

Effect of internal surface curvature on steady axisymmetric shock waves

Alessandro Antonio Filippi

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Declaration

I declare that this dissertation is my own, unaided work, except where otherwise acknowledged. It is being submitted for the degree of Master of Science in Engineering in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other university.

Signed this 11th day of May 2017



Alessandro Antonio Filippi

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Abstract

The cardinal aspects of supersonic and hypersonic propulsion intake design involve understanding the internal shock wave structures forming therein. A study was conducted to explore the effects of internal surface curvature and entry deflection angle on steady axisymmetric shock waves. Very little is known about these influences with only Curved Shock Theory, produced by Mölder, providing analytical insight directly after a curved shock wave.

The shock waves and accompanying flow fields which were generated were studied via experimental and numerical means. Radius normalised internal radii of curvature of 1, 1.5 and 2 with entry deflection angles of 0° , 4° and 8° were investigated between a Mach number range of Mach 2.4 and 3.6. Experimental results were produced using a blow down supersonic wind tunnel facility and were captured via shadowgraph and schlieren flow visualisation techniques. The numerical simulations were validated using the experimental results.

A self similar curved shock wave shape equation was presented with an empirical model which uses flow Mach number and internal radius of curvature in order to produce the resulting curved shock shape. Curved Shock Theory streamlines were used to try predict the internal surfaces that produced the curved shocks but results did not correlate. This was due to extreme streamline curvature curving the streamlines when the shock angle approached the Mach angle. Very good agreement was however found between the theoretical and numerical streamlines at lower curvatures. The higher the internal surface curvature and entry deflection angle, the greater the flow fields were impacted. Steeper characteristics formed as a result, curving the shock wave more noticeably. Both the internal surface curvature and entry deflection angle were found to have an effect on the trailing edge expansion fans which then altered the shape of downstream shock wave structures. The highest curvature models produced steady double reflection patterns due the flow being turned in onto itself by the imposed internal surface curvature. The effects of conical and curved internal surfaces were explored for additional insight into the presence of flow-normal curvature and the curving of the attached shock waves.

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List of Symbols

The units of quantities defined by a symbol are indicated in square brackets following the description of the symbol. Quantities with no indicated units may be assumed to be dimensionless.

General Gas Dynamics

a_s	Local sound speed	[m/s]
A	Flow area	[m ²]
δ	Flow deflection angle	[°]
γ	Constant specific heat ratio	
M	Mach number	
μ	Mach angle	[°]
p	Absolute pressure	[kPa]
ρ	Density	[kg/m ³]
R	Gas constant	[J/(kg.K)]
s	Specific entropy	[J/(kg.K)]
θ	Shock angle	[°]
T	Absolute temperature	[K]
V	Velocity	[m/s]

Curved Shock Theory

D	Streamline curvature	[1/m]
Γ	Normalised vorticity	
J	Normalised pressure gradient influence coefficient	
L^*	Curved surface distance to the sonic region	
K	Streamline curvature influence coefficient	
$\mathbf{n}$	Normal vector to the streamline	[m]
P	Normalised streamwise pressure gradient	
$\Re$	Flow to flow-normal shock curvature ratio	
R_a	Radius of curvature of shock trace $a - a$	[m]

S_a	Flow plane shock curvature	[1/m]
S_b	Flow- normal plane shock curvature	[1/m]
σ	Tangential curved shock vector	[m]
ω	Vorticity	[1/s]
x_s	On shock streamline starting point x co-ordinate	[m]
y	Distance from the axis of symmetry	[m]
y_s	On shock streamline starting point y co-ordinate	[m]

Experimental Parameter Definitions

α	Wedge entry angle	[°]
L_w	Normalised ring wedge length	
MN	Calibration Mach number	
R_c	Normalised internal radius of curvature	
R_{le}	Leading edge radius	[m]
R_{te}	Trailing edge radius	[m]
VC	Veeder counter	

Curved Shock Wave Shape Equation

ε	Curve fitting constant
η	Curve fitting constant
λ	Curve fitting constant
ϕ	Curve fitting constant ratio
x_{max}	Horizontal shock length
x_s	Shock wave x coordinate
y_s	Shock wave y coordinate

Chapter 1 | Introduction

1.1 Shock Waves in Compressible Fluids

When a physical object is placed in a compressible fluid, such as air, the information about the object's location and presence is propagated upstream within the medium via a pressure or sound wave. The speed at which this information travels is at the local speed of sound of the fluid. Its magnitude is characteristic of the particular fluid or medium through which the information is propagating. Therefore any objects travelling below local sound speed allow the fluid to smoothly adjust to its presence as the information is able to propagate upstream. If the object is travelling faster than local sound speed its presence cannot be felt upstream as the information travels slower than the object hence the fluid adjusts to this boundary condition via a shock wave.

A shock wave is therefore a discontinuity in a fluid and causes a rise in pressure, temperature, density and entropy. This occurs to satisfy the governing equations which conserve the energy, momentum and continuity of a system. Shock waves can either be formed by objects moving faster than local sound speed through a stationary fluid or in the opposite case where stationary objects are placed in a fluid moving faster than sound speed. The resulting shock waves that form are identical, the difference being merely a matter of reference frame. This allows experimental work to be carried out with the use of wind tunnels while achieving results and confirming theories applicable to the flight of supersonic bodies.

1.2 Problem Statement

The research undertaken will observe and analyse the impact of internal surface curvature and entry angle of a ring wedge as is seen in Figure 1.1. The curving of the internal surface will result in the deflecting of the flow into itself. This will result in the formation of a curved axisymmetric shock wave. This is due to the geometric nature of the ring wedge and the

supersonic flow having to adjust to its presence. The changes in internal curvature and flow velocity will then have an impact on the generated shock wave. The same applies for the change in the entry angle due to the initial increase in flow deflection as the air enters the ring wedge. This allows the shock wave shape, curvature and downstream flow structures to be analysed and related to the independent parameters. Empirical models may then be established linking the shock wave shape to the self similar geometric and flow conditions that caused it.

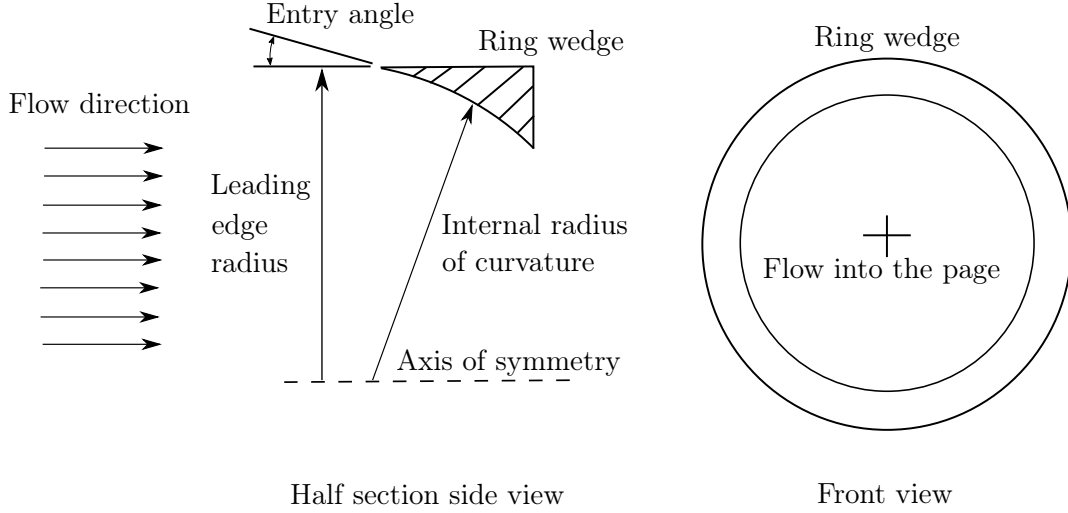


Figure 1.1: Ring wedge test model schematic

1.3 Background and Motivation

The field of steady shock wave dynamics has been well documented concerning both the resulting shock wave profiles and specific reflection types that form. Past research however has focused only on the application of straight planar and axisymmetric wedges without any imposed curvature on the internal surfaces of the models. Numerous literature examples can be found in [1] through [10] with the main target of the publications being applied in shock wave interaction and reflection type analysis. The effects of internal surface curvature on both axisymmetric and planar steady shock wave cases has been relatively undocumented. Thus an explorable gap exists in literature regarding both numerical and experimental results due to influencing internal curved surfaces.

The imposed curvature on the internal surface of an axisymmetric ring wedge results in the curving of the shock waves that are formed. This induced curving occurs both in the flow and flow-normal plane with respect to the incoming flow direction. A theoretical analysis technique has been proposed in [11] and [12] which allows for the calculation of flow property

gradients behind a curved shock wave. The applicability of this technique is however limited to conditions directly after the shock wave and not downstream. When considering the theory it is evident that an inverse model is dealt with in the sense that an initial shock wave shape or structure is chosen and internal surface that causes it may be theoretically calculated. This is the opposite case to a typical design problem where one would start with a given wall surface and work towards the shock wave shape and its subsequent properties. Experimental work has not been conducted in order to validate and correlate the proposed methods of [11] and [12]. Therefore a gap exists relating the effects of internal surface curvature and resulting shock waves in a "direct design" model working from an internal surface to the shock wave shape. Emphasis is placed on axisymmetric shock waves caused by the more common and structurally sound axisymmetric engine air intakes used for supersonic and hypersonic propulsion systems.

Regarding the practical applications of the research being undertaken there is possible scope for the results to be applied in the field of supersonic inlets for scramjet type aircraft engines. Scramjets are a specific family of propulsion systems that operate without the use of rotating turbomachine components such as those seen in conventional turbojet and turbofan engines. Conventional aircraft operate below the speed of sound unlike the case of aircraft using scramjet type engines which operate at multiple times the local sound speed. This is primarily due to the functioning of a scramjet. A schematic of an axisymmetric scramjet type engine can be seen in Figure 1.2.

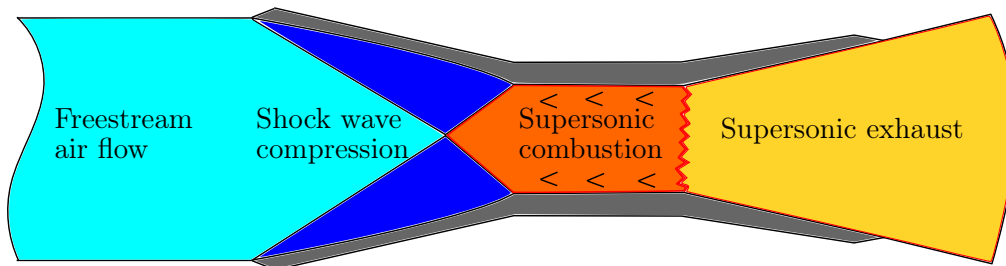


Figure 1.2: Scramjet type engine

Scramjets use the shock waves formed at engine inlets to compress and heat the air at supersonic or hypersonic speeds. After the shock wave induced compression and heating stage the air moves to the combustion section where it spontaneously combusts with injected fuel while still moving faster than the speed of sound. This transfers large amounts of energy into the moving air which is further accelerated via a supersonic nozzle in the exhaust phase to produce thrust and propel the aircraft forward at very high speeds. By studying the effects of internal surface curvature on the shock waves formed inside axisymmetric models one can improve and better understand the design of the inlets. The effects of varying scramjet inlet geometry to include the presence of axisymmetric internal curvature can then give engineers

more versatile tools when it comes to designing and optimising inlets. Requirements for combustion and propulsion can then be met by altering the shape and properties of the generated shock waves.

1.4 Chapter Overview

The following outlines the contents of the dissertation components. The current chapter provides an introduction to the field of compressible gas dynamics and an overview of the background and motivation for the research contained herein.

Chapter 2 outlines the literature relevant to the dissertation. The relationships and phenomena of both one and two dimensional gas dynamics are explored including shock wave reflection. Curved Shock Theory is introduced with its analysis method and both global and local flow detachment is described. Brief fundamentals in both flow visualisation and supersonic wind tunnel operation are mentioned.

Chapter 3 lists the research objectives derived from the problem statement in Section 1.2

Chapter 4 details the method used to obtain experimental results. The supersonic wind tunnel facility that was used is presented along with its testing procedure, precautions and testing models. The flow visualisation setup used to capture results is explained with its different shadowgraph and schlieren techniques.

Chapter 5 explains the method used to obtain the numerical results. The details behind the computational domain and mesh specification are presented. This included adaptive mesh refinement and a mesh grid independence study. The computational solver settings are listed should one wish to replicate results.

Chapter 6 presents and discusses the various aspects of the experimental and numerical results of this dissertation. Six main sections are outlined namely the numerical analysis validation, presentation of a curved shock wave shape equation, the utilising of Curved Shock Theory for wall profile prediction, the effects of varying internal surface curvature, the effects of varying wedge entry angle and finally a comparison between conical and curved internal surfaces.

Chapter 7 summarises and concludes the findings which were discussed in Chapter 6 and Chapter 8 provides recommendations for future work based on what was completed by this dissertation.

Chapter 2 | Literature Survey

2.1 Ideal Gas and Mach Number Relations

Compressible fluids such as air, more commonly referred to as a gas in this case, exhibit density variations depending on the pressure gradients and temperature states in which they inhabit. The relationship by which these properties can alter the gas's density can be described by and are assumed to follow the Ideal Gas law. This is due to the conditions and weak molecular interactions present in most compressible flow and engineering applications which follow the Ideal Gas model's assumption of negligible intermolecular forces [13]. The Ideal Gas law that relates the density, pressure and temperature of an ideal gas is given by Equation (2.1).

$$p = \rho RT \quad (2.1)$$

Where p is the absolute pressure, ρ the ideal gas's density, T the absolute temperature and R the gas constant specific to the type of gas in question. An information relaying sound wave that propagates and informs a compressible fluid about the presence of a disturbance does so at local sound speed. In an ideal gas flow, where entropy remains constant and no heat transfer occurs, local sound speed is expressed by Equation (2.2) [13].

$$a_s = \sqrt{\gamma RT} \quad (2.2)$$

Where γ is the ratio of constant specific heats of the ideal gas. If a gas or object is moving with a velocity V then its Mach number is defined by Equation (2.3) normalising it with respect to local sound speed.

$$M = \frac{V}{a_s} \quad (2.3)$$

Suppose a particle travels at a supersonic speed ($M > 1$ or $V > a_s$) then the information carrying sound waves trail behind it as it continues its motion. This means that they will never catch up with the particle as long as it continues at supersonic speeds, Figure 2.1 depicts this situation. The region where the particles presence is known will be behind it, in a region bound by the tangents of its circular sound waves. The tangent to said circular information carrying sound waves is known a Mach wave.

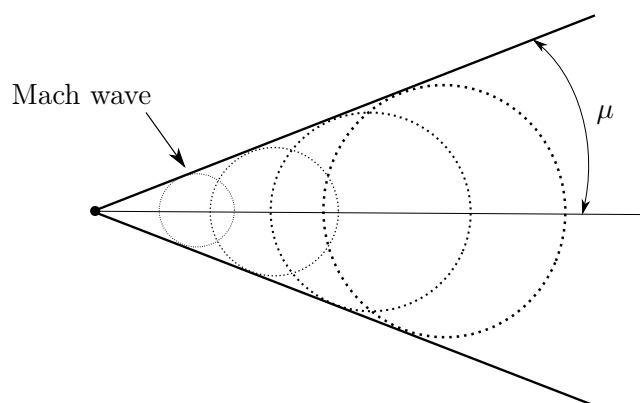


Figure 2.1: Supersonic particle and its trailing information carrying sound waves

The Mach angle between the line of motion of the particle and the Mach wave is given by Equation (2.4). Any particles within the compressible gas will not be informed about the travelling particle's presence until the Mach wave has passed through them. A Mach wave is the weakest form of a shock wave where it allows for the sudden adjustment of a very small disturbance travelling faster than local sound speed [13].

$$\mu = \sin^{-1} \left(\frac{1}{M} \right) \quad (2.4)$$

2.2 One Dimensional Gas Dynamics

Shock waves form as discontinuities to satisfy the boundary conditions of supersonic flow. Therefore across a shock wave, a compressible gas's properties change accordingly. A normal shock wave is one dimensional in nature and forms perpendicular to the oncoming flow direction. Figure 2.2 shows a normal shock wave that has formed downstream of the throat of a quasi-one dimensional nozzle with subsequent flow property changes across its boundary.

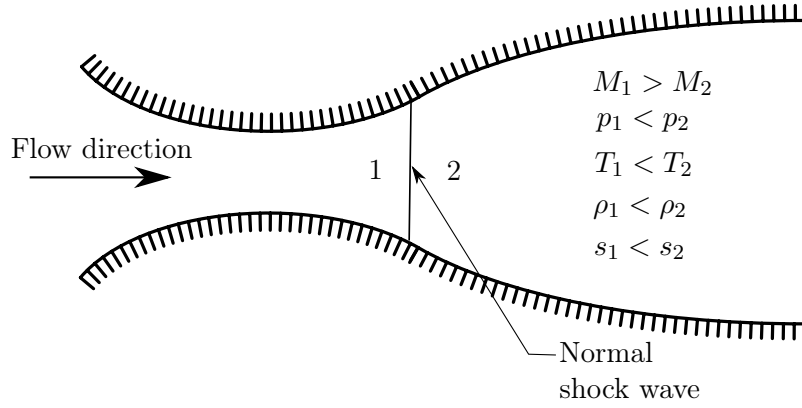


Figure 2.2: Normal shock wave with accompanying flow properties changes

Using the continuity, momentum and energy equations one can derive the relationships between the change in flow properties across a normal shock wave as a function of the oncoming flow Mach number [13]. This yields the Mach number after the shock and the property ratios listed in Equation (2.5) through Equation (2.7) following the same location notation used in Figure 2.2.

$$M_2 = \sqrt{\frac{1 + [(\gamma - 1)/2]M_1^2}{\gamma M_1^2 - (\gamma - 1)/2}} \quad (2.5)$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1) \quad (2.6)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_1^2}{2 + (\gamma - 1)M_1^2} \quad (2.7)$$

The relationships for temperature can then be solved for with the use of Equation (2.1) and the entropy increase across the shock can be formulated using the second law of thermodynamics applicable to ideal gases with constant specific heats [13].

2.3 Two Dimensional Gas Dynamics

2.3.1 Oblique Shock Waves

When supersonic flow is made to turn in onto itself and deflected from its original path an oblique shock wave forms. The function of the oblique shock wave is to discontinuously redirect the flow due to the presence of an obstruction or corner. The formation of an oblique shock wave with a shock angle θ is shown in Figure 2.3 where a wall bounded supersonic flow

is turned by a deflection angle δ . The flow streamlines are then deflected by the same angle δ through the oblique shock wave.

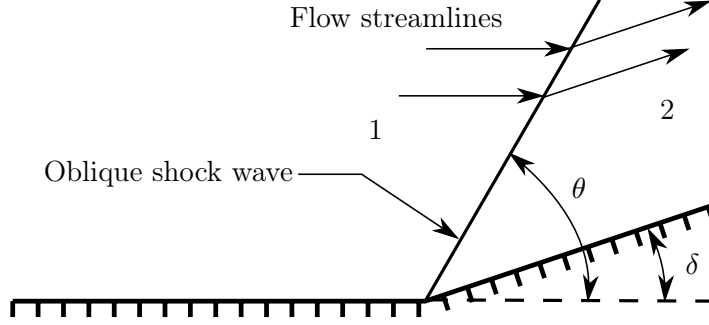


Figure 2.3: Formation of a oblique shock wave

The function describing the relationship between deflection, shock angle and flow Mach number is seen in Equation (2.8). The property magnitude inequality relationships follow the same as those for a normal shock in Figure 2.2 but their mathematical formulations differ. To find the property values across an oblique shock wave Equation (2.6) and Equation (2.7) may be used but M_1 is replaced by the normal component of the flow $M_1 \sin \theta$. The Mach number after the oblique shock wave is calculated using its normal component determined from Equation (2.5) and trigonometric relations using the angle between the oblique shock wave and the deflection [13]. Therefore it can be seen that a normal shock wave is just an oblique shock wave upper limiting case where the shock angle $\theta = 90^\circ$ and hence $\delta = 0^\circ$. Conversely the weakest form of an oblique shock wave would exist when $\theta = \mu$, the Mach angle, resulting in a Mach wave.

$$\tan \delta = 2 \cot \theta \left[\frac{M_1^2 \sin^2 \theta - 1}{M_1^2 (\gamma + \cos 2\theta) + 2} \right] \quad (2.8)$$

A graphical representation of Equation (2.8) is displayed in Figure 2.4 for Mach numbers ranging from $M_1 = 1.2$, on the left, to $M_1 = 6$, on the right. A local maximum exists for each variation of Equation (2.8) where the derivative of the deflection, δ , with respect to the shock angle, θ , is taken for the Mach number in question. This maximum value of the deflection, δ_{max} , represents the maximum value for which an attached shock exists at that particular Mach number. Deflections greater than δ_{max} will cause a curved detached shock to form ahead of the obstruction or corner. The dashed blue curve in Figure 2.4 connects the various δ_{max} values for the Mach number range. The same dashed blue curve also separates the graphs into two regions known as the strong and weak shock regions. The strong shock angle region lies above the blue dashed curve whereas the weak shock wave region lies below. The difference between the two regions being the Mach number after the oblique shock wave.

For a strong oblique shock wave $M_2 < 1$ or subsonic in nature but for a weak oblique shock wave $M_2 > 1$ [13].

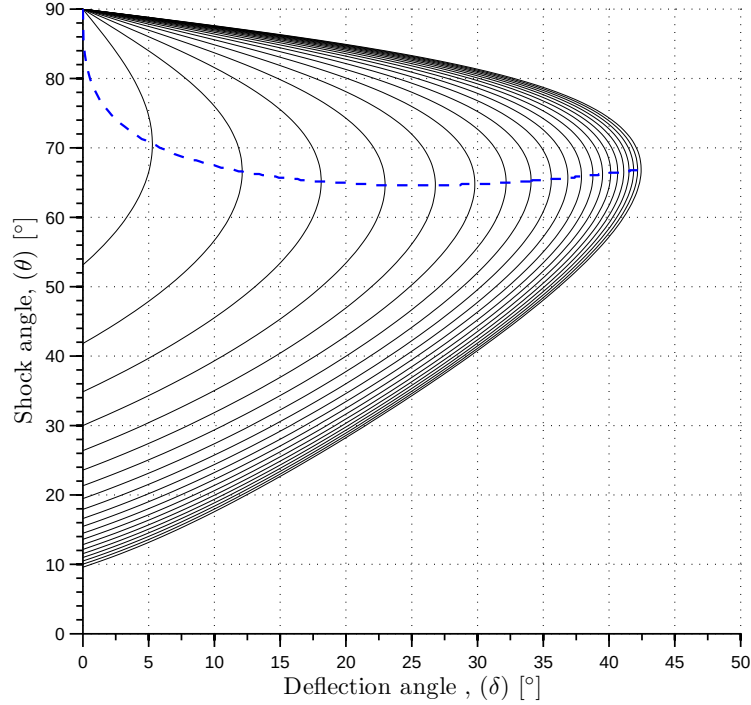


Figure 2.4: Graphical $\delta - \theta - M$ relationship

2.3.2 Expansion Waves

When supersonic flow is made to turn away from itself it results in the formation of expansion waves as seen in Figure 2.5. Expansion wave fans result in a continuous change of flow properties as they turn the flow. This is due to the fact that they are comprised of an infinite number of Mach waves such that the expansion process is isentropic as the change in entropy across a Mach wave is zero [13].

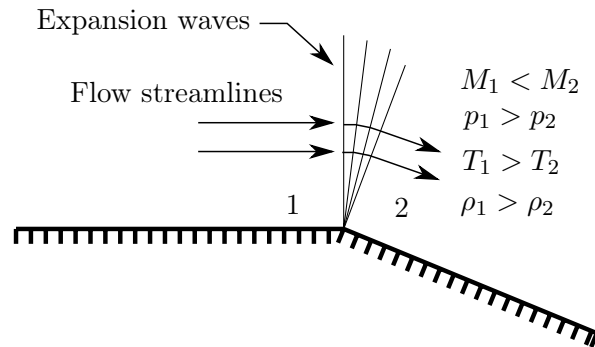


Figure 2.5: Formation of expansion waves

2.4 Shock Wave Reflection

2.4.1 Regular and Mach Reflection

If two wedge-like objects are placed in a supersonic flow, their generated oblique shock waves will interact with one another. Incident oblique shock waves form at the leading edge of each wedge in order to redirect the flow to a direction parallel with the wedge's inner surface. If the wedges are geometrically and spatially identical, and thus in planar symmetry, the shock waves will be of the same strength. The incident oblique shock waves then interact at the plane of symmetry which serves as a reflecting surface for the shock waves. This causes a reflected shock wave to form in order to redirect the flow leaving the incident shock wave back. The reflection that occurs is not similar to that of light rays in nature but is generated to discontinuously redirect the supersonic flow to meet boundary conditions.

The flow after the incident shock wave is formed following the laws of Equation (2.8) and thus deflects the flow into a post shock region with a decreased Mach number. Since the flow is now parallel with the inner surface of the edge it approaches the reflecting surface and plane of symmetry with the same deflection angle. If the deflection angle is less than δ_{max} at the decreased post shock Mach number then the resulting reflection is called a Regular Reflection, conversely if it is greater than δ_{max} it is called a Mach Reflection [3]. Figure 2.6a and Figure 2.6b display the shock structures of a Regular and Mach Reflection respectively.

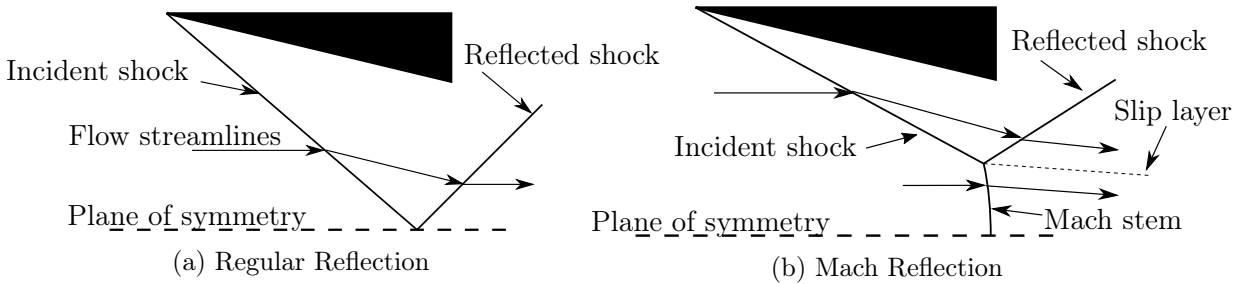


Figure 2.6: Regular and Mach Reflection of a straight wedge in planar symmetry

The characteristic three shock structure of a Mach Reflection is made up of an incident shock, a reflected shock and a third shock wave known as the Mach stem. These three shock waves then meet at the Mach reflection triple point. The flow behind the Mach stem is subsonic in nature and has a different velocity to the flow exiting the reflected shock. These differing velocities, even though they are in the same direction, form a slip layer between the two flows that originates from the triple point. The flow across the slip layer is isobaric but each side has differing velocities, densities and temperatures. Due to the geometries of the trailing edge

of the wedges in Figure 2.6a and Figure 2.6b an expansion wave fan would form to turn the flow around the corner. This has been neglected graphically for the sake of clarity but its presence would curve the reflected shock and the slip layer as it protrudes into the flow field.

2.4.2 Axisymmetric Shock Wave Reflection

In an axisymmetric case the only reflection that is possible is a Mach reflection due to the fact that the incident shock wave steepens towards the triple point, exceeding the Von Neumann angle [10]. The Von Neumann angle limit at the triple point is a limiting shock incidence angle where Regular Reflection no longer becomes possible and a transition to Mach Reflection is said to occur [1]. Due to symmetry about a central axis the Mach stem in the planar symmetry case of Figure 2.6b now forms what is known as a Mach disk. The Mach stem therefore becomes the radius of the Mach disk and the incident and reflected shocks form cones in an axisymmetric Mach Reflection such as that in Figure 2.7.

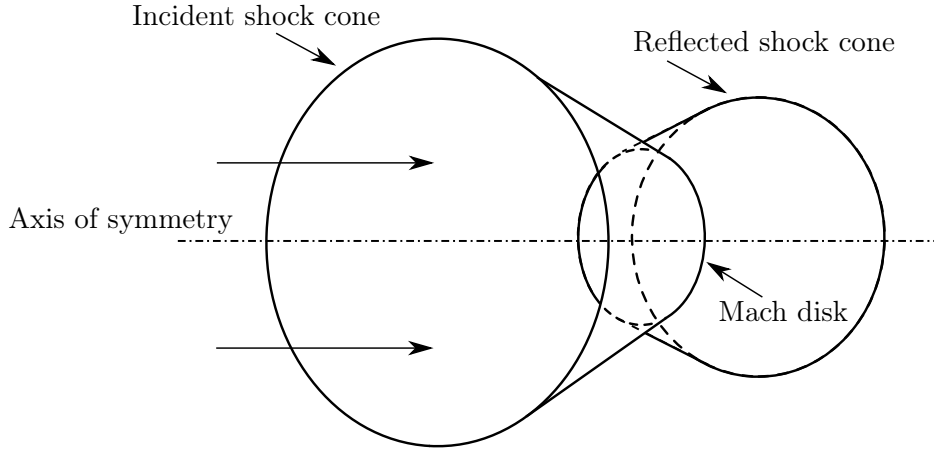


Figure 2.7: Axisymmetric Mach Reflection

2.5 Curved Shock Theory

Curved Shock Theory (CST) is proposed by Mölder [12] as a means to analytically calculate and relate the pressure gradients, flow streamline curvatures and vorticity immediately behind a curved shock wave that is planar or axisymmetric. This allows the analysis of a curved shock wave given the shock shape and hence its shock curvatures. For a steady supersonic flow entering a stationary shock element in Figure 2.8, the shock traces in the flow and flow-normal directions can be defined as $a - a$ and $b - b$ respectively. The shock curvatures along

said shock traces being S_a and S_b . The radii of curvature being $R_a = -1/S_a$ and $R_b = -1/S_b$.

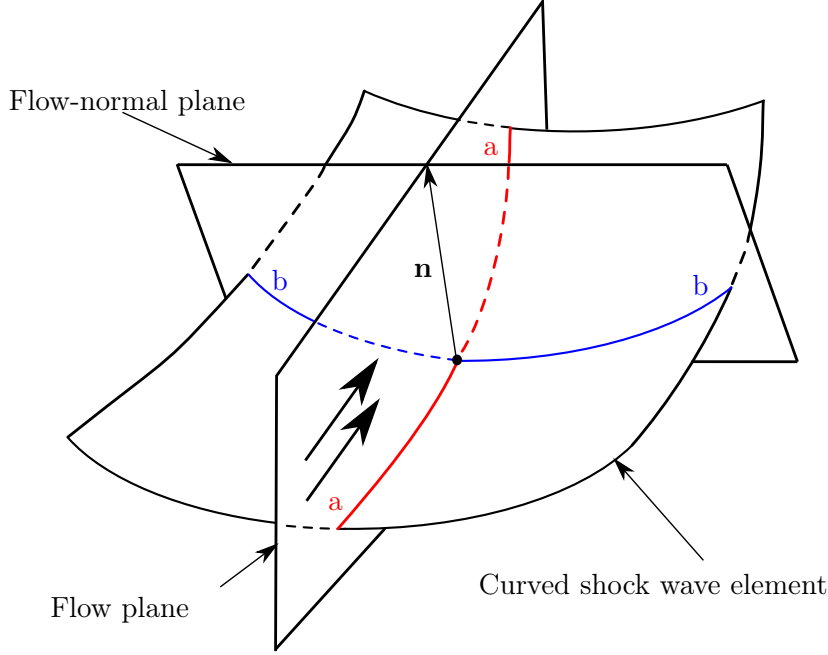


Figure 2.8: Steady curved shock wave element (adapted from [12])

Consider a supersonic flow entering the curved shock wave element passing through flow plane shock trace $a - a$ such as that in Figure 2.9. At pre-shock conditions the flow enters with a Mach number, M_1 , at an initial angle of δ_1 with the x -axis. The point of entry giving rise to the normal vector to the streamline, $\mathbf{n}$, and distance along the streamline, $\mathbf{s}$, after the intersection of pre-shock flow vector and the shock wave. The tangent of the curved shock being σ . The post-shock flow then leaves with Mach number, M_2 , at an angle of δ_2 with the x -axis. Through the curved shock wave the flow is deflected by angle δ , where $\delta = \delta_2 - \delta_1$. The aerodynamic shock angle θ is the angle between the tangent of the curved shock wave, at the point where the flow enters, and the original vector path of the pre-shock flow. The geometric shock angle, θ_1 , is the angle of the shock measured with respect to the x -axis such that $\theta_1 = \theta + \delta_1$ [12].

In an axisymmetric case the vertical distance from the axis of symmetry to any point of the curved shock wave is y . The resulting flow-normal plane shock curvature is then defined as $S_b = -\cos\theta_1/y$. The flow plane curvature would be the geometric curvature of the curved shock wave, dependant on the function that describes the shock shape. By dividing the flow plane shock curvature by that of the flow-normal plane, the shock curvature ratio is $\Re = S_a/S_b$. When the supersonic flow entering an axisymmetric curved shock wave is non-diverging or converging, the pre-shock flow angle, δ_1 , becomes zero. This is a result of the flow being aligned and parallel with the central axis of symmetry. Consequently δ becomes

only a function of δ_2 . The geometric shock angle, θ_1 , becomes equal to the aerodynamic shock angle, θ , as the δ_1 term disappears from the relation [12].

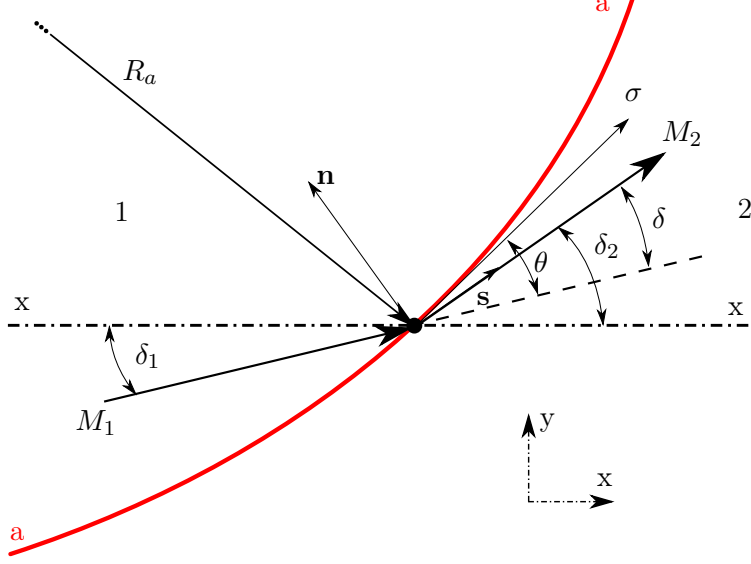


Figure 2.9: Curved shock element trace $a - a$ [12]

By solving the conservation equations of energy, mass, momentum, state and vorticity for a curved shock wave with normal and tangential components, Mölder [12] developed the Curved Shock Equations in Equation (2.9) and Equation (2.10).

$$A_1 P_1 + B_1 D_1 + E_1 \Gamma_1 = A_2 P_2 + B_2 D_2 + C S_a + G S_b \quad (2.9)$$

$$A'_1 P_1 + B'_1 D_1 + E'_1 \Gamma_1 = A'_2 P_2 + B'_2 D_2 + C' S_a + G' S_b \quad (2.10)$$

The coefficients for Equation (2.9) and Equation (2.10) are listed as follows:

$$\begin{aligned} A_1 &= \frac{2 \cos \theta ((3M_1^2 - 4) \sin^2 \theta - (\gamma - 1)/2)}{\gamma + 1} & B_1 &= \frac{2 \sin \theta ((\gamma - 5)/2 + (4 - M_1^2) \sin^2 \theta)}{\gamma + 1} \\ E_1 &= \frac{-2 \sin^3 \theta ((\gamma - 1)M_1^2 + 2)}{\gamma + 1} & A_2 &= \frac{\sin \theta \cos \theta}{\sin(\theta - \delta)} \\ B_2 &= \frac{-\sin \theta \cos \theta}{\cos(\theta - \delta)} & C &= \frac{-4 \sin \theta \cos \theta}{\gamma + 1} \end{aligned}$$

$$\begin{aligned}
G &= \frac{4 \sin^2 \theta \cos \theta \sin \delta_1}{(\gamma + 1) \cos(\theta + \delta_1)} & A'_1 &= M_1^2 \cos \delta \cos^2 \theta - (M_1^2 - 1) \cos(2\theta + \delta) \\
B'_1 &= -\sin(2\theta + \delta) - M_1^2 \sin \delta \sin^2 \theta & E'_1 &= -(2 + (\gamma - 1)M_1^2) \sin \delta \sin^2 \theta \\
A'_2 &= \frac{(1 + (M_2^2 - 2) \sin^2(\theta - \delta))(\sin \theta \cos \theta)}{\sin(\theta - \delta) \cos(\theta - \delta)} & B'_2 &= -\sin(2\theta) \\
C' &= \frac{-\sin(2\delta)}{2 \cos(\theta - \delta)} & G' &= \frac{\sin \theta \sin \delta_1 \sin(\theta + \delta) - \sin \theta \cos \theta \sin \delta_2 \tan(\theta - \delta)}{\cos(\theta + \delta_1)}
\end{aligned}$$

$$\text{where } M_2^2 = \frac{(\gamma + 1)^2 M_1^4 \sin^2 \theta - 4(M_1^2 \sin^2 \theta - 1)(\gamma M_1^2 \sin^2 \theta + 1)}{[2\gamma M_1^2 \sin^2 \theta - (\gamma - 1)][(\gamma - 1)M_1^2 \sin^2 \theta + 2]}$$

The P , D and Γ terms in Equation (2.9) and Equation (2.10) denoting the normalised stream wise pressure gradient, streamline curvature and normalised vorticity Their algebraic definitions being those stated in Equation (2.11), Equation (2.12) and Equation (2.13). Following the same numbering scheme throughout of 1 denoting flow upstream and 2 denoting flow downstream of the shock wave .

$$P = \frac{\partial p / \partial s}{\rho v^2} \quad (2.11)$$

$$D = \frac{\partial \delta}{\partial s} \quad (2.12)$$

$$\Gamma = \frac{\omega}{V} \quad (2.13)$$

By solving Equation (2.9) and Equation (2.10) for the downstream stream wise pressure gradient, P_2 , and streamline curvature, D_2 , Equation (2.14) and Equation (2.15) are obtained.

$$P_2 = J_p P_1 + J_d D_1 + J_g \Gamma_1 + J_a S_a + J_b S_b \quad (2.14)$$

$$D_2 = K_p P_1 + K_d D_1 + K_g \Gamma_1 + K_a S_a + K_b S_b \quad (2.15)$$

Equation (2.14) and Equation (2.15) are known as the influence coefficient form of the Curved Shock Equations where the influence coefficients are denoted by J and K . The value of the influence coefficients show the effect of the upstream flow properties and shock wave curvatures on P_2 and D_2 [12]. If the flow entering a curved shock wave is uniform and parallel with the axis of symmetry, Equation (2.14) and Equation (2.15) can be simplified such that the upstream flow property terms and δ_1 are reduced to zero. Figure 2.10 and Figure 2.11 show the effects of the influence coefficients, J and K , on the post shock pressure gradient and streamline curvature terms respectively for a Mach 3 air flow where $\delta_1 = 0$.

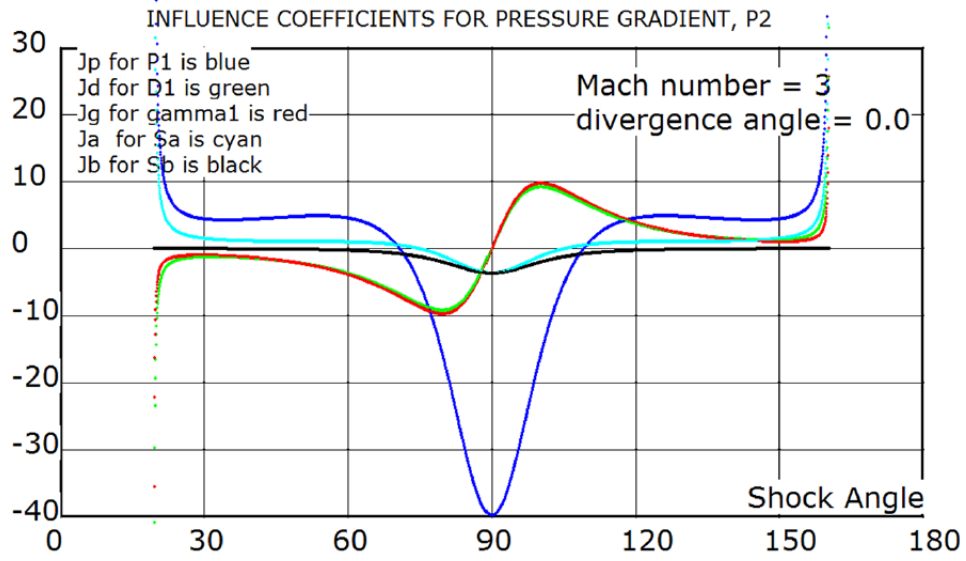


Figure 2.10: Pressure gradient influence coefficients [12]

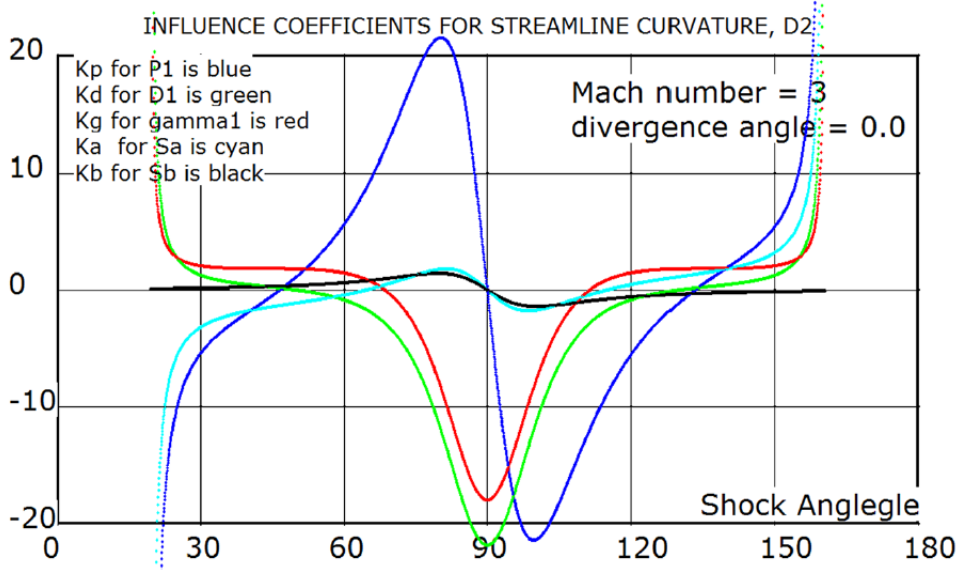


Figure 2.11: Streamline curvature influence coefficients [12]

By using the streamline curvature, D_2 , the equation of the streamline shape can be calculated from a Taylor series approximation resulting in Equation (2.16) as proposed by Mölder et al. in [14] and Mölder in [15].

$$y = y_s + (x - x_s) \tan \delta_2 + \frac{(x - x_s)^2}{2} D_2 \quad (2.16)$$

Where (x_s, y_s) denote the starting co-ordinates of the streamline on the curved shock wave.

2.6 Shock Wave Detachment

2.6.1 Global Flow Detachment

When a stream of supersonic flow passes through an area reduction, such as that of a typical supersonic diffuser in Figure 2.12, it decelerates to a lower Mach number. This follows the mathematical relationship set up by the Hugoniot area-velocity relation in Equation (2.17).

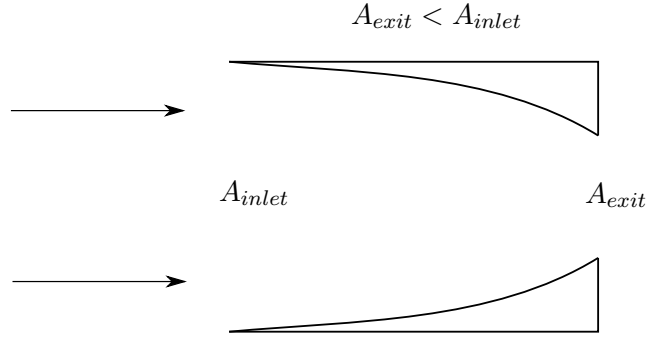


Figure 2.12: Typical supersonic diffuser schematic

$$\frac{dA}{A} = (M^2 - 1) \frac{dV}{V} \quad (2.17)$$

Thus it can be deduced that the lowest possible Mach number that a supersonic flow can be decelerated to via area reduction is $M = 1$ due to the area reaching a minimum value as the area differential approaches zero [13]. Similarly the highest Mach number a subsonic flow can be accelerated to via area reduction is $M = 1$. The contraction ratio (A_{exit}/A_{inlet}) for producing sonic flow (flow where $M = 1$) at the exit area from a supersonic inlet Mach number can be derived from the isentropic nozzle relation in Equation (2.18).

$$\left(\frac{A}{A_{sonic}} \right)^2 = \frac{1}{M^2} \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{(\gamma + 1)/(\gamma - 1)} \quad (2.18)$$

Once sonic conditions have been reached at the minimum area, the mass flow rate through the area reduction cannot change and a condition known as "choked flow" is now present [13]. Therefore for an area contraction ratio, A_{exit}/A_{inlet} , that is greater than that dictated via isentropic relations, supersonic flow through the area reduction is possible. Should A_{exit}/A_{inlet} be less than that allowed by isentropic relations, supersonic flow would not be possible in the contraction and the flow will have extra mass which it would need to dispose.

In the case of a supersonic diffuser or area reduction, an attached shock under these conditions is not possible at the incoming mass flow rate and Mach number so becomes it globally choked. A detached shock, called a bow shock, then forms ahead of the inlet as depicted in Figure 2.13. The bow shock forms such that it brings the flow to subsonic speeds. The flow can then pass through the contraction to generate sonic conditions at the minimum area. Excess mass is then "spilled out" from the inlet to flow elsewhere as it cannot pass through the choked conditions of the supersonic diffuser. This type of flow is known as "unstarted flow" as the shock wave has not been swallowed into the contraction. The "started flow" case then permitting supersonic flow within the contraction during its attached shock wave state.

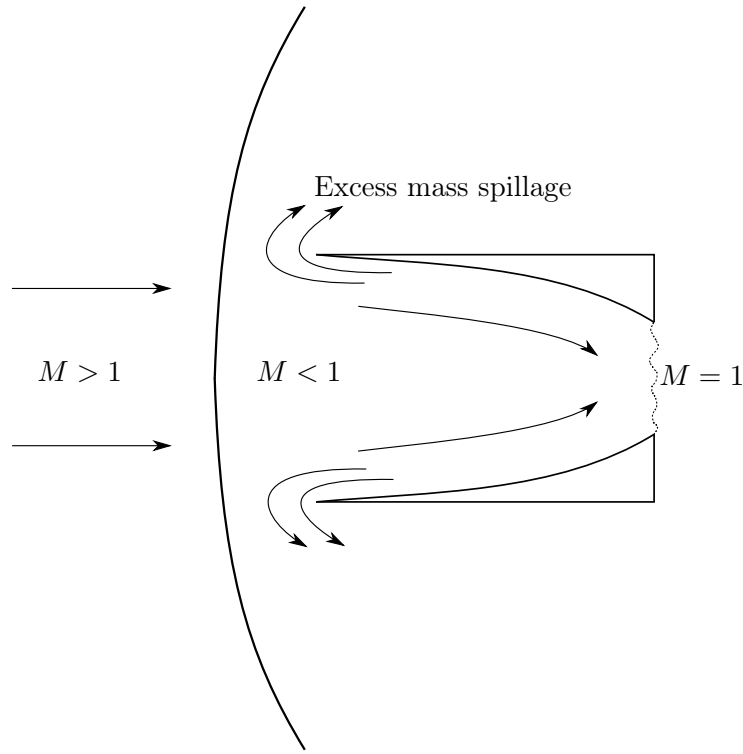


Figure 2.13: Detached bow shock

Kantrowitz and Donaldson [16] suggested that flow through a supersonic diffuser or area reduction would spontaneously swallow its shock and start if the contraction ratio, A_{exit}/A_{inlet} , was greater than that defined in Equation (2.19). The value being defined as the Kantrowitz limit for a fully enclosed duct.

$$\left(\frac{A_{exit}}{A_{inlet}} \right)_K = \left[\frac{\gamma - 1}{\gamma + 1} + \frac{2}{(\gamma + 1)M_1^2} \right]^{-1/2} \left[\frac{2\gamma}{\gamma + 1} - \frac{\gamma - 1}{(\gamma + 1)M_1^2} \right]^{-1/(\gamma - 1)} \quad (2.19)$$

Referring to Figure 2.14, below the Kantrowitz limit line any reduced area one dimensional

flow will continue in an attached and steady state but the swallowing of the shock or starting would not be spontaneous [16]. Below the isentropic relation limit an attached shock is not possible and a detached bow shock will form. Therefore an attached shock will exist for a contraction ratio above that of the isentropic relation limit. For a contraction ratio above that of the Kantrowitz limit the shock swallowing would be spontaneous [16].

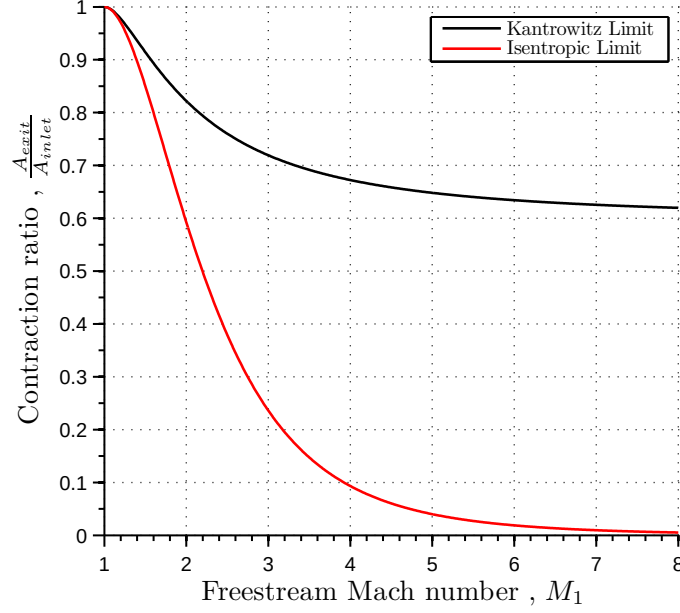


Figure 2.14: Kantrowitz and isentropic relation area contraction limits

2.6.2 Local Flow Detachment

A mechanism for shock detachment from curved planar or axisymmetric surfaces has been suggested by Mölder et al [17]. By using CST in order to calculate the flow gradients, an analytical model for the detachment of shock waves from sharp and curved surfaces has been derived. The mechanism behind the process of local choking and shock detachment occurs when a sonic region forms after the attached shock wave at the leading edge and before the trailing edge. The area reduction is then not able to accept the additional mass flow which enters the shock wave between the leading edge and the sonic region behind the shock [17]. Just like global flow detachment, mentioned previously in Section 2.6.1, the excess mass is spilled out from the inlet as the attached shock wave then detaches and forms a bow shock.

The approximate analytical model developed in [17] is used to predict the geometric conditions necessary for shock detachment by local choking during axisymmetric flow using a "Unit Ring Wedge" (URW). An annular ring wedge where the leading edge is at a radius

$y = 1$ and hence everything is normalised with regards to it. Similarly to how a nozzle cannot decelerate or accelerate a flow past sonic conditions past the minimum area, so is the method developed on the preposition that the flow cannot be contracted further once the sonic region is on the inside of the URW [17]. Figure 2.15 displays a wedge with an internal curvature, D_2 , initial deflection, δ , and surface distance to the sonic region, L^* , placed in supersonic flow.

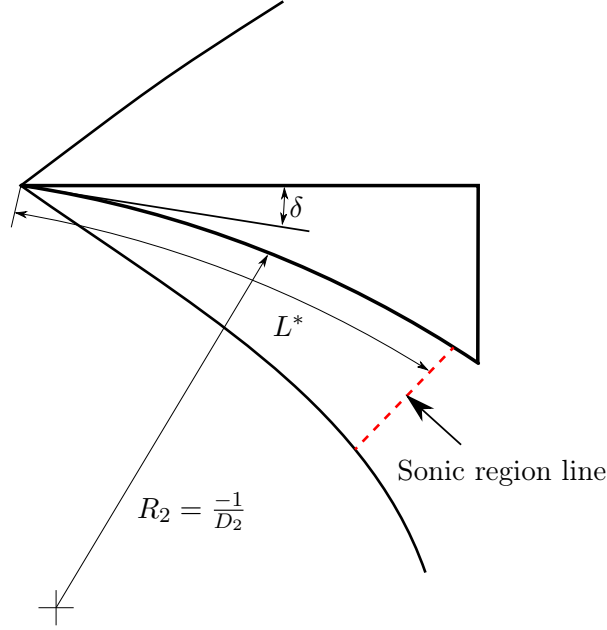


Figure 2.15: Wedge with internal surface curvature D_2 [17]

The flow is said to locally choke and detach from the wedge if the curved surface length of the wedge is greater than L^* [17]. The expression for the surface distance to the sonic region line, L^* , in an uniform and axisymmetric flow that is parallel to the axis of symmetry is given by Equation (2.20).

$$L^* = \frac{\frac{b_1}{a_1}(M_2^2 - 1)}{[2 + (\gamma - 1)M_2^2][D_2 - (b_1 \frac{a_2}{a_1} - b_2) \cos \theta]} \quad (2.20)$$

The post shock Mach number, M_2 , and shock angle, θ , is calculated just after the attached shock region at the leading edge. The constants a_1 , a_2 , b_1 and b_2 are comprised of the CST coefficients listed in Section 2.5 and can be calculated as follows:

$$a_1 = \frac{B_2 C' - B_2' C}{A_2 B_2' - A_2' B_2} \quad a_2 = \frac{B_2 G'}{A_2 B_2' - A_2' B_2}$$

$$b_1 = \frac{CA'_2 - C'A_2}{A_2B'_2 - A'_2B_2}$$

$$b_2 = \frac{-A_2G'}{A_2B'_2 - A'_2B_2}$$

2.7 Flow Visualisation

Gas dynamics related phenomena such as shock waves change the density of the compressible gas in which they are formed. As the density of the gas changes so does its refractive index thereby affecting the way light passes through it. Density-sensitive flow visualisation utilises these density changes in order to visualise the flow field. The flow itself, however, is not being observed but rather the disturbances within it that are manipulating the light [18]. The shadowgraph method only allows one to visualise variations in density gradients, equivalent to the second derivative of density, therefore allowing it to only detect discontinuous density zones such as shock waves or slip layers [18]. In order to visualise gradual density changes, equivalent to the first derivative of density, the schlieren method is used in order to detect regions such as expansion wave fans [18]. Figure 2.16 displays a typical two mirror monochromatic schlieren optical system where light passes perpendicularly through the region of interest developing the schlieren field. To convert a schlieren optical system to one of the shadowgraph variety, the camera side knife edge is removed. Monochromatic schlieren may be changed to colour schlieren by switching out the camera side knife edge for a colour filter.

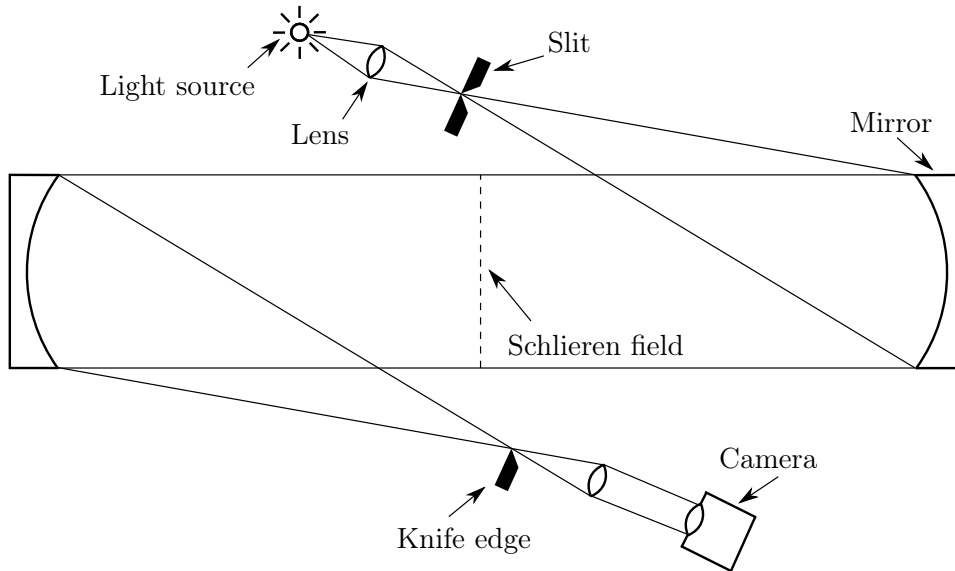


Figure 2.16: Typical two mirror monochromatic schlieren optical system (adapted from [19])

2.8 Supersonic Wind Tunnels

A supersonic wind tunnel is an apparatus that is used to generate and study steady supersonic flow phenomena. A typical blowdown supersonic wind tunnel schematic is displayed in [Figure 2.17](#). The layout is constructed with four main sections namely: a pressure reservoir, a nozzle section, a test section and finally a diffuser section. The pressure reservoir feeds the nozzle which is used to accelerate the working gas (air in most cases) to develop steady supersonic flow conditions in the test section. The gas is then decelerated in the diffuser section.

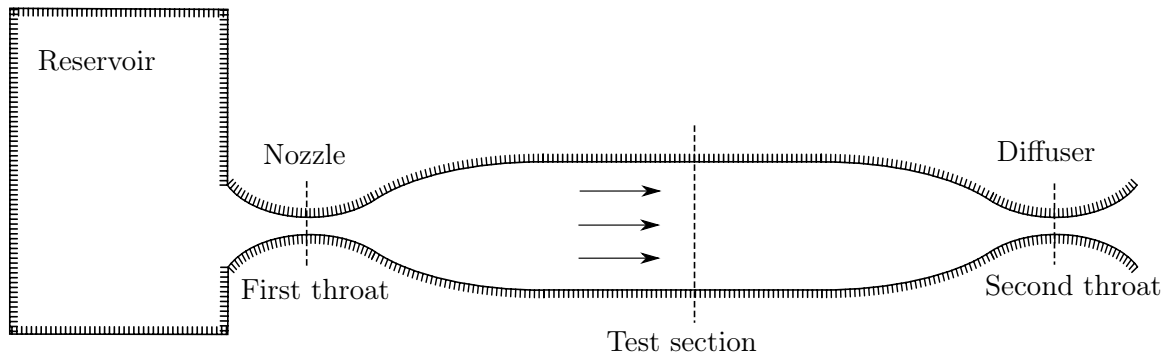


Figure 2.17: Typical blowdown supersonic wind tunnel schematic

In order to start the wind tunnel and achieve supersonic conditions in the test section certain flow requirements need to be met. Firstly the second throat in the diffuser is required to be larger than the first throat in the nozzle section. This is such that it can absorb the mass flow passing through the wind tunnel and ensure the second throat does not choke before the first. When gas is released by a valve into the nozzle, it is accelerated to sonic conditions at the first throat. As the feed pressure increases from the reservoir while the control valve opens, a shock will develop in the diverging section of the nozzle. With further increase the shock will move downstream until it sits at the nozzle exit. For the flow to adjust to boundary conditions, given the fixed throat areas and increasing pressures during the valve being opened, a shock in the constant area test section would not be possible. Hence with increasing feed pressure from the valve, the shock known as the starting shock, moves from the nozzle exit to the diverging section of the diffuser and then further upstream. This initiates steady supersonic flow in the test section where the wind tunnel starting shock ultimately remains in its weakest state at the second throat for optimum conditions. This occurs once the valve is fully opened and stationary thereby providing constant stagnation conditions during operation.

Chapter 3 | Objectives

Proceeding from the problem statement outlined in Section 1.2 the following research objectives were formulated and explored through experimental and numerical simulation based techniques:

- Determine the effects of internal surface curvature on steady axisymmetric shock waves generated by axisymmetric ring wedges.
 - The internal surface curvature of ring wedges will be varied along with varying flow Mach number for each ring wedge.
- Determine the effects of varying ring wedge inlet entry angles with the varying flow Mach numbers and curvatures on the resulting shock wave profiles and properties.
 - The inlet entry angles will be varied for the selected internal curvatures and tested under identical flow Mach numbers in the previous objective.
- Compare experimental results with those obtained in computational fluid dynamics simulations in order to validate the numerical analysis.

Chapter 4 | Experimental Method

4.1 Introduction

In order to validate the numerical analysis, experimental tests were conducted on manufactured ring wedges. Experimental work was completed with the use of the supersonic wind tunnel facility within the North-West Engineering Barloworld Laboratory located at the University of the Witwatersrand, Johannesburg. Shadowgraph and schlieren flow visualisation was used to capture features in the flow field and provide qualitative flow data. The details about the experimental facility, optical flow visualisation setup and procedure is contained in the subsequent sections herein.

4.2 Experimental Facilities

4.2.1 Supersonic Wind Tunnel Facility

The blowdown supersonic wind tunnel (SSWT) facility at the University of the Witwatersrand allowed for the completion of the experimental work that was used to validate the numerical analysis. The design and fabrication of the SSWT was carried out by Aerolab and a schematic layout of the facility is shown in Figure 4.1 with component labels listed in Table 4.1. The test section measures $4'' \times 4''$ ($101.6 \text{ mm} \times 101.6 \text{ mm}$) with achievable flow speeds of Mach 2 to 3.6. The pressure storage vessel which feeds the SSWT was manufactured by Kies & Travers Ltd with a volume of 9.04 m^3 and nominal operating pressure of 1.5 MPa. This gives the SSWT an approximate steady flow blow time of 14 seconds. The air is supplied to the pressure storage vessel by an electronically powered model CSA 4 18.7 kW reciprocating compressor manufactured by Reavell & Co. A Serex regenerating dryer and Tegnon heater make up the air dryer unit which is used to reduce the moisture in the air being pumped into

the storage vessel. This eliminates the formation of ice crystals in the SSWT during operation due to the extremely low flow expansion temperatures leaving the supersonic nozzle.

During operation the pneumatically controlled pressure supply control valve opens, allowing the passage of air to the settling chamber. The pressure supply control valve then controls the air flow to maintain constant stagnation conditions in the settling chamber. The settling chamber itself serves as a flow stabiliser in order to bring the flow to rest and reduce turbulence before entry into the supersonic nozzle. The flow is then accelerated through the supersonic nozzle and sent into the constant area test section for observation. The sliding section is used to close up the SSWT during operation with the fixed area second throat sitting within it. Finally a silencer and diffuser are used to decelerate the flow before sending it to vent to atmosphere via the laboratory roof.

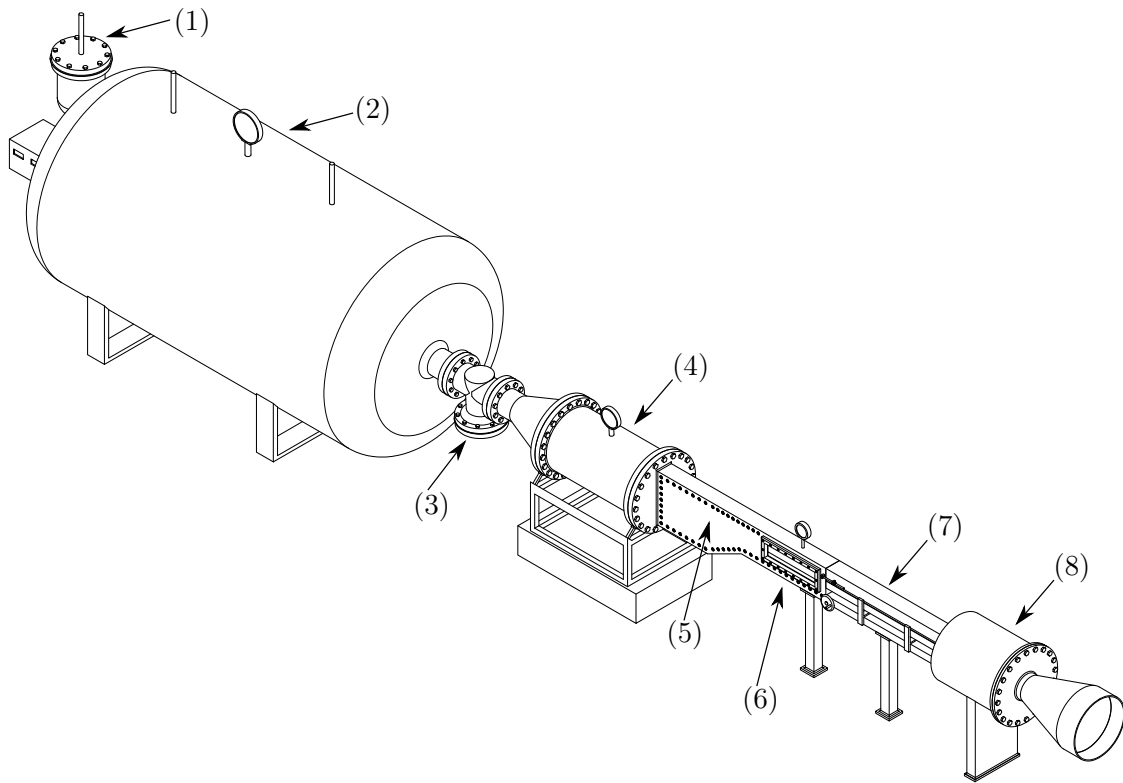


Figure 4.1: Supersonic wind tunnel facility schematic layout

Table 4.1: Supersonic wind tunnel facility component labels

Label Number	Component	Label Number	Component
(1)	Air Dryer Unit	(5)	Supersonic Nozzle Section
(2)	Pressure Storage Vessel	(6)	Test Section
(3)	Pressure Supply Control Valve	(7)	Sliding Section
(4)	Settling Chamber	(8)	Silencer and Diffuser Unit

An adjustable asymmetric supersonic nozzle, (Figure 4.2), is used to vary the Mach number in the test section. The first throat area is altered by moving a lower sliding block while the upper block remains fixed thereby changing the exit Mach number. This is completed by rotating an adjustment handle which changes the displayed value on the veeder counter. The veeder counter is used to track the position of the adjustable lower sliding block.

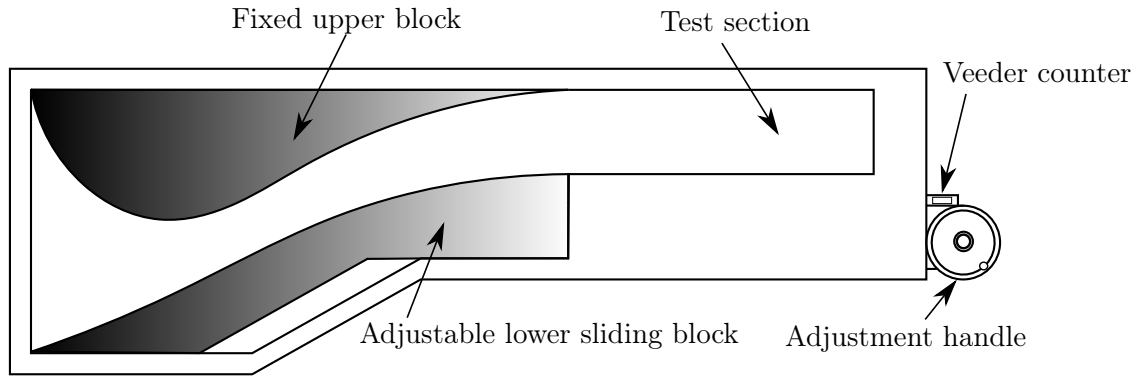


Figure 4.2: Supersonic nozzle assembly schematic

The veeder counter value is calibrated to give a specific test section Mach number. Optical techniques were used in order to measure the Mach number within the test section. A small strip of packaging tape was placed on the floor of the test section which generated a Mach wave. The angle between the Mach wave and a horizontal datum was measured and the test section Mach number was then calculated using Equation (2.4). The calibration curve in Figure 4.3 was produced using the above mentioned method.

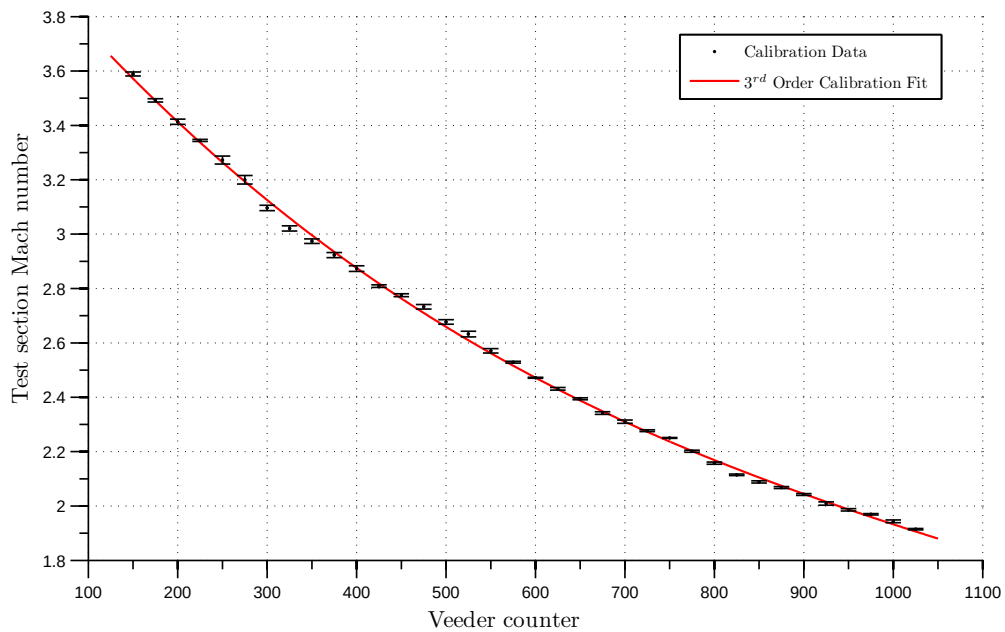


Figure 4.3: Veeder counter calibration curve

The calibration curve fit that best described the trend of the calibration data was of the 3rd order. Its function is shown in Equation (4.1) where MN denotes Mach number and VC the associated veeder counter value;

$$MN = -6.992 \times 10^{-10}VC^3 + 2.514 \times 10^{-6}VC^2 - 0.004VC + 4.118 \quad (4.1)$$

The veeder counter values listed in Table 4.2 were used to achieve the experimental test section Mach numbers.

Table 4.2: Experimental veeder counter values and Mach numbers

Veeder Counter	Mach Number
141	3.6
206	3.4
239	3.3
273	3.2
309	3.1
348	3.0
388	2.9
432	2.8

4.3 Experimental Test Pieces

A new test rig and experimental test pieces were designed in order to carry out the experimental component of the research. The ring wedge entry angles and internal constant radii of curvature were the experimental variables that were altered. All the dimensions of the ring wedge have been normalised with respect to the leading edge radius, R_{le} , and are a ratio thereof. Table 4.3 contains the specifications of the manufactured and tested ring wedge models with accompanying schematic, Figure 4.4, where R_c denotes the normalised internal radius of curvature, L_w the normalised ring wedge length and α the wedge entry angle.

The leading edge radius was set to $R_{le}=20$ mm and the wedge length to 10 mm ($\therefore L_w = \frac{10}{R_{le}}=0.5$) after initial parameter studies were undertaken. Thereby keeping the maximum cross-sectional area of the model less than 9% of the second throat. This conservative design constraint ensures supersonic flow through the test section and compensates for boundary layer effects in the flow field. The boundary layer efficiently increases the maximum cross-sectional area in the test section thereby increasing the chance of wind tunnel choking. The constrained dimensions provided a reasonably sized and resolved flow field to be captured for visualisation purposes given the limiting test section size and maximum area.

Table 4.3: Experimental model specifications

Model Number	α [°]	R_c	L_w
(1)	0	1	0.5
(2)	0	1.5	0.5
(3)	0	2	0.5
(4)	4	1	0.5
(5)	4	1.5	0.5
(6)	4	2	0.5
(7)	4	2	0.75
(8)	8	1.5	0.5
(9)	8	2	0.5
(10)	18.1	∞	0.5

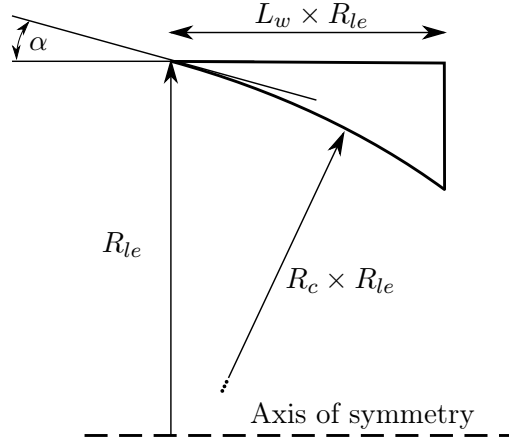


Figure 4.4: Experimental parameter schematic

Regarding additional test cases, models (7) and (10) were included for comparative purposes. Model (7) was designed with an increased wedge length to obtain insight on the length's influence. In order to compare the difference between a straight and a curved internal surface, model (10) was designed with a conical interior and the same exit area as model (8). This joins the leading and trailing edge of model (8) with a straight surface rather than a curved wall with $R_c=1.5$.

The brass ring wedge models were attached to two high tensile silver steel rods which secured them to the test rig. This provided a rigid support while keeping the flow field unobscured thus allowing for the shock structure and features behind the ring wedge to be captured. The test rig was then bolted in place and secured to the test section. Figure 4.5 displays the final designed SSWT model assembly. Manufacturing drawing used to produce all the ring wedge models and test rig components can be found in Appendix A.

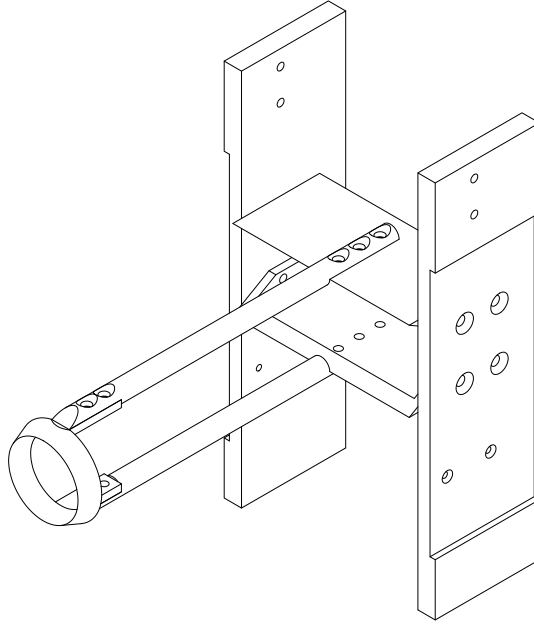


Figure 4.5: SSWT test model assembly

4.4 Experimental Procedures and Precautions

4.4.1 Air Storage Vessel Pressurising Procedure

To be able to operate the SSWT and achieve supersonic speeds within the test section it is required that the pressure storage vessel pressure be between the range of 0.8 - 1.4 MPa. Without this available pressure the SSWT is unlikely to start and achieve the required Mach numbers. Therefore before each testing session the pressure storage vessel needs to be pressurised as follows:

1. Ensure the air dryer unit is switched off.
2. Close the air dryer bleed valves, starting off with the receiver tank valve then the heater valve.
3. Switch on the solenoid for the pressure vent system.
4. Switch on the water sump supply pump.
5. Open the cooling water supply valve.
6. Switch on the air compressor.

Once the pressure storage vessel has reached a pressure of 1.4 MPa the compressor will automatically shut down to fall within safe operating limits. In order to be able to prepare and run the SSWT, the pressurising system must be shut down as follows:

1. Switch off the air compressor.
2. Close the cooling water supply valve.
3. Switch off the water sump supply pump.
4. Switch off the solenoid for the pressure vent system.
5. The air dryer unit receiver tank pressure must be bled. This is completed by slowly opening the receiver tank valve until its gauge reads 0 lb/in². Once this has been completed the heater valve may be opened.

4.4.2 Pre-test Procedure

1. Switch on the SSWT controller PC
2. Switch on the power supply to the warning siren, data acquisition unit, light source trigger box and the light source.
3. Pressurise the storage vessel to 1.4 MPa as mentioned previously.

4.4.3 SSWT Testing Procedure

Once the pre-test and pressuring procedures have been completed, the SSWT can be setup for experimental testing. The operating procedure is outlined below:

1. Open SSWT.exe application on the controller PC
2. Complete the required testing details fields in the SSWT.exe application.
3. Ensure the model assembly is securely fastened and place it in the SSWT test section. This must be done carefully as not to scratch the test section glass windows or damage the leading edge of the test piece.
4. Once the model assembly is inside the test section it must be bolted to the upper member of the SSWT. The sliding section can then be shut and secured in place using the threaded rod and nuts attached to the SSWT.

5. Set the required value on the veeder counter using the rotating adjustment handle. This value is dependant on the required Mach number which can be obtained from Figure 4.3 or the SSWT.exe application.
6. Switch on the wind tunnel control box and open the stagnation pressure supply valve until the gauge reads 250 *kPa*.
7. Complete the pre-test checklist within the SSWT.exe application to allow the SSWT to be started and fill out the testing details in the SSWT logbook.
8. Arm the light source trigger box to allow for the optical system to capture an image of the flow field and switch on the camera.
9. Switch off the lights in the SSWT control room in order for the optical system to capture an image. If it is the first time running a test with a new model in the test section, the first run must be completed with the lights on. No image may be captured under these conditions. This is too ensure the model does not move excessively and is securely fastened.
10. Sound the warning siren for 6 seconds.
11. Wait 6 seconds before starting the SSWT to allow for people around the laboratory to prepare hearing protection.
12. Once the 6 seconds is over the SSWT can be started by clicking the "START" button in the SSWT.exe application.
13. While the test is running the optical system may be triggered to capture an image of the flow field. The optimum time for this occurs when the pressure tracer plot for the test section displays a flat constant pressure line. This indicates steady state conditions in the test section.
14. After the set test time is passed the SSWT will automatically close the pressure supply to the stagnation chamber. Final test details and pressure traces can then be saved in the SSWT.exe and recorded in the SSWT logbook.
15. The SSWT can then be prepared for another run my clicking the "NEXT TEST" button in the SSWT.exe application.
16. Steps 2 through 15 can be repeated for each successive test.

4.4.4 SSWT Shut Down

Once the storage vessel's pressure is less than 0.8 MPa it is required to be repressurised. This is completed as follows:

1. Reduce the stagnation pressure at the wind tunnel control box to 0 kPa and switch it off.
2. The power to the light source and camera may be switched off.
3. Pressurise the storage vessel as outlined in Section [4.4.1](#).

If the day's testing has been completed then the SSWT can be prepared for the next day's session as follows:

1. Reduce the stagnation pressure at the wind tunnel control box to 0 kPa and switch it off.
2. The end of day check list must be completed in the SSWT.exe application after which the controller PC can be shut down and switched off along with the data acquisition system.
3. All optical equipment such as the camera and mirrors must be covered to prevent dust collection.
4. Switch off the light source and camera. The camera's memory card may now be removed in order to retrieve the flow fields images and increase memory space for the next testing session.
5. Open the sliding section of the SSWT.
6. Open both air dryer unit bleed valves.
7. Switch on the air dryer unit.

4.4.5 Precautions

The following precautions must be taken while operating and setting up the SSWT:

- Hearing protection must be worn at all times during the SSWT blow down and while bleeding the air dryer unit to prevent hearing damage.
- The test section must be clear of all debris before being shut closed for testing.
- The optical system must not be moved during testing.
- Care must be taken not to scratch the optical mirrors and test section windows. Any damage or anomalies may be captured in the optical system and could provide nucleation sites for crack propagation.

- Ensure the compressor is switched off before testing as it will automatically begin to pressurise once the storage vessel falls below 1.4 MPa during operation.
- The use of the warning siren is vital as the SSWT vents past populated sections of the laboratory.
- The SSWT sliding test section must be shut tightly and rechecked after every test to ensure operating vibrations do not loosen the nuts.

4.5 Flow Visualisation

To validate the numerical method, experimental flow field images were captured through shadowgraph and schlieren optical techniques. Due to the axisymmetric nature of the ring wedges the flow field within the wedge could not be visualised therefore images were captured behind the models. This ensured the resulting shock structure features were made present for comparison purposes. The optical system used in the SSWT control room was designed to hang from the ceiling with a pre-aligned gantry system. This was to eliminate the effect of the large vibrations passing through the grounded foundations of the SSWT during operation while compensating for space constraints.

The optical system used was of the u-configuration type and is displayed in Figure 4.6. The original optical system's continuous halogen light source was replaced with a manually triggered Xenon flash lamp. This was done in order to reduce the light exposure time while capturing the flow field in order to produce more detailed and refined images. The light source was triggered with a constantly open camera shutter rather than the previously used system where the camera shutter was triggered with a continuous light source. This decreased the exposure time to $1.5 \mu\text{s}$. A converging condensing lens was used to focus the light source onto a double knife-edge cut-off slit. The cut-off slit was placed at the focal length (1200 mm) of the first parabolic mirror in order to simulate a light point source. The light was then reflected to the first planar mirror which passed it through the test section normal to the direction of the incoming flow. After capturing the details of the flow field a second planar mirror reflected the light onto the second parabolic mirror. The incoming light from the parabolic mirror's direction was changed using a 45° planar mirror in order to feed it into the axially aligned camera rail system. The second focal point provided the location for the knife edge or colour mask. Finally the light passed through a focusing converging lens before being captured by a 8-Megapixel Canon EOS 350D camera using ISO settings of ISO800 and ISO1600.

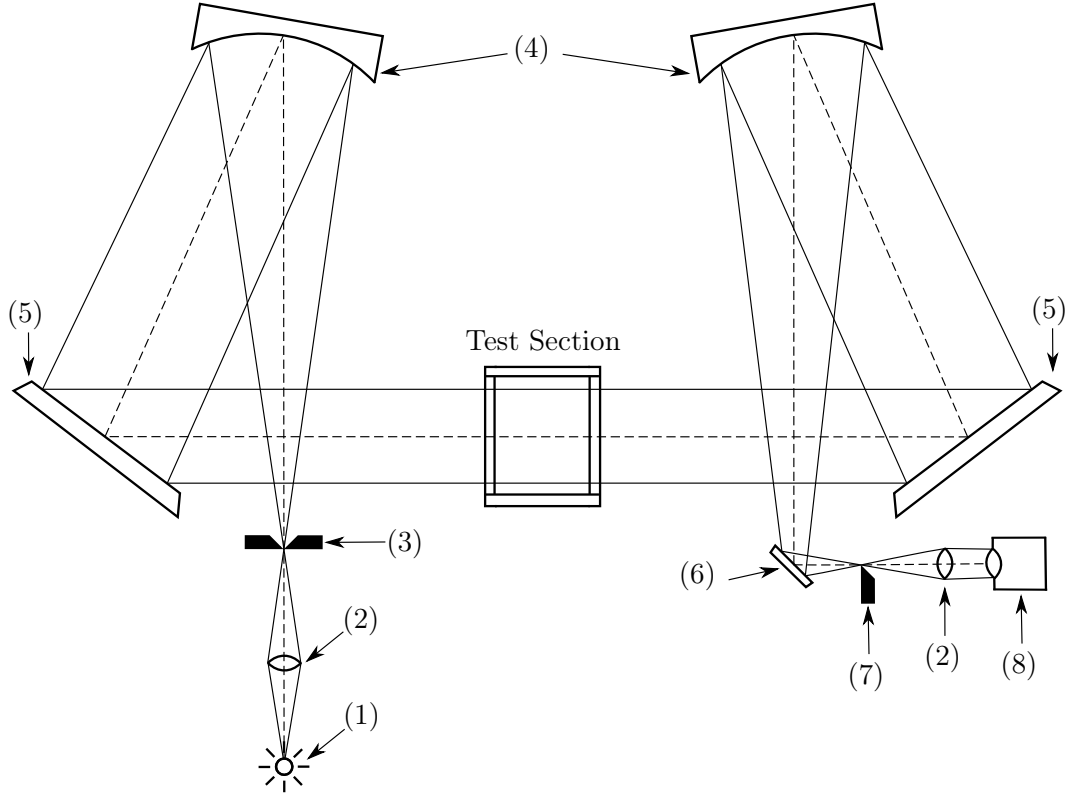


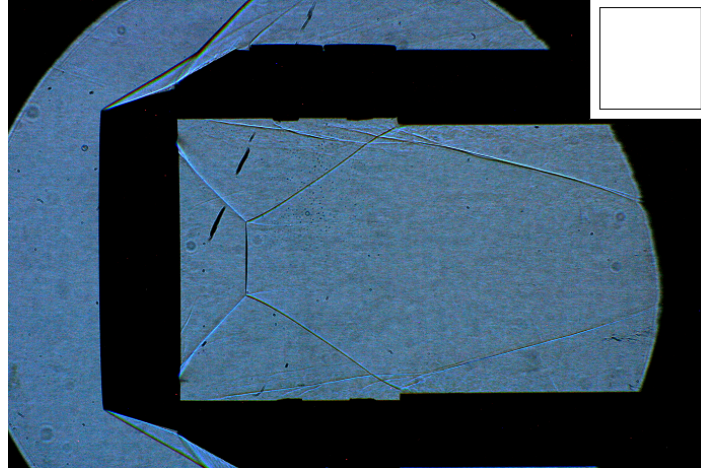
Figure 4.6: Flow visualisation optical setup schematic

Table 4.4: Optical system component labels

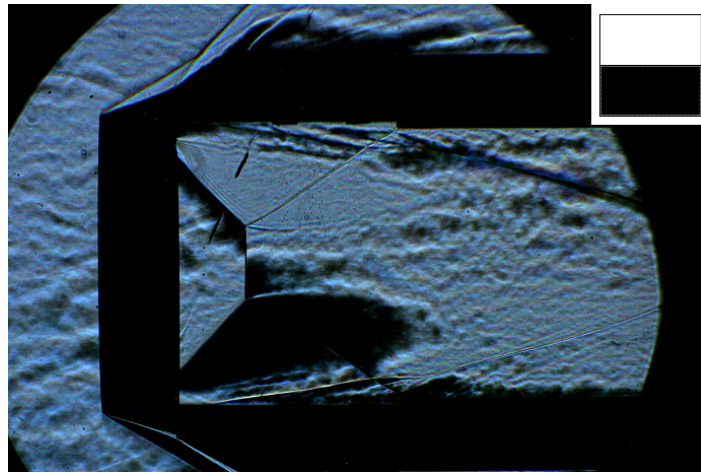
Label Number	Component	Label Number	Component
(1)	Xenon Light Source	(5)	Planar Mirror
(2)	Converging Lens	(6)	45° Planar Mirror
(3)	Cut-off Slit	(7)	Knife Edge/Colour Mask
(4)	Parabolic Mirror	(8)	Camera

Both shadowgraph and schlieren optical techniques were explored. A horizontal knife edge and a band lattice colour mask was used for schlieren imaging. This provided insight on the density gradients present within the flow field. The shadowgraph and two schlieren optical techniques are shown in Figure 4.7.

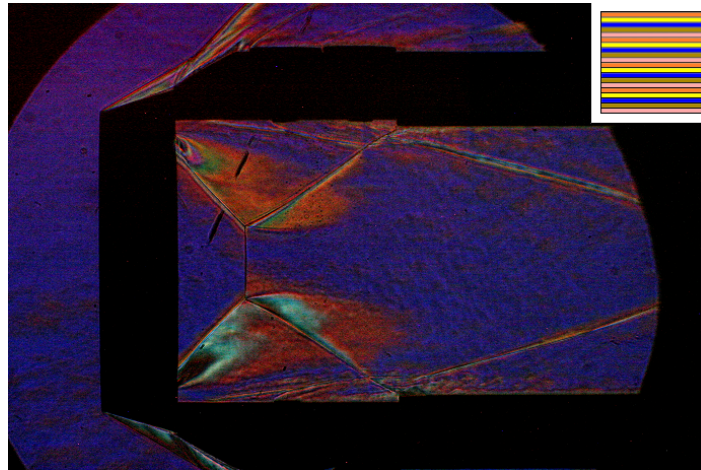
Regarding the impact of the upstream flow quality in Figure 4.7b, density gradients are seen ahead of the leading edge. These gradients, however, are caused by the turbulence being captured in the boundary layer on the SSWT test section windows. The sensitivity of the schlieren setup that was used was quite strong as shown by the extremely dark region of the lower reflection. This increased sensitivity then captured the turbulence which would not have been seen in a flow visualisation schlieren setup with lower sensitivity.



(a) Shadowgraph with no mask used



(b) Horizontal knife edge monochromatic schlieren



(c) Horizontal banded lattice colour mask schlieren

Figure 4.7: Flow visualisation techniques using the $R_c=1.5$, $\alpha=8^\circ$ model at Mach 3.4

Chapter 5 | Numerical Method

5.1 Introduction

Due to the physical axisymmetric nature of the ring wedge model it is not possible to visualise what is occurring within it. Only the flow field behind the ring wedge can be captured as the internal flow field is hidden from view. By using numerical methods one can analyse the shock structure and flow properties as a direct result of the internal surface curvature. The numerical method results were obtained using ANSYS Fluent 15, a commercially available Computational Fluid Dynamics (CFD) package. The simulation results were then post-processed using Tecplot 360 EX 2014 Release 1 and numerical code scripts were developed with the use MATLAB R2009b. All the computational processing was completed within the Flow Research Unit. Details on the employed numerical method are contained herein.

5.2 Computational Fluid Dynamics

5.2.1 Domain Specification

The computational domain was modelled to physical experimental scale using Autodesk Inventor 2015. Two different domain variations were originally tested namely a full domain and a partial domain model as seen in Figure 5.1. This was done to try improve the time efficiency of the simulation while still maintaining numerically identical results. After comparing the shape of the resulting shock structure it was found that their position and form did not change. Therefore the partial model was used as it was more computationally efficient. The simulation of the wake in the full domain model was found to cause solution convergence issues which too prompted the domain change. Differences in the density contour lines in Figure 5.1 are due to the different upper and lower density bounds caused by the presence of

the wake behind the ring wedge.

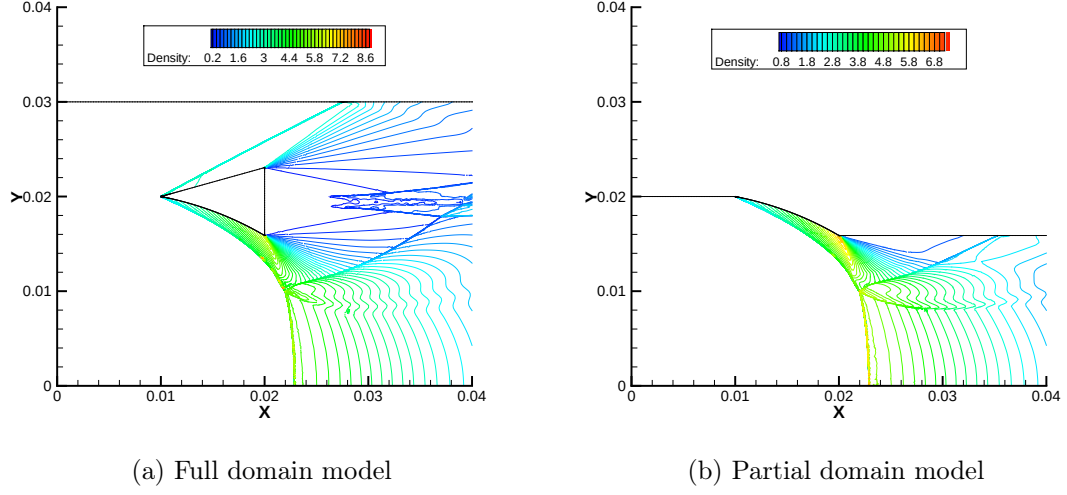


Figure 5.1: Initial domain variation

The final two dimensional computational domain was based on the previously mentioned partial domain model and is displayed in Figure 5.2. The computational advantage of an "axis" boundary condition was used in order to eliminate the use of three dimensional axisymmetric domains while producing identical results. The upper side boundaries were set to "pressure far field" in order to stop the formation of shock internal reflections emanating from the resulting shock structure thereby matching the experimental conditions. Only the internal surface of the ring wedge model is classified as a "wall" boundary condition.

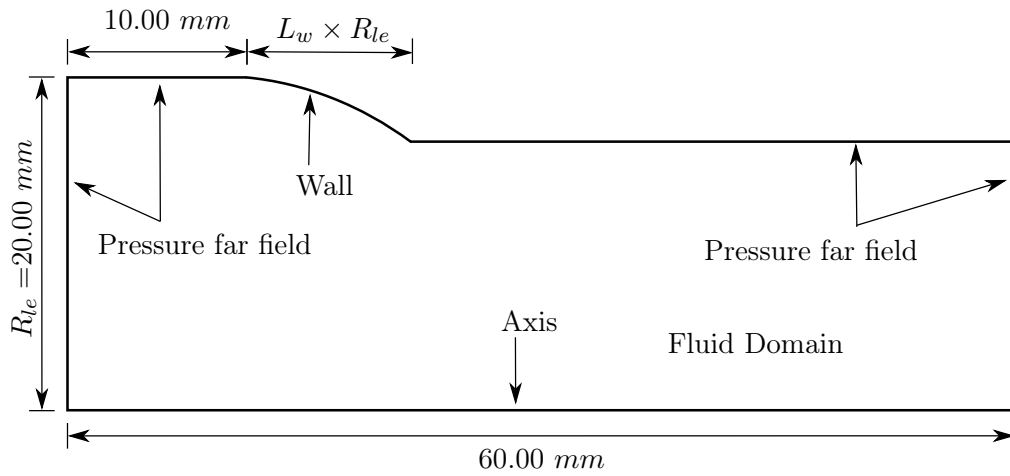


Figure 5.2: Final computational domain and boundary conditions

5.2.2 Mesh Specification

The computational mesh was generated using ANSYS Workbench 15.0 after the domain was imported into the application. Two dimensional facing size methods were used to establish a uniformly sized mesh grid. Adaptive mesh refinement methods were used in order to refine the shock wave shape structure and accurately position them within the flow field. The influence of adaptive mesh refinement can be see in Figure 5.3 at the location of a continuously curved shock wave segment. Both the shock wave shape and presence of the trailing edge expansion wave fan are refined and captured.

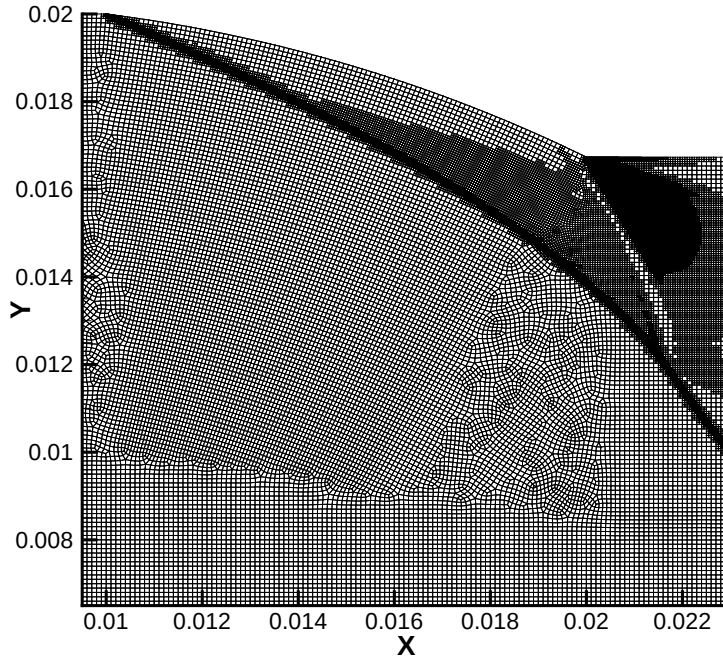


Figure 5.3: Refined mesh region of a continuously curved shock wave

The adaptive mesh refinement flow property parameters were set to temperature, pressure, velocity magnitude and density. The refinement and coarsen thresholds were set to 0.015 and 0.0075 respectively after testing various values in order to refine the shock wave structure. Six levels of refinement were completed at 5000 iteration increments using normalised space gradient methods.

To find the influence of initial grid sizing three quadrilateral based mesh cell sizes were tested namely 0.4 mm, 0.2 mm and 0.1 mm. For each curvature set ($R_c=1, 1.5$ and 2) the model with the smallest exit area was tested at highest testing speed of Mach 3.6. These selected tests where chosen as they were expected to experience the highest flow energy changes due to

their increased induced flow turning. The previously mentioned adaptive mesh refinement was included in the grid sensitivity study as the simulations were set to use it irrespective of the final mesh sizing. The pressure along a horizontal line through the domain at a location of 10 mm from the axis of symmetry was measured to capture the effect of both the incident and reflected shock waves. The results for the mesh grid sensitivity study are displayed in Figure 5.4.

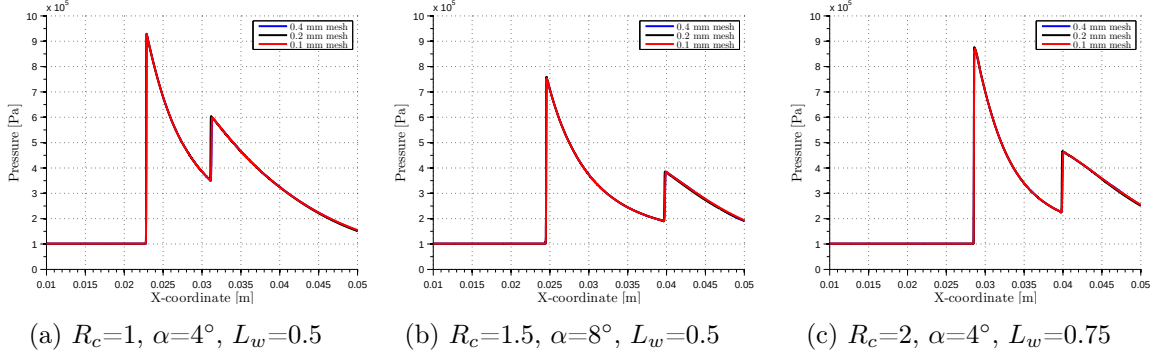


Figure 5.4: Selected mesh grid sensitivity results at Mach 3.6

The data for each selected scenario's different mesh sizing lies on the same path. The reason for this is due to the included adaptive mesh refinement. For sake of computational efficiency the 0.4 mm mesh should have been chosen but the simulations were completed with the 0.1 mm mesh instead. The reason for this was the refinement of the shock wave thickness for image processing and locating purposes. The smaller mesh size would result in a more accurate base solution from which the simulation would adapt, reducing the chance of solution instability and divergence. Solution instability was an issue that was experienced in preliminary simulations as a result of the high flow energy changes due to the curvatures involved. Starting off with an initial mesh of 0.1 mm and after all six rounds of mesh adaptation were completed, a final mesh size of $1.5625 \mu\text{m}$ was present at locations of large flow property gradients.

5.2.3 Solver Settings

The solver setup and solution settings used in the CFD simulations are listed as follows:

5.2.3.1 Solution Setup

- Solver type: Density based
- Velocity formulation: Absolute
- Time: Steady

- 2D Space: Axisymmetric
- Energy model: On
- Viscous model: Inviscid
- Fluid: Air, density formulation set to ideal gas
- Operating conditions: Operating pressure = 0 Pa
- Pressure far field boundary conditions:
 - Mach number = Desired flow speed
 - Gauge pressure = 101325 Pa,
 - Temperature = 300 K

5.2.3.2 Solution Settings

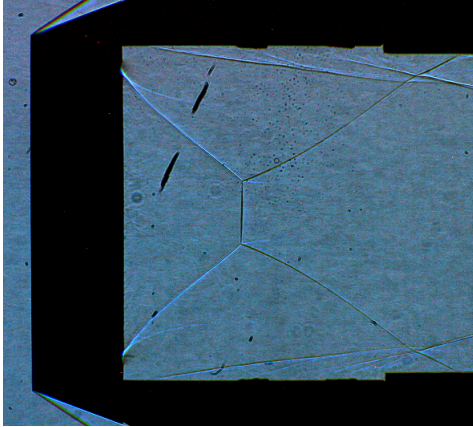
- Solution formulation: Explicit
- Flux type: Roe-FDS
- Spatial discretisation:
 - Gradient: Least squares cell based
 - Flow: Second order upwind
- Courant number: 1
- Residual convergence monitors:
 - Continuity absolute criteria = 1×10^{-4}
 - X-velocity absolute criteria = 1×10^{-4}
 - Y-velocity absolute criteria = 1×10^{-4}
 - Energy absolute criteria = 1×10^{-6}
- Solution initialisation method: Standard
- Solution reference frame: Absolute
- Initialisation computed from: Pressure far field
- Dynamic adaptive mesh refinement:
 - Levels of refinement = 6
 - Refinement interval = 5000 iterations
- Number of iterations: 34999

Chapter 6 | Results and Discussion

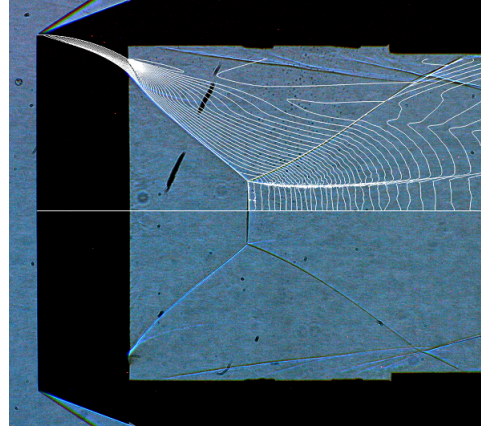
6.1 Numerical Analysis Validation

6.1.1 Comparison of Experimental and Numerical Results

In order to validate the numerical method comparison needs to be drawn between the experimental and numerical simulation based results. This was done by qualitative means, assessing the correlation between the shock structures produced in various cases. Figure 6.1 displays experimental and numerical results for the $R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ model at Mach 3.6. Figure 6.2 displays experimental and numerical results for the $R_c=1.5$, $\alpha=4^\circ$, $L_w=0.5$ model at Mach 3.2. Figure 6.3 displays experimental and numerical results for the $R_c=2$, $\alpha=8^\circ$, $L_w=0.5$ model at Mach 2.8.

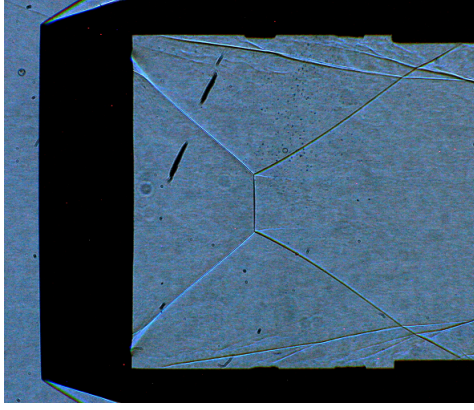


(a) Experimental shadowgraph results

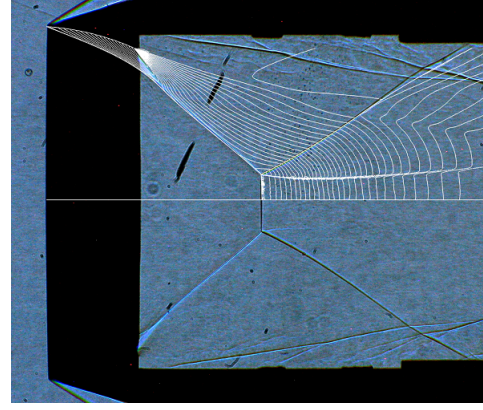


(b) Experimental with overlaid CFD results

Figure 6.1: $R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.6

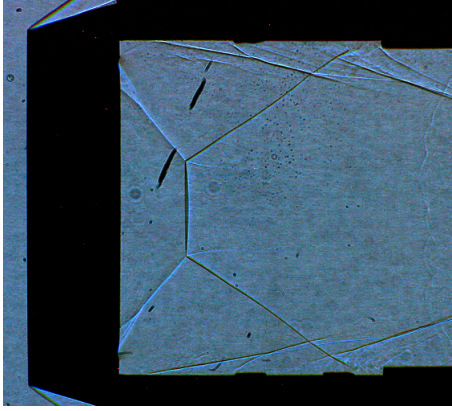


(a) Experimental shadowgraph results

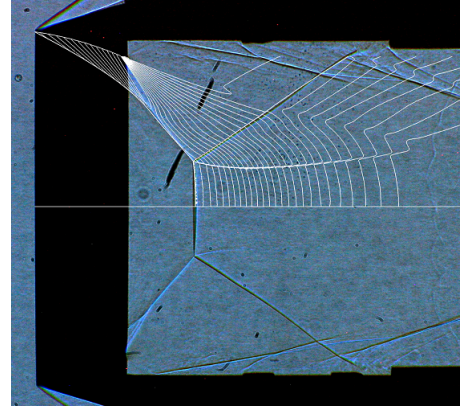


(b) Experimental with overlaid CFD results

Figure 6.2: $R_c=1.5$, $\alpha=4^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.2



(a) Experimental shadowgraph results



(b) Experimental with overlaid CFD results

Figure 6.3: $R_c=2$, $\alpha=8^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 2.8

The experimental and numerical results are in good agreement with one another. The differences that are more apparent are in the region of the reflected shock wave that originates from the Mach reflection triple point. As the reflected shock stretches from triple point it is altered by the trailing edge expansion wave fan and shock wave interactions emanating from the test rig. Said flow features divert and curve the reflected shock wave very slightly. Seeing as the image plane is in the same plane as the ring wedge model rod supports, their flow interactions and subsequent effects on the reflected shock wave are captured.

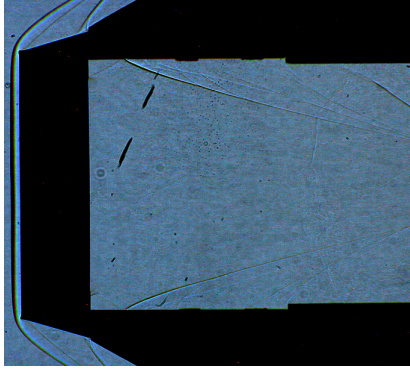
Regarding the thickness of the Mach stem in the experimental results, this is due to its strength and three dimensional nature. Seeing as a Mach stem is a normal shock, the flow property gradients across it change more harshly in comparison to an oblique shock, causing it to appear darker. The three dimensional nature of the Mach stem accompanied by slight concavity causes the shadowgraph technique to produce a thicker, more defined shock wave. The numerical results do not exhibit this three dimensional optical feature as they are just

a two dimensional slice of the flow field. Although the flow properties in both cases change through a large degree across the Mach stem, the numerical results produce a thinner shock wave due to the concentration of the mesh in its location.

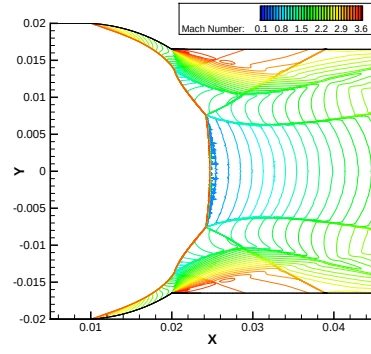
Finally, there are very minor differences between the positions of the incident shock wave in the experimental and numerical results. This can be due to the presence of a boundary layer in the experimental results. The numerical simulations were completed with an inviscid model thereby eliminating the effect a viscous boundary layer as it is non-existent. The experimental results bare the effect of the working fluid's viscosity thereby meaning that a boundary layer would be present on the internal surface of the ring wedge models. When taking into consideration the short wedge lengths involved in the experimental tests, the boundary layer does not develop to a influential size. Therefore its existence does not cause a detrimental impact on the use of an inviscid model for the numerical simulations.

Validation of the numerical method is crucial to visualising and categorising the internal flow patterns of the ring wedge models. The axisymmetric physical nature of the experimental test models hide the flow field from view therefore the direct impact of the internal surface curvature can only be visualised through numerical means. It can be concluded using the above mentioned reasoning and experimental evidence that the experimental method sufficiently validates the numerical method.

Some issues were however experienced with the lower exit area models. Four of the ten experimentally tested models, namely models (4), (7), (8) and (10), experienced detached shock regimes when according to the numerical simulations attached curved shock waves should have been present. Figure 6.4 displays experimental and numerical results for the $R_c=1$, $\alpha=4^\circ$, $L_w=0.5$ model at Mach 3.4. Figure 6.5 displays experimental and numerical results for the $R_c=1.5$, $\alpha=8^\circ$, $L_w=0.5$ model at Mach 3.3. Figure 6.6 displays experimental and numerical results for the $R_c=\infty$, $\alpha=18.1^\circ$, $L_w=0.5$ model at Mach 3.3. Figure 6.7 displays experimental and numerical results for the $R_c=2$, $\alpha=4^\circ$, $L_w=0.75$ model at Mach 3.6. The selected Mach numbers that were chosen for display were based on highest test Mach number at which a detached shock wave was experimentally present. In all four numerical cases it can be seen that the Mach reflection is quite developed and recedes downstream. This is contrary to the experimental results which depict a bow shock at the leading edge, characteristic of a detached shock flow regime as described in Section 2.6.1. The mechanism for the detachment process and the cause has yet to be determined, although a suggestion is presented later

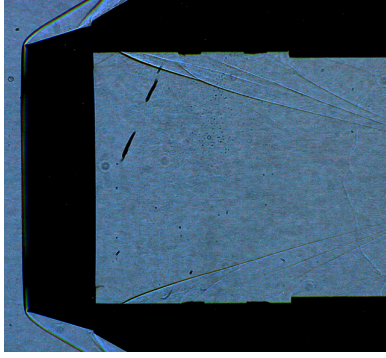


(a) Experimental shadowgraph results

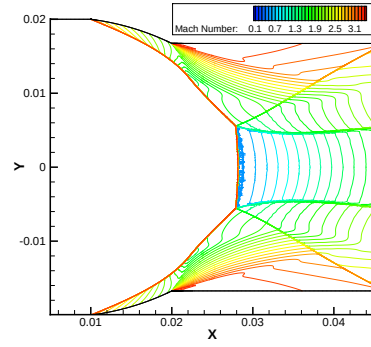


(b) Numerical Mach number contour lines

Figure 6.4: $R_c=1$, $\alpha=4^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.4

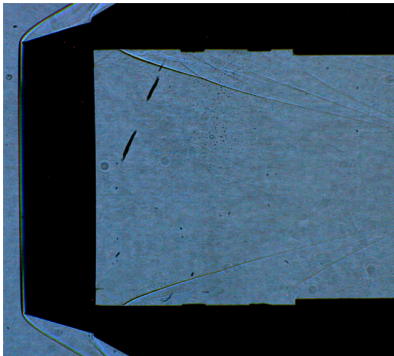


(a) Experimental shadowgraph results

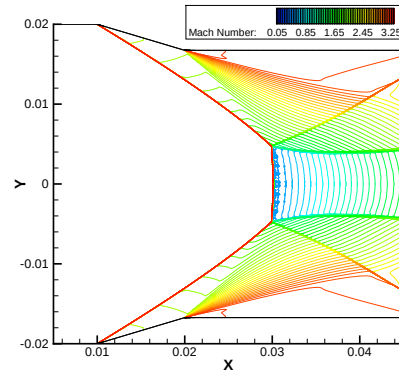


(b) Numerical Mach number contour lines

Figure 6.5: $R_c=1.5$, $\alpha=8^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.3

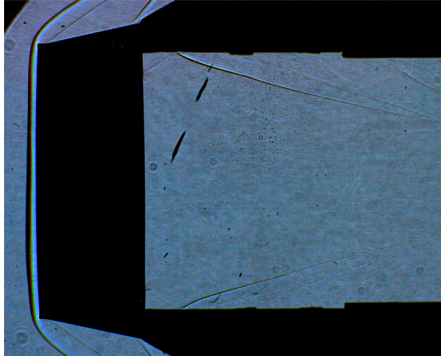


(a) Experimental shadowgraph results

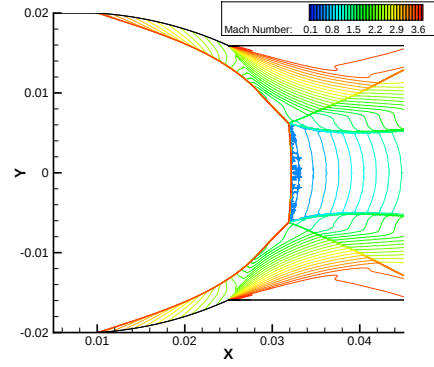


(b) Numerical Mach number contour lines

Figure 6.6: $R_c=\infty$, $\alpha=18.1^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.3



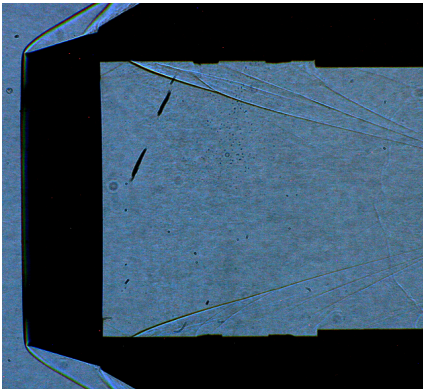
(a) Experimental shadowgraph results



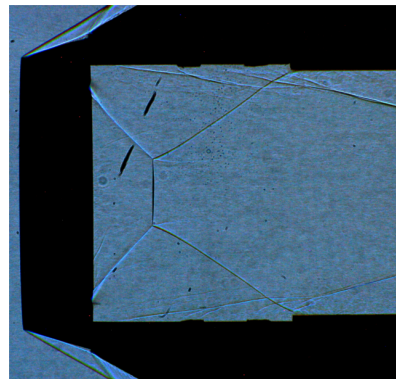
(b) Numerical Mach number contour lines

Figure 6.7: $R_c=2$, $\alpha=4^\circ$, $L_w=0.75$ model experimental and numerical comparison at Mach 3.6

Figure 6.8 displays the $R_c=1.5$, $\alpha=8^\circ$, $L_w=0.5$ with a detached shock wave at Mach 3.3 and an attached shock wave at Mach 3.4. Even though the two states are a mere Mach 0.1 apart, their flow regimes are very different. Referring to Figure 6.8b, the Mach reflection has formed almost an entire wedge length downstream of the trailing edge. This means the flow should be far from the possibility of detachment due to the incident shock wave reflecting off the internal surface. The Mach disk movement during a speed decrease of Mach 0.1 should not be significant enough to warrant it as the cause. Considering the case of local choking flow detachment it can be seen from Figure 6.5b that the only subsonic flows are behind the Mach disk in the axisymmetric stream tube bounded by the slip layer. Since a sonic region is not present on the curved internal surface of the ring wedge, the source of detachment cannot be initiated by local choking mechanisms.



(a) Mach 3.3



(b) Mach 3.4

Figure 6.8: $R_c=1.5$, $\alpha=8^\circ$, $L_w=0.5$ experimental shadowgraphs with detached and attached shock waves

The suspected issue was the start up process of the SSWT and the flow initiated by the starting shock wave moving from the first to the second throat as detailed in Section 2.8. In order to investigate this issue a Photron FASTCAM SA5 high speed camera was used to visualised the movement of the starting shock. The camera was operated with a frame rate of 5000 frames per second and a shutter speed of 1/61000 seconds. Model (8), ($R_c=1.5$, $\alpha=8^\circ$, $L_w=0.5$), was used as it had a the largest exit area ($A_{exit}=879.52 \text{ mm}^2$) of the four models that experienced the same detached shock phenomena. Figure 6.9 displays the first high speed camera frame during the Mach 3.2 start-up.

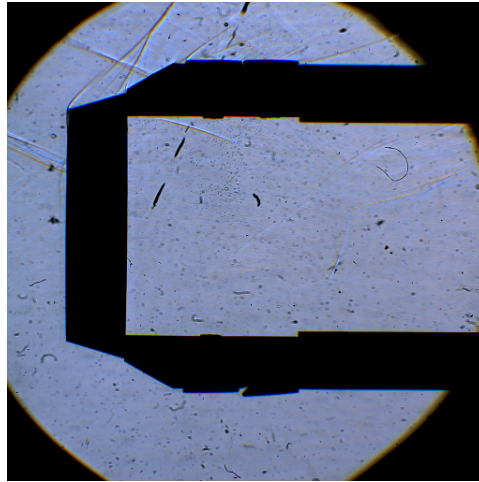


Figure 6.9: First high speed camera frame at Mach 3.2

It is evident in Figure 6.9 that supersonic flow already exists in the test section by the presence of attached shock waves on the upper leading edge. This means that the starting shock has already passed but has not been captured during the high speed footage. Due to the lack of any shock wave features near the lower leading edge, subsonic flow must be present. The reason this occurs is shown in Figure 6.10.

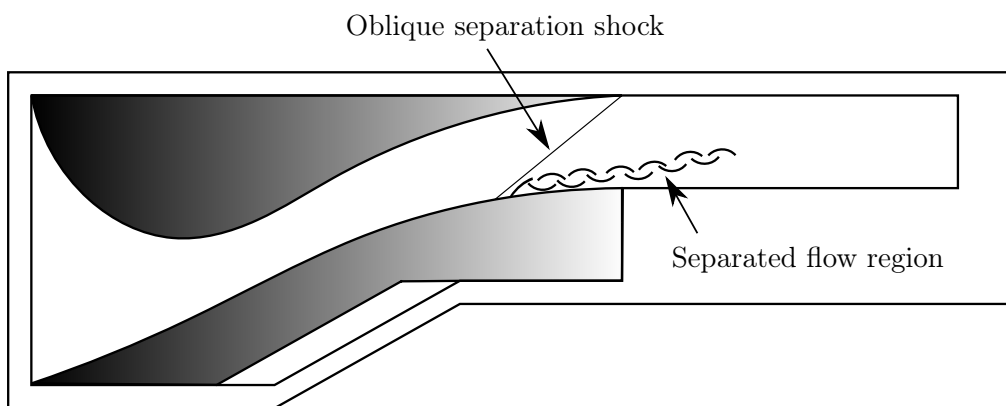


Figure 6.10: Separated supersonic nozzle flow schematic

During start up the asymmetric supersonic nozzle must be experiencing a separated flow region on its lower surface. This causes a velocity gradient to be present in the test section as supersonic air is leaving the upper nozzle region but subsonic flow is leaving the separated flow region. This separated flow region would result in an oblique shock that forms at its location as the supersonic flow is deflected upward. With start-up progression and the adverse pressure gradients involved, the separation zone and oblique shock would move further downstream, bringing steady supersonic flow throughout the test section as it passes by. The movement of this oblique separation shock was captured with high speed camera footage and is displayed in Figure 6.11 as it interferes with the creation of attached shock regimes.

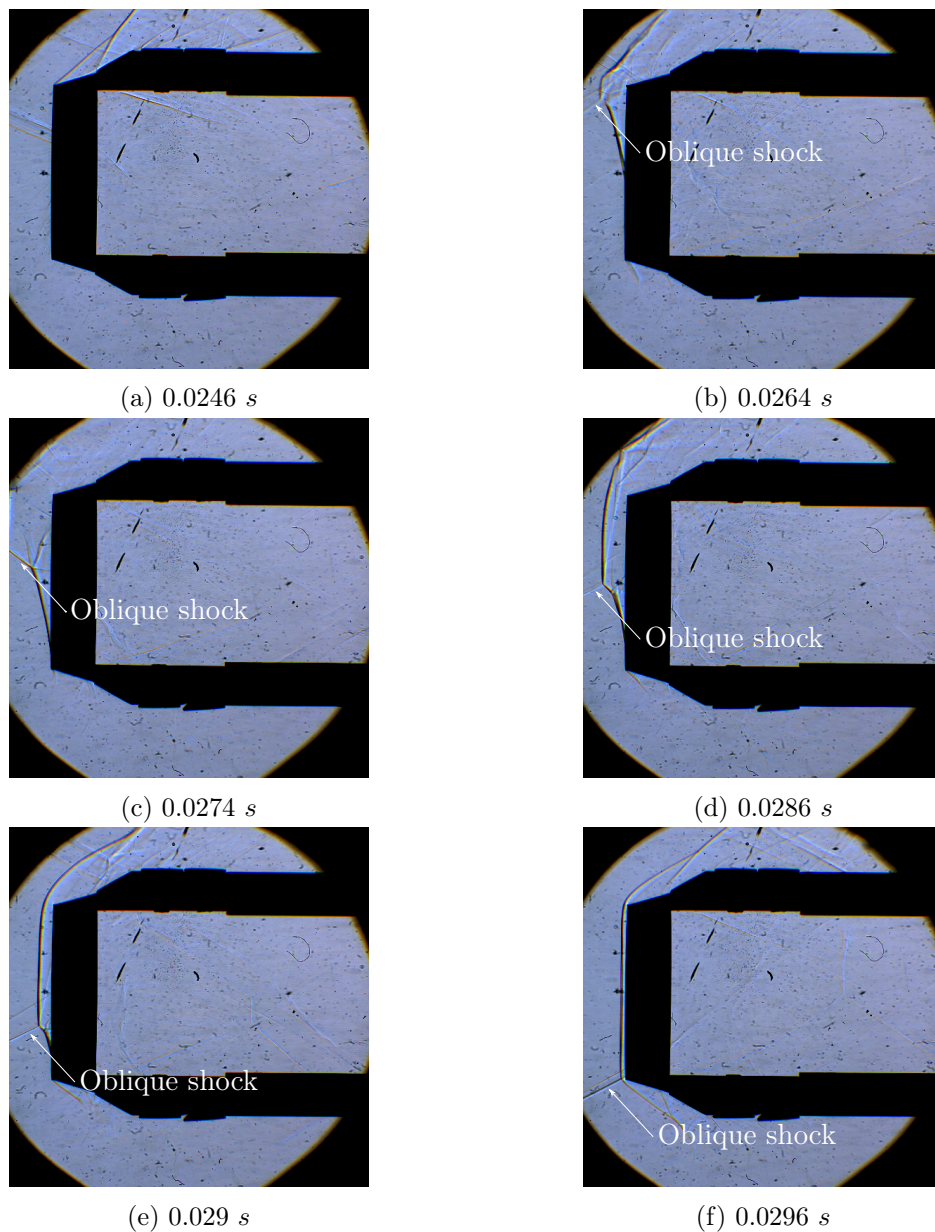


Figure 6.11: High speed oblique shock progression for the $R_c=1.5$, $\alpha=8^\circ$, $L_w=0.5$ model at Mach 3.2

When considering the flow field in Figure 6.11a, even though uniform supersonic flow has not been initiated throughout the test section, the top outer leading edge of the ring wedge model already carries a shock wave. This means that the test section experiences a velocity gradient from the upper to the lower surface during start up as previously mentioned. The reason for this is the asymmetry of the supersonic nozzle and the flow separation shown in Figure 6.10. This causes the air leaving it and entering the test section to have different velocities depending on vertical positioning.

As the oblique shock passes through it first detaches the shock from the top outer leading edge as seen in Figure 6.11b. At this point uniform supersonic flow has begun to initiate further into the ring wedge, below the oblique shock and in the bottom internal and external ring wedge sections. In Figure 6.11c the previously attached shock wave is detached and out of view but the oblique shock has progressed further down into the test section. An attached shock wave is visible at the bottom leading edge. When the oblique shock moves further down and into the test section it appears to pull the attached bottom shock wave away from the leading edge. This occurs as the detached bow shock comes back into view in Figure 6.11d. The detached bow shock wave begins to settle in Figure 6.11e and Figure 6.11f with further progression of the oblique shock, eliminating the formation of an attached Mach reflection.

This uneven detachment and reattachment process is thought to be the cause of previously mentioned ring wedges experimentally experiencing detached bow shocks rather than the numerically simulated attached Mach reflections. The flow field experiences an asymmetric velocity distribution and oblique shock due to the asymmetric supersonic nozzle and separated flow region. Therefore the exit area may be too small to compensate for this variation and allow for an attached shock to exist at the given Mach number resulting in global choking. The attached bottom leading edge shock is then pulled further forward with the oblique shock progression to encourage the formation of a detached flow regime. This uneven velocity distribution does not favour the possibility of attached shock waves to form within the bottom internal flow field given the area limitations. In the numerical simulations the attached shock waves are formed in a uniform flow field without the presence of excess turbulence and an oblique shock progression as experienced in the SSWT. Hence the flow separation causes some instability within the test section prohibiting the formation of a steady attached shock wave structure. Even though the flow Mach number should theoretically be sufficient to result in such a system given the exit area, the induced instability contributes negatively promoting shock detachment.

To further examine the effects of the oblique shock, the same ring wedge model was tested under conditions where an attached shock structure existed both experimentally and numerically. Figure 6.12 displays the oblique separation shock progression at Mach 3.4.

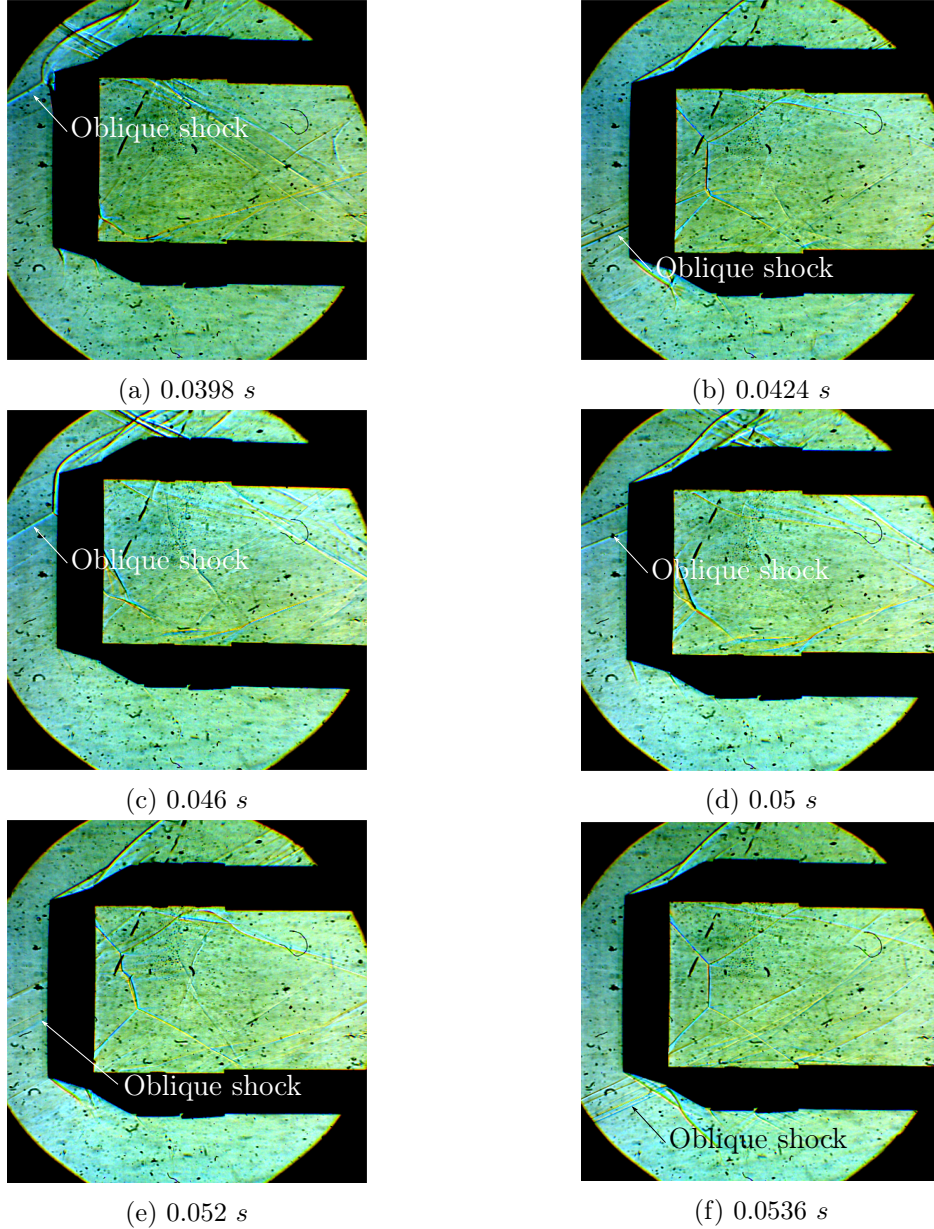


Figure 6.12: High speed oblique shock progression for the $R_c=1.5$, $\alpha=8^\circ$, $L_w=0.5$ model at Mach 3.4

Starting off at Figure 6.12a, a resemblance to Figure 6.11b can be seen. The oblique shock detaches the already attached upper outer leading edge shock wave in the supersonic region of the test section. The oblique shock progression seems to be quite far along in Figure 6.12b as most of the flow field is fully developed after it has passed. Although the oblique shock progression process was near completion it was found that it retracted back in Figure 6.12c. This caused the attached shock in the lower region to become unstable and a detached shock wave was beginning to form, similar to Figure 6.11e. The instability of the separated flow region does not favour a stable and continual progression of its oblique shock wave.

Since the flow velocity was increased to Mach 3.4, the area limitations were overcome and the oblique shock then reattached the top leading edge shock wave. This promoted stronger formation of attached shock waves in the lower flow regions as seen in Figure 6.12d. From Figure 6.12e it can be seen that even though the lower region of the ring wedge contains a fully formed shock structure, the upper region's stability is questionable. This can be due to possibly different supersonic velocities being present as the passage of the oblique shock is still undergoing completion. Finally the oblique shock progressed further down and into the test section, developing the attached shock structure visible in Figure 6.12f and creating a uniform supersonic flow field.

Therefore it can be noted that the asymmetry of the supersonic nozzle and separation region is producing relatively unstable flow field dynamics and a test section that experiences a velocity gradient from supersonic to subsonic flow before the oblique separation shock has passed. Since a portion of the flow field is already supersonic before the passage of the oblique shock, attached shock waves form. These are then detached during the oblique separation shock progression and interfere with the reattachment and formation of shock structures throughout the flow field. Coupling this idea with the area contractions in a non-uniform and turbulent flow field, detached shock waves may be forming under experimental conditions due to apparatus design parameters.

With a large majority of experimental results falling in agreement with numerical simulations and the remaining detached shock situations being accounted for, it can be concluded that the numerical analysis has been validated. Additional comparison cases are displayed in Appendix B. This applies to testing regions where detached bow shocks were present as a result of the supersonic nozzle asymmetry and separation induced oblique shock progression. The numerical analysis can thus be used to provide insight and information on areas where the experimental results are limited and not suitable.

6.1.2 CST MATLAB Code Validation

A CST MATLAB code was developed in order to implement CST on the various curved shock waves using shock wave shape equations. To validate the numerical code script, the influence coefficients mentioned in Section 2.5, are calculated in the code and are compared to those present in Figure 2.11 and Figure 2.11 which are available from [12]. These comparisons can be seen in Figure 6.13b and Figure 6.14b. The influence coefficients produced by the CST code were calculated under the same parameters using a specific heat ratio of $\gamma=1.4$, a flow speed Mach 3 and a pre-shock divergence angle of $\delta_1=0$ as detailed in [12].

Since the influence coefficients are independent of shock wave shape and only on shock angle, they can be plotted for comparison. The influence coefficients produced by the CST code, in both cases, show no discernible differences from the available data published in [12]. The only differences are due to the vertical axis scale in Figure 2.11. Therefore the CST code script's structure is valid and only dependent on the shock wave shape function that is used to calculate the various shock angle and curvature terms.

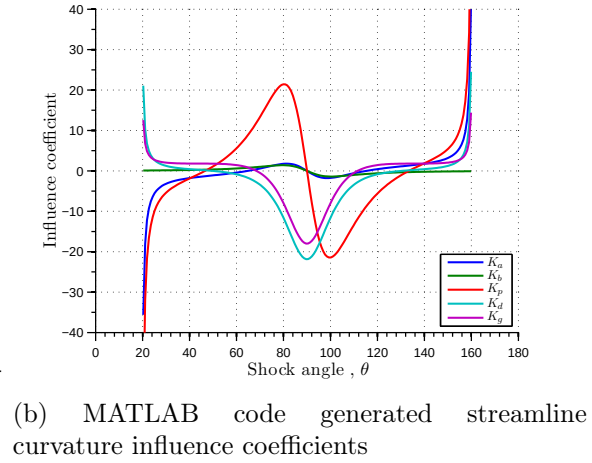
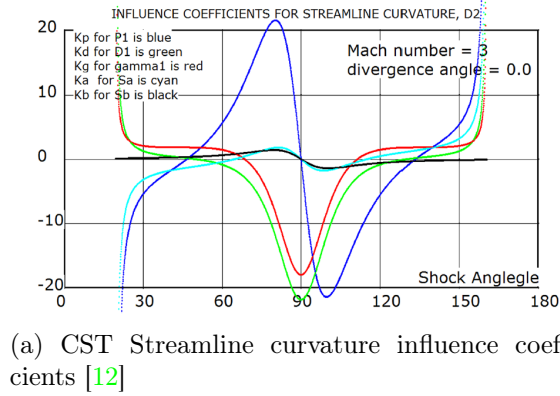


Figure 6.13: Streamline curvature influence coefficient comparison

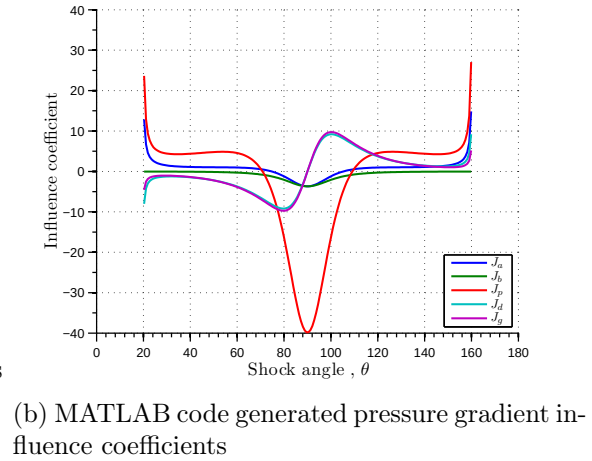
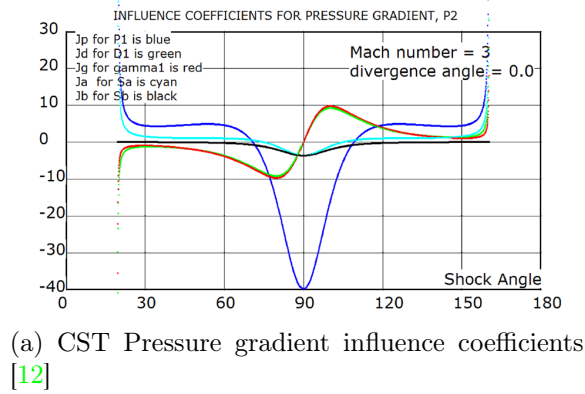


Figure 6.14: Pressure gradient influence coefficient comparison

6.2 Curved Shock Wave Shape Equations

To allow for the curved shock wave shape to be used for analysis purposes, a function was required to be fitted to the curved shock wave segment. Said continuously curved segment is bounded by the leading edge of the ring wedge and the influence of the trailing edge expansion

or initial reflection triple point. Since the numerical method was validated, the CFD results could be utilised to find the curved shock wave shapes. The physical axisymmetric limitations of the experimental ring wedge models blocked the internal flow features from view and hence hid the continuously curved shock wave segment.

To curve fit a function that would describe the shock wave shape, imaging processing codes and techniques were developed. The CFD results were first displayed as density contours within Tecplot which allowed the shock wave shape to be displayed. Using the first frontal contour line which propagated from the leading edge and into the Mach reflection, the outline of the incident shock wave structure was defined. The continuously curved shock wave segment could then be marked off manually for each case, exported as a high resolution image of 10833×9633 *px* from Encapsulated Postscript Vector (EPS) format and isolated using image editing. Once this was completed the image was fed into MATLAB for image processing and curve fitting. Since all the images were saved under the same pixel ratio and coordinates, static features such as the domain origin, bounds and leading edge could be used to develop pixel conversion factors. These were used to convert the pixel points which constructed the shock wave into coordinates of physical dimensions which then matched the coordinate locations from the CFD results. Once this was achieved all the shock wave coordinates, were normalised with respect to the leading edge radius. The coordinate system origin was then placed at the leading edge and flow normal plane such that the origin sat on the axis of symmetry, and in-line with the leading edge, whose tip touches the y -axis as opposed to 0.01 m ahead as seen in the CFD results.

After the image processing and coordinate normalisation was completed the points, (x_s, y_s) , which made up the shock wave were ready to be curved fitted to a particular function. The shock wave shape curve fitting was completed with the use of numerical scripts compiled in MATLAB. Due to the nature of MATLAB's regression and curve fitting operations it was found that a curve fits could more easily be determined if the shock wave points were plotted in reflected form as y_s vs x_s , where x_s and y_s are the shock wave's x and y coordinate respectively. The mathematical function for the shape in their final x_s vs y_s format could be found by simply taking the inverse of the original curve fit in order to transform it from its reflected domain. Originally the shock waves were fitted using a second order polynomial function. However in some cases the curvature toward the end of the continuously curved shock wave segment was being over-compensated for as the left opening parabola approached its local maximum, straying away from the shock wave. The parabolic curve fit was thus deemed as unsuitable. The final curved fit that described the shock wave shape very accurately in all cases was determined to be a power law. In the y_s vs x_s domain the selected power law fit was calculated to be of that in Equation (6.1). Once its inverse was taken, thereby reflecting it back to the x_s vs y_s domain, the final calculated function was that in Equation (6.2) where ε ,

η and λ are constants specific to each curve fit. Therefore a general equation was determined that accurately describes the shape of the continuously curved shock wave segments generated in axisymmetric internal flow fields, that are curved in both the flow and flow-normal planes. The unit-less constants ε , η and λ then vary depending on the flow Mach number, internal surface radius of curvature and wedge entry angle.

$$y_s = \varepsilon x_s^\eta + \lambda \quad (6.1)$$

$$y_s = \left(\frac{x_s - \lambda}{\varepsilon} \right)^{\frac{1}{\eta}} \quad (6.2)$$

6.2.1 Wedge Entry Angle $\alpha=0^\circ$

Figure 6.15a, 6.15b and 6.15c show the varying values of ε , η and λ respectively for the wedge entry angle, $\alpha=0^\circ$, with increasing Mach number. Figure 6.15d shows the maximum x -axis value which bounds the continuously curved shock wave segment for the $\alpha=0^\circ$ models, giving the horizontal shock wave length.

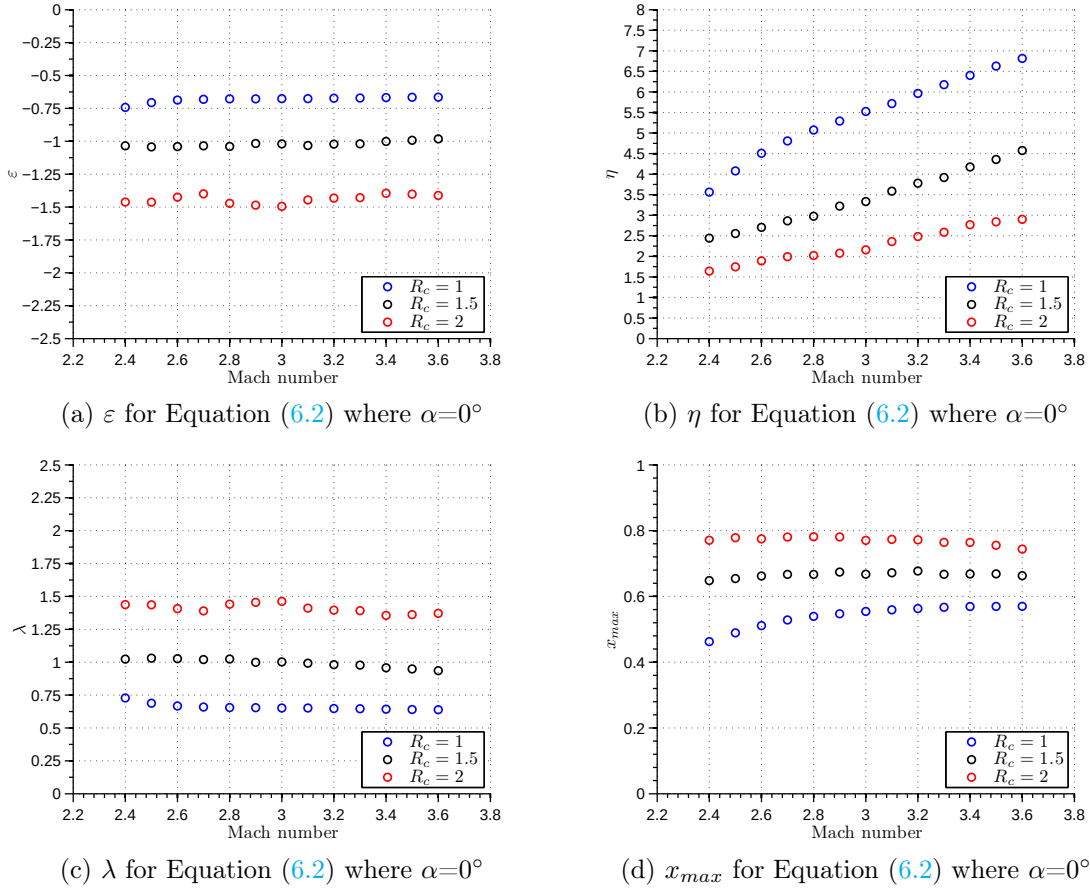
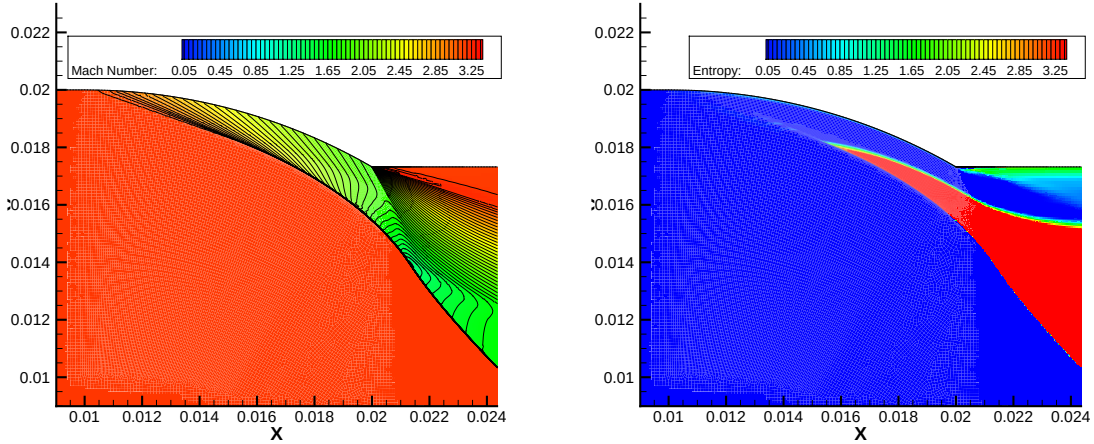


Figure 6.15: ε , η , λ and x_{max} from Mach 2.4 to 3.6 where $R_c = \{1, 1.5, 2\}$ and $\alpha=0^\circ$

Seeing as $\alpha=0^\circ$, a shock wave is not fully formed from the leading edge of the ring wedge. The curved internal surface rather forms an isentropic compression, consisting of a successive series of Mach waves, which eventually coalesce to create a shock wave. This feature is displayed in Figure 6.16a which displays Mach contours for the $R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ model at Mach 3.2. As can be seen the Mach lines at the front of the compression do not touch and until further into the ring wedge. Once they begin coalescing a shock wave is formed. This is confirmed by Figure 6.16b which displays the entropy generation for the same model and conditions.

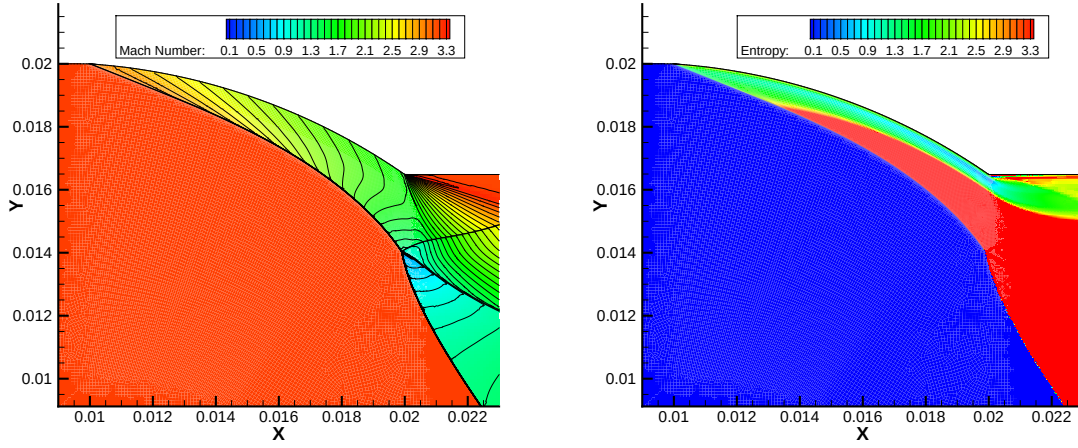


(a) $R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ Mach number contours (b) $R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ entropy contours

Figure 6.16: Mach number and entropy contours for the $R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ model at Mach 3.2

Before the Mach waves form a shock wave the internal flow region has a constant blue colour contour until at the point of coalescence, characteristic of an isentropic compression where entropy does not change. A dramatic contour shift then occurs thereby depicting an entropy production and the formation of a shock wave. The constant zero entropy region then trails downstream and does not stop abruptly at the coalescence point as its isentropic properties carry downstream along their streamlines and are not affected by the formation of the shock wave. Since expansion wave fans are isentropic they do not change the entropy of the flow passing through them, keeping blue contours constant after the trailing edge and until after its influence. There is a slightly different coloured entropy contour region along the internal surface of the ring wedge in Figure 6.16b. This is brought about by the transverse curvature of the model, curvature induced due by the axisymmetric nature of the model, causing slight entropy generation. The values of entropy being shown are not of importance, only the existence of zero and non-zero entropy regions.

Comparatively in Figure 6.17a a shock wave is known to exist due to the wedge entry angle being increased to $\alpha=4^\circ$ for the same flow speed and internal radius of curvature. This causes an initial flow deflection and hence starts of the formation of a shock wave from the leading edge. In Figure 6.17b the entropy generation is depicted for the same model and conditions as Figure 6.17a. Starting from the leading edge of the ring wedge an entropy production is already noticed as the contours differ from the blue zero entropy conditions of the upstream uniform flow field. This is expected as a shock wave is present. Therefore the differences can be seen between cases that exhibit the progress from isentropic compression to shock wave formation and those which bare a shock wave from the leading edge.



(a) $R_c=1$, $\alpha=4^\circ$, $L_w=0.5$ Mach number contours (b) $R_c=1$, $\alpha=4^\circ$, $L_w=0.5$ entropy contours

Figure 6.17: Mach number and entropy contours for the $R_c=1$, $\alpha=4^\circ$, $L_w=0.5$ model at Mach 3.2

Equation (6.2) for the $\alpha=0^\circ$ models rather describes the frontal compression boundary caused by the ring wedge which eventually develops into a shock wave further downstream. The larger the value of R_c , the more gradual the curvature and hence flow turning. This results in the isentropic compression coalescence being delayed as the Mach waves that form along the internal curved surface do so with a more gradual and similar inclination in comparison to the previous upstream waves. They therefore then catch up with one another at a location further downstream to form a shock wave since the change in flow deflection is decreased with constant horizontal distance and increasing R_c . As this occurs the shock wave shape becomes less curved as Equation (6.2) describes a segment which is flat towards the leading edge, due to delayed coalescence, and then begins to curve more after shock wave formation. This means that the curve fitting that is carried out has to try fit a region of the power law which has a more constant gradient due to the straight nature of the Mach waves involved. The constants that are then calculated become more erratic as less curvature is used to define

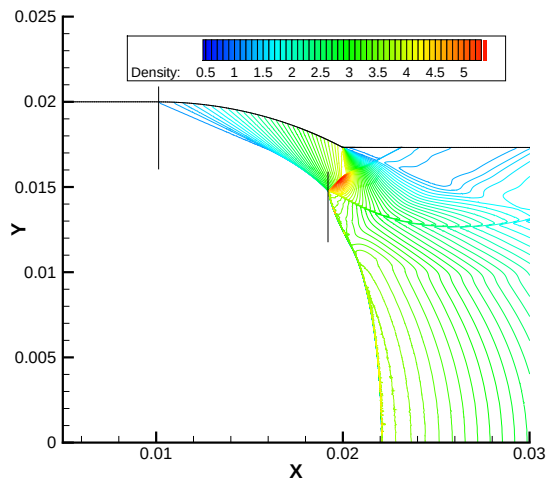
the fit. A flat surface becomes the predictor of the shock shape and not a characteristic curved shape, unique to particular conditions for each shock wave. This relationship can be seen in each plot in Figure 6.15. The data from the $R_c=2$ model does not follow the consistency of that of the $R_c=1$ model as the points are more vertically scattered. This occurs well into the Mach number range until the shock wave formation starts occurring sooner and a more curved shape is fed into the curve fitting algorithms within MATLAB.

Theoretically the transverse or flow-normal curvature mentioned in Section 2.5, S_b , caused by the axisymmetric nature of the model, would curve the flow field and hence the Mach waves. Its effects though would be extremely slight at large radii of curvature. It would only be noticeable as the flow contracts, causing the distance to the axis of symmetry to decrease and the transverse curvature to become larger. By the time this point is reached the flow plane curvature, S_a , that is induced by the curved internal surface, has taken over and curved the shock wave by a more surpassing and notable extent.

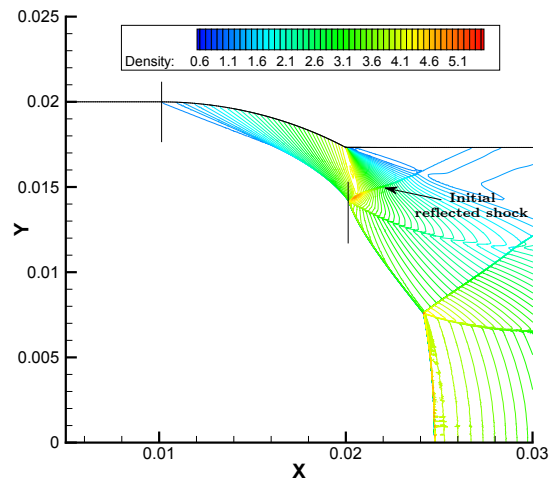
The constants plotted in Figure 6.15a, 6.15b and 6.15c follow particular patterns in each case. Both the values of ε and λ seem to remain almost constant with a very gradual linear relationship as Mach number increases. The values of η however follow a sharper gradient bearing a linear increasing relationship. It can be seen that the first four Mach number data points for the $R_c=1$ series in all constant cases follow a differing relationship to the remaining data points. They increase in the case of ε and η or decrease in the case of λ more rapidly in comparison to the data points ahead of them before joining a more of a linear type trend. The reason this occurs is due to their stage in shock wave progression. For the $R_c=1$ model, Mach 2.4 is near the lowest Mach number before the flow detaches and globally chokes. This initial shape progression is depicted in Figure 6.18 using density contours. The two vertical black lines in Figure 6.18a through Figure 6.18d represent the lower and upper bounds of the curve fit governed by Equation (6.2).

Figure 6.18a is the lowest Mach number simulated before which the $R_c=1$ model would experience a detached flow regime. Thus it was the lowest Mach number used for numerical data gathering for all the $\alpha=0^\circ$ models, for comparison purposes. Comparing Figure 6.18a and Figure 6.18b it can be seen that the shock wave shape has changed quite significantly due to the movement of the small reflected shock that is present. The increase in Mach number has too begun to curve the flow to a more noticeable extent. The curve fit constants would therefore progress and differ the more the shock wave shape differs between flow Mach numbers. The constants then begin to settle off and follow a similar future trend after Mach 2.8. The disappearance of the small initial reflected shock from Figure 6.18b to Figure 6.18c and Figure 6.18d shows that the curve fit used to describe the shock shape is now being fed more consistent data as its downstream bound is determined by the expansion fan interference. The general shape bounded by the two vertical black lines remains the same and therefore the

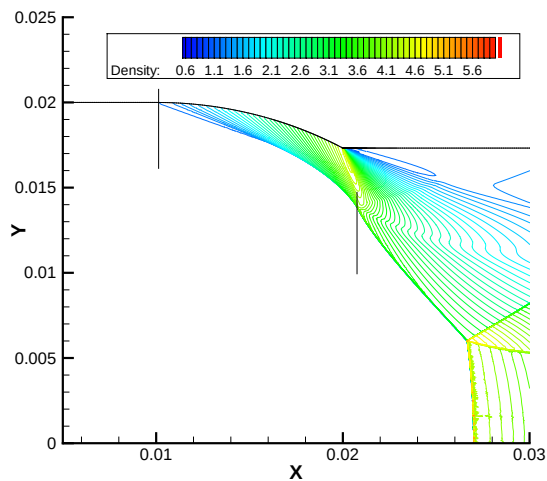
curve fit does not change dramatically. Hence ε , η and λ start to settle off with a more linear type pattern with Mach number progression. The tip of the shock wave, x_{max} , that describes the end of the continuous curvature and bounds the curve fit follows the same pattern in Figure 6.15d. The values of x_{max} for the $R_c=1$ model increase as the shock wave shape progresses and then begin to settle. This occurs when the upper bound becomes determined by the interference of the trailing edge expansion fan and no longer by the small initial reflected shock that was present and then dissipated.



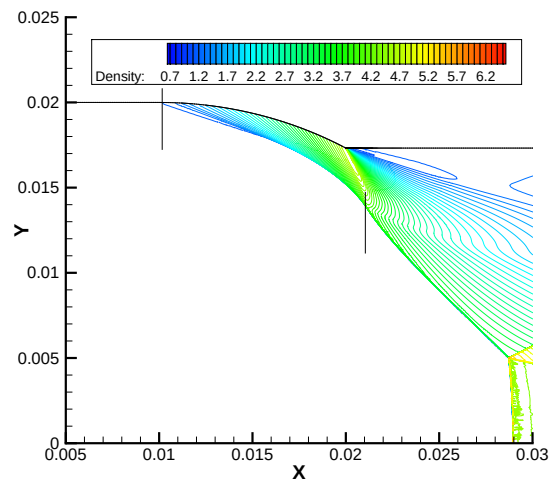
(a) Density contours at Mach 2.4



(b) Density contours at Mach 2.6



(c) Density contours at Mach 2.8



(d) Density contours at Mach 3.0

Figure 6.18: Density contours for the $R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ model at Mach 2.4, 2.6, 2.8 and 3.0

6.2.2 Wedge Entry Angle $\alpha=4^\circ$

With the wedge entry angle, α , increased to 4° , Equation (6.2) describes the full length of a continuously curved shock wave. Figure 6.19a, 6.19b and 6.19c show the varying values of ε , η and λ respectively for the wedge entry angle, $\alpha=4^\circ$. Figure 6.19d displays the maximum x -axis value and horizontal shock length for the $\alpha=4^\circ$ models. Due to α increasing, the exit area of the model decreases and the internal flow turning increases. This means the $R_c=1$ model which previously choked below Mach 2.4 now chokes below Mach 2.9. Therefore the data point values on the Mach number axes now range from Mach 2.9 to 3.6. The upper Mach number limit was chosen on the basis of the limitations of the SSWT being used.

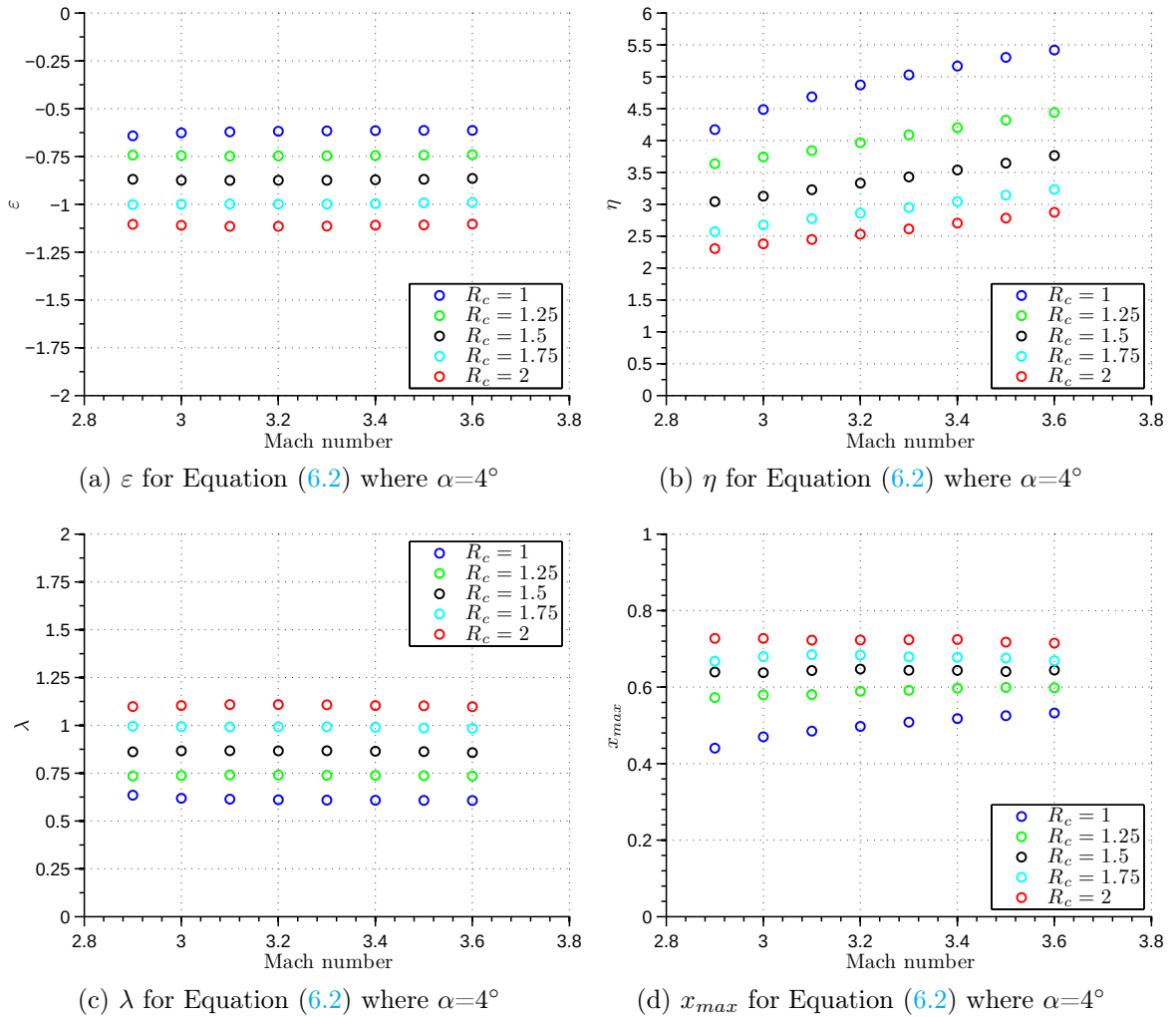


Figure 6.19: ε , η , λ and x_{max} from Mach 2.9 to 3.6 where $R_c=\{1, 1.25, 1.5, 1.75, 2\}$ and $\alpha=4^\circ$

As a result of being able to capture data for all the values of R_c with $\alpha=4^\circ$, and with a full shock wave being present, additional CFD cases were simulated. Normalised internal radii of

curvature ratios of $R_c=1.25$ and $R_c=1.75$ were selected to fit between the original data sets. This was done in order to confirm the relationship between the constants ε , η and λ versus Mach number in an attempt to normalise them in future analyses.

With the $R_c=1$ model being close to choking at Mach 2.9, the same shock progression pattern is seen in the constants as in the $R_c=1$, $\alpha=0^\circ$ data between Mach 2.4 and 2.8. The effect is less pronounced though in the $R_c=1$, $\alpha=4^\circ$ data between Mach 2.9 and 3.1. In Figure 6.19a, 6.19b and 6.19c the change in the constants, before they settle off and join a final trend, is less noticeable and seems only visible in Figure 6.19d. This could be due to the increase in α meaning that a shock exists from the leading edge and not an isentropic compression such as that seen in the $\alpha=0^\circ$ models. Therefore a more consistent shape exists from which the remaining shock structure extends and so the constants that determine the shock wave shape are more similar.

Comparing the constants of the $R_c=2$ model for the $\alpha=0^\circ$ and $\alpha=4^\circ$ cases it can be seen that the data has become less vertically scattered and more consistent with a uniform trend. This is a result of a shock wave being present from the leading edge and not an isentropic compression. Since the internal curved surface is placed at an angle, due to the increase in α , the flow is initially deflected through a larger angle. A shock wave is present so the decrease in flow Mach number behind the initial shock compression is larger than in the isentropic compression case. The decrease in flow speed then means that the following compression occurs at a steeper angle, a relationship displayed by Figure 2.4. For a lower Mach number and constant deflection, the shock angle increases. Therefore the infinitesimally present compressive Mach waves or characteristics, which alter the shape of the shock wave, occur with a steeper angle. The shape of the curved shock wave formed through this mechanism becomes more regular than in the case of an isentropic compression. The determination of the shape and curve fitting data becomes more consistent and hence the constants follow suit with a uniform trend.

This change in consistency for the values of ε , η and λ between the $\alpha=0^\circ$ and $\alpha=4^\circ$ models is only noted in the highest R_c case of $R_c=2$. The more gradual flow turning present in higher radii of curvature mean that in the isentropic compression to shock wave coalescence is delayed further downstream. So the difference in the consistency between $\alpha=0^\circ$ and $\alpha=4^\circ$ increases with decreasing R_c as shock wave formation occurs sooner further upstream. The curve fitting data for the $\alpha=0^\circ$ models then resemble that of the $\alpha=4^\circ$ models more closely as the leading edge attached shock case is approached when the isentropic compression coalescence point travels further upstream. It must be noted that the consistency that is being referred to is not the similarity between the numerical values in either case but rather a uniform trend relationship with minimal scatter along the y -axis. The values of ε and λ exhibit constant value relationships as Mach number increases with a very slight linear slope. The values of η

however experience a steeper gradient while maintaining a linear trend with increase in Mach number.

6.2.3 Wedge Entry Angle $\alpha=8^\circ$

Figure 6.20a, 6.20b and 6.20c show the varying values of ε , η and λ respectively for the wedge entry angle, $\alpha=8^\circ$. Figure 6.19d displays the maximum x -axis value and horizontal shock length for the $\alpha=8^\circ$ models.

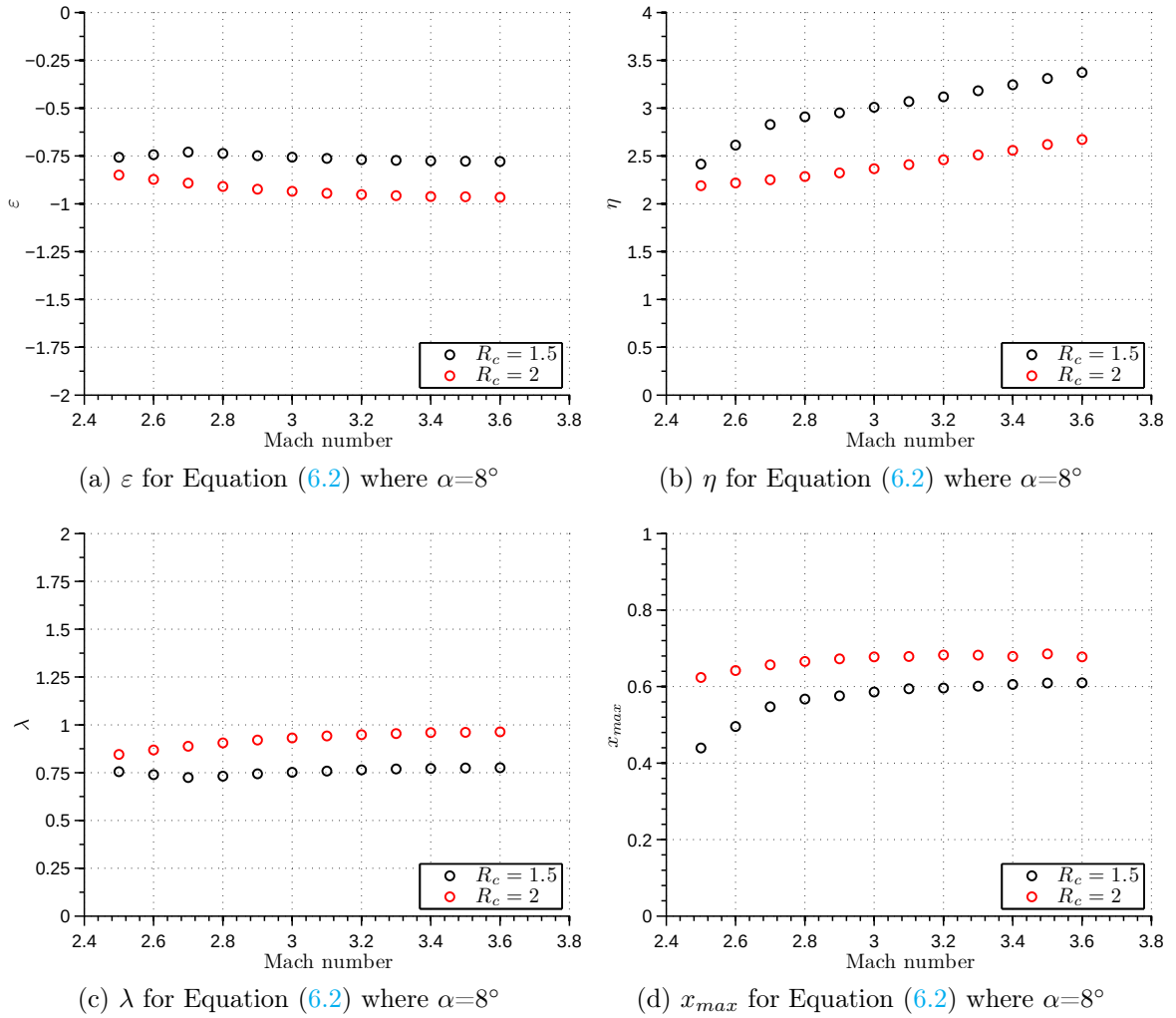


Figure 6.20: ε , η , λ and x_{max} from Mach 2.5 to 3.6 where $R_c=\{1.5, 2\}$ and $\alpha=8^\circ$

For the $\alpha=8^\circ$ models, the $R_c=1$ case experienced such extreme flow deflection and exit area contraction to the point where even at Mach 3.6 an attached shock was not possible and choking occurred. Seeing as Mach 3.6 was the upper limit for the SSWT, simulations were not carried out for the $R_c=1$ case as an attached shock was only possible above Mach 4.2. This

meant that the other $\alpha=8^\circ$ ring wedges would have to be simulated at much higher Mach numbers to allow for comparisons where the possibility of solution divergence increased due to the higher temperatures involved. Therefore only the $R_c=\{1.5, 2\}$ cases were simulated from the $R_c=1.5$ choking Mach number of 2.6 to 3.6. Unlike $\alpha=4^\circ$ models additional internal radii of curvature were not simulated for normalisation purposes as the $R_c=1$ case, which would be used for normalisation, experienced choking below Mach 4.2.

Since the $R_c=1.5$ model is now the model closest to choking, from which the Mach number range started, comparison can be drawn between the constants which experienced that same states of shock progression. Between the Mach 2.5 and 2.9 the data points in Figure 6.20a through Figure 6.20d, for the $R_c=1.5$ model, displayed the same pattern where they change rapidly before settling off, as those of the $R_c=1$, $\alpha=\{0^\circ, 4^\circ\}$ models. Therefore the constants exhibit that same pattern regardless of change in α or R_c and so the curved shock waves shape must progress through the same initial reflected shock stage. Their continuous shape then becomes governed by the trailing edge expansion fan's influence. Figure 6.21 displays the lower and upper Mach number density contour plots during the progression where constants change before settling.

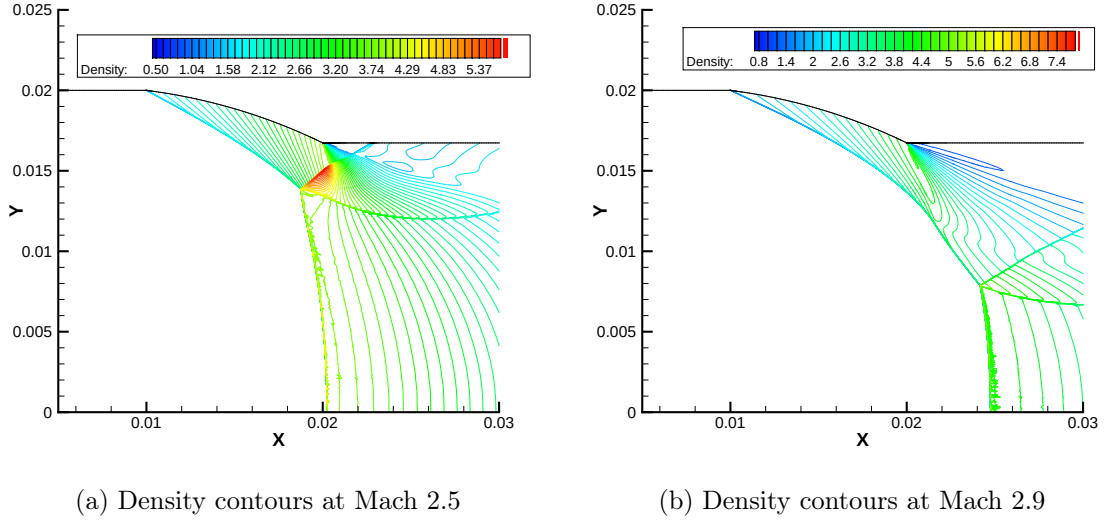


Figure 6.21: Density contours for the $R_c=1.5$, $\alpha=8^\circ$, $L_w=0.5$ model at Mach 2.5 and 2.9

At Mach 2.5 the effects of the expansion fan on the attached leading edge shock wave are preceded by the initial reflected shock wave. For curve fitting the continuously curved region, its bounds would be the reflection point as previously mentioned. Eventually, with an increase in Mach number the shape remains consistent as a flow field, such as that at Mach 2.9 is approached, and the curve fit stops where the trailing edge expansion interferes. Appendix C

contains all the values for ε , η and λ for Equation (6.2) and as well x_{max} and curve fit R^2 data for each case.

6.2.4 Curved Shock Wave Characterisation

In order to find an underlying relationship between the experimental parameters and the curved shock wave shape they produce, an attempt was made to characterise the constants used in Equation (6.2). Only the $\alpha=0^\circ$ and $\alpha=4^\circ$ cases contained R_c values of 1 through 2. The $\alpha=0^\circ$ case however did not represent a true shock wave but rather the frontal surface of an isentropic compression which later transitioned to a shock wave. Therefore only the $\alpha=4^\circ$ was considered for characterisation purposes as an attached leading edge shock wave was present with available simulation data below Mach 3.6. Additional simulations at $R_c=1.25$ and $R_c=1.75$ were completed, as previously mentioned, to provide additional datasets for relationship determination.

Referring to Figure 6.19a, 6.19b and 6.19c it is evident that self similar trends are present with each value of R_c . Each trend looks to show the same general relationship and is only separated by a particular ratio as R_c increases. Since this occurs an attempt was made to collapse the curves and possibly characterise them by a set of functions which could describe the curve shock wave shape given a set of input parameters. To try to collapse the curves for each set of constants, they were normalised with respect to those of the $R_c=1$ case. Therefore a set of curve fitting constant ratios were created, symbolised by ϕ , where the numerator represents ε , η or λ for the set $R_c=\{1, 1.25, 1.5, 1.75, 2\}$ and the denominator ε , η or λ for $R_c=1$. Said ratios are depicted in Equation (6.3), Equation (6.4) and Equation (6.5).

$$\phi_\varepsilon = \frac{\varepsilon_{R_c}}{\varepsilon_1} \quad (6.3) \quad \phi_\eta = \frac{\eta_{R_c}}{\eta_1} \quad (6.4) \quad \phi_\lambda = \frac{\lambda_{R_c}}{\lambda_1} \quad (6.5)$$

Since each ϕ value will be coupled to a particular Mach number they must be averaged for each R_c dataset to find their correlation with change in R_c . The standard deviation in the mean could then be calculated using the technique outlined in [20]. Using Equation (6.6), where σ is the variance derived standard deviation and n the number of data points used to calculate the average ϕ value for each R_c case, the standard deviation in the mean is $\sigma_{\bar{x}}$. This provides values to be used in the uncertainty error bounds. Figure 6.22, 6.23 and 6.24 displays the average ϕ_ε , ϕ_η and ϕ_λ values respectively for each R_c case where $\alpha=4^\circ$. The vertical error bars were calculated using a 99% confidence interval where the bounds encompass the range $[\phi - 3\sigma_{\bar{x}}, \phi + 3\sigma_{\bar{x}}]$ [20]. Since the values of ϕ were normalised with respect to that of the $R_c=1$ datasets, the error bars have 0 magnitude for the $R_c=1$ data points.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad (6.6)$$

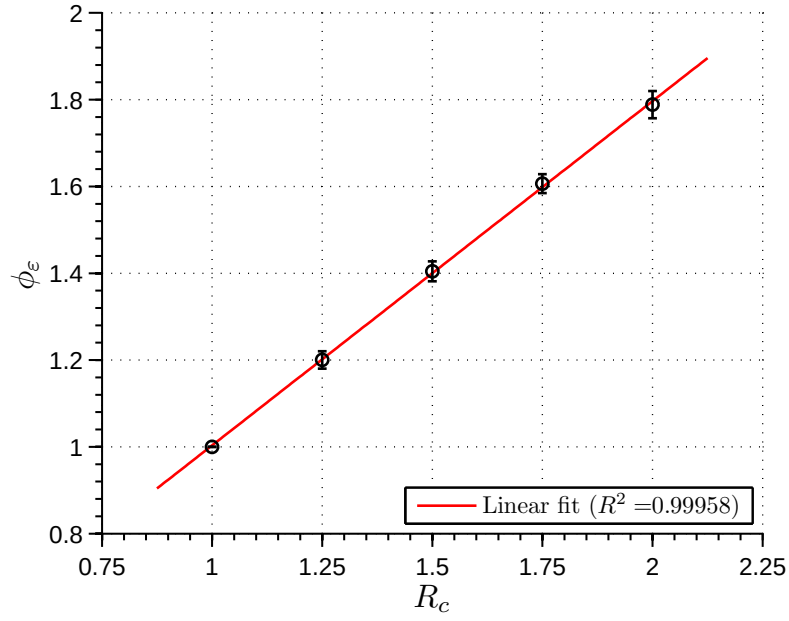


Figure 6.22: Average values of ϕ_ε for each $\alpha=4^\circ$ case

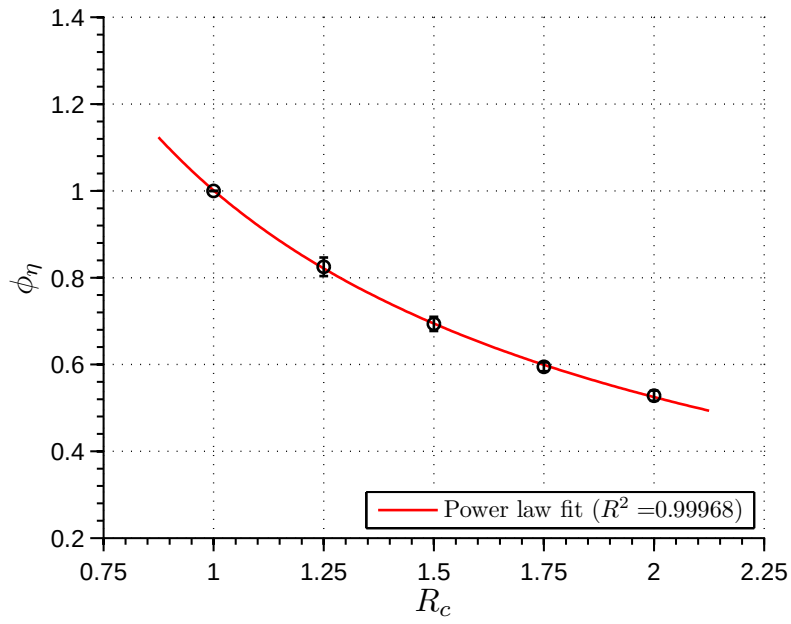


Figure 6.23: Average values of ϕ_η for each $\alpha=4^\circ$ case

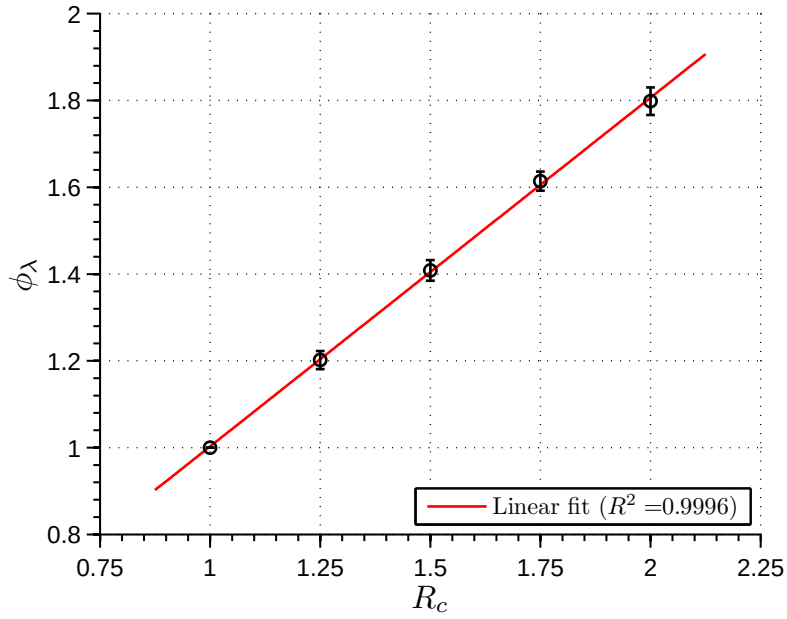


Figure 6.24: Average values of ϕ_λ for each $\alpha=4^\circ$ case

Curve fits were applied to the data in Figure 6.22, Figure 6.23 and Figure 6.24. Each fit's regression R^2 values appear on the legend of each plot. The data points in Figure 6.22 and Figure 6.24 were described very closely with linear fits where as the Figure 6.23 data points better followed that of a power law trend. A 2^{nd} order fit was originally applied to Figure 6.23 as it described the trend with a very high correlation R^2 value. The there was an issue however in the underlying nature of a 2^{nd} order fit. Even though the dataset was captured accurately, its extrapolated data points would be incorrectly predicted. The reason for this being the inherent local minimum involved in a 2^{nd} order function meaning that at some point the fit would change direction and increase the values of ϕ_η as R_c increased. This hypothesis was proven when the fit set approach larger values of R_c during data processing. A power law fit was then seen as more suitable due to its lack of a local minimum and asymptotic nature. The regression fit equations for ϕ_ϵ , ϕ_η and ϕ_λ are shown in Equation (6.7), Equation (6.8) and Equation (6.9) respectively.

$$\phi_\epsilon = 0.7934R_c + 0.2100 \quad (6.7)$$

$$\phi_\eta = 1.2131R_c^{-0.7194} - 0.2118 \quad (6.8)$$

$$\phi_\lambda = 0.8037R_c + 0.1991 \quad (6.9)$$

The regression fits which match the relationship of each ϕ fall well within the 99% confidence interval bounds and seem to pass very closely to the centre of each data point. Since the values of ϕ are generated by normalising the curve fitting constants by those of $R_c=1$ it implies that

the regression fits' lower limits are that of the $R_c=1$ data point. Any model with $R_c<1$ would immediately experience choking due to the reduction in flow exit area and increased flow turning as a result of a smaller internal radius of curvature. It would therefore be impossible for an attached shock wave to exist at a Mach number for models where $R_c<1$ if one is not present at $R_c=1$. Predicting values of ϕ via extrapolating the regression fits would not be valid unless done so where $R_c \geq 1$ and $\alpha=4^\circ$.

By applying regression fitting methods to the relationships between ε , η and λ against Mach number for the $R_c=1$ produces Equation (6.10), (6.11) and (6.12). This supplies a means to estimate the shock shape curve fitting constants where $R_c=1$ and $\alpha=4^\circ$.

$$\varepsilon_1 = \frac{1.688 - 0.6084M}{M - 2.781}, \quad R^2 = 0.9997 \quad (6.10)$$

$$\eta_1 = \frac{7.092M - 16.54}{M - 1.937}, \quad R^2 = 0.999 \quad (6.11)$$

$$\lambda_1 = \frac{0.603M - 1.685}{M - 2.801}, \quad R^2 = 0.9989 \quad (6.12)$$

The regression fits for ϕ_ε in Equation (6.7), ϕ_η in Equation (6.8) and ϕ_λ in Equation (6.9) can be multiplied by the functions that predict the values for ε in Equation (6.10), η in Equation (6.11) and λ in Equation (6.12). This provides a series of functions which can calculate ε , η and λ where $R_c \geq 1$ for Mach numbers greater than 2.9. Then substituting in Equation (6.2), ultimately producing the shape of the continuously curved shock wave region via empirical methods, Equation (6.13) is produced.

$$y_s = \left(\frac{x_s - \phi_\lambda \lambda_1}{\phi_\varepsilon \varepsilon_1} \right)^{\frac{1}{\phi_\eta \eta_1}}, \quad \text{for } R_c \geq 1 \quad \& \quad M \geq 2.9 \quad \text{where } \alpha = 4^\circ \quad (6.13)$$

Equation (6.13) therefore determines the shape of the continuously curved shock wave as a function of internal radius of curvature, R_c , and Mach number, M . The simulation model wedge lengths used to develop the relationship were however only half the length of the leading edge radius ($L_w=0.5$). This implies another constraint on Equation (6.13) to curved internal surfaces where $L_w \leq 0.5$. The applicability of the empirical model can be tested by examining its ability to predict the curved shock wave shape produced by the simulation model where $L_w=0.75$. Figure 6.25 displays the comparison between empirical fit used for $R_c=2$, $\alpha=4^\circ$, $L_w=0.5$ model being extrapolated downstream to meet shock length conditions of the $R_c=2$, $\alpha=4^\circ$, $L_w=0.75$ model at Mach 3.4 and the exact curve fit determined from CFD results. Figure 6.25a contains the exact $R_c=2$, $\alpha=4^\circ$, $L_w=0.5$ shock shape curve fit obtained from CFD results plotted with the shock shape produced using Equation (6.13) with $R_c=2$ and $M=3.4$ as inputs. As can be seen they have very close resemblance as the curves lie upon

one another. Using the same empirical model inputs Figure 6.25b compares the $R_c=2$, $\alpha=4^\circ$, $L_w=0.5$ empirical fit being extrapolated to the x_{max} shock length conditions for the CFD results of the $R_c=2$, $\alpha=4^\circ$, $L_w=0.75$ model.

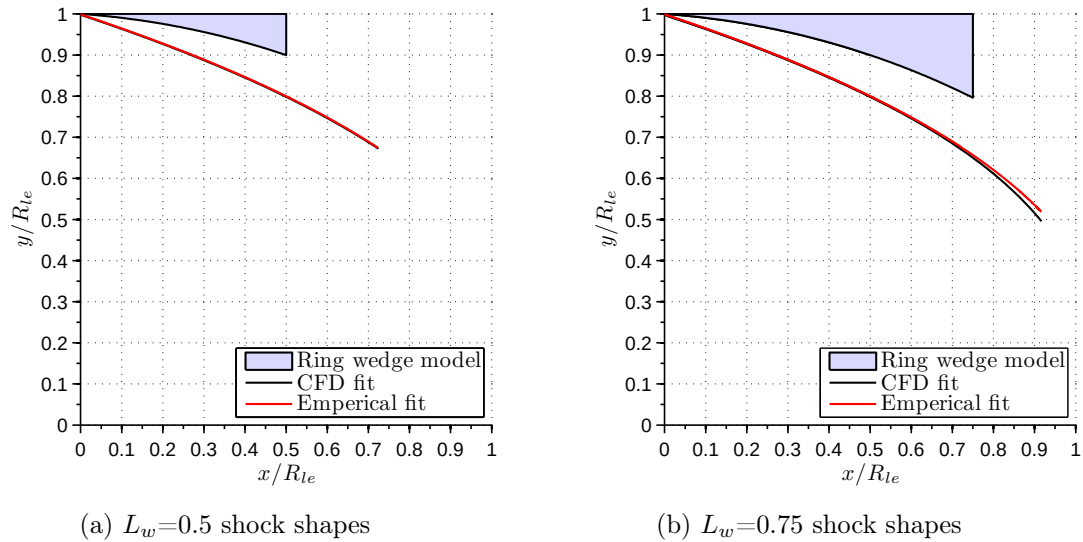


Figure 6.25: Comparison between the CFD and empirical fit shock shapes for the $R_c=2$, $\alpha=4^\circ$, $L_w=0.5$ and $L_w=0.75$ models at Mach 3.4

Moving along the shock shape from the leading edge, the empirical fit in Figure 6.25b resembles the CFD result very closely. This is because they experience the same curvature conditions. As one moves further downstream near the trailing edge the differences begin to become apparent and toward the end of the shock shape clear differences are evident. The R_c of each model is kept constant in each case with a value of $R_c=2$ so the same flow plane and flow normal (transverse) curvature is imposed up until $L_w = 0.5$. After this point the empirical model assumes the same conditions and continues downstream with the shock shape being extrapolated. The CFD results however include the continued effects of the flow plane and transverse curvature on the shock wave. This curves the shock wave toward the axis of symmetry to a further extent and so the CFD shock bends away from the empirical model near the end of the shock shape. The reason this occurs towards the end of the shock shape and not further upstream is due to the orientation of the compressive Mach lines or characteristics which affect the shape of the shock wave. These characteristics are angled downstream and are slightly curved due to the axisymmetric effects of transverse curvature. Their influence is only felt by the shock wave when they merge with it. This occurs at an angled downstream location so the majority of their influences are seen toward the end of the shock wave. Their strength is non-linear in nature due to the curvatures involved and so their influence becomes more pronounced downstream where the effects of transverse curvature increases as the distance

towards the axis of symmetry decreases. Therefore the empirical model only serves to predict curve shock wave shapes where $L_w \leq 0.5$.

Interpreting the trends seen in the values of ϕ one can gain a sense of self similarity evident in the changing values of ε , η and λ . The linear fits of Equation (6.7) and Equation (6.9) show how the values of ε and λ change by equal incremented amounts for each R_c dataset. Therefore as R_c increases, the values of ε and λ will change by the same constant proportion, which can be seen in Figure 6.19a and Figure 6.19c. The power law fit of Equation (6.8) is expected as the η makes up part of the exponential component of Equation (6.2). It gives an indication to the role of R_c in determining η and with increasing R_c , the power law fit in Figure 6.23 seems to approach an asymptotic value of ϕ_η .

The effects of changing the ϕ values on the shock wave shape, based on how R_c changes, gives an indication of the influence of ε , η and λ in determining the overall shock shape. By changing values of ε , η and λ while keeping two of the three constants unchanging provides insight on the role they play in affecting the curved shock wave shape. Linking these changes to the trends followed by ϕ_ε , ϕ_η and ϕ_λ points to the involvement of physical parameters on curved shock wave shape. To provide a base curved shock wave shape from which to progress the base values for ε , η and λ are calculated using the empirical model where $R_c=1.5$ and $M=3.6$. This yields $\varepsilon_{1.5}=-0.8586$, $\eta_{1.5}=3.7715$ and $\lambda_{1.5}=0.8540$ and, where according to CFD, $x_{max}=0.6447$. The value for R_c being selected for it being the mid-range radius of curvature and the Mach number being selected to maximise the length of the shock wave. Figure 6.26, 6.27 and 6.28 display the variation of the respective curve fit constants ε , η and λ .

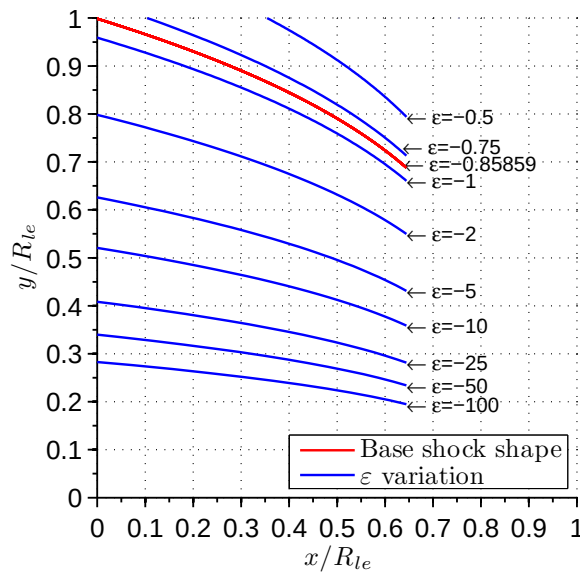


Figure 6.26: Variation of ε with constant η and λ for $R_c=1.5$ and $M=3.6$

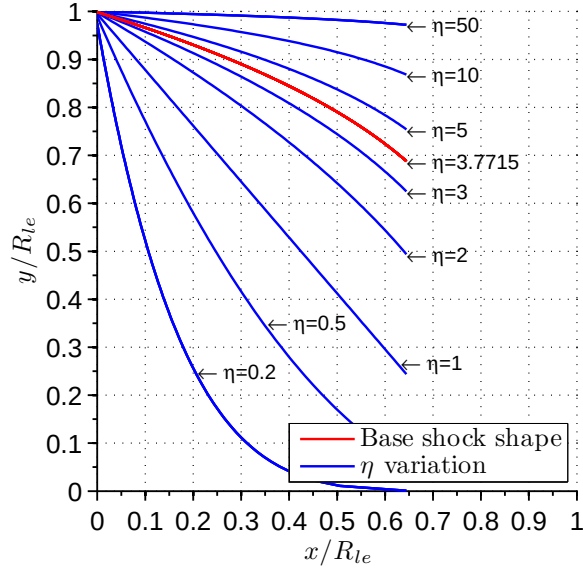


Figure 6.27: Variation of η with constant ε and λ for $R_c=1.5$ and $M=3.6$

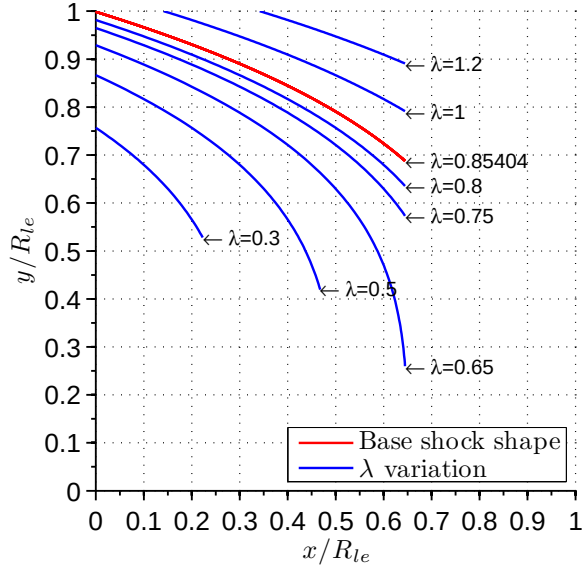


Figure 6.28: Variation of λ with constant η and ε for $R_c=1.5$ and $M=3.6$

Firstly, referring to Figure 6.26, as ε decreases while η and λ remain constant, shock wave shape flattens and translates further down. This is expected as ε plays the part of the denominator in Equation (6.13). The larger the denominator becomes, the more the function is pushed to resemble that of a constant y value as it overpowers the influence of the other curve fit constants, η and λ . Linking this to the relationship of ϕ_ε versus R_c in Figure 6.22, as R_c increases so does ϕ_ε . Therefore the larger the internal radius of curvature, the larger the negative value of ε causing the shock wave to flatten out as the imposed curvature is decreasing. In Figure 6.27, as η decreases while ε and λ remain constant, the shock wave

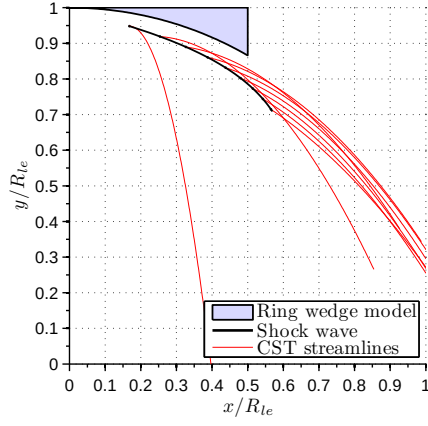
shape loses its curvature flow plane. Referring to Figure 6.23 as R_c increases, ϕ_η decreases therefore the value of η is set to decrease due to the normalising ratio involved in calculating ϕ_η . The larger the value of R_c , the less flow turning and curvature that is imposed on the shock wave shape which matches Figure 6.27 as the shock wave shape becomes less curved with decreasing η . For this specific input parameter set any values of $\eta < 1$ while keeping ε and λ constant would produce a shock shape that is convex and fictitious as a concave internal surface cannot produce a convex shock wave. Finally Figure 6.28 shows decreasing values of λ which seem to impose more curvature on the shock shape and translate it to the left while ε and η remain constant. This correlates with Figure 6.24 as ϕ_λ increases with increasing R_c when the shock shape would appear to be less curved. With decreasing R_c , ϕ_λ would decrease, curving the shock shape to a larger extent as more curvature is imposed since curvature is the inverse of radius of curvature. The above analysis of the variation in ε , η and λ was merely completed to show a basic relationship between R_c and how it influences the shock wave shape. The curve shock wave shape constants are all coupled by the underlying physics within the flow field. One cannot select a value of η to obtain a shock shape in Figure 6.27 and select a Mach number thereby calculating a value of R_c for which to apply to an internal surface. ε and λ are coupled in the calculation and would not produce the intended shock wave shape.

The extent to which ε , η and λ are coupled with each other and to physical parameters such as R_c , α , L_w and Mach number is hidden in the governing physics of the flow fields and mathematical relationships that determine them. Section 6.2 was intended to provide a means to calculate curve fits for the continuously curved shock wave regions for future analyses and show a sense of self similarity between the experimental parameters and results they produced. To determine a concrete relationship for the shock wave shapes, the physics and mathematics involved would require far larger data sets across a broad spectra of governing parameters and simulation cases. Equation (6.13) merely exists to display the evident patterns of self similarity involved in curved shock waves and provide a starting point for further developments. Its applicability is only limited to cases where $R_c \geq 1$, $M > 2.9$, $\alpha = 4^\circ$ and $L_w \leq 0.5$.

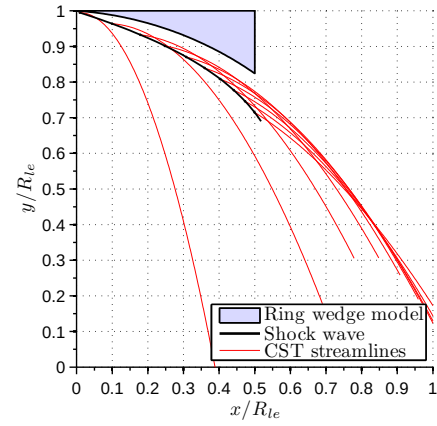
6.3 Curved Shock Theory Streamlines

In an attempt to mimic the curved internal surface which generated the curved shock wave using only the shock shape, CST was employed using a self developed MATLAB script. A shock shape in each case is needed which is supplied by Equation (6.2). By using the approximation for a streamline behind a curved shock wave, Equation (2.16), one can theoretically estimate the wall profile using the shape of the streamline closest to the wall. This gives an indication of the applicability of the CST model in predicting downstream conditions and

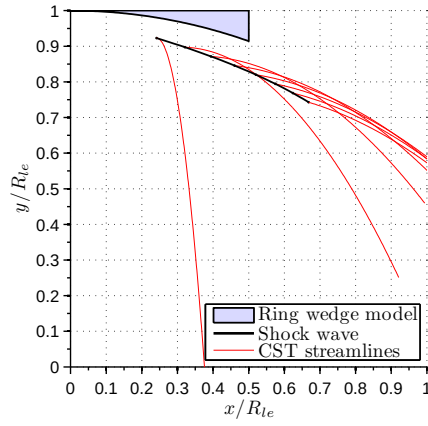
parameters which define the shock wave shape. Figure 6.29 displays the CST streamlines at Mach 3.4 for the $\alpha=\{0^\circ, 4^\circ\}$ models.



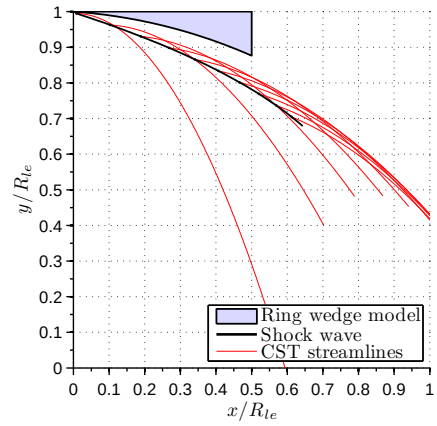
(a) CST streamlines where $R_c=1$, $\alpha=0^\circ$



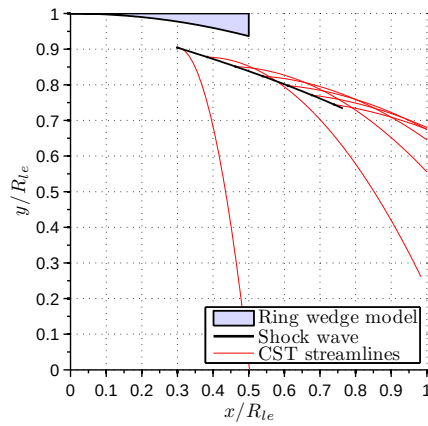
(b) CST streamlines where $R_c=1$, $\alpha=4^\circ$



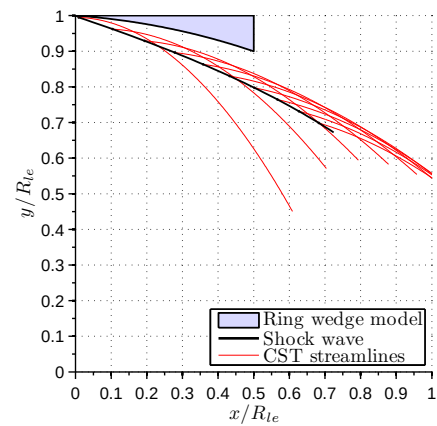
(c) CST streamlines where $R_c=1.5$, $\alpha=0^\circ$



(d) CST streamlines where $R_c=1.5$, $\alpha=4^\circ$



(e) CST streamlines where $R_c=2$, $\alpha=0^\circ$



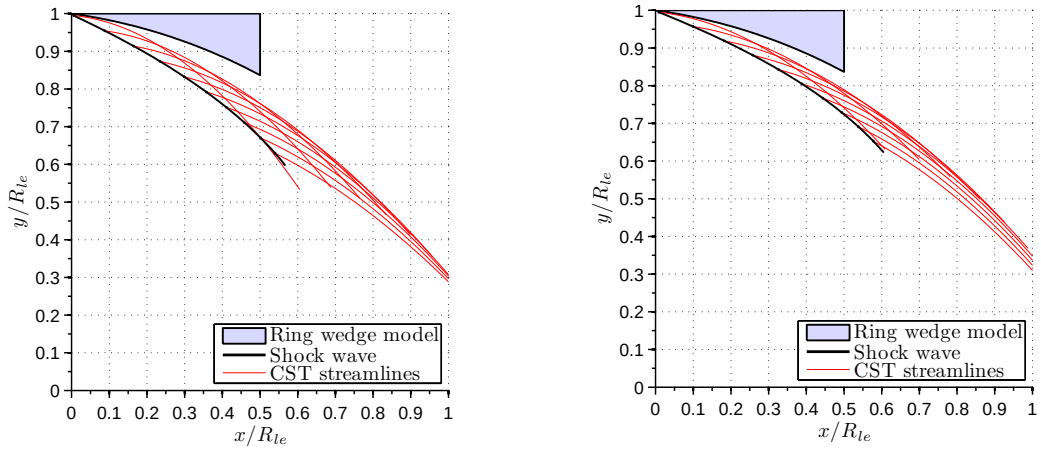
(f) CST streamlines where $R_c=2$, $\alpha=4^\circ$

Figure 6.29: CST predicted streamlines at Mach 3.4

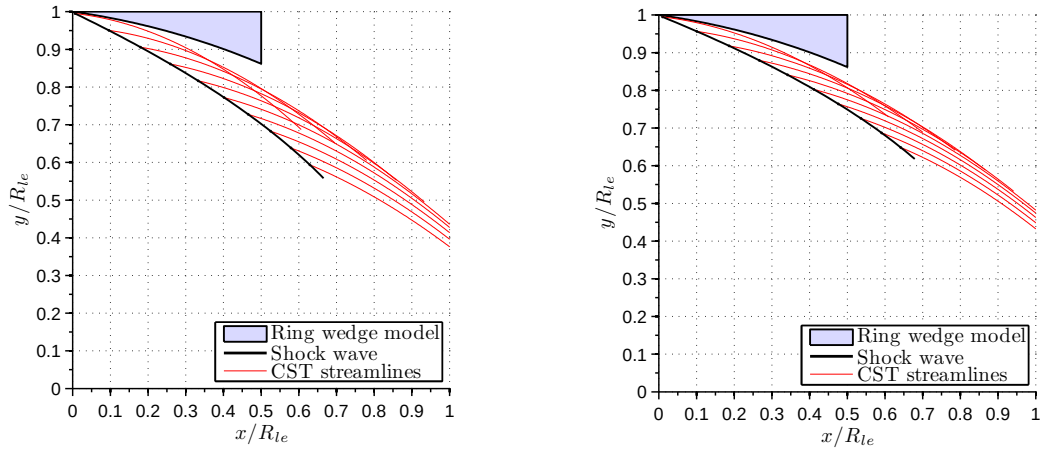
In the first image column in Figure 6.29 (models where $\alpha=0^\circ$) it can be seen that their shock waves start at some point after the leading edge. This is due to the initial isentropic compression present in the $\alpha=0^\circ$ cases which only form shock waves downstream. Therefore the shock wave is only drawn from the location where the shock angle surpasses the Mach angle for the given flow conditions signifying the formation of a shock wave. CST is only applicable to shock waves and so it can only be applied once the shock angle is greater than the flow's Mach angle, calculated by Equation (2.4). The second image column in Figure 6.29 (models where $\alpha=4^\circ$) contain shock waves from the leading edge due to their initial leading edge flow deflections. As can be seen in Figure 6.29 the streamlines do not behave appropriately and cross which is not possible when their y -coordinates approach negative infinity. This is more pronounced in the $\alpha=0^\circ$ models. The reason this occurs is due to the shock angle being very close to the Mach angle. Referring to Figure 2.11 the influence coefficients, which control the nature of the CST streamline curvature term D_2 , approach infinity near the Mach angle. This feature is then carried through when the streamlines are drawn as D_2 is used to calculate their describing functions. Hence the closer one is to the Mach angle, the more prone the streamlines are to curving excessively and crossing.

With α being increased to 4° the initial few streamlines do not curve as drastically as their $\alpha=0^\circ$ counterparts although they still cross. This is due to the shock angle moving away from the Mach angle and so the influence coefficients which control D_2 become more stable. As one moves down the shock wave, with the shock angle deviating further from the Mach angle, the streamlines approach shapes which one would expect to be more reasonable. The curvature of the streamlines becomes less prominent with increasing R_c . This is expected because as the internal surface becomes less curved, less curvature is imposed on the shock wave and flow field. This is evident in the $\alpha=4^\circ$ models where as R_c increases, the streamlines toward the end of the shock wave do not curve as excessively toward the axis of symmetry. For the $\alpha=\{0^\circ, 4^\circ\}$ models CST fails to predict the profile of the internal surface used to generate their accompanying particular shock shape. The streamline curvature term D_2 causes the streamlines to curve excessively the closer one is to the Mach angle. Since the streamline closest to the wall experiences a shock angle closest to the Mach angle, it's curvature is more affected by large D_2 values and hence it does not follow the wall profile.

To examine the effect of Mach number on the streamlines and surface prediction two different Mach numbers were explored for the $\alpha=8^\circ$ models namely Mach 2.8 and 3.4. The maximum α cases were used due to them being the furthest away from possible Mach angle proximity effects for each Mach number. Figure 6.30 displays the $\alpha=8^\circ$ models at Mach 2.8 and 3.4.



(a) CST streamlines where $R_c=1.5$, $\alpha=8^\circ$, $M=2.8$ (b) CST streamlines where $R_c=1.5$, $\alpha=8^\circ$, $M=3.4$



(c) CST streamlines where $R_c=2$, $\alpha=8^\circ$, $M=2.8$ (d) CST streamlines where $R_c=2$, $\alpha=8^\circ$, $M=3.4$

Figure 6.30: CST predicted streamlines at Mach 2.8 and 3.4

Now that α has increased to 8° in Figure 6.30, the effect of the infinity approaching influence coefficients near the Mach angle has decreased. This improvement in behaviour of D_2 has begun to have an effect on how closely the streamlines resemble the wall profile which can be seen in Figure 6.30. Each initial shock wave streamline in Figure 6.30a through Figure 6.30d now follows the wall profile until it begins to deviate and curve toward the axis of symmetry downstream. This confirms what was shown in Figure 6.29 where with increasing α , the streamlines become less prone to excessive curvature. The increase in R_c has become more apparent now that the first streamline in each case follows the wall profile. With constant Mach number the first streamline of $R_c=2$ models break away from the wall further downstream in comparison to those of the $R_c=1.5$ model. Therefore the larger the radius of curvature applied to the internal surface, the more accurately the initial streamline paths predicted by CST follow the wall profile, decreasing their tendency to cross.

Comparing the increase in Mach number between Figure 6.30a and Figure 6.30b and between Figure 6.30c and Figure 6.30d, it can be seen that the larger Mach number case of $M=3.4$ contains initial streamlines that break away from the wall further downstream. This effect however is not as pronounced as the effect due to the change in R_c values. The reason the streamlines follow the wall profile more closely at higher Mach number is due to the change in shock angle. Since α is being kept constant and the Mach number is increasing, via Figure 2.4, the shock angle is said to decrease. This decrease in shock angle then affects the value for D_2 via the coefficients used in Equation (2.9) and Equation (2.10) as the value of θ , the shock angle, varies. The higher the Mach number, the lower the shock angle and the lower the value of D_2 . For numeric comparison when $M=3.4$, $\theta=(180^\circ-158.65^\circ)=21.35^\circ$ and $D_2=-1.2901$ for the first streamline as opposed to when $M=2.8$, $\theta=(180^\circ-154.16^\circ)=25.84^\circ$ and $D_2=-1.3310$. Therefore the excessive curvature imposed on the streamline is decreased and hence it follows the wall profile more closely.

For this exercise the shock angle and curvature terms are extracted from the shock shape curve fit of Equation (6.2) using simple calculus. Therefore the change in δ_2 , the post shock deflection, can be observed when given the accompanying flow conditions. Although with increasing flow Mach number, the value of δ_2 increases, its impact is reduced due to it being used in the tan function in Equation (2.16). The influence of the change in D_2 therefore outweighs the impact of the change in δ_2 so the streamline function is not adversely affected to the extent where they counteract one another. Therefore the higher the Mach number and wedge entry angle, α , the closer CST comes to predicting the internal wall profile.

To examine the applicability of the CST drawn streamlines they can be compared to those produced from the CFD simulations in the same locations. Two cases are explored for comparison purposes namely the $R_c=\{1.5, 2\}$, $\alpha=8^\circ$ models at Mach 3.6. These parameters were chosen as streamlines at higher values of α and Mach number were shown to produce more reasonable streamline shapes as previously mentioned. Figure 6.31 and Figure 6.32 display the comparison between the CST and CFD streamlines at Mach 3.6 for the above stated cases. For the sake of clarity only six streamlines were drawn from a normalised height of $y/R_{le}=0.95$ to $y/R_{le}=0.7$. The dashed black line depicts the start of the trailing edge expansion fan. Its location was calculated by tracking pressure along each streamline and determining the point where the pressure began to decrease, signifying the start of the expansion fan. The expansion fan is what changes the direction of the CFD streamlines gradually, converting them from a concave to a convex profile. Although the expansion fan is curved in nature, its location has been approximated using only the six displayed streamlines therefore its shape may not be extremely accurate. But it is enough to approximate the bounding expansion region between the ring model trailing edge and end of the curved shock wave segment.

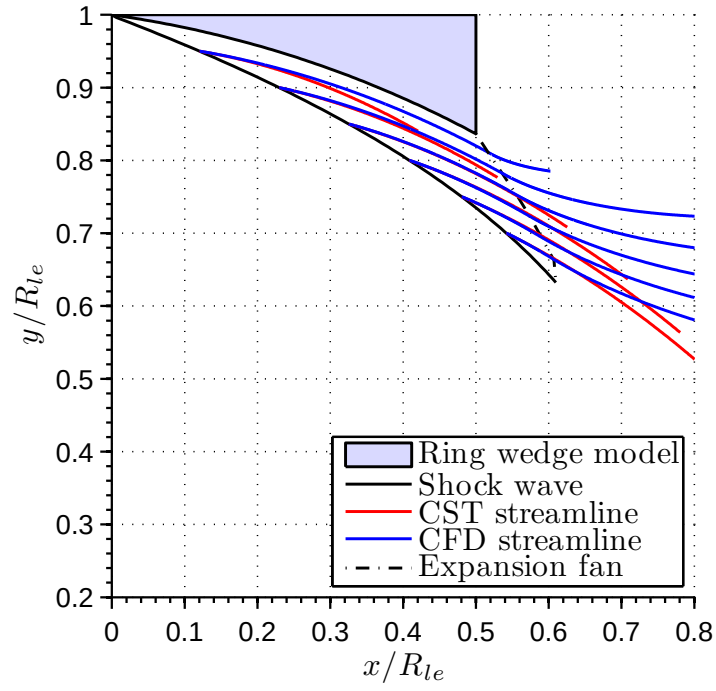


Figure 6.31: Comparison between CFD and CST predicted streamlines at Mach 3.6 for the $R_c=1.5$, $\alpha=8^\circ$ model

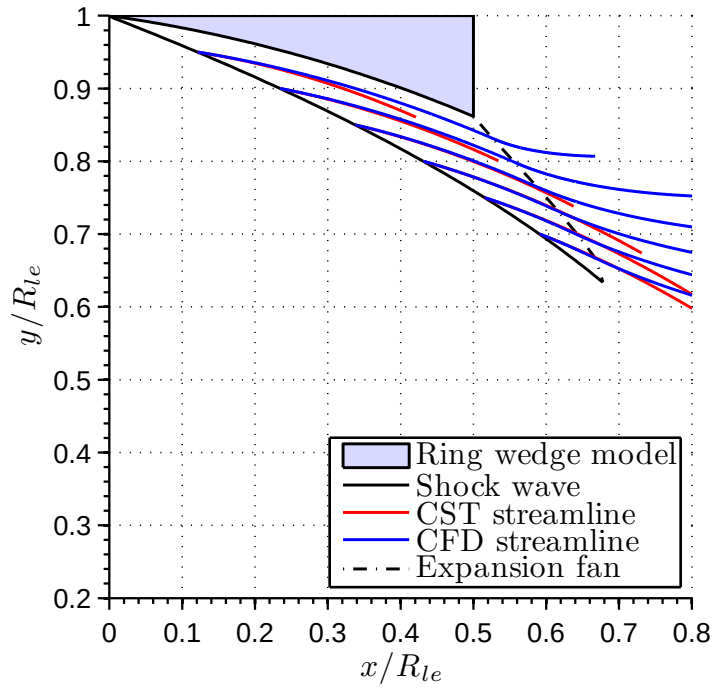


Figure 6.32: Comparison between CFD and CST predicted streamlines at Mach 3.6 for the $R_c=2$, $\alpha=8^\circ$ model

The further along the shock wave one travels, the more in agreement the CST and CFD shock waves are with one another. The streamlines in each case are calculated independently with the only common factor being the shock shape which is curve fitted from CFD results. The CFD streamlines are extracted from numerical results whereas the CST streamlines are predicted by theoretical means as suggest in [15]. CST is only applicable directly after the shock wave however using its calculated coefficients and streamline approximation it may be extrapolated further downstream. This extrapolation is only valid when the streamline curvature term D_2 takes on values where it does not cause the streamlines to cross, which is not physically possible. It is in this domain that the CFD and CST streamlines bare extremely close resemblance. The value of D_2 is affected by the curvature imposed by the internal surface which was shown above to be dependent on R_c . For increasing values of R_c , D_2 imposes less excessive curvature which produces more accurate results. This is evident when comparing Figure 6.31 and Figure 6.32. The streamlines predicted by CST follow the shape of the CFD streamlines more closely further downstream in Figure 6.32 where $R_c=2$ as opposed to Figure 6.31 where $R_c=1.5$, causing them to break away further upstream.

Focusing on the more agreeing case of Figure 6.32, the more accurate CST streamlines seem to only break away from those of the CFD results when the influence of the expansion fan comes into play. CST fails to take this into consideration as the presence on the trailing edge, from which the expansion fan originates, cannot be accounted for. Therefore the CST streamline shape approximation causes them to continue along a concave path. The shock angle's influence on the CST streamlines can now be visualised in comparison to the actual path predicted via CFD results. It was concluded previously that the inability for the wall profile to be predicted via CST streamlines is due to the proximity of the leading edge shock angle to the flow Mach angle. This proximity is what causes the CST streamline to deviate from the actual physical result. Now that the CFD results have been overlaid, one can see how a streamline close to the internal surface follows the wall profile to a certain extent.

If one notes the vertical distance between the start of the first CFD streamline and the internal surface at the shock wave then at the trailing edge, a difference can be seen. The first distance appears to be slightly larger than the second distance meaning that the streamline does not follow the wall exactly. The internal surface curvature is constant meaning that if the streamline where to predict the wall profile exactly it too would be constant, however it is not. The reason this occurs is due to the initial flow deflection through the curved shock wave. This causes the streamline to initially point in a direction governed by Equation (2.8) as a curved shock wave is essentially comprised on infinitely small oblique shock waves. As the streamline continues further downstream the internal surface curvature begins to turn the flow and streamlines. This outcome is not immediate due to the angle of the characteristics governing this procedure and so the streamlines close to the wall only follow it's resembling

profile downstream. This is confirmed in Figure 6.33 which displays a close up of the streamlines near the internal surface of the $R_c=2$, $\alpha=8^\circ$ model at Mach 3.6.

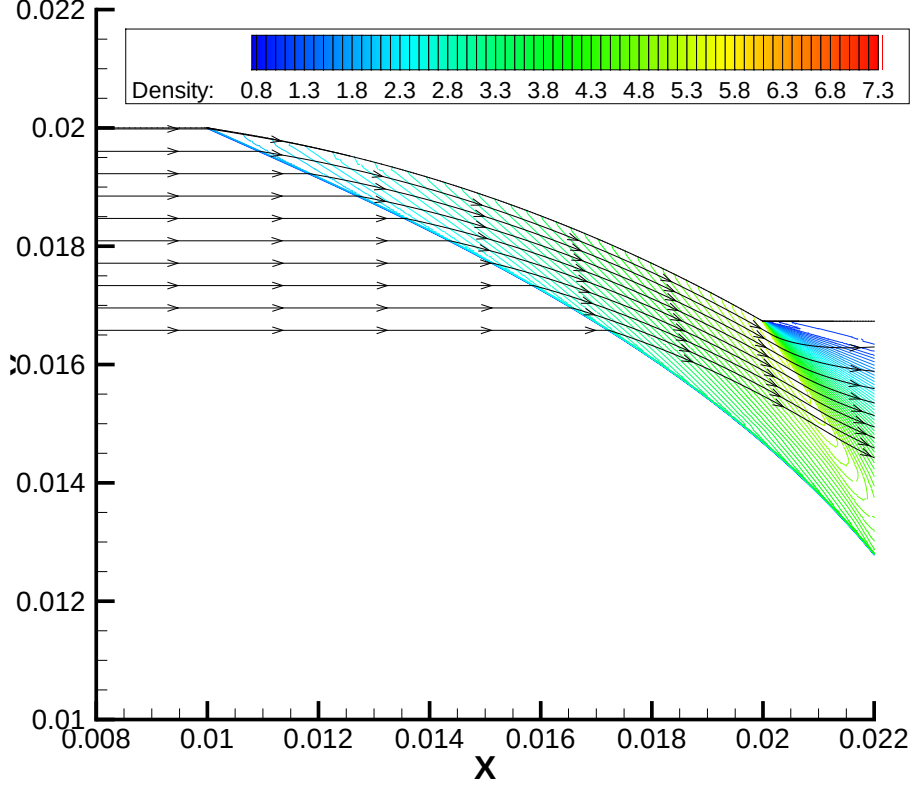


Figure 6.33: CFD streamlines near the internal surface of the $R_c=2$, $\alpha=8^\circ$ model at Mach 3.6

The streamlines in Figure 6.33 are more vertically spread the further upstream they are. They begin to bunch up the closer they approach the trailing edge where the influence of the wall curvature is at a maximum. This gap spacing difference means that they do not initially exactly follow wall profile but do so with increasing accuracy the further downstream the streamlines propagate. Hence a constantly curved wall surface does not guarantee a constantly curved streamline. Close to the wall, the effects of transverse curvature are at a minimum due to the maximum distance to the axis of symmetry. As a result only the streamlines further down the shock wave appear to be constantly curved for the entire distance they cover. This is due to the greater imposed transverse curvature combating the effects of the initial flow deflection at the start of the streamlines. This influence of transverse curvature does not account for the differences between the CFD and CST streamlines closer to the leading edge. The CST model takes into consideration the effects of transverse curvature due to the distance from the axis of symmetry via the inclusion of its S_b term. The streamlines further down the

shock wave experience the more developed wall surface's characteristics which emanate from further down the internal surface. These characteristics are steeper due to the curvature and post shock Mach number involved at the wall with increasing horizontal distance and therefore turn the flow and streamlines with a stronger effect. This causes the streamlines to appear to have a constant curvature, like the wall profile, the further along the shock wave they are as the characteristics initially rotate the post shock flow to a larger extent.

The resemblance between the results produced using CFD and CST further down the shock wave, away from the leading edge, is extremely close. Due to the relative infancy of CST, coupled with few people currently working on the flow fields behind steady curved shock waves, published data is not widely or publicly available for comparison purposes.

6.4 Effect of Varying Internal Surface Curvature

For clarity sakes when referring to general flow features produced by the axisymmetric ring models Figure 6.34 is displayed. Figure 6.34 contains the naming convention used and assigned to certain flow features mentioned throughout the remainder of Section 6.4 and Section 6.5. The different labelled features will be discussed in the subsequent sections.

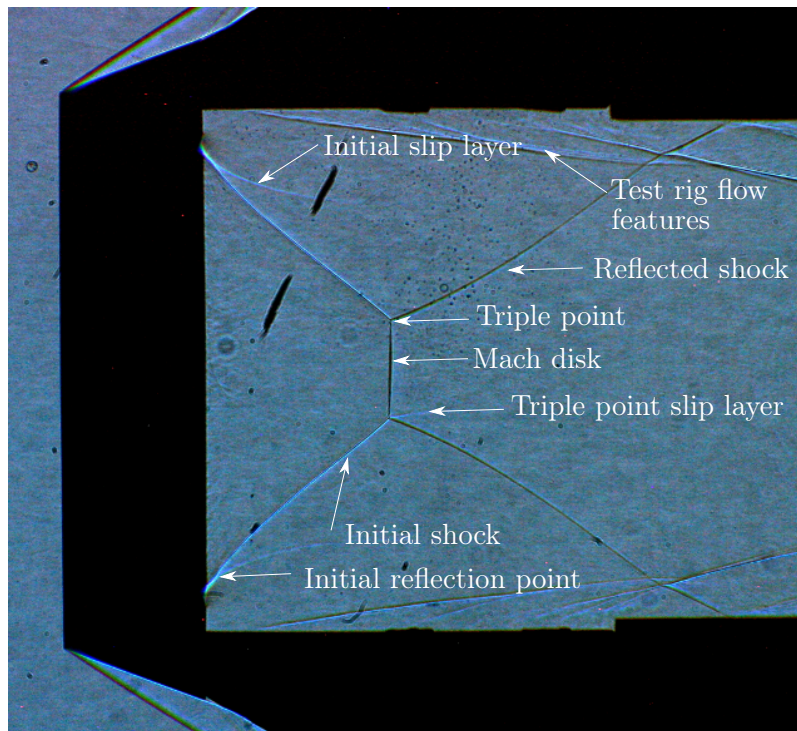


Figure 6.34: General flow feature naming convention

6.4.1 Wedge Entry Angle $\alpha=0^\circ$

When $\alpha=0^\circ$ the flow field and shock waves that form are purely controlled by the internal surface curvature. A shock wave does not exist from the leading edge and is replaced by the presence of an isentropic compression which coalesces into a shock wave downstream. The characteristics from the internal surface fan inward and cause the shock wave to curve as each interact with the shock structure. The angle of these characteristics is determined by the flow speed and deflection caused by the internal surface. Since an axis of symmetry is involved the flow field is three dimensionally curved. This causes the characteristics which emanate from the wall to be curved so their influence cannot be predicted easily using analytical methods. Due to the entropy changes involved in three dimensionally curved flow fields such as these past analytical techniques such as methods of characteristics become extremely complicated and arduous.

Considering the change in internal surface curvature, as R_c increases the internal surface curvature decreases since the radius of curvature increases. This affects how the shock wave in the internal flow field of ring wedge models form due to the characteristics that control their shape being driven by the surface curvature of the wall. Figure 6.35, Figure 6.36 and Figure 6.37 display the shadowgraphs of the $R_c=1$, $R_c=1.5$, and $R_c=2$ models respectively where $\alpha=0^\circ$ at Mach 3.0.

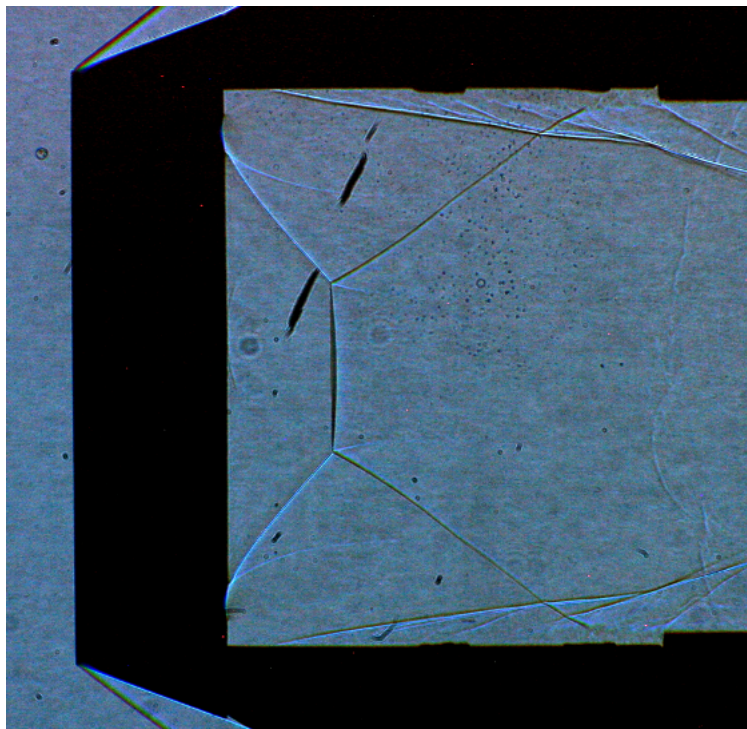


Figure 6.35: $R_c=1$, $\alpha=0^\circ$ model at Mach 3.0

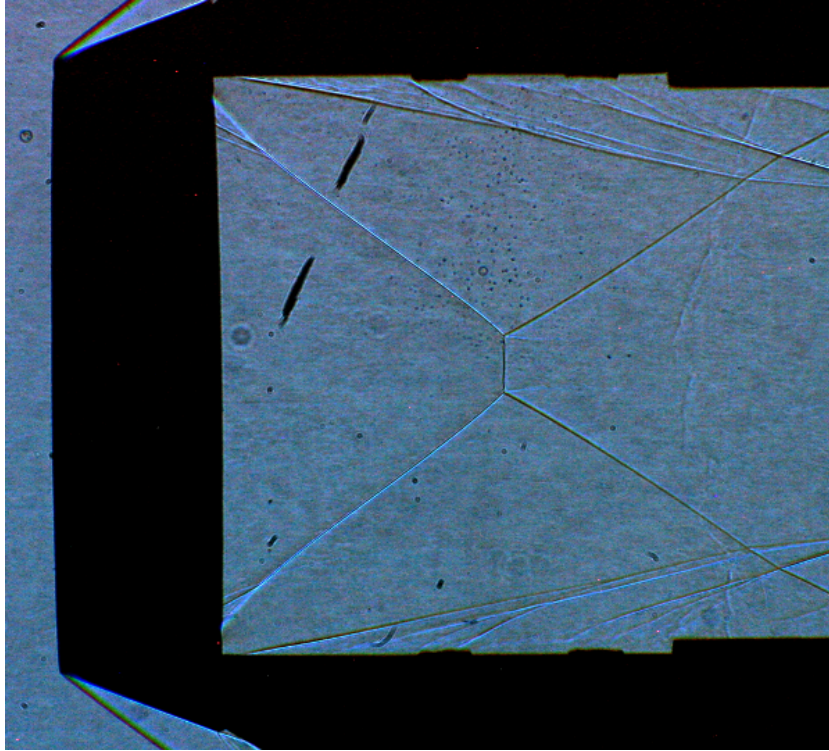


Figure 6.36: $R_e=1.5$, $\alpha=0^\circ$ model at Mach 3.0

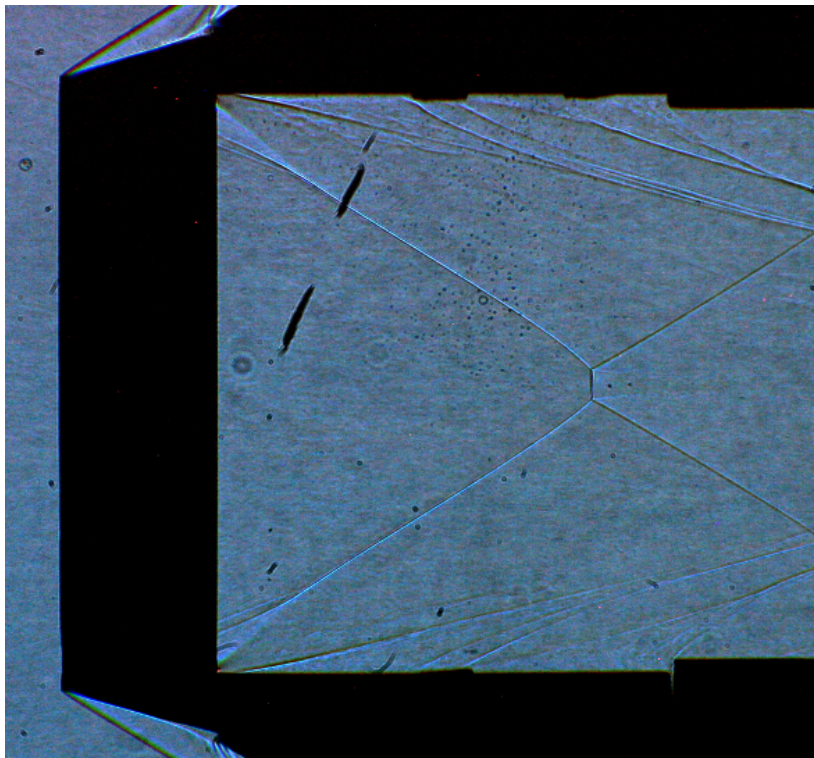
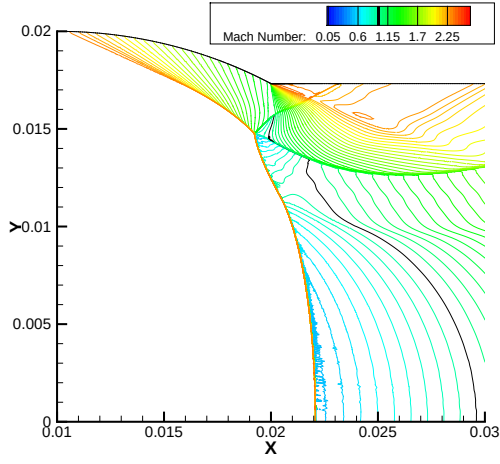


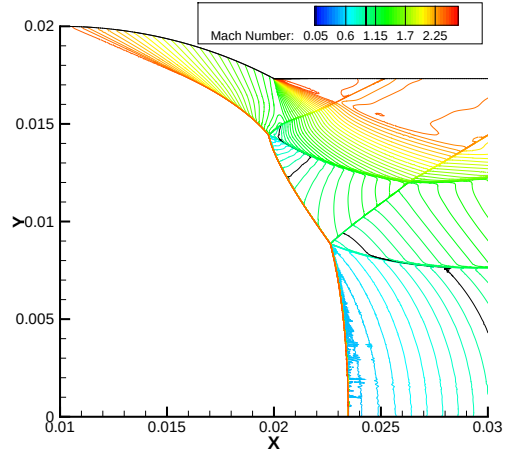
Figure 6.37: $R_e=2$, $\alpha=0^\circ$ model at Mach 3.0

Many differences are apparent between the different surface curvature cases. Firstly, examining the existence of an initial slip layer forming on the initial shock it can be seen that it is at its strongest in the $R_c=1$ case. Increasing R_c to 1.5, the visibility of the initial slip layer is low however it can still be seen very faintly with magnification into the image. Finally at $R_c=2$ the slip layer is non-existent. The strength of the slip layer is governed by the ratio of flow velocities across it. The higher the value of R_c the more the shock wave and flow field is curved. This causes the post shock flow speed to decrease further than it would at higher R_c values. The stronger the influence of the curvature and the trailing edge expansion fan, the greater the velocity ratio across the slip layer so the stronger and more pronounced it is. The origin of the slip layer is explored in Figure 6.38.

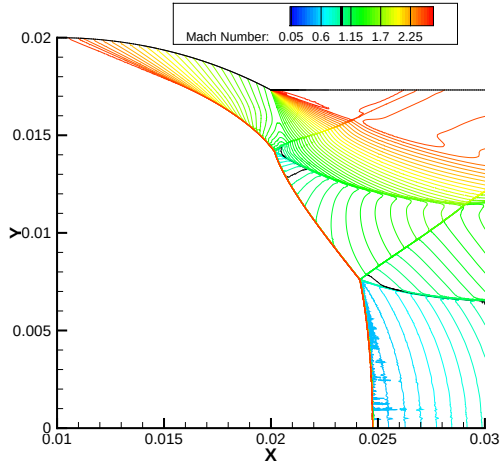
Figure 6.38 displays the Mach number contour plots for $R_c=1$, $\alpha=0^\circ$ model at various stages of the Mach number progression. The black line present in some plots depicts the sonic line. Contours surrounded by it are subsonic whereas contours past it show supersonic regions. Starting off in Figure 6.38a, the initial reflection point is present in the flow field, out of the influence of the trailing edge expansion fan. Although the main triple point has not formed as of yet it can be seen that a triple point slip layer is starting to form as the Mach contours bunch around a height of approximately $y=0.01$ m. The initial reflection forms to readjust the flow from turning into itself at the current Mach number. In Figure 6.38b the Mach number has increased to Mach 2.5. A double reflection pattern is now visible with the initial reflection occurring further up the shock wave and the triple point Mach reflection forming further downstream. As the expansion passes through the slip layer it still causes convexity in the initial shock before the triple point. This causes the flow to turn and be readjusted by the Mach reflection. Four subsonic regions are now present behind each reflected shock and under each slip layer. At Mach 2.6 and 2.7 in Figure 6.38c and in Figure 6.38d respectively the strength of the initial reflection weakens. This can be seen due to the decrease in the size of the subsonic regions behind the initial reflection. The decrease in strength comes about as the shock angle decreases when the Mach number is increased with the deflection of the flow remaining constant due to unchanging wall profile. This decreases the initial reflected shock strength, as seen in the contours which make up the initial reflected shock whose thickness and discontinuity begins to decrease. As the Mach number is increased to Mach 2.8 and Mach 3.0 in Figure 6.38e and Figure 6.38f respectively, the double reflection pattern has begun to vanish. A very small subsonic region is still present at Mach 2.8 in Figure 6.38e meaning that a very weak shock still exists at that location. The initial slip layers are still present at both Mach 2.8 and Mach 3 even though a reflected shock is not visible. Its formation must therefore be as a result of the internal curvature.



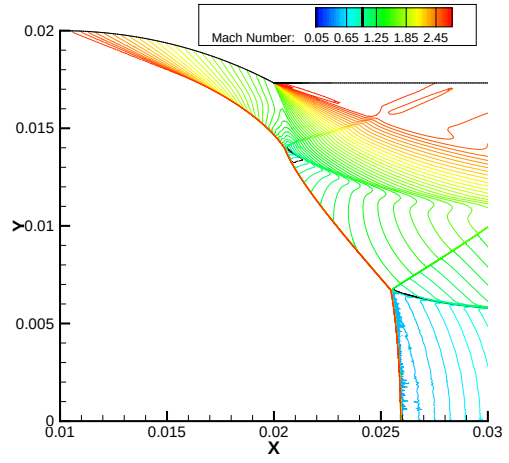
(a) Mach 2.4



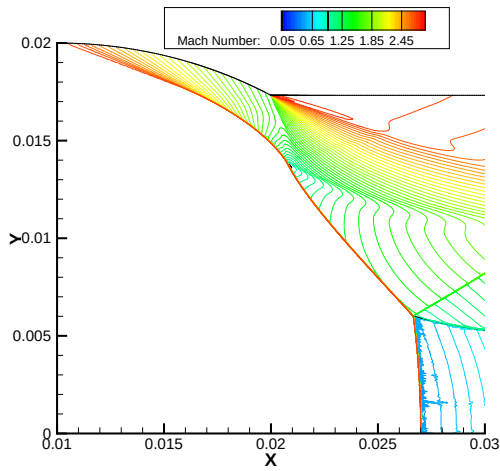
(b) Mach 2.5



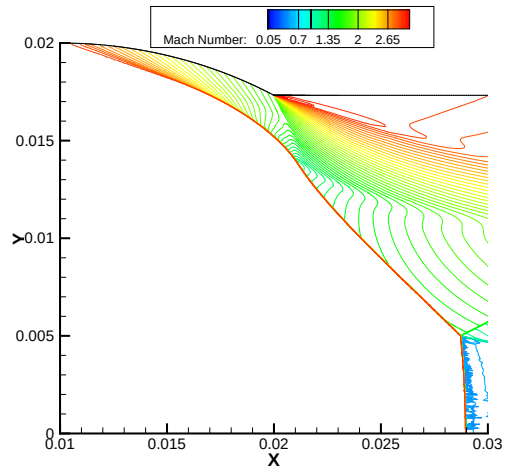
(c) Mach 2.6



(d) Mach 2.7



(e) Mach 2.8



(f) Mach 3.0

Figure 6.38: Mach number contours for the $R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ model at Mach 2.4, 2.5, 2.6, 2.7, 2.8 and 3.0

At Mach 2.8 the initial reflection point begins to enter the expansion fan's influence as the shock moves further upward and downstream with increasing Mach number. This could be another reason as to why the presence of the initial reflected shock is not seen. The interference of the expansion fan begins to affect the formation of the initial reflection and readjustment of the flow field. At Mach 3.0 it is clear that the continuously curved shock wave segment's length is now determined by the location where the trailing edge expansion fan begins to introduce convexity to the initial shock wave. Since the post curved shock's Mach number is supersonic, information about the presence of the expansion fan ahead of it cannot be relayed upstream. The flow slows and adjusts to the curved wall via the curved shock wave until it meets the expansion fan where it is accelerated. The slip layer that forms at this point is very weak, as can be seen by the weak pattern formed by the Mach number contours in Figure 6.38f, but it is still present. This is validated in Figure 6.35 where the experimental shadowgraph results show the same ever present slip layer. Since the varying strength of the slip layer can be seen in Figure 6.35 through Figure 6.37 by its decreasing visibility, its formation must be linked to the change in internal surface curvature. This is the the only changing parameter between the models in three separate shadowgraph result sets.

The additional flow features in Figure 6.35, at the top and bottom of the model, out of the reflection zone, are formed by the test rig. These test rig flow features are created due to wake forming behind the test rig and interacting with the connecting mounts. The expansion fan which is reflected in onto itself by the arms that extend out the trailing edge surface, which connect to the rod supports, now becomes compressive in nature and form additional test rig flow features.

Returning to the comparison between the different curvatures involved, the differences in location on the Mach disk is as a result of the angle of the initial shock pointing downstream. The flow leaving the ring model is deflected through a greater angle as R_c decreases. Therefore the incident shock of the Mach reflection is steeper. This will then affect the progression of the Mach disk further downstream. The change in R_c will additionally change the size of the angle through which the flow must adjust via the expansion fan. The smaller the value of R_c , the larger the internal surface curvature. This means the angle between the tangent of the internal surface and back surface of the trailing edge is larger as the tangent becomes steeper with decrease in R_c . This increase in angle through which the flow expands via the expansion fan will correlate to a larger pressure ratio across the expansion fan, increasing its strength and influence. The difference in the expansion wave fan strength for $\alpha=0^\circ$ models at Mach 3.4 can be seen in Figure 6.39, Figure 6.40 and Figure 6.41 using colour mask schlieren techniques. The expansion fan is more clear on the bottom trailing edge.

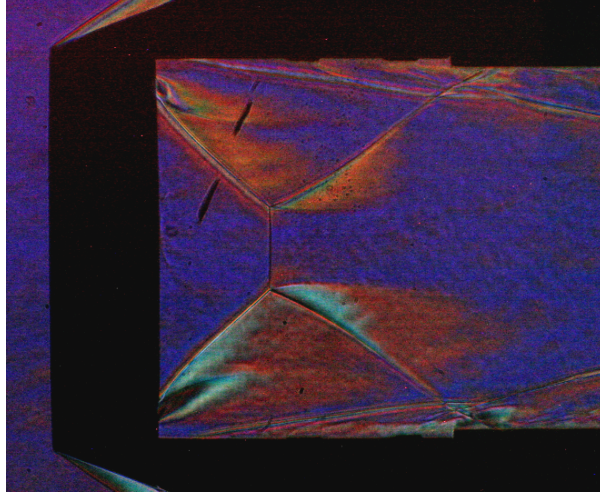


Figure 6.39: Colour mask schlieren for the $R_c=1$, $\alpha=0^\circ$ model at Mach 3.4

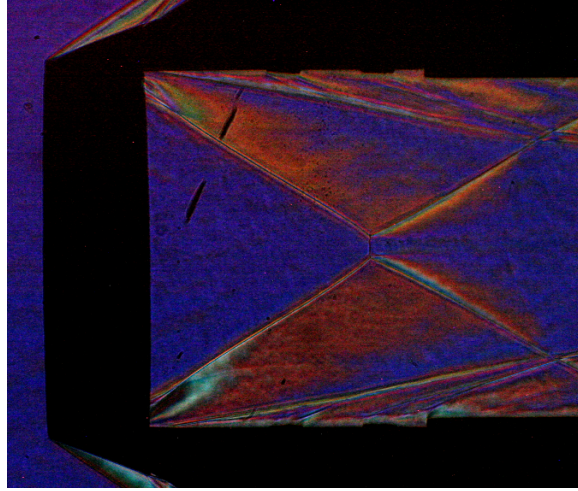


Figure 6.40: Colour mask schlieren for the $R_c=1.5$, $\alpha=0^\circ$ model at Mach 3.4

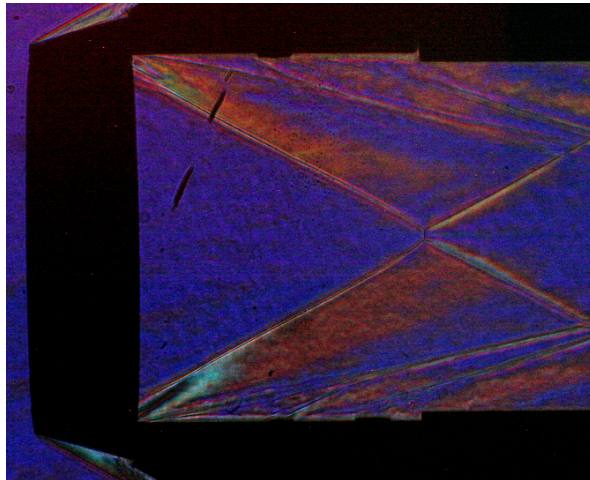


Figure 6.41: Colour mask schlieren for the $R_c=2$, $\alpha=0^\circ$ model at Mach 3.4

In Figure 6.39, R_c is at the minimum value of 1 therefore the internal surface has more curvature. This would mean that the flow has to pass around the largest angle of the three cases at the trailing edge resulting in the strongest expansion fan. This is confirmed by examining the relative size of the expansion fans and density gradients displayed in Figure 6.39, Figure 6.40 and Figure 6.41. The larger the value of R_c , the weaker the expansion and therefore a smaller sized affected region is seen in the density gradients. Since the internal surface curvature directly controls the deflection, it governs what speed the flow exits the ring wedge. This then controls the angle of the first expansive characteristic Mach line, which makes up part of the expansion fan, and controls what direction it originally starts at. The slower the flow, the steeper the angle, causing the expansion fan to interfere with the initial shock sooner. Additional convexity is then introduced into the initial shock wave, curving it, therefore inhibiting its ability to propagate further downstream.

The comparative density gradients behind the initial shocks in each R_c case can be visualised by the amount a region deviates from the purple background into the orange and red bands of the horizontal lattice mask used in the colour mask schlieren setup and shown in Figure 4.7c. The relative strength between the initial shock waves can be seen by the size of the red and orange regions behind the initial shock wave. This dictates how much light it refracted by the changes in air density causing it to pass through the different colour bands. For the $R_c=1$ case one would expect the shock wave leaving the internal ring wedge section to be steeper due to the increased flow deflection it experiences in comparison to the $R_c=1.5$ and $R_c=2$ cases. Its strength would then be comparatively higher which can be seen as the flow field behind the initial shock in Figure 6.39 is filled by more red and orange colour regions relative to Figure 6.40 and Figure 6.41.

With the change in R_c manipulating the nature of the expansion fan, the effect of transverse curvature is hampered. Transverse curvature is brought about by the axial symmetry of the flow field causing it to turn flow toward the central axis of symmetry. In the $R_c=1$ case the expansion fan causes the initial shock to become convex, evident in Figure 6.39. This convexity bends the shock away from the axis of symmetry thereby counteracting the effects of transverse curvature which rely on proximity to said central axis. The further downstream the reflection moves the more apparent the effect of transverse curvature. With distance from the trailing edge the influence of the expansion fan is weakened and so it loses its ability to combat and oppose the onset of concavity in the shock wave due to transverse curvature. Its effects then become more pronounced. Figure 6.42 and Figure 6.43 display shadowgraph images of the $R_c=1.5$ and $R_c=2$ $\alpha=0^\circ$ models respectively at Mach 3.6 where their reflections will be at the furthest point downstream.

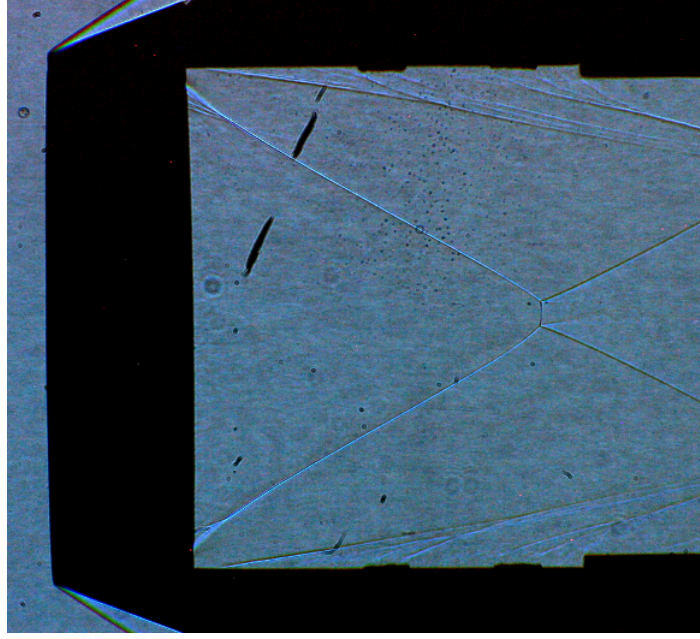


Figure 6.42: $R_c=1.5$, $\alpha=0^\circ$ model at Mach 3.6

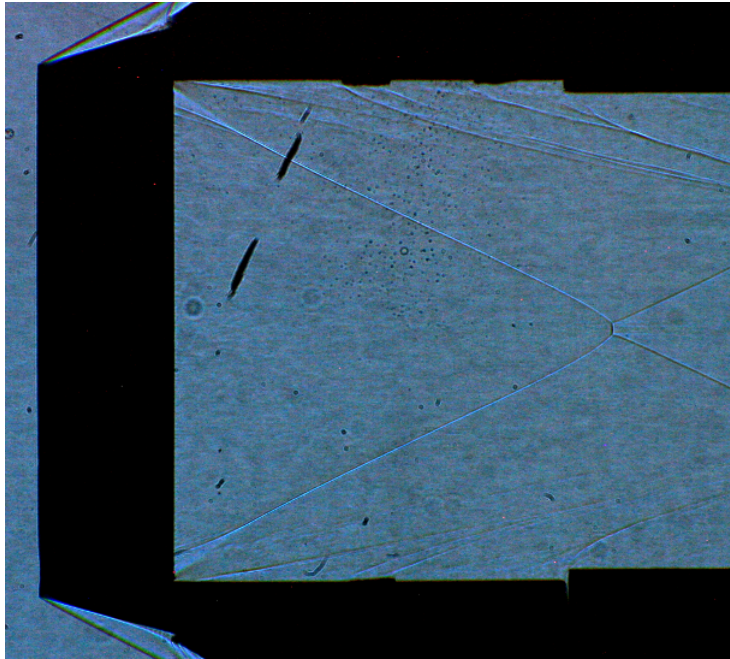


Figure 6.43: $R_c=2$, $\alpha=0^\circ$ model at Mach 3.6

In Figure 6.42 the convexity induced by the expansion can be seen to slightly affect the start of the incident shock wave. The shock wave then straightens out after the influence of the expansion has weakened and begins to bend inward toward the triple point due to transverse curvature effects. In Figure 6.43 the expansion fan's induced convexity is not visible due to its weakened effects as R_c has increased. Since the reflection has moved further downstream, the triple point of the Mach reflection has moved closer to the axis of symmetry. Transverse

curvature is dependant on the vertical distance between the shock and the axis of symmetry so the closer one gets the more abrupt the induced curvature. This is why the initial shock wave near the triple point for the $R_c=2$ case turns more rapidly in comparison to that of the $R_c=1.5$ case. The stronger the influence of transverse curvature, the more rapid the shock rotation toward the axis of symmetry.

The progression of Mach number does not have as great an impact on the formation of the shock wave structures in comparison to varying the internal surface curvature. Since the internal surface remains constant for each ring wedge, by increasing Mach number all that is achieved is the variation of the angles of the characteristics that curve the shock wave. The higher the Mach number the more swept back they are. This causes the shock to curve to a greater extent further downstream, creating a more gentle angled initial shock wave, sending the Mach reflection to a more downstream position.

6.4.2 Wedge Entry Angle $\alpha=4^\circ$

With the the value of the wedge entry angle, α , being increased to a non-zero value, an initial deflection exists at the leading edge. Therefore an attached leading edge shock exists for the wall characteristics to curve. Examining the initial reflection slip layer that was previously mentioned in Section 6.4.1, Figure 6.44 and Figure 6.45 display shadowgraphs for the $R_c=1$ and $R_c=1.5$, $\alpha=4^\circ$ models respectively at Mach 3.6.

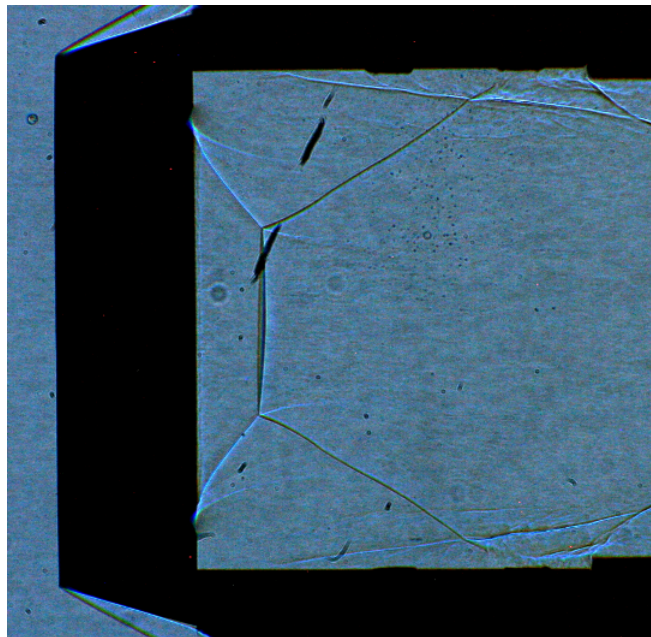


Figure 6.44: $R_c=1$, $\alpha=4^\circ$ model at Mach 3.6

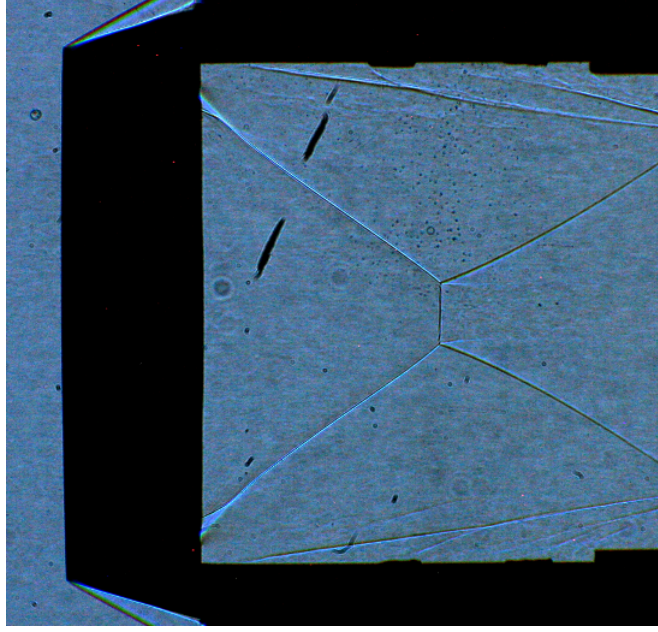


Figure 6.45: $R_c=1.5$, $\alpha=4^\circ$ model at Mach 3.6

For the $R_c=1$ model in Figure 6.44 the initial slip layers are still very pronounced whereas in Figure 6.45 it is not visible for the $R_c=1.5$ case. Their presence must therefore be a function of R_c as previously suggested. Since the initial slip layer is present in a case where $\alpha=4^\circ$ it is not a product of the isentropic compression to shock wave transformation of the $\alpha=0^\circ$ case either. Its formation must then be linked to the way the flow is deflected in each R_c separate case. A shock wave reflection exists to realign supersonic flow therefore the excess deflection, caused by lower values of R_c , increasing the internal surface curvature, causes the flow to want to turn in onto itself. This then must be corrected so a reflection forms. This can be confirmed by examining the shapes of the characteristics emanating from the wall, the characteristics in question, by nature, being weak pressure waves.

The pressure contours for the $R_c=1$ and $R_c=1.5$, $\alpha=4^\circ$ models at Mach 3.6 are displayed in Figure 6.46. These two cases match Figure 6.44 and Figure 6.45 where a slip layer is present for the $R_c=1$ model, meaning a very weak shock must exist. A slip layer itself is isobaric so no pressure difference exists across its boundary hence they are not visible in Figure 6.46. The only reason a slip layer like feature exists in Figure 6.46a at the Mach reflection triple point is due to numerical instabilities that arise from a Kelvin-Helmholtz instability forming and being refined by mesh adaptation. The location of the slip layer is given by the placement of a weak initial reflected shock in Figure 6.46a. The characteristics which are represented by the pressure contours show how they shape and turn a curved shock wave with progression along the internal curved surface. The closer one moves down the initial shock toward the first reflection point, the characteristics can be seen to change direction and point upstream slightly. This is most pronounced right before the initial reflection point as they turn from

pointing downstream and begin to bend upstream meaning the flow is beginning to turn in on itself. Therefore the large flow deflection and curvature imposed by the $R_c=1$ internal surface coupled with transverse curvature is causing the characteristics to bend in on themselves and form a weak reflection and slip layer.

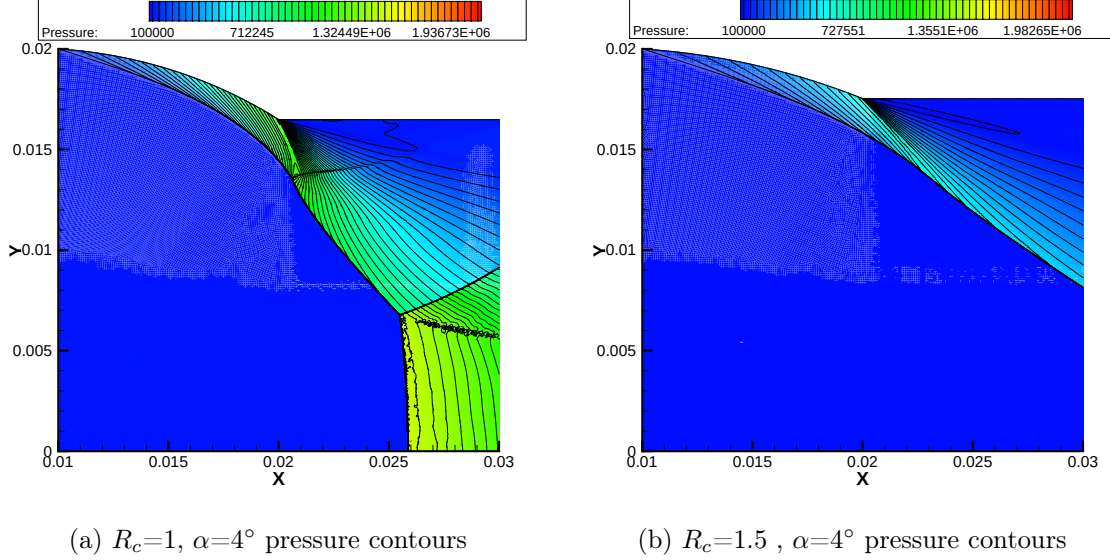


Figure 6.46: Pressure contours for the $R_c=1$ and $R_c=1.5$, $\alpha=4^\circ$ models at Mach 3.6

In comparison the characteristics of Figure 6.46b only extend downstream. They do not bend inward onto themselves causing no reflection to form as the imposed curvature is not high enough. This explains the absence of a slip layer and weak reflected shock given the Mach number of the post curved shock flow.

The remaining differences between any flow features at $\alpha=4^\circ$ have been covered in Section 6.4 and omitted here.

6.4.3 Wedge Entry Angle $\alpha=8^\circ$

With α at its maximum value of 8° the leading edge shock is now at its steepest. Due to the increased exit area contraction, the $R_c=1$, $\alpha=8^\circ$ model was not possible to experimentally or numerically test, given the current study's Mach number range. Issues outlined in Section 6.1.1 hindered the experimental gathering of results below Mach 3.4 for the $R_c=1.5$, $\alpha=8^\circ$ model. Figure 6.47 and Figure 6.48 display colour mask schlieren results for the $R_c=1.5$ and $R_c=2$, $\alpha=8^\circ$ models respectively at Mach 3.4.

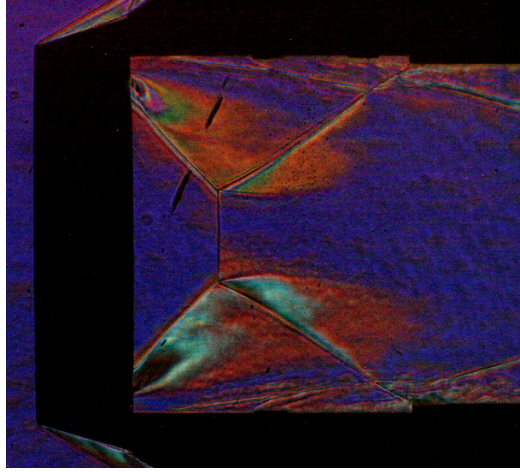


Figure 6.47: Colour mask schlieren for the $R_c=1.5$, $\alpha=8^\circ$ model at Mach 3.4

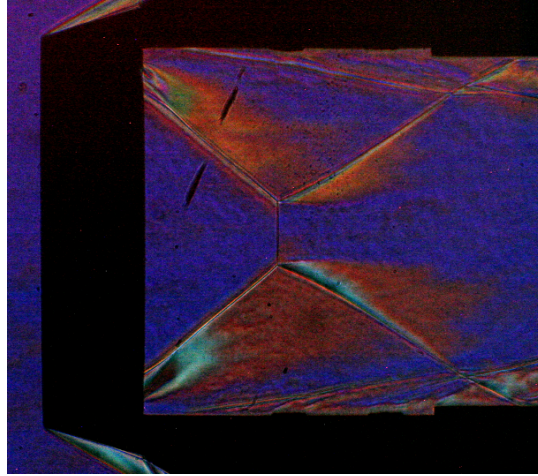


Figure 6.48: Colour mask schlieren for the $R_c=2$, $\alpha=8^\circ$ model at Mach 3.4

With the $R_c=1.5$ case imposing a larger curvature on the curved shock wave than that of the $R_c=2$ case, the angle of the initial shock wave is steepened. The same curvature induced expansion wave strength mechanics that were previously mentioned can be seen to apply in Figure 6.47 and Figure 6.48. In Figure 6.47 where $R_c=1.5$, the flow leaving the internal surface passes with a greater angle around the trailing edge so the pressure ratio and strength of the expansion fan is greater in comparison to Figure 6.48 where $R_c=2$. Due to the additional convexity imposed on the initial shock it becomes more three dimensionally curved. That is why the pre-initial shock density gradients in Figure 6.47 seem larger in size as opposed to those in Figure 6.48.

The influence of the size of the Mach disk on the density gradients can be seen in Figure 6.47 and Figure 6.48. For the $R_c=1.5$ case the Mach disk is larger as a result of the steepened initial shock and more upstream positioning. The different density gradients regions behind the Mach

disk are larger in Figure 6.47 than in Figure 6.48 and so the strength must differ between the two cases. The larger the Mach disk, the greater the tendency for it to curve therefore creating larger density gradients within the triple point slip layer bounded streamtube behind it. Although the curvature is very slight and not easily discernible, small differences in curvature can result in larger density gradients. This is due to nature of a Mach disk being a very strong normal shock wave as the shock angle approaches 90° at the axis of symmetry.

6.5 Effect of Wedge Entry Angle

6.5.1 Normalised Internal Radius of Curvature $R_c=1$

Figure 6.49 and Figure 6.50 contain the shadowgraph results for the $\alpha=0^\circ$, $\alpha=4^\circ$ $R_c=1$ models respectively at Mach 3.6. The curvature of the Mach disk can now be seen prominently in Figure 6.50 due to its size and thickness. A double image of the Mach disk seems to have formed but this occurs due to its curved three dimensionality and the shadowgraph technique picking up both the downstream curved shock surface and the upstream frontal surface of the Mach disk. In Figure 6.49 the initial shock is not as steep as the leading edge is tangent to the flow and the internal surface curvature does not curve the shock to as great an extent. This means the Mach reflection forms further downstream and is smaller in size. The curvature of Mach disk in Figure 6.49 is therefore more difficult to visualise in comparison to Figure 6.50.

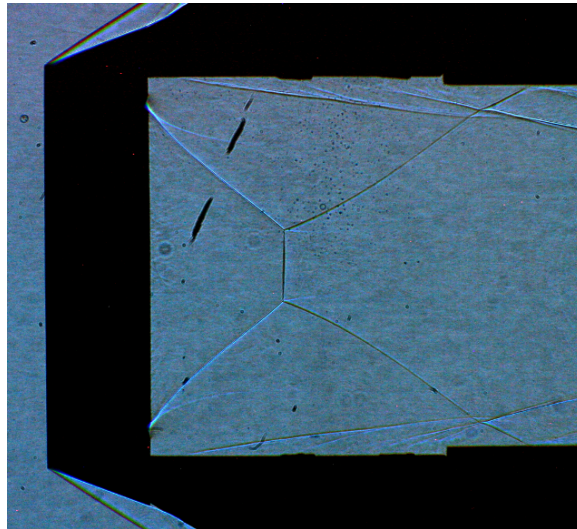


Figure 6.49: $R_c=1$, $\alpha=0^\circ$ model at Mach 3.6

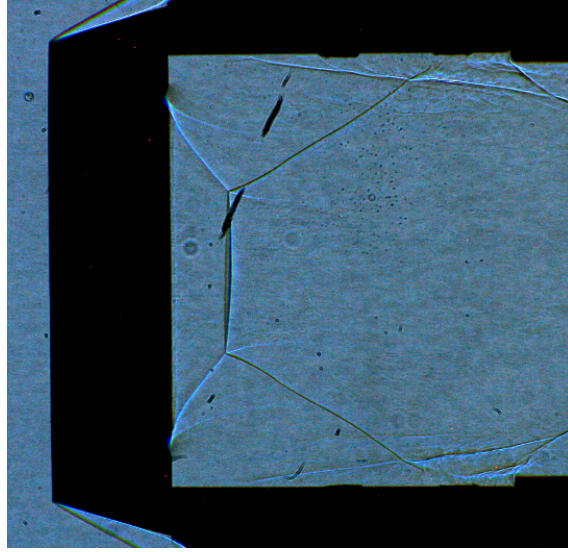


Figure 6.50: $R_c=1$, $\alpha=4^\circ$ model at Mach 3.6

Since the advent of an increase in α to a non-zero value causes the attached shock to form, the flow bounding the internal surface is slower in velocity than that of an isentropic compression. Therefore the flow that enters and leaves the initial reflection is at a lower velocity compared to the case where an isentropic compression initially exists and then coalesces into a shock wave. The velocity ratio across the slip layer would then be larger and the appearance of the slip layer would be more prominent and longer. This can be seen when comparing Figure 6.49 and Figure 6.50. Firstly the initial reflection slip layers in Figure 6.49 are slightly longer than those in Figure 6.50 where $\alpha=4^\circ$. Secondly the initial reflection slip layer near the lower trailing edge seems to be more prominent in Figure 6.49 than in Figure 6.50. This difference in strength causes the slip layer to appear as more prominent and longer. It does not dissipate as soon in the shadowgraph images where $\alpha=0^\circ$ and its discontinuity effects are still captured.

6.5.2 Normalised Internal Radius of Curvature $R_c=1.5$

With the full array of experimental and numerical results available with varying α values at $R_c=1.5$, its overall influence can be more clearly observed. Figure 6.51 displays the progression of an internally curved wedge along a constant radius arc. When the wedge's leading edge is tangent to the flow, in the case of $\alpha=0^\circ$, it sits at the top of the arc. As α increases to some non-zero value, α_1 , the location of the wedge moves along the arc from (1) to (2). Finally when $\alpha_2 > \alpha_1$, the wedge is moved to location (3) further along the constant radius arc. During this progression the angle between the tangents to the internal curved surface at the leading and trailing edge remain constant as the wedge moves from (1) to (2) and eventually to (3). The net flow deflection along the internal surface then remains constant

as α increases due to the constant radius along the arc. The characteristics that curved the shock wave therefore have to deflect the flow through the same net deflection in each case.

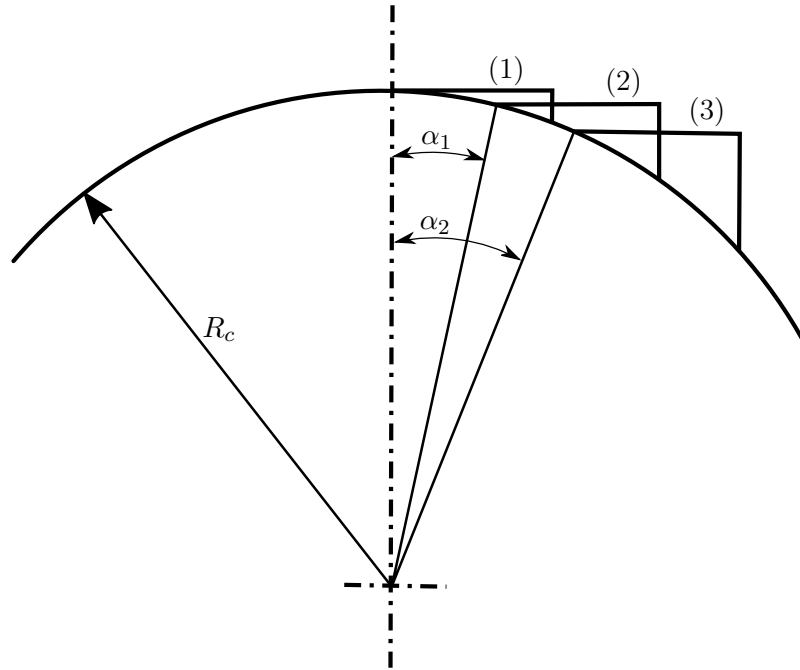
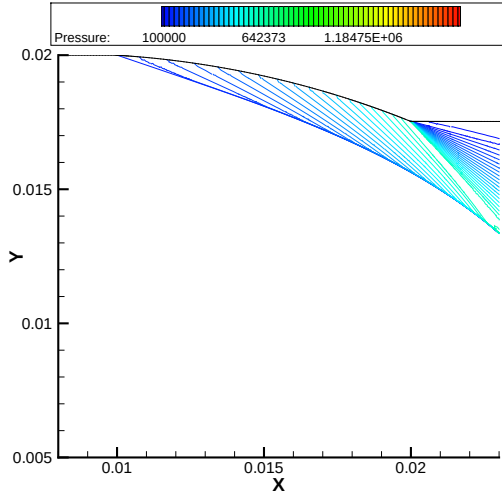
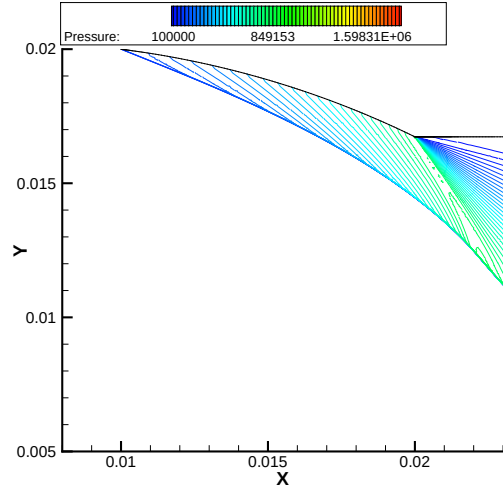


Figure 6.51: Progression of an internally curved wedge along a constant radius arc

Since the increase in α causes a shock to form at the leading edge, the leading edge post curved shock Mach number is different in each case. Therefore the angle of the characteristics along the internal curved surface will differ as they are reliant on the Mach number of the flow in order to readjust it to the wall's presence. The lower the value of α , higher the post shock Mach number, the more swept back the characteristics. This means they only curve the shock further downstream and so the imposed curvature on the curved shock wave segment is delayed and the shock does not curve as much. The higher the value of α , the lower the post shock Mach number, the less swept back and more steeper the characteristics. As a result the curved shock segment is influenced by the characteristics more rapidly and so it experiences a more curved profile. This effect can be seen in Figure 6.52 which displays the characteristics near the internal surface for both the $\alpha=4^\circ$ and $\alpha=8^\circ$, $R_c=1.5$ models at Mach 3.4. The characteristics displayed in Figure 6.52b are seen to be less swept back than those in Figure 6.52a. This is more pronounced closer to the trailing edge where the wall bound flow's Mach number is at its lowest and the characteristics at their steepest. Therefore through the same net flow deflection the influence of post shock Mach number curves the shock wave to considerably differing extents.



(a) $\alpha=4^\circ$, $R_c=1.5$, pressure contours



(b) $\alpha=8^\circ$, $R_c=1.5$, pressure contours

Figure 6.52: Pressure contours for the $\alpha=4^\circ$ and $\alpha=8^\circ$, $R_c=1.5$ models at Mach 3.4

Referring to Figure 6.51 as the ring wedge moves along the constant radius arc with increasing α , the change in the trailing edge reflex angle can be seen. The further down the arc one moves, the steeper the tangent to the curve. This increases the angle between the flat back surface of the trailing edge and the tangent at the trailing edge. Therefore the flow has to navigate around a larger angle with increasing α . This increases the strength of the expansion fan and pressure ratio across its bounding expansive Mach lines. Figure 6.53, Figure 6.54 and Figure 6.55 display colour mask schlieren results for the $\alpha=0^\circ$, $\alpha=4^\circ$ and $\alpha=8^\circ$, $R_c=1.5$ models respectively at Mach 3.4.

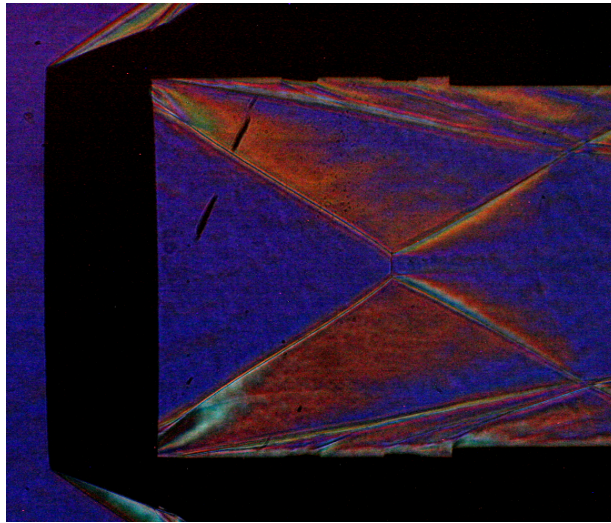


Figure 6.53: Colour mask schlieren for the $\alpha=0^\circ$, $R_c=1.5$ model at Mach 3.4

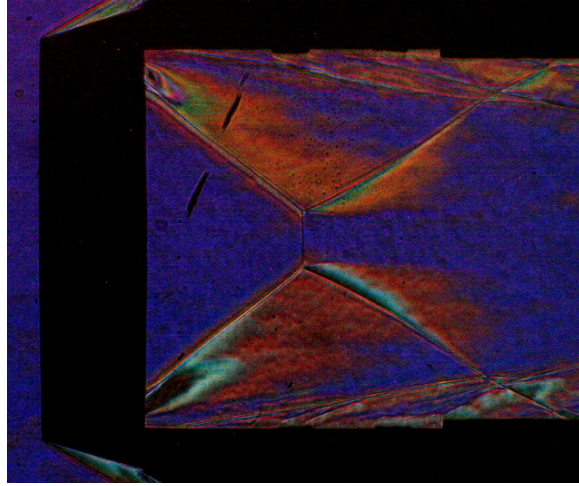


Figure 6.54: Colour mask schlieren for the $\alpha=4^\circ$, $R_e=1.5$ model at Mach 3.4

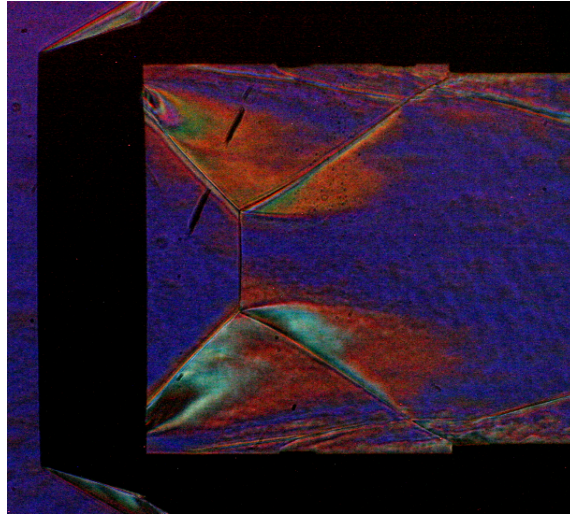


Figure 6.55: Colour mask schlieren for the $\alpha=8^\circ$, $R_e=1.5$ model at Mach 3.4

The differences in the size and strength of the expansion fans affecting the density gradients can be easily seen with increasing α . They are more prominent on the bottom trailing edge due to the bands of the colour mask lattice which are used to colour the refracted light. A similar result was discussed in Section 6.4. The same effect was produced due to the varying internal curvatures producing differing flow exit angles and trailing edge corner angles thereby changing the influence of the expansion fan. The different corner angles at the trailing edge is not the only reason as to why the expansion fan's influence was effected. As previously mentioned different flow speeds exist along the internal surface due to the leading edge shock angle and post shock conditions. This means that the leading expansive Mach line in each trailing edge case is at a different angle. In turn the expansion fan's direction is deflected further downstream the faster the internal surface bounded flow speed.

With α effecting the angle of the leading edge shock wave, the angle of the initial shock is changed especially after the continuously curved shock wave segment. The flow is deflected by this shock wave towards the axis of symmetry and with progression downstream is curved by the trailing edge expansion fan. To redirect the flow away from itself the reflected triple point shock is present. The angle of said reflected shock is then governed by the extent to which the flow needs to be deflected and pre-shock flow conditions. It is therefore directly effected by the change in α as is evident by the differing orientations of the reflected triple point shock wave in Figure 6.53, Figure 6.54 and Figure 6.55. The steeper the initial shock wave, the greater the angle by which the flow is directed towards the axis of symmetry and so the steeper the reflected triple point shock wave. This too increases its strength as shown by the variation in size of the post shock density gradients.

6.5.3 Normalised Internal Radius of Curvature $R_c=2$

The differences between the change in α are repeated in the $R_c=2$ cases and therefore have been omitted as they have already been discussed. The only essential difference between the results of the $R_c=2$ and $R_c=1.5$ cases are the rates of flow deflection which are different due to varying internal surface curvatures. The variation of α is more prominent when $R_c=1.5$ than in $R_c=2$ as the net flow deflection between the leading and trailing edge is greater. Therefore as α varies when $R_c=2$, the changes in the expansion fan and downstream conditions are repeated but to a less noticeable extent.

6.6 Conical and Curved Internal Surfaces

To explore the differences between the flow fields caused by curved and conical internal surfaces additional numerical and experimental tests were completed. A conical model was created such as that in Figure 6.56 where the leading edge radius, R_{le} , and the trailing edge radius, R_{te} , where the same as those generated by the $\alpha=8^\circ$, $R_c=1.5$, $L_w=0.5$ model. The conical surface then joined the leading and trailing edge creating parameters where $\alpha=18.1^\circ$ and $R_c=\infty$. No flow plane curvature would then be imposed on the shock wave as the internal surface is straight but transverse curvature would still be present due to the ring wedge model's axisymmetric nature.

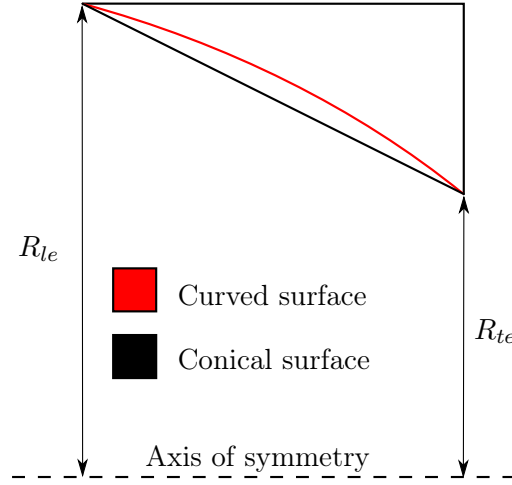
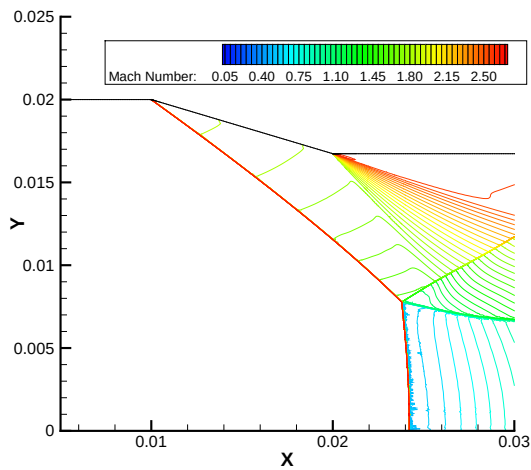
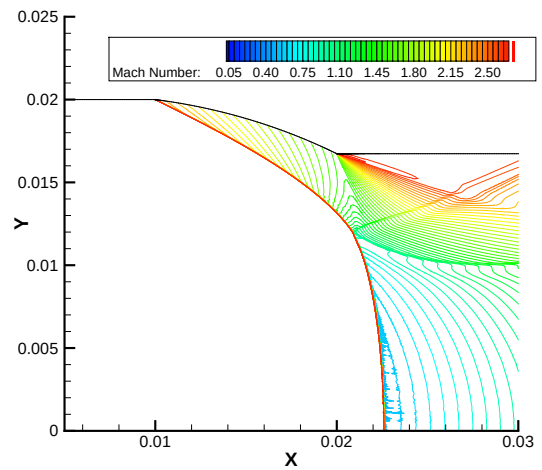


Figure 6.56: Conical and curved internal ring wedge surface

Figure 6.57 displays the Mach number contour plots for the $\alpha=18.1^\circ$, $R_c=\infty$ and $\alpha=8^\circ$, $R_c=1.5$ models at Mach 2.7. The resulting flow fields in either case are extremely different. Firstly in Figure 6.57a although the internal surface is conical a Mach number variation exists between the attached initial shock wave and the internal surface. This exists due to the transverse curvature that is present which slightly curves the initial shock wave, increasing the shock angle thereby reducing the post shock Mach number. The influence of the transverse curvature is known to increase as the distance to the axis of symmetry decreases. This can be seen in Figure 6.57a as the spacing between Mach number contours decreases on approach to the triple point meaning the shock angle is increasing with progression along the shock wave.



(a) $\alpha=18.1^\circ$, $R_c=\infty$ Mach number contours



(b) $\alpha=8^\circ$, $R_c=1.5$ Mach number contours

Figure 6.57: Mach number contours for the $\alpha=18.1^\circ$, $R_c=\infty$ and $\alpha=8^\circ$, $R_c=1.5$ models at Mach 2.7

Since nothing is interfering with the initial shock wave due to the supersonic flow's nature inhibiting pressure waves to travel upstream and the current location of the triple point, transverse curvature must be the only influence. In Figure 6.57b the leading and trailing edge are now joined by a curved surface and so characteristics readjust the flow to its presence and curve the shock wave. A large Mach reflection is present, however, the reflected triple point shock wave is very weak in comparison to that in Figure 6.57a but a clear kink in the shock wave is present after the curved shock segment. Since this weak shock reflection stage exists it could point to a transition region for the shock's geometry. During this point the shock wave could shift from being continuously curved from the leading edge to the axis of symmetry to developing a kink and Mach reflection. This transition region would occur between very small Mach number increments near Mach 2.7 and be very different to produce experimentally due to possible hysteresis effects. With the Mach reflection forming further upstream in Figure 6.57b than in Figure 6.57a due to the increased shock angle at the triple point, the height of the Mach stem is increased. The increased height promotes its tendency to curve to a larger extent in comparison to the Mach stem produced in the conical case.

Comparing orientation of the expansion fans in Figure 6.57a and Figure 6.57b it is evident that the internal surface curvature plays a considerable role in its size and direction. For the conical case the corner reflex angle between the internal surface and the back of the trailing edge surface is smaller than the reflex angle in the curved internal surface case. This means the strength of the expansion fan is decreased as the flow has to navigate less of an expansion. The Mach number of the flow at the trailing edge of the curved internal surface is less than the conical case due to the internal curvature continually deflecting the flow. As previously mentioned this causes the leading Mach line of the expansion to form at a steeper angle which results in the influence of the expansion fan to be felt further upstream. The expansion fan in Figure 6.57b covers a larger region and is steeper than that in Figure 6.57a.

Monochromatic schlieren results for the $\alpha=18.1^\circ$, $R_c=\infty$ and $\alpha=8^\circ$, $R_c=1.5$ models are shown in Figure 6.58 and Figure 6.59 respectively at Mach 3.4. Although colour mask schlieren results were obtained under the same conditions, monochromatic results have been displayed instead. The reasoning for the switch being the differences between the two cases being more apparent using monochromatic schlieren. This was particularly the case behind the Mach disk and regions before the initial shock wave.

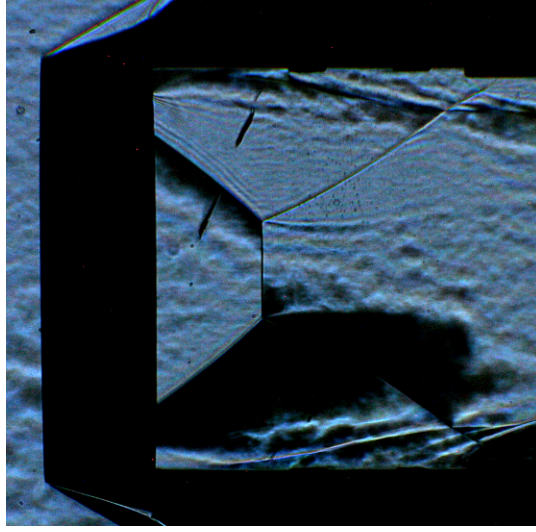


Figure 6.58: Monochromatic schlieren for the $\alpha=18.1^\circ$, $R_c=\infty$ model at Mach 3.4

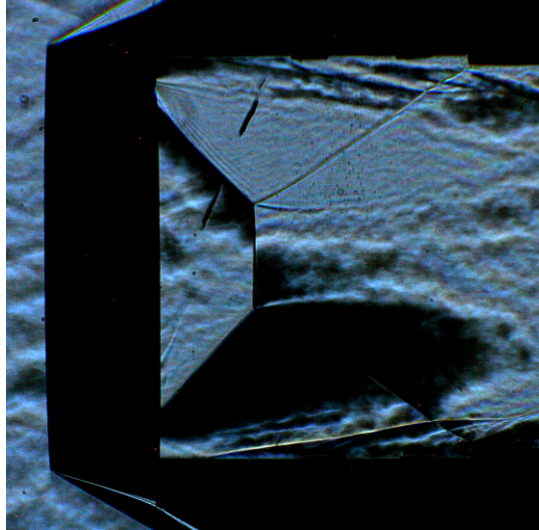


Figure 6.59: Monochromatic schlieren for the $\alpha=8^\circ$, $R_c=1.5$ model at Mach 3.4

With the Mach number being increased to Mach 3.4 in Figure 6.58, the transverse curvature induced concavity of the shock wave can be more easily noticed. Similarly in Figure 6.59 the expansion fan induced convexity in the initial shock wave is recognisable to what has been seen in previous models, now that the triple point has moved further downstream. The banded presence of the upper expansion fan can be seen in the post shock flow field with its direction orientated toward the initial shock wave. The three dimensionality of the flow fields is what generates the dark patch ahead of the upper initial shock wave in each case. This dark patch, formed by the light blocked by the horizontal knife edge, is more uniform in Figure 6.58 than in Figure 6.59. This could be as a result of the expansion fan induced shock wave convexity interfering with the concave curvature of the initial shock wave in Figure 6.59 as one progresses along its length. In Figure 6.58 the shock wave is only curved due to transverse curvature as

the expansion fan is seen to be angled further downstream. The lack of conflicting curvature sources then allow for a more uniform density gradient to form.

Even though the Mach disks in Figure 6.58 and Figure 6.59 are of the same relative size, they produce considerably different density gradients in their exhausts. Therefore they must possess a three dimensional difference in curvature. Seeing as there is a difference in the initial shock wave that forms the Mach reflection in either case, one being more concave and the other more convex, the conditions that form the reflection will differ. As a result the formation of the Mach disk and triple point slip layer will not be identical.

Chapter 7 | Conclusions

An experimental and numerical study was completed to examine the effects of internal surface curvature and wedge entry angle on the shock waves produced in steady axisymmetric flow fields. Test models with leading edge radius normalised internal radii of curvature of $R_c=\{1, 1.5, 2\}$ and wedge entry angles of $\alpha=\{0^\circ, 4^\circ, 8^\circ\}$ were manufactured and tested. The Supersonic Wind Tunnel (SSWT) facility at the University of the Witwatersrand was used to capture shadowgraph and schlieren results between Mach 2.8 and 3.6. Numerical CFD simulations were completed for the range of models using ANSYS Fluent between Mach 2.4 and 3.6 and validated using experimental results. Below are the conclusions drawn from each result section discussed in Chapter 6. All experimental and numerical data including manufacturing drawings have been given to the supervisor as a digital appendix and are listed in Appendix D.

7.1 Numerical Validation Results

Experimental and numerical results were compared for validation purposes and were found to be in good agreement thereby validating the numerical method. Issues that caused some experimental models to experience detached flow regimes were found to be caused by the asymmetry of the SSWT supersonic nozzle and the mechanics of the oblique separation shock's progression. The CST influence coefficients plotted by a self-developed CST MATLAB script for analysis purposes matched those published by [12] thereby validating the code script.

7.2 Curved Shock Wave Shape Equation Results

A general curved shock wave shape was determined for the continuously curved shock wave segment bounded by the leading edge and downstream influence effects. The curve-fitted shock shape relationship is presented in Equation (6.2). The curve fit constants which are used in

the curved shock wave shape equation were found to be self similar in nature with varying experimental parameters. The relationship for the $\alpha=4^\circ$ models was then characterised into an empirical model represented by Equation (6.13) which is a function of R_c and Mach number. The empirical model can accurately predict the curved shock wave shape when $R_c \geq 1$, $M > 2.9$, $\alpha=4^\circ$ and $L_w=0.5$. The curve fit constants that form part of the shape relationship are all coupled and dependant by governing compressible flow mechanics. They therefore cannot be selected independently of one another to form desired shock shapes.

7.3 Curved Shock Theory Streamline Results

The internal surfaces that formed the curved shock wave shapes were not able to be predicted using wall bounded CST streamlines. This was due to the proximity of the shock angle to the flow Mach angle at the leading edge affecting the influence coefficients causing extremely large streamline curvature values. As a result the streamlines crossed as they were excessively curved. The larger the value of α and R_c , the smaller the streamline curvature and the more closely the wall bounded streamline began to resemble the internal surface profile. The theoretical CST streamlines in the most accurate cases were compared with those produced by CFD results to determine applicability. Very good agreement was found the further one progressed down the shock path as they began to match up and align with one another. By examining the CFD streamlines near the internal surface it was found that their curvature was not constant throughout their progression in the post curved shock flow field. This was due to the varying conditions of the characteristics imposing the curvature and initial post shock flow deflection.

7.4 Varying Internal Surface Curvature Results

The higher the internal surface curvature, the more the flow is caused to turn. For the highest curvature case the rotating of the characteristics occurred to such an extent that with the influencing flow plane and transverse curvature they began to bend inward. This caused the flow to want to turn in onto itself. As a result an initial reflection formed before the main Mach reflection, creating a slip layer and reflected shock that dissipated in strength with Mach number progression. With increasing internal surface curvature, the trailing edge corner angle, around which the expansion fan rotates the flow, increases. This causes the strength and pressure ratio across the expansion to increase, changing its direction and influence on downstream shock structures. The strength of the initial shock wave was shown to increase for increased internal surface curvatures due to it becoming steeper. This resulted in the Mach

reflection forming closer to the trailing edge, reducing the impact of transverse curvature due to the influence of the expansion fan.

7.5 Varying Wedge Entry Angle Results

With α increasing from a flow tangent scenario to a non-zero value, the flow dynamics at the leading edge change from an isentropic compression to an attached leading edge shock wave. This steepens the initial shock angle increasing the size of the Mach disk. The change in α essentially means the internal surface progresses in position along a constant radius arc. The net flow deflection between the leading and trailing edge remains constant but the post shock flow is slowed more greatly as α increases. This steepens the angle of the characteristics and causes the leading edge shock to be curved to a greater extent. The corner angle is affected with variation in α as the angle between the back trailing edge surface and internal surface trailing edge's tangent changes. As a result the expansion fan strength is influenced by the same means as varying the internal surface curvature, affecting the downstream shock structure and flow conditions. The first Mach line of the expansion fan will be steepened by the slower wall bounded flow's velocity further affecting its direction. The strengths of the initial and reflected Mach reflections shocks were seen to vary with changing α , as it controlled the direction and curvature applied to the leading edge curved shock wave segment.

7.6 Conical and Curved Internal Surface Comparison Results

Since the trailing edge of the conical model's corner experienced a smaller corner angle and faster wall bounded flow, the influence of the expansion fan was directed downstream and not towards the initial shock wave. This allowed the effects of transverse curvature to be more effectively visualised concerning the shock shape and Mach number variation. Large differences were noted in the flow field produced by both cases due to the imposed curvature involved, changing the shock angle for the curved model. The transverse curvature applied in the conical case produced more uniform pre-shock three dimensional density gradients due to the lack of conflicting expansion wave induced convexity. Since the initial shock waves were so varied in each case, the conditions of the resulting Mach reflections vary, producing different Mach disk strengths.

Chapter 8 | Recommendations

The presented work explored a variety of experimental and numerical parameters. Given the current field of steady curved shock waves being relatively new, many opportunities exist for further research. Future work can be explored from what was completed with the recommendations listed below:

- The double steady reflection phenomenon caused by higher internal curvatures can be further explored using higher internal radii of curvature and more sensitive flow visualisation. The flow feature was only captured at lower Mach numbers in numerical results as the experimental facility was not capable of achieving Mach numbers where a stronger reflection is present.
- Given the size and choking restrictions of the SSWT facility, the influence of varied wedge length was not fully explored. By scaling up models and increasing the leading edge diameter, one allows more room for diversifying physical geometric parameters within less constrained manufacturable limits.
- Constant radii of curvature were used to describe the internal surfaces of the ring wedge models. To explore the mechanics behind the characteristics and shock waves that form in axisymmetric flow fields, differing internal wall profiles can be researched. These could possibly include parabolic or elliptic surfaces to observe the effects of internal surface curvature which differs along the wedge length.
- An empirical model used to describe the shape of the continuously curved shock wave segment was presented. Seeing as a self similar pattern was uncovered, underlying hidden relationships that control the determination of the curved shock wave shape are present. Extensive research in both the fields of compressible flow dynamics and applied mathematics could lead to discovering the parameters which make each shape self similar yet unique to its parameters.

- Higher wedge entry angles and internal surface curvatures were found to produce CST streamlines that more accurately described the internal wall profile that produced the given shock shape. Exploring models where these parameters are set to more suitable limits could yield results that match the wall profile very accurately. Comparison between theoretical CST streamlines and those produced from numerical simulation showed strong correlation where the streamlines were not excessively curved.
- The weakness of the reflected shock present in the conical and curved wall comparison could indicate a region where an axisymmetric shock can exist as a continuously curved shock wave without a kink. Should this be possible, the Mach number where the transition point occurs would exist in very small variable flow speed limits. This is easily achievable using the accuracy of numerical methods but will be considerably difficult to experimentally reproduce given the resolution of most supersonic wind tunnel facilities. A possible continuously curved shock wave was simulated at Mach 1.85 during preliminary ring wedge parameters studies and is displayed in Figure 8.1
- Finally the testing of curved wedges to generate shock waves with planar symmetry has been recently completed within the University at an undergraduate honours level. If further postgraduate research is undertaken in order to expand on what has been completed, comparison between the axisymmetric and planar symmetry cases may be made. This will shed more light on the influence of transverse curvature in the formation of the shock wave structure and trailing edge effects.

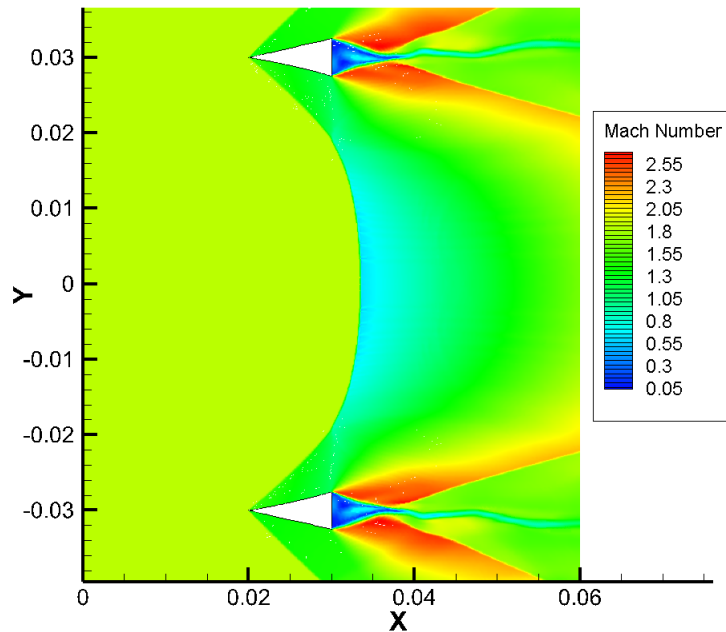


Figure 8.1: Possible continuously curved shock wave at Mach 1.85

References

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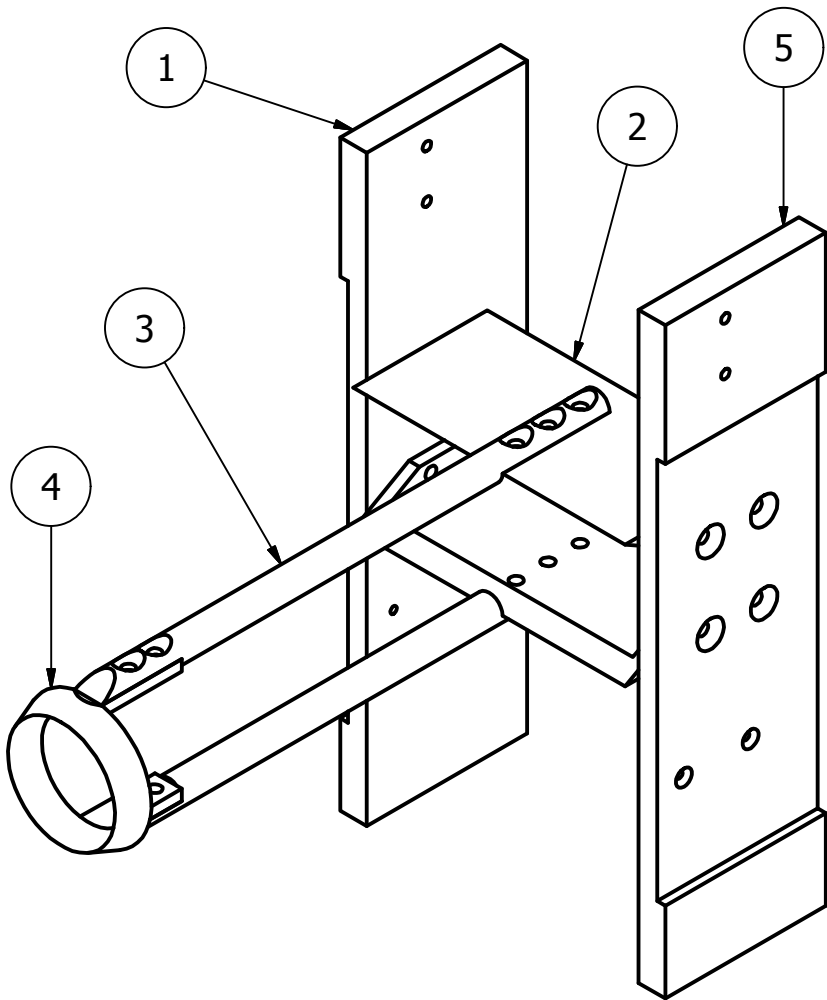
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Chapter A | Experimental Manufacturing Drawings

Table A.1 lists the experimental manufacturing drawings in order of appearance.

Table A.1: Manufacturing drawing numbers and descriptions

Appearance number	Description
1	Experimental test rig assembly
2	$R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ test model
3	$R_c=1.5$, $\alpha=0^\circ$, $L_w=0.5$ test model
4	$R_c=2$, $\alpha=0^\circ$, $L_w=0.5$ test model
5	$R_c=1$, $\alpha=4^\circ$, $L_w=0.5$ test model
6	$R_c=1.5$, $\alpha=4^\circ$, $L_w=0.5$ test model
7	$R_c=2$, $\alpha=4^\circ$, $L_w=0.5$ test model
8	$R_c=1.5$, $\alpha=8^\circ$, $L_w=0.5$ test model
9	$R_c=\infty$, $\alpha=18.1^\circ$, $L_w=0.5$ test model
10	$R_c=2$, $\alpha=8^\circ$, $L_w=0.5$ test model
11	$R_c=2$, $\alpha=4^\circ$, $L_w=0.75$ test model
12	Test model rod supports
13	Left side plate of the test rig
14	Right side plate of the test rig
15	Test rig support plates



PARTS LIST			
ITEM	QTY	PART NUMBER	DESCRIPTION
1	1	5	Left side plate
2	2	7	Rod support connecting plate
3	2	4	Ring wedge rod supports
4	1	1	20 mm internal radius of curvature ring wedge
5	1	6	Right side plate

REVISION HISTORY

NOTES:
SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
DIMENSIONS IN mm.
DO NOT SCALE.
ALL THREADS MEDIUM FIT.

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DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: **Assembly**

REV NO.



SHEET 1 OF 1

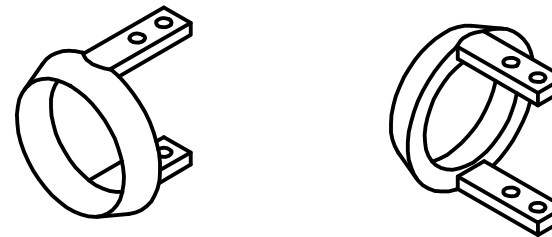
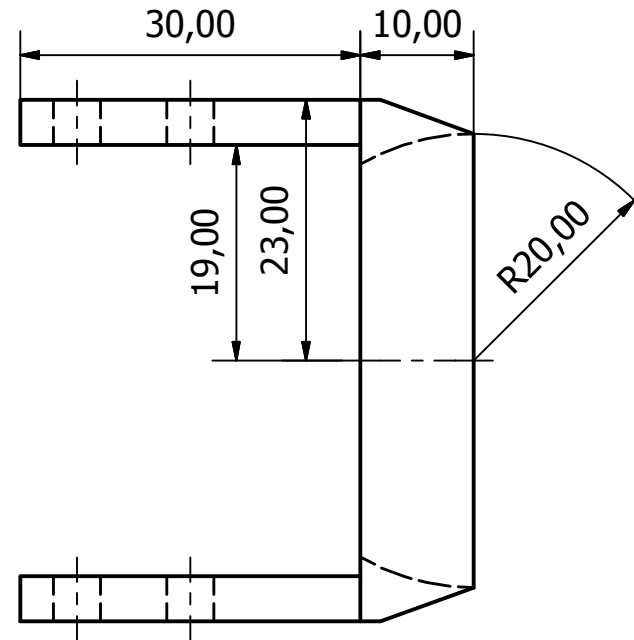
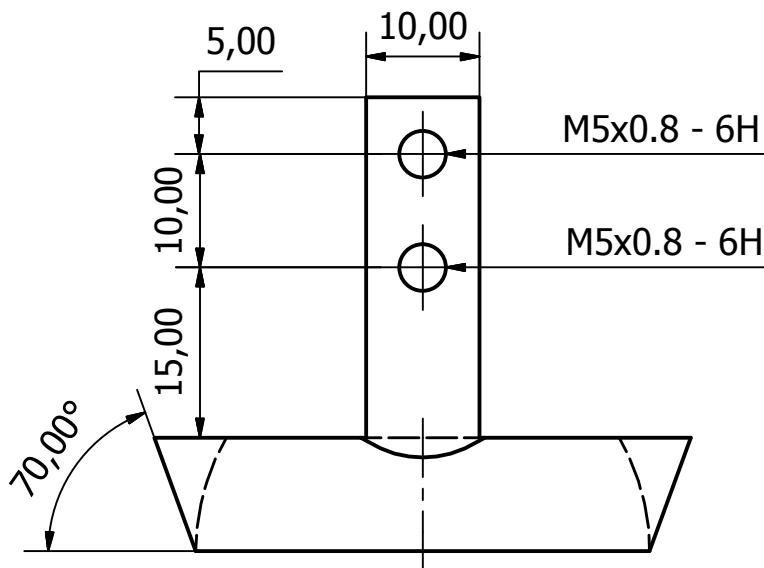
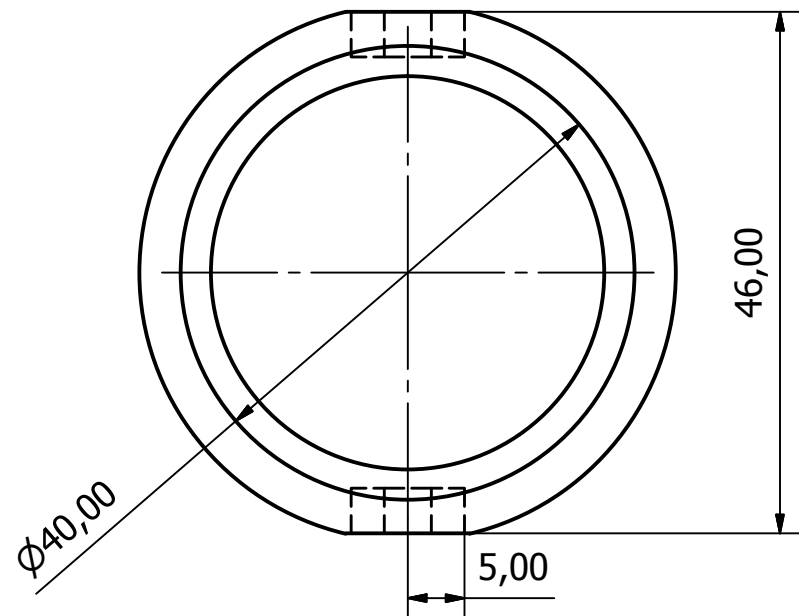
DESCRIPTION: Windtunnel model assembly

MATERIAL:



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REVISION HISTORY

Scale: 1.5 : 1

NOTES:
 SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
 DIMENSIONS IN mm.
 DO NOT SCALE.
 ALL THREADS MEDIUM FIT.

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CHECKED:

DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: 1

REV NO.



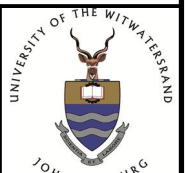
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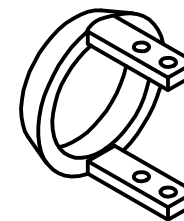
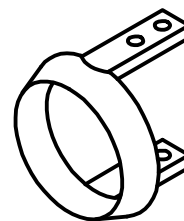
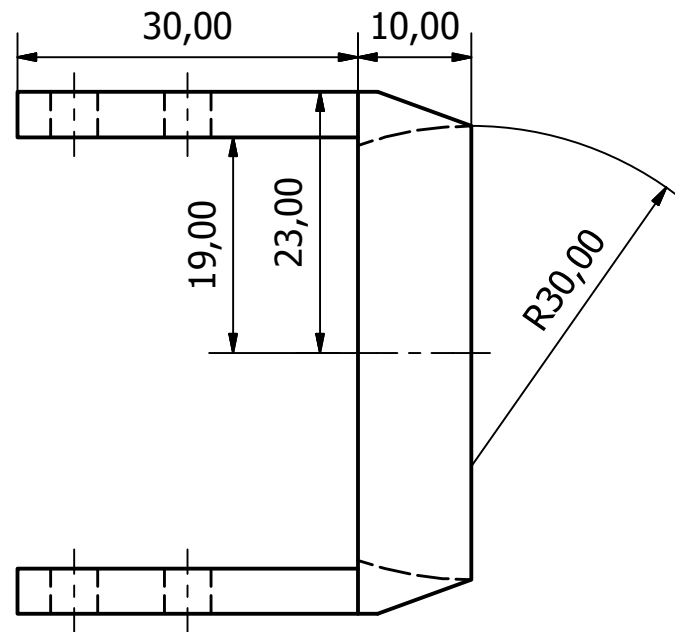
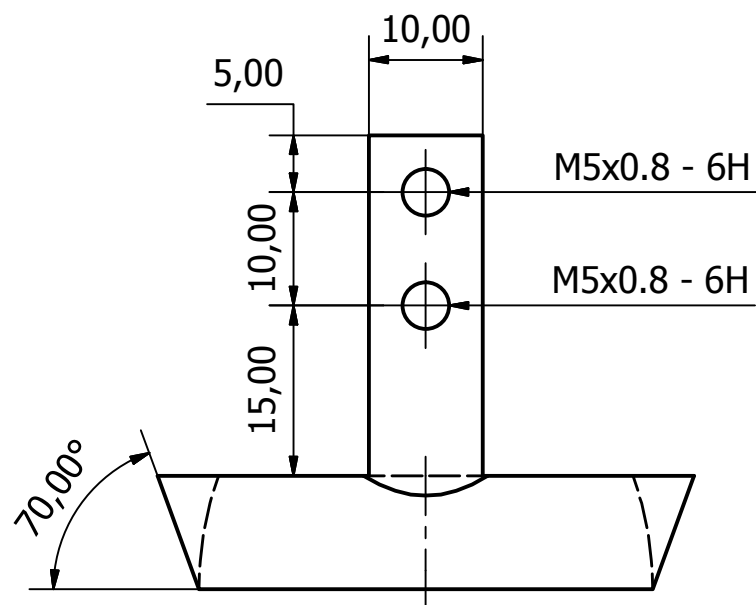
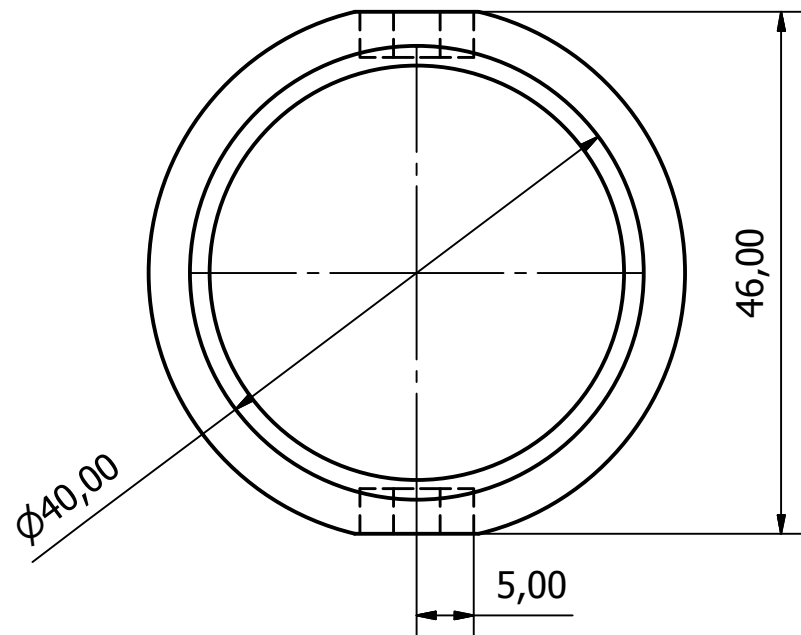
DESCRIPTION: 20 mm internal radius of curvature ring wedge

MATERIAL: Brass



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REVISION HISTORY

Scale: 1.5 : 1

NOTES:
 SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
 DIMENSIONS IN mm.
 DO NOT SCALE.
 ALL THREADS MEDIUM FIT.

APPROVED:

CHECKED:

DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: 2

REV NO.



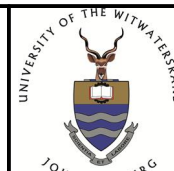
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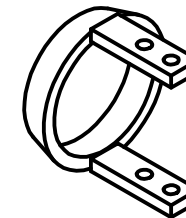
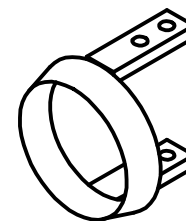
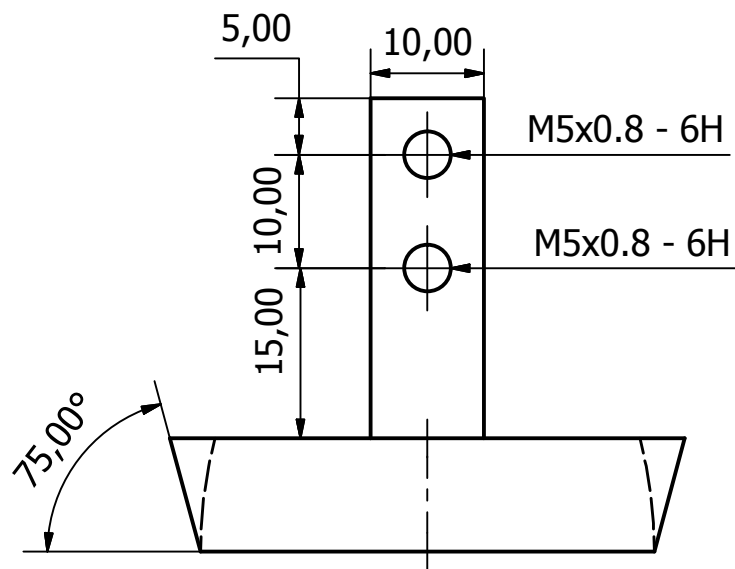
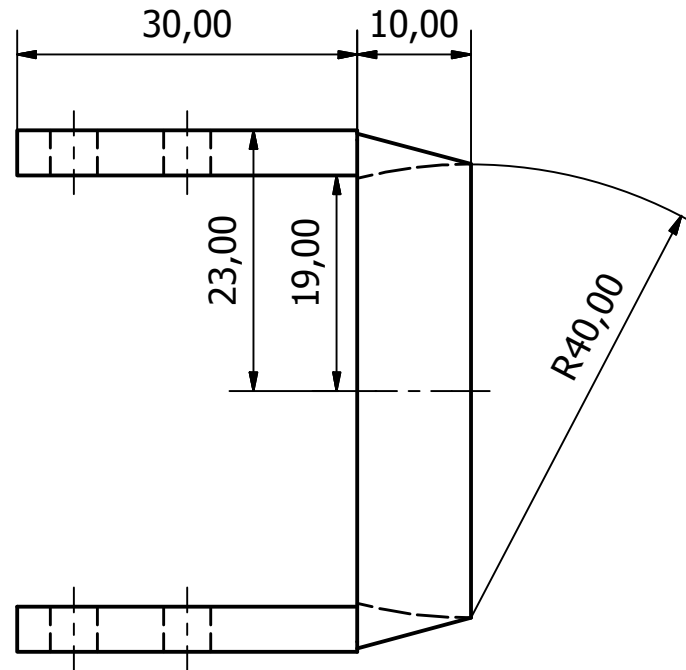
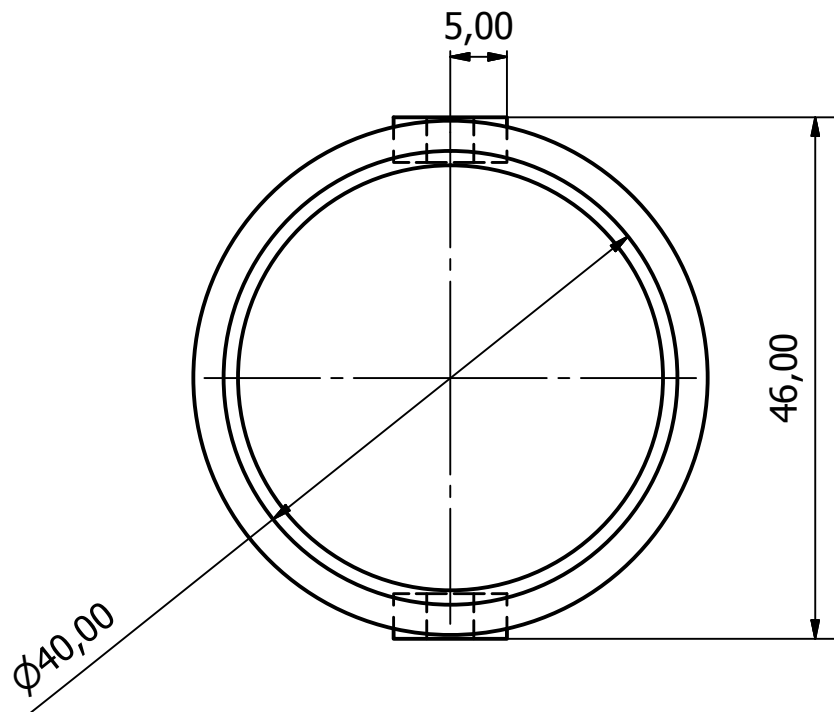
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MATERIAL: Brass



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REVISION HISTORY

Scale: 1.5:1

NOTES:
 SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
 DIMENSIONS IN mm.
 DO NOT SCALE.
 ALL THREADS MEDIUM FIT.

APPROVED:

CHECKED:

DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: 3

REV NO.



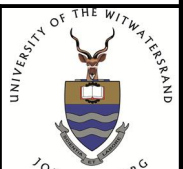
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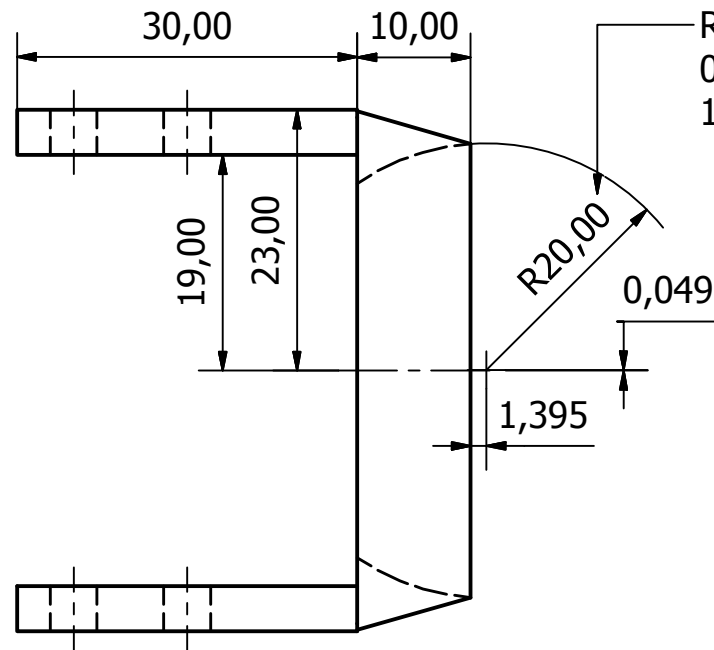
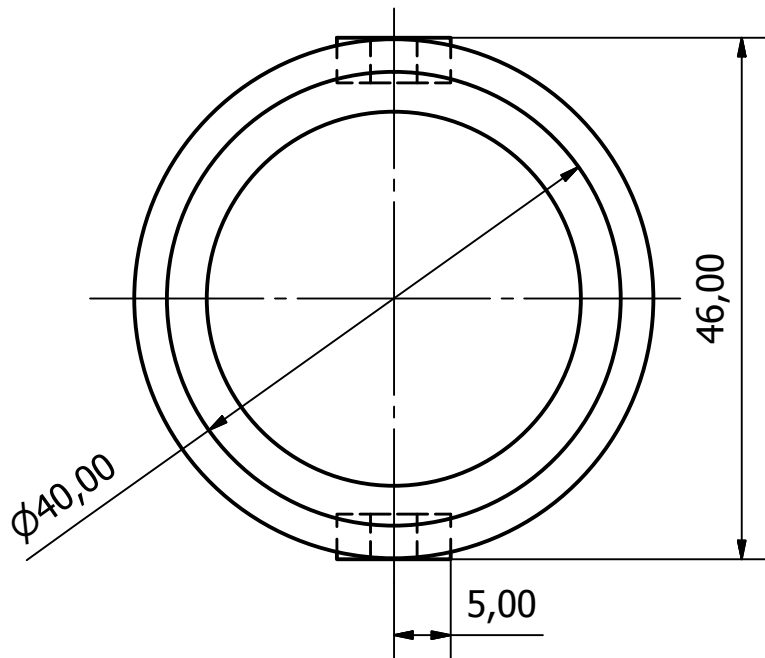
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MATERIAL: Brass



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Radius centre:
0.049 mm above centreline
1.395 mm to the right

REVISION HISTORY

Scale: 1.5: 1

NOTES:
SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
DIMENSIONS IN mm.
DO NOT SCALE.
ALL THREADS MEDIUM FIT.

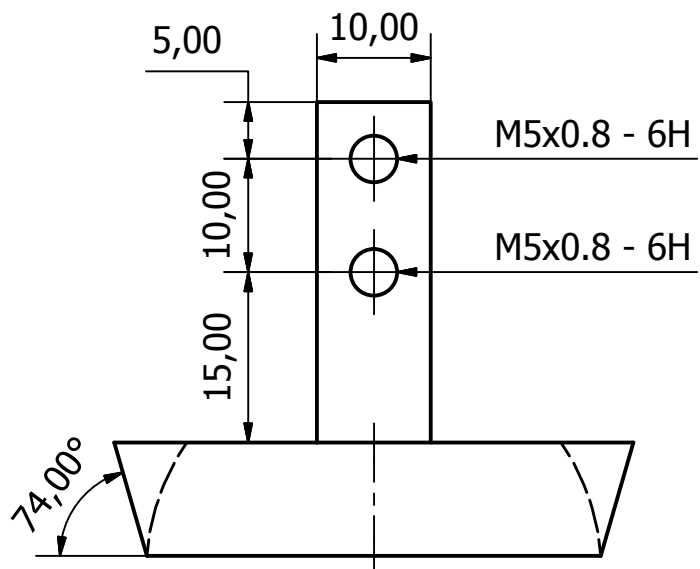
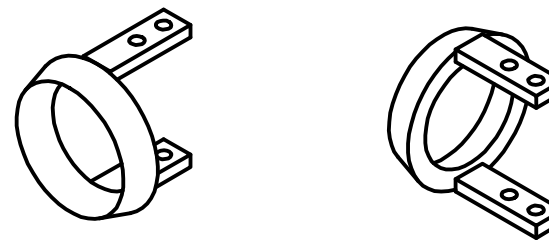
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DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4



DRG NO: 1

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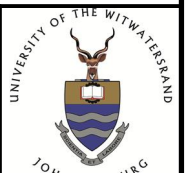
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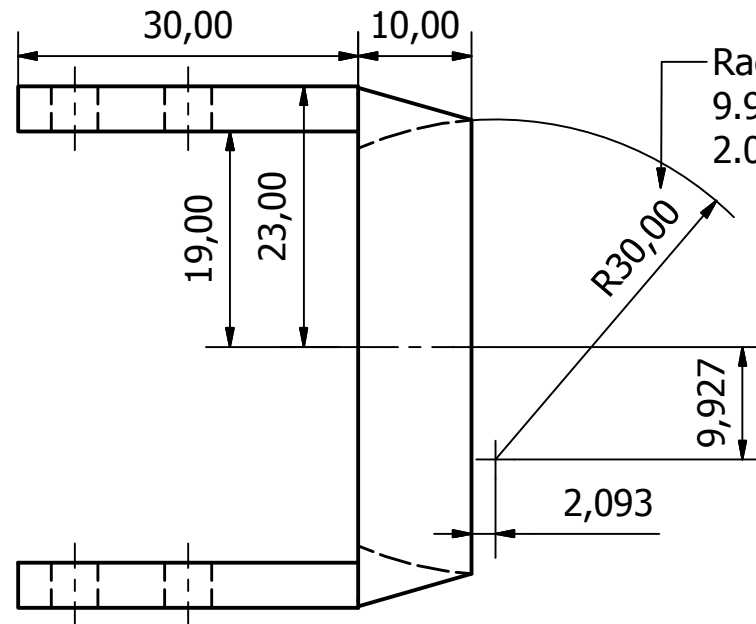
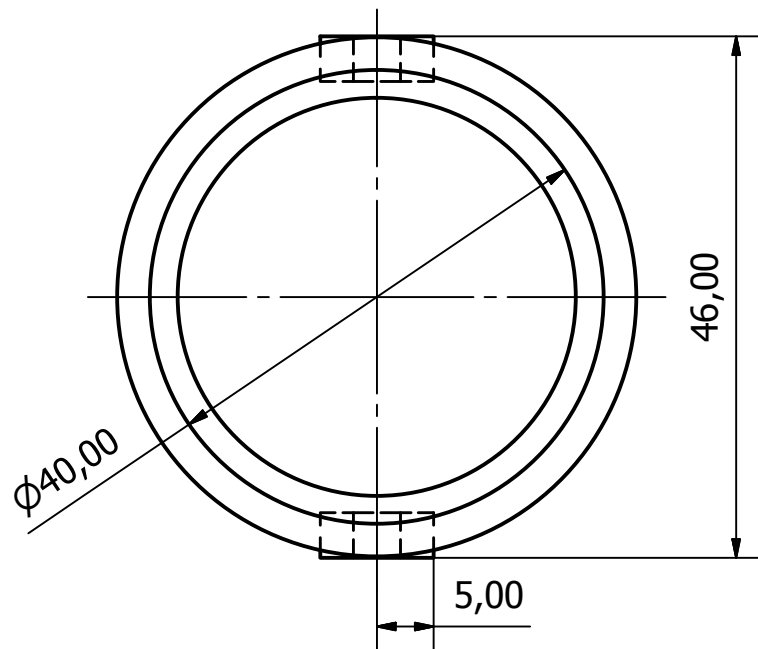
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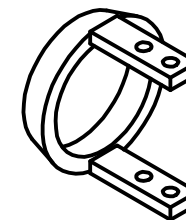
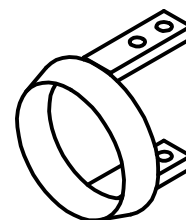
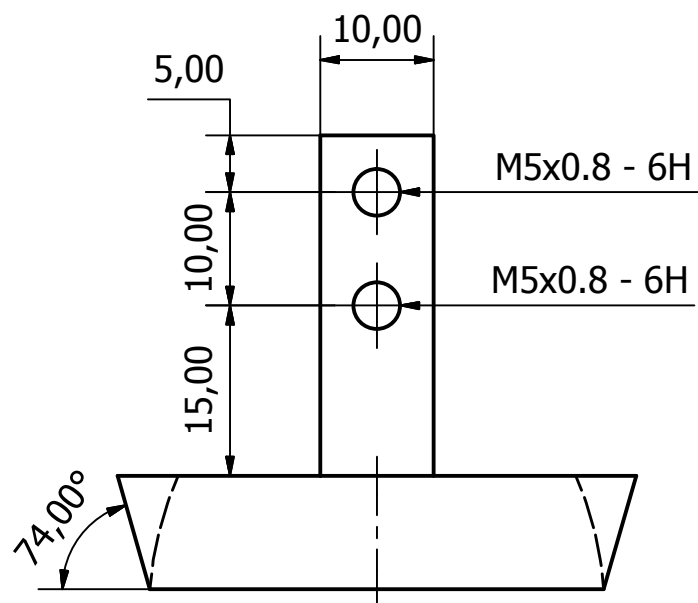


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Radius centre:
9.927 mm below centreline
2.093 mm to the right



REVISION HISTORY

Scale: 1.5:1

NOTES:
SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
DIMENSIONS IN mm.
DO NOT SCALE.
ALL THREADS MEDIUM FIT.

APPROVED:

CHECKED:

DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: 2

REV NO.



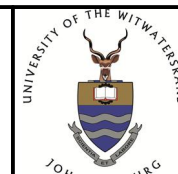
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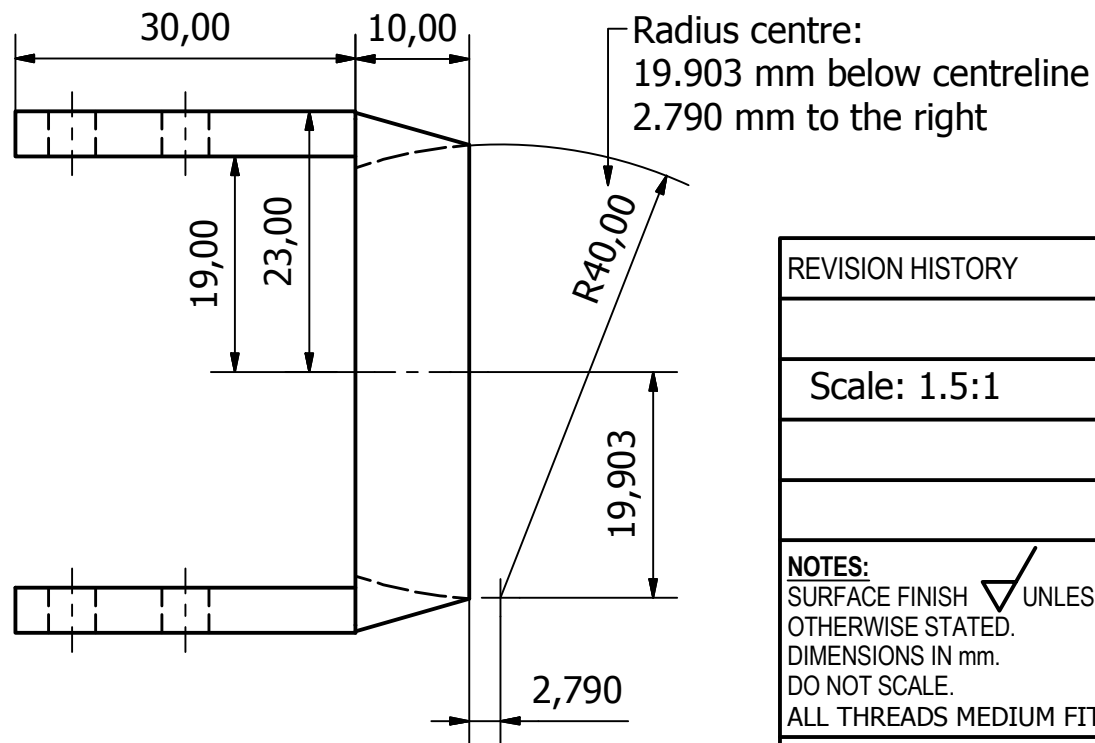
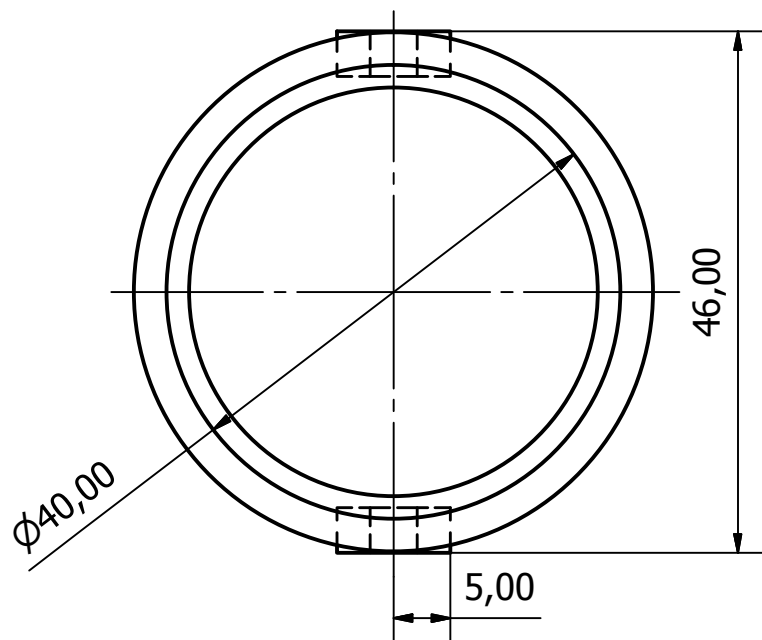
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MATERIAL: Brass



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NOTES:
 SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
 DIMENSIONS IN mm.
 DO NOT SCALE.
 ALL THREADS MEDIUM FIT.

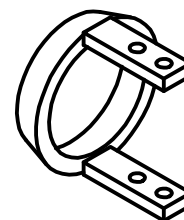
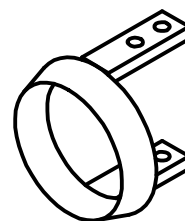
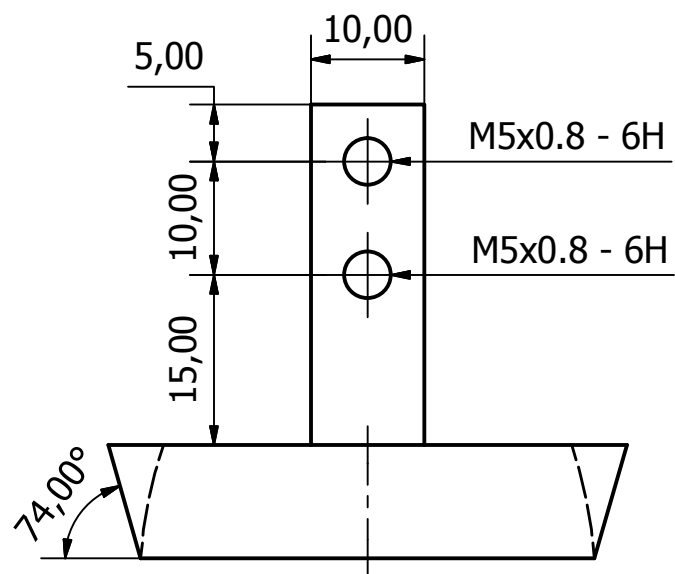
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DATE:

DRAWN BY: FRU-Alessandro

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DRG NO: 3

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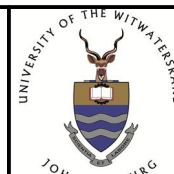
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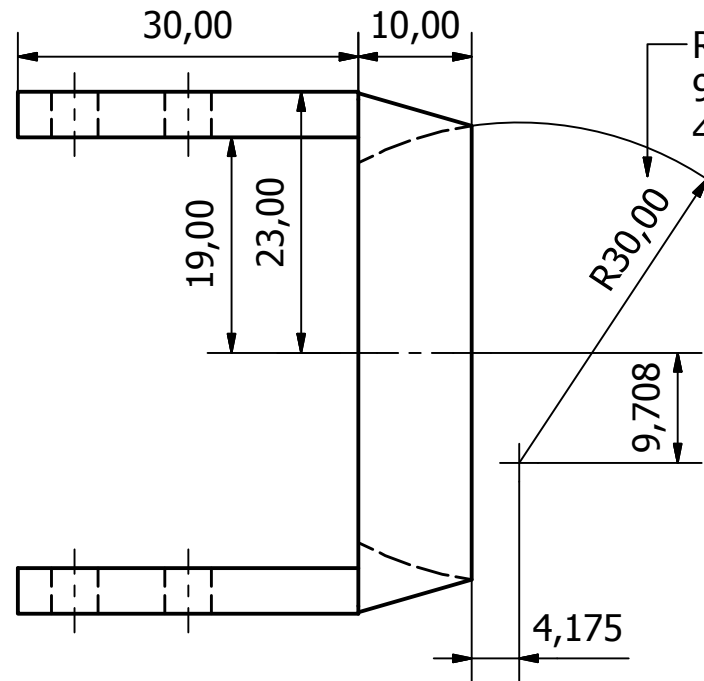
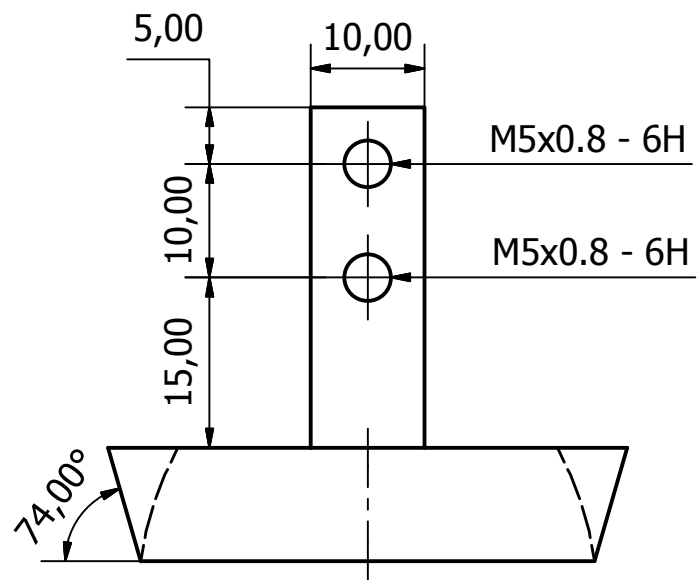
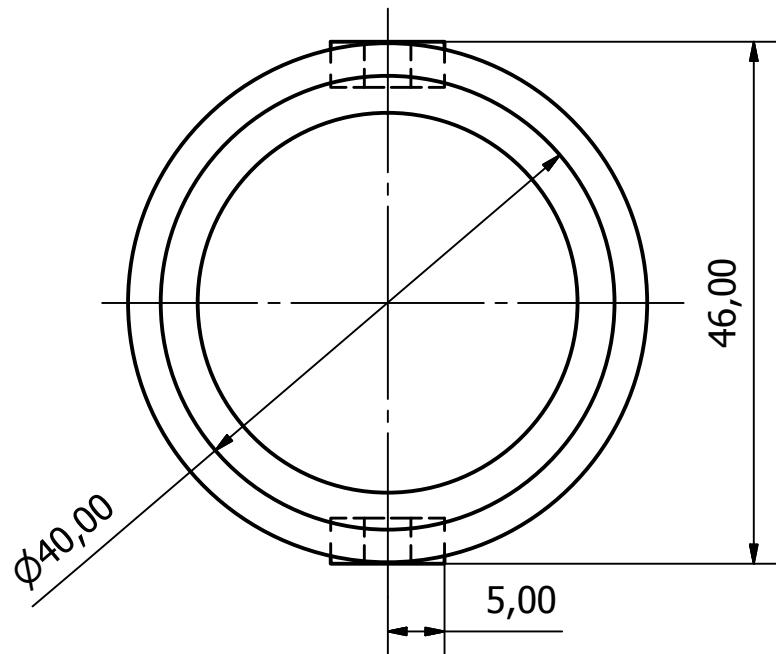
DESCRIPTION: 40 mm int. rad. of curv. w/ 4 deg leading edge

MATERIAL: Brass



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Radius centre:
9.708 mm below centreline
4.175 mm to the right

REVISION HISTORY

Scale: 1.5:1

NOTES:
SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
DIMENSIONS IN mm.
DO NOT SCALE.
ALL THREADS MEDIUM FIT.

APPROVED:

CHECKED:

DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: 4

REV NO.



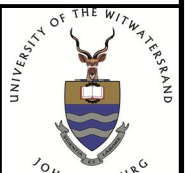
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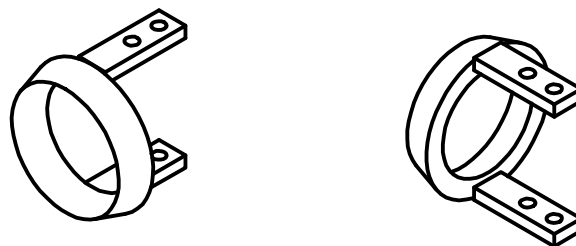
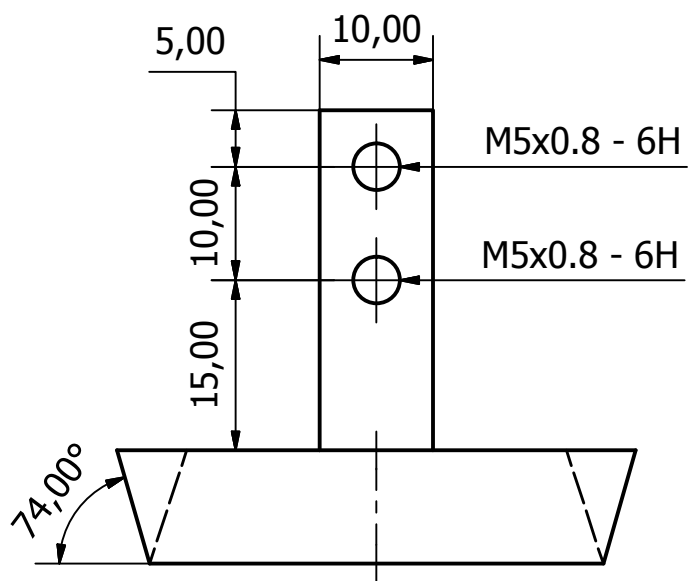
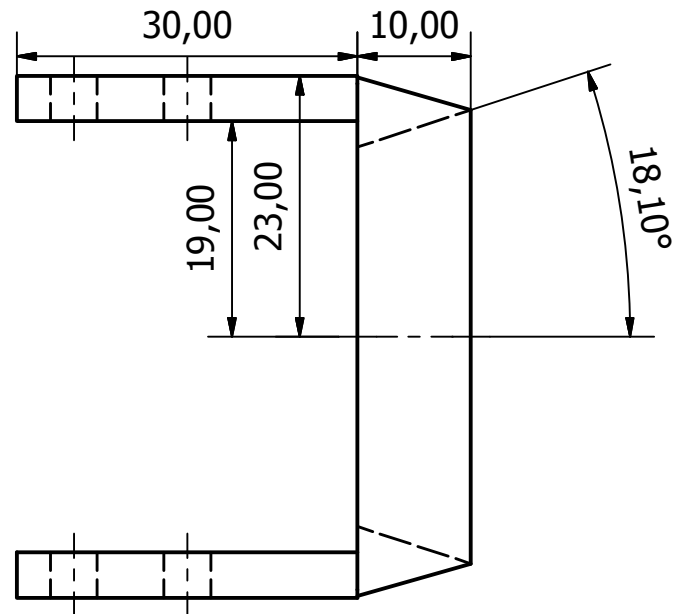
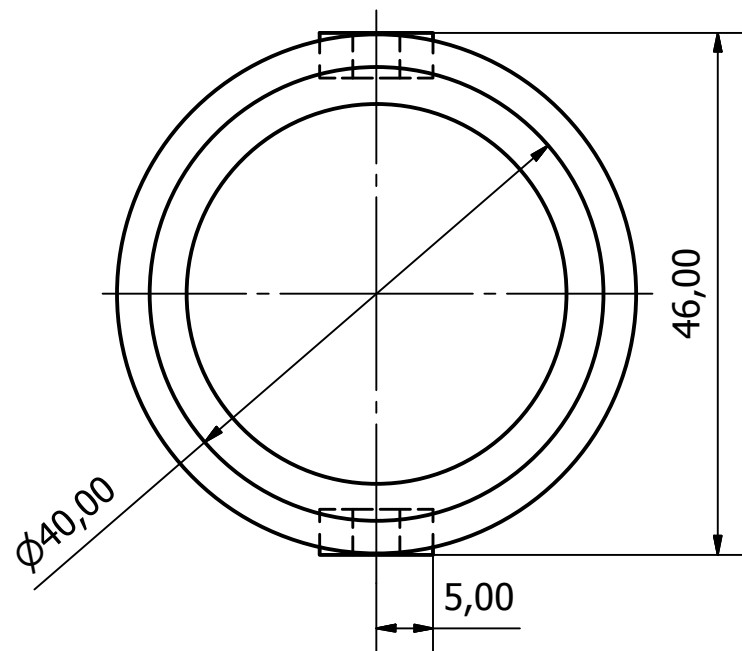
DESCRIPTION: 30 mm int. of rad. curv. w/ 8 deg leading edge

MATERIAL: Brass



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REVISION HISTORY

Scale: 1.5:1

NOTES:
 SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
 DIMENSIONS IN mm.
 DO NOT SCALE.
 ALL THREADS MEDIUM FIT.

APPROVED:

CHECKED:

DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: 5

REV NO.



SHEET 1 OF 1

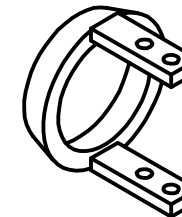
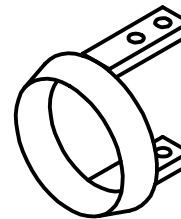
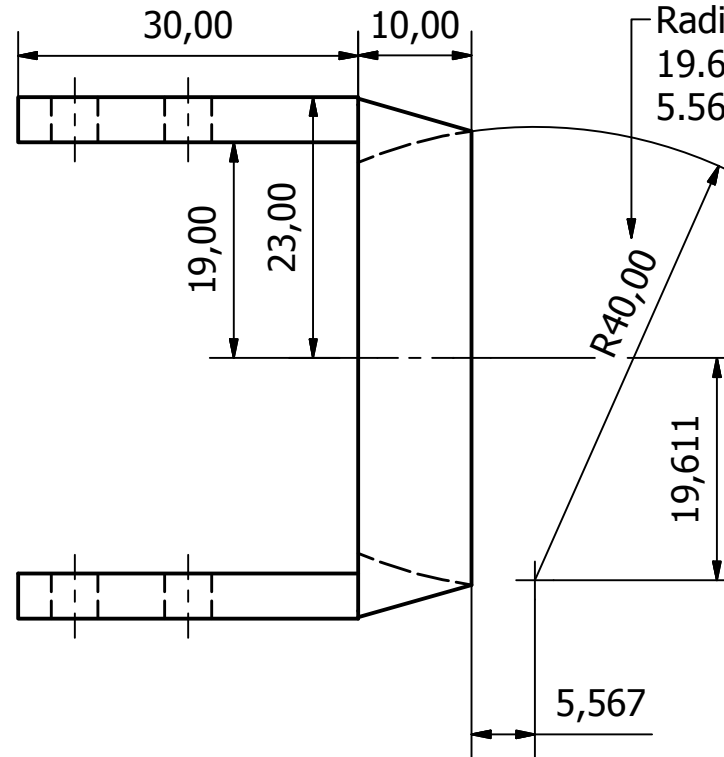
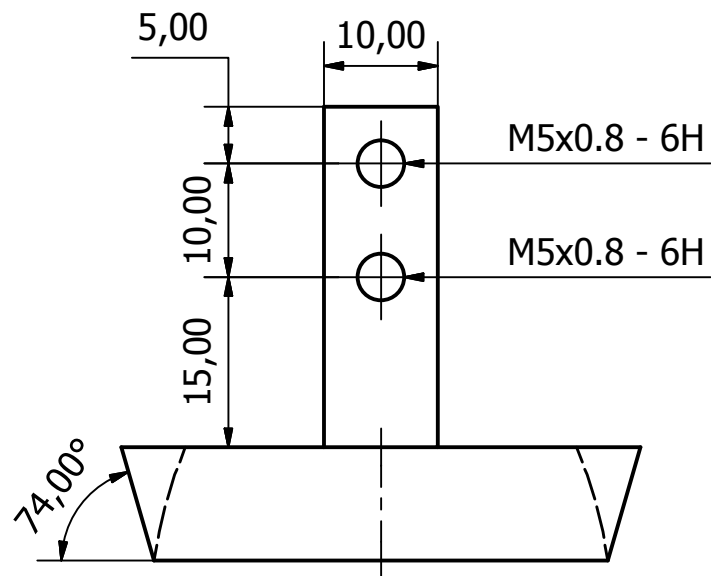
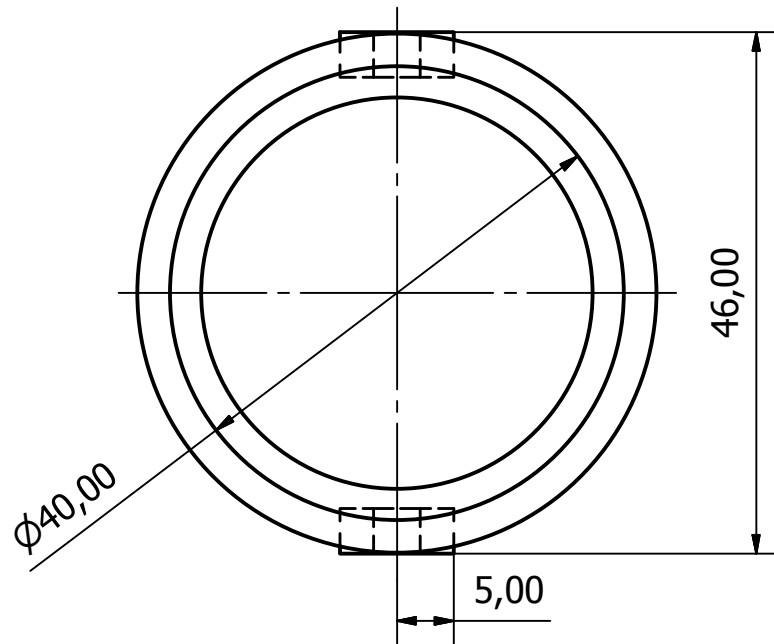
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MATERIAL: Brass



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REVISION HISTORY

Scale: 1.5:1

NOTES:
 SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
 DIMENSIONS IN mm.
 DO NOT SCALE.
 ALL THREADS MEDIUM FIT.

APPROVED:

CHECKED:

DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: 6

REV NO.



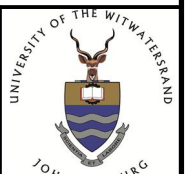
SHEET 1 OF 1

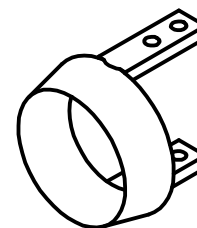
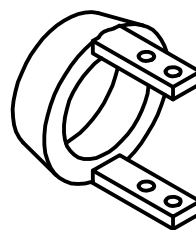
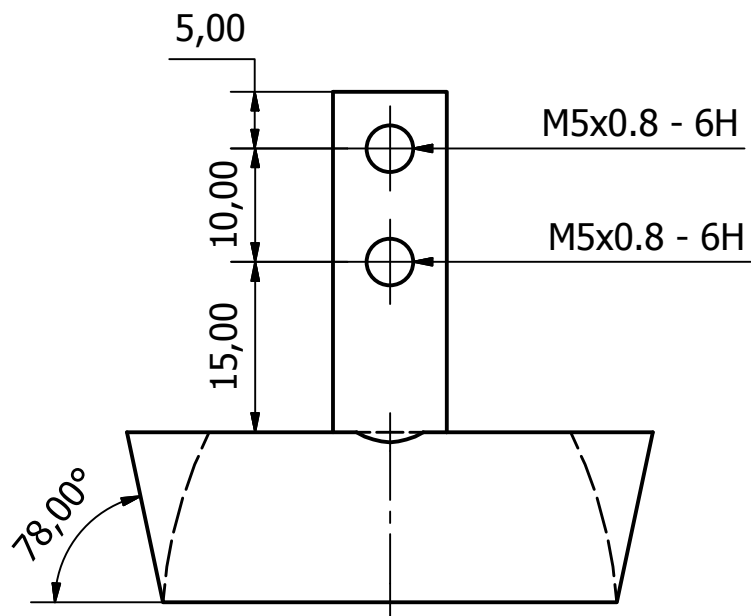
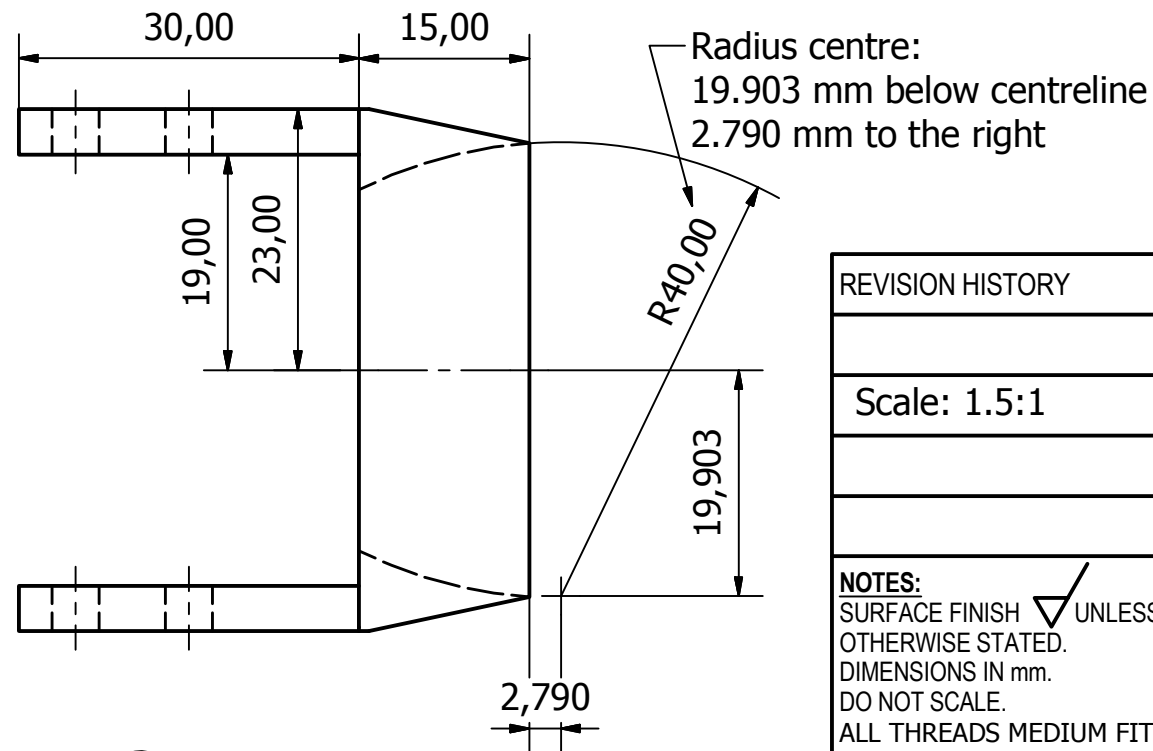
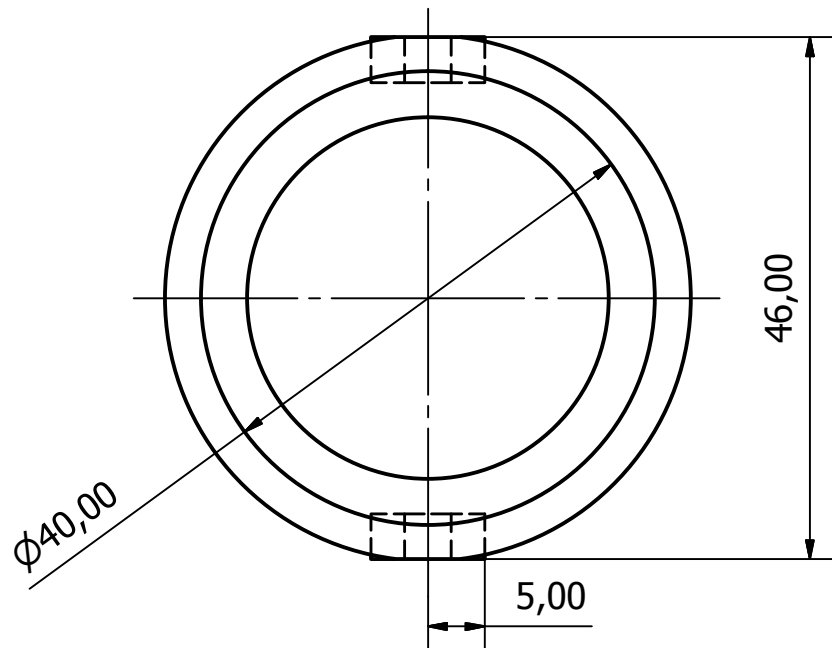
DESCRIPTION: 40 mm int. rad. of curv. w/ 8 deg leading edge

MATERIAL: Brass



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REVISION HISTORY

Scale: 1.5:1

NOTES:
 SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
 DIMENSIONS IN mm.
 DO NOT SCALE.
 ALL THREADS MEDIUM FIT.

APPROVED:

CHECKED:

DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: 7

REV NO.



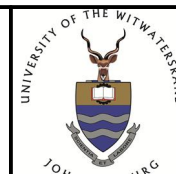
SHEET 1 OF 1

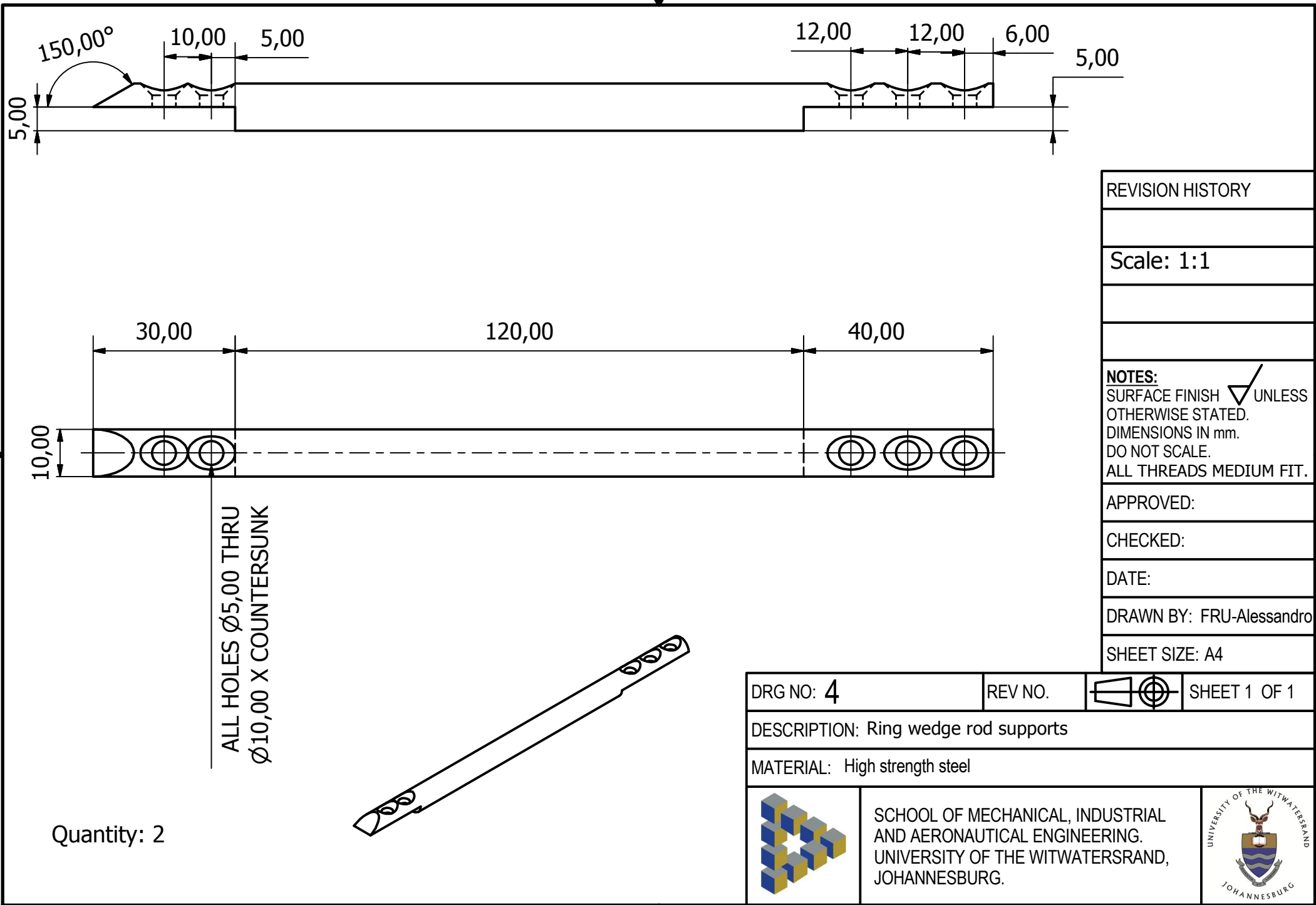
DESCRIPTION: Length 15 mm, 40 mm int. rad. of curv. w/ 4 deg l.e.

MATERIAL: Brass

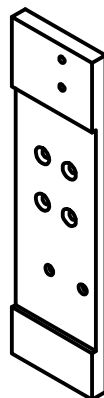
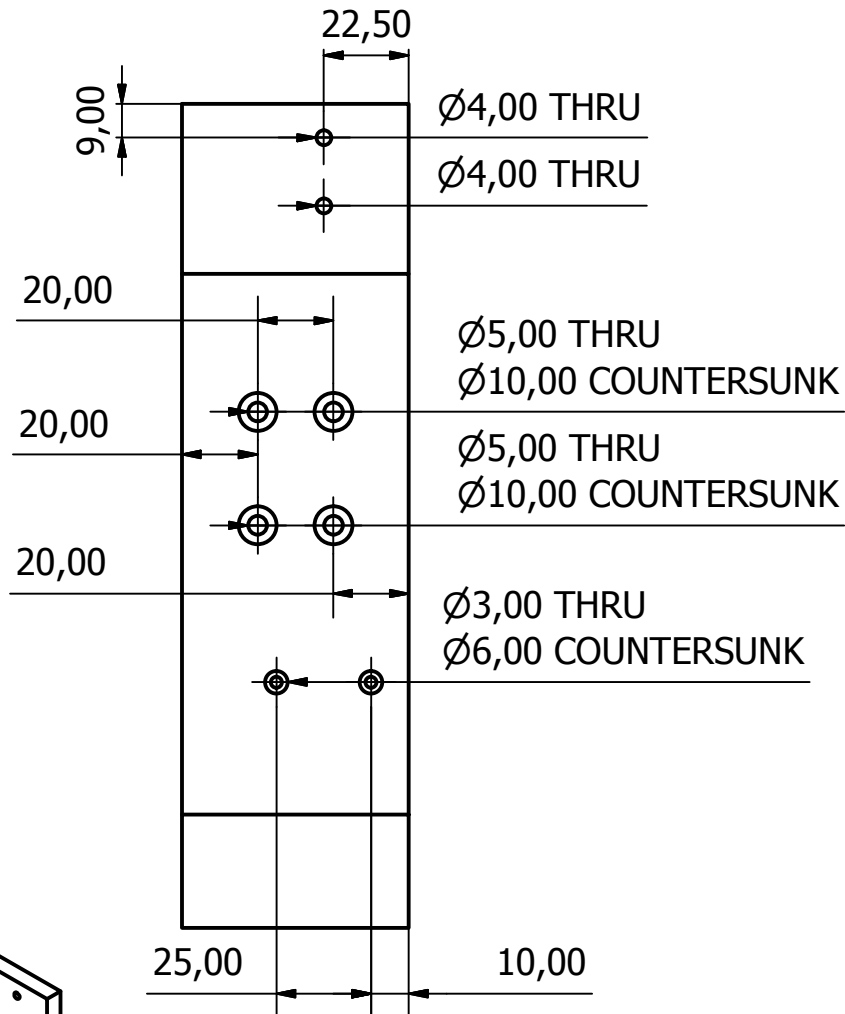
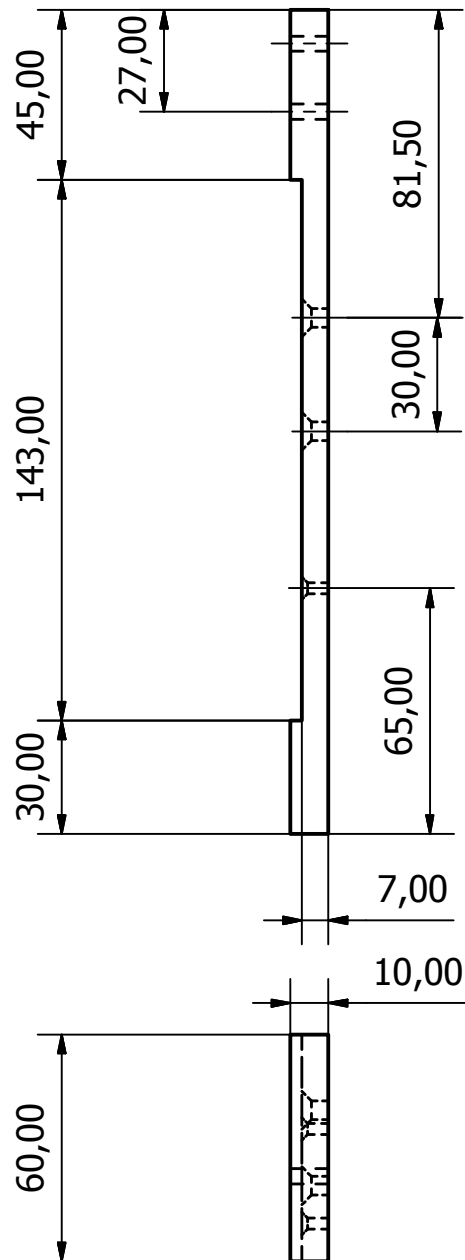


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REVISION HISTORY
Scale: 1:1
NOTES: SURFACE FINISH ✓ UNLESS OTHERWISE STATED. DIMENSIONS IN mm. DO NOT SCALE. ALL THREADS MEDIUM FIT.
APPROVED:
CHECKED:
DATE:
DRAWN BY: FRU-Alessandro
SHEET SIZE: A4



REVISION HISTORY

Scale: 1:2

NOTES:
 SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
 DIMENSIONS IN mm.
 DO NOT SCALE.
 ALL THREADS MEDIUM FIT.

APPROVED:

CHECKED:

DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: 5

REV NO.



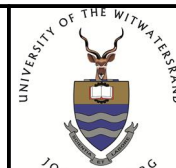
SHEET 1 OF 1

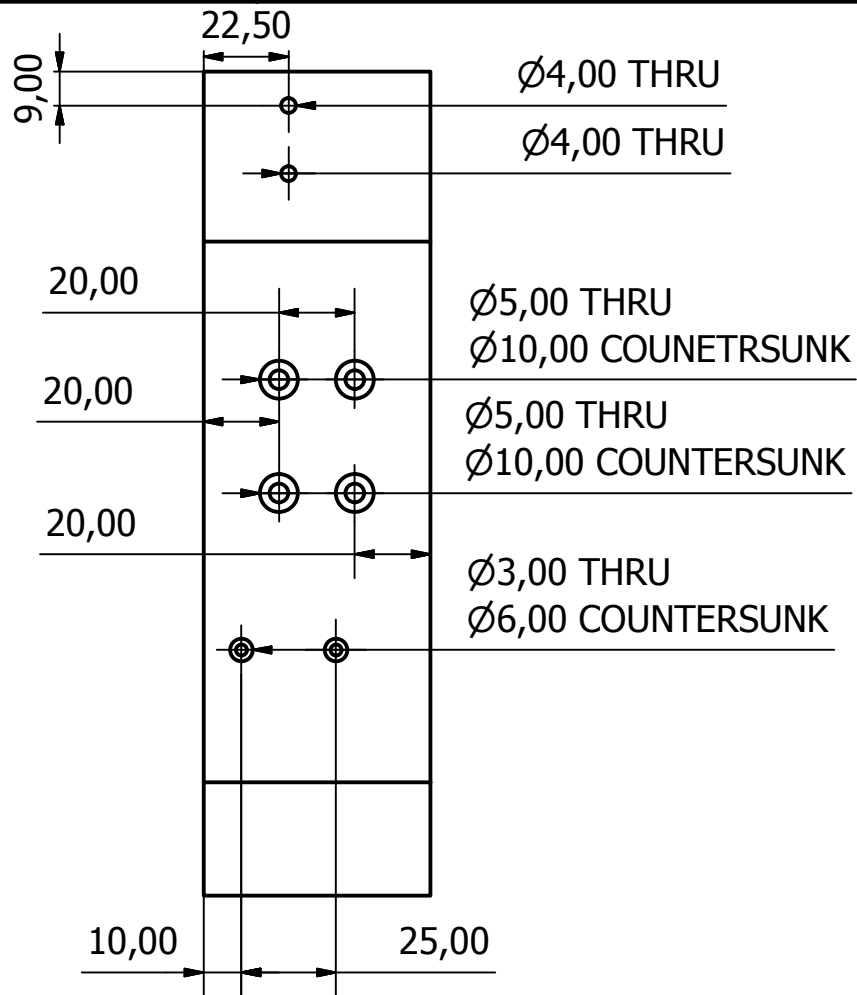
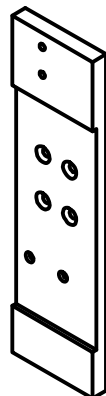
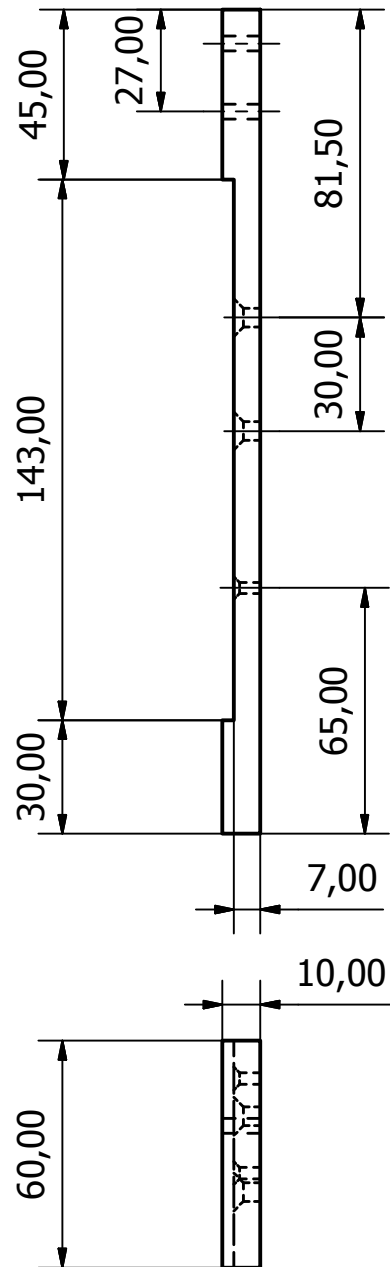
DESCRIPTION: Left side plate

MATERIAL: Aluminium



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REVISION HISTORY

Scale: 1:2

NOTES:
 SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
 DIMENSIONS IN mm.
 DO NOT SCALE.
 ALL THREADS MEDIUM FIT.

APPROVED:

CHECKED:

DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: 6

REV NO.



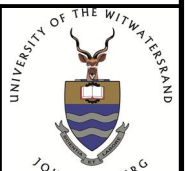
SHEET 1 OF 1

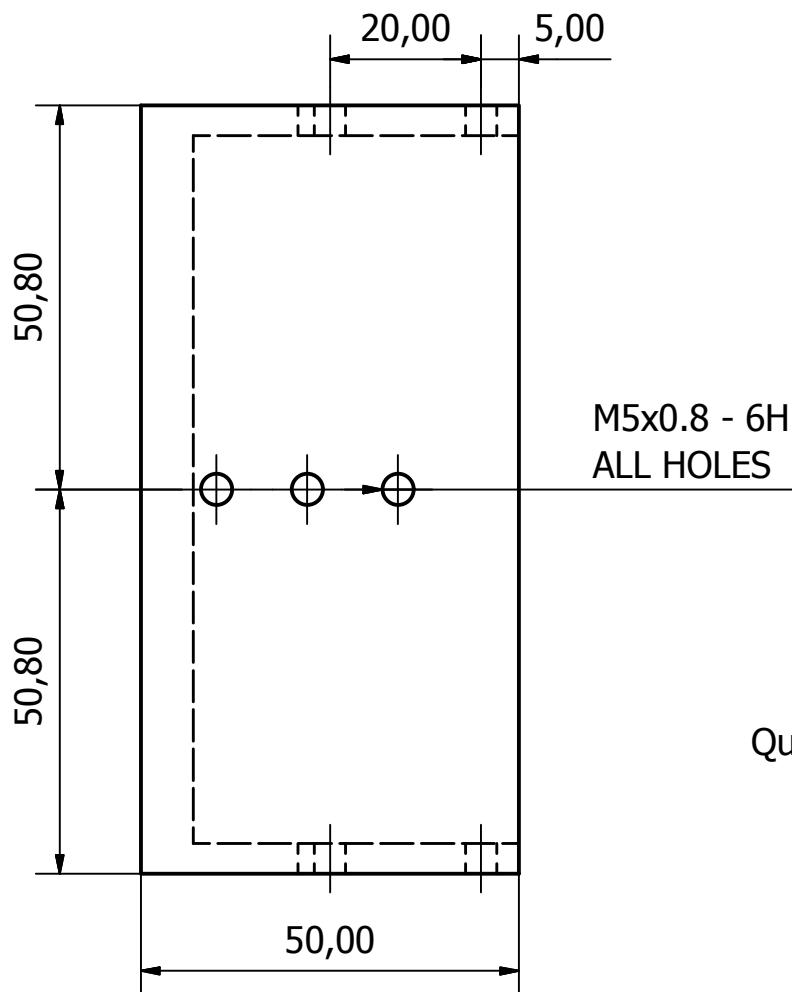
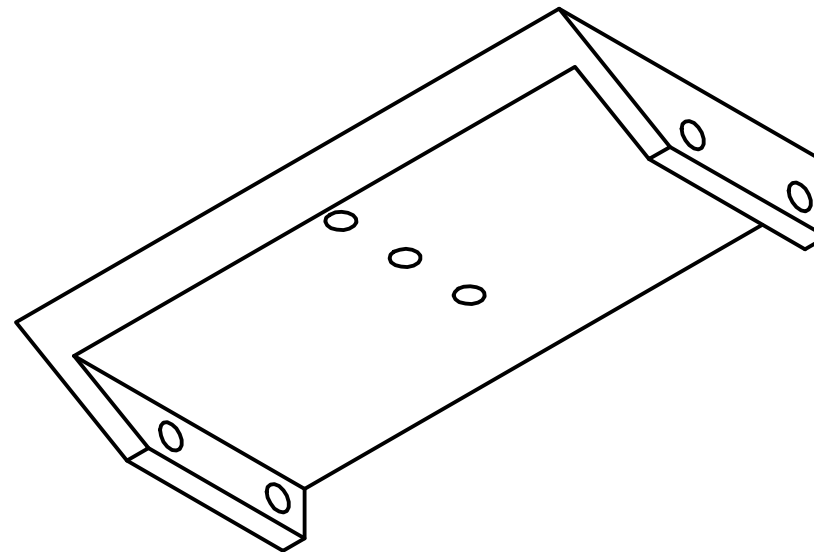
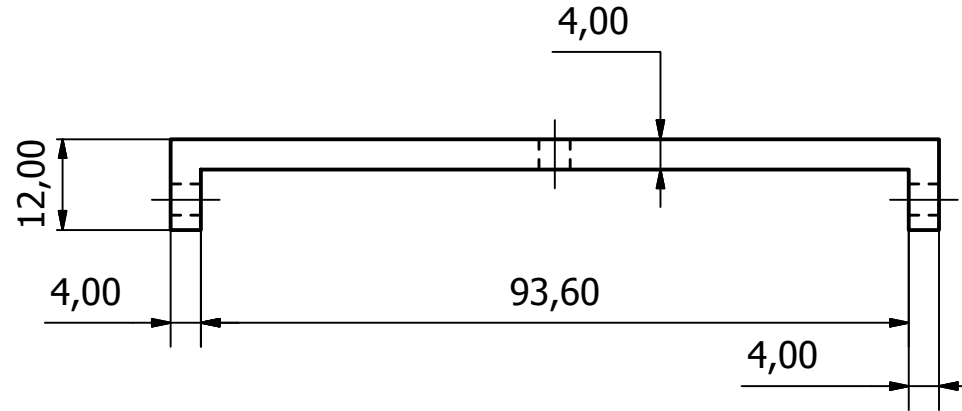
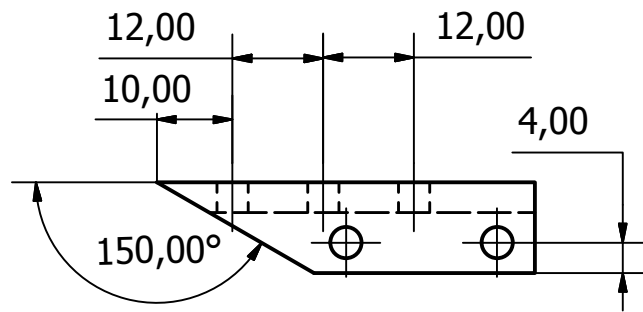
DESCRIPTION: Right side plate

MATERIAL: Aluminium



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Quantity: 2

REVISION HISTORY

Scale: 1:1

NOTES:
 SURFACE FINISH ☒ UNLESS OTHERWISE STATED.
 DIMENSIONS IN mm.
 DO NOT SCALE.
 ALL THREADS MEDIUM FIT.

APPROVED:

CHECKED:

DATE:

DRAWN BY: FRU-Alessandro

SHEET SIZE: A4

DRG NO: 7

REV NO.



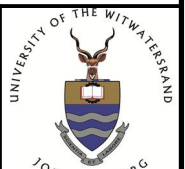
SHEET 1 OF 1

DESCRIPTION: Rod support connecting plate

MATERIAL: Brass

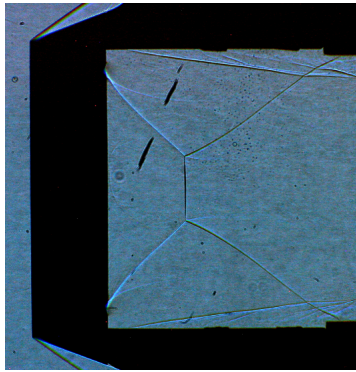


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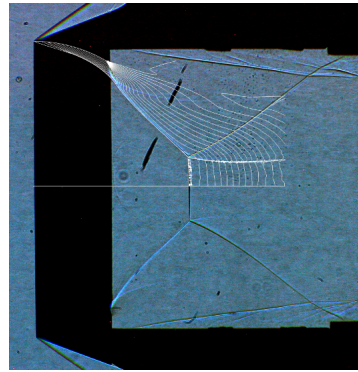


Chapter B | Additional Experimental Comparisons

Additional experimental and numerical result comparisons are displayed below with a broad varying experimental parameter range. All the numerical and experimental results are obtainable from the supervisor via submitted digital appendix.

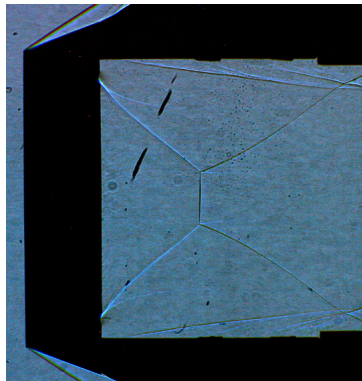


(a) Experimental shadowgraph results

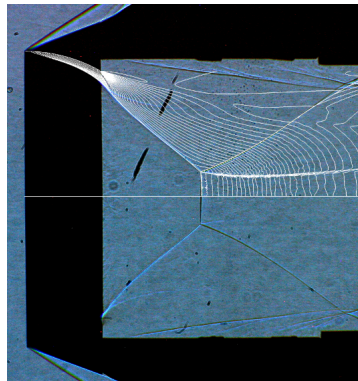


(b) Experimental with overlaid CFD results

Figure B.1: $R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.3

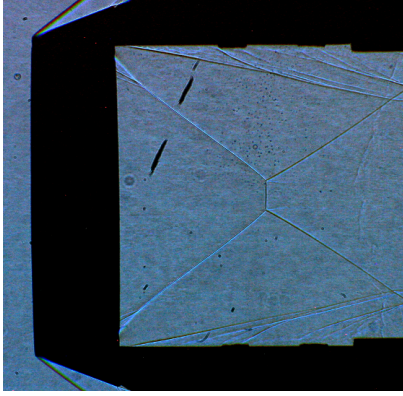


(a) Experimental shadowgraph results

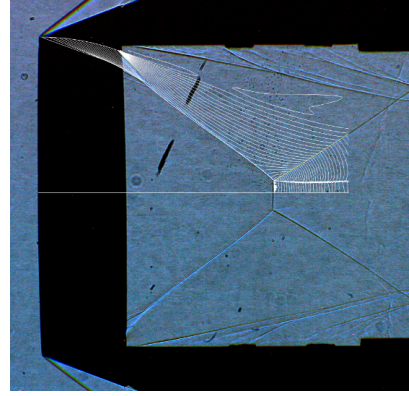


(b) Experimental with overlaid CFD results

Figure B.2: $R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.6

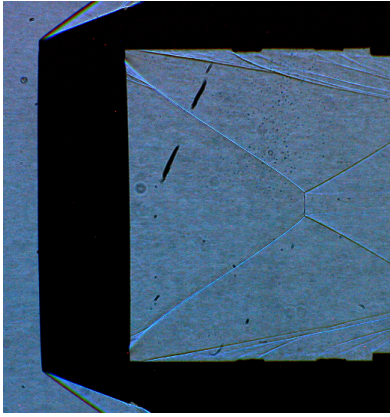


(a) Experimental shadowgraph results

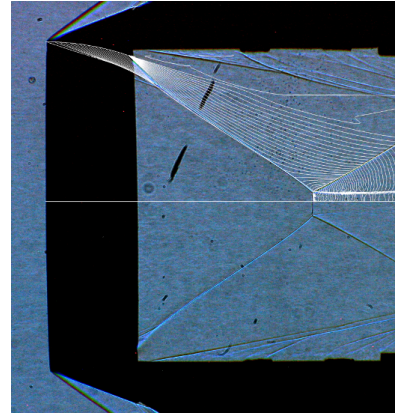


(b) Experimental with overlaid CFD results

Figure B.3: $R_c=1.5$, $\alpha=0^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.0

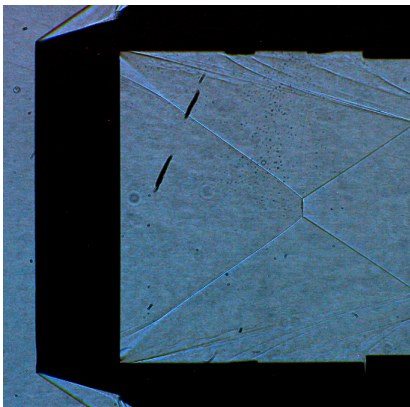


(a) Experimental shadowgraph results

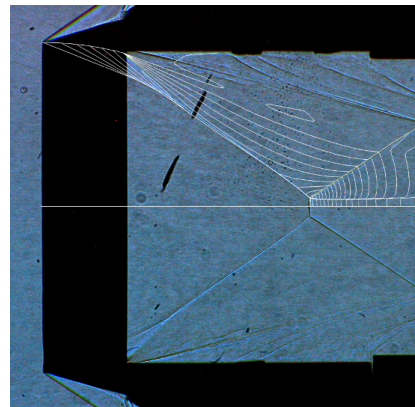


(b) Experimental with overlaid CFD results

Figure B.4: $R_c=1.5$, $\alpha=0^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.2

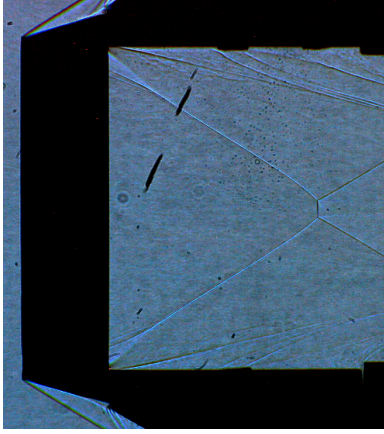


(a) Experimental shadowgraph results

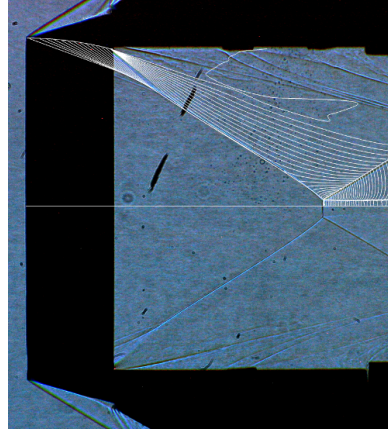


(b) Experimental with overlaid CFD results

Figure B.5: $R_c=2$, $\alpha=0^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 2.8

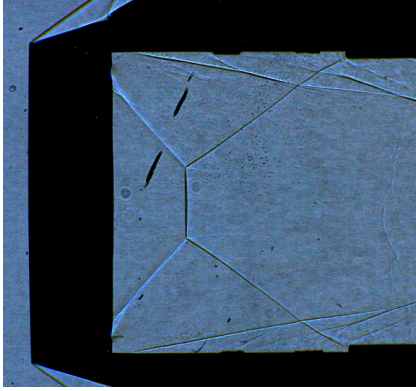


(a) Experimental shadowgraph results

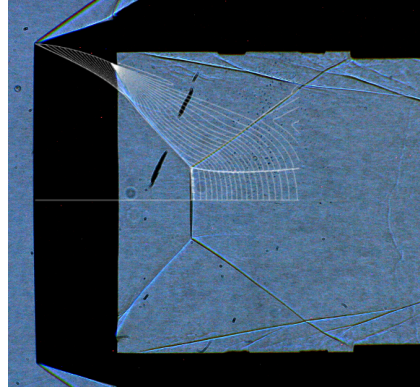


(b) Experimental with overlaid CFD results

Figure B.6: $R_c=2$, $\alpha=0^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.0

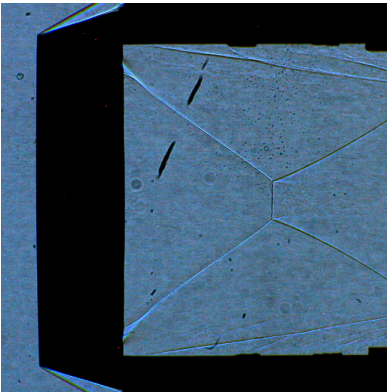


(a) Experimental shadowgraph results

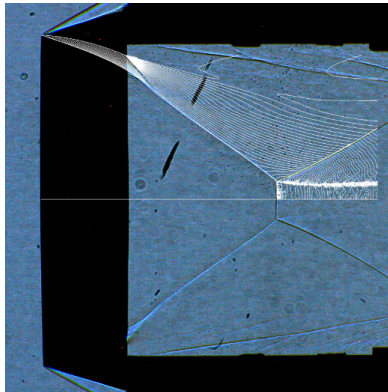


(b) Experimental with overlaid CFD results

Figure B.7: $R_c=1.5$, $\alpha=4^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 2.8

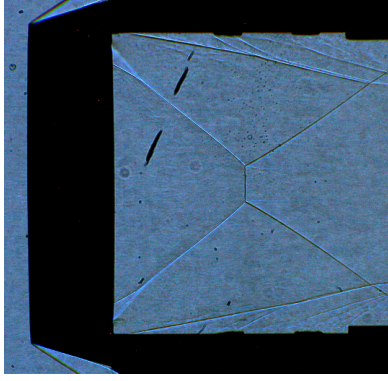


(a) Experimental shadowgraph results

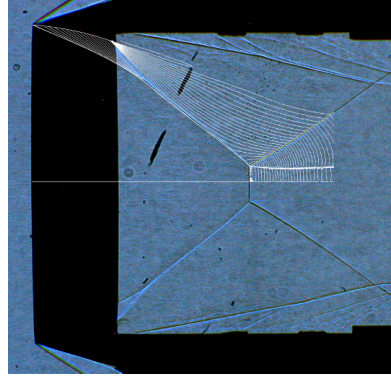


(b) Experimental with overlaid CFD results

Figure B.8: $R_c=1.5$, $\alpha=4^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.6

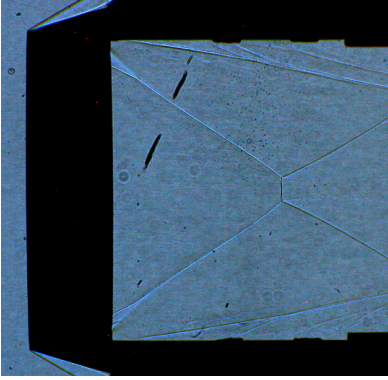


(a) Experimental shadowgraph results

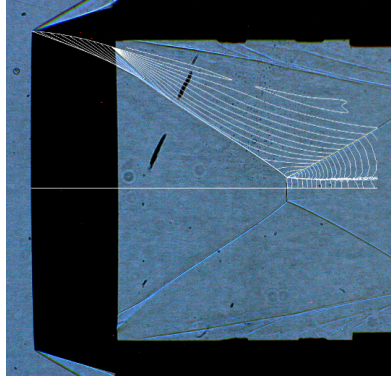


(b) Experimental with overlaid CFD results

Figure B.9: $R_c=2$, $\alpha=4^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.0

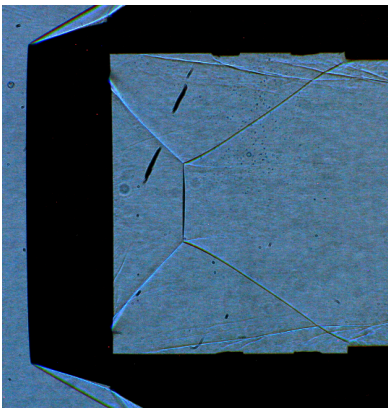


(a) Experimental shadowgraph results

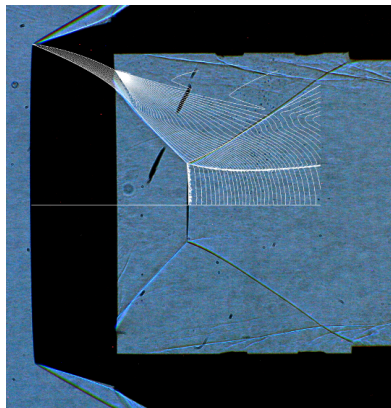


(b) Experimental with overlaid CFD results

Figure B.10: $R_c=2$, $\alpha=4^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.4

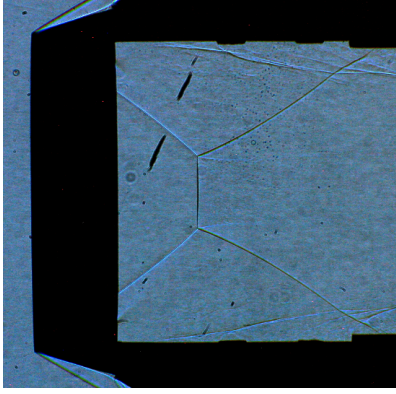


(a) Experimental shadowgraph results

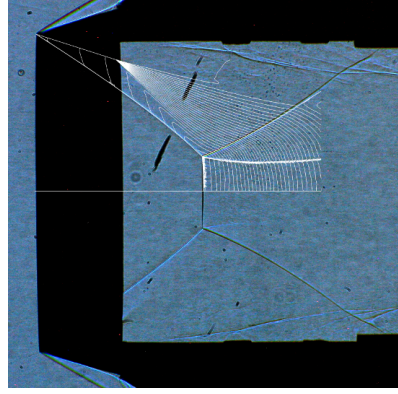


(b) Experimental with overlaid CFD results

Figure B.11: $R_c=1.5$, $\alpha=8^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.4

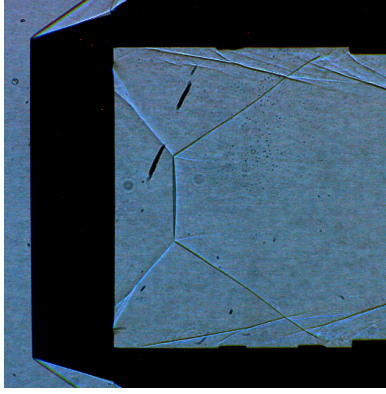


(a) Experimental shadowgraph results

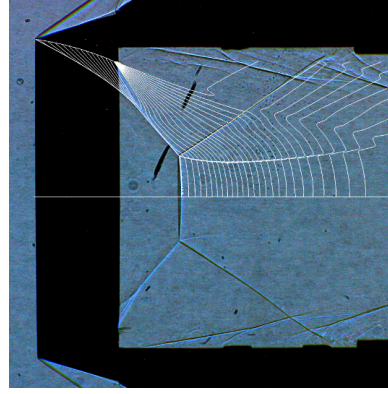


(b) Experimental with overlaid CFD results

Figure B.12: $R_c=\infty$, $\alpha=18.1^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.4

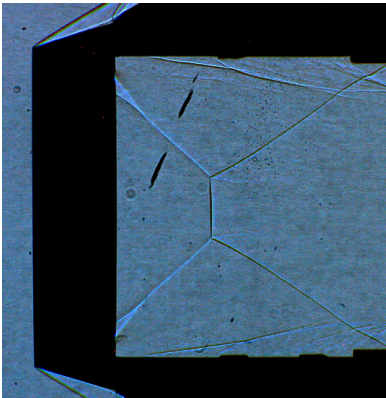


(a) Experimental shadowgraph results

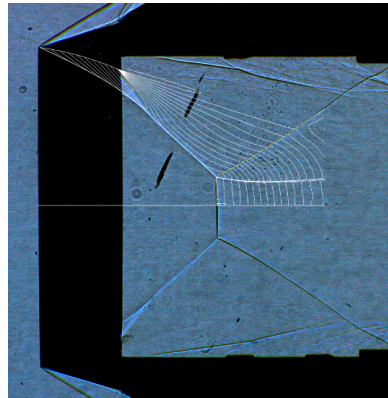


(b) Experimental with overlaid CFD results

Figure B.13: $R_c=2$, $\alpha=8^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 2.8



(a) Experimental shadowgraph results



(b) Experimental with overlaid CFD results

Figure B.14: $R_c=2$, $\alpha=8^\circ$, $L_w=0.5$ model experimental and numerical comparison at Mach 3.2

Chapter C | Shock Wave Curve Fit Data

Table C.1 through Table C.11 contain all the values of the curved shock wave shape curve fit constants that were obtained and used during the various analyses.

Table C.1: Curve fit constants for the $R_c=1$, $\alpha=0^\circ$, $L_w=0.5$ model

M	ε	η	λ	R^2	x_{max}
2.4	-0.742301	3.561566	0.728291	0.997790	0.462481
2.5	-0.705918	4.080055	0.688473	0.997638	0.489042
2.6	-0.686962	4.506286	0.666910	0.997816	0.511144
2.7	-0.680738	4.807971	0.658919	0.997905	0.528232
2.8	-0.678126	5.072988	0.655020	0.998110	0.539376
2.9	-0.677977	5.292059	0.654152	0.998279	0.547177
3.0	-0.676128	5.524829	0.651874	0.998427	0.554235
3.1	-0.676357	5.715894	0.651549	0.998478	0.559250
3.2	-0.673015	5.963139	0.648108	0.998681	0.563150
3.3	-0.670957	6.177203	0.645913	0.998788	0.566865
3.4	-0.667645	6.402498	0.642960	0.998957	0.569279
3.5	-0.665270	6.627200	0.640059	0.998935	0.569465
3.6	-0.665008	6.814868	0.638634	0.998700	0.569837

Table C.2: Curve fit constants for the $R_c=1.5$, $\alpha=0^\circ$, $L_w=0.5$ model

M	ε	η	λ	R^2	x_{max}
2.4	-1.034540	2.443924	1.023198	0.998482	0.648031
2.5	-1.043081	2.555302	1.030831	0.998486	0.654346
2.6	-1.040054	2.706340	1.026485	0.998471	0.662147
2.7	-1.034998	2.865470	1.020174	0.998473	0.666976
2.8	-1.039491	2.976486	1.024094	0.998458	0.666790
2.9	-1.016677	3.222618	0.999289	0.998519	0.674220
3.0	-1.020114	3.334933	1.002319	0.998491	0.667719
3.1	-1.032053	3.586120	0.992319	0.997614	0.671805
3.2	-1.021983	3.779406	0.981048	0.997118	0.677192
3.3	-1.019413	3.917261	0.977847	0.997534	0.667162
3.4	-1.001093	4.175563	0.957590	0.997591	0.668276
3.5	-0.992831	4.356093	0.948336	0.997488	0.668648
3.6	-0.981599	4.575964	0.935863	0.997462	0.662890

Table C.3: Curve fit constants for the $R_c=2$, $\alpha=0^\circ$, $L_w=0.5$ model

M	ε	η	λ	R^2	x_{max}
2.4	-1.462456	1.642727	1.437883	0.998606	0.770617
2.5	-1.462620	1.744424	1.436105	0.998499	0.778603
2.6	-1.424401	1.890285	1.407170	0.998496	0.774889
2.7	-1.399565	1.992613	1.389630	0.999008	0.780646
2.8	-1.471594	2.021039	1.440820	0.998401	0.781575
2.9	-1.486020	2.079087	1.454967	0.998372	0.780832
3.0	-1.494578	2.157697	1.462430	0.998309	0.770431
3.1	-1.446127	2.361200	1.410906	0.998079	0.773403
3.2	-1.432350	2.483104	1.395980	0.998019	0.771917
3.3	-1.428674	2.587081	1.391197	0.997877	0.764116
3.4	-1.395480	2.771240	1.355901	0.997841	0.764116
3.5	-1.401767	2.839879	1.361925	0.997803	0.755386
3.6	-1.411674	2.900230	1.371568	0.997850	0.743685

Table C.4: Curve fit constants for the $R_c=1$, $\alpha=4^\circ$, $L_w=0.5$ model

M	ε	η	λ	R^2	x_{max}
2.9	-0.642041	4.171348	0.635123	0.999442	0.440379
3.0	-0.626449	4.486041	0.618867	0.999558	0.469911
3.1	-0.621213	4.684389	0.614216	0.999632	0.484770
3.2	-0.618013	4.871692	0.610815	0.999677	0.497214
3.3	-0.616008	5.029078	0.609027	0.999715	0.508358
3.4	-0.615003	5.168193	0.608239	0.999749	0.517831
3.5	-0.613783	5.303266	0.607412	0.999792	0.525260
3.6	-0.613311	5.419164	0.607376	0.999808	0.532318

Table C.5: Curve fit constants for the $R_c=1.25$, $\alpha=4^\circ$, $L_w=0.5$ model

M	ε	η	λ	R^2	x_{max}
2.9	-0.742926	3.636365	0.734642	0.999608	0.572623
3.0	-0.745305	3.743422	0.737429	0.999647	0.579681
3.1	-0.748226	3.842418	0.740593	0.999686	0.580609
3.2	-0.747375	3.964240	0.739959	0.999711	0.588967
3.3	-0.746363	4.089070	0.738929	0.999748	0.591753
3.4	-0.745141	4.201227	0.738252	0.999765	0.597325
3.5	-0.743388	4.319253	0.736736	0.999775	0.599183
3.6	-0.740958	4.439131	0.734715	0.999812	0.598254

Table C.6: Curve fit constants for the $R_c=1.5$, $\alpha=4^\circ$, $L_w=0.5$ model

M	ε	η	λ	R^2	x_{max}
2.4	-0.823907	2.612279	0.815886	0.999504	0.584324
2.5	-0.841228	2.675787	0.833306	0.999567	0.598626
2.6	-0.849992	2.772140	0.841883	0.999595	0.614227
2.7	-0.858521	2.859128	0.850462	0.999632	0.624814
2.8	-0.866996	2.938611	0.859088	0.999673	0.628900
2.9	-0.869545	3.041846	0.861624	0.999694	0.639487
3.0	-0.874289	3.126611	0.866683	0.999722	0.637816
3.1	-0.874941	3.228280	0.867429	0.999733	0.643388
3.2	-0.874392	3.332716	0.866968	0.999752	0.647288
3.3	-0.874408	3.429252	0.867258	0.999766	0.643759
3.4	-0.871938	3.537974	0.865000	0.999784	0.643574
3.5	-0.869224	3.646004	0.862429	0.999792	0.640788
3.6	-0.864496	3.764575	0.857787	0.999812	0.644688

Table C.7: Curve fit constants for the $R_c=1.75$, $\alpha=4^\circ$, $L_w=0.5$ model

M	ε	η	λ	R^2	x_{max}
2.9	-1.001666	2.570954	0.995274	0.999773	0.667533
3.0	-0.999669	2.675118	0.993189	0.999769	0.679606
3.1	-0.998421	2.775854	0.991659	0.999778	0.684621
3.2	-0.999042	2.861773	0.992508	0.999784	0.683692
3.3	-0.999190	2.947541	0.992852	0.999794	0.678863
3.4	-0.996427	3.043863	0.990120	0.999796	0.677749
3.5	-0.992566	3.142803	0.986307	0.999801	0.675706
3.6	-0.989776	3.233169	0.983803	0.999803	0.669019

Table C.8: Curve fit constants for the $R_c=2$, $\alpha=4^\circ$, $L_w=0.5$ model

M	ε	η	λ	R^2	x_{max}
2.4	-1.027520	2.015541	1.021034	0.999748	0.695579
2.5	-1.048406	2.072030	1.041773	0.999761	0.710067
2.6	-1.069592	2.118959	1.063136	0.999781	0.715639
2.7	-1.086380	2.171854	1.080092	0.999794	0.718796
2.8	-1.095662	2.242039	1.089272	0.999800	0.724740
2.9	-1.104971	2.306464	1.098562	0.999799	0.727340
3.0	-1.110161	2.379317	1.103809	0.999806	0.727340
3.1	-1.114928	2.448987	1.108748	0.999820	0.722697
3.2	-1.115114	2.529722	1.108983	0.999821	0.723068
3.3	-1.113952	2.613251	1.107643	0.999819	0.723626
3.4	-1.109409	2.705477	1.103225	0.999827	0.724368
3.5	-1.108084	2.783709	1.102094	0.999829	0.717682
3.6	-1.103436	2.874210	1.097343	0.999823	0.715082

Table C.9: Curve fit constants for the $R_c=1.5$, $\alpha=8^\circ$, $L_w=0.5$ model

M	ε	η	λ	R^2	x_{max}
2.5	-0.757162	2.415131	0.754919	0.999853	0.439450
2.6	-0.743147	2.613099	0.739617	0.999825	0.495542
2.7	-0.729667	2.828558	0.724361	0.999750	0.547363
2.8	-0.736996	2.909566	0.731539	0.999756	0.567236
2.9	-0.748830	2.950306	0.743709	0.999801	0.575780
3.0	-0.756527	3.009620	0.751546	0.999827	0.585810
3.1	-0.762824	3.068993	0.758085	0.999848	0.594354
3.2	-0.769387	3.118445	0.765058	0.999867	0.596211
3.3	-0.773110	3.181920	0.768999	0.999883	0.601040
3.4	-0.775989	3.243824	0.772182	0.999890	0.605498
3.5	-0.777834	3.309571	0.774216	0.999904	0.609027
3.6	-0.779252	3.372085	0.775995	0.999910	0.609770

Table C.10: Curve fit constants for the $R_c=2$, $\alpha=8^\circ$, $L_w=0.5$ model

M	ε	η	λ	R^2	x_{max}
2.4	-0.822798	2.165501	0.818238	0.999799	0.600669
2.5	-0.849681	2.189021	0.845233	0.999829	0.623700
2.6	-0.872743	2.217607	0.868431	0.999852	0.642088
2.7	-0.892194	2.251239	0.888026	0.999871	0.656761
2.8	-0.909504	2.284844	0.905571	0.999887	0.665676
2.9	-0.923938	2.322346	0.920246	0.999898	0.672363
3.0	-0.935105	2.366583	0.931567	0.999907	0.677749
3.1	-0.944998	2.410108	0.941634	0.999914	0.678492
3.2	-0.951855	2.459789	0.948670	0.999918	0.682392
3.3	-0.957437	2.510611	0.954383	0.999923	0.682207
3.4	-0.962212	2.559066	0.959427	0.999925	0.678863
3.5	-0.963475	2.619956	0.960710	0.999927	0.685364
3.6	-0.965955	2.671576	0.963394	0.999928	0.677749

Table C.11: Curve fit constants for the $R_c=2$, $\alpha=4^\circ$, $L_w=0.75$ model

M	ε	η	λ	R^2	x_{max}
2.6	-1.120158	1.979853	1.116044	0.999844	0.648960
2.7	-1.099267	2.133689	1.093803	0.999816	0.698737
2.8	-1.084616	2.278132	1.077686	0.999788	0.743685
2.9	-1.072399	2.418680	1.064292	0.999774	0.788819
3.0	-1.063519	2.551565	1.054274	0.999773	0.832652
3.1	-1.057647	2.673700	1.047529	0.999787	0.878158
3.2	-1.058601	2.761749	1.048366	0.999813	0.903789
3.3	-1.062311	2.831556	1.052317	0.999842	0.914933
3.4	-1.066042	2.894122	1.056625	0.999862	0.916790
3.5	-1.068109	2.959736	1.059363	0.999880	0.917533
3.6	-1.069420	3.026550	1.061104	0.999885	0.914376

Chapter D | Digital Appendix

The submitted digital appendix consists of the following sections:

- D1-Steady SSWT Images
- D2-Unsteady Highspeed Images
- D3-CFD Simulations
- D4-Manufacturing Drawings
- D5-Numerical Code Scripts and Curve Fit Data