Analysing geometry in the Classroom Mathematics and Mind Action Series mathematics textbooks using the van Hiele levels

by

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DECLARATION

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Science at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

Richard Nicholas Ian McIntyre

19th day of August in the year 2017

ABSTRACT

In this study, the geometry chapters in two textbook series (Mind Action Series and Classroom Mathematics) have been analysed and compared using the van Hiele levels of geometric thought. They were analysed using the matrix presented by Hoffer (1981), and the units of analysis that were used were from Foxman (1999), i.e. Explanation, Kernel and Exercise. The chapters leading into three-dimensional shapes were excluded due to the idea of them leading into Optimization in Calculus, more than Euclidean Geometry. The results of the research revealed that both textbook series were weighted heavily towards Exercises. This creates an issue since they require teacher supplementation, but many teachers are unable to do so. In addition to these results, it was discovered that neither textbook series followed the progression suggested by the van Hiele levels. There was a distinctive absence of Order (Level 3) classifications which is the precursor to formal geometric deduction. This could explain why learners struggle with proof-style questions. The textbooks also have a cognitive gap when looking at the progression of each unit of analysis. The Kernel classifications are predominantly Recognition (Level 1), the Explanation classifications are a mix between Recognition (Level 1) and Deduction (Level 4) and the Exercise classifications are dominated by Deduction (Level 4). A correction of the progressions could have positive results for weaker and "textbook bound" teachers.

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GLOSSARY OF TERMS AND ABBREVIATIONS

CAPS – Curriculum and Assessment Policy Statement

CM – Classroom Mathematics textbooks

DBE – Department of Basic Education

DOE – Department of Education

MA – Mind Action Series mathematics textbooks

NSC – National Senior Certificate

Paper 3 – At the end of high school, there have always been 2 compulsory exams. An optional additional exam was added called Paper 3.

RNCS – Revised National Curriculum Statement

CHAPTER 1 – INTRODUCTION

1.1 Significance of the study

Between 2008 and 2013, Geometry was left out of the compulsory Revised National Curriculum Statement (RNCS) curriculum and placed into an optional Paper 3 examination (Department of Education (DoE), 2005). In 2014, Geometry was brought back into the curriculum through the Curriculum and Assessment Policy Statement (CAPS) (Department of Basic Education (DBE), 2011d). It was once again examined in Paper 2 as it was before and it became compulsory again. Due to this change in the curriculum, as a country, we sit in the scenario where there are some teachers who are capable of teaching geometry, some teachers who struggle with teaching geometry since they do not have a solid grasp of it themselves, and some teachers that were never taught geometry in school, but now must teach geometry in their own classrooms (Dr. J. Basson, personal communication, 2 May 2015).

Studies in South Africa have looked at the competency of teachers and learners in the classroom (Bennie, 1998; Feza & Webb, 2005). They found that the van Hiele levels attained were very low. Bowie (2013) suggests that geometry is not even taught at some schools at all. If the van Hiele levels of the teachers were very low, it is not a surprise that the van Hiele levels of the learners were as well. Studies from Hopf (1977), as cited in Haggarty & Pepin (2002, p. 571), mention that 48% of teachers only use the textbook in preparation for their lessons and that about 50% of teachers are "textbook bound". To improve this, the learners could look for another source of information besides the teacher – the textbook! If the teachers are "textbook bound", could this problem be alleviated if the textbook that was used in schools could be followed prescriptively, since then the level of teaching would be set, regardless of the teacher? Even in the Quintile 1, 2 or 3 schools where resources and highly skilled teachers are lacking, could the textbooks be the answer to the skills shortage?

The Diagnostic Report on the 2014 National Senior Certificate (NSC) exams states that "Learners need to be told that success in answering Euclidean Geometry comes from regular practice, starting off with the easy and progressing to the difficult. There is no short-cut to this process." (DBE, 2015, p. 130). This statement is aimed at helping learners develop what Godfrey and Siddons (1910), cited in Fujita & Jones (2002, p. 16), refer to as the "geometrical eye". This is when the learner can "see" the geometry they are required to do.

In 2014, the first year when geometry was a compulsory topic again, the percentage of learners passing their matric mathematics exams fell by 5.6% (Department of Basic Education, 2015) and in 2015, fell below 50% for the first time in four years (Department of Basic Education, 2016). In 2016, the number of students writing mathematics was the highest it has been since 2009 at 265 810 (Department of Basic Education, 2012; Department of Basic Education, 2013, Department of Basic Education, 2017). With the numbers of learners writing mathematics increasing and the percentage of learners passing mathematics not following this trend, there needs to be an efficient and large-scale solution to the geometry problem.

Whether it is in South Africa or another country around the world, textbooks are used in education and in the classroom. Pepin (2008) tells us that textbooks are an educational tool in classrooms around the world and they have a huge impact on the way that the learners perceive the subject. Lui and Leung (2012) mention that "textbooks play an important role in the daily school activities" (p. 56). They say that they are "a bridge between the intended curriculum and the implemented curriculum" (Ibid, p. 56). Therefore, textbook analysis is crucial in all educational systems around the world to make sure that that the best educational research and pedagogical practices are being implemented in the creation of textbook materials.

Lui and Leung (2012) also make a point of mentioning that the textbooks are created as the authors' interpretation of the curriculum, where this interpretation is not necessarily the "embodiment of the intended curriculum" (p.56). Hersberger and Talsma (1991) build on this idea by stating that "careful attention must be paid to pedagogical ideas that enhance and facilitate the attainment of newly developed curricular goals" (p. 192). Teachers should be able to use the textbook as a resource; however, when they are not able to, we need to make sure that the textbook is well enough designed to just be followed. Luke and Luke (1989) argue that "the school textbook holds a unique and significant social function: to represent to each generation of students an officially sanctioned, authorised version of human knowledge and culture" (p. vii). Howson (1999) affirms agreement with this point of view by stating that "despite the obvious powers of the new technology, it must be accepted that its role in the vast majority of the world's classrooms pales into insignificance when compared with that of textbooks and other written materials" (p. 21). Therefore, textbooks are an integral part of teaching in any classroom.

1.2 Context of the study

Geometry has been a troublesome section in the curriculum for each of my classes, through each year. This is not only from the perspective of the learners trying to understand it, but also from my perspective of trying to teach it.

At the school where I taught, The Mind Action Series mathematics textbook was the textbook of choice. In addition to this, one of the authors of the Mind Action Series mathematics textbooks had a workshop that I attended. In this workshop, he explained his methods for teaching geometry and they seemed to correlate closely to the van Hiele levels of geometric thought. After implementing his methods with my Grade 11 class, half way through the geometry topic, I found that my learners were more responsive and had a significantly better understanding and recall of the work I had taught them. Even the following year when those learners were in Grade 12, many commented to me that they remembered their Grade 11 geometry theorems far better than their counterparts in their classes. Without the workshop, I would not have made the change in teaching style.

1.3 Purpose of the study

The purpose of this study is to evaluate the geometry chapter(s) in the Class Mathematics and Mind Action Series mathematics textbooks, from Grade 8 through to Grade 12, according to the van Hiele levels of geometric thought. The report will establish how closely these textbooks follow the theory as a series from Grade 8 through to Grade 12. In performing this research, it will bring light to the state of available resources that could be used to aid the teachers that are text bound, as mentioned previously. Therefore, the study will be used to the answer the research question:

> *"How closely do the Classroom Mathematics and Mind Action Series mathematics textbooks follow the van Hiele levels of geometric thought as a series from Grade 8 – Grade 12?"*

There are a few reasons for choosing these textbooks. Firstly, the idea of changing my teaching style according to the van Hiele levels was introduced by an author of the Mind

Action Series mathematics textbook. Therefore, I am hoping that these ideas he has proposed have come through in the textbook. Secondly, the Classroom Mathematics has been around for many years and could be the most well-known mathematics textbook on the market. Both textbooks appear on the LTSM (Learning and Teaching Support Material) booklist and are available for schools to use as the textbook of choice. Finally, my school currently uses Classroom Mathematics as the textbook for Grade 8 and Grade 9 and Mind Action Series mathematics textbooks for Grade 10, Grade 11 and Grade 12. Since the Mind Action Series have recently released a Grade 8 and Grade 9 mathematics textbook, my school is looking to make the choice of mathematics textbook the same through the grades, i.e. either Mind Action Series or Classroom Mathematics. This research will help in the decision-making process.

1.4 Outline of chapters

The following is a brief outline of the chapters that will be present in this research:

1.4.1 Chapter 1 – Introduction

Chapter one will introduce the context of mathematics teaching in South Africa with respect to geometry. This will include a look at the curriculum changes, influence of these changes on teachers and the Grade 12 matric exam results trends.

1.4.2 Chapter 2 – Literature review

This section is dedicated to bringing together relevant literature on document and textbook analysis, as well as discussing previous studies using the van Hiele levels of geometric thought. Several frameworks will be discussed; however, the van Hiele levels will be used as my conceptual framework for the study and therefore will be discussed in more detail.

1.4.3 Chapter 3 – Methodology

In this chapter, the working of how the study will be conducted are explained. The Hoffer matrix (Hoffer, 1981) will be used as an instrument to analyse the chapters in the textbooks, as well as a set of units of analysis that will be used. These units will analyse a different dimension of the textbooks, but will give greater context when linked to the van Hiele levels for analysis. The reliability and validity of the research, as well as the lack of ethical concerns will be elaborated on.

1.4.4 Chapter 4 – Results

The results that are shown in this chapter of the research will be summarised through graphs, with the tables of values inserted into the appendices. The graphs will show interesting and unexpected results to do with the units of analysis, as well as an overall view of the van Hiele levels. A deeper view of the van Hiele level composition will be attained through looking at the spread of the levels per unit of analysis.

1.4.5 Chapter 5 – Discussion

Taking the results and interpreting them in the correct context is crucial for the success of research. Therefore, in this chapter, the results obtained will be compared with the results that were expected to be observed and where there are differences, possible reasons for these differences will be discussed and related to existing literature.

1.4.6 Chapter 6 – Conclusion

In this final chapter of the research, I will summarise the research and results obtained, discuss implication for research in practice and mention possible avenues for future research based on this current study.

1.5 Conclusion

In this introductory chapter, the South African educational landscape has been elaborated upon, as well as the struggles for teachers in teaching and the learners in performing well in the matric exam's geometry questions. A brief introduction to how this research became of interest to me was explained and a brief outline of the rest of the research report has been included.

CHAPTER 2 – LITERATURE REVIEW

This chapter presents a review of a selection of pertinent literature and discusses the conceptual framework used in the research. The chapter begins with a review of literature regarding document analysis. Thereafter, literature on textbook analysis is described. Finally, geometric frameworks are examined and more detail is given on the van Hiele levels of geometric thought.

2.1 Document Analysis

Textbook analysis falls under the document analysis umbrella of research. Document analysis is defined to be "a systematic procedure for reviewing or evaluating documents" (Bowen, 2009, p. 27) or as "the analysis of documents that contain information about the phenomenon we wish to study" (Bailey, 1994 as cited in Mogalakwe, 2006, p. 221). Mogalakwe (2006) tells us that this choice of research is not as popular as others, such as questionnaires/surveys and interviews, and if it is used, it will often only be used in a supplementary manner to support other more major research methods. Document analysis has been an accepted method of research for many years. There is evidence that social theorists such as Karl Marx (the father of Marxism) and Emile Durkheim (one of the founding fathers of sociology) have used document analysis as early as 1841 (Mogalakwe, 2006).

Just like any research method, document analysis has advantages and disadvantages. The advantages, as per Bowen (2009), are that it is an efficient method, there is often good availability of the documents required (since many are in the public domain), it is a cost-effective research method, there is a lack of reactivity of the documents through the research process and the details included in the documents are exact. Bowen (2009) also brings some disadvantages to the forefront, for example even though the details in the documents are exact, they may not be comprehensive enough and it is not possible to ask the document for more information. In addition to this, some documents may be very difficult to obtain or the documents chosen may have only been chosen to further a biased selectivity of the researchers.

2.2 Textbook Analysis

Pepin (2008) tells us that textbooks are an educational tool in classrooms around the world and they have a huge impact on the way that the learners perceive the subject. This links to the idea that the learning of the mathematics by the learner and textbooks have a very important link, according to Dickson and Adler (2001). They argue that due to the nature of mathematics being abstract, it will lack form until seen in a written version.

Pepin (2008) also argues that the need for textbook analysis is to make sure that the offering to the learners is “rich coherent and a connected learning experience” (p. 1). Mathematics textbooks are defined by Rezat and Straesser (2013) as “a book about mathematics presented in a way which is considered to support its teaching and learning” (p. 51). The support that the textbook will be expected to give could come in different forms. The support could differ from generation to generation. This means that textbook analysis would be necessary to determine if the textbook is still supporting the teaching and learning of the current generation as well as it did for the previous generation. Castell, Luke and Luke (1989) argue that “the school textbook holds a unique and significant social function: to represent to each generation of students an officially sanctioned, authorised version of human knowledge and culture” (p. vii). In addition to the generational change issue, in South Africa in the last 20 years, we have had five curriculum changes. The Apartheid era curriculum, i.e. NATED curriculum, was phased out in 1997 and then Curriculum 2005 (C2005) was introduced (DoE, 1997). It was then replaced by the Revised National Curriculum Statement (RNCS) in 2002 (DoE, 2002). In 2006, the higher grade, standard grade and lower grade Mathematics was implemented with Mathematics and Mathematical Literacy in 2006 (DoE, 2003a, 2003b). In 2009, the syllabus for Grades 1-6 was reformed (DoE, 2008) and finally in 2012, the CAPS syllabus came into effect (DBE, 2011a, 2011b, 2011c, 2011d, 2011e). With so many curriculum changes, the textbooks that are used by the teachers would need to be checked to see if they are compliant with the new curriculum. This reason of analysis brings into focus another reason for textbook analysis, which is how textbooks are used in the classroom. This will be elaborated on further in the Methodology section. A final reason for textbook analysis would be to check that if the textbooks, given a section of the curriculum, are presenting the mathematical works in a way that will be compliant to theoretical frameworks of progression. Hersberger and Talsma (1991) say that “careful attention must be paid to pedagogical ideas that enhance and facilitate the attainment of newly developed curricular goals" (p. 192). Similarly, Haggarty and Pepin (2002) mention an idea from Schmidt *et al*. (1997) where the rationale for textbook analysis is that they are likely to reflect official intentions of the curriculum, since the textbook would not be viable commercially if it did not follow the curriculum (p. 127).

2.3 Frameworks for textbook analysis

There have been many studies performed focusing on tertiary education and how learners interact with textbooks at this level (Sikorski *et al*., 2002; Clump, Bauer & Breadley, 2004; Durwin & Sherman, 2008). I will be focusing on secondary level textbooks and therefore will not focus on these previous studies.

One of the first aspects that could be analysed is the language or discourse used in the textbook. A researcher could use Dowling's (1998) Domains of practice or even Bloom's Taxonomy; however, a problematic aspect of trying to analyse geometry chapters through the grades would be that since these are not geometry (or even mathematics) based frameworks, they may not capture the intricacies of geometry in mathematics.

Jones and Fujita (2003) performed a study where they used an analytical framework and coding table, from Valverde *et al* (2002, p. 184-187), to analyse the geometry chapters in Mathematics textbooks for learners aged 12-14 from Scotland and Japan. They wanted to determine if the one series developed reasoning skills in geometry better than the other one. This coding classified the chapters into *block type, content, performance expectation* and *perspective*. It was discovered that the Japanese textbook formed the notion of proof better than its counterpart; however, it did maintain a narrower view of geometric knowledge that the learner should be exposed to. This type of study focused more on individual books, compared to a large spread of grades and the progression thereof. Other authors have also used the TIMSS framework, or an adapted verson of the TIMSS framework from Valverde *et al* (2002), such as Delil (2006) or Bowie (2013). A large scale study performed by Haggarty and Pepin (2002) compared textbooks used in British, French and German classrooms. They argued that the exposure and access to the mathematics is different in each of the textbooks and that this could be attributed to the cultural context in which they are being used.

A similar issue arises with the frameworks used by Aineamani and Naicker (2013) in their study. They use a mixture of the five strands of mathematical proficiency, that was formalized by Kilpatrick, Swafford, and Findell (2001), and Variation theory that was taken from Marton *et al.* (2004). Kilpatrick, Swafford, and Findell (2001) state that to learn mathematics successfully, the learner must develop Conceptual understanding, Procedural fluency, Strategic competence, Adaptive reasoning and Productive disposition, whereas Marton *et al.* (2004) look at the differences in the dimensions of a particular topic. Both frameworks would be good for analysis of a single textbook, but are not set up to show progressions.

Dimmel and Herbst (2015) performed a two-stage survey. The first stage was to create a semiotic catalogue based on the geometric diagrams, and the second phase was to analyse over 2000 diagrams from different textbooks, spanning the 20th century. This is a good framework for the progression that I am looking for; however, it is very restrictive in the sense of only looking at the diagrams. Even though geometry can be diagrammatic by nature, only looking at the diagram will likely end up in missing evidence that would be useful to my analysis.

Khasawneh, Al- Omari, and Tilfah (2000) describe that "in the NCTM 1998 discussion draft of principles and standards for school mathematics, the four standards emphasise that teaching and learning geometry is integrated with the geometric thought and the model of teaching and learning geometry suggested by van Hiele" (p. 1). This made me believe that the van Hiele levels would be a good framework to use for my research.

2.4 The van Hiele levels of geometric thought

The van Hiele theory was developed by Pierre van Hiele and Dina van Hiele-Geldof in the 1950s (Usiskin, 1982). Concurrently, Jean Piaget was developing a framework for geometric thinking, but it proved to be less effective than the van Hiele levels (Pusey, 2003). Burger and Shaughnessy (1986) inform us that the van Hieles seemed to feel that their learners passed through several stages or levels of reasoning about geometric concepts. Using their own experiences, they developed their five-level theory about geometric thought and understanding. Their work has been claimed to be one of the most influential theories for teaching and learning geometry at a high school level by Yazdani (2007).

Even though Pierre van Hiele and Dina van Hiele-Geldof originally labelled the levels from level zero through to level four, Wirszup (1976) was the first person to rename them to be levels one through to five and Hoffer (1979) was the first person to give the levels the names that we know today. This opened the door for Clements and Battista (1990) to introduce what they called the *prerecognition* level. This is a level of geometric thinking that will fit in before the first traditional van Hiele level. This could be characterised by the learner not being able to recognize everyday shapes.

The five levels are shown below and an example of how the geometric thought would take form has been included in the descriptions. These definitions and ideas about the levels come from Usiskin (1982), Fuys, Geddes, and Tischler (1988), van Hiele, P. M., & van Hiele-Geldof, D. (1958):

- ❖ Level 1 is *Recognition* or *Visualisation*. This is where the learner can learn names of figures and recognizes a shape as a whole, e.g. squares and rectangles seem to be different. This would be noticed by a teacher as the learner recognising shape that relates to a real-world object. The learner may recognise a square since it looks like a CD case, or the given shape is a circle since it looks like a CD, or even think of a triangle as a mountain. It is important to remember that this is a purely visual skill without any deductive or inductive skills.
- ❖ Level 2 is *Analysis*. Once the learner progresses from Level 1 to Level 2, they will be able to identify properties of figures, e.g. rectangles have four right angles, circles have no right angles, or any other properties of the shapes. The learner has the ideas of the properties; however, they are in isolation. This means that there will not be any linkages between the fact that a square and a rectangle both have 4 right angles. The identification of the shapes is now taken as a known skill that is developed.
- ❖ Level 3 is *Order* or *Informal Deduction*. Once attained, the learner can logically order figures and relationships, but does not operate within a mathematical system. This means that simple deduction can be followed, but proof is not understood. This means that inter-relationships between shapes are now a reality, for example putting two CD cases together (two squares) to form a rectangle. Therefore, if the rectangle is made of squares, there could be inter-linking of properties. This is also a very important stage since it is the beginning of seeing proofs; however, this is informal deduction. Informal deduction means that the learner can use properties of the shapes or other geometrical concepts and put

them together. This will not be in the form of a Statement-Reason column breakdown; however, it is still a very important level to attain since without it, there is no exposure to deduction and joining ideas together to obtain other information. The learner may be able to follow a given proof, but they will not be able to write and structure it themselves. By the time that the learner finishes Grade 9, they should have attained Level 3 (Order) (Carine Steyn, personal communication, 19 January 2017) because from Grade 10 onwards, formal proofs make an appearance in the curriculum (DBE, 2011d).

- ❖ Level 4 is *Deduction*. This is where formal deduction takes place and the learner understands the significance of deduction and the roles of postulates, theorems, and proof, i.e. proofs can be written with understanding. This means that the learner must be able to structure and write up formal proofs in the Statement-Reason format. By the time that the learner finishes Grade 12, they should have attained Level 4 (Deduction). This is since the next level is based on post-matric mathematics courses.
- ❖ Finally, Level 5 is *Rigor*. As mentioned above, this would be the level of post-matric courses at university-level where the learner understands the necessity for rigor and can make abstract deductions. The learner could challenge axioms since in the system in which the axioms are used, these axioms may break down. An example of this would be the axiom about the sum of angles in a triangle equaling 180°. This holds if we are looking at Planar Geometry; however, it does not hold if we are using Spherical Geometry. Therefore, Non-Euclidean geometry can be understood.

The van Hieles also identified five properties or characteristics of the levels that were named and described later by Usiskin (1982, p. 5). The first property is that the van Hiele levels have a "*fixed sequence*", i.e. a learner cannot be at van Hiele level *n* without having gone through level *n-1*. This means that a learner would be unable to move from Level 1 (Recognition) straight to Level 3 (Order) or Level 4 (Deduction) without mastering Level 2 (Analysis) first. Similarly, the next property is that the van Hiele levels have "*adjacency*" which means that at each level of thought, what was intrinsic in the preceding level becomes extrinsic in the current level. Therefore, moving from Level 1 (Recognition) to Level 2 (Analysis), the learner would see that the CD case has right angles, even though the learner does not know what right angles are. The third property is that there is “*distinction*” between the levels, meaning that each level has its own linguistic symbols and its own network of relationships connecting those symbols. This means that as the school grades and van Hiele levels increase, the necessity for formal mathematical writing becomes more prominent. The next property is that there is “*separation*” between different levels, i.e. two persons who reason at different levels cannot understand each other. If two parties are trying to communicate and one party is on one level and the other party is on another level, the means of communication will be different (causing difficulty). The two parties could be a teacher and a learner, a textbook and a learner, or even two learners. Finally, the last property is that “*attainment*” of the van Hiele levels will only occur through achievement of five phases, namely inquiry, directed orientation, explanation, free orientation and integration.

The van Hiele levels are a popular framework to use in the classroom to measure geometrical thought in learners and/or teachers (Jones, 1998); however, using the van Hiele levels to analyse a textbook is far less common. There is much literature on the benefits of the van Hiele levels, but there is also literature discouraging the use of it for the purposes of textbook analysis and even disproving some of its characteristics (Gutiérrez & Jaime, 1998; Bowie, 2013). There is even literature suggesting that the five-level van Hiele model should be condensed into a three-level model. Teppo (1991) suggests that Level 1 (Recognition) can stay by itself in a “visual” category, Level 2 (Analysis) and Level 3 (Order) can be combined into a “descriptive” category and finally Level 4 (Deduction) and Level 5 (Rigor) can be combined into a “theoretical” category. This idea of restructuring the van Hiele levels has gained acclaim by Lawrie (1998), as cited in Pegg (2014, p. 36). I can understand the grouping; however, I feel that the grouping dissolves the influence of Level 3 to join the pre-deduction and post-deduction levels.

One of the first applications of the van Hiele levels was to the Soviet curriculum. Using the van Hiele levels, they radically revised their geometry curriculum to follow the van Hiele levels more closely (Fuys, Geddes, and Tischler, 1988). Comparatively, one of the first major studies done using the van Hiele levels was by Usiskin (1982) and the team in the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project. They created a test to assess the levels of almost 2700 17-year-old learners from 13 schools in the US during a one-year geometry course. The team discovered some very interesting results that put the van Hiele levels structure into question, such as discovering from their data that Level 5 (Rigor) is easier to attain than Level 4 (Deduction), and therefore, does not act like it was suggested in the theory. In addition to this, the classification of the van Hiele level of the learner was dependent on the decided cut-off points, which made the classification very subjective. The most important point that they discovered through their research is that the van Hiele levels is a "very good predictor" of the performance of the learners on a standard geometry assessment that they would have normally done in class. It did not matter if the test was content based or proof based; however, the correlation was stronger with the proof based tests. Sadly, the team found that the learners in the classes that studied proof still had very low van Hiele levels and would therefore not gain much success in geometric proof. They concluded that the learners would not know the required geometrical notions upon leaving high school and are therefore "not versed in the basic terminology and ideas of geometry" (p. 87).

One of the earliest studies that used the van Hiele levels as a framework for textbook analysis was by Fuys, Geddes, and Tischler (1988). They investigated three very popular textbook series in the United States of America from Grade K – Grade 8. They used the van Hiele levels of geometric thought to analyse the topics per grade, the interaction with the curriculum and if there was the correct progression in levels as a series. They discovered that the textbooks started showing Level 2 (Analysis) from about Grade 3 onwards, but there was very little Level 3 (Order) classifications. The occurance of any Level 3 (Order) classifications only started to appear in Grade 7 and Grade 8. They discovered that average learners would not need to think above Level 1 (Recognition) for almost all of the their geometric experience, up to the end of Grade 8. Another key result they discovered is that the "exposition", as they call it, is of a *higher* level than that of the exercise questions. This creates a future problem for the learner since if the learner can progress through tests that are easier than the instructional material, the learner possibly would not improve their level to the exposition level. This will cause them to encounter difficulty when moving to a higher grade. At that stage, the results of the Second International Mathematics Study had been released four years prior and found that American learners were in the bottom 25% to do with achievement in geometry.

A more recent example of textbook analysis using the van Hiele levels is by Khasawneh, Al- Omari, and Tilfah (2000). This team of researchers decided to analyse the national

Jordanian mathematics textbooks from Grade 6 to Grade 9. The team mentions that the major focus of researchers is to measure van Hiele levels of the learner using certain content. Fewer researchers would teach according to the van Hiele levels and then test post-teaching. Finally, the rarest version is using the van Hiele levels to analyse textbooks. Since this framework is seldomly used with textbooks, it could be seen in one of two ways: either that the framework is unsuitable for textbook analysis; or that textbook analysis is not very popular (as mention previously) and therefore would not be high on the list of application for a well known framework. Khasawneh and his team discovered that the textbook series did have a progression in the van Hiele levels and that the amount of Level 3 (Order) was on average about a quarter of the classifications. It did not vary very much compared to the other levels that decreased or increased as expected

2.5 Conclusion

In this chapter, I have presented background on document analysis and textbook analysis. I have also drawn attention to different studies that have used textbook analysis and the frameworks used in these studies. Finally, I have chosen to use the van Hiele levels of geometric thought for my research and I have elaborated and described these levels. In the following section, the research methodology is presented.

CHAPTER 3 – METHODOLOGY

This section outlines the methodology used to conduct this research. Firstly, the research methodology and design will be discussed. This will be followed by the research instrument to be used and an elaboration on the units of analysis. Finally, a discussion on the validity, reliability and ethical considerations of this study will conclude this chapter.

3.1 Research methodology and design

The term "methodology" and the term "method" are two different concepts. Opie (2004) refers to methodology as "the theory of getting knowledge, to the consideration of the best ways, methods or procedures, by which data that will provide the evidence basis for the construction of knowledge" (p. 16). This definition is elaborated on to distinguish between methodology and methods. Opie (2004) says that methodology is more concerned about describing and analysing the means of collecting the data and methods, compared to the actual application of data collection which refers to the methods or procedures.

The proposed research is a quantitative research model, where I will be tallying classifications and calculating proportions. The choice of chapters to use in the textbook was made with the idea in mind that they would lead towards Euclidean Geometry in Grade 10, 11 and 12. In doing this, it became apparent that some of the chapters will not be relevant to my study, for example volume and surface area of 3-dimensional shapes. This chapter will not come into my research since this topic leads towards an algebra topic (i.e. Optimization in Calculus). The remaining sections about lines, angles and two-dimensional shapes will be the focus of my research since they flow into the Euclidean Geometry sections of Grades 10, 11 and 12. For the Mind Action Series, the chosen chapters are chapters 8, 9, 10 and 13 for Grade 8, chapters 10, 11, 12 and 13 for Grade 9, chapter 7 for Grade 10, chapter 8 for Grade 11 and chapter 8 for Grade 12. For the Classroom Mathematics series, the chosen chapters are chapters 11, 12, 13 and 14 for Grade 8, chapters 11, 12, 13 and 14 for Grade 9, chapters 9 and 14 for Grade 10, chapter 10 for Grade 11 and chapter 11 for Grade 12.

3.2 Data analysis and interpretation

Previous work on textbook analysis using the van Hiele levels that I will be drawing from was done by Khasawneh, Al- Omari and Tilfah (2000). In their paper, they used a truncated version of the 5 x 5 Hoffer matrix (Hoffer, 1981) to analyse the national textbooks for Grade 6 through to Grade 9. The original matrix involves five geometric skills and five van Hiele levels; however, the researchers only used four of the van Hiele levels. Since my research is only on the van Hiele levels, I will not be using the geometric skills outlined in the matrix. In addition to this adaption, I will also be including the last van Hiele level in my analysis, in case the challenge questions fall into this category. Since the study will be dealing with high school textbooks, the assumption was made that the learners would have achieved at least Level 1 (Recognition) and therefore, the textbook classification would start with Level 1, not Level 0 (*prerecognition*) like Clements and Battista (1990) introduced.

The Hoffer matrix (Hoffer, 1981) is useful for classification of the van Hiele levels since it not only has descriptions of each van Hiele level split up by geometric skill, but it also has a sample question per van Hiele level and per geometric skill. This enables researchers to have a more reliable idea and understanding of the requirements per van Hiele level.

A drawback of the research by Khasawneh, Al- Omari and Tilfah (2000) is in the method they used to allocate the levels to the units of analysis. They decided to allocate the highest van Hiele level that was represented within the unit of analysis. This can create very misleading results since if there is a huge majority of level 3 (Order) questions and then 1 single level 4 (Deduction) question, then the level would be classified as level 4. I will be allocating each unit a van Hiele level and record the individual classifications. This will eliminate the issue of misleading results.

I will be analysing the current series of Grade 8 - Grade 12 textbooks in both textbook series. The publishers of the Mind Action Series mathematics textbook released a geometry workbook in 2016. After looking at them, it was decided that it would be an unfair comparison to compare these purely geometry workbooks for the Mind Action Series to the geometry chapters in the Classroom Mathematics textbooks. There were two reasons for this decision. The first reason was that in a classroom situation in an archetypical school, textbooks would be a more common sight, compared to an additional workbook over and above the recommended textbook. This means that the analysis done would not be as far reaching for the Mind Action Series workbook since it would be used more infrequently, as compared to the textbook. The second reason is more of an economic one in that the publishers would have to include something different in the workbooks, compared to the textbooks. This means that they could be (and after having a quick analysis of the books were indeed) differently arranged, could have different levels of questions and thus create a different progression, as compared to the textbook levels, questions and progression.

In this study, I will be collating tallies of each unit of analysis, as well as tallies for each of the van Hiele levels. Since each unit of analysis will be given a van Hiele classification, I will also be looking at the van Hiele proportions per unit of analysis to see if there are any interesting trends. Once the classification stage is complete, the tallies will be presented as percentages in tables, where one table shows the progression of the van Hiele levels per grade, another shows the proportions of units of analysis and finally, the last table will show the progression of van Hiele levels for each unit of analysis per grade. Comparative graphs will be plotted, examined and discussed.

3.3 Units of analysis

The last component of the research that needs to be defined are the units of analysis. The conceptual framework has been defined as the van Hiele levels, the instrument from Hoffer (1981) has been selected to apply the conceptual framework, the textbooks have been selected, and the chapters in the textbooks have been selected. The final question that needs to be answered by the units of analysis is "What in the chapters in the textbooks will the instrument be used on to classify?"

There have been studies performed that analyse different breakdowns of components of textbooks. Love and Pimm (1996) mention that textbooks can be used for exposition. This means that they will explain concepts necessary for the learner to learn. These necessary concepts have been named as "Kernels" by van Dormolen (1986). A wider breakdown of textbooks can be seen in Rezat (2006), where he splits the many German mathematics textbooks into "Introductory Tasks, Exposition, Kernels, Worked Examples and Exercises" (p. 483). Rezat (2006) does give characteristics of each of these classifications; however, the characteristics are not mutually exclusive. By this statement, I mean that definitions and theorems were put under exposition and kernels.

There was no extra explanation or definition of how to know which classification to choose. Therefore, this breakdown of the textbooks was not as useful as I had hoped.

Similarly, Haggarty and Pepin (2002) analysed British, French and German mathematics textbooks. They found that the structures were rather different, in the sense that the British textbooks had far more straightforward questions that appear before the worked examples, whereas the French textbooks had activities and different levels of exercises and the German textbooks had introductory exercises, a main idea and then an extensive collection of exercise questions. The three classification model that Haggarty and Pepin (2002) discuss, to do with the German textbooks, seemed to be a far more mutually exclusive classification system. This system was confirmed by Foxman (1999). Foxman (1999) cites Howson (1993), where the breakdown of the textbook has three classifications, i.e. "Kernels", "Explanations" and "Exercises" (p. 3). These three classifications have well defined and mutually exclusive definitions, shown below (Ibid):

- "Kernels" are defined to be "theorems, rules, definitions, procedures, notations and conventions which have to be learned as knowledge".
- "Explanations" are defined as something that will "prepare the students for the kernels" and the explanations are not kernels themselves.
- "Exercises" are the the exercises and problems used as an assessment tool for the learner.

It is important to note that the exercises could be seen throughout the chapter, not necessarily the summative exercise at the end of the chapter. In addition to this, the explanations are used to aid the learner in acquiring the kernels, but they could appear before or after the kernel, if it is an aid to the kernel.

3.4 Issues of reliability, validity and objectivity

Wellington (2000) defines validity as "the degree to which a method, a test or a research tool actually measures what it is supposed to measure" (p. 201). This is referring to the instrument that is being used from Hoffer (1981). This instrument has been used in previous research, as mentioned above. Besides the previous research, it is specifically designed for textbook analysis which means that it should deliver the information that is required to answer the research question.

Maxwell (1992) believes that there are five categories of validity for research. These categories are Descriptive Validity, Interpretive Validity, Theoretical Validity, Generalisability and Evaluative Validity. Descriptive validity is described as the factual accuracy of the researcher's data. I believe that there will be no problem with the Descriptive Validity in my study since all the content has already been written in the textbooks and the textbooks do not have a pitch or stress of voice like live participants. Therefore, I will only be creating secondary data. The Theoretical Validity remains intact since the instrument I will be using has been taken from Hoffer (1981) which is believed to be a reliable source since it is one the most cited resources for the van Hiele levels. Through use in studies in several countries already mentioned, the instrument has been shown to be a good reflection of the van Hiele levels. When looking at Generalisability, I do not believe that the research will be generalisable since it is based on specific textbooks. Other textbooks could possibly be different and have a different progression. Lastly, I do not believe that Interpretive Validity or Evaluative Validity are relevant to my research since neither really apply to quantitative data.

Validity is only one side of the Rigour coin that must be examined. Reliability is the other side of the coin that cannot be ignored. Wellington (2000) defines reliability as "the extent to which a test, a method or a tool gives consistent results across a range of settings, and if used by a range of researchers" (p. 200), which is akin to the definition of Bell (1999), which is "the extent to which a test or procedure produces similar results under constant conditions on all occasions' (p. 103). Since the research being conducted is on textbooks, and textbooks are not affected by an interaction with the researcher, the data collection will not be influenced like data collection in a classroom would be influenced. This will aid in one aspect of reliability. The other aspect of reliability, besides physical data collection, would be data classification and the objectivity of them (Opie, 2004). How am I sure that my classification of the work will be reliable? I will be asking my supervisor to classify the work that I have classified.

3.5 Ethical considerations

Ethical considerations can be broken down into three categories, as specified by Durrheim and Wassenaar (2002). These three categories are "Autonomy", "Non-maleficence" and "Beneficence" (p. 66).

Firstly, autonomy refers to the need for researchers to obtain the consent from all participants and for them to be willing participants in the study. Since this research is only using textbooks, not live participants, the issue of autonomy is a moot point. Secondly, non-maleficence describes that requirement that the study will cause no harm to the participants. This refers more to the publishing of personal information, which would be a breach of confidentiality. Since textbooks are in the public domain, there is no issue with publishing details about them. Finally, beneficence refers to the study being useful and of benefit to the research participants, other researchers or even society. For this study, there is beneficence since this could inform a revision of the textbooks and possibly inform the Department of Basic Education on a means by which the standard of geometry teaching could be improved. Therefore, all ethical considerations have been cleared. Since there are no live participants, no ethical clearance is needed.

3.6 Conclusion

In this chapter, the difference between research methodology and research methods was discussed. The difference was that the research methods is a subsection of the research methodology. The 5 x 5 Hoffer Matrix from Hoffer (1981) was discussed and will be the instrument used to analyse the chosen chapters. This instrument will be applied to the three units of analysis from Foxman (1999), i.e. “Explanations”, “Kernels” and “Exercises”. Issues of validity, reliability and objectivity has been investigated. A process of reclassification by my supervisor has been implemented to make sure that the classifications are replicable. Since I am analysing textbooks, my ethical considerations are not pertinent. I am not dealing with any live participants in my study, e.g. learners, teachers, parents and the Department of Basic Education.

CHAPTER 4 – RESULTS

This chapter outlines the results from the study. The results are broken into three parts. The first part will comprise of a comparison of the units of analysis. After that, a global view of the van Hiele levels and their progression will be investigated. Finally, a deeper examination of the van Hiele progression per unit of analysis will be done. This will conclude the chapter.

4.1 Units of analysis overall

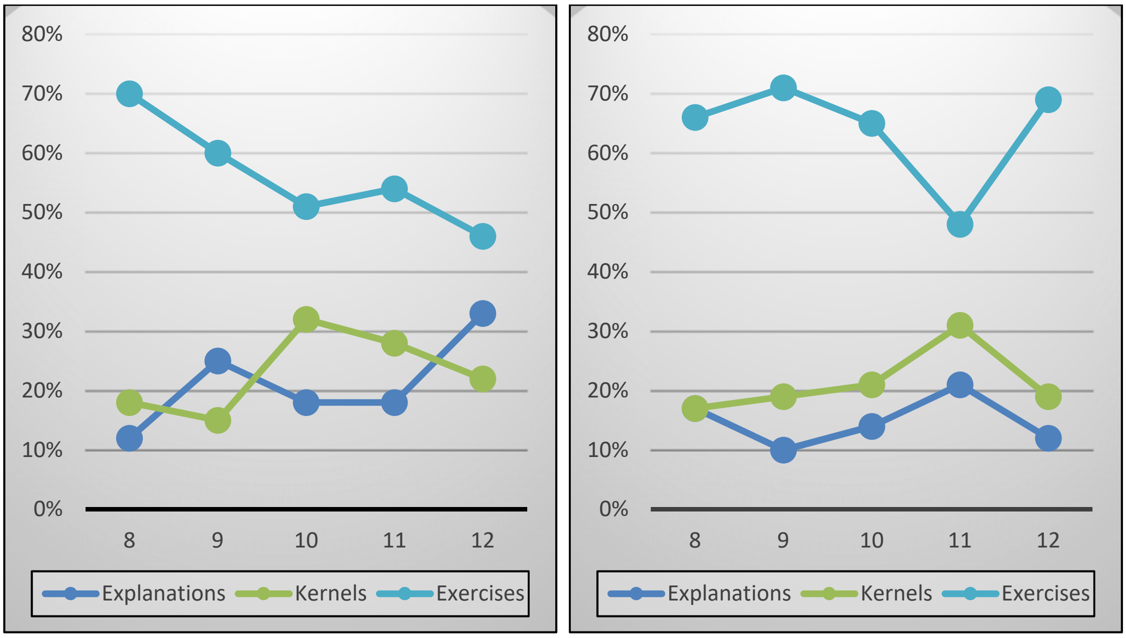


Figure 4.1: The prevalence of Explanations, Kernels and Exercises in the Mind Action Series (left) and Classroom Mathematics (right).

The results of the analysis using the units of Explanation, Kernel and Exercise is shown in Figure 4.1 with a comparison of both texbooks. We can see that there is a very distinct prevalence of Exercises through both series of textbooks. If the average percentages are taken for each unit of analysis, it becomes apparent in the Mind Action Series there is around 44% instructional material (Explanation and Kernel); however, only 36% is instructional in the Classroom Mathematics series. The prevalence of Explanation and Kernel are fairly similar to each other. This means that the textbooks are intending to get the learner to practice as much as possible, but not providing them with much Explanation to aid the acquisition of the Kernel (the knowledge that they will need to be successful at the Exercises).

Another interesting observation is that the proportions for Exercise generally follow a downward trend, casing the other two proportions to rise on average. However, in Grade 12, there are sudden spikes in Explanation (for the Mind Action Series) and in Exercise (for the Classroom Mathematics series). Possible reasons for this will be discussed in the next chapter.

4.2 The van Hiele levels overall

The results of the actual progression can be seen in Figure 4.2 with a side-by-side comparison of the Mind Action Series (on the left) and Classroom Mathematics (on the right) textbooks.

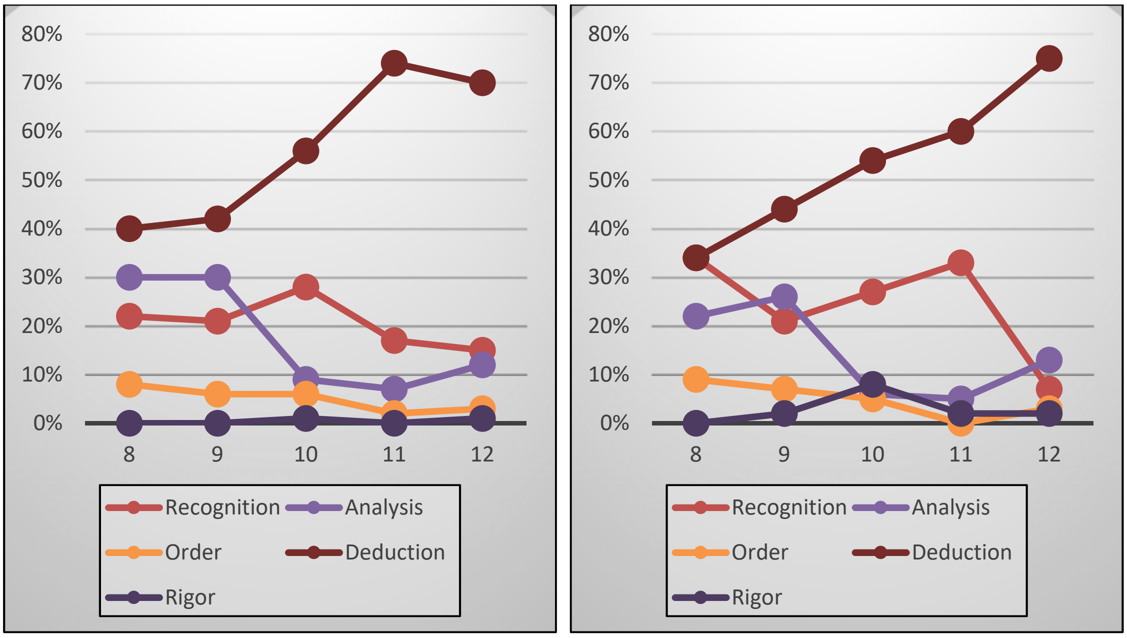


Figure 4.2: The prevalence of the van Hiele levels in the Mind Action Series (left) and Classroom Mathematics (right) overall.

The first and most clear result is the dominance of the Deduction (Level 4) level, even from Grade 8. We know that Deduction (Level 4) should have the highest prevalence by the end of Grade 12, but not from as early as Grade 8. The next very interesting result is the high prevalence of Recognition (Level 1). Surprisingly, for much of both textbooks, Recognition (Level 1) is more prevalent than Analysis (Level 2). We would expect that Recognition (Level 1) would be the least prevalent level in the FET textbooks; however, it is the most prevalent level after Deduction (Level 4).

A far larger issue with the progression of the learner is the scarcity of the Order (Level 3) level. At no point throughout either textbook series was the proportion of Order (Level 3) above 10%.

4.3 The van Hiele levels per unit of analysis

Sometimes when results are looked at overall, small intricacies can be missed. Therefore, results of the van Hiele progression have been calculated per unit of analysis. The analysis will start with Explanation, then move on to Kernel and then finish with Exercise.

4.3.1 The van Hiele levels of Explanation

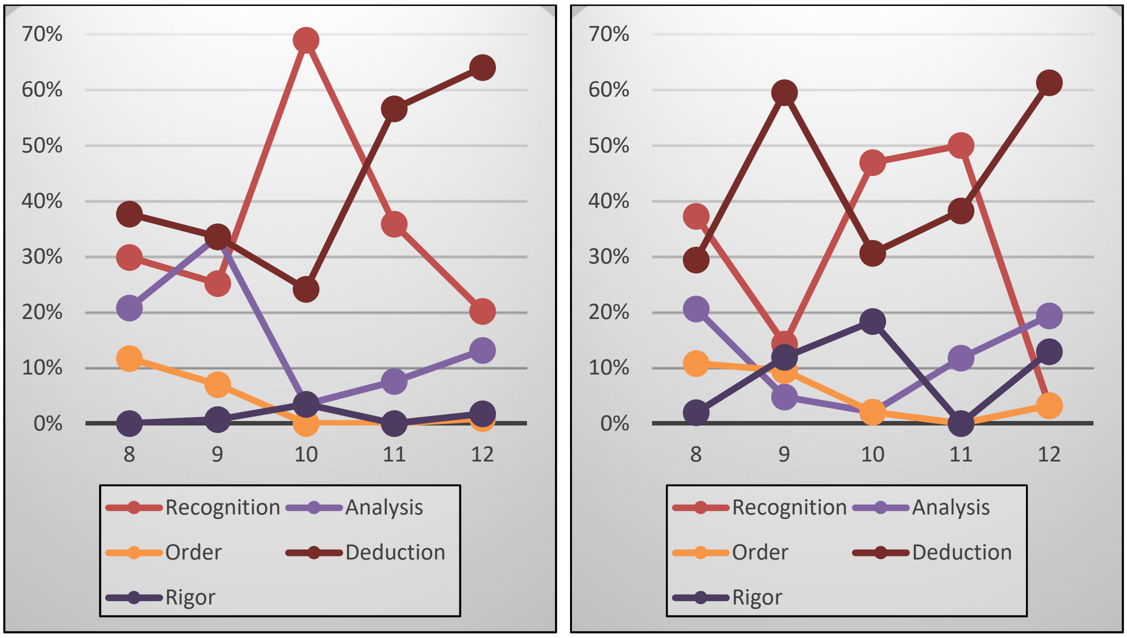


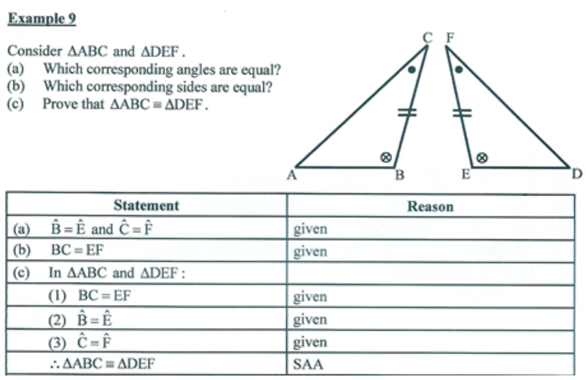
Figure 4.3: The prevalence of the van Hiele levels in the Mind Action Series (left) and Classroom Mathematics (right) for the Explanations.

In Figure 4.3, we can see that there is a definite dominance of Recognition (Level 1) and Deduction (Level 4) when looking at Explanation. They are intermixed through each textbook series. This is different from the expected results of having far more Order (Level 3) explanations involved.

It is also interesting to note other increases or decreases that were unexpected, such as in the Mind Action Series, Deduction (Level 4) decreased from Grade 8 through to Grade 10, when it should have increased. In the Classroom Mathematics series, the progression was far more up-and-down. Recognition (Level 1) was the most prominent van Hiele level in three of the grades, but Deduction (Level 4) peaked at Grade 9 and Grade 12.

We should note that each of these grades are the end of a phase in the curriculum, however it does not help the learners to assist them at Recognition (Level 1) level for two of the years and then jump to Deduction (Level 4) level in the final year of the phase. There is a more noticeable absence generally of Analysis (Level 2) and Order (Level 3) explanations throughout both textbook series.

Explanation extracts



Example 9

Consider ΔABC and ΔDEF.
(a) Which corresponding angles are equal?
(b) Which corresponding sides are equal?
(c) Prove that $\Delta ABC \equiv \Delta DEF$.

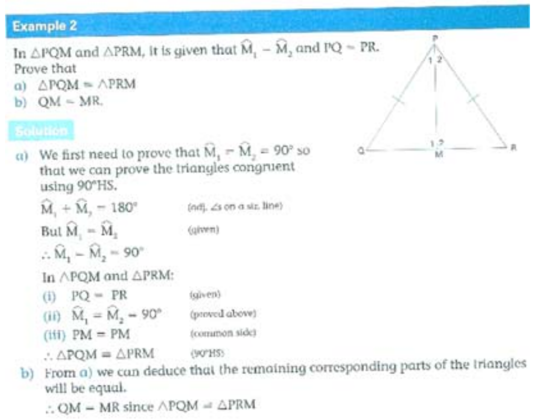
Statement	Reason
(a) $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$	given
(b) $BC = EF$	given
(c) In ΔABC and ΔDEF:	
(1) $BC = EF$	given
(2) $\hat{B} = \hat{E}$	given
(3) $\hat{C} = \hat{F}$	given
$\therefore \Delta ABC \equiv \Delta DEF$	SAA

Extract 4.1: Extract of an Explanation at Recognition (Level 1) level (Part a-b) and Deduction (Level 4) level (Part c) from Mind Action Series Grade 9.

When dealing with a right-angled triangle, the first step is to identify the hypotenuse.

The hypotenuse is the longest side of a right-angled triangle and is directly opposite the 90° angle.

Extract 4.2: Extract of an Explanation at Recognition (Level 1) level from Classroom Mathematics Grade 9.



Example 2

In ΔPQM and ΔPRM, it is given that $\hat{M}_1 = \hat{M}_2$ and $PQ = PR$.
Prove that
a) $\Delta PQM \equiv \Delta PRM$
b) $QM = MR$.

Solution

a) We first need to prove that $\hat{M}_1 = \hat{M}_2 = 90°$ so that we can prove the triangles congruent using 90°HS.
$\hat{M}_1 + \hat{M}_2 = 180°$ (adj. ∠s on a str. line)
But $\hat{M}_1 = \hat{M}_2$ (given)
$\therefore \hat{M}_1 = \hat{M}_2 = 90°$
In ΔPQM and ΔPRM:
(i) $PQ = PR$ (given)
(ii) $\hat{M}_1 = \hat{M}_2 = 90°$ (proved above)
(iii) $PM = PM$ (common side)
$\therefore \Delta PQM \equiv \Delta PRM$ (90°HS)
b) From a) we can deduce that the remaining corresponding parts of the triangles will be equal.
$\therefore QM = MR$ since $\Delta PQM \equiv \Delta PRM$

Extract 4.3: Extract of an Explanation at Deduction (Level 4) level from Classroom Mathematics Grade 9.

4.3.2 The van Hiele levels of Kernel

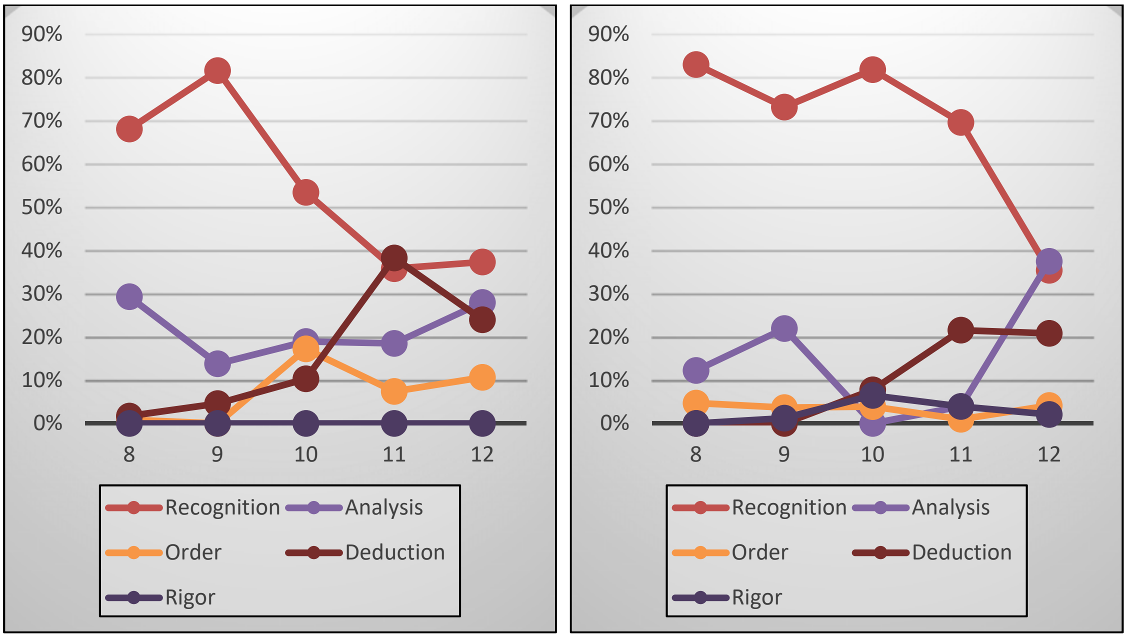
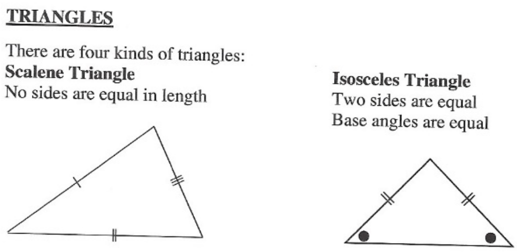


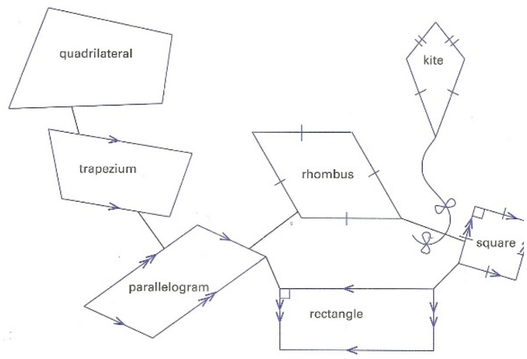
Figure 4.4: The prevalence of the van Hiele levels in the Mind Action Series (left) and Classroom Mathematics (right) for the Kernels.

Moving on to the Kernels, in Figure 4.4 we can see that Kernel is dominated by Recognition (Level 1) classifications. There is a very noticeable lack of Deduction (Level 4). This is not a huge surprise, due to the nature of what kernels are. An interesting note is that the textbooks are almost entirely Recognition (Level 1) Kernel classifications, but the explanations are at Recognition (Level 1) and Deduction (Level 4). This is the start of a mismatch that will be discussed in the next chapter. Following the Recognition (Level 1) classifications, we can see that they are on a decline from Grade 9 in the Mind Action Series; however, the first major decline in the Classroom Mathematics series is from Grade 11 to Grade 12.

Kernel extracts



Extract 4.4: Extract of a Kernel at Recognition (Level 1) level from Mind Action Series Grade 10.



Extract 4.5: Extract of a Kernel at Recognition (Level 1) level from Classroom Mathematics Grade 10.

4.3.3 The van Hiele levels of Exercise

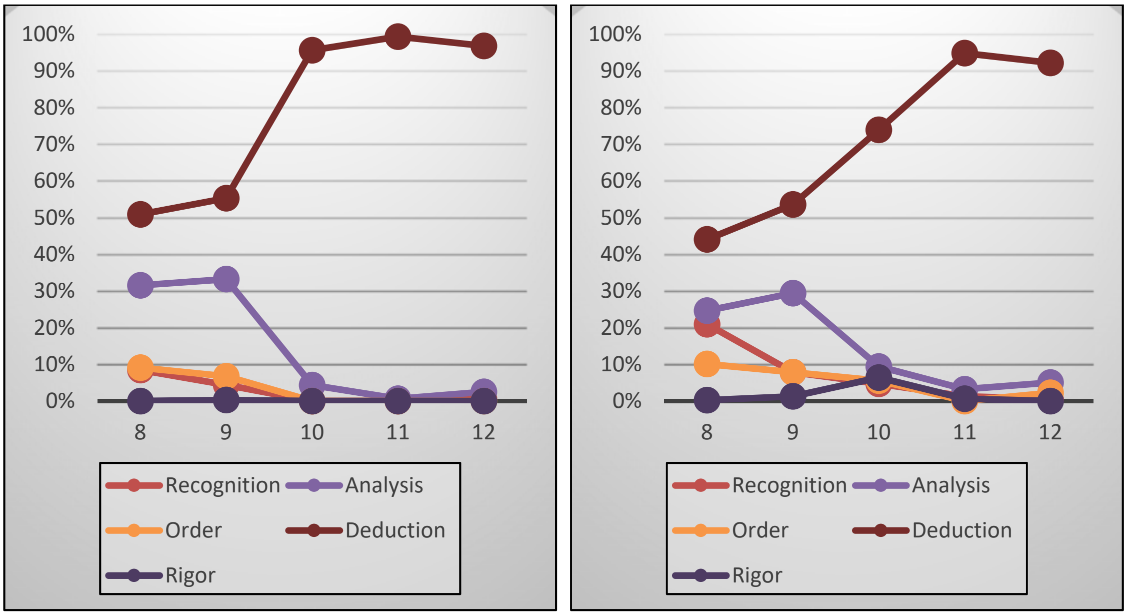
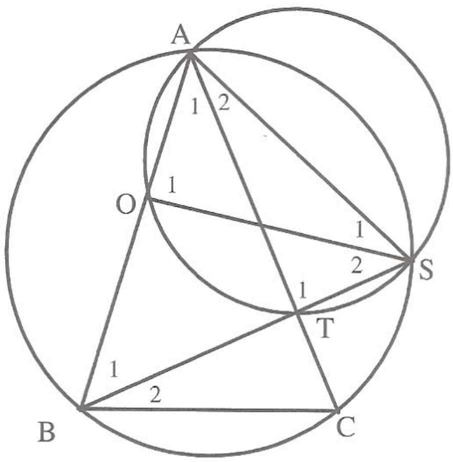
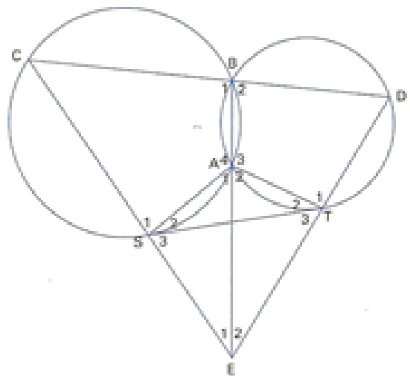


Figure 4.5: The prevalence of the van Hiele levels in the Mind Action Series (left) and Classroom Mathematics (right) for the Exercises.

Finally, having a look at Figure 4.5, it is the most clear-cut result obtained, as well as the most surprising. From Grade 8 level, the learners are being tested at Deduction (Level 4) level for the most part. Even more unexpected is how all the other levels drop off in Grade 10 in both series. Therefore, the level that the textbooks are assessing at is almost exclusively Deduction (Level 4), irrespective of the levels of Explanation and Kernel in those grades.

<table>
<tr><th colspan="2">Exercise extracts</th></tr>
<tr>
<td>7. Circles OAS and ABCS intersect at A and S. SB bisects $A\hat{B}C$. Chord AT is produced to C. Prove that $SA = SO$


Extract 4.6: Extract of an Exercise at Deduction (Level 4) level from Mind Action Series Grade 11.</td>
<td>4. Two circles cut at A and B. CBD, BAE, CSE and DTE are straight lines.
Prove that:
a) SATE is a cyclic quadrilateral
b) STDC is a cyclic quadrilateral.


Extract 4.7: Extract of an Exercise at Deduction (Level 4) level from Classroom Mathematics Grade 11.</td>
</tr>
</table>

4.4 Conclusion

The results show us two main points:

1) There is a cognitive gap between the units of analysis since the kernels are presented on a predominantly Recognition (Level 1) level, the explanations are presented on a mix of Recognition (Level 1) and Deduction (Level 4) levels, but the exercises are totally dominated by Deduction (Level 4) level questions.
2) If the learner moving into Grade 10 has not mastered Order (Level 3), not only are they at a disadvantage, but they will have very little opportunity to practice other levels besides Deduction (Level 4) level questions.

CHAPTER 5 – DISCUSSION

This chapter will involve discussion of the results generated from the study. The results are broken into two parts. The first part will comprise interpretation of the units of analysis results obtained, whereas the second part will involve a contextualization and further interpretation of the results of the van Hiele analysis, both globally and split by unit of analysis.

5.1 Discussion of the units of analysis

With the average proportions of instructional content in the textbooks analysed being low, and keeping in mind the studies from Hopf (1977), as cited in Haggarty & Pepin (2002, p. 571), about how a large proportion of teachers are textbook bound, there seems to be a problematic cycle. The cycle involves the textbook bound teacher relying on the textbook as the resource for teaching, but the textbook does not have enough instructional content for it to be the sole resource. Therefore, the teacher would have to supplement the textbook, but since the teacher is textbook bound, they would not be able to. This property is present in every textbook, from Grade 8 through to Grade 12. This problem relates to the authority that is present in the classroom. Does the teacher have authority over the knowledge in the classroom, or is it the textbook? Maybe it is a combination of both? Luke and Luke (1989) mentions that the "teacher and text can be seen to co-constitute a domain of knowledge" (p. 258). However, they go on to mention that teachers can relinquish some of their authority simply through how they refer to the textbook. If the teacher is textbook bound, their authority will be diminished, meaning that the learners will look more towards the textbook for the knowledge they seek. This hinders the mediation of knowledge transfer that the teacher would provide (Rezat, 2006).

In our country, a predominance of Exercises could lead to a problem in the classroom since the composition of the textbooks can be seen to be interconnected with the skills and knowledge level of the teacher. If we compare these results with results of international textbook studies, we see that that there are a large variety of compositions. This is since the actual use of the textbook changes from country to country, as well as the view of its role in the classroom. Howson (1999) states that he does not see a feasible and reliable way of comparing textbooks between 40 educational systems and in 30 different languages. This could be due to the cultural and linguistic differences that are present in the textbooks, as seen by Haggarty & Pepin (2002). They mention that in France, textbooks are authored by mathematics inspectors and therefore would represent the best pedagogical practices. All the textbooks from France that were analysed followed the pattern of Explanation, Kernel and then Exercise. The exercises are designed to push the learner to extend their thinking. However, in Switzerland, most textbooks are slim texts and are only exercise questions (Foxman, 1999). Therefore, the teacher does not rely on the textbook at all for instructional material. Howson (1999) explains that thicker textbooks will "probably have a greater range of materials" (p. 11) and therefore, would play a more significant role in the classroom. Therefore, it appears that the textbooks are designed to cater for the needs of the teacher. However, the textbooks used in my research do not cater for the skill levels of many teachers in practice since they do not have much instructional content. In South Africa, it does not seem like the needs of teachers are being met due to the imbalance between the lack of instructional content and the large need by unskilled teachers for it.

The decrease of the proportion of Exercise classifications from Grade 8 to Grade 12 in both textbook series is an interesting result, since it seems that as the cognitive level of the Mathematics increases through the grades, the amount of practice the learner is afforded decreases. Learners often rely on the exercises for support in their understanding. In this sense, the predominance of the Exercises in both textbook series can be of benefit to the learner. This is supported by Rezat (2006) who mentions that learners can be impatient and skip over the explanations and go to the Kernels and Exercises. With Explanations/Kernels in the Exercise part of the chapter, there could be a higher chance of the impatient learners still being exposed to them. The Classroom Mathematics textbook series did this often in their textbooks, which could be why the proportions for Exercise in the Classroom Mathematics textbooks were higher than the Mind Action Series textbooks on average.

5.2 Discussion of the van Hiele levels

As with any progression, a starting point and ending point are known, and then the steps in between are set. In the case of geometry and when using the van Hiele levels, the progression has been explained and research by several authors on the levels is shown in the literature review chapter. According to the ideas of progression that have been presented by the van Hieles and other authors, we would expect that as the grades increase, by the end of Grade 12 the following should happen:

- Recognition would decrease to become the least occurring level of question
- Analysis would decrease to the second least occurring level of question
- Order would be the second most prevalent level of question (mastered end of Grade 9)
- Deduction would be on the increase to become the most prevalent level of question (mastered end of Grade 12)
- Rigor would be focused more on post matric work. (Not many matric level students would attain this level)

The results in this study have shown that Deduction (Level 4) is the most prominent level, even from Grade 8. This deviates from the presented literature of the van Hiele levels; however, it does follow more closely the approach chosen in Japanese textbooks. Jones and Fujita (2003) examined geometry in Japanese textbooks and Scottish textbooks at the level of Grade 8. They discovered that the Japanese textbooks are more theoretical and focus on teaching geometric proof, whereas the Scottish textbooks focused more on measuring, drawing and finding angles. In comparison, the breadth of the Japanese textbooks was not as comprehensive as the Scottish textbooks. Looking at the TIMSS 2015 results, Japanese Grade 8 learners were ranked fifth in mathematics. This shows that the Japanese are succeeding in Mathematics using their style of textbooks.

A study by Healy and Hoyles (1999) reports that in the UK, academically strong Grade 8 to Grade 9 learners show poor performance in constructing proofs. They say that the learners "are likely to focus more on measurement, calculation and the production of specific (usually numerical) results, with little appreciation of the mathematical structures and properties, the vocabulary to describe them, or the simple inferences that can be made from them' (Healy & Hoyles, 1999, p. 166). Could they be focusing on those other sections because the textbooks are not helping them to attain the required level to be able to appreciate the other items?

A larger issue with the progression of the van Hiele levels in this study is the scarcity of the Order (Level 3) level classifications. This is of concern since by the end of Grade 12, learners are expected to have mastered Deduction (Level 4) and to do this, they must have mastered Order (level 3). If the learners are not properly exposed to Order (level 3), they will not be able to master it and thus, will not be able to move on to Deduction (Level 4). One of the properties of the van Hiele progression ("*fixed sequence*" as named by Usiskin (1982, p. 5)) is that for a particular learner, if Level *n-1* is not mastered, the learner would not be able to progress to Level *n*. Therefore, with the missing Order (Level 3) classifications, the learner will be stuck on Analysis (Level 2) and fail to master Deduction (Level 4). For these learners, it may seem as though the teacher is speaking a foreign language (Atebe & Schäfer, 2008b).

The results of the van Hiele breakdown of the Explanation unit of analysis shows an approach that is distinctly different to the theory on the progression. It appears that during a phase (Grade 8, Grade 10 and Grade 11), there is a dominance of Recognition (Level 1) and at the end of the phase, there is a dominance of Deduction (Level 4). In Grade 9, the learners should only have mastered Order (Level 3), not Deduction (Level 4). Grade 10 is the year where formal theorems start to make an appearance in the curriculum, which makes it curious that the most common level of Explanation is Recognition (Level 1), far away from the Deduction (Level 4) that is expected with formal proofs. The help that Explanations are giving is not helping the progression with 2 years of Recognition (Level 1) in Grade 10 and Grade 11, then 1 year of Deduction (Level 4) in Grade 12. Gutiérrez *et al*. (1991) argues that the attainment of a level could take months or even years, but fail to specify which of the van Hiele levels could take a longer or shorter time.

Having a look at the van Hiele level spread for the Exercise classifications, we see that they are clearly at Deduction (Level 4) level from as early as Grade 8. This is a huge problem since the learner is not meant to have mastered Deduction (Level 4) by Grade 8 or even Grade 9. Therefore, a dominance of this level of question will very likely have learners in the class feeling as though they can understand the work in class, but not achieve answers in the homework exercises. This could be seen to the learner as inadequacy, causing them to doubt their mathematical abilities, when it is the textbook that is at the incorrect level.

With too much of a cognitive jump for the learner to bridge, Explanations are crucial mediators between the Kernels and the Exercises. By definition, Explanations aid the learner in supplementing the Kernel to a point that the learner will henceforth be able to master the Kernel. Without Explanations and Kernels on a consistent level, these Kernels will be far harder to assimilate in the learner's mind. Kernels were defined as "theorems, rules, definitions, procedures, notations and conventions which have to be learned as knowledge" (Foxman, 1999, p.3) and these are the items that a learner will be using to answer the Exercises. Since the most prevalent level was Recognition (Level 1) in the Kernels and the most prevalent level was Deduction (Level 4) in the Exercises, the exercises are set to a higher level and therefore, theoretically, the learner will not be able to complete the Exercises.

Therefore, in summary, there is a cognitive gap through both textbook series that the Explanations are trying to bridge. The cognitive gap is between the majority Recognition (Level 1) Kernels and majority Deduction (Level 4) Exercises. As mentioned previously, if the learner skips the explanations, they will miss this bridge between the levels and if the Kernels to be learnt are only presented at a lower level, the learner will not be exposed to the training of how to look at the work at a higher level. Teachers that are "textbook bound" would be transferring this cognitive gap to the learners and propagating the malformed "geometric eye" (Godfrey (1910), as cited in Fujita & Jones (2002), p. 16) of the learners, which is defined as "the power of seeing geometrical properties detach themselves from a figure". However, if the teacher's van Hiele level is higher than the learner, Clements and Battista (1992) say that "Teachers can "reduce" subject matter to a lower level, leading to rote memorization, but students cannot bypass levels and achieve understanding". This also does not benefit the progression of van Hiele levels of the learner and hinders the development of their "geometric eye".

5.3 The van Hiele levels as a framework for textbook analysis

Battista (2007) mentions that a "considerable amount of research has established the van Hiele theory as a generally accurate description of the development of students' geometry thinking" (p. 846). Therefore, since the van Hiele levels are accepted as an accurate measure on learners, the only question that must be answered is if this measure can be transferred to measure the development of textbooks' geometry progression.

Khasawneh, Al- Omari, and Tilfah (2000), who studied Jordanian textbooks using the van Hiele levels, answered this question by stating that “math textbooks should be built carefully with an acceptable structure of geometric standard in regard of van Hiele geometric levels” (p. 160). I have found that the use of the van Hiele levels in analysis has helped me to discover why the learners struggle to “see” geometry and formulate proofs, i.e. the lack of informal deduction through Order (Level 3) classifications.

There are researchers that believe that the van Hiele levels could be combined to form three levels. This has been suggested by Pegg and Davey (1998) and has been favoured by Lawrie (1998), as cited by Pegg (2014, p.36). Pegg and Davey (1998) suggest that Recognition (Level 1) become the “Visual” level, Analysis (Level 2) and Order (Level 3) combine into a “Descriptive” level and then Deduction (Level 4) and Rigor (Level 5) combine into a “Theoretical” level. The “Visual” level is defined as decisions that are guided by a visual network, the “Descriptive” level is defined as elements and relations being described and the “Theoretical” level is where there is deductive coherence and geometry generated according to Euclid is considered. If this was put into practice in my research, I believe the strength of the results would have been weaker since the same levels would have come out as dominant or absent, but we would not have been able to narrow down if the problem was a lack of Analysis (Level 2) or Order (Level 3) and if the dominant level was Deduction (Level 4) or Rigor (Level 5).

The progression of the van Hiele levels for each of the Units of Analysis was an invaluable addition to this research. It aided me to see trends, in particular, parts of the textbook that would not have been possible otherwise. Other researchers, such as Khasawneh, Al- Omari, and Tilfah, (2000), performed textbook analysis and even used the Hoffer matrix as well; however, they did not look at the progression per unit of analysis. The deeper issues that have been illuminated using this have been shown and it shows a far deeper problem than just a lack of Order (Level 3) classifications. Each of the three units of analysis should have a progression like the textbook series as a whole.

As for usability, the van Hiele levels and the Hoffer matrix were easy to use due to their explicit nature. The progression that was presented by the matrix was very logical and clear. I did find it difficult to classify the “Constructions” chapters; however, I believe that was just due to the nature of the content. The question that needs to be asked is, “Why do the textbook series deviate so much from the framework? There are only two possible reasons, which are that the book was not designed to follow the van Hiele progression or that the van Hiele levels are not appropriate for textbook analysis. As mention previously, the van Hiele levels are a suitable measure, therefore the other option must apply. Since the textbook was not designed according to the van Hiele levels, I would hope that another appropriate framework was used in their creation.

5.4 Conclusion

In this chapter, a discussion of the results of the units of analysis has been presented, followed by a more in-depth discussion on the van Hiele levels overall and the levels linked to each of the units of analysis. Comparisons of obtained results and the literature were made and suggestions for the reasons why particular differences to literature were presented.

CHAPTER 6 – CONCLUSION

6.1 Overview of the study and results

This research was aimed at analysing the Classroom Mathematics and Mind Action Series mathematics textbooks from Grade 8 through to Grade 12. The van Hiele levels were used as an analytical framework. It was decided to focus on three units of analysis, i.e. Explanations, Kernels and Exercises, where Kernel was defined as all items that a learner would have to learn as knowledge, Explanation was an aid to the learner in the attainment of the Kernel and Exercise was the assessment the learner could use to check their knowledge.

If the textbooks did follow the van Hiele levels, then the results that were expected were that there would be the least classifications of Recognition (Level 1) which were decreasing across the grades, then Analysis (Level 2) with a slower decrease, then Order (Level 3) on the increase and finally Deduction (Level 4) with a faster increase. We did not need to be concerned about Rigor (Level 5) since that would be mastered only post-matric. The results we observed contradicted these expectations.

Firstly, the research found that the three different units of analysis had vastly different spreads of the van Hiele levels. Secondly, it appeared that in both textbook series up to Grade 12, there was a predominance of Recognition (Level 1) for Kernel, Recognition (Level 1) and Deduction (Level 4) for Explanation and Deduction (Level 4) for Exercise. This will create a cognitive gap that is not being filled by Analysis (Level 2) and Order (Level 3). There was a low proportion of Order (Level 3) classifications throughout all the units of analysis. This might hinder the smooth progression of geometric understanding and could cause a problem for learners when attempting to answer a Deduction (Level 4) assessment question. They may struggle to perform at that required level since they have not progressed sufficiently and have not fully developed Godfrey's (1910) idea of the "geometric eye", as cited in Fujita and Jones (2002, p. 16).

Finally, most research on the van Hiele levels have been on teachers' or learners' levels, whereas this research was about textbooks using van Hiele levels (which was far more uncommon). This has a filled a gap in research for the South African context, although there have been international studies performed on textbooks using the van Hiele levels, such as Khasawneh, Al- Omari, and Tilfah (2000) and Haggarty and Pepin, 2002.

6.2 Strengths, weaknesses and ideas for future research

The textbook series used are very prominent in the schooling system and on the LTSM booklist. However, the strength of the results and conclusions drawn from the data could be strengthened if more textbook series from the LTSM booklist and beyond would have been used. Future research would include a worthy comparison of international textbooks with South African textbooks for Grade 8 through to Grade 12.

Another means of strengthening the results, with regards to reliability, could have been attained with more researchers. It is possible that a deeper understanding of the van Hiele levels may have been obtained through discussion of the levels and classifications. With only me performing the classifications, the understanding has been left to be my understanding only. To aid in this issue, I did get my supervisor to compare his classifications with mine, but thankfully, there was total agreement with regards to the classifications.

This research could be extended in two important directions. The first direction would be to have a look at the progression of the van Hiele levels within each textbook, meaning from the beginning of a chapter to the end, or if there are multiple chapters, from the beginning of the first one to the end of the last one. As a learner works through the textbook, they are meant to be increasing their competency and taking a step closer to mastering their current van Hiele level. It would be interesting to see if the textbooks aid in this progression of skill through the chapters in a particular book, instead of the books a series. Rezat (2006) states that "the utilization of mathematics textbooks by students has scarcely been investigated" (p.486). This statement leads off from the previous point where it could be very beneficial to observe a learner using the textbooks and determine if the barriers that have been identified from the text book research are manifested in the learners, or if it is just academic speculation.

The second direction would be to determine if particular topics within the geometry chapter(s) follow the van Hiele levels better than others, for example, if the sections on triangles and congruency follow the van Hiele levels more closely that the sections on lines, line segments and angles. This sort of research could aid in becoming more explicit on where the issues would be, optimise the amount of work required to fix these issues and help the people fixing them by looking at the chapters that do follow the van Hiele levels as a guide.

6.3 Implications for practice

Bischoff, Hatch, and Watford (1999, as cited in Pusey, 2003, p. 72) support the idea that textbooks should not be relied upon to direct the teaching, but rather, the teacher should be responsible for directing the pace and sequence through a progression of reasoning levels. However, the DBE are always looking for means to help under skilled teachers. Many teachers feel that educational theory is something that someone with more status and prestige can apply to their classes (Elliott, 1991); however, if it is included in the textbook, I believe it will be more easily accepted and accessible. Textbooks could be a viable solution predominantly since it is a sustainable one. The Cockcroft Report (1982), as cited in Foxman (1999, p. 26), mentions that "textbooks provide support for teachers in the day-to-day work of the classroom". Looking at this solution economically, it could be cheaper than upskilling and it is also easier to get coverage in schools (including more rural ones). Teachers would not have to travel to a workshop where they may not be able to afford the transport, cannot organise transport, the school does not pay for transport, or that they do not have the time on that particular day. Textbooks can be used at any hour of the day, even at home. This is not solving the problem of teachers being textbook bound, but it is giving a better resource for those teachers to use to help them to provide the best teaching that they can. Souza (2014) supports my argument by saying that "Developing teacher expertise is a long process that needs to be supported until teachers can actively sustain their own development" (p. 5).

Even though the generation of learners in school now is different to what it was many years ago, the textbooks have not changed in their teaching of geometry. This new generation will have new needs for learning and teaching and these needs will need to be addressed. With textbooks that can address these needs and the inequality in the South African Educational landscape, the differences in learning needs and rampant inequality could be dealt with more effectively and aided more easily than the upskilling workshops could.

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APPENDIX A - Instrument from Hoffer (1981)

Below is the instrument I will be using, as seen in Hoffer (1981):

TABLE I
Basic Skills in Geometry

SKILL \ LEVEL	I Recognition	II Analysis	III Ordering	IV Deduction	V Rigor
VISUAL	Recognizes different figures from a picture. Recognizes information labeled on a figure.	Notices properties of a figure. Identifies a figure as part of a larger figure.	Recognizes interrelationships between different types of figures. Recognizes common properties of different types of figures.	Uses information about a figure to deduce more information.	Recognizes unjustified assumptions made by using figures. Conceives of related figures in various deductive systems.
VERBAL	Associates the correct name with a given figure. Interprets sentences that describe figures.	Describes accurately various properties of a figure.	Defines words accurately and concisely. Formulates sentences showing interrelationships between figures.	Understands the distinctions among definitions, postulates, and theorems. Recognizes what is given in a problem and what is required to find or do.	Formulates extensions of known results. Describes various deductive systems.
DRAW-ING	Makes sketches of figures accurately labeling given parts.	Translates given verbal information into a picture. Uses given properties of figures to draw or construct the figures.	Given certain figures, is able to construct other figures related to the given ones.	Recognizes when and how to use auxiliary elements in a figure. Deduces from given information how to draw or construct a specific figure.	Understands the limitations and capabilities of various drawing tools. Pictorially represents nonstandard concepts in various deductive systems.
LOGICAL	Realizes there are differences and similarities among figures. Understands conservation of the shape of figures in various positions.	Understands that figures can be classified into different types. Realizes that properties can be used to distinguish figures.	Understands qualities of a good definition. Uses properties of figures to determine if one class of figures is contained in another class.	Uses rules of logic to develop proofs. Is able to deduce consequences from given information.	Understands the limitations and capabilities of assumptions or postulates. Knows when a system of postulates is independent, consistent, and categorical.
APPLIED	Identifies geometric shapes in physical objects.	Recognizes geometric properties of physical objects. Represents physical phenomena on paper or in a model.	Understands the concept of a mathematical model that represents relationships between objects.	Is able to deduce properties of objects from given or obtained information. Is able to solve problems that relate objects.	Uses mathematical models to represent abstract systems. Develops mathematical models to describe physical, social, and natural phenomena.

Table A.1: The van Hiele levels definitions split by geometric skill

TABLE 2
Sample Problems for the Skill Areas

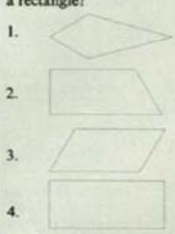
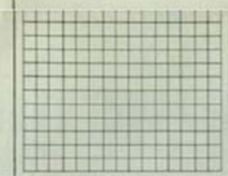
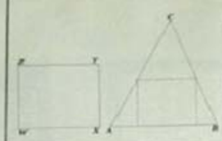
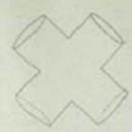
SKILL \ LEVEL	I Recognition	II Analysis	III Ordering	IV Deduction	V Rigor
VISUAL	Which figure below is a rectangle? 1. 2. 3. 4. 	How many different lines of symmetry does a rectangle have?	Can you find the cross section of a tetrahedron that has the shape of a rectangle?	A rectangular piece of paper can be rolled to form the lateral surface of a right circular cylinder. What shape of paper is needed to make an oblique circular cylinder?	Does there exist a rectangle in taxicab geometry whose diagonals are not congruent?
VERBAL	For rectangle $ABCD$, 1. What are the segments $\overline{AC}$ and $\overline{BD}$ called? 2. Which angle is opposite $\measuredangle ABC$? 3. Which sides are adjacent to $\overline{BC}$?	List as many properties as you can of a rectangle.	Write a careful and brief definition of the word *rectangle*.	Which of these is a definition? Which is a postulate? And which is a theorem? 1. A rectangle is a parallelogram with a right angle. 2. The area of a rectangle is equal to the product of the lengths of two adjacent sides. 3. A rectangle with perpendicular diagonals is a square.	Explain why rectangles do not exist in non-Euclidean geometry.
DRAW-ING	Use grids as shown to draw a rectangle $WXYZ$ with side lengths 4 and 7 units. 	Construct a rectangle given the length of one side and of one diagonal.	Given $\triangle ABC$ and rectangle $WXYZ$. Using compasses and straightedge, inscribe a rectangle in $\triangle ABC$ that is similar to $WXYZ$. 	Two cylinders are the same size and shape. Draw a picture of the region that these cylinders have in common. 	Given a circle, is it possible using only compasses and straightedge to construct a rectangle equal in area to that of the circle?
LOGICAL	If a rectangle is turned as shown, is the new figure also a rectangle? 	Is the area of a rectangle determined by its perimeter? If two rectangles have equal perimeters, are their areas also equal?	Which are true and which are false? 1. Each rectangle is a square. 2. Each square is a rectangle. 3. If the diagonals of a parallelogram are congruent, the figure is a rectangle.	Prove or disprove: If the diagonals of a quadrilateral are congruent, the figure is a rectangle.	Since rectangles do not exist in non-Euclidean geometry, how are the areas of figures determined?
APPLIED	Describe the rectangular shapes you see in this classroom and on the athletic field.	If a rectangular field is 100 meters long, what is the smallest size paper you need to draw a map of the field on a scale of 1:1000?	What is the area of the largest rectangle that can be inscribed in a given triangle?	Design a plug that you can push completely through all three holes without leaving any gaps. 	What is the actual shape on earth of a rectangular region on a map of the earth?

Table A.2: Examples of questions for the van Hiele levels split by geometric skill

APPENDIX B – Summary of units of analysis per grade

	Explanations		Kernel		Exercise	
Grade	MA	CM	MA	CM	MA	CM
8	12%	17%	18%	17%	70%	66%
9	25%	10%	15%	19%	60%	71%
10	18%	14%	32%	21%	51%	65%
11	18%	21%	28%	31%	54%	48%
12	33%	12%	22%	19%	46%	69%
Average	21.2%	14.8%	23%	21.4%	56.2%	63.8%

Table B.1: Units of analysis per grade

APPENDIX C - Summary of van Hiele levels per grade

	Recognition		Analysis		Order		Deduction		Rigor	
Grade	MA	CM	MA	CM	MA	CM	MA	CM	MA	CM
8	22%	34%	30%	22%	8%	9%	40%	34%	0%	0%
9	21%	21%	30%	26%	6%	7%	42%	44%	0%	2%
10	28%	27%	9%	6%	6%	5%	56%	54%	1%	8%
11	17%	33%	7%	5%	2%	0%	74%	60%	0%	2%
12	15%	7%	12%	13%	3%	3%	70%	75%	1%	2%

Table C.1: Van Hiele levels per grade

APPENDIX D – Summary of van Hiele levels per unit of analysis per grade

		Explanation		Kernels		Exercises	
Grade	van Hiele levels	CM	MA	CM	MA	CM	MA
8	1 - Recognition	37%	30%	83%	68%	21%	8%
	2 - Analysis	21%	21%	12%	29%	25%	32%
	3 - Order	11%	12%	5%	1%	10%	9%
	4 - Deduction	29%	38%	0%	2%	44%	51%
	5 - Rigor	2%	0%	0%	0%	0%	0%
9	1 - Recognition	14%	25%	73%	82%	8%	4%
	2 - Analysis	5%	34%	22%	14%	29%	33%
	3 - Order	10%	7%	4%	0%	8%	7%
	4 - Deduction	60%	34%	0%	5%	54%	55%
	5 - Rigor	12%	1%	1%	0%	1%	0%
10	1 - Recognition	47%	69%	82%	53%	5%	0%
	2 - Analysis	2%	3%	0%	19%	9%	4%
	3 - Order	2%	0%	4%	17%	6%	0%
	4 - Deduction	31%	24%	8%	10%	74%	96%
	5 - Rigor	18%	3%	6%	0%	6%	0%
11	1 - Recognition	50%	36%	70%	36%	1%	0%
	2 - Analysis	12%	8%	4%	19%	3%	1%
	3 - Order	0%	0%	1%	7%	0%	0%
	4 - Deduction	38%	57%	22%	38%	95%	99%
	5 - Rigor	0%	0%	4%	0%	1%	0%
12	1 - Recognition	3%	20%	35%	37%	1%	1%
	2 - Analysis	19%	13%	38%	28%	5%	3%
	3 - Order	3%	1%	4%	11%	2%	0%
	4 - Deduction	61%	64%	21%	24%	92%	97%
	5 - Rigor	13%	2%	2%	0%	0%	0%

Table D.1: Van Hiele levels per unit of analysis per grade