

Non-local skyrmions as topologically resilient quantum entangled states of light

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In the early 1960s, inspired by developing notions of topological structure, Tony Skyrme suggested that sub-atomic particles can be described as natural excitations of a single quantum field. Although never adopted for its intended purpose, the notion of a skyrmion as a topologically stable field configuration has proven to be highly versatile, finding application in condensed-matter physics, acoustics and more recently, optics, but it has been realized as localized fields and particles in all instances. Here we report the first non-local quantum entangled state with a non-trivial topology that is skyrmionic in nature, even though each individual photon has no salient topological structure. We demonstrate how the topology makes such quantum states robust to smooth deformations of the wavefunction, remaining intact until the entanglement itself vanishes. Our work points to a nascent connection between entanglement classes and topology, opens exciting questions into the nature of map-preserving quantum channels and offers a promising avenue for the preservation of quantum information by topologically engineered quantum states that persist even when entanglement is fragile.

In the early 1960s, Tony Skyrme proposed a nonlinear meson field theory to describe sub-atomic particles as excitations of a single fundamental field—the pion^{1–4}. To accomplish this, he used the inherent mathematical structure of the basic pion theory to postulate topologically non-trivial pion field configurations, now called skyrmions. A skyrmion is a topologically stable field configuration, characterized by an integer topological invariant—the Skyrme number. A skyrmion cannot be smoothly deformed into another field configuration with a different Skyrme number. The generality of this definition and its associated topologically conserved quantity allowed the notion to be extended beyond its initial intent^{5,6}. In particular, skyrmions have been instrumental in advances in magnetism and spintronics, where their topological stability makes them ideal candidates for information storage and transfer^{7–13}. Here the quantum properties of these localized magnetic textures have been theoretically studied^{14–19} and suggested as a basis for quantum information processing²⁰, where their macroscopic quantum tunnelling and energy-level quantization are indicative of quantum behaviour, with some quantum dynamics already observed in magnetic systems²¹.

Beyond magnetism, skyrmions have been revisited in the context of nuclear physics to resolve long-standing debates on the nucleon–nucleon spin–orbit potential²² and have been observed in atomic matter²³, chiral liquid crystals²⁴ and acoustics²⁵. Optical realizations have only recently been explored, including observations in evanescent waves²⁶, in focused orbital angular momentum (OAM) beams at sub-wavelength scales²⁷, in certain classes of full Poincaré beams^{28–30} and even as toroidal pulses³¹. Tracking the trajectories of individual pseudospin states with propagation exposes a mapping to the four-dimensional hypersphere, realizing the Skyrme field as a Hopf fibration^{32,33}.

All these impressive advances are local realizations of skyrmions as particles and fields, without any essential role for entanglement (Fig. 1a). Creating skyrmions as quantum entangled states with non-local quantum correlations would allow the potent applications of spatially structured topologies to quantum topological photonics³⁴. Despite being very much in its infancy, the merging of topological and quantum structure holds great promise for information robustness even in non-ideal quantum systems, including stable

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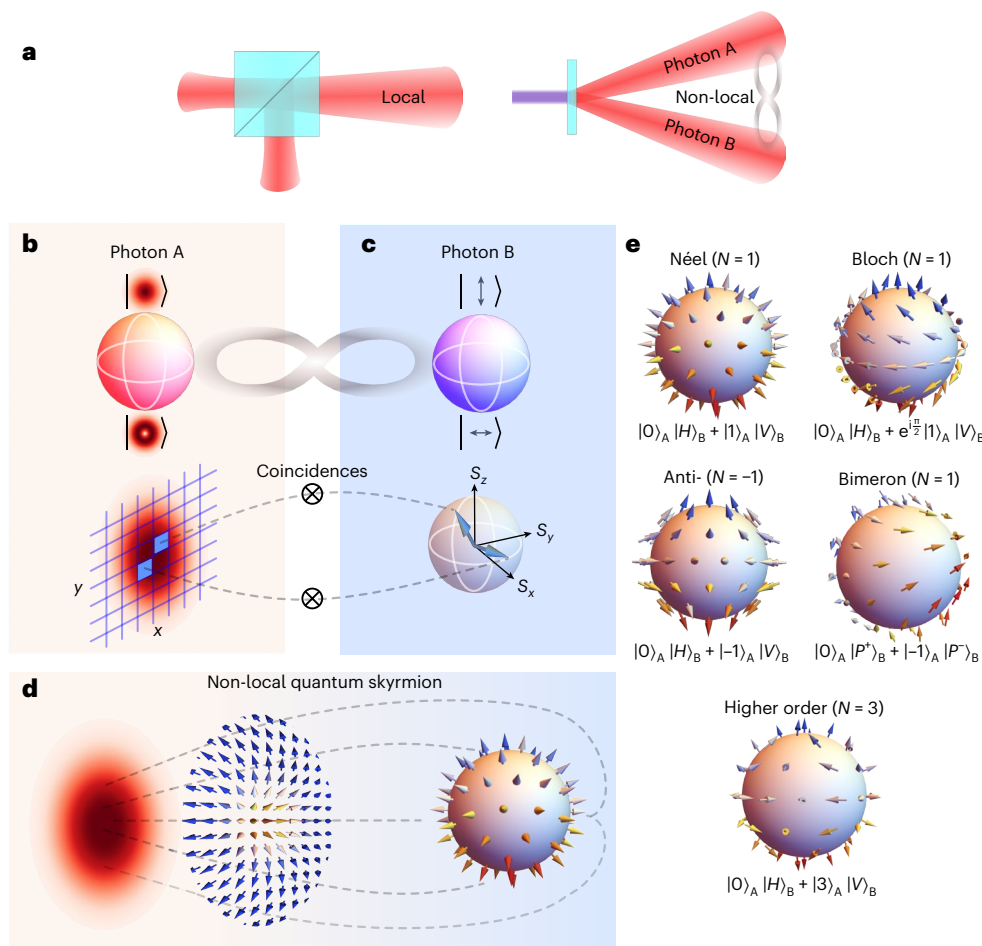


Fig. 1 | Non-local quantum skyrmions. **a**, A typical local field configuration giving rise to a skyrmion can be produced by superimposed, orthogonally polarized spatial modes. The non-local quantum skyrmion exists as a shared property of spatially separated photons, namely, A and B. **b, c**, Non-local skyrmionic configuration is formed in the entanglement between the two photons, with photon A carrying the spatial degree of freedom (DoF) expressed on an OAM Bloch sphere (**b**) and photon B carrying the polarization DoF expressed on the polarization Bloch sphere (**c**). A spatial measurement on photon A collapses photon B into some definite polarization state, non-locally linking space to polarization by a coincidence measurement (bottom).

A complete spatial mapping of photon A reveals all the polarization possibilities in photon B. **d**, Entanglement between the photons is now skyrmionic, as denoted by the vectorial arrows in space, which can be stereographically mapped to an abstract sphere that holds spatial information (coverage) and the skyrmionic state (arrows and location). **e**, Full control over the quantum wavefunction and particularly the relative OAM of the constituent components of the wavefunction (Methods provides the rules to control the topology) allows for the manipulation of the texture and precise topology of the non-local skyrmion, thereby giving access to a plethora of topological structures.

quantum emitters^{35–37}, robust transport of quantum states through quantum circuits^{38,39} and entanglement storage through light–matter interactions⁴⁰.

Here we report the first non-local quantum entangled state with a non-trivial topology that is skyrmionic in nature. Intriguingly, the non-trivial topological structure does not exist in the properties of the individual (local) photons in the two-photon entangled state, but rather, it emerges from the (non-local) entanglement between them. We demonstrate how the topology of the quantum wavefunction remains intact under a smooth deformation (with entanglement decay as an example), only vanishing once the entanglement itself vanishes; here we report the first demonstration of this invariance. Our approach allows the topology of these quantum wavefunctions to be fully controlled, which we outline theoretically and demonstrate experimentally across a wide variety of skyrmion types, and allowing us to distinguish entanglement classes according to their quantum topology. Our work leverages on topological photonics and quantum state engineering, offering a promising avenue for the preservation of quantum information by topologically engineered quantum states that persist even when entanglement is fragile.

Concept

An optical skyrmion can be represented as a topologically protected spin-textured field, where every point on the Poincaré sphere (S^2) is in correspondence with a point in the two-dimensional transverse spatial plane ($\mathcal{R}^2$). The optical skyrmion is a mapping from the two-dimensional transverse spatial plane onto the space of polarization states, that is, a mapping of $\mathcal{R}^2 \rightarrow S^2$, with the Skyrme number characterizing the number of times $\mathcal{R}^2$ wraps S^2 . We create this by entanglement, by engineering two entangled photons (A and B) to occupy the quantum state $|\Psi\rangle = \frac{1}{\sqrt{2}} (|\ell_1\rangle_A |H\rangle_B + e^{i\gamma} |\ell_2\rangle_A |V\rangle_B)$, where ℓ_1 and ℓ_2 denote the OAM of $\ell_1\hbar$ and $\ell_2\hbar$ per photon, respectively; H and V are the orthogonal horizontally and vertically polarized states, respectively; and γ allows for a rotation of the state vector and can be extended to include the effects of the quantum Gouy phase⁴¹ on the state, which merely alters γ in propagation (the ‘Topological resilience against quantum Gouy phase’ section in the Supplementary Information provides the complete details). The reduced state of photon A is an incoherent mixture of OAM, whereas that of photon B is unpolarized; therefore, individually, neither has any salient topological structure. Quantum correlations

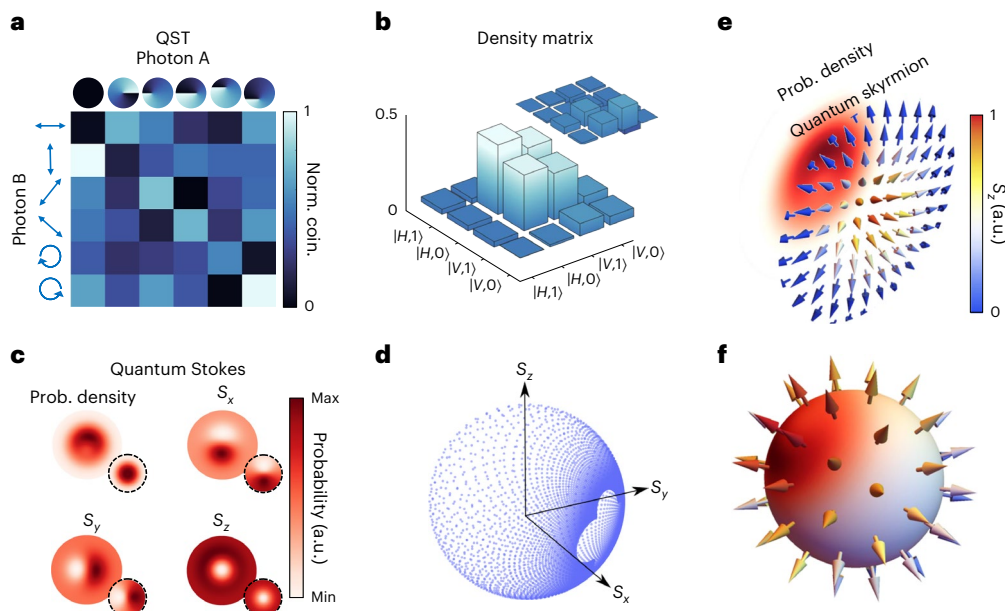


Fig. 2 | Experimental quantum skyrmion. **a**, A QST was performed on the selected entangled state, that is, $|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_A |H\rangle_B + |0\rangle_A |V\rangle_B)$, by spatial measurements on photon A (columns) and polarization measurements on photon B (rows), with coincidences collected for all the outcomes shown in false colour from low (black) to high (white). **b**, Real and imaginary (inset) parts of the density matrix for the reconstructed state, indicative of the desired state. **c**, Quantum Stokes projections directly measured from the tomography data. **d**, Reconstruction of the coverage on the Poincaré sphere, with the experimental

data shown as blue dots. **e**, Experimental probability density of finding a photon in space, given by the spatial structure of photon A, and the experimentally reconstructed skyrmion as a spin-textured field with a defined topology (Methods provides the rules to compute the Skyrme number). **f**, Stereographic mapping of the experimental data showing spatial information (coverage) and probability of detection (colour), with the skyrmionic state depicted by the direction of arrows and their location, a direct outcome from the joint state of photons A and B.

between the two photons imply a rather different picture: the collapsed state of one photon is determined by the measurement choice on the other; therefore, a joint measurement on both reveals a non-local quantum topological structure. Intriguingly, each localized position of photon A in real space $\mathcal{R}^2$ is associated with a polarization state, occupied by its entangled twin (photon B) and parametrized on a sphere, S^2 . We mathematically expand on this later in the context of a wider meaning. Consequently, photon A holds the possibility for every spatial position, shown in Fig. 1b as both a spatial-mode Bloch sphere (top)⁴² and the corresponding spatial probability distribution (bottom). Photon B, meanwhile, holds the possibility for every polarization state shown in Fig. 1c as a polarization Bloch sphere (top) and the corresponding Poincaré sphere (bottom). A spatial measurement on photon A collapses photon B into a particular polarization state, producing a mapping from space to the Poincaré sphere, revealed by joint measurements in coincidence; therefore, we have indeed obtained the mapping $\mathcal{R}^2 \rightarrow S^2$ as desired. Although the topological structure of each individual photon is always trivial and with a zero Skyrme number, the non-local Skyrme number (we call it the quantum Skyrme number), denoted by N , can be tailored as desired, with the skyrmion existing in the entanglement between the photons themselves. This non-local quantum entangled skyrmion is graphically depicted in Fig. 1d. The skyrmion topology is shown as vectorial arrows in space, the position and direction of which are derived from the joint state of photons A and B. Photon A also contributes the probability of detection, spatially varying, shown as a false-colour density plot. This information can be holistically visualized by a stereographic mapping to an abstract sphere whose spatial coverage is given by photon A and the skyrmion topology (position and directions of the arrows) by the joint state of photons A and B. Furthermore, with complete control over the quantum wavefunction, the topology and texture can be modified at will, examples of which are given in Fig. 1e. Here the topology (skyrmion, anti-skyrmions and higher-order skyrmions) is largely dictated by the OAM values

selected, whereas the texture (Néel, Bloch and bimeron) is determined by the relative phase between the states and polarization basis of the states, that is, a change from $\{|H\rangle, |V\rangle\}$ to $\{|P^+\rangle, |P^-\rangle\}$ (where $|P^+\rangle$ and $|P^-\rangle$ are any orthogonal polarization states) also changes the texture⁴³.

Creating and detecting quantum skyrmions

To verify this concept, we prepared our entangled state from an initial spontaneous parametric downconversion source and performed a unitary operation on the OAM of photon A while performing a spatial-to-polarization conversion, exchanging the spatial information of photon B for polarization information; the ‘Experiment’ section of the Supplementary Information provides the complete experimental details. This altered the typical spontaneous parametric downconversion OAM spiral spectrum to the desired asymmetric spiral bandwidth for a skyrmionic two-photon quantum wavefunction. The non-local two-photon wavefunction was then analysed by projective measurements with a spatial light modulator on photon A and polarization projections on photon B, measured in coincidence. The results from this quantum state tomography (QST) are shown in Fig. 2a for an example state with $\ell_1 = 1$, $\ell_2 = 0$ and $\gamma = 0$, where the detected coincidences for each projective measurement have been normalized against the maximum coincidences detected. The top-left 2×2 matrix partially depicts the behaviour of the generated state, that is, when projecting onto $|\ell\rangle = |0\rangle$ for photon A, we only detect coincidences for vertically polarized photon B; similarly, when projecting onto $|\ell\rangle = |1\rangle$, we only detect coincidences for horizontally polarized photon B. This is consistent with our expected state behaviour, a measurement of photon A revealing $\ell = 0$ (1) collapses photon B into the state $|V\rangle$ ($|H\rangle$). Further projections are performed to build an over-complete QST (the ‘Quantum state tomography’ section in the Supplementary Information provides further details), which enabled the reconstruction of the density matrix for our skyrmionic quantum state (Fig. 2b). The real and imaginary (inset) parts contain all the information of our quantum

state. Compared with the expected pure theoretical state, the experimentally generated state revealed a fidelity of $F = 95.0\%$. Additionally, the QST contains the information necessary to reconstruct the non-local quantum Stokes parameters for the state (Fig. 2c) after local normalization, with the theoretical quantum Stokes parameters given in the insets. The probability density is given by $\langle I_A \otimes I_B \rangle$ and depicts the probability of finding photon A in a particular position and photon B in the corresponding polarization state. Furthermore, the remaining quantum Stokes parameters are calculated as the expectation values of the Pauli matrices, with further detail on how this information is extracted from the QST given in the Supplementary Information. The quantum Stokes parameters reveal a mapping of each point in space to a position on the Poincaré sphere such that our complete set of spatial measurements on photon A reveals a collapse of photon B into 97.3% of all the possible polarization states parametrized by the Poincaré sphere. This coverage is shown in Fig. 2d. In Fig. 2e, the true skyrmionic nature of the generated two-photon state is depicted. The probability density and spatial information is extracted by the measurement of photon A and in coincidence, the polarization information is extracted from the measurement of photon B, revealing a Néel-type configuration. A stereographic projection of the configuration (Fig. 2e) yields the hedgehog-like polarization texture (Fig. 2f) of the entangled state on the surface of a sphere. An experimental quantum Skyrme number of $N_{\text{exp}} = 0.972$, which is very close to the theoretical value of $N_{\text{th}} = 1$, provides the first realization of a non-local quantum entangled skyrmion.

Next, we traverse the skyrmionic quantum landscape, altering the texture and topology by controlling γ and N , respectively. We illustrate this in Fig. 3a, moving from anti-skyrmions ($N < 0$) through non-skyrmionic states ($N = 0$) to examples of Bloch-type ($N = 1, \gamma = \pi/2$) and Néel-type ($N = 1, \gamma = 0$) skyrmions and lastly to higher-order skyrmions ($N > 1$), all of which are faithfully produced with high fidelity when compared with maximally entangled states (Fig. 3b), and the measured Skyrme numbers (N ; Fig. 3c) is in excellent agreement with the theoretically predicted integer values. Supplementary Table 1 lists the exact fidelity and Skyrme number values for the generated states referenced in Fig. 3. In Fig. 3d, we show the experimentally inferred topology of the generated states as stereographic projections onto the surface of S^2 , whereas in Fig. 3e, we show selected experimental and theoretical topologies (with the experimental and theoretical probability densities of each state shown in the insets), in excellent agreement. Notice that when $\ell_2 = -\ell_1$, that is, opposite twists to the OAM components, as would be created in typical spin-orbit hybrid entanglement experiments⁴⁴, no quantum skyrmion is produced and $N = 0$. For $N = -1$, we see the characteristic hyperbolic texture embedded into the wavefunction, and for the Néel- and Bloch-type skyrmions, we see the characteristic hedgehog and spiral textures, respectively. Supplementary Table 1 provides the complete experimental datasets (Skyrme numbers, fidelities and concurrences), and Supplementary Fig. 7 (reconstructed density matrices), Supplementary Fig. 8 (reconstructed quantum Stokes parameters) and Supplementary Fig. 9 (reconstructed state topologies) provide additional information.

Topology and entanglement

A convenient basis of states in which to expand the wavefunction of photon A is provided by the position space states $|\mathbf{r}_A\rangle$. Using the explicit expression $|\ell\rangle = \int_{\mathcal{R}^2} |\text{LG}_{\ell}(\mathbf{r}_A)\rangle |\text{LG}_{\ell}(\mathbf{r}_A)\rangle d^2r_A$ for the OAM eigenstates, we write our entangled photon state as

$$|\Psi\rangle = \int_{\mathcal{R}^2} |\mathbf{r}_A\rangle (a(\mathbf{r}_A)|H\rangle_B + b(\mathbf{r}_A)e^{i\Theta(\phi_A)}|V\rangle_B) d^2r_A, \quad (1)$$

where $a(\mathbf{r}_A) \equiv |\text{LG}_{\ell_1}(\mathbf{r}_A)|$, $b(\mathbf{r}_A) \equiv |\text{LG}_{\ell_2}(\mathbf{r}_A)|$, $\Theta(\phi_A) = \Delta\ell\phi_A + \gamma$ and $\Delta\ell = \ell_2 - \ell_1$. The coefficient of $|\mathbf{r}_A\rangle$ above defines a state for photon B as

$$|\psi_{B|A}\rangle = \cos(\Theta(\mathbf{r}_A))|H\rangle_B + \sin(\Theta(\mathbf{r}_A))e^{i\Theta(\phi_A)}|V\rangle_B, \quad (2)$$

where the state $|\psi_{B|A}\rangle$ describes photon B's state having measured photon A at position $\mathbf{r}_A$. The $\Theta(\mathbf{r}_A)$ dependence arises since we have normalized the state for every position $\mathbf{r}_A$, that is, $a(\mathbf{r}_A)^2 + b(\mathbf{r}_A)^2 = 1$ for $\mathbf{r}_A \in \mathcal{R}^2$, which implies that we may set $a(\mathbf{r}_A) = \cos(\Theta(\mathbf{r}_A))$ and $b(\mathbf{r}_A) = \sin(\Theta(\mathbf{r}_A))$ for $\Theta(\mathbf{r}_A) \in [0, \pi/2]$. The polarization state space for photon B is a two-dimensional sphere S^2 . Using a stereographic projection, the original position state space $\mathcal{R}^2$ can also be identified as S^2 , that is, we can map it onto a sphere. In this way, by associating $|\mathbf{r}_A\rangle$ and $|\psi_{B|A}\rangle$, the entangled photon wavefunction defines a map from a sphere to a sphere: from the position state space S^2 (associated to photon A) to the polarization state space S^2 (associated to photon B).

Every map from S^2 to S^2 has an integer degree N measuring how many times the first sphere wraps the second. This degree is a topological invariant: it is unchanged by continuous deformation of the map. This rich topological structure of the maps from spheres to spheres is the source of the topological structure in our quantum two-photon wavefunctions. In the current context, the degree N is the quantum Skyrme number. An important corollary of the existence of a topological invariant is that the space of continuous maps is partitioned into equivalence classes⁴⁵. Any two maps in the same class can be continuously deformed into each other, whereas this is not possible for maps in distinct classes⁴⁵. The connection we have established between entangled photon wavefunctions and maps from S^2 to S^2 implies that the space of all possible wavefunctions is itself partitioned into equivalence classes, with each class labelled by its quantum Skyrme number.

To illustrate this, we show examples of experimentally measured mappings with various winding numbers (Fig. 4). All the entangled states with $N = 0$ map to rings on S^2 and hence are topologically equivalent, with the results shown in Fig. 4a confirming the mapping. This topology groups conventional hybrid entangled states, as is usually created by geometric phase approaches⁴⁶, into a single class. Note the deviation between the perturbation-free prediction (solid line in the inset) and the experimental points, which we return to shortly in the context of robustness. States with $N \neq 0$ wrap the entire S^2 space N times, as shown in Fig. 4b,c for $N = 1$ and $N = 3$, respectively. The experimentally measured coverage of approximately 97% ($N = 1$) and 299% ($N = 3$) come close to the theoretical values of 100% and 300%, respectively. A small deviation can be seen at the pole of the mapped sphere, a natural artefact of mapping from $\mathcal{R}^2$ to S^2 , where the data at spatially infinite distances cannot be recovered. Furthermore, we note that for any entanglement class, the spatial state space may be partitioned into N segments, each of which will be found to wrap the entire S^2 state space (Fig. 4c).

Having identified the origin of the topological structure in the two-photon quantum state, we now offer a connection between topology and entanglement. A non-trivial topology is not a property of either of the S^2 spaces on their own, but rather, it is a shared emergent property originating in the non-trivial global structure of the map between the two spaces. Entanglement itself is not a property of the state of either particle on its own, but rather, it is a shared emergent property, too. There is a quantitative relationship between the two: it is only the topologically trivial $N = 0$ class of two-photon wavefunctions that can have vanishing entanglement. Wavefunctions belonging to any other class are always entangled. This argument is supported by noting that since for the trivial $N = 0$ topology, the entangled states map onto rings on S^2 , and therefore, it is possible to continuously deform the map so that all the points on the first sphere map onto a single point on the second sphere. This map corresponds to a wavefunction for which every position state $|\mathbf{r}_A\rangle$ in equation (1) multiplies the same state from the polarization state space. This product state is a separable state and therefore not entangled. For any other winding number $N \neq 0$, the coefficient of different position space states range N times over the complete polarization state space; therefore, the corresponding state is entangled. The inescapable

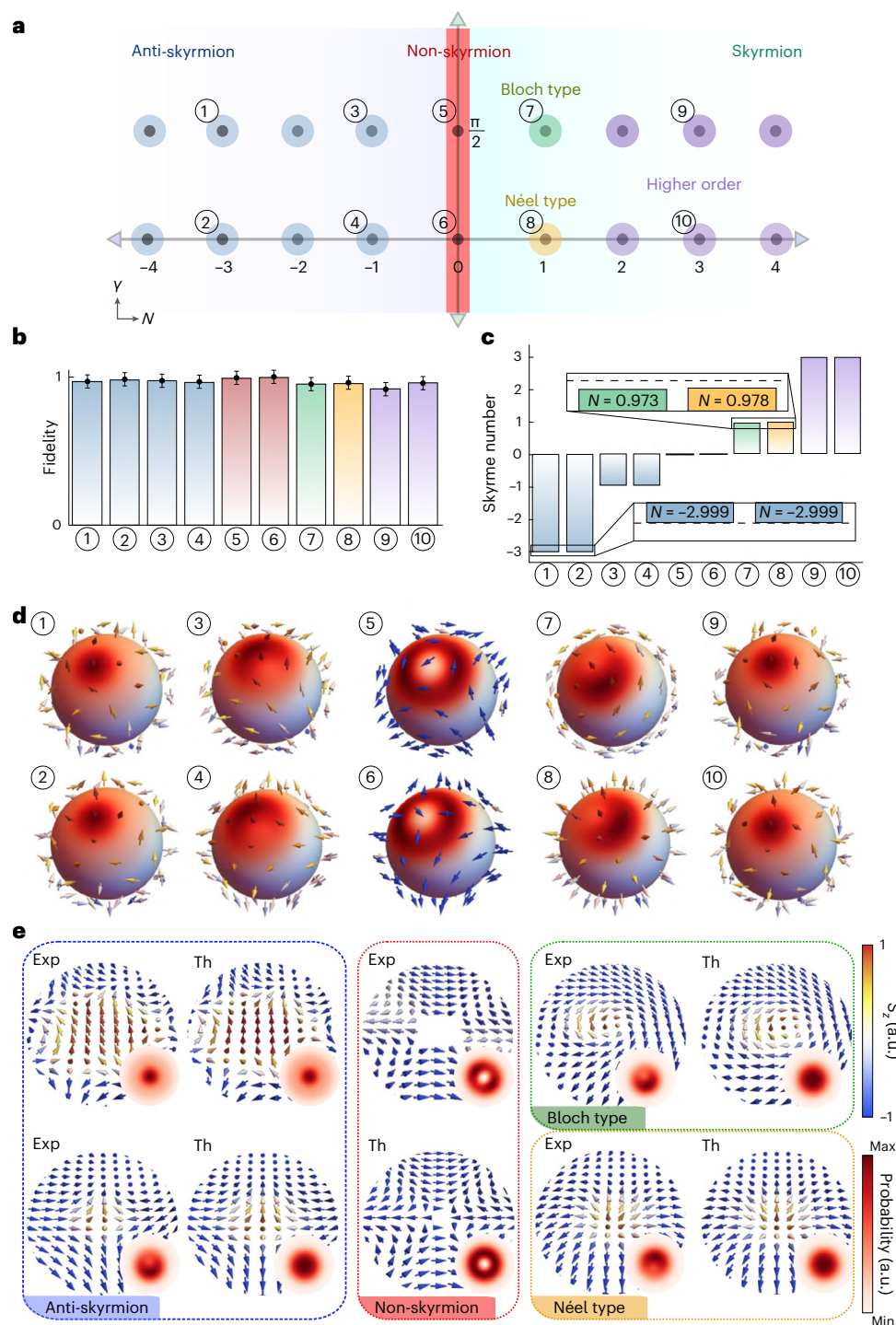


Fig. 3 | Traversing the quantum skyrmionic landscape. a, Varying textures (down the column) and Skyrme numbers (across the row) were created by controlling γ and the OAM difference ($\Delta\ell$) in the quantum state to indirectly control N . Example states are graphically shown and labelled with numbers 1 through 10, traversing the skyrmionic landscape from anti-skyrmions ($N < 0$) through non-skyrmionic states ($N = 0$) to Bloch-type ($N = 1$, $\gamma = \pi/2$), Néel-type ($N = 1$, $\gamma = 0$) and higher-order ($N > 1$) skyrmions. **b**, Experimental fidelities for

the example states when compared with theoretical maximally entangled pure states. The data are presented as mean values ± 0.05 , which were derived from Poissonian statistics. **c**, Experimental Skyrme numbers for example states. **d**, Examples of experimental stereographic projections. **e**, Example skyrmion topologies with varying textures shown as experimental (Exp) and theoretical (Th) reconstructions. The insets show the probability of detection as measured (Exp) and calculated (Th) from photon A.

conclusion is that non-trivial topology ($N \neq 0$) implies non-zero entanglement. On the other hand, trivial topology does not imply that the corresponding wavefunction has vanishing entanglement. Indeed, wavefunctions corresponding to maps that partially cover the polarization S^2 are entangled, but still topologically trivial with $N = 0$. The point, however, is that these maps can be continuously

deformed to a map that corresponds to the zero-entanglement wavefunction. For any other $N \neq 0$ class, it is not possible to reach the map of a zero-entanglement wavefunction by continuous deformations. In this sense, the topology of the wavefunction partitions the space of all the possible wavefunctions into entanglement classes.

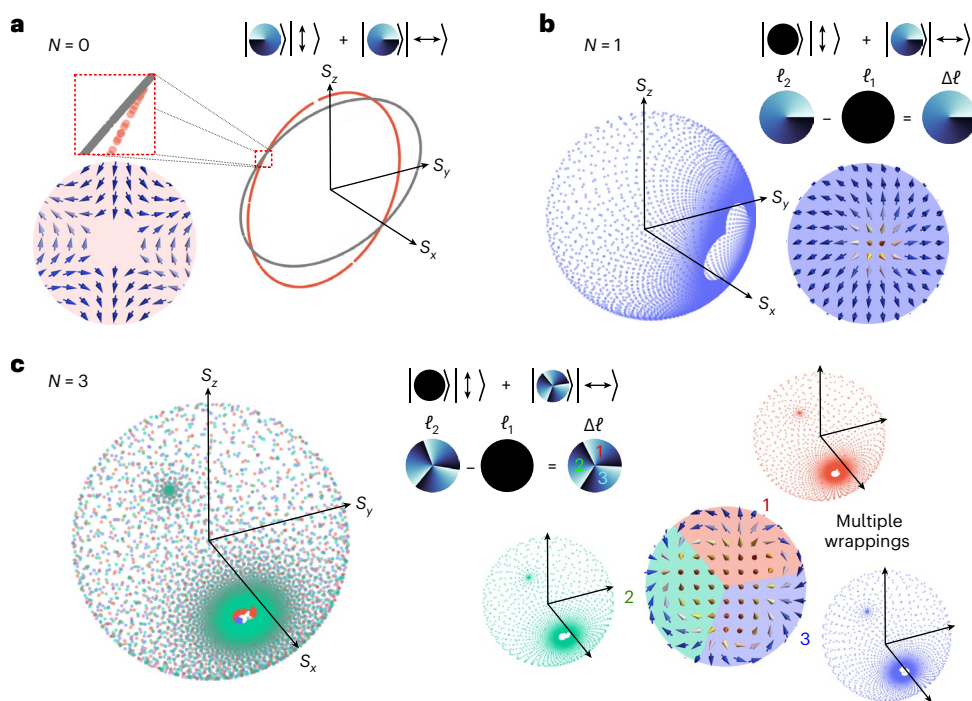


Fig. 4 | Topology of quantum entangled states. Two-photon entangled states can be classified according to their topology. **a**, Entangled states where $|\ell_1| = |\ell_2|$ have $N = 0$ and map to rings on S^2 with close to zero coverage and are topologically trivial. Experimental results (dots) together with the perturbation-free prediction (solid line) are shown in the inset. **b, c**, States with $|\ell_1| \neq |\ell_2|$ necessarily have $N \neq 0$ and map to the entirety of S^2 , through N wrappings. This is shown for

$N = 1$ (**b**) and $N = 3$ (**c**), with the experimental data shown as dots on the spheres. In **b**, the coverage is close to 100% for $N = 1$. **c**, For $N = 3$, the non-local field has three segments, each with a full coverage of S^2 , shown as red, green and blue dots (experimental). The entire state maps to S^2 with three windings for a coverage of 299%, close to 300%, illustrated in the composite sphere (left) with all three experimental datasets.

Topological resilience

Our connection between topology and entanglement suggests the following: (1) it is not possible to have no entanglement and a non-trivial topology ($N \neq 0$); (2) it is possible to have a trivial ($N = 0$) topology and no entanglement; (3) the local deformation-preserving topology suggests that so long as the entanglement persists, the topology is robust—a quantum form of topological resilience. The results reported in Fig. 5a support these statements. We introduced an amplitude-damping operation⁴⁷ applied to the wavefunction that resulted in entanglement decay (Methods and the ‘Topological resilience against entanglement decay’ section in Supplementary Information provide further details) from a maximally entangled state to no entanglement, all the while monitoring the topology through the quantum Skyrme number N . The results for $N = 1$ and $N = 3$ reveal a new form of topological resilience, with the quantum Skyrme number intact in the presence of decaying entanglement, satisfying points (1) and (3) above, falling off to zero only when the entanglement itself vanishes, satisfying point (2). To the best of our knowledge, this is the first observation of an invariance (N) to entanglement decay derived from the topology of the entanglement between particles, which holds exciting promise for topologically protected quantum information processing.

We now sketch a simple argument explaining how this occurs for quantum wavefunctions, which we believe is not to be found elsewhere. As the first step, can we understand what we gain by coding information into topology? Topology is a systematic approach to characterize those quantities that are insensitive to smooth deformation. Our observed topological resilience of the skyrmionic wavefunction can be explained by considering the entanglement decay operation as a smooth deformation of the state, which—in our problem—is the statement that the quantum Skyrme number is unchanged by a change in coordinates, that is, $(x, y) \rightarrow (x', y')$, for photon A (the ‘Quantum

topological invariance’ section of the Supplementary Information provides a proof). After a short derivation, we find that indeed

$$N = \frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Sigma_z(x, y) dx dy = \frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Sigma_z(x', y') dx' dy', \quad (3)$$

where

$$\Sigma_z(x, y) = \frac{1}{2} \epsilon_{pqr} S_p \frac{\partial S_q}{\partial x} \frac{\partial S_r}{\partial y}, \quad (4)$$

with $p, q, r \in \{x, y, z\}$; ϵ_{pqr} is the Levi-Civita tensor; $S_{p,q,r}$ are the quantum Stokes parameters (Methods gives the detailed derivation); and x' and y' are both arbitrary, smooth functions of x and y . This remarkable property is proved by noting that in moving from the first to second equality in equation (3), Σ_z picks up a factor of the inverse Jacobian, whereas the measure picks up a factor of the Jacobian; therefore, the product is invariant. This is the complete content of the statement that the quantum Skyrme number is invariant under smooth deformations.

As the second step, we must explain how this leads to resilience. It is simplest to phrase the discussion in a single spatial dimension, which we do with the aid of Fig. 5b. Consider a perturbation that modifies a wavefunction $\psi(x)$ into $\psi'(x)$. If the perturbation is modest, the equality $\psi'(x) = \psi(x')$ defines a change in coordinates from x to a new coordinate x' , as graphically shown in Fig. 5b. The topological invariant—our quantum Skyrme number—remains unchanged by this coordinate transformation; therefore, we are free to perform it. The transformation replaces $\psi'(x) \rightarrow \psi(x')$, that is, a simple coordinate change allows one to map from the distorted wavefunction to the original wavefunction. The return to our undistorted wavefunction by this simple coordinate transformation completes the demonstration of topological resilience.

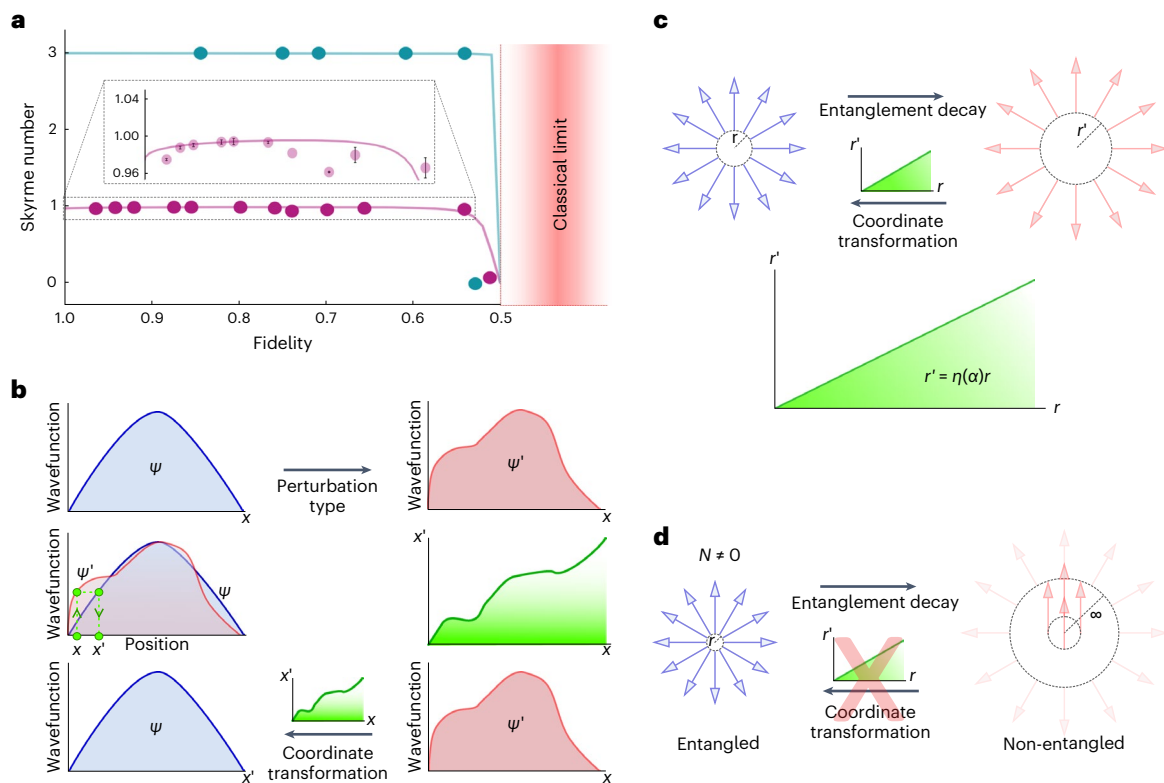


Fig. 5 | Quantum topological invariance. a, Measured quantum Skyrme number (N) as a function of state fidelity (F) against a maximally entangled state. Here F decreased by a deliberately induced decay in entanglement, from fully entangled ($F = 1.0$ and concurrence $C = 1$) to no entanglement ($F = 0.5$, $C = 0$). Results are shown for initial $N = 1$ and $N = 3$, with the invariance of N clearly revealing that the topology remains intact in the presence of decaying entanglement. Experimental data are shown as dots and theory as solid lines, in excellent agreement with one another. The errors calculated from Poissonian statistics for Skyrme number $N = 1$ are shown in the enlarged inset, with a maximal error of ± 0.01 . **b**, Coordinate transformation defined by the original and distorted wavefunctions. To obtain

the mapping $x \rightarrow x'$, we vertically follow point x up to ψ' , then horizontally traverse from ψ' to ψ and finally project down from ψ to x' . In this way, the distorted wavefunction is mapped back to the original wavefunction through a coordinate transformation. **c**, Entanglement decay operation results in a simple smooth deformation of the wavefunction, which can be physically seen as a scaling of the radial position of any set of polarization vectors that lie at some radial position. A simple linear radial coordinate transformation, therefore, returns the wavefunction to its original state. **d**, When the entanglement has vanished, so does the non-trivial topology; thus, there is no coordinate transformation that can be realized to return the state to its previous form.

This argument is a mathematical demonstration of the fact that the topology is a global property of the wavefunction, insensitive to the local changes induced by perturbations.

Returning to the experimental demonstration of the topological resilience to an amplitude-damping channel resulting in entanglement decay (Fig. 5a), we can now discuss this in terms of the arguments provided above. First, we note that our robustness argument will hold for any quantum channel that can be shown to be a smooth deformation of the wavefunction and thus is map preserving. The isomorphism between quantum channels and quantum states⁴⁸ suggests that our notion of topological entanglement classes can be used to characterize quantum channels, too: those that are smooth deformations and those that are not. In the present example, we can indeed observe the amplitude-damping operation as a smooth deformation of the entangled state as it manifests as a radial position coordinate scaling for polarization state vectors that lie at some r (Fig. 5c), so that a simple linear coordinate transformation of the form $r \rightarrow r'$, where $r' = \eta(\alpha)r$, yields the original entangled state (the 'Topological resilience against entanglement decay' section of the Supplementary Information provides full details of the coordinate transformation derivation). From Fig. 5a, it is evident that as we asymptotically approach vanishing entanglement, the topology abruptly changes from non-trivial ($N \neq 0$) to trivial ($N = 0$), emphasizing that the skyrmionic topology is a shared property of the entangled photons, which necessarily vanishes once there is no entanglement.

Because perturbing the wavefunction corresponds to deforming the map, information about the entanglement class of the wavefunction is naturally resilient to such a perturbation. Further, any topologically non-trivial ($N \neq 0$) class enjoys this resilience, whereas the topologically trivial class does not. An experimental demonstration of this can be visualized by the mappings shown in Fig. 4: since the topologically trivial class is mapped onto a ring (Fig. 4a), its orientation can be easily discerned. Indeed, in Fig. 4a, we can see the deviation from the ideal prediction of the ring (solid line) and the experimental reconstruction (data points), the latter always with some inherent perturbation. Not only can the ring alter in orientation but it can be deformed to a point since states with trivial topology ($N = 0$) could be entangled or not. Conversely, one cannot discern an orientation change due to perturbation when the entire sphere is mapped ($N > 0$), as is the case for all non-trivial classes with examples shown in Fig. 4b,c: full (or more) coverage always looks the same regardless of the perspective. As discussed earlier, the topology of such states remains intact so long as there is some entanglement in the system and hence can be considered robust even when entanglement is fragile.

Discussion and conclusion

We have reported the first realization of skyrmionic fields with non-local quantum correlations, which have hitherto been elusive. The topologically non-trivial two-photon entangled state has a Skyrme number that can be any non-zero integer, and this topological structure is not

derived from either local photon state, but rather, from the entanglement between them. We demonstrated experimental control over the topology of our two-photon entangled states showing exotic topologies ($N \in \{-3, -1, 0, 1, 3\}$) and textures (anti-skyrmions, Néel-type skyrmions and Bloch-type skyrmions). We have outlined a new tool for the classification of these entangled states according to their topology, from trivial classes that map onto rings on S^2 , to non-trivial classes with N wrappings of S^2 , shown up to $N = 3$. We have shown that such quantum textures imbue the state with topological resilience, preserving the topology until the entanglement itself vanishes and establishes that one may identify an entangled state's topology even when the entanglement is fragile. This invariance in the topology of the state could be combined with topological approaches to preserve entanglement³⁴, which would see the robustness of both information and entanglement by topology. The non-local nature of our topology lends itself to exciting advances, for instance, distributing topology across distant parties, topological teleportation, topology exchange between light and matter and topology transport down non-map-preserving channels by a judicious choice of the transported photon (for example, the polarization superposition photon through a single-mode fibre).

Evidently, an important outcome of this work is the emerging understanding of the link between quantum entangled states and their topology. Topology quantifies the failure of these spaces to factorize into a product, as entanglement does for states in Hilbert space. Our connection reveals a hitherto-hidden commonality in seemingly disparate states that allows them to be grouped into entanglement classes by their topology, exposing the potential for the topological prediction of entanglement properties. The isomorphism between quantum channels and quantum states⁴⁸ suggests that our notion of topological entanglement classes can be used to characterize quantum channels, too, grouping them into classes and negating the need to study each channel as a separate entity. We anticipate that the topology will remain intact in all the channels that can be shown to be smooth deformations of the state, an open challenge to be addressed.

Our results should inspire new directions in engineering topological quantum states, for instance, for a quantum version of vectorial holography⁴⁹, and could benefit from resonant metasurfaces for compact sources of such states⁵⁰. The fact that the Skyrme number can take any quantized integer value should also inspire the use of non-local topology as a high-dimensional encoding alphabet, which can be useful for quantum communication and information processing. Although fuelling new research avenues, we believe that our work holds exciting prospects for information robustness by topology even in non-ideal quantum systems that use entanglement as a resource.

Online content

Any methods, additional references, Nature Portfolio reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at <https://doi.org/10.1038/s41566-023-01360-4>.

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Methods

Experiment

The full experimental details are given in the ‘Experiment’ section of the Supplementary Information.

Concurrence and fidelity

To quantify the quality and degree of entanglement of our states, we used fidelity and concurrence as our figures of merit. The fidelity was calculated as

$$F = \left(\text{Tr} \left(\sqrt{\sqrt{\rho_T} \rho_M \sqrt{\rho_T}} \right) \right)^2, \quad (5)$$

where ρ_T is the target density matrix and ρ_M is the measured density matrix. The fidelity is 0 if the states are not identical or 1 when they are identical up to a global phase. The concurrence was used to measure the degree of entanglement between the hybrid entangled photons. It was calculated as

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (6)$$

where λ_i are eigenvalues of the operator $R = \text{Tr}(\sqrt{\sqrt{\rho} \tilde{\rho} \sqrt{\rho}})$ in the descending order and $\tilde{\rho} = \sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y$ with σ_y being the Pauli matrix defined by $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. The concurrence ranges from 0 for separable states to 1 for entangled states.

Quantum Stokes measurements

The aim is to derive the non-local Stokes parameters $\tilde{S}(\vec{r}) = \langle S_x(\vec{r}), S_y(\vec{r}), S_z(\vec{r}) \rangle$. Traditionally, the spatially resolved Stokes parameters of a vectorial field are measured from the Pauli matrices in the polarization degree of freedom, that is, $S_j(\vec{r}) = \langle \sigma_j \rangle(\vec{r})$, where $j \in \{x, y, z\}$ and σ_j are the usual 2×2 Pauli matrices.

Here we achieve this by extracting them from the reconstructed density matrix of the two-photon state. We can, therefore, compute the Stokes parameters as

$$S_j = \langle \vec{r} |_A \langle \vec{r} |_A \otimes \sigma_{B,j} \rangle = \text{Tr}(|\vec{r}\rangle_A \langle \vec{r}|_A \otimes \sigma_{B,j} \rho), \quad (7)$$

where $\text{Tr}(\cdot)$ is the trace operator. The general decomposition of the density matrix is given as

$$\rho = \sum_{pqst=1}^2 \tau_{pqst} |\ell_p\rangle_A \langle \ell_q|_A \otimes |e_s\rangle_B \langle e_t|_B, \quad (8)$$

where τ_{pqst} are coefficients and $|\ell_{p(q)}\rangle_A$ and $|e_{s(t)}\rangle_B$ are the OAM and polarization basis states of photons A and B, respectively. It follows that we can now express non-local Stokes parameters as

$$S_j = \sum_{pqst=1}^2 \tau_{pqst} \text{Tr}(|\vec{r}\rangle_A \langle \vec{r}|_A |\ell_p\rangle_A \langle \ell_q|_A) \text{Tr}(\sigma_{B,j} |e_s\rangle_B \langle e_t|_B). \quad (9)$$

Next, we apply the trace of photon A in the position basis $\{|\vec{r}\rangle_A | \vec{r} \in \mathcal{R}_A^2\}$, satisfying the orthogonality ($\langle \vec{r}_1 | \vec{r}_2 \rangle = \delta(\vec{r}_1 - \vec{r}_2)$) and the completeness relation ($\int |\vec{r}\rangle_A \langle \vec{r}|_A d^2r = \mathbb{I}_A$, with $\mathbb{I}_A$ being the identity matrix). By noting that the OAM eigenmodes can be projected onto the position basis $\langle \vec{r} | \ell \rangle = \text{LG}_\ell(\vec{r})$, we can perform the trace operation for photon A in equation (9), resulting in

$$S_j(\vec{r}) = \sum_{pqst=1}^2 \text{LG}_{\ell_p}(\vec{r}) \text{LG}_{\ell_q}^*(\vec{r}) \text{Tr}(\sigma_{B,j} |e_s\rangle_B \langle e_t|_B). \quad (10)$$

Quantum Skyrme number

To find the Skyrme number, we reconstructed the paraxial skyrmion field Σ_z using the quantum Stokes parameters as

$$\Sigma_z(x, y) = \frac{1}{2} \epsilon_{pqr} S_p \frac{\partial S_q}{\partial x} \frac{\partial S_r}{\partial y}, \quad (11)$$

where $p, q, r \in (x, y, z)$ and ϵ_{pqr} is the Levi-Cevita tensor. Since the Stokes parameters expressed in the position basis are spatially dependent on cylindrically symmetric Laguerre–Gaussian functions, the Skyrme number was calculated using

$$N = \frac{1}{4\pi} \int_0^\infty \int_0^{2\pi} \Sigma_z d\varphi dr, \quad (12)$$

which, after the substitution of appropriate Laguerre–Gaussian functions, can be shown to simplify down to $N = m\Delta\ell$, with

$$m = \begin{cases} 0, & |\ell_1| = |\ell_2| \\ 1, & |\ell_1| > |\ell_2| \\ -1, & |\ell_1| < |\ell_2|, \end{cases}$$

and $\Delta\ell = \ell_2 - \ell_1$. The quantities $\Delta\ell$ and m then dictate the rules that the non-local topology will follow. Supplementary Information provides further details and discussion.

Data availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Code availability

The code used to produce the results are available from the corresponding author upon reasonable request.

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Author contributions

P.O. and I.N. performed the experiment, and P.O., I.N. and R.M.K. contributed to the theory. All authors contributed to the writing of the manuscript and analysis of data. A.F. conceived of the idea and supervised the project.

Competing interests

The authors declare no competing interests.

Additional information

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