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GENERATOR EARTHING : A REVIEW AND ANALYSIS OF PAST AND CURRENT PRACTICE WITH SPECIAL REFERENCE TO PROTECTION AND SYSTEMS CONDITIONS

BY

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Submitted as part fulfilment for the degree of M.Sc. (Eng.) at the University of the Witwatersrand, Johannesburg

December 1974

DECLARATION

I hereby declare that this thesis is my own work and that it has not previously been submitted for a degree at any university.

J.P. HOLTZHAUSEN

23/12/74.

FOREWORD

Generator neutral earthing is only one aspect of the involved issue of power system neutral earthing and has been dealt with fully in the literature by authors like Wilheim and Waters [1] Mortlock [2] and others.

In this report the basic philosophy which relates to various earthing practices is summarised and discussed. A survey of current earthing practices for generators was undertaken and the results of the questionnaire are analysed. Finally, an earthing scheme, currently being used for new power stations, is presented and analysed. In particular, the possibility of neutral inversion due to ferroresonance is investigated by means of a describing function technique. A previously published criterion for occurrence of neutral instability is confirmed although certain aspects of the miniature system approach used are questioned.

The author would like to express his sincere appreciation and thanks to Professor W. Cormack for the assistance offered during the course of this project.

The author would also like to extend his thanks to the Electricity Supply Commission for their permission to conduct this research project and also to the University of Stellenbosch for the computing facilities made available.

Finally a word of thanks for Mrs Kay for typing the script.

LIST OF PRINCIPAL SYMBOLS

α	-	operator = $e^{j2\pi/3}$
A	-	amplitude of input to non-linearity
B	-	amplitude of second input to non-linearity
C	-	capacitance
D	-	output from non-linear element
E	-	applied voltage (e.m.f.) (phase to neutral)
e	-	instantaneous e.m.f.
f	-	frequency e = 2, 7183
G(jω)	-	frequency transfer function
I	-	current
i	-	instantaneous current
i	-	normalised current
k	-	polynomial coëfficients
L	-	inductance
N(A)	-	describing function (DF) with input A
N(A,B)	-	*dual input DF with inputs A and B*
n	-	order of harmonic
P	-	normalised Laplace operator
R	-	resistance
r	-	normalised resistance
s	-	Laplace operator
t	-	time
U	-	voltage drop
u	-	normalised voltage drop
u	-	instantaneous voltage drop
X	-	reactance
x	-	normalised reactance
α,γ	-	earthing coefficients
Z	-	impedance
α,β,θ	-	phase angles
Ψ	-	flux linkages
ψ	-	normalised flux linkages
ω	-	angular frequency (radians/second or p.u.)

Subscripts

A - relating to phase A
B - relating to phase B
C - relating to phase C
c - relating to capacitance
d - relating to a delta winding
E - relating to earth (capacitance)
e - relating to earth (fault)
F - relating to fault
G - relating to generator
h - relating to $\frac{1}{2}$ fundamental frequency
L - relating to inductance
m - relating to magnetising
n - relating to neutral
p - relating to phase (fault)
r - relating to reference values
t - relating to a transient change (in current)
T - relating to quantities measured at the generator terminals
The - Thèvenin
x - relating to extinction voltage
0 - relating to zero sequence quantities
1 - relating to positive sequence quantities or the fundamental wave
2 - relating to negative sequence quantities
3 - relating to the third harmonic

CONTENTS PAGE

FOREWORD 1

LIST OF PRINCIPAL SYMBOLS 2

CHAPTER 1 : BASIC OBJECTIVES OF GENERATOR NEUTRAL EARTHING 6

1-1 : Introduction

1-2 : Limitation of Stator Earth Fault Current

1-3 : Limitation of Fundamental Frequency Overvoltages during earth faults

1-4 : Suppression of Transient Voltages

1-4-1 : Surges due to switching

1-4-2 : Surges due to arcing faults

1-5 : Prevention of Ferroresonance Phenomena

1-5-1 : Series Ferroresonance

1-5-2 : Parallel Ferroresonance : neutral instability

1-5-3 : Method of Describing Functions applied to parallel ferroresonance phenomena

1-5-3-1 : Introduction

1-5-3-2 : DF analysis of neutral inversion

1-5-3-3 : Experimental Investigation

1-5-3-4 : Conclusion

1-6 : Selective and Sensitive earth fault protection

1-6-1 : Introduction

1-6-2 : The influence of third harmonic currents on stator earth fault protection

CONTENTS (proceed)

		PAGE
CHAPTER 2	: VARIOUS METHODS OF GENERATOR NEUTRAL EARTHING	43
2-1	: Introduction : Operation with isolated neutral	
2-2	: Direct Earthing	
2-3	: Resistance Earthing	
2-4	: Reactance Earthing	
2-5	: Resonant Earthing	
CHAPTER 3	: EARTHING PRACTICES OF VARIOUS COUNTRIES	50
CHAPTER 4	: EARTHING SCHEME FOR A UNIT-CONNECTED GENERATOR WITH A GENERATOR CIRCUIT BREAKER	53
4-1	: Introduction	
4-2	: Design criteria	
4-3	: Conclusion	
CHAPTER 5	CONCLUSIONS	58
APPENDIX A	: Power Station Electrical Layouts	59
APPENDIX B	: Fundamental frequency voltages of sound phases during a single phase to earth fault	61
APPENDIX C	: Transfer function of generator earthing scheme	64
APPENDIX D	: Analytical Derivation of Describing Functions	70
APPENDIX E	: Computer programs	78
REFERENCES		81

CHAPTER 1

BASIC OBJECTIVES OF GENERATOR NEUTRAL EARTHING

1-1 Introduction

The objectives of power system neutral earthing are twofold : On low voltage distribution networks the major objective is the safety of persons while on the main high voltage distribution networks the safety of the equipment is the deciding factor.

Generator neutral earthing falls into the latter category. In small power systems where distribution takes place at generator voltage the generator earth in fact represents the system earth. As outlined in APPENDIX A the so-called unit connected scheme has gained universal acceptance for application to power stations of an interconnected power system. In such a system the stator windings of generators and their connections up to and including the low voltage windings of the associated generator transformers are isolated electrically from the rest of the system as far as stator faults involving earth are concerned. When deciding on a value of neutral earthing impedance it is therefore a valid assumption to disregard the rest of the system. This report, therefore, focusses on unit connected generators - the general principles of power system earthing falling somewhat outside the scope of the present report.

The objectives dealt with in the following sections are conflicting in a certain sense in that, as will be shown, low fault currents lead to high overvoltages and visa versa.

1-2 Limitation of Stator Earth Fault Current

Consider a generator with its neutral connected to earth through an impedance Z_n ohm as shown in FIGURE 1-2-1 (a). A single phase to earth fault occurs on one of the phases at or near a phase terminal of the stator winding. In FIGURE 1-2-1 (b) the corresponding interconnected sequence networks are shown for this fault condition.

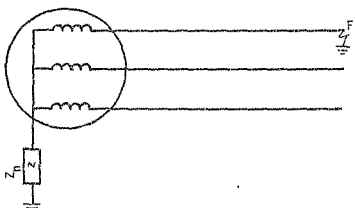

FIGURE 1-2-1 (a) : Single phase fault to earth at generator terminals

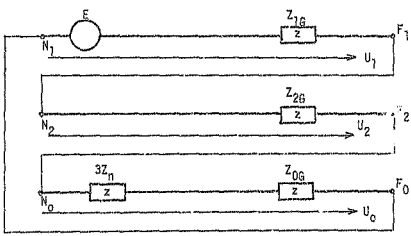

FIGURE 1-2-1 (b) : Sequence networks for FIGURE 1-2-1 (a)

The fault current is

$$I_e = \frac{3\,E}{Z_{1G} + Z_{2G} + Z_{0G} + 3Z_n} \qquad \text{----} \qquad (1\text{-}2\text{-}1)$$

For a three phase fault at the same position the corresponding fault current would be

$$I_p = \frac{E}{Z_{1G}} \qquad \text{----} \qquad (1\text{-}2\text{-}2)$$

From (1-2-1) and (1-2-2)

$$\frac{I_e}{I_p} = \frac{3}{1 + \left(\frac{Z_{2G}}{Z_{1G}}\right) + \left(\frac{Z_{0G} + 3Z_n}{Z_{1G}}\right)} \qquad (1\text{-}2\text{-}3)$$

For a synchronous machine the negative sequence impedance, although somewhat lower than the positive sequence impedance, is still of the same order of magnitude. The zero sequence impedance of a generator, however, is much lower than the positive sequence impedance [67]. The reason is that, due to the symmetrical arrangement of the three phases of the stator winding, the magnetomotive forces associated with zero sequence currents add up to zero in the air gap of the machine.

It is therefore clear that in the case of a generator which is solidly earthed ($Z_n = 0$) the ratio $\frac{I_e}{I_p}$ can become larger than 1. The dynamic current rating of a system is usually based on the three phase fault level and is therefore necessary to limit the earth fault current to a value equal to or less than the three phase fault current.

In general, however, it is advisable to limit the earth fault current to a much lower value. The type of stator earth fault that usually occurs is a puncture from the stator windings to the surrounding steel laminations in the stator slots. Such a fault causes burning of the stator laminations and welding together of adjacent laminations. It is obvious that both the magnitude and duration of the fault current have an effect on the degree of damage at the point of fault.

Pohl[3] published results of tests in 1930 to indicate that, providing earth fault current is limited to less than 10 A, the damage to the stator will be limited to local burning of the iron. This type of damage is easily repaired by replacing the faulty coil and removing traces of burning on the

iron. For a current of 5 A the arcing, which causes most damage, actually ceases after 4 tot 6 minutes. These tests were done on test samples utilising bonded mica flakes as insulation which is similar to insulation used in modern machines. The test voltage was 6 000 V.

Westphal [4] in a more recent publication also supported the idea that extensive damage of the stator iron only occurs for currents in excess of 20A and he indicated that for a 7,5 A fault current the damage do the stator laminations is still very localised despite fault durations as long as 10 minutes. Sustained earth faults however cause breakdown of the insulation in the slots and could lead to damaging interturn and phase faults.

Stalder[5] reported that welding of adjacent stator laminations occurs for fault currents as low as 6 A and durations as short as 0,5 s. He stressed the importance of rapid de-excitation following earth faults and indicated that the amount of damage to thr stator iron is quite acceptable even for fault currents as high as 40 A.

Berman, Kripsky and Skalka [6], at a recent CIGRE conference, described a theoretical and experimental investigation which lead to the conclusion that if repairs on the magnetic circuit are to be avoided earth fault current should be limited to a value as low as 1 A or 1,5 A. They also show that such low currents may flow for extended periods without causing deterioration of the stator insulation. In order to achieve fault currents as low as these and to prevent arcing at the point of fault resonant earthing is proposed by these authors. Resonant earthing will be discussed in detail in Chapter 2.

Summarising, it can be said that low valued earthing impedances lead to high fault currents and extensive damage to iron circuits, especially where fault durations are long. Repairs will be extensive and outages prolonged. For large generators it is therefore essential to limit the fault current to such a small value as practicable.

1-3 Limitation of fundamental frequency overvoltages during earth faults

Consider again the case of a single phase to earth fault at the generator terminals as shown in FIGURE 1-2-1. If the fault is on phase A, the voltages of the sound phases to earth are determined by the so-called earthing coefficients. These are defined as

$$x = \frac{X_0}{X_1} \quad \text{and} \quad y = \frac{R_0}{X_1}$$

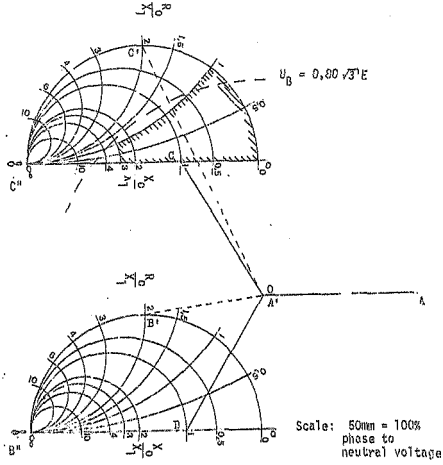

FIGURE 1-3-1 : Steady state overvoltage on phases B and C during a single phase to earth fault on phase A.

In APPENDIX B it is shown that the sound phase to earth voltages can be obtained from a chart such as that shown in FIGURE 1-3-1. For the specific case where $x = \frac{X_0}{X_1} = 0$ and $y = \frac{R_0}{X_1} = 2$ the voltages of phases B and C to earth are represented by OB' and OC' respectively. The voltage of phase A to ground is zero; the fault being on phase A. The highest phase to earth voltage will occur on phase C as indicated by OC'. If the generator neutral is not earthed full line-to-line voltage ($\sqrt{3}$ E) will appear on each of the sound phases. These voltages are OB" and OC".

The value of the phase to earth voltages on phase C will be less than 80% of the rated line-to-line voltage ($\sqrt{3}$E x 0,8), provided $\frac{R_0}{X_1} < 1$ and $\frac{X_0}{X_1} < 3$. These conditions are indicated by the shaded area in FIGURE 1-3-1. A system of this type is termed an effectively earthed system [7]. It is important to know whether a system is effectively earthed or not as lightning arrestors with a lower voltage rating (80% line-to-line voltage) can be applied on effectively earthed systems. On non-effectively earthed systems lightning arrestors capable of withstanding 100% line-to-line voltage are required. The insulation level on effectively earthed systems is lower and appreciable savings in capital cost ensue.[8].

Inspection of FIGURE 1-3-1 indicates that earthing resistors cause higher fundamental frequency overvoltages than earthing reactors. As will be shown in CHAPTER 1-4, resistance earthing is favoured for generators due to the improved damping of transient overvoltages. Consequently, unit connected generators are usually non-effectively earthed and insulation and surge divertors must be rated for line-to-line voltage.

Although single phase to earth faults are considered here it can be shown that two phase to earth faults cause overvoltages of a similar order of magnitude.

1-4 Suppression of transient voltages

Transient voltages may occur as a result of either lightning, switching or arcing. In the unit connected arrangement the generator transformer aids damping of surges that originate as a result of switching or lightning on the high voltage system. In addition, lightning arrestors are provided on the high voltage side of the generator transformer.

1-4-1 Surges due to switching

In the version of the unit connected scheme which employs a generator circuit breaker switching surges can pose problems and surge divertors near the generator terminals are a necessity.

Consider a generator with a single phase to earth fault at one of its terminals as shown in FIGURE 1-4-1. Interruption of the fault current takes place at or near a normal current zero. In the case of reactance earthing the fault current will lag behind the voltage of the faulted phase by about 90^{0}. As indicated in FIGURE 1-4-2 (a) a negative so-called extinction voltage pulse (u_x) will develop across the circuit breaker if the current is interrupted slightly before a natural current zero. This is due to the inductive energy ($\frac{1}{2} L i_t^2$) being converted to capacitive energy ($\frac{1}{2} C u_x^2$). This negative peak is followed by a positive going recovery voltage. It is clear that a large voltage peak is possible. In FIGURE 1-4-2 (b) the transient recovery voltage is shown for a resistance earthed generator. It will be noted that the extinction voltage is now followed by a small recovery voltage. The significance of this is that resistive earthing reduces switching overvoltages - firstly by improving the power factor and thus reducing the recovery voltage and secondly by virtue of increased damping. Earthing through reactors, on the other hand, leads to large transient overvoltages.

Witzke [9, p. 381] conducted a series of tests on an alternating current model network and concluded that $\frac{X_0}{X_1}$ should not exceed 4,0 if the transient voltages are to be limited to 250 per cent normal line-to-neutral peak voltage.

1-4-2 Surges due to arcing faults

An earth fault, such as that shown in FIGURE 1-4-1, can also be cleared by the self-clearing action of the arc, especially if earth fault currents are low as is the case when large earthing impedances are used. Under certain conditions these arcs reignite intermittently and excessively high overvoltages are possible.

When the first three phase distribution systems were put into operation they invariably operated with isolated neutral points. Such arrangements

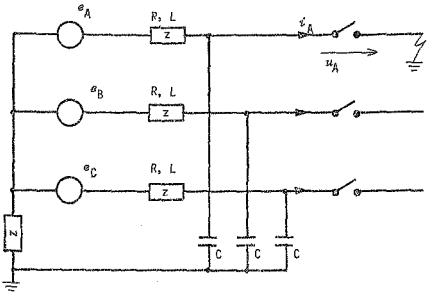

FIGURE 1-4-1 : Single phase to earth fault - equivalent diagram for transient analysis

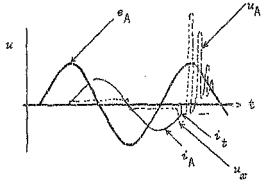

FIGURE 1-4-2 (a) : Recovery voltage across circuit breaker for reactor earthing

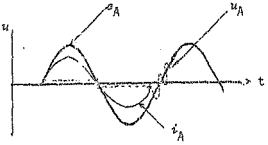

FIGURE 1-4-2 (b) : Recovery voltage across circuit breaker for resistive earthing

allowed continued operation during earth faults. As systems became larger trouble was soon experienced as the arc current was sustained by the system capacitance to earth.

Consider a single phase to earth fault on a three phase generator as shown in FIGURE 1-4-3 (a). Applying Thèvenin's theorem to this circuit, replacing the part of the circuit which excludes the arc (fault) by an equivalent voltage source, FIGURE 1-4-3 (b) is obtained.

It is clear that, in the case where R_n is infinitely large, the earth fault current is sustained only by the capacitances C. These capacitances are usually fairly small and the resulting fault current on an unearthed system is consequently also small. If the fault is an arc it is extinguished and reignites intermittently. Very high transient voltages are possible due to the intermittent arcing.

Peters and Slepian [10] proposed several theories of intermittent arcing (also called arcing grounds). In all these theories restriking of the arc occurs at a fundamental current zero and this leads to high frequency oscillations. Multiple restriking is possible and the theories differ only with respect to the exact instant of arc extinguishing.

Nowadays with the advent of solidly earthed transmission s[illegible] subject of arcing grounds has lost some of its importance. While [illegible]ed and high reactance earthed systems were still in use a considerable [illegible]ference of opinion existed as to whether arcing faults actually cause such high overvoltages as claimed [1, p. 115].

As pointed out in section 1-2 it is advisable to limit generator stator earth fault current to a small value and relatively high neutral earthing impedances are necessary. On generators arcing faults are therefore a possibility.

Consider the case of resistance earthing as shown in FIGURE 1-4-3. The total earth fault current is limited by the parallel combination of the phase to earth capacitances and the earthing resistor and therefore leads the driving voltage as shown in FIGURE 1-4-4. At point A the current is extinguished at a normal current zero thus leaving a "trapped voltage" equal to AB on the capacitances. This voltage will theoretically remain constant for an unearthed system but for a resistively earthed generator this voltage will decay exponentially as shown. At point C the system voltage reaches a maximum and the voltage across the arc is CD. If the arc were to reignite at this instant a high frequency transient would be established.

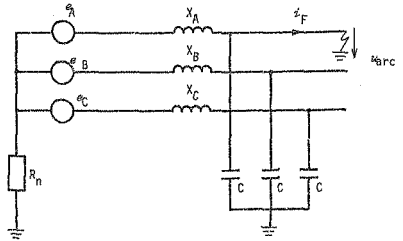

FIGURE 1-4-3 (a) : Sin à phase fault to earth on a resistively earthed generator

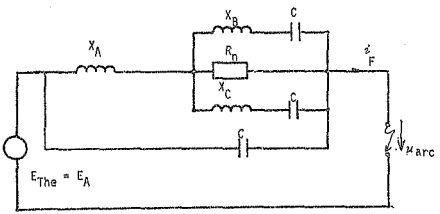

FIGURE 1-4-3 (b): Thèvenin equivalent circuit for current in and voltage across arc

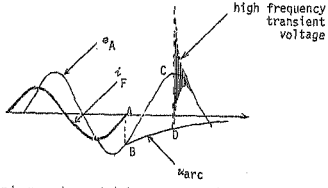

FIGURE 1-4-4 : Voltages and currents pertaining to FIGURE 1-4-3 (a)

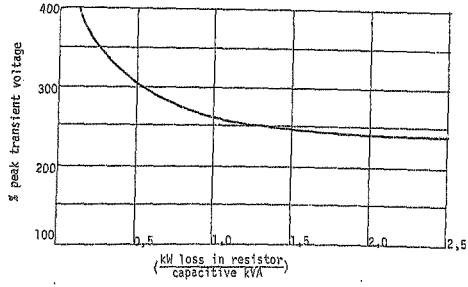

FIGURE 1-4-5 : Transient voltages for any number of restrikes in fault arc or cross circuit breaker contacts

In order to minimise the probability of reignition of the arc the resistor should be of such a low value that the "trapped voltage" decays to a small value before the generator voltage reaches its peak. If the time constant $3R_nC$ is made $\frac{1}{\pi}$ times the period of a half cycle $(\frac{1}{2f})$, it can be shown that this voltage decays to $e^{-\pi} = 0{,}0432$ of the initial value within half a cycle.

$$\therefore \quad 3R_nC = \frac{1}{\pi} \times \frac{1}{2f} = \frac{1}{2\pi f} = \frac{1}{\omega}$$

$$\therefore \quad R_n \leq \frac{1}{3\omega C} \qquad \text{---- (1-4-1).}$$

This inequality is also known as Peterson's rule [1, p. 209] and can be alternatively stated : "the kW loss in the resistor during a single phase to ground fault should be equal to or less than the capacitive kVA of the total capacitances to ground."

Marter and Johnson [11] report on tests conducted by means of a model network from which the curve shown in FIGURE 1-4-5 resulted.

From this curve it is clear that the transient voltage is not substantially reduced if the ratio of kW loss in the resistor to capacitive kVA exceeds unity. In fact it appears that unity is an optimum value for this ratio. The kW loss in the resistor is therefore made equal to the capacitive kVA which can also be stated in terms of Peterson's rule :

$$R_n = \frac{1}{3\omega C} \qquad \text{---- (1-4-2).}$$

1-5 Prevention of Ferroresonance Phenomena

1-5-1 Series Ferroresonance

Consider a generator earthed at its neutral point through a non-linear reactance. If the three phase circuit contains an asymmetry such as a broken phase conductor a Thèvenin equivalent circuit can be derived as shown in FIGURE 1-5-1 (a) and (b).

If the series reactance of the generator is negligible compared to the neutral impedance the equivalent circuit can be simplified to that shown in FIGURE 1-5-2.

Rüdenberg [12, p. 642] proposed a graphical method of analysis of circuits of this type. This method considers only the fundamental component of currents and voltages.

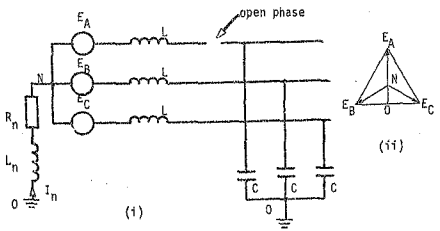

FIGURE 1-5-1 (a) : Broken phase conductor on generator: (i) Circuit diagram and (ii) phasor diagram for ON open circuited.

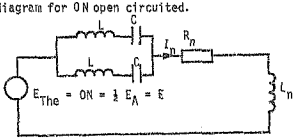

FIGURE 1-5-1 (b) : Thévenin equivalent diagram for FIGURE 1-5-1 (a)

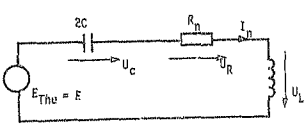

FIGURE 1-5-2 : Simplified version of FIGURE 1-5-1 (b) for L=0

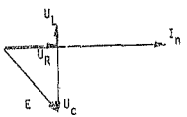

FIGURE 1-5-3 : Phasor diagram for FIGURE 1-5-2

The phasor diagram for above circuit is shown in FIGURE 1-5-3. Considering only the magnitudes of the voltage drops it is possible to write down the following equation

$$E = \sqrt{(I_n R_n)^2 + (U_L - U_c)^2}$$

$$\therefore \quad U_L = \pm \sqrt{E^2 - (R_n I_n)^2} + U_c$$

$$U_L = \pm \sqrt{E^2 - (R_n I_n)^2} + \frac{I_n}{\omega C} \qquad \text{---- (A)}$$

This equation can be analysed graphically. Consider firstly the case of $R_n = 0$ when equation (1-5-1) reduces to

$$U_L = \pm E + \frac{I_n}{\omega C} \qquad \text{---- (B)}$$

As stated previously, U_L is a *non-linear function* of I and can be represented by the magnetising curve of the reactor.

FIGURE 1-5-4 is a graphical representation of equation (B). As pointed out by Rüdenberg only points *A and C are stable*. As indicated by the dotted lines small capacitances (steep slopes) permit only a single operating point C' at a low value of voltage across the neutral impedance.

If a similar graphical analysis is carried out for the general case where $R_n \neq 0$ the term under the square root in equation (A) can be shown to represent an ellipse as shown in FIGURE 1-5-5.

From FIGURE 1-5-5 it is clear that two stable operating points A and C are possible. However, if R_n is made larger only one stable operating point A' at a low value of voltage is possible. Such a case is given by the dotted ellipse. As pointed out by Rüdenberg the addition of more resistance to the circuits eliminates the higher operating point. Under certain conditions a circuit will also "jump" from the lower operating point to the higher one.

From the preceding analysis it appears that the circuit of FIGURE 1-5-1 can go into series ferroresonance if a circuit phase unbalance such as an open phase were to occur. The possibility of an open stator phase on a large generator is remote but unbalance may be caused by a reluctant circuit breaker in one phase.

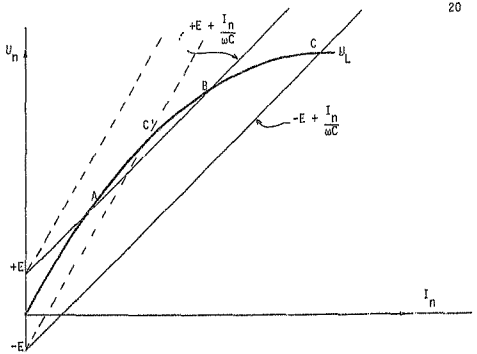

FIGURE 1-5-4 : Graphical Analysis of circuit with $R_n = 0$

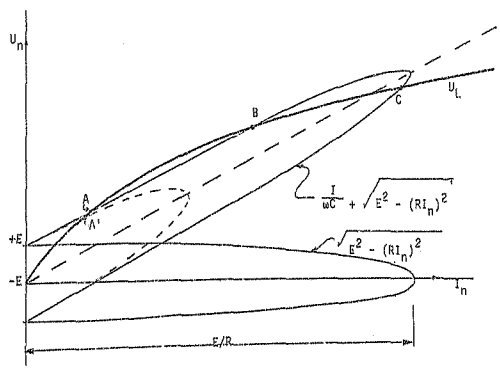

FIGURE 1-5-5 : Graphical analysis for $R_n \neq 0$

1-5-2 Parallel Ferroresonance : Neutral Instability

Shott and Peterson [13] pointed out that parallel ferroresonance is possible in a symmetrical circuit such as that shown in FIGURE 1-5-6. Here the actual neutral point is not earthed but the phases are tied to earth by means of three voltage transformers, the secondary windings of which are often connected in open delta for protection and indication purposes. The capacitances C are the capacitances to earth of the stator winding, busbars, etc. for each phase.

In each phase the capacitance C is in parallel with the magnetising reactance of the voltage transformer of that phase. The volt/ampere characteristic of the parallel combination can be derived as indicated in FIGURE 1-5-7 (a).

From this diagram it is clear that for voltages below U' the resulting current is leading while for values of U above U' the resultant current is lagging. Suppose for example that a temporary disturbance in the voltages of the system shown in FIGURE 1-5-6 establishes a voltage less than U' across the capacitor and voltage transformer of one phase (A) while the corresponding voltages on the other two phases are in excess of U'. The requirement is thus that the current I_A must be leading while the currents I_B and I_C must be lagging with respect to the corresponding voltages. From inspection of the phasor diagram of the circuit it is clear that this requirement can only be met if the neutral point of the system becomes inverted as shown in FIGURE 1-5-7 (b).

The above analysis is an oversimplification of a very complex problem but serves the purpose of illustrating that under certain circumstances very high fundamental frequency overvoltages are possible. Shott and Peterson [13] illustrated by means of tests on a miniature system that, apart from the fundamental frequency neutral displacement voltage, third harmonic and one half fundamental frequency ferroresonance modes are also possible. Their recommendations can be summarised as follows:

For line-to-line rated voltage transformers connected line-to-neutral corresponding to general practice in the application of such devices the range of abnormality is defined approximately by $0{,}012 < \frac{X_c}{X_m} < 0{,}6$ where X_c is the capacitive reactance to ground and X_m is the ratio of voltage to magnetising current of the voltage transformer measured at rated voltage. If a given circuit is found to fall within this range, normal behaviour can

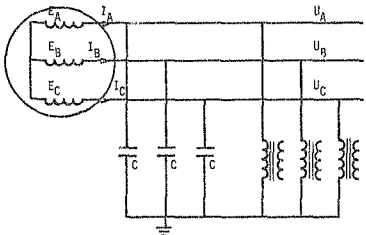

FIGURE 1-5-6 : Circuit for analysis of neutral instability

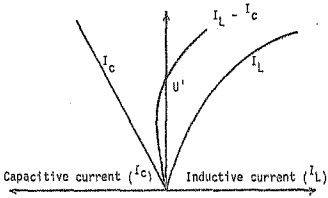

FIGURE 1-5-7 (a) : Parallel combination of capacitance and inductance of one phase

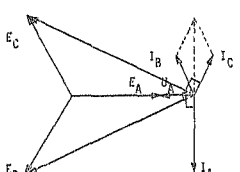

FIGURE 1-5-7 (b) : Phasor diagram for the circuit of FIGURE 1-5-6

be ensured by connecting a resistor of value equal to that of X_m across the open delta.

A complete picture of Shott and Peterson's findings is given in FIGURE 1-5-8.

Analysis of a system such as this is a formidable task since three non-linearities that operate under different voltage conditions are involved. It is therefore impossible to derive an equivalent single phase circuit for the analysis of the problem. In the following the method of describing functions will be used.

1-5-3 Method of Describing Functions applied to parallel ferroresonance phenomena

1-5-3-1 Introduction

In general the method of describing functions can be applied to a system with a block diagram as shown in FIGURE 1-5-9. The necessary conditions for the application of this method is that at the frequency under consideration (ω) the linear part of the open loop transfer function $G_1(j\omega)G_2(j\omega)$ must be of such a high order that all harmonics are filtered out. Irrespective of the harmonic content of the output (y) of the non-linearity, the input to the non-linearity therefore largely consists of a signal of the fundamental frequency (ω).

The describing function (DF) of a non-linearity is defined as the ratio between the fundamental frequency components of the output and input signals of the non-linear element.

$$\therefore \quad N(A) = \frac{D_1}{A} \underline{/\theta} \qquad \text{---- (1-5-1)}$$

At the frequency ω the non-linear element is therefore replaced by a gain $N(A)$.

A limit cycle is a self-sustained oscillation at frequency ω and amplitude A and will occur in FIGURE 1-5-8 if the input is zero when

$$y = (0 - yG_2(j\omega))G_1(j\omega)N(A)$$

$$= - yG_1(j\omega)G_2(j\omega)N(A)$$

$$\therefore \quad G_1(j\omega)G_2(j\omega) = - \frac{1}{N(A)} \qquad \text{---- (1-5-2)}$$

$$\therefore \quad G(j\omega) = G_1(j\omega)G_2(j\omega) = \frac{-1}{N(A)} \qquad \text{---- (1-5-3)}$$

The method of describing functions consists of plotting $G(j\omega)$ and $-\frac{1}{N(A)}$ in the complex plane and finding the value of A which corresponds with the required frequency ω.

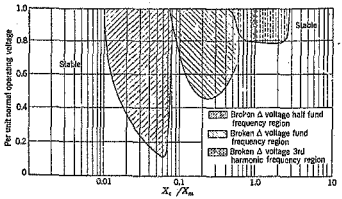

FIGURE 1-5-8 : Regions of possible neutral instability for the circuit of FIGURE 1-5-6

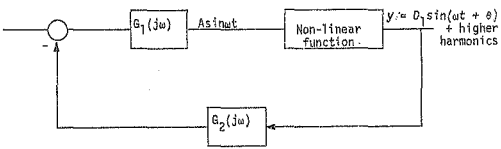

FIGURE 1-5-9 : Block diagram of system for DF analysis

West, Price and Livesley [14] developed the concept of describing functions to apply in the case where the input to the non-linearity takes the form $A_1 \sin(\omega t + \phi) + A_n \sin(n\omega t)$. The low frequency dual input DF is defined as

$$N_1(A,B) = \frac{D_1}{A_1} \angle \theta_1 - \phi \qquad \text{---- (1-5-4)}$$

and the high frequency dual input DF as

$$N_n(A,B) = \frac{D_n}{A_n} \angle \theta_n \qquad \text{---- (1-5-5)}$$

where D_1 and D_n are the amplitudes of the components of frequency ω and $n\omega$ respectively. θ_1 and θ_n are the phase shifts associated with the components of the two frequencies respectively.

There have been various attempts to use the method of describing functions for the analysis of ferroresonance phenomena. Several authors analysed series ferrosonance circuits using some or other form of DF method [15,16]. Swift [17] and Kumar et al [18] analysed parallel ferroresonance of a single phase circuit by means of an incremental DF method and indicated that jump resonance can occur.

In most of above mentioned analyses only single phase circuits are considered and the effect of hysteresis and eddy current losses are neglected often without being mentioned at all.

1-5-3-2 DF analysis of neutral inversion

In the present analysis an attempt is made to verify the neutral stability criteria of Shott and Peterson [13] by means of a DF method.

The three phase circuit diagram is shown again in FIGURE 1-5-10.

In Appendix C the basic equations for this circuit are derived together with the block diagram for zero sequence quantities shown in FIGURE 1-5-11.

The transfer function of the open loop is

$$G(P) = \frac{1}{P}\left(r + \frac{1}{P/x_c + 1/r_c}\right) = \frac{r\left(P/x_c + \frac{1}{r_E}\right) + 1}{P\left(P/x_c + 1/r_E\right)} \qquad \text{---- (1-5-6)}$$

It is clear that there are two poles and one zero. In order to be able to apply the DF method with reasonable accuracy it is preferable that the majority of the poles over the zeros should be two. However, in the special

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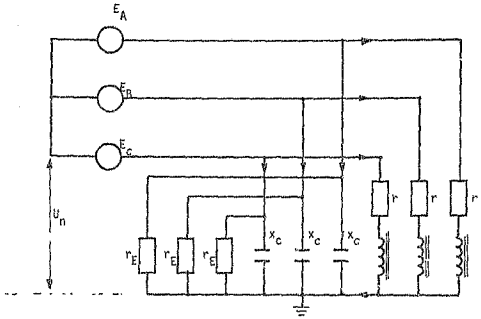

FIGURE 1-5-10 : Complete three phase diagram under analysis

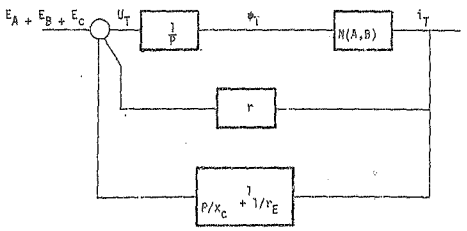

FIGURE 1-5-11 : Block diagram for zero sequence quantities

case where r = 0 there are two poles and no zeros, thus making the system suitable for DF analysis. It therefore appears that the system can be analysed by means of the DF method bearing in mind that r is usually fairly small in comparison with the magnetising reactance.

At a frequency of ω the operator P can be replaced by jω, thus giving the frequency transfer function of the linear part of the system:

$$G(j\omega) = \frac{r(j\omega/x_c + 1/r_E) + 1}{j\omega(j\omega/x_c + 1/r_E} \cong \frac{1 + r/r_E + j\omega\, r/x_c}{-\omega^2/x_c + j\,\omega/r_E} \qquad \text{---- (1-5-7)}$$

The condition for the existence of a self-sustained oscillation at an angular frequency ω is (from equation 1-5-2)

$$G(j\omega) = -1/N(A,B) \qquad \text{---- (1-5-8)}$$

where $A = E/j\omega$ and $B = 1/3\ U_T/j\omega$

The neutral displacement voltage $U_n = \frac{1}{3}U_T = \frac{1}{3}j\omega\dot{\psi}_T = j\omega B$ can under different circumstances consist of fundamental frequency, third harmonic or ⅓ fundamental frequency voltages. The normalised voltage U_T can be interpreted as the sum of the three phase voltages (to earth) which can be obtained by measuring the voltage across the open delta secondary windings of the earthing transformers.

The existence of self-excited oscillations at these frequencies can be verified in turn by plotting G(jω) for the specific frequency as a function of the system parameters and noting the regions of possible intersection with $-1/N(A,B)$.

The only information available regarding the transformers used in the tests done by Shott and Peterson is a rms magnetisation curve of the transformers. In the present analysis the non-linearity of each phase is represented by a polynomial of the form

$$i = k_1\psi - k_3\psi^3 + k_5\psi^5 \qquad \text{---- (1-5-9)}$$

The constants k_1, k_3 and k_5 are chosen such that under sinusoidal voltage excitation a rms curve as close as possible to that given in Shott and Peterson's paper. In FIGURE 1-5-12 the magnetising curve obtained which coincides with that of Shott and Peterson is given. In this derivation the effect of iron losses are neglected.

The describing functions for the various frequencies are derived analytically in APPENDIX D and in TABLE 1-5-1 the results are summarised.

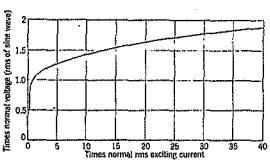

FIGURE 1-5-12: Magnetisation curve of transformer derived from equation $i = 0,7\psi - 1,1\psi^{3} + 0,713\psi^{5}$

A is the amplitude of the fundamental frequency supply flux linkages and B the amplitude of the neutral displacement flux linkages. The phase angle between the neutral displacement and the supply is denoted by β.

TABLE 1-5-1 : Describing Functions for ω = 1,3 and $\frac{1}{3}$ p.u.

Frequency ω (per unit)	
1	$N_1(A,B) = k_1 - \frac{3}{2}k_3A^2 - \frac{3}{4}k_3B^2 + \frac{15}{8}k_5A^4 + \frac{15}{4}k_5A^2B^2 + \frac{5}{8}k_5B^4 + \frac{5}{8}k_5A^3Be^{-j\beta}$
3	$N_3(A,B) = (\frac{1}{4}k_3A^3/B - \frac{5}{16}k_5A^5/B - \frac{5}{4}k_5A^3B)e^{-j3\beta} + k_1 - \frac{3}{2}k_3A^2 - \frac{3}{4}k_3B^2 + \frac{15}{8}k_5A^4 + \frac{15}{4}k_5A^2B^2 + \frac{5}{8}k_5B^4 - \frac{5}{8}k_5A^3Be^{j3\beta}$
$\frac{1}{3}$	$N_h(A,B) = k_1 - \frac{3}{2}k_3A^2 - \frac{3}{4}k_3B^2 + \frac{15}{8}k_5A^4 + \frac{15}{4}k_5A^2B^2 + \frac{5}{8}k_5B^4$

The locus of the negative inverse of the fundamental DF $N_1(A,B)$ for fixed values of A and B and variation of β between 0 and 60° is shown to be a circle (See APPENDIX D). The radii and centres of circles for A = 1,414 (corresponding to a rms supply voltage of 1 per unit) and various values of B are computed. (See APPENDIX E for computer programs). The results are presented in FIGURE 1-5-13 for the fundamental DF. On the same diagram the transfer function G(jω) is mapped as a function of x_c , r_E and r.

FIGURE 1-5-13 shows the family of circles for different values of B. It is clear that the circles converge to the origin for increasing values of ψ_T. Two loci of the frequency transfer function are shown for values of r_E = 5 and 0,33. The value of 5 represents the iron losses of the transformers and is based on the somewhat arbitrary assumption that the numerical value of the iron loss resistance in the transformer equivalent circuit is five

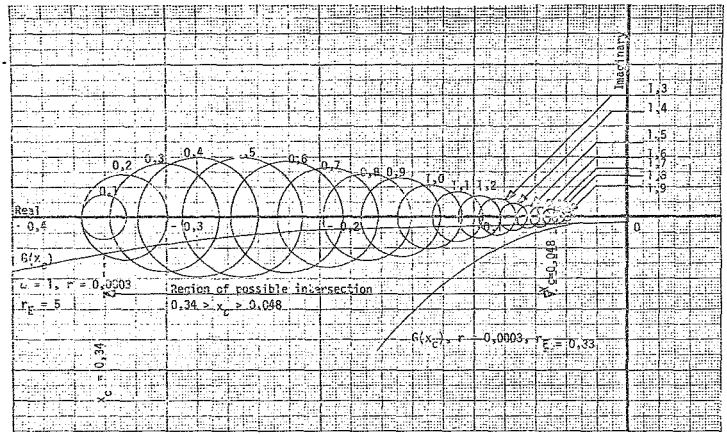

FIGURE 1-5-13 : fundamental frequency DF and frequency transfer function for $\omega = 1$. Values of B are indicated on the circles.

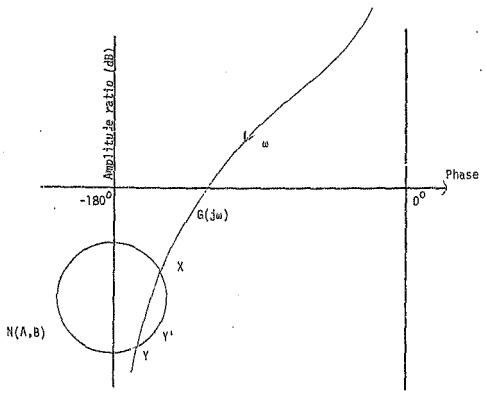

FIGURE 1-5-14 : Circular describing function characteristic and frequency transfer function in the amplitude ratio/phase plane

times that of the magnetising impedance. The value of r = 0,0003 for the series resistance of the transformer is given by Shott and Peterson [13].

It will be noted that there are two points of intersection with the frequency transfer function. Consider a general case represented on the amplitude ratio - phase plane as shown in FIGURE 1-5-14 [19]. The two possible stable operating points are X and Y. The stability of these states is checked by introducing a small perturbation when the system is operating at these states. Consider for example state Y. If the phase is increased slightly to Y' say then the amplitude is also increased. On the frequency transfer function, however, an increase in amplitude corresponds to a decrease in frequency which restores the phase angle. Y is thus a stable operating point. Likewise X can be shown to be an unstable state. In FIGURE 1-15-13 only the left hand intersections are therefore stable. Note that for high values of B no intersections are obtained. According to this diagram the range of values of x_c where neutral instability due to fundamental frequency ferroresonance can occur is $0{,}048 < x_c < 0{,}34$. The corresponding range obtained by Shott and Peterson (See FIGURE 1-5-8) is $0{,}075 < x_c < 0{,}55$. It will be noted that no stable operating points are obtained for $r_E = 0{,}33$. This is in keeping with Shott and Peterson's suggestion that a resistor of value 1,0 per unit, connected across the open delta of the transformers would suppress neutral instability.

The third harmonic describing function or rather $\frac{-1}{N_3(A,B)}$ is given in FIGURE 1-5-15. It will be noted that a family of skew ellipses is obtained. Once again only the point of intersection further from the origin is stable. The describing function corresponding to a value of B between 0,2 and 0,3 has its leftmost point of intersection with the negative real series at $-\infty$. Since we are dealing with the negative of the inverse of the DF the actual describing function is zero for this value of B. The significance of this value of B is that the third harmonic of the fundamental wave is cancelled by the third harmonic signal (Bsin3ωt) applied to the circuit. This condition in fact corresponds to the operation of three star-connected single phase transformers without the capacitors where third harmonic currents can not flow in the neutral connection and a third harmonic component appears in the flux and the phase voltages [9, p.416].

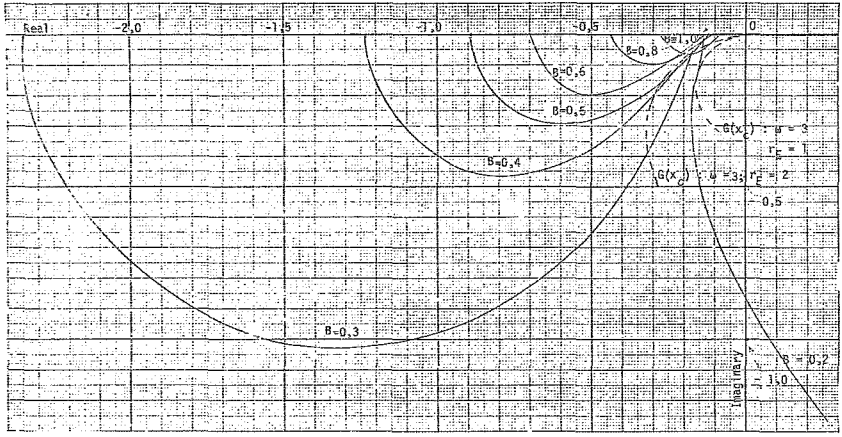

FIGURE 1-5-15 : Third harmonic DF and frequency transfer function for $\omega = 3$.

The frequency transfer function for $\omega=3$ is superimposed on the describing function. The transfer function is drawn for $r_E = 2$ and $r_E = 1$. The iron losses are frequency dependent and the lower value of r_E used here (compared to the fundamental frequency case) allows for this fact. Again this value of r_E gives a range of x_c where possible neutral instability can occur. At high values of B the ellipses become so small that no points of intersection are obtained.

From TABLE 1-5-1 it appears that the DF for $\omega = \frac{1}{2}$ contains only a real part. If this were so no points of intersection would be possible in the presence of even the slightest series or shunt resistance. This is obviously not correct and the inaccuracy is probably due to the fact that a component of frequency $\omega = \frac{3}{2}$ is generated due to the input of frequency $\omega = \frac{1}{2}$. The $\omega = \frac{3}{2}$ component was not taken into account in the analysis. This, together with the fact that the "low pass filter" action cannot distinguish between components $\omega = 1$ and $\omega = \frac{3}{2}$ explains why the DF model developed for $\omega = \frac{1}{2}$ is unacceptable. A more accurate model will have to be developed in order to study the $\frac{1}{2}$ fundamental frequency resonance in more detail.

1-5-3-3 Experimental verification

An experimental investigation of this phenomenon was carried out on a circuit similar to that shown in FIGURE 1-5-10. Three single phase transformers were used and a mean magnetisation curve was derived from a family of hysteresis curves which was obtained by measurement. In addition the iron losses were measured and an average value was obtained over the expected range of voltage.

From above measurements and based on a reference volage $U_r = 78,5$ V the following equation for the magnetisation curve was obtained

$$i = 2\psi - 0,75\psi^3 + 1,9\psi^5 \qquad \text{---- (1-5-10)}$$

During these tests with $x_c = 0,506$ per unit third harmonic resonance manifested itself as a jump resonance which occurred at a supply voltage of 1,67 p.u. (peak). The resulting third harmonic neutral displacement is shown in FIGURE 1-5-16. A third harmonic peak voltage of about 250 V (3,2 p.u.) was observed.

In FIGURE 1-5-17 the third harmonic DF for the circuit is shown for different values of B. On the same diagram point Q is indicated to represent $G(j\omega)$ for $\omega = 3$, $x_c = 0,506$ and $r_E = 0,7$. The value of r_E was obtained from the iron loss measurements.

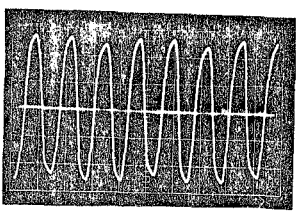

FIGURE 1-5-16 : Third harmonic neutral displacement voltage U_n.

Vertical scale : 1 cm = 1,27 p.u.

Horizontal scale : 1 cm = 5 ms

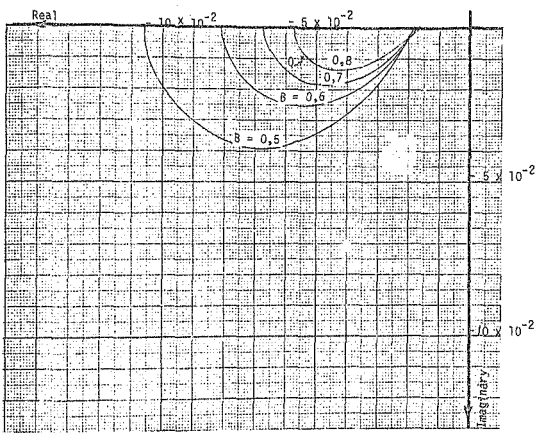

FIGURE 1-5-17 : Third harmonic DF for experimental set up. Complex value of $G(^{X}c)$ indicated by point Q.

From this diagram it is clear that Q corresponds to a value of B=0,8. The voltage associated with this value of B is $U_n = \omega B = 3B$. The neutral displacement voltage is therefore 3 x 0,8 = 2.4 p.u. This value deviates 25% from the measured 3,2 p.u. The reason for the large error can be sought in the presence of higher harmonics and inaccurate modelling of iron losses.

During these tests fundamental frequency resonance could also be established with some difficulty whilst it was not possible to establish the one half fundamental frequency resonance.

It was also established that a resistor having an ohmic value of the same order of magnitude suggested by Shott and Peterson would adequately suppress neutral instability.

1-5-3-4 Conclusions

Although originally intended as a verification of Shott and Peterson's neutral stability criteria only moderate success has been achieved at this stage.

From these qualitative results and analysis it does appear that Shott and Peterson's criteria are probably reasonable. It does however seem that iron losses have an important effect on the extent of the stability regions. From FIGURE 1-5-13 it also appears that the winding series resistance plays an important part in establishing the "low x_c" stability boundary.

In practical applications the following measures to preclude the possibility of neutral instability are suggested.

i) Adherence to Shott and Peterson's criteria.

ii) Choice of operating point very low on the transformer magnetisation curve.

1-6 Selective and Sensitive Stator Earth Fault Protection

1-6-1 Introduction

In general higher earth fault currents should be cleared faster than smaller currents. In the case of generators where the earth fault currents are limited to values of the same order as the phase fault currents, the phase fault protection such as overcurrent or differential protection would be adequate to clear earth faults.

In cases where earth fault current is limited to small values, however, sensitive earth fault protection should be provided and since currents are small high speed protection is not so important. Earth fault protection can consist of either a voltage operated relay connected parallel with the earthing

impedance or a current operated relay in series with the earthing impedance.

1-6-2 The influence of third harmonic currents on stator earth fault protection

The provision of sensitive earth fault protection is hampered by the presence of circulating third harmonic currents. These currents are caused by the third harmonic voltages generated in the stator winding due to the non-sinusoidal flux distribution in the air gap of the generator. Fractional pitch windings can be used to minimise these voltages but short chording normally concentrates on the fifth harmonic as third harmonics voltages can easily be eliminated from the system by suitable transformer connections. The third and multiply related harmonics of the three phases form a "zero sequence" set and therefore do not summate to zero at the star point. The tendency is therefore for these harmonic currents to follow the same paths as genuine fault currents.

If a generator is not connected to other equipment with earthed neutral points the flow of third harmonic currents is inhibited. In the unit-connected layout mentioned earlier the generator is connected to the delta winding of the generator transformer.

When several generators operate in parallel, third harmonic currents will tend to circulate among the neutrals of the parallel connected machines. Unless the third harmonic voltages which give rise to these currents happen to be of equal phase and magnitude provision must be made to earth only one generator at a time.

In cases where the earthing impedance has a value comparable with the capacitive reactance to earth of the stator windings, third harmonic currents will circulate through the neutral earthing impedance and the capacitive reactances even if the machine is operating isolated from the system. Moreover, these currents will be of the same order of magnitude as the fundamental frequency earth fault current. This places a limitation on the sensitivity of the earth fault protection as the setting of the protection must be chosen above the third harmonic current or voltage. A low pass filter is often embodied in the earth fault relay for the rejection of the third harmonics.

In the remainder of this section unit connected generators earthed through relatively high neutral impedances will be considered. The magnitude of the earth fault current is roughly proportional to the position of the

fault on the stator winding relative to the neutral point. If the fault is near the neutral point the fault current may be too small for the protection relays to detect - the relay of course having a finite setting. A certain portion of the stator winding near the star point will thus be unprotected. A further hazard of having such an unprotected zone is that a permanent fault in this zone would bypass the earthing impedance and a second fault on the stator would cause an excessively high earth fault current while the protection relay in the neutral circuit is short circuited.

Pohl [1, p. 89] suggested that a fundamental frequency unbalance voltage should be injected in order to cause the flow of fault current also for faults at or near the neutral point. Such a scheme is shown in FIGURE 1-6-1. In this scheme a deliberate voltage is injected at the neutral point and the capacitive current caused by this voltage during normal operation is prevented from causing relay operation by suitable compensation of the wattmetric relay [5].

Other schemes have been devised which depend on the third harmonic voltages generated within the generator to cause relay operation for faults near the neutral point. Under these circumstances the relay should not suppress third harmonics. In such schemes an artificial neutral point is established at the generator terminals by means of an earthing transformer and the actual generator neutral is left floating as shown in FIGURE 1-6-2. The operation of this scheme depends on the third harmonic content of the generated voltage. The third harmonic content in the voltage of modern generators is low as 2% [4] and the actual third harmonic voltage varies with load conditions. In the scheme shown in FIGURE 1-6-2 the terminal capacitances (C) tend to divert some of the third harmonic current away from the relay. These terminal capacitances represent the busbar and cable capacitances as well as surge capacitors which may be provided. Each scheme must therefore be analysed in detail to ensure that coverage for the full stator winding is obtained.

Berman [6] and Pazmandi [20] suggested a scheme which is shown diagrammatically in FIGURE 1-6-3. In these schemes the third harmonic voltage is measured both at the neutral point and at the generator terminals. As is shown in FIGURE 1-6-3 the third harmonic components of these voltages vary with fault location. The criterion for operation of the earth fault relay is that the ratio of $U_t(3\omega)$ to $U_n(3\omega)$ should exceed a preset value. This relay takes care of faults near the neutral point and is tuned for the third harmonic. For faults elsewhere on the winding a separate fundamental frequency relay is provided. The advantage of this type of scheme is that

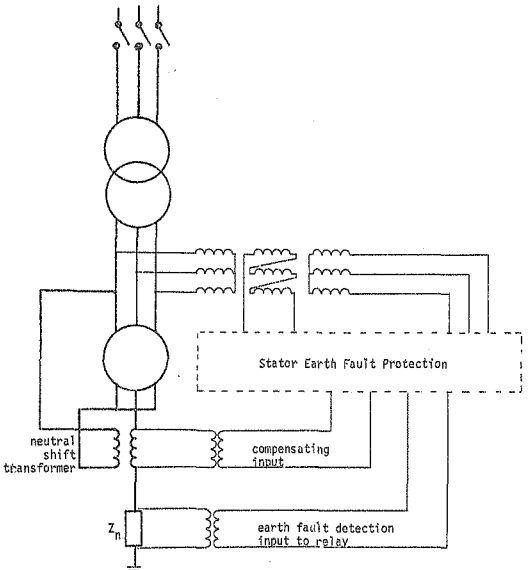

FIGURE 1-6-1 : Stator Earth fault protection scheme to ensure 100% coverage of stator winding

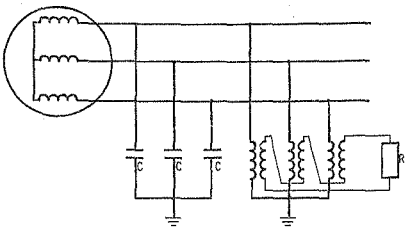

FIGURE 1-6-2 : Generator earth fault protection scheme utilising third harmonic voltage for 100% coverage.

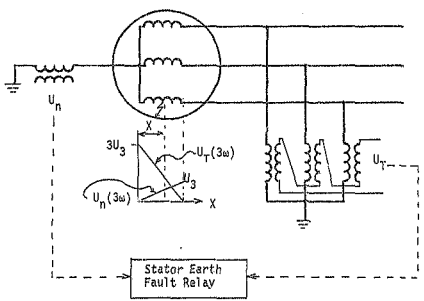

FIGURE 1-6-3 : Earth fault protection utilising both neutral and terminal third harmonic voltages for 100% stator earth fault protection.

since satisfactory operation depends on the ratio of the third harmonic voltages at the two locations, variations of the actual third harmonic voltage do not affect the sensitivity of the protection.

Further reference to protection schemes will be made in CHAPTER 2 when different earthing methods are discussed.

CHAPTER 2

VARIOUS METHODS OF GENERATOR EARTHING

2-1 Introduction : Operation with isolated neutral

From the previous chapter it appears that it is advisable to tie the generator neutral down to earth to some or other extent.

In transmission systems operation with isolated neutral has become obsolete due to the excessive overvoltages produced by arcing. The late Dr Bernard Price gave the following account of the effects of unearthed operation [21, p. 288] :

"With an insulated neutral heavy surging took place whenever a lightning flashover occurred because the system was not definitely held down to earth potential. The arrestors which were of the old fashioned horn gap water resistance type, on the occurrence of a flashover to earth on the line, discharged continuously until a second breakdown to earth or between phases occurred in the neighbourhood of the arrestors, thus causing the switches to operate. On inspection, afterwards, one stood in awe to see what appeared to be the terrifying effects of lightning, whereas it was really the culmination of prolonged discharging of arrestors."

A further disadvantage of unearthed operation is that earth faults are not easily and selectively detected. The usual method of fault detection is to have either a voltage transformer between the star point and earth or three single phase voltage transformers at the generator terminals with the secondary windings in open delta. The voltage transformers actually serve as earthing reactances of a high value and such a system cannot strictly be considered unearthed.

The absence of an intentional connection to earth on a low voltage section of a system means that this section can assume a voltage with respect to the high voltage system due to the capacitive coupling between the transformer windings as shown in FIGURE 2-1-1.

Similarly, a fault on the high voltage network would raise the average voltage to earth of the high voltage winding and the voltage to ground of the low voltage winding would be determined by the capacitive voltage divider consisting of C_1 and C_2. In the case of lightly earthed systems a fault on the high voltage system could thus cause maloperation of the generator earth

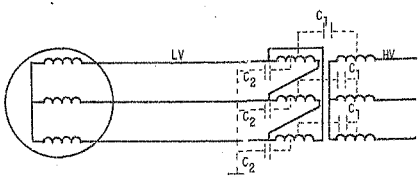

FIGURE 2-1-1 : Capacitive coupling between high and low voltage windings of generator transformer when HV neutral is not earthed.

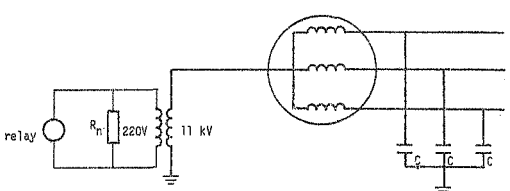

FIGURE 2-1-2 : High resistance earthing schcme for generator distribution transformer earthing.

fault protection relays. This is a particular hazard if the high voltage system is not effectively earthed.

2-2 Direct Earthing

Whereas solid earthing has become the accepted method of earthing high voltage transmission systems the susceptibility of generators to damage due to high earth fault currents necessitate the use of some or other form of neutral impedance. On generators the advantages of reduced insulation levels with effectively earthed systems are offset by the risk of high fault currents which, due to the time constant of the field circuit, cannot be interrupted quickly. In addition further problems are experienced with circulating third harmonic currents on directly earthed generators.

Solid earthing is therefore only used on small low voltage generators due to the smaller investment and lower voltage and fault current levels.

2-3 Resistance Earthing

Resistance earthing is probably the most popular type of generator earthing. As pointed out in section 1-4 the main advantage of resistance earthing is the fact that transients are suppressed.

When the actual ohmic value of earthing resistance is considered, two main schools of thought exist : a) low resistance and b) high resistance earthing.

In low resistance earthing a resistance value is chosen such that earth fault current is limited to 200-300 A [22, p. 31]. In larger power stations an earth fault current of this magnitude necessitates immediate shutdown of the generator and disconnection of the field. Although used extensively on unit connected generators it is the author's opinion that the value of fault current is excessive and no real advantages accrue due to this low value of resistor. The low resistance scheme is however particularly suitable for busbar connected generators which feed directly into a distribution system as the relatively high value of fault current facilitates grading of each fault relays on the system.

Another factor to be taken into account when choosing the earthing resistor is that the power loss in the resistor should not be too large in relation to the generated power on the system as excessive power swings could be initiated [23, p. 523].

In high resistance earthing the fault current is limited to 5-15 A. It has been shown in section 1-2 that currents as low as these cause damage of a limited extent to the stator iron. Bearing in mind that the generator voltages under consideration range from 11 - 20 kV it is clear that a fairly high ohmic value of resistance is required. In practice a resistor of a value 2 000 Ω to 3 000 Ω is required. Although high voltage resistors of this value rated at 5-15 A are available [24], a convenient way of realising such a resistor is to connect the primary winding of a single phase "distribution transformer" between the generator neutral and earth. The low voltage secondary winding is loaded with a low valued metallic resistor, having a relatively high current rating. A typical arrangement of this type is shown in FIGURE 2-1-2. Either a voltage operated relay in parallel or a current operated relay in series with the resistor is used for stator earth fault protection.

In order to ensure that overvoltages due to arcing earth faults and switching surges are suppressed it is necessary to choose a value of resistance such that the resistance referred to the HV winding of the distribution transformer (R'_n) is in accordance with Peterson's rule (See section 1-4-2).

$$R'_n \leq \frac{1}{3\omega C}$$

High resistance or distribution transformer earthing as described here has been used extensively in the United States of America, Canada and also in the Republic of South Africa. The Central Electricity Generating Board has recently decided to abandon its previous practice of low resistance earthing in favour of high resistance earthing when earthing of 500 MW generators is considered.

2-4 Reactance Earthing

As shown in section 1-2 a reactor could be used to limit the earth fault current to a suitable value. The steady state overvoltages would be less than those associated with a corresponding resistance while transient voltages would be worse on a reactor [9, p. 381].

The main advantage of reactor earthing is that there is only a small power dissipation in the reactor and a cheaper and more compact earthing installation is obtained. Cost is however not an important factor if the importance of a reliable earthing scheme is related to the cost of a large generator.

2-5 Resonant earthing

Resonant earthing is a form of reactance earthing where the reactor is chosen such that earth fault currents are suppressed. Consider the reactance earthed generator shown in FIGURE 2-1-3 (a). The corresponding interconnected sequence networks for a single phase to earth fault on one of the phases are shown in FIGURE 2-1-3 (b). In the zero sequence network the zero sequence capacitance is equal to the capacitance to earth (C) of one phase and the earthing inductance is multiplied by a factor of 3 [9, p. 11]. It is clear that, providing Z_0 is negligibly small, $3L_n$ and C are in parallel. If L_n is chosen such that parallel resonance is obtained between $3L_n$ and C in the zero sequence network at system frequency, earth fault current will be suppressed. In a practical circuit full compensation of the earth fault current will not be possible due to the resistance of the circuit. However, the fault current is limited to such an extent that the arc become unstable and is extinguished. The so-called arc suppression coil (also called ground fault neutraliser and Peterson coil) was invented by W. Peterson in 1916. The theory, principles and applications have been thoroughly investigated by various authors [1].

On transmission systems resonant earthing has been used extensively with considerable success in Europe, less frequently in the United States of America and also in South Africa by the Victoria Falls and Transvaal Power Company in the pre-war years [21]. For extensive and rapidly growing and changing systems periodic retuning of the coils are necessary and this together with other disadvantages detracts from the apparent ideal performance of the coil. For this reason resonant grounding has been largely superseded, especially on EHV systems.

The generator stator winding, the connections to the generator transformer and the low voltage winding of the generator transformer forms an isolated subsystem as far as earth faults are concerned. The capacitance to earth of this subsystem remains constant and the capacitive earth fault current can easily be neutralised by means of a Peterson coil. It is therefore surprising that this type of earthing is seldom used as it would seem an ideal application.

In 1953 Tomlinson [25] described two installations in America for unit connected generators. Among the advantages claimed are earth fault currents as low as 0,3 A and suppression of transient voltages. One interesting aspect mentioned in the discussion of this paper is that series resonance is possible in the zero sequence network, should an unintentional zero sequence generated

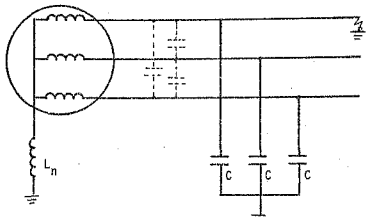

FIGURE 2-1-3 (a) : Resonant earthing of generator - Single phase to earth fault.

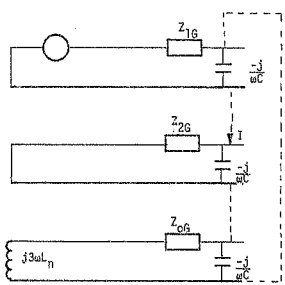

FIGURE 2-1-3 (b) : Interconnected sequence diagram for FIGURE 2-1-3 (a).

voltage exist, e.g. because of phase unbalance. An appreciable voltage can therefore appear across the earthing reactor during normal operation of the generator. This voltage can cause serious problems unless the X/R ratio (Q factor) of the coil is carefully chosen. Another expedient is to use an iron cored reactor, operating near its knee point. During overvoltage conditions the reactor goes into partial saturation and the consequent detuning suppresses the series resonance.

Gross et al [26] recently reported on the operation of 16 Peterson coil installations on unit connected generators, the largest being 667 MVA. Gross points out that at resonance the parallel combination of reactor and capacitor can be replaced by a very high resistance which represents the losses in the reactor and the capacitors. An earthing method similar to the distribution transformer high resistance scheme is used, the resistor being replaced by a reactor. A very favourable account is given of the performance of the existing installations. During 16 million kVA-years of experience no maloperations occurred and in fact no trips due to earth faults occurred. This is one of the main advantages of Peterson coil earthing, i.e. continued operation in the presence of earth faults. In addition, prolonged arcing cannot occur.

During the 1972 session of CIGRE Berman et al [6] reported on Czechoslovakian practice with regard to resonant earthing of generators. The main reason for using resonant earthing is the concern over the damaging effects of even low earth fault currents (See section 1-2). Encouraging operating experience is reported on sets as large as 235 MVA.

CHAPTER 3

EARTHING PRACTICES OF VARIOUS COUNTRIES

3-1 Introduction

A questionnaire was sent to several power generating authorities in various countries. Some of the principal findings of the questionnaire are discussed here also taking into account information on earthing practices of power companies not included on the circulation list of the questionnaire.

3-2 Earthing Method

The earthing method is often decided by the manufacturer of the generator. In other instances power authorities choose to design their own earthing schemes and to standardise throughout their supply area.

In TABLE 3-1 the earthing methods adopted in a number of countries are summarised. It is clear that high resistance (distribution transformer) earthing is by far the most general scheme. Reactance earthing is not used at all while voltage transformer earthing has been largely superseded and resonant earthing is making a comeback.

3-3 Surge Arrestors

Only two of the power companies questioned indicated that it is general practice to provide surge arrestors between line and earth at the generator terminals. Normally the generator transformer attenuates incoming surges from the system. Surge arrestors are usually only provided on the high voltage side of the generator transformer. In layouts where switching takes place at generator voltage surge divertors should be provided. In the Republic of South Africa the Electricity Supply Commission always provides surge arrestors at the generator terminals.

3-4 Occurrence of Earth Faults

Although stator earth faults are claimed to be the most common type of fault on a generator [5] they seldom occur. Only one power company reported

TABLE 3-1 : Principal Generator Earthing Methods in Various Countries

	High Resistance	Low Resistance	Voltage Transformer	Resonant Grounding	Reactance
Australia	G	P	S		
Belgium	G				
Czechoslovakia				G	
Denmark	G				
Germany	G				
Netherlands	G				
New Zealand	G				
Spain	N	N			
South Africa	G				
United Kingdom	N	G			
United States of America	G		S	P	

Legend : G : General Practice
S : Superseded Practice
N : General Practice for new installations
P : General Practice in Certain parts of country

that once in ten years stator iron is damaged due to earth faults.

3-5 Abnormal Voltage Conditions

All the firms questioned confirmed that no difficulties were experienced due to arcing earth faults, neutral inversion and switching surges.

3-6 Earth Fault Protection

Three of the firms questioned, all from Europe, indicated that special measures are taken to ensure 100% coverage of the stator winding. These measures apparently originate from the same European manufacturers of protection relays.

It appears to be general practice to use relays that eliminate third harmonic components from the input to the actual measuring relay.

As far as tripping sequences are concerned it is general practice to trip the high voltage circuit breaker of the generator transformer, the field switch and the steam valves. In at least one case shutdown is left to operator action. The Electric Supply Commission of South Africa using a distribution transformer earthing scheme, first trips the steam valves and afterwards the high voltage circuit breaker.

CHAPTER 4

EARTHING SCHEME FOR A UNIT-CONNECTED GENERATOR WITH A GENERATOR CIRCUIT BREAKER

4-1 Introduction

As mentioned in APPENDIX A there are definite advantages in using a unit-connected scheme with a circuit breaker interposed between the generator and generator transformer. Such a scheme is shown in FIGURE 4-1.

Being a unit-connected scheme a logical choice for an earthing method would be either high resistance or Peterson coil earthing. In the following an earthing scheme along the lines of high resistance distribution transformer will be considered.

Earthing of the generator neutral point only would leave section AB unearthed while the generator circuit breaker (CB_1) is open. As described in APPENDIX A this is a normal operating condition which occurs prior to synchronising of the generator. Some or other method of earthing AB during this condition is essential. The provision of an additional earth on this section, say at the unit transformer neutral point, would provide a low impedance circuit for circulating third harmonic currents (See section 1-6). Apart from the losses associated with these currents, earth fault protection is made difficult.

A scheme was therefore developed for use in Escom's power stations whereby duplicate earthing systems are provided, one on each side of the generator circuit breaker. This scheme is shown in FIGURE 4-2. Each earthing system consists of three single phase distribution type transformers (T_1 to T_3 or T_4 to T_6), each transformer having its primary connected in an open delta which is closed by means of a resistor R. A voltage operated protection relay is connected across each resistor.

When the generator circuit breaker is closed the two earthing systems operate in parallel. Since the voltage drop across the circuit breaker is negligible the primary terminals of two transformers on the opposite sides of the circuit breaker are in fact connected to equipotential points, thus excluding the possibility of circulating third harmonic currents. With the generator circuit breaker open the two parts of the system adjacent to the

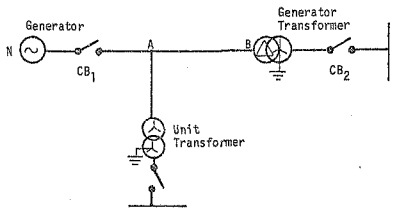

FIGURE 4-1 : Unit connected scheme utilising a generator circuit breaker.

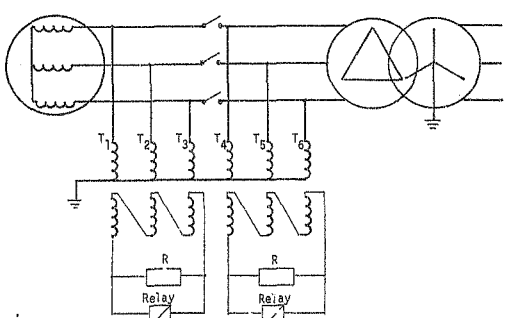

FIGURE 4-2 : Earthing scheme for generator with generator circuit breaker.

circuit breaker are separately earthed.

4-2 Design Criteria

This scheme is basically a high resistance earthing scheme and the same criteria apply as in the case of the star point earthed generator. The value of the resistor is chosen in accordance with Peterson's rule (Equation 1-4-1)

$$R_n \leq \frac{1}{3\omega C}$$

where C is the capacitance to earth per phase and R_n is the ohmic value of the neutral resistor. The rule is, however, modified in this instance. Consider one of the earthing systems as shown in FIGURE 4-3 (a) together with the relevant system capacitances. A single phase to earth fault occurs at F. The applicable sequence network interconnections are shown in FIGURE 4-3 (b). The resistance across the open delta (R_d) appears in the zero sequence network as $\frac{1}{3}R_d$ if we assume a 1:1 ratio for the earthing transformers. That this is so can be verified by injecting a zero sequence set of currents equal to I ampere into the transformer. A circulating current I A will flow in the open delta and cause a voltage drop of $R_d \times I$ volt across R. A third of this voltage will appear across each phase so that the zero sequence resistance = $\frac{1}{3} \times R_d \times I/I = \frac{1}{3}R$. In accordance with section 1-4-2 the time constant of the RC circuit must be less than $^1/_{\pi}$ of a period of a 50Hz wave (See section 1-4-2) $\therefore \frac{1}{3}R_d \leq \frac{1}{\omega C}$

$$\therefore \frac{1}{3}R_d \leq \frac{3}{\omega C} \qquad \text{---- (4-2-1)}$$

If the value of the resistor connected across the delta is chosen equal to this value the total resistance when the two systems operate in parallel will still conform to above inequality. It should however be borne in mind that there is also capacitance associated with section AB in FIGURE 4-1.

With reference to FIGURE 1-6-2 voltage generated in the stator can be used to provide protection for 100% of the stator winding.

The value of earthing resistance obtained by means of above rule is usually of the right order to suppress neutral instability. When specifying the transformer care should be taken that under maximum voltage conditions the earthing transformers are operating well below the knee point of their magnetisation curves. In an unearthed or lightly earthed system full line-to-line voltage can appear on the sound phases. On a 11 kV system the

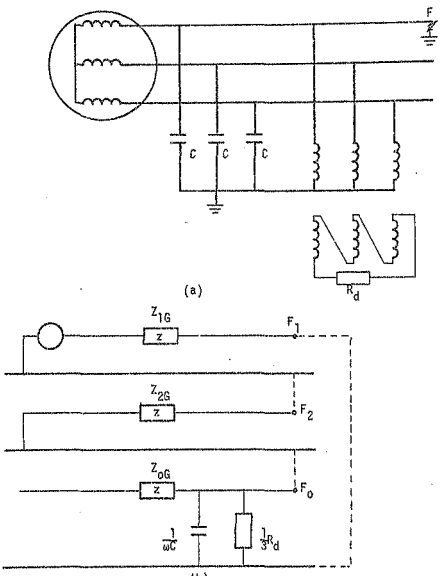

FIGURE 4-3 : Earthing scheme and sequence networks for a single phase to earth fault.

sound phases can therefore be subject to 11 kV. On such a system the magnetic circuits of the earthing transformers should be designed for $\sqrt{3} \times 11$ kV = 19,7 kV. According to Shott and Peterson [13] and investigations by the author (Section 1-5-3) these measures reduce the possibility of neutral instability adequately.

4-3 Conclusion

Earthing schemes based on these principles are used by Escom at various new power stations. On 30 MW sets at Van Eck Power Station similar schemes have been in operation for more than a year and good operational experience is reported.

Similar schemes are being used for Escom's new 500 MW generator installations.

CHAPTER 5

CONCLUSIONS

Generator earthing has been developed to a well understood science. Especially in the case of unit-connected generators the fact that the generator is electrically separate from the rest of the systems, as far as earth faults are concerned, allowed designers a certain amount of freedom in choosing an earthing impedance. This investigation revealed, however, that high resistance earthing is the most reliable technique. Resonant earthing can also be successfully applied.

The design of an earthing scheme for unit-connected generators with generator circuit breakers was discussed and the hazard of neutral instability due to ferroresonance in this scheme lead to the development of a describing function technique for analysis of this phenomenon. Further work in this field is necessary and in particular the effect of iron losses should be studied in detail. The block diagrams and techniques thus far developed should prove useful in a further investigation.

APPENDIX A

POWER STATION ELECTRICAL LAYOUTS

In relatively small systems where distribution is effected at generator voltage, the generators are paralleled at their terminals as shown in FIGURE A1. Such systems were widely used during the early stages of development of the power supply industries and are still used in small industrial power stations.

As distribution and transmission systems became more extensive, the scheme shown in FIGURE A2 was developed whereby each generator is connected directly to its step-up transformer. In this scheme, the so-called unit-connected scheme the generators are effectively paralleled on the high voltage side of the step-up or generator transformer. Unit-connected layouts are used in most new installations involving large turbogenerators. A unit transformer, connected between the generator and generator transformer supplies power for the auxiliaries of the generator, boiler and turbine. A station transformer is required for starting up purposes. The station transformer initially supplies power for the auxiliaries. Once the generator is running, the auxiliaries are however transferred to the unit transformer.

The scheme shown in FIGURE 4-1 is similar to the unit-connected scheme except that an additional circuit breaker is provided between the generator and the T-off point of the unit transformer. Synchronising takes place at this generator circuit breaker. This means that the unit transformer is energised via the generator transformer even before start-up and the provision of separate station transformers and a change-over routine is not necessary. Such a unit-connected scheme with a generator circuit breaker has been successfully used by Escom on the 30 MW sets installed at Van Eck Power Station. A similar scheme is also being used at Kriel Power Station which is still under construction. In this case air blast generator circuit breakers [27] which forms part of the isolated phase busbars are being used.

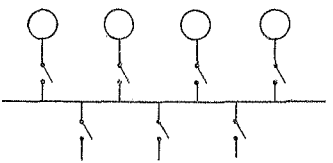

FIGURE A1 : Power station layout with generators paralleled at generator voltage.

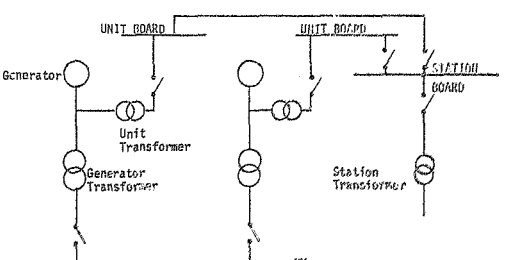

FIGURE A2 : Unit-connected power station layout.

APPENDIX B

FUNDAMENTAL FREQUENCY VOLTAGE OF SOUND PHASES DURING A PHASE TO EARTH FAULT

The interconnected sequence networks for a single phase to earth fault at the generator terminals are shown in FIGURE B1.

It is assumed that the positive and negative sequence impedances of the generator are equal and wholly inductive. The zero sequence resistance in fact constitutes the resistance of the neutral earthing device as the zero sequence resistance of the generator is assumed to be negligible.

The fault current for phase A is given by

$$I_A = I_1 + I_2 + I_o = \frac{3E}{2Z_1 + Z_o}$$

The fault currents in the sound phases are zero.

The sequence components of the voltage at the point of fault are given by

$$U_1 = E - I_1 Z_1 = E - \frac{Z_1}{2Z_1 + Z_o} E$$

$$U_2 = - I_2 Z_1 = - \frac{Z_2}{2Z_1 + Z_o} E$$

$$U_o = - I_o Z_o = - \frac{Z_o E}{2Z_1 + Z_o}$$

$$\therefore \quad U_A = U_1 + U_2 + U_o = E\left[\frac{Z_1 + Z_o}{2Z_1 + Z_o} - \frac{Z_1}{2Z_1 + Z_o} - \frac{Z_o}{2Z_1 + Z_o}\right] = 0$$

$$U_B = a^2 U_1 + a U_2 + U_o$$

$$= \left[\frac{a^2(Z_1 + Z_o)}{2Z_1 + Z_o} + \frac{a(-Z_1)}{2Z_1 + Z_o} + \frac{(-Z_o)}{2Z_1 + Z_o}\right] E$$

$$= \left[\frac{a^2(Z_1 + Z_o) - aZ_1 - Z_o}{2Z_1 + Z_o}\right] E \qquad \text{---- (B1)}$$

Prior to the incidence of the fault the voltage of phase B was

$$U_B' = a^2 E \qquad \text{---- (B2)}$$

The change in voltage due to the fault is therefore

APPENDIX B

FUNDAMENTAL FREQUENCY VOLTAGE OF SOUND PHASES DURING A PHASE TO EARTH FAULT

The interconnected sequence networks for a single phase to earth fault at the generator terminals are shown in FIGURE B1.

It is assumed that the positive and negative sequence impedances of the generator are equal and wholly inductive. The zero sequence resistance in fact constitutes the resistance of the neutral earthing device as the zero sequence resistance of the generator is assumed to be negligible.

The fault current for phase A is given by

$$I_A = I_1 + I_2 + I_0 = \frac{3E}{2Z_1 + Z_0}$$

The fault currents in the sound phases are zero.

The sequence components of the voltage at the point of fault are given by

$$U_1 = E - I_1 Z_1 = E - \frac{Z_1}{2Z_1 + Z_0} E$$

$$U_2 = - I_2 Z_1 = - \frac{Z_2}{2Z_1 + Z_0} E$$

$$U_0 = - I_0 Z_0 = - \frac{Z_0 E}{2Z_1 + Z_0}$$

$$\therefore \quad U_A = U_1 + U_2 + U_0 = E\left[\frac{Z_1 + Z_0}{2Z_1 + Z_0} - \frac{Z_1}{2Z_1 + Z_0} - \frac{Z_0}{2Z_1 + Z_0}\right] = 0$$

$$U_B = \alpha^2 U_1 + \alpha U_2 + U_0$$

$$= \left[\frac{\alpha^2(Z_1 + Z_0)}{2Z_1 + Z_0} + \frac{\alpha(-Z_1)}{2Z_1 + Z_0} + \frac{(-Z_0)}{2Z_1 + Z_0}\right] E$$

$$= \left[\frac{\alpha^2(Z_1 + Z_0) - \alpha Z_1 - Z_0}{2Z_1 + Z_0}\right] E \qquad \text{---- (B1)}$$

Prior to the incidence of the fault the voltage of phase B was

$$U_B' = \alpha^2 E \qquad \text{---- (B2)}$$

The change in voltage due to the fault is therefore

$$\Delta U = U_B' - U_B = \frac{\alpha^2 E - \alpha^2(Z_1 + Z_o) - \alpha Z_1 - Z_o E}{2Z_1 + Z_o}$$

$$= \frac{Z_o - Z_1}{Z_o + 2Z_1} E = \frac{\frac{Z_o}{Z_1} - 1}{\frac{Z_o}{Z_1} + 2} E = \frac{k - 1}{k + 2} E \qquad \text{---- (B3)}$$

Where $k = \frac{Z_o}{Z_1}$ ---- (B4)

Put $Z_1 = jX_1$ and $Z_o = R_o + jX_o$

$\therefore k = \frac{R_o + jX_o}{jX_1} = \frac{X_o}{X_1} - j\frac{R_o}{X_1} = x - jy$ ---- (B5)

where $x = \frac{X_o}{X_1}$ and $y = \frac{R_o}{X_1}$

Substitute (B5) in (B3) :

$$\Delta U = \frac{x - jy - 1}{x - jy + 2} E = \frac{x - 1 - jy}{x + 2 - jy} E$$

$$= \left[\frac{x + 2 - jy}{x + 2 - jy} - \frac{3}{x + 2 - jy}\right] E$$

$$\Delta U = \left[1 - \frac{3}{x + 2 - jy}\right] E \qquad \text{---- (B6)}$$

$$\therefore U_B = U_B' - \left[1 - \frac{3}{x + 2 - jy}\right] E = \left[\alpha^2 - 1 + \frac{3}{x + 2 - jy}\right] E \qquad \text{---- (B7)}$$

The loci of $\frac{3}{x + 2 - jy}$ for constant values of X and varying y can be represented by semi-circles such as MPN with diameters $\frac{3}{x + 2}$ as shown in FIGURE B 2.

Similarly a family of semi-circles can be obtained for constant values of y and variable x as indicated by MPQ. It is therefore clear that two families of orthogonal circles are obtained, each circle being identified with a particular value of x or y. If a line is drawn from the point of intersection of a x-circle and a y-circle the length of the line such as OP represents the voltage of phase B to earth.

Two similar orthogonal sets of circles can be drawn to obtain U_C. In this case the overvoltage will be OP .

The chart derived here is similar to the familiar Smith chart which is used in transmission line theory.

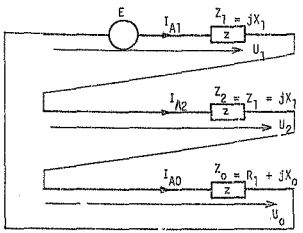

FIGURE B1 : Sequence networks for single phase to earth fault.

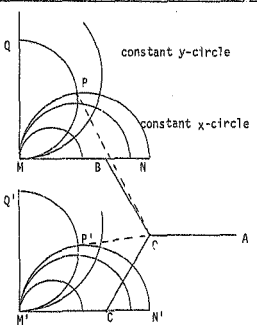

FIGURE B2 : Diagram for steady state overvoltages on sound phases during a single phase to earth faults.

APPENDIX C

TRANSFER FUNCTION OF GENERATOR EARTHING SYSTEM

Consider the general circuit shown in FIGURE C1. All currents and voltages refer to peak values of sinusoidal quantities.

The following mesh equations can be derived from this circuit

$$E_A - E_B - (R_1 + sL)(I_A' - I_B') = U_A - U_B \qquad \text{---- (C1)}$$

$$E_B - E_C - (R_1 + sL)(I_B' - I_C') = U_B - U_C \qquad \text{---- (C2)}$$

$$E_C - E_A - (R_1 + sL)(I_C' - I_A') = U_C - U_A \qquad \text{---- (C3)}$$

At point O all currents summate to zero. $I_A'' + I_B'' + I_C'' + I_A + I_B + I_C = 0$ also

$$I_A'' = U_A \left(Cs + \frac{1}{R_E}\right)$$

$$I_B'' = U_B \left(Cs + \frac{1}{R_E}\right)$$

$$I_C'' = U_C \left(Cs + \frac{1}{R_E}\right)$$

$$\therefore\ U_A\left(Cs + \frac{1}{R_E}\right) + U_B\left(Cs + \frac{1}{R_E}\right) + U_C\left(Cs + \frac{1}{R_E}\right) = -(I_A + I_B + I_C)$$

$$\therefore\ U_A + U_B + U_C = -\frac{\frac{1}{C}}{s + \frac{1}{R_E C}}(I_A + I_B + I_C) \qquad \text{---- (C4)}$$

C1 + C4 :

$$2U_A + U_C = E_A - E_B - (R_1 + sL)(I_A' - I_B') - \frac{\frac{1}{C}(I_A + I_B + I_C)}{s + \frac{1}{R_E C}} \qquad \text{---- (C5)}$$

C5 - C3 :

$$3U_A = 3E_A - (R_1 + sL)(2I_A' - I_B' - I_C') - \frac{\frac{1}{C}(I_A + I_B + I_C)}{s + \frac{1}{R_E C}}$$

$$= 3E_A - (R_1 + sL)(-I_A' - I_B \qquad \frac{I_A + I_B + I_C)}{s + \frac{1}{R_E C}}$$

$$= 3E_A - (R_1 + sL)3I_A' - \frac{\frac{1}{C}(I_A + I_B + I_C)}{s + \frac{1}{R_E C}}$$

$$= 3E_A - 3(R_1 + sL)(I_A + U_A(Cs + \frac{1}{R_E})) - \frac{\frac{1}{C}(I_A + I_B + I_C)}{s + \frac{1}{R_E C}}$$

$$U_A(3 + 3(Cs + \frac{1}{R_E})(R_1 + sL)) = 3E_A - 3(R_1 + sL)I_A - \frac{\frac{1}{C}(I_A + I_B + I_C)}{s + \frac{1}{R_E C}}$$

$$U_A = \frac{1}{1 + (Cs + \frac{1}{R_E})(R_1 + sL)}\left[E_A - (R_1 + sL)I_A - \frac{\frac{1}{3C}(I_A + I_B + I_C)}{s + \frac{1}{R_E C}}\right]$$

$$= \frac{\frac{1}{LC}}{s^2 + s(\frac{R_1}{L} + \frac{1}{CR_E}) + \frac{1}{LC}(1 + \frac{R_1}{R_E})}\left[E_A - (R_1 + sL)I_A - \frac{\frac{1}{3C}(I_A + I_B + I_C)}{s + \frac{1}{R_E C}}\right]$$

----(C6)

Likewise

$$U_B = \frac{\frac{1}{LC}}{s^2 + s(\frac{R_1}{L} + \frac{1}{CR_E}) + \frac{1}{LC}(1 + \frac{R_1}{R_E})}\left[E_B - (R_1 + sL)I_B - \frac{\frac{1}{3C}(I_A + I_B + I_C)}{s + \frac{1}{R_E C}}\right]$$

---- (C7)

and

$$U_C = \frac{\frac{1}{LC}}{s^2 + s(\frac{R_1}{L} + \frac{1}{CR_E}) + \frac{1}{LC}(1 + \frac{R_1}{R_E})}\left[E_C - (R_1 + sL)I_C - \frac{\frac{1}{3C}(I_A + I_B + I_C)}{s + \frac{1}{R_E C}}\right]$$

----(C8)

Further we have that

$$U_A' = U_A - I_A R \qquad \text{---- (C9a)}$$

$$U_B' = U_B - I_B R \qquad \text{---- (C9b)}$$

$$U_C' = U_C - I_C R \qquad \text{---- (C9c)}$$

Also

$$\Psi_A = \frac{1}{s} U_A' \qquad \text{---- C10a}$$

$$\Psi_B = \frac{1}{s} U_B' \qquad \text{---- C10b}$$

$$\Psi_C = \frac{1}{s} U_C' \qquad \text{---- C10c}$$

These equations can be used to draw a block diagram.

A block diagram which relates "zero sequence" quantities with one another can be obtained by forming the sum of the phase quantities derived above.

From C4:

$$U_T = U_A + U_B + U_C = \frac{-\frac{1}{C}}{s + \frac{1}{R_E C}} (I_A + I_B + I_C) = \frac{-\frac{1}{C}}{s + \frac{1}{R_E C}} I_T \qquad \text{---- (C11)}$$

This equation can also derived by adding C6, C7 and C8 while taking into account the fact that $E_A + E_B + E_C = 0$. Note that the terms containing $R_I + sL$ cancel out.

$$\text{Also } \Psi_T = \Psi_A + \Psi_B + \Psi_C = \frac{1}{s}(U_A' + U_B' + U_C') = \frac{1}{s} U_T' \qquad \text{-----(12)}$$

and

$$U_T' = U_A' + U_B' + U_C' = U_A + U_B + U_C - R(I_A + I_B + I_C)$$

$$\therefore U_T' = U_T - RI_T$$

Finally a function which relates I_T to Ψ_T is required. Consideration of FIGURE C1 indicates that

$$U_n = -E_A + U_A \qquad \text{---- (C14)}$$

$$U_n = -E_B + U_B \qquad \text{-----(15)}$$

$$U_n = -E_C + U_C \qquad \text{----(C16)}$$

From (C14) + (C15) + C16)

$$3U_n = -(E_A + E_B + E_C) + (U_A + U_B + U_C) = 0 + U_T$$

From C14, C15 and C16

$$U_A = E_A + U_n = U_A' = s\Psi_A \text{(Since } R_I + sL \text{ and } R \text{ are small with respect}$$

to the magnetising reactances)

$$\therefore \quad \Psi_A = \frac{1}{s}(E_A + U_n) = \frac{1}{s}(E + \frac{1}{3} U_T) \qquad \text{---- (C17a)}$$

where E is the peak value of the applied e.m.f.
Likewise

$$\Psi_B = \frac{1}{s}(\alpha^2 E + \frac{1}{3} U_T) \qquad \text{- -- (C17b)}$$

and

$$\Psi_C = \frac{1}{s}(\alpha E + \frac{1}{3} U_T) \qquad \text{---- (17c)}$$

If $A = \frac{E}{s}$ where α is the operator $= e^{j\frac{2\pi}{3}}$ and $B = \frac{1}{3}\frac{U_T}{s}$ it is clear that

Ψ_A, Ψ_B and Ψ_C are all defined in terms of A and B.

The non-linearity which relates I_T to Ψ_T can be represented by a DF

$N(\Psi_A, \Psi_B, \Psi_C)$ such that

$$I_T = N(\Psi_A, \Psi_B, \Psi_C)\Psi_T \qquad \text{---- (C18)}$$

From above reasoning :

$$N(\Psi_A, \Psi_B, \Psi_C) = N(A, B)$$

$$\therefore \quad I_T = N(A, B)\Psi_T$$

where $A = \frac{E}{s}$ and $B = \frac{1}{3}\frac{U_T}{s}$

Equations C11 to C18 can also be normalised with respect to a set of reference values ω_r, I_r, U_r and Z_r such that

$$U_r = \omega_r \Psi_r \text{ and } U_r = I_r Z_r.$$

Equation C11 becomes

$$u_T = \frac{U_T}{U_r} = \frac{-\frac{I_T}{I_r} Z_r \frac{I_r}{U_r}}{\frac{\omega_r C s Z_r}{\omega_r} + \frac{Z_r}{R_E}} = \frac{i_T}{\frac{p}{X_C} + \frac{1}{r_E}} \qquad \text{---- (C19)}$$

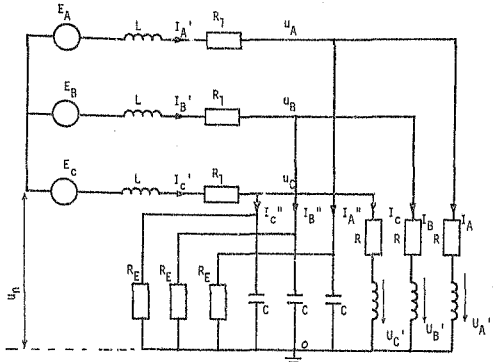

FIGURE C1 : Circuit diagram for three phase generator earthing scheme.

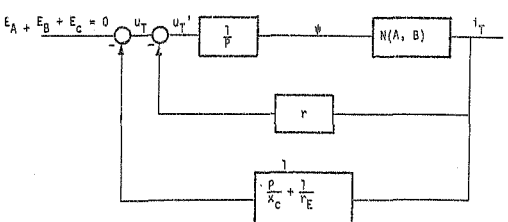

FIGURE C2 : Block diagram for "zero sequence" quantities.

Where $P = \frac{s}{\omega_r}$, $U_T = \frac{U_T}{U_r}$, $i_T = \frac{I_T}{I_r}$, $x_c = \frac{\frac{1}{\omega_r C}}{Z_r}$, $r_E = \frac{R_E}{Z_r}$

$$\psi = \frac{\Psi}{\Psi_r} \text{ and } r = \frac{R}{Z_r}$$

Likewise all other equations can be normalised and the block diagram shown in FIGURE 62 is obtained.

APPENDIX D

ANALYTICAL DERIVATION OF DESCRIBING FUNCTIONS

The non-linearity of each phase is represented by a polynomial of the form

$$i = k_1\psi - k_3\psi^3 + k_5\psi^5$$

Consider an input to the non-linearity of

$$\psi = A\sin(\omega t + \alpha - \beta) + B\sin n\omega t$$

The first term represents the fundamental input voltage while the second term corresponds to the neutral displacement of frequency $n\omega$.

Consider one phase.

$$i = k_1[A\sin(\omega t + \alpha - \beta) + B\sin n\omega t]$$
$$- k_3[A\sin(\omega t + \alpha - \beta) + B\sin n\omega t]^3$$
$$+ k_5[A\sin(\omega t + \alpha - \beta) + B\sin n\omega t]^5$$

This equation is of the form

$$i = k_1(x + y) - k_3(x + y)^3 + k_5(x + y)^5 \quad \text{where}$$

$x = A\sin(\omega t + \alpha - \beta)$ and $y = B\sin n\omega t$.

$$\therefore i = k_1(x + y)$$
$$- k_3(x^3 + 3x^2y + 3xy^2 + y^3)$$
$$+ k_5(x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5)$$

Put $\alpha + \beta = \gamma$ and consider these terms in turn.

$$x^3 = A^3\sin^3(\omega t + \gamma) = \frac{A^3}{4} - (3\sin(\omega t + \gamma) - \sin 3(\omega t + \gamma))$$

$$3x^2y = 3A^2B\sin^2(\omega t + \gamma)\sin(n\omega t)$$

$$= \frac{3}{2}A^2B(1 - \cos 2(\omega t + \gamma))\sin(n\omega t)$$

$$= \frac{3}{2}A^2B(\sin n\omega t - \sin n\omega t\cos 2(\omega t + \gamma))$$

$$= \frac{3}{2}A^2B(\sin n\omega t - \frac{3}{4}\sin((n + 2)\omega t + 2\gamma) - \frac{3}{4}\sin((n - 2)\omega t - 2\gamma)$$

$$3xy^2 = 3AB^2\sin(\omega t + \gamma)\sin^2 n\omega t$$

$$= \tfrac{3}{2}AB^2\sin(\omega t + \gamma)(1 - \cos 2n\omega t)$$

$$= \tfrac{3}{2}AB^2\sin(\omega t + \gamma) - \tfrac{3}{4}AB^2\sin[(1 + 2n)\omega t + \gamma] - \tfrac{3}{4}AB^2\sin[(1 - 2n)\omega t + \gamma]$$

$$y^3 = B^3\sin^3 n\omega t$$

$$= \tfrac{3}{4}B^3\sin n\omega t - \tfrac{1}{4}B^3\sin 3n\omega t$$

$$x^5 = A^5\sin^5(\omega t + \gamma)$$

$$= \tfrac{1}{16}A^5\{\sin 5(\omega t + \gamma) - 5\sin(\omega t + \gamma) + 5[3\sin(\omega t + \gamma) - \sin 3(\omega t + \gamma)]\}$$

$$= \tfrac{1}{16}A^5\{\sin 5(\omega t + \gamma) + 10\sin(\omega t + \gamma) - 5\sin 3(\omega t + \gamma)\}$$

$$= \tfrac{1}{16}A^5\sin 5(\omega t + \gamma) + \tfrac{5}{8}A^5\sin(\omega t + \gamma) - \tfrac{5}{16}A^5\sin 3(\omega t + \gamma)$$

$$5x^4y = 5A^4B\sin^4(\omega t + \gamma)\sin n\omega t$$

$$= \tfrac{5}{4}A^4B(1 - 2\cos 2(\omega t + \gamma) + \tfrac{1}{2}[1 + \cos 4(\omega t + \gamma)]]\sin n\omega t$$

$$= \tfrac{5}{4}A^4B\{\tfrac{3}{2} - 2\cos 2(\omega t + \gamma) + \tfrac{1}{2}\cos 4(\omega t + \gamma)\}\sin n\omega t$$

$$= \tfrac{5}{4}A^4B[\tfrac{3}{2}\sin n\omega t - 2\cos 2(\omega t + \gamma)\sin n\omega t + \tfrac{1}{2}\cos 4(\omega t + \gamma)\sin n\omega t]$$

$$= \tfrac{15}{8}A^4B\sin n\omega t - \tfrac{5}{4}A^4B\sin[(n + 2)\omega t + 2\gamma] - \tfrac{5}{4}A^4B\sin[(n - 2)\omega t - 2\gamma] + \tfrac{5}{16}A^4B\sin[(n + 4)\omega t + 4\gamma] + \tfrac{5}{16}A^4B\sin[(n - 4)\omega t - 4\gamma]$$

$$10x^3y^2 = 10A^3B^2\sin^3(\omega t + \gamma)\sin^2 n\omega t$$

$$= \tfrac{5}{4}A^3B^2[3\sin(\omega t + \gamma) - \sin 3(\omega t + \gamma)][1 - \cos 2n\omega t]$$

$$= \frac{5}{4}A^3B^2\{3\sin(\omega t + \gamma) - \sin3(\omega t + \gamma) - 3\sin(\omega t + \gamma)\cos2n\omega t$$

$$+ \sin3(\omega t + \gamma)\cos2n\omega t\}$$

$$= \frac{15}{4}A^3B^2\sin(\omega t + \gamma) - \frac{5}{4}A^3B^2\sin3(\omega t + \gamma)$$

$$- \frac{15}{8}A^3B^2\sin[(2n + 1)\omega t + \gamma] - \frac{15}{8}A^3B^2\sin[(1 - 2n)\omega t + 3\gamma]$$

$$+ \frac{5}{8}A^3B^2\sin[(3 + 2n)\omega t + 3\gamma] + \frac{5}{8}A^3B^2\sin[(3 - 2n)\omega t + 3\gamma]$$

$$10x^2y^3 = 10A^2B^3\sin^2(\omega t + \gamma)\sin^3n\omega t$$

$$= \frac{10}{8}A^2B^3[1 - \cos2(\omega t + \gamma)][3\sin n\omega t - \sin3n\omega t]$$

$$= \frac{5}{4}A^2B^3[3\sin n\omega t - \sin3n\omega t - 3\sin n\omega t\cos2(\omega t + \gamma)$$

$$+ \sin3n\omega t\cos2(\omega t + \gamma)]$$

$$= \frac{5}{4}[3\sin n\omega t - \sin3n\omega t - \frac{3}{2}(\sin[(n + 2)\omega t + 2\gamma] + \sin[(n - 2)\omega t - 2\gamma])$$

$$= \frac{15}{4}A^2B^3\sin n\omega t - \frac{5}{4}A^2B^3\sin3n\omega t - \frac{15}{8}A^2B^3\sin[(n + 2)\omega t + 2\gamma]$$

$$- \frac{15}{8}A^2B^3\sin[(n - 2)\omega t - 2\gamma] + \frac{5}{32}A^2B^3\sin[(3n + 2)\omega t + 2\gamma]$$

$$+ \frac{5}{32}A^2B^3\sin[(3n - 2)\omega t - 2\gamma]$$

$$5xy^4 = 5AB^4\sin(\omega t + \gamma)\sin^4n\omega t$$

$$= \frac{5}{4}AB^4[\frac{3}{2} - 2\cos2n\omega t + \frac{1}{2}\cos4n\omega t]\sin(\omega t + \gamma)$$

$$= \frac{5}{4}AB^4[\frac{3}{2}\sin(\omega t + \gamma) - 2\cos2n\omega t\sin(\omega t + \gamma) + \frac{1}{2}\cos4n\omega t\sin(\omega t + \gamma)]$$

$$= \frac{15}{8}AB^4\sin(\omega t + \gamma) - \frac{5}{4}AB^4\sin[(2n + 1)\omega t + \gamma] - \frac{5}{4}AB^4\sin[(1 - 2n)\omega t + \gamma]$$

$$+ \frac{5}{16}AB^4\sin[(1 + 4n)\omega t + \gamma] + \frac{5}{16}AB^4\sin[(1 - 4n)\omega t + \gamma]$$

$$y^5 = B^5\sin^5 in n\omega t$$

$$= \frac{1}{16}B^5\sin^5 n\omega t + \frac{5}{8}B^5 sinn\omega t - \frac{5}{16}B^5 sin3n\omega t$$

We therefore have that for one phase :

$$\begin{aligned}
i = {} & (k_1A - \tfrac{3}{4}k_3A^3 - \tfrac{3}{2}k_3AB^2 + \tfrac{5}{8}k_5A^5 + \tfrac{15}{4}k_5A^3B^2 + \tfrac{15}{8}k_5AB^4)\sin(\omega t + \alpha - \beta) \\
& + (\tfrac{9}{8}k_3A^2B - \tfrac{5}{4}k_5A^4B - \tfrac{15}{8}k_5A^2B^3)\sin[(n+2)\omega t - 2\beta + 2\alpha] \\
& + (\tfrac{9}{8}k_3A^2B - \tfrac{5}{4}k_5A^4B - \tfrac{15}{8}k_5A^2B^3)\sin[(n-2)\omega t + 2\beta - 2\alpha] \\
& + (\tfrac{1}{4}k_3A^3 - \tfrac{5}{16}k_5A^5 - \tfrac{5}{4}k_5A^3B^2)\sin 3(\omega t + \alpha - \beta) \\
& + (\tfrac{3}{4}k_3AB^2 - \tfrac{15}{8}k_5A^3B^2 - \tfrac{5}{4}AB^4)[\sin[(1+2n)\omega t + \alpha - \beta] + \sin[(1-2n)\omega t + \alpha - \beta]] \\
& + \tfrac{1}{16}k_5A^5\sin 5(\omega t + \alpha - \beta) \\
& + (k_1B - \tfrac{3}{2}k_3A^2B - \tfrac{3}{4}k_3B^3 + \tfrac{15}{8}k_5A^4B + \tfrac{15}{4}k_5A^2B^3 + \tfrac{5}{8}k_5B^5)sinn\omega t \\
& + (\tfrac{1}{4}k_3B^3 - \tfrac{5}{4}k_5A^2B^3 - \tfrac{5}{16}k_5B^5)\sin 3n\omega t \\
& + \tfrac{5}{16}k_5A^4B[\sin[(n+4)\omega t + 4\alpha - 4\beta] + \sin[(n-4)\omega t - 4\alpha + \beta] \\
& + \tfrac{5}{8}k_5AB^4[\sin[(1+4n)\omega t + \alpha - \beta] + \sin[(1-4n)\omega t + \alpha - \beta] \\
& + \tfrac{1}{16}k_5B^5\sin 5n\omega t \\
& + \tfrac{5}{32}k_5A^2B^3\{\sin[(3n+2)\omega t - 2\beta + 2\alpha] + \sin[(3n-2)\omega t + 2\beta - 2\alpha] \\
& + \tfrac{5}{8}k_5A^3B^2\{\sin[(3+2n)\omega t + 3\alpha - 3\beta] + \sin[(3-2n)\omega t + 3\alpha - 3\beta] \quad \text{----(D1)}
\end{aligned}$$

In the three phase arrangement under consideration we are interested in the relationship between the neutral displacement flux linkages ψ_T (which is represented by B) and the resulting current in the neutral conductor (i_T).

Equation D1 is valid for one phase, α being the phase angle of the driving voltage for that phase (A) with respect to ψ_T.

The currents for the three phases i_A, i_B and i_C can be obtained from this equation by putting $\alpha = 0$, 240^0 and 120^0 respectively. When forming

$i_T = i_A + i_B + i_C$ all terms containing a phase shift of kα vanish unless k is equal to or a multiple of 3 when these terms are trebled.

Thus

$$i_T = 3(\tfrac{1}{4}k_3A^3 - \tfrac{5}{16}k_5A^5 - \tfrac{5}{4}k_5A^3B^2)\sin3(\omega t - \beta)$$

$$+ 3(k_1B - \tfrac{3}{2}k_3A^2B - \tfrac{3}{4}k_3B^3 + \tfrac{15}{8}k_5A^4B + \tfrac{15}{4}k_5A^2B^3 + \tfrac{5}{8}k_5B^5)\sin n\omega t$$

$$+ 3(\tfrac{1}{4}k_3B^3 - \tfrac{5}{4}k_5A^2B^3 - \tfrac{5}{16}k_5B^5)\sin3n\omega t$$

$$+ 3(\tfrac{5}{8}k_5A^3B^2)\{\sin[(3 + 2n)\omega t - 3\beta] + \sin[(3 - 2n)\omega t - 3\beta]\}$$

$$+ 3(\tfrac{1}{16}k_5B^5)\sin5n\omega t.$$

We are interested in the describing function which relates the output of frequency nω to the relevant input $B\sin n\omega t$, taking into account the influence of the magnitude of the supply voltage (represented here by A) and its relative phase angle (β) with respect to $B\sin n\omega t$.

In TABLE D1 the relevant output terms are summarised for the cases $n = 1, 3$ and $\tfrac{1}{2}$.

TABLE D1 : Relevant output terms for various frequencies

n	Output of nω
1	$3(k_1B - \tfrac{3}{2}k_3A^2B - \tfrac{3}{4}k_3B^3 + \tfrac{15}{8}k_5A^4B + \tfrac{15}{4}k_5A^2B^3 + \tfrac{5}{8}k_5B^5)\sin\omega t$ $+ \tfrac{15}{8}k_5A^3B^2\sin(\omega t - 3\beta)$
3	$3(\tfrac{1}{4}k_3A^3 - \tfrac{5}{16}k_5A^5 - \tfrac{5}{4}k_5A^3B^2)\sin(3\omega t - 3\beta)$ $+ 3(k_1B - \tfrac{3}{2}k_3A^2B - \tfrac{3}{4}k_3B^3 + \tfrac{15}{8}k_5A^4B + \tfrac{15}{4}k_5A^2B^3 + \tfrac{5}{8}k_5B^5)\sin3\omega t$ $+ \tfrac{15}{8}k_5A^3B^2\sin(3\omega t + 3\beta)$
$\frac{1}{2}$	$3(k_1B - \tfrac{3}{2}k_3A^2B - \tfrac{3}{4}k_3B^3 + \tfrac{15}{8}k_5A^4B + \tfrac{15}{4}k_5A^2B^3 + \tfrac{5}{8}k_5B^5)\sin\tfrac{1}{2}\omega t$

The describing function for the fundamental frequency is in terms of phasors :

$$N_1(A,B) = 3(k_1B - \frac{3}{2}k_3A^2B - \frac{3}{4}k_3B^3 + \frac{15}{8}k_5A^4B + \frac{15}{4}k_5A^2B^3 + \frac{5}{8}k_5B^5)e^{j0} \pm \frac{15}{8}k_5A^3B^2e^{-j3\beta}$$

$$= k_1 - \frac{3}{2}k_3A^2 - \frac{3}{4}k_3B^2 + \frac{15}{8}k_5A^4 + \frac{15}{4}k_5A^2B^2 + \frac{5}{8}k_5B^4 + \frac{5}{8}k_5A^3Be^{-j3\beta}$$

$$= X_1 + Y_1e^{-j3\beta}$$

where $X_1 = k_1 - \frac{3}{2}k_3A^2 - \frac{3}{4}k_3B^2 + \frac{15}{8}k_5A^4 + \frac{15}{4}k_5A^2B^2 + \frac{5}{8}k_5B^4$

and $Y_1 = \frac{5}{8}k_5A^3B$

For analysis purposes we require - $1/N_1(A,B)$

$$\frac{1}{N_1(A,B)} = \frac{1}{X_1 + Y_1e^{-j3\beta}}$$

$X_1 + Y_1e^{-j3\beta}$ is a circle with centre at X_1 and radius Y_1.

From FIGURE D1 it is clear that - $\frac{1}{X_1 + Y_1e^{-j3\beta}}$ represents a circle with radius $\frac{Y_1}{X_1^2 - Y_1^2}$ and centre at $\frac{X_1}{X_1^2 - Y_1^2}$.

The describing function for the third harmonic $N_3(A,B)$ is

$$N_3(A,B) = \{3(\frac{1}{4}k_3A^3 - \frac{5}{16}k_5A^5 - \frac{5}{4}k_5A^3B^2)e^{-j3\beta}$$

$$+ 3(k_1B - \frac{3}{2}k_3A^2B - \frac{3}{4}k_3B^3 + \frac{15}{8}k_5A^4B + \frac{15}{4}k_5A^2B^3 + \frac{5}{8}k_5B^5)$$

$$+ (\frac{15}{8}k_5A^3B^2)e^{+j3\beta}\}/(3B)$$

$$= X_3e^{-j3\beta} + Y_3 - Z_3e^{+j3\beta}$$

where $X_3 = \frac{1}{4}k_3A^3/B - \frac{5}{16}k_5A^5/B - \frac{5}{4}k_5A^3B$

$Y_3 = k_1 - \frac{3}{2}k_3A^2 - \frac{3}{4}k_3B^2 + \frac{15}{8}k_5A^4 + \frac{15}{4}k_5A^2B^2 + \frac{5}{8}k_5B^4$

$Z_3 = \frac{5}{8}k_5A^3B$

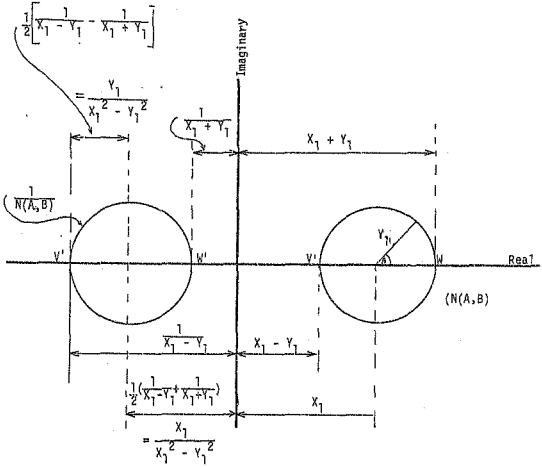

FIGURE DL : Locus of $-\frac{1}{N(A,B)}$ derived from N(A,B) in the complex plane (A and B constant and β variable).

$$N_3(A,B) = X_3(\cos 3\beta - j\sin 3\beta) + Y_3 - Z_3(\cos 3\beta + j\sin 3\beta)$$

$$= (X_3 - Z_3)\cos 3\beta + Y_3 - j(X_3 + Z_3)\sin 3\beta.$$

The negative inverse of $N_3(A,B)$ is obtained by computing $N_3(A,B)$ for various values of A,B and β and by obtaining the inverse of the complex values thus derived.

The describing function for $n = \frac{1}{2}$ is

$$N_h(A,B) = k_1 - \frac{3}{2}k_3A^2 - \frac{3}{4}k_3B^2 + \frac{15}{8}k_5A^4 + \frac{15}{4}k_5A^2B^2 + \frac{5}{8}k_5B^5.$$

APPENDIX E

COMPUTER PROGRAMS

a) Program to compute fundamental DF

```
10  LET  K1=  .7
20  LET  K3= 1.1
30  LET  K5=  .733
40  LET  A = 1.41399
50  FOR  B = 0 TO  3 STEP  .1
70  LET  X =K1-K3*(1.5*A*A+.75*B*B)+K5*(A^4*15/8+B*B*A*A*15/4+B4*5/8)
80  LET  Y=K5*A*A*A*B*5/8
90  LET  Z=Y*Y-X*X
100 LET  R=Y/Z
110 LET  C=X/Z
120 PRINT A,B,R,C
40  NEXT  B
150 END
```

b) Program to compute third harmonic DF

```
10  LET  K1= .7
20  LET  K3= 1.1
30  let  K5=  .733
40  LET  A = 1.41399
50  FOR  B = 1 TO  3 STEP  .1
5   FOR  H = 0 TO PHI(2/36)STEP PHI (2/36*12))
70  LET  X = K1-K (1.5*A*A+.75*B*B)+K5*(A^4*15/8+B*B*A*A*15/4+B4*5/8)
80  LET  Y = K3*A^3/(4*B)-K5*5*A^5/(B *16)-5*K5*A^3*B/4
90  LET  Z = -5*K5*A^3*B/8
100 LET' R = X+(Y+Z)*COS(3*H)
110 LET  I = (Z-Y)*SIN(3*H)
120 LET  V = R*R+I*I
130 LET  P = -R/V
140 LET  Q = -I/V
145 LET  H1=H*180/PHI(1)
150 PRINT B,H1,P,Q
155 NEXT H
156 NEXT B
160 END
```

c) Computer program to compute ½ fundamental frequency DF

```
5   CALL      I/U, 0, 2
10   LET  K1=  .7
20   'ET  K3= 1.1
30   LET  K5=  .733
40   LET  A = 1.41399
50   FOR  B = 0 TO  2  STEP  .1
70   LET  X =K1-K3*(1.5*A*A+.75*B*B)+K5*(A^4*15/8+B*B*A*A*15/4+B^4*5/8)
80   LET  R =-1/X
120  PRINT A,B,R
140  NEXT B
150  END
```

d) Program to compute frequency transfer function

```
2    LET  V(1) = 1
3    LET  V(2) = 3
4    LET  V(3) = 5
5    FOR  K= 1 TO  3 STEP  1
6    LET  W=V(K)
7    PRINT "W="V(K)
10   FOR I=-2 TO  1 SIEP  1
20   FOR N= 1 TO  9 STEP  1
30   LET X=N*10^I
35   FOR L1= 0 TO  10 STEP  5
36   LET R=L1*1.00000E-04
50   FOR M= 0  TO  1  STEP  1
60   LET G=M
70   LET A=1+R*G
75   LET B=W*R/X
80   LET C=-W*W/X
85   LET D=W*G
87   LET T=C*C+D*D
90   LET P=(A*C+B*D)/T
00   LET Q=-(A*D-B*C)/T
120  PRINT X,R,G,P,Q
121  NEXT M
123  NEXT L1
```

```
123  NEXT L1
124  NEXT N
125  NEXT I
126  NEXT K
130  END
```



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