THE ROLE OF THE TEACHER IN DEVELOPING LEARNERS' MATHEMATICS DISCOURSE AND UNDERSTANDING

By

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DECLARATION

I declare that this thesis is my own, unaided work. It is being submitted for the Degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

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(Signature of candidate)

5th day of April 2021 at Cape Town

Abstract

This study focuses on the role of the teacher in developing learners' mathematics discourse and understanding. Mortimer and Scott's (2003) framework of meaning making as a dialogic process was used in conjunction with variation theory, in order to gain an understanding of the role of the teacher in mathematics classrooms. I collected data in three phases: phase one was pre-observation interviews of 6 teachers at three high schools in one Province in South Africa; phase Two was through video recorded classroom observations of three teachers (selected from phase one), and phase three data collection was through post-observation interviews of the three teachers. Based on Mortimer and Scott's framework, variation theory, and other relevant literature, I developed an analytical framework which I used to analyse the data collected. The findings that emerged from the study indicate the intricate role of the teacher, with multiple layers embedded in the teacher's day-to-day tasks in the classroom setting. The role of the teacher in selecting an appropriate example set and enacting it in a way that provides opportunities which lead to developing learners' mathematics discourse and understanding was one of the findings in the study. Like some of the literature reviewed, the study found that everyday contexts in the example set may not enable learners to develop their mathematics discourse and understanding if not carefully selected and enacted. The communicative approach and patterns of discourse that were used by the teachers enabled different possibilities in the process of enacting the example set selected. The implication of the findings is that the example set selected by the teacher, whether pre-planned or spontaneous needs to be carefully selected by drawing on the intended object of learning. Variation theory provided a potent way of engaging with the teachers' example set to ascertain what the example set makes available to learn, through patterns of variation. As shown by the study, patterns of variations have great potential to guide the teacher in selecting an example set that brings the object of learning to the fore. A theoretical contribution of this study lies in extending and merging Mortimer and Scott's framework for meaning making and variation theory into describing, analysing and interpreting the role of the teacher in developing learners' mathematics discourse and understanding in functions classrooms. Methodologically, the study contributes to knowledge by providing a framework to analyse and interpret a mathematics example set to ascertain the opportunities that the example set provide for developing mathematics discourse and understanding.

Dedication

This thesis is dedicated to the following special people in my life.

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2. My treasured parents: ***Callist Byaba Wandera*** and ***Magdalene Kubirabe Wandera*** (who is late)
3. My beloved brother: ***Peter Kisembo***
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List of abbreviations/acronyms

LoLT: Language of Learning and Teaching

CAPS: Curriculum Assessment Policy Statement

IA: Interactive-Authoritative

NIA: Non-interactive-Authoritative

NID: Non-interactive-Dialogic

ID: Interactive-Dialogic

I-R-E: initiation-Response-Evaluation

MKI: Marking key ideas

RLE: Revoicing learner

LPK: Learner prior knowledge

LM: Learner misconceptions

AA-LU: Learner understanding before teaching

SS-TS: Separation teacher strategy

FF-TS: Fusion teacher strategy

CC-TS: Contrast teacher strategy

I-R-P: Initiation-Response-Prompt

DM-TS: Double move teacher strategy

MM-LU: Learner understanding after teaching

NCTM: National Council of Teachers of Mathematics

DBE: Department of Basic Education

DOE: Department of Education

NRC: National Research Council

Key words

Mathematics discourse; understanding, scientific discourse, everyday discourse, content, activities, example set, double move, fusion, contrast, separation, object of learning, intended object of learning, enacted object of learning, prior knowledge, assessing; communicative approach, patterns of discourse; interactive-dialogic, interactive-authoritative, non-interactive-authoritative, non-interactive-dialogic, evaluation; initiation, response.

CHAPTER 1: Introduction and background

1.1 Background and Rationale

In my previous study, I investigated learners' communication of mathematics algebraic reasoning in a mathematics multilingual classroom. I drew on Gee's (2005) notion of communication. Gee argues that language is situated, and this informed my study. I used Gee's notion of language as a theoretical framework for the study. The main aim of the study was to find out how learners used language as a resource to help them understand the concepts of algebra. The research employed a qualitative approach. Data was collected from a township school in South Africa. The learners were from different language backgrounds, hence the emphasis of the issue of multilingualism in the study. Methods of data collection were document analysis (i.e. analysing learner notebooks and textbook), and lesson observations. Results from the study indicated that learners preferred communicating their mathematics algebraic reasoning using informal algebraic language rather than using formal algebraic language. Further analysis of classroom observations revealed that the teacher did not encourage learners to communicate their algebraic ideas and concepts formally. In the interaction, learners' informal algebraic reasoning was not linked to the objective of the lesson and hence to the formal algebraic reasoning (see Aineamani, 2011; 2018). The main conclusion from my previous study was that although the learners were not able to communicate their mathematics reasoning in an appropriate way, the teacher in the study did not encourage them or probe them to communicate appropriately (Aineamani, 2011). Learners in the study did not use the informal ways of communicating to develop formal ways of communicating which would have enabled them to participate in the mathematics discourse and understand algebraic concepts that they engaged with in the lessons observed.

Within a mathematics classroom, learners need to be able to participate in the mathematics discourse appropriately because this is an integral part of learning mathematics concepts (Bennett, 2014; Stein, 2007; Moschkovich, 2003). Erath, Prediger and Quasthoff (2018) concur with Stein (2007) and Moschkovich (2003), and they argue that learning to communicate mathematics in an acceptable manner within the mathematics discourse is as equally important as learning the mathematics concepts themselves. It is through communication that one's understanding of concepts is assessed.

If learners are not able to use the appropriate language to communicate their mathematics reasoning, as shown in my previous study, they may struggle to engage with the mathematics concepts at a level that is acceptable. Communicating mathematics reasoning appropriately is an indication of a developed mathematics discourse and hence understanding of the mathematics concepts that are being communicated, and this is what Erath et al. (2018) refer to as discourse competence. Webb, Franke, Ing, Turrou, Johnson and Zimmerman (2017) argue that the teacher needs to actively support and invite learners to participate in the classroom discussion, in order to enable achievement.

Although the importance of developing mathematics discourse and hence understanding is acknowledged by many researchers (see Wachira, Pourdavood & Skitzki, 2013; Bell & Pape, 2012; Walshaw & Anthony, 2008), there is a scarcity of research that focusses on the role of the teacher in developing learners' mathematics discourse and understanding, especially in a South African context, hence the significance of my study. Although it was evident that the teacher in my previous study did not engage with the learner responses to enable them to communicate using the appropriate language, I was not able to unpack and discuss this because of the focus of my study. Therefore, my previous study became for me the springboard for my present study. For my study, I argue strongly that developing learners' mathematics discourse and understanding involves more than a focus on the language-in-use. The content and activities[1] that the teacher selects in the process of developing learners' mathematics discourse and understanding are crucial. However, research on developing learners' mathematics discourse and understanding has not given the content and activity selection the attention that it requires.

Many studies have engaged with and explored the use of examples for teaching new mathematical concepts (see Knuth, Zaslavsky & Ellis, 2019; Zaslavsky, 2019; Huang, 2017; Rissland, 1991; Mason & Pimm, 1984). However, there are not many studies that have

[1] Content and activities is used interchangeably with examples or example set, and content and activities make up the example set of a lesson in this study.

investigated the selection of content and activities, with a focus on how the content and activities selected play a role in helping the teacher to develop learners' mathematics discourse and understanding.

The mediation moves that are used by the teacher are also important, however, if the content and activities are not carefully selected, then the attention to what needs to be mediated is not in place. This is discussed in detail in Chapter 2 and 3. In the next section, I present a discussion that gives context to my argument of what mathematics discourse and understanding entails in relation to the study.

1.2 Mathematics discourse and understanding

Discourse is defined as talk that allows students to participate deliberately in a dialogic process where meaning making happens (Mortimer & Scott, 2003). Through discourse, learners obtain tools used to think about ideas. Mortimer and Scott (2003) define a dialogic process as a process where learners engage with their understanding of ideas and compare them with other ideas that are being discussed in a social setting. Mortimer and Scott's definition of discourse is defined within the science classroom as indicated in their work. For my study, it was crucial to give a working definition of mathematics discourse and understanding, and therefore, I drew on NCTM and National Research Council as presented below.

According to NCTM (1991), mathematics discourse is the mathematical communication – written or verbal and non-verbal, in an interaction between the teacher and learners, and between learners. National Research Council (2002) define understanding within the mathematics context as "comprehending mathematical concepts, operations, and relations – knowing what mathematical symbols, diagrams, and procedures mean" (p. 9).

The definitions above resonate with my understanding of mathematics discourse and understanding as used in the study. One of the main ways we can experience mathematics discourse is through talk (verbal), as indicated in the definition of discourse by Mortimer and Scott above, and the definition of mathematics discourse by NCTM above.

Manouchehri (2007) argues that focusing on discourse may lead to a move towards improving and hence developing mathematical language and concept development in a lesson. Therefore, as learners develop an appropriate mathematics discourse, they may participate productively in any mathematics classroom discussion and hence opportunities for mathematics understanding of concepts are availed to the learners. The teacher's obligation to mediate the process of developing the appropriate classroom discourse becomes critical in this regard (Williams & Baxter, 1996). The major challenge of teaching mathematics is how best to fulfil the obligation of bringing learners into the mathematics discourse so that they can appropriately participate in it (Williams & Baxter, 1996). This challenge was evident in my previous study as discussed earlier (see Aineamani, 2011; 2018). Following from my previous study, the learners' inability to use formal ways of communicating mathematically became for me the *raison d'être* for an investigation into how they (learners) are in fact enculturated into this practice. In what follows below, I discuss the move between formal and informal ways of communicating and argue for why this move is important in developing learners' mathematics discourse and understanding.

1.3 Informal and formal ways of communicating mathematics

Pimm (1991) argues that learners come to the classroom with informal mathematics language, which they then draw on to communicate mathematics ideas. Informal mathematics language that learners bring needs to be developed into formal mathematics language. Halliday, (1975) argues that part of learning mathematics entails mastery of the mathematics register. Halliday (1975) argues that mathematics register cannot be referred to as simply knowing terms and being able to define terms. He unpacks a mathematics register as:

> "a set of meanings that belong to the language of mathematics (the mathematical use of natural language) and that a language must express if it is used for mathematical purposes. We should not think of a mathematical register as constituting solely terminology, or of the development of a register as simply a process of adding new words" (p.65).

Halliday argues that mathematics register is not about being able to define terminology, as shown in the excerpt above. For learners to have control over the mathematics register of functions as a mathematics topic for example, they need to be able to define terms appropriately and to communicate procedures, conventions and justifications using the acceptable mathematics register of functions. Learners therefore need to develop mathematics discourse that enables them to talk and mean like mathematicians. William and Baxter (1996) argue that the learners would be referred to as members of the mathematics society once they can talk and mean like mathematicians.

Mortimer and Scott (2003) argue that every subject that is taught at school has two types of discourse: scientific discourse and the everyday discourse. Within the scientific discourse, scientific concepts are used while everyday concepts are used in everyday discourse (Mortimer & Scott, 2003). Mortimer and Scott (2003) provide an account of everyday and scientific concepts by drawing on Vygotsky's (1978) sociocultural theory to define the terms scientific and everyday concepts, and this is the account of scientific and everyday that I draw on for this study. Everyday concepts are acquired through interaction with the world, intuitive understanding of concepts is also developed at the everyday level. Scientific concepts are learnt at school and the environment in which they are learnt is not familiar to the learners (Mortimer & Scott, 2003). Everyday concepts may be used to understand scientific concepts and therefore, everyday concepts if used appropriately, can lay a foundation for learning scientific concepts. For my present study, scientific and everyday discourse is important, as indicated previously.

Scientific mathematics discourse draws on terms from the mathematics register. Taking an example of a quadratic equation, $3m^2 + 15m + 5 = 0$, 3 and 15 are referred to as co-efficients of x^2 and x respectively, and 5 is a constant. The terms co-efficient and constant are from the mathematics register and are therefore referred to as scientific mathematics discourse. Everyday mathematics discourse is when the everyday language (everyday context) is used to communicate mathematics concepts. For example, in the quadratic equation above, 3 and 15 may be referred to as numbers and not co-efficient of x^2 and x respectively, and 5 as a number with no letter. The move from informal ways to formal ways of communicating mathematics is crucial and may be used in the process of developing learners' mathematics discourse and understanding as argued by Hedegaard and Chaiklin (2005).

Hedegaard and Chaiklin (2005) discuss a 'double move' as a process where everyday ideas are used to develop scientific ideas, and scientific ideas are drawn on to clarify everyday ideas. Hedegaard and Chaiklin argue that for learning to be meaningful and powerful, teachers need to keep in awareness that everyday ideas and scientific ideas are important and that they complement each other. For Mortimer and Scott (2003), an understanding of a scientific discourse involves being able to move confidently between everyday discourse and scientific discourse as the context demands. Hedegaard and Chaiklin (2005) argue that the teacher may need to ensure that the double move is done in such a way that learners' thinking, and practices are transformed. This raised a question for me on what precisely the teacher should do to ensure that the double move is successful in relation to the learning objective of a given lesson.

Scientific mathematics discourse is the global mathematically accepted way of communicating (Neria & Amit, 2004) and it is also emphasised in the South African curriculum. The Curriculum and Assessment Policy Statement (CAPS) highlights specific skills that mathematics learners in the Further Education Training phase (FET) should focus on are:

- *Correct use of the language of mathematics*
- *Ability to communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams* (DoE, 2010, p. 8).

The South African curriculum emphasises the importance of learners' ability to understand the language of mathematics and the ability to communicate mathematics ideas and concepts. The main argument that makes my study important is that learners may not understand and develop the appropriate language of mathematics if left to coincidence and impulse and hence the need for guidance and mediation of the process in enabling learners' development of scientific mathematics discourse. Mediating scientific knowledge within a dialogic classroom is particularly hard as many researchers have found out (see Brenner, 1994; Moschkovich, 2019). And although many researchers have emphasised the challenge of developing learners' mathematics discourse and hence understanding, the role of the teacher in this regard has not attracted the attention that it deserves as discussed earlier in this Chapter. Drawing on my teaching experience of mathematics teaching, and the findings from my previous study, I wanted to unpack the role of the teacher, and therefore I embarked on the study, guided by the purpose of the study presented below.

1.4 Purpose of the study

The purpose of the study was to investigate the role of the teacher in developing learners' mathematics discourse and understanding, with a focus on Grade 10 functions mathematics lessons.

1.5 Research Questions

The main research question is broken down into three critical questions as stated below.

1.5.1 Main research question

What is the role of the teacher in developing learners' mathematics discourse and understanding?

1.5.2 Sub questions

a. What kind(s) of content and activities is/are selected by the teacher in the process of teaching concepts to learners in a mathematics functions lesson?

b. How do the content and activities shape the classroom engagement in a mathematics functions lesson?

c. What mediation moves does the teacher employ to explain the concepts to learners in a mathematics functions lesson?

In Chapter 2, I will engage with the research questions and link them to the problem of the study and restate the research questions in relation to the theoretical and analytical framework of my study.

1.6 Relevance of the study

The role of the teacher in developing mathematics discourse and understanding is acknowledged in mathematics education research, and my study hopes to contribute to this important area of mathematics education research, through a qualitative study of teaching of functions in classroom practices in South Africa. My goal is to both add to the field by identifying and illuminating what the required teaching strategies/roles are and considering how these are enacted in classroom practice.

All stakeholders involved directly or indirectly in mathematics education: curriculum specialists, mathematics teachers, mathematics subject advisers and heads of departments both at school, district, provincial and national level, would appreciate information on what teachers might emphasise and what they (teachers) ignore in the process of teaching and learning of mathematics, with a focus on functions. The study will provide insight into how appropriate content and activities may be selected by the policy makers, textbook writers and teachers, and finally how the content and activities are enacted by teachers, to enable learners develop mathematics discourse and understanding.

This study will highlight and acknowledge issues that are taken for granted in the process of mathematics instruction and yet if not focused on can have significant implications. In other words, this study will engage with the ‘taken for granted’ issues in teaching mathematics.

1.7 Outline of Chapters

This study is divided into eight chapters as shown in Figure 1 below.

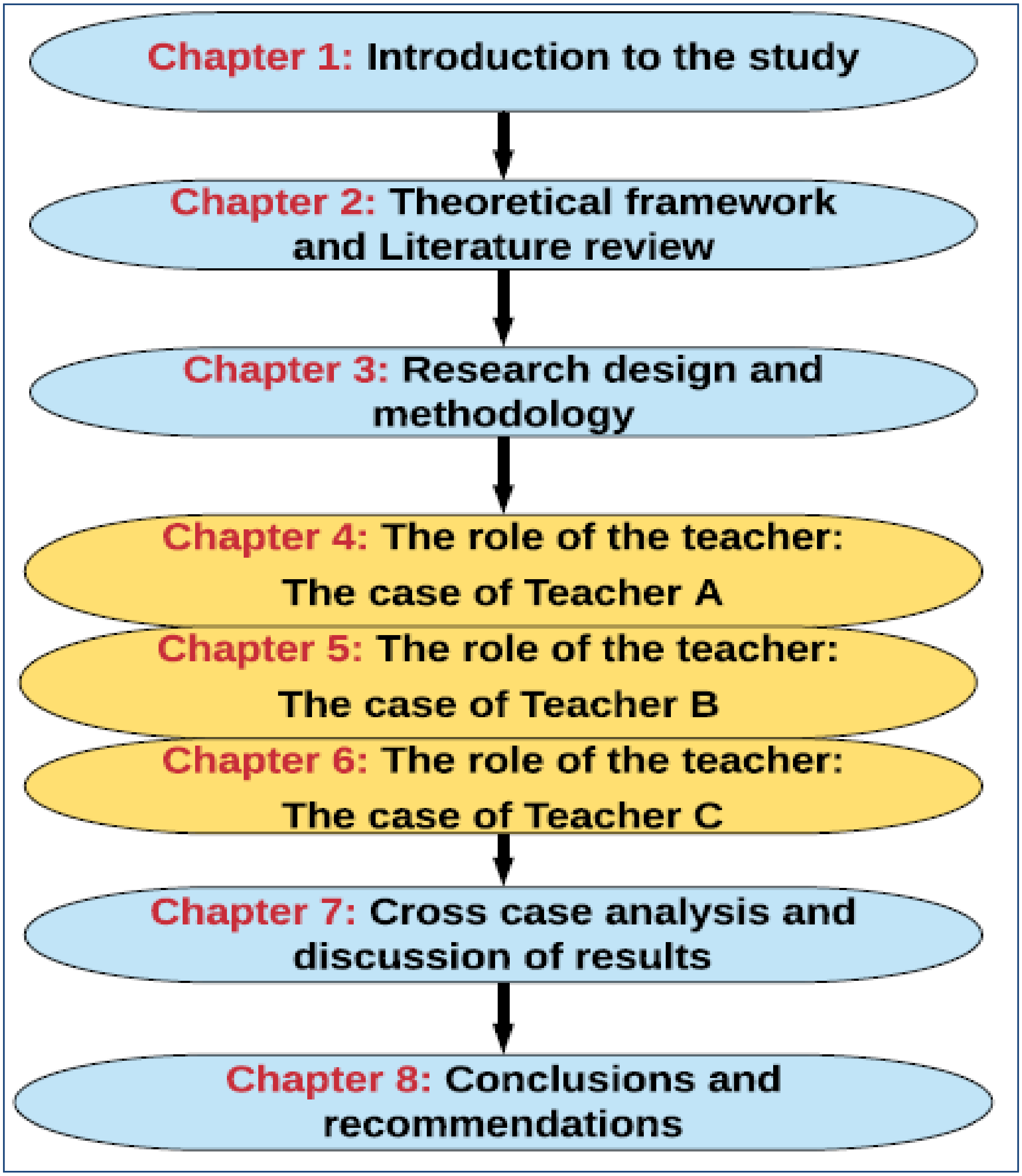


Figure 1: Structure of the thesis

1.8 Conclusion

In this Chapter, I have presented the rationale for the study, by drawing on my previous study and literature to explain mathematics discourse and its importance to learners' understanding of mathematics concepts. The focus of each of the following Chapters is presented below.

Chapter 2 theorises the role of the teacher in developing learners' mathematics discourse and understanding. Mortimer and Scott's (2003) framework for analysing meaning making in science classrooms is discussed in this Chapter. Variation theory by Marton, Runesson and Tsui (2004) and Watson and Mason (2006) among others is also presented in this Chapter as a complement to Mortimer and Scott's framework. The Literature of the study is also presented in Chapter 2 and focuses on the following aspects.

- Developing learners' mathematics discourse and understanding
- Language and interaction
- Teacher's awareness
- Terminology
- Participation in a classroom interaction
- Opportunities to learn
- Selecting appropriate examples to teach mathematics concepts.
- Engaging with essential parts of teaching mathematics
- Generalisation
- Random selection of examples
- Prior knowledge
- Cognitive conflict
- Counter-examples
- Everyday context in mathematics examples
- The concept of a function
- Definition of a function
- Facets of the concept of a function
- The importance of (and difficulties with) teaching the concept of a function,

In Chapter 3, a discussion of the paradigm, and design of the study is presented. The issues relating to population and research context, sampling, data collection and analysis are presented. Trustworthiness, credibility, and ethical considerations in relation to the data collection process are presented in this Chapter. The analytical framework and the codes and the code identification rules that were developed for the study are presented in Chapter 3.

In Chapter 4, findings of Teacher A are presented ranging from Teacher A's example set to how the example set was enacted in the lessons observed. The patterns of discourse and communicative approach that Teacher A used in the process of enacting the example set are also presented in Chapter 4. The discussion of results draws on existing literature.

Chapter 5 presents Teacher B's findings from the data. Teacher B's example set is presented, followed by how it was enacted, with a focus on the communicative approach and patterns of discourse that were evident in Teacher B's lessons.

Chapter 6 presents Teacher C's findings from the data. The findings presented in this Chapter range from the example set that Teacher C selected, to how the example set was enacted in the lessons observed. The patterns of discourse and communicative approach that Teacher C used are also presented in this Chapter. Discussion of results of Teacher C's findings is presented in this Chapter, with a link to existing literature.

In Chapter 7, a cross case analysis of the findings from Teachers A, B and C are discussed and links are made with regards to how the three teachers played their role of developing learners' mathematics discourse.

Chapter 8 reflects on the PhD journey, presenting the link between the findings of the study and implications. I use this Chapter to state the answer(s) to my main research question. The study's limitations and the recommendations are also presented in Chapter 8. I also present recommendations for further research. I present the significance of the study to relevant stakeholders. In this Chapter, I present the contribution to the field.

CHAPTER 2: Theoretical framework and Literature review

2.1 Introduction

In this chapter, I present the theoretical framework that underpins the study. As I present the framework, I will link it to my study. The literature review of the study is also presented, with a focus on particular aspects that link to the focus of the study.

2.2 Theoretical Framework

For the study, I draw on two frameworks: Mortimer and Scott (2003) framework for meaning making in a dialogic classroom, and variation theory.

2.2.1 Mortimer and Scott's framework

As mentioned above, one of the theoretical frameworks for my study is Mortimer and Scott's (2003) framework for meaning making in a dialogic classroom. Vygotsky's theory was used by Mortimer and Scott to develop their framework. The main argument that was drawn on by Mortimer and Scott is that learning originates from social situations and that ideas are rehearsed between people using actions such as gestures, talk, visual images and writing. Mortimer and Scott's framework was suitable for my study because of the focus on talk and learning through meaning making within the science discourse. As discussed in Chapter 1, my study focused on mathematics discourse and understanding. Mortimer and Scott's framework was developed in a science classroom, while my study was focused on mathematics classrooms. Therefore, one may argue that it would have been better for me to draw on frameworks which were developed in mathematics classrooms or setting. I engaged with some of the frameworks that were developed within the mathematics classrooms, however, I did not find any that was appropriate for my study. For example, Nardi, Biza and Zachariades (2012) developed a framework that focuses on analysis on mathematics arguments between the teacher and student in a mathematics classroom. Nardi et al. draw on Toulmin's model of structure and semantic content to describe informal arguments. I did not draw on Nardi et al.'s framework because it is focused on mathematics arguments between the teacher and students while my study is focused on the role of the teacher in developing learners' mathematics discourse and understanding.

Mortimer and Scott's (2003) framework provided me with 'aspects of analysis' such as patterns of discourse and communicative approach that explain how the teacher plays the role of making the scientific story available to learners (see Aineamani, 2014; 2018).

The main challenge on drawing on Mortimer and Scott's framework was how it relates to the mathematics classroom. I grappled with this in an attempt to relate the framework to the mathematics classroom.

Another challenge that I experienced with drawing on Mortimer and Scott's framework for my study was explaining how the teacher carefully selects content and activities in an attempt to develop learners' mathematics discourse and understanding. To overcome the challenge, I brought in variation theory, a posteriori, in order to be able to engage with how the content and activities are selected, an aspect which was not able to allow me to engage with using Mortimer and Scott's framework. I will discuss variation theory later in this Chapter. In what follows, I present Mortimer and Scott's 2003 framework.

Mortimer and Scott (2003) developed a comprehensive framework that is based on five aspects which are linked together, as shown in Table 1 below.

Table 1: Aspect of analysis (Mortimer & Scott, 2003, p. 25).

Focus	1. Teaching Purpose	2. Content
Approach	3. Communicative approach	
Action	4. Patterns of discourse	5. Teacher interventions

Focus

This part of the framework is what the lesson covers. This includes the topic objectives and the content that has to be covered in order for the objectives to be fulfilled. Teaching purpose and content are the main aspects of this part of the framework, as shown in the table above. The table below is a discussion of the teaching purpose.

Focus: Teaching purpose

The teaching purpose is specific to a phase in the lesson and can be placed under six categories, each category has its teaching focus (Mortimer & Scott, 2003), as shown in the table below.

Table 2: Teaching purpose (Mortimer & Scott, 2003, p. 29).

Teaching Purpose	**Focus**
Opening up the problem	Engaging students intellectually, and emotionally, in the initial development of the scientific story.
Exploring and working on students' views	Probing students' views and understandings of specific ideas and phenomena.
Introducing and developing the scientific story	Making the scientific meanings (including conceptual, epistemological, technological, social and environmental themes) available on the social plane of the classroom.
Guiding students to work with scientific meanings, and supporting internalisation	Providing opportunities for students to talk and think with new scientific meanings, individually, in groups or in whole-class situations. At the same time, supporting students in making individual sense of, and internalizing, those meanings.
Guiding students to apply, and expand on the use of, the scientific view, and handing over responsibility for its use	Supporting students in applying taught scientific meanings in a range of contexts and handing (Wood et al. 1976) over responsibility for using those meanings to the students.
Maintaining the development of the scientific story	Providing a commentary on the unfolding scientific story, to help students to follow its development and to see how it fits into the wider science curriculum.

Adaptations of Table 2 content to the Mathematics classroom

Mortimer and Scott (2003) developed the categories above based on observation of science lessons and also on the Vygotskian perspective of teaching and learning. I engaged with the content of Table 2 above and linked it to a mathematics classroom as shown below. I used the rows of Table 2 to link the explanations below to the contents in Table 2 above.

Row 1

Opening the problem in a mathematics classroom involves **assessing learners' prior knowledge.**

Row 2

Exploring and work on student's views in a mathematics classroom involves **checking learners' understanding of mathematics ideas** that are being discussed in the lesson.

Row 3

Introducing and developing in a mathematics classroom involves drawing on **everyday discourse** to explain mathematics concepts.

Row 4

Guiding students and allowing them to share their ideas while the teacher gives them feedback.

Row 5

Guiding students to apply in a mathematics classroom includes the **class activities, homework and any practice opportunities** that the teacher gives learners.

Row 6

Maintaining the development in a mathematics classroom includes linking the ideas in discussion to the ideas that will be discussed later or those that have already been discussed prior to the lesson.

Focus: Content

Mortimer and Scott (2003) argue that interactions in the classroom range from subject content knowledge to managerial and organizational issues but in their framework, Mortimer and Scott focus on content in relation to the science story. Mortimer and Scott (2003) came up with three categories of the content of the classroom interactions namely: everyday-scientific; description-explanation-generalization and empirical-theoretical.

Everyday-scientific

In the process of learning science, learners are introduced to a special type of language which Mortimer and Scott (2003) refer to as a scientific language. The everyday-scientific category brings in the aspect of language that is being used by either the teacher or the learners. The same learners are inducted into everyday language from the time they are born. Linking this to the mathematics classroom, learners are also introduced to a scientific language, also known as the mathematics register. Therefore, learners in the mathematics classroom have two languages available to them: everyday language and the scientific language.

Description-explanation-generalisation

Mortimer and Scott (2003) argue that within a science classroom, students have to provide an account of any phenomenon, the students have to import some form of theoretical models to give an account of the phenomenon, that is applicable to different contexts. Within the mathematics classroom, learners are expected to work with providing reasons and arguments for their answers and ideas. Learners are also expected to work with concepts that are not just specific to only one context but can be generalised. The concept of generalisation discussed by Mortimer and Scott is similar to one of the patterns of variation and this is further discussed under patterns of variation later in this Chapter.

Empirical-theoretical

Empirical description refers to what can be observed while theoretical descriptions refers to features that are not observable (Mortimer & Scott, 2003). I did not focus on this category in the study.

The everyday-scientific category was the main focus of my study. I used this category to unpack the difference between everyday mathematics discourse and scientific mathematics discourse, and how learners communicate their mathematics ideas. The relationship between everyday and scientific was thought about in relation to Hedegaard and Chaiklin's (2005) 'double move', as discussed in Chapter 1. Description-explanation-generalisation category was a focus in relation to generalisation as a pattern of variation.

Approach

Approach refers to the classroom interactions taking place in the process of achieving the lesson objectives. This includes the communicative approach. In the section below, I discuss what is involved in the communicative approach.

Approach: Communicative approach

Scott, Mortimer and Aguiar (2006), in their work that came after the framework was developed, emphasise the importance of communicative approach in meaning making. The categories are defined based on the talk between the teacher and the learners with a focus on two dimensions: 1) dialogic-authoritative discourse and 2) interactive-non-interactive. Mortimer and Scott argue that dialogic discourse allows for different perspectives, with an opportunity to explore ideas or what Bakhtin (1981) refers to as interanimation of ideas. Authoritative approach allows for only one viewpoint, which is always the viewpoint of the teacher, with no opportunity for explorations of any ideas that learners may have (Scott et al., 2006).

The two dimensions above are then further broken down to generate four categories as discussed below.

Interactive/dialogic: the teacher together with the learners explore ideas, genuine questions are asked by either the teacher or learners. There is an opportunity to listen to different viewpoints from either the teacher or learners.

Non-interactive/dialogic: the teacher reviews several viewpoints and emphasises the similarities or differences to the viewpoints being discussed.

Interactive/authoritative: the teacher engages learners by focusing them on a series of carefully selected questions, to get to a specific viewpoint.

Non-interactive/authoritative: the teacher highlights and presents only one specific viewpoint. Here the teacher is the dominant or only voice and learners are not given an opportunity to participate in the interaction. Table 3 below summarises the four categories.

Table 3: Four classes of communicative approach (Mortimer & Scott, 2003, p. 35).

	Interactive (*Many persons*)	**Non- interactive (*One person*)**
Dialogic (*Many viewpoints*)	Interactive- dialogic (*Many viewpoints considered by more than one person*)	Non-interactive-dialogic (*One person but many viewpoints considered*)
Authoritative (*One viewpoint*)	Interactive-Authoritative (*Many persons but one viewpoint discussed*)	Non-interactive-Authoritative (*One viewpoint and one- person contribution*)

According to Mortimer and Scott (2003), the main idea to look at during the process of analysing communicative approach in the classroom is the way that the teacher engages learners in an attempt to allow for a variety of ideas that are available during a phase in the lesson. Mortimer and Scott (2003) argue it is through teacher interventions and patterns of discourse that communicative approach comes into action in a lesson. In what follows, I discuss both of these constructs.

Action

This part of the framework engages with the steps that are involved in checking that learners have understood the content of the lesson, and steps that can be taken as the teacher mediates in the process of ensuring that learners develop the scientific story through the content of the lesson. In the section below, I discuss what is involved in the patterns of discourse and teacher interventions.

Action: Patterns of discourse

This refers to the forms of interactions which emerge during talk in the classroom (Mortimer & Scott, 2003). Mortimer and Scott (2003) discuss three patterns of discourse namely: I-R-E; I-R-F and I-R-F-R-F-.

The I-R-E pattern

In the classroom, the initiation (I) is from the teacher, the response (R) is from the learner and the evaluation (E) is from the teacher (Mortimer & Scott, 2003). This type of pattern leads to interactive-authoritative approach because the teacher initiation is done by posing a question, and if the learner gives a response which the teacher does not want, the teacher evaluates the learner's response immediately without probing the learner for further explanation (Mortimer & Scott, 2003).

The I-R-F and I-R-F-R-F- (I-R-P and I-R-P-R-P-) patterns

These patterns occur when the teacher does not evaluate the learner's response but instead allows learners to continue the discussion through probing (Mortimer & Scott, 2003). In these patterns, the 'F' stands for feedback while the 'I' and 'R' stand for initiation and response respectively (Mortimer & Scott, 2003). Scott, Mortimer and Aguiar (2006) discussed the I-R-F and I-R-F-R-F- pattern of discourse, where P stands for prompt, and F stands for feedback. For I-R-P-R-P-, the learner may respond to teacher's initiation and if the teacher encourages the learner to continue, the interaction is sustained (Mortimer & Scott, 2003).

According to Scott et al. (2006), different learners may respond to the initiation from the teacher before the teacher prompts and this leads to a pattern of discourse that is in the I-R_{S1}-R_{S2}-R_{S3}-R_{S4}- form, where R_{S1} stands for response from student 1, R_{S2} stands for response from student 2 and so on. This interaction is different from the patterns of interaction discussed above.

The patterns of interaction are important in this study because they enabled me to analyse how the teacher supported (or not) learners to communicate and clarify their ideas in the process of developing their mathematics discourse.

Action: Teacher interventions

This aspect of the framework focuses on the way the teacher mediates to help learners develop the scientific story (Mortimer & Scott, 2003). In this study, the aspect focused on the way the teacher intervened in the process of helping learners develop scientific mathematics discourse. Mortimer and Scott (2003) draw on Scott's (1997) scheme in which Scott developed six forms of teacher interventions as shown in Table 4 below. The six forms of intervention have a focus and action which the teacher might take, as shown in the table below.

Table 4: Teacher interventions (Mortimer & Scott, 2003, p. 60).

Teacher Intervention	**Focus of the intervention**	**Action the teacher might take**
1. Shaping Ideas	Working on ideas, developing the scientific story	Introduce a new term; paraphrase a student's response; differentiate between ideas
2. Selecting Ideas	Working on ideas developing the scientific story	Focus attention on a particular student's response; overlook a student's response
3. Marking key ideas	Working on ideas, developing the scientific story	Repeat an idea; ask a student to repeat an idea; enact a confirmatory exchange with a student; use a particular intonation of voice
4. Sharing ideas	Making ideas available to *all* the students in a class	Share individual student's ideas with the whole class; ask a student to repeat an idea to the class; share group findings; ask students to prepare posters
5. Checking student understanding	Probing specific student meanings	Ask for clarification of student ideas; ask students to write down an explanation; check consensus in the class about certain ideas
6. Reviewing	Returning to and going over ideas	Summarise the findings from a particular experiment; recap on the activities of the previous lesson; review progress with the scientific story so far

The teacher intervention occurs for different reasons as shown in Table 4 above. In an attempt to make the teaching purpose available to the learners, the teacher may draw on different interventions. In the table below, I present my adaptation of Mortimer and Scott's table above to show how it would look like in a mathematics classroom.

Table 5: Adapted Teacher interventions (Mortimer & Scott, 2003).

Teacher Intervention	**Focus**	**Action the teacher might take**
1. Shaping Ideas	Working on ideas, developing the mathematics concept	Assess what students already know, work with what students already know to introduce a new term and or concept; revoice a student's response; differentiate between ideas and ask learners to give explanations and justifications for their ideas. In my study, this involves **assessing learners' prior knowledge** and linking it to what is to be taught.
2. Selecting Ideas	Working on ideas, developing the mathematics concept	Focus attention on a particular student's response and engage with it; overlook a student response. In my study, this involves **choosing what to emphasise on and what to ignore in an interaction**.
3. Marking key ideas	Working on ideas, developing the mathematics concept	Repeat an idea; ask a student to repeat an idea; revoice an idea, enact a confirmatory exchange with a student; use a particular intonation of voice to show that the student's answer is right or wrong. In my study, this involves giving learners appropriate **feedback** in an interaction.
4. Sharing ideas	Making ideas available to *all* the students in a class	Share individual student's ideas with the whole class; ask a student to repeat an idea to the class; share group findings; ask students to prepare posters summarizing their views. In my study, this involves giving learners an opportunity to **share their ideas** with the teacher and peers during the lesson.
5. Checking student understanding	Probing specific student meanings	Give students classwork and homework to do individually and mark it; ask for clarification of student ideas; ask students to write down an explanation; check consensus in the class about certain ideas. In my study, this involves **assessing learners' understanding** through various activities.
6. Reviewing	Returning to and going over ideas	Summarize the findings from a particular discussion; ask learners to summarise ideas learnt in the lesson, recap on the activities of the previous lesson; review progress with the mathematics concept so far. In my study, this involves monitoring learners' progress.

Mortimer and Scott (2003) suggest teacher interventions as shown in the table above. The figure below is a summary of Mortimer and Scott’s framework and how it links to my study.

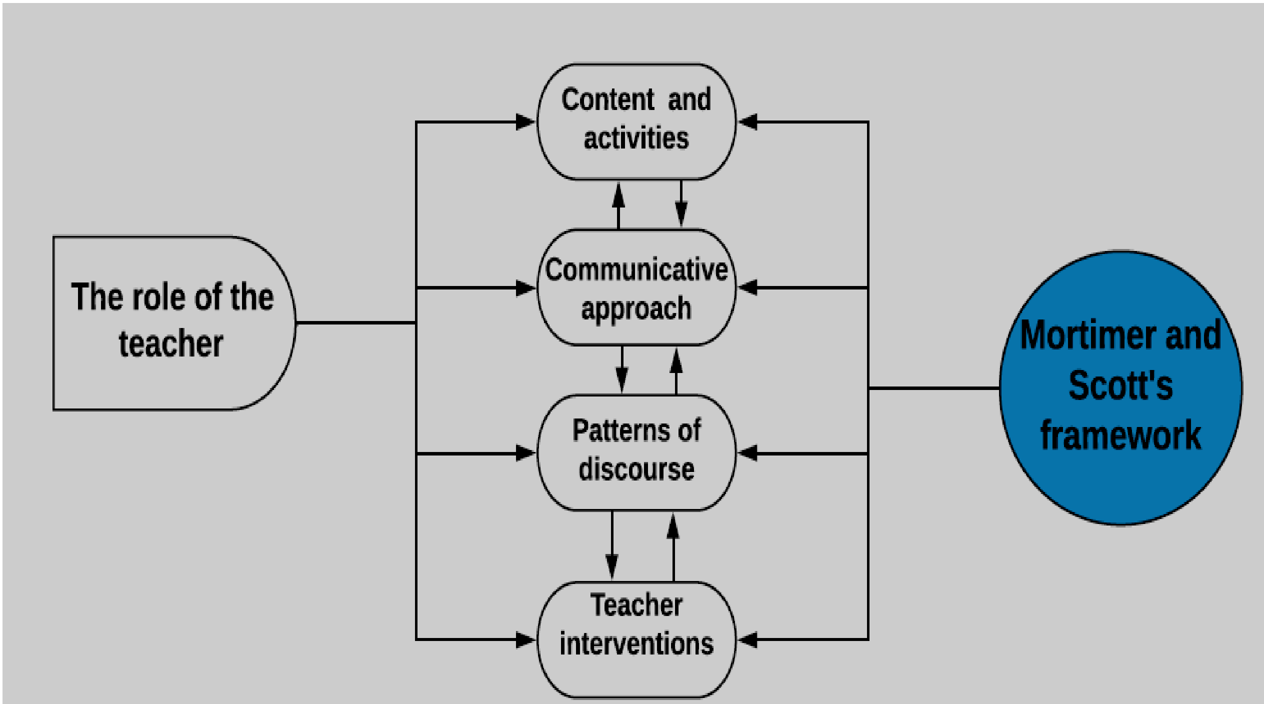


Figure 2: Mortimer and Scott's framework linked to the focus of the study.

The role of the teacher in this study was unpacked using the aspects from Mortimer and Scott’s framework. Selecting content, using the appropriate communicative approach and pattern of discourse, and choosing appropriate teacher interventions would fall under the role of the teacher. Choosing content and activities for teaching is important. Huang (2017) argues that learners’ understanding of any mathematics ideas is dependent on the examples that are selected by the teacher. This was confirmed by the data which I collected for the study. All the lessons observed were centered around the content and activities that the teachers selected. I refer to these as the example set of a lesson. Selecting an appropriate example set is an art in mathematics teaching (Michener, 1978; Huang, 2017).

As Huang (2017, p. 1152) puts it, “good examples serve as cultural tools that mediate the relationship between learners, mathematical concepts, and theorems”. Therefore, examples are fundamental to understanding abstract ideas. For my study, I wanted to go beyond simply stating the role of the teacher as selecting examples and be able to unpack what would make an example appropriate. I therefore decided to draw on variation theory as discussed below.

2.2.2 Variation theory

As discussed earlier, although Mortimer and Scott's (2003) framework has a section on content, it does not engage with the selection of content and activities of the lesson. In other words, the framework does not engage with what might be considered when selecting particular content and activities in the process of affording learners the opportunities for meaning making. The teacher interventions presented in Table 5 above would also require carefully selected activities for successful intervention, for example, the teacher would have to select appropriate content and activities to check learner understanding of a concept. The content and activities selected by the teacher during the lesson are crucial and these are referred to as the example set of the lesson. As such, I brought in Marton, Runesson and Tsui's (2004) variation theory to explain how and why content and activities may be drawn on by the teacher in the process of developing learners' mathematics discourse. As discussed earlier, the addition of variation theory to my framework was an *a posterior* addition, and this was informed by the preliminary findings from the data, during the initial data analysis process. It is important to note here that although I brought in variation theory at a later stage in my study, both Mortimer and Scott's framework and variation theory were central to my study. I will discuss variation theory below in detail.

Runesson (2005) argues that variation theory is a learning theory[2]. Learning is defined as the "process of becoming capable of doing something as a result of having had certain experiences" (Marton et al., 2004: 5). "Learning always has an object" (Runesson, 2006: 406) and the object is defined as "that which is the focus of attention" (Watson & Mason, 2006: 100).

Marton et al. (2004) argue that variation theory may not be used as a *silver bullet*, however, it can be drawn on to enable learners to experience the critical features of the object of learning. The object of learning has three categories, namely: intended (what is planned), enacted (what is experienced during the learning) and the lived (actual results of the learning) object of learning.

[2] Although variation theory is a theory of learning, I drew on it to engage with content and activities that were selected by teachers for teaching so that learning would take place through these content and activities.

Figure 3 below is a summary of the object of learning.

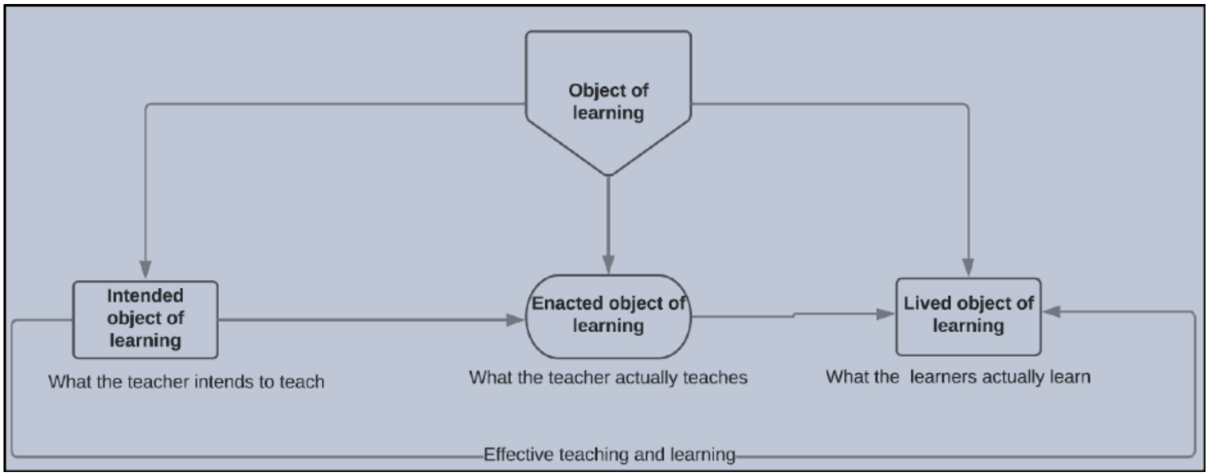


Figure 3: Object of learning.

The figure above is an illustration of the object of learning. As stated earlier, the aim of teaching is to provide opportunities for learners in an attempt to ensure that the intended object of learning is enacted in a way that enables learners to get to the desired lived object of learning.

Variation theory has four patterns, namely: contrast, generalisation, separation and fusion (Marton et al., 2004).

Contrast

Contrast refers to the comparison between the critical feature of the object of learning and the other aspects that are not critical to the object of learning (Marton et al., 2004). For example, in the context of a linear function, $y = 2x + 1$ can be contrasted with $y = 2x^2 + 1$ in order to discern the critical features of a linear function. In the two equations, learners' attention is drawn to the power of x. In order to understand how an equation of a linear function is written, learners need to experience how it is not written.

Generalisation

Generalisation refers to experiencing features of the object of learning that are varying in order to discern the irrelevant features. This occurs when learners are given various examples on a particular object of learning, so that learners can discern the critical features of the object of learning by comparing the examples to see which features are common and hence appreciate generality (Watson & Mason, 2006). For generalisation, the examples given should allow for attention to the invariant amongst variation and hence lead to generality. Watson and Mason (2006) argue that it is important for learners to be given several examples in order for them to get a general sense of what they are being taught. For example, in the context of linear functions, learners may be given different types of equations, but all equations of linear functions: $y = x + 1$; $y = 2x + 1$; $y = 3x + 1$. The three equations given show the invariant power of x, a critical feature of an equation to be referred to an equation. The constant term and the coefficient of x are irrelevant here because the learner's focus needs to be drawn to the invariant power of x. Generalisation is critical because it is through generalisation that the conceptual and structural similarities become apparent and learners focus on them rather than simply looking for cues in syntax, habit or context (Watson & Mason, 2006), and this is what makes mathematics powerful and at the same time difficult.

Separation

Separation refers to when certain features of the object of learning are kept invariant while varying the others. Separation occurs through controlled variation, where certain aspects are kept invariant while varying others (Watson & Mason, 2006; Marton et al., 2004). In the context of a linear function, to draw attention to the power of x, learners may be given two equations $y = 2x + 1$ and $y = 2x^2 + 1$. Allowing the power of x to change draws the learners' attention to it, and this is a critical feature of the equation of a linear function.

Fusion

Fusion occurs through synchronic simultaneity (Marton et al., 2004). Synchronic simultaneity refers to the ability of discerning more than one critical feature of the object of learning in a given example (Runesson, 2005). Fusion as a pattern of variation occurs when learners are given examples that have various critical features of the object of learning and are able to discern all of them simultaneously. For fusion to occur, the learners need to have sufficient previous knowledge and experiences to draw on (Marton et al., 2004). Discerning the critical features of the equation of a linear equation, which are the power of x, the coefficient of x, and constant term is as a result of fusion and experiencing changes simultaneously in the given equations.

Reformulated research questions in relation to the theoretical framework

The diagram below is an adaptation of Figure 2, to show how the frameworks: Mortimer and Scott and variation theory worked together to enable me to come up with comprehensive theoretical framework for the study.

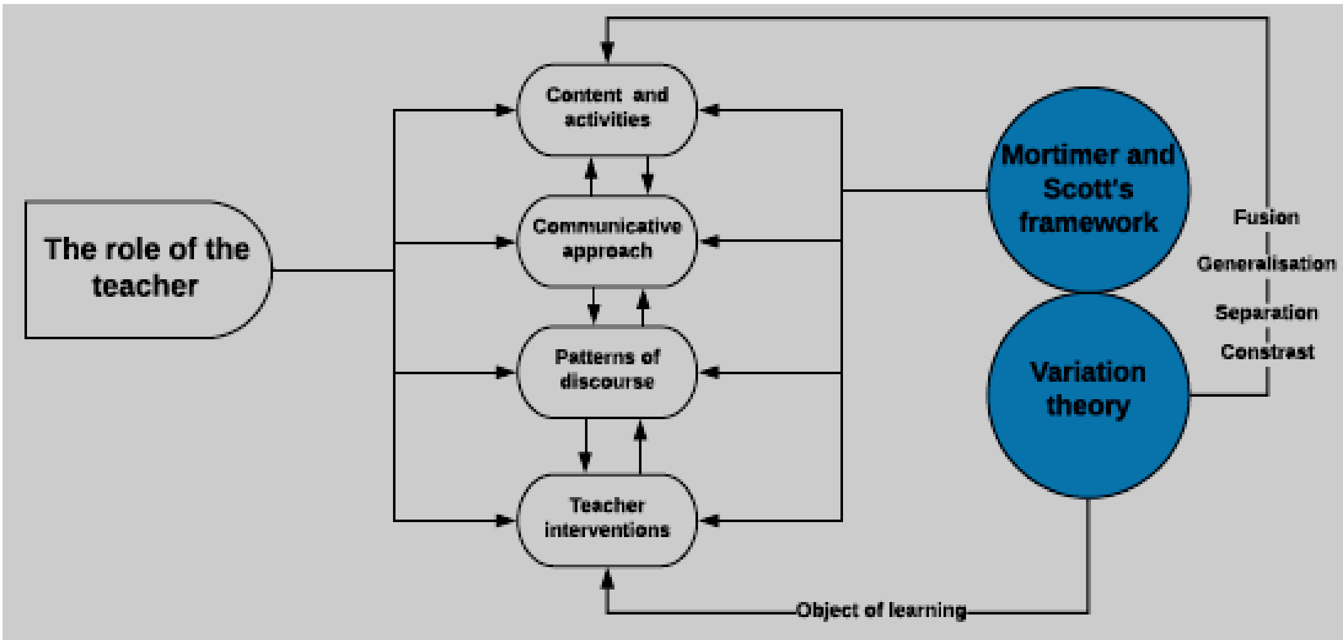


Figure 4: Theoretical framework.

Figure 4 above is an illustration of how the frameworks were linked together to engage with the role of the teacher in the study.

The teacher starts by identifying the content that has to be taught. In the process of selecting the content and activities or the example set, the teacher may use any pattern of variation and hence variation theory was considered to be a strategy in the study as shown in the figure above. The theoretical framework provided me with a lens for my study.

From Mortimer and Scott's (2003) framework and variation theory, I reformulated the research question of the study, as stated below.

Table 6: Reformulated questions.

Original questions	Reformulated questions
a. What kind(s) of content and activities is selected by the teacher in the process of teaching concepts to learners in a mathematics functions lesson?	a. What kind(s) of content and activities (or example set), in relation to the intended object of learning is/are selected by the teacher in the process of teaching concepts in a mathematics functions classroom?
b. How do the content and activities shape the classroom engagement in a mathematics functions lesson?	b. How does the communicative approach and pattern of discourse shape the enactment of the content and activities (example set) selected by the teacher in a mathematics functions lesson?
c. What mediation moves does the teacher employ to explain the concepts to learners in a mathematics functions lesson?	c. What strategies does the teacher employ to enable learners to discern the critical features of the object of learning in a mathematics functions lesson?

The research questions above focus on how the teacher plays the role of content and activities selection in the process of developing learners' mathematics discourse and understanding. In the next section, I present the literature review of the study. The literature is focused on aspects that came to light while engaging with the study's theoretical framework.

2.3 Literature review

The focus of the study is on the role of the teacher, and therefore, in this section, I will present existing literature on the role of the teacher in developing learners' mathematics discourse and understanding, with a focus on themes that stood out for me in the literature reviewed, in relation to my study.

2.3.1 Developing learners' mathematics discourse and understanding

Over the years, research in mathematics teaching and learning has grappled with issues of communication and interaction in the classroom (see Goizueta, 2019; Aineamani, 2018; Gellert, 2014; Douek, 2007; Sfard, 2001; Ernest, 1998; Voigt, 1995; Bauersfeld, 1980).

Walshaw and Anthony (2008) categorise three roles of teachers in the process of shaping the mathematics discourse: 1. determining specific mathematics ideas and tasks; 2. rectifying the mathematical language and conventions used by learners in the lesson, and 3. shaping mathematical discussion and interaction as it progresses in the lesson. The three roles identified by Walshaw and Anthony emphasise the role that the teacher plays in determining the example set that is enacted in the classroom, in an attempt to develop learners' mathematics discourse and understanding.

In order to participate in the classroom interaction, learners should acquire the appropriate language which allows them to engage with and communicate about important mathematics content, and mathematical thinking and reasoning as a result of productive mathematical conversations. In their 3-year study, Muller and Maher (2009) found that learners were not able to use a certain form of reasoning if they had not developed the appropriate language required in the form of mathematics reasoning. In any language, it is not enough that one reads the words. Within functions, simply presenting symbols and operations to learners may not be effective if learners are not able to relate to the meaning that the symbols and operations represent and explain their mathematics reasoning in relation to the activity. For example, giving learners an activity of determining the domain and range of the equation: $y = 2x^2 + 3$, if learners do not know what domain and range means in relation to the equation given, they may struggle to attempt the activity. Communicating mathematics reasoning is important within a mathematics discourse (Aineamani, 2018). In the example above, the learner should be able to explain the mathematics reasoning involved in determining the domain and range.

In a study that Stathopoulou and Kalabasis (2006) conducted in a mathematics classroom, they found that if learners feel left because of language or terminology, they may end up hating the subject. The language is what allows the learners to communicate their mathematics reasoning within the discourse (Aineamani, 2018). Pimm (1991) argues that mathematics itself is a specialised language. Mathematics has its own words, and it also borrows words from languages such as Latin and Greek to formulate words, for example parabola and co-efficient (Cuevas, 1984). Therefore, emphasising the first and second language of the learners is not enough since mathematics is a language which the learners have to master (Cuevas, 1984). A mathematics teacher may need to create meaningful opportunities for learners to practice mathematics language within the mathematics classroom, through classroom interactions that allow learners to explain and justify their mathematics communication. According to Temple and Doerr (2012), learners require enough time and support to use their first and second language to interact and learn the mathematics register.

Within the language complexities, I reviewed literature on three aspects: terminology and vocabulary, allowing learners to participate in the classroom interaction and providing learners opportunities to learn. One of the roles that the teacher has in any classroom setting is being conscious of the language complexities in that classroom. As learners and the teacher engage in a classroom interaction, the teacher may need to remain mindful of the ways in which learners use the mathematics vocabulary (Gay, 2008).

Terminology and vocabulary bring me to another important aspect of mathematics discourse in any mathematics classroom, the definition of terms. Mathematics definitions are important in developing learners' mathematics discourse and, hence understanding. Marzano (2004) argues that defining terms accurately should be emphasised by the teacher in the lesson. Vinner (2011) presented an introspective paper on "view of three central conceptual activities in the learning of mathematics: concept formation, conjecture formation and conjecture verification" (p. 247). One of the arguments is on the importance of concept definition in mathematics. Vinner (2011) argues that mathematics terms need to be defined accurately because in some contexts in mathematics, concept definitions are crucial for learners to be able to make concept images. If definitions are not accurately given, learners may find it difficult to understand the explanations and it may also be difficult for them to make cognitive shifts in their understanding (Vinner, 2011). In functions for example, if the teacher defines the concept of a function with a focus on a one-to-one relationship, when learners come across a scenario that has a many-to-one relationship, they will not be able to use the previous definition to make a concept image of the concept of a function in this scenario. In process of the developing learners' vocabulary within the mathematics discourse, interaction becomes crucial as shown by the literature below.

Within the classroom, developing learners' vocabulary may require giving learners an opportunity to interact and share their ideas. Collaboration and engagement should be encouraged so that all learners get a chance to share their ideas (Manouchehri & St. John, 2006). In the process of sharing their ideas, learners are given opportunities to explain and justify their mathematics reasoning. Individual responses might provide the teacher with better results of learners' understanding as shown in a study by Staples and Colonis (2007), where learners were reluctant to contribute and engage in classroom discussions when they were not required to give individual responses. For my present study, I focused on how the teacher would assist learners in developing their mathematics discourse and understanding, especially if they were faced such challenges as the ones discovered by Staples and Colonis. The approach (communicative approach) and action (pattern of discourse) that are discussed in the theoretical framework above may come in handy if they are used appropriately by the teacher in the process of interacting with learners.

Bell and Pape (2012) conducted a study on how teachers in a mathematics classroom constructed opportunities to learn within a lesson. They conducted interviews and they also videotaped classroom lessons. They used descriptive narrative to analyse their data. The findings from the study were that classroom discourse may contradict with learners' everyday discourse from their everyday life or contexts. Teachers need to focus on how the interaction unfolds in the classroom. Anghileri (2006) argues that within a mathematics classroom, the teacher may support learners by providing responsive and challenging strategies and allowing learners to verbalise their mathematics ideas in a conducive environment that is free of hierarchies and also encourage learners to collaborate in the classroom (Gay, 2008). For Gay, authoritative approach as discussed by Mortimer and Scott (2003) should not be explicitly employed in a classroom where interaction needs to be encouraged. Webb et al. (2017), in their paper that seeks to understand teacher practices which lead to productive classroom dialogue, argue that engaging and probing learners during interaction in the classroom is crucial. Engin (2017) argues that learners' expectation of the teacher's and peers' role may influence how they participate in the discussions.

It is important to note here that the use of appropriate language of mathematics would not be possible in the absence of examples. By examples, I mean the actual content that the teacher chooses to focus on in the lesson. The examples maybe in form of activities, worked examples and definition of concepts, as I will discuss further in Chapter 3. Wachira et al. (2013) argue that understanding needs to be emphasised and learners should be encouraged to focus on the reasoning behind the answers rather than focus on correct answers. For Wachira et al. (2013), concept development should be the focus of the teacher and therefore careful measures need to be taken to ensure that learners' understanding is assessed with appropriate examples so that interventions are put in place to ensure that learners develop their mathematics discourse and understanding. With that said, I will present the literature on selecting appropriate examples to teach mathematics concepts in the process of developing learners' mathematics discourse and understanding.

2.3.2 Selecting appropriate examples to teach mathematics concepts

Many researchers have discussed the role of examples in mathematics (Knuth, Zaslavsky & Ellis, 2019; Zaslavsky, 2019; Huang, 2017; Antonini, Presmeg, Alessandra & Zaslavsky, 2011; Zaslavsky, 2010; Ball, Thames, & Phelps, 2008; Zazkis & Chernoff, 2008; Rissland, 1991; Mason & Pimm, 1984). It is through examples that concepts are illustrated, demonstrated, and explained to learners. As Rissland (1991) states, teaching mathematics requires careful selection of examples. Zaslavsky (2010) emphasises the role of examples, and he states that "examples are essential for generalization, abstraction, and analogical reasoning" (p. 107). Zodik and Zaslavsky (2008) argue that the examples selected by the teacher may hinder or enable learning. It therefore becomes imperative to select appropriate examples.

One main aspect that teachers need to consider is that an example may exemplify various mathematical ideas (Antonini et al., 2011). Examples enable the teacher to provide learners with opportunities to engage with essential parts of teaching mathematics such as analogical thinking, generalisation, conceptualisation, argumentation, and abstraction (Yanuarto, 2016). Watson and Mason (2006) argue that an example is any item that is derived from a larger aspect, which can be used by learners to reason and come to a generalisation about a concept. Peled and Zaslavsky (1997) argue that while many examples may be used to illustrate a concept, some examples have more powerful pedagogical aspects than others. For example, when representing a linear function, giving an example such as $y = 1$ and $y = 2$ may not be as effective as giving learners the examples $y = 2x + 1$; $y = 3x + 1$ and $y = 2x$, to engage with the critical features such as the co-efficient and the y-intercept. In the example of equations above, the learners need to be able to see the general equation ($y = mx + c$) of a linear function, and also be able to identify the coefficient and y-intercept in any particular equation, by drawing on the general equation. Yanuarto (2016) argues that examples that allow for generalisation are clear and they do not distract learners with irrelevant features or patterns. In his study, he illustrated the argument with triangles. For learners to see that a triangle has three sides, the teacher does not need to focus on orientation or symmetry as these would be irrelevant with respect to what needs to be in focus (Yanuarto, 2016).

Three types of examples: specific, semi-general, and general examples, are discussed by Peled and Zaslavsky (1997). Through general examples, learners are given an opportunity to discern how to generate further examples from the given example (Zaslavsky, 2010). An example of a general example is the general formula of a linear equation: $y = mx + c$, where m is the gradient of the straight-line and c the y-intercept. From this example, learners are able to generate other examples if they know the gradient and y-intercept of a given line. Specific examples focus on certain aspects of the object of learning. For example, to draw learners' attention on a parabola with 1 as the co-efficient of x^2, the teacher may give learners the equations: $y = x^2 + 1$ and $y = x^2 + 2$. Zaslavsky (2010) argues that developing specific examples is challenging because the teacher to pay attention to what is brought to the fore by the example. For example, in the example: $y = x^2 + 1$ and $y = x^2 + 2$; the teacher may be illustrating the co-efficient and learners may end up focusing on the constant term. Zaslavsky emphasises the role of the teacher in guiding learners through any example that is given, in an attempt to ensure that learners are given an opportunity to see what the teacher wants them to see.

Yanuarto (2016) argues that the first examples that teachers present to learners in a lesson are very important, and if carefully selected, they help learners not to jump to the wrong conclusions about what is in focus. Mason and Pimm (1984) argue that undesirable issues arise when there is a misalignment between what the teacher intends and what the learners end up experiencing as a result of the example set given.

From the findings of their study on choice of examples, Rowland, Thwaites, and Huckstep (2003) argue that teachers are prone to making poor choices of examples if random selection of examples is done, where carefully selected examples are required, in line with the intended object of learning. The base knowledge of teachers should enable them to select examples ‘in the moment’ (Mason & Spence, 1999). Mason and Spence (1999) argue that learners may not draw on previous knowledge in unfamiliar context because their knowledge is compartmentalised, they are not able to apply knowledge when it is needed.

In order for an example set to be selected appropriately, learners’ prior knowledge needs to be in focus because learners come to any learning environment with prior knowledge on the skills and concepts that are presented to them (Bransford, Brown & Cocking, 1999; Campbell & Campbell, 2009). As Bransford et al. argue, the prior knowledge that learners bring has a considerable effect on what they ‘see’ and how they analyse new concepts and develop skills in the classroom.

Learners’ prior knowledge may be either partial or incorrect in relation to a given subject (Bransford et al., 1999). The role of selecting the appropriate examples in relation to learners’ prior knowledge cannot be over emphasised. Teachers should have an indication on some predictable preconceptions of learners on given concepts (Bransford et al., 1999; Bereiter & Scardamalia, 1989). In the absence of predictable preconceptions, for example if a teacher is teaching new learners, assessment of prior knowledge needs to be a starting point in any learning situation as suggested by many researchers (McInerney, Zumeta, Gandhi, & Gersten, 2014; Bransford et al., 1999; Bereiter & Scardamalia, 1989). The results from the assessment may then be used to focus and pay attention to learners' prior knowledge. As teachers pay attention to learners' prior knowledge, they need to use it to select examples that help learners develop their mathematics discourse and understanding. If learners' prior knowledge is ignored, this may impede understanding as the teacher's intentions may not be in line with what the learners experience as a result of what learners already know (Bransford et al., 1999), or do not know. For example, in the equation of a linear function discussed earlier, learners need to have prior knowledge of algebraic expressions and equations. Pintrich and Schunk (2002) argue that learners are more likely to make sense of what is being taught if they can relate to it, in relation to what they already know and hence retain this knowledge because it relates to familiar context(s) (Crawford & Witte, 1999).

One way of engaging with learners' prior knowledge is providing an overview of the new concept to be taught in a lesson. Campbell and Campbell (2009) argue that the teacher should elicit learners' prior knowledge by asking questions that will give insights on what learners know and what they struggle with.

Learners' prior knowledge should not be taken as negative effect on learning. Campbell and Campbell (2009) argue that effective learning begins from the known to the unknown. Therefore, deliberate engagement with learners' prior knowledge should be a focus in the process of selecting examples. When learners are presented with examples or an example set which contradict with their prior knowledge, this leads to what is known as cognitive conflict.

Zazkis and Chernoff (2008) discuss cognitive conflict and they define it as a situation when a learner is faced with a concept that contradicts with what the learner already knows, and this may cause inconsistencies in the learner's knowledge. Cognitive conflict has been confirmed as a strategy to remedy learners' misconceptions as discussed by Ernest (1996). Many researchers have conducted studies on cognitive conflict as a pedagogical strategy in a mathematics lesson (Zaslavsky & Ron, 1998; Peled & Zaslavsky 1997). Counter examples that show learners what a concept is and what it is not is a powerful way of illustrating concepts to learners (Zazkis & Chernoff, 2008). In variation theory terms, this is known as contrast (Marton et al., 2004). When learners are presented with counter examples that create a cognitive conflict, it is not sufficient to leave it to the learners to resolve their conflict. Research has shown that the learners may take the conflict as an exception and hence ignore it (Zazkis & Chernoff, 2008). One important point to note here is that cognitive conflicts cannot be pre-planned and therefore, teachers should choose appropriate spontaneous examples to engage learners in the process of resolving the conflicts (Zazkis & Chernoff, 2008).

Zodik and Zaslavsky (2008) identified two types of examples: pre-planned and spontaneous examples. Pre-planned examples are carefully planned and used as part of a planned lesson, while spontaneous examples are those examples which the teacher chooses in-the-moment, when faced with a completely new situation in the classroom. Zodik and Zaslavsky argue that if there is some evidence that the teacher included an example in the planning process of the lesson, then it is pre-planned. Zodik and Zaslavsky argue that are two reasons for choosing spontaneous examples: 1) to address learners' responses, especially when learners gave incorrect responses, and 2) response to limitations of pre-planned examples during the lesson. As shown by Zodik and Zaslavsky's study, choosing examples, whether pre-planned or spontaneous examples is not an easy task. My study will contribute to this discussion by investigating how the two types of examples become accessible in the context of my study.

Everyday contexts in examples selected is another aspect that is important to discuss in the process of selecting an example set. The examples with everyday context may be spontaneous or pre-planned. Jo (1993) argues that there has been a debate that suggests that including everyday context helps learners to learn mathematics better and may also help learners to relate abstract mathematics to real world problems. This debate has led to inclusion of everyday context into the teaching of mathematics at various levels, from materials to assessments and it is also evident in classroom teaching (Jo, 1993). The relation between including of everyday context has three main advantages: 1) recognise the requirements of the activity, 2) retrieve information from previous knowledge about the context that learners can relate to and 3) translate the information provided to suit the demands of the activity (Jo, 1993). If the three things do not happen, no matter how powerful the context, misconceptions may occur. One difficulty of everyday context that Jo highlights is the 'choice' of context to be used, because learners may have different contexts that they can relate to. The teacher may also have an adult perspective on contextual reality. This leads to a question of whose context is it? Learners interact with contexts in unexpected and different ways (Jo, 1993), mainly informed by their previous experiences. Therefore, Jo argues that before using real life context in examples, the teachers needs to consider the learners who will interact with the context that is selected.

Having presented existing literature on selecting appropriate examples, I will now focus the discussion and literature review on the topic which I focused on in the study, which is functions. The discussion above has argued for the importance of selecting appropriate examples. Selecting appropriate examples becomes even more important in topics such as functions as discussed in the next section.

2.3.3 The concept of a function

Even (1990) argues that the concept of functions is everywhere in mathematics, in geometry, algebraic operations, trigonometry among others. I therefore saw it necessary to engage with literature on the concept of a function.

According to Bradley, Campbell and McPetri (2012), a function f is defined by determining a set, say A, which is referred to as the domain of the function, f, and another set, say B, which is referred to as the range of f. A function is then referred to as a mapping and the descriptions of the function f is that f maps A to B and x to $f(x)$ (Bradley et al., 2012). According to Smith (2003), the definition above is a traditional view of the concept of a function. In contrast to the definition above, Smith and Confrey (1994) argue that a function should be examined in terms of x- and y- values and how the coordinated change occurs. In their studies, Smith and Confrey (1994) argue that learners were able to make sense of the concept of function using a covariation approach (see Smith & Confrey, 1994). Smith and Confrey (1994) argued that learners' first approach to functions is always covariational in nature because they start by comparing the change in x- and y- values. Ellis (2009) argues that the covariation approach gives learners a powerful way of discerning and hence understanding the concept of function. However, Carlson and Oehrtman (2005) argue that any understanding of the concept of a function should include more than using covariation approach. Therefore, learners should be able to understand multiple views of the concept of a function in order to be successful in mathematics.

Functions as a topic is a section under algebra in mathematics. A function is made up of the signs and symbols of algebra (Tirosh, Even & Robinson, 1998). A learner must understand algebraic language in order to interpret the definitions of a function given above. Just as a learner cannot read a literature text without understanding the language, similarly this applies to mathematics (Jamison, 2000). Mathematics terms have meanings that evolve as learners adapt their understanding of the terms to accord with new experiences and new ideas (Jamison, 2000). The language used to define and describe functions is precise and it should be interpreted in the same way by everyone who wants to use it (Bunt, Jones & Bedient, 1988). Bunt et al. (1988) argue that learners need to understand how things are said and what is being said so that they can have a chance to know why things are said in mathematics. Therefore, it is important for teachers to guide learners in an attempt to help them develop their mathematics discourse so that they acquire the rules required to participate in the discourse (Jamison, 2000).

Functions is one of the main topics in the South African mathematics Further Education and Training (FET) curriculum (DoE, 2010). The Curriculum and Assessment Policy Statement (CAPS) requires teachers to cover the following concepts of functions in Grade 10:

- *"Work with relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae).*
- *Include linear and some quadratic polynomial functions, exponential functions, some rational functions and trigonometric functions*
- *Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence generalise the effect of the parameter which results in a vertical shift and that which results in a vertical stretch and /or a reflection about the x axis.*
- *Problem solving and graph work involving the prescribed functions"* (DoE, 2010, p. 12).

The extract from the CAPS above on functions shows various facets that are emphasised such as representations and types of functions. Throughout the CAPS curriculum, the concept of function is drawn on for other concepts and hence used as a basis for cognitive development. DeMarois and Tall (1999) in their study question the suitability of functions to be used as a basis for cognitive development. DeMarois and Tall (1999) discuss the concept of a function as shown in Figure 5 below.

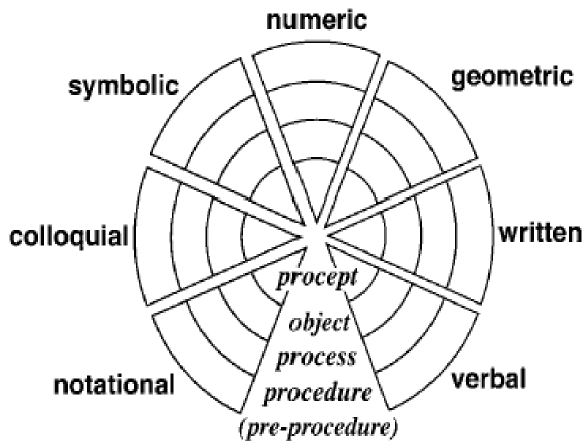


Figure 5: Facets and layers of the function concept (DeMarois & Tall, 1999, p. 256).

DeMarois and Tall argue that at pre-procedure layer, the learner has not yet attained procedural layer and therefore the learner may not be able to carry out procedures. A learner at the process level is able to carry out procedures without mental processes while a learner who see a function as an object is able to "treat the idea as a manipulable mental object to which a process can be applied" (DeMarois & Tall, 1999: 256). DeMarois and Tall provide a link between the facets of the concept of a function, as shown in Figure 6 below.

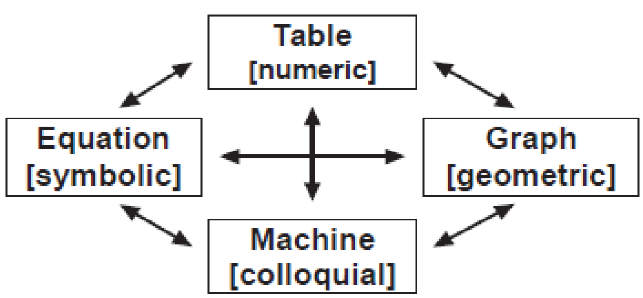


Figure 6: Possible links between function facets (DeMarois & Tall, 1999, p. 256).

Figure 6 shows the link between the different facets. There is a link between the symbolic and numeric facet of functions, and the numeric facet is linked to the graphing facet and so on, as shown in Figure 6 above. Developing learners' mathematics discourse and discourse in the context of functions requires that the learners acquire the facets in a way that they are able to make links between them as shown in Figure 6 above. Breidenbach, Dubinsky, Hawks and Nichols (1992) argue that for learners, there should be a move towards forming mental processes of the concept of a function in order for conceptual understanding to occur. They further argue that process conception does not occur spontaneously in human beings. DeMarois and Tall argue that not all facets are available in all learners' understanding of functions, and DeMarois argues that learners are good at hiding their difficulties in functions through a process referred to as "plug and chug", where learners carryout procedures mindlessly, without demonstrating an understanding of what is required. In a study conducted by DeMarois (1998), the complexity of the function concept was highlighted by the findings on how learners responded to the pre- and post- questionnaires. Some learners displayed an incomplete understanding of the concept, by attempting the tasks in a manner that the learners thought that the teacher desired, without making the required internal connections. Learners also displayed difficulties of moving from a function as a machine to other representations. The learners in this study also misinterpreted the function notation. In what follows, I engage with the importance of the concept of a function and the difficulties of the concept of a function.

2.3.4 The importance of (and difficulties with) the concept of a function

The concept of a function and how learners engage with it has been studied and discussed at length (see Ellis, 2011; Schliemann, Carraher & Brizuela, 2001; Stacey & MacGregor, 1995; Buck, 1995; Confrey & Smith, 1995; Monk & Nemirovsky, 1994; Even, 1990). Learners need to attain a deep understanding of the concept of a function because this understanding is critical for success in future mathematics topics (Farenga & Ness, 2005). For example, learners need to understand the concept of a function if they are to understand trigonometry. In various studies, it is evident that there is limited emphasis on the conceptual understanding of the concept of a function (Ellis, 2011; Cooney & Wilson, 1996). The limited emphasis on the understanding of functions leads to a situation of learners going through school without a solid understanding of the key concepts in the functions (see Carlson & Oehrtman, 2005; Ellis, 2009; Confrey & Smith, 1995).

Even (1990) argues that a function is defined as a unifying concept, although "functions appear and behave in different ways" (p. 531). Even further argues that the different uses of functions force us to approach the teaching of functions in a different way. Functions may be approached by a point-wise approach, where point-by-point plotting is done, and this emphasised in the curriculum as shown in the third bullet above. Another approach is the interval approach, where the local extremum is considered. The interval approach is not emphasised in the curriculum. The other approach is considering the function in global way, by looking at the behaviour of the function, where the effect of the parameters is emphasised in order to sketch the graphs of the functions, and this approach is emphasised by the curriculum as shown in the bullet points above. Even (1990) argues that all the approaches above cannot be appropriate for all situations, and sometimes more than one approach may be used, as shown in the curriculum above.

As discussed earlier, the interaction cannot be shaped in the absence of appropriate examples. The examples selected to teach the concept of a function needs to be done carefully so that the different approaches: point-wise approach, interval and global approach are used according to what needs to be brought to the fore in relation to the intended object of learning. As the intended object of learning is brought to the fore, the aim should be to develop learners' mathematics discourse and understanding.

2.4 Conclusion

In this chapter, Mortimer and Scott's (2003) framework has been presented in detail and linked to my study. Variation theory has been discussed in detail and the rationale for why variation theory was included in the study has been presented. The theoretical framework of the study allowed me to theorise the role of the teacher in developing learners' mathematics discourse and understanding. Both Mortimer and Scott's framework and variation theory provided me with a lens which enabled me to develop an analytical framework that I will present in the next Chapter. Literature review in relation to the study has also been presented in this Chapter, with a focus on the developing learners' mathematics discourse and understanding. Random selection of examples may lead to poor choices of examples which may not allow the intended object of learning to be brought to the fore and this may not enable the teacher to develop learners' mathematics discourse and understanding. In the next Chapter, I will present the research design and methodology of the study.

CHAPTER 3: Research Design and Methodology

3.1 Introduction

In this Chapter, I present the design and methodology of the study. The issues relating to population and research context, sampling and data collection and analysis are presented. Trustworthy and credibility and ethical considerations in relation to the data collection process are presented in this Chapter. The analytical framework and the codes, and the code identification rules of the study are presented in this Chapter.

3.2 Research paradigm

According to Crotty (1998, p. 10), "ontology is the study of being". Ontology as a paradigm focuses on the nature of reality and how it is constructed (Crotty, 1998). Ontology focuses on the norms that guide us in the process of determining whether something is real or not (Scotland, 2012). Ontological and philosophical norms that are based on the nature of reality were crucial for my study. Epistemology is a division of philosophy that focuses on knowledge and how knowledge is acquired. In order to delineate how one comes to know, we draw on epistemology (Crotty, 1998).

My study is located within the pragmatic paradigm, which was presented by philosophers (Alise & Teddlie, 2010; Creswell, 2003; Patton, 1990) who argued that the truth about the world is neither determined by relying on one scientific method (positivist paradigm) nor through social reality (interpretivist paradigm) but rather use methods that are appropriate to the phenomenon that is being studied. Therefore, I drew on the following characteristics of the pragmatic paradigm from the work of Creswell (2003), Patton (1990) and Martens (2015): not locating my study as either Positivist (post-positivist) paradigm or an Interpretivist (constructivist) paradigm, emphasis on what works within the context of my study, adoption of worldviews that allowed me use research design and methodologies that were best suited for investigating the role of the teacher in developing learners' mathematics discourse and understanding, choosing research methods based on the purpose of the study.

The epistemological basis for my study was *authoritative* knowledge and *empirical* knowledge. Authoritative knowledge is the knowledge that has been endorsed and accredited by experts in the field of study (Slavin, 1984). From an authoritative perspective, I reviewed existing literature that was written and endorsed by experts in the field. Empirical knowledge is derived from observations and experiences, and demonstration of facts that are considered objective (Slavin, 1984). From the empirical knowledge perspective, I drew on experiences, classroom observations, and objective facts that were demonstrated in the lessons which I observed.

3.3 Research design

I conducted a qualitative study. Brantlinger, Jimenez, Klingner, Pugach and Richardson (2005), define a qualitative research design as a "systematic approach to understanding qualities, or the essential nature, of a phenomenon within a particular research" (p. 195). Qualitative research was appropriate for my study because it allows the researcher to view reality as multilayered and interactive (Schumacher & McMillan, 1993). Opie (2004) also argues that a qualitative research design allows the researcher to understand the social phenomenon from the perspective of the participants of the study. The study took place in a real-life setting, the classroom, and a qualitative study allowed me to understand people in their real setting. There was no attempt by the researcher to influence the participant's behaviour during the study.

3.4 Population and research context

Data collection for the study was in three phases. I selected Grade 10, with a focus on functions as a topic. Functions is a crucial topic in mathematics as discussed in Chapter 2, and Grade 10 learners are introduced to the topic for the first time. Phase one comprised six teachers, phase two and three comprised three teachers, selected from the six teachers in phase 1. Phase 2 and 3 had two teachers from the same school and 1 teacher from another school. The teachers were teaching the concept of functions to Grade 10 learners. The teachers in the study were selected from three schools, based on availability and willingness to participate in the study.

3.5 Sampling and Data collection

In order to collect rich data for my study, I used a 3-phase approach. The sample selection was opportunistic because I had access to teachers who were part of a professional development course that was run by the University. The first phase of data collection was pre-observation teacher interviews, the second phase was classroom observations and the third phase was post-observation teacher interviews.

For this study, Cohen, Manion and Morrison's (2000) purposive sampling was used. As discussed earlier, the schools and the teachers who took part in phase one were selected from a group of available schools, but the participants for phase two and three in my study were selected purposefully. The teaching experience of the teachers and the qualification level did not vary in this study. In what follows, I discuss the 3-phase approach of how I selected the sample and how data was collected.

Phase 1: Pre-Observation interviews

In this phase, I conducted semi-structured interviews with the six teachers that I selected from the three schools. The aim of the pre-observation interview was to select three teachers from the six teachers for an in-depth study in phase 2 and phase 3. Interviews were important for my study as they provided information which participants may not display during observations, as discussed by Opie (2004). The questions I asked during the semi-structured interview are listed in the table below.

Table 7: Pre- observation Interview questions.

Question	**Justification for choosing the question**
1. For how long have you been teaching mathematics? Which grades do you teach and why?	This question gave me an insight into the teacher's experience because I did not want the experience of the teachers to vary in the study.
2. How do you explain concepts and ideas to learners in class?	This question enabled me to see how the teacher normally interacts with the learners during lessons and how the content is delivered to the learners

3. What do you consider as everyday knowledge in mathematics? How do you use learners' everyday knowledge to explain mathematics terms?	This question enabled me to find out if the teacher was aware of everyday concepts and if the teacher allowed learners to use the concepts to understand mathematics concepts
4. How do you move back to mathematics terms to explain to learners about their everyday knowledge?	This question helped me to find out if the teacher allows the double move in the classroom
5. There is a growing consensus that we talk to learn, that language enables access to mathematics, and that learning means being able to talk about what one has learnt. Do you agree? How do you think your learners learn to talk and talk to learn in your class?	This question helped me to identify the teachers who argue that learners need opportunities to talk about and share mathematics ideas in the classroom and teachers who argue that developing learners' mathematics discourse is of great importance.

The questions were not completely preplanned. I probed the interviewee for instances where I needed clarity on any issue discussed. Opie (2004) argues that the flexibility of semi-structured interviews may become problematic if not carefully handled. I conducted the interview for 20 minutes with each of the six teachers. Before the actual interviews, I carried out a pilot study in order to minimise ambiguities in the interview questions and to also check the length of the interview. The final interviews were audio recorded, and thereafter transcribed.

Selection of sample for Phase 2

In phase 1 of the data collection, I interviewed six teachers from three schools. Phase 1 was a random selection of teachers from schools that were available and willing to take part in the study. From the pre-observation interviews, I then selected three teachers purposefully to take part in the next phase of data collection. The three Grade 10 teachers that I selected from phase 1 to move to phase 2, indicated that they allowed learners to engage and interact in their lessons. It was evident that they were aware of the importance of mathematics discourse. Selecting the teachers was very important for my study because teachers who do not see the importance of developing learners' mathematics discourse in a dialogic classroom may not create a conducive environment for the development of learners' mathematics discourse and understanding to take place in their classrooms.

The sample for phase 2 was selected based on the pre-observation interviews conducted in phase 1 as discussed earlier. Therefore, the three teachers selected from phase 1 were my sample for phase 2 of the study. I selected three teachers whose interviews showed a great deal of intentionally developing learners' scientific mathematics discourse. The table below shows the profiles of the three teachers who were selected for phase 2 and 3 of the study.

Table 8: Profile of teachers selected for Phase 2 and 3.

Teacher	Years of experience	Grade taught over the years	Comment on importance of classroom talk
A	24	Grade 10 - 12	Very important and it is one way to monitor learners' understanding
B	18	Grade 10 - 12	Learners need to be allowed to share their ideas in class so that the teacher is able to intervene when they give an incorrect answer
C	15	Grade 10 - 12	Learner talk is important in teaching and learning of mathematics. This was emphasised in the course I did at University

Phase 2: Classroom observation

As stated in Chapter 1, the purpose of the study was to investigate the role of the teacher in developing learners' mathematics discourse and understanding. The main method of data collection was classroom observation because I wanted to study the participants' behavior in situ. Observations also allow the researcher to understand the content in the context it is being observed in. The actions that might be missed unconsciously in the absence of observation are availed to the researcher. For example, actions which the participants may not freely engage in during an interview or any other data collection method. Before I started the classroom observations, I spent one week visiting the schools to familiarise myself with the context. The one week was after the pre-observation interviews and three teachers had been selected for phase two and three. During the week of studying the context, I asked the teachers for their timetables to avoid having lessons occurring at the same time. I then planned according to the individual school timetables. Fortunately, all the three teachers' timetables had no clashes.

After one week, I was now ready to start the classroom observations. The teachers were video recorded teaching functions to Grade 10 learners. The focus of the observations was on the content and activities selected by the teachers, and the interactions that emerged from engaging with the content and activities during the lesson.

Data was collected in six weeks. All the lessons were approximately 50 minutes long, with an exception of three lessons from one of the teachers that lasted for 30 minutes. For anonymity and confidentiality, I refer to the three teachers with pseudonyms: Teacher A, Teacher B, and Teacher C. Teacher A was observed teaching 3 lessons; Teacher B was observed teaching 3 lessons and Teacher C was observed teaching 5 lessons. Teacher C had more lessons because three of her lessons were single lessons of 30 minutes. This gave me 11 video recordings of the whole study. As I recorded the lessons, I took field notes during the lessons in order to be able to capture as much information as possible.

As stated earlier, I used a video recorder in order to record all the lessons observed. A video recorder may have many technical problems such as poor quality of sound. To minimise the technical problems, I asked a video recorder expert to train me on how to operate the video recorder before using it. I then recorded a few pilot lessons before recording the actual data to check if the video recorder was functional and hence good for recording the data well. The pilot study did not focus on functions, but on a different topic in mathematics due to the school curriculum that prescribes that functions has to be taught at a specific time.

Phase 3: Post-observation interviews

I conducted a post-observation interview with the 3 teachers from Phase 2 in order to ask questions which I had as a result of the observation. The post-observation interviews were semi-structured, and they took place after I had transcribed my data and started analysis of the data. I wanted to make sure that I have all my questions in place before asking the teachers to avail themselves. The questions were formulated in such a way that they helped me confirm some issues which were not clear during the observation. During the post-interviews, I also asked teachers to explain some of the examples that I observed and the rationale behind the choice of examples in the lessons. All the post-observation interviews lasted about 20 minutes.

3.6 Data analysis

All the video recordings collected were transcribed in preparation for data analysis. Hatch's (2002) typological analysis was used to analyse the data. Predetermined typologies were developed. The choice of typologies was informed by the theoretical framework and literature review of the study that is presented in Chapter 2, the objectives of the study and researcher's experience (Hatch, 2002). The steps followed during data analysis are outlined in the table below.

Table 9: Steps in typological analysis (Hatch, 2002.)

Step	Activity
1	Classify the typologies e.g. assessing prior knowledge; content and activity selection; content selection of the lesson; scientific mathematics discourse, everyday mathematics discourse, double move, communicative approach, teacher interventions, patterns of discourse and patterns of variation
2	Read the data using the selected typologies
3	Read and record entries as per typology
4	Identify themes in the data
5	Code themes
6	Identify themes in the data and look for patterns
7	Determine relationships in the patterns
8	Describe patterns
9	Identify excerpts from the data to support the description

The analytical framework I discuss below was informed by the constructs mainly from the study's theoretical framework. The data collected and the literature reviewed also informed part of the analytical framework. I also drew on my 10-year experience as a mathematics teacher.

Drawing on Mortimer and Scott's framework, variation theory, literature review and my experience, I came up with the following categories:

- assess;
- content and activity selection;
- teacher strategies;
- monitor learners' progress.

I argue that these categories represent crucial steps taken by the teacher in the process of helping learners to access mathematics discourse and hence understand mathematics concepts. In what follows, I provide a discussion of the rationale for why I chose the categories above, using the literature and this study's theoretical framework and also discuss the code identification rules that I will use during my analysis.

3.6.1 Assess

As discussed by Campbell and Campbell (2009), any effective teaching begins with an assessment of learners' knowledge to determine what the learners already know (prior knowledge). Assessing prior knowledge involves asking learners questions that require them to give a definition or an explanation of a concept before they engage with the concept in any way. An example of assessing for prior knowledge is when learners are asked to define a function at the introductory part of a functions lesson (that is, before the definition is given to the learners). By doing this, the teacher is finding out what learners already know about what a function is. Assessing may also occur to uncover and address misconceptions that learners may have about the concepts to be taught (McInerney et al., 2014). Bereiter and Scardamalia (1989) argue that when teachers anticipate misconceptions and deliberately draw attention to them, better learning of concepts occurs. Assessment is also done to check for learner understanding of the concepts that have been discussed in the lesson. Assessment may be conducted using a diagnostic test, selected activities in class and or questions about the concepts being covered during the lesson.

Table 10 below shows the aspects which I focused on in the process of analysing the role of assessment in my study (as it concerns the category 'Assess'), the codes used to analyse assessment and the recognition rules for assessment aspects in the lesson transcripts. In Table 10 below, I used three codes: LPK to represent checking learner's prior knowledge; LM to represent checking learners' misconceptions and AA-LU to represent checking learners' understanding of concepts from previous lessons.

Table 10: Code identification rules for Assess.

Teacher activity	Aspect	Code	Code identification rule(s)
Assess	Checking learners' prior knowledge	LPK	• When the teacher asks learners to engage with a concept or procedure which has **NOT** yet been taught in the lesson. Phrases indicating LPK -Today's topic is functions. Who knows what a function is? Tell me anything you know about functions -Who can tell us what a function notation is*? (the first time the teacher introduces the function notation)* - Is there anyone who knows how to represent a set of ordered pairs here? *(the first time the teacher is engaging with ordered pairs in the lesson)* - Can you tell us any function notation you know? *(the first time the teacher is engaging with function notation in the lesson)* -Have you drawn a linear graph before? *(The first time the teacher is engaging with a linear graph and or any other graph in the lesson)* - Do you still remember the concept of a function? -Is there anyone who knows how to plot a parabola?
	Checking learners' misconceptions	LM	• When the teacher asks questions to check learners' thoughts and reasoning about a given concept being discussed in the class and or when the teacher presents learners with what a concept is **NOT** in order to see whether they know what it is. Phrases indicating LM: -I notice that some of you are writing an ordered pair as (*y; x*). Can I write an ordered pair like (*y; x*)? (*when the teacher is checking if learners have made an error or it is their actual interpretation of the concept, which makes it a misconception*) -Why this bracket and not that one? (*questioning a wrong procedure)* -Am interested in hearing your thinking about why you say this is a function (*for a situation where the relationship given is not a function)* -Why do you say that the gradient of this graph is positive? (*In a situation when the gradient is not positive)*

	Checking learners' understanding	AA-LU	When the teacher asks if learners understood a given concept that was discussed in previous lessons. Phrases indicating AA-LU From what we discussed yesterday, who can now tell us what a function is? -We have gone through the worked examples together. I want you to try this exercise on drawing graphs that we covered last week - Given these set of x-values, can you complete the table? - Are you able to see what is going on, from what we covered in the previous lesson? - What would be the equation of the hyperbola? -Why is the gradient of this graph negative? -Do you see what is going on here?

Table 10 above has an overlap between checking learner understanding (LU) and checking learner misconceptions (LM). This is because, in the process of the teacher checking for learner understanding, misconceptions may be uncovered, and these have to be dealt with accordingly. LU occurs every time a teacher wants to check if learners have understood what is being discussed during the lesson while LM occurs when a teacher recognises the need to confirm a learner's understanding due to a learner's incorrect explanation or definition of a concept. Therefore, after the teacher has checked for learners' prior knowledge and or understanding, and the teacher still needs to probe further to get clarity on learners' explanation, it becomes checking learners' misconceptions in my study.

3.6.2 Content and activity selection

For my study, content and activities refer to the example set that the teacher draws on to illustrate or demonstrate a concept or skill. In a lesson, examples are always embedded in a task, and this is why for my study, I used example set content and activity and interchangeably. From the literature review in Chapter 2, I defined an example set as a set of examples that the teacher selects to teach a concept.

Content and activities may be selected to address the concepts that learners do not know, struggle with, and or misunderstood from previous encounter. Therefore, the code identification rules for content and activity selection are embedded in the respective categories: assess, teacher strategy and monitoring learners' progress. The analytical task for content and activities was the example set of the teacher in the lessons observed.

3.6.3 Teacher strategy

Mortimer and Scott (2003) describe the teaching purpose of a lesson as guiding learners to work with mathematics meanings and supporting internalisation. This can be done through providing opportunities for learners to talk (see Table 2 in Chapter 2). For my analysis, I refer to this as teacher strategy. Under teacher strategy, the teacher uses the content and activities selected to teach and engage with the concepts of the lesson with learners. This can be enhanced by emphasising learner engagement through talk. To analyse teacher strategies, I draw on Mortimer and Scott's (2003)'s communicative approach and patterns of discourse (see Chapter 2). I also draw on the double move and variation theory in the process of analysing teacher strategies. The analytical task for teacher strategy was the interactions in the episodes in relation to the content and activity in focus.

Listening carefully during a lesson is important because the teacher is able to decide when to step in and when to step aside (NCTM, 1991). During teacher strategy, the teacher has to give learners space and opportunities to direct discussions and explore viewpoints (Mortimer & Scott, 2003). Learners should also be allowed to articulate their ideas (NCTM, 1991). Chazan and Ball (1999) argue that the teacher's role is not limited to telling but also to probing learners so that they engage during the lesson. Table 11 below is a discussion of the teacher strategies code identification rules. The codes for patterns of discourse and communicative approach were used as discussed in the theoretical framework of the study. The codes for patterns of variation, and double move that were used in Table 11 below are unpacked below.

SS-TS – Separation as a teaching strategy
CC-TS – Contrast as a teaching strategy
FF-TS – Fusion as a teaching strategy
DM-TS – Double move as a teaching strategy

Table 11: Code identification rules for Teacher strategies.

Focus	Elements	Code	Code identification rule(s)
Teacher strategy	Separation	SS-TS	• When the teacher uses controlled variation (keeping one aspect varying and the other(s) invariant) in the examples to focus learners' attention on what a concept is, in an attempt to bring about some form of generalisation. Phrase indicating SS-TS - *h(x), p(x) h(x)* are all function notations -Let us look at the gradients of the four graphs, you can see that… -The graphs we plotted, we can see that as slope increases, …… Now here we are only taking only one we, can't take all of them otherwise they will be too many -The gradient of these graphs is negative -The y-intercepts of these graphs… -The turning point of the graph is…
	Fusion	FF-TS	• When the teacher uses an example set that has more than one feature varying/invariant. Phrase indicating FF-TS: -Let us plot the graphs with the equations below. Can see the effect of the coefficient of x^2 and the effect of the constant term on the graph? $y = x^2$ $y = 2x^2$ $y = x^2 + 1$ $y = 2x^2 - 1$ $y = -x^2 + 2$-Let us determine at the domain, range and intercepts of the graphs… …..
	Contrast	CC-TS	• When the teacher uses examples of what something is and what it is not to explain what a concept is and what it is not. Phrase indicating CC-TS: -Consider the equations below: Which of the given equations is hyperbolic: $y = \frac{3}{x}$; $y = \frac{x}{3}$; $y = 3x^2$
	Communicative approach: Interactive-dialogic (ID);	ID	When the teacher allows learners to share their ideas and they are acknowledged in the lesson. Phrases indicating ID: -Did you hear that? What else? (*Teacher acknowledges*

	interactive-authoritative (IA) Non-interactive-dialogic (NID); non-interactive-authoritative (NIA)		*learners' contribution and calls on other learners to give their own contributions*) -Any other response that is different from this one? -All the definitions we have heard are correct.
		IA	When the teacher allows learners to share their ideas but does not engage with the contributions during the lesson. *Or when the teacher ignores what the learner has said and gives his/her own definition of a concept without engaging with what the learner said* Phrases indicating IA: -We do not represent it like that, we….
		NID	When the teacher does not allow learners to share their ideas, but more than one idea is considered in the lesson. Phrases indicating NID: -A function is… it can also be defined as….
		NIA	When the teacher does not allow learners to share their ideas and only one idea is considered in the lesson. Phrases indicating NIA: -A function is…
	Patterns of discourse	I-R-E	When the teacher initiates the talk, a learner gives a response and the teacher evaluates the learner's response. Phrases indicating IRE: Teacher: What is a function? Learner: A function is… Teacher: Very good, a function is…
		I-R-P	When the teacher initiates the talk, a learner gives a response and the teacher probes the learner. Phrases indicating IRP: Teacher: What is a function? Learner: A function is…. Teacher: Very good, can you elaborate more on the one-to-one and one-to-many relationship? What does that mean? When the teacher initiates the talk, a learner gives a response and the teacher probes the learner which leads to another response, and the teacher may probe further or evaluate the learners' response. An I-R-P-R-P-R-E example: Teacher: What is a function? Learner: A function is…. Teacher: Very good, can you elaborate more on the one to one and one to many relationship? What does that mean?

			Learner: One to one…. Teacher: Can you give us an example of such a relationship? Learner: An example of a one to one relationship is… Teacher: Correct.
	Double move	DM-TS	When the teacher uses everyday examples to explain the mathematics concept being discussed, then after relates the everyday situation back to the mathematics concept, and vice versa. Phrases indicating E-DM: - A situation in everyday life which you can say this one is similar to a function is … - Now that we have discussed what a relationship is in our everyday lives is, we can now define a relationship in mathematics as… -A factory setting is an example of a function.

3.6.4 Monitoring learners' understanding

The main reason for monitoring is for the teacher to find out if learners have understood or misunderstood the content that has been discussed. The teacher is expected to summarise main points and make sure that all learners understand them (Alsawaie, 2003). The main importance of monitoring is to ensure that learners are guided to what is required of them as participants in the mathematics discourse. Lorenz (1980) argues that there is an influence of the structure of mathematics on learner-teacher interaction that cannot be ignored in any mathematics classroom. This influence requires the teacher to use expert knowledge to provide appropriate guidance to learners towards acceptable mathematics approach and ways of thinking. Monitoring learners' understanding may involve marking learners' ideas and revoicing learners' responses among others. Monitoring can be done by assessing learners' understanding. The elements for monitoring are discussed in the table below.

Table 12: Code identification rules for monitoring.

Focus	**Elements**	**Code**	**Code identification rule(s)**
Monitoring	Marking key ideas	MKI	When the teacher legitimises with a learners' response. Phrases indicating MKI: - I like that… -Correct… -Very good… -Did you all hear that? Please repeat for the whole class

			- Do you see now?
	Revoicing of learner explanations and language used	RLE	When the teacher repeats what the learner says but corrects the use of language or the definition given. Or when the teacher repeats learners' explanation for audibility either in the same words or rewording or both
	Learner understanding	MM-LU	When the teacher gives learners activities to attempt after a concept has been covered during the lesson. This can also be in form of homework. Phrases indicating MM-LU - From what we have just discussed, who can now tell us what a function is? - Right let's move on did you get what the domain is? -Right that is a function notation are we clear on that one? • When the teacher asks learners to engage with and explain concepts that have already been covered in the lesson. For example: • -I want you to try this exercise on drawing graphs that we have covered today -Now that we have discussed a function notation, who wants to try and show us how to write a function notation? - Can you complete the table now? - Complete all tables whoever finishes put up your hand - Now I want you to label that graph using a pencil - Can we have somebody who can work out f(x) for us? - Are you able to see what is going on? - What would be the equation of the hyperbola? -Why is the gradient of this graph negative? -Do you see what is going on here? • When the teacher gives learners activities to attempt as homework.

In Figure 7 below, the analytical framework of the study is presented diagrammatically.

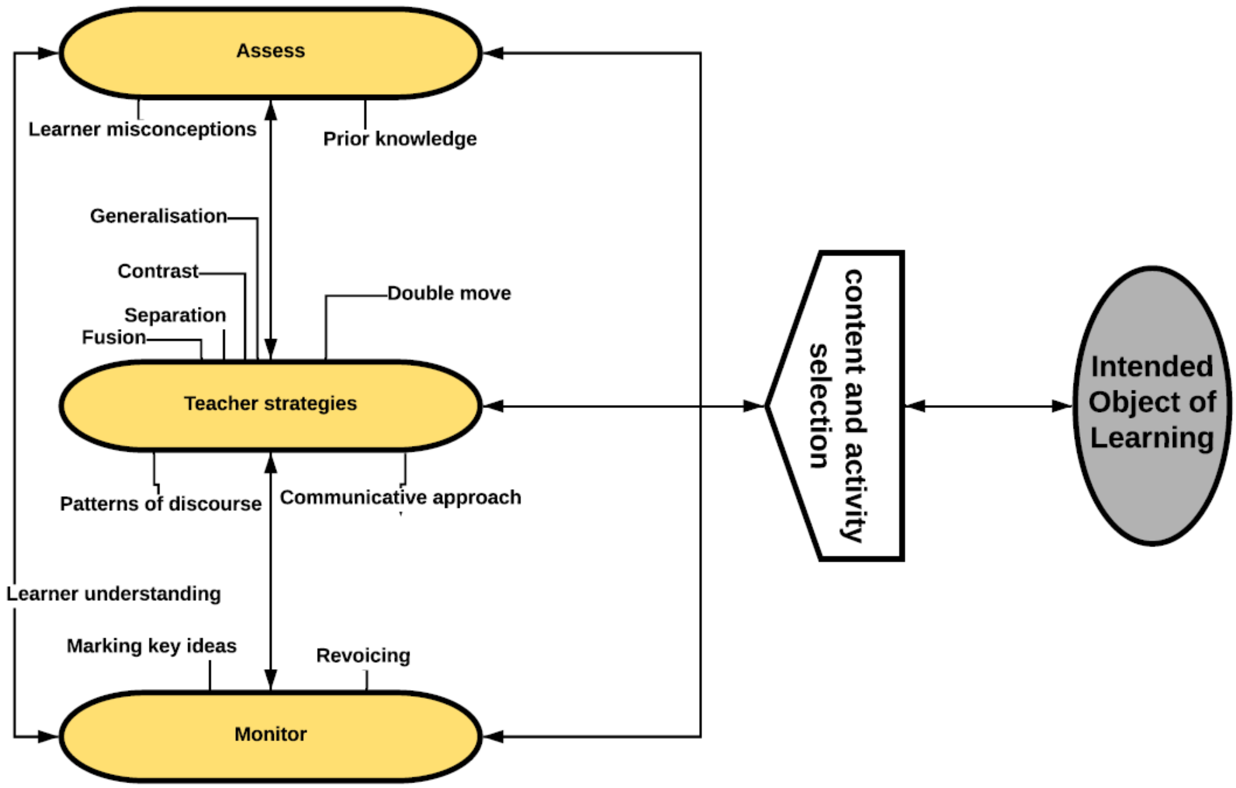


Figure 7: Analytical framework for the study.

Figure 7 is the analytical framework of the study. The intended object of learning is the starting point of the analysis, followed by the content and activity selection. The arrow between the intended object of learning and the content and activity illustrates that there is a back and forth movement between the two. The teacher may move back to confirm the intended object of learning after content and activities have been selected. In Figure 7 above, content and activity selection is illustrated in relation to assessing, monitoring and teacher strategy. The content and activities selected are crucial in the lesson, and they were considered as the second point in the analysis after the intended object of learning of the lessons observed in the study, as discussed in my next section. The teacher strategies are central to the analytical framework because it is through these teacher strategies discussed above that the teacher assesses and monitor's learners' understanding. In the process of content and activity selection, the teacher may also draw on some strategies such as the patterns of variation.

3.6.5 Analytical route and unit of analysis

My first unit of analysis was an episode in the lessons observed. I identified the episodes by paying attention to the shift in content focus. Each episode in the study is linked to introduction of a new activity. Dividing a lesson into episodes enabled me to consider the content and activities selected and utterances within the episodes which then became my final unit of analysis. I then analysed the content and activities related to the intended object of learning, and the accompanying teacher strategies.

Holmqvist, Tullgren and Brante (2011) argue the teacher defines the intended object of learning. For my study therefore, I planned to determine the intended object of learning from the lesson objectives (teacher's verbal announcements and statements written on the chalkboard as a topic (e.g. Functions) that the teacher sets up at the start of the lesson, and then link these to the Grade 10 CAPS curriculum (DBE, 2011). In what follows below, I discuss my point of departure after I identified my unit of analysis.

3.6.5.1 Point of departure for the analysis

As discussed earlier, the example set in my study is made up of the definitions, worked examples and activities that are presented in the lesson (see Table 12 above). My point of departure for the analysis was categorising the lessons into episodes. I then identified the intended object of learning in the episodes after which I identified the content and activities in the episodes, which I refer to as the example set, with respect to what was made available to learn in the lessons observed. Linking back to the analytical framework, assessing, and teacher strategy and monitoring, all depend on the example set, hence the importance of the content and activities selected for the lesson. The figure below is an illustration of the analytical route that I followed for my study.

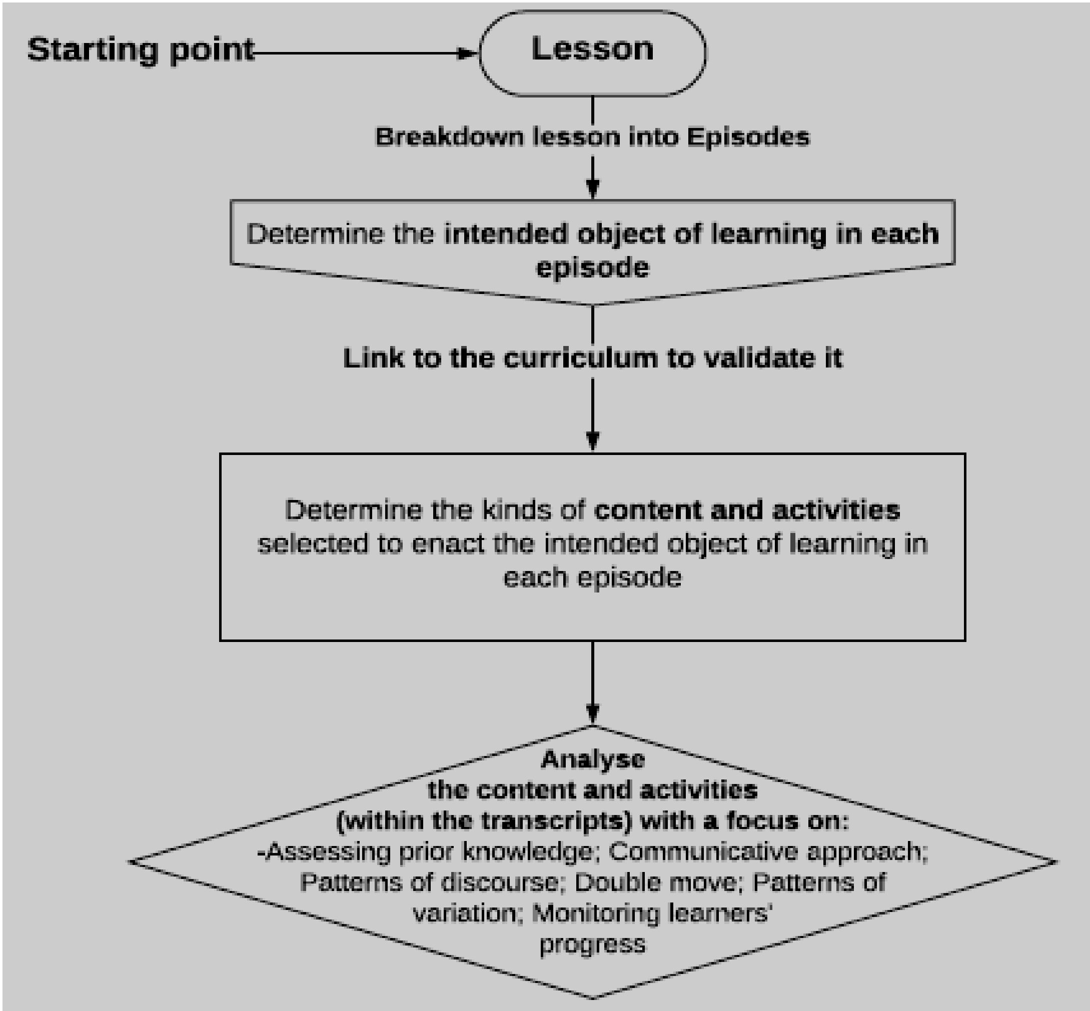


Figure 8: Analysis route.

From Figure 8 above, I illustrate the route that I followed to analyse my data. I started off my analysis by looking at the lessons to determine the intended object of learning. Then I linked the intended object of learning to the CAPS curriculum to validate it. I then moved on to determining the kinds of content and activities that the teacher selected in relation to the intended object of learning. After I had a list of the content and activities, I analysed them with a focus on how they were used to assess learners' prior knowledge, the communicative approach and patterns of discourse that was visible in the teacher-learner interactions, the everyday content and hence double move that was visible in the content and activities selected, the patterns of variation that were visible in the content and activities, and how monitoring learners' progress was carried out by the teacher in the lesson. The table below is an illustration of how I analysed an episode in my study.

Table 13: Illustration of Analysis of data.

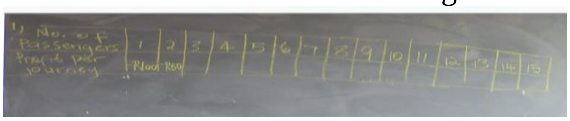
Episode	Transcription and example set	Code
Episode 1 **Intended object of learning:** Definition of a function **Example:** Completing a table of values	1. Tr: Let us look at the journey by a taxi driver with 15 passengers. I can represent the journey on a table. The price per passenger is twenty rand. … 7. Lnrs: Yes 8. Tr: And for two passengers? 9. Lnr2: Negative eighty 10. Tr: So you have negative eight rand loss because there is a loss [Writes -80 on board]. It is negative to show that there is a loss no profit has been made. If there are three passengers in the car, how much loss is Mr Shadrack making?  11. Lnr2: Six rand↓ 12. Tr: [Writes on black board] six rand loss right, so Mr. Shadrack is going to lose six rand. And if they are four passengers, what is the loss? 13. Lnrs: Forty rand↓	**DM-TS:** Everyday context used in the example **LPK**: Learners were expected to complete the table of values before engaging with it in the lesson. Interaction Line 1: *Initiation* … Line 7: *Prompt* Line 8: *Response* Line 9: *Prompt* Line 10: *Response* Line 11: *Evaluation*/*Prompt* Line 12: *Response* Communicative approach: **ID** Pattern of discourse: **I-R-P-R-P-R-E**

Table 13 above is an illustration of how I coded and categorised the data and coded it in the analysis process. The analysis illustration above shows that the intended object of learning in episode 1 was completing a table of values. The teacher selected an example with everyday context, and this is coded as DM-TS as shown above. The interaction is then coded in relation to the communicative approach (ID) and pattern of discourse (I-R-P-R-P-R-E) evident in the utterances in the transcript. The analytical framework enabled me to engage with my data and come up with a discussion that enabled me to argue for the role of the teacher in developing learners' mathematics discourse and understanding, as I will discuss in the next Chapters.

3.7 Validity and reliability issues

Maxwell (1992) and Opie (2004) argue that validity and reliability in qualitative research refers credibility of the results of the study. Any qualitative researcher should plan to ensure that reliability and validity concepts are catered for at every stage in the process of the study (Patton, 2002). Schumacher and McMillan (1993) argue that reliability in qualitative research refers to the "consistency of the researcher's interactive style, data recording, data analysis, and interpretation of participant meanings from the data" (p. 385). To ensure that the data collected is reliable, Schumacher and McMillan (1993) argue that the researcher should explain the social relationship, criteria, rationale and decision processes in the study. For my study, the social relationship of the researcher (non-participant observer) with the participants during data collection was stated earlier. Data analysis was also explained in detail, with the analytical framework that guided the study process.

Validity refers to the appropriateness of the research findings and conclusions (Maxwell (1992). For Maxwell (1992), validity threats have to be identified and dealt with in a qualitative research. Maxwell (1992) describes different types of validity from a qualitative perspective. These are descriptive, interpretive, theoretical, generalisation and evaluative validity. In what follows below, I describe the different types of validity. Maxwell argues that evaluative validity is not as important as the other types of validity, therefore, I did not discuss it below.

Descriptive validity refers to how activities are seen physically through behavioral events (Maxwell, 1992). The data was based on primary descriptive validity. To ensure that descriptive validity was in place, an analytical framework was developed to describe the data collected. The analytical framework provided me with a lens to resolve issues that would have led to any disagreements in the data description. All the video transcriptions were verbatim, and the researcher double checked to ensure that there was no omission or commission. I drew on statistically descriptive aspect to describe the number of content and activities that were selected in the lessons observed. To ensure an accurate account of record for the content and activities, I extracted all the content and activities, and created visibility of the description in Chapters 4, 5 and 6.

Interpretive validity focuses on the participant's perspective and not that of the researcher in the study (Maxwell, 1992). Schumacher and McMillan (1993) refer to interpretive validity as internal validity or credibility criterion. For interpretive validity, consistency in data collection process was maintained. The time taken and context to collect the data was stated and explained, together with the process of how data was collected and analysed.

Theoretical validity focuses on a theory within a given context (Maxwell, 1992). The validity of the theory I drew on for the study were checked by drawing similar theories to check for consensus in how concepts were defined within the field of study. I also ensured that the way I brought the concepts together was valid in relation to phenomenon being studied.

Another type of validity is external validity, which Maxwell (1992) refers to as generalisability. External validity, also known as transferability in qualitative research, refers to the extent to which the findings can be used in other settings or contexts (Maxwell, 1992; Golafshani, 2003). My study was a qualitative study, and therefore did not seek to provide transferable findings. Opie (2004) argues that a qualitative study does not aim at transferability.

3.8 Ethical considerations

Due to the sensitivity and nature of data collection procedures, I started by applying for ethics clearance from the University ethics committee (see Appendix A). I then sought permission from all the participants in the study as discussed below.

As stated earlier, six schools were selected for the pre-observation interviews and therefore, permission was sought from the six principals of the schools using a written consent form and an information sheet indicating what the study is about. After the principals had given me permission to go ahead with the research in their schools, I then sought permission from the teachers using written consent forms, accompanied with an information sheet with the purpose of the study, and a video consent form. After the teachers had agreed to take part in the study, I then sought permission from the Department of Education because the schools are public school. I started with the schools because from my previous experience, the school may reject the invitation to take part in the study because the Department of Education cannot force the school to take part in the study, so it is always better to start with the school and then get permission to do the research from the Department of Education after the school has accepted to take part in the study. After I had permission from the Department of Education, I then conducted the pre-observation interviews in order to select the teachers for the second and third phase of the study.

After I finished selecting the teachers, I then went to the classes which took part in the second phase of the study and gave learners consent forms and an information sheet with details about the study. Learners who were below the age of 18 were given consent forms to give to their parents. It was made clear to the participants that they could not be forced to take part in the study. When it came to video recording, I sought permission from the participants as this is a very sensitive issue.

I collected all the consent forms before beginning the process of data collection and captured the responses. All the learners in the study were willing to participate in the study. Only two learners did not want to be video recorded. I requested the teacher to strategically position learners as per their responses, and parents' responses in the consent forms so that they were out of the camera view. Since two learners (both from different classes) did not want to be part of the video recording, the teachers did not have to change the classroom settings. I ensured that the camera was positioned in a way that excluded the learners. I also emphasised the issues of anonymity and confidentiality to all the participants. In the analysis and write up of the study, all the participants were referred to using pseudonyms.

3.9 Conclusion

In this Chapter, I have explained the research design in relation to the focus of my study. I have also presented the sample of the study and the rationale for the sample selected. The data collection methods have been discussed in detail, with justification for using the methods in the study. The analytical framework that was used for the study has also been discussed in detail, with the focus being: content and activity selection, teacher strategies (patterns of variation; communicative approach; patterns of discourse; double move), assessing and monitoring learners' progress. In the three Chapters that follow, I present the findings and discussion of results from the study, as analysed using the analytical framework that has been presented in this Chapter.

CHAPTER 4: The role of the teacher: The case of Teacher A

4.1 Introduction

In this Chapter and the two Chapters that follow, I present findings of the study on the role of teachers in developing learners' mathematics discourse and understanding. In order to investigate the role of teachers in my study, I analysed pre-observation teacher interviews, lesson observations and post-observation teacher interviews, in an all-encompassing manner. As mentioned in Chapter 3, three teachers (Teacher A, Teacher B and Teacher C) were selected as participants in all three stages of data collection of the study. As discussed in Chapter 3, I drew on the analytical framework (see Figure 9 below) of the study to analyse the data.

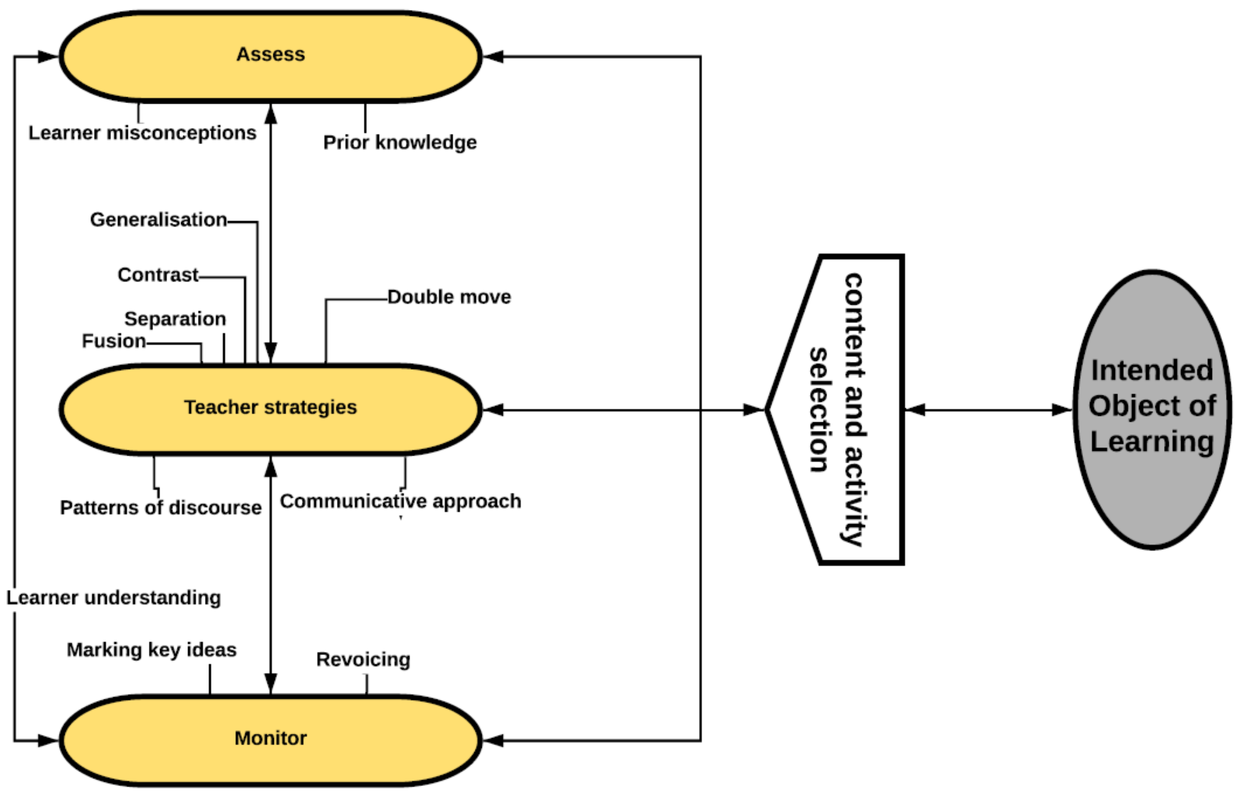


Figure 9: Analytical framework for the study.

Figure 9 above is the analytical framework of the study which was presented and discussed in detail in Chapter 3. As discussed in Chapter 3, the content and activities, in relation to the intended object of learning, that were selected by the teacher during the lesson became my starting point in the process of analysing the data. In the presentation of the content and activities the example set; I engage with my first sub-research question below:

a. What kind(s) of content and activities, in relation to the intended object of learning are selected by the teacher in the process of teaching concepts in the mathematics functions classroom?

As I present the findings, and discuss the results in relation to the question above, I will concurrently engage with the findings related to the next sub-question below:

b. How does the communicative approach and pattern of discourse shape the enactment of the content and activities selected by the teacher?

To present the findings in relation to Question b, I will engage with how the communicative approach and patterns of discourse shape the enactment of the content and activities selected by the teachers. As I engage with the findings of sub-questions a and b, I will present and discuss the findings of the last sub-question of my study concurrently:

c. What strategies does the teacher employ in an attempt to enable learners to discern the critical features of the object of learning?

To recap from Chapter 1, mathematics discourse is the mathematical communication – written or verbal and non-verbal, among the teacher and learners in the classroom (NCTM, 1991). According to the NRC (2002), understanding refers to “comprehending mathematical concepts, operations, and relations – knowing what mathematical symbols, diagrams, and procedures mean” (p. 9).

As presented in Chapter 1 and 2, my main research question is: What is the role of the teacher in developing learners' mathematics discourse and understanding?

In terms of the definitions of mathematics discourse and understanding above, my main research question focused on what the role of the teacher is in developing learners' mathematical communication (written; verbal and non-verbal), and comprehension of mathematical concepts, operations and relations (developing meaning of mathematical symbols, diagrams and procedures).

It was crucial for me to keep the above research question in mind for this Chapter and the next two Chapters as it was instrumental in guiding the trajectory of the analysis and hence the presentation of the findings and discussion. It is important here to emphasise the dual nature of the study. Answering the research question has two elements ultimately for my study, 1) what the roles of the teacher are and 2) how they were enacted by the teachers. It is the interaction of these two elements that fully answers the research question.

This chapter focuses on Teacher A. I observed Teacher A teaching three lessons which I referred to as: A1, A2 and A3 respectively. Lesson A1 was on defining the concept of a function, with Teacher A drawing on some examples of how functions can be related to scenarios from everyday contexts, as I will discuss later in this Chapter. In Lessons A2, Teacher A focused on linear functions, specifically on completing a table of values to determine a set of ordered pairs - to plot a straight-line graph using the point-by-point method. Lesson A3 focused on sketching, defining the domain and range of a given linear function, and how to determine the gradient and find the equation of a straight-line graph. In this Chapter, I will begin by presenting the kinds of content that Teacher A selected, and as I present the content and activities, I will present the findings and discussion of how the communicative approach and patterns of discourse shaped the enactment of the content and activities. I will then present the findings on the strategies that Teacher A employed in the lessons observed.

4.2 Kinds of content and activities selected by Teacher A

All the lessons observed for Teacher A lasted for approximately 50 minutes. Lesson A1 was the first lesson which the teacher conducted on functions. As an introductory lesson to functions, the focus was on the definition of a function, the domain, range and function notation. Lesson A2 focused on plotting straight-line graphs, using a table of values. Lesson A3 focused on determining the domain and range of a given straight-line and finding the gradient and the equation of a straight-line. As discussed in Chapter 3, for my study, the intended object of learning was crucial for me to engage with the content and activities in the episodes of the lessons observed. I therefore begin the analysis by determining the intended object of learning.

4.2.1 Intended object of learning

In Teacher A's lessons observed, the lesson objectives were announced in A1 only. A2 and A3 did not start with the teacher announcing the lesson objectives, and as such, I was not able to determine the lesson objectives and hence the intended object of learning from the teacher's announcements at the beginning of the lesson, as discussed in Chapter 3. It was also not in my design to access the teacher's lesson plans or engage him before the lesson. Because I had no access[3] to the teacher's intended object of learning prior to the lesson, and the intended object of learning was not announced, I relied on the lessons observed. Therefore, I inferred the intended object of learning from the lessons observed, the enacted object of learning. I found it useful to compare the inferred intended object of learning to the CAPS content clarifications, as a way of validating my inferences, as shown in Tables 14, 15 and 16 below. Lesson A1 below was different from the other lessons because Teacher A announced the lesson objectives and hence the intended object of learning, which I took as is and linked it to the CAPS for validation as stated above.

[3]I should have engaged the teacher prior to the lesson to find out the intended object of learning. However, I assumed that the teacher would have announced this in the lessons, which was not the case. Therefore, I had to rely on the lessons observed to infer the intended object of learning in lessons A2 and A3.

Table 14: Lesson A1'sinferred intended object of learning and a link to the CAPS.

Intended object of learning announced by Teacher A in lesson A1	Linking the intended object of learning from lesson A1 content to CAPS
Defining a function -What a function is -What a domain and range is -The function notation -How to represent a function.	**Curriculum statement**: [Define] "The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value). Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations" (DBE, 2011, p. 24). **Clarification comment from CAPS**: "A more formal definition of a function follows in Grade 12. At this level it is enough to investigate the way (unique) output values depend on how input values vary. The terms independent (input) and dependent (output) variables might be useful" (DBE, 2011, p. 24).

From Table 14 above, Teacher A's intended object of learning was in line with the curriculum statement, which is to define a function. Other aspects such as function notation were also discussed as shown in the table above. In the table below, I present the inferred intended object of learning of lesson A2.

Table 15: Lesson A2's inferred intended object of learning and a link to the CAPS.

Intended object of learning inferred from the content of Lesson A2	Linking the intended object of learning from lesson A2 content to CAPS
• Plot a straight-line graph on the Cartesian plane *Some content from the lesson* - Complete a table of values of a given straight-line function to get the ordered pairs -Plot points on the Cartesian plane	**Curriculum statement:** Point by point plotting of basic graphs (DBE, 2011, p. 24).

In lesson A2, Teacher A used the term 'sketch' instead of point-by-point plotting of graphs as per CAPS curriculum. However, the actual content that was covered during the process of enacting the object of learning, was in line with CAPS, and hence the overall intended object of learning that I stated in Table 15 above. The difference between plot and sketch in this case shows the importance of the teacher's attention to terminology used in the lesson, a key aspect of mathematics discourse. Sketching graphs is covered later in the curriculum and it is not the same as point-by-point plotting of graphs. Lesson A3's inferred intended object of learning is presented below.

Table 16: Lesson A3's inferred intended object of learning and a link to the CAPS.

Intended object of learning inferred from the content of Lesson A3	**Linking the intended object of learning from lesson A3 content to CAPS**
• Plot a straight-line graph (revision from previous lesson) • Determine the domain and range of a straight-line function • Determine the gradient of a straight line in order to find the equation of a given straight line graph	**Curriculum statement:** "Sketch graphs, find the equations of given graphs and interpret graphs" (DBE, 2011, p. 24). **Clarification comment from CAPS**: "Sketching of the graphs must be based on the observation of the effect of the parameters *a* and *q"* (DBE, 2011, p. 24).

From the table above, the intended object of learning was also in line with the CAPS. For lesson A3, three distinct intentions of the teacher were evident in the lesson as stated in the table above. I added the revision of plotting a straight-line graph in the intended object of learning of lesson A3 because of the time that Teacher A spent on it.

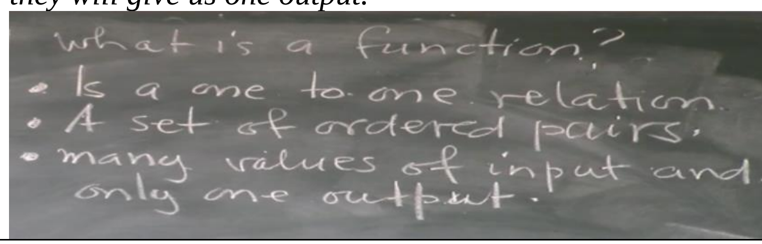
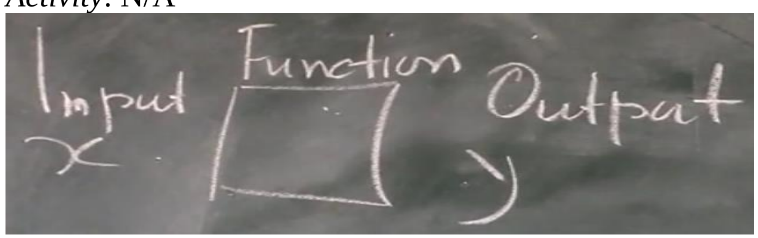
The inferred intended object of learning guided me in the analysis process as I engaged with the kinds of content and activities that were selected by Teacher A in the lessons observed. In what follows, I present the findings and discussion of the kinds of content and activities selected and how they were enacted in relation to the intended object of learning.

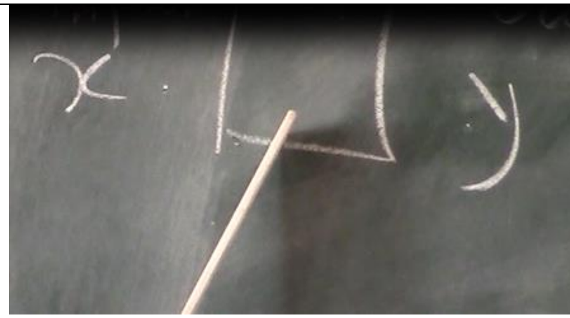
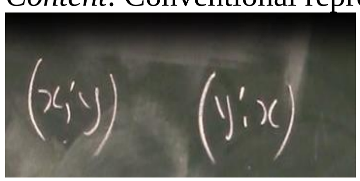
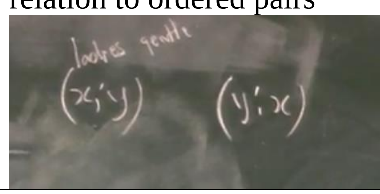
4.2.2 Content and activities selected to enact the intended object of learning

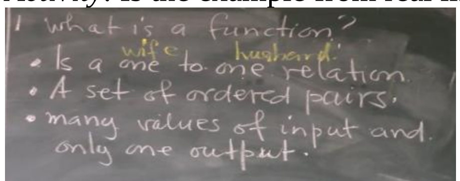
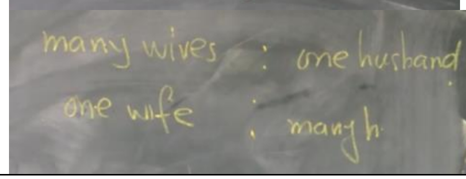
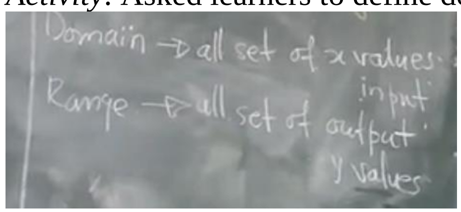
In an attempt to enact the intended object of learning, Teacher A selected content that learners were required to engage with. Some of the content and activities were pre-planned and taken directly from the textbook or teacher notebook, with Teacher A copying content and activities from the textbook/notebook to the board, and/or directing learners to open specific pages in the textbook. Other examples were ‘spontaneous’- in the moment decisions, as defined by Zodik and Zaslavsky (2008).

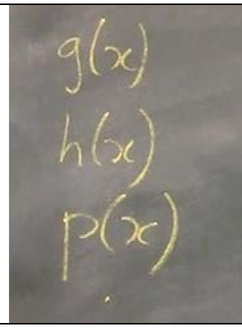
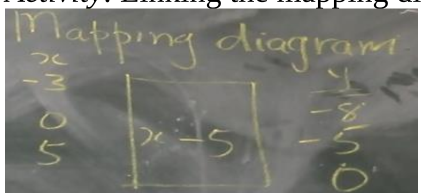
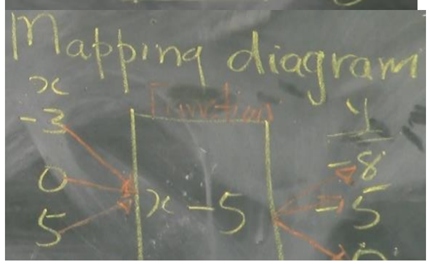
In the table below, I present all the content and activities, hence the example set that Teacher A selected across his three lessons observed. In the table below, AL1E1 stands for Teacher A’s first item in the example set, presented in lesson 1, AL1E2 stands for Teacher A’s second item in the example set, presented in Lesson 1, AL2E13 stands for Teacher A’s thirteenth item in the example set, presented in Lesson 2, AL3E15 stands for Teacher A’s fifteenth item in the example set, presented in Lesson 3, and so on. The examples have been numbered continuously from Lesson 1 to Lesson 3.

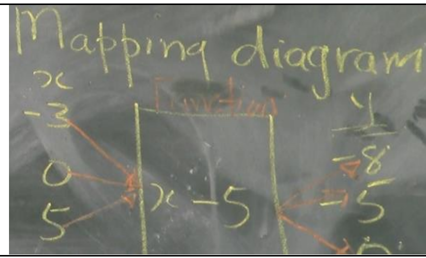
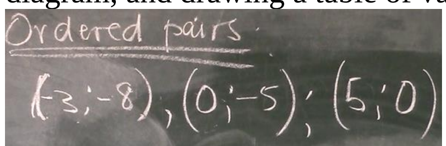
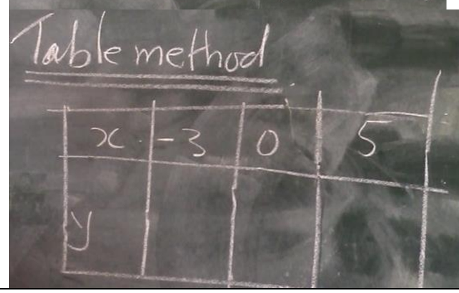
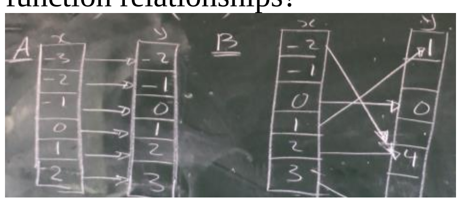
Table 17: Teacher A's content and activities in the lessons observed.

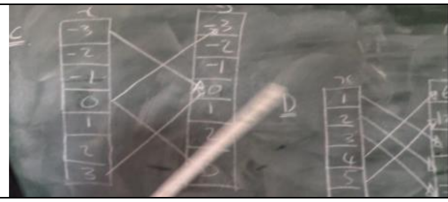
Intended object of learning	Item number	Content/activity	Key features from the lesson observed	Comment
1. Defining a function	AL1E1	*Content:* Definition of a function *Activity:* Learners asked to define a function What is a function? [These include verbal definitions] A function is a special relationship between input and output values, where every input value has got only one output value. *We can have a number of input values but they will give us one output.*  	-Relationship: one-to-one; many-to-one -Input- x-value -Output- y-value -variable	- **Scientific** content and activity: definition of a mathematics term/concept. - **Pre-planned** content and activity: Teacher A wrote down the lesson objectives at the start of the lesson, and this definition was one of the lesson objectives.
	AL1E2	*Content*: Diagram representation of a function relationship *Activity*: N/A  	-Input -Output -Rule	-**Scientific** content: Teacher used terms: input (x) and output (y), with a conventional box in the middle to represent the rule. -**Spontaneous** content: Teacher A drew diagram on the board to illustrate what was being discussed at that moment in the lesson.
	AL1E3	*Content*: Real life instance of a function: Factory setting as function in real life	-Factory setting -Product -Raw materials	-**Everyday** content: Teacher A used everyday context of a factory setting to explain the special relationship of a

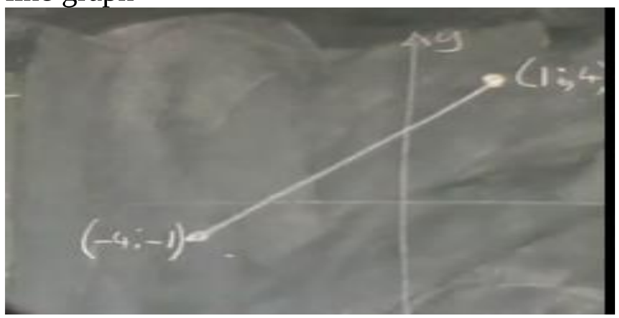
			-Process -Input -Output -Rule	function. Teacher A related *x* to raw materials, the box to what is done in the factory (the process), and *y* to the finished product. -**Spontaneous** content: Teacher A drew on the factory setting context to further illustrate what was being discussed at that moment in the lesson.
	AL1E4	*Content*: How to represent a set of ordered pairs *Activity*: Choose any numbers to make a set of ordered pairs (3; 6) and (3; 9) as sets of numeric ordered pairs	- *x* as the Input value -*y* as the output value -ordered pair	-**Scientific** content: Numeric representation of a set of ordered pairs. -**Spontaneous** content: Teacher asked learners for random numbers to come up with the two sets of ordered pairs.
	AL1E5	*Content*: Conventional represntation of Ordered pairs  *Activity*: Compare ordered pair to an everyday convention Ladies and gentlemen Vs gentlemen and ladies example in relation to ordered pairs 	- *x* as the Input value -*y* as the output value - order of the set of ordered pair	- **Scientific** content: standard mathematical convention of an ordered pair (*x*; *y*). -**Spontaneous** content and activity: Discussed the convention in relation to the discussion that was going on. -**Everyday** activity: relating *x* to ladies and *y* to gentlemen- ladies and gentlemen as a convention of greeting and

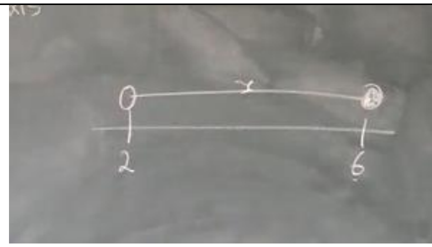
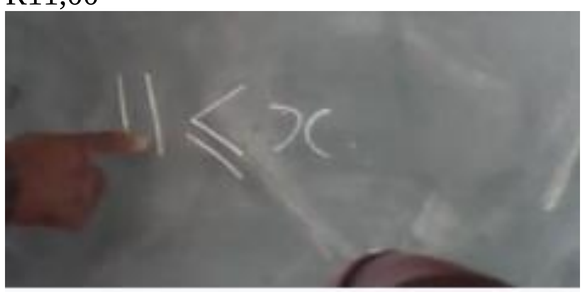
				not gentlemen and ladies – ($x; y$) and not ($y; x$).
	AL1E6	*Content*: Illustrating the one-to-one and many-to-one relationship using an everyday scenario of wife and husband *Activity*: Is the example from real life a function?  	- x as the Input value -y as the output value -one to one relationship -Many to one relationship -set of ordered pairs	-**Everyday** content: Using the wife and husband scenario to illustrate a one-to-one and many-to-one relationship, with wife being x and husband being y. -**Spontaneous** content: Teacher A drew on this example during the lesson in relation to the discussion that was transpiring.
	AL1E7	Content: Defining domain and range *Activity*: Asked learners to define domain and range 	- x as the Input value -y as the output value -rule -Domain -Range	-**Scientific** content: Teacher defined domain and range using mathematics terms and hence language – input, output. -**Pre-planned** content and activity: definition of domain was included in the lesson objectives at the start of the lesson.
	AL1E8	*Content*: Function notation: $f(x) = y$ $h(x) = y; p(x) = y; k(x) = y$ *Activity:* N/A	-Function notation $f(x) = y$	-**Scientific** content: Teacher presented mathematical symbols of function notation. -**Pre-planned** content: function notation was included in the lesson

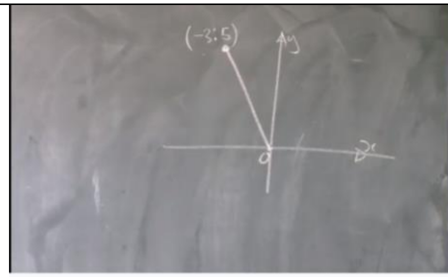
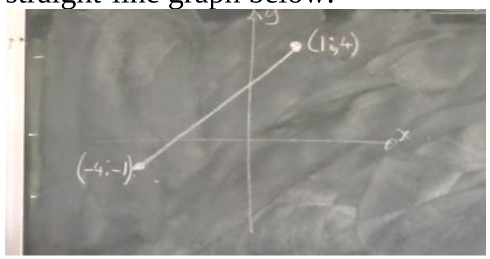
				objectives at the start of the lesson.
	AL1E9	*Content*: representing a function using a mapping diagram *Activity*: Linking the mapping diagram to a machine 	- *x* as the Input value -*y* as the output value -Rule -Ordered pair -Domain -Range -mapping diagram	-**Scientific** content: Mapping diagram is one mathematical way of representing a function. -**Everyday** activity: Linking the mapping diagram to a machine. -**Pre-planned** content- how to represent a function is included in the lesson objectives at the start of the lesson. -**Spontaneous** activity: Activity is constructed by the teacher and learners during the lesson.
	AL1E10	Activity: Taking sweets out of a jar- input and output	-Rule - *x* as the Input value -*y* as the output value	-**Everyday** activity: Taking sweets from a jar. -**Spontaneous** activity: Activity is constructed by the teacher and learners during

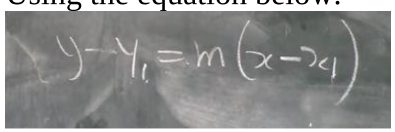
				the lesson.
	AL1E11	*Content*: Representing a function using a set of ordered pairs and in a table of values *Activity*: writing a set of ordered pairs from a mapping diagram, and drawing a table of values.  	-Rule - *x* as the Input value -*y* as the output value -Ordered pair -Domain -Range -Table of values	-**Scientific** content: mathematical convention of writing a set of ordered pairs, and drawing a table of values. -**Spontaneous** activity: ordered pairs were drawn from spontaneous activity that was designed by the teacher and learners during the lesson.
	AL1E12	*Content*: Representing a function using a mapping diagram *Activity*: Are the mapping diagrams below representing function relationships? 	-Rule - *x* as the Input value -*y* as the output value -Ordered pair -Domain -Range -mapping diagram	-**Scientific** content: Mapping diagram is one mathematical way of representing a function. **-Pre-planned** content: function notation was included in the lesson objectives at the start of the lesson. Teacher A copied the activities from his pre-

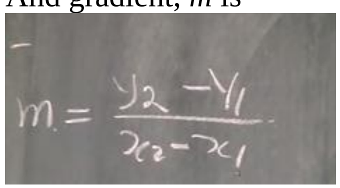
				planned notes that he was using in the lesson.
2. Plotting a straight-line graph	AL2E13	*Content*: Plotting a straight-line graph *Activity*: Completing a table of values and plotting the straight-line graph with the equation below: $y = f(x) = 2x$	- $y = f(x)$ -coefficient of x -Rule - x as the Input value -y as the output value -Substitution -Cartesian plane -y-intercept	-**Scientific** content and activity: $y = f(x) = 2x$ is a mathematics equation, written with mathematics symbols and operations. -**Pre-planned** content and Activity: Teacher A wrote $y = f(x) = 2x$ when he got to the class, before engaging the learners.
	AL2E14	*Content*: Point by point plotting of straight-line graphs *Activity*: Completing a table of values and then plotting the straight-line graphs with equations below: $f(x) = -2x$ $g(x) = x + 1$ $h(x) = -x - 1$ $j(x) = -x + 1$	-Set of ordered pairs -Plot points -coefficient of x -Rule - x as the Input value -y as the output value -Substitution -Cartesian plane -y-intercept -Function notation can	-**Scientific** content and activity: these are mathematics equations, written with mathematics symbols and operations. -**Pre-planned** Activity: Teacher A used his notes that he was referring to in the lesson.

			be written with any letter	
3. a) Domain and range of a straight line-graph b) Finding the equation of a graph.	AL3E15	*Content*: Domain and range *Activity*: Determining the domain and range of a straight-line graph  Domain $x \in [-4;1]$ $-4 \leq x \leq 1$ Range $y \in [-1;4]$	-Cartesian plane -Set of ordered pairs -Plot points -coefficient of *x* -Rule - *x* as the Input value -*y* as the output value -Substitution -*y*-intercept -Gradient -Types of gradients -Line parallel to the x-axis -Line parallel to the y-axis -Standard form of the equation -Domain of a function -Range of a function	-**Scientific** content and activity: mathematical representation of a straight-line graph sketch on the Cartesian plane. **-Pre-planned** content and Activity: Teacher A copied the activity from the textbook on to the board, from a Pre-planned page.
	AL3E16	Content: Domain and range Activity: determining the domain and range from the diagram below.	-set of ordered pairs -Domain of a function	-**Scientific** content: - mathematical representation of a straight-line graph, showing the domain and

		 $x \in (2;6]$	-Range of a function	range, conventional representation of domain and range. **Spontaneous** activity: Teacher A developed this activity during the lesson, as a result of how the classroom engagement was emerging, with a link back to learners' prior knowledge from Grade 9.
	AL3E17	*Content:* The concept of at least and at most *Activity*: Differentiating at least from at most, using a taxi to travel to Johannesburg and the taxi fare is R11,00 	-Domain of a function -Range of a function	-**Everyday** activity: Teacher used the taxi example to check learners' understanding of at least and at most. -**Spontaneous** content and activity: Teacher A drew on this content and activity as a result of how the classroom engagement was emerging.
	AL3E18	*Content*: Domain and range *Activity*: Determining the domain and range of a straight-line graph	-Domain of a function -Range of a function -Cartesian plane -*y*-intercept -Gradient	-**Scientific** content and activity: mathematical representation of a straight-line graph sketch on the Cartesian plane. **-Pre-planned** content and Activity: Teacher A copied the activity from the textbook

<table>
<tr><td></td><td></td><td>
</td><td></td><td>on to the board, from a Pre-planned page.</td></tr>
<tr><td></td><td>AL3E19</td><td>Content: Gradient and y-intercept of a straight line.
$y = mx + c$
Activity: Determining the gradient and equation of a straight-line graph below.


Using the equation below.


And gradient, m is


The equation of the graph as: y = x + 3.</td><td>-Substitution
-y-intercept
-Gradient
-Types of gradients
-Line parallel to the x-axis
-Line parallel to the y-axis
-Standard form of the equation</td><td>-Scientific content and activity: general equation of a straight-line. Formula for determining the gradient of a straight-line graph.

-Pre-planned content and Activity: Teacher A copied the activity from the textbook on to the board, from a Pre-planned page.</td></tr>
</table>

In Table 17 above, I have presented the content and activities that Teacher A selected during the lessons observed. At face value, Teacher A's example set in Table 17 above highlights the following:

- More than one activity was given for the same concept, and the content and activities given had the same key features brought to the fore (see AL1E1 - AL1E7 on defining a function) – this relates to the patterns of variation discussed in Chapter 2 and 3.
- Examples from everyday life were used frequently (see AL1E3; AL1E5; AL1E6; AL1E10 and AL3E17) – this suggests that the examples had potential for a double move strategy
- From AL3E15 - AL3E19, many concepts were embedded in few examples (see AL3E15 on domain and range, gradient and equation of a line) – activities suggested possibility for fusion and hence generalisation.

A closer analysis of the content and activities brought to light some aspects that I will present and discuss next. In the analysis, I categorised the content and activities selected by Teacher A as either pre-planned or spontaneous. Spontaneous examples are selected 'in the moment' while pre-planned examples are selected before the lesson (Zodik & Zaslavsky, 2008).

Teacher A drew on scientific and everyday context as shown in the example set presented in Table 17 above. Teacher A did not have any pre-planned/**Everyday** content or activities in his examples. Most of Teacher A's content and activities were pre-planned/**Scientific** as shown in Table 18 below. ✔ indicates that the type of content and activity was present while ***X*** indicates that the type of content and activity was not present.

Table 18: Kinds of content and activities selected by Teacher A in the lessons observed.

	Type of content and activity				Communicative Approach and Pattern of discourse
Example set Item number	**Spontaneous /Scientific**	**Spontaneous/ Everyday**	**Pre-planned /Scientific**	**Pre-planned /Everyday**	
AL1E1	*X*	*X*	✓	*X*	IA/I-R-E
AL1E2	✓	*X*	*X*	*X*	IA/I-R-E
AL1E3	*X*	✓	*X*	*X*	ID/ I-R-P-R-P-R-
AL1E4	✓	*X*	*X*	*X*	ID/ I-R-P-R-P-R-
AL1E5	✓	✓	*X*	*X*	ID/ I-R-P-R-P-R-
AL1E6	*X*	✓	*X*	*X*	ID/ I-R-P-R-P-R-
AL1E7	*X*	*X*	✓	*X*	IA/I-R-E
AL1E8	*X*	*X*	✓	*X*	IA/I-R-E
AL1E9	*X*	✓	✓	*X*	IA/I-R-E
AL1E10	*X*	✓	*X*	*X*	IA/I-R-E
AL1E11	✓	*X*	*X*	*X*	ID/I-R-E
AL1E12	*X*	*X*	✓	*X*	IA/I-R-E
AL2E13	*X*	*X*	✓	*X*	IA/I-R-E
AL2E14	*X*	*X*	✓	*X*	IA/I-R-E
AL3E15	*X*	*X*	✓	*X*	IA/I-R-E
AL3E16	*X*	*X*	✓	*X*	IA/I-R-E
AL3E17	*X*	✓	*X*	*X*	ID/ I-R-P-R-P-R-
AL3E18	*X*	*X*	✓	*X*	IA/I-R-E
AL3E19	*X*	*X*	✓	*X*	IA/I-R-E

AL1E5 and AL1E9 cut across two kinds of content and activities, and I will discuss them in detail below. I will now present the findings and discussion of the kinds of content and activities, by presenting the pre-planned, spontaneous, everyday and scientific content and activities that were evident in Teacher A's lessons.

4.2.2.1 Teacher A's Pre-planned/Scientific content and activities

As the word suggests, pre-planned refers to content and activities that are selected before the lesson (Zodik & Zaslavsky, 2008). Scientific content and activities are mathematical in nature, with mathematical conventions, symbols, diagrams, and procedures being used.

Teacher A had eleven pre-planned/scientific content and activities out of nineteen items in his example set. To unpack the pre-planned/scientific content and activities, I will present AL1E1 from Lesson A1. I selected AL1E1 because of the engagement that the teacher and learners had around this particular activity. AL1E1 is also an important content and activity in the entire example set due to its place in Teacher A's lessons. AL1E1 was the first content and activity of functions, that Teacher A's Grade 10 learners engaged with- their first encounter with functions as a topic.

At the start of the lesson, Teacher A wrote the statements below on the board.

What we must know on this lesson:

- What a function is
- What a domain and range is
- The function notation
- How to represent a function.

Teacher A then asked learners to define a function (AL1E1) before it was taught in the lesson. Learners were then expected to share their definitions in a whole class discussion. This activity of defining a function was discussed at length, and learners engaged actively in attempting it as shown in the excerpt below.

Table 19: Communicate approach and Pattern of discourse of Activity AL1E1.

Transcription	**Code**
30.Lnr5: A function is a relationship between two variables [learner reading from her notebook]. *31.Tr: A function is a relationship between two variables. Did you hear that↑?* *32.Lnrs: [Yes.* *33.Tr: What else?* *34.Lnr5: Any set of ordered pairs in which the first element is never repeated. That is the x is different from… from… for every y [learner reading what she wrote in her notebook].* *35.Tr: Okay…okay↑. Did you hear that↑?* *36.Lnrs: [Yes.* *37.Tr: Alright. Any other any other response which is different from that?...[points at one of the learners who raises up her hand]Yes?* *38.Lnr6: A function is a special relationship between input and output values, where every input value has got one output value. [Learner reading what she wrote in her book]* *39.Tr: Ok. Did you hear what she was reading?* *40.Lnrs: [Yes yeah no]* *41.Tr: Uhh. You did not hear?* *42.Lnrs: [Yes.* *43.Tr: Can you say it again louder ↑louder ↑* *44.Lnr6: A function is a special relationship between input and output values, where every input value has got only one output value [Learner reading what she wrote in her book]* *45.Tr: Hmm… Okay, we can have more than one input, but they must give us only one output. Did you hear that↑?*	Line 30: Learner responds to teacher's question *(Response)* Line 31: Teacher revoices and evaluates response *(Evaluation)* Line 33: Teacher asks for more on the definition *(Interactive-Dialogic, Evaluation)* Line 34: Learner responds to teacher's question *(Response)* Line 37: Teacher asks for different definition of function *(Interactive-Dialogic)* Lines 39 and 43: Teacher evaluates response *(Evaluation)* Line 45: Teacher revoices and changes learner's response to include many-to-one relationship *(Authoritative)*

In the excerpt in Table 19 above, learners' prior knowledge of the definition of a function was assessed. I coded this activity as assessing learners' prior knowledge (LPK). The teacher allowed learners enough time to engage with the key features of the object of learning such as input values, output values, and ordered pairs, among others. Teacher A engaged with learners' responses as shown in the excerpt above (see lines 30 - 48). Teacher A responded to all learners' responses, either by marking them as correct (MKI) (lines 31, 35, 37 and 48) or revoicing (RLE). In line 44, a learner gave a definition which Teacher A revoiced and changed the language that was used by the learner, to include the many-to-one relationship in line 45. Here, the teacher was evaluating learner's response authoritatively, which brings me to the communicative approach that was evident in the lesson. The excerpt above shows that learners interacted and responded to the teacher's question, and the teacher listened to their ideas, although he did not consider all learners' responses as shown in line 45. The teacher's responses to the learners show that the teacher had a pre-planned answer to the question he had posed to the learners, as shown in line 45, where he revoiced the learner's response and changed it to add many-to-one, without acknowledging that the learner's answer did not include 'many-to-one'. Although it seems like an interactive-dialogic approach in the excerpt above, the teacher's responses to the learners led to an Interactive-authoritative communicative approach (IA).

Teacher A gave learners an opportunity to interact and share ideas during the lessons, learners' ideas were not probed by the teacher (see Lines 38-45). Mortimer and Scott (2003) refer to this interaction as low interanimation of ideas. The probing could have given the teacher an indication of why the learner gave that response. Giving an authoritative response after a learner has given a response that is not sufficient may not be appropriate feedback to help the learner reflect on the response given. Teacher A would have used this instance to develop learners' discourse on the concept of a function, by explicitly engaging with what they already know (or do not know) hence developing their mathematics discourse. Brophy and Good (1986) argue that for effective learning, teachers need to encourage learners to give explanations, debate alternative approaches and justify their ideas before the teacher gives evaluative feedback in a mathematics classroom. In a mathematics lesson, both the teacher and the learner have a role of ensuring that learning takes place, as argued by Schifter (1998). Schifter calls this the learning that teacher and the learner engage in during the lesson.

Hufferd-Ackles, Fuson and Sherin (2004) concur with this argument. Learners' learning is on them sharing ideas and providing justifications for those ideas while the teacher's learning is on them listening to learners' responses and choosing the appropriate pedagogical strategies to provide learners with opportunities to consolidate their mathematics understanding (Sherin, 2002b). Simply allowing learners to interact may not lead to improved mathematics discourse and hence understanding (NCTM, 2000). The interaction should be used to help learners engage productively, for example, probing them to explain their mathematics reasoning. The teacher's responses to learners' mathematics explanation and ideas play a crucial role in determining the function of the interaction in the classroom (Li, Cao & Mok, 2016). Therefore, it can be argued that Teacher A's response to learners' incomplete definition was not an effective strategy to provide learners with opportunities to consolidate their mathematics discourse. The teacher could have provided a probing response to enable learners reflect on their definition and hence develop their mathematics discourse on defining the concept of a function.

The interaction on this activity was initiated by the teacher when he asked learners to define a function as shown in the excerpt above. Learners responded and the teacher evaluated learners' responses as shown in the table above. The pattern of discourse that was evident here is I-R-E (see Table 18). I will discuss the pattern of discourse in relation to the teacher's role in detail later. Teacher A asked the question, a learner responded, learner's response was evaluated by Teacher A (MKI, RLE). After the whole class discussion, Teacher A captured the main points from the discussion and consensus was sought as shown in the Figure below.

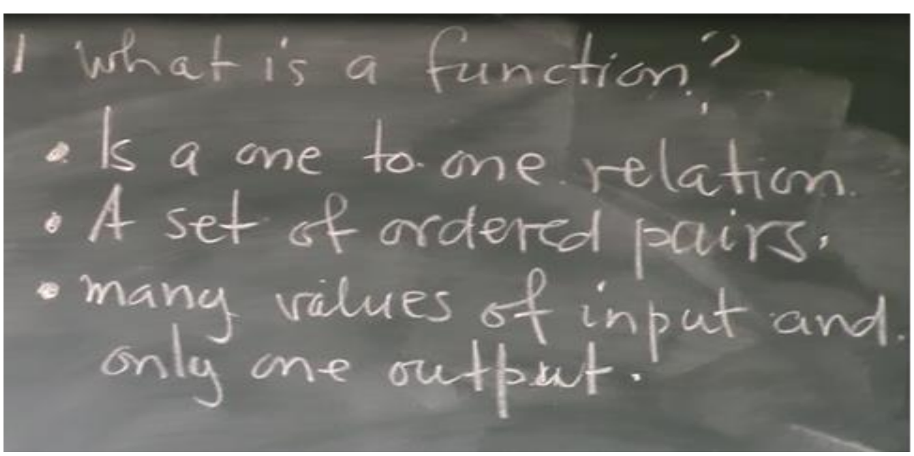


Figure 10: Definition of a function.

Figure 10 above shows the final answer that Teacher A gave to the questions he posed. From the activity in Lesson A1, Teacher A was able to capture the prior knowledge that learners displayed. As mentioned earlier, activity AL1E1 was an assessment of learners' prior knowledge, and therefore, learners' responses were crucial for the lesson as argued by Carr et al. (2009). Learners did not demonstrate prior knowledge on the many-to-one relationship in their responses. The ability to define terms is an important aspect in mathematics discourse (Vinner, 1983; Manouchehri, 2007), and hence one which requires the teacher's attention during the lesson. Teacher A simply changed the learners' response and wrote down one-to-one and many-to-one on the board, without probing learners or emphasising this gap in their prior knowledge. There was also no final comprehensive definition given in the lesson, and yet this would have been an appropriate opportunity for authoritative approach to be used to engage with learners' prior knowledge. Carr et al. (2009) argue that in the process of assessing learners' prior knowledge, if any issues are identified in the process, the role of the teacher is to address them in an explicit manner at the 'right time' in the lesson (Carr et al., 2009).

To summarise the findings on AL1E1, Teacher A presented it as a pre-planned activity to assess learners' prior knowledge. In the interaction, the teacher and learners used mathematics terminology to engage with AL1E1. I-R-E pattern of discourse was evident in the interaction. Mortimer and Scott (2003) argue that I-R-E pattern leads to interactive-authoritative communicative approach (IA) because the teacher initiates by asking a question, and if the learner gives a response which the teacher does not want, the teacher evaluates the learner's response immediately without probing the learner for further explanation. According to Cazden (1988), I-R-E pattern is the most common pattern in engagements that are initiated by the teacher. Cazden (1988) further argues that I-R-E is a "default" pattern and therefore it is the pattern that should be expected unless the teacher takes deliberate action to influence the pattern of discourse in the classroom. No deliberate action from Teacher A was seen to influence the pattern of discourse during the engagement on activity AL1E1. The findings from AL1E1 were not unique to this activity only, as they were seen in all the pre-planned/scientific content and activities in Teacher A's lessons. To substantiate this argument, I will present the findings that came out from analysing AL2E13.

In Lesson A2, AL2E13 was the pre-planned/scientific activity that Teacher A presented to the learners, as shown in the extract below.

Point-by-point plotting of *y* = *f*(*x*) = 2*x*

3. Tr: [writes on the board]

If you want to sketch this graph, what do you do? We want to see how we sketch. We want to sketch the graph of y is equal to f of x equal to 2x. Let's see what he does [Points at a learner to come to the board].

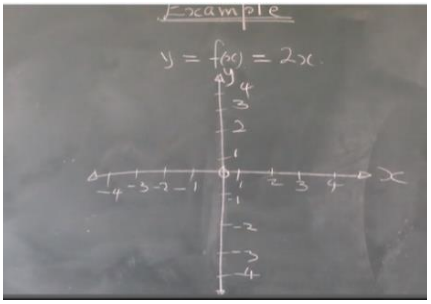
4. Lnr1: [Attempting the task of plotting a graph on the blackboard]

5. Tr: Now in your books please use your rulers. Where you not given rulers?

Extract 1

Teacher A selected one learner to come and plot the graph on the board (line 4), while the other learners were busy attempting the task in their books. Teacher A expected learners to attempt the point-by-point plotting before it was taught in the lesson (LPK). Teacher A's question required learners to demonstrate their mathematics discourse by explaining the steps for plotting the graph as shown in the extract above. Sketching and plotting in the mathematics language do not mean the same thing. However, in this lesson, Teacher A used the two words interchangeably. Walshaw and Anthony (2008) argue that the teacher's role is to rectify the mathematics language and conventions used in the lesson. The activity selected required learners to have prior knowledge on plotting a straight-line graph. Some of the critical features that learners need to engage with when plotting a straight-line graph are table of values, substitution, ordered pairs and the Cartesian plane among others. In this activity, learners' prior knowledge on many aspects was being assessed at the same time, such as being able to create a table of values and complete it in order to have a set of ordered pairs. The other aspect that was being assessed is learners' ability to plot a set of ordered pairs on a Cartesian plane. As observed in the lesson, a learner was called to the board and he drew the Cartesian plane on the board, but he did not continue with the activity, as shown in the extract below.

9. Lnr2: [Continues to sketch graph on the board]
10. Tr: [Walking around the class, checking on what learners are doing as one learner sketches the graph on the board]
11. Lnr2:[Pauses to see what he has sketched on the board so far]

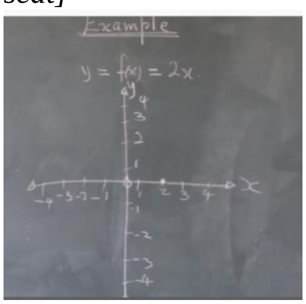


12. Tr: [Focuses attention on learner sketching graph on the board] So right, what we do now?
13. *Lnr2: [Makes a gesture of drawing a straight line and walks back to his seat]*

Extract 2

From Extract 2 above, the learner did not demonstrate an ability of being able to attempt a point-by-point plotting of a straight-line graph (lines 9, 11 and 13). However, the learner was aware that the Cartesian plane was needed to plot the graph, but he was not able to plot the graph. Another learner was given an opportunity to come to the board and attempt the activity as shown in the extract below.

20. Tr: Okay, so what do you need?
21. Lnr2: Sir
22. Tr: Yes? [Pointing at the learner]
23. Lnr2: [Walks to the board and plots a point on the Cartesian plane and walks back to his seat]

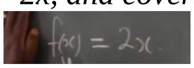
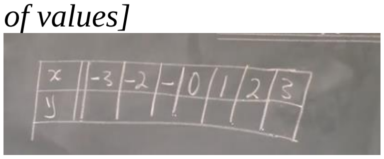


24. Tr: What is that?
25. Lnr2: Two zero
26. Tr: Alright? So if we up to here, it means you are stuck here, right?
27. Lnrs: Yes

Extract 3

The learner came to the board and plotted the point (2; 0) and walked back to her seat, as shown in the excerpt above. After learners struggled to attempt the activity as per Teacher A's expectations (see line 26), he intervened by splitting the activity into a step by step series of activities required for learners to plot a straight-line graph. The whole class engagement, which was enabled by the step-by-step series of activities brought to light the fact that learners had prior knowledge on ordered pairs and substitution and were able to complete a table of values. The excerpt below shows some questions that Teacher A asked learners.

Table 20: Probing learners on plotting a straight-line graph.

Transcript: Probing learners on plotting a straight-line graph	**Code**
28. Tr: [Walks to the board and points at the equation f(x) = 2x, and covers y with his hand]  *But then if we start here f of x is a function, right?* 29. *Lnrs: Yes↓* *30. Tr: Have you forgotten about ordered pairs?* *31. Lnrs: No* *32. Tr: Ordered pairs, do you still remember?* *33. Lnrs: Yes* *34. Tr: Table of values, do you still remember?* *35. Lnrs: Yes* *36. Tr: So where do you want to start with your x? [points at the 2x value in the equation]* *37. Lnrs: No response* *38. Tr: Suppose you want to have your table of values [draws table on the board] x there, let's say y here, and you want to start from negative three, negative two, negative one, zero, one, two, and lets end at three as well. Can you find out what will be your y here? [points below -3 on the table of values]*  *If your x is negative three, what is your y? Which is your f of x.*	Line 28: Probing Line 30, 32 and 34: Probing Line 36: Initiation Line 38: (Takes learners' silence as not knowing-evaluation)- authoritative

The excerpt above shows the interaction that transpired after Teacher A concluded that learners were stuck (see line 26 in extract 3 above). After that utterance (extract 3, line 26), the interaction continued with the teacher initiating using questions that were checking learners' prior knowledge. Teacher A reminded learners about ordered pairs (line 30), and he also reminded them about the table of values (line 34). Learners did not respond to the teacher's question in line 36, where he wanted to find out if they are able to determine the input. In line 38, Teacher A gave the learners the input values, without probing them to see if they were able to come up with them, which is an example of an interactive-authoritative approach. Although the learners were not able to answer the question of choosing the input values (Line 36 and Line 37), i.e. generating a table of values, given an equation, they demonstrated the ability of completing the table of values using substitution method, after the teacher suggested the input values for them. This highlighted the issue of selecting the suitable activity for learners to attempt in the time available during the lesson. Campbell and Campbell (2009) argue that for learning to occur, learners need to be given opportunities to link their prior knowledge to the topic and hence object of learning in the classroom. Teacher A's pre-planned activity in this instance took a lot of teaching time, as the teacher was trying to get learners to the level where they would be able to attempt the activity by linking it to their prior knowledge. The teacher's awareness and emphasis on assessing learners' prior knowledge is an important aspect in ensuring that learners' mathematics discourse is developed. Activity AL2E13 gives evidence that a pre-planned activity in the lesson may not work well if learners do not have the necessary prior knowledge to engage with it. In the interaction, activity AL2E13 led to IA and I-R-E as shown in Table 20 above.

Teacher A initiated the interaction (line 28) and asked learners some questions to guide them through attempting the activity, as shown in Table 20 above (see lines 30, 32, 34, 36 and 38), with an I-R-E pattern of discourse. He gave the correct answer to the question when learners did not respond as shown in line 38 above, which is IA approach.

Going back to the pattern of discourse and communicative approach evident in Teacher A's lessons discussed above, there was evidence of interanimation of ideas (Bakhtin, 1981) (see Table 20 above for example). However, interanimation of ideas in all the lessons was at a low level as learners gave different ideas and the teacher only revoiced learners' responses, without engaging with the ideas. Although Teacher A did not ignore learner's contributions, he gave the authoritative conclusion to any activity that was being discussed. Interanimation of ideas is important for developing learners' mathematics discourse as it allows the teacher to engage with learners' responses in a way that probes them to justify and reflect on their responses. Authoritative responses in a mathematics classroom may not provide enough opportunity for learners to reflect on their answers. The teacher needs to pose questions that allow learners to share their responses in a way that leads to exploration of the topic under construction (Wegerif & Mercer, 1997; Nassaji & Wells, 2000). Nassaji and Wells (2000) argue that in a classroom where there is a triadic dialogue, the teacher is likely to initiate the dialogue with a question that elicits learners' variety of perspectives. The most important aspect of this classroom setting is the follow-up that the teacher gives to the learners after they have responded to the initiation question, as argued by Nassaji and Wells (2000). If learners' responses are met with evaluative feedback from the teacher, learners tend to hold back on their contribution in the interaction. Nassaji and Wells therefore argue that the teacher should avoid evaluation and aim to probe learners for justifications or counterarguments to their responses, as this allows learners to 'self-select' (p.33), in the process of responding to the teacher's questions.

To summarise the findings on AL2E13, Teacher A presented it as a pre-planned activity to assess learners' prior knowledge on point-by-point plotting of straight-line graphs. In the interaction, the teacher and learners used mathematics symbols and conventions to engage with AL2E13. I-R-E pattern of discourse was evident in the interaction. IA was also evident in the interaction as discussed above - the teacher initiated the interaction by asking a question, and when learners gave no response, the teacher evaluated the learner's silence immediately without probing the learners to find out why they were not able to answer. No deliberate action from Teacher A was seen to influence the pattern of discourse during the engagement on activity AL2E13.

From the discussion above, it can be argued that Teacher A did not make any assumptions at the start of his lessons. He made a visible effort to ascertain learners' preconceptions and prior knowledge, using pre-planned content and activities. This was also evident in lesson A3 (see Table 17), where the teacher started the lesson by revising the work that was covered in the previous lesson, in an attempt to check learner's prior knowledge and understanding (MM-LU).
All pre-planned/scientific content and activities that Teacher A selected transpired in a similar manner as the two content and activities (AL1E1 and AL2E13) presented above. The content and activities that Teacher A pre-planned were familiar cases to learners, as they were able to relate

to them, although in some instances, learners needed further probing from the teacher. It was evident in Teacher A's pre-planned/scientific activities that the sequence of simple and familiar was considered in the process of choosing the content and activities, and this is in line with what Bills and Bills (2005) found in their study. Bills and Bills conducted a study with a focus on how teachers construct examples, and they found that the teachers construct content and activities, starting with simple and familiar ones, and then building to more complex activities as the lesson goes on. The activities selected in this category show the teacher's attempt to pre-determine learner preconceptions (see Table 21 below) and hence draw their attention to relevant features (Zodik & Zaslavsky, 2008) of the object of learning through the content and activities selected. This was evident in all the content and activities, especially in AL2E14, as shown below.

Table 21: Features of activity AL2E14.

Content/Activity	**Features**
AL2E14 $f(x) = -2x$ $g(x) = x + 1$ $h(x) = -x - 1$ $j(x) = -x + 1$	-A function notation can be represented as $f(x); g(x); h(x); j(x)$ And not just as the commonly used $f(x)$. -Coefficient of x (Variant) $f(x) = -2x$ $g(x) = 1x + 1$ $h(x) = -1x - 1$ $j(x) = -1x + 1$ -y-intercept (Invariant for $g(x)$ and $j(x)$) $f(x) = -2x + 0$ $g(x) = x + 1$ $h(x) = -x - 1$ $j(x) = -x + 1$

AL2E14, as an activity had a potential of bringing critical features of the object of learning to the fore. These critical features include representing function notation; how the co-efficient of x, which is also the gradient of the straight-line graph affects the graph, and the y-intercept of a straight-line graph. The features would have provided opportunity for the teacher and the learners to engage with the similarities and differences and hence make the necessary generalisations on plotting straight lines graphs, with respect to the co-efficient of x and the y-intercept. The opportunities for generalisation in activity AL2E14 would have been availed to learners by 1) completing one table of values for all four functions, 2) plotting the graphs on the same set of axes, 3) comparing the four graphs after plotting them. The opportunities that are availed by AL2E14 in respect to developing learners' mathematics discourse and understanding were evident in the lesson. As discussed in Chapter 2, Zaslavsky (2010) argues that it is the role of the teacher to guide learners through an activity in an attempt to unpack the activity's potential.

The findings on the pre-planned/scientific content and activities revealed the teacher's awareness of the role of assessing prior knowledge using content and activities that relate to what learners already know. Teacher A engaged learners in an interactive-authoritative approach in the process of enacting the pre-planned/scientific content and activities. Some content and activities were enacted in a way that enabled opportunities for learners to develop their mathematics discourse and understanding while other content and activities were enacted in a way that did not explore all available opportunities for developing learners' mathematics discourse and understanding. The next category I present is pre-planned/everyday.

4.2.2.2 Teacher A's Pre-planned/Everyday content and activities

Pre-planned/Everyday content and activities are selected before the lessons, and have context from everyday, in relation to the intended object of learning that the teacher wants to enact in the lesson. Teacher A did not have any pre-planned/everyday content and activities in the example set, as shown in Table 18 above. It is important to note here that Teacher A being an experienced teacher, it can be argued that perhaps all his everyday content and activities may have been pre-planned and that he may have used them over the years in his teaching. During the analysis, categorising the content and activities, especially for everyday content and activities was not a straightforward decision to make, as Zodik and Zaslavsky (2008) present it to be. My rationale for distinguishing the content and activities as either pre-planned (teacher referring to textbook or notes) or spontaneous (teacher selecting content and activity based on the interaction in the lesson) allowed me to categorise them as spontaneous because of how Teacher A[4] introduced them in the lessons observed. Therefore, drawing on my rationale for categorising, I did not 'see'

[4] During the post-observation interview, Teacher A alluded to the fact that he uses everyday examples all the time in his lesson – however, he did not point a specific everyday example that he uses all the time. I was therefore not able to make a conclusion that any of the everyday activities in his lessons were pre-planned.

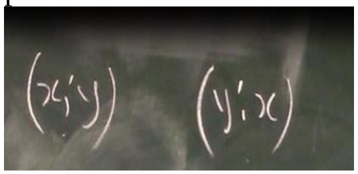
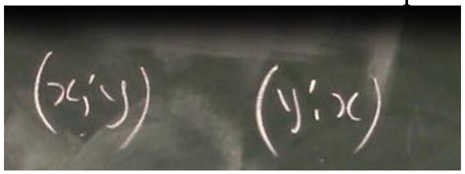
any pre-planned/everyday content and activity in Teacher A's example set. In what follows below, I discuss Teacher A's spontaneous/scientific content and activities.

4.2.2.3 Teacher A's Spontaneous/Scientific content and activities

As discussed in Chapter 2, spontaneous content and activities are those that are chosen 'in-the-moment, - during the lesson (Zodik & Zaslavsky, 2008). Teacher A had four spontaneous/scientific content and activities in the example set. In Table 22 below, I present AL1E4 and AL1E5 to show how Teacher A enacted these kinds of content and activities. I chose the two activities because of the role of the activities selected in relation to mathematics discourse, which was to draw learners' attention to the convention of representing an ordered pair. Convention within the mathematics register is important for developing mathematics discourse and understanding. Learners need to see and understand meanings in mathematics conventions in order to communicate and mean like mathematicians, as discussed by Pimm (1981).

Table 22: Enacting AL1E4 and AL1E5.

Content/Activity	Transcript	Comment
AL1E4 *Content*: How to represent a set of ordered pairs *Activity*: N/A (3; 6) and (3; 9) as sets of numeric ordered pairs	114.Tr: Very good. Now there is something very funny about what we are having here↑. A set of ordered pairs. Is there anyone who can write how to clearly represent a set of ordered pairs here? 115.Lnr: No. 116.Tr: A set of ordered pairs↑? 117.Lnrs: [Making noise] 118.Tr: Uhh…? Ok give me any value of *x* you think of. 119: Lnr: Three 120: Tr: Three. This one is saying three [writes 3 on the chalkboard] Why? 121: Lnr: It's a value of *x*. 122: Tr: Give me the value of… any value of *y* you think of. 123: Lnr: Six. 124: Lnr: Two. 125: Lnr: Five. 126: Lnr: Nine. 127: Lnrs: Yes. 128: Tr: Do you see that↑? [writes (3; 6) on the chalkboard] 129.Lnr: Yes. 130.Tr: Uuhh…? And this one is saying maybe I have got three and another one says what? 131.Lnr: Nine. 132.Tr: [Writes (3; 9) on the chalkboard] Says nine. Do you get that?	As set of ordered pairs came up in the interaction (Line 114) and Teacher A chose this activity. **Interaction** *Line 114: Initiation* *Line 115: Response* *Lines 116 and 118: Prompt* *Line 119: Response* *Line 120: Prompt* *Line 121: Response* *Line 122: Initiation* *Lines 123-127: Responses* *Line 128: Evaluation* *Line 130: Prompt* *Line 131: Response* *Line 132: Evaluation* Communicative approach: ID Pattern of discourse: I-R-P-R-P-R-E

AL1E5 *Content*: Conventional representation of Ordered pairs  	148.Tr: Can I write an ordered pair like…[writes (*y; x*) on the chalkboard]?  149.Lnrs: [*y, x*. 150.Lnrs: [Noooo↑↑. 151.Lnr: It's not allowed. 152.Tr: It's not allowed↑? 153.Lnrs: [Yes↑↑. 154.Tr: By who↑? 155.Lnrs: [Making noise] 156.Tr: It's not allowed↑? 157.Lnr: No 158.Lnr: By the maths language. 159.Tr: Maths language does not allow it↑? 160.Lnrs: [Yes↑.	In the discussion about ordered pairs, Teacher A checked whether learners knew the convention of writing an ordered pair. **Interaction** *Line 148: Initiation* *Line 149: Response* *Line 150: Prompt* *Line 151: Response* *Line 152: Prompt* *Line 153: Response* *Line 154: Prompt* *Line 156: Prompt* *Line 157 and 158: Response* *Line 159: Prompt* *Line 160: Response* Communicative approach: ID Pattern of discourse: I-R-P-R-P-R-

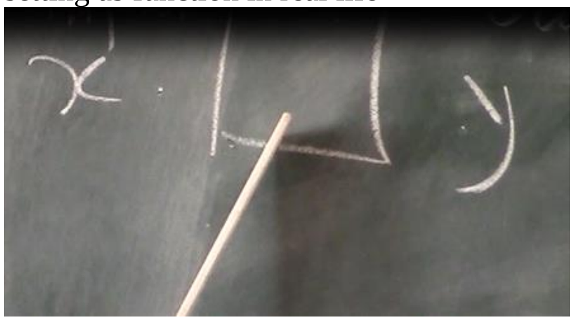
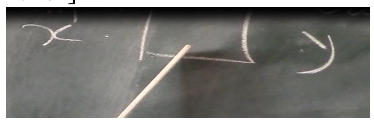
Teacher A selected spontaneous activities to draw learner's attention to relevant features AL1E4 and AL1E5 (and AL1E11 see Table 18 above) were selected to emphasise the convention of representing a set of ordered pairs. The three activities (all focusing on set of ordered pairs), allowed for generality on how a set of ordered pairs should be represented conventionally in mathematics. In AL1E5, Teacher A highlighted the general convention of representing a set of ordered pairs using variables and not numbers (see line 148). This was an illustration to learners that it does not matter what number you use, conventionally, the *x*-value comes first, and the *y*-value next. Within the mathematics discourse, conventions are important, and they need to be emphasised by the teacher when they arise in the lesson, especially when the pre-planned content and activities have limitations of making these conventions explicit to learners. AL1E11 (see Table 17) was used by Teacher A to further emphasise how ordered pairs are represented, and how a set of ordered pairs is then used to complete a table of values. These content and activities provided opportunity for learners to develop their mathematics discourse, with a focus on representing ordered pairs.

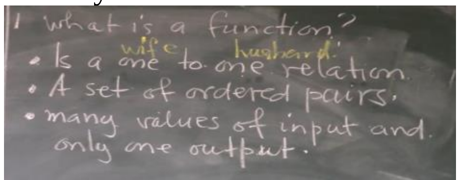
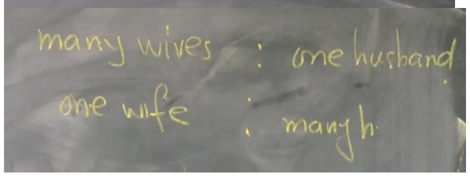
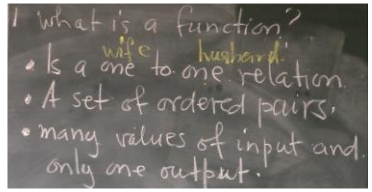
In the process of enacting the content and activities in this category, Teacher A interacted with the learners as shown in Table 22 above. The communicative approach was interactive-authoritative because learners were given an opportunity to give answers, and the teacher evaluated the learners' responses at the end of the discussion. He prompted learners to continue with the interaction - the teacher initiated (see lines 114 and 148), learners responded (see lines 115 and 119) to teacher's initiation and the teacher encouraged the learners to continue (see lines 122, 128 and 130), and then the interaction was sustained. The learner responses were one-word answers as seen in Table 22 above. The pattern of discourse was I-R-P-R-P-R-E, where Teacher A evaluated the learners' response(s) at the end of the interaction (see lines 159 and 128). Although the interaction was sustained, it was not aimed at probing learners for justification for their responses, hence opportunities for engaging learners in an attempt to develop their mathematics discourse and understanding through interactive-dialogic approach was not evident in the process of enacting this category of content and activities. In the next section, I present Teacher A'a spontaneous/everyday content and activities.

4.2.2.4 Teacher A’s Spontaneous/Everyday content and activities

Spontaneous/everyday content and activities are unplanned (Zodik & Zaslavsky, 2008) activities within the everyday context, which are selected by the teacher during the lesson. In the lessons observed, these kinds of content and activities mainly came about as a result of how learners responded to the content being discussed, and also learners’ lack of responses to questions posed. Teacher A had six spontaneous/everyday content and activities in his example set (see Table 18 above). To engage with how Teacher A enacted spontaneous/everyday content and activities, I discuss AL1E3 and AL1E6 from the example set of Teacher A, as presented in Table 22, with the transcription, to show the quality of interaction (in relation to developing learners’ mathematics discourse and understanding) that transpired during the lesson in relation to the content and activities. I chose AL1E3 and AL1E6 because of the everyday context (see discussion of the context below) that Teacher A selected to draw learners’ attention to the concept of a function.

Table 23: Enacting AL1E3 and AL1E6.

Content/Activity	Transcript	Comment
AL1E3 Content: Real life instance of a function: Factory setting as function in real life 	104.Tr: Factory here [points at the box between *x* and *y* on the chalkboard with a ruler]  Whatever is going to take place in the factory, we don't know. And then we have a product [shows a cold drink bottle to learners]. Can we see that? 105.Lnrs: [Yes. 106.Tr: Uuhh…? There was a raw material which was used here right? 107.Lnrs: [Yes sir. 108.Tr: And the factory plays a very important role there. [points on the box between *x* and *y* on the chalkboard with a ruler]. Then we have got the product the finished product.[He points at the y on the chalkboard with a ruler] Can you see that? 109.Lnrs: [Yes. 110.Tr: Now in real life situation, if I talk about a function, I must have raw material, must have a factory somewhere I must get my…? 111.Lnr: [Product. 112.Tr: Product. Is that clear? 113.Lnrs: [Yes.	-Teacher A linked the concept of a function to a factory setting: • Raw material is the input (Line 106 and 110) • The box is what the factory does to the raw material (Line 108 and 110) • Product is the output (Line 108 and 112) **Interaction** *Line 104: Initiation* *Line 105: Response* *Line 106: Prompt* *Line 107: Response* *Line 108: Prompt* *Line 109: Response* *Line 110: Initiation* *Line 111: Response* *Line 112: Evaluation and Prompt* *Line 113: Response* Communicative approach: ID Pattern of discourse: I-R-P-R-P-R-
AL1E6 Content: Illustrating the one-to-one and many-to-one relationship using an everyday scenario of wife and	180.Tr: We are not going to forget what a function is↑. You said one to one relationship right↑?	-Teacher used an everyday content of wife and husband to explain the one-to-one and many-to-one relationship

husband Activity: N/A  	181.Lnrs: Yes. 182.Tr: Good… I know you are not you are not going to like it↑↑. One↑ [writes wife and husband on the board as shown below]  183.Lnr1: Husband. 184.Lnr: One husband. 185.Tr: Do you see that↑? 186.Lnrs: [Laughing]Yes↑↑. 187.Tr: uuhuh… do you see↑? 188.Lnrs: Yes. 189.Tr: This one becomes a perfect function↑. We can also have [writes many wives on the chalkboard] many what↑? 190. Lnr2: Many ladies↑. 191. Lnrs: Many wives↑. 192. Tr: Many wives and↑? 193. Lnrs: [Yes↑. 194. Lnrs: [One husband↑. 195. Tr: One husband↑. 196.Lnrs: [Yes. 197. Tr: Uuhh…? You agree with this one now [points on the chalkboard]?  198. Lnrs: [Yes no] 199. Tr: We have this.. 200. Lnrs: [Many wives. 201. Tr: This one becomes a function. Can	**Interaction** *Line 180: Initiation* *Line 181: Response* *Line 182: Initiation* *Line 183: Response* *Line 184: Response* *Line 185: Prompt* *Line 186: Response* ……… *Line 193: Response* *Line 194: Response* *Line 195: Evaluation* *Line 196: Response* *Line 197: Prompt* *Line 198: Response* *Line 199: Initiation* *Line 200: Response* *Line 201: Prompt* *Line 202: Response* *Line 203: Initiation* *Line 204: Response* *Line 205: Prompt* *Line 206: Response* *Line 207: Response* *Line 208: Prompt* *Line 209: Response* *Line 210: Prompt* *Line 211: Response* *Line 212: Prompt* *Line 213: Response* *Line 214: Prompt* *Line 215: Response*

	you see that↑? 202. Lnrs: [Yes. 203. Tr: Are you going to forget this↑? 204. Lnrs: Nooooo↑↑ 205. Tr: Now will you say this one is a function? One wife [writes one wife on the chalkboard] 206. Lnrs: [Many husbands noooo↑↑ 207. Lnr4: It's impossible↑ 208. Tr: It's not a function, right↑? 209. Lnrs: [Yes. 210. Tr: You know what↓, here we are talking about legally married↓… You can't have a wife who is going to wed with me and wed with Mr Kgoso and wed with Mr Phontso. ↑Can you say I have seen this one with the wife of Mr so and so and so↑? 211. Lnrs: [Nooooo↑↑ [laughing] 212. Tr: Then they build their houses around one wife? 213. Lnrs: [Nooooo↑↑ [laughing] 214. Tr: It's not a function. Right? 215. Lnrs: [Yes.	Communicative approach: ID Pattern of discourse: I-R-P-R-P-R-

Table 23 above shows how Teacher A used everyday contexts to explain how mathematics concept(s) relate to our everyday lives in a factory setting, in a marriage, and I coded this as a double move (DM-TS). Teacher A used the double move as a strategy to ensure that learners discern the critical features of the object of learning (one-to-one and many-to-one relationship), and to show evidence of this, I engage with how AL1E3 and AL1E6 were enacted in the lesson observed.

The factory setting in relation to a function

AL1E3 was selected by Teacher A after the teacher and learners had finished defining what a function is, he went on to explain a function in relation to a factory setting (DM-TS) as shown in the table above. The factory setting example is what DeMarois and Tall (1999) refer to as colloquial in the facets of the concept of a function (see Chapter 2). Like all the everyday content and activities above, in Teacher A's lessons, activity AL1E3 was not pre-planned. Teacher A linked the concept of a function of an input and output to a product in a factory. In his explanation to the learners, he referred to the *x*-values as the raw materials and the *y*-values as the products in a factory situation. Ellis (2009) argues that learners struggle to perceive a functional relationship. This may be the reason why Teacher A drew on an activity such as this to demonstrate to learners a function relationship from everyday life. Learners have encountered factory setting in their everyday lives, and from the interaction in the lesson, they were aware of raw materials and products in relation to factories. This made the example seem like one which could help learners visualise an abstract concept of a function, by focusing their attention on the functional relationship between quantities (Ellis, 2009). Teacher A engaged with the activity and there was evidence of a double move as shown in the figure below.

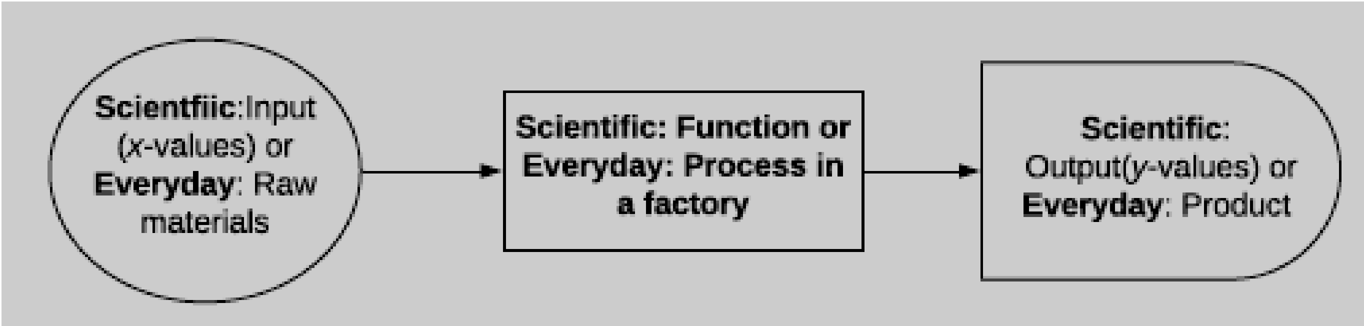


Figure 11: Functional relationship in a factory setting.

In activity AL1E3, the quantities that Teacher A drew learners' attention to are shown in Figure 11 above. The link between everyday and scientific was done explicitly by Teacher A and hence the double move, as shown in the figure above. During the enactment of this activity, Teacher A attempted a double move as shown in Figure 11 above. In the activity, Teacher A did not engage with the one-to-one and many-to-one relationships. This may have brought to light that in some factory settings, a one-to-many relationship is possible. During my analysis, I engaged with some scenarios in a factory setting where activity AL1E3 may not be the case, as discussed below.

There are instances in everyday life where one input (raw materials) can produce several outputs (products), for example crude oil after fractional distillation can become paraffin, petrol, gas, and so on. On the other hand, some distillations produce only one product (which would be a function). The fact that there is a counter-example to this scenario makes this activity inappropriate to be considered as an example of a functional relationship. A one-to-many relationship is not a function. The activity however could have been presented as a counter-example of what a function is not, to make it exemplary as discussed by Zazkis and Chernoff (2008). Content and activity appropriateness that is in line with the intended object of learning is one which cannot be stressed enough. According to Carlson and Oehrtman (2005), learners need to be exposed to multiple views of the concept of a function in order for them to understand it. However, all views that are exposed to the learners need to be accurate in relation to the concept of a function. The context also needs to be relatable to learners' real-life experiences. For this content and activity, Teacher A presented to learners an activity that had potential for them to 'visualise' the nature of a functional relationship, however, the content and activity was not accurate and hence had limitations to what was availed to learners. This brings me to the next activity that I will discuss and engage with whether it was exemplary in the lesson.

Functional relationship - legal marriage

In AL1E6, Teacher A used an example of a wife and husband to explain the concept of a function.

A wife represents *x*-value and the husband represents *y*-value, as shown in what Teacher A wrote on the board.

Figure 12: Legal marriage representation in relation to a function.

AL1E6 can be interpreted further as follows:

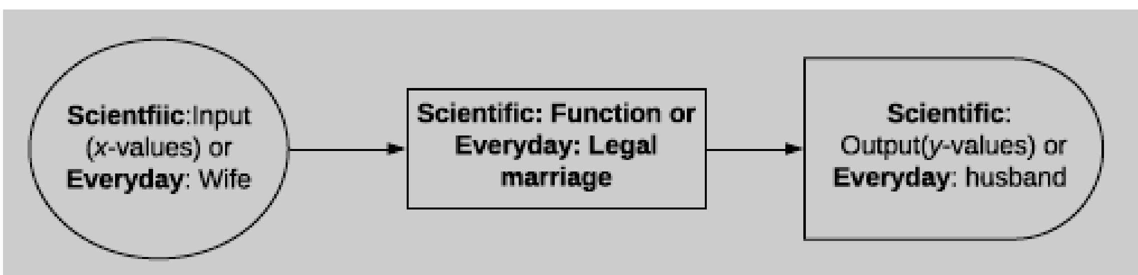


Figure 13: Legal marriage as a functional relationship.

From Figure 13 above, the wife or wives (for many-to-one) is the input, the legal marriage is the process and the husband is the output. Some learners agreed with the example while other disagreed (see excerpt in Table 22, line 198). The teacher continued to explain the concept of a function using the example of wife and husband, with a lot of excitement from learners (see excerpt in Table 22, lines 199-215). The example went on for a few minutes in the lesson. This example qualifies as an attempt to a double move (DM-TS) since the teacher linked the real-life scenario back to the mathematics lesson. This example was enacted by Teacher A in a way that made it exemplary in the lesson.

The teacher attempted to contextualise the example, in an attempt to eliminate scenarios which would render the example incorrect. He limited it to legal marriages only (see excerpt in Table 22, line 210) so that learners could understand where he was coming from, although it led to extreme excitement from the learners. Teacher A was aware of a scenario from real life that would render his example invalid and so he attempted to put limitations to the example. I interviewed Teacher A after the lesson observation, and an excerpt from the interview is presented below.

Interviewer*: How do you decide on the everyday contexts in your lessons? I am particularly interested in understanding your rationale for the wife-husband example when you defined as a function.*

Teacher*: I normally use such examples to help learners get excitement about mathematics since from my experience, some of them see it as a boring subject. I allow them to share their views about the examples so that they feel included in the examples and the contexts that I use to explain the mathematics.*

Interviewer*: Don't you think some of the examples end up not being appropriate especially in our diverse community settings and beliefs, and may lead to misconceptions?*

Teacher*: I think the debate that is created by everyday examples helps my learners not to forget the mathematics concepts we are discussing in the lesson. Yes, some of them do not like to hear some of the examples like the one of wife and husband but I explain the setting of the example. I have not had learners that got misconceptions in the examples I give for the years I have been teaching.*

In the excerpt above, Teacher A emphasised the role of everyday contexts in mathematics. From the examples on everyday contexts, Teacher A used everyday contexts as a strategy to show the link between everyday scenarios and mathematics concepts, hence the double move. While Hedegaard and Chaiklin (2005) support the use of a double move, where mathematics concepts are explained using everyday context and vice versa, Moschkovich (2003) argues that relating everyday meanings of concepts to the more restricted meaning of concepts within the mathematics register may lead to misunderstandings in classroom discussions. The example of the factory that the teacher selected may be interpreted differently based on what the factory manufactures. Therefore, for such an example, the learner who understands different types of factories, especially the ones which cannot be related to the concept of a function, may end up confused with the example selected and taught during the lesson. Cultural aspects from everyday situations may distract learners from discerning the concept that the teacher is trying to unpack during a double move attempt. In the example of wife and husband, many learners have encountered different 'legal' marriage scenarios where the description that was given by the teacher is not the norm. Therefore, this could lead to confusion for learners. During the post interview, Teacher A said that he did not see how his examples would lead to any confusion to the learners. He said that he had been using the examples for many years, and his learners have never displayed any misconceptions as a result of the examples given. AL1E10 is another example where the teacher's intention of explaining to learners the relationship between input and output did not come out as intended. Teacher A was attempting to explain to learners how to get *y*-values when given a rule, and he related this to a jar of sweets, which was not an explicit link that learners could relate to.

In example AL3E17, Teacher A was explaining the concept of 'at least' in relation to domain and range intervals. Learners related to the example, and they were excited because they were focusing on the value of money that was being discussed in the lesson (see lines 130 -134). Teacher A did not complete the double move as he did not link the activity on the taxi fare to the concept of domain and range. There was no visible connection created between AL3E16 (Mathematics example) and AL3E17 (Everyday example).

In the discussion, I have engaged with the kinds of content and activities that Teacher A selected to enact the intended object of learning. Pre-planned/scientific content and activities were predominant in the lessons observed while pre-planned everyday activities were not selected by Teacher A in all the lessons observed. Teacher A allowed learners to interact in the lessons observed and he responded to learners' responses evaluatively, hence the I-R-E pattern of discourse that was predominant in the lessons observed. Teacher A's spontaneous everyday content and activities were engaging as discussed above, and learners interacted, and the teacher probed them for further engagement hence the I-R-P-R-P- pattern of discourse. In what follows, I engage with the strategies that were evident in Teacher A's lessons observed.

Although Teacher A interacted with learners and allowed them to respond to questions during the lessons, the focus was mainly on vocabulary, definitions, and conventions, and this is at the lexical level. Teacher A did not create opportunities for comprehensive mathematical discussions during the lessons. There were very few instances of learners providing explanations and justifications to their responses.

4.3 Strategies in Teacher A’s lessons

In the presentation of findings and discussion of the role of Teacher A above, I engaged with the strategies that were evident in the lessons observed. In this section, I will further unpack these strategies that were evident in Teacher A’s lessons.

4.3.1 Stating the intended object of learning

Teacher A stated the intended object of learning, although not consistently as discussed earlier. In one of the lessons observed, he stated the intended object of learning and throughout the lesson, he kept referring to it as he engaged with the content and activities selected. In lessons A2 and A3, Teacher A did not state the intended object of learning. In these two lessons, Teacher A and the learners discussed many aspects during the lesson, and it was not an easy task to infer what the intended object of learning of the two lessons. In Lesson A2, the teacher and learners spent a lot of time completing the table of values, and less time on plotting the graphs. The effect of the parameters did not come into focus in the lesson, as discussed earlier. Stating the intended object of learning could have helped the teacher and learners to focus on the essential aspects and ignore the non-essential aspects in the lesson, and hence increasing learners’ motivation to learn (Brophy, 2004). For instance, where the intended object of learning was stated and kept in focus, it informed the content and activities that were selected to assess learners’ prior knowledge, as discussed below.

4.3.2 Assessing learners’ prior knowledge

In Teacher A’s lessons, content and activities were selected to assess learners’ prior knowledge as discussed above. As discussed earlier, assessing learners’ prior knowledge was identified as one of the strategies, and hence role of the teacher to ensure that learners discerned the critical features of the object of learning. Content and activities that Teacher A used to assess learners are shown in the table below.

Table 24: Content and activities selected to assess learners' prior knowledge.

Content/activity	Prior knowledge assessed by activity	Comment
AL1E1 *Content:* Definition of a function *Activity:* Learners asked to define a function	-Definition of a function: - Input value (x-value) - Output value (y-value) -one-to-one relationship -many-to-one relationship	In order for learners to be able to define a function, they had to have prior knowledge on terminology required to communicate the definition of a function
AL1E7 Content: Defining domain and range *Activiy*: Asked learners to define domain and range	-Definition of domain -Definition of range	Teacher A required learners to define domain and range before he taught or explained it to them in the lesson.
AL2E13 *Content*: Plotting a straight-line graph *Activity*: Completing a table of values and plotting the straight-line graph with the equation below: $y = f(x) = 2x$	-Interpreting the equation of a straight-line graph -Determining the x-values -Substituting in an equation to find y-values -Drawing and completing a table of values -Drawing and labeling a Cartesian plane -representing a set of ordered pairs -Point-by-point plotting on the Cartesian plane -Joining points to get a straight-line graph	Teacher A expected learners to plot the given straight-line graph, by going through all the necessary steps of point-by-point plotting when given an equation.

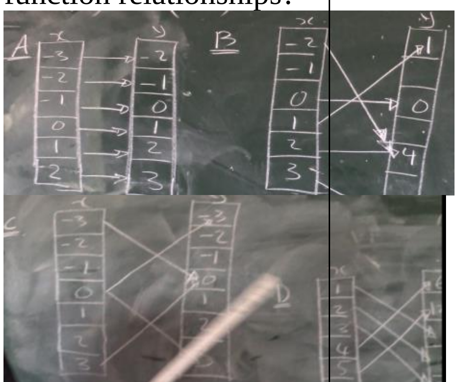
Table 24 shows content and activities that Teacher A used to assess learners' prior knowledge. These content and activities are assessing prior knowledge because, as discussed earlier, Teacher A expected learners to engage with the activities before they were taught in class. This strategy of assessing learners' prior knowledge is seen in all Teacher A's lessons. In lesson A1, Teacher A assessed learners' prior knowledge on whether they were able to define a function, as discussed earlier in this Chapter. Assessing learners' prior knowledge enabled Teacher A to start his lesson in such a way that allowed him to engage with what learners already knew. Some of the content and activities selected to assess learners' prior knowledge were enacted in such a way that they were appropriate for learners as the learners were able to engage and show their prior knowledge. Campbell and Campbell (2009) discuss the issue of the prior knowledge that learners come with to the classroom and how it may influence how they engage with the new content that is being taught. The experiences may aid learning by providing a foundation for learners to relate to the new learning. The new learning may also be thwarted if there is a conflict with what learners are being taught and what they already know, which maybe a misconception or misinformation. Carr et al. (2009) emphasise the role of assessing learners' prior knowledge, and how it enables teachers to teach better because there is an opportunity of identifying learner preconceptions and misconceptions through questioning. This was seen in Teacher A's lessons through learner responses to his questions. Given the literature presented in Chapter 2, the finding on assessing prior knowledge is not unexpected. The interesting finding is how the teacher dealt with this through the kinds of questions that the teacher asked to assess learners' prior knowledge in the lessons observed.

4.3.3 Patterns of variation as a strategy

In Teacher A's example set, there was evidence of content and activities that had patterns of variation, as shown in Table 25 below.

Table 25: Separation and Fusion content and activities in Teacher A's example set.

Content/activity	Aspects of variation	Comment
AL1E4 *Content*: How to represent a set of ordered pairs *Activity*: Choose any numbers to make a set of ordered pairs (3; 6) and (3; 9) as sets of numeric ordered pairs	-*x*-value is the same: 3 -*y*-value is different: 6 and 9	Code: SS-TS(Separation) • Teacher A used a set of similar examples to focus learners' attention on how to represent an ordered pair, in an attempt to bring about some form of generalisation.
AL1E8 *Content*: Function	-*y* is a constant $f(x) = y$	Code: SS-TS (Separation) • Teacher A used different symbolic

notation: $f(x) = y$ $h(x) = y$; $p(x) = y$; $k(x) = y$ *Activity:* N/A	$h(x) = y$ $p(x) = y$ $k(x) = y$ Letters *f*, *h*, *p* and *k* are used – but all have the same meaning.	representation to focus learners' attention on how to write function notation, in an attempt to bring about some form of generalisation around how function notation should look like.
AL1E12 *Content*: Representing a function using a mapping diagram *Activity*: Are the mapping diagrams below representing function relationships? 	Four different mapping diagrams provided to learners to determine which mapping diagram represents a function	Code: SS-TS (Separation) Teacher A gave four mapping diagrams, with controlled variation (x- and y-values) The x-values in A, B and C are invariant while the y-values are all varying.
AL2E14 *Content*: Point by point plotting of straight-line graphs *Activity*: Completing a table of values and then plotting the straight-line graphs with equations below: $f(x) = -2x$ $g(x) = x + 1$ $h(x) = -x - 1$ $j(x) = -x + 1$	All equations are for straight-line graphs The equations below have some various features of the object of learning: - Co-efficient of x is 1 and -1 for the two equations - y-intercept is 1 and -1 for $g(x) = x + 1$ $h(x) = -x - 1$ $j(x) = -x + 1$	Code: FF-TS (Fusion) Teacher A gave learners four equations of straight-line graphs to work with. The equations given do not have controlled variation but rather synchronic simultaneity where features of the object of learning are varying simultaneously. Coefficient of x and hence gradient: -2; 1; -1 y- intercept: 0; 1; -1 Code: SS-TS (Separation) g(x) and j(x) have the same y-intercept - invariant, and different coefficient of x - varying h(x) and j(x) have the same coefficient of x – invariant.

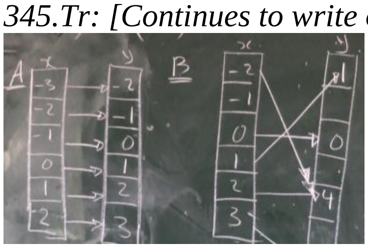
In Table 25 above, I have presented content and activities from Teacher A’s lessons observed which have separation and fusion aspects.

Activity AL1E4 shows that (3; 6) and (3; 9) were given as sets of ordered pairs, and the two examples of ordered pairs were written on the board for reference. For generality purposes, Teacher A could have varied the *x*-value in another example, or in the second set of order pairs, so that learners could see that the *x*-value does not have to always be 3.

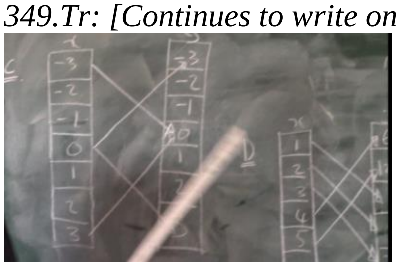
In AL1E12, learners were given four different mapping diagrams. Learners were asked to engage with the four mapping diagrams and check whether each mapping diagram is a representation of a function, using the definition of a function that was discussed in the lesson.

Which mapping diagram represents a function?

341.Tr: You need to tell which one is a function, which one represent a function. [writes on the chalkboard]
342.Lnrs: [Making noise]
343.Tr: Right. [cleans the chalkboard]
344.Lnrs: [Making noise]
345.Tr: [Continues to write on the board]



346. Lnrs: [Making noise]
347.Tr: Last one, don’t worry, don’t get lost↑
348.Lnr: Yes.
349.Tr: [Continues to write on the board]



The activity above provided opportunity for generalisation in such a way that it would allow learners to come up with a rule of what qualifies a mapping diagram to be a representation of a function. The set of *x* values were invariant in the three mapping diagrams and different *y*-values were given. Therefore, the rule for each mapping diagram was varying. In the next lesson, Teacher A did not continue with the activity. He moved on to another task without engaging with this. I therefore did not have access to learners' responses to the activity.

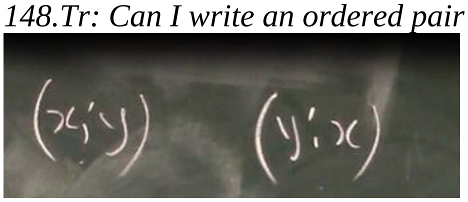
As discussed earlier in this Chapter, AL2E14 focused on: representing function notation; negative coefficient and how it relates to the gradient of the straight-line graph, and hence the link between gradient and the coefficient of the *x* in the equation of a straight line; *y*-intercept (see Table 25). This particular example was coded as both separation and fusion. If the entire example set is considered, it is a fusion activity as shown in the table above, however, if groups are made within the example set, separation is visible. Equations g(*x*) and *j*(*x*) have the same y-intercept (invariant) and different coefficient of *x* (varying) Equations *h*(*x*) and *j*(*x*) have the same coefficient of *x* (invariant) and different *y*-intercepts (varying).

Contrast content and activities

In Teacher A's example set, some content and activities were selected to enable learners see what something is or what it is not, hence contrast. In, AL1E5, Teacher A was attempting to contrast through the activity selected. Learners were being shown that an ordered pair is written as (*x; y*). The learners were also shown that an ordered pair cannot be represented as (*y; x*), as shown in the excerpt below.

Ordered pair

148.Tr: Can I write an ordered pair like…[writes (y; x) on the chalkboard]?



149.Lnrs: [y, x.
150.Lnrs: [Noooo↑↑.
151.Lnr: It's not allowed.
152.Tr: It's not allowed↑?
153.Lnrs: [Yes↑↑.
154.Tr: By who↑?
155.Lnrs: [Making noise]
156.Tr: It's not allowed↑?

157.Lnr: No
158.Lnr: By the maths language.
159.Tr: Maths language does not allow it↑?
160.Lnrs: [Yes↑.

In the excerpt above, Teacher A and the learners engaged in an interactive way, intended to make it explicit that the order of the *x* and *y* matters when representing a set of ordered pairs. Marton et al. (2004) argues that in the process of learning, we are more likely to experience what something is if we are presented with what it is not, so that we are able to make a comparison. From the excerpt above, the correct and incorrect order of the set was written on the board, and this provided learners with an opportunity to compare and hence experience the order for writing a set of ordered pairs. Although it can be argued that the above activity is not a strong example of a contrast activity, Teacher A's intention was to present learners with how an ordered pair is represented in the presence of how it is not represented. Therefore, for my study, I qualified this activity as a contrast activity.

Teacher A went on to give more explanation by drawing on an everyday scenario where the order of presentation matters. In the excerpt below, Teacher explained that when addressing people, we start with ladies (*x*) and then gentlemen (*y*). We cannot start with gentlemen and then ladies, as shown in the excerpt below. This is an example of what something is and what it is not, hence the emphasis of ordered pairs written as (*x; y*) and not (*y; x*). I coded AL1E5 as CC-TS.

How to represent an ordered pair

168.Tr: So↑, so the English, this English that says that you honour ladies is very important↑ right↑?
169.Lnrs: Yes↑.
170.Tr: Ladies and gentlemen. So in this case it looks like our x represent…?
171.Lnrs: [Ladies.
172.Tr: And this one [points at y as he on the chalkboard]?
173.Lnrs: [Gentlemen.
174.Tr: Always our ordered pairs must be x and…?
175.Lnrs: [y.
176.Tr: x and y.

From the excerpt above, Teacher A and the learners are seen to be engaging around the convention of writing an ordered pair, by focusing on what it is not. The example set for Teacher A did not have ample activities that would qualify as contrast activities. Teacher A engaged with concepts that would have been better presented and emphasised using contrast. For example, when Teacher A was discussing the equation of a straight line, it would have been a good opportunity for learners to see what an equation of a straight line is and what it is not. Another concept that would have been an opportunity for contrast is how gradient is calculated, by showing learners how it is not calculated.

In example, AL3E15:

$x \in [-4; 1]$
$-4 \leq x \leq 1$

Range
$y \in [-1; 4]$

It would have been worthwhile to provide opportunity for generality by engaging with the symbols and how the meaning would change if they are changed, for example how and why $-4 \leq x \leq 1$ is not $-4 \geq x \leq 1$. Teacher A however engaged with the meaning of the symbols by presenting AL3E17, as discussed earlier.

So far, I have presented separation and contrast in relation to the patterns of variation and how they provided for generality in the lessons observed. Fusion as a pattern of variation also leads to further generality (Marton et al., 2004). Other than example AL3E14, throughout the Teacher A's example set, there was no visible movement towards generality through fusion and therefore it became challenging for me to identify how fusion was used to bring about generality. The example AL3E14 had opportunity for fusion as shown in the table above, however, the enactment was not towards generality of the effect of the parameters in relation to how they were varying.

4.3.4 Double move as a strategy

Double move as a strategy was used by Teacher A, and this is discussed in detail in the section on spontaneous/everyday content and activities. Teacher A drew on everyday context as discussed earlier. In the post-observation interview, Teacher A confirmed this as a strategy he consciously draws on in his teaching as discussed earlier. Although this was a strategy that was evident in Teacher A's lessons, it was not always successfully done as shown in the factory setting content and activity discussed earlier. The double move as a strategy should aim to help learners to get to understand mathematics concepts in relation to the everyday context that is used, as discussed by Hedegaard and Chaiklin (2005). A double move that is not complete or where context does not fit into an accurate back and forth explanation, does not fit the purpose of developing learners' mathematics discourse and understanding.

4.3.5 Learner feedback

In order for feedback to be given, there has to be an interaction between the teachers and learners. Teacher A engaged learners in an I-R-E pattern of discourse as discussed in this Chapter. Teacher A did not use learner feedback to probe learners - to find out why learners responded the way they did. Feedback as a strategy in his lessons was used to give an authoritative response to learners, and not for him to understand what learners were communicating in the interaction.

4.4 Conclusion

In this chapter, I have presented the findings of Teacher A. Teacher A selected pre-planned/scientific, spontaneous/scientific and spontaneous/everyday content and activities in his lessons. Although Teacher A's lessons were not driven by variation theory, drawing on variation theory to analyse the lessons (see Chapter 3), I was able to conclude that most of Teacher A's example set provided opportunity for critical features of the object of learning to be brought to the fore, and hence provided opportunities for what was possible to learn in the lessons observed. This was done through selecting content and activities that was in line with the intended object of learning. However, a few of the content and activities were enacted in a way that did not bring the critical features of the object of learning to the fore. In Teacher A's lessons, the predominant communicative approach was Interactive Authoritative, with some instances of Interactive Dialogic, and this was evident when the teacher and learners were engaging with spontaneous/everyday content and activities. The teacher allowed learners to interact in all the lessons observed, and he gave authoritative feedback to learners' responses, and this did not enable learners to justify and reflect on their answers. Teacher A engaged learners during the lessons observed, and he ensured that learners were given an opportunity to attempt activities after concepts were covered. However, the engagement was at a low level, where learners gave chorus answers and one-word answers, and Teacher A did not probe them for further explanation.

I-R-E was evident in Teacher A's lessons, with the teacher initiating, learners responding, and the teacher evaluating learner responses without probing learners for further explanation of their responses. In all his lessons, Teacher A attempted to assess learners' prior knowledge and preconceptions before starting the lesson. Figure 15 below is a summary of the role of Teacher A in developing learners' mathematics discourse and understanding.

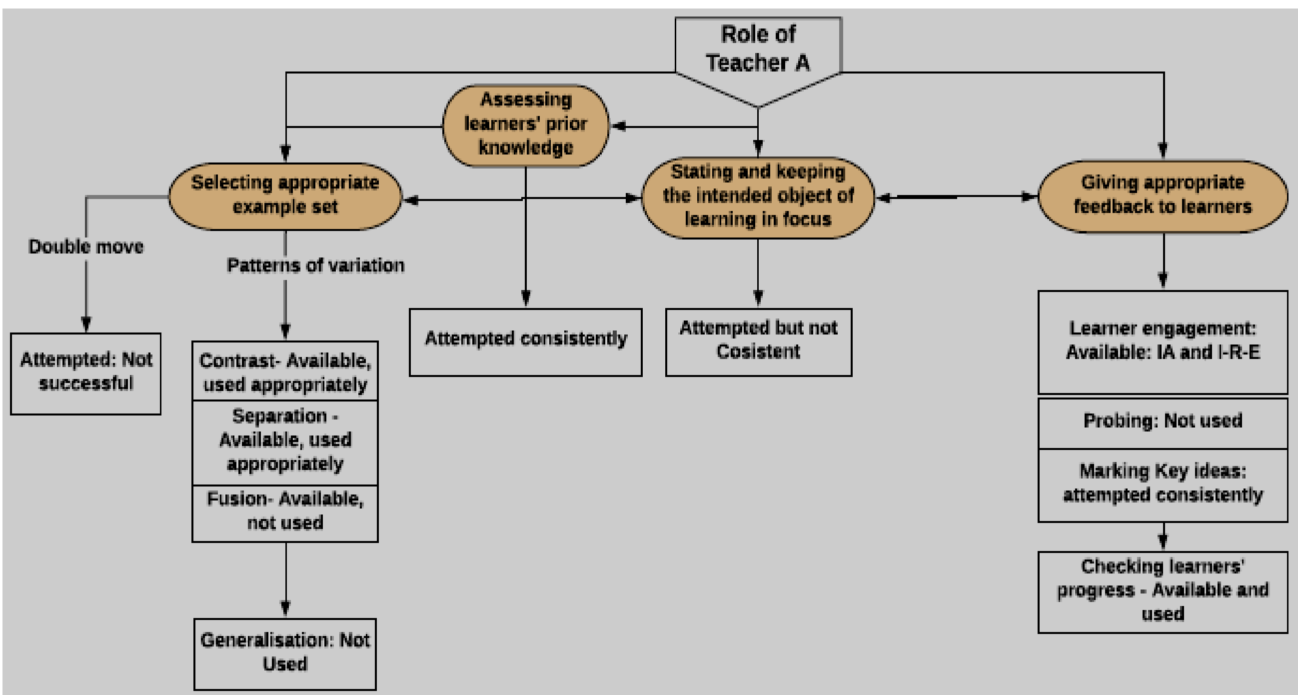


Figure 14: The role of Teacher A in developing learners' mathematics discourse and understanding.

Figure 14 above is a summary of the role of Teacher A in developing learners' mathematics discourse and understanding, as discussed in this Chapter. Stating the intended object of learning and keeping it in focus throughout the lesson is critical role for the teacher, and this ensures that the focus is kept on the important aspects of the object of learning. The focus of the lesson informs the other roles of the teacher, and the teacher assesses learners' prior knowledge based on the intended object of learning. This Chapter is a presentation of Teacher A's role in developing learners' mathematics discourse and understanding. In the next Chapter, I will present findings and discussion of Teacher B's role in developing learners' mathematics discourse and understanding.

CHAPTER 5: The role of the teacher: The case of Teacher B

5.1 Introduction

In this Chapter, I present the findings of Teacher B on the role of the teacher in developing learners' mathematics discourse and understanding. As with Teacher A, in order to investigate the role of teacher B in my study, I analysed pre-observation teacher interviews, lesson observations and post-observation teacher interviews, in a comprehensive manner. I observed Teacher B teaching three lessons which I referred to as: B1, B2 and B3 respectively. Each of the lessons observed lasted for approximately 45 minutes. Lesson B1 was on defining a function, Lesson B2 was on the effect of parameters a and q on a parabola, and Lesson B3 was on point-by-point plotting of a parabola.

As discussed in Chapter 4, I will engage with my research questions in the presentation and discussion of the findings. I will begin by presenting the kinds of content and activities that Teacher B selected, and as I present the content and activities, I will present the findings and discussion of how the communicative approach and patterns of discourse shaped the enactment of the content and activities selected by Teacher B. The strategies and hence the role of the teacher in ensuring that learners discern the critical features of the object of learning will be presented concurrently.

5.2 Kinds of content and activities selected by Teacher B

As mentioned above, before presenting the kinds of content and activities, I will begin by presenting the intended object of learning. Teacher B did not announce the lesson objective verbally or in written form before the lessons. Therefore, the intended object of learning of each lesson was inferred from the enacted object of learning. I then compared the inferred intended object of learning to the Curriculum and Assessment Policy Statement (CAPS), for validation purposes, as shown in Tables 26 -28.

5.2.1 Intended object of learning

The intended objects of learning that were covered in lessons B1, B2 and B3 respectively are presented below in Tables 26 – 28.

Table 26: Lesson B1's intended object of learning.

Intended object of learning inferred from the content of Lesson B1	**Linking the intended object of learning from lesson B1 content to CAPS**
➢ Define a function -Substituting the input values to get the output values -Describing the relationship between the input and output: the increment in the domain; the minimum value of the domain; the maximum value of the domain; the minimum value of the range; the maximum value of the range -Representing a set of ordered pairs *(x; y)*. -Define domain and range: Range is the set of all output values of the function; Domain is the set of all input values of the function -function notation: The symbol *f(x)* or *h(x)* or *g(x)* describes the elements of the range of a function for each element of the domain. ➢ Represent a function: Listing a set of ordered pairs; Table of values; Equations	**Curriculum statement**: [Define] "The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value). Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations" (DBE, 2011, p. 24). **Clarification comment from CAPS**: "A more formal definition of a function follows in Grade 12. At this level it is enough to investigate the way (unique) output values depend on how input values vary. The terms independent (input) and dependent (output) variables might be useful" (DBE, 2011, p. 24).

From Table 26 above, Teacher B's focus of the lesson, which was to define a function, was in line with the curriculum statement. In the Lesson B1, Teacher B engaged with various terms in an attempt to define a functional relationship. Different representations of functions were only listed in the lesson observed.

The intended object of learning in Lesson B2 is presented in Table 27 below.

Table 27: Lesson B2's intended object of learning.

Intended object of learning inferred from the content of Lesson B2	**Linking the intended object of learning from Lesson B2 content to CAPS**
➢ Investigate the effect of *a* and *q* on the parabola ➢ Determine the equation of a graph using the features of a parabola ➢ Interpret parabolic graphs	**Curriculum statement:** - "Investigate the effect of *a* and *q* on the graphs defined by $y = a.f(x) + q$, where $f(x) = x^2 + q$" (DBE, 2011, p. 24). **Clarification comment from CAPS**: "After summaries have been compiled about basic features of prescribed graphs and the effects of parameters *a* and *q* have been investigated: a: *a* vertical stretch (and/or a reflection about the *x*-axis) and *q* a vertical shift" (DBE, 2011, p. 24).

Teacher B conducted Lesson B2 as shown in Table 27 above. Learners were required to engage with the question and answer the questions that were given in the textbook (see Table 29 below). The link of the object of learning to the CAPS document is given in the table above. The effect of *a* and *q* on the parabola was the main focus of the lesson as per the CAPS. The intended object of learning in Lesson B3 is presented in Table 28 below.

Table 28: Lesson B3's intended object of learning.

Intended object of learning inferred from the content of Lesson B3	**Linking the intended object of learning from lesson B3 content to CAPS**
➢ Point by point plotting of a parabola to discover the domain (input values) and range (output values)	**Curriculum statement:** "Point by point plotting of basic graphs defined by $y = x^2$" (DBE, 2011, p. 24).

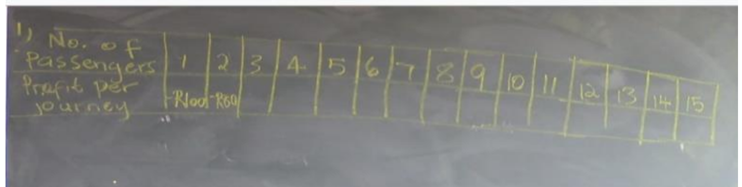
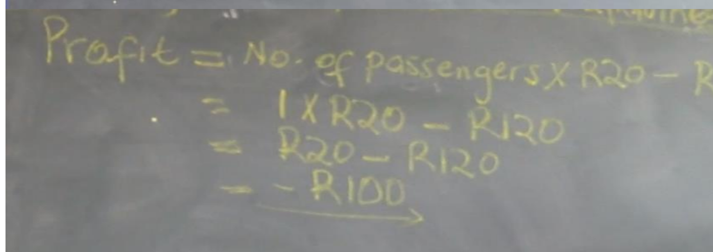
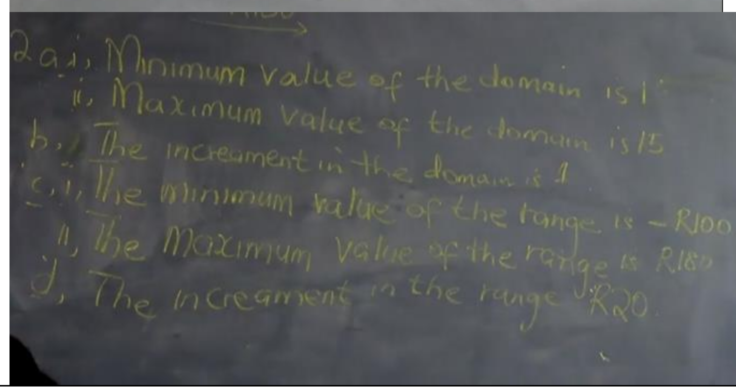
Lesson B3's intended object of learning was point-by-point plotting of a parabola, and this was according to the curriculum statement as shown in Table 28 above.

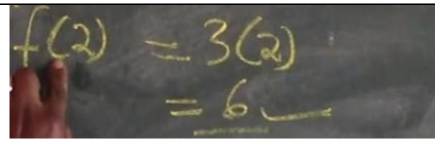
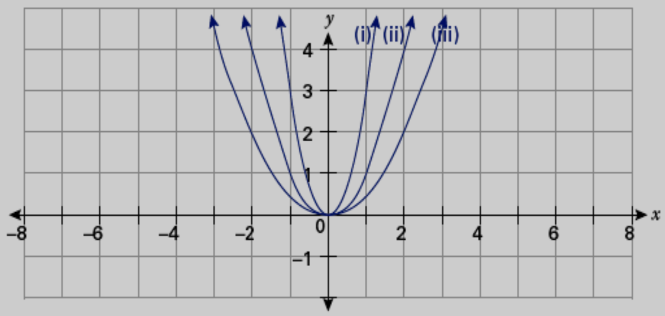
The tables above show that the object of learning shifted from the definition to key features of the object of learning and then to plotting. In this chapter, I look at the enacted lessons, strategies and the role of Teacher B in developing learners' mathematics discourse and understanding In what follows, I present the example set that was used by Teacher B in an attempt to enact the intended object of learning in each lesson.

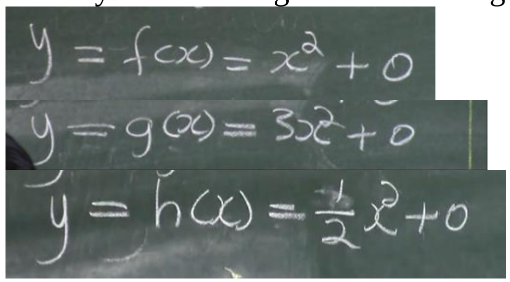
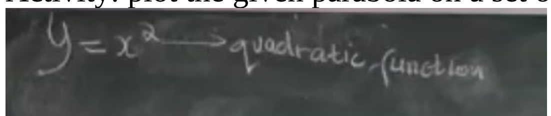
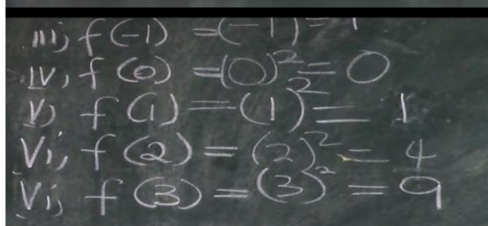
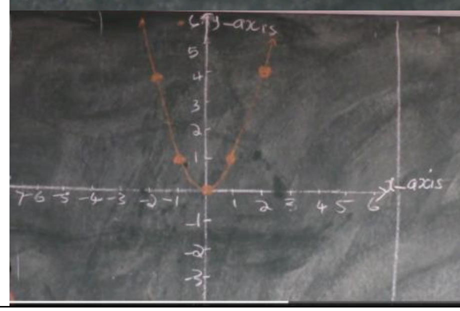
5.2.2 Content and activity selection to enact the intended object of learning

In an attempt to enact the intended object of learning, pre-planned and spontaneous content and activities were selected by Teacher B during the lessons. Most of the examples were taken directly from the textbook. Only one example - in Lesson B1, was an example from a real-life scenario. Teacher B did not bring in any ‘in-the-moment’ (spontaneous) examples or illustrations, as defined by Zodik and Zaslavsky (2008). In the table below, I present all the content and activities, hence the example set that Teacher B selected in her three lessons observed. The examples below have been numbered continuously from Lesson 1 to Lesson 3, for example, for BL1E1, B stands for Teacher B, L1 stands for an example in Lesson 1, E1 stands for the first example in Teacher B’s example set.

Table 29: Example set and the key features of the object of learning evident in Teacher B's lessons.

Intended object of learning	**Item number**	**Content/activity**	**Key features from the lesson observed**	**Comment**
Completing a table of values Domain and range	BL1E1	*Content*: Representing the journey of a taxi driver in a table of values. *Activity*: Complete the table on the board. What is the profit for one passenger? 	-Relationship: One-to-one; -Input -Output -Ordered pair -Domain -Range -Minimum value -Maximum value	- **Pre-planned** content and activity: Teacher B started with this activity at the start of the lesson, as an introduction to functions -**Everyday** content: Teacher B used everyday context of a taxi driver and passengers to generate the table of values. Teacher B related *x* to number of passengers, and *y* to the profit per journey.
	BL1E2	Content: Substitution Activity: Determining the value of *y*, given the value of *x* – using substitution	-substituting -*x*-value -*y*-value	-**Spontaneous** content: Teacher B used this activity to explain to learners how to substitute.

		 		-**Scientific** content: Teacher B presented mathematical symbols of function notation.
Investigate the effect of *a* and *q* on the graphs defined by $y = a.f(x) + q$, where $f(x) = x^2$	BL2E3	*Content*: Investigate the effect of *a* and *q* on the graphs *Activity*: Match the graphs with the defining equation; describe how the graphs differ from the basic parabola. **1.** The graphs of three parabolas are given: $y = f(x) = x^2$ $y = g(x) = 3x^2$ $y = h(x) = \frac{1}{2}x^2$.  **a)** Match the number **(i)**, **(ii)** or **(iii)** on each graph with its defining equation. **b)** Describe how the graphs of $y = g(x) = 3x^2$ and $y = h(x) = \frac{1}{2}x^2$ differ from the graph of the basic parabola $y = f(x) = x^2$. Source: Classroom mathematics Grade 10 Learner book (p.159) (Pike et al., 2011)	-Cartesian plane -coefficient of *x* -Rule -Substitution -*y*-intercept -Gradient -Average gradient -Standard form of the equation of a parabola -Domain and range of a function -Effect of *a* and *q* on a parabola	- **Pre-planned** content and activity: Teacher B started the lesson by opening the page in the textbook and asking learners to engage with the activity. The learners were expected to complete the activities in their notebooks and thereafter, the teacher engaged with learners' responses. - **Scientific** content and activity: The activity given is mathematical in nature with terms, symbols and operations used.
	BL2E4	Content: Effect of parameters on parabola	-Constant term	-**Spontaneous** content: Teacher B

		Activity: Determining a constant in a given equation  	-Effect of $q = 0$	used this activity to show learners a constant. - **Scientific** content and activity: The activity given is mathematical in nature with terms, symbols and operations used.
Point by point plotting of a parabola to discover the domain (input values) and range (output values)	BL3E5	Content: Point-by-point plotting of a parabola Activity: plot the given parabola on a set of axes.    	-Input value -Output value -Substitution -Cartesian plane -Point by point plotting -Axis of symmetry	- **Pre-planned** content and activity: Teacher B started the lesson by opening the page in the textbook and asking learners to engage with the activity. - **Scientific** content and activity: The activity given is mathematical in nature with terms, symbols and operations used.

From Teacher B's example set in Table 29 above, at face value, I came up with the following conclusions:

- Teacher B had four pre-planned content and activities, and one spontaneous activity.
- A pre-planned content and activity from everyday context was selected in Lesson 1. BL1E1 had potential for a double move, where the context of passengers and the taxi could be used to explain a linear function.
- From BL2E3, many concepts were embedded in the content and activity (determine equation of a parabola, describe the effect of parameter *a* on a parabola, describe parabolic graphs using its features) – content and activity suggested possibility for fusion.

The content and activities selected by Teacher B were mostly pre-planned the content and activities were all selected before the lesson. Teacher B drew on mathematics context and everyday context as shown in the example set presented in Table 29 above. Table 30 below is a summary of the content and activities that Teacher B selected. ✔ indicates that the type of content and activity was present while ***X*** indicates that the type of content and activity was not present.

Table 30: Kinds of content and activities selected by Teacher B in the lessons observed.

	Type of content and activity				**Communicative approach and pattern of discourse**
Example set Item number	**Spontaneous/Scientific**	**Spontaneous/ Everyday**	**Pre-planned /Scientific**	**Pre-planned /Everyday**	
BL1E1	***X***	***X***	***X***	✔	ID and I-R-P-R-P-R-E
BL1E2	✔	***X***	***X***	***X***	IA and I-R-E
BL2E3	***X***	***X***	✔	***X***	IA and I-R-E
BL2E4	✔	***X***	***X***	***X***	IA and I-R-E

BL3E5	***X***	***X***	✔	***X***	IA and I-R-E

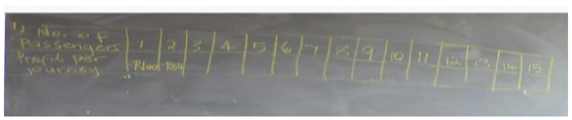
In Table 30 above, I have presented the kinds of content and activities that were selected by Teacher B in her lessons. I will now present the findings and discussion of these kinds of content and activities, by presenting the pre-planned, spontaneous, everyday, and scientific content and activities that were evident in Teacher B's lessons.

5.2.2.1 Teacher B's Pre-planned/Everyday content and activities

Teacher B had only one pre-planned/everyday content and activity out of five items in her example set. BL1E1 is an important content and activity in the entire example set due to its place in Teacher B's lessons. BL1E1 was the first content and activity of functions that Teacher B's Grade 10 learners engaged with during their first encounter with functions as a topic.

Teacher B started Lesson B1 by drawing a table on the board and asking learners questions about the table, she expected learners to complete the table, without highlighting the objectives of the activity, or the lesson, as shown below.

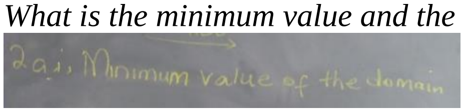
Table 31: Enacting BLIE1.

Item number	Transcription	Code
BL1E1	14. Tr: Let us look at the journey by a taxi driver with 15 passengers. I can represent the journey on a table. The price per passenger is twenty rand. 15. Lnrs: Yes 16. Tr: Let us complete the table on the board. What is the profit for one passenger? 17. Lnr1: Loss 18. Tr: Negative 100 [writes -100 in the table]. 19. Lnrs: Yes 20. Tr: And for two passengers? 21. Lnr2: Negative eighty 22. Tr: So you have negative eight rand loss because there is a loss [Writes -80 on board]. It is negative to show that there is a loss no profit has been made. If there are three passengers in the car, how much loss is Mr Shadrack making?  23. Lnr2: Six rand↓ 24. Tr: [Writes on black board] six rand loss right, so Mr. Shadrack is going to lose six rand. And if they are four passengers, what is the loss? 25. Lnrs: Forty rand↓ 26. Tr: Forty rand. Negative forty rand [Writes on board]. If there are five passengers… is there any loss made? ↑ 27. Lnrs: Yes	LPK: Learners were expected to complete the table of values before engaging with it in the lesson. Interaction Line 1: *Initiation* Line 2: *Response* Line 3: *Prompt* Line 4: *Response* Line 5: *Evaluation* Line 6: *Response* Line 7: *Prompt* Line 8: *Response* Line 9: *Prompt* Line 10: *Response* Line 11: *Evaluation/Prompt* Line 12: *Response* Line 13: *Evaluation/Prompt* Line 14: *Response* Communicative approach: ID Pattern of discourse: I-R-P-R-P-R-E

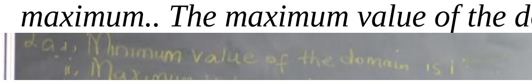
From the excerpt above, Teacher B expected learners to complete the table given and find the profit per journey of 15 passengers. The learner's response was not probed further (see lines 3 and 4), although Teacher B prompted learners by asking more questions, which they responded to. In the engagement above, Teacher B did not link the example to the topic of functions in anyway, as the table was being completed. The communicative approach that was evident while the teacher and learners were completing the table of values was Interactive-dialogic (ID) because Teacher B acknowledged learner responses and based on the responses, she asked other questions (see lines 7-14). Teacher B initiated the interaction and she also prompted the learners throughout the discussion (see Table 31 above). She also evaluated learner responses. Therefore, the pattern of discourse that was evident during the enactment of BL1E1 was I-R-P-R-P-R-E – Teacher B initiated, learners responded, Teacher B prompted learners and she evaluated their responses. The communicative approach and pattern of discourse enabled learners to contribute during the lesson, and although the learners were engaged in the lesson, they gave one-word responses. This was evident in the entire lesson as shown in the excerpt below.

Definition of domain

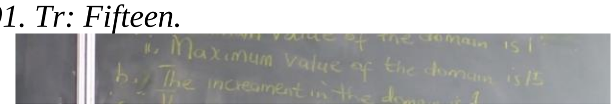
85. Tr: profit, right, but if he doesn't make any profit is there anything that he is losing or getting?
86. Lnrs: Losing
87. Tr: He's losing. This is the money that Mr. Shadrack will lose per trip if he is just taking one passenger to the city, right? [Checks learner's book]. The next question that you were having was… can I see the next questions? Use the table of values to answer the questions that follow. What is the minimum value and the maximum value of the domain? [Writes on the board]



Firstly, we want the minimum value of the domain. What is the minimum value of the domain?
88. Lnr5: One
89. Tr: One, [Writes 1 on the board] so the minimum value of the domain is one, We need the maximum.. The maximum value of the domain. [points at learner] Tobeka



90. Lnr5: Fifteen
91. Tr: Fifteen.



what do you understand by the minimum value [Points to the word "Minimum" on the board]
92. Lnrs: Less …
93. Tr: Less …

94. *Lnr2: Smallest*
95. *Tr: The smallest value of the domain. And what do you understand by the maximum value … the maximum value [Points to learner] …*
96. *Lnr4: The highest*
97. *Tr: The highest or biggest value. What is the domain, how do you explain the word domain … to your fellow learner? … If your friend comes and says, my friend I have got a problem I don't know what this domain stands for … how do you explain domain? [Pointing at learner]*
98. *Lnr5: The input*
99. *Tr: The input … still maybe this person does not understand↑, what you mean by input? [Points at another learner]*
100. *Lnr5: The set of all inputs values*
101. *Tr: The person does not understand what input values are, how are you going to help give guidance to this learner↑, your friend in the class? [Points to another learner]*
102. *Lnr6: I think you get the values and you calculate*
103. *Tr: And where do you get those numbers. …, Katlego, how do you explain domain? Yes.*
104. *Tr: Still your friend does not understand him↑, what do you mean by input values? ↑*
105. *Lnr5: The numbers that you start with.*
106. *Tr: The numbers that…*
107. *Lnr2: You start with*
108. *Tr: We start with…for example,*
109. *Lnr5: one, two,*
110. *Tr: Zero…*
111. *Lnr2: One, two*
112. *Tr: One, two … all the x values that you use in order to get the corresponding y values, right?*
113. *Lnrs: Yes*

From the excerpt above, there are two things to note. The first one is how Teacher B asked learners what domain means and how she linked this to the table of values that was completed during the first half of the lesson (see lines 97 - 103). The second is how Teacher B required learners to take up a role of clearly explaining the term domain to their peers (see lines 97, 99, 101 and 104). I use the word 'clear' here because Teacher B did not agree with a learner who answered the question in line 98, by saying that domain is input. Teacher B probed the learner for further explanation of what input means. Although the learners had prior knowledge on the meaning of domain, Teacher B expected a lot more from them, by asking them to explain the term to another person in a way that would help the other person make sense of it. A learner defined domain as the number that you start with (see line 107), and the teacher agreed with her. However, the definition that Teacher B agreed with does not provide all the necessary conditions for the definition of domain in functions. Drawing from the definition of domain by Martínez-

Planell and Trigueros (2009), domain is the set of all possible *x*-values of a function which give output of real *y*-values. The definition given by the learner in line 107 does not include the aspect of domain being a set of all possible values, and therefore, for such a definition, the learner may not be in position to give the set of possible domain values when required. Also, the definition does not include the aspect of 'real values', because it says all corresponding values. Therefore, the learner may not be able to discern the limitation of the domain and range when determining the set of *x*-values for any given function. The definition of domain could have been used to help learners unpack the functional relationship, in relation to the object of learning, the concept of a function. For a set of values to be called the domain of a function, the *x*-values and *y*-values should either be in a one-to-one or many-to-one relationship. Therefore, Teacher B's definition of domain: "*One, two … all the x values that you use in order to get the corresponding y values"*, could have been fine tuned to read as: *One, two … all the* ***possible*** *x values that you use in order to get the corresponding* ***real*** *y values,* ***of a function****.* The definition of domain that Teacher B gave (see line 112) did not take the definition of a function into consideration, as discussed above. Definitions of abstract concepts enable learners to make concept images, as discussed by Vinner (1983). Vinner argues that referring to formal definitions in some contexts in mathematics is critical for a learner. Functions as a topic is one of those contexts in mathematics where definitions are critical for engaging and performing tasks. Correct definitions of terms in this context need careful attention from the teacher so that learners have an accurate point of reference when engaging with tasks in functions. The teacher prompted the learners in an I-R-P-R-E pattern of discourse, until the learners gave the response that Teacher B wanted as shown in lines 105 – 113 in the excerpt above. The communicative approach was interactive-authoritative (IA) because Teacher B did not attend to learners' responses in form of probing and hence enabling them to develop their discourse, by moving them from what they knew (on definition of domain of a function) to what they needed to know (the formal definition of domain of a function) as discussed above. This is evident in the engagement of the definition of range in the excerpt below.

Definition of range

125. Tr: This Saake in Grade 10, he does not know what is meant by range. How will you help him to understand what range is? Naomi, Naomi, how?

126. Lnr1: Negative 100
127. Tr: Thank you, at least you are paying attention you are alert. Naomi, Sandile does not understand what range is, can you help him?
128. Lnr1: A product of a certain process
129. Tr: A product…
130. Lnr2: Of a certain process
131. Tr: Of a certain process, in this case what is your process,
132. Lnr2: (inaudible)
133. Tr: Come on be loud so I can hear please, How, you need to talk to Sandile so Sandile can be able to hear what you say because you are trying to help him …
134. Lnr1: The product you get after a certain process has taken place
135. Tr: Eeh Saake still does not understand, is there anyone who can help us … how do you explain what range is to him, yes [points at learner]
136. Lnr3: Range is the biggest number
137. Tr: The biggest number … his still not understanding what you said,
138. Lnrs: (learners' murmurings)
139. Tr: Come on guys come guys, Lovemore…
140. Lnr3: The range is the profit
141. Tr: The profit, in this case.

From the excerpt above, Teacher B was engaging with prior knowledge of learners on the definition of range (see lines 125-127). Learners responded to the teacher's question using the activity on profit and loss that was being discussed in the lesson. One learner responded by giving a specific value of 100 as the range. In line 136, another learner responded with an inaccurate definition of range. The definition of range as the biggest value in line 136 was not probed by the teacher and hence another missed opportunity, of engaging with learners' pre-conceptions in an attempt to enable them to develop their mathematics discourse. Carr et al. (2009) argues that the role of assessing prior knowledge is to address any prior misunderstandings, and hence help the learners construct new understandings. Teacher B explained range in relation to the activity of the lesson (see lines 140 and 141), without probing or querying the response in line 136. Teacher B's definition of range was specific to the activity being discussed and hence another missed opportunity. Learners need to be provided with an opportunity to be able to define range as a set of all possible *y*-values that can be produced by a given function (Martínez-Planell & Trigueros, 2009). The learner response in line 136 could have been used by Teacher B to unpack where the learner was coming from in saying the range is the product of certain processes. As discussed earlier, Teacher B was using an authoritative approach because Teacher A responded to learners' answers with an evaluation and she ignored some of the answers, with no interanimation of ideas. The teacher therefore did not probe the learner, neither did she gradually build on the inaccurate definition to arrive at a more mathematically acceptable definition, hence to a better understanding of range. Lines 140-141 show that Teacher B had a predetermined answer- she accepted the answer that the learner gave in line 140 because that is what she was looking for. The limitation of the definition given in line 140 above is that it is not one that can be used in another scenario as it is very specific to the example that was being discussed. If learners are given another task where they need to determine range of a function, they may be seen looking for profit, if they use the definition given by the learner and confirmed by Teacher B in the excerpt above. The activity of defining a term did not give learners an opportunity to explain or justify their answers. Communication was limited to one-word responses as shown in the transcript above. Teacher B had a focus on definition, an indication that she considered it as an important aspect of mathematics discourse.

Taking a global look at Teacher B's enactment of activity BL1E1, as discussed earlier, there was no announcement from Teacher B about the objective of the lesson to contextualise the activity presented to the learners. The interaction suggested that the activity was focused on determining profit and loss, with a focus on substitution. The excerpt below from enacting BL1E2 shows the procedural discussion that Teacher B engaged learners in to find out if they were able to identify profit or loss.

Profit and Loss

114. Tr: [Writes on board] negative hundred rand.

Why is it negative hundred and what does it mean?

115. Lnr8: A loss
116. Lnr5: No profit
117. Tr: Mr. Shadrack is not making any…?
118. Lnrs: profit
119. Lnrs: Yes
120. Tr: profit, right, but if he doesn't make any profit is there anything that he is losing or getting?
121. Lnrs: Losing
122. Tr: He's losing. This is the money that Mr. Shadrack will lose per trip if he is just taking one passenger to the city, right? [Checks learner's book]. The next question that you were

having was… can I see the next questions? Use the table of values to answer the questions that follow. What is the minimum value and the maximum value of the domain? [Writes on the board]

From the excerpt above, Teacher B emphasised the concept of profit and loss. Teacher B explained profit and loss to learners, following from their responses (see 114-122). As mentioned earlier, prior to this stage in the lesson, Teacher B had not yet linked the activity of the lesson to functions. The point to note here is the time that Teacher B spent with the learners completing the table of values. Many of the learners were disengaged after the first four values were completed. The activity took far too long based on the task that was required from the learners. The time spent on this task highlights the role of the teacher in ensuring that appropriate time on task is maintained in the lesson, and an explicit link to the intended object of learning is made during the lesson. In mathematics classrooms, time is always a fixed construct because specific time is allocated for a mathematics lesson (NCTM, 2014). Therefore, it is crucial that the allocated time be maximised to give learners the best opportunities to engage in meaningful mathematics (NCTM, 2014). Maximising the time is dependent on the teacher clearly articulating the mathematical learning goal of the lesson (NCTM, 2014). The role of the teacher in ensuring that time on task enables learners to engage with meaningful activities, which is linked to the intended object of learning cannot be overemphasised. If learners are not exposed to appropriate activities within the time available in a lesson, developing their mathematics discourse and hence understanding may not be possible.

As discussed above, the intended object of learning, which was defining the concept of a function, was not made explicit when activity BL1E1 was being discussed by the teacher and learners. Holmqvist et al. (2011) argue that the teacher's upfront intentions concerning the object of learning are crucial to any lesson. This gives the teacher's perspective on what is to be learnt in the learning situation. As such, it is the teacher's role to delimit the object of learning in the lesson, and hence choose appropriate tasks to engage with the intended object of learning. It should be possible to get a view of the intended object of learning by what the teacher presents to the learners (Holmqvist et al., 2011). This was not possible during the enactment of BL1E1. One could have inferred that the intended object of learning is to learn how to substitute or determine profit and loss. This was not the case in Teacher B's lesson observed. The everyday example was used throughout Lesson B1, and Teacher B only linked to the concept of the function towards the end of the lesson. As stated earlier, I found it challenging to follow the focus of the lesson based on the teacher and learner interactions in Lesson B1. In the post-observation interview, I asked Teacher B about the intended outcome of Lesson B1 and her rationale for the choice of example of the taxi and the passengers. Teacher B said that she always uses this example to start her lesson on functions as shown in the interview excerpt below.

Using everyday contexts

***Interviewer**: When using contextualised examples like the one you used in the classroom about the taxi…*
***Tr**: Yes*
***Interviewer**: Where you were showing them how to get the taxi fare and how it increases with the number of passengers that come in the taxi…*
***Tr**: Yes..*
***Interviewer**: Did you pre-determine the example or you thought about it when you came to the classroom? So what informed the decision to use the kinds of examples that you used in the lesson?*
***Tr**: Uhmm…the area where we live, most of them are using taxis in everyday life.*
***Interviewer**: Okay.*
***Tr**: To come to school, to go to the shops, you know taxi business is the one that is booming in the townships. So I knew they have this. If we talk about getting a ride or taking a taxi to town or to the shopping centre or coming to school from home using a taxi, it's easier for them. I will use that example. This is something that they do daily. If one is catching a taxi to come to school, they do understand that to say the driver will get the amount will be increasing every time a new passenger gets into the car. If like local, they pay six rand. If the driver started with…no at first from where he started maybe he started from home, there was no passenger. The first passenger he gets, now he knows I am having a six rand. The second one comes in we say plus six, the third one…so as a passenger is coming in, then we are making a set of ordered pairs, or we start first on the number line, drawing in the table of values. We have the number of passengers the driver started with which is zero and then was he given any amount by anyone? No. Why? Because there was no passenger. He drives along the route that he is using, then he hoots and then there is a person who is saying I am going local or town. He gets in, he knows he is giving him that amount of money, which is six rand, the first person is your x-value and the amount that he pays is your y-value. The second one gets in up to maybe when the taxi is full. Now we have all the coordinates. If it's a fourteen-seater, we have the coordinates of fourteen points. Now, let's go and plot these points on the graph and then after that they draw and after drawing, what do you notice? Uuh mum, the points are forming a straight line. Why are these points forming a straight line? What do you think makes this graph to be a straight-line graph? Then they will say because ma'am they*

are paying the same amount of money. There is no one who is going to pay extra or less because if you have less, you can't take a taxi. If you have more, you are going to get your change. So everybody in the taxi is going to pay the same amount. And now let's look at the x, where the graph is cutting the y-axis and where the graph is cutting the x-axis. That's when I will introduce the x-intercepts and y-intercepts and now they are seeing clearly, it's from an everyday situation that they are familiar with.

From the excerpt above, Teacher B explained the everyday example that she used to start her lesson on functions, to show what informed her decision of the example. In the example BL1E1, the cost per person is the same (R20 per person), it is not varying. The number of passengers is varying. The variable, x, is the number of passengers (the input); the output is the fare that passengers pay. The profit is the result of a formula based on the fixed costs for the driver. This is a linear function, with the equation: Profit = $20x - 100$. There is immense potential for learning in the example, which is the relationship between taxi profits and the number of passengers when the fare is constant is a linear function. However, Teacher B did not exploit the patterns of variation in the example, and how the functional relationship is seen through the taxi driver and passenger fare, which could have led to some form of generalisation about linear functional relationships. The possibilities in this example were not fully realised by the teacher during the lesson, and her explanation in the post-interview is not the same as what was observed in the lesson.

Teacher B did not engage with the back and forth discussion between everyday context in the example and mathematics context (linear function) that was embedded in the example. From the post interview, I gained insight into why Teacher B selected this particular example to start off her lesson on functions to Grade 10 learners, who were encountering the concept of a function for the first time. However, during the lesson observed, it was not clear as to why this example was selected. The double move (DM-TS) was not apparent in the first part of the lesson, which was three quarters of Lesson B1. As mentioned earlier, my analysis of the double move for Teacher B focused on only example BL1E1, presented above in Table 30, as this was the only example from everyday context in the observed lessons. The explanation given in the interview does not reflect what was enacted in the lesson. Focusing on the excerpt below from Teacher B's post interview:

Now, lets go and ***plot these points on the graph*** *and then after that they draw and after drawing,* ***what do you notice?*** *Uuh mum, the points are forming a* ***straight line****. Why are these points forming a straight line? What do you think makes this graph to be a straight-line graph? Then they will say because ma'am they are paying the same amount of money. There is no one who is going to pay extra or less because if you have less, you can't take a taxi. If you have more, you are going to get your change. So everybody in the taxi is going to pay the same amount. And now let's look at the x, where the* ***graph is cutting the y-axis*** *and where the* ***graph is cutting the x-axis****. That's when I will introduce* ***the x-intercepts and y-intercepts*** *and now they are seeing clearly, it's from an everyday situation that they are familiar with.*

From the excerpt above, Teacher B had intentions of giving learners an opportunity of investigating the functional relationship from the example given. Teacher B also intended to allow learners to plot the points so that they could see that a straight line is formed from the points. She intended to ask the questions below, as she stated in the interview.

Why are these points forming a straight line?
What do you think makes this graph to be a straight-line graph?

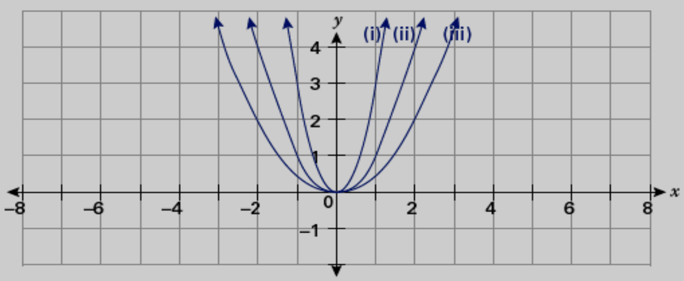
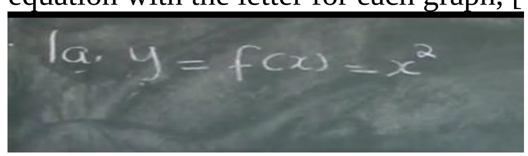
The questions above were not asked in the lesson, and the lesson did not go as far as learners plotting the straight line graph and interrogating the relationship, and this could have been a result of the amount of time spent of aspects such as substituting and determining profit and loss as discussed above.

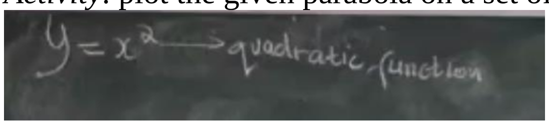
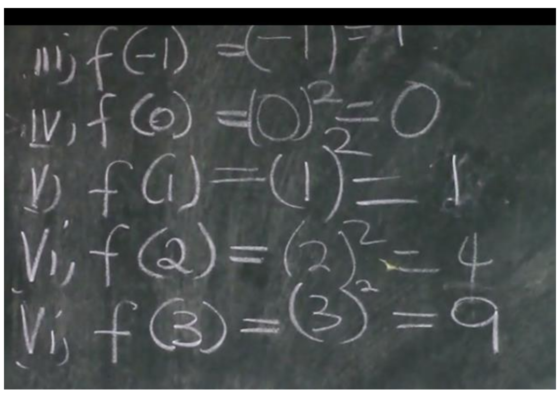
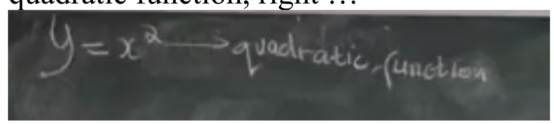
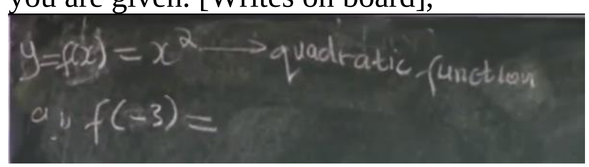
To summarise the findings on BL1E1, Teacher B presented it as a pre-planned activity from everyday context. The activity was not enacted appropriately in relation to the intended object of learning – as confirmed by the post-observation interview of Teacher B above. In the interaction, the teacher and learners focused on substituting, determining profit and loss and defining domain and range. I-R-P-R-P-R-E was predominant in the interaction, with I-R-E pattern of discourse evident in some instances in the interaction. IA was predominant in the interaction, with ID communicative approach evident, though at a very low level. As seen in the interaction, no deliberate action from Teacher B was seen to influence the pattern of discourse during the engagement on activity. Although BL1E1 is an everyday content and activity, there was no established evidence of a double move in the lesson observed.

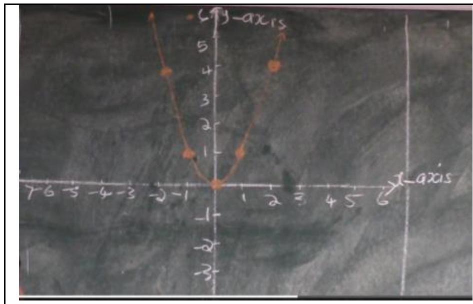
5.2.2.2 Teacher B’s Pre-planned/Scientific content and activities

Teacher B had two pre-planned/scientific content and activities out of five items in her example set. In the table below, I present the content and activities that were selected by Teacher B in this category.

Table 32: Pre-planned/scientific content and activities selected by Teacher B.

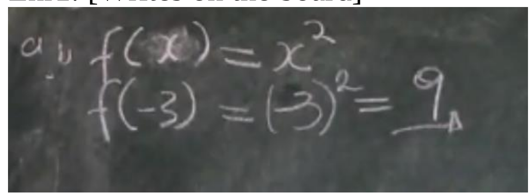
Content/activity	**Transcript**	**Comment**
BL2E3 *Content*: Investigate the effect of *a* and *q* on the graphs *Activity*: Match the graphs with the defining equation; describe how the graphs differ from the basic parabola. **1.** The graphs of three parabolas are given: $y = f(x) = x^2$ $y = g(x) = 3x^2$ $y = h(x) = \frac{1}{2}x^2$.  **a)** Match the number **(i)**, **(ii)** or **(iii)** on each graph with its defining equation. **b)** Describe how the graphs of $y = g(x) = 3x^2$ and $y = h(x) = \frac{1}{2}x^2$ differ from the graph of the basic parabola $y = f(x) = x^2$. Source: Classroom mathematics Grade 10 Learner book (p.159) (Pike et al., 2011)	1. Tr: [Looking at textbook] You did not finish this, what was yesterday's work? We will do that after we have finished, right? what was yesterday's work? 2. Lnr1: Page one fifty-nine 3. Tr: Page one five nine page one five nine …[Reading from text book] The graphs of three parabolas are given, right? We have *y* is equal to *f* of *x*. Where is your textbook? Textbook? *y* is equal to *f* of *x*, We are on page one five nine, page one five nine, [Pacing around class] You need to see the graphs that are drawn on the page, in order to be able to answer the question … right? You have three parabolas and parabolas that are drawn. The first is *y* is equal to *f* of *x*, which is *x* squared, *y* is equal to *g* of *x* which is equal to three *x* squared. The last one is *y* is equal to *h* of *x* which is equal to half of *x* squared. The question says, match roman figure number one, roman figure number two or roman figure number three on each graph with its defining equation, so you need to match the defining equation with the letter for each graph, [Writes on board.  Right, you will look at the drawn graphs, which graph represents the graph of *y* is equal to *f* of *x* which is equal to *x* squared, [Looking at text book] Is it roman figure number one or roman figure number two or roman figure number three? Please raise up your hands. The graph that represents [pointing at equation on the black board] the graph of *y* is equal to *f* of *x* which is equal to *x* squared [pointing at learner with raised hand]	Teacher B started Lesson 2 by looking at previous lesson content and activity. Teacher used the textbook to introduce the activity of the lesson. Although this activity was given previously, it was the main activity in Lesson 2. Interaction *Line 1: Initiation* *Line 2: Response* *Line 3: Initiation* *Line 4: Response* *Line 5: Evaluation* *Line 6: Response* *Line 7: Prompt* *Line 8: Response* *Line 9: Evaluation/prompt* *Line 10: Response* Communicative approach: IA Pattern of discourse: I-R-E

	4. Lnr2: Roman figure number two 5. Tr: Roman figure number two, correct 6. Lnr2: Yes 7. Tr: [Asking learners] is it correct? Okay, let’s see if she’s right. [Looking at textbook] If *x* is one, come on, if *x* is one↑, one squared, one squared will give us what? 8. Lnrs: One 9. Tr: One, do you see the point one and one, the point one and one lies on the graph of roman figure number two, right? 10. Lnrs: Yes	
BL3E5 *Content*: Point-by-point plotting of a parabola *Activity*: plot the given parabola on a set of axes.   	1. Tr: [Writes on board] You are told that this is called a quadratic function, right …  Right, yes that is question one. Then the question says, calculate the values of, remember we call this a quadratic function, right? 2. Lnrs: Yes 3. Tr: Remember we said a function is a relationship where each element of the domain is related to only one element of the range, right? 4. Lnrs: Yes 5. Tr: Ok, let’s see… We are required to find … this is what you are given. [Writes on board],  The question says we need to calculate the value of *f* of negative three. Can we get somebody who can work this one out for us, *f* of negative three, if we substitute *x* by negative three, can we get the value of *y*? … We have *f* of *x* is [writes	Teacher A took the activity from the textbook and engaged learners in a discussion in order to attempt the activity. Interaction *Line 1: Initiation* *Line 2: Response* *Line 3: Initiation* *Line 4: Response* *Line 5: Initiation* *Line 6: Response* *Line 7: Evaluation* *Line 8: Response* *Line 9: Prompt* *Line 10: Response* *Line 11: Evaluation/prompt* Communicative approach: IA Pattern of discourse: I-R-E

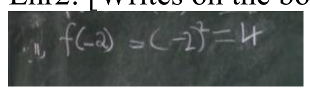


on the black board], remember we have *f* of *x* is equal to *x* squared, if we are now going to substitute *x* by negative three, can you finish up to get the value of *f* of negative three?...

6. Lnr1: [Writes on the board]



7. Tr: Thanks that's good. If *x* is negative three, negative three squared is equal to positive nine or negative three times negative three is equal to nine, well done. We need someone who will get [writes on the blackboard] *f* of negative two, *f* of negative two, if *x* is negative two, if *x* is negative two. Let's see what you are thinking. You do exactly the same, there is nothing new, just substitute *x* by negative two …
8. Lnr2: [Writes on the board] …

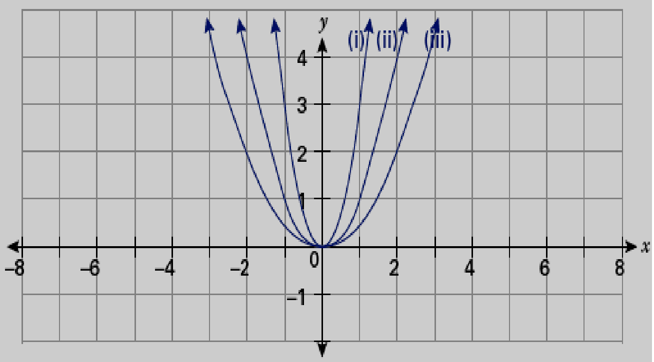
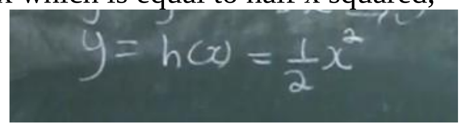


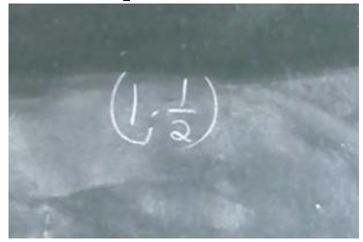
9. Tr: Thank you, is the answer correct?
10. Lnrs: Yes
11. Tr: Well done. If *x* is negative four, *y* is equal to, if *x* is negative two, sorry, negative two raised to exponent two is equal to positive four.
    [Writes *f(-1)* = on the board] Can we get someone who will work out for us *f* of negative one, if *x* is negative one, Boitumelo. Substitute *x* by negative one …. Boitumelo… do you get what we are doing? Come on guys, Boitumelo, Boitumelo you are not helping? You are scared of the chalk? The chalk has never beaten anyone … Please sit properly, are you able to see what's happening there?

BL2E3 was the first content and activity in Lesson B2. As shown in Table 32, Teacher B did not announce the intended object of learning for the lesson. She started by presenting the activity to the learners as shown above. As presented in Table 27 above, I inferred the intended object of learning from the context of activity BL2E3, which was 1) to investigate the effect of parameters a and q on the parabola; 2) to match representations, given an equation and a graph using the characteristics of a parabola, and 3) to interpret parabolic graphs. The intended object of learning then guided my analysis, to ascertain whether activity BL2E3 provided opportunities for learners to develop their mathematics discourse and understanding of parabolic functions.

In activity BL2E3, learners were required to engage with three types of parabolic functions, with equations and corresponding graphs. The three functions are similar because they are all parabolic, with different coefficients of x^2 in the three functions (1; 3 and $\frac{1}{2}$), and function notations ($f(x)$; $g(x)$ and $h(x)$). These two varying aspects (coefficient and notation) in the three equations were good examples to afford learners an opportunity for generalising that function notation is not just $f(x)$, it can also be written as $g(x)$ or $h(x)$ for different functions, and change in coefficient of x^2 (1; 3 and $\frac{1}{2}$) affects the shape of the graph. In other words, focusing on variance amidst invariance, the x-intercept or turning points are invariant, as is the degree of the equation and hence quadratic function. In addition to notation, what is varying is the coefficient of x^2 and this makes it possible to generalise only about parameter a, for a positive value of a, the graph opens upwards, and as parameter a increases, the parabola is vertically stretched towards the y-axis. The table below shows the separation aspects in activity BL2E3.

Table 33: Separation aspects in activity BL2E3.

Example number	**Example**	**Transcription**	**Code**
BL2E3	1. The graphs of three parabolas are given: $y = f(x) = x^2$ $y = g(x) = 3x^2$ $y = h(x) = \frac{1}{2}x^2$.  a) Match the number (i), (ii) or (iii) on each graph with its defining equation. b) Describe how the graphs of $y = g(x) = 3x^2$ and $y = h(x) = \frac{1}{2}x^2$ differ from the graph of the basic parabola $y = f(x) = x^2$. Source: Classroom mathematics Grade 10 Learner book (p. 159) (Pike et al., 2011)	38. Tr: So Sandra is right. The answer is [Writes on board], the graph represented by y is equal to g of x is equal to three x squared is from roman figure number, one We have the function y [Writing on the black board] is equal to h of x which is equal to half x squared,  *y* is equal to *h* of *x* and this is equal to half of *x* squared… Leanne, which graph represents the graph of y is equal to h of x which is equal to half of x squared? 39. Lnr1: Roman figure number two, 40. Tr: Roman figure number two, two, let's see if it's correct. If *x* is one, one squared is one and what is half of one, half of one? 41. Lnr1: Zero-point five … 42. Tr: What is half of one? 43. Lnrs: Zero coma five, 44. Tr: Zero coma five is equivalent to what? Zero coma five is equivalent… to? 45. Lnr2: One over two 46. Tr: Is equivalent to, one over two, just check the point one and half, one for *x* half for *y* is this point lying on the graph of roman figure number two?… You look at the graph of roman figure number two, the point that you are looking for half is one and half, one for x half for y. Is this point lying on the graph of roman figure number two? No, 47. Lnr2: Of course not 48. Tr: One for *x*, half for y, do you see this point on roman figure number two? Boys stop dreaming look at the graph. 49. Lnr3: It's almost right.	**SS-TS** – Teacher B used a set of similar examples ***Separation aspects*** -All equations given are quadratic -Values of parameter *a* are varying while the values of q is invariant and is equal to zero: $y = 1x^2$ $y = 3\,x^2$ $y = \frac{1}{2}\,x^2$ -Value of $q = 0$

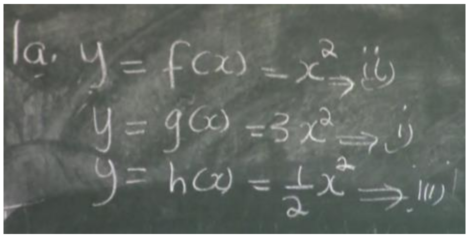
		50. Tr: uh uh, it cannot be almost↑, it’s either right or wrong not almost, not almost↑. The point that you are looking for is [writes on board] a one and half.  Is it right on roman figure number two? 51. Lnrs: No, Yes↑↑ 52. Tr: Mark you said that roman figure number two is the graph that is representing y is equal to *h* of *x*, 53. Lnrs: Yes, yes. 54. Tr: Roman figure number two.	

Questions *a* and *b* in activity BL2E3 required learners to engage with the key features of the object of learning: to match the graphs with the defining equations, and then describe the differences between the graphs. In Tables 32 and 33, I presented the excerpts of the interaction that transpired in the classroom. Teacher B guided learners on how to go about matching the equation to the graph (see line 3 in Table 32). The learners were not able to match the graph to the equation (see lines 39 to 51 in Table 33). Teacher B continued to probe the learners in an I-R-E pattern of discourse until they were able to match all the graphs to their defining equations. The excerpt below is the final engagement on question *a*, of matching the graphs to their defining equations. It is important to note here that learners struggled to engage with the activity that the teacher selected in the lesson. Learners displayed a gap in prior knowledge that was required for them to attempt the activity.

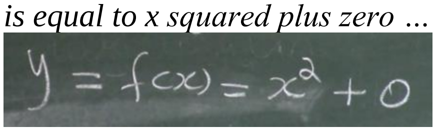
After probing, Teacher B and the learners matched all the graphs to their defining equations. The object of learning, which was inferred from the lesson (see Table 27 above), as the effect of parameter *a* on the parabola, was not emphasised by Teacher B. Separation and contrast in the equations (see Table 33) were not used to make learners aware of the opportunities for generalising. Learners were not guided to see how the graph changes with a change in the coefficient of x^2. This is made clear by the answers which learners gave for question b of the activity as shown in the excerpt below, where the teacher was asking a learner to attempt question b on the difference between the graphs.

Comparing parabolic equations

83. Lnr4: Roman figure number two is greater than roman figure number one↓.
84. Tr: Roman figure number two… what do you mean by greater? …
85. Lnr3: Roman figure number two is more bigger↓.
86. Tr: When you say it's more bigger↑, what are you looking at that makes it to be bigger … what are you looking at↑, that shows you that this one is bigger?.... Tshepo, you have something to say? …
87. Lnr4: Roman figure number two has a constant
88. Tr: People↑, roman figure number…one and number two does it have a constant↑?



89. Lnr4: It has a constant.
90. Tr: What is the constant in roman figure number two↑?
91. Lnrs: One
92. Tr: One, people what is the constant there … in roman figure number two? Where is the constant because she has brought in the constant, where is the constant … or what do you understand by the word constant, in math?
93. Lnr5: No changing
94. Tr: Something that does not change↑.
95. Lnr7: Yes
96. Tr: Do we have something that does not change there?
97. Lnrs: Yes, nooo
98. Tr: Its zero, right? He is right, the constant in each of the three graphs is..?
99. Lnr5: Zero
100. Tr: Zero, right, meaning that you are having [writes on board] y is equal to f of x which is equal to x squared plus zero …



This is the constant [points at zero on the board], it does not change. It is not accompanied by any value, right?
101. Lnrs: Yes

From the excerpt above, Teacher B tried to get learners to focus on how the three equations were different (see line 87-88). From the learners' responses, they were looking at the numbers, and it was not clear what learners were referring to as a constant in line 89. The interaction in the lesson led to a spontaneous/scientific content and activity (see BL2E4 in Table 28). Teacher B showed the learners how the constant in the given equation in line 100 is represented, after one of the learners brought in the constant to identify what was different (see lines 92-100). The teacher showed learners another way of how to represent an equation when we have a constant of zero. She went to on to explain to the learners as shown below.

What is the difference in the equations given?

104. . Tr: Clear? [Pointing at the board] So in all the three graphs, the y intercept is zero, even when you go and check the graph, all the three graphs are passing through the point zero and zero because they are having the y intercept which is zero. So I don't know what *you were talking about the constant, what were you referring to? …What were you referring to? … They are all having the same constant, which is?*

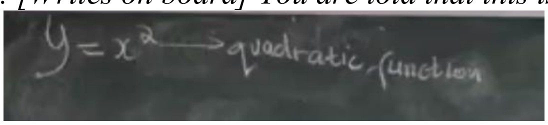
105. *Lnrs: Zero*
106. *Tr: Zero, that is not what is different. It is the same in each of the three graphs … What is different in roman figure number one and roman figure number two?… Come one guys …[points to learner] What is different? … Please talk aloud remember we are having a video in class, your answer must be captured. Sandile*
107. *Lnr1: Roman figure number two has two points joined together*
108. *Tr: Two points joined together?*
109. *Lnrs: Yes*
110. *Lnr4: Roman figure number one has separate points.*
111. *Tr: No… No. Is there anyone who can tell us what he or she sees as the difference, between roman figure number one and roman figure number two?...Please let's pay attention and not disturb other people. Wrong … what do you see as the difference? Sabelo? These two boys here, you are troublesome*

In the excerpt above, Teacher B was starting to dismiss learners' answers and giving them instant evaluation (I-R-E) about their answers (see lines 106 and 111). Teacher B was starting to get disturbed by the learners' responses (see line 111). The discussion on what is different went on until the lesson came to an end, without learners giving the correct answer. It became clear in Lesson B2 that the learners did not discern the features of the graphs and equations presented and hence did not see the similarities and differences (contrast) when presented with equations of the parabola. From the interaction in the lesson, learners were not able to discern the features of a parabolic function, and the lesson ended without learners responding to the correctly to the teacher's questions. Unfortunately, the transcript of Lesson 2 ended abruptly due to recording technical issues. I therefore relied on my field notes for the last five minutes of the lesson. Teacher B could have afford learners opportunities to discern the critical features of a parabolic function, by starting with naming the terms in an equation because learners did not know what a constant is, and other than Teacher B bringing in a spontaneous content and activity to illustrate how a constant is represented, she did not engage with the names of the other terms of the equation of a parabola. Breaking down the activity further in spontaneous activities from activity BL2E3 or by giving different activities could have helped learners to discern critical features of the intended object of learning. For example, discussing characteristics of a parabola could have been a good starting point to engage learners in the lesson observed. Highlighting characteristics such as the basic equation, the shape, the maximum and minimum value, the turning point, and the intercepts could have been made explicit in the lesson before learners were asked to compare the equations. Unfortunately, the lesson ended without the teacher assisting the learners to 'see' what they needed to see in order for them to be able to identify the similarities and differences in the equations given. In the next lesson, Teacher B, went straight to point-by-point plotting of a parabola, without concluding the activity from the previous lesson. The activity in Lesson B3 (BL3E5) was the other pre-planned/scientific content and activity that Teacher B selected.

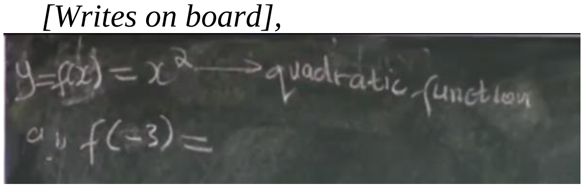
Activity BL3E5 was selected by Teacher B from the textbook and presented to learners at the start of Lesson B3. The excerpt below shows the interaction that emerged in the lesson.

Point-by-point plotting of a parabola

1. *Tr: [Writes on board] You are told that this is called a quadratic function, right …*



   Right, yes that is question one. Then the question says, calculate the values of, remember we call this a quadratic function, right?
2. *Lnrs: Yes*
3. *Tr: Remember we said a function is a relationship where each element of the domain is related to only one element of the range, right?*
4. *Lnrs: Yes*
5. *Tr: Ok, let's see for one two one. We are required to find … this is what you are given. [Writes on board],*



The question says we need to calculate the value of f of negative three. Can we get somebody who can work this one out for us, f of negative three, if we substitute x by negative three, can we get the value of y? … We have f of x is [writes on the black board], remember we have f of x is equal to x squared, if we are now going to substitute x by negative three, can you finish up to get the value of f of negative three?...

From the excerpt above, Teacher B defined a function as a one-to-one relationship (see line 3). This definition is incomplete, and especially on this activity, as the learners, together with the teacher determined the values of *y* from the domain {-3; -2; -1; 0; 1; 2; 3}, and for the set of *x*-values (-3; -2; -1; 0; 1; 2; 3), there was a many-to-one relationship, also shown in the graph that was plotted on the board.

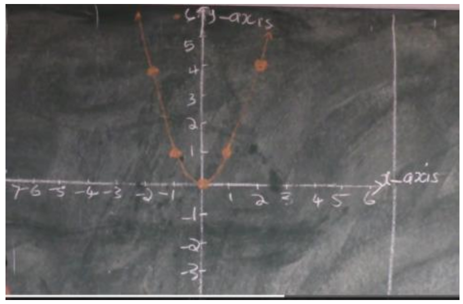


Figure 15: Parabola.

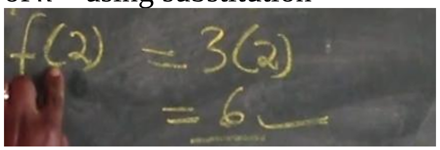
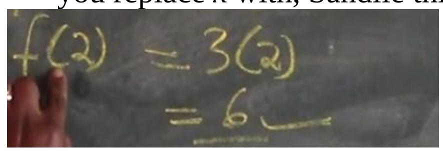
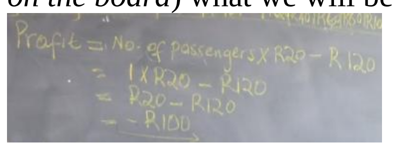
From Figure 15 above, there is a many-to-one relationship: many *x*-values map to one *y*-value. However, neither the teacher nor the learners highlighted the definition that Teacher B gave in line 3 above. Plotting the graph in Figure 15 above in a way that highlights the critical features such as the relationship between the input and output values, would have highlighted the definition of a function that Teacher B gave learners in an attempt to guide them in the plotting process.

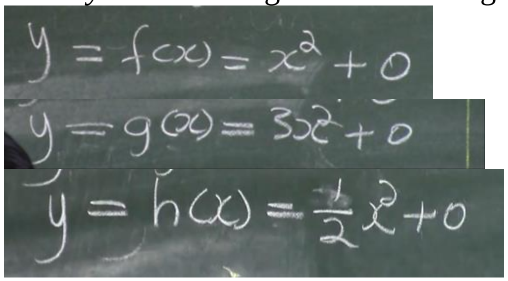
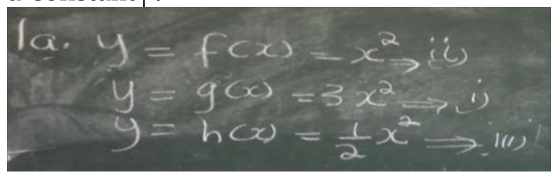
Teacher B's pre-planned activities highlighted three points, which are: 1) choosing the appropriate content and activity to develop mathematics discourse that is not focused on only one aspect of vocabulary; 2) sequencing appropriately to ensure meaningful engagements and 3) using learners' 'in-the-moment' responses to design effective spontaneous activities. Points 2 and 3 also bring in the aspect of how Teacher B used language to enable understanding. On choosing the appropriate content and activities, in the activities selected, Teacher B was not able to get learners to engage in a meaningful way in an attempt to enact the intended object of learning (see discussion of BL2E3 and BL3E5 above). Teacher B's activities were mainly focused on only terminology, without providing learners opportunities to explain and justify their responses. The effect of parameters was taught after point-by-point plotting of the graphs, and Teacher B did not link the two aspects to each other in any way hence the sequencing of the concepts did not allow learners to engage with the activities meaningfully. Even (1990) argues that diverse uses of functions inform the approach of teaching of functions. Sometimes teaching functions may start by dealing with pointwise, which involves plotting of points and dealing with specific points and defining a function on a discrete set (Even, 1990). Teaching functions may also involve looking at intervals rather than pointwise (Even, 1990). Even (1990) further argues that there are times when teaching of functions starts by considering a function in a global way and looking at the behavior of functions. The argument that Even (1990) makes is that all the alternative approaches the teaching of functions are different, and each alternative is appropriate for a specific situation, with some approaches more appropriate than others. The curriculum document that is followed by the teacher may guide how the concept of a function is taught. While engaging with the CAPS curriculum during the analysis of the intended object of learning, I noted that the CAPS does not have a coherent guidance of how to sequence the functions topic in order to present the concepts in a meaningful way. DeMarois and Tall (1999) also argue that there are different facets of the concept of a function that need to be in place in order for learners to be able to engage with functions appropriately (see Chapter 2). I will discuss this further in Chapter 7. In the process of enacting pre-planned/scientific content and activities, Teacher B was faced with numerous instances where she needed to bring in spontaneous (in-the-moment content and activities). I will discuss the spontaneous category later in this Chapter. In the section, I present Teacher B's pre-planned/everyday content and activities.

5.2.2.3 Teacher B's Spontaneous/scientific content and activities

On the issue of 'in-the-moment' and hence spontaneous content and activities, Teacher B was faced with numerous instances where spontaneous content and activities could have helped learners to engage with the content and activities, by linking the discussions to what they already knew. Teacher B did not use many spontaneous activities, as shown in Table 30 above. Two spontaneous activities were evident in Teacher B's lessons. BL1E2 was used to show learners the procedure of substituting and BL2E4 was used by Teacher B to illustrate to learners how to represent a constant term of zero, as shown in the table below.

Table 34: Teacher B's spontaneous/scientific content and activities.

Content/activity	Transcript	Comment
BL1E2 *Content*: Substitution *Activity*: Determining the value of *y*, given the value of *x* – using substitution 	144. Tr: The answer, you get each time you substitute your domain into the given relationship, right, the relationship that we are having here is profit again you get it by substituting the number of passengers and the amount that each person has to contribute, right? Remember what we have been doing, right? What we have been doing in this case, class … you are having a function (*points at black board*) where *f* of *x* is equal to three times *x*, right. (*points to black board*) The two that you replace *x* with, Sandile this two (*pointing at the board*)  You are going to replace *x* by it. The two will be your domain. And if you replace two into this (*points at black board*) eh, relation or function, then you get your answer as six, so two in this case is your input or your domain and six is the range or the output value. What Naomi was saying is the answer of the product you get after a certain process has taken place, the process will be (*pointing at the board*) defined by the equation or the function that you are "*given*". In this case this is (*points at equation on the board*) what we will be using to get the output or the range, clear?  OK (*looking at textbook*), Ehh the question was, what is minimum and the maximum value of the range? What is the increment between each value of the range? [writes question on the black board]… What is the increment in the range [points at student]	Teacher B selected this activity to show learners how to substitute. Interaction The communicative approach used here was IA- Teacher B explained to learners in a lecture mode as shown in the excerpt.
BL2E4 *Content*: Effect of parameters on parabola	85. Tr: Roman figure number two… what do you mean by greater? … 86. Lnr1: Roman figure number two is more bigger↓. 87. Tr: When you say it's more bigger↑, what are you looking at that	Teacher B selected this activity to show them how a constant is represented in

<table>
<tr>
<td>Activity: Determining a constant in a given equation

</td>
<td>makes it to be bigger … what are you looking at↑, that shows you that this one is bigger?.... Tsepo, you have something to say? …
88. Lnr4: Roman figure number two has a constant
89. Tr: People↑, roman figure number…one and number two does it have a constant↑?


90. Lnr1: It has a constant.
91. Tr: What is the constant in roman figure number two↑?
92. Lnrs: One
93. Tr: One, people what is the constant there … in roman figure number two? Where the constant is because she has brought in the constant, where is the constant … or what do you understand by the word constant, in math?
94. Lnr2: No changing
95. Tr: Something that does not change↑.</td>
<td>the equations given.
Interaction
Line 85: Prompt
Line 86: Response
Line 87: Prompt
Line 88: Response
Line 89: Prompt
Line 90: Response
Line 91: Prompt
Line 92: Response
Line 93: Prompt
Line 94: Response
Line 95: Evaluation</td>
</tr>
</table>

Activity BL1E2 was selected by Teacher B to show learners how to substitute. Teacher B did all the talking on this activity. She did not probe learners in anyway on this activity. She explained the procedure of substituting in an authoritative manner. After the explanation, she moved back to the activity that was being discussed in the class. Here, Teacher B saw it necessary to be explicit about the procedure of substituting. She used the spontaneous activity to do that.

Activity BL2E4 was used by Teacher B to engage learners after one learner said that the difference between the equations is the constant (see line 90 in Table 34 above). Teacher B prompted learners to explain further on what a constant means (see lines 91-95). Again here, Teacher B was focusing on terminology as an important aspect of developing learners' mathematics discourse. She evaluated the learner's response as correct in line 95. As discussed earlier, the response given by the learner should have given Teacher B a clue on what learners knew or did not know with regards to terminology, especially terminology that they needed to engage with the activity at hand. Marzano (2004) argues that terminology is vital in the mathematics discourse. In the absence of appropriate terminology, learners may not be able to engage and hence develop their mathematics discourse. However, terminology is not the only aspect that is required to develop learners' mathematics discourse as argued by Walshaw and Anthony (2008). Teacher B's lesson had a lot of focus on terminology as shown in the activities discussed.

Although Teacher B interacted with learners and allowed them to respond to questions during the lessons, the focus was mainly on vocabulary, definitions, and conventions, and this is at the lexical level. Teacher B did not rectify or correct conventions that were used by the learners. Opportunities for comprehensive mathematical discussions during the lessons were not evident in Teacher B's lessons. There were no instances of learners providing explanations and justifications to their responses.

5.2.2.4 Teacher B's Spontaneous/Everyday content and activities

In the lessons observed, Teacher B did not have any spontaneous content and activities with everyday contexts, in relation to the intended object of learning that the teacher wanted to enact in the lesson.

5.3 Strategies in Teacher B's lessons

In Teacher B's lessons, certain strategies were visible (or not) in the process of the teacher selecting and enacting the intended object of learning in ensuring that learners develop their mathematics discourse and understanding. In this section, I will engage with the strategies that I identified during the analysis of data for Teacher B.

5.3.1 Stating the intended object of learning

Teacher B did not state any intended object of learning in all her lessons observed. In lesson B1, Teacher B engaged with an everyday content and activity, and the intended object of learning was not clear. The lesson did not go according to plan and this was confirmed by the post-observation interview, where Teacher B explained what she intended to accomplish in the lesson.

Stating the intended object of learning could have helped the teacher and learners to focus on the essential aspects and ignore the non-essential aspects in the lesson. If Teacher B had stated the intended object of learning and kept it in focus, it would have informed the content and activities that were selected to assess learners' prior knowledge, as discussed below.

5.3.2 Assessing learners' prior knowledge

Teacher B assessed learners' prior knowledge by asking learners questions on concepts that were covered in previous lessons as shown in the discussion above. However, Teacher B's engagement with learners' prior knowledge did not enable learners to develop their understanding through the probing and teacher involvement. The extract below is one example where prior knowledge of the learner was not used to develop on what was being discussed.

104. Tr: Clear? [Pointing at the board] So in all the three graphs, the y intercept is zero, even when you go and check the graph, all the three graphs are passing through the point zero and zero because they are having the y intercept which is zero. So I don't know what you were talking about the constant, what were you referring to? …What were you referring to? … They are all having the same constant, which is?
105. Lnrs: Zero

Extract 1

In the excerpt above, Teacher B did not give learners an opportunity to tell her what they were talking about when they referred to a constant. This would have helped to clarify the distinction between a number as a coefficient and as a constant and she would not have had to move on to $q = 0$ and hence off the focus on the effect of a, that which was varying.

As discussed in Chapter 3, assessing learners' prior knowledge gives the teacher guidance on how to proceed with the lesson. Instructional practices, learner interactions, and feedback to learners' responses need to be focused on mathematical ideas central to the learning goal (NCTM, 2014), which I refer to as the object of learning. This brings me to the next focus of analysis, the content and activities that Teacher B selected for teacher strategies in an attempt to enact the intended object of learning.

5.3.3 Patterns of variation as a strategy

In the discussion above, I engaged with the aspects of variation that were evident in Teacher B's example set, although Teacher B's lesson was not driven by variation theory. In this section, I highlight some important points that came out from the analysis of Teacher B's example set. In order to discern something, one has to be able to discern the critical features of that object (Marton & Booth, 1997). For example, when learners are required to engage with a straight line, they may have no difficulty in visualising it if they already have knowledge of it, and the key features that are associated with a straight-line such as the general form of the equation of a straight line. Thus, to be able to discern something, one has to have experienced variation in an equivalent facet of the aspect (Holmqvist et al., 2011). That is, to be able to discern a straight-line graph, you need to discern other types of functions. In Teacher B's example set, she discussed two types of functions: a linear function and a quadratic function. However, opportunities for learners to compare the two types of functions were not afforded to the learners by the teacher. Teacher B had limited examples that she used to bring the key features of the object of learning to the fore (see Table 28). Teacher B had one example in Lesson B1, and so this means that only one example of a function – a linear function, was presented to the learners. However, there were a range of inputs and hence outputs, which led to some sort of variation, as the example gave different number of passengers but kept the price of each passenger constant (see Table 29). Although Teacher B had some activities that had similarity pattern, this was not used a strategy to draw learners' attention to what was varying, with reference to the intended object of learning. This was evident in activity BL2E3, where aspects were varying (the co-efficient of x) and could have helped learners to engage with the differences, as discussed above.

5.3.4 Double move as a strategy

Teacher B had only one everyday context (see example BL1E1). The everyday example was used throughout Lesson B1, and only linked to the concept of the function towards the end of the lesson. As stated earlier, I found it challenging to follow the focus of the lesson based on the teacher and learner interactions in Lesson B1. In the post-observation interview, I asked Teacher B about the intended outcome of Lesson B1 and her rationale for the choice of example of the taxi and the passengers. Teacher B said that she always uses this example to start her lesson on functions as shown in the interview excerpt above. Teacher B alluded to the fact that she uses everyday context as a strategy, however, the enactment of the everyday content and activity was not successfully done as discussed above., the double move as a strategy for Teacher B was available but not used effectively as discussed earlier in this Chapter.

5.3.5 Learner feedback

As discussed in Chapter 4, in order for feedback to be given, there has to be an interaction between the teachers and learners. Teacher B engaged learners in an I-R-E pattern of discourse as discussed in this Chapter. Teacher B's feedback to learners was mostly evaluative. Feedback as a strategy in her lessons was used to give an authoritative response to learners and not to understand what learners were communicating about their level of mathematics discourse and understanding in the interaction, as discussed above.

5.4 Conclusion

In this Chapter, I have presented findings related to Teacher B 's role of developing learners' mathematics discourse and understanding. From the lessons observed and interview data, it became apparent that Teacher B relied on the textbook to deliver her lessons. In some instances, it was challenging to follow Teacher B's lessons because the intended object of learning was not stated upfront in the lesson. Learners were given ample opportunities to engage in the lesson, although the engagement was teacher-led. Throughout the lesson, learners did not ask any questions. Teacher B engaged learners in an interactive-authoritative (IA) communicative approach, with some occurrences of interactive-dialogic (ID). The role of Teacher B, as discussed in this Chapter, is summarised in the figure below.

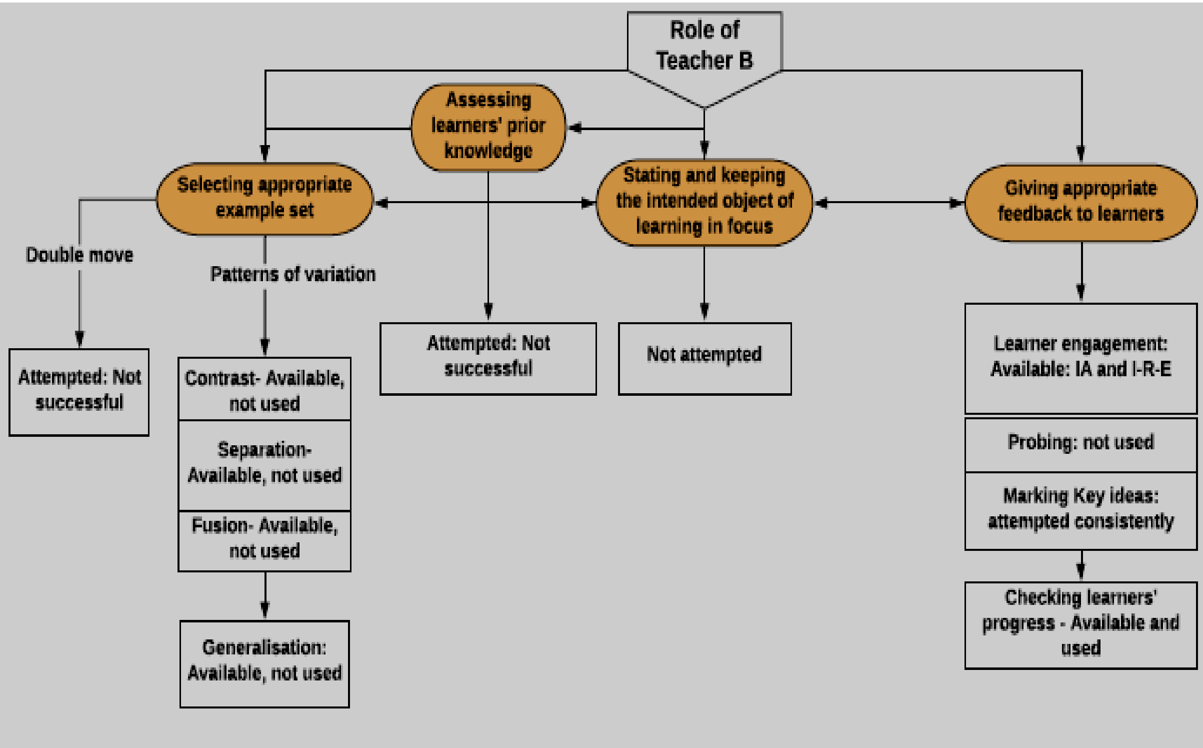


Figure 16: The role of Teacher B in developing learners' mathematics discourse and understanding.

Figure 16 above is a summary of the role of Teacher B in developing learners' mathematics discourse and understanding, as discussed in this Chapter. Stating the intended object of learning and keeping it in focus throughout the lesson is critical role for the teacher, and this was absent in Teacher B's lessons. Feedback that was given to learners was authoritative. Teacher B's selection and enactment of the example set did not provide opportunities for learners to develop their mathematics discourse and understanding, as I have discussed in this Chapter. In the next Chapter, I will present findings of Teacher C.

CHAPTER 6: The role of the teacher: The case of Teacher C

6.1 Introduction

In this Chapter, I present findings of Teacher C on the role of developing learners' mathematics discourse and understanding. I observed Teacher C teaching five lessons which I referred to as: C1 - C5. Two of the lessons lasted about 50 minutes while the other three lasted about 30 minutes. The lessons were focused on different aspects: lesson C1 was focused on defining a function, lesson C2 and C3 focused on the effect of a and q respectively, on the straight-line graph - with general formula $y = ax + q$. In lesson C4, Teacher C focused on drawing a linear graph using gradient method and intercept method. Lesson C5 focused on determining the equation of a straight-line graph.

As discussed in Chapter 4 and 5, in the present Chapter too, I will engage with my research questions in the presentation and discussion of the findings. Like the previous Chapters, I will begin by presenting the intended object of learning, followed by the kinds of content and activities that Teacher C selected. As I present the content and activities, I will present the findings and discussion of how the communicative approach and patterns of discourse shaped the enactment of the content and activities selected by Teacher C. The strategies that Teacher C employed in ensuring that learners discern the critical features of the object of learning will be presented in this Chapter.

6.2 Kinds of content and activities selected by Teacher C

Before presenting the kinds of content and activities, I will begin by presenting the intended object of learning. Teacher C did not announce the lesson objectives in all the lessons observed. Therefore, the intended object of learning of each lesson was inferred from the enacted object of learning – the lessons observed. I then compared the inferred intended object of learning to the Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011), for validation purposes, as shown in Tables 35-39.

6.2.1 Intended object of learning

In Tables 35-39, the intended object of learning of each lesson was inferred from the content that was being discussed during the lesson. I then drew on the Curriculum and Assessment Policy Statement (CAPS) to ascertain if Teacher C's content and activity selection, and hence intended object of learning had a correlation with the curriculum. In what follows, I discuss the five lessons in relation to the intended object of learning that was evident during the lesson observations, and in relation to the CAPS curriculum. It is important to note here that the discussion that follows below is a brief discussion in an attempt to establish the intended object of learning in each lesson. A detailed discussion of the lessons, with a presentation and analysis of the example set that was created by Teacher C in her lessons will follow in the section after this one.

Lesson C1 was the first lesson on functions that Teacher C presented. In this lesson, learners were given a pre-test that lasted for 35 minutes and the last 15 minutes were used to define a function. The intended object of learning in Lesson C1 is presented in Table 35 below.

Table 35: Lesson C1's intended object of learning.

Intended object of learning inferred from the content of Lesson C1	**Linking the intended object of learning from lesson C1 content to CAPS**
➢ Definition of a function: one to one and many to one relationship ➢ Naming different in which to represent a function: Table, graph, words, equation, symbols, flow diagram	**Curriculum statement**: "The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value). Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations" (DBE, 2011, p.24). **Clarification comment from CAPS**: "A more formal definition of a function follows in Grade 12. At this level it is enough to investigate the way (unique) output values depend on how input values vary. The terms independent (input) and dependent (output) variables might be useful" (DBE, 2011, p.24).

From Table 35 above, Teacher C's focus of the lesson was in line with the curriculum statement, which is to define a function. Different representations of functions were named in the lesson observed.

In C2, Teacher C recapped the definition of a function that was discussed in the previous lesson before moving on to the general formula of a straight-line graph. The main focus of the lesson was the effect of *a* on the straight-line graph, with general formula [5]$y = ax + q$.

The intended object of learning in Lesson C2 is presented in Table 36 below.

Table 36: Lesson C2's intended object of learning.

Intended object of learning inferred from the content of Lesson C2	**Linking the intended object of learning from lesson C2 content to CAPS**
Effect of *a* on the straight-line graph, with general formula $y = ax + q$: when a is positive, the graph increases. When a is negative, the graph decreases	**Curriculum statement:** "Investigate the effect of *a* and *q* on the graphs defined by $y = a.f(x) + q$, where $f(x) = x$" (DBE, 2011, p. 24). **Clarification comment from CAPS**: "After summaries have been compiled about basic features of prescribed graphs and the effects of parameters *a* and *q* have been investigated: a: *a* vertical stretch (and/or a reflection about the *x*-axis) and *q* a vertical shift" (DBE, 2011, p. 24).

As stated in Table 36 above, lesson C2's intended object of learning was to investigate the effect of *a* on the straight - line graph, and this was in line with the curriculum statement as shown in Table 36 above. Teacher C focused on when the graph increases or decreases based on the sign of *a*. In the clarification comment in the table above, a vertical stretch was stated as the effect of *a* on the graph. Teacher C emphasised the effect of *a* on the steepness of the graph, which is linked to a vertical stretch (graph moving towards the *y*-axis) or horizontal stretch (line moving towards the *x*-axis).

Lesson C3 was the third lesson on functions. Teacher C started by assessing learners' prior knowledge on the effect of *a* on the straight-line graph from the previous lesson. The lesson focused on the effect of *q* on the straight-line graph, with general equation $y = ax + q$. The intended object of learning in Lesson C3 is presented in Table 37 below.

[5] *a* and *q* is used interchangeably with *m* and *c* to get $y = mx + c$

Table 37: Lesson C3's intended object of learning.

Intended object of learning inferred from the content of Lesson C3	Linking the intended object of learning from lesson C3 content to CAPS
Effect of *q* on the straight-line graph, with general formula $y = ax + q$: when the value of *q* changes from zero to another number, the graph shifts	**Curriculum statement:** "Investigate the effect of *a* and *q* on the graphs defined by $y = a.f(x) + q$, where $f(x) = x$" (DBE, 2011, p. 24). **Clarification comment from CAPS**: "After summaries have been compiled about basic features of prescribed graphs and the effects of parameters *a* and *q* have been investigated: a: *a* vertical stretch (and/or a reflection about the *x*-axis) and *q* a vertical shift" (DBE, 2011, p. 24).

Lesson C3's intended object of learning was to investigate the effect of *q* on the straight-line graph, and this was according to the curriculum statement as shown in Table 37 above. Teacher C focused on how the graph shifts when the value of *q* changes from zero to any number.

In Lesson C4, Teacher C gave learners two trigonometric graphs and asked them to determine the coordinates of certain points, and also determine the period and amplitude of each graph. Learners were then asked to check when the given graphs are decreasing or increasing. This activity on trigonometric graphs was given as a quick activity to begin the lesson. Teacher C went on to revise the effect of *a* and *q* on a straight-line graph from previous lessons. The main focus of the lesson was on sketching a linear graph without a table of values. The intended object of learning in Lesson C4 is presented in Table 38 below.

Table 38: Lesson C4's intended object of learning.

Intended object of learning inferred from the content of Lesson C4	Linking the intended object of learning from lesson C4 content to CAPS
Drawing a linear graph without a table of values: gradient method and intercept method	**Curriculum statement:** "Sketch graphs, find the equations of given graphs and interpret graphs" (DBE, 2011, p. 24). **Clarification comment from CAPS**: "Sketching of the graphs must be based on the observation of the effect of the parameters *a* and *q*" (DBE, 2011, p. 24).

In lesson C4, Teacher C focused on two methods of sketching a graph without a table of values. The curriculum statement refers to this as sketching a graph while Teacher C refers to it as drawing a graph. During the lesson, the teacher and the learners were all sketching the graph because they were all using freehand to make basic drawings of straight lines.

Lesson C5 began with Teacher C assessing learner's prior knowledge on gradient and *y*-intercept method of sketching graphs. Teacher C then moved on to the focus of the lesson, which was determining the equation of a given graph, as shown in Table 39 below.

Table 39: Lesson C5's intended object of learning.

Intended object of learning inferred from the content of Lesson C5	**Linking the intended object of learning from lesson content to CAPS**
Determining the equation of a given graph.	**Curriculum statement:** "Sketch graphs, find the equations of given graphs and interpret graphs" (DBE, 2011, p.24). **Clarification comment from CAPS**: "Sketching of the graphs must be based on the observation of the effect of the parameters *a* and *q"* (DBE, 2011, p.24).

Lesson C5 focused on finding the equation of a straight-line graph. This was in line with the curriculum statement as shown in Table 39 above.

In the discussion above, I engaged with the intended object of learning of Teacher C's lessons observed. In what follows, I present the content and activities, which is the example set that was used by Teacher C in an attempt to enact the intended object of lesson in each lesson.

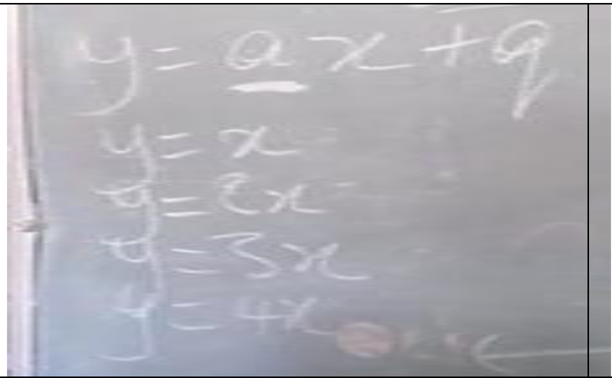
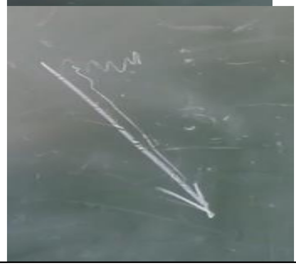
6.2.2 Content and activity selection to enact the intended object of learning

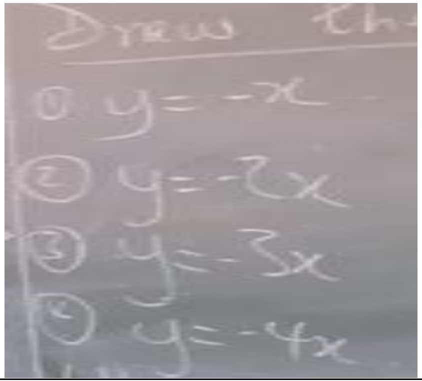
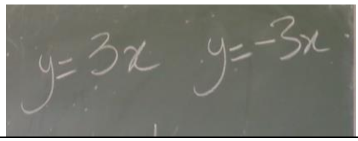
Teacher C selected content and activities that learners were required to attempt and engage with in the lessons. In the table below, I present all the content and activities, hence the example set that Teacher C selected in the five lessons observed. The key features of the object of learning as per mathematics discourse are also presented in the table. Teacher C's content and activities are categorised as either pre-planned or spontaneous, as discussed in

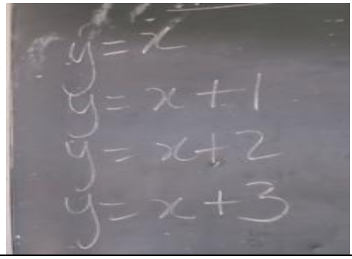
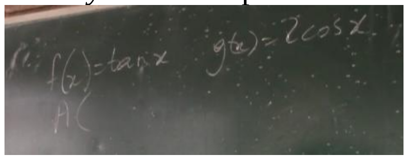
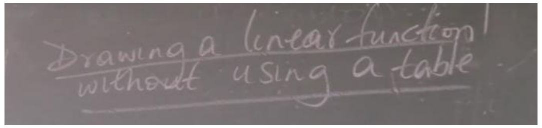
Chapters 4 and 5. The content and activities were further categorised as either scientific or everyday, as shown in the table below. The examples below have been numbered continuously from lesson 1 to lesson 5, for example, for CL1E1, C stands for Teacher C, L1 stand for an example in lesson 1, E1 stands for the first example in Teacher C's example set.

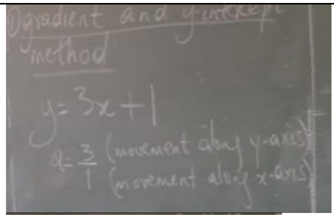
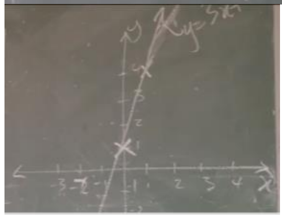
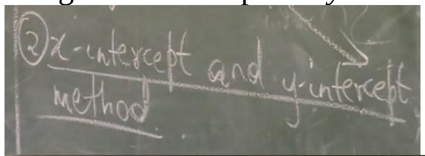
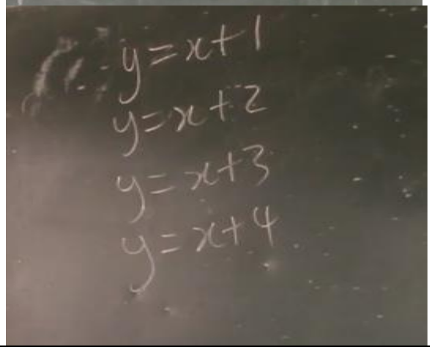
Table 40: Example set and the key features of the object of learning evident in Teacher C's lessons.

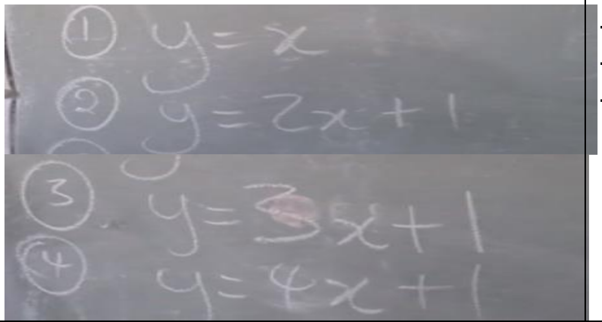
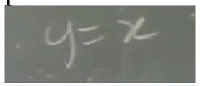
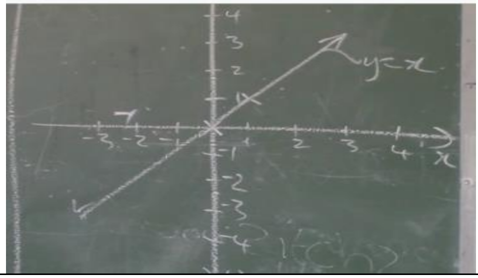
Intended object of learning	**Item number**	**Content/activity**	**Key features from the lesson observed**	**Comment**
1. Defining a function	CL1E1	*Content:* Definition of a function *Activity:* What is a function?	-One to one -many to one -*x* value -*y* value -relationship -input value -output value	- **Pre-planned** content and activity: Teacher C started with this activity at the start of the lesson, as an introduction to functions - **Scientific** content: Teacher C and the learners defined a function using terms from the mathematics register
	CL1E2	*Content:* Naming Five different representations of functions *Activity*: What are the five representations of functions?	-Flow diagram - words -symbols -Graphs -Table	- **Pre-planned** content and activity: Teacher C started with this activity at the start of the lesson, as an introduction to functions - **Scientific** content: Teacher C and the learners used mathematics terms to name the representations of a function
2. Effect of parameter *a* on a straight-line graph	CL2E3	*Content:* Sketch straight line graphs, to investigate the effect of a positive gradient (the value of *a* in the linear function) *Activity:* What is a linear function? What is the effect of parameter *a* on a linear graph?	-Gradient -positive gradient -negative gradient -Cartesian plane	- **Pre-planned** content and activity: Teacher C started with this activity at the start of the lesson - **Scientific** content: Teacher C and the learners used mathematics terms to describe the effect of parameter *a*

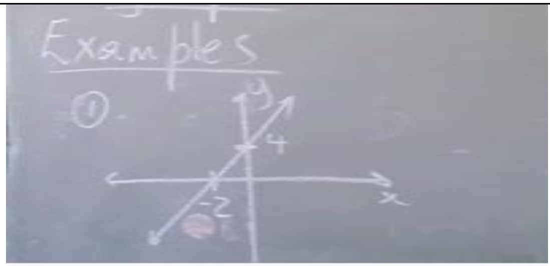
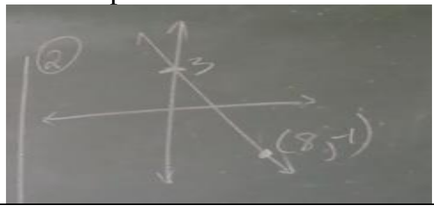
				
	CL2E4	*Content:* Increasing vs decreasing graph *Activity:* How do you notice a decreasing or increasing graph?  	-positive gradient -negative gradient -steeper	-**Spontaneous** content: Teacher C formulated this activity during the lesson to illustrate an increasing and decreasing graph -**Scientific** content: Teacher C used mathematical representation
	CL2E5	*Content:* Effect of positive *a* on a straight-line graph *Activity:* Comparing to writing convention		-**Spontaneous** content: Teacher C formulated this activity during the lesson to illustrate the effect of *a* on a straight-line graph

		to how a graph is affected with a positive a		**-Everyday** content: Teacher C used everyday context of writing convention
	CL2E6	*Content:* Investigate the effect of a negative gradient (the value of *a* in the linear function) *Activity:* Draw straight line graphs using a table of values, to investigate the effect of a negative gradient (the value of *a* in the linear function) 	-domain -range -table of values -Cartesian plane -Decreasing graph -negative gradient	**-Pre-planned** content and activity: Teacher C copied the activity from her notebook on to the board **-Scientific** content: Teacher C used mathematical representation and symbols – equations of straight lines
	CL3E7	*Content:* Effect of parameter a on a straight-line graph *Activity:* Draw the graphs to compare the effect of *a* 	-positive gradient -negative gradient -Cartesian plane	-**Spontaneous** content: Teacher C formulated this activity during the lesson to illustrate the effect of the sign on parameter *a* **-Scientific** content: Teacher C used mathematical representation and symbols
3. Effect of	CL3E8	*Content:* Plot straight line graphs	-*y* intercept	- **Pre-planned** content and activity:

parameter *q* on a straight-line graph		*Activity:* Plot graphs on a graph paper and investigate the effect of *q*  	-table of values -*y*-intercept -gradient	Teacher C copied the activity from her notebook on to the board **-Scientific** content: Teacher C used mathematical representation and symbols –equations of straight lines
	CL4E9	*Content:* Interpreting trigonometry graphs *Activity:* Effect of parameters  	-trigonometry -coordinates -increasing -decreasing -period -amplitude	-**Pre-planned** content and activity: Teacher C came to class with worksheets of the activity and gave them to the learners **-Scientific** content: Teacher C used mathematical representation and symbols
4. Sketch a linear function without a table of values	CL4E10	*Content:* Sketch a linear function without using a table of values *Activity:* Sketch a linear function using the Gradient and *y*-intercept method  	-Cartesian plane -gradient -*y*-intercept -*x*-intercept -coordinates -substitute -plot	-**Pre-planned** content and activity: Teacher C copied the activity from her notebook on to the board **-Scientific** content: Teacher C used mathematical representation and symbols – equations of straight lines, and graphs

		 		
	CL4E11	*Content:* Sketch a linear function without using a table of values *Activity:* Sketch the straight line graphs using the *x*-intercept and *y*-interept method  	-Cartesian plane -gradient -*y*-intercept -*x*-intercept -coordinates -substitute -plot	-**Pre-planned** content and activity: Teacher C copied the activity from her notebook on to the board **-Scientific** content: Teacher C used mathematical representation and symbols
	CL4E12	*Content:* Sketching straight line graphs	-Cartesian plane	-**Pre-planned** content and activity:

		Activity: Sketch the graphs without a table of values 	-gradient -*y*-intercept -*x*-intercept -coordinates -substitute -plot	Teacher C copied the activity from her notebook on to the board **-Scientific** content: Teacher C used mathematical representation and symbols
4. Determine the equation of a straight-line graph	CL5E13	*Content:* Equation of a straight line graph *Activity:* Plot the graph on the Cartesian plane  	-Cartesian plane -gradient -*y*-intercept -*x*-intercept -coordinates -substitute -plot -equation -variable -equal sign	-**Pre-planned** content and activity: Teacher C asked learners to refer to work that was given previously **-Scientific** content: Teacher C used mathematical representation and symbols
	CL5E14	*Content:* Equation of a straight line graph *Activity:* Determine the equation of a straight line graph when given the intercepts.	-Cartesian plane -gradient -*y*-intercept -*x*-intercept -coordinates -substitute -plot	-**Pre-planned** content and activity: Teacher C copied the activity from her notebook on to the board **-Scientific** content: Teacher C used mathematical representation and symbols – sketches of straight-line

			-equation -variable -equal sign	graphs
	CL5E15	*Content:* Equation of a straight line *Activity:* Determine the equation of a straight line when given the y-intercept and another point on the line. 	-Cartesian plane -gradient -*y*-intercept -*x*-intercept -coordinates -substitute -plot -equation -variable -equal sign	-**Pre-planned** content and activity: Teacher C copied the activity from her notebook on to the board **-Scientific** content: Teacher C used mathematical representation and symbols

The content and activities selected by Teacher C were mostly pre-planned. Teacher C drew on scientific context mostly to illustrate and demonstrate concepts as shown in the example set presented in Table 40 above. Table 41 below is a summary of the content and activities that Teacher C selected. ✔ indicates that the type of content and activity was present while ***X*** indicates that the type of content and activity was not present.

Table 41: Kinds of content and activities selected by Teacher C in the lessons observed.

	Type of content and activity				
Example set Item number	**Spontaneous/ Scientific**	**Spontaneous/ Everyday**	**Pre-planned /Scientific**	**Pre-planned /Everyday**	**Communicative approach/Pattern of discourse**
CL1E1	***X***	***X***	✔	***X***	IA and I-R-E
CL1E2	***X***	***X***	✔	***X***	IA and I-R-E
CL2E3	***X***	✔	✔	***X***	IA and I-R-E
CL2E4	✔	***X***	***X***	***X***	IA and I-R-E
CL2E5	***X***	✔	***X***	***X***	IA and I-R-E
CL3E6	***X***	✔	✔	***X***	IA and I-R-E
CL3E7	✔	***X***	***X***	***X***	IA and I-R-E
CL4E8	***X***	***X***	✔	***X***	IA and I-R-E
CL4E9	***X***	***X***	✔	***X***	IA and I-R-E
CL4E10	***X***	***X***	✔	***X***	IA and I-R-E
CL4E11	***X***	***X***	✔	***X***	IA and I-R-E
CL5E12	***X***	***X***	✔	***X***	IA and I-R-E
CL5E13	***X***	***X***	✔	***X***	IA and I-R-E
CL5E14	***X***	***X***	✔	***X***	IA and I-R-E
CL5E15	***X***	***X***	✔	***X***	IA and I-R-E

The table above is a summary of the kinds of content and activities that were selected by Teacher C in her lessons. Teacher C had 15 examples in the example set, two were spontaneous/scientific, three were spontaneous/everyday activities, twelve activities were pre-planned/scientific, and there were no pre-planned/everyday activities. I will now present the findings and discussion of the kinds of content and activities, by presenting the pre-planned, spontaneous, everyday and scientific content and activities that were evident in Teacher C's lessons.

6.2.3 Teacher C's Pre-planned/Scientific content and activities

Teacher C had twelve pre-planned/scientific content and activities out of fifteen items in her example set. Teacher C's pre-planned activities were mostly to check learners' prior knowledge. As stated in the earlier Chapters, assessing learners' prior knowledge is important and gives the teacher an opportunity to identify learner preconceptions and misconceptions (Carl et al., 2009). In all Teacher C's observed lessons, learners were asked questions about the concepts to be covered, before they were discussed as shown in the lesson excerpt of CL1E1 engagement below. CL1E1 is a pre-planned activity and is categorised as assessing learners' prior knowledge (LPK). CL1E1 was presented in Lesson C1, and this was the first lesson on functions. As stated earlier, Teacher C gave learners a 35-minute test to check their prior knowledge on the topic. She then collected the test for marking. After learners had written the test, Teacher C then selected CL1E1, an activity from the test and engaged the learners as shown in the except below. CL1E1 was given to learners to assess their prior knowledge on the concept of a function.

Table 42: Communicative approach and Pattern of discourse of Activity CL1E1.

Transcript: Definition of a function	**Code**
1. *Tr: Class, so today we are starting our topic on functions, I want you to write this test that I told you about last week. Write the answers in your books. We do not have a lot of time so, Katlego….[Teacher distributing the tests]* 2. *Lnrs: [writing the test individually in silence]* 3. *Tr: [Walking around the class, and after 35 minutes she asks learners to stop writing] So time is up. I want you to collect your books. What is a function? What is a function?* 4. *Lnr1: A function is a relationship between two variables where one x value has a corresponding y value.* 5. *Tr: Does that make sense?* 6. *Lnrs: Yes↑* 7. *Lnrs: Nooo↑↑* 8. *Tr: Who can define for us a function?* 9. *Lnr2: A function is a relationship between the input and output value, where every input has an output value or where more than one input may have one output value.* 10. *Tr: Beautiful, beautiful…does that make sense? We can have a…. a one to one relationship or a many-to-one relationship. For one-to-one, the relationship is between an input and an output value. For the many-to-one, many input values have only one output value. Does that make sense?* 11. *Lnrs: Yes↑* 12. *Tr: What are the five representations of functions?...what did you write in your books?* 13. *Lnrs: Flow diagram* 14. *Lnr3: words* 15. *Lnr3: equation* 16. *Lnr4: Table* 17. *Tr: Table, very good very good. What else did you write?* 18. *Lnr2: Graph* 19. *Tr: very good, graph. And we have different types of graphs. So in our next lesson we will look at what a linear graph is. For your homework, go and find out what a linear graph is. We will continue from here in our next lesson. Collect the books for marking.*	Line 3: Initiation Line 4: Response Line 5: Prompt Line 6: Response Line 7: Response Line 8: Evaluation/Prompt Line 9: Response Line 10: Evaluation Communicative approach: IA Patterns of discourse: I-R-E

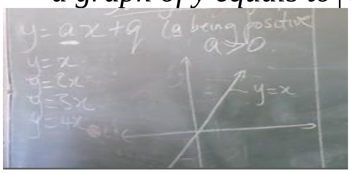
From the excerpt above, Teacher C assessed learners' prior knowledge with a written test. Unfortunately, I was not able to get hold of the test in order to engage with the kinds of activities that were given to the learners. I only had access to the activities that the teacher engaged with in the lesson: CL1E1 and CL1E2 (see Table 42 above). In CL1E1, Teacher C asked learners to share their definitions of functions (Line 3). In line 4, a learner defined a function- with a focus on only one-to-one relationship. Without evaluating the learner's response, Teacher C probed the other learners to analyse the definition of the learner (see line 5, 6 and 7). Teacher C asked the question again (line 8). The teacher's question in line 8 was an evaluation of the definition given in line 3. Without explicitly providing feedback to the learner, Teacher C marked the definition as incorrect (MKI), by asking for someone else to define a function. Another learner defined a function, with a focus on both one-to-one and many-to-one relationship. Teacher C marked the definition as correct (MKI) and revoiced the learner's response (RLE) for emphasis, and also asked learners to confirm if they agree with the definition (see line 9). From the definition of a function in line 4, learners were comparing the change in *x*- and *y*- values, and this is similar to what Smith and Confrey (1994) found in their study as discussed in Chapter 2. According to Even (1990), a function is "any set of ordered pairs of elements such that if $(a, b) \in f$, $(c, d) \in f$, and $a = c$, then $b = d$" (p.531). Through assessing of prior knowledge, Teacher C had an opportunity to check learners' preconceptions or if learners had any misconceptions that needed her attention. The definitions given by the learners above were focused on the special subset of Cartesian products and it did not engage with the form of correspondence or assignment between two sets or variables.

Teacher C also assessed if the learners were able to name the different ways of representing functions (see CL1E2 in Table 42 above). From the responses in lines 13 – 18, it is clear that learners had prior knowledge of different representations of functions. The responses from the learners provided important information for the teacher, especially around how to proceed with the lesson in relation to what learners already knew. With regards to representation of functions, the CAPS required more engagement than simply stating the different ways of representing functions, and as the lessons went on, Teacher C engaged with working with the representations by giving content and activities on plotting the graph with a table of values and determining the equation of a given graph, as I will discuss later in this section.

In the excerpt above, the teacher initiated the interaction with a question (lines 3, 8 and 12), the learners answered the question (see lines 4, 9, 11, 13-16 and 18) and the teacher evaluated the learners' responses (see lines 10, 17 and 19). Therefore, the interaction above was an I-R-E pattern of discourse. Teacher C allowed learners to share their ideas (interactive). However, learners' ideas were not considered – the teacher's responses to learners' ideas showed that she was looking for a specific answer to her question (see line 8, 10, 17 and 19). Therefore, interactive-authoritative (IA) was evident in the interaction on CL1E1 and CL1E2.

The next pre-planned/scientific content and activity I discuss is CL2E3. As stated earlier, Lesson C2's intended object of learning was the effect of parameter *a* on a straight-line graph. Before discussing the effect of *a*, Teacher C engaged with the key features of the object learning such as *a* being positive, *a* being the gradient of the straight line and *a* being negative. Teacher C then moved on to the effect of parameter *a* as shown in the excerpt below.

Table 43: Communicative approach and Pattern of discourse of CL2E3.

Transcript: **Effect of positive parameter a on the straight-line graph**	**Code**
38. *Tr: [Writes on the board] y equals to three x, draw a graph of y equals to↑?*  39. *Lnrs: Four x* 40. *Tr: Four x, now I want you to tell me, as a grows bigger and bigger, what happens to our graph, what happens to our straight-line graph?* 41. *Lnr1: It shifted up.* 42. *Tr: You say, did it shift up, [writes on board] OK, so let's say that's our first graph y equals to x, right, so the second graph is it going to be on that side (points to the left of the line y = x in the first quadrant) or it's going to be on that side, up down?* 43. *Lnrs: Up↑* 44. *Lnr: Yes↑* 45. *Tr: OK, [writes on the board] so it's going to be up, so y equals to two x, and then [draws the line y = 2x on the board]*	Line 38: Prompt Line 39: Response Line 40: Initiation Line 41: Response Line 42: Prompt Line 43: Response Line 44: Response Line 45: Evaluation Communicative approach: IA Pattern of discourse: I-R-E

From the excerpt above, Teacher C assessed learners' prior knowledge on the effect of *a* on the straight-line graph by giving them equations: $y = x$ and $y = 2x$ to engage with.

Hailikari, Nevgi and Lindblom-Ylänne (2007) argue that any assessment should bring value to both parties, the assessor and the learner. In other words, the assessment should be informative and have enough detail to guide and support instruction (Dochy, 1996). Hailikari et al. (2007) also argue that a good prior knowledge assessment should distinguish between different types of knowledge, the knowledge of 'what', 'how' and 'why'. In the prior knowledge assessment for Teacher C, the knowledge of 'what' was emphasised, as shown in the excerpt above – the other types were not assessed. Valencia, Stallman, Commeyras, Pearson, and Hartman (1991) argue that various forms of prior knowledge assessments have to be used in an attempt to capture a holistic view of learners' prior knowledge. Teacher C's prior knowledge assessment was used to guide the lesson.

After Teacher C had assessed learners' prior knowledge, she then continued to enact the intended object of learning, with a basis on the prior knowledge feedback that was received during the lessons. Holmqvist et al. (2011) argue that teachers need to take careful attention when selecting the opportunities that are given to learners in relation to what is already known and the aspects of the object of learning that are offered in the lessons taught. This was done well by Teacher C's pre-planned/scientific content and activities.

Although learners gave chorus answers, they were actively engaged in the lessons and they answered all Teacher C's questions, and Teacher C evaluated their responses authoritatively, hence interactive-authoritative (IA). The communicative approach that was evident in this lesson was I-R-E as shown in the table above.

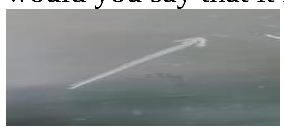
6.2.3.1 Teacher C's Pre-planned/Everyday content and activities

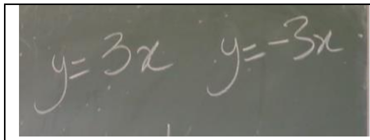
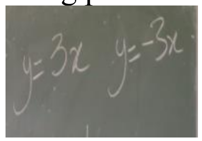
As discussed in Chapter 4, pre-planned/Everyday content and activities are selected before the lessons, and have context from everyday, in relation to the intended object of learning that the teacher wants to enact in the lesson. Teacher C did not have any pre-planned/everyday content and activities in the example set, as shown in Table 41 above.

6.2.3.2 Teacher C's Spontaneous/Scientific content and activities

As discussed in Chapter 4 and 5, spontaneous content and activities are those that are selected 'in-the-moment', - during the lesson. Teacher C had two spontaneous/scientific content and activities in the example set. In Table 44 below, I present CL2E4 and CL3E7 to show how Teacher C enacted these kinds of content and activities.

Table 44: Enacting CL2E4 and CL3E7.

Content/Activity	**Transcript**	**Comment**
CL2E4 *Content:* Increasing vs decreasing graph *Activity:* How do you notice a decreasing or increasing graph?  	62. Tr: OK so this will get closer to the *y* axis, right, so in other words the description, the two things that, they should be in your mind as you think about the gradient, the value of *a*, right, when *a* is positive, OK, first thing what happens when we write like as uuhmm… writes on the board we right from left to right, OK, unless you stay Afghanistan where they write from, OK, from right to left, but in South Africa when we write we write from left to right, OK, so when you look at that line [writes on board] how would you, how would you describe that, how would you say that it is increasing or decreasing?  63. Lnrs: Increasing 64. Tr: This is increasing [writes on board], OK, but if I draw a line like that?  65. Lnrs: Decreasing 66. Tr: That should be decreasing [writes on board] going down, right, so what should you say if *a* is positive what type of (*inaudible*) graph do we have, what does it do?	Line 62: Initiation Line 63: Response Line 64: Prompt Line 65: Response Line 66: Evaluation Communicative approach: IA Pattern of discourse: I-R-E
CL3E7 *Content:* Effect of parameter a on a straight-line graph *Activity:* Draw the graphs to compare the effect of *a*	. Tr: [Writes on board] Closer to the *x* axis, moves or away from *y* axis, it's the same thing, away from *y* axis closer to the *x* axis, right. OK, so, now uhm let's say I give you two graphs and I say, OK, [Writes on board] *y* is equal to *3x* right, and I say *y* is equal to negative three *x* and then I draw right, so I want you to draw just a graph uhm just a small graph, I want you to show me the difference between those two graphs just draw, just draw them, yes in your exercise books, just draw uhm don't draw a table, I just	Line 66: Initiation Line 67: Response (written) Line 68: Prompt Line 69: Response (written) Line 70: Evaluation Communicative approach: IA

	want to see if you uhm, if you get the picture of *a* being negative and *a* being positive …  Lnrs: [Drawing the graphs in their books] Tr: Right, just want, I just want a rough sketch, very small uhm, OK, just a rough sketch, I want to see if you get the difference between the two … guys that's a one-minute thing … OK, raise up your hand for marking … OK, please don't, don't write the numbers, don't write the numbers just draw two small lines and then you draw the graphs, I just want to figure out if you get the difference between *y* equals to minus three *x* … [marking learner's books] … Lnrs: [drawing the graphs in their books] Tr: You are supposed to label that. OK, just quickly guys, here uhm what Asipe has done, right. OK, Asipe in both cases uhm, two graphs in both cases right, so [Writes on board] this one and this one, right, so that is uhm, I think that is brilliant. Right so she first drew this graph and then she says that *y* equals to minus *x*. [Writes on board] And then she drew the second graph, then she says *y* equals to minus three *x*. 	Pattern of discourse: I-R-E

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The content and activities in Table 44 above were unplanned. Spontaneous/scientific content and activities were selected during the lesson. They were mathematical in nature, with mathematical conventions, symbols, diagrams, and procedures being used (both verbally and written) in relation to the intended object of learning that the teacher wanted to enact in the lesson. Teacher C selected CL2E4 to illustrate a decreasing and increasing graph, and this was done during the interaction on effect of parameter a. CL3E7 was selected by Teacher C to illustrate the effect of *a*. From the table above, it is evident that the spontaneous activities led to an interactive lesson. But the type of communicative approach was interactive-authoritative because the teacher had a pre-planned answer to the questions she asked. She prompted learners to continue with the interaction and this led to I-R-P-R-P- pattern of discourse - the teacher initiated (see lines 62 of CL2E4 and 66 of CL3E7), learners responded (see lines 63 and 65 of CL2E4; and lines 67 and 69 of CL3E7) to teacher's initiation and the teacher encouraged the learners to continue (see lines 64 of CL2E4 and 68 of CL3E7), and then the interaction was sustained. Teacher C evaluated the learners' response(s) at the end of the interaction (see lines 66 of CL2E4 and 70 of CL3E7) and this was interactive authoritative communicative approach (IA). In the next section, I present Teacher C's spontaneous/everyday content and activities.

6.2.3.3 Teacher C's Spontaneous/Everyday content and activities

Teacher C had only one content and activity in this category, which is CL2E5. As discussed earlier, Teacher C compared an increasing graph to the left – to – right, right-to-left writing convention (see line 62 in Table 44 above). This explanation that Teacher C gave to learners, would help learners to remember what an increasing graph looks like. This brings in the aspect that was discussed by Adler and Ronda (2015) on the importance of both word use and explanatory communication, and how mathematics is substantiated or legitimised through the two. For Teacher C, the mathematics she was communicating through the explanation was not legitimised during the enactment. Although Teacher C interacted with learners and allowed them to respond to questions during the lessons, the focus was mainly on vocabulary, definitions, and conventions, and this is at the lexical level. Teacher C did not create opportunities for comprehensive mathematical discussions during the lessons. There were no instances of learners providing explanations and justifications to their responses.

6.3 Strategies in Teacher C's lessons

In the presentation of findings and discussion of the role of Teacher C above, I engaged with the strategies that were evident in the lessons observed. In this section, I will further unpack these strategies, and discuss the role of Teacher C in developing learners' mathematics discourse and understanding through the strategies below.

6.3.1 Stating the intended object of learning

Teacher C did not state any intended object of learning in all her lessons observed. For example, in Lesson 4, one of the lessons observed, she started the lesson by giving learners trigonometry graphs (see CL4E9 in table 40). The teacher and the learners engaged with CL4E9 – after which they switched to back to the straight-line graphs in the same lesson. Learners seemed unsure on what they needed to do with the activity that Teacher C presented to them.

6.3.2 Assessing learners' prior knowledge

In Teacher C's lessons, content and activities were selected to assess learners' prior knowledge. As discussed earlier, assessing learners' prior knowledge was identified as one of the strategies that Teacher C used in her lessons to ensure that learners discerned the critical features of the object of learning. All pre-determined content and activities (CL1E1; CL1E2; CL2E3; CL3E6; CL4E8; CL4E9; CL4E10; CL4E11; CL5E12; CL5E13; CL5E14 and CL5E15) (see Table 41 above) that Teacher C selected were presented to learners in form of assessing their prior knowledge. Teacher C started by posing questions based on the activity, she expected learners to respond to her questions using their prior knowledge on the activity. In the table below, I present CL1E1 and CL1E2, two of the activities that were selected to assess learners' prior knowledge.

Table 45: CL1E1 and CL1E2 selected to assess learners' prior knowledge.

Content/activity	**Prior knowledge assessed by activity**	**Comment**
CL1E1 *Content:* Definition of a function *Activity:* Learners asked to define a function	-Definition of a function: - Input value (x-value) - Output value (y-value) -one-to-one relationship -many-to-one relationship	In order for learners to be able to define a function, they had to have prior knowledge on terminology required to

		communicate the definition of a function
CL1E2 *Content:* Naming Five different representations of functions *Activity*: What are the five representations of functions?	-Flow diagram - words -symbols -Graphs -Table	Teacher C asked learners to name the five different representations of functions she taught them

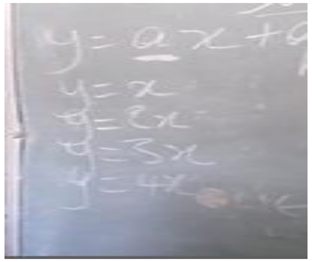
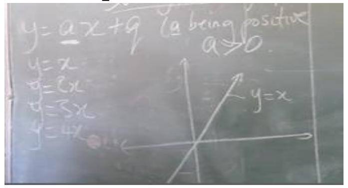
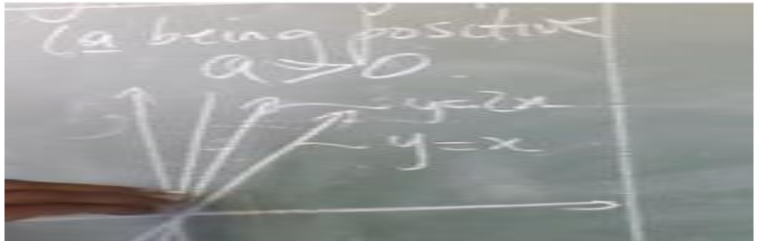
Table 45 shows some of the content and activities that Teacher C used to assess learners' prior knowledge. These content and activities were assessing prior knowledge because, as discussed earlier, Teacher C started with learners' prior knowledge to build on to the lesson content. This strategy of assessing learners' prior knowledge is seen in all Teacher C's lessons. Assessing learners' prior knowledge enabled Teacher C to start her lessons in such a way that allowed her to engage with what learners already knew. All the content and activities selected to assess learners' prior knowledge were appropriate for learners as they were able to engage and show their prior knowledge.

6.3.3 Patterns of variation as a strategy

In what follows below, I engage with how Teacher C's example set provided opportunity for generality through separation and contrast in the example set.

I identified five examples and I will engage with the content and activities, in order to provide a detailed analysis of each separation example. I will begin with example CL2E3. In this lesson, Teacher C was teaching the effect of parameter *a* on a straight-line graph.

Table 46: Analysis of CL2E3.

Content/activity	Transcription	Code
CL2E3 Draw graphs of the following equations, when *a* is greater than zero. 	45. Tr: It is positive, OK, so if I don't want to write a being positive then uuhmm…writes *a* is greater than zero, so our focus is when *a* is greater than zero right now [writes on the board] and then draw a graph of *y* equals to two *x*, then you draw a graph of what? 46. Lnrs: Three *x* 47. Tr: [Writes on the board] *y* equals to three *x*, draw a graph of *y* equals to↑?  48. Lnrs: Four *x* 49. Tr: Four *x*, now I want you to tell me, as *a* grows bigger and bigger, what happens to our graph, what happens to our straight-line graph? 50. Lnr4: It shifted up. 51. Tr: You say, did it shift up, [writes on board] OK, so let's say that's our first graph *y* equals to *x*, right, so the second graph is it going to be on that side (points to the left of the line $y = x$ in the first quadrant) or it's going to be on that side, up down? 52. Lnrs: Up↑ 53. Lnr2: Yes↑ 54.Tr: OK, [writes on the board] so it's going to be up, so *y* equals to two *x*, and then [draws the line $y = 2x$ on the board]  55. Lnrs: Yes 56. Tr: So and then *y* equal to?	**SS-TS** – Teacher C used a set of separation examples to focus learners' attention on what a concept is, in an attempt to bring about some form of generalisation – on the effect of parameter a on a straight-line graph. *Separation aspects (Controlled variation)* -All equations given are linear -Values of parameter *a* are varying: $y = 1x$ $y = 2x$ $y = 3x$ $y = 4x$ -Value of parameter *q* is zero for all equations. $y = 1x + 0$ $y = 2x + 0$ $y = 3x + 0$ $y = 4x + 0$ - Parameter *a* is positive for all equations. **CC-TS** – Teacher C used an example to explain an increasing and a decreasing

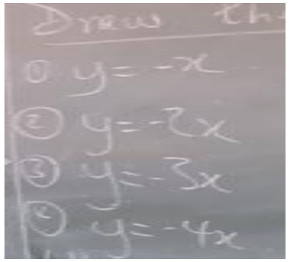
	57. Lnrs: Three *x* 58. Tr: Three *x*,  [Draws the line of *y* = 3*x* on the board] So now I want you to tell me, so if I say, if I say to draw me a graph of *y* equal to eight *x*, is that graph going to get closer to the *y* axis or it is going to run away from there, it's going to move away from the *y* axis? 59. Lnrs: Get closer to the *y*, 60. Tr: Get closer to the *y* axis [writes on board], 61. Lnrs: *y* axis	function in lines 62 and 64. **FF-TS** – CL2E3 has only one key feature of the object of learning that is varying/invariant simultaneously- the co-efficient of *x*

Table 46 shows how Teacher C enacted CL2E3. Teacher C enacted the intended object of learning by selecting four equations of straight-line graphs, with varying parameter *a (a = 1; 2; 3 and 4)*, and constant (invariant) parameter q (q = 0). The intended object of learning from this example was for learners to discern that parameter a affects the slope of the graph, and hence generalise the effect of $a > 0$ on a straight-line graph. For $a > 0$, the straight-line graph increases from the left to the right of Cartesian plane. As a becomes larger in value, the value of the function increases. The varying and invariant parameters made this set of examples a separation example. As discussed by Runesson (2005), example CL2E3 opened a space of variation that offered possibilities for learners to discern specific aspects of the object of learning in order to generalise about the effect of a. In line 49, Teacher C asked learners to explain the behaviour of the graph as a becomes larger and larger. A learner responded that it shifted up. The teacher revoiced the response and rephrased the question as shown in line 51 in the table above, with a description of the behaviour in relation to quadrants of the Cartesian plane (see line 51). Teacher C, together with the learners engaged with the four graphs, with $y = x$ being considered as the first graph (see line 51). There was everyday talk in the interaction as shown in lines 49 – 54. As the teacher discussed the behaviour of the function, words such as shift up; up down (line 51) and going to be up (line 54) were used. Teacher C linked the informal talk to the illustration of the behaviour of the graph using a visual (see line 55 in the table above).

Teacher C illustrated what an increasing graph looks like and juxtaposed this with what a decreasing graph looks like (see lines 62-65 in Table 43 presented earlier). This provided an opportunity for learners to see what an increasing graph is and what it is not, hence contrast. The interaction was geared towards the mathematics register of an increasing and decreasing graph. Teacher C did not use the word increasing and the legitimation was based on how the graph ‘looks’ like, its visual representation, and not its mathematical meaning. The interaction was focused on when the graph is increasing because as x increases in value the values of the function (the related y values) are increasing. So, when x is 2, $y = x$ gives $y = 2$; when $x = 2$, $y = 2x$ gives $y = 4$ and so on, the y values increase as the values of x increase, and this is what an increasing curve is. For a decreasing graph, as values of x increase, values of the function decrease. If this relational idea was established then it would be possible to look at trigonometric graphs globally and ask learners about when the graph is increasing or decreasing, however, the interaction did not focus on this idea of increasing and decreasing aspect of the graphs.

Teacher C went on to relate the increasing and decreasing to writing from left to right (see line 70 in Table 44 presented earlier). This explanation that Teacher C gave to learners would help learners to remember what an increasing graph looks like because writing from left to right is part of the everyday activities and they are familiar with this context and hence can make a link between what Teacher C was referring to and the context of writing from left to right. The focus of the example CL2E3 was on parameter *a* being positive, as shown in line 46 above. Teacher C then moved on to parameter *a* being negative as shown in the table below.

Table 47: Analysis of CL2E6.

Content/activity	**Transcription**	**Code**
CL3E6 *Content:* Investigate the effect of a negative gradient (the value of *a* in the linear function) *Activity:* Draw straight line graphs using a table of values, to investigate the effect of a negative gradient (the value of *a* in the linear function) 	100. Tr: Right uhm, quiet, a minute, almost very few have not drawn the first graph. Why calculate *x*? What do you notice in this graph as compared to when *a* is positive? what is… what is… let's talk about the first graph *y* is equal to negative *x*, how is it behaving, increasing or decreasing? 101. Lnrs: Decreasing 102. Tr: It is decreasing right 103. Lnrs: Yes 104. Tr: So it's like you're going down the mountain, OK, so it is decreasing, [continues to mark learner's work], let's make sure you label your graphs so to know which one is which one … [writes on board] right, when *a* is, when *a* is what, let's write negative, how do we write negative, when *a* is? 105. Lnr8: Less than zero 106. Tr: [Writes on the board] Less than zero, awesome, when *a* is less than zero, the straight-line graph does what? 107. Lnr6: Is decreasing 108. Tr: [Writes on the board] Is decreases … right, the straight-line graph decreases, OK, so uhm, ok so please just finish up the other graphs the few of you have drawn the others, and Andani has actually finished all … 109. Lnrs: ohhh 110. Tr: Ok, so I want you to finish drawing them and then you look at their behaviour remember that, right, when *a* was positive and then when *a* increases the graph becomes steeper and it gets closer and closer to the *y* axis, so I want you to look at what happens with these ones. Now I want to ask you again, is *a* increasing here from negative one to negative two to negative three, is *a* increasing? 111. Lnrs: No 112. Tr: It is doing what? 113. Lnrs: Decreasing 114. Tr: It is decreasing, remember that negative two, negative two is smaller than negative one, are you getting it? 115. Lnrs: Yes	**SS-TS** ***Separation aspects*** -All equations given are linear -Values of parameter *a* are varying: $y = -1x$ $y = -2x$ $y = -3x$ $y = -4x$ -Value of parameter *q* is zero for all equations. - Parameter *a* is negative for all equations.

Teacher C enacted the intended object of learning by selecting four equations of straight-line graphs, with varying parameter *a (a = -1; -2; -3 and -4)*, and parameter q ($q = 0$) (invariant for all equations). The intended object of learning from the example above was for learners to discern that parameter a affects the slope of the graph. For $a < 0$, the linear function increases. As parameter becomes smaller in value, the linear function decreases. When $a = 0$, the straight-line graph is horizontal and parallel to the x – axis. Teacher C did not engage with the $a = 0$ scenario in the lesson. Teacher C, together with the learners engaged with the four graphs, with $y = - x$ being considered as the first graph (see line 100).

From the engagement in the lesson observed, the discussion was on the effect of a negative a being a decreasing graph. Teacher C did not engage with how a negative parameter a affects the steepness of the graph. This was a missed opportunity because in the previous lesson, Teacher C emphasised the steepness of the graph and how it moves closer to the y-axis. In the CL2E6, it would have been a useful engagement with how a positive parameter a differs from a negative parameter a for a straight-line graph. The varying and invariant parameters made this example a similarity example. Examples CL2E3 and CL2E6 above made it possible for the learners to experience the effect of a positive a and a negative a on a straight-line graph, and the numerical value of parameter a.

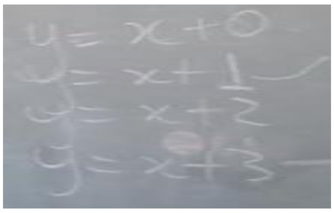
Example CL3E7 in lesson C3 is similar to examples CL2E3 and CL2E6. Teacher C was still emphasising the effect of parameter a on a straight-line graph. In this example, she used two similar equations: $y = 3x$ and $y = -3x$. Both equations are linear, with values of parameter a varied by the sign (3 and - 3). The value of parameter q is zero for both equations. In the lesson, both graphs were sketched on the same set of axes as $y = x$ *and* $y = -x$ as shown in Table 46 above. Teacher C selected the two equations to allow learners to compare the difference between parameters as shown in the excerpt in Table 47 above, hence contrast.

From the excerpt in Table 47 above, Teacher C wanted learners to see the effect of a negative parameter a and a positive parameter a. She used 3 and only changed the sign and hence contrast in the example – to get 3 and -3. Controlled variation was used here because every other key feature of the object of learning in the activity was invariant, for example the value of q was zero for both equations. Teacher C allowed learners to sketch the graphs in their books and she marked some of the books during the lesson. The example gave learners an opportunity to determine the effect of *a*, when the sign changes. Visual representations of the graphs were drawn on the board and referred to in the whole class discussion.

The intended object of learning from example CL3E8 was for learners to discern that parameter *q* affects the point where the straight-line graph cuts the *y*- axis. Teacher C enacted the intended object of learning by selecting four equations of straight-line graphs, with varying parameter *q (q = 0; 1; 2 and 3)*, and invariant parameter *a* (*a* = 1). When *q* is increased in value the graph shifts vertically upwards, so from *q* = -4 to q = -2 is a vertical shift up. From the analysis of example CL3E8, Teacher C gave this example as a prior knowledge assessment to the learners. The discussion continued in lesson C4 as shown in the excerpt below.

Assessing learners' prior knowledge

112. Tr: Now, we will be drawing the graphs so the graph I gave you last time, I gave you y equal to x, [writes on the board] y equal to x plus one, y equal to x plus two, y equal to x plus three. So what do you notice with these graphs?

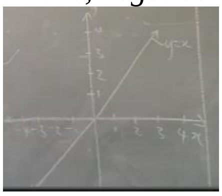


Graph y equal to x, what's the y intercept? ...
113. Lnrs: Zero
114. Tr: Its zero, good, the y intercept is zero. Ok?
115. Lnrs: Yes
116. Tr: So y equal to x is the same as y equal to x plus zero, ok. And then for this graph, what's the y intercept?
117. Lnrs: One
118. Tr: What's the y intercept for this one [points at y = x + 1]
119. Lnrs: One↑
120. Tr: It is one. Right, so for this one? [points at y = x + 2]?
121. Lnrs: Two
122. Tr: Two, for this one [points at y = x + 3]
123. Lnrs: Three
124. Tr: Right, now, has the gradient changed in these graphs?
125. Lnrs: Nooo
126. Tr: No, the gradient has remained the same, and what is the gradient?
127. Lnrs: One
128. Tr: One, good, so the gradient is one. But has the y intercept changed?
129. Lnrs: Yes

From the excerpt above, in Lesson C4, Teacher C engaged with example CL3E8. She asked learners some questions to check if they were able to see what was changing in the equations and what was the same (see lines 116 - 129). Teacher C went on to discuss the effect of parameter *q* with the learners as shown below.

Effect of Parameter q

132. Tr: They are increasing, because a is positive, so they are increasing. Right, now, the y intercept is equal to zero for the first one, so y equal to x [labels the straight line], ok? And then we have one, we have two, we have three, we have four [puts a scale on the y axis] and then y. And x is x axis. And then we have negative one, negative two, negative three, negative four, one two, three, four.



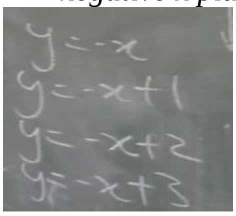
So we use the graph papers all the time, right. So because it makes us see the true picture, right. Now, so when you draw the second graph [draw a line on the Cartesian plane], does it look like this graph?

133. Lnrs: Yes
134. Tr: Its does, right. y is equal to x plus one. So what has changed in that graph? The y intercept changed from being what?
135. Lnrs: Zero
136. Tr: To being what?
137. Lnrs: One
138. Tr: One, so the rest of the graph. The whole graph has shifted how many times?
139. Lnrs: One
140. Tr: Once, upwards, can you see that?
141. Lnrs: Yes
142. Tr: So there has been a shift. Which means that when you change the y intercept, what happens to the graph?
143. Lnrs: Shifts
144. Tr: It shifts, ok. So remember that as we were looking at a, ok, the y intercept remained but the graph moved, but the y intercept remains the same. But now when we change the y intercept, the whole graph does what?
145. Lnrs: Shifts

From the excerpt above, for the example CL3E8, Teacher C emphasised the effect of parameter q on a straight-line graph. The effect of parameter q being a shift in the graph came to the fore of the discussion as shown in lines 134 – 145. Teacher C also referred to parameter *q* as the *y*-intercept of the graph during the discussion. Example CL3E7 was for increasing graphs. Teacher C gave learners an example of decreasing graphs as shown below.

Decreasing graph

230. Tr: Decreasing graph. That's a decreasing graph. Ok, so uhm now, I want you, just like you drew this, without a table and in the same axis, ok, I want you to draw those graphs y equal to negative x, y equal to negative x plus one, y equal to negative x plus two, y equal to negative x plus three,



Draw them quickly, roughly, don't draw a table. Just draw them now

231. Lnrs: Where… where?

232. Tr: Where you drew those graphs [points at the graphs of y = x; y = x + 1; y = x + 2 and y = x + 3]. In the same, yes in the same axis, in the same Cartesian plane.

233. Lnrs: [Drawing in their books]

234. Tr: Right uhm, now, just before you drew that, so we need to have a conclusion. Because we had a conclusion when is positive, the graph is increasing, right?

235. Lnrs: Yes

236. Tr: When a is negative, the graph is?

237. Lnrs: Decreasing

238. Tr: Decreasing, right? Now when a increases and a is positive, what happens to the straight line? It moves closer to the…?

239. Lnrs: y axis

240. Tr: To the y axis. It moves away from…?

241. Lnrs: The x axis

242. Tr: The x axis, right?

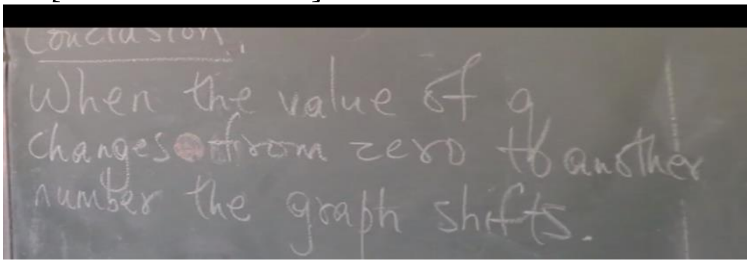
243. Lnrs: Yes

244. Tr: So, when a is negative, right and when as decreases, what happens to the graph? It gets… closer and closer to the?

245. Lnrs: y axis

In the excerpt above, Teacher C engaged with the effect of a negative parameter *a*, and gave a conclusion as shown in lines 234 – 238. A conclusion on the effect of the parameter *q* was also given in the lesson as shown below.

254. Tr: To the y axis, right. That was our conclusion. But now, here we want a conclusion for the y intercept. So when q, ok, so when the value of q [writes on the board], right, when the value of q changes, what happens to the graph?
255. Lnr2: It shifts
256. Tr: It shifts. Yes. When the value of q changes from being zero, right, so the graph shifts [writes on the board]



We can see the shifts in Geogebra in the computer, but right now there is a new part now, a new part but it is still linear functions. Can you just pause to the drawing that we will have to do today, where we have been drawing a graph using a table, right? But now, we are going to draw a graph, so it is drawing a linear function without using a table [writes on the board]

From the excerpt above, Teacher C summarised the effect of parameters by giving conclusions based on what was observed in the examples provided. The conclusion given in the excerpt above does not state the type of shift that the graph undergoes. As discussed in Chapter 2, it is crucial for learners to get an accurate summary especially for generalisation purposes in functions. The conclusion given by the teacher should have stated that the shift is a vertical shift because there is a vertical and a horizontal shift of the graph.

In the excerpt above (line 256), Teacher C announced the next object of learning, which was drawing a linear function without using a table of values. CL5E13 (Figure 17) below was given to the learners, and they were asked to draw the graphs using two methods: the gradient and *y*- intercept method and the *x*-intercept and *y*-intercept method.

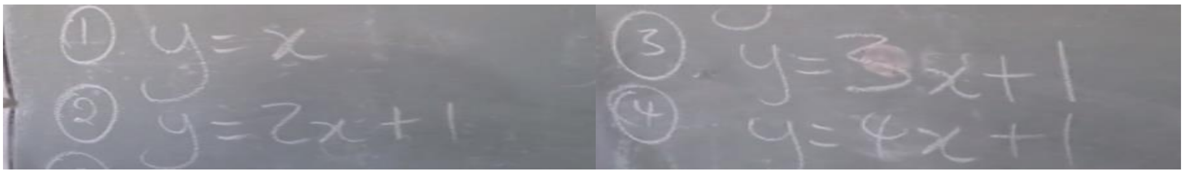
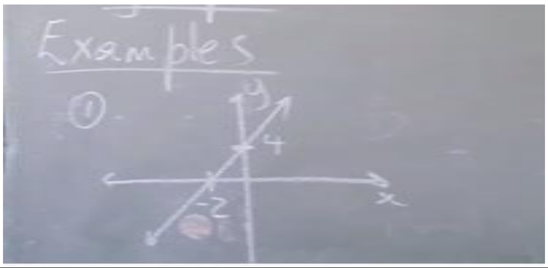
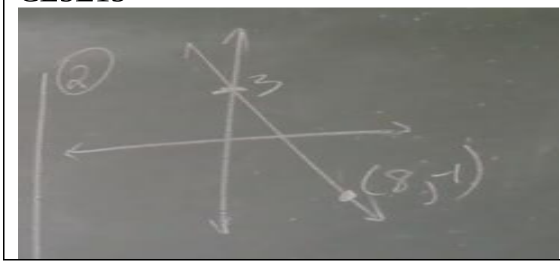


Figure 17: Homework activity.

The equations above are for straight lines. Parameter *q* is 1 for the three equations. Parameter *a* is varying but positive for both equations, hence separation aspects. Teacher C gave the example as a homework activity. In the next lesson, there was no engagement or discussion about the example. Therefore, there is no transcription for this example from the lessons observed. I was therefore not able to analyse the process of how Teacher C engaged with the example in class.

Teacher C gave two examples, CL5E14 and CL5E15, as shown in Table 40, for the object of learning that followed after the one discussed above. The two examples that were selected by Teacher C to determine the equation of a straight-line are not similarity content and activities because of the variation in the examples as shown in the table below.

Table 48: Key features of examples CL5E14 and CL5E15.

Example	**Key features of the object of learning**
CL5E14 	• Positive parameter *a* • Increasing graph • *y*-intercept is 4 • *x*-intercept is -2 • No other point through the point is given
CL5E15 	• Negative parameter *a* • Decreasing graph • *y*-intercept is 3 • *x*-intercept is unknown • Another point (8; -1) through the line is given

The key features of the object of learning in the examples given were varied as shown in the table above. The two examples required different methods for determining the equation of the straight-line graph as per information given. CL5E14 required the intercept methods while example CL5E15 required the gradient and *y*-intercept method. Therefore, there was no opportunity for generalisation because each example given required a different method. The two examples can be categorised as fusion examples because there is more than one feature of the object of learning varying/invariant simultaneously, as shown in Table 48 above. However, during the enactment, the pattern of variation was not evident in what the teacher focused on in the lesson.

The examples I presented in this section are what afforded me to determine the patterns of variation that were evident in Teacher C's lessons. Teacher C's content and activities had potential to bring the critical features of the object of learning to the fore are shown in the analysis above.

In the example set discussed above, key features of the object of learning were varied, while keeping other features invariant. There was a variation between the parameters (a = 1; 2; 3; 4 and q = 0; 1; 2; 3; 4), and then there was another variation on the signs of the parameters (a = -1; -2; -3; -4 and q = 0; -1; -2; -3; -4). The variations provided opportunities for the learners to discern how the straight-line graph is affected by the parameters a and q. Runesson (2005) argues that the way examples are composed affect what it is possible to learn in any learning situation. In the example set presented above, there was a particular variation present. Teacher C's example set had a rich set of content and activities with aspects of variation as presented at length above. Possibilities for discerning the critical features of the object of learning were availed to learners through the rich example set that Teacher C selected in the lessons observed. However, Teacher C did not exhaust the possibilities during the enactment of the example set as presented above.

6.3.4 Double move as a strategy

Opportunities for double move were present in Teacher C's lessons. She used everyday context in her interaction in Lesson 2 and 3, in examples CL2E3 and CL3E6.

In Lesson 2, Teacher C compared the writing convention to representing an increasing graph (see line 62 in Table 44). Teacher C gave this comparison in passing and she did not illustrate the back and forth movement between the writing convention and the representation of the increasing graph. Therefore, the double move strategy in this instance was not successful. In Lesson 3, Teacher C was explaining a decreasing graph, as shown in the excerpt below.

Decreasing graph

101. Lnrs: Decreasing
102. Tr: It is decreasing right
103. Lnrs: Yes
104. Tr: So it's like you're going down the mountain, OK, so it is decreasing, [continues to mark learner's work], let's make sure you label your graphs so to know which one is which one … [writes on board] right, when a is, when a is what, let's write negative, how do we write negative, when a is?

Line 104 above shows the everyday context that Teacher C used to explain a decreasing graph. She used the context of going down the mountain to illustrate a decreasing graph. This is a double move. The nature of the context of going down a mountain and the slope of a decreasing graph allow for a back and forth comparison between the two contexts, and example afforded learners an opportunity to visualise a decreasing graph and I can argue that the double move in this instance had potential to help learners with developing their mathematics discourse and understanding.

6.3.5 Learner feedback

In order for feedback to be given, there has to be an interaction between the teachers and learners. Teacher C engaged learners in an I-R-E pattern of discourse as discussed in this Chapter. Teacher C gave learners effective feedback, she probed learners - to find out why learners responded the way they did. Feedback as a strategy in her lessons was used to give an authoritative response to learners, and also to understand what learners were communicating about their level of mathematics discourse and understanding in the interaction.

6.4 Conclusion

In this chapter, I presented the findings of Teacher C. Teacher C's example set provided opportunity for key features of the object of learning to be brought to the fore, and hence providing opportunities for what is possible to learn. Through the analysis, I looked at the example set that Teacher C selected. The example set was rich and Teacher C's example set had opportunities for a double move and the examples also had patterns of variation present although some were not used appropriately in developing learners' mathematics discourse and understanding. In all her lessons, Teacher C attempted to assess learners' prior knowledge and preconceptions during lessons observed. Teacher C engaged learners during the lessons observed, and she ensured that learners were given an opportunity to attempt activities after concepts were covered by giving learners classwork and homework.

In Teacher C's lessons, learners were allowed to share their ideas and hence many voices were present in the lesson (interactive). Teacher C asked learners some questions and gave them an opportunity to come to the board and share their ideas about the content and activities selected during the lessons observed. Teacher C expected learners to respond to her questions during the lessons observed. Learners contributed during the lessons, although in chorus form throughout lessons observed. Although Teacher C did not ignore learner's contributions, she gave the authoritative conclusion to any activity that was being discussed. I-R-E pattern was evident in all Teacher C's lessons. All initiations in the lessons were from the teacher. The teacher initiated the talk, learners gave chorus answers and the teacher probed the learners which led to other responses. In some instances, Teacher C probed further or evaluated the learners' responses. Figure 19 below is a summary of the role of Teacher C in developing learners' mathematics discourse and understanding – as discussed in this Chapter.

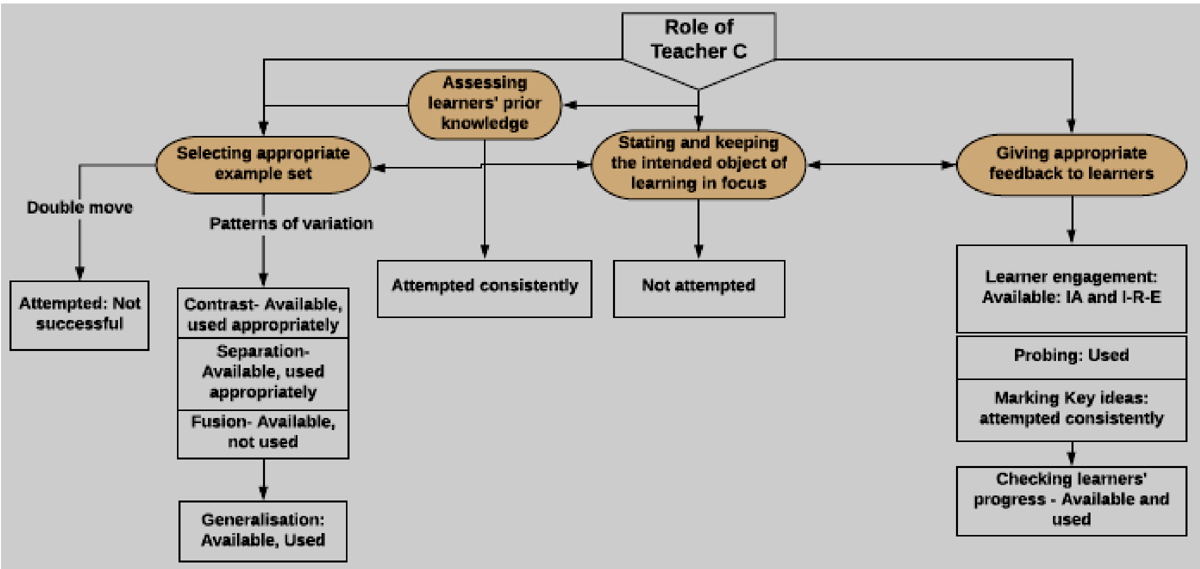


Figure 18: The role of Teacher C in developing learners' mathematics discourse and understanding.

Figure 18 above is a summary of the role of Teacher C in developing learners' mathematics discourse and understanding, as discussed in this Chapter. Stating the intended object of learning and keeping it in focus throughout the lesson is a critical role for the teacher and this was not available in Teacher C's lessons. The teacher assessed learners' prior knowledge based on the intended object of learning. The example set was selected based on the intended object of learning. Feedback was given to learners based on the focus of the lesson. In conclusion, Teacher C's example set, and enactment had great potential to develop learners' mathematics discourse and understanding, with some missed opportunities of course, as discussed above. In the next Chapter, I present a synthesis of the three teachers in relation to the role of the teacher in developing learners' mathematics discourse and understanding.

CHAPTER 7: Cross case analysis and discussion of results

7.1 Introduction

In Chapters 4, 5 and 6, I presented Teacher A, B and C's findings and discussion of results. As discussed in the previous Chapters, I analysed the teachers' data using the study's analytical framework below.

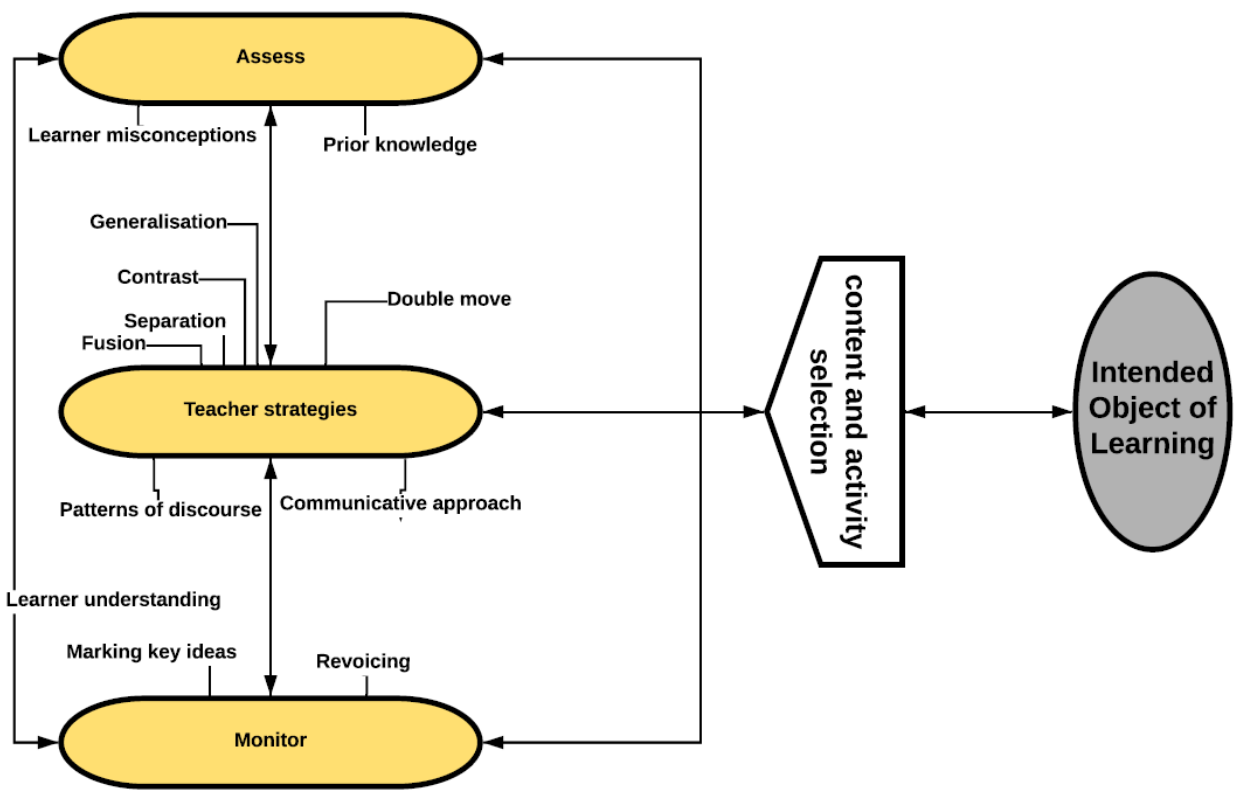


Figure 19: Analytical framework for the study.

Using the analytical framework above, I analysed each teacher's data in relation to content and activities selected, assessing learners' prior knowledge, teacher strategies, and monitoring learners' understanding, in relation to the intended object of learning. From the analysis of these categories, I further discussed teacher strategies and hence roles that emerged from data. The strategies discussed in the previous three Chapters are: stating the intended object of learning, assessing learners' prior knowledge, and selecting and enacting an example set. The figure below is a recap of the teacher strategies that were discussed in the previous Chapters.

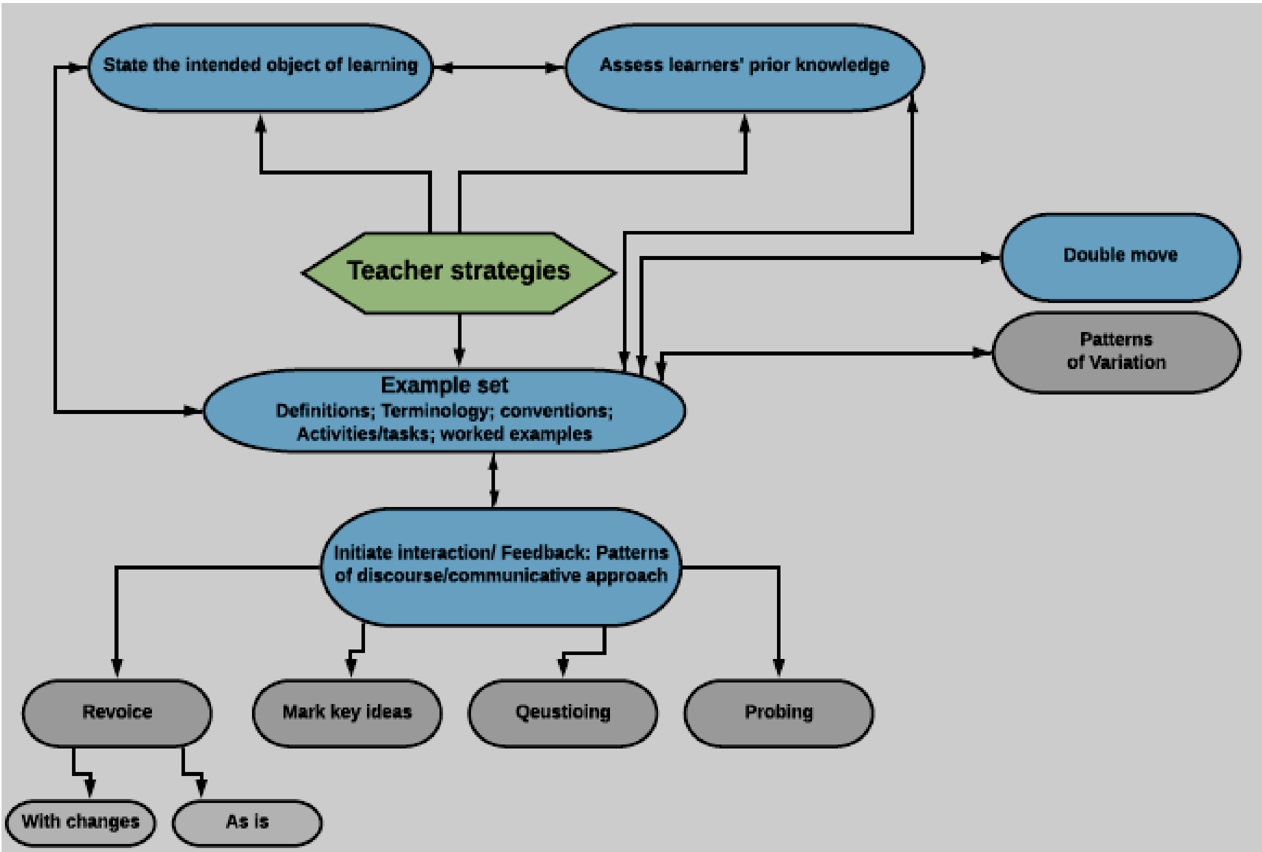


Figure 20: Teacher strategies in ensuring that learners discern the critical features.

As stated above, Figure 20 is a summary of the teacher strategies that I discussed in the previous Chapters. There were three main strategies, with sub strategies stemming out of the main strategies as shown in the figure above.

In the present Chapter, I will pullout the common strategies that occurred across the three teachers and discuss them, with an ephemeral comparison across the three teachers. I will then relate the strategies to existing literature to understand the results of the study further, so that I can argue more strongly for the role of the teacher, especially in developing learners' mathematics discourse and understanding. The strategies I have pulled out for discussion in this Chapter are: 1) stating the intended object of learning and keeping it in focus throughout the lesson; 2) assessing learners' prior knowledge; 3) selecting the example set; 4) double move, and 5) Patterns of discourse.

7.2 State the intended object of learning

Out of the 11 lessons observed, only one lesson had a clearly stated intended object of learning, and this was Teacher A's Lesson A1, as discussed in Chapter 4. Teacher B and C did not state the intended object of learning in all lessons observed. The teachers introduced the lessons by either asking learners to open their textbooks and engage with an activity or focus on the activity written on the board by the teacher. As obvious as it may seem that any lesson should start with the teacher introducing and announcing the lesson objectives and hence intended object of learning, this was not the case in the study. The three teachers observed were experienced teachers and had been teaching for over 10 years at the time of the study, however, they did not demonstrate an awareness of the role of stating the intended object of learning and keeping it in focus throughout the lesson. The implications of this finding were evident in the lessons observed. It was difficult to follow what was going on in the lessons and what the teacher and the learners were aiming to achieve from the interaction and engagements that transpired in the lessons observed. Literature on the role of stating the lesson objectives and keeping them in focus emphasises the importance of this strategy as discussed below.

Holmqvist et al. (2011) argues that the intended object of learning is the teacher's standpoint on what needs to be learnt. In Mortimer and Scott's (2003) terms, this is the focus of the lesson, and it entails the teaching purpose and the content, as discussed in Chapter 2. In lessons where the teacher does not state the intended object of learning, it is difficult to focus learners' attention on that which the teacher intends them to focus on, and this was evident in the lessons observed. The selection and enactment of the example set was also affected because there was no intended object of learning to guide the selection and enactment of the example set. In my study, it was evident that without stating the intended object of learning and keeping it in focus throughout the lesson, the teacher and learners may end up focusing on the non-critical features in the example set.

Watson and Mason (2006) argue that simply providing examples to learners is not enough for them to be able to focus on only the essential aspects of the intended object of learning. Learners they are likely to focus on the non-essential ones too if their focus is not guided. This was evident in the lessons observed as discussed in the previous chapters. I maintain therefore that the teacher should guide learners' focus to the essential aspects by stating the intended object of learning and keeping it in focus as examples are enacted in the lesson. Stating the intended object of learning influences our awareness and guides us to that which is in focus.

Lo (2012) argues that learners may not learn successfully if they do not share the same awareness as the teacher. In a lesson where objectives are clearly stated, the teacher and the learners work in a structured sequence, with a purpose of achieving the stated objectives. Stating the intended object of learning gives direction to the lesson as argued by Dean, Hubbell, Pitler and Stone (2012). Pintrich and Schunk, (2002) also argue that setting and communicating objectives is fundamental to directing and guiding the learning process. Dean et al. (2012) argue that a lesson with no direction for learning is comparable to taking a purposeless trip to a city that is not familiar to the traveler. To ensure that learners' journeys within a lesson are purposeful, the teacher needs to identify and communicate objectives, and this enables the learners to make connections between what is being focused on in the lesson and what they are supposed to learn (Dean et al., 2012). Lesson objectives that are clearly communicated reduce anxiety on the learners since they are given an opportunity to determine what they need to pay attention to (Dean et al., 2012). An important point to note here that is argued by Dean et al. is that clearly stated lesson objectives also enable learners to identify aspects of the object of learning where they may need help from their teacher or from peers. In the figure below, I summarise the importance of stating the intended object of learning that I have discussed above. The list below is not exhaustive, it captures the issues I have discussed above.

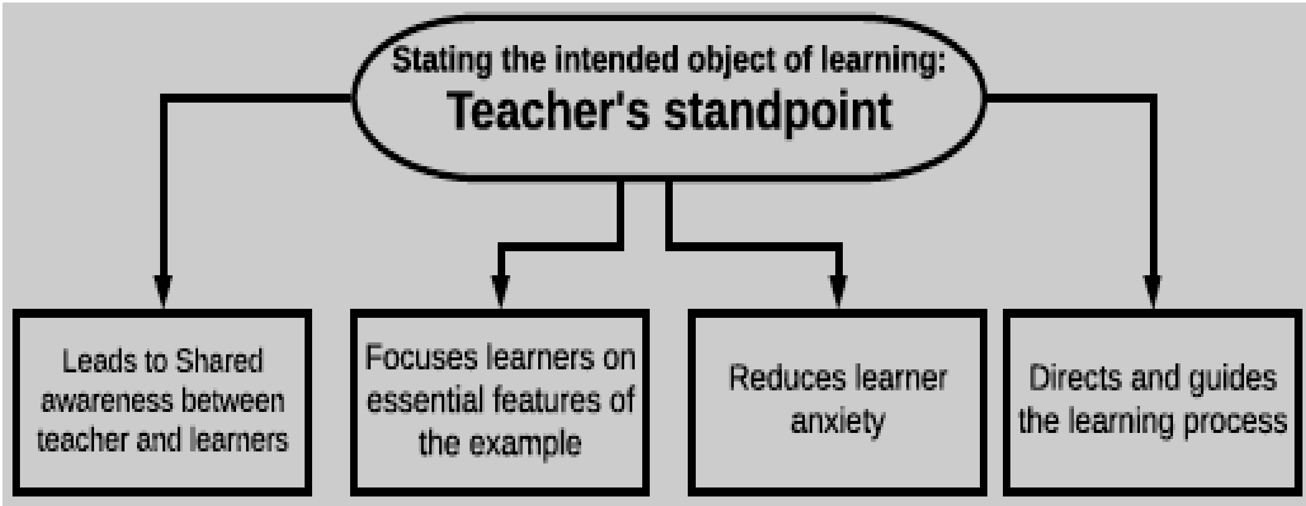


Figure 21: The importance of stating the intended object of learning.

Having discussed the importance of stating the object of learning above, a question comes to mind: why did the teachers leave out this strategy if the discussion above argues for its fundamental part in teaching and learning? As stated earlier, this strategy may be taken for granted. From experience as a mathematics teacher, this is one strategy that is emphasised in teacher training. Also, many teaching and learning materials that teachers use as resources highlight the importance of stating the lesson objectives in teaching and learning. The findings of the study dispute the 'taken for granted' strategies and shows that unfortunately, nothing can be left to 'common sense'. No matter how important a strategy is, as discussed above, teachers may not use it in their lessons.

One may argue against stating the intended object of learning, especially in a lesson that is designed for discovery through activities. Blair (2007) argues that lessons that are led by objectives may be taught in an artificial way and learners may be inactive in the lesson. Reed (2012) disagrees with this notion and argues that a lesson with carefully selected objectives that are referred to in the lesson may lead to better learner engagement and interaction. Therefore, I argue that whether the lesson is aimed at discovery or explicit discussion around a concept, the intended object of learning needs to be communicated in one way or another, depending on the purpose of the lesson. Dean et al. (2012) argue that learning objectives of a lesson need to be communicated explicitly, in a learner friendly language, and in various forms, verbally or in writing. The teacher should then draw learners' attention to the objectives throughout the lesson.

It is not easy to conclude that the teachers were not aware of the role of state the intended object of learning in the lessons. I also cannot say that the teachers struggled with formulating the intended object of learning, because they did not develop any, except for one lesson as stated above. My conclusion is that the teachers were not aware of the impact of not stating the intended object of learning in their lessons and keeping it in focus. And by impact, I refer to Figure 21 above to argue that in all the lessons where the intended object of learning was not stated, the important issues presented in Figure 21 were not present in the lessons observed. Therefore, the study reveals that for teachers, there is a need for re-engagement from a professional development perspective to emphasise the importance of stating the intended object of learning, especially in developing learners' mathematics discourse and understanding. In a heavily prescriptive curriculum, lesson objectives are often given and so it is the role of the teacher to ensure that these prescribed objectives are adhered to.

Before I move on to the next strategy, I would like to contextualise my discussion on stating the intended object of learning and keeping it in focus in a functions lesson. Functions is a broad topic as discussed in Chapter 2, and learners need to engage with many concepts, for example, the teacher may present an activity where learners need to sketch a graph, using the gradient and intercept method. If the intended object of learning is not clearly stated, learners may end up using a table of values to complete the task. In this case, the intended object of learning would not have been achieved. However, if the intended object of learning is not stated, the teacher may not be able to draw learners' attention to it, and learners may not be in a position to anticipate the intended object of learning from an activity provided by the teacher with an instruction of sketching a graph. Also, in functions, the terminology and conventions form a large part of the language that is used in the topic. In a lesson of interpreting graphs as the intended object of learning, if learners do not have prior knowledge on terminology, this might become the focus of the lesson, and the teacher may not focus learners' attention back to interpreting graphs if it is not clearly stated as an intended object of learning. So, in such a lesson, the teacher and the learners will focus on the terminology and not engage with interpreting the graphs. For effective linking between the intended object of learning and what learners already know, the teacher may need to be aware of what learners already know from previous learning. This brings me to the next strategy that I identified as crucial for developing learners' mathematics discourse and understanding in my study, which is assessing learners' prior knowledge.

7.3 Assess learners' prior knowledge

Teachers A, B and C all assessed learners' prior knowledge in the lessons observed, as discussed in Chapters 4, 5 and 6 respectively. In Teacher A's lesson where the intended object of learning was communicated, the link between the intended object of learning and the assessment of prior knowledge was observable. In the other lessons of all three teachers, assessing prior knowledge was not focused on any intended object of learning because the teacher did not state clearly what the intended object of learning was. This finding implies that the assessing of prior knowledge was not effective because there was no focus on exactly what was to be assessed. From the data, teachers were aware of the importance of assessing learners' prior knowledge. My findings contradict with what Campbell and Campbell (2009) argue that when teachers are planning for a lesson, the focus is mainly on what is going to be taught, and less planning is focused on accessing learners' prior knowledge in relation to what is going to be taught. Research has shown that engaging learners and asking learners questions about the key features of the object of learning before teaching them in the lesson increases achievement (Thompson & Zamboanga, 2003; Portier & Wagemans, 1995; Alexander, Pate, Kulikowich, Farrell, & Wright, 1989; Glaser, 1984).

Therefore, it goes without saying that one powerful way of informally diagnosing learners' baseline knowledge is by engaging with their prior knowledge (Campbell & Campbell, 2009). From this finding, I therefore argue that starting from where learners are, helps the teacher to make informed decisions about the content to be taught, and hence appropriate decisions are made on how to enact the intended object of learning. But, the teacher needs to be clear on the intended object of learning, and state it in the lesson as discussed earlier, in order to engage with appropriate assessment of learners' prior knowledge. As discussed in Chapter 3, one of the decisions that is informed by the results of the prior knowledge assessment is the content and activities, which is the example set that the teacher draws on to enact the intended object of learning. From my study, I argue for the importance of drawing on the intended object of learning and prior knowledge assessment results, in the process of selecting and enacting an example set in the lesson, which is another strategy that the teacher can use to ensure that learners discern the critical features of the object of learning.

7.4 Selecting and enacting the example set

As discussed in Chapters 4, 5 and 6, Teachers A, B and C all selected different kinds of content and activities to enact the intended object of learning in all the lessons observed. The examples selected by the three teachers were either spontaneous or pre-planned. Spontaneous examples were selected 'in-the-moment' as defined by Zodik and Zaslavsky (2008), while pre-planned examples were selected before the lesson. The teachers drew on scientific and everyday contexts as shown in the example sets presented in Chapters 4, 5 and 6. In the diagram below, I recap the kinds of content and activities that were selected by the teachers.

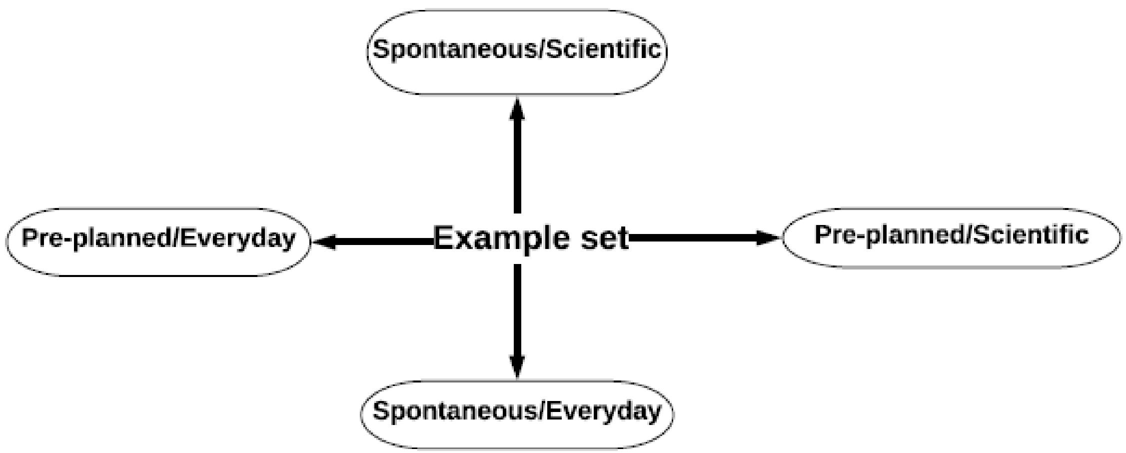


Figure 22: Kinds of content and activities.

As stated above, Figure 22 is a summary of the kinds of content and activities that teacher selected. In the table below, I present a summary of the number of the kinds of content and activities that each teacher selected during the lessons.

Table 49: Content and activities selected by Teacher A, B and C.

Type of content/activity	**Teacher A**		**Teacher B**		**Teacher C**	
Spontaneous/Scientific	✔	4	✔	2	✔	2
Spontaneous/Everyday	✔	6	X	0	✔	3
Pre-planned/Scientific	✔	11	✔	2	✔	12
Pre-planned/Everyday	X	0	✔	1	X	0

Table 49 above is a summary of the results from the analysis of the data, showing the kinds of content and activities and the number of each of the content and activities that made up the example set for each teacher. Teacher B had a significantly low number of content and activities compared to Teacher A and C. The reasons for Teacher B's few content and activities were discussed in Chapter 5. To recap some of the reasons here, Teacher B spent a considerable time (35 minutes of the lessons) completing a table of values with the learners and this activity did not allow for other activities to be covered in the lesson. In Teacher B's lessons, the content and activities that were selected to assess learners' prior knowledge were not attempted by learners. Teacher B had to engage learners for a longtime and in some instances, the teacher and the learners ended up discussing concepts that were not part of the inferred intended object of learning.

As discussed in Chapters 4, 5 and 6, the kind of content and activities that the teacher selected, and how they were enacted, did not provide opportunities for learners to explain and justify their responses in the lessons and hence develop mathematics discourse. The teachers were aware of the role of examples in teaching and learning; however, the teacher-learner engagement was not geared towards fostering a comprehensive mathematics discourse. In what follows below, I draw on literature to discuss how examples played out in the lessons.

Rissland (1991) argues that examples are a fundamental part of mathematics and a substantial component of proficient knowledge. Without drawing on examples in a mathematics lesson, we may not be able to generalise and illustrate abstract concepts, and present mathematical analogical reasoning (Zaslavsky, 2010). Zaslavsky (2010) makes an important point about the kinds of examples that the teacher uses in a lesson. Selecting and using examples in a lesson, be it spontaneous or pre-planned examples, is not a means to the end. To recap my findings with respect to Zaslavsky's argument, in the lessons observed, some content and activities had great potential to bring the intended object of learning to the fore. However, this potential was not realised as discussed in Chapters 4, 5 and 6. The implication of this finding is that the teacher may have to take further effort to ensure that the examples selected are pedagogically useful (Zaslavsky, 2010), and enacted in a way that allows for the usefulness to be made explicit. In what follows below, I unpack the meaning of a pedagogically useful example - this is the first time 'pedagogically useful' is appearing in the thesis because I came across it when I was reviewing literature in order to interpret my findings. It interested me to discuss 'pedagogically useful' because I was able to relate my findings to it and further build an argument on what the data showed me as discussed below.

According to Bills, Dreyfus, Mason, Tsamir, Watson and Zaslavsky (2006), there are two main characteristics that make an example pedagogically useful. The first is transparency of the example and the second is the ability for examples to be generic and hence foster generality.

By transparency, Bill et al. (2006) refer to an example that has features that make it exemplary to the target audience. One way of making the example exemplary is using everyday context that learners can relate to. I will engage with the everyday context in the double move strategy that I discuss later in the Chapter. Zaslavsky and Peled (1996) argue that unless learners are exposed to a rich example set, they may be attracted to the noise in the examples given. Watson and Mason (2006) concur with Zaslavsky and Peled (1996), and they argue that examples need to be extensive for connections to be made. It is worthwhile to provide learners with several examples from which to get an overall sense of what is being taught and hence generalisation. Looking at Table 49 above, I was tempted to conclude that based on the number of content and activities in Teachers A and B's example sets, generalisation may have been possible. However, it is not as simple as that, and I will engage with generality in the discussion below.

On the notion of generality, Mason and Pimm (1984) argue that generic examples are examples that allow the target audience to see the general through the particular. It is therefore not startling that learners focus their attention on the particular and may remain unaware of the general. When engaging with the point-by-point of a parabola, Teacher B provided a specific example and hence did not give an opportunity for generality. The example has to be presented in a way that allows one to stress and ignore various key features in the process of perceiving the message delivered by the example. One challenge that teachers face is that it is not possible to tell if learners are stressing and ignoring in the same way as the teacher. Mason and Pimm (1984) caution about a misalliance that may occur between what the teacher intends and what the learners end up understanding. Therefore, the role of the teacher is to present learning opportunities that involve a variety of "suitable examples" to address the varied needs and individualities of the learners, through pre-planned and or spontaneous content and activities. In the figure below, I present the analysis and comparison of the three teachers in relation to how content and activities were selected based on the discussion above. In the figure below, I chose the aspects of example set, everyday context, transparency, and generality to illustrate the discussion I have presented above in relation to the teachers in my study.

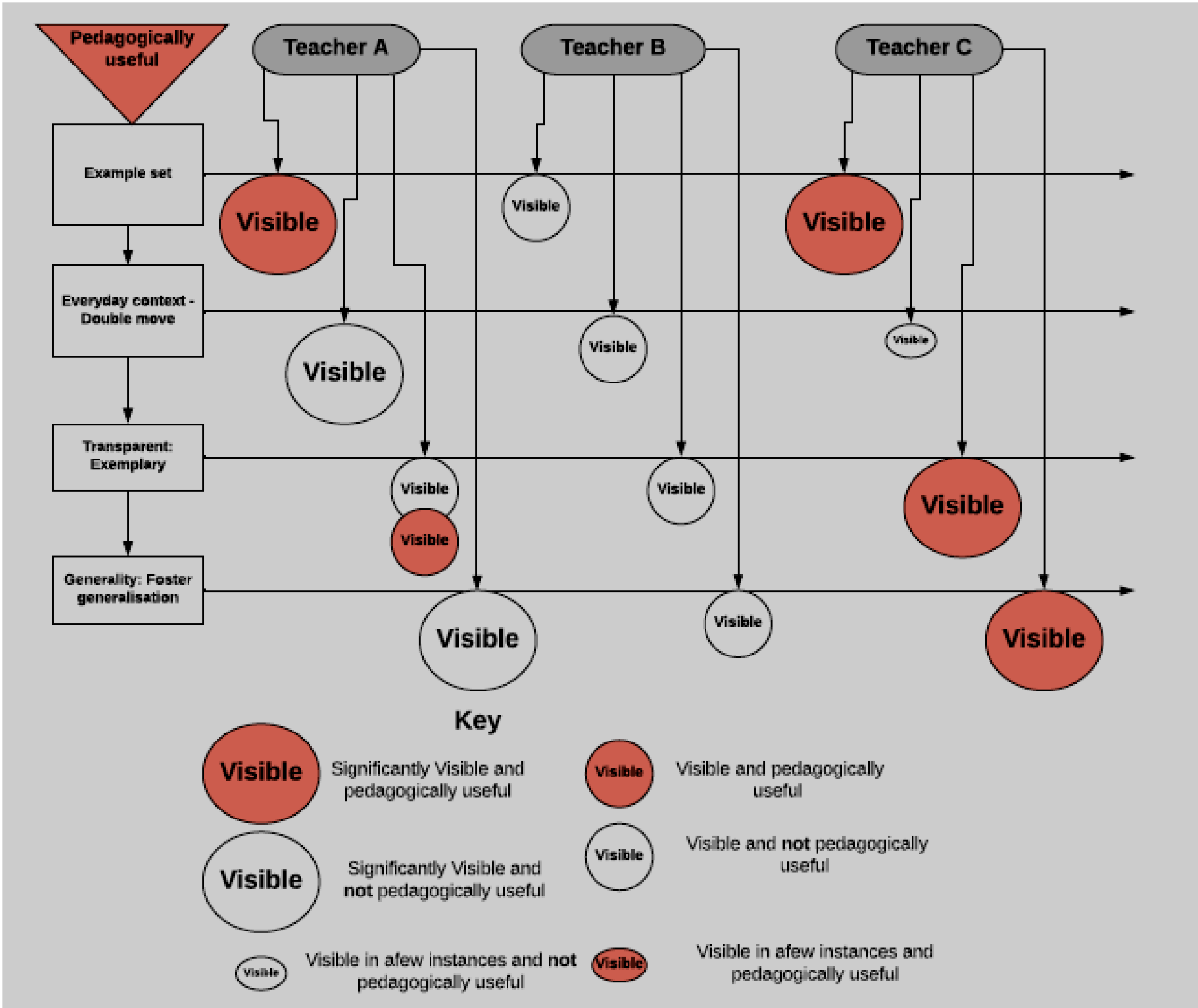


Figure 23: Teacher A, B and C's content and activity selection.

Figure 23 above is a summary of how pedagogically useful the teachers' example sets were in developing learners' mathematics discourse and understanding, with a focus on functions. In presenting the figure above, I am showing that the information presented in Table 49, if taken on face value maybe deceiving when it comes to concluding on how pedagogically useful the content and activities selected were, as seen in Teacher A's case above.

Within the teachers' example sets, there were content and activities that attempted a double move, Teacher A's double move being the most visible and Teacher C's the least visible. However, all the three teachers' double moves were not pedagogically useful and as mentioned earlier, this will be unpacked in the next strategy. With regards to the example set being transparent and being exemplary to the target audience, Teacher A and B's example sets were not transparent as seen in the content and activities presented to the learners. For example, the activity of a factory in relation to the concept of a function is not exemplary and hence not transparent, and therefore it was not pedagogically useful. Some of Teacher A's content and activities were transparent and learners were able to relate to them, hence the two circles in Figure 23 under Teacher A. Teacher C selected an example set that was transparent in relation to what she was teaching on the concept of functions. However, transparency for Teacher C did not mean drawing on the everyday context to make the examples exemplary, she drew on the scientific context in her example set as shown in Table 48 above. On the possibility to foster generality, Teacher A and C's example sets had possibilities for generality, with Teacher C's example set being pedagogically useful. Teacher B's example set did not foster generality as shown in the discussion in Chapter 5. The discussion above has highlighted the complexity of selecting and enacting an example set. Yes, it is the teacher's role to ensure that the example set selected is as transparent as possible, by drawing on content and activities that are exemplary to the target group and also in relation to the intended object of learning. However, it is not an easy task due to the complex nature of selecting an appropriate example set. In what follows below, I engage with what might be involved in the process of selecting an example set, to understand further the role of the teacher in this process.

In the process of selecting an example set, certain aspects such as curriculum interpretation have to be considered and carefully addressed. The figure below shows what may influence the teacher's choice of content and activities, hence the example set for a particular lesson. To conceptualise the figure below, I drew on my experience as a mathematics teacher and the empirical evidence of the study to present what might influence selecting an example set for any mathematics lesson.

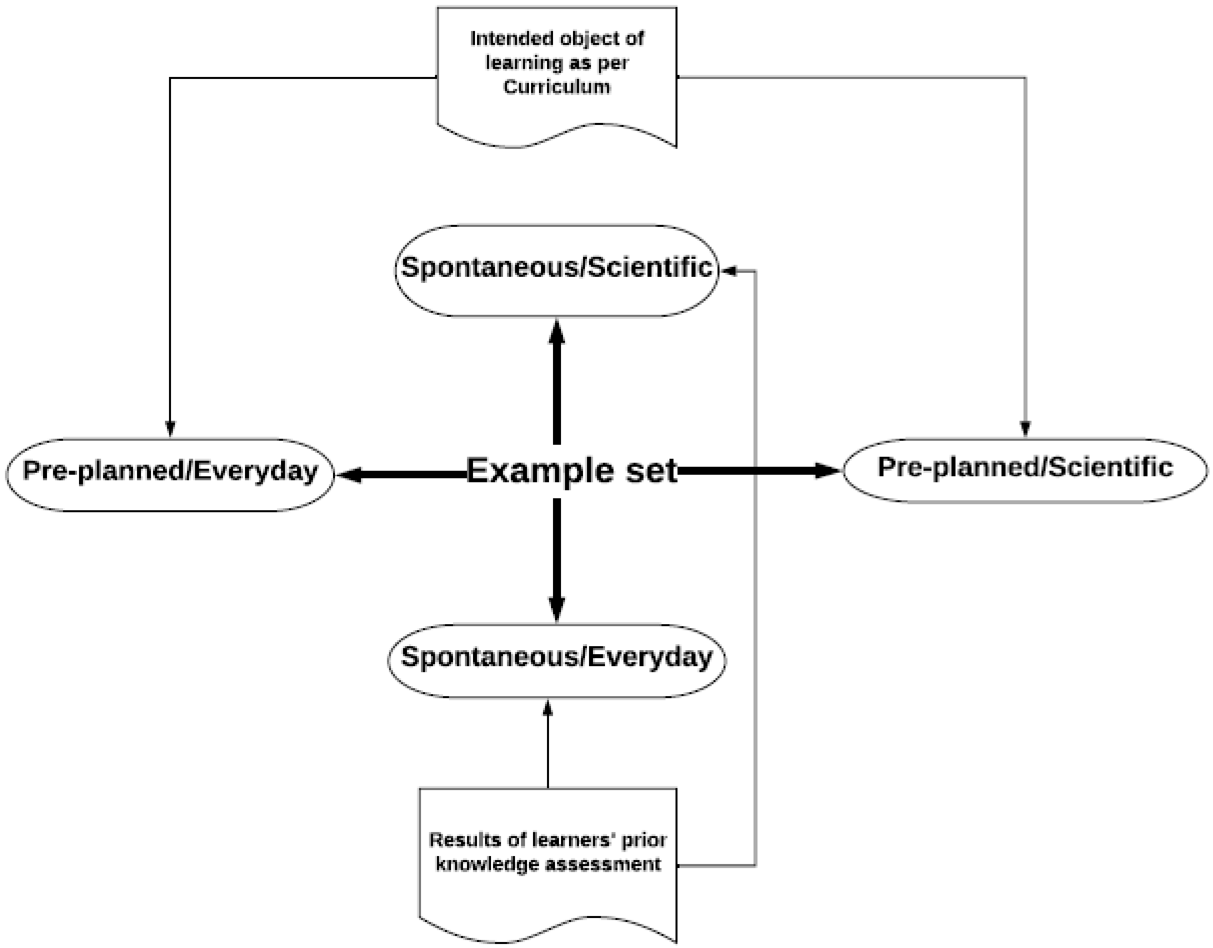


Figure 24: Rationale for teacher's choice of content and activities in a lesson.

Figure 24 above shows two aspects that may influence the choice of the teacher's content and activities. In the previous chapters, I engaged with how I linked the curriculum to the intended object of learning for the lessons observed. To build my argument further on the teacher's role in selecting and enacting an appropriate examples set, I will engage with literature that talks about the intended object of learning, to unpack the strategy of selecting an appropriate example set and what it means in practice.

Valverde, Bianhi, Wolfe, Schmidt and Houang (2002) discuss the role of the curriculum and how it should be interpreted. Valverde et al. (2002) emphasise the significance of this translational process in enacting the curriculum's intended object of learning. But whose role is it to interpret and enact the curriculum's intended object of learning?

I argue that the teacher is at the heart of the process of curriculum enactment and hence plays a vital role in this process. Drawing on literature to substantiate my argument, Remillard and Heck (2014) argue that the role of the teacher is to draw on the designated curriculum, and available resources to prepare for a lesson. In this process, the teacher has to interpret the curriculum and make decisions on how the lessons will be conducted. Remillard and Heck argue that this process leads to what they call the '*teacher-intended curriculum*'. Gueudet and Trouche (2009) maintain that the teacher-intended curriculum is the one that is designed with specific learners in mind at a particular moment, and the teacher designs it in such a way that it is ready to come alive in that specific classroom. Therefore, from the curriculum interpretation, the teacher may be able to draw the intended object of learning for a lesson.

Marton et al. (2004) discuss the intended, enacted and lived object of learning as presented in Chapter 2. In my study, I inferred the intended object of learning from the enacted object of learning (the lessons observed) (see Chapters 4, 5 and 6). The content and activities, and hence example set may be informed by the intended object of learning as shown in the figure below.

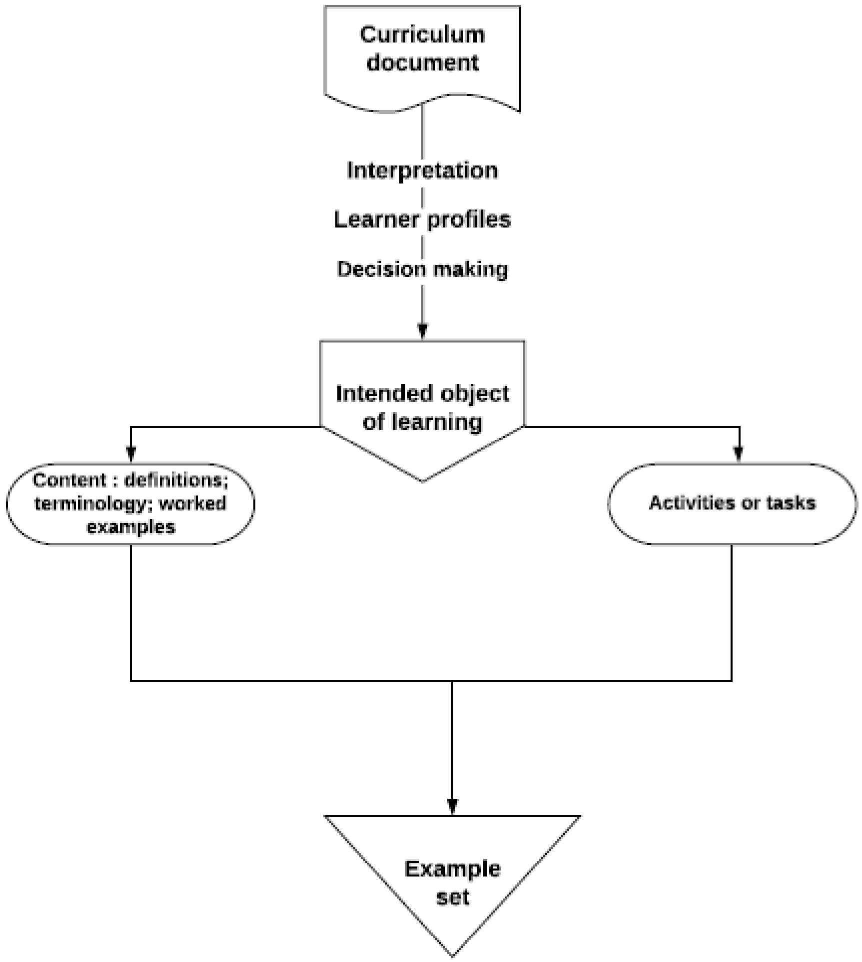


Figure 25: Selecting the example set.

In Figure 25 above, the teacher may decide on the intended object of learning by interpreting the curriculum and making decisions as to what the curriculum document means. The intended object of learning includes content and activities or tasks, worked examples showing procedures and conventions, and definitions of terminology, which are required in the lesson. All these components then feed into the example set of the planned lesson. From the illustration of the selection of the example set above, it is important to note the role of the teacher in interpreting the curriculum document, which is often not as emphasised as it should be. If the interpretation and decisions made by the teacher are not appropriate, this will have a significant effect on the intended object of learning and hence example set selected for the lesson. But is it enough to interpret the curriculum and come up with the intended object of learning? The answer to this question is no. The teacher still has to enact the intended object of learning. In process of enacting the intended object of learning, if learner profiles were not considered in the process of deciding on the intended object of learning, then issues arise because the learners are not on the same page as the teacher, as I discussed earlier in this Chapter.

Marton et al. (2004) define the enacted object of learning as what the teacher actually teaches in a lesson. From a mathematics perspective, the enacted curriculum, similar to the intended object of learning in variation theory terms, is made up of four aspects that are discussed by Remillard and Heck (2014), namely: 1. mathematics – the nature of the topic covered, the specific content, how it is presented and the engagement and practices that are highlighted and valued; 2. instructional interactions – the teacher and learner interactions in relation to the example set in the lesson, and the norms that direct the interactions; 3. teacher pedagogical moves – teacher moves, both intentional and unintentional such as the feedback given to learners, that influence the mathematics that is presented in the lesson; 4. tools and resources – the physical, technological, verbal and cognitive tools that both the teacher and learners may draw on in the lesson. These four aspects are crucial for the role of the teacher in developing learners' mathematics discourse and understanding.

According to Zodik and Zaslavsky (2008), content and activity selection sometimes requires decisions that are made 'in-the-moment' during the classroom interactions. The teacher, therefore, has to be as flexible as possible in relation to the 'in-the-moment 'interactions (Leikin and Dinur, 2007). In his Mathematics Teaching Cycle, Simon (1995) argues that there often a call for spontaneous actions during the classroom interactions. Such instances may occur when the intended object of learning is met with learners who do not have prior knowledge of the teacher's pre-planned content and activities that are selected to enact the intended object of learning.

In my study, spontaneous content and activities were evident during the enacted object of learning. In some lessons, for example in Teacher B's lesson where learners were not able to determine the constant in the equations given, there was as a shift from what the teacher had pre-planned in the teacher-intended curriculum, to meet learners where they were, as revealed by the interaction in the lesson. The modification of the content and activity was not successfully done as discussed in Chapter 5. This highlighted an important aspect of selecting an activity or task that is at the level of the learners so that they are able to engage productively, and also linked to DeMarois and Tall's (1999) facets of the concept of the function (see Chapter 2).

Stein, Grover and Henningsen (1996) describe three levels of mathematical tasks that have to be taken into consideration when engaging with the curriculum in relation to the dynamics of the classroom - mathematical tasks informed by the curriculum, tasks designed by the teacher in the classroom, and tasks implemented by learners in the classroom. Remillard and Heck (2014) argue that the features and level of cognitive demand of tasks and activities change as they go through the process of curriculum enactment. The teacher and learner engagements, and learners' habits and characters, play an important role in shaping the enacted curriculum (Goodlad, Klein & Tye, 1979). The enacted curriculum is therefore dependent on the teacher's pedagogical moves in the lessons, in response to nature of the learners' engagement with the object of learning.

From the discussion above, I have positioned the process of selecting and enacting an example set as one that is not straight forward. The approach requires the teacher to engage with various aspects - from engaging with curriculum documents, to understanding the profile of learners with regards to their prior knowledge and ability, to interpreting classroom engagement in order to react 'in-the-moment' with pedagogically useful content and activities.

As discussed in Chapter 3, drawing on variation theory (see Marton et al., 2004), there are patterns of variations that are valuable for organising lessons to make the features of the object of learning explicit. From my study, I analysed the teachers' example sets, with a focus on how they employed the patterns of variation: similarity, contrast and fusion. To recap my rationale for variation theory, although the lessons were not driven by variation theory, I argue in Chapter 3 that it was necessary for me to have a yardstick to 'describe and interpret' what an example that will provide potential for learning looks like. I therefore drew on the patterns of variation: separation, contrast, generalisation and fusion to analyse the example sets of the teachers.

All the teachers had separation content and activities, with varying features against an invariant feature, for example, equations of functions with the different co-efficients of *x* and the same *y*-

intercept, to highlight the effect of a on a given function. Adler and Ronda (2015) argue that similarity in a set of examples leads to possibilities for generalisation. Contrasting examples also make available opportunities for generality (Adler & Ronda, 2015). Teachers in my study had some examples that could qualify as counter examples. Counter-examples may be useful in the process of highlighting relevant features and irrelevant features in a given object of learning, however, Zaslavsky and Ron (1998) argue that counter-examples may confuse learners who are unable to understand how or what they counter.

There were many instances where separation and contrast could have been employed better to enhance the choice and enactment of examples. For example, on the activity of functions related to a factory setting, Teacher A could have used this as a contrast example, to show learners what a function is not. From this finding, I argue that the teachers were not conscious of patterns of variation and how they could possibly be used to develop an example set. Therefore, it is not surprising that the teachers in my study did not employ the strategy of patterns of variation. Bills and Rowland (1999) argue that the choice of examples and how contrast is employed in the example set is imperative in the process of providing learners with the opportunities to develop generalisations of structures, in an attempt to develop learners' mathematics discourse and understanding. Therefore, the complex nature of selecting and enacting an appropriate example set may be moderated by using patterns of variation to enable learners to discern the critical features of the object of learning. Another strategy that I argue for in this Chapter is the double move, which I will discuss next.

7.5 Double move

Double move attempts were evident in all the teachers' lessons, with Teacher A having the most attempts. I use the word 'attempts' because the way content and activities that had potential for a double move did not provide opportunities for developing scientific mathematics discourse as discussed in Chapters 4, 5 and 6. Teacher A used everyday context, categorised as spontaneous - to exemplify the concept of a function. The learners related to it in the lesson observed- although the context was not accurate as discussed in Chapter 4. Teacher B also used everyday context to exemplify a linear function, and the example she used was a pre-planned example, she copied the activity onto the board from her notes. Teacher B's content and activity was not effectively enacted as discussed in Chapter 5. Teacher C had three instances of everyday context in her example set, two of which were pedagogically.

Everyday examples and context are valuable in a lesson as discussed earlier. One criticism of everyday context is that there are not always relevant to the learners (Jo, 1993). In some cases, the contexts are a lot more real to the teacher rather than the learners, as seen in Teacher A's example of wife and husband. The teacher therefore has to be aware of how to select the appropriate context to make the example exemplary and hence transparent (see discussion above on pedagogically useful), so that learners are able to relate to it.

Bringing everyday context into an example set has advantages (if learners have previous experience of the context and can relate to what is being exemplified) and disadvantages (if learners do not have previous experience of the context and cannot relate to what is being exemplified), and the teacher has to be aware that some learners may not experience the example as per the teacher's intentions (Zaslavsky, 2010). Everyday context may introduce the aspect of irrelevancy in an example, which Skemp (1971) call the noise of an example. Learners may be attracted to the noise of the example instead of the features that the teacher intends to direct their attention to. In the example presented by Teacher A about one wife- one- husband and many-wives- one- husband, learners may be attracted to the context of relationships as it excites them and hence miss the important information of relating the context to one-to-one and many-to-one functions.

The double move strategy only becomes useful if enacted in a way that allows learners to discern the critical features of the object of learning. The double move is not an easy task as seen in the study. The teachers attempted it, but it was not as successful as they would have wanted it to be (as argued in the analysis).

Van den Heuvel-Panhuizen (2001) argues that compared to most mathematics context questions; everyday context problems offer the learners an opportunity to show their capabilities because contexts contribute to the transparency of the problem. In the lessons observed, learners were sharing their ideas on these content and activities (everyday context content and activities). If the content and activities had been pedagogically useful, it would have been a great potential to develop learners' mathematics discourse and understanding through the interaction that transpired during the lessons. Interaction in the study was linked to two aspects: communicative approach and patterns of discourse. I focused on patterns of discourse as a strategy because of the findings around the teachers' 'default' pattern, as I discuss below.

7.6 Patterns of discourse in the classroom

From the discussion in Chapters 4, 5 and 6, the kinds of content and activities led to specific communicative approaches and patterns of discourse as shown in the figure below.

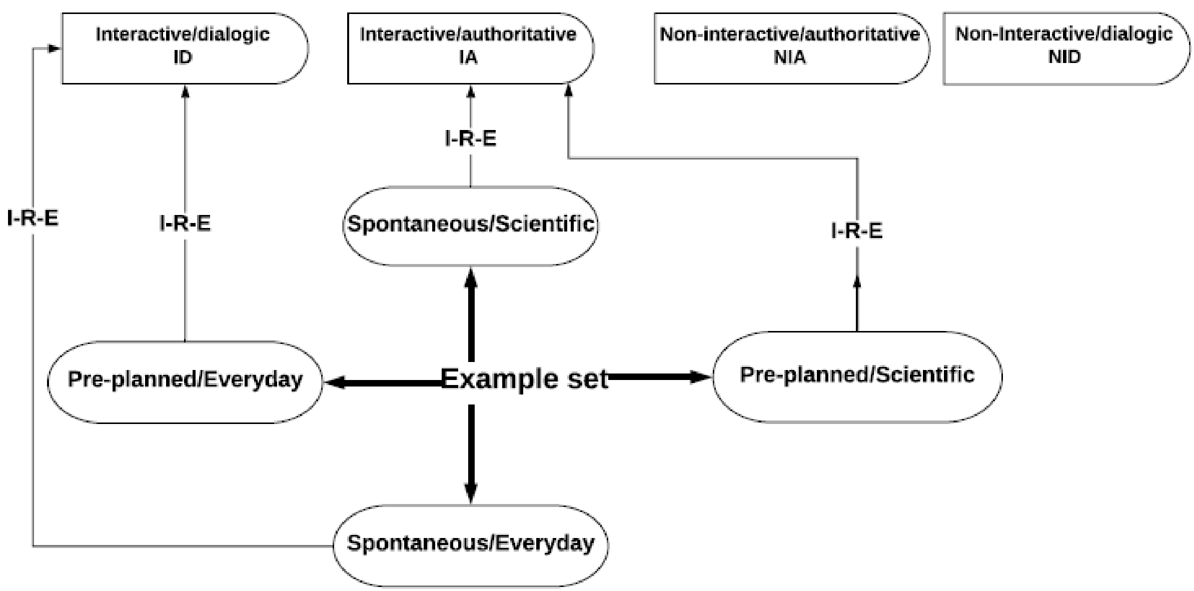


Figure 26: Example set and the communicative approach and pattern of discourse visible.

From Figure 26 above, interactive-dialogic (ID) was visible when teachers were enacting content and activities that had everyday context while interactive-authoritative was visible when teachers were enacting content and activities that had scientific context. When everyday context was used in the lessons, learners were seen to be more active and engaged in responding to the teachers' questions, as discussed above. I-R-E was the predominant pattern of discourse that was observed in all the lessons observed, hence my focus on it in this section.

In the lessons observed, the example set was enacted in such a way that teachers initiated the classroom interactions by posing a question related to the intended object of learning, for example ‘What is a function?’ (Initiation). The teachers’ questions were always followed with a response from the learners (response), which was acknowledged by the teachers (evaluation). Enacting the intended object of learning was affected by the pattern of discourse because this pattern of discourse did not enable the teachers to engage with learner responses in a way that would provide opportunities to develop their (learners) mathematics discourse. In all the lessons observed, learner engagement was limited to chorus answers, one-word answers, without the teacher probing for explanation or justification of the answers from the learners. Mortimer and Scott’s framework allowed me to analyse the interaction in the lessons, in relation to the patterns of discourse and communicative approach. The teachers worked with mainly the learners who were engaged in the lessons, but the teachers did not probe learners to justify their answers.

Although the teachers were selected in the study because they alluded to the fact that they engage learners and probe them to justify their answers, I wanted to understand the common pattern in the teachers’ lessons of not engaging learners who were not active in the lessons. I also wanted to unpack the observation of not asking learners to explain or justify their answers. Therefore, I drew on literature to understand what this means in practice, as discussed below.

Mortimer and Scott (2003) argue that I–R–E pattern of discourse is characteristic in many classrooms, and especially in authoritative interactions. Cazden (1988) argues that I-R-E pattern is the most common pattern in engagements that are initiated by the teacher. Cazden further argues that I-R-E is a "default" pattern and therefore it is the pattern that should be expected unless the teacher takes deliberate action to influence the pattern of discourse in the classroom. No deliberate action from all the teachers was seen to influence the pattern of discourse in the lessons observed. All the teachers accepted one-word answers to the questions asked, without probing learners to find out the why or the how part of the single word answers. All the teachers accepted and welcomed chorus answers, without probing for reasoning behind the answers that learners gave. The implication of this finding is that if teachers are not aware of ‘default’ patterns in the process of teaching and learning, they are not in a position to take deliberate action to influence them, in an attempt to develop learners’ mathematics discourse and understanding.

Therefore, the role of the teacher is to have an awareness of the default pattern of classroom discourse and be in a position to take deliberate action to influence the pattern of discourse during a teacher-learner interaction. The pattern of discourse used by the teacher should provide opportunities for the teacher to enact the intended object of learning in a way that brings its critical features to the fore.

7.7 Conclusion

In this Chapter, I have presented the five teacher strategies and hence roles of the teacher that emerged from the study. I have recapped the findings from Chapters 4, 5 and 6 in the process of arguing for the critical roles of the teacher. The figure below is a summary of the findings in relation to the five strategies that I discussed in this Chapter.

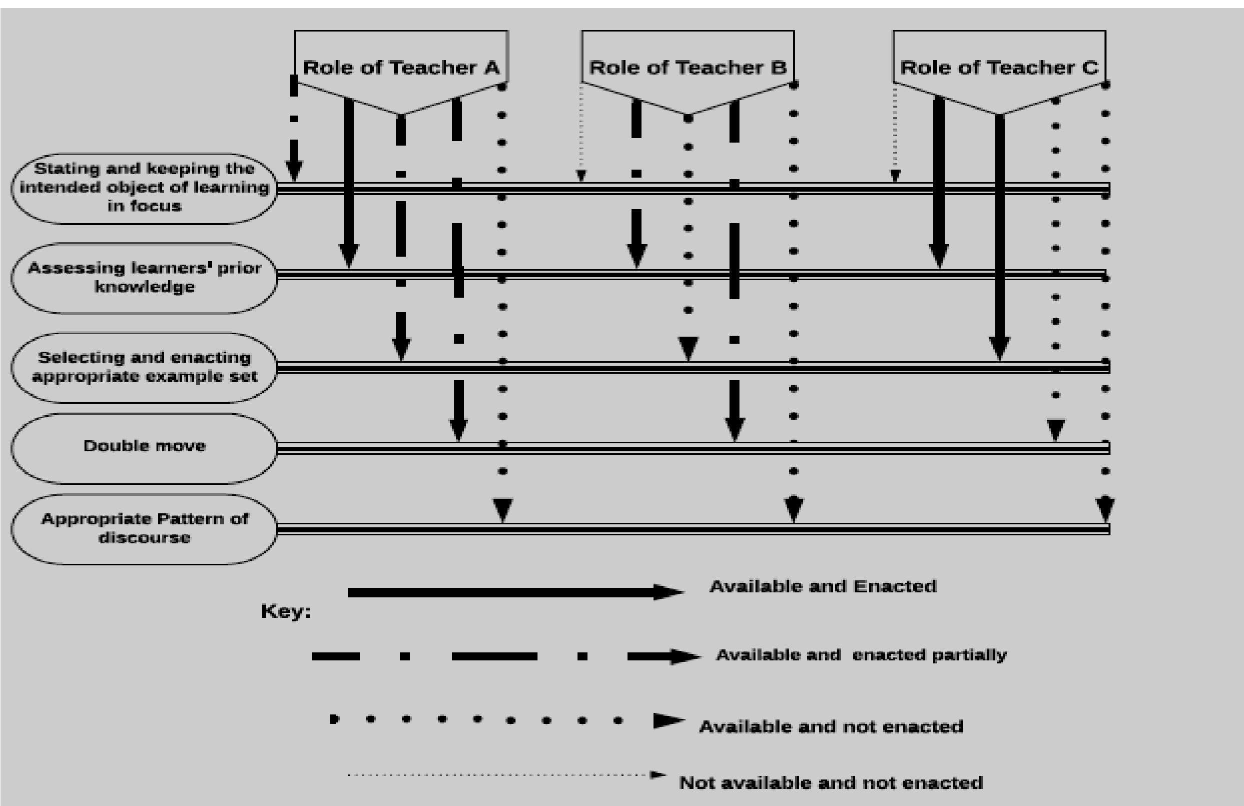


Figure 27: Cross case analysis of the role of the teacher.

Figure 27 above is a summary of the findings of the study. From the figure above, it is evident that Teacher B's lessons had the least impact in developing learners' mathematics discourse and understanding. Teacher A and C's lessons had potential to develop learners' mathematics discourse and understanding, although there were aspects that needed attention in the teachers' lessons as discussed in the respective Chapters.

In conclusion, I have argued for what the role of the teacher is in relation to developing learners' mathematics discourse and understanding. At one level, one may argue that the five roles I have discussed above are obvious, and I would agree with the argument. However, in this chapter, I have used the findings of the study and existing literature to move beyond the obvious nature of the role of the teacher and argue for how complex it is to play this role. Therefore, there is need for re-engagement in enabling teachers to play their role in a way that provides learners with opportunities to develop their mathematics discourse and understanding. In the next chapter, I will present the contribution, limitations, recommendations, and suggestions for future research in relation to the study.

CHAPTER 8: Conclusion and recommendations

8.1 Introduction

As I discussed at the beginning of this thesis, the primary motivation for my study stemmed from a previous study that I conducted. In my previous study, the focus was on learners, and the finding was that learners were not able to communicate their mathematics reasoning appropriately and the teacher in the study did not challenge the learners in any way that enabled them to communicate better. This was of great concern for me, especially due to my experience as a mathematics teacher. However, I was not able to unpack the teacher's role in my previous study because my focus was on learners. This led to the purpose of this study, which was an investigation into the role of the teacher in developing learners' mathematics discourse and understanding. I embarked on the journey to investigate the role of the teacher in a mathematics functions classroom. I conducted the study in Grade 10 classrooms because this is the first time the learners encounter the formal approach to functions. In previous grades, learners would have encountered functions informally, and linear functions is covered in Grade 9. According to the CAPS curriculum, most of the mathematics terms and concepts in functions are introduced in Grade 10 (see DBE, 2011).

The study focused on the teacher's role in developing learners' mathematics discourse – mathematical communication (both verbal and written) (NCTM, 2010) and understanding - "comprehending mathematical concepts, operations, and relations – knowing what mathematical symbols, diagrams, and procedures mean" (National Research Council, 2002, p.9). The research question and sub research questions that have been answered by this study are reiterated below.

What is the role of the teacher in developing learners' mathematics discourse and understanding?

a. What kind(s) of content and activities is/are selected by the teacher in the process of teaching concepts in the mathematics functions classroom?
b. How does the communicative approach and pattern of discourse shape the enactment of the content and activities selected by the teacher?

c. What strategies does the teacher employ to enable learners to discern the critical features of the object of learning?

To be able to answer the research questions above, I embarked on a qualitative study, and employed appropriate data collection methods. I developed an analytical framework to guide me in the data analysis process. I was able to answer the research questions as discussed in Chapter 7. In what follows, I present the study's contribution to research.

8.2 Contribution to research

The main contribution of my study is the analytical framework which I developed to analyse the role of the teacher in developing learners' mathematics discourse and understanding. The analytical framework below was developed over two years, and this allowed me to engage and comprehensively capture all aspects of what the role of the teacher entails.

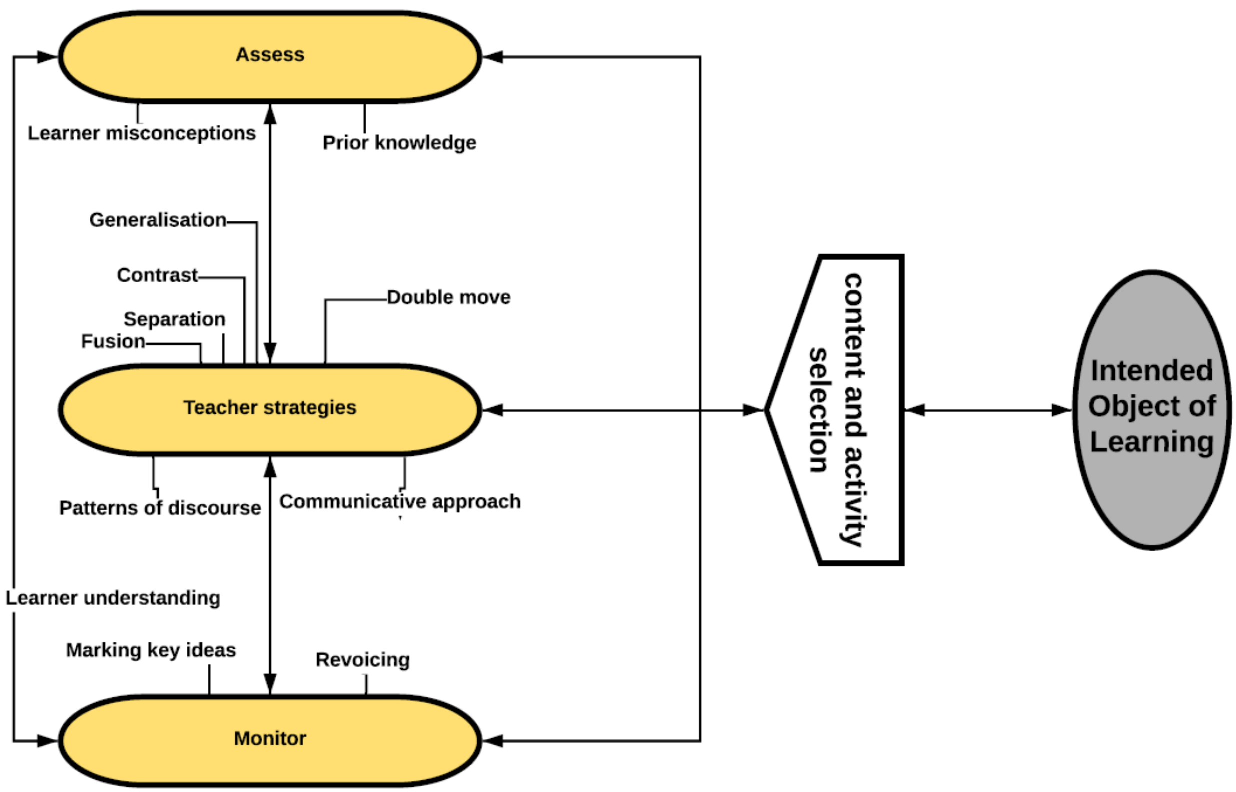


Figure 28: Analytical framework for the study.

In the process of developing the analytical framework, I had to draw on Mortimer and Scott's (2003) framework for analysing meaning making in science classrooms, and variation theory. This was because, at the time of developing this framework, there was no framework developed within the context of mathematics teaching and learning that I could draw on. The analytical framework that I present as a contribution to the field of mathematics instruction has been developed by drawing on empirical findings from real mathematics classrooms.

In the process of developing the analytical framework, I was able to add to the discussion and arguments on the role of assessing learners' prior knowledge, a field that has been extensively researched in mathematics teaching and learning. My study adds a voice to the discussion in the field, with a perspective from a different context, by focusing on how content and activity selection may contribute to effective prior knowledge assessment.

Another aspect that the study has contributed to research is the role of the teaching in selecting content and activities to enact the intended object of learning. I have contributed to the discussion in the field, with a focus on variation theory and how it can be used to select and enact content and activities to provide opportunities for learners to develop their mathematics discourse and understanding.

The final aspect that came out through the development of the analytical framework is that of the double move, and how I have engaged with it in my study, to argue for when and how everyday content and activities can be pedagogically useful in the process of developing learners' mathematics discourse and understanding. The study has contributed to the research on everyday contexts in mathematics classrooms and how it may enable (or not) learners to develop their mathematics discourse and understanding.

8.3 The analytical framework developed for the study

As discussed in Chapter 3, I developed an analytical framework for the study. The analytical framework was informed by Mortimer and Scott's (2003), variation theory (Marton et al., 2004), literature review and my experience as a mathematics teacher. Some of the literature that informed the analytical framework was Hedegaard and Chaiklin's (2005) literature on the double move, and Carr et al's (2009) literature on assessing learners' prior knowledge. My experience as a mathematics teacher, and the existing literature on teaching functions and choosing examples allowed me to bring an awareness to the study in relation to the challenges faced by teachers especially when teaching functions. The analytical framework was a powerful tool that acted as a lens through which I viewed my data, guided by the recognition rules which I developed as part of the analytical framework. The focus of the analytical framework was on the content and activities (example set) selected including their relation to the object of learning, assessing prior knowledge, teacher strategies and monitoring learners' progress. The analytical framework was therefore a powerful tool that enabled me to engage with my data in a meaningful way, in the process of investigating the role of the teacher in developing learners' mathematics discourse and understanding. Without the analytical framework that I developed for my study; I would not have been able to come to the conclusions that I present in this thesis. The amalgamation of the two frameworks is a powerful methodological contribution to the field. The two frameworks allowed me to engage with both the kinds of examples selected by the teacher and the interaction that the example set selected enabled (or not). Using my analytical framework, I was able to come up with recommendations for what the role of the teacher should be in relation to the findings of the study, as presented below.

8.4 Recommendations

From the discussion of results in Chapters 4-7, I came up with five recommendations based on: stating the intended object of learning; assessing prior knowledge, patterns of discourse that develop mathematics discourse, and example set.

The first recommendation that arises from my study is on stating the intended object of learning. Every mathematics lesson should start with the teacher stating the intended object of learning in relation to the purpose of the lesson. If the lesson is a discovery lesson, the teacher should find an appropriate way of ensuring that the intended object of learning is embedded in the discovery activity and hence kept in focus during the lesson. Learners are also brought on the same page with the teacher in the lesson. A lesson which begins without any form of announcement on what the intended object of learning is may lead to confusion on what learners need to focus on and what they need to ignore, as seen in the study. Therefore, mathematics teachers need to be made aware of the impact of starting a lesson without announcing their intention, on the development of learners' mathematics discourse and understanding. This awareness may be raised through in-service professional development or built into the pre-service teacher training and re-emphasised in policy documents and any curriculum teaching and learning materials.

The second recommendation that arises from my study is on assessing prior knowledge in mathematics classrooms. The role of assessing learners' prior knowledge in any lesson cannot be overemphasised. In Chapter 7, I discussed how prior knowledge informs teacher strategies within the lesson. Effective teaching and learning may not occur if the teacher is not aware of what learners already know about a topic or concept before they are introduced to a new topic or concept in the mathematics classroom. Therefore, my recommendation is that mathematics lessons should be conducted with a solid understanding of learners' prior knowledge on the concept being discussed. Assuming that learners have been taught a concept previously and hence are ready to move on should not be a norm in any mathematics classroom. The results from the assessment of prior knowledge should be used to determine the example set that is enacted in the lesson. Assessment of prior knowledge does not have to be written tests, teachers could creatively come up with different forms of assessing learners' prior knowledge that will give effective results to guide the learning process, as discussed by Hailikari et al. (2007). For example, if procedural knowledge is required, the teacher could come up with a task that will help to get the appropriate results for this kind of knowledge.

The third recommendation from my study is that mathematics teachers need to be made aware of the patterns of discourse and communicative approach that provide (or not) learners with opportunities to develop their mathematics discourse and understanding. Teachers need to be made aware of their role in taking deliberate action to influence the interaction in the mathematics classroom. Once teachers' awareness is raised on the issue of patterns of discourse and communicative approach, they may be in position to look out for these actions and take measures to influence and change the patterns or actions to ones that will enable learners develop their mathematics discourse and understanding.

The fourth recommendation from the study is that mathematics teachers need to be supported in developing a more multifaceted understanding of mathematical language, register, and discourse that goes beyond vocabulary, definitions, and conventions to focus more on discursive practices that allow learners to communicate their mathematics reasoning by explaining or justifying their mathematics reasoning.

The fifth recommendation from my study is on the example set and how the teacher may develop it for it to be effective. There is need for re-emphasis of the notion of example set in the teaching and learning of mathematics, especially in critical topics such as functions, to raise teacher awareness on how to develop an example set before and during the lesson, and what should be considered when developing the example set. Curriculum documents and the role that the teacher plays in interpreting and making decisions about how to implement the intended object of learning needs to be emphasised. Strategies such as double move, patterns of variation, and learner feedback need to be emphasised and linked to how teachers play a vital role in selecting and enacting the example set to develop leaners' mathematics discourse and understanding.

8.5 Reflections

My PhD journey has been a long one, with valuable lessons learnt along the way. This focus of the study was closer to my reality than I envisioned when I embarked on the study. The role of the teacher for me has been redefined in a way that is more complex than I thought or ever imagined. With 10 years' experience of teaching mathematics, and four years of mathematics textbook publishing, I thought that I knew it all in some way. However, I came to learn so many things that I was not aware of.

From the time I started thinking about my problem purpose statement, I kept wondering what my findings would look like. Sometimes I felt like I might not find anything about the role of the teacher, except what I already knew, and so I would question the reason for my study. But I soldiered on, and I am glad I did not give up, as I would not have been able to be where I am today in terms of my understanding of the role of a mathematics teacher in developing learners' mathematics discourse and understanding.

When I started reviewing literature, I was fascinated by what I found. One main discovery from my study that I would like to share, and highlight was variation theory. At the start of my project, variation theory was not part of the proposed frameworks that would inform my study. However, during the data collection process, I realised that I would need to engage with the kinds of content and activities that the teachers were selecting in the lessons. In one of my master's courses, I had engaged with variation theory. Therefore, I decided to read further on it, in order to understand how it would enable me analyse the examples that teachers were selecting in their lessons. I could not stop reading literature on variation theory. I had many ideas of how this could be incorporated in teaching mathematics, especially how it could be used to inform development of teaching and learning materials, since teachers use these to select examples to prepare for their lessons. This then led me to reviewing some teaching and learning materials to see what was available to teachers. I was shocked at what I was able to see in relation to variation theory. Prior to this discovery, I would not have been able to analyse a 'good example' in relation to patterns of variation. This discovery enabled me to engage with all teaching and learning materials that I was publishing in a different way. My biggest delight was how teachers received the materials that were developed based on variation theory. I presented on variation theory at numerous workshops and at mathematics conferences. Teachers related to my presentation and they were excited and could not wait to receive the teaching materials that had been developed by drawing on patterns of variation.

The other discovery was teacher awareness. In the pre-observation interviews, I was amazed by what the teachers were saying about their teaching, and by this time, I had already reviewed numerous literature and I was not in the same head space as I had started out at the start of the PhD. Therefore, when teachers were responding to interview questions and showing awareness of some aspects that I had reviewed in literature, I could not wait to go to their classes and see them in action. All the teachers I interviewed seemed confident about what they were doing, they believed that they knew everything they needed to know in order to carry out their role as mathematics teachers successfully, and this rang a bell, as this is what I thought when I started out on this journey as a mathematics teacher with 10 years of teaching experience and having published numerous textbooks and other teaching and learning materials. Learner interaction was one question I asked the teachers. I wanted to know their perception and how they engage learners in their lessons. Only three teachers in phase one of the data collection phase stated that they were aware of their role in ensuring that learners engage in the lessons and communicate their mathematics understanding. This made my selection of phase two participants straight forward. The other three teachers shared their experiences of how it never works when learners are actively involved because they were worried of completing the curriculum requirements on time.

In phase two of data collection, I was saddened by the reality in the classrooms. During the lesson observations, I kept wondering whether this is what the teachers referred to as learner interaction and engagement. Maybe the knowledge I had acquired through the literature review and engaging with developing my analytical framework was making me see things that the teacher would not be able to see, and this is how I would console myself during the lesson observations. In some lessons, I had to hold myself back and not intervene because of the way the teacher engaged with opportunities where a spontaneous activity was needed. In other instances, I felt like talking to the teachers about variation theory and how they could use it in their teaching. I did not engage the teachers until I had finished data collection.

What do I do with these findings and recommendations from my study? This is one question that I have had from the time I started to see emerging findings as a mathematics teacher, and I feel that all mathematics teachers should be made aware of the findings of this study. In my role as mathematics materials developer, I have already used this knowledge and the platform available to me, to influence how content and activities are developed in relation to my findings, especially with regards to the patterns of variation. I developed a framework, which I workshopped to my department, with a focus on how to create an example set in mathematics textbooks that will allow teachers to have ‘carefully’ thought out content and activities that provide opportunities for the intended object of learning to be brought to the fore.

I presented preliminary results of my study at various conferences. I feel obliged to share my findings with every mathematics teacher as much as possible because the role of the teacher is not as straight forward, and this has been highlighted by my study.

The study brought to my awareness that there are many mathematics teachers out there who are not aware of the complexity of their role in developing learners' mathematics discourse and understanding. This is a reality because many teachers may be aware of the theoretical perspective of their role. However, when it comes to putting theory into practice, the teachers may not do this successfully, as shown in my study.

As mentioned earlier, this has been a long journey, with very many lessons, both professionally and socially. I have had to learn how to manage my time the hard way. From having a full-time job that demanded my undivided attention to having two young children who could not imagine a mother who ignores their cry for attention. The experiences I have been through on this journey are priceless and I cannot exchange them for anything. I can comfortably say that I am not in the same head space that I was in at the start of this journey. I will need a whole thesis just to write down my entire reflection of this journey as it has been filled with valuable lessons that are worth sharing. For purposes of this thesis, I will stop here with my reflection and move on to limitations of the study.

8.6 Limitations of the study

The first limitation of the study is that I had to infer the intended object of learning from the enacted object of learning. During the analysis, I realised that I needed to be aware of the intended object of learning by asking the teachers prior to the lessons observed. The realisation came post-lesson observation and after the post-observation interviews. Therefore, there was a missed opportunity of incorporating questions to engage the teachers on the intended object of learning in the post-observation interviews. Therefore, I had to rely on the lessons observed and CAPS document as stated earlier to determine the intended object of learning.

The second limitation is that due to the nature of my study, I was not able to engage with the lived object of learning. It is important to note that although I highlight this as a limitation of my study, it was not part of the focus of the study. My study was focused on the teacher and not on the learner. Therefore, I could not focus on the learners, to find out how the intended object of learning was aligned to the lived object of learning. This aspect is one that can be suggested for future research as I will discuss in the next section.

8.7 Directions for future research

In my study, I realised that in the process of the teacher performing the roles identified, the textbook plays a vital role. I did not focus on how the textbook or any teaching and learning resource that is available to the teacher enables (or not) the role of the teacher in developing learners' mathematics discourse and understanding, and this is one aspect that requires further research.

The other aspect that requires further research is investigating the intended, the enacted and the lived object of learning. In my study, I was not able to engage with the lived object of learning. I was able to argue that opportunities were provided that would enable learners to experience the intended object of learning in ways that would allow them to get to the appropriate lived object of learning. Further research could focus on the role of the teacher in ensuring that the intended object of learning aligns with the lived object of learning, and this would require a comprehensive framework to analyse how to determine the link between intended and lived object of learning.

Another aspect I would suggest for further research is how to effectively develop spontaneous content and activities, and how the teacher can effectively develop them in the time available in the lesson. There were many instances in my study where the teachers would have developed a spontaneous content and activity, but they did not. Further research can focus on that and find out how to enable teachers to make these 'in-the-moment' decisions as discussed by Zaslavsky (2010). Spontaneous activities that draw on everyday contexts and how the teacher chooses them is another aspect that would suggest for further research.

The major contribution of this study is the analytical framework that I developed. A suggestion for research is a study in other topics of mathematics to further strengthen the framework, and thus moving the study forward.

The final suggestion is at a policy level. there is need to investigate how the policy, especially the curriculum enables (or not) the teacher to play a role of developing learners' mathematics discourse and understanding.

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10. Appendices

10.1Appendix A: Ethics clearance letter, information letters and consent forms

University Ethics letter

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

10 November 2014

Student Number: 386418

Protocol Number: 2014ECE015D

Dear Bernadette Aineamani

Application for Ethics Clearance: Doctor of Philosophy
Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

The role of teachers in developing learners' mathematics discourse

The committee recently met and I am pleased to inform you that **clearance was granted**. However, there were a few small issues which the committee would appreciate you attending to before embarking on your research.

The following comments were made:

- You still have not put title of project onto consent forms.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

MMabeta

Wits School of Education

011 717-3416

Cc Supervisor: Dr Anthony Essien

Consent forms

LETTER TO THE PRINCIPAL

DATE:

Dear Sir/Madam,

My name is Benadette Aineamani. I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing research on the role of teachers in developing learners' mathematics discourse, with a title: The role of the teacher in developing learners' mathematics discourse and understanding.

My research involves working with mathematics teachers and learners in Grade 10 mathematics classrooms to administer research instruments. Data collection methods will be teacher interviews and lesson observations. The whole process is expected to take not more than three weeks. I wish to assure you of my commitment to ensure minimal disruptions of the operation of the school and of the classroom where the data will be collected.

I am inviting your school to participate in this research.

The reason why I chose your school is because the Grade 10 mathematics teachers in your school aim to develop learners' mathematics discourse in their classrooms.

Consent will be sought from the teacher and learners and their guardians. The teacher and learners will be told that participating will be on voluntary basis and that those who will agree to participate will be guaranteed confidentiality and anonymity and an option to withdraw during the course of the exercise. I promise to abide by the school regulations during the period of data collection.

The research participants will not be advantaged or disadvantaged in any way. They will be informed that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.
Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely
SIGNATURE:
NAME: Benadette Aineamani
ADDRESS:
EMAIL:
TELEPHONE NUMBERS:

INFORMATION SHEET LEARNERS

DATE:

Dear Learner,

My name is Benadette Aineamani and I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing research on the role of teachers in developing learners' mathematics discourse, with a title: The role of the teacher in developing learners' mathematics discourse and understanding.

My investigation involves working with mathematics teachers and learners in Grade 10 mathematics classrooms to administer research instruments. This entails videotaping some lessons in your class. The whole process is expected to take not more than three weeks. I wish to assure you of my commitment to ensure minimal disruptions of the operation of the school and of the classroom where the data will be collected.

I was wondering whether you would mind if I invite you to be a participant in my research. I need your help as a participant as I observe the teacher during the mathematics lesson. Remember, this is not a test, it is not for marks and it is voluntary, which means that you don't have to do it. Also, if you decide halfway through that you prefer to stop, this is completely your choice and will not affect you negatively in any way.

I will not be using your own name but I will make one up so no one can identify you. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join us in the study.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

SIGNATURE:
NAME: Benadette Aineamani
ADDRESS:
EMAIL:
TELEPHONE NUMBERS:

LEARNER CONSENT FORM

Please fill in the reply slip below if you agree to participate in my study called:

My name is: ________________________

Permission to observe you in class

I agree to be observed in class. YES/NO

Permission to be videotaped

I agree to be videotaped in class. YES/NO

I know that the videotapes will be used for this project only. YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped, photographed and/or videotaped
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____________________________ Date___________________________

INFORMATION SHEET PARENTS

DATE:

Dear Parent

My name is Benadette Aineamani and I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing research on the role of teachers in developing learners' mathematics discourse, with a title: The role of the teacher in developing learners' mathematics discourse and understanding.

My research involves working with mathematics teachers and learners in Grade 10 mathematics classrooms to administer research instruments. This entails videotaping some lessons in your child's class. The whole process is expected to take not more than three weeks. I wish to assure you of my commitment to ensure minimal disruptions of the operation of the school and of the classroom where the data will be collected.

I was wondering whether you would mind if I video record your child during the mathematics lesson.

The reason why I have chosen your child's class is because the teacher is interested in developing learner's mathematics discourse.

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

Your child's name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.
Thank you very much for your help.

Yours sincerely,

SIGNATURE

NAME: Benadette Aineamani
ADDRESS
EMAIL:

PARENT'S CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called:

I, ________________________ the parent of ______________________

Permission to observe my child in class

I agree that my child may be observed in class. YES/NO

Permission to be videotaped

I agree my child may be videotaped in class. YES/NO

I know that the videotapes will be used for this project only. YES/NO

Informed Consent

I understand that:

- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- he/she does not have to answer every question and can withdraw from the study at any time.
- he/she can ask not to be audiotaped, photographed and/or videotaped
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____________________________ Date___________________________

INFORMATION SHEET TEACHERS

DATE:

Dear Teacher,

My name is Benadette Aineamani and I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing research on the role of teachers in developing learners' mathematics discourse, with a title: The role of the teacher in developing learners' mathematics discourse and understanding.

I would like to invite you to be my participant in this study.

My research involves working with mathematics teachers and learners in Grade 10 mathematics classrooms to administer research instruments. This entails videotaping some lessons in your class and conducting a 20-minute interview with you. The interviews would be audiotaped to ensure accuracy. The whole process is expected to take not more than three weeks. I wish to assure you of my commitment to ensure minimal disruptions of the operation of the school and of the classroom where the data will be collected.

The reason why I have chosen your class is because you are interested in developing learner's mathematics discourse.

You will not be advantaged or disadvantaged in any way. You will be reassured that you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.
Thank you very much for your help.

Yours sincerely,

SIGNATURE

NAME: Benadette Aineamani
ADDRESS
EMAIL:

TEACHER'S CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to be a participant in my voluntary research project called:

I, ________________________ give my consent for the following:

Circle one

Permission to observe you in class

I agree to be observed in class.	YES/NO

Permission to be audiotaped

I agree to be audiotaped during the interview or observation lesson	YES/NO
I know that the audiotapes will be used for this project only	YES/NO

Permission to be interviewed

I would like to be interviewed for this study.	YES/NO
I know that I can stop the interview at any time and don't have to answer all the questions asked.	YES/NO

Permission to be videotaped

I agree to be videotaped in class.	YES/NO
I know that the videotapes will be used for this project only.	YES/NO

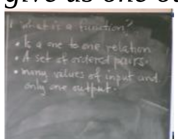
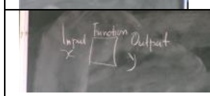
Informed Consent

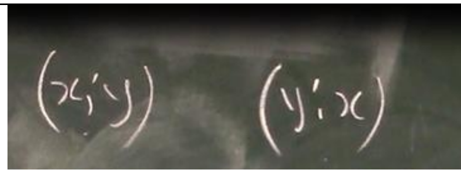
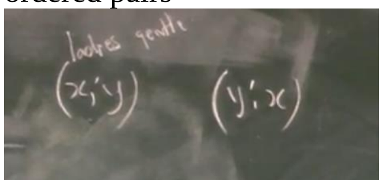
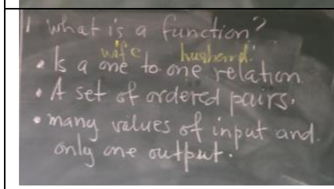
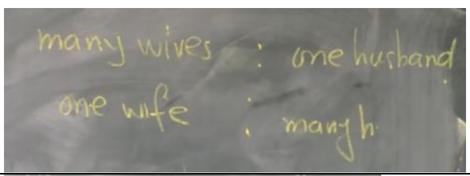
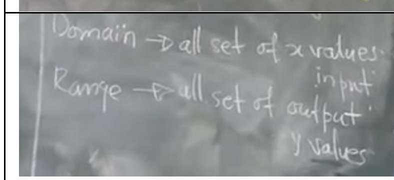
I understand that:

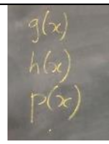
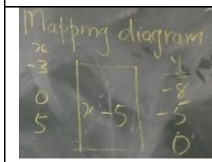
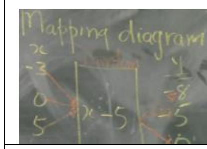
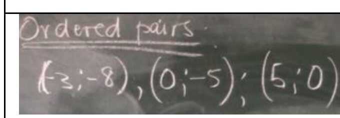
- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped, photographed and/or videotaped
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

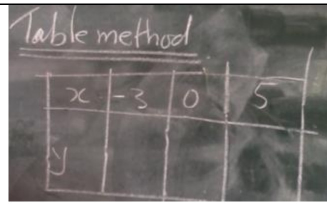
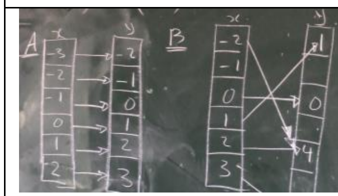
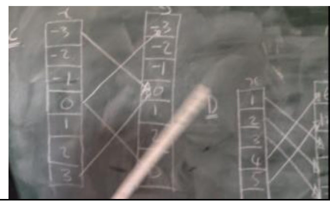
Sign_____________________________ Date___________________________

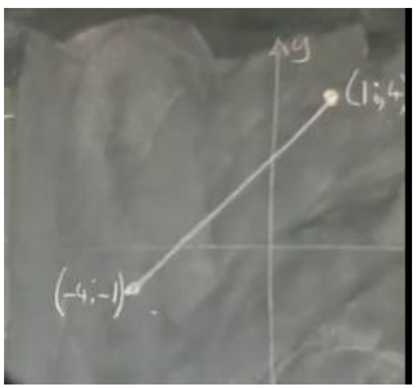
10.2 Teacher A's lessons summarised

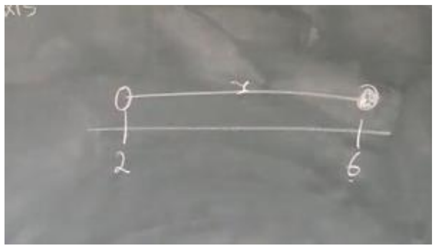
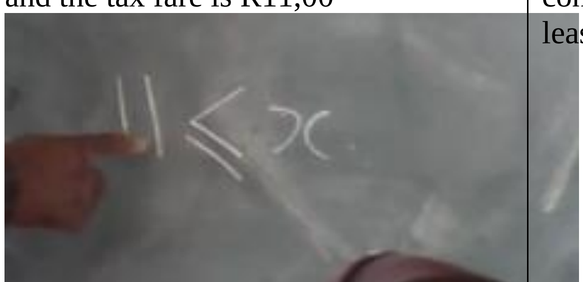
Episode	**Content/ activity**	**Teacher activity**	**Learner activity**	**Comment**
1. Defining a function	What is a function? A function is a special relationship between input and output values, where every input value has got only one output value. *We can have a number of input values but they will give us one output.* 	Walking around the class and checking learners' definitions Asks learners to share their definitions	Writing down the definition of a function in their books Learners share their definition in a whole class discussion	This was the first activity given in class. The teacher asked learners to engage with the concept of a function before it is taught in this lesson **- Code: LPK** The Teacher evaluated learners' responses- **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback - **Code: IA**
		Asking learners questions about the input and output values	Learners responding in chorus form	Teacher illustrated the concept of a function – **Code: SS-TS** The Teacher evaluated learners' responses- **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback - **Code: IA**
	Factory setting as function in real life	Linking the concept of input and output to a factory setting	Learners responding in chorus form	Teacher used a factory setting to illustrate the concept of a function- **Code: DM-TS** The Teacher evaluated learners' responses **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback - **Code: IA**
	(3; 6) and (3; 9) as sets of ordered pairs	Asks learners to give any numbers and he writes them on the board	Learners identifying the input and output in the set of ordered pairs	Teacher explained the convention of an ordered pair – **Code: SS-TS** The Teacher evaluated learners' responses **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback - **Code: IA**

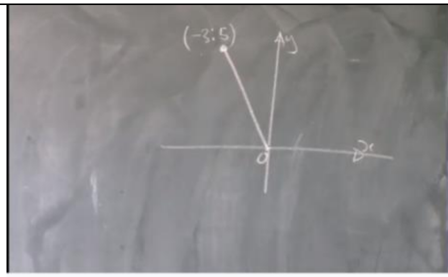
		Explains to learners the convention of writing the ordered pair	Responding to the teacher on how to write an ordered pair	Teacher continued to explain the convention of an ordered pair – **Code: CC-TS** The Teacher evaluated learners' responses **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback - **Code: IA**
	Ladies and gentlemen Vs gentlemen and ladies example in relation to ordered pairs 	Explains to learners the convention of writing the ordered pair	Responding to the teacher's questions	Teacher continued to explain the convention of an ordered pair, using an everyday convention of ladies and gentlemen – **DM-TS** The Teacher evaluated learners' responses- **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback - **Code: IA**
	 	Introduces an everyday example to the learners	Learners responding to teacher's questions	Teacher explained the concept of a function using a legal marriage example – **DM-TS** The Teacher evaluated learners' responses- **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback - **Code: IA**
		Asks learners to define domain and range – writes on the board	Learners respond to teacher's questions	Teacher asked leaners to define domain and range before they were taught in class – **Code: LPK** The Teacher evaluated learners' responses- **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback - **Code: IA**

	 Function notation: *f(x) = y* *h(x) = y; p(x) = y; k(x) = y*	Asks learners to give examples of function notation	Learners respond to teacher's questions	Teacher asked learners to give different ways of writing a function notation – **Code: SS-TS** The Teacher evaluated learners' responses- **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback **- Code: IA**
	 	Teacher asks learners to give input values and explains to learners a mapping diagram	Learners give the teacher input values	The teacher explained to learners the mapping diagram as one way of representing a function notation – **Code: SS-TS** The Teacher evaluated learners' responses- **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback **- Code: IA**
	Taking sweets out of a jar	Teacher explains to learners about taking sweets from a jar in relation to a mapping diagram	Learners responding to teacher's questions	Teacher A used an example of taking sweets out of a jar in relation to a mapping diagram – **Code: DM-TS** The teacher allowed learners to interact, and gave authoritative feedback **- Code: IA**
		Teacher explains to learners a table of values	Learners responding to teacher's questions	The teacher explained to learners the table of values as one way of representing a function notation – **Code: SS-TS** The Teacher evaluated learners' responses- **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback **- Code: IA**

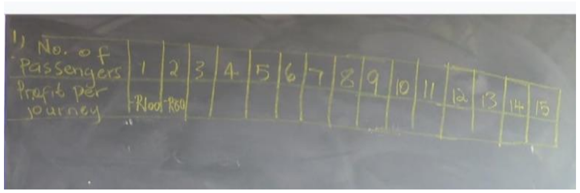
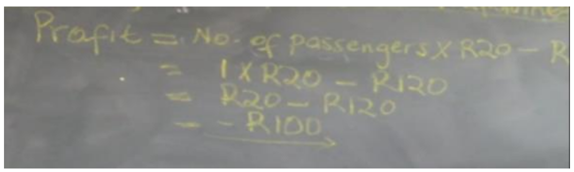
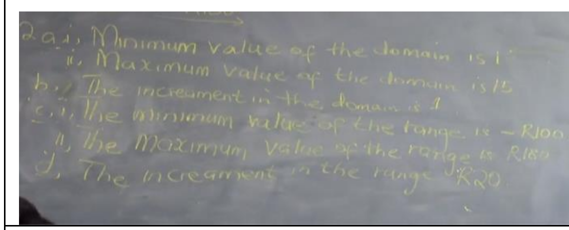
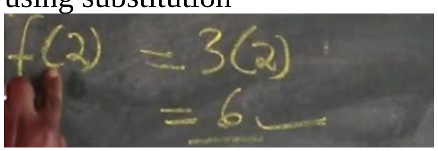
				
	 	Teacher gives learners homework to attempt	Learners copying work in their books	Teacher A gave learners different mapping diagram for them to determine which representation of a function and which is not – **Code: DM-TS** The Teacher evaluated learners' responses- **Code: I-R-E** The teacher allowed learners to interact, and gave authoritative feedback - **Code: IA**
2. Plotting a straight line graph	$y = f(x) = 2x$	Writing on the board. Asking learners to attempt this activity Guides the learners to create a table of values and get ordered pairs as a starting point for plotting.	Attempting the activity on the board in a whole class discussion	This was the first activity given in class. The teacher expected learners to plot the graph but he referred to sketching the graph **Code: LPK** The teacher and the learners in the class patiently waited for the learner attempting the activity on the board, and the teacher gave an authoritative feedback. **Code: IA** The Teacher evaluated learners' responses- **Code: I-R-E**
	$f(x) = -2x$ $g(x) = x + 1$	Writing on the board.	Attempting the activity of creating a table	This activity has similar examples given to learners by the teacher, but there was no discussion of the similarities. **Code: SS-TS**

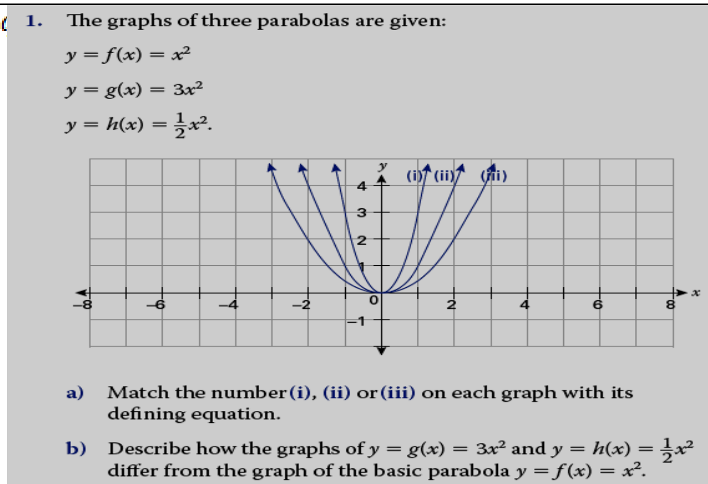
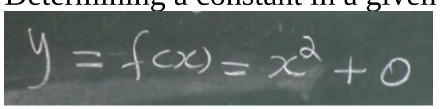
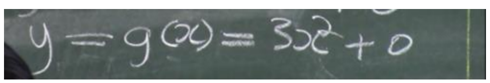
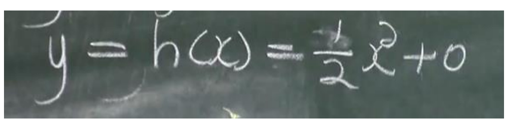
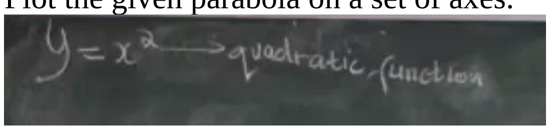
	$h(x) = -x - 1$ $j(x) = -x + 1$	Asks learners to get a table of values for the functions given Completes a section of the table of values. Walks around the class helping some of the learners	of values in their books	Learners were asked to attempt the activities as a class activity and later as homework. **Code: MM-LU** The teacher asked learners if they understood and were able to complete a table of values. **Code: AA-LU** This activity continued as corrections in the next lesson. However, the teacher attempted only $f(x) = -2x$ and then moved on to the next concept. **Code: MKI** The Teacher evaluated learners' responses- **Code: I-R-E**
3. a) Finding the equation of a graph. b) domain and range of a straight line graph	 Domain $x \in [-4; 1]$ $-4 \leq x \leq 1$	Writing on the board. Asking learners questions about domain and range and asking learners how to find the equation of a straight line	Learners responding to teacher's questions in chorus form	The teacher asked learners to refer to their books to see this example. The focus was on domain and range. The learners were asked to find the equation of the graph. Here, learners needed to know how to find the gradient of the graph. The question was asked in such a way that the teacher expected learners to have prior knowledge on the concept of gradient and y-intercept. **Code: LPK** The teacher asked learners questions about the domain and range, and engaged with their responses, with an authoritative voice. **Code IA** The Teacher evaluated learners responses- **Code: I-R-E**

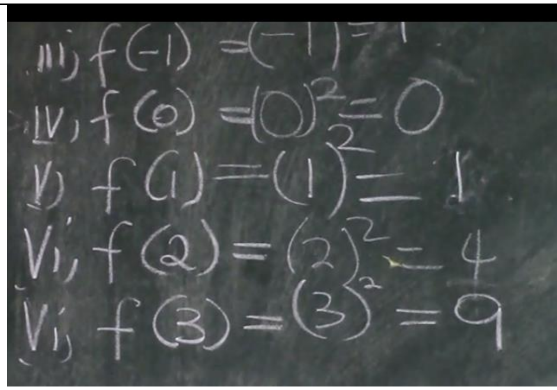
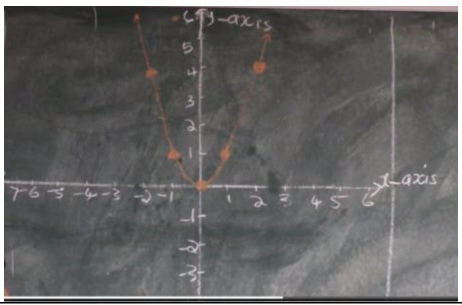
	Range $y \in [-1;4]$ Equation of the straight line			
	 $x \in (2;6]$	Explaining the domain and range, using the open and closed notations on the example	Learners responding to teacher’s questions in a chorus form	This is a spontaneous question that the teacher drew on in the class to explain the domain of a function and how it is represented. The teacher relates this question to the previous question. **Code: SS-TS** The teacher used terms such as banana bracket (and telephone bracket] and it was very well received and acceptable language in this class. **Code: DM-TS** Learners respond to teacher questions in chorus form and the teacher used learners’ responses to talk about the next concept. **Code: ID; I-R-E**
	Using a tax to travel to Johannesburg and the tax fare is R11,00 	Explaining to learners the concept of at least and atmost	Responding to teacher’s questions in chorus form	This is an example from everyday explaining the less or equal to concept. The teacher used the example to explain the domain and range. **Code: DM-TS** Learners respond to teacher questions in chorus form and the teacher used learners’ responses to talk about the next concept. Teacher evaluate learners’ responses authoritatively. **Code: ID; I-R-E** The teacher asked learners if they understood how to determine the domain and range of a straight-line graph. **Code: AA-LU**
		Writing on the board	Responding to teacher’s	This is a second example that the teacher gave learners to find the domain, range and equation of

		Asking learners questions	questions in chorus form	the graph. **Code: SS-TS** Learners respond to teacher questions in chorus form and the teacher used learners' responses to talk about the next concept. Teacher evaluate learners' responses authoritatively. **Code: ID; I-R-E**
	Gradient and y-intercept of a straight line. $y = mx + c$ $y - y_1 = m(x - x_1)$	Writing on the board Asking learners questions. Explaining what y-intercept is and how to find the gradient of a straight line.	Responding to teacher's questions in chorus form	Here, the teacher explained to learners how to find the gradient, using the formula given, and the general equation of a straight line. The example required learners to engage with various concepts, such as y-intercept, gradient. **Code: FF-TS** Learners respond to teacher questions in chorus form and the teacher used learners' responses to talk about the next concept. Teacher evaluate learners' responses authoritatively. **Code: ID; I-R-E**

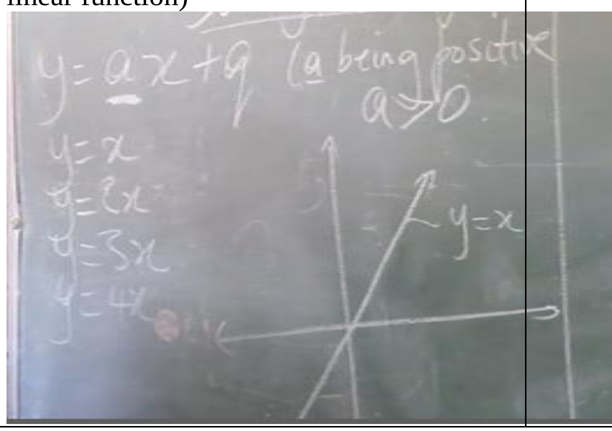
10.3 Teacher B's lessons summarised

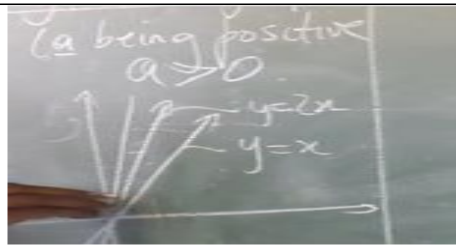
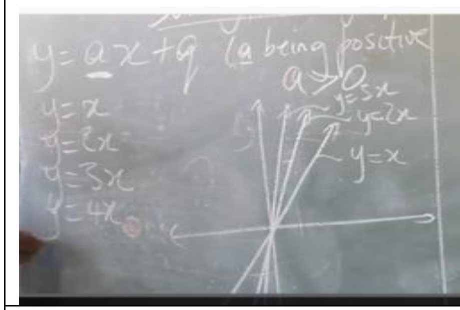
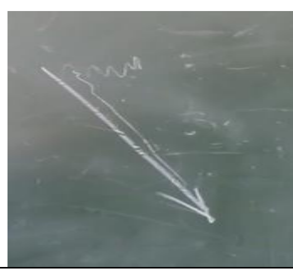
Episode	Content/activity	Teacher activity	Learner activity	Comment
5. Defining a function	Representing the journey of a taxi driver on a table of values.   	Asking learners questions about the values to complete the table and questions that follow on domain and range	Learners responding to teacher's questions in chorus and individually	Teacher B started with this activity at the start of the lesson, as an introduction to functions – **Code: LPK** Teacher B used everyday context of a taxi driver and passengers to generate the table of values. Teacher B related *x* to number of passengers, and *y* to the profit per journey – **Code: DM-TS** Teacher allowed learners to interact and she evaluated their responses authoritatively- **Code: I-R-E; IA**
	Determining the value of *y*, given the value of *x* – using substitution 	Teacher writes on the board and asks learners questions	Learners respond to teacher's questions	Teacher B used this activity to explain to learners how to substitute – **Code: AA-LU** Teacher allowed learners to interact and she evaluated their responses authoritatively- **Code: I-R-E; IA**
6. Investigate the effect of *a* and *q* on the graphs defined by	Investigate the effect of *a* and *q* on the graphs. Match the graphs with the defining equation; describe how the graphs differ from the basic parabola.	Teacher writes on the board and asks learners questions	Learners respond to teacher's questions	Teacher B gave learners this activity from the text. There are various aspects being covered, with variant and invariant aspects- **Code: SS-TS; FF-TS**

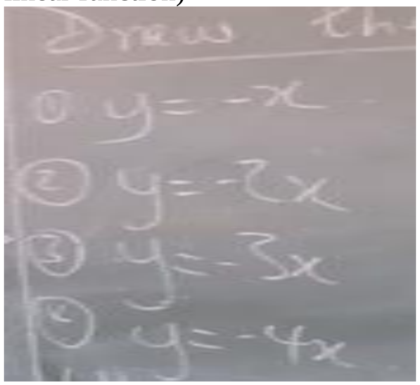
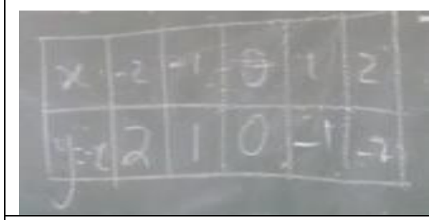
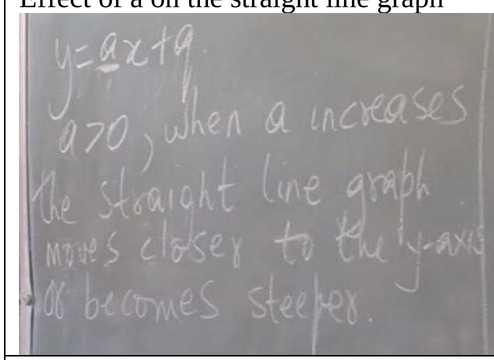
$y = a.f(x) +$	 1. The graphs of three parabolas are given: $y = f(x) = x^2$ $y = g(x) = 3x^2$ $y = h(x) = \frac{1}{2}x^2$. a) Match the number (i), (ii) or (iii) on each graph with its defining equation. b) Describe how the graphs of $y = g(x) = 3x^2$ and $y = h(x) = \frac{1}{2}x^2$ differ from the graph of the basic parabola $y = f(x) = x^2$. Source: Classroom mathematics Grade 10 Learner book (p. 159) (Pike et al., 2011)			Teacher allowed learners to interact and she evaluated their responses authoritatively- **Code: I-R-E; IA**
	Determining a constant in a given equation    	Teacher writes on the board and asks learners questions	Learners respond to teacher's questions	Teacher B used this activity to show learners a constant – **Code: SS-TS; CC-TS** Teacher allowed learners to interact and she evaluated their responses authoritatively- **Code: I-R-E; IA** Teacher allowed learners to interact and she evaluated their responses authoritatively- **Code: I-R-E; IA**
7. Point by point plotting of a parabola to discover the domain (input values) and	Plot the given parabola on a set of axes.  	Teacher writes on the board and asks learners questions	Learners respond to teacher's questions	Teacher B gave learners one graph plot.

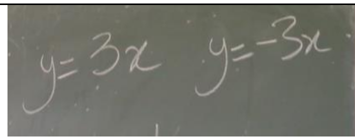
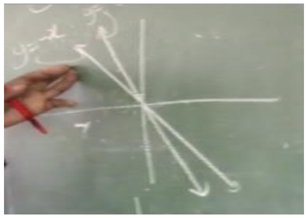
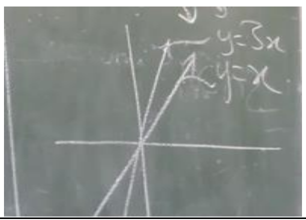
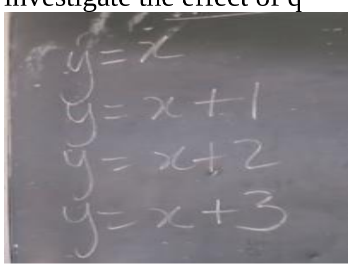
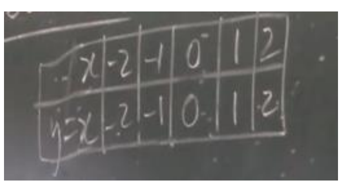
range (output values)	 			

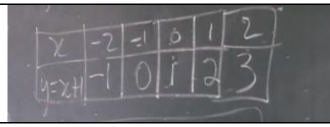
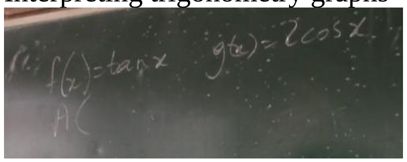
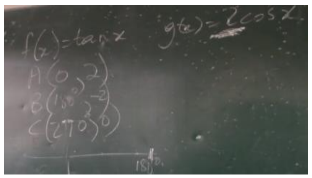
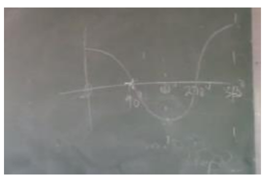
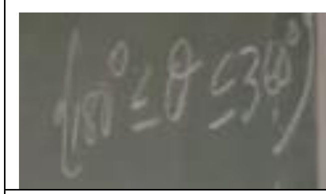
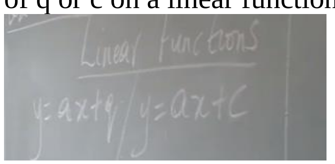
10.4 Teacher C’s lessons summarised

Episode	Content/activity	Teacher activity	Learner activity	Comment
1. Defining a function	Define a function Five representations of functions	Walking around the class as learners attempt a test. Asks learners to define a function and give types of representations	Writing down the definition of a function in their books Learners share their definition in a whole class discussion and share the different types of representations	The teacher asked learners to engage with the concept of a function before it is taught in this lesson - **LPK** The teacher asked learners questions engaged with their responses, with authoritative responses. **Code: IA** Learners’ responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**
2. Effect of a and q on a straight-line graph	Draw straight line graphs using a table of values, to investigate the effect of a positive gradient (the value of a in the linear function) 	Teacher writes on the board and asks learners questions	Learners respond to teacher’s questions	The teacher asked learners to engage with the effect of a before it is taught in this lesson - **LPK** The equations given have variant and invariant aspects– **Code – SS-TS** The teacher asked learners questions engaged with their responses, with authoritative responses. **Code: IA** Learners’ responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**

	 			
	Increasing vs decreasing graph  	Teacher writes on the board and asks learners questions	Learners respond to teacher's questions	Two illustrations given – **Code: CC-TS** The teacher asked learners questions engaged with their responses, with authoritative responses. **Code: IA** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**

	Draw straight line graphs using a table of values, to investigate the effect of a negative gradient (the value of a in the linear function)  	Teacher writes on the board and asks learners questions	Learners respond to teacher's questions	The equations given have variant and invariant aspects– **Code – SS-TS** The teacher asked learners questions engaged with their responses, with authoritative responses. **Code: IA** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**
	Effect of a on the straight line graph 	Teacher writes on the board as she summarises the effect of a	Learners listen to the teacher	The teacher gives a conclusion on the effect of a– **Code – FF-TS**
	Draw the graphs to compare the effect of *a*	Teacher writes on the board and asks learners questions	Learners respond to teacher's questions	The equations given have variant and invariant aspects– **Code – SS-TS**

	  			The teacher asked learners questions engaged with their responses, with authoritative responses. **Code: IA** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**
	Plot graphs on a graph paper and investigate the effect of q  	Teacher writes on the board and asks learners questions	Learners respond to teacher's questions	The teacher asked learners to engage with the effect of q before it is taught in this lesson - **LPK** The equations given have variant and invariant aspects– **Code – SS-TS** The teacher asked learners questions engaged with their responses, with authoritative responses. **Code: IA** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**

				
	Interpreting trigonometry graphs    	Teacher gives learners papers and writes on the board as she asks learners questions	Learners read the handouts given by the teacher as they answer questions	The equations given have variant and invariant aspects and the functions are different from straight-line graphs – **Code – SS-TS; CC-TS** The teacher asked learners questions engaged with their responses, with authoritative responses. **Code: IA** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**
	Linear functions: investigating the effect of q or c on a linear function 	Teacher writes on the board and asks learners questions	Learners respond to teacher questions	The equations given have variant and invariant aspects – **Code – SS-TS** The teacher asked learners questions, and engaged with their responses, with authoritative responses. **Code: IA** Learners' responses were probed for

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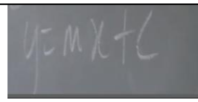

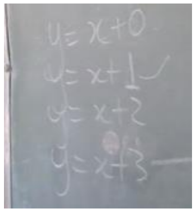

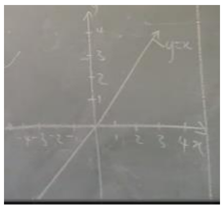


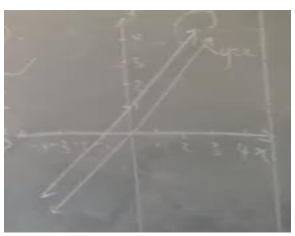

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<td>further explanation and teacher gave evaluative response. Code: I-R-E</td>
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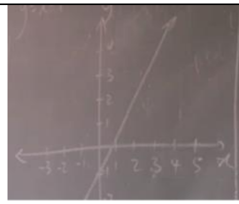
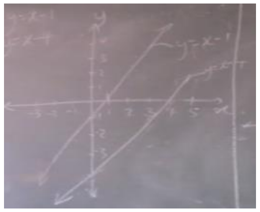
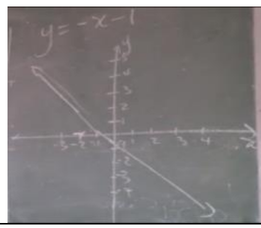
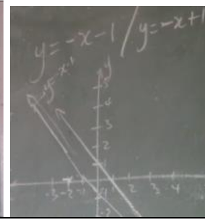
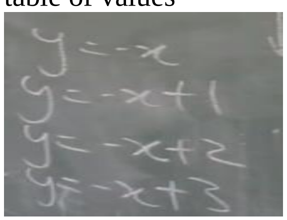
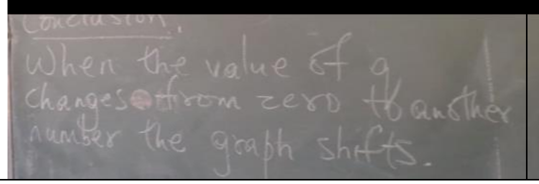
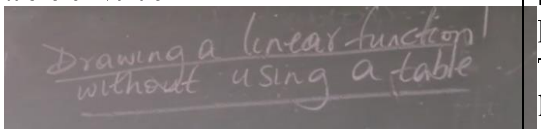
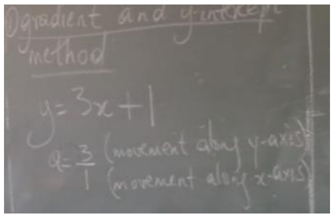
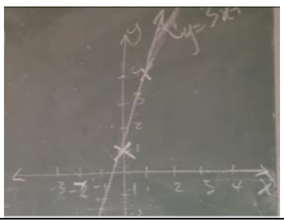
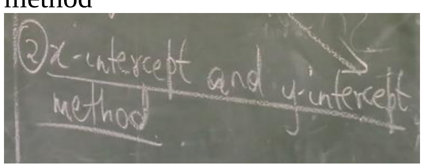
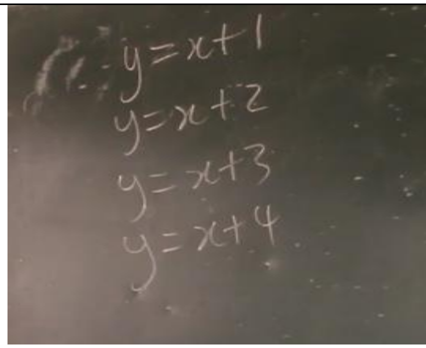
	  			
3. Sketch straight-line graphs	Sketch the graphs below without using a table of values  Conclusion on effect of q on linear graphs 	Teacher writes on the board and asks learners questions	Learners responding to teacher's questions	The equations given have variant and invariant aspects – **Code – SS-TS** The teacher asked learners questions, and engaged with their responses, with authoritative responses. **Code: IA** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**

	Sketch a linear function without using a table of value  Method 1: Gradient and *y*-intercept method  	Teacher writes on the board while asking learners questions. Teacher was writing learners' responses on the board, with changes made to learner responses to suit Teacher's expected answer	Learners responding to teacher's responses	Teacher introduced this method while asking learners questions and evaluation them. The teacher asked learners questions about the gradient and *y*-intercept, and engaged with their responses, with authoritative responses. **Code: IA** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**
	Method 2: *x*-intercept and y-interept method 	Teacher writes on the board while asking learners questions. Teacher was writing learners' responses on the board, with changes made to learner responses to suit Teacher's expected answer	Learners responding to teacher's responses	Teacher introduced this method while asking learners questions and evaluation them. The equations given have variant and invariant aspects – **Code – SS-TS** The teacher asked learners questions about the gradient and *y*-intercept, and engaged with their responses, with authoritative responses. **Code: IA** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**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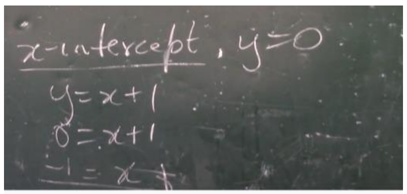
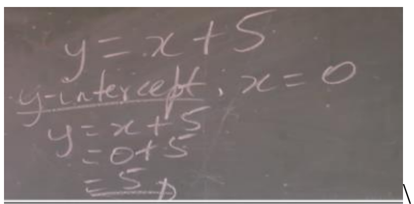
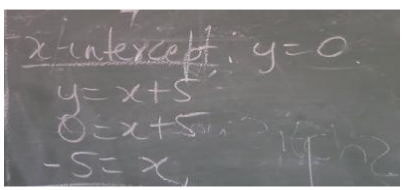
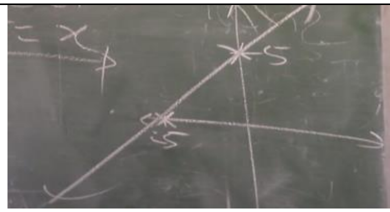
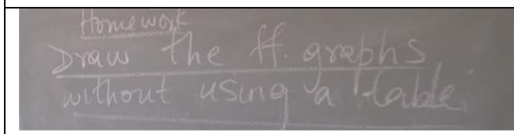
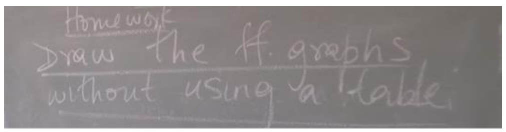
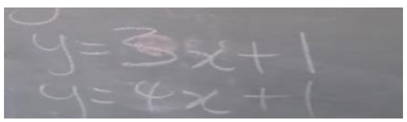
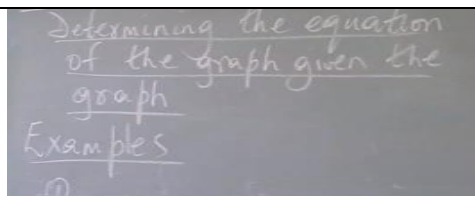
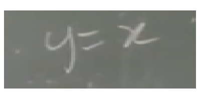
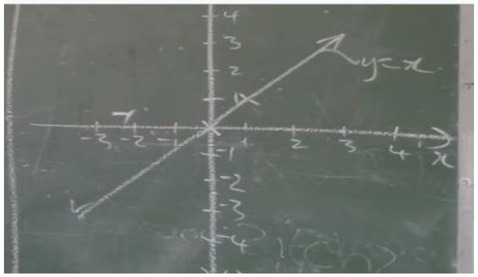
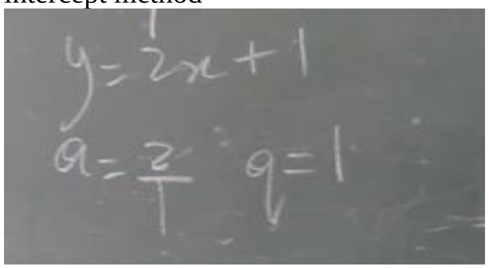
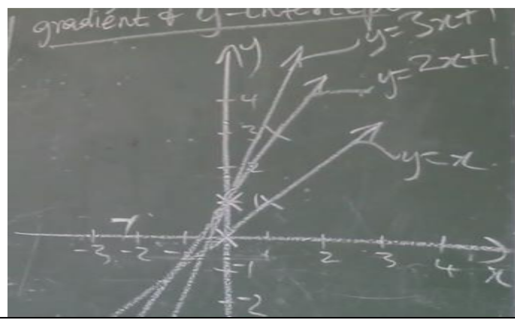
y=x+2
y=x+3
y=x+4

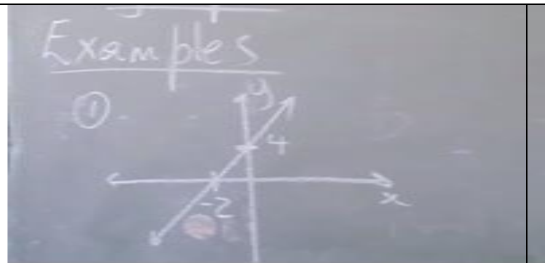
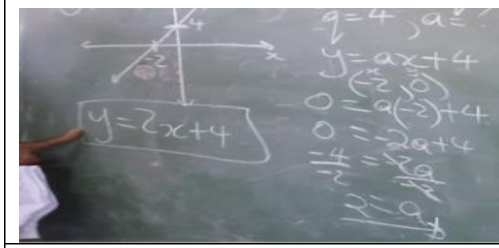
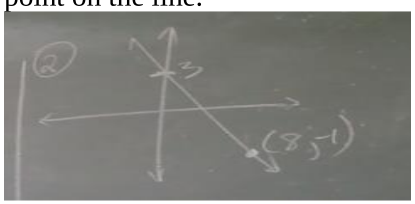
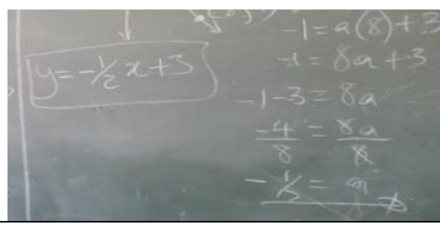

x-intercept, y=0
y=x+1
0=x+1
-1=x


y=x+5
y-intercept, x=0
y=x+5
=0+5
=5


x-intercept, y=0
y=x+5
0=x+5
-5=x

				
	  			
	 	Teacher writes on the board as she asked learners questions	Learners respond to teacher's questions	Teacher gave learners an activity to attempt before she have them different equations to sketch – **Code – SS-TS; FF-TS** The teacher asked learners questions about the gradient and *y*-intercept, and engaged with their responses, with authoritative responses. **Code: IA** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**

				
	Sketch a graph using the gradient and y-intercept method  	Teacher writes on the board and asks learners questions	Learners respond to teacher's questions	Teacher C gave learners more than one equation to be plotted - **Code: SS-TS; FF-TS** The teacher asked learners questions about the gradient and *y*-intercept, and engaged with their responses, with authoritative responses. **Code: IA** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**
4. Determine the equation of a straight-line	Determine the equation of a straight line graph when given the intercepts.	Teacher writes on the board and asks learners questions	Learners respond to teacher's questions	The teacher asked learners questions about the gradient and *y*-intercept, and engaged with their responses, with authoritative responses. **Code: IA** This was one of the content and activities given to learners on this concept: **Code:**

	 			**SS-TS** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**
	Determine the equation of a straight line when given the y-intercept and another point on the line.  	Teacher writes on the board and asks learners questions	Learners respond to teacher's questions	Teacher C gave learners an activity to attempt – when given two points, learners were asked to attempt the activity. This was one of the content and activities given to learners on this concept: **Code: SS-TS** The teacher asked learners questions about the gradient and *y*-intercept, and engaged with their responses, with authoritative responses. **Code IA** Learners' responses were probed for further explanation and teacher gave evaluative response**. Code: I-R-E**