It is in general a formidable problem to calculate G_{kk}. As a first step the interaction of a nucleon pair, in free space is considered. The effect of the Pauli exclusion principle is neglected and the treatment is restricted to s partial waves.

Then

$$\Delta E(k) = g_{kk} = \frac{\hbar^2}{m} \iiint \Phi_k(\underset{\sim}{r})V(\underset{\sim}{r})\Psi_k(\underset{\sim}{r})\, d^3r$$

$$= \frac{4\pi\hbar^2}{m} \int r^2\, \Phi_k(\underset{\sim}{r})\, \Psi_k(\underset{\sim}{r})\, dr$$

Introducing the radial wavefunctions :

$$S_k(\underset{\sim}{r}) = r\, \Phi_k(\underset{\sim}{r}) = \frac{\sin kr}{k}$$

$$R_k(\underset{\sim}{r}) = r\, \Psi_k(\underset{\sim}{r}) \tag{9.36}$$

Then

$$g_{kk} = \frac{4\pi\hbar^2}{m} \int S_k(\underset{\sim}{r})\, V(\underset{\sim}{r}) R_k(\underset{\sim}{r})\, dr \tag{9.37}$$

Now S_k is the solution of the Schrodinger equation when no interaction potential is present.

$$\frac{-\hbar^2}{2m}\, \nabla^2\, S_k = E_o\, S_k$$

or $$-\nabla^2\, S_k = k^2\, S_k \tag{9.38}$$

R_k is the solution of the Schrodinger equation in the presence of an interaction potential V.

$$-\nabla^2 R_k + VR_k = k^2 R_k \tag{9.39}$$

In order to normalize the wavefunction correctly, suppose that the wavefunction is subject to the boundary condition R(b) = 0 where b is large compared to the interaction range and the de Broglie wavelength. $S_k(b)$ must also vanish and we have from eqn 9.36, that both R and S are zero at r=o.

From the normalization conditions

$$S_k(r) \underset{r\to b}{=} \frac{\sin kr}{k} \tag{9.40}$$

where $kb = n\pi$

If we introduce the interaction V, $S_k(r)$ goes over into $R_k(r)$. However we require R to vanish at r=b and this necessitates a slight shift in the momentum. Thus

$$R \sim \text{const}\ \frac{\sin (k'r + \delta(k'))}{k'} \tag{9.41}$$

where $k'b + \delta(k') = kb = n\pi$ (9.42)

The Schrodinger equation obeyed by R, equation 9.39, thus becomes :

$$-\nabla^2 R_k + VR_k = k'^2 R_k \tag{9.43}$$

Now multiply equation 9.43 by S_k and equation 9.38 by R_k. We then take the difference and integrate from 0 to b. This gives

$$(k'^2-k^2) \int_0^b R_k\, S_k\, dr + \int_0^b (S_k\nabla^2 R_k - S_k V R_k - R_k \nabla^2 S_k)\, dr = 0$$

$$\int_0^b S_k\, V\, R_k\, dr = (k'^2-k^2) \int_0^b R_k\, S_k\, dr + \int_0^b (S_k\nabla^2 R_k - R_k\nabla^2 S_k)\, dr$$

$$= (k'^2-k^2) \int_0^b R_k\, S_k\, dr + S_k \nabla R_k \Big|_0^b - \int_0^b \nabla S_k\, \nabla R_k\, dr$$

$$- R_k\, \nabla S_k \Big|_0^b + \int_0^b \nabla R_k\, \nabla S_k\, dr$$

$$= (k'^2-k^2) \int_0^b R_k\, S_k\, dr \tag{9.44}$$

By the normalization condition $\langle \Phi_k | \Psi_k \rangle = \langle \Phi_k | \Phi_k \rangle$

Therefore $\int_0^b R_k S_k \, dr = \int_0^b S_k^2 \, dr$

$$= \int_0^b \frac{\sin^2 kr}{k^2} \, dr \quad \text{from equation 9.39}$$

$$= \frac{b}{2k^2} \tag{9.45}$$

Also from equation 9.42

$$k'^2 - k^2 \sim -2k \frac{\delta(k)}{b} \tag{9.46}$$

Therefore $\int_0^b S_k V R_k \, dr = -2k \frac{\delta(k)}{b} \frac{b}{2k^2}$

$$= -\frac{\delta(k)}{k} \tag{9.47}$$

The g matrix (from equation 9.35) is therefore

$$g_{kk} = \Delta E(k) = \frac{-4\pi\hbar^2}{m} \frac{\delta(k)}{k} \tag{9.48}$$

If we allow for the presence of partial waves other than δ waves in the nucleon-nucleon interaction, this expression can be generalized to

$$\Delta E = \frac{-4\pi\hbar^2}{m} \sum_\ell (2\ell+1) \frac{\delta_\ell(k)}{k} \tag{9.49}$$

It is thus possible to relate the energy shift experienced by an interacting nucleon pair in free space to the phase shift experienced by the nucleons in the interaction.

IX.7 The Integration in Momentum Space

To derive the interaction energy density, $\varepsilon(k_{F1}, k_{F2}, K)$, the integration $\int_{F1} \int_{F2} \frac{\delta(k)}{k} \, d^3k_1 d^3k_2$ was performed analytically.

It was found that a careful choice of the integration variables could greatly simplify the integration. In

It was found that a careful choice of the integration variables could greatly simplify the integration. In

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partic[illegible] choice of variables k_1, k_2 as shown in Fig. 9.5 was found to give rise to mutually cancelling divergent terms in the integration, which is undesirable for an actual calculation. Therefore the integration variables that were chosen, k_2

majo[illegible] difficulties arose.

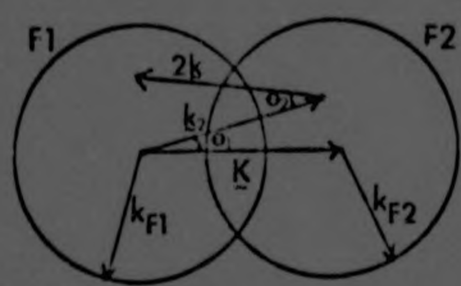

Fig. 9.6. Integration variables k_2 and k.

To treat the problem in phase-shift approximation all that is now required is an expression for $\delta(k)$ as a function of k where δ is the phase shift experienced by nuclear matter colliding with nuclear matter with relative momentum k. What is well known is the phase shift experienced in nucleon-nucleon collisions. We make use of this information and use a statistical averaging to approximate nuclear matter interaction.

For proton-proton (p-p) scattering and neutron-neutron (n-n) scattering only singlet states are possible. However in neutron-proton (n-p) scattering both singlet and triplet states are allowed. The triplet states are weighted 3:1

against the singlet states for the n-p scattering. The average phase shift $\bar{\delta}$ is therefore

$$\bar{\delta} = \frac{1}{4}\,\delta^{s}_{pp} + \frac{1}{4}\,\delta^{s}_{nn} + \frac{1}{8}\,\delta^{t}_{np} + \frac{3}{8}\,\delta^{t}_{np} \tag{9.50}$$

where s denotes singlet states

and t denotes triplet states

Below are shown the phase shifts for nucleon-nucleon scattering as given by Arndt and McGregor [35]

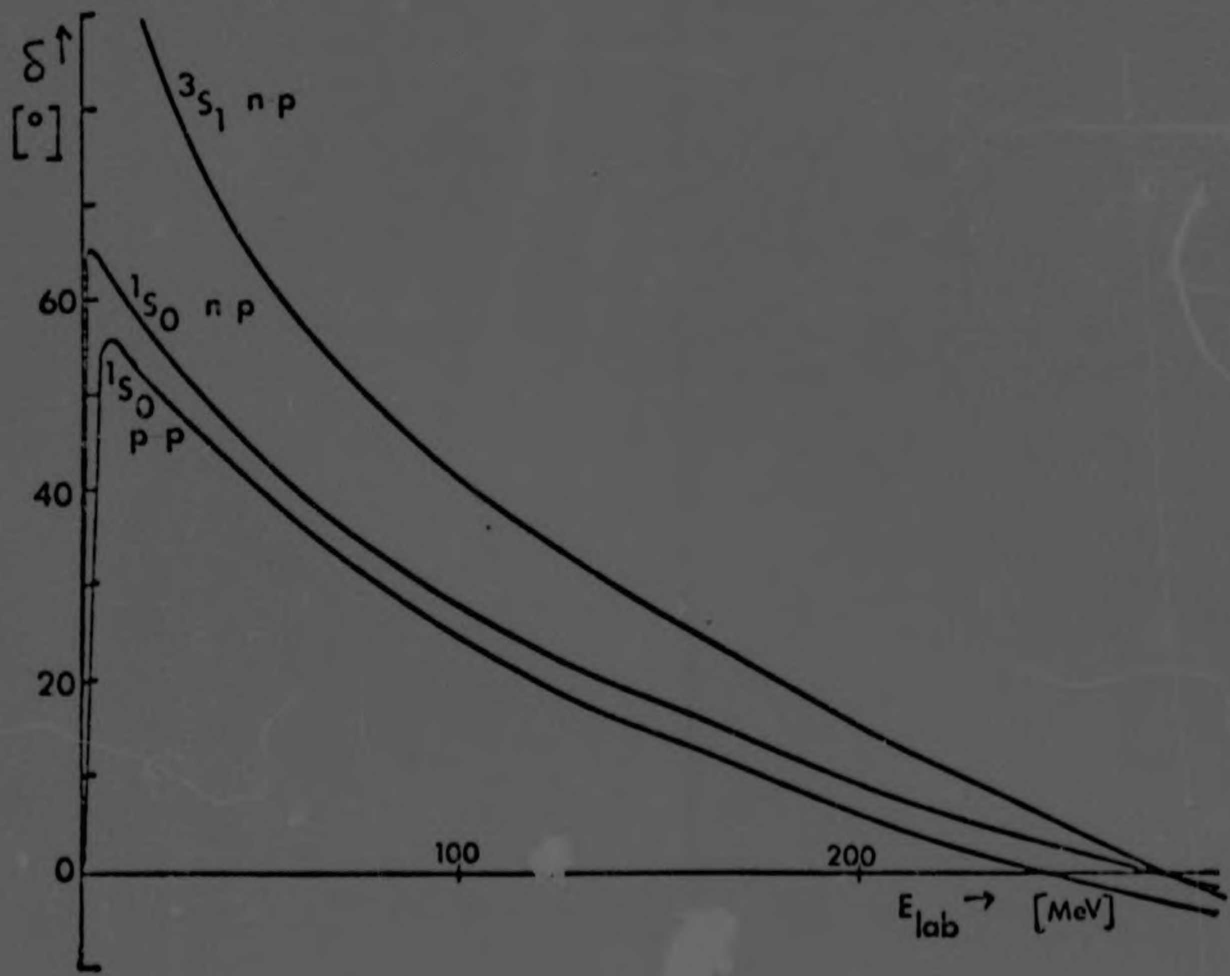

Fig. 9.7. Nucleon-nucleon phase shift data from Ref (35).

It is possible to construct a good fit to these curves using an expression of the form $\delta = E + Ak(e^{-Bk}+Cke^{-Dk})$ where B,C,D and E are parameters which can be adjusted to give the closest fit to the experimental curves. This form was chosen to facilitate the integrations in momentum space. The expression is given the correct low energy behaviour by the condition that :

$$A = \lim_{k \to o} \tan(\delta(k)) = -a \tag{9.51}$$

where a is the scattering length for the nucleon-nucleon interaction being parametrized. Equation 9.35 becomes

$$\varepsilon(k_{F1},k_{F2},K) = \frac{1}{(\Delta v)^2} \frac{\rho_1}{\rho_{C.D}} \frac{\rho_2}{\rho_{C.D}} \int_{F1} \int_{F2} \left[\frac{E}{k}+A(e^{-Bk}+Cke^{-Dk})\right] d^3k_1 d^3k_2 \tag{9.52}$$

IX.8 The Reaction Matrix for Bound Nucleons

A pair of particles in the nucleus does not scatter in the same way as a pair of free particles. The Pauli principle prohibits any particles from scattering into states which are already occupied by other particles. A method has been proposed [36] by Scott and Moszkowski for constructing an effective two-body interaction in nuclear matter.

They note that a realistic s-state interaction for a pair in nuclear matter has a short-range repulsion and a long-range attraction. If the total s-wave phase shift is attractive, which it is up to about 250 MeV, they propose splitting the interaction into two components. These components are a short-range part V_s which contains the short range repulsion and enough attraction to give zero s wave phase shift and a long range part V_L which is the remaining attractive potential.

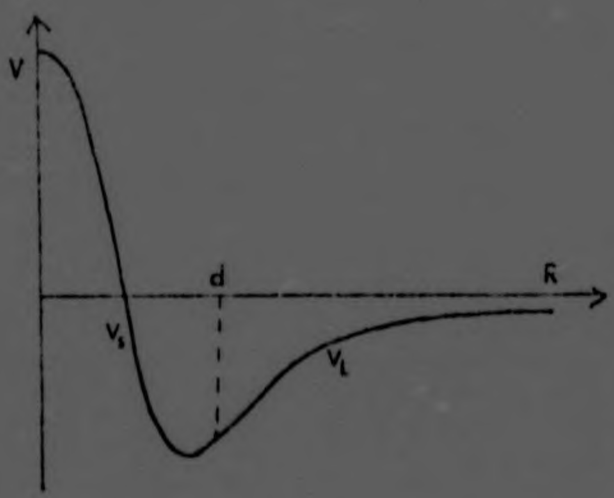

Fig. 9.8. Separation of nucleon-nucleon potential into short range, V_S, and long range, V_L, components.

The reaction matrix G satisfies the integral equation

$$G = V - V \frac{Q}{e} G$$

Using perturbation theory Scott and Moszkowski showed that the reaction matrix can be expanded in terms of the short range reaction matrix for particles in free space, g_s^F, and the long range interaction potential V_L as

$$G = g_s^F + V_L - g_s^F \left(\frac{Q}{e} - \frac{1}{e_o}\right) g_s^F - 2g_s^F \frac{Q}{e} V_L - V_L \frac{Q}{e} V_L + \ldots \tag{9.53}$$

The first term is just the term we have previously considered and approximated in the phase shift approximation. For low values of the relative momentum the phase shift approximation gives much too large an effect. In nuclear matter the exclusion principle prohibits the large phase shift that would occur for free particles and thus cuts down the interaction.

We compare the phase shift approximation with the first order terms, $G = g_s^F + V_L$, of the true reaction matrix in Fig. 9.9

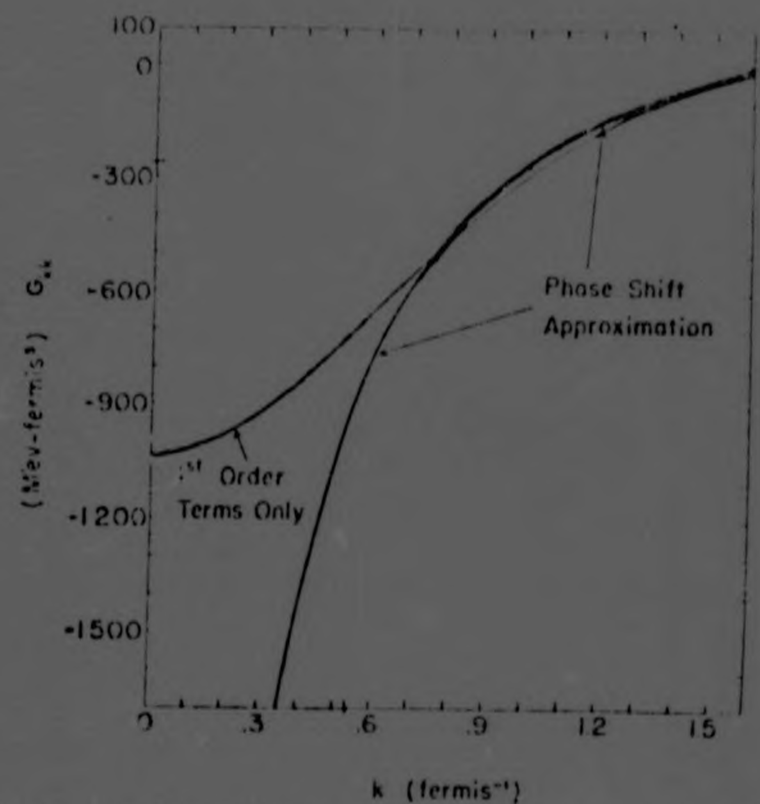

Fig. 9.9 A comparison of the nuclear reaction matrix element using the phase shift approximation with the correct result.

Scott and Moszkowski tabulate the values for the first order terms in the reaction matrix in Ref. (36). We can again parametrize the interaction in the form

$$g_s^F + V_L = \frac{E}{k} + A\ (e^{-Bk}+Cke^{-Dk})$$

IX.9 Analytical Solution of the Integral

We illustrate the technique adopted to solve the energy density integral (Equation 9.52) by considering the integration of the second term of equation 9.52 for the case illustrated by Fig. 9.3b

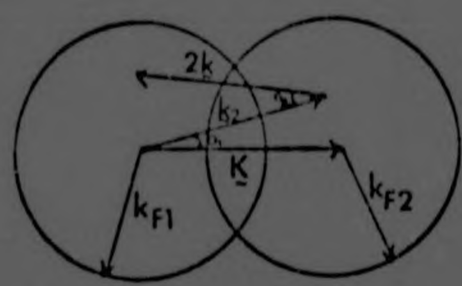

Fig. 9.10 Case 9.3b

The vector $\underset{\sim}{k}_2$ can take on all values for which its endpoint lies inside Fermi sphere F_2 and the endpoint of the vector $2\underset{\sim}{k}$ must lie inside Fermi sphere F_1. We must therefore perform a six-fold integration, two of which are trivial owing to cylindrical symmetry arising from our choice of integration variables.

$$\int_{F1}\int_{F2} e^{-Bk}\, d^3k_1 d^3k_2$$

$$= \int_{k_{F1}}^{k_{F2}+K} 2\pi k_2^2 dk_2 \int_{\frac{k^2+k_2^2-k_{F2}^2}{2k_2K}}^{1} d\cos\theta_2 \int_{k_2-k_{F1}}^{k_2+k_{F1}} 2\pi k^2 \int_{\frac{k_2^2+K^2-k_{F1}^2}{2k_2K}}^{1} d\cos\theta_1\; e^{-Bk} dk$$

$$+\int_{k_{F2}-K}^{k_{F1}} 2\pi k_2^2\, dk_2 \int_{\frac{k^2+k_2^2-k_{F2}^2}{2k_2K}}^{1} d\cos\theta_2 \left\{ \int_{0}^{k_{F1}-k_2} 4\pi k^2 e^{-Bk} dk + \int_{k_{F1}-k_2}^{k_{F1}+k_2} 2\pi k^2 \int_{\frac{k_2^2+K^2-k_{F1}^2}{2k_2K}}^{1} d\cos\theta_1\; e^{-Bk}\, dk \right\}$$

$$+\int_{0}^{k_{F2}-K} 2\pi k_2^2 dk_2 \int_{-1}^{1} d\cos\theta_2 \left\{ \int_{0}^{k_{F1}-k_2} 4\pi k^2 e^{-Bk}\, dk + \int_{k_{F1}-k_2}^{k_{F1}+k_2} 2\pi k^2 \int_{\frac{k_2^2+K^2-k_{F1}^2}{2k_2K}}^{1} d\cos\theta_1\; e^{-Bk}\, dk \right\} \qquad (9.54)$$

A lengthy but straightforward calculation leads to the following expression :

$$\int_{F2} d^3k_2 \int_{F1} d^3k_1 \; e^{-\beta k} = \frac{\pi^2}{\beta^3}\Big(\frac{16}{3}k_{F1}^3 + \frac{16}{3}k_{F2}^3 - 4k_{F2}^2K - 4k_{F1}^2K$$

$$+ \frac{2}{3}K^3 + \frac{4k_{F1}^2k_{F2}^2}{K} - \frac{2k_{F2}^4}{K} - \frac{2k_{F1}^4}{K} + \frac{16K}{\beta^2} - \frac{16k_{F2}^2}{K\beta^2} - \frac{16k_{F1}^2}{K\beta^2} + \frac{48}{K\beta^4}\Big)$$

$$+ \frac{2\pi^2}{\beta^3}e^{-\beta(k_{F1}+k_{F2}+K)}\Big(\frac{2k_{F1}k_{F2}^2}{K\beta} + \frac{2k_{F1}^2}{K\beta^2} + \frac{2k_{F1}}{\beta^2} + \frac{2}{\beta^3} + \frac{2k_{F1}k_{F2}}{\beta}$$

$$+ \frac{2k_{F2}}{\beta^2} + \frac{12k_{F1}k_{F2}}{K\beta^2} + \frac{12k_{F2}}{K\beta^3} + \frac{12k_{F1}}{K\beta^3} + \frac{2k_{F1}^2k_{F2}}{K\beta} + \frac{12}{K\beta^4} + \frac{2k_{F2}^2}{K\beta^2}\Big)$$

$$+ \frac{2\pi^2}{\beta^3}e^{-\beta(k_{F1}+k_{F2}-K)}\Big(\frac{2k_{F1}k_{F2}}{\beta} + \frac{2k_{F2}}{\beta^2} + \frac{2k_{F1}}{\beta^2} + \frac{2}{\beta^3} - \frac{2k_{F2}^2}{K\beta^2} - \frac{2k_{F1}^2}{K\beta^2}$$

$$- \frac{2k_{F1}k_{F2}^2}{K\beta} - \frac{2k_{F1}^2k_{F2}}{K\beta} - \frac{12k_{F1}k_{F2}}{K\beta^2} - \frac{12k_{F1}}{K\beta^3} - \frac{12k_{F2}}{K\beta^3} - \frac{12}{K\beta^4}\Big)$$

$$+ \frac{2\pi^2}{\beta^3}e^{-\beta(k_{F1}-k_{F2}+K)}\Big(\frac{2k_{F1}k_{F2}}{\beta} - \frac{2k_{F1}}{\beta^2} + \frac{2k_{F2}}{\beta^2} - \frac{12}{K\beta^4} + \frac{12k_{F1}k_{F2}}{K\beta^2} - \frac{2}{\beta^3}$$

$$- \frac{2k_{F1}k_{F2}^2}{K\beta} + \frac{2k_{F1}^2k_{F2}}{K\beta} - \frac{2k_{F2}^2}{K\beta^2} - \frac{2k_{F1}^2}{K\beta^2} + \frac{12k_{F2}}{K\beta^3} - \frac{12k_{F1}}{K\beta^3}\Big)$$

$$+ \frac{2\pi^2}{\beta^3}e^{-\beta(k_{F2}-k_{F1}+K)}\Big(\frac{2k_{F1}k_{F2}}{\beta} + \frac{2k_{F1}}{\beta^2} - \frac{2k_{F2}}{\beta^2} - \frac{12}{K\beta^4} + \frac{12k_{F1}k_{F2}}{K\beta^2} - \frac{2}{\beta^3}$$

$$+ \frac{2k_{F1}k_{F2}^2}{K\beta} - \frac{2k_{F1}^2k_{F2}}{K\beta} - \frac{2k_{F2}^2}{K\beta^2} - \frac{2k_{F1}^2}{K\beta^2} - \frac{12k_{F2}}{K\beta^3} + \frac{12k_{F1}}{K\beta^3}\Big)$$

Similar calculations were performed for the other terms in this case and for all terms in the other cases as well (Appendix A). To check that the integrations had the correct form it was noted that for symmetrical cases the result was invariant for interchange of k_{F1} and k_{F2}. Also the different integrals for different cases were seen to yield the same result at their mutual boundaries.

In order to avoid numerical difficulties these integrations were also performed for the limiting case K=0.

The expressions which were thus generated are totally general and it is only when a particular parameter choice has been made that these expressions reflect a particular interaction theory.

We have thus been able to derive analytic expressions for the interaction energy density for two interacting nuclei. What remains to derive the total interaction is to sum over all volume elements in coordinate space

$$V = \int \varepsilon(k_{F1}, k_{F2}, K)\, d^3r \qquad (9.56)$$

This calculation was performed numerically using Gaussian quadrature. We are thus able to derive an interaction potential between heavy-ions which displays an energy dependance and is parameter free.

IX.10 The Calculations in Phase Shift Approximation

We have in phase shift approximation that

$$g_{kk} = \frac{-4\pi\hbar^2}{m} \sum_{\ell} (2\ell+1) \frac{\delta_\ell(k)}{k} \qquad (9.57)$$

In Fig. 9.11 we show the phase shifts for various partial waves for p-p scattering. It is clear that at low energies the dominant contribution is the s wave phase shift.

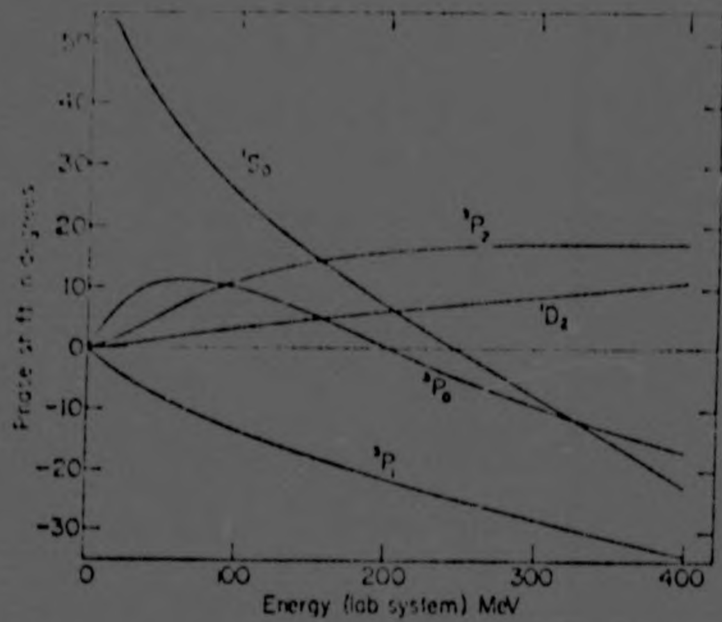

Fig. 9.11 Empirical p-p scattering phase shifts for S,P and D waves vs. energy. From Ref (35).

We have therefore restricted the treatment to s partial waves. Arndt and McGregor (35) give extensive data for the $^1S_0$ p-p scattering and for the $^3S_1$ n-p scattering. A reasonable fit to the experimental data was found with an expression having the functional form $E+Ak(e^{-Bk}+Cke^{-Dk})$ where A was determined by the scattering length. Choosing the following values for the parameters provided a fit as shown in Fig. 9.12

	A[fms]	B[fms]	C[fms]	D[fms]	E
$^1S_0$	7.8163	.910	.004	-2.0	0
$^3S_1$	-5.400	2.30	-.370	2.5	2π

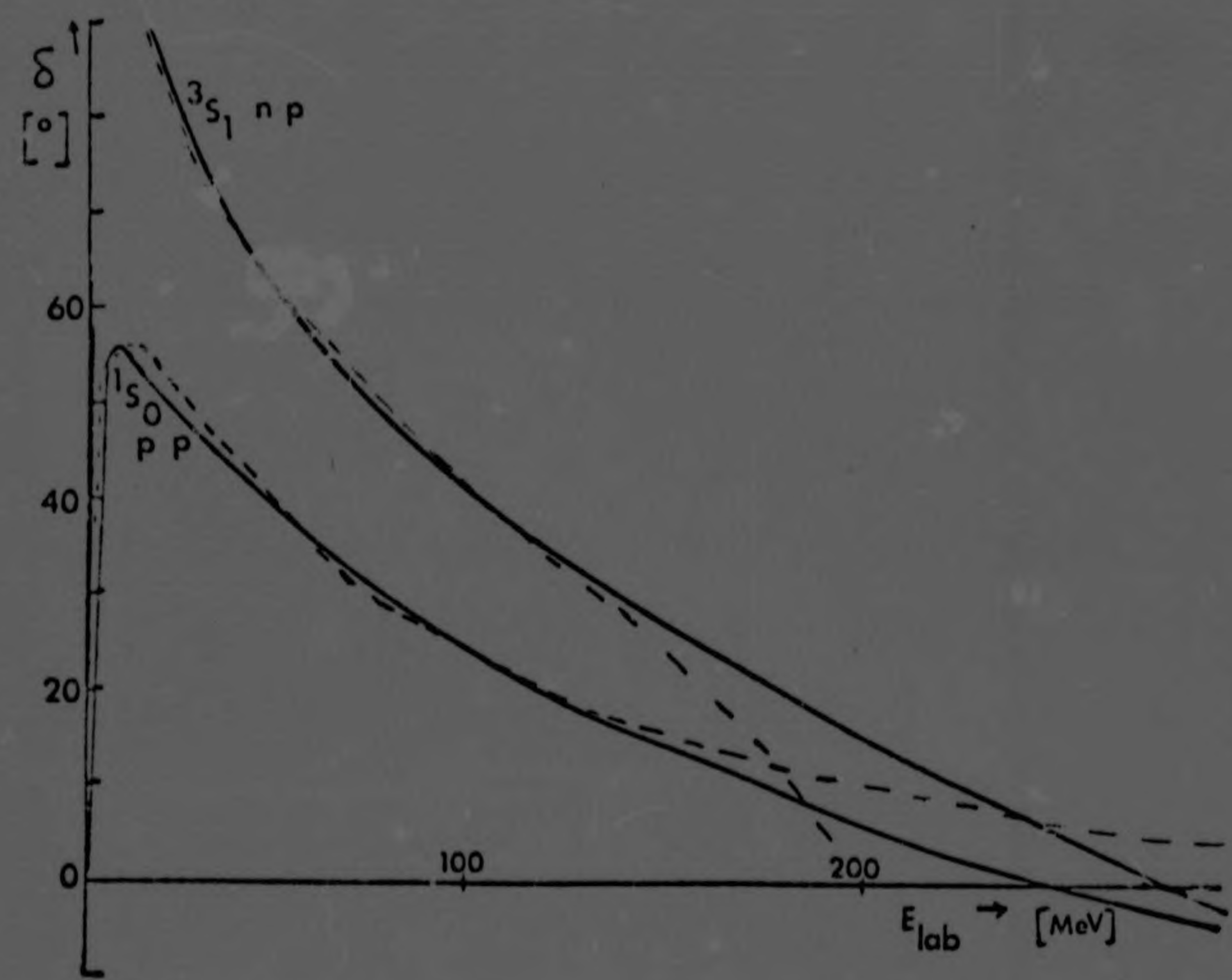

Fig. 9.12 Empirical fits to phase shift data. The experimental curves are the full lines

The integration $\int \epsilon(k_{F1},k_{F2},K)\, d^3r$ was then performed numerically where $\epsilon(k_{F1},k_{F2},K)$ had the analytical form determined by the integration 9.52. The form of $\epsilon(k_{F1},k_{F2},K)$ is illustrated in Fig. 9.13 for a fixed value of K. A family of such surfaces for different values of K was otained.

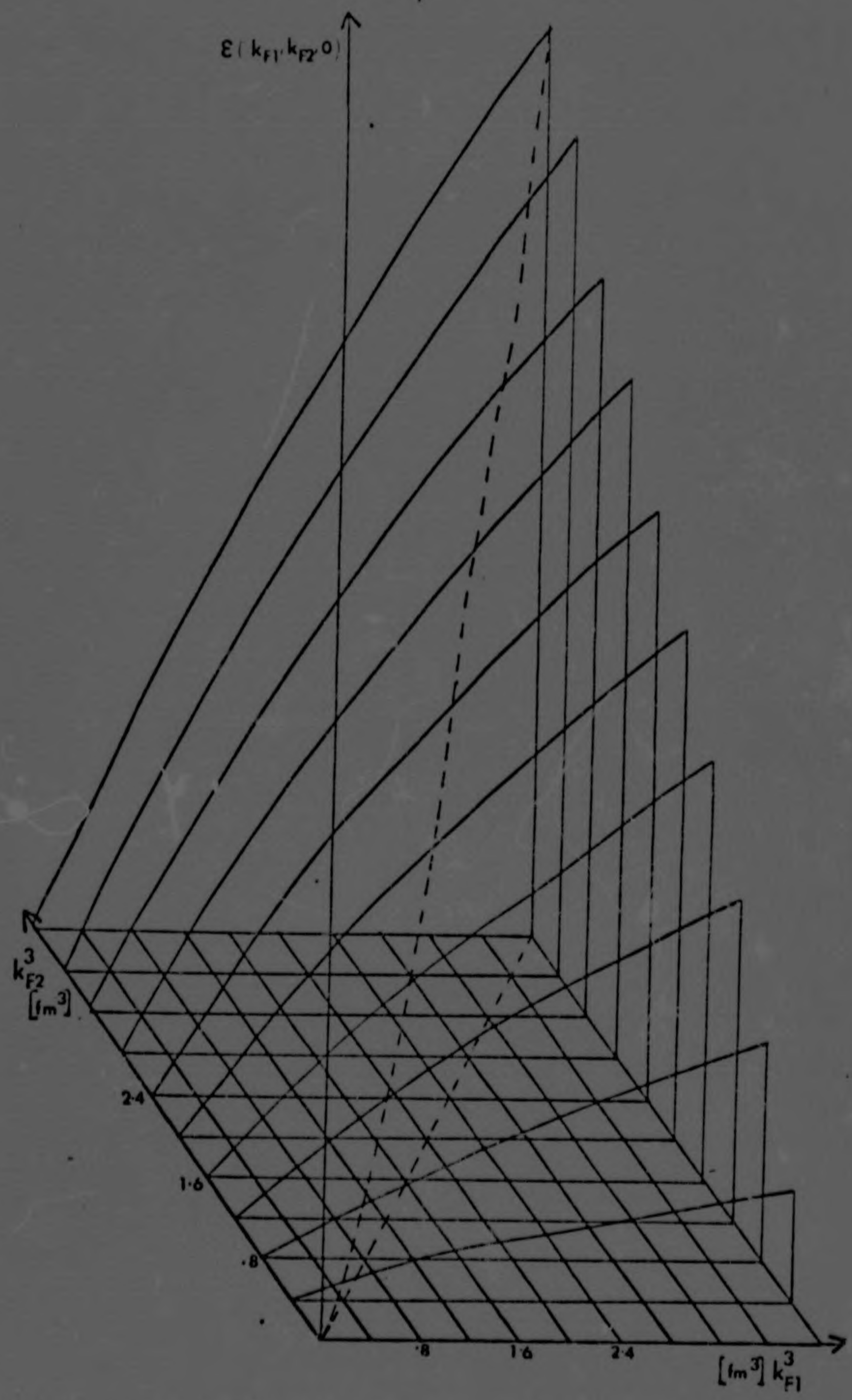

Fig. 9.13 Energy density surface for K=0. The dashed line indicates $k_{F1}=k_{F2}$.

These values of $\varepsilon(k_{F1},k_{F2},K)$ are determined by the nucleon-nucleon phase shift data and are thus valid for all ion-ion collisions. They were used to calculate the potentials for ^{16}O-^{16}O scattering.

The compound density form for the density distribution gives that $k_{F1}=k_{F2}$ for all points in coordinate space. In this case only the values of $\varepsilon(k_{F1},k_{F2},K)$ which lie in the diagonal region, denoted by the dashed line in Fig. 9.14, need be used for the integration.

The procedure used was to first consider the problem in coordinate space. A compound density and consequently the Fermi momenta k_{F1} and k_{F2} were associated with every point in coordinate space. The nucleus-nucleus relative momentum $A\underset{\sim}{K}$ was calculated for a given separation distance R, from the centre-of-mass energy $E_{C.M.}$ and the total effective potential at that separation distance. These values of k_{F1}, k_{F2} and K determined the value of $\varepsilon(k_{F1},k_{F2},K)$, the integrand. These values of the integrand were then inserted into a Gaussian quadrature routine and the potential was calculated (Appendix B).

The nucleus-nucleus potential was thus determined as a function of energy without any free parameters. The form of the potentials and their energy dependance is illustrated in sudden approximation in Fig. 9.14

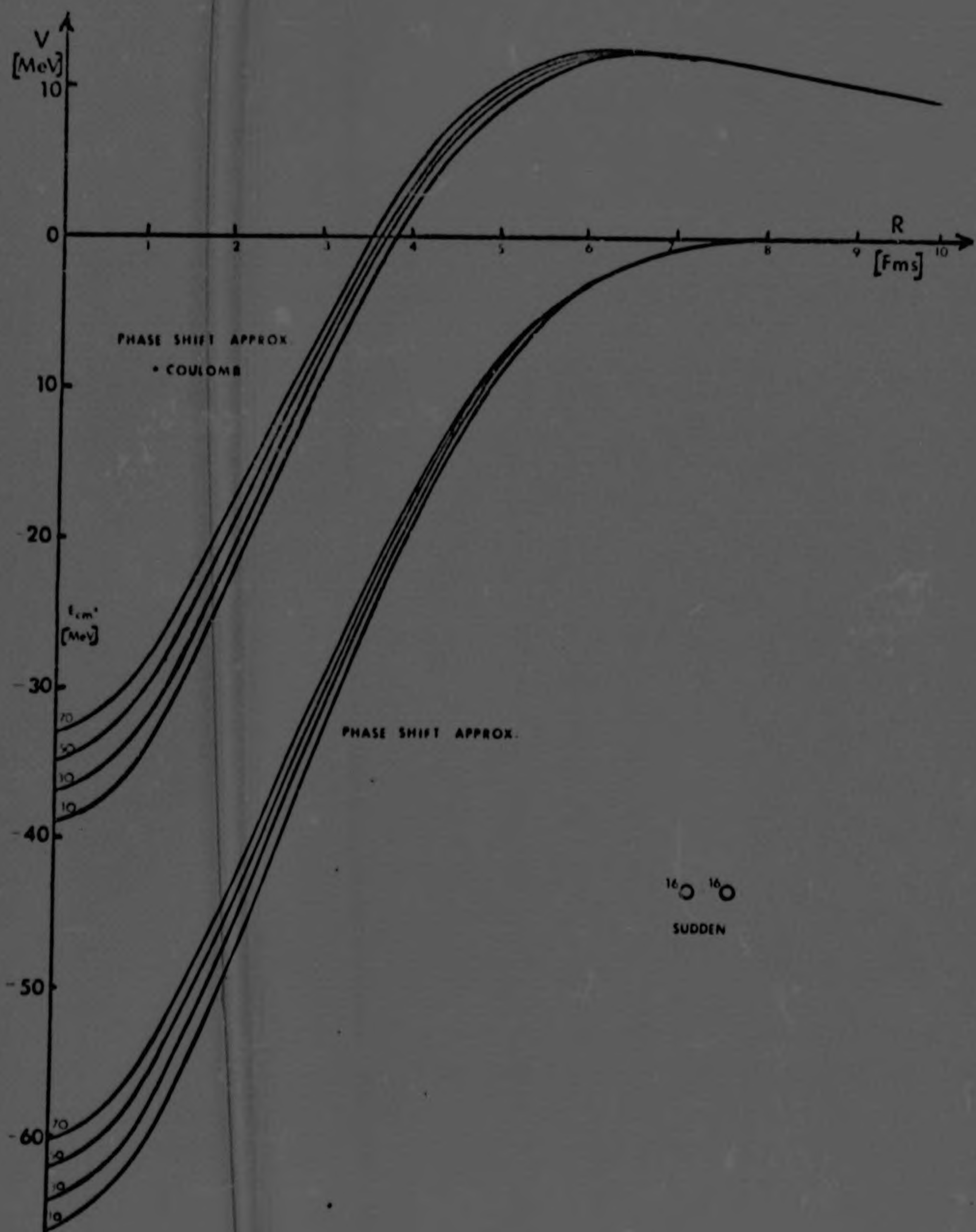

Fig. 9.14 The ^{16}O-^{16}O real potential in sudden approximation showing their energy dependance. The potentials with the Coulomb contribution added are also shown. These potentials are calculated in phase shift approximation.

The overall shape of the potential is seen to have the same form as the overlap density. The potentials become shallower with increasing energy reflecting the decreasing overlap in momentum space.

The potentials were also determined in adiabatic approximation. In adiabatic approximation the magnitude of the compound density is restricted and consequently the value of $\varepsilon(k_{F1},k_{F2},K)$ is reduced increasingly with increasing overlap. The effect of this can be seen in Fig. 9.15 where the adiabatic and sudden potentials are compared. The potentials with the Coulomb term added are also illustrated.

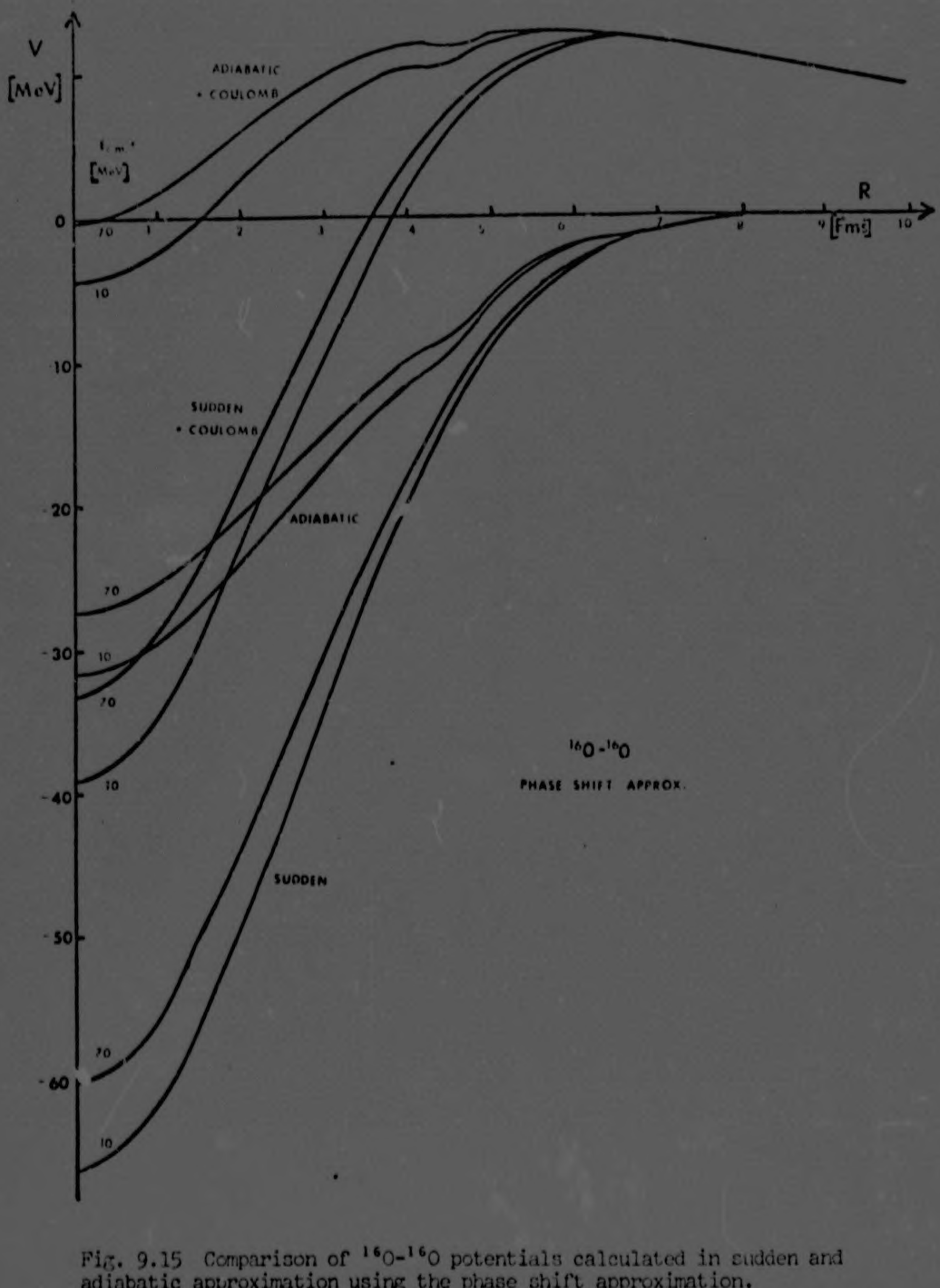

Fig. 9.15 Comparison of ^{16}O-^{16}O potentials calculated in sudden and adiabatic approximation using the phase shift approximation.

IX. 11 The Calculations using the Separation Method

In the separation method we have for the reaction matrix G_{kk}

$$G_{kk} = g_s^F + V_L + g_s^F \left(\frac{Q}{e} - \frac{1}{e_o}\right) g_s^F - 2g_s^F \frac{Q}{e} V_L - V_L \frac{Q}{e} V_L + \ldots$$

We have considered the contribution of the first order terms only, as considerations of higher order terms require a treatment of the Pauli operator which gives rise to more complicated boundary conditions for the integration in momentum space.

It was again possible to parametrize the reaction matrix by an expression with the functional form

$$g_s^F + V_L = \frac{E}{k} + A\,(e^{-Bk} + Cke^{-Dk})$$

The parameters found were :

A[fms]	B[fms]	C[fms]	D[fms]	D
113.618	1.860	2.250	2.950	0

These provided a fit as shown in Fig. 9.16 below

IX. 11 The Calculations using the Separation Method

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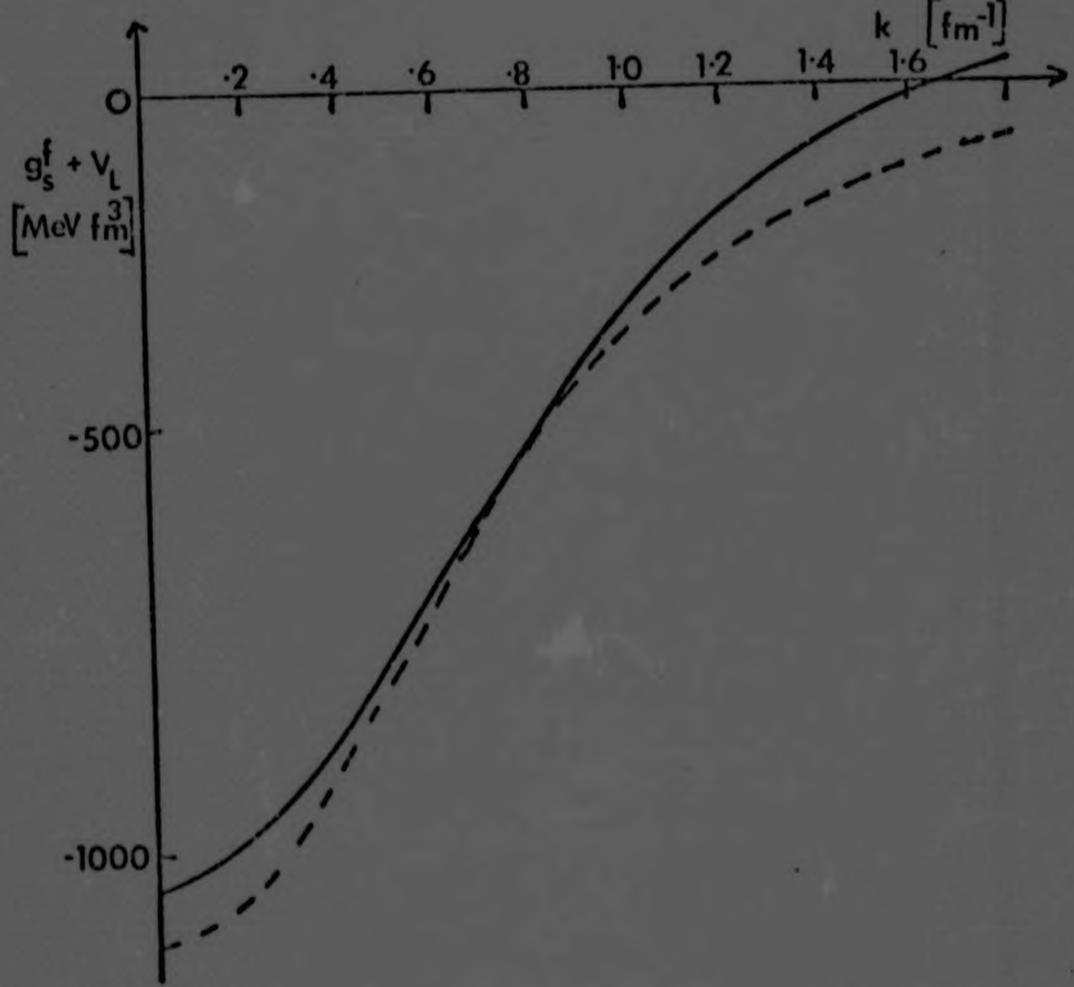

Fig. 9.16 The dashed line is an empirical fit to the first order interaction energy.

The values of $\varepsilon(k_{F1},k_{F2},K)$ were again calculated in this approximation.

The potentials for ^{16}O-^{16}O scattering were then calculated in sudden and adiabatic approximations. The potentials found in adiabatic approximation are illustrated in Fig. 9.17 and are compared with those found in phase shift approximation.

The inhomogeneity correction term was then added and the corresponding potentials calculated (Fig. 9.18)

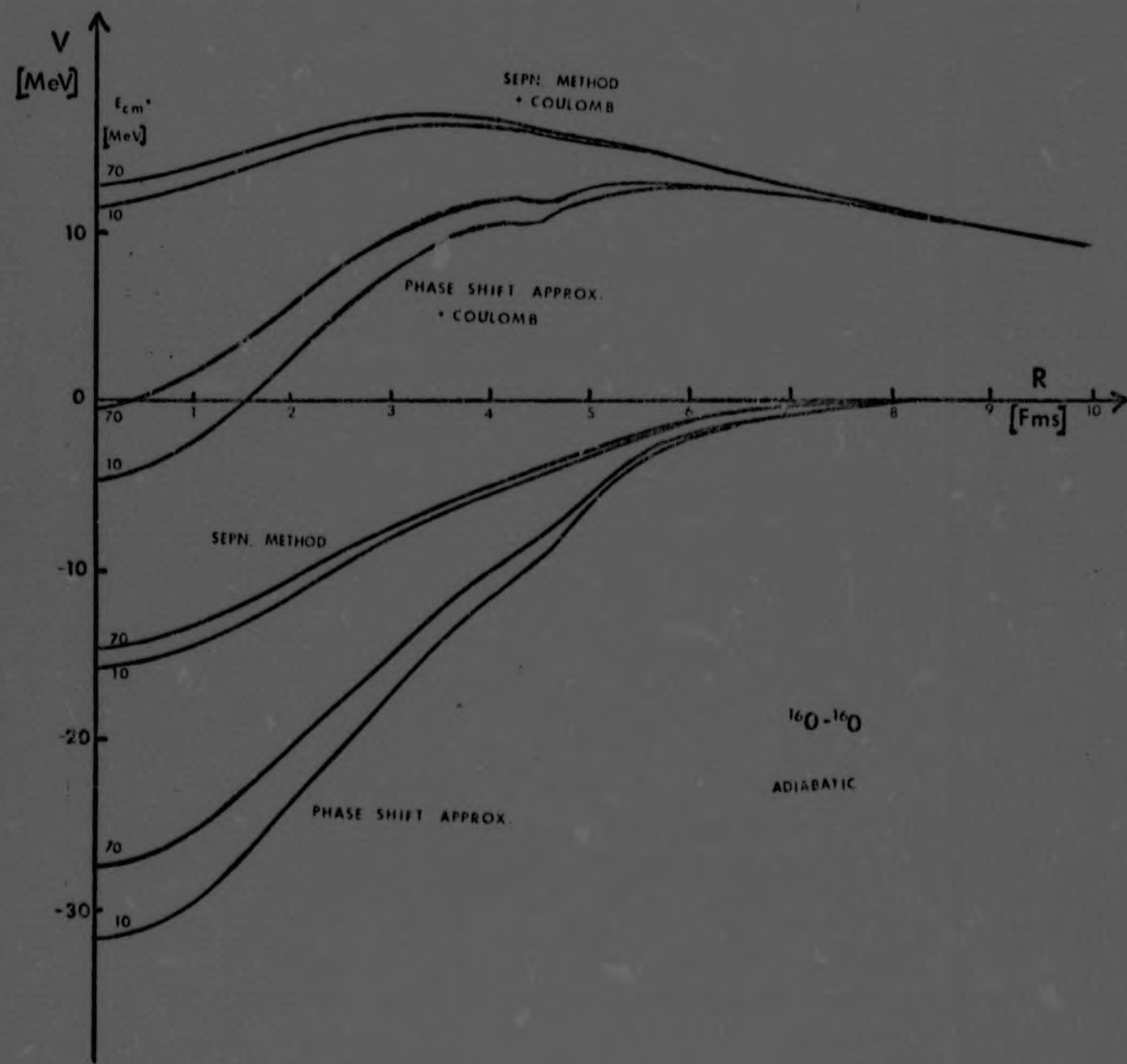

Fig. 9.17 Comparison of $^{16}O-^{16}O$ adiabatic potentials as calculated in phase shift approximation and using the separation method.

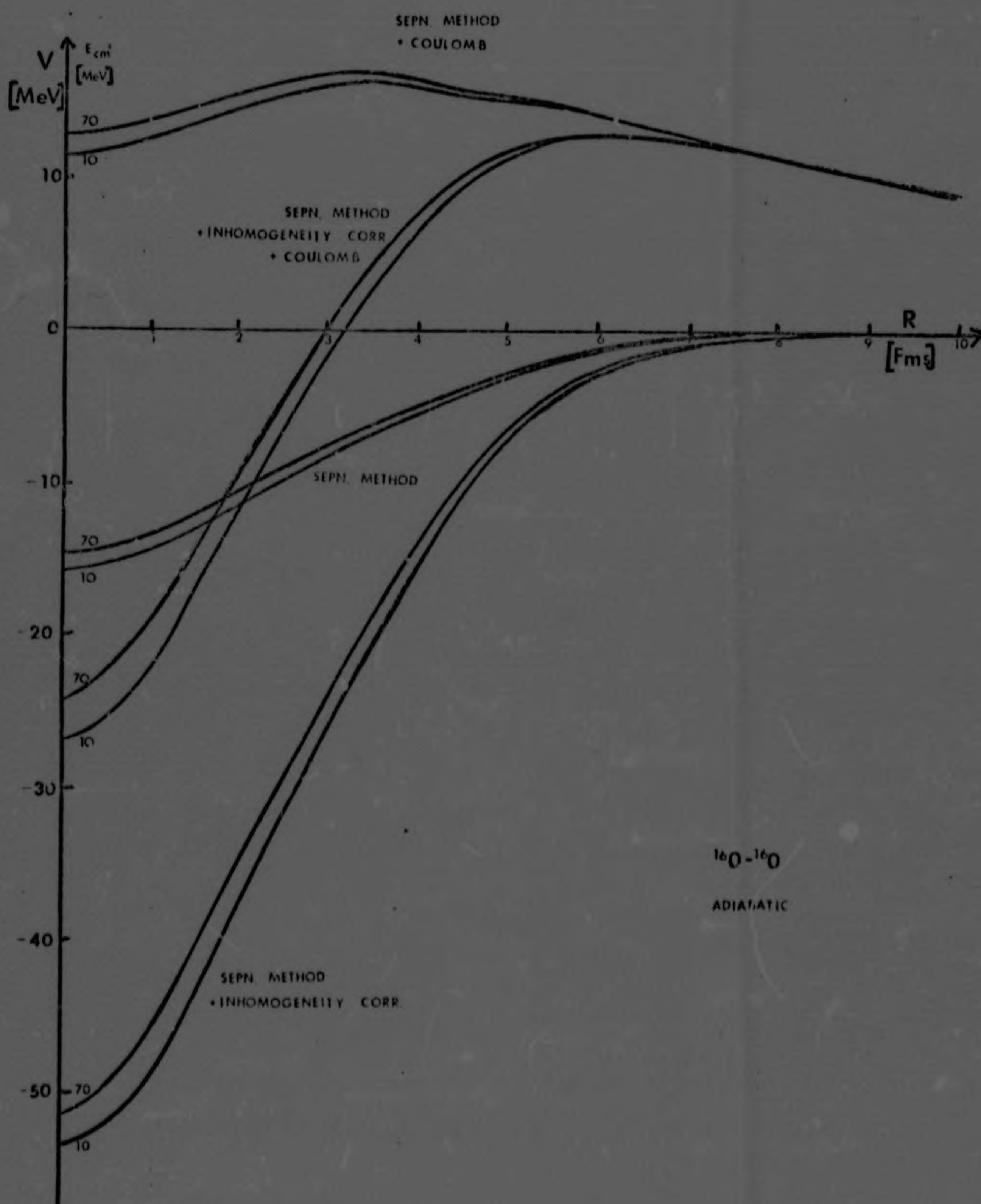

Fig. 9.18 ^{16}O-^{16}O adiabatic potentials using the separation method both with and without inhomogeneity corrections.

The effect of adding an inhomogeneity correction term can be seen from a plot of the difference of the potential with the correction term to the potential without the correction term (Fig. 9.19)

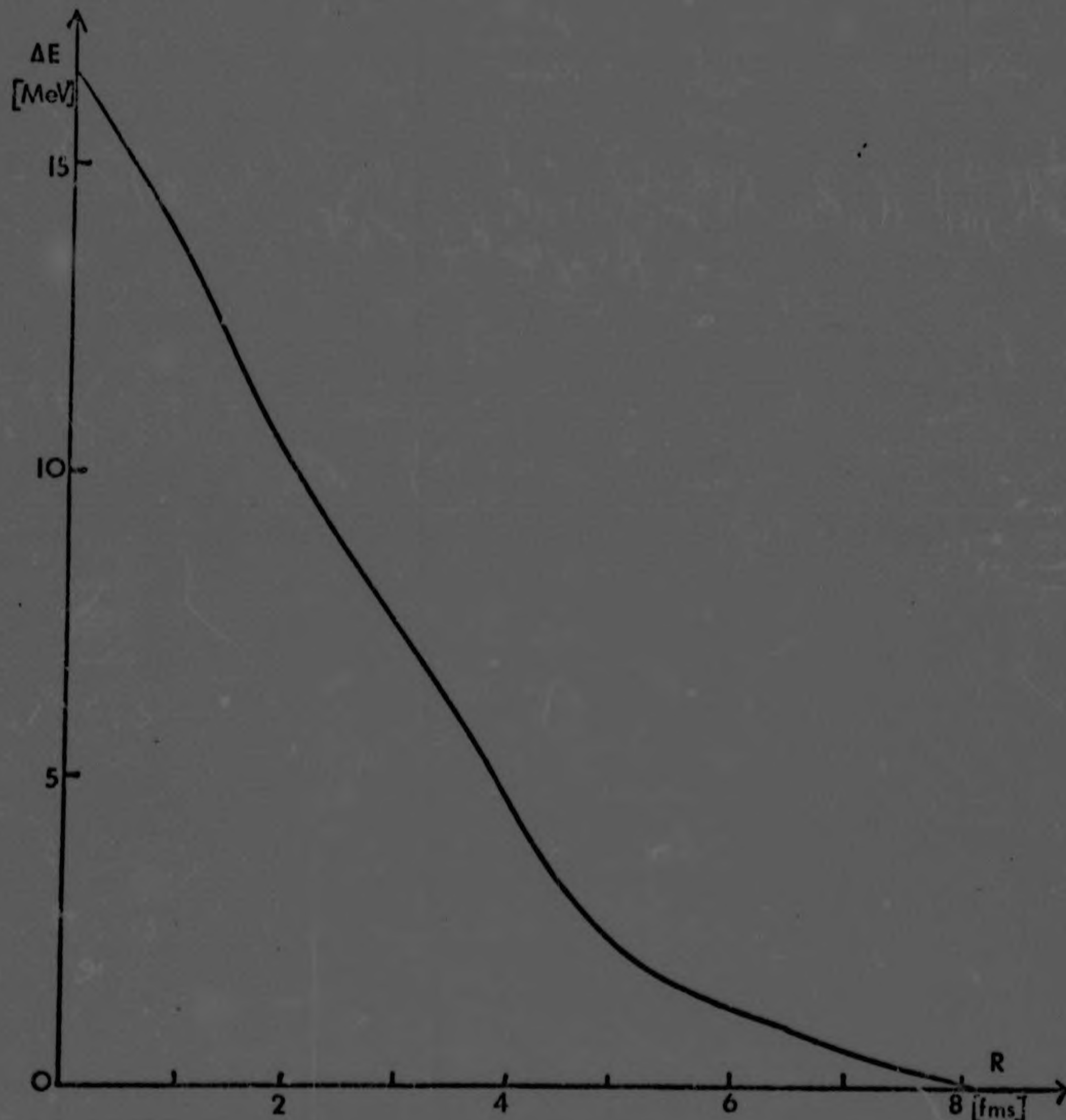

Fig. 9.19 Plot of difference in potential with and without inhomogeneity correction using the separation method.

The inhomogeneity correction increases the depth of the potential everywhere since it always gives a positive contribution to the size of the Fermi spheres and has an increasing effect for diminishing overlap.

IX. 12 Calculation of the Cross-Section

The real potentials which we calculated were used with the imaginary potentials calculated in Ref (27) to determine the cross-section for $^{16}O-^{16}O$ scattering. As the imaginary potentials calculated in this method are energy dependant and since the wave number at any separation distance depends on the effective potential at that separation, the form of the real potential modifies the form of the imaginary potential. The imaginary potentials that were obtained in the adiabatic approximation with inhomogeneity correction and using the separation method parameters are illustrated in Fig. 9.20

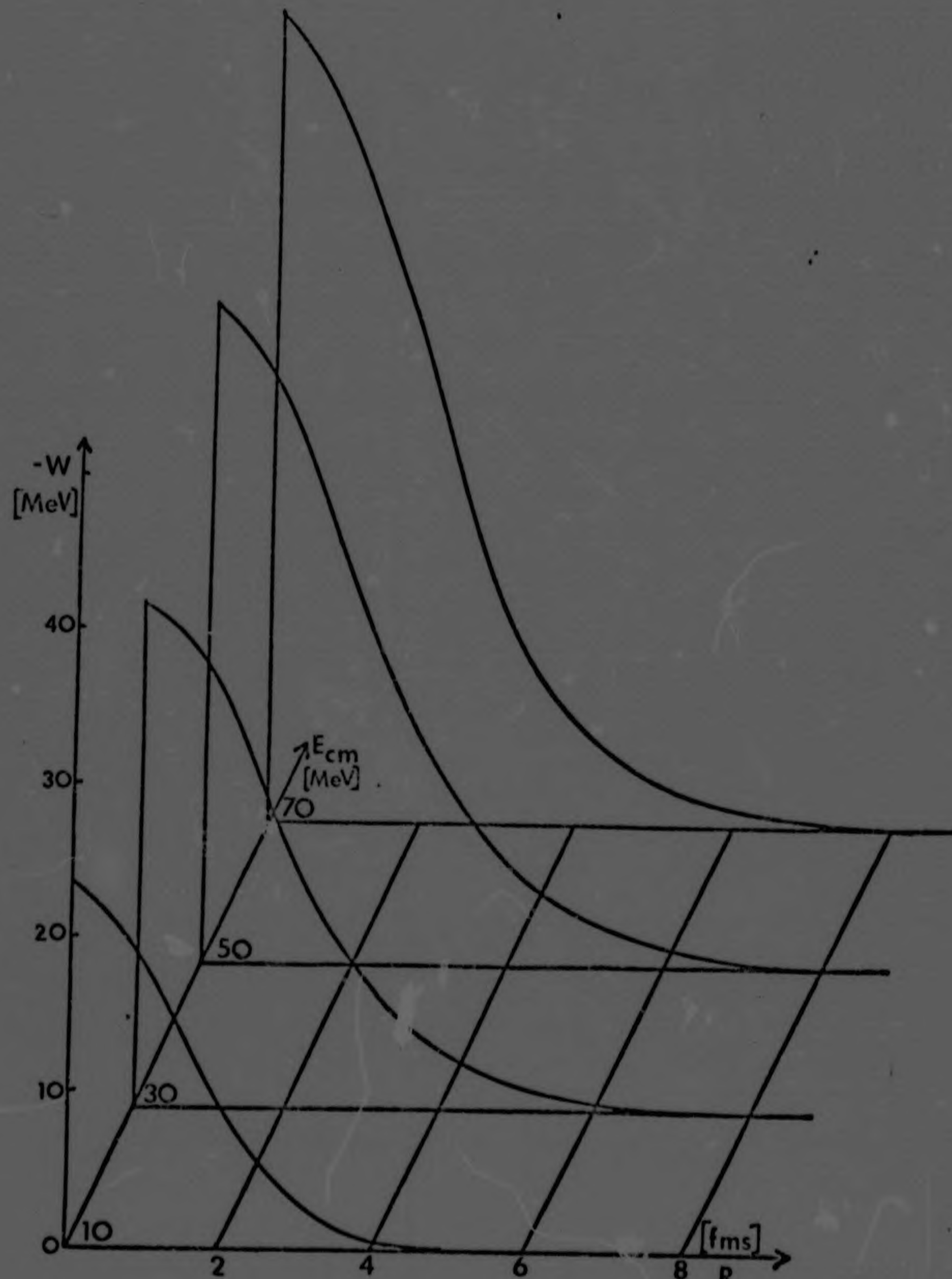

Fig. 9.20 Imaginary potential for ^{16}O-^{16}O in adiabatic approximation.

These potentials were then used in the computer program of H.J. Fink and T. Morovic to generate the cross-sections for $^{16}O-^{16}O$ scattering. The predicted cross-sections are compared to the experimentally observed cross-section in Fig 9.21. The values of the mass distribution parameters were taken as C=2.9 fms and b=0.43 fms for ^{16}O, and C=3.97 fms and b=0.59 fms for ^{32}S.

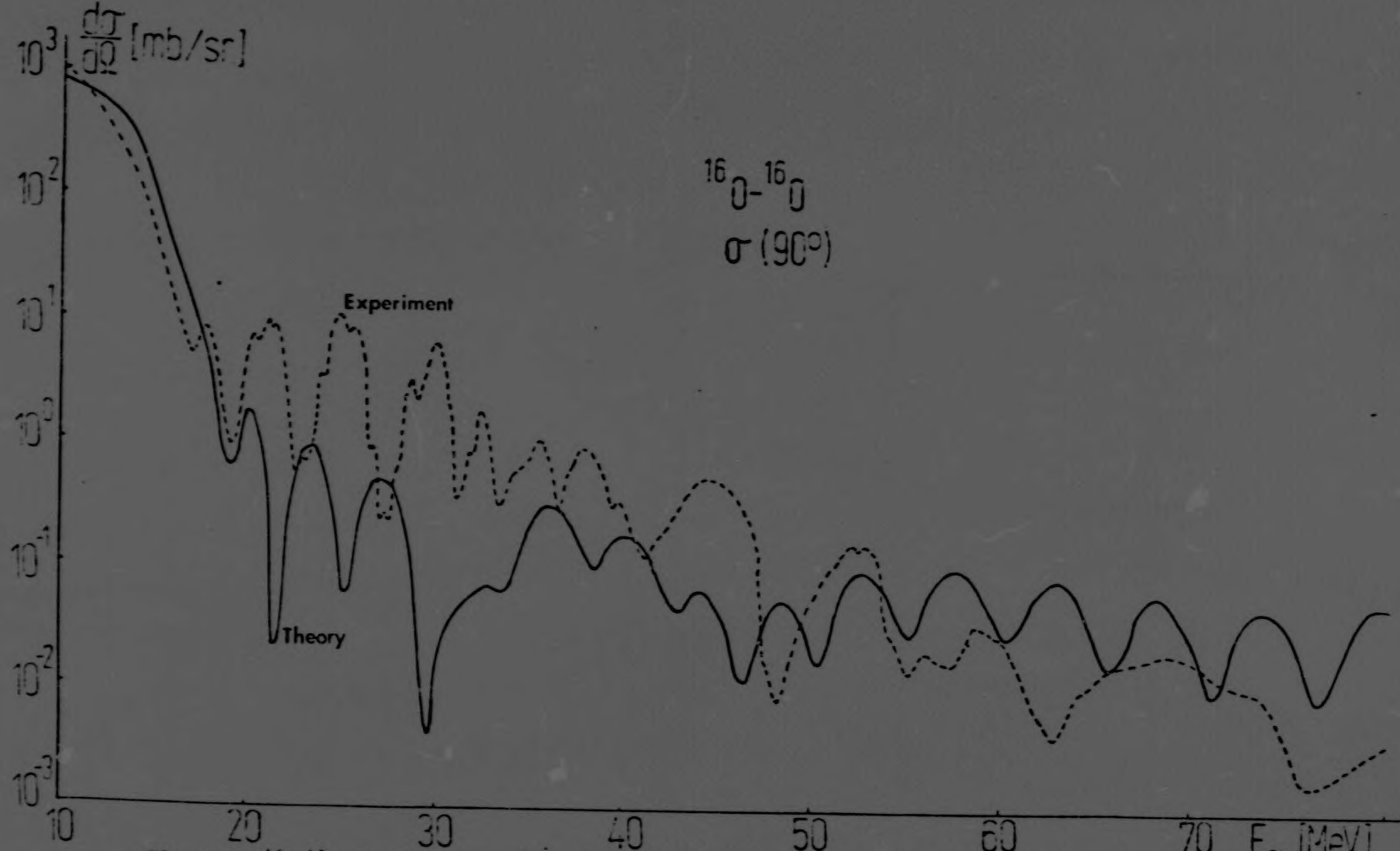

Fig. 9.21 $^{16}O-^{16}O$ cross section using adiabatic energy-dependant potentials with no free parameters

Although there are no free parameters in our treatment at all, the excitation function can be fairly well reproduced over a range of several orders of magnitude.

Fig. 9.22 shows the behaviour of the amplitude of the various partial waves as a function of energy. The effective potentials of the first twenty-two partial waves are also plotted.

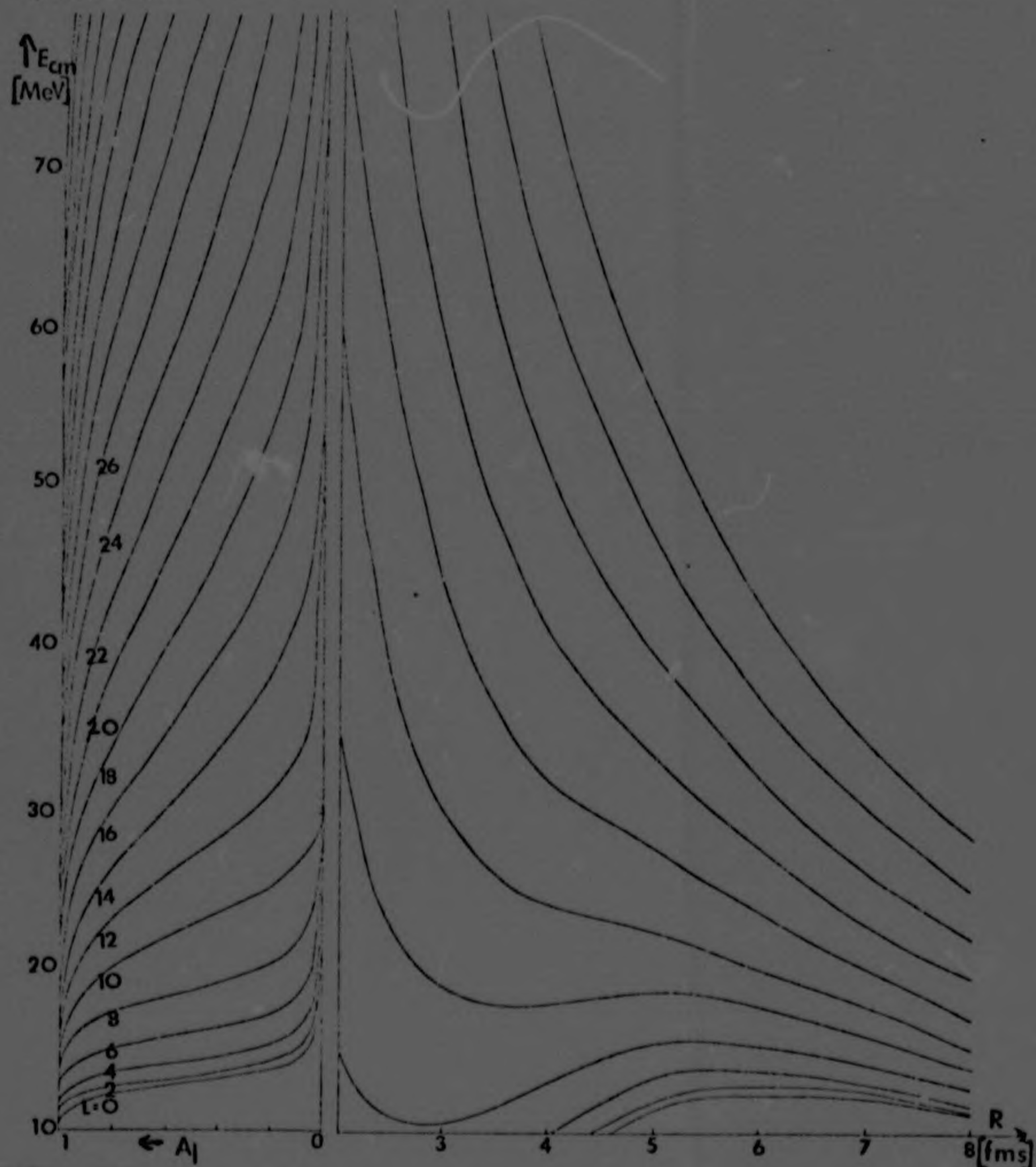

Fig. 9.22 Amplitudes and potentials for the various partial waves.

The rather regular structure of the amplitude forms above about 50 MeV is seen to result in regular oscillations in the cross-section above 50 MeV. Gobbi [37] claimed that below 35 MeV the oscillations in the excitation function were caused by an individual partial wave. In contrast we find that the amplitudes of the partial waves are such that there is seldom only one partial wave contribution.

APPENDIX A

The solutions of the various integrals in momentum space are given below

CASE a. $k_{F1} + k_{F2} < K$.

$$\int_{F2} d^3k_2 \int_{F1} d^3k_1 \; e^{-\beta k} = \frac{2\pi^2}{\beta^3 K} e^{-\beta(k_{F1}+k_{F2}+K)} \Big(\frac{2k_{F2}^2}{\beta^2}$$

$$+ \frac{2k_{F1}^2}{\beta^2} + \frac{2k_{F1}K}{\beta^2} + \frac{2k_{F2}K}{\beta^2} + \frac{2k_{F1}k_{F2}^2}{\beta} + \frac{2k_{F1}^2 k_{F2}}{\beta} + \frac{12k_{F2}}{\beta^3} + \frac{12k_{F1}}{\beta^3}$$

$$+ \frac{2k_{F1}k_{F2}K}{\beta} + \frac{12k_{F1}k_{F2}}{\beta^2} + \frac{12}{\beta^4} + \frac{2K}{\beta^3} \Big)$$

$$+ \frac{2\pi^2}{\beta^3 K} e^{-\beta(k_{F}-k_{F2}+K)} \Big(\frac{2k_{F1}^2 k_{F2}}{\beta} - \frac{2k_{F2}^2}{\beta^2} - \frac{2k_{F1}^2}{\beta^2} - \frac{2k_{F1}k_{F2}^2}{\beta}$$

$$- \frac{2K}{\beta^3} + \frac{2k_{F1}k_{F2}K}{\beta} - \frac{2k_{F1}K}{\beta^2} + \frac{2k_{F2}K}{\beta^2} + \frac{12k_{F1}k_{F2}}{\beta^2} + \frac{12k_{F2}}{\beta^3} - \frac{12k_{F1}}{\beta^3} - \frac{12}{\beta^4} \Big)$$

$$+ \frac{2\pi^2}{\beta^3 K} e^{-\beta(K-k_{F1}-k_{F2})} \Big(\frac{2k_{F1}k_{F2}K}{\beta} - \frac{2k_{F1}k_{F2}^2}{\beta} - \frac{2k_{F1}^2 k_{F2}}{\beta} + \frac{2k_{F2}^2}{\beta^2}$$

$$+ \frac{2k_{F1}^2}{\beta^2} - \frac{2k_{F2}K}{\beta^2} - \frac{2k_{F1}K}{\beta^2} + \frac{12k_{F1}k_{F2}}{\beta^2} + \frac{2K}{\beta^3} - \frac{12k_{F2}}{\beta^3} - \frac{12k_{F1}}{\beta^3} + \frac{12}{\beta^4} \Big)$$

$$+ \frac{2\pi^2}{\beta^3 K} e^{-\beta(k_{F2}-k_{F1}+K)} \Big(\frac{2k_{F1}k_{F2}^2}{\beta} - \frac{2k_{F2}^2}{\beta^2} - \frac{2k_{F1}^2}{\beta^2} - \frac{2k_{F1}^2 k_{F2}}{\beta} - \frac{2K}{\beta^3}$$

$$+ \frac{2k_{F1}k_{F2}K}{\beta} - \frac{2k_{F2}K}{\beta} + \frac{2k_{F1}K}{\beta} + \frac{12k_{F1}k_{F2}}{\beta^2} - \frac{12k_{F2}}{\beta^3} + \frac{12k_{F1}}{\beta^3} - \frac{12}{\beta^4} \Big)$$

CASE a. $k_{F1} + k_{F2} < K$.

$$\int_{F2} d^3k_2 \int_{F1} d^3k_1 \left(k e^{-Dk}\right) = \frac{2\pi^2}{D^3K} e^{-D(K-k_{F1}-k_{F2})} \Bigg(\frac{4k_{F1}^2 k_{F2}^2}{D}$$

$$+ 2\frac{2k_{F2}^2}{D^3} - 2\frac{2k_{F1}k_{F2}^2}{D^2} - \frac{2k_{F1}K^2}{D^2} + \frac{2K^2}{D^3} - 4\frac{k_{F1}^2 k_{F2} K}{D} + 2\frac{4k_{F1}k_{F2}K}{D^2}$$

$$- 2\frac{4k_{F2}K}{D^3} - 4\frac{k_{F1}k_{F2}^2 K}{D} + 2\frac{k_{F1}k_{F2}^3}{D} + \frac{4k_{F2}^2 K}{D^2} - \frac{2k_{F2}^3}{D^2} + 2\frac{k_{F1}k_{F2}K^2}{D} - 2\frac{k_{F2}K^2}{D^2}$$

$$+ 4\frac{k_{F1}^2 K}{D^2} - 2\frac{4k_{F1}K}{D^3} + \frac{24K}{D^4} - 2\frac{2k_{F1}^2 k_{F2}}{D^2} + 2\frac{k_{F1}^3 k_{F2}}{D} + 8\frac{4k_{F1}k_{F2}}{D^3} - \frac{84k_{F2}}{D^4}$$

$$+ 2\frac{2k_{F1}^2}{D^3} - \frac{2k_{F1}^3}{D^2} - 8\frac{4k_{F1}}{D^4} + \frac{84}{D^5}\Bigg)$$

$$+ \frac{2\pi^2}{D^3K} e^{-D(k_{F1}+k_{F2}+K)} \Bigg(4\frac{k_{F1}^2 k_{F2}^2}{D} + 22\frac{k_{F1}k_{F2}^2}{D^2} + 2\frac{2k_{F1}^2 k_{F2}}{D^2} + \frac{2k_{F1}k_{F2}^3}{D}$$

$$+ \frac{2k_{F1}^3 k_{F2}}{D} + 4\frac{k_{F1}^2 k_{F2} K}{D} + 2\frac{2k_{F1}^2}{D^3} + 22\frac{k_{F2}^2}{D^3} + 2\frac{k_{F1}K^2}{D^2} + 2\frac{k_{F2}K^2}{D^2} + \frac{2K^2}{D^3}$$

$$+ 24\frac{k_{F1}k_{F2}K}{D^2} + 24\frac{k_{F2}K}{D^3} + 2\frac{4k_{F1}K}{D^3} + \frac{2k_{F1}^3}{D^2} + 2\frac{k_{F2}^3}{D^2} + 4\frac{k_{F2}^2 K}{D^2}$$

$$+ \frac{4k_{F1}^2 K}{D^2} + 2\frac{k_{F1}k_{F2}K^2}{D} + 2\frac{4K}{D^4} + 8\frac{4k_{F1}k_{F2}}{D^3} + 8\frac{4k_{F1}}{D^4} + 84\frac{k_{F2}}{D^4} + \frac{84}{D^5}\Bigg)$$

$$- \frac{2\pi^2}{D^3K} e^{-D(k_{F2}-k_{F1}+K)} \Bigg(4\frac{k_{F1}^2 k_{F2}^2}{D} - 2\frac{2k_{F1}k_{F2}^2}{D^2} + 22\frac{k_{F1}^2 k_{F2}}{D^2} + \frac{22k_{F2}^2}{D^3}$$

$$+ 2\frac{2k_{F1}^2}{D^3} + \frac{2K^2}{D^3} - 2\frac{k_{F1}K^2}{D^2} + \frac{2k_{F2}K^2}{D^2} - 24\frac{k_{F1}k_{F2}K}{D^2} + 4\frac{k_{F1}^2 k_{F2} K}{D} - 4\frac{k_{F1}k_{F2}^2 K}{D}$$

$$+ 24\frac{k_{F2}K}{D^3} - 24\frac{k_{F1}K}{D^2} - 2\frac{k_{F1}k_{F2}^3}{D} - \frac{2k_{F1}^3 k_{F2}}{D} + \frac{2k_{F2}^3}{D^2} - \frac{2k_{F1}^3}{D^2} + \frac{4k_{F2}^2 K}{D^2}$$

$$+ 4\frac{k_{F1}^2 K}{D^2} - 2\frac{k_{F1}k_{F2}K^2}{D} + \frac{24K}{D^4} + 8\frac{4k_{F2}}{D^4} - 8\frac{4k_{F1}}{D^4} - 84\frac{k_{F1}k_{F2}}{D^3} + \frac{84}{D^5}\Bigg)$$

$$-\frac{2\pi^2}{D^3 K}\, e^{-D(k_{F1}-k_{F2}+K)} \left(\frac{4k_{F1}^2 k_{F2}^2}{D} - \frac{22 k_{F1}^2 k_{F2}}{D^2} + \frac{22 k_{F1} k_{F2}^2}{D^2} + \frac{2K^2}{D^3} \right.$$

$$+ \frac{22 k_{F1}^2}{D^3} + \frac{22 k_{F2}^2}{D^3} - \frac{2 k_{F2} K^2}{D^2} + \frac{2 k_{F1} K^2}{D^2} - \frac{24 k_{F1} k_{F2} K}{D^2} + \frac{4 k_{F1} k_{F2}^2 K}{D}$$

$$- \frac{4 k_{F1}^2 k_{F2} K}{D} + \frac{24 k_{F1} K}{D^3} - \frac{24 k_{F2} K}{D^3} - \frac{2 k_{F1}^3 k_{F2}}{D} - \frac{2 k_{F1} k_{F2}^3}{D} + \frac{2 k_{F1}^3}{D^2} - \frac{2 k_{F2}^3}{D^2}$$

$$\left. + \frac{4 k_{F1}^2 K}{D^2} + \frac{4 k_{F2}^2 K}{D^2} - \frac{2 k_{F1} k_{F2} K^2}{D} + \frac{24 K}{D^4} + \frac{84 k_{F1}}{D^4} - \frac{84 k_{F2}}{D^4} - \frac{84 k_{F1} k_{F2}}{D^3} + \frac{84}{D^5} \right)$$

CASE a. $\quad k_{F1} + k_{F2} < K$

$$\int_{F2} d^3k_2 \int_{F1} d^3k_1 \, \frac{E}{k} = \frac{16}{9} \pi^2 \frac{E}{K} k_{F1}^3 k_{F2}^3$$

CASE b. $\quad k_{F1} \leq k_{F2} + K$

$\quad k_{F1} + k_{F2} > K$

$$\int_{F2} d^3k_2 \int_{F1} d^3k_1 \, e^{-Bk}$$

solution on page 110.

CASE b $\qquad k_{F1} \leq k_{F2} + K$

$\qquad\qquad k_{F1} + k_{F2} > K$

$$
\begin{aligned}
\int_{F2} d^3k_2 \int_{F1} d^3k_1\, k e^{-Dk} = \; & \frac{2\pi^2}{D^3 K}\left(\frac{40K^2}{D^3} - \frac{40k_{F2}^2}{D^3} - \frac{40k_{F1}^2}{D^3} + \frac{168}{D^5} \right. \\
& \left. + \frac{6k_{F1}^2 k_{F2}^2}{D} - \frac{3k_{F2}^4}{D} - \frac{3k_{F1}^4}{D} - \frac{6k_{F2}^2 K^2}{D} - \frac{6k_{F1}^2 K^2}{D} + \frac{K^4}{D} + \frac{8k_{F2}^3 K}{D} + \frac{8k_{F1}^3 K}{D} \right) \\
& + \frac{2\pi^2}{D^3 K} e^{-D(k_{F2} - k_{F1} + K)} \left(\frac{22k_{F1}k_{F2}^2}{D^2} - \frac{22k_{F1}^2 k_{F2}}{D^2} - \frac{4k_{F1}^2 k_{F2}^2}{D} - \frac{22k_{F1}^2}{D^3} - \frac{22k_{F2}^2}{D^3} \right. \\
& - \frac{2K^2}{D^4} + \frac{2k_{F1}K^2}{D^2} - \frac{2k_{F2}K^2}{D^2} + \frac{24k_{F1}k_{F2}K}{D^2} - \frac{4k_{F1}^2 k_{F2}K}{D} + \frac{4k_{F1}k_{F2}^2 K}{D} + \frac{24k_{F1}K}{D^3} \\
& - \frac{24k_{F2}K}{D^3} + \frac{2k_{F1}k_{F2}^3}{D} + \frac{2k_{F1}^3 k_{F2}}{D} + \frac{2k_{F1}^3}{D^2} - \frac{2k_{F2}^3}{D^2} - \frac{4k_{F2}^2 K}{D^2} - \frac{4k_{F1}^2 K}{D^2} - \frac{24K}{D^4} \\
& \left. + \frac{2k_{F1}k_{F2}K^2}{D} + \frac{84k_{F1}}{D^4} - \frac{84k_{F2}}{D^4} + \frac{84k_{F1}k_{F2}}{D^3} - \frac{84}{D^5} \right) \\
& + \frac{2\pi^2}{D^3 K} e^{-D(k_{F1} - k_{F2} + K)} \left(-\frac{22k_{F1}k_{F2}^2}{D^2} + \frac{22k_{F1}^2 k_{F2}}{D^2} - \frac{4k_{F1}^2 k_{F2}^2}{D} - \frac{22k_{F2}^2}{D^3} \right. \\
& - \frac{22k_{F1}^2}{D^3} - \frac{2K^2}{D^3} - \frac{2k_{F1}K^2}{D^2} + \frac{2k_{F2}K^2}{D^2} + \frac{24k_{F1}k_{F2}K}{D^2} + \frac{4k_{F1}^2 k_{F2}K}{D} - \frac{4k_{F1}k_{F2}^2 K}{D} - \frac{24k_{F1}K}{D^3} \\
& + \frac{24k_{F2}K}{D^3} + \frac{2k_{F1}k_{F2}^3}{D} + \frac{2k_{F1}^3 k_{F2}}{D} - \frac{2k_{F1}^3}{D^2} + \frac{2k_{F2}^3}{D^2} - \frac{4k_{F2}^2 K}{D^2} - \frac{4k_{F1}^2 K}{D^2} + \frac{2k_{F1}k_{F2}K^2}{D} \\
& \left. - \frac{24K}{D^4} - \frac{84k_{F1}}{D^4} + \frac{84k_{F2}}{D^4} + \frac{84k_{F1}k_{F2}}{D^3} - \frac{84}{D^5} \right)
\end{aligned}
$$

$+$

$$+\frac{2\pi^2}{D^3K}\,e^{-D(k_{F1}+k_{F2}+K)}\Bigg(\frac{4k_{F1}^2k_{F2}^2}{D}+\frac{22k_{F1}k_{F2}^2}{D^2}+\frac{22k_{F1}^2k_{F2}}{D^2}+\frac{2k_{F1}k_{F2}^3}{D}$$

$$+\frac{2k_{F1}^3k_{F2}}{D}+\frac{22k_{F2}^2}{D^3}+\frac{22k_{F1}^2}{D^3}+\frac{2k_{F1}K^2}{D^2}+\frac{2k_{F2}K^2}{D^2}+\frac{4k_{F1}k_{F2}^2K}{D}$$

$$+\frac{4k_{F1}^2k_{F2}K}{D}+\frac{2K^2}{D^3}+\frac{24k_{F1}k_{F2}K}{D^2}+\frac{24k_{F2}K}{D^3}+\frac{24k_{F1}K}{D^3}+\frac{2k_{F1}^3}{D^2}+\frac{2k_{F2}^3}{D^2}$$

$$+\frac{4k_{F1}^2K}{D^2}+\frac{4k_{F2}^2K}{D^2}+\frac{2k_{F1}k_{F2}K^2}{D}+\frac{24K}{D^4}+\frac{84k_{F1}k_{F2}}{D^3}+\frac{84}{D^5}+\frac{84k_{F1}}{D^4}+\frac{84k_{F2}}{D^4}\Bigg)$$

$$+\frac{2\pi^2}{D^3K}\,e^{-D(k_{F1}+k_{F2}-K)}\Bigg(-\frac{4k_{F1}^2k_{F2}^2}{D}-\frac{22k_{F1}^2}{D^3}-\frac{22k_{F2}^2}{D^3}-\frac{2K^2}{D^3}-\frac{2k_{F1}K^2}{D^2}$$

$$-\frac{2k_{F2}K^2}{D^2}+\frac{4k_{F1}^2k_{F2}K}{D}+\frac{4k_{F1}k_{F2}^2K}{D}+\frac{24k_{F1}k_{F2}K}{D^2}+\frac{24k_{F2}K}{D^3}+\frac{24k_{F1}K}{D^3}$$

$$-\frac{2k_{F1}k_{F2}^3}{D}-\frac{2k_{F1}^3k_{F2}}{D}-\frac{2k_{F1}^3}{D^2}-\frac{2k_{F2}^3}{D^2}+\frac{4k_{F2}^2K}{D^2}+\frac{4k_{F1}^2K}{D^2}-\frac{2k_{F1}k_{F2}K^2}{D}$$

$$+\frac{24K}{D^4}-\frac{84k_{F1}}{D^4}-\frac{84k_{F2}}{D^4}-\frac{84}{D^5}-\frac{84k_{F1}k_{F2}}{D^3}-\frac{22k_{F1}^2k_{F2}}{D^2}-\frac{22k_{F1}k_{F2}^2}{D^2}\Bigg)$$

CASE b. $\quad k_{F1}\le k_{F2}+K$

$\qquad\qquad k_{F1}+k_{F2}>K$

$$\int_{F2}d^3k_2\int_{F1}d^3k_1\,\frac{E}{k}=\frac{\pi^2}{E}\Bigg(\frac{8}{9}\frac{k_{F1}^3k_{F2}^3}{K}-\frac{4}{9}k_{F1}^3K^2-\frac{4}{9}k_{F2}^3K^2$$

$$+\frac{4}{3}k_{F1}^3k_{F2}^2+\frac{4}{3}k_{F1}^2k_{F2}^3-\frac{4}{15}k_{F1}^5-\frac{4}{15}k_{F2}^5-\frac{k_{F1}^4k_{F2}^2}{2K}-\frac{k_{F1}^2k_{F2}^4}{2K}$$

$$+\frac{k_{F1}^4K}{2}+\frac{k_{F2}^4K}{2}+\frac{1}{18}\frac{k_{F2}^6}{K}-k_{F1}^2k_{F2}^2K+\frac{k_{F1}^2K^3}{6}+\frac{k_{F2}^2K^3}{6}-\frac{K^5}{90}\Bigg)$$

CASE c. $\quad k_{F1} > k_{F2} + K$

$$\int_{F2} d^3k_2 \int_{F1} d^3k_1 \; e^{-\beta k} = \frac{32}{3}\frac{\pi^2}{\beta^3} k_{F2}^3$$

$$+\frac{2\pi^2}{\beta^3} e^{-\beta(k_{F1}-k_{F2}-K)} \left(\frac{2k_{F1}k_{F2}}{\beta} + \frac{2k_{F2}}{\beta^2} - \frac{2k_{F1}}{\beta^2} - \frac{2}{\beta^3} - \frac{2k_{F2}^2}{K\beta^2} \right.$$

$$\left. -\frac{2k_{F1}^2}{K\beta^2} - \frac{2k_{F1}k_{F2}^2}{K\beta} + \frac{2k_{F1}^2k_{F2}}{K\beta} - \frac{12}{K\beta^4} - \frac{12k_{F1}}{K\beta^3} + \frac{12k_{F2}}{K\beta^3} + \frac{12k_{F1}k_{F2}}{K\beta^2} \right)$$

$$+\frac{2\pi^2}{\beta^3} e^{-\beta(k_{F1}+k_{F2}+K)} \left(\frac{2k_{F1}k_{F2}}{\beta} + \frac{2k_{F1}}{\beta^2} + \frac{2k_{F2}}{\beta^2} + \frac{2}{\beta^3} + \frac{2k_{F2}^2}{K\beta^2} + \frac{2k_{F1}^2}{K\beta^2} \right.$$

$$\left. + \frac{2k_{F1}k_{F2}^2}{K\beta} + \frac{2k_{F1}^2k_{F2}}{K\beta} + \frac{12}{K\beta^4} + \frac{12k_{F1}}{K\beta^3} + \frac{12k_{F2}}{K\beta^3} + \frac{12k_{F1}k_{F2}}{K\beta^2} \right)$$

$$+\frac{2\pi^2}{\beta^3} e^{-\beta(k_{F1}-k_{F2}-K)} \left(\frac{2k_{F1}k_{F2}}{\beta} - \frac{2k_{F1}}{\beta^2} + \frac{2k_{F2}}{\beta^2} - \frac{2}{\beta^3} + \frac{2k_{F2}^2}{K\beta^2} \right.$$

$$+\frac{2k_{F1}^2}{K\beta^2} + \frac{2k_{F1}k_{F2}^2}{K\beta} - \frac{2k_{F1}^2k_{F2}}{K\beta} + \frac{12}{K\beta^4} + \frac{12k_{F1}}{K\beta^3} - \frac{12k_{F2}}{K\beta^3}$$

$$\left. - \frac{12k_{F1}k_{F2}}{K\beta^2} \right)$$

$$+\frac{2\pi^2}{\beta^3} e^{-\beta(k_{F1}+k_{F2}-K)} \left(\frac{2k_{F1}k_{F2}}{\beta} + \frac{2k_{F1}}{\beta^2} + \frac{2k_{F2}}{\beta^2} + \frac{2}{\beta^3} - \frac{2k_{F1}^2}{K\beta^2} \right.$$

$$\left. - \frac{2k_{F2}^2}{K\beta^2} - \frac{2k_{F1}k_{F2}^2}{K\beta} - \frac{2k_{F1}^2k_{F2}}{K\beta} - \frac{12}{K\beta^4} - \frac{12k_{F1}}{K\beta^3} - \frac{12k_{F2}}{K\beta^3} - \frac{12k_{F1}k_{F2}}{K\beta^2} \right)$$

CASE c. $k_{F1} > k_{F2} + K$.

$$\int_{F2} d^3k_2 \int_{F1} d^3k_1 \, k e^{-Dk} = \frac{32\pi^2}{D^4} k_{F2}^3$$

$$+ \frac{2\pi^2}{D^3 K} e^{-D(k_{F1}+k_{F2}+K)} \Bigg(4\frac{k_{F1}^2 k_{F2} K}{D} + 4\frac{k_{F1} k_{F2}^2 K}{D} + \frac{4 k_{F1}^2 k_{F2}^2}{D}$$

$$+ \frac{22 k_{F1} k_{F2}^2}{D^2} + \frac{22 k_{F1}^2 k_{F2}}{D^2} + \frac{22 k_{F2}^2}{D^3} + \frac{22 k_{F1}^2}{D^3} + \frac{2 k_{F1} K^2}{D^2} + \frac{2 k_{F2} K^2}{D^2} + \frac{2K^2}{D^3}$$

$$+ \frac{24 k_{F1} k_{F2} K}{D^2} + \frac{2 k_{F1} k_{F2}^3}{D} + \frac{2 k_{F1}^3 k_{F2}}{D} + \frac{4 k_{F2}^2 K}{D^2} + \frac{4 k_{F1} K^2}{D^2} + \frac{2 k_{F2}^3}{D^2} + \frac{2 k_{F1}^3}{D^2} + \frac{24 K}{D^4}$$

$$+ \frac{2 k_{F1} k_{F2} K^2}{D} + \frac{24 k_{F1} K}{D^3} + \frac{24 k_{F2} K}{D^3} + \frac{84 k_{F1} k_{F2}}{D^3} + \frac{84 k_{F1}}{D^4} + \frac{84 k_{F2}}{D^4} + \frac{84}{D^5} \Bigg)$$

$$+ \frac{2\pi^2}{D^3 K} e^{-D(k_{F1}-k_{F2}+K)} \Bigg(4\frac{k_{F1}^2 k_{F2} K}{D} - \frac{4 k_{F1} k_{F2}^2 K}{D} - \frac{22 k_{F1} k_{F2}^2}{D^2} + \frac{22 k_{F1}^2 k_{F2}}{D^2}$$

$$- \frac{22 k_{F2}^2}{D^3} - 22\frac{k_{F1}^2}{D^3} - \frac{2 k_{F1} K^2}{D^2} + \frac{2 k_{F2} K^2}{D^2} - \frac{2K^2}{D^3} + 24\frac{k_{F1} k_{F2} K}{D^2} + \frac{2 k_{F1}^3 k_{F2}}{D} + \frac{2 k_{F1} k_{F2}^3}{D}$$

$$- \frac{4 k_{F1}^2 K}{D^2} - \frac{4 k_{F2}^2 K}{D^2} + \frac{2 k_{F2}^3}{D^2} - \frac{2 k_{F1}^3}{D^2} + \frac{2 k_{F1} k_{F2} K^2}{D} - \frac{24 k_{F1} K}{D^3} + \frac{24 k_{F2} K}{D^3} - \frac{24 K}{D^4}$$

$$+ 84\frac{k_{F1} k_{F2}}{D^3} - \frac{84 k_{F1}}{D^4} + \frac{84 k_{F2}}{D^4} - \frac{84}{D^5} \Bigg)$$

$+$

$$+\frac{2\pi^2}{D^3K}\, e^{-D(k_{F1}-k_{F2}-K)}\left(\frac{4k_{F1}^2k_{F2}K}{D}-\frac{4k_{F1}k_{F2}^2K}{D}+\frac{4k_{F1}^2k_{F2}^2}{D}+\frac{2K^2}{D^3}\right.$$

$$+\frac{22k_{F1}k_{F2}^2}{D^2}-\frac{22k_{F1}^2k_{F2}}{D^2}+\frac{22k_{F2}^2}{D^3}+\frac{22k_{F1}^2}{D^3}+\frac{2k_{F1}K^2}{D^2}-\frac{2k_{F2}K^2}{D^2}+\frac{24k_{F1}k_{F2}K}{D^2}$$

$$-\frac{2k_{F1}k_{F2}^3}{D}-\frac{2k_{F1}^3k_{F2}}{D}-\frac{4k_{F2}^2K}{D^2}-\frac{4k_{F1}^2K}{D^2}+\frac{2k_{F1}^3}{D^2}-\frac{2k_{F2}^3}{D^2}-\frac{2k_{F1}k_{F2}K^2}{D}$$

$$\left.-\frac{24k_{F1}K}{D^3}+\frac{24k_{F2}K}{D^3}-\frac{24K}{D^4}-\frac{84k_{F1}k_{F2}}{D^3}+\frac{84k_{F1}}{D^4}-\frac{84k_{F2}}{D^4}+\frac{84}{D^5}\right)$$

$$+\frac{2\pi^2}{D^3K}\, e^{-D(k_{F1}+k_{F2}-K)}\left(\frac{4k_{F1}^2k_{F2}K}{D}+\frac{4k_{F1}k_{F2}^2K}{D}-\frac{4k_{F1}^2k_{F2}^2}{D}-\frac{22k_{F1}k_{F2}^2}{D^2}\right.$$

$$-\frac{22k_{F1}^2k_{F2}}{D^2}-\frac{22k_{F2}^2}{D^3}-\frac{22k_{F1}^2}{D^3}-\frac{2k_{F1}K^2}{D^2}-\frac{2k_{F2}K^2}{D^2}-\frac{2K^2}{D^3}+\frac{24k_{F1}k_{F2}K}{D^2}$$

$$-\frac{2k_{F1}k_{F2}^3}{D}-\frac{2k_{F1}^3k_{F2}}{D}+\frac{4k_{F2}^2K}{D^2}+\frac{4k_{F1}^2K}{D^2}-\frac{2k_{F1}^3}{D^2}-\frac{2k_{F2}^3}{D^2}-\frac{2k_{F1}k_{F2}K^2}{D}$$

$$\left.+\frac{24k_{F1}K}{D^3}+\frac{24k_{F2}K}{D^3}+\frac{24K}{D^4}-\frac{84k_{F1}k_{F2}}{D^3}-\frac{84k_{F1}}{D^4}-\frac{84k_{F2}}{D^4}-\frac{84}{D^5}\right)$$

CASE C. $\qquad k_{F1} > k_{F2}+K$.

$$\int d^3k_2 \int d^3k_1\, \frac{E}{k} = \pi^2 E\left(\frac{8}{3}k_{F1}^2k_{F2}^3-\frac{8}{9}k_{F2}^3K^2-\frac{8}{15}k_{F2}^5\right)$$

APPENDIX B

The calculation of the potential was done in two steps. Firstly the program INTEGRAND was executed. This program calculated the numerical value of the energy density as a function of k_{F1}^3, k_{F2}^3 and K using the analytical expressions calculated for the energy-density. The grid used covered values of k_{F1}^3, k_{F2}^3 from 0 to 5 fm^{-3} with spacing of .2 fm^{-3} and values of K from 0 to 1.0 fm^{-1} in steps of 0.1 fm^{-1}.

The program POTENTIAL then used these values of ε to calculate the real potential. For separation distances of the nuclear centres less than twice the radius parameter, the coordinate space was subdivided into two regions as illustrated below

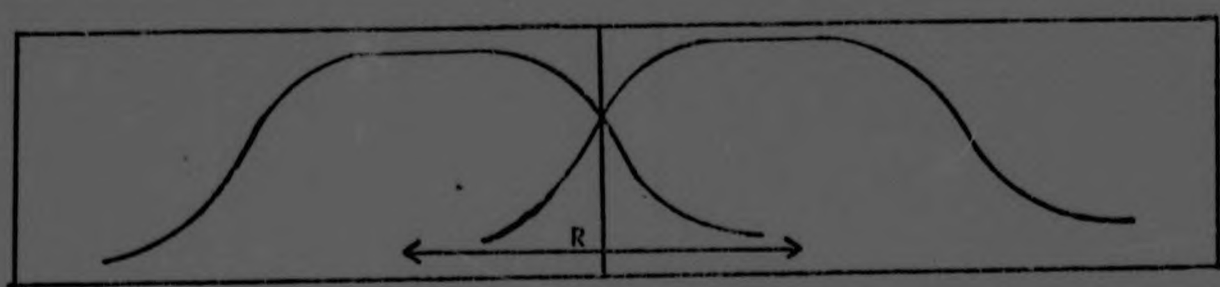

Fig. B1. Separation for R<2C

Three-dimensional Gaussian quadrature was then performed in each region using 11-point Gaussian formulae. The Gaussian n-point quadrature formula is of the form :

$$\int_{-1}^{1} f(t)\, dt = \sum_{i=1}^{n} \omega_i f(t_i) \qquad (B.1)$$

where ω_i are weights

and t_i are the corresponding points

For the integration $\int_a^b f(x)\,dx$, substitution of

$x = \frac{1}{2}\,(b-a)t+\frac{1}{2}(b-a)$

gives

$$\int_a^b f(x)\,dx = \frac{b-a}{2}\int_{-1}^{1} F(t)\,dt$$

For three-dimensional integration we have :

$$\int_{z_\ell}^{z_u}\int_{y_\ell}^{y_u}\int_{x_\ell}^{x_u} f(x,y,z)\,dxdydz = \frac{(x_\ell-x_u)}{2}\,\frac{(y_\ell-y_u)}{2}\,(z_\ell-z_u)\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\omega_{ijk}F(x_i,y_j,z_k) \qquad (B.2)$$

where $x_i = \frac{1}{2}\,(x_u-x_\ell)t_i+\frac{1}{2}(x_u+x_\ell)$

$y_i = \frac{1}{2}\,(y_u-y_\ell)t_i+\frac{1}{2}(y_u+y_\ell)$

$z_i = \frac{1}{2}\,(y_u-z_\ell)t_i=\frac{1}{2}(z_u+z_\ell)$

For separation distances of the nuclear centres greater than twice the radius parameter, the coordinate space was subdivided into six regions for increased accuracy as the numbers involved became smaller. The division is illustrated below

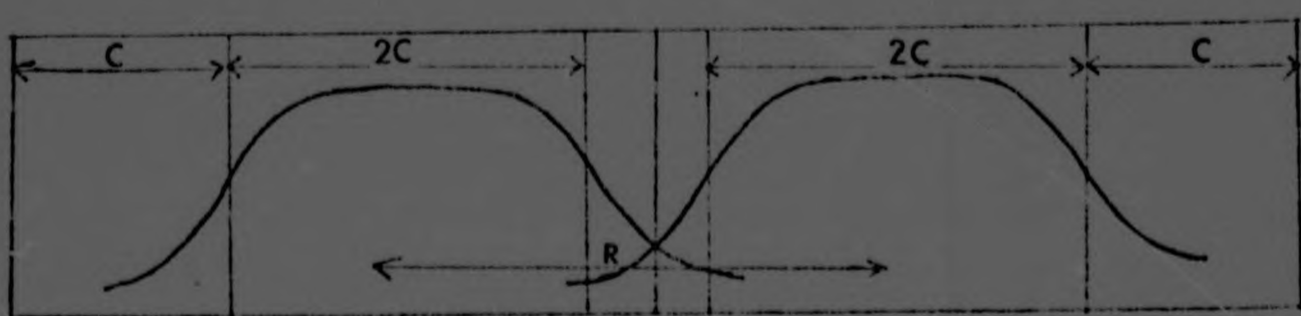

Fig. B.2 Separation for R>2C

At each point the density of each nucleus and the compound density (in either sudden or adiabatic approximation) were calculated. The inhomogeneity correction was calculated in the function subroutine SURF. This gave the Fermi momenta for each separation distance and as the local nucleus-nucleus relative momentum was calculated for each separation distance the momentum space situation was fixed. The values of the functions $F(x_i,y_j,z_k)$ were then taken by interpolation from the 3-dimensional array generated by the program INTEGRAND. The summation B.2 was performed and the potentials thus realized.

Using 11-point Gaussian quadrature the value of the potentials could be calculated with a numerical error of about 5 per cent.

INTEGRAND

```
      IMPLICIT REAL*8(A-H,O-U,W-$)
      REAL*4 V(30,30,30)
      PI=3.141592653589793 2+D+00
      PS=PI*PI
      READ(5,62)RODELT,RTDELT,RMDELT
62    FORMAT(3F10.3)
      READ(5,40)B,D,ACONS,CCON,ECON
40    FORMAT(5F10.3)
      WRITE(6,46)
46    FORMAT(1H1)
      WRITE(6,19)
19    FORMAT(3X,'PHASE SHIFT FIT = E/K+A*(EXP(-B*K)+C*K*EXP(-D*K))')
      WRITE(6,140)
140   FORMAT(5X,'B',9X,'D',9X,'A',9X,'C',9X,'E')
      WRITE(6,40)B,D,ACONS,CCON,ECON
      WRITE(6,41)RODELT,RTDELT,RMDELT
41    FORMAT(2X,'RODELT=',F5.2,'RTDELT=',F5.2,'RMDELT=',F5.2/)
      BS=B*B
      BC=BS*B
      BF=BC*B
      DS=D*D
      DC=DS*D
      DF=DC*D
      DFI=DF*D
      PSDBC=PS/BC
      RM=0.0
      I=1
25    CONTINUE
      J=1
70    CONTINUE
      IF(RM.LT.0.1E-05)RM=0.1E-05
101   RMS=RM*RM
      RMC=RMS*RM
      RMF=RMC*RM
      RMB=RM*B
      RMBS=RM*BS
      RMBC=RM*BC
      RMBF=RM*BF
      PSDDR=PS/(DC*RM)
      ROO=0.
1001  CONTINUE
      RTO=ROO
      K=J
1000  CONTINUE
      IF(ROO.LT.RTO)GO TO 81
      RO=ROO
      RT=RTO
      GO TO 82
81    RT=ROO
      RO=RTO
82    CONTINUE
      RC=RO**(1./3.)
      RD=RT**(1./3 )
      RE=RC*RC
      RF=RD*RD
      RG=RE*RF
```

```
      RH=RF*RF
      WPO=-B*(RC+RD+RM)
      WPT=-B*(RC+RD-RM)
      WPR=-B*(RC-RD+RM)
      WPF=-B*(RD-RC+RM)
      WPG=-B*(RM-RC-RD)
      WPH=-B*(RC-RD-RM)
      XPO=-D*(RC+RD+RM)
      XPT=-D*(RC+RD-RM)
      XPR=-D*(RC-RD+RM)
      XPF=-D*(RD-RC+RM)
      XPG=-D*(RM-RC-RD)
      XPH=-D*(RC-RD-RM
      EC=PSDBC
      EO=2.DO*EC*DEXP(WPO)
      ET=2.DO*EC*DEXP(WPT)
      ER=2.DO*EC*DEXP(WPR)
      EF=2.DO*EC*DEXP(WPF)
      EG=2.DO*EC*DEXP(WPG)
      EH=2.DO*EC*DEXP(WPH)
      FC=PSDDF
      FO=2.DO*FC*DEXP(XPO)
      FT=2.DO*FC*DEXP(XPT)
      FR=2.DO*FC*DEXP(XPR)
      FF=2.DO*FC*DEXP(XPF)
      FG=2.DO*FC*DEXP(XPG)
      FH=2.DO*FC*DEXP(XPH)
      IF(RM.LT.0.01)GO TO 141
      SOT=RC+RD
      STM=RD+RM
      IF(SOT.LT.RM)GO TO 131
      IF(STM.LE.RC)GO TO 132
130   CONTINUE
      COMP=16.*(RO+RT)/3.-4.*RM*(RE+RF)+2.*RMC/3.+(4.*RE*RF-2.*(RG+RH))/
     1RM+16.*RM/BS-16.*(RF+RE)/RMBS+48./RMBF
      CO=2.*RC*RD/B+2.*(RC+RD)/BS+2./BC
      CT=(2.*(RE+RF)+12.*RC*RD)/RMBS+12.*(RC+RD)/RMBC+2.*(RC*RF+RD*RE)/R
     1MB+12./RMBF
      CR=2.*RC*RD/B-12./RMBF+(12.*RC*RD-2.*(RE+RF))/RMBS-2./BC
      CF=2.*(RD-RC)/BS+2.*(RE*RD-RC*RF)/RMB+12.*(RD-RC)/RMBC
      P=EC*COMP+EO*(CO+CT)+ET*(CO-CT)+ER*(CR+CF)+EF*(CR-CF)
      DOMP=80.*(RMS-RE-RF)/DC+336./DFI+2.*(6.*RE*RF-3.*RH-3.*RG-6.*RF*RM
     1S-6.*RE*RMS+RMF+8.*RO*RM+8.*RT*RM)/D
      DO=4.*RM*(RE*RD+RF*RC)/D+24.*RM*RC*RD/DS+4.*RM*(RE+RF)/DS+24.*RM*(
     1RC+RD)/DC+24.*RM/DF
      DT=(4.*RE*RF+2.*RO*RD+2.*RC*RT)/D+(2.*RMS*(RD+RC)+2.*(RO+RT)+22.*(
     1RE*RD+RC*RF))/DS+(22.*(RE+RF)+2.*RMS+84.*RC*RD)/DC+34.*(RC+RD)/DF+
     234./DFI+2.*RMS*RC*RD/D
      DR=(2.*(RC*RT+RD*RO)-4.*RE*RF+2.*RC*RD*RMS)/D+(24.*RC*RD*RM-4.*RM*
     1(RE+RF))/DS+(84.*RC*RD-22.*(RE+RF)-2.*RMS)/DC-24.*RM/DF-84./DFI
      DEF=4.*RM*(RE*RD-RC*RF)/D+(22.*(RE*RD-RC*RF)+2.*(RT-RO)+2.*RMS*(RD
     1-RC))/DS+24.*RM*(RD-RC)/DC+84.*(RD-RC)/DF
      Q=FC*DOMP+FO*(DO+DT)+FT*(DO-DT)+FR*(DR+DEF)+FF*(DR-DEF)
      O=8.*RO*RT/(9.*RM)-4.*RO*RMS/9.-4.*RT*RMS/9.+4.*RO*RF/3.+4.*RT*RE/
     13.-4.*RG*RC/15.-4.*RH*RD/15.-RG*RF/(2.*RM)-RE*RH/(2.*RM)+RG*RM/2.
     2+RH*RM/2.+RO*RO/(18.*RM)+RT*RT/(18.*RM)-RE*RF*RM+RE*RMC/6.+RF*RMC/
     36.-RMC*RMS/90.
      O=PS*O
```

```
      GO TO 135
131   CONTINUE
      COT=2.*(RF+RE+RC*RM)/BS+2.*RC*RF/B+12.*RC/BC+2.*RM/BC+12./BF
      CTT=2.*(RD*RM+6.*RC*RD)/BS+2.*(RE*RD+RC*RD*RM)/B+12.*RD/BC
      CRT=2.*(RF+RE)/BS-2.*RC*RF/B-2.*RC*RM/BS+2.*(RM-6.*RC)/BC+12./BF
      CFT=2.*(RE*RD-RC*RD*RM)/B+2.*(RD*RM-6.*RC*RD)/BS+12.*RD/BC
      P=EO*(COT+CTT)+ER*(CTT-COT)+EG*(CRT-CFT)-EF*(CRT+CFT)
      FOT=2.*(RC*RT+RO*RD+2.*RE*RD*RM+RC*RD*RMS)/D+2.*(11.*RE*RD+RD*RMS+
     112.*RC*RD*RM+RT)/DS+(24.*RD*RM+84.*RC*RD)/DC+84.*RD/DF
      FTT=(4.*RE*RF+4.*RC*RF*RM)/D+(22.*RC*RF+2.*RC*RMS+2.*RD+4.*RF*RM+4
     1.*RE*RM)/DS+(22.*RE+22.*RF+2.*RMS+24.*RC*RM)/DC+(24.*RM+84.*RC)/DF
     2+84./DFI
      FRT=2.*(RC*RT-2.*RE*RD*RM+RO*RD+RC*RD*RMS)/D+2.*(12.*RC*RD*RM-11.*
     1RE*RD-RT-RD*RMS)/DS+12.*(7.*RC*RD-2.*RD*RM)/DC-84.*RD/DF
      FFT=(4.*RE*RF-4.*RC*RF*RM)/D+(4.*RE*RM-2.*RO+4.*RF*RM-22.*RC*RF-2.
     1*RC*RMS)/DS+(2.*RMS+22.*RF-24.*RC*RM+22.*RE)/DC+(24.*RM-84.*RC)/DF
     2+84./DFI
      Q=FO*(FOT+FTT)+FR*(FOT-FTT)+FG*(FRT+FFT)+FF*(FRT-FFT)
      O=16.*RO*RT/(9.*RM)
      O=PS*O
      GO TO 135
132   CONTINUE
      IF(RM.LT.0.1)GO TO 141
      COMPR=32.*RT*PSDBC/3.
      COR=(2.*RC*RD/B)+2.*(RC+RD)/BS+2./BC
      CTR=2.*(RC*RF+RE*RD)/RMB+2.*(RF+RE+6.*RC*RD)/RMBS+12.*(RC+RD)/RMBC
     1+12./RMBF
      CRR=2.*(RC*RD)/B+2.*(RD-RC)/BS-2./BC
      CFR=2.*(RC*RF-RE*RD)/RMB+2.*(RF+RE-6.*RC*RD)/RMBS+12.*(RC-RD)/RMBC
     1+12./RMBF
      P=COMPR+EO*(COR+CTR)+ET*(COR-CTR)+EH*(CRR+CFR)+ER*(CRR-CFR)
      DOMPR=32.*PS*RT/DF
      DOR=(4.*RE*RD*RM+4.*RC*RF*RM)/D+(24.*RC*RD*RM+4.*RF*RM+4.*RE*RM)/D
     1S+(24.*RC*RM+24.*RD*RM)/DC+24.*RM/DF
      DTR=(4.*RE*RF+2.*RC*RT+2.*RO*RD+2.*RC*RD*RMS)/D+(22.*RC*RF+22.*RE*
     1RD+2.*RC*RMS+2.*RD*RMS+2.*RT+2.*RO)/DS+(22.*RF+22.*RE+2.*RMS+84.*R
     2C*RD)/DC+(84.*RC+84.*RD)/DF+84./DFI
      DRR=4.*(RE*RD*RM-RC*RF*RM)/D+(24.*RC*RD*RM-4.*RF*RM-4.*RE*RM)/DS+(
     124.*RD*RM-24.*RC*RM)/DC-24.*RM/DF
      DFR=2.*(RC*RD*RMS-2.*RE*RF+RC*RT+RO*RD)/D+(22.*RF*RD-22.*RC*RF-2.*
     1RC*RMS+2.*RD*RMS-2.*RO+2.*RT)/DS+(84.*RC*RD-22.*RF-22.*RE-2.*RMS)/
     2DC+(84.*RD-84.*RC)/DF-84./DFI
      Q=DOMPR+FO*(DOR+DTR)+FT*(DOR-DTR)+FR*(DRR+DFR)+FH*(DRR-DFR)
141   CONTINUE
      IF(RM.LT.0.1)RMS=0.0
      O=8.*RE*RT/3.-8.*RT*RMS/9.-8.*RF*RT/15.
      O=PS*O
      IF(RM.GE.0.1)GO TO 135
      COMPR=32.*RT*PSDBC/3.
      COR2=RC*RF+5.*RC/BS+(RE+RF)/B+5./BC
      CTR2=5.*RC*RD/B+5.*RD/BS+RE*RD
      EO2=8.*PS*DEXP(-B*(RC-RD))/BC
      ET2=8.*PS*DEXP(-B*(RC+RD))/BC
      P=COMPR+EO2*(COR2-CTR2)-ET2*(COR2+CTR2)
      DOMPR=32.*PS*RT/DF
      DOR2=(36.*RC*RF+4.*RO)/D+120.*RC/DC+36.*(RE+RF)/DS+120./DF+8.*RE*R
     1F
      DTR2=4.*(RC*RT+RO*RD)+120.*RC*RD/DS+(4.*RT+36.*RD*RE)/D+120.*RD/DC
```

```
      FO2=2.*PS*DEXP(-D*(RC-RO))/DC
      FT2=2.*PS*DEXP(-D*(RC+RO))/DC
      Q=DOMPR+FO2*(DOR2-DTR2)-FT2*(DOR2+DTR2)
135   CONTINUE
      V(I,J,K)=ECON*Q+ACONS*(P+CCON*Q)
      RTO=RTO+RTDELT
      K=K+1
      IF(RTO.LE.5.)GO TO 1000
      ROO=ROO+RODELT
      J=J+1
      IF(ROO.LE.5.)GO TO 1001
      RM=RM+RMDELT
      I=I+1
      IF(RM.LE.1.05)GO TO 25
      JMAX=J-1
      WRITE(8,40)B,D,ACONS,CCON,ECON
      WRITE(8,45)JMAX
45    FORMAT(2X,I3)
      WRITE(8,52)RODELT,RTDELT,RMDELT
      DO 64 I=1,11
      DO 65 J=1,JMAX
      WRITE(8,1003)(V(I,J,K),K=J,JMAX)
1003  FORMAT(8F10.2)
65    CONTINUE
64    CONTINUE
      DO 164 I=1,11
      DO 165 J=1,JMAX
      WRITE(6,3001)(V(I,J,K),K=J,JMAX)
3001  FORMAT(8F10.2)
165   CONTINUE
164   CONTINUE
      STOP
      END
```

POTENTIAL

```
      DIMENSION W(12),WZ(12),X(12),Y(12),XN(12),YN(12),Z(12),ZN(12),XS(1
     12),YS(12),A(12,12,12),V(30,30,30),VA(50),VB(50)
      MO=1
      PI=4.*ATAN(1.)
      PS=PI*PI
      RANGE=10.0
      READ(5,40)B,D,ACCNS,CCCN,ECCN
40    FORMAT(5F10.3)
      READ(5,45)JMAX
45    FORMAT(2X,I3)
      READ(5,62)RODELT,RTDELT,RMDELT
62    FORMAT(3F10.3)
      DO 64 I=1,11
      DO 65 J=1,JMAX
      READ(5,1003)(V(I,J,K),K=J,JMAX)
1003  FORMAT(8F10.2)
65    CONTINUE
64    CONTINUE
      WRITE(6,168)RODELT,RTDELT,RMDELT
168   FORMAT(/2X,'RODELT=',F5.2,'RTDELT=',F5.2,'RMDELT=',F5.2/)
      DO 164 I=1,11
      DO 165 J=1,JMAX
      DO 166 K=J,JMAX
166   V(I,K,J)=V(I,J,K)
      WRITE(6,167)(V(I,J,K),K=1,JMAX)
167   FORMAT(8F10.2)
165   CONTINUE
164   CONTINUE
      READ(5,711)FA,BETA
711   FORMAT(2F10.3)
      READ(5,712)FEN
712   FORMAT(F10.3)
      READ(5,18)FA
18    FORMAT(2X,E14.7)
      READ(5,14)CL,BL,CR,BR,ZL,XNL,AL,ZR,XNR,AR
14    FORMAT(10F8.2)
      CMAX=CL
      ZULO=2.*CMAX
      ZLLO=-ZULO
      XUL=ZULO
      YUL=XUL
      XLL=ZLLO
      YLL=XLL
      READ(5,10)M,MZ
10    FORMAT(2(2X,I2))
      WRITE(6,46)
46    FORMAT(1H1)
      WRITE(6,19)
19    FORMAT(3X,'PHASE SHIFT FIT = E/K+A*(EXP(-B*K)+C*K*EXP(-D*K))')
      WRITE(6,140)
140   FORMAT(9X,'B',9X,'D',9X,'A',9X,'C',9X,'E')
      WRITE(6,40)B,D,ACCNS,CCCN,ECCN
      EIAV=10.0
      ENCM=EIAV
      NPOT=RANGE/FA+0.1
      N=(M+1)/2
```

```
      ZENCI=1.5*PS*RHO(0.,AR,CR,BF)
      WRITE(6,44)M,MZ
44    FORMAT(/2X,'USE ',I2,' BY ',I2,' GAUSS QUAD.')
      DO 8 I=1,MZ
      READ(5,7)Z(I),WZ(I)
7     FORMAT(3X,E14.7,2X,E14.7)
8     CONTINUE
      DO 2 I=1,M
      READ(5,3)X(I),W(I)
3     FORMAT(3X,E14.7,2X,E14.7)
      XN(I)=((XUL-XLL)*X(I))/2.+(XUL+XLL)/2.
      XS(I)=XN(I)*XN(I)
      YS(I)=XS(I)
2     CONTINUE
      WRITE(7,1099)FEN
1099  FORMAT(F10.3)
      WRITE(7,1004)FIAV
1004  FORMAT(F10.3)
      WRITE(7,1002)NPCT,HA,BETA
1002  FORMAT(I10,5F10.4)
25    CONTINUE
      RB=RANGE
      VNUC=0.0
      L=1
      IZ=0
      WRITE(6,41)ENCM
41    FORMAT(/2X,'C.M. ENERGY =',F6.2)
      WRITE(6,55)
55    FORMAT(/12X,'SEP.DISTANCE',9X,'NUCL.POTEN.',9X,'NUCL+COULOMB'/)
70    CONTINUE
      IZ=IZ+1
      IF(RB.LE.5.)GO TO 71
      VCOL=64./RB*1.4398
      GO TO 72
71    VCOL=64.*(3.-(RB/5.)*(RB/5.))/10.*1.4398
72    CONTINUE
      ENLO=ENCM-(VNUC+VCOL)
      IF(ENLO.LE.0.)ENLO=0.0
      RM=SQRT(4./FA*ENLO/16.)
      KA=1
      SUM=0.0
      ZLL=ZULO+RB/2.
      IF (RB.GT.2.*CMAX)GO TO 73
      ZLL=0.0
      GO TO 83
99    ZUL=ZLL
      ZLL=ZLLO-RB/2.
      GO TO 83
73    ZLL=RB/2.+CL-2.*BL
      GO TO 183
199   ZUL=ZLL
      ZLL=RB/2.-CL+2.*BL
      GO TO 183
198   ZUL=ZLL
      ZLL=0.0
      GO TO 183
197   ZLL=-(RB/2.-CL+2.*BL)
      ZUL=0.0
```

```
      GO TO 183
196   ZLL=ZLL
      ZLL=-(RB/2.+CL-2.*BL)
      GO TO 183
195   ZLL=ZLL
      ZLL=ZLLO-RB/2.
83    CONTINUE
183   CONTINUE
      VCL=(ZUL-ZLL)/2.*(XLL-XLL)/2.*(YLL-YLL)/2.
      ICC=0
      DO 4 I=1,MZ
      ZN(I)=((ZUL-ZLL)*Z(I))/2.+(ZLL+ZLL)/2.
      ZS=(ZN(I)-RB/2.)*(ZN(I)-RB/2.)
      ZSS=(ZN(I)+RB/2.)*(ZN(I)+RB/2.)
      ZV=ZN(I)
      DO 5 J=N,M
      YVS=YS(J)
      DO 6 K=J,M
      XVS=XS(K)
      RSQ=XS(K)+YS(J)+ZS
      RSSQ=XS(K)+YS(J)+ZSS
      R=SQRT(RSQ)
      RS=SQRT(RSSQ)
      ROO=1.5*PS*RHO(R,AL,CL,BL)
      RTO=1.5*PS*RHO(RS,AL,CL,BL)
      ROTT=ROO+RTO
      IF(ROTT.LE.1.E-05)ROTT=1.E-05
      IF(MO.NE.0)GO TO 23
      IF(ICC.NE.0)GO TO 20
      IF(ROTT-ZENDI) 20,20,22
22    ICC=1
      RCUT=RB
20    IF(ROTT-ZENDI) 23,23,24
24    CICN=CR*(1.-RB**2/RCUT**2+CL/CR*RB**2/RCUT**2)
      BICN=BR*(1.-RB**2/RCUT**2+BL/BR*RB**2/RCUT**2)
      RO=1.5*PS*RHO(R,AL,CICN,BICN)
      RT=1.5*PS*RHO(RS,AL,CICN,BICN)
      ROTT=RO+RT
      F3=5./108.*SURF(RO,RT,RB,ZV,XVS,YVS,AL,CICN,BICN)
      F2=ROTT**(2./3.)
      GO TO 702
23    F3=5./108.*SURF(ROO,RTO,RB,ZV,XVS,YVS,AL,CL,BL)
      F2=ROTT**(2./3.)
702   RO=(F2+F3)**1.5
      RT=RO
92    CONTINUE
      IL=RM/RMDELT+1
      IU=IL+1
      JL=RT/RTDELT+1
      JU=JL+1
      KL=RO/RODELT+1
      KU=KL+1
      DII=RM/RMDELT+1-IL
      DIJ=RT/RTDELT+1-JL
      DIK=RO/RODELT+1-KL
      XDER=DII*(DIK*(V(IL,JL,KU)-V(IL,JL,KL))+(1.-DIK)*(V(IU,JL,KL)-V(IL
     1,JL,KL)))
      YDER=DIJ*(DII*(V(IL,JL,KL)-V(IL,JL,KL))+(1.-DII)*(V(IL,JU,KL)-V(IL
```

```
     1,JL,KL)))
      ZDER=DIK*(DIJ*(V(IL,JL,KL)-V(IL,JL,KL))+(1.-DIJ)*(V(IL,JL,KU)-V(IL
     1,JL,KL)))
      XDR2=DII*(DIK*(V(IL,JL,KL)-V(IL,JL,KL))+(1.-DIK)*(V(IU,JU,KL)-V(IL
     1,JU,KL)))
      YDR2=DIJ*(DII*(V(IL,JL,KU)-V(IL,JL,KL))+(1.-DII)*(V(IL,JU,KU)-V(IL
     1,JL,KU)))
      ZDR2=DIK*(DIJ*(V(IL,JL,KL)-V(IL,JL,KL))+(1.-DIJ)*(V(IL,JL,KU)-V(IL
     1,JL,KL)))
      XDER=DIJ*XDR2+(1.-DIJ)*XDER
      YDER=DIK*YDR2+(1.-DIK)*YDER
      ZDER=DII*ZDR2+(1.-DII)*ZDER
      A(I,J,K)=VOL*PI/180*(XDER+YDER+ZDER+V(IL,JL,KL))
      A(I,J,K)=A(I,J,K)*RCC*R1C/((ROC+R1C)*(RCC+R1C))
6     CONTINUE
5     CONTINUE
4     CONTINUE
      NL=N+1
      DO 13 I=1,MZ
      SUM=SUM+A(I,N,N)*WZ(I)*W(N)*W(N)
      DO 11 J=NL,M
      SUM=SUM+4.*(A(I,N,J)*WZ(I)*W(N)*W(J)+A(I,J,J)*WZ(I)*W(J)*W(J))
11    CONTINUE
      MN=M-1
      DO 12 J=NL,MN
      JN=J+1
      DO 12 K=JN,M
12    SUM=SUM+8.*A(I,J,K)*WZ(I)*W(J)*W(K)
13    CONTINUE
      KA=KA+1
      IF(RB.GT.5.)GO TO 113
      IF(KA.EQ.2)GO TO 99
      GO TO 114
113   IF(KA.EQ.2)GO TO 199
      IF(KA.EQ.3)GO TO 198
      IF(KA.EQ.4)GO TO 197
      IF(KA.EQ.5)GO TO 196
      IF(KA.EQ.6)GO TO 195
114   CONTINUE
      SUM=SUM*2.*FA/((2.*PI)**5)
      IF(L.GT.1)GO TO 53
      RENORM=-SUM
      L=L+1
53    CONTINUE
      VNUC=-SUM-RENORM
      VB(IZ)=VNUC+VCOU
      WRITE(6,51)RB,VNUC,VB(IZ)
51    FORMAT(16X,F4.1,8X,F14.5,9X,F14.5)
      RB=RB-FA
      IF(RB.GT.0.05)GO TO 70
      IMAX=NPUT
      DO 52 I=1,IZ
      II=IZ-I+1
      VA(II)=VB(I)
52    CONTINUE
      WRITE(7,1010)(VA(I),I=1,IMAX)
1010  FORMAT(8F10.3)
      ENOM=ENOM+FEN
```

```
      IF(ENCM.LE.80.5)GO TO 25
      STOP
      END
```

RHO

```
      FUNCTION RHO(RR,A,C,B)
      PI=3.14159265
      RHO=0.
      IF((RR-C)/B.GT.1.E02) RETURN
      RHO=3.0*A/(4.0*PI*C**3*(1.0+(PI/C*B)**2)*(1.0+EXP((RR-C)/B)))
      RETURN
      END
```

SURF

```
      FUNCTION SURF(RO,RT,RB,Z,XS,YS,A,C,B)
      PI=4.*ATAN(1.)
      PS=PI*PI
      RO=RO/(1.5*PS)
      RT=RT/(1.5*PS)
      ZU=Z+RB/2.
      ZL=Z-RB/2.
      R1=SQRT(ZU*ZL+XS+YS)
      C1=SQRT(XS+YS)
      R2=SQRT(ZL*ZL+XS+YS)
      EX1=EXP((R1-C)/B)
      EX2=EXP((R2-C)/B)
      IF(EX1.GT.1.E+05)GO TO 10
      IF(EX2.GT.1.E+05)GO TO 11
      GRR=-(RO*C1*EX2/(B*R2*(1.+EX2))+RT*C1*EX1/(B*R1*(1.+EX1)))
      GRZ=-(RO*ZL*EX2/(B*R2*(1.+EX2))+RT*ZU*EX1/(B*R1*(1.+EX1)))
      GO TO 14
10    IF(EX2.GT.1.E+05)GO TO 12
      GRR=-(RO*C1*EX2/(B*R2*(1.+EX2)))
      GRZ=-(RO*ZL*EX2/(B*R2*(1.+EX2)))
      GO TO 13
11    GRR=-(RT*C1*EX1/(B*R1*(1.+EX1)))
      GRZ=-(RT*ZU*EX1/(B*R1*(1.+EX1)))
13    CONTINUE
      GRHO2=GRR*GRR+GRZ*GRZ
      SURF=GRHO2/((RO+RT)*(RO+RT))
      GO TO 14
12    SURF=((C1*R1+C1*R2+ZL*R1+ZU*R2)/(2.*B*R1*R2))**2
14    CONTINUE
      RETURN
      END
```

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Statement of Originality

This dissertation has not previously been submitted for any degree.

3rd March 1975.