

Abstract

Forward and inverse spectral problems concerning Sturm-Liouville operators without discontinuities have been studied extensively. By comparison, there has been limited work tackling the case where the eigenfunctions have discontinuities at interior points, a case which appears naturally in physical applications. We refer to such discontinuity conditions as transmission conditions. We consider Sturm-Liouville problems with transmission conditions rationally dependent on the spectral parameter. We show that our problem admits geometrically double eigenvalues, necessitating a new analysis. We develop the forward theory associated with this problem and also consider a related inverse problem. In particular, we prove a uniqueness result analogous to that of H. Hochstadt on the determination of the potential from two sequences of eigenvalues. In addition, we consider the problem of extending Sturm's oscillation theorem, regarding the number of zeroes of eigenfunctions, from the classical setting to discontinuous problems with general constant coefficient transmission conditions.