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2008-03-20T10:24:13Z
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CONTRIBUTIONS TO SIMPLE RANDOM WALKS ENUMERATION

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A thesis submitted to the Faculty of Science, University of the Witwatersrand, Johannesburg, in fulfillment of the requirements for the degree of Philosophy.

Johannesburg 2007

ACKNOWLEDGMENT

I wish to express my sincere gratitude to my supervisors, Professor H. Prodinger and Professor A. Knopfmacher for their encouragement and unfailing aid till the completion of this thesis.

My acknowledgment is also due to:

- The University of Venda, NRF and Mellon Foundation Scholarship for their financial support.
- My darling wife, Emily, for her tremendous encouragement and the care she gave to our three children (Hope, Lutendo and Lufuno) during my stay at Wits University.

DECLARATION

I hereby declare that this is entirely my own work and it is being submitted for the degree of Doctor of Philosophy in the University of the Witwatersrand, Johannesburg. This work has not been submitted before for any degree to any other university.

S.N. Mavhungu

.....day of.....2007

ABSTRACT

We study how often different types of lattice paths (or simple random walks) of length n such as Dyck, unrestricted Dyck, Knödel, Motzkin, and unrestricted Motzkin random walks reach a nonnegative height r as well as on the number of returns to the origin by such walks. This work is motivated by a paper presented by Katzenbeisser and W. Panny [14] where the results were achieved probabilistically.

We use the method of combinatorial constructions to get a generating function corresponding to each of these five simple random walks. The famous Cramer's rule for solving matrices plays a key role in deriving these generating functions. Using singularity analysis of generating functions given in [8] each of these generating functions are expanded, asymptotically, around a dominant singularity to get moments and their corresponding variances. All our asymptotic expansions are done using computer algebra softwares such as Maple and Mathematica . In each case we consider both closed and open ended walks.

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