_{jk} r_{jk} + (f_k + 1) \quad \sum_{m=1,\ m \neq k}^{K} p_{jm} \beta_{jjm} r_{jm} \left(\psi_{jmjk}^2 J_{jjm}^2 \right)} \tag{5-1}$$

It can be observed from the above expression that the interference resulting from pilot contamination cannot be eliminated as N→∞. Pilot contamination always limits the SINR regardless of the number of antennas if non-orthogonal pilot sequences are used i.e. $\psi_{bmjk} \neq 0$

The expression in Eq. (5-1) can be simplified to

$$\mathrm{SINR}_{jk,\infty}^{MRC} = \frac{\beta_{jjk} p_{jk}}{(f_k + 1) \quad \sum_{m=1}^{K} p_{jm} \beta_{jjm} \left(\psi_{jmjk}^2 J_{jjm}^2 \right) - \beta_{jjk} p_{jk}} \tag{5-2}$$

Based on Eq. (5-2) and the uplink power control rule given by $q_{bm} \beta_{jbm} \leq 1$, can be simplified to

$$\mathrm{SINR}_{jk,\infty}^{MRC} = \frac{p_{jk}}{(f_k + 1) \quad \sum_{m=1}^{K} p_{jm} \left(\psi_{jmjk}^2 \right) - p_{jk}} \tag{5-3}$$

$$\mathrm{SINR}_{jk,\infty}^{\mathrm{MRC}} = \frac{p_{jk}}{(f_k + 1)\mathrm{tr}(\mathbf{s}_{jk}^{T}\mathbf{SPS}^{T}\mathbf{s}_{jk}) - p_{jk}} \tag{5-4}$$

Where $\mathbf{S}$, $\mathbf{P}$ are block matrices given by $\mathbf{P}=\mathrm{diag}[p_{j1}, \dots, p_{j2}, \dots, p_{jK}]$, $\mathbf{S} = \mathrm{diag}\,[\mathbf{S}_{j1}, \dots, \mathbf{S}_{j2}, \dots, \mathbf{S}_{jK}]$.

Making use of Eq. (5-4) above we obtain the following expression

$$\sum_{m=1}^{K} \frac{1 + \mathrm{SINR}_{jm,\infty}^{\mathrm{MRC}}}{\mathrm{SINR}_{jm,\infty}^{\mathrm{MRC}}} = \sum_{m=1}^{K} \frac{1}{p_{jm}} (f_{jm} + 1)\mathrm{tr}(\mathbf{s}_{jk}^{T}\mathbf{SPS}^{T}\mathbf{s}_{jk}) \tag{5-5}$$

We next define $t_{jm} = (f_{jm} + 1)$, T= $\mathrm{diag}(\ t_{j1}, \dots\ t_{jm}, \dots\ t_{jK})$ and $\mathbf{S}^{T}\mathbf{S} = G_s$. We then expand Eq.(5-5) as

$$\begin{aligned} \sum_{m=1}^{K} \frac{1 + \mathrm{SINR}_{jm,\infty}^{\mathrm{MRC}}}{\mathrm{SINR}_{jm,\infty}^{\mathrm{MRC}}} &= \mathrm{tr}(\mathbf{P}^{-1}\mathbf{S}^{T}\mathbf{SPTS}^{T}\mathbf{S}) \\ &= \mathrm{tr}(\mathbf{P}^{\frac{-1}{2}}\mathbf{S}^{T}\mathbf{SPTS}^{T}\mathbf{SP}^{\frac{-1}{2}}) \end{aligned} \tag{5-6}$$

We expand the trace in Eq.(5-6) and obtain its lower bound as

$$\begin{aligned} \sum_{m=1}^{K} \frac{1 + \mathrm{SINR}_{jm,\infty}^{\mathrm{MRC}}}{\mathrm{SINR}_{jm,\infty}^{\mathrm{MRC}}} &= \mathrm{tr}(\mathbf{P}^{\frac{-1}{2}}\mathbf{S}^{T}\mathbf{SP}\mathrm{T}\mathbf{S}^{T}\mathbf{SP}^{\frac{-1}{2}}) \\ &= K + \sum_{u=1}^{K}\sum_{v>u=1}^{K}\left(\frac{t_{ju}p_{ju}}{t_{jv}p_{jv}} + \frac{t_{jv}p_{jv}}{t_{ju}p_{ju}}\right)\psi_{jujv}^{2} \\ &\geq K + \sum_{u=1}^{K}\sum_{v>u=1}^{K} 2\,\psi_{jujv}^{2} = \mathrm{tr}(\mathbf{G}_s\mathbf{G}_s) \end{aligned} \tag{5-7}$$

where the inequality is due to $\left(\frac{t_{ju}p_{ju}}{t_{jv}p_{jv}} + \frac{t_{jv}p_{jv}}{t_{ju}p_{ju}}\right) \geq 2$. The matrix $\mathbf{G_s}$ has eigen decomposition $\mathbf{UD}_G\mathbf{U}^{T}$, where U is a unitary matrix and $\mathbf{D}_G$ is a K×K diagonal matrix, $\mathbf{D}_G = \mathrm{diag}(d_1, \dots, d_K)$. It can be note that the first τ elements in the main diagonal of U have values greater than zero and the rest are zeros. From the definition given above we also have the condition $\sum_{i=1}^{\tau} d_i = K$. Then we have

$$\mathrm{tr}(G_s G_s) = \mathrm{tr}(\mathbf{U}\mathbf{D}_G\mathbf{U}^T)$$
$$= \sum_{i=1}^{\tau} d_i^2$$
$$= \frac{\left(\sum_{i=1}^{\tau} d_i\right)^2}{\tau}$$
$$= \frac{K^2}{\tau} \tag{5-8}$$

Substituting Eq. (5-8) into Eq. (5-7), we obtain

$$\sum_{m=1}^{K} \frac{1 + \mathrm{SINR}_{jm,\infty}^{MRC}}{\mathrm{SINR}_{jm,\infty}^{MRC}} \geq \frac{K^2}{\tau} \tag{5-9}$$

The number of selected users is related to the pilot length, τ and the SINR is described by Eq.(5-10) obtained by rearranging Eq. (5-9).

$$K_s \leq \sqrt{\tau \sum_{k=1}^{K} \frac{1 + \mathrm{SINR}_{jk,\infty}^{MRC}}{\mathrm{SINR}_{jk,\infty}^{MRC}}} \tag{5-10}$$

In massive MIMO systems there exist a power allocation vector p_j =[$p_{j1}, \ldots\ p_{jK}$] such that the SINR_{jk} is greater than a predefined minimum rate $\mathrm{SINR}_{jk,min}^{MRC}$ for each user k. Hence we can assume that $\mathrm{SINR}_{jk,\infty}^{MRC} \geq \mathrm{SINR}_{jk,min}^{MRC}\ \forall$ k. Hence the desired inequality follows

$$\sum_{k=1}^{K} \frac{1 + \mathrm{SINR}_{jk,\infty}^{MRC}}{\mathrm{SINR}_{jk,\infty}^{MRC}} \leq \sum_{k=1}^{K} \frac{1 + \mathrm{SINR}_{jk,min}^{MRC}}{\mathrm{SINR}_{jk,min}^{MRC}} \leq \sum_{k=1}^{K} \frac{1 + C_{jk,min}}{C_{jk,min}} \tag{5-11}$$

The last inequality comes from the assumptions that $\log_2\left(1 + \mathrm{SINR}_{jk,min}^{MRC}\right) = C_{jk,min} \leq \mathrm{SINR}_{jk,min}^{MRC}$ for the high SINR region. The preposition below describes how the number of selected users is related to the pilot length, τ and the minimum sum rate per user.

Preposition 5.1 :If K are the number of users in cell j, then the maximum number of users which can be served simultaneously through the uplink transmission such that the cell sum rate is maximized is given by,

$$K_s \leq \sqrt{\tau \sum_{k=1}^{K} \frac{1 + C_{jk,min}}{C_{jk,min}}} \tag{5-12}$$

It is clear that the number of selected users is proportional to the pilot length. If the pilot length decreases the number of admissible users also decrease.

User selection is a very important tool in multi user multi cell massive MIMO systems to maximize sum rate. In this solution the K_s users that have a high SINR values assuming equal power allocation are selected from the K users in the cell of interest. The scheduling factor, ξ_{jk} in Eq. (5-21) below is used to select the users to be scheduled. The users are sorted in ascending order of ξ_{jk}, and K_s users with the highest values of ξ_{jk} are chosen.

5.5 Problem Formulation

The problem is formulated as a sum rate maximization problem under user specific QoS constraints as

$$\max_{\{p_{jk}, r_{jk}\} \in \mathbb{R}} \sum_{k=1}^{K_s} \left(1 - \frac{V}{S}\right) \left\{\log_2 \left(1 + \frac{p_{jk}\beta_{jjk}}{I_1 p_{jk} + I_2 + z_{jk}}\right)\right\}$$

subject to:

$$A: \ p_{jk} \leq p_{max}, \forall \, j,k$$

$$B: p_{jk} \geq 0, \forall \, j,k$$

$$C: r_{jk}\left(1 - \frac{V}{S}\right)\left\{\log_2 \left(1 + \frac{p_{jk}\beta_{jjk}}{I_1 p_{jk} + I_2 + z_{jk}}\right)\right\} \geq C_{k,min}, \forall \, j \tag{5-13}$$

The Lagrangian function from Eq. (5-14) is given by

$$L(\{s_{jk}\},\{r_{jk}\},\nu,\eta,\lambda,\theta) =$$

$$\sum_{k=1}^{K_s} H\left\{\log_2\left(1+\frac{p_{jk}\beta_{jjk}}{(I_1 p_{jk}+I_2)}\right)\right\} + \sum_{k=1}^{K_s} \nu_{jk}\,(p_{max} - s_{jk}) +$$

$$\sum_{k=1}^{K_s} \lambda_{jk}\left(r_{jk} H\left\{\log_2\left(1+\frac{p_{jk}\beta_{jjk}}{(I_1 p_{jk}+I_2+z_{jk})}\right)\right\} - C_{k,min}\right) \quad (5\text{-}15)$$

Where ν_{jk} , λ_{jk} are the Lagrangian multipliers for the constraints A and C respectively. The dual function can be formulated as [72]

$$g(\nu,\eta,\lambda,\theta) = \inf_{\{s_{jk},r_{jk}\}\in\mathbb{R}} L(\{p_{jk}\},\nu,\lambda) \quad (5\text{-}16)$$

The subsequent dual problem can be expressed as

$$\min_{\nu,\eta,\lambda,\theta} L(\nu,\eta,\lambda,\theta) \quad (5\text{-}17)$$

$$\text{Subject to: } \nu,\eta,\lambda,\theta \geq 0.$$

The optimal solution of the sub problems can be obtained as

$$\frac{\partial L\{\ldots\}}{\partial p_{jk}} \begin{cases} = 0, p_{jk} > 0 \\ < 0, p_{jk} = 0 \end{cases} \quad \forall j,k \quad (5\text{-}18)$$

By taking the partial derivatives of the Lagrangian with respect to the primal and dual variables, the KKT conditions can be expressed as

$$\frac{\partial L\{\ldots\}}{\partial p_{jk}} = \frac{p_{jk}\beta_{jjk}(1+\lambda_{jk})(I_2)}{\ln 2\ (I_2+I_1 p_{jk})\,(p_{jk}\beta_{jjk}+I_2+I_1 p_{jk})} - v_{jk} \quad (5\text{-}19)$$

The above optimization problem can be solved by the multi-level water filling approach. Note the effective power allocation is dependent on the large scale fading parameter β. The equation below is obtained by equating Eq. (5-19) to zero.

$$p_{jk} = \left(\frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}\right)^{+} \tag{5-20}$$

Where $a = \left(I_1{}^2 + \beta_{jjk} I_1\right)$

$$b = \beta_{jjk} I_2 + 2I_2 I_1$$

$$c = \left(I_2{}^2 + z_{jk}{}^2 - \frac{\beta_{jjk}(1+\lambda_{jk})(I_2)}{\nu_{jk} \ln 2}\right)$$

5.6 Two Stage Algorithm User Selection and Power Allocation

The two stage algorithm estimates K_s using the cell maximum user capacity equation in Eq. (5-12). To select the K_s users a lot of factors have to be taken into account such as channel fading parameters of the users and the total interference from other users. We select the K_s users with the highest $\mathrm{SINR}_{jm,\infty}^{const}$ given in assumption 5.1 below. To fairly choose the number of users selected we assume constant power allocation where all users are allocated the same power P. Hence ξ_{jk} , the user selection parameter is dependent on the large scale fading of both the user k to be selected β_{jjk}, and the large scale fading of the other users β_{jbm}. It is also takes into consideration pilot sequences. The following assumption can therefore be made,

Assumption 5.1: The user selection parameter ξ_{jk} can be calculated by assuming equal power allocation

$$\xi_{jk} = \frac{\beta_{jjk}}{\sum_{b=1}^{B} \sum_{m=1\ (b,m)\neq(j,k)}^{K} \beta_{jbm}(\psi_{bmjk}^2 J_{jbm}^2) + \frac{\alpha_{jk}}{N} \overline{R_{jk}}} \tag{5-21}$$

Where $\overline{R_{jk}} = \sum_{b=1}^{B} \sum_{m=1}^{K} \beta_{jbm} + \sigma_w^2$ $\alpha_{jk} = \sum_{\substack{t=v \\ u=z}} \psi_{tujk}^2 J_{jtu}^2 + \sigma^2$ and $J_{jbm} = \sqrt{q_{bm}\beta_{jbm}}$

Proof: By substituting $p_{jm} = P\ \forall\, m$ and $r_{jm} = 1\ \forall\, m$ in Eq. (4-12) where P is an arbitrary constant power.

After selecting users per cell using ξ_{jk} above, the users in the cell are allocated power using the multi-level water filling approach. The two stage algorithm entails that users are scheduled on a per cell basis and the inter cell interference is modelled as described in Section 4.3. After user selection the users in the cell are then allocated power. The complexity of the two stage algorithm is much lower than the complexity of the branch and bound and hence it is much cheaper to optimally allocate resources using the two stage algorithm.

Two stage algorithm

Input: I, p_{max}, c_{min} , $\beta = [\beta_{j11}, \ldots, \beta_{jbk}, \ldots, \beta_{jBK}]$, N_j

Initialize: $p_j \leftarrow 1_K$, $\lambda_j \leftarrow 0_K$,$\nu_j \leftarrow 0_K$

Return: p=[p_{11} …, p_{1k}, …p_{1K}]

for the cell of interest j

Calculate K_s using Eq. (5-12)

K_s =min (K, K_s)

Calculate $\xi_j = [\xi_{j1}, \ldots, \xi_{jk}, \ldots, \xi_{jK}]$ $\forall$ k and sort in descending order

Choose the first K_s users with the highest ξ_{jk}

Repeat

  for k = 1 to K do

    sum_rate_max←0

    Calculate f_{jk} according to Eq. (4-17)

    Calculate I_1 and I_2 according to Eq. (4-20)

    Update p_{jk}^{i+1} according to Eq. (5-20)

    Calculate sumrate

    if sumrate >= sum_rate_max

      sum_rate_max= sumrate

    end

    Update λ_{jk},ν_{jk} according to Eq. (4-42) and Eq. (4-41) respectively

  End for

21. Until convergence

5.7 Complexity analysis of two stage algorithm

The two stage algorithm is divided into two stages; in the first stage a maximum number of users to be scheduled is calculated using the derived scheduling equation. A multi-level water filling approach is then used to optimize the power in the resource allocation optimization problem. The user selection stage has a total computational complexity of O (K). Assuming that the power allocation stage needs Δ iterations to converge, the complexity of the power allocation stage is a polynomial function of K and hence has a complexity of O (K). The total complexity of the two stage algorithm is therefore O (K).

5.8 Simulation and Results

5.8.1 Simulation Model

The two stages algorithm is compared with the BnB algorithm for different correlation values. Firstly we evaluate how the cell sum rate and the number of users selected varies with pilot sequence correlation. Next, we check how the sum rate and the number of users selected varies with p_{max}. We also evaluate sum rate and the number of users selected as $C_{jk,min}$ varies. Lastly we investigate the effect of varying the number of antennas on cell sum rate and the number of users selected per cell. The simulation parameters are set as shown in Table 1 above

5.8.2 Results and Discussion

The two stage algorithm is compared to the BnB algorithm firstly by assessing the effect of pilot sequence correlation on sum rate. It can be observed in Figure 5-1 that the sum rate decreases with an increase in pilot sequence correlation. Therefore if orthogonal pilots are used the solution to the resource allocation problem will give better cell sum rate than when non orthogonal pilots are used. It can also be observed that although the BnB algorithm has a higher complexity than the two stage algorithm it gives a higher sum rate for the same pilot sequence correlation, maximum power and minimum rate. Figure 5-2 shows the effect of pilot sequence correlation on the number of users selected per cell in both BnB algorithm and the two stage algorithm. In general, the two stage algorithm selects more users than the BnB algorithm. It can therefore be deduced that the two stage algorithm selects users that interfere more with each other.

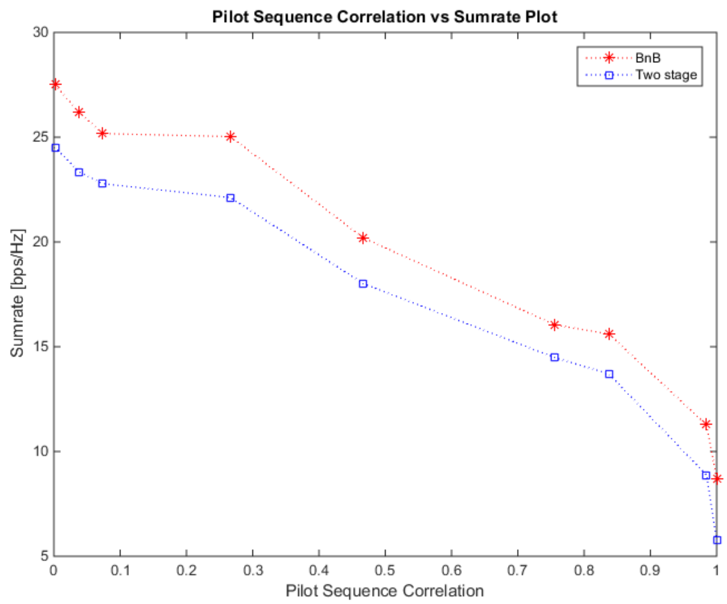

Figure 5-1: Effect of pilot sequence correlation on sum rate after BnB and two stage algorithm

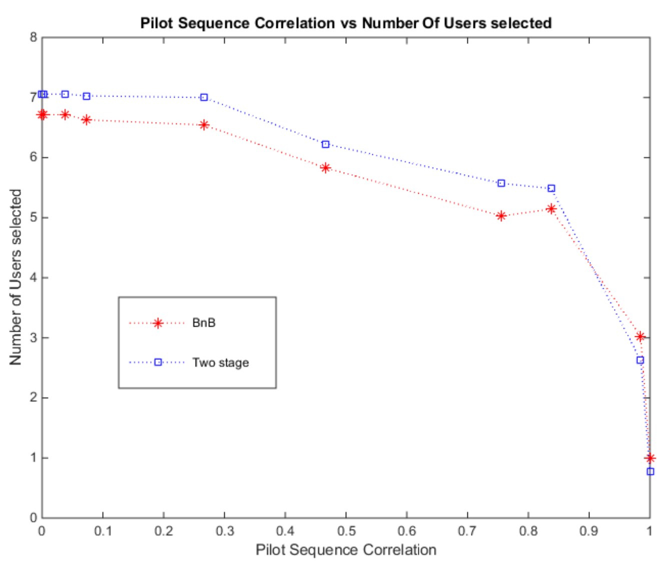

Figure 5-2: Effect of pilot sequence correlation on the number of users selected for both BnB and two stage algorithms

As can be seen in Figure 5-3, there is an optimal pmax value at which the algorithms optimally maximize the cell sum rate. This is mainly because as the desired signal power is increased iteratively from zero, the interfering power also increases. However the rate of increase of the interfering power is more than the rate of increase of the desired signal when pilot contamination is more pronounced. The graphs in Figure 5-3 have peaks where the interference starts having a detrimental effect on cell sum rate and the total cell sum rate starts decreasing. It can also be observed that for pilot correlation values closer to zero (i.e. orthogonal or almost orthogonal pilots) there is no much difference between the two stage algorithm and the BnB algorithms. However for pilot correlation values closer to one, the BnB has a higher total cell sum rate than the two stage algorithm. It can be observed in Figure 5-4 that for low values of pmax the BnB algorithm selects more users than the two stage algorithm. This is because the BnB algorithm uses the branch and bound technique to select users to be scheduled, hence it tries to select users with low interference contributions to other users. At low pmax values interference is small hence the BnB can select more users than the two stage algorithm. However when pmax is increased interference becomes significant and the users scheduled in the BnB becomes less than the ones scheduled in the two stage algorithm.

Figure 5-5 summarizes the effects of minimum rate constraint on cell sum rate after both BnB and two stage algorithms for different correlation values. The two stage algorithm derived equation for maximum user scheduling which is dependent on the minimum rate and pilot sequence correlation value. Hence the number of users scheduled for the two stage algorithm is directly dependent on minimum rate and pilot sequence correlation. Figure 5-5 shows that for correlation values closer to zero (orthogonal pilot sequences) the two stage algorithm performs slightly better than the BnB algorithm over all minimum rate values with a constant pmax=10dB and number of antennas, N=40000. However when the sequence correlation values are closer to one (pilot sequences are non-orthogonal) the BnB performs better than the two stage algorithm. This is mainly because when the pilots used are non-orthogonal the pilot length, τ in Eq. (5-12) is small.

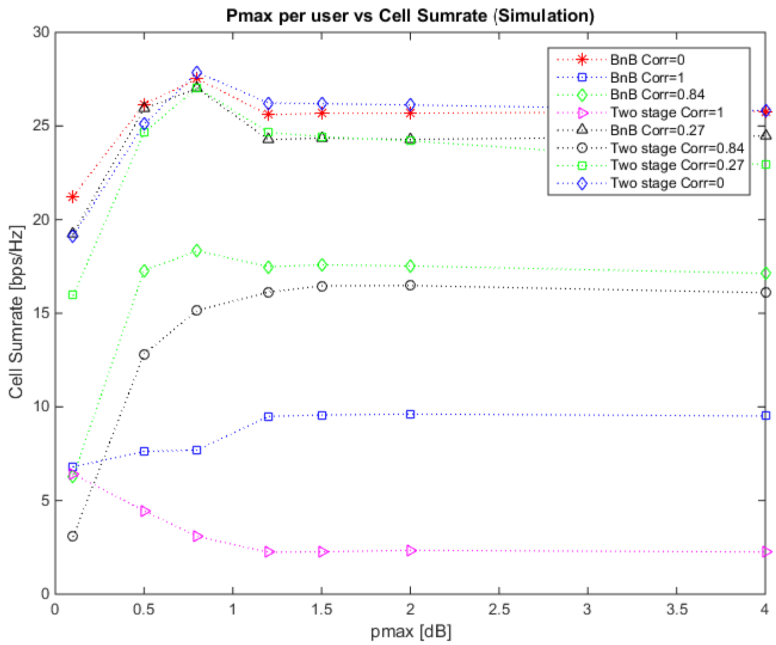

Figure 5-3: Effect of pmax constraint on sum rate in both the BnB and two stage algorithms

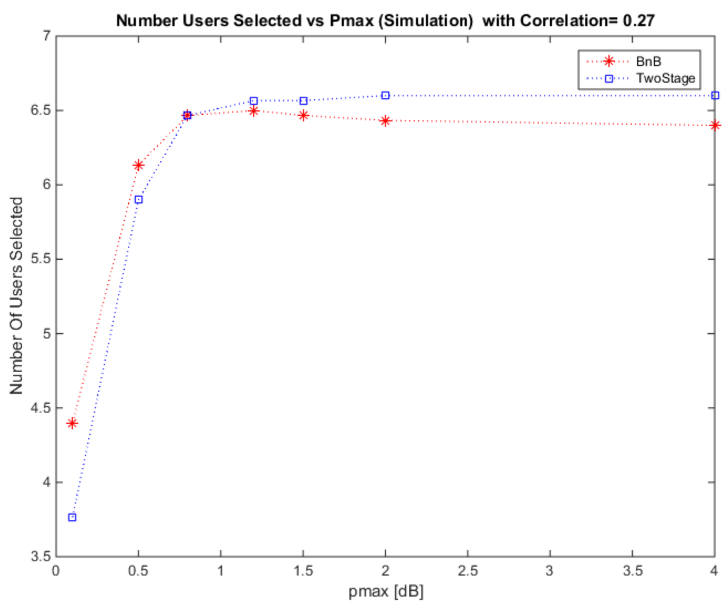

Figure 5-4: Effect of pmax constraint on the number of users selected per cell for both BnB and two stage algorithms

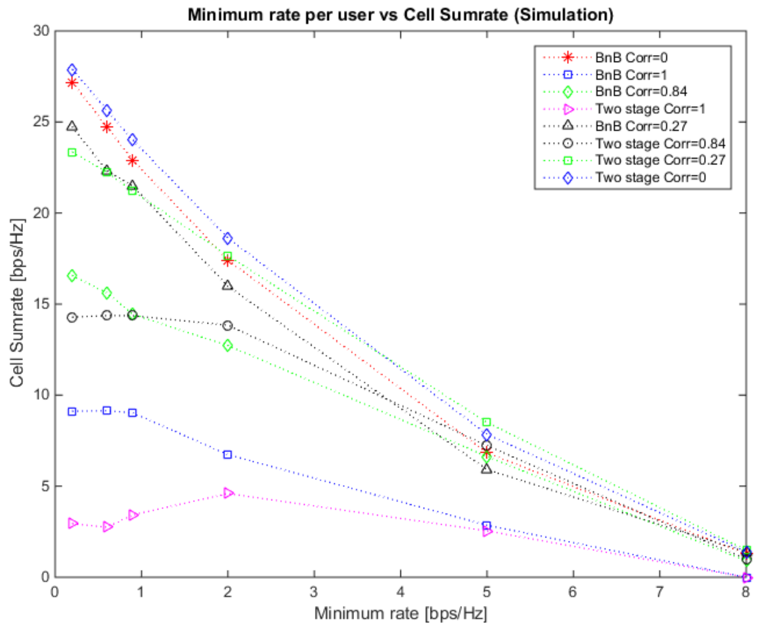

Figure 5-5: Effect of minimum rate constraint on the sum rate for both the BnB and two stage algorithms

Figure 5-6 shows the effect of minimum rate on number of users selected for a pilot sequence correlation = 0.27. It can be observed that there are more users selected for the two stage algorithm than the BnB. This is because the pilot sequence correlation is low, consequently the pilot length, is high τ.

Figure 5-7 shows the effect the antenna size has on cell sum rate for both BnB algorithm and two stage algorithm. The graphs in Figure 5-7 show that when the antenna size is small the cell sum rate is independent of the pilot sequence correlation. It can be seen from the results that there is a significant increase in sum rate when antenna size increases beyond 1000 antennas this is because the Gaussian noise and a part of the interference that is not dependent on pilot contamination decreases, hence the individual user capacities increase. The results are obtained by increasing the number of antennas whilst maintaining a minimum rate constraint and a maximum power constraint of 0.4 bps/Hz and 10dB respectively. The results show that for orthogonal pilot sequences the sum rates obtained for the BnB algorithm and the two stage algorithms are comparable. However as the pilot sequence correlation increases the BnB algorithm produces a higher sum rate. Figure 5-8 shows the effect of number of antennas on the number of users selected. It can be observed that the two stage algorithm selects more users than the BnB algorithm. However the resultant sum rates from the optimization problem after both the BnB and the two stage algorithms are roughly the same. This is because the two stage algorithm selects users with the highest constant power allocation SINR values, hence it choses users that have a higher potential to interfere with each other, hence a sum rate comparable to BnB sum rate.

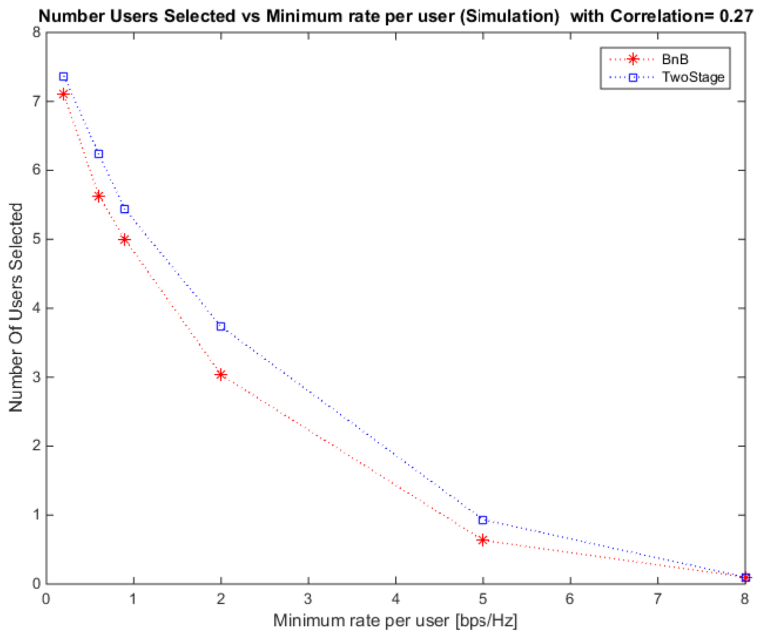

Figure 5-6: Effect of minimum rate on the number of users selected per cell for BnB and two stage algorithms

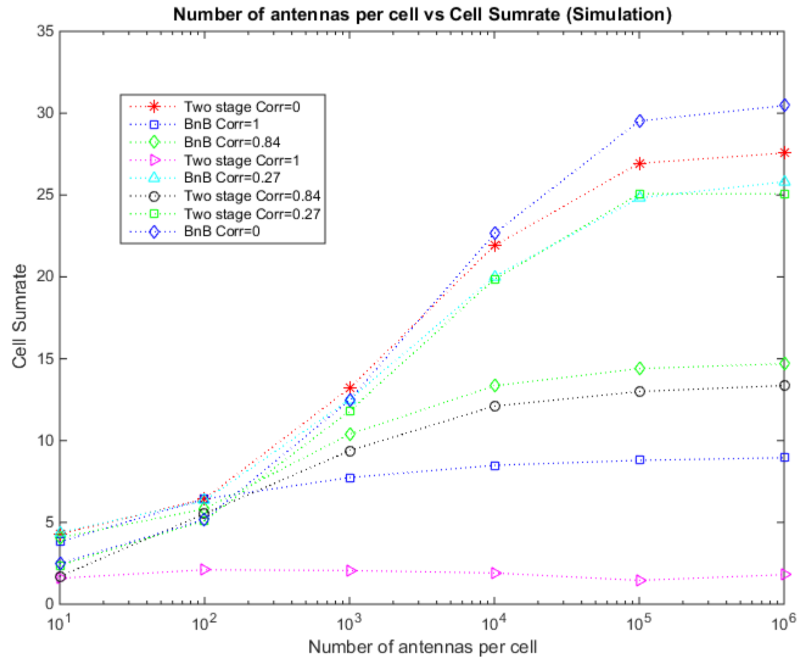

Figure 5-7: Effect of number of antennas on sum rate for BnB and two stage algorithms

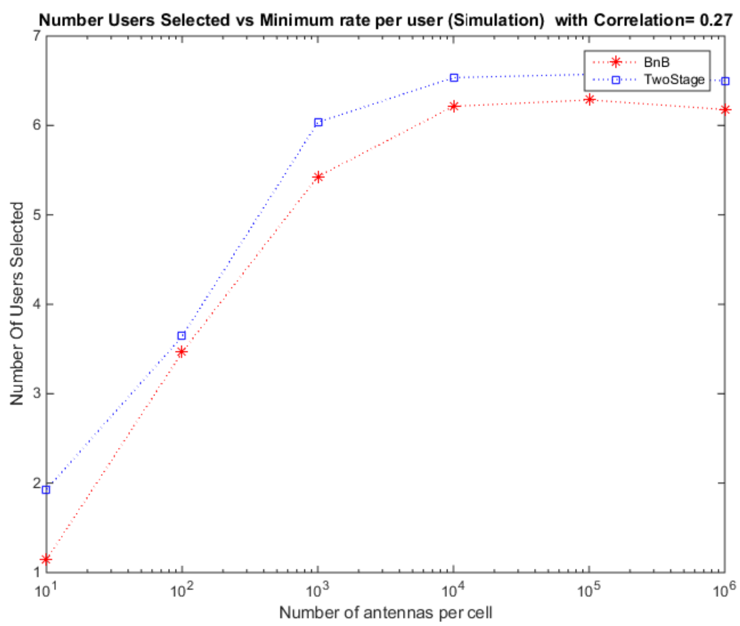

Figure 5-8: Effect of number of BS antennas on the number of users selected for BnB and two stage algorithm for pilot sequence correlation 0.27

5.9 Chapter Conclusion

This chapter proposes a two stage resource allocation algorithm for a particular cell of interest in a multi cell massive MIMO system. The proposed algorithm is divided into two parts; the user selection part and the power allocation part. The user selection part derives an expression for the maximum number of users which can be served simultaneously through the uplink transmission per cell. The K_s users that have a high SINR value assuming equal power allocation are selected from K users. Finally the multi-level water filling technique is used to allocate powers to the selected users.

Simulation results demonstrate that the BnB algorithm performs better than the two stage algorithm. Although the BnB algorithm has a higher complexity than the two stage algorithm it gives a higher sum rate for the same pilot sequence correlation, maximum power and minimum rate. In general, the two stage algorithm selects more users than the BnB algorithm. It can therefore be deduced that the two stage algorithm selects users that interfere more with each other. The simulation results prove that better performance is obtained when the orthogonal pilot sequences are used. The simulation results also show that there is an optimal pmax value at which the algorithms optimally maximize the cell sum rate. This is mainly because as the desired signal power is increased iteratively from zero, the interfering signal power also increases. However the rate of increase of the interfering signal power is more than the rate of increase of the desired signal power when pilot contamination is more pronounced. The designed two stage algorithm performance is improved if a large number of antennas is used (from 1000 antennas and above).

CHAPTER 6

6 HYBRID RESOURCE ALLOCATION ALGORITHM

6.1 Introduction

The BnB and two stage algorithms presented in Chapters 4 and 5 respectively allocate resources in the cell of interest by making statistical assumptions to model the inter-cell interference. This chapter proposes another algorithm, the hybrid algorithm which combines the two algorithms described above. In the hybrid algorithm the cell of interest allocates resources in the interfering cells using the two stage algorithm described in Chapter 5. The cell of interest first performs both user selection and power allocation in the interfering cells (to obtain near optimal resource allocation values) using the low complexity two stage algorithm. The cell of interest then uses these near optimal values to perform its own resource allocation using the BnB algorithm. The two stage algorithm is chosen for resource allocation in the interfering cells because it has a much lower complexity compared to the BnB algorithm. The BnB algorithm is chosen for resource allocation in the cell of interest because it gives higher sum rate in a sum rate maximization problem than the two stage algorithm. Simulation results prove that the proposed algorithms perform better than the solutions in literature.

The rest of the chapter is organized as follows: Section 6.2 presents interference modelling. Section 6.3 discusses the problem formulation. Section 6.4 presents simulation based performance evaluation for various resource allocation techniques, and finally concluding remarks are presented in Section 6.5.

6.2 System Model

The system model follows the model described in Chapter 4.2.

6.3 Resource Allocation in the Interfering Cells

The cell of interest j estimates the resource allocation in the interfering cells using the two stage algorithm. The interfering cells' resource allocation values is estimated with the data available to the cell of interest. We assume that the resource allocation values obtained after the cell of interest allocates resources in the interfering cells is approximately equal to the resource allocation values obtained after the interfering cells allocate resources in their own cells.

The user selection parameter ξ_{zi} for user i in cell z after the cell of interest j performs resource allocation in cell z,

$$\xi_{zi} = \frac{\beta_{jzi}}{\sum_{b=1}^{B}\sum_{m=1\ (b,m)\neq(z,i)}^{K}\beta_{jbm}(\psi_{bmzi}^{2}J_{jbm}^{2}) + \frac{\alpha_{zi}}{N}\overline{R_{zi}}} \tag{6-1}$$

Where $\overline{R_{zi}} = \sum_{b=1}^{B}\sum_{m=1}^{K}\beta_{jbm} + \sigma_w^2$ $\alpha_{zi} = \sum_{\substack{t=v\\u=z}}\psi_{tuzi}^{2}J_{jtu}^{2} + \sigma^2$ and $J_{jbm} = \sqrt{q_{bm}\beta_{jbm}}$

After selecting users per cell using ξ_{zi} above, the users in the cell are allocated power using the multi-level water filling approach in Eq. (5-20).

6.4 Inter-cell Interference Modelling in the Cell of Interest

In Section 4.3 we used statistical approximations to estimate the inter cell interference. Two near optimal resource allocation algorithms were designed in Chapters 4 and 5 for multi cell massive MIMO systems. Simulations proved that the BnB algorithm performs better than the two stage algorithm but with higher complexity. In the proposed hybrid algorithm, the cell of interest allocates resources in the interfering cells and then uses the resource allocation values obtained from the interfering cells to model the inter-cell interference. From Eq. (4-14), the interference can be modelled as:

$$I_{jk} = \sum_{b=1}^{B} \sum_{m=1\ (b,m)\neq(j,k)}^{K} p_{bm}\beta_{jbm}r_{bm}(\psi_{bmjk}^2 J_{jbm}^2) + \frac{\alpha_{jk}}{N}\left(\sum_{b=1}^{B}\sum_{m=1}^{K} p_{bm}\beta_{jbm}r_{bm}\right) \tag{6-2}$$

Which can then be separated into the inter-cell and intra-cell interferences.

$$I_{jk} = \overbrace{\sum_{m=1,\ m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm}\left(\psi_{jmjk}^2 J_{jjm}^2\right) + \frac{\alpha_{jk}}{N}\sum_{m=1}^{K} p_{jm}\beta_{jjm}r_{jm}}^{\text{intra-cell interference}} + \overbrace{\sum_{b=1,\ b\neq j}^{B}\sum_{m=1}^{K} p_{bm}\beta_{jbm}r_{bm}\left(\psi_{bmjk}^2 J_{jbm}^2\right) + \frac{\alpha_{jk}}{N}\sum_{b=1,\ b\neq j}^{B}\sum_{m=1}^{K} p_{bm}\beta_{jbm}r_{bm}}^{\text{inter-cell interference}} \tag{6-3}$$

The two stage algorithm is used to optimally allocate the resources in each of the interfering cells, hence I_{jk} in Eq. (6-3) above can be simplified to

$$I_{jk} = \overbrace{\sum_{m=1,\ m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm}\left(\psi_{jmjk}^2 J_{jjm}^2\right) + \frac{\alpha_{jk}}{N}\sum_{m=1}^{K} p_{jm}\beta_{jjm}r_{jm}}^{\text{intra-cell interference}} + \overbrace{\sum_{b=1,\ b\neq j}^{B}\sum_{m=1}^{K} p_{bm}^{*}r_{bm}^{*}\beta_{jbm}r_{bm}\left(\psi_{bmjk}^2 J_{jbm}^2\right) + \frac{\alpha_{jk}}{N}\sum_{b=1,\ b\neq j}^{B}\sum_{m=1}^{K} p_{bm}^{*}r_{bm}^{*}\beta_{jbm}}^{\text{inter-cell interference}} \tag{6-4}$$

where r_{bm}^{*} and p_{bm}^{*} are the resource allocation values for user m in cell b obtained after the cell interest estimates the inter-cell interference using the two stage algorithm described in Chapter 5. Eq. (6-4) above can be further simplified to

$$I_{jk} = I_1 p_{jk} r_{jk} + I_2 \tag{6-5}$$

Where I_1 is $\frac{\alpha_{jk}}{N}\beta_{jjk}p_{jk}r_{jk}$ and I_2 is:

$$
\begin{aligned}
I_2 &= \overbrace{\sum_{m=1,\ m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm}\left(\psi_{jmjk}^{2}J_{jjm}^{2}\right)+\frac{\alpha_{jk}}{N}\sum_{m=1,m\neq k}^{K} p_{jm}\beta_{jjm}r_{jm}}^{\text{intra-cell interference}} \\
&+ \overbrace{\sum_{b=1,b\neq j}^{B}\sum_{m=1}^{K} p_{bm}^{*}r_{bm}^{*}\beta_{jbm}r_{bm}\left(\psi_{bmjk}^{2}J_{jbm}^{2}\right)+\frac{\alpha_{jk}}{N}\sum_{b=1,b\neq j}^{B}\sum_{m=1}^{K} p_{bm}^{*}r_{bm}^{*}\beta_{jbm}}^{\text{inter-cell interference}}
\end{aligned}
\tag{6-6}
$$

6.5 Problem Formulation and Hybrid Algorithm Design

The maximization problem is formulated as a convex optimization problem exactly as in Section 4.5. The only difference between the hybrid algorithm and the BnB algorithm described in Section 4 is in the interference modelling, hence I_1 and I_2 expressions are different for two algorithms. The hybrid algorithm entails that the cell of interest j uses the statistical approximation described in Section 4.3 to obtain optimal user selection, $r_b^* = [r_{b1}^*, \dots, r_{bm}^*, \dots, r_{bK}^*]$ and power allocation vectors, p_b^* =$[p_{b1}^*, \dots, p_{bm}^*, \dots, p_{bK}^*]$ using the two stage algorithm .The algorithm then allocates resources in the cell of interest j using BnB algorithm whilst the inter-cell interference is calculated using the near optimal r_b^* and p_b^* values. Consequently hybrid algorithm aims at achieving a tradeoff between complexity and performance.

Hybrid algorithm

1. **Input**: I, p_{max}, c_{min} , β, N
2. **Initialize**: $p^* \leftarrow 1_{B\times K}$, $r^* \leftarrow 1_{B\times K}$ $\lambda \leftarrow 0_{B\times K}$,$\nu \leftarrow 0_{B\times K}$
3. **Return:** p=$[p_{1,1} \dots p_{1,K} \dots p_{2,1} \dots p_{2,K} \dots \dots p_{B,K}]$
4. **for z=1 to B, j ≠ z**
   1. **Calculate** K_s using equation Eq. (5-12)
   2. K_s =min (K, K_s)
   3. Sort ξ_z =$[\xi_{z1}, \dots, \xi_{zi} , \dots, \xi_{zK}]$ obtained using Eq.(6-1) in descending order
   4. Choose the first K_s users with the highest ξ_{zi}
   5. **Repeat**
   6. **for** $i = 1$ to K do

sum_rate_max←0

Calculate f_{bk} according to Eq. (4-17)

Calculate I_1 and I_2 according to Eq. (4-20)

Update $p{^*}_{zi}^{i+1}$ according to Eq. (5-20)

Calculate sumrate

if sumrate >= sum_rate_max

sum_rate_max= sumrate

end

Update λ_{zi},ν_{zi} according to Eq. (4-42) and Eq. (4-41) respectively

End for

Until convergence

End for

for b=1=j

Calculate I_1 and I_2 using Eq. (6-5)

$\widehat{r_j} \leftarrow \emptyset_k$

Repeat

Compute $r_j{}^*$ and $p_j{}^*$, Using the dual decomposition algorithm

If $r_j{}^* \in \{0,1\}\ \forall$ k

Exit

end

Select maximum fraction, r_{jk} from relaxed $r_j{}^*$

Branch: $\widehat{r_{jk}}$← 1

Compute $r_{j\_one}{}^*$ and $p_{j\_one}{}^*$, Using the dual decomposition algorithm

Calculate sumrate_one($r_{j\_one}{}^*$, $p_{j\_one}{}^*$)

Branch: $\widehat{r_{j\imath}}$← 0

Compute $r_{j\_zero}{}^*$ and $p_{j\_zero}{}^*$, Using the dual decomposition algorithm

Calculate sumrate_one($r_{j\_zeo}{}^*$, $p_{j\_zero}{}^*$)

If sumrate_one >= sum_rate_zero

$(r_j^* , p_j^*)=(r_{j\_one}^* , p_{j\_one}^*)$

else

$(r_j^* , p_j^*)=(r_{j\_zero}^* , p_{j\_zero}^*)$

end

until $r_{jk}^* \in \{0,1\} \ \forall k$

end

6.6 Complexity analysis of hybrid algorithm

In the hybrid algorithm, the cell of interest first performs both user selection and power allocation in the interfering cells using the low complexity two stage algorithm to obtain sub-optimal resource allocation values. Since there is (B-1) interfering cells in the multicellular system the computational complexity of the interfering cells is O (BK). The cell of interest then uses these sub optimal values to perform its own resource allocation using the BnB algorithm. The computation complexity of the BnB algorithm in the cell of interest is O (K^2 +K). The total computational complexity of the hybrid algorithm is O (K^2 + BK). It can be easily seen that the computational complexity of the hybrid algorithm increases with the number of cells.

6.7 Simulation and Results

6.7.1 Simulation Model

The hybrid algorithm is compared with the BnB, two stage and existing algorithms for different correlation values. Firstly we evaluate how the cell sum rate and the number of users selected varies with pilot sequence correlation. Next, we check how the sum rate and the number of users selected varies with p_{max}. We also evaluate sum rate and the number of users selected as $C_{jk,min}$ varies. Lastly we investigate the effect of varying the number of antennas on cell sum rate and the number of users selected per cell.

The authors in [13] have modeled this interference as a constant interference by allocating equal power to all the inter cell interfering users. These authors assume that all the interfering cells are transmitting with power P regardless of user selection. Since the research is on both user selection and power allocation in a multi cell massive MIMO system, we are going to compare the solutions in literature with the hybrid algorithm above. The solutions in literature will be modified to incorporate user selection, by using equation Eq. (5-12) to select the maximum number of users that can be scheduled K_s. Different solutions for constant interfering power P=p_{max} , P=0.5p_{max} and P=0 are obtained and compared with the proposed hybrid algorithm.

6.7.2 Results and Discussion

In this section, simulations are used to compare the proposed hybrid algorithm with the two stage, BnB algorithm and the existing solutions in literature. Setting the interfering powers in neighboring cells to zero corresponds to a single cell resource allocation model.

Figure 6-1 shows the effect of varying pmax on sum rate for the proposed algorithms. As can be seen in Figure 6-1there is an optimal pmax value at which the algorithms optimally maximize the cell sum rate. This is mainly due to the fact that as the desired signal power is increased iteratively from zero, the interfering power also increases. However for the case where the inter-cell interference is zero (when the power of interfering users in the other cells is zero), and the pilot correlation is close to zero, interference is negligible hence the sum rate is limited only by the pmax constraint. Figure 6-1 shows the best results for the maximization problem are obtained when the inter cell interference is zero (i.e. a single cell scenario). Figure 6-1also proves that the hybrid algorithm performs better than both the two stage algorithm and BnB algorithm. The proposed algorithms give better results than assuming average power or maximum power for the interfering inter-cell users. The graphs in Figure 6-1 show that when pmax is increased from 0.2 the sum rate also increases to a peak maximum, after that increase in pmax will cause a decrease in sum rate. This is because the rate of increase of the interfering power is more than the rate of increase of the desired signal when pilot contamination is more pronounced or when non orthogonal pilot sequences are used. Figure 6-2 shows how the pmax constraints affects the number of users selected. It is clear that more users are selected when the inter-cell interference is set to zero. More users are selected after resource allocation using hybrid algorithm than assuming average or maximum interfering power. This is because the hybrid algorithm does its power allocation after receiving optimum resource allocation values from the low complexity two stage algorithm.

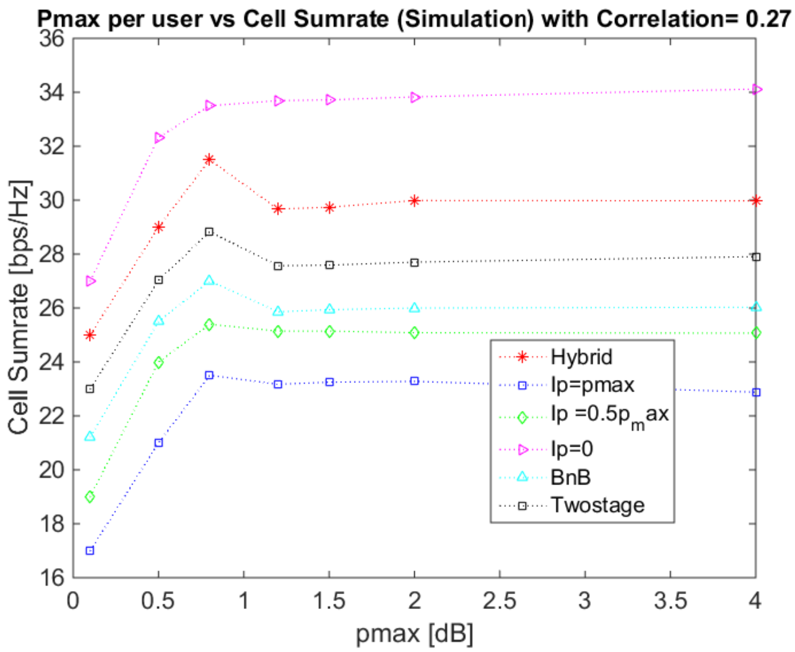

Figure 6-1: Effect of pmax on sum rate for all the proposed resource allocation algorithms.

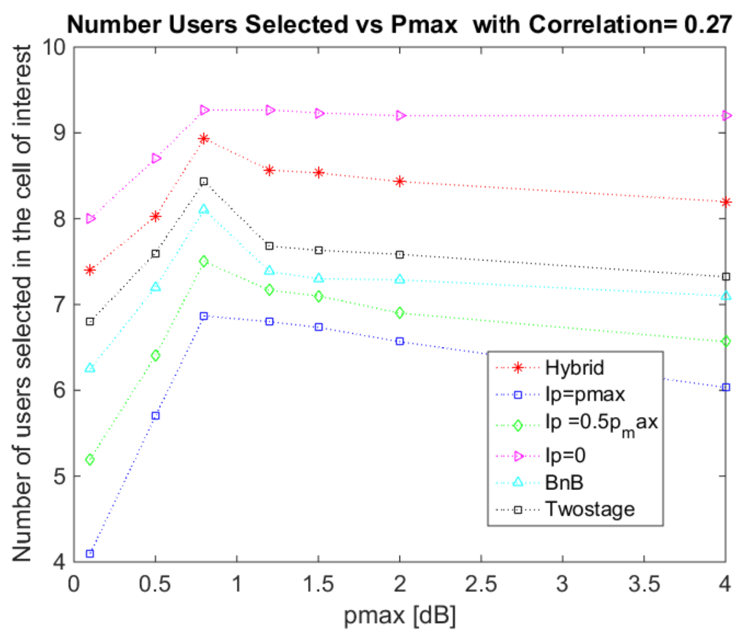

Figure 6-2: Effect of pmax on number of users scheduled per cell

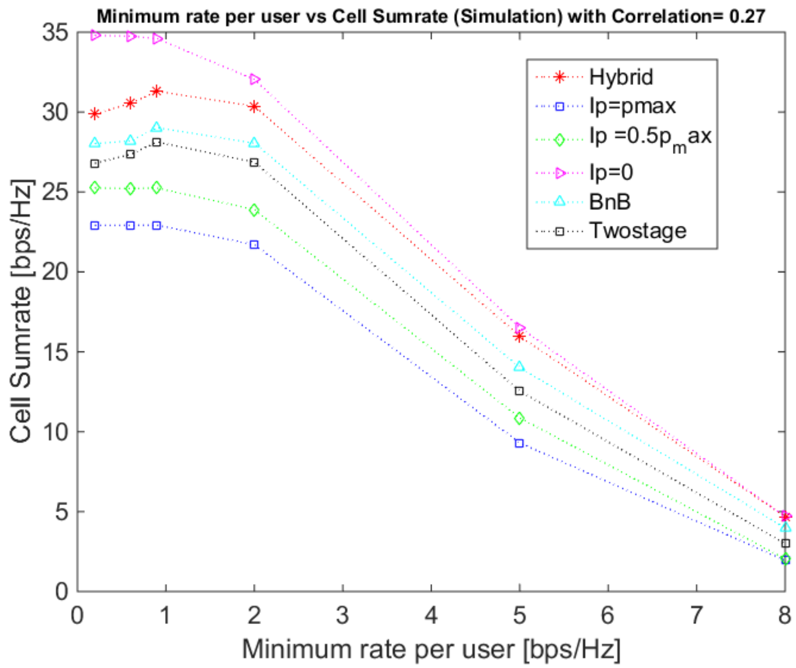

Figure 6-3: Effect of varying the minimum rate constraint on the cell sum rate for all the proposed algorithms

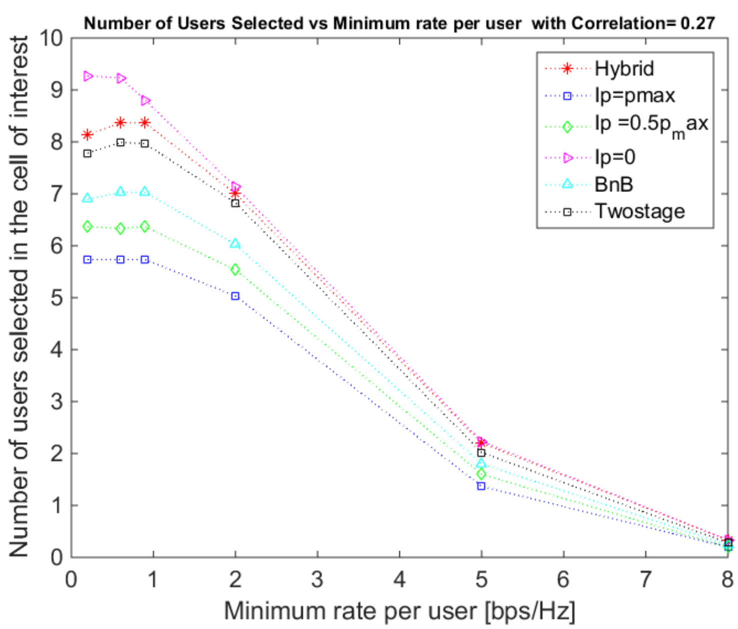

Figure 6-4: Effect of varying the minimum rate constraint on the number of users scheduled per cell for all the proposed algorithms.

A seen in Figure 6-3 increasing the minimum rate constraint decreases the sum rate because fewer users are selected as some users will not meet the QoS constraint. This becomes even more pronounced if non orthogonal pilots are used, the significant interference will have a detrimental effect on the SINR. Due to zero inter-cell interference in the single cell case, more users meet the required QoS condition and hence more users can be scheduled. More users are scheduled for the hybrid algorithm case because resource allocation in the cell of interest is done using optimal resource allocation values in neighboring cells. The algorithms with average and maximum interfering power have lower sum rates because their users have a lower SINR and hence some do not meet the QoS requirement. Figure 6-4 shows the effect of varying the minimum rate constraint on the number of users selected. It can be clearly seen that the number of users scheduled in the cell of interest is higher for the single cell case because of the zero inter-cell interference. The hybrid algorithm gives a fairly high number of scheduled users in the cell of interest because sub-optimal interfering cells resource allocation values are used to model the inter-cell interference. The two stage and BnB curves show a fairly high number of scheduled users because they model the inter-cell interference better than assuming constant power in the interfering cells. The algorithms with average and maximum power have a fairly low number of selected users because of their significant inter-cell interference. Generally the number of users scheduled decreases with an increase in minimum rate because as the minimum rate increases most users will not meet the minimum rate constraint.

In general Figure 6-5 and Figure 6-6 show that as the number of BS antennas increase, the cell sum rate and the number of scheduled users per cell also increase respectively. This is mainly because the Gaussian noise and the interference decreases as the number of antennas increase. The single cell user model has the highest sum rate because inter cell interference is zero and if orthogonal pilots are used in the cell of interest the total interference will be negligible. However the interference due to pilot contamination does not disappear when the number of antennas increase and hence the hybrid algorithm and the average and maximum interfering power models have lower sum rates and number of scheduled users. However the proposed hybrid, two stage and BnB algorithms perform better than the average and maximum interfering power models.

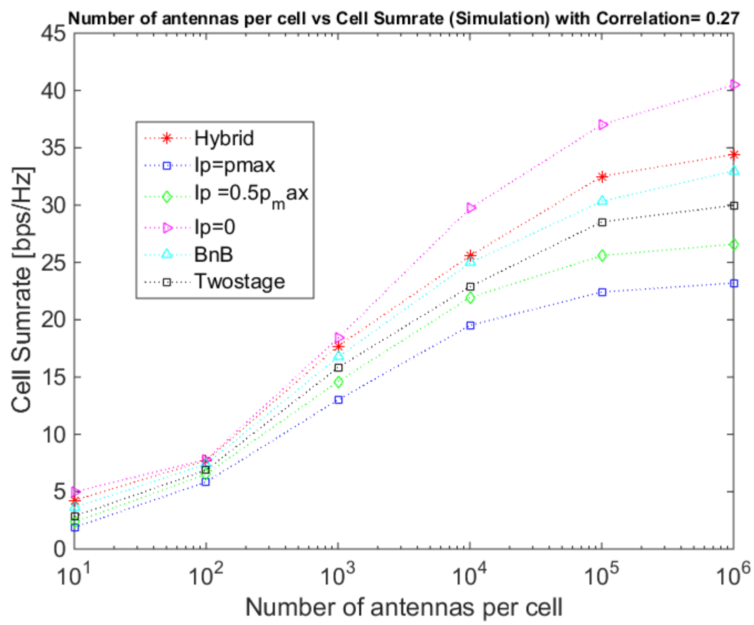

Figure 6-5: Effect of varying the number of BS antennas on the cell sum rate for all the proposed algorithms

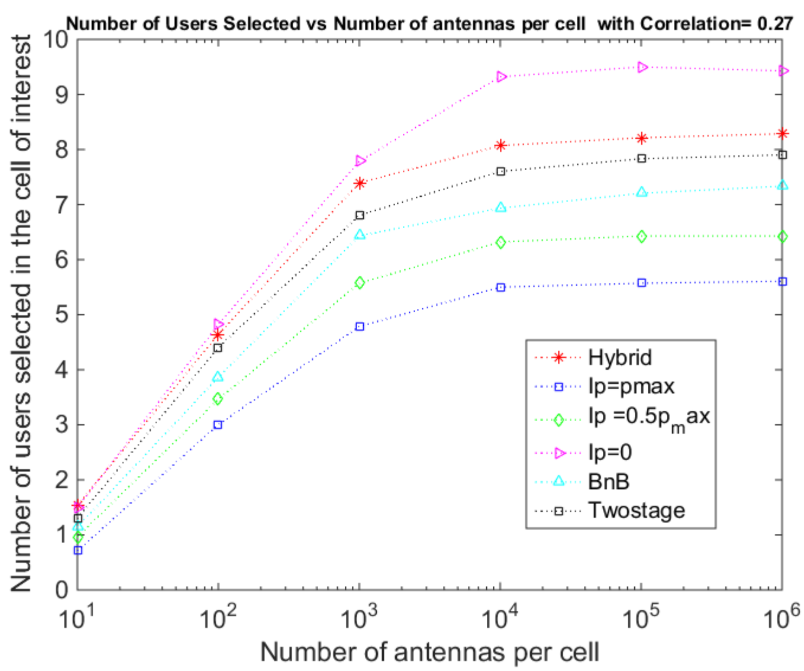

Figure 6-6: Effect of varying the number of BS antennas on the number of users scheduled per cell for all the proposed algorithms

6.8 Chapter Conclusion

The chapter presented a hybrid algorithm which combines the two algorithms described above. In the hybrid algorithm the cell of interest allocates resources in the interfering cells using the two stage algorithm. The cell of interest first performs both user selection and power allocation in the interfering cells using the low complexity two stage algorithm to obtain sub-optimal resource allocation values. The cell of interest then uses these sub optimal values to perform its own resource allocation using the BnB algorithm. The two stage algorithm is chosen for resource allocation in the interfering cells because it has a much lower complexity compared to the BnB algorithm. The BnB algorithm is chosen for resource allocation in the cell of interest because it gives higher sum rate in a sum rate maximization problem than the two stage algorithm.

Simulation results show that the best results for the maximization problem are obtained when the inter cell interference is zero (i.e. a single cell scenario). Simulation results also proves that the hybrid algorithm performs better than both the two stage algorithm and BnB algorithm. The proposed algorithms give better results than assuming average power or maximum power for the interfering inter-cell users. However for the case where the inter-cell interference is zero (single cell case), and the pilot correlation is close to zero, interference is negligible hence the sum rate is limited only by the pmax constraint. It is clear that more users are selected when the inter-cell interference is set to zero. More users are selected after resource allocation using hybrid algorithm than assuming average or maximum interfering power. This is because the hybrid algorithm does its power allocation after receiving optimal resource allocation values from the low complexity two stage algorithm

Simulation results show that increasing the minimum rate constraint decreases the sum rate because fewer users are selected as some users will not meet the QoS constraint. It was also proved that for the designed hybrid algorithm, if non orthogonal pilots are used, the significant interference will have a detrimental effect on the SINR. The algorithms with average and maximum power have a fairly low number of selected users because of their significant inter-cell interference. The results show that the designed hybrid algorithm performs better if a large number of BS antennas are used

CHAPTER 7

7 CONCLUSION AND FUTURE WORKS

7.1 Concluding Remarks

This chapter summarizes the main conclusions of the dissertation, which provides insights on the design of user selection and power allocation algorithms in multi cell massive MIMO systems. The performance of multi cell multiuser massive MIMO systems greatly depends on optimal resource allocation. However it becomes practically impossible to allocate resources on a global level because of its high complexity. To reduce the complexity, this research considers per cell optimal resource allocation with interference modelling. The focus is on designing algorithms that maximizes a particular cell's sum rate capacity taking into consideration the interference from other cells. The interference is introduced when non-orthogonal pilots are used for channel estimation, resulting in pilot contamination. To maximize the sum-rate capacity there is need to optimally allocate power and select the optimal number of users to be scheduled. To tackle the problem sum-rate expressions are derived to enable power allocation and user selection algorithm analysis. However resource allocation values from interfering cells are unknown hence the inter-cell interference had to be approximated which in turn depends on channel information acquisition and pilot contamination.

Chapter 2 presents a comprehensive description of the massive MIMO principle. The chapter introduces massive MIMO basic theories and principles which includes channel estimation, signal detection in UL, precoding techniques in downlink, pilot contamination and mitigation techniques.

Chapter 3 details the work that has been done by other researchers highlighting the recent works and accomplishments on resource allocation in multi cell massive MIMO systems. The previous research in multi cell massive MIMO systems were therefore summarized with more emphasis on power allocation and user selection. The major problem is the high complexity of obtaining a global resource allocation solution as the number of cells increase. It was established that there is a need for efficient inter-cell interference modelling schemes to efficiently optimize resource allocation in massive MIMO systems.

The main objective in Chapter 4 is to model the inter-cell interference so that global resource allocation problem can be converted to a per cell resource allocation problem. To model the inter cell interference it is assumed that the statistical properties of the intra-cell interference in the cell of interest are the same as in the other interfering cells. Therefore the inter-cell interference of the cell of interest can be modelled in terms of the intra-cell interference multiplied by a correction factor which takes into consideration pilot transmission, large scale fading and also the distance the users causing inter-cell interference are from the user of interest k in cell j. The problem is modelled as a non-convex optimization problem which is nondeterministic in polynomial time because the user selection variable can only take binary integer values. To convert the non-convex optimization problem into a convex optimization problem the user selection variable is relaxed. The Lagrangian frame work and the Newton Raphson method is employed to determine the optimal solution. The BnB technique is then used to obtain the optimal solution. In general the results show that for optiml recourse allocation in multi cell massive MIMO system, it is preferable to assign orthogonal pilots to users. It was also observed that the maximum power allocated to a user should be chosen carefully so as to minimize interference and maximize cell sum rate. Simulation results also proved the sum rate of a massive MIMO system is significantly increased if the number of antennas is increased.

In Chapter 5, a low complexity two stage algorithm is proposed. The algorithm is divided into two parts; the user selection part and the power allocation part. The user selection part derives an expression for the maximum number of users which can be served simultaneously through UL transmission per cell, K_s. The K_s users that have a high SINR value assuming equal power allocation are selected from K users. Finally the multi-level water filling technique is used to allocate powers to the selected users Simulation results demonstrate that although the BnB algorithm has a higher computational complexity it performs better than the designed two stage algorithm. In general, the two stage algorithm selects more users than the BnB algorithm. It can therefore be deduced that the two stage algorithm selects users that interfere more with each other.

In Chapter 6, the hybrid algorithm is presented. In the hybrid algorithm the cell of interest allocates resources in the interfering cells using the two stage algorithm. The cell of interest first performs both user selection and power allocation in the interfering cells using the low complexity two stage algorithm to obtain sub-optimal resource allocation values. The cell of interest then uses these sub optimal values to perform its own resource allocation using the BnB algorithm. The two stage algorithm is chosen for resource allocation in the interfering cells because it has a much lower complexity compared to the BnB algorithm. The BnB algorithm is chosen for resource allocation in the cell of interest because it gives higher sum rate in a sum rate maximization problem than the two stage algorithm. Performance analysis and evaluation of the developed algorithms have been presented, mainly through extensive simulations. The designed algorithms have also been compared to existing solutions. In general the presented results demonstrate that the proposed hybrid, BnB and two stage algorithms perform better than existing solutions

7.2 Future Directions

There is an endless road of improvements to extend this work for future research. The resource allocation solution has been structured in such a way that the intra-cell interference is updated iteratively until the solution converges to an optimal solution. Updating the intra-cell iteratively has a high complexity therefore there is need to model the intra-cell interference in such a way that the complexity of the solution is greatly reduced.

The work in this research focused on MRC receive beamforming, however the performance of the designed algorithm need to be tested against beam formers such as ZF and MMSE. The research work also focused on UL, the solution can also be extended to the DL. In this research it was assumed that the user is served by the BS in its cell, the solution can be extended so that users can be served by many BSs or the BSs closest to it.

Furthermore the work in this research focused mainly on power allocation and user selection, however optimal resource allocation also involves determining the optimal number of activated antennas. The pilot sequences assigned to users greatly determine the overall sum rate of the massive MIMO system due to pilot contamination. The work in this research can be extended to include pilot sequence design which aims at mitigating the effects of pilot contamination.

APPENDIX A

A BNB ALGORITHM

A1 Channel Estimation

The approach in [2] was adopted to simplify the LS estimator. The channels are independent hence $\mathbf{h}_{\mathrm{jbl}}\mathbf{h}_{\mathrm{jbm}}{}^{\mathrm{H}} = 0$

$$\begin{cases} \mathbf{h}_{\mathrm{jbk}}\mathbf{h}_{\mathrm{jbm}}{}^{\mathrm{H}} = 0 \\ \mathbf{h}_{\mathrm{jbk}}\mathbf{h}_{\mathrm{jbk}}{}^{\mathrm{H}} = 1 \end{cases} \tag{A-1}$$

Using the property that $\mathbf{S}_{\mathrm{jk}}^{\mathrm{T}}\mathbf{S}_{\mathrm{jk}}^{\mathrm{T}} = \mathbf{I}_{\mathrm{N}}$, the LS estimator can be simplified to

$$\begin{aligned} \widehat{\mathbf{h}_{\mathrm{jjk}}} &= \mathbf{S}_{\mathrm{jk}}^{\mathrm{T}}\mathbf{Y}_{\mathrm{j}} \\ &= \mathbf{S}_{\mathrm{jk}}^{\mathrm{T}}\left(\sum_{\mathrm{b=1}}^{\mathrm{B}}\sum_{\mathrm{m=1}}^{\mathrm{K}} \mathrm{r}_{\mathrm{bm}}\sqrt{\mathrm{q}_{\mathrm{bm}}\beta_{\mathrm{jbm}}}\,\mathbf{h}_{\mathrm{jbm}}\mathbf{S}_{\mathrm{bm}} + \mathbf{N}_{\mathrm{j}}\right) \\ &= \sum_{\mathrm{b=1}}^{\mathrm{B}}\sum_{\mathrm{m=1}}^{\mathrm{K}} \mathrm{r}_{\mathrm{bm}}\psi_{\mathrm{bmjk}}\sqrt{\mathrm{q}_{\mathrm{bm}}\beta_{\mathrm{jbm}}}\,\mathbf{h}_{\mathrm{jbm}} + \mathrm{N}_{\mathrm{j}}\mathbf{S}_{\mathrm{jk}}^{\mathrm{T}} \\ &= \mathrm{r}_{\mathrm{jk}}\,\mathbf{h}_{\mathrm{jjk}} + \sum_{\mathrm{b=1}}^{\mathrm{B}}\sum_{\mathrm{m=1},(\mathrm{b,m})\neq(\mathrm{j,k})}^{\mathrm{K}} \mathrm{r}_{\mathrm{bm}}\psi_{\mathrm{bmjk}}\sqrt{\mathrm{q}_{\mathrm{bm}}\beta_{\mathrm{jbm}}}\,\mathbf{h}_{\mathrm{jbm}} + \mathbf{N}_{\mathrm{j}}\mathbf{S}_{\mathrm{jk}}^{\mathrm{T}} \end{aligned} \tag{A-2}$$

A2 Simplifying Expectaction in the SINR Expression

The UL effective SINR for user k in cell j given by Eq. (4-9) for the system model given by Eq.(4-7) can be simplified one at a time as shown below

Simplifying $|\hat{\mathbf{h}}_{\mathrm{jjk}}^{\mathbf{h}}\mathbf{h}_{\mathrm{jbm}}|^2$ let $\mathrm{J}_{\mathrm{jbm}} = \sqrt{\mathrm{q}_{\mathrm{bm}}\beta_{\mathrm{jbm}}}$

$$\begin{aligned} |\hat{\mathbf{h}}_{\mathrm{jjk}}^{\mathrm{H}}\mathbf{h}_{\mathrm{jbm}}|^2 &= \left(\hat{\mathbf{h}}_{\mathrm{jjk}}^{\mathrm{H}}\mathbf{h}_{\mathrm{jbm}}\right)^{\mathrm{H}}\left(\hat{\mathbf{h}}_{\mathrm{jjk}}^{\mathrm{H}}\mathbf{h}_{\mathrm{jbm}}\right) \\ &= \left(\sum_{\mathrm{t=1}}^{\mathrm{B}}\sum_{\mathrm{u=1}}^{\mathrm{K}} \psi_{\mathrm{tujk}}\mathrm{J}_{\mathrm{jtu}}\mathbf{h}_{\mathrm{jtu}}^{\mathrm{H}}\,\mathbf{h}_{\mathrm{jbm}} + \mathbf{S}_{\mathrm{jk}}\mathrm{N}_{\mathrm{j}}^{\mathrm{H}}\mathbf{h}_{\mathrm{jbm}}\right)^{\mathrm{H}} \times \end{aligned} \tag{A-3}$$

$$\left(\sum_{V=1}^{B}\sum_{z=1}^{K} \psi_{vzjk}J_{jvz}\mathbf{h}_{jvz}^{H}\,\mathbf{h}_{jbm} + \mathbf{S}_{jk}\mathbf{N}_{j}^{H}\mathbf{h}_{jbm}\right)$$

$$=\sum_{t,u}\sum_{v,z}\psi_{tujk}\,\psi_{vzjk}J_{jtu}J_{jvz}\mathbf{h}_{jbm}^{H}\mathbf{h}_{jtu}\mathbf{h}_{jvz}^{H}\mathbf{h}_{jbm}+$$

$$\sum_{t=1}^{B}\sum_{u=1}^{K}\psi_{tujk}J_{jtu}\mathbf{h}_{jbm}^{H}\,\mathbf{h}_{jtu}\mathbf{S}_{jk}\mathbf{N}_{j}^{H}\mathbf{h}_{jbm} +$$

$$\mathbf{h}_{jbm}^{H}\mathbf{N}_{j}\mathbf{S}_{jk}^{H}\sum_{v=1}^{B}\sum_{z=1}^{K}\psi_{vzjk}J_{jvz}\mathbf{h}_{jvz}^{H}\,\mathbf{h}_{jbm} +$$

$$\mathbf{h}_{jbm}^{H}\mathbf{N}_{j}\mathbf{S}_{jk}^{H}\mathbf{S}_{jk}\mathbf{N}_{j}^{H}\mathbf{h}_{jbm}$$

Then $\mathbb{E}\left[|\hat{\mathbf{h}}_{jjk}^{H}\mathbf{h}_{jbm}|^{2}\right]$

$$= \mathbb{E}\left[\sum_{t,u}\sum_{v,z}\psi_{tujk}\,\psi_{vzjk}J_{jtu}J_{jvz}\mathbf{h}_{jbm}^{H}\mathbf{h}_{jtu}\mathbf{h}_{jvz}^{H}\mathbf{h}_{jbm}\right]+ 0+0+$$

$$\mathbb{E}[\mathbf{h}_{jbm}^{H}\mathbf{N}_{j}\mathbf{S}_{jk}^{H}\mathbf{S}_{jk}\mathbf{N}_{j}^{H}\mathbf{h}_{jbm}]$$

$$=\psi_{bmjk}^{2}J_{jbm}^{2}\;\mathbb{E}[\mathbf{h}_{jbm}^{H}\mathbf{h}_{jbm}\mathbf{h}_{jbm}^{H}\mathbf{h}_{jbm}] + \sum_{\substack{t=v\neq b\\ u=z\neq m}}\psi_{tujk}^{2}J_{jtu}^{2}\;\mathbb{E}[\mathrm{tr}\{\mathbf{h}_{jbm}^{H}\mathbf{h}_{jtu}\mathbf{h}_{jtu}^{H}\mathbf{h}_{jbm}\}] + \mathbb{E}[\mathrm{tr}\{\mathbf{h}_{jbm}^{H}\mathbf{N}_{j}\mathbf{S}_{jk}^{H}\mathbf{S}_{jk}\mathbf{N}_{j}^{H}\mathbf{h}_{jbm}\}] \qquad \text{(A-4)}$$

It can be observed that $\mathbf{h}_{jbm}^{H}\mathbf{h}_{jbm}$ has a gamma distribution Γ(N,1). Hence $\mathbb{E}[\mathbf{h}_{jbm}^{H}\mathbf{h}_{jbm}\mathbf{h}_{jbm}^{H}\mathbf{h}_{jbm}]$ simplifies to $(N^{2} + N)$.

This simplifies $\mathbb{E}\left[|\hat{\mathbf{h}}_{jjk}^{H}\mathbf{h}_{jbm}|^{2}\right]$ to

$$\psi_{bmjk}^{2}J_{jbm}^{2}\,(N^{2} + N) + N\sum_{\substack{t=v\neq b\\ u=z\neq m}}\psi_{tujk}^{2}J_{jtu}^{2} + N\sigma^{2}$$
$$=N^{2}\psi_{bmjk}^{2}J_{jbm}^{2} + N\alpha_{jk} \qquad \text{(A-5)}$$

where $\alpha_{jk}= \sum_{\substack{t=v\\ u=z}}\psi_{tujk}^{2}J_{jtu}^{2} + \sigma^{2}$

Simplifying $|\mathbf{v}_{jk}|^2$

$$= \hat{\mathbf{h}}_{jjk}^{H}\hat{\mathbf{h}}_{jjk}$$

$$= \left(\sum_{t=1}^{B}\sum_{u=1}^{K}\psi_{tujk}J_{jtu}\mathbf{h}_{jtu}^{H} + \mathbf{S}_{jk}\mathbf{N}_{j}^{H}\right) \times \left(\sum_{v=1}^{B}\sum_{z=1}^{K}\psi_{vzjk}J_{jvz}H_{jvz} + \mathbf{N}_{j}\mathbf{S}_{jk}^{H}\right)$$

$$=\sum_{t,u}\sum_{v,z}\psi_{tujk}\psi_{vzjk}J_{jtu}J_{jvz}\mathbf{h}_{jtu}^{H}\mathbf{h}_{jvz} + \sum_{t=1}^{B}\sum_{u=1}^{K}\psi_{tujk}J_{jtu}\mathbf{h}_{jtu}^{H}\mathbf{N}_{j}\mathbf{S}_{jk}^{H} + \tag{A-6}$$

$$\mathbf{S}_{jk}\mathbf{N}_{j}^{H}\sum_{v=1}^{B}\sum_{z=1}^{K}\psi_{vzjk}J_{jvz}\mathbf{h}_{jvz}^{H} + \mathbf{N}_{j}\mathbf{S}_{jk}^{H}\mathbf{S}_{jk}\mathbf{N}_{j}^{H}$$

Then $\mathbb{E}\,|\mathbf{v}_{jk}|^2$ simplifies to

$$=\sum_{t,u}\sum_{v,z}\psi_{tujk}\psi_{vzjk}J_{jtu}J_{jvz}\mathbb{E}[\mathbf{h}_{jtu}^{H}\mathbf{h}_{jvz}] + 0 + 0 + \mathbb{E}[\mathrm{tr}\{\mathbf{N}_{j}\mathbf{S}_{jk}^{H}\mathbf{S}_{jk}\mathbf{N}_{j}^{H}\}]$$

$$=\sum_{\substack{t=v\\u=z}}\psi_{tujk}^{2}J_{jtu}^{2}\,\mathbb{E}[\mathrm{tr}\{\mathbf{h}_{jtu}\mathbf{h}_{jtu}^{H}\}] + \mathbb{E}[\mathrm{tr}\{\mathbf{N}_{j}\mathbf{S}_{jk}^{H}\mathbf{S}_{jk}\mathbf{N}_{j}^{H}\}]$$

$$=N\sum_{\substack{t=v\\u=z}}\psi_{tujk}^{2}J_{jtu}^{2} + N\sigma^{2} \tag{A-7}$$

$$=N\alpha_{jk}$$

It can easily be deduced that $\mathrm{Var}\left|\mathbf{v}_{jk}{}^{H}\mathbf{h}_{jjk}\right| = N\alpha_{jk}$.

Substituting Eq.(A-5) and Eq. (A-7) into Eq. (4-9)

$$\mathrm{SINR}_{jk}^{MRC} = \frac{r_{jk}\beta_{jjk}p_{jk}N^{2}}{\beta_{jjk}p_{jk}r_{jk}N\alpha_{jk} + \sum_{b=1}^{B}\sum_{m=1\,(b,m)\neq(j,k)}^{K}p_{bm}\beta_{jbm}r_{bm}\left(N^{2}\psi_{bmjk}^{2}J_{jbm}^{2} + N\alpha_{jk}\right) + N\alpha_{j}} \tag{A-8}$$

The above equation can be further simplified to

$$\mathrm{SINR}_{jk}^{MRC} = \frac{\beta_{jjk}p_{jk}r_{jk}}{\sum_{b=1}^{B}\sum_{m=1\,(b,m)\neq(j,k)}^{K}p_{bm}\beta_{jbm}r_{bm}\left(\psi_{bmjk}^{2}J_{jbm}^{2}\right) + \frac{\alpha_{jk}}{N}\overline{R_{jk}}} \tag{A-9}$$

Where $\overline{R_{jk}} = \sum_{b=1}^{B}\sum_{m=1}^{K}r_{bm}p_{bm}\beta_{jbm} + \sigma_{w}^{2}$

A3 Interference Modelling

The inter-cell interference in Eq.(4-15) is written in terms of the intra-cell interference as shown in Eq.(A-10) below

$$\begin{aligned}\sum_{b=1,\ \ b\neq j}^{B}\sum_{m=1}^{K} p_{bm}\beta_{jbm}r_{bm}\left(\psi_{bmjk}^{2}J_{jbm}^{2}\right)\\ +\frac{\alpha_{jk}}{N}\sum_{b=1,\ \ b\neq j}^{B}\sum_{m=1}^{K} p_{bm}\beta_{jbm}r_{bm}\\ = f_{jk}\left(\sum_{m=1}^{K} p_{jm}\beta_{jjm}r_{jm}\left(\psi_{jmjk}^{2}J_{jjm}^{2}\right)\right.\\ \left.+\frac{\alpha_{jk}}{N}\sum_{m=1}^{K} p_{jm}\beta_{jjm}r_{jm}\right)\end{aligned} \tag{A-10}$$

To obtain a fair estimate of f_{jk} we assume that $p_{bm} = P,\ \forall\ b, m.$ We also assume that all the users are chosen i.e., $r_{bm} = 1, \forall\, b, m.$ Using the above assumption Eq.(A-10) above simplifies to

$$\begin{aligned}\sum_{b=1,\ \ b\neq j}^{B}\sum_{m=1}^{K} \beta_{jbm}\left(\psi_{bmjk}^{2}J_{jbm}^{2}\right)+\frac{\alpha_{jk}}{N}\sum_{b=1,\ \ b\neq j}^{B}\sum_{m=1}^{K}\beta_{jbm}\\ = f_{jk}\left(\sum_{m=1}^{K} \beta_{jjm}\left(\psi_{jmjk}^{2}J_{jjm}^{2}\right)+\frac{\alpha_{jk}}{N}\sum_{m=1}^{K}\beta_{jjm}\right)\end{aligned} \tag{A-11}$$

Solving for f_{jk} we get

$$f_{jk} = \frac{\sum_{b=1,\ \ b\neq j}^{B}\sum_{m=1}^{K}\beta_{jbm}\left(\psi_{bmjk}^{2}J_{jbm}^{2}\right)+\frac{\alpha_{jk}}{N}\sum_{b=1,\ \ b\neq j}^{B}\sum_{m=1}^{K}\beta_{jbm}}{\sum_{m=1}^{K}\beta_{jjm}\left(\psi_{jmjk}^{2}J_{jjm}^{2}\right)+\frac{\alpha_{jk}}{N}\sum_{m=1}^{K}\beta_{jjm}} \tag{A-12}$$

REFERENCES

[1] W. Fan *et al.*, "A Step Toward 5G in 2020: Low-cost OTA performance evaluation of massive MIMO base stations.," *IEEE Antennas and Propagation Magazine*, vol. 59, no. 1, pp. 38–47, Feb. 2017.

[2] "Guest Editorial Special Issue on 5G Wireless Systems With Massive MIMO," *IEEE Systems Journal*, vol. PP, no. 99, pp. 1–3, 2017.

[3] E. G. Larsson, D. Danev, M. Olofsson, and S. Sorman, "Teaching the Principles of Massive MIMO: Exploring reciprocity-based multiuser MIMO beamforming using acoustic waves," *IEEE Signal Processing Magazine*, vol. 34, no. 1, pp. 40–47, Jan. 2017.

[4] A. G. Gotsis and A. D. Panagopoulos, "On user association and multiple access optimisation in 5G massive MIMO empowered ultra dense networks," *Trans. Emerging Tel. Tech.*, Jan. 2016.

[5] J. G. Andrews *et al.*, "What Will 5G Be?," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 6, pp. 1065–1082, Jun. 2014.

[6] K. N. R. S. V. Prasad, E. Hossain, and V. K. Bhargava, "Energy Efficiency in Massive MIMO-Based 5G Networks: Opportunities and Challenges," *IEEE Wireless Communications*, vol. PP, no. 99, pp. 2–10, 2017.

[7] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang, "An Overview of Massive MIMO: Benefits and Challenges," *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, no. 5, pp. 742–758, Oct. 2014.

[8] Y. Zhang, B. Golkar, E. S. Sousa, and Q. Zhang, "Efficient User Selection for Downlink Zero-Forcing Based Multiuser MIMO Systems," in *Vehicular Technology Conference (VTC Fall), 2011 IEEE*, 2011, pp. 1–5.

[9] J. Zuo, J. Zhang, C. Yuen, W. Jiang, and W. Luo, "Energy-Efficient Downlink Transmission for Multicell Massive DAS With Pilot Contamination," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 2, pp. 1209–1221, Feb. 2017.

[10] A. Pitarokoilis, S. K. Mohammed, and E. G. Larsson, "Uplink Performance of Time-Reversal MRC in Massive MIMO Systems Subject to Phase Noise," *IEEE Transactions on Wireless Communications*, vol. 14, no. 2, pp. 711–723, Feb. 2015.

[11] K. Guo, Y. Guo, and G. Ascheid, "Energy-Efficient Uplink Power Allocation in Multi-Cell MU-Massive-MIMO Systems," presented at the European Wireless 2015; 21th European Wireless Conference; Proceedings of, 2015, pp. 1–5.

[12] C. S. Chabalala, R. V. Olst, and F. Takawira, "Optimal Channel Selection and Power Allocation for Channel Assembling in Cognitive Radio Networks," in *2015 IEEE Global Communications Conference (GLOBECOM)*, 2015, pp. 1–6.

[13] W. Liu, S. Han, C. Yang, and C. Sun, “Massive MIMO or small cell network: Who is more energy efficient?,” in *2013 IEEE Wireless Communications and Networking Conference Workshops (WCNCW)*, 2013, pp. 24–29.

[14] N. Akbar, N. Yang, P. Sadeghi, and R. Kennedy, “Multi-Cell Multiuser Massive MIMO Networks: User Capacity Analysis and Pilot Design,” *IEEE Transactions on Communications*, vol. PP, no. 99, pp. 1–1, 2016.

[15] E. Pakdeejit, “Linear Precoding Performance of Massive MU-MIMO downlink System,” M.Sc. Thesis, Linköping University, Sweden, 2013.

[16] Hien Quoc Ngo, *Massive MIMO: Fundamentals and System Designs*, vol. Volume 1642 of Linköping Studies in Science and Technology. Dissertations. Linköping University Electronic Press, 2015.

[17] R. B. Nielsen and P. C. Buelga, “Near Field Channel Modelling through Large Aperture Massive Array Systems and Low-Complexity Pre-processing Robust to Local Mobility,” 2014.

[18] S. V. Hanly, I. B. Collings, Z. A. Shaikh, and P. Whiting, “Law of large numbers analysis of antenna selection aided downlink beamforming in massive MISO under RF chains constraint,” in *2016 Australian Communications Theory Workshop (AusCTW)*, 2016, pp. 163–168.

[19] E. Björnson, E. G. Larsson, and M. Debbah, “Massive MIMO for Maximal Spectral Efficiency: How Many Users and Pilots Should Be Allocated?,” *in IEEE Transactions on Wireless Communications*, vol. 15, pp. 1293–1308, Feb. 2016.

[20] A. Adhikary, A. Ashikhmin, and T. Marzetta, “Uplink Interference Reduction in Large Scale Antenna Systems,” *IEEE Transactions on Communications*, vol. PP, no. 99, pp. 1–1, 2017.

[21] J. Jose, A. Ashikhmin, T. L. Marzetta, and S. Vishwanath, “Pilot Contamination and Precoding in Multi-Cell TDD Systems,” *IEEE Transactions on Wireless Communications*, vol. 10, no. 8, pp. 2640–2651, Aug. 2011.

[22] Emil Björnson, “Multiantenna cellular communications channel estimation, feedback, and resource allocation,” Doctoral Thesis in Telecommunications, Linköping University, Sweden, 2011.

[23] E. G. Larsson, T. L. Marzetta, and E. Björnson, “Massive MIMO: ten myths and one critical question,” *IEEE Journals & Magazines*, no. 54, pp. 114–123, 2016.

[24] T. L. Marzetta, “Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

[25] Hien Quoc Ngo, E. G. Larsson, and T. L. Marzetta, “Energy and Spectral Efficiency of Very Large Multiuser MIMO Systems,” *IEEE Transactions on Communications*, vol. 61, no. 4, pp. 1436–1449, Apr. 2013.

[26] J. Hoydis, S. ten Brink, and M. Debbah, “Massive MIMO in the UL/DL of Cellular Networks: How Many Antennas Do We Need?,” *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 2, pp. 160–171, Feb. 2013.

[27] Y. C. Liang, G. Pan, and Z. D. Bai, "Asymptotic Performance of MMSE Receivers for Large Systems Using Random Matrix Theory," *IEEE Transactions on Information Theory*, vol. 53, no. 11, pp. 4173–4190, Nov. 2007.

[28] T. V. Chien, E. Björnson, and E. G. Larsson, "Joint Power Allocation and User Association Optimization for Massive MIMO Systems," *IEEE Transactions on Wireless Communications*, vol. 15, no. 9, pp. 6384–6399, Sep. 2016.

[29] E. Larsson, O. Edfors, F. Tufvesson, and T. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Communications Magazine*, vol. 52, no. 2, pp. 186–195, Feb. 2014.

[30] G. C. Ferrante, G. Geraci, T. Q. S. Quek, and M. Z. Win, "Group-Blind Detection for Uplink of Massive MIMO Systems," *IEEE Transactions on Signal Processing*, vol. 65, no. 5, pp. 1272–1286, Mar. 2017.

[31] H. Q. Ngo and E. G. Larsson, "EVD-based channel estimation in multicell multiuser MIMO systems with very large antenna arrays," in *2012 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2012, pp. 3249–3252.

[32] L. Cottatellucci, R. R. Muller, and M. Vehkapera, "Analysis of Pilot Decontamination Based on Power Control," in *2013 IEEE 77th Vehicular Technology Conference (VTC Spring)*, 2013, pp. 1–5.

[33] R. R. Müller, L. Cottatellucci, and M. Vehkaperä, "Blind Pilot Decontamination," *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, no. 5, pp. 773–786, Oct. 2014.

[34] Y. Li, Y.-H. Nam, B. L. Ng, and J. Zhang, "A non-asymptotic throughput for massive MIMO cellular uplink with pilot reuse," in *2012 IEEE Global Communications Conference (GLOBECOM)*, 2012, pp. 4500–4504.

[35] K. Appaiah, A. Ashikhmin, and T. L. Marzetta, "Pilot Contamination Reduction in Multi-User TDD Systems," in *2010 IEEE International Conference on Communications*, 2010, pp. 1–5.

[36] F. Fernandes, A. Ashikhmin, and T. L. Marzetta, "Inter-Cell Interference in Noncooperative TDD Large Scale Antenna Systems," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 2, pp. 192–201, Feb. 2013.

[37] J. C. Chen, C. K. Wen, S. Jin, and K. K. Wong, "A Low Complexity Pilot Scheduling Algorithm for Massive MIMO," *IEEE Wireless Communications Letters*, vol. 6, no. 1, pp. 18–21, Feb. 2017.

[38] A. Khansefid and H. Minn, "Asymptotically optimal power allocation for massive MIMO uplink," in *2014 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, 2014, pp. 627–631.

[39] H. V. Cheng, E. Björnson, and E. G. Larsson, "Uplink pilot and data power control for single cell massive MIMO systems with MRC," in *2015 International Symposium on Wireless Communication Systems (ISWCS)*, 2015, pp. 396–400.

[40] L. Zhao, H. Zhao, F. Hu, K. Zheng, and J. Zhang, "Energy Efficient Power Allocation Algorithm for Downlink Massive MIMO with MRT Precoding," in *Vehicular Technology Conference (VTC Fall), 2013 IEEE 78th*, 2013, pp. 1–5.

[41] Y. Guo, J. Tang, G. Wu, and S. Li, “Power allocation for massive MIMO: Impact of power amplifier efficiency,” in *2015 IEEE/CIC International Conference on Communications in China (ICCC)*, 2015, pp. 1–6.

[42] S. Zarei, W. Gerstacker, and R. Schober, “A low-complexity linear precoding and power allocation scheme for downlink massive MIMO systems,” in *2013 Asilomar Conference on Signals, Systems and Computers*, 2013, pp. 285–290.

[43] J. Zhang, Y. Jiang, P. Li, F. Zheng, and X. You, “Energy Efficient Power Allocation in Massive MIMO Systems Based on Standard Interference Function,” in *2016 IEEE 83rd Vehicular Technology Conference (VTC Spring)*, 2016, pp. 1–6.

[44] Z. Zhou, S. Zhou, J. Gong, and Z. Niu, “Energy-efficient antenna selection and power allocation for large-scale multiple antenna systems with hybrid energy supply,” in *2014 IEEE Global Communications Conference*, 2014, pp. 2574–2579.

[45] Y. Hao, Z. Song, S. Hou, and H. Li, “Energy- and spectral-efficiency tradeoff in massive MIMO systems with inter-user interference,” in *2015 IEEE 26th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*, 2015, pp. 553–557.

[46] Q. Zhang, S. Jin, M. McKay, D. Morales-Jimenez, and H. Zhu, “Power Allocation Schemes for Multicell Massive MIMO Systems,” *IEEE Transactions on Wireless Communications*, vol. 14, no. 11, pp. 5941–5955, Nov. 2015.

[47] E. Björnson, E. G. Larsson, and M. Debbah, “Optimizing multi-cell massive MIMO for spectral efficiency: How Many users should be scheduled?,” in *2014 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, 2014, pp. 612–616.

[48] T. M. Nguyen, V. N. Ha, and L. B. Le, “Resource Allocation Optimization in Multi-User Multi-Cell Massive MIMO Networks Considering Pilot Contamination,” *IEEE Access*, vol. 3, pp. 1272–1287, 2015.

[49] T. M. Nguyen and L. B. Le, “Joint pilot assignment and resource allocation in multicell massive MIMO network: Throughput and energy efficiency maximization,” in *2015 IEEE Wireless Communications and Networking Conference (WCNC)*, 2015, pp. 393–398.

[50] W. Liu, A. Zappone, C. Yang, and E. Jorswieck, “Global EE optimization of massive MIMO systems,” in *2015 IEEE 16th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2015, pp. 221–225.

[51] Y. Zhang, H. Gao, F. Tan, and T. Lv, “Resource Allocation of Energy-Efficient Multi-User Massive MIMO Systems,” in *2016 IEEE Globecom Workshops (GC Wkshps)*, 2016, pp. 1–6.

[52] K. Guo, Y. Guo, G. Fodor, and G. Ascheid, “Uplink power control with MMSE receiver in multi-cell MU-massive-MIMO systems,” in *2014 IEEE International Conference on Communications (ICC)*, 2014, pp. 5184–5190.

[53] H. Q. Ngo, M. Matthaiou, and E. G. Larsson, “Massive MIMO With Optimal Power and Training Duration Allocation,” *IEEE Wireless Communications Letters*, vol. 3, no. 6, pp. 605–608, Dec. 2014.

[54] Z. Shen, R. Chen, J. G. Andrews, R. W. Heath, and B. L. Evans, "Low complexity user selection algorithms for multiuser MIMO systems with block diagonalization," *IEEE Transactions on Signal Processing*, vol. 54, no. 9, pp. 3658–3663, Sep. 2006.

[55] S. Kaviani and W. A. Krzymien, "User Selection for Multiple-Antenna Broadcast Channel with Zero-Forcing Beamforming," in *IEEE GLOBECOM 2008 - 2008 IEEE Global Telecommunications Conference*, 2008, pp. 1–5.

[56] B. Zhou, B. Bai, Y. Li, D. Gu, and Y. Luo, "Chordal Distance-Based User Selection Algorithm for the Multiuser MIMO Downlink with Perfect or Partial CSIT," in *2011 IEEE International Conference on Advanced Information Networking and Applications*, 2011, pp. 77–82.

[57] A. Bayesteh and A. K. Khandani, "On the User Selection for MIMO Broadcast Channels," *IEEE Transactions on Information Theory*, vol. 54, no. 3, pp. 1086–1107, Mar. 2008.

[58] N. fang, J. Zeng, X. Su, and Y. Kuang, "Effective user selection algorithm for quantized precoding in massive MIMO," *ICST Transactions on Scalable Information Systems*, vol. 2, no. 4, p. e3, Feb. 2015.

[59] H. Liu, H. Gao, S. Yang, and T. Lv, "Low-Complexity Downlink User Selection for Massive MIMO Systems," *IEEE Systems Journal*, vol. PP, no. 99, pp. 1–12, 2015.

[60] Q. Ye, O. Y. Bursalioglu, H. C. Papadopoulos, C. Caramanis, and J. G. Andrews, "User Association and Interference Management in Massive MIMO HetNets," *IEEE Transactions on Communications*, vol. 64, no. 5, pp. 2049–2065, May 2016.

[61] D. Liu, L. Wang, Y. Chen, T. Zhang, K. K. Chai, and M. Elkashlan, "Distributed Energy Efficient Fair User Association in Massive MIMO Enabled HetNets," *IEEE Communications Letters*, vol. 19, no. 10, pp. 1770–1773, Oct. 2015.

[62] Y. Xu and S. Mao, "User Association in Massive MIMO HetNets," *IEEE Systems Journal*, vol. PP, no. 99, pp. 1–1, 2015.

[63] D. Bethanabhotla, O. Y. Bursalioglu, H. C. Papadopoulos, and G. Caire, "User association and load balancing for cellular massive MIMO," in *Information Theory and Applications Workshop (ITA), 2014*, 2014, pp. 1–10.

[64] E. Bjornson, T. V. Chien, and E. G. Larsson, "Downlink Power Control for Massive MIMO Cellular Systems with Optimal User Association," *ICCC proceedings*, Jul. 2016.

[65] J. Li, E. Björnson, T. Svensson, T. Eriksson, and M. Debbah, "Joint Precoding and Load Balancing Optimization for Energy-Efficient Heterogeneous Networks," *IEEE Transactions on Wireless Communications*, vol. 14, no. 10, pp. 5810–5822, Oct. 2015.

[66] W. Dinkelbach, "On Nonlinear Fractional Programming," *Management Science*, vol. 13, no. 7, pp. 492–498, 1967.

[67] J. C. Shen, J. Zhang, and K. B. Letaief, "User capacity of pilot-contaminated TDD massive MIMO systems," in *2014 IEEE Global Communications Conference*, 2014, pp. 3713–3718.

[68] A. Khansefid and H. Minn, "On Channel Estimation for Massive MIMO With Pilot Contamination," *IEEE Communications Letters*, vol. 19, no. 9, pp. 1660–1663, Sep. 2015.

[69] J. C. Shen, J. Zhang, and K. B. Letaief, "Downlink User Capacity of Massive MIMO Under Pilot Contamination," *IEEE Transactions on Wireless Communications*, vol. 14, no. 6, pp. 3183–3193, Jun. 2015.

[70] K.-N. Chang and K.-D. Lee, "Capacity of multicell CDMA/shared-TDD system," in *14th IEEE Proceedings on Personal, Indoor and Mobile Radio Communications, 2003. PIMRC 2003.*, 2003, vol. 1, p. 891–895 Vol.1.

[71] V.E Idigo, K.A Akpado, and I.I Nsionu, "Estimating the Capacity of a Multi-cell System Using Concentric Circle Geometry," *International Journal of Electrical & Telecommunication Systems Research*, vol. 1, pp. 19–22, 2006.

[72] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[73] H. Zhang, C. Jiang, N. C. Beaulieu, X. Chu, X. Wen, and M. Tao, "Resource Allocation in Spectrum-Sharing OFDMA Femtocells With Heterogeneous Services," *IEEE Transactions on Communications*, vol. 62, no. 7, pp. 2366–2377, Jul. 2014.

[74] D. Axehill and M. Morari, "Improved complexity analysis of branch and bound for hybrid MPC," in *49th IEEE Conference on Decision and Control (CDC)*, 2010, pp. 4216–4222.

[75] M. Maesoumi and M. A. Masnadi-Shirazi, "Efficient low complexity partial crosstalk cancellation algorithm by using successive linear convex relaxation and branch and bound approaches in VDSL networks," in *2008 Second International Conference on Communications and Electronics*, 2008, pp. 306–310.