Case I: Classification with respect to 3D Lie algebras

- 3D Lie algebra: $A_1 \oplus A_2$ (e)

```
Get["C:\Documents and Settings\molati\My Documents\YaLie.m"]
```

```
======================================================

                 YaLie

          Version 2.0 (2000)

              @ JM Díaz

      Departamento de Matemáticas

        Universidad de Cádiz

======================================================
```

```
eq1 = D[α[t, x], t] + α[t, x] D[u[t, x], x] + u[t, x] D[α[t, x], x] == 0;
eq2 = ρ (D[u[t, x], t] + u[t, x] D[u[t, x], x]) + D[P[α[t, x]], x] + R[α[t, x], u[t, x] ] == 0;
VD = {α, u};
VIs = {t, x};
VP = {D[α[t, x], t], D[u[t, x], t]};
EDs = YaLie[{eq1, eq2}, VD, VIs, VP];
DFormat[True];
EDs
```

$$\Big\{u\,\alpha\,(Xi[1]_u) - \alpha\,(Xi[2]_u) - \alpha^2\,(Xi[1]_\alpha) == 0,\ -\frac{u\,(P_\alpha)\,(Xi[1]_u)}{\rho} + \frac{(P_\alpha)\,(Xi[2]_u)}{\rho} + \frac{\alpha\,(P_\alpha)\,(Xi[1]_\alpha)}{\rho} == 0,$$

$$-\frac{R[\alpha,u]\,(Phi[1]_u)}{\rho} + u\,(Phi[1]_x) + \alpha\,(Phi[2]_x) + \frac{\alpha\,R[\alpha,u]\,(Xi[1]_x)}{\rho} + Phi[1]_t == 0,$$

$$Phi[1][t,x,\alpha,u] + \alpha\,(Phi[2]_u) - \alpha\,(Phi[1]_\alpha) + 2\,u\,\alpha\,(Xi[1]_x) - \alpha\,(Xi[2]_x) + \alpha\,(Xi[1]_t) == 0,$$

$$Phi[2][t,x,\alpha,u] - \frac{(P_\alpha)\,(Phi[1]_u)}{\rho} - \frac{u\,R[\alpha,u]\,(Xi[1]_u)}{\rho} + \frac{R[\alpha,u]\,(Xi[2]_u)}{\rho} + \alpha\,(Phi[2]_\alpha) +$$
$$\frac{\alpha\,R[\alpha,u]\,(Xi[1]_\alpha)}{\rho} + \left(u^2 + \frac{\alpha\,(P_\alpha)}{\rho}\right)(Xi[1]_x) - u\,(Xi[2]_x) + u\,(Xi[1]_t) - Xi[2]_t == 0,$$

$$\alpha\,(P_\alpha)\,(Xi[1]_u) - u\,\alpha\,\rho\,(Xi[1]_\alpha) + \alpha\,\rho\,(Xi[2]_\alpha) == 0,\ -\frac{(P_\alpha)^2\,(Xi[1]_u)}{\rho} + u\,(P_\alpha)\,(Xi[1]_\alpha) - (P_\alpha)\,(Xi[2]_\alpha) == 0,$$

$$Phi[2][t,x,\alpha,u]\,(R_u) + Phi[1][t,x,\alpha,u]\,(R_\alpha) - R[\alpha,u]\,(Phi[2]_u) - \frac{R[\alpha,u]^2\,(Xi[1]_u)}{\rho} +$$
$$(P_\alpha)\,(Phi[1]_x) + u\,\rho\,(Phi[2]_x) + u\,R[\alpha,u]\,(Xi[1]_x) + \rho\,(Phi[2]_t) + R[\alpha,u]\,(Xi[1]_t) == 0,$$

$$Phi[1][t,x,\alpha,u]\,(P_{\alpha\alpha}) - (P_\alpha)\,(Phi[2]_u) - \frac{2\,R[\alpha,u]\,(P_\alpha)\,(Xi[1]_u)}{\rho} +$$
$$(P_\alpha)\,(Phi[1]_\alpha) + 2\,u\,(P_\alpha)\,(Xi[1]_x) - (P_\alpha)\,(Xi[2]_x) + (P_\alpha)\,(Xi[1]_t) == 0,$$

$$\rho\,Phi[2][t,x,\alpha,u] + (P_\alpha)\,(Phi[1]_u) - u\,R[\alpha,u]\,(Xi[1]_u) + R[\alpha,u]\,(Xi[2]_u) - \alpha\,\rho\,(Phi[2]_\alpha) -$$
$$\alpha\,R[\alpha,u]\,(Xi[1]_\alpha) + \left(u^2\rho + \alpha\,(P_\alpha)\right)(Xi[1]_x) - u\,\rho\,(Xi[2]_x) + u\,\rho\,(Xi[1]_t) - \rho\,(Xi[2]_t) == 0\Big\}$$

```
LXi = {-t + c[1], c[2]};
LPhi = {c[3] α, u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\{True, True, True, True, True, True, True,$$
$$2\,R[\alpha,u] == u\,(R_u) + \alpha\,c[3]\,(R_\alpha),\ (-2 + c[3])\,(P_\alpha) + \alpha\,c[3]\,(P_{\alpha\alpha}) == 0,\ True\}$$

```
LXi = {-t, 0};
LPhi = {k α, u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\{True, True, True, True, True, True, True,\ 2\,R[\alpha,u] == u\,(R_u) + k\,\alpha\,(R_\alpha),\ (-2 + k)\,(P_\alpha) + k\,\alpha\,(P_{\alpha\alpha}) == 0,\ True$$

$k \neq 0$ since we require that $P \neq 0$.

```
DSolve[k α P''[α] + (-2 + k) P'[α] == 0, P, α]
```

$$\left\{\left\{P \to Function\left[\{\alpha\},\ \frac{1}{2}\,k\,\alpha^{2/k}\,k[3] + k[4]\right]\right\}\right\}$$

```
DSolve[ 2 R[α, u] == u D[R[α, u], u ] + k α D[R[α, u], α ], R, {α, u}]
```

$$\left\{\left\{R \to Function\left[\{\alpha, u\},\ \alpha^{2/k}\,\Gamma\left[u\,\alpha^{-1/k}\right]\right]\right\}\right\}$$

Created with Wolfram *Mathematica* 7.0

Case I: Classification with respect to 4D Lie algebras

- 4D Lie algebra containing 3D Lie algebra $A_1 \oplus A_2$ (e).

```
eq1 = D[α[t, x], t] + α[t, x] D[u[t, x], x] + u[t, x] D[α[t, x], x] == 0;
eq2 = ρ (D[u[t, x], t] + u[t, x] D[u[t, x], x]) + D[(1/2 k α[t, x]^(2/k) K[3] + K[4]), x] +
    α^(2/k) Γ[u[t, x] α[t, x]^(-1/k)] == 0;
VD = {α, u};
VIs = {t, x};
VP = {D[α[t, x], t], D[u[t, x], t]};
EDs = YaLie[{eq1, eq2}, VD, VIs, VP];
DFormat[True];
EDs
```

$$\Big\{u\,\alpha\,(Xi[1]_u) - \alpha\,(Xi[2]_u) - \alpha^2\,(Xi[1]_\alpha) == 0,$$

$$-\frac{u\,\alpha^{-1+\frac{2}{k}}\,K[3]\,(Xi[1]_u)}{\rho} + \frac{\alpha^{-1+\frac{2}{k}}\,K[3]\,(Xi[2]_u)}{\rho} + \frac{\alpha^{2/k}\,K[3]\,(Xi[1]_\alpha)}{\rho} == 0,$$

$$-\frac{\alpha^{2/k}\,\Gamma[u\,\alpha^{-1/k}]\,(Phi[1]_u)}{\rho} + u\,(Phi[1]_x) + \alpha\,(Phi[2]_x) + \frac{\alpha^{1+\frac{2}{k}}\,\Gamma[u\,\alpha^{-1/k}]\,(Xi[1]_x)}{\rho} + Phi[1]_t == 0,$$

$$Phi[1][t, x, \alpha, u] + \alpha\,(Phi[2]_u) - \alpha\,(Phi[1]_\alpha) + 2\,u\,\alpha\,(Xi[1]_x) - \alpha\,(Xi[2]_x) + \alpha\,(Xi[1]_t) == 0,$$

$$Phi[2][t, x, \alpha, u] - \frac{\alpha^{-1+\frac{2}{k}}\,K[3]\,(Phi[1]_u)}{\rho} - \frac{u\,\alpha^{2/k}\,\Gamma[u\,\alpha^{-1/k}]\,(Xi[1]_u)}{\rho} + \frac{\alpha^{2/k}\,\Gamma[u\,\alpha^{-1/k}]\,(Xi[2]_u)}{\rho} +$$
$$\alpha\,(Phi[2]_\alpha) + \frac{\alpha^{1+\frac{2}{k}}\,\Gamma[u\,\alpha^{-1/k}]\,(Xi[1]_\alpha)}{\rho} + \left(u^2 + \frac{\alpha^{2/k}\,K[3]}{\rho}\right)(Xi[1]_x) - u\,(Xi[2]_x) + u\,(Xi[1]_t) - Xi[2]_t == 0,$$

$$\alpha^{2/k}\,K[3]\,(Xi[1]_u) - u\,\alpha\,\rho\,(Xi[1]_\alpha) + \alpha\,\rho\,(Xi[2]_\alpha) == 0,$$

$$-\frac{\alpha^{-2+\frac{4}{k}}\,K[3]^2\,(Xi[1]_u)}{\rho} + u\,\alpha^{-1+\frac{2}{k}}\,K[3]\,(Xi[1]_\alpha) - \alpha^{-1+\frac{2}{k}}\,K[3]\,(Xi[2]_\alpha) == 0,$$

$$\left(-1 + \frac{2}{k}\right)\alpha^{-2+\frac{2}{k}}\,K[3]\,Phi[1][t, x, \alpha, u] - \alpha^{-1+\frac{2}{k}}\,K[3]\,(Phi[2]_u) - \frac{2\,\alpha^{-1+\frac{4}{k}}\,K[3]\,\Gamma[u\,\alpha^{-1/k}]\,(Xi[1]_u)}{\rho} +$$
$$\alpha^{-1+\frac{2}{k}}\,K[3]\,(Phi[1]_\alpha) + 2\,u\,\alpha^{-1+\frac{2}{k}}\,K[3]\,(Xi[1]_x) - \alpha^{-1+\frac{2}{k}}\,K[3]\,(Xi[2]_x) + \alpha^{-1+\frac{2}{k}}\,K[3]\,(Xi[1]_t) == 0,$$

$$-\frac{u\,\alpha^{-1+\frac{1}{k}}\,Phi[1][t, x, \alpha, u]\,\Gamma'[u\,\alpha^{-1/k}]}{k} + \alpha^{\frac{1}{k}}\,Phi[2][t, x, \alpha, u]\,\Gamma'[u\,\alpha^{-1/k}] - \alpha^{2/k}\,\Gamma[u\,\alpha^{-1/k}]\,(Phi[2]_u) -$$
$$\frac{\alpha^{4/k}\,\Gamma[u\,\alpha^{-1/k}]^2\,(Xi[1]_u)}{\rho} + \alpha^{-1+\frac{2}{k}}\,K[3]\,(Phi[1]_x) + u\,\rho\,(Phi[2]_x) + u\,\alpha^{2/k}\,\Gamma[u\,\alpha^{-1/k}]\,(Xi[1]_x) +$$
$$\rho\,(Phi[2]_t) + \alpha^{2/k}\,\Gamma[u\,\alpha^{-1/k}]\,(Xi[1]_t) == 0,\ \rho\,Phi[2][t, x, \alpha, u] + \alpha^{-1+\frac{2}{k}}\,K[3]\,(Phi[1]_u) -$$
$$u\,\alpha^{2/k}\,\Gamma[u\,\alpha^{-1/k}]\,(Xi[1]_u) + \alpha^{2/k}\,\Gamma[u\,\alpha^{-1/k}]\,(Xi[2]_u) - \alpha\,\rho\,(Phi[2]_\alpha) - \alpha^{1+\frac{2}{k}}\,\Gamma[u\,\alpha^{-1/k}]\,(Xi[1]_\alpha) +$$
$$\left(u^2\,\rho + \alpha^{2/k}\,K[3]\right)(Xi[1]_x) - u\,\rho\,(Xi[2]_x) + u\,\rho\,(Xi[1]_t) - \rho\,(Xi[2]_t) == 0\Big\}$$

```
LXi = {0, c[2] Exp[x]};
LPhi = {c[3] Exp[x] α, c[2] Exp[x] u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\Big\{True,\ True,\ e^x\,u\,\alpha\,(c[2] + c[3]) == 0,\ True,\ True,\ True,\ True,\ \frac{e^x\,\alpha^{-1+\frac{2}{k}}\,(k\,c[2] - c[3])\,K[3]}{k} == 0,$$
$$\frac{e^x\left(k\,(u^2\,\rho\,c[2] + \alpha^{2/k}\,c[3]\,K[3]) - k\,\alpha^{2/k}\,c[2]\,\Gamma[u\,\alpha^{-1/k}] + u\,\alpha^{\frac{1}{k}}\,(k\,c[2] - c[3])\,\Gamma'[u\,\alpha^{-1/k}]\right)}{k} == 0,\ True\Big\}$$

```
LXi = {0, c[2] Exp[x]};
LPhi = {k c[2] Exp[x] α, c[2] Exp[x] u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\left\{\text{True}, \text{True}, e^x (1+k)\, u\,\alpha\, c[2] == 0, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}, e^x c[2]\left(u^2\rho + k\,\alpha^{2/k} K[3] - \alpha^{2/k}\,\Gamma\left[u\,\alpha^{-1/k}\right]\right) == 0, \text{True}\right\}$$

We must have k=-1 since c[2]≠0.

```
k = -1;
eq1 = D[α[t, x], t] + α[t, x] D[u[t, x], x] + u[t, x] D[α[t, x], x] == 0;
eq2 = ρ (D[u[t, x], t] + u[t, x] D[u[t, x], x]) + D[((1/2) k α[t, x]^(2/k) K[3] + K[4]), x] +
    α^(2/k) Γ[u[t, x] α[t, x]^(-1/k)] == 0;
VD = {α, u};
VIs = {t, x};
VP = {D[α[t, x], t], D[u[t, x], t]};
EDs = YaLie[{eq1, eq2}, VD, VIs, VP];
DFormat[True];
EDs
```

$$\Big\{-\frac{u\,K[3]\,(\mathrm{Xi}[1]_u)}{\alpha^3\rho} + \frac{K[3]\,(\mathrm{Xi}[2]_u)}{\alpha^3\rho} + \frac{K[3]\,(\mathrm{Xi}[1]_\alpha)}{\alpha^2\rho} == 0,\; u\,\alpha\,(\mathrm{Xi}[1]_u) - \alpha\,(\mathrm{Xi}[2]_u) - \alpha^2\,(\mathrm{Xi}[1]_\alpha) == 0,$$

$$-\frac{\Gamma[u\,\alpha]\,(\mathrm{Phi}[1]_u)}{\alpha^2\rho} + u\,(\mathrm{Phi}[1]_x) + \alpha\,(\mathrm{Phi}[2]_x) + \frac{\Gamma[u\,\alpha]\,(\mathrm{Xi}[1]_x)}{\alpha\rho} + \mathrm{Phi}[1]_t == 0,$$

$$\mathrm{Phi}[1][t, x, \alpha, u] + \alpha\,(\mathrm{Phi}[2]_u) - \alpha\,(\mathrm{Phi}[1]_\alpha) + 2\,u\,\alpha\,(\mathrm{Xi}[1]_x) - \alpha\,(\mathrm{Xi}[2]_x) + \alpha\,(\mathrm{Xi}[1]_t) == 0,$$

$$\mathrm{Phi}[2][t, x, \alpha, u] - \frac{K[3]\,(\mathrm{Phi}[1]_u)}{\alpha^3\rho} - \frac{u\,\Gamma[u\,\alpha]\,(\mathrm{Xi}[1]_u)}{\alpha^2\rho} + \frac{\Gamma[u\,\alpha]\,(\mathrm{Xi}[2]_u)}{\alpha^2\rho} +$$

$$\alpha\,(\mathrm{Phi}[2]_\alpha) + \frac{\Gamma[u\,\alpha]\,(\mathrm{Xi}[1]_\alpha)}{\alpha\rho} + \left(u^2 + \frac{K[3]}{\alpha^2\rho}\right)(\mathrm{Xi}[1]_x) - u\,(\mathrm{Xi}[2]_x) + u\,(\mathrm{Xi}[1]_t) - \mathrm{Xi}[2]_t == 0,$$

$$-\frac{K[3]^2\,(\mathrm{Xi}[1]_u)}{\alpha^6\rho} + \frac{u\,K[3]\,(\mathrm{Xi}[1]_\alpha)}{\alpha^3} - \frac{K[3]\,(\mathrm{Xi}[2]_\alpha)}{\alpha^3} == 0,\; \frac{K[3]\,(\mathrm{Xi}[1]_u)}{\alpha^2} - u\,\alpha\rho\,(\mathrm{Xi}[1]_\alpha) + \alpha\rho\,(\mathrm{Xi}[2]_\alpha) == 0$$

$$-\frac{3\,K[3]\,\mathrm{Phi}[1][t, x, \alpha, u]}{\alpha^4} - \frac{K[3]\,(\mathrm{Phi}[2]_u)}{\alpha^3} - \frac{2\,K[3]\,\Gamma[u\,\alpha]\,(\mathrm{Xi}[1]_u)}{\alpha^5\rho} +$$

$$\frac{K[3]\,(\mathrm{Phi}[1]_\alpha)}{\alpha^3} + \frac{2\,u\,K[3]\,(\mathrm{Xi}[1]_x)}{\alpha^3} - \frac{K[3]\,(\mathrm{Xi}[2]_x)}{\alpha^3} + \frac{K[3]\,(\mathrm{Xi}[1]_t)}{\alpha^3} == 0,$$

$$\frac{u\,\mathrm{Phi}[1][t, x, \alpha, u]\,\Gamma'[u\,\alpha]}{\alpha^2} + \frac{\mathrm{Phi}[2][t, x, \alpha, u]\,\Gamma'[u\,\alpha]}{\alpha} - \frac{\Gamma[u\,\alpha]\,(\mathrm{Phi}[2]_u)}{\alpha^2} - \frac{\Gamma[u\,\alpha]^2\,(\mathrm{Xi}[1]_u)}{\alpha^4\rho} +$$

$$\frac{K[3]\,(\mathrm{Phi}[1]_x)}{\alpha^3} + u\rho\,(\mathrm{Phi}[2]_x) + \frac{u\,\Gamma[u\,\alpha]\,(\mathrm{Xi}[1]_x)}{\alpha^2} + \rho\,(\mathrm{Phi}[2]_t) + \frac{\Gamma[u\,\alpha]\,(\mathrm{Xi}[1]_t)}{\alpha^2} == 0,$$

$$\rho\,\mathrm{Phi}[2][t, x, \alpha, u] + \frac{K[3]\,(\mathrm{Phi}[1]_u)}{\alpha^3} - \frac{u\,\Gamma[u\,\alpha]\,(\mathrm{Xi}[1]_u)}{\alpha^2} + \frac{\Gamma[u\,\alpha]\,(\mathrm{Xi}[2]_u)}{\alpha^2} - \alpha\rho\,(\mathrm{Phi}[2]_\alpha) -$$

$$\frac{\Gamma[u\,\alpha]\,(\mathrm{Xi}[1]_\alpha)}{\alpha} + \left(u^2\rho + \frac{K[3]}{\alpha^2}\right)(\mathrm{Xi}[1]_x) - u\rho\,(\mathrm{Xi}[2]_x) + u\rho\,(\mathrm{Xi}[1]_t) - \rho\,(\mathrm{Xi}[2]_t) == 0\Big\}$$

```
k = -1;
LXi = {0, c[2] Exp[x]};
LPhi = {k c[2] Exp[x] α, c[2] Exp[x] u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\left\{\text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \frac{e^x c[2]\,(u^2\alpha^2\rho - K[3] - \Gamma[u\,\alpha])}{\alpha} == 0, \text{True}\right\}$$

Case II: Classification with respect to 3D Lie algebras

• 3D Lie algebra: $A_1 \oplus A_2$ (f)

```
eq1 = D[α[t, x], t] + α[t, x] D[u[t, x], x] + u[t, x] D[α[t, x], x] == 0;
eq2 = ρ (D[u[t, x], t] + u[t, x] D[u[t, x], x]) + R[α[t, x], u[t, x] ] == 0;
VD = {α, u};
VIs = {t, x};
VP = {D[α[t, x], t], D[u[t, x], t]};
EDs = YaLie[{eq1, eq2}, VD, VIs, VP];
DFormat[True];
EDs
```

$$\Big\{u\,\alpha\,(\mathrm{Xi}[1]_u) - \alpha\,(\mathrm{Xi}[2]_u) - \alpha^2\,(\mathrm{Xi}[1]_\alpha) == 0,$$
$$-\frac{R[\alpha,u]\,(\mathrm{Phi}[1]_u)}{\rho} + u\,(\mathrm{Phi}[1]_x) + \alpha\,(\mathrm{Phi}[2]_x) + \frac{\alpha\,R[\alpha,u]\,(\mathrm{Xi}[1]_x)}{\rho} + \mathrm{Phi}[1]_t == 0,$$
$$\mathrm{Phi}[1][t,x,\alpha,u] + \alpha\,(\mathrm{Phi}[2]_u) - \alpha\,(\mathrm{Phi}[1]_\alpha) + 2\,u\,\alpha\,(\mathrm{Xi}[1]_x) - \alpha\,(\mathrm{Xi}[2]_x) + \alpha\,(\mathrm{Xi}[1]_t) == \text{C}$$
$$\mathrm{Phi}[2][t,x,\alpha,u] - \frac{u\,R[\alpha,u]\,(\mathrm{Xi}[1]_u)}{\rho} + \frac{R[\alpha,u]\,(\mathrm{Xi}[2]_u)}{\rho} + \alpha\,(\mathrm{Phi}[2]_\alpha) + \frac{\alpha\,R[\alpha,u]\,(\mathrm{Xi}[1}{\rho}$$
$$u^2\,(\mathrm{Xi}[1]_x) - u\,(\mathrm{Xi}[2]_x) + u\,(\mathrm{Xi}[1]_t) - \mathrm{Xi}[2]_t == 0,\ -u\,\alpha\,\rho\,(\mathrm{Xi}[1]_\alpha) + \alpha\,\rho\,(\mathrm{Xi}[2]_\alpha) == 0,$$
$$\mathrm{Phi}[2][t,x,\alpha,u]\,(R_u) + \mathrm{Phi}[1][t,x,\alpha,u]\,(R_\alpha) - R[\alpha,u]\,(\mathrm{Phi}[2]_u) - \frac{R[\alpha,u]^2\,(\mathrm{Xi}[1]_u)}{\rho} -$$
$$u\,\rho\,(\mathrm{Phi}[2]_x) + u\,R[\alpha,u]\,(\mathrm{Xi}[1]_x) + \rho\,(\mathrm{Phi}[2]_t) + R[\alpha,u]\,(\mathrm{Xi}[1]_t) == 0,$$
$$\rho\,\mathrm{Phi}[2][t,x,\alpha,u] - u\,R[\alpha,u]\,(\mathrm{Xi}[1]_u) + R[\alpha,u]\,(\mathrm{Xi}[2]_u) - \alpha\,\rho\,(\mathrm{Phi}[2]_\alpha) -$$
$$\alpha\,R[\alpha,u]\,(\mathrm{Xi}[1]_\alpha) + u^2\,\rho\,(\mathrm{Xi}[1]_x) - u\,\rho\,(\mathrm{Xi}[2]_x) + u\,\rho\,(\mathrm{Xi}[1]_t) - \rho\,(\mathrm{Xi}[2]_t) == 0\Big\}$$

```
LXi = {a[u], -x + b[u]};
LPhi = {c[α, u], d[u]};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\Big\{u\,\alpha\,(a_u) == \alpha\,(b_u),\ \frac{R[\alpha,u]\,(c_u)}{\rho} == 0,\ \alpha + c[\alpha,u] + \alpha\,(d_u) == \alpha\,(c_\alpha),\ \frac{u\,\rho + \rho\,d[u] + R[\alpha,u]\,(-u}{\rho}$$
$$\text{True},\ d[u]\,(R_u) + c[\alpha,u]\,(R_\alpha) == R[\alpha,u]\left(\frac{R[\alpha,u]\,(a_u)}{\rho} + d_u\right),\ u\,\rho + \rho\,d[u] + R[\alpha,u]\,(b_u) == u$$

$$\text{LXi} = \left\{a[u],\ -x + C[2] + a[u]\,u - \int a[u]\,du\right\};$$

```
LPhi = {c[α], d[u]};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\Big\{\text{True},\ \text{True},\ \alpha\,(c_\alpha) == \alpha + c[\alpha] + \alpha\,(d_u),\ u + d[u] == 0,\ \text{True},$$
$$d[u]\,(R_u) + c[\alpha]\,(R_\alpha) == R[\alpha,u]\left(\frac{R[\alpha,u]\,(a_u)}{\rho} + d_u\right),\ \rho\,(u + d[u]) == 0\Big\}$$

$$\text{LXi} = \left\{\text{a[u]},\ -\text{x} + \text{C[2]} + \text{a[u] u} - \int \text{a[u]}\, d\text{u}\right\};$$

```
LPhi = {c[α], -u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\left\{\text{True, True, c}[\alpha] == \alpha\,(c_\alpha),\ \text{True, True},\ \frac{\text{R}[\alpha,\ \text{u}]^2\,(a_u)}{\rho} + \text{u}\,(R_u) == \text{R}[\alpha,\ \text{u}] + \text{c}[\alpha]\,(R_\alpha),\ \text{True}\right\}$$

$$\text{LXi} = \left\{\text{a[u]},\ -\text{x} + \text{C[2]} + \text{a[u] u} - \int \text{a[u]}\, d\text{u}\right\};$$

```
LPhi = {C[3] α, -u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\left\{\text{True, True, True, True, True},\ \frac{\text{R}[\alpha,\ \text{u}]^2\,(a_u)}{\rho} + \text{u}\,(R_u) == \text{R}[\alpha,\ \text{u}] + \alpha\,\text{C[3]}\,(R_\alpha),\ \text{True}\right\}$$

```
a'[u] ≠ 0; C[3] = c[7];
```

```
LXi = {u, -x + u^2/2};
LPhi = {c[7] α, -u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\left\{\text{True, True, True, True, True},\ \frac{\text{R}[\alpha,\ \text{u}]^2}{\rho} + \text{u}\,(R_u) == \text{R}[\alpha,\ \text{u}] + \alpha\,\text{c[7]}\,(R_\alpha),\ \text{True}\right\}$$

$$\text{DSolve}\left[\frac{\text{R}[\alpha,\ \text{u}]^2}{\rho} + \text{u D[R}[\alpha,\ \text{u}],\ \text{u}\,] == \text{R}[\alpha,\ \text{u}] + \alpha\,\text{c[7] D[R}[\alpha,\ \text{u}],\ \alpha\,],\ \text{R},\ \{\alpha,\ \text{u}\}\right]$$

$$\left\{\left\{\text{R} \to \text{Function}\left[\{\alpha,\ u\},\ -\frac{\rho}{-1 + e^{\rho\,\text{G}\left[u\,\alpha^{\frac{1}{\text{c[7]}}}\right]}\,\alpha^{\frac{1}{\text{c[7]}}}}\right]\right\}\right\}$$

```
c[7] = 0;
```

```
LXi = {u, -x + u^2/2};
LPhi = {0, -u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\left\{\text{True, True, True, True, True},\ \frac{\text{R}[\alpha,\ \text{u}]^2}{\rho} + \text{u}\,(R_u) == \text{R}[\alpha,\ \text{u}],\ \text{True}\right\}$$

$$\text{DSolve}\left[\frac{\text{R}[\alpha,\ \text{u}]^2}{\rho} + \text{u D[R}[\alpha,\ \text{u}],\ \text{u}\,] == \text{R}[\alpha,\ \text{u}],\ \text{R},\ \{\alpha,\ \text{u}\}\right]$$

```
{{R → Function[{α, u}, - (u ρ)/(e^(ρ G[α]) - u)]}}

a'[u] = 0;

LXi = {C[1], -x + C[2]};
LPhi = {C[3] α, -u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]

{True, True, True, True, True, R[α, u] + α C[3] (R_α) == u (R_u), True}

C[3] = c[8];

DSolve[u D[R[α, u], u ] - c[8] α D[R[α, u], α ] - R[α, u] == 0, R, {α, u}]

{{R → Function[{α, u}, α^(-1/c[8]) G[u α^(1/c[8])]]}}

c[8] = 0;

LXi = {0, -x};
LPhi = {0, -u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]

{True, True, True, True, True, R[α, u] == u (R_u), True}

LXi = {0, x};
LPhi = {0, u};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]

{True, True, True, True, True, R[α, u] == u (R_u), True}

DSolve[u D[R[α, u], u ] - R[α, u] == 0, R, {α, u}]

{{R → Function[{α, u}, u G[α]]}}
```

Case II: Classification with respect to 4D Lie algebras

- 4D Lie algebra containing 3D Lie algebra $A_1 \oplus A_2$ (f)

```
eq1 = D[α[t, x], t] + α[t, x] D[u[t, x], x] + u[t, x] D[α[t, x], x] == 0;
eq2 = ρ (D[u[t, x], t] + u[t, x] D[u[t, x], x]) + α[t, x]^(-1/c[8]) G[u[t, x] α[t, x]^(1/c[8])] == 0;
VD = {α, u};
VIs = {t, x};
VP = {D[α[t, x], t], D[u[t, x], t]};
EDs = YaLie[{eq1, eq2}, VD, VIs, VP];
DFormat[True];
EDs
```

$$\Big\{u\,\alpha\,(Xi[1]_u) - \alpha\,(Xi[2]_u) - \alpha^2\,(Xi[1]_\alpha) == 0,$$

$$-\frac{\alpha^{-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,(Phi[1]_u)}{\rho} + u\,(Phi[1]_x) + \alpha\,(Phi[2]_x) + \frac{\alpha^{1-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,(Xi[1]_x)}{\rho} + Phi[1]_t == 0,$$

$$Phi[1][t, x, \alpha, u] + \alpha\,(Phi[2]_u) - \alpha\,(Phi[1]_\alpha) + 2\,u\,\alpha\,(Xi[1]_x) - \alpha\,(Xi[2]_x) + \alpha\,(Xi[1]_t) == 0,$$

$$Phi[2][t, x, \alpha, u] - \frac{u\,\alpha^{-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,(Xi[1]_u)}{\rho} + \frac{\alpha^{-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,(Xi[2]_u)}{\rho} +$$

$$\alpha\,(Phi[2]_\alpha) + \frac{\alpha^{1-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,(Xi[1]_\alpha)}{\rho} + u^2\,(Xi[1]_x) - u\,(Xi[2]_x) + u\,(Xi[1]_t) - Xi[2]_t == 0,$$

$$-u\,\alpha\,\rho\,(Xi[1]_\alpha) + \alpha\,\rho\,(Xi[2]_\alpha) == 0,\ Phi[2][t, x, \alpha, u]\,G'\left[u\,\alpha^{\frac{1}{c[8]}}\right] + Phi[1][t, x, \alpha, u]$$

$$\left(-\frac{\alpha^{-1-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]}{c[8]} + \frac{u\,G'\left[u\,\alpha^{\frac{1}{c[8]}}\right]}{\alpha\,c[8]}\right) - \alpha^{-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,(Phi[2]_u) - \frac{\alpha^{-\frac{2}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]^2\,(Xi[1]_u)}{\rho} +$$

$$u\,\rho\,(Phi[2]_x) + u\,\alpha^{-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,(Xi[1]_x) + \rho\,(Phi[2]_t) + \alpha^{-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,(Xi[1]_t) == 0,$$

$$\rho\,Phi[2][t, x, \alpha, u] - u\,\alpha^{-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,(Xi[1]_u) + \alpha^{-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,(Xi[2]_u) - \alpha\,\rho\,(Phi[2]_\alpha) -$$

$$\alpha^{1-\frac{1}{c[8]}}\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,(Xi[1]_\alpha) + u^2\,\rho\,(Xi[1]_x) - u\,\rho\,(Xi[2]_x) + u\,\rho\,(Xi[1]_t) - \rho\,(Xi[2]_t) == 0\Big\}$$

```
LXi = {k[1] Exp[t], k[2] u Exp[t]};
LPhi = {k[3][α^(1/c[8]) u] Exp[t] α, Exp[t] (k[4] u)};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\Big\{e^t\,\alpha\,k[2] == 0,\ \frac{e^t\,\alpha\left(\rho\,k[3]\left[u\,\alpha^{\frac{1}{c[8]}}\right] - G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,k[3]'\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)}{\rho} == 0,$$

$$\frac{e^t\,\alpha\left(c[8]\,(k[1] + k[4]) - u\,\alpha^{\frac{1}{c[8]}}\,k[3]'\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)}{c[8]} == 0,$$

$$\frac{e^t\,\alpha^{-\frac{1}{c[8]}}\left(G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,k[2] + u\,\alpha^{\frac{1}{c[8]}}\,\rho\,(k[1] - k[2] + k[4])\right)}{\rho} == 0,\ True,$$

$$\frac{1}{c[8]}\,e^t\,\alpha^{-\frac{1}{c[8]}}\left(G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\left(c[8]\,(k[1] - k[4]) - k[3]\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right) +\right.$$

$$\left. u\,\alpha^{\frac{1}{c[8]}}\left(\rho\,c[8]\,k[4] + \left(c[8]\,k[4] + k[3]\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)G'\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)\right) == 0,$$

$$e^t\,\alpha^{-\frac{1}{c[8]}}\left(G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\,k[2] + u\,\alpha^{\frac{1}{c[8]}}\,\rho\,(k[1] - k[2] + k[4])\right) == 0\Big\}$$

```
LXi = {k[1] Exp[t], 0};
LPhi = {k[3][α^(1/c[8]) u] Exp[t] α, Exp[t] (k[4] u)};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\Big\{\text{True},\ \frac{e^{t}\,\alpha\left(\rho\,k[3]\left[u\,\alpha^{\frac{1}{c[8]}}\right]-G\left[u\,\alpha^{\frac{1}{c[8]}}\right]k[3]'\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)}{\rho}==0,$$
$$\frac{e^{t}\,\alpha\left(c[8]\,(k[1]+k[4])-u\,\alpha^{\frac{1}{c[8]}}\,k[3]'\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)}{c[8]}==0,\ e^{t}\,u\,(k[1]+k[4])==0,$$
$$\text{True},\ \frac{1}{c[8]}\,e^{t}\,\alpha^{-\frac{1}{c[8]}}\left(G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\left(c[8]\,(k[1]-k[4])-k[3]\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)+\right.$$
$$\left.u\,\alpha^{\frac{1}{c[8]}}\left(\rho\,c[8]\,k[4]+\left(c[8]\,k[4]+k[3]\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)G'\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)\right)==0,\ e^{t}\,u\,\rho\,(k[1]+k[4])==0\Big\}$$

```
LXi = {k[1] Exp[t], 0};
LPhi = {k[3][α^(1/c[8]) u] Exp[t] α, Exp[t] (-k[1] u)};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\Big\{\text{True},\ \frac{e^{t}\,\alpha\left(\rho\,k[3]\left[u\,\alpha^{\frac{1}{c[8]}}\right]-G\left[u\,\alpha^{\frac{1}{c[8]}}\right]k[3]'\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)}{\rho}==0,\ \frac{e^{t}\,u\,\alpha^{1+\frac{1}{c[8]}}\,k[3]'\left[u\,\alpha^{\frac{1}{c[8]}}\right]}{c[8]}==0,$$
$$\text{True},\ \text{True},\ \frac{1}{c[8]}\,e^{t}\,\alpha^{-\frac{1}{c[8]}}\left(G\left[u\,\alpha^{\frac{1}{c[8]}}\right]\left(-2\,c[8]\,k[1]+k[3]\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)+\right.$$
$$\left.u\,\alpha^{\frac{1}{c[8]}}\left(\rho\,c[8]\,k[1]+\left(c[8]\,k[1]-k[3]\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)G'\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)\right)==0,\ \text{True}\Big\}$$

```
LXi = {k[1] Exp[t], 0};
LPhi = {0, Exp[t] (-k[1] u)};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\Big\{\text{True, True, True, True, True},\ e^{t}\,\alpha^{-\frac{1}{c[8]}}\,k[1]\left(-2\,G\left[u\,\alpha^{\frac{1}{c[8]}}\right]+u\,\alpha^{\frac{1}{c[8]}}\left(\rho+G'\left[u\,\alpha^{\frac{1}{c[8]}}\right]\right)\right)==0,\ \text{True}\Big\}$$

```
DSolve[-2 G[z] + z (ρ + G'[z]) == 0, G, z]
```

$$\{\{G\to\text{Function}[\{z\},\ z\,\rho+z^{2}\,K]\}\}$$

```
eq1 = D[α[t, x], t] + α[t, x] D[u[t, x], x] + u[t, x] D[α[t, x], x] == 0;
eq2 =
  ρ (D[u[t, x], t] + u[t, x] D[u[t, x], x]) + α[t, x]^(-1/c[8]) (u[t, x] α[t, x]^(1/c[8]) ρ + (u[t, x] α[t, x]^(1/c[8]))^2 K) ==
   0;
VD = {α, u};
VIs = {t, x};
VP = {D[α[t, x], t], D[u[t, x], t]};
EDs = YaLie[{eq1, eq2}, VD, VIs, VP];
DFormat[True];
EDs
```

$$\{u\alpha(Xi[1]_u) - \alpha(Xi[2]_u) - \alpha^2(Xi[1]_\alpha) == 0,$$

$$\left(-u - \frac{K u^2 \alpha^{\frac{1}{c[8]}}}{\rho}\right)(Phi[1]_u) + u(Phi[1]_x) + \alpha(Phi[2]_x) + \left(u\alpha + \frac{K u^2 \alpha^{1+\frac{1}{c[8]}}}{\rho}\right)(Xi[1]_x) + Phi[1]_t == 0,$$

$$Phi[1][t, x, \alpha, u] + \alpha(Phi[2]_u) - \alpha(Phi[1]_\alpha) + 2u\alpha(Xi[1]_x) - \alpha(Xi[2]_x) + \alpha(Xi[1]_t) == 0,$$

$$Phi[2][t, x, \alpha, u] + \left(-u^2 - \frac{K u^3 \alpha^{\frac{1}{c[8]}}}{\rho}\right)(Xi[1]_u) + \left(u + \frac{K u^2 \alpha^{\frac{1}{c[8]}}}{\rho}\right)(Xi[2]_u) + \alpha(Phi[2]_\alpha) +$$

$$\left(u\alpha + \frac{K u^2 \alpha^{1+\frac{1}{c[8]}}}{\rho}\right)(Xi[1]_\alpha) + u^2(Xi[1]_x) - u(Xi[2]_x) + u(Xi[1]_t) - Xi[2]_t == 0,$$

$$-u\alpha\rho(Xi[1]_\alpha) + \alpha\rho(Xi[2]_\alpha) == 0, \left(\alpha^{-\frac{1}{c[8]}}\left(\frac{2Ku^2\alpha^{-1+\frac{2}{c[8]}}}{c[8]} + \frac{u\alpha^{-1+\frac{1}{c[8]}}\rho}{c[8]}\right) - \frac{\alpha^{-1-\frac{1}{c[8]}}\left(Ku^2\alpha^{\frac{2}{c[8]}} + u\alpha^{\frac{1}{c[8]}}\rho\right)}{c[8]}\right)$$

$$Phi[1][t, x, \alpha, u] + \alpha^{-\frac{1}{c[8]}}\left(2Ku\alpha^{\frac{2}{c[8]}} + \alpha^{\frac{1}{c[8]}}\rho\right) Phi[2][t, x, \alpha, u] +$$

$$\left(-Ku^2\alpha^{\frac{1}{c[8]}} - u\rho\right)(Phi[2]_u) + \left(-2Ku^3\alpha^{\frac{1}{c[8]}} - \frac{K^2u^4\alpha^{\frac{2}{c[8]}}}{\rho} - u^2\rho\right)(Xi[1]_u) + u\rho(Phi[2]_x) +$$

$$\left(Ku^3\alpha^{\frac{1}{c[8]}} + u^2\rho\right)(Xi[1]_x) + \rho(Phi[2]_t) + \left(Ku^2\alpha^{\frac{1}{c[8]}} + u\rho\right)(Xi[1]_t) == 0,$$

$$\rho\, Phi[2][t, x, \alpha, u] + \left(-Ku^3\alpha^{\frac{1}{c[8]}} - u^2\rho\right)(Xi[1]_u) + \left(Ku^2\alpha^{\frac{1}{c[8]}} + u\rho\right)(Xi[2]_u) - \alpha\rho(Phi[2]_\alpha) +$$

$$\left(-Ku^2\alpha^{1+\frac{1}{c[8]}} - u\alpha\rho\right)(Xi[1]_\alpha) + u^2\rho(Xi[1]_x) - u\rho(Xi[2]_x) + u\rho(Xi[1]_t) - \rho(Xi[2]_t) == 0\}$$

```
LXi = {Exp[t], 0};
LPhi = {0, -u Exp[t]};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

{True, True, True, True, True, True, True}

$$\text{FullSimplify}\left[\alpha[t,x]^{-\frac{1}{c[8]}}\left(u[t,x]\,\alpha[t,x]^{\frac{1}{c[8]}}\rho + \left(u[t,x]\,\alpha[t,x]^{\frac{1}{c[8]}}\right)^2 K\right)\right]$$

$$u\left(\rho + K u \alpha^{\frac{1}{c[8]}}\right)$$

```
c[8] = 0;
```

```
eq1 = D[α[t, x], t] + α[t, x] D[u[t, x], x] + u[t, x] D[α[t, x], x] == 0;
eq2 = ρ (D[u[t, x], t] + u[t, x] D[u[t, x], x]) + u[t, x] G[α[t, x]] == 0;
VD = {α, u};
VIs = {t, x};
VP = {D[α[t, x], t], D[u[t, x], t]};
EDs = YaLie[{eq1, eq2}, VD, VIs, VP];
DFormat[True];
EDs
```

$$\{u\,\alpha\,(\text{Xi}[1]_u) - \alpha\,(\text{Xi}[2]_u) - \alpha^2\,(\text{Xi}[1]_\alpha) == 0,$$
$$-\frac{u\,G[\alpha]\,(\text{Phi}[1]_u)}{\rho} + u\,(\text{Phi}[1]_x) + \alpha\,(\text{Phi}[2]_x) + \frac{u\,\alpha\,G[\alpha]\,(\text{Xi}[1]_x)}{\rho} + \text{Phi}[1]_t == 0,$$
$$\text{Phi}[1][t, x, \alpha, u] + \alpha\,(\text{Phi}[2]_u) - \alpha\,(\text{Phi}[1]_\alpha) + 2\,u\,\alpha\,(\text{Xi}[1]_x) - \alpha\,(\text{Xi}[2]_x) + \alpha\,(\text{Xi}[1]_t) == 0,$$
$$\text{Phi}[2][t, x, \alpha, u] - \frac{u^2\,G[\alpha]\,(\text{Xi}[1]_u)}{\rho} + \frac{u\,G[\alpha]\,(\text{Xi}[2]_u)}{\rho} + \alpha\,(\text{Phi}[2]_\alpha) + \frac{u\,\alpha\,G[\alpha]\,(\text{Xi}[1]_\alpha)}{\rho} +$$
$$u^2\,(\text{Xi}[1]_x) - u\,(\text{Xi}[2]_x) + u\,(\text{Xi}[1]_t) - \text{Xi}[2]_t == 0,\ -u\,\alpha\,\rho\,(\text{Xi}[1]_\alpha) + \alpha\,\rho\,(\text{Xi}[2]_\alpha) == 0,$$
$$G[\alpha]\,\text{Phi}[2][t, x, \alpha, u] + u\,\text{Phi}[1][t, x, \alpha, u]\,(G_\alpha) - u\,G[\alpha]\,(\text{Phi}[2]_u) - \frac{u^2\,G[\alpha]^2\,(\text{Xi}[1]_u)}{\rho} +$$
$$u\,\rho\,(\text{Phi}[2]_x) + u^2\,G[\alpha]\,(\text{Xi}[1]_x) + \rho\,(\text{Phi}[2]_t) + u\,G[\alpha]\,(\text{Xi}[1]_t) == 0,$$
$$\rho\,\text{Phi}[2][t, x, \alpha, u] - u^2\,G[\alpha]\,(\text{Xi}[1]_u) + u\,G[\alpha]\,(\text{Xi}[2]_u) - \alpha\,\rho\,(\text{Phi}[2]_\alpha) -$$
$$u\,\alpha\,G[\alpha]\,(\text{Xi}[1]_\alpha) + u^2\,\rho\,(\text{Xi}[1]_x) - u\,\rho\,(\text{Xi}[2]_x) + u\,\rho\,(\text{Xi}[1]_t) - \rho\,(\text{Xi}[2]_t) == 0\}$$

```
LXi = {k[1] Exp[t], 0};
LPhi = {0, Exp[t] (-k[1] u)};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\{\text{True}, \text{True}, \text{True}, \text{True}, \text{True}, e^t\,u\,(\rho - G[\alpha])\,k[1] == 0, \text{True}\}$$

```
eq1 = D[α[t, x], t] + α[t, x] D[u[t, x], x] + u[t, x] D[α[t, x], x] == 0;
eq2 = ρ (D[u[t, x], t] + u[t, x] D[u[t, x], x]) + ρ u[t, x] == 0;
VD = {α, u};
VIs = {t, x};
VP = {D[α[t, x], t], D[u[t, x], t]};
EDs = YaLie[{eq1, eq2}, VD, VIs, VP];
DFormat[True];
EDs
```

$$\{u\,\alpha\,(\text{Xi}[1]_u) - \alpha\,(\text{Xi}[2]_u) - \alpha^2\,(\text{Xi}[1]_\alpha) == 0,$$
$$-u\,(\text{Phi}[1]_u) + u\,(\text{Phi}[1]_x) + \alpha\,(\text{Phi}[2]_x) + u\,\alpha\,(\text{Xi}[1]_x) + \text{Phi}[1]_t == 0,$$
$$\text{Phi}[1][t, x, \alpha, u] + \alpha\,(\text{Phi}[2]_u) - \alpha\,(\text{Phi}[1]_\alpha) + 2\,u\,\alpha\,(\text{Xi}[1]_x) - \alpha\,(\text{Xi}[2]_x) + \alpha\,(\text{Xi}[1]_t) == 0,$$
$$\text{Phi}[2][t, x, \alpha, u] - u^2\,(\text{Xi}[1]_u) + u\,(\text{Xi}[2]_u) + \alpha\,(\text{Phi}[2]_\alpha) + u\,\alpha\,(\text{Xi}[1]_\alpha) +$$
$$u^2\,(\text{Xi}[1]_x) - u\,(\text{Xi}[2]_x) + u\,(\text{Xi}[1]_t) - \text{Xi}[2]_t == 0,\ -u\,\alpha\,\rho\,(\text{Xi}[1]_\alpha) + \alpha\,\rho\,(\text{Xi}[2]_\alpha) == 0,$$
$$\rho\,\text{Phi}[2][t, x, \alpha, u] - u\,\rho\,(\text{Phi}[2]_u) - u^2\,\rho\,(\text{Xi}[1]_u) + u\,\rho\,(\text{Phi}[2]_x) + u^2\,\rho\,(\text{Xi}[1]_x) + \rho\,(\text{Phi}[2]_t) + u\,\rho\,(\text{Xi}[1]_t)$$
$$0,\ \rho\,\text{Phi}[2][t, x, \alpha, u] - u^2\,\rho\,(\text{Xi}[1]_u) + u\,\rho\,(\text{Xi}[2]_u) - \alpha\,\rho\,(\text{Phi}[2]_\alpha) -$$
$$u\,\alpha\,\rho\,(\text{Xi}[1]_\alpha) + u^2\,\rho\,(\text{Xi}[1]_x) - u\,\rho\,(\text{Xi}[2]_x) + u\,\rho\,(\text{Xi}[1]_t) - \rho\,(\text{Xi}[2]_t) == 0\}$$

```
LXi = {Exp[t], 0};
LPhi = {0, -u Exp[t]};
EDs = Simplify[SInfinitesimales[EDs, LXi, LPhi]]
```

$$\{\text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}\}$$