

PLANE STRAIN FRACTURE TESTING
OF CEMENTED MATERIALS

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DECLARATION BY CANDIDATE

I, Mark Gavin Alexander, hereby declare that the work presented in this dissertation is my own and that it has not been submitted for a degree of another university.

M. G. Alexander

December, 1974

ABSTRACT

This dissertation examines the problem of plane strain fracture testing of cemented material of construction. Two types of cemented materials were selected for study, namely a Portland cement mortar, and a gap-graded asphalt mix. While much work has been done in the metallic field on the development of suitable test criteria for valid fracture testing of a material, no uniform approach to the fracture testing of non-metallic materials has been developed, with the result that differing and often divergent techniques of fracture testing have been used. In addition, the direct and logical application of Fracture Mechanics fundamentals to cemented materials is severely lacking. The work of this dissertation aims at studying the basic fracture mechanisms and properties of cemented materials of construction, and applying the results to the development of initial, tentative test criteria for the valid fracture testing of such materials. An understanding of the fracture mechanisms of a material and the factors influencing such mechanisms is the first logical step towards applying fracture testing methods in the development of new and improved materials for construction purposes.

A broad background study of the Fracture Mechanics approach to the strength of materials is first presented, followed by a brief literature review of applications of this approach to common cemented materials. A theoretical and diagrammatic treatment of crack propagation in various model materials then follows, and the various concepts are used to explain the fracture and inherent strength mechanisms of cemented materials. With the background of concepts already put forward in treatises on the fracture testing of high strength metallic materials, a laboratory test program was initiated in order to distinguish and isolate the different variables affecting the fracture behaviour of the cement mortar specimens selected for study. The tests were conducted on edge-notched bend specimens, and the results were then analysed to produce the relevant fracture parameters (K_{Ic} , G_c , $\bar{\gamma}$). A major portion of the work was the development of

suitable testing apparatus and adequate instrumentation for the tests. Six series of tests were conducted in order to study the various effects on the measured fracture parameters of specimen dimensions, testing machine set-up and loading system, and rate of load application during a test. It was apparent that, in addition to the strain rate used for a test, and the degree of testing machine stiffness, the state of the stress/strain field ahead of the crack tip in a fracture specimen profoundly influenced the measured fracture toughness from the test. The test results allowed the more truly plane strain state of testing to be distinguished from the predominantly plane stress state. The use of different notches and loading techniques also allowed an investigation of the post cracking behaviour of the beam specimens.

The test results stated that the fracture parameters evaluated from the tests were valid estimates of the fracture toughness of cemented materials in the presence of natural or artificial notches, provided the state of the stress field in the material was predominantly one of plane strain. Plane stress specimens yielded fracture toughnesses that were considerably higher than the lower limit plane strain values. Lower strain rates used during the tests generally gave reduced fracture toughness values in comparison to 'fast' tests. This fact has application in the fracture testing of materials to be used in situations where the deformations and loadings are time-dependent, such as road pavements. The post-cracking behaviour of fracture specimens of cemented materials is profoundly affected by the stiffness of the loading system, and unrepresentative load-deflection curves are obtained where 'soft' test set-ups are used which can feed energy back into a specimen at the critical point of unstable crack propagation.

Initial, tentative criteria are suggested for the valid fracture testing of specimens similar to the cement mortar specimens studied in the tests. These criteria relate to minimum dimensional ratios for the specimens, as well as a dimensional ratio involving a parameter thought to describe the pseudo-plastic zone size in a specimen. The development of suitable fracture test criteria for cemented materials would probably involve separate criteria being developed for each particular class of cemented material, due to the wide range of behaviour of such materials. Many problems remain unresolved, and the field of fracture testing of cemented materials and suitable applications to practical problems is still largely unexplored.

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NOMENCLATURE

$A_{F,\delta}$	Total Work Required to Produce Fracture, i.e. Area under the Load-Deflection Curve
A_f	Area of New Fracture Surfaces Produced
b	Beam Width
c	Crack Half-Length (Internal Crack) Crack Depth (Surface Crack)
c_o	Initial Crack Length
c_c	Critical Crack Length at Fracture Instability
d	Beam Depth
E	Young's Modulus
F	Total Load on the Beam
G	Strain Energy Release Rate, or Crack Extension Force
G_c	Critical Strain Energy Release Rate
h	Uncracked Depth of Beam (= $d-c$)
K	Stress Intensity Factor
K_c	Critical Stress Intensity Factor, or Fracture Toughness
L	Beam Compliance, defined by $F = (1/L) \delta$
L_i	Initial Beam Compliance
L_o	Unnotched Beam Compliance
M	Bending Moment in the Beam
R	Plastic Zone Size Factor
S	Beam Span
S_1	Major, Support Span
S_2	Minor, Loading Span
U	Energy (Strain Energy)

γ	Surface Energy (True Free Surface Energy)
$\bar{\gamma}$	Effective Fracture Surface Energy
δ	Beam Deflection Under Load Points
ϵ	Strain
ϵ_L	Longitudinal Beam Strain
ϵ_T	Transverse Beam Strain
ν	Poisson's Ratio
σ	Stress or Strength
σ_L	Limiting (Theoretical) Strength
σ_n	Nominal Stress at Notch Root
σ_t	Tensile Strength (Modulus of Rupture)
σ_{ys}	Yield Strength

Note on the Accuracy of Numerical Figures Produced from the Laboratory Tests.

Numerical quantities such as compliances, elastic moduli and notch depth ratios, are quoted to a maximum of three decimal places (or alternatively a maximum of five significant figures) when reported in the tables in the main text. This was done in order to minimise the accumulation of errors in the analysis, but the various quantities should not be regarded as being significant beyond the first decimal place (or alternatively the third significant figure). For this reason, values quoted in the main text from the tables contain only those figures estimated to be significant.

CHAPTER 1

INTRODUCTION: THE FRACTURE MECHANICS APPROACH IN MATERIALS ENGINEERING

1.1 Fracture Mechanics and Materials Engineering

Materials Science concerns itself with attempting to understand why various materials behave as they do, and then applying this knowledge both in the better and more efficient use of existing materials, and in their modification and improvement in order to make better materials. It therefore brings together the ideas of chemistry and physics, which often centre on the molecular scale, with those of engineering, dealing with structures on a macroscopic scale. It is fairly simple to describe how a material behaves, by subjecting it to various tests, but the questions as to the reasons why different materials have different properties need to be answered to help the engineer in the proper use of his materials. In the last few decades, much new light has been shed on these questions by new approaches such as the Linear Elastic Fracture Mechanics approach, with its related concepts of stress raisers and crack propagation. Knowledge about the molecular structure of materials, including the chemical and physical bonds between atoms and molecules, is not sufficient on its own to explain the behaviour of engineering materials.

When a force is applied to any material, the individual atoms and molecules react to oppose the force by means of their own interatomic bond forces. Theoretically, there is a limit to the interatomic forces preventing separation of the atoms in a material under tension, and when this limiting interatomic force is exceeded by the applied tension, the material will 'break' or cleave or fail. A simplified consideration of the forces binding atoms together in a solid gives us an estimate of the theoretical tensile strength of a material, viz

$$\sigma_t = \frac{E}{10} \quad (1.1)$$

This means that theoretically ideal materials should fail under a tensile stress approximately equal to one-tenth of the Young's Modulus of the material, or at a tensile strain of 10 per cent.

This analysis approaches the subject using the strength of chemical bonds, and we might expect the engineering strengths of materials to be proportional to the chemical bond strengths existing between the atoms. However, this is demonstrably not so, actual strengths often being orders of magnitude lower than theoretical strengths. Table 1.1 below shows how the actual tensile strengths of some common engineering and other materials fall far below their theoretical strengths.

Table 1.1 Some Typical Values of Predicted and Actual Strengths of Materials

	Young's Modulus E (MPa)	Predicted Strength (MPa)	Actual Strength (MPa)
Iron	200 000	20 000	205
Iron Whiskers	200 000	20 000	13 110
Mild Steel	210 000	21 000	410
High Tensile Steel	210 000	21 000	1 550
Asformed Steel Wire	210 000	21 000	3 100
Annealed Copper	120 000	12 000	140
Drawn Copper Wire	120 000	12 000	550
Aluminium	70 000	7 000	70
Cement & Concrete	15 000	1 500	4
Hardwood, along grain	14 000	1 400	100
Hardwood, across grain	14 000	1 400	3
Bulk Glass	70 000	7 000	35 - 170
Parallel Fibreglass	35 000	3 500	1 050
Polyester	1 400	140	3
Bone	30 000	3 000	140

Furthermore, there is no definite relationship between chemical and mechanical strengths of a material. Linked to all this is the concept of ductile and brittle failure of materials, allowing materials to be grouped into two broad classes:- those materials which exhibit considerable deformation and plastic flow before failure,

i.e. ductile materials; and those materials which fail with little or no plastic deformation, i.e. brittle materials. Whether a material fails in a ductile or brittle fashion depends upon temperature, strain-rate, and basic fracture properties such as notch-sensitivity. In general, the two fracture mechanisms of plastic ductile flow and brittle cracking are always competing to break a material, and the material will succumb to whichever mechanism is the weaker; if it yields before it cracks, the material is ductile; if it cracks before it yields it is brittle. This dissertation essentially deals with the brittle fracture of materials and the fracture tests related to the study of the brittle behaviour of such materials.

1.1.1 Griffith's Contributions

Early in this century, A.A. Griffith¹, in asking questions such as why solids were not as strong as they ought to be used the concept of energy conservation to relate the strain-energy stored in a loaded specimen to the surface energy of the fracture surfaces which are produced when the solid fractures. He proposed that the strain-energy is converted into surface energy when a brittle material fails. He showed that the stress required to just separate two layers of atoms x cm apart in the material was

$$\sigma_t = \sqrt{\frac{E\gamma}{x}} \quad (1.2)$$

Using the equation, the strength of ordinary materials is predicted to be about ten to one hundred times greater than their actual bulk strength. During his classic work on the tensile strength of thin glass fibres, Griffith found that the thinner the fibre he tested, the greater was its strength. In fact, the extrapolated strength value for fibres of near zero diameter very closely approximated the theoretical strength predicted by his tensile strength equation. After thus demonstrating that the theoretical strength could be approximated experimentally in at least one case, he went on to postulate why the bulk strength of glass fell far below its ideal strength.

Griffith realised that the strengths of common materials were dependent, not primarily on their interatomic bond strengths, but rather on weakening mechanisms inherent in the material, such as

microscopic cracks and flaws of minute dimensions. While interior cracks could exist, it was Orowan who showed conclusively that the more important cracks causing the weakening of the material lay on the surface of the material. (See ref. 2, pp. 115 - 117)

Any such flaw or sharp re-entrant causes severe stress concentrations around its tip, and the local bonds at the crack tip become over-stressed long before the whole material is severely stressed. These over-stressed bonds break one by one as the crack grows. In a brittle material then, a crack is really a mechanism which enables a weak external force to break even the strongest chemical bonds one by one.

The crack runs through the material until total failure occurs, at stress levels far below the theoretical strength. Such weakening mechanisms as cracks are easy to visualise in a brittle material, which will usually fail catastrophically. However, not all materials fail in this manner, by the spread of a crack from some local defect. Another mechanism of fracture already mentioned is that of plastic flow, where the material fails by flowing in shear. At normal temperatures and for materials that are brittle at that temperature, the stress needed to fracture a material by flow may be very high, and thus materials like glass, ceramics and cement pastes are more susceptible to brittle fracture at normal temperatures.

1.2 The Mathematical and Physical Basis of the Fracture Mechanics Approach

It is possible to enumerate the physical principles governing the occurrence and spreading of flaws and stress raisers in a material, and express these in mathematical terms. Physically, two conditions must be fulfilled for a crack to propagate. Firstly, it must be energetically desirable, and secondly there must be a molecular mechanism by which the energy transformation can occur.

1.2.1 Energetic Desirability for a Crack to Propagate

This criterion is mainly related to crack length. A crack will propagate only when the free energy of the system is being continually lowered by the propagation, i.e. the energy released by the propagation is sufficient to make fracture possible. This criterion leads to a relationship between crack length and critical stress required for

Propagation, and includes Griffith's criterion as a special case (see below). In addition, this criterion is necessary, but not always sufficient to ensure fracture. It becomes sufficient, however, when the task of parting the atoms at the crack tip is accomplished by some form of molecular mechanism. The two types of energy that need to be considered, strain energy and surface energy of the crack, are both dependent on crack size: the former decreasing, and the latter increasing, with increasing crack length. The crack propagates only when the release of strain energy more than compensates the increase in surface energy.

Consider the Griffith case represented in Fig. 1.1(a), that of a surface crack in a solid under a macroscopically applied tensile stress σ . A fairly simple analysis utilising the above criterion gives that the stress required to just cause the crack to propagate, that is, the breaking stress or bulk tensile strength of the material, is

$$\sigma_t = \sqrt{\frac{2E\gamma}{\pi c}} \quad (1.3)$$

(Griffith Formula for Plane Stress)

Note: The same result applies to an internal crack as in Fig. 1.1(b).

The equation is dependent on a characteristic crack length, c .

It is important to stress the assumptions involved in deriving

Griffith's Formula:-

- (1) The material is a perfectly elastic, solid Hookean material. In particular, it is linearly elastic right to the point of brittle failure.
- (2) The volume of destressed material due to the presence of the crack is equal to half the volume of an ellipse of major axis $4c$.
- (3) The surface energy of the material is a linear function of the crack length c .

Equation (1.3) is best explained by means of Fig. 1.2, in which the surface energy requirement and the elastic strain-energy release are plotted as a function of c .

The main points are summarised below:-

- (a) The release of strain energy is proportional to the square of the crack depth, while the surface energy increases in direct proportion to the crack depth. Therefore, the rate of release of strain energy increases with increasing crack depth, while the rate of demand or increase of surface energy is constant.

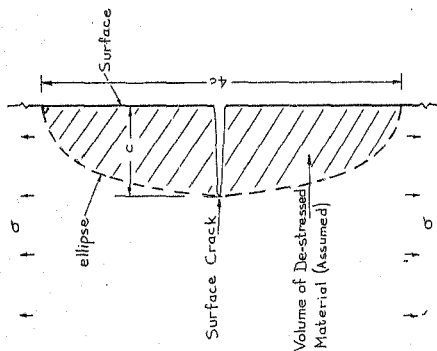


FIG. 11(c) SURFACE CRACK, GRIFFITH CASE

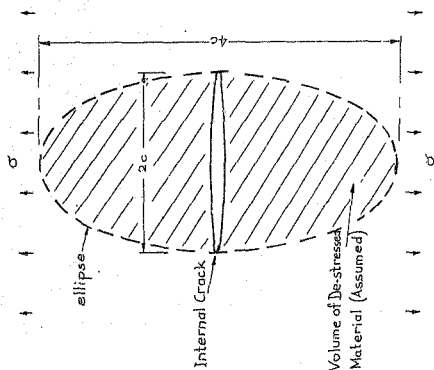


FIG. 11(b) INTERNAL CRACK, GRIFFITH CASE

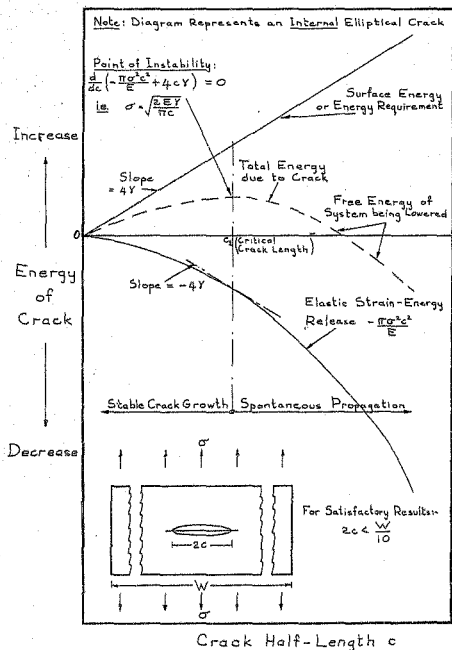


FIG. 12 CRACK PROPAGATION FOR THE IDEAL GRIFFITH CASE

(b) Hence, a shallow crack consumes more energy as surface energy than it releases as strain energy, until a critical crack length c_1 is reached, called the point of instability, where the crack begins to release more energy than it consumes. Stable crack growth occurs for $c < c_1$, whereas unstable, spontaneous crack propagation occurs for $c > c_1$.

(c) There is a critical Griffith crack length for each stress level in the material. Critical crack lengths are often very small, and concrete has a relatively large critical crack length, of the order of a few millimetres.

At levels of stress which are of interest in engineering, all but the smallest cracks have an energetic incentive to propagate if they can. The question now is, do they have a mechanism for doing so? Is there some way of actually converting the energy from one form into another? What distinguishes a tough from a brittle material is the mechanism for implementing the energy change. This mechanism is described below.

1.2.2 Mechanism of Crack Propagation

The mechanism for crack propagation is the stress concentration, which at the tip of a crack is approximately given by

$$K = 2\sqrt{\frac{c}{r}} \quad \left(K = \frac{\sigma_{max}}{\sigma} \right) \quad (1.4)$$

where c = the depth of a crack proceeding inwards from a surface, or the half-length of an internal elliptical crack, and r = the tip radius. In a typical brittle material, r remains constant whatever c is. As the crack grows, c increases, the stress concentration increases, and the energy balance becomes more and more favourable for rapid crack propagation. Therefore, under sustained load, and provided the mechanism for crack propagation exists, which is usually the case in brittle materials, the crack rapidly accelerates to failure.

1.2.3 The Effective Fracture Surface Energy Concept

The Griffith Criterion (1.3) in the form $\sigma\sqrt{c} = \text{constant}$ holds for a great variety of materials, both brittle and semi-brittle metals and non-metals. However, equation (1.3) is dependent on the assumption that the material is linearly elastic right to the point of failure,

i.e. a perfectly brittle material. This is not generally true since most materials exhibit some ductility (plastic or pseudo-plastic deformation) before failure. The energy balance then actually exists between the elastic energy release and the total work done in propagating the crack³. This total work includes the surface free energy of the fracture surfaces, as well as the plastic work irreversibly consumed at the crack tip during propagation, and any other energy-absorbing mechanisms unavoidably linked with fracture^{4,5}. Mathematically, we can write

$$\bar{\gamma} = \gamma + \gamma_p \quad (1.5)$$

where $\bar{\gamma}$ represents the total work of fracture, and is called the effective fracture surface energy; γ represents the free surface energy of the material, and γ_p represents plastic work plus any other work required for fracture. With $\bar{\gamma}$ substituted for γ in equation (1.3), $\sigma\sqrt{c} = \text{constant}$ shows good agreement with experimental evidence for both metals and non-metals⁶. Usually, $\gamma_p \gg \gamma$, particularly for metals, and this shows up the inadequacy of using purely the surface energy approach. Hence, depending on the ability of the material to deform plastically or pseudo-plastically, we can write $\bar{\gamma} \approx \gamma_p$, and changes in surface energy often have very little effect on fracture toughness. Brittle materials with their lack of plasticity cannot relieve localised high stress concentrations at crack tips, and are less tough than ductile materials.

1.2.4 Evaluation of the Griffith Approach

Assuming random distribution of flaws and cracks of microscopic size in a material, Griffith predicts that the material will fracture at a critical stress level with a corresponding critical energy level, both dependent on a characteristic flaw size for the material. The Griffith Formula (1.3) with the general interpretation of $\bar{\gamma}$ has become the starting point for the engineering mechanics of fracture toughness, since it leads to a property of the material describing its fracture toughness in the presence of flaws and stress-raisers. Reasons for accepting Griffith's concept are:-

- (1) It yields a correct functional relationship between stress at fracture and critical flaw size ($\sigma\sqrt{c} = \text{const}$).
- (2) It predicts a theoretical cohesive strength for materials of the right order of magnitude (0.1E), verified by experiments on

i.e. a perfectly brittle material. This is not generally true since most materials exhibit some ductility (plastic or pseudo-plastic deformation) before failure. The energy balance then actually exists between the elastic energy release and the total work done in propagating the crack³. This total work includes the surface free energy of the fracture surfaces, as well as the plastic work irreversibly consumed at the crack tip during propagation, and any other energy-absorbing mechanisms unavoidably linked with fracture^{4,5}. Mathematically, we can write

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- (2) It predicts a theoretical cohesive strength for materials of the right order of magnitude (0.1E), verified by experiments on

whisker crystals.

Reasons against accepting Griffith's concept are:-

- (1) The elusiveness of a suitable value for γ or $\bar{\gamma}$.
- (2) Some ductile materials do not obey the rules, implying that Griffith must be a special case.
- (3) It represents an oversimplification of a series of much more complicated phenomena.

Nevertheless, the very simplicity of the Fracture Mechanics approach of Griffith has led to its acceptance and advance.

1.3 Fracture Mechanics Parameters G_c , K_c , $\bar{\gamma}$

The Griffith Formula with the general interpretation of $\bar{\gamma}$ has become the starting point for the engineering mechanics of fracture toughness. It provides design engineers with a logical basis for finding limiting stresses, to avoid fracture, in terms of a property of the material called its 'fracture toughness', and a knowledge of notches or other stress-raising flaws in the structure. The three fracture parameters of interest here are:-

- (1) The Fracture Toughness K_c
- (2) The Critical Strain Energy Release Rate G_c
- (3) The Effective Fracture Surface Energy $\bar{\gamma}$

1.3.1 The Fracture Toughness, or the Critical Stress Intensity Factor, K_c

The concept of K_c depends on the stress analysis of cracks in linear elastic bodies. K_c is measured in terms of the opening or cleavage mode stress intensity factor K , defined by an equation such as (1.6) below, where the coordinates and symbols are defined in Fig. 1.3.

$$\sigma_y = K \frac{\cos \theta / 2}{\sqrt{2r}} \left[1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] \quad (1.6)$$

K has the units of stress $\times$ length^{1/2}, and is alternatively designated K_I to indicate that the opening mode of fracture is being considered. Correspondingly, the alternative designation of K_c in such a situation is K_{Ic} . The stress intensity factor K is related only to the loading

and geometry of the structural element. At the critical point of unstable crack propagation, K becomes K_c , the critical stress intensity factor. K_c is also known as the Fracture Toughness of the material, and measures the ability of the material to resist crack growth. In appropriate circumstances, K_c of a material could be used to calculate the load that a structural member containing a flaw of known size could sustain without fracture. The important distinction between the mathematical quantity K and the material property K_c is similar to the distinction between stress and strength. An important property of K is that the total K value due to superimposed stress fields is a linear addition of the K -values of the individual stress fields. It thus has application to problems involving a combination of loadings.

Investigators in the metallic field (eg. Srawley, Ref.14) have recognised that the evaluation of K_c assumes a high degree of elastic constraint to plastic flow of the material at the crack tip. A plastic zone size factor is used as a criterion for this condition. Where lower constraint occurs such as in a thin plate, the effective toughness of the material can be substantially greater than K_c . This is basically a plane stress / plane strain problem, and the fracture toughness as far as metallic materials are concerned can alter enormously according as the elastic-plastic stress field ahead of the crack approximates to plane stress or plane strain conditions. K_c , therefore, implies plane strain conditions, i.e. a heavy section with adequate dimensions. Most investigators in the non-metallic field have assumed plane strain conditions exist during tests on notched specimens, and no quantitative evaluation of plane stress/strain conditions on K_c values for non-metallic materials seems to have been done to date. This dissertation attempts to briefly investigate this problem among others (see later Section 4.2.5). K_c therefore represents a lower bound strength estimate, and can be regarded as a basic material property. An example of how K_c can vary in metals, depending on plane stress or plane strain conditions, is given in Fig. 1.4.

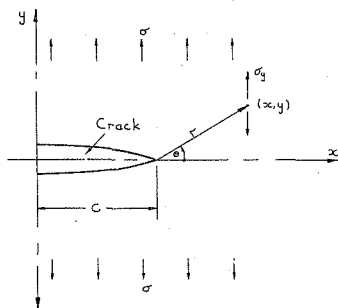


FIG.13 COORDINATE SYSTEM AT CRACK TIP

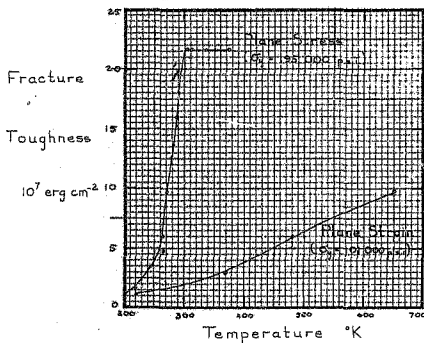


FIG.14 FRACTURE TOUGHNESS OF STEELS FOR
PLANE STRESS AND PLANE STRAIN CONDITIONS
(From Ref. 29)

1.3.2 The Critical Strain Energy Release Rate G_c

For a crack, either advancing from the surface or propagating internally in a material, strain-energy is released by the crack, in extending to a length of c . (See Fig. 1.2)

For the case of a surface crack, this strain-energy release is given by

$$U = \frac{\pi \sigma^2 c^2}{2E} \quad (1.7)$$

Now, the rate of release of strain energy with crack advance is

$\frac{\partial U}{\partial c}$, and is denoted G , i.e.

$$G = \frac{\pi \sigma^2 c}{E} \quad (\text{plane stress}) \quad (1.8)$$

Note that the units of G are either Force per unit length, or Work per unit area.

At the critical point of unstable crack propagation, G becomes G_c , the critical strain-energy release rate, or the critical crack-extension force. G is thus regarded as the driving force motivating crack extension. Note that from equations (1.3) and (1.8), G is numerically equal to 2γ , but this only has significance in the limiting case, when we can write $G_c = 2\bar{\gamma}$. The relationship between G and G_c is the same as that between K and K_c : G is primarily a function of loading and geometry, while G_c is a material property. Linear Elastic Fracture Mechanics leads to simple relationships between the two quantities G and K , viz.

$$\begin{aligned} K^2 &= EG & \text{for plane stress} \\ K^2 &= \frac{EG}{(1-\nu^2)} & \text{for plane strain} \end{aligned} \quad (1.9)$$

where E is Young's Modulus and ν is Poisson's Ratio.

(Note:- Some investigators introduce $1/\pi$ in the above relations).

Most reported G_c values refer to the opening mode of crack extension, and then G_c is alternatively designated as G_{Ic} .

1.3.3 The Effective Fracture Surface Energy $\bar{\gamma}$

This parameter has been dealt with previously. It is that parameter describing the amount of work required to produce unit area of fracture

surface, and includes true free surface energy as well as other energy-absorbing factors unavoidably linked with fracture.

1.4 Non-Metallic Materials of Construction:- Brief Review of Applications of the Fracture Mechanics Approach to Civil Engineering Materials

1.4.1 Diagrammatic Models of Crack Propagation

The Griffith formula (1.3) was derived from considering the balance between two main types of energy: the release of strain-energy $\pi\sigma^2 c^2/E$, and the surface energy of the crack surfaces $4\gamma c$. With $\bar{\gamma}$ substituted for γ , the second term is generalised to the energy requirement term, representing the demand for energy to make the crack continue to grow. This energy requirement for crack propagation is consumed, not only in surface energy of the new crack surfaces, but also in plastic work, and even in sonic and kinetic energy. The energy requirement term and the energy release term can both be represented by curves of energy versus crack length c . It is also instructive to plot these curves in terms of energy demand and release rates versus c .

Case 1 : The Ideal Elastic and Homogeneous Material (The Griffith Case)

Fig. 1.2 shows this case, which has already been discussed in Section 1.2.1. This material, being ideal, is linear elastic right up to the point of brittle fracture. The energy requirement curve is a straight line of slope 4γ while the energy release curve is a parabola in c . Beyond the point of instability at c_1 , any small increase in crack length will cause more energy to be released than is required for crack propagation, and the crack propagates spontaneously. It is also instructive to refer to Fig. 1.5. The surface energy demand rate is a constant 4γ and independent of crack length for this ideal case. The rate of strain energy release varies linearly with c , and increases with increasing stress level. A material with a crack of length c_1 , under increasing stress would remain stable until the stress level reached σ_f , at which the energy balance for

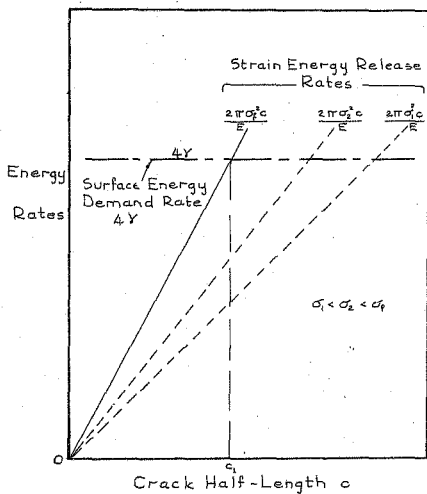


FIG.1.5 ENERGY RATE CURVE FOR THE
IDEAL GRIFFITH CASE

spontaneous crack propagation is satisfied, and fracture would then occur. The energy balance for continued crack propagation is then possible even under decreasing stress. This ideal case is probably never obtained in practice, so we extend the concept further.

Case 2 : The Visco-Elastic and/or Non-Homogeneous Material

Common construction material such as concretes, mortars, asphalts, cements, soil lime and other stabilised materials fall into this category. The surface energy term γ is difficult to interpret, since it might include some plastic flow, or be modified by multiple micro-cracking, or refer to fracture in a bi-or multi-phase material. Fig. (1.6) represents the probable energy balance for a crack propagating in such a material⁷. As the crack grows, so too does the area of the highly stressed zone immediately ahead of the crack tip. In this area, extensive micro-cracking can occur in addition to the main crack, for non-ideal, brittle materials. (Similarly for ductile metals, the volume affected by plastic flow increases with increased crack length). The energy-requirement curve is therefore an increasing function of c and no longer a straight line. It is conceivable that the curve becomes linear at point A, where the highly stressed zone reaches its maximum size, determined amongst other things by the geometry of the specimen.

Consider now an initial flaw or crack of length c_0 in a material under macroscopic tensile stress σ_0 . The crack begins to grow but is checked at c_1 , because the energy demand has now increased. The stress must be raised to σ_1 before the crack continues to grow to c_2 , where it is again checked by the increase in energy demand. This whole process is repeated under increasing stress until the crack reaches point A, which represents the point of instability, or the critical energy level. Both energy-requirement and energy-release curves reach their critical slopes at this point, and since there is no further increase in slope of the energy demand curve, any further increase in crack length causes unstable crack propagation to occur. Of course, the process is not stepped as in Fig. (1.6), but continuous under increasing stress. The continuous process is shown by the dotted line in the figure. Fig. (1.6) uses the concept of $\bar{\gamma}$, which varies with crack length, and is significant only at the onset of instability.

Consider also Fig. 1.7, which is a re-plot of Fig. 1.6 in terms of energy rates. The energy demand rate increases rapidly, and then reaches a uniform value at A. The process of slow crack growth occurs under increasing stress level, and no unstable crack propagation can occur on the ascending portion of the energy demand rate curve. At point A, spontaneous crack propagation can begin and proceed even under decreasing stress levels. Fig. 1.7 very clearly explains the process of slow crack growth in a notched beam under increasing flexure stress, and this will be discussed in more detail in a later section. (see Section 5.3.1).

Case 3 : Crack Propagating in a Bi-or Multi-Phase Material

Such materials consist of two or more distinct constituents or phases, each one having its own distinct energy-requirement curve, represented by different slopes, such as in Fig. 1.8. The curve is non-continuous, and the steep portion represents the energy demand of the "tougher" phase of the material. A crack of length c_0 propagating in the weaker phase would progress under stress σ_0 to a length c_1 . At this point, it intersects the tougher phase of the material, represented by a sharp rise in energy demand. The crack is checked, and will only propagate further if the stress is raised sufficiently to overcome the energy demand of the tougher phase. The tougher phase acts as a crack arrestor, and causes the toughness of the composite material to increase. This crack-arrest mechanism does not necessarily have to be a discrete phase in the material, but could also be weak interfaces the crack must intercept in order to progress. This is the basic concept of a material like fibre-glass in which the two phases themselves are fairly weak and brittle, but when combined give a tough material whose toughness and strength directly stem from interface crack stopping.

1.4.2 Fracture Mechanics of Cemented Materials

What follows applies to cemented materials in general, but concrete is used as a specific example of the fracture behaviour of such materials. These materials do not strictly obey the laws of elasticity, but fracture mechanics concepts can readily be applied to them.

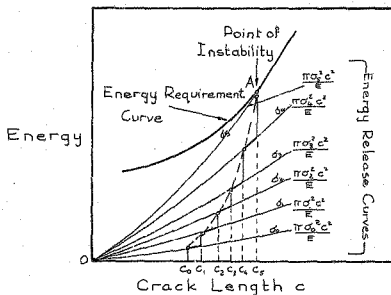


FIG.1.6 CRACK PROPAGATING IN A NON-HOMOGENEOUS BRITTLE MATERIAL UNDER MONOTONICALLY INCREASING TENSION
(From Ref.7)

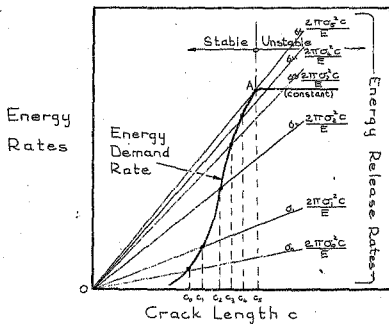


FIG.1.7 ENERGY RATE CURVE CORRESPONDING TO FIG.1.6

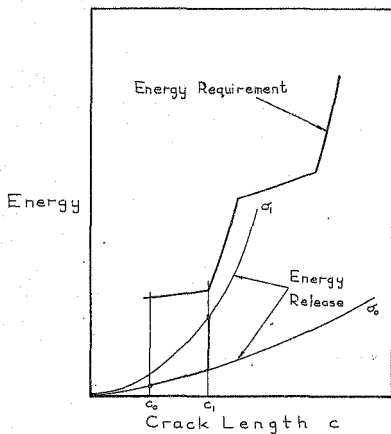


FIG.1.8 CRACK PROGRESSING IN A
BI-PHASE MATERIAL
 (From Ref. 7)

Concrete is a heterogeneous, brittle material whose strength derives almost directly from its heterogeneity and the phenomenon of microcracking. Fracture surface area and energy can refer to the hardened cement paste, or the aggregate, or the aggregate-paste interface. Cracking in concrete is not limited to one critical crack, but involves extensive microcracking in the highly stressed zone ahead of and surrounding the main crack⁹. An important consequence of this is that the area of newly formed fracture surfaces may be many times larger (up to ten or twelve times) than the nominal fracture area⁹. The weak bond in concrete is the aggregate-paste bond, and cracks form here where the energy demand is least, and then spread, eventually bridging the paste between aggregate particles. This causes concrete to act very much as a bi-phase material (Fig. 1.8) The aggregate-paste interface and the cement gel exhibit essentially brittle behaviour, and thus the strain energy released during fracture is transformed mainly to surface energy. The aggregates in concrete modify the three-dimensional elastic stress field and impose a crack-arresting action on the concrete, causing the fracture resistance of concrete to be greater than that of an equivalent paste¹⁰. Also, in addition to microcracking of the cement paste, cracks meander around aggregate particles, and extensive side cracking can occur. The crack-arresting function of the aggregates and the mechanism of microcracking all give to concrete an apparent property of pseudoplasticity, since all of these phenomena increase the energy demand for crack propagation.

1.4.3 Applications to Civil Engineering Materials

This section reviews some of the work done by other investigators in evaluating fracture mechanics parameters. Direct applications of this work to Civil Engineering problems are few and far between, and sometimes the basis of the application is very questionable. This section also deals with the assumptions and modifications required in order to apply fracture mechanics principles to construction materials. Materials considered will be concrete and cement mortars, asphalts and asphalt cements, soil cements, and other road pavement materials.

(1) Concrete and Cement Mortars

(a) Assumptions and Modifications

Application of fracture mechanics theory to concrete is based on the following two assumptions:-

- (i) The Laws of Elasticity for homogeneous materials can be applied to concrete (which is heterogeneous and discontinuous due to the presence of micro-cracks.)
- (ii) The values of the elastic modulus E and Poisson's ratio ν are constant throughout the material. This criterion is satisfied by assuming an average E and ν for the concrete. In addition, the surface energy term γ must be interpreted as resulting from the total work of fracture, and $\bar{\gamma}$ is the relevant parameter.

(b) Tensile Fracture in Concrete

Under tensile stress, concrete behaves perfectly elastically until the stress $\sigma = \sqrt{2E\gamma/\pi c}$ is reached. Beyond this point, both stress and crack grow slowly, as depicted in Fig. 1.6, and the stress strain curve is no longer linear. Due to decrease of the cross-section, the stress grows faster than the external load, and the strain energy release rate $\pi\sigma^2 c'$ grows even faster until it reaches a value G_c where rapid crack propagation occurs, and the stress-strain curve terminates abruptly.

(c) Compression Fracture in Concrete (Homogeneous Uniaxial Compression)

Flaws and cracks in concrete under compression can lead to tensile stresses being developed near crack tips, depending on the shape and orientation of the cracks. Crack propagation can only occur in planes parallel to the direction of compression, and not necessarily in a plane containing the initial crack. Assuming that concrete has numerous initial flaws and cracks, the weakest of these will begin propagating under compression. However, due to the crack arrest mechanisms of micro-cracking and aggregate zones of high strength, these cracks would be checked by the rapid increase in energy demand. This permits a further increase in load, and another crack, next in order of weakness, starts propagating. The new crack in turn is soon arrested and another begins growing. This process is termed the "progressive cracking mechanism" by Glucklich⁷.

The process of progressive cracking eventually brings about total destruction, either because the material can no longer resist shear stresses in its disintegrated state, or the value of the critical strain energy release rate G_c is attained, and the particular crack propagating at that instant runs to failure. In tension, the stress field σ_{tens} increases rapidly with the crack propagation because of the decrease in cross-section; however, in compression, this is not so. Also, the progressive cracking mechanism, which effectively eliminates nuclei of fracture just as plastic flow does in metals, provides a mechanism of energy dissipation that constitutes an alternative to fracture. These two factors make compression fracture of concrete far more stable than tension fracture.

(d) Brief Review of Work by other Investigators

Kaplan¹¹ was one of the first investigators in the non-metallic field. He performed tests on notched concrete beams in order to determine the critical strain-energy release rate G_c . He found that smaller 75x100 millimetre beams gave somewhat lower G_c values than 150x150 millimetre beams, but concluded that the concept of the critical strain-energy release rate being a condition for rapid crack propagation was applicable to concrete. Kaplan neglected the effects of slow crack growth prior to failure, which caused his G_c values to be underestimated. He also suggested a tentative application of his fracture mechanics parameters to the failure of a simply supported plain concrete slab. However, it seems more logical in the case he uses to simply apply bending theory to the failure of the slab. It is obvious that some more direct and logical application of fracture mechanics principles to the failure of concrete members must be sought. Moavenzadeh and Koguel⁹ also used notched-beam specimens to study the fracture of cement paste, mortar and concrete. They found that the effective fracture surface energy $\bar{\gamma}$ was greater for mortars and concretes than for pure cement pastes, due to the introduction of solid particles and the corresponding increase in microcracking and heterogeneity. Using the technique of quantitative microscopy, however, they were able to show that $\bar{\gamma}$ for cracks propagating through the aggregate paste interface was less than the $\bar{\gamma}$ for cracks propagating directly through the cement paste. They also ignored the effect of slow crack growth prior to failure.

Naus and Lott¹⁰ calculated the fracture toughnesses of pastes, mortars and concretes in which they varied water-cement ratio, sand cement ratio, gravel-cement ratio, age of test, air content and gradation and type of coarse aggregate. Although they ignored the effect of slow crack growth, they employed the ideas of Lott and Keslar¹², who realized that the presence of aggregate particles near the tip of a crack modified the stress field of the crack, and increased the degree of microcracking. The latter authors argued that when concrete was analysed as a homogeneous material, a pseudo-fracture toughness K'_c resulted which was the summation of the fracture toughness of the cement paste and an arresting action of the aggregates on crack growth. They found that concretes had higher fracture toughnesses than equivalent mortars.

Glucklich¹³ did a considerable amount of work in studying the propagation of fatigue cracks in mortar, using both notched and unnotched beams. He introduced a compliance-crack length relationship in order to interpret measured strains in terms of crack length, and hence overcome the problem of slow crack growth prior to failure. He found that the value of G_c was approximately constant for both notched and unnotched beams, and $\sigma\sqrt{c}$ was constant for beams with equal notch depths. Generally G_c measured in fatigue tests was lower than G_c measured in static tests, which he attributed to elastic creep. Glucklich was one of the first to point out that the object of a notch was not to simulate a crack, but to predetermine the cross-section of failure. In fact, investigators in the metallic field use the standard practice of growing a sharp fatigue crack from a notched specimen before fracture testing the specimen¹⁴.

Welch and Haisman^{15,16} evaluated fracture toughness parameters of a wide range of concretes, mortars and pastes, using notched and unnotched beams in flexure. They also considered the influence of different types of notches. Over the range of compressive strengths they were considering, they found that fracture toughness parameters tended to increase in proportion to concrete strength properties such as compressive strength, water-cement ratio, indirect tensile (splitting) strength, and solid volume fraction of cement paste. Investigating the effect of notch sharpness (i.e. root radius), sharper notches produced flatter load displacement curves and lower ultimate loads, than blunt notches. However, if slow crack growth

was taken into account, the notch type did not influence fracture toughness values by more than about 10 per cent. They point out the important fact that fracture toughness values are affected considerably by the assumptions used to estimate the effect of slow crack growth, the stress concentration at the root of the notch, and values of Young's Modulus. Three sets of assumptions were proposed in order to provide a better comparison of results by different research workers. These assumptions are:-

"Assumptions A" : The original notch depth, c_0 , is used to compute the nominal stress σ_n at the root of the notch, and for substitution in the remainder of the equation. A nominal value of E, based on dynamic measurements on representative specimens for each concrete class is adopted.

"Assumptions B" : The original notch depth, c_0 , is used as in "Assumptions A", but the value of E is calculated directly from the load deformation curve in each case. Because of non-linear relationships in most cases, the secant modulus is adopted between zero and seventy per cent of ultimate load, and a correction is also made for shear deflection. It may be considered that the adoption of this lower value of E takes into account the slow crack growth and creep effects during loading. Consequently, it is suggested that it would be incorrect to use both this value of E and to allow for slow crack growth in addition.

"Assumptions C" : The critical crack depth, c_c , is used to compute the nominal stress σ_n at the root of the notch, and for substitution in the remainder of the equation. The critical crack depth is the original notch depth plus slow crack growth prior to the onset of unstable crack propagation, as determined from compliance - notch depth relationships. The dynamic value of E is used, as in "Assumptions A".

It is necessary here to point out that in the present dissertation, a different set of assumptions was used to calculate fracture toughness parameters. The assumptions used were identical to "Assumptions C" above, except that E was determined for the particular class of asphalt or mortar from the linear portion of the load-deformation curves of unnotched specimens, since it was found that under the conditions of test adopted, creep and rate of strain had a significant effect on all parameters measured. The assumptions and procedure are fully outlined in Section 3.2.4.

Welch and Haisman are the first to have pointed out the need to adopt more standard practices and assumptions for the fracture testing of non-metallic materials, and this is commendable when one considers the often divergent range of techniques used up to the present time. Much of the work of this dissertation aims at developing standardised procedures for fracture testing of non-metallic materials, similar to the type of work that has been done in the metallic field. Finally, Brown¹⁷ used two methods to measure the fracture toughness K_{Ic} of cement pastes and mortars. The first was the usual notched-beam technique, combined with compliance measurements to measure the slow crack growth prior to instability. He measured the change of toughness for separate increments of crack growth as the crack propagated. The second method, using a double-cantilever beam (DCB), avoids the slow crack growth problem by making a specimen of variable web width such that the length of crack front increases with and exactly compensates for the effect of crack growth. He found that the fracture toughness of cement paste was independent of crack growth, but that the toughness of mortar increased as the crack propagated, which ties in with the concept of Fig's 1.6 and 1.7 very well. For both materials, the stress intensity required to initiate crack growth was less than that to maintain crack growth at the loading rates used.

(2) Asphalts and Asphalt Cements

Asphalt is a visco-elastic material whose properties depend on the shear rate and temperature, as well as the degree of ageing. At low temperatures, asphalt behaves as a brittle material, but at normal service temperatures, it can undergo considerable plastic deformation, leading eventually to cracking and permanent deformation in the form of rutting. In an asphalt mixture, it is the fracture behaviour of the asphalt that is the primary controlling factor in the cracking of the mixture. Some investigators have used fracture mechanics principles to determine fracture parameters for different asphalts. Others have attempted to use these parameters in a study of the fatigue life of asphalts.

(a) Assumptions in Applying Fracture Mechanics to Asphalt

As with concrete, fracture mechanics can be applied to asphalt with some modifications and assumptions. Fracture mechanics concepts are

based on the assumption that the material is linear elastic and homogeneous. Asphalts display linear elastic behaviour at low temperatures, and most investigators have studied asphalts in the low temperature range. An important point to note here is that only at low temperatures causing brittle fracture, and under plane strain conditions can the parameters G_c and K_{Ic} be regarded as material constants. At higher temperatures where the plastic zone size ahead of the crack tip increases, K_{Ic} is larger and corresponds more to a plane stress state.

(b) Variables Affecting the Fracture Mechanics Parameters

The following variables were recognised as affecting the fracture mechanics parameters:-

- (i) Temperature. This has an influential effect on the brittle behaviour of asphalts. Elastic behaviour only occurs at low temperatures of around minus ten Celsius and lower, and above zero Celsius there is a marked increase in the rate at which the material can absorb energy with increasing temperature.
- (ii) Rate of Loading. Fracture toughness of asphaltic mixtures generally increases with increasing load rate.
- (iii) Age of the Asphalt. In general, an aged asphalt shows higher values of the fracture parameters than an unaged asphalt, indicating increasing resistance to brittle fracture. However, this trend has not been adequately justified in the existing literature, and seems out of phase with practical experience in road pavements. It is possible that this trend represents a thixotropic effect.
- (iv) Different Rheological Properties. Higher toughnesses are obtained in asphalts by increasing the asphalt content, or using softer, low consistency grades of asphalt cements. Also, mixes of higher unit weights have higher fracture toughnesses, and higher effective fracture surface energies.

(c) Brief Review of Previous Work

Moavenzadeh¹⁸ used the notched beam technique to study the fracture susceptibility of asphalts at low temperatures, varying such parameters as rate of loading, temperature and depth of notch.

Bahgat and Herrin¹⁹ extended the work by including in their fracture testing an impact test to evaluate the brittle resistance of asphaltic mixtures.

Majidzadeh et al²⁰ attempted to predict the fatigue life of a paving mixture in terms of material constants, geometry, boundary conditions and the state of stress, and tested simply supported beams and beams on elastic foundations. They used a crack propagation law derived by Paris, which, includes as one of its parameters the stress-intensity factor K .

Blight³ used the concept of the strain energy released when a pavement cracks due to shrinkage stresses to predict crack spacings in pavements. In addition, he also showed that the maximum wheel load to cause flexural failure of a pavement could be obtained in principle. Finally, using the concept that failure occurs when a limiting, critical or saturation quantity of plastic work has been performed on a material, he was able to derive an expression predicting the number of cycles to failure, and from this the plastic deformation the pavement can tolerate before cracking. Thus, the fatigue problem is viewed in terms of tolerable permanent deformation.

(3) Soil Cement

The classic work carried out on the application of fracture mechanics to the failure of soil cement was done by George²¹. He used very similar methods in determining the parameters G_c and K_c as those used for concrete or asphalt, and found that soil type, temperature and loading rate all affected these parameters. He then used G_c and K_c to evaluate the crack propagation potential in a pavement base. He used model studies of a soil-cement base, and concluded that K_c rather than G_c was the parameter governing the rate of crack propagation. Crack propagation rate increased with a decrease in K_c . He also considered the spacing and configuration of cracks in a soil-cement base, using the principle of minimum potential energy. His findings that the crack pattern should be one of random orthogonal polygons with a bias towards hexagons were borne out by his model studies.

(4) Lime Stabilised Soil

Blight³ carried out both beam bending and unconfined compression tests on soil-lime, in order to find the effective fracture surface energy γ ,

i.e. the actual work of fracture. Age and lime content had very little effect on $\bar{\gamma}$. He also plotted, for a number of different cemented materials, values of $\bar{\gamma}$ versus strain at failure, on a log-log plot. The $\bar{\gamma}$ term contains plastic work done to cause the material to fracture, and since plastic work is accompanied by strain, the plastic work increases as the strain to failure increases. The results of Blight's log-log plots are a series of straight lines of approximately constant slope, each line representing a particular material. These plots seem to indicate a relation between $\bar{\gamma}$ and ϵ_{fail} .

Blight calculated the effective fracture surface energy of his cemented materials, $\bar{\gamma}$, by equating the work done by the applied load to produce fracture (i.e. the area under the load-deflection curve) to the area of fracture surface produced, i.e.

$$\bar{\gamma} = \frac{A_{P,\delta}}{A_f} \quad (1.10)$$

where $A_{P,\delta}$ = measured input energy = work done in fracturing the specimen = area under P,δ curve, and A_f = area of new fracture surfaces. (Note: a crack opens up two new fracture surfaces, so A_f includes both of the new surfaces).

$\bar{\gamma}$ can be interpreted as the energy absorption of crack propagation, and equation (1.10) immediately relates it to both the surface free energy plus any other energy-absorbing processes that may be inseparably connected with fracture. In an ideally brittle material, the fracture surface area would be equal to twice the cross-sectional area of the specimen. However, as mentioned previously, multiple micro-cracking in the highly stressed zone can occur, as well as meandering crack paths (eg. as in concrete), and the actual true fracture surface area may be many times larger than the effective surface area represented by the cross-sectional area. In this case, one may compute the effective fracture surface energy as

$$\bar{\gamma} = \frac{A_{P,\delta}}{2A_0} \quad (1.11)$$

where A_0 is the nominal cross-sectional area. To compute the actual fracture surface energy would require a knowledge of A_f . The true A_f can be obtained from a technique such as quantitative microscopy, which studies the cracks on the fracture surface.

Blight presumed that $\bar{\gamma}$ was made up of two components:- the true free surface energy of the material, and the plastic work component required to cause fracture, and included the following factors in the plastic work component:-

- (a) Work unaccounted for by the inability to accurately measure the true surface energy of the fracture surfaces.
- (b) Work done in dilatancy and particle reorientation near failure
- (c) Work done in producing microcracks that do not form part of the visible fracture surface
- (d) Actual plastic deformation of the material

Blight also suggested that the true fracture surface energy of a cemented material is represented by the surface energy of the cementing component, and the effective fracture surface energy $\bar{\gamma}$ exceeds the true surface energy γ by the amount of plastic work required to cause fracture. The engineer invariably requires $\bar{\gamma}$ for his calculations, as long as $\bar{\gamma}$ represents the failure mode likely to be experienced in practice. This means that testing procedures must be linked to actual service conditions for the material.

1.5 Introductory Remarks on Fracture Testing

Over the last decade or so, fracture mechanics has had a rapid development with most of the impetus coming from problems of the fracture of high-strength metallic materials. Research in the metallic field has been stimulated largely by recent requirements of military and aerospace structures to be made of high-strength, lightweight materials. The questions to be resolved centered around how to evaluate the strength of metals in the presence of cracks or crack-like defects, since the average stresses at which failure due to catastrophic crack propagation occurred were way below the yield strength of the material. The development of laboratory tests and analytical techniques to permit a measure of fracture toughness has been based on the Griffith-Irwin linear elastic fracture mechanics theory.

Current studies in the metallic field have been aimed at measuring the fracture toughness or crack tolerance of a material, that is, the maximum size of defect, whether natural or artificial, that can be tolerated in a material under a given stress level. It is unrealistic to depend upon the total absence of crack-like defects

in a structure, be it a concrete beam or a rocket motor-case. Therefore, it is necessary to know something of the crack tolerance of materials in order to predict the quality of their service life. In the metallic field, fracture mechanics has found an immediate application in those situations where plane strain fractures occur causing the material to fail without excessive plastic deformation. This has led to considerable work being done in the metallic field on the development of suitable methods of fracture testing.

However, in the non-metallic field, the application of fracture mechanics is still a relatively new field, and no uniform approach seems to have been developed. This is strange, since it is in the non-metallic field that the traditional "ideal brittle" materials occur, to which the Griffith-Irwin theory has direct application. Up to this stage, few direct or logical applications of fracture mechanics to non-metallic materials have been made. One exception has been a fracture mechanics-based study of the cracking of road pavements³. Non-metallic materials are generally far more notch-sensitive than metallic materials, and this means that fracture testing of brittle non-metals has its own specific problems associated with it. While the fundamentals of the fracture testing of metals or non-metals are the same, the detailed application differs. Also, no material conforms to the ideal elastic-brittle type of failure assumed in linear elastic fracture mechanics, and it is, therefore, necessary to develop specifications for valid fracture testing, whether of metals or non-metals.

1.6 Scope of This Dissertation

This dissertation borrows extensively from the ideas already put forward in treatises on the fracture testing of high strength metallic materials (eg. refs. 22 and 23), and develops these ideas in relation to non-metals, attempting to arrive at some logical and uniform method of fracture testing of non-metallic materials, specifically those cemented materials in common use in Civil Engineering construction. In order to do this, an extensive laboratory test program was initiated, in which notched and un-notched beams of mortar and asphalt were fracture tested in flexure. This allowed the different variables affecting the fracture parameters obtained from the tests to be isolated and evaluated. In particular, the rate of strain

and testing machine hardness were found to have a significant effect on the fracture behaviour of the beams. The use of different notches also allowed an investigation of the post-cracking behaviour of the beams. In order to correctly measure those quantities affecting the fracture parameters, it was necessary to pay close attention to adequate instrumentation of the tests. The fracture properties of the cemented materials studied were then used to attempt an evaluation of the fracture testing of non-metallic materials. This led finally to proposals of some tentative criteria for valid fracture testing of non-metallic materials.

CHAPTER 2
FUNDAMENTAL REQUIREMENTS FOR THE DESIGN AND TESTING
OF FRACTURE SPECIMENS

The fundamental requirements for the design and testing of fracture specimens apply to any material, whether metallic or non-metallic. The detailed requirements for the fracture testing of any particular material may differ, however, from other materials. For instance, the plane-strain fracture testing of metals often leads to a phenomenon called 'pop-in' being observed, which is not observed in the fracture testing of other construction materials. On the other hand, non-metallic materials often exhibit a far more brittle type of fracture, leading to problems of how to monitor the load and deflection at the critical point of instability and subsequent period of rapid crack propagation, than do the more ductile metals. Since the fracture testing of metallic materials is more advanced in technique and in application than that of non-metallic materials, it has been advantageous in considering the fundamental requirements for non-metallic materials fracture testing, to both borrow and develop concepts and techniques already advanced in the metallic field. Thus, much of the content of this chapter is by no means original, but it is necessary to include it in order to assist in developing the discussion of subsequent chapters.

The linear-elastic theory of Fracture Mechanics can be applied to specimens provided the plastic or highly-stressed zone ahead of the crack tip is below a critical size in relation to the specimen size, thereby causing the localised plastic or pseudo-plastic deformation at the root of the notch (or crack) to be accommodated within purely elastic surroundings. The fracture toughness of a material depends primarily upon the ductility, for metals, and analogously micro-cracking for brittle non-metals, at the roots of the notches. Investigators in the metallic field^{1,4} have defined the Plastic Zone Size Factor as being $R = (K_c / \sigma_{YF})^2$. When R is small in relation to specimen dimensions, plane-strain conditions prevail, and the test

is a valid K_{Ic} fracture test. When R is large, plane-stress conditions prevail, and the fracture toughness measured can be much greater than the lower limit represented by the plane-strain value. (see Fig. 1.4) All of this implies that fracture test specimens must meet with certain dimensional and shape requirements. In particular, a sample for fracture testing must have some minimum dimensions if the results are to be meaningful. This requirement has been recognised and applied in the metallic field, but somewhat ignored in the non-metallic field.

2.1 Requirements for a Satisfactory Fracture Toughness Test

Basically, a fracture toughness test involves loading a specimen which contains some type of pre-crack or notch until the crack becomes unstable and extends abruptly. Such specimens may be loaded in pure tension, or in different ways of beam bending, such as pure bending, or bending combined with shear. At the critical point of unstable crack propagation, the energy balance becomes favourable for fracture, and it is possible to measure a K_{Ic} or a G_{Ic} value. The fracture parameters measured are not only a function of specimen geometry and loading, but also of such variables as temperature, strain-rate and related creep. All fracture testing leads to a wide scatter of results, and for design purposes it is usually best to accept the lower confidence limits.

The output from a fracture test is usually in the form of a curve of applied load versus some measurement of deflection which can later be interpreted in terms of the extension of the crack. Such deflection measurements could be the *deflection of the load points*, or the relative displacement of two points located symmetrically on opposite sides of the crack plane. The type of common cemented materials examined as part of this dissertation exhibit the behaviour of slow crack growth prior to catastrophic fracture (which is somewhat analogous to the 'pop-in' phenomenon in metals). Here, the crack extends slowly until it reaches a critical length at which the load on the specimen causes rapid propagation, and it is imperative to be able to measure this slow crack growth by some type of deflection measurement. The method adopted here was to derive experimentally

and theoretically a relationship between beam compliance and crack-length^a. (See later Section 3.2.4). An autographic recording of the load-deflection curve is the most satisfactory means of conducting a test provided the autographic plotter is sufficiently sensitive to pick up the transient effects at catastrophic failure or unstable crack growth. (See later Section 3.3) Hence, a satisfactory fracture toughness test should meet two requirements:-

- (1) The specimen dimensions, and loading arrangement should allow the fracture parameters (eg. G or K) to be calculated accurately at any stage of the test at which the values of load and crack dimensions are known.
- (2) The values of load and crack dimensions at the point of instability of crack extension (G_c or K_{Ic}) should be accurately measured.

2.2 Consideration of the Simplest Type of Fracture Model

The simplest type of fracture model is that of an axially symmetric crack inside a body sufficiently large that boundary surface effects on the stress field of the crack are negligible. The specimen is tested by steadily increasing the gross tensile stress σ applied remote from and normal to the crack plane. The opening mode stress intensity at every point around the crack border is given by:-

$$K = 2\sigma \sqrt{\frac{c}{\pi}} \quad (2.1)$$

($2c$ = effective crack diameter, and for metals, c must be increased by a plane-strain plastic zone correction term for matching an equivalent elastic crack to an elastic-plastic crack).

Another alternative simple fracture model is the original Griffith model, that of a wide plate of width W under uniform uniaxial tension σ , and containing a straight ideal crack of length $2c$, less than $W/10$, in the center and normal to the direction of the applied stress. (See Fig. 1.2)

^a Beam Compliance is defined as the inverse of the slope of the linear portion of the load-deflection curve, i.e. a deflection per unit load when the load is expressed as a load per unit beam width.

The stress intensity factor for this case is given by:-

$$K = \sigma \sqrt{\pi c} \quad (2.2)$$

In such a model, only the stress field immediately around the crack is disturbed, and hence the highly-stressed zone ahead of the crack is located in purely elastic surroundings. It has been shown in thick metallic specimens that plane-strain conditions prevail in the middle part of the thickness, and plane-stress conditions near the faces, i.e. free faces impart a constraint-relieving influence to the crack front, and therefore the thickness of the specimen should be greater than the lower limit where this influence extends entirely through the thickness. This limiting thickness must usually be found by experiment. Common construction materials are usually not used in the form of thin sheets, and the assumption has always been that plane-strain conditions prevail. Nevertheless, fracture tests currently being conducted on glass have shown results that differ widely depending on whether a deep beam or flat slab was being tested. The plastic zone size factor R in metals increases rapidly as the general yield strength of the material decreases, and therefore, the minimum specimen dimensions increase rapidly as the yield strength decreases. Such an effect might very well also apply to non-metallic materials.

The fracture behaviour of these simple models can be shown by means of idealised load-deflection curves. An ideal, perfectly brittle material would show no increase in crack length right up to the point of abrupt fracture, and a linear load-deflection curve would result. (See Fig. 2.1(a)). This case is the classic Griffith case, also represented by Fig. 1.2, which is never obtained in practice. Cemented materials usually show slight extension of crack front during the last stages of loading, and therefore the displacement per unit load or compliance increases near the peak of the curve. (See Fig. 2.1(b)). This non-linearity near the maximum load can also be due to plastic or pseudo-plastic deformation around the crack front which manifests itself as a virtual crack extension (eg. microcracking of a brittle matrix), and creep and other time-dependent deformations. In both of the above cases, K_{Ic} or G_c is calculated from the value of the maximum load and the measured crack length. However, when non-linearity of the curve becomes

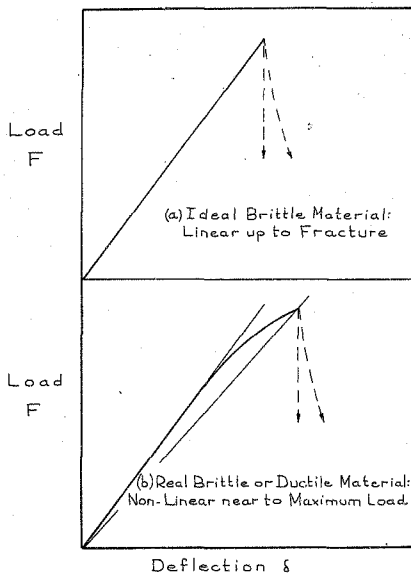


FIG. 2.1 TYPES OF LOAD-DEFLECTION CURVES
RELATED TO THE FRACTURE OF
SIMPLE IDEALIZED MODELS

excessive, it may be difficult to infer a K_{IC} or G_c value. Investigators of metals have limited the amount of non-linearity allowable by specifying the limits that the inverse of the secant slope between zero and maximum load can exceed the compliance of the initial straight line portion¹⁴.

The simple fracture models discussed above are straightforward in principle, but are inefficient as to the volume of material required and the load required. Plate or beam specimens, which are conceptually more complicated (and have more complicated expressions for K) have therefore been developed.

2.3 Criterion of Fracture Instability

It is necessary to make some precise, unambiguous definition of that point in a monotonically-loaded fracture test at which unstable crack extension occurs. Most tests have as the independent variable the deflection of the machine loading heads and this can usually be taken as the specimen deflection δ . Thus, load F is not the independent variable in the test. Then following on the practice of investigators of metals, the point of instability of crack extension is defined as that point at which the load-deflection relationship reaches a maximum or a point of zero slope, i.e.:-

$$\frac{dF}{d\delta} = 0 \quad (2.3)$$

(Note: This is the operational definition adopted by the ASTM Special Committee on Fracture Testing. See ref. 22, p.138)

At this point, the ability to control the load is lost, at least temporarily.

G at the point of instability can be calculated from measurements of the load and the instantaneous crack length at that point, and is designated G_c - a measure of the fracture toughness of the material. Other definitions of G_c do exist, such as that value of G at the onset of rapid crack propagation, but this can be a bit vague, and the definition above is preferred.

2.4 Catastrophic, Semi-Stable and Stable Fractures

While this subject is discussed in greater detail in a later section, with specific reference to actual tests conducted, it is necessary at this stage to mention that a satisfactory fracture test is often dependent on whether a catastrophic, a semi-stable or a stable fracture is obtained. This problem is more important in the fracture testing of brittle non-metals such as cemented materials than that of ductile metals, and the subject is not usually mentioned by investigators of metals. The problem is one of the controlled growth of a crack in a fracture specimen, and is vitally related to the elastic energy stored in the specimen and the machine system at the point of unstable crack propagation.

Materials may fracture in different ways: brittle non-crystalline cracking, as in glass; cleavage in crystalline solids; plastic shear or grain-boundary sliding; separation of atoms by evaporation or chemical dissolution; fracture through brittle constituents or along brittle interfaces in multi-phase materials such as concrete. These different processes lead to fractures that are either catastrophic, semi-stable or stable, where these terms are defined by means of characteristic load-deflection diagrams in Fig. 2.2.

At the point of instability in catastrophic fracture, all control over the growth of the crack is lost, whereas semi-stable and stable fractures allow some measure of control over crack growth throughout the test. Most brittle materials show catastrophic fracture in a bending test, with the load dropping instantaneously to zero, as in curve (A) of Fig. 2.2. The energy stored in the specimen at the peak load is sufficient to cause the crack to run right through the specimen to failure. In addition, if a soft machine loading system is being used, energy might actually be fed back into the specimen by the machine at the point of failure. Such a fracture then contains excess energy over and above the energy required to form new crack surfaces. This excess energy is expended in such energy-absorbing processes as kinetic energy of the fracture pieces, sonic energy accompanying fracture, heat energy, and energy associated with the velocity of crack propagation. In using the simple formula (equation (1.10)) for calculating $\bar{\gamma}$, viz. $\bar{\gamma} = A_{P,0}/A_F$ where $A_{P,0}$ is the

energy expended, an incorrect or unrealistic measure of $\bar{\gamma}$ will be obtained if $A_{F,0}$ comes from a catastrophic test record. Therefore, the test must be designed to produce a $\bar{\gamma}$ that will include surface free energy, plastic flow, and any other energy-absorbing processes inseparably connected with fracture. Those effects not connected with realistic fracture must be eliminated. The way to do this is to produce a stable or semi-stable fracture in the fracture test, by controlling the crack growth. This means that external work must continually be done to keep the crack moving. It is then possible to measure the amount of energy required to make it grow. A stable or semi-stable fracture is achieved by limiting the amount of elastic energy stored in the specimen and machine loading system at the moment of fracture initiation, thereby preventing spontaneous crack propagation, due to insufficient energy being available to cause total fracture. The stored elastic energy causes the crack to propagate, but drops off so quickly that crack growth is checked (see curves (B) and (C) of Fig. 2.2). Further external work is required before final fracture occurs. Thus, the total external work energy, as calculated from the area under the load deflection curve, is transformed into fracture energy without excess energy. The amount of elastic energy available for crack propagation at the critical peak load is governed by the strain energy stored in the specimen at that instant, as well as the strain energy stored in the machine. In order to produce an acceptable fracture then, attention should be given to the following two factors:-

- (1) The use of a rigid test machine and a hard load cell. This limits the amount of energy stored in the machine loading system and produces a more stable fracture.
- (2) Shaping the test specimen so that only a small load is required to initiate fracture (energy proportional to the product of the load and the deflection). Crack-notching the specimen is the most suitable way of limiting the stored elastic energy in the specimen. The benefits of a crack-starter notch are firstly that it reduces the peak load required for fracture by reducing the cross-section at the notch and by providing a high stress concentration, and secondly that it pre-determines the cross-section of failure in the beam. Furthermore, a small

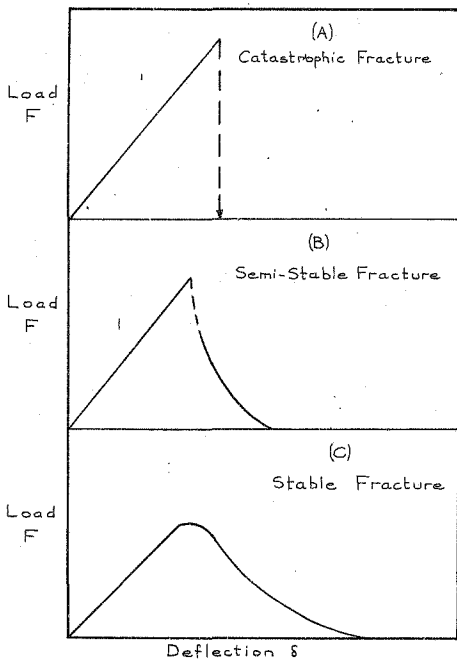


FIG.2.2 DEFINITION SKETCHES FOR CATASTROPHIC, SEMI-STABLE OR STABLE TYPES OF FRACTURE

load at the onset of crack growth ensures that energy is not expended on permanently damaging any of the remainder of the specimen.

CHAPTER 3

PRACTICAL SPECIMEN TYPES, FORMULAE AND INSTRUMENTATION

Chapter 2 described the simple type of fracture model for fracture testing, either that of the Griffith model, or the more general three-dimensional axially symmetric circular crack in a large body. These fracture models do not lend themselves readily to practical fracture testing, due to the volume of material required, the difficulty of fabrication and testing, and the load requirements of the specimen. For this reason, many different types of practical specimens for fracture testing have been devised, and once again the impetus here has come primarily from the metallic field. This chapter briefly reviews the different practical specimen types devised for opening mode fracture, and then discusses in more detail the particular crack-notched beam bending specimens selected for the current testing program. In addition, the formulae for calculating the fracture parameters obtained from a test are presented, and the instrumentation involved in the testing is discussed.

3.1 Brief Review of Practical Specimen Types

The specimen types documented below are given as an indication of the wide variety that are available for fracture testing. They are discussed in far greater depth in references 22 and 23, to which the reader is referred for more detail. Many of these fracture specimens, particularly the direct tensile test specimens, are not very suitable for fracture testing of non-metallic materials.

3.1.1 Symmetrical Plate Specimens

Plate specimens are tested in direct tension, and have either a central transverse crack of initial length $2c_0$ equal to about $0.3W$ where W is the width of the plate, or equal transverse edge cracks of initial length c_0 equal to about $0.15W$. These specimens are just a

modification of the wide plate specimens such as the simple Griffith model. The main difference is that the elastic strain energy field in the vicinity of the ends of the crack is appreciably influenced by the proximity of the crack to the free edges of the specimen. The simple expression for G , viz $EG = \pi a^2 c$, no longer holds, and some other form of expression for G is necessary, such as the tangent form proposed by Irwin^{24, 25}, i.e. $EG = \sigma^2 W$ can ($\pi c/W$). Parameters which affect the accuracy of the G_c measurement in this test are the plastic zone correction term (for metals), the width of the plate, the length of the plate, and the initial crack length. In addition, the thickness of the plate has a profound effect on whether plane-stress or plane-strain conditions predominate, and therefore on the value of G_c .

3.1.2 Single Edge-Notched Tension Specimens

This type of specimen can be derived from either type of symmetrically cracked plate specimen by bisecting along the longitudinal centreline. This specimen has the advantage of requiring less material and considerably less load to determine G_c than the symmetrically cracked specimen. It is necessary to impose limitations^{22, 23} on specimen dimensions and loading arrangement in order to produce a reliable G_c value in testing.

3.1.3 Surface-Cracked Plate Specimens

These specimens were originally introduced in the metallic field in order to investigate directly the effects of cracks similar to those from which fractures had often originated in service. The specimen consists of a plate loaded directly in tension, with a surface or part-through crack, approximately semi-elliptical in shape with the major axis at the surface, formed transverse to the direction of tension. Limitations are imposed^{22, 23} on the crack depth and the plate width.

3.1.4 Circumferentially-Notched Round Bars

Tension testing of notched round bars has an extensive history, and it was natural that this type of specimen should have been one of the earliest used for G_c measurements. However, the round notched bar

requires a considerably greater amount of material and considerably more loading capacity than any of the other types of specimens, and in addition is not suitable for fracture testing of brittle non-metals.

3.1.5 Crack-Notched Bend Specimens

These specimens generally consist of a rectangular section bend specimen with some form of crack-notch introduced in the tension face at the centre of the span. This was one of the earliest types of specimens to be used for fracture toughness testing. Earlier investigators in both the metallic and non-metallic fields did not at first appreciate the fact that a fabricated notch in itself does not represent an actual crack in the material. For testing of metals, it is customary to grow a sharp, natural crack from the starter notch by fatigue stressing before testing¹⁴. In the testing of non-metallic materials, specifically cemented materials of construction, the phenomenon of slow stable crack growth prior to catastrophic failure occurs, and so the crack becomes naturally sharp before the point of instability is reached. Glucklich¹³ has pointed out that a notch in a beam serves the following purposes:-

- (1) It enables one to know in advance the cross-section of failure.
- (2) It serves as a sufficient stress concentration to allow a natural crack to grow from it in a stable manner before unstable crack propagation occurs. This also facilitates more readings and better accuracy.
- (3) It allows for deformations measured across the notch to be the deformations at the critical area of crack growth, leading to better accuracy.
- (4) The notch can serve as a convenient trough for ink if a staining technique is employed.

The notched bend specimen has the distinct advantage of requiring lower loads in general to produce fracture than other specimens of identical dimensions. The test procedure itself leads to a simple beam-bending set-up that can be used in a variety of test machines. This facilitates investigating the effect of different parameters, such as test machine hardness and strain rate, on the fracture parameters. The practical convenience of testing a bend

specimen also greatly outweighs the problems of testing brittle non-metallic materials in direct tension. One disadvantage of the notched bend specimen is that the accuracy of G_c measurements is inherently lower than for any other type, because the sensitivity of the calculated value of G to a small error in c is greater than for any other type.

There are two possible ways of loading beam bending specimens:— either in three-point bending, or in four-point bending. An illustration of the two loading methods and the specimen shape is given in Fig. 3.1. Three-point bending complicates the failure by the existence of a shearing stress in the beam which changes from a positive to a negative value at the centre loading point. Four-point bending produces a uniform bending moment in the central span of the beam, with no shearing stress present — a state of pure bending. The extraneous effect of shearing stress increases with a decreasing span to depth ratio. It is reasonable to expect that somewhat more accurate G_c measurements could be made with a bend specimen in four-point loading than in three-point loading.

Different notch depths up to 50 per cent of the beam depth were selected for study in the present work. These notches were rectangular, and the notch-depth ratio was found to profoundly influence the shape of the load-deflection curves obtained in the test (see later Section 4.2.4).

One other type of notch shape was studied — that of a 'Vee' notch where the sharp apex of the notch was on the tension face, and the crack was continually guided between crack-guiding grooves. Such a chevron-shaped notch is often used in the metallic field as a starter notch from which a naturally sharp fatigue crack is produced, but in the current work it was mainly used to produce a completely stable fracture (see Section 2.4) as a comparison with the other rectangular notches used.

3.2 Formulae and Procedure for Determining Fracture Parameters

Investigators in the metallic field have devoted much time and effort in arriving at a reliable and suitable method for measuring the plane strain fracture toughness of a material K_{Ic} . The results are well summarised in an article by Srawley¹⁴, in which he says that K_{Ic} (or more strictly, K_{Ic}) can be regarded as a basic material property, the lower bound of effective fracture toughness, and that

the related quantity $G_c = K_c^2 (1-\nu^2)/E$ is the appropriate plastic work term for generalisation of the Griffith concept. He also stressed the requirement of adequate and correct specimen dimensions to determine K_c . All testing should be designed with actual practice conditions in mind, and the test itself should be a model of expected practice conditions.

Furthermore, an operational definition of K_c was suggested as follows:- K_c is that stress intensity at which the crack reaches an effective length two per cent greater than at the beginning of the test. This is to accommodate material specimens that do not fracture completely when the stress intensity reaches the K_c level, in particular those metallic specimens which exhibit "pop-in" of the central crack front before overall advance of the crack occurs, due to the higher effective toughness near the plate surface where the constraint is relaxed. A three-point bend test on a chevron-notched specimen is used to determine K_c . The test record must meet certain requirements before the calculated value of K_c is accepted. This value of K_c is then generally used in computations of the strength of a structure^a.

The stress intensity factor K in a test specimen is equal to the applied load multiplied by some function of the specimen dimensions, including the crack length, which depends on the specimen design. Any particular fracture test specimen has a relationship between K (or alternatively G) and its relevant dimensions and loads. Because of the difficulty of complete three-dimensional stress analysis, the fracture specimens are usually treated as two-dimensional plate specimens with through-thickness cracks. The most general method of relating K or G to the dimensions and load characteristics of a particular specimen is an experimental method derived by Irwin and Kies²⁶. The relevant equation is:-

$$G_c = \frac{1}{2} P^2 \frac{dL}{dC} \quad (3.1)$$

where L is defined as the compliance of the specimen in the presence of a notch, $F = \frac{1}{L}$ (i.e. the reciprocal of stiffness)

^a Note:- The article by Srawley represents a draft recommended practice for K_{Ic} tests with bend specimens adopted by ASTM Committee E-24 on Fracture Testing of Metals.

The method involves taking measurements of the compliance of a specimen with a narrow notch of varying depths, expressing the specimen compliance as a function of crack length and then obtaining the derivative of this function with respect to crack length. In this method, the finite width notch is used to simulate a crack during the initial linear-elastic behaviour of the specimen under load, and it is possible that the compliance of a natural crack of given length will not be exactly the same as that of a finite width notch of the same length. However, Welch and Haisman¹⁵ report that in the testing of their mortar and concrete beams, there was no significant difference between the compliance of beams with wide sawn notches and those with notches cast with rubber formers or special razor blade inserts. They therefore conclude that the initial compliance of the linear-elastic portion of a load-deflection curve for cemented construction materials is independent of notch sharpness, and a general relationship exists between compliance and crack length, in the absence of slow crack growth. (See later Section 3.2.4)

Brown and Srawley²³ also suggest that it is an advantage to use as large a specimen as possible for compliance measurements, firstly because any difference between the compliance of a natural and artificial notch will be minimised, and secondly because the displacements will be proportionately large and, therefore, good accuracy of measurement will result. Accurate compliance calibrations require sensitive, accurate gauges and careful attention to detail, and this is discussed further in the next section (3.3.2). As pointed out by Brown and Srawley²³, the main advantage of the compliance calibration method is that the actual configuration and load distribution of a fracture test specimen can be closely modelled by the compliance calibration specimens. Furthermore, since the fracture toughness relationship is based on mathematical stress-analysis models, the design of the specimen should be compatible with the mathematical model on which the fracture toughness relationship is based.

In the following sub-sections, formulae for the fracture parameters are presented. The relationships apply to the centre-notched bend specimens used in the tests described later. The K-relationship is derived from a compliance calibration method as described above, and as reported by Brown and Srawley²³. The G-relationship uses an

analytical stress-analysis procedure of the notched beams to arrive at the relevant formula.

3.2.1 Formula for K_c

K_c has previously been defined as the fracture toughness parameter, or the stress-intensity factor at failure. Brown and Srawley²³ have reported a relationship between K , the opening mode stress intensity factor, and the specimen dimensions and loading for a centre-notched beam specimen in pure bending (four-point bending) and in three-point bending, as follows:- (see Fig. 3.1)

$$K = Y \frac{6Mc}{bd^2} \quad (3.2)$$

where $Y = A_0 + A_1(c/d) + A_2(c/d)^2 + A_3(c/d)^3 + A_4(c/d)^4$

M is the bending moment in three-point or four-point bending.

Y is a function of a fourth-degree polynomial in the notch-

to c/d. K is accurate to within 0.2 per cent for all values

c/d up to 0.6. The coefficients A have the following values, dependent on the type of bending in the specimen:-

Table 3.1 Values of the Coefficients A for calculating K
(from Ref. 23)

	A_0	A_1	A_2	A_3	A_4
Pure Bending	+ 1,99	- 2,47	+ 12,97	- 23,17	+ 24,80
Three-Point: $S/d=8...$	+ 1,96	- 2,75	+ 13,66	- 23,98	+ 25,22
$S/d=4...$	+ 1,93	- 3,07	+ 14,53	- 25,11	+ 25,80

S = major span

Also, four-point bending is considered to be equivalent to pure bending if the ratio of the minor span to specimen depth is not less than two (Note:- For the beams used in this study,

$$\frac{\text{minor span}}{\text{depth}} = \frac{8}{4} = 2, \text{ therefore satisfactory})$$

Finally, if the ratio of the major, support span to specimen depth, S/d , is less than about four, in either three-point or four-point bending, then substantial errors in measurement can be introduced due to specimen indentation and friction at the supports.

(Note:- For this case, $\frac{S}{d} = \frac{24}{4} = 6 > 4$, therefore satisfactory).

3.2.2 Formula for G_c

Apart from the simple relationship between G_c and K_{Ic} , derived from linear-elastic fracture mechanics (see equation (1.9), another relationship for G_c in a notched beam specimen can be produced by a stress analysis procedure. This results in a mathematical relationship for G in terms of the specimen dimensions, depth of crack, applied load and modulus of elasticity. G_c is then the value of G in the test at that stress level which causes rapid crack propagation.

The strain energy release rate due to a surface crack propagating inwards is, for plane strain conditions:-

$$G = \frac{\pi(1-\nu^2)\sigma^2 c}{E} \quad (3.3)$$

For shallow notches ($a \ll d$), σ may be regarded as the maximum tensile bending stress at the extreme fibre of the beam.

Thus, for a rectangular beam of width b :-

$$\sigma = \frac{6M}{bd^2} \quad (3.4) \quad (M = \text{Applied moment})$$

The nominal stress at the root of the notch is:-

$$\sigma_n = \frac{6M}{bh^2} \quad (3.5)$$

where $h = d - c$

$$\text{Therefore} \quad \sigma^2 = \sigma_n^2 \frac{h^4}{d^4} \quad (3.6)$$

Substituting in (3.3) gives:-

$$G = \frac{\pi(1-\nu^2)}{E} \sigma_n^2 c \frac{h^4}{d^4} \quad (3.7)$$

Now, $h = d - c$, and so we can express this as follows:-

$$\begin{aligned} \frac{ch^4}{d^4} &= c \frac{(d-c)^4}{d^4} = c \left(\frac{d-c}{d} \right)^4 = c \left(\frac{d-c}{d} \right) \left(1 - \frac{c}{d} \right)^3 \\ &= h \frac{c}{d} \left(1 - \frac{c}{d} \right)^3 \end{aligned} \quad (3.8)$$

Therefore, G may be written as:-

$$\begin{aligned} G &= \frac{(1-\nu^2)}{E} \sigma_n^2 h \left[\pi \frac{c}{d} \left(1 - \frac{c}{d} \right)^3 \right] \\ \text{or } G &= \frac{(1-\nu^2)}{E} \sigma_n^2 h f \left(\frac{c}{d} \right) \end{aligned} \quad (3.9)$$

where $f \left(\frac{c}{d} \right) = \pi \frac{c}{d} \left(1 - \frac{c}{d} \right)^3$

This formula only applies to shallow notches of $c/d < 0.15$

For deeply notched beams, the effects of the reduced cross-section and the non-uniformity of stresses must be taken into account.

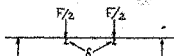
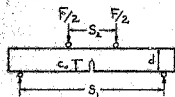
The most useful solution for this is the $f \left(\frac{c}{d} \right)$ curve proposed by Winne and Wundt²⁷, which incorporates work by both Neuber and Bueckner. This curve is shown in Fig. 3.2. It is seen that for deep notching, i.e. $c/d > 0.5$, $f \left(\frac{c}{d} \right)$ tends to a value of 0.521.

3.2.3 Formula for $\bar{\gamma}$

The formula used to calculate the effective fracture surface energy of a material is that presented previously in equation (1.10), viz.

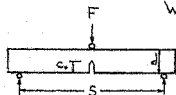
$$\bar{\gamma} = \frac{A_{F,\delta}}{A_f}$$

where $A_{F,\delta}$ equals the area under the load-deflection curve, and A_f is the area of new fracture surfaces (conventionally taken as twice the cross-sectional area). (Refer to Section 1.4.3 (4) for a full discussion of this.) In addition, $\bar{\gamma}$ is related to G_c by means of linear-elastic fracture mechanics theory, viz. $G_c = 2\bar{\gamma}$. It is important to point out at this stage that an evaluation of $\bar{\gamma}$ for any particular test specimen is critically dependent on the assumed area under the load-deflection curve and hence on the measured shape



a) Four-Point Bending (Pure Bending)

Width 'b' in both cases



b) Three-Point Bending

FIG.3.1 EDGE NOTCHED BEND SPECIMENS

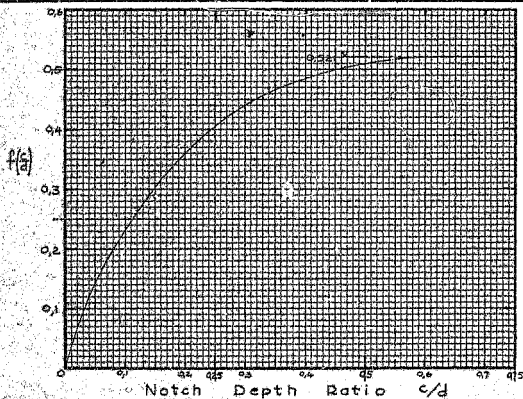


FIG.3.2 CURVE FOR CALCULATING G FOR NOTCHED BEND SPECIMENS (From Ref.27)

of this curve. This is very much an instrumentation problem, since for those beams which fracture catastrophically when G_c is reached, the transient effect of rapid crack growth and drop in load after failure is difficult to measure. Most investigators have assumed the load to drop instantaneously to zero at failure, but by creating stable and semi-stable fractures, it will be shown later that a definite tail portion to the curve exists even when a nominally brittle material is tested. Unless the area under this tail portion is included in the term $A_{p,d}$, $\bar{\gamma}$ may be substantially underestimated. The shapes of the load-deflection curves for the beams studied in the present work are discussed more fully in a later section (Section 4.2.4)

3.2.4 Assumptions and Procedure

The set of assumptions used in order to evaluate the fracture parameters for a test specimen are re-iterated here. Section 1.4.3 (1)(d) contains previous mention of the various assumptions used by different investigators, but the circumstances prevailing in the present tests made it necessary to slightly modify a set of assumptions suggested by Welch and Haisman¹⁵. These assumptions have to do with obtaining the quantities c_c , the critical crack depth at failure; σ_n , the nominal stress at the root of the notch at failure; and E , the elastic modulus of the material. These quantities are then substituted into the expressions for K_c or G_c , to arrive at the fracture parameters.

The set of assumptions adopted here are:-

- (1) The critical crack depth, c_c , as computed from the specimen compliance at failure by means of a compliance-crack depth relationship for the specimen, is used in order to account for the effects of slow crack growth from the notch prior to failure.
- (2) The nominal stress at the root of the notch, σ_n , is computed from c_c , on the basis of a linear stress distribution in the beam over the reduced beam depth h at the notch cross-section.
- (3) The elastic modulus of the beam, E , is determined as the average flexural modulus from the linear portions of the load-deflection curves of unnotched specimens. This modulus is adopted to account for the effects of creep and rate of strain.

The critical crack depth, c_c , is computed as mentioned previously from a suitable compliance-crack depth (L - c) relationship. The compliance of a beam for the present tests is defined as the inverse of the slope of the initial linear portion of the load-deflection curve, where the load is expressed as the load per unit width of the beam, i.e. :-

$$L = \left(\frac{\delta}{F/b} \right) \text{ linear portion} \quad (3.10)$$

Such a relationship can be obtained in two ways : either by experiment, or by theoretical derivation. The experimental method involves measuring the compliance L , for the particular material, on beam specimens tested with progressively deeper notches. In this way, the notch is used to simulate a natural crack, and as long as no slow crack growth occurs (i.e. the initial linear portion of the load-deflection curve is used to measure L), this is quite acceptable. In the theoretical method, the compliance-notch depth relationship is derived from simple linear-elastic fracture mechanics principles. The equations forming the basis of the derivation are Brown and Srawley's formula for K_c (equation (3.2)), Irwin's equation for G_c in terms of the rate of change of compliance with crack depth (equation (3.1)), and the simple relationship between G_c and K_c for the plane-strain case (equation (1.9)). Using these equations, a relationship between compliance and crack depth can be derived. The more general and acceptable form of derivation, however, is a relationship between L/L_0 and c/d , where L_0 is the compliance of unnotched specimens (zero notch depth). This form eliminates the need to evaluate E , and makes the relationship a general one for any material, dependent only on the value of Poisson's ratio, ν , for the material. Fig. 3.3 shows the theoretical plot of L/L_0 versus c/d , for the three values of ν of 0, 1; 0, 15; 0, 2. The detailed derivation of the relationship is given in Appendix A.

The procedure to determine the crack length at the critical point of instability is to measure the compliance (secant value) at the onset of unstable crack propagation, directly from the load-deflection curve, and then to estimate the corresponding critical crack depth from the compliance-notch depth curve. This crack depth value is then used in the evaluation of the fracture parameters

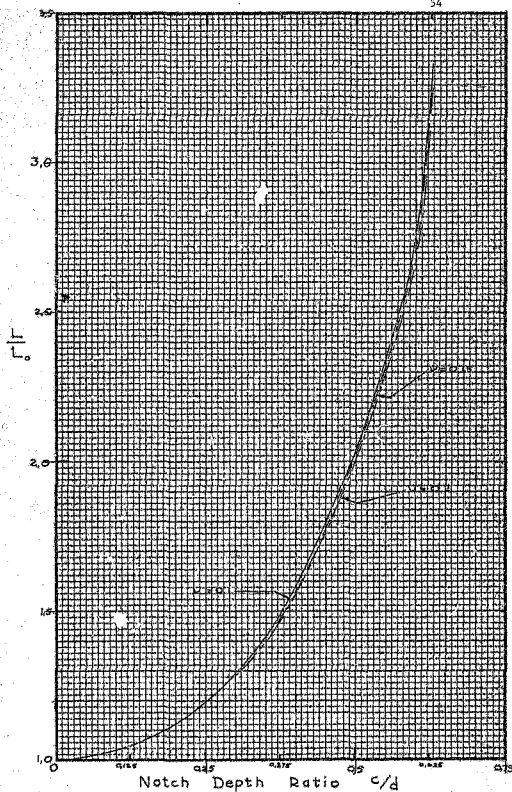


FIG. 3.3 COMPLIANCE - NOTCH DEPTH CURVE
(Theoretical)

G_c and K_{Ic} , using the adopted set of assumptions. It is important to note, though, that the calculation of G_c or K_{Ic} involves an assumption about the instantaneous value of the crack depth c_c , and that uncertainty about the value of c_c at instability is possibly the largest single source of error in fracture measurements.

A detailed set of calculations of G_c and K_{Ic} illustrating this procedure is given in Appendix B.

3.3 Instrumentation

The basic output requirement from a fracture test is an accurate record of load versus deflection, which can later be interpreted in terms of load at failure and crack length at failure. The load-deflection curve should also be a true representation of the actual load-deflection characteristics of the specimen after failure. This calls for a sensitive and accurate autographic recorder, able to respond to the transient effects following failure. Once again, fairly sophisticated methods of measuring deflections have been developed for the testing of metallic materials, but many of these methods are not applicable to non-metallic materials (eg. Electrical Potential Measurements). Only those methods suitable for fracture tests on brittle non-metals are mentioned below. Details of the various electrical transducers used in the test program are contained in Appendix D.

3.3.1 Measurement of Load

The easiest and most suitable method of measuring load is by means of an electronic load-cell. The cell is included between the test machine platen and the loading rig on the specimen, and under load produces an electrical output in the form of a voltage proportional to the load. The voltage output is then fed into a suitable autographic recorder.

3.3.2 Measurement of Deflection

Three methods are worth mentioning prior to the more detailed discussion of the method of deflection measurements used for the present tests. The first is the use of a staining fluid introduced into the notch or crack before starting the test. While not very suitable for use

in metals, this method has been tried and proven for non-metals. Glucklich¹³ used it to measure slow crack growth from a notch prior to failure, and the same technique was employed as a part of the current laboratory test program (see later Section 4.2.2). Providing care is taken in the selection of a suitable ink, and in the time allowed for penetration, the method is quite acceptable. The second method is the use of cinematography, whereby a frame by frame record of the plane surface of the notched specimen, from a ciné camera, is synchronised to, say, the load record for the test. Care needs to be given to the resolution of the crack, and lighting. After the test, the film is examined frame by frame and the crack length directly measured. This photographic method has been widely employed by the aircraft industry in tests on very wide thin-sheet specimens containing long through-thickness slots, but to date, no references to the use of this method in tests on non-metals or construction materials have been encountered. The third method measures the relative displacement of two points located symmetrically on opposite sides of the crack plane. A gauge which combines high sensitivity with linearity of output is a simple double cantilever beam gauge which is mounted on either side of the notch, and has electrical resistance strain gages bonded onto the cantilever arms and connected in a bridge circuit. Details of the gauge are given in Ref. 14.

For the present tests, the vertical displacement of the beam at the loading points was measured by means of two linear variable differential transformers (LVDT), mounted in a suitable frame in order to eliminate unwanted extraneous deflections. Details of the testing apparatus are given in Appendix D. These LVDT gauges combine a highly linear response to displacement with high sensitivity and adequate range, and hence are very suitable for displacement measurements on the type of materials examined for this dissertation. Outputs from the two gauges were electrically summed and then fed into the autographic recorder being used. It was absolutely essential to ensure that only valid deflections of the beams under the loading points were measured. Unwanted deflections such as local plastic crushing under the load platens or over the support points were eliminated by the use of suitable mounting rigs. A series of tests using load platens of 12.5 mm bright steel rod on 100 mm mortar cubes cut from beams and tested in direct compression were done, in order to quantitatively evaluate the effect of local crushing under the load

points. Virtually all the deflection of the load platens measured in these tests resulted from plastic crushing, since the load on the platens was less than two per cent of the crushing strength of the cubes. At the levels of load experienced in actual testing, it was found that crushing could account for up to 40 per cent of the total measured deflection of the beam, if the deflection was monitored through the load platens. The original series of beam tests had the displacement measured through the load platens, before it was appreciated that local crushing could have such a significant effect. The subsequent series of tests eliminated local crushing deflections. Some representative curves of the effect of local crushing under the load platens are shown in Fig. 3.4(a). From these curves, it is evident that very little elastic recovery occurs, showing that the major component of deflection in these tests was plastic crushing, and that neglecting to eliminate the unwanted deflections can lead to any or all of the following effects:-

- (1) A great variability in results, depending on the area of contact of the platen and the surface, and the density of the surface.
- (2) A local creep effect dependent on the time of loading.
- (3) The inevitability of the linear portion of the load-deflection curve from a beam test containing a hidden linear component of local crushing. Thus, the apparent load-deflection curve can differ substantially from the true one, as shown schematically in Fig. 3.4(b).

Hence, accurate gauges of adequate range, and careful attention to the details of measurement, are essential to record valid displacements.

3.3.3 Autographic Recorders

Two types of autographic recorder were used in the test program, each of which could accept a continuous output from load and displacement transducers. Details of the recorders and their performance are given in Appendix D.

- (1) Watanable X-Y Recorder, Load was monitored on the Y (vertical) axis, and displacement on the X (horizontal) axis. This recorder is most suitable for recording the initial ascending portion of a

Details of Test

Load Platens: 27mm Bright Steel Rod
 Mortar Cubes: 100 mm cube
 From Beam
 Deflection Measured by two LVDT's

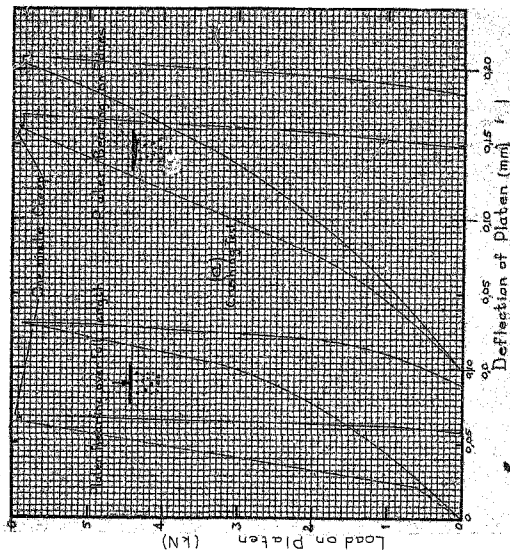
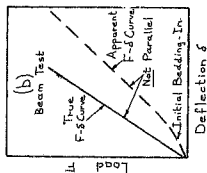


FIG.3.4 CURVES SHOWING EFFECT OF LOCAL CRUSHING UNDER LOAD PLATENS (100mm MORTAR CUBES)

load-deflection curve, but it cannot adequately follow the catastrophic crack growth portion after peak load. The particular recorder used had a frequency response of only about one Hz which was insufficient to monitor the transient effects of rapid crack propagation. Where the fracture was entirely stable, however, no problem was encountered in defining the full load-deflection curve. Also, this instrument has the great advantage of producing an immediately interpretable record. An example of how the response of the plotter differed from the true shape of the load-deflection curve after failure (i.e. the tail portion of the curve) is given in Fig. 3.5, showing that it is highly inaccurate to accept the record from such a plotter after failure in a catastrophic fracture specimen.

(2) N.E.P. Ultra-Violet Recorder (U.V.) This instrument was brought into use when it was realised that a definite tail portion existed for most of the load-deflection curves being recorded, and that the X-Y recorder could not adequately measure this tail portion if failure was catastrophic. Sensitivity and frequency response to transients were much higher on the U.V. recorder than on the X-Y recorder, in fact the increase of sensitivity to transients on the U.V. recorder over the X-Y recorder was twenty to twenty five times higher. The use of the U.V. recorder also allowed a simultaneous, autographic record of the output from any electrical resistance strain gauges cast into the specimen to be obtained.

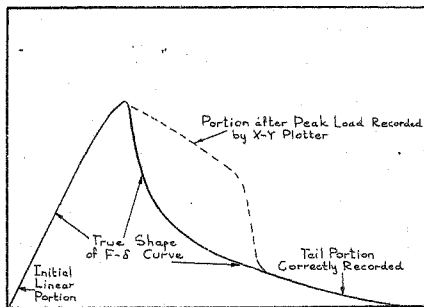


FIG.3.5 COMPARISON OF TRUE LOAD-DEFLECTION
CURVE AND APPARENT CURVE AFTER FAIL-
URE FROM X-Y RECORDER

CHAPTER 4

LABORATORY TEST PROGRAM AND RESULTS

4.1 Introduction

An extensive laboratory test program was initiated in order to study the variables affecting the fracture properties of cemented materials of construction, and to distinguish the characteristics of a valid fracture test on such materials. A total of approximately one hundred and twenty beam and slab specimens were tested in flexure in a variety of test machines and test set-ups, in order to investigate the various effects of strain rate, creep, testing machine stiffness and plane stress/plane strain. The majority of specimens tested were 101x101x610 mm beams (i.e. a nominal size of 4x4x24 inch). In addition, 101 mm cube tests were conducted as a control on beam strengths. The development of adequate instrumentation for the tests was a significant part of the work involved. Two types of cemented materials were studied:- the first was a sand-cement mortar using Rapid-Hardening Portland Cement and a sand residual from weathered granite; the second was a gap-graded asphalt mix to B.S.594. The mortar specimens were designed to have a compressive strength of 30 MPa at the time of test, which varied from five to seven days after casting. The asphalt beams were stored at room temperature in the laboratory for approximately sixty days before being tested. Details of the mix designs, specimen preparation and curing of the mortar beams are presented in Appendix C.

Six series of tests were conducted, and each series included both notched and unnotched bend specimens. The unnotched beams were used to evaluate unnotched compliances, and equivalent time-dependent Young's Moduli for the various test series. The other specimens contained cast-inches in the case of the cement mortar beams, and sawn notches in the case of the asphalt beams. Two types of notches were used:- either rectangular through-thickness notches of varying notch-depth ratios or 'vee' notches forming a chevron-shaped notch in the specimen, with the apex of the notch on the tension face. All the cast-notches were formed with a thirty degree root angle, produced by steel notch-

formers machined from 3 mm plate. Details of the notch-formers are given in Fig. 4.1.

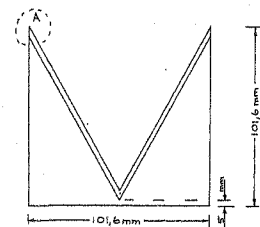
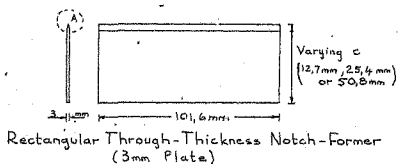
The output from each test was in the form of a load-deflection curve, produced autographically on either the Watanabe X-Y Recorder or the N.E.P. Ultra-Violet Recorder. These load-deflection curves were then analysed and the relevant fracture parameters evaluated. Some tests in Series 5 also monitored the longitudinal and transverse strains just above the notch root, by means of cast-in electrical resistance strain gauges mounted on thin copper I-Sections. In this case, the U.V. recorder was used to produce a simultaneous output of load, deflection and strain. The copper I-sections had provision for end anchorage into the specimen, and also provided a longer gauge length for the strain gauges. In addition, the rate of strain for the tests in Series 3 to 6 was measured and was expressed as a rate of deflection under the load points (i.e. $\mu\text{m/sec}$). In Series 4 and 5, the total machine load as well as the load on the specimen was measured. The ratio of the specimen load to the total machine load was called the 'load ratio', and this allowed a measure of the machine stiffness to be obtained.

The test program was designed with the object of arriving at some logical and uniform method of fracture testing of cemented materials, and this necessitated isolating the different variables affecting the fracture parameters. Hence, Series 1 and 2 comprised an initial evaluation of the fracture testing of cemented materials, and highlighted some of the important variables affecting the fracture properties, as well as the need for suitably designed and adequate instrumentation. Series 3 and 4 studied respectively the effects of controlled strain rate and testing machine stiffness, while Series 5 attempted a very brief, introductory evaluation of plane stress/plane strain effects. Finally, Series 6 comprised a short series of tests on asphalt beams as a comparison with the mortar beams used in Series 1 to 5. Details of all the apparatus used in the tests, including the autographic recorders, are given in Appendix D.

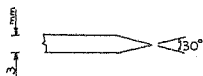
4.2 Laboratory Tests

4.2.1 Series 1 : Initial Study of Notched Beams

Series 1 was conducted as an initial study of notched mortar beams in flexure in order to distinguish the different variables of fracture



'Vee' - Notch-Former
(3mm Plate)



Detail (A): Notch-Former Apex (Enlarged)

FIG. 4.1 DETAILS OF RECTANGULAR AND
'VEE' NOTCH - FORMERS

testing. A total of twenty five beams were tested for Series 1. The subsequent Series 2 to 5 were then designed in order to isolate and study in greater detail those variables that were recognised as being significant from Series 1. It is important to stress here that Series 1 employed specimens whose deflections under the load points during testing were monitored through the load platens, and therefore contained unwanted extraneous deflection components due to localised plastic crushing of the specimen surface under the load platens. (See previous Sec. 3.3.2). Hence, the deflections are all overestimated and the results are unreliable to a greater or lesser extent. No satisfactory basis for converting the deflection measurements back to the true values for the deflections of the beam itself was available, and so the load-deflection curves from the tests were accepted and analysed without correction. However, the fracture parameters calculated from the tests cannot be accepted as valid, and the test results are presented in order to show the need for adequate and well-designed instrumentation measurements.

(1) Test Set-Up and Procedure

The test set-up is shown photographically in Fig.'s 4.2 and 4.3, and diagrammatically in Fig. 4.4, which also shows the specimen dimensions, i.e. nominal 101x101 mm mortar beams tested in four-point bending with a major span $S_1 = 610$ mm and a minor span $S_2 = 203$ mm (nominal). Note the deflections of the LVDT plungers which were measured through the load platens (Fig. 4.3). The rig holding the LVDT's is supported on the specimen over the support points, and this eliminates extraneous deflections due to crushing over the support points. The load is transferred to the specimen by means of 12,5 mm bright steel rod load platens mounted in a 203 mm span load spreader rig (See details in Appendix D).

The testing machine used for Series 1 was a Macklow-Smith hydraulic testing machine of max capacity 340 kN. The machine was capable of loading at varying rates. The load-cell and LVDT outputs from all the tests in Series 1 were fed into the Watanabe X-Y Recorder. It was found that this recorder could not adequately measure the crack growth after failure of those specimens that fractured catastrophically. Nevertheless, where possible the shapes of the load-deflection curves

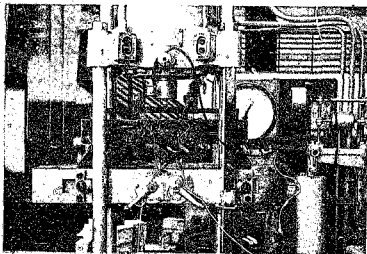


FIG.4.2 GENERAL TEST SET-UP, SERIES 1,
SHOWING MACKLOW-SMITH HYDRAULIC
MACHINE

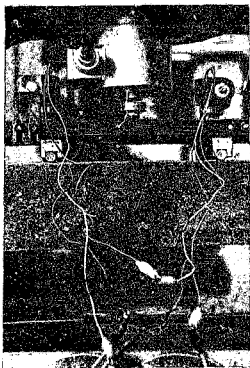


FIG.4.3 DETAILED TEST SET-UP, SERIES 1
(Note Deflections being Monitored Through the
Load Platens)

were inferred from later tests so that values of the effective fracture surface energy $\bar{\gamma}$ could still be obtained. Fig. 4.5 shows the X-Y recorder producing a load-deflection record during a test.

Although strain rates were not measured for Series 1, it is considered that the majority of the tests were 'fast' tests, i.e. the rate of load application was sufficient to cause the rate of deflection under the load points (hereafter called the 'strain-rate') to exceed $1.0 \mu\text{m/sec}$. (See discussion of this later in Series 3).

Ref. 23 p.14 contains specifications for fracture specimen dimensions. The first specification relates to the required dimensions in order to produce pure bending in the central minor span of the specimen. Four-point bending is considered to be equivalent to pure bending if the ratio of the minor span to specimen depth is not less than two. For the beams in Series 1, $\frac{S_2}{d} = \frac{203}{101} = 2$ and was therefore

satisfactory. The second specification relates to specimen indentation and friction at the supports. Substantial errors will not be introduced provided the ratio of the major, support span to specimen depth is greater than about four. For this case, $\frac{S_1}{d} = \frac{610}{101} = 6 > 4$ and was therefore

satisfactory.

(2) Test Results

The results from the tests for Series 1 are presented in Tables 4.1(a) and (b). Table 4.1(a) represents the results and the analysis of the load-deflection curves for those beams with no notches, or with rectangular cast-in notches. Notch depths were 12.7 mm, 25.4 mm, 38.1 mm (only one beam), and 50.8 mm. Table 4.1(b) shows the results for the 'vee'-notch beams. Columns 1 to 6 of Table 4.1(a) represent the results directly from the load-deflection curves of the beams, while columns 7, 8 and 9 represent the analysis of the results in order to arrive at the notch-depth ratio at failure, c_c/d , and hence the notch depth at failure c_c , as well as the value of $f(c/d)$ from Fig. 3.2 to calculate G_c . The method of finding the notch-depth ratio at failure makes use of the change in compliance ratio $\Delta L/L_o$ at failure, and is explained fully in Appendix B. L_o is the average of the unnotched compliances for the prevailing strain rate which is assumed to have been 'fast' (see Fig. 3.3). Column 10 shows the values of

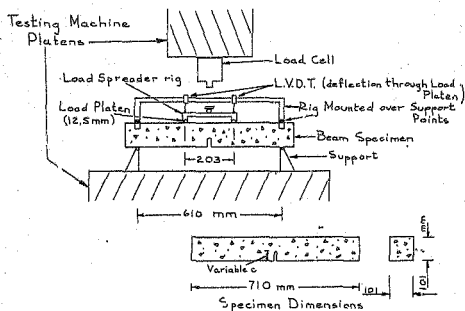


FIG. 4.4 TEST SET-UP FOR SERIES 1
(DIAGRAMMATIC)

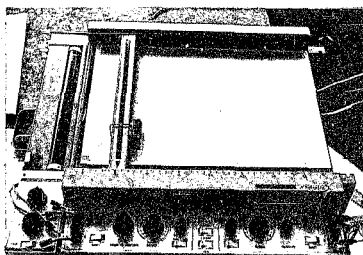


FIG. 4.5 X-Y RECORDER PRODUCING A TEST OUTPUT

Table 4.1 (c) Rectangular Notches : Results for Series 1: Initial Tests

Loading System Specimen No. (Cube Str. MPa)	1	2	3	4	5	6	7	8	9	10
	Initial Notch Depth (mm)	Strain Rate	Max. Load at Failure (kN)	Initial Compliance L_i ($\frac{\text{mm}^2}{\text{GN}}$) (E: 68 GPa)	Compliance at Failure L_f ($\frac{\text{mm}^2}{\text{GN}}$)	$\Delta L = L_f - L_i$ ($\frac{\text{mm}^2}{\text{GN}}$)	Notch Depth Ratio at Fail. $\frac{c_c}{d}$	Notch Depth From Fracture at Fail. c_c (mm)	$f(c/d)$	Total Fracture Energy $A_{2.5}$ (Nm)
1.1 (29.53)	No Notch		7,337	3,264 (E=12,281)	3,434	0,170	0,125	12,70	0,268	0,945
1.2 (31.39)			6,359	3,261 (E=12,292)	3,493	0,232	0,149	15,14	0,300	0,779
1.3 (31.39)			6,069	3,544 (E=11,31)	3,690	0,146	0,118	11,99	0,257	0,679
1.4 (28.77)			4,530	2,868	3,521	0,653	0,277	28,14	0,424	0,442
1.5 (28.77)	12,7 mm Notch		4,329	4,455	4,696	0,241	0,198	20,12	0,358	0,533
1.6 (28.77)			4,426	3,976	4,293	0,317	0,216	21,95	0,375	0,984
1.7 (29.55)			4,324	3,448	3,994	0,546	0,259	26,31	0,412	0,649
1.8 (31.39)			3,619	3,930	4,396	0,466	0,243	24,69	0,399	-
1.9 (31.39)			3,909	4,349	-	-	-	-	-	-

Table 4.1(d) (Cont'd)

	1	2	3	4	5	6	7	8	9	10
1.10 (33.97)	25.4 mm Notch	Fast	3.560	5.126	5.822	0.696	0.345	35.05	0.464	0.739
1.11 (33.97)			3.440	4.711	5.257	0.546	0.327	33.22	0.454	0.660
1.12 (33.97)			3.360	4.292	4.989	0.697	0.346	35.15	0.465	0.556
1.13 (28.54)			2.742	5.079	5.410	0.331	0.300	30.48	0.440	0.498
1.14 (29.55)			2.778	5.291	5.961	0.670	0.342	34.75	0.463	0.601
1.15 (31.39)			2.688	4.509	4.849	0.340	0.301	30.58	0.441	0.419
1.16 (28.77)			1.508	4.653	8.826	4.173	0.626	63.6c	0.521	0.300
1.17 (28.77)			1.508	6.768	9.567	2.799	0.608	61.77	0.521	0.193
1.18 (29.55)			1.331	12.247	14.282	2.035	0.589	59.84	0.520	0.299
1.25 (29.55)			2.387	5.776	-	-	-	-	-	0.464
	38.1									

'Fast' Strain Rate $\geq 1.0\text{mm/sec}$ 'Slow' Strain Rate $< 1.0\text{mm/sec}$

Note Figures in all Tables should not be regarded as being significant beyond the first decimal place.
See general note on numerical significance at end of Nomenclature

Table 4.1(b) 'Vee' Notches : Results for Series 1:

Initial Tests

		1	2	3
Loading System Specimen No. (Cube Str. MPa)		Strain Rate	Max. Load at Failure (kN)	Total Fracture Energy $A_{F,\delta}$ (Nm)
UNSTIFFENED MACKLOW-SMITH	1.19 (28,54)	'E A S I'	1,038	0,180
	1.20 (28,54)		1,146	0,099
	1.21 (28,54)		1,647	0,321
	1.22 (28,54)		1,082	0,225
	1.23 (29,55)		2,084	0,292
	1.24 (29,55)		-	0,250

'Fast' Strain Rate $\geq 1,0 \mu\text{m/sec}$ 'Slow' Strain Rate $< 1,0 \mu\text{m/sec}$ Note 'Vee'-Notch Beams for Series 1 were tested on a610 mm span

total fracture energy $A_{F,q}$ for each specimen, i.e., the energy required to produce fracture, represented by the area under the load-deflection curve for the specimen.

The average flexural Young's Modulus for the unnotched beams of Series 1 was 12,0 GPa, and the average unnotched compliance was 3,4 m^2/GN . (m^2/GN is dimensionally equivalent to $(GPa)^{-1}$) These values, as with all other values involving deflection, are 'apparent' values because of the extraneous deflection components included in them. Initially, half-beams from fracture tests on beams of 610 mm span were tested on a 305 mm span with centre-point loading, but the results from these tests were so erratic as to be unusable and are therefore not presented here.

Using the test results of Table 4.1, the fracture parameters G_c and K_c and $\bar{\gamma}$ were calculated using the procedure outlined in section 3.2.4, and are presented in Table 4.2. An example of a detailed calculation is given in Appendix B.

Summary curves of the results and the fracture parameter $\bar{\gamma}$ are presented in Fig.'s 4.6 and 4.7. Characteristic load-deflection curves of the beams with various notch depths are not presented here because of the inaccurate deflection measurements which produced a greater inherent variability and unreliability in comparison to those beams in later test series that had valid deflection measurements. A comparison between 'valid' and 'invalid' load-deflection curves is shown schematically in Fig. 3.4(b). For the same reason, a thorough discussion of the results and the fracture parameters they yield is not presented here. Series 3 and 4 contain a more detailed study of the effect of different variables on the fracture parameters. Nevertheless, two points are worthy of note here. The first is that Fig. 4.6 shows that while the experimental L/l_o versus c/d might be displaced laterally from the theoretical line (which is reproduced from Fig. 3.3, $\nu=0.1$), the slopes of the two curves are very similar. This allows the theoretical curve to be used in order to find the critical crack length at failure, c_c , since it is the change in compliance at failure, Δl , produced by slow stable crack growth from the initial known notch depth, that is used to find c_c . Secondly, Fig. 4.7 shows that the value of $\bar{\gamma}$ can vary depending on the notch depth used to calculate it. This is an effect introduced by the type of fracture present in the

Table 4.2 Fracture Parameters for Series 1:Initial Tests

		1	2	3	4	5
Loading System Specimen No. (Cube Str. MPa)		Strain Rate	Initial Notch Depth (mm)	G_c (N/m)	K_c ($MN/m^{1/5}$)	$\bar{\gamma}$ (N/m)
UNSTIFFENED MACKLOW-SMITH	1.1 (29,55)	P A S T	No Notch	61,2	0,89	45,8
	1.2 (31,39)			55,9	0,84	37,7
	1.3 (31,39)			38,9	0,71	32,9
	1.4 (28,77)		12,7 mm Notch	65,4	0,86	24,5
	1.5 (28,77)			37,0	0,67	29,5
	1.6 (28,77)			43,3	0,72	54,5
	1.7 (29,55)			53,8	0,79	35,9
	1.8 (31,39)			34,2	0,63	-
	1.10 (33,97)		25,4 mm Notch	59,5	0,81	47,7
	1.11 (33,97)			50,1	0,74	42,6
	1.12 (33,97)			53,3	0,76	35,9
	1.13 (28,54)			27,4	0,55	32,2
	1.14 (29,55)			35,6	0,62	38,8
	1.15 (31,39)			26,5	0,54	27,1

Table 4.2 (Cont'd)

		1	2	3	4	5
UNSTIFFENED MACKLOW-SMITH	1.16 (28,77)	'P A S T'	50,8 mm Notch	64,4	0,81	29,1
	1.17 (28,77)			55,9	0,75	18,7
	1.18 (29,55)			37,7	0,62	29,0
	1.19 (28,54)		'Vee' - Notch	-	-	21,2
	1.20 (28,54)			-	-	11,7
	1.21 (28,54)			-	-	37,8
	1.22 (28,54)			-	-	26,5
	1.23 (29,55)			-	-	34,4
	1.24 (29,55)			-	-	29,5
	1.25 (29,55)			-	-	36,0
			38,1	-	-	

'Fast' Strain Rate $\geq 1,0 \mu\text{m/sec}$ 'Slow' Strain Rate $< 1,0 \mu\text{m/sec}$

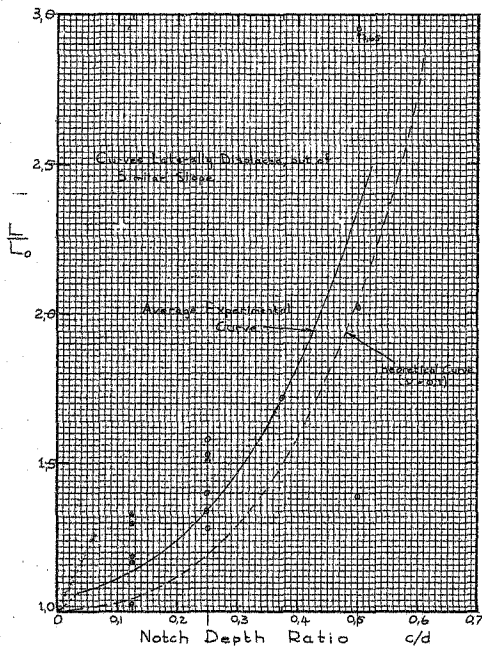


FIG. 4.6 THEORETICAL AND EXPERIMENTAL COMPLIANCE - NOTCH DEPTH CURVE FOR SERIES 1

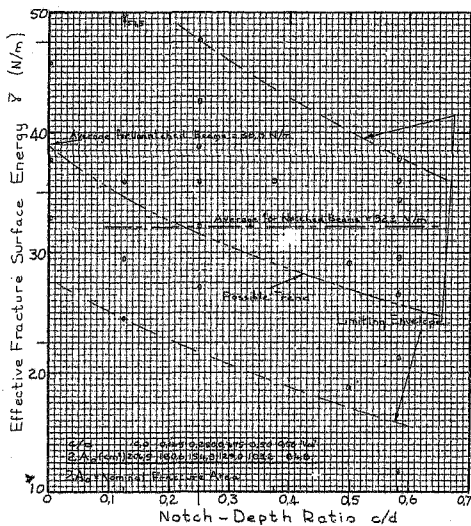


FIG. 4.7 γ VERSUS c/d CURVE FOR SERIES 1

specimen, i.e. whether the fracture is stable or catastrophic (see Sec. 2.4), and is discussed more fully later in Sec. 4.2.4(2)(c). Fig. 4.7 also tabulates the nominal fracture area ($2A_0$) for the different c/d ratios. This allows $\bar{\gamma}$ for 'vee'-notches to be included in Fig. 4.7, by assuming an effective rectangular notch-depth ratio for 'vee'-notches of 0,582, i.e. the notch-depth ratio corresponding to the nominal fracture area of the 'vee'-notch.

For Series 1, the average values of the fracture parameters are:-

$$\begin{aligned} G_c &= 4,1 \text{ N/m} \\ K_c &= 0,72 \text{ MN/m}^{3/2} \\ \bar{\gamma} &= 32,7 \text{ N/m for all the beams} \\ \bar{\gamma} &= 26,4 \text{ N/m for the 50,8 mm and 'vee'-notch beams.} \end{aligned}$$

In comparison to the valid fracture parameter values presented later, G_c for Series 1 is about 50 per cent higher than for later series, but K_c and $\bar{\gamma}$ are closely comparable.

4.2.2 Series 2 : Tests on Hand-Jack Set-Up

(1) Test Set-Up and Procedure

Series 2 was originally designed to provide a test set-up that would give a greater degree of control over the deflection of the beam specimen during loading. A mechanical screw-jack which was hand operated was mounted on a loading frame and used to load the specimens from below. It was hoped that the use of this system would allow the deflection being imparted to the specimen by the jack to be halted at the critical point of unstable crack propagation, and thereby produce a more stable type of fracture. The critical effect of loading machine stiffness, which governs the strain energy stored in the machine system, had not been fully appreciated at this stage of the test programme. The result was that the hand-jack set-up never produced more stable fractures in the tests on notched and unnotched beams than the other systems with less control over the specimen deflection. This was firstly because the strain energy stored in the loading frame and jack at the critical point of failure instability was sufficiently large to cause catastrophic failure in all except the 50,8 mm notched beams, and secondly because the Watanabe X-Y Recorder was used to monitor the

tests, and as explained previously, this recorder was unable to respond sufficiently to transient effects at failure. It is important to note that other investigators (eg. Glucklich¹³) have failed to record tail portions to their load-deflection curves after the critical peak load, and the reason is very possibly that the strain energy of the various test set-ups was being fed back into the specimen during failure, preventing a stable type of fracture being produced. (See Sec. 2.4) Undoubtedly, Series 2 of the present tests encountered this problem.

A total of fourteen beam specimens 101x101x610 mm (nominal) were tested with varying notch depths in the hand-jack set-up, using four-point loading. The test set-up is shown photographically in Fig's 4.8 and 4.9. The major and minor spans of the beams as well as the basic testing apparatus were all as in Series 1, and as presented in detail in Appendix D. The deflections of the beam above the loading points were measured by the LVDT's mounted in a rig above the specimens. The plungers of the LVDT's were secured to the top surface of the specimens (i.e. the opposite face to the loading face) and were therefore unaffected by any localised crushing under the load platens. However, some extraneous effects might have been introduced by local crushing under the support points, since the LVDT rig rested over the support points.

(2) Ink-Staining Technique

Series 2 also employed an ink-staining technique in order to attempt to produce a more accurate compliance-notch depth relationship. Since the beams were loaded from below, the notches were on the top faces of the beams and this allowed the notches to be used as convenient troughs for the ink during the staining process. The great variability in results as well as the fact that it was virtually impossible to halt the crack in the beam after reaching the critical peak load (except for 50.8 mm notched beams) because of the 'soft' loading set-up made the results from the staining process unrepresentative, and they were therefore not used. Nevertheless, the procedure of the method is outlined below for the sake of completeness.

A notched beam is carefully loaded so as to avoid unstable or catastrophic crack growth in the beam. The object is to produce an extension of the crack front from the notch without total failure occurring, and then to allow a suitable ink to penetrate the extent of

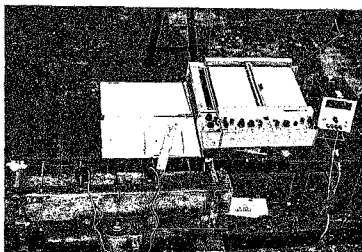


Fig.4.8 GENERAL TEST SET-UP, SERIES 2
(Note Specimen being Loaded from Below)

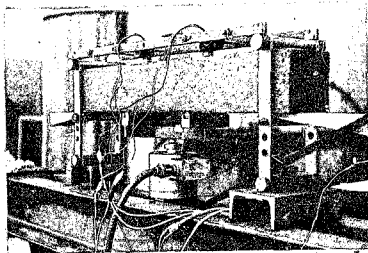


Fig.4.9 DETAILED TEST SET-UP, SERIES 2
(Hand-Jack is under Load Cell, obscured
by Loading Frame)

this slow crack growth, while the beam is kept under load. Thereafter, the remaining ink in the notch-trough is sucked out, and the beam is unloaded. The beam is then re-loaded to failure, and the prevailing initial compliance is noted. This compliance is then attributed to the notch depth as measured after failure by the extent of the ink penetration. By using this method, a number of discrete points can be plotted in order to produce a comprehensive compliance-notch depth curve. The method, however, was not employed successfully for Series 2, and subsequent series showed that it was acceptable to use the theoretical compliance-notch depth curve of Fig. 3.3. The work of Welch and Haisman¹⁵ has also shown that the prevailing initial compliance of a beam is not affected by the type of notch in the beam, and it is therefore possible to produce an experimental compliance-notch depth curve merely by plotting the initial compliances for the specimens versus their nominal initial notch depths. (See Fig.'s 4.6 and 4.17)

Fig. 4.10 shows a beam during the ink-staining process. A period of approximately twenty minutes was allowed for the ink to penetrate the newly formed crack while the beam was kept under load. Green India ink appeared to be the most suitable ink for staining, since other inks did not penetrate the crack sufficiently due to their higher viscosities.

(3) Test Results

The benefit of Series 2 was that it clearly indicated that strain rate and creep effects have a very significant effect in fracture tests on cemented construction materials. It was impossible using the hand-jack to apply a uniform strain rate during a test, nor was it possible to quantify the difference in strain rates between different tests. The excessive variability and scatter of the results of Series 2 made it virtually impossible to satisfactorily analyse the tests and evaluate the fracture parameters, and for this reason, the only results presented here are characteristic load-deflection curves from the tests, shown in Fig.'s 4.11 to 4.13. Although many of the load-deflection curves were uninterpretable in terms of the fracture parameters, they did indicate the way in which initial compliances, failure loads and areas under the load-deflection curves varied with varying strain rate. Apparent Young's Moduli as low as 4 to 5 GPa were produced in those beams in which excessive creep during rust periods occurred. A summary of the

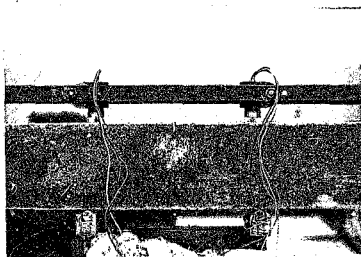


FIG. 4.10 BEAM DURING INK-STAINING PROCESS
(A Wax Plug Keeps the Ink in the Trough)

main characteristics of these curves is presented here:-

Fig. 4.11 This figure shows the load-deflection curves of two unnotched beams, the one (curve A) using an approximately constant rate of load application, and the other (curve B) allowing rest periods during the load application. Each rest period was approximately one minute. The apparent flexural Young's Modulus, taken from the most nearly straight portion of each curve, was 6,5 GPa for A, and 4,6 GPa for B. In the case of A, the maximum load was approached with approximately constant strain rate, but for B, the value of the load represented by point M was reached, and a rest period to allow creep deflections to occur was commenced. After a short period (less than thirty seconds) of stable creep deflection, the creep rate accelerated to spontaneous failure without further load application. No tail portion to the load-deflection curve for either of the beams could be recorded.

Fig. 4.12 is included in order to show the large effects of creep on beam deflections. Beam A was loaded with approximately constant strain rate, and when the load application was stopped at point M, first stable and then accelerating creep occurred until final failure. Beam B was loaded with a slow strain rate and one minute rest periods at intervals. Beam B also failed due to accelerating creep during a rest period. The failure compliance of beam A was $4,7 \text{ m}^2/\text{GN}$, while that of B was $10,7 \text{ m}^2/\text{GN}$. Once again, no tail portions to the curves were recorded, although tests in Series 4 have shown that a tail portion does exist for 12,7 mm notched mortar beams. The increase in compliance of the beams during loading seems to be almost totally attributable to creep rather than slow crack growth.

Fig. 4.13 is the load-deflection curve for a 50,8 mm notched beam which employed the ink-staining technique. The point of maximum load was carefully approached, and the beam was then left under load for about twenty minutes after ink was introduced into the notch. During this period, the compliance of the beam increased due to slow crack growth produced by the stored elastic energy in the beam, as well as creep. After twenty minutes, the remaining ink was sucked out, and the beam unloaded. It was then re-loaded to failure, and examination of the beam showed ink penetration of 17 to 18 mm from the 50,8 mm notch, i.e. a total effective crack depth of 68 to 69 mm. The re-loading compliance was $18,0 \text{ m}^2/\text{GN}$, while the

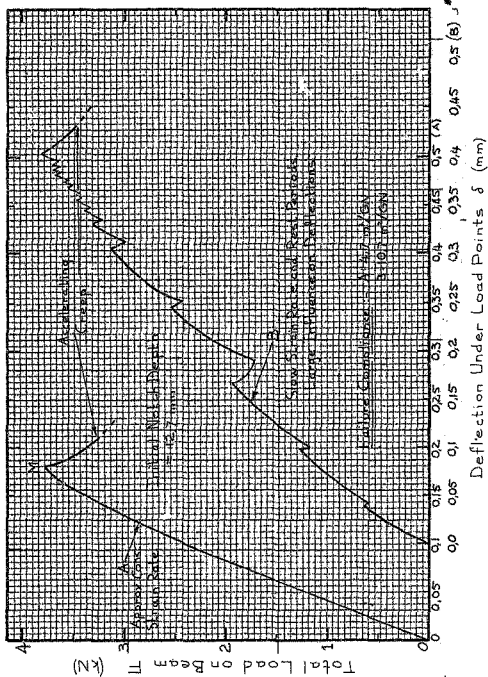


FIG. 4.12 LOAD-DEFLECTION CURVES FOR 12.7mm
NOTCHED BEAMS, SERIES 2

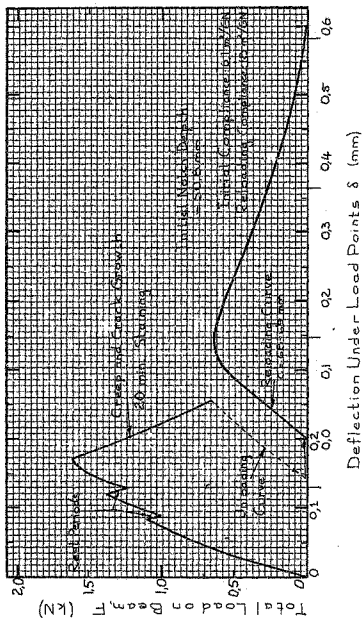


FIG. 4.13 LOAD-DEFLECTION CURVE FOR A 50.8 mm
NOTCHED BEAM EMPLOYING THE INK -
STAINING TECHNIQUE, SERIES 2

initial compliance was $6.1 \text{ m}^2/\text{GN}$.

Hence, Series 1 and 2 immediately indicated that both rate of load application during a test, and stiffness of the testing machine set-up, were important factors in fracture tests on cemented construction materials. In order to quantify these two effects more accurately, Series 3 and 4 of the laboratory test programme were designed, and the details and results are presented below.

4.2.3 Series 3 : Controlled Strain-Rate Tests

Series 3 of the laboratory tests were designed in order to isolate and study in greater detail the effect of the rate of strain applied to a specimen during a fracture test, on parameters such as the failure load, the apparent Young's Moduli and compliances, and the fracture parameters. A total of twenty eight beams were tested, and the ratio of strain rates selected for the tests spanned a range of approximately 3 000, some tests running for four to six hours, and others taking only a few seconds. All the deflection measurements in these tests, as well as in the subsequent Series 4 to 6, were monitored using a redesigned deflection rig, which was directly fixed to the specimen at its nominal neutral axis, and thus measured only the true elastic deflection of the beam under load. The results presented in this and subsequent sections can probably be regarded as valid estimates of the fracture parameters of the materials used in the laboratory tests.

(1) Test Set-Up and Procedure

Series 3 was conducted on a triaxial testing machine (manufactured by Leonard Farnell). This machine, shown photographically in Fig. 4.14, allowed controlled rates of deflection of the beams under the load points to be chosen by means of a gear-selector system. The machine had a capacity of 100 kN. Fig. 4.15 shows the detailed set-up for Series 3, while Fig. 4.16 shows details of the improved deflection measurement rig and set-up used for Series 3 to 6. Appendix D contains full diagrammatic details of the deflection rig. As usual, load was measured by an electronic load cell, and deflection by LVDT's suitably mounted in the deflection rig, and the outputs were fed into the Watanabe X-Y Recorder, which was used to produce autographic recordings of all the tests in Series 3. The specimen dimensions and

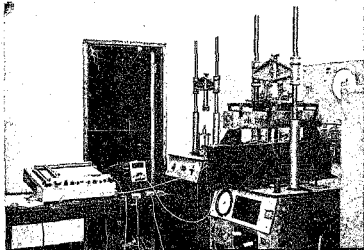


FIG. 4.14 GENERAL TEST SET-UP, SERIES 53

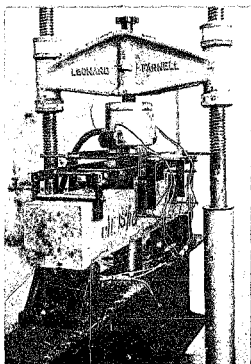


FIG. 4.15 DETAILED TEST SET-UP, SERIES 3

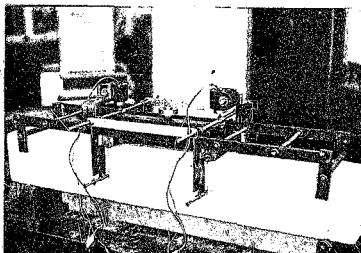


FIG. 4.16 DETAILS OF RE-DESIGNED DEFLECTION
MEASUREMENT SET-UP

method of loading were identical to Series 1, and are as set out in Sec. 4.2.1(1). The specifications for specimen dimensions as set out in that section also fully applied to the beams in Series 3 with the exception of those beams which contained 'vee'-notches. Tests on 610 mm span 'vee'-notch beams in Series 1 indicated that handling of such beams without damage was extremely difficult, due to the high stress concentration at the apex of the notch. For this reason, 'vee'-notch beams in Series 3 were cast in 350 mm lengths with a central notch, and tested on a 305 mm major span with a minor loading span of 101 mm (nominal). Although such beams did not meet up with the specifications of Ref. 23, nevertheless the only parameter being sought from such tests was $\bar{\eta}$, and the total load required to fail the specimen was small (less than 3 kN) due to the notch shape. Provided the test output indicated a stable-type fracture, the test record was accepted. However, later analysis indicated the need to adhere to the specifications of Ref. 23.

(2) Test Results

The results from the tests for Series 3 are presented in Table 4.3(a) and (b). Table 4.3(a) shows the results and analysis of the load-deflection curves for those beams with no notches, or with rectangular cast-in notches. Notch depths used for Series 3 were 25,4 mm and 50,8 mm. Values of the total fracture energy $A_{F,\delta}$ are included in Table 4.3(a) only where the true shape of the load-deflection curve could be either accurately measured by the recorder, or inferred from the test record. The triaxial test set-up was a relatively 'soft' loading system, and the X-Y recorder was therefore unable to record the tail portions of many of the curves. Where the true shape of the load-deflection curves could not be inferred, the value of $A_{F,\delta}$ was not evaluated. The method of finding notch-depth ratios at failure for the rectangular notched beams is that set out in Appendix B. Table 4.3(b) shows the results for tests on the 305 mm span 'vee'-notch beams. Note from Table 4.3(a) that as the initial notch-depth ratio increases, so too does the slow stable crack growth from the notch before the point of instability represented by the peak load is reached. This is shown by the change in compliance ΔL , which increases as the c/d ratio increases. Completely stable fractures were obtained for 50,8 mm notches, while unnotched beams exhibited catastrophic failure.

Table 4.3 (a) Rectangular Notches : Results for Series 3: Controlled Strain Rate Tests.

Loading System Specimen No. (Cube Str.)	1 Initial Notch Depth (mm)	2 Strain Rate (mm/sec)	3 Max. Load at Failure (kN)	4 Initial Compliance I_i (m^2/GN) (E=69 GPa)	5 Compliance at Failure I_f (m^2/GN)	6 $\Delta L = L_f - L_i$ (mm)	7 Notch Depth Ratio at Fail. $\frac{c_c}{d}$	8 Notch Depth at Fail. c_c (mm)	9 $f(c/a)$ From Fig. 3.2	10 Total Fracture Energy $A_F \cdot \delta$ (Nm)
3.1 (31,77)		15,7	7,497	2,030 (E=19,741)	2,358	0,328	0,230	23,37	0,388	0,762
3.2 (31,77)		9,2	8,143	2,241 (E=17,883)	2,538	0,297	0,220	22,35	0,397	0,945
3.3 (31,77)		1,7	7,574	2,096 (E=19,126)	2,407	0,311	0,223	22,66	0,383	0,813
3.4 (31,77)		0,18	5,816	2,490 (E=16,099)	2,774	0,284	0,198	20,12	0,358	0,562
3.5 (31,77)		0,17	7,135	2,412 (E=16,616)	2,741	0,328	0,211	21,44	0,370	0,793
3.6 (31,77)		0,10	7,135	2,329 (E=17,207)	2,617	0,288	0,194	19,71	0,354	0,791
3.7 (26,54)		0,0083	5,248	3,864 (E=10,372)	-	-	-	-	-	0,560
3.8 (31,19)		0,0078	5,936	2,844 (E=14,091)	3,023	0,179	0,130	13,21	0,275	0,625

TRIAXIAL MACHINE

Table 4.3 (a) (Cont'd)

	1	2	3	4	5	6	7	8	9	10		
TRIAXIAL MACHINE	3.9 (29,52)	FAST	25,4 mm Notch	15,2	2,904	3,005	4,338	1,333	0,462	46,94	0,502	-
	3.10 (31,77)			13,3	3,474	2,945	3,691	0,746	0,395	40,13	0,483	-
	3.11 (29,52)			1,6	2,853	2,580	3,486	0,906	0,415	42,16	0,489	-
	3.12 (29,52)			0,45	2,853	3,009	3,835	0,826	0,396	40,23	0,484	-
	3.13 (29,52)	SLOW	25,4 mm Notch	0,16	2,615	2,230	3,380	1,150	0,422	42,86	0,492	-
	3.14 (29,52)			0,037	2,497	3,503	4,557	1,054	0,400	40,64	0,485	-
	3.15 (31,19)			0,0082	2,670	3,541	4,262	0,721	0,349	35,46	0,465	0,516
	3.16 (30,13)			0,0052	2,446	3,385	4,083	0,698	0,342	34,75	0,462	-
	3.17 (29,52)	FAST	50.8	L ₁ 7,8								
				6,4 48,5	1,596	4,450	6,576	2,126	0,620	62,99	0,521	0,296

Table 4.3(a) (Cont'd)

TRIAL MACHINE	1	2	3	4	5	6	7	8	9	10
3.18 (29.52)	50.8 mm Notch	FAST $\dot{\epsilon}_F$	1,287	6,717	7,981	1,264	0,591	60,03	0,520	0,243
3.19 (29.87)			90,038	1,253	5,073	6,617	1,544	0,601	61,06	0,521
3.20 (29.87)		SLOW $\dot{\epsilon}_S$	55,35	1,131	5,634	7,815	2,181	0,616	62,59	0,521
3.21 (29.87)			35,0035	1,223	5,354	7,228	1,874	0,608	61,77	0,521
3.22 (31.48)		$\dot{\epsilon}_F$	35,21.5	1,172	5,676	7,143	1,467	0,593	60,25	0,520
3.23 (31.48)			072,51	1,162	4,403	7,318	2,915	0,622	63,20	0,521

'Fast' Strain Rate $\geq 1,0\text{mm/sec}$ 'Slow' Strain Rate $< 1,0\text{mm/sec}$

Note The Strain Rate for $\dot{\epsilon}_F$ is taken as the total measured deflection during the test divided by the total time for the test.

Table 4.3(b) 'Vee' Notches : Results for Series 3:

Controlled Strain Rate Tests

		1	2	3	
Loading System Specimen No., (Cube Str. MPa)		Strain Rate ($\mu\text{m/sec}$)	Max. Load at Failure (kN)	Total Fracture Energy $A_{F,6}$ (Nm)	
T R I A X I A L M A C H I N E	3.24 (32,48)	'FAST'	8,3	2,303	0,410
	3.25 (29,87)		4,6	2,629	0,415
	3.26 (31,48)		2,7	2,018	0,292
	3.27 (29,87)	'SLOW'	0,21	2,364	0,291
	3.28 (29,87)		0,19	2,232	0,377

'Fast' Strain Rate $\geq 1,0 \mu\text{m/sec}$ 'Slow' Strain Rate $< 1,0 \mu\text{m/sec}$ All beams tested on 305 mm span

Semi-stable fractures occurred for intermediate notch depths. It will be shown later that the unnotched beams used in the present tests could only fail catastrophically, from a consideration of the effective fracture surface energy $\bar{\gamma}$ of the material. (See later Sec. 4.2.4(2)(c) and Fig. 4.29).

Table 4.4 sets out the fracture parameters evaluated for the beams. Note that the beam reference numbers are consistent in Tables 4.3(a) and (b) and Table 4.4, i.e. beam 3.1 in Table 4.3(a) is the same as beam 3.1 in Table 4.4.

Summary curves of the results and the fracture parameters are presented in Fig.'s 4.17 to 4.22. Discussion of these results relevant to this chapter is presented here:-

Fig. 4.17 shows the experimentally derived L/L_0 versus c/d curve for all those specimens in Series 3 and 4 that were tested at a 'fast' strain rate, i.e. $> 1.0 \mu\text{m/sec}$ (see later Fig. 4.18). The figure also shows the theoretical curve for $\nu=0.1$ which was used in the analysis of the fracture parameters, reproduced from Fig. 3.3. Once again it is obvious that while the experimental curve is displaced laterally from the theoretical line, the slopes of the two curves are very similar. It is therefore quite acceptable to use the theoretical curve as a uniform basis for evaluating the fracture parameters.

Fig. 4.18 shows a plot of both average Young's Modulus E and average initial unnotched compliance L_0 versus strain-rate, for the unnotched beam specimens of Series 3. The results for the unnotched beams of Series 4 were also included in order to evaluate the average E and L_0 for 'fast' strain rates. The envelope showing the scatter of results is also shown. The L_0 curve is plotted as the reciprocal of the average E curve, since a direct relationship exists between L_0 and E for an unnotched beam. Both E and L_0 are derived from the slope, P/δ , of the initial straight line portion of the load-deflection curve. The relationship between L_0 and E is:-

$$L_0 = \left(\frac{5}{324} \frac{S_1^3 b}{I} \right) \left(\frac{1}{E} \right) \quad (4.1)$$

where S_1 = the major span (three times the minor span) and I = the second moment of area of the beam.

It is seen that the curve levels out and approaches a constant value

Table 4.4 Fracture Parameters for Series 3:
Controlled Strain Rate Tests

		1		2		3		4		5	
Loading System Specimen No. (Cube Str. MPa)		Strain Rate ($\mu\text{m}/\text{sec}$)		Initial Notch Depth (mm)		G_c (N/m)		K_c ($\text{MN}/\text{m}^{3/2}$, 5)		$\bar{\gamma}$ (N/m)	
TRIAXIAL MACHINE	3.1 (31,77)	'FAST'	15,7	No Notch		81,3		1,26		36,9	
	3.2 (31,77)		9,2			90,1		1,33		45,8	
	3.3 (31,77)		1,7			79,7		1,25		39,4	
	3.4 (31,77)	'SLOW'	0,18			47,1		0,90		27,2	
	3.5 (31,77)		0,17			77,3		1,14		38,4	
	3.6 (31,77)		0,10			72,4		1,09		38,3	
	3.7 (26,54)		0,0083			-		-		27,1	
	3.8 (31,19)		0,0078			41,2		0,73		30,3	
	3.9 (29,52)	'FAST'	15,2	25,4 mm Notch		46,3		0,90		-	
	3.10 (31,77)		13,3			44,8		0,90		-	
	3.11 (29,52)		1,6			33,8		0,76		-	
	3.12 (29,52)	'SLOW'	0,45			33,5		0,74		-	
	3.1 (29,52)		0,16			35,2		0,72		-	
	3.14 (29,52)		0,037			32,3		0,65		-	
	3.15 (31,19)		0,0082			33,4		0,61		33,3	
	3.16 (30,13)		0,0052			28,9		0,55		-	

Table 4.4 (Cont'd)

TRIAXIAL MACHINE		1		2	3	4	5	
		'FAST'	G_c, K_c	$\bar{\gamma}$				
	3.17 (29,52)	'FAST'	6,4	48,5	50,8 mm Notch	41,2	0,84	28,7
	3.18 (29,52)		1,2	8,7		22,4	0,62	23,7
	3.19 (29,87)		,90	038		23,1	0,61	19,6
	3.20 (29,87)		,53	,35		21,8	0,58	18,0
	3.21 (29,87)	'SLOW'	,35	0035		24,7	0,61	19,0
	3.22 (31,48)		,35	1,5		20,2	0,56	18,0
	3.23 (31,48)		,072	,51		28,2	0,61	17,7
	3.24 (31,48)		8,3			-	-	48,3
	3.25 (29,87)	'FAST'	4,6		'Vee'-Notch	-	-	48,9
	3.26 (31,48)		2,7			-	-	34,4
	3.27 (29,87)	'SLOW'	0,21			-	-	34,3
	3.28 (29,87)		0,19			-	-	44,4

'Fast' Strain Rate $\geq 1,0 \mu\text{m/sec}$ 'Slow' Strain Rate $< 1,0 \mu\text{m/sec}$

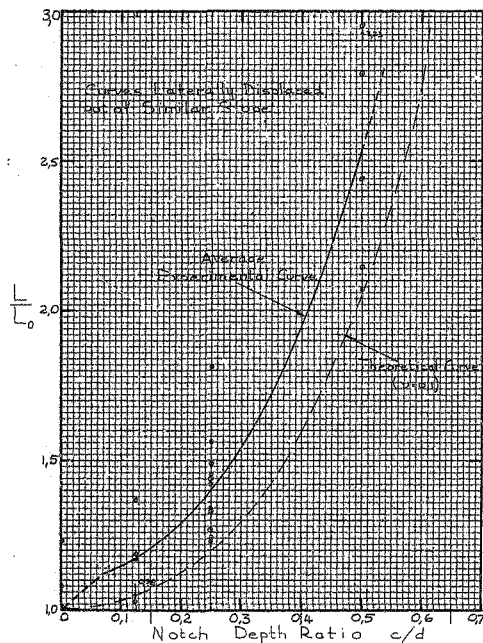


FIG. 4.17 THEORETICAL AND EXPERIMENTAL COMPLIANCE NOTCH DEPTH CURVE FOR 'FAST' TESTS OF SERIES 3 AND 4

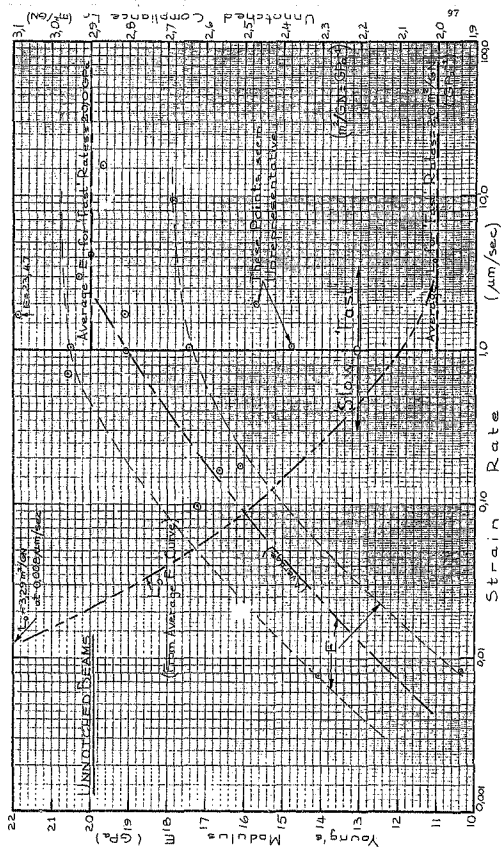


FIG. 4.18 E AND L_0 VERSUS STRAIN RATE, SERIES 3 & 4

of E for strain rates greater than about 1,0 $\mu\text{m/sec}$. This is the basis for defining a 'fast' strain rate as being a strain rate greater than 1,0 $\mu\text{m/sec}$, and all tests with a strain rate greater than 1,0 $\mu\text{m/sec}$ have had the fracture parameters evaluated using the average values of E and L_0 derived from 'fast' tests on unnotched beams. These average values are:-

$$\begin{aligned} E &= 20,0 \text{ GPa} \\ L_0 &= 2,0 \text{ mm}^2/\text{GN} \quad (\text{GPa}^{-1}) \end{aligned}$$

Those tests with strain rates less than 1,0 $\mu\text{m/sec}$ had the fracture parameters evaluated using the time-dependent values of E and L_0 for their particular strain rates. These values of E and L_0 were read off directly from the average curves of Fig. 4.18. The range of E values encountered for the tests in Series 3 was from as low as 10,4 GPa for a 'slow' test to as high as 19,7 GPa for a 'fast' test. E values for the 'fast' tests of Series 4 were as high as 23,5 GPa.

Other investigators do not seem to have encountered any significant effects introduced by the strain rate during a test. Brown¹⁷ stated that substantial variations of loading rate had a negligible effect upon the load-deflection plots for his tests. However, he obtained the unnotched compliance L_0 for his beams using a loading rate of about 5 $\mu\text{m/sec}$, and the lowest loading rate he used was about 0,5 $\mu\text{m/sec}$. Fig. 4.18 shows that within this range of strain rates, any significant reduction in L_0 or E for the slower strain rates would not be seen as a distinct effect within the limits of experimental variation of the results. Hence, it seems reasonable to assume that most reported fracture tests on cemented materials have used 'fast' strain rates for testing, and therefore no significant effects of strain rate upon fracture parameters have been noticed. Nevertheless, Fig. 4.18 shows very clearly the effect of strain rate on E and L_0 .

Fig. 4.19(a) and (b) shows the effect of strain rate on the failure load for all the beams tested in Series 3 and 4. This curve is derived from Table 4.3(a) and (b) for Series 3, and Tables 4.5 to 4.7 for Series 4. Note that the tests in Series 4 were all 'fast' tests. The average curve is shown as well as the limiting envelopes in order to show the scatter of results. The failure

loads of the unnotched beams were more affected by strain rate than the failure loads of the notched beams. The 'vee'-notch beams exhibit no definite trend of failure load with strain rate. The curves also indicate clearly the effect of the stress concentration at the roots of the notches on the failure load of the beams. The cross-section of the 50,8 mm notched beams was 50 per cent of that of the unnotched beams, but the ratio of the average failure loads for 'fast' rates was only 17 per cent. The 'vee'-notch beams exhibit a higher failure load than the 50,8 mm notched beams due to the fact that they were tested on a shorter 305 mm span,

Characteristic load-deflection curves for the unnotched and rectangular-notched beams in Series 3 are not presented, because the true shape of these curves after the point of unstable crack propagation was often unobtainable on the Watanabe X-Y recorder. This was because the triaxial test set-up was a much 'softer' testing machine system than that used for Series 4. The beams of Series 3 tended to fracture more catastrophically than those of Series 4, since the strain-energy stored in the triaxial set-up was greater than that stored in the set-up for Series 4. This also made it difficult to evaluate accurately the total energy of fracture $A_{P,3}$, represented by the area under the load-deflection curves. Fig.'s 4.11 and 4.12 presented previously have indicated the effect that strain rate can have on the shape of the load-deflection curves. True shapes of the load-deflection curves for the various rectangular-notched beams are presented later in Fig. 4.27.

Fig. 4.20 shows a characteristic load-deflection curve for a 305 mm span 'vee'-notch beam of Series 3. (Specimen 3.26 of Table 4.3(b).) The 'vee'-notch beams fractured in a stable manner due to the high stress concentration at the root of the notch. It is seen that as the crack grows initially, the specimen actually gets stiffer as shown by the increase in slope of the curve near the beginning. As the crack grows in depth, so the crack front increases in width, causing the specimen to become stiffer before the point of maximum load is approached. The crack width continues to increase as the crack grows, and a stable-type fracture is produced.

The variation of G_c , K_{Ic} and $\bar{\gamma}$ with initial notch-depth ratio for 'fast' strain rates is presented in a later section (See Sec. 4.2.4(2)(c)), where the results for Series 3 and 4 are combined in Fig.'s 4.28 and 4.29. These figures use the results in Table 4.4 and in the later Table 4.8.

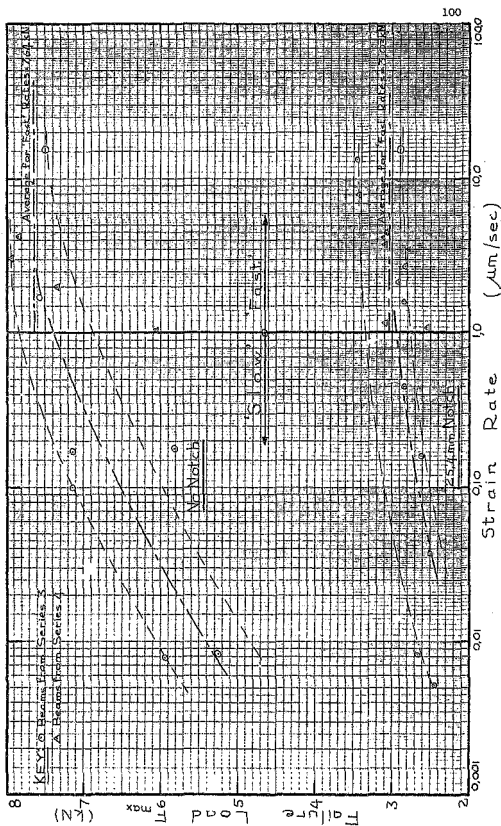


FIG. 4.19(a) FAILURE LOAD VERSUS STRAIN RATE, SERIES 3 & 4

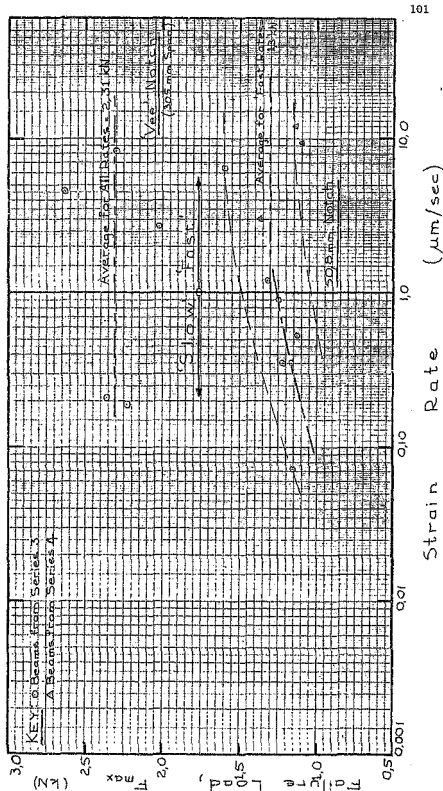


FIG. 4.19(b) FAILURE LOAD VERSUS STRAIN RATE,
SERIES 3 AND 4

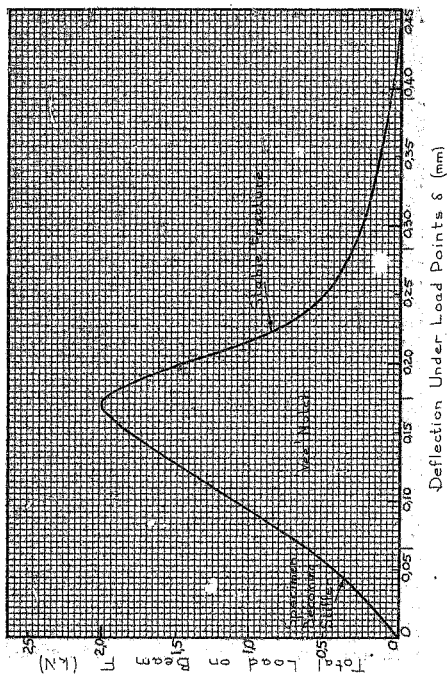


FIG. 4.20 CHARACTERISTIC LOAD-DEFLECTION
CURVE FOR A 'VEE' NOTCH BEAM/305
mm SPAN, SERIES 3

Fig. 4.21(a) and (b) shows the variation of G_c and K_{Ic} with strain rate, for the unnotched beams and the 25,4 mm notched beams of Series 3 and 4. Note that all the beams in Series 4 were tested at a 'fast' strain rate. The fracture parameters were evaluated using the time-dependent values of E and L_0 for strain rates less than 1,0 $\mu\text{m}/\text{sec}$, and the average values from 'fast' tests for strain rates greater than 1,0 $\mu\text{m}/\text{sec}$. The diagrams show the average curves for G_c and K_{Ic} as well as the limiting envelopes. It is evident that a large scatter of test results occurred. However, the fracture parameters show a general trend of increasing with increasing strain rate, despite the fact that the time-dependent Young's Modulus and unnotched compliance were used in their evaluation. The curves also show a marked drop in the values of G_c and K_{Ic} for the 25,4 mm notched beams in comparison to the unnotched beams. This is discussed later in Section 4.2.4(2)(c). The average values of G_c and K_{Ic} for the 'fast' tests are shown in Fig. 4.21, and it is clear that, for the mortar beams tested here, the fracture toughness can be considerably reduced as the total time of loading increases.

Fig. 4.22 shows the plot of $\bar{\gamma}$ versus strain rate, for the 50,8 mm notched beams, and the 'vee'-notch beams. Results from Series 3 and 4 are included. This figure is derived from Tables 4.4 and 4.8. Results of $\bar{\gamma}$ for 25,4 mm notched beams are not included due to the lack of results for 'slow' tests on these beams. In addition, results for unnotched beams are not included due to the fact that $\bar{\gamma}$ is usually overestimated when evaluated from a test on an unnotched beam. This is discussed more fully in later Section 4.2.4(2)(c). Figs 4.22 and 4.21 indicate that, for the 50,8 mm and 'vee'-notch beams plotted, $\bar{\gamma}$ is not as affected by strain rate as are G_c and K_{Ic} . It is difficult to infer a general trend of $\bar{\gamma}$ with strain rate from Fig. 4.22, due firstly to the fact that only a few test results were available, and secondly because the plotted values show no great variation with strain rate. More tests would be required to confidently define the actual variation of $\bar{\gamma}$ with strain rate.

Average values of the fracture parameters G_c , K_{Ic} and $\bar{\gamma}$, and how these are related, are discussed later in Sec. 4.2.4(2)(c).

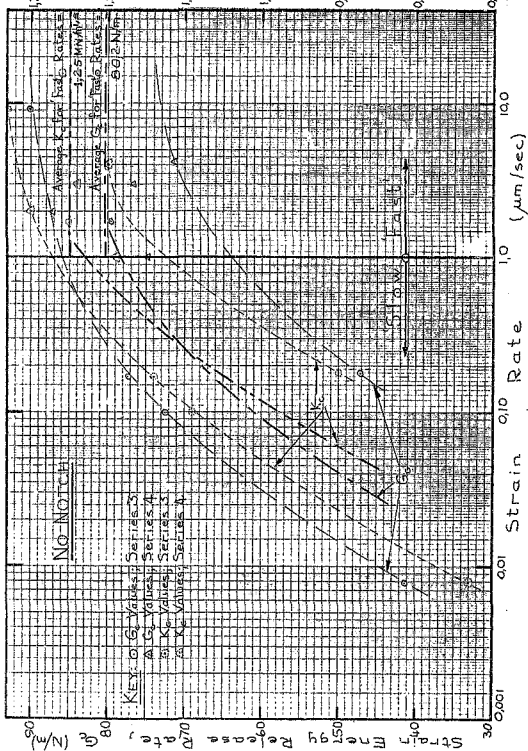


FIG. 4.21(a) G_c AND K_c VERSUS STRAIN RATE, SERIES 3 & 4

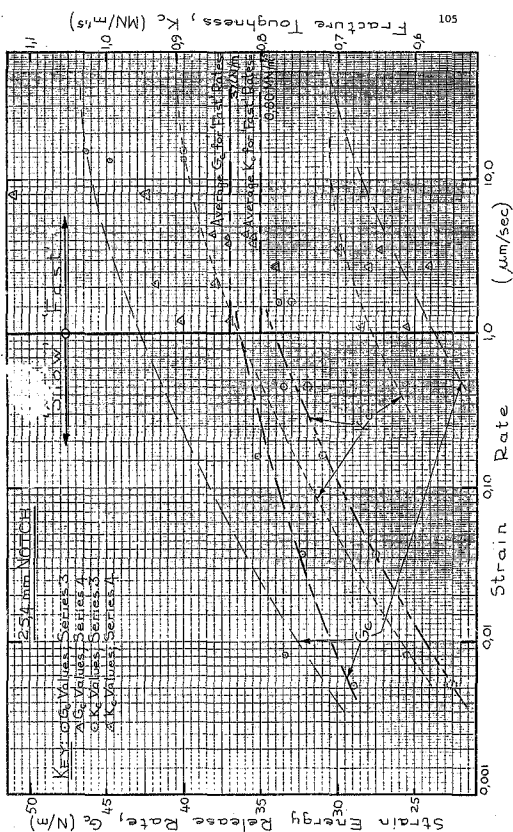


FIG. 4.21(b) G_c AND K_{Ic} VERSUS STRAIN RATE, SERIES 3 & 4

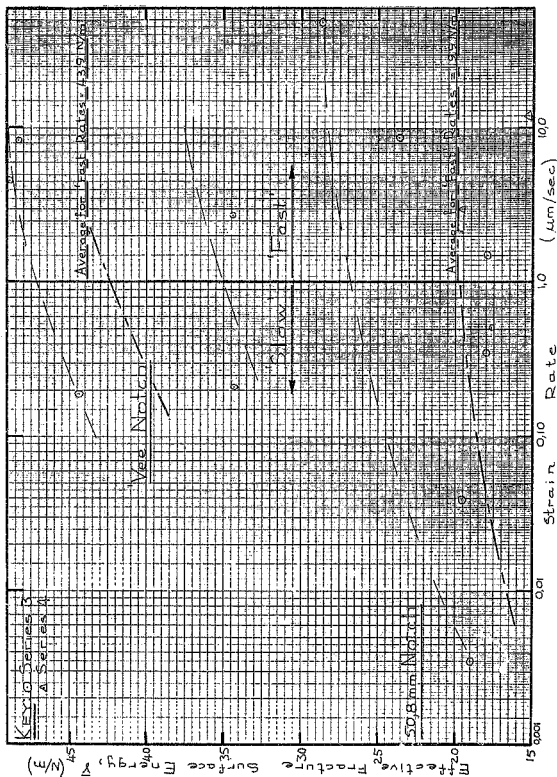


FIG. 4.22 γ VERSUS STRAIN RATE, SERIES 3 AND 4

4.2.4 Series 4 : Tests with Stiffened Machine

Series 4 of the laboratory test program was designed when it was realised from the preceding series that the stiffness of the loading machine system could have a profound effect on the fracture behaviour of a notched beam, particularly on the post-cracking behaviour after the onset of unstable crack propagation. The basic factor to be considered here is the strain energy stored in the machine system at the critical point of failure, and the balance between the strain energy in the system and the strain energy stored elastically in the beam at failure. If the energy stored in the machine system can be minimised, then a more stable type of fracture can be obtained. This allows the true shapes of the load-deflection curves to be recorded. If, however, a large amount of energy, relatively speaking, is stored in the machine system at the critical point of unstable crack propagation, then this energy can be fed back into the specimen during failure, causing a less stable type of fracture to occur due to the excess of energy available in the machine/specimen system to produce fracture. All of the notched beams studied in the present tests had insufficient elastic energy stored in them at the point of failure to produce a catastrophic fracture, but if energy was fed back into the beam from the machine during the period of fast crack propagation, then a catastrophic fracture could be produced. Stiffening up the machine loading system ensures that the failure load of the specimen is a small proportion of the total machine load capacity, and prevents the loading heads of the machine from 'following-up' when the load on the specimen suddenly drops off during fast crack propagation. In general, the present tests showed that a hydraulic testing system was better than a mechanical system. The hydraulic system could not 'follow-up' the specimen deflection during failure as quickly as could the mechanical systems.

A total of twenty four notched and unnotched beams were tested for Series 4. All the strain rates used for the tests were 'fast', i.e. the loading rate on the specimen was greater than $1.0 \mu\text{m/sec}$. This meant that the average values of E and L_0 as shown in Fig. 4.18 could be used for analysis of the results and evaluation of the fracture parameters. (The results of unnotched beams in Series 4 are included in Fig. 4.18). In addition, the 'fast' strain rates used would have no significant effect on the results of Series 4. Finally, it should be

noted that the deflection measurements for Series 4 were monitored correctly using the newly designed deflection rig.

(1) Test Set-Up and Procedure

The Macklow-Smith hydraulic testing machine used previously for Series 1 was selected as the most suitable machine that could be stiffened up for the tests of Series 4. Different machine stiffnesses were obtained by modifying the basic set-up, which was that of a steel I-beam (RSJ) included in series between the testing heads of the machine and mortar beam specimen. The test set-up, with a 200x100 mm I-beam being used, is shown photographically in Fig. 4.23. The basic test set-up is shown in detail in Fig. 4.24. The span of the I-beam was 0,75 m. In order to vary the stiffness of the loading system, three I-beams of nominal size 150x75 mm, 200x100 mm, and 250x125 mm were selected for inclusion in the test set-up. The load ratio at failure, defined as the maximum specimen load at failure divided by the total load on the machine at failure, could be varied from about 2 per cent to 13 per cent. Some specimens were tested without any I-beam stiffeners, and the load ratio for such a case was 100 per cent.

Towards the end of the tests in Series 4, it was realised that sufficient energy might be stored at failure in the loading rig, which comprised a 25,4 mm deep solid section, to prevent the occurrence of a more stable type of fracture in the beams. It was therefore decided to increase the section of the loading rig to 50,8 mm deep, thus stiffening it up by a factor of about eight. A few tests were then run on 12,7 mm and 25,4 mm notched beams to provide a comparison with previous tests which did not employ the stiffened load rig. Appendix D gives full details of both the original and the modified loading rig.

The tests of Series 4 were monitored using either the Watanabe X-Y recorder, or the N.E.P. Ultra-Violet recorder. The use of the U.V. recorder allowed the true load-deflection curves of the beams to be obtained, since the recorder could respond sufficiently to the transient effects following failure. A photograph of the N.E.P. U.V. recorder is shown in Fig. 4.25. The type of beam specimens used, and the loading spans, were as in Series 1. Unnotched and rectangular-notched beams were tested in Series 4, and the nominal notch depths used were 12,7 mm, 25,4 mm and 50,8 mm. The X-Y recorder was able to faithfully record the load-deflection curves for the 25,4 mm and 50,8 mm notched beams, provided the load ratio was less than about

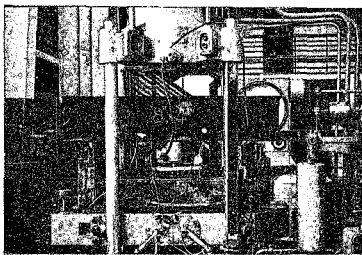


FIG. 4.23 GENERAL TEST SET-UP, SERIES 4
(I-Beam Span = 0.75 m)

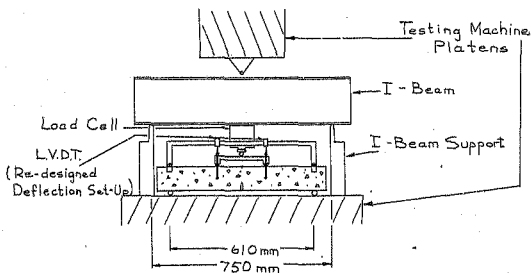


FIG. 4.24 DETAILED TEST SET-UP, SERIES 4

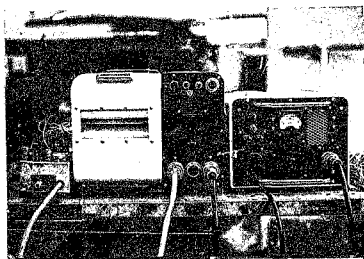


FIG.4.25 N.E.P. ULTRA-VIOLET RECORDER
(Bridge Box at Far Left)

5 per cent. However, when the stiffened load spreader rig was used, a distinctly more stable type of fracture occurred, allowing even 12,7 mm notched beams to be recorded on the X-Y recorder. When the machine system was unstiffened, the X-Y recorder was unable to faithfully record the load-deflection curves of all except the 50,8 mm notched beams. Usually, the recorder was able to pick up the extremity of the tail portion of the curve (see Fig. 3.5), but the intermediate shape of the curve after failure was lost. This intermediate portion represented the period of rapid crack propagation.

(2) Test Results

(a) Constant 25,4 mm Initial Notch Depth; Different Machine Stiffnesses

The first group of tests in Series 4 attempted to show that the degree of catastrophic failure of a notched specimen was influenced by the degree of testing machine stiffness. The X-Y plotter was used for this group of tests. The results of the tests are presented in Table 4.5. It is most instructive, however, to examine the load-deflection curves obtained for the 25,4 mm notched beams with different machine stiffnesses. These curves are presented in Fig. 4.26:-

Each curve shows schematically in outline the true load-deflection curve for a 25,4 mm notched beam (obtained from tests described later.) Superimposed on the true outline is an indication of the actual curves that were recorded by the Watanabe X-Y plotter, for the particular machine stiffness used. Curve A refers to an unstiffened system, curve B to a 150x75 mm I-beam stiffened system, curve C to a 200x100 mm I-beam stiffened system, and curve D to a 250x125 mm I-beam stiffened system. It is clearly seen that as the machine system is stiffened up, so the X-Y plotter can more easily record the true shape of the load-deflection curve, indicating that a more stable type of fracture has been produced.

From later tests in groups (b) and (c), it seems very possible that a completely stable type of fracture could have been obtained on a 25,4 mm notched beam if the modified stiffened load spreader rig had been used in conjunction with the 250x125 mm I-beam. The portion of the load-

Table 4.5 Rectangular Notches: Results for Series 4: Machine-Stiffened Tests
(a) Constant Notch; Different Machine Stiffnesses

Loading System Specimen No. (Cube Str.)	1	2	3	4	5	6	7	8	9	10	11
MPa	Initial Notch Depth (mm)	Strain Rate (mm/sec)	Load Ratio (per cent)	Max. Load at Failure (kN)	Initial Compliance L_i (mm ² /GN)	Compliance at Failure L_f (mm ² /GN)	$\Delta L = L_f - L_i$ (mm ² /GN)	Notch Depth Ratio at Fail. $\frac{c}{d}$	Notch Depth at Fail. c_c (mm)	$f(c/d)$ From Fig. 3.2	Total Fracture Energy $A_{F,6}$ (Nm)
4.1 (32.65)	A L L 25,4 mm	8,0	100	3,418	2,667	3,621	0,974	0,423	42,98	0,493	0,438
4.2 (32.65)		4,4	12,9	3,077	3,097	3,949	0,852	0,409	41,55	0,488	0,512
4.3 (32.65)		3,8	13,1	3,098	2,762	3,568	0,806	0,402	40,84	0,486	0,489
4.4 (33,33)		4,2	9,1	3,108	3,252	4,054	0,802	0,401	40,74	0,486	0,569
4.5 (33,33)		2,7	7,9	2,833	2,778	3,705	0,927	0,418	42,47	0,490	0,442
4.6 (34,88)		2,1	5,1	2,914	3,012	4,170	1,158	0,444	45,11	0,498	0,475
4.7 (34,88)		1,2	3,9	3,098	2,795	3,709	0,914	0,416	42,27	0,489	0,507

'Fast' Strain Rate $\geq 1,0$ mm/sec

'Slow' Strain Rate $< 1,0$ mm/sec

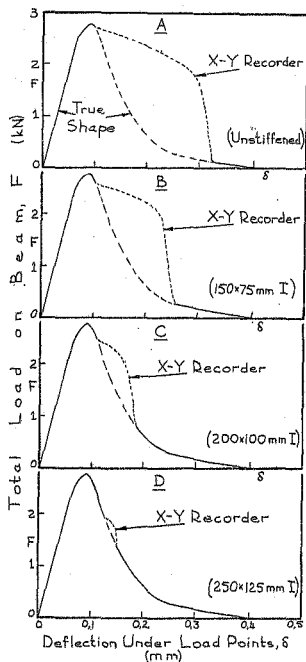


FIG. 4.26 RECORDED SHAPES, USING X-Y RECORDER,
OF F- δ CURVE FOR 25.4 mm NOTCHED BEAM
INFLUENCED BY STIFFNESS OF TEST SET-UP
(SERIES 4)

deflection curve after the peak is profoundly affected by the strain energy level in the specimen and the loading system at the peak load, and the specimen/machine interaction as the load drops off, i.e. whether the machine feeds energy back into the specimen or not. In the un-stiffened system, the machine and load rig are feeding energy back into the specimen during failure, and promoting a more catastrophic type of failure. When the machine loading system is sufficiently stiffened, however, the energy used for crack propagation is due to the stored elastic energy in the specimen itself, and the crack therefore propagates more stably.

One other factor that influenced the type of fracture produced was the strain rate of the specimen immediately prior to failure. A faster strain rate before the critical point of instability produced a less stable type of fracture after instability than did a slower rate. For this reason, the tests of Series 4 had the strain rate reduced just prior to the peak load being reached.

The values of the fracture parameters G_c , K_{Ic} and $\bar{Y}$ for group (a) are summarised later in this section.

(b) Constant Machine Stiffness; Different Notches

The second group of tests in Series 4 attempted to define the borderline between those notch depths that would produce a stable or semi-stable type of fracture, and those notch depths for which the beam always failed catastrophically, with the load dropping instantaneously to zero at failure. In order to do this, a constant machine stiffness was used for all the tests:- the 250x125 mm I-beam stiffened Macklow-Smith hydraulic machine. In addition, the stiffened load spreader rig was used for one of the 12,7 mm notched beams in this group. This test was conducted at a later stage when it was realised that the load rig itself was preventing more stable type fractures due to the strain energy stored in it. Unnotched beams and beams with initial notch depths of 12,7 mm, 25,4 mm and 50,8 mm were used for the tests. The results of the tests are presented in Table 4.6. It was found that the 25,4 mm and 50,8 mm notched beams produced a completely stable type of fracture. The 12,7 mm beams without the stiffened load spreader rig produced a semi-stable type of fracture, with more spontaneous crack propagation than for the 25,4 mm or 50,8 mm notched beams. The Watanabe X-Y recorder, however, was unable to

Table 4.6 Rectangular Notches : Results for Series 'Machine-Stiffened Tests'
(b) Constant Machine Stiffness; Different Notches

Loading System Specimen No. (Cube Str. MPa)	1 Initial Notch Depth (mm)	2 Strain Rate ($\mu\text{m}/\text{sec}$) (per cent)	3 Load Ratio	4 Max. Load at Failure (kN)	5 Initial Compliance L_i (m^2/GN) ($E=20,562$)	6 Compliance at Failure L_f (m^2/GN) ($E=15,69$)	7 $\Delta L = L_f - L_i$ (m^2/GN)	8 Notch Depth Ratio at Fail. $\frac{c_c}{d}$	9 Notch Depth at Fail. c_c (mm)	10 $I(C/d)$ From Fig. 3.2	11 Total Fracture Energy $A_{p,d}$ (Nm)
4.8 (26,54)	No Notch	1,0	3,5	6,063	1,949	2,478	0,529	0,288	29,26	0,432	0,632
4.9 (26,54)		2,0	4,5	7,337	2,555	2,952	0,397	0,249	25,30	0,403	0,904
4.10 (26,54)		1,3	2,7	3,903	2,478	2,944	0,466	0,293	29,77	0,436	0,594
4.11 (26,54)		1,8	3,0	3,316	2,857	3,732	0,875	0,372	37,80	0,476	0,507
4.12 (31,97)	12,7 mm Notch	1,5	4,8	4,056	2,436	2,833	0,397	0,278	28,24	0,427	0,570
4.13 (29,89)		1,1	2,0	2,558	3,374	4,191	0,817	0,403	40,94	0,487	0,402
4.14 (26,54)	25,4	3,0	3,3	1,365	4,308	6,719	2,411	0,624	63,40	0,521	0,202
None	50,8										

'Fast' Strain Rate $\geq 1,0\text{m}/\text{sec}$

'Slow' Strain Rate $< 1,0\text{m}/\text{sec}$

completely follow the load-deflection curve after initial failure, but it was quite evident that a purely catastrophic failure was not occurring. However, the 12,7 mm notched beams tested with the stiffened load-spreader rig exhibited a completely stable type of fracture, with the X-Y recorder being able to faithfully record the whole curve except for a tiny portion just after peak load. The true shape of the load-deflection curve was immediately apparent from the test record, and was confirmed by tests conducted using the U.V. recorder (group (c) later). The unnotched beams, however, fractured catastrophically. The 12,7 mm and 25,4 mm notched mortar beams used in the present tests seemed to be on the critical borderline between stable and semi-stable/catastrophic fractures, and the type of fracture produced by these beams was critically dependent on the machine energy at failure and on the strain rate at failure. It will be shown later under group (c) that it was impossible for an unnotched beam to fracture stably or semi-stably, from a consideration of the effective fracture surface energy $\bar{\gamma}$ of the mortar beams and the strain energy stored in the beam at failure.

For a summary of the values of the fracture parameters G_c , K_{Ic} and $\bar{\gamma}$ from the tests of group (b), see later Table 4.8 and Fig.'s 4.28 and 4.29. It should be pointed out here that where it was possible to infer the true shape of the load-deflection curve of a beam from the test record and from the characteristic curves presented in Fig. 4.27, then the total work of fracture for that beam as represented by the area under the load-deflection curve was evaluated. Where this was not possible, however, the value was neglected. The load was assumed to drop instantaneously to zero at failure in the case of the unnotched beams.

(c) Tests on the 250x125 mm I-beam Stiffened System
Together with the N.E.P. Ultra-Violet Recorder.

The N.E.P. U.V. Recorder was used for the tests in group (c) in order to determine confidently and accurately the true shapes of the load-deflection curves of the unnotched and rectangular notched mortar beams used in the laboratory tests. As mentioned previously, this instrument had a frequency response to transients at failure that was twenty to twenty five times higher than the X-Y recorder, and it was found that the sensitivity of the U.V. was quite sufficient to accurately monitor the true load-deflection curves. The results of the tests are

presented in Table 4.7. Characteristic load-deflection curves for unnotched beams, and for 12,7 mm, 25,4 mm, and 50,8 mm notched beams are presented in Fig. 4.27. The curve for the unnotched beam corresponds to specimen 4.16 in Table 4.7, the 12,7 mm notched beam to specimen 4.12 in Table 4.6, the 25,4 mm notched beam to specimen 4.19 in Table 4.7, and the 50,8 mm notched beam to specimen 4.21 in Table 4.7. It should be noted that specimens 4.18 to 4.21 used the modified stiffened load spreader rig, and that specimen 4.12 was actually recorded on the X-Y recorder using the stiffened rig. A discussion of Fig. 4.27 appears below:-

The unnotched mortar beams used in the present tests always fractured catastrophically, with the load dropping instantaneously to zero just after the peak was attained.

The $A_{F,0}$ terms for unnotched beams contain excess energy not expended on producing new fracture surfaces. The notched beams exhibit a progressively greater degree of post-cracking ductility as the initial notch depth increases. Thus, the load on the 12,7 mm notched beam drops off more quickly after the peak than does the load on the 50,8 mm notched beam. The beam deflection at final failure also increases as the initial notch depth increases. Furthermore, deep notched beams experience a greater degree of slow, stable crack growth prior to failure than do shallow-notched beams. The important fact to note from the curves in Fig. 4.27 is that even a nominally brittle material like cement mortar, when fracture tested in flexure, can and does exhibit a considerable amount of post-cracking 'ductility', and the load-deflection curve for notched beams is a continuous line without any discontinuities, provided the fracture is not catastrophic as for an unnotched beam. This phenomenon of beam ductility will be demonstrated later by means of load-cycling a specimen during testing. (See tests in group (d) of Series 4.)

Included at this point is the summary of the values of the fracture parameters for the first three groups of tests of Series 4, in Table 4.8. These parameters are derived from the previous tables 4.5, 4.6 and 4.7. Together with the values of the fracture parameters for the 'fast' strain rate tests of Series 3 (see Table 4.4), the values in Table 4.8 appear diagrammatically in Fig.'s 4.28 and 4.29.

Table 4.7 Rectangular Notches : Results for Series 4: Machine-Stiffened Tests
(c) 250x125 mm I-Beam together with U.V. Recorder

Loading System Specimen No. (Cube Str. MPa)	Initial Notch Depth (mm)	1	2	3	4	5	6	7	8	9	10	11
		Initial Notch Depth (mm)	Strain Rate (mm/sec)	Load Ratio (per cent)	Max. Load Failure (kN)	Initial Compliance L_i (mm ² /GN) (E=20,47)	Compliance at Failure L_f (mm ² /GN)	$\Delta L = L_f - L_i$ (mm ² /GN)	Notch Depth Ratio at Fail. $\frac{c}{d}$	Notch Depth at Fail. c (mm)	f(c/d) From Fig. 3.2	Total Fracture Energy $A_p \delta$ (Nm)
None	4.15 (32,03)	No Notch	4,2	9,8	7,850	2,004 (E=20,00)	2,239	0,235	0,195	19,81	0,355	0,786
	4.16 (32,03)	No Notch	3,0	7,7	7,926	1,708 (E=23,47)	1,958	0,250	0,203	20,62	0,363	0,775
	4.17 (32,03)	12,7	2,2	5,9	3,804	2,032	2,190	0,158	0,204	20,73	0,365	0,521
	4.18 (32,03)	12,7	2,7	3,9	2,808	2,557	3,063	0,506	0,360	36,58	0,470	0,331
Stiffened Load Rtg	4.19 (32,03)	25,4 mm Notch	3,5	3,7	2,789	2,639	3,315	0,676	0,385	39,12	0,479	0,369
	4.20 (32,03)	25,4 mm Notch	9,4	1,3	1,098	5,796	7,542	1,746	0,510	61,98	0,521	0,144
	4.21 (32,03)	50,8 mm Notch	12,0	1,4	1,136	5,080	6,082	1,002	0,577	58,62	0,520	0,158
	4.21 (32,03)	50,8 mm Notch	12,0	1,4	1,136	5,080	6,082	1,002	0,577	58,62	0,520	0,158

Fast Strain Rate $\geq 1,0$ mm/sec

Slow Strain Rate $< 1,0$ mm/sec

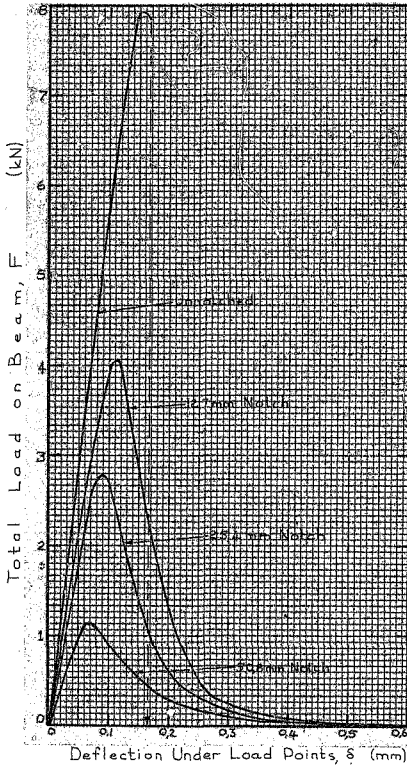


FIG. 4.27 CHARACTERISTIC LOAD-DEFLECTION CURVES FOR NOTCHED AND UNNOTCHED MORTAR BEAMS

Table 4.8 (Cont'd)

MACKLOW-SHULTZ HYDRAULIC MACHINE			1	2	3	4	5	6
Stiff.	250x125 mm I Beam	None	'A I L F A S T'					
4.12 (31,97)	1,5	4,8		12,7	31,7	0,78	31,6	
4.13 (29,89)	1,1	2,0		25,4	25,5	0,67	26,0	
4.14 (26,54)	3,0	3,3		50,8	31,1	0,73	19,6	
4.15 (32,03)	4,2	9,8		No Notch	71,4	1,20	38,1	
4.16 (32,03)	3,0	7,7			76,7	1,24	37,5	
4.17 (32,03)	2,2	5,9		12,7	17,8	0,60	28,8	
4.18 (32,03)	2,7	3,9		25,4	24,1	0,66	21,4	
4.19 (32,03)	3,5	3,7		25,4	27,3	0,70	23,8	
4.20 (32,03)	9,4	1,3		50,8	18,0	0,55	14,0	
4.21 (32,03)	12,0	1,4		50,8	15,1	0,51	15,3	

'Fast' Strain Rate $\geq 1,0 \mu\text{m/sec}$ 'Slow' Strain Rate $< 1,0 \mu\text{m/sec}$

Fig. 4.28 shows G_c and K_c versus the initial notch depth ratio c/d for the 'fast' tests of Series 4, while Fig. 4.29 shows $\bar{\gamma}$ plotted against c/d , again for the 'fast' tests of Series 3 and 4.

Fig. 4.28 shows that both G_c and K_c are affected by the initial notch-depth ratio of the beams used to measure their values. The measured fracture toughness of an unnotched beam is considerably higher than the fracture toughnesses of notched beams. Two trends are possible from Fig. 4.28. The first is that the fracture toughness values continue to decrease as the notch-depth ratio increases. However, an examination of the average values of G_c or K_c for notched beams shows that, within the limits of experimental scatter, a discontinuity might exist in the curve of G_c or K_c between notched beams and unnotched beams. The average values for the various notch-depth ratios are tabulated below in Table 4.9. Also shown is the dimensionless ratio $K^2(1-\nu^2)/EG$, which should have a value of unity according to the Fracture Mechanics relationship of equation 1.9.

Table 4.9 Average Values of G_c and K_c

	G_c (N/m)	K_c (MN/m ^{3/2})	$K^2(1-\nu^2)/EG$
Unnotched	80,2	1,25	0,966
$c/d = 0,125$	29,4	0,74	0,923
$c/d = 0,25$	37,0	0,80	0,857
$c/d = 0,50$	25,6	0,65	0,818
Average for Notched Beams	30,7	0,73	0,861

The grand average of G_c for the notched beams is 30,7 N/m, and that of K_c is 0,73 MN/m^{3/2}. These values are plotted in Fig. 4.28, indicating an average value of the fracture parameters for notched beams.

Fig. 4.28 suggests that a basic difference might exist between the modes of fracture of notched and unnotched beams. In an unnotched beam, the crack propagates from a natural surface defect in the beam, and in pure bending, this propagation can theoretically occur anywhere within the central loading span. The possibility therefore exists that numerous

microcracks might begin growing from the surface during the test, and it is only those microcracks which satisfy the relationship between critical crack length and stress level in the beam at the point of failure that can then propagate spontaneously to cause catastrophic failure of the beam. The multiplicity of tiny microcracks in the central portion of the beam in themselves represent an energy-absorbing mechanism, and could therefore impart an apparently higher fracture toughness to the beam when compared to an artificially deep-notched beam. This fact is also borne out by the results of Fig. 4.19(a) and (b), which show that strain rate has a more marked effect on the failure load of unnotched beams than on notched beams. For low-strain rates, more extensive multiple microcrack growth in a stable fashion from the surface of an unnotched beam could occur than for higher strain rates. However, for a notched beam, the critical crack is compelled to initiate at the notched section, and the degree of microcracking in the beam over the central loading span would be limited in extent to the notched cross-section. This isolated crack area would not be able to exercise as great an influence on the failure load of a notched beam with varying strain rates. Thus, 'vee'-notch beams representing a high stress concentration at the notch exhibit failure loads that are apparently unaffected by strain rate, and the effect of strain rate on failure load increases as the notch-depth ratio decreases.

Hence, artificially notched beams limit the area of fracture damage to the immediate vicinity of the notch cross-section, while unnotched beams have no such limiting factor to their fracture behaviour. This leads to a marked difference in the measured fracture toughnesses of notched and unnotched beams. This fact is further demonstrated by the variation of $\bar{\gamma}$ with c/d , discussed below.

Fig. 4.29 Again, two trends of the variation of $\bar{\gamma}$ with c/d are possible from this figure. The one is that $\bar{\gamma}$ decreases continuously as c/d increases, and the other is that unnotched beams and notched beams exhibit basically different types of fracture. As will be shown later in this section, the unnotched beams in the present tests could never fracture in a semi-stable or stable fashion. However, all the notched beams were capable of exhibiting semi-stable or stable type fractures, depending on such variables as loading machine stiffness and strain rate at failure. The average

values of $\bar{\gamma}$ for the various notch-depth ratios are tabulated below in Table 4.10. Also shown is the dimensionless ratio $G_c/\bar{\gamma}$, which should have a numerical value of 2 according to the Fracture Mechanics relationship in Section 1.3.2.

Table 4.10 Average Values of $\bar{\gamma}$

	$\bar{\gamma}$ (N/m)	$G_c/\bar{\gamma}$
Unnotched	38,9	2,062
c/d = 0,125	30,4	0,967
c/d = 0,25	29,2	1,267
c/d = 0,50	19,9	1,286
'vee'-notch (effective c/d = 0,582)	43,9	-
Average for rect. notched beams	26,5	1,158

The grand average of $\bar{\gamma}$ for the rectangular notched beams is 26,5 N/m. This value is plotted in Fig. 4.29. The average $\bar{\gamma}$ value for the 'vee'-notch beams is considerably higher than for the rectangular notched beams, and seems to indicate a marked departure from the trend of Fig. 4.29. Although these notches produced a stable type fracture, the beams were tested on a 305 mm span rather than the 610 mm span of the other beams. The central loading span was only 101 mm. According to the specifications of ref. 23, p.14, pure bending exists in the central minor span if the ratio of the minor span to specimen depth is not less than two. For the case of the 'vee'-notch beams, this ratio (101/101) had a value of only one. In addition, the ratio of the major support span to specimen depth was three which is less than the minimum value of four recommended by ref. 23 in order to avoid substantial errors due to specimen indentation and friction at the supports. Hence, it seems that the 'vee'-notch beams, despite exhibiting a stable type fracture, produced results that were unrepresentative due to the effects of shear and friction. These results are a clear indication of the need to both develop and adhere to suitable criteria for valid fracture testing.

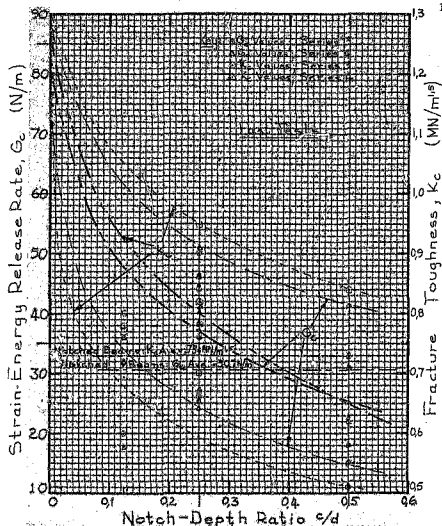


FIG. 4.28 G_c AND K_{Ic} VERSUS c/d , SERIES 3 & 4 ('FAST TESTS')

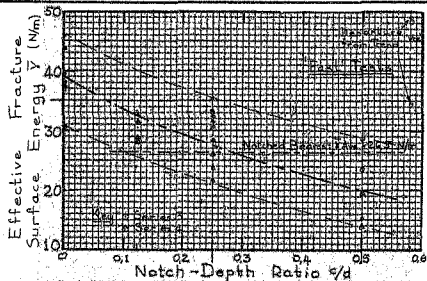


FIG. 4.29 γ VERSUS c/d , SERIES 3 & 4 ('FAST TESTS')

As the c/d ratio increased, so the type of fracture became increasingly more stable and the crack growth less spontaneous. This was because the energy stored in the specimen was limited by the presence of a notch, and thus the energy available for spontaneous fracture decreased as the c/d ratio increased. The deep-notched beams (50,8 mm) required external work to be done on the specimen in order to make the crack grow, while the shallow-notched specimens exhibited a degree of spontaneous crack growth under the energy level present in the beam. It is therefore conceivable that a greater degree of microcracking surrounding the main crack could occur in unnotched and shallow-notched beams during the course of crack propagation than in deep-notched beams. This microcracking would constitute a mechanism of energy absorption which would increase as the notch-depth ratio decreased. The fracture of a shallow-notched beam is therefore more 'violent' than that of a deep-notched beam. This phenomenon is critically dependant on the energy level in the specimen, which in turn is dependent on the notch-depth ratio. Fig. 4.29 indicates that the measured value of $\bar{\gamma}$ can vary depending on the type of fracture exhibited by the beam, and reflects the need for the fracture test to be designed to be an accurate model of actual fracture conditions in practice. The lower value of $\bar{\gamma}$ represented by the deep-notched beams is probably the more acceptable value for design purposes, especially in those situations such as road pavements where the fracture is a time-dependent process.

Catastrophic Fracture of Unnotched Beams

As seen from Fig. 4.28, the value of $\bar{\gamma}$ depends on the notch depth in the beam. Unnotched beams give higher $\bar{\gamma}$ values than notched beams. This is a result of the catastrophic/stable fracture phenomenon discussed previously. An unnotched beam, which fractures spontaneously and catastrophically, has sufficient strain energy stored in it at the point of failure to cause the crack to run right through the specimen, and excess energy is then used up in kinetic energy of the fracture pieces, sonic energy and heat energy. The area under the load-deflection curve contains the component of total energy required to cause complete fracture, as well as the excess energy component. When evaluating the total work of fracture $A_{F,\delta}$ using the area under the load-deflection curve, $A_{F,\delta}$ is overestimated for an unnotched beam, and therefore $\bar{\gamma}$ is overestimated. Immediately a specimen

is notched, however, the energy level in the specimen at failure drops significantly, and there is insufficient elastic energy in the specimen at failure to cause the crack to run right through the specimen. External energy must be fed into the specimen to make the crack grow to final failure. Hence, the $A_{F, \delta}$ term from such a specimen is a more accurate estimate of the true effective fracture surface energy $\bar{\gamma}$ of the material.

Using the average $\bar{\gamma}$ value of 26,5 N/m for the rectangular notched beams from Fig. 4.28, and the average $A_{F, \delta}$ value of 0,80 Nm for the unnotched beams from Tables 4.3(a), 4.6 and 4.7, it is possible to show that an unnotched beam in the present tests could never produce anything but a catastrophic type fracture. The theoretical fracture surface area for the unnotched beams is found by dividing $A_{F, \delta}$ for the unnotched beams by the true $\bar{\gamma}$ from the notched beams, i.e.

Theoretical fracture surface area, unnotched beam

$$\begin{aligned}
 &= \frac{A_{F, \delta}}{\bar{\gamma}} \\
 &= \frac{0,80 \times 10^4}{26,5} \\
 &= 301,9 \text{ cm}^2
 \end{aligned}$$

However, the available fracture surface area for an unnotched beam is only 206,46 cm² (see Fig. 4.7), and therefore the unnotched beams must fracture catastrophically. Using the average $\bar{\gamma}$ for notched beams, 26,5 N/m, the predicted areas and the actual average areas, $A_{F, \delta}$, under the load-deflection curves for the beams of Series 3 and 4 are compared in Table 4.11. It is seen that the notched beams should all produce stable or semi-stable type fractures, as opposed to the catastrophic type fracture of the unnotched beams.

Table 4.11 Comparison of Predicted and Actual Areas under Load-Deflection Curves

	Nominal Fracture Surface Area (cm ²)	Approx. Predicted A _{F,δ} (Nm)	Range of Actual Areas A _{F,δ} (Nm)	Catastrophic (C) or Stable (S)
Unnotched	206,46	0,55	0,8-0,9	C
12,7 mm	180,64	0,48	0,5-0,6	Semi-S
25,4 mm	154,84	0,41	0,4-0,5	S
50,8 mm	103,22	0,27	0,2-0,3	S

(d) Tests on Notched Beams Using a Load-Cycling
Technique to obtain Stable Crack Growth

Tests using the Macklow-Smith hydraulic testing machine were done on two 25,4 mm and one 50,8 mm notched beams, using a load-cycling technique in order to produce stable crack growth in the beam. The technique, which has also been used by Brown¹⁷, involves loading the beam until the critical point of fracture instability is reached, and then quickly removing the load before rapid crack propagation can occur. By using successive cycles in which the load is first applied and then 'dumped' before fracture instability can occur, the crack can be made to grow in stable increments through the beam until final failure occurs. An example using a 25,4 mm notched beam is shown in Fig. 4.30. This figure represents the actual output from the test. As the crack front advances with each successive loading cycle, so the prevailing compliance increases. The shape of the complete load-deflection curve is immediately apparent from the test output, and is shown drawn in on the figure in dotted lines. Note that the load-deflection curve is shown in two portions, the lower portion following on from point A at the end of the upper portion. Three points are worth noting from the figure:-

- (i) With each successive loading cycle, there is an increase in the permanent set in the beam, as shown by the shift of the zero load point successively to the right. This indicates there is

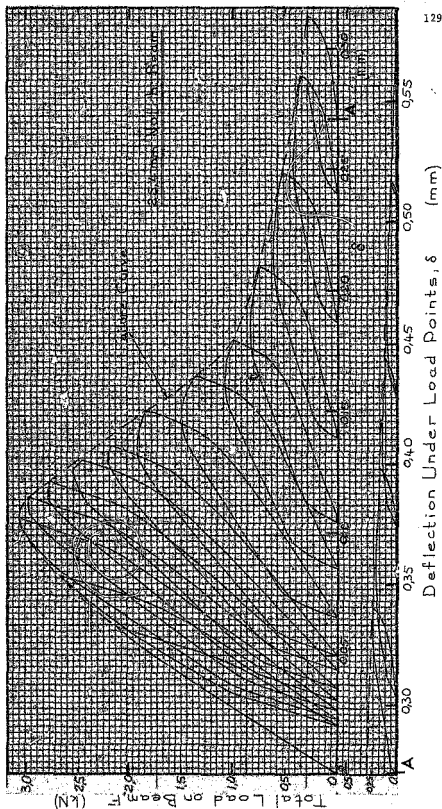


FIG. 4.30 LOAD-CYCLING TECHNIQUE ON 25.4 mm NOTCHED
BEAM, SERIES 4

- some positive resistance to crack closing on unloading. In addition, at the start of each successive re-loading cycle, the specimen seems to offer positive resistance to crack opening, as shown by the steeper slopes at the start of each load cycle.
- (ii) Beyond the initially steeper portion of the curve for each load cycle, there is a markedly linear portion to the curve, from which a prevailing compliance can be calculated, and hence the crack length at that point evaluated. This indicates that only the area immediately ahead of and surrounding the main-crack is over-stressed, and that the remainder of the specimen is still in the elastic range, i.e. the crack is accommodated within purely elastic surroundings. This is one of the basic premises of Linear Elastic Fracture Mechanics.
- (iii) The load-cycling technique graphically demonstrates the post-cracking ductility of the nominally brittle mortar beams used in the tests. The area under the load-deflection curve after the peak load is the major component of the term $A_{F,\delta}$, and cannot be ignored when calculating $\bar{\gamma}$.

4.2.5 Series 5 : Plane Stress/Strain Tests on Cement-Mortar Specimens

Series 5 of the laboratory test programme was designed to attempt a brief, initial study of the effect on the fracture properties of the state of the stress field ahead of the crack tip in a notched specimen. Investigators in the metallic field have recognised that the measured fracture toughness of a thin sheet section, in which plane stress conditions prevail, can be considerably higher than the measured fracture toughness of a thick slab, in which plane strain conditions prevail^{23,29}. The fracture toughness of a material, K_{IC} , is defined as the plane strain fracture toughness, the lower effective limit to the fracture strength of the material. Under plane strain conditions, K_{IC} (and the corresponding value of G_c), can be regarded as basic material properties. However, under plane stress conditions, the measured K_{IC} can not be regarded as a basic material property. Investigators in the non-metallic field have assumed that plane strain conditions have prevailed in their fracture specimens, and the present series of tests attempted to verify or disprove this assumption.

The stress field ahead of the crack tip in a fracture specimen changes from plane stress to plane strain as the dimensions of the specimen change

from a thin sheet to a thick plate of heavy section. In a thin sheet, the constraint-relieving effect of the free faces extends throughout the specimen thickness, whereas a thick plate imposes considerable restraint on the lateral strain at the crack tip. For this reason, the cement-mortar specimens studied in the present series varied from wide, thick slabs to narrow, deep beams. Out of a total of seven test specimens, two had electrical resistance strain gauges cast into the beam just above the notch root, in order to monitor the longitudinal and transverse strains in the specimen. The controlling variable was taken as the ratio of specimen breadth (b) to specimen depth (d), i.e. the b/d ratio, which was varied from 4.0 down to 0.295. The limiting dimensions in order for plane strain conditions to prevail can usually not be found by theoretical analysis alone, but must be established by experiment²³.

(1) Test Set-Up and Procedure

All the tests in Series 5 were performed using the Macklow-Smith hydraulic testing machine described earlier in section 4.2.1. The loading rates used for the tests were all 'fast', i.e. greater than 1.0 $\mu\text{m/sec}$. This allowed the average values of E and L_0 from Fig. 4.18 for 'fast' rates to be used, viz:-

$$E = 20,0 \text{ GPa}$$

$$L_0 = 2,0 \text{ m}^2/\text{GN for beams with } d = 101,6 \text{ mm}$$

$$L_0 = 10,2 \text{ m}^2/\text{GN for the slab with } d = 59 \text{ mm}$$

$$L_0 = 16,0 \text{ m}^2/\text{GN for the slab with } d = 50,8 \text{ mm}$$

(For specimen dimensions, see Table 4.12)

Note that L_0 , the unnotched compliance, varies with beam or slab depth, but is independent of beam or slab width. This is consistent with the definition of compliance in equation 3.10. The L_0 for the particular specimen depth is calculated from the relationship of equation (4.1), with $E = 20,0 \text{ GPa}$.

Those tests which measured the longitudinal and transverse strains in the beams in addition to the load and deflection, were monitored on the N.E.P. Ultra-Violet recorder. This permitted recording of a simultaneous record of load, deflection and strain, which could afterwards be re-plotted in the form of load-deflection curves and strain-deflection curves. The other tests employed the stiffened load-spreader

rig and the 250x125 mm I-beam stiffened test machine, except for the two slab specimens. This allowed the Watanabe X-Y plotter to be used for the remaining tests, since stable type fractures were produced by all the specimens. A photograph of the test set-up for a slab test is shown in Fig. 4.31. It should be noted that all the deflection measurements in Series 5 were valid measurements.

All the specimens were tested with a major, support span of 0,610 m, and a minor loading span of 0,203 m. The basic testing apparatus was the same as for previous series, and is fully described in Appendix D. The dimensions of the specimens are set out at the foot of Table 4.12, and comply with the specifications of Ref. 23, p.14.

(2) Test Results

The results of the tests on the slab and beam specimens of Series 5 are presented in Table 4.12. Initial notch depths varied from 6,4 mm to 13 mm, and b/d ratios from 0,295 to 4,0. Specimens 5.5 and 5.6 had cast-in strain gauges which measured the longitudinal and transverse strains. The gauges were situated approximately 6 mm above the notch roots, i.e. a total of approximately 13 mm from the tension face of the beam. The strain gauges were mounted at the centre of thin copper I-sections designed to extend the gauge length of the 2 mm long strain gauge. These copper I-sections were not considered to affect the fracture behaviour of the beam before the critical point of instability was reached. However, after initial fracture, the copper I-sections acted as tensile reinforcement to the beam, and affected the post-cracking behaviour. For this reason, the true effective fracture surface energy for specimens 5.5 and 5.6, could not be obtained, and the load-deflection curve after peak load was unrepresentative of the actual fracture.

The notch-depth ratios at failure for the slabs were calculated according to the method outlined in Appendix B, except that the relevant L_0 for the particular slab depth had to be used (see sec. 4.2.5 (1)). It was still permissible, however, to use the compliance-notch depth relationship of Fig. 3.3, since this relationship is in terms of dimensionless variables. Table 4.12 shows that the slabs (specimens 5.1 and 5.2) were far more compliant than the beams (specimens 5.3 to 5.7). Nevertheless, the calculated notch-depth ratios at failure

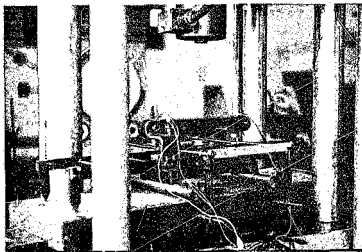


FIG. 4.31 SLAB TEST SET-UP, SERIES 5

Table 4.12 Rectangular Notches: Results for Series 5: Plane Stress/Strain Tests

Loading System Specimen No. (Cube Str. MPa)	1 Initial Notch Depth (mm)	2 b/d Ratio	3 Strain Rate (µm/sec)	4 Load Ratio	5 Max. Load Failure (kN)	6 Initial Compliance L_i (mm ² /GN)	7 Compliance at Failure L_F (mm ² /GN)	8 $\Delta L = L_F - L_i$ (mm ² /GN)	9 Notch Ratio at Fail. $\frac{c}{d}$	10 Notch Depth at Fail. c (mm)	11 $f(c/d)$ Depth From Fig. 3.2	12 Total Fracture Energy $A_{2,6}$ (Nm)
5.1 (31.97)	8.0	4.0	6.5	100	1,798	17,142	20,043	2,901	0.304	15.44	0.443	0.340
5.2 (32.42)	13.0	3.429	6.0	100	2,095	13,318	15,829	2,511	0.325	19.18	0.454	0.371
5.3 (31.97)	13.0	0.739	10.0	5.7	2,553	5,540	5,875	0.335	0.263	26.72	0.414	0.547
5.4 (32.42)	12.0	0.679	2.7	3.5	2,334	2,559	2,960	0.401	0.278	28.24	0.427	0.332
5.5 (33.13)	6.35	0.620	-	-	4,080	1,826	2,378	0.552	0.300	30.48	0.440	-
5.6 (33.13)	6.35	0.335	-	-	2,490	1,782	2,526	0.744	0.340	34.54	0.461	-
5.7 (32.42)	12.7	0.295	4.4	2.4	1,070	2,210	2,683	0.473	0.297	30.18	0.437	0.125

Dimensions of Specimens

5.1 b = 203.2 mm d = 30.8 mm (slab)
 5.2 b = 203.2 mm d = 59 mm (slab)
 5.3 b = 75 mm d = 101.6 mm
 5.4 b = 69 mm d = 101.6 mm

5.5 b = 63 mm d = 101.6 mm
 5.6 b = 34 mm d = 101.6 mm
 5.7 b = 30 mm d = 101.6 mm

Values of L_i : Specimen 5.1 and 5.2: $L_i = 12.6$ mm²/GN (average)
 Specimen 5.3 to 5.7: $L_i = 2.0$ mm²/GN

'Fast' Strain Rate ≥ 1.0 µm/sec
 'Slow' Strain Rate < 1.0 µm/sec

of the slabs and beams were comparable.

The fracture parameters calculated from the specimens of Series 5 are shown in Table 4.13. The initial notch-depth ratios of the specimens, which were all notched, varied from 0,063 to 0,157. On the basis of the average curves of G_c and K_{Ic} versus c/d of Fig. 4.28, it was assumed that any variation of G_c or K_{Ic} for the present tests was due to the variation of the b/d ratio, and not to the variation of the c/d ratio. This implies that the variation of G_c or K_{Ic} for the present tests was due to a change in the stress field ahead of the crack, from a predominantly plane strain condition in the case of the slabs to a predominantly plane stress condition in the case of the deep beams. Diagrams of the relevant results and fracture parameters are given in Fig.'s 4.32 and 4.33.

Fig. 4.32 shows the variation of G_c and K_{Ic} with b/d ratio. Included in the plot are the average values of G_c and K_{Ic} from Series 3 and 4, for $b/d = 1,0$. The figure shows the average curves for G_c and K_{Ic} , as well as the limiting envelopes. The average K_{Ic} value varies from $1,36 \text{ MN/m}^{3/2}$ for $b/d = 0,35$ to $0,52 \text{ MN/m}^{3/2}$ for $b/d = 4,0$, while the corresponding values for G_c are $84,0 \text{ N/m}$ to $14,5 \text{ N/m}$. It should be stressed, however, that the curves are derived from tests on only seven specimens, besides the average values from Series 3 and 4. In particular, the results should be viewed with caution for b/d ratios greater than 1,0, and less than about 0,5. For b/d greater than 1,0, only two test results were available, and for b/d less than 0,5, difficulty was experienced in monitoring the tests accurately. This is shown by the increasing scatter in results as the b/d ratio decreases. Nevertheless, on the basis of the rather limited results available, there does seem to be a trend of both G_c and K_{Ic} with b/d ratio, and the thinner beams seem to produce a higher measured fracture toughness than the thicker slabs. Series 5 was intended to merely provide a tentative initial evaluation of the effects of plane stress/strain, and much more work would be required to confidently define a definite relationship between the fracture parameters and specimen dimensions. The curves of Fig. 4.32 cannot be regarded as being more than assumptions of the possible trend of the fracture parameters with b/d ratio. It is clear, though, that specimen dimensions exercise

Table 4.13 Fracture Parameters for Series 5:
Plane Stress/Strain Tests

			1	2	3	4	5	6	7	
Loading System			Initial	b/d	Strain	Load	G _c	K _c	γ̄	
Specimen No.			Notch	Ratio	Rate	Ratio				
(Cube Str.			Depth							
MPa)			(mm)		(μm/s c)	(per cent)	(N/m)	(MN/m ^{1.5})	(N/m)	
MACKLON-SMITH	I-Beam Stiff	250x125 mm I None	5.1 (31,97)	8,0	4,00	6,5	100	14,5	0,52	19,5
		None	5.2 (32,42)	13,0	3,429	6,0	100	14,2	0,51	19,8
		None	5.3 (31,97)	13,0	0,738	10,0	5,7	21,1	0,64	41,2
		None	5.4 (32,42)	12,0	0,679	2,7	3,5	22,8	0,66	26,8
		None	5.5 (33,13)	6,35	0,620	-	-	94,5	1,33	-
	I-Beam Stiff	None	5.6 (33,13)	6,35	0,335	-	-	151,1	1,66	-
		None	5.7 (32,42)	12,7	0,295	4,4	2,4	28,1	0,73	23,4
		None								
		None								
		None								

For Details of Specimen Dimensions, see Table 4.12

'Fast' Strain Rate $\geq 1,0 \mu\text{m/sec}$

'Slow' Strain Rate $< 1,0 \mu\text{m/sec}$

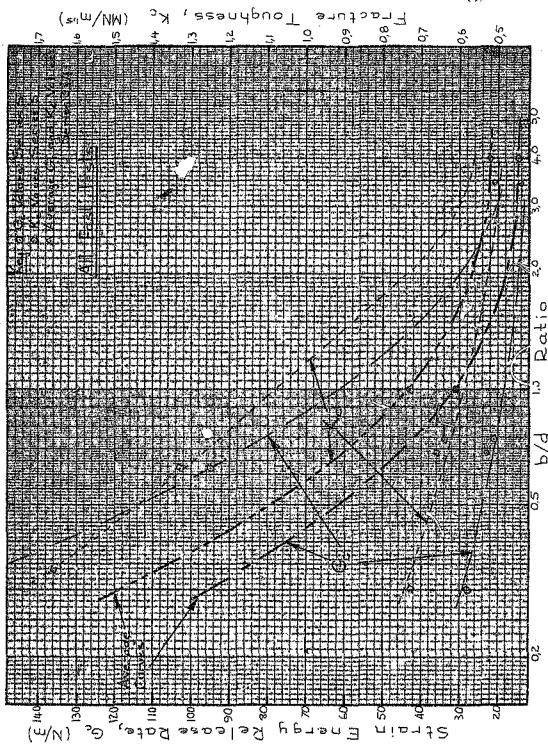


FIG. 4.32 G_c AND K_{Ic} VERSUS b/d , SERIES 5

a marked influence on the measured fracture values, and that some restrictions will need to be placed on allowable specimen dimensions for a valid fracture test on cemented materials. Generally, the thicker sections give lower values of K_{Ic} and G_c , indicating that a more truly plane strain condition applies in the specimen.

Fig. 4.33(a) and (b) shows the load-deflection curves and the strain-deflection curves for the two beams 5.5 and 5.6 of Series 5 respectively. The b/d ratio of beam 5.5 was 0.62, and that of beam 5.6 was 0.335. The post-peak portions of the load-deflection curves are shown, but they were affected by the cast-in copper I-sections on which the strain gauges were mounted, and are therefore unrepresentative. The following points should be noted from the curves:-

(i) The load-deflection curve deviates from true linearity at about 50 per cent of the peak load. This agrees with observations by Glucklich¹³.

(ii) The failure strain (longitudinal), represented by the strain at the peak load, of beam 5.5 was 80 microstrains, and that of beam 5.6 was 198 microstrains. Note, however, that as the crack propagates so the strain becomes concentrated in the crack, and therefore increases rapidly. The strains were monitored in the notch cross-section, and therefore represent the fracture strains of the beam.

(iii) The failure strain (transverse) of beam 5.5 was 57 microstrains and that of 5.6 was 68 microstrains.

However, the measured fracture toughness of beam 5.5 was $K_{Ic} = 1.33 \text{ MN/m}^{3/2}$, while that of beam 5.6 was $1.66 \text{ MN/m}^{3/2}$. It is possible that a small change in the stress/strain field ahead of the crack tip could account for a large change in measured fracture toughness.

(iv) The measured Poisson's Ratio $\nu (= \epsilon_T/\epsilon_L)$ for the initial linear elastic portion of the curves varied from an average of 0.08 for beam 5.6 to an average of 1.02 for beam 5.5. However, it is possible that the ϵ_L in beam 5.5 was not accurately monitored, since the measured failure strain was only 80 microstrains.

Inaccurate measurement could result from slippage between the gauge and the copper I-section.

Characteristic load-deflection curves for the specimens were very similar to those presented previously in Fig. 4.27, i.e. all the beams exhibited

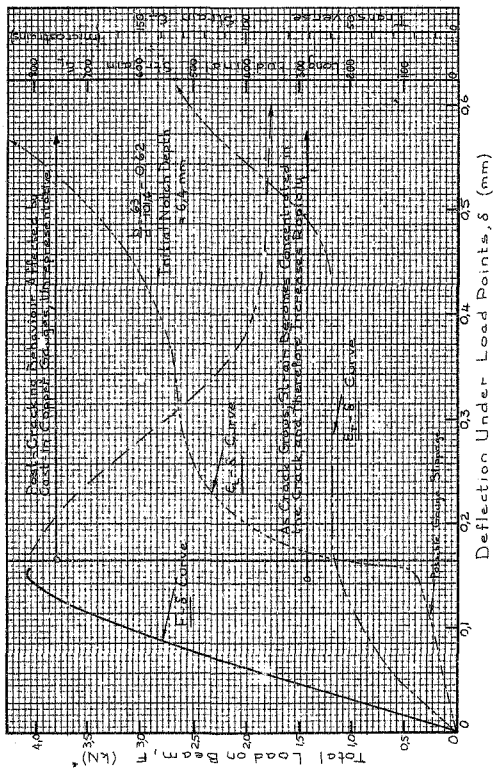


FIG. 4.33 (a) LOAD-DEFLECTION AND STRAIN-DEFLECTION
CURVES FOR BEAM 55, SERIES 5
 (Cast-in Strain Gauges)

an initial straight line portion which became more compliant at failure due to slow crack growth from the cast-in notches. Stable type fractures were recorded for all the specimens. The slab specimens were considerably more compliant than the beam specimens, and they also exhibited a completely stable-type fracture despite the fact that the unstiffened testing machine was used in order to test them.

From Table 4.13, $\bar{\gamma}$ does not show as great a variation with b/d as does G_c or K_{Ic} . For this reason, as well as due to the lack of results for $\bar{\gamma}$, no plot of $\bar{\gamma}$ versus b/d is shown. Nevertheless, Table 4.13 indicates that the measured effective fracture surface energy seems to increase with decreasing b/d ratio, i.e. $\bar{\gamma}$ increases as the constraint on the specimen decreases.

To conclude, the dimensions of a fracture specimen seem to significantly affect the measured fracture toughness of the specimen. Tests in Series 5 on a cemented material indicated that plane strain conditions are more truly approached in thick slabs than in thin, deep beams. However, much work still needs to be done to arrive at necessary minimum dimensions for fracture tests on cemented materials, and the field is still largely unexplored.

4.2.6 Series 6 : Study of Notched Asphalt Beams

Series 6 was the briefest test series of the laboratory test programme, and was aimed at providing a very brief, introductory study of the fracture behaviour of notched asphalt beams in the low temperature range, as a comparison with the fracture behaviour of the cement-mortar specimens of Series 1 to 5. Five asphalt beams of nominal size 101x101x610 mm were tested at a temperature of about minus twenty five degrees Celsius, in order to exhibit brittle fracture. This temperature was well below the glass transition temperature, T_g , of the asphalt, which was estimated to have been a few degrees below freezing. (Attempts to test beams at temperatures above T_g failed due to the excessive visco-elastic deformations of the beams under self-weight). One beam was unnotched, three beams had initial notch depths of 12,7 mm, and one beam had an initial notch depth of 25,4 mm. All the notches were sawn into the beams after casting. The asphalt mix was a gap-graded mix designed to B.S. 594, and employed mine sand as fine aggregate and crushed granite as coarse aggregate.

(1) Test Set-Up and Procedure

The tests on the asphalt beams were designed to show the variation of fracture parameters with strain rate for a constant notch depth of 12,7 mm, as well as the variation of fracture parameters with notch depth for a constant strain rate. Due to the limited number of asphalt specimens available, only one unnotched beam was tested, and no variation of E or L_0 with strain rate could be established. It was therefore assumed that for strain rates greater than 1,0 $\mu\text{m}/\text{sec}$, the fracture behaviour of the beams would be unaffected by strain rate, as was the case for the mortar beams of Series 3 and 4.

The tests of Series 6 were done on the triaxial machine set-up used for the tests of Series 3. This allowed controlled rates of deflection to be applied to the beam specimens. The machine characteristics and basic apparatus are as described in section 4.2.3 (1) and Appendix D. Each specimen was sheathed in a plastic bag before freezing, in order to prevent the beam from warming up too quickly during the test, which was done at room temperature. The longest test ran for about twenty minutes, and the beam showed no signs of excessive visco-elastic deformation during this period as did beams above the glass transition temperature. It was therefore assumed that the specimens remained below T_g for the duration of the tests.

The Watanabe X-Y recorder was used to record the tests of Series 6. Although less stable type fractures of the beams were produced on the triaxial machine than would have been produced if, say, a hydraulic testing machine had been used, nevertheless, the X-Y recorder was capable of recording the load-deflection curves for the specimens.

(2) Test Results

Two assumptions had to be made in order to analyse the test records and evaluate the fracture parameters:-

- (a) The value of E (and the corresponding L_0) as obtained in the 'fast' test on the unnotched specimen could be applied to all strain rates in the tests. This assumption had to be made in the absence of an experimental relationship between Young's Modulus and strain rate. The effect of this assumption was to reduce the apparent values of K_{IC} and G_c for the lower strain rates where E

and L_0 can vary significantly from those values measured in 'fast' tests.

(b) The theoretical compliance-notch depth curve of Fig. 3.3 with $v=0,1$ applies to the asphalt beams. Although Fig. 3.3 theoretically applies to any material, it is conceivable that, for a highly visco-elastic material like asphalt, where creep and other time-dependent deformations can markedly influence the measured compliance, considerable errors might be introduced into the calculated values of the fracture parameters by using Fig. 3.3.

Hence, the reported results and fracture parameters for Series 6 should be regarded with caution. Generally, the results from those beams tested at lower strain rates are probably less accurate than those tested at higher strain rates, but no quantifiable estimate of the errors could be obtained. The results of the tests for Series 6 are presented in Table 4.14. The E and L_0 values assumed for analysis of the results were:-

$$\begin{aligned} E &= 9,4 \text{ GPa} \\ L_0 &= 4,3 \text{ m}^2/\text{GN} \quad (\text{i.e. } \text{GPa}^{-1}) \end{aligned}$$

One point should be noted from Table 4.14. The change in compliance at failure, ΔL , for specimen 6.4, far exceeded the changes in compliance for the other specimens. This was because specimen 6.4 was loaded at a relatively low strain rate, and experienced excessive visco-elastic deformation during testing. Fracture Mechanics theory assumes that such a change in compliance is attributable to slow crack growth prior to failure, but in the case of the asphalt beams, this was not necessarily true. The ΔL for specimen 6.4 was probably mainly due to visco-elastic deformation rather than slow crack growth. For beam 6.4, the apparent crack depth at failure, as calculated from ΔL , was therefore very large (66,0 mm) whereas the actual crack depth at failure was probably not much larger than the initial notch depth of 12,7 mm. Also, the strain rate did not have a large influence on the failure load of the specimen, as seen from column 3 of Table 4.14.

The fracture parameters, evaluated using the assumptions mentioned previously, are shown in Table 4.15. These values, however, should be viewed with caution. Specimen 6.4, for instance, has fracture toughnesses whose values far exceed those of the other specimens. This is directly due to the visco-elastic deformation of the specimen under the low loading rate. Provision for the separate effects on failure compliance

Table 4.14 Rectangular Notches : Results for Series 6: Asphalt Beams

Loading System Specimen No.	1 Initial Notch Depth (mm)	2 Strain Rate ($\mu\text{m}/\text{sec}$)	3 Max. Load at Failure (kN)	4 Initial Compliance I_i (m^2/GN) ($\text{E}=25\text{ GPa}$)	5 Compliance at Failure I_f (m^2/GN)	6 $\Delta I = I_f - I_i$ (m^2/GN)	7 Notch Depth Ratio at Fail. $\frac{c_c}{a}$	8 Notch Depth at Fail. c_c (mm)	9 $f(c/d)$ From Fig. 3.2	10 Total Fracture Energy $A_{F,8}$ (Nm)
TRIAXIAL MACHINE Slitted Load Spreader Rig	6.1 No Notch	21.8	5,019	4,255 ($\text{E}=9,44$)	4,975	0,720	0,233	23,67	0,391	1,577
	6.2 12.7 mm Notch	8.8	3,441	4,846	5,931	1,085	0,303	30,78	0,442	1,740
	6.3	2.4	3,006	5,690	6,912	1,222	0,318	32,31	0,450	1,050
	6.4	0.61	5,477	6,202	17,753	11,551	0,650	66,04	0,521	5,172
	6.5	7.1	3,638	5,524	6,854	1,340	0,379	38,51	0,478	1,149

'Fast' Strain Rate $\geq 1,0 \mu\text{m}/\text{sec}$ 'Slow' Strain Rate $< 1,0 \mu\text{m}/\text{sec}$

Table 4.14 Rectangular Notches : Results for Series 6: Asphalt Beams

Loading System Specimen No.	1	2	3	4	5	6	7	8	9	10
Initial Notch Depth (mm)	Initial Strain Rate (µm/sec)	Max. Load at Failure (kN)	Initial Compliance L_i (mm ² /GN) (E: GPa)	Compliance at Failure L_f (mm ² /GN)	$\Delta L = L_f - L_i$ (mm ² /GN)	Notch Depth Ratio at Fail. $\frac{c_c}{d}$	Notch Depth at Fail. c_c (mm)	f(c/d) From Fig. 3.2	Fracture Energy $A_{T,6}$ (Nm)	Total Fracture Energy (Nm)
6.1 No Notch	Fast	21.8	5,019	4,255 (E=9,44)	4,975	0.720	0.233	23.67	0.391	1,577
		8.8	5,441	4,846	5,931	1.085	0.303	30.78	0.442	1,740
		2.4	3,006	5,690	6,912	1.222	0.318	32.31	0.450	1,050
		0.61	5,477	6,202	17,753	11,551	0.650	66.04	0.521	5,172
6.5 12.7 mm Notch	Slow	7.1	3,638	5,524	6,854	1.340	0.379	38.51	0.478	1,149
		25.4								

'Fast' Strain Rate ≥ 1.0 µm/sec'Slow' Strain Rate < 1.0 µm/sec

Table 4.15 Fracture Parameters for Series 6:Asphalt Beams

			1	2	3	4	5
Loading System Specimen No.			Strain Rate ($\mu\text{m}/\text{sec}$)	Initial Notch Depth (mm)	G_c (N/m)	K_c ($\text{MN}/\text{m}^{1,5}$)	$\bar{Y}$ (N/m)
TRIAXIAL MACHINE Stiffened Load Pig	6.1	'Fast'	21,8	No Notch	78,6	0,85	76,4
	6.2		8,8	12,7 mm Notch	139,1	1,11	96,3
	6.3		2,4		46,2	0,64	58,1
	6.4	'Slow'	0,61		1312,0	3,21	286,3
	6.5	'Fast'	7,1	25,4	95,1	0,90	74,2

'Fast' Strain Rate $\geq 1,0 \mu\text{m}/\text{sec}$ 'Slow' Strain Rate $< 1,0 \mu\text{m}/\text{sec}$

of slow crack growth and creep and other time-dependent deformations has not yet been made in the basic Fracture Mechanics relationships for materials that exhibit such extraneous deformations. The result is that a pseudo-fracture toughness is evaluated for such specimens, which is not a measure of the true fracture toughness of the material. For example, specimen 6.4 has a pseudo-fracture toughness of $3.21 \text{ MN/m}^{1/2}$, which is well above the average for the other beams. Nevertheless, a material like asphalt by virtue of its greater ductility in comparison to other cemented materials, exhibits higher values of fracture toughness than the cement mortar used in Series 1 to 5. For instance, the average K_{IC} value for specimens 6.1 to 6.3 and 6.5 is $0.88 \text{ MN/m}^{1/2}$, compared to the average K_{IC} value for the notched mortar specimens of $0.73 \text{ MN/m}^{1/2}$. In addition, the asphalt beams were less notch-sensitive than the mortar beams, as shown by the relatively small effect of notch depth on failure load of the specimen. The notch in an asphalt beam does not represent as intense a stress concentration as does the notch in a mortar beam, and the greater ductility of the asphalt allows local high stress concentration to be relieved under load by flow of the material.

Curves of the fracture parameters versus the notch-depth ratio and strain rate are not presented due to the lack of available results. Characteristic load-deflection curves are presented in Fig.'s 4.34 and 4.35. Asphalt, being a more 'ductile' material than mortar, exhibited stable type fractures, even in the case of the unnotched beam. Generally, deflections of the asphalt beams were greater than those of the mortar beams.

Fig. 4.34 shows the characteristic load-deflection curve for the unnotched beam 6.1, superimposed on the load-deflection curve of a typical unnotched mortar beam, reproduced from Fig. 4.27. The greater ductility of the asphalt beam is clearly shown by the existence of a considerable tail portion to the curve.

Fig. 4.35 shows the load-deflection curve for the 12.7 mm notched specimen 6.4, in order to give an idea of the excessive deflections produced by asphalt beams loaded with low strain rates. A characteristic load-deflection curve for a 12.7 mm notched mortar beam is also shown in Fig. 4.35, as a comparison with the asphalt specimen. The mortar beam was selected from the tests of Series 4 and had a

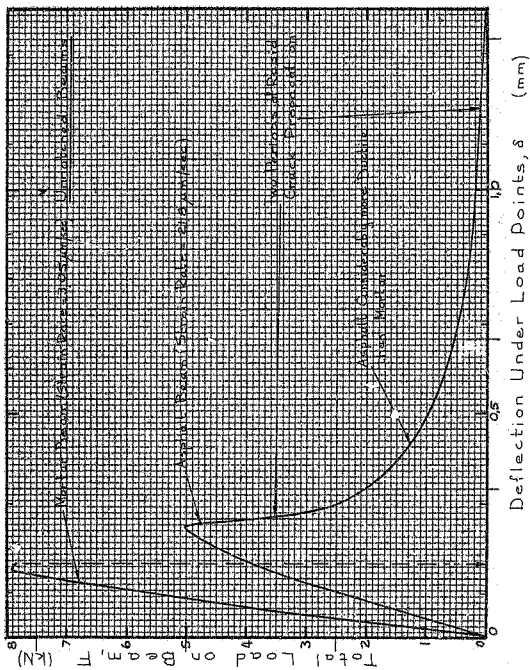


FIG. 4.34 LOAD-DEFLECTION CURVES FOR UNNOTCHED ASPHALT AND MORTAR BEAMS

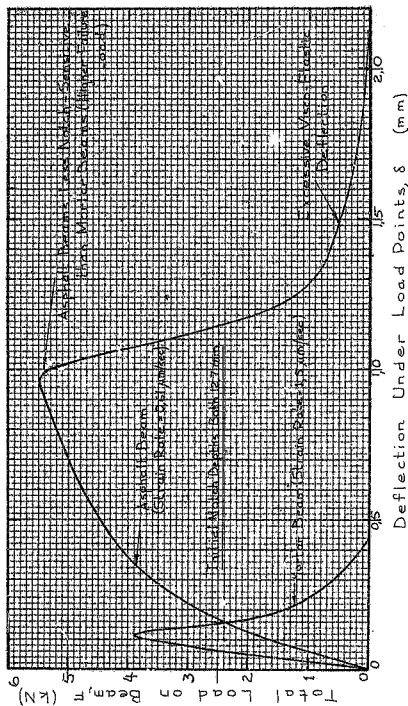


FIG. 4.35 LOAD-DEFLECTION CURVES FOR 12.7 mm NOTCHED ASPHALT AND MORTAR BEAMS

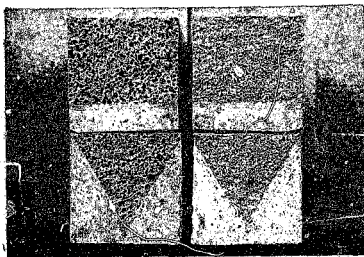
strain rate of 1,5 $\mu\text{m/sec}$, comparable to the strain rate of 0,61 $\mu\text{m/sec}$ of the asphalt beam.

All the asphalt beams in Series 6 showed two periods of rapid crack propagation during testing, one at the point of instability at peak load, and the other just prior to final failure. This implies that the Griffith energy balance is satisfied both at peak load instability, and just before final fracture.

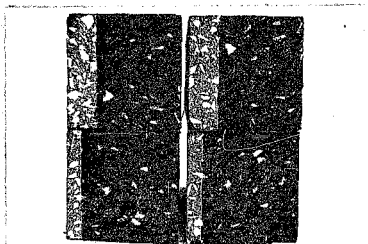
The basic test results of Chapter 4 indicated that specimen dimensions and loading machine set-up had significant effects on the fracture parameters measured in fracture tests on cemented materials. These results allowed the suggestion of initial, tentative test criteria in order to produce a valid fracture test on materials like cement mortars. These criteria, and further conclusions drawn from the tests, are presented in Chapter 5.

It was stated in sections 1.4.1 and 1.4.2 that fracture surface area and energy of cemented materials could refer to the hardened cement paste, or the aggregate, or the aggregate-paste interface. Cracking in such materials is not limited to one critical crack, but involves extensive micro-cracking in the highly stressed zone ahead of and surrounding the main crack. The fracture surfaces produced by the mortar and asphalt specimens tested in the laboratory confirmed these ideas. In both cases, the main crack was not limited to the cementing component alone, but was observed to have advanced through and around the aggregate particles, as well as propagating through the cement paste.

Fig. 4.36 shows photographs of the fracture surfaces produced by the mortar and asphalt beams. In the case of the mortar beams, the fracture surfaces of the two left-hand specimens were ink-shaded before photographing, to show up the cleavage surfaces of the crushed granite aggregate where the crack had propagated through the aggregate itself (both feldspars and quartz particles). The white marks on the fracture surfaces of the asphalt beams show where the crack propagated through the coarse aggregate granite chips. In addition, the occurrence of extensive micro-cracking surrounding the main crack was demonstrated by the ease with which the material could be flaked away from the fracture surfaces.



MORTAR BEAMS (Left-Hand Specimens
Ink-Shaded)



ASPHALT BEAMS (Sawn Notches)

FIG. 4.36 FRACTURE SURFACES OF ASPHALT
AND MORTAR BEAMS

CHAPTER 5

TENTATIVE CRITERIA FOR FRACTURE TESTING OF CEMENTED MATERIALS

The purpose of this chapter is to attempt to summarise and draw together the conclusions from the previous chapters, specifically Chapter 4. That chapter highlighted the need for further thorough investigation of effects such as creep and other time-dependent deformations, strain rate and specimen dimensions on the measured fracture parameters, in order to arrive at some logical and uniform method of fracture testing of cemented materials. A fracture test, in order to produce reliable results, must be designed and performed with practical conditions in mind, and the link between experimental testing and practical application must not be neglected. This chapter therefore puts forward some tentative criteria for the valid fracture testing of cemented materials on the basis of the experimental work described in this dissertation. These criteria are concerned with specimen size requirements, test set-up requirements and analysis of test records. Ideas have been freely borrowed from the metallic field in developing these tentative criteria. No hard and fast rules for fracture testing of cemented materials can be laid down at this stage, as in the metallic field, due to the relatively recent application of Fracture Mechanics principles to cemented materials. Nevertheless, the tests described in Chapter 4 have clearly indicated that certain restrictions must be imposed on the fracture testing variables in order to conduct a valid fracture test. Comments and criteria in this chapter will relate to the cement mortar specimens of Series 1-5 in Chapter 4. Insufficient tests on asphalt specimens were available to allow any definite conclusions to be made.

5.1 Specimen Size Requirements

Previous tests indicated that the dimensions of the fracture specimen could exercise a significant influence on the measured fracture toughness of the specimen. The calculation of the fracture toughness is based on a mathematical model of the actual stress/strain field ahead of the crack tip in a specimen, and the reliability of the K_{Ic} or G_c value obtained

from a test is dependent on the accuracy with which this mathematical model describes the actual stresses and strains inside the fracture zone. Linear Elastic Fracture Mechanics is based upon the assumption that the highly stressed zone in a fracture specimen is small in relation to the specimen size. The mathematical model of the stress intensity factor gives an exact representation only in the limiting case of zero plastic or pseudo-plastic strain. The smaller the highly stressed zone is in relation to the specimen size, the more accurate will be the analytical expression for the fracture toughness, since the elastic stress field surrounding the plastic zone will be closely approximated by the stress-intensity factor.

As the dimensions of a fracture specimen are increased, so the region around the crack tip in which the elastic stresses are adequately described by the analysis for K increases. The object of criteria for valid fracture testing is to restrict the specimen dimensions to some limiting values in order to ensure that the fracture toughness parameter (K) adequately describes the fracture process. Such criteria in terms of specimen size requirements can usually only be established experimentally. The pertinent specimen dimensions to be considered are crack length, c , specimen width (or thickness), b , and uncracked length, h . (Note that $h = d - c$). For the sake of consistency with Chapter 4, it is convenient to express these quantities as dimensionless ratios of the specimen depth, i.e. c/d ratio, b/d ratio, and h/d ratio. It should be realised, though, that absolute size requirements are important in addition to the relative size requirements expressed by the above ratios, but an insufficient range of absolute specimen sizes have been studied to arrive at reliable conclusions concerning absolute sizes for fracture specimens of cemented materials.

Investigators in the metallic field have defined the Plastic Zone Factor as being $R = (K_c / \sigma_{ys})^2$, where σ_{ys} is the yield strength of the materials¹⁴. (See previous sec. 1.3.1 and introduction to Chapter 2). They have stressed that when R is small in relation to specimen dimensions, plane strain conditions prevail, and the test is a valid K_c fracture test. Since R is a characteristic dimension of the plastic zone, investigators of metals have used the value of R to estimate limiting specimen dimensions, specifying that the dimensions should exceed a certain multiple of R , these multiples to be determined by an adequate number of trial K_c tests. The lower limits of the dimensions for which

K_c remains constant can be expressed in terms of R to produce useful 'working limits' for specimen dimensions. The parameter R was produced from theoretical considerations of the size of the plastic zone in a fracture specimen. It is suggested here, tentatively, that a corresponding factor R' might be defined in order to provide a useful basis for estimating the limiting dimensions for fracture tests on cemented materials. The significance of R' for cemented materials would depend on a definition rather than a theoretical derivation based on Linear Elastic Fracture Mechanics, and would be no more than a direct follow-on from current practice in the metallic field. R' must contain some parameter describing the strength of the material, and it is suggested that, for the flexure test specimens used for this dissertation, the most logical strength parameter would be the modulus of rupture of the material, i.e. the flexural tensile strength of the material, σ_c . Following on from the metallic field, R' , the parameter describing the pseudo-plastic zone in cemented materials, could then be defined as follows:-

$$R' = \left(\frac{K_c}{\sigma_c} \right)^2 \left(\begin{array}{l} \text{dimensions} \\ \text{of length} \end{array} \right) \quad (5.1)$$

R' can now be used to provide an estimate of the necessary specimen dimensions in order to produce a valid fracture test on cemented materials.

5.1.1. Crack-Length Requirement (c/d Ratio)

Refer to Figures 4.28 and 4.29 which show that G_c , K_c , and $\bar{\gamma}$ are all affected by the initial notch-depth ratio of the beams used to measure their values. Notched beams gave lower values of the fracture parameters than did unnotched beams. As suggested in section 4.2.4 (2)(c), this was possibly because a basic difference exists between the fracture behaviour of notched and unnotched beams. A notched beam causes the area of fracture damage and crack growth to be limited in extent to the notched cross-section, while an unnotched beam has no such limiting factor to its fracture behaviour. The fracture toughness of a cemented material depends primarily upon the micro-cracking and other energy-absorbing mechanisms at the notch root, and these mechanisms are limited in notched beams. It is interesting to note that fracture tests on steels reported in ref. 23 show that fracture specimens with extremely shallow notches produce higher effective fracture toughnesses

than specimens with deeper notches. The trend of K_c values reported in this reference is very similar to the trend of the average lines showing a constant value for K_c (or G_c) in Figure 4.28, with a discontinuity in the curve at very small notch-depth ratios, indicating a rapid increase in the value of K_c for unnotched and extremely shallow-notched beams. The results of Glucklich¹³, however, indicated no significant difference between the fracture values yielded by notched beam tests as opposed to unnotched beam tests.

All the specimens represented in Figures 4.28 and 4.29 were 101,6 x 101,6 mm beams in section. The initial notch depths, c_o , varied from 12,7 mm (shallow) to 50,8 mm (deep-notched beams). Assuming that the fracture parameters as evaluated from tests on notched beams represented a more valid estimate of the 'true' mean K_c of the material, the values of c/d and c/R' are shown below in Table 5.1 for the different notch depths. (Note, the average value of σ_t for the unnotched mortar beams of Series 1, 3 and 4 ('fast' tests) was $\sigma_t = 4,19$ MPa (10 beams)).

Table 5.1 Values of R' and c/R' for Cement Mortar Beams

	$\frac{c}{d}$	K_c (MN/m ^{3/2})	R' (mm)	$\frac{c}{R'}$
$c_o = 12,7$ mm	0,125	0,74	31,1	0,408
$c_o = 25,4$ mm	0,25	0,80	36,4	0,698
$c_o = 50,8$ mm	0,5	0,65	24,0	2,117
Ave. for notched beams	-	0,73	30,3	-
Ave. for unnotched beams	0	1,25	88,8	-

(where $R' = (K_c/\sigma_t)^2$)

The curve of $\bar{\gamma}$ versus c/d ratio (Figure 4.29) seems to indicate that $\bar{\gamma}$ decreases continuously as c/d increases (see the discussion of this in section 4.2.4 (2)(c)). Nevertheless, an average value for the notched beams could be inferred from the curve. Figures 4.28 and 4.29 indicate that the measured values of the fracture parameters are dependent on the type of fracture in the specimen. Catastrophic type fractures give higher values of fracture parameters than stable type fractures. Notched beams which fracture stably or semi-stably give consistent results for the fracture parameters, and the lower values of the fracture parameters, as represented by the lower limits for the

notched beams, are probably the more acceptable values for design purposes. It is suggested that, provided a beam specimen fractures in a stable or semi-stable fashion, the fracture parameters yielded from the tests are valid estimates of the 'true' mean lower limits. This criterion is critically dependent on the initial notch depth in the beam. For the cemented materials studied in the present tests, the above suggested criterion is satisfied when the c/d ratio is greater than about 0.1, and the c/R ratio is greater than about 0.4. Ratios of c/d greater than about 0.5 are also undesirable because the compliance-notch depth curve (Figure 3.3) rises very steeply at high c/d values. Under these circumstances, small errors in measured crack lengths can have undesirably large effects on the calculated values of the fracture parameters. The critical effect of testing machine stiffness must also be considered, however, and this is discussed later in section 5.2.1.

5.1.2 Specimen Width Requirement (b/d Ratio)

Refer to Figure 4.32, which shows the variation of G_c and K_{Ic} with b/d ratio. All the specimens represented in Figure 4.32 were notched specimens with the majority having c/d ratios greater than 0.1. Hence, it is assumed that the variation of G_c or K_{Ic} in Figure 4.32 was due to the variation of the b/d ratio, and not to the variation of the c/d ratio. As the stress field ahead of the crack varied due to a change in the b/d ratio, so the measured fracture parameters varied. Investigators in the metallic field have observed a marked increase in the fracture toughness of a material as the fracture changed from a plane stress, slant type of fracture in a thin sheet specimen to a plane strain, square type of fracture in a thick plate specimen. Figure 4.32 seems to indicate a similar effect for the cement mortar specimens investigated in the present tests. However, it must be stressed here again that Figure 4.32 was derived from tests on only seven specimens, besides the average values from Series 3 and 4. The curves can therefore not be regarded confidently as true indications of the variation of G_c and K_{Ic} with b/d ratio. They should rather be interpreted as showing the possibility that the b/d ratio can exert a marked influence on the measured fracture toughness. The wide scatter of results in Figure 4.32 also indicates the need for a more thorough investigation of this subject.

The results of Series 5 indicate that wider specimens in the form of

notched beams, are probably the more acceptable values for design purposes. It is suggested that, provided a beam specimen fractures in a stable or semi-stable fashion, the fracture parameters yielded from the tests are valid estimates of the 'true' mean lower limits. This criterion is critically dependent on the initial notch depth in the beam. For the cemented materials studied in the present tests, the above suggested criterion is satisfied when the c/d ratio is greater than about 0.1, and the c/R' ratio is greater than about 0.4. Ratios of c/d greater than about 0.5 are also undesirable because the compliance-notch depth curve (Figure 3.3) rises very steeply at high c/d values. Under these circumstances, small errors in measured crack lengths can have undesirably large effects on the calculated values of the fracture parameters. The critical effect of testing machine stiffness must also be considered, however, and this is discussed later in section 5.2.1.

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The results of Series 5 indicate that wider specimens in the form of

thick slabs give lower estimates of G_c and K_c than narrower specimens in the form of thin, deep beams. The fracture values approach a lower limit for b/d ratios greater than about 1,0. It is assumed that this lower limit represents a valid estimate of the 'true' plane strain values for the K_c of the mortar. Using the average curve of K_c from Figure 4.32, the values of b/R' for specimens spanning the range of b/d ratios are shown in Table 5.2 below:-

Table 5.2 Values of R' and b/R' for Mortar Beams

Specimen No.	$\frac{b}{d}$	b (mm)	Average K_c From Fig. 4.32 (MN/m ^{1/2})	R' (mm)	$\frac{b}{R'}$
5.1	4,0	203,2	0,52	15,4	13,20
Average for Sa. 3 and 4	1,0	101,6	0,73	30,3	3,35
5.5	0,62	63,0	0,97	53,5	1,18
5.6	0,335	34,0	1,39	109,8	0,31

(where $R' = \left(\frac{K_c}{\sigma_c}\right)^2$)

R' , the parameter describing the pseudo-plastic zone, increases, and b/R' decreases as the b/d ratio decreases. On the basis of Figure 4.32, the following tentative criteria are suggested for tests on cemented materials such as cement mortars:-

$$\begin{aligned} b/d \text{ ratio} &\geq 1,0 \\ b/R' \text{ ratio} &\geq 3,35 \end{aligned}$$

These criteria are open to improvement as more results from tests similar to those in Series 5 become available. Other similar criteria would presumably have to be developed for tests on other types of cemented materials such as asphalts.

5.1.3 Requirements for Uncracked Length of Specimen (b/d Ratio)

The uncracked length of a rectangular notched bend specimen, h , is defined by $h = d - c$. The value of h governs the shape of the load-deflection curve produced in a test, and the applicability of the expressions for the fracture parameters to the specimen. An examination of the curves in Figure 4.27 indicates that, as h decreases, so the deviation of the curves from linearity increases before the critical point of instability, indicating that extensive slow crack growth is occurring. Slow crack growth in the 50,8 mm deep-notched beams initiated at low stress levels

due to the high stress concentration of the notch. The average values of the fracture parameters for the deep-notched beams were lower than for the shallow notched beams. The presence of a deep notch considerably reduces the energy level in the specimen at the point of instability. It is possible that this results in unrepresentatively low values of the fracture parameters being recorded, due to the relatively non-violent type of fracture produced in such beams. Deep-notched beams also produce flatter load-deflection curves with a less well-defined peak at the critical point of rupture (see Figure 4.27) than for shallow-notched beams. This leads to uncertainty and inaccuracy in defining the exact point of fracture instability. The estimated compliance at failure critically affects the calculated notch-depth ratio at failure, and hence the evaluation of the fracture parameters. In addition, deep-notched beams, with their distinctly non-catastrophic type of fracture, require work to be done on the specimen even at the critical point of peak load, in contrast to shallow-notched beams which exhibit a degree of spontaneous crack propagation after the peak load under the stored elastic energy in the specimen. This implies the possibility that the deep-notched mortar beams (50,8 mm) used in the present study never attained a true state of critical crack propagation i.e. the energy requirement for brittle fracture as expressed by equation 1.3 (Griffith's equation) was never satisfied. This could explain the lower measured fracture values for the deep-notched beams.

For specimens similar to the mortar beams used in this study, it is suggested that h/d should not be less than 0,5, and preferably not less than about 0,625, on the basis of the 38,1 mm notched beam tested in Series 1 which still exhibited a degree of spontaneous crack propagation.

5.1 Requirements for Loading Spans

The requirements for the loading spans are taken directly from the specifications given in ref. 23, p.14. These specifications are concerned with the type of bending in the specimen, and the degree of specimen indentation and friction at the supports. The expressions for K_c and G_c presented in equations (3.2) and (3.9) respectively can be applied to bend specimens provided the following ratios of loading spans to specimen depth are adhered to:-

$$\frac{\text{Minor Span } S_2}{d} \geq 2 \quad \text{ensures that four-point bending is equivalent to pure bending.}$$

$\frac{\text{Major Span } S_1}{d} > 4$ ensures that errors due to specimen indentation and friction at the supports are negligible.

As an example of the need to adhere to these loading span requirements, refer to Figure 4.29 which shows that the 'vee'-notch beams used in Series 3 produced results for $\bar{\gamma}$ that showed a distinct departure from the trend of the diagram. These 'vee'-notch beams were tested using a smaller major support span and a smaller minor loading span than the remainder of the beams which met up with the requirements for loading spans. The 'vee'-notch beams are a case in point indicating the need to develop and adhere to fracture testing criteria.

5.1.5 Summary of Suggested Size Requirements

For specimens similar to the cement mortar beams studied for this dissertation, restrictions on the requirements for crack length and specimen size are summarised below in Table 5.3. These requirements are at present only in the form of relative size ratios, rather than absolute size requirements, which still need to be established. The reliability of the size requirements from columns 2 and 4 of the table below depends upon the validity of using the definitions of R' adopted here as a means of estimating specimen size requirements. This approach definitely needs further careful attention.

Table 5.3 Summary of Suggested Size Requirements for Cement Mortar Bend Specimens

	1	2	3	4	5	6	7	
	<u>Crack Length</u>		<u>Width</u>		<u>Uncracked Length</u>	<u>Loading Spans</u>		
	c/d	c/R'	b/d	b/R'	h/d	S ₁ /d	S ₂ /d	
{ Notched Bend Specimen	> 0,1	> 0,4	≥ 1,0	≥ 3,35	> 0,625	> 4	≥ 2	
								6:1 Major Span/depth
								2:1 Minor Span/depth
where R' = (K _{rc} /σ _{rc}) ²								

The 12,7 mm and 25,4 mm rectangular notched specimens of Series 1 to 4 complied with the above requirements. However, the 'vee'-notch beams did not comply with the loading span requirements, and the 50,8 mm notched beam did not comply with the uncracked length requirement. Only the slab specimens in Series 5 complied with the width requirement.

The practical situation for which the fracture tests are being designed

must continually be borne in mind. The type of fracture test will vary greatly depending on whether the fracture strength for a cement-treated road base (plane strain situation) is being sought as opposed to, say, the fracture strength for a thin notched concrete pylon (plane stress situation).

5.2 Requirements for Test Set-Up in Fracture Tests on Cemented Materials

The two primary factors that can be controlled when designing a test set-up for fracture testing are the loading rate of the machine and the stiffness of the machine/loading system. Both of these factors affect the values of the fracture parameters measured in a test, and the type of fracture produced. Implications from the results of tests in Series 3 and 4 are presented below.

5.2.1 Requirements for the Strain Rate in Fracture Tests on Cemented Materials

Refer to Figure 4.21 (a) and (b). This figure shows that strain rate has a marked effect on the values of the fracture parameters measured in a test. Assuming the 25,4 mm notched beams in Figure 4.21 (b) to have produced a valid estimate of the fracture parameters, it is seen that the average K_{Ic} for a strain rate of 0,01 $\mu\text{m/sec}$ can be as low as 0,59 $\text{MN/m}^{3/2}$ - a reduction of 27 per cent. The reduced values of K_{Ic} and G_c were calculated using the time-dependent Young's Moduli and compliances from Figure 4.18. The curves for the unnotched beams in Figure 4.21 (a) show an even more distinct reduction in fracture toughness values for low strain rates than do the notched beams of Figure 4.21 (b). A discussion of this appeared earlier in section 4.2.4 (2)(c).

Important effects in cemented materials of construction linked to strain rate are creep and other time-dependent visco-elastic deformations. Especially in asphalt specimens, the visco-elastic deformations can give a pseudo-fracture toughness that is well above the true fracture toughness for the particular notch depth in the beam (see previous section 4.2.6). The strain rate selected for a particular fracture test will depend on the practical situation being modelled in the test. Situations such as road pavements in which the deformations are time-dependent might require a low strain rate in order to estimate the time-dependent fracture toughness of the pavement. However, the fracture toughness under a transient wheel load in the same pavement might be considerably higher

due to the higher strain rate, and this higher fracture toughness would need to be measured in a 'fast' test. For the cement-mortar specimens of Series 3, 4 and 5, no significant change in the values of the fracture parameters occurred for strain rates greater than about $1.0 \mu\text{m/sec}$ (i.e. 'fast' tests). Insufficient results were available for the asphalt beams in Series 6 to make any conclusions about the limiting strain rate above which the fracture parameters were invariant.

On the basis of the test results, it is suggested that, for cemented materials such as mortars and concretes, a strain rate in the region of $1.0 \mu\text{m/sec}$ is the borderline between those tests which produce approximately constant values of the fracture parameters, and those tests which show a marked reduction in the measured values of the fracture parameters.

5.2.2 Requirements for Testing Machine Stiffness

The tests of Series 4 indicated that the stiffness of the loading machine system had a critical effect on the post-cracking behaviour of fracture specimens after the onset of rapid crack propagation. The balance between the strain energy stored in the loading set-up and the elastic energy stored in the specimen at the point of failure must be considered. This balance of energies governs the interaction between loading system and specimen during critical crack propagation. If the loading system is a relatively 'soft' one, energy can be fed back into the specimen during the period of rapid crack propagation. This influences the type of fracture produced by the specimen. A 'soft' system can produce a catastrophic type fracture in a notched specimen that would otherwise exhibit a stable or semi-stable type fracture. Hence, the stiffness of the loading set-up critically affects the recorded shape of the load-deflection curve of a fracture specimen after peak load, and therefore affects the measured value of $\bar{\gamma}$. Most investigators have neglected the effects of machine stiffness, and have often failed to record a tail portion to their load-deflection curves. Nevertheless, for the beams tested in Series 4, it was shown that a considerable degree of post-cracking 'ductility' was present in the specimens.

The stiffness of both the basic testing machine itself and the loading apparatus need to be considered when designing a loading set-up for fracture tests on cemented materials. Generally, it was found that a hydraulic testing system was better than a mechanical system, due firstly to the lower energy level stored in the hydraulic system under

load, and secondly to the fact that the hydraulic system could not 'follow-up' the specimen deflection during failure as quickly as could the mechanical systems. It is usually possible to stiffen up a testing machine system by including some sort of load-absorber (e.g. a steel I-beam) in series between the testing machine platens and the fracture specimen (see Figure 4.24). The loading apparatus normally consists of a loading yoke or load-spreader rig for transferring the load from the machine to the specimen and a specimen support system. Tests in Series 4 indicated that the stiffness of the loading yoke exercised a greater influence on the type of fracture in the specimen than did the stiffness of the machine system, provided a large-capacity hydraulic machine system is being used. For the 'softer' mechanical systems of Series 3 and 4, the loading machine itself was probably the major contributing factor to the catastrophic type fractures that were recorded. The principle is to stiffen up both the testing machine and the loading apparatus sufficiently so that energy is not fed back into the specimen during failure to cause an unrepresentatively catastrophic type fracture. The energy stored in the specimen itself at failure must be considered in order to predict whether a stable or catastrophic type fracture should result (see Table 4.11). Finally, the autographic recorder used to monitor the fracture test should be compatible with the testing set-up, i.e. if a 'softer' loading set-up is unavoidable, then the recorder should be capable of sufficient response at failure to the transient effects of load and deflection.

5.3 Analysis of Load-Displacement Curves

The values of the fracture parameters K_{IC} and G_c are computed on the basis of the load corresponding to a well-defined unstable advance of the crack. Section 2.1 outlined the requirements for a satisfactory fracture test. These requirements were that the test record should allow the values of load and crack extension at any point during the test, and specifically at the point of instability of crack extension, to be accurately measured. In order for the energy balance of equation 1.3 (Griffith's equation) to be satisfied, sufficient elastic strain energy must be stored in a specimen at the critical point of failure to 'drive' the process of crack extension, i.e. there must be a degree of spontaneous crack propagation in the specimen. If this condition is not satisfied, then the load-deflection curve becomes difficult to interpret in terms

of the actual point of instability. Referring to the characteristic load-deflection curve for a deep-notched beam ($c/d \geq 0.5$) (see Figure 4.27), it is seen that the change in compliance of the specimen in the region of the maximum load is considerable, and it is therefore difficult to infer a true failure compliance for such a specimen. In addition, it is doubtful whether such a deep-notched specimen ever experiences spontaneous crack propagation, due to insufficient elastic energy being stored in the specimen at failure. The characteristic load-deflection curves for the shallow and intermediate notched beams (Figure 4.27) are easily interpretable in terms of the instantaneous values of load and crack extension at the onset of unstable fracture. This requirement for a pronounced peak to the load-deflection curve is closely linked to the uncracked length of the specimen (see earlier section 5.1.3). Curves with ill-defined peaks are difficult to interpret and can lead to substantial errors in the calculation of the fracture parameters. It might be necessary in the future development of suitable criteria for fracture tests on cemented materials to limit the amount of non-linearity allowable in a test record, similar to the limits imposed on fracture specimens in the metallic field. (For a full discussion of the limits of non-linearity in metallic specimens, see Ref. 14).

5.3.1 Analysis of the Load-Deflection Curve of a 25.4 mm Notched Bend Specimen to Produce the Energy Rate Curve

Section 4.2.4 (d) reported tests on notched beams using a load-cycling technique in order to obtain stable crack growth. The beam is loaded until the critical point of fracture instability is reached, and the load is then rapidly 'dumped' before rapid crack propagation can occur. In this way, it was possible to make the crack grow in stable increments through the beam until final failure occurred. A further example of this technique is given in Figure 5.1, which shows a 25.4 mm notched beam load-cycled to failure. The figure represents the actual test output from the beam, and the failure curve is shown drawn in dotted lines. For a fuller discussion of the shape of the load-deflection curve, see section 4.2.4 (d). One extra point to notice, however, is that the re-loading compliance which is taken from the most nearly straight portion of the curve for each cycle is identical to the failure compliance of the previous cycle at the point of unstable crack propagation when the load was dumped. This is shown by the series of parallel lines in the

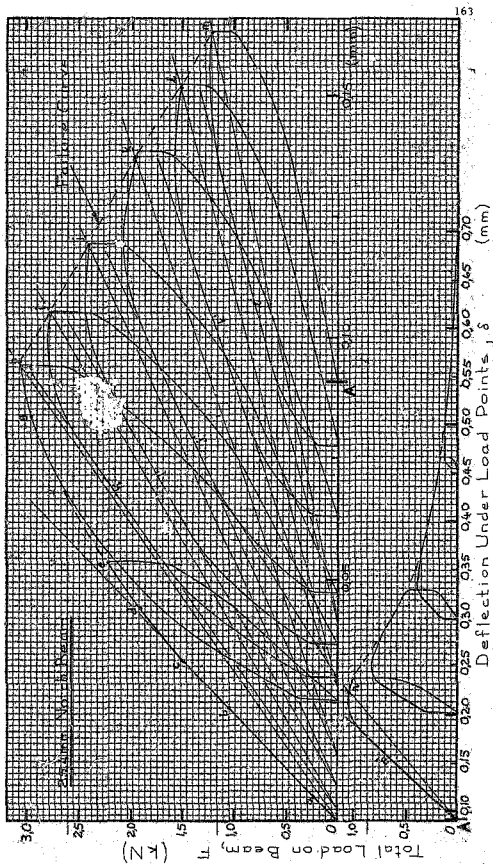


FIG. 5.1 LOAD-CYCLING TECHNIQUE TO PRODUCE ENERGY RATE CURVE

figure. For example, the reloading compliance of i' is identical to the compliance at failure for the previous cycle, represented by point i on the failure curve. The zero load points for each successive cycle are found by extrapolating the straight line portions of the reloading curves backwards to intersect the zero load axis.

Figure 5.1 was analysed in order to produce an energy rate curve for the beam, similar to the schematic representation shown in Figure 1.7. An energy rate curve is basically a plot of the energy demand rate as the crack grows versus crack extension. For a non-homogeneous multiphase material like cement mortar or concrete, the energy demand rate increases initially and then attains the critical value of G_c , when spontaneous fracture can occur. (See previous discussion of this in section 1.4.1 Case 2)). Before the point of instability, the energy demand rate is numerically equal to the energy release rate.

A number of discrete points were chosen on the failure envelope of Figure 5.1, and the strain energy release rates and values of instantaneous crack length were evaluated at each point. These chosen points are listed a to n in Figure 5.1. The results and analysis are shown in Table 5.4. Using this table, a plot of the energy demand rate versus crack length can be drawn (assuming energy demand rate equals energy release rate before instability). Before this plot can be discussed, the current concept of the growth of resistance to crack extension during a test must be explained. This concept is more fully discussed in Ref. 22 p.138 ff. The essence of the concept is that, as the crack extension force (or strain energy release rate) G is increased during a test, it is opposed by an increasing resistance to crack extension (usually called R by investigators in the metallic field, but not to be confused with the plastic zone size term.) Equilibrium between G and R is maintained up to the point of instability. (The crack extension resistance, R , may be thought of as analogous to the increasing resistance to plastic deformation due to work hardening in metals, which opposes the applied stress in an ordinary tension test.) By definition, G_c is equal to the value of R at instability and beyond this point, G increases more rapidly with crack growth than does R . G and R are equal up to the point of instability, but they represent distinctly different physical entities and have a different functional relationship to the test variables c and σ . For the notched bend specimens being considered, $G = \pi(1-\nu^2)\sigma^2 c/E$, but the function $R = f(\sigma, c)$ is not known.

Table 5.4 Rectangular Notches : Results for 25.4 mm Notched Beam to Produce Energy Rate Curve

	1	2	3	4	5	6	7	8
Loading System; to Point corresp. to Figure 5.1 (Obse Str. = 31.58 MPa)	Load at Point (kN)	Compliance at Point L_1 (m^2/GN)	Compliance at last Point L_2 (m^2/GN)	$A L = L_2 - L_1$ (m^2/GN)	Notch Depth Ratio $\frac{c}{d}$	Morch Depth c (mm)	$f(c/d)$ From Fig. 3.2	Str. En. Rel. Rate G at Point (N/m)
a	0,204	2,288	2,288	0	0,25	25,4	0,404	0,07
b	1,039	2,288	2,288	0	0,25	25,4	0,404	1,76
c	1,488	2,288	2,288	0	0,25	25,4	0,404	3,61
d	1,916	2,370	2,288	0,082	0,273	27,69	0,422	6,85
e	2,232	2,455	2,370	0,085	0,293	29,77	0,435	10,43
f	2,662	2,609	2,455	0,152	0,327	33,22	0,454	18,00
g	2,965	2,870	2,609	0,263	0,373	37,90	0,475	28,83
h	3,067	3,134	2,870	0,264	0,408	41,40	0,486	37,40
i	2,776	3,778	3,134	0,645	0,479	48,67	0,507	47,00
j	2,417	4,739	3,778	0,961	0,550	55,88	0,518	56,51
k	1,961	6,418	4,739	1,679	0,624	63,40	0,521	64,11
l	1,529	8,270	6,418	1,852	0,677	68,80	0,521	61,46
m	1,225	10,197	8,270	1,928	0,714	72,50	0,521	56,85

MACKLOW-SMITH HYDRAULIC AND STIFFENED LOAD RIG

Consider now Figure 5.2, which shows a plot of the strain energy release rate G versus crack length c , for the beam of Figure 5.1. This curve is derived directly from columns 6 and 8 of Table 5.4. As the crack grows, so the value of G increases, until a peak is reached at point k (represented also by point k in Figure 5.1). According to the expression for G in equation (1.8), the straight lines representing $\pi(1-v^2)\sigma^2 c/E$ are drawn in for each discrete point a to m in the figure, and the effective value of the slope $\pi(1-v^2)\sigma^2/E$ is shown on each straight line. This value is a measure of the macroscopic stress level surrounding the crack at that particular point, and is evaluated according to section 3.2.2 in order to take account of the effects of the reduced cross-section and the non-uniformity of stresses. The points of intersection between the straight lines and the G -curve represent the process of crack growth through the beam (i.e. the failure curve), as discussed fully in section 1.4.1.

It is suggested (see ref. 22) that for a particular crack length in the beam, there is a corresponding R . For such hypothetical R -curve is shown in Figure 5.2. As the stress level in the beam increases, so the value of G increases, and the value of R increases equally with G . However, at point X , a critical point of instability is reached, the value of G exceeds R , and an increment of crack growth occurs, which is then checked by an increase in resistance to crack extension represented by another R -curve for this new instantaneous crack length. Once the peak value of G at point k is exceeded, the crack can continue growing under decreasing stress levels (points l and m).

It is interesting to note that the peak load point in Figure 5.1 (point h) does not represent the peak G -value of Figure 5.2, represented by point k in Figure 5.1. This is in agreement with the results of Brown ¹⁷, who, in using the load-cycling technique, found that the fracture toughness increased with crack extension after the initial point of unstable crack growth, reached a maximum, and then tailed off. The G -curve of Figure 5.2 is very similar to the K -curves produced by Brown. Brown found, however, that the fracture toughness of pure cement paste was approximately constant with crack growth, as opposed to cement mortar whose toughness seems to increase with crack growth. Figure 5.2 indicates that the effective toughness of cement mortar can increase significantly (e.g. by as much as 60 per cent) as the crack grows through the specimen, and this phenomenon in itself constitutes a crack arrest mechanism in the beam.

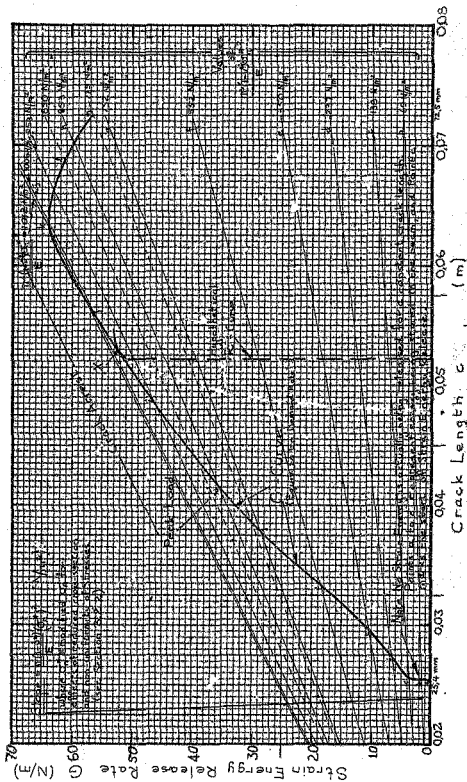


FIG. 5.2 ENERGY RATE CURVE FOR BEAM
OF FIG. 5.1

Figure 5.2 is a graphic illustration of the growth of a crack through a notched specimen, and lends considerable support to the hypothetical curves of Figures 1.5 and 1.7.

5.4. Concluding Summary

The investigations involved in the preparation of this dissertation were primarily aimed at identifying and isolating those test variables which had a significant effect on the measured fracture parameters from a fracture test. The particular material selected for testing was a Portland cement mortar, and the conclusions drawn from the tests strictly apply only to this material. (A few asphalt beams were also tested, but their results were inconclusive.) Following on from the various series of fracture tests on notched beam specimens of the mortar, initial tentative criteria for valid fracture testing of cemented materials could be suggested. These criteria were concerned primarily with specimen size requirements and the requirements for a satisfactory test set-up.

The three fracture parameters evaluated from the tests were the critical strain energy release rate (or crack extension force) G_c , the fracture toughness (or critical stress intensity factor) K_{Ic} , and the effective fracture surface energy $\bar{\gamma}$. It was found that initial notch-depth ratios (c/d), dimensional ratios of the specimen (b/d), loading rates used during the tests, and testing machine stiffness all had significant effects on the measured values of the fracture parameters. In addition, notched beams had lower values of fracture toughness than unnotched beams, which was in agreement with reported tests in the metallic field. The average fracture toughness value (K_{Ic}) for notched beams was $0.73 \text{ MN/m}^{1/2}$, while that for unnotched beams was $1.25 \text{ MN/m}^{1/2}$. The actual state of the stress/strain field ahead of the crack tip in a notched specimen seemed to critically influence the measured fracture toughness, depending on whether a predominantly plane stress or plane strain condition was prevalent in the beam. Wider, thicker sections such as slabs gave lower values of the fracture toughness than narrower, thinner sections such as deep beams. The slab sections were considered to approximate more truly to the plane strain state than the deep beams.

Considerable work is still needed before comprehensive criteria for valid fracture testing of cemented materials can be drawn up. In particular, the plane stress/plane strain effect (which is dependent on the relative specimen dimensions) and the necessary absolute sizes of specimens for valid fracture testing need to be studied in depth. It is probable

that each particular class of cemented materials (e.g. hydrated cements, asphalt cements, thermoplastic cements) will require a different set of test criteria specifically applicable to that class. This is because of the wide range of behaviour of different cemented materials under stress. Nevertheless, the concepts of strain energy release rate and fracture toughness as developed in the metallic field are applicable to cemented materials, and give a measure of the fracture susceptibility of such materials in the presence of natural or artificial notches. The development of suitable test criteria for the valid fracture testing of cemented materials is the first logical step towards applying fracture testing methods in the development of new and better materials of construction.

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APPENDIX ATHEORETICAL COMPLIANCE - NOTCH DEPTH RELATIONSHIP

The theoretical compliance - notch depth relationship is derived from three basic equations, viz.:-

The expression ²³ for K : $K = Y \frac{6Mc^{\frac{1}{2}}}{bd^{\frac{3}{2}}}$ (A.1)

where $Y = A_0 + A_1(c/d) + A_2(c/d)^2 + A_3(c/d)^3 + A_4(c/d)^4$

The expression for G in terms of the rate of change of compliance with crack length ¹¹ :

$$G = \frac{1}{2} \left(\frac{F}{b} \right)^2 \frac{dL}{dc} \quad (A.2)$$

and the relationship between K and G :

$$K^2 = \frac{EG}{(1-\nu^2)} \quad (A.3)$$

The values of the coefficients A are to be found in Table 3.1 in the main text, for the particular type of bending being considered.

The compliance L is defined as :

$$L = \frac{\delta}{F/b}$$

where δ = beam deflection under the load points. Refer to Figure 3.1.

The bending moment in the beam, M, for the case of four-point loading being considered (i.e. pure bending), can be written as:-

$$\begin{aligned} M &= \frac{F}{2} \left(\frac{S_1 - S_2}{2} \right) \\ &= \frac{F}{4} (S_1 - S_2) \end{aligned} \quad (A.4)$$

The compliance - notch depth relationship is then derived as follows:-

From Equation (A.2), $\frac{dL}{dc} = \frac{2b^2}{F^2} G$

Substituting for G from equation (A.3), and then for K from equation (A.1):-

$$\frac{dL}{dc} = \frac{2b^2}{F^2} \frac{(1-v^2)}{E} \left[Y \frac{6Mc}{bd^2} \right]^2$$

which, by substituting for M, reduces to :

$$\frac{dL}{dc} = \frac{9(1-v^2)(S_1 - S_2)^2}{2E} \frac{Y^2 c}{d^4} \quad (A.5)$$

Integrating with respect to c:-

$$L = \frac{9(1-v^2)(S_1 - S_2)^2}{2E d^4} \int Y^2 \left(\frac{c}{d} \right) dc \quad (A.6)$$

The integral $\int Y^2 \left(\frac{c}{d} \right) dc$ is an integral of a polynomial function in the variable (c/d), i.e. the notch depth ratio.

$$\text{Let } \int Y^2 \left(\frac{c}{d} \right) dc = Z \quad (A.7)$$

From elementary beam bending theory, an expression for E can be obtained for an unnotched beam in four-point bending (see Figure 3.1). The expression is:-

$$E = \frac{1}{48} \left(\frac{F}{\delta} \right) \frac{(S_1 - S_2)^2 (S_1 + 2S_2)}{I} \quad (A.8)$$

where I = the second moment of area of the section, and $\left(\frac{F}{\delta} \right)$ is the slope of the initial linear portion of the load-deflection curve for the unnotched beam.

Writing the unnotched compliance, L_0 , as:-

$$L_0 = \left(\frac{\delta}{F} \right) b, \text{ equation (A.8) can be re-written as follows:-}$$

$$E = \frac{1}{48} \left(\frac{b}{L_0} \right) \frac{(S_1 - S_2)^2 (S_1 + 2S_2)}{I}$$

Since $I = bd^3/12$, this reduces to:

$$2Ed^3 = \frac{(S_1 - S_2)^2 (S_1 + 2S_2)}{2 L_0} \quad (A.9)$$

From equations (A.6) and (A.7), a relationship between compliance L and crack depth c, where c is included in the integral Z, can now be

written down as follows:-

$$L = \frac{9(1-\nu^2)(S_1 - S_2)^2}{2Ed^3} Z \quad (\text{A.10})$$

Alternatively, a more convenient form of expression which eliminates E is a relationship between L/L_0 and c/d , which by using equation (A.9) can be written as follows:-

$$L = \frac{9(1-\nu^2)(S_1 - S_2)^2}{(S_1 - S_2)^2(S_1 + 2S_2)/2L_0} Z$$

which finally reduces to

$$\frac{L}{L_0} = \frac{18(1-\nu^2)}{(S_1 + 2S_2)} Z \quad (\text{A.11})$$

Equations (A.10) and (A.11) represent the required compliance-notch depth relationships.

The integral Z depends on the specimen dimensions, including the crack depth ratio c/d , and on the type of bending in the specimen. It also includes a constant which can be evaluated from boundary conditions. In particular, the constant includes the value of Poisson's ratio, ν , and equations (A.10) and (A.11) also show that the compliance-notch depth relationship is dependent on ν .

For the particular test geometry:-

$$S_1 = 0,610 \text{ m (24 in)}$$

$$S_2 = 0,203 \text{ m (8 in)}$$

$$d = 0,102 \text{ m (4 in)}$$

Using this, the values of Z as functions of c/d and ν , as obtained from integrating the expression in equation A.7 and inserting the boundary conditions, appear in Table A.1 below.

Table A.1 Values of Z

$\frac{c}{d}$	Values of Z		
	$\nu = 0,1$	$\nu = 0,15$	$\nu = 0,2$
0,0	0,0570	0,0577	0,0588
0,125	0,0597	0,0604	0,0615
0,25	0,0681	0,0688	0,0699
0,375	0,0845	0,0852	0,0863
0,5	0,1162	0,1169	0,1180
0,625	0,1858	0,1865	0,1876
0,75	0,3659	0,3666	0,3677

With the values of the integral Z evaluated, it is now possible to evaluate the complete compliance-notch depth relationship (A.11), and the values of L/L_0 for various c/d ratios and different ν values appear below in Table A.2.

Table A.2 Compliance-Notch Depth Relationship

$\frac{c}{d}$	Values of L/L_0		
	$\nu = 0,1$	$\nu = 0,15$	$\nu = 0,2$
0,0	1,0	1,0	1,0
0,125	1,047	1,047	1,046
0,25	1,195	1,192	1,189
0,375	1,482	1,477	1,468
0,5	2,039	2,026	2,007
0,625	3,260	3,232	3,191
0,75	6,419	6,354	6,254

Table A.2 appears in graphical form in the main text as Fig. 3.3.

APPENDIX BSAMPLE CALCULATION FOR K_c , G_c AND $\bar{\gamma}$

G_c and K_c are calculated according to the assumptions and procedure outlined in the main text in section 3.2.4. The relevant equations to be used are as follows:-

Calculation of K_c

$$K = Y \frac{6M/c}{bd^2} \quad (B.1)$$

where $Y = 1.99 - 2.47(c/d) + 12.97(c/d)^2 - 23.17(c/d)^3 + 24.80(c/d)^4$.

Calculation of G_c

$$G = \left(\frac{1-\nu^2}{E} \right) \sigma_n^2 h f(c/d) \quad (B.2)$$

Calculation of $\bar{\gamma}$

$$\bar{\gamma} = \frac{A_{F,\delta}}{A_F} \quad (B.3)$$

where $A_{F,\delta}$ = area under the load-deflection curve, i.e. the total work done to produce fracture,

and A_F = area of new fracture surfaces produced, nominally taken as twice the cross-sectional area of the specimen at the notch cross-section, i.e. $2A_0$.

The relevant curves to be used in calculating the fracture parameters are as follows:-

Fig. (3.2) which shows the function $f(c/d)$ versus the notch-depth ratio c/d .

Fig. (3.3) which shows the theoretical L/L_0 versus c/d relationship for the specimens.

It is important to point out here that the theoretical rather than the

experimentally derived compliance-crack depth relationship is used to determine the fracture parameters. This theoretical curve is assumed to apply for all strain rates, since at this stage the quantifiable effect of strain rate, as distinct from slow crack growth, on compliance is not known. The theoretical curve is also assumed to apply to any material (provided the relevant Poisson's ratio value ν is used) and to either the beam or slab specimens used in the laboratory tests. It was found that while the experimentally derived compliance-notch depth curve might be displaced laterally from the theoretical line, nevertheless the slopes of the two curves are very similar. Therefore, the theoretical line adequately represents the change in compliance, ΔL , with crack growth. The method of calculating G_c and K_{Ic} involves using this ΔL together with the known initial notch depth of the specimens to arrive at the fracture parameters, and it is thus acceptable to use the theoretical curve. In addition, using the theoretical curve provides for a uniform basis of analysis for all the tests. (See Ref. 17 for further discussion of this subject.)

It was stressed in section 3.2.4 that the calculation of G_c or K_{Ic} involves an assumption about the instantaneous value of the crack depth c_c , and that uncertainty about the value of c_c at instability is possibly the largest single source of error in fracture measurements.

Sample calculations of the fracture parameters for the 25,4 mm notched test specimen represented by beam 4.5 of Table 4.5 in Chapter 4 are presented below. The average values of E and L_0 assumed for the calculation are as follows:-

$$E = 20,0 \text{ GPa}$$

$$L_0 = 2,0 \text{ m}^2/\text{GN} \quad (\text{Note that m}^2/\text{GN is dimensionally}$$

equivalent to $(\text{GPa})^{-1}$).

Calculation of K_{Ic}

Bending Moment M

Refer to Fig. 3.1. The relevant specimen dimensions are:-

$$S_1 = 0,610 \text{ m}$$

$$S_2 = 0,203 \text{ m}$$

$$b = 0,102 \text{ m}$$

$$d = 0,102 \text{ m}$$

$$\text{Initial Notch depth } c_0 = 25,4 \text{ mm}$$

The Bending Moment in the central span is:-

$$M = \frac{F}{4} (S_1 - S_2) \quad (B.4)$$

where F = the total load on the specimen.

Note that $6M/bd^2$ (equation B.1) is equivalent to the nominal tensile stress at the extreme fibre of the beam, σ .

Critical Crack Length

For the 25,4 mm notched beam being considered:-

Initial Compliance $L_i = 2,8 \text{ m}^2/\text{GN}$

Compliance at Failure $L_f = 3,7 \text{ m}^2/\text{GN}$

Change in Compliance $\Delta L = L_f - L_i = 0,9 \text{ m}^2/\text{GN}$.

Taking L_o , the average unnotched compliance, as $2,0 \text{ m}^2/\text{GN}$ (see Fig. 4.18):-

$$\frac{\Delta L}{L_o} = 0,45.$$

From Fig. 3.3, the compliance ratio L/L_o for a 25,4 mm initial notch is:-

$$\frac{L}{L_o} = 1,195 \quad (c/d = 0,25).$$

Therefore, the compliance ratio at failure is:-

$$\begin{aligned} \frac{L_f}{L_o} &= \frac{L}{L_o} + \frac{\Delta L}{L_o} \\ &= 1,195 + 0,45 \\ &= 1,65. \end{aligned}$$

Using Fig. 3.3, the notch-depth ratio at failure is:-

$$\frac{c_c}{d} = 0,418$$

and the critical crack length is therefore $c_c = 42,5 \text{ mm}$.

Evaluation of Y

From equation (B.1), and using $c_c/d = 0,418$:-

$$\begin{aligned} Y &= 1,99 - 2,47(0,418) + 12,97(0,418)^2 - 23,17(0,418)^3 + 24,80(0,418)^4 \\ &= 2,289. \end{aligned}$$

Finally, using the failure load $P = 2,83 \text{ kN}$, the fracture toughness K_{Ic} is

obtained from equation (B.1):-

$$K_c = Y \sigma \sqrt{c} \\ = 0,78 \text{ MN/m}^{3/2}$$

Calculation of G_c

Evaluation of σ_n , the nominal stress at the root of the notch:-

From equation (3.6) $\sigma_n^2 = \sigma^2 \frac{d^4}{h^4}$

where $h = d - c_c = 0,059 \text{ m}$

and $\sigma = \frac{6M}{bd^2}$

Evaluation of $f(c/d)$

For $c_c/d = 0,418$, $f(c/d)$ from Fig. 3.2 is:-

$$f(c/d) = 0,49$$

Taking Poisson's Ratio, ν , as 0,1, the Strain Energy Release Rate G_c is then calculated from equation (B.2):-

$$G_c = 34,0 \text{ N/m.}$$

Calculation of $\bar{\gamma}$

From Table 4.5, specimen 4.5, the total work done to produce fracture, represented by the area under the load-deflection curve for the beam is:-

$$A_{F,\delta} = 0,44 \text{ Nm.}$$

Now, A_F for a 25,4 mm notched beam is taken as twice the cross-sectional area at the notch, i.e.

$$A_F = 154,84 \text{ cm}^2$$

The Effective Fracture Surface Energy $\bar{\gamma}$ is then calculated from equation (B.3):-

$$\bar{\gamma} = 28,5 \text{ N/m.}$$

APPENDIX C

MIX DESIGN AND SPECIMEN PREPARATION FOR THE CEMENT MORTAR SPECIMENS

Mix Design

A cement mortar rather than a concrete was chosen for the laboratory test programme, because of its greater degree of homogeneity and therefore a more simplified type of fracture. The materials used in the mix were as follows:-

Cement Rapid Hardening Portland Cement.

Sand Washed sand derived from a residual granite pit. (Halfway-House granite dome). The sand was relatively coarse with a Fineness Modulus of 3,35 and a Relative Density of 2,68.

Water Potable standard and free from contamination.

No additives or admixtures were used in the mix.

Two mixes were used during the course of the laboratory programme. Both were designed according to the Portland Cement Institute method in order to have adequate workability for use in the laboratory moulds. The design compressive strength of the mixes, when measured on 101 mm cubes, was 30 MPa at the time of test which was nominally six days after casting. The use of two mixes was necessary because the laboratory test programme spanned a period of five months, commencing in winter and terminating in summer. The higher rate of hydration of the cement in the warmer weather allowed a reduction in the paste content of the mix. The proportions per cubic metre of the two mixes are given below in Table C.1.

Table C.1 Mix Proportions for Cement Mortar

	Winter Mix	Summer Mix
	(kg)	(kg)
Rapid Hardening Cement (c)	504	476
Water (w)	280	280
Sand (s)	1 500	1 524
c/w ratio	1,8	1,7
s/c ratio	2,976	3,202

Mixing and Casting Technique

The mortar was mixed in a rotary pan mixer. The time of mixing was approximately four minutes. The steel moulds used for casting the beam specimens consisted of channel sections bolted together to form a mould with nominal dimensions of 101 x 101 x 710 mm. The specimens were compacted in the steel moulds by placing them on the vibrating table of a 'Veebee' Consistometer for approximately four minutes, until the entrapped air had been expelled. In the case of the notched beams, the steel notch-formers (see Fig. 4.1) were fixed onto a vertical side of the mould in order that a smooth flat surface for loading and support would be produced. Care was taken to properly compact the mix around the notch-formers. No problems with workability, segregation or bleeding were encountered during mixing and casting. In addition to the beams cast from a mix, three cubes (101 mm) per mix for control on compressive strength were also cast.

Stripping and Curing of Specimens

After casting, the specimens were covered with damp cheesecloth and left for twenty-four hours to set. Great care had to be exercised in stripping and removing the specimens from the moulds, in order to prevent damage at the notched cross-section. The specimens were then submerged in a water bath and left to cure until the time of test. The water baths were situated in the laboratory and no control over water temperature was exercised. The ambient temperature in the laboratory during winter was between 5 to 15°C, and during summer between 15 to 25°C. The specimens were removed from the water bath immediately prior to testing, and the tests were conducted with the specimens in a surface wet condition. For the protracted tests in Series 3, the specimens were sheathed and sealed in plastic bags prior to testing to prevent them from drying out during testing. Any extraneous effects due to shrinkage were thus excluded from the test results.

APPENDIX DDETAILS OF APPARATUS USED IN THE LABORATORY TEST PROGRAMMEApparatus Developed in the LaboratoryLoad Spreader Rig (Loading Yoke)

(a) Original Rig. The original load spreader rig is shown in Fig. D.1 (full lines), and was used for the tests of Series 1, 2, 3 and part of Series 4. It consisted of a 25,4 mm square section steel beam with 12,7 mm bright steel rod load platens in suitable mountings at either end so as to produce a loading span of 203 mm.

(b) Modified Rig. When it was realised that the original rig was preventing the occurrence of more stable type fractures due to the strain energy stored in it at the point of fracture instability, the rig was modified by increasing the section of the loading beam to a 25,4 mm wide by 50,8 mm deep steel section (see Fig. D.1, dotted lines). This effectively stiffened up the rig by a factor of about eight, and considerably reduced the strain energy which could be stored in the rig. The modified rig was used for certain tests of Series 4, and all the tests in Series 5 and 6.

Deflection Rig

(a) Original Set-Up. The deflection rig was designed in order to support the LVDT's over the load points. It was made from 12 x 3 mm flat steel sections bolted together, as in Fig. D.2(a). The rig rested on the top face of the specimens over the specimen support points by means of 12,7 mm support bars. This excluded extraneous deflections due to local crushing over the specimen support points from the results. The LVDT's were mounted in suitable clamps which were centred over the loading points, and the plungers of the LVDT's rested on the ends of the load spreader rig. This set-up applied to the tests of Series 1. The deflections were thus measured through the load platens, and the deflection measurements were all unreliable to some degree. (See also

Fig. 4.3 in the main text). The LVDT clamps had provision for zeroing the LVDT's before testing.

(b) Re-designed Set-Up. The deflection measuring set-up was re-designed for Series 3 to 6. This modified set-up was designed in order to eliminate unwanted deflections due to local crushing under the load platens from the deflection readings. A second rig was designed which straddled the rig described in (a) above and could clamp onto the specimen under the load points at the nominal neutral axis. The plungers of the LVDT's then rested on the cross-bars of this clamping rig (see Fig. D.2(b) and Fig. 4.16 in the main text). The LVDT's themselves continued to be held in the original rig which rested over the support points. This allowed only the true elastic deflections of the beam under the load points to be measured, and all extraneous deflections were excluded from the results.

Electronic Testing Apparatus

Electronic Load Cell

Made by Kyowa Electronic Instruments Co. Ltd., Tokyo, Japan.

Type : Kyowa Strain Gauge Based Load Cell, Compression/Tension,
Series No. LU-2TE.

Capacity : 20 kN.

Bridge Voltage : 10.0 V D.C.

Output Voltage Sensitivity : 2 mV/V $\pm$ 0.2 per cent.

Linear Variable Differential Transformers (LVDT's)

Two LVDT's were used, made by G.L. Collins Corp., Long Beach, California.

Transducer Length : 20 mm.

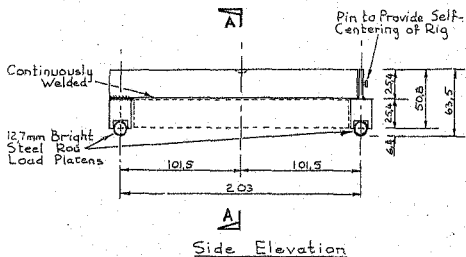
Transducer Diameter : 10 mm.

Linear Range : $\pm$ 1.0 mm.

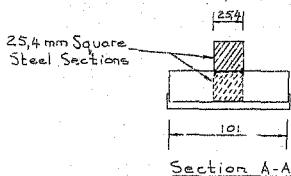
Linearity : approx. 0.80 per cent Full Scale.

Excitation : 5.53 V D.C.

The outputs from the two LVDT's were electrically summed before being fed into the autographic recorder being used.



All Dimensions in mm.



Dashed Lines Represent Modification to Rig Welded Steel Construction

FIG.D.1 DETAILS OF ORIGINAL AND MODIFIED LOAD-SPREADER RIG

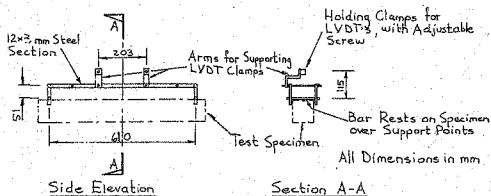


FIG. D.2(a) DETAILS OF DEFLECTION RIG
SUPPORTING THE LVDT'S

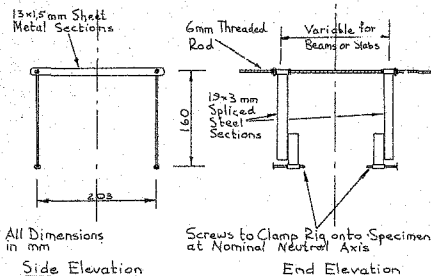


FIG. D.2(b) DETAILS OF CLAMPING RIG

Power Supply Modules

A separate power supply module was used to power the load cell and the LVDT's.

Type : Load Cell Module : Operational Amplifier (OA2) Module

LVDT Module : Type IC 100/6.

Made by Coutant Electronics Ltd., Reading, England.

Powered by 220 V A.C. mains voltage.

Autographic Recorders

(a) Watanabe X-Y Recorder. Made by Watanabe Instruments Corp., Tokyo, Japan.

Model WX 411, Floating Differential Input.

Effective Writing Area : X scale : 380 mm, Y scale : 250 mm.

Response of Recorder :

Band Width = 1.0 Hz for X axis

Band Width = 1.25 Hz for Y axis

(where B.W. is defined as the frequency response at 70 per cent of maximum amplitude).

Maximum Frequency Response at Maximum Amplitude:-

0.6 Hz for X axis; 0.8 Hz for Y axis.

Rise time in seconds:-

X axis : 0.35 sec; Y axis : 0.28 sec.

(b) N.E.P. Ultra Violet Recorder. Made by New Electronic Products Ltd., London, England.

Type 1050, 12 galvanometer channels recording on Ultra Violet sensitive paper on a chart roll.

Response of Recorder :

Undamped Natural Frequency of Galvanometers : .30 to 40 Hz.

Band Width taken as 60 per cent of 40 Hz, i.e. 24 Hz.

Rise time in seconds:- 0.014 sec.

Strain Gauges Mounted on Copper I-Sections

The copper I-sections, with provision for end anchorage into the specimen, were made of 0.5 mm annealed copper sheet and had variable gauge lengths depending on the width of specimen into which they were cast. The strain gauges were bonded on to the centre of the copper sections,

and were of the following type:-

Type : Kyowa Electrical Resistance Foil Gauges, Unidirectional.

Gauge Length : 2 mm

Gauge Factor : 2,12

Gauge Resistance : 120 Ω .

The gauges were temperature compensated by including them electrically in a bridge circuit with a dummy gauge bonded onto a separate copper strip. The dummy gauges were identical to the active gauges. However, they were not cast into any mortar specimens. It was found that the gauges took about half an hour to come to a temperature equilibrium and allow the electrical output to stabilise before a test. After the tests it was realised that a better procedure would have been to bond the dummy gauges at right angles to the active gauges on the same copper I-section, and then to correct the test output later for the Poisson's Ratio effect of the dummy gauge.

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Claims regarding this component must be submitted within 90 days from date of purchase.

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Excess wear and tear charges will apply.

Excess wear and tear charges will apply.

Author Alexander Mark Gavin

Name of thesis Plane Strain Fracture Testing Of Cemented Materials. 1974

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