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A CRITICAL EXAMINATION OF THE USE OF PRACTICAL PROBLEMS AND A LEARNER-CENTRED PEDAGOGY IN A FOUNDATIONAL UNDERGRADUATE MATHEMATICS COURSE

Catherine Jane Le Roux

A thesis submitted to the Wits School of Education, Faculty of Humanities, University of the Witwatersrand in fulfilment of the requirements for the degree of Doctor of Philosophy.

Johannesburg, 2011

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ABSTRACT

This study is a located in a foundational undergraduate mathematics course designed to facilitate the transition from school mathematics to advanced mathematics. The focus of the study is on two innovations in the course; the use of practical problems that make links to non-mathematical practices and a learner-centred pedagogy. While these innovations are part of the discourse of the mathematics education community in terms of access to school mathematics, this study investigates the relationship between these innovations and access to advanced mathematics.

The texts of three practical problems from the course and texts representing the verbal and non-verbal action of 17 students as they worked collaboratively in small groups on these problems were analyzed. The analysis of these texts is used to describe and explain, firstly, how the practical problems in the foundational course represent the practice of foundational undergraduate mathematics and its relationship to other practices in the educational space (for example, school mathematics, calculus reform, advanced mathematics, and non-mathematical practices). Secondly, the students' enabling and constraining mathematical action on the practical problems is described and explained.

Answering the empirical questions in this study has required theoretical work to develop a socio-political perspective of mathematical practice. This theoretical perspective is based on Fairclough's social practice perspective from critical linguistics, but has been supplemented with recontextualized theoretical constructs used by Morgan, Moschkovich and Sfard in mathematics education. These constructs are used to conceptualize the notion of mathematical discourse and action on mathematical objects in this discourse. The methodological work of this study has involved supplementing Fairclough's method of critical discourse analysis with Sfard's method of focal analysis to analyze mathematical, discursive, social and political action in a socio-political mathematical practice.

The central finding of this thesis is that foundational mathematical practice represents both continuities and disruptions in its relationship to other practices in the space. As a result, participation in the foundational practice is complex, requiring control over the how and when of boundary crossings across practices, social events and texts. On the basis of this complexity, innovative foundational practice is positioned paradoxically in the higher education space. On the one hand, it represents an alternative to the dominant representation of mathematical practice and positioning of the foundational student in higher education. On the other hand, the complexity of foundational practice makes access to advanced mathematics problematic and foundational practice thus reproduces the dominant ordering.

Keywords: access, critical discourse analysis, focal analysis, foundational mathematics, learner-centred pedagogy, mathematical discourse, practical problems, socio-political practice perspective, undergraduate calculus

DECLARATION

I declare that this thesis is my own, unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other university.



Catherine Jane Le Roux

8th day of June in the year 2011.

To my two families, the Bennie family and the Le Roux family …

for supporting me and giving me the space to grow

PAPERS AND PRESENTATIONS EMANATING FROM THIS RESEARCH

Bennie, K., & Tobias, B. (2007*). Shifting the lens on contextualised mathematical problems: Two researchers compare notes*. Paper presented at the Fourteenth International Conference on Learning, 26 - 29 June, Johannesburg.

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Le Roux, K. (2008). Relevance and access in undergraduate mathematics: Using discourse analysis to study mathematics texts. In J. F. Matos , P. Valero, & K. Yasukawa (Eds.), *Proceedings of the Fifth International Mathematics Education and Society Conference* (pp. 340-351). Centro de Investigação em Educação, Universidade de Liboa and the Department of Education, Learning and Philosophy, Aalborg University.

Le Roux, K. (2008). *Adopting a socio-political perspective of learning: Reflections on the choice of appropriate analytic tools in a study of learning in undergraduate mathematics*. Paper presented at the Higher Education Close-up 4 Conference, 26 - 28 June, Cape Town.

Le Roux, K. (2009). Flu viruses, a lucky community and cosine graphs: The possibilities opened up by the use of a socio-political perspective to study learning in an undergraduate access course in mathematics. *African Journal of Research in Mathematics, Science and Technology Education*, *13*, 44-63.

Le Roux, K. (2010). "I was thinking the wrong thing" / "I was looking in a particular way": In search of analytic tools for studying mathematical action from a socio-political perspective. In U. Gellert, E. Jablonka, & C. Morgan (Eds.), *Proceedings of the Sixth International Mathematics Education and Society Conference* (pp. 307-318). Berlin: Freie Universität Berlin.

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CHAPTER 1
INTRODUCTION TO THE PROBLEM SPACE

1.1 Introduction to this study

Foundational mathematics courses at South African universities are a key strategy for facilitating the transition from school mathematics to advanced mathematics for students disadvantaged by an inequitable schooling system. The question is, "What should such a course look like if it is to fulfil this aim?" Various models of foundational provision have been attempted in South Africa in the past 25 years, as summarized in Pinto (2001) and Rollnick (2010). This provision may be "backward-looking" in the sense that it revises school mathematics, "forward-looking" by unpacking university-level mathematics, or a hybrid of the two (Allie, 2010, p.9). Provision may be in the form of extra tutorials in a mainstream course or in the form of stand-alone, formalized courses. Foundational provision may reproduce the pedagogic approach used in a mainstream undergraduate mathematics course but at a slower pace, or may be set up as a different practice by adopting a pedagogy that is quite different from the mainstream. Foundational provision is not only a South African phenomenon; writing from an Australian perspective Wood (2001) argues that foundational courses provide the opportunity for innovation:

> Bridging programmes allow lecturers to be innovative because they are not part of the mainstream mathematics degree programmes and therefore not contingent on the same constraints. (p.89)

This study is located in a foundational undergraduate mathematics course (called the *Foundational Course*[1]) at a South African university and focuses on this course as a strategy to provide *epistemological access* (Morrow, 2009, p.77) to advanced mathematics for students identified as disadvantaged by their schooling.[2] In this thesis I examine what this Foundational Course looks like by investigating two related innovations in the Course; the use of problems that make links to objects that take on meaning in everyday and disciplinary practices other than mathematics (which I refer to *practical problems*[3]) and a learner-centred pedagogy in which students are assigned agency to work in small groups to solve these problems. I lectured on this Course for a five-year period, and while my observations suggested that many students are able to solve these problems when working collaboratively with their peers, what matters in terms of the role that these innovations play in facilitating the school/advanced mathematics transition was not visible to me. These innovations are certainly part of the discourse of the mathematics education community, particularly at school level, but not in ways that issues of access to advanced mathematics at university are visible. How can what is substantive in this respect be made visible?

This thesis investigates how the practical problems in this Course represent the practice of foundational undergraduate mathematics and what enables and constrains students' mathematical action on practical problems when solving these problems collaboratively in small groups. Yet this thesis is as much about answers to the empirical questions of what this practice looks like and what it means to participate in this practice as it is about the development of a theoretical perspective and the associated methodological tools that allow

[1] I have made the ethical decision not to use the official course codes used by the institution to name this and the other undergraduate mathematics courses discussed in this thesis. The course material from these courses referred to in the discussion is thus not included in the Reference List.

[2] Morrow (2009, p.77) distinguishes between *epistemological access* to higher education which involves "learning how to become a successful participant in an academic practice", and *formal access* which is gaining entry to the institution. I use these two terms throughout this thesis.

[3] I explain my choice of the term *practical problems* in Section 4.2.5.

me to view (or to talk about[4]) mathematical, discursive, social and political action within a socio-political mathematical practice.[5] In the background of the empirical and theoretical answers presented in this thesis is three years of ongoing work between the empirical and theoretical spaces of the study, a period during which the two spaces have co-constituted one another.

1.2 The empirical and theoretical spaces

In this section I use research texts from this study to introduce both the empirical and theoretical spaces to the reader. The descriptions of these spaces serve only as an introduction and more detail is provided in the chapters that follow. After providing the background and rationale for this study in Sections 1.3 and 1.4. I bring the empirical and theoretical spaces together in research questions in Section 1.5.

1.2.1 A "problem" evaluating the limit of a derivative function

The extract from the Flu Virus Problem in Figure 1.1 is a practical problem from the material for the Foundational Course. Students work in small groups of four to five students to solve these problems (with support from a tutor) in a weekly afternoon workshop. As a lecturer on the Course at the time of this study, I played the role of tutor to a workshop class of approximately thirty students. Worked solutions to the problems (as in Figure 1.2) are given to students after the workshop. Evaluating the limit $\lim_{t\to\infty} P'(t)$ in question (g) requires that the student adopt an operational view of the limit and move to and fro between

[4] In Section 3.2 I make the distinction between using a theoretical perspective to *view* the empirical space and using a theoretical perspective to *talk about* this space.

[5] The socio-political perspective of mathematical practice used in this study is inclusive of discursive, social, political and mathematical action. The theoretical work of this study, as described in Chapter 4, has involved moves in which I attend specifically to the different types of action within this perspective. So where appropriate in the discussion, I distinguish between discursive, social, political and mathematical action.

the function and its derivative, the graphical representations of these functions, and the meaning of these functions in terms of the spread of the flu virus in the community.

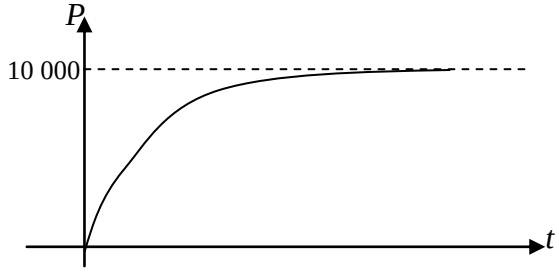
Figure 1.1: Question (g) of the Flu Virus Problem, Question 6, Workshop 10, Foundational Course Resource Book, p.54

A flu virus has hit a community of 10 000 people. Once a person has had the flu he or she becomes immune to the disease and does not get it again. Sooner or later everybody in the community catches the flu. Let $P(t)$ denote the number of people who have, or have had, the disease t days after the first case of flu was recorded.

g) What is $\lim_{t\to\infty} P'(t)$? Give a reason for your answer.

Figure 1.2: Worked solution for question (g) of the Flu Virus Problem

g) $\lim_{t\to\infty} P'(t) = 0$. Eventually the number of people who have caught the flu becomes (very nearly) constant at 10 000, so the rate of new infections is 0 (see graph).



Five students in Group 2 (Bongani, Lungiswa, Mpumelelo, Siyabulela and Vuyani) are seated together at a table as they solve question (g) of the Flu Virus Problem. Four of the five students are enrolled for the foundational programme in the science faculty and all their science courses are foundational courses. Vuyani is in a mainstream[6] programme and has recently changed to the Foundational Course in mathematics on the basis of his

[6] I use the term *mainstream* to refer to undergraduate programmes and courses that have traditionally been offered at universities.

performance in the mainstream first-year mathematics course in the first six weeks of the academic year.

Prior to the action re-presented[7] in Transcripts 1.1 and 1.2 the students evaluated the limit $\lim_{t\to\infty} P(t)$ in question (f) with relative ease, and this solution was enabled by the link the students made to the graph of the function $P(t)$. In their initial discussions about evaluating the limit $\lim_{t\to\infty} P'(t)$ in question (g), the students identify the derivative function $P'(t)$ as distinguishing the limit expression $\lim_{t\to\infty} P'(t)$ from the limit expression in question (f). Siyabulela describes question (f) as "easier", and Mpumelelo starts to talk about his difficulty evaluating the limit in question (g) in line 704 of Transcript 1.1.

Transcript 1.1: The Flu Virus Problem, question (g), Group 2, lines 704 to 709[8]

704	Mpumelelo:	Cause this ... like this derivative ((*Looking at the question in the Resource Book*)) like when you use a like when we don't work with this one ... infinity ... we usually give the exact time ((*Using his pen to demonstrate at the point*)) ... right↑
705	Siyabulela:	Ja ((*Nodding his head*)) (other?) problem ja I hear you
706	Mpumelelo:	So now ... as *t* approaches infinity ...
707	Siyabulela:	Ja ... and we're interested in a particular point then↑
708	?	Ja
709	Siyabulela:	That boy ... is is sitting there ((*Leaning forward and looking across the room at the Tutor who is helping another group*))

The students want assistance from the Tutor[9], but as suggested by Siyabulela's comment in line 709, the Tutor is helping another group of students. Lungiswa suggests they move on to the next problem while they wait (line 729 of Transcript 1.2).

[7] I explain the choice of the word *re-presented* in Section 5.4.

[8] The transcription notation is presented in Appendix A.

[9] I use the word *Tutor* with an upper case "T" as a proper noun to name the tutor who participated in this study. The term *tutor* is used for the common noun.

Transcript 1.2: The Flu Virus Problem, question (g), Group 2, lines 729 to 740

729	Lungiswa:	Let's go to the next question
730	Vuyani:	((*He chuckles*)) Yes (unclear) but that guy's going to ask us [... er what did you do?]
731	Lungiswa:	[Yeah he's gonna ask us]
732	Siyabulela:	((*Putting his book down on the floor again*)) Then because if we [[(unclear)]]
733	Vuyani:	[[What are we going to]] tell him↑... in order for him to
734a	Siyabulela:	Simple …
734b	Siyabulela:	we were waiting for you while you were sitting there ... so ... just ... ((*Vuyani laughs, Siyabulela is turning over the page as he speaks*))
734c	Siyabulela:	tell him that
735	((*Lungiswa, then Vuyani, then Siyabulela glance across the room to the Tutor*))	
736	Vuyani:	I mean like ... I mean like he is going to ask us ... what did we do↑ ((*Looking at Siyabulela*))
737	Lungiswa:	[If you skip it]
738	Siyabulela:	[Ja] ... oh oh ... what did you do?
739	Vuyani:	Ja ... did you [[you you↑]]
740	Siyabulela:	[[No ...]] we just tell what are our ideas ... ja

In Transcript 1.1 the students are talking and making gestures about mathematical objects.[10] Mpumelelo uses the mathematical terms "derivative" and "infinity" to talk about the symbols in the limit expression (lines 704, 706). He accompanies this talk with gestures such as pointing to the symbols in the expression and representing mathematical objects with his hand in the air (line 704). Siyabulela and Mpumelelo are looking at parts of the limit expression, the symbols $t \to \infty$ ("t approaches infinity", line 706) and $P'(t)$ ("we usually give the exact time", line 704; "we're interested in a particular point", line 707). These descriptions of symbols in words identify the symbols as representing mathematical objects with particular meaning, for example, the derivative function as an instantaneous rate of change. They are attending to what happens as time passes ("t approaches infinity", line 706), suggesting that they are viewing the limit as a process. However, they cannot reconcile this view with their identification of the derivative function as the instantaneous

[10] I use the term *mathematical object* for the theoretical objects that are acted on in a mathematical practice, objects that are represented using mathematical symbols, graphs, etc. I discuss the ontology of these objects in Section 4.2.1.

rate of change (what Siyabulela calls their "problem" (line 705)). They did not have the same "problem" when acting on the "easier" limit $\lim_{t\to\infty} P(t)$ in question (f).

The students' use of the inclusive pronoun "we" suggests that they view the action of answering question (g) as a collective one (lines 704, 709). Siyabulela is listening to Mpumelelo and gives him positive feedback and prompts him to continue by saying, "… ja I hear you" and by nodding his head (line 705). Siyabulela also builds on Mpumelelo's text to clarify their difficulty (line 707). Mpumelelo talks about what they "usually" do when working with derivatives, suggesting he is drawing on the texts from other events in the Foundational Course (line 704).

Yet the students' collective action and their textual links do not enable them to move beyond talking and making gestures about what Siyabulela later calls their "problem" (line 705). The students identify the Tutor as the authority who can assist them (line 709). While waiting for the Tutor, Lungiswa identifies herself as a student who manages the pace of the group by suggesting that they move on to the next problem (line 729). In responding to her suggestion, Vuyani and Siyabulela identify themselves differently in terms of their relationship with the Tutor. Vuyani identifies himself as a student who wants to be able to respond to the question "what did you do?" (line 730) from the Tutor, a concern that he states three times in Transcript 1.2. He identifies the Tutor as a facilitator in a learner-centred pedagogy who is going to ask them about their solution, but also as an authority to whom he must respond. In contrast Siyabulela identifies himself as being confident about interacting with the Tutor, an interaction he describes as "simple" in line 734a. Using a joking tone, he positions himself as having some power over the Tutor by calling the Tutor "boy" (line 709) and ordering him to help in line 734b.

1.2.2 Talking about the students' "problem" evaluating the limit of a derivative function in this study

The student's action as described in Section 1.2.1 is mathematical in that the students are acting on mathematical objects such as functions and limits. This action is also discursive, social and political, for example, they have particular ways of talking and using gestures, ways of making links across texts and events, ways of evaluating one another's work, ways of identifying themselves and others, and ways of interacting. Taken together, this action is constraining as they do not move beyond their "problem" and identify the Tutor as the authority who can move them forwards.

In acting in this way, Lungiswa, Mpumelelo, Siyabulela and Vuyani are, in interaction with the Flu Virus Problem, giving meaning to what it means to do foundational mathematics and to be a foundational mathematics student. This study is about describing how the students' action on the Flu Virus Problem during a workshop class represents foundational mathematics and explaining their enabling or constraining action in relation to their mathematical action on the one hand, and their discursive, social and political action on the other. It is also about developing a theoretical perspective and analytic tools to talk about the mathematical, discursive, social and political action of the students on the micro-level of the classroom.

Yet, as students in a Foundational Course in mathematics at a South African university, Lungiswa, Mpumelelo, Siyabulela and Vuyani are participating in a particular mathematical practice, which I refer to as *foundational practice*. This practice can be regarded as *socio-political* in the sense that it is a relatively stable form of action or way of doing things in which certain activities, participants or *subjects*, socio-political relations, objects, and discourse are given value (Fairclough, 2003, p.205).[11] The subjects in

[11] The term *discourse* here is used for written and spoken language as well as other forms of semiosis such as body language and visual images (Fairclough, 2003, p.205). I explain my choice of the word *subject* rather than *participant* in Section 4.2.3.

foundational practice are students, lecturers and tutors. They engage in particular types of activities like attending lectures, tutorials and workshops, work with particular mathematical objects such as functions and limits, and solve certain types of problems by sketching graphs, operating on mathematical objects, explaining answers etc. They use tools such as scientific calculators and are expected to write and talk in particular ways. Engaging in these activities the subjects take on certain identities and relate to one another in particular ways. From this perspective, Lungiswa, Mpumelelo, Siyabulela and Vuyani's action as described in Section 1.2.1 is both enabled and constrained by their location within the socio-political foundational practice; they are enabled to act on condition they act within the constraints of how a foundational students should act (Fairclough, 2001). This does not mean, however, that the student action is completely determined by the constraints of the practice, for in Transcript 1.1 we see Siyabulela exercising agency in terms of his relationship to the Tutor.

Yet the foundational practice does not exist in isolation, but is part of a *network* (Fairclough, 2003, p.24) of overlapping socio-political practices each with particular valued ways of acting. For example, foundational practice is networked with other mathematical practices such as mainstream first-year mathematics, advanced mathematics, and school mathematics, teaching practices at school and at university, mathematics education research, etc.[12] In addition, through the use of practical problems, the foundational practice makes links to non-mathematical practices whether everyday or disciplinary (for example, chemistry, epidemiology or economics). Power relations are at work in this network. For example, at a university there will be particular valued ways of recontextualizing non-mathematical practices into undergraduate mathematics, valued ways of teaching first-year

[12] Where necessary I divide mainstream undergraduate mathematics into *first-year mathematics* and *advanced mathematics*. The terms *advanced mathematics* and *undergraduate mathematics* are often used interchangeably, since advanced mathematics is what has traditionally been studied at undergraduate level. However, certainly at universities in South Africa, the first year of undergraduate studies in mathematics is generally considered as preparation for advanced mathematics which is introduced formally in the second year of mainstream study.

mathematics, and ways of defining students such as Bongani, Lungiswa, Mpumelelo, Siyabulela and Vuyani as belonging in particular mathematics courses. Historically in South Africa, foundational undergraduate mathematics is a relatively new practice in this network and represents a move in this network, with possible consequences for the power relations in the network.

So Lungiswa, Mpumelelo, Siyabulela and Vuyani are foundational students acting within a particular socio-political practice, as part of a wider network of practices in which power is at work. This study is thus also about explaining their enabling or constraining action in terms of the location of this action in this network of practices. It is also about describing the move that the foundational practice represents in this network, with consideration of the implications for access to the dominant practices. In terms of theoretical work, this aspect of the study involves identifying a theoretical perspective and associated analytic tools that allow me to talk about the relationship between the micro-action of the students and the macro-space and the power relations between socio-political practices in the network.

Not only are the five foundational students located in a wider network of socio-political practices, but as a researcher and lecturer I am also located in this network, a positioning that both enables and constrains my action in these roles. So this study also entails its location within a socio-political practice of doctoral research in mathematics education. Locating this study in this way requires that I set out "who I am" (or in the terms used by Valero (2004), "who we are" (p.6)), in the form of a personal history in Section 1.3, a section which also serves as background to the study.

1.3 Teacher, lecturer and researcher

1.3.1 Becoming and being a school mathematics teacher

My decision to follow a career in mathematics education was made during my years at high school, and based on my own rewarding experience of learning school mathematics. As a White South African growing up during the apartheid years in South Africa, I had the

opportunity to attend a (marginally) multi-racial independent secondary school for girls, yet my place at the school was based on an academic scholarship rather than the position of my family within a privileged socio-economic class. My experience of learning school mathematics can be described as more traditional[13], and I enjoyed the algorithmic and routine nature of the subject. My initial university education involved studying both pure mathematics and languages, followed by preparation to become a teacher (when I became aware of the limited nature of my experience of learning school mathematics). My days as a university student during the final and turbulent apartheid years alerted to me the enormity of the challenges faced by the country in moving forward.

In the first eleven years of my career in mathematics education I worked in primary and secondary schools in South Africa, mainly as a classroom teacher but also as a project worker for a non-governmental organisation involved in curriculum development and teacher support work. In my role as project worker I participated in the research component of the project and completed a masters degree in mathematics education by course work and dissertation. My exposure to research paradigms generally mirrors what was happening in mathematics education research at the time, beginning with an ontological/psychological perspective and moving to perspectives within the *social turn* (Lerman, 2002, p.23), perspectives that are discussed in Chapter 3. During this time the school curriculum in South Africa was undergoing rapid and radical change (as described in Section 2.3), and through my membership in a professional organization for mathematics teachers I was able to participate in this development process. My work in mathematics education during these eleven years was driven by a commitment to redress and access; a strong belief that all students should have opportunities to learn meaningful mathematics, a vision that the teaching and learning of mathematics could be better than what was taking place in my own classroom and classrooms that I visited, and a commitment that as someone who, on the strength of my race, was privileged by the apartheid system, I have a duty to contribute to building a post-apartheid South Africa.

[13] I explain my use of the term *more traditional* in Section 2.3.2.

1.3.2 Becoming a university mathematics lecturer

In 2004 I began work as a mathematics lecturer on a foundational programme at a historically White South African university. The foundational programme is designed for Black and Coloured students who do not gain acceptance to science-related studies at the university on the strength of their school-leaving examination results. Yet they are identified as having been disadvantaged by the schooling system as well as having the potential for the study of science at university level and are thus granted *formal access* (Morrow, 2009, p.77) into the institution. The Foundational Course in mathematics that is the focus of this study, together with the foundational course that follows, is recognized by the institution as equivalent to the mainstream first-year mathematics course. A pass in the two foundational courses provides formal entry into the mainstream second-year course in advanced mathematics. The two foundational courses also serve as service courses for other science-related disciplines. The focus of this study is on the role of the Foundational Course in providing epistemological access to advanced mathematics.

In deciding to make the career move from school mathematics education to mathematics education in higher education I was attracted on a personal level by the principles of redress and development underlying the foundational programme well as by the opportunity to begin a career as an academic, both as teacher and researcher. I served as convenor of the Foundational Course from 2005 to 2008 and as an advisor to the students on the foundational programme from 2006 to 2008.[14]

1.4 Defining the research problem

Adler and Lerman (2003, p.446) argue that quality research should "count" both for the participants ("locally") and for the mathematics education community ("globally"). In this section I define the research problem locally at the level of practice, but also globally by identifying the problem as also located in the mathematics education community.

[14] The data for this study was collected during the 2007 academic year.

1.4.1 The emergence of a research problem in my practice

When I began teaching on the Foundational Course in 2004, this introductory calculus Course had a ten-year history of development, a wealth of accumulated course material, and a relatively established pedagogy, all a product of that history. During the first weeks of the Course the focus is on the revision of topics in school mathematics relevant to calculus, for example, functions and trigonometry. Students then study limits, differentiation and integration. The Course is taught in English, although more than half of the students in the Course are learning mathematics in a language other than their home language (Visser, 2006).[15] The pedagogic approach used in the Course differs from my own experience of using an algebraic approach to learning calculus at undergraduate level, and my observations suggested that this pedagogy differs from that used in the mainstream first-year course at this university. This Course places an explicit emphasis on "understanding", which is communicated to the students as being about understanding the reasoning behind mathematical operations, being able to explain the meaning of the mathematical objects of calculus, and making links between these objects (in mathematical and non-mathematical contexts). The Course also promotes flexibility in moving between different representations of mathematical objects and the solving of practical problems that make links to everyday and disciplinary practices other than mathematics, for example, chemistry and economics. The extract from the Flu Virus Problem in Figure 1.1 is an example of such a problem (the three selected practical problems in this study are given in Appendices B to D and Appendix Q[16]). The Course has some features of a learner-centred

[15] I use *home language* to refer to the language spoken by the student at home, while acknowledging the complexity of the language landscape in South Africa and hence the difficulty in classifying languages in this way. For example, for many of these students the official medium of instruction for most of their schooling was English, but they report their mathematics teachers using both English and the home language of the students. Yet many university students would consider English as their main language as they use this on a daily basis in the educational space and in their place of residence while studying.

[16] Appendix Q is a foldout of the three practical problems in the back cover of this thesis, provided for the convenience of the reader.

pedagogy[17] in that students are encouraged and provided with the space to work collaboratively in small groups.

A small research study conducted in 2004 using a sample of Foundational Course material (as reported in Bennie, 2005), provided me with an initial description of these practical problems. The material was described as rich in "information transfer", which I defined as questions that require students to apply their knowledge across certain boundaries. Three types of transfer were identified in the tasks (with most questions requiring more than one type of transfer); transfer between mathematical representations, transfer between mathematical concepts, and transfer between mathematical and real-world contexts.[18] Furthermore, the contexts used in the problems requiring information transfer (the practical problems) were classified. Three types of contexts were identified, namely, "context not new"; "new context but identifiable mathematics"; and "new context but initially unidentifiable mathematics". A large proportion of the problems were classified as having a new context, but requiring identifiable mathematics for their solution. For example, a number of questions require students to interpret the graph of the derivative of a function. These questions have a variety of contexts such as the queue for a concert, water flow in a tank, population change, and purely mathematical contexts. Although working in different contexts, students use the same mathematics to solve all these problems. The study recommended that further research be undertaken into the use of real-world contexts and language-based problems in the Course.

[17] I use the term *learner-centred pedagogy* broadly here, and elaborate on the use of this term in Section 2.3.2.

[18] In this, my first research study on undergraduate mathematics, I used words such as *mathematics*, *transfer*, *boundary*, *context* and *real-world* rather uncritically. Given that I am reporting on this early study here, I retain their use. Conducting this doctoral research has challenged me to interrogate my language use and the assumptions underlying my initial choice of terms. Elsewhere in this thesis I clarify my use of terms and present their meaning in terms of my overall theoretical orientation.

As a new lecturer on the Foundational Course, it was the practical problems (used in a workshop setting that promotes a learner-centred pedagogy) that caused a sense of unease. I observed that students were solving these problems (some, with assistance) in a workshop class. Furthermore, the official pass rate for the Course as a whole was deemed appropriate by the university. Yet it was common knowledge (also reflected in university statistics) that while most students passed the two foundational mathematics courses, few of those who actually attempted further study in advanced mathematics succeeded.[19] In other words, the statistics were suggesting that passing the foundational courses did not facilitate the boundary crossing between school mathematics and advanced mathematics. Rather, it seemed that the foundational courses simply represented an additional boundary in the school/advanced mathematics transition. Not only was the Course promoting a different pedagogy to the mainstream courses, but through the use of practical problems the Course introduced an additional boundary into this space, that is, the boundary between mathematical practices and non-mathematical practices. Furthermore, I was concerned that making this boundary crossing placed particular demands on the students (many of whom do not have English as a home language) in terms of reading and writing. [20]

In summary, the design of the Foundational Course was based on assumptions that the use of practical problems and a learner-centred pedagogy are enabling for students. My unease about the use of these innovations in the Course stemmed from the fact that the role that these two innovations were playing in terms of providing epistemological access to advanced mathematics was not visible to me in my practice. What is required of students when moving between mathematical and non-mathematical practice in the practical problems, and how does this relate to vertical movement towards abstraction within mathematical practice? What is required of students when crossing the school/foundational

[19] Data on students who have passed the Course suggest that the majority of the students proceed to graduate with degrees in the chemical and molecular sciences and in computer science (Buffler & Davidowitz, 2006).

[20] A key feature of the theoretical journey I have made while conducting this doctoral research study lies in broadening the view of language that I held at the start of this study.

mathematics boundary, and how does this relate to the mainstream first-year and second-year courses?

1.4.2 What the mathematics education community says about the research problem

The use of practical problems and a learner-centred pedagogy in the Foundational Course can be located in mathematics curriculum reform efforts both in school mathematics and in calculus reform at undergraduate level. Certainly, what the mathematics education research was saying about the use of these innovations suggested that my personal sense of unease, as described in Section 1.4.1, was not idiosyncratic. Firstly, a number of studies in school mathematics (conducted in South Africa and elsewhere) suggest that the use of practical problems may limit access to mathematical practice (e.g. Dowling, 1998; Moschkovich, 2002; Nesher & Hershkovitz; 1997; Sethole, 2005).[21] Secondly, it is argued that certain students may be marginalized from practical problems (e.g. Tobias, 2009) and that this marginalization may be related to social class (e.g. Cooper & Dunne, 2000; Lubienski, 2000). Thirdly, the link between interpretations of learner-centred pedagogy and access to mathematical practice has been problematised (e.g. Adler, 1997; Davis, 2001). Lastly, the implications of using a reform-oriented pedagogy in multilingual classroom in South Africa have been considered (e.g. Adler, 1997; Setati, 2005).

Yet while the mathematics education community was talking about practical problems and learner-centred pedagogy in terms of access to mathematical practice, I argue in the rest of this section (and elaborate on this argument in detail in Chapter 3) that there is a gap in the literature since the community does not talk about the two innovations in a way that makes the problem that arose in my practice visible. In this section I draw on the description of the empirical and theoretical spaces set out in Section 1.2, as this sets out what I need to talk about in this study.

[21] Various terms are used in this literature for problems that make links to practices other than mathematics, for example, *realistic problems*, *real-world problems* and *applications.* In this initial discussion I am using *practical problems* as a broad term for these problems. Developing a clearer description of the practical problems in the Foundational Course is a key part of this study.

The research reviewed briefly here talks about practical problems and learner-centred pedagogy in relation to access to school mathematics practice. Yet my study is about the transition from school mathematics to advanced mathematics at university. From a socio-political perspective of practice, these are different (yet related) practices.

Furthermore, the innovations that are of interest in this study are not in view in the research on advanced mathematics (e.g. the edited volume by Tall, 1991a). Firstly, this can be attributed to the nature of advanced mathematics as a vertical mathematical practice aimed at abstraction. So this research does not have a need to talk about how students evaluate the limit in question (g) of the Flu Virus Problem by crossing the boundary between foundational mathematics and the non-mathematical practice of epidemiology.[22] Secondly, the research on advanced mathematics has predominantly been conducted from an ontological/psychological perspective. In the literature review in Chapter 3 I will provide examples from this research to suggest that this perspective is strong in terms of how it allows me to talk about how Siyabulela and Mpumelelo act on the mathematical objects when evaluating the limit in Transcript 1.1. Yet this perspective does not allow me to talk about the discursive, social and political aspects of the students' action, and it does not allow me to talk about this action with reference to the wider socio-political space and how the students recruit ways of acting from other practices. Lastly, the process of vertical mathematization is viewed in terms of mental reconstructions made by the individual to overcome cognitive obstacles, a perspective that may lead to the description of individual students as being in deficit (Lerman & Zevenbergen, 2004).

Although calculus reform curricula give prominence to practical problems and the role of social relationships and reading and writing in learning, the research on this reform has either been restricted to experimental studies that compare traditional and calculus reform

[22] I refer to the non-mathematical practice of epidemiology broadly here. Part of this study has involved developing a more nuanced description of the non-mathematical practices represented by the practical problems.

pedagogies (e.g. Garner & Garner, 2001), or has tended to adopt the theoretical tools used in research on advanced mathematics (e.g. Bowie, 2000), a study that is located in the Foundational Course). As result, the calculus reform research is disconnected from the concerns of this study.

What then of other theoretical perspectives for talking about my research problem? Given the concern about relevance in school mathematics reform, research on school mathematics does talk about the boundary between mathematical and non-mathematical practices. The school mathematics research reviewed briefly at the beginning of this section can be located in shifts away from an ontological/psychological perspective to discursive, social, and political perspectives. In Chapter 3 I review literature that suggests that research conducted within these three turns allows me to talk about the discursive, social and political action of Lungiswa, Mpumelelo, Siyabulela and Vuyani. It also allows me to link this action to the wider socio-political practices in which it is located and to talk about the power relations between these practices. Furthermore, sociological and post-structuralist researchers talk about the movement between practices, either in terms of recognizing the boundary in the case of the former, or as discursive shifts in the case of the latter. Yet, these research perspectives are weak in a key respect in terms of talking about my research problem; Valero & Matos (2000, p.398) note that that going "deeply" outside of mathematics to talk about social, political and cultural aspects of mathematics education results in the mathematics tending "to vanish or to be questioned".

This initial discussion (and one that is developed in detail in Chapter 3) suggests then that the role that innovative pedagogies in the Foundational Course play in facilitating the transition from school to advanced mathematics is not visible to the mathematics education community. I argue that a key contribution of this study is in developing a theoretical perspective and associated methodological tools that make this research problem visible. Certain recent studies on undergraduate mathematics (Wistedt & Brattström, 2005) and the transition between school and undergraduate mathematics (Jooganah & Williams, 2010) recognize the dilemma of trying to keep both the mathematical and social action in view

and address this by drawing on both psychological perspectives and perspectives located within the social turn. I take an alternative way forward in this study; I use the socio-political perspective of practice used by Fairclough (1992, 1995, 2001, 2003, 2006) in critical linguistics, but recontextualize certain theoretical constructs from mathematics education for use within Fairclough's perspective. Such an approach requires attention to the movement of meaning in these theoretical constructs for use from the particular ontological and epistemological perspective used by Fairclough. I argue that this theoretical work allows me to talk about the mathematical action as well as the discursive, social, and political action of Lungiswa, Mpumelelo, Siyabulela and Vuyani, to relate this micro-action to the macro socio-political space in which power relations are at work, and to talk about boundary crossings. Talking about these features of the space allows me to address the research problem in a way that "counts" (Adler & Lerman, 2003, p.446) on the level of practice for the participants and more broadly in the mathematics education community. The problem addressed in this study is thus simultaneously empirical and theoretical, with the two problems co-constituting one another during the three-year journey as I worked between the empirical and theoretical spaces.

1.5 The research questions

I end this introductory chapter by bringing together, in the form of research questions, the empirical and theoretical spaces sketched here. These questions point to what I will be saying about the research problem and how I will be talking about this problem. Each of the two questions below begins with a proposition, the first emerging from the theoretical space (as described in Section 1.2.2), and the second emanating from my practice as a university lecturer (as described in Section 1.4.1). After the presentation of the research questions below, I explain these propositions in relation to the research problem. I revisit these research questions in more detail in Section 4.6.

Research Question 1:

The practical problems give meaning to the practice of foundational mathematics and set up subject positions for the students.

(a) What relationships between this practice and other practices, both mathematical and non-mathematical, are represented?

(b) What do these problems represent as the valued mathematical action in this practice?

(c) What socio-political relationships and social identities do these problems construe for the subjects and who has power in the discourse?

(d) What continuities and/or disruptions in the movement of meaning across texts, events and practices are represented?

(e) How can this representation be explained with reference to the wider socio-political space, that is, what discourse types, genres and styles do the problems draw on?

(f) What continuities and/or disruptions does the foundational practice represent in the wider order of discourse, and with what implications for access to dominant mathematical practices and change in the higher education space?

Research Question 2:

The student mathematical action on the practical problems both enables and constrains the adoption of the valued subject positions in the practice of foundational mathematics.

(a) What mathematical action do students use when solving the practical problems?

(b) Does this mathematical action enable or constrain their occupation of the subject positions set up for them in the practice (as described in research question 1)? In particular, do they control the movement across texts, events and practices, both mathematical and non-mathematical, as required in the practice?

(c) In what ways is this mathematical action enabled or constrained by the discourse types, genres and styles that the students recruit and/or the socio-political interaction in the classroom?

This study is about the transition from school to advanced mathematics, and the role of the foundational practice in facilitating this transition. The first proposition recognizes that the

foundational practice represents a move in the wider network of socio-political practices. The foundational practice itself and the innovative pedagogies within this practice introduce additional boundaries into the network of mathematics education practices at school and university. The sub-questions of research question 1 are designed to describe and explain this practice and its relationship to other mathematical and non-mathematical practices in terms of what it means to move across these boundaries.

If students such as Lungiswa, Mpumelelo, Siyabulela and Vuyani are to gain access to advanced mathematics via the foundational practice as represented in the answers to research question 1, they need to be able to participate in this practice to the extent that they pass the Foundational Course. Such participation requires making the necessary boundary crossings between school and foundational mathematics and between mathematical and non-mathematical practices. The second proposition recognizes that the students' mathematical, discursive, social and political action may constrain (as in Transcript 1.1) or enable this participation. The sub-questions of research question 2 are designed to describe this enabling and constraining action with reference to the action in the classroom and to how the students recruit ways of being a mathematics student in the wider socio-political space.

1.6 The structure of this thesis

In this introductory chapter I have presented the empirical and theoretical space of the study and introduced the reader to the research problem. In the next five chapters I expand on this initial elaboration of the empirical and theoretical spaces. In Chapter 2 I describe the socio-political space in which the study is located. In Chapter 3 I present a literature review; this literature review focuses on what the mathematics education community says about the research problem and how it talks about the problem. The discussion of various theoretical perspectives in Chapter 3 provides the grounds for my theoretical and methodological choices which I set out in Chapters 4, 5 and 6. In Chapter 4 I present a detailed elaboration of the socio-political perspective of mathematics practice and in Chapters 5 and 6 I present

and illustrate the use of the analytic tools that allow me to operationalize this theoretical perspective. Chapter 6 also attends to issues of quality in this study. In Chapters 7 to 11 I present the analysis, firstly of the three selected practical problems in Chapter 7 and then of the student action on these problems in Chapters 8 to 11. This analysis allows me to answer research questions 1 and 2. I conclude in Chapter 12 with a summary of the findings of this study, discussion of the limitations of the study, and recommendations for practice and further research.

CHAPTER 2
THE WIDER SOCIO-POLITICAL SPACE OF SCHOOLING AND HIGHER EDUCATION

2.1 Introduction to this chapter

Valero (2004) is critical of research reports that mention the "context" (p.17) of the study but then, in her opinion, forget the role of context in understanding the analysis of the research. She argues that that a key feature of a socio-political perspective of mathematics education research is:

> … finding ways of knitting together the micro contexts on which mathematics education research normally concentrate – such as a community of learners in the classroom – with the multiple layers of contexts in which that micro context is inserted, with the aim of finding significant revelations about the social and political essence of the educational practices of mathematics. (p.17)

Morgan (2006, p.221) also points to importance of the researcher taking into account both the current action (the *situation context*) and the broader context in which this action is embedded (the *context of culture*). In this study I use the term *wider socio-political space* to describe this second meaning of context used by Morgan.

The research questions presented in Section 1.5 signal my intention to explain the practical problems and the student action on these problems in a foundational classroom with reference to the location of the classroom in a wider network of socio-political practices of schooling and higher education. In this chapter I describe this socio-political space. This description plays two roles in this study. Firstly it provides, in the traditional sense, the

"context" of the study. Secondly, the description of the space specifies the resources that I use in the explanation stage of the analysis presented in Chapters 7 to 11.

I describe this socio-political space by identifying the discourses at work in this space. I use the term *discourse* here to indicate how the world is represented and how social relations and identities are construed in the language of a social practice.[23] For example, a curriculum document may represent mathematics as being unrelated to non-mathematical practices and a research-based overview may identify mathematics as being for students of particular educational backgrounds.

2.2 Choices about what resources to use in a description of the space

Morgan (2006) argues that recognizing the relationship between micro- and macro-level practices raises two methodological questions; "how much of the context it is necessary to consider and what means to use to describe the context" (p.239). Fairclough (2001, p.126) raises a concern related to Morgan's first question when he warns of a tendency in linguistics to "delimit" the context and to "constrain the vastness of context". Valero (2007, p.227) suggests a possible way forward in addressing these methodological challenges; she suggests "digging" in the network of socio-political practices and discussing "existing research literature and policy documents" for insight. I present the results of this "digging" in this chapter and in Chapter 3. I use a variety of resources for my description of the space of schooling and higher education; official policy and curriculum documents (including textbooks), research-based overviews of the space produced by both the state and the private sector, interviews with the participating students, and my personal experience of the empirical space.

My choice of resources for this chapter is driven, firstly, by the location of the study in the Foundational Course in mathematics at the intersection of schooling and higher education.

[23] This use of *discourse* is consistent with Fairclough's (2003, p.214) use of *discourse* for the language aspect of a social practice, as presented in Section 4.2.2.

Secondly, I consider the resources drawn on as "key" texts in the sense that they are either widely cited in the relevant socio-political practices of schooling or higher education, or have been identified for me by participants located in these practices. For example, the school textbooks discussed were identified by the students participating in the study, and the sources related to higher education were identified by a key participant in the shaping of higher education policy on teaching and learning. This consultation aside, I am conscious of the operation of my own resources in this selection, resources that are both enabled and constrained by my location within the space, and I recognize that these texts alone cannot be regarded as representative of or as exhaustive of the wider practices.

2.3 The schooling system

In this section I provide an overview of South African schooling, focusing on mathematics education at the secondary level. The overview moves from a description of intended changes to a discussion of how these changes have played out in practice.

2.3.1 Changes to schools in post-apartheid South Africa

Prior to 1994 the state schooling system in South Africa was structurally fragmented along racial lines.[24] Reddy (2006, p.393) reports that participation and performance in school Mathematics[25] was racially differentiated; participation in Mathematics in the final year of

[24] Less than 5% of schools in South Africa are classified as *independent* (Organisation for Economic Co-operation and Development [OECD], 2008, p.20)). Prior to 1994 these schools were accepting students of all races. Since state schools are the major feeder to university education in this country, I focus on state schools in this description. However, some of the students who participated in the study completed their schooling at independent schools. I do not consider this a problem as the students' descriptions of their schooling, provided in individual interviews, point to consistencies in the students' experiences of learning school mathematics at independent and former White state schools.

[25] I use the term *school Mathematics* with an upper case "M" for the school subject in South Africa, and reserve the term *school mathematics* for the practice in general.

schooling was 64% for White students and 24% for Black students, with 97% passes for White students vs. 15% passes for Black students.[26]

In the fifteen years since the first democratic elections in South Africa in 1994, schooling has undergone major restructuring, in terms of structure, policy, and curriculum, aimed at increasing both formal and epistemological access to schooling. These changes, which came about through interventions by government, the private sector and non-governmental organizations, and have been contested:

> But while constitutional imperatives directed the trajectory of change, the chosen policy mechanisms were often the result of intense political contestation between various educational and other interest groups, especially big business and organised labour. Policy was also influenced by international educational perspectives and by global economic trends.
>
> (Organisation for Economic Co-operation and Development [OECD], 2008, p.75)

Structurally, schooling now falls under one national department and nine provincial education departments. Admission requirements have been changed with the opening of all state schools to all races. Ensor (1997) locates school curriculum reform within the establishment of the South African National Qualifications Framework; this policy aims to combine education and training systems and to facilitate movement of students between these systems. Ensor (1997) argues that this Framework is based on assumptions that school knowledge, workplace knowledge and everyday knowledge are equivalent.

A new outcomes-based school curriculum was phased in from Grades R to 12 on an ongoing basis from 1997 to 2008. This curriculum replaced a content-driven curriculum

[26] In this study I use the terms *Black*, *Coloured*, *Indian* and *White* for the race groups in South Africa, since this terminology is consistently used in reporting educational participation and performance in South Africa. I use the term *Black* for *Black African*. The figure for Blacks here does not include data for the former homelands and self-governing states.

which, prior to 1994, differed across racial education departments.[27] Harley and Wedekind (2004) argue that the reform of the school curriculum in South Africa "was of a scale arguably unparalleled in the history of curriculum change" (p.195). This new curriculum positions itself in three roles in the new nation; upholding the values of the constitution (such as human rights, non-racism and non-sexism), redressing past inequities, and catering for the development needs of the country.

2.3.2 A new outcomes-based curriculum for school Mathematics

Prior to the new outcomes-based curriculum the school subject Mathematics was compulsory for the first nine years of formal schooling. Students choosing to study Mathematics in their last three years of schooling studied the subject either on higher grade or standard grade, with higher grade demanding more mathematical reasoning and insight than standard grade. Higher grade Mathematics served as a filter to formal access into science-, engineering- and business-related studies at most higher education institutions.

The new outcomes based curriculum requires every student to study either Mathematics (no longer divided into higher and standard grades) or a new subject called *Mathematical Literacy* in the final three years of schooling. The official curriculum documentation for Mathematics foregrounds how mathematics has been used in the past as "a filter to block access to further or additional learning" not just in mathematics and mathematically-related careers, but also to careers "unrelated to Mathematics" (Department of Education [DoE], 2003, p.62).[28] In contrast, the new subject Mathematics is constructed as a "pump" that provides access to "a wide variety of learning" (DoE, 2003, p.62).

[27] I use the terms *old content-based curriculum* and *new outcomes-based curriculum* to distinguish between the two curricula.

[28] The curriculum document for the subject Mathematics does not distinguish between the school subject and the practice of mathematics as a discipline.

Two related features of this curriculum are pertinent to this study. The first is the notion of *relevance* by which I mean making links between school mathematics and everyday, workplace and disciplinary (other than mathematics) practices. The second feature is the pedagogic concept of *learner-centredness*. The discussion in the rest of this section focuses on the school Mathematics curriculum for the final three years of schooling.

The subject Mathematics is constructed as having "power" (DoE, 2003, p.62), with its power lying in its value both as a discipline itself and as a tool for making sense of society in general:

> Mathematics enables creative and logical reasoning about problems in the social world and in the context of mathematics itself. (p.9)

The construction of school Mathematics represents a weakening in the boundary between the academic and the everyday (Ensor & Galant, 2005), and assumes transfer across a range of practices:

> Mathematics is an essential element in the curriculum of any learner who intends to pursue a career in the physical, mathematical, computer, life, earth, space and environmental sciences or in technology. Mathematics also has an important role in the economic, management and social sciences.
> …
> Mathematics is being used increasingly as a tool for solving problems related to modern society. The financial aspects of dealing with daily life are informed by mathematical considerations. Mathematical ways of thinking are often evident in the workplace. (DoE, 2003, p.11)

Furthermore, the requirement that the content and process skills of school Mathematics "where possible, be embedded in contexts that relate to HIV/AIDS, human rights, indigenous knowledge systems, and political, economic, environmental, and inclusivity issues" (DoE, 2003, p.12) points to an assumption that the contextualization of school Mathematics facilitates transfer across practices. This assumption is a commonly held belief about the use of practical problems in school mathematics, along with other beliefs,

for example that these problems motivate students and are appropriate for weaker students since they are less abstract (Boaler, 1993).

The new outcomes-based school curriculum promotes certain values, such as problem-solving, productive group work, effective communication, personal responsibility for learning, and sensitivity to diversity and the needs of others (DoE, 2003, p.2). The teacher is described as a "mediator of learning" (DoE, 2003, p.5). These values are promoted by a particular type of pedagogy that is described in formal curriculum documents as "learner-centred" and "activity-based" (DoE, 2003, p.2), a pedagogy that represents a weakening in the boundary between the participants in the classroom (Ensor & Galant, 2005).

The features of the new outcomes-based school curriculum described in this section can be linked to international reforms in mathematics education and what I choose to call *reform-oriented pedagogy*.[29] In describing this type of pedagogy I draw on the work of Adler (1997), Ball (1991), and Boaler (2002a): The students are working together on mathematically rich tasks (including practical problems), and participating in mathematical practices such as explaining procedures, providing conjectures and proofs, communicating strategies and results verbally and in writing (rather than focusing on content). The use of appropriate problem-solving strategies and the provision of multiple solution strategies are encouraged. Students are identified as having agency in terms of negotiating meaning with one another and evaluating one another, with the teacher facilitating the students' participation in school mathematics (a pedagogy commonly referred to as *learner-centred pedagogy*).

A reform-oriented pedagogy can be contrasted with a *more traditional pedagogy*, described as follows (again drawing on the work of Adler (1997), Ball (1991), and Boaler (2002a)):

[29] Instead of using the terms *reform pedagogy* and *traditional pedagogy*, I choose to use the terms *more traditional pedagogy* and *reform-oriented pedagogy*. This choice avoids setting up a duality, with the selected terms suggesting a range of possible pedagogies, the two extremes of which are described in detail by Boaler (2002a).

The teacher explains methods on the board and the students watch/make notes. Then students practice the methods, usually using textbooks. The teacher and the textbook as the authorities hold access to the answers to the mathematical problems. Students work individually and in silence. The emphasis is on learning content, getting the right answers and obtaining these answers using standard methods.

In summary, school reform in post-apartheid South Africa assigns schooling the multiple roles of redress, development and nation-building. Structural changes suggest that a student can gain formal access to schooling, irrespective of his race and the racial group that a school formerly served.[30] The educational space represents a flattening of boundaries between school, everyday and workplace practices and between mathematical and non-mathematical practices. School Mathematics is powerful knowledge and knowledge that, through the adoption of reform-oriented pedagogy, can be accessible to all.

2.3.3 Schooling in South Africa post-1994

The discursive space representing the intended school reform in South Africa is overlaid by the actual practice of school reform since 1994, a discursive space that I describe in Sections 2.3.3 to 2.3.5.

The demographic profile of schools suggests that there has been a shift from former Black schools into English-medium former Coloured, Indian, and White schools, with former Black schools still populated almost entirely by Black students (Reddy, 2006; Soudien, 2004). These former Black schools are the poorest and suffer backlogs in the provision of physical infrastructure and educational resources (OECD, 2008). A number of empirical studies point to the fact that formal access is not only an issue of race, but also of related issues such as socio-economic class, geographical location and language. For example, Reddy (2006) argues that access is determined by the ability to pay school fees, and by residential and transport arrangements in accessing areas from which people of colour were

[30] For consistency and ease of reference I refer to the student as male in this thesis.

previously excluded. Soudien (2004) suggests that dominant race groups, classes (such as the middle class) and languages (such as English) have made space for others in schools, but that this is achieved on the terms of the dominant group and requires assimilation into the dominant values.

In terms of epistemological access to school Mathematics, Reddy (2006) argues that "… the gap in the performance of the different schools is not changing" (p.411). Although participation in school Mathematics increased in former Black schools from 1990 to 2003, participation in higher grade Mathematics decreased in these schools during this period (Reddy, 2006).[31] Reddy (2006) notes that in the 2003 Trends in International Mathematics and Science Study (TIMSS), Grade 8 students in South Africa had the widest distribution of scores on mathematics, with students in former Black schools (also the poorest schools) scoring the lowest and with the mean performance of learners in former White schools close to the international mean.

So while the intention of opening state schools to all races was aimed at enabling formal access, the reality suggests that access to the English-medium, middle class schools regarded as enabling epistemological access to school Mathematics is constrained by related factors of race, language, socio-economic class and geographical location.

2.3.4 The implementation of the new outcomes-based school curriculum

Practical constraints related to the large-scale school curriculum reform meant that the new outcomes-based curriculum was implemented over a period of twelve years. Given the sense of urgency that accompanied this reform, attempts were made to infuse some of the underlying ideas of the new curriculum into existing content-driven curricula. After the 1994 elections the syllabi from the pre-1994 racially segregated education departments were consolidated and this included interim "cosmetic changes" (OECD, 2008, p.79). For

[31] As noted in Section 2.3.2, this is the more demanding grade that provides entry into science-, engineering-, and business-related studies in higher education.

example, *social aims* and *teaching and learning aims* that display similarities to those in the new outcomes-based curriculum were included (Department of Education and Culture, Province of the Eastern Cape, n.d., p.1). Some students followed the new outcomes-based curriculum for the first nine years of their schooling, but then completed their schooling using the content-driven curriculum. Teachers were given guidelines on infusing the philosophy of outcomes-based education into the original content-based curriculum. Yet the learning materials in use during this interim period were designed with a content-based curriculum in mind.

Thus during the gradual implementation of the new outcomes-based curriculum both the discourse of reform-oriented pedagogy and more traditional pedagogy were represented in the discursive space of schooling.

2.3.5 Grounding the experience of schooling in South Africa

In this section I ground the description of the discursive space of schooling provided so far by drawing on the descriptions of schooling provided by 15 students who participated in the study. An individual interview was conducted with each student during their first year at the university. While these narratives were created within the genre of the research interview and do not form part of the practice of foundational mathematics which is the focus of this study, they nonetheless illuminate the view of schooling already presented in this chapter.

Most of the students who participated in this study completed their final year of schooling in South African schools in 2006, and thus belong to the cohort of students who experienced both the old and the new school curricula, as described in Section 2.3.4.[32] So

[32] Two students whose work forms part of this study were not interviewed, as they withdrew from the Foundational Course before an interview could be arranged. Four of the fifteen students interviewed attended independent schools (they studied the same official curriculum, but wrote a different school-leaving examination) for their final years of schooling, and one completed the Cambridge school curriculum in a country neighbouring South Africa. Although I have focused on the state schooling system so far in this

their schooling took place in a discursive space in which the discourse of reform-oriented pedagogy and the discourse of more traditional pedagogy were represented.

Schooling is represented in two different ways in the students' descriptions. Some students attended former-White state schools or independent schools. They studied Mathematics in classes of 15 to 30 students, all of whom were studying the subject on the higher grade. A number of these students attended extra classes for Mathematics, lessons offered on a commercial basis by private service-providers. These students studied Mathematics in English, with the Black students learning in a language that is not their home language. The second group consists of Black students who attended former Black township schools. Although the medium of instruction in these schools was officially English, these students reported that their teachers switched to the students' home language(s) when explaining mathematical content and that students used their home languages when learning with peers. These students reported being in Mathematics classes of 45 to 70 students. Although these particular students were doing Mathematics on the higher grade, almost all the students in their classes were studying Mathematics on the standard grade. The teachers were reported to teach on the standard grade level, expecting the handful of higher grade students to work on their own, or to attend classes after school. Students reported participating in study groups with their peers and enlisting the help of other township students who had completed school and were studying further.

These representations point to what Mathematics is studied at different types of schools. Most students who attend former-White or independent schools study higher grade Mathematics, while this is not the case in former-Back schools where the students who take this grade are the high-performers and are in the minority. These high performers in the former-Black schools exercise agency in organizing additional learning support and are

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chapter, I include the descriptions of these students for two reasons. Firstly, there are continuities across the experiences of the students who attended independent schools and those who attended former White schools. Secondly, these students' experiences form part of the resources that these students recruit in the study.

identified by their schools as having the agency necessary to fill the gaps between standard and higher grade Mathematics. The students at the former-White or independent schools are represented as needing Mathematics tuition in addition to their regular classes, with access to extra tuition depending on a student's financial resources. Black students at the former-White or independent schools have to speak English rather than their home languages, whereas at former-Black schools the varied home languages are valued as a resource for learning. These language practices have implications for students' transition to English-medium universities, with students from former-Black schools reporting difficulties with the level of English used for teaching and learning at these institutions (Bangeni & Kapp, 2007; Cross, Shalem, Backhouse, Adam & Baloyi, 2010).

While the students' narratives point to two different representations of schooling in South Africa, all students in this study reported similar pedagogies in their Mathematics classrooms, an approach that can be classified as a more traditional pedagogy. Teachers are reported to have begun the lesson with some form of explanation on the board. This was followed by students working through exercises, either on their own or in informal pairs or groups, with the work completed for homework. All the students, irrespective of the school attended, reported a reliance on textbooks either to fill the gaps in the understanding that they did not get during scheduled Mathematics classes or for test and examination preparation. Below I present a short summary of the pedagogic approach promoted in the two textbooks identified by the students, textbooks that were widely used in school Mathematics classrooms at this time. Given the students' reported reliance on these two texts, this summary is used as one view of the kind of learning experiences the students had in their final years of schooling.

Both textbooks follow a consistent pattern; an information section (formulae, definitions, theorems, steps to follow when solving problems), followed by annotated worked examples, and sets of exercises and examination-type questions for the student to complete. The second text has more explanatory text and developmental activities than the first. In terms of content, in both textbooks the text on functions focuses on the classification of

selected functions (mainly linear, quadratic, and cubic) by their properties. The majority of the tasks require students to move between representations of functions in the form of equations and graphs by (a) deriving the properties of functions from a given equation, and (b) finding the equation of a function given certain properties. The second textbook makes some links to non-mathematical practices after extensive work in mathematical contexts, while the first restricts itself to purely mathematical practices. In both textbooks, differential calculus can be divided into three sections: finding derivatives using the limit definition and the rules for differentiation, working with cubic functions as described above under functions, and solving application problems in the form of maximum/minimum problems. The second textbook presents an intuitive approach to limits in preparation for differential calculus.

In summary, the discursive space of intended school reform is overlaid by a discursive space representing the practice of schooling during the period of reform. The latter space is characterized by sometimes contradictory representations about who attends different types of schools and which of these schools enable epistemological access to school Mathematics. Formal access to schools and epistemological access to school Mathematics, it seems, are not just determined by race but a complex array of factors such as socio-economic class (and financial access to extra classes), language, geographical location, pedagogic approaches and individual agency of students.

2.4 The higher education space

In this section I describe the discursive space of state higher education in South Africa, beginning with an overview and then locating this in a description of specific programmes and courses at the university at which this study was conducted.[33]

[33] The term *state higher education* includes but is not restricted to universities and I specify where the discussion refers exclusively to universities.

2.4.1 Access and success in higher education

Prior to 1994 all state higher education institutions in South Africa were designated for a particular race. Historically White English-medium higher education institutions, such as the university at which this study is located, were circumventing the legal criteria for formal access from the 1980s, and by 1993, 38% of the enrolment at these institutions was Coloured, Indian or Black (Council on Higher Education [CHE], 2004, p.60). All institutions were opening to all races from 1990.

Since 1994 the state higher education system has undergone many structural and policy changes. Like schooling, this system has been tasked with the dual roles of social redress and national development. Scott, Yeld and Hendry (2007) argue that in higher education "the need for representativity and social inclusion is interwoven with the need for competence" (p.6). During this period higher education institutions developed policies to promote transformation (Soudien et al., 2008).

In terms of formal access there has been growth in the intake into higher education as a whole (CHE, 2009). People of colour make up an increasing proportion of the student population; in 1994 almost half of students in state higher education were White, a figure that decreased to a quarter in 2006, while the proportion of Black students increased from 40% in 1994 to 61% in 2006 (OECD, 2008, p.70, p.341).

However as early as 1995, Scott (1995) was warning that a more nuanced view of the achievements with respect to access was required. Firstly, in terms of formal access, participation rates of different race groups are not representative. For example in 2006, 60% of Whites in the 20-24 age group were enrolled in higher education compared to only 12% of Coloureds and 12% of Blacks in this age group (Scott et al., 2007, p.10). In addition, growth in Black enrolment has tended to take place in the humanities and social sciences, and not in scientific and technological disciplines (Scott, 1995). Secondly, in terms of epistemological access, the recent cohort study reported in Scott et al. (2007) indicates that, while the educational process is not working for many students at contact

universities, the situation is worst for Black students; approximately one-third of Black students graduate with a 3-year Science degree within 5 years, compared to approximately two-thirds of White students. Even though Black enrolment may exceed that of Whites, the completion rate for Whites in the Sciences is almost two times higher than that of Blacks (Scott et al., 2007, p.16).

Scott et al. (2007) identify a number of factors that affect performance in higher education, all of which give a sense of the discourse around who belongs in higher education. The first factor is the quality of the schooling sector. In Section 2.3 it was argued that access to quality schooling is related to race, socio-economic status and language. In fact, the issue of student finance is a factor identified by Scott et al. (2007) as constraining formal access to higher education. The third factor affecting performance is the response of higher education to the problems in the schooling system and to the widening of formal access to higher education to a diverse population. It is generally agreed that mainstream higher education curricula at former White universities were designed for a white, middle-class student body (Cross et al., 2010; Scott, 1995), suggesting that such curricula may not articulate well with the schooling experienced by Black students. Scott (1995) refers to this lack of linkage between a student's schooling and higher education as *articulation failure* (p.4).

The discussion in this section points to a representation of the university as being for students who have quality schooling and a certain level of financial resources. In reality, a small proportion of Black students achieve formal access to university but this does not necessarily translate into epistemological access as Black students are less likely to graduate with a science degree than students of other races. The discourse of who belongs and succeeds in higher education has implications for what I choose to call *social access*, that is, whether students see themselves as belonging in higher education. Scott et al. (2007) identify affective factors as the fourth aspect affecting performance. Students from former Black schools identify themselves as "other" in the higher education space, a space representing "whiteness" and "Englishness", and as different to those Black students who attended former White schools (Bangeni & Kapp, 2007; Cross et al., 2010). Soudien et al.

(2008) and Cross et al. (2010) argue that there is a disjuncture between institutional policy on transformation at higher education institutions and the everyday experiences of students, with students identifying the space as racist.

The discourses around who has social and epistemological access to higher education have implications for how a student engages in the space. For example, a student from a former Black school (identified as a high achiever in that school) may feel that he does not belong at the institution on the basis of his socio-economic class, language or race and he may see other students with his background failing. In the terms used by Skovmose (2005, p.6), such a student's ruined *foreground*, that is what she perceives as the opportunities provided by this space, can be an obstacle to learning.

2.4.2 Addressing the transition from school to higher education

Responses to the *articulation failure* (Scott, 1995, p.4) between schooling and higher education in South Africa represent shifting discourses about where students identified as being affected by the gap between the two levels of the education system belong in higher education and what should be done to facilitate the transition. Scott (1995) points to arguments that suggest that responding to articulation failure is not the core business of a university, for example, that such a response is too costly for the sector and will compromise the standards of the university, that addressing the gap between school and higher education is "remedial" work for which university lectures are not equipped or not interested in, and that it should be the responsibility of independent providers or community colleges.

Yet South African higher education institutions have a history of providing foundational provision[34] to equip students who gain formal access to these institutions but are identified as needing additional support in order to succeed. Since the 1980s various models of

[34] The term *foundational provision* is used in official documentation of the Department of Education in South Africa (see Scott et al., 2007, p.43)

foundational provision have been attempted (see Pinto (2001) and Rollnick (2010) for summaries). In some early programmes, all students were admitted to the mainstream and additional support in the form of extra tutorials was provided to some students. Other programmes required the foundational students to complete non-credit-bearing *bridging courses* in preparation for entry into mainstream first-year level courses. A more recent model, termed *extended curricula* or *augmented courses*, also segments out the foundational students but provides students with additional contact and learning time in credit-bearing courses that form part of a students' degree programme. Foundational provision may be "backward-looking" in that it tries to fill the gaps by revising school work, or may be "forward-looking" by focusing on the conceptual development and skills required for higher education, or may be a combination of the two approaches (Allie, 2010, p.9).

During the 1980s and 1990s foundational provision was seen as separate from the business of mainstream first-year teaching, and received no state funding. Since 2004 this form of provision has received some official recognition through state funding. More recently, increased enrolments into higher education and recognition of the poor performance of the sector as a whole have foregrounded debates about who foundational provision is for. Scott et al. (2007, p.47) note that foundational provision has traditionally reached only those students who do not meet minimum standard entry criteria into higher education, and is not usually made available to students who qualify for the mainstream but are underprepared for this. Proposals for higher education institutions to move from a three-year degree (traditionally regarded as the norm) to a four-year degree (currently provided for students on extended degree progammes) point to shifts in thinking about the nature of foundational provision and who it is for. Boughey (2007) identifies a shift in thinking about foundational provision from locating the "disadvantage" (p.7) in the individual Black student who needs to adapt to an unchanged institution to focusing on the development of the institution to meet the changed demography of the student population" (p.8).

2.4.3 A foundational programme in science

Foundational provision for science students has been a feature of the university at which this study was conducted since 1986, with the extended curriculum model in place since 1999. Consistent with the goals of the university as a whole, this foundational programme has an overall goal of social redress. Currently students targeted for the programme are Black students in South Africa and Coloured students in the province in which the university is located, suggesting that social redress is linked to race. These students gain "differential entry" (Allie, 2010, p.29) to the institution since they do not have the necessary school-leaving results for entry into the mainstream but are "deemed to have the potential" (Allie, 2010, p.5) to succeed in higher education. Each year a small number of Black and Coloured students whose marks place them on the borderline for entry into the mainstream are given the choice to join the foundational programme. Enrolment figures suggest that this programme is where Black students belong; the approximately 120 students accepted into the foundational programme each year represents approximately 80% of the first-year intake of Black students into this faculty (Allie, 2010, p.22).

The criteria for entry into the foundational programme suggest that certain students, mainly Black students, are identified as possibly belonging in the institution, but belonging in a programme that is separate from the mainstream. However, Allie (2010) points to a concern that the foundational programme in science has increasingly catered for Black and Coloured students with middle class backgrounds who have attended former White schools, rather than students of colour from working class backgrounds and disadvantaged schools. This concern points to a shift in thinking about the relationship between racial redress and social redress, a relationship that is currently hotly debated at the university and in the media (e.g. University of Cape Town, 2010).

The structure of the foundational programme in science suggests that the students admitted to this programme need different support to those in the mainstream. The foundational students' first-year courses are spread over 18 months or two years (as in the case of mathematics). The programme thus allows for "differential pace" (Allie, 2010, p.29) in the

sense that students should complete a three-year science degree programme over four years (hence the term *extended curriculum programme*). The courses in the programme are regarded as intensive since they have the same contact time (lectures, tutorials etc.) as the first-year courses in the mainstream. On successful completion of these foundational courses, students gain entrance to mainstream programmes by enrolling for second-year courses in the mainstream.[35]

While the foundational programme is a route for students of colour to gain formal access to studies in science at the university, the throughput rates for this programme suggest that this does not necessarily translate into epistemological access. Allie (2010, p.24) indicates that the average graduation rate in science for students who begin their studies in the foundational programme is 32%, and these students are less likely to graduate in the faculty than mainstream students (where approximately 50% graduate). Yet, the statistics also suggest that the foundational programme may be the place for Black students if they are to have a chance of graduating in the science faculty; Allie (2010, p.27) notes that for the graduation years 1996 to 2000, approximately half of the Black graduates in the faculty utilized foundational courses.

2.4.4 The Foundational Course in mathematics

I begin this section by locating the Foundational Course in the institution at which this study was conducted. This is followed by discussion of the initial development of the Course and a description of the Course at the time that the study was conducted.

The location of the Foundational Course at the university

Two mathematics courses form part of the foundational programme in science described in Section 2.4.3. The first half-course, which runs for the full first-year of study, is compulsory for all students in the programme (all students in the faculty require at least one

[35] The model of foundational provision described here is specific to the science faculty in which this study is located. The university has extended curricula programmes in all faculties, but the specific form of these programmes and the entrance criteria vary across faculty.

half-course in mathematics). The second half-course runs for the second year of study. Taken together, the two half courses are considered by the faculty to be equivalent to the mainstream mathematics course (see Section 2.4.5 below). In this section I focus on the first, compulsory course as it is in this Course that this study is located. Most of the students enrolled for this Course have traditionally scored from 40% to 60% in the final higher grade Mathematics examination at school, although some students who scored above 70% on the standard grade examination have been admitted. These entrance criteria suggest that these foundational students may be affected by the lack of articulation between schooling and higher education discussed in Section 2.4.1.

The foundational programme in science of which this Course forms part recognizes that many school-leavers do not have a "clear and informed idea of the area of study they wish to pursue" (Allie, 2010, p.11) and thus has a stated goal to provide "flexible entry" (p.5) into a variety of programmes in science. This flexibility is represented in the Foundational Course which caters for students who are potential mathematics majors and those students who need mathematics in the service of other scientific disciplines. Students who pass both foundational mathematics courses gain formal access into the mainstream second-year mathematics course in advanced mathematics. However, the data on students who have passed the Foundational Course suggests that this Course is for non-mathematics majors; the majority proceed to graduate with degrees in the chemical and molecular sciences and in computer science. Of the students in the faculty who graduate in the mathematical sciences[36] in 2005, only 4% began their degree on the foundational programme (Buffler & Davidowitz, 2006, p.3).

While the Foundational Course serves all students in the foundational programme, it also caters for students who do meet the traditional entry requirements for the mainstream programmes in science, but who are identified after six weeks of the academic year as not

[36] *Mathematical sciences* refers to the cluster of disciplines pure mathematics, applied mathematics, physics and statistics.

actually belonging in the mainstream. Each year fifty to one hundred of these students enroll for the Foundational Course. Within timetabling constraints, an effort is made to integrate the two groups of students as soon as possible, so that they attend classes together.

In terms of content, the Foundational Course looks back to school Mathematics by focusing initially on topics in the school Mathematics curriculum that are regarded by the lecturers as important in the learning of undergraduate mathematics, for example, functions and trigonometry. After the first six weeks of the academic year the Foundational Course shifts to looking forward to the content of first-year calculus, that is, limits, differentiation and integration.

The initial Foundational Course and calculus reform

The initial development of the Foundational Course during the 1990s was influenced by the ideas of calculus reform. This turn to a calculus reform curriculum suggests who this Course is for; initial arguments for this approach in undergraduate mathematics suggested that a traditional algebraic approach to teaching calculus was not promoting access to undergraduate mathematics, particularly for an increasingly diverse student body, with some students needing calculus for application in science and engineering rather than for further study in mathematics (Bowie, 2000; Douglas, 1986; Tall, 1996).

While there is certainly no clear consensus on what is valued in a calculus reform curriculum, I would argue that value is placed both on students' gaining proficiency in operating on mathematical objects and on understanding these objects. This can be achieved through the use of multiple representations of mathematical concepts (geometric, numerical and algebraic), technology, student talk (preferably in groups) and writing, and "applications". My use of inverted commas for the word "applications" here is deliberate, since calculus reform texts give significance to various relationships between mathematical and other practices. As an illustration I discuss the preface of a key calculus reform text for

students, that is, *Calculus*, by Hughes-Hallet et al. (1994, p.vii).[37] This text references a horizontal relationship in the sense that calculus is represented as having "practical value" and as a powerful tool "to illuminate questions in mathematics, the physical sciences, engineering, and the social and biological sciences". Yet "practical problems" (which are "usually … real world applications") are also represented as playing a role in vertical mathematization. For example, Hughes-Hallet et al. (1994) identify one of the basic principles for teaching calculus using a reform approach as "formal definitions and procedures evolve from the investigation of practical problems" (p.vii). Requiring students to explain what an answer means in "practical terms" (p.vii) is regarded as a way of reinforcing algebraic approaches to calculus through strengthening the meaning attached to the mathematical symbols.

The Foundational Course at the time of this study

Over the years the Foundational Course has been adapted into what the current convenor of the Course terms a "modified reform calculus approach" (Allie, 2010, p.15). The textbook that exemplifies the reform approach to teaching calculus that was prescribed in the early years of the Course has been replaced by one that represents a more traditional pedagogy. This replacement is the same textbook as that used in the mainstream first-year and second-year courses. However, many students indicate that they do not use this text, but rely solely on the Foundational Course Resource Book that has been constructed by the lecturers on the Course. The Resource Book contains course information, tutorial and workshop material, notes (some of which are based on the prescribed textbook), and past test papers. So while the mainstream first-year course and the Foundational Course have the same prescribed textbook, the repackaging of material in this textbook in the form of Course notes identifies the foundational student as not being able to navigate the original text. In this section I use material from this Resource Book to describe the Foundational Course at the time that this study was conducted. This description is designed to give the reader a

[37] This text was, for a number of years, the prescribed textbook for students in the Foundational Course.

sense of the Course as a whole, and how the practical problems and learner-centred pedagogy that are the focus of this study feature in the overall Course.

The page headed "Course Content" in the Foundational Course Resource Book summarizes the mathematical content of the Course in a list of topics and also includes a list of the aims of the Course:

> We would like all students to:
> 1. Perform mathematical procedures appropriately and accurately
> 2. Demonstrate an understanding of the relevant concepts by explaining them in a meaningful way
> 3. Apply this mathematical knowledge, skills and understanding appropriately
> 4. Complete simple proofs logically and with understanding
> 5. Present their work neatly and logically
> 6. Develop appropriate study skills that they will be able to apply in their future academic careers, for example, working consistently, making suitable use of feedback, and working with fellow students.
>
> (Foundational Course Resource Book, 2007, p.5)[38]

Points 1, 2 and 3 from the above extract point to the influence of calculus reform in terms of the value placed on both operations on mathematical objects and understanding of these objects, as well on the ability to apply mathematical knowledge and skills. These three ways of acting can be identified in the three consecutive examination questions in Figure 2.1. These past examination questions, which are typical of the teaching and learning material and assessment material used in the Course, are provided in the Resource Book to help students prepare for assessments.

[38] The name of the Course has been removed and the bulleted points in the original have been changed to numbered points for ease of discussion in this section.

Figure 2.1: Questions 8, 9 and 10, Final Examination, Foundational Course

8. Given the function $f(x)=\begin{cases}1-x^2 & \text{if } x<1\\ -1 & \text{if } x=1\\ e^{x-1}-1 & \text{if } x>1\end{cases}$

(a) Is the function $f(x)$ continuous at $x = 1$? Show all your working and explain your reasoning clearly.

(b) Draw a sketch graph of $f(x)$, clearly labeling any important points and any intercepts with the axes.

9. The temperature in a room can be modelled by the function $H(t)=4\sin\left(\frac{\pi}{50}t\right)+16$, where $H(t)$ is the temperature measured in °C and t is the time in minutes since midnight.

(a) What is the maximum temperature in the room?

(b) What is the minimum temperature in the room?

(c) How many minutes will elapse between when the temperature in the room is a maximum, and when the temperature is a minimum?

(d) When will the temperature in the laboratory be 14°C? Do not use your calculator and include all possible answers.

(e) Calculate $H'(12)$, giving the correct units for your answer (answer rounded to 2 decimal places).

(f) Give the everyday meaning of your answer in (e).

10. Find the value of $\lim_{x\to 0}\frac{\cos x-1}{x\sin x}$, showing all working.

Question 10 in Figure 2.1 requires that the foundational student evaluate the limit of the function using L'Hospital's Rule. In question 8 the student should apply his understanding of the definition of continuity of a function in a mathematical context and explain his reasoning. This question also illustrates the "strong numerical, graphical, algebraic and verbal descriptive approach" (Allie, 2010, p.15), with the student working with both algebraic and graphical representations of the function. Question 9 is a practical problem that makes a link to the temperature in a room, requiring the student to operate on mathematical objects when differentiating and solving equations and apply his knowledge of the derivative in the non-mathematical practice. Practical problems (like in question 9)

are included in the Course material when students study the meaning of functions, derivative functions, and integrals. Practical problems in the form of "applications", these being related rates and optimization problems, are included at the end of the section on differentiation.

Returning to the list of aims of the Foundational Course quoted on page 45. The "simple proofs" identified in point 4 may be in the form of using a definition to prove that a given function has a particular property, for example, proving that a function is differentiable or continuous (as in Question 8(a) of Figure 2.1). Proofs of the rules for differentiation are also studied in the Course. Points 5 and 6 in the list of aims refer to valued actions in a learner-centred pedagogy; effective communication, taking responsibility for one's own learning, and co-operating with other students. These two points also talk to the achieving student in a former Black school who has demonstrated agency by taking responsibility for his learning and achieving in such a school.

The introductory Foundational Course material not only sets out what mathematical content is to be studied in the Course, but also makes explicit how the foundational student should behave when participating in the practice of foundational mathematics.

In terms of pedagogy, the Foundational Course represents features of both a more traditional pedagogy and a reform-oriented pedagogy. Timetabling constraints, class size and the physical layout of the institution mean that students attend a forty-five minute lecture in a steeped lecture venue four times a week. Most of the time in lectures is spent with the lecturer explaining new work and doing examples on the board. These examples are usually similar to questions in other Course material. Within the confines of the lecture format, the lecturers do encourage student engagement and interaction, setting small tasks for students to work on, either individually or in pairs, at intervals during a lecture.

The students attend a weekly forty-five minute tutorial, the focus of which is practising the mathematical skills presented in lectures. Question 10 given in Figure 2.1 is typical of a

tutorial question. Every Monday afternoon of the academic year is dedicated to a 105 minute workshop. Students work in self-selected groups of four to six students to solve a variety of mathematics problems; these include problems requiring mathematical operations such as solving equations and finding derivatives and integrals, but also application-type questions. These applications may be in purely mathematical contexts (as in Question 8 in Figure 2.1), or in the form of practical problems (for example, Question 9 in Figure 2.1 and the Flu Virus Problem in Figure 1.1). Most of the work done in a workshop will have been encountered by the students in lectures prior to the workshop. One tutor (or in some cases a lecturer) is assigned to each workshop class of approximately 30 students. Instructions about group interactions provided in writing to students in the first workshop of the academic year make explicit how the foundational student should behave in the workshop class, in this case placing value on the actions in a learner-centred pedagogy:

> What should you be doing in your group?
> - Making suggestions about strategies to solve a problem
> - Explaining answers
> - Asking questions about solutions
> - Asking for further explanation
> - Criticizing **ideas**, not people
> - Encouraging one another to keep going/to participate
> - Congratulating one another.
>
> (Foundational Course Resource Book, 2007, Workshop 1, p.16)

A tutor is given full responsibility for the functioning of his class of 30 students for the academic year. These responsibilities thus include controlling the pace at which students work through the prescribed work for the afternoon and making decisions about dealing with students who arrive late or students whose action may be disruptive. Lecturers and tutors meet on a weekly basis to discuss the upcoming workshop material; such discussions are characterized by the lecturers indicating what work has been done in lectures and talking about the role of the tutor in a workshop class. For example, tutors are reminded to emphasize the valued group skills on a regular basis (and not just at the beginning of the year) and encouraged to engage with the small groups of students by listening and asking

questions, rather than simply attending to individual students who raise their hands to call for assistance. The expectations of tutors assigned to the workshop classes in this Course differ from what is expected of tutors in mainstream mathematics courses at the university (as discussed in Section 2.4.5). Not only is the tutor in the Foundational Course identified as a participant and authority in the practice of undergraduate mathematics, but he is also viewed as an authority on how to behave in a workshop class and as a facilitator in a learner-centred pedagogy.

All formal instruction in the Foundational Course is in English, the medium of instruction at the university. Discussion in some of the small groups in the workshop class takes place in languages other than English. Students are also encouraged to work collaboratively with one other outside of formal teaching and learning time, and students have access to a tutorial venue next to the lecturers' offices where this type of activity can take place.

The Course is intensive and lecturers work closely with the students on a daily basis, often assisting them individually or in small groups. Lecturers thus have an opportunity to get to know many of the students during the academic year, and annual lecturer evaluations for the course consistently reflect students' appreciativeness of the level of interest that the lecturers show in them as individuals and in their progress at the university. The foundational student is thus visible in this Course, a feature of the course that talks back to schooling where a student is "constantly being watched" (Cross et al., 2010, p.63).

In summary, the Foundational Course is designed to serve two groups of students at the university; those who receive differential entry to the science faculty from the start and those that are initially identified as belonging in the mainstream but whose positioning is revised in the first weeks of first-year university. In practice, Black students wanting to graduate with a science degree belong in this Course. In intent, the Course is designed both for potential mathematics majors and students in other disciplines, but in practice the Course services students who do not pursue studies in mathematical sciences. As noted in Section 2.4.3, in 2005 only 4% of students graduating in the mathematical sciences began

their science degrees in the foundational programme (Buffler & Davidowitz, 2006). While the content of the Course initially talks back to school Mathematics, and then forward to mainstream first-year mathematics, the promotion of the discourse of relevance and learner-centredness draws on reforms in school mathematics and undergraduate calculus.

2.4.5 Mainstream undergraduate mathematics

Consistent with the distinction made in Section 1.2.2, I divide mainstream undergraduate mathematics into *first-year mathematics* and *advanced mathematics*. In the science faculty at the university at which this study was conducted, the two foundational courses are officially regarded as equivalent to the mainstream first-year course and should, theoretically, provide preparation for advanced mathematics.

A mainstream first-year mathematics course

In this section I draw on course material to provide a brief description of the mainstream first-year course to which the two foundational courses are considered equivalent. The course is prescribed for students pursuing studies in mathematical and statistical sciences, physics, chemistry and actuarial science. The course serves a different population of students to the Foundational Course, certainly in terms of performance in the school-leaving examinations; students registered for this course will have scored over 60% in their final higher grade Mathematics examination at school. Those students who enter the course on the basis of these higher marks but who are identified as not coping with the demands of mainstream first-year mathematics are encouraged to change to the Foundational Course after six weeks of the academic year.

The syllabus for this course is presented as a list of mathematical topics, for example:

> Terminology related to real-valued functions of one real variable. Limits and derivatives. Rules for differentiation, implicit and logarithmic differentiation, linear approximations.

This representation specifies the mathematical content to be studied in the course and there is an absence in the course material of explicit instructions on how a mainstream first-year mathematics student should behave in the mathematics classroom. Given that the topics addressed in this course are presented in one year, as compared to two years in the Foundational Course, the pace in the mainstream course is much faster than that in the Foundational Course.[39] Knowledge of school Mathematics is assumed and students are expected to make extensive use of the prescribed calculus textbook (this is the same textbook, representing a more traditional pedagogy, as that prescribed for the Foundational Course). A reading of tutorial and assessment material for this mainstream course suggests that an algebraic approach to calculus is most valued, with some attention paid to graphical representations. Related rates and optimization problems are typical of the practical problems in the course.

Weekly tutorial groups are generally larger than those in the Foundational Course, but with more than one tutor assigned to a group. There is no explicit attention to developing productive group work in these sessions, with the tutor responding to requests by individuals or small groups as required.

A mainstream second-year course in advanced mathematics

Tall (1991b) describes doing advanced mathematics as working with abstract objects which are constructed deductively from formal definitions. Advanced mathematical practice has a vertical structure, with the goal of doing mathematics "vertical growth" (Harel and Kaput, 1991, p.93) by "abstracting from mathematical situations" (Dreyfus, 1991, p.34). Advanced mathematics includes, but is not restricted to, the topic of calculus which is the mathematical focus of this study. Tall (1996) notes that calculus is "both a climax of school mathematics and a gateway to further theoretical developments" (p.289). He presents the topic of calculus in a vertical hierarchy, from intuitive, enactive representations related to

[39] In the individual interviews conducted in this study, the students who had moved from this mainstream first-year course to the Foundational Course identified the pace as the feature that distinguished these two courses.

everyday calculus, to elementary calculus built with numeric, symbolic and visual representations, to formal mathematical analysis which deals with formal definitions and theoretical proofs (which he terms "a higher level of mathematical representation" (p.289)). While the focus of advanced mathematics is on abstraction, the practice may involve solving "applied problems" (Dreyfus, 1991, p.33). This is a horizontal movement in which the objects of advanced mathematics are employed to solve problems in non-mathematical practices.

The syllabus for the second-year advanced calculus course at the university at which this study is conducted is represented as a list of mathematical topics:

> Differentiable functions, independence of order of repeated derivatives, chain rule, Taylor's theorem, maxima and minima, Lagrange multipliers. Curves and surfaces in three dimensions, change of coordinates, spherical and cylindrical coordinates. Line integrals, surface integrals. Stokes' theorem. Green's theorem, divergence theorem.

Like the syllabus for the mainstream first-year course, this syllabus positions the advanced calculus student as someone who does mathematics, and not someone who needs to be told explicitly in course material how to behave. The prescribed textbook for this course is the same as the calculus text prescribed for the Foundational Course and the mainstream first-year course, so the chapters addressing the mathematical topics set out in the syllabus do contain "applied problems" (Dreyfus, 1991, p.33). However, a reading of the tutorial and assessment material for this second-year course points to an absence of such problems. There is a focus in this material on algebraic activity and explanations drawn from within mathematics. Pedagogy in this course is more traditional and weekly tutorials are run in a similar way to those in the first-year mainstream course, as described in the previous section.

In summary, the mainstream first-year course caters for students with higher school Mathematics results than the students in the foundational programme. Yet the formal structures (the opportunity to change from the mainstream course to foundational Course)

represent a recognition that not all of the students with good school results will succeed in mainstream first-year mathematics. The mainstream first- and second-year courses differ in terms of their use of "applied problems" in calculus, but there is continuity in other respects. For example, a mainstream student is identified as a student of mathematics who does not need instructions on how to behave but is expected to be "self-reliant, resourceful, motivated, and 'get on with it'" (Cross et al., 2010, p.76). The pedagogy in the two mainstream courses points to a more traditional pedagogy and a more traditional algebraic approach than the pedagogy represented in the Foundational Course.

2.5 Summary of this chapter

In this chapter I have begun to sketch the social context of the study by using key documents to construct a description of the socio-political practices in which the research site is located. By describing both the schooling system and the higher education landscape in South Africa, I have constructed foundational programmes, and the Foundational Course in mathematics used in this study in particular, as an attempt to bridge the "articulation gap" between school Mathematics and advanced mathematics practice. Certain discourse are at work in this socio-political space. Reforms in mathematics education in South Africa and internationally (mainly at, but not restricted to, school level) point to discourses of relevance and learner-centredness. These two discourses can be linked to others that circulate in the historical context of South Africa, for such reforms are seen to play a role in redress and economic development. Both the school and higher education spaces represent discourses of formal and epistemological access; these are about who has access to education and where different students belong in educational institutions. In the next Chapter I elaborate more on the socio-political space in which this study is located by reviewing selected theoretical and empirical studies that relate to the research problem.

CHAPTER 3
LITERATURE REVIEW

3.1 Introduction to this chapter

Maxwell (2006, p.28) argues that a dissertation literature review is a review "for" research since the literature is used to support and inform choices made in the study. "Relevant" literature is that which has "important implications for the design, conduct, or interpretation of the study" (p.28). Krathwohl and Smith (2005, p.50) describe a dissertation literature review as a select group of studies that provide a "foundation" for the project, presented in enough detail to suggest their relevance, their contribution to the study, and how the study itself moves beyond these.

In this chapter I review selected mathematics education research with a focus on what the research says about the research problem and on what theoretical tools are used to talk about this problem. This literature review serves three functions. Firstly, it serves as a further elaboration of the socio-political space described in Chapter 2, this time drawing on selected mathematics education research literature at school and undergraduate level. The description of this space provides the resources for the interpretation of the research texts. Secondly, this review provides the grounds for my theoretical and methodological choices, that is, how I choose to talk about the research texts. Thirdly, I use this review to identify gaps in the research from both an empirical and theoretical perspective and to locate this study in such a way that it "counts" (Adler & Lerman, 2003, p.446) both for the participants and for the mathematics education research community.

3.2 Choices about what to include in this literature review

The empirical space of this study is the practices of school mathematics and undergraduate mathematics and the innovative pedagogies of relevance and learner-centredness in these practices. This literature review is expansive in this respect. However, the research problem relates specifically to the transition between practices and issues of access. The review is thus selective in terms of focusing on literature that attends to boundary crossings between mathematical practices and between mathematical and non-mathematical practices, and to issues of epistemological access to mathematical practice in innovative pedagogies.

Yet what a research study says about the research problem cannot be separated from the theoretical perspective of the study. Lerman (1998) describes a theoretical perspective metaphorically as a *lens* (p.67); the zoom of the lens allows the researcher to bring some aspects of the space into view and not others. He argues that a language of description has to be informed by the empirical space but also "needs to take account of relations of power, of voice and of silence of any theory" (p.166). Valero (2008) also suggests that the choice of a theory allows one to answer certain questions about the empirical space and not others:

> A choice of theoretical and methodological approach in mathematics education research (or in any research in general) is not an accidental act. … different possibilities are opened and closed by different approaches. (p.56)

On an ontological level, Lerman's metaphor of a lens suggests that the selected theoretical perspective reflects what is in the zoom of the lens or in view. Yet Barwell (2009, p.255) argues that the research process as a whole (including the theories used) is a discursive one; research is not a simple reflection of some external reality, but conducting research involves the interpretation and production of spoken and written texts, for example, interviews, transcripts, conference presentations and journal articles. So the research

process itself discursively construes theoretical perspectives.[40] A number of researchers argue that constructs used in mathematics education research are discursive in nature, for example, *mathematical thinking* (Barwell, 2009), *gender* (Boaler, 2002b), *power* (Valero, 2004) and *success* and *failure* of students (Zevenbergen & Flavel, 2007).[41] Consistent with these arguments I choose not to refer to what a theoretical perspective allows the researcher to *view* in the space, but rather to how a perspective enables one to *talk about* the space. This chapter thus serves both as a review of what existing research says about the research problem and of what the theoretical perspectives used in this research are able to talk about.

3.3 Research on advanced mathematics

Research on the learning of advanced mathematics[42] is included in this review for two reasons. Firstly, foundational practice in this study should theoretically provide epistemological access to advanced mathematics in the second year of undergraduate study. Secondly, the theoretical perspective from which this research is conducted and which came to prominence in the 1980's still dominates mathematics education research at undergraduate level (e.g. Biza & Zachariades, 2010; Habre & Abboud, 2006; Maharaj, 2010; Semadeni, 2008).

[40] Barwell's (2009) argument is consistent with the socio-political perspective of doctoral research practice that I set out in Section 5.2. I use the word *construe* rather than *construct* here as it is consistent with the critical realist ontology on which this perspective is based (as discussed in Section 4.2.1).

[41] What these researchers have in common is that they talk about these constructs as discursive construals. However, they do not necessarily agree on the ontological status of what these construals talk about. I specify the ontology and epistemology used in this study in Section 4.2.1.

[42] Advanced mathematics is commonly referred to as *advanced mathematical thinking* in the research literature (e.g. the edited volume by Tall, 1991a). In this thesis I use the term *advanced mathematics* for this practice.

3.3.1 An ontological/psychological perspective

Research on advanced mathematics draws theoretically on both the ontology of mathematics, that is "the nature of mathematical entities" (Sfard, 1991, p.8), and on constructivist psychology and how these entities are "perceived by the thinker" (p.8). A number of theoretical tools were developed and used extensively in the 1980s and 1990s to study both mathematical objects and what thinking is required of a student in order to understand and use these objects.[43] Commonly used tools are *APOS (Action, Process, Object, Schema) theory* (Dubinsky, 1991), the theory of *reification* (Sfard, 1991), *procept* (Gray & Tall, 1994), *concept definition* and *concept image* (Vinner, 1991), and *epistemological obstacle* (Sierpinska, 1992). Common to these tools is an underlying ontological perspective that distinguishes mathematical objects and an individual's mental representations of these objects.[44] Dreyfus (1991) describes mental representations as "internal frames of reference or frames of reference which a person uses to interact with the external world" (p.31). Also common to these tools is a perspective that learning and doing mathematics involves working with these mental representations (variously called *objects*, *processes*, *conceptions*, *schema* etc. in the literature) and making cognitive shifts in order to move from one mental representation to another.

3.3.2 The transition from school calculus to advanced calculus as overcoming cognitive obstacles

Tall (1996) argues that the vertical movement from intuitive approaches to elementary calculus to formal analysis, "requires significant constructions and reconstructions" (p.293). So the vertical movement that characterizes the transition to advanced mathematics involves mental reconstructions on the part of the individual student. The research

[43] Some of the theorists whose work is mentioned in this section have made other theoretical turns. For example, the work of Anna Sfard is mentioned here, but also later in this thesis with reference to her contributions to the social and discursive turns.

[44] The word *concept* is used in some of the literature reviewed here but I have chosen to use *object*, consistent with the definition given in Section 1.2.1 (while acknowledging possible differences in views on the ontology of these objects).

foregrounds the difficulties students have in developing the required mental constructions when working with the objects of calculus, such as variable, function, limit, derivative and integral (e.g. Artigue, 2000; Eisenberg, 1991; Cornu, 1991, Sierpinska, 1992).[45] In particular the move from intuitive methods (as in school and first-year university calculus) to a logical-deductive system (typical of advanced calculus) is identified as a "difficult transition" (Tall, 1992, p.495). Given the ontological/psychological perspective that informs this research, the boundary is viewed as a "conceptual gulf" (Tall, 1996, p.296) and crossing this boundary involves making cognitive shifts. Hence the representation of this boundary crossing as overcoming *cognitive obstacles* (these may be *psychological*, *didactical* or *epistemological obstacles*) (Cornu, 1991, p.158). In the rest of this section I refer to selected research, conducted using a variety of the tools listed in Section 3.3.2, that talks about the mathematical objects in view in this study.

Process/object view of mathematical objects

Sfard (1991, p.4) argues that a mathematical object has *ontological duality* in that it can be interpreted both as a process or action (the *operational conception*) and as an object with a static structure (the *structural conception*). She argues that these two conceptions are "complementary" (p.4) and that having both conceptions is "indispensable for a deep understanding of mathematics" (p.5). Sfard (1991) argues that an operational conception of a mathematical object should precede the structural conception. She also suggests that students who do not experience an operational conception may develop a *pseudostructural conception*, in which case a representation such as a graph or symbolic notation comes to stand for the object itself (Sfard, 1992, p.74).

In a similar way Gray and Tall (1994, p.121) use the term *procept* to refer to the duality of process and object. The notions of limit, function, derivative, and integral in calculus are

[45] The work of Orton (1983a, 1983b) on students' understanding of differentiation and integration is frequently cited for examples of students' difficulties with calculus. My reading of Orton suggests that his work is descriptive and not informed by theory from advanced mathematics.

identified by Gray and Tall (1994) as procepts. For example, the symbols $\lim_{x \to a} f(x)$ represent both the process of tending to a limit and the value of the limit itself, and the function notation $f(x) = x^2 - 3$ represents both how to calculate the value of the function for a particular value of *x* and the object of a function for a general value of *x* (Gray & Tall, 1994, p.120).

The absence of a dual view of mathematical objects is used to explain students' difficulties with the concepts of calculus, for example, students not identifying functions as equivalent if they are defined by different computational processes and students believing that functions must display some regularity in terms of process (Sfard, 1992). Eisenberg (1991) argues that students do not identify an integral as a function, since they have not developed an object view of a function. In constrast, Tall (1992) refers to an argument by Schwingendorf and Dubinsky (1990) that if a student experiences graphs of functions by looking at graphs drawn on the backboard or by entering expressions into a symbolic computer system that performs calculations for him, he may overlook the function as a process.

Concept image and concept definition

Vinner (1991) distinguishes between *concept definition* which is the formal mathematical definition of an object and *concept image* which is a mental structure consisting of experiences and impressions an individual has of an object. For example, when a student hears the word "function" he might recall a particular visual representation or the algebraic formula for a specific function, say $y = \sin x$ (Vinner, 1991, p.68). Vinner (1991) uses the term *evoked concept image* (p.68) for the image that is evoked in a particular situation.

Many of the difficulties students have with the concepts of calculus have been explained in terms of a students' concept image not matching the related concept definition, for example, the view that functions not given by an algebraic rule are not functions unless they are given a name or special notation by the mathematical community, and the belief

that the graph of a function should be "regular, persistent, reasonably increasing" (Vinner, 1991, p.75). Tall (1996) identifies other concept images from the research on functions and calculus, for example, that a function should be given by an algebraic formula, that a function of x must include an x in the formula, and that a graphical representation of a function should have a recognizable shape (eg. polynomial, trigonometric, exponential etc.). Tall (1992) also refers to research by Markovits, Eylon and Brickheimer (1988) suggesting that students easily evoke concept images of linear graphs. Eisenberg (1991) attributes students' failure to link different representations of a function to the fact that they do not have the concept image of a function as a graph.

Focusing on the mathematical object of limit, Cornu (1991) suggests that the use of everyday words to describe this object results in concept images that can conflict with the formal definitions required in advanced mathematics. For example, the term "tends to" can be interpreted as "to approach … without reaching it", or "to approach … just reaching it", or "to resemble". The term "limit" can be regarded as "an impassable limit which is reachable", or "impassable limit which is impossible to reach", or as "a maximum or minimum" (Cornu, 1991, p.154). Artigue (2001) argues that the everyday limit suggests the limit is a "barrier" (p.211).

The research presented here points to a number of valued relationships within mathematical practice itself; these are links between mental representations of mathematical objects. Firstly, a student should be able to switch between operational and structural conceptions. Secondly, a student should link mental representations of different mathematical objects, for example the integral as a function or the integral and derivative as inverse processes. Thirdly, "switching" or "integrating" across symbolic representations (for example symbolic notation, graphs and words) is valued as necessary for the vertical process of abstraction (Dreyfus, 1991, p.32). Overall, successful use of a concept in problem solving requires multiple mental links, that is, "the various representations are correctly and strongly linked" in "rich mental representations of concepts" (Dreyfus, 1991, p.32).

3.3.3 Links between mathematical and non-mathematical practices

The focus in advanced mathematics on vertical progression and abstraction means that when the research talks about non-mathematical practices, significance is given to the one-way movement from non-mathematical to mathematical practices. Crossing the mathematical/non-mathematical boundary involves making cognitive shifts. As noted in Section 3.3.2, Tall (1996) describes the difference between practical calculations or manipulation of symbols in calculus and formal analysis in calculus as "a wide conceptual gulf" (p.296). Vinner (1991, p.73) frames this in terms of different "thought habits" in different practices; he argues that "everyday life thought habits" are different from those of "technical contexts" such as mathematics.

The second relationship between mathematics and non-mathematical practices is a horizontal one. Dreyfus (1991) refers to the process of "translating" between representations, or what he describes as "going over from one formulation of a mathematical statement or problem to another one" (p.33). The example he provides is "applied problems" (p.33). This process of "translating" is represented as different from the process of modelling in applied mathematics (p.34).[46] Dreyfus (1991) provides an example of such an applied problem:

> … a second order linear differential equation with constant coefficients can be presented as an oscillation problem, possibly with friction; its solution then may be discussed in terms of permanent and transient states". (p.33)

The use of the word "presented" in this description suggests that the mathematical object (the differential equation) can be represented unproblematically as "oscillation". This perspective does not take into account any movement of meaning across the mathematical/non-mathematical boundary. For the student, crossing the boundary requires

[46] In this thesis I do not refer to the extensive literature on mathematical modelling. The analysis of the practical problems as presented in Chapter 7 suggests that these problems share features with mathematical word problems and not modelling problems. This was identified in the pilot for this study, as reported in Le Roux (2008a).

"the construction of the appropriate mental schemata", a mental process of translating that is a "difficult task" for students (Dreyfus, 1991, p.33). The teacher, for whom the correspondence is assumed to be "obvious" (p.33), should make this explicit to students. The assumption that the difficulties students have crossing the mathematical/non-mathematical boundary can be ameliorated by specific pedagogical strategies is also suggested by Eisenberg (1991) when he discusses word problems; he argues that the rephrasing of simple word problems can "greatly" (p.150) affect student performance.

3.3.4 What does research on advanced mathematics have to offer my study?

In this discussion I draw on the student action on the Flu Virus Problem presented in Section 1.2. The ontological/psychological perspective used in research on advanced mathematics talks about mathematical objects, the relationships between these objects, and student action on these objects. In the terms used by Adler and Lerman (2003), "the zoom of the lens is tightly on the mathematics" (p.445). So this perspective has something to offer my investigation in terms of talking about how Siyabulela and Mpumelelo act on the function, derivative function and limit in Transcript 1.1.

However, in the rest of this section I argue that other aspects of the research problem in this study are not in view (or in my terms, talked about) when adopting an ontological/psychological perspective.[47] Firstly, the conceptualization of the boundaries between practices as cognitive obstacles potentially leads to a particular positioning of the student who fails to make the necessary mental reconstructions. So Siyabulela and Mpumelelo's difficulties evaluating the limit $\lim_{t\to\infty} P'(t)$ could be attributed to their not having the correct cognitive thinking ability or "mathematical attitude" (Dreyfus, 1991, p.27), or not having the motivation to struggle when overcoming ontological shifts (Sfard,

[47] I stress at this point that my focus here is on the limitations of the ontological/psychological perspective for talking about my research problem and is not specific to the researchers who are quoted. Again, I use the example of Sfard who has taken different theoretical turns, more recently using theoretical perspectives that allow me to talk about the problem in a way that moves beyond the ontological/psychological perspective.

1991). My choice to avoid such a positioning of these students in this study is located within my argument about the quality of this research and making it "count" (Adler and Lerman, 2003, p.446) for the participants (and discussed further in Section 6.4). Adler and Lerman (2003) warn that the selection of a perspective might produce a description of failure as located in students, and hence not only harm the individuals but also the space in which they act.

Secondly, given the focus within advanced mathematics on vertical progression and abstraction, there is little talk within the research about the relationship between mathematical and non-mathematical practices, suggested by Zevenbergen's conclusion to a review of the use of "contextualized" problems in undergraduate mathematics in 2001; "… there appears to be no research conducted at the tertiary level" (p.22). So this perspective does not allow me to talk about how the students link the function and derivative function to the spread of the flu virus in the community when solving the Flu Virus Problem.

Thirdly, in terms of language, the research on advanced mathematics talks about students' use of vocabulary and symbolic notation as signifiers for their underlying mental conceptions. This perspective does not allow me to talk about the students' language-use in Transcripts 1.1 and 1.2 more broadly in terms of other forms of semiosis such as gestures and ways of arguing and evaluating within a particular mathematical practice. It also does not allow me to talk about how the students link different texts, for example in Transcript 1.1 Mpumelelo makes links to other texts in the Foundational Course when he suggests how they "usually" act on derivatives; "we usually give the exact time" (line 704). This research also does not allow me to talk about how the students use language to identify themselves as certain types of mathematics students, for example how Siyabulela identifies himself as a student who challenges the authority of the Tutor in Transcript 1.2.

Fourth, the ontological/psychological perspective talks about the social in a particular way. While the mathematical practices are both social and historical, learning is an individual cognitive activity and the social provides the context for this activity, that is, the social

provides "a spark that generates or stimulates an individual's internal meaning-making activity" (Lerman, 2000, p.23) or is regarded as "mechanisms for learning" (Sfard, 1998, p.7). My reading of more recent research on advanced mathematics suggests that, while the social is talked about more, much of this research talks about the social in a way that is consistent with the perspective described here (e.g. Harel & Sowder, 2007; Schwarz, Dreyfus & Hershkovitz, 2009). Such a perspective does not allow me to talk about the action of the students in Transcripts 1.1 and 1.2 as part of a social practice in mathematics, action that gives meaning to and is given meaning by its location in a network of social practices.

Lastly, although the research on advanced mathematics does not refer explicitly to issues of power, I argue that the literature points to a view of mathematics and mathematics learning as having "intrinsic power" (Valero, 2008, p.46), that is, advanced mathematics is seen as being powerful knowledge that is worth having. For example, Tall (1991c) argues that, "*Advanced Mathematical Thinking* has played a central role in the development of human civilization for over two millennia" (p.xiii, emphasis in the original) and motivates for the need for more research on the learning of advanced mathematics so that "the average student in an advanced mathematical course" (p.xiii) can gain access to it. As noted in Section 3.3.3, the teacher is assumed to possess this powerful knowledge and has the role of making it explicit to the student. Lerman (2001) points to the limitations of perspectives from mathematics and psychology, that is, they do not allow us to address the view of schooling as reproduction and the role of culture and power in the mathematics classroom. The research on advanced mathematics does not allow me to explain Siyabulela and Mpumelelo's action with reference to the power relations in the classroom and the power relations in the wider socio-political space.

In summary, the ontological/psychological perspective commonly adopted in research on advanced mathematics gives me a partial and particular view of the student action on the practical problems. While this perspective allows me to talk about the student action on mathematical objects, it does not allow me to talk about action in a mathematical practice

that is also discursive, social and political or about the boundary between mathematical and non-mathematical practices. This perspective locates a student's difficulty acting mathematically in the mental conceptions of the student and does not explain this difficulty with reference to the classroom action or the wider socio-political space.

3.4 Research on calculus reform

The research on calculus reform is singled out in this review for two reasons. Firstly, as noted in Chapter 2, calculus reform ideas were drawn on extensively in the conceptualization of the Foundational Course. Secondly, as suggested by the description of calculus reform in those chapters, calculus reform as a practice represents some discontinuities with respect to advanced mathematics. Calculus reform curricula give significance to the horizontal movement between mathematical and non-mathematical practices, the relationship between representations of the mathematical objects of calculus, social relationships between students when learning mathematics, and the role of reading and writing in learning. The question is whether the research on calculus reform expands on or shifts the resources used in the research on advanced mathematics when talking about these features of the reform.

Robert and Spear (2001) argue that the manner in which calculus reform curricula were designed was pragmatic and not research driven. Referring to the Foundational Course in which this study is located, Bowie (2000) notes that the adoption of the reform ideas in the Course was initially driven by "intuitive appeal" (p.2). Robert and Spear (2001) suggest that research conducted into the teaching and learning of calculus using the reform approach was dominated by experimental studies that evaluate reform teaching approaches by comparing them to traditional approaches to teaching calculus. For example, Garner and Garner (2001) compare the understanding and retention of students who experienced traditional and reform calculus classes. Four of the ten test items used in their study make links to non-mathematical practices. Each of these problems is described in terms of the valued mathematical thinking, with this thinking being classified as "procedural" or

"conceptual". In a "conceptual" test item that requires the student to make a link between a function, its derivative and the fuel consumption of a car travelling on a freeway, the description of the required thinking points to a to-and-fro movement between non-mathematical and mathematical approaches:

> … it requires students to think about the sign and the magnitude of the derivative of a function from everyday-life. Students needed to realize that the slope of $f(x)$ was positive for all positive x, and to make the inference that the motor home used more gas, and thus its graph would be above the other graph. (Garner & Garner, 2001, p.174)

This description of a to-and-fro movement across the mathematical/non-mathematical boundary contrasts to the one-way movement from the non-mathematical to the mathematical that I have argued in Section 3.3.3 dominates the talk about boundary crossings in the advanced mathematics research.

Garner and Garner (2001) report on different outcomes in the reform and traditional classes, an argument that suggests that different mathematical action may be valued in the two curricula; the reform students retained "better conceptual knowledge" (p.180) and "seemed more confident in trying to explain things, talked more about applications of calculus, and used graphical explanations more" (p.179). Students' "imprecise descriptions of mathematical ideas" (p.177) are explained in terms of a lack of vocabulary to adequately describe their thinking. However, given the absence of a theoretical perspective to talk about these observations, this study by Garner and Garner (2001) remains descriptive. Robert and Spear (2001) argue that evaluation studies serve only to identify interesting phenomena to be pursued in more detail.

Given the emergence of calculus reform during the dominance of the research on advanced mathematics, the research that does pursue the detail mentioned by Robert and Spear

(2001) is informed by the ontological/psychological perspectives described in Section 3.3.[48] Thus although calculus reform represents discontinuities in relation to the practice of advanced mathematics, the research does not offer more in terms of how to talk about the reforms that are of interest in this study. To support this argument I discuss research that was conducted in the initial calculus reform version of the Foundational Course (as described in Section 2.4.4).

Bowie (2000) uses APOS theory from Dubinsky (1991) and the concept of reification from Sfard (1991) to analyze foundational students' errors. She argues that the students try to build an understanding of calculus on a pseudostructural approach to algebra, evidenced by students treating letters as objects, treating symbols as objects while ignoring the processes behind the symbols, and manipulating symbols with no regard for their meaning. Bowie (2000) argues that, with only "pseudo-objects" (p.9) to work with, students develop rules for solving problems, and that these rules have no meaningful link to the mathematical objects on which they are performed. As a result links between objects, for example between the integral and the derivative, can only be based on surface features. She argues that this leads students to over-generalize, for example, expecting all graphs to be smooth and regular and assuming that a shape should be represented in a standard orientation. Students also make connections on the basis of textual cues or on the basis of the form of the written symbols.

Since Bowie (2000) draws on Sfard's theory of reification in her study, she locates the students' difficulties in the students themselves, pointing to the "motivation" and "determination, stamina, and intellectual discipline" required to develop a meaningful understanding of calculus concepts (Bowie, 2000, p. 13, quoting from Sfard, 1992, p.84). Bowie's (2000) view of the "social" here refers to the social context as a mechanism for

[48] It is not always possible to identify research on undergraduate mathematics conducted in the 1980s and 1990s, for example that involving multiple representations and the use of technology, as lying firmly within calculus reform. So in this review I focus on research papers in which the researchers explicitly locate their work in the reform calculus movement.

learning; she argues for "the kind of learning environment that encourages students to persevere with the intellectual effort that is required to achieve this" (p.13). Bowie (2000) attends briefly to issues of language in the study, and consistent with the research in advanced mathematics, views language as representing the students' inadequate mental structures. For example, she argues that "… the difficulty students have in expressing themselves clearly could also be interpreted as reflecting an inadequate grasp of the subject matter" (p.12) and particularly the rule-bound nature of this thinking. These difficulties are also attributed to the fact that English is not the first language of students. While her writing on language focuses on the notion of explanation in "the discourse of mathematics" (p.12), Bowie (2000) does not explore in any detail the language use particular to the foundational practice. Bowie's (2000) study does not attend specifically to the practical problems in the Course, yet she expresses a concern that the action she has identified in her study will not allow students to "link, extend and apply" (p.13) their knowledge. This phrase could be interpreted as referring to links to non-mathematical practices.

In this section I have reviewed two studies related to the learning of calculus in curricula informed by the ideas of calculus reform; one is an evaluation study that compares two groups of students while the other has a stronger theoretical orientation which draws on research in advanced mathematics. I end this section by referring to an overview paper on calculus reform. Having reviewed the research literature that I have discussed in Section 3.3, Ferrini-Mundy and Graham (1991) provide possible areas for further research. I present some of these areas for research as examples of how these authors talk about the research problem in this study and to reinforce my argument that the research on calculus reform does not offer anything new in terms of talking about this problem. Firstly, questions related to the link between mathematical and other practices suggest that, on the one hand, practical problems are used in the service of a vertical movement into mathematical practice ("Do physical examples help in the learning of concepts?", p.633). On the other hand mathematics is presented as a tool to be applied in other disciplines ("Is a strong conceptual base adequate for applying the ideas of calculus in new contexts within the sciences and engineering?", p.33). Social interaction, for example working in groups, is

expressed as a possible mechanism to enhance learning ("Can students learn calculus by working in pairs or groups?" p.633). The text-related questions talk about reading and writing as tools in the service of learning rather than as giving meaning to the practice ("How do students use calculus textbooks?", and "How does the use of writing enhance calculus learning?" p.633).

In summary, calculus reform curricula represent a move with respect to the practice of advanced mathematics in terms of the emphasis on a horizontal movement between mathematical and non-mathematical practices, multiple representations of mathematical objects, social interaction, and using reading and writing for learning. However, the research on this reform remains within an ontological/psychological frame, with the result that mathematical rather than discursive, social and political action is in view, difficulties are located in students' conceptions and motivation, and the boundaries between practices are not in view. In the next section I consider a perspective from within school mathematics that recognizes the mathematical/non-mathematical boundary.

3.5 School mathematics: The ideas of Freudenthal and Realistic Mathematics Education

The ideas of Freudenthal (1973) and the take-up of these ideas in mathematics education at school level are included in this review for what they say about the relationship between mathematical and non-mathematical practices. Freudenthal (1973) proposes that mathematics learning will only be retained if the mathematics that is taught is "fraught with relations" (p.79). He distinguishes between relations within mathematics and relations outside of mathematics, the latter relations suggesting that the mathematical/non-mathematical boundary is visible in Freudenthal's work. Freudenthal (1973) argues that "reality is the framework to which mathematics attaches itself" (p.77). This argument suggests that mathematics can reflect reality, and thus that the boundary crossing is unproblematic. Freudenthal (1973) distinguishes two types of "reality"; he stresses the importance of making connections between mathematics and what he terms the *lived-*

through reality (p.77) of the learner, a reality that can be distinguished from *dead-mock reality* (p.78) that has been invented with the only purpose to serve as an example in a mathematics problem.

In the following quote Freudenthal (1973) identifies "barriers" between mathematics and its applications, but argues that these barriers have been constructed by mathematicians and can be removed by mathematics teachers ("we"):

> If we wish to teach the pupil to apply mathematics we should make it easier for him to apply it. We should break the barriers surrounding mathematics. We should apply mathematics as much as needed in other sciences, in order to teach how it is applied. … it is fair to turn again to the mathematicians who have isolated mathematics. (pp.72-73)

This quote represents mathematics as a tool for solving applications; mathematics is learned first, and then the "concrete problem" comes later as an application (Freudenthal, 1973, p.132).

However, Freudenthal gives particular significance to the movement from the everyday to mathematics:

> Negative numbers should start at the lever if they should be applied to the lever. Logarithms should start with the slide rule or with air pressure, or with the hyperbola if it should be applied there, the derivative with velocity, density, and acceleration, and the linear function with all those proportionately in nature and society that everybody must become acquainted with. (p.133)

Freudenthal (1973) uses the word "mathematizing" for the process of organizing mathematical or non-mathematical matter into "a structure that is accessible to mathematical refinements" (p.133).

Freudenthal's work has been used as the basis for a theory of teaching and learning school mathematics called *Realistic Mathematics Education*. According to Treffers (1987), reality

is regarded as the "source of concept formation" (p.246) in the realistic approach. Instructional activities begin within a "concrete context" (p.248), giving the student an opportunity to develop intuitive notions as a basis for concept formation. Learning is a process of progressive mathematization in which students engage in both *horizontal* and *vertical mathematization* (p.247). Horizontal mathematization is about crossing the mathematical/non-mathematical boundary; this involves "transforming a problem field into a mathematical problem" using "model formation, schematizing, symbolizing" (p.247). Vertical mathematization involves "processing within the mathematical system" (p.247).

The theory of Realistic Mathematics Education makes the mathematical/non-mathematical boundary visible. Yet the boundary crossing is presented as unproblematic for the student (with support from the teacher). For example, Barnes and Venter (2010) describe horizontal mathematization using the metaphor of "a bridge that assists one in crossing to the other side" (p.7). This metaphor suggests that horizontal mathematization is indispensable for and enables vertical mathematization. Barnes and Venter (2010) also propose that the ideas of Realistic Mathematics Education should be adopted in higher education.

Arcavi (2002) critiques the idea that "mathematization appears to be a one-way from the everyday to the academic" (p.22). He argues for an additional process, *contextualization*, which runs in the opposite direction to mathematization:

> In order to make sense of a problem presented in academic dress, one can remember, imagine, or even fabricate a context for that problem in such a way that the particular features for that context provide a scaffolding for and expand one's understanding of the mathematics involved. (p.22)

Arcavi argues that both processes, mathematization and contextualization, are "necessary when one wants to connect the academic meaningfully with the everyday" (p.22) and that the two processes complement one another. Although Arcavi does not problematize the movement of meaning between practices and limits the non-mathematical contexts to those that one can "remember, imagine, or even fabricate", his argument that the

mathematical/non-mathematical boundary crossing is not necessarily a one-way process is revisited in the results of this study.

So the ideas of Freudenthal (1973) and Realistic Mathematics Education offer something other than what is available as a resource in advanced mathematics and calculus reform research. For this work makes the boundary between mathematical and non-mathematical practices visible. However, it considers the boundary crossings as unproblematic, an issue that is taken up in the literature reviewed in Section 3.6.

3.6 School mathematics: A general "turn" away from an ontological/ psychological perspective

In this section I review reform-oriented school mathematical practice and related research that emerged from the late 1980s onwards. While the main focus of this study is not on school mathematics, attention to this practice is necessary for two related reasons. Firstly, as noted in Chapter 2, it is in reform-oriented versions of the school mathematics curriculum that the boundaries between mathematical practices and other practices have become blurred. Secondly, it is in mathematics education research at school level that *turns* have been made away from the ontological/psychological perspective.[49] Here I point to three related turns that have been identified in research since the 1980s; the *social*, *discursive* and *political* turns. Thirdly, unlike the work of Freudenthal (1973) and Realistic Mathematics Education (e.g. Barnes & Venter, 2010; Treffers, 1987) reviewed in Section 3.5, the research on practical problems located within these turns problematizes mathematical/non-mathematical boundary crossings

I acknowledge that attempting to name and describe these turns in a linear text such as this is problematic. For it represents the perspectives as fixed and sets up unnecessary

[49] I note, however, that much of the research at school level (including research that focuses on reforms) still talks from an ontological/psychological perspective.

boundaries between them. For example, the sociological perspective of Bernstein (1996, 2000) discussed in this chapter represents aspects of all three turns. In addition this brief discussion certainly does not do justice to the contested debates within mathematics education research around the nature of these turns. However, for the purposes of this review a brief elaboration of my interpretation of the three turns is required.

Lerman (2000) identifies the emergence in the late 1980s of the *social turn* (p.23) in mathematics education research out of three disciplines anthropology, sociology and cultural psychology; a trend which saw a move away from a focus on learning as the acquisition of knowledge by the individual to "theories that see meaning, thinking and reasoning as products of social activity" (Lerman, 2000, p.23). Social practices are not just the context for learning, but learning mathematics involves participating in a mathematical practice and taking on the mathematical identity of the practice.

Barwell (2008), Lerman (2009), Morgan (2006), and Sierpinska (2005) identify a perspective on language that is variously known as the *discursive* / *language* / *linguistic turn* in mathematics education research. This perspective talks about language-in-use or discourse. Mathematics is regarded as a discourse or a number of discourses, learning involves coming to participate in a particular mathematical discourse by using the "meanings of communication, the patterns of communication, and the genre of the discourse" (Sierpinska, 2005, p.211).

Lastly, consistent with Valero (2004), I refer to a *political turn* in mathematics education research. Lerman (2000) suggests that many researchers were receptive to the social turn precisely due to concerns about the role of culture and politics in schooling. So in this sense the word "social" also has a political dimension. While Valero (2004) does not dispute this, she argues that the social turn does not necessarily include a conception of *power*. Studies of mathematics education located within this turn tend to draw on Marxist and/or Foucouldian perspectives of power. These studies focus on power relations in the network of socio-political practices in which the mathematics classroom is located (e.g. Valero,

2007) and/or the power relations between participants within the classroom (e.g. Lerman & Zevenbergen, 2004).

In the sections that follow I focus on specific theoretical and empirical work on the use of practical problems in school mathematics that are conducted from one or more of the three perspectives described here. The selection is based on what the research says about my research problem and on what the underlying theoretical perspective allows the researcher to say about the research problem.

3.7 Sociological perspectives that foreground boundaries between practices

In this section I present pertinent aspects of the work of the educational sociologist Basil Bernstein (1996, 2000) and of Paul Dowling (1998, 2009) who has applied and developed Bernstein's work specifically for mathematics education. I also refer to selected empirical studies on the use of practical problems[50] that use and develop the work of these two theorists.[51]

3.7.1 Relationships between practices as recontextualization

Bernstein (2000, p.157) identifies esoteric or academic knowledge (such as academic research mathematics) as *vertical knowledge discourse* and everyday, context specific knowledge as *horizontal discourse.* From this perspective, academic research mathematics and everyday practices are structured differently, have different organizing principles, have

[50] These studies name these problems variously as *realistic items*, *realistic problems*, etc. I refer to them consistently as practical problems, that is, mathematics problems that make links to non-mathematical practices.

[51] These empirical studies draw on various versions of Bernstein's work. In this section I refer mainly to the relevant theoretical concepts as explained in Bernstein (1990), Bernstein (1996) and Bernstein (2000).

different social relations, and are acquired differently.[52] School mathematics and undergraduate mathematics are, according to Bernstein (2000, p.33), *pedagogic discourses* and they differ from the discourse of academic research mathematics. The relationship between discourses is one of *recontextualization* (p.33), since knowledge in one discourse is delocated from the principles that underlie that discourse and is relocated in the recontextualizing discourse, subject to the principles of the latter discourse. The concept of recontextualization problematizes boundary crossings between practices (whether mathematical or non-mathematical), since there is a movement of meaning during this boundary crossing. In Section 4.2.4 I present the perspective of Fairclough (2003, 2006) on the boundary crossing, which draws on Bernstein's work as described here.

Dowling (1998) uses the concept of recontextualization (Bernstein, 1990, 2000) to expose as *myths* the notions of *relevance* and *participation* that are espoused in reform-oriented curricula. According to Dowling (1998) the *myth of relevance* constructs mathematics as being "about something other than itself" (p.4) and hence powerful in the sense that it can be exchanged for a range of other activities. The *myth of participation* constructs mathematics as being "for something else" (p.9) and is presented as "a necessary condition for optimizing of participation in apparently non-mathematical practices" (p.33). These notions are exposed as myths by the principle of recontextualization, since the everyday is recontextualized under the *gaze* (p.121) of school mathematics. From this perspective, the practical problems in this study are problems in a pedagogic practice in mathematics. They are not for participation in non-mathematical practices, but due to their location in the Foundational Course should, in theory, promote participation in advanced mathematics.

3.7.2 Power between practices

Bernstein (2000, p.6) argues that discourses are defined by the spaces or boundaries between them. He uses the term *strong classification* for strong insulation between

[52] Bernstein's (1990) use of the term *acquisition* broadly in relation to the rules of pedagogic discourse differs from the way that the term is commonly used in education to refer to an accumulation of knowledge (e.g. Sfard, 1998).

discourses and *weak classification* for discourses that are less specialized (p.7). Using this terminology, the practical problems represent foundational practice as weakly classified. The description of the second-year advanced mathematics course in Section 2.4.5 suggests that this practice is strongly classified with respect to school mathematics and non-mathematical practices. Bernstein (2000, p.7) argues that it is power that preserves the insulation, so "classifications, strong or weak, always carry power relations" (p.7). When there is a change in the strength of classification, for example, a weakening that creates a new discourse, ideology is at work; "Everytime a discourse moves, there is a space for ideology to play" (p.9). Since foundational practice represents a "move" in the discourse of undergraduate mathematics practice, I need to be able to talk about the power relations at the macro-level when addressing the research problem.

3.7.3 Controlling the movement of meaning between practices as recognizing the boundary

Straehler-Pohl (2010) uses Bernstein's work to analyze the teacher-student interaction in a high school mathematics classroom in which the students are solving a practical problem. The problem refers to the border between two farms, but this serves as an application of linear equations in school mathematics. Straehler-Pohl (2010, p.449) identifies the classification (Bernstein, 2000) between the mathematical and non-mathematical as strong; the teacher is aiming for vertical mathematization (as soon as possible), and the valued action involves making mathematical arguments and using mathematical language. Staehler-Pohl (2010) uses Bernstein's term *framing* to describe the pedagogic practice in the classroom where this problem is solved; framing refers to "how meanings are to be put together, the forms by which they are to be made public, and the nature of the social relationships that go with it" (Bernstein, 2000, p.12). Straehler-Pohl (2010) argues that in the first part of the discussion the teacher's framing is weak as he uses everyday language and does not make the boundaries between the everyday and the mathematics explicit. Thus the rules for recognizing the boundary and the differences between the discourses are not explicit. Straehler-Pohl (2010) identifies some students as not possessing the *recognition rules* (Bernstein, 2000, p.17) that allow them to move into the vertical discourse; one

student relies on a visual representation of the non-mathematical context, one student uses a practical argument that would be of use to the farmers, and another asks for more information about the non-mathematical context as she is having difficulty imagining the real situation. Straehler-Pohl (2010) identifies students who get "stuck" (p.454) in the everyday.

Straehler-Pohl (2010) uses this analysis to argue that "a crucial condition" for solving a practical problem in the mathematics classroom is "the ability to *recognize* the boundary and its strength between the vertical and horizontal discourse" (p.449, emphasis in original). In this study I will be arguing that knowing *how* to make this boundary crossing involves more than just recognizing the boundary, and that a student also needs to know *when* to cross the boundary. I will also be problematizing Straehler-Pohl's (2010) representation of a one-way movement from the non-mathematical to the mathematical (which was also suggested by Freudenthal (1973)). Rather, I argue that solving the practical problems in this study involves a to-and-fro movement across the mathematical/non-mathematical boundary. In this study I regard this movement as a particular way of making links between practices which is specific to the discourse of foundational mathematics practice. Knowing that solving a practical problem in the practice of foundational mathematics involves this to-and-fro movement could also be described as possessing the *realization rules* (Bernstein, 2000, p.16) of the practice, that is, "how we put the meanings together" in the practice.

3.7.4 Representing the non-mathematical in pedagogic texts in mathematics

Bernstein's (1990, 2000) work on classification has been used in mathematics education to describe school mathematics texts. In his earlier work, Dowling (1998) analyzed texts according to both the strength of classification of content (the signifieds) and the strength of

classification of expression (the signifiers).[53] In his more recent work, Dowling (2009) replaces classification with the concept of *institutionalisation* which is "the extent to which a practice represents an empirical regularity that marks it out as recognizably distinct from other practices" (p.81). I choose to use the original term classification in this literature review. Dowling (1998, p.141) argues that the regulating principles of the domain of school mathematics are located in the *esoteric domain*, but that the "initial hailing" of the student has to take place outside of this domain. Pedagogic action should then proceed "via the construction of metonymic chains" (p.141) into the esoteric domain by making accessible the regulatory principles of this domain. This action is described as a one-way movement from the non-mathematical to the mathematical. The esoteric domain of school mathematics thus casts a *gaze* (p.136) on non-mathematical practices and recontextualizes these practices; "recontextualizing entails the subordination or partial subordination of the forms of expression and/or contents of the practices of one activity to the regulatory practices of another" (p.136).

This recontextualization results in three other domains of school mathematics, depending on the strength of classification of content and expression. Dowling (1998, pp.135-137) is thus able to locate school mathematics texts in one of four domains of school mathematics; the *esoteric domain* (the most specialized), the *public domain* (the least specialized with weak classification of content and expression), the *expressive domain* (strong classification of content, but weak in terms of expression) and *descriptive domains* (weak classification of content, but strong in terms of expression). Dowling's (2009) attention to both content and expression suggests that movement across the mathematical/non-mathematical boundary is not just about the movement of content over this boundary, but also involves a discursive shift.

[53] Dowling (1998) uses the term *content* (p.132) for what is signified. I use the term *task context* (adapted from Ernest (2004, p.76)) for the non-mathematical practice referenced in the problem (acknowledging the recontextualized nature of this practice).

In terms of access to school mathematics, Dowling (1998, 2009) identifies the public and descriptive domains as problematic. Although all four domains are domains in school mathematics (a product of the mathematical gaze during the process of recontextualization), the regulatory principles of school mathematics are not explicit in these two domains; "We might say that school mathematics has no explicit methodology in its constitution of its descriptive and public domains" (Dowling 2009, p.206). I discuss the implications of this for access to mathematics further in Section 3.7.5. However, in this section I argue that this feature of Dowling's framework also makes the classification of mathematics texts in terms of the four domains problematic. I provide examples from the practical problems used in this study to develop this argument.

An initial analysis of the text of the Flu Virus Problem (see Appendix B and Appendix Q) suggests that some of the sentences are weakly classified in terms of expression. For example, no mathematical notation is used in the first sentence, "A flu virus has hit a community of 10 000 people". In addition, an answer that explains the meaning of a function in "practical terms" with no mathematical terms or symbols (for example, "4 days after the first recorded person got flu, 1200 people had the flu" in question (c) of the Flu Virus Problem), is weakly classified in terms of expression. Yet other sentences in the Flu Virus Problem (such as, "Let $P(t)$ denote the number of people who have, or have had, the disease t days after the first case of flu was recorded") use the specialized function notation of school and undergraduate mathematics and are thus strongly classified in terms of expression.

Yet identifying the content in the Flu Virus Problem as weakly or strongly classified is problematic. Consistent with the examples that Dowling (1998, 2009) provides, since the practice of epidemiology[54] referenced in the problem is non-mathematical, the content is weakly classified. Hence the various sentences in the text of the Flu Virus Problem and the

[54] I provide a more nuanced analysis of this representation of the non-mathematical practice in Section 7.2. The detailed analysis is not required for the critique of Dowling's work in this chapter.

required answers to the problem can be located in the public and descriptive domains. However, the spread of the flu virus is based on the concept of a function, which is strongly classified content in the domains of school and undergraduate mathematics. In their analysis of a school mathematics word problem, Gellert and Jablonka (2009) also problematize Dowling's notion of strong/weak classification of content for the analysis of school word problems.[55] They discuss a word problem that refers to a man rowing upstream and downstream in a river, but which is used for the student to practise solving simultaneous linear equations in two unknowns. Gellert and Jablonka (2009) argue that, while the link to the non-mathematical content suggests weak classification of content, the "still water" (p.47) in the word problem represents the strongly classified mathematical concept of continuity.

The difficulty classifying mathematics texts according to Dowling's four domains seems to lie in the weakness in his framework in terms of classifying the strength of the mathematical gaze; while the non-mathematical is recontextualized under the gaze of the mathematical (even in the public domain), the dichotomy of strong/weak classification of content does not allow one to talk about the strength of gaze. Dowling (2009) himself points to this problematic in his recent writing about these domains and he suggests why this occurs, yet he does not develop his framework to take into account the strength of the mathematical gaze:

> *School* mathematics, however, might be described as recruiting to its *esoteric domain* regions of mathematical activity, which are then used in the mathematising of the world to create descriptive and *public domains* (…). School mathematics, then, does constitute empirical referents (to do with shopping etc.). But the 'precision' of the statements that it makes about these referents does not so much depend on the strength of the esoteric domain syntax as on the strength of the syntax of the *gaze* whereby non-mathematical

[55] The focus of Gellert and Jablonka's (2009) critique of Dowling is the extent to which his domains of school mathematics can be used to distinguish between traditional word problems and problems that require mathematical modelling. Nonetheless, their discussion of word problems is appropriate for the discussion of Dowling's work in this chapter.

> objects are consumed by mathematical language as *descriptive domain* text. By and large, this syntax would seem not to be explicit. (p.206, emphasis in the original).

Two studies, one located in South Africa and the other in Lesotho, have developed the work of Dowling (1998) for classifying school mathematics texts. These studies are relevant for this discussion since they both investigate practical problems that represent innovation in the mathematics curriculum.

Sethole (2005) also critique's Dowling's (1998) notion of classification of content, but his focus is on weak classification of content, which he argues fails to take into account "different types of the everyday" (p.41) in terms of how a task context resonates with a student's experiences. In his study of the implementation of the new outcomes-based school curriculum in South Africa, Sethole (2005) refine's Dowling's work to describe four types of "everyday". Firstly, he links his notion of *inauthentic* task context to Freudenthal's (1973, p.78) "dead-mock reality" described in Section 3.6, describing this as an everyday reference that is highly unlikely or impossible. An *authentic* task context (likened to Freudenthal's (1973, p.77) "lived reality" of a student) refers to "a genuine or not far-fetched use of the everyday" (p.41). Furthermore, according to Sethole (2005, p.42), a task context, whether authentic or inauthentic, can be *near* in the sense that a student can relate to the use of the everyday by virtue of it being near in terms of space and time. In contrast, a task context could be *far* (p.42) from a students' experience. Yet Sethole's classification does not talk about how a task context positions the student within the everyday practice, an issue taken up by the post-structuralists (as discussed in Section 3.8) and one which I take up in the presentation of the results.

Gellert and Jablonka (2009) also allude to notions of authenticity and nearness when they argue that "a semantic distance" (p.46) between the student and the non-mathematical practice may facilitate the students' recognition of a problem as residing in the public domain of school mathematics rather than in the non-mathematical practice itself. For they suggest that solving a word problem involves "a two-fold recognition" (p.47), that is,

recognizing that the text is public domain text and recognizing the esoteric domain knowledge that drives the recontextualization of the non-mathematical practice. So in the case of the Flu Virus Problem a student should recognize (a) that the problem is located in the practice of foundational mathematics rather than in epidemiology, and (b) that the spread of the flu virus represents the mathematical concept of function. Gellert and Jablonka (2009) suggest that the first recognition process is easier when the "situation is not part of students' everyday experience" (p.47). Such a situation could be described as far in the language used by Sethole (2005, p.42). Yet while Sethole (2005) argues that a near task context facilitates the movement from the non-mathematical to the mathematical, Gellert and Jablonka seem to be suggesting that recognition of the public domain is easier if the task context is far.

Nyabanyaba (2002, p.207), on the other hand, moves beyond the notion of authenticity in his investigation of the use of practical problems in summative assessments in Lesotho (in the form of a UK-based O-level examination). Beginning with the notions of *familiar* and *authentic* task contexts, he replaces these with three descriptors based on the extent to which the mathematical demands are evident in a problem; *mundane* (the mathematical demands are not obscured), *deep* (the mathematical demands are obscured) and *ritualistic* (the item has become so familiar that the problem becomes ritualized). Nyabanyaba (2002) argues that, "one of the conditions for a realistic context to be accessible is that the context does not obstruct its mathematical demands" (p.214), an argument that refers to the recognition of the esoteric domain knowledge identified by Gellert and Jablonka (2009). In this study I draw on Nyabanyaba's work when classifying the practical problems.

3.7.5 Recontextualization and the positioning of students

Bernstein (2000) suggests that students are positioned by practices when he makes the theoretical argument that the recontextualization of the everyday into school subjects results in social differentiation, in terms of who this recontextualization is for and what form it takes. For example, the recontextualization of the everyday is usually aimed at

facilitating access for the "less able" and is limited to a "procedural or operational level of a subject" (p.169).

Focusing on practical problems in two secondary school mathematics texts, Dowling (1998) suggests that one text constructs the student as a potential participant in the esoteric domain of school mathematics through an initial "interpellation" (p.141) in the public domain, followed by pedagogic action that shifts the student from weakly classified expression and content to strongly classified expression and content. The second text identified by Dowling (1998) constructs the student in a "dependent position" (p.141) as the student is restricted to the public domain and is not given access to the regulative principles of the esoteric domain. Given that the public domain is a recontextualization of the everyday into school mathematics and thus represents a different practice to the everyday, the student is denied access to both the horizontal and the vertical discourse. Dowling (1998) notes that the second text that restricts the student to the public domain identifies the student as having a "short attention span, inability to cope with complex situations and difficulty in following instructions" (p.34).

Dowling's (1998) analysis of the two school mathematics texts raises three questions about the practical problems in this study. Firstly, of interest in this study is whether the practical problems (which I have, with difficulty, located in the public and descriptive domains of Dowling's framework in Section 3.7.4) enable the foundational student to proceed from these domains into the esoteric domain of undergraduate mathematics. Dowling (2009) argues that when students do not have access to the principles of the esoteric domain of school mathematics (which is the case in the public and descriptive domains), they can only deploy "tricks" (p.100) when doing mathematics. He suggests that investigative approaches to school mathematics (which can be located in reform-oriented versions of school mathematics) may be reduced to such tricks. Secondly, Dowling's (1998) analysis suggests that certain practical problems may not only restrict access to the esoteric domain, but also deny access to the non-mathematical practices that they represent. Certainly, Gellert and

Jablonka (2009) suggest that a question requiring a student to explain a mathematical object in "practical terms" is problematic in terms of its relationship to the non-mathematical:

> By the very virtue of the recontextualization principle, which refocuses and selectively appropriates context specific discourse, any meaning within the horizontal discourse is disrupted. Given this disruption, it is highly problematic to have students evaluate their numerical solutions of word problems in terms of the local situational context described in the problem (p.47).

Thirdly, Dowling's analysis of the second school mathematics text points to the need to investigate how the practical problems in this study position the foundational student. In the analysis in Chapter 7 I will argue that the practical problems position the foundational student in contradictory ways, simultaneously constructing him as able/not able to deal with the complexity of first- and second-year undergraduate mathematics.

Bernstein (2000) suggests that the differential positionings that arise from the recontextualization of the everyday into school subjects are related to social class. He argues that "certain distributions of power give rise to different social distributions of recognition rules", and that as a result, children from "marginal classes" (p.17) may not possess the recognition rules for successful participation in school. In the context of assessment, Cooper and Dunne (2000) investigate the performance of 10 to 14 year-old children on practical problems and problems with purely mathematical contexts. They use the concepts of recognition and realization rules from Bernstein (2000, p.16, p.17) and the notion of *habitus* or "feel for the game" from Bourdieu (1987/1990, p.9) to conclude that "appropriate reading of contexts and items is a necessary (though not sufficient) condition for success in tests and that these readings may not be distributed randomly across social structure" (Cooper & Dunne, 2000, p.66). Their empirical work suggests that working and intermediate class children perform less well on the practical problems than service-class children, in comparison with performance on the purely mathematical problems, with the former classes more likely to respond in an inappropriate manner on the practical problems.

Swanson (2005) uses the work of Bernstein and Dowling, not in relation to practical problems in school mathematics, but to investigate a foundational programme designed to give Black students from former Black township schools in South Africa entry into an elite independent school. Like the foundational students in this study, these students are segmented out of the mainstream at the independent school. Swanson (2005) uses the concepts of *instructional* and *regulative discourse* from Bernstein to suggest how race, social class, language and culture on the level of social structure come to figure in students' experience of school mathematics. According to Bernstein (1996, p.48) pedagogic discourse embeds instructional discourse (such as school mathematics) in the regulative discourse of the social order (such as the social order of schooling), that is, how one should behave. Bernstein (1996) argues that every pedagogy is socially regulated; "… all pedagogic discourse creates a moral regulation of the social relations of transmission/acquisition, …" (p.184).

Swanson (2005) argues that texts within the independent school (texts that drew on wider socio-political discourses) recruited markers of social difference such as language and culture in positioning the foundational students as different and as low ability with respect to mathematics. Her analysis of the power relations between discourses in the space suggests that mainstream mathematics was strongly classified in relation to other instructional discourses, including the foundational programme, and that mainstream mathematics held a privileged position in the hierarchy of instructional discourses at the school. Swanson (2005) argues that the social difference and socially constructed disadvantage led to "pedagogic disadvantage" (p.270) as the foundational students were denied access to the regulating principles of the esoteric domain of school mathematics. Since the foundational programme did not provide students with access to the realization and recognition rules of mainstream mathematics, constructions of difference were reinforced.

In this study I am interested in the relationship between foundational mathematics practice and undergraduate mathematics practice at undergraduate level, the positioning of the

foundational student in this space and the implications for access to advanced mathematics. Certainly, my description of different undergraduate mathematics courses in Sections 2.4.4 and 2.4.5 suggests that the Course material for the Foundational Course makes explicit how the foundational student should behave when doing the required mathematics. Such explicit social regulation is an absence in the course material for the mainstream undergraduate courses as this material only lists the mathematical content that is covered in a course and positions the mainstream student as independent and not requiring social regulation.[56]

Bernstein's (2000) theoretical work and the empirical work of Cooper and Dunne (2000) and Swanson (2005) focus on the positioning of the student on the structural level of social class, language etc. This work does not talk about the identity work of the student that takes place at the level of social event within social practices.

Nyabanyaba (2002), on the other hand, takes the sociological perspective presented here forward in a way that is productive for this study. Nyabanyaba draws on Bernstein's (2000, p.16, p.17) concepts of recognition and realization rules and on the use of these concepts by Cooper and Dunne (2000), but adapts them for use in a post-colonial setting. Nyabanyaba (2002) argues that gaining epistemological access to mathematics involves not only having the recognition rules to recognize the non-mathematical/mathematical boundary, but also about recognizing how mathematical competencies in general have been recontextualized. He gives the example of items in the imported O-level examinations that require students to "explain" and "show". Both are mathematical practices that he argues are not common in Lesotho schools. This suggests that the movement of meaning across practices is also about a movement in ways of acting mathematically.

Secondly, Nyabanyaba (2002) addresses criticisms of sociological perspectives as deterministic by using Bourdieu's (1987/1990, p.9) notion of habitus. He identifies students who choose not to engage in the deep realistic items at all as they aimed to maximize their

[56] I set out how I deal with the notion of social regulation and Bernstein's (1996) concepts of instructional and regulative discourse in Section 4.5.3.

performance in an imported examination. Those students who made these choices, he argues, were enabled by their socio-economic status. Nyabanyaba (2002) thus talks about both structure (socio-economic status) which enables certain students to make choices, and individual agency in terms of students making choices to act in a certain way. Nyabanyaba (2002) also suggests that engaging with the boundary between practices is a matter of positioning; these students were positioning themselves in relation to the higher education practices in their foreground.

3.7.6 What does the sociological research located in school mathematics have to offer my study?

The sociological studies reviewed here have the boundary between mathematical and non-mathematical practices in view. Unlike the work of Freudenthal (1973) and Realistic Mathematics Education (e.g. Barnes & Venter, 2010; Treffers, 1987), these studies problematize this boundary crossing. However, unlike some studies within the social turn which Evans (2000) argues represent the boundary crossing between practices as "hopeless" (p.80), the studies I have reviewed in this section use the concept of recontextualization (Bernstein, 2000) to point to the movement of meaning as one practice is relocated in another practice. In this study I talk about how meaning moves between school, foundational, and advanced mathematics and between mathematical and non-mathematical practices. The sociological perspective does have power in view at the macro-level, that is, in the boundaries between practices. This allows me to talk about how the foundational practice is positioned by the move that it represents in the educational space. These sociological studies also provide me with some tools for classifying the task contexts of the practical problems.

Yet the sociological perspective presented here also has its limits with respect to how it allows me to talk about my research problem. Firstly, in the terms used by Lerman (1998), the zoom of the "lens" (p.67) is broad. Lerman (2000, p.28) argues that sociological perspectives provide "resources for identifying the macro-social issues that bear on schooling", but do not generally provide the resources for linking these issues to the micro-

social issues of the classroom such as the action of individual students or a group of students. In terms of my study, these perspectives do not have the action of Siyabulela, Mpumelelo, Vuyani and Lungiswa as re-presented in Transcripts 1.1 and 1.2 in view.

On the whole, the research reviewed here (with the exception of Nyabanyaba (2002)) talks about the positioning of students at the structural level of social class and it does not allow me to talk about (a) the power relations between Siyabulela and Vuyani, and (b) the agency of Siyabulela in challenging the authority of the Tutor in Transcripts 1.1 and 1.2. Secondly, this research is limited in terms of talking about how these students move between the task context of the flu virus and the mathematics and how they make links to school mathematics; these studies suggest that it is mainly a matter of their recognizing the boundaries between these practices. Lastly, in contrast to the ontological/psychological perspective, the sociological perspective does not allow me to talk about how Siyabulela and Mpumelelo act on the mathematical objects of function, derivative function and limit in Transcript 1.1.

In critiquing the way that these sociological studies talk about the positioning of students, I am conscious that a researcher's choice of empirical data has implications for what one is able to talk about. In his earlier work on school mathematics, Dowling (1998) makes explicit his decision to focus on the analysis of school mathematics texts alone, and not to include attention to the readings of these texts by students and teachers. He argues that the analysis of text results in the production of the author and model reader. These productions, he argues, differ from the empirical author and reader that would emerge from an analysis of the use of these texts in the classroom.[57] In my study I draw on both types of texts, that is, the texts of the practical problems and the texts representing the student action on these texts. I certainly do not consider the author and reader emerging from the analysis of these

[57] In his more recent work, Dowling (2009) takes a broader view on the notion text. However, much of his discussion of school mathematics refers to written texts, for example, the secondary school textbooks referred to in his 1998 work, mathematical investigations, and assessment items from the TIMSS website.

two texts as the same. Like Dowling, I distinguish between the reader in the two texts; by analyzing the practical problem texts I am able to identify what subject positions are set up for the students, and by analyzing the action of the students on the problems I am able to investigate how the students occupy the positions set up for them in the practice. This choice of data thus allows me to talk about student agency and positioning.

3.8 Post-structuralist perspectives that talk about the movement of meaning between discursive practices and individual positioning

In this section I draw on two aspects of post-structuralist perspectives in order to address some of the limitations of the sociological perspective discussed in Section 3.7.6. Firstly, post-structuralist perspectives talk about practices as discursive practices, suggesting that discourse gives meaning to, or has a "constitutive effect" (Evans, 2000, p.97) on practices. As suggested by Walkerdine (2000), "… signifiers are made to signify when united with a signified within a particular practice, from which they take meaning" (p.53). From this perspective the movement of meaning across boundaries between practices is a relationship of *intertextuality*, that is, a term in one discourse might recall a similar term in another discourse (Evans, 2000, p.96). Walkerdine (2000) gives as an example how the signifier "more" takes on different meanings in school mathematics and in everyday practice. In this study, the Flu Virus Problem can be located in foundational mathematics practice, yet the term *flu virus* takes on particular meanings within epidemiological and everyday practices, and the function and limit notation in the problem is also used in school mathematics. In the following quote Walkerdine (1988) describes how everyday practices are incorporated into school mathematics in a series of discursive transformations, a description that points to the movement of meaning during boundary crossings:

> ... non-mathematics practices become school mathematics practices, by a series of transformations, which retain links between the two practices. This is achieved not by the same action on objects, but rather by the formation of complex signifying chains, which facilitate the move into new relations of signification which operate with written symbols in which the referential content of the discourse is suppressed. (p.128)

Walkerdine (1988) argues that the move from the everyday should not be conceptualised as a movement from the concrete to the abstract, but rather as a move from one discursive practice to another. She provides an example in which a teacher worked with a group of children who were having difficulty; she assisted the children not by making the action more concrete but by changing the metaphor that allowed the task to be located in a familiar discursive framework. Evans (2000) also points to the role of an authority in the particular mathematical practice who can cast a mathematical gaze on other practices and follow "relations of signification" (p.100) in the movement across the boundary. Chapman (1995) uses the concept of intertextuality to talk about spoken language in school mathematics classrooms. She argues that the teacher and students use text connecting practices when developing mathematical meaning on the topic of functions.

The concept of discursive practices in a post-structuralist perspective is related to a particular view of the student within this perspective. Walkerdine (2000) argues for the need to "abandon our view of the pre-given subject with skills and pre-social models of human cognition altogether" (p.50). She suggests that the notions of context and transfer can be viewed differently by viewing subjectivity as located in practices and by examining the discursive means by which an individual becomes subjected. Such a view, she argues, allows us to bring the social and historical into view. Walkerdine (2000) suggests that the individual is positioned in particular ways in a discursive practice. For example, in the incorporation of an everyday practice such as shopping into school mathematics a student is no longer positioned as a shopper, but is now positioned as a school mathematics student. She also suggests that the positioning differs for different students.

A post-structuralist perspective has something to offer my study in two respects; talking about the boundary crossing in terms of intertextuality, and talking about the discursive positioning of individual students at the micro-level of action in a particular practice and how this may change in the boundary crossing.

3.9 Word problems as genre

Within the practices of school and undergraduate mathematics, various terms are used to refer to mathematics problems that make links to non-mathematical practices, for example, *real-world problems*, *applied problems*, *applications*, *mathematical modelling*, *word problems*, *non-standard problems*, and *authentic problems*. In spite of the various names used, research into the use of such problems in school mathematics suggests that the majority of these problems do not actually require mathematical modelling-type activity and are designed merely for students to practise mathematical algorithms (Gerofsky, 2004; Gravemeijer, 1997; Sierpinska, 1995; Verschaffel, Greer & De Corte, 2000). In fact, most of these problems can be identified as traditional word problems, as classified by Gerofsky (2004), the International Commission for Mathematics Instruction (2002), and Verschaffel, Greer, and De Corte (2000). In Section 1.1 I signaled my intention to talk about the problems in this study as *practical problems*. I will be arguing in Chapter 7 that these problems share some, but not all, features with mathematical word problems. Hence the need to review the research literature on word problems. In this review I focus on Gerofsky's (2004) work on word problems; her work is located within the discursive turn as it represents word problems as a genre, a genre that exists within school mathematics and, in some cases, undergraduate mathematics.

Gerofsky (2004) classifies mathematical word problems as a *genre* in the sense that they are a "culturally recognizable convention" (p.17). A genre is not just the physical text but also the expectations that go with the text about what action is appropriate. She identifies the recognizable features of the genre of mathematical word problems. For example, a mathematical word problem has a three-component structure: the *set-up* which establishes the location and the characters but which is often not necessary to the solution to the problem, the *information* needed to solve the problem (such as what is given and sometimes the operations to be performed), and the *question* or goals (p.27). Gerofsky (2004) points to boundaries between genres when she argues that the word problem structure is derived from algebraic or arithmetic problems and not from the conventions of

story-telling. This suggests, too, that the task context of a word problem can be replaced by another, provided that the mathematical structure remains the same.

Gerofsky (2004, p. 33) suggests that a word problem refers to the everyday in a "cursory way", that is, the *locutionary force* or literal meaning is indeterminate. She draws on the work of Lamarque and Olsen (1994, p.61) to argue that these problems use *pretence*.[58] Gerofsky (2004), points to three aspects of pretence when solving a word problem; the student should "pretend *that* a particular story situation exists" (p.35, emphasis in the original), pretend that the situation exists under instruction from the writer, and pretend that someone is telling them about the story situation. The work of Lamarque and Olsen (1994, p.61) points to how the student is positioned in this pretence; they argue that pretending that something is the case need not involve "projecting oneself into the pretence" (p.61), something that is required when pretending *to be* something. Gerofsky (2004) identifies the inconsistent use of tenses as one of the features of word problems. Yet since the assumption of the genre is that the problem is only a pretence of the real world, this inconsistency is not considered a problem by teachers and students who are familiar with the genre.

The *illocutionary* force, or what the text intends (but does not state literally), is obvious only to a student who is familiar with the genre and its assumptions (Gerofsky, 2004, p.33). For example, the instruction in a word problem such as "Solve" or "Find" comes with a number of assumptions about the valued action, for example, that the problem is solvable and can be solved using mathematics that the student has access to, that all necessary information for solving the problem is provided and no extraneous information need be sought, that the problem is provided to practise an algorithm recently presented in the mathematics class, that the problem can be reduced to mathematical form and the role of the student is to uncover the algebraic or arithmetic formulation that has been "dressed up" (p.33) by attending to some features and not others, and that there is one correct

[58] Gerofsky (2004) uses an "s" in her spelling of *pretense*, but I choose to use the word *pretence* as used by Lamarque and Olsen (1994).

mathematical interpretation, one right answer, and the teacher can judge the accuracy. So according to Gerofsky (2004), trying to make sense of a word problem in terms of everyday life or identifying deficiencies or contradictions in the task context are not part of the illocutionary force. Word problems do not contain explicit instructions on what aspects of the real world to attend to and which to ignore. Rather, students are meant to draw generalizations about how to solve word problems after seeing many such examples, in other words, they should follow "a pedagogy of tacit generalization over the course of many problems" (Gerofsky, 2004, p.57).

Referring to her empirical work, Gerofsky (2004) argues that undergraduate students were able to identify features of the genre of mathematical word problems when given these problems in an interview situation. They identified that such a problem is structured by mathematics and "is a problem looking for words" (p.92). For example, an undergraduate engineering student identified a group of related rates problems as "train problems" (p.96), noting that the train as task context could be replaced by a car. This student noted that the problem was structured around a mathematical concept recently learnt in class:

> … they have to be cooked up to a certain degree to represent what you've just learned. If you're looking at them in a textbook, there'll be one topic that it will cover, say one concept that you have to demonstrate knowledge of. (Gerofsky, 2004, p.96)

While the interviews discussed by Gerofsky (2004) suggest that some students, such as the undergraduate engineering student quoted, have access to the conventions of the genre of mathematical word problems, she does warn that the "culturally and historically encoded intentions" (p.49) may not be the same as those of the student. However, she does not foreground positioning with respect to social class or other forms of social difference in terms of access to word problems.

Like Dowling (1998), Gerofsky (2004) identifies mathematical word problems as part of school mathematics and not about reality. She argues that while the author might intend to refer to reality, this intention is "subverted by the generic form of the word problem"

(p.50). Again, the question for this study is, do the practical problems in the Foundational Course (which I argue share features with word problems) provide access to advanced mathematics? Gerofsky (2004) suggests that talking about word problems as a genre enables one to explore the boundaries between genres. Crossing the mathematical/non-mathematical boundary involves students being exposed to many problems from the genre and making generalizations about the tacit assumptions. She suggests that teachers can facilitate this movement by focusing attention on the genre and by exploring its boundaries (p.142). This perspective suggests that boundary crossing involves controlling the movement in genre and the implicit assumptions of a genre.

3.10 Studies that talk about reform-oriented pedagogy from an equity and access perspective

The sociological, post-structural, and discursive work reviewed so far focuses on the nature of and relationship between different practices, both mathematical and non-mathematical. Yet some of the studies discussed in Sections 3.7 to 3.9 suggest that understanding the use of practical problems requires that we also talk about what happens in the particular pedagogic practice. For example, Straehler-Pohl (2010) attributes the students' difficulties moving from the horizontal to the vertical discourse to the weak framing and resulting confusion caused by the teacher. Both Walkerdine (1988) and Evans (2000) point to the important role of an authority in facilitating the movement between discursive practices and the need to talk about how students are positioned within a pedagogic practice. Gerofsky (2004) also gives the teacher a role in facilitating boundary crossings. My choice of what research literature to review in this section is driven by the research problem. The innovations of relevance and a learner-centred pedagogy in the Foundational Course that are the focus of this study can be located in reform-oriented pedagogy as a whole and there is a wealth of research literature related to this reform. However, my particular interest in these innovations is in terms of whether they facilitate the transition between mathematical practices. Thus in this review I focus on research literature that investigates reform-oriented pedagogy from the perspective of equity and access to mathematical practice.

3.10.1 Reform-oriented pedagogy: The Lubienski-Boaler debate

Lubienski (2000) focuses on problem-solving curricula typical of a reform-oriented pedagogy. Using a sociological perspective she argues that, contrary to expectations that the use of open, "contextualized" problems will improve equity in relation to the learning of school mathematics, lower class students may be disadvantaged by such curricula.

Lubienski (2000) argues that the lower socio-economic status (SES) seventh-graders in her study found the general lack of direction and explorative nature of the open-ended problems frustrating and wanted clearer explanations from the teacher.[59] She suggests that these students were more passive and less confident about how to proceed than the higher SES students and wanted to get an answer rather than grappling with understanding the mathematical ideas. Lubienski (2000) suggests that the lower SES students' difficulties with the open-ended nature of the tasks contributed to their difficulties dealing with the "contextualized" problems:

> The tendency for the lower SES students to focus on the immediate context, as well as their desire for more specific teacher direction for their mathematical thinking and less intrinsic interest in understanding the mathematics for its own sake, seemed to contribute to the lower SES students' tendencies to becomes "stuck" in the contexts. (p.477)

Lubienski (2000) identifies differences in students' reasoning on the contextualized problems; she argues that the higher SES students had more mathematically focused reasoning compared to the lower SES students who drew more on common sense, contextualized arguments or reasoning based on rules learned previously in school mathematics. Lubienski (2000, p.467) also identifies differences in students' use of language; she argues that the higher SES students used more "generalized language" (language that refers "only to the numbers without contextual attachment") than the lower SES students who tended to use "contextualized" language that referenced objects in the task context.

[59] Lubienski (2000) uses the term *socio-economic status* (SES) to refer to individual students in her study, and uses the term class more generally for large societal groups.

The work of Lubienski (2000) points to the need to talk about students' ways of acting mathematically in the classroom, for example, she refers to the nature of their reasoning and their ways of talking. Yet, like most of the sociological studies reviewed in Section 3.7, this study talks about the students in a deterministic way in terms of social class and does not talk about the identity work of individual students in the mathematics classroom. In the discussion of the results of this study I will be problematizing Lubienski's (2000) description of the language of the students and their positioning as "stuck" (p.477) in the horizontal discourse.

Boaler (2002c) acknowledges the need to interrogate assumptions behind reform-oriented pedagogy, yet she engages critically with Lubienski's work as well as other critiques of such a pedagogy. She warns of adopting a deficit view of students and proposes a different way of talking that recognizes the location of the classroom in a wider network of practices:

> This requires a shift in focus away from what **students cannot do –** for example, cope well with open-ended problems – to what **school can do** to make the educational experience more equitable. (Boaler, 2002c, p.241, emphasis in the original)

In her empirical work Boaler uses a situated theoretical perspective; social practices are regarded not just as the context for learning, but that learning mathematics "*is* participation in social practices" (Boaler & Greeno, 2000, p.172, my emphasis). So knowledge cannot be separated from the practice in which it is located, as the practices of learning mathematics itself define the knowledge that is produced. Different pedagogies are not just vehicles for more or less knowledge, "they shape the nature of the knowledge produced and define the identities students develop as mathematics learners through the practices in which they engage" (Boaler 2002a, p.132).

Boaler (2002a) uses her empirical work to argue that what she calls *reform* and *traditional pedagogies* (p.3) produce different knowledge and students develop different mathematical identities in the different practices. The mathematics lessons at a school named Amber Hill are described as traditional as they were content-based and students were mainly engaged

in working through short, closed, procedural questions from a textbook, with structured support from the teacher in the form of procedures to follow. She suggests that students responded to non-mathematical cues (such as what the teacher or the textbook required) to judge mathematical demands. Boaler (2002a) describes the pedagogic practice in the Amber Hill mathematics classrooms as a specialized community of practice which was unrelated to others. She argues that reproducing standard procedures was specific to the mathematics classroom and the students developed limited identities within this practice. Applying their mathematical knowledge in non-mathematical practices was not part of the practice:

> If a question required some real-world knowledge or non-mathematical knowledge, students would stop and ask for help. They would be able to answer the question if prompted; they would probably be able to answer the question if they were in a science classroom or if they were at home, but their expectation of the knowledge they should use in the mathematical classroom stopped them from answering such questions. (Boaler, 2002a, p.46)

In contrast the mathematics lessons at the second school in Boaler's (2002a) study, Phoenix Park, integrated mathematical content and process. The students worked together on open-ended projects (which were not traditional word problems) over a number of weeks, with the teacher providing support by helping students understand the task context, introducing any necessary mathematical content, and developing students' mathematical process skills. She suggests that students at Phoenix Park developed different knowledge and identities to the students at Amber Hill. Boaler (2002a) argues that, since the Phoenix Park students were able to establish personal meaning in the mathematics they were learning, they were able to apply their knowledge more flexibly in different contexts. Although Boaler (2002a) foregrounds the application of school mathematics in other practices, she does suggest that her language allows one to talk about transfer across practices more generally, for example, in vertical mathematization:

> …if we want students to consider mathematical situations and flexibly make use of mathematical knowledge in the real world or in examinations of higher mathematics, we need to engage students in similar practices in the

> classroom. It is through such practices that students develop identities as mathematics problem solvers (p.179)

Boaler's work is useful for the purposes of this study in that it points to the importance of looking beyond curriculum and attending to pedagogic practice. Her attention to the identities that are developed in a particular practice suggests that boundary crossing involves a movement in an individual's positioning. This perspective also avoids a deficit view of the students. Yet I argue that her perspective is weak in two respects. Firstly, in setting up reform and traditional pedagogies as dichotomies, there is an assumption that the student is positioned in a consistent way by each pedagogy. She does not talk about pedagogies that identify the student in contradictory ways, which I argue is the case in the foundational practice that is the topic of this study. Secondly, her perspective is weak in terms of how it talks about the power relations between the student and teacher in the reform classroom. Drawing on the work of Adda (1989), Boaler (1993) argues that traditional pedagogy reinforces the notion of mathematical truth and objectivity and the positioning of the teacher as the source of this truth. In contrast, at Phoenix Park the students determine the direction of their investigations and work together in groups over extended time, while the teacher is involved in "organizing" (Boaler, 2002a, p.22). Yet what is included in Boaler's (2002a) description of the practice at Phoenix Park but not given significance in her discussion is the skilled work of the teacher in designing and implementing the pedagogy, for example, the design of the tasks, the unpacking of the text-based activities before students engaged with them in groups, introducing the required mathematical content at an appropriate time, and developing students' process skills.

3.10.2 Learner-centred pedagogy and the new outcomes-based school Mathematics curriculum in South Africa

Like Lubienski (2000), Brodie and Pournara (2005) problematize representations of traditional pedagogy and reform pedagogy as a dichotomy, representations that are common in curriculum documentation. They argue that the notion of "group work" as promoted in the new outcomes-based school curriculum in South Africa needs to be

interrogated if teachers are to be enabled to "achieve new ways of working in their classrooms" (p.32). Brodie and Pournara use a review of the literature on the use of group work in school mathematics to identify five different "approaches" to group work. Each approach is informed by a particular theoretical perspective on learning, the nature of interactions, the nature of the tasks, and the roles for students and teachers. Three of these approaches (the socio-cultural, the situated, and the socio-political) can be located in the theoretical turns mentioned in Section 3.6. Of interest to this study is that all three approaches foreground the role of an authority (usually the teacher) who can facilitate or mediate students' access to mathematical discourse.

In this section I review two studies conducted during the early implementation of the new outcomes-based school curriculum in South Africa; both of these studies highlight the challenging yet critical role that the teacher has to play in a learner-centred pedagogy.

Davis (2001) is highly critical of interpretations of learner-centred pedagogy that position the teacher as a "facilitator" and suspend "the teacher's evaluative knowledge with respect to the quality of the 'knowledge' statements produced by students" (p.2). He argues that the notion of *evaluation* as used by Bernstein (1996, p.56) does not only refer to tests, projects, examinations etc., but also teacher-student interactions, and that for Bernstein, pedagogic practice is "heavily saturated with evaluative acts that are continually being performed by teacher and students" (p.2). Davis argues that evaluative "judgements on knowledge" (p.11) have to be inserted into learner-centred pedagogy and warns that "uncritical affirmation … of the enunciations of students" as an interpretation of the role of the teacher as facilitator "ultimately serves to negate pedagogic judgement as well as mathematics" (p.11).

Adler (1997) draws on a sociocultural perspective in her empirical work which focuses on the communicative aspects of what she calls a *participatory-inquiry approach* (p.235) and

the mediating role required of the teacher in facilitating this communication.[60] Adler (1997) talks about three aspects of language in a multilingual classroom; access to the language of learning, access to the language of mathematics and access to classroom cultural processes. In this study I argue that calculus reform texts introduce a fourth aspect of language into the mathematics classroom, that is, when students "explain verbally what their answers mean in practical terms" (Hughes-Hallet et al., 1994, p.vii).

Adler (1997) uses her empirical work to suggest that a teacher's attempts to acknowledge and validate students' informal contributions in an attempt to build a meaningful culture of inquiry as valued in a participatory-inquiry approach may mitigate against the teacher making use of rich opportunities to develop students' mathematical understanding. The participatory-inquiry approach thus introduces a tension into the instructional role of the teacher (in Adler's study the teacher Sue is in view):

> While the **withdrawal** of the teacher as continual intermediary and reference point for pupils enables Sues' participatory classroom culture, her **mediation** is essential to improving the substance of communication about mathematics and the development of scientific concepts. That is, both are required, and managing the tension is the challenge! (p.255, emphasis in the original).

Adler (1997) thus points to an additional boundary crossing in a learner-centred pedagogy and one that has not been discussed so far in this literature review; in this case the teacher moves to and fro across the boundary between using the language of a facilitator in a participatory pedagogic practice and using the language of mathematics required for access to mathematical practice. Adler (1997) argues that the teacher needs to recognize and work with this boundary. This perspective on language provides me with a way to talk about the moves of the tutor in interaction with the students in this study.

[60] Adler (1997) uses the term *participatory-inquiry* (p.235) in the same way that I use the term *learner-centred*.

3.10.3 What do the studies on reform-oriented pedagogy in school mathematics have to offer my study?

All of the studies in this section point to the need to talk about more than just the boundaries between practices and students' recognition of these boundaries. Rather, it is also necessary to talk about what happens in the pedagogic practice (school mathematics is the focus in the studies reviewed in this section). Acting mathematically is not just about the relationships between practices, but is also involves particular ways of talking, ways of evaluating, ways of arguing, and specific roles for the teacher and for the students. These studies suggest therefore that it is necessary to talk about more than just the mathematical texts used in the classroom, as Dowling (1998) does. Rather, there is a need to talk about the action in the classroom when students solve these problems in interaction with one another and with the teacher.

3.11 Summary and a way forward in this study

The focus of this literature review has been on what mathematics education research says about my research problem and on what theoretical perspectives are used to talk about this problem. This review points to the fact that all of the perspectives referenced in this chapter have both strengths and weaknesses in terms of my study, and that none of the perspectives reviewed, taken in isolation, are adequate for what I need to talk about in this study. For example, while an ontological/psychological perspective allows me to talk about the students' mathematical action, it may lead to a deficit view of the students. The sociological perspective talks about the macro-level social and political issues, but not the micro-action within socio-political practices.

The theoretical work of this study involves developing a theoretical perspective and associated analytic tools in such a way that the research "counts" (Adler & Lerman, 2003, p.446) for the participants and makes a contribution to the mathematics education community. This review suggests that, if this research study is to count in these two respects, the theoretical perspective in this study should, firstly, talk about student action on

mathematical objects on the micro-level of the classroom. Secondly, I need to talk about the social, political, and discursive action on this micro-level. This micro-level action should be spoken about in a way that avoids a deficit view of students. Thirdly, I need to talk about the micro-level action in relation to the wider social practices in which it is located, practices that are held in place at the macro-level by power relations. Fourth, linking the macro- and micro-levels in this way I need to be able to talk about both structure and agency. Lastly, my theoretical perspective should talk about the relationships between practices and what it means to move between practices. The theoretical perspective presented in Chapter 4 responds to the five needs described here; it is in the composition described in that chapter that I am able to adequately talk about foundational mathematics practice and the enabling and constraining conditions for participation in this practice.

CHAPTER 4 THE RESEARCH PROCESS THEORETICAL FRAMEWORK

4.1 Introduction to this chapter

Crotty (2003) argues that once a researcher has identified a problem or an interest, two descriptions are necessary. First, what methodology and methods will I use in this research? Secondly, how do I justify these choices? The latter question relates both to my theoretical perspective (what I am able to say about the world and my assumptions about the world) and to the related epistemology (my views on the nature of knowledge and what kind of knowledge can be constructed through this research study). So in the view of Crotty (2003), the research process can be divided into four related parts; theoretical perspective, epistemology, methods, methodology. The theoretical perspective is the focus of this chapter.

In presenting the theoretical framework for this study, what I call a socio-political perspective of mathematical practice, I respond to the challenges of selecting a theoretical framework as discussed in Chapter 3. In concluding that chapter I argued that making the description "count" (Adler & Lerman, 2003, p.446) in relation to the participants and the mathematics education research community requires a theoretical perspective and associated analytic tools that allow me to talk about the micro-action in the mathematics classroom (the mathematical, discursive, social and political aspects of acting mathematically) in relation to the practices of the wider socio-political space in which this action is located. It should be noted that the selection and development of this theoretical framework did not emerge from the literature review as presented in Chapter 3 alone, but emerged in the interaction between this literature (the theoretical space) and the empirical

space of this study. Brown and Dowling (1998) describe the research process as "the construction of the theoretical and empirical as increasingly coherent and systematically organized and related conceptual spaces" (p.11).

This chapter is divided into three sections. Firstly, I draw on critical linguistics and present a socio-political perspective of mathematical practice. Secondly, I present the work necessary to adapt this perspective specifically for the study of mathematical discourse. Lastly, I revisit my research question presented in Chapter 1.

4.2 A social perspective of practice

I begin the section by presenting a *social* perspective of practice. I draw on the work of Fairclough in critical linguistics. While the ideas are not unique to Fairclough (he draws on the work of a number of social theorists, for example, Archer, Bakhtin, Bernstein, Bourdieu and Foucault)[61], I have chosen to use his explication of the concepts and his associated analytic tools, a choice that I motivate in Chapters 4, 5 and 6. I focus specifically on the concept of power as it relates to Fairclough's social practice perspective in Section 4.3.

Fairclough uses a number of social contexts (including education, but not specifically mathematics education) to illustrate his perspective. However, the notion that mathematics and mathematics education are both social and political practices has been used within mathematics education (e.g. De Freitas & Zolkover, 2009; Lerman & Zevenbergen, 2004; Morgan, 2006, 2009; Valero, 2008). In Sections 4.2 and 4.3 I draw on the description of the discursive space of schooling and undergraduate mathematics in Chapters 2 and 3 to populate Fairclough's perspective with examples specific to the mathematical practices in this study. I attend to the detail of mathematical discourse within the socio-political mathematical practice perspective in Section 4.5.

[61] Fairclough uses theoretical constructs from the work of these social theorists to develop a social of perspective of practice, referencing the theorists in detail where necessary. Since I use Fairclough's perspective in this study, I do not reference the individual social theorists on whom Fairclough draws.

4.2.1 Three levels of abstraction in social analysis

Fairclough (2006, p.30) distinguishes three levels of abstraction within social analysis. On the most concrete level is *social event*, the action of everyday life or what actually happens. A social event has related aspects, for example, activities, subjects, objects, position in time and space, and text. The word *text* refers to the semiotic aspect of a social event, and is used broadly with reference to writing, speech, visual images, body language, gesture, facial expression etc. I use the word *action*[62] here as a broad term for what happens in a social event. Much of the action in a mathematics classroom is discursive or text-based. In this study I investigate two related discursive events in the Foundational Course; the texts of practical problems and the action of students on these problems (as re-presented in written transcripts).

At the most abstract level of social analysis is *social structure*, representing for Fairclough (2006, p.30), "the most general and enduring characteristics of society", for example, gender relations, class structure, economic systems, and languages such as English. The constructs of social class and race that I have alluded to in my description of the socio-political space of this study in Chapters 2 and 3 are located at this level. Social structures define a "set of possibilities" (Fairclough, 2003, p.23) or have "generative powers" (Chouliaraki & Fairclough, 1999, p.19). For example certain ways of combining words in a sentence are possible in English (Fairclough, 2003), and social class can be theorized as defining a set of possibilities for success in formal education (Bernstein, 2000).

Yet the relationship between a social event and social structure "is a very complex one" and not everything that is possible on the level of social structure actually happens on the level of social event (Fairclough, 2003, p.23).[63] In between these two levels is the level of *social*

[62] I use the word *activities* for what subjects physically engage in, an aspect of *action* in a social event. My use of the term action is also distinct from my use of the term *practice*, which I introduce later in this section as "a habitual way of acting" (Chouliaraki & Fairclough, 1999, p.22).

[63] Fairclough adopts an ontological position of critical realism and distinguishes between the *real*, the *actual*, and the *empirical* (Fairclough, Jessop & Sayer, 2001, p.5). The real are "objects, their structures or natures

practice, which mediates or is the "point of connection … between 'society' and people living their lives" (Chouliaraki & Fairclough, 1999, p.21). Social practices are defined as "the way things are generally done or happen in particular areas of social life" (Fairclough, 2003, p.30). A social practice consists of a number of elements which are both discursive and non-discursive, namely material activities, mental activities, the material world, interaction, social relations, subjects[64], beliefs/values/desires, institutions/rituals, and discourse (Chouliaraki & Fairclough, 1999, p.21; Fairclough, 2003, p.25). The word *discourse* here is used as an abstract noun to refer to written and spoken language as well as other forms of semiosis such as body language and visual images (Fairclough, 2003).[65]

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and their causal powers and liabilities" (p.5), while the actual is what happens when these powers are activated. Although the actual (for example, political debates) may change the real (for example, political institutions), the real is not reducible to the actual. The empirical is what actors observe or experience of the real and the actual. Yet the real has an independent existence, irrespective of whether it is experienced, observed or represented (even if not actualized). Symbolic (including discursive) elements are also regarded as real as they have the power to have material effects (Chouliaraki & Fairclough, 1999). Although discourses contribute to the representation of the real, these discourses are not the same as the objects themselves (Fairclough, 2006). Whether discourse has the effect of changing the construction of objects depends on the context, for example, the way social reality is and who is construing it (Fairclough, 2003). Fairclough (2003) describes his ontological perspective as a "moderate version of the claim that the social world is textually constructed" (p.9). I would argue that Fairclough's descriptions of objects as real can include mathematical objects such as functions and limits; while these abstract objects can be regarded as having an independent existence, we use visual and symbolic representations to represent them in text.

[64] I explain my choice of the word *subject* rather than *participant* in Section 4.2.3.

[65] Fairclough subscribes to an epistemology of *constructivist structuralism* (developed from Bourdieu) which emphasizes the structure/action dialectic (Chouliaraki & Fairclough, 1999, p.32). Fairclough (2006) recognises the social and discursive construal of objects and structures, but within constraints. Firstly, a social practice controls the selection of possibilities within certain areas of social life (Fairclough, 2003). Secondly, this construal is both enabled and constrained by non-discursive conditions, for example, although an educational practice such as a university lecture is mainly semiotic and derives meaning from language, it is "co-produced by mental, social and material" structures (Fairclough, Jessop & Sayer, 2001, p.5). I would include mathematical objects in these structures. Both the discursive and non-discursive are real and the discursive is not privileged (Chouliaraki & Fairclough, 1999).

A social practice is associated with particular organisations or institutions, and can occur at different levels, for example, education as a whole can be conceptualized as a social practice, as can undergraduate mathematics or school mathematics. Within the practice of school mathematics, reform-oriented and more traditional pedagogies can be conceptualized as practices.[66] For example, the practice of a more traditional pedagogy in school mathematics may be defined by a characteristic spatial layout such as the desks arranged in rows, activities like a teacher explaining on the blackboard and students sitting quietly and working alone on textbook problems, mathematical objects like functions and representations such as graphs, tools such as calculators, texts such as word problems, and certain roles and forms of interaction for the subjects, such as the student acting operationally to produce an answer and the teacher evaluating this answer.

Social practices are networked together in an institution or organisation, a network that is constantly shifting (Fairclough, 2003). Valero (2007, p.226) uses the term *network of mathematics education practices* to refer to the many social practices where mathematics teaching and learning is given meaning, for example, mathematics classroom activity at different levels, policy making, teacher education and research. In addition, I include in this network other mathematical practices such as professional research mathematics and mathematics that is applied in the workplace, for these practices also give meaning to teaching and learning. In this study I use the detailed analysis of the practical problems and the student action on these problems to elucidate *foundational practice in mathematics* as a social practice and its relationship to other practices in higher education and mathematics education (a network that is described in Chapters 2 and 3).

[66] In this paragraph I name and classify educational practices. Firstly, this naming has the effect of fixing these practices in time and space and of setting up boundaries between practices. Yet, by definition, practices are overlapping and changing. This dilemma is noted by Moschkovich (2007) in her naming of different *mathematical Discourse practices* (p.26). Secondly, my classification is influenced by my understanding of the social space (which I have described in Chapters 2 and 3).

So how does a social practice mediate between social structure and a social event? A social practice "controls the selection of certain structural possibilities … and the retention of these selections over time, in particular areas of social life" (Fairclough, 2003, p.23-24). In turn, social practices partly shape social events by defining particular ways of participating in activities, interacting, etc. in certain areas of social life. Fairclough (2001) argues that the concept of social practice both enables and constrains action on the level of social event; people "are enabled through being constrained: they are able to act on condition that they act within the constraints of a type of practice" (p.23). Yet Fairclough (2003) emphasizes that social events are "partly" (p.25) shaped by social practices since the actual social event may diverge from what is expected as a result of it cutting across different social practices and of the activity of social agents. Fairclough (2003) emphasizes that subjects have agency; "Agents have 'causal powers' which are not reducible to the causal powers of social structures and practices" (p.22).

This dialectic between structure and action in Fairclough's three levels of social analysis allows me to talk about both the "macro-social issues" (Lerman, 2000, p.28) that bear on foundational mathematics practice and the "micro-social issues of the classroom such as the action of individual students or a group of students" (p.28). In this study I am interested in both causal effects. Firstly, I investigate how the student action derives meaning from the foundational practice and the wider network of practices of higher education and of mathematics education. Secondly, I consider how this action derives meaning from the creative action of subjects in the classroom.

4.2.2 How language figures in the three levels of abstraction

In Section 4.2.1 I have set out a general description of social practice and its relationship to social events and social structure. In this section I focus on language (and broader semiosis) in these three levels. This is necessary since much of the action in education is discursive. According to Fairclough (2003), language is an aspect of the social at all three levels. On the most abstract level, we have languages such as English or isiXhosa which define what

linguistic elements can or cannot be combined in a particular language. In the rest of this section I discuss how language figures at the level of social practice and social event.

Discourse and order of discourse

The term *discourse* is used for the language aspect of social practice which is related to (and not privileged over) other elements of the practice (Chouliaraki & Fairclough, 1999; Fairclough, 2003). On this level discourse is not only text, but "the whole process of social interaction of which a text is just part", and this includes the processes of production and interpretation (Fairclough, 2001, p.20). Fairclough (2003) argues that discourse figures in social practice in three ways. Firstly, language is used to represent some area of the social world from a particular perspective. Fairclough (2001, p.24) uses the word *discourse* (as a count noun) or *discourse type* to describe such a representation. I choose to use the term discourse type to emphasize this concept as different from the abstract use of discourse. The discourse type of mainstream undergraduate mathematics and the discourse type of foundational mathematics are both representations of how mathematics is taught and learnt at undergraduate level. These may represent the same "area of the world", but not necessarily from the same perspective or position (Fairclough, 2003, p.26). Secondly, we use language to act communicatively or to interact, and *genre* is a valued way of using language to interact in a particular social practice (Fairclough, 1995, p14; 2003, p.26). For example, the lecture as a way of communicating at university can be regarded as a genre. Thirdly, we use language as a resource for identifying ourselves and others as particular types of people. *Style* is a way of being in a particular social practice (Fairclough, 2003, p.26). For example, in undergraduate mathematical practice there are certain styles for or ways of using language as a lecturer or as a student.

The concepts of discourse type, genre, and style are not distinct, but are dialectically related (Fairclough, 2003). This is a complex relationship, but I give one example of a possible relationship related to foundational practice; the discourse type of foundational mathematics is a particular way of representing the activity of teachers and students at a university, but it also specifies particular ways of communicating discursively (genres), for

example, how lectures, tutorials, additional support is given in these programmes. This discourse type also sets out specific ways for the students and teachers to identify themselves and be identified through language (styles). On the other hand, ways of communicating discursively (such as interactive lectures, workshop-type classes, practical problems in the course material) and ways of using language to identify oneself (such as explaining the meaning of mathematical objects in "practical terms") give meaning to the discourse type of foundational provision.

Fairclough (2003, p.31) argues that in an institution or organisation the three aspects of discursive practice (discourse types, genres and styles) will be arranged or ordered in particular ways. He uses the term *order of discourse*[67] to describe the ordering of the set of discursive practices in an institution as well as the boundaries and relationships between these practices (Fairclough, 1995, p.12). The relations between the elements in an order of discourse may be complementary or they may be conflicting. For example, at a university there may be different genres related to teaching mathematics at undergraduate level and there may be different styles of being an undergraduate mathematics student. These genres and styles may be in conflict. For example, in her empirical work on school mathematics, Morgan (2006) identifies the discourse types of investigation and of assessment as being in tension. Where there are conflicting alternatives, one ends up being dominant (Fairclough, 2006). Fairclough (2001, p.33) argues that an order of discourse is held in place by power relations which represent particular ideological assumptions, an argument that I develop in Section 4.3.1 below. Boundaries between orders of discourse and within an order of discourse are constantly shifting, and these shifts are part of social change (Fairclough, 1995, p.13).

In this study I conceptualize foundational mathematics as a new practice in higher education, designed to facilitate the transition from school to advanced mathematics. The concept of order of discourse indicates that I cannot simply compare the knowledge

[67] This term is from Foucault, but Fairclough has adapted it for use in critical linguistics.

developed in the different practices, as Garner and Garner (2001) do when comparing traditional and calculus reform pedagogies and Boaler (2002a) does in her study of reform and traditional pedagogies in school mathematics. Rather, I need to talk about the social practice of foundational mathematics (as represented in the practical problems) in relation to other social practices in the order of discourse of undergraduate mathematics (as described in Chapters 2 and 3) and the power relations that maintain the boundaries within this order of discourse. This is necessary if I am to produce an adequate description of the conditions that enable or constrain access to different mathematical practices.

Text as both repetition and creation

The three elements discourse type, genre and style as discussed in the previous section are the discursive features of social practice, but they also feature in texts at the level of social event, that is, in discursive events. Fairclough (2003) draws on but adapts work from Systemic Functional Linguistics to identify three ways in which meaning is *construed*[68] by text.[69] Firstly, *representation* refers to how text represents aspects of the physical, social and material world. *Action* refers to how text interacts by enacting relations between participants and between other texts. Since I am using the term action in a different way to identify what happens in a social event, I choose to call this meaning *interaction*, since it involves relations. Thirdly, *identification* refers to how text identifies people and their values. These three meanings are "difficult to pull apart", as when interacting discursively, "people do not represent the world abstractly but in the course of and for the purposes of their social relations with others and their construction of social identities" (Chouliaraki & Fairclough, 1999, p.41). These three types of meaning are linked to the three elements of discursive practice; discourse type, genre and style are relatively stable ways of representing, interacting, and identifying respectively (Fairclough, 2003).

[68] Fairclough (2003, p.8) uses the word *construe* rather than *construct* in order to distinguish his ontological view from that of social constructivism.

[69] Fairclough (2003) reorganizes Halliday's (1973) classification of the three functions of language, that is, the *ideational*, *interpersonal* and *textual* functions of language.

Consistent with the structure/agency dialectic discussed in Section 4.2.1, Fairclough (2003) argues that text is shaped by two causal "powers" (p.22); on the one hand by the social practice in which it is located (and in particular the order of discourse of the institution) and social structure, and on the other hand by the agency of the subjects. So text is in part *repetition* and in part *creation* (Fairclough, 1995, p.7). For example, the producer of a practical problem in the Foundational Course will draw on the discourse types, genres and styles available in the order of discourse.[70] But the producer as a social agent may draw on the order of discourse in creative and possibly contradictory ways. Fairclough (2001, p.20) regards text as a *product* of the process of production. He suggests that the processes of production involve an interplay between the text itself and the *members' resources* of those involved in production. According to Fairclough (2001) members' resources are socio-cognitive processes; they are cognitive in the sense that the individual has various representations stored in long-term memory which serve as "prototypes" (p.9) for various things such as "the shape of words, the grammatical forms of sentences, the typical structure of a narrative, the properties of types of object and person, the expected sequence of events in a particular situation type" (p.9), and they are social in that the nature of these cognitive resources and when to use them is "socially generated" (p.20).

Not only is a text a product, but it is also a resource in the process of interpretation on the level of discursive practice (Fairclough, 2001). Like the process of production, the process of interpretation involves interplay between the text itself and the socio-cognitive resources of the interpreter. So when interpreting the text of a practical problem a student will attend

[70] Fairclough (2003, p.30) argues that a text does not occur in isolation, but is part of a network or *chain* of other texts. For example, a practical problem used in a foundational undergraduate mathematics Course might originate in a commercially available textbook, be selected and then adapted by the lecturer for inclusion in printed Course material, and solved by students in a tutorial. So there may be one or more *producer* of a text. In this study I refer to the producer of a practical problem as all those involved in the chain of texts that resulted in the situating of the problem in the printed Course material (I discuss the notion of chains of text in more detail in Section 4.2.4). I use the term *interpreter* for the foundational student who solves the problem.

to the actual text as a resource, but will also draw on his members' resources related to what should happen when solving such a problem in a mathematics classroom.

Fairclough (1995) argues that heterogeneity of texts (in the sense that they draw creatively on different discourse types, genres and styles) is an indication of social contradictions and possible change. In this study I investigate how the texts of the practical problems and the texts representing student action on these problems draw on different discursive practices and whether this indicates any change with respect to the wider order of discourse.

4.2.3 Subjects and identity from a social practice perspective

So far I have referred to the participants in social events and the associated social practices as *subjects*. Fairclough (1995) uses the term to emphasize that "discourse makes people, as well as people make discourse" (p.39). As I have noted in Section 4.2.1, Fairclough (2001, 2003) emphasizes that participants have agency, and it is only through being constrained by the conventions of a social practice and the order of discourse that they are able to exercise this agency. From Fairclough's (2003, p.32) perspective, the conventions of a social practice and the order of a discourse are the resources that subjects draw on (they are *subjects* in the sense that they are passive and shaped), and these may be drawn on in creative ways (they are *subjects* in that they are active and involved in action). So a social practice sets up *subject positions* (Fairclough, 2001, p.32) for participants by identifying them in certain ways. Yet participants are not passive, they actively position themselves in relation to these subject positions.

In Section 2.4.4 I identified an educational discourse of who does and does not belong in higher education. Students who enter university via the foundational programme are positioned as possibly not belonging and needing different support from mainstream students. Students who enter the mainstream but then change to foundational courses within a few weeks of the academic year are positioned as students who the institution regards as initially, but possibly no longer, belonging in higher education. Such subject positions may have implications for affect, resulting for example in a lack of motivation.

Fairclough (1992) notes that a text constitutes subjects in the sense that text producers *interpellate* (p.134) interpreting subjects who are capable of making the relevant assumptions about the text and hence a coherent reading of the text. So the producer of a practical problem in a foundational undergraduate mathematics course will have had a particular student in mind when setting the problem. For example, if the foundational student is positioned as not belonging in higher education, then he might require regulation in the form of reminders of how to behave or problem-solving steps to follow. Yet it is through the students occupying the subject positions set up in a practical problem that the practice is given meaning (in occupying the positions the student may repeat the practice or may develop it in creative ways).

Since social practices set up particular subject positions, we can conclude that the positioning of a student may differ from one educational practice to another. So the subject positions set up for a student in the Foundational Course in mathematics and how he assumes these positions may be different from his positioning in the practice of school mathematics or in other disciplinary practices in higher education. In addition, certain subject positions may be construed as having more value than others within the order of discourse of a university, an issue I address in Section 4.3.

Fairclough (2003, p.223) uses this notion of subject to identify two aspects of *identity*. On the one hand *social identity* is defined by both one's circumstances and early socialization (for example, one's gender identity) and one's later socialization into particular subject positions (for example, the role of mathematics student). On the other hand there is *personal identity* or *personality*. The development of an individual's identity is a result of the dialectical relationship between social and personal identity, that is, it depends on the individual personally investing in the subject positions on offer, and thus becoming an active social agent. Fairclough (2003, p.223) regards the concept of style or "way of being" on the level of discursive practice as the language aspect of identity (as opposed to the bodily aspect).

In this study I use the concepts of identification (on the level of text) and style (on the level of discourse) to consider, firstly, how the practical problems used in the Foundational Course set up subject positions for the student. As suggested by Walkerdine (2000) and discussed in Section 4.2.4 below, this positioning may change in the movement between practices. Secondly, I am interested in how the student occupies these positions when solving the problems. If foundational practice is to facilitate the transition from school to advanced mathematics, the student should be enabled, firstly, to adopt the subject position of foundational student, and secondly, to adopt the subject position of an advanced mathematics student.

4.2.4 Crossing the boundaries between social practices as a process of mediation

Fairclough (2006, p.34) gives significance to the boundary between practices when he talks about what is *inside* and *outside* practices in a network of socio-political practices. A relationship is set up between practices when the discourse types, genres and styles of the outside come "into contact" (p.34) with the inside. In this study foundational mathematics (the inside) sets up a relationship to a number of outside practices such as first-year undergraduate mathematics, advanced mathematics, school mathematics, and non-mathematical practices. In this section I consider what it means, from a socio-political perspective, to cross the boundaries between these practices.

Fairclough (2006) uses the work of the sociologist Bernstein as described in Section 3.7.1 to talk about the relationship between what is inside and outside a practice as a "relationship of recontextualization" (p.34). Particular networks of social practices and their associated genres have specific recontextualizing principles that specify how discourse types, genres and styles considered outside a practice are incorporated into a practice (Fairclough, 2003). These elements do not simply flow from one practice to another, but are actively appropriated or "filtered" (Fairclough, 2003, p.139) by the recontextualizing practice. This process of appropriation is not neutral, but is shaped by "circumstances, histories, trajectories, strategic positions and struggles within these contexts" (Fairclough, 2006, p.167). The recontextualized discourse types, genres and styles may or may not be

operationalized in the recontextualizing practice, and if operationalized, this might be done in unpredictable or unmanageable ways. For example, the foundational practice may draw in contradictory ways on both school and first-year undergraduate mathematics, or may draw from non-mathematical practices in ways that are inconsistent with the discourse types, genres and styles in these non-mathematical practices. Fairclough thus gives significance to the fact that the movement of meaning between practices may represent continuities or disruptions between what is the inside and outside the practices. This perspective suggests that the movement between mathematical and non-mathematical practices may not be as straightforward as suggested by Barnes and Venter (2010), Freudenthal (1973) and Treffers (1987) in their work on school mathematics and by Dreyfus (1991) in advanced mathematics.

However, Fairclough (2006) avoids conceptualizing the boundary crossing as impossible by viewing recontextualization from a post-structuralist perspective as a discursive process, that is, recontextualization is "led by discourses" (p.167). This perspective allows him to talk about movement across the boundaries, not as performing individual mental reconstructions (Tall, 1992) or as possessing the recognition rules to recognize the context (Bernstein, 2000), but as the movement of meaning across texts, social events and social practices. Rather than talking about transfer, Fairclough (2003, p.30) uses the term *mediation* from Silverstone (1999, p.13) for this movement of meaning.

On the level of social practice, mediation or the movement of meaning occurs along *genre chains* or genres that are regularly linked together (Fairclough, 2003, p.31). For example, in mathematics education at school level one could argue that what is valued mathematical knowledge moves along a genre chain from the written curriculum document, to prescribed textbooks, to implementation by the teacher in the classroom, to assessments, to students' written answers.

On the level of social event, mediation involves the movement of meaning along a chain of texts. For example, in the Foundational Course the lecturer may draw on both mathematical

and non-mathematical texts to produce a practical problem for the Course material. The foundational student then attempts to produce a written solution to the problem, moving between the mathematical and non-mathematical aspects of the text and getting verbal feedback from the tutor. A similar practical problem may be reproduced in subsequent Course material or in a written assessment. According to Fairclough (2003), the recontextualizing principle means that texts may be linked together in a regular way, as specified for a particular social practice or order of discourse, or they may be combined in contradictory ways. This, since the recontextualizing principle specifies what aspect of a social event should be included/excluded or given significance/not given significance in the movement across the boundary. For example, a social event like a chemical reaction may be represented differently in an undergraduate chemistry textbook and in a practical problem in a mathematics course.

In this study I am interested in the transition from school mathematics to advanced mathematics, a transition that foundational practice is designed to facilitate. The practical problems, as a means for the transition, introduce additional boundaries into this space. I use the analysis of these practical problems and the student action on the problems to develop a description of the movement of meaning involved in the boundary crossings between these practices. Control over this movement has implications for the transition (and hence access to advanced mathematics); Fairclough (2003) describes the capacity to influence or control this movement of meaning as "an important aspect of power in contemporary society" (p.31). In Section 4.3 I use Fairclough's concept of power to suggest that this capacity exists both at the level of order of discourse (hence in this study I consider the location of foundational practice in the order of discourse) as well as at the micro-level in terms of how the student controls the movement of meaning across texts, events and practices (hence in this study I attend to the micro-action of the classroom).

4.2.5 Talking about practical problems in this study

My choice of the term *practical problems* to talk about the collection of problems in the Foundational Course that make links to objects in practices other than mathematics is

derived from the theoretical perspective on practice presented so far in Section 4.2. In order to explain this choice I begin by giving the reader some examples of the non-mathematical practices referenced in these problems; the spread of a flu virus, the income and expenditure of a company, the travel distances and speed of cars, the shadow made by a spotlight on the ground, the production of chemicals in a chemical reaction, and the queue at the ticket office of a soccer stadium. As noted in Section 3.7.4, I refer to these non-mathematical practices as the *task contexts* of the practical problems. I also talk about the objects in these practices, for example, the chemicals, the cars, and the flu virus as *non-mathematical objects.* In this study I use the terms non-mathematical practice and non-mathematical object, yet recognizing the recontextualized nature of these practices and objects when they are appropriated into the practical problems in the foundational mathematics practice (Dowling, 1998).

These problems may be called *real-world problems*, a term that has been criticized by Roth (1996, p.488) who argues that mathematics lessons themselves constitute the real world for mathematics students. A similar criticism can be directed at the term *everyday problems*; referring more generally to the concept of Discourse practices, Moschkovich (2007) argues that school and academic mathematics can be considered *everyday practices* (p.26) for the participants.[71]

4.3 A socio-political perspective of practice

In his articulation of social practice as I have set out in the previous section, Fairclough (1995) argues that social practice has "various orientations" (p.66), whether these be economic, political, cultural or ideological. He suggests that the political orientation is the overarching one. The discussion in Section 4.2.2 points, too, to the discursive nature of social practices. So Fairclough's use of the word *social* in social practice implicitly includes the notions of *political* and *discursive*. From this point in this thesis I refer to Fairclough's

[71] My use of the term *everyday practice* drawn from Moschkovich (2007) in Chapter 7 takes this concern into account.

perspective as a *socio-political perspective of practice*. In Section 4.5 I develop this theoretical perspective to include mathematical aspects of practice within this perspective.

In this section I present Fairclough's concepts of power in discourse and power behind discourse to indicate why a social practice is also political and to argue that power is a significant part of this study.

4.3.1 Power behind discourse

The concept of *power behind discourse* refers to the relations of power that hold in place networks of socio-political practices and the discursive aspects of these networks, that is, the orders of discourse (Chouliaraki & Fairclough, 1999; Fairclough, 2001). In an institution control over the order of discourse is asymmetrical; those with power give value to particular discourse types, genres and styles and the conventions of the valued ordering are imposed on all those involved. For example, at a university there will be particular valued ways of recontextualizing non-mathematical practices into undergraduate mathematics, valued ways of teaching undergraduate mathematics, ways of defining who an undergraduate mathematics student is (perhaps in terms of who belongs in a mathematics course and, if so, in which mathematics course a particular student belongs), and what action is appropriate for such a student. By valuing certain discourse types, genres and styles in an order of discourse, power maintains boundaries between practices and between discourses and controls the relationships of recontextualization.

Fairclough (2001) argues that power behind discourse is discursive, since the valued conventions of the order of discourse of an institution are held in position not by physical power, but by ideology.[72] For Fairclough (1992, 1995), *ideology* is defined as discursive

[72] Marxist conceptions of power in mathematics education have been criticized for emphasizing the "destructive" effect of power which suggests that it is not possible to break the oppressive power exercised by one class over another (Valero 2008, p.52). However, Fairclough adopts a neo-Marxist perceptive and discusses issues of overdeterminism in explaining his choice of perspective. Firstly, Fairclough (1992) is critical of perspectives that overstate the ideological constitution of subjects while understating the agency of

constructions of reality that contribute to the (re)production and transformation of relations of domination or asymmetrical power relations. These constructions become naturalized in the sense that subjects are not aware of them. Fairclough (1995) uses the example of turn-taking practices in the classroom which are regarded as normal, with the teacher and students generally unaware of how they are positioned by such practices.

This concept of power behind discourse figures in two ways in this study. Firstly, foundational practice represents a new practice in the higher education space, with particular relationships to other practices in the space. Any innovation that such a practice represents in the existing order of discourse cannot be considered in isolation from the shifting power relations in the space (Chouliaraki & Fairclough, 1999). Secondly, since power maintains boundaries between discourses, the question of access to the dominant discourse types, genres and styles becomes significant (Fairclough, 2001). Such access can be regarded as *cultural capital*[73] which may also lead to socially powerful subject positions and social goods such as wealth and good jobs (Fairclough, 2001, p.52). Fairclough (2001)

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subjects to engage critically with ideological practices. He argues rather that "subjects are ideologically positioned, but they are capable of acting creatively to make their own connections between the diverse practices and ideologies to which they are exposed" (p.92). Secondly, Fairclough (1995) suggests that a Marxist conception of power (as opposed to a Foucauldian perspective) is necessary if one is to analyze asymmetries in power relations, and in his case how discursive constructions of the world in particular reproduce such asymmetries. Thirdly, Fairclough (1995) looks beyond class relations to other aspects of social structure; he argues that the social system cannot be restricted to class relations alone, but that class relations should be considered along with other relations such as gender and ethnic relations. In selecting Fairclough's perspective on power in this study, I acknowledge that other conceptions of power could be adopted, for example, Valero (2008) proposes the use of a Foucauldian view of power in mathematics education and Janks (2010) draws on both Marxist and Foucauldian perspectives in her work on literacy. There is certainly potential to explore the use of these perspectives of power in the research reported in this thesis.

[73] Fairclough draws on Bourdieu in his use of the notion of *capital* here. According to Chouliaraki & Fairclough (1999), certain languages, cultures, or social positions will be endowed with power and legitimacy in a particular network of socio-political practices and corresponding order of discourse.

notes that access is usually regarded as an individual achievement (such as individual hard work), and masks the social constraints on access. He identifies literacy as a prestigious practice in society, and I would argue that successful participation in mathematical practices at higher and higher levels is regarded in the same way. Foundational practice is designed to give students, disadvantaged by their schooling, access to advanced mathematics, a prestigious/dominant practice. The question to be answered in this study is whether this practice represents continuities or disruptions in the existing order of discourse and the implications for access to the dominant practices. Furthermore, Fairclough's concept of power behind discourse allows me to talk about access not as located in individuals, but in wider ideological struggles at the level of order of discourse, for example, about who belongs and does not belong in higher education.

4.3.2 Power in discourse

Power is not only at work between practices and discourses, but discourse itself is "a place where relations of power are exercised and enacted" (Fairclough, 2001, p.36). So the concept of *power in discourse* "is conceptualized in terms of asymmetries between subjects in discourse events".[74] It is also conceptualized in terms of "unequal capacity to control how texts are produced, distributed and consumed" (Fairclough, 1995, p.2).[75] Fairclough (2001) identifies four different but related constraints or forms of control; on the content or what is said/done (in this study this includes whether the content may be mathematical or non-mathematical), on the language form used, on the social relations that the participants enter into, and on the subject positions that are available for participants. With respect to the latter two constraints, subjects may be positioned in relation to one another in such a way that some are able to "incorporate the agency of others into their own actions" and so

[74] I interpret the term subjects broadly here, for example, the producer of a mathematics problem may not be present in the mathematics classroom, but through a text can constrain the discursive action of the students on the problem.

[75] In his earlier work, Fairclough uses both the terms *distribution* and *consumption*, but later uses *interpretation* only, which is what I am adopting in this study.

reduce the agency of the latter (Chouliaraki & Fairclough, 1999, p.24). In this study I will argue that control of the action in foundational practice (power in discourse) requires that the student also control the movement of meaning between this and other practices in the network.

While some subjects in a practice may be more powerful than others, on the strength of these two forms of control, Fairclough (2001) stresses that all subjects are constrained by the conventions of the discourses and the order of discourse that is being drawn on. Yet he suggests that the powerful subject has more freedom to select which discourse types are drawn on, may draw in a more cavalier way on these conventions, and control the way in which the non-powerful participants draw on these conventions (by putting pressure on them to occupy the subject positions set out for them and to act in certain ways). Nyabanyaba (2002) argues that the students in his study who chose not to engage at all with the deep practical problems tended to be students who had a socio-economic advantage. I would argue that these students had the power (seemingly a result of the advantages of their socio-economic status) to challenge the convention of using deep practical problems in the imported summative examination.

4.4 What Fairclough's socio-political perspective of practice allows me to talk about

In Sections 4.2 and 4.3 I presented Fairclough's socio-political perspective of practice, supplemented with examples of mathematical practices. Fairclough's three levels of abstraction, with a focus on language within each of these levels, allows me to talk (on the micro-level) about texts in the foundational mathematics practice and to talk (on the macro-level) about these texts as moments in a foundational practice which is located in a network of socio-political practices with a particular order of discourse. The practices in this network are related by recontextualization and crossing the boundaries between practices (and also events and texts) involves a movement of meaning. The concepts of power in discourse and power behind discourse indicate that asymmetries in control over meaning

and the movement of meaning are present at both the macro- and micro-levels. This theoretical perspective thus enables me to investigate the practice of foundational mathematics not only in terms of structure and how it relates to other socio-political practices, but also in terms of the agency of subjects acting in the practice. This is a theoretical contribution of the study; the socio-political perspective of mathematical practice allows me to reconfigure undergraduate mathematical practice in a way that neither constructs deficits nor ignores boundaries.

4.5 The mathematical work of this study

4.5.1 The challenge of talking about mathematical action in a socio-political practice

Fairclough's interest is in certain aspects of social change (such as capitalism and globalization), but his work has been used in mathematics education for analyzing interaction in school mathematics classrooms (e.g. De Freitas & Zolkover, 2009; Thornton & Reynolds, 2006). Yet I argue that this work talks about the discursive, social and political action on the micro-level (for example how language and other forms of semiosis in the classroom function in positioning the participants) and how this relates to macro-aspects of mathematics education. However, this use of Fairclough's work in mathematics education is weak in terms of talking about the mathematical objects and action on these objects. In the terms used by Adler and Lerman (2003), the use of Fairclough's perspective in mathematics education represents a "zooming out" (p.443) in which the talk is not "tightly on the mathematical activity" (p.445). Valero and Matos (2000) acknowledge the absence of talk about mathematical action in socio-political perspectives, noting that going "deeply" outside of mathematics results in the mathematics tending "to vanish or to be questioned" (p.398). Sierpinska (2005) warns that perspectives focusing on the social dimensions of learning run the risk of "discoursing the mathematics away" (p.229).

The literature talks about the challenge of selecting a theoretical framework that allows the researcher to talk about both the discursive, social and political action in a mathematical

practice on the one hand, and the mathematical action in such a practice on the other. However, the need for such a theoretical framework in this study only emerged for me in the interaction between the empirical data and my developing understanding of Fairclough's perspective. In Section 1.2, I presented transcripts from the study to illustrate how this challenge plays out in the empirical data.[76]

In this study the "construction of the theoretical" (Brown & Dowling, 1998, p.11) involves recontextualizing theoretical constructs from Morgan (1998), Moschkovich (2004, 2007) and Sfard (2000, 2001, 2007, 2008) in mathematics education, in interaction with the empirical data, in order to supplement the socio-political perspective of mathematical practice presented in Sections 4.2 and 4.3. This process, which I call the *mathematical work* of this study, has two parts; firstly, developing the notion of *mathematical discourse* as the language aspect of a socio-political mathematical practice in a way that is consistent with Fairclough's perspective, and secondly, attending specifically to mathematical objects and the action on these objects within this wider concept of mathematical discourse.

4.5.2 Choices involved in the mathematical work of this study

Before presenting the notion of mathematical discourse used in this study I discuss the choices that were made when appropriating theoretical constructs used by Morgan, Moschkovich and Sfard from mathematics education into Fairclough's socio-political practice perspective.[77] I emphasize that the choices about how to talk about mathematical

[76] I was alerted to this challenge during the preparation and delivery of Le Roux (2008b); while the discursive, social and political action of the students on questions (c), (d) and (e) of the Flu Virus Problem is similar, the students' action on three different mathematics objects (a function as well as the average and instantaneous rates of change of this function) in these questions differs.

[77] The mathematical work of this study has involved the recontextualization of the theoretical constructs used by Morgan, Moschkovich and Sfard within a perspective from critical linguistics that adopts a critical realist ontology. Such a recontextualization involves the movement of meaning across practices, and hence requires consideration of the effects of this movement on these constructs. Morgan, Moschkovich and Sfard, like Fairclough, subscribe to a view that objects are discursively construed. However, there are some differences

discourse in a way that is consistent with Fairclough's perspective and is suitable for the purposes of this study were made in interaction with the empirical data and the selection of analytic tools for the analysis.

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with respect to the ontology of these objects and the constraints on the discursive construal of meaning for these objects.

Morgan (1998, 2006) aligns herself with Fairclough and her use of language in the discussion of such objects suggests that she views them as having an existence independent of what actually happens and our representations of them. For example, she distinguishes between *representational objects* (Morgan, 1998, p.83) such as tables, graphs and symbols, and "their referents" (p.89) which are other objects like numbers, shapes, products, and patterns. These abstract referents are given a "concrete" (p.92) form by such representations. Morgan, like Fairclough, regards the discursive representation of such objects as constrained both by discursive and non-discursive conditions, for example, she argues that production of a text involves choice, a choice that is constrained by the writer's "place in the world, physically, cognitively, socially, culturally, conceptually" (Morgan, 1998, p.79, quoting Kress, 1993, p.172).

Moschkovich draws her notion of *Discourse* from Gee's work in sociolinguistics, and I use Gee's perspective on the constructive nature of language to infer that Moschkovich distinguishes between objects and the discourses about them. Gee (1990, p.8) distinguishes between "'reality' (experience, facts)" and "ideas" or "theories" that we use to describe this reality. The latter, he argues, "partially help to create, to constitute" (p.8) the former. Further, Gee (2005) argues that institutions and language about institutions are reciprocally related and the one cannot pre-exist the other; an institution exists because it is spoken of in a certain way, but if the institution did not already exist then discourse about it would have no meaning.

My recontextualization of Sfard's work draws on both her earlier work from an ontological/psychological perspective and on her more recent work within the social and discursive turn; she regards the latter work as a development on the former (personal communication, 22 September 2008). In her more recent work, Sfard (2000) ascribes to an ontological position that Fairclough (2003) would classify as *social constructivism* (p.8). This since she regards language as constitutive of reality, giving mathematical objects an "externality and an apparent ontological status" (Sfard, 2000, p.297). She argues, "Instead of merely being helpful in constructing and sharing knowledge of preexisting mathematical objects, communication and its demands must now be regarded as the primary cause of their existence" (p.297). Sfard (2008, p.173) suggests that it is not possible to separate mathematical objects and their representations; she describes "symbolic artifacts" as "the fabric of which these objects are made". I choose, rather, to follow the critical realist position suggested by Fairclough (2006), that is, that these discursive objects are subject to both discursive and non-discursive constraints. I refer to my recontextualization of Sfard's analytic tools when discussing the methodology in Section 5.3.

Within the linguistic turn in mathematics education notions of mathematical language/discourse have been developed. Much of this work draws on Halliday's concept of *mathematics register* (1978, p.195) and his functional perspective of language from systemic functional linguistics (1973, 1985). While some of this work focuses on the grammatical features of the register (e.g. Chapman, 1993; Pimm, 1987; Veel, 1999), other work extends this by attending to the multisemiotic nature of mathematical discourse (e.g. O'Halloran, 2000), or by attending to the socio-political aspects of the discourse (e.g. Herbel-Eisenmann & Wagner, 2007, 2010; Herbel-Eisenmann, Wagner & Cortes, 2010; Morgan, 1998, 2006, 2009).

The detailed work of Morgan (1998) on the discourse of written mathematics texts is suitable for this study in three respects. Firstly, Morgan's classification of mathematical discourse considers the grammatical features of the discourse identified in seminal work such as that of Pimm (1987) and Halliday (1985).[78] However, her concept of discourse is broader than suggested by the definition of mathematical register alone, since Morgan (like Fairclough) draws on Halliday's (1973) ideational, interpersonal and textual functions of language. Her work is thus consistent with Fairclough's notion of discourse as constituted by discourse types, genre, and style and allows me to talk about social, political and discursive action as part of mathematical discourse. Lastly, Morgan does focus on mathematical objects in written mathematics, either as processes or as objects in their own right.

Fairclough's socio-political practice perspective, supplemented with Morgan's classification of mathematical discourse, initially appeared adequate for the purposes of investigating the written texts of the practical problems in this study.[79] Morgan's work focuses on the analysis of written texts in mathematics (e.g. Morgan, 1998, 2005, 2006,

[78] Morgan (1998) acknowledges the limits of her own perspective in terms of how it addresses the visual forms of mathematical text. I address this issue in relation to my study in Section 4.5.3.

[79] As noted, my initial results using this perspective were published in Le Roux (2008a).

2009). She argues, however, that although classroom interaction is likely to have different "generic characteristics" (Morgan, 2006, p.237) from written mathematics, her methodological tools can also be applied to this interaction. A key part of this study involves investigating the classroom interaction as students solve the practical problems, action that involves writing, talking, representing, and using gestures such as pointing, looking, and tracing representations. Morgan (1998) shows how the process/object duality of mathematical objects figures in mathematical writing, for example, in nominalization. However, in this study I need to talk about action on mathematical objects in the broader semiotic action re-presented in face-to-face interaction. The work of Sfard (2000, 2008) and of Moschkovich (2004) enabled me to take my work forward in this respect.[80]

Firstly, Sfard (2008) argues that communication about abstract mathematical objects depends on our talk and on "what we see" (p.146). She uses the concept *pronounced focus* to refer to the words, drawings or gestures the student uses to identify "the object of her or his attention" (2000, p.304). Where Sfard (2000) supplements Morgan's work is in her use of the concept *attended focus* for what the student is "looking at, or listening to" when talking (p.304).[81] The latter concept suggests that mathematical discourse is about ways of attending, and not just ways of talking, writing and representing.[82]

[80] Herbel-Eisenmann and Wagner (2007, 2010) and Herbel-Eisenmann, Wagner & Cortes (2010) use critical discourse analysis to analyse in detail both textbooks and transcripts representing classroom interaction. However, this work focuses on the socio-political positioning of the student, rather than on action on mathematical objects. Reference is made to *content* (Herbel-Eisenmann & Wagner, 2007, p.9), but this is not the focus of their analyses.

[81] Sfard has developed the notions of pronounced focus and attended focus in her 2008 work, but in a way that is consistent with her 2000 work. I discuss the links in Section 5.3.2.

[82] In this discussion I consider the attended focus and pronounced focus as aspects of mathematical discourse, or ways of acting mathematically. In Chapter 5 I discuss how these concepts are also used as analytic tools in this study.

Moschkovich (2004, p.50) identifies *perspectives* or *ways of seeing* as one of the four features of mathematical discourse, thus supporting my argument made using Sfard (2000, 2008). In addition, Moschkovich's work enables me to make a key theoretical move in this study with respect to talking about mathematical action in a socio-political practice, that is, to link the notion of ways of acting mathematically to the ontological/psychological research in advanced mathematics. In Section 3.3.4 I argued that this research is strong in terms of how it talks about mathematical action, but does not talk about the discursive, social and political aspects of mathematics education. Using the concept of ways of seeing, Moschkovich (2004) argues that an object such as a linear function can be viewed both as an object and a process. Yet from her perspective, viewing an object as a process or as an object is a mathematical practice (or what I call ways of acting mathematically in discourse, as part of a socio-political mathematical practice) and does not refer to individual mental structures or "conceptions" (Sfard, 1991, p.3) as in the ontological and psychological research. In fact, in her more recent work Sfard (2008) provides a discursive perspective on her concept of reification (discussed is Section 3.3.1); she argues that reification is a discursive process and "involves the replacement of talk about processes with talk about objects" (p.171).

The concept of mathematical discourse used in this study is thus based on Morgan's work, but is supplemented with the related notions of attended focus and ways of seeing from Sfard and Moschkovich respectively. In the presentation of this concept in Section 4.5.3, I also discuss how the work of Sfard and Moschkovich overlaps with Morgan's work in other respects, thus strengthening my argument about the nature of mathematical discourse.

4.5.3 Mathematical discourse as the language aspect of a socio-political mathematical practice

Sfard (2000, 2007, 2008), Morgan (1998) and Moschkovich (2004, 2007) agree that there is something distinctive that we can call *mathematical discourse*, while acknowledging that this is in constant flux, has no fixed boundaries, and is used in a variety of practices such as school mathematics, undergraduate mathematics, or professional research mathematics. I

have used the classifications provided by these three researchers, in interaction with the empirical research texts, to develop the concept of *ways of acting mathematically in discourse* that can be used in this study. My use of discourse as the language aspect of social practice, which takes the form of particular discourse types, genres and styles, is consistent with Fairclough (2003).

The ways of acting mathematically in discourse, which respond to the challenge of talking about the discursive, social, political and mathematical action in a mathematical practice, are as follows:

- Ways of talking and writing about objects[83] and ways of representing objects
- Ways of making links between objects, texts, events and practices
- Ways of endorsing arguments about mathematical objects
- Ways of evaluating the pronouncements of other subjects
- Ways of attending (ways of looking at mathematical objects and their representations, and ways of listening to talk about objects)
- Ways of operating on mathematical objects
- Ways of identifying oneself and others
- Ways of interacting socio-politically

In the rest of this section I motivate for each of these ways of acting, with reference to the work of Morgan, Moschkovich and Sfard and, since these constructs have been developed in interaction with the empirical data, using appropriate extracts from the three practical problems used in the study (the latter is provided for illustration at this stage, with the detail of the analysis provided in Chapters 7 to 11). The linear presentation of the ways of acting does not represent the interrelated nature of these ways.

[83] I clarify the reference to both *objects* and *mathematical objects* in the discussion of the ways of acting mathematically that follows.

Ways of talking and writing about objects and ways of representing objects

Morgan (1998) provides the detail of these ways, which are consonant with what Sfard (2007, 2008) calls *word use* and *visual mediators* and what Moschkovich (2004) terms *meanings for utterances.* Since my study is located in a pedagogic practice in mathematics, I draw on Mercer (1995, p.80) to distinguish between *educated* ways of talking, writing and representing that signal participation in mathematical practice and *educational* ways of talking as part of the pedagogic practice of mathematics, for example, patterns of exchange between students and the teacher.[84] In this study the language of learning adds a third dimension to the ways of talking and writing (Adler, 1997); the medium of instruction at the university is English, although some students in the study use isiXhosa, Sesotho and Setswana when interacting in the small groups.[85]

The educated ways of talking, writing and representing are mainly about mathematical objects. These objects may be *basic objects* such as numbers and shapes, objects derived from these basic objects (for example, areas), or *relational objects* such as patterns

[84] Mercer (1995) uses the terms *educated discourse* and *educational discourse*, using the word *discourse* in a way that is consistent with my use in this study, that is, as "language as it is used to carry out the social and intellectual life of a community" (p.79). I use Mercer's concept more specifically to refer to ways of talking, writing and representing.

[85] I have chosen not to make the student's talk in languages other than English an analytic focus of this particular study. This choice was made for two reasons. Firstly, one group of students was selected for the mix of languages that they used in their small group discussions. However, they spoke English during the first video-recording session (action they said they thought appropriate for the research project). During the second session (when I reassured them that use of other languages was allowed), they used their home languages (isiXhosa, Sesotho and Setswana) more than I had observed them to do during ordinary workshop classes. Neither recording thus re-presents what I had observed during workshop classes in which no video recordings were made. Secondly, the social practice perspective of language that I adopt in this study looks more broadly at language use in social practices for all students, and not just those students who are learning in a language that is not a home language. Having made these choices, I acknowledge that there is scope to broaden the ways of talking and writing used in this theoretical perspective to include the student's use of languages other than English.

(Morgan, 1998, p.83). Sfard (2000) argues that since mathematical objects do not have concrete referents, we need symbols, graphs, diagrams etc. to represent them. A distinguishing feature of mathematical discourse is its use of symbolic notation to represent mathematical objects (Morgan, 1998). [86]

The Flu Virus Problem (see Appendix B and Appendix Q) contains variables, a function in one variable, the derivative of this function, and the limits of both functions. The variables are represented using the symbols *P* and *t*. The functional relationship is represented by the notation *P*(*t*) and no algebraic formula is provided. The instruction to the student in question (a) places value on a "rough sketch of the graph" for representing the functional relationship. The term "rough sketch" has a particular meaning in the Foundational Course, that is, the student constructs the graph by attending only to the values that are cued in the text and to the shape of the graph (whether it is increasing or decreasing and the concavity), and not by plotting individual points.

Morgan (1998) identifies specific features of the language that are used to talk about mathematical objects, for example, specialist vocabulary or colloquial vocabulary that takes on specific meaning within the discourse and new grammatical structures or existing grammatical structures used in new ways. The word "function" and the symbol "lim" for limit in the Flu Virus Problem are everyday words that take on specific meanings within mathematics. Morgan identifies the imperative "Let" as in "Let *P*(*t*) denote the number of people …" in the Flu Virus Problem as typical of school mathematics texts. Sfard (2008, p.135) argues that in a sentence such as "Let … denote …", which is typical of advanced

[86] Morgan (1998, p.83) includes in her list of mathematical objects *representational objects* such as graphs, diagrams and table. However, I have noted that Morgan distinguishes these objects from "their referents" (p.89) in a way that is consistent with the critical realist ontological perspective used in this study. There is scope beyond this study to investigate the representational aspects of mathematical discourse in more detail, for example, drawing on the work of O'Halloran (2000).

mathematical discourse, the naming word (for example $P(t)$ in the Flu Virus Problem) is a pointer to the existence of some object.[87]

Yet Morgan (1998) recognizes the need to go beyond talking about mathematical objects in terms of their naming, and to attend to whether mathematical objects are talked about as processes or objects in their own right (with the grammatical feature of nominalization signaling the talk about objects themselves). In her recent work on reification, Sfard (2008) distinguishes between talk about processes and talk about objects. I refer further to the process/object duality of mathematical objects in the discussion of ways of looking at mathematical objects.

The discourse of relevance in school mathematics and undergraduate calculus reform suggests that the educated ways of talking, writing and representing can also be about non-mathematical objects.[88] This talk is from the perspective of the recontextualizing mathematical practice. For example, the Flu Virus Problem references the spread of a flu virus in a community in such a way that these non-mathematical objects appear not real. The spread of the flu virus is represented in a sketch graph which represents some features of the task context and not others. In addition mathematical discourse may involve talking and writing about mathematical objects in everyday words. This is a feature of calculus reform texts; "… we continually ask students to explain verbally what their answers mean in practical terms" (Hughes-Hallet et al., 1994, p.vii). This way of writing is also required in the Flu Virus Problem and the Chemical Reaction Problem (see Appendix D and Appendix Q) in the Foundational Course. The term "practical terms" is interpreted in the Course to signal that no mathematical words like "derivative", "rate" etc. should be used. In this study I refer to this way of talking or writing as the *practical terms genre*.

[87] As noted in Footnote 78, Sfard and Fairclough disagree on the ontological status of this object to which the name refers.

[88] I use the term *non-mathematical object* while acknowledging that this is actually a recontextualized object that has been appropriated into mathematical practice.

The educational ways of talking will depend on the pedagogy. For example, in a more traditional pedagogy the student may be providing final answers which are evaluated by the teacher, with an absence of student interaction. In a learner-centred pedagogy the students may be talking to one another. For example, the workshop material provided to students in the Foundational Course specifies the ways that a student should be talking in their small groups, for example, "making suggestions about strategies to solve a problem", "explaining answers", "asking questions about solutions", "encouraging one another to keep going / to participate" (Foundational Course Resource Book, 2007, p.16). Furthermore, a learner-centred pedagogy can influence what type of educated talk and representation occurs; when a student is explaining or making suggestions verbally in a small group he may read mathematical symbols from left to right as if it were linear text (Morgan, 1998), use gestures to point to mathematical objects, and represent these objects by tracing graphs in the air. Sfard (2008) distinguishes between the linear nature of spoken text about mathematical objects and symbolic representations of these objects, with the latter "even more likely than its spoken or written counterpart to become the object of metadiscursive activity" (p.159).[89]

Educational ways of talking may not be about mathematical objects and their relationships alone, but can be explicitly about how to behave in the mathematical practice. In this type of educational talk the social regulation in the pedagogy is explicit. For example, a student may regulate the pace of work in a small group by suggesting that the students proceed to the next question while waiting for assistance from the teacher. The teacher may instruct the students to follow particular instructions in the text of a problem or to work faster through a set of problems.[90] In a learner-centred pedagogy the instructions might be about

[89] Sfard (2008) identifies reasoning, abstracting, and objectifying as metadiscursive activities.

[90] As noted in Section 3.7.5, Bernstein (1996) argues that every pedagogy is socially regulated, with the instructional discourse embedded in the regulative discourse. So in the terms used by Bernstein (1996, p.48), this type of educational talk could be referred to as making the regulative discourse explicit. In this study it has not been necessary to bring Bernstein's work in this regard into my theoretical framework, but I do draw on his insight that every pedagogy is socially regulated. Consistent with the socio-political perspective of

how to act in such a pedagogy, for example, the teacher may remind the students to explain their answers or to compare answers with one another. The description of undergraduate mathematics in Sections 2.4.4 and 2.4.5 suggests that such social regulation is made explicit in the Course material for the Foundational Course, but is an absence in the course material of the first- and second-year mainstream courses. I discuss this further in the discussion of ways of identifying oneself and others and ways of interacting socio-politically.

Ways of making links between objects, texts, events and practices

Morgan (1998, p.80) draws on Halliday (1985, p.123-124) to identify relational processes as a key feature of scientific writing. In particular, these processes are either attributive (attributing a property to an object or an object to a class of objects) or identifying (setting up an identity between objects). Sketching the graph of $P(t)$ in the Flu Virus Problem requires the student to attribute the property of increasing to the graph of the function $P(t)$ and to identify the derivative function $P'(t)$ with the gradient on the function $P(t)$. Both Morgan (1998) and Sfard (2008) identify the role that the equals sign plays in stating identities between objects.

Relational processes form part of what Sfard (2007) calls the *narratives* of mathematical discourse, in this case, stories about mathematical objects. She defines narratives as "… any sequence of utterances framed as a description of objects, of relations between objects, or of processes with or by objects that is subject to *endorsement* or rejection with the help of discourse-specific substantiation procedures" (2008, p.134, emphasis in the original).[91] I

practice, I choose to use the concept of talk about how to behave/talk that makes the social regulation explicit. This talk identifies a student in a particular way, that is, as a student who needs to be told how to behave, and thus uses the concepts of identification on the level of text and style on the level of discourse from Fairclough (2003).

[91] Sfard (2008) distinguishes between narratives about mathematical objects and higher level metadiscursive narratives about the discourse itself. In developing the ways of acting mathematically in discourse for this

refer to the nature of the endorsement of the utterances in the discussion of ways of arguing that follows.

The discourse of relevance in school mathematics and undergraduate calculus reform suggests that links can also be made between non-mathematical and mathematical objects. Much of the literature in Chapter 3 suggests that in both school and advanced mathematics, a one-way movement from the non-mathematical to the mathematical is valued. I will be arguing in this study that the foundational practice values a to-and-fro movement over the mathematical/non-mathematical boundary.

In developing the notion of ways of making links in this study, I combine the focus on properties of and relationships between objects from Morgan and Sfard with Fairclough's notion of mediation and making links between texts, events and practices. Sketching the graph of the function $P(t)$ in the Flu Virus Problem requires the student to identify the mathematical objects with their meaning in the task context, for example, that the derivative function $P'(t)$ represents the rate of change of the number of people who have or have had the disease with respect to time. Not only does this relationship represent a movement between mathematical and non-mathematical objects, but it also sets up a

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study, in interaction with the empirical data, I did not find the need for the latter metadiscursive narratives in the theoretical framework for this particular study.

Sfard's (2008) description of mathematical discourse also includes the concept of *routines* or the metarules that describe the repetitive action/patterns of action in a discourse. In terms of the socio-political perspective of practice used in this study, I see the notion of routine as used by Sfard (2008) as featuring at a higher level than the individual ways of acting mathematically in discourse presented in this section. So a particular routine of foundational mathematics could consist of the rules for how and when to make links between mathematical and non-mathematical practices, and what form of endorsement is considered appropriate in this border crossing. Consistent with Fairclough's (2001) notion that a socio-political practice is both enabling and constraining, Sfard (2008) argues that mathematical routines "are both confining and indispensible" (p.221) in that they set the ways of behaving for a community but also allow creativity. The description of foundational practice in this study could be regarded as the beginning of an attempt to describe the routines of this practice, but this task extends beyond the work presented in this thesis.

relationship between practices, in this case mathematical practice and the non-mathematical practice of the task context. Recognizing the assumptions behind making a "rough sketch" of the graph of $P(t)$ requires that the student make a link to other events in the Foundational Course (such as lectures) and the texts used in these events. In Chapter 7 I will also be arguing that the successful foundational student is also able to make links within the text of a practical problem itself.

Ways of endorsing arguments about mathematical objects

As noted in the previous section, Sfard (2008) indicates that mathematical discourse has specific ways of endorsing the narratives about mathematical objects. She suggests that "colloquial mathematical discourses" (Sfard, 2008, p.223) are often endorsed with reference to empirical evidence. Both Sfard (2007, 2008) and Morgan (1998) indicate that academic mathematics uses deductive reasoning based on definitions and theorems from within the discourse for endorsement. The description of advanced mathematics provided in Section 3.3 suggests that this is also the valued way of arguing in this practice.

In the Flu Virus Problem the student is required to use the properties of mathematical objects as endorsement. For example, the derivative function $P'(t)$ can never be negative since the function $P(t)$ is always increasing. Yet for other arguments in this problem the endorsement lies in the task context, for example, the graph of $P(t)$ is always increasing since $P(t)$ is defined as the number of people who have or have had the disease at time t.

Ways of evaluating the pronouncements of other subjects

This way of acting has three parts. Firstly, it involves *who* evaluates. Sfard (2008) argues that for academic mathematics who subscribe to a platonic ontology, human agency is absent, with the authority residing in the mathematical practice itself. Sfard (2008) argues that for a child, mathematical decisions may ultimately depend on other people and on the power relations between them. Although Sfard (2008) speaks generally of "the child" (p.234) in making this argument, I would argue that another subject acts as the authority for evaluation in a pedagogical mathematical practice. For example, in a more traditional

pedagogy the teacher is assigned authority to evaluate students' answers, while in a learner-centred pedagogy students are given some agency for evaluating one another. The text of the Flu Virus gives the authority for evaluation of a student's sketch graph to the tutor, by stating, "… **do not continue until you have had your graph checked by a tutor**" (emphasis in the original). Yet the workshop material also assigns the role of evaluator to the student when the text indicates that the student should be "asking questions about solutions" (Foundational Course Resource Book, 2007, p.16). The notion of *who* evaluates is related to issues of power in discourse, which I discuss under ways interacting socio-politically.

Secondly, ways of evaluating includes the *form* of the evaluation, for example, this may be positive or negative, it may be content free or contain content in the form of explanation of why a pronouncement is correct or incorrect, a rewording of a pronouncement, or an alternative pronouncement. This Foundational Course Resource Book (2007) promotes evaluation that is positive ("congratulating one another" (p.16)) and non-personal ("criticizing ideas, not people" (p.16)).

Thirdly, ways of evaluating involves the *authority* for the evaluation, that is, what is recruited in the evaluation. An evaluation may recruit the non-mathematical or the mathematical. In academic mathematics or advanced mathematics the authority is mathematical, and resides in the mathematical practice. An evaluation may recruit the power of the evaluator himself and not recruit from a mathematical practice at all, for example a teacher who says, "Just do as I say". An evaluation may recruit a specific text, for example, a student may refer to the instruction to use "practical terms" in the Flu Virus Problem. In Chapter 8 I will argue that the Tutor in this study recruits the pedagogic practice of the Foundational Course itself; evaluating the students' action on the Car Problem he gives a negative evaluation to a student's use of an intuitive method to solve the problem on the basis that following the prescribed method will allow the student to practise the method for implicit differentiation.

Ways of attending (ways of looking at mathematical objects and ways of listening to talk about objects)

This way of acting draws on the concept of focus of attention from Sfard (2000). Firstly, there are ways of looking at mathematical objects (or from Moschkovich (2004), ways of seeing) as a feature of mathematical discourse. As noted in Section 4.5.2, this way of acting mathematically is a recontextualization of Sfard's earlier work conducted from an ontological/psychological perspective. Sfard (1991) describes a structural conception as "recognizing an idea at a glance" (p.4) in which an object has a "static structure, existing somewhere in space" (p.4). An operational conception means regarding an object as a "potential rather than actual entity, which comes into existence upon request in a sequence of actions" (Sfard, 1991, p.4). In this earlier work, Sfard views these conceptions as individual mental structures. In her more recent work Sfard (2008, p.144, p. 171) presents a discursive perspective on the duality of mathematical objects; she refers to structural utterances (or talk about objects) and processual utterances (or talk about processes).

In this study, I do not refer to structural and operational views as mental conceptions, but rather as ways of acting in mathematical discourse, that is ways of *looking structurally* and *looking operationally.*[92] This draws on the argument by Moschkovich (2004) that these ways of seeing are mathematical practices. As discussed in Section 6.3, I use a student's talk about mathematical objects to identify their ways of looking at these objects.

In question (d) of the Flu Virus Problem the student should look structurally at the expression $\frac{P(7)-P(4)}{7-4}$ and identify this as representing the average rate of change of the function $P(t)$ with respect to time. In this study I argue that identifying the properties of the graph of the function $P(t)$ in the Flu Virus Problem requires that the student look operationally at the function and consider the spread of the disease in the community over

[92] In the analysis in Chapters 7 to 11 I reference Sfard's (1991, 1992) ideas on the dual role of mathematical thinking, but from this recontextualized perspective, that is, that structural and operational views are ways of acting mathematically within mathematical discourse rather than mental conceptions.

time. This way of looking can be contrasted to a structural view of the function $P(t)$ which might result in a student viewing the function in the static form of a "straight line graph" or a "cosine graph".

In this study I have, however, defined *ways of attending* as more than *ways of looking* but also as *ways of listening*. The educational ways of talking in a learner-centred pedagogy, for example, "asking questions about solutions" (Foundational Course Resource Book, 2007, p.16) imply particular ways of listening. The discourse of relevance in school mathematics and calculus reform suggests that this involves listening to talk about both mathematical and non-mathematical objects. In this study I will argue that the students listen to one another's verbal discourse in a way that enables them to collectively construct answers by repeating and rewording one another's talk.[93]

Ways of operating on mathematical objects

I have identified "looking operationally" as a particular way of looking mathematically that forms part of mathematical practice. Sfard (1991, p.4) argues that a process view is about the *actions* on mathematical objects (alternatively, *processes* or *algorithms*). Since I use the term action more broadly in this study, I choose here to refer to *operations*[94] on mathematical objects.

Certain ways of acting operationally can be identified in the Car Problem (see Appendix C and Appendix Q), for example, differentiating (implicitly) functions of one variable in the

[93] Chapman (1995, p.244) uses the concept of *intertextuality* to argue that learning mathematics involves text-connecting practices. The text-connecting practices in talk suggest that there are valued ways of listening in mathematical practice.

[94] So I use the term *operations* for what Sfard (1991, p.44) talks about as *actions*, *processes*, or *algorithms*. Davis (2010) uses the term *operations* as it is used within the practice of mathematics (Davis calls this *Mathematics*, with an upper case "M"). Davis argues that, "The study of Mathematics is, amongst other things, a study of the operatory possibilities and their inter-relations, and not merely the study of mathematical objects" (p.380).

equation $x^2 + y^2 = z^2$, substituting numerical values into both the Pythagorean and the derivative equation $2x.\frac{dx}{dt} + 2y.\frac{dy}{dt} = 2z.\frac{dz}{dt}$, and rearranging the subject of the formula in the derivative equation to solve for $\frac{dz}{dt}$. Acting operationally may be represented in talk; in Chapter 8 I will describe how students in this study verbalize the operation of substitution in the Car Problem.

Ways of identifying oneself and others and ways of interacting socio-politically

Morgan (1998) argues that mathematical discourse identifies subjects in certain ways and sets up particular relationships between these subjects. For example, the author of a mathematical text may speak with authority and as a participant in a particular mathematical practice. In contrast, a student who is required to explain the meaning of a mathematical object to people who are not specialists (using the practical problem genre as is required in both the Flu Virus and Chemical Reaction Problems) is not identified as a participant in the mathematical practice. Morgan's (1998) analysis of written mathematics texts within the practice of assessment points both to power relations at the level of the mathematics classroom (and hence the notion of interacting *socio-politically*) and to power in relation to the ordering of specific genres of assessment in the wider network of practices.

These two ways of acting cannot be separated from the other six ways of acting mathematically discussed so far. For example, talking about mathematical objects in "practical ways" identifies the student as not participating in a mathematical practice, moving to-and-fro between mathematical and non-mathematical objects (as opposed to a vertical movement from the mathematical to the non-mathematical) or using the task context to support an argument (rather than using deduction) may identify a student as not participating in advanced mathematics. A mathematical text instructing a student to have his work checked by the teacher sets up the teacher as an authority in the ways of evaluating in a particular mathematical practice, whereas a learner-centred pedagogy that

identifies the student as evaluating the pronouncements of others sets up different power relations between participants.[95] A teacher's explicit verbal instructions to students about how to behave in a small group, for example talk about the pace at which the students are working or the need to compare answers within the group, identifies the students as needing to be socially regulated.

4.6 Revisiting my research questions

In Section 1.5 I presented two propositions that address the research problem in this study. These propositions emerged from the empirical and theoretical spaces, as introduced in Chapter 1. In this section I indicate how the socio-political perspective of mathematical practice presented in detail in this chapter allows me to elaborate on these propositions in the form of detailed critical questions. The research questions are repeated below, followed by discussion of these questions.

Research Question 1:

The practical problems give meaning to the practice of foundational mathematics and set up subject positions for the students.

(a) What relationships between this practice and other practices, both mathematical and non-mathematical, are represented?

(b) What do these problems represent as the valued mathematical action in this practice?

(c) What socio-political relationships and social identities do these problems construe for the subjects and who has power in the discourse?

(d) What continuities and/or disruptions in the movement of meaning across texts, events and practices are represented?

[95] In this study I argue that certain participants are identified by others and/or identify themselves as authorities in the foundational practice. By *authority* here, I mean someone who participates successfully in the practice. The identification of such an authority in the mathematics classroom has implications for the power relations, since this participant may control the action, for example, by controlling who talks, what is talked about, and how this content is talked about.

(e) How can this representation be explained with reference to the wider socio-political space, that is, what discourse types, genres and styles do the problems draw on?
(f) What continuities and/or disruptions does the foundational practice represent in the wider order of discourse, and with what implications for access to dominant mathematical practices and change in the higher education space?

Research Question 2:

The student mathematical action on the practical problems both enables and constrains the adoption of the valued subject positions in the practice of foundational mathematics.

(a) What mathematical action do students use when solving the practical problems?
(b) Does this mathematical action enable or constrain their occupation of the subject positions set up for them in the practice (as described in research question 1)? In particular, do they control the movement across texts, events and practices, both mathematical and non-mathematical, as required in the practice?
(c) In what ways is this mathematical action enabled or constrained by the discourse types, genres and styles that the students recruit and/or the socio-political interaction in the classroom?

The first research question focuses on the practical problems as products of the processes of production and is based on the dialectical relationship between text and social practice (Fairclough, 2001). On the one hand the texts of the practical problems in the study give meaning to the practice of foundational mathematics; questions 1(a) to 1(d) are designed to identify how the problems represent the ways of acting mathematically, interact by setting up socio-political relations between subjects and discursive relations to other texts, discursive events and practices, and identify the subjects of the practice in particular ways. Since this study is about access to mathematical practices and focuses on practical problems, a focus of these questions is the relationships between foundational practice and other mathematical and non-mathematical practices.

Yet on the other hand, consistent with Fairclough (1995), the meaning that these problems create is constrained by their location in a network of socio-political spaces with a particular order of discourse, that is, they repeat certain discourse types, genres, and styles from within this network. Question 1(e) allows me to explain, with reference to the wider discursive space described in Chapters 2 and 3, the representation of the foundational practice described in questions 1(a) to 1(d). Lastly this study is about foundational practice as it relates to access to dominant mathematical practices in higher education and about innovation in this space, issues that I explore in question 1(f).

Research question 2 focuses on the mathematical action of students on the practical problems. This question serves two purposes in this study. Firstly, the students draw on the problems as a resource and thus also produce the meaning of the foundational practice (Fairclough, 1995, 2001). The descriptive question 2(a) allows me to identify this meaning (the need to focus on both the problem texts and the action on these texts was identified in Section 3.7.6). Secondly, this study is about students' transition between mathematical practices, that is, from school to foundational to advanced mathematics. In question 2(b) I investigate whether students are able to cross the school/foundational boundary, that is, whether they are able to adopt the style of foundational mathematics students. In question 2(c) I explain the student action with reference to both the socio-political action at the micro-level of the classroom and the constraints afforded by the wider socio-political practices.

4.7 Summary of this chapter

In this chapter I have presented a socio-political perspective of mathematical practice as the theoretical perspective for this study. Fairclough's perspective provides the overall framework, a perspective that I have suggested allows me to talk about the socio-political aspects of mathematical practice at the micro-level of classroom events and the macro-level of the network of socio-political practices. This perspective has been supplemented with recontextualized theoretical constructs from Morgan, Moschkovich and Sfard in

mathematics education, constructs that allow me to talk about ways of acting mathematically in discourse, with a focus on action on mathematical objects. In Chapters 5 and 6 I consider the implications of this choice of perspective for the remaining three parts of the research process identified by Crotty (2003), that is, epistemology, methodology and methods.

CHAPTER 5 THE RESEARCH PROCESS METHODOLOGY AND METHODS (PART 1)

5.1 Introduction to this chapter

In Chapter 4 I introduced the theoretical framework for this study, a socio-political perspective of mathematical practice, as one of four elements of the research process (Crotty, 2003). I begin this chapter by considering what it means to talk about doctoral research practice as a socio-political practice, and the implications of this perspective for the production of knowledge in this study (the epistemology of this study). I then focus on the methodology and method as the remaining two elements of the research process (Crotty, 2003). I present the analytic framework and describe how I collected the data to which these tools are applied. In Chapter 6 I illustrate the use of the tools on this data, and end with a discussion of issues of quality and ethics. The linear presentation of the research process in Chapters 5 and 6 is necessary if I am to justify my choices, but does not represent how the research process played out in practice.

5.2 A socio-political perspective of doctoral research practice

The concept of research as a socio-political practice is part of the discourse of mathematics education research (e.g. Valero, 2004). In this section I reconceptualize this notion specifically for doctoral research in mathematics education in a way that is consistent with Fairclough's epistemology of constructivist structuralism used in this study, and described in Section 4.2.

The socio-political perspective of practice presented in Sections 4.2 and 4.3 can be used to describe doctoral research practice in mathematics education. Such research has

characteristic activities (for example, reviewing literature, collecting data), objects (both material and theoretical), position in time and space, subjects (doctoral researchers, supervisors, research participants, other cited researchers, etc.), socio-political relations between these subjects, values in terms of want counts as doctoral research (criteria for rigour and the production of new knowledge), and discourse. In Section 3.2 I have used the work of Barwell (2009) to point to the discursive nature of this practice; participating in practice involves interpreting texts (for example, the text of a practical problem) and producing texts (such as conference papers and this thesis).

In addition, the practice of doctoral research in mathematics education is located in a network of socio-political practices related to mathematics education and research. This network is characterized by power struggles at the level of order discourse, for example, over what theoretical perspectives are used to talk about data, what ethical procedures govern the relationship between the researcher and the research participants, and what principles of rigour should be applied.

So what are the implications of adopting such a perspective on research in terms of the knowledge that can be produced in this study? Firstly, the description of the background to this study in Chapter 1 locates me as a subject in a number of overlapping practices in the network of socio-political practices in mathematics education, for example, as a former school teacher, a foundational lecturer, a doctoral researcher etc. The description of my positioning in Section 1.3 is not just in the interests of "transparency" (Valero, 2004, p.19), but about acknowledging "the dialogical, political and social nature of our task as researchers" (p.19) and that "…what we choose to research and the ways in which we carry out that research are constructions determined, among other things, by who we are and how we choose to engage in academic inquiry" (p.6). However, using Fairclough's epistemology of constructivist structuralism which emphasizes the structure/action dialectic (Chouliaraki & Fairclough, 1999), I interpret Valero's (2004) use of the term *construction*

(p.6) in a particular way.[96] The empirical (in this case what I can observe about the empirical space) is not the same as the real, which has an independence irrespective of whether it is observed or not (Fairclough, Jessop & Sayer, 2001). Although I represent the empirical in discourse and am able to exercise agency in my research practice, this discursive construal is constrained in two respects. Firstly, this construal is constrained by non-discursive conditions (Fairclough, Jessop & Sayer, 2001). Secondly, it is constrained by my positioning within the practice of doctoral research in mathematics education which has particular valued ways of acting; according to Fairclough (2001), subjects "are enabled through being constrained: they are able to act on condition that they act within the constraints of a type of practice" (p.23).

5.3 Methodology

Fairclough (2006) describes *methodology* as a way "of tackling a topic in a theoretically coherent and systematic way" (p.11). Part of the methodological work of this study has been selecting analytic tools that operationalize a socio-political perspective of mathematical practice. These tools specify what I attend to when talking about ways of acting mathematically in discourse (which involves discursive, social, political and mathematical action) on the micro-level of text, and how I make links between these ways of acting and the wider discursive space on the macro-level. In this section I begin by arguing that critical discourse analysis, supplemented with focal analysis recontextualized from mathematics education, provides the appropriate analytic tools for this study, and I

[96] Valero, like Fairclough, talks about the discursive construction of objects. However, I argue that Valero's use of language suggests that she subscribes to stronger version of the ontological claim that "the social world is textually constructed" (Fairclough, 2003, p.9). For example she states that, "… research creates discourses about phenomena and objects which do not necessarily exist as such, but that exist in as much power/knowledge of the scientific endeavour has phrased them and, therefore, created them" (Valero, 2008, p.45). Although Valero (2008) acknowledges that the objects of mathematics education research are not arbitrary constructions, I follow the more strongly stated view of Fairclough (2006) that these constructions are subject to discursive and non-discursive constraints.

refer to the transcripts in Section 1.2 in this discussion. I then present the analytic framework for this study.

5.3.1 The choice of critical discourse analysis and focal analysis

Fairclough (2003) suggests that critical social research begins with questions such as:

> … how do existing societies provide people with the possibilities and resources for rich and fulfilling lives, how on the other hand do they deny people these possibilities and resources? (p.202)

My interest in this study is in the practice of foundational undergraduate mathematics which exists to provide students with the "possibilities and resources" to make a successful transition from school mathematics practice to undergraduate mathematical practice. This practice as a pedagogic practice in mathematics is discursive since it predominantly takes place in language. Text, such as the text of a practical problem or the transcript representing student action, is a both a *mode of representation* since it reflects wider socio-political practices, but it is also a *mode of action* as it gives meaning to these practices by constituting them (Fairclough, 1992, p.64). Gee (2005) points to text as action when he argues that "there is no institution (socio-political practice) unless it is enacted and reenacted moment-by-moment in activities (texts), and the identities connected to them" (p.1, my additions in parenthesis). Yet these arguments by Fairclough and Gee describe the relationship between text and socio-political practice theoretically; Fairclough (2003) says that, "without detailed analysis one cannot really *show* that language is doing the work one may theoretically ascribe to it" (p.204, emphasis in the original). Discourse analysis is a methodology that allows one to perform this detailed analysis of the text, that is, it provides the tools to operationalize the theoretical perspective presented by Fairclough and Gee.

But why, in particular, is *critical* discourse analysis (CDA) necessary in this study? Firstly, the texts of the practical problems and the student action are the discourse moments of social events, that is, they are "instances of language in use" (Fairclough, 2006, p.9). Fairclough (2006) describes CDA as the analysis of such instances, with particular interest

in relations between discourse and other elements of social practices. Given the dialectical relationship between text and socio-political practices, the choice about what to analyze is, for Fairclough (2003), not an "either/or" (p.3) choice:

> On the one hand, any analysis of texts which aims to be significant in social scientific terms has to connect with theoretical questions about discourse (e.g. the socially 'constructive' effects of discourse). On the other hand, no real understanding of the social effects of discourse is possible without looking closely at what happens when people talk and write. (p.3)

In presenting my use of CDA in this chapter and in Chapter 6 I will argue that the tools provided by Fairclough are productive for this study since they allow me to identify the socio-political work that the texts do on the micro-level of social event. For example, in the discussion of Transcripts 1.1 and 1.2 in Section 1.2 I use the tools of CDA to talk about how the students talk about and represent mathematical objects, make links between texts, evaluate one another, and position themselves as certain types of people in relation to others in the classroom. Furthermore, the three stages of CDA allow me to link these texts to the macro-level socio-political practices.[97]

Secondly, Fairclough (2001) argues that we legitimize and deligitimize power relations through discourse, yet since discourse is "opaque" (p.33), we do this without being conscious of it. Fairclough argues that the specifically *critical* aspect of discourse analysis involves helping people become conscious of what appears opaque. I will argue that Fairclough's tools allow me to talk about power at work on the micro-level of interaction in

[97] Part of the theoretical journey I have taken in this study has involved identifying discourse analytic tools that allow me to make links between the micro-level action of the classroom (represented in text) and the wider space. I began this journey by using Fairclough's tools as part of my participation in a Master's level module on academic literacies and discourse analysis (as presented in Bennie and Tobias (2007) and Le Roux (2008a)). This was followed by pilot work using Gee's (2005) method for discourse analysis (as reported in Le Roux, 2008c). Fairclough's tools were then chosen on the strength of the possibilities they provide for providing the micro-macro link.

the mathematics classroom as well as in the boundaries between foundational mathematics practice and other practices in the wider order of discourse.

Yet the discussion of Transcripts 1.1 and 1.2 in Section 1.2 suggests that, if I am to talk about the student action, I need to be able to talk about how the students act on the mathematical objects. This is where the tools of CDA fall short in terms of my study, as they do not operationalize the ways of acting that are specific to acting on mathematical objects, for example, the ways of looking at mathematical objects. This dilemma is also recognized in the mathematics education literature. Ernest (1998, p.80) argues that theoretical perspectives traditionally regarded as being "outside" of mathematics education may not have developed the appropriate analytic tools to study what he calls the "unique characteristics of mathematics". Of note for the purposes of this study is the view of Sfard (2000) that, while discourse analysis has been used to study the "rules and norms constituting mathematical practices" (p.298), little attention has been given to using the methodology for the study of mathematical content and in particular for studying mathematical objects. Sfard (2000, 2007) points to the methodological challenges of studying mathematical objects, challenges which are certainly not specific to discourse analysis. She argues that, since mathematical objects are abstract entities and do not have concrete referents, we use language and representations to talk and write about them. Yet it is hard (if not impossible) to distinguish the object itself from the language and other forms of semiosis that we use to represent it. In this chapter and in Chapter 6 I will argue that Sfard's (2000, 2001) tools for *focal analysis* can be recontextualized into the wider framework provided by Fairclough for the purposes of talking about the student action on mathematical objects in this study.[98]

[98] In Section 4.5.2 I have noted differences in the ontological positions adopted by Sfard in her recent work and Fairclough. Both theorists view objects as discursive constructions (but differ as to whether the objects are real), and so the challenge of studying mathematical objects remains. I argue that the tools of focal analysis proposed by Sfard (2000, 2001) can be used alongside the tools of CDA, irrespective of differences in the ontology of the objects that are the discursive focus.

5.3.2 The analytic framework for this study

The analytic framework used in this study is presented in Table 5.1. The tools in this framework were selected in the interaction between (a) my work with the empirical data, (b) my reading of the mathematics education literature, and (c) my evolving understanding of the meaning of a socio-political perspective of mathematical practice. I illustrate my use of these tools in Chapter 6, after I have described the method used to collect the data to which these tools are applied.

Fairclough's three meanings of text

As discussed in Section 4.2.2, Fairclough (2003, p.27), identifies three ways in which meaning is construed by text, and these are represented in the three rows of the framework in Table 5.1. Since my interest in this study is in mathematical practice, the three meanings in this table are based on the description of what it means to act mathematically in discourse, as presented in Section 4.5.3. Firstly, *representation* refers to how text represents aspects of the physical, social and material world. In this study I am interested in how the text represents mathematical practice and its relationship to other practices. For example, a particular way of attending to or representing functions or a certain way of presenting an argument may represent the practice as being school mathematics. *Interaction* refers to how text interacts by enacting relations between participants and between other texts. For example, a pedagogic text in mathematics may construe the author of the text as an authority who commands the student to act in a certain way. An undergraduate mathematics tutorial may enact links to a textbook or lecture notes in the undergraduate course or a practical problem may enact links to word problems in the practice of school mathematics. Thirdly, *identification* refers to how text identifies people and their values. A mathematical text may set up the author of the text as being an authority in the particular mathematical practice or a pedagogic text in mathematics may identify the student as being a weak student who needs to perform certain operations on mathematical objects.

Table 5.1: Analytic framework

Fairclough's three meanings of text

Fairclough's three stages for critical discourse analysis

Level of socio-political practice (explanation)	**Meaning of the text (interpretation)**	**Identifying features of the text (description)**
Discourse type as a relatively stable way of representing	Representation: How is mathematical practice represented? 1. What objects are included / excluded / given significance? 2. What ways of talking and writing about objects are included / excluded / given significance? 3. What ways of representing objects are included / excluded / given significance? 4. What ways of making links between objects, texts, events and practices are included / excluded / given significance? 5. What ways of endorsing arguments are included / excluded / given significance? 6. What ways of evaluating the pronouncements of others are included / excluded / given significance? 7. What ways of looking at mathematical objects are included / excluded / given significance? 8. What ways of listening to talk about objects are included / excluded / given significance? 9. What ways of operating on mathematical objects are included / excluded / given significance? 10. What is the intended focus, that is, what meanings are given to mathematical objects?	Critical discourse analysis: For example, naming, pronouns, reference relations, mood, modality (see Appendix E) Focal analysis: 1. attended focus 2. pronounced focus
Genre as a relatively stable way of interacting communicatively	Interaction: What action is the text performing in constituting relations (both socio-political and discursive)? 1. What ways of interacting socio-politically are included / excluded / given significance? 2. What ways of making discursive links are included / excluded / given significance?	
Style as a relatively stable way of being	Identification: How does the text identify people, and their attitudes and values? 1. How do participants identifying themselves and others?	

The three meanings of text are not distinct but are dialectically related. They are only separated for analytic purposes as given in Table 5.1. For example, a text that represents the practice of foundational mathematics will also construe particular relations between the participants in this practice. On the other hand, a particular way of identifying oneself as a mathematics student in a foundational mathematics classroom gives meaning to the representation of the foundational practice.

The three meanings of text described here can be linked to the three elements on the level of discursive practice; *discourse type*, *genre* and *style* are relatively stable ways of representing, interacting, and identifying respectively (Fairclough, 2003, p.28). These elements are presented in column 1 of Table 5.1. The notions of *attended focus*, *pronounced focus*, and *intended focus* included in this framework are explained later in this section.

Fairclough's three stages for critical discourse analysis and where focal analysis figures in the framework

Fairclough (2001, p.21-22) identifies three stages in the process of CDA, with each stage related to a particular conception of discourse. Each stage is presented in a column of Table 5.1.[99]

The *descriptive* stage (the right-hand column of Table 5.1) relates to the formal properties of the text. Fairclough (2003) provides a list of textual features and explains the function that each feature serves in terms of one or more of the three meanings of text. In my analysis I have selected, in interaction with the data, appropriate features from this list. For example, naming and renaming signals a particular way of representing the world, intonation is used by a speaker to identify himself/herself as hesitant, confident etc., and the use of the definite article "the" allows referencing within a text (interaction). I have also drawn on the work of other researchers to aid my understanding of the grammatical features

[99] Fairclough (2001, p.21) has presented these three stages in his *three dimensional framework*.

used by Fairclough and to supplement Fairclough's list for the purposes of my study of mathematical action. For example, Janks (2010) has interpreted Fairclough's work for research students who are new to CDA, and the work of Morgan (1998) and Pimm (1987) refers specifically to grammatical features of mathematics texts. For example, Morgan (1998) suggests that the use of specialist vocabulary allows the speaker to talk with authority and an absence of pronouns suggests distancing and a formal relationship between subjects. The list of textual features is provided in Appendix E, together with the function each feature plays in the three meanings of text.

Two of the three tools of what Sfard (2000, 2001) calls *focal analysis* that allow me to operationalize action on mathematical objects are included in the descriptive stage of CDA. Firstly, the *pronounced focus* refers to the words the student uses when identifying "the object of her or his attention" (Sfard, 2000, p.304). This could also refer to other forms of semiosis such as a student's drawing or gesture. In the examples Sfard (2000, 2001) provides to demonstrate her use of focal analysis, she identifies the actual object of attention with the pronounced focus, for example the pronounced focus may be the "the green ones", "the slope" or "the intercept". In this study I identify the full text in which the object figures as the pronounced focus. This is necessary, for in this study I am not only interested in action on mathematical objects, but I also use CDA on the pronounced focus to analyze the discursive, social and political action (as described in detail in Section 6.3.1). For example, when solving question (a) of the Flu Virus Problem the student Mpumelelo claims, "It won't it be a cos graph?". My reading of Sfard's (2000, 2001) work suggests that she would identify the "cos graph" as the pronounced focus. However, in this study I identify the full question as stated by Mpumelelo as the pronounced focus.

The second tool of focal analysis used in the descriptive stage is the *attended focus* or what the student is "looking at, listening to" when speaking (Sfard, 2000, p.304). A student may look at certain words in the written text of a practical problem, listen to a statement by another student, or look at a particular feature of a graph. The extent to which one can

identify the attended focus in the analysis depends on the nature of the video recording, and Sfard (2000) notes that this may be speculative.

The *interpretation* stage is given in the centre column of Table 5.1. According to Fairclough (2001), analysis of a social event involves focusing on the discursive processes of text production and interpretation. The text described in the descriptive stage is a product of these processes. The textual features identified in descriptive stage serve both as *traces* of the process of production as well as *cues* for the interpretation (Fairclough, 2001, p.20). Thus I can use the textual features identified in the descriptive stage of the analysis to consider what meanings these give to the text. Specific questions related to each of the three meanings of text have been developed through interactive work between the data and my conception of mathematical practice as particular ways of acting mathematically. These questions relate not only to what is included, but also to what is excluded and to what is given significance in what is included (Fairclough, 2003). Given my interest in this study in recontextualization, such questions about inclusion, exclusion, and significance allow me to consider how a particular social event may be represented in different practices.

Sfard's (2000) third tool for focal analysis, the *intended focus*[100], figures in the interpretation stage. Sfard (2001, p.34) describes the intended focus as "a collection of experiences and discursive potentials" which are evoked by the pronounced focus and the attended focus, and which determine what the student is able to do with and say about the object identified in the pronounced focus. Since the intended focus is largely private, it is only possible to use the pronounced focus and the attended focus as clues to suggest what this "discursive potential" (Sfard, 2001, p.34) may be. In this study I give a particular meaning to the intended focus; where possible I use the pronounced and attended foci to identify what meaning the subject appears to be giving to a mathematical object.

[100] In her 2008 work Sfard uses the term *realization of a signifier* (p.154) for the intended focus. Instead of referring to the pronounced and attended foci, Sfard (2008, p.155) refers to the *verbal* and *visual* realizations of a signifier respectively. For the purposes of this study I have found the original analytic tools of focal analysis from Sfard (2000, 2001) productive.

Fairclough (2001) reminds us that CDA does more than focus on textual features, for the processes of production and interpretation are socially constrained, firstly, by the members' resources (p.20), that is, the internalized social structures and conventions that individuals bring to the setting. Secondly, they are socially constrained by the specific socio-political practices of which the members are part. So this brings me to the third stage of analysis, the *explanation* level represented in the left-hand column of Table 5.1. Fairclough (2003, p.28) provides the tools for making the link to the wider socio-political practices, for he states that discourse type, genre and style are relatively stable ways of representing, acting, and identifying respectively that operate on the level of socio-political practice. It is thus possible to make a link to the wider socio-political practices by asking which discourse types, genres and styles are articulated in the text. In making these links I draw on the description of the wider socio-political space of school and higher education described in Chapters 2 and 3.

Fairclough (2001) emphasizes that all three stages are, in fact, the "analysis" (p.22), but that the nature of this analysis changes from stage to stage. The descriptive stage involves identifying and labeling the textual features according to the three different meanings. According to Fairclough (2001), the interpretation and explanation stages involve the analysis of less concrete objects, like the cognitive processes of individuals (in interpretation) and the relationships between a social event and "more durable social structures" (p.22) (in explanation). Yet Fairclough (2001) warns us that none of these stages are unproblematic. For example, the text with which one works in the descriptive stage is a transcript of the verbal and non-verbal action, and the production of this transcript itself has a history. This warning refers to the validity of the analysis, which I deal with in more detail in Section 6.4.1.

5.4 Methods

Before illustrating how the analytic tools presented in Section 5.3 are applied to the data in this study, I describe and justify the methods used to collect this data. My choice of methods must, of course, be consistent with the other three elements in the research process (epistemology, theoretical framework and methodology). Yet as this section proceeds, the reader will be alerted to the fact that my decision-making was also enabled and constrained by my role as both lecturer and researcher in the space.

From this point in the dissertation I choose to use the term *research text* rather than *data*, to talk about the empirical and to give significance to the discursive nature of data. Terms such as *data collection* and *data capture* suggest that data unproblematically reflects the real and they background how the research process involves selections that are made in the light of the selected theoretical framework and the researcher's interest (selections that are constrained, as I have argued in Section 5.2). Setati (2003) uses the term *re-presentation* to foreground the relationship between such selections and the research process as a whole; "… re-presentation of data is a selective process informed by research questions, the tools of analysis and the purposes of representing the data" (p.294). Setati (2003) argues that an acknowledgement of data as a re-presentation is crucial. The first is a political argument; she notes that a researcher's texts are used to make interpretations and conclusions about the teaching and learning of mathematics classrooms (in the terms used by Adler and Lerman (2003, p.446), the research "counts" locally). Yet the re-presentation itself shapes these interpretations. Secondly, she notes that re-presentation is related to the validity of the research, an issue I discuss in Section 6.4.1.

5.4.1 Selecting research texts that re-present language use in the practice of foundational mathematics

My interest in this study is in the use of two innovative aspects of pedagogy in the Foundational Course; practical problems and a learner-centred pedagogy in the weekly

workshop class. Since CDA is the analysis of "language in use" (Fairclough, 2006, p.9), I selected texts from the Foundational Course itself.

Selecting practical problem texts from the Course material

I selected three practical problems texts[101] from the Resource Book for the Foundational Course (refer to Table 5.2). These texts are given in Appendices B to D and Appendix Q. This selection had to take into account both the "labour intensive" (Parker & Burman, 1993, p.156) nature of CDA and the need to have sufficient research texts to get the description "right" (Adler & Lerman, 2003, p.446) when answering my research questions.

Table 5.2: Matrix of the three practical problems selected for the study

Problem name[102]	Broad topic	Task context	Mainstream / Reform Calculus	Timing
The Flu Virus Problem	rates of change	spread of a flu virus in a community	foundational	May 2007
The Car Problem	related rates	speed / distance/ time problem involving cars	foundational and mainstream	August 2007
The Chemical Reaction Problem	integration	formation of a product in a chemical reaction	foundational	October 2007

The three practical problems were selected from the Course material used in 2004, 2005 and 2006 (with some small changes to facilitate the collection of research texts in the classroom). The problems are structured around a variety of mathematical topics typical of first-year undergraduate calculus, for example functions, rates of change, limits, differentiation and integration. The selected problems are used in the Course over a period

[101] For brevity, I refer to *practical problems* rather than *practical problem texts* from this point.

[102] A problem is named according to the task context, that is, using the non-mathematical practice referenced in the problem.

of five months from May to October of the academic year (which starts in February).[103] These problems require varied mathematical action on the part of students, for example, constructing a visual representation of a relationship vs. working with a given visual representation, constructing a mathematical expression from a verbal description and acting operationally on this expression vs. explaining the practical meaning of an expression. The three problems have a variety of task contexts (such as a chemical reaction, the spread of a flu virus, the travel distances and speed of cars). The Car Problem is similar in structure to problems used in the course material for the mainstream first-year mathematics course described in Section 2.4.5 and to problems in the textbook prescribed for this mainstream course and the Foundational Course in the year that this study was conducted. The other two problems appear in the material for the Foundational Course only, and share many features with problems appearing in calculus reform texts, as described in Section 2.4.4.

Selecting research texts that re-present the student action on the practical problems

Fairclough (2003) says that when researching meaning-making, "one needs to look at interpretations of texts as well as texts themselves, and more generally at how texts practically figure in particular areas of social life, …" (p.115). In my study I not only analyze a selection of the practical problems in the Foundational Course as examples of "language in use" (Fairclough, 2006, p.9), but also texts representing the students' mathematical action on these problems in regular workshop classes (where students work in self-selected groups, with the help of a tutor or lecturer who takes responsibility for a class of 20 to 30 students).[104] The semiotic aspect of a social event (such as students solving a

[103] The characterization of the practical problems used for this initial selection process was based, not on a formal analysis of the practical problems in the Course material, but on my knowledge of the Foundational Course and the problems themselves, gained when lecturing on the Course. A detailed, theoretically informed analysis of these problems is what this study is designed to elucidate.

[104] In Section 3.7.6 I discussed Dowling's explicit intention in his 1998 work to study the texts from school mathematics textbooks alone, and not to investigate the student interpretation of these texts within the practice of school mathematics. I agree with Dowling's contention that the model producer and interpreter in of a textbook are different to the empirical producer and interpreter when the textbook is used in the classroom.

practical problem) is not just about what is said or written in a social event, but also includes other forms of semiosis such as representations, body language and gestures (Fairclough, 2006). A pilot study conducted as part of my participation in the Masters level course on academic literacies and discourse analysis suggested that video recordings would be the best way to record these forms of semiosis. In order to ensure quality video recordings, I hired two technical staff from the television services at the university at which the study was conducted to provide both the equipment (a radio microphone placed on the desk at which the students were seated and a video recorder) and to make the actual recordings of the action. The technical staff were briefed (both before and following a pilot recording session) to record both the verbal and non-verbal student action by zooming in on the written work of individual students and discussions between two or three students and by zooming out to record the group working as a whole. Students' written work was collected and copied after each recording session and these texts were used to supplement action as re-presented in the video recordings.

Individual interviews as background information

This study aims not only to describe the student action on the practical problems, but also to explain this action with respect to the wider socio-political space by identifying what resources students may draw on when participating in the Foundational Course. To obtain information on the participating students' positioning within this space I conducted an individual interview with each student between May 2007 and October 2007 when the video recordings were being made. During an interview lasting 30 to 45 minutes I asked the student to describe his/her experience of learning school Mathematics as well as his/her initial experiences of studying mathematics at university (the interview questions are provided in Appendix F). These interviews were transcribed by two paid transcribers.

Yet in my study the choice to also investigate the student action on the practical problems is not only related to my research interest but also about the validity of my research. In Chapter 6 I will argue that the analysis of the students' enabling and constraining action as well as the Tutor's intervention played a key role in identifying the valued mathematical for solving the practical problems.

Unlike the texts of the practical problems and the video recordings discussed in this section so far, these interview transcripts do not represent "language in use" (Fairclough, 2006, p.9) in the Foundational Course, but rather "language in use" (p.9) in an interview setting. The content of these transcribed interviews was used to provide the student perspective of schooling in South Africa in Section 2.3.5. This perspective grounds the descriptions based on the large-scale studies described in Chapter 2. Taken together, these descriptions serve as the resources I use to identify what discourse types, genres and styles students draw on when answering the practical problems, hence making a link between the micro-level classroom action and the macro-level space.

5.4.2 Selecting students to participate in the study

Decisions about how many and which students should participate in this study were based on my research interest, on the theoretical perspective that I had selected to talk about this interest, on my choice of methodology, and on practical decisions related to my role as both lecturer in and researcher of the Foundational Course. This selection of potential participants was made on three levels; selection as individuals, as members of a group of five to six students, and as a workshop class from all the students registered for the Course.

Selecting a workshop class

The video recordings were made in the workshop class for which I was responsible as a lecturer on the Foundational Course. This choice of location was motivated, firstly, by the need for me to balance my dual roles. Secondly, I recognized the possibility that verbal and non-verbal interaction could be an absence in the research texts if the students did not co-operate in their small groups as intended in the Foundational Course; collecting the research texts from within my own workshop class would give me an opportunity in the weeks prior to the collection sessions to develop the small group skills intended in the learner-centred pedagogy of a workshop class.[105]

[105] My experience as a convenor of this Foundational Course suggests that the task of developing productive group skills is a difficult one for some of the tutors, many of whom have not experienced such a pedagogy in

The selection of the workshop class was made at the beginning of the academic year when, in my role as convenor of the Course, I allocated students enrolled in the foundational programme to one of six workshop classes of 20 to 30 students, and assigned a tutor to each class. Using the completed Course Enrolment Forms, I identified a block of students in the alphabetical list that gave me some diversity in terms of secondary school attended, home language, grade 12 score for school Mathematics, year completed school, form of accommodation at the university, and whether the student was repeating the Foundational Course. Since a key aim of the study is to identify and explain students' enabling and constraining mathematical action on the practical problems, it was necessary to select a class that represented in some way the diversity of subject positions available in the practices that the students were likely to draw on as resources.

Yet this initial identification of a workshop class was made in the knowledge that the composition of the class would change in the weeks prior to the recordings. For example, at the beginning of the academic year the class list from which I made my initial selection was unstable; after six weeks students having difficulty in the mainstream would be joining the Course. The inclusion of these students in the alphabetical list would influence the diversity of the workshop class in terms of race, and consequently, as suggested by the descriptions in Chapter 2, also in terms of language and educational background. As a researcher my initial reaction was to anticipate the possible effects of these changes on who would ultimately participate in my study by increasing the size of my particular workshop class. Yet ethically as a lecturer on the Course I could not justify creating one workshop class that was larger than the others.

At the time of the first recording session in May the workshop class contained 33 students (some from the foundational programme, others in mainstream programmes but enrolled

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their own undergraduate studies. My intention to support students in developing the intended skills did not play out as intended in practice, largely due to my having to attend to other roles in the Course.

for the Foundational Course). This class was larger than the 24 to 25 students in the other five workshop classes, but as a lecturer I did not want to shift the regular students between groups during the academic year.

Selecting groups of students from the workshop class

From the workshop class of 33 students, I selected three of the six small groups on the basis that the students in these three groups (a) represented some of the diversity according to the criteria used in the initial identification of a workshop class, and (b) were interacting productively (in the sense that their language use could be re-represented in a research text). However, conscious of the disruption that the video recordings might cause in the regular running of the workshop class, I chose to record two rather than three of the small groups in one recording session. The video recordings were made according to the plan given in Table 5.3 (this plan was adapted from an earlier version to accommodate students who were absent from class).

Table 5.3: Summary of the video-recording sessions

	Flu Virus problem (May 2007)	Car Problem (August 2007)	Chemical Reaction Problem (October 2007)
Group 1:	✓	✓	
Group 2:	✓		✓
Group 3:		✓	✓

Negotiating student participation in the study

The selections described in the previous two sections were made with the knowledge that participation still had to be negotiated with the individual students in the three selected groups, and that the students' personal decisions would play a role in the final selection. Since I was playing the role of both lecturer and researcher, the ethical implications of students' participation in this study had to be taken into account, an issue I discuss in Section 6.4.2.

Students in the three selected groups were invited to attend a lunchtime meeting to discuss the research. At this meeting information about the study was communicated to the students verbally and in writing (the Information Sheet for Students is given in Appendix G). The study was set out as a description and explanation of how the students solve the practical problems in the Foundational Course, with the overall aim to use the knowledge gained in the study to improve the Course in future. Students were told what participation would involve (video recording of the groups during selected workshop classes and interviews) as well as the timelines.[106] Ethical information regarding participation, for example voluntary participation, the opportunity to withdraw participation, and the relationship between participation in the study and in the Course was discussed. Details regarding the use of the research texts during and after the study and issues of anonymity and confidentiality were also discussed. The students were asked to submit the Student Consent Form (see Appendix H) when they had made a decision. Some of the students signed this form at the meeting, while others submitted later. In the case of students who did not attend the initial meeting, I met individually with the students in my office in order to brief them about the study. Three of the twenty students approached chose not to participate.[107] A broad description of the three groups following the negotiation process is given in Table 5.4. This description locates the students in the wider space sketched in Chapter 2. Consistent with the ethical negotiations, the names of the students have been changed.[108]

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[106] The information provided to students points to my original intention to conduct interviews with participating students after the video-recording sessions. While conducting the study my developing understanding of a socio-political perspective of practice led me to remove these interviews from the study as they do not represent "language in use" (Fairclough, 2006, p.9) in the Foundational Course.

[107] During the video recording of the student action these students joined other groups of students in the workshop class.

[108] Ismail (2008) refers to the "politics of representation" (p.5) when noting that the research writing process constructs identities for the research participants. The pseudonyms used for seventeen student participants have been selected to acknowledge my admiration for these students and my gratitude for their willingness to take part in the study; Jane (gracious), Lulama (gentle and kind), Darren (great), Hanah (grace), Shae (gift), Jeff (gift of peace) , Bongani (thanks), Mpumelelo (success), Siyabulela (thanks), Vuyani (happy), Lungiswa

Table 5.4: Description of the three groups of students selected for the study

Group	Names of students	Description
Group 1	Darren Hanah Lulama Jane Jeff Shae	Two of the six students were enrolled for the foundational programme, with the others located in mainstream programmes. Three of the students had completed their schooling at independent schools (two at the same school), and another attended a former White high school. The sixth student, who identified his home language as isiZulu, attended a former Black high school. Jane missed the second recording session in August due to illness.
Group 2	Bongani Lungiswa Mpumelelo Siyabulela Vuyani	Only one student in this group had joined the Course from the mainstream course. Four of the five students completed their schooling at former Black schools, with the fifth having attended a former White high school as a hostel student. None of these students reported their home language as English (the home languages in this group are isiXhosa, isiZulu, Sesotho, and Setswana). My decision to include this group in the study was influenced by my observation that the students interacted in English and in their home languages.[109] Mpumelelo left the class early on the day of the October recording session for his group (with my permission).
Group 3	Akbar Kelsa Lwazi Ndumiso Nqobile Thokozile	Two students joined the Foundational Course from the mainstream, while two of the foundational students were repeating the Course, having failed in the previous year. Four students attended former White high schools, one student completed his schooling at an independent religious high school, and another had followed the Cambridge curriculum at a school in a neighbouring country to South Africa.[110] Of those who attended former White high schools, three identified isiXhosa as their home language, with the fourth giving this as isiZulu. Nqobile and Akbar withdrew from the Foundational Course prior to the October recording session and before their individual interviews could be conducted.

(perfect), Akbar (great), Kelsa (brave), Lwazi (knowledge), Ndumiso (praise), Nqobile (conquer), and Thokozile (happy).

[109] Collecting research texts that recorded this interaction proved problematic, as discussed in Section 4.5.3.

[110] The inclusion of a student who completed her schooling outside of South Africa resulted from the selection of groups of students (rather than individual students) after the students from the mainstream first-year mathematics course had joined the foundational class.

The description of the students in Table 5.4, together with the description of the schooling and higher education spaces in Chapter 2, points to the diversity in the group of seventeen students who participated in the study. These students bring different cultural, social, and linguistic capital[111] to the university. They are also differentially positioned as mathematics students in the university, for example some are students in the foundational programme, some have changed to the Foundational Course from the mainstream first-year course, while others are repeating the Course. This positioning also changes, for example, some students withdrew from the Course while the study was underway. Traces of how these differences in capital and positioning play out in the mathematics classroom emerge in this study, and where appropriate in the analysis I note these as of interest. However, this is not the focus of the study and it is not possible to assign any causality in this respect, given the nature of the sample and how the groups were set up.

5.4.3 Selecting a tutor to participate in the study

During the workshop classes in which video recordings were made there was a conflict between my interest in researching the action in the selected groups and my duty as a lecturer to all the students in the class. For this reason I employed a tutor to co-tutor with me during the recording sessions. Mindful that the students in the class as a whole should not be harmed by the research, I wanted to employ the best possible tutor for this role. The selected postgraduate student (called the *Tutor* in this study) was someone who had previously tutored on the Course and who had studied both mathematics and mathematics education at post-graduate level at the university at which the study was conducted. The Tutor's brief was to act as tutor to all students in the workshop class (while I would assist only those students not participating in the study). The Tutor co-tutored a workshop class prior to the recording sessions so that the students were familiar with his tutoring style when these sessions began.

[111] I use the notion of *capital* consistent with Fairclough's use of the work of Bourdieu, as discussed in Section 4.3.1.

The initial negotiation for participation with the Tutor stressed that, while the Tutor's interaction with the students would be part of the research texts, the focus of the study was on the student action (the Information Sheet for the Tutor and the Tutor Consent Form are given as Appendices H and I respectively). However as I began to analyze the research texts, it became clear that I had under-estimated the role that the Tutor played in the groups, mainly by enabling (and sometimes constraining) the student action. This involved a renegotiation of the Tutor's involvement in the study to include him as a participant in the study.

5.4.4 Re-presenting video recordings in written transcripts: transcription and translation

Once the video recordings had been made, the recordings were transcribed into written text re-presenting both what the students said and what they did (this includes their gestures, writing, body language) in these recordings. This transcription was done using Transana (Woods & Fassnacht, 2007). An adapted version of Jefferson notation (see Appendix A) was used. Transcripts 1.1 and 1.2 in Chapter 1 are examples of the final product of this transcription process.

I initially transcribed two of the six video recordings (the action of Groups 1 and 2 on the Flu Virus Problem); in interaction with my initial analysis of the transcripts I was able to determine how much detail was required and select a transcription notation that was appropriate for the study. This interactive process also allowed me to check the initial transcription a number of times.

Once I was satisfied that the transcripts were completed in enough detail and that the application of the analytic tools in the analysis of these transcripts was productive, I employed a postgraduate student to produce the first draft of the transcripts for the remaining four video recordings. This transcriber was recommended to me by colleagues who were also analyzing video recordings of mathematics classroom interaction. I then produced a second draft of each transcript based on these first drafts and my review of the

recordings. At times during the analysis it was necessary to refer back to the video recordings, with the result that the transcripts were refined even further during the analysis process.

Since isiXhosa is the home language of the transcriber, she was also able to transcribe and translate those aspects of the discussion in Group 2 that took place in isiXhosa. The small amount of discussion in Sesotho and Setswana in this Group was transcribed and translated by a second translator. Since I am not familiar with the language group to which Sesotho and Setswana belong, I had the first transcription and translation of the relevant sections checked by a third translator.

5.5 Summary of this chapter

I began this chapter by presenting a socio-political perspective of doctoral research in mathematics education, with a focus on what such a perspective means for knowledge production in this study. In discussing the methodology I argued that operationalizing a socio-political perspective of mathematical practice involves identifying analytic tools that allow me to talk about discursive, social, political and mathematical action. I presented the analytic framework for this study, which derives its structure from Fairclough's methodology for CDA, but has been supplemented with recontextualized tools from Sfard's focal analysis. Before illustrating the use of these analytic tools on the research texts (which is set out in Chapter 6), I explained the re-presentation of the research texts to which the analytic tools are applied and motivated for the selections that were made in this respect.

CHAPTER 6 THE RESEARCH PROCESS METHODOLOGY AND METHODS (PART 2)

6.1 Introduction to this chapter

In Chapter 5 I presented and motivated for the choice of analytic framework for this study (Table 5.1). This framework is used to analyze the two types of research texts in this study; the practical problems and the written transcripts re-presenting the student action on these problems. In this chapter I use extracts from these texts to illustrate how the analytic tools are used. I end this chapter by discussing issues of quality in this study, with a focus on validity and ethics.

6.2 Analyzing the texts of the practical problems

In this section I demonstrate the use of the three stages of critical discourse analysis on the four introductory sentences and question (a) of the Flu Virus Problem and the written solution for this question (see Appendices B and Q). The analysis illustrated here is used to answer the first research question as it allows me to describe how the practical problems represent the practice of foundational undergraduate mathematics and its relationship to other practices, and to explain this meaning with reference to the wider order of discourse.

6.2.1 The description stage

The description stage involves working sentence by sentence through the text to identify the formal properties of the text (provided in Appendix E). The tools pronounced focus and attended focus are not used in this stage of the analysis, an issue I discuss in Section 6.2.2.

Sentence 1 names “a flu virus”, which is renamed as “the flu” and “the disease” in the sentences that follow. The virus is represented in general terms, suggested by the use of the

indefinite article “a”, the absence of a specialist term to name the virus, and the renaming of the virus (also in non-specialist terms). The definite article “the” used in “the flu”, “the disease” etc. provides a textual link to “a flu virus” that is introduced for the first time in Sentence 1, suggesting that these terms refer to the same virus. In Sentence 2 the word “immune” is restated in everyday words as “does not get it again”, the reference pronoun “it” linking the second clause of the sentence to “the flu” in the first clause.

The indefinite article “a” in “a community” in Sentence 1 suggests some timeless general group of people, a group that is not named and not given a location in time and space. The “people” in the community are identified impersonally and as part of a collective “10 000 people”, although the pronouns “he or she” suggest that the people can be male or female. These people are also passive in relation to the spread of the flu virus as the community is “hit” by the virus and they all catch the flu in the absence of resistance. The verbs in Sentence 3 point to material processes taking place in the non-mathematical practice, for example, the disease “has hit” the community, and a person “catches” the flu and then “becomes immune”.

Time is described in general terms as “sooner or later” in Sentence 3. The spread of the flu virus in the community is given in the present tense; the virus “has hit” the community, a person “becomes immune” to the virus, and everyone “catches” the flu. This use of the present tense presents the information about the flu virus as ongoing and as fact. This certainty is reinforced by the declarative mood of the statements in Sentences 1 to 3, which present the facts with no supporting evidence.

The first three statements are followed by two instructions to the student reader. Sentence 4 is in the imperative mood and commands the student to act in a certain way (“Let…”). This sentence introduces a mathematical object, a function, represented using the symbols $P(t)$. The verb “denote” sets up a relational (identification) process between the letters P and t and the flu virus, standing for the “number of people who have, or have had, the flu” and time “in days after the first case of flu was recorded” respectively (again not locating the flu

virus in a specific time). Implicit in the arrangement of the symbols $P(t)$ is a relationship between mathematical objects; the lower case t in the bracket representing the independent variable and the upper case P representing the dependent variable. No mathematical formula is provided to represent this mathematical relationship.

The next instruction to the reader (in the form of the imperative "Draw…") is numbered with the letter "(a)". In this instruction the relationship between the variables P and t is named as a functional relationship, the word "function" taking on a particular meaning in mathematics. The definite article "the" in "the graph of P" suggests that there is only one possible graph (a representation that is reproduced in the worked solution in which one possible graph is provided). What is required in question (a) is a "sketch graph", a term that takes on a particular meaning in the Foundational Course. Firstly, this meaning is suggested by the second clause in this sentence, linked to the first by a comma, which gives significance to one value ("the maximum number of people who get infected"), and not others. Secondly, the graph in the solution represents only the initial and final value of $P(t)$ (and no other points) and the shape of the graph. In the second clause of the instruction in question (a) the "number of people who have or have had the disease" is reworded as the "number of people who get infected". In question (a) the text producer addressed the student personally as "you" and the pronoun "your" gives the student personal ownership of his graph.

Certain features of question (a) place an emphasis on the instructions to the student. In the second clause the adverb "clearly" reminds the reader to show the relevant information on the graph, and the bold text in the third clause reminds the reader to have his graph "checked by a tutor".

6.2.2 The interpretation stage

I describe this stage in two parts; the first develops on the textual clues identified by the CDA in Section 6.2.1 to identify the valued ways of acting mathematically represented in

the text, while the second addresses the challenge of identifying the valued action on mathematical objects in the text of a practical problem.

The interpretation stage (part 1)

The textual features identified in the description stage are the clues for answering the questions in the central column of Table 5.1. The answers allow me to identify the three meanings of the text (representation, interaction, and identification) and hence the valued mathematical ways. The interpretation stage goes beyond the sentence level to working with a practical problem as a whole.

The "flu virus" represented in Sentences 1 to 3 of the Flu Virus Problem is an object in the practice of epidemiology. However, there is an absence of the specialist terminology of this practice. Rather, the wording represents the disease using everyday words and the student is identified as someone needing to have terms such as "immune" explained, and as having an interest in rather than as being a participant in the practice of epidemiology.

While the spread of the disease is represented in everyday language, the disease and the people affected by the disease are represented as not real in the everyday practice, since they are not named and identified in time and space. Sentence 4 reinforces this representation of the everyday objects as not real by giving significance to a mathematical object, that is, a function that represents the spread of the flu virus over time. This function is represented using symbols, and the arrangement of the symbols signifies the relationship between the variables t and P. This suggests that the Flu Virus Problem is a mathematical text rather than a text about epidemiology or everyday life.

In addition, certain textual features represent the text as a pedagogic text in mathematics. This is suggested by the overall structure of the Flu Virus Problem; the first four sentences provide factual information and then the numbered question called "(a)" instructs the reader to produce a certain type of mathematical representation, and only one graph is valued. The graph has to be evaluated by an authority. Since no algebraic formula is provided to

represent the function, the text does not give significance to operational action on a formula for constructing the graph.

The pedagogic text in mathematics identifies the student in a subordinate relationship, firstly, to the author of the text who provides the facts and defines what is to be done, and secondly, to the tutor who is given the power to evaluate the student's graph. The student is identified personally in question (a), but in Sentence 1 to 4 he is an unnamed student who accepts the facts without question.

The text points to the type of mathematical action required of a student who successfully participates in the foundational practice. The student should identify the everyday objects such as the disease and the community as being not real, and recognize the text as a pedagogic text in mathematics. The student should also identify the textual cues that link the parts of the text itself, for example, the use of reference pronouns to identify the flu virus. Knowing what the term "sketch graph" means requires that the student link to other social events in the Foundational Course (such as lectures) and recognize that he should not plot individual points, but should only attend to the key points and the shape of the graph.

This text positions the student in two ways. On the one hand the student is identified as someone with a diverse range of ways of acting, for example, working with the boundary between the mathematical and everyday practice and between social events in the Course, following the textual cues within the text itself, and constructing the graph without acting on an algebraic formula. Yet on the other hand the student is positioned as needing to be told how to behave (in the form of a reminder to show the relevant information on the graph) and needing to have his graph evaluated by a tutor.

The interpretation stage (part 2)

In the analysis presented in the previous section (part 1) I have drawn on the textual features used in CDA to identify certain ways of acting mathematically, for example, how objects are represented, the valued links between practices, events and texts, how subjects

are identified, and the socio-political interaction between these subjects. Yet there is a silence in this interpretation as I have not answered the questions in Table 5.1 that refer to the action on mathematical objects. This silence points again to the methodological challenge of analyzing action on mathematical objects. In the interpretation of the Flu Virus Problem so far I have argued that the absence of an algebraic formula representing the function suggests that the student is not required to act operationally on a formula to identify the properties of the graph. Yet the analysis of the Flu Virus Problem text and solution using the tools for CDA in Table 5.1 does not allow me to talk in any more detail about this mathematical action, that is about the ways of looking and ways of making links involved in constructing the graph.

In Section 5.3.2 I have argued that Sfard's tools for focal analysis allow me to identify these ways of acting in texts re-presenting student action on mathematical problems (and this is illustrated in Section 6.3). As this study proceeded it became clear that the silence with respect to talking about the action on mathematical objects in my analysis of the practical problem texts and the solutions could be filled by studying the actual interpretation of these texts, that is, by analyzing the action of a variety of subjects on the problems. In this study I use three sources for these interpretations. Firstly, I use the analysis of the student action on the three practical problems in interaction with the tutor (as described in Section 6.3). This analysis suggests that at times a group of students does not make progress until interaction with the Tutor suggests a productive way forward, at other times one group of students arrives at the required solution with ease while another group of students does not, or perhaps some students in a group move ahead, while others in the same group do not. This varied action is used to identify the mathematical action that appears to enable the correct solution. Secondly, I draw on my experience of teaching the Foundational Course over a five-year period and particularly on my knowledge of how similar practical problems are tackled in lectures for the Course. My third source is the interpretation of a colleague at my university (both a first-year lecturer on a mainstream mathematics course and a research mathematician) who solved the three practical problems in discussion with me.

Using these three sources I construct what I call the *valued mathematical action* for solving a problem. This valued mathematical action is described in the form of a narrative that describes the mathematical action of a successful student in the Course (who I have named Mihlali[112]). It should be noted that my identification of the valued mathematical action on a practical problem is not exhaustive of the possible ways of solving the problem.

In the rest of this section I return to the interpretation of the Flu Virus Problem and present an extract from Mihlali's action to illustrate how the analysis of the interpretation of the text enriches the analysis of the text of the Problem described in the previous section (the full narrative for this problem is provided in Appendix B).

In Extract 6.1 Mihlali identifies the initial and final value and the increasing property of the graph of $P(t)$ by making links between the function $P(t)$, its graphical representation and the meaning of the function in the task context. He moves to-and-fro across the boundary between the mathematical and non-mathematical practices, adopting an operational view of the function as he considers what happens as the virus spreads over time.

Extract 6.1: The valued mathematical action on question (a) of the Flu Virus Problem

> Firstly, Mihlali attends to the functional relationship $P(t)$ and identifies this with its meaning in the task context, that is, the number of people who have or have had the disease at time t. He adopts an operational view of the function as he considers what happens as the disease spreads over time. Initially (at time $t = 0$ days) no-one has the disease, so (0,0) is a point on the graph. Since there are 10 000 people in the community and "sooner or later" everyone catches the flu, the graph of $P(t)$ reaches a maximum value of 10 000 at some time t. Since $P(t)$ is by definition the number of people who have or have had the disease at time t, the graph of $P(t)$ is always increasing.

I use the narrative in Extract 6.1 to supplement the original analysis using the tools from CDA in a way that allows me to talk about the ways of making links between the

[112] Consistent with my choice of pseudonyms for the students in this study, as presented in Section 5.4.2, I name this fictional student with an isiXhosa name meaning "joy".

mathematical object function (represented using the symbols $P(t)$), the meaning of this function in the task context, and the graphical representation of the function. It also allows me to talk about how the student is looking at the function, that is, adopting an operational view and considering the properties of the graph as the value of the independent variable changes.

6.2.3 The explanation stage

In the explanation stage I link how the text represents mathematical practice, interacts between subjects and texts, and identifies subjects (represented in the centre column of Table 5.1, and described in Section 6.2.3) to certain discourse types, styles and genres respectively in the wider discursive space (represented in the left-hand column of Table 5.1). This allows me to explain how the meanings of the text "come to be articulated at a particular moment" (Pennycook, 1996, p. 116). The description of the discursive space in Chapters 2 and 3 are the resources that I draw on for this explanation.

The Flu Virus Problem is a pedagogic text in mathematics that links to the practice of epidemiology. This link suggests that this practical problem draws on the discourse of relevance promoted in school and undergraduate mathematics reforms. The labeling of the people in the community affected by the disease as male or female suggests that the text draws on the notion of inclusivity promoted in the new school curriculum in South Africa.

The link in particular to a scientific discipline other than mathematics points to the role that the Foundational Course plays as a service course for the study of a variety of scientific disciplines at the university (and not just for the study of advanced mathematics). Yet the representation of the objects in epidemiology in everyday terms suggests that, while the student may have an interest in a scientific practice such as epidemiology, he is not a participant in this practice.

One the one hand the pedagogic text draws on the discourse of relevance in reform-oriented curricula, but through its use of the mathematical word problem genre as identified by

Gerofsky (2004), it also draws on a more traditional pedagogy. For example, the overall structure of the practical problem has a set-up and information section followed by an instruction to the student to produce an answer the teacher has the authority to judge, and the task context is represented as not real. The link to this genre suggests that the student should adopt the style of a school mathematics[113] student solving a word problem when deciding how meaning should move across the boundary between the mathematical and non-mathematical practice.

Yet certain features of the Flu Virus Problem diverge from the genre of mathematical word problems that students would have experienced at school. For example, Gerofsky (2004) argues that a word problem assumes that the student should be practising an algorithm recently practised in class, yet the student is not able to act operationally on an algebraic formula in the Flu Virus Problem. In addition, the literature on the use of practical problems in school mathematics and in advanced mathematics (e.g., Dreyfus, 1991; Freudenthal, 1973; Gerofsky, 2004; Straehler-Pohl, 2010) points to a one-way movement from the everyday to the vertical mathematical practice. Yet, the description of how Mihlali constructs the graph in Extract 6.1 suggests that a to-and-fro movement between the task context and the mathematical objects and their representations is the valued action in this problem. This divergence from the genre of mathematical word problems suggests that the student should, at certain times, not adopt the style of a school mathematics student solving a word problem.

The text draws in contradictory ways on the representations of the foundational student in the higher education space. On the one hand the student is identified as needing support in the form of reminders and feedback from an authority to solve the Flu Virus Problem. For this positioning of the student the problem draws on the higher education discourse that

[113] In the analysis I refer to the students as subjects in the general practice of *school mathematics*, and not specifically in the school subject Mathematics in South Africa (which is part of the practice of school mathematics). I do, however, use the name of the subject when discussing the school textbooks that the students in the study used for their study of school Mathematics.

foundational students need different support to mainstream students. In contrast the student is identified as being able to control how and when meaning moves between practices, as drawing on social events within the Course, and as following the textual cues within the text itself. In this sense the foundational student is positioned as having the potential to succeed in higher education, a positioning that challenges the identification of such a student in the higher education space.

6.3 Analyzing the text re-presenting student action on the practical problems

For continuity with Section 6.2 I have selected an extract of the transcript re-presenting the initial action of the students in Group 2 on question (a) of the Flu Virus Problem (Transcript 6.1 on the next page). I use the analysis illustrated here to answer the second research question by describing the enabling and constraining mathematical action used by the students when solving the practical problems and by explaining this action with reference to the socio-political action in the classroom and the discourse types, genres and styles that the students recruit from the wider discursive space. In Section 6.2.2 I noted that this analysis of the students' interpretation of the practical problems (in interaction with the Tutor) also plays a role in answering questions about the practical problems themselves.

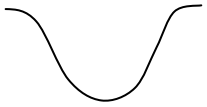
6.3.1 The description stage

The description stage, represented in the right-hand column of Table 5.1, involves a detailed analysis of the features of the text using both focal analysis and CDA (see Appendix E for the textual features).

During the transcription process each speech turn is given a line number, for example, in the line numbered 17 Mpumelelo's question represents one speech turn. This is followed by another speech turn, that is, Lungiswa's prompt "uhm↑" which is numbered as line 18. However, some of the more lengthy turns that represent more than one speech function (for example, a statement followed by an explanation) are divided into sub-lines using the letters a, b, c etc. In line 25 Mpumelelo makes two causal statements ("so") in one speech

turn, and these are labeled lines 25a and lines 25b. Splitting the transcript into lines and sub-lines in this way allows me to focus in detail on the textual features of a speech function.

Transcript 6.1: The Flu Virus Problem, question (a), Group 2, lines 17 to 25

17	Mpumelelo:	It won't it be like a cos graph?
18	Lungiswa:	Uhm↑
19	Mpumelelo:	It won't it be like a cos graph? ((*Using his pen to demonstrate a full wave of a cosine graph, starting and ending at its maximum value*)) 
20	Lungiswa:	Why do you say so?
21	Mpumelelo:	Because they say we must we must also solve ... the ... the <u>number</u> ((*Using his hand to emphasize*)) of people who get the disease right?
22	Lungiswa:	Uh huh↑
23	Mpumelelo:	So it's like they won't get it ... at the same time ((*Bongani and Siyabulela are reading the question, Vuyani is looking at Mpumelelo as he speaks*))
24	Lungiswa:	Ja ((*Nodding her head*))
25a	Mpumelelo:	So it's like there are the others that get it ((*Holding his pen in the air as he speaks*))
25b	Mpumelelo:	so ... ... it is from that thousand of the community ((*Raising his hand for "1 000", alternating between looking at Siyabulela, Vuyani and Lungiswa, Bongani does not make eye contact, seems to be reading*))

In the rest of this section I present in detail the analysis of lines 17 to 22. The first tools to be applied to each line or sub-line are the two tools of focal analysis; where possible I identify the pronounced focus and the attended focus. In line 17 the pronounced focus is what Mpumelelo says, "It won't it be like a cos graph?" At this stage his attended focus is not clear. I then use CDA on the text of this pronounced focus. Mpumelelo's use of the preposition "like" suggests a relational process, attributing the required graph to the class of cosine graphs. His question is in the declarative mood, suggesting that this is a tentative attempt at an answer and one on which he wants feedback. This tentative tone is reinforced by his use of a negative modal auxiliary verb "won't". He begins with the reference pronoun "It", as if he is to claim "It will be a cos graph", the use of the modal verb "will" here would suggest certainty about his prediction. However, he chooses to use the negative "won't" which suggests less certainty. Although Mpumelelo is the first student in Group 2

to attempt an answer verbally (however tentative), the absence of the first person pronoun "I" (such as in "I think…") in his talk suggests that he is not claiming personal responsibility for this hypothesis.

In line 19 Mpumelelo repeats this pronounced focus, but adds to the focus by tracing a standard cosine graph in the air with his pen. He traces the shape of the graph in the air rather than committing to a sketch graph in his answer book. Since Mpumelelo has traced a cosine graph that starts and ends at its maximum value, it is possible that his attended focus is the word "maximum" in the text of question (a) of the Flu Virus Problem.

Lungiswa has been listening to Mpumelelo and watching his tracing in the air (her attended focus) and pronounces, "Why do you say so?" (line 20). Here she is asking for an explanation. Her use of the second person pronoun "you" suggests that she is addressing Mpumelelo and assigning to him personal responsibility for the claim about the cosine graph.

Mpumelelo, in turn, is attending to Lungiswa's query and starts to explain. The pronounced focus in line 21 is "Because they say we must we must also solve ... the ... the number ((*Using his hand to emphasize*)) of people who get the disease right?" The use of the causal conjunction "because" identifies Mpumelelo's response to Lungiswa as an explanation. In this explanation he attends to the text of question (a) of the Flu Virus Problem. This attended focus is suggested by his naming of the text as an authority (named as "they") and he supports his argument using the authority of the text. His use of the third person pronoun "they" to describe the text suggests that he sees himself as distanced from this authority and subject to it (the modal verb "must" suggests that his action is a necessity). In contrast to his identification of the text as authority, his use of the first person pronoun "we" is inclusive of his peers in the group and suggests that he sees the task of answering the question as a collective one. Although Mpumelelo is attending to the text of question (a) in making his argument, he renames the instruction to, "draw a rough sketch" with the word "solve". He then reproduces the text of question (a) as "the number of people who get ...",

but interchanges "infected" with "the disease". In making this change he reproduces a feature of the problem text in which the words "the number of people who have, or have had, the disease" are used interchangeably with "the number of people who are infected". Once again, Mpumelelo makes a tentative claim, the word "right" and his rising tone suggest that he is asking for feedback.

Lungiswa pronounces, "uh … huh↑" in response to Mpumelelo's explanation (her attended focus). This response is in the form of positive, content-free feedback and the rising tone serves as a prompt for Mpumelelo to continue his explanation, which he does in line 23.

6.3.2 The interpretation stage

Interpretation on the level of Episode

While the description stage requires that I focus on each line or sub-line of the transcript, the interpretation stage requires me to ask questions about the textual features (in the central column of Table 5.1) over larger sections of the transcript, and this is where I use Episodes. I refer to Transcript 6.1 to illustrate this concept. The analysis presented in Section 6.3.1 suggests that the *discursive focus* (Sfard, 2000, p.303), or what the students are attending to and talking about/representing, is Mpumelelo's choice of a cosine graph and his explanation of this choice. This discursive focus is maintained in the lines that follow, up to line 76 of the transcript. This constitutes one *Episode*, in this case Episode 2 for question (a). In line 77 Siyabulela asks a question about the function values, that is, whether the cosine graph can take on negative values. This and the discussion that follows represents a change in discursive focus and hence a new Episode. I use this grouping of lines into Episodes to divide the full transcript (the transcript representing the action of Group 2 on question (a) of the Flu Virus Problem is 334 lines long) into smaller sections. In the interpretation stage I ask questions (the central column of Table 5.1) about the textual features in each Episode in order to identify the three meanings, representation, interaction, and identification. Taken as a whole, these three meanings allow me to write a narrative that describes the students' mathematical action in that Episode.

Although I have not presented the full transcript for Episode 2 in Section 6.3.1, I use the lines of Transcript 6.1 to illustrate this stage of the analysis (the analysis presented here is consistent with the rest of the Episode, which is presented in Chapter 10).

Having identified Mpumelelo's pronounced and attended focus in lines 17 and 19 I can speculate what meaning he is trying to communicate (the *intended focus*), in this case, "The required graph is a cosine graph that starts and finishes at its maximum value". Mpumelelo is attributing the required graph to a class of functions (cosine functions in trigonometry in school mathematics and in the Foundational Course) and attributing this graph to other possible classes is excluded. By naming the graph as a "cos graph" he appears to be looking at the function as a static object belonging to a class of graphs, rather than adopting an operational view of the function and considering the shape of the graph over time. His selection of the standard representation of the cosine graph starting and ending at the maximum value of the function excludes other versions of the cosine graph and becomes the significant graphical representation of the cosine graph in the rest of the Episode. I have suggested that this choice of representation for the cosine graph results from a textual link to question (a) and Mpumelelo's attending to the word "maximum" as a cue in this text.

Mpumelelo makes another textual link to question (a) when he explains his choice of graph; he makes a link to the task context and supports his argument using the authority of the text. Mpumelelo's stress on the word "number" in his explanation points to his *intended focus*; he seems to be arguing that his cosine graph represents this "number", that is, "the number of people who get infected". What is absent from this explanation (and also in lines 23 and 25) is an explicit and accurate link between his description of the task context and the properties of his selected cosine graph. Mpumelelo's initial explanation, together with what follows, suggests that he is not in control of the movement in meaning of the objects across the mathematical/non-mathematical boundary. In addition, Lungiswa does not attend to this movement in meaning, suggested by her content-free feedback in which she does not challenge Mpumelelo's argument.

Although Mpumelelo identifies himself as a student who is prepared to make his answer public, his confidence is tempered by his requests for feedback. It is also tempered by his identification of the text as an authority in his action, and hence that as a student he is subject to this authority. Mpumelelo is involved in the activity of tracing the shape of the graph in the air and the sketching of this graph in his answer book is an absence, again pointing to his hesitancy. Mpumelelo's renaming of the instruction in the text from "draw a rough sketch" to "solve" in his explanation in line 21 suggests that he identifies mathematical action as "solving".

In her responses Lungiswa identifies herself as the student who encourages the other students to talk and to explain their answers. She gives content-free feedback to Mpumelelo and does not attend to how he is controlling the movement of meaning between the task context and his graphical representation.

Interpretation of the enabling and constraining mathematical action

The interpretation of an Episode as described in the previous section is applied to all Episodes in the research texts and used to write a narrative[114] describing the mathematical action of each group of students on a practical problem (the practical problems solved by each group is summarized in Table 5.3). Taken as a whole, these narratives point to similarities and differences in the action of individual students, groups of students, and the Tutor. For example, in both Groups 1 and 2 students select named graphs from school to identify the graph in question (a) of the Flu Virus Problem and have difficulty proceeding beyond these choices. In both groups it is the Tutor who models the enabling ways of looking and making links required for constructing the graph. Four of the students in Group 1 make enabling links between the variables and the task context of the Car Problem, yet one student does not make these links and is constrained from doing so by the talk in which these links are not made explicit and by the social relations in the group.

[114] Here I use the word *narrative* for a written description of the ways of acting mathematically, and not as an aspect of mathematical discourse as used by Sfard (2008).

Comparisons between students and groups of students, in interaction with the Tutor, in all Episodes allow me to identify the mathematical action as enabling or constraining.

6.3.3 The explanation stage

Working on the explanation level, I start to explain the enabling and constraining actions by asking what discourse types, styles and genres are articulated in the text. The description of the discursive space in Chapter 2 and 3 are the resources that I draw on for this explanation. Although it is not possible to perform this stage of the analysis on the few lines of Transcript 6.1, I use my knowledge of the bigger set of data to make some suggestions about these few lines.

Mpumelelo's representation of the action of mathematics as "solving" and his choice of a cosine graph suggests that he adopts the style of a school mathematics student and draws on the valued mathematical action of classifying functions in school mathematics. His choice of a standard cosine graph also links to other pedagogic texts in school mathematics and in the Foundational Course in which the cosine graph is presented in this way. The absence of control over, and lack of attention to, the movement of meaning between the task context and the graph suggests that Lungiswa and Mpumelelo are treating this problem as if it belongs to the genre of mathematical word problems and that they "pretend that" (Gerofsky, 2004, p.35) the flu virus exists. The link to this genre is also supported by Mpumelelo's attention to a cue ("the maximum") in the task context which may suggest to him which mathematical graph to select. Lastly, Lungiswa's use of prompts and positive feedback as encouragement to her peers to talk and to explain suggests that she is adopting the style valued in a learner-centred pedagogy and in the Course material which reminds students that they should be "asking questions about solutions", "encouraging one another to keep going / to participate" (Foundational Course Resource Book, 2007, p.16).

6.4 Quality in this study

Silverman (2010) identifies four criteria for evaluating the quality of qualitative research, criteria that address "methodological, theoretical and practical issues" (p.293). Firstly, whether the research makes use of the theoretical constructs of the social sciences and contributes to the field. Secondly, whether the research is reliable and valid. Thirdly, whether the methods are carefully selected, with alternatives in mind. Lastly, how valid and reliable studies that are "conceptually well-defined" (p.294) contribute to practice and policy.

Referring specifically to mathematics education, Adler and Lerman (2003) would refer to Silverman's first point as making the description "count" (p.446) for the mathematics education community. According to Silverman (2010), this involves talking with and to the community. Adler and Lerman (2003) adopt a broader view on how the study contributes to practice (Silverman's fourth point); they refer to making the description "count" (p.146) for the participants, in terms of the ethics of the research practice and how the results are used. Silverman's (2010) remaining two points could be called getting the description "right" (Adler & Lerman, 2003, p. 446). Yet Silverman's four points (or getting the description "right" and making it "count" in Adler and Lerman's (2003) terms) are related. For example, in this study the methods I use to collect research texts are related to the use of CDA (and focal analysis) as a methodology, which in turn is consistent with a socio-political perspective of practice; my theoretical constructs have to be valid if they are to make a contribution to the mathematics education research community; and providing rich descriptions of the space in the interests of validity has ethical implications in terms of the visibility of participants.

In this section I focus on issues of validity and reliability and on the ethics of this study. I continue the discussion of the quality of the study when I present the contributions of this research in Chapter 12.

6.4.1 Validity and reliability in this study

Attention to validity and reliability are important in establishing the quality of qualitative research (Silverman, 2010). In this section I attend to these issues in relation to the study as a whole and more specifically in relation to the use of CDA as a methodology, a methodology that has been criticized for its subjectivity. For example, Widdowson (1998) argues that the interpretations made in CDA are subjective, suggesting that the commitment to social justice that underlies the methodology results in certain aspects of text being spoken about and not others. Fairclough (2001, p.22) acknowledges that texts cannot be "mechanically described without interpretation", for analysts who use CDA "cannot help themselves engaging with human products in a human, and therefore interpretive way".

In this discussion I use a critical realist notion of validity from Maxwell (1992), as this provides a productive link to the ontology and epistemology adopted by Fairclough. Maxwell (1992, p.283) talks about the validity of an *account* rather than the validity of a procedure or of data. He argues that validity of an account is about the relationship between this account and what it is about (or "something outside of the account", p.283). From a critical realist perspective, "we can have no direct knowledge of the objects of our accounts and thus no independent entity to which to compare these accounts" (Maxwell, 1992, p.283). Maxwell (1992, p.283) suggests that the relationship between an account and the object it talks about is based on *contiguity* (rather than *similarity*), that is, "on the implications and consequences of adopting and acting on a particular account".

Fairclough (2003) notes that the representation (or in Maxwell's terms the *account*) of a social event, such as the student action on practical problems, cannot be compared to some "truth" (p.136) about the event. The representation of the real is discursive; Maxwell (1992, p.284) notes that an account cannot be independent of a particular perspective. Yet these discourses are not the same as the objects themselves (Fairclough, 2006). Rather, we can only compare different representations of "the same or broadly similar events" (Fairclough, 2003, p.136). This does not, however, mean that all representations are "equally useful, credible, or legitimate" (Maxwell, 1992, p.183).

Maxwell (1992) identifies five types of validity in qualitative research; descriptive validity, interpretative validity, theoretical validity, generalizability, and evaluative validity. However, these five types were developed for qualitative research more generally, and not specifically for CDA. Thus for the purposes of this study I reinterpret where appropriate Maxwell's types of validity in a way that is consistent with Fairclough's method for CDA.[115] I also draw on the work of Gee (2005) on the validity of discourse analysis. I refer to issues of reliability where appropriate within the discussion of validity that follows.

Descriptive validity

Descriptive validity is about the "factual accuracy" (Maxwell, 1992, p.285) of the account, that is, that the researcher is not making up what was said or done in the social event. According to Maxwell (1992) questions about the accuracy of a description of what was said or done can be settled with reference to the data.

This form of validity figures in two respects in this study. Firstly, what Maxwell (1992, p.286) calls *primary descriptive validity* refers in CDA to the accuracy of the production of the research texts to be analyzed. Referring to the descriptive stage of CDA, Fairclough (2001) argues that the "'object' of description, the text, is often seen as unproblematically given" (p.22). He argues that such a view is problematic, citing the example of spoken discourse which has to be transcribed in text; "there are all sorts of ways in which one might transcribe any stretch of speech, and the way one *interprets* the text is bound to influence how one transcribes it" (p.22, emphasis in the original). In Section 5.4.1 it was noted that the video recordings in this study are a re-presentation of the actual social events in which the students solved the practical problems. Thus in this study primary descriptive validity relates to the accuracy of the written transcripts (and where necessary the translations) as re-presentations of these video recordings. Consider for example Mpumelelo's verbal and non-verbal action re-presented in line 19 of Transcript 6.1; "It

[115] Maxwell's (1992, p. 295) fifth type of validity listed here, *evaluative validity*, is not appropriate for the study; in this study I describe and explain social events, and do not evaluate them.

won't it be like a cos graph? ((*Using his pen to demonstrate a full wave of a cosine graph, starting and ending at its maximum value*))". Is this what Mpumelelo said and did in the video recording?

I have addressed this first form of descriptive validity in this study by successively revisiting the relationship between the written transcripts and the video recordings; this took place both during the transcription itself and during the analysis when the video footage was revisited for clarity. Since I am not familiar with the language group to which Setswana and Sesotho belong, the first translations have been checked by a second translator.

Maxwell (1992) notes, however, that no descriptive account can include everything and that accuracy may be relative to the purposes of the study. So when re-presenting Mpumelelo's action as in line 19 I may have omitted a gesture or tone that is relevant to the study. I have addressed this aspect of descriptive validity in this study by doing the initial transcription of two out of the six video recordings personally, in interaction with the initial development of appropriate analytic tools for the study. As I developed the tools I was able to decide what detail was required in the transcripts and develop a consistent transcription notation.

This attention to the relationship between the transcripts and the video recordings in the interests of descriptive validity goes some way to addressing reliability issues, that is, whether I apply my analytic tools consistently across transcription. Silverman (2010) suggests that careful transcription of recordings in a theoretically informed way can contribute to such consistency.

Addressing the descriptive validity in this study becomes more problematic when one wants to make links between the action re-presented in the transcripts and the wider socio-political space on the macro-level (in the explanation stage of CDA). For this requires a description of this macro-level space. Such a description is an issue of *secondary descriptive validity* (Maxwell, 1992, p.286), since it represents a context that is, in a sense,

observable. Blommaert (2004) identifies "the *framing* of discourse in particular *selections of contexts"* (p.15, emphasis in original) as a particular problem in CDA, since the relevance of the selection is established by the researcher and is not made an object of study in itself. I respond to this aspect of descriptive validity in this study in two ways. Firstly, I dedicate Chapters 2 and 3 to the description of the wider socio-political space of the study, thus identifying and delineating the space that I draw on. Secondly, I note that the choice of sources for the description of the wider space is not arbitrary. On the contrary, from a socio-political perspective of practice these selections are constrained by my participation and the participation of others who were consulted in the selection process in the practices of schooling and higher education. For Fairclough (2001) argues that people "are enabled through being constrained: they are able to act on condition that they act within the constraints of a type of practice" (p.23). In both Chapters 2 and 3 I dedicate a section to explaining the choices that I have made.

Interpretive validity

According to Maxwell (1992, p.290) *interpretive validity* relates to the relationship between the account of a participants' meanings and the perspective of the participant whom the account is of. This form of validity is not appropriate for CDA, although claims have been made that this methodology lacks interpretative validity of this type since it fails to consult the producers and interpreters of the texts that are analyzed (e.g. Widdowson, 1998). However, such a criticism shows a lack of understanding of CDA as a methodology. This methodology analyzes "language in use" (Fairclough, 2006, p.9) or "how meanings come to articulated at a particular moment" (Pennycook, 1996, p. 116). A participant's perspective on the language use at this "particular moment" (Pennycook, 1996, p.116) represents a different moment. Furthermore, the work that language does, for example in legitimizing certain power relations, is opaque to the subjects of a social practice and CDA aims to identify this work in texts (Fairclough, 2001, p.33).

I argue that interpretative validity figures at two levels of the CDA that I use in this study. Firstly, it applies to the relationship between the account of a participant's meanings and

the text that is being analyzed (as opposed to the perspective of participant, as suggested by Maxwell (1992)). In Fairclough's method for CDA this refers to using the textual features (identified in the descriptive stage) to identify what meanings these give to the text (the interpretation stage). Fairclough (2001) argues that what one is analyzing is "less determinate" (p.22) in the interpretation stage than in the description stage (as discussed under primary descriptive validity in the previous section). For in the interpretation stage one is analyzing the cognitive processes of individuals and "offering (in a broad sense) interpretations of complex and invisible relationships" (p.22). In Sfard's method of focal analysis (at the interpretation stage), interpretive validity refers to using the pronounced focus and attended focus to identify the intended focus. For example, in Transcript 6.1 I use the pronounced focus and attended focus in the text to identify Mpumelelo's meaning as, "The required graph is a cosine graph that starts and finishes at its maximum value". Is such an interpretation valid?

Ensuring interpretive validity at this level requires attention to the "linguistic detail" (Gee, 2005, p.114) that is identified in the descriptive stage of CDA (represented in the right-hand column of Table 5.1):

> Part of what makes a discourse analysis valid then, is that the analyst is able to argue that the communicative functions being uncovered in the analysis are closely linked to grammatical devices that manifestly can and do serve these functions, according to the judgments of "native speakers" of social languages involved and the analyses of linguists. (Gee, 2005, p.114)

In Appendix E I specify what meaning each textual function performs. For example, Fairclough (2003) indicates that the choice of scientific or everyday terms is a way of representing the world, Morgan (1998) suggests that nominalization transforms a process into an object in mathematics, and Janks (2010) indicates that the present tense indicates a timeless truth and absolute certainty. These meanings are not idiosyncratic. Rather, I regard Fairclough, Morgan and Janks as the "native speakers" referred to by Gee (2005, p.114).

The questions related to each of the three meanings of the text at the interpretation stage (the centre column of Table 5.1) also play a role in establishing interpretative validity. From a critical realist perspective, it is not possible for me to compare an account of Mpumelelo's action with some "truth" (Fairclough, 2003, p.136). Yet Fairclough (2003) suggests that by considering what is included/excluded and given significance/not given significance in a representation, my account can be compared with other representations. The questions in the interpretation stage are designed for this purpose. For example, I use the question, "What ways of talking and writing about objects are included/excluded/given significance?" to consider what Mpumelelo did not talk about or how he could have talked about the graph in a different way. For example, he did not plot individual points on the graph, or draw an irregular function, or talk about what happens to the population as time passes.

Gee (2005, p.113) argues that both *convergence* and *coverage* are important in considering the validity of accounts produced using discourse analysis. In terms of my analytic tools, *convergence* means that the answers to all the questions related to representation, interaction and identification at the interpretation stage should converge in a way that provides "*compatible* and *convincing* answers" (p.113, emphasis in the original). *Coverage* refers to being able to apply the same tools to "related sorts of data" (Gee, 2005, p.114); in this study I apply the same tools across three practical problems, each solved by two groups of students. In Section 6.3.2 I have described how I asked the same questions across all six transcripts, identifying patterns in the answers in such a way that I am able to identify the enabling and constraining conditions.

I turn for a moment to interpretive validity and focal analysis. Can my interpretation of Mpumelelo's intended focus in Transcript 6.1 be regarded as valid? Again, it is not possible for me to infer some "truth" (Fairclough, 2003, p.136) about Mpumelelo's meaning. In fact, in developing the tools for focal analysis, Sfard (2001) acknowledges that the intended focus is largely private. However, the pronounced focus and the attended focus serve as

clues to Mpumelelo's intended focus. Since Mpumelelo's talk and gestures are re-presented in the transcript, these transcripts serve as the evidence for my claims about his meaning.

Yet interpretative validity not only needs to be established at the description and interpretation stages of my analysis, but also at the explanation stage (the left-hand column of Table 5.1). At this level, interpretive validity applies to the relationship between the account of a participant's meanings and the wider socio-political space. As is the case with the interpretation stage, the objects of analysis in the explanation stage are "less determinate" (Fairclough, 2001, p.22) than those in the description stage, for again the relationships are "complex and invisible" (p.22). For example I may claim that, in viewing the function in the Flu Virus Problem as a static object and severing the link to the task context in his explanation, Mpumelelo is drawing on his experience of classifying functions and solving word problems in school. Is such an explanation valid? This explanation draws as a resource on the description of the wider socio-political space that I have delineated in Chapters 2 and 3 and in particular on the description of school mathematics and school mathematical word problems within this space. This description, I have argued in the previous section, has descriptive validity.

Theoretical validity

Theoretical validity refers to the validity of an account as a theory of something (Maxwell, 1992, p.291). Maxwell (1992) divides theoretical validity into two types. Firstly, it is about the relationship between the theoretical constructs and the objects to which they are applied. In this study this refers to how I identify a particular way of acting mathematically (a theoretical construct) in the text. I have addressed this first aspect of theoretical validity in a number of ways in this study. Firstly, the analytic tools in Table 5.1 are designed to operationalize the theoretical concepts of ways of acting mathematically. The overall analytic framework is based on Fairclough's theory of socio-political practice and is thus consistent with this theoretical perspective. Where I have recontextualized analytic tools from mathematics education I have considered a possible movement of meaning in these tools. Secondly, I have demonstrated in detail in Sections 6.2 and 6.3 how the analytic tools

are applied. For example when analyzing the action re-presented in Transcript 6.1, the pronounced focus allows me to identify Mpumelelo's talk. Then I use the tools of CDA to look at the features of this talk, for example, his naming of the graph and the relational process in which he attributes the required graph to a class of trigonometric graphs. The concept of pronounced focus also points me to the standard cosine graph he traces in the air.

Thirdly, Gee (2005, p.113) suggests that *agreement* is important for establishing the validity of the relationship between theoretical constructs and the objects they describe. Answers to the questions asked in discourse analysis are more convincing when other participants in the community agree with the use of the constructs (Gee, 2005). Throughout this research journey I have had opportunities for feedback from a variety of researchers "who share our basic theoretical assumptions" (p.114); ongoing and regular feedback from my doctoral supervisor, regular scrutiny of my work by fellow mathematics education doctoral students, discussions with some of the theorists whose work I have drawn on (for example, Morgan, Sfard, and Valero), presentation of work-in-progress at conferences (representing mathematics education, language and higher education communities), and peer review for journal publication (representing mathematics education and language communities). Presentation of this thesis for examination is another step in the process of establishing agreement on the validity of relationship between my theoretical constructs and the objects of study.

The second aspect of theoretical validity refers to the "postulated relationships" (Maxwell, 1992, p.291) between the theoretical constructs themselves. For example, I may argue that Mpumelelo's pronounced focus (the text) suggests that he is looking at the function in the Flu Virus Problem as a static object rather than viewing it operationally (the representation) and relate this to his experience of classifying functions in school mathematics (the discourse type). Is such a set of relationships between the three levels of language valid? In this study the link between the three columns of Table 5.1 are based on the concept of language in Fairclough's (2003) socio-political perspective of practice, that is, that when

we use language we represent the world in certain ways, and there are relatively stable ways of representing or discourse types at the level of socio-political practice.

In the presentation of my results in Chapters 7 and 11 I present extracts from the practical problems and the transcripts re-presenting the student action as evidence for my arguments. Thus not only is the interpretive and theoretical validity of the accounts up for scrutiny, but also whether I have applied the analytic tools consistently (reliability).

Generalizability

This form of validity refers to the extent to which an account of a particular social event can be extended to other "persons, times or settings than those directly studied" (Maxwell, 1992, p.293). However, it is not possible for me to make claims to any insights that are generalizable across practices, since a socio-political perspective of practice means that the accounts produced in this study are cases that cannot be separated from the practices in which they are produced. From this perspective accounts are not transferred unproblematically across practices, but are recontextualized in a process that involves a movement of meaning across practices. Indeed, Cotton and Hardy (2004) warn of the normalizing that may occur in any search for generalizable knowledge. My response to the challenge of generalizability, therefore, is to provide a detailed description of the network of socio-political practices in which the study is located (as is done in Chapters 2 and 3) and to use the results of this study to point to issues in the foundational practice and its relationship to other practices in the space that are worthy of attention. Based on this presentation, another researcher should be able to consider the possibility of recontextualization in a different practice and/or network of practices.

6.4.2 Ethics in this study

In this section I consider the quality of this study in terms of what it might mean for the research to "count" (Adler & Lerman 2003, p.446) for participants. I focus here on how the research might "count" in the sense that (a) the students and Tutor are not harmed by the

study, and (b) their participation is respected and valued. I refer to how the research might "count" with respect to practice in Chapter 12.

In the negotiations for participation with the students and the Tutor ethical considerations such as informed consent, voluntary participation, anonymity, confidentiality, the use of the data, and (in the case of the students) the relationship between participation in the study and enrolment in the Foundational Course were addressed (see Appendices G to J). Yet this study presents particular ethical challenges which I discuss in the rest of this section.

Informed and voluntary consent

Obtaining informed and voluntary consent from students and the Tutor to participate in this study involved more than simply getting the relevant signature on consent forms. Firstly, the developing and changing nature of the qualitative research process means that the researcher cannot always know in advance the exact path that the research may take (Van den Hoord, 2002). During this study the Tutor's role was reconceptualized from being a replacement tutor for myself in the workshop class (as presented in the Information Sheet for the Tutor in Appendix I) to being a participant in the study, a process that required the renegotiation of the tutor's participation.

Secondly, adopting a socio-political perspective of doctoral research practice (as conceptualized in Section 5.2) requires sensitivity to "the complexity of the evolution of power relationships in the research process" (Valero & Zevenbergen, 2004, p.143). Given that I was both the lecturer and convenor of the Foundational Course in the year that the study was conducted, one could question whether a student's decision to take part in the study could, in fact, be regarded as voluntary. Although I needed consent from individuals for their participation in the study, I was in fact negotiating for participation of already existing groups of students, of which the individual students formed part. There could be a conflict between the rights of the individual and the rights of the collective or group of students (Van den Hoord, 2002), with the result a function of the power relations in the group. In this study I chose to make these power relations explicit in my initial negotiations

with the students. Three students chose not to participate, suggesting that they felt empowered to make this decision.

Confidentiality and anonymity

The typical strategy of providing pseudonyms of participants (as I have done in this study) is not a guarantee of anonymity in qualitative research. Firstly, only seventeen students and one tutor participated in this study and the video recordings were made during workshops in the Foundational Course. The participants could, therefore, be recognizable to one another, to other students in the workshop class and to the larger foundational class. Secondly, in an effort to provide quality research, I present a rich description of the context and detailed transcriptions of the student action, all of which make the participants visible or recognizable.

Furthermore, in presenting this detail detailed and rich data to the wider research community both during and after the study, I am placing the data in the public domain and the data is no longer confidential. This is so, despite my adopting the traditional approach of raising issues of confidentiality with the audio-visual personnel, transcribers and translators, and ensuring that video recordings were only been viewed by myself, the transcriber of the videos and the translators.

Harm and risk

Graven (2002) argues that recognizability is not necessarily problematic, unless the data is of a sensitive nature. In deciding what might be sensitive in this study I have turned to my research question and the underlying assumptions that inform this question; I aim to document and explain the action of students (and the Tutor) in the light of the socio-political action of the class and the wider socio-political space, and in particular to avoid adopting a deficit view of individuals. Doing no harm to the participants from a research perspective involves being consistent to this aim. Yet doing no harm as a researcher cannot be divorced from the ethics of my teaching and the rights and needs of all students on the

Foundational Course (for example, making sure that the other students had access to a tutor during the recording sessions).

Balancing the ethics of both teaching and researching required ongoing decision making during this study. For example, during a recording session I made the decision to allow a participating student to leave the session early so that he could see to his funding application for the following academic year, knowing that this would impact on my collection of research texts. In an attempt to separate my role as both lecturer and researcher I undertook not to mark the participating students' assessments during the year in which the study was conducted, yet practically this proved difficult to implement when marking supplementary examination scripts and assessments in the subsequent foundational mathematics course where I was the sole lecturer.

My interest in some students in a workshop class and not others (for the purposes of this study) meant that the participating students were visible to me as a lecturer. My knowledge of the participating students had to be managed carefully. For example, as a student advisor to the foundational students I had to present a participating student's case to a faculty committee which was considering his appeal against exclusion on academic grounds. Here I was bound by the faculty process to make an argument based solely on the students' written submission, and could not draw on my knowledge of the student gained during the study. Yet on another occasion my knowledge of a participating student led me, in the year following the collection of research texts, to refer the student for counseling on a personal matter.

Doing no harm also requires attention to the ways of writing in this thesis, an issue that is significant for a study conducted in South Africa.[116] My description of the discursive space in Chapters 2 and 3 indicates that some of the participating students had already been

[116] Graven (2002) and Adler and Lerman (2003) argue that, while certain ethical issues may be illuminated in research conducted in South Africa, these issues are not unique to this country and apply more widely.

labeled "educationally disadvantaged" by the institution and placed in a foundational programme, suggesting that they had already been "harmed" by the complex history of the country. Others, who had started their studies in the mainstream, had already had to face failure in their mathematics course and change to the "decant" Foundational Course. Referring to teacher education research conducted in the context of rapid and complex change in South Africa, Adler and Lerman (2003) note that any description of inadequacy can be used for a political agenda. In my writing I have had to pay constant attention to *how* I write about students to avoid adopting a deficit view of the students (something which our language, so steeply rooted in an individualistic portrayal of performance, makes it easy to do). I have also made the political decision to adopt pseudonyms for these students that portray them in a positive light. Yet doing no harm in terms of writing about students also requires attention to *which* students are described; in Section 8.3 I note that decisions about what to present in the results was driven, in part, by an ethical decision to re-present the action of all participating students.

Other participants in the study

I have discussed the ethical issues related to the obvious "participants" in the study, these being the students and the Tutor. Yet my colleagues in mathematics education who designed the Foundational Course and developed the Course material (such as the practical problems) are, indirectly, also participants in this study (as producers of texts). In Section 1.1 I set out my intention not to name the Foundational Course and the related mainstream courses. Yet the rich description of the space in which this study is located and my personal positioning in this space has implications for the recognizability of these courses. So in considering what it means for the research to count "locally" (Adler & Lerman, 2003, p.446) it is also necessary to attend to the ethical implications of this recognizability.

Reform-oriented curricula that call for relevance and learner-centred pedagogy generally have, in the words of Nyabanyaba (2002), "noble intentions" (p.51). For example, Ensor and Galant (2005) argue that educational reforms in South Africa have been driven by an "agenda of educational empowerment" (p.282). Yet Ensor and Galant (2005) argue that this

should not prevent us from investigating how these reforms play out in practice, since they may have "unintentionally produced contradictory effects" (p.282). Fairclough (2001) suggests that educational discourse is opaque, in the sense that the more abstract aspects of social structure, for example, social class, are "hidden" (p.33). Thus it is possible that through educational discourse one might be reproducing particular power relations without being aware of it. Fairclough (2001) thus presents a moral argument for CDA when he argues that this methodology can have social effects by "raising people's self-consciousness" (p.33).

My understanding of the development of the Foundational Course in which this study is located suggests that this development was based on "noble intentions" (Nyabanyaba, 2002, p.51), in particular to provide access to higher mathematics for students who have been disadvantaged by the inequitable educational system in South Africa. This study in no way sets out to challenge these intentions. Rather, the purpose of this study is to use research to make visible what is not visible (at the level of the Course and the location of the Course in the wider higher education space), and to consider whether the innovations adopted in the Course may contradict the underlying agenda of redress and development that underlies the Course.

6.5 Summary of this chapter

In the first part of this chapter I have demonstrated how the tools in the analytic framework (Table 5.1) are applied to the research texts, that is, the practical problems and the transcripts re-presenting the student action. The detail given here serves to demonstrate the interpretive and theoretical validity of my accounts of the texts, aspects of validity that I respond to in Section 6.4. This section also attends to the ethics of the study, with a focus on making the research "count" (Adler & Lerman, 2003) for the participants from an ethical perspective. This section contributes to my discussion of the quality of the study as a whole, which I develop further in Chapter 12. Having described the research process in Chapters 5 and 6, I present the results of the study in Chapters 7 to 11.

CHAPTER 7 RESULTS (PART 1)
THE TEXTS OF THE PRACTICAL PROBLEMS

7.1 Introduction to this chapter

In this chapter I present an analysis of the texts of the Flu Virus Problem, the Chemical Reaction Problem and the Car Problem (see Appendices B to D and Appendix Q). This analysis was conducted using the process demonstrated in Section 6.2.

The analysis of the practical problems serves two functions in this study. Firstly, viewing the problems as the product of the processes of production on the level of discursive practice (Fairclough, 2001), the analysis is used to answer the first research question. I describe how the problems represent the practice of foundational mathematics and set up subject positions for the student. I also explain this description with reference to the wider network of socio-political practices described in Chapters 2 and 3. As noted in Section 6.2.2, the actual student interpretation of the problems also informs this analysis.

The second function of this analysis is related to the student action on the three problems and answering the second research question. For a practical problem is not only the product of the processes of production, but also a resource for the interpretation of the text on the level of discursive practice (Fairclough, 2001). When interpreting a practical problem the student will attend to the text of the problem as a resource, but will also exercise agency and draw on his member's resources (Fairclough, 2001, p.20) with respect to what should happen when solving such a problem in a mathematics classroom. So the analysis of the practical problems provides background to the analysis of the student action presented in

Chapters 8 to 11 where I consider how the students invest in the subjects positions set up for them by these problems.

I present the analysis of all three practical problems in this chapter, since they were selected as representing the variety of practical problems in the Foundational Course. The analysis is organized around the research sub-questions; the relationship between the foundational practice and other practices, the mathematical action that is given value in this practice, and the construal of social relationships and identities for the subjects. I end this chapter by considering the location of the foundational practice in the wider order of discourse, and the power between the practices in this space.

7.2 Relationships between foundational mathematics practice and other practices

In this section I describe and explain the relationship between the foundational practice and other practices, both mathematical and non-mathematical.

7.2.1 Enacting a link to non-mathematical disciplinary practices

The objects represented in the introductory sentences of the Flu Virus Problem and the Chemical Reaction Problem make links to disciplinary practices other than mathematics. In the Flu Virus Problem the spread of a "flu virus" in a community sets up a relationship to the study of epidemiology. The first sentence of the Chemical Reaction Problem introduces the reader to two "chemicals" that are being mixed together in a "reaction chamber" to form a new chemical "product", thus setting up a link to the practice of chemistry.

The inclusion in the practical problems of objects that take on particular meanings in non-mathematical disciplinary practices points to a weakening of the boundary between the mathematical and non-mathematical and suggests that the problems draw on the discourse of relevance in school mathematics and undergraduate calculus reform. Foundational mathematics is represented as relevant and powerful since it is "for something else", that is,

for participation in non-mathematical practices (Dowling, 1998, p.9). The students who solve these two problems are identified as science students interested in chemistry and epidemiology, pointing to the discourse that the Foundational Course is for students aiming at a science specialization other than mathematics. The task contexts of the two problems can be described as near (Sethole, 2005, p.42) to the students in the sense that they resonate with students' experience of studying science courses and their career aspirations in science.

7.2.2 Representing the non-mathematical disciplinary practices using everyday discourses

The objects such as the "flu virus" and the "chemical" in the two problems set up links to non-mathematical disciplinary practices. However, these objects are not represented using the specialist discourse of these practices, but using everyday discourse (Moschkovich, 2007, p.27), such as would be used outside of the disciplinary practices and their related pedagogical practices. The "flu virus" is not named and is replaced elsewhere in the Flu Virus Problem with other everyday terms such as "the flu" and "the disease". In Sentence 3 of the problem the term "immune" is renamed in everyday language as "does not get it again". The "chemicals" in the Chemical Reaction Problem are named with arbitrary symbols, "A", "B" and "X", and the quantities of the chemicals are not given. In contrast to the two problems discussed so far, the Car Problem does not make a link to a disciplinary practice, and the objects (the "cars" and the "starting point") and the measurements (distance and speed) are represented in everyday discourse.[117]

The verbs describing the action in the everyday practices refer to material processes. The flu virus "has hit" the community and the people in the community "catch" the disease, the cars are "moving" and they "travel", and the chemicals "are mixed" and they "react".

[117] In Section 7.3.5 I argue that the valued mathematical action for the Car Problem sets up a link to problem-solving methods for solving motion problems in school or introductory undergraduate physics. It is not, however, the naming of the objects that suggests this link.

These material processes can be distinguished from the relational processes that characterize scientific writing (Morgan, 1998, p.18, citing Halliday, 1985, p.123-124).

The use of everyday discourse in the practical problems points to an attempt to make the practices of chemistry and epidemiology accessible to someone who is outside of these practices. So foundational mathematics is not really for participation in non-mathematical practices. Rather, the link to objects in non-mathematical practices serves the function of motivating the student with an interest in this practice to participate in foundational mathematics. This suggests that the problem draws on a particular view within the discourse of relevance, that is, relevance for motivation to participate in mathematical practice (Boaler, 1993).

While the student is identified as having an interest in these non-mathematical disciplinary practices, he is not identified as a participant in these practices. This analysis points to an absence in Sethole's (2005, p.42) classification of a task context as being near, since it does not give significance to the style of the student in the near practice. For the student's degree programme and career interest might suggest that the task context is near, yet he may not be positioned by the problem as a participant in this practice.

7.2.3 Representing everyday objects as not real in time and space

While the three texts enact a relationship to everyday objects, a number of the textual features suggest that the everyday objects do not really exist in the practices they represent. Firstly, the everyday objects such as the "flu virus", the "community", the "cars", and the "chemicals" are named in general terms. In particular, the individuals who act in the everyday practice are not made visible. In the Flu Virus Problem the "people" are passive in relation to the disease as they are "hit" by the flu. Eventually everyone "catches" the flu, and there is an absence of resistance to the disease. The "people" are not identified as individuals, but collectively as part of the "community". In the Chemical Reaction Problem the chemist working with the chemicals is not named, the chemicals are simply "mixed" and "removed" by an unnamed agent. No human agency is evident in the Car Problem as it

is the cars that “travel”. Secondly these everyday objects have no location in time and space. The “point” from which the cars depart is named generally as “some” point and the timeless definite article “a” is used for the “community” and the “reaction chamber”. The time when everyone “catches” the flu is described vaguely as “sooner or later”. In all three problems the use of the present tense suggests that the action is ongoing. For example, the chemicals A and B “react”, people “catch” the flu and “become” immune to it, and the cars “start moving” and “travel”.

Analysis of the worked solutions for the Flu Virus Problem suggests that the accurate use of tenses is not given attention in these practical problems. The function value $P(t)$ is defined in Sentence 4 as “the number of people who have, **or have had**, the disease t days after the first case of flu was recorded” (my emphasis in bold). So the value $P(t)$ includes those members of the community who currently have the flu (at time t) as well as those who have had the flu (up to time t) and recovered from it. Yet the worked solution that gives the meaning of the expression $P(4) = 1\,200$ in “practical terms” (question (c)) does not take into account this latter group of flu sufferers. This solution states, “4 days after the first recorded person got flu, 1 200 people **had** the flu” (my emphasis in bold), the verb “had” here suggesting that the value 1 200 only refers to those people with the flu at that particular time.

In addition, certain aspects of the action represented in the practical problems challenge our everyday experience of the practices represented. For example, it is highly unlikely that two cars would move at a constant speed and at right-angles to one another. In the Chemical Reaction Problem the graph representing the reaction process suggests that initially the mass of product X being removed from the chamber exceeds the mass of the product formed.

This representation of the everyday objects as not being real in the practices that they represent and the inconsistent use of tenses suggests that the practical problems draw on the genre of mathematical word problems as described by Gerofsky (2004). Interacting within

this genre, the student should "pretend *that*" (Gerofsky, 2004, p.35, emphasis in the original) these objects exist. Since the social events such as the spread of the flu virus and the chemical reaction are highly unlikely in the non-mathematical practice, the task contexts in these practical problems can be classified as dead-mock reality (Freudenthal, 1973, p.78) or as inauthentic (Sethole, 2005, p.41).

7.2.4 Representing practical problems as pedagogic texts in mathematics

Foregrounding mathematical objects and mathematical representations of these objects

Following the initial reference to everyday practices in each practical problem and expression that is weakly classified in terms of expression (Dowling, 1998), mathematical words, symbolic notation and/or graphical representations are used to describe mathematical objects. The text is populated with relational processes setting up links between the mathematical objects, their representations, and the task contexts of the problems. The particular mathematical objects (variables, functions, derivative functions, limits and integrals) and their representations feature in the practice of school mathematics and/or undergraduate mathematics. The objects "function" and limit (in the abbreviation "lim") are named using words borrowed from everyday discourse, but which take on specific meanings in mathematical practice (Pimm, 1987).

In the Flu Virus Problem the spread of flu virus over time is identified with a mathematical function in one variable and the variables "number of people" and "time" are represented by the symbols P and t respectively. In the Chemical Reaction Problem the rate at which the product X is formed is identified with the derivative function $m'(t)$, the variables "mass" and "time" represented by the symbols m and t, and a named sketch graph (the "parabola graph" is a name that is used in school mathematics and the Foundational Course). The use of mathematical symbols in the text points to expression that is strongly classified (Dowling, 1998). In both these problems, the choice of certain letters of the alphabet as symbols suggests a link to the words that represent the everyday objects (Pimm, 1987). Pimm (1987) notes that differences in alphabet and fonts in the mathematics register

indicate conceptual differences; in the Flu Virus Problem the lower-case *t* represents the independent variable and the capital *P* in *P*(*t*) represents the dependent variable, yet this notation is not used consistently in the Chemical Reaction Problem where the dependent variable has a lower case *m*.

The relationship between the symbols also represents particular mathematical relationships (Pimm, 1987). In the Flu Virus Problem the function *P*(*t*) represents a relationship between the independent variable (identified by the brackets) and the dependent variable, that is, *P*(*t*) represents the number of people who have or have had the disease as time changes. Yet a mathematical relationship is less obvious in the Car Problem where the distance variables *x*, *y*, and *z* are implicit functions of time. Pimm (1987) notes that the punctuation symbol ′ in *m*′(*t*) modifies the symbols in the functional relationship *m*(*t*); *m*′(*t*) is also a function, one which is obtained from *m*(*t*) by the process of differentiation.

The function *P*(*t*) in the Flu Virus Problem is not represented using an algebraic formula or a graph, a contrast to the typical presentation of functions in the school Mathematics textbooks used by the students in this study. In the Chemical Reaction Problem the function *m*′(*t*) identifies the derivative function with a particular graphical representation. It is possible to derive an algebraic formula for the function from this representation, however this is only required of the students in question (d), the sequencing suggesting that the formula is not required in the preceding questions.

This foregrounding of mathematical objects and their representations and the representation of the non-mathematical practice as not real suggests that foundational mathematics is "about something other than itself" (Dowling, 1998, p.4), rather than "for something else" (p.9). This representation suggests that foundational mathematics practice has *exchange value* (Dowling, 1998, p.6) as it can be exchanged for the various non-mathematical practices (and hence is powerful, with the range of activities it can replace acting as a measure of the power of the practice). This notion of exchange value can be identified in the set of practical problems in the Foundational Course material. For example, the spread

of the flu virus is replaced by the growth of bacteria and the flow of water in other practical problems that foreground functions and rates of change. The chemical reaction is replaced in other practical problems that foreground functions and integrals, but using the task contexts of financial earnings, the flow of water into a dam, and the movement of people standing in a queue at a ticket office. In fact, when interacting with the students in this study the Tutor exchanges the chemical reaction for the flow of rainwater into a dam. This exchange of task contexts suggests that the practical problems draw on the genre of mathematical word problems, for Gerofsky (2004, p.35) argues that the story part of the word problem can be replaced by other stories.

Recontextualizing of non-mathematical practices by mathematical practices

The representation of the mathematical objects in the practical problems suggests that these problems cast a mathematical gaze (Dowling, 1998, p.24) on the non-mathematical practices. For example, since the cars in the Car Problem move at right angles to one another the student is enabled to represent the motion in a right-angled triangle and relate the variables using the Theorem of Pythagoras. In the worked solutions for the Flu Virus Problem the $\lim_{t\to\infty} P'(t) = 0$ is explained as "the number of people who have caught the flu becomes (**very nearly**) constant at 10 000" (my emphasis in bold). This answer backgrounds the link to the task context in which "sooner or later everybody in the community catches the flu", and foregrounds the approach to limits used in school Mathematics textbooks and the intuitive definition of the limit used in the Course (Figure 7.1). The latter definition is taken from Stewart (2006), the calculus textbook that is prescribed for both the foundational and mainstream undergraduate mathematics course.

Figure 7.1: Definition of the limit at infinity (Foundational Course Resource Book, 2007, p. 145, my emphasis in bold)

Let $f(x)$ be a function defined on the interval $(a; \infty)$. Then $\lim_{x\to\infty} f(x) = L$ means that values of $f(x)$ **can be made as close to *L* as we like** by making x sufficiently large.

Further examples of a mathematical gaze can be identified in the text of the Chemical Reaction Problem. Firstly, the formation of the new chemical is modelled using a parabola function, a function from school mathematics and the Foundational Course. As noted in Section 7.2.3 the graphical representation of the chemical reaction is not consistent with its meaning in the task context, yet the reaction as represented by this graph can be modeled using an integral.

The mathematical gaze in these problems points to "a relationship of recontextualization" (Fairclough (2003, p.167) between the mathematical and non-mathematical practice. Aspects of chemistry, epidemiology and the motion of cars have been appropriated into the mathematics texts, and this appropriation has been shaped by the valued mathematical action in the foundational practice, for example, relating variables in a mathematical equation and applying implicit differentiation, using an integral to represent a functional relationship, and evaluating limits. Gellert and Jablonka (2009, p.47) argue that one aspect of the recognition of the mathematical/non-mathematical boundary involves recognizing the esoteric domain knowledge that drives the recontextualization. The analysis in this section, together with that in Section 7.4, suggests that the boundary crossing is not just about recognizing the relevant mathematical object, but also about controlling the movement of meaning in the action on this object across the boundary.

This analysis exposes the representations of the foundational practice as "for something else" (Dowling, 1998, p.9) or "about something other than itself" (p.4) as *myths* (p.24). Foundational mathematics is represented as powerful, since the representation of the flu virus, the cars, and the chemical reaction is done according to the valued mathematical ways of the foundational practice.

Yet the mathematical gaze is not used consistently in the practical problems in this study, for the worked solution $\lim_{t \to \infty} P(t) = 10\,000$ for question (f) locates the explanation in the everyday practice; "Eventually after a long time everyone gets the flu".

Representing the three problems as pedagogic texts in mathematics, with some features of word problems

The analysis presented so far represents the practical problems as mathematical texts. In this section I argue that the problems represent particular genres of mathematical texts. The information provided in the first sentences of a problem is represented as fact with no supporting evidence or references, either in declarative statements without supporting evidence, as "given" by an unidentified authority, or as given in the form of the imperative "let". The imperative "let" is identified by Morgan (1998) as a feature that is common in school mathematics texts, and hides the human voice of the author. The text producer is thus identified as an authority, that is, the giver of the information (Fairclough, 2001).

The initial statements and instructions are followed by interrogative questions beginning with "what" or imperative instructions such as "write down", "explain", "find the equation", and "draw a rough sketch". These instructions and questions ask the reader to act in certain ways, thus representing the texts as pedagogic texts. This representation is reinforced by the repetitive structure of the Flu Virus Problem and Chemical Reaction Problem, with the listing of the instructions and questions as numbered sub-questions. Both the use of imperatives and a repetitive structure are identified by Morgan (1998) as features of school mathematics texts.

Further, as noted in the analysis so far, certain features of the problems represent these texts as belonging to a particular genre of pedagogic texts, that is, the genre of mathematical word problems. Here I provide further evidence to support this argument. In terms of overall structure, the first sentences of each problem can be regarded as a combination of what Gerofsky (2004, p.37) calls the *set-up* of a word problem ("establishing the location and the characters") and the *information* part of a word problem ("the givens" needed to solve the problem). The questions and instructions that follow are the third component of the word problem structure, setting out the goals of the problem (Gerofsky, 2004).

Gerofsky (2004) identifies a number of assumptions that are implicit in the genre of mathematical word problems, for example, that the problem is solvable, and that it is solvable with the available information, that there is only one right answer, that trying to make sense of the task context in terms of everyday understanding is not required, and that the teacher has the authority to decide on the accuracy of a student's answer. These are assumptions in the sense that they are not stated explicitly, but which would be familiar to someone who is familiar with the genre. The practical problems provide some textual cues to these assumptions. For example in both the Flu Virus Question (sub-question (a)) and the Chemical Reaction Problem (sub-question (f)) the definite article "the" in the "the graph" suggests that there is only one possible graph. In the same sub-question of the Flu Virus Problem the student is instructed (in bold text) not to continue with the rest of the sub-questions "**until you have had your graph checked by a tutor**". This text invests the tutor with the authority to evaluate foundational students' answers.

Viewing the practical problems as texts in the word problem genre further exposes as a myth (Dowling 1998, p.24) the referential nature of the foundational practice. For Gerofsky (2004) argues that word problems do not represent real life, but rather imitate other word problems. This suggests that the practical problems can be grouped into certain categories of word problems according to the underlying mathematical ways. For example, the Car Problem is classified as belonging to the group "related rates problems". A task context may play a naming role for a particular category of word problems. For example, in the Foundational Course the lectures talk about problems like the Chemical Reaction Problem as "Joe Problems", a reference to the first problem of this type in the Course which references the earnings and savings of a fictional person called "Joe". In this study the naming of a problem like the Chemical Reaction Problem as a "Joe Problem" is reproduced by the students in Group 2. In her study of word problems in undergraduate mathematics, Gerofsky (2004) refers to an interview with an engineering student who identified certain problems as "train problems" according to similarities in their underlying mathematical structure; "If you're looking at them in a textbook, there'll be one topic that it will cover, say one concept that you have to demonstrate knowledge of" (p.96).

7.3 Discussion: Relationships between foundational practice and other practices

Solving the practical problems requires that students cross related boundaries between (a) school mathematics practice and foundational mathematics, and (b) mathematical and non-mathematical practices. Crossing these boundaries requires, firstly, control over the movement of objects across practices, for example, the movement of the mathematical object "function" from school to foundational mathematics or the movement of a non-mathematical object "disease" into foundational mathematics. Secondly, crossing these boundaries is about reading the genre and the assumptions of the genre, for example, pedagogic texts in school mathematics suggest that the information about the disease should be regarded as unquestionable facts and the genre of mathematical word problems indicates that the unrealistic nature of the disease should not be considered a problem. Thirdly, the boundary crossing requires control over the movement of ways of acting on mathematical objects across the school / foundational boundary. For example, the school textbooks reviewed in Chapter 2 suggest that school students act on functions that are represented by algebraic equations, yet the function in the Flu Virus Problem has no such representation. In his discussion of Basotho students' unfamiliarity with questions requiring explanations in an imported O-level examination, Nyabanyaba (2002) talks about this movement of meaning as a recontextualization of mathematical competencies. I provide further evidence for this particular aspect of the boundary crossing in Section 7.3. Fourth, in support of Walkerdine (2000, p.53), I argue that having control of content, ways of acting mathematically, and genre involves the student controlling the movement in his positioning across practices. For example, the student solving the Flu Virus Problem is positioned as a mathematics student (as opposed to a student of epidemiology) and sometimes he should adopt the style of a school mathematics student but at other times should not act in this way. This example also points to the need for the student to control the timing of the movement of objects, genre, ways of acting on mathematical objects, and positioning.

So crossing the boundary between foundational practice and other practices in the space involves control over *how* to cross a boundary and *when* to cross a boundary.[118] The students in the study by Nyabanyaba (2002) knew when to attend to non-mathematical practice in a summative exam if they were to gain access to higher study; they exhibited agency in choosing not to solve the "deep" realistic items in the examination.

I describe the two boundary crossings represented in the practical problems as "related", for how a non-mathematical practice is operationalized in the foundational practice is related to movement of genre across the school/foundational boundary. For example, the analysis presented so far suggests that the assumptions of the genre of mathematical word problems which students would have experienced at school drives the gaze on the non-mathematical objects.

The representation of foundational practice described here suggests that the problems draw on a number of discourses in the wider socio-political space, discourses that may be contradictory. The problems talk back to school mathematics in terms of mathematical objects, and to some extent with regard to the valued action on mathematical objects. In setting up relationships to non-mathematical practices the problems talk to two representations of the discourse of relevance in reform versions of school and undergraduate mathematics. On the one hand the relationship is represented as being for participation in scientific disciplines other than mathematics and hence the Foundational Course is for non-mathematics majors in science. On the other hand the relationship serves as a motivation for studying mathematics itself, with the foundational student identified as a mathematics student. Yet the practical problems also talk to the word problem genre typical of more traditional pedagogy in both school and undergraduate mathematics.

[118] This argument about boundary crossings in foundational mathematics has the potential to be developed using the concept of *routines* from Sfard (2008), a concept that I have noted is not being used in this study. Sfard (2008, p.215) argues that a routine is defined not only by the *how* or "the course of actions that they prescribe", but also the *when* of following this course of actions.

I have argued that the practical problems represent foundational mathematics as being powerful in the sense that it either empowers the student to act in non-mathematical practices or that it casts a gaze on these non-mathematical practices. I revisit this discussion in Section 7.9 where I consider the power between this practice and other practices in the wider order of discourse.

7.4 The valued mathematical action in the foundational practice

In this section I develop the argument presented in Section 7.3 that the boundary crossing between school and foundational mathematics involves control over the movement in ways of acting mathematically on objects. This is the action required to produce the worked solutions of the problems. This description draws on the textual analysis of the practical problems and accompanying worked solutions as well as on the analysis of the actual interpretations of the practical problems by participants in the space of undergraduate mathematics (as described in Section 6.2.2). A full description of the valued mathematical action for each problem is provided in Appendices B to D.

7.4.1 Looking at mathematical objects structurally and operationally

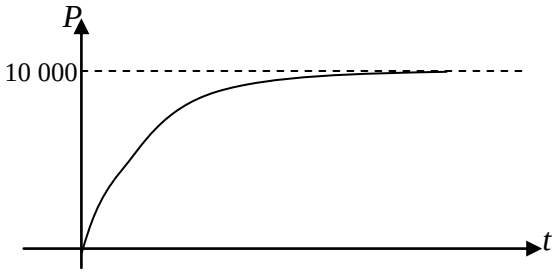
In this section I present three examples to illustrate the varied ways of attending to mathematical objects in the practical problems. Representing the functions graphically in the Flu Virus Problem (question (a)) and the Chemical Reaction Problem (question (f)) requires that the student adopt an operational view of the functions and consider what is happening in the task context over time. This can be contrasted to selecting a static graph like a "straight line graph" or an "exponential graph", which suggests a structural view of a function (Tall, 1992, citing Schwingendorf & Dubinsky, 1990). I illustrate this argument by presenting the valued mathematical action of the fictional student Mihlali on the Flu Virus Problem (Extract 7.1). In viewing the function $P(t)$ operationally Mihlali identifies the properties of the graph as the value of the independent variable increases.

Extract 7.1: The valued mathematical action on question (a) of the Flu Virus Problem

Firstly, Mihlali attends to the functional relationship $P(t)$ and identifies this with its meaning in the task context, that is, the number of people who have or have had the disease at time t. He adopts an operational view of the function as he considers what happens as the disease spreads over time. Initially (at time $t = 0$ days) no-one has the disease, so (0,0) is a point on the graph. Since there are 10 000 people in the community and "sooner or later" everyone catches the flu, the graph of $P(t)$ reaches a maximum value of 10 000 at some time t. Since $P(t)$ is by definition the number of people who have or have had the disease at time t, the graph of $P(t)$ is always increasing.

Mihlali also attends to a second functional relationship, that is, the derivative function $P'(t)$. He identifies the relationship between the two functions; $P'(t)$ is the rate of change of the function $P(t)$. He also links the two functional relationships graphically, that is, the function $P'(t)$ represents the gradient of the graph of $P(t)$. Linking both functions to their meaning in the task context, Mihlali identifies $P'(t)$ as the rate of change of the number of people who have or have had the disease with respect to time.

He adopts an operational view of the derivative function and attends to what happens to this rate of change as the disease spreads over time. Initially, there are many people who can catch the disease, so the rate at which people are catching the disease is high (and linking the functions graphically, the gradient of the graph of $P(t)$ is steep). But as time passes, more people have caught the disease and there are fewer people to catch it, so the rate $P'(t)$ decreases (and the gradient of the graph is less steep). This way of looking allows Mihlali to identify the graph of $P(t)$ as concave down.[119]



[119] I note at this point that, at the time the students solve the Flu Virus Problem in the Foundational Course, they have not yet formally studied the second derivative of a function and its relationship to the concavity of the function. So the operational view of the function and the links to the task context as described here in the work of Mihlali are necessary for solving the problem.

The second example refers to using "practical terms" to explain the meaning of the equation $\frac{P(7)-P(4)}{7-4}=350$ in the Flu Virus Problem (question (d)). Firstly, it can be argued that this representation encourages a structural view of the equation. The equal sign in the equation is used to identify an identity rather than to signal that an operation should be performed on the objects, thus pointing to a static relation between the expressions (Sfard, 1991). Furthermore, in order to identify the expression $\frac{P(7)-P(4)}{7-4}$ as an average rate of change the student should look structurally at the expression as an object.

However, in order for the student to explain the meaning of this object in "practical terms" and without using the word "rate", he should look operationally at the object and consider the meaning of the separate parts, as suggested by Mihlali's approach in Extract 7.2.

Extract 7.2: The valued mathematical action on question (d) of the Flu Virus Problem

> He begins by looking at the symbols on the denominator of the expression and links these to their meaning in the task context; the subtraction on the denominator indicates the three-day time period from t = 4 days to t = 7 days. He then looks at the numerator and identifes the subtraction as representing the change in the number of people who have or have had the disease over this three year period. Attending to the division in the expression $\frac{P(7)-P(4)}{7-4}$ enables him to identify the expression as representing the daily change in the number of people who have or have had the disease.

The final example is related to evaluating the limit $\lim_{t\to\infty} P(t)$ in question (f) of the Flu Virus Problem. Gray and Tall (1994) argue that a limit expression such as this can be viewed both as a process of tending to a limit and as the value of the limit itself. In the Flu Virus Problem the student can view the object $\lim_{t\to\infty} P(t)$ structurally and identify this object as the limit at infinity of $P(t)$, and in terms of the task context, the number of people who "sooner or later" have or have had the disease. Yet this question can also be answered by adopting an operational view of the limit and considering what happens to the function as time

passes (enabled by looking at the graph of $P(t)$); as time passes the number of people who have or have had the disease increases and will eventually reach 10 000.

Sfard (1991) argues that mathematical objects can be viewed both structurally and operationally and that both ways of looking are necessary for understanding in mathematics. The analysis of the practical problems suggests that they afford opportunities for the student to look both operationally and structurally at mathematical objects, but that certain questions encourage an operational view, while others can be solved by viewing the object either structurally or operationally thus providing the student with more than one way of acting.

7.4.2 Operating on mathematical objects

Certain parts of the practical problems require that the foundational student operate on mathematical objects. The boxed text of the Car Problem constructs certain operations as valued, for example, point 5 instructs the student to "differentiate" (meaning implicit differentiation) and "complete the question" (which involves substitution and rearranging the subject of the formula). However, I will argue in Section 7.4.3 that successful completion of the Car Problem requires more than these valued operations.

The Chemical Reaction Problem values certain operations on objects, and when this is required may or may not be made explicit. In question (a) the student identifies that the total mass of the product in the reaction chamber can be obtained by subtracting the amount removed from the amount formed. The amount removed can be represented using the integral $\int_0^t 3.dt$ and this object can be operated on to produce the anti-derivative $3t$. However, since there is no formula for the function $m'(t)$ (as yet), the integral $\int_0^t m'(t).\,dt$ cannot be operated on and is viewed structurally as the amount formed. The final solution can be obtained by subtracting the two objects and using the law for the sum of integrals to

obtain $\int_0^t m'(t)\,dt - 3t$, an expression that is viewed structurally as representing the total mass of the product in the reaction chamber.

The Chemical Reaction Problem contains textual cues that indicate whether the student must act operationally or not. In question (e) the word "hence" provides a link to an earlier question (in this case (d)), suggesting that the student operate on the algebraic formula for the parabola when calculating the required mass. The ways of operating in this case involve finding anti-derivatives and substituting. However, in question (f) the student should not operate on the cubic equation of the function $m(t)$ when sketching the graph of the function. This is indicated by the instruction to draw a "rough sketch" (which takes on a particular meaning in the Course) and to attend to the given time values.

The Flu Virus Problem does not require the student to operate on mathematical objects, since no algebraic formula is provided for the function $P(t)$. Graphs and numerical answers can only be obtained by making links between the mathematical objects and the task context, as described in Section 7.4.3.

Gerofsky (2004) identifies one of the assumptions underlying word problems as that they are provided to practise an algorithm recently presented in the mathematics course. Yet the analysis in this section indicates that the operation on mathematical objects is valued in some but not all parts of the practical problems and that some answers can only be obtained by setting up a relationship between mathematical objects and the task context. The practical problems thus draw selectively on the school mathematics and the genre of word problems in this practice.

7.4.3 Moving within mathematical practice and between mathematical and non-mathematical practice

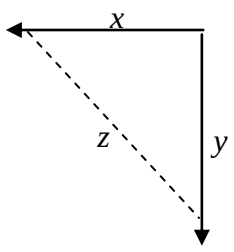
In this section I present evidence to suggest that answering the three practical problems, including those parts that give value to operating on mathematical objects, involves (a)

moving within mathematical practice and making links between mathematical objects and their representations, and (b) moving to-and-fro across the boundary between these mathematical objects and the task context.

Firstly, I refer to the valued mathematical action in Extract 7.1. Mihlali does not have an algebraic formula with which to construct the graph of the function $P(t)$ in question (a) of the Flu Virus Problem. Sketching the graph (using an operational view of the function, as I argued in Section 7.4.2) requires that the student work with a number of relationships concurrently. Firstly, the relationship between the function and the derivative function, that is, $P'(t)$ is the rate of change of the function $P(t)$. Secondly, the relationship between the graphical representations of these functions, that is, $P'(t)$ represents the gradient of $P(t)$, Thirdly, the relationship between the two functions and their meaning in the task context, that is, $P(t)$ represents the number of people who have or have had the disease at time t, and $P'(t)$ represents the rate of change of the number of people who have or have had the disease at time t. In addition, the movement between the mathematical practice and the task context is a to-and-fro movement as Mihlali uses the spread of the disease over time to identify the property of the graph of $P(t)$ at different times.

My second illustration focuses on Mihlali's action on the Car Problem (Extract 7.3). In Extract 7.3a Mihlali identifies the cues in the task context and uses this to draw a mathematical diagram.

Extract 7.3a: The valued mathematical action on the Car Problem



Mihlali identifies that the "diagram" required in point 1 of the boxed text is a mathematical representation that includes only the necessary information for solving the problem. In constructing this diagram, he attends to two features of the task context; the "same point" as the starting point for the two cars and the direction of travel ("south" and "west"). When deciding how to "define variables" in point 1 he considers what is changing in the task context, that is, the distance of each car from the starting point and from one another. Mihlali uses this information to draw a right-angled triangle

(the right angle is at the starting point), with the hypotenuse representing the distance between the cars at a particular time and the other two sides representing the distance travelled by each car from the starting point at a particular time. He attends to the hint in Sentence 4 of the Car Problem to use the variable *z* as the distance in km between the two cars after time *t*, and labels the hypotenuse z. Linking to what is usually done when solving related rates problems in the Course, he assigns the variables *x* and *y* to the other two sides of the triangle.

This action to "undress" (Gerofsky, 2004, p.33) the algebraic formulation that has been "'dressed up' in words" (p.33) is valued when solving mathematical word problems. However, I argue that the valued ways of operating in the Car Problem are enabled by the student making links to the task context at other key moments during the problem-solving process, and not just for identifying the appropriate formula $x^2 + y^2 = z^2$ in point 4. For example, identifying that the implicit differentiation of this formula should be performed with respect to time requires that the student make the link between the speed in km/h given in the problem text, the rate of change of distance (x) with respect to time and the symbolic notation $\frac{dx}{dt}$. These links also signal to the student what numbers can be substituted into the derivative equation $2x.\frac{dx}{dt} + 2y.\frac{dy}{dt} = 2z.\frac{dz}{dt}$ and the original Pythagorean equation $x^2 + y^2 = z^2$ when "completing the question" (Extract 7.3b).

Extract 7.3b: The valued mathematical action on the Car Problem

Having completed the implicit differentiation, Mihlali again revisits what he wrote under "To find". In point 5 of the boxed text, "completing the question" involves operating on the derivative equation $2x.\frac{dx}{dt} + 2y.\frac{dy}{dt} = 2z.\frac{dz}{dt}$ by substituting values into this equation, and rearranging the subject of the formula to solve for $\frac{dz}{dt}$. Mihlali attends to the time of 2 hours. Since the speed at which the cars are travelling is constant, he substitutes 75km/h and 100km/h for $\frac{dx}{dt}$ and $\frac{dy}{dt}$. He cannot identify values for the distances *x*, *y* and *z* directly in the problem text, but he makes a link to the task context to calculate the distances *x* and *y* after 2 hours; the car travelling at 75km/h will travel 75km in 1 hour and 150km in 2 hours. Mihlali then uses his

Pythagorean equation $x^2 + y^2 = z^2$ to calculate the distance z. When he has made $\frac{dz}{dt}$ the subject of the formula and used his calculator to arrive at a final answer, he makes a link to the task context to identify the units as km/h.

This representation of a to-and-fro movement between the mathematical and non-mathematical practice in the practical problems contrasts with literature reviewed in Chapter 3 that represents this movement as a one-way movement from the horizontal non-mathematical practice to the mathematical practice. This one-way action is captured in the focus on "vertical growth in mathematical ideas" (Harel and Kaput, 1991, p.93) in advanced mathematics, in the notion of vertical mathematization (Treffers, 1987, p.247), in the description by Gerofsky (2004) of having to "uncover" (p.33) the mathematical form of a problem, in descriptions of certain students getting "stuck" in the task context of a practical problem (Lubienski, 2000, p.477; Straehler-Pohl, 2010, p.454) and in the description by Dowling (1998) of the movement from the initial "interpellation" (p.141) of the student in the task context proceeding "via the construction of metonymic chains" (p.141) into the mathematical, that is, the esoteric domain. This analysis also points to the limitations of the classification of students' language as either contextualized or generalized (Lubienski, 2000, p.467), as the analysis presented here suggests that both forms of language are required as the foundational student moves to and fro across the boundary.

Arcavi (2002), on the other hand, points to a to-and-fro movement in his argument that mathematization and contextualization are complementary processes. However, for Arcavi (2002) the task context is one that the student can "remember, imagine, or even fabricate" (p.22), whereas in this study the task context remains the same. As noted in Section 3.4, a to-and-fro movement between the mathematical practice and the same task context is presented in the description of practical problems in calculus reform by Garner and Garner (2001). Yet the focus of the research on calculus reform means that the nature of this movement and how it might differ from the valued movement across the boundary in other mathematical practices is not foregrounded.

7.4.4 Locating arguments in both mathematical and non-mathematical practices

In this section I consider the three types of explanations required in the practical problems. Firstly, certain questions in the Flu Virus Problem explicitly instruct the student to explain why a value has been assigned to a particular mathematical object, for example, to explain why the derivative function is positive in question (e) and to explain the evaluation of the limits in questions (f) and (g). The worked solutions suggest that these explanations are located in the task context. Using the task context to endorse an argument may require a specifically mathematical gaze as in question (g), which foregrounds the intuitive definition of the limit used in the Foundational Course and at school rather than an everyday definition of a limit. However, the argument may not require a mathematical gaze, as in the limit in question (f).

Secondly, the Course material that sets out how the foundational students should interact in small groups indicates that explanations are valued as part of the educational talk of a learner-centred pedagogy, that is, students should be "explaining answers" (Foundational Course Resource Book, 2007, p.16) and "asking for further explanations" (p.16) even when this is not explicitly stated in the practical problem. In Section 7.4.3 I have argued that solving the three practical problems requires that the student make links between the mathematical objects, their representations and their meaning in the task context. This means that the arguments in support of the solutions are supported by these relationships. For example, in Extract 7.1 Mihlali considers the concavity of the graph of $P(t)$. He recruits the task context to argue that initially there are many people in the community to be infected resulting in the rate of infection being high. Yet the explanation for the gradient of the graph of $P(t)$ being steep is endorsed using the mathematical relationship between the function and its derivative and their graphical relationships, that is, the rate of change $P'(t)$ of the function $P(t)$ is the gradient of the graph of $P(t)$.

Thirdly, both the Flu Virus Problem and the Chemical Reaction Problem require the student to explain the "practical" meaning of objects. As noted, this term is used in the

Foundational Course to signal that the student must not use any mathematical terminology, suggesting that the task context should be recruited for endorsement. In Section 4.5.3 I have named the way of writing or talking in such an explanation, the *practical terms genre*. This genre can also be identified in calculus reform textbooks such as Hughes-Hallet et al. (1994). In the terms used by Dowling (1998), the expression in such an explanation is weakly classified.

The three types of explanations described in this section suggest that the support for these arguments lies in the relationships between the mathematical objects, their representations and the task context. This is not the deductive reasoning based on mathematical definitions and theorems that is valued in academic and advanced mathematics (Morgan, 1998; Sfard, 2007, 2008). Drawing on Adler (1997) it is possible to identify three ways of interacting in discourse in the foundational practice; writing answers using English (the medium of instruction), writing answers using the educated ways of writing and representing in the foundational practice, and interacting in small groups using the educational ways of talking in a learner-centred pedagogy. Yet the analysis in this section points to a fourth way of interacting in discourse, that is, talking and writing about mathematical objects using the practical terms genre.

7.4.5 Moving between mathematical practices and events

I have argued that the practical problems talk back to school mathematics for objects, action on these objects, genre and positioning. However, this talking back is selective and the foundational practice may represent discontinuities in its relationship to school mathematics. Answering question (d) of the Chemical Reaction Problem requires the student to draw on his knowledge of quadratic functions from school mathematics, as Mihlali does in Extract 7.4.

Extract 7.4: The valued mathematical action in question (d) of the Chemical Reaction Problem

> The words "parabola part" provide a link to Sentence 3 where it is stated that the graph is a parabola up to the point $t = 4$. Reading about the "quadratic function" and the "parabola" in this question cues Mihlali to make a link to school mathematics (and work that is revisited in the first weeks of the Course). He identifies two possible quadratic functions from school and links each to the points of a parabola graph; the equation $y = a(x - x_1)(x - x_2)$ is used when the x-intercepts and one other point on the graph are given, the equation $y = a(x - p)^2 + q$ is used when the co-ordinates of the turning point and one other point on the graph are given.

Yet other questions in the practical problems, while talking about the same objects as those used in school mathematics, do not require the student to draw on the valued ways of acting on these objects in that practice. For example, I have noted that both the Flu Virus Problem and the Chemical Reaction Problem do not require the student to use an algebraic formula for a function to identify the properties of the required graph. These problems also do not require the student to name the graph as a static graph from school mathematics.

The practical problems mainly represent continuity in the movement between social events within the practice of foundational undergraduate mathematics itself, for example, lectures or other Course material. I have already suggested that the term "rough sketch" in the Flu Virus Problem and the Chemical Reaction Problem takes on a particular meaning in the Course, that is, the student should attend to the shape of the graph (increasing/decreasing and concavity) and the important points cued in the text, for example, the maximum value of 10 000 in the Flu Virus Problem and the points $t = 2$, 4 and 5 hours in the Chemical Reaction Problem. The student is not required to plot the graph point-by-point and to scale as might be required in school mathematics. Both these problems require the student to use the practical terms genre, and as stated to students in lectures, not using words like "rate", "rate of change", "instantaneous", and "derivative". However the practical terms genre is not used consistently in the practical problems, for example, the worked solutions to questions (c) and (d) of the Chemical Reaction Problem make use of the word "rate".

The boxed text that precedes the actual Car Problem text sets out the steps to be followed when solving a "related rates problem". Prescriptions such as this for solving such problems can also be found in undergraduate calculus textbooks, for example in Stewart (2006), the prescribed textbook for both the Foundational Course and the mainstream first- and second-year courses at the time that this study was conducted. The worked solutions for the Car Problem place value on following the problem-solving steps and operating on functions using implicit differentiation, in spite of the fact that it is possible to solve this problem intuitively, without using calculus. This suggests that the Car Problem is drawing on the genre of mathematical word problems in undergraduate mathematical practice. The problem- solving steps represented in the boxed text also set up a link to the steps for solving motion problems in school and introductory undergraduate physics. In this study a student identifies such a link to these pedagogic practices in physics when he likens the problem-solving steps to "the Physics approach" (Jeff, action of Group 1 of the Car Problem, line 14).

7.4.6 Following textual links within the text of a practical problem

The Chemical Reaction Problem and the Flu Virus Problem differ structurally from the Car Problem (excluding the boxed text) in that the former two problems contain a number of short sub-questions while the Car Problem addresses one question to the student. Yet in this section I will identify a variety of textual features that provide implicit support to the student in answering the sub-questions or main question. These features also provide coherence to the description of the task context.

The action of answering the main question in the Car Problem is given meaning by the position of this problem immediately after the boxed text and the naming of the problems that follow this boxed text as "related rates problems". This textual link in the boxed text is followed by an assertion in bold and upper case that "**EVERY**" question should be answered using a set of five steps. So the action of solving the main question is constructed in a sequence of instructions about what to draw, what to write down, and what operations to perform.

The sub-questions in the Flu Virus Problem and the Chemical Reaction Problem are sequenced and linked (either implicitly or explicitly) in such a way that they provide scaffolding for the student. In the Chemical Reaction Problem the word "hence" in question (e) suggests that the student should draw on earlier answers. The repetition of the wording "**total** mass of product X" suggests a link to the integral produced in question (a), but without the removal of the product, and this integral suggests that the equation of the function $m'(t)$ derived in question (d) can be used. In question (f) the instruction to draw a graph of $m(t)$ that shows the points $t = 2$, 4 and 5 hours sets up links to question (c), the graph of $m'(t)$ where these values for time are labeled, and the text in Sentence 3.

Yet in other cases the link between the individual sub-questions is less explicit, for example, in the Flu Virus Problem students answer the limit questions (e) and (f) with reference to the graph in (a), but there is no textual cue (other than the sequencing) to do so.

The textual links do not only refer to relationships between sub-questions in these two problems, but also links within sentences and across sentences. Some links provide coherence in the narrative of the task context. For example, the repetition of the name "product X" in Sentence 2 of the Chemical Reaction Problem provides an explicit link between the product described in this sentence with the chemical described in Sentence 1. The narrative about the spread of the flu virus in the community in the first four sentences of the Flu Virus Problem is linked using the interchangeable terms "the disease", "the flu virus" and "the flu", and the reference pronoun "it".

Certain textual links are required to set up relationships between mathematical objects, their representations and their meaning in the task context. For example, in Sentence 2 of the Chemical Reaction Problem the punctuation symbol "," and the conjunction "where" after the first clause introduce a second clause that elaborates on the function notation $m'(t)$. The second clause "where m is the mass of the product formed, in grams," identifies the symbol m in $m'(t)$ with their meaning in the task context. The additive conjunction "and",

also in Sentence 2, is used to identify the symbol t in the function notation $m'(t)$ with its meaning in the task context and the units of measurement. While the repetition of the symbols m and t in this sentence provides an explicit link to the function named $m'(t)$, the student is still required to make use of the conjunctions in the text in order to identify this link.

The derivative function $m'(t)$ is linked to its graphical representation through repetition of the function notation $m'(t)$ in Sentence 3 and as a label on the vertical axis of the graph, and also by the proximity of the Sentence 3 to the graph below. The textual description provided in Sentence 3 also links the "graph of $m'(t)$" to the graph below; firstly students are required to link the word "parabola" to the shape of the graph from $t = 0$ to $t = 4$, and secondly to identify that the wording "after which it is zero" is telling them that after time $t = 4$ ("which") the graph ("it") has equation $m'(t) = 0$ (which in turn has to be linked to the horizontal line on the t-axis of the graph below the text).

Adler (1997) identifies access to the language of learning as one of the three aspects of language at play in a multilingual classroom in which a learner-centred pedagogy is given value. The analysis presented in this section suggests that all foundational students, including those for whom English is a home language, are required to read (in the language of instruction) cues suggested by textual features such as repetition, renaming, reference pronouns, conjunctions, and sequencing .

7.4.7 Interacting with other students and the tutor

In this section I draw on Foundational Course material more broadly to discuss how the student is expected to interact with other students and with the tutor when solving the practical problems. As noted in Section 2.4.4, the Resource Book contains a list of mathematical content to be studied in the Course, a list that identifies the student as studying the mathematical objects of calculus. In addition, the Course material for the first workshop of the academic year foregrounds educational ways of talking that are typical of a

learner-centred pedagogy, for example, "making suggestions about strategies to solve a problem", "explaining answers", "asking questions about solutions", and "encouraging one another to keep going / to participate" (Foundational Course Resource Book, 2007, Workshop 1, p.16). These expectations about how to act in a foundational workshop classroom are explicit, identifying the foundational student as needing to be told how to behave in the workshop. The Foundational Course material also recruits student comments from evaluations of the course in previous years to reinforce the value of the learner-centred interaction:

> Here are some past students' comments about the workshops:
> "We get help from tutors and out fellow classmates when we experience problems. Discussing different problems helps a lot."
> "… you get to discuss your answers with other people and you help each other so it's good. …"
>
> (Foundational Course Resource Book, 2007, Workshop 1, p.16)

This explicit attention to how a foundational student should behave in a workshop can be contrasted with introductory material in the first- and second-year mainstream mathematics courses (as described in Sections 2.4.5); the material for the latter courses explicitly names the mathematical content in the course but there is an absence of explicit instructions on how to behave in the undergraduate mathematics classroom. This course material identifies the mainstream student as a mathematics student, working independently and taking responsibility for his own behaviour.

In describing the valued interaction between the student and the tutor in the Foundational Course I draw on training material that I used (in my role as convenor of the Course) with the tutors in the year in which the study was conducted. This material identifies the tutor in a number of roles. Firstly, by "answering questions" the tutor is an authority in the practice of foundational mathematics and the educated ways of talking, writing and representing in this practice. Secondly, the tutor is identified as a facilitator in a learner-centred pedagogy, for example when interacting with students seated in a small group the tutor should be "asking questions" and "listening to explanations by students/discussions between

students". Adopting the style of a facilitator in a learner-centred pedagogy may involve instructing a student how to behave, as suggested by the training material; "working on the productivity of the groups (e.g. encouraging students to compare answers)". Thirdly, a tutor on the Foundational Course is given responsibility for running a workshop for thirty students over a full semester, identifying the tutor as responsible for other forms of student behaviour, for example, dealing with students who arrive late, controlling the pace at which students work through the prescribed problems for an afternoon workshop, deciding which students may leave the workshop early, etc.

This valued interaction for students and the tutor represented in this section differs from the students' descriptions of their school experience (as described in Section 2.3.5) and also differs to what is valued in the more traditional pedagogy adopted for tutorials in the mainstream first- and second-year courses (as described in Section 2.4.5). In the mainstream the tutorial groups are larger than the workshop classes in the Foundational Course and may take place in tiered venues. Each large tutorial group is supported by a group of tutors (sometimes with the support of a lecturer), and the tutors "answer questions" about undergraduate mathematics. The mainstream tutors are not given the responsibility to develop productive group skills or to regulate the pace and attendance of the students.

The analysis in this section suggests, like Swanson (2005), that foundational and mainstream students (in this case at undergraduate level) are construed differently by the discourses in the educational space.

7.5 Discussion: The valued mathematical action in the foundational practice

The analysis of the valued mathematical ways in Section 7.4 supports the argument that crossing the school/foundational and mathematical/non-mathematical boundaries requires control over the movement of objects, ways of acting mathematically on these objects, genre, and positioning. The identification of the continuities and disruptions in the

movement from school to foundational mathematics points again to the need for control over the timing of this movement. For example, when solving the Car Problem the student should adopt the style of a school mathematics student solving a word problem, using the cues in the text to construct a mathematical representation and operating on the mathematical objects, while not attending to the unrealistic nature of the task context. However, the student should act like a student in a calculus reform course and move to and fro between the mathematical objects, their representations and the task context when deciding how to operate on these objects.

The analysis in this section also suggests that movement of meaning occurs not only between practices, but also between social events within the foundational practice and texts used in these events (mainly as both continuities but with some disruptions) and within the text of a practical problem itself. The presence of continuities and disruptions suggests that "text-connecting" may not necessarily cohere in the way that produces mathematical meaning in the manner suggested by Chapman (1995).

While the problems may talk to school mathematics and calculus reforms at undergraduate level, the valued mathematical ways described in Section 7.4 raise questions about the extent to which the foundational practice talks forwards to advanced mathematics. Firstly, the to-and-fro movement between the mathematical and non-mathematical practice (as opposed to a one-way vertical movement) represents a disruption in the representation of this movement in the literature on advanced mathematics. Secondly, questions that require the student to use the practical terms genre do not provide opportunities to move into the esoteric domain of undergraduate mathematics. This is an argument made by Gellert and Jablonka (2009) but in relation to the esoteric domain of school mathematics. Thirdly, the opportunities to solve the practical problems using an operational view of mathematical objects suggest that the student may not have sufficient opportunity to look structurally at mathematical objects as required in advanced mathematics. Fourth, the valued arguments in the practical problems are located in the relationship between mathematical objects, their

representations and the task context, rather than in the definitions and theorems of abstract mathematics.

Yet while the foundational practice represents disruption in its relationship to advanced mathematics, it represents continuity in a key respect. The representation of the foundational practice in Sections 7.2 and 7.4 points to the complexity of this practice, and that participation in this practice requires flexible action on the part of the student in knowing how and when to make boundary crossings across practices (both mathematical and non-mathematical), social events, and texts. This action is not about having the connected mental representations and flexibility to move between these representations that the ontological/psychological research on advanced mathematics talks about (e.g. Dreyfus, 1991), but having flexibility in terms of controlling the movement of objects, ways of acting mathematically, genre, positioning and timing in a variety of boundary crossings. Sfard (2008) argues that creativity and new developments in mathematics occur when "a familiar course of action is transplanted into new discursive contexts" (p.221). This suggests that a participant in the practice of advanced mathematics moves flexibly across boundaries, or in the words used by Sfard (2008), is able to "step in and step out of discourses" (p.220). The analysis of the foundational practice presented in this section suggests that this practice provides the foundational student with the opportunity to act flexibly in this way.

This analysis suggests that the problems draw on a number of discourses in the wider socio-political space, and again in ways that may be contradictory. The significance given to a learner-centred pedagogy points to similar reforms in school and undergraduate mathematics. Yet the movement of meaning across the mathematical/non-mathematical boundary is driven by both the word problem genre (a one-way movement) and the discourse of calculus reform (a to-and-fro movement). The boxed text for solving the Car Problem draws on mainstream undergraduate texts, but the learner-centred pedagogy and the explicit instructions in the Course material about how to behave suggest that

foundational students are different to mainstream students. The contradictory positioning of the student is discussed further in Section 7.6.

7.6 The construal of social identities and social relationships

The representation of foundational practice as described in Sections 7.2 and 7.4 points to the positioning of the subjects (the text producer, the tutor, and the student) and the valued social relations between these subjects. In this section I develop the discussion presented so far and attend to the power in discourse as represented in the problems.

7.6.1 Positioning of the text producer and the tutor

In Section 7.2.4 I argued that the producer of a pedagogic text in mathematics (and by implication the lecturer) is positioned as an authority in the foundational practice. For example, in the set-up and information part of a practical problem the producer controls what is said about the non-mathematical practice and how the non-mathematical objects are talked about. The text producer also controls the valued mathematical action by giving instructions and questions that specify how the foundational student should act and by producing worked solutions. The relationship of power between the text producer and the student is thus an asymmetrical one.

As noted, the Flu Virus Problem sets up the tutor as an authority who can evaluate a student's graphical representation. In this case, the tutor is identified as an authority in the educated ways of representing in the foundational practice. In addition, the tutor training for the Foundational Course identifies the tutor as an authority in promoting the educational ways of talking in a learner-centred pedagogy as well as an authority who makes other aspects of behaviour explicit, for example, how fast students should be working and what students should be talking about at different times, what operations they should be using, etc. The relationship of power between the tutor and the student is thus an asymmetrical one.

However, the valued style of a tutor interacting in the manner of a facilitator in a learner-centred pedagogy and the agency assigned to the student in this pedagogy suggests a symmetrical relationship between the tutor and the student. On the one hand the student is "asking questions about solutions", "asking for further explanation" and "encouraging one another to participate" (Foundational Course Resource Book, 2007, p.16), while on the other hand the tutor training material identifies the tutor as "asking questions", "asking the student to explain the solution" and "involving other members of the group in the discussion".

7.6.2 Positioning the foundational student as different from a mathematics major in the mainstream

The text producer as authority addresses the individual student personally as "you" when instructing him to have his graph checked in Flu Virus Problem. This instruction also assigns this student responsibility for his graph which is named as "your graph". However, elsewhere in the practical problems the student is not named, but assumed to be the recipient of the information and the instructions, a recipient who accepts the information about the task context unquestioningly and follows the instructions.

In all three practical problems certain text is highlighted using bold, underlining or upper case text. In the Flu Virus Problem bold text emphasizes that he must not proceed until his graph has been "**checked by a tutor**". In the Chemical Reaction Problem the word "**rate**" in bold emphasizes that the notation $m'(t)$ and the graph represent the derivative function, and the naming of the graph as a "**parabola**" graph makes a link to a class of functions studied in school mathematics. Bold text is also used for the words "**total**" and "**in**" to foreground the difference between finding the total mass in the reaction chamber and the total mass formed during the reaction. As noted, the problem-solving steps in the boxed text before the Car Problem also feature in mainstream undergraduate calculus textbooks, identifying the student as an undergraduate mathematics student. Yet the use of the underlined, upper case and bold text in these instructions distinguishes this Course material from the text in these textbooks.

Bracketed text provides hints and reminders to the student. In the Car Problem the introduction of the variable *z* in the bracketed text provides a hint on how to represent the task context mathematically, and through the use of the variable *z*, provides a link to other similar problems solved in the Course. The bracketed text in question (c) of the Flu Virus Problem serves as a reminder to the student of what is meant by "practical meaning" in the Course, suggesting that the student might not do this correctly without such a reminder.

These textual features provide scaffolding for the foundational student by reminding him to act in a certain way. Yet these features also position him as someone needing a reminder to act this way and needing additional hints that are not given in mainstream undergraduate mathematics. This positioning of the foundational student reproduces the discourse about these students in higher education; although the foundational student has gained formal access to higher education, he needs different support to a student in the mainstream. This support takes the form of additional reminders and hints. In his analysis of school mathematics texts, Dowling (1998) argues that a text that identifies a student as having difficulty following instructions constructs him as low ability. Swanson (2005) analyzes a broad set of texts at an independent school in South Africa and points to the positioning of foundational students as different to those in mainstream and as low ability in terms of school mathematics.

The Foundational Course material contains, along with a list of mathematical content to be covered, explicit instructions on the valued interaction when the foundational student interacts with others in small groups. This positions the student as needing explicit instructions on how to behave. In Section 7.4.7 I have argued that such social regulation is absent in the course material of the first- and second-year mainstream courses in which students are given a list of mathematical content but are expected to act independently. In contrast, the boxed text in the Car Problem sets up a relationship between the foundational student and students in the first-year mainstream course who learn problem-solving procedures and solve related rates problems using implicit differentiation. Yet this positioning is in itself contradictory as the foundational student is further identified as

needing a reminder to use the prescribed steps and who needs to practise the problem-solving steps rather than rely on his intuitive methods.

I have argued that the practical problems represent some discontinuity in the relationship to advanced mathematics in the opportunities afforded the foundational student to act operationally rather than structurally and the need to move to and fro between mathematical and non-mathematical practice, rather than move vertically into the esoteric domain. These discontinuities suggest that the foundational student is not positioned as a potential mathematics major. Rather, he is positioned as having an interest in non-mathematical scientific practices such as chemistry and epidemiology and thus as a science student needing a course in mathematics for degree purposes. Yet this identification of the student as a science student is in itself contradictory as the science student is not positioned as participating in these non-mathematical scientific practices. This analysis suggests, like Dowling (1998) with reference to school mathematics, that the practical problems may deny epistemological access to the practice of advanced mathematics and access to the non-mathematical practices referenced in the problems.

7.6.3 Identifying the student as having agency and as a boundary-crosser

In contrast to the identification of the foundational student as needing reminders, hints and explicit instructions on how to behave, the introductory Foundational Course material identifies this student as displaying agency in term of controlling his own learning, for example, he is expected to work "consistently" and to make "suitable use of feedback" (Foundational Course Resource Book, 2007, p.5). In addition, as noted in Section 7.6.1, the foundational student is assigned some of the same tasks as the tutor when interacting with his peers in the small group. He is expected to display agency by supporting the learning of his peers, for example, by "making suggestions about strategies for solving problems", "criticizing **ideas**", "encouraging one another to keep going", etc. (Foundational Course Resource Book, 2007, p.16, emphasis in the original).

Furthermore, the foundational practice as described in Sections 7.2 and 7.4 positions the powerful foundational student, that is the participating student, as controlling the how and the when of multiple boundary crossings. This is a complex movement since it represents both continuities and disruptions in the movement of meaning across practices, events within the foundational practice, and within the texts of the practical problems. The flexibility required of the foundational student in controlling this movement identifies this student with the style of a participant in advanced mathematics.

7.7 Discussion: The construal of social identities and relationships

In Section 7.6 I argued that the foundational student is positioned in contradictory ways by the practical problems. On the one hand the student is positioned as lacking power in the practice of undergraduate mathematics and thus needing support that is different from that offered to mainstream students. This support is characterized by explicit instructions and reminders to act in certain ways. In this representation the practical problems reproduce the view in higher education that foundational students should be segmented out of the mainstream for different support. On the other hand the practical problems appear to draw on a view in higher education that the foundational students have the potential to succeed in higher education and are capable of taking responsibility for and of controlling the complex movement across practices, social events and texts.

From a situated theoretical perspective, Boaler (2002a) argues that the pedagogy of a practice shapes the knowledge and identities that are developed in that practice. In her empirical work Boaler (2002a) juxtaposes the knowledge and identities produced in what she calls reform and traditional pedagogies (p.3). Yet the analysis presented in this section suggests that the practical problems position the foundational student in contradictory ways, and that the identities construed in the two pedagogies are not as clear cut as suggested by Boaler. The question that Boaler's empirical work does not answer is, what knowledge is developed in a practice that identifies the student in contradictory ways, and

what are the implications for the student's transition to other practices such as advanced mathematics?

7.8 Summary of the argument so far

The analysis of the three practical problems in this chapter points to what it means to participate in the practice of foundational mathematics. In this section and Section 7.9 that follows, I begin to build an argument that answers my research questions and which also serves as a reference for Chapters 8 to 11 where I present the analysis of the student action in the foundational practice.

Boundary crossing in the foundational practice involves, firstly, knowing *how* to boundary cross; this involves controlling the movement of objects (both mathematical and non-mathematical), ways of acting mathematically, genre and positioning across the boundary. Secondly making these boundary crossings is a matter of timing and knowing *when* to boundary cross.

A successful foundational student controls the movement of meaning between the mathematical practice of school mathematics (both reform-oriented and more traditional versions of this practice) and foundational mathematics. This student also moves between mathematical and non-mathematical practices, a boundary crossing that draws selectively on the genre of mathematical word problems in more traditional versions of school and undergraduate mathematics for interpretation of the non-mathematical. This movement may be a one way movement from the horizontal to the vertical (as in genre of mathematical word problems), it may be a to-and-fro movement (as in the practice of calculus reform), or it may end in the practical terms genre with no movement into the esoteric domain of undergraduate mathematics (also, as in calculus reform).

Participating in the foundational practice also requires that the student control the movement of meaning (both the how and the when) across social events within this practice

itself and also follow the textual links that explicitly or implicitly cue the movement of meaning within the text of a practical problem.

The valued ways of acting mathematically represent both continuities and disruptions in terms of how they talk to the mathematical practices of school mathematics, reform calculus, mainstream first-year mathematics and advanced mathematics. In the rest of this section I summarize these valued ways of acting.

The successful foundational student talks and writes in English (the medium of instruction) and presents his answers using the educated ways of talking and writing in foundational mathematics. This may involve talking and writing about mathematical objects using the practical terms genre. He also interacts with his peers in the small group using the educational ways of talking in a learner-centred pedagogy.

The foundational practice requires that the student make links at the level of object, that is, between mathematical objects, the representations of these objects, and the meaning of these objects in the task context. As noted above, participating in the practice also requires that the student make links between practices (mathematical and non-mathematical), social events within the foundational practice, and texts in a particular social event. The movement across the mathematical/non-mathematical boundary may be a one-way movement from the horizontal to the vertical or a to-and-fro movement between the two domains (which may end in the practical terms genre within the horizontal rather than vertical).

The foundational student is afforded opportunities to view mathematical objects both operationally and structurally, although some problems encourage an operational view while others provide the student with the possibility of using either view. Some questions in the practical problems require the student (explicitly or implicitly) to act operationally on mathematical objects, while others require no operational action.

The need to provide an argument in the foundational practice may be signalled by an explicit instruction in the text of a practical problem or may be an implicit assumption of a learner-centred pedagogy. At times, an argument may be endorsed with reference to the properties of the mathematical objects and the relationships between these objects. However, at other times, the argument is located in the task context alone.

On the one hand the foundational student is positioned as lacking power in the foundational practice. He is identified as different to a mathematics student in the mainstream and needing instructions and reminders about how to act in the practice. There is an asymmetrical relationship between the student and the tutor, with the latter constructed both as a mathematical authority in the foundational practice and as responsible for ensuring particular social behaviours on the part of the student. Yet on the other hand, the student is positioned as powerful; he is expected to take on the responsibilities of a student in a learner-centred pedagogy and is able to control the complex movement of meaning across practices, events and texts as required in the foundational practice.

7.9 The location of the practice of foundational mathematics in the wider order of discourse

Historically, foundational undergraduate mathematics is a new practice in the higher education space. In Section 2.4 I described the dominant ordering of the discourse types, genres and styles in this discursive space as one that foregrounds the boundary between foundational and mainstream undergraduate practice. Foundational practice is segmented out of the mainstream practice, with the specific role of facilitating the school/undergraduate transition for students who traditionally would not have gained formal access to university. The foundational student is identified, in this dominant ordering, as a Black student aiming at a non-mathematics major in science. I also argued that current debates around a four-year degree and admissions policies in higher education represent alternative conceptions of how the order of discourse could be arranged.

The boundaries within an order of discourse are held in place by power, that is, power behind discourse (Fairclough, 2001). Fairclough (1995) notes that these boundaries are constantly shifting and that shifts are part of social change. The description in this chapter of the mix of continuities and disruptions in the relationship between foundational practice and other practices in the space suggests that the foundational practice represents an alternative in the current order of discourse. In this section I consider whether this alternative practice challenges the power relations and represents a shift in the dominant order of discourse.

On the one hand, I argue that the representation of foundational practice and accompanying positioning of the foundational student challenges the dominant order of discourse. This challenge lies, firstly, in the continuities and disruptions in the relationship between this practice and other practices in the network of mathematics education practices. The foundational practice talks back to the practice of school mathematics and draws selectively on aspects of both reform-oriented and more traditional pedagogies. The foundational practice also draws selectively on undergraduate mathematics practice, setting up links both to traditional word problems and to calculus reform. I have suggested that the valued ways of arguing, looking at mathematical objects, and making links to non-mathematical practices may represent a discontinuity in the transition from foundational to advanced mathematics. Secondly, the challenge to the dominant order of discourse lies in the positioning of the foundational student as having power in the sense that he takes responsibility for and controls the complex movement of meaning across practices, events and texts. I have suggested that the flexibility in these movements is also valued in advanced mathematics. This construal of the student contrasts with the dominant positioning of the foundational student as someone with a weak schooling background who needs different support from a mainstream student.

Yet while the foundational practice introduces an alternative conception of both mathematical practice and the student into the higher education space, I argue that this practice does not represent a shift in this dominant ordering, precisely because of the mix of

continuities and disruptions that it represents. For while the practice positions the foundational student as having power in some respects, it also sets up a contradictory positioning that construes the student as lacking in power in the sense that he is a non-mathematics major and needs guidance and reminders on how to act. This positioning reproduces the dominant belief that a foundational student is different from a mainstream student and thus should be in a course that is segmented out of the mainstream.

The practical problems represent foundational mathematics as powerful in that it either empowers the student to act in non-mathematical practices or that it casts a gaze on these non-mathematical practices. Yet I argue that the problems represent foundational practice as lacking power in terms of how it facilitates the transition from school to advanced mathematics. Firstly, the selective appropriation of aspects of both more traditional and reform-oriented pedagogies in school mathematics into the foundational practice suggests that some students might not be able to control the complex movement of meaning required for success in the foundational practice itself (the student action is explored in detail in Chapters 8 to 11). An absence of this control has implications for formal access to the second-year advanced mathematics course and could reproduce the dominant view that the student does not actually belong in higher education. Secondly, I have suggested that the valued mathematical ways of the foundational practice represent some disruption in terms of the relationship to advanced mathematics, with implications for epistemological access to the dominant mathematical practice. It is possible that it is precisely this lack of power in terms of facilitating the transition to advanced mathematics that results in the practice remaining marginalized in the wider order of discourse.

Fairclough (2001) argues that an important aspect of power behind discourse is the notion of access and who has access to the powerful discourses and associated subject positions (and hence cultural and social capital). In her work on literacy, Janks (2010) identifies what she calls the *access paradox* (p.24) in the power relations between literacy practices:

> If we provide access to dominant forms, this contributes to maintaining the dominance of these forms. If, on the other hand, we deny students access, we perpetuate their marginalisation in a society that continues to value and importance of these forms. (p.24)

The discussion in this section points to a particular version of the access paradox (Janks, 2010, p.24) in the power relations at work between the mathematical practices in this study. On the one hand, the foundational practice, on the basis of the continuities and discontinuities to other practices that it represents, represents an alternative to the dominant ordering. In particular I have suggested that, given the complexity of the foundational practice, it has the potential to develop in the foundational student the flexibility in moving across boundaries that is valued in advanced mathematics. Yet if this new practice is to be recognized as an alternative to the dominant mathematical practices of the mainstream, it needs to provide students with epistemological access to these dominant practices. Providing this access, paradoxically, reproduces the dominant ordering. This analysis suggests that it is precisely the difference that this foundational practice represents in relation to mainstream practice that makes its role in facilitating formal and epistemological access to advanced mathematics problematic. Since the foundational practice is not seen to promote access to the dominant mathematical practice in higher education, it remains marginalized in the space and the innovative pedagogy is not recognized as an alternative. I revisit this discussion of the access paradox in Section 12.2, after the analysis of the student participation in the foundational practice in Chapters 8 to 11.

A CRITICAL EXAMINATION OF THE USE OF PRACTICAL PROBLEMS AND A LEARNER-CENTRED PEDAGOGY IN A FOUNDATIONAL UNDERGRADUATE MATHEMATICS COURSE
Volume 2

Catherine Jane Le Roux

A thesis submitted to the Wits School of Education, Faculty of Humanities, University of the Witwatersrand in fulfilment of the requirements for the degree of Doctor of Philosophy.

Johannesburg, 2011

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CHAPTER 8: RESULTS (PART 2a)
"… THESE KINDA LIKE WORDS SUMS"

8.1 Introduction to this chapter

In Chapter 7 I have argued that the practical problems represent foundational mathematics as an alternative practice in the higher education space, since the practice sets up both continuities and disruptions in its relationship to school mathematics, mainstream undergraduate mathematics and advanced mathematics. In addition, the practice makes links to non-mathematical practices. These continuities and disruptions mean that the boundary crossing required for participation in the foundational practice is a complex process requiring control over the how and when of the movement of meaning across boundaries.

If a student is to gain access to advanced mathematics via the foundational practice as described in Chapter 7, he needs to be able to adopt the subject position of foundational student set up for him and participate in the foundational practice by using the valued mathematical ways of acting. A student's action on the practical problems will be both enabled and constrained by the foundational practice and his positioning in this practice (Fairclough, 2001). In the one hand the practical problem texts are a resource that the student uses when solving the problems (Fairclough, 2001). Yet the student's action on the problems may differ from the valued mathematical ways as a result of the practice cutting across a variety of other practices as described, and through the agency of the student in adopting the valued subject positions (Fairclough, 2003).

I begin this chapter by providing an overview of the participating students' progress on the three practical problems selected for this study, as well as their overall progress in the

Foundational Course and beyond in the university. I then discuss the selection of extracts from the transcripts for detailed presentation in this thesis. In the rest of this chapter (as well as Chapters in 9 to 11 that follow) I present the detailed analysis of the student action on the practical problems, conducted using the process described in Section 6.3. This analysis is used to answer the second research question; I describe the student action, identify whether this action enables or constrains the students' occupation of the valued subject positions of the practice, and explain the action with reference to the socio-political interaction in the classroom and the discourse types, genres and styles that the students recruit.

8.2 Summary of the student action on the practical problems and more generally

8.2.1 Performance on the three practical problems

Before looking at the detail of the student action on selected problems, I provide the reader with an overall picture of the progress made by the participating students when solving the three practical problems used in this study (with a more detailed description provided in Appendices K to P).

When solving the Car Problem the students follow the five prescribed steps and make progress as they draw and label a diagram, relate the variables using a formula, operate on this formula using implicit differentiation, and use substitution. The Tutor's role is minimal; he only reminds them to follow the prescribed steps as they begin the question. The students also make links to other social events in the Foundational Course where related rates problems are solved using the five steps. Ten out of 11 of the students arrive, with relative ease, at the correct answer, although some of these students do not have full control over the movement across the mathematical/non-mathematical boundary. One student in Group 1 does not control the boundary crossings between practices and events, and is not enabled by the discussion in the group; he writes down the correct answer, but with incorrect working. One student in each of the two groups uses an intuitive method that

does not follow the prescribed steps. However, these students are encouraged to abandon the intuitive method by another subject (a student in Group 1, the tutor for Group 3).

The students' progress on practical problems that either require no operational action (the Flu Virus Problem) or require a mix of operational and non-operational action (the Chemical Reaction Problem) is more varied. Both these problems have sub-questions and the students may or may not miss the implicit or explicit textual links between the sub-questions which provide cues on how to act. For example, when evaluating the limit of the function in question (f) of the Flu Virus Problem the students link to the graph of the function in question (a), but this link is not made for the limit of the derivative function in question (g).

Most of the students perform the necessary operational action (finding anti-derivatives, finding the equation of a quadratic function) required in the Chemical Reaction Problem, sometimes with assistance from the Tutor or another student. However, in question (d) the socio-political interaction in Group 1 constrains certain students from using the input of an authority in the foundational practice.

The students have difficulty drawing sketch graphs that model functions (question (a) of the Flu Virus Problem, question (f) of the Chemical Reaction Problem), and are constrained by recruiting ways of acting on functions from school mathematics and their difficulties controlling the movement of meaning across the mathematical/non-mathematical boundary. Most of the students eventually sketch the required graphs, but only after intervention from the Tutor. This intervention takes the form of the Tutor modelling the links between the mathematical functions, their representations and their meaning in the task context. The Tutor actually sketches the required graphs for the students in Group 2, and the students end by simply choosing one of the Tutor's sketches for their own answers.

When using the practical terms genre (questions (c) to (e) of the Flu Virus Problem and questions (b) and (c) of the Chemical Reaction) the students' progress varies according to

the mathematical object acted upon. The use of the practical terms genre is achieved when talking about a function, but is problematic when talking about rates of change, and particularly the average rate of change. While students are able to describe these mathematical objects using mathematical terms such as "rate", they have difficulty when trying to avoid these terms. Some students expend considerable time on this, while others ignore the requirement to avoid mathematical terms and/or joke about it. The value that the Tutor places on the use of this genre varies; at times he gives the wording to students, while at other times he accepts the use of mathematical terms.

8.2.2 Performance on the Foundational Course and beyond

In this section I provide a summary of the overall progress of the 17 students in this study; I refer to their progress in the Foundational Course in the year that the data was collected (2007), in other mathematics courses at the university, and in their undergraduate degrees more generally. The information in Table 8.1 is not representative of the foundational class. Rather, it serves as additional background to the classroom action that is described in detail in this chapter and in Chapters 9 to 11. The first row of Table 8.1 gives the performance of the 17 participating students in the Foundational Course in 2007. The performance of these students as they proceed (or do not proceed) to other mathematics courses and eventually to an undergraduate degree can be traced by reading down the columns.

Table 8.1 indicates that nine out of the 17 participating students passed the 2007 Foundational Course, with two of these students passing the Course on their second attempt in 2007. In 2008 six of these students registered for the second foundational course that follows, with five students eventually passing and thus gaining formal access to the mainstream second-year course in advanced mathematics. The three students who took this second-year course failed on their first attempt, with one student passing the half-course version on his second attempt and another in the process of repeating the course at the time of writing this thesis.

Table 8.1: Summary of progress of the 17 participating students

	Progress through undergraduate programmes				
Foundational Course used in this study (2007)	7 students passed (first attempt in 2007), with a 52% to 65% range in marks	2 students passed (second attempt in 2007)	5 students qualified for the supplementary exam, all 5 students failed this exam	1 student failed	2 students dropped out of the Course during the year
Foundational course that follows the Course investigated in this study	4 students passed (first attempt) 1 student passed (second attempt) 1 student did not qualify to write exam				
Other mainstream first-year mathematics courses		1 student changed to commerce degree and passed first-year mathematics course for commerce students	1 student repeated Foundational Course in 2008 and did not qualify to write exam 1 student passed mainstream first-year course on 3rd attempt 1 student passed mathematics course for life sciences		
Mainstream second-year course in advanced mathematics	3 students attempted and failed the advanced mathematics course (1 student passed the half-course on repeat and 1 student repeating course at the time of writing this thesis)				
Undergraduate degrees at the institution	1 student excluded[120] after first year of study 1 student excluded after second year of study 1 student excluded after third year of study 1 student excluded after fourth year of study 1 student graduated in chemical and molecular sciences 2 students still active in undergraduate system (chemical and molecular sciences, mathematics and applied mathematics)	1 student graduated with a degree in commerce 1 student excluded from the institution after third year of study	1 student graduated in computer science 1 student graduated in chemical and molecular sciences 1 student graduated with degree in humanities 1 student excluded after first year of study 1 student excluded after second year of study	1 student excluded after first year of study	2 students excluded after first year of study (1 returned later in a different faculty)

[120] A student who has not passed the required number of course for a particular year of study is excluded from the science faculty on the basis of academic performance. A student excluded on these grounds may apply for admission to another faculty at the university or reapply for admission to science after gaining appropriate credits at another university.

By the end of 2010, five students had graduated at the institution, three with science degrees (chemical and molecular sciences, computer science) and one each with a degree in humanities and commerce. Three of the original 17 students were still registered for undergraduate study in 2011, with one of these students aiming for a specialization in mathematics. The remaining nine students had been excluded from the science faculty on the basis of academic performance (either after their first, second, third or fourth year of study), and had not returned to the university.

8.3 Selecting evidence

The six research texts re-presenting the action of each group on two of the three practical problems constitute 4 000 lines of transcript (the summary of the problems solved by each group is given in Table 5.3). Since the detailed analysis of these lines serves as the evidence for my argument, it was necessary to select extracts from these transcripts for presentation in Chapters 8 to 11. To facilitate this selection I began by writing a vignette describing the student action in each of the six transcripts (see Appendices K to P) and a summary of the enabling and constraining action for each transcript. The detail in these summaries was used to construct an answer to the second research question.

Extracts from the six transcripts that serve as evidence for this answer were selected with the aim of saturating the data in four respects. Firstly, the extracts were selected for their richness in making the overall argument. Secondly, action in an extract had to be identifiable as enabling or constraining on the basis of comparisons between students, groups of students or the students and the tutor. Thirdly, the extracts were selected in such a way that the action of all three groups is represented. This is an ethical decision; since the students agreed to participate I choose to give them a voice in the study. The inclusion of a variety of voices not only points to the range of action, but means that certain students cannot be consistently foregrounded.[121] Fourth, the extracts were selected such that they

[121] I was alerted to the possibility that certain students in the study might be foregrounded when writing a journal article based on an initial analysis of the action of Group 2 on question (a) of the Flu Virus Problem

represent the variety of the practical problems in the Foundational Course. The problems and groups selected as evidence are presented in Table 8.2. The term *support* in brackets in the table means that the action of the relevant group is included to support or contradict an argument about the action in the group that is the focus of the discussion.

Table 8.2: Summary of the transcripts presented in detail in this thesis

	Flu Virus Problem (a) (May 2007)	**Flu Virus Problem (f) & (g) (May 2007)**	**Flu Virus Problem (c) to (e) (May 2007)**	**Car Problem (August 2007)**	**Chemical Reaction Problem (d) (October 2007)**
Group 1:	✓ (support)	✓	✓	✓	
Group 2:	✓	✓ (support)	✓		✓
Group 3:				✓ (support)	✓

For each extract selected, the presentation of the results is structured around a particular enabling or constraining action. Yet these actions are not discrete, for example, all the transcripts presented as evidence can be used to describe the students' ways of talking and their socio-political interaction. By revisiting the mathematical action across sections within a chapter and across chapters I aim to provide the reader with a sense of patterns in this action (or where appropriate, to signal a discontinuity).

(see Le Roux, 2009). In this article my focus on Mpumelelo, whose action is visible in the particular transcripts selected for the article, could be interpreted as constructing him as being in deficit, despite my explicit intention to avoid a deficit view of the student.

8.4 Student action on the Car Problem (Group 1)

In Chapter 7 I argued that the Car Problem belongs to a set of related rates problems in the Foundational Course and the mainstream first-year course. While certain mathematical operations are valued in the prescribed steps for solving this problem, successful completion of the problem also requires a to-and-fro movement across the mathematical/non-mathematical boundary at appropriate times in these five steps.

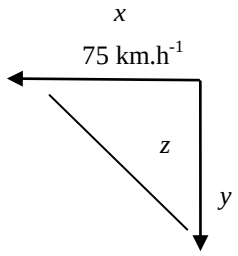
The student action on the Car Problem (as described in Appendices M and N) was selected for what it says about how students operate on mathematical objects and how they control the movement of meaning across the mathematical/non-mathematical boundary. In this section I present the action of the five students in Group 1 (Lulama, Darren, Hanah, Shae and Jeff) as evidence. Since there are many similarities between the action of these students and the students in Group 3 (Kelsa, Lwazi, Ndumiso, Nqobile, Akbar and Thokozile), I draw on the action in Group 3 as additional evidence.

8.4.1 Attending to the five steps in the boxed text as enabling

As the students start the Car Problem the Tutor approaches their table and queries whether they have started the problem, as presented in line 3 of Transcript 8.1.

Transcript 8.1: The Car Problem, Group 1, lines 1 to 22

1	((*Shae is using his ruler to draw perpendicular lines in his answer book, Darren and Shae are still working on the previous question*))	
2	Tutor:	So you guys starting with number 2 yet?
3	Shae and Hanah:	I'm just drawing the diagram
4	Tutor:	So have you read the thing in the box? Okay
5	Shae:	Yes ((*Nodding his head*))
6	Tutor:	Have you taken it to heart? ((*Darren looks up at the Tutor as the Tutor speaks to Shae. Then Darren looks at his Resource Book*))
7	Shae:	I just don't like the drawing the diagram part ((*Pointing to the Resource Book, then looking up at the Tutor, smiling*))

8	Tutor:	Okay but drawing the diagram … you see how it's got stuff underlined … it's got stuff in capitals … it's got stuff in bold
9	Shae:	I just find it easier if you just ... the given… and that
10	((*Hanah is writing in her answer book, Jeff is talking as he completes the previous question*))	
11	Tutor:	Oh yeah I mean ((*Shrugging his shoulders, looking at Shae*)) I I think that ... personally when I was going through this tut actually drawing it helped a lot ((*Hanah looks up at the Tutor as he speaks*)) … and uhm I think it can help avoid confusion↑
12	((*Shae looks up at the Tutor and nods*))	
13	Tutor:	>But just do what they say< ((*Using his hand for emphasis and moving away from the desk*))
14	Jeff:	Take the physics approach ((*Shae chuckles*))
15	Jeff:	[((*Jeff is reading from the problem text*)) Two cars start moving from the same point … one travels south↑ ((*He draws a vertical line*)) ... and the other travels to west ((*He draws a horizontal line, and is writing something above it, possibly 75km/h*)) ... at what rate is the distance between the cars increasing two hours later? 
16	[((*Darren is reading quietly to himself and takes his ruler out of his pencil case and draws a vertical line in his book, Shae is also reading from the problem text*))]	
17a	Jeff:	So *y*, *x* ((*Adding the variable names to his diagram))*
17b	((*Jeff draws in the hypotenuse, starts writing* $x^2 + y^2 = z^2$ *below his diagram in line 15*))	
18	Lulama:	So … we are using Pythagoras Theorem↑
19	Jeff:	uhm … [Ja]
20	Shae:	[Ja]
21	Jeff:	So then↑ ((*Mumbling*))
22	Lulama:	But you will have a problem

The Tutor attends to the boxed text and questions whether the students have read the steps, named as "the thing in the box" (line 4). He makes a number of pronouncements to convince the students to attend to the instructions in the box, thus reproducing the value placed on these instructions by the text producer. Firstly, he addresses the students directly as "you" (line 8) and draws on the bold text, capital letters etc. to give significance to the five steps; "you see how it's got stuff underlined … it's got stuff in capitals … it's got stuff

in bold" (line 8). Secondly, he locates his argument for pursuing the prescribed method in his personal experience "of going through this tut" (line 11) where "drawing it helped a lot" (line 11), thus positioning himself as someone who also solves practical problems in the foundational practice. Thirdly, he presents his personal opinion ("I", line 11) that attending to the boxed text "can help avoid confusion" (line 11). This opinion may relate to his position as someone who has solved the problem. However, by not naming who can "avoid confusion" (line 11), the Tutor may be identifying the students as subjects who might get confused if they do not follow the given steps. Lastly, the Tutor positions both himself and the text ("they", line 13) as controlling how the students behave in the foundational practice by simply telling them to ">just do what they say<" (line 13). However, this instruction seems to sit uneasily with the Tutor, suggested by the increase in his pace as he delivers the instruction and his leaving the table soon thereafter.

The Tutor presents his argument for using the boxed text to Shae, who makes eye contact with him and responds to him. The Tutor may be assuming that the other students are listening to this two-way discussion, as suggested when Darren and Hanah look up at the Tutor (lines 6 and 11) and when Jeff joins the discussion (line 14).

The Tutor's introduction of the Car Problem to Group 3 is similar to that given to Group 2, as he emphasizes the importance of attending to the boxed text. He also gives significance to operating on objects when he describes the related rates problems as being "all about differentiation and implicit differentiation" (Group 3, line 1a) and he makes a link to the genre of mathematical word problems by describing them as "kinda like word sums" (Group 3, line 1b).

In Group 1, Shae expresses his personal opinion ("I") to the Tutor that he doesn't "like the drawing the diagram part" (line 7). Yet in his action he adopts the subject position of a student who does follow the prescribed steps; he pronounces that he is drawing the diagram (line 3) and he uses his ruler to draw the required triangle (line 1). His pronouncement in line 7 that drawing the diagram is a "part" and his description of other steps ("the given …

and that", line 9) suggests that he views solving the Car Problem as following the steps in the boxed text. Jeff describes the approach discussed by Shae and the Tutor as "the physics approach" (line 14), suggesting that he links the steps to another practice in which he participates, that is, the pedagogic practice of undergraduate physics.

In Transcript 8.1 the students are drawing mathematical diagrams and writing down mathematical equations. This activity appears to be enabled by having the five steps to follow (including an instruction to differentiate) and by the students attending to cues in the text, as they might do when solving mathematical word problems. For example, in line 15 Jeff's reading of the words "south" and "west" are cues to draw a vertical and horizontal line respectively. The right angled triangle constructed in this way is a cue that "the formula linking the variables" in point 4 of the boxed text is the Theorem of Pythagoras (see Jeff and Lulama, lines 17b and 18). The students' choice of the variables *x* and *y* (see Jeff, line 17a) as labels for the sides of the triangle may be cued by the hint to use the last letter of the alphabet, *z*, in the bracketed Sentence 4 of the Car Problem text. This choice may also suggest that the students recognize the Car Problem as similar to other related rates problems about motion solved in lectures in the Foundational Course where these letters of the alphabet are used. By relating the variables *x*, *y* and *z* in the Pythagorean equation $x^2 + y^2 = z^2$ the student activity is enabled in the sense that they can act on this equation and apply the operation of "differentiation" as instructed in point 5. Without much discussion in the group, all the students proceed to differentiate the equation implicitly. I argue that the decision to use implicit differentiation and how they perform this differentiation is enabled by the students making links to the other related rates problems about motion in the Course.

When pronouncing "So … we are using Pythagoras Theorem↑" in line 18 of Transcript 8.1, Lulama identifies himself as a student who makes his claims public by using the name of a mathematical theorem. Yet the rising intonation at the end of his statement suggests that he is tentative about this claim and is requesting feedback. He receives this feedback in the form of content-free "Ja" from both Jeff and Shae (lines 19 and 20), who in responding

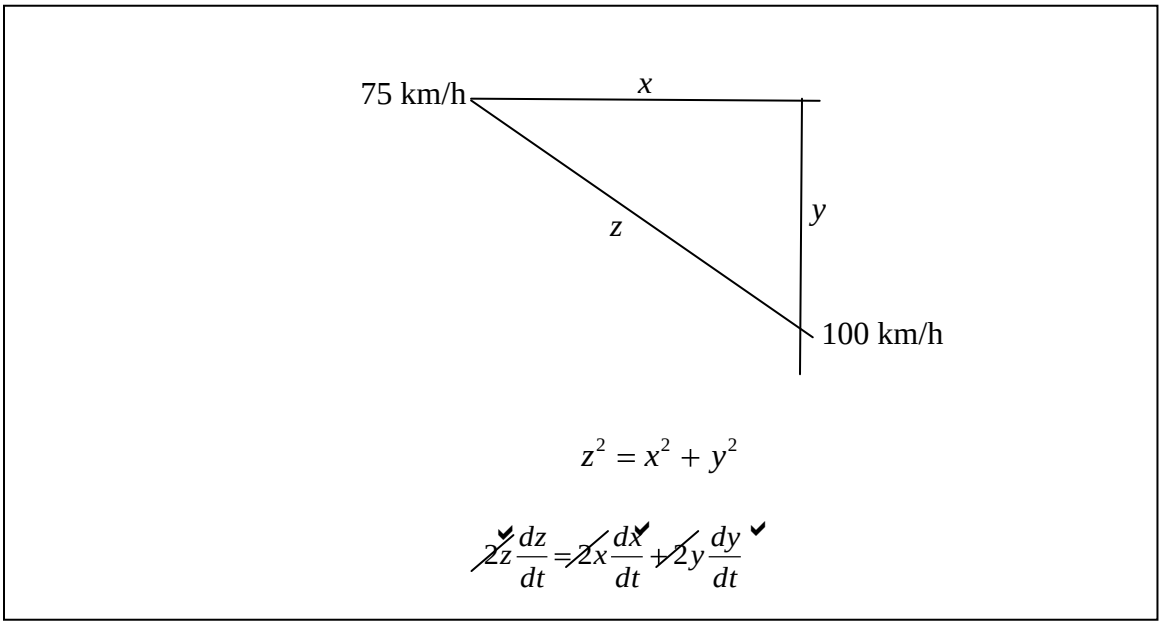
identify themselves as authorities the foundation practice who evaluate the pronouncements of other students in the group.

In this section I have argued that the students adopt the style of a foundational student by following the five steps in the boxed text as instructed. Their action is enabled by following these steps and being able to act operationally using implicit differentiation. The students are enabled to control the one-way movement of meaning from the task context to the mathematical drawing and formula by adopting the style of a school student identifying the cues in a word problem. They are also enabled in following the five steps by drawing on the valued mathematical action in other related rates problems involving motion in the Foundational Course.

8.4.2 Severing the links to the task context as constraining

At the end of Transcript 8.1 Lulama makes a statement in which he identifies "a problem" (line 22) that he foresees them as having (the general pronoun "you" suggests they will all have this problem). This concern is not attended to by the other four students, but emerges again in the discussion when the students get to the step of "completing the question" in point 5 of the boxed text. As represented in Figure 8.1, Lulama has drawn a diagram, written down an equation and differentiated this equation implicitly with respect to *t*. The ticks inserted above some of the symbols in the derivative equation suggest that he is attending to what can be substituted for the symbols. Lulama summarises his difficulty as not having a value to substitute for the object he names using the symbols "*dz* over *dt*"; "But we don't have a a [...] a *dz* over *dt* what are we going to do?" (line 32). His repeated use of the pronoun "we" here suggests that Lulama sees the problem-solving process as a collective one.

Figure 8.1: Lulama's written answer for the Car Problem[122]



I argue that Lulama's "problem", which continues after he pronounces it in line 22, is linked to the absence in the verbal discourse of an explicit link between the objects that the students are operating on (represented by the symbols *x*, *y*, *z*, $\frac{dx}{dt}$, $\frac{dy}{dt}$, and $\frac{dz}{dt}$) and the meaning of these symbols in the task context, which the students are instructed to set up in steps 1 to 3 of the boxed text. In the discussion of Transcript 8.1 I have argued that the students draw on their experience of solving other related rates problems in the Course when naming the sides of the triangle using the letters *x*, *y* and *z*. Yet they do not make explicit (either verbally or in writing) the meaning of these letters as representing functions with a particular meaning in the task context, for example, that *x* represents "the distance travelled by car A from the starting point after time *t*". These links are not pronounced in Lulama's solution in Figure 8.1, and there is no evidence of such pronouncements in the written work of the other students (later, Shae and Hanah write down the distances after two hours as x = 200km and y = 150km, but these are not given meaning as functions).

[122] The students' hand-written work has been re-presented as typed text to protect the anonymity of the students.

There is also an absence of attention to the meaning of these symbols in the task context in the verbal discussion, as suggested by the talk in Transcript 8.2.

Transcript 8.2: The Car Problem, Group 1, lines 34 to 54

34	Shae:	We're trina ... no we get we get given the two different values we get given the↑ ((*Pointing with his pen in his Resource Book*))
35	((*Darren is saying something quietly*))	
36	?	Yes we do
37	Jeff:	[*dx* over *dt* is ... 75 { $\frac{dx}{dt} = 75$ }]
38	Shae:	[I just made mine *ds* over *dt* and *dw* over *dt* { $\frac{ds}{dt}$ and $\frac{dw}{dt}$ }]
39*	[[…]]	
40	Jeff:	[[*dw*↑... *ds*↑
41	Shae:	over *dt*]]
42	Shae:	okay ... ...[ja]
43*	[…]	
44	Shae:	Do your do your things as *ds* and *dw* just to make it easier...[for like the whole thing] ((*Looking across at Darren's answer book, and then down at his own again*))
45	Lulama:	[*ds*↑]
46	Shae:	You know for the south car and the west car↑ ((*Looking across at Darren, and then across at Lulama as he speaks*))
47	Lulama:	I've used *x* and *y*
48	?	[Yeah *x* and *y*]
49	Jeff:	[>Ja just use *x* and *y*< ((*Looking across at Lulama*))]
50	Shae:	Okay yeah sorry … okay ((*Erases from his book and re-writes*))
51	Jeff:	Because you can just use *dx* over uh *dt* { $\frac{dx}{dt}$ }↑
52	Darren:	Once you use different things it becomes ...
53	((*Hanah is writing and rubbing out*))	
54	Shae:	Then we find *dz* ja ((*Looking across at Darren*))

* The Tutor is addressing the rest of the class about a different question.

In this discussion Shae suggests that the other students follow his lead and use the letters *s* and *w* instead of *x* and *y* (lines 38 and 44). His linking of these letters to the "south car and

the west car" (line 46) in the task context suggests that Shae identifies the letters as labels for the cars, rather than as representing functions, in this case, distance as a function of time. Darren's response to Shae's proposal suggests that other students in the group are viewing their alternative choice (the letters *x* and *y*) in a similar way and have chosen these on the basis of their common use in the Course. Making his argument that they should all be using the same letter, Darren locates his argument in the need for them to communicate in the group (line 52), and does not challenge the meaning assigned to these letters.

Figure 8.1 also indicates that Lulama has skipped steps 2 and 3 of the boxed text, steps in which he is required to make links between the mathematical symbols representing the derivatives $\frac{dx}{dt}$, $\frac{dy}{dt}$, and $\frac{dz}{dt}$ and the speeds in the task context. The other students do make these links in their written work, but the links between the naming of the derivative symbols "*dx* over *dt* { $\frac{dx}{dt}$ }" (line 37) and the meaning in the task context, that is, as the rate of change of distance with respect to time or the speed, is not given significance in the student talk. In line 54 Shae has shortened this derivative to "*dz*". In line 37 Jeff assigns the value 75 to "*dx* over *dt* { $\frac{dx}{dt}$ }" without naming the units of the value (km/h), a naming which would make a link to the speed in the task context. In line 35 Shae is assigning values to his symbols, but this is done by pointing to parts of the problem text and to his written answer rather than by pronouncing the links verbally.

In the sections that follow I provide evidence suggesting that some of the students have made implicit links between the mathematical objects and the task context and make these links explicit verbally when challenged to do so, but that this is not case for the student Lulama. In addition, I provide evidence that the students do not exercise complete control over these links, but are enabled in producing the correct answer by following the action used when solving other related rates problems in the Foundational Course.

8.4.3 Talking about operations on objects as constraining

I begin this section by presenting two transcripts as evidence of the significance given to talk about operations on mathematical objects in the students' verbal text. I then suggest why this talk may be constraining.

In Transcript 8.3, Jeff's verbal pronouncements in lines 58 and 69a link the distance functions *x*, *y* and *z* to their meaning in the task context. This is evidence that, having moved from the task context to the mathematical diagram and formula, he has now moved back into the task context to identify what distance values to substitute into the formula. In line 69a he introduces unnamed "guys" into the task context, reproducing the representation in the text of the task context as not real.

Transcript 8.3: The Car Problem, Group 1, lines 55 to 70

55	Jeff:	Okay so now we are gonna do the equation and plug in for two hours↑ ((*Attending to the equation* $z^2 = x^2 + y^2$))
56	Darren:	*d*↑ ((*He is writing in his answer text, Hanah and Shae are writing*))
57	Lulama:	Why↑ why do you plug for 2 hours? ((*Looking up at Jeff who is looking at the problem text*))
58	Jeff:	Because you've got to measure the distance *z* has got to be the distance <at 2 hours> [...] from their starting time↑ ((*Still looking at the problem text, Shae is looking at him*))
59	Lulama:	[yes]
60	Lulama:	Yes↑ ... ja I understand that ((*Looking down at his answer text again*))
61a	Jeff:	So it'll be like ... 200↑ and... 150↑ square uh squared ... each↑ then add them together and that will give you the *z*↑ ((*Looking up from the problem text, raises his eyebrows and looks to the others for feedback*))
61b	Jeff:	Wouldn't it?
62	((*Darren, Hanah and Shae are writing as Jeff speaks to Lulama, Lulama looks down at his answer book as he is listening*))	
63	Shae:	Uh ... for ... if you gonna use Pythagoras you get you get that equation [then you derivatise it↑]
64	Jeff:	[Yeah but because ja] but you got to do it after 2 hours so ... it'll be double ... double their speed ((*Seems to be looking at his answer book*))]
65	Shae:	The the thing will be ((*Writing in his answer book as he speaks*)) *z* squared [[is equal to <*x* squared plus *y* squared { $z^2 = x^2 + y^2$ }>]]
66	((*Lulama has a diagram and equations written in his answer book, see Figure 8.1*))	
67	Jeff:	[[So it will be ... 200 and 150↑]]
68	Shae:	[And then]

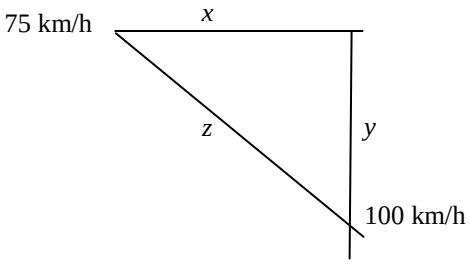
69a	Jeff:	[>Because after 2 hours] they would have travelled that far so < ... the guy travelling south would have travelled 200kms in 2 hours↑ ... the guy travelling west would have travelled 150k's ... travelling west↑
69b	Jeff:	Then you would use Pyth Pythagoras using those values ... wouldn't you?
70	((*While Jeff is speaking, Lulama is adding to his writing, he substitutes the speeds into the Pythagoras equation:* $z^2 = x^2 + y^2 \Rightarrow \quad 75^2 + 100^2 \quad \Rightarrow$))	

Yet much of the talk in Transcript 8.3 involves talking about the operations, with no link to the task context. For example, in lines 61a and 65 Jeff and Shae respectively describe the substitution into the Pythagoras equation $z^2 = x^2 + y^2$ in words and Shae uses the word "derivatising" (line 63) to talk about the operation of finding the derivative of this equation. In addition, the students use reference pronouns in their pronouncements without clarifying verbally what a pronoun is referring to. For example, in lines 64 and 67 Jeff appears to use the personal pronoun "it" to refer to the distances, and Shae names the Pythagoras equation as "the thing" (line 65). In some cases the pronouns are accompanied by the student pointing to what is being referred to, but this is not always the case; in line 69a Jeff uses the demonstrative pronoun "those" to provide a textual link to the values he named in his explanation in line 69a.

Transcript 8.3 is similar to much of the rest of the transcript representing Group 1's action on the Car Problem in that Shae and Jeff dominate the talk. Much of this involves talking about the different steps in their calculations and is presented with a tentative tone. For example, in line 61a the rising intonation at the end of the Jeff's statement and the raised eyebrows as he makes eye contact with the others suggests that Jeff wants feedback. Yet by talking frequently and by responding to the queries by other students, Shae and particularly Jeff identify themselves as mathematical authorities in the group. Other students identify these two students as authorities in the foundational practice by asking them questions, as Lulama does in line 57.

Transcript 8.4 provides further evidence of the significance given to talk about the mathematical operations, this time between Darren and Lulama (Shae and Jeff are involved in a parallel discussion).

Transcript 8.4: The Car Problem, Group 1, lines 89 to 107

89	Lulama:	[(unclear)]
90	Darren:	((*Looking across at Lulama*)) Are you lost?
91	Lulama:	So so what what are we going to put in? ((*Referring to the derivative equation* $2z\frac{dz}{dt}=2x\frac{dx}{dt}+2y\frac{dy}{dt}$ *as in Figure 8.1*))
92	Darren:	in *dz dt* { $\frac{dz}{dt}$ }?
93	Lulama:	Ja
94	Darren:	We're finding *dz* ((*Pointing to* $\frac{dz}{dt}$ *in the derivative equation in Lulama's book, see line 91*))
95	Lulama:	Okay yes↑... ((*Looking at Darren*)) and you do have *z* … you do have *x* and you do have *y*↑ ((*Pointing to his answer book and using his hand in the air for emphasis*))
96	Darren:	And you'll find *z* using Pythagoras↑ ((*Pointing at the z on Lulama's diagram as in Figure 8.1*)) 
97	Lulama:	Pythagoras yes↑ ... ... but how about *dx* over *dt* { $\frac{dx}{dt}$ }?
98	Darren:	We have *dx* over *dt* { $\frac{dx}{dt}$ } (unclear as Shae and Jeff are also talking) 100 km/h and 75 km/h ((*Pointing to the Resource Book in front of Lulama*))
99	Lulama:	That is *x* and *y*↑ it's not ...
100	Darren:	No ... that is your *d* that is your *dx* [[over *dt* { $\frac{dx}{dt}$ }]]((*Using his pencil on the page for emphasis*))
101	?	[[over *dt*]]
102	Darren:	When you're comparing rates ... rates is a comparison between two variables ((*He has his pencil on the 100km/h in the problem text*)) this is distance ((*He circles km in 100km/h*)) and this is time ((*He circles h in 100km/h*))
103	Lulama:	So ... okay ... we're gonna put this here ... then how about this?

104a	Darren:	This we're gonna take↑ ... 75 and 100 ((*He points to the circled 100 in problem text*)) and we're gonna go ((*He writes "× 2" on the problem text*))...
104b	Darren:	because it is 2 hours (unclear) 100km/h so 100km for 1 hour ... 200↑
105	Lulama:	So that's going to be ... 200↑ ... ... are you sure? ((*He is looking at his answer text*))
106	Darren:	Yeah I'm sure↑ ... pretty sure↑
107	((*Lulama starts writing. He is still using the value of z found by substituting the speed values for the distance variables:* $z^2 = x^2 + y^2 \Rightarrow \quad 75^2 + 100^2 \quad \Rightarrow \quad 15\,625$. *He substitutes the value of* z^2 *rather than 2z:* $15625\frac{dz}{dt} =$))	

When explaining to Lulama, Darren demonstrates that he moves over the boundary between the mathematical objects and their meaning in the task context to decide what values to substitute into his mathematics formula. For example, in lines 98 to 102 he links the symbol $\frac{dx}{dt}$, the mathematical term "rate" and the speed (with units) of 100km/h. He also identifies the units "km/h" with "distance" and "time". Yet the attention in this discussion focuses on "finding *dz*" (line 94) and having to "find *z*" (line 96) with an absence of naming these as distance or speed, and of identifying certain symbols with numbers, for example, whether the 100 and 75 are "*x* and *y*" (Lulama, line99) or $\frac{dx}{dt}$ and $\frac{dy}{dt}$. Lulama talks about the symbols "*x*" and *y*" (line 99) and "*dx* over *dt*" (line 97) and the values like "200" (line 105), which he refers to using demonstrative pronouns "this" (line 103) and "that" (lines 99 and 105). There is an absence in his talk of the everyday objects like distance and speed, suggesting that he has not moved back into the task context.

Although the talk of Jeff, Shae and Darren (and later Hanah) gives significance to the operations they are performing and lacks some clarity in the use of reference pronouns, their explanations for these operations suggest that they have some control over (a) the movement of meaning between social events in the Course, and (b) over the to-and-fro movement of meaning between the task context and the mathematical objects.

Yet I argue that such talk is constraining in three respects. Firstly, it slows down the communication between the students since time is taken establishing common

understanding of the focus of attention (in Transcript 8.4 Lulama and Darren have difficulty establishing whether they are talking about the function or its derivative). Secondly, although the student Lulama moves from the task context to a mathematical diagram and uses the "Pythagoras Theorem" (line 18), he does not establish links between the mathematical objects and their meaning in the task context as required in steps 1, 2 and 3. As a result he has difficulty moving back into the task context when required. Although the other students are "explaining answers" (Foundational Course Resource Book, 2007, Workshop 1, p.16) as required in the learner-centred pedagogy promoted in the Course, the predominance of talk about operations and lack of clarity in references appears to constrain Lulama from gaining control over this movement. Following input from Shae and Jeff as represented in Transcript 8.3, Lulama substitutes the speed values for the distance variables in the Pythagorean equation (line 70). Although this is productive when following an intuitive approach to solving the Car Problem, the discussion in Transcript 8.4 suggests that Lulama is following the prescribed five-step method. It appears that the nature of the talk is preventing Lulama from gaining access to the required control over the movement of meaning across practices and events. In Transcript 8.5 below I present additional evidence for Lulama's difficulty in accessing this control in a discussion in which answers in the form of values only are exchanged.

Transcript 8.5: The Car Problem, Group 1, lines 190 to 201

190	Lulama:	132? ((*He has written the following:* $125\frac{dz}{dt} = 200(75) + 150(100)$ $= 1500 + 15000$ $= \sout{1.056\ km/h}$ $= 132 km/h$))
191	Jeff:	Ja 125
192	Lulama:	125↑ ... what did you get?((*Looking across at Jeff*))
193	Jeff:	125
194	Lulama:	132 ((*Looking down at his book again, seems puzzled*))
195	((*Darren is talking quietly through his calculation*))	
196	Jeff:	Okay
197	Darren:	>125< ((*Pushing his calculator forward as he finishes*))

198	Lulama:	125↑
199	Jeff:	Good good↑
200	Shae:	(unclear) a very sexy question
201	((*Lulama is writing in his book: he crosses out his answer of 132 in line 190, and writes 125, without changing his calculation.* $125\frac{dz}{dt} = 200(75) + 150(100)$ $= 1500 + 15000$ = ~~1.056 km/h~~ = 1~~3~~25km/h))	

Having attended to the explanations of his peers, Lulama produces an answer of 132km/h, which he pronounces without the units, and appeals for feedback. Jeff provides positive feedback "Ja", but then produces an alternative answer of "125" (line 191). Lulama sounds puzzled when he repeats the value "125↑" (line 192) with rising intonation at the end and appeals to Jeff again. Jeff responds by simply repeating the answer "125" (line 193). This exchange of answers, which constrains Lulama's access to Jeff's methods, suggests that the students may be drawing on their experience of a more traditional pedagogy in which value is placed on the final answer. The only way Lulama can respond is to repeat his earlier answer of "132" (line 194). Jeff's "okay" in line 196 suggests he is closing the discussion and moving on. However, in line 197 Darren joins the conversation and provides an answer in support of Jeff's "125". Lulama seeks help once again by repeating this answer in a puzzled way, "125↑" (line 198). However, Jeff sees Darren's pronouncement as enough evidence to move on ("good good↑", line 199), and he enters a conversation with Shae about the difficulty of the question. In the absence of a discussion about their methods, Lulama returns to his answer book and replaces the value of 132 with the value 125 (line 201).

So far the description of the interaction between Lulama, Shae, Jeff and Darren suggests that Lulama participates in the group discussion, either by making statements or by appealing to Shae, Darren and particularly Jeff for assistance. Yet Lulama does not appeal to the Tutor in the same way, possibly as he perceives the power relations between himself

and the Tutor as different from those between him and his peers. This difference may also be due to Shae's positioning of himself as the spokesperson for the group, and the Tutor seemingly identifying him in this role. I noted in the discussion of Transcript 8.1 that the Tutor talks to Shae about the boxed text (lines 2 to 13) and may assume that the other students are listening to this two-way discussion. The absence of contributions from the other students to this discussion positions Shae as the student who talks to the Tutor. Later, shortly after the interaction represented in Transcript 8.4, the Tutor approaches the desk and asks, "How is it going?" (line 122), and it is Shae who responds positively that it is "going good" (line 123). The Tutor asks Shae to "show me" (line 127). Shae explains by making links between the mathematical objects and the task context, while the other students work individually, either writing or using their calculators. The Tutor's "okay" provides positive feedback to Shae (line 132). Then the Tutor shifts attention to the other students by moving to a different position next to the table and asking them whether they "are all on the same wavelength" (line 134). Jeff and Darren respond with content-free positive responses ("Ja") and continue working (line 136). Lulama does not signal to the Tutor that he needs help.

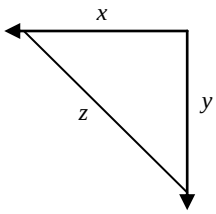
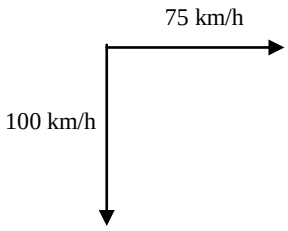
In this interaction with the Tutor, Shae's "wavelength" is represented as being that of the group as a whole. The Tutor takes the students at their word, thus positioning them as students in a learner-centred pedagogy who take responsibility for their own learning and request help when required. He may also be balancing his duty to the students in Group 1 and his role as a tutor responsible for all students in the class who does not have the time to check the working of individual students.

In this section I have described the student talk about their operational action and have argued that this talk, firstly, slows down the students as they establish shared communication and, secondly, constrains Lulama from establishing productive links between the mathematical objects and the task context. I make a third point related to this operational talk in Section 8.4.4.

8.4.4 Links across practices as enabling and constraining

I have argued that four of the five students in Group 1 have some control over the to-and-fro movement between the mathematical and non-mathematical (usually implicit). However, the analysis suggests that these students are enabled to produce the correct answer of 125km/h in the absence of full control over this movement by making links to the texts of other related rates problems in the Foundational Course. The interaction in Transcript 8.6 takes place as the students write down what is "given" for step 2 of the boxed text.

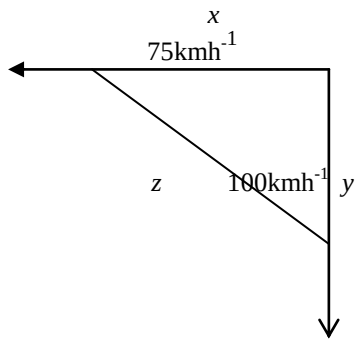
Transcript 8.6: The Car Problem, Group 1, lines 23 to 30

23a	Jeff:	But now would you … would you take the speeds as negative↑
23b	Jeff:	Because they are moving away from the point or what? ((*Looking across at Shae as he finishes speaking*))
24	Darren:	Why?
25	? (2 students)	Uhm
26	Jeff:	[Why did he make them negative in class?] ((*Looking across at Darren*))
27	Lulama:	[I mean it's positive]
28	Darren:	[[Because it's going towards]]((*His fingers are pointing inwards and moving towards one another*)) 
29a	Shae:	[[No, no no it's not ((*Tapping Jeff on the shoulder*))]] it's going to be positive
29b	Shae:	you've got to look at this final at this final thing … and that's increasing ((M*oving his hands outwards along the hypotenuse of the triangle in his book to show the increasing, then looks across at Jeff*)) 
29c	Shae:	so it's going to be a positive value that you want
30	Jeff:	Okay ... ... so↑ ((*Holding his pen above his answer text and looking at the text*))

Jeff is attending to point 2 of the boxed text and linking the derivatives $\frac{dx}{dt}$ and $\frac{dy}{dt}$ with the "speeds" (line 23a) 75km/h and 100km/h in the problem text. In attending to the sign of the derivatives he makes a link to the texts of related rates problems solved in lectures where this was discussed by the lecturer ("he", line 26). Jeff and Darren argue that the derivatives will be negative, recruiting the task context and attending to the direction of the cars. Jeff makes a claim public in a tentative way; he argues that the derivatives are negative since "they (the cars) are moving away from the point" (line 23b). Darren argues the opposite to Jeff, suggesting that "it's (the cars?) going towards", using his fingers to demonstrate on his diagram (line 28). Shae provides negative feedback and pronounces an alternative explanation; "… it's going to be positive you've got to look at this final at this final thing … and that's increasing" (line 29), his hand movement on the hypotenuse of the triangle suggests that he is attending to the increase in the distance between the cars. Darren appears to be using the reference pronoun "it's" in line 28 for the cars, but Shae uses this same pronoun for the derivative/speed in line 29. Although the students use the task context to support their arguments as required, they do not attend to the definitions of the distance variables *x* and *y* as functions of time. Indeed, as argued in Section 8.4.2, they do not seem to have defined the variables in this way. Lulama states that the derivatives are positive, but without a reason, supporting my argument that he is not making links to the task context.

Further evidence of an absence of control over the mathematical/non-mathematical boundary crossing comes from Jeff's written work, presented in Figure 8.2.

Figure 8.2: Jeff's written answer for the Car Problem



$\frac{dx}{dt} = 75$ (A)*

$\frac{dy}{dt} = 100$ (B)

$x^2 + y^2 = z^2$ (C)

$2z\frac{dz}{dy} = 2x\frac{dx}{dy} + 2y\frac{dy}{dx}$ (D)

$2(250)\frac{dz}{dt} = 2(150)(75) + 2(200)(100)$ (E)

$\frac{dz}{dt} = \frac{(150)^2 + (200)^2}{500}$ (F)

$\frac{dz}{dt} = 125\text{km.h}$ (G)

* Line labels A to G have been added.

In the "given information" in lines A and B, Jeff links the speeds given in the problem text (75km/h and 100km/h) to the derivative of the distance with respect to time ($\frac{dx}{dt}$ and $\frac{dy}{dt}$). Yet his operations on the variables *x*, *y* and *z* that follow suggest that he is not attending to these as functions of the variable time and to the speed as the derivative of the distance with respect to time. In line D Jeff differentiates *z* with respect to *y* ($\frac{dz}{dy}$), *x* with respect to *y* ($\frac{dx}{dy}$), and *y* with respect to *x* ($\frac{dy}{dx}$). He replaces the derivative $\frac{dz}{dy}$ with $\frac{dz}{dt}$ in line E and substitutes the values 75km/h and 100km/h for $\frac{dx}{dy}$ and $\frac{dy}{dx}$ respectively. He continues to obtain the correct answer of 125km/h. It seems that Jeff is enabled to produce this correct answer by following the steps used in other related rates problems in the Course, even

though he does not have full control of the movement of meaning in the objects across the mathematical/non-mathematical boundary.

8.4.5 Adopting the position of a student who follows the instructions in the text (Group 1)

Transcript 8.7 is another transcript re-presenting the students' discussion about answers. This time, the discussion proceeds in a different way to that discussed in Section 8.4.3.

Transcript 8.7: The Car Problem, Group 1, lines 141 to 156

141	Lulama:	1.056↑
142	Hanah:	I got... 1,4 but I think that is totally wrong
143	((*Darren picks up his calculator and starts doing calculations, he is talking quietly through the calculation as he punches in the numbers*))	
144	Shae?:	What did you get?
145	Hanah:	I think I did mine totally differently to yours
146	Jeff:	125↑
147	((*Hanah moves her calculator off her book and looks at her page*))	
148	Shae:	I calculated mine wrong ((*Looking across at Jeff's book and then using his calculator*))
149	Hanah:	Isn't↑... 125 just what you get if you... [put the] things into Pythagoras? ((*Underlining something in her book and then looking across at Jeff*))
150	Jeff:	[°Yes°]
151	Jeff:	pretty much↑
152	((*Someone sighs loudly, Shae is using his calculator*))	
153	Hanah:	Cause that is what I got the first time the first time round and I thought it was a very ... easy way of doing it ((*Looking across at Jeff, she twists her pen around in her fingers as she speaks*))
154	Jeff:	Maybe maybe it's not that difficult↑
155	Hanah:	((*Jeff and Hanah laugh*)) But we're making it worse than it is
156	Jeff:	See what Darren gets here

In Line 141 Lulama pronounces an answer of "1.056" (see line 190, Transcript 8.5). The content of Lulama's pronouncement is not attended to, but seems to encourage Hanah (who has not taken part in the verbal discussion so far, as if she only has the power to join the conversation when she has an answer) to pronounce her answer of "1,4" (line 142). She

positions herself as unsure; without pausing after saying her answer she pronounces that she thinks her answer is "totally wrong" (line 142) and that her method is "totally" (line 145) different to Shae's. Jeff then offers the number "125" as an answer (line 146). Shae evaluates the value on his calculator screen against Jeff's answer and concludes "I calculated mine wrong" (line 148). Hanah attends to Jeff's answer of "125" and attends to an earlier calculation she did, "Isn't↑... 125 just what you get if you ... put the things into Pythagoras?" (line 149). Here she talks about the substitution ("put the thing into Pythagoras") and names the values as "the things" rather than pronouncing the meaning of the values in the task context. Jeff agrees with her ("Yes", line 150) and they agree that "maybe" the question is "not that difficult" (Jeff, line 154) and "we're making it worse than it is" (Hanah, line 155).

In this discussion the students position Jeff as an authority in the foundational practice, for example, Shae assumes that Jeff's answer pronounced in line 146 is correct and Hanah discusses her alternative method with Jeff. On the one hand Jeff identifies himself in this way in his response to Hanah, but on the other hand he identifies himself as needing confirmation for his answer of 125, and hence the turn of attention to Darren's work. Secondly, it is not possible to tell (due to Hanah erasing her initial written solution and an absence of video footage of her earlier work) whether Hanah's "first time around" (line 153) involved using an intuitive method and the constant speeds or whether she has mistakenly substituted the speeds for the distance in the Pythagoras equation (as Lulama did in line 107 of Transcript 8.4). Either way, in lines 149 and 150 of Transcript 8.7 Hanah and Jeff have identified that there is a shorter method than the prescribed five steps. Yet they do not attend to why this alternative method may work. Rather, they follow the instructions in the text (and reinforced earlier by the Tutor) to follow the steps in the boxed text. In discussion with Shae, Hanah proceeds to use implicit differentiation and substitution to produce a written answer of 125km/h.

8.4.6 Adopting the position of a student who follows the instructions in the text (Group 3)

In Section 8.4.5 I have indicated that Hanah in Group 1 may begin by using an intuitive approach to solving the Car Problem, although this may be an error on her part when following the five steps. In this section I present evidence that Nqobile in Group 3 uses an intuitive approach using a distance, speed and time formula rather than implicit differentiation.

The talk about the Car Problem in Group 3 takes place between five students; Akbar, Thokozile, Ndumiso, Lwazi, Kelso. This action follows a similar pattern to that in Group 1 as students talk about the operations like differentiation and substitution, without an explicit link between the mathematical objects and the task context. Nqobile, who started her academic year in the mainstream mathematics course, is a quiet student who usually interacts on a one-on-one basis with Kelso sitting next to her. Nqobile's written solution for the Car Problem is re-presented in Figure 8.3.

Figure 8.3: Nqobile's written solution for the Car Problem

$s = \frac{d}{t}$	$s = \frac{d}{t}$	(A)*
$100 = \frac{d}{2}$	$75 = \frac{d}{2}$	(B)
$d = 200$	$d = 150$	(C)
$d =$		(D)
$z^2 = x^2 + y^2$		(E)
$z = \sqrt{22500 + 40000}$		(F)
$z = 250\text{km}$		(G)
$s = \frac{d}{t}$		(H)
$= \frac{250}{2}$		(I)
$= 125\text{km/h}$		(J)

* Line labels A to J have been added.

As shown in Figure 8.3, Nqobile has made use of the speed, distance and time formula $s = \frac{d}{t}$ (probably learnt at school) to calculate the distances *x* and *y* (she calls them both "*d*") after two hours (lines A to C). Hence she calculates the distance *z* after two hours to be 250km (lines E to G), and gets positive feedback on this answer from Kelso.

In Transcript 8.8 Nqobile describes her final step to the Tutor. Since she has the distance travelled in two hours (250km in line G) she can "divide by 2 hours↑ to find the speed↑" (line 224), the rising intonation suggesting that she wants feedback from the Tutor. Her verbal and written text indicates that she is distinguishing between distance and speed in her calculation. The Tutor gives negative feedback to this solution, but he tempers this by suggesting that it does make some sense to him, "I can see ... what you kinda getting at" (line 225a) and kneeling down next to the desk so that he is at the same eye-level as Nqobile (line 225b). He then proceeds to name the speed she is attending to as "average speed", thus supplementing the description of the task context in the problem text. He does not, however, attend to the fact that the speed of the cars is constant. As happened in the discussion between Hanah and Jeff in Section 8.4.5, the opportunity to explore why an alternative method to the method of using implicit differentiation works is shut down, this time by the Tutor. In her shyness and tendency to work quietly on her own, Nqobile has positioned herself outside of the group, and as a result her alternative method is not made public for discussion in the group. The Tutor then names the workshop as being "for … implicit differentiation" (line 225b), attending to the operation to be practised in a related rates problem and linking to the genre of mathematical word problems which are "to get the student to practice an algorithm recently presented in their maths course" (Gerofsky, 2004, p.33).

Transcript 8.8: The Car Problem, Group 3, lines 224-228

224	Nqobile:	((*Explaining to the Tutor what she has written, she points to her diagram and to the equation* $s = \frac{d}{t}$)) Then you divide by 2 hours↑ to find the speed↑
225a	Tutor:	Uh ... uh I can see ... what you kinda getting at cause you're saying that is an average speed … we're not asking for an average speed over the two hours ... ...
225b	Tutor:	okay so have you have you done any ... I mean this is a ... a workshop for ... uhm differentiation implicit differentiation have you done any? ((*The Tutor has got down on his haunches next to Nqobile and is looking at her*))
226	Nqobile:	Ja I haven't no ((*Looking across at the Tutor next to her and shaking her head, still holding her hand in front of her face*))
227	Tutor:	Okay ... ((*Looking across at Akbar and Thokozile and then Ndumiso and then back at Nqobile*)) but the other people in the group? ... ... have they?
228	((*Nqobile looks across the table, she holds her hand in front of her mouth*))	

In line 226 of Transcript 8.8 the Tutor positions himself as a facilitator in a learner-centred pedagogy by encouraging Nqobile to work with the other students. However, seeing her hesitancy (line 228), he identifies himself as a tutor who is sensitive to the needs to his students and he engages in a one-on-one interaction with Nqobile in which he sets up a link between the speeds Nqobile has attended to and the derivative symbols $\frac{dx}{dt}$ and $\frac{dy}{dt}$. Nqobile attends to the Tutor's talk by answering and asking questions. Yet she does not erase her original answer in Figure 8.3, suggesting that she chooses to adopt a subject position outside of the valued subject position of a student who follows the instruction to use implicit differentiation.

8.5 Discussion of the student action on the Car Problem

With the exception of Nqobile, the students occupy their positioning as students who follow the five prescribed steps in the boxed text. Yet they attend more to some steps than others. Their attention to "draw a diagram", "write down a formula linking the variables" and "differentiate" in the boxed text enables their action. In both groups the students are busy drawing, writing, differentiating and substituting from the beginning, action that contrasts with that on other problems, for example when drawing the graph for the Flu

Virus Problem students talk about and use gestures to represent their answers rather than writing and drawing (this action on the Flu Virus Problem is presented in Chapter 10). In this initial action on the Car Problem the students act like school students solving word problems as they control the one-way movement from the task context to the mathematics by transforming "the words back into the … algebra that the writer was thinking of" (Gerofsky, 2004, p.33) and practise "an algorithm recently presented in their maths course" (p.33).

The students pay less explicit attention to steps that require them to set up links between the mathematical functions and the task context. I have argued that Jeff, Shae, Darren (and later Hanah) do have some control over these links and they can make these explicit when required to explain, but they do not have complete control. Rather, they are enabled to produce a correct answer by making a link to other social events in the Course and the texts of other related rates problems in these events. They follow the valued operational action used for other "Car Problems" in the group of related rates problems, the solving of which is modelled by the lecturers. It is possible that the students have become so familiar with following this action that their solving of related rates problems has become ritualized (Nyabanyaba, 2000).

I have argued that, after the initial movement required to draw the diagram and construct the formula using the Theorem of Pythagoras, Lulama does not control the movement of meaning across the mathematical/non-mathematical boundary. It is as if he gets "stuck" in the mathematical operations, rather than in the task context. Getting "stuck" in the task context is a problem identified by both Straehler-Pohl (2010, p.454) and Lubienski (2000, p.477) in their empirical work at school level. Lulama also does not control the movement of the ways of acting mathematically across social events in the Course as the other students appear to do. Furthermore, the socio-political interaction constrains Lulama from gaining this control. Shae and Jeff position themselves as authorities in the foundational practice, by dominating the speech turns (although making tentative claims) and by responding to requests for feedback from other students. The other students identify Shae

and Jeff in this way by asking them for feedback. Jeff and Shae thus control the talk, both within the group and between the students and the Tutor. This control involves, firstly, who talks and when. For example, Lulama participates in the sense that he makes claims and asks questions in the group. Sometimes the other students respond, and other times they do not attend to his talk. Also, Hanah's alternative method is not pursued as Jeff shuts this conversation down. Secondly, Jeff and Shae's control over the talk also involves controlling what is spoken about and how this content is talked about. I have suggested that the prevalence of talk about the operations involving symbols and values (with no units) as well as the ambiguous use of reference pronouns (a) slows down the communication in the group as the students try to establish some shared meaning, (b) constrains students' gaining full control over the links to the task context, and (c) constrains Lulama from establishing productive links to the task context. In this case the students' language that refers "only to numbers without contextual attachment" (Lubienski, 2000, p.467), without the necessary links to the task context, is constraining rather than enabling as Lubienski would argue.

The students in Group 1 adopt the valued style of students in a learner-centred classroom in that they ask questions about and explain solutions. Yet the analysis suggests that this does not necessarily promote participation in the valued mathematical ways of the practice for all students. Rather, the asymmetrical power relations mean that some students control who talks and when, as well as what is talked about and how this content is talked about.[123]

In both groups one student provides an alternative method for solving the Car Problem. However, the reason why the alternative method works is not followed up. Firstly, this can be attributed to the students' occupying the subject position of students who follow the textual instructions. Secondly, following up why the alternative method works involves moving back into the task context to consider the speeds of the cars as constant. It is

[123] I note here that both Jeff and Shae have English as a home language and attended a former White and independent school respectively. Lulama, on the other hand, attended a former Black school and identified isiZulu as his home language. However, it is not possible from this study to draw conclusions about the role of language and educational background in the interaction.

possible that Jeff and Hanah in Group 1 are following the one-way movement from the task context to the mathematical representations, a typical movement in the genre of mathematical word problems. Moving back to the task context would disrupt the valued way of acting in this genre. Thirdly, the two students who provide alternative solutions position themselves outside of the group discussions by working individually. Both students follow the lead of students who occupy more powerful positions in the foundational practice (Jeff in the case of Group 1 and the Tutor in the case of Group 3) and change to using the prescribed method. Yet Nqobile's failure to erase her first attempt suggests that she is, to a certain extent, resisting her positioning as a foundational student.[124]

In both Groups 1 and 3 the Tutor identifies himself as an authority in the foundational practice in the sense that he reproduces the imperatives in the text to follow the prescribed method. He also promotes the link to the genre of word problems by naming the problems as "kinda like word sums" (line 1b, Group 3) and encouraging the students to use the valued operation, that is, implicit differentiation. In discussion with Nqobile in Group 3 the Tutor shuts down the opportunity to move back into the task context to investigate her method. Yet in his interaction with Nqobile and Shae the Tutor positions himself as a facilitator in a learner-centred pedagogy by asking about their work and listening to their explanations. He also identifies the students as able to evaluate themselves and to request assistance without him having to assess their individual solutions. Yet it seems the asymmetrical power relations between the Tutor and the group and between the students themselves constrain Lulama from occupying this positioning.

8.6 The developing argument about the student action

The first detailed analysis of student action presented in this thesis highlights issues related to the following ways of acting mathematically (and the complex interplay between them);

[124] Nqobile is one of the two students who withdrew from the Foundational Course before the end of the academic year.

the ways of operating on mathematical objects, the movement of meaning across the mathematical/non-mathematical boundary and between social events within the foundational practice, the socio-political interaction, and the educational ways of talking in a learner-centred pedagogy.

This analysis suggests that students are enabled to participate in the foundational practice when they have steps to follow and mathematical operations to perform. I have suggested that the students control the initial one-way movement from the task context to the mathematical diagram and formula, and are enabled by acting like students solving mathematical word problems. Yet there is an absence of full control over the to-and-fro movement required throughout the problem-solving process. Without this full control, the students are still enabled to solve the problem by making a link to the texts of related rates problems in other social events in the Course and by following the ritualized action of solving related rates problems used in these events. The possibility of solving the problem using this ritualized action, together with the shutting down of the use and discussion of alternative problem-solving methods to the five prescribed steps, suggests a discontinuity to the valued action in advanced mathematics.

The students' positioning of themselves and one another contributes to asymmetrical power relations between the five students in Group 1 and between the students and the Tutor. On the one hand, two students identify themselves and are identified by others as authorities in the foundational practice and these students control who talks (in the group and in interaction with the Tutor), when participants talk, the content of the talk, and how this content is talked about. On the other hand some students position themselves as hesitant to participate in the group, and their challenge to the valued mathematical ways of acting in the foundational practice is not taken up.

The students explain their answers using the educational talk of a learner-centred pedagogy. Yet the nature of this talk (for example, the verbal left-to-right descriptions of the formulae and the verbal descriptions of the operational action), in interaction with the asymmetrical

power relations in the group, means that the pedagogy does not enable access to foundational mathematics for all students.

The discussion so far indicates that it is not possible to talk about one way of acting as either enabling or constraining of participation in foundational practice. Rather, the student action is a complex interplay of the different ways of acting mathematically presented here. Not only is participation in foundational practice complex for the student, but this analysis also points to how the Tutor balances different roles; on the one hand a prescriptive role in making the students use the prescribed method, and on the other hand a facilitative role that recognizes students as taking responsibility for their learning.

CHAPTER 9: RESULTS (PART 2b)
"…WE USED THIS IN GRADE 12 GUESS IT'S THE SAME THING"

9.1 Introduction to this chapter

In this chapter I present the student action on question (d) of the Chemical Reaction Problem (see Appendices D and Q for the problem and Appendices O and P for the summary of the student action). Solving this problem does not require the student to cross the mathematical/non-mathematical boundary and make links to the task context of the chemical reaction. Rather, this question represents continuity in the movement of meaning between school mathematics and foundational practice in terms of the mathematical object (the quadratic function) and its graphical and algebraic representations, and in terms of the valued ways of acting mathematically on the quadratic function. These ways of acting involve making links between symbols in the general algebraic formula and properties of the parabola graph in the problem text, and acting operationally using substitution into the general formula for a quadratic function. In this chapter I present the action of both Groups 2 and 3 on question (d), as the contrast between the two groups in terms of their control over the timing of the link to school mathematics and over the movement of meaning in the valued mathematical ways allows me to identify action as enabling or constraining.

9.2 Summary of the student action on question (d) of the Chemical Reaction Problem

In Group 2 some of the students do not make an initial link to school mathematics, but the student Siyabulela makes the link to the general quadratic formula $y = ax^2 + bx + c$ and the

operational action of solving simultaneous linear equations at school. He explains to the group how to set up two linear equations by making clear links between the points on the parabola graph and the variables x and y in the general formula. The other students listen to Siyabulela and are enabled to answer the question. They listen to, but do not draw on, the assistance provided by the Tutor.

In contrast to Group 2, more than one student in Group 3 controls the timing of the initial link to the content of school mathematics when they identify three equivalent quadratic formulae from this practice. However, a disagreement about which formula to use results in the students severing this link to school mathematics and adopting a method for finding the equation of the parabola that is used in other social events in the Foundational Course. They do not, however, have sufficient control over the movement of meaning across these events to solve the problem using this method. They re-establish the link to school mathematics following the Tutor's validation of the use of the algebraic formula $y = a(x - r_1)(x - r_2)$ from this practice, and with assistance from the Tutor who makes links between the points on the graph and the variables in the formula. However, the socio-political interaction between the students constrains some students from using the Tutor's suggested method correctly.

9.3 Student action on question (d), Chemical Reaction Problem (Group 2)

9.3.1 Attending to some properties of the graph and not others as constraining

Initially, Lungiswa and Bongani pronounce answers of "$-t^2$", suggesting firstly that they are attending to the independent variable as time (t), the representation of a parabola graph as a quadratic equation (thus adopting the subject position of students who follow the relevant textual cues), and the maximum turning point of the graph. They are not, however, attending to the vertical and horizontal translations of the graph. In Transcript 9.1 Siyabulela, who has been working alone, enters the discussion and evaluates Lungiswa's next step. In this discussion the Siyabulela and Lungiswa shift between talking in English, Sesotho (lines 632, 637a, 637b, 639, 640, 643d and 643e) and Setswana (lines 643a and

643b). Since there are similarities between Sesotho and Setswana, some of the talk in lines 639 and 643c can be identified as being in either of these languages.

Transcript 9.1: Chemical Reaction Problem, question (d), Group 2, lines 628 to 645

628	Lungiswa:	(unclear, minus 2 *t* {-2*t*}… ja minus 2 *t?*)
629	Siyabulela:	Ja … minus 2 *t* squared {-2t^2} ja↑ (unclear) ja minus 2 t squared ((*He has written m′(t) = -2t^2 under his answer of t^2 that is crossed out*))
630	Lungiswa:	2 *t* squared {2t^2}?
631	Bongani:	2 *t* squared {2t^2}? Why? ((*Looking at Siyabulela*))
632	Siyabulela:	Ha wa bona ha ba e beha moo… ha ho na *t* square {t^2}? (Didn't you see when they put it there … there was no *t* square {t^2}?)
633	Lungiswa:	*t* squared {t^2}and then derivative (unclear) … gradient (unclear)
634	Lungiswa:	Hmmm ((*Looking at the graph in the Resource book*))
635	Siyabulela:	(unclear, Sesotho) our function ((*Stretching across and pointing at the graph in Lungiswa's Resource Book*))
636	Lungiswa and Bongani: Ja	
637a	Siyabulela:	Ea le tsena ke tsona di functions … e re 'na ke di nke (Ja these are our functions … let me take them) ((*Picking up his answer book and then putting it down again*))
637b	Siyabulela:	wena u re *t* ena ke e behe mona … e tlo re fa straight line (if you say we put a *t* here … it is going to give a straight line) ((*Pointing to the graph in Lungiswa's Resource Book*)) the derivative second derivative of this ((*Tapping his finger on the graph*)) the straight line↑ ((*Showing a straight line with his finger on Lungiswa's book*))
637c	Siyabulela:	… …((*Looking at Lungiswa*)) this thing is ((*Pointing to the graph in Lungiswa's Resource Book*))
638	((*Mpumelelo is watching Siyabulela as he speaks in line 637*))	
639	Lungiswa:	Ena ke gradient ja (this is gradient yes) the gradient yeah this thing ((*Pointing to the graph with her pencil, and then to something in her answer text*)) but ketlameile ke drawe (but I must draw) for <u>this</u> one ((*Circling over graph with her pencil, then looking up at Siyabulela*)) … which is
640	Siyabulela:	[No no no ka re wena <u>this</u> one u e batlileng (I'm saying you found this one) ((*Pointing to -2t in Lungiswa's answer book?*)) is the derivative of that ((*Pointing to the parabola graph in Lungiswa's Resource Book?*))]
641	Bongani:	[ja ja … ja ja ((*Listening to Siyabulela*))]
642	Lungiswa:	Mm?
643a	Siyabulela:	Ha ke itse kgore o irile right↑ (This one I don't know whether you did it right↑) ((*Pointing to the equation -2t in Lungiswa's answer book?, then pointing to the graph*))
643b	Siyabulela:	Ha u nna (if you do it) this way it will turn out different … highest power of 2 there … so when you differentiate you're gonna get highest power of 1 … so this↑ ((*Pointing to -2t in Lungiswa's answer book?*)) is the derivative of this ((*Pointing to the parabola graph in Lungiswa's Resource Book?*))

643c	Siyabulela:	so etjena ke (if it's like this it is) a second derivative of that thing ((*Pointing to Lungiswa's book*))
643d	Siyabulela:	so wena o batla equation ea this thing … e tshwanetse e be square (so you are finding the equation of this thing .. it must be a square) i-question ((*Running his finger over the parabola graph*)) is squared
643e	Siyabulela:	re tshwanetse re e batle ka di piece piece (we should do it in pieces)
644	((*While Siyabulela is talking in line 643 Bongani is rubbing out in his answer book, and then watching Siyabulela again. Vuyani is watching Siyabulela and then looking down at his answer book*))	
645	Lungiswa:	((*Pointing to the graph briefly*)) right … mm mm ((*Nodding her head*))

Lungiswa now pronounces an answer of "minus 2 t {$-2t$}" (line 628). She is looking operationally at the derivative function and interprets the punctuation symbol $'$ in $m'(t)$ as an instruction to operate on the function $m'(t) = -t^2$ by differentiation. This is suggested by her description of a sequence of actions linked by "and" in "t squared {t^2} and then derivative" (line 633). Her reference to "gradient" in line 633 suggests that she is linking the operation of differentiation on the algebraic formula to finding the gradient of the graph.

Siyabulela has been using the general quadratic formula $y = ax^2 + bx + c$ from school mathematics and announces the first term in his quadratic formula as "minus 2 t squared {$-2t^2$}" (line 629). The different pronouncements in lines 628 and 629 enable a comparison of the methods, in which students adopt the position of students in a learner-centred pedagogy by "explaining answers" and "criticizing ideas" (Foundational Course Resource Book, 2007, p.5). Siyabulela evaluates Lungiswa's pronouncement and provides feedback by linking the two different algebraic expressions $-2t$ and $-t^2$, their graphical representations and the mathematical names of the functions that they represent. Siyabulela's talk suggests that he views the functions structurally as objects, for example, he talks about more than one function in "our functions" (line 637a). In line 637a he says "e re 'na ke di nke (let me take them)"; it is not possible to tell whether his reference pronoun "them" refers to his answer book (which he picks up) or to the actual functions. In the latter case one could argue that he is viewing the functions structurally. Siyabulela's structural view is further evidenced by his naming of the objects and their representations; in line

637b he identifies the algebraic expression $-2t$ with a "straight line" graph and the words "second derivative". Adopting a structural view enables him to distinguish between the second derivative/straight line graph/algebraic expression $-2t$, and the first derivative/parabola graph/algebraic expression $-t^2$. Yet Siyabulela also adopts an operational view of the two functions by talking about the operation of differentiation that links the first and second derivative functions; "… highest power of 2 there … so when you differentiate you're gonna get highest power of 1 … so this↑ ((*Pointing to –2t in Lungiswa's answer book?*)) is the derivative of this ((*Pointing to the parabola graph in Lungiswa's Resource Book?*)) (line 643b). Lungiswa's response (line 645) to Siyabulela's explanation that adopted first a structural and then operational view of the function suggests that she is enabled by this explanation.

In Transcript 9.1 Siyabulela is addressing Lungiswa and uses a mixture of English, Sesotho and Setswana, using English for mathematical terms, for example, the word "straight line" in line 637b. In their interviews for this study Siyabulela identified Setswana as his home language and Lungiswa identified Sesotho as her home language. Yet these two students have a wider audience; Vuyani and Mpumelelo's body language suggests they are listening and Bongani responds by erasing his earlier attempt. Vuyani, whose home language is isiZulu, indicated in his interview that he understands some Sesotho and asks the other students in the group if he does not understand. Mpumelelo and Bongani are both isiXhosa speaking. Their knowledge of other African languages was not discussed in their individual interviews. The discussion that follows Transcript 9.1 suggests that Vuyani, Mpumelelo and Bongani are not constrained by Siyabulela and Lungiswa's talk in their home languages.[125]

[125] As noted in Section 4.5.3, the analytic framework in this study does not allow a more in depth investigation of the students' use of more than one language in their discussions. However, I note at this point that many Black students are able to understand a variety of spoken African languages, particularly those that belong in the same language groups, for example, isiXhosa and isiZulu or Sesotho and Setswana.

9.3.2 Links to objects and ways of acting mathematically in school mathematics as enabling

Having explained to Lungiswa where she has made her mistake, Siyabulela returns to his earlier pronouncement of "minus 2 *t* squared $\{-2t^2\}$" (line 629) and begins to explain his method (his full written answer is provided in Figure 9.1). He accompanies his verbal explanation with gestures in which he points to the graph in the text of the Chemical Reaction Problem, and for this graph the reader is referred to the foldout Appendix Q.

Transcript 9.2: Chemical Reaction Problem, question (d), Group 2, lines 651 to 676

651a	Siyabulela:	(unclear) initially it is *a x* squared plus *b x* plus *c* $\{ax^2 + bx + c\}$
651b	Siyabulela:	(unclear) but the *y*-intercept is nought
652	Lungiswa:	Uh…hum↑
653	Siyabulela:	So when *x* is 2 *y* is 8
654	Lungiswa:	8
655a	Siyabulela:	Ja … so when is 4 *y* is nought ((*Pointing to the t-intercept at 4 and then moving his pen along the horizontal axis to the origin*))
655b	Siyabulela:	so you solve a simultaneous equation
655c	Siyabulela:	unclear
656	Lungiswa:	O re formula ke eng? (What do you say is the formula?) ((*Preparing to write it down in her book*))
657	Siyabulela:	*a x* squared plus *b x* plus *c* $\{ax^2 + bx + c\}$
658	Bongani:	Siyabulela andikuva skhokho (I can't hear you man)
659	Siyabulela:	Oh … no I was talking Japanese there I know ((*Smiling at Bongani*))
670	Lungiswa:	[((*Laughing*))]
671	Siyabulela:	[((*Stretching across and pointing at Lungiswa's* graph)) That … you know that general formula] ... Japanese gape ne (again) ... the formula of this thing … *a* squared plus *x* plus *c* $\{a^2 + x + c\}$ is ((*Dropping his pen but quickly picking it up again*))
672	Bongani:	Ja
673	Siyabulela:	(unclear) so you are going to need your *a* and *b* so you can ((*Using his pen and points to graph in Lungiswa's book*))… ja find the equation of that thing … so the best way is you know when *x* is 2 or when *t i*s 2 ja then your *y* is 8 ((*Shows reading off of 8 on Lungiswa's graph*))
674	Bongani:	Uthi (you say) the equation is what? ((*Writing as he looks at graph and talks to Siyabulela*))… *a x* squared $\{ax^2\}$↑
675	Siyabulela:	Write in terms of *t a t* squared … plus *bt* plus *c* $\{at^2 + bt + c\}$
676	((*Bongani starts to write* at^2 *is his answer book*))	

Siyabulela begins by pronouncing the general form of the quadratic function ("it") as "*a x* squared plus *b x* plus *c* $\{ax^2 + bx + c\}$" (line 651a), suggesting that he controls the movement of content between the school and foundational mathematics practices. His instruction to "solve a simultaneous equation" (line 655b) suggests that he also controls the movement of the valued way of acting operationally across the boundary between these practices. He explains the operational action of substitution into the general formula by linking the two representations of the function, that is, the variables *x* and *y* in the algebraic formula (variables which he then changes to *t* and $m'(t)$) and the points on the graph. He makes these links by pointing and sketching lines on the graph to show how to read off the points on the graph as in lines 655a and 673. In this sense Siyabulela adopts the subject position of a student who links different representations of a function, a valued action in school mathematics and in calculus reform. Siyabulela's explanation is given in English, possibly as a response to Bongani's comment isiXhosa; "Siyabulela andikuva skhokho (I can't hear you man)" (line 658).

Figure 9.1: Siyabulela's written answer for question (d), Chemical Reaction Problem

$$m'(t) = at^2 + bt$$

when $t = 2$: $m'(t) = 8$

$$8 = 4a + 2b$$
$$b = 4 - 2a \quad \ldots (1)$$

when $t = 4$: $m'(t) = 0$

$$0 = 16a + 4b$$
$$0 = 16a + 4(4 - 2a)$$
$$0 = 16a + 16 - 8a$$
$$-16 = 8a$$
$$a = -2$$
$$b = 4 - 2(-2)$$
$$b = 8$$

$$\therefore m'(t) = -2t^2 + 8t$$

The other four students in Group 2 appear to be enabled by Siyabulela's explanation and take up his way of operating using simultaneous equations. They pursue this method even

in the light of the Tutor's proposal of the algebraic formula $y = a(x - r_1)(x - r_2)$ which is equivalent to the formula used by Siyabulela and also used in school mathematics ("There is a formula that uhm might help ((*Writing down the formula involving the x-intercepts on a piece of paper silently, all the students are leaning forward to look*)) … … do you remember this↑ from school↑" (line 689)). Bongani questions the Tutor's use of *x* rather than *t* for the independent variable, a correction that the Tutor acknowledges without changing his written formula. The Tutor explains the substitution into his proposed formula by making links between the variables in the formula and the parabola graph in a similar manner to that used by Siyabulela. In contrast to the Tutor's insistence that the students use the prescribed steps in the Car Problem, in this question the Tutor identifies himself as a facilitator who suggests a possible method; he represents his proposed formula using the *x*-intercepts as one possible formula to be used in this question ("I'm not saying that's the only way to do it", line 709).

9.3.3 Links to the operation of solving simultaneous linear equations in two variables from school mathematics as enabling

Having settled on using the ways of acting mathematically as proposed by Siyabulela, the students explicitly link the action to their final year of school mathematics, for example, Lungiswa states, "…we used this in matric[126] guess it's the same thing" (line 721) and Vuyani notes, "I remember the simultaneous equation from grade 12 ((*Bongani and Vuyani laugh*)) grade 12 ..." (line 747a). This link enables them to talk about the operations on the two equations of the form $y = ax^2 + bx + c$. In the following statement, which is structured in English, but draws on some words from isiXhosa, Vuyani describes the operational action on one of the linear equations; "umultiplaya (you multiply) i-equation (the equation) number 1 ngo (with) 2 in order to get 4*b*" (line 731). Elsewhere his description of the operations is in English only; "okay here we have to multiply by ... by 2 in order to get 4*b* and we have 4*b* here and then you will subtract there in order to get *a* … ((*Pointing to the equations in Bongani's book*))" (line 747b).

[126] "Matric" is another word for grade 12.

When performing the necessary operational action, the students attend mainly to the operation of substituting the points from the graph into the formula (which is enabled by pointing to the graph) and on what to "multiply" and "subtract" as suggested by Vuyani's descriptions in line 747. They do not attend much to the conventions for writing mathematical symbols, for example Bongani writes, "when $t = 2$, $a4 + 2b = 8$", and Lungiswa writes her equations in a similar way. Although Vuyani corrects Bongani's writing verbally, Bongani does not correct his written work. All of the students proceed, using the ways of acting mathematically proposed by Siyabulela, to obtain the required formula $m'(t) = -2t^2 + 8t$.

9.4 Discussion of the student action on question (d) of the Chemical Reaction Problem (Group 2)

In Section 9.2 I have suggested that, initially, some of the students are constrained by attending to some properties of the parabola graph and not others and by not controlling the timing of a link to school mathematics. In addition, Lungiswa is constrained by adopting an operational view of the derivative function $m'(t)$. Siyabulela's enabling explanation of her difficulty draws on both a structural and operational view of derivative functions, supporting Sfard's (1992) argument about the importance of a student being able to switch between the two ways of looking when doing mathematics.

Siyabulela controls both the how and the when of the movement of meaning across the boundary between school and foundational mathematics; using the general formula for a quadratic function $y = ax^2 + bx + c$, setting up the two linear equations by making links between the algebraic and graphical representations, and operating on the equations using multiplication and subtraction. In this action he adopts the required subject position of a school mathematics student. His action enables him to produce a correct solution and he explains his action in a manner that enables the other students. In this interaction Siyabulela's identifies himself as an authority in the foundational practice, both in evaluating Lungiswa's action and in explaining his own action. The Tutor identifies himself

as an authority in the practice in a similar way to Siyabulela by suggesting a formula from school mathematics and explaining its use. Instead of insisting that the students use his suggested method (as is the case with the Car Problem), the Tutor offers it as an alternative way of acting to Siyabulela's way of acting and he recognizes the agency of the students to choose from the two methods. This result supports the significance given by Brodie and Pournara (2005), Adler (1997) and Davis (2001) to the need for a mathematical authority in a learner-centred pedagogy; in this case both the Tutor and Siyabulela suggest a way forward and Lungiswa, Vuyani, Bongani and Mpumelelo follow the lead of Siyabulela.

9.5 Student action on question (d), Chemical Reaction Problem (Group 3)[127]

9.5.1 Attending to some properties of the graph and not others as constraining

Thokozile and Ndumiso begin their work on question (d) in a similar way to the students in Group 2 as they name an expression "negative *x* squared $\{-x^2\}$" (lines 232 and 233). They attend to the textual cues about the derivative function in the problem text that link Sentence 3, the graph and question (d). They also attend to the maximum turning point of the graph, suggested by the emphasis on the adjective "down" in Thokozile's explanation; "because the parabola is facing down" (line 232). Unlike the students in Group 2, they do not attend to the independent variable as *t* (rather than *x*).

The pronounced answer of "negative *x* squared $\{-x^2\}$" is rejected by Kelsa who uses the operation of substitution to support her evaluation. She speaks slowly, attending to the point (2,8) on the graph and substituting $x = 2$ into the expression $-x^2$ to get a value of 4; "<2 corresponds to 8 … if you say negative *x* squared ... 2 squared is 4>" (line 236). The use of substitution is a consistent feature of Kelsa's action, used either as a checking mechanism or as a way of building a formula, as discussed further in Section 9.5.4.

[127] The students Akbar and Nqobile referred to in Chapter 8 had withdrawn from the Foundational Course by October when the action on the Chemical Reaction Problem was recorded. The transcripts used in this section re-present the action of four students in Group 3; Thokozile, Ndumiso, Lwazi and Kelsa.

9.5.2 Severing initial links to quadratic formulae from school mathematics as constraining

Following the negative feedback from Kelsa in line 236 of Transcript 9.2, Ndumiso proposes another possible quadratic formula in Transcript 9.3.

Transcript 9.3: Chemical Reaction Problem, question (d), Group 3, lines 237 to 258

237	Ndumiso:	Don't you use that equation? … ((*Looking at Kelsa*)) *y* is equal to *a* then *x* minus *x* 1 *x* minus *x* 2 {$y = a(x - x_1)(x - x_2)$}?
238	Thokozile:	No but now for
239	Kelsa:	*y* is equals to *ax* plus *b* {*ax* + b} ax squared plus *b* squared [plus *c* {$ax^2 + b^2 + c$}?]
240	Thokozile:	[plus *c*? ((*Frowning*))]
241	Kelsa:	That's the equation for parabola↑
242	Ndumiso:	What's this equation [[for?]] ((*Getting ready to write something at the top of his page*))
243	Thokozile:	[[for the parabola? ((*Frowning*))]]
244	Thokozile:	No man isn't it? ... what's the *x* squared for? ((*Writing something in her book*))
245	Ndumiso:	Don't you know this equation here? *a x* minus *x* 1 […] *x* minus *x* 2 {$y = a(x - x_1)(x - x_2)$} ((*Writing* $y = a(x - x_1)(x - x_2)$ *at the top of his page*)) isn't it for uh
246	Thokozile:	[oh ja]
247	Thokozile:	*x* squared {x^2} ja ((*Both Thokozile and Kelsa sit up, Thokozile glances to the side briefly*))
248	Lwazi	There is an equation like that
249	Ndumiso:	For what though? ((*Looking at Lwazi*))
250	Lwazi:	But that's not it though
251	((*Ndumiso and Kelsa laugh, Thokozile is looking seriously at Lwazi*))	
252	Lwazi:	[No no really↑] the *a* is right at the beginning↑ but the stuff in the middle isn't ((*Pointing to Ndumiso's equation* $y = a(x - x_1)(x - x_2)$))
253	Thokozile:	[You know what though? the equation ja ((*Tapping her pen on the equation in Ndumiso's book*))]...
254	Ndumiso:	It IS
255	Lwazi:	It's not
256	Ndumiso:	I know it is
257	Lwazi:	It's not … I'm telling [you it's not]
258	Thokozile:	[Okay guys uhm↑] … … ((*Tossing her pen around above Ndumiso's answer book*)) the equation for a parabola isn't it just *x* squared {x^2}?

Ndumiso pronounces "that equation" $y = a(x - x_1)(x - x_2)$ by naming the symbols from left to right in line 237, a formula he will have encountered at school (he makes this link explicit in line 523) and in the Foundational Course. He receives negative, content-free feedback from Thokozile in the form of "No" (line 238) (she is interrupted by Kelsa) and a frown (line 243). Kelsa's negative feedback in line 239 takes the form of an alternative quadratic formula ($y = ax^2 + bx + c$) from school mathematics and the Foundational Course. She supports this choice by stating it as fact with no supporting evidence; "That's the equation for parabola↑" (line 241). Although the rising intonation at the end of Kelsa's statements in lines 239 and 241 may represent a request for feedback, the emphasis on the reference pronoun "that's" in line 241 represents her version as the only one. Her emphasis on this equation persuades Ndumiso that his proposed equation represents something other than a parabola; in line 242 he asks, "What's this equation for?" and later he asks Lwazi, "For what though?" (line 249).

The students have made a link to two quadratic formulae from school mathematics (or from the Foundational Course), but there is an absence of a link between these formulae as representations of the same type of graph. This absence is evidenced by Lwazi's evaluation of Ndumiso's proposed formula which he represents as a statement of fact with no supporting argument; he claims that, "there is an equation like that" (line 248) but "that's not it though" (line 250). In these statements he is using emphasis on the reference pronouns to distinguish Ndumiso's formula $y = a(x - x_1)(x - x_2)$, ("that"), from the quadratic formula for the parabola ("it"). It emerges in a later discussion between Lwazi and the Tutor that Lwazi is identifying a third quadratic formula from school mathematics, that is, the formula $y = a(x - p)^2 + q$.

Transcript 9.3 ends with Ndumiso using emphasis and personal opinion to support his choice of the formula $y = a(x - x_1)(x - x_2)$, for example, "it IS" (line 254) and "I know it is" (line 256). This is accompanied by Lwazi's negative evaluation, also based on emphasis and personal opinion, for example, "it's not" (line 255) and "It's not … I'm telling you it's not" (line 257). This exchange is interrupted by Thokozile ("[okay guys uhm↑]" (line 258)

who repeats her earlier pronouncement of "*x* squared {x^2}" (line 258) as the formula. From this point the two quadratic formulae from school mathematics proposed in Transcript 9.3 are an absence in the group discussion until the Tutor reintroduces them (see Section 9.5.7).

There are a number of possible actions that constrain the students from pursuing these initial links to school mathematics. Firstly, the students do not identify the two formulae as representing the same function (a quadratic function) and graph (the parabola graph). It is possible that the students are viewing the function operationally in that the different operations represented in each formula suggest that the functions are different. Since the students do not link the symbols in the formulae and the points of the parabola graph, they do not view the formulae as representing the same function. Sfard (1992) suggests that an operational view of a function may constrain students from identifying formulae as equivalent, since the different formulae represent different computational processes. Secondly, the students may be adopting the style of school mathematics students in a more traditional pedagogy in which it is assumed that there is only one possible method for solving the problem. Since they cannot settle the debate about which formula in Lwazi's words, is "it" (line 250), they sever the link to school mathematics and attend rather to Thokozile's alternative suggestion. Thirdly, the arguments put forward by Kelsa, Ndumiso and Lwazi to support their claims and evaluations are located in personal opinion, statements of fact or alternative claims. There is no expectation in the group that an argument be based on the mathematical properties of the function and this constrains the development of a shared basis for debate.

Lastly, the severing of initial links to school mathematics can be explained with reference to the socio-political interaction in Group 3. The discussion represented in the full transcript for Group 3's work on the Chemical Reaction Problem suggests some individual competition between Kelsa, Lwazi and Ndumiso as they identify themselves as being able to solve the problems. This socio-political interaction may prevent students from investigating the links between their formulae. For example, when Lwazi and Ndumiso request the help of the Tutor, Kelsa responds with "we don't <u>have</u> a problem" (line 447),

referring to the two females in the group, herself and Thokozile. Later in his interaction with Ndumiso and Lwazi, the Tutor confirms Lwazi's claim that there is a third form of the quadratic formula ($y = a(x - p)^2 + q$) and Lwazi identifies himself excitedly as a student who does know the quadratic formula for the parabola equation; "yes↑ … yes↑ … yes↑ … that's the one" (line 533). However, Ndumiso identifies Lwazi personally ('you") as someone who is not able to use this equation; "But you can't even use it" (line 537).

9.5.3 Making a link to other social events in the Foundational Course as constraining

In Transcript 9.4 the students pursue Thokozile's suggestion of "*x* squared {x^2}" (line 258) for the quadratic formula.

Transcript 9.4: Chemical Reaction Problem, question (d), Group 3, lines 259 to 262

259	Ndumiso:	Parabola?
260	Lwazi:	But that's moved ... 4 units to the [left↑]
261	Ndumiso:	[That's a] that's a general parabola↑ that's the easy one … [[this has moved]] ((*Showing shift with his fingers over the graph, looking at Thokozile as he speaks*))
262	Thokozile:	[[Hang on]] then we gonna do the whole movement thing ((*Demonstrating movement with her hand on the* graph)) where you gonna say [minus 4]

In lines 260 to 262 Lwazi, Ndumiso and Thokozile attend to the required parabola graph as a graph that has "moved" (lines 260 and 261) from the position of the "general parabola" (line 261) of the form $y = x^2$. Here they are looking at the function structurally as an object that can be "moved", suggested by the reference pronouns "this" and "that" used to distinguish between the "general parabola" (p.261) and the parabola graph in the problem text. In attending to the function in this way they are making a link to a method of operating on functions using transformations (named "the whole movement thing" by Thokozile, line 262) that is used in other social events in the Foundational Course. This approach, which the students would not have encountered in their study of school Mathematics, is introduced in the first week of the Foundational Course and involves finding quadratic functions of the form $y = a(x - p)^2 + q$ (where a is +1 or −1 only) using translations and

reflections.[128] A problem from the Course material that uses this approach is provided in Figure 9.2.

Figure 9.2: Question 6, Workshop 1, Foundational Course Resource Book, 2007, p.19

6) On the same system of axes sketch **as quickly as possible** the graphs of the following functions , indicating where each graph cuts the *x* and *y*-axes (label them clearly):

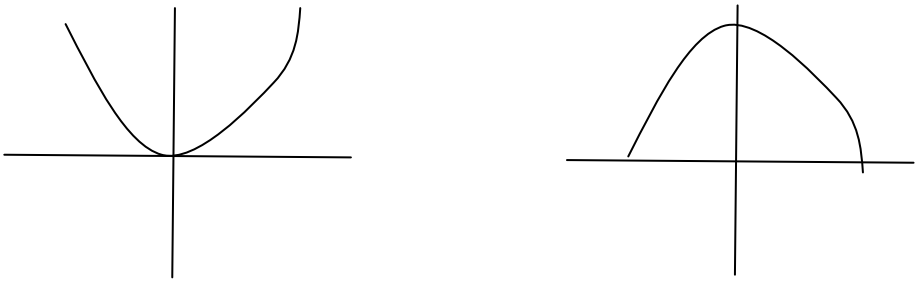
$f(x) = x^2$ $f(x) = x^2 - 4$ $f(x) = (x-4)^2$

$f(x) = -x^2$ $f(x) = x^2 + 10x + 25$

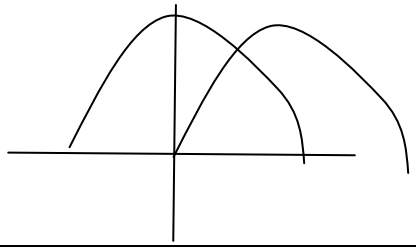
(For this last graph, think of the **quickest** way to draw the graph)

The discussion that follows in Transcript 9.5 suggests that Thokozile's action is constrained by an absence of full control over the movement of meaning of the valued mathematical ways from other social events in the Course where "the movement thing" (line 262) is used.

Transcript 9.5: Chemical Reaction Problem, question (d), Group 3, lines 269 to 272

269	Thokozile:	I know ((*looking at Ndumiso*)) but I'm saying we gonna <u>start</u> from the general one okay … let's say there … is going to be a *x* squared {x^2} right? [((*Drawing the rough sketch on the left below*)) … … but then it's negative]… wait …so it's gonna be the other way round ((*Drawing, quickly, the sketch on the right*)) 
270	Kelsa:	[but then we working with <u>zero</u> to 4↑]
271	Kelsa:	[[But <u>this</u> graph it <u>increases</u> and then it <u>decreases</u>]]

[128] This way of operating on functions using transformations features in the new outcomes-based curriculum for school Mathematics in grades 10 to 12. This method is not part of the old content-based school curriculum that the students in this study followed in their final three years of schooling (see Sections 2.3.4 and 2.3.5), and my experience of lecturing these students suggests that they encountered the method of transformations for the first time in the Foundational Course.

272	Thokozile:	[[And then the whole shift ((*Pointing to the graph in front of Ndumiso*))]] … hey? ((*Looking at Kelsa and then frowning)*)… but then we are going to do the whole shift↑ to ... 4 units to the right ((*Demonstrating the shift on Ndumiso's graph, he is watching*)) … I think↑… then you should get this graph ((*Drawing the graph below on the page*)) … … which is gonna be plus 4↑… so it's *x* squared … plus 4 {$x^2 + 4$}↑ minus *x* squared plus 4 {$-x^2 + 4$ or $-(x^2 + 4)$}↑ 

In her pronouncement of the function $y = x^2$ as the "general one" (line 269) Thokozile is assuming that the value of the coefficient *a* of the squared term in the quadratic formula $y = a(x - p)^2 + q$ is 1, as was the case for the parabolas in problems such as that in Figure 9.2. Thokozile draws rough sketches to represent the transformation of this function, reproducing the method used by lecturers in the Foundational Course. She attends to the maximum turning point of the parabola graph in the problem text and pronounces that the coefficient of x^2 ("it's") is "negative" (line 269). In her second sketch in line 269 she has reflected the graph of $y = x^2$ horizontally, but does not attend to the line of reflection and thus the position of the turning point. Her reflection includes a vertical translation. This constrains her from quantifying the vertical translation as she does for the horizontal translation. In line 272 she attends to the horizontal translation, but it appears that she attends to the *t*-intercept of (4,0) rather than the turning point (2,8) and thus does not quantify this translation correctly, pronouncing that the graph has been translated "4 units to the right". In her formula, stated verbally in a tentative tone and not written down, as "minus *x* squared … plus 4 {$-x^2 + 4$ or $-(x^2 + 4)$}↑" (line 272), she uses the word "plus" incorrectly to represent the horizontal shift to the right. Furthermore, the verbal presentation of this formula does not allow Thokozile to attend to the appropriate use of symbols such as brackets for representing the transformations. I return to Thokozile's formula after a discussion of Kelsa's evaluation of this formula.

9.5.4 Kelsa's use of substitution as constraining

The students interpret Thokozile's verbal description of the quadratic formula in line 272 in writing as $y = -x^2 + 4$. Transcript 9.6 begins with Kelsa evaluating this formula using the mathematical operation of substitution of the value $x = 2$ into the formula (line 276). Her emphasis on the *y*-value of "zero" (line 276) serves as a negative evaluation of Thokozile's formula.

Transcript 9.6: Chemical Reaction Problem, question (d), Group 3, lines 276 to 295

276	Kelsa:	No it doesn't↑ … minus 2 squared plus 4 gives you zero $\{-2^2 + 4 = 0\}$
277	Thokozile:	Minus 2 squared ((*Looking across briefly at Kelsa*)) the *x* is ... ... this minus is here ((*Pointing to her equation*)) ja minus *x* squared $\{-x^2\}$
278	Kelsa:	So why don't you just say *x* squared plus... ... plus 4 $\{x^2 + 4\}$? … cause your minus putting your minus there means negative 1 times *x* squared $\{-1 \times x^2\}$ … … so you must just make it *x* squared plus 4 $\{x^2 + 4\}$↑
279	((*Ndumiso stretches and yawns as Kelsa is speaking in line 278*))	
280	Thokozile:	But you can't say it's *x* squared plus 4 $\{x^2 + 4\}$because it's ... ... ((*Seems to be talking to herself quietly as she thinks*))
281	Lwazi:	Ja it is *x* squared plus 4 $\{x^2 + 4\}$
282	Thokozile:	[It can't be *x* squared plus 4 $\{x^2 + 4\}$]
283	Kelsa	[Ja it is *x* squared plus 4 $\{x^2 + 4\}$]
284	Lwazi:	Because if you substitute 2 you gonna get 8
285	Thokozile:	I
286	Ndumiso:	Can't be *x* squared $\{x^2\}$
287	Thokozile:	((*Looking briefly at Ndumiso in line 286, and then at Lwazi*)) Ja when you are looking at the graph *x* squared plus 4 $\{x^2 + 4\}$doesn't make sense
288	Kelsa:	[It does]
289	Thokozile:	[Cause the]((*Looking up at Kelsa briefly*)) [[graph is facing downwards ((*Tracing over the graph with her pen*))]]
290	Kelsa:	[[It's facing this way]]
291	Lwazi:	[It should be negative↑]
292	Thokozile:	[When it's facing downwards] it's negative
293	Ndumiso:	Yeah minus *x* squared plus 4 $\{-x^2 + 4\}$
294	((*Kelsa is working on her calculator, Thokozile is looking at the graph in front of Ndumiso*))	
295	Thokozile:	Hey guys I don't know

Kelsa then attends only to the point (2,8) on the graph and adapts Thokozile's formula to "*x* squared plus... ....plus 4 {$x^2 + 4$}?" (line 278). This adaptation is suggested by her use of the word "just" in "you just say" and "just make it" in line 278. However, Ndumiso and Thokozile are attending to the maximum turning point of the graph; Thokozile agrees with Ndumiso that Kelsa's version of the formula "doesn't make sense" (line 287). She uses an argument based on the property of the graph to support her evaluation; "when it's facing downwards it's negative" (line 292). Here she uses the reference pronoun "it's" for both the parabola graph and the coefficient of the squared term x^2 in her formula. This lack of clarity may constrain the students attention to the value of *a*, the coefficient of the squared term in the quadratic formula.

Kelsa on the one hand and Thokozile and Ndumiso on the other cannot agree on the formula as they are attending to different properties of the graph (either the point (2,8) **or** the maximum turning point). Lwazi appears to be attending to both and agreeing with both sides in the argument. For example, in line 284 he provides evidence for Kelsa's pronouncement, "because if you substitute 2 you gonna get 8". Yet his use of and emphasis on the modal auxiliary verb "should" in his claim, "it <u>should</u> be negative↑" (line 291) suggests that he is committing himself to Thokozile and Ndumiso's argument.
At the end of this exchange Thokozile identifies herself ("I") as not knowing what action to use; "Hey guys I don't know" (line 295). She repeats this identification after using her method of transformations further; "Hey guys °I don't know°" (line 329), with the "I don't know" said quietly, and calls for the Tutor (line 331). The Tutor is busy with another group, so the students try a few more possibilities and continue to call for the Tutor at intervals as they work, suggesting that the students cannot proceed productively without the Tutor's assistance.

Throughout the exchange, even when the Tutor is present, Kelsa uses the operation of substitution for two purposes; for evaluation of formulae and for reworking these formulae. This suggests that she is viewing the function operationally, with the formula representing

operations to be performed. She evaluates Thokozile's attempt $y = -(x^2 - 4) + 8$ by talking through the substitution aloud, "you get look … zero minus four you get minus four {0 – 4 = –4} ((*Pointing to the terms in Thokozile's formula and substituting in x = 0*)) … I'll give you that plus 8↑… but then because you have done that ((*Pointing to the coefficient of* x^2 as *–1*)) you get 12 … " (line 438). Here she identifies herself as the evaluator by saying "I'll give you that". She ends by suggesting how Thokozile's formula can be adapted so that the substitution works, "… your thing works just take out the minus" (line 438). The use of the pronouns "I" and "your" suggests that Kelsa assigns ownership of the formulae to individual students and does not view the action as communal. Later, while the Tutor is working with Lwazi and Ndumiso, Kelsa and Thokozile settle on the formula $y = x^2 - 4 + 8$" (line 443 and 445). In this case the substitution of the point (2,8) works, but that of (4,0) does not, which is not pronounced. However, Kelsa introduces a new x-value, that is $x = 1$, and suggests that the corresponding y-value of 5 obtained using the formula is "realistic"; she uses her pencil on the graph to read off a value that corresponds to $t = 1$ on her graph and then pronounces, *"*... realistically this could be five right↑" (line 473).

9.5.5 The absence of links between properties of the graph identified by Lwazi and an algebraic formula as constraining

In Section 9.5.4 I have argued that Lwazi attends to and provides positive feedback on both mathematical arguments related to the maximum turning point of the graph and the substitution of the point (2,8) into the formula. His pronouncements suggest that he is also attending to other properties of the graph. For example, following Thokozile's second admission that "I don't know" (line 329), Lwazi attends (he does this twice) to the method of transformations by attending to the vertical translation; "the graph has moved 8 up … we forgot that ((*Looking at the graph and then up at Thokozile*))" (line 334). This pronouncement enables Thokozile to adapt her formula by including a "plus 8" (line 336). She produces a written expression (and not a formula) of $-(x - 4)^2 + 8$ (line 367) and then another of $-(x^2 - 4) + 8$ (line 431). These two answers suggest that she is attending to the maximum turning point of the parabola, the horizontal translation to the right (which she

still identifies as 4 units) and the vertical translation eight units up. She is, however, assuming that the coefficient of the squared term is –1 and not attending to the differences in the structure of these two expressions. She relies on substitution of the point (2,8) for evaluation; in this case she assesses the expression in line 367 on her own, but relies on Kelsa to tell her that the expression in line 431 "… doesn't work" (line 433). Lwazi does not make a link between Thokozile's expression in line 367 and the formula he has memories of from school but which he has not yet been able to pronounce, that is, the formula $y = a(x - p)^2 + q$.

Lwazi continues to use Kelsa's method of substitution to test his formulae, as suggested in Transcript 9.7.

Transcript 9.7: Chemical Reaction Problem, question (d), Group 3, lines 402 to 415

402	Lwazi:	Yes… minus 2 *x* squared … plus 4↑ … plus 8↑ {$-2x^2 + 4 + 8$} will give you the answer
403	Kelsa:	((*Laughing*)) Why would you have an equation that says plus *c* plus *c* {$+c + c$}?
404	Lwazi:	((*Looking down at his answer book and reading his equation again*)) Minus 2 *x* squared plus 4 *x* plus 8 {$-2x^2 + 4x + 8$}
405	Kelsa:	[Oh … ((*Chuckling*)) … this is sad]
406	[((*Ndumiso is stretching and yawning*))]	
407	Lwazi:	It'll give you the answer↑ I promise you
408	Thokozile:	Ja I ((*Stops working on her calculatorand sits back*))
409	Kelsa:	Which one? ((*Looking aside at Thokozile*))
410	Thokozile:	I got like minus 2 (unclear)
411	Ndumiso:	(Tutor's name)
412	Thokozile:	[2 … 2] times 2 is 4 {$2 \times 2 = 4$}, times 2 {$\times 2$} is minus *x* {$-x$} that's 8, that cancels [[that]] ((*Pointing to the terms of her expression and substituting mentally*))
413	Kelsa	[Wait] … … [[wait]]
414	((*Thokozile is talking to herself quietly as she works, something about "original graph"*))	
415	Lwazi:	And it makes sense when you look at the gradient ((*Running his pen over the graph quickly*)) because it's wider 2's gonna make it wider ((*Holding his hand out in the shape of a wide graph with minimum turning point*))
416	Kelsa:	You are wrong
417	Lwazi:	Why am I wrong?

418	Kelsa:	Ask me why
419	Lwazi:	Why? why? ((*Leaning forward, smiling, looking at Kelsa's book*))
420	Kelsa:	Because we have this point as well …. ze 4 zero … <mine works out yours doesn't>

In line 402 Lwazi pronounces a possible expression of "minus 2 *x* squared… plus 4↑ … plus 8↑ {$-2x^2 + 4 + 8$}". Kelsa evaluates this negatively, this time attending to the format of the expression "plus c plus c {$+c + c$}" (line 403) and linking to the formula $y = ax^2 + bx + c$ she introduced initially. Her evaluation identifies Lwazi as doing something silly, suggested by her laugh and her phrasing of her evaluation as a rhetorical question in line 403. This negative feedback prompts Lwazi to read his expression again, changing the middle term, "Minus 2 *x* squared plus 4 *x* plus 8 {$-2x^2 + 4x + 8$}" (line 404). He defends his argument in two ways, firstly, by promising (with emphasis) that it is correct, "It'll give you the answer↑ I promise you" (line 407). Based on his earlier mental calculations, I argue that the substitution of the point (2,8) has given him the confidence to make this promise. Secondly, in line 415 Lwazi draws on the gradient and the "wideness" of the parabola graph. This is the first time in the discussion that attention is given to this property of the graph, with the students having assumed so far that the value of *a* in the formula $y = ax^2 + bx + c$ is either 1 or −1. Yet this pronouncement is not taken up by the other students, possibly since the students do not identify a link between the shape of the graph and the value of the coefficient *a*. This absence of attention to Lwazi's pronouncement may be due to the socio-political interaction; Kelsa has already identified herself as an authority to make such evaluations and identified Lwazi's contributions as laughable, the students claim ownership of their answers, with Kelsa distinguishing between "mine" and "yours" (line 420), and they evaluate individual efforts rather than a collective effort (Kelsa says, "You are wrong" (line 416)).

9.5.6 The negative evaluation of Ndumiso's use of a formula from school mathematics constrains the necessary link to this practice

Ndumiso decides to pursue his use of the formula $y = a(x - x_1)(x - x_2)$ which he proposed in line 237. He pronounces this, "Let me try my method … well who said I'm wrong

anyway?" (line 313). He claims personal ownership of this formula with his emphasis on the personal pronoun "my" and in the question at the end of this pronouncement he identifies himself as choosing to ignore the earlier evaluation of his peers. He works quietly on his own and then pronounces that he has been working on his earlier equation, which he names as "that *x* minus *a* thing" (line 377). He points to his written work as in Figure 9.3.

Figure 9.3: Ndumiso's solution for question (d), Chemical Reaction Problem

$y = -4(x)(x - 4)$	(A)*
$= -4(x^2 - 4x)$	(B)
$= -4x^2 + 16x$	(C)
$= -x^2 + 4x$	(D)

* Line labels A to D have been added.

Ndumiso has made the link between the symbols x_1 and x_2 in the formula $y = a(x - x_1)(x - x_2)$ and the *t*-intercepts of the parabola graph (line A). However, it is not clear where the value $a = -4$ in line A comes from. In lines B and C he operates on the right-hand side of the expression by multiplying. In line D he is viewing the formula as an equation with $y = 0$ and divides the right-hand side expression by a common factor of four.

Giving negative feedback, the other students do not attend to Ndumiso's choice of formula, his links to the graph, or his operational error. Rather, they evaluate it by substituting the point (2,8) and pronouncing that this formula has already been evaluated; "we did that" (Thokozile, line 380) and "you're meant to get 8" (Thokozile, line 383). Ndumiso concludes, "I give up" (line 384).

9.5.7 The Tutor's link to school mathematics as enabling for some students

The students have made attempts to call the Tutor to their desk a number of times, although as noted in Section 9.5.2 the socio-political interaction between the students leads to some debate as to whether they do need his help (lines 440 to 466). The interaction between the Tutor and the group suggests some humor and teasing, for example, the Tutor mimics their calling him, identifying himself as a "boy" who can be ordered around; "come here boy

((*They laugh*)) we want you" (line 440). Acting like a Tutor in a learner-centred pedagogy the Tutor encourages the students to discuss their work in the group. He identifies the girls ("you") as being able to explain, "you girls are clever … are so clever you can explain it" (line 460). However, following Lwazi's argument that he and Ndumiso do not trust the girls (line 463), the Tutor changes his strategy and works with Lwazi and Ndumiso. Kelsa and Thokozile work quietly together during this time, suggesting that they do not identify the Tutor as an authority in the foundational practice in the same sense that Lwazi and Ndumiso do.

The Tutor begins by reading question (d) aloud and then looking at the expression Lwazi has written in his book, that is, $-2x^2 + 4x + 8$. Transcript 9.8 begins with the Tutor questioning Lwazi about this formula.

Transcript 9.8: Chemical Reaction Problem, question (d), Group 3, lines 472 to 491

472a	Tutor:	Where did you get that from?
472b	Tutor:	Did you just make it up?
473*	...	
474	Lwazi:	[((*Mumbling something in response to the Tutor in line 472, pointing to his equation at the bottom of his page*))]
475	Tutor:	[Didn't you do something uhm like … you did at school? ((*Moves some books away from in front of him, looking at Lwazi*))]
476*	…	
477	Lwazi:	Yes we did but I forgot the formula there… it's got an *a* it's got an *l* … [you know what I'm talking about it's got a bracket ((*Gesturing in the air, looking at the Tutor as he speaks*))]
478	Ndumiso:	[It's *y* equals to *a x* minus x_1 *x* minus x_2 $\{y = a(x - x_1)(x - x_2)\}$↑ ((*Nodding his head in time as he says each term*))]
479	Lwazi:	No ((*Shaking his* head)) it's not [[it's]]
480	Tutor:	[[YES ((*Looking at Ndumiso*))]]
481	Ndumiso:	Exactly
482	Tutor:	What is x_1? ((*Pointing at Ndumiso*))
483	Ndumiso:	x_1 is going to be your first intercept↑ ((*Pointing to something on his graph*))
484	Tutor:	x_2? ((*Still looking at Ndumiso*))

485	Ndumiso:	Your second one
486	Tutor:	Cha … why you don't have a problem↑ … ((*Lifting up Lwazi's answer page and putting it back in front of Lwazi*)) why he's got it all worked out ((*Smiling at Ndumiso*))
487	Ndumiso:	I told you ((*Tapping his fist on Lwazi's shoulder*))
488	Lwazi:	((*Looking at Ndumiso's working in Figure 9.3 and tapping his finger on it, grinning*)) Right idea right idea
489	Thokozile:	[So what's happening?]
490	Kelsa:	His equation was right and Lwazi led us off on the wrong track
491	Ndumiso:	And Lwazi … … ((*Turning to look at Kelsa*)) and why did you trust Lwazi in the first place?

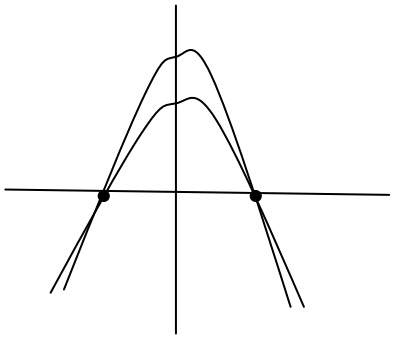
* An overlapping conversation between Kelsa and Thokozile.

Not giving Lwazi time to respond to his first question in line 472a, the Tutor identifies Lwazi as a student who might "make it up" (line 272). This identification is consistent with the teasing relationship that the Tutor has established with the students. The Tutor's link to "something uhm like … you did at school?" (line 475) enables Ndumiso and Lwazi to return to their earlier discussion about the different quadratic formulae from school. In line 477 Lwazi pronounces his version verbally, attending to the individual symbols such as "*a*", "*l*" and "a bracket". Ndumiso also pronounces his earlier formula ($y = a(x - x_1)(x - x_2)$) verbally in line 478.

Although Ndumiso, on the prompting of the Tutor, has started to pronounce verbal links between his formula and the parabola graph (line 483), the interaction between the students does not suggest that they have moved beyond the operational view of the function and the informal competition described in Section 9.5.2. For example, Lwazi's use of the pronoun "it's" for the required equation suggests that he still views the two formulae as representing different functions; "No ((*Shaking his head*)) it's not it's" (line 479). The Tutor's validation of Ndumiso's formula $y = a(x - x_1)(x - x_2)$, an evaluation that resides in his personal authority rather than a mathematical argument, together with the absence of further attention to Lwazi's formula constrains any exploration of links between the two formulae. The interaction turns into personal accusations about responsibility, for example, Kelsa identifies Lwazi as the student who "led us off on the wrong track" (line 490).

The Tutor's validation of one formula from school mathematics does not alone enable the students to solve the problem. The Tutor has a further role to play in making links between the symbols in the formula and the points on the graph. Ndumiso starts talking about his substitution, "so your x will be zero right?" (line 495), asking for feedback by ending his statement with "right"? The Tutor responds by attending to the clarity of Ndumiso's language, "Well… which one will be zero?" (line 496). When Ndumiso links the number zero to the "x intercept" (line 498) the Tutor intervenes. In Transcript 9.9 the Tutor makes links between the symbols in the formula $y = a(x - x_1)(x - x_2)$, different points on the graph and different sketches of parabola graphs.

Transcript 9.9: Chemical Reaction Problem, question (d), Group 3, lines 513 to 523

513	Tutor:	What is x 1 $\{x_1\}$?
514	Ndumiso:	It's going to be zero
515	Tutor:	What is x 2 $\{x_2\}$?
516a	Ndumiso:	It's 4
516b	Ndumiso:	x is zero $\{x = 0\}$ and y is zero $\{y = 0\}$ ((*Pointing to the equation in Lwazi's book*))
517a	Tutor:	<That is not gonna work> … you were right up until there … cause you have just … basically you are substituting the same point in twice … ((*Looking at Ndumiso, who looks down at the graph*)) you need a different point on the parabola … see if you just ... if you have just got the roots if you only have the roots ((*Pointing to the two t-intercepts of the graph*)) … uhm … how are gonna get the uhm ...
517b	Tutor:	I'm gonna sketch somewhere … ((*Stretching across and taking a page from Ndumiso*)) I'm gonna make a mess of your page so just give me a page that you don't mind me destroying … okay I can draw it small […] are you sure? This is a big ... ((*Smiling at Ndumiso who is looking serious*))
517c	Tutor:	… … just look ((*Drawing on the page*)) ... look at these two parabolas have … this one has a root here and root here ((*Marking the t-intercepts*))… now you have this parabola ((*Drawing another parabola with the same t-intercepts, but the second is narrower than the first*)) 

517d	Tutor:	if the only thing you substitute in this formula is the root ((*Picking up the paper and putting it in front of Lwazi, then pointing to the t-intercepts on the graph in the Resource Book*)) … how is the formula supposed to know which parabola you are talking about? How are you supposed to know [how steep it is? ((*Demonstrating a graph with a maximum turning point in the air with his hand*))]
518	Ndumiso:	((*Ndumiso tears off a new page and passes it to him*))[okay]
519	Lwazi:	So you must use the maximum the local maximum or minimum↑ ((*Looking at the Tutor*))
520	Tutor	You can use any other point and you've got another point there ((*Pointing to the local maximum on the graph in the Resource Book))*
521	Ndumiso:	[Oh ja]
522	Tutor	[You do need] to use different points though ((*Putting the lid on his pen and looking across the classroom*))
523	Ndumiso:	I was in matric when I did this … it just sort of ((*Waving his hand in the air*))

In lines 514 and 516b of Transcript 9.9 Ndumiso is substituting the x-value zero for both the symbols x_1 and x_2 in his formula $y = a(x - x_1)(x - x_2)$. This suggests that in making the first substitution he is not viewing the x-intercept as the ordered pair (0,0). The Tutor gives negative feedback, "that is not gonna work" (line 516), which he accompanies with a mathematical argument about the need to use an additional point on the graph in the substitution. He places emphasis on certain words, for example, "you are substituting the same point in twice …" (line 517a) and links to two sketch graphs of parabolas with the same t-intercepts but different turning points (line 517c). The Tutor frames the links between the points and the formula as a conversation between the student "you" and a personified "formula" about the parabola graph (line 517d). He also attends to how "steep" the graph is (517d), a reference which links to Lwazi's earlier talk about the "wideness" of the graph in line 415. However, no explicit link is made between the steepness of the graph and the value of the symbol a in the formula $y = a(x - x_1)(x - x_2)$. The Tutor's talk enables Lwazi to identify the "the local maximum or minimum" (line 519) as an additional point to be substituted. The Tutor's development of Lwazi's contribution by pointing to the parabola graph and emphasizing that any other point can be used (line 520) serves as positive evaluation. Ndumiso reinserts himself into the conversation by making a link to school mathematics, "I was in matric when I did this" (line 523).

The Tutor's intervention as described here enables Ndumiso to produce the formula

$y = -2x^2 + 8x$ using appropriate written working. Kelsa uses the formula $y = a(x - x_1)(x - x_2)$, but her working in Figure 9.4 suggests that she has difficulty making the appropriate links between the points on the graph and the variables in her equation.

Figure 9.4: Kelsa's initial attempts to use the formula $y = a(x - x_1)(x - x_2)$, question (d), Chemical Reaction Problem

~~x~~ *t*	~~y~~ *m'*
2	8
4	0

$a(x - x_1)(x - x_2)$	(A)*	$a(x - 2)(x - 4)$	(D)
$a(x - 0)(x - 4)$	(B)	$a(x^2 - 2x - 4x + 8)$	(E)
$a(x^2 - 4x + 4)$	(C)	$a(x^2 - 6x + 8)$	(F)

* Line labels A to F have been added.

In the table in Figure 9.4 Kelsa attends to the meaning of the variables in the Chemical Reaction Problem, replacing the variable x with t and the variable y with m'. She makes two attempts at substituting the x-intercepts into the expression $a(x - x_1)(x - x_2)$. In line B she substitutes the x-intercepts correctly, but the absence of an actual equation with the dependent variable on the left-hand side constrains her from moving forward. In line D she substitutes two x-values, irrespective of whether they are x-intercepts or not. She requests help from Ndumiso (she actually picks up his page, line 565), identifying him as a mathematical authority now that his formula has been validated by the Tutor. Ndumiso responds by linking the variables in the equation and the points on the graph, as the Tutor did for him earlier. Having listened to this explanation and looked at Ndumiso's work, Kelsa starts to use an equation of the form $y = a(x - x_1)(x - x_2)$ and proceeds to the correct answer with appropriate working.

Lwazi also starts using Ndumiso's formula $y = a(x - x_1)(x - x_2)$. In line B of Figure 9.5 he substitutes the x-intercepts correctly, but his substitution of the value 8 for y and no corresponding value for x constrains further progress. He skips steps in line C and appears to copy the expression $-2x + 8x$ in line D from the other students.

Figure 9.5: Lwazi's final written answer for question (d), Chemical Reaction Problem

$y = a(x - x_1)(x - x_2)$	(A)*
$8 = a(x - 0)(x - 4)$	(B)
$=$	(C)
$\therefore -2x + 8x$	(D)

* Line labels A to D have been added.

It is possible that Lwazi's focus on identifying himself by the formula $y = a(x - p)^2 + q$ and his defense of this formula to the other students constrains him from attending to the discussion with the Tutor. After the Tutor has helped them with Ndumiso's formula, Lwazi continues his debate with Ndumiso, "there's another formula though" (line 525), and the two students continue to use emphasis to support their arguments. The Tutor hears Lwazi and writes down the equation $y = a(x - p)^2 + q$ in his book. Lwazi is excited about being proved right, "yes↑ … yes↑ … yes↑ … that's the one" (line 533). Again, two personal evaluations from Ndumiso suggest that Lwazi "can't even use it" (line 532 and 537).

Thokozile does not produce any written work for question (d), but proceeds to use the correct formula $y = -2x^2 + 8x$ (obtained by looking at Ndumiso's solution) in the next question.

9.6 Discussion of the student action on question (d) of the Chemical Reaction Problem (Group 3)

Initially the students in Group 3 propose a basic form of the quadratic formula, but they soon make a link to school mathematics and pronounce two quadratic formulae (Lwazi is attending to a third quadratic formula from school). This suggests that the students control the movement of the quadratic function and its algebraic representations over the school mathematics/foundational mathematics boundary and also control the timing of this movement. However, the students set aside these formulae relatively quickly in favour of a method for finding the equation of a parabola used in other social events in the Foundational Course. I have argued that, since the students do not control the movement of

the valued mathematical ways for acting on the quadratic function, they sever the initial link to school mathematics. For example, they view the function operationally with the result that the three formulae are identified as representing different operations to be performed (Sfard, 1992), they do not make links between the symbols in these formulae and the points on the graph, and they adopt the style of students in a more traditional pedagogy searching for the formula that is "it" (Lwazi, line 250). Together, these actions prevent them from identifying the three quadratic formulae as equivalent. The socio-political interaction also prevents the students from either investigating this equivalence or pursuing one of the three options; there is personal competition between the students as they identify themselves as students who can solve the problem, and their choices are supported by personal opinion and emphasis rather than mathematical arguments.

Having severed the link to school mathematics, the students move to other social events in the Foundational Course in which they have found the equation of parabola graphs. Yet the students do not have complete control of the movement of ways of acting mathematically across these social events and this constrains their construction of a formula by viewing the function structurally and using transformations. For example, they do not attend to a key assumption of the method of transformations as used in other Course material. The students' tendency to use verbal rather than written descriptions of the formulae constrains attention to their use of symbols in these formulae. Students attend to a variety of properties of the graph, for example, individual points on the graph, the maximum turning point, the translations and reflections, and the "wideness" of the graph. However, they do not have a general formula to which they can link these properties.

The students' action is also constrained by the socio-political interaction within the group. Kelsa identifies herself as an authority in the ways of evaluating in the foundational practice, evaluating the proposed formulae using the operation of substitution. Although the other students attend to Kelsa's evaluation, they do not seem to identify her as an authority in the practice with respect to her choice of the quadratic formula $y = ax^2 + bx + c$ from school mathematics. I have indicated that Ndumiso and Lwazi both pursue their

school formulae and Thokozile appears to give up when her method of transformations from the Course does not work. This interaction differs from that in Group 2 in which the students identify Siyabulela as the authority in the practice (with respect to his evaluation, his link to school mathematics, and his ways of acting on the mathematical objects), and they follow his proposed solution with confidence.

In contrast to the action in Group 2, the students in Group 3 do not make progress until the Tutor acts as an authority in the practice by both validating Ndumiso's formula from school mathematics and making links between this formula and the points on the graph. Yet the Tutor's contribution is enabling for some but not all students. The difference may lie in the power dynamics in the group in that the Tutor has validated one of the three pronounced formulae, one which is regarded as "owned" by Ndumiso. Lwazi continues to identify himself by his formula $y = a(x - p)^2 + q$, and Kelsa and Thokozile identify themselves as not needing help from the Tutor. Kelsa eventually seeks help from Ndumiso, but still identifies Lwazi as being responsible for their lack of progress.

9.7 The developing argument about the student action

The analysis in this chapter allows me to further develop the argument about the ways of operating on mathematical objects, the socio-political interaction, and the educational ways of talking in a learner-centred pedagogy that I began in Section 8.6. This analysis also talks to the movement of meaning between school mathematics practice and foundational mathematics practice and links between different representations of a function. As in Section 8.6 it is not possible to talk about these ways of acting mathematically in isolation. Rather, it is the complex interplay between the ways of acting that enables or constrains student participation in the practice.

Since the movement of meaning across the school mathematics/foundational practice boundary in question (d) represents continuity between the two practices, a successful boundary crossing requires full control of the how (the quadratic function, the ways of

acting mathematically on the function, the student positioning) and the when of this crossing. The student Siyabulela in Group 2 has this control. He is identified as an authority in the foundational practice by the other students in the group and his explanation in the style of a student in a learner-centred pedagogy enables the other students to control the how of the boundary crossing to school mathematics. In contrast, the students in Group 3 control some but not all aspects of the movement of meaning across the school mathematics/foundational practice boundary; they control the timing and the movement of the algebraic representation of the function, but sever the link to the school mathematics before acting on the function.

In the absence of a link to school mathematics, the students in Group 3 make a link to other social events in the foundational practice. However, unlike the description of the action on the Car Problem (in Chapter 8) in which I have argued that such a link is enabling, the link to the method for finding the equation of the quadratic function in the Course in question (d) is constraining. This link is constraining since the students do not control the movement in the ways of acting mathematically across these events. The difficulty of the students in Group 3 is compounded by their viewing the quadratic function operationally, the verbal description of algebraic formulae that are not written down, a problematic way of evaluating these formulae, and an absence of links between the different properties of the graphs.

The analysis in this chapter points to how the relations of power in discourse, that is in the classroom interaction, may be enabling or constraining. Siyabulela in Group 2 identifies himself as an authority in the various ways of acting in the foundational practice, for example, in evaluating the work of others, viewing the quadratic function both structurally and operationally, controlling all aspects of boundary crossing between school mathematics and foundational practice, and explaining his action. The other students in the group recognize Siyabulela's authority in these respects and are enabled by attending to his action. A number of students in Group 3 identify themselves as authorities in the foundational practice by recruiting quadratic formulae from school mathematics. Yet they

do not position one another as authorities with respect to this movement of meaning, and the interaction becomes a competition between individuals. In contrast, the students do position Kelsa as an authority in terms of the ways of evaluating in the practice, and reproduce her way of acting in this respect.

The Tutor identifies himself as an authority in the foundational practice, in this case suggesting but not prescribing formulae from school mathematics and modelling the required links between the algebraic formula and the parabola graph. The students in the two groups position the Tutor differently. In Group 2 the students choose to follow the lead of Siyabulela rather than the Tutor. In contrast, the students in Group 3 use the authority of the Tutor to validate their "personal" formulae, in this way implicating him in the power relations at work in the group. As a result only some of the students in Group 3 are enabled by the Tutor's input.

CHAPTER 10: RESULTS (PART 2c)
"IT WON'T IT BE LIKE A COS GRAPH?"

10.1 Introduction to this chapter

In this chapter I present the student action on question (a) of the Flu Virus Problem (see Appendices B and Q for the problem and Appendices K and L for the summary of the student action). In contrast to the problems discussed in Chapters 8 and 9, this question does not require operational action, since no algebraic formula is provided for the function. Rather, sketching the graph requires that the foundational student adopt an operational view of a function and work with a number of relationships concurrently, that is, linking the mathematical objects $P(t)$ and $P'(t)$, linking these objects and their graphical representations, and moving to and fro between each of these objects and the task context for the spread of the flu virus. In contrast to question (d) of the Chemical Reaction Problem that is the focus of Chapter 9, this question represents both continuity and disruption with respect to school mathematics practice. I present the action of Group 2, but also refer to the action of Group 1 as additional evidence for the argument.[129] The initial analysis of the action of Group 2 on question (a) was published in Le Roux (2009), and I draw on the review feedback and this published work in the presentation of the results in this chapter.

10.2 Summary of the student action on question (a) of the Flu Virus Problem

In both Groups 1 and 2 the students begin by making links to functions in school mathematics (which are also revised in the Foundational Course). The students name the

[129] Unlike the action of Group 2 on the Chemical Reaction Problem (the action on question (d) is represented in Section 9.3), almost all the action of this group on the Flu Virus Problem took place in English, with some isiXhosa words.

straight line, exponential and cosine graphs as possible answers. The students recruit ways of acting on functions from school mathematics when they identify the required population graph in question (a) with these static, named graphs. They do not view the function operationally and sketch the graph by considering what is happening in the task context as time passes.

The students attend to some properties of their proposed graphs and not others. In both groups the students attend to the increasing/decreasing nature of the graph, and recruit the task context to explain their arguments in this respect. However, these arguments suggest that some students do not have full control of the movement of meaning across the mathematical/non-mathematical boundary. The students appear to follow the assumptions of the genre of mathematical word problems; they identify cues in the task context, distinguish additional information about the task context that is not required for the mathematical problem, and move from the non-mathematical to the mathematical (and not to and fro across this boundary). Prior to the interaction with the Tutor, only one student appears to attend to the concavity of the graph. The student in Group 1 attends to this property early in the interaction, and her talk enables the students to move beyond the choice of a straight line graph (to an exponential graph). However, this property is not the focus in the rest of the student discussion in Group 1. In both groups there is a hesitancy to draw sketch graphs and the students use gestures to trace the shape of their graphs.

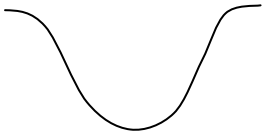
The students in Group 1 and Group 2 do not proceed beyond their static, named graphs from school mathematics. The Tutor acts as an authority in the foundational practice by modelling the required links between the functions $P(t)$ and $P'(t)$, their representations, and the task context. This action is enabling for four of the five students in Group 1. In his interaction with Group 2, the Tutor is concerned about the students' pace on the Flu Virus Problem and he controls the action to the extent that he draws graphs for the students.

10.3 Student action on question (a), Flu Virus Problem (Group 2)

10.3.1 Links to ways of acting on functions in school mathematics as constraining

The students begin question (a) by reading the problem text. Mpumelelo is the first student to pronounce an answer verbally, as in line 17 of Transcript 10.1.

Transcript 10.1: The Flu Virus Problem, question (a), Group 2, lines 17 to 40

17	Mpumelelo:	It won't it be like a cos graph?
18	Lungiswa:	Uhm↑
19	Mpumelelo:	It won't it be like a cos graph? ((*Using his pen to demonstrate a full wave of a cosine graph, starting and ending at its maximum value*)) 
20	Lungiswa:	Why do you say so?
21	Mpumelelo:	Because they say we must we must also solve ... the ... the <u>number</u> ((*Using his hand to emphasize*)) of people who get the disease right?
22	Lungiswa:	Uh huh↑
23	Mpumelelo:	So it's like they won't get it ... at the same time ((*Bongani and Siyabulela are reading the question, Vuyani is looking at Mpumelelo as he speaks*))
24	Lungiswa:	Ja ((*Nodding her head*))
25a	Mpumelelo:	So it's like there are the others that get it ((*Holding his pen in the air as he speaks*))
25b	Mpumelelo:	so ... ... it is from that thousand of the community ((*Raising his hand for "1 000", alternating between looking at Siyabulela, Vuyani and Lungiswa, Bongani does not make eye contact, seems to be reading*))
26	Lungiswa:	Ja↑
27	Mpumelelo:	So
28	Vuyani:	You mean ((*Stretching so that his pen is on the Resource Book Mpumelelo is using*))
29	Lungiswa:	Oh ja ... I get what you are saying
30	Mpumelelo:	So it's like we have that ... this this maximum now, right? ((*Tracing a circle in the Resource Book*))
31	Lungiswa:	Uh hum↑
32	Bongani:	Ten thousand
33a	Mpumelelo:	Ja it's then thousand

33b	Mpumelelo:	but then when I get there the ... number of who get it ... that would like the minimum. ((*Tracing decreasing part of cosine graph with his hand*)) 
34	Bongani:	The minimum
35	((*Vuyani is tapping his eraser on the table*))	
36	Lungiswa:	Cause eventually they are all gonna get it
37	Mpumelelo:	[Ja]
38	Bongani:	[Ja]
39	Lungiswa:	[[Ja]]
40	Mpumelelo:	[[But the ones]] who get it first↑ ... they won't get it again … that is what the statement says

Mpumelelo names the graph as a "cos graph" (line 17) and then repeats his claim in line 19, following a query from Lungiswa. He links the required population graph with a graph from school mathematics and the Foundational Course, that is, the cosine graph. By naming the graph as a "cos graph" he appears to be looking at the function as a static object belonging to a class of graphs, rather than adopting an operational view and considering the changing shape of the graph over time. In line 19 Mpumelelo also pronounces a specific cosine graph by tracing in the air with his pen a standard version of the cosine graph that he will have encountered in his school Mathematics textbooks and which is reproduced in the notes in the Resource Book for the Foundational Course. This action of tracing the shape of the graph in the air rather than drawing it on paper is a common feature of the student action on question (a).

Mpumelelo's pronouncement about the cosine graph is tentative and he identifies himself as wanting feedback; his questions in lines 17 and 19 are in the declarative mood and his use of the negative modal auxiliary verb "won't" suggests that he is not certain about his claim. Before providing feedback, Lungiswa asks for an explanation, "Why do you say so?" (line 20), thus creating an expectation in the group that explanations are provided. Vuyani also identifies himself as a student wanting clarification from Mpumelelo ("you") when he attempts to enter the conversation in line 28. However, his contribution in this

instance, and at some other times during the discussion of question (a), is not attended to by the other students. For example, later when Mpumelelo is explaining his choice of graph the students do not attend to Vuyani's attempt to interject, "[[the thing is]]" (line 53). This absence of attention to Vuyani's interjections may be due to the fact that he was a relatively new member of Group 2 as he had recently moved to the Course from the mainstream first-year mathematics course.[130]

Following Lungiswa's prompt, Mpumelelo sets out to explain his choice of graph. He reproduces the authority of the text, using this to give authority to his own explanations. For example, in line 21 he identifies the producer as "they" and in line 40 he reinforces his description of the task context by recruiting "what the statement says". His pronouncement that the text is instructing them to "solve" (line 21) rather than "graph" represents mathematics as being about "solving", a representation he may have drawn from school mathematics. Mpumelelo crosses the boundary to the task context to support his claim about the cosine graph. However he does not control the movement of meaning across the boundary. For example, his argument that "they won't get it ... at the same time" (line 23) suggests that he is attending to how many people in the community "get it" rather than to the meaning of the function $P(t)$ in Sentence 4, that is, "the number of people who have, or have had the disease t days after the first case of flu was recorded". The other students do not attend to whether Mpumelelo has made a meaningful link across the boundary; rather he gets encouragement from Lungiswa to continue when she says "Ja" and nods her head (line 24). And so he continues to recruit the task context in a way that does not support his choice of a cosine graph; "it's like there are the others that get it" (line 25a).

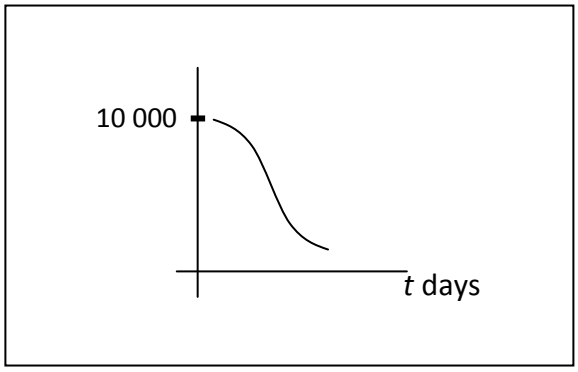
Lungiswa's evaluations are characterised by content-free, positive feedback, as in lines 24, 26 and 29. Lungiswa is adopting the position of a student in a leaner-centred pedagogy;

[130] In his interview for this study Vuyani indicated that, when joining Group 2, he felt that the students might identify him as having being "beaten" by the mainstream first-year mathematics course. However, he suggested that he had only felt welcomed by the other four students and that they were always willing to answer his questions.

she is listening to her peers, and is "asking questions about solutions", "asking for further explanation" and "encouraging one another to keep going/to participate" (Foundational Course Resource Book, 2007, Workshop 1, p.16). Other students also provide positive content-free feedback such as this to their peers, for example the responses of "Ja" in lines 37 and 38. I have noted that Vuyani is not given a voice in the initial discussion of the Flu Virus Problem, yet there is a change in the interaction as the discussion proceeds. The encouraging talk in the group may enable Vuyani to identify himself as a student with a voice. Yet this positive, content-free feedback is also constraining in that the only content-related feedback provided to Mpumelelo is Bongani's correction of his use of "thousand" to "ten thousand" (line 32). It is possible that the nature of the feedback and Lungiswa's prompts contribute to Mpumelelo's proposed graph (the "cos graph") and his accompanying tracing of this graph in the air becoming the attended and the pronounced focus for much of the interaction, to the exclusion of other possible graphs. This is despite the tentative manner in which Mpumelelo has represented his claims and supporting arguments.

Mpumelelo's explanation suggests he is drawing on the genre of mathematical word problems from school and identifying cues in the problem text. In line 25b he attends to the value of ten thousand (he pronounces this as "thousand", but is corrected by Bongani in line 32) in Sentence 1 of the Flu Virus Problem and in line 30 he attends to the word "maximum" in the instructions for question (a). It is possible that his attention to these cues has led him to represent the standard cosine graph that begins and ends at its maximum value (line 19). Certainly, the students attend to a maximum value of 10 000 on the vertical axis as they begin their sketches (see Lungiswa's first attempt in Figure 10.1). The attention to the maximum value also cues Mpumelelo to introduce a new

Figure 10.1: Lungiswa's first sketch graph, question (a), Flu Virus Problem



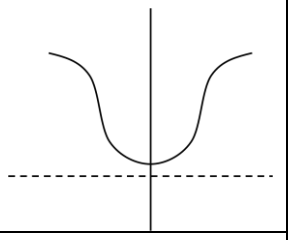
feature into the task context, that is, the minimum number of people in the community in line 33b. This suggests that Mpumelelo introduces meaning from the mathematical object and its representation into the task context.

The students make links to other named graphs from school (I present another example from Group 2 in Section 10.3.5). To end this section I identify a similar action by students in Group 1. Shae begins the discussion of question (a) of the Flu Virus Problem by asking his peers for help, "How do we do it?" (line 16a). He is viewing the function operationally and identifying points on the graph to plot; he attends to the value "ten thousand" given in Sentence 1 of the problem text, but cannot identify other points to plot; "We don't have values for it we've only got ... 10 000 people↑" (line 16b). Shae's method of plotting a graph point-by-point is valued in school mathematics, but discouraged in the Foundational Course. The other students respond by proposing static, named graphs from school mathematics and the Course, for example, Hanah names the graph as "a straight line" (line 17), a name which Jane later renames to "a line" (line 29). After some discussion Jeff pronounces that, "it should actually be more of an exponential thing" (line 53).

10.3.2 Attending to some properties of the function and its representation and not others as constraining

In Section 10.3.1 I provided evidence that the students in Group 2 attend to the maximum and minimum values of the graph, possibly cued by the word "maximum" in the text of question (a). They also attend to where the graph may be increasing or decreasing as suggested in Transcript 10.2.

Transcript 10.2: The Flu Virus Problem, question (a), Group 2, lines 55 to 69

55	Mpumelelo:	So first it get ... the few of them.
56	Lungiswa:	Uh huh↑
57	Siyabulela:	So it's going to be [something like this] then↑
58	Lungiswa:	[As time]((*Drawing a dotted horizontal line below the minimum point, working from left to right*)) 
59	Mpumelelo:	Ja
60	Lungiswa:	[And <u>then</u> it goes like]
61	Siyabulela:	[down and down]
62	Lungiswa	It goes down and down until ((*Tracing down with her pen on the graph in line 58*))
63	Mpumelelo:	Until it get ... [all of them]
64	Lungiswa:	[Ja] ... ja.
65	Siyabulela:	[[Oh]]
66	Lungiswa:	[[Ja]]
67	Siyabulela:	So this is ... the one lucky the community
68	?	Ja ((*They laugh*))
69	?	Okay ... ja ja ja

The three students Mpumelelo, Siyabulela and Lungiswa co-construct an argument about the properties of the graph by giving one another content-free feedback in the form of "ja", by repeating what others say, and by building on what others say. Mpumelelo begins in the task context, suggesting that at first the disease ("it") only affects "a few" of the people in the community (line 55). Siyabulela develops this with "So" and makes a suggestion about the graph "it" (line 57). In lines 60 and 61 Lungiswa and Siyabulela both start their descriptions, which Lungiswa combines in line 62. They do not use the mathematical terminology "decreasing" but talk about the graph in everyday language as "going down". Mpumelelo completes the description by moving back into the task context and referring to the disease ("it") affecting everyone in the community (line 63). Although the students have

moved across the boundary (from the task context, to the properties of the graph, and back to the task context), their choice of a decreasing graph to model the increasing number of infected people suggests that they do not have full control of this boundary crossing.

Lines 55, 57, 60 and 62 are evidence that the students move, relatively quickly, from talking about the "cos graph" as Mpumelelo did in lines 17 and 19 to talking about the graph using the reference pronoun "it". I argue that the use of this pronoun reinforces a structural view of the function and may prevent the students from attending critically to Mpumelelo's initial choice of this graph.

When tracing the cosine graph in the air in lines 18 and 33b of Transcript 10.1, Mpumelelo does not attend to the positioning of the graph on co-ordinate axes. The same can be said of Lungiswa's sketch of the graph in line 58 of Transcript 10.2. In her graph in Figure 10.1 (line 88) she changes the positioning of the graph on the axes but this change is not discussed. The only suggestion that they are attending to the positioning of the graph comes when Siyabulela asks, "It will never go to the negative side will it?" (line 77). Here he is referring to the graph using the reference pronoun "it" but it is not clear whether the "negative side" refers to the independent variable t being negative or to the dependent variable (the number of people) being negative. However, Siyabulela's explanation which follows this question suggests that he is attending to the dependent variable as he draws on the task context to argue, "You can't have a negative number of the population of people" (line 79). This explanation indicates that Siyabulela is controlling the movement of meaning from the task context to the graph. However, his description of the function $P(t)$ as the "population of people" suggests that he may not be attending in detail to the specific meaning of this function represented in Sentence 4 of the problem text, that is, "the number of people who have, or have had, the disease t days after the first case of flu was recorded".

In this section I have argued that the students attend mainly to the maximum and minimum values of the graph and to the increasing/decreasing nature of the graph. They do not attend

to the concavity of the graph, something which the Tutor alerts them to, as I describe in Section 10.3.4.[131]

10.3.3 Links to the genre of mathematical word problems as enabling and constraining

I have argued that the students in Group 2 use cues in the problem text to drive the choice of function and that they draw on the task context to support this choice. However, some students do not have complete control over the movement across the mathematical/non-mathematical boundary. It may be that the students are adopting the style of school students solving mathematical word problems, since trying to make sense of the problem in terms of the non-mathematical practice is not part of this genre (Gerofsky, 2004, p.34).

However, there is evidence that most students draw productively on the genre of mathematical word problems, in particular on the assumption that no "extraneous information" (Gerofsky, 2004, p.33) about the task context need be sought. For example, in line 67 of Transcript 10.2 Siyabulela describes the community using the adjective "lucky", the laughter from the other students in line 68 suggesting that they interpret this statement as a joke and not part of the problem-solving process. Siyabulela's representation of the people in the community in terms of "luck" differs from the impersonal representation of these people in the text of the Flu Virus Problem. It also differs from how the students represent the people when identifying a graph in question (a), for example, explaining his choice of the "cos graph" Mpumelelo also represents the people impersonally as "people who get the disease" (line 21, Transcript 10.1) and "the others who get it" (line 25a, Transcript 10.1).

[131] As noted in Section 7.4.1, the students had not yet studied the second derivative formally in the Foundational Course, although the changing concavity of graphs had been explored informally in other practical problems.

I turn for a moment to the action of Group 2 on question (c) of the Flu Virus Problem for further discussion of how the students deal with the movement of the mathematical word problem genre into foundational mathematics.[132] Two strands of talk develop in the students' interaction; one which draws only on the information in the task context necessary to solve the mathematical problem and another in which jokes are made about the "extraneous information" (Gerofsky, 2004, p.33). In the first strand of talk the students attempt to describe the meaning of the equation $P(4) = 1\,200$ in "practical terms" and the talk involves stating verbal answers only with no explanations and presenting these answers with certainty. For example, Vuyani simply states his answer, "After 4 days … 1200 are infected" (line 365), an impersonal statement (he does not claim ownership and say, "I think the answer is …", or "My answer is …"). In the second strand of talk Lungiswa, Vuyani and Siyabulela exclaim about the large number of people infected, question why this may be the case, and talk about a disease spreading in South Africa. This discussion is characterized by the use of full sentences, the expression of personal opinions, explanations for these opinions, and lots of laughter. For example, Siyabulela supports his view that the flu should come to South Africa by explaining that "There are lots of unwanted people in South Africa" (line 373), but his smile suggests that he is identifying himself as a joker. Lungiswa's indignation as well as her amusement is expressed by her high pitched "Ja" (line 372), her laughter and her repetition of the question, "What do you mean? What do you mean?" (line 375a).

In this section I have provided evidence that the students in Group 2 make jokes about the "extraneous information" (Gerofsky, 2004, p.33), suggesting that they are not regarding the task context as real and thus acting like school students solving word problems. In Group 1 the discussion of "extraneous information" (p.33) in the Flu Virus Problem is dealt with in a different way, that is, it is ignored by those students who are positioned as authorities in

[132] The analysis of the student action on question (c) of the Flu Virus Problem was first presented in Le Roux (2008b).

the practice. The students have decided that the population graph is an increasing "exponential graph". However, Lulama introduces "extraneous information" (Gerofsky, 2004, p.33) into the task context when he supports his claim with the task context, "But they immunize people … I mean" (line 63). This argument is ignored, which is the case with a number of Lulama's statements (as discussed in the action of Group 1 on the Car Problem in Chapter 8).

The difference in the action of Siyabulela and Lulama with respect to this "extraneous information" (Gerofsky, 2004, p.33) may be due to their positioning in their respective groups. Overall, the analysis suggests that Siyabulela controls the movement of meaning in the genre of word problems from school mathematics and the accompanying movement of meaning across the mathematical/non-mathematical boundary that this entails. As noted in Section 9.3, Siyabulela is also identified in the group as an authority in the foundational practice. In contrast, Lulama has difficulty controlling the movement of meaning across these boundaries and the socio-political interaction in Group 1 constrains his access to this control. This supports the argument by Fairclough (2001) that, while all subjects are constrained by the conventions of a practice, the more powerful subjects may draw in a more cavalier way on these conventions. In this study Siyabulela's power, which resides in his control over the movement of meaning across practices, enables him to make jokes about the task context.

10.3.4 The Tutor's modelling of links between the functions, their representations and the task context as enabling

I have argued so far that the students in Group 2 are constrained by selecting and attending only to a static, named graph from school mathematics and attending to some properties of this graph and not others. Although the link to the genre of word problems from school is enabling in some respects, the fact that the students do not regard the task context as real constrains them from supporting their choice of properties with relevant arguments located in this context. The Tutor plays the role of authority by modelling the necessary

relationships between the functions *P(t)* and $P'(t)$, their representations, and the task context.

Initially, the Tutor does not attend explicitly to the graphs the students have drawn. Rather, he switches the focus by gaining the attention of the students with "okay" and then focusing their attention on the axis labels, "Okay ... I think the thing … so remember what is on your one axis and what is on your other axis?" (line 103). He evaluates the students' verbal pronouncements by drawing on the text in Sentence 4 and using "okay", repetition, rewording and variation in pace. For example, he responds to Vuyani's description of the function *P*(*t*); "Okay … so *P* is the number of people … but uhm ... it's not just the number of people ... it's the number of people who <have the flu↑...or who have had the flu>" (lines 108a and 108b).

In Transcript 10.3 the Tutor attends to what is happening in the task context over time. He identifies the value of the variable "*t* equal to zero" (line 112) with "the first day that the first person gets the virus" (line 113) in the task context. Remaining in the task context, he asks about the function value *P*(0), "… how many people have it?" (line 115a). Before the students answer he provides two possible everyday adjectives as answers, "a lot or a little" (line 115b).

Transcript 10.3: The Flu Virus Problem, question (a), Group 2, lines 112 to 117

112	Tutor:	But ... where it says like at *t* equal to zero ((*Pointing to the graph in Lungiswa's book*))…
113	Tutor:	That's on ... that's the first day that the first person gets the virus … right? the flu ((*Using his hands to demonstrate the first (as if at the intersection of the axes)*))
114	Lungiswa:	Uh huh↑
115a	Tutor:	So ... ((*Resting his hands on the desk again*)) at *t* equal to <u>zero</u>, how many people have it?
115b	Tutor:	Is it a lot or a little?
116	Siyabulela and Vuyani:	((*Mumbling*)) °a little°
117	Lungiswa:	It's a ... little

In line 127d of Transcript 10.4 the Tutor links "*t* equal to zero" (line 122) and "a little" (lines 115b to 117, Transcript 10.3) to the "starting" point of the graph. By focusing attention on different points on the graph of $P(t)$ he is modelling an operational view of the function. Using the pronoun "we" (line 127d) he represents the action as a communal. However, as before, he controls the talk by presenting a choice of two answers, this time numbers rather than everyday descriptions; "ten thousand" and "zero" (line 127e).

Transcript 10.4: The Flu Virus Problem, question (a), Group 2, lines 127d to 143

127d	Tutor:	Where ... are we starting the graph from?
127e	Tutor:	Are we starting from ten thousand? Or are we starting from ... zero?
128	Bongani:	Fro ... starting [from zero]
129	Lungiswa:	[From ten thousand]
130	Vuyani:	[From zero]
131	Mpumelelo:	Zero
132a	Tutor:	But why why ten thousand? ((*Looking down at Lungiswa*))
132b	Tutor:	I don't understand↑
132c	Tutor:	We're not we're not graphing how many haven't had it … we're graphing how many people ... [have it]
133	((*Tutor has been standing up, now gets down on his haunches next to the desk, between Bongani and Lungiswa*))	
134	Lungiswa:	[Have it]
135	Vuyani:	[Have it]
136	Siyabulela:	So it's gonna make the graph's gonna maybe like uh ... increase (unclear) [Something like sss ... so ((*Showing increasing graph with his hand, the graph he traces is wavy*))]
137	[((*Bongani also demonstrates a wavy, increasing graph with this hand*))]	
138a	Tutor:	Yeah … because as people get it like more people <have it or have had it>
138b	Tutor:	so you can't ... you can't go down↑((*Using his hands to demonstrate, dropping his hand down*))
139	((*The Tutor stands up again, resting his hands on the table*))	
140	Siyabulela:	Oh ... so eventually you are going to affect the whole community then?
141	((*Bongani, Mpumelelo, Siyabulela, and Lungiswa are looking to the Tutor, Vuyani is writing in his book*))	
142	Tutor:	Yeah ((*Nodding his head)*) no exactly ((*They all laugh*))
143	Siyabulela:	That's one lucky the community that one

The Tutor responds to Lungiswa's incorrect response of "ten thousand" (line 129), firstly asking for an explanation "why" (line 132a) and then suggesting that there may be something wrong with his personal ("I") understanding; "I don't understand↑" (line 132b). The rising intonation at the end of his statement in line 132b suggests that he wants a response. Yet he immediately explains what is wrong with her reasoning by linking the task context and using emphasis on the words "have" and "haven't" (line 132c) and the decreasing graph he demonstrates with his hand (line 138b). He switches from the collective "we" (line 127d) to the singular personal pronoun "you" (line 138b) giving the students personal ownership of the decreasing graphs. By implicating the student personally in the action ("you can't go down", line 138b) the Tutor is representing mathematics as a material process, or about doing something (Morgan, 1998, p.80).

The Tutor's link between the task context and the graph enables the students to identify the graph as increasing (lines 136 and 137). However, Bongani and Siyabulela's graphs are also fluctuating as they increase, suggesting that they are still attending to the cosine graph. The Tutor provides positive feedback on the increasing property of the graph by crossing the boundary to the task context (line 138a), again attending to the meaning of the function $P(t)$ stated in Sentence 4 of the problem text. However, there is an absence in his feedback of attention to the fluctuating nature of their graphs. It is possible that the students' tracing the graphs in the air rather than drawing the graphs in their answer books and the Tutor's attention to Lungiswa's difficulty identifying the graph as increasing are preventing the Tutor from attending to this additional property of the graph.

The Tutor continues his discussion with Lungiswa, making links between an increasing/decreasing graph and its meaning in the task context. He gives negative feedback on Lungiswa's graph, but softens this by suggesting that Lungiswa's decreasing graph could represent a different aspect of the task context, "like at the way you've drawn it […] ((*Pointing to Lungiswa's book*)) it looks like to me like at time equal zero ... maybe this is 10 000 people do not have the virus↑" (line 151b). He assigns personal ownership to her graph ("you've") but suggests that he is interpreting it from his personal perspective ("it

looks to me"). This discussion is enabling for Lungiswa; her exclamation, "Oh" (line 153), suggests she identifies her difficulty, she makes an appropriate link to the task context, "So we are looking at the people who are getting the virus? ... not? ... ((*Looking up at Tutor as she speaks*))" (line 150), and she draws increasing graphs from this point on.

The end of Transcript 10.4 provides further evidence that the students do not view the task context as being real, but as a joke. In line 142 they laugh when the Tutor emphasizes ("no exactly") that the flu virus will affect the whole community. Then Siyabulela repeats his statement about the community being "lucky" (line 143).

In this discussion the Tutor identifies himself as an authority in the foundational practice as he models the valued mathematical links between the mathematical objects, their representations and the task context. The interaction re-presented Transcript 10.4 suggests that the Tutor has difficulty balancing his roles in the workshop class. At times the Tutor acts like a facilitator in a learner-centred pedagogy as he requires the students to explain their answers (line 132a). Yet at other times he controls the content of the talk by presenting possible answers to his questions (line 127e). The students identify the Tutor in the latter role when they respond by selecting answers from the options provided, by completing the Tutor's sentences (lines 134 and 135), and by giving prompts like "Uh … huh" (line 114, Transcript 10.3) as the Tutor speaks. During this interaction the students do not evaluate one another (as they do when working without the Tutor), but identify the Tutor as the evaluator in foundational practice.

10.3.5 The Tutor's attention to the concavity of the graph as enabling

After the discussion between the Tutor and Lungiswa in Transcript 10.4, Mpumelelo makes a tentative pronouncement (both verbally and by tracing in the air) that suggests that he has also been enabled by this discussion to identify the graph as increasing. However, he is still attending to his original cosine graph and to Siyabulela and Bongani's tracings in lines 136 and 137; he pronounces, "Okay ... [so our graph will it will be like fluctuating upward↑ ((*Moving his hand up from left to right*))]" (line 156).

Responding to Mpumelelo the Tutor does not control the content of the talk as he does in Transcript 10.4. Rather, he identifies himself as a facilitator in a learner-centred pedagogy by asking why the graph is "oscillating" (line 158a) and requesting (twice) an explanation, "I mean try and like 'kay say something and explain why it would look like that ... " (line 160a). He also makes the time to listen to Mpumelelo's explanation. Mpumelelo responds by replacing his pronouncement focus in line 156 with a different static, named graph from school mathematics and the Foundational Course, this time, "a straight line" (line 160b). It appears that Mpumelelo has not been enabled by the Tutor's intervention to view the function operationally and to consider what is happening to the graph as time passes. Challenged by the Tutor to explain the choice of a "straight line" (line 160b), Mpumelelo supports his argument by recruiting the task context; "Well it's because ... as it as the days get on ... more people get it↑" (line 162). Bongani draws on the property of the graph as "always increasing" (line 161). These two students are linking a straight line graph with an increasing graph, suggesting that they are not attending to the concavity of the graph. In Mpumelelo's case, his argument is further evidence that he does not control the movement of meaning across the mathematical/non-mathematical boundary, in spite of the Tutor's intervention as described in Section 10.3.4.

In line 165a of Transcript 10.5 the Tutor gives a negative evaluation of the students naming of the graph as a straight line. His use of the modal auxillary "wouldn't necessarily" (line 165a) enables him to avoid dismissing the claim with certainty. The Tutor has been listening to the students' explanations, and then responds as in Transcript 10.5.

Transcript 10.5: The Flu Virus Problem, question (a), Group 2, lines 165 to 175

165a	Tutor:	Okay ...<it wouldn't necessarily> go as a straight line
165b	Tutor:	[cause think about uhm ... like say for example ... uh] when when <u>almost</u> everybody has had the virus like if 9 999 people have had the virus so it's <u>almost</u> everybody has been ... infected … there's only one more person that can be infected … right?
166	[((*Bongani uses his finger and makes the shape of a concave down increasing curve in his Resource Book, he then moves his pen to his book and draws the same shape, he then looks up at the Tutor again*))] 	
167	Mpumelelo:	Ja
168	((*Vuyani is drawing in his answer book and then rubbing out, Lungiswa is writing in her answer book, the others are looking at the Tutor as he speaks*))	
169a	Tutor:	So ... the <<u>rate</u> of new infections must be ... very low> because [there's not that many people to infect↑
169b	Tutor:	But earlier on there's more people to ... be infected so the rate can be .... higher↑] right? ((*Using his hand for emphasis, standing and leaning on the table*))
169c	Tutor:	[[So ... if you think about it like that]] the rate is not guaranteed to be the same ... all the time
169d	Tutor:	[[but if it was the same ... then you would have a ... straight line↑]]
169e	Tutor:	So I mean like wh ... [if I have said it like that then what do you think the graph will] … look like?
170	[((*Bongani uses his finger to trace the shape of the concave down increasing graph he has drawn in his book (see line 166), then looks up a Tutor again, all the others are looking at the Tutor as he speaks)*)]	
171	[[*Siyabulela puts his hand in different positions in the air as he listens to the Tutor*))]]	
172	[[((*Bongani is rubbing out in his Resource Book*))]]	
173	[[((*Lungiswa has traced the shape of a concave up increasing graph in her book*))]] 	
174	Mpumelelo:	It's like [...] we'll have ... some curves↑ ((*Tracing something with his pen on his book*))
175	Siyabulela:	[curves]

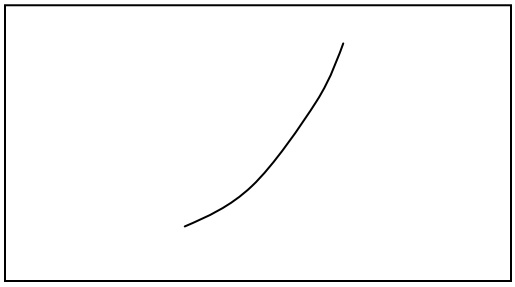
Once more, the Tutor identifies himself as an authority in the foundational practice as he models the valued relationships. He adopts an operational view of the function by considering a particular time in the task context; "when <u>almost</u> everybody has had the virus like if 9 999 people have had the virus" (line 165b). This time he introduces the word "rate", still working in the task context as he talks about the "rate of new infections" (line

169a) and describes this rate at different times (lines 169a and 169b). He presents this mathematical action as a mental process or way to "… think about it" (line 169d). The Tutor's attention to rate in the task context enables the students to move beyond the straight line. For example, Siyabulela's different hand positions in lines 171 suggest he is attending to the gradient of a curve at different points and Lungiswa and Bongani's pronouncements suggest they are attending to the concavity (lines 166, 170 and 173). Mpumelelo adopts Siyabulela's naming of the graph as "curves" (line 174).

In Group 1, it is one of the students (rather than the Tutor as in Group 2) who focuses attention on the concavity of the graph. In Section 10.3.1 I have noted that initially the students in Group 1 suggest that the required graph is a straight line. Jane then questions whether the graph ("it") is "a line" (earlier she uses the word "line" interchangeably for a "straight line"); "But is it a line? ... or is it a?" (line 34). In a similar way to the students in Group 2, Lulama and Jeff use the property of the graph as increasing to argue that it must be a straight line. Lulama draws on the authority of the problem text and the task context; "Because they say sooner or later everybody in the community catches ... ((*Reading from the problem text, then looks up at the others*))" (line 35). Jeff's agreement with Lulama suggests that he also equates the "increasing" property of the graph with a "straight line"; "[So it <u>should</u>]… ja … it <u>should</u> increase … ja ((*Looking at Lulama as he speaks*))" (line 37). However, his use of the modal auxiliary verb "should" does not reflect certainty about this claim.

However, Jane is attending to the rate of change when she argues (without drawing on the task context) that this does not mean the graph is increasing at a "constant rate" and that the graph could be "curvey" (line 40). She demonstrates an increasing concave up graph in the air (see Figure 10.2). In my description in Chapter 8 of the

Figure 10.2: Jane's "curvey" graph, question (a), Flu Virus Problem



work of Group 1 on the Car Problem (when Jane was absent), I argued that Shae and Jeff identify themselves and are identified by the others in the group as authorities in the foundational practice. This also seems to be the case even when Jane is present. Her second attempt at making her attention to the concavity of the graph explicit in line 40 is dismissed by Shae who draws on the authority of the problem text ("they"); "No, but they don't specify that … so we just have to put a general ... increase" (line 43). It is possible that Shae is treating the Flu Virus Problem as a mathematical word problem; since the rate of increase is not "specified" he may view it as "extraneous information" (Gerofsky, 2004, p.33) and thus does not attend to it.

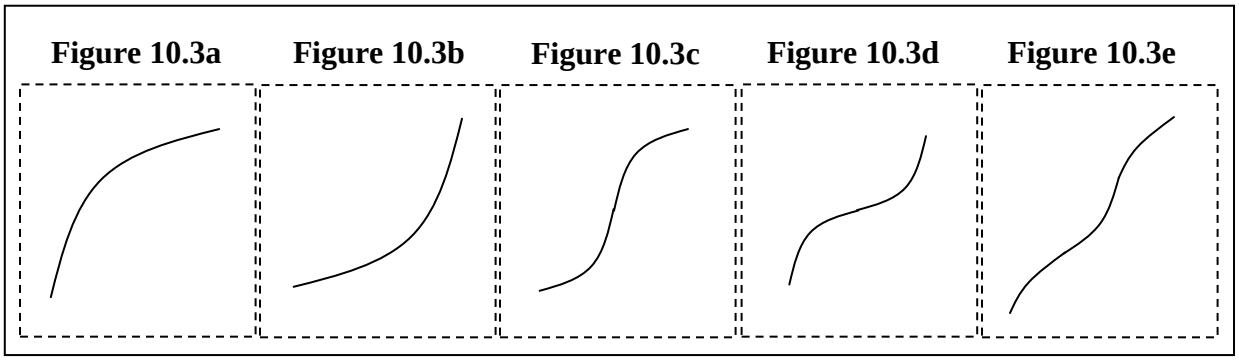
It requires the authority of Jeff to make Jane's ("she") earlier suggestion significant; "But as she said then it should it should actually be more of an exponential thing ..." (line 53). Jeff makes a meaningful link to the task context to explain the exponential growth; "[Because one person] comes into contact with two … then two … then ddd ((*Moving his hands outwards and upwards)*)" (line 56). The others agree with his argument and do not attend to what happens to the change in the rate of infections as more people in the community of 10 000 become infected. In line 56 Jeff is adopting an operational view of the function and attending to what is happening in the population over time. However, his naming the graph as "exponential" (line 53) which suggests a structural view of the function, may constrain him from considering the properties of the graph as more people are infected. Lulama questions the collective ("we") choice of the exponential graph, "So we agree it is an … exponential?" (line 100), but as I have argued in Chapter 8, the socio-political interaction in Group 1 means that Lulama's pronouncements tend to be ignored. Although the students are hesitant to draw the "exponential" graph that Jeff pronounced by naming and tracing in the air, they decide to call the Tutor to have it "checked", thus reproducing the authority of the Tutor as represented in the problem text.

10.3.6 Gestures and severing links to the task context as constraining

I return to the description of the student action in Group 2. The Tutor leaves the group when the students start attending to the concavity of the graph as in Transcript 10.5. The

students draw a variety of increasing graphs, some concave up, some concave down, and some that change concavity (see Figure 10.3). Some of these are drawn on axes and others not, but the positioning on the axes is not attended to explicitly.

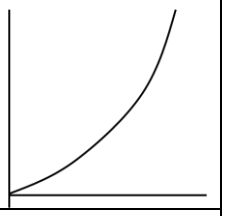
Figure 10.3: Various increasing graphs for Group 2, question (a), Flu Virus Problem

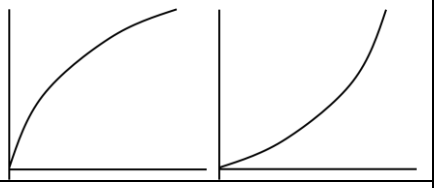


The students talk about the graph as "increasing" (Bongani uses isiXhosa to pronounce that his graph is increasing, "iya-increasa" (line 193)). However, most of the discussion involves pointing to different graphs, using gestures, using reference pronouns and drawing sketches rather than describing the properties of the graphs in words and linking these properties to the task context as modelled by the Tutor. For example, Vuyani gives negative feedback to Lungiswa's sketch in figure 10.3e; "No like ... like like this ((S*tretching across to Lungiswa's book with his pencil in his hand, drawing an increasing, concave up graph as in Figure 10.3b on the axes*))" (line 191b). Lungiswa points to the curve he has drawn in her answer book and asks, "Why do want to draw it like this?" (line 192b). It is possible that the absence of verbal descriptions of the properties of their graphs is preventing the students from attending to the necessary properties.

However, Siyabulela's verbal and visual pronouncements differ from those of his peers in that he talks about the properties of the graphs and makes links to the task context, as suggested in Transcript 10.6.

Transcript 10.6: The Flu Virus Problem, question (a), Group 2, lines 218 to 245

218a	Siyabulela:	But ... how <u>this</u> mos if the graph's like this ((*Drawing an increasing concave down graph, with small tangent lines*)) 
218b	Siyabulela:	... that means that the gradient here is much more steeper here than there … isn't that right? ((*Looking at Vuyani and Lungiswa*))
218c	Siyabulela:	[So if it's like this ((*Drawing an increasing, concave up graph*)), then what's the difference then?] 
219	[((*Mpumelelo is watching Siyabulela as he speaks, Bongani is writing and using his hand to show an increasing curve that starts concave down and changes to concave up, as in Figure 10.3d*))]	
220	Lungiswa:	((*Laughing*)) [Yeah that's the same thing↑]
221	Mpumelelo:	[Ja that's the same] ... that's the same ((*Looking at the graphs in Siyabulela's book*))
222	Siyabulela:	There <u>has</u> to be a difference ... somehow ... I just don't know
223	Lungiswa:	Ja
224	Vuyani:	But I <u>do</u> believe that there <u>is</u> a difference ((*Reaching across to Siyabulela's book and using his pen for emphasis*))
225	((*Mpumelelo and Siyabulela laugh*)	
226	Vuyani:	Really ... because if there was no difference ... [they still] ((*Pointing to the graphs on Siyabulela's page*))
227	Siyabulela:	It has to do like with the with the rate of how much they get affected by time or something
228	Bongani:	The thing is if like ... uh
229	Lungiswa:	Is the rate <u>increasing</u>?
230	Bongani:	They increasing and decreasing.
231	Lungiswa:	[[Do you remember]] the do you remember Workshop <u>1</u>? ((*Looking at Bongani*))
232	[[((*Vuyani looks as if he is going to say something, but does not*))	
233	Bongani:	Ja
234	Lungiswa:	Like the population things ... the popula ... i the graph's like this↑ ((*Drawing an increasing graph going from concave up to concave down, as in Figure 10.3c*))]

235	[((*While Bongani and Lungiswa are talking Siyabulela draws a pair of small axes on his page, and a graph that is increasing and concave down (the graph on the left), he then draws another set of small axes and draws an increasing graph that is concave up (the graph on the right)*))] 	
236	Lungiswa:	[[Do you remember the ... what's the difference? ((*Talking to Bongani and then looking across at Siyabulela as he speaks*))]]
237	Siyabulela:	[[Ah but you know what is difference↑ ((*Looking across at Lungiswa*))]]
238	((*Vuyani looks across at Siyabulela's book*))	
239	Siyabulela:	The difference is ... with this one ((*Pointing to the concave down graph in line 235*)) like the [...] like uh ... as time progresses ... a a a large number a large number of people will be infected ... infected
240	Bongani:	[But I think]
241	Vuyani:	Ja ja ja
242	Lungiswa:	[Uh..huh↑]
243	Vuyani?:	[Ja]
244	Siyabulela:	<u>This</u> one ((*Pointing to his concave up graph in line 235*)) first time [[a small number]] of people will be infected but it is going to increase anyway↑ So I think ((*Putting a star "*" next to the concave up graph in line 235*))
245	Vuyani?:	[[a small number]]

In line 218 Siyabulela attends explicitly to the gradient of the graph at different points by drawing tangent lines at different points on his graph (line 218a) and talking about the "gradient" in line 218b. Lungiswa and Mpumelelo do not attend to the concavity when they describe Siyabulela's two graphs as "the same" (lines 220 and 221). In line 224 Vuyani suggests that there is a difference between the two graphs, locating his argument in his personal belief. Siyabulela makes a tentative argument that draws on the task context and "the rate of how much they get affected by time or something" (line 227). His use of the term "rate" enables Lungiswa to consider the concavity (suggested by the emphasis on "increasing") when she asks, "Is the rate <u>increasing</u>?" (line 229). It is possible that Bongani is also attending to the concavity, but his use of the reference pronoun "they" in line 230 is not clear. The attention to the increasing/decreasing rate enables Lungiswa to make a link to a population graph ("population things", line 234) in another social event in the Foundational Course, that is Workshop 1 where the second derivative was explored

informally using a population graph with concavity as in Figure 10.3c. She does not use the task context to explain her choice. Rather, she tries to "remember" (line 236) the graph from Workshop 1. Siyabulela, on the other hand, does try to explain using the task context by adopting an operational view of the function and considering what happens over time in the task context ("as time passes", line 239). However, his use of tense and reference pronouns constrains him from communicating a meaningful link to the other students. In Siyabulela's first attempt in line 239 it seems that the absence of attention to tense (typical in mathematical word problems (Gerofsky, 2004, p.38)) prevents him from making a meaningful verbal link; his use of "will be" rather than "will have been" results in him making a pronouncement about the increasing property of the graph rather than the concavity. In line 244 it is not clear whether he uses the reference pronoun "it" for the number of people or for the rate of infection, but the latter would have suggested that he is attending to the concavity.

So Siyabulela is the only student who makes links to the task context to reason about the concavity of the graph, yet his language use prevents him from communicating the link meaningfully. The other students ground their arguments in personal choices and in a link to a graph done in previous Course material.

There is a sense in the student talk about the various graphs (in Figure 10.3) that one of the many sketch graphs they have drawn must be right, and in this they are reproducing the view presented in question (a) of the Flu Virus Problem that there is only one possible solution. They may be acting like students in a more traditional pedagogy that values one answer. For example, Vuyani attends to their strategy, "Now ((*laughing*)) now we have to find which way are we going to ... draw this thing. So like this or like this?" (line 199). At one point Lungiswa challenges the view that only one graph is correct. Responding to the graph that Vuyani has drawn in her answer book she draws attention to her sketch in Figure 10.3e; "But you can also draw it like this↑ ((*Pointing to the graph with her pen*))" (line 192a). Yet her view that there is more than one possible answer does not seem to be shared by the others in the group. The Tutor reproduces the representation of one possible graph in

the problem text, for when he returns to Group 2 he asks, “Guys ... do you have a do you have ... an agreement on a graph yet?” (line 254). This leads the students to select a graph. Mpumelelo pronounces his personal choice (“I”); “I think guys ... we must take this one” (line 257). Again, the students do not describe the graph they are selecting, for when the others ask Mpumelelo for clarification he looks at Lungiswa and names the choice as, “The one you were talking about” (line 260) and they link Lungiswa’s choice to a graph from Workshop 1 earlier in the year. Vuyani identifies the graph he is selecting by pointing, “This one ((*Pointing to one of the graphs on Siyabulela’s page*))” (line 263).

10.3.7 The Tutor’s interaction with the students as both enabling and constraining

When the Tutor returns to Group 2 (as promised in line 176), he controls the pace of the students when he says, “... well you have to move on now okay↑” (line 267). In his evaluation of the students’ method for choosing a graph as described at the end of Section 10.3.6, he identifies himself as an authority in the ways of arguing in the practice of foundational mathematics; “It’s not about that we have a democracy that you have to vote for a graph” (line 271a). In this evaluation he includes the students (“we”) as participants in this practice.

The Tutor proceeds to recap their previous discussion (as in Transcript 10.5) and links to the rate in the task context; “We said remember I said as lots of people have been infected then the rate of infection will be ... small because there was ... very few people to be infected↑ right?” (line 273). In the interaction that follows the Tutor does not act like a facilitator in a learner-centred pedagogy who asks students to explain etc. Rather, he controls the content by providing two possible graphs, naming them the “basic” graph and the more “sophisticated” option (line 290a) (see Figures 10.4 and 10.5 respectively). So although he has recently reinforced the text’s reference to one possible answer (line 254), here the Tutor produces two possible answers. The Tutor thus exercises his agency in adapting the Course material and improving on the valued answer in the Course material.

As a subject with some power in the foundational practice, he has some freedom in his action (Fairclough, 2001).

Figure 10.4: The Tutor's "basic" graph, question (a), Flu Virus Problem

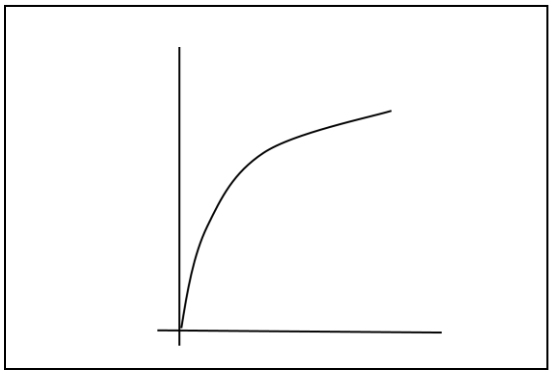
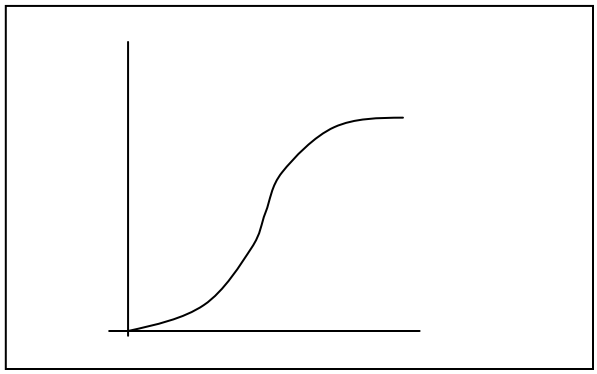


Figure 10.5: The Tutor's "sophisticated" graph, question (a), Flu Virus Problem

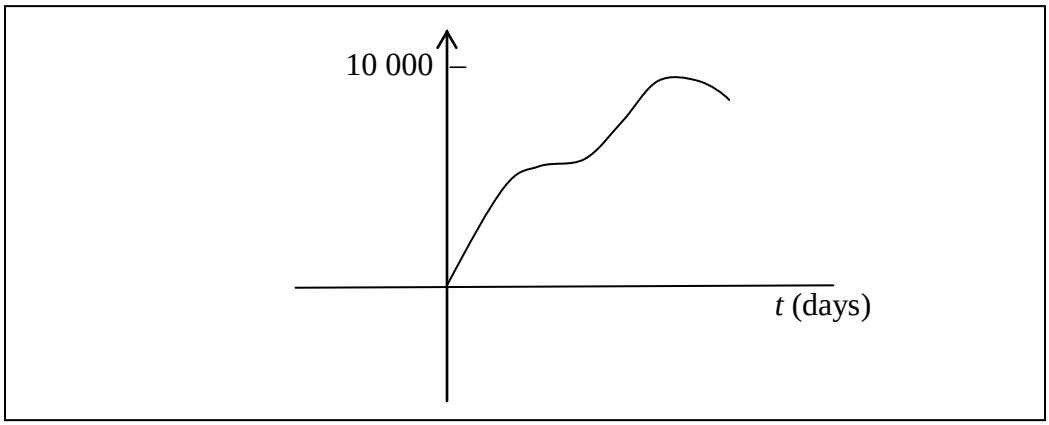


The Tutor then borrows a pencil from a student and starts to draw in Siyabulela's answer book. As he draws he makes verbal links between the properties graph he is drawing and the task context. For example, as he draws the graph in Figure 10.4 he says, "cause say one day has passed ... and at the start of the day a few people had it and the rate is high↑ then at the end of the day ... many people would have it ... so ... then this slope is gonna be ... very steep↑ (lines 279b and 279c). The Tutor attends to the maximum of 10 000 but does not include this on the graph he draws (he holds his hand horizontal in the air instead). During this extended explanation the students' participation is restricted to providing content-free feedback and prompts ("okay" as he nods his head, Mpumelelo, line 280), providing a one-word answer of "ten thousand" to the Tutor's question about the number of people who can be infected, and completing the Tutor's sentence (when the Tutor explains, "because ... basically because [[everybody has had it]]" (line 286g), Lungiswa joins him in completing his statement, "[[everybody has it]]" (line 289)).

So while the Tutor is playing a necessary role of modelling the to-and-fro movement between the graph and the task context, a movement that I have argued the students have difficulty controlling, he controls the action by drawing the graph for the students. Once the

Tutor has modelled the boundary crossing and drawn the two possible graphs in Siyabulela's book, the students do not explore the links to the task context any further. Mpumelelo, Vuyani and Bongani simply choose one of the two options, Siyabulela leaves the graphs the Tutor has drawn in his book, and Lungiswa has revisited and extended the graph in Figure 10.3e, with a graph that eventually starts decreasing (see Figure 10.6). The discussion in 10.3.6 suggests that Siyabulela has some control over the to-and-fro movement across the mathematical/non-mathematical boundary. However, there is not enough evidence to conclude that the Tutor's intervention has enabled the other students in Group 2 to gain the necessary control over this boundary crossing.

Figure 10.6: Lungiswa's final sketch graph, question (a), Flu Virus Problem



10.4 Discussion of the student action on question (a) of the Flu Virus Problem

The description of the student action in this chapter suggests that the students make little progress sketching a graph when the Tutor is absent. I have provided evidence to suggest that this action is constrained by how the students control the movement of meaning across the school mathematics/foundational practice boundary. In both Groups 1 and 2 the students identify the graph with static, named graphs from school mathematics and the Foundational Course. While these functions and their graphical representations form the content of both practices, the movement of meaning in the ways of acting mathematically on these functions is not continuous across the school mathematics/foundational practice

boundary. The students' choice of static graphs with recognisable shapes suggests a structural view of the function (Tall, 1992, 1996), while an operational view of the function and attention to the graphs as the value of the independent variable increases is valued in the Flu Virus Problem.

The students' choice of a static graph is constraining as they proceed to attend to some properties of the graph and not others. In particular, as noted in Section 7.4.1, the student needs to adopt an operational view if he is to attend to the concavity of the graph of $P(t)$. The students' educational talk about this function in the group also constrains the attention to the necessary properties. For example, the students talk about the graph using reference pronouns such as "it", "that" and "this", and they give one another positive, content-free feedback and prompts. When talking the students represent the graphs using gesture rather than in sketches. Furthermore, the socio-political interaction in a group may constrain students who are attending to the necessary properties of the graph from gaining a voice. This analysis supports the argument by Adler (1997) that a learner-centred pedagogy may mitigate against the development of mathematical understanding. However, in this study this argument is based on student interaction, rather than on teacher-student interaction as in Adler's study. In this study the learner-centred pedagogy valued in the Foundational Course workshops in which students talk about their work, evaluate one another, and encourage one another to participate appears to constrain their attention to the properties of the mathematical function and its representation.

Yet on the other hand the talk in such a learner-centred pedagogy may also enable the mathematical action. For example, I have demonstrated how the students co-construct answers as they repeat, reword and extend one another's statements. I also suggested that Vuyani develops a voice in Group 2 precisely because of the supportive relationships set up by the students as they position themselves as students in a learner-centred pedagogy.

I have argued that the students recruit the genre of mathematical word problems and act like school mathematics students solving word problems. This link to the genre of

mathematical word problems is both enabling and constraining. On the one hand the students draw on the genre productively to identify "extraneous information" (Gerofsky, 2004, p.33) not necessary for the mathematical action. However, the students' identification of cues in the task context is constraining, either since it is not possible to identify enough points to sketch a graph (Shae), or since it is used to introduce additional information into the problem (Mpumelelo). In addition, although the students do support their answers using the task context, the absence of attention to the task context as being real (Gerofsky, 2004) means that they do not attend to the consistency in the movement of meaning across the boundary between the mathematical and non-mathematical. Finally, when the Tutor is absent the students do not give significance to the to-and-fro movement across this boundary in support of the properties of the graph; the severing of the link to the task context by the students in Group 2 (as described in Section 10.3.6) appears to be constraining rather than enabling as suggested by Straehler-Pohl (2010) and Lubienski (2000).

I have also argued that the student Lungiswa does not control the movement of meaning across social events in the Foundational Course where increasing and decreasing rates are represented graphically. Although she seems to remember a static graph and the task context of population growth from Workshop 1, the absence of links between the task context and the graph in each event constrains her productive use of this link.

The students in both groups are not able to move beyond their choice of static graphs from school mathematics in the absence of an authority in the foundational practice who can model the required ways of acting on the function. This role of authority is adopted by the Tutor who models the necessary relationships between the functions, their graphs, and the task context. This action appears to enable four out of five of the students in Group 1 to attend to the concavity of their graphs and to complete the question. Yet the extent to which the Tutor is able to support the students in Group 2 is constrained by his having to occupy multiple subject positions in the workshop class. On the one hand the Tutor acts like a facilitator in the learner-centred pedagogy valued in the Foundational Course by asking

students for explanations. Yet his concern about the pace at which the students in Group 2 are working results in him controlling what the students talk about (he provides possible answers to his questions) and whether they act at all (he eventually draws two possible graphs for them). This control is enabling in some respects, for example, in enabling students to identify the graph as increasing. It is also more enabling to some students than others, for example, Siyabulela's talk suggests that he is making similar links to those made by the Tutor. Since the Tutor draws the solution for the students in Group 2, it is not possible to identify his modelling of the valued mathematical action as enabling or constraining for these students.

10.5 The developing argument about the student action

The analysis in this chapter further develops the representation in Chapters 8 and 9 of the complex nature of participation in the foundational practice. Again, it is not possible to identify a particular way of acting mathematically as enabling or constraining, and the analysis points to the complex interplay between the various ways of acting in the practice.

When solving question (a) of the Flu Virus Problem the students have difficulty working with the continuity and disruption that this problem represents in its relationship to school mathematics. They control the movement of the mathematical object function across the school mathematics/foundational practice boundary, but they also recruit a way of acting mathematically on functions from school mathematics, a way of acting that is not valued in the foundational practice.

While sketching the graph in the foundational practice requires an operational view of the function, the students are constrained by their ongoing attention to their static graphs from school mathematics (which suggests a structural view of the function). A number of other ways of acting mathematically identified in this analysis appear to contribute to the students' ongoing attention to their cosine, straight line and exponential graphs. For example, I have argued that the students are constrained by the links they make to other

practices and to other social events within the foundational practice. As in the action on the Car Problem presented in Chapter 8, the students draw productively on certain aspects of the genre of mathematical word problems that they encountered in school mathematics, yet the assumptions of this genre constrain the students from making the necessary to-and-fro movement across the mathematical/non-mathematical boundary. A student in Group 2 makes an unproductive link to a text in another workshop in the Foundational Course. In addition, constraints are located in the nature of the educational talk, their way of evaluating one another in a learner-centred pedagogy, and in the students' use of gesture to represent the function when talking.

I have suggested that the students' talk (in the style of students in a learner-centred pedagogy) is constraining with respect to their attention to the properties of the required graph. Yet the analysis of the action of Group 2 in this chapter suggests that this style may be enabling, for example, in building textual responses and in promoting productive socio-political interaction.

Since the students are not able to move beyond their choice of graphs from school mathematics, this analysis points to the need for an authority in the foundational practice who can model both the valued mathematical action on the function and the necessary to-and-fro movement across the mathematical/non-mathematical boundary. Yet the Tutor's attempt to act as such an authority in Group 2 points to challenge of adopting the style of a tutor in the foundational practice; the Tutor models the valued action for sketching the graph, yet in his concern about controlling the pace of the students he eventually sketches graphs for the students.

CHAPTER 11: RESULTS (PART 2d)
"DOES THE MAXIMUM VALUE EQUAL YOUR LIMIT?"

11.1 Introduction to this chapter

In this chapter I present, firstly, the student action on questions (f) and (g) of the Flu Virus Problem (see Appendices B and Q for the problem and Appendices K and L for the summary of the student action). These questions require the student to evaluate both limits at infinity $\lim_{t\to\infty} P(t)$ and $\lim_{t\to\infty} P'(t)$ and to "Give a short reason for your answer". In Chapter 7 I have argued that these two questions can be solved by viewing the limit either operationally or structurally, in both cases by making a link to the task context. When viewing the limit operationally, the student can make a link within the problem text itself and use the graph of the function $P(t)$ in question (a). Since no algebraic formula is provided for the function, the students cannot act operationally on mathematical objects.

I focus on the action of Group 1 in Sections 11.3 to 11.5. Where appropriate I also refer to the action of Group 2 as additional evidence or to contrast the action in the two groups. The initial analysis of the student action on question (g) was published in Le Roux (2010), and I draw on the review feedback and this published work in presentation of the results that follow. The action of the students in both groups on question (f) and (g) includes attempts to talk about mathematical objects using the practical terms genre. To supplement these results, I end this Chapter with a discussion of the action of Groups 1 and 2 on questions (c) to (e) of the Flu Virus Problem, an initial analysis of which was reported in Le Roux (2008b).

11.2 Summary of the student action on questions (f) and (g) of the Flu Virus Problem

In both Groups 1 and 2 the students correctly evaluate, with ease, the limit $\lim_{t\to\infty} P(t)$ in question (f). They are enabled by making an intertextual link to the graph of $P(t)$ in question (a), by viewing the limit operationally, and by attending to what is happening to the number of people infected by the flu virus as time passes. They explain the answer of "10 000" with reference to the maximum number of 10 000 people in the community.

In Group 1 the incorrect solution of "does not exist" for question (g) is driven by Shae and Jeff (who work ahead of the other students), and later by Darren who listens to the initial discussion. Shae and Jeff represent the derivative function $P'(t)$ in the expression as a vertical straight line graph (presented using a gesture) and this graph becomes the focus of attention in the group. Darren joins the discussion and the three students agree that it is not possible to find the derivative of a vertical line, and hence the limit $\lim_{t\to\infty} P'(t)$ "does not exist". The other students accept this answer. The endorsing arguments made by Shae, Jeff and Darren do not recruit the task context, but are mathematical arguments that draw on the limit definition of the derivative that was discussed in the Course lecture on the same day. The students' use of unclear reference pronouns, the absence of a sketch of the derivative graph and a name for this graph, their severing of links to the task context, and the socio-political interaction between the students constrain their identification of the problematic nature of this argument.

When evaluating the limit $\lim_{t\to\infty} P'(t)$ in question (g), the students in Group 2 are constrained by not being able to reconcile the meaning they give to the symbols $t \to \infty$ (t approaching infinity) and the meaning they give to the derivative $P'(t)$ (the derivative at a particular time). The students are not able to progress any further on the problem and wait for help from the Tutor.

In both groups, the Tutor acts as the authority in the foundational practice by modelling the valued mathematical action; he focuses attention on the graph of the function $P(t)$ in question (a), the gradient of the graph, and the meaning of the function and its derivative in the task context. His explanations enable the students to correctly evaluate the limit $\lim_{t\to\infty} P'(t)$ in question (g) as "0".

When giving "a reason for your answer" as required by the problem text (and the Tutor when interacting with Group 1), students in both groups attempt, unsuccessfully, to use the practical terms genre in their explanations. The students in Group 2 in particular invest considerable time testing for verbal pronouncements that sound right, based on other practical problems they have encountered in the Foundational Course.

11.3 Student action on question (f), Flu Virus Problem (Group 1)

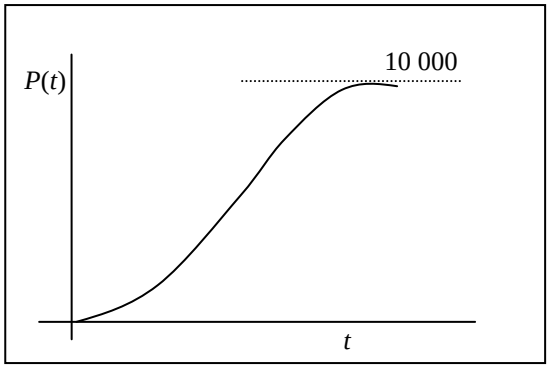
11.3.1 Attending to the task context and the graph of *P*(*t*) as enabling for evaluating the limit

Shae and Jeff work ahead of the other four students (Darren, Hanah, Jane and Lulama). So the discussion moves to and fro between questions (f) and (g) as the other students ask Shae and Jeff for feedback.

Shae and Jeff both pronounce answers of "10 000" (lines 322 and 324) for the limit $\lim_{t\to\infty} P(t)$ in question (f). They recruit the task context in their written explanations; "it is the max number of people in the community" (Shae), and "10 000 cause maximum amount of people in town" (Jeff). Shae's pronoun "it" in his written answer possibly references the answer 10 000. The answer suggests that they are attending to the value "10 000" and the word "maximum" in the problem text and to the function $P(t)$ in the limit expression $\lim_{t\to\infty} P(t)$ in question (f). By working ahead and verbally pronouncing answers, the two students identify themselves as authorities in the practice of foundational mathematics.

Later, Shae and Jeff occupy these positions of authority by responding to the query from Jane, "What's the reason?" (line 410). Their explanations suggest they are attending to the graph of $P(t)$ in question (a), a graph that Darren names as "the s-shaped graph" (line 445). Shae draws attention to his graph by making a verbal pronouncement and turning to the page in his answer book where he drew the graph, "Cause like have you have seen the graph↑ like ((*Turning back in his answer book*))" (line 444). Shae seems to use the reference pronoun "it" for the graph; "Yes … this is going to be the line 10 000 is the maximum it can get to↑ ((*Pointing to the page where the graph in Figure 11.1 is drawn*))" (line 446). In Jeff's explanation the first reference pronoun "it" appears to refer to the graph, with the second being used to identify the maximum value of 10 000; "... there are there <u>are</u> only 10 000 people in the community… <u>so</u> it <u>can't</u> exceed it ((*Tracing an s-shaped graph with his pen*)) (line 448). Jeff is reproducing Jane's earlier argument that the value of 10 000 cannot be "exceeded" (line 414). The material processes suggested by the verbs in "get to" (line 446) and "exceeds" (line 448) and Jeff's tracing of the graph in the air in line 448 suggest that the students are looking operationally at the limit and considering what is happening to the graph over time.

Figure 11.1: Shae's graph, question (a), Flu Virus Problem



11.3.2 Attending to the "maximum" in the task context as possibly constraining in mathematical discourse

Darren poses two questions of clarification to Jeff in lines 450 and 452 of Transcript 11.1, and Shae and Jeff identify themselves as authorities in the foundational practice by responding. In line 452, Darren questions the relationship between the two objects which he names as "the maximum value" and "your limit". Using the pronoun "your" in line 452, he gives ownership of this relationship to Jeff rather than to himself. I use Transcript 11.1 as evidence that (a) the representation of the spread of the disease in the task context drives

this relationship, and (b) the students have difficulty reconciling this representation with the intuitive definition of the limit to infinity used in the Foundational Course (see Figure 7.1, Section 7.2.4).

Transcript 11.1: The Flu Virus Problem, question (f), Group 1, lines 450 to 461

450	Darren:	[So the] maximum value will be 10 000? ((*Looking at Jeff*))
451	Jeff:	Ja
452	Darren:	So does the maximum value equal your limit though? ((*Looking at Jeff*))
453	Shae:	[Ja]
454a	Jeff:	[Ja], well it is the limit
454b	Jeff:	you can't go more than it
454c	Jeff:	and it says everyone gets infected doesn't it? ((*Looking back at the question*))
455	Hanah and Shae:	[Ja]
456	Jeff:	So it has to go up to to 10 000 … it can't go above it mustn't be below ((*Demonstrates an s-shape with his pen in his hand, Jane, Lulama and Darren are watching, Hanah is looking at her Resource Book*)) 
457	Jane:	((*Looking at Darren*)) It's tending towards 10 000
458	Shae:	°It actually gets there it doesn't tend° ((*Looking at Jane*)).
459	Jane:	Oh ... right huh huh ((*Laughing and looking at Shae*))
460	Jeff:	Okay
461	Shae:	°It does … ja tend to ... ° ((*Looking at his answer book, Jeff and Jane are looking at Shae*))

Both Shae (line 453) and Jeff (line 454a) give positive feedback to Darren. The verb "is" in line 454a suggests that Jeff is identifying a relationship between the two objects. Jeff's explanation in line 454c suggests that he recruits the problem text ("it", line 454c) and the task context to support this relationship, arguing that the number of people infected does reach the maximum value of 10 000. He sets up a causal relationship ("so") to conclude; "So it has to go up to to 10 000" (line 456). Jeff's description of a material process of

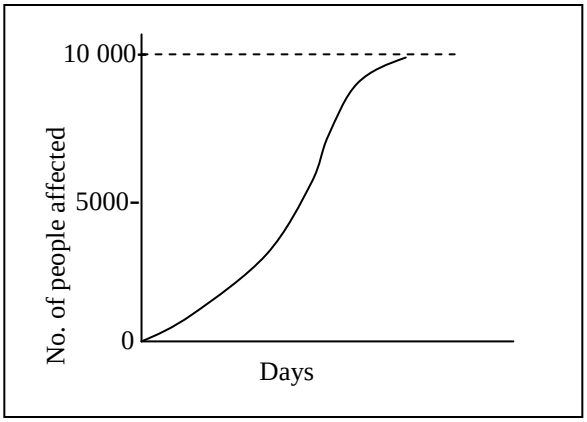
movement ("go", lines 454b and 456) and his tracing of the graph in line 456 is further evidence that he is adopting an operational view of the limit.

This identification of the limit with the everyday object of "maximum" is a relationship noted by Cornu (1991, p.154) in the empirical research in advanced mathematics. Jane and Jeff's pronouncements suggest they identify the limit as a "barrier" (Artigue, 2001, p.211) that cannot be "exceeded" (lines 414 and 448).

Darren does not ask any more questions, suggesting that he accepts Jeff's explanation. Jane's tone in line 458 suggests that her pronouncement is an addition (rather than a challenge) to Jeff's explanation, this time using the term "tending towards" as would be used at school and in the Foundational Course. Shae, however, identifies her pronouncement as different from Jeff's. He gives a negative evaluation (but one which is softened by talking more quietly than usual) by recruiting the task context as Jeff does, distinguishing between "gets there" and "tends" (line 459). Jane agrees with Shae (line 460). However in line 461, Shae quietly contradicts his own statement in line 458. Although Jeff and Jane's glancing at Shae suggests they are attending to this new statement, they do not attend to it verbally and continue with the next question. It may be that the students do not attend to this further, as they do not know how to reconcile the representation of the disease in the task context and the intuitive limit definition they have used in the Foundational Course. On the other hand they may not attend to it simply because they are able to answer question (f) without attending to it (and this is confirmed by the worked solution for question (f), see Appendices B and Q).

The students in Group 2, on the other hand, give significance to the meaning of the limit emphasized in the Course (and at school). Although they acknowledge that 10 000 people will be infected" (Lungiswa, line 619), they describe the number of infections over time as "comes closer to 10 000" (Lungiswa, line 611) and "tends to 10 000" (Bongani, line 625). It is possible that the students' attention to the "asymptote" named and sketched by Vuyani promotes their use of this terminology (Figure 11.2[133]). The students identify with the style of students solving practical problems in a mathematics classroom in that they adopt a mathematical gaze (Dowling, 1998, p.121) on the task context.

Figure 11.2: Vuyani's graph, question (a), Flu Virus Problem



11.3.3 Attending to the graph of *P*(*t*) and an operational view of the limit as enabling (Group 2)

In this section I use the action of Group 2 on question (f) to provide further evidence that attention to the graph of *P*(*t*), together with an operational view of the limit $\lim_{t\to\infty} P(t)$, is enabling. The students in Group 2 begin by describing the expression "physically" (Lungiswa, line 563) or "not mathematically" (Siyabulela, line 562), as illustrated in Transcript 11.2. It seems that they are still attending to the instructions in questions (c) to (e) of the Flu Virus Problem, thus reproducing the practical terms genre of the foundational practice.

[133] It is not possible to tell from the copy of Vuyani's written work whether his graph actually touches the horizontal asymptote.

Transcript 11.2: The Flu Virus Problem, question (f), Group 2, lines 592 to 598

592	Mpumelelo:	It's like as days ((*Looking up from the problem text and across at Vuyani*))
593	Lungiswa:	Uh huh ... goes by
594	Mpumelelo:	goes by ... the number of people ... ((*Pausing and looking at Siyabulela*))
595	Siyabulela:	Who were infected (unclear) ((*Holding his hand in front of his mouth, looking at the problem text and then glancing sideways at Mpumelelo*))
596	Mpumelelo:	Were … … ((*Raising his eyebrows*)) more people [were infected↑] ((*All the students are looking at him*))
597	Lungiswa:	[More .. … more people] are infected↓
598	Mpumelelo:	Ja ((*Looking across at Lungiswa and then sideways at Siyabulela))*

In lines 592 and 593 Mpumelelo and Lungiswa attend to the symbols $t \to \infty$ and co-construct a description of this as "as days go by". Then Mpumelelo and Siyabulela attend to the changing function values $P(t)$, described in everyday language as "the number of people who were infected" (lines 594 to 595). Looking operationally at the limit expression in this way the students identify that "more people are infected" (lines 596 and 597). Although the students, prompted by Lungiswa, have attended briefly to the meaning of the function $P(t)$ in Sentence 4 of the problem text (line 579 to 591), they do not attend to this in Transcript 11.2 and in the rest of the discussion.

The students in Group 2 have yet to evaluate the limit numerically. Siyabulela proposes, first, that $P(t)$ tends to infinity (line 600). This is followed by a reworked pronouncement by Siyabulela and Lungiswa that "it has to end" (lines 603 and 605), but without an actual value. This evaluation is enabled by Vuyani's attention to his graph in question (a) and his sketching of a dotted horizontal line at 10 000 on the vertical axis, a line he names the "asymptote" (line 606) (refer to Figure 11.2). This link to the graph enables Vuyani, Siyabulela, Lungiswa to evaluate, with ease, the limit as 10 000.

The students then finalize their written answers which are variations of "As days progress/go by the number of people infected got close to/tends to/approaches 10 000". Although the students' use of "practical terms" to describe the limit expression (as in

Transcript 11.2) enables the link to the task context and possibly an operational view of the expression, these pronouncements involve considerable debate about appropriate everyday wording. For example, the students compare the phrases "as days go by" (Mpumelelo, line 632) and "as the days progress" (Lungiswa, line 642). There is also some joking about this action. Siyabulela's smile and the laughter of the other students as he supplements the terms in lines 632 and 642 with, "… when they go back" (line 638), suggest that they do not take this attempt seriously. Siyabulela laughs as he evaluates one of his attempts at using "practical terms" as "too much physics" (line 628b). Siyabulela's action provides further evidence for the argument presented in Section 10.3.3 that he identifies himself as a joker and, as a student who has some power in the foundational practice, draws in a more cavalier way on the conventions of this practice (Fairclough, 2001). I discuss the action of describing mathematical objects in "practical terms" further in Sections 11.5.6 and 11.7.

11.4 Discussion of the student action on question (f) of the Flu Virus Problem

I have presented evidence that the students in both groups are enabled to evaluate the limit $\lim_{t\to\infty} P(t)$ with ease by attending to the graph of the function $P(t)$ and by viewing the limit operationally and considering the change in the number of infections over time in the task context.

The analysis of the students' "short reasons" points to the extent of their control of the movement of meaning across the mathematical/non-mathematical boundary. In Group 2 the students invest considerable time attending to non-mathematical wording in the practical terms genre, with debates that end in the non-mathematical practice rather than in the mathematical practice. The latter practice is the valued destination in the study of calculus in advanced mathematics (Tall, 1996). However, the students use of terms like "tends to" and "closer to" and an absence of attention to whether the function values actually reach 10 000, suggests that they do adopt a mathematical gaze (Dowling, 1998, p.121) on the task context and use the intuitive view the limit promoted in the Foundational Course.

In contrast, I have argued that the students in Group 1 use the task context to drive the relationship between the everyday object "maximum" and the limit. There is an absence of attention to the discrepancy between the representation of the disease in the task context as reaching the maximum value of 10 000 and the intuitive definition of the limit in the Foundational Course in which the graph "tends to" the value of 10 000. As suggested by Cornu (1991, p.154) in the ontological/psychological research in advanced mathematics, the significance given by these students to the relationship between the limit and everyday objects such as the "maximum" and a "barrier" may conflict[134] with the formal definition of the limit required in advanced mathematics.

11.5 The student action on question (g), Flu Virus Problem (Group 1)

11.5.1 Linking the maximum value of the derivative function and the limit as constraining

Shae is the first student to pronounce an answer (in the form of a question in the declarative mood) for value of the limit $\lim_{t\to\infty} P'(t)$; "Wouldn't g be 10 000 as well↑ ((*Looking at Jeff*))" (line 342). His pronouncement of a question and use of the modal auxiliary verb "wouldn't" identifies him as uncertain about the value 10 000, and by looking at Jeff he is identifying Jeff as a student who can give him feedback. Jeff, who is reading the question, responds negatively with "[Nooo]" (line 345). This evaluation overlaps with Shae's explanation (using the conjunction "cause" to signal a reason) which he locates in the task context, "[Cause it could be 10 000 people that catch it per day]" (line 346). Shae's reference to the number of people who catch the flu "per day" (line 346) indicates that he is attending to the derivative function $P'(t)$ in the expression $\lim_{t\to\infty} P'(t)$ and identifying this function with its meaning in the task context. Shae uses the reference pronoun "that" to connect his next statement, "That would be the maximum amount" (line 347), to the

[134] For Cornu (1991) this would be a cognitive conflict, but from the theoretical perspective adopted in this study it would be a conflict in the ways of defining within mathematical discourse.

"10 000" in line 346. His emphasis on "maximum" (line 347) suggests that he is identifying the required limit with the maximum value, this time, the maximum daily rate of infection. Although, as I have argued in Section 11.3.2, the link between the maximum value of the function and the limit enables the students to evaluate the limit in question (f), this is not the case with the link between the maximum value of the derivative function and the limit in question (g).

When answering question (g), the student Bongani in Group 2 also evaluates the limit $\lim_{t\to\infty} P'(t)$ as "10 000" (line 679). However, unlike Shae, Bongani has not attended to the derivative function $P'(t)$ in the expression $\lim_{t\to\infty} P'(t)$. The other students draw his attention to this part of the expression by attending to the difference between the limit expressions in questions (f) and (g) ("That only answered number f ((*Pointing to the problem text with his pen*))", Siyabulela, line 681) and by naming the symbols $P'(t)$ as "the derivative" ("We have the derivative now ((*Looking at Bongani*))", Vuyani, line 687).

11.5.2 Using reference pronouns for mathematical objects and gestures to represent graphs as constraining

Returning to the action in Group 1, Jeff and Shae continue attending to the derivative function $P'(t)$ and represent this function graphically in Transcript 11.3. Although the students do not explicitly name their graphs as representing the derivative function at this stage, their attention to this function is confirmed later in interaction with the Tutor, as discussed in Section 11.5.5. Jeff and Shae do not make a link within the problem text to the graph of $P(t)$ in question (a), as is valued in the worked solution.

Transcript 11.3: The Flu Virus Problem, question (g), Group 1, lines 348 to 358

348	Jeff:	That would mean an infinity↑... gradient wouldn't it↑
…*		
353	Shae:	[You couldn't have a graph for that...][[that will just be a dot ... it will just be a dot↑]]
354	Jeff:	[[That will be like]] ((*Holding his hand vertically*))
355	Shae:	((*Looking at Jeff*)) Ja a straight line
…*		
358a	Jeff:	((*Addressing Shae*)) Or not ... okay it won't be ... because it is one to 10 000 so ... ja it will be relatively a straight line ...
358b	Jeff:	so it will tend towards [infinity↑] ((*Shae is tapping his pen as he listens*))

*The other students in the group are answering different questions.

Jeff uses gesture (line 354) and naming (line 358a) to represent the derivative function as a vertical straight line going from "one to 10 000" (line 358a) on the vertical axis. Shae describes the graph as "a dot" (line 353). Neither student draws a graph. The students are attending only to the derivative function $P'(t)$ in the expression $\lim_{t\to\infty} P'(t)$ and identifying this with the instantaneous rate of change. Hence the graph exists at only one value of the variable t on the horizontal axis. They do not attend to the symbols $t \to \infty$ in the limit expression (I provide further evidence for this claim in Section 11.5.5). In Jeff's pronouncement in line 358b he is using his graph to conclude ("so") that "it" will tend towards infinity. It is not clear whether this reference pronoun refers to the limit $\lim_{t\to\infty} P'(t)$, the derivative graph that he has traced in the air or the "gradient" (line 348) of this graph at the selected value of t.

Hanah, on the other hand appears to be attending to the symbols $t \to \infty$ in the limit expression, suggested by the plural "days" in her interjection; "[But they are talking about the days ... time]" (line 366). However, her pronouncement is not attended to by the other students. As noted in Chapter 8, Hanah usually acts individually and tends not to identify herself as a subject who makes pronouncements in the discussion. Lulama also interjects,

but is not attended to and does not complete his pronouncement, "[But infinity]" (line 367). This absence of attention is also a pattern in the socio-political interaction in Group 1.

11.5.3 Linking to a lecture in the Foundational Course and an absence of a link to the task context as constraining

Although Jeff has not drawn the vertical straight line graph he has pronounced using gesture and has not named it as representing the derivative function $P'(t)$, this graph and tangents to this graph become the focus of attention until the students arrive at a solution. Darren is prompted to talk by attending to Shae and Jeff's interaction in Transcript 11.3, rather than by his own attempts to answer question (g). It is only one hundred speech turns later that he appears to attend to the limit expression $\lim_{t\to\infty} P'(t)$ that Jeff and Shae are currently attending to; after looking at the two limit expressions in questions (f) and (g) and pronouncing that they are "the same thing↑" (line 426), he exclaims ("Oh", line 428) and identifies the "derivative" (line 428) in question (g).

Darren responds to Shae and Jeff by giving a negative evaluation, recruiting what is generally done in mathematics (with the pronoun "you"); "You can't have infinity ... as the derivative" (line 362). All his arguments that follow reference mathematical objects and their representations, with no links to the task context and to particular functions $P(t)$ and $P'(t)$ in the Flu Virus Problem. Darren is interpreting the reference pronoun "it" in line 458b as referring to the gradient of Jeff's vertical line graph. In Transcript 11.4 he sets up a relationship with another social event in the Foundational Course, that is, the lecture that took place "today" (line 367b). The lecturer used the definition of the derivative $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ to identify points at which a function may not be differentiable, for example, at a discontinuity or a where the tangent to the graph is vertical.

Transcript 11.4: The Flu Virus Problem, question (g), Group 1, line 367

367a	Darren:	[No, no ... you see that's the] the whole thing with the gradient ... ...
367b	Darren:	it can't be … like like we said today … it can't be ...
367c	Darren:	like too greatly positive or too greatly negative …
367d	Darren:	otherwise it will have point of turn turning
367e	Darren:	like you have a point of intersection which turns↑
367f	Darren:	And you can't have … you can't find an instantaneous a a a figure↑ at a point ((*Looking across at Jeff and Shae as he talks, using his hand to show lines, first positive going towards the vertical, then negative going towards the vertical*))

Darren's attended and pronounced foci in Transcript 11.4 suggest he is arguing that, as the tangent (representing the gradient, and pronounced using his hand in the air, line 367f) approaches the "point" (line 367f) from the right, the gradient will become larger and larger positive ("too greatly positive", line 367c). Similarly, as the tangent approaches from the left ("too greatly positive", line 367c). The gradient thus changes (or "turns", line 367e) at the "point".

Darren then turns to his lecture notes, "Like I'll show you" (line 378). While Darren looks for his notes, Jeff uses Darren's argument so far to make a tentative statement in line 382 of Transcript 11.5 about "it" not existing. It may be that the reference pronoun "it" refers to the derivative function (in which case the limit in $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ does not exist) or to the actual limit $\lim_{t \to \infty} P'(t)$ in question (g).

Transcript 11.5: The Flu Virus Problem (g), Group 1, lines 382 to 405

382	Jeff:	It doesn't exist↑
383	Hanah:	((*Looking at her book*)) Does f exist↑
384	Darren:	((*Placing his answer book in the middle of the desk, seems he has gone back to a graph from lectures*)) See they are not differentiable ...
385	Jeff:	[Ja]
386	Darren:	[If you look at] the tangents ... are at like 90 degrees ((*Pointing to his book with his pen and showing a line that would be vertical on his page. The other students are looking at his book*))

387	Jeff:	It's tending towards infinity ja.
388	Darren:	So it's tending towards infinity where [*x* will tend]
389	Jane:	[But if you took] the tangent here it would exist ((*Reaching out with her pen and pointing to Darren's page, others are watching*)) ... or here
390	Darren:	Ja, but [if you look] we are looking at the point ((*Looking at Jeff and then across at Jane*)) of where it will tend to infinity ... and because ((*Using his pen in his hand to emphasize*)
391	Jeff:	[Ja]
392	Jeff:	Because this graph is going to be ((*Reaching across and pointing with his pen on Darren's page*)) ... it's gonna be like ... [like a] ((*Sitting back and holding his hand vertically, then pointing to his book with his pen*)) ... almost 90 degree line because it's so high because it is 1 to 10 000
393	Shae:	[Like this] ((*Tracing a very steep line in the air with his pen and nodding his head*))
394	((*All the student look at Jane, who starts paging back in her book*))	
395	Shae:	Right so it doesn't exist↑ ((*Writing in his book, Hanah also starts writing*))
396	Jane:	((*Looking back*)) Okay ((*Writing in her book*))
397	((*Darren takes his answer book back and pages forwards to where he was writing*))	
398	Lulama:	And then what about the short ... a reason↑
399	Jane:	Is that because the tangent would be ... vertical↑
400	Darren:	Basically ((*Looking at Jane and nodding his head*)) … and it's ... and the graph at that point is not differentiable
401	Jane:	Mmm ((*Nodding her head, carries on writing*))
402	((*Hanah is writing, Shae and Jeff listen to the exchange between Jane and Darren, and then write*))	
403	Lulama:	But then ... again ... it's not differentiable and and infinity is not a number ((*Looking at Darren who is writing*))
404	Darren:	Uh hum ((*Agreeing with Lulama as he writes*))
405	((*Jane, Hanah, Shae and Jeff are all writing*))	

In line 384 of Transcript 11.5 Darren draws attention to the graphs he has drawn in his lecture notes that morning with the imperative "see". He pronounces that the functions ("they") are "not differentiable" (line 384), using mathematical terminology from the lecture and supporting his argument with reference to the vertical tangents of the graphs (line 386). Both Jeff and Darren conclude that the tangents ("it") are "tending towards infinity" (lines 387 and 388). Jeff is now supporting Darren by co-constructing his argument, and Darren starts to appeal to Jeff for support by looking at him (line 390). Darren's pronouncement "where *x* will tend" (line 388) suggests that he might be attending

to the independent variable in the limit expression $\lim_{t\to\infty} P'(t)$. However this claim is not supported by his explanation to Jane in line 388; "we are looking at the point of where it will tend to infinity". His use of the reference pronoun "it" suggests that he is not focusing on what is tending to infinity. Jane, however, is considering different values of the independent variable *t* when she attends to tangents to the graph at other values of *t* in line 389. Jane's argument is not ignored, but Darren provides his own mathematical argument against it; in making mathematical arguments (with support from lectures), Darren has identified himself as the authority in the foundational practice and is now receiving support from another authority, Jeff. Shae repeats Jeff's earlier conclusion that, "it doesn't exist↑" (line 395), again the reference pronoun is unclear. The students then start writing, suggesting that they agree with the evaluation of the limit and the explanation collectively presented by Darren, Jeff and Shae.

In his written solution Jeff pronounces that the limit $\lim_{t\to\infty} P'(t)$ "DNE" (which stands for "does not exist), with his "short reason" being that "the graph at that point is non diffable".[135] This reason is appropriate for "the graph", that is, the vertical straight line. However, the link to the evaluation of the limit $\lim_{t\to\infty} P'(t)$ as "DNE" is not clear; it is possible that Jeff is attending, rather, to the limit $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ introduced in the link to the morning lecture. I argue that Jeff's difficulty is related to the unclear use of reference pronouns and the absence of a drawing and name for the vertical straight line graph that is the focus of attention. The written answers of other students are evidence of the constraining nature of this action. In her written answer, Jane agrees that the limit "DNE" but pronounces the "tangent" as "not differentiable" in her reason; "because the tangent to that point is vertical therefore not differentiable". Lulama repeats an earlier verbal pronouncement about infinity (line 403) for which he received positive feedback from Darren (line 404) as his reason; "infinity is not a number".

[135] The term *diffable* is used interchangeably with *differentiable* in the Foundational Course.

11.5.4 Attending to different parts of the limit expression as constraining (Group 2)

I have argued so far in Section 11.5 that the three students who control the action in Group 1 are not attending to the symbols $t \to \infty$ in the limit expression. In Section 11.3.3 I provided evidence that, when answering question (f), the students in Group 2 attend to both the symbols $t \to \infty$ and the function $P(t)$. For the discussion of the action of these students on question (g) I draw on the interaction re-presented in Transcripts 1.1 and 1.2 in Section 1.2. The students also attend to both parts of the limit expression $\lim_{t \to \infty} P'(t)$ in question (g), as suggested by the words "derivative" and "infinity" in Mpumelelo's, "Cause this ... like this derivative ((*Looking at the Resource Book*)) like when you use a like when we don't work with this one .... infinity ... we usually give the exact time ((*Using his pen to demonstrate at the point*)) ... right↑" (line 705). Yet attending to these two objects in the limit expression is not enabling in itself. What the students call "the problem" (Vuyani, line 692) is that their attention to "infinity" (line 705) means that "you can't … restrict the days" (Mpumelelo, line 702), yet their identification of the "derivative" (line 705) as the instantaneous rate of change suggests that they should "give the exact time" (line 705).

The students are not able to resolve "the problem" (Vuyani, line 692) and wait for help from the Tutor. The discussion about how to proceed points to how the students represent their relationship with the Tutor. Both Vuyani and Lungiswa are concerned about what to say when the Tutor arrives; "But that guy's going to ask us ... er what did you do?" (Vuyani, line 730). Siyabulela, on the other hand, identifies himself as planning to resist the position identified by his peers; "we were waiting for you while you were sitting there ... so … just" (line 734b) and "No ... we just tell what are our ideas ... ja" (line 740). His use of the adverb "just" represents this interaction as "simple" (Siyabulela, line 734a). In the interaction with the Tutor that follows Siyabulela is identified by the others as the student who communicates these 'ideas" (line 740) to the Tutor, a position that he occupies.

11.5.5 The Tutor's reading of the limit expression in words and attention to the graph of $P(t)$ as enabling

I return to the action of Group 1 on question (g). Having concluded that the limit in question (g) "does not exist" (Shae, line 472, Transcript 11.6), the students have moved onto the next question in the Resource Book. The Tutor approaches the group and reads the students' written answers for question (g) aloud, as in line 471.

Transcript 11.6: The Flu Virus Problem, question (g), Group 1, lines 471 to 483

471	Tutor:	Okay, I see <u>two</u> answers saying that ... *P* <u>dash</u>...*t* as *t* tends to infinity ((*Reading from limit expression* $\lim_{t\to\infty} P'(t)$)) is ... not defined or [does not exist] ((*Stretching across and pointing to Shae's Resource book*))
472	Shae:	[Ja, it does not exist] ((*Looking up at the Tutor who is standing at his shoulder*))
473	Tutor:	Okay [well] do you all have that?
474	Shae:	[Because it'll] ...
475	Lulama and Hanah:	Ja ((*Lulama, Darren and Hanah are nodding their heads*))
476	Tutor:	What is your reasoning behind that?
477	Jeff:	Because the graph ... it is s ... such a steep graph that it's tending more towards infinity ... than ... ((*All the others look at Jeff and then at the Tutor, who has got down on his haunches next to the desk)*)
478	Tutor:	Okay ... well can I … where is the graph?
479	Jeff:	Do we have to go and draw it? ((*Turns his page back*))
480	Tutor:	No ... you have already drawn it.
481	Shae:	No but that's of that is not of the dash ((*Looking at his graph in question (a)*))
482	((*Jane, Lulama and Darren are watching*))	
483a	Tutor:	No no okay <u>this</u> is the graph of the number of ... people okay …
483b	Tutor:	so wh ... where do you get this thing that it is getting so steep ((*Showing steep gradient with his hand*)) as as *t* tends to infinity?

As described in Chapter 8, Shae identifies himself as the spokesperson in the group by responding to the Tutor (line 472). The Tutor does get a response from the others this time (line 475), however, in the action that follows Shae and Jeff identify themselves as mathematical authorities in the foundational practice by responding to Tutor's questions. The Tutor identifies himself as a facilitator in the learner-centred pedagogy valued in the

Course by asking for the students' "reasoning" (line 476) and positioning himself as a listener next to the students at the table (line 477). Jeff's explanation with reference to his vertical straight line graph in line 477 is further evidence of his unclear use of reference pronouns. Lines 478 to 483a are evidence that the Tutor and the students are attending to different graphs. The Tutor is attending to the graph of $P(t)$ that the students "have already drawn" (line 480) in question (a), a graph that the Tutor links to the task context; "the graph of the number of ... people" (line 483a). However, Shae and Jeff's vertical straight line graph with the steep gradient (line 483b) that they have not drawn (line 479) is the graph "of the dash" (line 481) or the derivative function $P'(t)$.

In the rest of this section I provide evidence that the Tutor's attention to the graph of $P(t)$ with an accompanying link to the task context and his description of the limit expression in words (and his reading of "as t tends to infinity" (line 483b) in particular) enables some of the students. Responding to the Tutor's request for an explanation in line 483(b), Shae develops a response over a number of speech turns in Transcript 11.7.

Transcript 11.7: The Flu Virus Problem, question (g), Group 1, lines 486 to 499

486	Shae:	Well it's bec ... it's because like if you think about it ... uhm ... it will be P … it will the amount of ... the amount of people over time will be the ... that will be the increase the ((*Looking at his Resource Book and also looking up at the Tutor who is standing at his shoulder*))
…*		
489	Shae:	[The *P* dash *t* { $P'(t)$ }]
…*		
492	Tutor:	Okay ... just explain that to me again I didn't catch it↑ ((*Looking at Shae*))
493	Shae:	So what this is saying is that ((*Picking up his Resource Book, holding it at an angle towards himself, and pointing on the page with his pen. The Tutor is looking at the page. The other students cannot see, so go back to their own working in their books*)) ... uhm ... this is equal to the amount of people over time ... that is the increase ...°per day°
... *		
499a	Shae:	Ja so it means that when time increases to infinity
499b	Shae:	... it means it's going to be the number of people over time and as time increases ... it means there is going to be a ...

* Speech turns not related to Shae's argument have been removed.

In line 486 Shae is attending to the function "$\underline{P}$" and the graph of this function in his Resource Book. His pronouncement of the "amount of people over time" (lines 486 and 493) and "time increases" (lines 499a and 499b) is evidence that he is attending to the Tutor's expression of "as *t* tends to infinity" in line 483b. He uses this to describe the meaning of "The *P* dash *t*" (line 489) in the task context; "the increase ...°per day°" (line 493). The body language of the Tutor and other students indicates that the discussion takes place between the Tutor and Shae only.

Jeff also attends to the Tutor's expression of "as *t* tends to infinity" in line 483b; Jeff is about to respond to the Tutor's challenge to explain his graph "We thought that … okay … it will be" (line 484a) when he pauses and repeats the Tutor's wording from line 483b; "... °*t* tends to infinity° ((*looking up at the ceiling*)) okay wait … I'm thinking of the wrong thing ((*Grinning*))" (line 484b). While the Tutor and Shae are talking as in Transcript 11.7, Jeff starts to write in his book, but interjects in the discussion with a verbal pronouncement of the answer in line 502, "It should actually be nought ((*looking at Shae*))". Prompted to explain his answer by both Shae and the Tutor, Jeff uses the graph of $P(t)$ ("it") for support; "Because as it gets ... it it gets to 10 000 then it will just stay constant ((*Uses pen in hand to trace the graph of P*(*t*) *in the air*))" (line 505).

Hanah has been enabled by listening to the discussion, as she pronounces, "[Well] ... the rate of change at infinity is zero" (line 509a). Her naming of the limit $\lim_{t\to\infty} P'(t)$ as "the rate of change at infinity" suggests that she is viewing this limit structurally as an object. She is, however, having difficulty explaining her argument ("because") in the task context; "because it already has ... everything ... °has already happened°" (line 509b), the quieter tone suggesting that she is not confident about this argument. The Tutor evaluates this by rewording Hanah's explanation in the task context (the pronoun "we" suggests that this is a collective action), "So we are saying no new people are getting infected" (line 513), but adding units "per day" with emphasis and pausing for a response from the students; "... per day ... that means that the ..." (line 513).

11.5.6 Attention to using the practical terms genre as constraining

Completing the Tutor's explanation as prompted in line 513, the students introduce the word "rate of change" (lines 514 and 516). The Tutor attends to the students' use of this mathematical term, and focuses their attention on question (e); it seems that he plans to draw on the students' description of the derivative function $P'(t)$ in "practical terms" in this question to help them with their explanation for question (g). However, this plan does not play out as intended as the students' pronouncements for question (e) indicate that they have had difficulty using the practical terms genre as required. For example, Jane pronounces "The instantaneous infection rate" (line 530) and Lulama says, "It's the rate of change" (line 537). The Tutor identifies himself as an authority in the foundational practice by reproducing the practical terms genre and evaluating these attempts as "mathsy" (lines 532 and 538), suggesting rather that they explain "to somebody on the street" (line 533).

The Tutor's attempt to correct the answers in question (e) leads to a discussion between the Tutor, Darren, Lulama and Jane about whether the derivative function $P'(t)$ represents the instantaneous or average rate of change, with the Tutor eventually verbally pronouncing the answer for (e) in using the practical terms genre. The Tutor leaves the group, without them having revisited the explanation for question (g), as he balances his responsibility to these students with others in the workshop class.

For Hanah and Jeff using the practical terms genre for the explanation in question (g) is constraining. Hanah attends to the Tutor's evaluation of statements as "mathsy" (line 538a) in Transcript 11.8. In line 538b the Tutor suggests possible wording for the time $t = 4$ in question (e), his use of "after 4 days" reproducing the valued wording in the Foundational Course.

Transcript 11.8: The Flu Virus Problem, question (g), Group 1, lines 538 to 545

538a	Tutor:	Jaaa ... that's also quite mathsy ((*Shaking his body slightly to the side*))
538b	Tutor:	[so like ... uhm ... after 4 days]
539	Hanah:	[The number of people getting infected]
540	Jane:	Ja ((*Nodding his head across the table at Hanah*))
541	Hanah:	After infinity at infinity ... is zero ((*Looking at the Tutor, all the others are looking at the Tutor for his reaction*))
542	((*Darren is talking to himself as he attempts at answer, then talking louder*))	
543	Tutor:	Okay ((*Nodding his head*)) ... uhm ((*Looking and pointing slightly at Jane and then at Darren*)) ... but even at infinity okay but we are talking about ... ... [at]
544	Jeff:	[Forever] it did it until forever ((*Looking and grinning at the Tutor. Jane, Hanah, Darren and the Tutor laugh*))
545	Tutor:	Ja ... that's that's fine … but we are talking about they are actually doing question e now [[...]]

Hanah attempts to apply this phrase when rewording her earlier pronouncement of, "the rate of change at infinity is zero" (line 509a). She replaces "the rate of change" (line 509a) with a description in the task context; "The number of people getting infected" (line 539). Then, following the Tutor, she changes the preposition from "at" to "after" in "after infinity" (line 541). However, her immediate return to the preposition "at" suggests that she is using the sound of her verbal pronouncement to evaluate herself. The Tutor's "Okay" in line 543 suggests that he agrees with Hanah's evaluation, but his emphasis on "at" in "even at infinity" (line 543) signals a negative evaluation. Jeff interjects with a possible everyday wording, replacing "at infinity" with "until forever" (line 544). This is interpreted by all as a joke, and the Tutor's evaluation "that's fine" (line 545) suggests he is not taking this phrasing seriously and wants to continue the discussion about (e) with the other students ("they").

In their written answers, both Hanah and Jane exercise agency by resisting the instruction to use the practical terms genre in their explanations; Hanah uses "the rate of change per day at ∞ is 0", and Jane pronounces, "at infinity 10 000 people had already been infected so the growth rate is 0". The other students simply avoid referring to the derivative, for example Jeff writes, "when it reaches 10 000 no more people can be infected".

11.6 Discussion of the student action on question (g) of the Flu Virus Problem

I have argued that, prior to the Tutor's intervention, the student action in Group 1 is driven, firstly, by Shae and Jeff who are identified as and identify themselves as authorities in the foundational practice. This action is then developed by Darren who uses mathematical arguments from a Course lecture to identify himself as an authority in the practice. His authority is given significance through support from Jeff and Shae. Positioned as the students who make pronouncements (verbally and using gesture) and who evaluate the interjections of the other three students, Darren, Shea and Jeff control the content of these pronouncements, the valued mathematical action, and the timing of links to other practices and social events.

Shae and Jeff attend to the derivative function $P'(t)$ in the expression $\lim_{t\to\infty} P'(t)$. They represent the graph of $P'(t)$ as a vertical straight line and do not make a link within the text of the Flu Virus Problem to the graph of $P(t)$, a link that is identified in the worked solutions as enabling. The representation of the derivative graph is constrained by their identification of this function as representing the instantaneous rate of change, corresponding attention to one value of the independent variable t on the horizontal axis, and an absence of attention to the change in t, represented by the symbols $t \to \infty$ in the limit expression. They are attending to one object (the derivative function) within the limit expression, rather than viewing the limit expression structurally as an object or operationally by considering what happens to the derivative function as the time variable changes. Although both Jane and Hanah attend to the changing time variable, the socio-political interaction in the group means that their pronouncements either do not receive attention or are given a negative evaluation by Jeff, Shae or Darren. The students in Group 2, on the other hand, attend to both objects $t \to \infty$ and $P'(t)$ in the limit expression, but are constrained from viewing the limit operationally by their identification of the derivative as the instantaneous rate of change.

In Group 1, Shae's link to the task context when identifying of the derivative function as the daily rate of change in the number of infections is severed once a graphical representation of this function has been produced. This suggests that the students are following the one-way movement across the mathematical/non-mathematical boundary typical of the mathematical word problem genre (Gerofsky, 2004). They only revisit the task context when prompted to do so by the Tutor.

The vertical straight line graph traced by Jeff and tangents to this graph are the focus of the attention for the rest of the student action. The mathematical argument that develops is based on an ill-timed link to the morning lecture in the Foundational Course. This argument is not explicitly linked to the limit expression $\lim_{t\to\infty} P'(t)$ that should be in focus. However, the students' use of unclear reference pronouns and the absence of a sketch of the derivative graph and a name for this graph constrain their identification of the problematic nature of this argument. The students do not control the movement of meaning across social events in the Foundational Course, and the verbal discussions that characterize the interactions valued in the learner-centred pedagogy promoted in the Course do not enable the students to evaluate this movement of meaning. This analysis supports Adler's (1997) argument, but this time with evidence from the student interaction, that a learner-centred pedagogy may constrain the development of mathematical understanding.

The Tutor plays the role of authority in the foundational practice by focusing the students' attention on the limit expression in question (g) (and particularly by reading the symbols in words), on the graph of $P(t)$ in question (a), and on the meaning of this function in the task context. This, together with the opportunity provided by the Tutor for the students to explain their initial answer, is enabling for Jeff, Shae and Hanah. Jeff and Shae are enabled as they evaluate the limit correctly by adopting an operational view of the limit and use the graph from question (a). Hanah identifies herself as viewing the limit structurally. The written answers of Lulama, Darren and Jane suggest that they are enabled to attend to the change in the number of people infected over time in the task context. This change suggests

that, prior to the Tutor's intervention, the students getting "stuck" in the mathematics was constraining, in contrast to the claims of Straehler-Pohl (2010, p.454) and Lubienski (2000, p.477) that getting 'stuck" in the task context is constraining. The mathematical authority of the Tutor was required to get the students "unstuck" from the mathematics and to facilitate the to-and-fro movement between the task context and the mathematical objects.

However, the link to the practical terms genre is not necessarily enabling. The Group 2 students' use of this genre in question (f) enables a necessary link to the task context and possibly an operational view of the expression $\lim_{t\to\infty} P(t)$. However, the difficulties experienced by the students in Group 1 using this genre for the limit expression $\lim_{t\to\infty} P'(t)$ suggests that the nature of mathematical objects themselves may determine the difficulty of this action (an argument I develop further in Section 11.7). This analysis suggests that the Tutor and the students invest considerable time, sometimes without success, describing mathematical objects using the practical terms genre, even when they have produced appropriate mathematical descriptions of these objects. Some students represent the use of practical terms genre as a joke or eventually resist it. Yet some students are investing time in a genre that is not valued in advanced mathematics and has the potential for them to get "stuck" in the task context.

11.7 The student action on questions (c) to (e) of the Flu Virus Problem

In this section I discuss further the use of the practical terms genre. I identify similarities and differences in the action of the students in Groups 1 and 2 in order to make claims about the enabling and constraining actions. The students correctly identify the mathematical objects (the function and both rates of change), but have particular difficulty expressing the rates of change using this genre. They address this difficulty either by ignoring the genre or by investing time attending to their everyday wording, with varying success.

11.7.1 Attending to the problem text and to similar problems in the Course as enabling in question (c)

The students draw on the wording used in similar problems in the Course for the wording "After 4 days" and make a link within the problem text between the function $P(t)$ in question (d) and earlier text. Their verbal pronouncement may not be complete, for example, Siyabulela in Group 2 pronounces the number 1 200 by pointing rather than by saying it; "… after 4 4 days ... these ((*Circling with his end of his pen in the Resource Book*)) number of people are infected" (line 356b). However, they are enabled by this talk to produce full written answers. Some of these written answers suggest that the students are attending to the meaning of the function in Sentence 4 of the problem text, for example Jane in Group 1 pronounces, "After 4 days 1 200 people will have it or have had it". Some of the students use the word "infected" in the text of question (a), but their use of tense suggests that they are attending to the meaning of the function in Sentence 4, for example, both Hanah and Jeff in Group 1 indicate that, "After 4 days 1 200 have been infected". However, some students use other tenses, for example, Siyabulela in Group 2 changes from using "are infected" in his verbal pronouncement in line 356b to "will be infected" in his written answer, an answer that is also produced by Lulama in Group 1. It is not possible to tell from these pronouncements whether the students are not attending to the meaning of the function in Sentence 4, or whether they are attending to this sentence but are following the genre of mathematical word problems and not attending to the inconsistencies in their use of tenses (Gerofsky, 2004).

11.7.2 Looking structurally and attending to the units in the practical terms genre as enabling in question (d)

The first full verbal pronouncement in Group 2, characteristically by Siyabulela, uses the practical terms genre; "Ja from four to seven days ... 350 people were ... infected" (line 387). He attends to the denominator of the expression $\frac{P(7)-P(4)}{7-4}$ and identifies this as the change in time. Siyabulela's use of the wording "from four to seven days" suggests he is drawing on wording used in other problems in the Foundational Course. He also attends to

the function $P(t)$ on the numerator, but does not identify the numerator as a whole as representing the change in the number of people infected. He is not looking structurally at the expression as a whole as representing an average rate of change.

However, Vuyani is looking structurally at the expression $\frac{P(7)-P(4)}{7-4}$ and makes a link to other problems in the Foundational Course where the word "average" was used; "Aren't we supposed to include the word ... average?" (line 392). Vuyani's use of the negative auxiliary verb "aren't" and the declarative mood of his statement suggest that he wants feedback on this pronouncement. The use of reference pronouns "this one" and 'this" in the following pronouncements (speakers are unidentifiable in the video footage) suggest that the students are viewing the expression as an object; "It's the average this one" (line 397) and "This is the average" (line 398).

The students then try out "practical" wording by inserting the phrase "on average" into Siyabulela's pronouncement in line 387. They are attending to the meaning of the function $P(t)$ itself, rather than to the rate of change of this function. For example, Lungiswa adds "on average" to her written answer; "from 4 to 7 days on average 350 people will be infected" (line 430). Bongani attends to the number of infected people in the problem text; "… between that period of 4 and 7 ((*Using his hands to show interval from 4 to 7*)) ... there were like … how many people infected? ... ((*Looking briefly at the Resource Book*)) ja like 350 people ... on average average" (line 418). The students do not attend to the instruction in question (d) of the problem text to "Give the correct units", which might focus their attention on the rate of change.

Pronouncements in which students attend to the rate of change are either not attended to ("the average for increasing", unidentified, line 396) or receive a negative evaluation; Bongani's pronouncement, "the number of people were increasing that were infected by 350" (line 420), which suggests that he is attending to the change on numerator of the expression is dismissed by Siyabulela as "the derivative" (line 421). The students do not

know how to proceed, suggested by Bongani's exclamation, "Aa ... rgh ((*Laughing loudly*))" (line 426). Not knowing how to proceed, they attend to the practical terms genre and debate the choice of preposition ("from" or "between") and whether what is being said, in Siyabulela's words, is "bad English" (line 432b).

The Group 2 students' attention to "the word ... average?" (Vuyani, line 392), linked to other social events in the Foundational Course, and their attempts to include this word in their "practical" wording can be likened to their action for question (b) of the Chemical Reaction Problem. They identify the point on the graph where $t = t_1$ with the word "maximum", this time linked to a particular practical problem (called the "Joe Problem") used in lectures. This is followed by attempts to remember what wording was used by the lecturer, with no attention to the "maximum" of what.

I return to the action of Group 2 on question (d) of the Flu Virus Problem. Mpumelelo is viewing the expression $\frac{P(7)-P(4)}{7-4}$ structurally as an object ("this") when he pronounces "[This is a rate of change]" (line 450). He also uses the term "average rate" in his "practical" wording; "So the average rate of people ... to getting ... infected was 350" (line 440), but does not attend to the units. Mpumelelo's pronouncement in line 450 overlaps with Siyabulela's pronouncement about division in "[And divide by the change in (unclear) so]" (line 451), a pronouncement suggesting that he is looking operationally at the expression and identifying the rate of change. They agree that the expression represents "An average rate of change … ja" (Bongani, line 466).

They then attend to the practical terms genre to avoid using the word "rate" which is named as a "mathematical term" (Siyabulela, line 469b). However, they do not communicate the rate of change in their written answers of, "From 4 to 7 days on average 350 people were infected", and they do not attend to the units of the value 350. Siyabulela's laughing when he attends to the practical terms genre in line 469b suggests he does not really give

significance to its use and Lungiswa is anxious to move on, identifying herself in her role of controlling the pace, "Okay guys let's go on let's go on" (line 470).

In contrast to Group 2, the first verbal pronouncement in Group 1, this time by Lulama, identifies the expression $\frac{P(7)-P(4)}{7-4}$ ("this") as, "This is the rate of change" (line 216). This is followed a few turns later by Jeff identifying the expression ("that") as, "And that's the average ... it's the average" (line 222). Both these pronouncements suggest that the students are looking structurally at the expression as an object.

However, successful use of the practical terms genre does not follow immediately in Group 1. For example, Hanah gives positive feedback to Lulama and poses a question in the declarative mood to Jeff, "Ja … does it mean that from 4 days to 7 days ... 350 people are infected↑((*Looking at Jeff*))" (line 217). This pronouncement does not attend to the rate of change. Jane's attempt attends to the average, but not to the rate of change; "The average ... people who will be infected is 350 ... from" (line 223).

However, the students' use of the practical terms genre is enabled, firstly, by Hanah's attention to the units; she pronounces, "Oh … the average amount ... per ... per day" (line 224). Secondly, Lulama may be attending to the problematic use of this genre by his peers when he asks, "...is this an average or an average rate of change?" (line 227). Lulama looks at Hanah as he talks, and both Shae and Hanah respond with "average rate of change" (lines 229 and 230). This action enables the students to write down answers within the practical terms genre. For example, Hanah writes, "From 4 to 7 days the average number of people infected per day are 350 people". In contrast to the socio-political action in Group 1 described so far, the action of both Lulama and Hanah is attended to and appears to be enabling to most students in the group. However, Jane's use of "practical terms" suggests that she inserts the words "average" and "per day" and attends to the wording in Sentence 4 of the problem text, but does not communicate the change on the numerator; "The average people have it or have had it between day 4 and 7 is 350 people per day".

In contrast to the action in Group 2, the students in Group 1 do not give explicit attention to avoiding mathematical terms such as "rate" in their use of the practical terms genre. Shae chooses to ignore the practical terms genre; "from 4 to 7 days the average rate of change is 350".

11.7.3 Attending to the derivative function as the instantaneous rate of change as enabling in question (e)

In Group 1 Hanah and Jeff's verbal pronouncements together enable them to identify the derivative function $P'(t)$ in question (e) as an object representing the instantaneous rate of change. For example, Hanah begins with "day 4 the rate of change ... of the number of people getting infected is 400↑" (line 314). In this case Hanah's pronouncement is attended to and developed by Jeff; he corrects her use of "day 4" (suggested by "that") when he pronounces, "No no well ja okay at that point the instantaneous ave the instant ... the instantaneous infection rate was 400 people per day" (line 318).

This identification enables these two students, together with Jane and Shae, to write answers such as "It is the infection rate (400) at 4 days" (Shae), and "At day 4 the rate of change/growth rate/instantaneous rate of change of the number of people getting infected is 400 people per day" (Hanah). The students' use of the preposition "at" in "at 4 days" signals their attention to the instantaneous rate of change, and does not reproduce the valued use of "after 4 days" in the Foundational Course. Jeff uses the two prepositions interchangeably. The students do not attend to their use of terms such as "instantaneous" and "rate" that would be regarded as mathematical in the Foundational Course.

Lulama and Darren in Group 1 are not enabled by the discussion between the other four students as described; they do not appear to be attending to this discussion at all and they briefly talk to one another only. In their written answers they both identify the derivative function as representing the average rate of change.

Unlike the action in Group 1, the students in Group 2 do not use any mathematical terms

when discussing or writing answers. Siyabulela takes the lead in Group 2 by identifying the derivative function $P'(t)$ in question (e) as an object representing the derivative. However, he does not name the function in this way but makes a link to his earlier discussion about the derivative with Bongani in lines 420 and 421 (see Section 11.7.2); "this comes to Bongani, then" (line 481). What follows are attempts to pronounce correct "practical" wording. For example, evaluating Lungiswa's first attempt of, "After each 4 days 400 people have been infected ((*Looking at Siyabulela*)) (line 486), Siyabulela spends time correcting her use of "after each 4 days" rather than attending to the absence of the rate of change, drawing on the wording "after 4 days" (line 499) used in the Foundational Course. Siyabulela pronounces a verbal answer that attends to the rate of change and the units, but not to the appropriate use of tense; "...after four days ... uh ... the number of people ... who were infected were increasing by ... 400 ... per ... day" (line 506). The rest of the action involves the students verbally co-constructing answers such as this and evaluating the word order used by one another, for example, the word order in Lungiswa's pronouncement, "After 4 days the number of people who were infected per day is increasing by 400" (lines 525 and 527).

11.8 Discussion of the student action on questions (c) to (e) of the Flu Virus Problem

This analysis suggests that the students look structurally at the objects representing rates of change in questions (d) and (e) and correctly identify them as the average rate of change and the instantaneous rate of change respectively. There is also evidence that an operational view of the expression $\frac{P(7)-P(4)}{7-4}$ enables the identification of the object as an average rate of change in question (d). The students have greater difficulty using the practical terms genre to talk about the rates of change of the function than the function itself (although the lack of attention given to tense in the case of the function makes it difficult to ascertain the extent of their control of this genre in question (c)). When students have difficulty with the rates of change (and with the average rate of change in particular), the students either resist

the use of the genre (as in Group 1) or invest time in identifying everyday wording that sounds right. The latter identification of everyday wording may involve just inserting a word into an existing statement or trying to remember wording from other social events in the Foundational Course. During the action on question (e) the students in Group 2 provide "practical terms" as valued in the worked solutions, but in doing so do not move out of the non-mathematical practice (that is, the recontextualized version of this practice). Dowling (1998) and Gellert and Jablonka (2009) identify this as a problematic feature of certain texts in school mathematics.

The action of Hanah and Lulama in Group 1 as presented here reinforces my argument that the socio-political interaction between the students can be enabling or constraining. I have argued elsewhere that these two students can be identified by others or by themselves as excluded from mathematical arguments. However, both Hanah and Lulama participate in the discussion about question (d) and attention to their pronouncements is enabling for the group as a whole. This is also the case for Hanah in question (e), but not for Lulama who is constrained by not participating.

11.9 The developing argument about the student action

In analysis presented in Chapters 8 to 10 has pointed to the complexity of controlling the how and when of boundary crossings between practices (both mathematical and non-mathematical) and social events within the Foundational Course, and controlling the intertextual links within the practical problems. The analysis of student action on questions (c) to (g) of the Flu Virus Problem in this chapter points to the necessity of talking about the nature of the mathematical objects in these boundary crossings, for the action in both groups differs when the mathematical objects change. The students control the movement from the mathematical to the non-mathematical when using the practical terms genre for the function in question (c), yet there is an absence of this control when attempting to use the practical terms genre for the rates of change in questions (d) and (e). Evaluating the

limit $\lim_{t\to\infty} P(t)$ in question (f), the students in both groups control the movement of meaning between the function and its meaning in the task context and within the text itself to the graph in question (a). However, the derivative function in the limit $\lim_{t\to\infty} P'(t)$ in question (g) introduces more complexity. The students do not control the boundary crossings, severing links to the task context, not making a link within the text to the graph in question (a), and making an unproductive link to lectures.

The analysis in this section provides additional evidence that the socio-political interaction in the small group can be enabling or constraining. When using the practical terms genre for the average rate of change, the students are enabled by the contributions made by Hanah and Lulama. However, these students' contributions when evaluating the limit in question (g) are not attended to, and Shae, Darren and Jeff control the interaction in a way that is constraining. While Siyabulela in Group 2 acts as a mathematical authority and enables the other students during action on other problems (for example, in question (d) of the Chemical Reaction Problem, as discussed in Section 9.3), Siyabulela's "problem" (line 705) in question (g) prevents him from playing this role when answering question (g). For both groups, the intervention of the Tutor who acts as an authority in the foundational practice is necessary for them to move forward in question (g). When evaluating the limit $\lim_{t\to\infty} P'(t)$ in question (g), the authority of Shae, Darren and Jeff in Group 1 is replaced by the authority of the Tutor. Hanah, Jeff and Shae in Group 1 are enabled by the opportunity the Tutor provides for them to act like students in a learner-centred pedagogy and to explain their answers. When using the practical terms genre to explain the evaluation of the limit, the students in Group 1 do not defer to the authority of the Tutor to the same extent. The students who choose not to use the practical genre appear to resist their positioning as foundational students who use this genre.

The action of the students on these questions points to disruptions with respect to the valued action of advanced mathematics. Firstly, the students who defer to the authority of the Foundational Course when using the practical genre end up getting "stuck" in the task

context and do not make the movement from the horizontal to the vertical. Secondly, the students in Group 1 are enabled in the foundational practice by identifying the limit with the maximum value of a function, a definition of the limit that is valued neither in the foundational practice nor in advanced mathematics.

CHAPTER 12
CONCLUSIONS

12.1 Introduction to this chapter

I begin this final chapter by providing the central argument or finding of this thesis. I then summarize the theoretical, methodological and empirical work that has been done in order to arrive at this point in the study. I also use this summary to argue for the quality of this study in terms of how it talks with and to the mathematics education community and thus makes a contribution to this community (Adler & Lerman, 2003; Silverman, 2010). In Section 12.5 I consider what the results of this study mean for policy and practice, that is, I consider what the study means for participants (Adler & Lerman, 2003; Silverman, 2010). I end by discussing the limitations of this study, with suggestions for further research.

12.2 The central argument of this thesis

This study is about the transition from school to advanced mathematics, and the role of foundational mathematics in facilitating this transition. I have considered the introduction of two innovations, the use of practical problems and a learner-centred pedagogy, in a Foundational Course at a South African university. This Course represents foundational practice as a move in the wider network of socio-political practices since it introduces new boundaries into the space. These boundaries relate to movements across mathematical practices in the school/foundational/advanced mathematics transition and across the mathematical/non-mathematical boundary. I have investigated this move, firstly, by describing the practice and, secondly, by describing how students participate in this practice.

Wood (2000) argues that foundational mathematics courses at undergraduate level can be innovative as they are not subject to the same constraints as mainstream courses. This study indicates that practical problems and a learner-centred pedagogy as innovations in foundational practice position this practice paradoxically in relation to other mathematical practices in higher education. This paradox emerges from the continuities and disruptions that the foundational practice represents in relation to other practices in the educational space. I have suggested that this paradoxical positioning is a particular version of the access paradox identified by Janks (2010, p.24) in the field of literacy.

On the one hand foundational practice represents an alternative mathematical practice in the higher education space, and thus challenges the boundaries of the existing order of discourse in which mainstream mathematical practices are dominant. The practice represents an alternative, since in the innovations it draws selectively on a variety of practices, for example, reform-oriented and more traditional versions of school mathematics, as well as more traditional and reform versions of undergraduate calculus. I use the word "selective" here, since boundary crossings between these practices involve the how (in terms of objects, ways of acting mathematically, genre, and positioning) and the when of the movement in meaning across the boundaries. These continuities and disruptions represent participation in foundational practice as complex. In this complexity the practice also challenges the dominant construal of the foundational student in higher education. Rather than being a student who entered the university with lower school results than mainstream students and who needs different support from these mainstream students, the foundational student is positioned as able to control a complex movement of meaning across practices, events, and texts, and within texts themselves. Rather than being a student who needs additional support, the foundational student is someone who can take on the responsibility to work consistently and make use of feedback and to work productively with his peers in a learner-centred pedagogy, sometimes acting as an authority in the ways of acting mathematically in the practice in the small group. I argue that this style of learner is valued in advanced mathematics.

On the other hand foundational practice, as an alternative in the space, does not challenge the dominance of mainstream mathematical practices in the order of discourse of higher education. The practice is not able to challenge this dominance and remains marginalized because it does not provide access to advanced mathematics. Foundational practice is not able to provide such access precisely because of the continuities and disruptions that it represents in its relationship to other mathematical practices. The analysis of the student action in Chapters 8 to 11 points to the complexity of participating in foundational practice; if students cannot deal with this complexity and thus make the transition from school to foundational mathematics, they do not even gain formal access to advanced mathematics. Furthermore, this study has highlighted a number of discontinuities in the relationship between the foundational practice and advanced mathematics; students who are accepted into the second-year mainstream course may not have epistemological access to the valued ways of acting mathematically in advanced mathematics.

So innovative foundational practice simultaneously challenges and reproduces the dominant order of discourse in which mainstream mathematical practices are dominant. In Section 12.5 I consider the implications of this finding for practice. Before this, however, I summarize the work required in order to arrive at this finding.

12.3 The theoretical and methodological work of this study

A key challenge of this study has been to develop a theoretical perspective that allows me to focus "tightly on the mathematics" (Adler & Lerman, 2003, p.445) while simultaneously zooming out in two respects, firstly, to talk about the discursive, social, and political action of the students at the micro-level, and secondly, to talk on the macro-level about relationships between socio-political practices (for example, between the foundational practice, school mathematics, advanced mathematics and non-mathematical practices), and what it means to cross the boundaries between these practices.

The literature review in Chapter 3 suggests that different theoretical perspectives used by the mathematics educations community allow me to talk about particular aspects of the research problem in this study. However, no perspective from this community, taken in isolation, does the work that this study requires. How to resolve this "dilemma" (Valero & Matos, 2000, p.398) is part of the discourse of the mathematics education community, and recent research on the school/university transition (e.g. Jooganah & Williams, 2010) addresses the dilemma by drawing on both psychological and social perspectives. Part of the theoretical journey that I have taken in this study has involved identifying, firstly, how this dilemma plays out in relation to the research problem and to the research texts in particular and, secondly, how it can be resolved. This description of the journey points to how the theoretical and empirical have co-constituted one another in this study.

12.3.1 A socio-political perspective of mathematical practice

In this study I use Fairclough's socio-political perspective of practice from critical linguistics. Fairclough's work allows me to talk about mathematical practice as a socio-political practice, a practice that is largely discursive. Using Fairclough's three levels of abstraction, with a focus on language within each of these levels, I talk (on the micro-level) about texts in a Foundational Course, that is, texts of the practical problems and texts re-presenting student action on these problems. On this level Fairclough provides the tools to talk about discursive, social and political action as re-presented in these texts. Yet I also talk (on the macro-level) about these texts as moments in a foundational practice which is located in a network of socio-political practices with a particular order of discourse.

The foundational practice is networked with other mathematical practices such as school mathematics and advanced mathematics, and with non-mathematical practices such as chemistry and epidemiology. Since these practices are related by recontextualization, I talk about crossing the boundaries between these practices as a movement of meaning.

The relationship between the micro- and macro-levels in Fairclough's perspective is productive in two respects. Firstly, I describe how the two types of texts give meaning to

the foundational practice by drawing on different practices (such as school mathematics and non-mathematical practices) and in terms of the agency of subjects in the practice. Secondly, I explain this practice in terms of structure and how the process of text production at the micro-level is constrained by its location in a network of socio-political practices.

Fairclough's concepts of power in discourse and power behind discourse point to asymmetries in control over the boundaries between practices and the movement of meaning at both the macro- and micro-levels. Thus I talk about power relations between the students and between the students and the tutor at the level of text, but also at the macro-level in terms of how the foundational practice positions itself in relation to the dominant mathematical practices in higher education.

Yet Fairclough's perspective has gaps in terms of how it can talk about mathematical practice, and resolving this "dilemma" (Valero & Matos, 2000, p.398) is what I call the mathematical work of this study. Rather than using more than one perspective, I recontextualize constructs from the work of Morgan, Moschkovich and Sfard in mathematics education for use within Fairclough's perspective. This is done in such a way that I can talk about research conducted from other perspectives, for example, the ontological/psychological research in advanced mathematics. Developing Fairclough's theoretical perspective by recontextualizing theoretical constructs from mathematics education has required attention to the movement of meaning in these constructs for use within a perspective that adopts a critical realist ontology.

The mathematical work of this study has involved, firstly, developing the notion of mathematical discourse as the language aspect of a socio-political mathematical practice. Secondly, this work has involved focusing on mathematical objects and the action on these objects within this wider concept of mathematical discourse. The latter work has been attended to in written mathematics (and from a socio-political practice perspective) (e.g. Morgan, 1998). However, in this study I talk about action on mathematical objects in the

broader semiotic action represented in face-to-face interaction, and this is where the work of Sfard and of Moschkovich is used in a way that both supplements and complements Morgan's work.

This mathematical work has produced the concept of ways of acting mathematically in discourse to respond to the challenge of talking about mathematical, discursive, social and political action in a mathematical practice. This concept combines what Fairclough, Morgan, Sfard and Moschkovich have to offer for talking about the semiotic action in the mathematical practice that is the focus of this study. For example, I talk about how the students talk about and represent mathematical objects, how they look at mathematical objects and operate on them, how they make discursive links across texts, events, and practices, and how they interact socio-politically. This concept allows me to talk to the literature on advanced mathematical thinking, not in terms of individual mental "conceptions", but in terms of ways of looking at mathematical objects and ways of operating on mathematical objects.

12.3.2 Operationalizing a socio-political perspective of mathematical practice

Yet the dilemma of how to talk about mathematical, discursive, social, and political action in a socio-political mathematical practice cannot be resolved at the level of theoretical perspective alone. This is also a methodological challenge, one that is recognized by the mathematics education community (e.g. Ernest, 1998; Sfard, 2000). Operationalizing a socio-political perspective of mathematical practice in this study has involved further mathematical work.

The theoretical framework presented in Table 5.1 derives its structure and much of its detail from CDA, as used by Fairclough. The three stages of CDA allow me to make links between the text, social practice, and the wider network of socio-political practices. In terms of analyzing text, I use the textual features identified by Fairclough, but supplemented with the extensive work that has been done on the mathematical register (e.g. Morgan, 1998; Pimm, 1987). Yet for the purposes of operationalizing a socio-political

perspective of mathematical practice, the tools of CDA do not operationalize the ways of acting that are specific to acting on mathematical objects, for example, the ways of looking at mathematical objects. In this study I recontextualize Sfard's (2000) method of focal analysis into the overall CDA framework for this purpose.

This study investigates how the texts of the practical problems represent foundational practice. Yet this study demonstrates that obtaining a detailed description of this practice also requires an analysis of the interpretation of these texts, that is, how students solve these problems. As noted by Dowling (1998), I cannot assume that the interpreting student is the same in the two social events. Rather, I use the analysis of the student action to identify what actions enable and constrain the adoption of the subject position of successful student in the practice, that is, the student who produces the worked solutions. Since focal analysis is appropriate for investigating student action, this additional source provides, in particular, insight into the action on mathematical objects.

12.3.3 The challenge of demonstrating validity in a study that uses critical discourse analysis

In addressing validity in this study, I have drawn on Maxwell's (1992) critical realist notion of validity in qualitative research. However, it has been necessary to consider how Maxwell's types of validity (descriptive validity, interpretive validity, theoretical validity and generalizability) apply in CDA and more specifically in the particular theoretical perspective, methodology, and methods used in this study. For example, I use primary descriptive validity in relation to the production of the text that is analyzed in the descriptive stage of CDA and secondary descriptive validity for the description of the network of socio-political practices in the macro-space. Interpretive validity is an issue in two of the three stages of CDA; the interpretation stage (in the relationship between the text and my account of the participant's meanings) and the explanation stage (in the relationship between my account of a participant's meanings and the wider socio-political space. I argue that the work done to reinterpret Maxwell's types of validity for a study that uses CDA can be used to demonstrate the validity of this study.

12.4 The empirical work of this study

The literature review for this study indicates that the relationship between practical problems and/or learner-centred pedagogy and equity and access in school mathematics is part of the discourse of the mathematics education community. However, I have argued in Chapter 3 that the relationship between these innovations and the transition from school mathematics to advanced mathematics (and how foundational practice may figure in this transition) is not in the discourse of this community, and certainly not in a way that makes the issues that arose in my practice as a lecturer visible. The theoretical and methodological tools summarized in Section 12.3 were designed precisely to allow me to talk about these issues. In this section I summarize the answers to the two research questions.

12.4.1 The foundational practice (Research Question 1)

This study began with a proposition that the foundational practice itself and the innovative pedagogies within this practice introduce additional boundaries into the school/advanced mathematics transition. In this respect, the practice represents a move in the wider network of socio-political practices. The theoretical perspective summarized in Section 12.3 enables me to talk about the nature of the move that this practice represents in the wider order of discourse.

Solving a practical problem in the Foundational Course is about boundary crossing at the level of practice or social event within a practice, and within and across texts. Crossing these boundaries is, firstly, about control over how to cross a boundary. This study points to four ways in which meaning moves over a boundary, that is, there may be a movement of objects, ways of acting mathematically, genre, and positioning (the how of boundary crossing). Secondly, boundary crossing is about control over the timing of this crossing (the when of boundary crossing), for objects, ways of acting, genre and positioning in a particular practice, event or text may not be operationalized at all in a particular social event in the foundational practice. Lastly, the boundary crossing is not necessarily

unidirectional from the horizontal to the vertical, but may be a to-and-fro movement between mathematical and non-mathematical practices.

This conceptualization of boundary crossing is more detailed than talking about boundary crossing in terms of recognizing the boundary (e.g. Cooper & Dunne, 2000; Gellert & Jablonka, 2009; Straehler-Pohl, 2010), making individual mental reconstructions (e.g. Tall, 1996; Vinner, 1991), following chains of signification across discursive practices (e.g. Evans, 2000), or exploring the genre boundaries (Gerofsky). This conceptualization also challenges the predominant view that solving practical problems is a one-way movement from the non-mathematical to the mathematical (e.g. Dowling, 1998; Freudenthal, 1973; Lubienski, 2000; Straehler-Pohl, 2010; Treffers, 1987) and suggests that contextualization (Arcavi, 2002, p.22) in which the task context remains the same is necessary when solving these problems. The results of this study suggest that boundary crossing (or controlling the how and when of links between practices, social events and texts and within texts themselves) is a particular of way of acting mathematically in discourse in the foundational practice.

The practical problems recruit a number of discourses in the wider socio-political space in such a way that the relationship between foundational practice and other mathematical practices is one of both continuity and disruption. The problems talk back to school mathematics and across to the mainstream first-year course at times, but not at other times. There is little talk forwards to advanced mathematics. The problems talk to the discourse of learner-centred pedagogy and the discourse of relevance in school and calculus reform, but also to the mathematical word problem genre typical of more traditional pedagogy in both school and undergraduate mathematics. The student is thus positioned in different, often contradictory ways. Rather than the student being positioned simply as low ability (e.g. Dowling, 1998; Swanson, 2005), the foundational student is construed simultaneously as needing additional support and as an independent student who is able to control the complexity of foundational practice. The relationship of continuity and disruption between

the foundational and other practices points to why the boundary crossings that the foundational practice introduces into the space are so complex.

This representation of foundational practice speaks to what the mathematics education community says about innovative pedagogies. Firstly, the relationship of continuity and disruption between this and other practices and the accompanying contradictory subject positions suggests that studying innovation in a mathematics course is not just about locating a course within a reform or traditional pedagogy and attending to the corresponding identities that the students develop within these pedagogies (e.g. Boaler 2000a). Secondly, the complexity of the boundary crossings and the disruptions with respect to advanced mathematics challenges Wood's (2001) claim about the opportunities for innovation in a foundational mathematics course at undergraduate level, for such a course is subject to the constraints of the dominant order of discourse.

12.4.2 Participating in the foundational practice (Research Question 2)

If students are to gain access to advanced mathematics via the foundational practice as described in the answers to research question 1, they need to participate in this practice by adopting the contradictory subject positions set up for them by these problems. This study began with a proposition that the students' mathematical, discursive, social and political action both constrains and enables this participation. The theoretical perspective summarized in Section 12.3 enables me to talk about this action in detail, and to explain this action with reference to the action in the classroom and what students recruit from the wider network of practices (rather than with reference to their individual cognitive ability).

The complex interplay between the ways of acting mathematically in foundational practice

The results presented in Chapters 8 to 11 point to the complexity of participating in foundational mathematics and suggest that, taken in isolation, a way of acting mathematically cannot be identified as enabling or constraining of participation in the practice. Solving these practical problems is not just about recognizing the

mathematical/non-mathematical boundary (Cooper & Dunne, 2000; Gellert & Jablonka, 2009), or just about having a teacher who facilitates this recognition (Straehler-Pohl, 2010), or just about moving from the horizontal to the vertical (Dowling, 1998; Lubienski, 2000). Neither is solving the practical problems just about having both a structural and operational view of a mathematical object (Sfard, 1991), or just about having the motivation to struggle when acting on mathematical objects (Bowie, 2000). Neither is solving the practical problems just about being of a particular race or social class or having a particular home language (Cooper & Dunne, 2000; Swanson, 2005).

Rather, solving the practical problems involves a complex interplay between (a) the ways of acting mathematically, (b) what students recruit from other practices, social events in the foundational practice and within the texts of the practical problems themselves, (c) the timing of these links to other practices, social events and texts, and (d) what happens when students solve the problems in small groups in the workshop class. In this section I provide two examples from the results to illustrate this complex interplay.

When solving question (d) of the Chemical Reaction Problem the students in Group 3 time a boundary crossing to school mathematics and control the movement of the quadratic function and three algebraic formulae for this function over the school mathematics/foundational practice boundary. Yet they are constrained from identifying these algebraic representations as equivalent by their operational view of a function, the absence of links between the symbols in these formulae and the points on the graph, the style of students in a more traditional pedagogy searching for one correct formula, and the competitive socio-political interaction in the group in which students take ownership of the different formulae. Furthermore, while the students control the movement of the quadratic function between social events in the Foundational Course, they do not control the movement of ways of acting on this function across these events. I argue that this absence of control is related to their tendency to use verbal rather than written descriptions of the formulae as they talk about their answers, and Kelsa's method of evaluation of these descriptions which is recruited by other students in group. Some of the students are enabled

by the intervention of the Tutor, who positions himself as an authority in the practice by evaluating the link to school mathematics and modelling the links between an algebraic formula from this practice and the points on the graph. However, the students use this authority of the Tutor to validate their "personal formulae", with the result that some students are not enabled by his input.

When solving the Car Problem, Lulama is enabled by having steps to follow and instructions to operate on mathematical objects, for example, differentiating and substituting. It is possible that his boundary crossing to Course lectures in which related rates are solved is enabling this operational activity. He is also enabled by recruiting the genre of mathematical word problems as he uses the task context to construct a mathematical diagram. Yet he is also constrained by following this genre; following his initial movement from the non-mathematical to the mathematical, he severs links to the task context and gets "stuck" in the mathematical practice. Although the students in his group adopt the valued style of students in a learner-centred classroom, their action does not facilitate a to-and-fro crossing across the mathematical/non-mathematical boundary for Lulama. For the students in the group that do have some control over this boundary crossing control Lulama's access to it, firstly through their talk (using reference pronouns ambiguously and verbalizing their operational activity), and secondly by dominating the interactions in the group and with the Tutor. The Tutor, in turn, identifies the students as having agency in terms of taking responsibility for their learning and does not identify Lulama as needing support.

In the sections that follow I focus in detail on certain aspects of participation in foundational practice that are highlighted in the analysis. However, these sections should be read with the interplay between the various ways of acting in the background.

Controlling the how and when of boundary crossings in the foundational practice

The analysis of the practical problems has enabled me to conceptualize what it means to cross boundaries between practices, social events and texts, and within texts themselves in

foundational practice (as summarized in Section 12.4.1). Yet the analysis of the student action in Chapters 8 to 11 illuminates the complexity of making the required links in this practice. For example, when solving question (d) of the Chemical Reaction Problem students in each group time the link to school mathematics. However, timing alone is not enough; Siyabulela in Group 2 accompanies this with control over the movement of the object quadratic function and the way of acting mathematically on this function (the how). The students in Group 3 control the movement of this function across the school mathematics/foundational practice boundary and within social events in the Course, yet in both cases there is an absence of control over the ways of acting mathematically in the movement of meaning.

For a second example I consider mathematical/non-mathematical boundary crossings in the Car Problem and the Flu Virus Problem. Crossing this boundary requires control of genre (the how), as the recruitment of the genre of mathematical word problems from school is enabling for students when it comes to dealing with "extraneous information" (Gerofsky, 2004, p.33) and making the initial one-way move from the task context to a mathematical drawing in the Car Problem. However, the analysis has pointed to many instances when the students recruit this genre in a way that is constraining, for example, they identify unproductive cues in the text and at certain times the students do not move to-and-fro across the mathematical/non-mathematical boundary. This suggests that they do not control the timing (the when) of the use of this genre (the how).

I turn now to discussion of the practical terms genre. When the mathematical/non-mathematical boundary crossing involves recruiting this genre (from other social events in the Foundational Course), some students adopt the required subject position of foundational student and reproduce this genre (with implications for epistemological access to advanced mathematics). At times, students attempt to use the practical terms genre in their explanations, even when this genre is not prescribed in the text. However, other students resist the positioning afforded by this genre. These results point to the importance of considering the movement in students' positioning across boundaries (e.g. Walkerdine,

2000; Evans, 2000) and the agency exercised by students in adopting the subject positions of a practice (e.g. Nyabanyaba, 2002).

Learner-centred pedagogy as both enabling and constraining

There is much in this study to suggest that the learner-centred pedagogy promoted in the workshops in the Foundational Course constrains rather than enables participation in foundational practice (and in particular the valued ways of acting on mathematical objects). I have argued that the educational talk in a learner-centred pedagogy in which students read formulae from left to right and give verbal descriptions of their operational action, together with the gestures for representing mathematical objects that accompany this talk, constrains attention to the mathematical objects themselves. The encouraging nature of this educational talk seems to discourage critical evaluation of the educated talk. This result supports Adler's (1997) argument, but this time with data from student interaction, that a learner-centred pedagogy may militate against the development of mathematical understanding.

In addition the socio-political interaction means that certain students are able to control the educational talk in a learner-centred pedagogy, with the result that they control who speaks and when, what is spoken about, and what content is spoken about. Lastly, the analysis points to instances where the student talk alone does not produce the required solution, and the students are not able to proceed without the intervention of an authority in the foundational practice, an issue I discuss in the next section.

Yet on the other hand the analysis of the action in Groups 1 and 2 suggests that the educational talk of a learner-centred pedagogy can enable participation in the foundational practice. For example, the students use the talk to repeat and reword their answers in such a way that they build the educated text together, the uncritical nature of the interaction means that students are supported to develop a voice in the group, and students may be enabled when explaining their solutions to the Tutor and when interacting with the Tutor in the small group.

Interaction with an authority in the foundational practice as enabling

The results presented in Chapters 8 to 11 suggest that solving the practical problems requires interaction with an authority in foundational practice. For example, in question (a) of the Flu Virus Problem the Tutor models the links between the function and its derivative, the graphical representations of these objects, and the meaning of these objects in the task context. Yet a foundational student can also act as an authority in this way. For example, in question (d) of the Chemical Reaction Problem the student Siyabulela produces a correct solution with ease and explains his action to the other four students in such a way that they are enabled to complete the problem. This result supports the significance given to the need for a mathematical authority in a learner-centred pedagogy by Brodie and Pournara (2005), Adler (1997) and Davis (2001), and also points to a gap in how Boaler (2002a) talks about the reform pedagogy in her study.

Yet the results of this study also problematize the role of this authority. Firstly, in this study the Tutor has difficulty balancing his multiple roles, for example, as an authority in the foundational practice who can model the valued ways of acting mathematically and as a facilitator in a learner-centred pedagogy who gives students the space to talk and identifies students as taking responsibility for their own learning. This challenge was identified by Adler (1997) when researching learner-centred pedagogy at school level. This study, however, points to additional positioning of the foundational tutor that challenges his role as an authority in ways of acting mathematically in the practice, for example, as a tutor who controls the pace of work in the workshop class, and as a tutor who should support all the groups in the class. Secondly, the action of Groups 1 and 3 suggests that role of the tutor as an authority in the practice cannot be considered in isolation from the socio-political interaction between the students themselves.

The boundary crossing to advanced mathematics

The analysis identifies a number of disruptions in the relationship between foundational and advanced mathematics practice, with implications for the transition. Firstly, the practical problems require a to-and-fro movement between the mathematical and non-mathematical,

rather than the one-way movement promoted in the literature on advanced mathematics (e.g. Dreyfus, 1991; Harel & Kaput, 1991). In particular, those students who adopt the positioning of a foundational student who uses the practical terms genre end up working in the public domain (a movement that is problematized by Dowling (1998) and Gellert and Jablonka (2009) in their analysis of school mathematics texts). Secondly, the foundational practice gives value to arguments that are located in the task context rather than in the mathematical practice itself. Students are able to solve question (f) of the Flu Virus Problem by recruiting an everyday meaning of the limit, rather than a mathematical one. Thirdly, while foundational students are given opportunities to use a structural view of mathematical objects, some of the practical problems require an operational view or provide the student with the possibility of using either a structural or an operational view. Fourth, the agency demonstrated by students in using alternative methods for solving related rates problems is not valued, either by the problem text or the Tutor.

Yet, while foundational practice may not talk to advanced mathematics in the above mentioned respects, it does talk to this practice in ways that are less visible (and would certainly not be made explicit in course outlines for undergraduate courses in advanced mathematics). Firstly, I have argued that boundary crossing is a valued way of acting in the foundational practice, a way of acting that is regarded as necessary for creative developments in mathematics (Sfard, 2008). Secondly, the foundational student is positioned as resourceful in that he takes responsibility for his learning and works productively with his peers.

12.5 Implications for policy and practice

The central argument of this thesis is that innovative foundational practice, on the strength of the complexity that it represents, is positioned paradoxically in the higher education space. This positioning is unavoidable and the question for policy and practice is how to respond to this positioning.

Firstly, one could argue that, since innovation in foundational practice makes access to the dominant practices problematic, this practice should avoid innovation in general and simply reproduce the mainstream, for example, by adopting the extra tutorials or slower stream models. Yet these models in themselves are problematic, as noted by Allie (2010) and Rollnick (2010).

Secondly, a response could focus on the specific innovations that are the focus of this study, that is, practical problems and learner-centred pedagogy. Given that students have difficulty controlling the complex mathematical/non-mathematical boundary crossing when solving practical problems and what this study says about the differences between this boundary crossing and the valued vertical movement in advanced mathematics, one could argue that the solution is simply to remove the practical problems from the Foundational Course. This action removes the additional boundary that the practical problems introduce into the space. Yet the valued action for solving the practical problems indicates that participating in foundational practice is about a lot more than having control over the movement of meaning across this mathematical/non-mathematical boundary. So simply removing the boundary does not necessarily reduce the complexity of the practice.

Since this study has problematized the relationship between a learner-centred pedagogy and access to mathematical practice, one could argue that the use of this pedagogy in workshops in the Foundational Course should be halted. Yet there is evidence in this study to suggest that students can be enabled by the interactions in the small groups and with the Tutor.

This study is restricted to the two innovations in the Foundational Course, and does not talk about other parts of this Course. Yet these innovations serve as cases which highlight the challenges of designing a Foundational Course. Even without these innovations, foundational practice represents a move in the higher education space, with particular relationships to other practices in the educational space. Simply avoiding innovation or particular innovations does not resolve the paradoxical positioning of foundational practice.

I argue in this section for the need, firstly, to accept this paradoxical positioning and to recognize boundary crossing as a valued way of acting mathematically in foundational practice. Secondly, I argue that this positioning should be engaged with constructively at a number of levels, from texts and social events within the Foundational Course, to the Course itself, to the level of socio-political practice and the network of practices. I identify possibilities at each level, but argue that responding to what this study says about the research problem requires action at all these levels.

12.5.1 Responding within the Foundational Course itself

On the level of the practical problems, I have argued that simply removing the practical problems is not a solution. Yet changes could be made to the problems. For example, the results of this study provide strong evidence that the use of the practical terms genre should be removed from the practical problems, and attention could be paid to consistency with respect to the mathematical gaze on the non-mathematical practice. Textual strategies that position the foundational student as needing hints and reminders and as different from mainstream students should also be reconsidered.

On the level of social events within the Foundational Course, the complexity of participating in foundational practice and the action of the Tutor in this study suggests that students may be enabled to participate if the lecturer or tutor makes explicit the differences and similarities between practices, for example, by tracking the movement of meaning in objects, ways of acting mathematically, and genre on functions across the school mathematics/foundational practice boundary and by modelling the valued ways of acting mathematically in the foundational practice itself. Both Evans (2000) and Gerofsky (2004) have suggested the need for exploring the discursive shifts across boundaries, yet the description of the boundary crossings in foundational practice suggest that additional aspects of the movement need to be made explicit. Certainly, since the need for this explicit work started to emerge in this study, this action has become part of my practice as a lecturer.

However, the analysis of the student-tutor interaction in this study also points to the challenges of making the links explicit and modelling the valued boundary crossings. Firstly, this study talks about the difficulties that the tutor has in fulfilling his role as an authority in the ways of acting mathematically in foundational practice as he juggles his multiple roles in this practice. Secondly, if these strategies are employed in other social events in the course, for example in lectures, a boundary crossing is still required in the move to the workshop class. This study suggests that students may or may not control the movement of meaning across such boundaries.

With respect to the role of the foundational tutor, this study has illuminated the challenges of balancing the multiple roles required of a tutor acting in foundational practice. These challenges are certainly not specific to foundational practice. Given the differences in the positioning of a foundational tutor and a tutor in a mainstream mathematics course and the fact that many tutors are employed as tutors in both practices, one could argue that foundational practice should simplify the role of tutor and adopt the mainstream model. However, I argue rather that there is a need to support tutors in adopting the subject position of foundational tutor, for this represents an opportunity to develop tutors and (potential) lecturers in a way that can be enabling to students (as indicated by certain interactions between the Tutor and students in this study) and in a manner that challenges the dominant construal of the undergraduate tutor and lecturer.

I have argued so far that innovation (including the mathematical/non-mathematical boundary introduced by the practical problems) and the complexity that it represents should be retained and worked with constructively at the level of social event by making links explicit and supporting tutors to do the modelling work required. Furthermore, I argue that this innovation should be retained for the opportunity it affords at a meta-level, that is, for supporting students to become boundary crossers as required in advanced mathematics. This is more than simply making the nature of boundary crossings explicit at the level of social event. Rather, it is about recognizing boundary crossing as a valued way of acting in both foundational and advanced mathematics practice and being deliberate about practising

boundary crossing in the social events of the practice. In other words, foundational mathematics is represented as a practice where this valued way of acting can be practised.

12.5.2 Responding beyond the Foundational Course

Being deliberate about boundary crossing in the foundational practice also involves engagement beyond the Course level with other mathematical courses in the higher education space. Certainly, this study suggests that the design of a foundational mathematics course cannot be done in isolation from the dominant mathematical practices in this space. Engagement of lecturing staff across the boundaries should involve, on the one hand, how foundational practice can be strengthened in talking forwards to advance mathematics (and this study points to aspects of the Course that should be addressed), and on the other hand, how foundational practice has something to offer in terms of the valued mathematical ways of boundary crossing and working independently as valued in advanced mathematics. Engagement at this level recognizes the dominant positioning of mainstream mathematical practice, but also provides the possibility of boundary crossings across mathematical practices and change within the wider order of discourse.

Furthermore, this study talks to the policy level in a university by suggesting that what a foundational practice can do in terms of widening access in higher education cannot be considered in isolation from the power relations within the higher education space and how foundational practice is located in this space. Decisions about what a foundational practice should look like have to engage with the power relations that hold in place the boundaries around the dominant mainstream mathematics practices and control what it means to participate in undergraduate mathematics and who participates. This study suggests that innovative foundational practice has the potential to challenge the dominant constructions.

12.5.3 Responding on a personal level

I began this thesis by locating the research problem in my practice as a lecturer in the Foundational Course. The discussion in this section points to how this research study has illuminated what was not visible in my practice, and in particular how it has looked beyond

my practice in the Foundational Course to relationships at different levels of the university. Yet it has only been possible to answer the empirical problem by simultaneously embarking on a journey to solve the theoretical problem of this study. Not only have a I grown personally as both a lecturer and a researcher in mathematics education, but the journey presented in this thesis points to the role that socio-political research practice can play in contributing to practice and policy in the higher education space.

12.6 Challenges and limitations of this study

In this section I discuss the challenges of conducting this study, and related to this, the limitations of the study. I use this discussion to point to opportunities for future research.

This study focuses on one particular model of foundational provision in undergraduate mathematics and other models in use, certainly at higher education institutions in South Africa, are not in view. In addition, this study does not investigate the Foundational Course as a whole, but focuses on the two innovations related to relevance and a learner-centred pedagogy. However, I argue that the two innovations in this Foundational Course can be regarded as cases in that they foreground issues related to conceptualizing foundational practice, a practice which represents a move in the wider space. While the results of this study have much to say about the use of practical problems and a learner-centred pedagogy in terms of access to advanced mathematical practice, this study identifies issues in foundational practice that need to be attended to, even in the absence of such innovations. Nonetheless, the theoretical tools developed in this study could certainly be used in research on other features of the Course and on other models of foundational provision.

One of the methodological challenges of this study has been answering two related questions; “How much data?”, and “How much depth?” Using the tools of CDA is labour intensive (Parker & Burman, 1993) and I have had to make choices about how much data I need to make a case about the role of these two innovations in the Course. Looking for patterns in the enabling and constraining student action has involved analyzing the verbal

and non-verbal action of three groups of students, each on two practical problems. Yet having to analyze almost 4 000 lines of transcript has implications for the level of detail used in this analysis. In Chapter 4 I make explicit my decision not to focus in depth on the graphical aspects of mathematical discourse and the use of different languages in Group 2. So this study does not talk about certain aspects of the student action. Yet there is potential to revisit the transcripts prepared for this study with the tools that the mathematics education community has to offer in these two areas.

A further challenge arose out of my ethical decision to avoid a deficit view of students. I was able to address the issue on a theoretical level, but responding to this challenge at a methodological level and in my writing has proved difficult. On the one hand, rich descriptions of the student action on different practical problems serve as evidence for my arguments and are necessary if I am to demonstrate the interpretative and theoretical validity of this study. Yet on the other hand, these rich descriptions have the potential to foreground certain students and, inadvertently, identify these students as deficit. While explicit attention was paid to addressing this when selecting extracts from the transcripts as evidence in Chapters 8 to 11 (see Section 8.3), I am conscious that, since visibility was not the only criteria for selection of evidence, descriptions of certain students in this study may be interpreted as deficit positioning. While avoiding such a positioning within a thesis of this length has been a challenge, it becomes even more problematic when writing articles from this study.

A key challenge of this study has been balancing my roles as both researcher and lecturer in the Foundational Course. In Chapters 5 and 6 I have described choices that had to be made, choices that related to the ethics of research, but also the ethics of my practice as a lecturer. By adopting a socio-political perspective of doctoral research practice, I have foregrounded what my positioning within the network of mathematics education practices may mean for the study and responded, for example, by making this positioning explicit, by delineating my description on this space in Chapter 2 for the purposes of interpretation, and by

revisiting the ethics of my research practice on an ongoing basis during this research journey.

The discussion in this section points to further possibilities for research, involving both empirical and theoretical work. Firstly, there is potential to develop the theoretical framework and analytic tools, in interaction with the texts used in this study, in order to investigate in detail aspects of the student action that are not visible in this study, for example, the multisemiotic aspects of this action or the use of isiXhosa, Sesotho and Setswana in interaction with English. I also have an interest in exploring the conceptions of power offered by Janks (2010) and Valero (2008) in interaction with the texts in this study. Secondly, there is potential to apply the theoretical framework and analytic tools developed for this study (with necessary developments) to other aspects of the Foundational Course, courses based on other models of foundational provision, and mainstream courses.

Thirdly, in my introduction of the students in Chapter 5 I have noted the varied capital and positioning that these students bring to the foundational mathematics classroom. Certain points of interest in this respect have arisen in this study, for example, the student who resists the positioning of the foundational student who follows prescribed steps withdrew from the Course, and it is the White English-speaking students who dominate the talk in Group 3. However, the conceptualization of this study does not allow me to make any claims in this regard. Given the complexity of foundational practice as identified in this study, it is possible that this practice acts in reproductive ways, and there is certainly space for a study that investigates how the student capital and positioning may play out in the foundational classroom.

Lastly, there is potential to pursue some of the recommendations for practice presented in Section 12.5, but from a research perspective. For example, researching what it means to make boundary crossings explicit in a course or to make boundary crossings as a way of acting more deliberate in a course.

12.7 Summary of this chapter

I began this chapter by presenting the central argument of this thesis; the continuities and disruptions that innovative foundational practice represents in relation to other practices in the educational space positions this practice paradoxically in higher education as it simultaneously challenges and reproduces dominant undergraduate mathematical practices.

I have argued that it was only possible to arrive at this finding by embarking on a research journey in which the theoretical and empirical problems have co-constituted one another. Solving the theoretical problem in this study has involved the development of a socio-political perspective of mathematical practice and associated analytic tools that both draws on and contributes to the mathematics education community. The answers to the empirical research questions talk to the relationship between innovative pedagogies and access to mathematical practice, not at school level, but in terms of the school/advanced mathematics transition and the role of foundational practice in this transition.

In making recommendations for practice, I have argued that a productive way forward is to recognize the paradoxical positioning of innovative foundational practice in higher education. Working with this positioning at the Course level, in interaction with the dominant mathematical practices, and at policy level is a constructive response to the paradox. Such a response affords opportunities to provide access to the dominant practices for foundational students, but also to change these dominant practices.

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APPENDIX A

ADAPTED JEFFERSON NOTATION USED FOR TRANSCRIPTION

1. non italicized text for verbal interaction
2. ((*italics*)) for non-verbal descriptions
3. (unclear) for unclear text, or (translated text) for verbal text that is translated into English
4. [text] indicates overlapping text. Where this occurs consecutively I alternate between [text] and [[text]]. In cases where two extended conversations take place simultaneously, I use this notation, but transcribe the text consecutively, giving line references where necessary.
5. {mathematical symbols} for symbols verbalized from left to right, e.g. { $\frac{dx}{dt} = 75$ } when a student says "*dx* over *dt* is ... 75".
6. Up arrow ↑ indicates rising intonation
7. An obvious question is indicated by a question mark ?
8. <text> indicates text said slower than normal.
9. >text< indicates text said faster than normal.
10. Underlined text indicates emphasis or stress.
11. °Degree text° indicates text said quieter than normal.
12. UPPER CASE text indicates text said louder than normal.
13. Short pauses indicated by three dots ..., longer pauses indicated by six dots ... ... No time intervals are given.

APPENDIX B THE FLU VIRUS PROBLEM

The Flu Virus Problem

A flu virus has hit a community of 10 000 people. Once a person has had the flu he or she becomes immune to the disease and does not get it again. Sooner or later everybody in the community catches the flu. Let $P(t)$ denote the number of people who have, or have had, the disease t days after the first case of flu was recorded.

a) Draw a rough sketch of the graph of P as a function of t, clearly showing the maximum number of people who get infected, and **do not continue until you have had your graph checked by a tutor.**

h) What are the units of $P'(t)$?

i) What does $P(4) = 1\,200$ mean in practical terms? (Your explanation should make sense to somebody who does not know any mathematics.)

j) What does $\frac{P(7)-P(4)}{7-4} = 350$ mean in practical terms? Give the correct units.

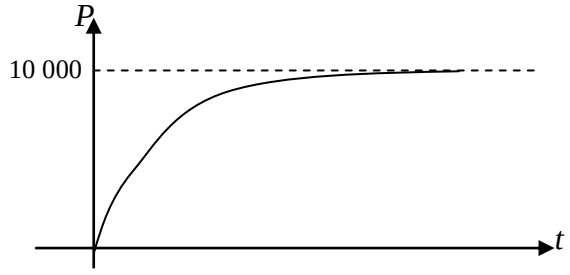
k) What does $P'(4) = 400$ mean in practical terms? Explain why $P'(t)$ can never be negative.

l) What is $\lim_{t\to\infty} P(t)$? Give a short reason for your answer.

m) What is $\lim_{t\to\infty} P'(t)$? Give a reason for your answer.

Worked solutions for the Flu Virus Problem

a)



b) $P'(t)$ units: people per day.

c) 4 days after the first recorded person got flu, 1200 people had the flu.

Worked solutions for the Flu Virus Problem continued

d) From the 4th to the 7th day after the first recorded person got flu, the number of people on average who had the flu was increasing by 350 people per day.

e) 4 days after the start of the epidemic, the number of people who had the flu was increasing by 400 people per day. ($P'(t) > 0$ since the total number of people with the flu or who have had the flu can only increase.)

f) $\lim_{t \to \infty} P(t) = 10\,000$. Eventually after a long time everyone gets the flu.

g) $\lim_{t \to \infty} P'(t) = 0$. Eventually the number of people who have caught the flu becomes (very nearly) constant at 10 000, so the rate of new infections is 0 (see graph).

The "valued mathematical ways" for the Flu Virus Problem

(a) Mihlali begins by drawing and then labelling the axes for the graph. By looking at the function notation $P(t)$ and at the description of the variables given in Sentence 4, he identifies time as the independent variable and "the number of people who have or have had the disease" as the dependent variable. So he plots the variable time on the horizontal axis and the number of people on the vertical axis.

Mihlali draws on his knowledge of how the word "rough sketch" is used in the Course, that is, he should attend to the shape of the graph and include only the values that are pronounced in the text.

Firstly, Mihlali attends to the functional relationship $P(t)$ and identifies this with its meaning in the task context, that is, the number of people who have or have had the disease at time t. He adopts an operational view of the function as he considers what happens as the disease spreads over time. Initially (at time t = 0 days) no-one has the disease, so (0,0) is a point on the graph. Since there are 10 000 people in the community and "sooner or later" everyone catches the flu, the graph of $P(t)$ reaches a maximum value of 10 000 at some time t. Since $P(t)$ is by definition the number of people who have or have had the disease at time t, the graph of $P(t)$ is always increasing.

Mihlali also attends to a second functional relationship, that is, the derivative function $P'(t)$. He identifies the relationship between the two functions; $P'(t)$ is the rate of change of the function $P(t)$. He also links the two functional relationships graphically, that is, the function $P'(t)$ represents the gradient of the graph of $P(t)$. Linking both functions to their meaning in the task context, Mihlali identifies $P'(t)$ as the rate of change of the number of people who have or have had the disease with respect to time.

He adopts an operational view of the derivative function and attends to what happens to this rate of change as the disease spreads over time. Initially, there are many people who can catch the disease, so the rate at which people are catching the disease is high (and linking the functions graphically, the gradient of the graph of $P(t)$ is steep). But as time passes, more people have caught the disease and there are fewer people to catch it, so the rate $P'(t)$ decreases (and the gradient of the graph is less steep). This way of looking allows Mihlali to identify the graph of $P(t)$ as concave down

(b) Mihlali links the functions $P(t)$ and $P'(t)$; he attends to the "dash" symbol and the independent variable t in the derivative function to identify that the function $P'(t)$ represents the rate of change of the function $P'(t)$ with respect to time. In addition he makes a link to the task context and attends to the meaning of the variables t and $P(t)$ in the task context to identify the derivative function as the rate of change of the number of people who have or have had the disease with respect to time. He also attends to the difference between the meaning of the variable (say "time" for t) and the units of measurement of this variable (days).

(c) Mihlali reads that the meaning must be given in "practical terms" and makes a link to how this term is used in the Course, that is, he should make a link to the task context and not use any mathematical terms in his description. He confirms the need to avoid mathematical terms when he reads the text in the bracket.

As in question (b), Mihlali attends to Sentence 4 and identifies the meaning of the variables $P(t)$ and t with their meaning in the task context (as well as the units associated with each variable). So the "4" in $P(4)$ refers to "4 days after the first case of flu was recorded", and $P(4)$ refers to the number of people who "have or have had the disease" 4 days after the first case of flu was recorded.

The answer in the worked solutions does not draw consistently on the text in Sentence 4 as Mihlali does. The answer makes a link to Sentence 4 for the meaning of the variable t, that is, "days after the first case of flu was recorded". However, the text does not use the wording in Sentence 4 to describe the meaning of P(t). Rather it refers to the number of people who "had the flu", a representation that does not attend to the people who had had the flu up to that point as the descipriton of tuhe function in Sentence 4 suggests.

(d) Mihali looks at the expression $\frac{P(7)-P(4)}{7-4}$ in different ways. He begins by looking at the symbols on the denominator of the expression and links these to their meaning in the task context; the subtraction on the denominator indicates the three-day time period from t = 4 days to t = 7 days. He then looks at the numerator and identifes the subtraction as representing the change in the number of people who have or have had the disease over this three year period. Attending to the division in the expression $\frac{P(7)-P(4)}{7-4}$ enables him to identify the expression as representing the daily change in the number of people who have or have had the disease. This identification is key in enabling Mihlali to avoid using the word "rate" in his answer and it also enables the identification of the appropriate units, that is, "people per day".

Attending to the equal sign and the positive value of "350" on the right hand of the eqaution, he identifes that this change is an increase of 350 people per day. In writing his answer, Mihlali makes a link to the elsewhere in the Course where practical

problems on average rate of change have been solved. He repeats the wording used in these problems, for example, “from 4 to 7 days after the first recorded person got flu...”.

(e) When Mihlali looks at the symbols in the expression $P'(4)$, he uses the “dash” symbol and his knowledge of the meaning of the variable t in the task context to identify this expression as representing the instantaneous rate of change, in this case, at time $t = 4$ days. He links this function $P'(4)$ to the task context and identifies it as representing the rate of change of the number of people who have or have had the disease 4 days after the first case of flu was recorded. However, Mihlali knows from elsewhere in the Course that “practical terms” means he should avoid using terms like “rate” and “instantaneous” in his answer. He identifes the instantaneous rate of change as the change in the number of people per day at the time $t = 4$ days. Attending to the equal sign and the positive value of “400” on the right hand of the eqaution, he identifes that on the 4^{th} day after the disease began spreading, this change is an increase of 400 people per day.

In writing his answer, Mihlali makes a link to the elsewhere in the Course where practical problems on instantaneous rate of change have been solved. He repeats the wording used in these questions, for example, “after 4 days” to identify the instantaneous rate of change.

Mihali can explain in two ways why “ $P'(t)$ can never be negative” in the second part of question (e). On the one hand he attends to the function $P(t)$ and links this to its meaning in the task context; since it represents the number of people who have or have had the flu over time, $P(t)$ is always increasing. Then he links the function $P(t)$ to the derivative function $P'(t)$; if the function is always increasing, then its derivative is always positive. His second way of responding involves attending to the graph in question (a) and also linking the meaning of the functions $P(t)$ and $P'(t)$ graphically.

Since the function $P'(t)$ represents the gradient of the increasing graph of $P(t)$ at different values of t, he sees that the gradients are always positive.

(f) Mihlali looks at the interrogative "what" in question (f) and identifies that this requires him to evaluate the expression $\lim_{t\to\infty} P(t)$. He attends to two parts of the expression. He looks at the function $P(t)$ in the limit expression, and links this with both its meaning in the task context (the number of people who have or have had the disease at time t) and its graphical representation in question (a). He also looks at the symbols $t \to \infty$ in the limit expression and links this to the time passing in the task context. Mihlali views the limit expression operationally by considering what happens to the function as time passes; as time passes the number of people who have or have had the disease increases and will eventually reach 10 000 (he can see this by looking at his graph in question (a)).

Yet Mihlali can also evaluate the limit expression by viewing the object $\lim_{t\to\infty} P(t)$ structurally; he identifies the object as representing the limit at infinity of $P(t)$, and in terms of the task context, the number of people who, "sooner or later" will have or have had the disease.

(g) As in question (f) Mihlali looks at the interrogative "what" in question (g) and identifies that this requires him to evaluate the expression $\lim_{t\to\infty} P'(t)$. As in the previous example, he looks at two parts of the limit expression. He looks at the symbols $t \to \infty$ and identifies this as time passing in the task context. He also looks at the derivative function $P'(t)$ in the limit expression, and identifies this with its meaning in the task context, that is, the rate at which the number of people who have or have had the disease is changing. Attending to the derivative function allows Mihlali to distinguish the limit expression in question (g) as different to that in question (f).

Although the function identified here is the derivative function $P'(t)$, Mihlali identifies that he can use the graph of $P(t)$ in (a). He identifies the function $P'(t)$ as the gradient of the graph of $P(t)$ at different points. Hence, by adopting an operational view of limit expression and considering what happens to the gradient of the graph of $P(t)$ at different points he sees that, as time passes, the gradient is decreasing. When everyone (10 000 people) has caught the disease the gradient will be horizontal and so the rate of change will be zero. Hence he writes $\lim_{t\to\infty} P(t) = 0$.

Yet Mihlali can also evaluate the limit expression by viewing the object $\lim_{t\to\infty} P'(t)$ structurally; he identifies the object as representing the limit at infinity of $P'(t)$, and in terms of the task context, the number of people who, "sooner or later" will have or have had the disease.

APPENDIX C THE CAR PROBLEM

Boxed text preceding the Car Problem

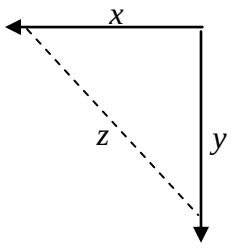
The following questions are related rates problems. These MUST be set up correctly. Follow these steps for **EVERY** question:

1. Draw a diagram and define variables.
2. Write down what is given, using the correct notation.
3. Write down what is to be found.
4. Write down a formula linking the variables.
5. Differentiate and complete the question

The Car Problem

Two cars start moving from the same point. One travels south at 100km/h and the other travels west at 75km/h. At what rate is the distance between the cars increasing two hours later? (Let the distance between the cars after a time t be z km).

Worked solution for the Car Problem



Let x = distance covered by car A
Let y = distance covered by car B
Let z = distance between car A and car B

Given: $\frac{dx}{dt} = 75$ and $\frac{dy}{dt} = 100$

To Find: $\frac{dz}{dt}$ when t = 2 hours

$x^2 + y^2 = z^2$ (Pyth)

$\therefore\ 2x.\frac{dx}{dt} + 2y.\frac{dy}{dt} = 2z.\frac{dz}{dt}$

When t = 2 hours, x = 150 km and y = 200 km and $z = \sqrt{150^2 + 200^2}$ = 250 km

$\therefore\ 150 \times 75 + 200 \times 100 = 250.\frac{dz}{dt}$

So $\frac{dz}{dt} = \frac{1}{250}(150 \times 75 + 200 \times 100) = 125$ km/h

The "valued mathematical ways" for the Car Problem

Mihlali attends to the instructions in the boxed text and the naming of the Car Problem as a "related rates problem" and makes a link to similar problems in the Course. Mihlali identifies that the "diagram" required in point 1 of the boxed text is a mathematical representation that includes only the necessary information for solving the problem. In constructing this diagram, he attends to two features of the task context; the "same point" as the starting point for the two cars and the direction of travel ("south" and "west"). When deciding how to "define variables" in point 1 he considers what is changing in the task context, that is, the distance of each car from the starting point and from one another. Mihlali uses this information to draw a right-angled triangle (the right angle is at the starting point), with the hypotenuse representing the distance between the cars at a particular time and the other two sides representing the distance travelled by each car from the starting point at a particular time. He attends to the hint in Sentence 4 of the Car Problem to use the variable *z* as the distance in km between the two cars after time *t*, and labels the hypotenuse z. Linking to what is usually done when solving related rates problems in the Course, he assigns the variables *x* and *y* to the other two sides of the triangle. Mihlali also writes down the meaning of each of the variables *x*, *y*, and *z* in the task context (including the units km, which he identifies in the problem text).

Following the instruction in point 2 of the boxed text, Mihlali looks at the values 75km/h and 100km/h in the problem text and uses the units (km/h) to identify, in the task context, that these represent the constant speed at which each of the cars travel. He identifies these given speeds as the rate of change of distance with respect to time, and the derivative notation $\frac{dx}{dt}$ and $\frac{dy}{dt}$. He also makes a link between the variable *x* and the derivative $\frac{dx}{dt}$; since the distance travelled by the car (*x*) increases as time passes, the rate $\frac{dx}{dt}$ is positive (similarly for $\frac{dy}{dt}$ being positive). Hence he is enabled to "write down" what is "given".

In identifying what "is to be found" for point 3 of the boxed text, Mihlali looks at Sentence 3. He identifies the rate at which the "distance between the cars is increasing two hours later" with the instantaneous rate of change with respect to time and the notation $\frac{dz}{dt}$ at $t = 2$ hours. Hence he is enabled to "write down" what is "to be found".

Mihlali attends to the naming of the Car Problem as a "related rates problem" and the instruction to find a "formula" in point 4 of the boxed text. The right-angled triangle in his diagram provides a cue to use the Theorem of Pythagoras to link the variables *x*, *y* and *z*. In deciding how to "differentiate" his equation $x^2 + y^2 = z^2$ as instructed in point 5 of the boxed text, Mihlali reads what he wrote down under "To find"; since he is finding the rate of change of distance over time ($\frac{dz}{dt}$), the differentiation must be performed with respect to time. Since he has identified each of the variables *x*, *y* and *z* as functions of *t*, he recognizes that he should operate on the formula using implicit differentiation, and use the chain rule for differentiation for each function of *t*.

Having completed the implicit differentiation, Mihlali again revisits what he wrote under "To find". In point 5 of the boxed text, "completing the question" involves operating on the derivative equation $2x.\frac{dx}{dt} + 2y.\frac{dy}{dt} = 2z.\frac{dz}{dt}$ by substituting values into this equation, and rearranging the subject of the formula to solve for $\frac{dz}{dt}$. Mihlali attends to the time of 2 hours. Since the speed at which the cars are travelling is constant, he substitutes 75km/h and 100km/h for $\frac{dx}{dt}$ and $\frac{dy}{dt}$. He cannot identify values for the distances *x*, *y* and *z* directly in the problem text, but he makes a link to the task context to calculate the distances *x* and *y* after 2 hours; the car travelling at 75km/h will travel 75km in 1 hour and 150km in 2 hours. Mihlali then uses his Pythagorean equation $x^2 + y^2 = z^2$ to calculate the distance z. When he

has made $\frac{dz}{dt}$ the subject of the formula and used his calculator to arrive at a final answer, he makes a link to the task context to identify the units as km/h.

APPENDIX D

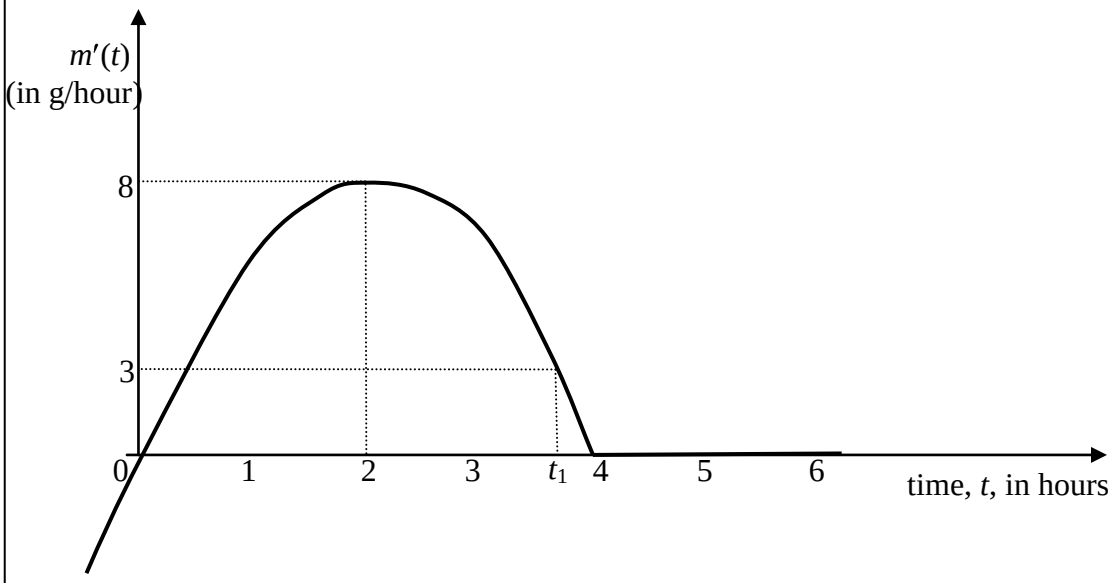
THE CHEMICAL REACTION PROBLEM

The Chemical Reaction Problem

Quantities of two chemicals A and B are mixed together in a reaction chamber, and they react to form a new product, X.

The **rate** at which the product X is formed is given by $m'(t)$, where m is the mass of the product formed, in grams, and the time t from the start of the reaction is measured in hours. The graph of $m'(t)$ is a **parabola** graph until time $t = 4$ hours, after which it is zero.

It is also given that, from the start of the reaction, some of the product X is removed from the reaction chamber at a constant rate of 3 g/hour.



a) Write down an expression involving an integral that gives the **total** mass of product X **in** the reaction chamber after a time of t hours.

b) Explain very clearly the significance/practical meaning of the time $t = t_1$ in the graph above.

c) Explain very clearly the significance/practical meaning of the local maximum in the graph of $m'(t)$ at time $t = 2$ hours.

The Chemical Reaction Problem continued

d) Find the equation of the parabola part of the graph – it will express $m'(t)$ as a quadratic function of t.

e) Hence find the **total** mass of product X formed in the 4 hours since the start of the reaction.

f) Draw a rough sketch of the graph of $m(t)$ for $0 \le t \le 5$ hours. Clearly indicate on your graph the times $t = 2$ hours, $t = 4$ hours, and $t = 5$ hours.

Worked solutions for the Chemical Reaction Problem

a) Total mass = $\int_0^t m'(t)\,dt - 3t$

b) At time $t = t_1$, the amount of product X in the reaction chamber is a maximum (or, after time $t = t_1$, the rate of formation of product X is less than the rate of removal of product X).

c) At $t = 2$ the rate of formation of product X is the greatest / reaction rate is the fastest.

d) $m'(t) = a(t-0)(t-4)$

$\therefore m'(t) = at(t-4)$

When $t = 2$, $m'(t) = 8$

So $8 = a(2)(2-4)$, $8 = -4a$ $\quad \therefore a = -2$.

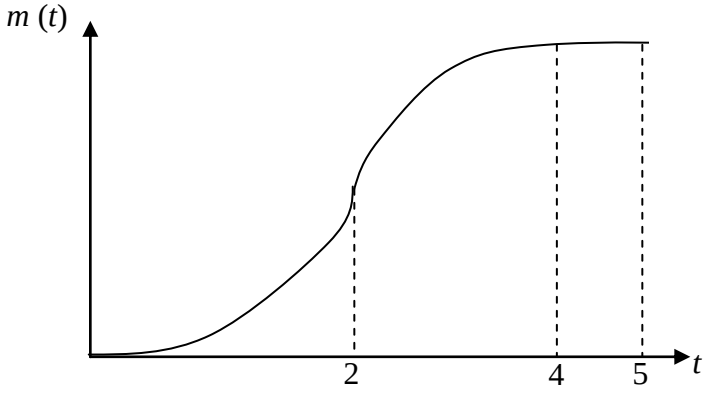
So the equation of $m'(t)$ is $m'(t) = -2t(t-4) = -2t^2 + 8t$

e) Total mass of product X formed = $\int_0^4 (-2t^2 + 8t)\,dt = \left[\frac{-2t^3}{3} + 4t^2\right]_0^4 = \frac{-2(64)}{3} + 64$

$= \frac{64}{3}$

$= 21\frac{1}{3}\,g$

f)



The "valued mathematical ways" for the Chemical Reaction Problem

(a) Mihlali attends to two aspects of the wording of question (a). Firstly, he attends to the phrase "**total** mass of the product X **in** the reaction chamber", with the bold text signalling that he must think carefully about the task context. Attending to the set-up of the problem (Sentences 1 to 4) he identifies that he needs to take into account both the mass of the product formed during the reaction and the mass of the product that is removed from the chamber, that is, the total mass of the product in the reaction chamber can be obtained by subtracting the amount removed from the amount formed. Secondly, he attends to the wording "after time t" to identify that his answer will be a function of t.

The instruction in question (a) to use an "integral" provides the cue to attend to the rate of formation and the rate of removal. Mihali links the total mass formed to the anti-derivative of the rate of formation $m'(t)$, and the total mass removed to the anti-derivative of the rate of removal (the constant 3g/hour identified in Sentence 4). Since no algebraic formula is provided for the function $m'(t)$ he cannot operate on a formula and views the object $\int_0^t m'(t)\,dt$ structurally as the total mass formed. He attends to the correct mathematical notation when writing this integral. Since the rate of removal is given in as 3g/hour, he operates on this constant to get $3t$. Drawing on the rule for the sum of integrals studied in the Course, Mihlali finds the total mass by subtracting the two integrals $\int_0^t m'(t)\,dt$ and $3t$.

(b) Reading the word "practical meaning" in question (b), Mihlali identifies from similar problems in the Course that he must make a link to the task context and use everyday language (avoiding the word "rate"). He recognises the word "significance" from similar problems in the Course, and remembers that this question requires more than just attending to the co-ordinates (t_1, 3) of the point and explaining the meaning of this point in the task context, for example, "the rate of formation of the product at time t_1 is 3g/hour" or "the rate of removal of the product at time t_1 is 3g/hour".

Mihlali looks at the graph and the dotted lines from t_1 and notices that the point (t_1, 3) lies at the intersection of two graphs; the parabola graph representing the rate of formation of the product, and the dotted horizontal line representing the rate of removal of the product (the value of 3 on the vertical axis sets up a link to the information about the rate of removal of the product in Sentence 4). Making the link between these two graphs and their meaning in the task context, Mihlali is enabled to write down the second solution in the worked solutions; the rate of removal of the product is equal to the rate of formation at $t = t_1$, and since the parabola graph drops below the horizontal line at time t_1, the rate of removal is greater after this point. This particular solution, although given value in the worked solutions, uses the word "rate". This wording is not valued in other problems in the Course when it comes to giving the "practical meaning".

The first solution provided in the worked solutions ("at time $t = t_1$, the amount of product X in the reaction chamber is a maximum") does not use the word "rate" and talks about the mass of the product rather than the rate. To produce such an answer Mihlali makes a link between the functions $m(t)$ and $m'(t)$, their graphical representations, and their meaning in the task context; if the function $m'(t)$ represents the rate of formation, the anti-derivative $m(t)$ represents the amount of product formed. Where the graph of $m'(t)$ is positive, the graph of $m(t)$ or the amount of product formed is always increasing. But since the product is being removed at a rate of 3g/hour, the mass of the product in the chamber is only increasing when the graph of $m'(t)$ is above the horizontal line at 3. The amount of the product in the chamber will start decreasing after $t = t_1$ when more of the product is being removed than is being formed. Hence the mass of the product in the chamber is at its maximum at the point (t_1, 3).

(c) As in question (b) the words "practical meaning" and "significance" in this question and used elsewhere in the Course signal to Mihlali that he must draw on the task context (avoiding the word "rate") and give an answer that goes beyond simply giving the meaning of the co-ordinates (2,8), that is, "the rate of formation of the product after 2

hours is 8g/hour". The inclusion of the mathematical term "local maximum" also signals to Mihlali that there is something of "significance" about this point.

Unlike question (b) where Mihlali had to attend simultaneously to the meaning of two graphs, he only attends to the parabola graph, $m'(t)$, in this question. Since this graph represents the rate at which product X is formed, the point $t = 2$ is the point where the rate of formation of the product is maximum. Again, the answer given in the worked solutions is one that would not be valued in the Course in general as it uses the mathematical term "rate".

(d) Mihlali attends to the wording of question (d). The words "parabola part" provides a link to Sentence 3 where it is stated that the graph is a parabola up to the point $t = 4$. Reading about the "quadratic function" and the "parabola" in this question cues Mihlali to make a link to school mathematics (and work that is revisited in the first weeks of the Course). He identifies two possible quadratic functions and links each to the points of a parabola graph; the equation $y = a(x - x_1)(x - x_2)$ is used when the x –intercepts and one other point on the graph are given, the equation $y = a(x - p)^2 + q$ is used when the co-ordinates of the turning point and one other point on the graph are given. Having reminded himself about these possibilities, Mihlali attends to the parabola graph in the problem text and looks to see what information he is given. He identifies that both general equations can be used, but settles on using
$y = a(x - x_1)(x - x_2)$. Before proceeding with the operations he attends to the fact that the independent variable in the Chemical Reaction Problem is t, and adapts the format of the equation to $y = a(t - t_1)(t - t_2)$. He then operates on this equation, substituting the t-intercepts (0,0) and (4,0) and the point (2,8) into this equation and rearranging the subject of the formula to find the value of a.

(e) Mihlali attends to the word "hence" in question (e) and notes that he must refer back to one or more of the previous questions. As in question (a), he attends to the bold text in "**total** mass of product X formed" and remembers that he must pay careful attention to

the task context; this question requires that he attend only to the amount of product formed and not to the removal of the product from the chamber. As in question (a) he links the total mass formed to the anti-derivative of the rate of formation $m'(t)$. This provides a link to his equation of $m'(t)$ in question (d) and he sets up an integral, paying attention to the appropriate notation for integrals. Mihlali then operates on this formula, finding the anti-derivative of each term and substituting values of t. In presenting his final answer he makes a link to the task context to consider that the units of the total mass will be "grams".

(f) Mihlali reads the term "sketch graph" in the question and makes a link to what is required in the Course; he should attend to the shape of the graph as well as the important points (these are identified in the question as being the times $t = 2$ hours, $t = 4$ hours, and $t = 5$ hours). He attends to these time values and notices that they are identified on the graph of $m'(t)$ and in some of the earlier questions. Mihlali also makes a link between the required function $m(t)$ and its meaning in the task context to identify that he only needs to attend to the formation of the product X and the graph of $m'(t)$ (and not the removal of the product from the chamber).

Mihlali begins to draw a sketch making links between the graph of $m'(t)$ and $m(t)$ and his understanding of the meaning of these functions in the task context. He adopts an operational view as he constructs the graph by attending to what is happening over time, that is, by considering the given points $t = 2$ hours, $t = 4$ hours, and $t = 5$ hours and what happens in between these times. For example, in deciding where the graph will start, Mihlali looks at the graph of $m'(t)$ to see that the rate of formation at time $t = 0$ is 0, so no product has been formed (he assumes the formation starts at $t = 0$). Since the rate of production is positive during the first 4 hours (suggested by the parabola graph being above the t-axis), the mass of the product is always increasing, hence the graph of the function $m(t)$ is always increasing and reaches a maximum mass at $t = 4$ hours. From $t = 0$ to $t = 2$ hours the graph of $m'(t)$ is increasing, that is the rate is

increasing, hence the graph of $m(t)$ is concave up. From $t = 2$ to $t = 4$ hours the graph of $m'(t)$ is decreasing, so the rate of formation is decreasing, hence the graph of $m(t)$ is concave down. Thus there is a point of inflection on the graph of $m(t)$ at $t = 2$ hours (where the rate of formation changes from increasing to decreasing). From $t = 4$ to $t = 5$ hours the rate of production is zero (suggested by the horizontal line on the t-axis). Since no product is formed during this time, the mass of the product will remain constant, hence the graph of $m(t)$ is horizontal from the maximum at $t = 4$ hours.

APPENDIX E

TOOLS FOR CRITICAL DISCOURSE ANALYSIS

Textual Feature (from Fairclough (2003) in interaction with the data, unless otherwise stated)	Meaning	Specific function within this meaning
Choice of wording		
Naming/renaming of objects nouns vs. reference pronouns everyday vs. scientific terms definite (*the*) vs. indefinite (*a*) article repetition, rewording mathematical terms (Pimm, 1987; Morgan, 1998) mathematical symbols (Pimm, 1987; Morgan, 1998) arrangement of symbols represents mathematical relationship (Pimm, 1987)	representation interaction	Specialist vocabulary allows one to speak with authority (Morgan, 1998); Definite article allows referencing forward and backwards, used for shared information (Janks, 2010); Indefinite article for object not mentioned before, for a general state of affairs (Crystal, 1988)
Naming/renaming of social actors nouns vs. pronouns individual vs. collective active vs. passive personal vs. impersonal inclusive vs. exclusive backgrounded vs. foregrounded	representation interaction identification	First person *I* for claiming personal responsibility (Morgan, 1998); First person *we* for generality or to spread responsibility or to derive weight of authority (Pimm, 1987); Second person *you* as personal direct address or indefinite pronoun for solidarity (Fairclough, 2001) or for general process. Active voice identifies "doers", passive voice identifies "done-to's" and deletes agency (Janks, 2010)
Alteration of meaning borrowed words in mathematics (Pimm, 1987)	representation	
Absence of naming Absence of pronouns	representation interaction identification	Absence of pronouns suggests distancing, formal relationship (Morgan, 1998).
Processes		
Processes in verbs material, mental, relational, verbal, existential	representation	Relational (identifying and attributing) common in mathematics (Morgan,1998).
Representation of processes nominalization	representation	Obscures agency, for generalizing and abstracting in scientific discourses; Transforms processes into objects in mathematics (Morgan, 1998).

Textual Feature (from Fairclough (2003) in interaction with the data, unless otherwise stated)	Meaning	Specific function within this meaning
Semantic relations between sentences and clauses		
Conjunctions causal reason (*because*) consequence (*so*) purpose (*in order*) conditional (*if*) temporal (*when*) additive (*and*) contrastive (*but*)	representation interaction	Causal for explanation in mathematics (Morgan, 1998).
Prepositions (*like*)	representation interaction	
Repetition Juxtaposition of sentences Sequencing of sentences Punctuation	representation interaction	
Reference relations (*the, this, that, he, she, it, they*)	representation interaction	
Reported speech	interaction identification	
Speech functions		
statement / restatement (content or procedure) rewording question explanation suggestion instruction request for feedback feedback (positive or negative) prompt placeholder (*okay*) interjection reading text aloud	interaction identification	Functions of "okay" (Gee & Green, 1998): asking for confirmation (with rising intonation); giving praise (with excitement); getting attention (at beginning of speech turn); placeholder, indicating that one is thinking but wishes to keep the speech turn (slowly, within a speech turn).
Mood		
Declarative Imperative exclusive/inclusive (Morgan, 1998) Interrogative (*wh… words, yes/no questions*).	interaction	Declarative: speaker is giver, addressee is receiver (Fairclough, 2001). Imperative: asking for something or making a suggestion (Fairclough, 2001); common in school texts (Morgan, 1998); speaking with authority (Morgan, 1998). Interrogative: asking for information/action (Fairclough, 2001).
Tense		
Tense for definiteness	representation	Present tense for timeless truths and absolute certainty (Janks, 2010).

Textual Feature (from Fairclough (2003) in interaction with the data, unless otherwise stated)	Meaning	Specific function within this meaning
Phonology and body language		
Stress Intonation Body language and gestures (eye contact, pointing, tracing in the air)	identification representation	
Modality		
Modal verbs (*should*, *must*, *will, won't etc.*) Hedges (*sort of, just, or something*) Tone	representation interaction identification	Suggests the level of commitment.
Textual structures		
Underlining, italics, upper case text Sequencing (beyond sentence level) Integration of visual verbal (Morgan, 1998)	representation identification	

APPENDIX F

INDIVIDUAL INTERVIEW WITH EACH STUDENT*

* The names of the courses and programme have been removed.

Introduction by the researcher:

As we have discussed, the focus of my study is on the use of problems with real-world contexts in the Foundational Course. I plan to investigate how you solve these problems. I want to be able to describe what practices you use when solving these problems, and to explain why you use these practices. In order to do this I need to know more about your educational background: your experiences of learning before coming to university and your reflections on your learning experiences in the few months that you have been at university.

Before we begin I want to remind you that all our discussions in this interview will be treated confidentially. The data collected in this interview will be used for my research project only.

Guiding interview questions:

I am going to begin with some questions about your schooling:

1. Where did you go to primary school? Where did you go to high school? What mark did you get for your final maths exam at high school? Were you satisfied with this mark? Why?
2. Please describe a typical maths classroom at your high school. For example, describe the furniture in the classroom, tell me where the students are sitting, etc.
3. Think of a typical mathematics lesson at your high school, and tell me about it. How did the lesson start? What did the teacher do? What did the students do? What books did you use? What type of problems did you solve?
4. In primary school/high school, what language/s did your teacher use? If the teacher used more than one language, when did s/he use each language? What language/s did you and your classmates use in the maths classroom? If you used more than one language, when did you use each language?
5. Think about your lessons in other subjects at school. Were these lessons similar/different to your maths lessons at school? In what way?

6. Please tell me about your study methods when you were at school. When and where did you do your homework? Did you do your homework alone or with other students? How did you study maths?

7. I notice from your Enrolment Form for the Foundational Course that you finished school in (give year). Please tell me what work or studying you have been doing in the years between finishing school and enrolling for this course.

Now I would like to ask some questions about your first few months at university.

8. Please think about the Foundational Course. Is this course similar/different to the way you learnt maths at school? In what way?

9. Tell me about the study methods you are using in your first year at university. Where do you study? Do you work alone or with other students? How do you study maths? Do you study in the same way for your other subjects?

10. Please tell me about your experience of working in a group in the Foundational Course workshop. What do you like or dislike about working in a group?

11. Where are you living while you are studying at university?

12. Do you have opportunities to speak your home language when you are doing maths at university? And in other subjects? If so, what languages do you speak, when do you use them, and why?

13. Why are you enrolled for a Science Degree?

14. How does your family feel about you coming to university in this city? Have other members of your family studied at university?

15. This Course is an access course which forms part of the foundational programme. How did you feel when you were told that you had been accepted for programme? Do you think that what you do in Foundational Course is different to what you would do in a mainstream maths course? In what way?

16. You changed to this Course from the mainstream first-year mathematics course. Why do you think you were struggling with the mainstream course? How did you feel about changing to the Foundational Course? Do you think that what you do in Foundational Course is different to what you did in the mainstream maths course? In what way?

Finally, is there anything else you would like to tell me about your educational background?

Thank you for your time. I look forward to working with you on this project.

APPENDIX G

INFORMATION SHEET FOR STUDENTS*

* The names of the courses and the contact person have been removed.

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24 April 2007

Dear (*student's name*)

Invitation to participate in PhD Research project

This is an invitation to you to take part in the research project that we discussed in the meeting on 24 April 2007). I look forward to working with you in the months ahead. I hope that the information in this letter may answer some of the questions you may have about your participation.

This project forms part of my research for my PhD degree in Mathematics Education. The focus of my study is on the use of problems with real-world contexts in the course and I plan to investigate how students on the course solve these problems. The broad aim of my study is to document what practices students use when solving these problems, and to explain why they use these practices. I hope that if I can document these practices, then it may help me (and other teachers) to determine which of these practices are helping students to learn maths and which are perhaps preventing students from performing well.

I have chosen to select three groups of students from your Workshop group – since I am the tutor for this Workshop group, I will be able to manage the data collection sessions more easily. Since I want to document the many different practices students use when solving real-world problems I have selected three groups of students that reflect some of the diversity in the class. For example, these groups include students who went to different schools, students who speak different languages at home, and students who either started the year in the foundation course or in the mainstream mathematics course.

What your participation will involve:

You will be involved in three ways.

What?	Why?	When?
1. **Interview 1:** One 45 minute individual interview. This interview will be arranged at a time that suits you and will be audiotaped.	The aim of this interview is to gather information on the educational background of each student taking part in the study. I will ask you questions about your past experiences of learning maths and about your initial learning experiences at university. This information will help me to explain why you solve the real-world problems in a certain way.	Starting May 2007
2. **Workshop observation:** This will take place during three Monday-afternoon Workshops. The observation will be of how you solve certain real-world problems when working in your group. You will be able to work as you usually do in a workshop, but we will record your group interaction using (a) a video-recorder on a tripod, and (b) an audiotape-recorder on the desk. I will not get involved in any way in what you are doing. I will move from group to group, observing the interaction and writing field-notes on what I see. At the end of the Workshop I will collect your written work and make copies of your solutions.	This observation will be used to gather information on how you solve the real-world problems in the Workshop.	May 2007 (1 workshop, 2 problems) August 2007 (1 workshop, 2 problems) October 2007 (1 workshop, 2 problems)

What?	Why?	When?
3. **Interview 2:** At least one 60 minute interview (individual or in a small group). This interview will be arranged at a time that suits you and it will be audiotaped.	In this interview I will ask you questions about what happened when you were solving the problems in (2). You will be able to watch the video to help you remember what happened. This interview will give me a chance to ask you any questions about what I see in the video and it will give you an opportunity to tell me whether my initial interpretation of how you solved the problems is appropriate.	After one of the sessions in May, August or October (to be advised)

Below I try to answer some of the questions you may have. Please ask any further questions, at any time.

1. **Question: Do I have to participate?**

   Answer: Participation is voluntary. You may choose whether to participate in this study or not. It may help to talk about your participation with your group members, but you must make the decision as an individual. If necessary, talk to me about any concerns you have.

2. **Question: What if I change my mind about participating?**

   Answer: You may withdraw from participation at any time during the research process. Please feel free to chat to me at any time if you are hesitant about participating. Also, if you prefer to not to answer certain interview questions, you may say so.

3. **Question: Will my participation / non-participation in the study affect the marks I get for the course?**

   Answer: Although I am your lecturer and the course convenor of the course, I will be playing the role of **researcher** during the data capturing sessions. As noted above, my aim in this study is not to assess you, but to gather information on **how** you solve the problems. You will have a different tutor to help you with your work during the three Workshops sessions when I collect the data. Furthermore, I will make sure that I do not mark any of your tests during the duration of the study, and will allocate these to a different marker.

4. **Question: What if I have questions about the study?**

   Answer: I will arrange discussion sessions with all participants during the process, but you may ask questions whenever you wish. If you want to talk to someone else about your involvement, you may contact (name) on (telephone number).

5. **Question: Will I have an opportunity to see the results of the study and to comment on these results?**

   Answer: Yes. During the interview after you have solved the problems I will ask you for feedback on my initial analysis. I will also ask for feedback when I am analysing the data in 2008.

6. **Question: Who else will see the data and the results of the study? Will other people know that I participated in the study?**

   Answer: Below are the steps I will take to ensure the anonymity of your participation and the confidentiality of the data.

   (a) All data will be stored securely during the research process, and will then be destroyed when it is no longer required.

   (b) All persons involved in the data collection (eg. the camera persons, the tutor, the transcriber etc.) will be required to commit to ensuring confidentiality of the data.

   (c) Only a few people will view the videotapes. I will view the footage in order to add information to the written transcript of the audiotape made during the Workshop. The students involved in the study will also view selected video clips (see "Interview 2" above). All students who view these clips will be required to commit to ensuring confidentiality of this data.

   (d) The work in his study will be presented to the wider community in the following four ways (in all such reports of the study, your name will be changed to ensure anonymity).

      (i) While busy with the study I will present my initial work to my PhD supervisors, to my fellow PhD students and at academic conferences so that I can get feedback on my progress.

      (ii) The results of the study will be presented in my final PhD dissertation.

      (iii) The results of the study will also be presented to other researchers in the form of academic journal articles.

      (iv) I also plan to present my work in journals for practising teachers.

You will need to complete the following forms before we begin the study:

- *Student consent form: Participation in PhD research project*
- *Student consent form: Data collection and data usage*

Please complete and return the signed forms to me by 12h00 on Monday 30th April.

Feel free to visit me in my office if you have any further questions now, or during the study. I look forward to working with you on this study.

APPENDIX H STUDENT CONSENT FORM

Student consent form: Participation in PhD research project

I, ……., consent to participate in this research project.

I am aware that participation will involve:

(a) The observation of how my group solves selected problems during three Workshops.

(b) One individual interview for gathering background information.

(c) At least one individual or group interview after one of the observation sessions.

Student initial: ……...

I am satisfied that the aims of the study and my role in the study have been explained at the beginning of the project, and that there will be ongoing discussion during the process.

Student initial: ……...

I am aware that I can withdraw from participation in the study at any time during the process.

Student initial: ……...

Signature of Student: …… Date: …….

Signature of Witness: ……. Date: …….

Student consent form: Data collection and data usage

I consent to be interviewed in the Individual Interview (Interview 1) and for this interview to be audiotaped.

Student initial: ……...

I consent to be interviewed in the Individual/Small Group Interview after the observation sessions (Interview 2) and for this interview to be audiotaped.

Student initial: ……...

I consent to the audiotaping of my work when I solve selected real-world problems during three Workshop sessions.

Student initial: ……...

I consent to the videotaping of my work when I solve selected real-world problems during three Workshop sessions.

Student initial: ……...

I consent to selected video clips of the group problem-solving being used in individual or small group interviews (Interview 2).

Student initial: ……...

I undertake to keep all information discussed and viewed on the video clips during Interview 2 confidential.

Student initial: ……...

I am aware that the initial and final results of the study will be presented (anonymously) as part of the researcher's studies, at academic conferences, in journal articles and in the PhD dissertation. I consent to the results being used in this way.

Student initial: ……...

Signature of Student: …… Date: …….

Signature of Witness: ……. Date: …….

APPENDIX I

INFORMATION SHEET FOR THE TUTOR*

* The name of the course has been removed.

26th April 2007

Dear (*tutor's name*)

Invitation to act as tutor during PhD Research project

This is an invitation to you to play the role of tutor in the research project that we discussed in the meeting on 26th April 2007. I look forward to working with you in the Course Workshops in the months ahead. I hope that the information in this letter may answer some of the questions you may have about your participation.

This project forms part of my research for my PhD degree in Mathematics Education. The focus of my study is on the use of problems with real-world contexts in the course and I plan to investigate how students doing the course solve these problems. The broad aim of my study is to document what practices students use when solving these problems, and to explain their use of these practices. I hope that if I can document these practices, then it may help me (and other teachers) to determine which of these practices are helping the students to learn mathematics and which are perhaps preventing them from performing well.

The three groups of students participating in the study will be chosen from the Workshop group for which I am the tutor. I will collect the data in three Workshop sessions during the year: one each in May, August and October. In these sessions the interaction between each group of participants will be audiotaped and videotaped. I will not get involved in any way in the interaction, but will move from group to group observing and writing field-notes on what I see. If I am involved in the data collection process during these sessions, then I cannot act as tutor either to the students involved in the study or to the non-participating students in the Workshop group. Since you have experience tutoring students on this course, I am inviting you to fill my role as tutor during these data collection sessions.

What your participation will involve:

Your role will be as the tutor during each of the three Workshop sessions during which data will be collected. That means that you will tutor **all** the groups of students in the Workshop group – both those who are taking part in the study as well as those who are not taking part. I would also like you to co-tutor one Workshop session prior to each of the three data collection sessions with me, so that the students can get accustomed to your presence as a tutor.

Although the focus of this study is the students in the course, and not the tutor, your interaction with the student participants will also form part of the data used in the study. For example, one of the ways that students solve the problems is by asking the tutor for assistance I am interested in what questions the student ask the tutor, what type of help s/he gives the students, and how the student responds to this assistance.

Below I try to answer some of the questions you may have. Please ask any further questions, at any time.

1. **Question: Do I have to participate?**

   Answer: Participation is voluntary. You may choose whether to participate in this study or not. If necessary, talk to me about any concerns you have.

2. **Question: What if I want to withdraw my participation once the study has started?**

   Answer: You may withdraw at any time from participation in the process. Please feel free to chat to me at any time if you are hesitant about participating or about continuing your participation.

3. **Question: Will my participation / non-participation in the study affect my role as tutor in the Department?**

   Answer: My invitation to you to participate is independent of the process of appointing tutors in the Department, and I have been given the go-ahead from the Tutor Committee to approach you.

4. **Question: What if I have questions about the study?**

   Answer: Please ask questions whenever you wish.

5. **Question: Will I have an opportunity to see the results of the study and to comment on these results?**

   Answer: You will have an opportunity to see my analysis of any data involving your interactions with the students.

6. **Question: Who else will see the data and the results of the study? Will other people know that I participated in the study?**

   Answer: Below are the steps I will take to ensure the anonymity and confidentiality of your participation.

   (a) All data will be stored securely during the research process, and will then be destroyed when it is no longer required.

   (b) All persons involved in the data collection (eg. the camera persons, the transcriber etc.) will be required to commit to ensuring confidentiality of the data.

   (c) Only a few people will view the videotapes. I will view the videotapes in order to add information to the written transcript of the audiotape made during the Workshop. The students involved in the study will also view selected video clips during an interview about the problem-solving process. All students who view these clips will be required to commit to ensuring confidentiality of this data.

   (d) The work in his study will be presented to the wider community in the following four ways (in all such reports of the study, your name will be changed to ensure anonymity).

      (i) While busy with the study I will present my initial work to my PhD supervisors, to my fellow PhD students and at academic conferences so that I can get feedback on my progress.

      (ii) The results of the study will be presented in my final PhD dissertation.

      (iii) The results of the study will also be presented to other researchers in the form of academic journal articles.

      (iv) I also plan to present my work in journals for practising teachers.

You will need to complete the following forms before we begin the study:

- *Tutor consent form: Participation in PhD research project*
- *Tutor consent form: Data collection and data usage*

Please complete and return the signed forms to me by Monday 30th April.

Feel free to visit me in my office if you have any further questions now, or during the study. I look forward to working with you on this study.

APPENDIX J TUTOR CONSENT FORM

Tutor consent form: Participation in PhD research project

I, ……., consent to participate in this research project.

I am aware that participation will involve:
(a) Tutoring in each of the three Workshops when data is collected for the research project.
(b) Tutoring in each of the Workshops prior to the data collection sessions.

Tutor initial: ……...

I am satisfied that the aims of the study and my role in the study have been explained at the beginning of the project and that there will be ongoing discussion during the process.

Tutor initial: ……...

I am aware that I can withdraw from participation in the study at any time during the process. **Tutor initial:** ……...

Signature of Tutor: …… Date: …….

Signature of Witness: ……..…….. Date: …….

Tutor consent form: Data collection and data usage

I consent to the audiotaping of my interaction with the participating students during the Workshop sessions.

Tutor initial: ……...

I consent to the videotaping of my interaction with the participating students during the Workshop sessions.

Tutor initial: ……...

I undertake to keep information on my interaction with the participating students confidential.

Tutor initial: ……...

I consent to selected video clips of my interaction with the students being used in interviews with the student participants.

Tutor initial: ……...

I am aware that the initial and final results of the study will be presented (anonymously) as part of the researcher's studies, at academic conferences, in journal articles and in the PhD dissertation. I consent to the results being used in this way.

Tutor initial: ……...

Signature of Tutor: …….. Date: ……

Signature of Witness: …….. Date: ……

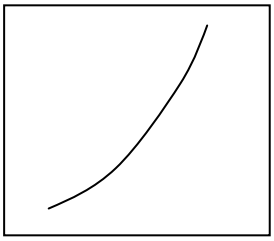
APPENDIX K

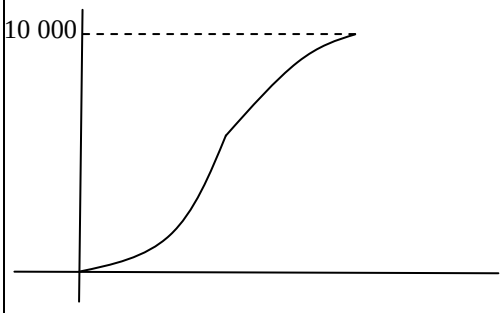
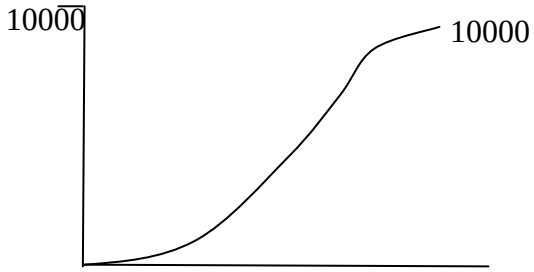
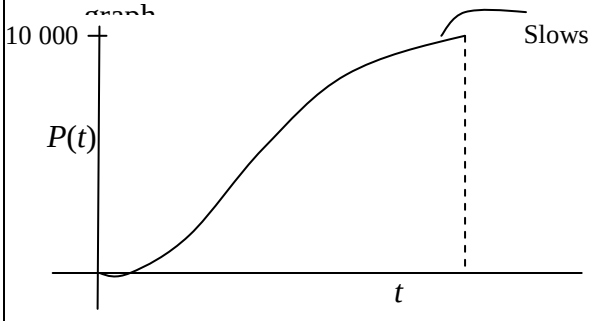
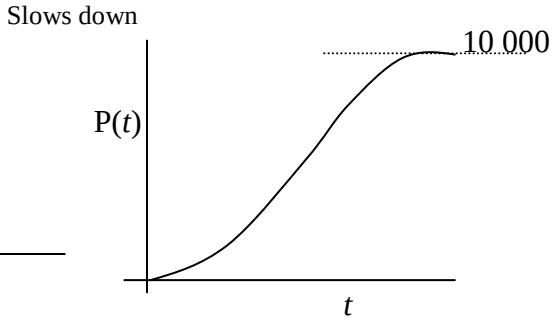
GROUP 1: ACTION ON THE FLU VIRUS PROBLEM

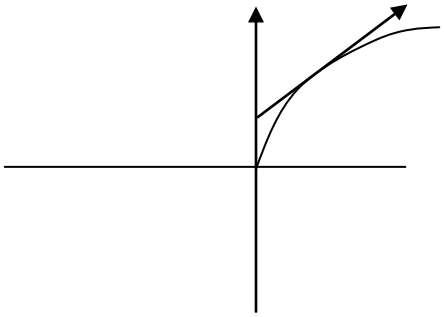
A flu virus has hit a community of 10 000 people. Once a person has had the flu he or she becomes immune to the disease and does not get it again. Sooner or later everybody in the community catches the flu. Let $P(t)$ denote the number of people who have, or have had, the disease t days after the first case of flu was recorded.

(a) Draw a rough sketch of the graph of P as a function of t, clearly showing the maximum number of people who get infected, and **do not continue until you have had your graph checked by a tutor.**

Episode	Description
1 (1-33)	Shae attends to the value of 10 000 in the problem text and is concerned that they do not have any values for drawing the graph. Hanah pronounces that it is going to be a "straight line", thus identifying the required graph with a known, named graph. Jane agrees and names the straight line graph as "just a line". The students are attending to the gradient of the graph; Some of the students (Lulama, Shae) use the word "gradient", while Hanah demonstrates positive and negative gradients with her hand in the air. Shae then proposes they draw an increasing straight line graph, which the others seem happy with for the moment. Jeff has proposed that the graph will "climb" and then "drop", but his pronouncement about a decrease is not attended to by the other students. In this interaction both Hanah and Shae use their hands in the air to demonstrate the graph. Lulama seems to want to draw on the task context to explain, but his sentences are incomplete and are not attended to by the other students. The students use the pronoun "it" interchangeably to reference the graph, the gradient of the graph, and the problem text, yet seem to understand one another.
2 (34-52)	Jane is questioning whether the graph is "a line" (by this she seems to mean a straight line). Both Lulama and Shae recruit the task context to explain; Lulama argues that sooner or later everyone catches the flu and Jeff uses this argument to agree that the graph must be increasing (they only attend to why the graph might be increasing, and not to why it might be a straight line). But Jane is attending to the rate of change when she argues (without drawing on the task context) that this does not mean the graph is increasing at a "constant rate"; the graph could be "curvey" (she demonstrates an increasing concave up graph in the air).

	Shae dismisses this idea; he thinks the nature of the increase has not been made explicit in the text ("they don't specify that… so we just have to put a general ...increase"). Hanah reminds the students that they have to show the "maximum" (she has been reading the problem text), and Shae and Jeff refer appropriately to the set-up to explain why this maximum value is 10 000. Hanah sums up their current answer in her question to the group, "we just draw a straight line and say like a 1 000?" 
3 (53-62)	Then Jeff refers back to Jane's suggestion in Episode 2 about the "curvey" graph and pronounces that it could be "exponential", thus identifying the required graph with a known, named graph. Both he and Shae use the task context to explain that the number of people being infected will increase, using their hands in the air (Jeff traces a concave up, increasing graph). It appears that these two students are considering what is happening as time passes. They are not considering that the rate will decrease towards the end. Lulama introduces the idea of immunization, but this is not followed up. The students give one another positive and negative feedback and their explanations recruit the task context. At this stage they do not seem to have drawn any graphs, but have only traced graphs in the air.
4 (63-82)	Jane begins by suggesting they draw the graph, and then get it checked (attending to the instruction in the problem text). But Jeff returns to his earlier comment in Episode 1 that the graph could "drop". Jane, Hanah and Shae give negative feedback by saying "but" and explaining ("because") by recruiting the task context; they use the wording of the text to argue that the number of people includes those people who have had the flu (Shae links this explicitly to the function notation $P(t)$). Lulama seems to want to add something related to the task context, but once again is not given a voice. At this stage the students have not drawn a graph, only having sketched the axes for the graph.
5 (83-104)	Jane makes the suggestion again that they "draw it". But they all seem hesitant to draw anything (despite the fact they trace the shape of the graph in the air, quite readily). Jeff says he is "not sure" and seems to want to check with the Tutor first (so does Jane). Hanah demonstrates an increasing concave up graph. But Shae says he is going to go ahead and draws an increasing, concave up graph. His action seems to spur the others on to start drawing sketch graphs, with the exception for Lulama who is not drawing, but asks whether they are agreed that "it is an exponential". At this stage the students do not attend to the maximum value of 10 000 on the graph.
6 (105-133)	The students attend to the instruction in the problem text to have "your graph checked by a tutor", and do not want to move on. The Tutor is busy with another group of students, and these six students are frustrated at having to wait.
7 (134-142)	When the Tutor arrives, Jeff poses a question. He begins by attending to the need to include the maximum number of the people, and draws on the task context to explain why this value will be 10 000 (there is no indication so far how the students have dealt

	with this on their increasing, concave up graphs). He then refers to the "shape" of the graph and asks whether this will be an "exponential graph" (explaining this with reference to the growth of the population). The Tutor gives positive feedback, showing that he values Jeff's "reasoning" and indicating that the graph "starts off being exponential". This pronouncement by the Tutor sparks Jeff to consider what might happen after some time has passed and he returns to his earlier argument that the graph will "drop" (his use of the task context is not appropriate as he is not using the fact that $P(t)$ includes the number of people who have or have had the disease). Lulama tries to introduce something about people being "cured", but his comment is not attended to.
8 (143-166)	The Tutor does not attend to Jeff's problematic use of the task context in his reasoning, but suggests a way of "thinking"; he refers to "rate of number of people being infected" and considers what will be happening after the 9 999th person has the disease. He then links this to the rate by talking about "it" slowing down towards the end (he and the students seem to interchange the use of the reference pronoun "it" for the graph and for the rate of change of the disease). When talking the Tutor links the task context to his tracing of the graph in the air. The Tutor explains the "slowing down" by referring to the limit on the number of people in the community. The students respond by describing the shape of the required graph in words (Shae) and drawing it. It appears that the students attend to the shape of the graph first, and then include the value of 10 000 as the maximum. Jane, Hanah, Shae and Jeff represent the "slowing down" referred to by the Tutor in their graphs. Lulama does not update his original graph following the discussion with the Tutor.  Jane's sketch graph  Jeff's sketch graph  Hanah's sketch graph  Shae's sketch graph

	 Lulama's sketch graph

(b) What are the units of $P(t)$?

Episode	Description
1 (134a, 167-179	Both Shae and Jeff read the problem aloud; they do not pay attention to saying the symbol "*P* prime" correctly in words but it seems that they are attending to the derivative. Jeff is the first to make a claim public; he tries a few possibilities ("people per hour", "people versus time", "*t* is days") which suggest that he is attending to the derivative but is making different pronouncements about the variable *t* and its units. Hanah attends first to the meaning of the derivative and the task context (as often valued in the Course): "It is the rate at which people get infected". Lulama attends to both the meaning of the derivative and the units, "People per time ... rate of change".
2 (180-189)	Jane's action moves the students beyond the description of the derivative when she attends to the need for "units" in question (b); "But what are the units?" Hanah is enabled to pronounce immediately, "people per day", thus attending to the units, and suggesting that she associates the word "per" with the units of the derivative. This gets positive feedback in the form of "Ja" and repetition by Jeff. Lulama revisits the problem text to confirm that time is in days (but his written answer refers to "people per time"). Shae's written answer also refers to "people/time", suggesting that the students are not attending to the difference between the meaning of the variables in the task context and to the units of these variables.

(c) What does $P(4) = 1\ 200$ mean in practical terms? (Your explanation should make sense to somebody who does not know any mathematics.)

Episode	Description
1 (190-214)	Shae takes the lead by reading the question aloud; he pronounces "12 000" instead of "1 200", but Jane and Lulama just reword this to 1 200 in their attempts that follow. Jane makes an initial attempt; she pronounces just the answer, "After 4 days … it will be 1 200". Lulama builds on this by adding "people" to the 1 200. Both Jane and Lulama use the reference pronoun "it", without pronouncing anything about the "number of people". Yet the others give positive feedback to Jane in the form of repetition and by proceeding to write an answer. In pronouncing their answers the students attend to the use of the word "after" in the Course discourse, the numbers 4 and 1 200 in the expression and the meaning of these values in the task context. The rest of the Episode involves Shae making a joke about having to explain to someone who does not know any mathematics, thus attending to the instruction in question (c). This is taken as a joke by all the other students, they clearly know what type of description is required by the Course. While this joke is made verbally, the students are writing. The written answers of Jane, Hanah, Shae and Jeff take into account the meaning of the function $P(t)$ given in the task context, for example, Jane writes, "after 4 days 1200 people will have it or have had it" and Hanah writes, "After 4 days, 1200 people have been infected by the flu virus". But Lulama's answer of "will be infected" and one of the options provided by Shae ("are infected") does not take this into account (they are not explicitly attending to the tense in their answers).

(d) What does $\frac{P(7)-P(4)}{7-4}=350$ mean in practical terms? Give the correct units.

Episode	Description
1 (215-216)	Lulama views the expression in (d) as an object when he begins by identifying "this" (the expression in (d)?) as "a rate of change".
2 (217-221)	Although she gives Lulama positive feedback about the "rate of change" (with "Ja"), Hanah still talks about "350 people are infected" as if she is attending to the function $P(t)$ on the numerator of the expression $\frac{P(7)-P(4)}{7-4}$, and not viewing the expression as an object representing a rate of change. The students agree on her use of "from 4 to 7 days" (attending to the denominator of the expression). They give one another positive feedback through "Ja" and by repetition of the phrase in subsequent action.

3 (222-226)	Jeff then makes a key contribution when he identifies "that" (meaning the whole expression in (d)?) as an "average", but he does not say an "average" of what. Jane and Hanah attend to Jeff's introduction of the term "average" to build their answers; Jane references the "average people who will be infected", but it does not seem that she is attending to the rate of change, but rather just adding the word "average". Hanah is attending to the rate of change when she identifies the units as "per day".
4 (227-234)	Lulama identifies that the students are not being explicit about their use of the word "average" when he asks whether they are talking about "an average" or "an average rate of change". He seems to be attending to his earlier argument that the expression in (d) represents a rate of change, but it is not clear from his pronouncement what the other "average" might be, possibly the average number of people that the others are pronouncing. In their response, "average rate of change", Shae and Jeff seem confident and carry on writing. Jane tries again to word her answer out loud, not using the rate of change but using everyday terms; "the ... average ... of the people who get infected." Here she does not attend to the fact that this should be "per day", so it is not clear whether she is explaining the average rate of change appropriately. Lulama indicates that something about her use of "average" is incorrect, but this is not explained.
5 (235-251)	Darren arrives late; he introduces himself to the students he does not know. Lulama, Hanah, Shae and Jeff are moving on to question (e). Hanah and Jeff 's written answers refer to the average rate of change without using the term "rate", for example Hanah writes, "From 4 to 7 days the average number of people infected per day are 350 people". Both Lulama and Shae use the term "average rate of change", but only Lulama's answer suggests that he can also use "practical terms"; "The average rate of change between 4 - 7 days the, will be 350 people infected per day"
6 (272-273)	Lulama, Hanah, Shae and Jeff have moved on to question (e), but Jane is still finishing writing her answer for (d). She queries her use of units with Jeff, asking whether it is "350 people per day". It is not clear at this stage how she has incorporated the "rate of change" and the "average" into her earlier writing (see written answer given below), but her statement of the units suggests that she is attending to the "rate of change" discussed earlier. Jeff gives positive feedback by repeating the "350 people per day" part, but adds "average of 350 people …". Jane's written answer suggests that she is just adding "average" and "per day" and is not attending to the daily change; "The average people have it or have had it between day 4 and 7 is 350 people per day".

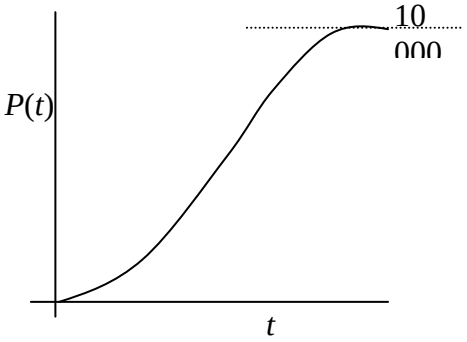
(e) What does $P'(4) = 400$ mean in practical terms? Explain why $P'(t)$ can never be negative.

Episode	Description
1 (252-265)	When reading the problem text Shae makes explicit mention of the derivative as "*P* dash". Again Jeff is the first student to make an answer public. He is attending to the derivative when he uses the words "growth" and "rate of infection" interchangeably (also linking mathematical terms to the task context). He is also attending to the difference between the expressions in (d) and (e) when he changes from "average" growth to "instantaneous" rate of infection. (Here he successfully incorporates the "average" into the rate of change by using "average growth", something he did differently in (d).)
2 (266-311)	Jeff and Jane attend to the task context (Shae claims, "there is always someone to be infected", and "there aren't people being cured"; Jane revisits her earlier argument about $P(t)$ being "cumulative" and pronounces that $P(t)$ means "They are counting the number of people who have had the flu as well"). Although they do not make the argument explicit, it seems that they are arguing that, since $P(t)$ is always increasing, $P'(t)$ is positive. Shae also draws on the context ("you can't have minus people being infected"), but he makes the argument that $P(t)$ cannot be negative. The students do not give negative feedback to the number of times he makes a comment of this type. He eventually resorts to "Well it just, it just can't happen ... you just can't have". It seems that Lulama may be attending to the increasing/decreasing nature of the graph when his wording and gesture suggests that he is referring to the gradient: "But it can be negative, to mean that it is decreasing". But he gets negative feedback (which draws on the meaning of $P(t)$ in the task context) from both Jane and Shae.
3 (312-329)	Shae and Jeff have moved on to questions (f) and (g). (Shae uses the word "rate" in his written answer, "It is the infection rate (400) at 4 days and can't be negative because you can't have a negative amount of people getting infected". Jeff also uses the word "rate"; "after 4 days the infection rate was 400 people per day. $P(t)$ cannot be negative there are always people that can be infected and $P(t)$ does not represent the number of people being cured." Jane, Darren and Hanah (and possibly Lulama) are still completing the first part of question (e). Both Hanah and Jane appeal to Jeff for feedback on their answers. Hanah attends to the time ("day 4"), and the derivative ("rate of change of the number of people getting infected"), but she does not attend to the units for 400 ("people per day") and does not attend to the requirement to use "practical terms" in the task context. Jeff gives feedback by rewording her attempt, attending to the time (at that point), the "instantaneous rate of infection" and the units "per day". Neither of these students attend to their use of the word "rate" interspersed with the everyday language. A few lines later Jane also asks Jeff for clarification by attending to the units "400 people per day". Again, Jeff attends to the particular time, but this time tags the instantaneous on at the end only, as if just as an explanation (as earlier he corrects himself from "average" to "instantaneous").

	Jeff's feedback has been in the form of repeating the girls' statements and adding to or rewording these statements. In both cases they use his responses and continue writing. Jane settles on the following written answer, "The ~~instantaneous infection rate~~ at day 4 is 400 people per day got infected. No it could never be negative because it is cumulative because you are counting the people who have it." Hanah writes, "At day 4 the rate of change / growth rate / instantaneous rate of change of the number of people getting infected is 400 people per day".
4 (330-331; 339-351)	Darren identifies the work with what they did in class that day and pages back in his answer book to find a population example. He attends to the time "at day 4", but includes the phrase "on average", suggesting that he is not identifying the derivative $P'(t)$ with the instantaneous rate of change emphasized by Jeff. His written answer also refers to "average"; "this means that on average per day 400 people were infected". To both Darren's responses that include the word "on average", Lulama responds with a positive, content-free feedback, "Ja". (Lulama's response for question (e) also includes the word "average"; "after 4 day the average rate of will be 400 people per day will be infected".
5 (332-338)	The Tutor arrives and asks, "How is it going here are you↑... finding it alright now↑" Based on the students' replies of "we are going fine", "yes" etc., he says, "good" and leaves the group.

(f) What is $\lim_{t\to\infty} P(t)$? Give a short reason for your answer.

Episode	Description
1 (322-334)	Jeff and Shae have started working on question (f). Jeff takes the lead and pronounces an answer of "10 000", suggesting that he is attending to the maximum value in the problem text and the function $P(t)$ in the limit expression. He gives no verbal explanation. He gets positive, content-free feedback from Shae, writes down the value and a written explanation of "it is the max number of people in the community", and continues with (g). Shae has a similar explanation (which is also not pronounced verbally); "10 000 cause maximum amount of people in town".
… (335-409)	Action on other questions.
2 (410-425)	Jane revisits question (f); she is attending to and asking about the reason for the answer. Jeff starts to explain question (g), but is corrected by Shae. Both Jeff and Shae give her the answer of "10 000", she gives positive feedback by nodding her head and "I know", but is actually interested in the reason. She begins by saying "it can't exceed" suggesting that the graph cannot go above 10 000. Jeff recruits the task context to explain that "it" (10 000) is the "maximum amount of people in the community". Jane seems satisfied with this and continues writing. (It appears that she writes, "10 000" and an explanation as an answer, but this is changed to "0" later, after the later discussion about (g). Her reason for her answer of 0 is, "Because at infinity 10 000 people had already been infected so the growth rate is 0". (She may be confusing the labeling of her questions and she intends this to be her answer for question (g)).

3 (426-443)	Darren is attending to the limit expression $\lim_{t\to\infty} P'(t)$ in question (g) for the first time (in spite of his contribution to the earlier discussion, see question (g), Episode 5). Initially, he pronounces that the limit expressions in questions (f) and (g) are the same, but then attends to the derivative in the symbolic expression $P'(t)$ in (g) and notices his error. Again, there is some confusion about which answer they are talking about; Jeff starts to explain question (g) again, but Shae draws his attention back to question (f). Shae claims that the answer for question (f) is 10 000 because 10 000 is the maximum value for "it" (referencing either the graph or the number of people infected).
4 (444-451)	Shae then attends to the graph that he drew for question (a). This is the first time they have attended to this graph for questions (f) and (g). He shows the maximum of 10 000 that is written on his graph (a light horizontal line is drawn across the top of his graph). Jeff links the task context and the graph and uses the word "exceed" (possibly drawing on Jane's use of the word in Episode 2). Darren also attends to the graph in (a) by tracing the shape of the "s-shaped" graph in the air. 
5 (452-456)	Darren then raises an issue that has not been discussed explicitly before when he asks whether the maximum value is equal to the limit. Hanah, Shae and Jeff all give positive feedback, "Ja", and Jeff explains by drawing on the task context. He argues that the graph/number of people ("it") "has to go up to 10 000".
6 (457-461)	But in the next line Jane pronounces that "it" is "tending towards 10 000". Shae gives negative feedback by pronouncing an alternative argument which supports that of Jeff, "It actually gets there, it doesn't tend". Although Jeff and Jane agree with Shae, Shae then contradicts himself, a verbal pronouncement that is attended to, but not followed up.
Refer to more discussion of (f) below under discussion of (g), Episode 7 onwards.	

(g) What is $\lim_{t\to\infty} P'(t)$? Give a reason for your answer.

Episode	Description
1 (342, 345-347)	Shae makes a claim public that the answer for question (g) will be the same as the answer for question (f), that is 10 000. He is attending to the maximum value of 10 000 in the task context and to the derivative $P'(t)$ in the limit expression in question (g) when he refers to both the task context and the rate in his explanation; "Cause it could be 10 000 people that catch it per day". His additional explanation, "that would be the maximum amount" suggests that he is identifying the required limit with the maximum rate of infection per day.

2 (348-359)	Both Shae and Jeff then visualize what this might look like graphically (they do not attend to the graph they drew for question (a)). Although they do not make it explicit here, it emerges later in the discussion with the Tutor that Shae considers the graph here to be the graph of the derivative function. Both students attend to a particular time (one day?, although this is not named and they do not draw on the task context to explain, other than to use the values 0 and 10 000, they seem to be attending to the instantaneous rate of change). Shae proposes "a dot", and Jeff proposes a vertical line (he demonstrates with his hand) that possibly goes from 0 to 10 000 on the vertical axis. They both pronounce Jeff's graph to be "straight line". Jeff is not explicit about whether he identifies his vertical straight line graph as the graph of the function or as the graph of the derivative function. He pronounces that "it" (possibly the derivative / gradient?) "will tend towards infinity". Jeff does not seem to attend to $t \to \infty$ in the limit expression.
3 (360-364)	Darren attends to Jeff's comment that "it will tend towards infinity↑" and possibly the vertical straight line that he has traced in the air. Darren pronounces that it is not possible for a derivative to be infinity. He does not draw on the task context. Shae repeats his earlier argument about the maximum possible derivative being 10 000.
4 (365-366)	Lulama tries to enter the conversation to say something about infinity, but this is not attended to. Hanah also tries to make a contribution by referring to the days in the task context, but also, this is not attended to by the others. It may be that she is attending to the symbols $t \to \infty$ in the limit expression.
5 (367-381)	Darren then gives negative feedback to Jeff by using a mathematical argument and attending to what appears to be discussion about the derivative in lectures that day; he pronounces that "it is not possible" to find the gradient that is too greatly positive or too greatly negative (he demonstrates with his hand in the air). It seems that he identifies the derivative function $P'(t)$ in the limit expression in question (g) but attends to the gradient tending to infinity, rather than t tending to infinity? It is possible that he is attending to the "cusp" that was discussed in lectures (this is supported when Jeff demonstrates something in the air and tries to give it a name, Darren refers to it as a "tangent" which the other seem happy with, nodding heads and saying, "Ja"). Others add non-content positive feedback, repeating "it is not possible".
6 (382-415)	Shae pronounces that "it does not exist"; his use of the pronoun "it" makes it unclear whether he is attending to the limit expression $\lim_{t\to\infty} P'(t)$ or the derivative function $P'(t)$ only. Darren appears to attend to the derivative when he says "they are not differentiable". He uses "like 90 degrees" to describe the vertical tangents and argues, like Jeff, that "it's tending towards infinity (possibly the tangents / gradients?). Jane presents an alternative by attending to other tangents on the graph where the derivative would exist, but Darren argues that they are only interested in the point where "it will tend to infinity", possibly using "it" for gradients /derivative. Lulama attends to the requirement in the question to provide a reason for the answer, and Jane uses a mathematical argument on which she wants feedback; "the tangent would be vertical". Darren adds to this by arguing that the graph is not "differentiable at that point". (Although Darren has had considerable input from a mathematical

	perspective into this discussion, it seems that he has not attended to the limit expression $\lim_{t\to\infty} P'(t)$ in question (g) at all. He only does this in the next Episode. It seems that in making this contribution here, he was only attending to Shae's earlier discussion about "infinity" and 'gradient" and Shae's tracing of a vertical line in the air.) Shae concludes that the graph is not "differentiable"; he is attending to his vertical line graph here. Lulama provides positive feedback, with an inappropriate mathematical argument, "infinity is not a number". Jeff replaces the term "differentiable" with the term "diffable" which is used as a synonym for "differentiable" in the Course.
7 (462-479)	The Tutor comes to the table when the students are working on the next question, and refers them back to their answer for the Flu Virus Problem. He looks first at question (f) in Shae's answer book and pronounces the answer, giving the meaning using a mixture of the task context and mathematical terms, "the number of people that have been infected (unclear) infinity is ten thousand". He then notices that "two" students have the answer of "not defined" for question (g), "saying that...*p* dash...*t* as *t* tends to infinity is...not defined or [does not exist]". In this pronouncement he gives the meaning of the symbols in the limit expression in words. He asks whether the others have that as an answer and they all nod. Jeff starts to explain, and the Tutor asks about their "reasoning", showing again that he values their reasoning about the answers. Jeff starts to explain; he is attending to the vertical straight line "it" that he demonstrated earlier when he indicates that "the graph ... it is s ... such a steep graph that it's tending more towards infinity ... than ...".
8 (480-483)	The Tutor asks to see the graph (he is attending to the graph for question (a) here), but Jeff is attending to the vertical line graph he demonstrated with his hand only (the students did not consider the graph in question (a) in relation to answering question (g) earlier). Shae refers to the graph of question (a) that the Tutor is attending to and says "but that's of, that is not of the dash". It seems that, certainly for Shae, he was seeing the vertical line graph as the graph of the derivative function of $P'(t)$.
9 (484-485b)	The Tutor begins by explaining the meaning of the graph in question (a) in Shae's book using the task context, and then asks for an explanation of the "steep" graph pronounced by Jeff and gives negative feedback by saying, "I don't understand". He asks, "Where do you get this thing that it is getting so steep ((*Showing a steep gradient with his hand*)) as, as *t* tends to infinity?"
10 (486-487)	Jeff starts to respond to the Tutor's challenge in Episode 9 and then pauses, attending to the phrase "*t* tends to infinity" in the Tutor's pronouncement; it seems that Jeff has not been attending to the symbols $t \to \infty$ in the limit expression in question (g) (he argues "I'm thinking of the wrong thing").
11 (488-502)	While Jeff pauses, Shae picks up his Resource Book and points to something for the Tutor; in a roundabout way he is attending to the derivative function $P'(t)$ and pronouncing that it is the "increase" "over time". Although Shae has not paid much attention to what Jeff said in Episode 10, it seems that he, too, is attending to the symbols $t \to \infty$ for the first time, too (when he says "over time"). In the meantime Jane asks the Tutor if they are wrong, and he confirms this in a joking

	way. The Tutor then returns to focus on Shae and asks him to explain further. Shae makes pronouncements about the amount of people "over time" and refers to the "increase… per day", suggesting that he is attending to the meaning of the derivative. The Tutor rewords this by referring to "the <u>difference</u> in the number of people infected" and rewording Shae's use of "over time" to "per day". Shae makes a pronouncement, "when time increases to infinity", suggesting again that he is attending to the symbols $t \to \infty$ in the limit expression, and then gives the meaning in the task context, "the number of people <u>over time</u> and as time increases". It seems that he is attending to the meaning of the derivative in the task context, as time passes, but is not yet attending to the limit of the derivative as required in question (g).
12 (503-509)	Jeff appears to have been thinking while Shae and the Tutor were talking in Episode 11. He then makes a pronouncement about the limit in question (g), saying that it should be "nought". Shae attends to this claim and asks for clarification by repeating "nought↑" and the Tutor asks Jeff to explain. In his explanation he attends to the concave down part of the graph ("it"?)(he traces the shape with his hand in the air) and indicates that "as it gets ... it it gets to 10 000, then it will just stay constant" . The pronouncement "as it gets … it gets to 10 000" suggests that he is attending to what happens over time. The Tutor gives positive feedback, "Okay, ja, that's kind of what I'm looking for". Again, Jeff says, "I was thinking of the wrong thing".
13 (510-521)	The Tutor works with Shae on the meaning of the derivative, as he attends to the wording for question (e); in doing this he links the symbols for the derivative and their mathematical meaning. But before Shae responds, Hanah provides an answer for question (g) (she also gives the mathematical meaning of the symbols in words), "the rate of change at infinity is zero". Her naming of the expression suggests that she is attending to the object as a whole, and is possibly also drawing on Jeff's earlier attempt in Episode 12. To explain this she recruits the task context, arguing that "it" (the number of people who have or have had the disease/$P(t)$?) "already has everything" and suggesting that the spread of the disease is complete. The Tutor attends to her argument, developing it in terms of the task context, by saying that no new people can be infected (he emphasizes the rate in a repetition when he says "per day").
14 (522-532)	The Tutor gets up to walk away but returns to look at Shae's answer for question (e), wanting to check whether he is "thinking about it … in the right way". He gives positive feedback on Shae's answer for question (e). He asks the others whether they are okay with (g), and Jane asks "But how do you put that in words?". In answering her question he asks about her wording for question (e), which she begins as "instantaneous infection rate".
15 (533-540)	The Tutor attends to her use of "instantaneous" by pronouncing that what she says is too "mathsy"; he attends to the instruction to use "practical terms" in the problem text and tells them not to use words like "instantaneous" and "velocity". Jane tries again using "infection rate at day 4". The Tutor gives tentative positive feedback ("okay") and shakes his head slightly. Lulama then adds a phrase, "it's the rate of change", a pronouncement which is typical of his attempts to make short contributions to the ongoing discussion. But the Tutor also classifies this as "too mathsy" and gives the

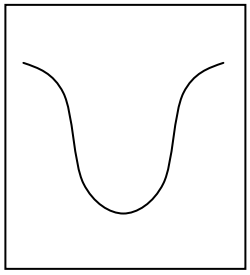
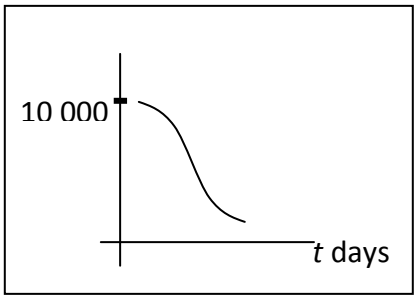
	students a suggestion of where to start, "at 4 days", thus drawing on the Course discourse.
16 (541-546)	Hanah then enters the conversation, but it seems as if she is trying to explain question (g) in everyday words (the "after" confuses her as she rewords her earlier attempt from "at infinity" to "after infinity"). Jeff attempts to reword "at infinity" in everyday terms when he pronounces "until forever", which all the other students find amusing.
17 (547-564)	The Tutor gives positive feedback to Hanah in Episode 16 but emphasizes that he wants to focus on question (e) (it seems he wants to use this to assist the students with question (g)). Darren has been speaking quietly to himself; while the Tutor is talking he provides an answer for question (e) to Jeff; "the average per day is 400 people infected↑". He is attending to his written answer. Jeff gives positive feedback "that sounds better, ja", but does not give negative feedback on the use of "average". The Tutor then pronounces a full alternative that does not make use of the word "instantaneous; "let's say after 4 days...uhm...<the rate of infection is 400 per...day>", but he still uses "rate of infection". Shae then rewords this for Jane, without using the word rate; "It means that on that <u>day</u>, 400 people <u>will</u> get <u>infected</u>". The Tutor attends to this answer, when he repeats this a few lines later, "After 4 days...400 people got infected on that ... day". In the meantime Darren and Shae are exclaiming and joking about the number of people getting the flu.
18 (565-574)	Jane suggests that the Tutor's pronouncement is "not true" because "it's" (the derivative or the 400?) is "just average", suggesting that she has not identified the object of the derivative as the instantaneous rate of change. The Tutor responds by emphasizing through his tone and repetition that it is the rate of infection for "<u>that</u> day". Jane then queries again whether this is an "exact" amount (possibly linking "exact" with instantaneous) or an average rate of change. Jane nods agreement with the Tutor, but she does not change her answer for question (g) after the discussion with the Tutor, and leaves it at "DNE because the tangent to that point is vertical therefore not differentiable". She gives the correct written answer for question (g) under question (f).

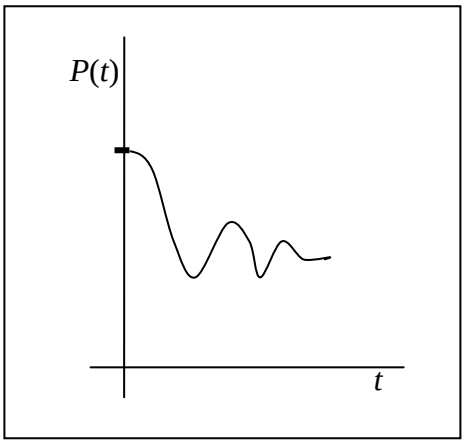
APPENDIX L

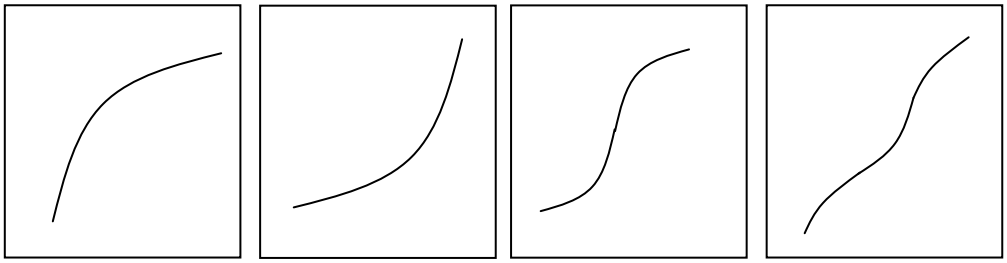
GROUP 2: ACTION ON THE FLU VIRUS PROBLEM

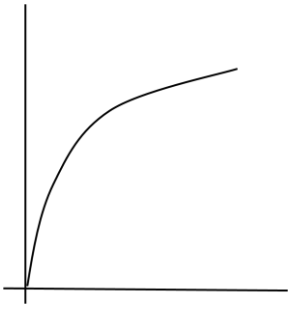
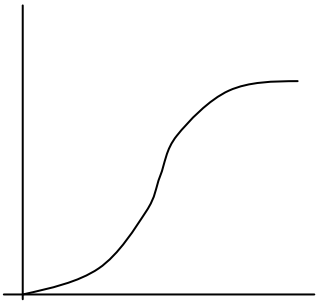
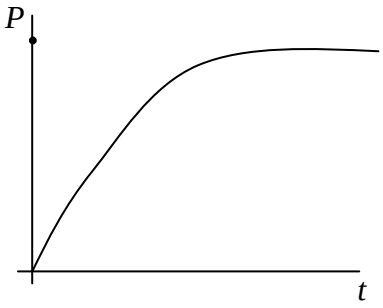
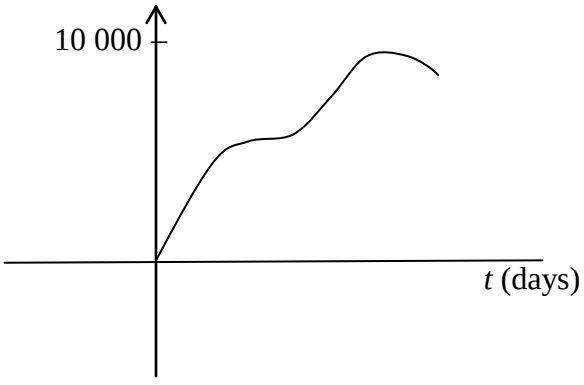
A flu virus has hit a community of 10 000 people. Once a person has had the flu he or she becomes immune to the disease and does not get it again. Sooner or later everybody in the community catches the flu. Let $P(t)$ denote the number of people who have, or have had, the disease t days after the first case of flu was recorded.

(a) Draw a rough sketch of the graph of P as a function of t, clearly showing the maximum number of people who get infected, and **do not continue until you have had your graph checked by a tutor.**

Episode	Description
1 (1-17)	The students read the question (aloud and silently).
2 (17 to 76)	Mpumelelo identifies the required graph as a "cos graph", and represents this (by tracing the shape in the air) using one full wave of a standard cosine graph, starting and ending at its maximum. The students attend to the maximum and minimum values of the graph, where it is increasing / decreasing, and not to the concavity of the graph. Mpumelelo tries to explain the shape of the graph by linking to the task context, but this is not done in a meaningful way. The students trace the shape of the graph in the air, but do not draw it. The students give one another positive, content-free feedback. Lungiswa encourages explanations and prompts students to speak. Vuyani's attempts to contribute to the conversation are not taken up. 
3(77-93)	Siyabulela questions whether the graph can be negative, and they discuss this, explaining using the task context appropriately. Lungiswa and Bongani are starting to draw the graph, and attend to the maximum value of 10 000 on the axis. Lungiswa's graph decreases from a maximum of 10 000. At times their use of reference pronouns and feedback is not clear, but there seems to be a shared understanding between the students of the meaning of one another's talk. 

4 (94-95)	The students are still working with the cosine graph proposed by Mpumelelo in Episode 1, which they now refer to using the reference pronoun "it". Lungiswa questions whether her graph in Episode 3 will increase again after decreasing. Mpumelelo mixes his description of the appearance of the graph ("it's like going to ... fluctuate") with a description of the task context; "until it get ... all all the people around ... in the population" He uses the task in his description. Bongani's sketch graph supports this idea of a fluctuating graph. 
5 (96-129a)	The Tutor approaches the group. He creates the expectation that the students discuss their responses in the group. He then engages the students in two relational processes. The first attends to the labels on the axes and identifies these labels with the mathematical variables *P* and *t*. Here he corrects the students' description of the function *P*(*t*) (as "the number of people") to "it's the number of people who <have the flu↑... or who have had the flu>", thus reproducing the wording from the problem text. In the second relational process the Tutor assigns a particular value to the variable *t*, that is, he makes *t* equal to zero, and questions students about the context, that is, the number of people with the flu at this particular time (his use of the description of the function *P*(*t*) from the problem text is not consistent). The interaction between the Tutor and the students involves the students giving non-content feedback (when the Tutor provides the opportunity) and prompting the Tutor to continue, and answering his questions using the choice of words he provides. The students are not required to explain their answers or to give full sentences. The Tutor acts as the evaluator, and the students do not evaluate one another. The Tutor's language is animated; he varies the pace, puts the emphasis on certain words and uses his hands to reference points in the air, on the students' axes and for general emphasis.
6 (129b-136)	Having established an understanding of the task context, that initially the number of people with the flu will be "a little", the Tutor then provides a link to the graph and asks where the graph will begin. Again, he provides two possible choices, of "zero" or "10 000". Bongani, Mpumelelo and Vuyani give the correct answer, but Lungiswa gives the incorrect answer of "from 10 000". The Tutor asks for an explanation, but before Lungiswa can respond he explains by clarifying what they are graphing; "how many people have it" and not "how many haven't had it".
7 (137-155)	Siyabulela responds to the discussion by proposing that the graph is "increasing", and traces an increasing, but wavy graph in the air. This tracing is also done by Bongani. The Tutor provides positive feedback, and links Siyabulela's argument to the task context; "because as people get it like more people <have it or have had it>". Siyabulela subsequently develops this with a tentative comment; "so eventually you are going to affect the whole community then?" The Tutor does not attend to Siyabulela's gesture

	suggesting that the graph will be "wavy". Siyabulela makes a joke about the context, but the Tutor continues to link the increasing property of the graph to the task context. Lungiswa tries to clarify, and the Tutor goes on to explain that the sketch represents those who have the disease.
8 (156-164)	Mpumelelo draws on the previous discussion to develop his suggestion in Episode 4; he suggests that the graph is "fluctuating upward", demonstrating this in the air and on his book. The Tutor challenges him to explain why it is "oscillating". Instead of explaining Mpumelelo proposes an alternative name, "a straight line". Again, the tutor requires an explanation. Mpumelelo explains the increasing part by referring to the task context; "as it as the days get on ... more people get it↑". This explanation does not attend to why the line would be straight.
9 (165-177)	The Tutor then evaluates Mpumelelo's argument, saying it "<it wouldn't necessarily> be a straight line". He proceeds to explain by attributing the value of 9 999 to $P(t)$ and linking to the task context; "when when almost everybody has had the virus like if 9 999 people have had the virus so its almost everybody has been ... infected. There's only one more person that can be infected right?" He also introduces the mathematical notion of "rate" of new infections but talks about this in terms of the task context, for example, the rate will be "very low" at the end (since there are not many people to infect). Both Bongani and Siyabulela respond by proposing increasing, concave down graphs (although Lungiswa proposes increasing and concave up). The students are now attending to the concavity of the graph.
10 (178-202)	When the Tutor leaves the group the students start to draw different, increasing graphs, some concave up, some concave down, and some a combination. There is a sense that one of the graphs must be right (reinforcing the problem text). They talk about the graph as "increasing"(also in isiXhosa), but the rest of the discussion involves pointing to different graphs and using gestures rather than describing the properties of the graphs in words, for example, Lungiswa argues, "But you can also draw it like this↑ ((*Pointing to the graph she has drawn with her pen*)). 
11 (203-217)	Lungiswa attends to the labels on the axes, asking for clarification from the others what variables and units are used on each axis.
12 (218-253)	Siyabulela is attending to the concavity when he compares the two increasing graphs. He uses words like "gradient" and "rate" in his statements. Bongani and Mpumelelo do not agree with Siyabulela and Vuyani that there is a difference between the two graphs. Lungiswa attends to Siyabulela's discussion about the "rate" and introduces a discussion

	about the increasing/decreasing rates, linking this to a problem they did in the Course earlier in the year. Siyabulela selects an increasing, concave up graph and starts to explain using the task context, and Bongani uses the context of shopping, but his argument is not clear.
13 (254-334)	The Tutor returns, suggesting they must agree on a graph, thus reinforcing the suggestion in the problem text that there is only one correct answer. This prompts the students to choose a possible graph (which Lungiswa and Mpumelelo link to Workshop 1) from those they have been discussing. The Tutor does not ask them to explain their graphs, but begins by relating back to the task context and what they had decided during their previous discussion, that if there are many to infect, the rate would be high. He then provides two options for the graph (which he draws in Mpumelelo's answer book), calling them the "basic" graph and the more "sophisticated" option. Here he relates the task context to the steepness of the graph at different points. The Tutor attends to the maximum value of 10 000 but does not include this on the graph he draws (he holds his hand horizontal in the air).  The Tutor's "basic" graph  The Tutor's "sophisticated" graph Siyabulela does not redraw a new graph, and leaves the two graphs the Tutor has drawn in his book. Vuyani and Bongani settle on the "sophisticated" answer. Mpumelelo draws the graph on the left (below). None of the students include a horizontal line at 10 000, but their graphs suggest that they are attending to the maximum of 10 000 people in the community. However, Lungiswa's graph begins to decrease (see graph on the right below).  Mpumelelo's graph  Lungiswa's graph

(b) What are the units of $P(t)$?

Episode	Description
1 (324, 327, 335-349)	Mpumelelo reads the question aloud, but refers to the function, rather than the derivative function as in question (b). This seems to cause some initial confusion, but the responses that follow suggest that there is an implicit understanding that they are finding the units of the derivative function. They attend to the units of $P(t)$ (the number of people) and t (days), and to the fact that the derivative function is a rate (suggested by the repetition of the word "per"). Bongani seems to be confused about the meaning of the symbols P and t and the units for these symbols, but his written answer is not discussed (initially he writes, "people per time", but during the discussion changes this to "people per day"). All the other students have written answers of "number of people per day".

(c) What does $P(4) = 1\ 200$ mean in practical terms? (Your explanation should make sense to somebody who does not know any mathematics.)

Episode	Description
1 (350-380)	Both Siyabulela and Lungiswa attend to the need for "practical terms" and the hint provided in brackets. They both make possible answers public, attending to the meaning of the symbols $P(t)$ and t in the task context. Other than Siyabulela's initial tentative attempt, all statements are said with certainty, and with no explanations. This is followed by the other students repeating parts of these statements, and in particular the use of the phrase "after 4 days" used in the Course (Bongani does not attend to this in his writing, just indicating "4 days"). Feedback is in the form of repetition ("after 4 days") and responses like "uh…huh" and "ja". Some of the verbal pronouncements are not complete, for example, reference to the number "1 200" or to the units "people" of this number may be left out. At times the students rely on pointing to fill these gaps, for example, Siyabulela circles the number "1 200" in his Resource Book when he says, "these number of people". Yet in spite of the incomplete verbal answers, the student write down answers in full, for example, Vuyani writes "After four days 1 200 people will be infected". The students do not use the wording from the problem text, for example, that t refers to days "after the first case of flu was recorded" and $P(t)$ to the "the number of people who have, or have had, the disease". Rather, in the latter case they use the term "infected" that is used in question (a). However, their lack of attention to tense means that it is not clear from their writing whether they are attending to the meaning given in the set-up. For example, Siyabulela writes, ""After four days 1 200 people will be infected". While the students finalize an answer to this question, as described above, another conversation takes place between Siyabulela and Lungiswa in which they go beyond what is required of the task context to answer the question. They attend to and exclaim about the size of the number of people infected after only four days. This interaction differs from the other parallel discussion in that the two students joke with one another and demand explanations for one another's statements.

(d) What does $\frac{P(7)-P(4)}{7-4}=350$ mean in practical terms? Give the correct units.

Episode	Description
1 (381-391)	The start of the action on this question follows a similar pattern to the start described for question (c); Lungiswa attends to need to use "practical terms" and both she and Siyabulela try out the wording aloud. Both students attend to the denominator of the fraction and link this to the change in time in the task context (they do not give explicit attention to the wording "between" or "after" at this stage). Siyabulela appears to be attending to the use of function $P(t)$ on the numerator and the value "350" on the right-hand side of the expression when he argues, "Ja from four to seven days ... 350 people were ... infected."
2 (392-404)	Vuyani enters the conversation; he is viewing the object structurally and linking to similar problems in the Course when he pronounces that they need to use "the word … average" (although he does not explain this argument). The students seem to agree when they repeat the word "average" and agree that "this one" is "the average", again possibly linking the word with similar objects studied in the Course. They then try to include this word in their earlier phrase "from 4 to 7 days", for example, "from four days to seven days … on average". They link the object to the word "average" and focus on getting wording that sounds right, rather than identifying the object as a rate of change.
3 (405-417)	There is a short interlude when Mpumelelo revisits the wording, "from 4 to 7 days". Characteristically, he asks for feedback by repeating an earlier statement. He gets positive feedback from Lungiswa who attends to the wording of the problem text to indicate that they must use "practical" rather than "mathematical" terms.
4 (418-422)	Bongani is developing his answer from the pronouncement "from 4 to 7 days". He attends to Siyabulela's earlier claim about the number of people infected, and adds the word average; he pronounces that from 4 to 7 days "350 people on average" are infected. He does not pronounce people "per day", suggesting that he is not identifying the expression $\frac{P(7)-P(4)}{7-4}$ as representing a rate of change.
5 (423-434)	Bongani then rewords his answer, suggesting that he is now identifying the numerator of $\frac{P(7)-P(4)}{7-4}$ as representing the change in the number of people; "the number of people were increasing that were infected by 350". However, he gets negative feedback from Siyabulela who links his statement to the "derivative". Lungiswa is not identifying the rate of change, and has simply added the word "average" to her wording; "from 4 to 7 days on average 350 people will be infected".

6 (435-440)	What follows is a revisiting of the phrase "from 4 to 7 days"; there is some discussion about the choice of preposition, whether this should be "from" or "between" and whether what is being said is, in Siyabulela's words, "bad English". There is also confirmation that the expression refers to the "average". Bongani has revisited his earlier attempt at the rate of change and concludes that "the average is 350". He is not attending to the instruction in the problem text to "give the correct units", as no units are given.
7 (441-479)	Mpumelelo is attending to the expression $\frac{P(7)-P(4)}{7-4}$ as an object when he pronounces, "So the average rate of people ... to getting ... infected was 350" and "This is a rate of change" (he does not explain this claim). The students are making a link to the expectations about the meaning of "practical terms" communicated in the Course, for example when Siyabulela pronounces, "If we say something about rate then that is a mathematical term". The students agree that the expression represents a rate of change, yet in their struggle to explain this in "practical terms", they settle with an answer that does not use "rate" and also does not describe a rate of change, for example, Mpumelelo writes, "From 4 - 7 days on average 350 people were infected"; Lungiswa is encouraging them to move on. They do not attend explicitly to the requirement in the problem text to "give the correct units", which might have enabled them to deal with their dilemma about "rate". Nor do they attend to the tense; Lungiswa uses "will be infected" whereas the others use the past tense, for example, "were infected".

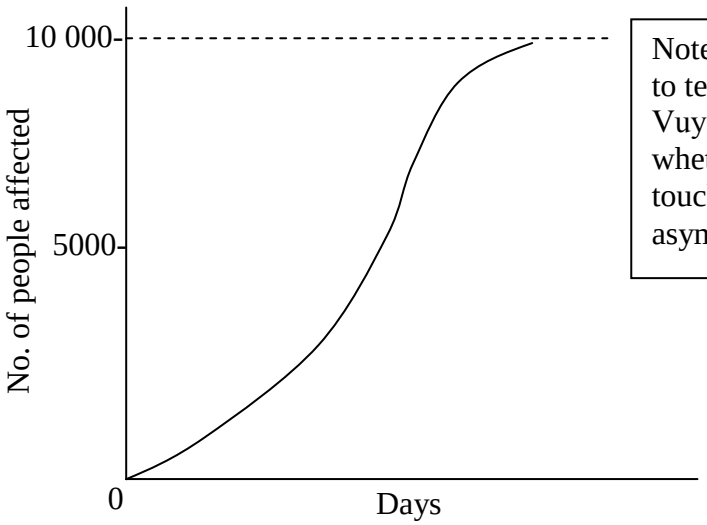
(e) What does $P'(4) = 400$ mean in practical terms? Explain why $P'(t)$ can never be negative.

Episode	Description
1 (480-485)	Both Siyabulela and Bongani attend to the derivative symbol $'$ in $P'(4) = 400$ when they make a link to Bongani's attempt at describing the rate of change using the "increase" in question (d) (Episode 5).
2 (486-501)	Lungiswa is the first to pronounce an answer verbally; she does not appear to be attending to the link Siyabulela and Bongani have made in Episode 1. She attends to the time, the wording valued in the course, and the meaning of the function $P(t)$ when she pronounces, "After each 4 days 400 people have been infected". Siyabulela gives negative feedback, but attends only to her wording for the description of the time; "After each four days". He does this by repeating the phrase, as if for clarification and then rewording this to, "After four days". There is some interaction in Sesotho; this is for interaction about procedures, for example, a request to repeat a statement, and not for mathematical answers.
3 (502-555)	Lungiswa is attending to the difference between questions (c) and (e). Siyabulela appears to be attending to Bongani's response to question (d) (see Episode 5) and to the symbol $'$ as the instantaneous rate of change when he pronounces, "after four days ... uh ... the number of people ... who were infected were increasing by ... 400 ... per ... day". Here he does attend to the need to include units (this is not an explicit requirement in this particular problem). Bongani co-constructs an answer with Siyabulela, and Lungiswa is giving feedback and prompts like 'uh ... huh".

	Lungiswa then pronounces a correct answer that is in a slightly different order, "After 4 days the number of people who were infected per day is increasing by 400". Siyabulela gives her negative feedback by wanting to add on "people per day" at the end, but Bongani exclaims and gets Lungiswa to repeat her answer. What follows is repetition of the different parts of the answer until they are happy with the sound. Siyabulela emphasizes the importance of the units at the end, and Mpumelelo is the only one who does not have units in his answer that suggest the rate of change ("After 4 days the number of people was increasing by 400"). The students do not attend explicitly to the tenses in the written or verbal answers, using "were infected", "are infected", 'being infected", "to be infected", suggesting that they are not attending to the meaning of the function $P(t)$ given in the problem text.

(f) What is $\lim_{t\to\infty} P(t)$? Give a short reason for your answer.

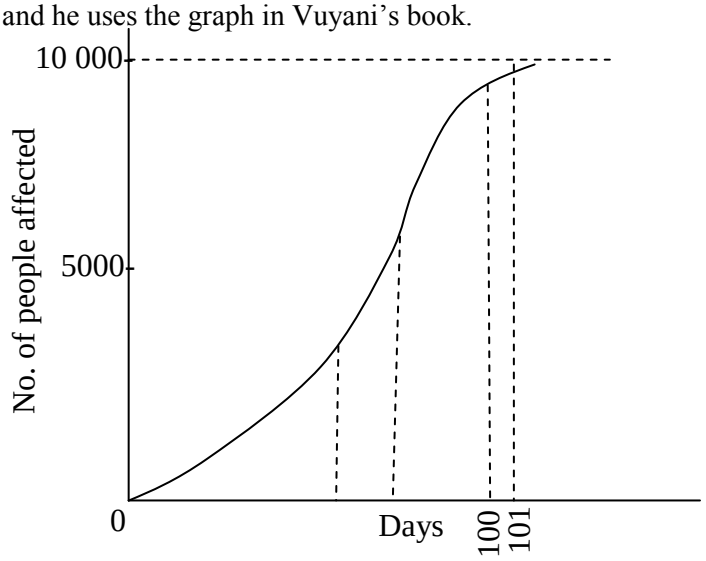
Episode	Description
1 (556-571)	Siyabulela is the first to attempt an answer aloud; his "as days progress" suggests that he is attending to the symbol $t \to \infty$ in the limit expression $\lim_{t\to\infty} P(t)$ and the meaning in the task context. What follows is attention to whether they need to explain using mathematical terms (Vuyani refers to this as "mathematically") or everyday terms (Siyabulela pronounces this as "physically" and "in the streets"). There is some humour in this discussion.
2 (572-578)	Lungiswa then rewords Siyabulela's original idea, "as days go by", thus attending to using the correct "practice terms". Siyabulela then adds to the first phrase, "then the number of people who were infected got ... larger", suggesting that he is using the task context to considering what is happening to the function $P(t)$ over time. His first answers are presented tentatively for feedback. Bongani and Lungiswa repeat parts of his statement.
3 (579-591)	Lungiswa is attending to the function $P(t)$ in the limit expression $\lim_{t\to\infty} P(t)$ and looks at Sentence 4 to identify the meaning in the task context, "... the number of people…". She also attends to the text "who have, or have had, the disease" when she pronounces slowly "who have … or who haven't been infected". Siyabulela challenges this by repeating her text "haven't been infected" which results in her rewording her initial pronouncement as "people who are … infected who are not yet infected". This discussion seems to prompt Mpumelelo attend to the meaning of $P(t)$ in the problem text.
4 (592-599)	Mpumelelo then attempts a pronouncement, and Lungiswa and Siyabulela add phrases as he speaks. He settles on "As days go more people were infected". They do not explicitly attend to the wording of the problem text as in Episode 3.
5 (600-605)	Siyabulela then asks a question, "But does it tend to infinity", his pointing at the problem text suggesting that he is attending to the limit expression $\lim_{t\to\infty} P(t)$ (the word "it" is taken to mean the function/the number of people infected, as discussed in

	Episodes 2 and 3). He gets positive feedback from Mpumelelo, on the grounds that "infinity is a large number". But Siyabulela and Lungiswa argue against this when they claim that "it" (the number of people) "has to end", suggesting that they are attending to the number of people in the community in the problem text.
6 (604-629)	Vuyani enters the discussion and makes a link to the graph they have drawn in (a). He draws in a dotted, horizontal line at 10 000 and identifies this as the "horizontal asymptote". This enables Lungiswa to conclude that the number of people infected "comes closer to 10 000s". They also use the terminology "tends to". In responding to Bongani's query about the increasing graph, Lungiswa does state in this discussion that "Till 10 000...is infected↑" suggesting that she attends to the fact that the graph will reach 10 000. Yet the students still explicitly state that the number of people "get closer to" / "tends to" 10 000. They do pay attention to writing about the meaning in practical terms, and Siyabulela comments on his own attempt ("As a function of days the more people are infected ... they... they tend to 10 000") as "using too much physics".  Note: It is not possible to tell from copy of Vuyani's written work whether his graph touches the horizontal asymptote or not.
7 (630-667)	Siyabulela starts the statement from the beginning again, and this time rewords it slightly to "as time progresses". What follows is a discussion about using "go by" vs. "progresses", suggesting they are attending to the need to use everyday language. They then repeat parts of the statement until settling on variations of "As days progress/go by the number of people infected got close to/tends to/approaches 10 000". They do not revisit Lungiswa's earlier attempt (Episode 3) to use the meaning of the function $P(t)$ given in the problem text.

(g) What is $\lim_{t\to\infty} P'(t)$? Give a reason for your answer.

Episode	Description
1 (665b, 668-688)	Bongani is the first student to pronounce an answer, pronouncing that $\lim_{t\to\infty} P'(t)$ is 10 000 (the same answer as was agreed on for question (f)). He is not attending to the derivative function $P'(t)$ in the limit expression in question (g). When challenged to explain his answer by Lungiswa, he identifies the limit with "getting closer to 10 000". He does not explicitly identify the 10 000 with its meaning in the task context, and the task context is not drawn on in the rest of this Episode. Mpumelelo gives negative feedback by reminding Bongani that they are now on question (g). However, this feedback is not followed up on immediately as Lungiswa asks Bongani for an explanation and gives him positive feedback and prompts in the form of phrases like "uh ... huh" and by repeating Bongani's answer "10 000" as Bongani explains. Siyabulela (and then Lungiswa and Vuyani) then give explicit negative feedback by indicating that Bongani has presented the answer to question (f). Bongani pronounces that the answers are the same for (f) and (g), but these three students attend to the derivative function $P'(t)$ in the limit expression in (g) and pronounce that it is the "derivative" that makes the expressions different. They also point to the problem text as they talk.
2 (689-708)	Siyabulela identifies the "derivative" with the instantaneous rate of change when he pronounces that they need a vertical asymptote so that they can talk about "infinities" and a "particular point". It seems he is having difficulty reconciling finding the limit as t tends to infinity with his definition of the derivative as the rate of change at a particular point. Vuyani agrees that "the problem here is is that derivative". Siyabulela identifies the variable t with its meaning in the task context when he pronounces that they "still" take days, attributes a particular value to it (10 days) and indicates that people ("they") are going to be infected after 10 days. Mpumelelo provides negative feedback in the form of a statement about the content (and by shaking his head); he is attending to the limit to infinity when he makes the contrastive statement that, "you can't restrict the days". Without prompting he goes on to explain that "we" usually give the exact time when using the derivative without the limit, associating himself with what is done in the maths community. He is attending to both the $t \to \infty$, and $P'(t)$ in the limit expression. Like Siyabulela, he identifies the derivative as being associated with a particular point (using his hand to gesture a point in the air), but argues that this cannot be the case when the limit is tending to infinity. Lungiswa hints that by focusing on particular days it may be possible to reconcile the two when she says, "it's not like ... it's still infinity". But this is not followed up on in this Episode. At the end of the Episode, Siyabulela and Mpumelelo agree on their dilemma: they pronounce that "t approaches infinity" and "we're interested in a particular point". (Siyabulela gives positive feedback by nodding and using "ja" and "I hear you".)

3 (709-753)	Much of this Episode involves the students deciding how to proceed, a discussion that is conducted with some humour and laughter. Mpumelelo looks back in his book, Siyabulela takes a book out of his bag on the floor, and Vuyani looks at the problem text. Yet Siyabulela and Bongani want to get help from the Tutor who is busy with a different group of students; they glance across the room and make statements about the Tutor being busy elsewhere. Bongani goes beyond the task context by introducing "911" in his search for help; this leads to questioning and responses from Lungiswa. Lungiswa suggests they move on to the next question while they wait, but Vuyani expresses concern about having an answer when the Tutor arrives, a concern that Lungiswa shares when she repeats, "Yeah he's gonna ask us". Vuyani and Lungiswa thus set up the Tutor as an authority. Siyabulela does not share the concern, suggesting in a humorous way that they tell the Tutor they have been waiting for him and then share their "ideas". Yet Bongani and Lungiswa do not seem to regard these ideas as their own and question him about these "ideas". It is Siyabulela, with some help from Mpumelelo, who pronounces what he regards as the group's current ideas; they argue that the derivative refers to a particular point, but the problem is "the infinity". In pronouncing these ideas Siyabulela uses references like "this thing" rather than the mathematical terminology and it is Mpumelelo who provides the mathematical term "infinity".
4 (754-764a)	The Tutor joins the group and questions the students about their progress. Siyabulela is put forward by the students to explain to the Tutor. His statement mirrors his earlier focus and is located in the mathematical discourse, with no reference to the task context; they have the derivative at a point, but the limit to infinity is causing difficulty. He does seem to attend to a graph when he pronounces something about reading off on the *y*-axis and having a vertical asymptote, but this is not observed in the video. The Tutor is listening to Siyabulela, saying "okay", possibly to prompt him to continue explaining.
5 (764b-772)	The Tutor then tries to get Siyabulela to relate what he is saying to "your graph" and points to Vuyani's graph. Mpumelelo responds by pronouncing that as the days progress, even if we have a particular day like 4 days, so the number of people getting the disease will be increasing. In this response he attends to the meaning of the function $P(t)$ in the task context, to the derivative as the instantaneous rate of change ("even if we have a particular day") and to some visual image he has of a graph (although he points to the problem text and not Vuyani's graph, he does trace the shape of an increasing graph with his hand in the air). This statement of content varies from earlier ones by students on this question, as they had not related the time to the number of people with the disease. The Tutor initially attends to and corrects Mpumelelo's wording for the meaning of the function, by attending to the wording in Sentence 4 of the problem text, as he reminds Mpumelelo "You have to be careful about the way you say it".
6 (773-793a)	The Tutor then focuses on the content of Mpumelelo's response and suggests that Mpumelelo's answer is appropriate for question (f) rather than (g). Here is attending to the function in (f) and the derivative function in (g). To highlight the differences he asks the students what the derivative ("this one") refers to, and before they can answer he identifies it with the term "rate", a word that Siyabulela immediately rewords to the

	mathematical "derivative". Siyabulela starts to ask a question, "Does it mean er ... okay" which he does tentatively, holding his hand over his mouth, but he does not develop this. The Tutor then proceeds to talk about question (f). He speaks using a mixture of statements about the content, instructions and suggestions about the procedure, and questions addressed to the students. In his pronouncements he sets up a number of relations; between the expression in the problem text and the graph in Vuyani's answer book, between the limit as *t* tends to infinity and its meaning in the task context, and between the everyday meaning of the function *P*(*t*) (the number of people infected) and variable *t* as this becomes larger. Bongani responds to the Tutor's question by using the everyday language "closer and closer" for the limit and attributes this to the value 10 000 from the problem text. The Tutor continues setting up relationships (at times asking Bongani to repeat his answer, or repeating it himself) between the graph and the task context (as *t* becomes larger), and then between the limit expression in (f) of the problem text and its meaning in the task context. In the latter case he also relates the limit expression in (f) with its meaning in words, "it says what is the limit as *t* tends to infinity". The Tutor gives positive feedback to the responses from Bongani and Siyabulela, telling them that that is their answer for question (f). The students (Siyabulela and Vuyani) respond by saying they have already answered that question. There is no explicit discussion of whether the graph will reach the value of 10 000.
7 (793b-819)	The Tutor indicates that they must also attend to the graph in (a) when answering question (g), and he uses the graph in Vuyani's book.  The Tutor's additions to Vuyani's graph He begins by attributing small time values (4 days and 5 days, drawn on Vuyani's graph) and considering how many people have been infected at different times ("after four days 5000 people have are infected↑... and have been infected okay↑", also attending to the wording from the problem text). He then considers bigger values of *t*, for example, 100 days and 101 days, and questions "how many more how many people

	have been infected in that passage of time". He also mentions the "rate of infection" at different times, and considers the "increase" at different times. He also refers to the gradient of the graph at different points and relates this to the task context, that is, how many people are infected at different times. The Tutor finishes by relating what he has said back to the limit expression $\lim_{t\to\infty} P'(t)$ which he identifies with its meaning in the task context as "the rate of change ... in infections". He also makes explicit that the students should use the graph. During the Tutor's explanation Siyabulela, Bongani and Vuyani respond to the Tutor's questions using single words ("zero") or short sentences ("The increase is very slight").
8 (820-859)	When the Tutor leaves, Mpumelelo asks for clarification of the Tutor's use of the word "rate". Lungiswa responds by indicating that it is the "rate of infection", but also giving an answer which she also explains using the task context, "The rate of ... uhm … the rate of infection ... is ... it's getting closer to zero … …. cause as the days ... uhm progress … … less people are getting infected". During this attempt, which she builds gradually, there is some comment on her use of word "progress". Mpumelelo repeats his earlier question, "So this is a rate of↑", and Lungiswa responds by repeating the first part of her answer. While this interaction between Lungiswa and Mpumelelo takes place, Vuyani is concerned about "this whole thing", which he circles in the problem text (but it is not clear what he is attending to). He seems to be concerned about the use of the term "rate of change" in their answer. Lungiswa and Mpumelelo do not attend to this in their written answer, for example, Lungiswa writes, "the rate of infection is getting closer to zero", but Vuyani avoids using "rate" in his answer, which refers to the function rather than the derivative function ("As the days progress the number of infected people gets closer to zero".) Bongani just writes $\lim_{t\to\infty} P'(t) = 0$ without relating the answer to the meaning in the task context, and Siyabulela gives no written answer.

APPENDIX M

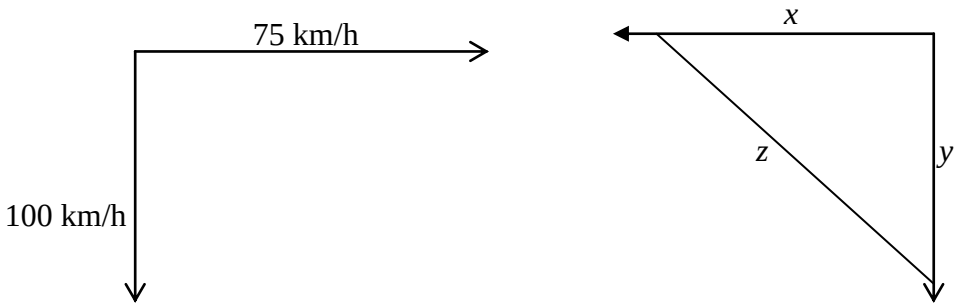
GROUP 1: ACTION ON THE CAR PROBLEM

The following questions are related rates problems. These MUST be set up correctly. Follow these steps for **EVERY** question:

1. Draw a diagram and define variables.
2. Write down what is given, using the correct notation.
3. Write down what is to be found.
4. Write down a formula linking the variables.
5. Differentiate and complete the question

Two cars start moving from the same point. One travels south at 100km/h and the other travels west at 75km/h. At what rate is the distance between the cars increasing two hours later? (Let the distance between the cars after a time *t* be *z* km).

Episode	Description
1 (1-14)	Shae has started on the car problem and is drawing perpendicular lines in his book, suggesting that he is attending to the instruction in point 1 of the boxed text to "draw a diagram" and to the information in the problem text (that one car is travelling "south" and the other "west"). He expresses the personal opinion that he does not like drawing the diagram, and prefers to work on point 2 of the boxed text ("the given and that"); it seems that Shae sees the problem-solving process as being divided into parts. The Tutor arrives at the table to ask whether the group has started the Car Problem, and then enters into a conversation with Shae. The Tutor encourages Shae to pay attention to what is in the boxed text and particularly to what is emphasized in this text ("it's got stuff underlined … it's got stuff in capitals … it's got stuff in bold"). He attends to Shae's expressed dislike of the diagrams by locating his argument, first, in his personal experience of doing the problem ("personally when I was going through this tut actually drawing it helped a lot"), then in the utility of using the box ("I think it can help avoid confusion"), and then as an authority who simply tells them to "just do what they say". The third approach seems to sit uneasily with him as he speaks faster than normal. Jeff enters the conversation by linking the need for a diagram to "the Physics approach".
2 (15-22)	The students draw right-angled triangles, suggesting that they are attending to both the instruction in the boxed text to "draw a diagram" and the direction in which the cars are travelling ("south" and "west") in the problem text. (Although Shae does not complete

	his triangle with the hypotenuse – see written solution). They may also be attending to similar problems done in lectures. Jeff attends to point 1 to "define variables" using *x* and *y* (as is done in similar problems in the Course), but does not actually pronounce or write down what these mean as instructed. Lulama, Darren, Hanah and Jeff all use the variables *x*, *y* and *z* on their diagrams. (Shae does not put the variables on his diagram, but uses them in his text.)  Shae's drawing Jeff's drawing The right-angled triangle seems to be a cue to Shae and Jeff to use Pythagoras. Jeff writes the equation $x^2 + y^2 = z^2$ in his book. Lulama makes a statement in the form of a question and actually names the theorem: "so … we are using Pythagoras Theorem↑". (The students seem to be attending to point 4 in the boxed text about using a formula.) Both Shae and Jeff give Lulama positive, content-free feedback on his query. When Lulama suggests that there may be "problem" with the use of this theorem, they do not attend to his answer.
3 (23-31)	The students have not attended to Lulama's concern about using Pythagoras in the previous Episode. Now Jeff asks whether the "speeds" will be negative; it appears he is attending to the speeds 75km/h and 100km/h in problem text, and is possibly writing down the values for *dx/dt* and *dy/dt* in the "given" part as required in point 2 of the boxed text. It is also possible that he us thinking about a lecture on this work as he asks "Why did he make them negative in class?" Shae uses the task context to explain that the speeds might be negative because "they" (the cars) are moving away from the point; he is not attending to what is happening to the distance in deciding whether the derivatives/speeds will be positive or negative. Darren gives negative feedback to Jeff by suggesting that the cars ("it's") are going towards one another; here he is drawing on the task context inappropriately. Lulama also gives negative feedback, claiming that "it's positive", but with no explanation and his response is not followed up on. Shae also gives negative feedback, saying that the speed ("it's") will be positive and arguing that one needs to look in a certain way, that is, look at the outcome. He pronounces that they must look at "this <u>final</u> thing"(moves his hands away from each other along the hypotenuse of his diagram). Since "that's" increasing, "it's" positive. Shae is not challenged on this (Jeff gives positive feedback in the form of "Okay"), and they continue. During this exchange the students use "it's" frequently – either for the speed (Lulama, Jeff and Shae) or the cars (Darren). They do not talk explicitly about the distance.

4 (32-37)	The students have used implicit differentiation to differentiate the Pythagoras expression $x^2 + y^2 = z^2$. They are attending to the instruction in point 5 of the boxed text to "differentiate". There is no discussion of this step; discussion only begins when they are trying to substitute into the differentiated expression (see also Episode 5). Lulama is concerned that "they" don't have "*dz* over *dt*", giving the mathematical symbols in words rather than the meaning of the derivative in the task context, that is, the rate at which the distance between the two cars is changing. He seems to be attending to the derivative equation $2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$ he has written in his book, and has placed "ticks" above some of the symbols. He addresses his concern to students Shae and Jeff. Shae responds; he seems to imply that they are trying to find *dz/dt* (but does not say it explicitly), but says they have "the two different values", which he does not pronounce to be *dx/dt* and *dy/dt*. Jeff makes a pronouncement about *dx/dt* being 75 (again, giving the name of the symbols in words).
5 (38-54)	Shae then tries to introduce the use of *s* and *w* as the variables, "You know for the south car and the west car". He does not appear to be attending to the meaning of the variables (this has not been pronounced verbally or made explicit in the writing), that is, as the distance travelled by each car. The *s* and *w* just seem to be symbols used to name the cars according to the direction of travel. He does not make any pronouncements about the actual letters *s* and *w* and the term "variable", but talks about "*ds*" and "*dw*". The other students give negative feedback, suggesting the use of *x* and *y*. Darren argues that it is important for them to use the same "things". In this discussion there is no reference to the meaning of the symbols that they are talking about, either mathematical or in the task context.
6 (55-70)	The speech in this Episode is dominated by Jeff who makes public claims (yet tentative, he asks for feedback and his body language suggests uncertainty). Jeff speaks about "doing the equation", by which he seems to mean using implicit differentiation. He also speaks about substituting (pronounced "plug in") 2 hours, suggesting he is attending to the need to find the rate of change after 2 hours in the problem text. But it is not clear where it should be substituted. Lulama queries the substitution and asks for an explanation; Jeff explains by referring to the meaning of *z* in the task context, that is, "z has got to be the distance <at 2 hours>... from their starting time↑". This is the first explicit reference to the meaning of the variable *z*. Following positive feedback (content-free) from Lulama, Jeff continues, but this time pronouncing the operational action for calculating *z*, "So it'll be like ... 200↑ and... 150↑ square uh squared ... each↑ then add them together and that will give you the z↑". This is a tentative claim, as he asks for feedback "Wouldn't it?" after the pronouncement (also suggested by his body language). Jeff is talking about the numbers he is substituting, and he links the meaning of the "200" and the "150" to the task context. Yet his language is not clear; he uses "it" for the distance without making the meaning explicit. He refers impersonally to the people in the task context ("they" and "the guy"). Shae has been writing in his book, but responds by confirming the use of "the equation"

	and then talking about the need to "deriviatise" this equation. Lulama is adding to his text; he does not seem to be attending to Jeff's words, as he circles the "100" on his diagram and then substitutes 75 and 100 for *x* and *y* in his Pythagoras equation and writes: $z^2 = x^2 + y^2 \Rightarrow \quad 75^2 + 100^2 \Rightarrow$ Here he is not distinguishing between the distances and the speeds.
7 (71-88)	Darren pronounces that you "differentiate it in this function ... is the two", attending to the operation to be performed and the instruction to "differentiate" in the boxed text. Shae has written $z^2 = 2(x^2 + y^2)$ in his answer text, with the 2 and brackets lightly in pencil (possibly added later); it seems he is attending to the multiplication by two referred to by Jeff in Episode 6, but has applied it incorrectly. Lulama argues that, "it doesn't work out", but it is not clear what he is referring to and his comment is not attended to by the others. Darren refers to "multiplying everything by 2", it seems he is attending either to Jeff's description in Episode 6 or to the multiplication by 2 that results from the implicit differentiation in his equation $2z\frac{dz}{dt} = 2y\frac{dy}{dt} + 2x\frac{dx}{dt}$. Shae gives negative feedback, "no no" and talks through the the operations; "you first get the equation" (attending to Pythagoras equation), then "you derivatize it" to get "2 *z* times *dz* over *dt* is equal to 2 *x* times *dx* over *dt* ... plus ... 2 *y* times *dy* over *dt*↑", and then the need to "plug" in values into this expression. Shae's operations are correct, but no meaning is attached to these; he simply identifies objects by pointing (e.g. to the values of *dx/dt* and *dy/dt* written under "Given") and by using the symbols (*x*, *y*). Darren attends to Shae's description, building on this by talking about the operation of substitution, "those will be your *x* and *y* values that you plug in".
8 (89-107)	Darren attends to Lulama's comment (unclear). Initially, Darren clarifies that they are finding *dz/dt* (attending to where Lulama is pointing in his answer book?). Lulama then talks about what they are given, pointing to and naming the variables in his derivative equation, "and you <u>do</u> have *z* … you <u>do</u> have *x* and you <u>do</u> have *y*↑". He is corrected by Darren who reminds him about calculating *z* using Pythagoras (he points to the right-angled triangle in Lulama's book). Lulama then raises a third issue related to his derivative equation, "how about *dx* over *dt*?", and Darren responds by linking *dx/dt* and *dy/dt* to the values 100km/h and 75km/h in the problem text (by pointing). But Lulama disagrees, linking the 100 and 75 to "*x* and *y*" (he made this link in Episode 6). This is followed by a correction by Darren who says, "that is your *d* that is your <u>*dx*</u> over *dt*". In this interaction between Darren and Lulama they talk about the symbols "*dx* over *dt*", "*x*" and "*y*" etc., but without any reference to the meaning of these symbols in the task context. Although Darren seems to be making the links implicitly, for example, that *dx/dt* is the speed 100km/h, Lulama does not appear to be making links between the symbols and their meaning. In order to explain, Darren introduces the word "rate" (for the first time in this interaction); "When you're comparing rates ... rates is a comparison between two variables" At the same time he points to the speed 100km/h in the problem text, and then

	identifies the two variables ("distance" and "time") by pointing to the units "km/h" in the text. He then explains the operation of substituting into the Pythagoras equation; some of his language is vague "we're gonna take↑ ... 75 and 100", but he is also pointing to parts of the problem text and writing on the problem text. He then explains where the "200" comes from, using the distance "100km for 1 hr", and attending to the units of the numbers. Lulama continues to talk more vaguely, "that's going to be ...200↑", appealing to Darren for confirmation. Lulama returns to his writing, but he is using the z^2 value he incorrectly calculated in Episode 6, and then substituting z^2 as 15625 (instead of substituting for 2*z*): $15625\frac{dz}{dt} =$ (The 15625 he obtained from his earlier use of Pythagoras: $z^2 = x^2 + y^2 \quad \Rightarrow$ $75^2 + 100^2 \Rightarrow 15\,625$.)
9 (108-121)	{Overlap with 89-107} Shae and Jeff are talking at the same time as Darren and Lulama (Episode 8). Jeff indicates that he has not followed Shae's earlier explanation and Shae sets out to explain. He begins with explaining that "I've just done it the other way round"; it seems he used implicit differentiation for "the main equation", and then did the substitution into the Pythagoras equation to find the value of *z* afterwards (in the latter case he links what he is saying explicitly to Jeff's earlier explanations). He and Jeff then talk about the operation of substitution. Shae does not mention only numbers, but sometimes gives a description, for example, "the 150km they would have travelled by 2 hours" or by including the units of the numbers ("y is equal to 15km"). But they still use "*z*" and the incomplete "*dz*" when they talk about the mathematical operations, and do not give the everyday meaning of these symbols.
10 (122-136)	The Tutor approaches the desk and asks, "How is it going?". Shae responds that it is "going good". A discussion between the two begins as the Tutor asks Shae to "show me". All the other students are working individually, either writing or working on their calculators. Shae starts to explain, and he points to things in his answer text and the problem text as he speaks. He attends first to the derivative equation in his answer book ($2z.\frac{dz}{dt} = 2x.\frac{dx}{dt} + 2y.\frac{dy}{dt}$). He then draws on the context, I used "their speeds to find out at exactly what distance they would be at that time of 2 hours↑", and links this to his substitution for "*x*" and "*y*" in his Pythagoras equation. He then refers to the substitution into the derivative equation to find "*dz* over *dt*", this time referring to the mathematical operations and with no meaning in the task context. The Tutor gives the students positive feedback in the form of "okay" and then asks all the students whether they "are all on the same wavelength". They respond with simple responses "Ja" and continue working.

11 (137-155)	Lulama pronounces his answer as 1.056 (as noted in Episode 8 he did not correct his value for *z* after Darren's explanation). $125\frac{dz}{dt} = 200(75) + 150(100)$ $= 1500 + 15000$ = 1.056 km/h The content of Lulama's pronouncement is not attended to, but seems to encourage Hanah (who has not taken part in the verbal discussion so far) to announce her answer of "1,4". She identifies herself as unsure here, indicating that she thinks her answer is "totally wrong" and that her method is different to Shae's ("I think I did mine totally differently to yours"). Jeff then offers the number "125" as an answer. Shae compares this with what he has on his calculator and says, "I calculated mine wrong". Jeff's written answer is as follows (his use of the derivative suggests that he is not attending to the meaning of the derivatives, for example, he shifts from *dz/dy* to *dz/dt*): $x^2 + y^2 = z^2$ $2z\frac{dz}{dy} = 2x\frac{dx}{dy} + 2x\frac{dy}{dx}$ $2(250)\frac{dz}{dt} = 2(150)(75) + 2(200)(100)$ $\frac{dz}{dt} = \frac{(150)^2 + (200)^2}{500}$ $\frac{dz}{dt} = 125\text{km.h}$ Hanah attends to Jeff's answer of 125 and thinks back to an earlier calculation she did, "Isn't↑... 125 just what you get if you ... put the things into Pythagoras?" (she does not pronounce the values or their meaning). Jeff agrees with her and they both agree that maybe the question is "not that difficult" (Jeff) and "you make it worse than it is" (Hanah). But they do not pursue this any further, neither do the other students attend to this; they are possibly following the prescribed method, preventing them from engaging with the meaning of the task.
12(140)	While Lulama, Hanah and Jeff are talking in Episode 11, Darren is writing in his answer book, using his calculator and talking quietly through the operations. He makes an error with his substitution, substituting the speeds for *x* and *y*, rather than the distances after 2 hours: $\frac{2(2\times100)100 + 2(2\times75)75}{2\sqrt{(75^2 + 100^2)}}$. He also cancels the value of "2" on the denominator and numerator.

13 (156-177)	Jeff wants to know what answer Darren has; Darren pronounces this as "250" and both Shae and Jeff pronounce their answer of "125". In these pronouncements only the number is attended to, and not the units (km/h). Jeff then questions Darren about the square root on the denominator of his equation (see equation in Episode 12); Jeff is attending to the error Darren has made in his substitution. Darren responds by explaining the operation, that is, that he is finding "z". Jeff then attends to the numbers 75 and 100 under the root sign and indicates that Darren has substituted the wrong figures. Darren acknowledges his mistake and crosses out and changes the figures to 150 and 200 in his calculation: $\frac{\cancel{2}(2\times100)100+\cancel{2}(2\times75)75}{\cancel{2}\sqrt{(150^2+200^2)}}$
14 (178-189)	Hanah appeals to Shae for assistance. She is attending to the numbers and the operations in both her and Shae's work when she says, "I've got those figures ... but I'm not using them in the same place as you↑". She also pronounces these numbers, "I've got 150↑... 200 and ... 250", but is not attending to their meaning. Shae responds by pointing to the right-angled triangle he has in his book and linking the calculations with their meaning in the task context, for example, "after 2 hours how far, what would the <u>distance</u> be that you travel" and "so you have 2 hours times by that↑… 2 times that is 200↑" . This seems to help Hanah who notes that they then substitute the values into the derivative equation, "we put it in there↑", although again her talk does not draw on the task context. Hanah has corrected her answer (by erasing her earlier calculation?) After 2 hours $x = 150$km $y = 200$km $z^2 = (200)^2 + (150)^2$ $z = 250$ $2\frac{dz}{dt}z = 2y\frac{dy}{dt} + 2x\frac{dx}{dt}$ $\frac{dz}{dt} = \frac{2(200)100 + 2(75)150}{2(250)}$ $= 125$ km.h^{-1} This is very similar to Shae's answer: $2\frac{dz}{dt}z = 2y\frac{dy}{dt} + 2x\frac{dx}{dt}$ $x = 150$km $y = 200$km $z = 250$km

	$\therefore \frac{dz}{dt} = \frac{2(200)100 + 2(150)(75)}{2(250)}$ $\frac{dz}{dt} = 125\,\text{km/h}$ During the exchange between Hanah and Shae, Jeff attends to Darren's calculator work (his use of memory).
15 (190-209)	Lulama then appeals for feedback from Jeff when he presents an answer of 132. Jeff responds by giving his answer of "125". Lulama clarifies this by repeating it, and then looks down at his answer book again, repeating his answer of 132. In this exchange the students only attend to one another's answers (pronounced without units) and no meaning is attached or explanations used. Lulama seems puzzled when he looks at his work again. Then Darren has completed working on his calculator and pronounces the same answer as Jeff, "125". Lulama repeats this value, with a rising intonation at the end as if he is appealing for feedback. But the other students joke about the problem and then continue with the next question. Lulama returns to his book and replaces his answer of 132 with 125, but the working for this answer remains unchanged: $125\frac{dz}{dt} = 200(75) + 150(100)$ $= 1500 + 15000$ = ~~1.056 km/h~~ = 13~~2~~5km/h

APPENDIX N

GROUP 3: ACTION ON THE CAR PROBLEM

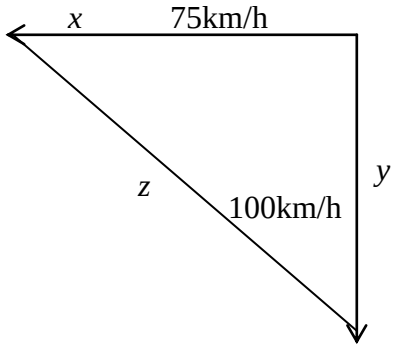
The following questions are related rates problems. These MUST be set up correctly. Follow these steps for **EVERY** question:

1. Draw a diagram and define variables.
2. Write down what is given, using the correct notation.
3. Write down what is to be found.
4. Write down a formula linking the variables.
5. Differentiate and complete the question

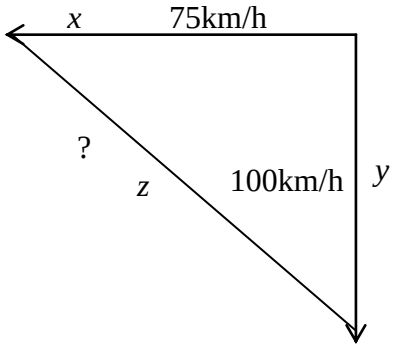
Two cars start moving from the same point. One travels south at 100km/h and the other travels west at 75km/h. At what rate is the distance between the cars increasing two hours later? (Let the distance between the cars after a time *t* be *z* km).

Episode	Description
1 (1-3)	The Tutor addresses the group (makes eye contact with each one) and describes the problems as being about "differentiation" and "implicit differentiation" (making a link to the mathematical operation) and "kinda like word sums" (making a link to the discourse of word problems in mathematics). He also reminds them to attend to the text in the box as this tells them how to "set it up". He encourages them to "write down" what is given, giving this value by indicating that they will find this useful in tests.
2 (4-22)	The Tutor leaves the group and the students are drawing the right-angled triangle in their books, attending to the Tutor's instructions, Point 1 of the boxed text, and the words "west" and "south" in the problem text. The discussion takes place between students Akbar, Thokozile and Ndumiso. Ndumiso associates the direction "south" with "going down" on his page. Akbar and Thokozile discuss how to assign the variables *x, y* and *z* to the sides of their triangles (Akbar appeals to Thokozile for assistance); they are attending to the "variables" in point 1 of the boxed text, but do not actually "define" them. They refer only to the letters "*x*", "*y*", and "*z*" and add them to their drawings ("that's *x*", "this is *y*"), without linking these letters to distance in the task context. Thokozile associates the variable "*y*" with the direction "south". In assigning "*z*" they are probably looking at the instruction given in brackets in the problem text, and/or to the use of this variable in this way in lectures.

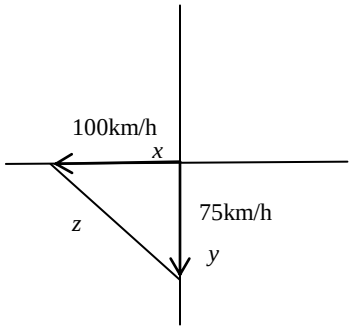
Akbar, Thokozile and Lwazi have diagrams as follows:

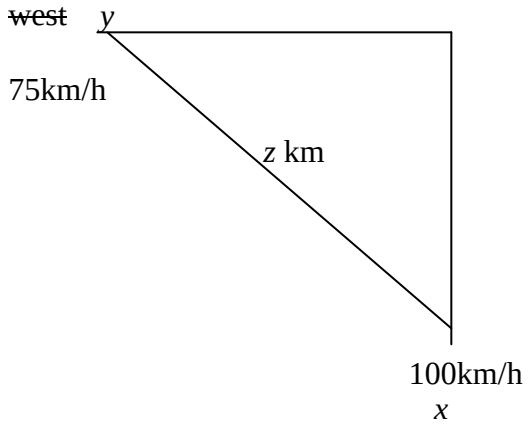


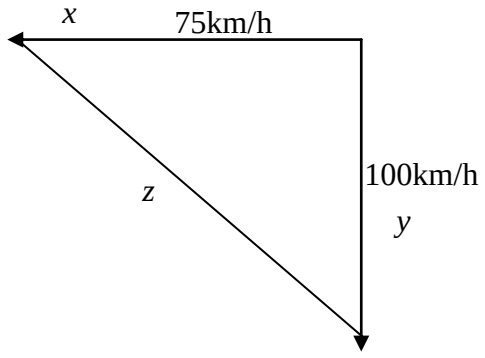
Nqobile's diagram looks similar, but she has put a question mark on the hypotenuse.:



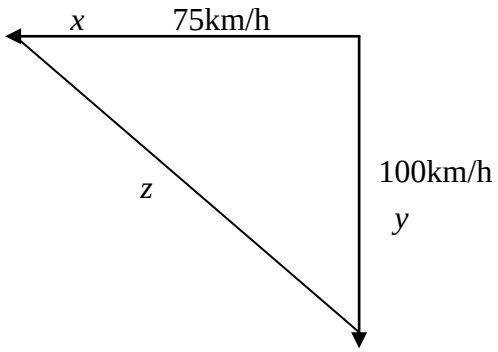
Ndumiso has placed his triangle on a set of axes:



	Kelsa places her labels at the vertices of the triangle: 
3 (23-52)	Thokozile asks a question related to the operation for differentiating, that is, whether they will be differentiating with respect to time, suggesting that she is attending to the meaning of the derivative. It seems she is writing down what is "given" (using the correct notation) and what is "to find" as required by points 2 and 3 of the boxed text, and this has made her think about how to write the derivatives. Ndumiso gives positive feedback to her in the form of "hmm". Ndumiso also seems to be attending to points 2 and 3 of the boxed text as he uses the language "to find" and "we are given". There is an interruption when Lwazi's cellphone rings and they tease him about this. Thokozile then indicates that she personally ("I") does not "understand" because they do not have "*dx* over *dt*" or "*dy* over *dt*". Here she names the symbols but does not give the meaning in the task context, that is, the speed. It seems that she is looking for possible values in the problem text as she seems to be looking at the "two hours" in the text when she says, "that is just time". Ndumiso seems to want to respond, but goes back to the text as the authority, "what do they say?". Kelsa gets the attention of Thokozile and Ndumiso when she says, "and you have *dy* over *dt*" (also using the symbols as names, rather than the meaning in the task context). She makes the link to the problem text using the word "the" and the speeds (100km/h and 75km/h).
4 (53-70)	Thokozile is attending to Kelsa's statement about the derivatives because she develops this with "and then". She asks what "*x*" and "*y*" will be, using the variable names rather than their meaning in the task context (distance). This suggests that she had assigned the 100km/h and 75km/h identified as *dx/dt* and *dy/dt* by Kelsa to *x* and *y* – this is confirmed by looking at her written work in which she has substituted these values for *x* and *y* in her Pythagoras equation and got *z* = 125: $75^2 + 100^2 = z^2$ $\therefore\ 125 = z$ Ndumiso asks the same question about "*x*" and "*y*": "what's your *x* and what's your *y*? ". Kelsa suggests they use "Pythagoras", identifying the operation by giving the name of a

	theorem. It is not clear whether she is referring to the query from Thokozile and Ndumiso about finding "*x*" and "*y*", or whether she is talking more generally about the method for the problem. But Ndumiso gives negative feedback by indicating the Pythagoras is used to calculate "z" (naming only, not giving the meaning). Thokozile clarifies aloud that they are finding *dz/dt*, to which Ndumiso gives positive feedback. He then tries to give advice to Thokozile, pointing to Thokozile's Pythagoras equation, but giving conflicting information; he tells her to substitute 75 and then tells her she probably won't need the equation. In the meantime Thokozile has started to use implicit differentiation with her Pythagoras equation, and brushes Ndumiso's pen away telling him in isiXhosa to "wait". But when Ndumiso sees the "2*x*" in her equation he points to it and repeats the question, "what is *x*?". Thokozile seems frustrated, "exactly that's what I'm asking", pointing to something she calls "this" in the problem text (it seems she is still deciding how to assign the numbers 75 and 100 to her variables). The Episode ends with Lwazi pronouncing that "we're trina find *dz* over *dt*↑", the rising intonation at the end suggesting that he is wants feedback.
5 (71-79)	The Tutor has approached the desk and possibly heard Lwazi's comment about finding *dz/dt*. So he asks, "what are you trying to find?", attending to the student's discussion as well as to the boxed text requiring that this be made explicit. The students respond immediately with "*dz/dt*" and the Tutor responds by asking, "what is *z*"; it is not clear what type of answer he requires, but the students simply point to the side of the triangle that they have labeled as "*z*". There is no verbal description of "*z*" but only using reference words like "that's" and "this one here" and gestures pointing to the diagrams. Ndumiso suggests a way forward by referring to an operation; "you just square" (and pointing to Lwazi's book, possibly attending to the Pythagoras equation).
6 (80-85)	Kelsa is also attending to the drawing in Lwazi's book, but corrects the students, indicating that the speeds 75km/h and 100km/h are "rates" (using the mathematical term), indicating that what Lwazi has called "*x*" is actually "*dx/dt*". Once again there is no reference to the meaning of these symbols.  This prompts Thokozile and Ndumiso to repeat their earlier query about what values to assign to "*x*" and "*y*". Lwazi wants "values" to assign to "*x*" and "*y*", but does not inquire about the meaning of these variables.

7 (86-101)	Since the students are still asking about "*x*" and "*y*", the Tutor intervenes with, "may I ask?" and asks "what is *x*?", not indicating whether he wants a value or the meaning. He does indicate that this is not the same as *dx/dt* by saying "not *dx/dt*". He then gives a hint that he wants them to make a link to the meaning of the variable in the task context when he asks for the "units". Thokozile and Kelsa are able to respond immediately with "kilometers", which the Tutor develops to give the meaning of the variable as "the distance". There is a pause, and the Tutor picks up that they are expecting a follow up question. The Tutor then does follow up be asking the students again what they are trying to find. When Thokozile and Kelsa respond with "*dz* over *dt*" as they did earlier, the Tutor emphasizes that it is not distance (although he does not develop that it is the speed).
8 (102-110)	Thokozile seems to agree with the discussion about finding *dz/dt*, but she attends to the derivative equation in her answer book ($2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$) to argue to the Tutor, "we are going to need values for *x* and for *y* which we don't have↑". Here she talks about the operation of substitution and the variables *x* and *y*, but with no meaning in the task context. The Tutor validates this query with "okay" and gives a suggestion regarding strategy, indicating that the information is not explicit in the problem text, but that there is enough information in the text to calculate *x* and *y*. This encouragement by the Tutor seems to cue Ndumiso to look at the problem text and he finds the value of 2 hours (this is the value they have not used yet); consistent with his earlier approach he makes a pronouncement (with confidence) that "it" is 2 hours. Both Thokozile and Kelsa query whether Ndumiso means that *x* is 2 hours, to which he responds positively by trying to explain this. He refers to *dx* over *dt* as being the "rate of change" (possibly attending to developing Kelsa's text about the "rate" in Episode 6) with respect to time, so *x* must be 2 hours. Here he does not link to the problem text, and does not consider the meaning of the variables (the discussion in Episode 7 was about *x* being a distance, but Ndumiso is assigning this a value of 2 hours).
9 (111-122)	The Tutor has heard the discussion about the "2 hours", and asks what the "2 hours" can be used for; this is a shift from attending to the meaning of the variables to how a number can be used. Kelsa responds immediately, making a link to the task context; "to find the distance ... travelled after 2 hours". The Tutor gives positive feedback and develops this response, linking the "distance" to the variable *x*, and prompting Kelsa to complete his statement, linking the "distance" to the variable *y*. Both Akbar and Kelsa then make suggestions about the values of *x* and *y*: Akbar suggests, "So *x* is 150↑" and Kelsa pronounces, "So for the one it's gonna be a 100 and for the other it's gonna be 150?", both wanting feedback from the Tutor. However, Akbar corrects Kelsa himself, changing the "100" to "200" as required. In all cases they give the values only, with no units or link to the meaning in the task context. The Tutor gives positive feedback, but tries to suggest that they must evaluate themselves.

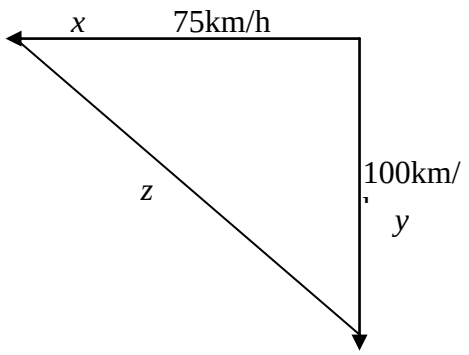
10 (123-147)	At the end of Episode 9 Akbar and Kelsa are using the correct values of 150 and 200, but do not pronounce the units or make explicit links to the meaning in the task context. In this Episode Akbar assigns the value of 150 to *dx/dt*, appealing to the Tutor for feedback; "so *dx* by *dt* is ... like 2 to 150?". The Tutor instructs him not to confuse the "distance" with the "rate" (giving meaning, but not using "speed" yet). He then asks Akbar, "what you are telling me?", and Akbar responds by suggesting the *dx/dt* is the "rate", this time giving the meaning (again, appealing to the Tutor for feedback). But the Tutor again asks "what" is *dx/dt*, seemingly wanting the value of 75km/h, but the student again gives the meaning, "the rate at which this car is moving ... after two hours". Nqobile suggests two numbers, "150" and "200", but this is not followed up. The Tutor reworks his question by asking for "the answer", which seems to signal to Akbar that a value is required, to which he responds with the same value as earlier, that is, 150 (with no units). The Tutor gives negative feedback be repeating the student's answer as a question, "150?", and again asking him to explain. The student seems at a loss, looking at his diagram and giving the units "kilometres per". Again, the Tutor requires an explanation, and the student responds by posing his answer as a question to the Tutor, "is it travelling at 75km/h?" Here he is using the units kilometers per hour. The Tutor gives positive feedback in the form of "okay" and makes a link between rate and the symbol *dx/dt*. Akbar continues to describe the car in the context, "it travels" seems to cue him to think about the distance (150): "after 2 hours it travels oh the distance is 150? Akbar is then able to conclude, "So *dx* by *dt* is equal to >75 kilometres per hour?< ". Akbar seems happy with this conclusion, and looks to see what Thokozile is doing. Yet the Tutor continues to try to get him to reflect on the importance of not getting the distance and the "rate" confused, and refers back to his initial introduction in which he encouraged the students to "write down". He emphasises the importance of writing down the units, which he deems to be key in alerting the students to the meaning of the mathematical symbols. In his last reminder he uses the word "speed" rather than "rate", but does not explicitly link the two terms.
11 (45)	While the Tutor and Akbar are talking, Lwazi is writing in his answer book. He performs the mathematical operations (implicit differentiation on the Pythagoras equation and rearranging the formula to make *dz/dt* the subject): $2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$ $\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$ 

	And only then does he write down "Given" below his working and the diagram: Given: $\frac{dx}{dt} = 75km/h$ $\frac{dy}{dt} = 100km/h$ He pauses when he gets to "To find": To find: *d*
12 (148-173)	Kelsa suggests the answer of 250 for "*z*". In the discussion that follows the students refer only to the symbol "*z*" without its meaning and give no units to the "250". Thokozile pronounces that she also got an answer of "250". Thokozile is also explaining the operation for the next step, that is, "you gonna differentiate according to time ... in that equation and then you gonna substitute *z*". She seems to be talking to Akbar and Ndumiso next to her. Ndumiso pronounces his answer of "125". He goes on talk through the operation he used; "75 squared plus 100 squared". Both Thokozile and Kelsa correct him talking through the same operation, but just changing the numbers to "150" and "200". Ndumiso queries why Thokozile is using the values of 150 and 200 in the Pythagoras equation, but before she can explain (she starts by referring to "75"), he answers his own question, "after two hours" suggesting that they must use the distance after 2 hours. Lwazi then asks Kelsa "what is *z*?", again using the symbol with no meaning. He does not evaluate the answer, but challenges Kelsa, "how right are you?" to which she replies without giving an explanation. Thokozile is doing calculations on her calculator. Nqobile appeals to Kelsa, referring only to "250", to which Kelsa responds with positive feedback.
13 (174)	Thokozile has proposed an answer to the group of "125", suggesting that this is her answer for *dz/dt*, but not naming it in any way or giving units (she pronounces "it" only). There is no immediate response.
14 (175-179)	Akbar appeals to Thokozile, asking her how she got "the 250" (attending to her answer for *z*). Thokozile responds to this by giving the operation "substituting" and using the symbols "*x*" and "*y*" with no value or meaning in the task context. Nqobile has calculated *z* correctly: $z^2 = x^2 + y^2$ $= 22500 + 40000$ $\sqrt{z} = 250$
15 (180-191, 196-197)	Lwazi pronounces, "I get 125↑", the rising intonation suggesting that he wants feedback. But he does not indicate whether this is the answer for *z* or for *dz/dt*. Thokozile has attended to this and asks for clarification, "Did you get 125?". But she also does not indicate what the value means or give the units. It is only when Kelsa asks the students to "please share that with the group↑" , that Lwazi repeats himself, but this time identifying *z* with his "125". Thokozile and Ndumiso interpret this as referring to *z*, the distance after 1 hour, (although the word "distance" is not used), and give him negative

	feedback, for example, Ndumiso says "that's after an hour↑". A few lines later (Akbar and Thokozile are now having a discussion, see Episode 17) Lwazi asks again, "Why did you guys get 250?" Ndumiso responds by attending to the number of hours; he reminds Lwazi that "z you get is for just after 1 hour … remember here it says two hours".
16 (192-194)	Nqobile is talking quietly to herself, and checking from her diagram that the units for her value of *z* ("distance") are km. She has revisited her working in Episode 14; she has crossed out the root sign for $\sqrt{z}$ and written: $z = \sqrt{22500 + 40000}$ $z = 250$ She then adds "km" to her answer of 250, suggesting that she is attending to the meaning of *z* as distance.
17 (187, 193-195, 198, 200, 203-220)	Akbar continues to question Thokozile about her Pythagoras equation, asking twice, "what did you put in here?" She describes the operation for finding the distance after 2 hours, linking to the meaning in the task context ("it"): "because it said after 2 hours it said per hour it travels 75 kilometres right? So when you want for↑ after 2 hours so you double that well times it by 2 basically". She starts to give Akbar negative feedback about his use of the variables *x* and *y* (it seems that they have assigned the numbers 150 and 200 to different variables), but then she halts herself. Akbar repeats the operation verbally as she speaks, wanting feedback after each phrase, like "plus↑... 200 squared". Akbar continues asking her for feedback and she is explaining the operation of substitution into the Pythagoras equation and correcting him as he speaks; "150 squared ((*Glancing up briefly*)) ... plus 200 squared". They talk through the operations only, with no meaning or units given to the numbers. Akbar pronounces again, "and how do you get 250?". Thokozile is getting impatient, suggesting by her exclamation, "hayibo" and further explanation; "using the calculator and doing this thing↑" (she is attending the finding the square root of *z*).
18 (221-223)	While Akbar and Thokozile are talking, Lwazi is explaining to Ndumiso and Kelsa what he has done (the Tutor started to question Kelsa, but she indicated that she was listening to the discussion in the group). He takes personal responsibility for his explanation, an explanation that focuses on the operations and symbols/numbers, for example, "I went... *dy*↑ that by that (unclear) *dz* by *dt* on this side ...". He also uses gesture to point to where he is substituting ("here"). He uses an incorrect value of *z* (125), but the others do not challenge him. At the end of his explanation, Kelsa asks him to repeat what he did after "you got 250 for *z*".

19 (224-227)	Nqobile is explaining her method to the Tutor. She has calculated $z = 250$ correctly (see Episode 16), and then substitutes this into her distance/speed/time formula to get 125km/h. $$s = \frac{d}{t}$$ $$= \frac{250}{2}$$ $$= 125\text{km/h}$$ She explains this to the Tutor, describing the operations but also giving the units of the numbers (hours and distance): "Then you divide by 2 hours↑ to find the speed↑". The Tutor gives negative feedback but tempers this (also kneeling down next to the desk so that he is at the same eye-level to her); he names what she has done (referring to "average speed") and says he can see what "you kinda getting at", but that this is not what is required. He reminds her that this is a workshop about "implicit differentiation" so she should be using this operation. He also suggests that they should be working together in the group.
20 (228-253)	However, when he sees Nqobile look shyly across the room he discusses further just with her. He begins by working with what she has written, the Pythagoras equation, which the Tutor names by linking it to distance; "equation for the distances". He then attends to the "speed", linked to 75km/h. He seems to want the student to give the symbol (*dx/dt*) for the speed 75km/h; "what is a way of writing down the speed ... uh ... in terms of ... like you know thinking thinking differentiation", but it takes a while for Nqobile to establish what is required as she tries different mathematical words (at no stage does he ask for the "symbol"). She begins by referring to the "rate of change", an answer he reinforces by repeating it and adding to it ("of the distance"). His wording thus suggests he is looking for another word for rate of change, which Nqobile gives as "the derivative". Again positive feedback from the Tutor through repetition, but adding, "with respect to?", and Nqobile completing the sentence. He links the four terms speed, rate of change of distance, derivative of the distance, and 75km/h. But he is still in search of the symbol, something he has not made explicit. He has to return to the use of *x* on her diagram, and asks how one "writes" the "derivative with respect to *t*". Nqobile then proceeds as required by the Tutor and writes $\frac{dx}{dt} = 75$. The Tutor then asks for the value (75) associated with the *dx/dt* that is given by "they" (the text). The Tutor then wants her to associate the 100km/h with the symbol *dy/dt*, but again it takes a while for Nqobile to determine from the Tutor's wording and pointing at the 100km/h what is required. The student's responses are tentative each time, requesting feedback, but she is not afraid to ask for clarification. By the end the student is talking about *dz/dt*, with no meaning attached, but is using the required symbols. The Tutor moves away in response to a call from Ndumiso, but instructs her to use differentiation on her equation and to work on her own now.

21 (254-266)	While the Tutor and Nqobile are talking, Thokozile is explaining to Akbar (following on from calculating the value of *z* in Episode 17); "And then you know this equation, right? ... ... the Pythag equation ... you're gonna differentiate this ... using ... time differentiate according to time"). Kelsa is attending to what Thokozile says and she asks why they are differentiating *z* squared and not *z*. Both Thokozile and Ndumiso respond by explaining that they only have the Pythagoras equation for relating the variables *x*, *y* and *z*, suggesting they are attending to the instruction in the boxed text to find a "formula linking the variables" and to the fact that these problems are called "related rates problems". Initially Ndumiso suggests there is "no relationship" between *x*, *y* and *z*, but then refers to the Pythagoras equation as the only way to relate the variables.
22 (267-277)	Lwazi pronounces just an answer of "2.33". There is confusion for Thokozile and Ndumiso who are not sure what this answer is for, they think it is *z*, but Lwazi has *dz/dt* written in his book: Following on from his equation (see Episode 11) he has: $\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$ $= \frac{150 + 200}{125}$ $= 2{,}33 \text{ km.h}^{-1}$ Lwazi spends some time looking at his work, and then pronounces aloud that he knows what he has done.
23 (278-286)	Lwazi turns to Kelsa next to him for clarification on the numbers she has squared, and she explains using the distance/speed/time formula and the meanings "speed" and "distance". Lwazi seems confused by Kelsa's inpuy; she intends him to use the formula she has described to find the distances after two hours, but he thinks she intends him to find speed (which he claims is "given"). Again there is confusion about the symbols *z* and *dz/dt*; Lwazi seems to think he is finding *z*, while Kelsa talks about finding *dz/dt*. While Akbar is using his calculator, he and Thokozile are discussing what problem in the Resource book to work on next

24 (287-300)	Lwazi then turns to Ndumiso to ask what he got for "*z*". Ndumiso responds with an answer of 250 with no units, claiming that 125 is "after 1 hour". He does not indicate that these are distances. Ndumiso then points to Lwazi's diagram and calculations, linking the speeds on the diagram (by pointing) to the distances (by pointing) in the calculation: $\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$ $= \frac{150 + 200}{125}$ $= 2{,}33 \text{ km.h}^{-1}$  Ndumiso indicates that this is "after 2 hours", but does not use the meaning of the numbers (the distances and speeds). For example, he talks about the distance as "it": "now we're trying to get it after two hours ... so times by two ... remember you were trying to get it for two hours" . Lwazi claims to follow what Ndumiso has said, but Ndumiso sounds skeptical; "I hope so". In his written answer Lwazi has crossed out the $\frac{150 + 200}{125}$ and written: $\frac{150 \times 75 + 200 \times 100}{250}$, giving an answer of "125k".
25 (297, 301-337)	Akbar pronounces his answer of 125km/h aloud, and includes the units for speed. He asks Lwazi if he also got "125". Ndumiso appeals to the Tutor (who has been working with Nqobile) for the answer. The Tutor replies by asking them what they got; Thokozile and Ndumiso both give 125, and then Akbar adds the units of km/h. The Tutor repeats the answer of 125km/h and then confirms that it is correct when he looks at his "magic solution". The students are pleased with the positive feedback, but the Tutor appeals to them to work together as a group. Akbar, Thokozile and Ndumiso have working that looks the same: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$ $2(150)(75) + 2(200)(100) = 2(250)\frac{dz}{dt}$ $22\,500 + 40\,000 = 500\frac{dz}{dt}$ $125\text{km/h} = \frac{dz}{dt}$ (Akbar has just *dz* in this line)

	The Tutor checks again on Nqobile. Nqobile then asks Kelsa, "Is this what you did? Can I see your number 2?", and Kelsa starts explaining the operations she has used. In her final written answer Nqobile has left her calculation shown in Episode 14, but has continued with: $\cancel{\frac{dz}{dt}}(z^2) = \cancel{\frac{dz}{dt}}(x^2 + y^2)$ $2z.\frac{dz}{dt} = 2x.\frac{dx}{dt} + 2y.\frac{dy}{dt}$ $2(250)\frac{dz}{dt} = 2(150)(75) + 2(200)(100)$ $\frac{dz}{dt} = 125\text{km/h}$

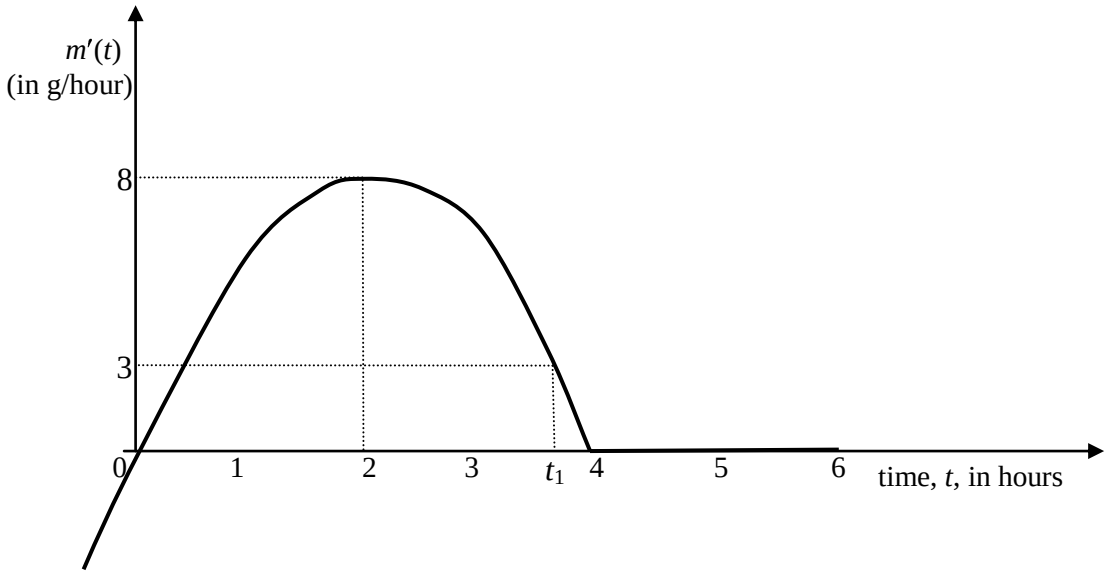
APPENDIX O GROUP 2

ACTION ON THE CHEMICAL REACTION PROBLEM

Quantities of two chemicals A and B are mixed together in a reaction chamber, and they react to form a new product, X.

The **rate** at which the product X is formed is given by $m'(t)$, where m is the mass of the product formed, in grams, and the time t from the start of the reaction is measured in hours. The graph of $m'(t)$ is a **parabola** graph until time $t = 4$ hours, after which it is zero.

It is also given that, from the start of the reaction, some of the product X is removed from the reaction chamber at a constant rate of 3 g/hour.



(a) Write down an expression involving an integral that gives the **total** mass of product X **in** the reaction chamber after a time of t hours.

Episode	Description
1 (1-49)	Some of the students are starting on the Chemical Reaction Problem, but Vuyani and Mpumelelo are completing the previous question. Lungiswa is reading the problem text aloud and is underlining in her book. She comments on the "Haber process" and links this to something the lecturer said about this in class. Bongani expresses surprise at having the "Haber process" in Maths; "Haber process↑ In Maths↑". Bongani, Vuyani and Mpumelelo are talking a mixture of Xhosa and English as they write and read. Most of this involves jokes beyond the task context (for example, Bongani is talking about joining the police, and they are referring to a character in a soap opera). There is also some reference to the camera being used for the research, and a discussion between Mpumelelo and Bongani about Mpumelelo leaving class early to go the post office; Mpumelelo has asked the researcher if he can leave early to attend to something concerning his financial aid application for the following year. Bongani accuses him of lying to the researcher, to which Mpumelelo responds (in isiXhosa) that he is going to post his future.
2 (50-52)	Bongani changes his focus to talk about the problem, and asks Lungiswa for her answer for (a). She has written an integral suggesting that she links the total mass with the anti-derivative of the rate, and she gives the process for calculating this. She also identifies the integral as a function of *t*. Yet she does not make an appropriate link to the context, giving the total mass of the product formed and not the amount of product in the chamber as required by the problem text ("the **total** mass of product X **in** the reaction chamber"): $\int_0^t m'(t) = m(t) - m(0)$ Siyabulela has the same expression as Lungiswa in his answer book.

(b) Explain very clearly the significance/practical meaning of the time $t = t_1$ in the graph above.

Episode	Description
1 (53-96)	Lungiswa is reading the text of question (b). Bongani refers to a particular problem termed the "Joe problem" that was done in the class and he asks Lungiswa what she remembers from the class. Both students pronounce the number "10", and Lungiswa also pronounces "15". They are attending to the horizontal line at 3 on their graph and linking this to a horizontal line (at 15, indicating the rate of spending money) that was used in the "Joe problem". Lungiswa seems to remember the problem from class, but she does not volunteer information easily. Bongani and Lungiswa are talking in isiXhosa for this general discussion about procedure, for example, referring to the lecture in which the "Joe problem" was discussed and trying to remember what the lecturer said when the problem was solved in lectures. Some of the mathematical content words are borrowed from English, for example, "ipoint" and 'igraph", but the sentences are given their structure by the isiXhosa. Lungiswa suggests that the point where t_1 cuts the graph is the maximum, "ufuna ukuthi (you want to say) this is the maximum", but she does not develop this into the maximum <u>of what</u>. She pronounces terms, like 'i-endpoint" and "point of inflection",

	but all are rejected with a "no" by Bongani as he searches through his papers for the Joe problem. Both Bongani and Lungiswa are looking for something they wrote down about "the point" in lectures. Lungiswa then pages back and finds her lecture notes on the Joe problem – she says something about "starts to decrease". In this Episode Lungiswa makes a start at statements that could be appropriate, but does not develop them any further in relation to the context.
2 (97-107)	Vuyani appeals to Lungiswa for help. He pronounces that at t_1, 3 grams is being removed (he is attending to horizontal line at 3, Sentence 4 of the set-up, and linking the graph to its meaning in the task context). He queries whether this should be included in the "explanation" for (b); "do we have to include the ... the rate ... the constant rate ... which is 3g per hour?" Lungiswa gives positive feedback, rewording the statement and making a link to the task context, "t_1 the rate at which the rust is being removed is ... ja". This is an appropriate answer, but the students seem to think that there should be more (possibly making a link to similar problems in the Course).
3 (108-126)	Bongani is still making reference to what was done on the "Joe problem" in lecturers and is looking back in his answer book. There is some banter in isiXhosa about Bongani not focusing in class and discussion about his social life (it seems form the interaction that Bongani and Lungiswa are currently in a relationship). He eventually pronounces in isiXhosa that he is just going to write what he thinks. In the meantime Lungiswa is writing in her book. Her first attempt is "After t_1 hours the rate at which the product A is formed is 3g / hour". This suggests that she is looking at the parabola graph and the meaning in the task context when t_1 is read off on the graph (rather than looking at the relationship between the parabola and the horizontal straight line graph as required in the solution). She does not attend to her use of "rate" in the answer requiring "practical terms".
4 (127-182)	Mpumelelo starts talking; he is attending to the time and is speaking in a mixture of isiXhosa and English. Bongani pronounces an answer, avoiding the use of "rate" and "rate of change" and using the word "changing" or "iyatshintsha" instead, for example, "The number of product is changing by ... by 3". He describes what is changing in various ways, either "it", or "the mass" or the "number of product" or "the mass of the product". In doing this he is attending to the problem text requirement that they use "practical terms" (Vuyani attends to this explicitly). Bongani, like Lungiswa, is attending to the meaning of the parabola graph only. He does not give any units for the value "3". He has not yet written anything down, all his attempts so far are verbal, and Siyabulela accuses him (in Sesotho) of talking too much. Mpumelelogives him positive, content-free feedback. But he is critical of Bongani's use of the word "change" which he does not regard as "practical terms", and is proposing the word "moves" (-muva") instead. Lungiswa simply repeats her initial answer about "rate at which the product is formed". Bongani pronounces another attempt which tries to avoid the use of rate, this time he attends to the rate of change, but again leaves out the units: "so the mass of the product itshintsha phayana (it changes there) by 3 to 1". At one stage Mpumelelo pronounces the units of the value 3 as "hours".

5 (183-248)	Mpumelelo, Vuyani and Siyabulela are attending to the problem text and the statement about the removal of the product. For example, Mpumelelo says, "At a constant rate of 3" and actually points to the problem text. Siyabulela pronounces (tentatively), "Maybe that's when the the ... that X has been removed↑ ... at t_1↑". It seems that these students are attending to the horizontal line at 3 (and making the link to Sentence 4), and not the parabola graph (as Lungiswa and Bongani are). Both Vuyani and Mpumelelo argue using a mixture of isiXhosa and English. Lungiswa goes back to reading the question about "significance" and they start looking around the classroom for the Tutor again. While they are waiting for the Tutor, Mpumelelo is getting anxious about leaving early (in isiXhosa). Siyabulela, Vuyani and Bongani attend to the word "rate" in Sentence 4, and the rate on the vertical axis ($m'(t)$) and agree that they are both "rates". Bongani uses this to argue that "it is zero", but this is not followed up. They do not seem to know how to go on from here.
6 (249-265)	Lungiswa is now shifting the attention away from the time t_1 on the graph to "the start", and is arguing that the product is removed from the start; "from the start of it ... some of the product is ... ja because ... from the ... start". Bongani agrees. There is discussion in isiXhosa about getting the attention of the Tutor. Lungiswa has revisited her first written answer in Episode 3 and crosses most of it out: "After t_1 hours the ~~rate at which the product A is formed is 3g/hour"~~.
7 (266-271)	Now Siyabulela attends to the parabola graph and "the rate" (he is not specific about the rate <u>of what</u>), and he demonstrates positive and negative gradients with his hand on the graph. He argues that "the rate" is increasing before 2 and decreasing after 2. Lungiswa gives positive feedback in the form of, "ja … it's increasing at a decreasing rate", but she is not explicit about what the "it's" refers to.
8 (272-288)	The Tutor arrives and the students indicate they need help with question (b). He does not ask what they have done, but makes a link to the task context rather than the graph (he also links to a similar attempt he made in the previous question). He models the chemical reaction using words and gestures; "we're imagining a real situation ... like they're making product X so we are imagining something that looks like a stove maybe and ((*Tutor is looking at Bongani and makes a round shape with his hands to illustrate the stove*)) … there is more and more stuff appearing in the stove ((*using hands to demonstrate*)) uhm". The Tutor rereads the question aloud, and then traces his pen over the parabola graph and pronounces the names of the graph. Then he seems to change his strategy and attends to the task context again. He asks the students what two processes are going on. He waits for an answer, indicating through his encouraging body language that he is listening. Siyabulela seems to want to say something, but does not. Lungiswa, Bongani and Vuyani all pronounce that the "product is formed", attending to the formation of the product X as described in the problem text. The Tutor gives positive feedback using "okay" and repeating the wording of the students and adding to it. But he also gives negative feedback by indicating it is only "half" of what he wants, and he pronounces, "the other process is that we are ... sucking ... product X out↑" and uses gesture to demonstrate the task context. The students give positive feedback to the Tutor in the form of "ja" and by nodding.

9 (289-303a)	The Tutor then introduces the word "rate" in relation to these two processes. He attends to the problem text (Sentence 4) to pronounce that, "we suck ... product X out at a constant rate", and to the parabola graph to pronounce that "the ... rate that we create the new stuff ... is not the same ... ... it it speeds up". He explains "constant rate" by rewording in everyday terms. He questions where the rate of formation is the fastest, and Siyabulela is able to respond by attending to the graph at $t = 2$. The Tutor indicates that "it speeds up" before and "it slows down" after 2 hours (using the pronoun "it" to reference the rate of formation). He explains this by switching to another context, this time rain falling during a thunderstorm, and attends to the difference between where the "rate or rainfall" will be maximum (in the middle of the storm) and where the maximum rain will have fallen. So far the Tutor has been answering the questions he has posed, but in the last example he gives two possible answers, ("in the middle of the storm" or "at the end of the storm") for the students to choose from. Bongani illustrates his answer of "the end" by pointing to his graph. The Tutor gives positive feedback and develops his explanation using the rainfall context, he uses everyday terms to distinguish between the rate of rainfall and the amount of rainfall; "at the end of the storm it doesn't really matter ... uhm ... that the rain is falling slowly↑ ... cause you are still adding more and more ... rain↑ "right↑". He develops the context further by indicating that the pond could have a leak.
10 (303b-304)	The Tutor's next step is to link (using gestures and words) the rainfall context introduced in Episode 9 to the chemical reaction context and the graph; "so ... the parabola ((*Tracing over the curve in Mpumelelo's problem text*)) represents ... that's that's the equivalent of the pond filling up ((*Using both hands to show the water level rising*))... we're creating ... product X↑"
11 (305-308)	The Tutor then attends to what is happening at t_1 as required in question (b) and reintroduces the term "rate" (referring to the label on the vertical axis). For the first time he names this "the rate of change of the mass". He begins by indicating that at t_1 the rate at which the product is formed is 3g / hour, with an emphasis on the units and getting students to complete his sentence. He then questions them about the rate of removal at t_1.
12 (309-318)	The Tutor then switches back to the pond context and asks, "is the pond ... raising its level↑ or ... decreasing its level↑", suggesting as in his other questions of this type that these are the only possible answers. Mpumelelo responds with "decreasing", indicating that he is still attending to the removal of the product only as in Episode 4. The Tutor softens the feedback, indicating that this is correct, but reminding him that the product is also being formed at 3g / hour at the point t_1.

13 (319-326)	Bongani makes a contribution, pronouncing that at the point t_1, they are "adding and removing 3", suggesting that he is attending to the intersection of the two graphs. The Tutor attends to this response and questions about the "level" (with reference to the pond). Bongani responds with a gesture of nodding his head an holding his hand horizontal to suggest that the level is not changing. The Tutor interprets this in words as, "It stays the same↑".
14 (327-350)	Mpumelelo is still attending to the text about the removal of the product in Sentence 4. The Tutor responds by indicating that this removal is happening all the time. He then attends to the graph at $t = 2$ and reads numbers off the graph to argue that the rate is 5g / hour. He then generalizes, indicating that when the parabola is above the straight line through 3, more is being added than is being removed. He traces parts of the parabola graph as he speaks, and models the task context of the level rising with his hands. At times he pronounces what is increasing, that is, "the total amount in the chamber" or the level of the pond or "the stuff", but at other times he does not pronounce this, for example, "it is level" or "it is decreasing". Towards the end of this Episode the Tutor is looking mainly at Bongani who is responding positively by nodding and giving mainly content-free feedback. At the end he refers back to Mpumelelo on his left, "you do not seem convinced", as in encouraging him to seek help. Mpumelelo's body language suggests he is not convinced, but another group of students call the Tutor and he leaves, encouraging the students to "talk about it".
15 (351-399)	There is a discussion (mainly between Bongani and Mpumelelo) about the merits of the Tutor's explanation. This take place mainly in isiXhosa. Lungiswa also indicates (in Sesotho) that the Tutor went on too long, Bongani agrees but says he "got it". Bongani is attending to different times and both graphs when he makes pronouncements about subtracting 2 from 3. But he does not specify that the "3" and "2" represent rates. He then pronounces an answer and explains it, "It doesn't the rate doesn't change ... ... you remove three and take three". Lungiswa requests that he give the answer in "English now". Vuyani queries what will be happening at other points, suggesting at $t = 4$. Bongani talks about "it" decreasing, and sometimes indicates that "irate" is increasing or decreasing. He talks about "it" being at a "standstill" and about the "maximum point". He explains this to Vuyani by subtracting values on the vertical axis (8 minus 3 and 3 minus 2). He gives no meaning to these numbers nor to why he is subtracting; it seems he is reproducing what the Tutor did earlier. He also uses the rainfall context used by the Tutor. It seems that Bongani is distinguishing between the rate of formation and the amount of the product, suggesting that product can be formed, although at a decreasing rate. However, his language is not clear. He also refers to increase before and decrease after the point, and the point at a standstill, but not being clear about the "rate".
16 (400-428)	Lungiswa identifies the lack of clarity in the language when she asks, "so it's the maximum point of what?" Bongani continues to use reference pronouns that are not clear and he is no longer linking to the task context, "serious it's a medium point between i-increase and decrease". Lungiswa appears to be attending to Bongani's suggestion that something is changing

	from increasing to decreasing, when she pronounces the term "point of inflection" (she is attending to where the rate changes from increasing to decreasing, but has not linked to the task context). Both Bongani and Mpumelelo respond negatively, still using unclear language and talking about "it" changing. The students seem to be getting frustrated, particularly Bongani who keeps trying to explain in a mixture of isiXhosa and English. But his language is still unclear, he is talking about the rate being "stable" and "not changing" and adding and removing "3" as if it is a mass. Mpumelelo wants to write an answer, and starts calling for the Tutor. Bongani is repeating his earlier statements, and settles on, "u-t_1 there is no change in the formation of ilantuka (the thing), iproduct of X", this time making some link to the task context.
17 (429-441)	Siyabulela then joins the conversation, and presents an answer for feedback, "Oh so that does it mean that the rate that you put in is the same as the rate that it comes out?". He is attending to the rates represented by the parabola graph and the horizontal straight line graph, making a link to the task context, and specifying what the rate is. Bongani responds positively, but still using "it" in "it is equal". Lungiswa rewords this as "the rate is equal", and is irritated with Bongani for telling them "stories" (in isiXhosa). Mpumelelo rewords Siyabulela's answer appropriately, linking it further to the task context, "the rate of formation is the same as the rate of removal".
18 (442-468)	Lungiswa is now writing in her book and talking aloud as she does this. Although the students had appropriate verbal answers in Episode 17, now that they are writing down they seem to be avoiding the word "rate". This also points to their confusion about what is equal. For example, after some thought about the second part of her answer Lungiswa writes, "The rate at which the product is formed is equal to the amount of product removed". In her written answer she has added another version which does not make use of the word "rate" at all; "Amount of product added is the same as the amount of product removed". Mpumelelo also comments about "rate" and "practical terms" suggesting that he is attending to the demand in the Course not to use rate for everyday terms and the instruction in the problem text. He comments on the English being complicated, and settles on an answer of, "The amount of product formed is equal to the amount removed". Bongani is talking about "… the amount of formation" of the product and he suggests writing something down so that he can look at it. He writes, "the amount of product that is added is = to the amount of product that is removed". Only Vuyani and Siyabulela attend to the rate in their answers, for example, Vuyani writes, "When $t = t_2$ the rate of the mass added is equal to the rate of the mass removed". Mpumelelo is talking about leaving early again.

(c) Explain very clearly the significance/practical meaning of the local maximum in the graph of $m'(t)$ at time $t = 2$ hours.

Episode	Description
1 (469-503)	Siyabulela begins by making a possible solution public, although in tentative way and asking for feedback. He uses the term "rate of production", but then questions, in Sesotho, what is happening at 2. He and Lungiswa then discuss possible answers, while Vuyani and Bongani are talking. Siyabulela is attending to the parabola graphs and the rate that is "increasing" before $t = 2$ and then "decreasing" after. He uses this to settle on "The rate at which … maybe the product is formed … is maximum", requesting feedback from Lungiswa. Lungiswa attends to the meaning of the parabola graph by reading across from the point $t = 2$ on the graph to the vertical axis and confirms, "this is the rate at which it's produced".
2 (469-503)	Vuyani is attending to the word "maximum" in the question for (c) and to the point t_1 on the graph which they earlier decided was a "maximum point" (this maximum refers to the maximum amount of the product X, but this was not made explicit). Bongani argues consistently that the rate for (c) is a "maximum rate" (he does not pronounce the maximum of "what"), and Vuyani continues to question what, then, is the maximum in (b). This discussion takes place mainly in isiXhosa, but with mathematical terms borrowed from English, like "i-maximum" and "i-rate". Mpumelelo suggests that the use of the word "maximum" by the Tutor for question (b) is not "a good word" (they have not used this word in their answers for (b)). But Bongani refers to the rain context again. He points to the graph and also uses his hands to model the amount of water; he distinguishes between the rate and the most water. As he ends he switches back to the chemical context and distinguishes between the rate of production and the "maximum product".
3(504-522)	Mpumelelo attempting an answer, and Vuyani is listening. He seems to be avoiding the use of the word maximum and using "the most"; "production is formed the most" and "high amount of production formed". Is he using "amount of production" for rate? Bongani responds negatively. It seems he is interpreting what Mpumelelo is saying as the maximum amount of product, as he refers to $t = 4$, and repeats the word "formation" (he is attending to the meaning of the parabola graph). He does not indicate what the "maximum" is of. Vuyani tries to enter the conversation but does not get the space. Mpumelelo argues with Bongani; it seems he is attending to the rate, but making pronouncements that Bongani interprets as the mass of the product. Bongani refers him back to the rain example used by the Tutor, to which Mpumelelo immediately responds that this refers to a different point, by pointing at the maximum turning point? Both Bongani and Mpumelelo are talking in isiXhosa for their discussion about the answer, but with English for the mathematical terms like "i-rate" and "i-maximum" and "formation". The Tutor is in the background, listening to the discussion. At one point Bongani requests his help, but The Tutor is not given space as Bongani and Mpumelelo continue debating.

4 (523-542)	Bongani calls the Tutor and refers him back to his use of the rain context. He points to two different places on the graph ("there" and "here", possibly for $t = 2$ and $t = t_1$) and says that the one refers to the where the storm is "heavier" (attending to the rate?) and the other where the "water" is at its maximum. Bongani argues that at $t = 2$ the rate is "high". Siyabulela interrupts and argues that they (he and Lungiswa?) already have this.
5 (543-558)	Mpumelelo is questioning the Tutor. He wants to know whether the point $t = 2$ is "the rate" or the "maximum height". The Tutor questions him about his use of the word "height", and Mpumelelo tries to reword, referring first to "maximum point where the product is formed" and then to "highest amount of product is formed" (the "is formed" suggesting he is thinking of rate?) The Tutor rewords to ask whether Mpumelelo is referring to the mass, "the amount, the total amount of X", to which Mpumelelo agrees, but immediately goes back to talking about the "highest production". The Tutor has identified Mpumelelo's confusion and puts it to him, "You can say the highest production, the highest rate↑ of production, or the highest total amount of mass see … now do you understand the difference between the two?" Here the Tutor has reworded Mpumelelo's "highest production" to "highest rate of production", thus reintroducing the rate again. In his statement, the Tutor is explicit about the rate <u>of what</u> and the amount <u>of what</u>. While the Tutor and Mpumelelo are talking Lungiswa, Bongani and Vuyani are writing in their answer books.
6 (559-568d)	Siyabulela talks to the Tutor; he clarifies (his wording is not always clear) that the "rate" (but not "rate" of what) is maximum at $t = 2$ and runs his finger over the parabola graph. The Tutor gives positive feedback, emphasizing the word "rate" in his repetition (but not the rate <u>of what</u>); "Yes, the total <u>rate</u> is at the maximum". Siyabulela goes on to talk about the "amount of product X" increasing at a decreasing rate after $t = 2$, indicating the decreasing part of the parabola graph with his hand. The Tutor then takes a pencil from a student and draws a line from the point $t = 3$ on the horizontal axis to the graph, and attending to the difference between the rate and the amount of the product, "*t* equals 3 we will have more of product X in the chamber than we had when *t* was 2 ((*moves his pencil to the maximum turning point at t = 2 on Mpumelelo's graph*))… because the rate of production is still higher than the rate of extraction" At one stage he asks for feedback from Mpumelelo; it seems he is looking at Mpumelelo to see his response, and then carries on with what "I'm saying". He now introduces "speed that we are adding the product" for rate of formation. He then makes the link to the rain context; distinguishing between the rate (the "heaviest" rainfall) and the amount of water in the pond.
7 (568e-580)	Again, the Tutor asks for a response from Mpumelelo, but seeing his response he decides to work one-on-one with Mpumelelo and moves around the table to sit next to him. He encourages Mpumelelo to explain what he understands, but the student looks embarrassed. The Tutor then asks about the point $t = 2$; "Do you think over here that is when you the amount of the amount of mass in the chamber should be at its highest?" Mpumelelo responds with, "No" and the discussion ends; Mpumelelo looks at the graph

	again and the Tutor is looking at him, waiting for a response. The Tutor seems at a loss as to how to help Mpumelelo, "I want to help ... I just have to understand what the problem is".
8 (581-612)	Siyabulela, Bongani and Lungiswa have started working on Question (d). The Tutor is now listening to Vuyani. Mpumelelo is still getting agitated about having to leave early. Vuyani is explaining his understanding of the context to the Tutor; he seems to be attending to the first line of the problem text and the "reactants" A and B and indicating that at $t = 2$ the two products A and B are reacting. The Tutor questions a bit to clarify what Vuyani is saying. The Tutor argues, by pointing to the graph, that the reaction is taking place "all the time". He then starts a longer explanation, using numbers to indicate the amount of each product formed each unit of time, suggesting the use of a "golf ball" to assist with visualization of the volume of the products, and pointing to the graph at different points. He does not refer to "rate" at all, but to "particles combining per second" And he points to different parts of the parabola graph.
9 (613-626)	The Tutor returns to working with Mpumelelo on his left and asks if "that way of thinking" helped him. Mpumelelo responds with content-free, positive feedback, "Ja". The Tutor leaves the group. While the other students are working on question (d) Mpumelelo asks for clarification, wording a possible answer (in isiXhosa) that the production is highest at the point $t = 2$. He gets non-content based feedback, "ja" from Vuyani. Mpumelelo's written answer does make an accurate pronouncement about the "rate" (a word that he eventually does use); "The rate of production is at a maximum.". Siyabulela and Lungiswa also use the word "rate" in their answers (Siyabulela does not describe this as a maximum, but rather as "high"). Both Vuyani and Bongani avoid the word "rate" and run into difficulty. Bongani writes, "The maximum point where the product is formed:, and Vuyani has, "When $t = 2$ hrs there is a higher amount of reactants mixing to give the product X".

(d) Find the equation of the parabola part of the graph – it will express $m'(t)$ as a quadratic function of t.

Episode	Description
1 (577-585)	Lungiswa and Bongani have moved on to question (d). Bongani begins by attending to the shape of the parabola as one with a maximum turning point and linking this to a quadratic equation with a negative coefficient. He names the parabola as "the negative parabola". He then suggests "it" is minus two x. Lungiswa is attending to the independent variable when she corrects the x to t for time and assigns this to the rate $m'(t)$ and writes $m'(t) = -2t$ in her answer book (initially she had written $f(t)$ suggesting that she is attending to the mathematical terminology for a function, and not the specific name in the Chemical Reaction Problem) . Bongani does not write anything down yet. The students are not attending to the general form of a "quadratic equation" named in question (d) of the problem text.
2 (586-624)	A discussion takes place between Siyabulela and Lungiswa, with Bongani listening, but this cannot be heard. Lungiswa now has an answer of $-t^2$, but it is not clear how she arrived at this answer from the original $-2t$ in Episode 1. Bongani is asking for

	clarification from Lungiswa; his speech is not specific, talking about "the thing" for equation, and "a negative" (but not indicating negative of what). Siyabulela has started writing. Initially he has $m'(t)= t^2$ suggesting that he is attending to the graph as a rate, to the variable time, and the parabola as a quadratic function. Yet he is not attending to the maximum turning point as Bongani does, nor to the axis of symmetry of the graph which is not $t = 0$.
3 (627-645)	Siyabulela has an answer of $-2t^2$. Lungiswa and Bongani question Siyabulela why his equation has the "*t* squared", attending to the form of the equation (linear or quadratic). Lungiswa talks aloud; it seems that she has $-t^2$ for the equation for the parabola, but then, since it is labeled $m'(t)$ she acts operationally and finds the derivative (she says, "*t* squared and then derivative"). Siyabulela responds in a mixture of Sesotho, Setswana and English, he points to the graph and argues that the derivative of $m'(t)$ (which he names as the second derivative), is linear (a "straight line"). He also links the two expressions $-t^2$ and $-2t$ indicating that the latter is the derivative of the former. Here he is attending to the mathematical terms, the form of the equation and the graphical representation ("straight line"). He points to Lungiswa's equation which he emphasises has "highest power of 1", and compares that to the graph (pointing) which has a "highest power of 2" and is squared.
4 (646-676)	Siyabulela goes back to his original answer of $-2t^2$; Bongani and Lungiswa now seem to be happy with the quadratic part, but query the coefficient of 2. Siyabulela then explains the operation he has used (he talks about "simultaneous equations"). He has crossed out his earlier answer and written the general form of a quadratic equation $y = ax^2 + bx + 0$, indicating that he is now looking at the *y*-intercept of 0. He then attends to two points (2, 8) and (4, 0) and substitutes for *x* and *y* in his general equation: $y = ax^2 + bx + 0$ $8 = 4a + 2b$ $0 = 16a + 4b$. In his explanation he makes links between the general form of the equation and the points on the graph (to which he points). Bongani asks for clarification, and Siyabulela talks him through it again. Siyabulela is also attending to the independent variable *t*, replacing *x* with *t* in the general equation (in his written and verbal communication).
5 (677-683)	[The tape ends]. When it resumes the students are writing, and the Tutor is in the background. Siyabulela is explaining the mathematical operations to Mpumelelo, using the equation and pointing to the graph. Mpumelelo starts writing.
6 (684-716)	The Tutor has been listening to and watching the students, but he now sits down next to Mpumelelo and asks if they need help. They have not appealed for his help. So he introduces a formula from school "that might help"; it appears from the discussion that he has introduced something like $y = a(x - r_1)(x - r_2)$. The students indicate that they recognize it, but Bongani corrects his use of *x* as the variable and the Tutor changes this to the variable *t*. Bongani questions the Tutor about what the *r*'s mean and the Tutor explains making a link to the graph and pointing. At the moment he is talking about the

	variable *m* rather than *m* prime as required. Bongani responds by saying he has been trying a different method, possibly attending to Siyabulela's method of using simultaneous equations (Bongani uses this method in his written work). The Tutor responds by saying that his suggested equation is not the only method, but describes it as "nice to remember", and makes a link to its use in school which he has encountered in his tutoring of grade 12 learners.
7 (717-750)	When the Tutor leaves, Bongani and Lungiswa continue talking about the operation of solving simultaneous equations. Although the Tutor has proposed an alternative equation, they all return to Siyabulela's method as described in Episode 4. Lungiswa identifies the equation $y = ax^2 + bx + c$ that they are using with work from school. Vuyani has been writing in his answer book and describes his work to Lungiswa. There is some confusion when he points to Bongani's work and he cannot read the writing; Vuyani is attending to the use of the notation $4b$ rather than $b4$ (which Bongani has used). This is clarified and then Vuyani talks him through the subtraction of the two equations, pointing to the coefficients of *a* and *b* as he talks about them. The talk is purely operational, for example, they talk about "adding", "multiplying". Vuyani identifies the method of simultaneous equation with school mathematics. Bongani has written the following (suggesting he is attending to the meaning of the points on the graph, but not attending to the use of the mathematical terminology): $at^2 + bt + c = 0$ when $t = 2$ $a4 + 2b = 8$ when t = 4 $a16 + 4b = 0$ $a12$
8 (751-780)	Lungiswa is now getting help from Siyabulela, who pronounces that he has the answer. He begins by attending to her general equation and focuses on the fact that this is the equation of $m'(t)$ and the use of the variable *t* (she has made the general equation equal to zero). He then talks her through her two equations that she has already set up correctly, linking to the graph and emphasizing each time the right hand side is $m'(t)$. Some of this takes place in Sesotho. Lungiswa has the following in her book: She has $at^2 + bt + c = 0$ written at the top of her page, but then crosses out the 0 and replaces it with $m'(t)$. Her first equation is $a4 + 2b = 8$ (this is called equation 1) Her second equation is $a16 + 4b = 0$ Mpumelelo then asks Siyabulela for help, talking about "the first one" which seems to mean the first equation, which Siyabulela is not sure of at first. Siyabulela talks Mpumelelo through the process of setting up the equations, making links to reading from the graph.

9 (781-791)	Bongani pronounces an answer of "8" for the value of *b* aloud, asking for feedback (but he does not make it explicit that it is *b*). It seems that Vuyani responds with "8". He then substitutes back into his general equation to get a final equation for (d) of $y = -2t^2 + 8t$. His working is as follows: He has removed the "*a*12" from Episode 7, and has multiplied the first equation by 2 and written it under the second, so it now looks like this: $a16 + 4b = 0$ $a8 + 4b = 16$ $a8 + 0 = -16$ $a = -2$ $-16 + 4b = 16$ $4b = 32$ $b = 8$ Lungiswa is looking at his working, and they joke about how his looks different to hers.
10 (792-806a)	The Tutor is now standing between Mpumelelo and Siyabulela. Siyabulela talks the Tutor through the operations for finding the equation in (d), linking his equations to the graph. The Tutor listens, giving positive feedback "yeah" at intervals. Siyabulela's final answer is as follows (all the other students have used his method): $m'(t) = at^2 + bt$ when $t = 2$: $m'(t) = 8$ $8 = 4a + 2b$ $b = 4 - 2a$….(1) when $t = 4$: $m'(t) = 0$ $0 = 16a + 4b$ $= 16a + 4(4 - 2a)$ $0 = 16a + 16 - 8a$ $-16 = 8a$ $a = -2$ $b = 4 - 2(-2)$ $= 8$ $\therefore m'(t) = -2t^2 + 8t$

11 (823-838)	Bongani and Siyabulela have moved on to question (e). Lungiswa is finishing the calculations for (d) and writes down the correct answer. Mpumelelo queries his working in the simultaneous equations with Siyabulela; "How did you get the 16 in this one?". Siyabulela explains by referring to the points on the graph and substitution into the general equation. Bongani asks Lungiswa about her answer and she reads the equation aloud, "-2*t* squared plus 8*t*". He gives positive feedback and they joke in isiXhosa about the authority to decide what is correct.

(e) Hence find the **total** mass of product X formed in the 4 hours since the start of the reaction.

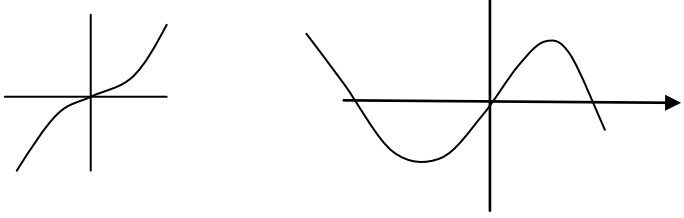
Episode	Description
1 (806b – 809)	Siyabulela is questioning the Tutor about question (e) and the requirement to find the "total mass" formed in 4 hours and queries whether this is the same as finding the area under the graph he points to. The Tutor gives positive feedback and emphasizes "total mass of the product". Although Siyabulela pronounces that it is the mass "formed after 4 hours", the Tutor does not pronounce what mass is required, that is, the total mass of the product formed rather than the total mass of the product in the chamber which was required in question (a)).
2 (810-822)	Bongani and Siyabulela then have a discussion, mainly in isiXhosa, but using English words for the mathematical terms like "i-total mass". This discussion involves making links between the task context ("total mass of product X formed", Siyabulela is the only student explicit about the mass of what) and the area under the graph of $m'(t)$, between the graph of $m'(t)$ and the quadratic equation $m'(t) = -2t^2 + 8t$, and between the area under the graph and the definite integral from 0 to 4.
3 (839-844)	Lungiswa has now started working on question (e). Bongani has written an integral for (e): $\int_0^4 -2t^2 + 8t \ dt$ He then asks Siyabulela what the "integration" for 2*t* is, using the name of the process for anti-derivative of 2*t*. He writes: $\frac{-2t^3}{3} + \frac{8t^2}{2} + c \Big]_0^4$ The students are then working individually, doing the calculations for question (e) in their books; they are attending to the operations and using the correct notation some of the time.

	For example, Lungiswa has written: $\int_0^4 m't \,.\, dt = \int_0^4 -2t^2 + 8t \;\; dt$ $= -2\int_0^4 t^2 - 4t$ $= -2\left(\frac{t^2}{2} - 2t^2\right) \;\Big]_0^4$ Siyabulela carries the process through to get the correct answer: $\int_0^4 (-2t^2 + 8t)\, dt$ $\frac{-2t^3}{3} + \frac{8t^2}{2} \;\Big|_0^4$ $\left[\frac{-2(64)}{3} + \frac{8(16)}{2}\right] - 0$ $\approx$ 21,3 mass There is no attention to the difference between the product being formed and the amount of product in the chamber.
4 (845-898)	Siyabulela announces an answer of 21,3 for feedback. He does not attend to the units initially, but then adds "mass" later (suggesting that he is not attending to the difference between the meaning of the variable and the units of the variable). Bongani then produces an answer of 74 (no units pronounced), and he and Siyabulela check their working. Following on from his initial attempt in Episode 3 he has written: $\frac{-2t^3}{3} + \frac{8t^2}{2} + c \;\Big]_0^4$ $-\frac{2}{3}t^2 + 4t^2 \;]_0^4$ $-\frac{2}{3}(4)^2 + 4(4)^2 - [0]$ $-\frac{2}{3}(16) + 4(16)$ = 74.6 Bongani has made small errors, copying incorrectly from line 1 to line 2 above, and not including the negative in the answer. Siyabulela identifies these by pointing to them with his pen.

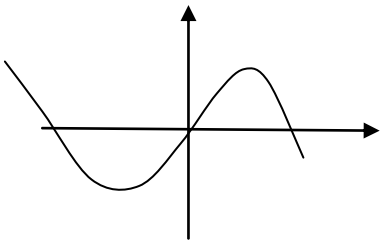
	Lungiswa also identifies an error in the first term of her anti-derivative and corrects it (She also has an answer of 21.3): $-2\left(\frac{t^3}{3}-2t^2\right)\Big]_0^4$ The rest of the Episode consists of Mpumelelo talking about needing to leave class.
5 (908-918)	Siyabulela and Lungiswa have moved on to question (f), but Bongani is still doing the calculations for question (e). He is attending to the answer and asks Siyabulela for this. Both Siyabulela and Lungiswa respond with "21.3". Bongani looks surprised, repeating the answer of 21. He has the answer "42.666666667" on his calculator (this is for the first term of $-\frac{2}{3}(64)$) and he goes back to using his calculator (attending to decimal places). Siyabulela's feedback is that he, Bongani, needs lessons on using his calculator.

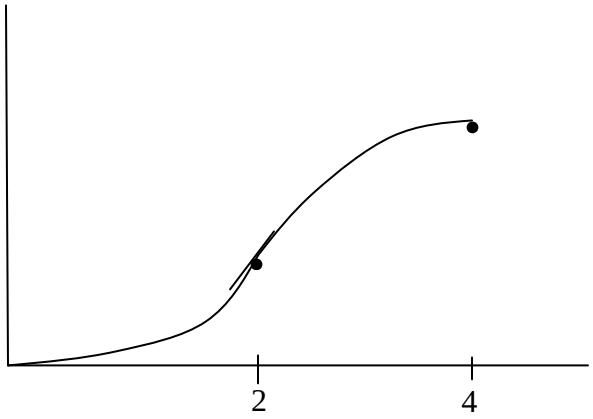
(f) Draw a rough sketch of the graph of $m(t)$ for $0 \leq t \leq 5$ hours. Clearly indicate on your graph the times t = 2 hours, t = 4 hours, and t = 5 hours.

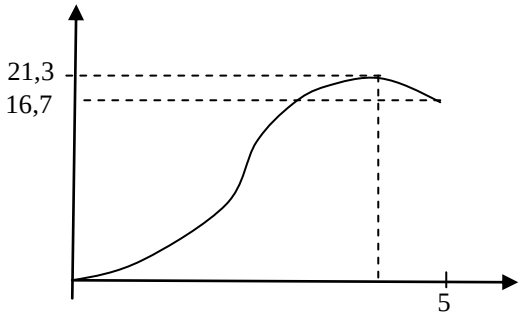
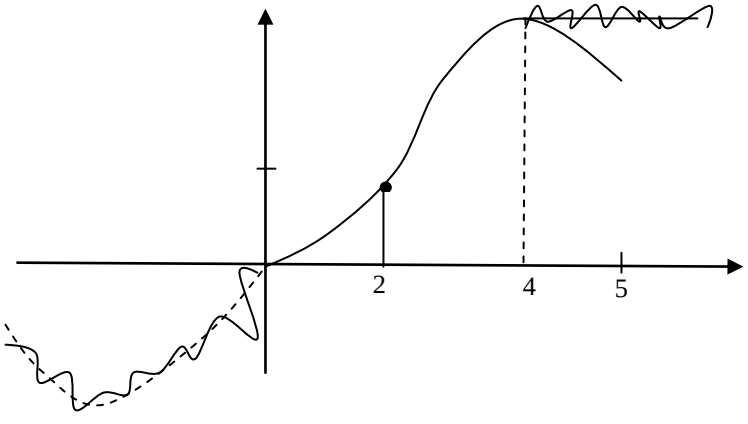
Episode	Description
1 (899-907)	Siyabulela has started on question (f). He attends to (by circling) the cubic expression he worked with in (e), linking this to the equation $m(t)$, the graph of which he must draw in question (f). He then attends to parts of this equation (by pointing with his pen); the cubic term and the coefficient of the cubic term. He then draws two small sketches at the top of his answer book – one is a cubic with a positive coefficient of the cubic term, the other the negative.  Lungiswa is also attending to the cubic equation used in (e) as she copies the equation from her previous page to the top of the page on which she is answering question (f): $-2\left(\frac{t^3}{3}-2t^2\right)=m(t)$
2 (915-933)	Siyabulela attends to the need to draw a "rough sketch" in the text of question (f) and describes his attempts in Episode 1 in this way. He and Lungiswa are talking a mixture of Sesotho and English, but they use English for the mathematics terms like "cubic" and "turning point". They appear to be attending to the coefficient of the cubic term (-2) in deciding on one of the two cubic graphs (Siyabulela selects "decreasing"). Lungiswa traces the shape of the two graphs in the air. No link is made to the task context. Lungiswa then names the graph as a "straight line", but quickly corrects herself.

	Siyabulela points at her answer text and talks about "i-cubic function"; it is possible that he is dividing a cubic graph into "parabolas" and attending the local maxima and minima. Lungiswa draws rough sketches of cubic graphs: 
3 (934-943)	Most of this conversation takes place in isiXhosa. Lungiswa refers to Bongani's answer for (e), and he responds that he has "got it". He then pronounces that for question (f) he is going to "reverse it", possibly intending that he must find the anti-derivative (he attends to the fact that they need to find $m(t)$). In response, Lungiswa then points to the cubic equation in Bongani's book, and asks, "asingo m? (is it not m?)". $\frac{-2t^3}{3}+\frac{8t^2}{2}+c\Big]_0^4$
4 (944-983)	Siyabulela and Lungiswa are drawing possible sketches and talking about what they are doing as the draw. Bongani asks questions at times. They are attending to the y- and t-intercepts of their cubic graph. Siyabulela has done a rough calculation to find the t-intercept which he pronounces as $\frac{4}{3}$: $-2\left(\frac{t^3}{3}-\frac{4t^2}{2}\right)$ $t^2\left(\frac{t}{3}-\frac{4}{2}\right)$ $\frac{4}{\cancel{2}}\times\frac{\cancel{2}}{3}$ They also attend to the shape of the graph in terms of how many turning points there will be, and which way it will "face", for example, "upwards". Siyabulela sketches the following possibilities:  Siyabulela emphasizes that it is a "rough sketch", making a link to the emphasis on this in the Course. The discussion takes place in a mixture of English and Sesotho, and then some isiXhosa when Bongani is talking part. The demonstrations take place by sketching and by tracing the shape of the graph in the air.

5 (984-1006)	Bongani questions (in isiXhosa) whether Siyabulela's second version of the graph will go down again. Bongani also instructs Siyabulela to speak English, rather than Sesotho. The debate is about whether the graph looks like the one on the left (Siyabulela) or the one on the right (Bongani) below:  Siyabulela is convinced that his choice is correct, but can only support his argument by repeating statements like, "it must go down" and it "must retain its shape" (suggesting he is attending to his representation of a cubic graph). The descriptions of the graph are done by sketching, pointing and tracing the shape in the air.
6 (1007-1034)	This Episode involves Siyabulela and Bongani linking different graphs (either demonstrated in the air or sketched) with the words "happy graph" and "unhappy graph". They have sketched a variety of graphs (in different orientations) at the top of the page in Bongani's answer book:  In making these links, they are using a description of a parabola graph (with maximum or minimum turning points) that is used in school mathematics (and in the Course). They are now trying to use it to describe cubic graphs (and also graphs with x^4). It seems that Siyabulela is labeling graphs as a whole as "happy or "unhappy" (and a "happy" graph is one that ends increasing). However, Bongani seems to be looking at different parts (parabolas) of the graphs and labeling different parts as "happy" or "unhappy". Vuyani is listening in.
7 (1035-1090)	Vuyani refers Siyabulela back to the calculation of the *t*-intercepts, and Siyabulela indicates that there should be two *t*-intercepts. They are attending to the cubic equation for $m(t)$ that was calculated in (e) and the operation for finding the *t*-intercepts. It takes a while for them to establish a common understanding of which equation they are talking about; Siyabulela is pointing to the integrand for (e) and Vuyani is reading it aloud. Vuyani makes pronouncements about the mathematical operations of integrating and differentiating. They attend to Siyabulela's calculations for question (e) and make links between the equations, their mathematical names ("integral" and "derivative") and the symbols ($m(t)$). Vuyani checks which graph they are required to draw (looking at the problem text) and then goes back to talking about finding the *t*-intercepts. They disagree on one of the *t*-intercepts; Siyabulela has $\frac{4}{3}$ and Vuyani has 6. Vuyani is also attending to the problem text, and pronounces that he finds the requirement to show particular times on the graph (like $t = 2$) "confusing". In his response, Siyabulela attends to the cubic equation and suggests they can find the value

	by substitution (he is not attending to these time-values as representing points with any particular "significance"). While Siyabulela and Vuyani are talking, Bongani is questioning Lungiswa about her graph (unclear, but using mixture of isiXhosa and English, for the mathematical terms), he is talking about the gradient. Then they are talking about "negative" and "positive". Bongani grabs Siyabulela's book and points to something, using the words "negative" and "gradient". Bongani is pointing to Siyabulela's graph and asking, in isiXhosa, what "it" is, both Siyabulela and Lungiswa respond with "positive", but it is not clear what they are referring to. Siyabulela responds to some of the queries simply by arguing that only a "rough sketch' is required (thus attending to the wording of the question in the problem text and not attending to the different values that are proposed by Vuyani).
8 (1091-1127a)	The Tutor joins the group and asks to see the sketch for question (f). Siyabulela shows his third attempt:  The Tutor does not ask any questions about this solution, but proceeds to explain a possible solution to the students. He uses a scrap piece of paper to draw the graph. He makes links between the symbols m and m', their meaning on the task context and different parts of the graph in the Resource Book. He is encouraging the students to consider what is happening to the mass at different times (from $t = 0$ to $t = 4$). He considers what the graph they need would look like if the "speed" were constant (a "straight line") or if the speed changes ("getting faster and faster"). The Tutor then asks for the equation of $m(t)$ which he wants to write at the top of his page. He has to clarify with the students that he wants the equation of the function $m(t)$ and not the equation of $m'(t)$. The Tutor then starts drawing the graph of $m(t)$, considering what is happening at different times (from 0 to 2, at 2, and from 2 to 4, thus attending to values in the question) and as he draws he describes the properties of "it", for example, using the mathematical terms "increasing" and "concave up". He does not explain the "increasing" at all, but gives a mathematical explanation for the "concave up", referring to the graph as the first derivative, but says we can also see the second derivative, because the rate is increasing (this is not linked explicitly to why the graph would be concave up). No link is made to the task context in this explanation. He describes the maximum turning point of the graph of $m'(t)$ as the "point of inflection" (a mathematical explanation, where the second derivative is zero), but does not make it explicit that it is the point of inflection of $m(t)$, nor as to why the second derivative is zero. He does then make a link to the task context at t equal to 2 – this is where the rate at which the product is created is the maximum – with a link to the "biggest slope", and he draws in a

	tangent line at the point of inflection. Describing what happens from 2 to 4 he refers to the "rate slowing down" and draws the concave down increasing curve but does not verbalize the link between his words and the mathematical properties of the graph he is drawing. From $t = 4$ he uses the task context to note that no more is being created. During this explanation he makes no link to the cubic equation he has written down at the top of the page. 
9 (1127b-1144)	Siyabulela queries whether they will get the same graph if they use the cubic equation (but only with positive values of *t*, they do not link to the task context to consider why this is the case). The Tutor says yes, but says the question does not require them to use the formula, but rather to "imagine"; he then proceeds to elaborate what he means by "imagine" when he describes what is happening in the task context and on the graph as time passes; "when we start off we hardly have any... the rate is very slow ((*Running his pen over the beginning of the graph of m*(*t*) *he has drawn*)) the rate is very close to zero ((*Running his pen over the graph of m'(t) in Siyabulela's Resource Book*)) … so we ... we add mass very slowly ((*Pointing to beginning of graph of m(t) again*)) and we getting we add it faster and faster and faster ((*Running his pen along the graph of m(t)*)) that's why it's sloping up like this ((*Running his pen along concave up part of graph of m(t)*))". He also refers to the thunderstorm context used earlier. Siyabulela is still attending to the cubic function, and asks whether sketching the graph of this function will give the same concavity as the graph the Tutor has presented. The Tutor gives positive feedback and talks through the operation of finding the second derivative and making it equal to zero. But he then encourages Siyabulela to "imagine" how it relates to the "physical situation" (either the chemicals or the pond example).
10 (1145-1152)	Siyabulela admits that the graph that he has drawn is wrong, but then queries whether the graph of $m(t)$ will decrease after $t = 4$. The Tutor responds by explaining that they are only drawing the graph of "*m*" and not considering the extraction. In his explanation he links the symbols, the parabola and straight line different graphs in the original graph and the meaning in the task context.

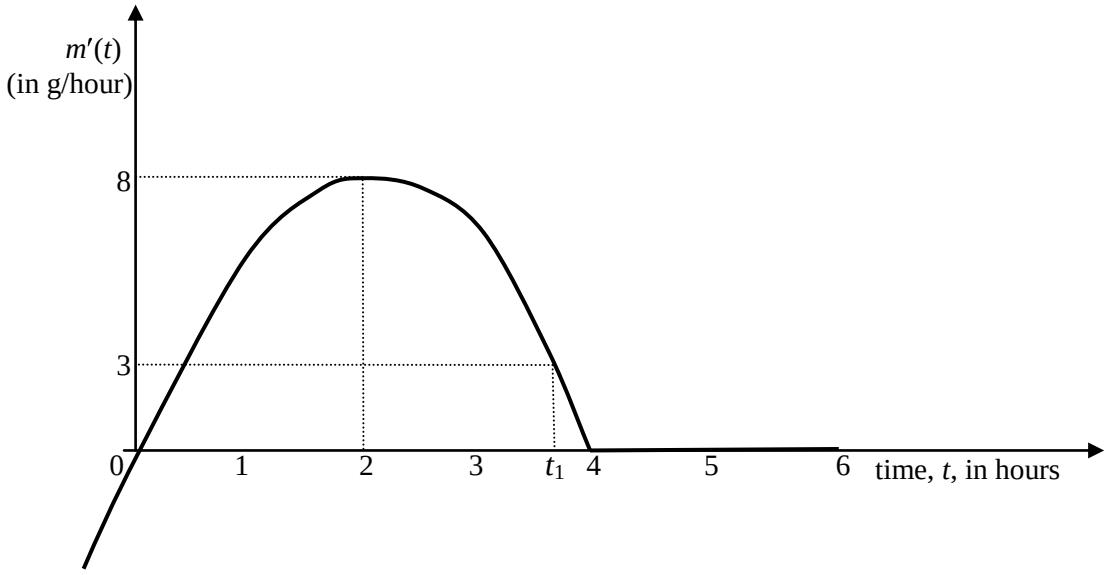
11 (1153-1195)	Both Siyabulela and Bongani are attending to the problem text and wanting values of $m(t)$ for the values of $t = 2$, 4 and 5 as required. Siyabulela gives Bongani the operation for finding the value at $t = 5$ (substitution into the cubic equation). Siyabulela includes these corresponding values in his graph (also using the answer of 21.3 from question (e)). He has not attended to the Tutor's response to his question about the graph decreasing after $t = 4$.  There is more discussion about what the corresponding values should be. Vuyani and Bongani argue that "it" is decreasing after 4 (they do not seem to have taken into account what the Tutor explained to Siyabulela earlier, or to the fact that the parabola equation only applies from zero to four). But Lungiswa has noticed that the parabola is only defined up to $t = 4$. But both Siyabulela and Bongani argue that they are working "from zero to five" (it may be that they are attending to the requirement in the question (f) to draw the graph from zero to five) Although they attend to the different endpoints, they do not attend to the bold line on the horizontal axis of the graph of $m'(t)$ and the meaning of this line. The rest of the discussion is about finding the correct corresponding value for $t = 5$. Lungiswa has revisited her earlier graph and crossed out some parts:  Bongani's graph simply stops at $t = 4$. They do not attend to what should be happening after $t = 4$.

APPENDIX P GROUP 3
ACTION ON THE CHEMICAL REACTION PROBLEM

Quantities of two chemicals A and B are mixed together in a reaction chamber, and they react to form a new product, X.

The **rate** at which the product X is formed is given by $m'(t)$, where m is the mass of the product formed, in grams, and the time t from the start of the reaction is measured in hours. The graph of $m'(t)$ is a **parabola** graph until time $t = 4$ hours, after which it is zero.

It is also given that, from the start of the reaction, some of the product X is removed from the reaction chamber at a constant rate of 3 g/hour.



(a) Write down an expression involving an integral that gives the **total** mass of product X **in** the reaction chamber after a time of t hours.

Episode	Description
1 (1-17)	When Lwazi first looks at the problem he exclaims, "Hey, this is chemistry↑", thus linking the task context to one of his university subjects. Thokozile is the first student to make her answer public. She links this question to a previous one, "aren't we doing the same thing?" She pronounces her answer as a string of symbols by talking aloud as she writes down her answer of $\int_0^6 m'(t).dt$. She is attending to the *t*-values from 0 to 6 on the horizontal axis when she constructs her integral (and not the instruction "for any time *t*" in the problem text). Her answer suggests that she links the "total mass" with the anti-derivative, but that she is not viewing her expression as a function of *t*. She is attending to the formation of the product and not the instruction to work with the total product in the chamber ("total" is given in bold in the problem text). Ndumiso has an answer of $\int_0^t m'(t).dt - 3$, suggesting that he is attending to the requirement to work with the "total mass of the product X in the chamber" and Sentence 4 of the set-up text which describes the removal of the product at a rate of 3g/hour. He evaluates Thokozile's answer and explains that she is missing "minus 3". He does this by linking to the task context ("Because they say it's <u>removed</u> from the chamber … ((*Pointing at the problem text*)) at a constant rate"). However, he does not attend to Thokozile's use of $t = 6$. This, and his use of the constant "-3" in his expression suggests that he, too, may not be viewing the expression as a function of *t*. Both students are talking through the symbols for the integrand, but there is confusion about where to put the "minus 3". For example, Ndumiso says, "*m* of *t dt* minus 3" and Thokozile responds with "minus 3 <u>*t*</u>". Also, Thokozile tries to correct her written answer by placing a "-" in different places. For example, she writes $\int_0^6 m'(t).^{-}dt -$ and then settles on: $\int_0^6 m'(t) - 3\,dt$. There is no explicit discussion about where the "minus 3" should go, and it is as if they know there should be a "minus 3", and play around to find the correct position. Kelsa has also been trying and writes 5 gives $\int m'(t)$ ~~*dt*~~ -3*dt*. She has not attended to the change in time.

(b) Explain very clearly the significance/practical meaning of the time $t = t_1$ in the graph above.

Episode	Description
1 (18-26)	It seems that Lwazi and Kelsa are still working on question (a), but Thokozile and Ndumiso have started on question (b). Ndumiso immediately links the question to what was done in lectures, "I thought we did this with (lecturer's name) man … the significance." He is attending to the word "significance" and to the required point on the graph and identifies this with the word "maximum" (although he does not pronounce the "maximum" of what). Thokozile identifies the point with "it cannot increase anymore" possibly linking to the maximum, and she tries a few options all related to the task context, for example, she says "it can't increase or decrease anymore". In all cases she uses the word "it" but is not explicit about what "it" refers to. Ndumiso sums up his difficulty, "I don't know how do you explain it".
2 (27-30)	Kelsa is attending to the horizontal line at 3 on the graph (she traces along this line, from the graph to the vertical axis). She links this to the task context, "it's been removed at the rate of 3 grams per hour". (This is possibly her answer.)
3 (31-63)	Ndumiso calls the Tutor. There is some banter about whether they have the Tutor's name correct. The Tutor begins by reading the question aloud (he emphasizes "very clearly" with a grin). He reminds the students that the graph represents a real-world situation (using gestures to model the situation); "remember this is a graph that represents °something happening in° … the real world … it's some big ... furnace or something … reaction chamber… where these two things are combining". He then wants the students to make a link between the graph and the task context when he asks "what is happening" when the graph is "sloping upwards". Ndumiso responds with a mathematical description of the graph; "increasing at a decreasing rate" (his tracing of the graph suggests he is attending to the gradient at different points on the increasing graph). It is not clear what is "increasing". But Lwazi makes a link to the task context when he refers to "the mass of product". The Tutor is wanting them to talk about the mass and he introduces this when Ndumiso talks about "it increasing". The Tutor gives positive feedback, but poses a further challenge; "the amount of mass is increasing … but… uh… it's not just increasing … … the mass". It seems that he wants the students to attend to the horizontal line graph as well (representing the removal of the product), but they are attending to the parabola only and continue to give appropriate answers for this graph. For example, Lwazi says, "it's increasing at a decreasing rate it's slowing down". The Tutor's feedback like "uhm" and "so" suggests to the students (Ndumiso, Lwazi and Kelsa) that they are not correct. They simply try to reword the same idea, "It's increasing slowly and getting slower and slower" (Lwazi pronounces his intention to use "layman's terms"). In all these attempts they are using "it", but without clarifying what this means.

4 (64-106)	The Tutor says what they are saying is not "especially clear". He then refers them back to the point where t equals t_1 on the graph and asks what is "important" about it. Ndumiso repeats what he said at the beginning of Episode 1, that it has something to do with the "maximum". The Tutor gives positive feedback to his use of the word, and then returns to the graph; he points to both graphs (the parabola and the horizontal line) and links them to their meaning in the task context. He uses gestures to model the chemical reaction. Sometimes he allows the students to respond to his questions, but other times he answers his own question. In responding to Tutor's question about what is happening at the point where t equals t_1 , Lwazi is attending to meaning of the horizontal line only ("taking the product out"). But Kelsa seems to be attending to both graphs when she suggests, "What's taken out is equals to what's there". The Tutor repeats the part that she has correct (the removal) and provides a sentence for them to complete: "what's taken out is equals to?" Thokozile's answer of "3" suggests that she is attending to the horizontal line graph only. The Tutor does not attend to this and answers his question himself by equating "what's taken out" with "<The rate that it's being↑ made>". He then rewords his answer (using his hands to model the reaction); "so at that point… if you're taking out as much as you're [… …] putting in↑" and asks what "it" means. The students then seem to enter a guessing game where they are trying to find out what is required by the Tutor. He attends to their answers, only to evaluate, but does not indicate why they may be incorrect. Their answers are vague; "there's nothing there", and "it's an equilibrium". In the absence of appropriate answers, he starts to give an answer, but then encourages the students to attend to what is happening just before the point where t equals t_1. Lwazi is still attending to the parabola and the rate of change as in Episode 3. Again the Tutor's feedback relates to the clarity. The Tutor then proceeds to answer his own question (with some prompting and questioning from Kelsa); "At point t_1 … that point that's when the amount the total amount is at a maximum … after that time… then we start sucking out more than↑we create". The Tutor emphasizes the value of understanding "why". Kelsa seems happy with the Tutor's explanation, but the others are silent. The Tutor appears aware of their continued difficulties. Lwazi plans to read the question again.
5 (107-133)	Lwazi has gone back to reading his problem text (both he and Ndumiso complain of being tired). Ndumiso is writing in his answer book. It seems he is also attending to what Kelsa is saying as he is writing. She gives the answer in words, but also gives a verbal explanation when giving Ndumiso feedback; "The total amount of x is at a maximum ((*Pointing to her graph as she speaks*)) we know this because after this point the amount of product that is taken out is more than what's being generated and before that the amount of product is being taken out is more ... was less than what's been taken". Her inclusion of an explanation suggests she is attending to Tutor's call for understanding "why" in Episode 4 (although she does not write the explanation down). She writes, "At t_1 the total amount product X is at a maximum".

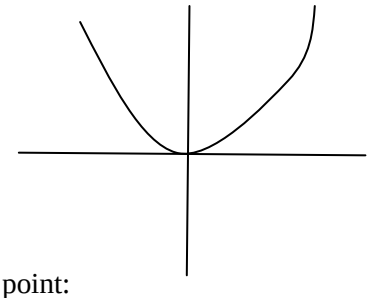
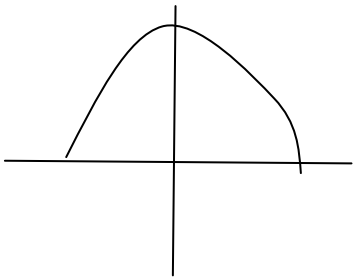
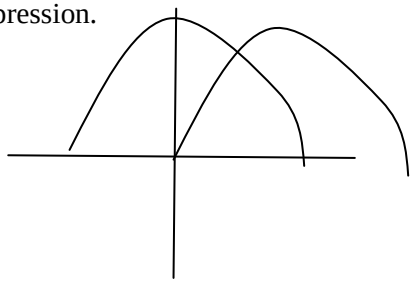
6 (134-195)	Ndumiso and Kelso are starting question (c). Lwazi is still confused about question (b); "okay now I've read the whole question … but I <u>still</u> don't understand what he was talking about↑". Kelsa tries to help. She begins by outlining the context of the chemical reaction. She identifies the graph as showing "how much product x is being formed", suggesting that she is not actually attending to the graph as the graph of $m'(t)$. This is confirmed when she points to the graph which she also calls "m", but then pauses when she places her finger next to the $m'(t)$ on the vertical axis. She resolves this by simply calling it "the graph". Kelsa then guides Lwazi through identifying the "corresponding value" for t_1 and its meaning in the task context. Lwazi then identifies the discussion with the "example we did today" (in lectures). (This is followed by a discussion of attendance in lectures and Thokozile admits that she has not been to lectures for a week as she has overslept.) Lwazi then pronounces that he understands (b), that it is "a maximum" (his written answer is "It a maximum"). No one follows up on this, and the discussion turns to getting a good class record for Maths, which then turns to the situation in Economics.

(c) Explain very clearly the significance/practical meaning of the local maximum in the graph of $m'(t)$ at time t = 2 hours.

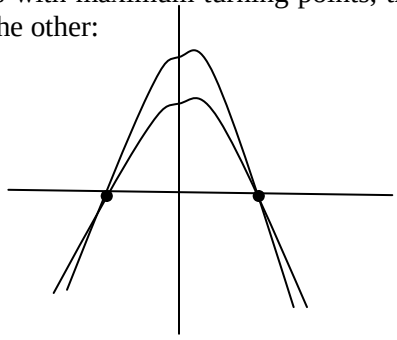
Episode	**Description**
1(196-215)	The students start by attending to the meaning of t = 2 on the horizontal axis, identifying this as "after two hours" (Lwazi). Kelsa gives the meaning of the point on the graph by attending to the time on the horizontal axis and the corresponding of value of 8 on the vertical axis, and identifying the graph with the rate of change; "I mean … 8 grams per hour is being created". Yet she seems to think there is more to it, since the question asks for the "significance" of the "local maximum". Thokozile tries by attending to the property of the parabola graph ("the thing") as increasing and then decreasing, with no reference to the context. Kelsa attempts a link to the task context, and correctly identifies the maximum rate (she says, "the most is being created", but does not use the word "rate"). But then she rewords, this time referring to the maximum amount of the product; "where the net … amount … is maximum". Both these students are not specific about the object they are referring to, using "the most" or "the thing" or "the net amount". Possibly due to the lack of clarity in the discussion, Lwazi links Kelsa's use of "maximum" to their answer for question (b); although the other students identified the "amount" in their answers for (b), Lwazi has pronounced the "maximum" only in his written answer in which he wrote, "It a maximum". There is a short pause but Kelsa tries again, this time she is clearer about the rate of change; "the amount of product X being generated is at a maximum". Students 2 and 3 are writing down what she says, without any debate, and correctly using the rate of change. Lwazi attempts the answer verbally, but it is not clear if he is describing the amount or the rate of change; "it's the maximum amount of product that's being generated". But his written answer refers to the "maximum rate" (he does not attend to the need to avoid using the word "rate"); 'Maximum rate the product is generated.

(d) Find the equation of the parabola part of the graph – it will express $m'(t)$ as a quadratic function of t.

Episode	Description
1 (216-231)	Thokozile and Kelsa (who is yawning) are clarifying what the question means; they focus on "the part of the parabola" in the question. There is lots of pointing to parts of the graph. They settle on "zero to four", attending to the t-values for where the graph is a parabola.
2 (232-236)	Thokozile pronounces that the formula is $-x^2$, and grounds her argument in the appearance of the graph: "because the parabola is facing down". She is attending to the parabola with a maximum turning point and the format of the formula as a quadratic formula. This receives negative feedback from Ndumiso (he repeats her answer as a question). Kelsa also gives negative feedback, one the basis of her test using substitution; she substitutes a point from the graph into Thokozile's formula.
3 (237-257)	Ndumiso pronounces, for feedback, the general formula for a parabola ("y is equal to a then x minus x_1 x minus x_2?"). This is a formula they used at school and elsewhere in the Course. He does not explain his choice. Kelsa pronounces an alternative ("ax squared plus b squared plus c?"), seemingly meaning $y = ax^2 + bx + c$). After Ndumiso's repetition and having written his suggested formula down ($y = a(x - x_1)(x - x_2)$), Thokozile seems to agree. Lwazi joins the conversation, firstly giving positive feedback to Ndumiso by saying that there "is such an equation", but secondly arguing that this is not "it". He identifies the "a" at the beginning as correct, but not "what is in the middle". Then there follows an exchange where Ndumiso and Lwazi argue about whether Ndumiso's option is correct; no reasons are given only "it IS" versus "it is not".
4 (258-262)	Thokozile then returns to her earlier suggestion in Episode 2 ("just x squared", without the minus this time). She is attending to the parabola, but not to the horizontal and vertical shifts. Ndumiso identifies her formula as the "easy one" and "the general one", thus making a link to the use of transformations as a method for finding the equation of a quadratic function in the Course (he demonstrates the transformation by moving his fingers over the graph). Lwazi refers to the graph when he pronounces the transformation; "that's moved … 4 units to the left↑". Thokozile recognizes what they are saying and names it "the whole movement thing", thus linking to the method used in the Course.
5 (263-268)	Kelsa interrupts, claiming that x squared "is not a parabola". This is met with negative feedback, either "it is" or Ndumiso's provision of a more complicated example; "x squared plus 4x minus 3 is also a parabola↑".

6 (269-275)	Thokozile responds to Ndumiso's example, saying they have to start with the "general equation" (she names x^2 "general"). She draws a "general" one as:  But she corrects herself to give a maximum turning point:  She then talks through the transformation process in a way that would be done in the Course, linking her sketches to the format of the expression that she gives verbally as $-x^2+4$ (it is not clear whether there are brackets). She is attending to the *t*-intercept of 4 when adding the 4 in this expression.  She then supports this by using Kelsa's method of substituting in the co-ordinates of the local max (see Episode 2). But she attends only to the x^2+4 for this substitution, and gets the required answer of 8. Meanwhile Lwazi is presenting different variations of the formula (for example, "but shouldn't it be 2*x* squared plus 4x?"). He has written $-2x^2 + 4x + 8$ in his book. The other students are not responding to these attempts.
7 (276-284)	Kelsa attends to Thokozile's mistake with the minus, and pronounces that the substitution does not work. So she suggests "just make it *x* squared plus 4"; she is not attending to the shape of the graph, but only to what works for the substitution of the point (2,8). Lwazi agrees with Kelsa, using the substitution for endorsement; "because if you substitute 2 you gonna get 8".

8 (285-294)	Both Ndumiso and Thokozile give negative feedback in the form of "it can't be", but do not explain. Then Thokozile makes a link between the formula and the graph; using gesture and describing the graph as "facing downwards", she argues that "it's negative" (supported by Lwazi and Ndumiso). They use "it's" interchangeably for the graph, the equation and the coefficient of x^2.
9(295-427)	The students then seem at a loss, and Thokozile admits, "Hey guys I don't know". Kelsa is continuing to substitute the point (2,8) into the formula, using her calculator to check, and trying to find what "works". Both Thokozile and Lwazi suggest alternative formulae; they are trying to construct formulae that will fit the substitution, but are not attending to the other features of the graph. For example, Lwazi pronounces, "*x* squared plus 4 *x* … then it's gonna work". Kelsa is testing the substitution as they pronounce possibilities ("trying to get 8"), and they adjust the formula as they go. Thokozile decides to seek help from the Tutor, but the Tutor is busy elsewhere. In the next lines they appeal to the Tutor a few times, but continue trying possibilities when they have no success. They only seem to attend to the properties of the graph at one stage when Lwazi pronounces, "the graph has moved 8 up … we forgot that", and they try to incorporate this into their formulae. On another occasion he attends to the "gradient" or how wide the graph is when he tries to support his argument that the value of a is -2, a value he obtained by testing the substitution. On two occasions they evaluate the pronounced formula on the basis of what the formula for a parabola should look like, that is, it should have an x^2 and it can only have one constant ("Kelsa asks, "why would you have an equation that says plus c plus c?"). The rest of the time they base their arguments on the substitution of the point (2,8) and claims like "I promise" (Lwazi). The students seem aware of their difficulty; Kelsa comments "this is sad", Ndumiso declares "I give up", and Lwazi claims "there's something we are doing".
10 (428-439)	Thokozile announces she has "got it", with her answer of $-(x^2 - 4) + 8$ (earlier she had $-(x - 4)^2 + 8$). She attends to the horizontal and vertical shifts, but she is assuming that the coefficient of x squared is either 1 or -1. She is still testing by substitution, and there is no attention to the different structure of these two expressions.
11 (440-466)	The Tutor responds to Ndumiso's call. The Tutor arrives and sits down next to Lwazi. Kelsa is still adapting Thokozile's formula and testing by substitution to find something that "works". So when Ndumiso pronounces that they have a "groot (great) problem" (using Afrikaans), she disagrees. There is some banter, with Lwazi and Ndumiso claiming that they need help from the Tutor, even if the others do not. The Tutor encourages the "clever" girls to explain to Ndumiso and Lwazi.
12 (467-491)	The Tutor asks Lwazi where he got his answer, suggesting that he "just made it up". Lwazi only mumbles something back and points to something at the bottom of his page. The Tutor makes a link to "something like you did at school". Lwazi responds in a way that suggest he is attending to the symbols in/format of the formula only, "We did but I forgot the formula there … it's got an *a* it's got an *l* … you know what I'm talking about it's got a bracket". Ndumiso gives a verbal version of the formula that makes use of the

	two *x*-intercepts (the one he suggested in Episode 3) and gets positive feedback from the Tutor, but not from Lwazi. The Tutor then attends to the symbols Ndumiso has used and asks for their meaning, which Ndumiso is able to give and link to the points on the graph. For example, in response to the Tutor's, "What is x_1?", he responds, "x_1 is going to be your first intercept↑ ((*Pointing to something on his graph*))". While the Tutor is talking with Lwazi and Ndumiso, Kelsa is explaining to Thokozile; she is working with the expression $(x^2 - 4) + 8$ that she and Thokozile are convinced is correct. She still tests using substitution, this time pointing to the values on the graph. The substitution works for the point (2, 8), but not the point (4, 0) (she has made a calculation error here, which is not identified by Thokozile). For $t = 1$ she gets 5 which she says is "realistic" on the graph (she is now identifying points that look right on the graph to support her substitution method). Once the Tutor has suggested what formula they can use, there is some banter about Lwazi leading them "off track", but they have not attended to why Ndumiso still has not got the correct formula.
13 (492-516a)	Ndumiso tries the substitution; he is substituting an *x*-intercept for *x* in his equation of $y = a(x - x_1)(x - x_2)$: "so your *x* will be zero right?". The Tutor gives him negative feedback in the form of, "Okay now you have to take it back a little bit". Then Lwazi interrupts and asks for the format of the equation which he writes down as Ndumiso gives it to him verbally: $y = a(x - x_1)(x - x_2)$. Ndumiso then proceeds to tell Lwazi what to do ("substitute"). The Tutor scaffolds this by asking in turn for the value of x_1 and x_2 (there is an implicit understanding that he wants the values, and not the meaning).
14 (516b-524)	But then Ndumiso also substitutes the point (0,0) for *x* and *y*, and the Tutor interrupts to tell him that "will not work" as he cannot use the same point twice. He is attending to the actual points and what they represent on the graph. The Tutor then draws a sketch of two parabolas with maximum turning points, that have the same *x*-intercepts but one is steeper than the other:  He asks Ndumiso and Lwazi: "<u>If</u> the only thing you substitute in this formula is the root ((*he picks up the paper and puts it in front of 4, then he points to the roots on the graph in the Resource book*)) … how is the formula supposed to know which parabola you are talking about? How are you supposed to know how steep it is?" Lwazi immediately attends to the local maximum. The Tutor gives positive feedback and emphasizes that it could be any point, as long as it is "different" and points them to the local maximum on their graph. Ndumiso links this to work from "matric".

15 (525-542)	Lwazi pronounces that there is "another equation", possibly attending to the formula $y = a(x – p)^2 + q$ from school and the Course. The Tutor overhears and suggests it is the one with the "turning points" and writes the equation in Lwazi's book: $y = a(x – p)^2 + q$. Lwazi is excited as he recognizes it (as does Kelsa). The Tutor admits that he cannot remember if the sign is plus or minus, but Lwazi and Kelsa confirm the minus. Ndumiso is not convinced; firstly he argues there is no such thing, and then says Lwazi can't even use it.
16 (543-569)	Thokozile has moved on to question (e). Kelsa is waiting while Ndumiso and Lwazi complete question (d). Ndumiso comes up with the correct answer; he has been writing and talking to himself as he writes: $y = a(x – x_1)(x – x_2)$ $8 = a(2 – 0)(2 – 4)$ $8 = –4a$ $–2 = a$ $y = –2(x – 0)(x – 4)$ $y = –2(x^2 – 4x)$ $y = –2x^2 + 8x$ Kelsa takes his page to look at it. In her own work she has attended to the two points (as we saw with her ongoing substitution earlier). She has then made two attempts at substitution, which suggest that she is not attending to the meaning of the points; she substitutes the x-intercepts 0 and 4 as required, and then she substitutes 2 and 4 for x_1 and x_2 respectively: ~~x~~ *t* | ~~y~~ *m'* 2 | 8 4 | 0 $a(x – x_1)(x – x_2)$ $a(x – 0)(x – 4)$ $a(x^2 –4x + 4)$ $a(x – 2)(x – 4)$ $a(x^2 – 2x – 4x + 8)$ $a(x^2 – 6x + 8)$ She is only attending to the right hand side of the equation, thus seeing it as an expression only. Kelsa requests help and Ndumiso explains what the symbols in his equation $y = a(x – x_1)(x – x_2)$ mean; "these are your <u>intercepts</u> x_1 and x_2 are your <u>intercepts</u> … y and (unclear)((*pointing to x*)) are just any point on the graph↑".

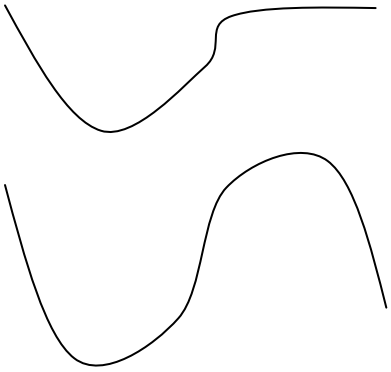
	For the first time Kelsa pronounces that zero and 4 are the *x*-intercepts and points to the second line of Ndumiso's working: $8 = a(2 - 0)(2 - 4)$. She then crosses out her working above and gets the correct equation (she is the only student to attend to the independent variable of *t* (rather than *x*) for her final answer): $y = a(x - x_1)(x - x_2)$ $8 = a(2 - 0)(2 - 4)$ $8 = a.-4$ $a = -2$ $y = -2(x - 0)(x - 4)$ $y = a(x - x_1)(x - x_2)$ $y = -2(x - 0)(x - 4)$ $y = -2(x^2 - 4x)$ $\underline{y = -2x^2 + 8x}$ $y = -2t^2 + 8t$
17 (570-579)	Thokozile's written work does not show any working for (d) – her earlier attempts were on a rough page. She has started working on question (e) but when she needs an expression for this question she attends to Ndumiso's two attempts and asks which equation is the answer. He shows her the correct equation, which she writes down for as the integrand in question (e): $\int_0^4 -2x^2 + 8x.\,dt$ In spite of his excitement about the "turning point" equation in Episode 15, Lwazi starts the working with the intercept formula. He does not complete his working, but writes down (incorrectly) the answer from Ndumiso. $y = a(x - x_1)(x - x_2)$ $8 = a(x - 0)(x - 4)$ $=$ $\therefore -2x + 8x$ He is excited that the coefficient of *x* squared is -2, as he earlier claimed, and he admits this was a "good guess".

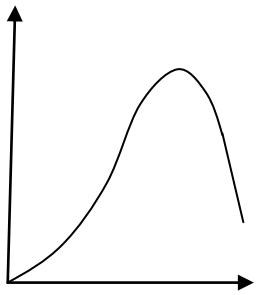
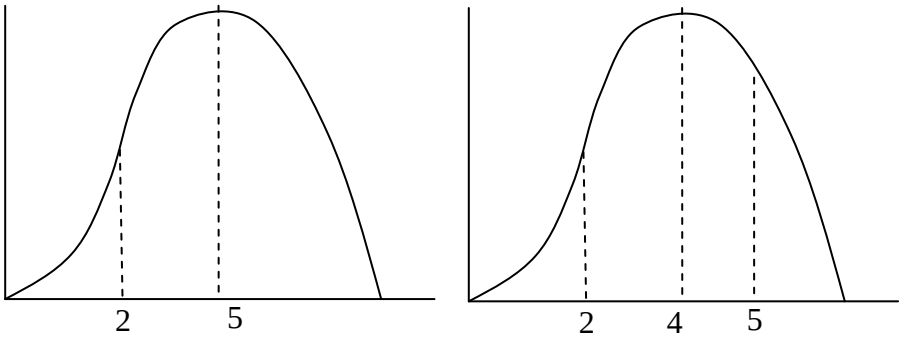
(e) Hence find the **total** mass of product X formed in the 4 hours since the start of the reaction.

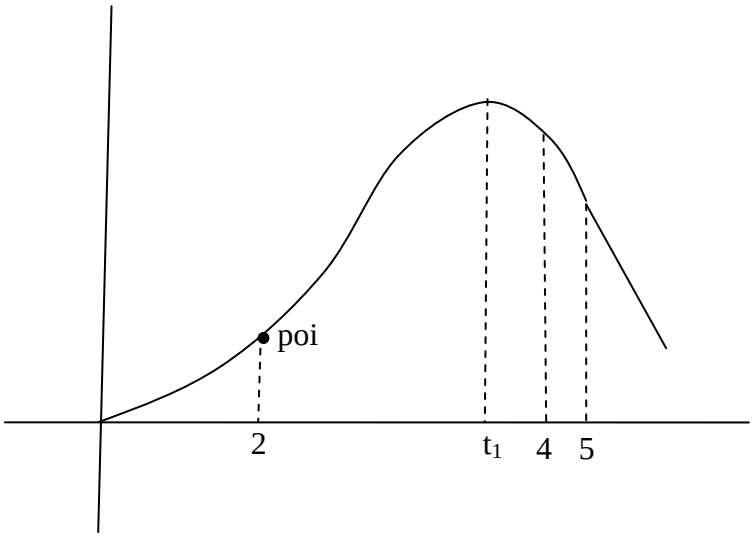
Episode	**Description**
1 (544, 555, 571,580-596)	Thokozile and Ndumiso started working on question (e) while the others were completing (d). Thokozile links the "total mass" required by the question with the graph by pronouncing "we're just gonna calculate the <u>areas</u> for this one". Ndumiso immediately identifies the total mass with the integral and he writes down $\int_0^4 -2x^2 + 8.dx$ as he talks through the symbols (he makes a copying error in the second term). Kelsa also identifies the "total mass" with the "area under the graph" and the integral; "so the total mass is the area under the graph basically? … so the integral". (There is no explicit discussion of whether they should subtract the "3" or not). During these students' first attempts, Lwazi is telling them about a dream he had.

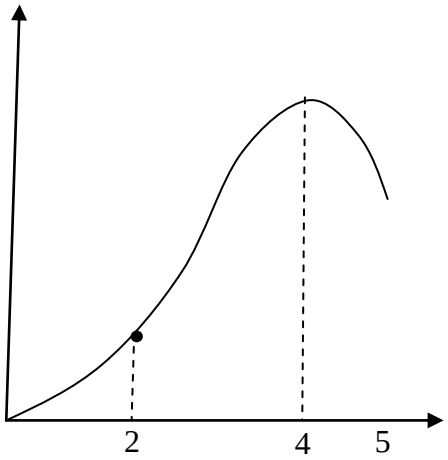
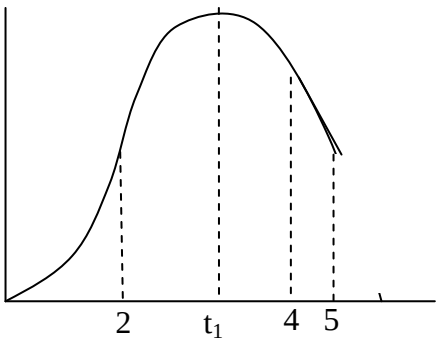
2 (597-611)	Ndumiso is finding the anti-derivative and does this correctly, using the correct notation: $\left[-\frac{2x^3}{3}+8x\right]_0^4$. But when he compares his working with Thokozile (she has written $-\frac{2x^3}{3}+\frac{8x^2}{2}\Big]_0^4$ and has not yet cancelled down the coefficient in the second term), he identifies the copying error he made in Episode 1 and corrects it. He changes the term "8" to "8*x*" in his integrand and changes his anti-derivative: $\left[-\frac{2x^3}{3}+4x^2\right]_0^4$. Thokozile has not yet cancelled down the fraction in her second term and initially does not see the link between her term and what Ndumiso has written. But they both check their answers using differentiation (Thokozile and Ndumiso do this on a rough paper). Kelsa reminds her, "you must simplify your answer". Kelsa begins by writing $\int_0^4 2x^2+8x.dx$, but then rewrites it, changing the independent variable from "*x*" to "*t*", and correcting the first term of the integrand by including the negative.
3 (612-672)	Lwazi is talking about his social life. The others are using their calculators. Thokozile has written the substitution step: $\left(\frac{-2(4)^3}{3}+\frac{8(4)}{}4(4)^2\right)-\left(\frac{-2(0)^3}{3}+4(0)^2\right)$. Both Thokozile and Kelsa both pronounce an answer of $21\frac{1}{3}$ (Kelsa has included the units, grams), an answer that they both seem happy with. Ndumiso has not pronounced his written answer of 21.33 aloud, and has continued with question (f). Thokozile is concerned about whether she has to include the constant "*c*" in her anti-derivative, but Ndumiso corrects her by naming the type of integral as a "definite integral". Lwazi is now talking about changing faculty. He has been writing as he speaks. He has found the anti-derivative correctly, but does not attend to the correct notation : $\int_0^4 2x^2+8x.dx$ = $-\frac{2x^3}{3}+4x^2$ (he has crossed out the "*c*"). He does not do any substitution.

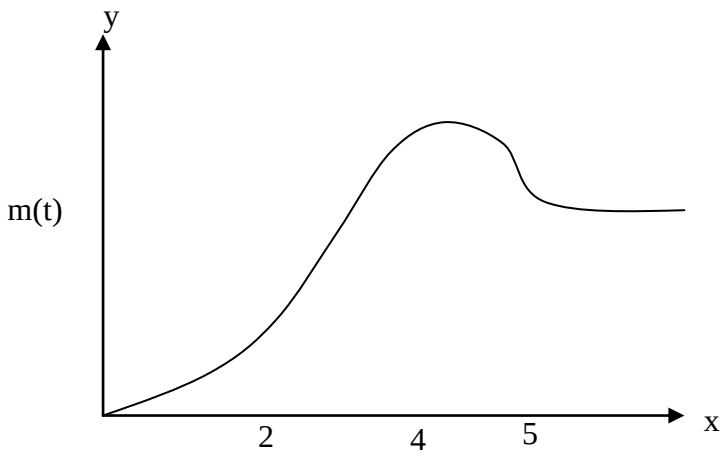
(f) Draw a rough sketch of the graph of $m(t)$ for $0 \le t \le 5$ hours. Clearly indicate on your graph the times $t = 2$ hours, $t = 4$ hours, and $t = 5$ hours.

Episode	Description
1 (658,671, 673-716)	Kelsa is the first to make her attempt public. She is attending to the requirement to include $t = 2$ on her graph, and she links this to their answer for (c); "that's when the graph thing is at a maximum". She is attending to the problem text, thus suggesting that the "graph" here is the graph of $m'(t)$. She then pronounces that "it" will "increase until two" and starts drawing. Thokozile gives a negative feedback, identifying that this will be the same as $m'(t)$. Kelsa then pronounces two possible graphs by tracing them in the air:  In her second attempt she traces over the graph of $m'(t)$ in the problem text. Her description of the properties suggests that she is attending to where her proposed graph is increasing/decreasing and the concavity (she uses the word "point of inflection"). Lwazi has been finishing question (e) (see his working in Episode 3 of question (e)). He and Ndumiso are also talking about the party Lwazi attended the previous evening. Ndumiso also encourages him to work on the graph for question (f);; he says that "you came up with the last one". In contrast, Thokozile identifies herself as someone who "can't draw graphs". In what follows the four students propose different options, usually requesting feedback from the others. For example, Lwazi names the graph as a whole, "It's a happy graph↑" They attend to the properties of the graph (intercepts, increasing/decreasing, concavity), for example, Thokozile suggests, "then it's increasing and then decreasing". They are also attending to different time values (as suggested by the text of question (f)); "what happens at 4?" (Thokozile). In all this discussion the students use the pronoun "it" as a reference, and it is not made explicit whether they refer to the graph of $m'(t)$ or of $m(t)$. For example, in response to Thokozile's question about what is happening a "4", Kelsa replies, "at 4 it hits zero", suggesting that she is attending to the graph of $m'(t)$. But Thokozile emphasizes that she is attending to the "original one" (the graph of $m(t)$).

	Lwazi seems to be attending to the cubic equation of $m(t)$ from (e) to help him sketch the graph; he is finding an x-intercept: $x^2\left(\frac{2}{3}x+4\right)$ $x=-4\times\frac{3}{2}$ $x=-6$ But the others seems to be attending to the graph of $m'(t)$ in the problem text. Students are suggesting different possibilities (by tracing in the air, by sketching, or by describing in words). Feedback involves identifying which graph is being spoken about. Ndumiso rejects Thokozile's answer on the basis that "it looks wrong".
2 (717-729)	The students are now drawing possible graphs. Ndumiso has the correct shape for the first part of the graph, but is not attending to different time values as required.  Kelsa pronounces that she has a possible graph ("that's my graph"). She is attending to what the graph looks like at different t values, and corrects herself as she goes;  Ndumiso evaluates Kelsa's graph as wrong on the basis that it is the same as the "original" ($m'(t)$). He is having difficulty and pronounces, "I can't ((*Shaking his head*)) think of a graph", suggesting that getting an answer involves a mental process.

3 (730-762)	Kelsa appeals to the Tutor, emphasizing as in the text that her attempt is a "rough sketch". Lwazi evaluates her graph suggesting that she has her graph "the wrong way round"; he is still referring to the "happy graph" he pronounced in Episode 1 (he traces the shape in the air). The Tutor is attending to Kelsa's second graph in Episode 2; he points to the label for the local maximum and says, "what is this?" It is not clear whether the Tutor is referencing her labels or the shape of the graph; Kelsa thinks it is the former and responds by changing back to her earlier version where the local maximum was at $t = 5$. The Tutor gives negative feedback simply by asking for "other possibilities". He attends to Thokozile's attempt which he says, in a joking manner, is "also wrong" (Ndumiso says her attempt looks like "a bird"). At this stage there is only evaluation from the Tutor with no explanations. There seems to be competition between the students; criticizing one another's answers, but not in a constructive mathematical way.
4 (763-779)	The Tutor interrupts this and refers students to their answer for question (b), which was the significance of the meaning of the point t_1. He is thus making links to the task context (required in (b)) and attending to a different point to those attended to so far by the students and mentioned in the text. Kelsa responds, "that's when the amount of product is at a <u>maximum</u>". The Tutor then makes a link to what graph they are trying to draw and both Thokozile and Kelsa link to the task context in their description; "We're graphing the total amount of product?" The Tutor and the students are not attending to the difference between the amount of product formed (required for question (f)) and the amount of product in the reaction chamber (as in question (b)). The Tutor then refers them to what is happening (in the task context) after t_1. Kelsa is able to describe using the context; "the <u>amount</u> of product is decreasing↑". The Tutor makes the link to the graph and stresses the need to include the point t_1. Kelsa returns to her original graph, relabels the horizontal axis, and links the graph to the task context, "ja … cause it's gonna <u>decrease</u> afterwards". She gets positive feedback "yeah" from the Tutor. 

5 (780-789)	Ndumiso then presents a graph to the Tutor for feedback (he pronounces it as "so this is wrong then?", suggesting that he has been attending to the discussion in Episode 4). Ndumiso has the same shape as his graph in Episode 2, but has now added labels:  The label for $t = 2$ is correct (confirmed by Tutor), but he has $t = 4$ for the local maximum. The Tutor begins by correcting him on the label for the local maximum. He is attending to Ndumiso's graph; he has changed the label of "5" to "4" and gives positive feedback on the shape of graph from the local maximum to the new point $t = 4$. He then relates the appearance of the graph between $t = 4$ and $t = 5$ to what is happening in the task context; "If you're adding something or subtracting something at a constant rate what are you expecting the graph to look like?" Both Ndumiso and Lwazi settle on a horizontal straight line, which they demonstrate in the air. Yet, Kelsa argues for "decreasing", which the Tutor builds on to argue that the graph will be a decreasing straight line. Lwazi and Kelsa thus change their original graphs to reflect the straight line between $t = 4$ and $t = 5$: 

6 (790-817)	Thokozile has a different graph, and is attending to her horizontal straight line (and not the label for the local maximum):  The Tutor responds, linking the task context to the steepness of the graph; "because …. the thing is … like … uhm the total amount of mass … is gonna be decreasing at it's fastest rate … at it's fastest rate after *t* equals to 4 … here your decreasing suddenly is leveling out". Again, he is attending to the total mass in the chamber, and not the mass formed as required. In her final answer, Thokozile crosses out her horizontal straight line after $t = 5$. The others are talking about their social life and preparation for the next maths test.

APPENDIX Q

FOLDOUT OF THE THREE PRACTICAL PROBLEMS

The Flu Virus Problem

A flu virus has hit a community of 10 000 people. Once a person has had the flu he or she becomes immune to the disease and does not get it again. Sooner or later everybody in the community catches the flu. Let $P(t)$ denote the number of people who have, or have had, the disease t days after the first case of flu was recorded.

a) Draw a rough sketch of the graph of P as a function of t, clearly showing the maximum number of people who get infected, and **do not continue until you have had your graph checked by a tutor.**

b) What are the units of $P'(t)$?

c) What does $P(4) = 1\,200$ mean in practical terms? (Your explanation should make sense to somebody who does not know any mathematics.)

d) What does $\dfrac{P(7)-P(4)}{7-4}=350$ mean in practical terms? Give the correct units.

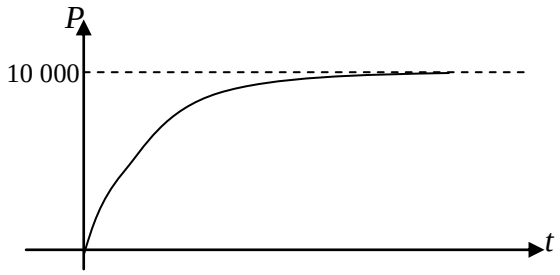
e) What does $P'(4) = 400$ mean in practical terms? Explain why $P'(t)$ can never be negative.

f) What is $\lim\limits_{t\to\infty} P(t)$? Give a short reason for your answer.

g) What is $\lim\limits_{t\to\infty} P'(t)$? Give a reason for your answer.

Worked solution for the Flu Virus Problem

a)



b) $P'(t)$ units: people per day.

c) 4 days after the first recorded person got flu, 1200 people had the flu.

d) From the 4th to the 7th day after the first recorded person got flu, the number of people on average who had the flu was increasing by 350 people per day.

e) 4 days after the start of the epidemic, the number of people who had the flu was increasing by 400 people per day. ($P'(t) > 0$ since the total number of people with the flu or who have had the flu can only increase.)

f) $\lim\limits_{t\to\infty} P(t) = 10\,000$. Eventually after a long time everyone gets the flu.

g) $\lim\limits_{t\to\infty} P'(t) = 0$. Eventually the number of people who have caught the flu becomes (very nearly) constant at 10 000, so the rate of new infections is 0 (see graph).

The Car Problem

Boxed text preceding the Car Problem

The following questions are related rates problems. These MUST be set up correctly. Follow these steps for **EVERY** question:

1. Draw a diagram and define variables.
2. Write down what is given, using the correct notation.
3. Write down what is to be found.
4. Write down a formula linking the variables.
5. Differentiate and complete the question

The Car Problem

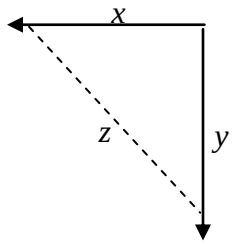
Two cars start moving from the same point. One travels south at 100km/h and the other travels west at 75km/h. At what rate is the distance between the cars increasing two hours later? (Let the distance between the cars after a time t be z km).

Worked solution for the Car Problem

Let x = distance covered by car A

Let y = distance covered by car B

Let z = distance between car A and car B



Given: $\frac{dx}{dt} = 75$ and $\frac{dy}{dt} = 100$

To Find: $\frac{dz}{dt}$ when t = 2 hours

$x^2 + y^2 = z^2$ (Pyth)

$\therefore\ 2x.\frac{dx}{dt} + 2y.\frac{dy}{dt} = 2z.\frac{dz}{dt}$

When t = 2 hours, x = 150 km and y = 200 km and $z = \sqrt{150^2 + 200^2}$ = 250 km

$\therefore\ 150 \times 75 + 200 \times 100 = 250.\frac{dz}{dt}$

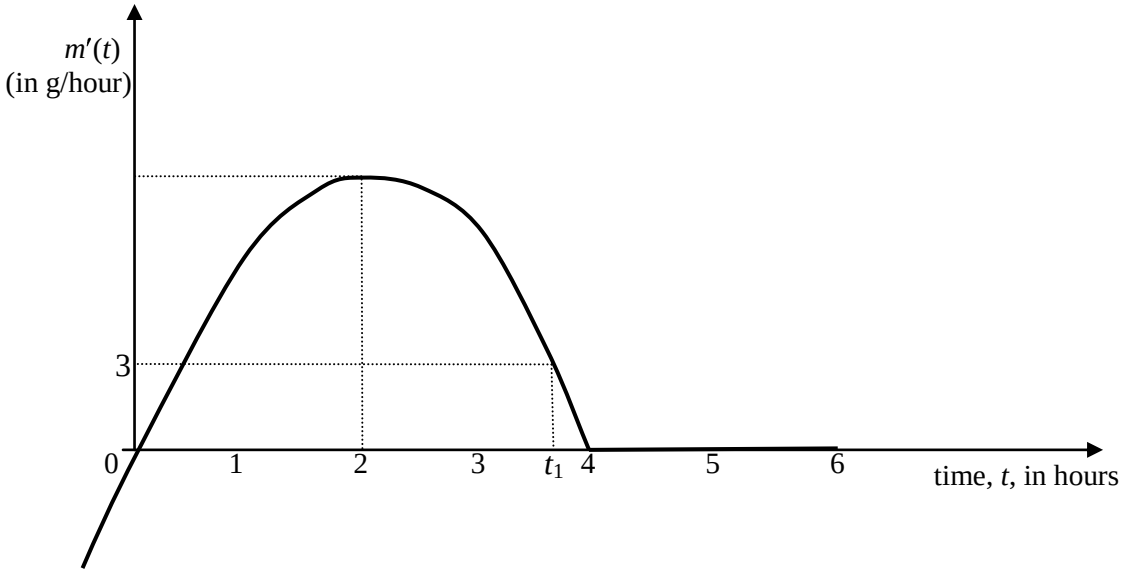
So $\frac{dz}{dt} = \frac{1}{250}(150 \times 75 + 200 \times 100) = 125$ km/h

The Chemical Reaction Problem

Quantities of two chemicals A and B are mixed together in a reaction chamber, and they react to form a new product, X.

The **rate** at which the product X is formed is given by $m'(t)$, where m is the mass of the product formed, in grams, and the time t from the start of the reaction is measured in hours. The graph of $m'(t)$ is a **parabola** graph until time $t = 4$ hours, after which it is zero.

It is also given that, from the start of the reaction, some of the product X is removed from the reaction chamber at a constant rate of 3 g/hour.



(a) Write down an expression involving an integral that gives the **total** mass of product X **in** the reaction chamber after a time of t hours.

(b) Explain very clearly the significance/practical meaning of the time $t = t_1$ in the graph above.

(c) Explain very clearly the significance/practical meaning of the local maximum in the graph of $m'(t)$ at time $t = 2$ hours.

(d) Find the equation of the parabola part of the graph – it will express $m'(t)$ as a quadratic function of t.

(e) Hence find the **total** mass of product X formed in the 4 hours since the start of the reaction.

(f) Draw a rough sketch of the graph of $m(t)$ for $0 \le t \le 5$ hours. Clearly indicate on your graph the times $t = 2$ hours, $t = 4$ hours, and $t = 5$ hours.

Worked solutions for the Chemical Reaction Problem

(a) Total mass = $\int_0^t m'(t)\,dt - 3t$

(b) At time $t = t_1$, the amount of product X in the reaction chamber is a maximum (or, after time $t = t_1$, the rate of formation of product X is less than the rate of removal of product X).

(c) At $t = 2$ the rate of formation of product X is the greatest / reaction rate is the fastest.

(d) $m'(t) = a(t-0)(t-4)$

$\therefore m'(t) = at(t-4)$

When $t = 2$, $m'(t) = 8$

So $8 = a(2)(2-4)$, $8 = -4a$ $\quad \therefore a = -2$.

So the equation of $m'(t)$ is $m'(t) = -2t(t-4) = -2t^2 + 8t$

(e) Total mass of product X formed = $\int_0^4 (-2t^2 + 8t)\,dt = \left[\frac{-2t^3}{3} + 4t^2\right]_0^4 = \frac{-2(64)}{3} + 64$

$= \frac{64}{3}$

$= 21\frac{1}{3}g$

(f)

