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TEACHERS' USE OF EXAMPLES TO PROMOTE MATHEMATICAL PROFICIENCY IN STATISTICS

A Research Report submitted in partial fulfilment of the requirements for the degree of Master of Education

University of the Witwatersrand

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July 2019

DECLARATION

I declare that this Research Report is my own, unaided work, except as indicated in the text and the references. It is being submitted for the Degree of Master of Education at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

ZULU S

SIGNATURE **DATE: 04 JULY 2019**

ABSTRACT

Examples are a prevalent and a powerful tool used in the teaching and learning of Mathematics. In statistics, where data sets used are normally extracted from real life situations, the use of examples is indispensable. This study examines what examples are used and how teachers use them to promote (or fail to) mathematical proficiency in statistics as the outcome of the learning process. Variation theory is employed through the patterns of variation to assess whether the use of examples leads to learning. To ascertain that the use of examples results in the attainment of mathematical proficiency, the strands of mathematical proficiency by Kilpatrick, Swafford and Findell, (2001), are incorporated in the theoretical framework. This study involves observing lessons interactions between Grade 10 teachers and learners, and interviewing teachers after observing the lessons at a school in Johannesburg. An analytical framework derived from variation theory and strands of mathematical proficiency is then used as an assessment tool. Findings show that in and of themselves examples do not necessarily help to achieve the intended object of learning nor mathematical proficiency. More poignant, whatever the teacher's intention to convey the object of learning through the use of examples, and no matter how the teacher enacts it, learners could still fail to grasp the lived object of learning and attain mathematical proficiency.

ACKNOWLEDGEMENTS

This report would not have been completed without my supervisor, Professor Anthony Essien's incisive and painstaking supervision. In the midst of other functions he performs and students he supervises, he was generous with his time and engaged with my work without a whimper. He offered invaluable advice that challenged me but left room for my thoughts and innovations to be embedded in my study. I have really grown in confidence and my research skills in general. For these, I am grateful to him.

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CHAPTER 1: INTRODUCTION

It is quite common to observe mathematics teachers using examples to teach mathematics. Learners are also used to learning mathematics through a series of examples. According to Rowland (2008), and Bills, Dreyfus, Mason, Tsamir, Watson and Zaslavsky (2006), evidence from historical records shows that examples play a central role in the teaching and learning of mathematics.

In the teaching of statistics, it is quite difficult to think of ways of teaching other than using examples. For instance, concepts such as measures of central tendency (mean, median and mode) can be defined and explained easily. However, examples are necessary to illustrate how these concepts work in real life and how they differ from one another. The equation used to calculate the mean: $\bar{x} = \frac{\sum f(x)}{n}$ cannot be used and fully understood without an example consisting of a data set to use to do the calculations.

According to Leinhardt, Zaslavsky and Stein (1990), constructing a mathematics lesson is highly demanding and often relies on the use of examples. Examples must be carefully selected because examples have the potential to either support or impede learning inadvertently. For example, in statistics a data set without the proper context could be presented, which could be misconstrued as just a set numbers without instead of it being construed as information about something real.

Presenting sets of numbers such as {2, 4, 6, 8,11} to calculate the measures of central tendency defeats the purpose of using an example to promote learning because the focus would only be on 'how to do' and not 'why and how'. Examples like these only facilitate procedural understanding. Granted, there many advantages to this approach. However, it delays the learners' understanding of a concept, and therefore hinders their ability to solve application problems. Further, learners could easily forget the procedures when their conceptual understanding is under-developed (Skemp, 1976).

In reaction to approaches that may impede learning, this study focuses on the use of examples to facilitate mathematical proficiency in the teaching of statistics. Milgram (2007) defines mathematical proficiency as the ability to understand mathematical concepts, use them in application, and represent problems mathematically.

Why statistics

North and Zewotir (2006) assert that according to Curriculum 2005 (C2005) of South Africa, learners must be able to work with data in different contexts to make logical judgements and conclusions. Learners must also be able to collect, represent, summarise and analyse data (North & Zewotir, 2006). According to Batanero, Burrill, and Reading (2011), C2005 focuses on the critical analysis of collected and represented data and on drawing conclusions and predictions from the data. This can be achieved through appropriate pedagogical practices and through the use of tools that can facilitate the process of achieving curriculum outcomes of C2005.

The focus or scope of this study is in the teaching of statistics in Grade 10. According to Batanero et al. (2011), the content taught in Grade 10 and 11 is a requisite for the content to be taught in Grade 12. This implies that the way in which statistics is taught in Grade 10 and 11 has a great influence on learners' performance in Grade 12. For this reason, the National Senior Certificate (NSC) Diagnostic Report (Department of Basic Education, 2017) suggests that for learners' performance in statistics to improve, statistical concepts must be thoroughly explained and illustrated. It is through the use of examples that mathematics teachers can explain statistical concepts in a way that learners can relate to and grasp more sustainably. This study, thus, looks at the way in which examples could be used in the teaching of statistics to ensure statistical proficiency.

Problem statement

In my experience of teaching statistics to a Grade 10 class, examples were a tool often used to facilitate the learners' ability to conduct procedures necessary to obtain the correct answers. Examples in these lessons were often used without much explanation of the underlying mathematical concepts. Most learners would master certain routines or certain types of questions without them knowing the broader usefulness of the mathematical tool they have that could be used in different circumstances. Very few learners could relate to, and see sense in, the examples and how/where the techniques they had learned may be applicable. This indicates that the way in which examples are used in the teaching of statistics has different, and sometimes negative, impacts on learners.

Batanero et al. (2011) argue that statistics in South Africa is in its infancy, and thus, there is a need for teachers to be prepared to teach the broad statistics curriculum in order to promote

statistical proficiency. The teaching of statistics to promote proficiency is influenced by the pedagogical knowledge and content the teacher has. This affects the way in which knowledge is transmitted to learners and the breadth and depth to which they will understand. Teachers need to be exposed to different teaching strategies that may assist in enhancing statistical proficiency as the ultimate outcome of their lessons.

1.1 Purpose of the study

Many studies have been conducted on the use of examples in the teaching and learning of mathematics (Zaslavsky & Zodik, 2007). But, there is only a few, especially recent ones, which focus on how teachers use examples in mathematics to promote mathematical proficiency. This study focuses on how teachers use examples in mathematics classrooms successfully to promote learners' understanding, interest, and seeing sense in mathematics, particularly in statistics.

The study, therefore, aims to investigate the extent to which examples used in Grade 10 mathematics classrooms promote mathematical proficiency. It also aims to uncover how specific teacher choices of examples assist in promoting learning and how they could be used in mathematics classrooms in a way that allows learners to relate to the underlying mathematical concepts. This could be, for instance, in the introduction of a concept or during an explanation of a mathematical concept.

While examples in mathematics in and of themselves may not facilitate mathematical proficiency, the different ways in which teachers use them may have a great impact in promoting mathematical proficiency in statistics. The following questions guide the study:

1. What examples do teachers use to teach statistics in Grade 10?
2. How do the examples used by the teachers when teaching statistics in class promote (or not) mathematical proficiency in learners?

To address these questions, the research for the study focuses on the way different teachers make use of examples and whether the way in which learners respond shows evidence of any strand of mathematical proficiency.

Context

The context of this study is the multilingual mathematics classrooms in a school with learners from different backgrounds. The language of instruction in the school is English. Mathematics is, thus, taught in English, with limited code switching.

As noted above, the main focus group is Grade 10 learners. In accordance with the Curriculum and Assessment Policy Statement (CAPS), they would have done statistics in Grade 8 and 9 mainly covering the use of statistical terms and the computational skills of the basic concepts (Department of Basic Education, 2011). The classrooms in which this study was conducted consisted of a minimum of 40 learners per classroom, with two different teachers in terms of experience and background. Both teachers are not fully competent in the learners' home languages, and, therefore, mainly stuck to English; only code switching when it was necessary.

Significance of the study

The significance of this study is that it will contribute to mathematics education research on how statistics is best taught in secondary schools to enhance conceptual understanding. This is useful not only to ensure better pass rates but also for those who may wish to enrol at tertiary institutions.

The study examines how examples could become an essential tool in the teaching of statistics and how they may be used to facilitate deeper learning. This will certainly add to the different teaching strategies necessary for teachers to teach statistics more successfully. It will also assist in devising ways to improve teachers' pedagogical skills in the teaching of statistics. Chick and Pierce (2008) suggest that it is essential for the teacher to have appropriate pedagogies to teach statistics. It will also raise awareness on how learners best learn statistics in a way that enhances deeper understanding of the concepts and how they are apply in their everyday lives.

Recommendations from this study may also be of use to teachers, principals, and relevant departments of education as they seek to improve learners' performance in statistics from lower Grades up to Grade 12. Further, implementing the recommendations will also help learners to develop a better foundation that will facilitate better and varied career choices.

Conclusion

This chapter outlines the background of the study and provides reasons for focusing on statistics. The problem statement asserts that statistics is a section of mathematics that lacks

full attention from teachers and learners. Statistics, thus, ends up being taught in a way that hinders understanding and therefore affecting learners' performance in higher grades.

Further, the chapter defines the purpose of the study and the research questions to be addressed in it. The context in which the study will be conducted is also described. Lastly, the significance of the study in the teaching and learning of statistics is provided.

Chapter 2 outlines the theoretical framework used. It incorporates variation theory to explore ways in which teachers could use examples and strands of mathematical proficiency (Kilpatrick, Swafford and Findell, 2001) and how examples facilitate these. For a theoretical understanding of the use of examples in promoting mathematical proficiency when teaching statistics to Grade 10s, chapter 2 delves in the literature review. This focuses on examples and their role in teaching mathematics; what the literature says about teaching and learning statistics; and, what the role of examples in the teaching and learning of statistics is.

Chapter 3 outlines the research design and methodology. These focus on the methodology of the study, the settings of the school in which the research was conducted, the sampling, the methods used to conduct the research, methodological limitations and the analytical framework used as a guiding tool for analysis.

Chapter 4 presents the analysis of the use of examples in the lessons conducted by teachers A and B. The analysis focuses on the intended, the enacted, and the lived object of learning per teacher. It examines the teachers' intentions behind using a particular set of examples, the way they use or enact those examples in class and how the learners experience the object of learning from how the teacher enacts the examples.

Chapter 5 consists of the conclusion drawn from data analysis, the limitations of the study, ethical considerations and the recommendations arising from the study.

CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

This chapter outlines this study's theoretical framework, which is based on the variation theory of learning. The literature review, thus, focuses on the use of examples in a mathematics classroom to promote mathematical proficiency through the lenses this theory provides.

Theoretical framework

According to Mun Ling and Marton (2011), variation theory can assist teachers in planning lessons that will promote learning in the classroom. When applied to mathematics, Watson & Mason (2006) argue that variation theory offers ways of describing learning in relation to mathematical structures and the way in which learning is experienced by learners as compared to the way it is intended by the teacher. To assess whether learning actually takes place, strands of mathematical proficiency, as propounded by Kilpatrick et al. (2001), will be employed.

Variation theory describes learning as a change in the way something is seen, experienced, discerned and understood. In this context, experience is more about the relationship between the object and the subject: the object being what is meant to be learnt and the subject being who is meant to learn (Marton & Pang, 2006; Runesson, 2005). Learning is also about developing capabilities to experience a concept in a different way from before and, thus, becoming capable of discerning aspects of a concept one has not been able to discern previously (Marton & Booth, 1997). According to Marton, Tsui, Chik, Ko, & Lo, (2004), learning is the acquisition of knowledge about 'something' that should always be specified and clear. Learning is also that capability meant to be acquired in the process of learning, which Marton et al. (2004) refer to as the object of learning.

There are different approaches to learning. They emanate from the different experiences one has had, there being variation with respect to what the object of learning is experienced to be and what acts create conditions that are conducive to learning (Marton & Booth, 1997). In their terms, Marton and Booth (1997) describe an object of learning as a composite of different ways in which a concept can be experienced.

Marton and Pang (2006) argue that the object of learning has two aspects none of which can exist without the other. These are, namely: the direct and indirect objects of learning. The direct object of learning is the concept as selected by the teacher. It could be about learning how, for

example, to work with quartiles/percentiles in statistics. The indirect object of learning constitutes the capabilities that learners are expected to develop.

When Marton et al. (2004) describe the object of learning as a capability; they suggest that it has two aspects: the general and the specific aspect. The general aspect is the nature of the capability, which could be discerning, remembering, interpreting or viewing; all being acts of learning the concept. The specific aspect is about the concept or subject in which the acts of learning are being carried out. In addition, they assert, the general aspect is about the acts of learning and is considered as the indirect object of learning, while the specific aspect is about what is acted on and is considered as the direct object of learning.

In keeping with this argument, Marton and Booth (1997) support that learning has two aspects, that is, the 'what' and the 'how'. The 'what' aspect focuses on the principal object to be learned, and the 'how' aspect has to do with the way learners experience learning and the type of capabilities they are trying to master. Marton et al. (2004) argue that the teacher's focus is on both aspects, that is, how learners learn, and what they are learning. Thus, teachers work on the object of learning, while the learners' focus is on what they must be learning (Marton et al., 2004).

The object of learning can be countenanced as either intended, enacted or lived; and these can be linked to the three types of variation Mun Ling (2011) builds upon. Runesson (2005) suggests that the intended object of learning can be viewed from the teacher's perspective as it is about what the teacher wants learners to learn. Marton et al (2004) also support this view as, according to them, only the teacher is aware of the intended object of learning and it might take a serious turn to something else during the lesson. Thus, according to Marton and Pang (2006), the intended object of learning does not result in what learners learn in itself; but, together with the enacted and lived objects of learning, they do. The intended object of learning can be linked to what Mun Ling (2011) describes as using variation as a guiding principle of pedagogical design, where teachers have to decide on how they deal with a certain concept through the use of examples or exercises to ensure that certain critical features will be discerned.

What learners learn is affected by the enacted object of learning, which Runesson (2005) argues can be viewed from the researcher's perspective. The enacted object of learning is what learners encounter (Marton et al., 2004) and it determines what is possible for learners to learn. It is linked to Mun Ling's (2011) idea of variation in the teachers' understanding and the way they deal with the object of learning. In support, Marton et al. (2004) and Marton and Pang (2006)

argue that the enacted object of learning is the researcher's point of view about what necessary conditions are provided in the classroom for the discerning of critical features and for the adoption of the object of learning therein.

The teacher's knowledge of content and students (Shulman, 1986) is essential. The best way to conduct a lesson that will assist learners to acquire knowledge depends on the teacher's knowledge of the learners and the way in which they understand a concept to allow them to present it in different ways. The teacher's understanding of the object of learning helps them to address the direct object of learning effectively to allow learning to take place. The indirect object of learning is, thus, realised (Marton & Pang, 2006).

The discerning of critical features leads to learning and what learners actually learn is the lived object of learning (Marton et al., 2004). Poignantly, Marton and Pang (2006. p.195) define the lived object of learning as the learners' "initial mastery of the object of learning". In support, Marton et al. (2004) argue that the lived object of learning is the result of learning as seen from the learners' point of view. The lived object of learning can, thus, be described as the variation in the way in which learners understand the object of learning (Mun Ling, 2011).

According to variation theory, the key to learning is in the patterns of variation in the examples or exercises used in the classroom (Watson & Mason, 2006). It is through that variation of examples or exercises that learners can see meaning from what they are learning and not from the sameness of the examples or exercises (Mun Ling & Marton, 2011). As variation theory boldly asserts, one cannot discern through 'sameness' (Marton & Booth, 1997).

Providing learners an opportunity to make meaning in a mathematics classroom promotes mathematical proficiency; this being a phrase used by Kilpatrick et al. (2001) to capture all aspects of expertise, competence and knowledge in mathematics. In support, Schoenfeld (2007) describes mathematical proficiency as the knowledge of mathematics and the ability to use the knowledge in applications.

The ability to use mathematical knowledge in applications may indicate that the learner has developed new 'ways of seeing', a phrase used in variation theory to describe observable patterns that a learner has learnt with respect to a particular concept. To facilitate the new 'way of seeing' which can also be described as the discerning of critical features, Marton et al. (2004) suggest that close attention must be given to what varies and what is invariant in a classroom. This assists in understanding what is possible to learn in the classroom and what is not. In

support, Mun Ling and Marton (2011) argue that when teaching a certain concept, some aspects of that concept must be kept invariant while leaving other aspects variant to allow critical features to be discerned. The notion in this strategy is that, in the case of using examples in the teaching of mathematics, the teacher must be very careful in selecting the examples with both invariant and variant features so that learners can learn through them.

Keeping some aspects of a concept variant and others invariant in a classroom can be achieved through the employment of patterns of variation. Marton et al. (2004) identify four patterns of variation, namely: contrast, generalization, separation, and fusion. These four patterns can be incorporated in a mathematics classroom through the use of examples. They are about how a learner experiences the object of learning.

Contrast is about exposing learners to different aspects of a concept in order for them to be able to compare. As Marton et al. (2004) argue, for a learner to experience something, they must experience something else to allow them to compare.

Generalization is about the varying appearances of something to allow learners to generalize. Marton et al. (2004) make an example about understanding what the number 'three' is. They argue that for this to happen, a learner must encounter different appearances of the number 'three', as in three bananas, three sweets, and so forth.

When it comes to separation, Marton et al. (2004) argue that in order for a learner to experience something and be able to separate it from other things, some aspects of it must vary while others remain invariant. Fusion suggests that if a learner is to discern the critical features of a concept, they must experience several critical aspects of that concept simultaneously (Marton et al., 2004); that is, they must experience the variant and the invariant aspects at the same time. In this study, patterns of variation, as defined above, are used to investigate ways in which teachers use examples in a mathematics classroom to facilitate mathematical proficiency in statistics.

The use of the patterns of variation in this study resonates with the revised patterns of variation by Adler and Ronda (2015) who revised them to three patterns of variation, which are; similarity, contrast, and fusion. According to them, in the use of examples generalization is achieved through similarity and contrast. For this reason, they work with the notion of variation amid invariance by Watson and Mason (2006), which focuses on variant and invariant aspects, and replaces separation in the four patterns of variation à la Marton, et al. (2004). This suggests

that focusing on the incorporation of variance and invariance in the use of examples covers separation, while, as argued above, generalization is covered by similarity and contrast. Fusion is kept as the third pattern of variation and is defined as having more than one aspect being kept variant and invariant in the object of learning (Adler & Ronda, 2015).

Adler and Ronda (2015) further argue that similarity alone does not cover all the aspects of a concept, such as clarifying what it is not. Contrasting examples helps in bringing attention to different aspects of a concept and, therefore, leading to generalizing. In support, Marton and Booth (1997) maintain that awareness of a single feature cannot exist without awareness of the differences between features.

As fusion is about simultaneously varying and keeping some aspects across an example set invariant (Adler & Ronda, 2015), Marton & Booth (1997) argue that a learner is said to have learnt a concept when they are capable of being simultaneously and focally aware of other aspects of that concept. This leads to what Marton, Dahlgren, Svensson and Saljo (1977, p. 142) refer to as "a change in the eyes through which we see the world", which is an indication of seeing sense or making meaning of mathematical concepts from different experiences.

Diagrammatically, variation theory as a learning theory and a good guiding tool for the teaching of mathematics is outlined below as an integral part of this study's theoretical framework. The diagram explains several key aspects: how the object of learning is observed in this study; where the intended object of learning will be observed from the set of examples chosen; and, finally, how the patterns of variation employed in the example will be observed. It should also be kept in mind that Marton et al. (2004) argues that what the teacher intends for learners to learn may not be what leaners actually learn. Notwithstanding, what the teacher intends may affect the way in which the teacher presents or enacts the object of learning during the lesson.

The enacted object of learning will be observed through interactions or talks between the learners and the teacher as the teacher verbally or otherwise transmits 'something' to the learners. To a considerable extent, the enacted object of learning does have an effect on what is to be learned. The way the teacher delivers the lesson and interacts with the learners affects the way learners experience the object of learning. The experience of the object of learning entails what learners learn and is, thus, the lived object of learning. Learner responses are observed to discern how learners understand and react to the teacher instructions in verbal and in written formats.

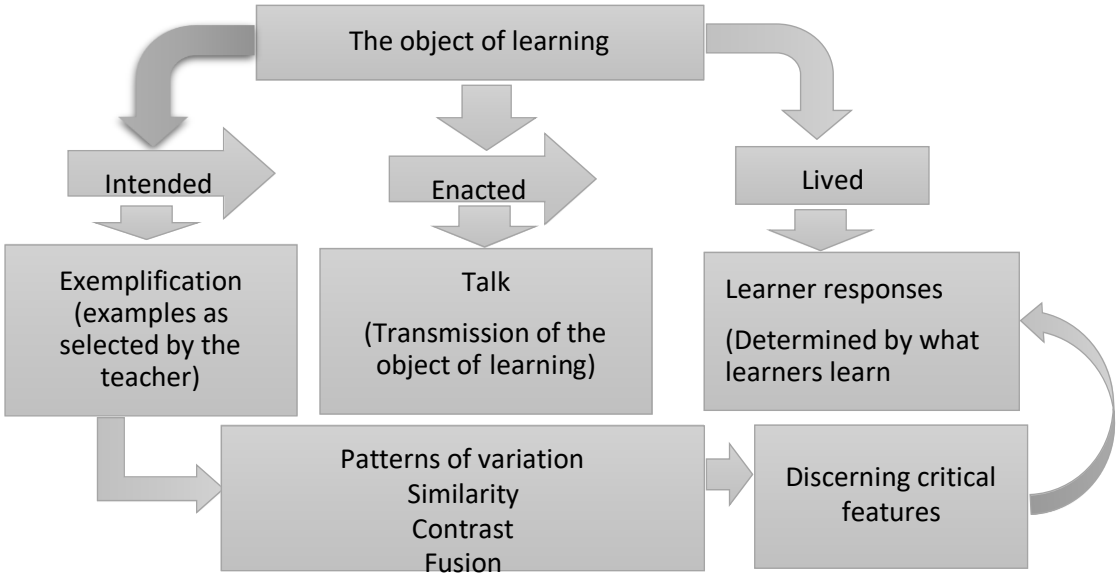


Figure 1: Variation theory

According to Adler and Ronda (2015), examples provide a good way to observe what teachers do in mathematics classrooms. However, examples do not give a full idea of how they bring about the object of learning to be experienced by learners. This is why this study observes the intended, the enacted and the lived objects of learning to understand how teachers transmit through verbal communication (talk) what needs to be known (Adler & Ronda, 2015). It is mainly the 'talk' that determines how learners live the object of learning. As noted above, this is observed from learner responses, which Adler and Ronda (2015) define as the opportunities learners are given by their teachers to show their mathematical reasoning verbally. In this study, learner responses are understood as the learners' ability to express themselves both in writing and verbally.

For the purposes of this study, variation theory will be engaged to determine the teacher's use of examples (as a learning tool) in the classroom to promote mathematical proficiency in learners. Samuelsson-Brown (2010) defines mathematical proficiency as the understanding of mathematical concepts, the ability to conduct calculations and the ability to display selfregulated learning skills such as internal and instrumental motivation. To evaluate proficiency in mathematics from learning through examples, the study will engage the five strands of mathematical proficiency (Kilpatrick et al., 2001).

Kilpatrick et al. (2001) argue that the five strands are necessary and sufficient for one to be declared mathematically proficient. Further, these strands are interdependent and represent different aspects of a complex whole. The five strands of mathematical proficiency are, namely:

- *Conceptual understanding*
- *Procedural fluency*
- *Strategic competence*
- *Adaptive reasoning*
- *Productive disposition*

According to Kilpatrick et al. (2001), conceptual understanding is an integrated and an efficient grasp of mathematical ideas. It concerns the ability to see relationships and make connections between mathematical concepts. Procedural fluency is about the knowledge of procedures (Kilpatrick et al., 2001). According to Rittle-Johnson, Siegler, & Alibali (2001), procedural knowledge is that which demonstrates the ability to perform action sequences in order to solve mathematical problems. Schoenfeld (2007) argues that knowing mathematics in terms of exhibiting the ability to produce facts, definitions, and conduct procedures is not enough. Accordingly, learners need to reason, make sense, and strategically apply the mathematics they learn in the classroom when solving problems in context.

Learners in a mathematics classroom also need to have problem solving strategies such as working backwards, re-formulating problems, making use of analogies, generalizing, and exploiting solutions to related problems (Polya, 1945). These strategies relate to strategic competence, another strand of mathematical proficiency, which has to do with the learners' ability to formulate, represent a problem mathematically and solve it. According to Kilpatrick et al. (2001) this strand is the basis of problem solving.

In the process of problem solving, learners need to be able to formulate mental representations of the solution strategy for the mathematical problem they are faced with. This will allow them to connect mathematical ideas and see relationships in the concepts (Kilpatrick et al., 2001). In this process, flexibility, the most important characteristic of problem solving, is practised. This characteristic can be associated with adaptive reasoning, a strand of mathematical proficiency where a learner displays their capacity to think logically about ideas and to see relationships in different mathematical concepts and situations (Kilpatrick et al., 2001).

According to Kilpatrick at el. (2001), one of the major characteristics of adaptive reasoning is the learner's ability to justify their solution strategy through reasoning. Further, they argue that adaptive reasoning interrelates with the other strands because, for a learner to solve a

mathematical problem successfully, they need to rely on their strategic competence. This enables them to formulate and represent mathematical problems symbolically and formally.

The fifth strand of mathematical proficiency, which is as important as the other four in a mathematics classroom, is productive disposition. This entails a learner seeing mathematics as sensible, useful and worth learning (Kilpatrick et al., 2001). In this respect, a learner can relate to mathematics concepts and see where and how they are applicable in context. This has a great impact on the learners' beliefs and attitude towards mathematics, which in turn has a big impact on their performance in mathematics. Schoenfeld (2007) supports this notion by arguing that beliefs and dispositions play a major role in mathematical proficiency for both learners in the classroom and professional mathematicians. To support this argument, Ma and Kishor (1997) have discovered that there is a relationship between the beliefs and attitude learners have towards mathematics and their achievement in mathematics as a subject.

The five strands of mathematical proficiency can be achieved through the use of variation theory by teachers. Figure 2 below provides an overview of how variation theory and its patterns of variation could assist teachers in planning lessons that lead to mathematical proficiency being attained by the learners.

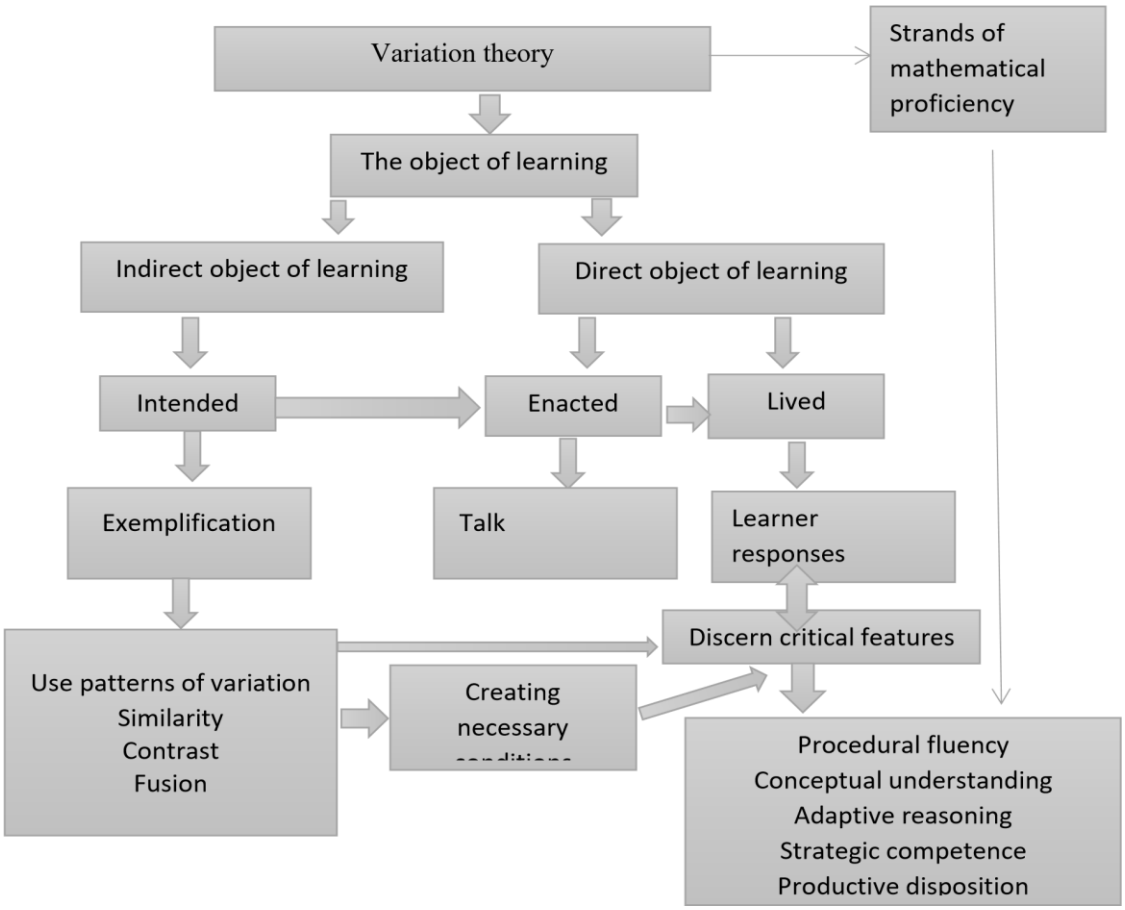


Figure 2: Theoretical framework of study

The next section provides an outline of how examples are used by teachers in mathematics classrooms (as a teaching tool) to help learners in becoming mathematically proficient. The section offers a breakdown of what the literature says about examples and the teaching of statistics, together with the role of examples in teaching mathematics.

Literature review

Examples in mathematics

An example is "a specific instantiation of a general principle, chosen in order to illustrate or explore that principle" (Claydon & Chick, 2005, p. 5). Bills, Dreyfus, Mason, Tsamir, Watson and Zaslavsky (2006) refer to an example as anything that is used as raw material to facilitate generalising, assist in seeing relationships, and that leads to inductive reasoning.

Courant (1981) states that examples allow mathematicians to see the general through the intimate experience of the particular when they encounter problems that are not immediately obvious. Zodik and Zaslavsky (2008) define an example as a certain item of a larger concept,

from which one can reason and generalize. Further, they explain that an example is a set of facts viewed through the use of a certain lens, and it carries some features that are intended to be illustrated and others that are immaterial. Skemp (1971) regards the immaterial features of an example as 'noise', which must always be kept low because the greater it is, the lesser the success of the goal intended to be achieved by the example.

In a mathematical context, examples are meant to satisfy certain mathematical conditions depending on the mathematical concept they are meant to demonstrate (Zaslavsky & Zodik (2007). Further, they then argue that a mathematical object becomes an example to illustrate a mathematical concept if there is an interaction between the mathematical object and the person (the teacher) presenting the example. In support, Rowland (2008) argues that for a mathematical object to be an example, it must be general in character to complement the fact that "examples are particular instances of the generality" (Rowland, p.150, 2008).

The role of examples in the teaching and learning of mathematics.

Studies in mathematics education have shown that examples have always been used for pedagogical reasons in mathematics from ages ago and continue to be used regularly by teachers (Chick, 2009; Rowland, 2008). Further, in teaching, examples are used to symbolize abstract concepts, characterize general procedures and to ease abstraction.

Rowland's (2008) assertions are in line with Zaslavsky and Zodik's (2007) argument that examples are an important component of expert knowledge in mathematics as they assist in generalizations, abstraction and analogical reasoning. They argue, further, that examples play an essential role in teaching mathematics as they are a communication device used to explain mathematical discourse. Bills et al. (2006) uphold this argument by affirming that all kinds of examples are one of the important devices used to communicate and illustrate concepts in the teaching and learning of mathematics.

The use of examples in the teaching and learning of mathematics is for different purposes, too. Rowland (2008) suggests that examples are used in exercises so that learners can make connections between them. Further, these examples are merely used to promote fluency in learners. Bills et al. (2006) argue that examples can be used as exercises and they differentiate among three types of exercise-examples in the teaching and learning of mathematics, namely:

- Generic example: examples of concepts or procedures

- Counter-example: usually needs an assertion to counter, and is, thus, often used in the context of an example of a concept or procedure.
- Non-example: examples used to clarify limitations. This can be done for a concept or in a case where a procedure may not be applicable.

Generic examples can be described as the actual examples, which are represented in a way that brings out the intention of the general idea (Mason & Pimm, 1984). Counter-examples, as described by Zaslavsky and Zodik (2008), are examples intended to illustrate why a specific procedure may not work, or why a certain statement is not true. Suggested by Thompson (1984), non-examples can be used by teachers to probe and clarify misconceptions in the learners.

According to Watson and Mason (2006), exercises play a critical role in the pedagogical design for teachers. Further, they argue that allowing learners to generate their own examples (which are often exercise examples) promotes learning unlike presenting ready-made examples. Allowing learners to generate their own exercise examples facilitates generalising (Watson & Mason, 2006) and helps learners to understand a concept as they enquire deeply for themselves.

Another type of example, which is said to be good in promoting learning, is the worked out example. Such an example is the basis of teaching as it is a common instructional method for teachers in addition to being used to demonstrate problem-solving procedures (Mevarech & Kramarski, 2003). The latter argue, further, that worked out examples are a very important tool in teaching as they can be used to promote cooperativeness in learners. This is done by facilitating interactions in learners within small groups, where learners try to understand worked out examples using mutual reasoning, or learners try to understand an example individually explaining to those within the group who do not understand.

While exercise examples and worked out, examples are good tools for pedagogical design. There are examples that come up during lessons, which are used to elaborate more or as exercises to validate the learners' understanding. These are spontaneous examples, which Zodik and Zaslavsky (2008) define as examples which show evidence that they were chosen at that moment. These examples, they argue further, are often observed in the classroom when the teacher generates an example in response to a learner's question or comment.

It is the teacher's role to select examples that will assist learners in generalising and not examples with a lot of 'noise' as referred to by Skemp (1971). Zodik and Zaslavsky (2008)

note that teachers are faced with the challenge of having many considerations when generating examples. This is because a specific example chosen by a teacher may promote or hinder learning.

Rowland (2008) argues that the choice of examples made by teachers is affected by the diversity of knowledge teachers have. Three characteristics of teacher knowledge relate to the ability to generate examples in mathematics teaching (Zodik & Zaslavsky, 2008): the knowledge of mathematics; the knowledge of the student's learning abilities; and pedagogical content knowledge. The three aspects of knowledge of a teacher allow them to use the notion of variation, as suggested by Watson and Mason (2006), to structure sense making when generating an example where learners are probed to notice structure and make generalizations.

Teaching and learning statistics.

According to Cobb and Moore (1997), statistics is a discipline that is methodological and provides tools for dealing with data in other fields of study. In support, Garfield and Ahlgren (1988) argue that statistics has elements of reasoning which have become necessary for many other fields of study. Garfield (1995) claims that learning statistics is about learning to solve statistical problems, using statistical language and drawing conclusions, which can be supported with logical thinking.

According to Garfield and Ben-Zvi (2008), statistical literacy is about understanding and having the ability to use basic statistical tools and language; that is, the knowledge of what basic statistical terms mean; being able to use the basic statistical symbols meaningfully; and, being able to recognise and interpret data represented in different ways. Wallmans (1993) describes statistical literacy as being able to understand and critically evaluate statistical results from real life problems, also appreciating the contributions made by statistical thinking in public and personal decision-making. Wessels (2009), cited in Batanero, Burrill and Reading (2011), argues that in mathematical literacy, statistics focuses on facilitating informal inferential reasoning and critical thinking.

According to Chick and Pierce (2008) the teaching of statistics at school level is important for three reasons, namely: it plays an instrumental role in the other disciplines of mathematics and some other subjects in school; it develops critical thinking and logical reasoning; and, it is a useful tool in everyday life outside school. In addition, Garfield and Ahlgren (1988) argue that

there is great demand for relevant statistics to be introduced at school level because it forms part of basic literacy in mathematics.

Statistical literacy in mathematics can be achieved through collaborative classrooms (Garfield & Ben-Zvi, 2008) where discussions are part of the classroom culture to enhance learners' ability to use statistical language and tools meaningfully. According to Moore (1993), the task of a teacher in a collaborative mathematics classroom is to guide and encourage learners in constructing correct mathematical understanding. Further, the telling approach in the teaching of statistics does not help learners in becoming statistically literate.

In a statistics lesson, learners must be involved through talks and writing to improve the mastering of statistical language, the correct use of statistical symbols and to improve their ability to interpret data (Garfield & Ben-Zvi, 2008). In support, Moore (1993) maintains that learners must be active participants in the learning of statistics to ensure conceptual understanding. Conceptual understanding could also be enhanced, according to Garfield and Ahlgren (1988), if the teaching of statistics is based on problem solving, which introduces the application of statistical tools.

Chick and Pierce (2008) note that it is critical for a teacher to have appropriate pedagogical content knowledge to ensure successful teaching of statistics. They argue, further, that teachers must also be capable of recognising and taking advantage of everyday situations and using them to expose learners to real life statistical problems. Routine processes like calculating the measures of central tendency and drawing graphs are important, but statistical literacy is the ability to think critically and reason logically when given real world data (Chick & Pierce, 2008). The use of appropriate examples which expose learners to real life problems can facilitate statistical literacy greatly.

The role of examples in the teaching and learning of statistics.

The teaching and learning of statistics is mainly about facilitating and achieving statistical literacy where the teacher creates an environment for learning through setting of tasks and encouraging learners to express their thinking and conclusions (Moore, 1993). To promote statistical literacy, teachers must develop the habit of using examples designed to facilitate generalization (Chick, 2009).

Statistical literacy also has to do with the ability to use statistical tools to compute procedure (Garfield & Ben-Zvi, 2008). In this case, teachers can employ the use of examples, which,

according to Rowland (2008), exposes learners to different types of problems they may encounter. When using examples to teach statistics, teachers must avoid using cases that may distract learners from the main goal of the lesson but rather focus on those that enhance understanding (Rowland, 2008). In support, Zaslavsky and Zodik (2008) assert that examples must be transparent enough to make it easy for learners to focus their attention on the target features of a particular concept.

Research has shown that learners often do not learn what teachers intend for them to learn (Garfield, 1995). In the case of statistics, it could be because learners cannot relate to the problems they are dealing with. To avoid this, Garfield (1995) suggests the use of activities, that is, a lot of exercises in small groups to help learners overcome their misconceptions through cooperative learning. This argument is in line with Mevarech and Kramarski (2003) assertions that examples are a good teaching tool, which can be used to promote cooperativeness in learning.

Regardless of the teacher's ability to explain concepts, learners will, according to Garfield (1995), only understand the meaning of a concept after they have constructed their own meaning in terms of what they are learning. This notion supports Watson and Mason (2006) as they argue that learners generalize and make meaning when they are allowed to generate their own mathematical examples, which pushes them to investigate deeper about the concepts. This allows learners to relate to mathematical concepts as they would then not all seem abstract.

The teaching and learning of statistics must take place in a way that concepts are introduced through activities and simulations, and not through foregrounding abstraction (Garfield & Ahlgren, 1988). Abstraction can be obtained through the use of other relevant examples, as Zodik and Zaslavsky (2008) argue, designed to ease it.

This chapter has outlined the theoretical framework of this study, which is the use of variation theory to promote the five strands of mathematical proficiency. The chapter also looks at literature around examples, the role of examples in the teaching and learning of statistics and the teaching and learning of statistics. The next chapter explores the methods and tools used in this study.

CHAPTER 3: DESIGN AND METHODOLOGY

Introduction

This chapter focuses on the methodology and data analysis tools employed in this study. It presents justifications for specific methods chosen. It also provides the outline of data collection, organisation and analysis. Further, it presents limitations of data collection methods and ethical considerations taken. The chapter also explains the sampling method, namely: the participants, their characteristics, and the setting of the research in terms of the school that is the source of the data.

Methodology

To explore the use of examples by teachers in the classroom, the study makes use of the interpretive approach, which focuses on social practices and assumes that all human actions are meaningful (Scott & Usher, 1996). This is a qualitative study because it employs the two major characteristics of qualitative research, namely: participant observations and in-depth interviewing (Bogdan & Biklen, 1997). This approach enables the discovery of underlying motives, values and perceptions using a number of relevant techniques (Salkind, 2011). According to Johnson and Onwuegbuzie (2004), extracting information from individual teachers in this manner constitutes qualitative research. Another feature of qualitative research that is used in this study is concern with process, which Bogdan and Biklen (1997) describe as being concerned with how things happen rather than the outcome.

Data from the use of examples by two qualified teachers were analysed together with their transcribed interview recordings to unravel the intended purpose of using the chosen examples and how they promote mathematical proficiency in learners. The use of the interpretive approach allows sense seeking to take place. Each example used in each lesson with the intention to promote mathematical proficiency in statistics is analysed to decipher sense in it.

To discover how learning was facilitated through examples, the teachers' intention of the use of examples was examined by observing patterns of variation in the example sets and asking interview questions that revealed those intentions.

Research setting

The school at which this research was conducted had about 1900 registered learners in 2018, with 67 staff members. There were 12 Grade 10 classes. All of them had enrolled for

mathematics regardless of the stream of subjects they had chosen. The school is located in an informal settlement in Johannesburg with most of the learners residing in this settlement. They speak different home languages. The school has IsiZulu, Sepedi and Setswana as home language subjects, with English as a first additional language and the language of instruction, too. This research focused only on two of the Grade 10 classes with a variety of learners in terms of background and home languages.

Sampling

The participants of this study were two Grade 10 mathematics teachers and their learners who had done statistics in Grades 8 and 9. Two teachers were chosen for this research because the researcher had a convenient relationship with them and they (the two teachers) are from the same school. The two have a particular set of similar and dissimilar characteristics. They both majored in mathematics and have teaching methods (mathematics) in their qualifications. Teacher A has more than five years of teaching mathematics while teacher B has less than five years of teaching experience.

None of the teachers speaks any of their learners' home language, and therefore, they use less code switching.

The classes taught by the two teachers are different. One is a science class, doing Physical Sciences, Life Sciences and Mathematics as their majors. The other is a humanities class with History, Geography and Mathematics as their majors.

The study, therefore, makes use of a non-probability sampling approach, which occurs when the sampled participants from the larger population do not have equal chances of being selected because they have different characteristics. (Altman, 1974). Figure 2 provides the sampling design of this study.

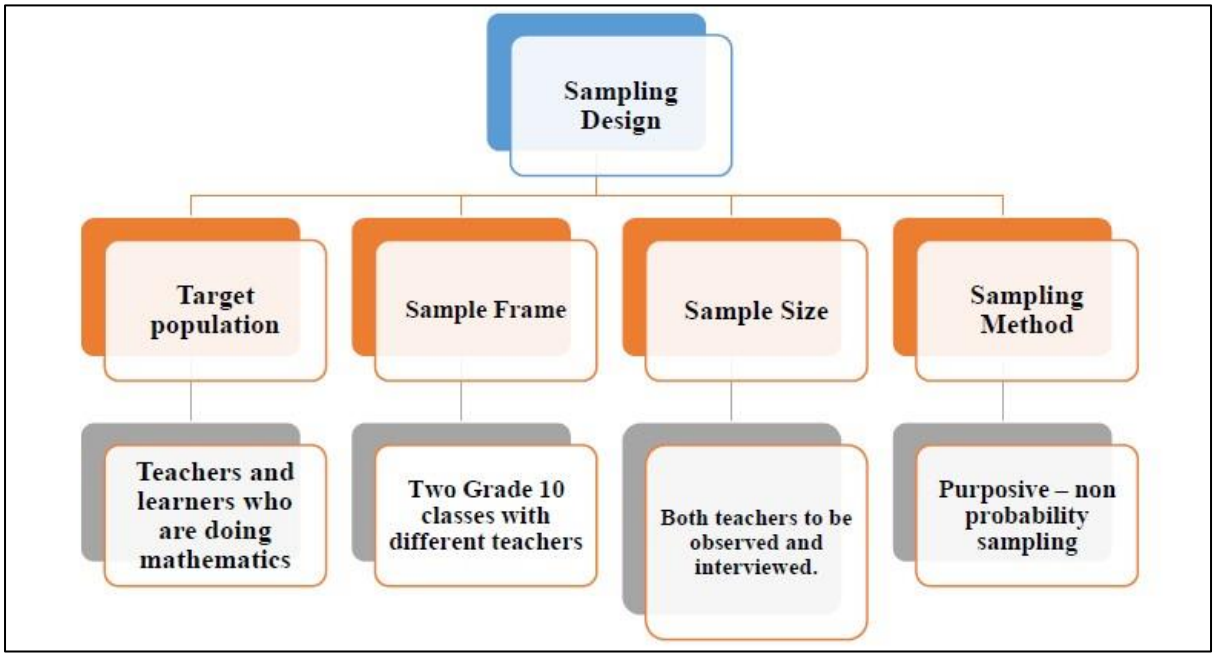


Figure 3: Sample design.

Data collection methods

In this study, data were collected through the use of field notes and audio recorded classroom observations and interviews. The focus of observations was on how the two teachers used their chosen examples in the teaching of statistics and what the nature of those examples was. During the observations and interviews, audio recording was used as a way to enhance the notes taken. Bell (2005) argues that audio recording assists in capturing some important features of observation. Theses audio recordings were then transcribed to assist in the process of data analysis.

The classroom observation transcripts were organised into episodes. Adler and Ronda (2015) argue that episodes are a way of organising transcripts in terms of the content focus and the object of learning. Episode 1 is about introduction to statistics and the measures of central tendency. Episode 2 is about representation of data (graphs, frequency tables, tally marks, and using equations for calculations). These episodes were designed based on how the teachers presented the objects of learning.

After observing and audiotaping the lessons, unstructured interviews, which consisted of questions that gradually lead to the desired information (Morgan, 1997), were conducted with the teachers. This was done after all lessons had been observed. The interviews focused on discovering the purpose of the examples used in class and how they assist in promoting

mathematical proficiency in statistics. The observation and interview schedules are attached as appendices A and B, respectively.

Classroom structured observation

Observations allow data to be gathered by engaging in natural behaviour in a natural environment (Bogdan & Biklen, 1997). Structured observations are used because they have a focus (Bell, 2005). The focus is on the use of examples by teachers in the teaching of statistics to facilitate mathematical proficiency. Good (1988) argues that observations in research assist researchers to describe what takes place in the classroom in order to address the research purpose. Lessons covering statistics from the beginning to the end of the topic were observed for teachers A and B. In these observations, teacher engagement with the topic and learners through the use of examples was recorded. Learners' written and verbal responses were also observed. The classroom observation schedule is attached as appendix A.

Audio recording

Taking notes as classroom interactions unfold has limitations. As noted earlier, audio recording was then used as a way to enhance the notes taken during observations and to ensure that important features are not lost (Bell, 2005). The quality of the audio recording is clear and the records are easy to use.

Unstructured interview

Unstructured interviews do not involve a detailed interview guide. Questions asked gradually lead to respondents giving the desired information (Morgan, 1997). Bogdan and Biklen (1997) refer to unstructured interviews as in-depth interviews, which use open-ended questions to obtain as much information as possible. An interview schedule was prepared together with the main questions (see appendix B). Follow up questions were used to probe further. In support of this method, Mason (1996) argues that qualitative interviewing is a loosely structured form of interviewing which opens up a broadened conversation during an interview.

Transcribing data

According to Matheson (2007) transcription is an important aspect of managing data for the analysis process. In this study, the classroom observations and interview recordings were transcribed to help manage data for analysis.

Limitations of data collection methods

Waxman (2013) notes that it is not easy to conduct classroom observations because of the permission required to gain access to the school and classrooms. Data collection was done during term 3 when learners had already started writing their cycle tests. This affected some lessons which were to be observed. The two teachers had one of their mathematics lessons taking place at the same time. In this instance, audio recording was used in one class, while the other class was being observed. Without observation, some valuable information may have been missed.

Another drawback of classroom observations mentioned by Waxman (2013) is that of the observer effects. The participants may not behave naturally because of the presence of the observer. The observer's presence during observations might have affected the teachers' and the learners' behaviour and thereby affecting the data. This was also noted in teacher A's comments when she said the class had been well behaved in the observer's presence.

Audio recordings also have their limitations. Transcribing has its own challenges, with the recording running too fast to transcribe comfortably. The transcript should not be cleaned to remove foul language, slang and grammatical errors (McLellan, MacQueen, & Neidig, 2003). Grammatical errors may interfere with the process of analysis, leading to more time taken to make sense of what participants are saying.

Validity and Reliability

Golafshani (2003) describes validity as a way to determine whether the research instrument used in the research measures is what is intended to be measured. According to Maxwell (1992), there are different forms of validity.

The first form of validity is descriptive validity, which provides an account of the accuracy of the researchers' facts. For example, whether the researcher did not simply make up what they claim to have observed during an interview or lesson. During note taking, the participants' words were written down as accurately as possible. The examples used in the lessons were carefully noted. Some examples had the numbering changed because of some ethical considerations regarding the real sources of the examples. This did not change the context of the examples.

The second form of validity proposed by Maxwell (1992) is interpretive validity. Interpretive validity seeks to understand phenomena from the participants' point of view and not the researcher's (Maxwell, 1992); which is germane to this study. Interviews were used as a tool to find out about what the teachers' words and actions meant in terms of the use of examples and how they promote mathematical proficiency. Learners' responses were also carefully noted in order to understand what they meant.

The third form of validity, which will be the last to be considered in this study, is theoretical validity. This form of validity is about the validity of the application of the underlying theory to the focus of the study. It is also about the validity of the relationships among the concepts explored in the study (Maxwell, 1992). This study's theoretical framework is valid because it focuses on learning and how learning can be facilitated through the teachers' use of examples. Flowing from the theoretical framework, this study's analytical framework focuses on aspects of the teachers' lessons that show how examples facilitate mathematical proficiency.

Joppe (2006) defines reliability as the degree to which the results are consistent. To ensure reliability in this study, two teachers who teach learners in the same Grade and teach the same topic were used. The same main questions used during the interviews also ensured the reliability of the results. To analyse data from lessons, the same analytical framework was used.

According to Fraenkel and Wallen (1996), reliability is the consistency of the results that come out of using an instrument. The results of this study are consistent because the observed teachers are both qualified to teach mathematics and the Grade. The data collection methods are reliable because audio taping was used to capture information that may have been missed during observations. These audio recordings together with the interviews were accurately transcribed. If they were to be transcribed again, the same transcripts would be produced.

Ethical considerations

Prior to the process of data collection, participants were approached and were informed of the nature of the research. Data collection was only done after permission had been granted by the stakeholders and the participants, namely: the school principal, teachers, learners and their parents. Permission was also sought from the Gauteng Department of Education and the Ethics Committee of the University of the Witwatersrand, Johannesburg.

Data analysis

Data analysis emanates directly from the theoretical framework described in chapter 2. Variation theory and its patterns of variation are used to analyse the way in which teachers use examples in the classroom to promote mathematical proficiency in statistics. Mathematical proficiency was observed in the enacted and lived objects of learning, which were observed from the teachers' use of examples and learners' responses in the classroom, respectively. The five strands of mathematical proficiency (Kilpatrick et al., 2001) were used.

The analysis is organised into two sections, namely: episodes 1 and 2. In each episode there are two sub-sections. The first sub-section focuses on the intended object of learning. The focus in this respect is the set of examples used by each teacher per episode and how the patterns of variation are used in that set of examples. The types of examples observed under the intended object of learning are the worked-out examples and exercise examples, all predetermined by the teacher.

The second sub-section covers the enacted and the lived objects of learning. It focuses on how the teacher uses the set of examples chosen to promote mathematical proficiency in statistics and how learners experience the examples. The way in which learners experience the examples determines the attainment of mathematical proficiency, which was observed from verbal and written responses. In this sub-section, the three types of examples discussed in this study (worked-out, exercise and spontaneous examples) are analysed. Data analysis also involved the analysis of the transcribed interviews for teacher A and teacher B. The analysis of the interviews was to uncover and clarify the teachers' intentions for using the examples. Figure 4 below, provides a schematic diagram of the analysis process which was done through the use of the analytical framework, as presented thereafter.

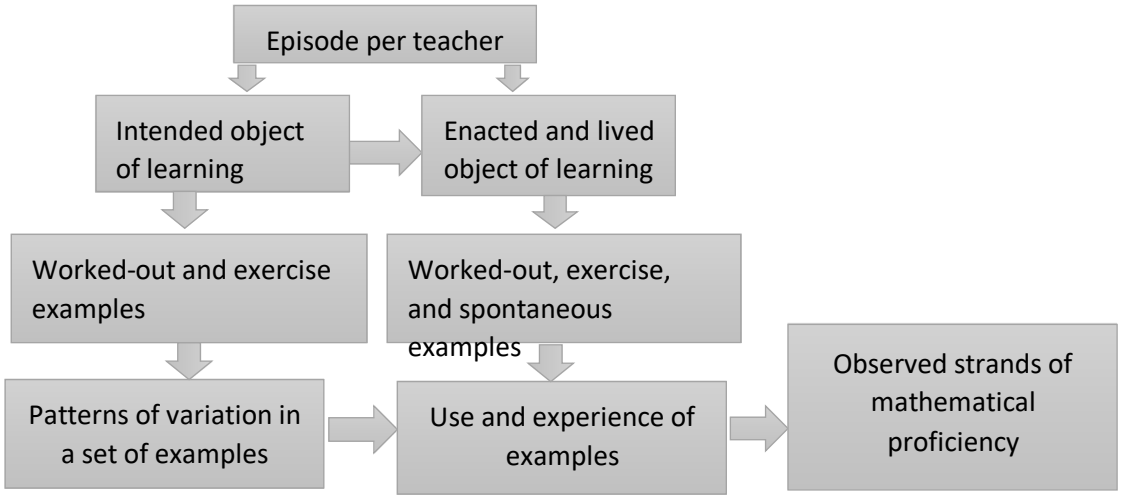


Figure 4: Analysis process

Table 1: Recognition guidelines for framework

1. From the teachers' perspective ***Choice of example***			
The object of learning *The focus of attention at a point in time.* *(content)*	**Examples**	**Pattern of variation employed in the example**	**Descriptor**
Intended	Worked out Exercise	Similarity	Intends to expose learners to a similar set of examples from which they can generalize and see a concept for what it is through the invariant features of that concept. Teacher ensures that some features of the concept are kept the same throughout the set of examples. Having examples that ask similar questions. • In each example, there is a question that asks learners to find the mean, median, mode, or to draw a graph, frequency table, use tally marks and so forth. • Examples are represented in a similar structure; tables, data entries, pictures, and etc.

		Contrast	Intends to contrast examples by varying some features to expose learners to a different aspect of a concept from which they can generalise Examples are contrasted to ensure that different aspects of the object of learning are brought to the attention of the learners. Teacher uses examples which are different in structure and format such as diagrams, pictures and wording. • Contrast may be seen when out of similar examples asking about calculating the measures of central tendency, one or two examples are asking different questions like drawing a graph or a frequency table. • One or two examples may appear different in representation.
		Fusion	Intends to simultaneously expose learners to varying and invariant aspects of the object of learning using a set of examples. • Having one or more examples in an example set which have both the similar and contrasted features of the object of learning. Like one example may ask learners to organise data in a frequency table and calculate the measures of central tendency. While other only ask one of these questions. • One or two examples are presented in a way that it looks similar to the way other examples are represented, and also has a different feature in the way it is represented.
2. From the researcher's perspective *Use of example*			

How the teacher uses the examples with the patterns of variation to facilitate mathematical proficiency in statistics				
The object of learning	**Example**	**Strand of proficiency promoted**	**Descriptor** ***(Using same set of examples with the patterns of variation)***	**Pattern of variation**
Enacted	Worked out Spontaneous Exercise	Conceptual understanding	With the help of the patterns of variation, the teacher uses examples to make learners understand the importance of mathematical ideas and in which contexts they are useful. • Teacher shows connections between concepts and real world. • Teacher makes statements that show how the measures of central tendency are used in application, or how they can use the frequency table to organise real life information.	Similarity Contrast Fusion
		Procedural fluency	Teacher uses example set with patterns of variation to show learners how to solve a problem or answer a question. Enhances learners' knowledge of procedures and skill of performing them. • Teacher may be showing learners how to find the median, see the mode, or calculate the mean using an equation.	
		Strategic competence	Teacher encourages learners to formulate and represent mathematical problems in order to solve them. • Teacher asking learners to participate in the creation of data from which they will have questions to answer. Like	

			creating data and then calculating the measures of central tendency.	
		Adaptive reasoning	Teacher encourages learners to think logically about relationships between concepts, and think critically to make conclusions. • Asking learners to analyse data and explain what it means.	
		Productive disposition	Teacher tries to get learners to see mathematics as sensible, useful, and worthwhile. Promoting a sense of one's own efficacy. • Teacher makes use of real life examples which learners can relate to. Teacher also encourages learner participation and confidence.	

3. Learning

Proficiency in statistics observed in learners during the lesson

Learning involves acquisition of knowledge by the learners through the teachers' use of examples in a way that suits and meets the conditions for, or space of, learning.

Kilpatrick et al (2001) are of the view that mathematical proficiency is demonstrated through an integrated acquisition of all the five strands of mathematical proficiency.

In terms of the variation theory, to acquire the capabilities that may lead to mathematical proficiency, a learner must be exposed to a certain pattern of variation.

	Strands of mathematical proficiency	**Example**	**Experience** ***(evidence of gained knowledge as seen from learners' responses)***
The lived object of learning	**Observed in learner participation**		
	Conceptual understanding	Worked out Spontaneous Exercise	Through the experience of different ways to express a concept, the learners show that they comprehend the operations and relations around that concept. • The learners are seen to be able to connect new ideas to old ideas. Like being able to summarize large data in a frequency table so that they can calculate the measures of central tendency.
	Procedural fluency		Learners showing the ability to flexibly carry out procedures, for example learners can do drawings by sticking to the rules. Learners can fuse different aspects of a concept to solve a problem. • Learners' responses show knowledge of estimating results. • Learners can use a procedure to solve a problem by following steps. For example, following steps on how to calculate the mean using an equation.

	Strategic competence		Learners can formulate, represent and solve the statistical problems by using different methods of solving and representing the problems. • Learners are involved in working with practical data from real life. • They can solve statistical problems using different methods
	Adaptive reasoning		Based on the learners' exposure to different concepts, learners can logically justify and explain their thoughts. • Learners can explain the methods they used to solve problems.
	Productive disposition		Leaners developing a sense of own efficacy. Statistics seems to be sensible as learners start to participate confidently because they can relate to the concepts. • Learners show the ability to use statistical tools to solve real life problems. • Learners show a positive attitude and believe towards learning statistics.

Table 1: Recognition guidelines for framework

Conclusion

This chapter provides a description of the design and the methodology of this study. It focuses on the research settings, sampling method, data collection methods and their limitations, issues of validity and reliability of the study, the ethical considerations of the study, the analysis process followed in the study together with the analytical framework designed. The next chapter contains the data analysis of the study and results obtained from it.

CHAPTER 4: DATA ANALYSIS AND FINDINGS

Introduction

This study seeks to investigate examples teachers choose and how the teachers' use of examples facilitate (or not) mathematical proficiency in statistics. Classroom observations together with interviews of teachers A and B were analysed to address the following questions:

1. What examples do teachers use to teach statistics in Grade 10?
2. How do the examples used by the teachers when teaching statistics in class promote (or not) mathematical proficiency in learners?

The three types of examples which are the focus of this study are analysed based on objects of learning. In the intended object of learning, analysis focuses on the worked-out and exercise examples which are often prepared before the lesson and from which the teachers' intentions can be observed. The enacted and lived objects of learning focus on all three types of examples, the worked-out, spontaneous, and exercise examples from which the process of knowledge transmission and how learners experience it can be observed.

According to Marton and et al. (2004) the intended object of learning may not be lived as the teacher intends for the learners to live it (Marton et al., 2004), and this is influenced by the enacted object of learning. Analysis will also look at how the enacted object of learning influences the lived object of learning.

Data is analysed per episode, where in each episode there are two sections, namely: the intended object of learning, and the enacted and lived objects of learning. In the intended object of learning section, analysis focuses on the set of examples presented by teachers A and B and how the patterns of variation are involved in the set of examples. The example sets are presented and then analysed for each teacher. Examples in the sets are then numbered in the order in which they were presented per episode. The enacted and lived objects of learning section also starts by examining how teacher A makes use of the examples, and the way in which learners respond to the way examples are presented. This is followed by an analysis of teacher B's lessons.

Episode 1

As indicated previously, the data was subdivided into episodes. An episode starts when the focus of the lesson changes in terms of content (the object of learning). The object of learning in episode 1 was the introduction to statistics; that is, raw data and how to arrange it, and calculating the measures of central tendency. Analysis will focus on how examples were used to facilitate learners' understanding of statistics as a concept.

Teacher A's intended object of learning

The first episode was observed in two lessons, where the teacher strictly focused on arranging data and calculating the measures of central tendency. The analysis of the intended object starts by first engaging with all the examples (worked-out examples presented in class and exercise examples) which the teacher used in class to introduce statistics. In this episode, the following set of examples were presented.

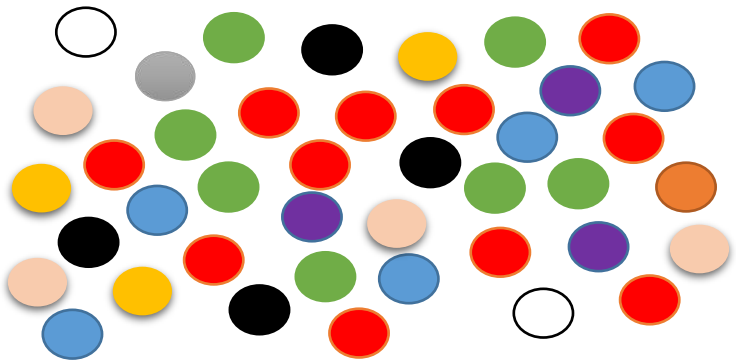
Worked out example

1. Collecting learners' shoe sizes for 10 learners: 7,4,4,6,3,5,4,5,5,5
2. Test marks out of 50: 18,10,19,35,02,49,10,10

Exercise example given in class

3. Favourite colours in Grade 10 L

Below is the information about the favourite colours in Grade 10 L.



I. What colour is least favoured in Grade 10L?
II. How many learners like the colour green?
III. Which colour do learners like the most?
IV. How many learners like this colour?

Example set A1.

In terms of similarities, the three examples are similar in the sense that they are a collection of information that is empirical and can be presented in numbers (numerical) and arranged from the smallest to the largest. That is, the information collected can be transformed from raw data to organised data from which the range can be easily calculated. These three examples are intended to assist learners to recognize raw data in statistics, and, therefore, understand the concept of organising data. Considering the context of these mathematics classrooms, it is very important that these examples be all empirical so that learners can relate to them, thus, promoting productive disposition where learners see mathematics as sensible and worth learning (Kilpatrick et al., 2001), more specifically statistics.

Examining examples 1 and 2, it is noticeable that the use of data with a few entries was to allow the introduction of statistics to be experienced by learners with ease. Examples 1 and 2 also show the varying components of the data sets, which are meant to show that the entries of a data set are not necessarily a fixed number. They could be 8, less, or even more. Another varying aspect in these examples is that example 3 is different because learners cannot just arrange the data and start calculating the range, but they must count the pictures of each type of fruit for the data to be interpreted as numerical.

Example 3 is a good example of what Courant (1981) describes as an example that can be used to allow learners to see the general through intimate experience when they come across problems that are not so immediately obvious. Example 3 also varies from the other examples as it has a larger number of entries and learners could only realise this as they were counting the number of colours. Examining the way questions are asked in example 3, one can argue that they are set to allow learners an opportunity to feel like part of the problem solving rather than working on a set of dry procedures.

As noted above, the intended object of learning looks at the teacher's intentions for using a particular set of examples using patterns of variation. The examples used in this episode have similarity and contrast, which together can facilitate the teacher's intention of helping learners generalize and understand statistical concepts.

Teacher B's intended object of learning

Episode 1 for Teacher B was observed in all three lessons, with the teacher ensuring that in all examples used, the intended object of learning is made clear and the start is always about the measures of central tendency. In this episode, the teacher makes use of practical examples

where the learners are involved in formulating the examples. Most of the worked-out and exercise examples were created in class, but clearly planned prior to the lesson as the teacher was drawing the examples from her lesson plans. Some of the examples were extracted from the Classroom Mathematics textbooks. Below is the list of examples used in the first episode.

Worked out example

1. Guessing the teacher's age in a class of 37 learners:

18, 20,20,21,21,21,21,21,22,22,22,22,22,22,22,22,23,23,

23,23,23,23,23,24,24,24,25,25,25,26,26,26,27,27,27,28,31

Exercise example given in class

2. Create your own data by throwing an object into the air and count the number of claps you can do before you catch the object.
3. Conduct a survey where you ask one of your classmates few questions that will give data from which you can calculate the mean, median, and mode.
4. Siyanda conducted a survey in a classroom of 40 learners. She asked them how many days in a week they needed to keep the books they borrow from the library. Their responses are summarised in the table below:

Days	0	1	2	3	4	5	6	7
Number of learners (frequency)	16	2	0	6	5	7	3	1

Siyanda wants to figure out the average number of days learners need to keep books from the library with them in a week.

I. What is the mode?

II. Find the median, and explain the method you used.

III. Use the following formula to calculate the mean:

$$= \frac{total\ number\ of\ days\ learners\ keep\ textbooks\ \bar{x}}{number\ of\ learners}$$

IV. Explain what the calculated mode, median and mean tell Siyanda.

Example set B1: (see Classroom Mathematics Grade 10 exercise 13.3 for original example)

The similarities in all these examples are that they are all practical, learners can relate to them, and they are all numerical data sets. Another similarity is that they all require learners to work with the measures of central tendency, which is finding the mean, median and mode. Further, all of these examples promote productive disposition (Kilpatrick et al., 2001) as they help learners gain interest in statistics and see it as sensible and worthwhile. Examples 2 and 3 are more similar because learners are creating their own data sets from which they will calculate the measures of central tendency.

In terms of contrast, Example 3 is different from the other examples because learners must be careful in their questions not to create data that is not numeric, which may not allow them to do the task. Learners have to think logically about their survey questions and how those would help them. Another strand of proficiency that may be facilitated by this question is adaptive reasoning (Kilpatrick et al., 2001) where learners can logically justify their thoughts.

Example 4, which is from a textbook has both the variant and invariant features of the rest of the example set, simultaneously. Fusion is observed on this example because it has both similar and contrasted features. It has questions that require learners to calculate the measures of central tendency like the rest of the example set, while some questions require learners to reason and logically justify their thoughts like the questions on numbers (II) and (IV). These questions are in line with the argument made by Garfield and Ahlgren (1988) that statistics has elements of reasoning, which are necessary for many other fields of study. These questions can help learners in developing their reasoning as Garfield (1995) claims that learning statistics is about using statistical language to solve statistical problems and drawing conclusions that can be supported with logical thinking.

Having examined the example sets for teachers A and B, the examples used make the intentions clear by making use of patterns of variation. It was observed that example set A1 shows more of similarity and contrast, while example set B1 shows similarity, contrast, and fusion. It is, therefore, clear that there is variation in the examples used by both teachers in the first episode as patterns of variation are observed. The use of the patterns of variation exposes learners to learning as they help them discern the critical features of the object of learning. In support, Marton et al. (2004) claim that exposing learners to a certain pattern of variation results in learners gaining some capabilities, and discerning critical features of the object of learning. The capabilities that may be attained by the learners from these example sets are how to organise raw data and use it to calculate the measures of central tendency.

The enacted and lived objects of learning

The enacted object of learning was mainly observed during the lessons. The focus was on how the teachers used examples to promote mathematical proficiency in statistics. During these observations, learners' experience of the object of learning was also observed, and, thereby, observing the lived object of learning.

Teacher A's enacted and lived object of learning

In this episode, the examples used by the teacher are similar and allow learners to relate to the concept as they are working with something they know. This facilitates conceptual understanding. The teacher's use of these examples shows that the attempt was to help learners gain conceptual understanding to see the difference between raw data and organised data (See excerpt 1 for an illustration of how the teacher used one of the examples).

Line		**Teacher/learner statements**
1.	T	Now we are going to continue with our topic, data handling. We highlighted a few things on Friday. We said data is the information. If you collect or you are given information, like we did an example last time of collecting the shoe sizes that you are wearing, and we said information like that is the raw data. We said for every raw data to be interpreted easily, it needs to be organised. Then we said the first thing you do when you are organising data… let's say for marks out of 50 *(teacher writes test marks on the board)*. Suppose these are the test marks that some of you obtained for a test. So how many learners do we have? (Teacher counts) alright 8. So, like this can you see the marks we have. So, this is the raw data which we cannot be able to interpret it easily without us organising it, so tell me what we can do, what is the first step?
2.	L1	From the lowest to highest
3.	T	Yes, so the first step to do here is to do what? Is to arrange from the what? From the lowest. To the what? To the highest. So, help me to arrange it
4.		*Learners arguing on the background whether to start with 2 or 10.*
5.	T	But learner1 said we arrange from the lowest to the highest.
6.	L2	No mam.
7.	T	What were you saying?
8	L1	Aish, I said from the lowest to the highest, so we will start with 2.

Excerpt 1.

The way in which this example is used helps learners to understand that data can be information about anything. Learners are encouraged to generalise about statistics being using information at hand. This facilitates productive disposition where learners see statistics as sensible and worthwhile. The learners' responses show that procedural fluency is being embedded. The method used to arrange data is the lived object of learning at this point, while the teacher had intended to encourage learners to relate to statistics and understand it to be a useful subject in their everyday lives. Excerpt 2 elaborates on the teacher's intention.

Interviewer: *How did you choose the examples you used in the lessons?* ***Teacher A:*** *Normally I just choose the examples with the intention to portray mathematics as something they do in their daily basis and not something that is in isolation. I chose those examples so that they can see statistics as part of human king. Not for them to see statistics as something that is strictly taught at school and far from them.*

Excerpt 2

Marton and et al. (2004) argue that the intended object of learning is not always in line with the lived object of learning. One of the critical features to be discerned from the used examples is that statistics is information about anything in life and it requires that data are organised in order to find ease in working with them.

At some point, the teacher worked together with the learners to calculate the range, mean, median and mode for example 2 where she introduced a new concept using a spontaneous example. The transcript below shows how the spontaneous example was generated.

Line		Teacher/learner statement
1.	T	Ok, thank you, now let's move on, and let's find the mode. What is the mode here?
2.	Ls	The number that appears the most.
3.	T	Is the number that appears the most…ok and then which number appears the most in our data set?
4.	Ls	10
5.	T	That's 10, let's check how many 10s do we have? (Counts) we have 3, and the other values we have one, one, one…so 10 appears the most in our data set, so we say 10 is the what?
6.	Ls	Mode…

7.	T	Mode. So, it happens sometimes that you have two values, let's say our data set was like this… (new data set: 50: 18,10,19,35,02,49,10,10,49,49) . Now if it was like this, what would be our mode in this case?
8.	Ls	49…
9.	T	Its 49?
10.	Ls	and 10
11.	T	And 10. Why do you say its 49 and 10? Because both appear the most in the data set. We have how many 10s? 3, and how many 49s? 3. Now this data set has got two modes, we say it is a bimodal set. It is a what?
12.	Ls	bimodal
13.	T	Remember we know that this word bi means… bi means?
14.	L1	2…
15.	T	Bi means 2. A bicycle has two wheels, and bisexual plant that u learn about in other subjects has a female part and a male part. So, in short, the word bi means two. So, we say this is a bimodal set because how many modes do we have in our data set? We have two

Excerpt 3.

Spontaneous example 4. Test marks out of 50: 18,10,19,35,02,49,10,10,49,49

Part of example set A1.

In line 7 of excerpt 2, the teacher makes changes to example 2, the test marks. This is now a spontaneous example that is similar to the other examples because it is of the same concept as example 2 and is a modification thereof (Zodik & Zaslavsky, 2008). It is similar to the other examples because it is still a practical example with the question still focusing on finding the mode. It only varies because it has two modes compared to the other ones; and, because of this, it is contrasted.

The teacher modified this example to enable her to introduce the concept of bimodal data. In this example, the teacher also gets an opportunity to enhance the learners' use of mathematical language as she explains what 'bi' means, and in which other cases it is used. Enhancing the learners' statistical language facilitates statistical literacy as Garfield and Ben-Zvi (2008) argue that statistical literacy is about the ability to use basic statistical tools and language with

understanding. This explanation promotes conceptual understanding as it shows the connection or the link of the prefix 'bi' to other concepts, and the mathematical meaning of the prefix in the real world. This is demonstrated in line 15 where the teacher uses a bicycle to show learners the use of 'bi' in their everyday lives.

Learner 1's response in line 14 did not come out so confidently. It could have been a response emanating from the fact that they are talking about and working with bimodal data. The teacher's response to the answer validates the learner's answer, giving them confidence about their process of learning statistics. Thus facilitating productive disposition. This is also observed in the same lesson (excerpt 4), when they were calculating the median. The teacher is seen to be trying to facilitate productive disposition by prompting learners to participate.

Line		**Teacher/learner statements**
1.	T	…Now let us find the median. What is the median?
2.	Ls	Learners mumble…
3.	T	It is the….? It is the middle number. So, what is our middle number here?
4.	L3	10 and 18
5.	T	alright, its 10 and 18 (points at the second 10)
6.	Ls	No, this one. (Pointing at the 10 before 18)
7.	T	Alright, how do we get that?
8.	Ls	You check the middle…
9.	T	Alright, 49 and 2 (crossing out largest and smallest), so in the middle we are left with how many numbers?
10.	Ls	2 numbers.
11.	T	We are left with 2 numbers. So, if we have two numbers, we will add those two numbers which means we have 10+18 and we divide by 2. So, what is 10+18
12.	Ls	28
13.	T	and divide by 2
14.	Ls	14
15.	T	14, so our median is 14. And we said you will know when you have one median and two medians. We count our data set, if it is even you have two values in the middle, and if it is odd, that's when you 5 values or 7 values then you know you have one median.

Excerpt 4.

In lines 3, 7, and 9, the teacher ensures participation by asking questions and using prompts in a way that does not put learners under pressure or make them think they do not know what they are doing. Learners seem unsure about how they could explain what a median is. This is heard in their mumbling, some of them arguing in home language but not confident enough to explain to the teacher in English. They eventually become comfortable as the teacher gives them time by working together with them. As the lesson continues, the learners' responses show that they are getting involved and are thinking carefully about their answers. One of the learners' responses ("you check the middle") in line 8 of excerpt 3 shows that learners must have had a strategy they use to quickly see (as they quickly answered).

Polya (1945) argues that in a mathematics classroom, learners must have problem solving strategies. The learners in this classroom are showing their capability to solve statistical problems using their own methods. This shows that learners have acquired strategic competence, which is a strand the teacher is also trying to facilitate as shown in line 15 excerpt 3, where she continues to give them ideas on what to do and how to know when a data set has one or two medians. The teacher also promotes procedural fluency when showing the learners how to find the middle number by crossing out numbers.

When observing teacher A's episode 1, it is observable that the main strand of mathematical proficiency being facilitated is productive disposition. The main intention is to ensure that learners see statistics as sensible and start to participate confidently because they can relate to the concepts. This is also evident from the teacher's response as reflected in excerpt 2.

The teacher's intention to allow learners the opportunity to see that statistics is part of human endeavour shows that the teacher is more focused on giving learners real life problems. Research in statistics education emphasises the importance of exposing learners to real life problems rather than letting them work with theoretical problems only (Wallmans, 1993).

The focus on the use of examples learners can relate to has also given teacher A time to address some issues around language, too. As a discipline that requires a lot of critical thinking (Batanero, 2011), statistics requires that learners understand the language in which problems are set in, and the mathematical language itself. In excerpt 5 below, the teacher explains how she addressed issues of language when learners did exercises.

Interviewer: *So on language, you gave learners worksheets. Do you think they were coping with the language in the worksheet?*

Teacher A: *Not at all, that's why I was taking rounds to further explain to them as they work.*

Interviewer: *It seemed that some of them were not even reading the questions. This is why I wanted to ask as I saw in some cases the mean is at times asked as an average. Do you think learners would understand that?*

Teacher A: *They will not even understand 'average'*

Excerpt 5.

The next section is an analysis of how teacher B made use of her examples to facilitate mathematical proficiency in statistics; that is, how the object of learning was enacted. The section also looks at the lived object of learning for learners, which is focusing how they are responding to the examples and, thus, which strands of mathematical proficiency learners are being acquired.

Teacher B's enacted and lived objects of learning

Teacher B's lessons are a combination of episode 1 and 2. Introduction to statistics started with learners guessing the teacher's age (a very practical example), which was then used to calculate the measures of central tendency. During the interview as shown below, the teacher claimed to have considered the learners when generating this example.

Interviewer: *What did you consider when choosing the example?*

Teacher B: *I thought they were interested in it because they had been asking me for some time, so I was trying to use an example of something that is of their interest.*

Excerpt 6.

Considering learners' interests means the teacher was trying to facilitate productive disposition. However, it is difficult to observe the strand of mathematical proficiency that is being facilitated using this example in excerpt 7. One can claim that the teacher calling the measures of central tendency the "triple Ms" (line 7), is a way of getting learners to remember them, but that does not assist learners in gaining any capability or discerning of critical features about the measures of central tendency.

Line		**Teacher/learner comment**
1.	T	At the end of the previous lesson I asked you to guess my age.
2.		*Writes the guessed numbers on the board.*
3.	T	Have you heard of the mean, mode and median?

4.	Ls	Yes…
5.	T	Ok! *(writes notes on the board)* Mean: add all your data up together and then divide by the number of the data points you have. Median: the middle number when your data is arranged from smallest to largest Mode: most (number that appears the most, you can have more than one mode)
6.		*Learners writing notes.*
7.	T	Ok these are the triple M's we are going to use to analyse our data…

Excerpt 7.

From written notes, the teacher asked learners to arrange the data, by arranging it in a frequency table (episode 2) instead of arranging it in ascending or descending order without putting it in a table. The teacher then worked the example out with the learners. Excerpt 8 shows a part of the transcript where they were working on the median.

Line		**Teacher/learner statements**
1.	T	Ok, so you arrange your numbers from 18 to 31 and you cross out from smallest to largest until you get to the middle. But because we have a lot of numbers, it's going to be very hard to do, and that's why it's good to get your data into a frequency table. So, class, if we add up the numbers of our frequency (teacher adds up) so we have 37 guesses, so where is the middle number going to be? Can you find out where the middle number is going to be? (learners work out)
2.	L1	we must find the half of 37
3.	T	so, 37 divided by 2
4.	Ls	18.5
5.	T	So, your middle number is going to be somewhere between your 18th and 19th number. So, we go along this side … *(teacher adds frequencies from the top),* so you know that your middle number is going to be 23 because before we at 16 now we are on 23 which means the 19th number appears in the 23rd … in this group here (points at row age 23, frequency 7). But if you have something like 1, 2,3,4,5 it's very easy to calculate the median. You just do this (cancels out) largest, smallest, largest, smallest. Our median…
6.	L2	is 3

7.	T	Is 3. And if you have 1, 2, 3,4,5,6? (Cancels out) smallest, largest, smallest, largest and now you have two numbers in the middle. So, you just take an average of these numbers. So, if instead of 37 we had a 36, you have to take the average of the two middle numbers. (Teacher writes on the board)
8.	T	Ok, so does anyone have something you can throw in the air? Like a rubber, or a small pen?
9.	Ls	Yes…

Excerpt 8.

The way in which this lesson is presented suggests that more procedural fluency is being facilitated. From line 1 to line 5 of excerpt 8, the teacher is taking learners through the process of working out the median for the data set. The method used seems to be confusing for learners even though the teacher tries to get them involved. Noticing this, the teacher quickly shows the learners what they could do if they were dealing with smaller data (line 5). She instantly generates a spontaneous example by creating a simple data set: {1, 2, 3, 4, 5}. Zodik and Zaslavsky (2008) explain that teachers generate spontaneous examples in response to what learners are asking or claiming. This helps to correct the misconceptions or to enhance the learners' understanding.

In this case, the teacher is promoting procedural fluency by exposing learners to different methods they could use to find the median. Showing them two different methods could also result in the learners' improvement of their strategic competence.

In line 7, the teacher creates another spontaneous example to show learners the difference between a data set with an even number of entries, and a data set with an odd number of entries. Showing them what to do when there are two medians in a data set. The two spontaneous examples introduced in excerpt 8 are similar because of the way they are represented and the concept they focus on (the median).

Based on the worked-out example, the teacher then gives learners a practical exercise where they have to create their own data from which they would calculate the measures of central tendency. Watson and Mason (2006) commend the approach of allowing learners to generate their own examples as they argue that mathematics is a constructive activity when learners generate their own mathematical examples. This example is similar to the worked-out example. It is practical. The questions are about the measures of central tendency and are structured in the same way. Learners became very excited and participative as they did this task (everyone

throwing objects in the air). At this point, productive disposition seemed to have been achieved. Learners felt like they were part of their mathematics learning and that mathematics made sense.

In this lesson, the purpose of the example seems to have been achieved as the teacher stated during an interview that the key thing is to try to get learners' interest and for them to feel the self-worth as the do mathematics. Below are the teacher's words:

Interviewer: *What response did you expect from the learners when using this example in statistics?* ***Teacher B****: I was expecting them to see that data collection is something they do every day other than taking an example from a textbook which is a bit far from them and the becomes a dull thing to do.*

Excerpt 9.

Teacher B's approach is greatly supported by Watson and Mason (2006) who argue that when learners generate their own examples, they investigate and relate more with their mathematical knowledge even deeper than when examples are ready made and presented by the teacher.

Looking at the way learners responded to the exercise example which is example 4 in example set B1, most learners were struggling with the questions and they were confused by the way the data had been represented. It was contrasted from the rest of the example set to promote statistical literacy by being represented differently. Garfield and Ben-Zvi (2008) argue that the ability to understand statistical symbols, and to recognise and interpret data represented in different ways is statistical literacy.

As already noted, the pattern of variation in this example is fusion. The questions still require learners to calculate the measures of central tendency while they also think critically and show logic. Further, the example is presented differently.

Some learners saw two data sets without considering the 'frequency' written in brackets as an explanation of what the number of learners gives them. From observation, the teacher had to go around showing learners the simplest way they could deal with the information on the table.

Learners had serious difficulty dealing with this problem. They could not explain what the mean, median and mode meant for Siyanda. Most of them simply defined what they meant in terms of what they instruct you to do (explaining how to calculate the mean). The lived object of learning is how to calculate the measures of central tendency, and not what they mean in

application. This is because learners could not understand the statistical language and therefore could not use the statistical tools (Garfield and Ben-Zvi, 2008).

In observing episode 1 for teacher A and teacher B, the patterns of variation were observed in their examples to assist learners to discern the critical features of the object of learning. This concerned understanding the measures of central tendency, working them out and what they mean. This was the teachers' intention. Having observed the enacted and lived object of learning in this episode, one can easily agree with Marton et al. (2004) that the intended object of learning is not always the lived object of learning. Garfield (1995) supports this argument by claiming that learners do not learn what the teacher intends for them to learn.

In this episode, teacher B's examples are capable of helping learners understand statistical concepts. The problem was on the enacted object of learning, which then compromises the lived object of learning. In excerpt 9, the teacher claims to have selected practical examples with the intention to keep learners' interest and show them that mathematics is something they do every day. The intention here was to facilitate productive disposition. However, learners are seen to be acquiring procedural fluency more than anything else.

Teacher A tries to engage learners to make them feel part of problem solving as suggested by Garfield and Ahlgren (1988) that the teaching of statistics must be improved and focused on problem solving and exposing learners to application problems. Learners are seen to be interested and engaged in the examples with the teacher.

Episode 1 focused on introduction to statistics and the measures of central tendency. This episode was observed with the intention to find out how learners are introduced to statistics, and how the examples used in the lessons promote mathematical proficiency. The next section contains the analysis of episode 2.

Episode 2

The object of learning in episode 2 is about representation of data, drawing the frequency table, graphs, and using symbols and equations for statistical calculations. Again, analysis will be based on the types of examples used by the teachers and the patterns of variation seen in the examples. Classroom interactions will be analysed to see how teachers use the examples to facilitate mathematical proficiency.

Teacher A's intended object of learning

Moving from episode 1 to episode 2, the teacher ensures that learners see that they are still on the same topic and working with the same things, except that now, the information they have been working with in episode 1 may be, or needs to be, presented in different ways. To ensure that learners experience the move in a way they still relate to, the teacher uses some of the examples used in episode 1. The teacher informs learners what they are doing in that day's lesson is a way of making the intended object of learning clear to the learners (Runesson, 2005). The set of examples used in episode 2 is presented below.

Worked out example

1. Test marks out of 50: 18, 10,19,35,02,49,10,10. Draw a frequency table
2. The number of passengers transported by an airport shuttle bus on each of its 25 trips are as follows: 1,8,2,6,8,7,4,3,5,6,1,6,3,4,6,6,4,7,5,3,4,6,7,5,5
   i. Is this data discrete or continuous? ii. Draw a frequency table to represent this information
3. The ages of patients who visited the Alexandra clinic on Thursday are listed. 4,7,10,14,17,20,24,29,49,50,51,53,57,57,58,77,84,91,93 Draw a stem and leaf representing the ages of patients.

Exercise example

4. A sandwich survey was conducted in Grade 10 L. the following was picked up:

Ham – 14

Polonies – 10

Peanut butter – 5

Eggs mayo – 1

Chicken mayo – 5

Jam and butter – 8

a) Use this information to fill in the table below.

Type of sandwich	Frequency	Tally mark
ham		
Polonies		
Peanut butter		
Egg mayo		
Chicken mayo		

Jam and butter		

b) Draw a neatly labelled bar graph represe information.

c) What questions would you ask about thi information?

Example set A2.

These examples are all practical for learners in this classroom. This helps the focus to be more on the mathematics and less on what the examples are about. There are notable patterns of variation in these examples. All of them are similar because they are presented numerically, but mainly, examples 1, 2, and 4, which require the drawing of the frequency table and tally marks. The teacher's intention is to keep this feature invariant across the examples to enable learners to generalise and understand what a frequency table is and how it is drawn in different situations.

Example 3 and 4 have varying features. Example 3 is about drawing the stem and leaf, and example 4 has a question that requires learners to draw a bar graph. There is, too, a question that requires learners to develop their own questions about the information they are working with. These two examples are contrasted from the rest of the example set. Asking learners to develop their own questions in example 4, encourages learners to generate their own examples which Watson and Mason (2006) suggest is the best way to get learners to enquire more about a concept.

Examples 3 and 4 bring attention to different features of the object of learning. From this, learners can generalise and see that data can be represented in different ways (stem and leaf, graphs, and etc.) and not just in a frequency table. In support, Adler and Ronda (2015) argue that generality is triggered by exposing learners to contrasting examples, which brings attention to the different features of a concept.

Asking learners to generate their questions in example 4.c) gets learners to make connections between what they are working with now and what they have been learning in the previous lessons. It is clear that the teacher's intention for learners to see that the measures of central tendency are is still very much at play here; that is, the teacher's intention is to facilitate conceptual understanding where learners are able to connect new ideas to previously learned concepts.

Fusion is absent in this set of examples because, according to Adler and Ronda (2015), fusion is simultaneously having the variant and invariant features of the examples sets in one of the examples. There is no example that requires the drawing of a frequency table, bar graph, stem and leaf, or a question that requires logical thinking all at the same time.

The use of these examples to facilitate mathematical proficiency in statistics will be analysed as part of the enacted and lived objects of learning. Next, is the analysis of the intended object of learning for teacher B.

Teacher B's intended object of learning

As already explained in episode 1, teacher B's lessons were a combination of the two episodes. In observing her lessons, some aspects of episode 1 and 2 were observed in each and every lesson. In lesson 1, the teacher introduced statistics and also showed learners how to calculate the measures of central tendency using a frequency table. Although the teacher makes use of some of the examples from episode 1 to do representations of data, analysis will focus on the way questions are asked in those examples to decipher representation. The set of examples used is shown below.

Worked out example

1. Guessing the teacher's age in a class of 37 learners:

18, 20,20,21,21,21,21,21,22,22,22,22,22,22,22,22,23,23,
23,23,23,23,23,24,24,24,25,25,25,26,26,26,27,27,27,28,31

a) Draw a frequency table with tally marks to represent this information. b) Calculate the mean.

Exercise example

2. Conduct a survey where you ask one of your classmates few questions that will give you data from which you can calculate the mean, median, and mode. Draw a frequency table.
3. Ms Thula conducted a survey in her classroom to find out about the shoe sizes of learners in her Grade 10 class. The information is presented in the table below.

Shoe size	2	3	4	5	6	7
Number of learners (tally)						
Frequency	0	5	8	11	9	2

a) Use tally marks to fill in the number of learners.

b) Explain whether this data is discrete or continuous.

c) Use this information to calculate the mean, median and mode, and explain what each one means about the shoe sizes.

Example set B2.

The set of examples used by teacher B are very similar, they all require learners to work with the frequency and drawing the tally marks. The different ways in which the questions are presented is a strategy to help learners to answer different questions about the frequency table for them to be able to draw it or to obtain information from it.

A notable contrast from these examples is observed in example 3, where learners are asked to explain whether the presented data is concrete or discrete. This is a different question from the rest. Learners must know and think deeper about the difference. In example 3, learners are also asked to calculate the measures of central tendency from the information provided. This could be the teacher's way of getting learners to make connections and understand that it is still data as they were presented in the previous lessons.

Learners in teacher B's class seemed to have been struggling with a similar question in episode 1, where they were working with data presented in a table. However, the teacher brought up a

question with the same structure, perhaps to see if learners would then be able to answer the questions. This approach is in line with Garfield's (1995) suggestion that to facilitate understanding in statistics, teachers must use more activities/examples. The teachers is taking advantage of similarity as a pattern of variation across the two episodes to help learners discern the critical features.

After all, Marton et al. (2004) argue that for learners to gain certain capabilities, that is, to discern the critical features of the object of learning, they must be exposed to a certain pattern of variation. In this episode, the teacher relies more on similarity to help learners discern the generality of the statistical concepts. Adler and Ronda (2015) suggest that similarity alone, does not get the learners' attention of the boundaries around a concept. The bit that could help learners learn of a few boundaries in this example set is in example 2 of the exercise examples. The teacher gets learners involved in their own data collection.

There is not much of contrast used in the examples. Adler and Ronda (2015) recommend that contrast be used because it captures learners' attention to the different features of the object of learning. It is, therefore, clear that the teacher's intention was to rely more on the invariant features of the object of learning by using similar examples to help learners generalize about statistical concepts.

The next section of this episode is the analysis of the enacted and lived objects of learning for teacher A and teacher B. The focus is on how the presented examples were used to help learners understand the different representations of data and the use of symbols in statistical calculations.

The enacted and lived object of learning

In this analysis, the focus is on how the teachers enact the object of learning through the use of examples with patterns of variation to promote mathematical proficiency in statistics. The way representation and the use of symbols in statistics are communicated to learners will be analysed together with how learners respond verbally and using the written work on the activities.

Teacher A's enacted and lived objects of learning.

The teacher takes advantage of the comfort learners are showing with working on this data. She introduces the concept of discrete and continuous data. In line 7 and 8 of excerpt 10, the teacher

explains what discrete and continuous data are through the use of practical examples learners can relate to. In this explanation, the teacher is facilitating productive disposition, about which Kilpatrick et al. (2001) argue that learners with productive disposition are able to see the usefulness of mathematics in their lives.

Line		Teacher/learners statement
1.	T	We are still on data handling right? We are on how to represent our data so that it can easily be interpreted. Which involves the tally marks. Now let's look at this example here, alright lets listen you will have time to write. What we are given here says the number of passengers transported by an airport shuttle bus on each of its last 25 trips were as follows… so these are the number of passengers in the shuttle. Alright we are required to draw a frequency table to represent this right. And we said if you are having a raw data like this. It is difficult to interpret it so we need to organise it. So where do we start when we organising data?
2.	Ls	We arrange them.
3.	T	We organise them in the ascending order.
4.		*Teacher works with learners in arranging the data*
5.	T	So it's good to have a pencil when you do this So that you cancel every value as you arrange.
6.		*teacher works together with learners in arranging the data*
7.	T	Is this data discrete or continuous? What is the difference between discrete data and continuous data? Alright, we have not talked about it right? Let me explain. Ok it happens that when you are dealing with information sometimes… for example here you were counting the number of people, maybe you measure the length or the height. Say I bring measuring tapes and tell you to work in pairs and measure each other's height, or say I bring a scale for us to see the fattest learner in class, we get on it to measure our weight. So information like that whereby we are measuring, we call it the continuous data, as long as there is measurements.

8.	T	Teacher: ok, so now the discrete data is when you can count whatever you are dealing with, as long as that information is countable alright. For example here, we are talking about the number of passengers, this says that we can count the passengers. We are not measuring, so this information is the discrete data because we are able to count, but when we are measuring, we can call it the continuous data.

Using this example may help learners to generalise as the teacher continues to show learners how to do a frequency table after having used the test marks example to practice and remind them what a frequency table looks like and how to draw tally marks. These examples are similar. The invariant feature of the object of learning here is about representing data in a frequency table. Procedural fluency may be attained as Kilpatrick et al. (2001) assert that it is about computational procedures. It is very important for learners to be efficient in them so that they can be used accurately.

Kilpatrick et al. (2001) argue further that to be efficient and accurate, learners must be exposed to a lot of practice, which will help them maintain fluency. The teacher helps learners practice by taking them through the process of working out the procedure and using as many similar examples as possible. Her explanation of the number of examples she used can be seen in excerpt 11 below.

Interviewer: *how do you decide on the number of examples to do in class?*

Teacher A: *It depends on their response. Because if they struggle to understand from one example, it means I have to check/ generate another example, so if they struggle I have to bring it down, so the number of examples is influenced by their response because sometimes I can go there and just do one example, if they understand then its fine.*

Excerpt 11.

Teacher A's justification is that the number of examples she uses in class is more influenced by the way learners respond to each example used. Bills et al. (2006) notes that there are great benefits when the teacher works out an example to solve a problem along with the learners using prompts and questions. The teacher's use of prompts and questions when working out an example gives her an idea of how learners are experiencing the object of learning. Excerpt 12 below shows the teacher working out how to draw the frequency table together with the learners.

Line		Teacher/ learner statement
1.	T	So let's name this one, what can we say, is this the discrete data or continuous?
2.	Ls	discrete
3.	T	It's discrete because we are able to count the passengers we are not measuring ok? Right, now let's go to number 2. Number 2 says put this information on the frequency table. Ok, so we know that when we are doing a frequency table, we have got some columns…
4.	Ls	number of passengers
5.	T	Ok, and the next column?
6.		Frequency…
7.	T	It will be frequency. Ok remember we said we are moving step by step as we started from grade 9 whereby our frequency table involved tally marks. So now we are in grade…
8.		*Teacher and learners: 10*
9.	T	So now this one, we start here this passengers column here, let's say its x, and we know that this one for frequency, we know the symbol will be f for frequency. So in the next column we will have f(x), like you have 2x, so what does 2x mean? Are we adding or multiplying or dividing?
10.		*Learners mumbling…*
11.	T	We are multiplying, which is 2 times x. so if we have 2a, it would be as times a. can you see? Now if it is f(x) it means?
12.	Ls	f times x.
13.	T	So it means in this column now we will be multiplying the frequency by our x. ok? Right. Remember when we are doing this graphical representation we are doing a sort of a summary of the information so that everyone can interpret it, can easily read it right.

Excerpt 12.

In line 1, the teacher asks learners to use their knowledge of discrete and continuous data on the data they are working with. Learners respond positively as they have come to know the difference. The teacher then takes learners through the process of drawing a frequency table step by step, and introduces the use of symbols. Doing this facilitates strategic competence which Kilpatrick et al. (2001) assert is the ability to represent problems mathematically in order

to solve them. Using symbols in the frequency table is a mathematical way of representing the data, which may help when solving problems related to the data.

In line 9, the teacher explains the process of substituting the headings in a frequency table with symbols. The teacher also explains what f(x) means and what they will be doing in that column. By doing this, the teacher is facilitating strategic competence using the manner in which this example is contrasted. She is helping learners represent a problem mathematically in order to be able to use mathematics to solve the problem (Kilpatrick et al., 2001). In terms of variation theory learning is the change in the way something is seen and experienced (Runesson, 2005).

Learners mumble in the background to show discomfort in this new way of drawing the frequency table. The teacher then tries to get them comfortable by simply repeating what she had been saying. Eventually, learners allow the teacher to facilitate procedural fluency, and they started responding well to the procedure. The teacher also tries to explain why they are now arranging the data in a frequency table as seen in line 13. The attempt is to keep learners seeing mathematics as useful.

The teacher then shows learners how the frequency table is used to do mathematical calculations by keeping on referring to examples they had done previously when they were working with the measures of central tendency. The teacher is seen to be trying to get learners to gain conceptual understanding, which is the learners' ability to make connections between mathematical concepts (Kilpatrick et al., 2001). This is observed when the teacher reminds learners that what they were doing was not new as they also had to count the number of entries when they were working with the measures of central tendency (line 1 and 3 excerpt 13).

Line		**Teacher/learner statement**
1.	T	Alright. Like in the previous case when we were dealing with colours we had to count the total. So we are going to do the same thing here. We will add all our frequencies and they will give us a total. So what is the total here? Remember we must get the same total as the total given in our information which we are getting 25, and we are told there are 25 trips can you see?
2.	Ls	Yes.
3.	T	Right. Now let's move to the next column, even here we are going to add, and remember when we add, the answer we are going to get when we are adding is called the sum. Like for example, what is 20+4?

4.	Ls	24
5.	T	We say this 24 is the sum. The sum of 20 and 4. So if we say 20 times 4, what is 20 times 4?
6.	Ls	80
7.	T	Its 80. So the answer we get when we are multiplying is called the…
8..	T	The product. Ok, let me leave it there. So coming back here, we need to write the sum of all these values and these are the values we obtained when we were multiplying the frequency by the data item. Now we are going to use a mathematical symbol, remember I told you we are moving higher with our mathematics. Now this is the symbol (draws a sigma notation) for summation. Summation means the total that we added. So this is the summation of what (writes in symbols) it's the summation of f times x. Isn't it that we have f(x) at the top of this column? That's how we are going to differentiate between this total and that total (points at symbols). So we need to know this is the total for frequencies, and this is the total for f(x). They are both called summations, and
		to get the summation for f(x) we are going to add all the values for this column. Is that clear good people?
9..	Ls	yes
10.	T	(Works with learners to add the values of f(x) so it means the summation of f times x is 122. This is also a summation, but what summation is this for?

Excerpt 13.

From line 3, the teacher takes time to take learners through mathematical language. The teacher explains to learners what 'sum' means. In the school context, this is very important as the teacher is paving her way to explain what a 'summation' is. The teacher is using this example to inculcate conceptual understanding, which is mainly about comprehending a mathematical idea (Kilpatrick et al., 2001).

Following the introduction of the use of symbols in a frequency table, the teacher shows learners how to calculate the mean using the information they had gathered. This helps the teacher to introduce the equation used to calculate the mean. She then adds a spontaneous example:

Spontaneous example:

5. Calculate the mean number of passengers using this formula:

$$\bar{x} = \frac{\sum f(x)}{n}$$

Part of example set A2.

This example is similar to the examples used in episode 1. It requires that learners calculate the mean. The teacher then takes learners through the example and helps them to substitute the values which they had already calculated from the frequency table. Strategic competence and procedural fluency are being facilitated by this example.

Following this example, the teacher des a worked out example on how to draw a stem and leaf diagram. In this example, the teacher is seen to be attempting to ensure that learners understand the idea behind the way numbers are represented in a stem and leaf diagram. See excerpt 14 below for an overview of how the teacher used example 3 of the worked-out examples to get leaners to learn how to draw a stem and leaf diagram.

Line		Teacher/learner statement
1.	T	Right we are given this information which represents the number of patients who visited the Alexandra clinic on a particular day. So this is the information on the aged of the patients who which were observed in Alexandra clinic, let's say on Thursday yesterday. So we have to arrange this information, we could use a frequency table or a bar graph, but now we are talking about the stem and leaf. I believe from the previous grades down there, you know that basically in maths we are dealing with these digits, maths is all about these digits. Zero to…?
2.	Ls	9
3.	T	And all the other numbers come from these, zero to nine. We can check every number is made up these digits. So as u remember, 0,1,2,3,4,5,6,7,8,9 are the units, they are single units. Now if we manipulate these numbers, for example we come across a number which is 10, it is made up of two digits, what are those digits?
4.	Ls	1 and zero
5.	T	1 and 0, let's look at 56, how many digits form 56?
6.	Ls	2… 5 and 6

7.	T	alright lets read the two numbers (points and 10 and 56, then learners read them out) ok as you say it 'fifty six' so when you listen to yourselves you must realise you are calling out two numbers '50' and '6' in this case, each of these numbers have their own roles or value. The 6 represents 6 and the 5 represents 50. Do you see this good people?
8.	Ls	yes
9.	T	let's look at this one (writes 179 on the board and asks learners to read it out)
10.	Ls	one hundred and seventy nine
11.	T	(Emphasizes the answer), now let's look at the digits that make up this number, how many digits are there?
12.	Ls	three
13.	T	Yes there are three digits, its 1, 7, and 9. Can we look at the value of each number? We said one hundred and seventy nine. So 1 here represents…
14.	Ls	hundred
15.	T	And 7 for 70 shows that we are counting in 10s here, 10, 20, 30… and then we come to 9, it means here we are counting in what?
16.	Ls	In ones…
17.	T	so we said 1,2,3,4,5,6,7,8,9 so we can now explain this number and say, 9 represents the units or the ones, and 7 represents the 10, what is seven tens
18.	Ls	seventy
19.	T	So the value here is seventy. Then we come to 1, this one here represents the what?

Excerpt 14.

In line 3, the teacher emphasises how all numbers are made up of the digits from 0 to 9. Here, the teacher is facilitating adaptive reasoning, enhancing the learners' capability for logical reflections and explanations (Kilpatrick et al., 2001). The teacher is also helping learners to discern the critical features of the object of learning, which is to recognise the different ways in which statistical data can be represented. This example is contrasted from the rest of the example sets because the teacher wants to ensure that different aspects of the object of learning, (which is the representation of data) are paid attention to by the learners. Below is excerpt 15, which shows the teachers working out the example on how to do the stem and leaf diagram.

Line		Teacher/learner statement
1.	T	So in this stem and leaf that we are going to do, we separate the ones, and all the numbers will represent the stem, and the ones will represent the leaf or the leaves because it's going to be many of them. Then this one the tens will represent the stem. So it means we come here and… remember everything we do must be done in order. So we have to check on our information, what is the smallest number we have here?
2.	Ls	4,
3.	T	Right, is 4 having a stem? Remember the stem is the 10s, is there any tens in this number?
4.	Ls	no
5.	T	Now because 4 has no stem. We do know that zero represents empty right?
6.	Ls	yes
7.	T	So we going to write zero to show that this number does not have a stem. So which other number does not have a stem?
8.	Ls	seven
9.	T	Right, so we going to write seven on the leaf because it does not have a stem. Remember we put the number on the leaf, but in order as in ascending order.
		Now in the same way we continue to check, do we have any numbers which have a stem?
10.	Ls	Yes…
11.	T	So in order, what is the smallest stem?
12.	Ls	its 10
13.	T	So it's 10, then on the stem, we write the digit which represents the tens, which is 1. Then check all the values which have a stem that is one. So which numbers are those?

Excerpt 15.

As the teacher makes the drawing on the board, she explains how the numbers will be separated to make the stem and the leaf. She uses prompts and questions to trigger learners' interests and focus. In lines 3-9, the teacher facilitates conceptual understanding as she attempts to get learners to make connections between previously learned concepts and how they apply on a stem and leaf. In line 9, the teacher asks learners a question to get them involved, and the

learners' response shows that they have not grasped the concept yet. The answer in line 12 should have been 1, instead of 10. This might mean that learners see all the numbers as the stems. The teacher tries to clarify in line 13.

What is observed in excerpt 16 line 10 proves what Marton et al. (2004) argue that the intended object of learning does not become the lived in most cases. This is the reason for many misconceptions that arise in mathematics classrooms. Lucky for this teacher, some learners asked a few questions to clarify their misconceptions as shown in excerpt 16.

Line		Teacher/ learner statement
1.	T	Yes 10…. So what is the stem?
2.	L1	it's 10
3.	L2	it's 1
4.	T	So how many units have we got here?
5.	L1	there is 1
6.	L2	there is 2
7.	T	aaaah how so?
8.	Ls	there is 1 and its 1
9.	L1	Ma'am you said when there is zero it means nothing as in nothing.
10.	T	Ok so do you want to tell me that if zero means nothing so this 1 represents the units?
11.	L1	Ma'am why do you have 1 on the stem?
12.	T	The 1 here stands for the stem because it represents the tens. So how many units are there in this number?
13.	L1	two
14.	L1	1
15.	T	What is the unit here?
16.	L2	zero
17.	Ls	one
18.	L3	there are no units (learners arguing and showing confusion)
19.	T	Ok let's look at this number (points at number 12). What is this number?
20.	Ls	12
21.	T	Right its 12. Now which digit here represents the units or the ones

22.	Ls	2
23.	T	And what does this 1 here represent?
24.	Ls	tens
25.	T	The tens right? How many tens
26.	Ls	one

Excerpt 16.

The discussion presented in excerpt 16 shows some of the misconceptions arose during the explanation of the stem and leaf for a single digit in excerpt 16. Learner 2 in line 6 seems to be misunderstanding the difference between the number of units and the number of digits. In line 9, L1 seems to be justifying their misconception. In excerpt 15, 'zero means nothing' implied that a zero before a number greater than zero (without a comma) does not change value of that number, but a zero after any other number greater than zero changes the value of that number. Learners seemed to have taken this to literally mean nothing as in line 18 learner 3 argues that there are no units because zero is seen as nothing.

Nesher (1987) upholds the notion of misconceptions arising from what was earlier learned and adds that they are not created illogically. It is, therefore, the teacher's responsibility to make use of these misconceptions to enhance learning. The teacher uses a different number as an example to get learners to understand. However, she does not address the 'zero means nothing' problem. At this point, the teacher is focusing on promoting procedural fluency by putting more emphasis on the computational skills; that is, how to separate a number into units and tens, but, without clarifying concepts.

The focus turns out to be more on procedure as learners seemed to be more concerned about it, they started asking questions that clarify what they have to do in different situations. So, they were enquiring more about the 'how' and less about 'what' and 'why'. The few questions justifying this argument are seen in the following excerpt.

Line		Teacher/learner statement
1.	L1	Ma'am what if we have a number that has three digits like 100?
2.	T	Ok thank you so much, so if we had a patient which was about 103, what do you think we would do?
3.	L1	3 was going to be the leaf
4.	T	Yes, 3 was going to be a leaf, and then what would we have on the stem?

5.	Ls	10
6.	T	Thank you very much. Why do we say 10? If we are counting in tens, 10 tens would give us what?
7.	Ls	100
8.	T	great, so every time, we just separate the units and put them on the leaves, the rest goes to the stem
9.	L2	So, ma'am if it's a thousand what do we do?
10.	T	Yes that would be Abraham's age (loughs). So now what we would do is to identify the unit which is the last digit and is?
11.	Ls	zero
12.	T	Then we put that on the leaves, now we are left with?
13.	Ls	100

Excerpt 17.

As much as procedural fluency is important, the learners are also asking impractical questions which means they are disregarding the reality and practicality of the mathematical concepts they are working with. In line 9, learner 2 asks what would happen if they had 1000 on the data. This shows that the learner might have forgotten that they were working with real information about people's ages. The teacher point this out jokingly in line 10, but continues to address the question to ensure that misconceptions are avoided. The excerpt below shows what the teacher had to say regarding questions from these learners on a follow up question from the interview.

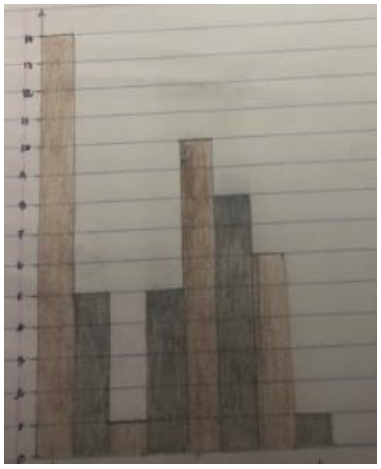
Interviewer: *About the questions asked on the stem and leaf, did you expect them? For instance you did the stem up to 9, and then some learner asked what would happen if it were a hundred and something. Did you expect it from the class?*
Teacher A: *I never expected that question from that class.*

Excerpt 18.

As much as the questions seemed to be more focused on the procedures, it seems, according to the teacher's response that this was a great response from the learners as they were showing interest. This was an achievement for the teacher as Moore (1993) suggest that teaching statistics is mainly about creating an environment for learning by setting tasks and encouraging learners to express their thinking and conclusions.

Encouraging learners to express their thinking and understanding of representation of data even from previous Grades, the teacher gave out an exercise example (example 4 of example set A2)

where learners were required to fill in a frequency table, draw a bar graph, and formulate their own questions regarding the information they are given. Learners could easily fill in the frequency table and tally marks, but drawing the bar graph was a bit of challenge to some learners. Some learners drew histograms, while some did not know where to put the headings and the variables. The picture below shows one of the learners' written response.



Learners also had a problem with creating the questions, with some learners only writing one or two questions which are very basic. One common question was 'how did you collect the data'. Learners did not realise they could ask questions about the measures of central tendency on this question. Again, the lived object of learning does not match the intended. This question could serve the purpose of facilitating adaptive reasoning and strategic competence to develop the ability to formulate mathematical problems and show logical thinking.

Having observed episode 2 of teacher A, one realises that there are a few challenges in trying to facilitate the five strands of mathematical proficiency when teaching the representation of data. One strand that gets more attention is procedural fluency. This is understandable because representing data is a more routine and method based with many instructions to follow and little to conceptualise other than the actual data at hand. The next section shows how teacher B makes use of her examples to promote proficiency.

Teacher B's enacted and lived object of learning

As already stated, teacher B's lessons all have some bits of data representation. The frequency table is used in every lesson to teach how to calculate the measures of central tendency. The teacher's worked out examples are similar in the sense that they require learners to draw a frequency table and do calculations using information in it.

Teacher B employs similarity as a pattern of variation by ensuring that all questions asked require learners to do the same thing (use frequency table to calculate the measures of central tendency). This is to help learners generalise statistical concepts.

Chick and Pierce (2008) argue that it is important for the teacher to have appropriate pedagogical knowledge when teaching statistics. This implies that it is important to have knowledge of content and of learners in the classroom (Shulman, 1987). The knowledge of learners assists in working with their prior knowledge to translate into new knowledge. The teacher gave learners an exercise example where learners were supposed to conduct a survey in the classroom asking questions that would give data they would present in a frequency table. Learners started moving around going from one peer to another. Some appeared unsure about what they were supposed to do. Excerpts 19 shows the teacher trying to give learners ideas.

Line		**Teacher/learner statement**
1.	T	Ok so a lot of the things that you need to include in your survey is like maybe you can have name, boy, girl (writes on the board) and like you can have your questions. Something like shoe size, and you can have different boxes like… What are your shoe sizes
2.	Ls	4…
3.	T	Ok 4, 5, 6, 7… then you can ask something like height, type of music, what you like doing in your free time, and you can ask your classmates. Do you understand?
4.	Ls	Yes!
5.	T	Ok so can I wipe this off?
6.	Ls	No!
7.	T	(Waits) you can do this in groups or pairs. (walks around praising and helping learners)

Excerpt 19.

The teacher is seen to be giving learners a clue on what they can work with on this survey. She also walks around encouraging learners and correcting them as they make mistakes. The teacher's use of this exercise promotes strategic competence, as learners learn to formulate their own problems, and represent them mathematically to solve them. Productive disposition is also observed as learners are encouraged to believe in diligence and own efficacy. (Kilpatrick et al., 2001)

After the survey exercise, the teacher then gives learners an exercise example which is about shoe sizes. In this exercise, learners have to fill in a frequency table and answer questions about the data. One of the questions requires learners to figure out if the data is discrete or concrete.

The teacher has not been observed explaining what these two terms mean. It may be that she is assuming that this was covered in the previous grades, although this may cause frustrations to some other learners leading them to develop a negative attitude. This is where the importance of the connection between the enacted and lived objects of learning is revealed. The way in which the teacher transmits knowledge of a concept does affect learners' understanding of a concept. Lastly, the teacher introduces learners to the use of an equation to calculate the mean. She makes use of a spontaneous example of a data set shown below:

Spontaneous example
1, 10, 12, 16, 5, 7

Part of example set B2.

The teacher makes this spontaneous example similar to the rest of the examples by ensuring that she works it out the same way they have been doing the rest of the examples; that is, drawing the frequency table and then calculating the measures of central tendency. The example is contrasted because it requires the use of an equation with symbols to calculate the mean. The teacher shows learners that the total number of data entries can be named, n, and the actual values (entries) can be named f(x). This did not seem to be so clear to the learners. But, the teacher quickly moves on to adding more symbols to the equation (sigma notation) and tells them it means the sum. Again, some learners may not have understood this completely. The teacher focuses more on facilitating procedural fluency and strategic competence because attention is mostly given to the different methods to calculate the mean.

See excerpt 20.

Line		**Teacher/learner statement**
1.	T	So this table makes it easier for you to calculate your mean mode and median.so if we count these numbers (adds the total of numbers and gets 6). So I am going to call this n, n=6, which means what we are calculating on mean, we going to divide by 6 because that's the number of data points that we have and we can also add up these numbers, so 1+10 =11, 11+12=23,23+16=39, 39+5= 44, 44+7=51. So while you are calculating it will be helpful if you do that. From here…
2.	Ls	51 divide by 6.
3.	T	So if this is f(x), we can call this the sum of f(x) (writes sigma notation) so this right here means the sum. And then the mode?
4	L1	The number that appears the most.

Excerpt 20.

Following the way the teacher uses the examples and the way learners respond to the use of examples, the teacher was asked whether learners' responses affected her next example choice. This is what she had to say:

Interviewer: *Do the ways in which learners respond to these examples influence your next choice of examples?* ***Teacher B****: Yes, I think because I teach Grade 9s as well, who are less enthusiastic. So then I give them something that suits them I guess different things work with different learners. So I will add some action.*

Excerpt 21.

The teacher's response indicates her awareness of learner diversity and that attention must be given to learners with various needs in the classroom. What makes this difficult for teacher B is that she uses the telling approach which Moore (1993) is not the best approach to be used in the classroom to enhance statistical literacy. The collaborative approach by Garfield & BenZvi, (2008) could have been useful here to promote statistical literacy. In this episode, teacher B focused mainly of facilitating procedural fluency, simply showing methods with scant explanation.

In conclusion, episode 2 shows teachers making use of a lot of similarity and contrast with the intention of helping learners see statistical concepts for what they are and what they are not. It was also observed in many cases that the teachers' intentions about trying to get learners to learn a certain concept were not always matched with what learners learned. The next section provides an overview of the findings from the data analysis.

Findings.

The examples used by teachers A and B employ patterns of variation, which make the intended object of learning easy to observe. . Fusion was not seen a lot in the example sets while contrast and similarity were used together to help learners generalise about statistical concepts. Marton et al., (2004) argue that exposing learners to a certain pattern of variation facilitates learning.

The ease with which learners learn is also affected by the sequence of the examples in an episode. The sequence seen in example set A1 shows that the teacher chose to start with simple examples with straightforward questions and later used examples that require the learners' thinking. The same approach can be observed in example set A2. Example set B1 starts with a big data set, which would make it a bit tedious to calculate the measures of central tendency

without organising it in a frequency table. The way in which example 1 was worked out in this data set already made the introduction of statistics to be procedural, with limited explanation of concepts. The same thing applies for example set B2.

It should be noted that the order of the examples in an example set do not necessarily affect the way in which the patterns of variation can be used. The order affects the way in which the object of learning is enacted and thereby affecting the lived object of learning, which is the way learners experience a concept. This is seen in the way teacher A makes use of her example set. In her step by step approach, the ease in the first examples allows the teacher to quickly move on to the next example sets. Teacher B faces some challenges in getting learners to work with big data in the beginning. This was observed when learners struggled to find the median using the frequency table. The order of the examples also affects learners' beliefs and attitude.

The enacted object of learning plays a major role facilitating the strands of mathematical proficiency in statistics. The way in which the teacher explains a concept has an effect in what learners understand about the concept. This is also affected by other factors such as the language of instruction. When asked how issues around language in the classroom are handled, teacher A admitted that language is a serious barrier in this classroom for her and for the learners. Firstly, they do not share the same home language. Secondly, learners also struggle with the understanding of mathematical terms that the teacher has to explain in English, which some of them, do not fully understand. This was also the case for teacher B. The teacher's explanation is shown on the excerpt below.

Interviewer: *How did you choose words in your use of examples so that learners understand the language used?*

Teacher A: *sometimes it's difficult for me because many a times, mathematics has got its own special language but sometimes we can code switch, but in a scenario like this one their language is a barrier to me because I cannot translate into any of their languages. So sometimes I just must bypass and leave a word like that because I can't explain it. If we were on the same page with the language (having the same home language), I would code switch better in their own language and then maybe they would be able to transfer back to English. Honestly, language is a barrier, I am experiencing it here in Alexandra and its better now as time goes on. But initially I would have a learner asking a question in their home language and I would hear nothing. Language in teaching and learning is a serious barrier.*

Excerpt 22.

The teacher's communication about how language interferes with the teaching and learning brings attention close to the way examples play a role in bridging the way between content and language. As Bills et al. (2006) affirms, examples are a very important device used to communicate and illustrate during the teaching and learning of mathematics. Through the use of examples, the teachers are able to facilitate the strands of mathematical proficiency.

Data analysis shows that in most cases, the teachers claim to intend to help learners to relate to mathematics; that is, to facilitate productive disposition, but out of this, learners focus on procedural understanding. For different reasons, learners get the definitions and procedures out of it. Kilpatrick et al. (2001) argue that a learner is said to be mathematically proficient when they have attained all five strands of mathematical proficiency. Not all strands of mathematical proficiency were given a chance in the two teachers' classrooms. Adaptive reasoning lacked the most.

In conclusion, this chapter presented the analysis of the intended object of learning, looking at how patterns of variation where used in example sets, and what the teachers' intentions had been. It also presented the enacted and lived objects of learning and findings from the study. The enacted and lived objects of learning focused on the way teachers used examples to facilitate mathematical proficiency in statistics, and the way in which learners responded to show their experience of the object of learning. The next chapter concludes this study.

CHAPTER 5: SUMMARY OF FINDINGS AND CONCLUSION

Introduction

The aim of this study was to investigate the examples used by the teachers and how they use them to facilitate mathematical proficiency in statistics. The study was motivated by my experience of teaching statistics where I used examples as a teaching tool. Therefore in conducting this study I was hoping to discover different ways in which examples can be used to promote proficiency in statistics. In this chapter, I delineate the findings from the study, draw conclusions from the study and also engage with the limitations of the study. I also make recommendations from the findings for future research and presents reflections and implications for teachers of statistics.

Summary of findings

The use of examples in the teaching of statistics does have the potential to facilitate mathematical proficiency. Not all strands of mathematical proficiency were observed in the teachers' episodes. This is for different reasons. The teachers had to present a few examples per episode because of time constraints. The lessons were observed when learners had already started writing cycle tests, which interfered with some of their mathematics lessons. The effect on the pressured mathematics lessons led to teachers using less examples as the focus was mainly to get learners to grasp the concepts. A few exercise examples were provided, with some lessons proceeding without exercises. According to my experience of teaching, the requirement of a pacesetter is to give out at least one exercise every lesson.

Generally, both teachers A and B had little time to address the topic. Mathematical proficiency may, thus have been affected adversely by the limited time teachers spent on preparing the lessons. The teachers, thus, opted for simple examples that would facilitate the basics of statistics. More time in preparing would most likely have led to patterns of variation being incorporated more thoughtfully in the examples and applied more effectively.

This is more evident in teacher B's lessons. Because of time constraints, she could not cover the stem and leaf, even though she organised an extra morning class to do it. Covering the concept in a morning class suggests she had to focus on showing learners how to do it (procedure) without explaining the underlying concepts.

Examples used and how they were used to facilitate mathematical proficiency.

Worked out examples

Mevarech and Kramarski (2003) argue that worked-out examples are a method of instruction in teaching mathematics. Part of their purpose is to explain all the steps of a problem solving method and provide a complete explanation of the sequence and actions taken (Mevarech & Kramarski, 2003). The teachers used most of the worked-out examples as an instruction tool (showing learners how to do a procedure) for the methods to be followed. Little explanation or reasons were given. The teachers used worked-out examples mostly to facilitate procedural fluency. There are some cases when teacher A attempted to promote conceptual understanding as seen in the table from excerpt 1 below. The teacher uses an example to help learners to understand the difference between raw and organised data, and that data can be information about anything. The teacher also encourages learners to generalise and see the connection between raw and organised data in statistics.

Line	**Teacher/learner statements**
T	Now we are going to continue with our topic, data handling. We highlighted a few things on Friday. We said data is the information. If you collect or you are given information, like we did an example last time of collecting the shoe sizes that you are wearing, and we said information like that is the raw data. We said for every raw data to be interpreted easily, it needs to be organised. Then we said the first thing you do when you are organising data… let's say for marks out of 50 *(teacher writes test marks on the board)*. Suppose these are the test marks that some of you obtained for a test. So how many learners do we have? (Teacher counts) alright 8. So, like this can you see the marks we have. So, this is the raw data which we cannot be able to interpret it easily without us organising it, so tell me what we can do, what is the first step?

Mevarech and Kramarski (2003) suggest that worked-out examples are more effective when they are used in cooperative setting. The teachers did not explore other teaching strategies like

letting learners work in small groups. Teacher B did use some of the examples to make learners interact with each other, but not innovatively to allow them to explain to one other or solve problems together. This could have promoted conceptual understanding and productive disposition. The use of worked-out examples tended to facilitate procedural fluency more than the other strands.

Spontaneous examples

According to Zodik and Zaslavsky (2008), teachers generate spontaneous examples because of the way in which learners respond to a problem. This was a feature in teacher A's class and was also observed a few times in teacher B's class. The teachers created examples upon discovering that the response from learners had not been positive. These examples were often made easier, but retained the same concept. This made similarity to be the most observed pattern of variation. Spontaneous examples also facilitated procedural fluency as teachers used them to clarify a method or procedure and not to broaden the concept.

Exercise examples

According to Garfield and Ahlgren, (1988), statistics must be introduced through activities and simulations, and not immediately as definitions of abstract concepts. The teachers gave out exercise examples as a tool for learners to practice and apply what they had learnt from the worked-out examples. Some of the examples required learners to create their own data sets, which Watson and Mason (2005) support. They argue that when learners are given a chance to create their own examples, they are empowered to create problems, which encourage them to enquire more about a concept. Letting learners generate their own examples facilitates strategic competence, productive disposition and adaptive reasoning.

Teachers used all three types of examples, which are the focus of this study, but not enough to cover all the strands of mathematical proficiency. Simply, for mathematical proficiency to be attained, all the strands must be intertwined and must work together (Kilpatrick et al., 2001).

The object of learning

The object of learning in episode 1 was the measures of central tendency. In episode 2 it was the representation of data and working with formulae to do calculations.

Intended object of learning

The analysis of the examples in the intended object of learning shows that teachers used similarity and contrast as patterns of variation, which could help them to achieve the goals of the lessons. Most of the examples were similar, using different data sets to ask the same questions. This was another way of getting learners to master procedure, but could also lead to other strands being attained depending on the object of learning being enacted. Contrast was observed when an example presented something different from the rest of the examples, or required a different way to solve the problem. The teachers used contrast to introduce learners to the different ways in which statistical questions had been asked.

Enacted and lived objects of learning

Teachers tried to enact the object of learning in a way that facilitated mathematical proficiency or promoted learning. Most of the enacted objects of learning resembled exam preparation, which depends on practicing questions. Learners learned how to execute methods and deal with questions. However, from all the observations, learners were not able to answer application questions such as 'what does the mean tell you'.

Runesson (2005) describes learning as a change in the way learners see something. The learners did not completely change the way they see statistics, as they did not seem to be able to move from procedure to conceptual understanding. The lived object of learning was mostly about how to work out a problem because of the way the teachers used or enacted the examples (enacted object of learning) to the learners.

Limitations of the study

The framework used in the study focuses mostly on teachers. Data collection was also focused on teachers more than the learners. While the teachers were interviewed, learners were not. Not enough data was gathered on how learners lived the object of learning. Learners could have been given a test to see how they had assimilated what had been taught. Both teachers did not cover all the content in statistics because of time constraints. This may lead to limitations in the findings.

Recommendations from the findings

After having observed the lessons on statistics and the time given to the subject, it is clear that statistics is not given the attention it deserves. Being amongst the last topics of the third term

affects its coverage. The department of education could consider moving it to the first or second term.

Statistics is a delicate topic requiring specific pedagogical content knowledge (Chick and Pierce, 2008). As noted in the introduction, it is difficult to teach statistics without the use of examples. Leinhardt, Zaslavsky and Stein (1990) argue that the explanation of any mathematical concept relies on the use of examples. Teacher workshops must, therefore, focus on the types of examples teachers can use to facilitate mathematical proficiency in statistics.

The workshops could assist teachers in using examples to facilitate each and every strand of mathematical proficiency and apply the patterns of variation when teaching statistics. Teacher B was faced with the challenge of using examples that facilitated more procedural fluency than any other strand. This could be because she has less experience in teaching statistics than teacher A. It could also imply that many other less experienced teachers are faced with a similar challenge. Teacher training must not stop after teachers obtain their qualifications.

To start with, there should be continuous training to assist entry level teachers. Further, the training should provide teachers with different and more innovative teaching methods for teaching statistics to promote mathematical proficiency in learners. Teachers must also be trained on how to use the learning theories such as variation theory in their lesson preparations and in preparing their examples. Further, they must be trained in how to ascertain whether all the strands of mathematical proficiency have been attained.

Reflections for my teaching

The theoretical and analytical framework of this study successfully facilitated the focus of this study to be on examples, and the way they are used by teachers. The strands of mathematical proficiency in the frameworks have helped as a tool to observe learning through the use of examples. The use of the frameworks have led to the conclusion that examples play a very important role in the teaching and learning of statistics. They are a good tool to communicate mathematical ideas. As seen in the analysis process, the teachers use examples to explain the meaning of mathematical terms. This shows that examples have a great potential if used effectively, to facilitate mathematical proficiency in statistics.

Having done this study, I have come to learn about the importance of the patterns of variation as a tool for teaching to facilitate learning. Examples can be very effective when used with the patterns of variation. This study was motivated by the difficulty I have had as a teacher in

teaching statistics, which most teachers also regard as a boring topic. This attitude is easily transmitted to learners. The study has made me enquire deeper about tools that can be used to teach statistics in a way that both the teachers and learners can relate to.

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APPENDIX A

CLASSROOM OBSERVATION

Class:

Teacher: A ☐ or B ☐

Object of learning: _____

Date: _____

Indicator: The use of examples in teaching statistics

Critical question: How do teachers use examples when teaching statistics?

Type of example used	**Yes**	**No**
Worked out example		
Spontaneous example		
Exercise example		

Elaboration (Field notes):

_____

_____

_____

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APPENDIX B

INTERVIEW SCHEDULE

SCHEDULE FOR FOLLOW – UP INTERVIEW WITH TEACHERS AFTER LESSON OBSERVATIONS.

Teacher:

Your choice of example

1. How did you choose the examples you used in the lesson? What did you have to consider:
2. How do you decide on the number of examples to do in class?
3. Do the ways in which learners respond to the examples used in class influence your next choice of examples?
    a. (If question is not answered adequately, ask: When do you come up with (generate) examples in class, and what leads to this?

Your use of examples in the class

1. How do you decide on the appropriate time to use an example (whether when introducing or during the lesson)?
2. What was the purpose of the specific example (name that example) used in the lesson?
3. (if necessary) What response did you expect from learners for the specific examples used?
4. Why did you use this number of examples?
5. In what ways are the examples connected?
6. How did these examples enable mathematical understanding in your opinion?
7. How did you choose words in your use of examples so that learners understand the language used?

APPENDIX C

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

21 June 2018

Student Number: 562694

Protocol Number: 2018ECE021M

Dear Sibongile Zulu

Application for Ethics Clearance: Master of Education

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

Teachers' use of examples to facilitate mathematical proficiency in statistics.

The committee recently met and I am pleased to inform you that **clearance was granted.** However, there were a few small issues which the committee would appreciate you attending to before embarking on your research.

The following comments were made:

- Section 6.1 does not show distinction between confidentiality and anonymity.
- Change learner consent to assent

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

M Mabeka

Wits School of Education

011 717-3416

Cc Supervisor: Dr. Tony Essien

APPENDIX D



GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**07 June 2018**
Validity of Research Approval:	**05 February 2018 – 28 September 2018** **2018/100**
Name of Researcher:	**Zulu S.**
Address of Researcher:	**Unit 267 Midriver Estate**
	16 Oranjeriver Dr
	Kempton Park, 1619
Telephone Number:	**076 487 4508**
Email address:	**nelle.maz@gmail.com**
Research Topic:	**Teachers' use examples to facilitate mathematics proficiency in statistics**
Type of qualification	**Masters of Education (mathematics Education)**
Number and type of schools:	**One Secondary School**
District/s/HO	**Johannesburg East.**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

11/06/2018

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
4. *A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

.....

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 11/06/2018

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
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APPENDIX E

LETTER TO THE PRINCIPAL

DATE:

Dear Mrs Lebelo

My name is Sibongile Zulu. I am a Masters of Education (mathematics education) student in the School of Education at the University of the Witwatersrand.

I am doing research on the teachers' use of examples to facilitate mathematics proficiency in statistics. My research involves working with Grade 10 teachers and learners, where I will be observing five lessons (per teacher) in statistics in two different classes. During the lesson observations, there will be audiotaping which will then be followed by teacher interviews to follow up on my observations. This will take place over a period of 1-2 weeks (one lesson per day per teacher), and there will be no lesson disruptions caused. The interviews would take place after school and would be for a maximum of 20 minutes per teacher.

The reason why I have chosen your school is because I have taught Grade 10 mathematics in your school, and have experience with working with the learners in your school. Knowing the contextual background of the school and the learners make me see it as part of my responsibility as a teacher to work on getting solutions to the learners' academic problems by trying to investigate different ways in which learners can be taught mathematics. I will do this by trying to investigate different ways in which learners can be taught mathematics.

I am inviting your school to participate in this research which will seek ways in which teacher can teach statistics in mathematics in a way that enhances understanding. Improvement in the teaching of statistics will assist teachers in improving their teaching in other areas of mathematics, which will lead to an improvement in the performance of mathematics.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

SIGNATURE

NAME: SIBONGILE ZULU

ADDRESS: 267 MIDRIVER ESTATE

EMAIL: nelle.maz@gmail.com

TELEPHONE NUMBERS:

INFORMATION SHEET LEARNERS

DATE: 28/05/18

Dear Learner

My name is Sibongile Zulu. I am a Masters of Education (mathematics education) student in the School of Education at the University of the Witwatersrand.

I am doing research on the teachers' use of examples to facilitate mathematics proficiency in statistics. The purpose of this research is to investigate the way in which teachers use examples in the teaching of statistics to help learners understand statistics.

My investigation involves observations and audiotaping of your mathematics lessons when your teacher is teaching statistics. I will observe and audiotape five lessons from your class after which I will work with your teacher in an interview discussing the lessons. These lesson observations will be done over a period of 1 - 2 weeks.

I was wondering whether you would mind if I conducted my research in your class with you being a participant in my research.

I need your help with getting the data (information) I need to conduct my research. You can help me get this information by being part of the learners in the classroom when I will be observing and audiotaping the mathematics lesson.

Remember, this is not a test, it is not for marks and it is voluntary, which means that you don't have to do it. Also, if you decide halfway through that you prefer to stop, this is completely your choice and will not affect you negatively in any way.

I will not be using your own name but I will make one up so no one can identify you. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join us in the study.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

SIGNATURE

NAME: SIBONGILE ZULU

ADDRESS: 267 MIDRIVER ESTATE

EMAIL: nelle.maz@gmail.com

TELEPHONE NUMBERS:

Learner Assent Form

Please fill in the reply slip below if you agree to participate in my study called: Teachers' use of examples to facilitate mathematics proficiency in statistics.

My name is: _____

Permission to observe you in class

I agree to be observed in class. YES/NO

Permission to be audiotaped

I agree to be audiotaped during the interview or observation lesson YES/NO

I know that the audiotapes will be used for this project only YES/NO

Informed Consent I

understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____ Date_____

INFORMATION SHEET PARENTS

DATE:

Dear Parent

My name is Sibongile Zulu. I am a Masters of Education (mathematics education) student in the School of Education at the University of the Witwatersrand.

I am doing research on the teachers' use of examples to facilitate mathematics proficiency in statistics. The purpose of this research is to investigate the way in which teachers use examples in the teaching of statistics to help learners understand statistics.

My investigation involves observations and audiotaping of Grade 10 mathematics lessons when their teacher is teaching statistics. I will observe and audiotape five lessons from two classes after which I will work with the teachers in an interview discussing the lessons. These lesson observations will be done over a period of 1-2 weeks.

The reason why I have chosen your child's class is because I will be observing their mathematics teacher's lessons. I was wondering whether you would mind if your child participates in my research by being in the classroom (part of the lesson) when I will be observing and audiotaping the lessons.

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

Your child's name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

SIGNATURE

NAME: SIBONGILE ZULU

ADDRESS: 267 MIDRIVER ESTATE

EMAIL: nelle.maz@gmail.com

TELEPHONE NUMBERS

Parent's Consent Form

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called: Teachers' use of examples to facilitate mathematics proficiency in statistics.

I, _____ the parent of _____

Permission to observe my child in class

I agree that my child may be observed in class. YES/NO

Permission to be audiotaped

I agree that my child may be audiotaped during interview or observations. YES/NO

I know that the audiotapes will be used for this project only YES/NO

Informed Consent I

understand that:

- My child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- He/she does not have to answer every question and can withdraw from the study at any time.
- He/she can ask not to be audiotaped.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____ Date_____

INFORMATION SHEET TEACHERS

DATE:

Dear _____

My name is Sibongile Zulu. I am a Masters of Education (mathematics education) student in the School of Education at the University of the Witwatersrand.

I am doing research on the teachers' use of examples to facilitate mathematics proficiency in statistics. The purpose of this research is to investigate the way in which teachers use examples in the teaching of statistics to help learners understand statistics. The aim of this research is to also investigate the role of examples in the teaching and learning of statistics.

My investigation involves observations and audiotaping of Grade 10 mathematics lessons when you are teaching statistics. I will observe and audiotape five lessons from two classes after which I will work with the teachers in an interview discussing the lessons. These lesson observations will be done over a period of 1-2 weeks. The interviews would take place after school and would be for a maximum of 20 minutes per teacher.

The reason why I have chosen your school is because I have taught mathematics to Grade 10 learners in your school, and I am familiar with the learners in the school.

I was wondering whether you would mind if you would be a participant in my research by allowing me to interview you after observing and audiotaping your mathematics lessons in statistics, where we will be discussing some aspects of the lessons which are relevant to my research. This will be an observation for research only, and not to critic your teaching.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

You will not be advantaged or disadvantaged in any way. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

SIGNATURE

NAME: SIBONGILE ZULU

ADDRESS: 267 MIDRIVER ESTATE

EMAIL:nelle.maz@gmail.com

TELEPHONE NUMBERS:

Teacher's Consent Form

Please fill in and return the reply slip below indicating your willingness to be a participant in my voluntary research project called: Teachers' use of examples to facilitate mathematics proficiency in statistics.

I, _____ give my consent for the following:

Permission to observe you in class

I agree to be observed in class. YES/NO

Permission to be audiotaped

I agree to be audiotaped during the interview or observation lesson YES/NO

I know that the audiotapes will be used for this project only YES/NO

Permission to be interviewed

I would like to be interviewed for this study. YES/NO

I know that I can stop the interview at any time and don't have to

Answer all the questions asked. YES/NO

Informed Consent I

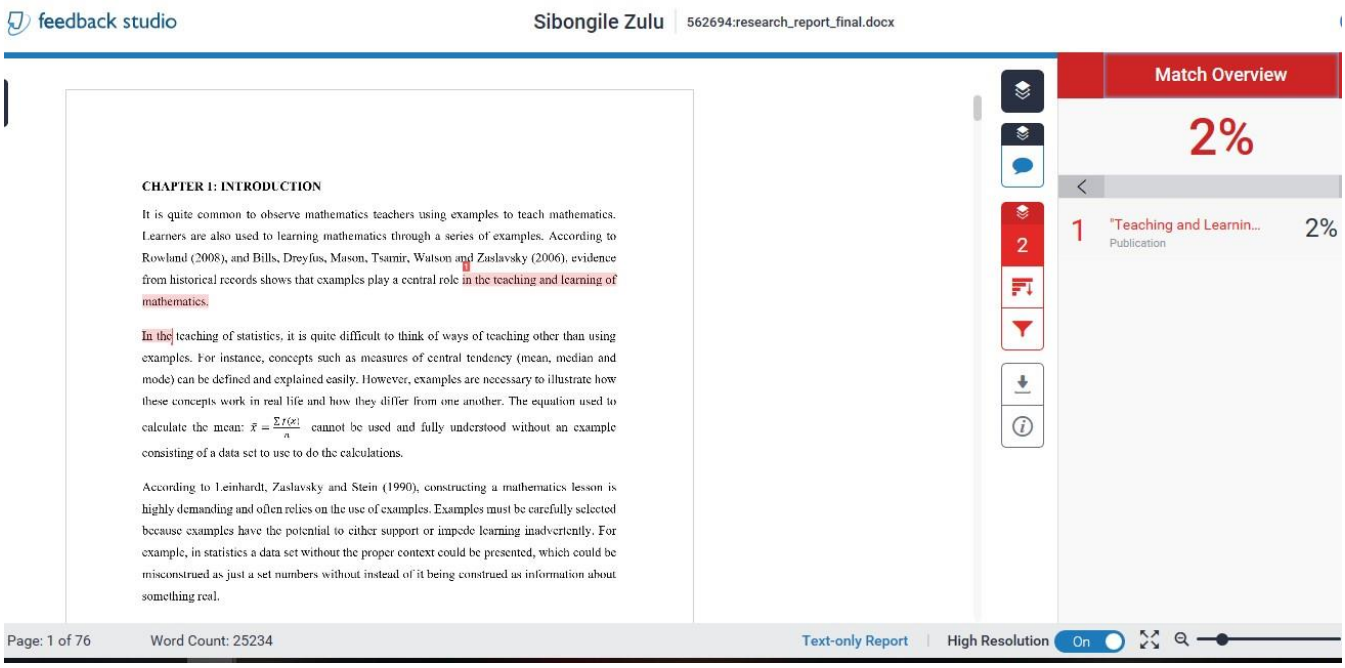
understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____ Date_____

APPENDIX F

Turn it in report



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