

OPTICAL TWEEZING AND MICROMANIPULATION

by

Melanie M^cLaren

A dissertation submitted to the Faculty of Science,
University of the Witwatersrand, Johannesburg, in
fulfilment of the requirements for the degree of Master
of Science.

May 27, 2009

Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

Signature..... .

Date.....27/05/2009

Abstract

Over the last 30 years, the ability to perform controlled manipulation of microscopic particles using only light, has become a prevalent technique. Many new and existing fields have benefited from the versatility that is offered by optical tweezers.

This dissertation provides a theoretical description of the interaction of light with microscopic matter within an optical tweezer. An experimental investigation into the construction of an optical tweezer is described in detail, along with experimental results verifying the assembly of a fully functional and calibrated optical tweezer.

The use of novel beam shapes including Bessel-Gauss, Laguerre-Gauss and super-Gaussian beams are applied to the tweezing setup, demonstrating additional manipulation methods. The procedures for generating each beam shape are discussed, together with the specific properties of the beam that provide further value to optical tweezing.

Acknowledgments

I would like to begin by thanking my supervisors Dr. Andrew Forbes and Prof. Elias Sideras-Haddad, who have both offered advice and support throughout my Masters. In particular, I'd like to thank Andrew for the almost daily discussions, the persistent pushes to get me to finish and of course for all the travels.

I've also made some valuable friends at the National Laser Centre, who have frequently helped me towards the end of a long day. I must thank Andrew Mori for forcing me to relax during the stressful periods and for providing many good laughs in old Pretoria. I'd like to thank my cubicle partner, Angela Dudley, for countless weeks spent working on the SLM - they are much appreciated. Also, thanks Ange for all the talks about our future plans, we're definitely going to conquer the next couple of years. Early mornings would not have been the same without my running partner, Hen-charl Strauss, who has mastered the art of coffee-making. Lastly, even though he only witnessed the first half of my Masters, Edward Bernhardi provided inspiration to work hard and continues to do so from afar.

While attending a few fun-filled conferences, I also met a number of people to whom I am tremendously grateful. Firstly, I must thank Carlos Lopez-Mariscal for all his advice at the start of my Masters, and then the continual motivational talks throughout the dissertation. I am also extremely grateful for the time and effort spent by Tristan Scott, who prepared and hand-delivered a sample of biological cells for my experiment, especially 'the skull'. I'd also like to thank Michael Nicolaou, who kept me smiling through all the tough times.

Finally, I'd like to thank my family for their support over the last two years. Only a few more years as a struggling student.

To my Mom

Contents

Declaration	i
Abstract	ii
Acknowledgments	iii
1 Introduction	2
2 Laser Beam Propagation	4
2.1 Gaussian Beams	4
2.1.1 Propagation Parameters	6
2.2 Bessel-Gauss Beams	12
2.2.1 Reconstruction of Bessel-Gauss Beams	15
3 Optical Manipulation	23
3.1 Fundamentals of Optical Tweezing	23
3.1.1 Mie Regime	24
3.1.2 Rayleigh Regime	28
3.2 Design Considerations	30
3.2.1 Choice of Laser	32
3.2.2 Objective Lens	32
3.2.3 Imaging System	35
3.3 Experimental Procedure	37
3.3.1 Experimental Setup	37
3.3.2 Experimental Results	43
3.4 Trapping with a Bessel Beam	57
3.4.1 Experimental Procedure	59
3.5 Applications	63
4 Rotational Tweezers	65
4.1 Spin Angular Momentum	67
4.1.1 Experimental Setup	71

4.1.2	Experimental Results	72
4.2	Orbital Angular Momentum	74
4.2.1	Experimental Setup	76
4.2.2	Experimental Results	80
4.3	Application in Microfluidics	82
5	Super-Gaussian Beams	84
5.1	Propagation Parameters	85
5.2	Intensity Gradient	90
5.3	Experimental Setup	91
5.4	Experimental Results	93
6	Conclusions and Outlooks	96
6.1	Summary	96
6.2	Future Work	98
A	Spatial Light Modulator	99
B	List of Publications	103

List of Figures

2.1	Intensity profile of a Gaussian beam.	6
2.2	Diagram illustrating the changing width of a propagating Gaussian beam, the most intense region is represented by yellow. The white lines denote the $1/e^2$ boundary of the beam.	8
2.3	Diagram illustrating the asymptotic behaviour far from the beam waist.	8
2.4	Diagram showing the change in the value for the radius of curvature as z changes. The dashed line represents the radius of curvature for a spherical wave.	9
2.5	Diagram of the phase shift of a Gaussian beam relative to a plane wave.	10
2.6	A ray is specified by a position r and angle θ	11
2.7	Intensity profiles of (a) a Bessel beam at $z \approx 0$ and (b) an annular ring at $z = 0.1$ m. The two beams are Fourier transforms of one another.	13
2.8	Diagram illustrating the finite propagation distance of a Bessel beam generated by an axicon of opening angle θ . The superposition of plane waves has wave vectors, which lie on the surface of a cone with opening angle γ	14
2.9	Diagram illustrating the projection of the obstacle in space by two conical waves, CW_1 and CW_2 . The shadow region where neither conical wave contributes has a minimum distance given by z_{min}	17
2.10	Diagram illustrating the finite propagation distance of a Bessel beam generated by an axicon of opening angle θ . The superposition of plane waves has wave vectors, which lie on the surface of a cone with opening angle γ	18

2.11	Experimental design for generating a Bessel-Gauss beam: a telescope with lenses $f_1 = 50$ mm, $f_2 = 250$ mm expands the beam before illuminating the axicon (A). The obstacle (O) was placed 45 mm from the axicon and was imaged onto a screen (S) by lens $f_3 = 25$ mm. The inserts represent the theoretical prediction (left), which was calculated using Mathematica 6 and the experimentally measured (right) Bessel-Gauss beam.	20
2.12	Experimental images of three different obstacle geometries within a generated Bessel beam, with the theoretical predictions of the shadow boundaries inset in the bottom left corners. The images illustrate the excellent comparison for the self reconstruction of a Bessel beam.	21
3.1	The more intense the ray, the greater the change in momentum resulting in a force acting on the particle in the opposite direction, that is, always towards the most intense region of the beam.	25
3.2	The refracted rays (gray) produce corresponding forces (green) on the bead, resulting in a net force toward the most intense region of the beam. In the axial direction, the gradient force must be greater than the scattering force to obtain three dimensional trapping.	25
3.3	Diagram illustrating how the forces in the ray optics regime are calculated.	26
3.4	Comparison of the forces at different incident angles, θ . The gradient force is greater than the scattering force for a range of incident angle values and reaches a maximum at $\theta \approx 70^\circ$	28
3.5	(a) Vector field plot of the illustrating the direction of the gradient force near the focus of the beam (0,0). The arrows clearly point to the origin, or beam focus, where the intensity is greatest. (b) Density plot of the magnitude of the gradient force. There is no gradient force at the origin (the beam focus), however there is a gradient force surrounding the central point, where the magnitude of the force increases steadily away from the centre.	29
3.6	Diagram of the experimental setup used to produce an optical trap. The beam is expanded before it is reflected off a dichroic mirror onto the back aperture of an objective lens. The sample is illuminated from below and imaged to a CCD camera placed above the system.	31

3.7	Diagram illustrating the effect of the size of the angle of the incoming rays. (a) θ is small, resulting in a weak intensity gradient. (b) θ is large, resulting in a strong intensity gradient as the beam is tightly focused.	34
3.8	Optimising the strength of the trap by making full use of the numerical aperture of the objective. (a) Beam underfills back aperture, resulting in a limited trapping spot, (b) Beam fills back aperture, (c) Beam overfills back aperture, resulting in a loss of intensity & strength of trap.	35
3.9	Diagram illustrating the imaging requirements for (a) a finite tube lens objective and (b) an infinity-corrected objective lens.	36
3.10	Diagram displaying Koehler illumination. The lamp, condenser iris and back focal plane of the objective form conjugate planes. While the collector iris and sample plane form another conjugate pair.	36
3.11	Image of the Gaussian beam from the diode laser.	37
3.12	Graph of the square of the measured beam widths at various distances from the lens along the z-axis.	39
3.13	Diagram of the experimental setup used to produce an optical trap. L_1 ($f_1 = 30$ mm) and L_2 ($f_2 = 100$ mm) expand the beam before it is reflected off a dichroic mirror onto the back aperture of a 100X objective lens. Mirror M_2 acts as the beam steering mirror. The sample is illuminated from below by a white light LED source and imaged to a CCD camera placed above the system.	40
3.14	Image of the completed home-built optical tweezing setup. The camera is situated at the top of the setup at position (1). Attached to the camera is the collecting lens or eyepiece lens situated in a tube lens holder at (2). The dichroic mirror is housed in a mount at (3) and directs the laser light into the objective lens (100X magnification) at (4) and onto the sample plane. The telescoping system is found at (5).	41
3.15	(a) Image of a 100 μm thick vinyl spacer stuck on a glass slide, where the diameter of the inner circle is 5 mm. (b) Diagram illustrating the sample depth. (c) Image of silica beads with a mean diameter of 4.32 μm as seen using an Olympus microscope. (d) Image of a trapped silica bead surrounded by untrapped beads as seen from the constructed microscope of the optical tweezers.	42

3.16	Images of tweezing of a 4 μm silica bead. (a) Bead is not yet trapped, (b) Bead is trapped in three dimensions and changes focus, (c)-(f) Bead is manipulated in a diamond-like pattern by moving the laser using the steering mirror.	44
3.17	The trapped particle is fixed at one position by the laser, while the background particles move towards the right as the stage is moved.	45
3.18	Images of tweezing of a 20 μm embryonic kidney cell. (a) Cell is not yet trapped, (b) Cell is trapped in three dimensions and changes focus, (c)-(f) Cell is manipulated in a diamond-like pattern by moving the laser using the steering mirror.	46
3.19	Lateral dimensions of the field of view of the trap.	47
3.20	Images demonstrating axial trapping. (a) Both the trapped bead (left) and untrapped bead (right) are in focus. (b) The stage is moved downwards causing untrapped bead to change focus, while trapped bead remains in focus. (c) The stage is moved even further in the axial direction and the untrapped bead is now completely out of focus.	48
3.21	Images of a trapped silica bead together with untrapped reference beads, at power $P = 25.4 \text{ mW}$. The trapped bead is held fixed, while the untrapped beads in are moved with the medium by moving the stage. (a) The initial position of a reference bead is determined just before the trapped bead escapes the trap. (b) The final position of the same reference bead is determined just as the trapped bead escapes from the trap, allowing the displacement of the reference bead to be calculated.	50
3.22	Graph illustrating the linear relationship of power versus the trapping force on the bead.	51
3.23	Graph illustrating the inverse relationship of power versus the variance of the position of a trapped bead.	53
3.24	Graph illustrating the linear relationship of power versus the spring constant of the trap.	54
3.25	(a) Density plot of the magnitude of the gradient force. (b) A particle placed in the conservative force field has potential energy, which is a minimum at the the beam focus.	55
3.26	Graph representing the distribution of bead positions. A best-fit graph is fitted to the measured data points using eq. (3.24).	56
3.27	Graph representing the potential energy calculated from the distribution of bead positions. A best-fit graph is fitted to the measured data points using eq. (3.25).	56

3.28	Experimental setup used in generating a Bessel beam trap. . .	57
3.29	Comparison of the intensity gradients for (a) Bessel beams and (b) Gaussian beams. There is an order of 26 difference between the maximum intensity gradient values.	58
3.30	(a) and (b) Gaussian beams diverge quickly after focusing, resulting in poor localization. (c) Bessel beams have a non-diffracting central maximum, allowing point-to-point guiding. .	58
3.31	The central maximum allows particles to be trapped at any point along the column. Together with the reconstructive property, Bessel beam traps allow more than one particle to be trapped at a time.	59
3.32	Diagram of the experimental setup to produce both a Bessel and Annular beam. Lenses L_1 ($f_1 = 50$ mm) and L_2 ($f_2 = 250$ mm) form a $5\times$ telescope. L_3 ($f_3 = 30$ mm) was used as an imaging lens when capturing the Bessel beam. The camera was moved to the focus of L_3 to capture an image of the Annular beam.	60
3.33	Comparison of the predicted Bessel and Annular beams, plotted using Mathematica 6, with the experimentally generated images. The Bessel beam was magnified by imaging onto a camera, whereas the Annular beam was produced at the focus of a lens placed within the Bessel field.	60
3.34	Experimental setup used in generating a Bessel beam trap. A Gaussian beam of width $\omega = 1$ mm was directed through an axicon of opening angle $\gamma = 5^\circ$ to produce a Bessel beam with a propagation distance of $z_{max} \approx 23$ mm and central spot size $r \approx 5.79 \mu\text{m}$	61
3.35	Images of 4 micron silica beads, manipulated by a Bessel beam trap at various points along the central column. (a)-(d) Bead is trapped at a certain point within the Bessel column, (e)-(h) The bead is moved further down the column and again trapped and manipulated, (i)-(l) The same bead is moved even further down the column and trapping is again achieved.	62
4.1	Beth's experimental design to detect angular momentum. Circularly polarized light is incident on a suspended $\lambda/2$ birefringent plate, transforming right-handed into left-handed circularly polarized light. The change in polarization results in a torque exerted on the plate.	68

4.2	Experimental setup used to observe the transfer of spin angular momentum from the beam to the particles, causing them to rotate. A quarter-wave plate is used to convert linearly polarized light into circularly polarized light, which carries spin angular momentum.	72
4.3	Images of birefringent calcite particles in ethanol. (a)-(d) Show the particle rotating in an anti-clockwise direction at a rate of $\omega = \pi/2$ rad/s.	73
4.4	Images of birefringent calcite particles in ethanol. (a)-(d) Show the particle rotating in a clockwise direction at a rate of $\omega = \pi/2$ rad/s.	74
4.5	'Twisted' light with helical wavefronts carries orbital angular momentum, unlike plane waves.	75
4.6	(a) The liquid crystal display of a spatial light modulator allows a phase pattern, shown in (b), to be encoded onto the device. The phase changes from 0 to 2π , denoted by white to black, respectively.	77
4.7	A fork phase pattern with 4 dislocations generates a vortex of order $l = 4$	78
4.8	Experimental setup used for rotating particles about a point. A spatial light modulator (SLM) is used to generate a vortex beam, which carries orbital angular momentum.	79
4.9	(a) An incident Gaussian beam is reflected off the SLM to produce a vortex beam. (b) The diffraction orders are reflected off the SLM.	80
4.10	Image of a vortex beam with $l = 1$, showing the intensity distribution. The spiral phase pattern was used to generate this beam.	81
4.11	A graphite particle of $1 \mu\text{m}$ diameter was exposed to the vortex of order $l = 1$. The particle rotated in an anti-clockwise direction at a frequency of approximately 0.37 Hz.	82
4.12	SEM micrograph of an actual microgear [60], has made this an exciting field of research.	83
5.1	Intensity profile of super-Gaussian beams of different orders. For $p = 2$, a Gaussian intensity plot is seen. As the order increases, the flatness of the peak intensity becomes more evident.	85
5.2	Intensity profile of a flattened Gaussian beam as it propagates along the z -axis.	88

5.3	Comparison of the propagation of two flattened Gaussian beams of order $N = 4$ and $N = 25$. It is clear that at the same z -values, the beams differ quite drastically. The larger the order, the faster the beam loses its shape.	89
5.4	A three dimensional plot of the intensity distribution of a SG beam. The intensity gradient along r is very steep, while along the z -axis, there is little to no change in the intensity.	90
5.5	The lateral intensity gradient increases as the order of the super-Gaussian beam increases. For $p = 10$, the gradient force is stronger than the Gaussian beam ($p = 2$) by two orders of magnitude.	91
5.6	Experimental setup used for rotating particles about a point. A spatial light modulator (SLM) is used to generate a vortex beam, which carries orbital angular momentum. L_1 and L_2 act as a 1 : 1 telescope, allowing the SLM to be placed further from the objective lens. The objective lens acted as a Fourier lens, thereby producing a super-Gaussian at the focus of the objective lens.	92
5.7	Calculated phase pattern for a super-Gaussian beam. The phase patterns were calculated and produced in Matlab. . . .	93
5.8	(a) Image of a circular flat-top beam of order 50. The intensity is uniform at the peak, and a steep gradient is evident from the immediate colour change, as compared to that of a Gaussian beam shown in (b).	94
5.9	The gradient force only acts in the lateral plane, so the trapped particle does not change focus. The stage was moved in a square-like pattern, causing the untrapped particle to move in the same pattern.	94
5.10	The trapped particle is moved towards the top of the image, while the untrapped particles remain fixed in position.	95
A.1	After passing through the two slits, light interferes in a constructive and destructive manner, producing bright spots at points of constructive interference.	100
A.2	SLMs cannot produce continuous phase patterns and therefore an approximation is made using a number of levels. The more levels used to approximate the function, the better the efficiency.	101
A.3	(a) A two level system. The first order has only 41% of the energy. (b) An ideal system where the first order has all of the energy, while the other orders have zero.	102

List of Tables

3.1	Measured beam parameters for the diode laser.	38
-----	---	----

Chapter 1

Introduction

There has always been a fascination around the idea of moving objects with seemingly no physical contact involved. Science fiction depicts rays of light ‘beaming up’ objects, moving them from one location to another. Today science fiction and reality meet in the field of optical tweezing, where microscopic particles are manipulated by a single beam of light.

Essentially, an optical tweezer makes use of a tightly focused laser beam to trap and manipulate particles in three dimensions. The technique provides a non-invasive method of investigating properties of particles, by generating piconewton forces powerful enough to manipulate particles in the size range from a few nanometers to hundreds of microns. Although this size range may seem limited, optical tweezing is an extremely useful technique when dealing with biological samples and has found numerous applications within this field.

Research into optical tweezing began over 30 years ago and has since been developed into a significant technique with far reaching applications, especially in the biological field. The first optical traps were built by Arthur Ashkin in 1970 at AT&T Bell Labs., where the acceleration of freely suspended particles due to forces from radiation pressure from laser light were first observed [1]. In these experiments, transparent spheres subjected to a beam of milliwatt power, were concurrently attracted to the centre of the beam and accelerated in the direction of the light propagation, until encountering a glass surface where they remained ‘trapped’. When the laser beam was blocked, the spheres drifted away under Brownian motion. Ashkin’s paper not only illustrated the first radiation trap, but also provided an explanation for the movement of the sphere both in the direction of beam propagation and toward the beam axis. Using these theoretical descriptions, Ashkin also demonstrated, in the same paper, the first ‘two-beam trap’, where two

counter-propagating beams were used to suspend a particle in space using only light, a method which is still used today [2, 3]. Ashkin then extended this idea the following year by producing a ‘levitation trap’ [4]. In analogy with a vertical fire hosepipe, a levitation trap makes use of the upward radiation pressure of the laser beam to balance the pull of gravity. This technique not only holds a particle in place, but also allows it to be manipulated laterally by moving the beam, which was demonstrated with charged and neutral liquid droplets [5].

Although this earlier work was crucial in the development of optical tweezing, none of the above mentioned experiments are considered optical tweezers as they do not achieve three dimensional trapping. It was not until 1986 that Ashkin and co-workers [6] used a single tightly focused laser beam to trap a transparent particle in three dimensions, known as a single-beam gradient force optical trap or ‘optical tweezer’. That is, a particle was trapped in a downward directed laser beam and manipulated not only laterally, but also along the beam axis. This ground-breaking achievement was followed by applying this tool to biological samples, which consequently led to the use of a Nd:YAG laser to avoid damage to live samples. This was just the beginning in a chain of experiments ranging from the cooling and trapping of neutral atoms to manipulating live bacteria and viruses.

Radiation pressure was an important and useful technique, particularly in the direction of laser cooling of atoms. For several years, research within this field thrived, producing numerous interesting results including the 1997 Nobel Prize in Physics shared by Steven Chu, Claude N. Cohen-Tannoudji, and William D. Phillips [7, 8, 9]. However, the development of optical tweezers in 1986 resulted in an upsurge of new ideas not only in laser physics, but in other new fields of research. This new technique allowed for a broader scope into researching the properties of microscopic particles, and consequently resulted in various exciting applications.

This dissertation provides a detailed explanation behind the elegant technique of optical tweezing and how such a tool is achieved practically. Different types of optical traps are generated and documented both theoretically and experimentally. Optical trapping groups around the world have already achieved many of these designs, however, to the best of our knowledge, we have constructed and demonstrated the first optical tweezing system in Africa. We have also extended this system by using super-Gaussian beams, which, as far as we know, have not yet been used in optical tweezing.

Chapter 2

Laser Beam Propagation

The ability to manipulate particles with light depends strongly on the understanding of the special characteristics of the propagation of laser beams through optical systems. Each laser has a different set a propagation parameters, specifying the beam shape, the quality of the beam, as well as how the beam will change after propagating a certain distance. These parameters can be determined experimentally, allowing a characterisation of the laser beam. This is an important aspect, as each component in an experimental setup depends on the parameters of the beam at that specific point. Often these parameters are stated by the manufacturer, however, every laser differs somewhat from the standard specifications thus experimental verification provides more accurate values of the parameters. It is also possible to alter the shape of the beam, thereby also changing the properties associated with it. Therefore by characterizing the propagation of the original beam shape, a comparison can be made with other beam shapes allowing for new applications of lasers.

2.1 Gaussian Beams

Gaussian beams are radially symmetric distributions whose Fourier transform is also Gaussian. These beams are therefore favourable in modelling and are commonly found in practical lasers.

The complex representation of a wavefunction describing a monochromatic wave is given by,

$$u(\mathbf{r}, t) = A(\mathbf{r}) \exp[i\Phi(\mathbf{r})] \exp(i\omega t), \quad (2.1)$$

where $A(\mathbf{r})$ is the amplitude, $\Phi(\mathbf{r})$ is the phase and ω is the angular

frequency. The wavefunction must satisfy the wave equation:

$$\nabla^2 u - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0, \quad (2.2)$$

where ∇^2 is the Laplacian operator. If only the time-independent complex amplitude, $u(\mathbf{r}) = A(\mathbf{r}) \exp[i\Phi(\mathbf{r})]$, is considered, the wave equation becomes,

$$\nabla^2 u + k^2 u = 0, \quad (2.3)$$

which is the Helmholtz equation, where $k = \omega/c$ is the wavenumber. A plane wave travelling in the z -direction modulated by a complex envelope $A(\mathbf{r})$ has a complex amplitude given by:

$$u(\mathbf{r}) = A(\mathbf{r}) \exp(-ikz), \quad (2.4)$$

where $k = 2\pi/\lambda$ is the wavenumber for a wavelength λ . For eq. (2.4) to satisfy the Helmholtz equation (eq. (2.3)), the complex envelope $A(\mathbf{r})$ must satisfy the paraxial Helmholtz equation [10]:

$$\nabla_T^2 A - i2k \frac{\partial A}{\partial z} = 0, \quad (2.5)$$

where $\nabla_T^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian operator. One solution to eq. (2.5) yields a paraboloidal wave, which when shifted by some amount ξ , is also a solution of the paraxial Helmholtz equation. Therefore, by setting $q(z) = z - \xi$, a paraboloidal wave centred about $z = \xi$ is given by,

$$A(\mathbf{r}) = \frac{A_1}{q(z)} \exp \left[-ik \frac{r^2}{2q(z)} \right], \quad (2.6)$$

where A_1 is a constant and $r^2 = x^2 + y^2$. This paraboloidal envelope leads to a Gaussian envelope when ξ is purely imaginary, $\xi = -iz_R$, where z_R is real and referred to as the Rayleigh range. The q -parameter is then defined as $q(z) = z + iz_R$, which can be separated into real and imaginary parts, such that,

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda}{\pi \omega^2(z)}, \quad (2.7)$$

where $R(z)$ and $\omega(z)$ are the radius of curvature and width of the beam, respectively. By substituting eq. (2.7) into eq. (2.6), the complex electric field amplitude of a Gaussian beam is given by [10]:

$$u(r, z) = u_0 \frac{\omega_0}{\omega(z)} \exp \left[- \left(\frac{r}{\omega(z)} \right)^2 \right] \exp \left(-i \left[kz - \phi(z) + \frac{kr^2}{2R(z)} \right] \right), \quad (2.8)$$

where $u_0 = A_1/iz_R$ is the peak amplitude, $\omega(z)$ is the beam width with ω_0 defined as the width at the beam waist, $k = 2\pi/\lambda$ is the wavenumber, $\phi(z)$ is the Guoy phase shift and $R(z)$ is the radius of curvature of the wavefronts. The functions $\omega(z)$, $\phi(z)$ and $R(z)$ are described in more detail later in this section, however, the intensity distribution is crucial in order to understand Gaussian beams fully.

The intensity of any beam is given by the square magnitude of the electric field amplitude, $I(r, z) = |u(r, z)|^2$. Therefore the intensity of a Gaussian beam is the square magnitude of eq. (2.8), described by

$$I(r, z) = |u(r, z)|^2 = I_0 \exp \left[-2 \left(\frac{r}{\omega(z)} \right)^2 \right]. \quad (2.9)$$

Here

$$I_0 = \frac{2P_0}{\pi\omega^2(z)} \quad (2.10)$$

with P_0 the total power of the laser and r is the radial co-ordinate. As illustrated in fig. 2.1, the most intense region of the beam is situated at the centre, which is an important characteristic of Gaussian beams for optical tweezing.

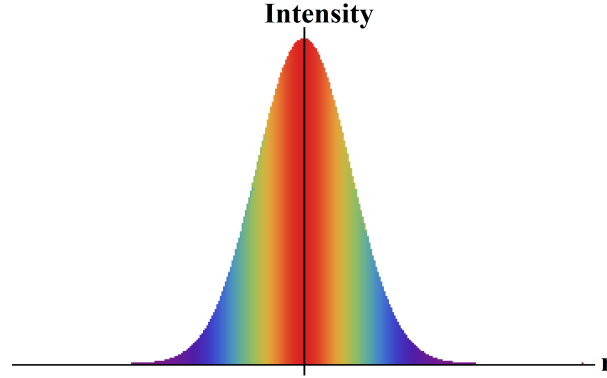


Figure 2.1: Intensity profile of a Gaussian beam.

2.1.1 Propagation Parameters

Ideal Gaussian propagation corresponding to the theoretical TEM_{00} mode, is an approximation, however, real lasers do not display this ideal profile,

although some are very good approximations. To accommodate for these imperfections, there exists a ratio of the beam parameter product (the beam waist multiplied with the beam's divergence) of a real beam to that of an ideal beam of the same wavelength. This ratio is known as the beam quality, M^2 , and for an ideal Gaussian beam $M^2 = 1$. For simplicity, the following parameters are explained under the assumption that an ideal beam is used. Typically, the paraxial approximation is assumed when considering Gaussian beam propagation, that is, the angle θ between the propagation axis and some ray is very small, $\theta \ll 1$ rad. Therefore the following approximations can be made; $\sin \theta \approx \tan \theta \approx \theta$.

Beam Width

The beam width, $\omega(z)$, is defined by the distance from the beam axis where the intensity of the beam decreases to $1/e^2$ of the maximum value. As Gaussian beams propagate through space, the width of the beam varies with propagation distance z as,

$$\omega(z) = \omega_0 \sqrt{1 + \frac{(z - z_0)^2}{z_R^2}} \quad (2.11)$$

where z_0 is the position of the beam waist and $z_R = \frac{\pi \omega_0^2}{\lambda}$ is the Rayleigh length. For $z < z_R$ the beam will propagate with minimal divergence.

Divergence

When considering the far-field, that is for $z \gg z_R$, the beam width approaches the value

$$\omega(z) \cong \frac{\lambda z}{\pi \omega_0}. \quad (2.12)$$

Since $z \gg z_R$, the boundary describing the beam width asymptotically approaches a cone with an angle of,

$$\theta \cong \frac{\omega(z)}{z} = \frac{\lambda}{\pi \omega_0}. \quad (2.13)$$

This angle is known as the divergence of a Gaussian beam and describes how the beam spreads out as it propagates.

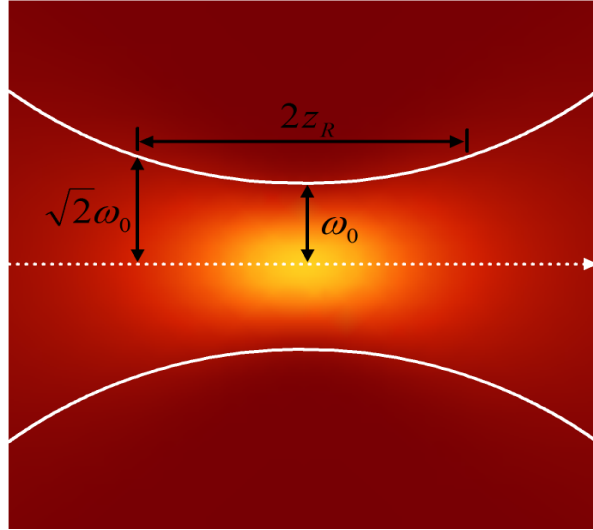


Figure 2.2: Diagram illustrating the changing width of a propagating Gaussian beam, the most intense region is represented by yellow. The white lines denote the $1/e^2$ boundary of the beam.

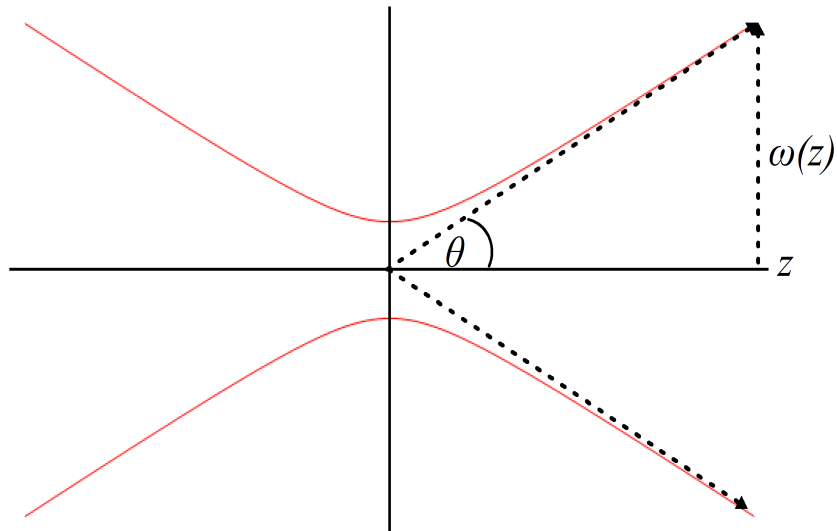


Figure 2.3: Diagram illustrating the asymptotic behaviour far from the beam waist.

It is clear that the divergence and beam waist are inversely proportional and therefore in order to obtain a well collimated beam, the beam width must be large.

Radius of Curvature

Thus far, only the real part of eq. (2.8) has been considered. The last term of the imaginary exponential function has a quadratic dependence on the radial component r , and within the paraxial approximation, describes a spherical wavefront of radius $R(z)$. The radius is defined as,

$$R(z) = (z - z_0) \left[1 + \left(\frac{z_R}{z - z_0} \right)^2 \right] \quad (2.14)$$

and is referred to as the radius of curvature of the wavefront of a Gaussian beam. The dependence on z is not immediately obvious. At $z = z_0$, the radius of curvature is infinity, resulting in a planar wavefront. Likewise as z tends to infinity, the wavefront is also planar. It therefore seems that there should be a point of maximum curvature as the beam propagates from the waist. This point lies at $z = z_0 + z_R$, the Rayleigh length, where $R(z)$ is a minimum signifying maximum curvature.

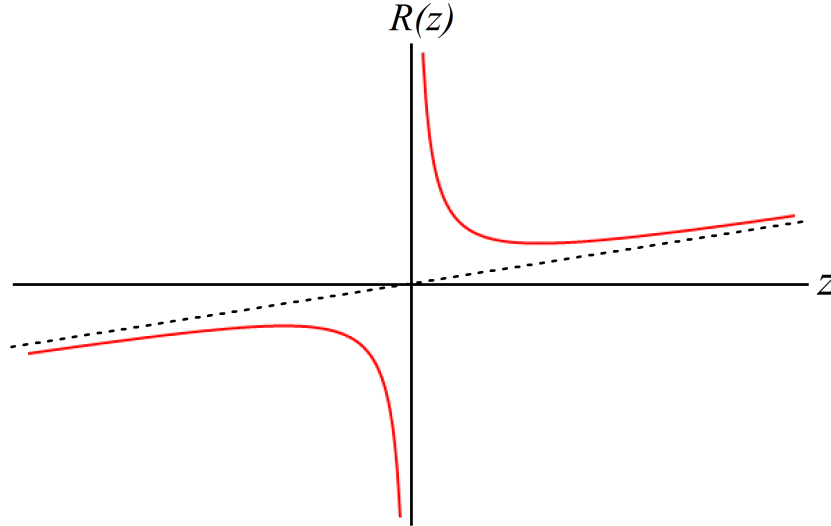


Figure 2.4: Diagram showing the change in the value for the radius of curvature as z changes. The dashed line represents the radius of curvature for a spherical wave.

Guoy Phase Shift

Another term associated with the imaginary exponential is the Guoy phase shift, $\phi(z)$. A Gaussian beam with the same optical frequency as a plane wave, attains a phase shift as it propagates, given by,

$$\phi(z) = \arctan\left(\frac{z}{z_R}\right). \quad (2.15)$$

This shift causes the distance between wavefronts to increase slightly as compared with that defined for a plane wave of the same frequency, and thus there is also an increase in the phase velocity of Gaussian wavefronts.

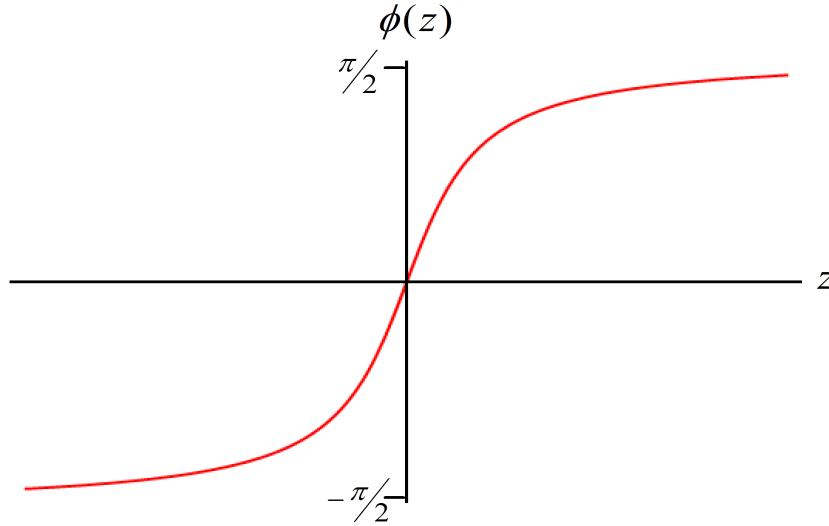


Figure 2.5: Diagram of the phase shift of a Gaussian beam relative to a plane wave.

ABCD Matrices and Complex q Parameter

An ABCD matrix is a 2×2 matrix corresponding to an optical element, which allows the effect of the element on the beam to be calculated. Fig. 2.6 shows a ray entering an optical element with position r and angle θ , and exiting with position r' and angle θ' :

For small enough angles for the paraxial approximation to be valid, there is a linear relationship between the r and θ parameters before and after an optical element, and can be written as,

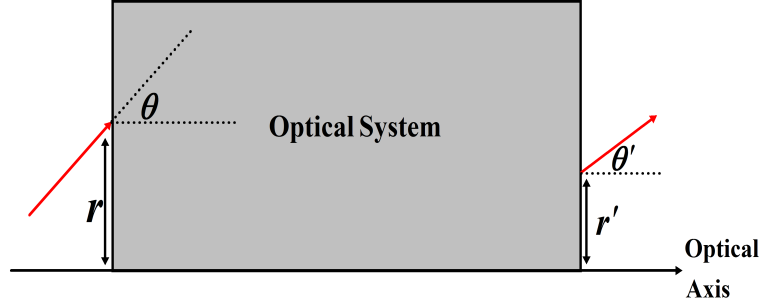


Figure 2.6: A ray is specified by a position r and angle θ .

$$\begin{pmatrix} r' \\ \theta' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r \\ \theta \end{pmatrix}$$

where the ABCD matrix represents some element and A, B, C and D are real numbers. For example, a thin lens with focal length f , has an ABCD matrix given by,

$$\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}.$$

This is a useful technique for evaluating various beam parameters after encountering optical elements. In particular, these matrices are useful in determining the complex q parameter. Information about both the beam width, ω , and the radius of curvature, R , are contained in this parameter by,

$$\frac{1}{q(z)} = -i \frac{\lambda}{\pi \omega^2(z)} + \frac{1}{R(z)}. \quad (2.16)$$

This parameter is transformed by the ABCD matrix as follows,

$$q' = \frac{Aq + B}{Cq + D}, \quad (2.17)$$

where q' refers to the beam after encountering the optical element. Eq. (2.17) is called the *ABCD* law and is used to describe the effect of any paraxial system on a Gaussian beam.

2.2 Bessel-Gauss Beams

Gaussian beams are generally used in both theoretical and experimental projects. They have a simple mathematical form and have therefore been well-documented. However, as the previous section displayed, Gaussian beams are subject to diffraction as they propagate, regardless of the divergence value. More generally, any field with initial radius r in the transverse plane and of wavelength λ , will experience diffractive spreading as it propagates away from that plane particularly after the Rayleigh range [11]. Although modern laser designs attempt to reduce this property of spreading, it is always present and therefore will always affect an experiment, often in a negative manner. There are however beams that do not experience diffractive spreading, referred to as ‘non-diffracting’ beams.

All diffractive occurrences can be described by the Helmholtz equation:

$$[\nabla^2 + \kappa^2]\Phi(r; \kappa) = 0 \quad (2.18)$$

where $\kappa = \omega/c$ is the wavenumber. [11] was the first to demonstrate that there exist solutions to this equation that are diffraction-free. In particular, these solutions described well-defined beams with narrow central beam spot radii, which do not display diffractive spreading.

Bessel beams (BBs) represent a class of so-called diffraction free solutions to the Helmholtz equation, and have been studied extensively since the seminal work of [12] in the late 1980s. One of the simplest solutions illustrated by [11], describes a monochromatic wave with an electric field envelope that is a zeroth-order Bessel function of the first kind, given by $J_0(k_r r)$, where k_r is the radial component of the wave vector k and r denotes the distance from the z -axis. This was the first mathematical concept of the Bessel beam, which ideally consists of an infinite number of rings of light, carrying an infinite amount of power in an infinite area. Experimentally, it is not possible to generate such a beam, and so an approximation is made in the form of a Bessel function enveloped by a Gaussian profile, thereby limiting the energy carried by the field to some finite value. The electromagnetic field $u(r, z)$ of a Bessel-Gauss (BG) beam propagating in freespace in the z direction is given by [13]:

$$u(r, z) = A \frac{\omega_0}{\omega(z)} \exp \left[i \left(k - \frac{k_r^2}{2k} \right) z - i\Phi(z) \right] J_0[k_r r / (1 + iz/z_R)] \\ \times \exp \left[\left(\frac{-1}{\omega^2(z)} + \frac{ik}{2R(z)} \right) \left(r^2 + \frac{k_r^2 z^2}{k^2} \right) \right] \quad (2.19)$$

where A is an amplitude factor, which may be complex, and the parameters z_R , $\omega(z)$, $R(z)$ and $\Phi(z)$ are defined as in Gaussian propagation. Eq. (2.19) can be simplified by considering the field at the plane $z = 0$, where the Gaussian dependence is more clearly seen:

$$u(r, 0) = A J_0(k_r r) \exp[-(r/\omega_0)^2]. \quad (2.20)$$

Fig. 2.7 illustrates how the intensity profile of eq. (2.20) changes along the axis of propagation (z -direction), from a Bessel-Gauss beam (a) to an annular ring (b).

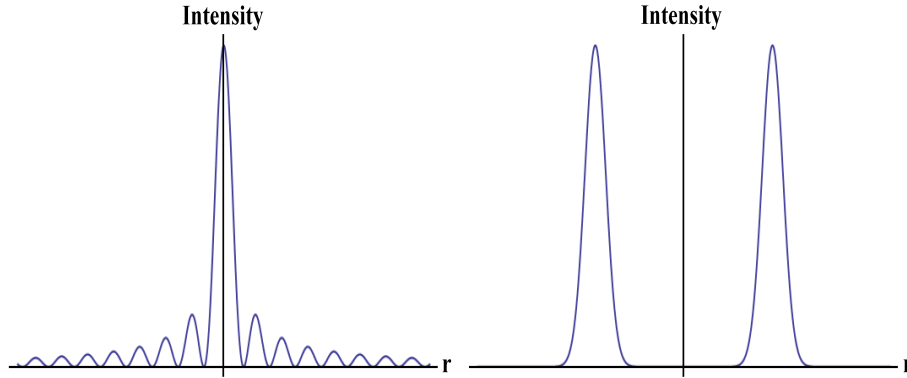


Figure 2.7: Intensity profiles of (a) a Bessel beam at $z \approx 0$ and (b) an annular ring at $z = 0.1$ m. The two beams are Fourier transforms of one another.

Of course, an important property of the Bessel beam is its non-diffracting nature. That is, for propagation in the z direction, the intensity satisfies,

$$I(x, y, z \geq 0) = I(x, y), \quad (2.21)$$

where $I(x, y, z) \propto J_0^2(k_r r)$ for a Bessel beam.

However, for a BG beam this equality is true only for a finite distance of z . This distance can be calculated by analysing the experimental generation of a BG beam. There are two well established methods for Bessel beam formation,

one of which makes use of an annular slit. A Bessel beam can be thought of as a superposition of plane waves whose wave vectors lie on a cone, which in k -space reveals itself as a ring. That is, the Fourier transform of a ring results in a Bessel beam, which allowed [12] to produce an approximation to a Bessel beam by illuminating an annular slit onto the back focal plane of a focusing lens. A more efficient method of Bessel beam generation makes use of an axicon. Fig. 2.8 illustrates how a BG beam is created by illuminating a Gaussian beam onto an axicon. The opening angle, θ , of the cone is given by,

$$\theta = (n - 1)\gamma, \quad (2.22)$$

where n is the refractive index of the axicon and γ is the opening angle of the axicon.

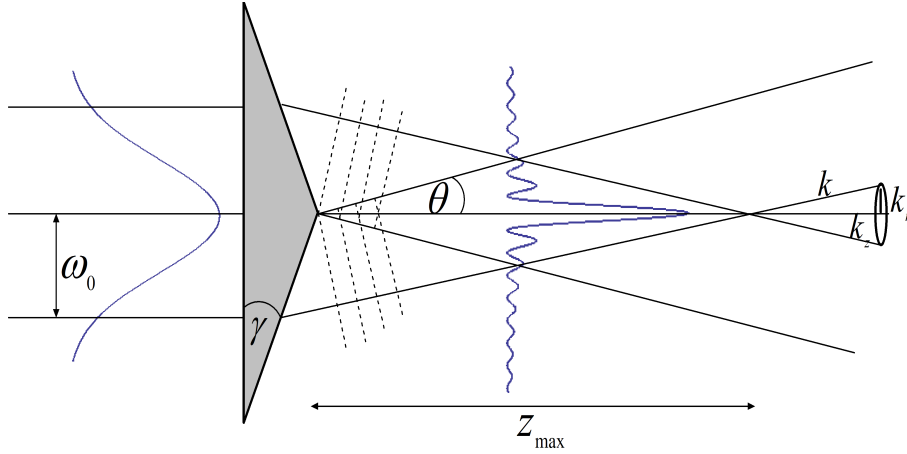


Figure 2.8: Diagram illustrating the finite propagation distance of a Bessel beam generated by an axicon of opening angle θ . The superposition of plane waves has wave vectors, which lie on the surface of a cone with opening angle γ .

The finite distance after the axicon in which a BG beam is found, is determined from the wave vectors of the Bessel beam that propagate on a cone. This maximum distance is given by,

$$z_{\max} = \frac{k}{k_r} \omega_0 \approx \frac{\omega_0}{\theta}. \quad (2.23)$$

Within this finite propagation distance, the properties associated with the ideal Bessel beam hold true for a BG beam. Using geometric optics,

further information about the beam can be revealed. The maximum width of the BG beam, r_{max} , is found at $z_{max}/2$:

$$r_{max} = z_{max} \tan \theta. \quad (2.24)$$

These two parameters together with the Bessel function parameter, $k_r = k \sin \theta$, allow the number of rings, N , at r_{max} to be solved:

$$N = \frac{r_{max}}{2} \frac{1}{\delta}, \quad (2.25)$$

where $\delta \approx \pi/k$ is the spacing between each ring.

Section 2.2.1 illustrates the reconstruction property that is preserved by Bessel-Gauss beams. For such beams, there is a minimum distance behind an obstacle of radius $D/2$ before reconstruction occurs,

$$z_{min} = D \frac{k_z}{2k_r} = \frac{D}{2 \tan \theta}. \quad (2.26)$$

This reconstructive property has been used in a variety of applications, including optical tweezing [14] and medical imaging [15], and is therefore an intriguing property worth investigating.

2.2.1 Reconstruction of Bessel-Gauss Beams

Reconstruction has been well explained by considering a Bessel beam as a superposition of plane waves, whose transverse amplitudes reform after encountering an obstruction. Theoretical studies of this property describe reconstruction under varying conditions, including in a nonlinear medium [16] and by wave packets due to spatial-temporal links [17]. The reliability of the periodical self-reconstruction, as well as the wavelength dependence of white light Bessel beams have been examined [18]. The reconstruction of Bessel beams has also been experimentally verified on a number of occasions by means of different methods.

Although extensive work has been devoted to this one property, the methods of modelling this characteristic are limited in accuracy and speed, and are yet to calculate Bessel fields after arbitrary obstructions. Conventionally, the Fresnel-Kirchoff diffraction integral together with Babinet's principle has been employed, which results in time consuming calculations. Recently, the phenomenon of reconstruction was eloquently explained by considering the

dynamics of the conical waves that form a Bessel beam [19]. Essentially, the cone of wave vectors previously mentioned, consists simultaneously of two conical waves, an incoming and an outgoing wave, which can be represented by Hankel functions of the second, $H_0^{(2)}(k_r r)$, and first, $H_0^{(1)}(k_r r)$, kind respectively:

$$H_0^{(1)}(k_r r) \exp(ik_z z) = [J_0(k_r r) + iN(k_r r)] \exp(ik_z z) \quad (2.27)$$

$$H_0^{(2)}(k_r r) \exp(ik_z z) = [J_0(k_r r) - iN(k_r r)] \exp(ik_z z) \quad (2.28)$$

where $N(k_r r)$ is the zero-order Newman function.

Essentially, the incoming conical wave transforms into the outgoing conical wave and the superposition of these two conical waves is crucial in obtaining propagation invariant beams. Within a Bessel beam, although locally the light travels parallel to the axis of propagation, the energy flux follows the direction determined by the cone of wave vectors. For this reason, reconstruction after an obstruction is observed. That is, the reconstructed beam results from light that was not obscured by the obstacle.

Despite this fluent explanation by [19], the numerical simulations demonstrated resorted back to solving the Helmholtz equation. Therefore, we propose a new model for predicting where and how BG beams reconstruct [20]. A description of the model together with experimentally results is presented here. The model is based on a geometrical approximation to the propagation of conical waves after obstacles. As we will show later, this approximation is amazingly resilient even when the obstacle sizes are in the order of the wavelength of the illuminating light, and so while our method cannot be considered a complete substitute for a full vector diffraction based calculation, it offers a very accurate first approximation.

Theoretical Model

Consider Fig. 2.9 where an arbitrary obstacle is placed off-centre in the path of a Bessel-Gauss beam.

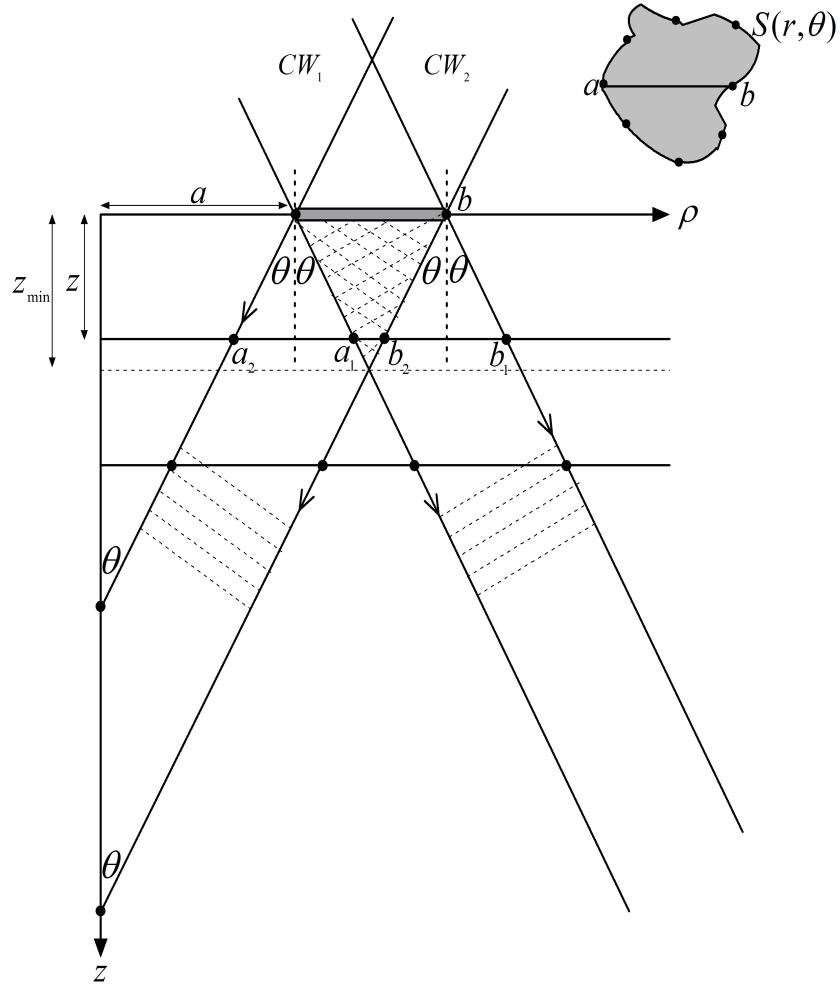


Figure 2.9: Diagram illustrating the projection of the obstacle in space by two conical waves, CW_1 and CW_2 . The shadow region where neither conical wave contributes has a minimum distance given by $z_{\min}$.

The obstruction cross-section due to this obstacle along one azimuthal direction is shown. The positive z -axis defines the direction of propagation of the Bessel-Gauss beam, which we assume to have a spatial extend of $2d$, as shown in fig. 2.10.

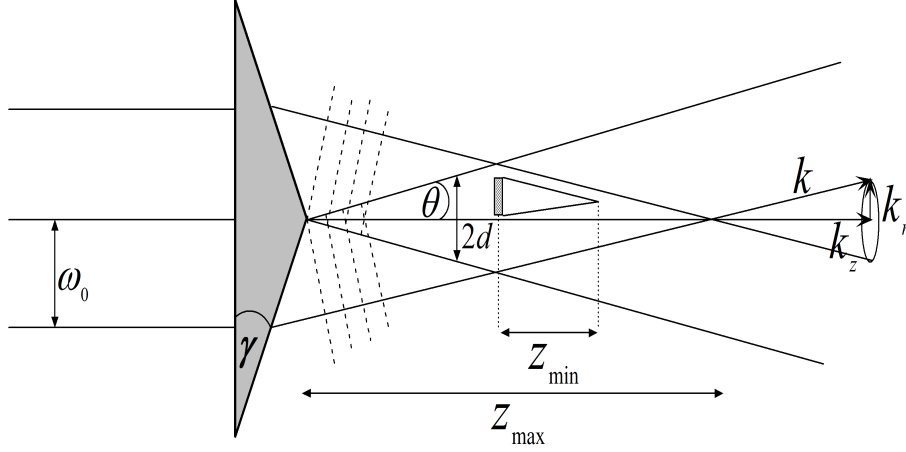


Figure 2.10: Diagram illustrating the finite propagation distance of a Bessel beam generated by an axicon of opening angle θ . The superposition of plane waves has wave vectors, which lie on the surface of a cone with opening angle γ .

Since only a cross-section is shown, there are only two conical waves (labelled CW_1 and CW_2 respectively) contributing to the Bessel-Gauss beam at any given point.

It is clear from Fig. 2.9 that the projection from these two conical waves results in distinct regions defined by one conical wave contribution only, a shadow region where neither conical wave contributes, and the Bessel-Gauss beam where both waves contribute. The boundaries that define these regions are mapped by the projection of the obstacle's boundaries along a given z plane. Without any loss of generality, consider an obstacle with cross-section shown in fig. 2.9 with full width D and that is placed some distance $\rho = a$ from the central axis ($\rho = 0$) of the Bessel-Gauss beam, so that the two extreme edges of the obstacle are defined by $\rho = a$ and $\rho = b$, with $D = b - a$. From the geometry one can derive the projection of the obstruction due to each conical wave. If the new positions of these edges due to CW_1 are labelled a_1 and b_1 , and similarly a_2 and b_2 due to CW_2 , then one can show that at some distance z after the obstacle we have:

$$a_1 = a + z \tan \theta \quad (2.29)$$

$$a_2 = a - z \tan \theta \quad (2.30)$$

$$b_1 = b + z \tan \theta \quad (2.31)$$

$$b_2 = b - z \tan \theta \quad (2.32)$$

Clearly the projection results in the creation of two zones defined by a single conical wave, with the boundaries of these zones moving farther apart at a rate of $2z \tan \theta$. The regions behind the obstacle are then defined as follows (for a given z propagation and a chosen azimuthal angle ϕ ; region (1), $a_1 < \rho < b_2$, where there is no contribution from either CW_1 nor CW_2 , resulting in a shadow region, region (2), $a_2 < \rho < a_1$ & $b_2 < \rho < b_1$, where only one of the conical waves contribute, and region (3): $-d \leq \rho \leq a_2$ & $b_1 \leq \rho \leq d$, where CW_1 and CW_2 add to form a complete Bessel-Gauss beam.

The description of these regions corresponds exactly with the equation given for z_{min} , that is, at z_{min} we require $b_2 = a_1$. Solving this equality gives,

$$\begin{aligned} b - z \tan \theta &= a + z \tan \theta \\ b - a &= D = 2z \tan \theta \\ z_{min} &= \frac{D}{2 \tan \theta} \quad , \end{aligned} \quad (2.33)$$

which is consistent with the expression in eq. (2.26), as expected. If the process described above is carried out for all points along the boundary of an obstruction, then the complete movement of the boundaries of each region can easily be computed. With this information the complete intensity pattern at any position after an obstacle, regardless of the complexity of the geometry of the obstacle, may readily be found.

It is worth pointing out some salient points in how these regions develop: firstly, the single conical wave regions move apart at a rate of $2z \tan \theta$. Thus in an off-centred geometry one expects the boundaries defining the Bessel-Gauss beam to move apart, in opposite directions, with one passing through the centre of the original Bessel-Gauss field. Secondly, any combination of points on the boundary of an obstacle that find themselves equidistant from the central propagation axis of the Bessel-Gauss beam will result in boundaries that evolve in identical fashion. This latter fact can be exploited to simplify the calculation for arbitrary obstacles.

Experimental Setup and Results

Fig. 2.11 shows the experimental setup used in generating the Bessel-Gauss beam. A CrystaLaser RCL-100-660 diode pumped solid state laser of wavelength 660 nm and power 100 mW was expanded through a $5\times$ telescope ($f_1 = 50$ mm & $f_2 = 250$ mm) before illuminating an axicon with an opening angle of $\gamma = 5^\circ$.

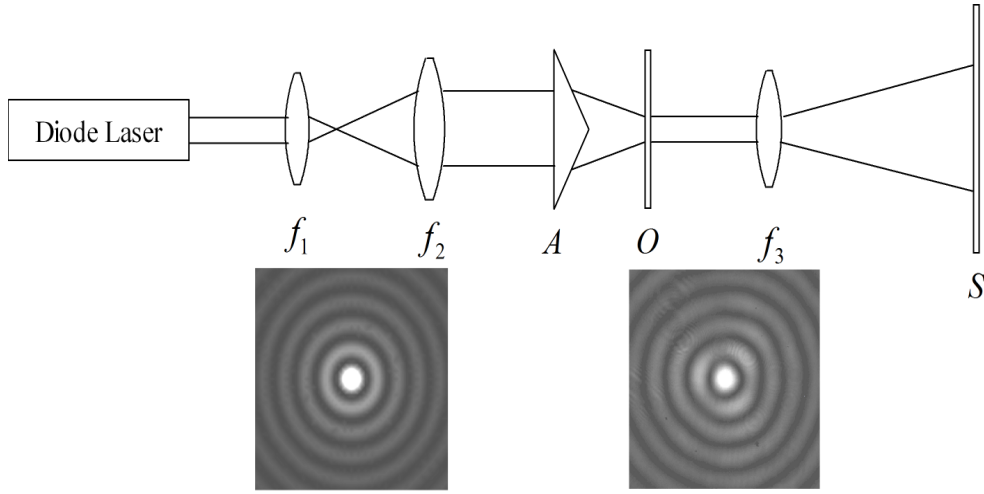


Figure 2.11: Experimental design for generating a Bessel-Gauss beam: a telescope with lenses $f_1 = 50$ mm, $f_2 = 250$ mm expands the beam before illuminating the axicon (A). The obstacle (O) was placed 45 mm from the axicon and was imaged onto a screen (S) by lens $f_3 = 25$ mm. The inserts represent the theoretical prediction (left), which was calculated using Mathematica 6 and the experimentally measured (right) Bessel-Gauss beam.

The obstruction was placed on a thin transparent cover slip and positioned within the Bessel beam at a distance of $1/2 z_{max}$ from the axicon. The resulting obstructed beam was imaged onto a screen with a suitable magnification ($57\times$) to observe the intensity variations during propagation. The image plane could be varied continuously by means of a translation stage so as to allow measurement of the beam from fully obstructed to fully reconstructed. The geometries of the three test obstructions - a centred circle (approximately), an off-centre oval and a vertical wire - are shown in fig. 2.12. In each case the obstruction is opaque so that immediately after the obstruction a dark shadow region is evident, surrounded by an unperturbed Bessel-Gauss beam.

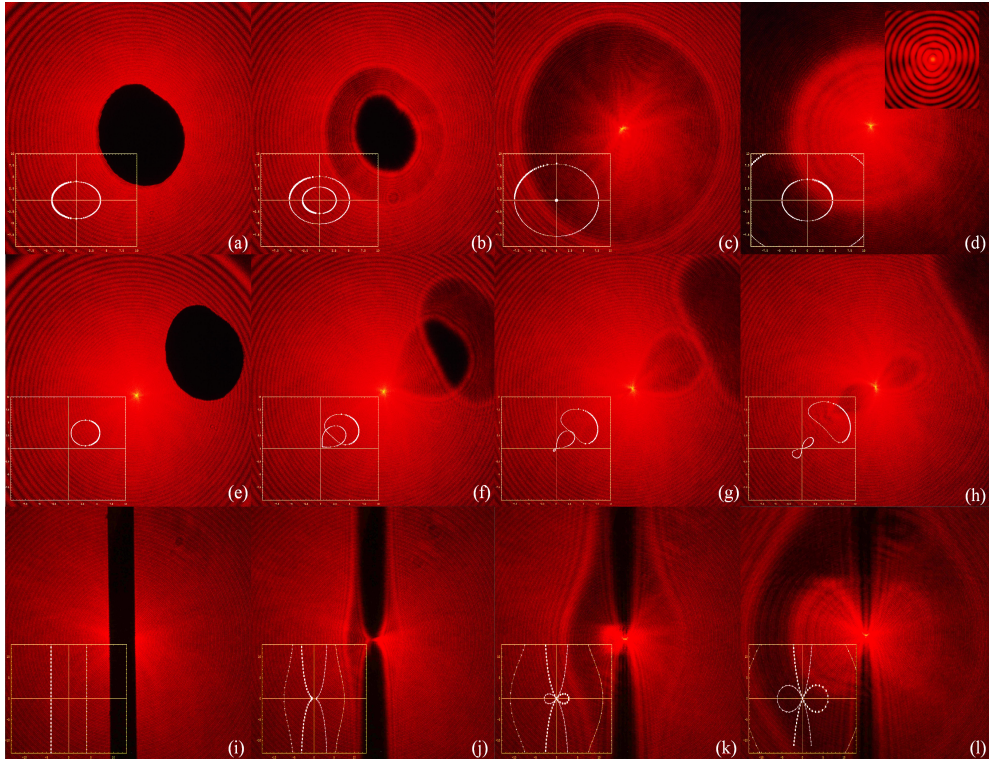


Figure 2.12: Experimental images of three different obstacle geometries within a generated Bessel beam, with the theoretical predictions of the shadow boundaries inset in the bottom left corners. The images illustrate the excellent comparison for the self reconstruction of a Bessel beam.

Fig. 2.12 shows a sequence of measured Bessel-Gauss beam intensities during propagation after the three obstructions, with insets to the experimental images depicting the calculated boundaries of the conical waves from the source. Fig. 2.12 (a)-(d) show the change in the Bessel-Gauss beam intensity during reconstruction when the starting obstacle is an approximation to a centred circle, a scenario with radial symmetry. As the propagation distance from the obstruction was increased, so the shadow regions appeared to move. Following the discussion in the previous section we noted that two boundaries appeared due to the projection of the obstacle by the two conical waves from the source. The intersection of the two boundaries defines the true shadow region, while the region between the two boundaries defines a single conical wave contribution. Elsewhere the original Bessel-Gauss beam is apparent due to the summation of both conical waves. This is evident in the sequence from (b) to (d). The final image at (d) was taken at a propagation distance that exceeded z_{min} , so that complete reconstruction had taken place. This is predicted by the calculation method and has been confirmed experimentally. Verification of the reconstruction at the centre of the field, where the original obstacle was, is clear by the zoomed inset of the centre of beam in (d).

A similar sequence is shown in fig. 2.12 (e)-(h) and fig. 2.12 (i)-(l). In these cases the radial symmetry is purposely broken. The sequence (e)-(h) illustrates the accuracy of the method for an arbitrary shaped obstacle that is placed off-centre in the Bessel-Gauss beam path. The prediction by the calculation method of the development of a ‘figure of eight’ boundary, as is observed experimentally, is striking. Likewise the vertical wire (fig. 2.12 (i)) is an interesting case, as here there is a region where no contribution from one of the conical waves is possible, as the obstruction extends across the entire Bessel-Gauss beam in the vertical plane following the wire. Again the agreement between calculation and experiment is very good.

From the experimental results, it is clear that our method correctly maps the boundaries of the reconstructed Bessel beam, regardless of the geometry or positioning of the obstacle [21]. The model calculates only the boundaries of each region, where the intensity pattern at any position can then be appropriately added. The progression of the shadow regions is evident in each of the experimental images, with the theoretical predictions matching exactly.

Chapter 3

Optical Manipulation

The key in using light as a tool for manipulation, is in the transfer of momentum. When light is incident on an object, the force exerted on the object is the rate of change in the momentum of the light. Each photon carries momentum $\vec{p}$, which for a given wavelength, has a magnitude of $|\vec{p}| = h/\lambda$. On the macroscopic level, it is difficult to imagine that only light, even extremely intense laser light, could be used for a mechanical task of lifting and maneuvering particles. However, on the microscopic level, the forces generated from the interactions of photons with matter can produce impressive results.

A theoretical description is given to explain the physical processes involved in tweezing, together with a thorough account of how an optical tweezer is built. The experimental results demonstrate the ease and effectiveness of manipulating particles with light, using a standard Gaussian beam as well as a Bessel-Gauss beam.

3.1 Fundamentals of Optical Tweezing

An optical tweezer is formed by tightly focusing a laser beam through an objective lens of high numerical aperture onto a particle. In 1970, Ashkin demonstrated the effect of the forces of radiation pressure on microscopic transparent particles. A horizontal laser beam was directed through a glass cell containing a solution of micron sized transparent latex spheres, such that the focus was on a single particle. He observed that the spheres were drawn into the beam and then accelerated toward the glass wall, where they remained trapped until the beam was blocked after which the spheres moved

away by Brownian motion. The movement and trapping of the particles along the beam axis were explained by considering two forces generated from the reflection and refraction of rays. The two forces identified (the gradient force and the scattering force) are still used today when explaining the dynamics of an optical trap. The latter force is governed by the reflection of light incident on a particle and is also referred to as radiation pressure. The incident light reflects off the particle and exerts a force per unit area on the particle. Thus the scattering force pushes particles in the direction along the direction of light propagation. The gradient force, however, acts in the direction of the spatial light gradient, both laterally and axially.

In order to obtain a stable three-dimensional trap, the gradient force in the axial direction must be greater than the scattering force, so as to overcome the radiation pressure. This requires a very steep intensity gradient, which is achieved by sharply focusing the laser beam to a diffraction-limited spot with the use of an objective of high numerical aperture. Therefore the equilibrium position, where a balance between the forces is obtained, occurs slightly beyond the focal point. There are a number of ways in which to visualise these forces. Two explanations are offered here, which are dependent of the size of the particle.

3.1.1 Mie Regime

For trapped particles larger than the wavelength of the light, $a \gg \lambda$, the conditions for Mie scattering are satisfied, and simple ray optics are used to calculate the forces involved in trapping. The gradient force can be explained in terms of conservation of momentum. As light is refracted through a transparent particle, there is a change in its direction and therefore a change in its momentum. Due to Newton's third law, the particle must undergo an equal and opposite change in momentum. The force that is experienced by the particle is simply the rate of change of the momentum, which is proportional to the intensity gradient of the beam.

Each refracted ray results in a force acting on the particle. Without an intensity gradient, these forces would cancel each other out, and the scattering force would dominate. However due to the intensity gradient of Gaussian beams, there is a net force acting toward the the centre of the beam where its intensity is greatest. This is illustrated in fig. 3.2, where the particle experiences a force to the right.

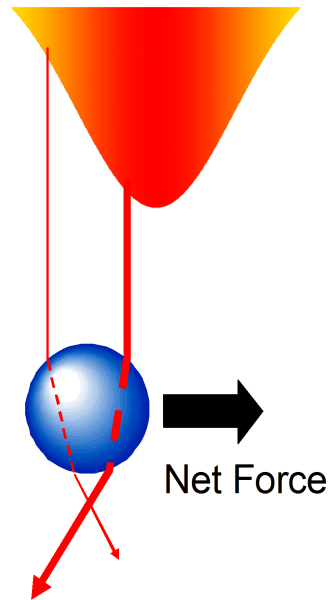


Figure 3.1: The more intense the ray, the greater the change in momentum resulting in a force acting on the particle in the opposite direction, that is, always towards the most intense region of the beam.

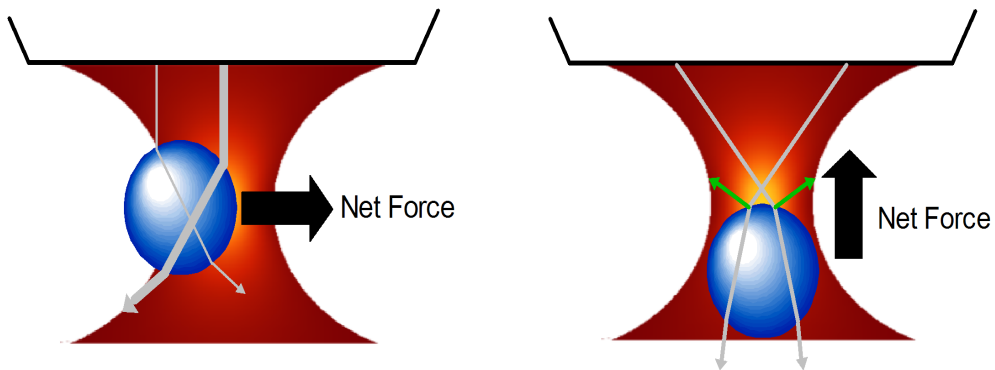


Figure 3.2: The refracted rays (gray) produce corresponding forces (green) on the bead, resulting in a net force toward the most intense region of the beam. In the axial direction, the gradient force must be greater than the scattering force to obtain three dimensional trapping.

The theory is then separated further by the index of refraction of the particle and its surrounding medium. If the index of refraction of the particle is greater than that of the surrounding medium, the force due to refraction acts in the direction of the intensity gradient. For a particle with refractive index lower than that of the medium, the force acts in the opposite direction of the intensity gradient and tweezing is not achieved.

These forces can be calculated directly from ray optic diagrams, see fig. 3.3, for a uniform sphere, where P is the incident power of a ray, PR is the reflected ray and PT^2R^n is the infinite set of refracted rays [22].

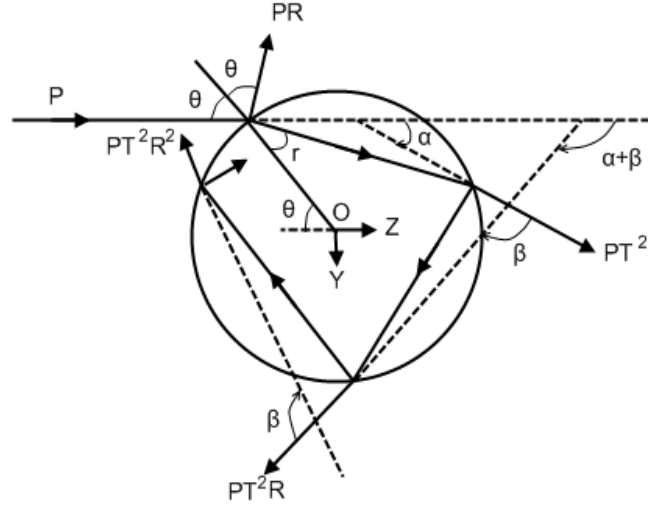


Figure 3.3: Diagram illustrating how the forces in the ray optics regime are calculated.

A ray of power P strikes a sphere at angle θ , after which it is partially reflected and refracted, resulting in a series of scattered rays of power PR , PT^2 , PT^2R , ..., PT^2R^n , The angles relative to the original propagation direction formed by these rays are $\pi + 2\theta$, α , $\alpha + \beta$, ..., $\alpha + n\beta$, ..., respectively. The total force in the z - direction is the net change in momentum per second in the z - direction from all the scattered rays. So,

$$F_z = \frac{n_m P}{c} - \left[\frac{n_m PR}{c} \cos(\pi + 2\theta) + \sum_{n=0}^{\infty} \frac{n_1 P}{c} T^2 R^n \cos(\alpha + n\beta) \right], \quad (3.1)$$

where $n_m P/c$ is the incident momentum per second in the z - direction. Likewise, the total force in the y - direction is:

$$F_y = 0 - \left[\frac{n_m P R}{c} \sin(\pi + 2\theta) - \sum_{n=0}^{\infty} \frac{n_1 P}{c} T^2 R^n \sin(\alpha + n\beta) \right]. \quad (3.2)$$

The Fresnel reflection and transmission coefficients, R and T respectively, vary with polarization resulting in polarization dependent forces. Using the geometric relations, $\alpha = 2\theta - 2r$ and $\beta = \pi - 2r$, the following formulae for the scattering and gradient forces are found:

$$F_z = F_s = \frac{n_m P}{c} \left(1 + R \cos 2\theta - \frac{T^2 [\cos(2\theta - 2r) + R \cos 2\theta]}{1 + R^2 + 2R \cos 2r} \right) \quad (3.3)$$

$$F_y = F_g = \frac{n_m P}{c} \left(R \sin 2\theta - \frac{T^2 [\sin(2\theta - 2r) + R \sin 2\theta]}{1 + R^2 + 2R \sin 2r} \right), \quad (3.4)$$

where $n_m P/c$ is the incident momentum per second of a ray of power P in a medium of refractive index n_m . The trapping efficiency, Q , is a dimensionless parameter describing the amount of momentum transferred by the beam to the particle. It can be determined experimentally by calibrating the trapping force against the Stokes drag,

$$Q \equiv \frac{F_{Drag}}{(n_m P/c)}, \quad (3.5)$$

where

$$F_{Drag} = 6\pi\eta R\nu, \quad (3.6)$$

and η is the viscosity of the fluid, R is the radius of the trapped particle and ν is the relative velocity of the fluid. Typical values for Q lie in the range $0 < Q < 2$ [22]. It has been noted that the Q -values measured in the axial direction are smaller than those measured in the transverse direction [23]. The above force equations can be analysed by considering their respective Q -values. The graph shown in fig. 3.4 illustrates how both the scattering and gradient forces vary with the angle of incidence, θ , as well as that of the total force $F_{mag} = (F_s^2 + F_g^2)^{1/2}$.

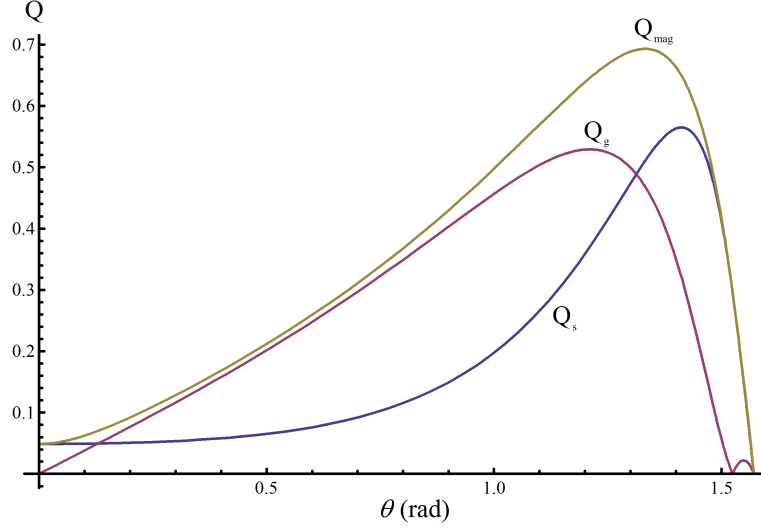


Figure 3.4: Comparison of the forces at different incident angles, θ . The gradient force is greater than the scattering force for a range of incident angle values and reaches a maximum at $\theta \approx 70^\circ$.

From the graph in fig. 3.4, the gradient force is greater than the scattering force for a range of incident angle values and reaches a maximum at $\theta \approx 70^\circ$.

The trapping efficiency can be improved in a number of ways by changing certain experimental parameters. These optimisations will be discussed in section 3.2.

3.1.2 Rayleigh Regime

For the second case where particles are smaller than the wavelength of the trapping laser beam, $a \ll \lambda$, the conditions for Rayleigh scattering are satisfied and particles are treated as point dipoles [6]. When light passes through a dielectric material, a set of fluctuating dipoles are generated by the inhomogeneous electric field. Just as a charge experiences a force in an electric field, an electric dipole experiences a force in a spatially varying electric field. That is, each charge of the dipole experiences different field strengths and therefore the force on the dipole is $\mathbf{p} \cdot \nabla \mathbf{E}$, where $\mathbf{p} = \alpha \mathbf{E}$ is the dipole field and α is the polarizability of the material. The induced dipoles of the particle are fixed, causing the particle to experience a force in the direction of the intensity gradient of the light. Thus, the strength of the gradient force is proportional to both the polarizability of the material and the intensity gradient of the beam at the focus. As shown in [24] for the case

of a sphere,

$$F_{Grad} = \frac{2\pi\alpha}{cn_m^2} \nabla I_0, \quad (3.7)$$

$$\alpha = n_m^2 R^3 \left(\frac{m^2 - 1}{m^2 + 2} \right), \quad (3.8)$$

where α is the polarizability of the sphere, I_0 is the intensity of the incident light, n_m is the index of refraction of the surrounding medium, m is the ratio of the index of refraction of the particle to that of the medium, and c is the speed of light.

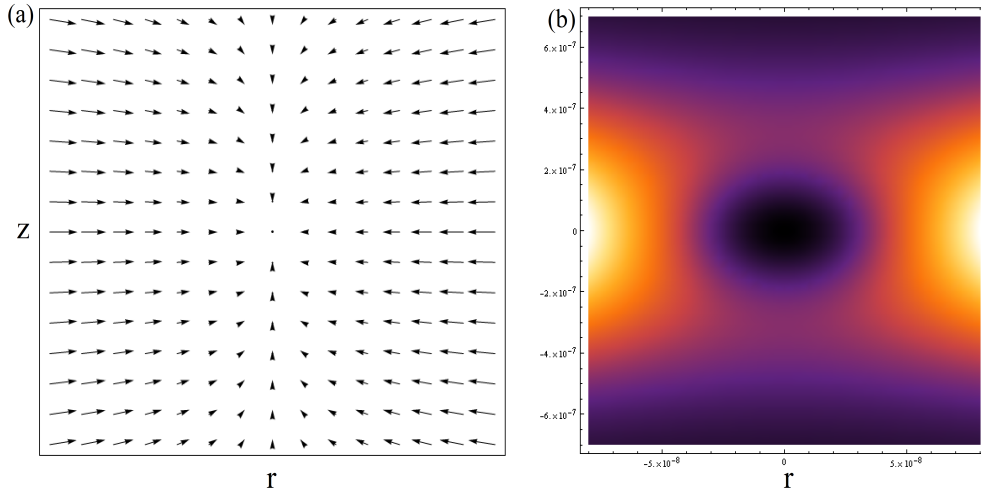


Figure 3.5: (a) Vector field plot of the illustrating the direction of the gradient force near the focus of the beam (0,0). The arrows clearly point to the origin, or beam focus, where the intensity is greatest. (b) Density plot of the magnitude of the gradient force. There is no gradient force at the origin (the beam focus), however there is a gradient force surrounding the central point, where the magnitude of the force increases steadily away from the centre.

The scattering force can also be described in terms of electric fields. A particle of radius R experiences a scattering force proportional to the intensity of the beam,

$$F_{Scatt} = \frac{I_0 \sigma n_m}{c}, \quad (3.9)$$

$$\sigma = \frac{128\pi^5 R^6}{3\lambda^4} \left(\frac{m^2 - 1}{m^2 + 2} \right)^2, \quad (3.10)$$

where σ is the scattering cross section of the sphere and λ is the wavelength of the laser beam. The scattering force acts in the direction of light propagation and is proportional to the intensity of the beam.

If however, the particle size is comparable to the wavelength of the laser, $a \sim \lambda$, as is the case for most practical applications, neither theoretical description is valid and a more complete electromagnetic theory is needed [25]. This regime is known as the Lorentz-Mie regime.

3.2 Design Considerations

The success of an optical tweezer strongly depends on the choice of apparatus. It is therefore necessary to examine the various components involved in setting up such a system. A diagram of a basic optical tweezer is shown in fig. 3.6.

The beam is directed through a series of mirrors and lenses before being directed downwards through the objective lens onto the sample. The lenses act as an expanding telescope so as to fill the back aperture of the objective. Mirror M_2 , acts as a steering mirror in that its adjustment repositions the beam within the sample. The sample is illuminated from below by a white light LED source. The sample is then imaged onto the camera using a collecting lens and a dichroic mirror, which reflects red light only and transmits wavelengths outside the red spectrum.

Over the years, various optical tweezing designs have been produced, some more complicated than others. A description of a basic, yet highly effective setup is discussed next. The particular design demonstrates the basics of tweezing while also displaying more versatile trapping using novel beam shapes.

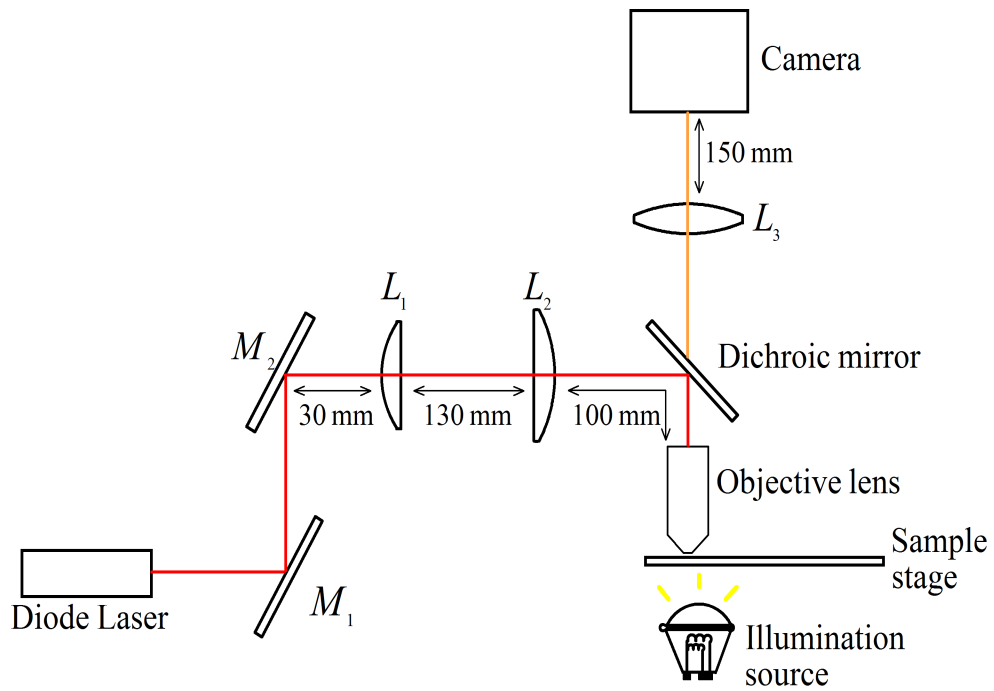


Figure 3.6: Diagram of the experimental setup used to produce an optical trap. The beam is expanded before it is reflected off a dichroic mirror onto the back aperture of an objective lens. The sample is illuminated from below and imaged to a CCD camera placed above the system.

3.2.1 Choice of Laser

When considering the type of laser to be used, there are five main features that need to be taken into account. That is, the shape of the output beam, the astigmatism associated with the beam, the polarisation of the beam, the power of the beam as well as its wavelength. An optical tweezer requires that the laser be tightly focused to a very small spot, thereby necessitating a single transverse output mode, for instance TEM_{00} . Of course, in order to attain a stable trap, there should be as few pointing fluctuations as possible, and thus lasers with very good pointing stabilities are essential. The use of a spatial filter is useful in producing a non-astigmatic output beam [26].

As mentioned in section 3.1.1, both the scattering and gradient forces are polarisation dependent. This however, does not restrict the beam to one particular polarisation, as trapping will be attained using either horizontally or vertically polarised light. Nevertheless, further research into the effect of different polarisation directions on the strength of the trapping force may produce interesting results.

The power of the laser determines the maximum achievable stiffness of the trap (defined in eq. 3.21), together with the strength of the forces involved. Therefore, to prevent any variations in the trap stiffness, a beam output with low power fluctuations is used. A typical stable trap can be attained using powers from 20 to 100 mW.

The wavelength of the beam is also important with respect to the available output power of the laser and is dependent on the transmission of the objective used. Also, biological samples are often damaged by the absorption of focused, visible light and thus infrared lasers are optimal for such experiments.

3.2.2 Objective Lens

The objective lens is the most critical component of an optical trap. It is responsible for obtaining the tightly focused laser spot needed in trapping. The trapping efficiency Q , that is the stiffness versus the input power, depends on both the numerical aperture, NA, and the transmittance of the objective. Therefore, the type of objective chosen will determine the efficiency of the trap. There are other features of an objective which must also be taken into consideration when designing a trap. The immersion

medium of the objective, either water, oil or glycerol, together with the working distance of the objective, will establish physical limitations on the depth to which particles can be trapped [24].

There are a variety of different types of objectives, some more expensive than others, which reduce spherical aberrations more effectively than their cheaper counterparts. [27], provides a comparison between a number of possible objective choices. However, the primary concern when selecting an objective is choosing one with a high numerical aperture. Objectives with a NA of 1.2 - 1.4 are sufficient for creating an intensity gradient required in obtaining a stable trap. The correlation between the NA of the objective and the angle of incidence, θ , of the beam is shown in eq. (3.11), where the NA determines the maximum allowed value of θ :

$$\text{NA} = n \sin \theta, \quad (3.11)$$

where n is the index of refraction of the medium surrounding the particle. Thus, a high NA results in a larger maximum angle of incidence, thereby producing a more tightly focused beam. This property is clearly illustrated in fig. 3.7. In (a), the z -component of the force associated with the reflected rays, r_1 & r_2 , acts in the positive z -direction as do the forces corresponding to the refracted rays, r_3 & r_4 . The bead is forced towards the focus of the rays. However in (b), the net force of the refracted rays acts in the negative z -direction, towards the focal point. If the force associated with the refracted rays is greater than that of the reflected rays, the bead will move towards the focus. That is, the net force of the refracted rays increases with increasing θ , whereas the net force associated with the reflected rays decreases with increasing θ .

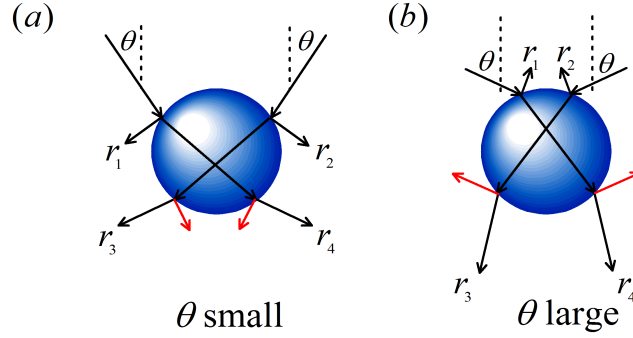


Figure 3.7: Diagram illustrating the effect of the size of the angle of the incoming rays. (a) θ is small, resulting in a weak intensity gradient. (b) θ is large, resulting in a strong intensity gradient as the beam is tightly focused.

A high NA is only beneficial for trapping if it is utilised correctly. The intensity gradient increases as the size of the trapping spot is decreased, assuming the power is kept constant. If the incoming beam diameter is less than the diameter of the back aperture of the objective lens, the trapping width will increase, thereby decreasing the intensity gradient. If however, the beam diameter is larger than that of the objective, not all light will be transmitted, thus despite having a minimum trapping width, the intensity of the beam will still not be optimal. Therefore the incoming beam diameter should be approximately the same as the diameter of the back aperture of the objective lens to obtain a minimum width as well as maximum intensity [26].

The trapping efficiency can be improved by expanding the laser beam so as to slightly overfill the back aperture of the objective lens. This is because the central rays are responsible for the scattering force, while the extremal rays contribute disproportionately to the axial gradient force, therefore the ratio of the gradient force to the scattering force is increased. To prevent heating and thermal expansion of the objective, the beam is expanded such that the $1/e^2$ intensity points correspond with the aperture.

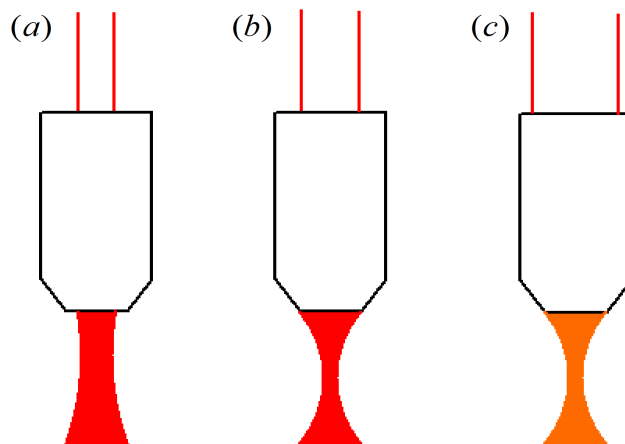


Figure 3.8: Optimising the strength of the trap by making full use of the numerical aperture of the objective. (a) Beam underfills back aperture, resulting in a limited trapping spot, (b) Beam fills back aperture, (c) Beam overfills back aperture, resulting in a loss of intensity & strength of trap.

3.2.3 Imaging System

The imaging setup is based on that of a microscope, where some groups actually incorporate a commercial microscope into their setup [28]. A conventional microscope setup was chosen rather than an inverted design to facilitate the low viscosity immersion oil placed between the objective and the sample. Although many groups use an inverted microscope setup for increased stability, no vibrational problems were observed with the conventional design. To document the results of successful trapping, a CCD camera should be used to record images of the particles. This is achieved by using the same objective lens that focuses the beam onto the sample, to image the sample plane to the camera situated above. To achieve this, a dichroic mirror is used to reflect red laser light and transmit all other wavelengths. Objective lenses have both an effective focal length and a working distance. The latter determines the point at which the sample plane must be located in order to be imaged correctly. There are many components within a compound microscope, which can be reproduced when building a microscope system. Both Nikon and Olympus have published microscope tutorials on their respective instruments [29, 30]. However, in the case of limited lenses, an effective imaging system can be produced by simply considering the image planes.

Most objective lenses are infinity-corrected, which means that light emerg-

ing from the lens is in parallel rays. A tube lens is therefore required to focus these rays to the eyepiece, which then collects the light onto the camera. If however, a finite tube lens objective is used, no tube lens is necessary as the objective focuses the light rays at a certain distance from the back aperture. This is more clearly seen in the following diagram, fig. 3.9:

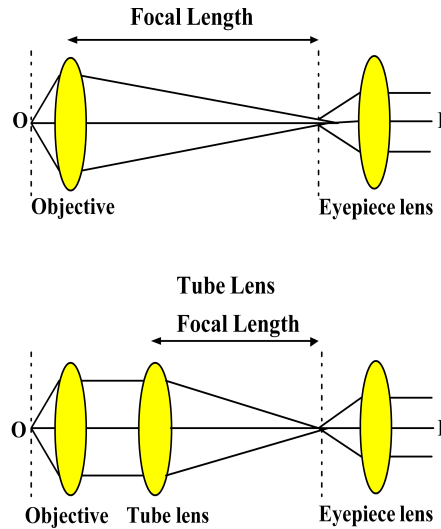


Figure 3.9: Diagram illustrating the imaging requirements for (a) a finite tube lens objective and (b) an infinity-corrected objective lens.

Of course, in order to view the trapped particle, the sample must be illuminated. Koehler illumination is used in most modern microscopes to produce a uniform field of illumination.

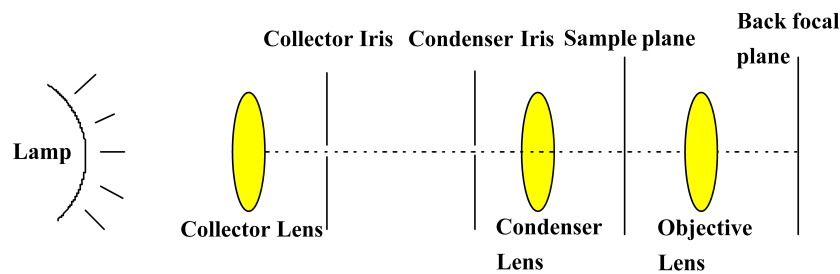


Figure 3.10: Diagram displaying Koehler illumination. The lamp, condenser iris and back focal plane of the objective form conjugate planes. While the collector iris and sample plane form another conjugate pair.

The condenser system should be housed in a tube to prevent the loss of stray light. The sample must be well illuminated as the source light passes through the dichroic mirror as well as a filter before reaching the camera. Alternatively, a commercial condenser can be purchased, or a low magnification objective lens can be used just as effectively and is simpler to use within a setup.

3.3 Experimental Procedure

3.3.1 Experimental Setup

A large portion of this dissertation, consisted of the design, construction and testing of an optical tweezing system. A CrystaLaser diode-pumped solid state laser of 660 nm, 100 mW was chosen purely for ease of alignment. Biological research was not the aim of this dissertation and therefore absorption by the sample was not a limiting factor.

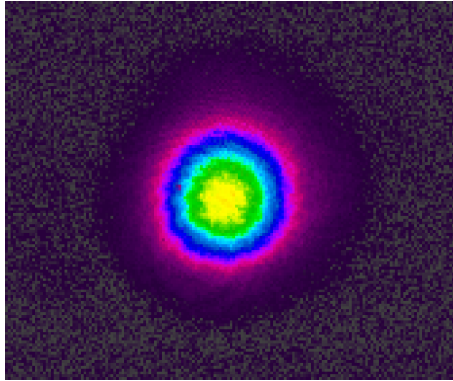


Figure 3.11: Image of the Gaussian beam from the diode laser.

Regardless of the laser, various parameters need to be measured to accurately calculate the optical design of the experiment.

As stated in section 2.1.1, real laser beams do not reflect the ideal Gaussian propagation behaviour. A beam quality factor, M^2 , must multiply the constant value obtained from the product of the beam waist and beam divergence:

$$\omega_0 \theta = \frac{M^2 \lambda}{\pi}. \quad (3.12)$$

A low beam quality value indicates a good beam quality, allowing for better end results.

A simple method to determine the quality of a laser beam makes use of a single positive lens and a camera. Once the laser is correctly aligned, it is focused through a thin positive lens of known focal length. The width of the beam after the lens is then measured at equally spaced intervals from the lens.

If eq. (2.11) is expanded into the familiar quadratic form, a second order polynomial can be fitted to the measured points, thereby providing the plotted equation. If eq. (2.11) is expanded a quadratic expression is obtained

$$\omega^2(z) = \frac{M^4\lambda^2}{\omega_0^2\pi^2}z^2 - \frac{2z_0M^4\lambda^2}{\omega_0^2\pi^2}z + \left(\frac{z_0^2M^4\lambda^2}{\omega_0^2\pi^2} + \omega_0^2\right). \quad (3.13)$$

This expression can then be compared with the fitted equation in order to determine the beam quality, beam waist, the z position of the waist and beam divergence:

$$M^2 = \frac{\pi\omega_0\sqrt{a}}{\lambda} \quad (3.14)$$

$$\omega_0 = \sqrt{c - \frac{b^2}{4a}} \quad (3.15)$$

$$z_0 = \frac{-b}{2a} \quad (3.16)$$

where a , b & c are the values obtained from the fitted polynomial.

The graph in fig. 3.12 shows how the beam width changed as it was focused by a lens of focal length, $f = 400$ mm.

A quadratic fit was plotted through the measured points and the calculated parameters are given in table 3.1.

Table 3.1: Measured beam parameters for the diode laser.

Parameter	Measured	Error
M^2	1.19	0.03
ω_0 (mm)	0.092	0.031
z_0 (mm)	328.27	0.44

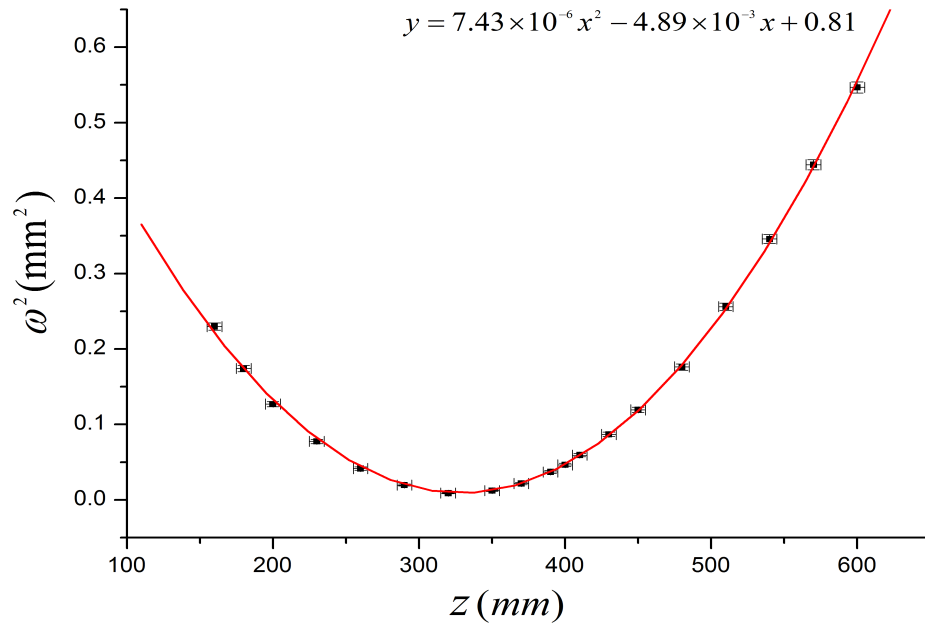


Figure 3.12: Graph of the square of the measured beam widths at various distances from the lens along the z -axis.

These characterisation results demonstrate that the diode laser produces a good beam. The beam quality factor, $M^2 = 1.24$, is considered satisfactory and compares well with that given by the manufacturer of $M^2 < 1.2$ ¹. The knowledge of these parameters indicates that the beam will not vary drastically as it propagates, which is ideal for elaborate systems.

The experimental design of the optical tweezer was based on previous reviews [24, 18, 31], where an intuitive understanding was obtained. However, the actual building took several attempts before a successful and versatile optical tweezer was produced.

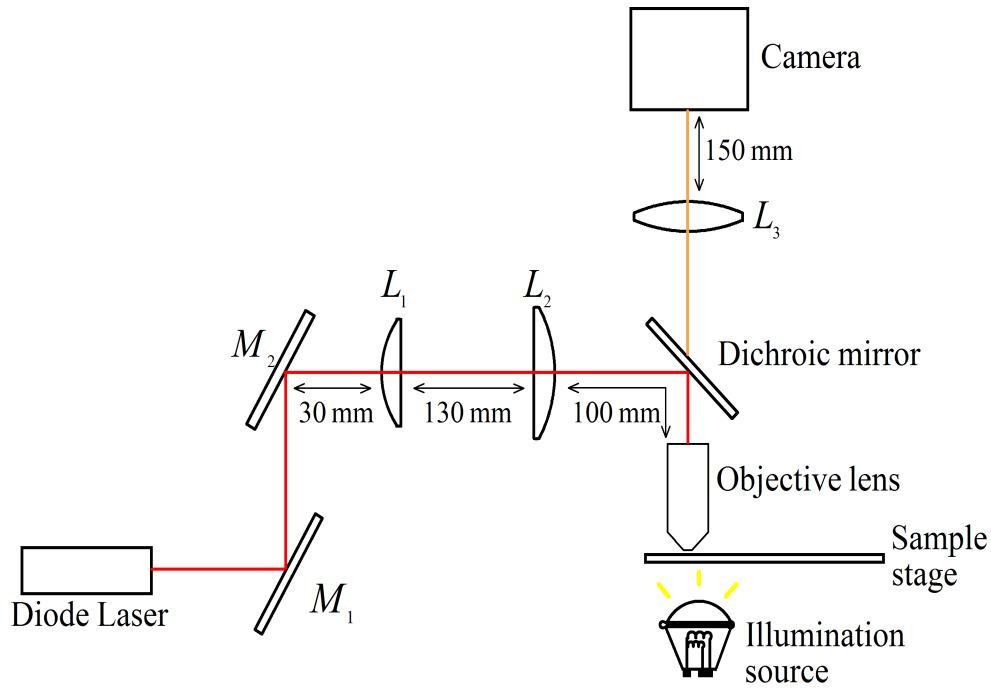


Figure 3.13: Diagram of the experimental setup used to produce an optical trap. L_1 ($f_1 = 30$ mm) and L_2 ($f_2 = 100$ mm) expand the beam before it is reflected off a dichroic mirror onto the back aperture of a 100X objective lens. Mirror M_2 acts as the beam steering mirror. The sample is illuminated from below by a white light LED source and imaged to a CCD camera placed above the system.

The beam was expanded through a $3\times$ telescope before it was reflected off the dichroic mirror onto the back aperture of the $100\times$ objective lens.

¹Specifications supplied by manufacturer on 06/12/2008.

The dichroic mirror transmits light between 455 and 650 nm, allowing the laser light ($\lambda = 660$ nm) to be reflected, while transmitting light from the white light LED illumination source. A $100\times$ LOMO objective (LA-MA-OB-EAB100) was used with a NA of 1.25, oil immersion, with a working distance of 0.17 mm and tube length of 160 mm. A field lens of focal length $f = 150$ mm was placed 160 mm above the objective lens, directing the image onto the camera. A Scorpion IEEE-1394 camera (model SCOR-20SO) was provided with the Spiricon data capturing software. The camera had a resolution of 1600×1200 with a pixel size of $4.4 \mu\text{m} \times 4.4 \mu\text{m}$. Fig. 3.14 shows the final design, which not only allows for tweezing with a Gaussian profile beam, but also several other profiles as mentioned in following chapters.

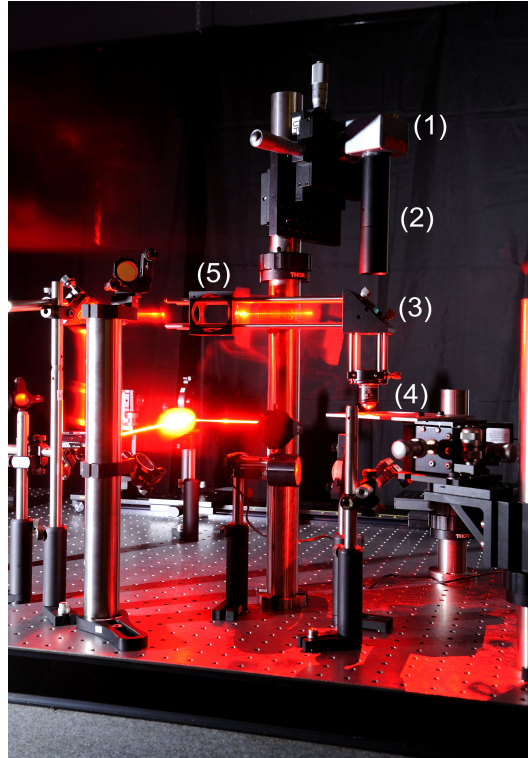


Figure 3.14: Image of the completed home-built optical tweezing setup. The camera is situated at the top of the setup at position (1). Attached to the camera is the collecting lens or eyepiece lens situated in a tube lens holder at (2). The dichroic mirror is housed in a mount at (3) and directs the laser light into the objective lens (100X magnification) at (4) and onto the sample plane. The telescoping system is found at (5).

The beam was reflected off an arrangement of mirrors such that the beam path through the telescope was higher than the height of the diode laser. This allowed enough space for the illumination components to be placed below the sample stage. The height also allowed the setup design to be altered slightly so as to perform trapping with a variety of beam shapes.

A sample of silica beads² with refractive index $n = 1.44$ placed in distilled water, $n_m = 1.33$, was prepared with a concentration of 0.5 mg/ml. The sample was confined to one region on the glass slide via a vinyl spacer. A picture of the spacer on a slide is shown in fig. 3.15.

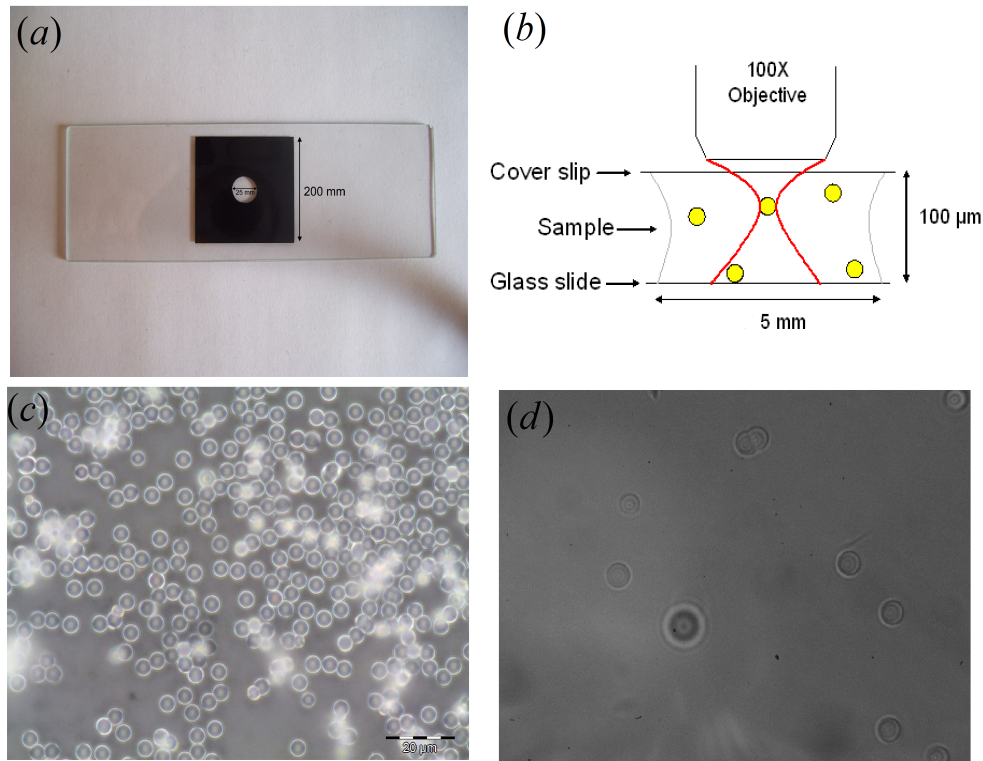


Figure 3.15: (a) Image of a 100 μm thick vinyl spacer stuck on a glass slide, where the diameter of the inner circle is 5 mm. (b) Diagram illustrating the sample depth. (c) Image of silica beads with a mean diameter of 4.32 μm as seen using an Olympus microscope. (d) Image of a trapped silica bead surrounded by untrapped beads as seen from the constructed microscope of the optical tweezers.

²Silica beads purchased from Bangs Laboratory, Inc., with a mean diameter given as 4.32 μm .

As is the case in most optical experiments, the alignment of the laser through the various optical components is crucial in obtaining good results. When aligning the optical tweezer, the microscope was initially built. The illumination light was initially placed below the sample stage to ensure sufficient light was incident on the sample. The objective, collecting lens and camera were then added. The camera was simply adjusted until only the sample was observed on the screen. Finer adjustments were then made to obtain sharp, focused images of the silica beads.

Once the microscope setup was aligned, the laser path through the various optical components was aligned. A breadboard containing pre-drilled holes was used for rough alignment of the components, while fine adjust was made using variable apertures and pinholes. When aligning a laser along a table, it is common practice to measure the height of the beam at the laser source and then accurately set both a variable aperture and pinhole at the same height. The aperture is placed close to the laser source, while the pinhole is placed a distance away. The aim is pass the laser through both the aperture and pinhole, which lie on the same line in both lateral and axial directions. This ensures that the laser path is not tilted in any direction and is parallel with the table. This method was performed not only along the table, but also between mirror M_2 and the dichroic mirror, by placing two pinholes between the two mirrors. The lenses were then inserted, ensuring that the beam directed onto the the centre of the pinhole as before. The final alignment of the dichroic mirror, required replacing the objective lens with a flat mirror, which reflected the beam back along the path to the laser. A correctly aligned path should then redirect the beam back to the laser source, although ideally not into the laser aperture as this can damage the diode. Once this was observed, the setup was ready to perform optical tweezing.

3.3.2 Experimental Results

Tweezing

The actual optical tweezing results are best viewed as videos, as the movement in the axial direction is more clear. However, the results presented here display the key frame shots illustrating successful tweezing of particles in three dimensions.

In fig. 3.16 (a), the circled bead clearly moves out of focus as it is trapped, indicating an axial movement into the focus of the beam, as seen in fig. 3.16 (b).

Once the bead is trapped, it is easily repositioned by simply changing the

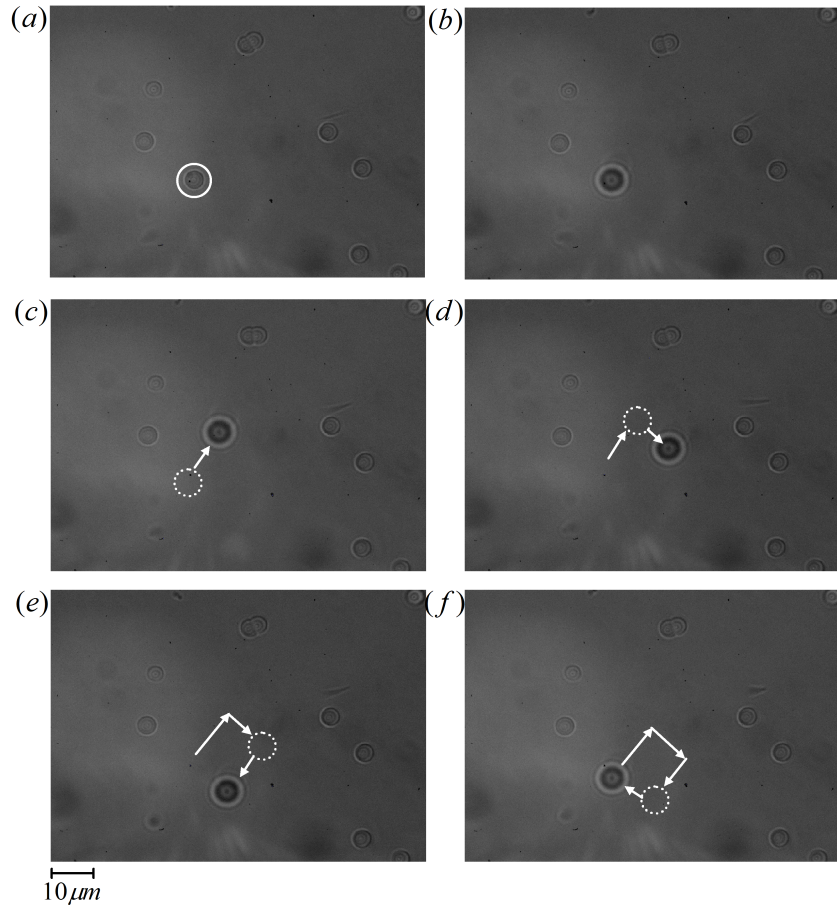


Figure 3.16: Images of tweezing of a $4\mu m$ silica bead. (a) Bead is not yet trapped, (b) Bead is trapped in three dimensions and changes focus, (c)-(f) Bead is manipulated in a diamond-like pattern by moving the laser using the steering mirror.

laser position using the steering mirror, M_2 , this is shown in fig. 3.16 (c) - (f) where the bead moves in a diamond-like pattern.

In fig. 3.16, the trapped bead is moved quickly and precisely within the solution, however, the bead can also be held in one position while the solution is moved by simply maneuvering the stage controls. Fig. 3.17 (a) shows a trapped particle together with several untrapped particles. The stage is moved, causing the untrapped particles to move with the stage. The untrapped particles move further away, toward the right of the images, from the trapped particle, shown in fig. 3.17 (b) - (d). This allows the bead to be relocated to any position within the sample.

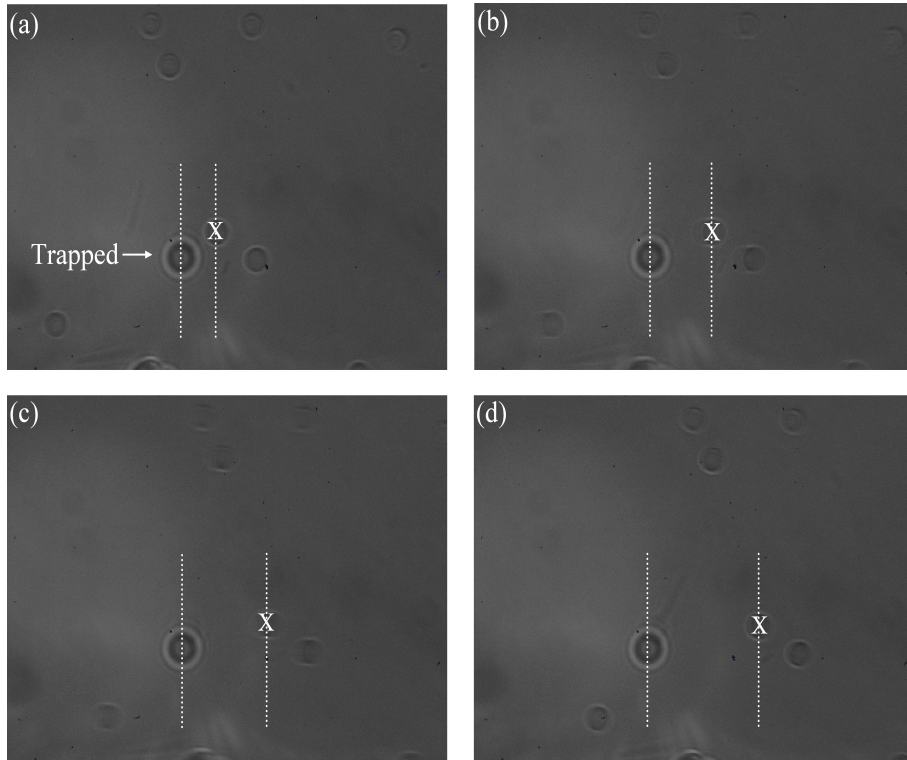


Figure 3.17: The trapped particle is fixed at one position by the laser, while the background particles move towards the right as the stage is moved.

Most of the experimental work dealt with solutions containing approximately $4\text{ }\mu\text{m}$ sized silica beads. They were chosen due to their refractive index, their uniform size and their long expiration period. However, for completeness, another solution was used to prove the optical tweezer was properly designed. As biological samples are often applied within tweezing, a

solution of human embryonic kidney cells of diameters ranging from 20 – 30 μm was used. The cells were grown in Dulbecco's Modified Eagle Medium (DMEM) and 10% Fetal Calf Serum (FCS). In order to lift the cells off the flask, the media were removed, placed in 2-4 ml of Phosphate buffer Saline (PBS) and then aspirated to remove them from the bottom. The cells were then diluted to working concentration of 1 in 10 and finally placed on the slide. A similar pattern of movement to that using the beads is shown in fig. 3.18.

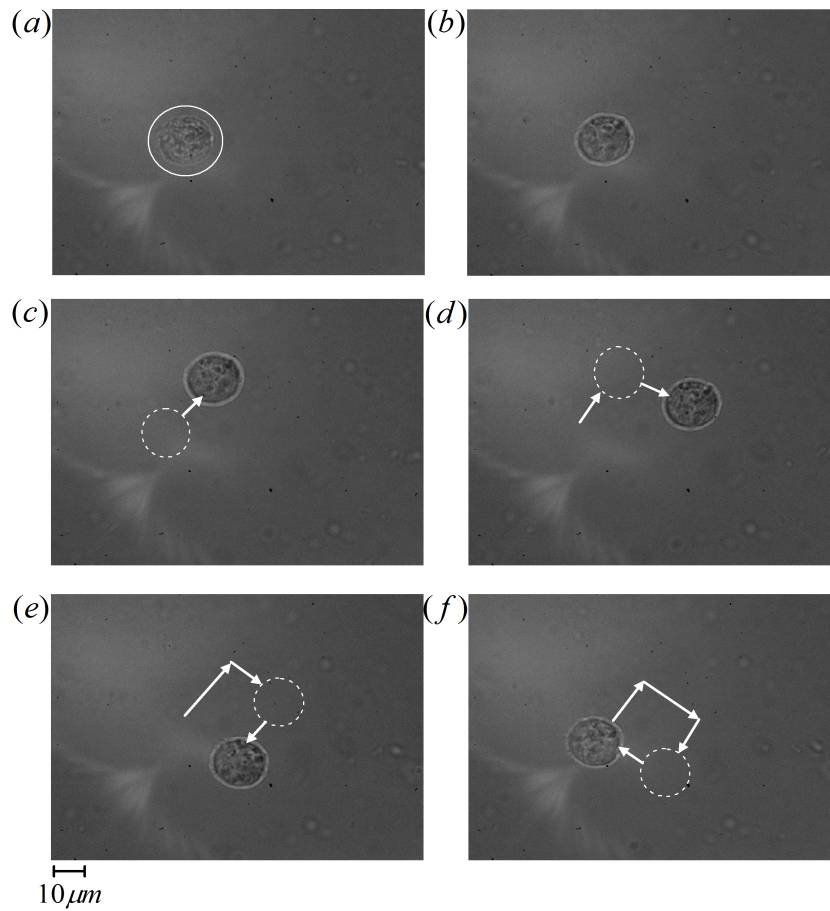


Figure 3.18: Images of tweezing of a 20 μm embryonic kidney cell. (a) Cell is not yet trapped, (b) Cell is trapped in three dimensions and changes focus, (c)-(f) Cell is manipulated in a diamond-like pattern by moving the laser using the steering mirror.

The ease of manipulation of the silica bead was emulated with the cell, despite the larger diameter of the cell. It was apparent that the movement

of the cell into the trap was somewhat slower than that of the silica bead, however this was expected as the surface area of the cell is larger than the bead, resulting in a stronger drag force acting on the cell. Although direct measurements to determine whether the cell was harmed by the laser light were not performed, a dead embryonic kidney cell displays several black spots within the cell. As this was not observed, it can reasonably be assumed that the cell was still alive after being optically tweezed, thereby illustrating the non-invasive feature of optical manipulation.

Calibrating the Trap

All recorded images of the trap were quantified by first calibrating the trapping dimensions. The actual size of the silica beads were measured on a commercial Olympus BX51M microscope system and using the FIVE Digital Imaging Solutions software, were determined to be $4.32\ \mu\text{m}$. This value can then be compared to the measurement of the bead as viewed on the video capturing software, so as to obtain a scaling factor. If the setup remains unchanged, all further measurements made on the camera image can be rescaled to determine the actual sample measurements, otherwise lateral position calibration must be performed after each alteration. The lateral scaling factor can be verified by moving the stage through a specified displacement while recording a fixed, reference bead. The displacement between the initial and final image frames should be proportional to that of the stage actuators.

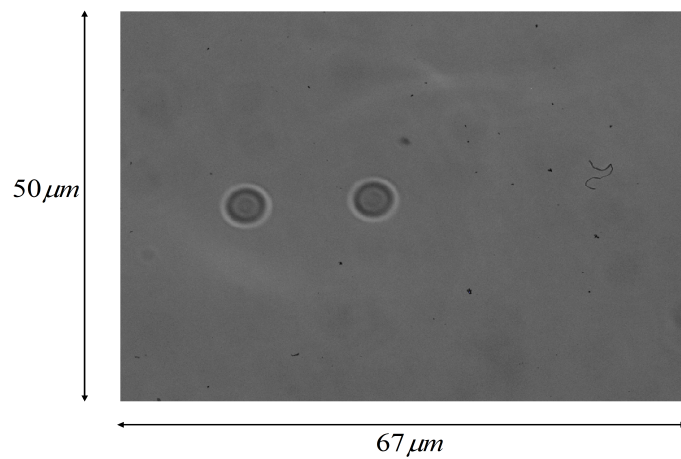


Figure 3.19: Lateral dimensions of the field of view of the trap.

Determining the axial position of the trapped bead is slightly more difficult than the lateral position, but just as important as the drag, β , on the bead varies nonlinearly with its height, h , above the surface of the glass slide. The change in the drag can be estimated by Faxen's law for the approximate drag on a sphere near a surface [25]:

$$\beta = \frac{6\pi\eta r}{[1 - \frac{9}{16}(r/h) + \frac{1}{8}(r/h)^3 - \frac{45}{256}(r/h)^4 - \frac{1}{16}(r/h)^5]}, \quad (3.17)$$

where η is the viscosity of the solution, r is the radius of the particle and v_c is the escape velocity of the particle.

One method of determining the axial position of the trap, which does not require additional detectors albeit somewhat less accurate, determines where the top and bottom surfaces are in relation to focused trapped bead. A bead is trapped and the stage is moved in the axial direction until the bead is released from the trap. The same is performed in the opposite direction. More often than not, it was found that the beads were located near the bottom of the solution. Ideally, the trapping region should lie midway between the two surfaces. A strong three dimensional trap can be recognised by observing the strength of the axial gradient force. That is, a trapped bead will remain in focus as the focus of the beam is changed vertically, while other beads will move out of focus. This was clearly observed, as shown in fig. 3.20.

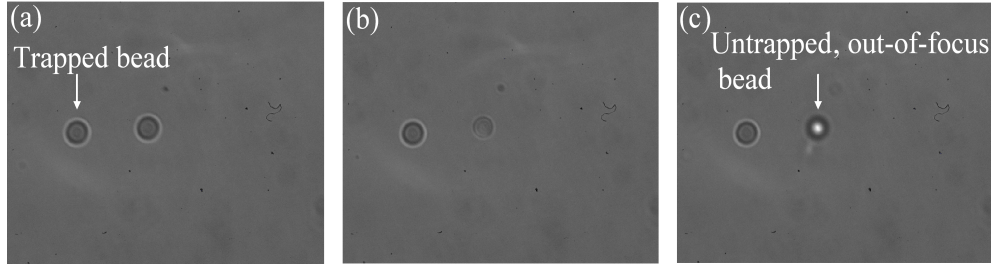


Figure 3.20: Images demonstrating axial trapping. (a) Both the trapped bead (left) and untrapped bead (right) are in focus. (b) The stage is moved downwards causing untrapped bead to change focus, while trapped bead remains in focus. (c) The stage is moved even further in the axial direction and the untrapped bead is now completely out of focus.

Trap Strength

There are several different methods available when determining the strength of the trap. The choice of method depends on availability of equipment and

the degree of accuracy required. The follow two methods were chosen based on equipment.

1. Drag Force Method

The first method takes into account that samples are predominantly viewed within a solution, and therefore any movement of a particle within the solution will experience a drag force in the opposite direction of motion. The drag force, a force equivalent to friction, of a solution is given by:

$$F_{Drag} = 6\pi\eta R\nu, \quad (3.18)$$

where η is the viscosity of the fluid, R is the radius of the particle and ν is the velocity of the fluid. This equation can be used to determine the trapping force by calculating the drag force which causes the particle to be released from the trap. That is, the velocity at which the particle escapes the ‘hold’ of the trap determines the drag force, which in turn determines the trapping force:

$$F_{trap} = 6\pi\eta R\nu_c, \quad (3.19)$$

where ν_c is the velocity of the fluid at which the particle escapes the trapping force.

Experimentally, once a bead was trapped, the stage was moved with increasing speed and the displacement of a reference bead, one that was stuck to the glass slide, was monitored. The displacement of the reference bead just as the trapped bead had escaped was then determined and by using the time between each frame, the velocity calculated.

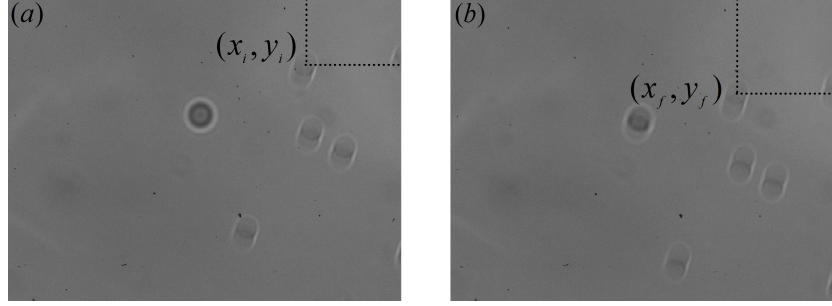


Figure 3.21: Images of a trapped silica bead together with untrapped reference beads, at power $P = 25.4$ mW. The trapped bead is held fixed, while the untrapped beads in are moved with the medium by moving the stage. (a) The initial position of a reference bead is determined just before the trapped bead escapes the trap. (b) The final position of the same reference bead is determined just as the trapped bead escapes from the trap, allowing the displacement of the reference bead to be calculated.

The frame rate of the camera was set at 15 frames/second, therefore the time difference between consecutive frames was approximately, $\Delta t \approx 66.7$ ms. The velocity of the fluid as the bead escaped was calculated using,

$$v_c = \frac{\Delta x}{\Delta t}. \quad (3.20)$$

From eq. (3.20), the escape velocity was calculated to be $v_c = 35.89$ $\mu\text{m/s}$, resulting in an escape force of $F_{trap} = 1.46$ pN.

This was performed at a number of different incident powers, where neutral density filters were used to reduce the power of the beam³.

³The actual power delivered to the trap was difficult to measure due to the fast divergence of the beam. Therefore relative power measurements were made here, by measuring the power before entering the objective.

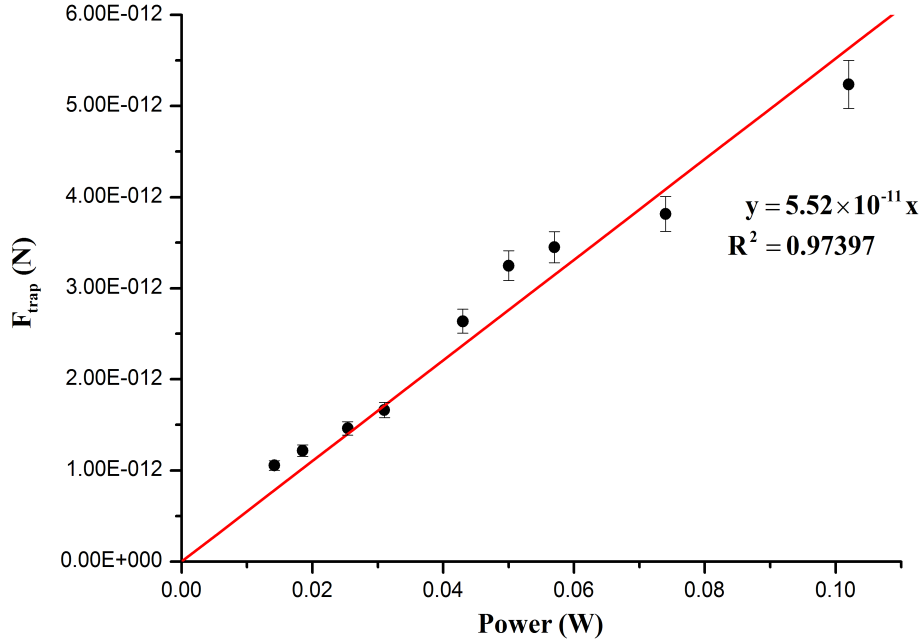


Figure 3.22: Graph illustrating the linear relationship of power versus the trapping force on the bead.

The maximum force acting on the bead for this particular method was calculated to be 5.24 ± 0.26 pN at a power of $P = 100$ mW. Although this is an acceptable value an optical tweezer, there is a great deal of uncertainty in this method. The stage was moved by hand, which lacks consistency in the acceleration of the stage at different power settings. More accurate and efficient results can be obtained using a regular triangular or sinusoidal pattern generated by a piezoelectric stage. A summary of the method can be found in the review by [24]. The objective used was also a fairly inexpensive make, where the transmittance of objectives is low even for high quality lenses, and so the trapping power may be significantly lower than the initial 100 mW. Measuring the transmittance of objective lenses is not easily achieved, and requires two identical objectives. A table of transmittance values of a number of lenses is given in a paper by [27].

2. Equipartition Method

The second method makes use of the equipartition theorem to infer the strength of the trap. The energy equipartition theorem states that the mean value of every independent quadratic term in the energy is equal to $\frac{1}{2}k_B T$, where k_B is Boltzmann's constant and T is the equilibrium temperature. A particle held at some distance away from the focus of trapping laser, experiences an attractive force towards the focus. This restoring force is proportional to the distance travelled by the bead, that is, the trapping force of an optical tweezer can be thought of as a harmonic oscillator, with Hooke's law:

$$\mathbf{F} = -\alpha \mathbf{x}, \quad (3.21)$$

where F is the applied force, α is a constant referred to as the trap stiffness and $\mathbf{x}$ is the displacement. The motion of a trapped particle of mass m , in a viscous medium at temperature T can be described by the theory of Brownian motion, which leads to a Langevin equation of the form [31],

$$m \frac{\partial^2 x}{\partial t^2} + \beta \frac{\partial x}{\partial t} + \alpha x = (2k_B T \beta)^{1/2} \eta(t). \quad (3.22)$$

The last term, $(2k_B T \beta)^{1/2} \eta(t)$, describes the random noise process at a temperature T and $\eta(t)$ represents the stochastic process of motion of the particle as a function of time. As this term is a result of thermal fluctuations, there is a characteristic time scale that must be considered [32]. If observations are made on scales much longer than the characteristic time, the measurements can be considered independent, and the last term averages out to zero, which is the case for these experiments.

For an undamped system, (ie. a vacuum), an oscillator would have a resonant frequency of:

$$f_{res} = \frac{1}{2\pi} \left(\frac{\alpha}{m} \right)^{1/2}. \quad (3.23)$$

For a realistic biological sample, this would result in a frequency of $f_{res} = 50$ kHz. However, most biological experiments are performed within an aqueous solution, resulting in a drag constant $\beta = 6\pi R\eta$. This damping term together with the stiffness of the trap, results in an over-damped system. Thus, there is a contribution of thermal energy to the system from the damping term, given by the product, $k_B T$. The bombardment of molecules of the surrounding medium produces a fluctuating thermal force, resulting in a variance in position $\langle x^2 \rangle$, which can be calculated from the equipartition of energy,

$$\frac{1}{2} \alpha \langle x^2 \rangle = \frac{1}{2} k_B T. \quad (3.24)$$

Using the Spiricon software, the centroid position of a trapped bead was recorded for 100 frames. The variance of the bead from its equilibrium position was then found and used in conjunction with eq. (3.24).

As with the previous method, the power of the laser was varied using neutral density filters and the variance of the bead was calculated at each power. The graphs in figs. 3.24 and 3.23 show the linear relationship between the trapping constant and the applied power.

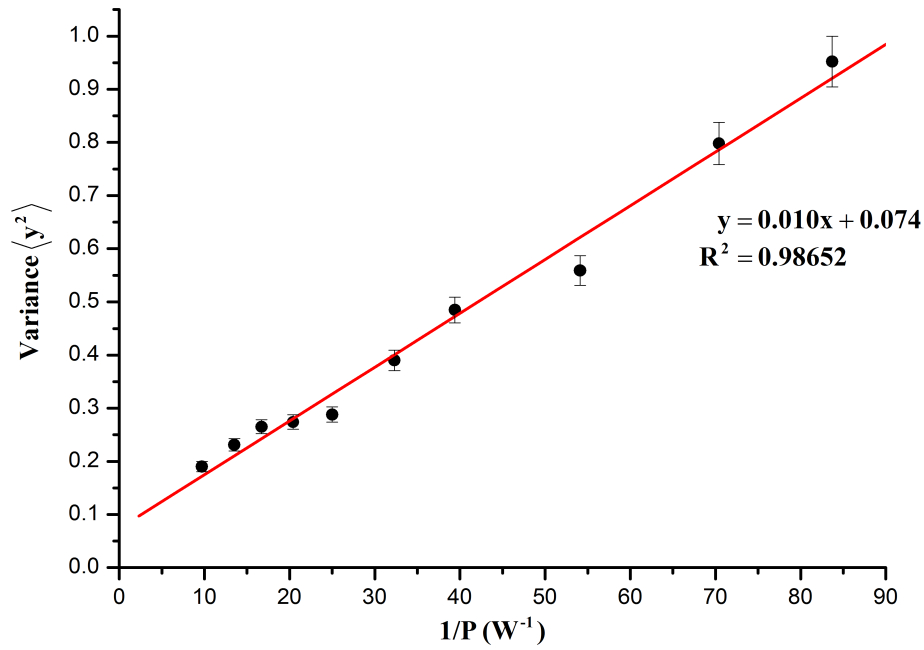


Figure 3.23: Graph illustrating the inverse relationship of power versus the variance of the position of a trapped bead.

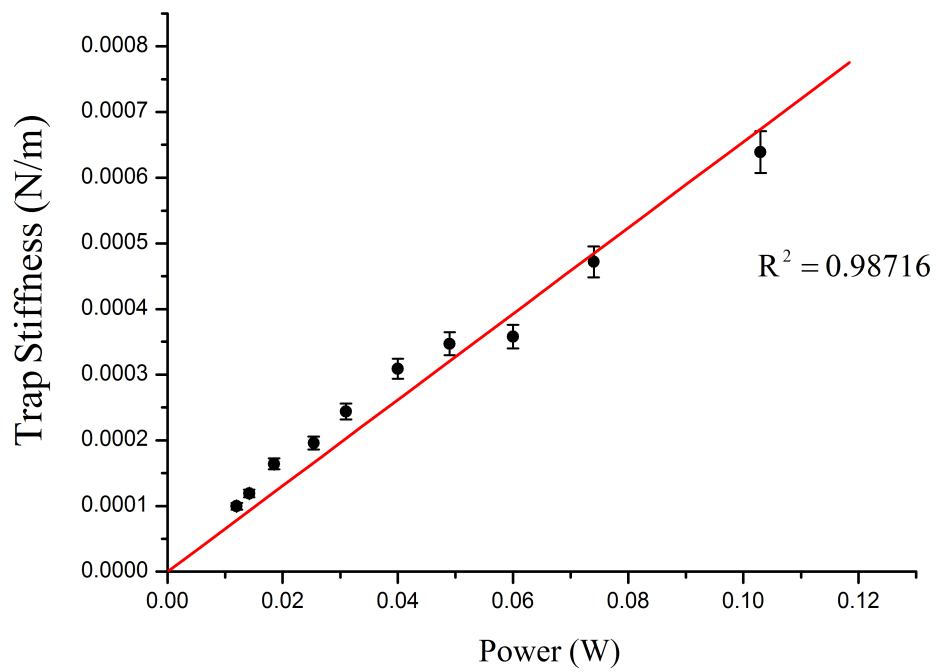


Figure 3.24: Graph illustrating the linear relationship of power versus the spring constant of the trap.

For the lowest power, a value of $9.9 \times 10^{-5} \pm 5 \times 10^{-6}$ N/m for the trap stiffness. The results obtained are in good comparison with theory, as well as results obtained in an explanatory paper by [32].

The equipartition theorem further illustrates that the complete distribution, not just the variance, of a particles position is given by Boltzmann's distribution,

$$P(x) \propto \exp\left(\frac{-U(x)}{k_B T}\right) = \exp\left(\frac{-\alpha x^2}{2k_B T}\right), \quad (3.25)$$

where $U(x)$ is the potential energy and $k_B T$ is the thermal energy. The gradient force can be shown to be a conservative force [22]. Therefore, as potential energy is the energy resulting from an object's position when it is acted on by a conservative force, a potential energy distribution is expected. Eq. (3.25) demonstrates the parabolic nature of the potential energy, which corresponds to the density plot of the gradient force.

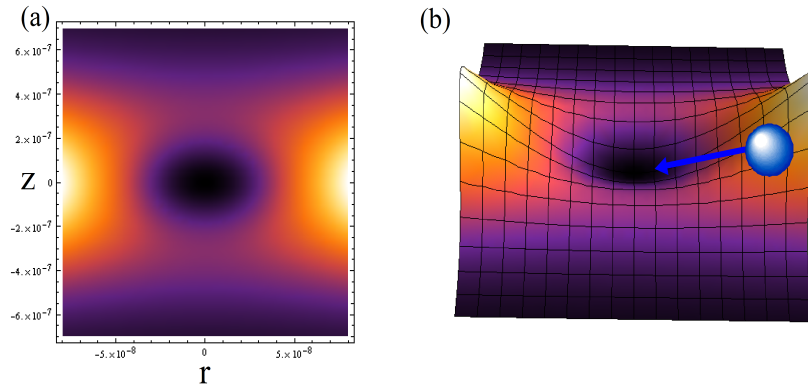


Figure 3.25: (a) Density plot of the magnitude of the gradient force. (b) A particle placed in the conservative force field has potential energy, which is a minimum at the the beam focus.

Using the previous data, the distribution of bead positions can be plotted with a fitted curve using eq. (3.25). The fitted curve allows the trap stiffness to be determined, where $k_B = 1.38 \times 10^{-23}$ m²kgs⁻²K⁻¹ and $T = 300$ K. In figs. 3.26 and 3.27, each plot point represents the probability of finding a range of bead positions about this point, where the range was chosen in such a way as to produce a meaningful graph. That is, if a large range, $0.2 \mu\text{m}$, was chosen from the equilibrium position, only one plot point would be

displayed. The error bars in figs. 3.26 and 3.27 illustrate the range chosen and not the spatial resolution, which is on the order of 44 nm.

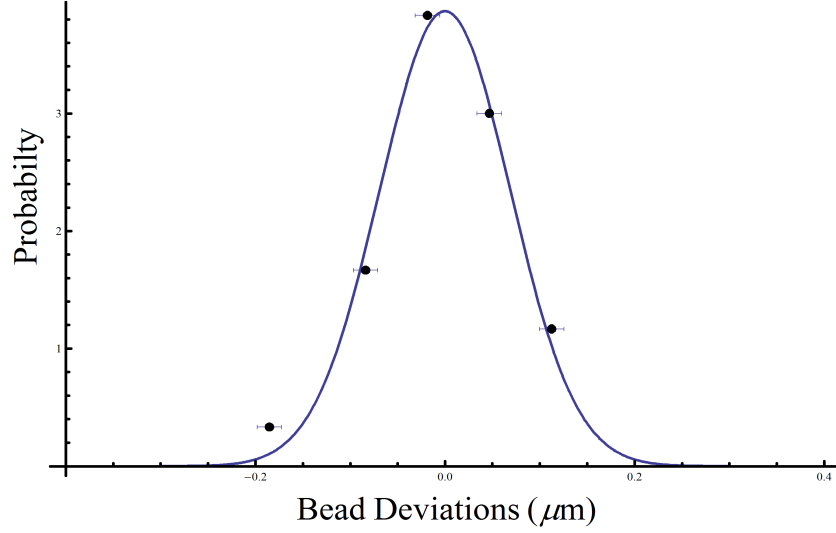


Figure 3.26: Graph representing the distribution of bead positions. A best-fit graph is fitted to the measured data points using eq. (3.24).

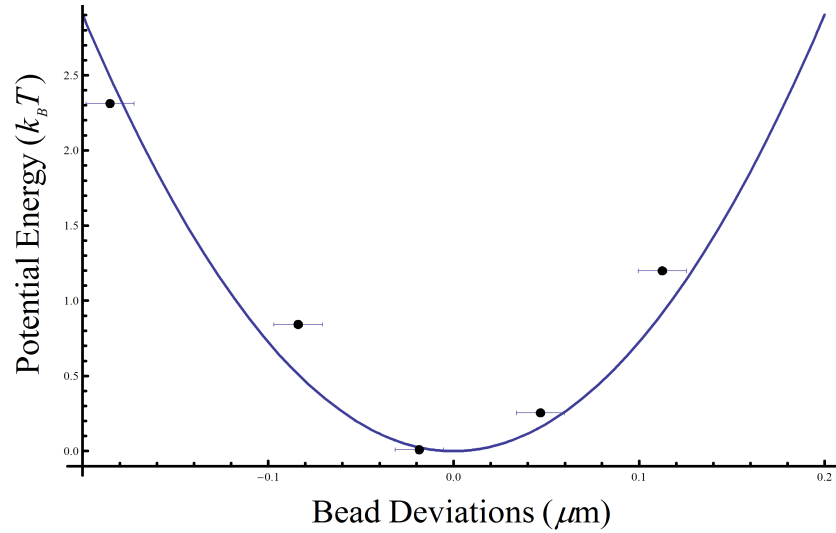


Figure 3.27: Graph representing the potential energy calculated from the distribution of bead positions. A best-fit graph is fitted to the measured data points using eq. (3.25).

The trap stiffness at the lowest measured power was determined to be $\alpha = 8.71 \times 10^{-5} \text{ N/m}$, which corresponds quite well with the value calculated at the same power using variance of $\alpha = 9.9 \times 10^{-5} \text{ N/m}$. The potential energy curve was obtained from the position distribution and clearly shows a parabolic shape. A trapped particle will therefore remain trapped at its equilibrium position unless it gains enough energy to escape the potential well.

3.4 Trapping with a Bessel Beam

Unlike the previous experimental design where the beam was focused onto the sample from above, with a Bessel-Gauss beam trap, the beam is directed onto the sample from below, as shown in fig. 3.28.

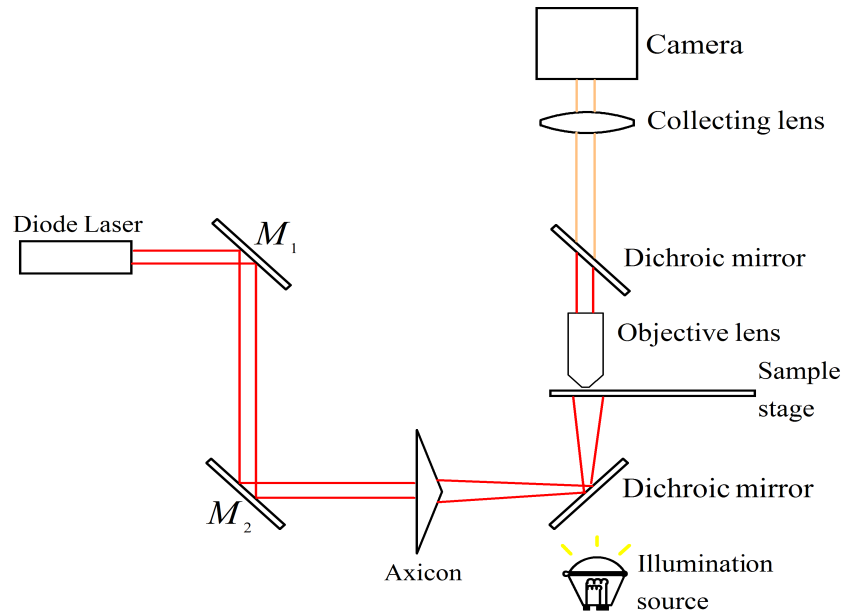


Figure 3.28: Experimental setup used in generating a Bessel beam trap.

Since the beam is not tightly focused through an objective, strong gradient forces are not present to counter the radiation pressure. Therefore, by directing the beam upwards, the radiation force acts against gravity in a ‘levitation’ trap, while the transverse gradient forces continue to attract the particle to the most intense region of the beam. This means that three dimensional tweezing is not possible with a Bessel beam. Nonetheless, the

principles are the same, that is, the particle is attracted to the most intense region of the beam. The strength of the Bessel trap is somewhat weaker than that of a Gaussian beam. The strength of the gradient force for the two beams is shown below.

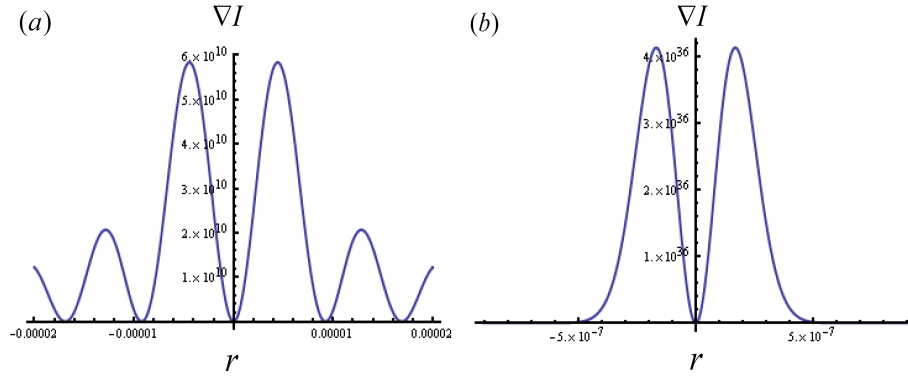


Figure 3.29: Comparison of the intensity gradients for (a) Bessel beams and (b) Gaussian beams. There is an order of 26 difference between the maximum intensity gradient values.

Particles can therefore still be manipulated using only light, and due to the non-diffracting central maximum of the Bessel beam, particles can be trapped at any point along the column. This is an additional feature of trapping that is not possible with Gaussian beams as the particle is trapped only in the beam focus [14].

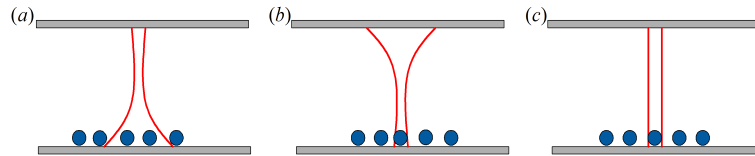


Figure 3.30: (a) and (b) Gaussian beams diverge quickly after focusing, resulting in poor localization. (c) Bessel beams have a non-diffracting central maximum, allowing point-to-point guiding.

The diagram below shows how Bessel beams can be used to trap multiple particles using one beam [33].

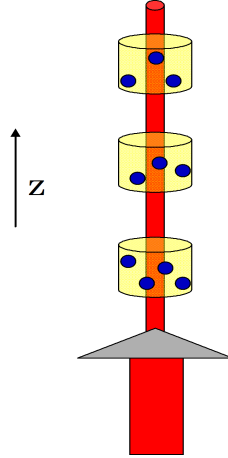


Figure 3.31: The central maximum allows particles to be trapped at any point along the column. Together with the reconstructive property, Bessel beam traps allow more than one particle to be trapped at a time.

3.4.1 Experimental Procedure

Experimental Setup

Producing a Bessel beam using an axicon requires some practice as the incoming Gaussian beam must pass through the centre of the axicon. A Gaussian beam of width $\omega = 1$ mm was expanded through a $5\times$ telescope before passing through an axicon of opening angle $\gamma = 5^\circ$. A Bessel beam of propagation distance $z_{max} \approx 115$ mm was magnified $27\times$ onto a camera using an imaging lens of focal length $f = 30$ mm. An Annular beam was also generated by simply moving the camera to the focus of the lens behind the axicon.

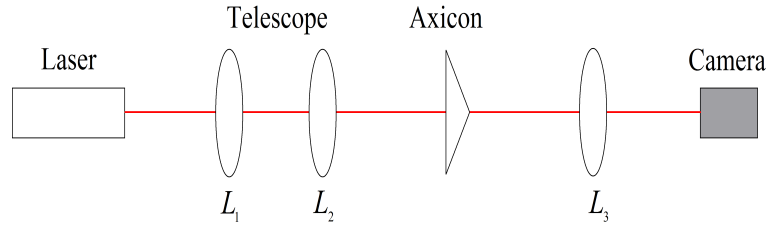


Figure 3.32: Diagram of the experimental setup to produce both a Bessel and Annular beam. Lenses L_1 ($f_1 = 50$ mm) and L_2 ($f_2 = 250$ mm) form a $5\times$ telescope. L_3 ($f_3 = 30$ mm) was used as an imaging lens when capturing the Bessel beam. The camera was moved to the focus of L_3 to capture an image of the Annular beam.

The images obtained from the setup in fig. 3.32 are shown in fig. 3.33 with the theoretical predictions as comparisons.

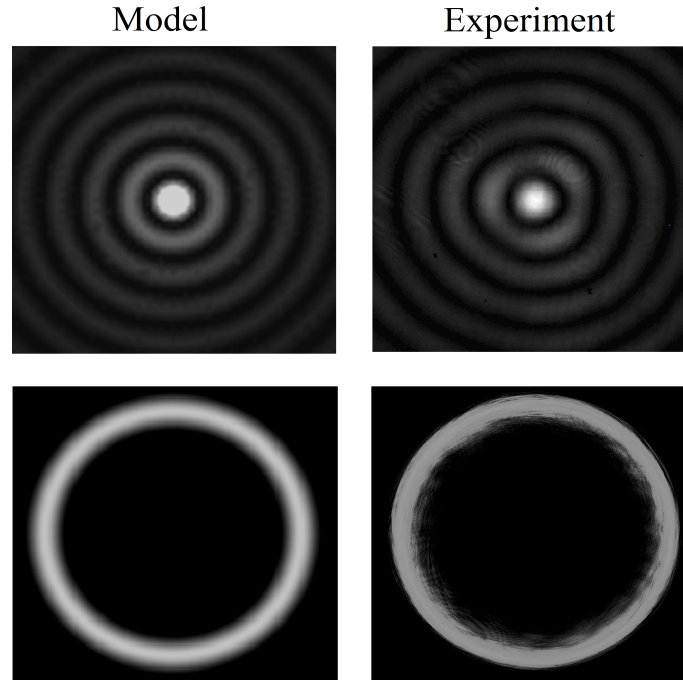


Figure 3.33: Comparison of the predicted Bessel and Annular beams, plotted using Mathematica 6, with the experimentally generated images. The Bessel beam was magnified by imaging onto a camera, whereas the Annular beam was produced at the focus of a lens placed within the Bessel field.

A similar setup was used when trapping with a Bessel beam. A Gaussian beam of width $\omega = 1$ mm was directed through an axicon of opening angle $\gamma = 5^\circ$ to produce a Bessel beam with a propagation distance of $z_{max} \approx 23$ mm.

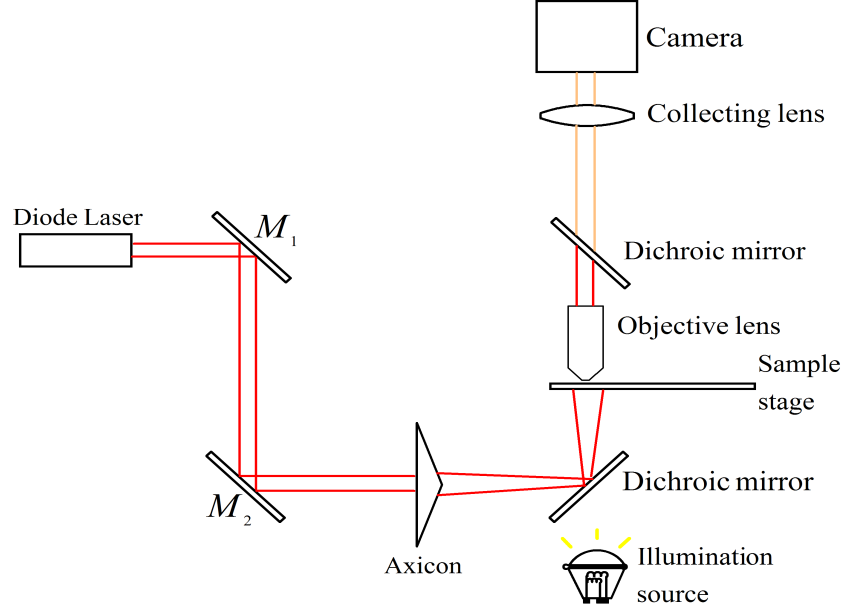


Figure 3.34: Experimental setup used in generating a Bessel beam trap. A Gaussian beam of width $\omega = 1$ mm was directed through an axicon of opening angle $\gamma = 5^\circ$ to produce a Bessel beam with a propagation distance of $z_{max} \approx 23$ mm and central spot size $r \approx 5.79 \mu\text{m}$.

The central maximum spot size was approximately $5.79 \mu\text{m}$, using the formula:

$$r \approx \frac{2.405}{k_r}. \quad (3.26)$$

A second dichroic mirror was placed below the sample, allowing the laser to be reflected upwards onto the sample, while still transmitted white light for illumination. The initial dichroic mirror served no purpose within the Bessel beam setup, however, it allowed for an easy change between Gaussian and Bessel beam trapping. The axicon was placed at a distance of $(1/2)z_{max} \approx 12.5$ mm from the sample, where the number of rings were at a maximum. The objective was used purely for imaging purposes, magnifying the image onto the camera as described before.

Experimental Results

The following results demonstrate successful trapping along the Bessel beam central maximum column. A silica bead was trapped at some z - position and manipulated in a lateral manner. The stage was then moved vertically downwards, where a bead at a different z - position, indicated by it's out-of-focus image, was trapped and manipulated. This was repeated for a third position. The images below show three different z - positions along the Bessel column, where the bead was manipulated each time.

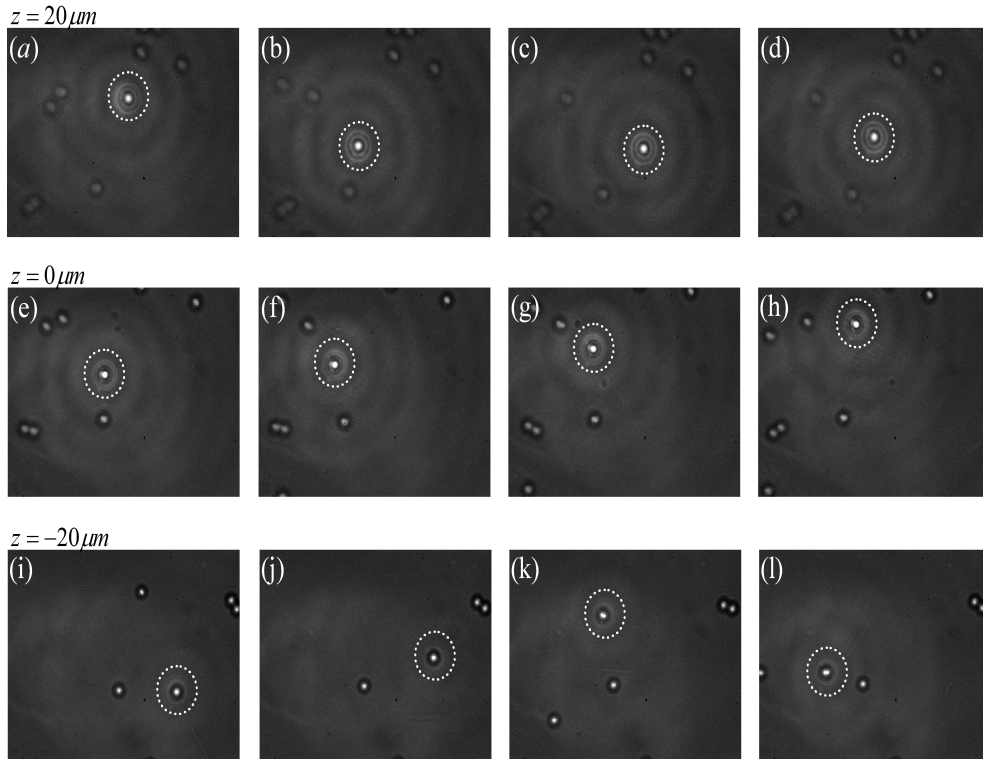


Figure 3.35: Images of 4 micron silica beads, manipulated by a Bessel beam trap at various points along the central column. (a)-(d) Bead is trapped at a certain point within the Bessel column, (e)-(h) The bead is moved further down the column and again trapped and manipulated, (i)-(l) The same bead is moved even further down the column and trapping is again achieved.

The propagation distance, $z_{max} \approx 23$ mm, indicates that the Bessel beam column will propagate throughout the sample as the vinyl spacer is only $80 \mu\text{m}$ thick. Reconstruction after a silica bead was not clearly seen, despite $z_{min} = 50 \mu\text{m}$, which is within the thickness of the vinyl spacer.

This may be attributed to the lack of flow within the sample, suggesting that many beads had settled on the glass slide and were unaffected by the forces of the Bessel trap. Nevertheless, the demonstration of trapping along the column, clearly shows the non-diffracting property of Bessel beams.

3.5 Applications

The continuous development of optical tweezers over the past 30 years has resulted in immense interest within a number of different research fields. In particular, the non-invasive quality of tweezers has attracted those dealing with biological materials. The ability to manipulate biological cells in a controlled manner without harming the cell in any way, offers biologists further insight into how various processes work. The pico-Newton forces of optical tweezers also allow the mechanical properties of molecular motors and biopolymers to be measured [34].

Biological research also makes use of trapped beads by attaching a biological structure to the trapped bead, using the bead as a handle. One such display by [35], used optical tweezers to tie a knot in a DNA strand in the aim of studying the curvature-dependent interactions with associated proteins.

Optical tweezers have also found use in stimulating neuronal growth [36, 37]. The gradient forces produced are not used to hold and move the neuron, but instead to induce a natural biological process.

Another field of research that is gaining a great deal of attention, is microfluidics. A microfluidic system often refers to a channel system where at least one dimension is on the order of micrometers. These systems are sometimes replica's of macroscale devices and therefore microfluidics is regularly termed as a 'lab-on-a-chip'. When working on such a small scale, certain properties alter making some experiments easier to perform than at macroscopic scales. That is, at the microscopic scale, the surface-to-volume ratio increases considerably causing the flows within the system to be almost laminar [38]. Therefore two different solutions can flow next to one another without mixing for as long as a few minutes. Optical tweezing has been used to trap a particle in one flowing solution and move it across to the neighbouring solution for the purpose of examining the effects of the various solutions on the particle [39]. The technique is efficient and reversible, allowing particles to be moved from one solution to another and finally returning to the original solution with ease. Chapter 4 discusses further how optical tweezers and beam shaping can play an important role

in microfluidics.

Optical tweezing is predominantly used in liquid solutions, however, it has also been exploited in the study of aerosols. The first demonstration of optically trapping single water droplet was performed by [40]. Droplets ranging from $4 - 14 \mu\text{m}$ were trapped for timescales of hours, which provided the means to examine the characteristics and evolution of each individual droplet. Since then, a variety of aerosol trapping experiments have been performed including characterisation of different liquid droplets [41], as well as solid aerosol trapping [42].

Interesting applications have also been found using Bessel beam trapping. [43] made use of the decreasing intensity profile to separate different sized particles within a solutions. The high intensity central maximum of Bessel beams, attracts larger particles to the centre of the beam, while the smaller particles are trapped by the outer, less intense rings. This provides a sorting mechanism, where the particles trapped in the centre column can be removed from the solution using microcapillaries. This technique was successfully performed on blood cells, where the larger lymphocytes (white blood cells) were collected in the centre while the smaller erythrocytes (red blood cells) remained on the outer rings.

Chapter 4

Rotational Tweezers

The key mechanism in optical tweezing, is the transfer of momentum. Thus far, only the transfer of linear momentum has been discussed. However, light also carries angular momentum, which can also be exploited within an optical trap to expand the application of tweezers. The angular momentum, $\mathbf{L}$, of a particle about a given origin is defined as,

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}, \quad (4.1)$$

where $\mathbf{r}$ is the position vector of the particle relative to the origin and $\mathbf{p}$ is the linear momentum of the particle. As in chapter 3, the force, $\mathbf{F}$, on an object is equal to the rate of change of its momentum, that is;

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}. \quad (4.2)$$

The total force can be written in terms of the Poynting vector, $\mathbf{S}$, defined as the energy per unit time, per unit area transported by the electromagnetic fields and given by,

$$\mathbf{S} \equiv \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B}), \quad (4.3)$$

where μ_0 is the permeability of free space and $\mathbf{E}$ & $\mathbf{B}$ are the electric and magnetic fields, respectively. Using eq.s (4.2) and (4.3) together with Maxwell's stress tensor, $\vec{\mathbf{T}}$, a general statement¹ for the conservation of momentum can be written as [44]:

$$\frac{d\mathbf{p}}{dt} = -\varepsilon_0\mu_0 \frac{d}{dt} \int_V \mathbf{S} d\tau + \oint_S \vec{\mathbf{T}} \cdot d\mathbf{a}, \quad (4.4)$$

¹A complete derivation is given by [44] pp. 351 – 353.

where ε_0 is the permittivity and ν is the volume. The first integral denotes the momentum stored in the electromagnetic field, while the second integral represents the momentum per unit time flowing in through the surface. By considering the first integral, the momentum density, $\vec{\rho}$, in the field is then,

$$\vec{\rho}_{em} = \mu_0 \varepsilon_0 \mathbf{S} = \varepsilon_0 (\mathbf{E} \times \mathbf{B}). \quad (4.5)$$

Using a more specific form of eq. (4.1), $\mathbf{j} = \mathbf{r} \times \vec{\rho}_{em}$, a definition for the angular momentum density, $\mathbf{j}$, associated with the transverse electromagnetic field is given as,

$$\mathbf{j} = \varepsilon_0 \mathbf{r} \times (\mathbf{E} \times \mathbf{B}). \quad (4.6)$$

Following [45], integrating this density over all space gives the total angular momentum, $\mathbf{J}$, of the field as,

$$\mathbf{J} = \varepsilon_0 \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) d\mathbf{r}. \quad (4.7)$$

This expression can be further expanded by considering the complex notation of two fields:

$$\mathbf{E}_{\text{tot}} = (\mathbf{E} \exp^{-i\omega t} + \mathbf{E}^* \exp^{i\omega t})/2, \quad (4.8)$$

$$\mathbf{B}_{\text{tot}} = (\mathbf{B} \exp^{-i\omega t} + \mathbf{B}^* \exp^{i\omega t})/2 \quad (4.9)$$

where $\mathbf{E}^*$ & $\mathbf{B}^*$ denote the complex conjugates of the electric and magnetic fields, respectively. Using the Maxwell equation, $\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$, it follows that,

$$\nabla \times \mathbf{E} = i\omega \mathbf{B}. \quad (4.10)$$

and

$$\mathbf{J} = \frac{\varepsilon_0}{2i\omega} \int \mathbf{r} \times [\mathbf{E}^* \times (\nabla \times \mathbf{E})] d\mathbf{r}. \quad (4.11)$$

An expanded expression is obtained by partial integration and using the transversality of $\mathbf{E}$,

$$\mathbf{J} = \frac{\varepsilon_0}{2i\omega} \int d^3r \sum_{j=x,y,z} E_j^* (\mathbf{r} \times \nabla) E_j + \frac{\varepsilon_0}{2i\omega} \int d^3r \mathbf{E}^* \times \mathbf{E}. \quad (4.12)$$

Within the paraxial approximation, the two terms in eq. (4.12) can be associated with the orbital and spin components of the total angular momentum.

However, [45] derived an expression for angular momentum for nonparaxial beams, which consists of an additional mixed term, of both spin and orbital contributions. The nonparaxial case applies to tightly focused beams and is discussed in section 4.2, however in general, angular momentum can be considered as a sum of the two components,

$$\mathbf{J} = \mathbf{l} + \mathbf{s}. \quad (4.13)$$

An analogy to illustrate the difference between the two can be made using planetary motion. Spin angular momentum can be compared with the rotation of the Earth about its axis, whereas the movement of the Earth around the Sun can be related to orbital angular momentum. Both components contribute to the rotation of a particle, however their contributions differ greatly, and are therefore considered separately.

Detecting angular momentum has been made possible with optical tweezers and has been presented by several groups [46, 47, 48]. The following chapter serves as a demonstration of the techniques used, which can then be applied in future work.

4.1 Spin Angular Momentum

The angular momentum of light associated with its polarization state is referred to as spin angular momentum. 1936 saw the first experimental results illustrating the transfer of spin angular momentum observed by [49]. He measured a torque on a birefringent half-wave plate as circularly polarized light passed through it, which was first inferred by [50]. Birefringent materials can act as wave-plates, therefore as light passes through such a material, its polarization changes. Consequently, there is a change in its spin angular momentum, resulting in a torque on the material, which causes it to rotate. The experiment involved the suspension of the plate by a fine quartz fiber located within a cylindrical vacuum copper chamber, 3 inches thick.

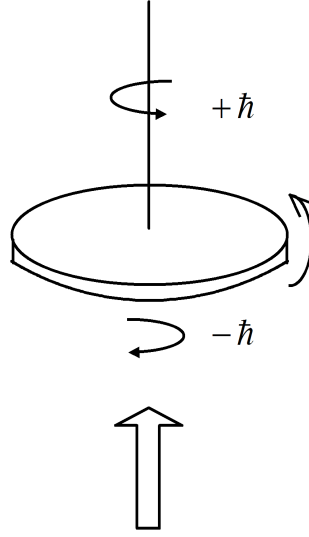


Figure 4.1: Beth's experimental design to detect angular momentum. Circularly polarized light is incident on a suspended $\lambda/2$ birefringent plate, transforming right-handed into left-handed circularly polarized light. The change in polarization results in a torque exerted on the plate.

A birefringent particle has refractive indices n_o and n_e for the propagation of the ordinary and extraordinary rays, respectively. The phase shift of a ray passing through a material of thickness d and refractive index n is $\Delta\varphi = kdn$. Light carries angular momentum of $\sigma\hbar$ per photon, where σ represents the polarization of light, that is $\sigma = 0$ for linearly polarized light and $\sigma = \pm 1$ for circularly polarized light. The value for circularly polarized light depends on the handedness of polarization, either left or right. As light passes through a birefringent half-wave plate, the handedness of polarization, and as a result, angular momentum is reversed. Each photon then transfers $2\hbar$ of angular momentum to the plate thereby exerting a torque on the plate.

The results obtained by [49] were significant in understanding the transfer of spin angular momentum, however an easier and more effective method of measurement is now possible with optical tweezers. A tutorial paper by [51] gives a follow-up of Beth's experiment. Instead of a birefringent plate, calcite particles are used as they possess the birefringent property required for rotation. [47], demonstrated and explained how transparent, non-absorbing particles could also rotate when exposed to circularly polarized light. As light passes through the particles, the ordinary and extraordinary components of the incoming light experience different phase shifts, resulting in a change in the angular momentum of the light. By comparing the angular momentum of

the beam before entering and after exiting the trapped particle, the amount of spin angular momentum transferred to the particle can be calculated. The change in angular momentum generates a torque exerted on the particle, which has been measured by a number of groups [48, 47]. [52] provide a thorough derivation for the torque, producing a general equation which can be used in determining specific cases. The subsequent derivation follows that of [52].

A beam propagating in the z -direction can be written as a plane wave in terms of two orthogonal components:

$$\mathbf{E} = (E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}}) \exp(ikz - i\omega t), \quad (4.14)$$

where E_x and E_y are complex amplitudes as they incorporate the phases of the two components. That is, for circularly polarized light the phase angle between the amplitudes is $\theta = \pm\pi/2$ and $E_y = E_x \exp(i\theta)$. The sign of the phase angle represents the helicity or handedness of the electric field, where $\theta = +\pi/2$ denotes positive helicity and the beam is called left circularly polarized. For $\theta = -\pi/2$, the beam is called right circularly polarized as it has negative helicity. So a circularly polarized beam is then written as,

$$\mathbf{E} = (E_c \hat{\mathbf{x}} \pm iE_c \hat{\mathbf{y}}) \exp(ikz - i\omega t) \quad (4.15)$$

$$= E_c (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}) \exp(ikz - i\omega t), \quad (4.16)$$

where E_c is the complex amplitude. Using the complex orthogonal basis vectors,

$$\begin{aligned} \hat{\mathbf{e}}_L &= \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) \\ \hat{\mathbf{e}}_R &= \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}}), \end{aligned} \quad (4.17)$$

together with the following relation:

$$\begin{aligned} E_L &= \frac{1}{\sqrt{2}}(E_x - iE_y) \\ E_R &= \frac{1}{\sqrt{2}}(E_x + iE_y), \end{aligned} \quad (4.18)$$

the electric field of any beam can be expressed as a sum of two circularly polarized components,

$$\mathbf{E} = (E_L \hat{\mathbf{e}}_L + E_R \hat{\mathbf{e}}_R) \exp(ikz - i\omega t). \quad (4.19)$$

For $E_L = 0$, a right circularly polarized beam is obtained, while $E_R = 0$ gives a left circularly polarized beam.

The irradiance of a beam is proportional to the square of the amplitude of the electric field [53], so the time-averaged irradiance is

$$I = \frac{c\varepsilon_0 E_L^* E_L}{2} + \frac{c\varepsilon_0 E_R^* E_R}{2} = I_L + I_R. \quad (4.20)$$

The energy of a photon is $\hbar\omega$, therefore the photon flux per unit area is,

$$N = \frac{I}{\hbar\omega} = \frac{I_L}{\hbar\omega} + \frac{I_R}{\hbar\omega}. \quad (4.21)$$

Since the angular momentum of left and right circularly polarized photons is $\pm\hbar$, the angular momentum flux per unit area is then,

$$L_z = \frac{I_L - I_R}{\omega} \quad (4.22)$$

$$= \frac{\sigma_z I}{\omega}, \quad (4.23)$$

where a coefficient of circular polarization has been defined as $\sigma_z = (I_L - I_R)/I$. A beam then has a net circularly polarised component with a power $|I_L - I_R|$, which contributes to the angular momentum of the beam. There is also a linearly polarised component, $I - |I_L - I_R| = 2\min(I_L, I_R)$, which does not contribute to the angular momentum of the beam. That is, when light is completely circularly polarised, $\sigma = 1$. The total power of the beam is found by integrating the irradiance over the whole beam, resulting in an average coefficient of circular polarization, $\sigma_z = (P_L - P_R)/P$, and therefore

$$L_z = \frac{\sigma_z P}{\omega}, \quad (4.24)$$

which is the total angular momentum flux of the beam. As previously stated, birefringent materials in general cause the polarization to change and correspondingly, the angular momentum of the beam will change. the torque is equal to the change in angular momentum flux given by,

$$\tau = \frac{(\sigma_{zin} - \sigma_{zout})P}{\omega}, \quad (4.25)$$

where σ_{zin} and σ_{zout} denote the incident and outgoing polarization coefficients, respectively. Although eq. (4.25) is general, it can always be used to

determine the torque assuming the polarization coefficients are known. For a specific case where a sample has thickness, d , the electric field incident on and emergent from the sample are used to calculate the polarization coefficients and in turn the torque. The dependence on thickness of the sample is seen in the final result for left circularly polarized light is,

$$\tau = \frac{c\varepsilon_0}{\omega} E_0^2 [1 - \cos(kd(n_o - n_e))]. \quad (4.26)$$

A similar result is obtained by [47], where the particle was assumed to have exactly known dimensions and birefringence. Both methods show that the particle experiences a constant torque due to circularly polarized light, where the maximum torque is achieved when $kd(n_o - n_e) = \pi$. In a viscous medium, a drag torque, $\tau_D = D\Omega$, is present which will balance the exerted torque, where D is the drag coefficient and Ω is the angular frequency of rotation. For a sphere rotating in a medium for low Reynolds number flows, the drag torque is given by,

$$\tau_D = 8\pi\mu a^3\Omega, \quad (4.27)$$

where μ is the viscosity of the medium and a is the radius of the sphere. As the exerted and drag torques are balanced, they can be equated to solve for any of the parameters which are unknown. For example, the viscosity of the medium is not always known or the medium itself is unknown, and so equating eq.'s (4.25) & (4.27) gives,

$$\mu = \Delta\sigma P / (8\pi a^3\Omega\omega). \quad (4.28)$$

The method for calculating these parameters is described in the next section. It should be noted that the demonstration of the basic principles can also be extended to more specific research.

4.1.1 Experimental Setup

For linearly polarized lasers, a quarter-wave plate must be inserted into the setup to change to circularly polarized light. A quarter-wave plate introduces a relative phase shift of $\Delta = \pi/2$ between the ordinary, o-, and extraordinary, e-, components of a wave. When linear light is incident on such a plate, the o- and e- components have equal amplitudes, converting linear light into circularly polarized light. This is the only additional optic added to the original tweezing design, however, to observe the transfer of spin angular momentum more easily, birefringent calcite particles suspended

in an ethanol solution replaced the silica bead sample. Calcite, with $n_o = 1.66$ and $n_e = 1.49$, acts as a microscopic wave plate, similar to that used in Beth's experiment, and are therefore expected to rotate according to the same principle. Fig. 4.2 shows the additional quarter-wave plate inserted before the telescope.

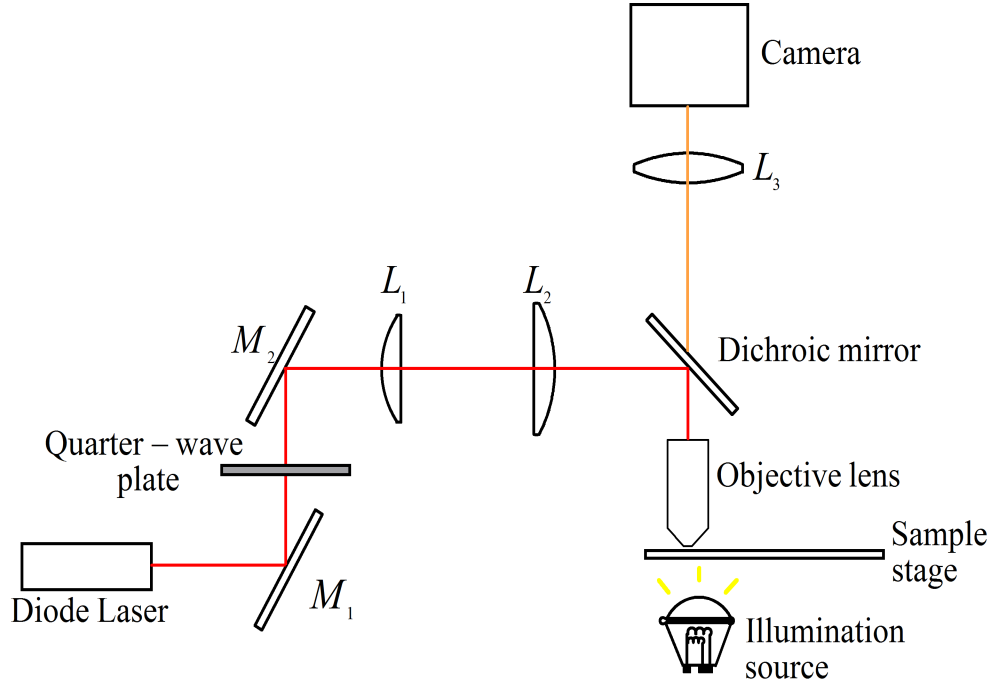


Figure 4.2: Experimental setup used to observe the transfer of spin angular momentum from the beam to the particles, causing them to rotate. A quarter-wave plate is used to convert linearly polarized light into circularly polarized light, which carries spin angular momentum.

The frame rate of the camera was set to 15 frames/s, which allows the time between a number of frames to be calculated.

4.1.2 Experimental Results

The slightly altered setup clearly demonstrated the expected results. A calcite particle exposed to circularly polarized laser light rotated in an anti-clockwise direction. The angular speed at which the trapped particle rotated was calculated using:

$$\omega = \lim \frac{\Delta\theta}{\Delta t}, \quad (4.29)$$

where $\Delta\theta$ is the change in angular position of the particle within time Δt . The time between each of the frames in fig. 4.3 is 1 s. It is clear that the particle rotated by angle of $\pi/2$ rad between each of the frames, therefore according to eq. (4.29), $\omega = \pi/2 \pm 0.09$ rad/s.

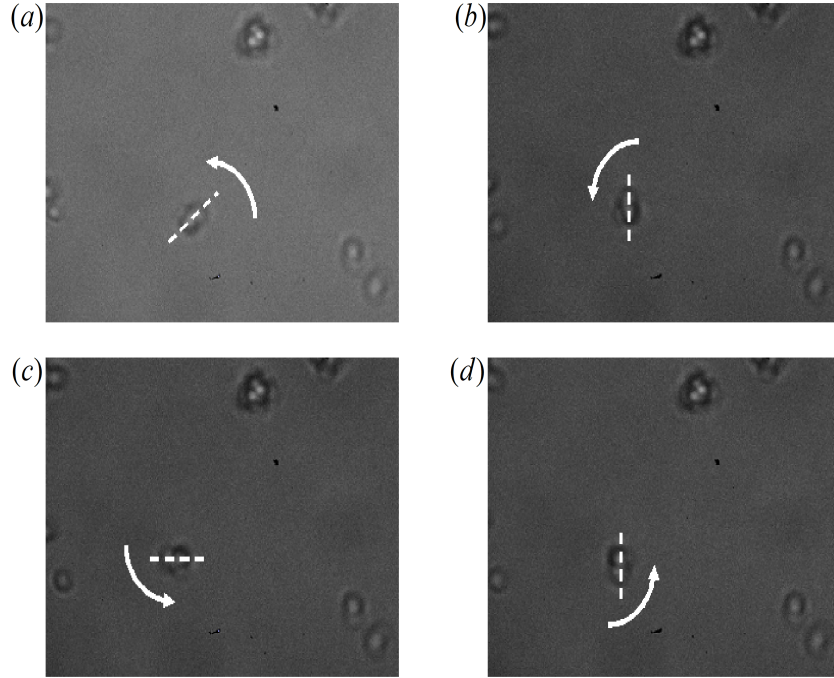


Figure 4.3: Images of birefringent calcite particles in ethanol. (a)-(d) Show the particle rotating in an anti-clockwise direction at a rate of $\omega = \pi/2$ rad/s.

The wave plate was then rotated by 90° , causing the polarization to change handedness and another particle was seen to rotate but in a clockwise direction, clearly illustrating the left- and right-handedness of spin angular momentum, shown in fig. 4.4. Again, the angular speed of rotation was calculated to be $\omega = \pi/2 \pm 0.09$ rad/s, which is expected as the direction of polarization was the only variable to change.

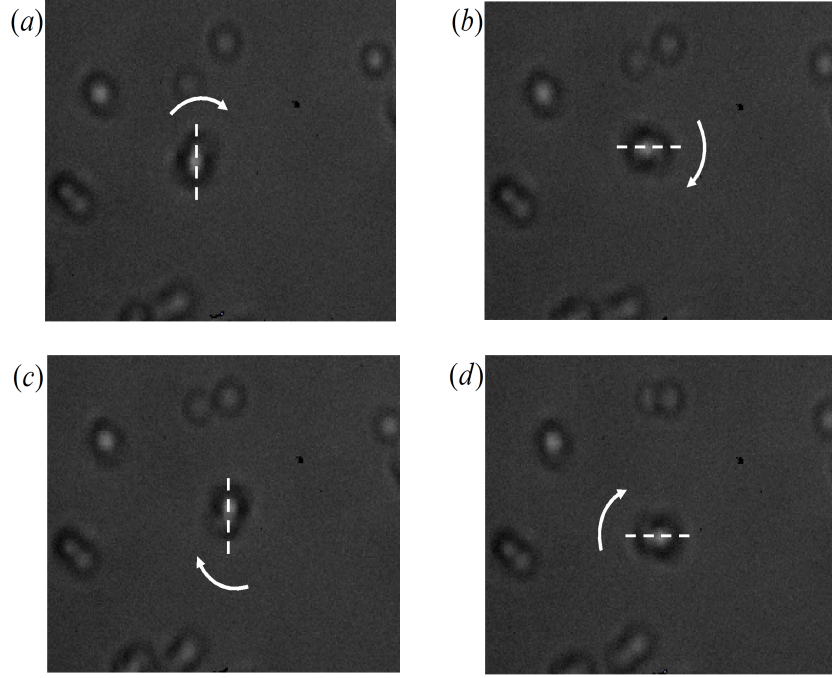


Figure 4.4: Images of birefringent calcite particles in ethanol. (a)-(d) Show the particle rotating in a clockwise direction at a rate of $\omega = \pi/2 \text{ rad/s}$.

Using eq. (4.27), it is possible to calculate the torque exerted on the calcite particle, as the viscosity of ethanol is $\mu = 1.1 \times 10^{-3} \text{ Nm}^{-2}\text{s}$, the radius of the particle is approximately $a = 1 \text{ }\mu\text{m}$ and the angular frequency of rotation is $\Omega = \omega/2\pi = 0.25 \text{ Hz}$. Using these values, a torque of $\tau = 7.5 \times 10^{-21} \pm 0.4 \times 10^{-21} \text{ Nm}$ was obtained.

4.2 Orbital Angular Momentum

The second contribution to the angular momentum of light is orbital angular momentum. Section 4.1 showed how spin angular momentum is associated to the polarization of light, in this section the orbital angular momentum is shown to be associated with the spatial distribution of the wave. That is, orbital angular momentum is carried by beams with ‘twisted’ or helical wavefronts, unlike plane waves.

In 1992, [54] showed that a Laguerre-Gaussian (LG) laser mode has a well-defined orbital angular momentum. The modes are commonly denoted LG_p^l ,

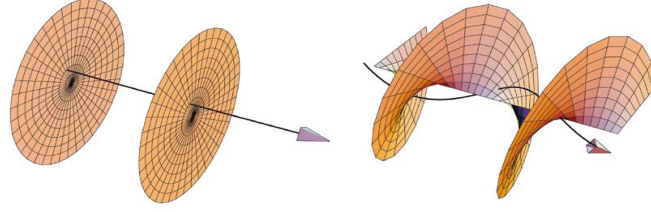


Figure 4.5: 'Twisted' light with helical wavefronts carries orbital angular momentum, unlike plane waves.

where l is the number of 2π cycles in phase around the circumference, known as the azimuthal mode index (or topological charge in a different context), and $(p+1)$ is the number of radial nodes. The amplitude of the LG mode has an azimuthal angular dependence of $\exp(-il\phi)$. The LG modes are thought of as the eigenmodes of the angular momentum operator L_z [54], and so, carry an orbital angular momentum of $l\hbar$ per photon.

[54] presented a method to measure the angular momentum of a beam by measuring the torque acting on suspended cylindrical lenses as a result from the reversal of helicity of a LG beam, a similar experiment to that performed by [49]. However, the first demonstration of the transfer of orbital angular momentum from a linearly polarized LG beam to absorbing particles, was shown by [46] by making use of an optical trapping setup. As the beam was linearly polarized with $\sigma_z = 0$, the rotation of the trapped particles caused by the transfer of angular momentum demonstrated the transfer of orbital angular momentum.

Within the paraxial approximation, the z -component of the angular momentum density for a LG beam has been shown to be [54],

$$L_z = \frac{l}{\omega} |u|^2 + \frac{\sigma_z r}{2\omega} \frac{\partial |u|^2}{\partial r}, \quad (4.30)$$

where u is the amplitude of the light field. This equation introduces an expression for the torque on an absorbing particle due to both the polarization and the helical wavefronts of the LG beam:

$$\Gamma = \frac{P_{abs}}{\omega} (l + \sigma_z), \quad (4.31)$$

where P_{abs} is the power absorbed by the particle. Eq.s (4.30) & (4.31) are both restricted to the paraxial approximation, which breaks down for highly focused light as is the case with optical tweezers. [45] re-examined the

workings by [54] for non-paraxial light beams and after a lengthy derivation found the torque to be,

$$\Gamma = \frac{P_{abs}}{\omega} \left[(l + \sigma_z) + \sigma_z \left(\frac{2kz_R}{2p + l + 1} + 1 \right)^{-1} \right]. \quad (4.32)$$

The mixed term in eq. (4.32) becomes more significant as the beam becomes more tightly focused. If however, the focused beam size to approximately one wavelength across, the contribution of the term would be at most 20% [55], and for linear polarized light, the total angular momentum of the beam will be $l\hbar$.

Therefore, a polarized LG mode can carry angular momentum of $(l + \sigma_z)\hbar$. It should be noted that although spin angular momentum can only assume values of $\pm\hbar$, l can accept an infinite number of integer values, which is of great interest as will be seen further within the chapter.

The preceding discussion focused on Laguerre-Gaussian beams, as they have been thoroughly studied over the years. However, in general, any beam with inclined wavefronts carries orbital angular momentum. At the centre of the helical phase structure, described by $\phi = l\theta$, the phase is undefined and the beam is said to have a phase singularity and the field amplitude vanishes, resulting in a ‘dark beam’ at the centre. Beams displaying this property are commonly called vortex beams, where the radius of the vortex increases linearly with topological charge l [56]. Vortex beams are being applied in a number of emerging research fields, including optical tweezers and quantum computing [57].

4.2.1 Experimental Setup

There are various methods to generate a vortex beam. Recently, the most commonly used method involves a spatial light modulator (SLM), which allows beam shaping to be performed quickly and efficiently.

A spatial light modulator consists of a liquid crystal display (LCD) of high resolution, which allows the phase and/or amplitude of the beam to be altered by a predetermined amount. The particular SLM used here was a HoloEye product, with a reflective display with a resolution of 1920×1080 pixels. The model was a phase-only SLM, with 256 (8 bit) encoding, so the phase which lies in the range 0 to 2π , is displayed as a gray-scale image, where white represents $\phi = 0$ and $\phi = 2\pi$ is displayed as black. The SLM was connected to a computer allowing the phase pattern on the LCD to be changed by simply displaying a new phase pattern on the computer screen.

Calibration of the SLM allowed the use of beams of different wavelengths. Fig. 4.6 shows the LCD of the SLM, which is then connected via a DVI input to a computer screen.

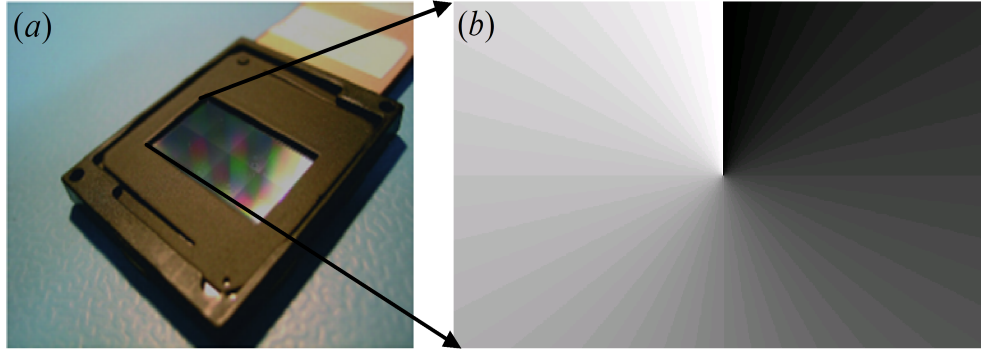


Figure 4.6: (a) The liquid crystal display of a spatial light modulator allows a phase pattern, shown in (b), to be encoded onto the device. The phase changes from 0 to 2π , denoted by white to black, respectively.

The angle between incident and reflected beams was made as small as possible to obtain optimal images of the generated beam. The incoming beam was also set at a specific size $\omega = 2.1$ mm, that is $3\omega_0 < R_{SLM}$, where R_{SLM} is the minimum radius of the SLM. The beam must be placed at the centre of the SLM, and was therefore mounted on a 3-axis translation stage.

The phase pattern shown in fig. 4.6 (b) represents a vortex beam of $l = 1$. The phase at the centre of this phase pattern is undefined and is referred to as a phase singularity, which results in the dark beam at the centre of the vortex beam. Another important feature of vortex phase patterns is the phase dislocation seen in fig. 4.6 (b) as the distinct line between 0 and 2π phase (white and black). This dislocation represents the order of the vortex beam. For this particular spiral phase pattern, only 1 dislocation is present. However, a different phase pattern has also been found to produce vortex beams, and is referred to as a fork hologram. Fig. 4.7 shows a fork phase pattern with 5 ‘prongs’ and 4 phase dislocations between these prongs.

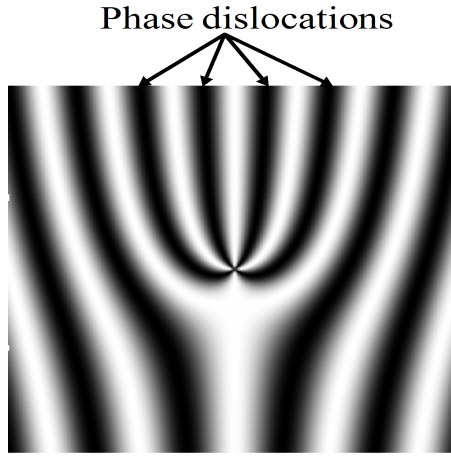


Figure 4.7: A fork phase pattern with 4 dislocations generates a vortex of order $l = 4$.

Fig. 4.8 shows the reflected beam passing through a lens, which focused the beam onto the back aperture of the objective lens and subsequently onto the sample. The $100\times$ objective lens above the sample stage was used to image the sample plane back to the camera.

The vortex beam was reflected off the SLM and focused by a lens L_1 ($f_1 = 100$ mm) into a $40\times$ objective lens, which then expanded the beam onto the back aperture of the objective lens. The vortex beam was approximately 5 mm in diameter before entering the objective.

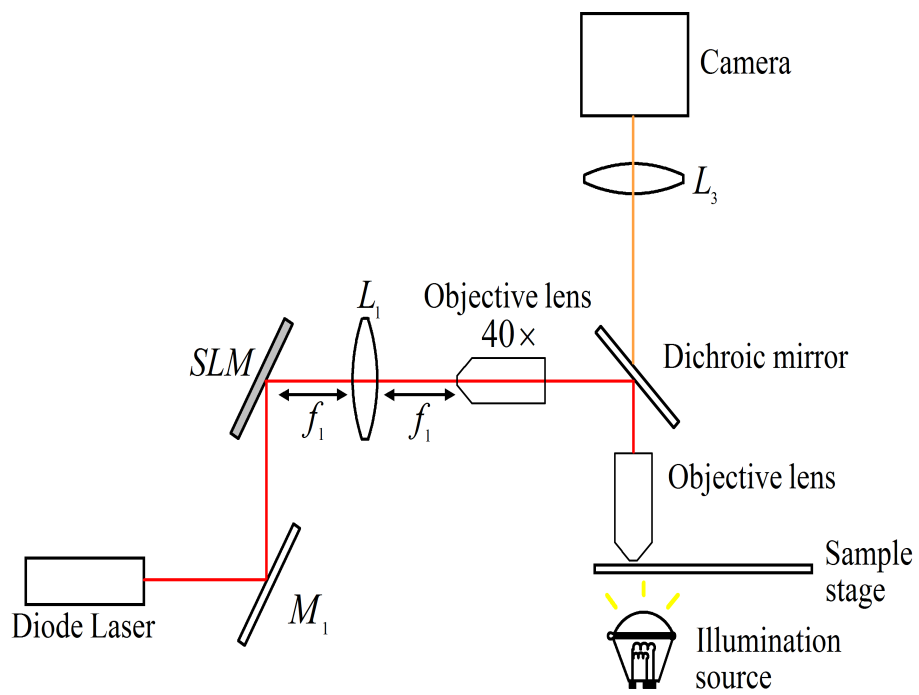


Figure 4.8: Experimental setup used for rotating particles about a point. A spatial light modulator (SLM) is used to generate a vortex beam, which carries orbital angular momentum.

4.2.2 Experimental Results

Initially, a vortex beam was generated and imaged onto a camera before placing it into the trap. Fig. 4.9 displays the setup used to produce a vortex beam. Due to the spacing between pixels and the phase changes of the phase pattern, many diffraction orders are reflected off the SLM, see appendix A.

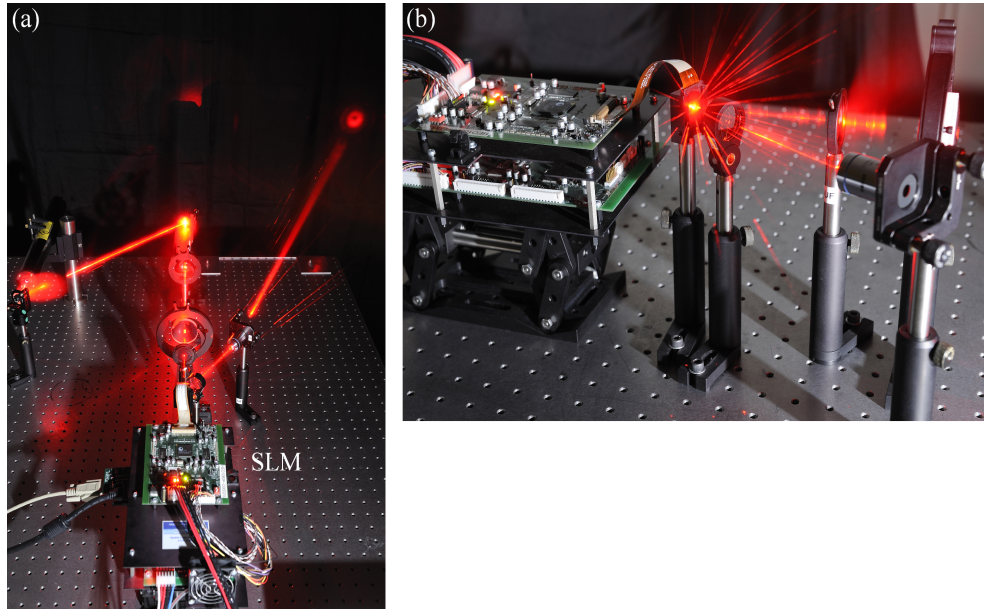


Figure 4.9: (a) An incident Gaussian beam is reflected off the SLM to produce a vortex beam. (b) The diffraction orders are reflected off the SLM.

The intensity profile of a vortex beam of order $l = 1$ is shown in fig. 4.10, where the dark centre is clearly seen.

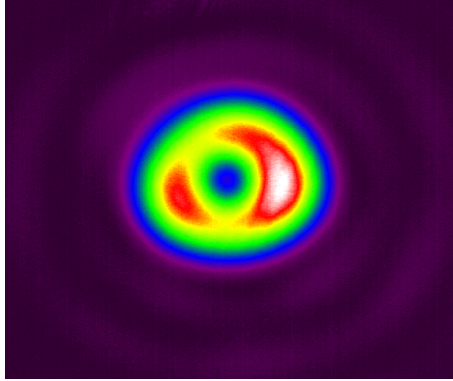


Figure 4.10: Image of a vortex beam with $l = 1$, showing the intensity distribution. The spiral phase pattern was used to generate this beam.

The same vortex beam in fig. 4.10 was then directed through the objective lens onto the sample. For this particular experiment, graphite particles of non-uniform size were suspended in distilled water and exposed to a vortex beam of order $l = 1$. The particles were trapped in the dark centre of the beam and subsequently began to rotate in an anti-clockwise direction. Fig. 4.11 shows a $1\ \mu\text{m}$ diameter graphite particle trapped in the centre of the beam and rotating in an anti-clockwise direction.

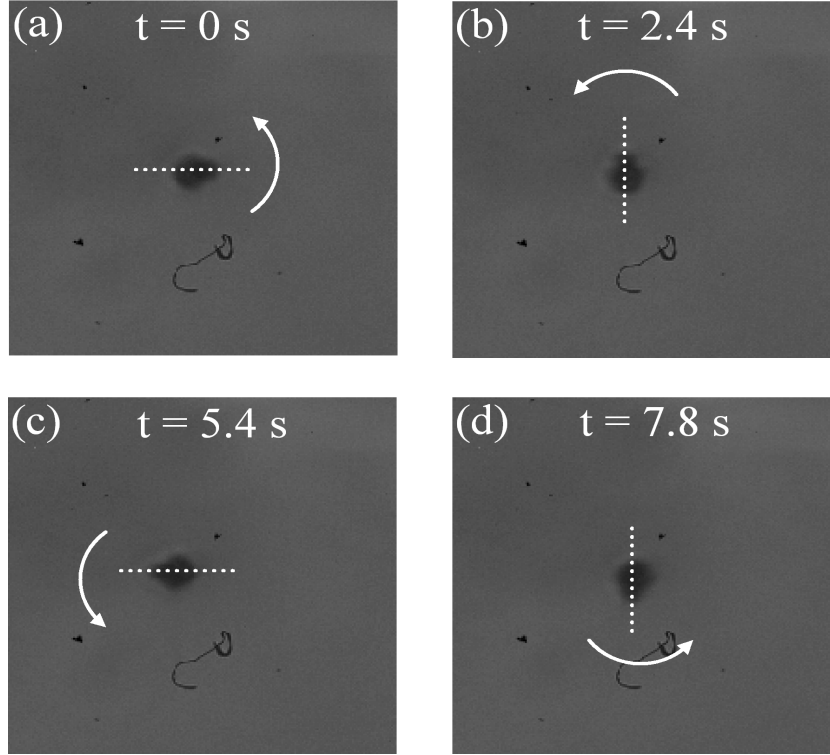


Figure 4.11: A graphite particle of $1\ \mu\text{m}$ diameter was exposed to the vortex of order $l = 1$. The particle rotated in an anti-clockwise direction at a frequency of approximately 0.37 Hz.

Using the method from section 4.1.2, the angular speed was calculated to be approximately $\Omega = 2.77\ \text{rad/s}$, giving a rotational frequency of $\Omega = 0.44\ \text{Hz}$. A torque of roughly $\tau = 1.2 \times 10^{-20} \pm 0.2 \times 10^{-20}\ \text{Nm}$ was calculated.

Further experiments can be performed with this particular setup to demonstrate faster rotation rates, rotation in the opposite direction, among other experiments. These results form the basis for future work.

4.3 Application in Microfluidics

The implementation of transferring angular momentum to particles within an optical trap has opened up a diverse range of applications within microfluidics. Often referred to as a ‘lab-on-a-chip’, a microfluidic system typically consists of channels, where at least one of the dimensions is on the order of microns. Optical tweezing is therefore a logical technique to explore within this field.

Spherical particles offer an effective method of determining unknown parameters of the fluid. As previously mentioned, by measuring the rotation frequency of a particle of known size and the change in polarization, the viscosity of the fluid can be calculated [58]. Similarly, the radius of the particle can be determined if the other parameters are known. Two spherical vaterite (CaCO_3) particles, trapped next to each other, have been rotated in opposite directions, thereby forming an optical micropump. A constant flow between the particles will form, forcing untrapped particles to follow the stream [59].

The technique has been extended to producing microstructures, which display birefringent behaviour despite the non-birefringent nature of the material. These structures are based on the principle of form birefringence [60]. Fabricated microgears of known birefringence allow two such gears to be rotated by placing the structures together and only optically rotating one gear. The teeth of the two gears interlock, forcing the second microgear to rotate in the opposite direction to the first. Fig. 4.12 shows such a microgear shown in a paper by [60].

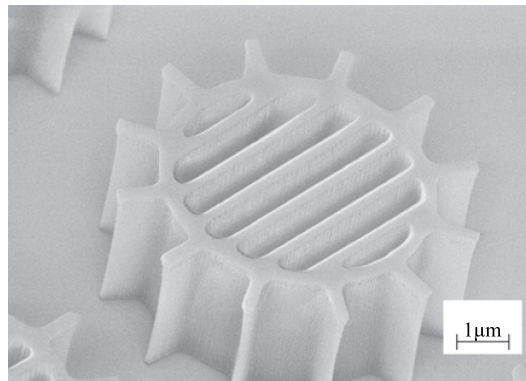


Figure 4.12: SEM micrograph of an actual microgear [60], has made this an exciting field of research.

The number of possible applications within the field of microfluidics using optical tweezer has made this an excited field of research.

Chapter 5

Super-Gaussian Beams

Optical tweezing is a versatile technique in that any beam can be directed onto the sample and depending on the properties of that beam, further information about various systems is revealed. A number of different beam shapes have been applied to tweezing, offering a number of particular uses, however super-Gaussian beams have not, to the best of our knowledge, been utilised in optical tweezing. The similarities between Gaussian and super-Gaussian beams suggests that the latter beam could also be used as an optical tweezing beam, which may offer additional, useful features. The properties, in particular the intensity gradient, of super-Gaussian beams are examined in this chapter in the aim of producing a novel trapping beam. These beams may offer increased radial trapped strengths, which can be adjusted by simply changing the order of super-Gaussian.

Super-Gaussian (SG) beams first gained attention within the examination of resonator design, where the propagation of the beams played an important role. The first description of these beams was given by [61], while looking at unstable resonators. The profile of the beam offered a number of benefits for resonator work, one being the simplicity of the of the equation defining the intensity profile of a SG beam:

$$SG_p(r, z) = A \exp \left[- \left(\frac{r}{\omega(z)} \right)^p \right], \quad (5.1)$$

where A is the amplitude of the beam, $\omega(z)$ is the width of the beam along the propagation axis in the z -direction, r represents the radial position and p is the order of the super-Gaussian beam. When $p = 2$, eq. (5.1) simply describes a Gaussian beam. Fig. 5.1 shows the intensity profiles for a 3 different values of p .

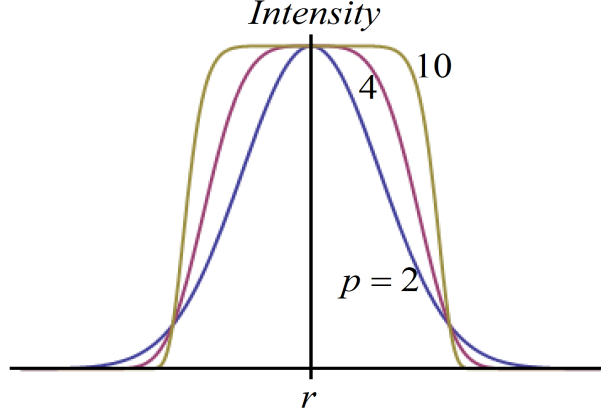


Figure 5.1: Intensity profile of super-Gaussian beams of different orders. For $p = 2$, a Gaussian intensity plot is seen. As the order increases, the flatness of the peak intensity becomes more evident.

Eq. (5.1) describes the shape of the intensity of a SG beam well, however, it does not accurately describe its propagation. A number of models were presented to approximate the beam [62, 63]. These approximate expressions demonstrate how quickly a SG beam loses its shape. A correlation between super-Gaussian and flattened Gaussian beams has been discussed [64].

Super-Gaussian beams have attracted interest from a variety of fields. In laser resonators, the intensity profile curve has less pronounced diffraction rings than a Gaussian beam while still filling the active medium [61]. The uniformly spread peak intensity offers a more efficient laser ablation method than the conventional Gaussian beam. Within optical tweezing, a super-Gaussian beam offers an increased radial trapping strength due to the steep intensity gradient, which increases with increasing order of the beam. That is, by simply altering the degree of super-Gaussian, the strength of the trap can be changed quickly and accurately.

5.1 Propagation Parameters

Although the super-Gaussian expression given by eq. (5.1) is very simple, however determining the propagation of such a field often leads to numerical integration, which is a tedious procedure. The propagation of flat-top beams are almost always solved numerically. Several different models have been

put forward, one of these being an expression given by [62]. The model makes use of a Laguerre-Gaussian modal expansion and derives expressions approximating the propagation of a SG beam. These numerical beams are referred to as flattened Gaussian beams (FGB). He begins by considering a circularly symmetric gaussian function $\exp(-r^2)$. By aiming to flatten a Gaussian beam to a constant value of one, Gori expands the factor $\exp(r^2)$ such that,

$$F_\infty(r) = \exp(-r^2) \sum_{n=0}^{\infty} \frac{r^{2n}}{n!} = 1. \quad (5.2)$$

This expression can be further approximated by truncating the series such that the function, $F_N(r)$, is approximately one for an interval, where after it smoothly decreases to zero. Further, the maximum of the function $\exp(-r^2)r^{2n}$ is $r = \sqrt{n}$, and so the width of $F_N(r)$ is $\sqrt{N}$. This prevents the width of $F_N(r)$ increasing with increasing N . Therefore, the rescaled approximation gives,

$$G_N(r) = \exp(-Nr^2) \sum_{n=0}^N \frac{N^n r^{2n}}{n!} = 1, N = 1, 2, \dots \quad (5.3)$$

At $N = 0$, a Gaussian curve is obtained. Eq. (5.3) describes a flattened Gaussian profile, which is the starting point in the derivation of an expression describing FGB propagation. The field distribution of FGB's at $z = 0$ is given in a form similar to that of Gaussian beams by including a beam spot size, ω_0 , while maintaining the rescaled terms:

$$U_N(r, 0) = A \exp(-Nr^2/\omega_0^2) \sum_{m=0}^N \frac{1}{m!} \left(\frac{Nr^2}{\omega_0^2} \right)^m = 1, N = 1, 2, \dots, \quad (5.4)$$

where A is a constant. The crux of Gori's model is that the propagation of FGBs can be studied by following them back to a superposition of Laguerre-Gauss beams, in particular, the circularly symmetric subset, which has a spatial distribution given by,

$$\begin{aligned} \Psi_h(r, z) = & \sqrt{\frac{2}{\pi}} \frac{1}{v(z)} \exp\{i[kz - (2h + 1)\Phi(z)]\} \\ & \times L_h \left(\frac{2r^2}{v^2(z)} \right) \exp \left[\left(\frac{ik}{2R(z)} - \frac{1}{v^2(z)} \right) r^2 \right], \end{aligned} \quad (5.5)$$

where $v(z)$ is the width of the beam defined in terms of v_0 , the spot size of the beam, and $z_R = kv_0^2/2$. The propagation parameters $v(z)$, $\Phi(z)$ and $R(z)$ are defined as with Gaussian beam propagation. The following formula is used by Gori in proving the validity of using Laguerre-Gauss beams,

$$x^{2m} = \frac{m!}{2^m} \sum_{n=0}^m (-1)^n \binom{m}{n} L_n(2x^2), m = 0, 1, \dots, \quad (5.6)$$

where L_n is the n th Laguerre polynomial. Eq. (5.6) together with eq. (5.5) ultimately produce an expression¹ describing the propagation of FGBs:

$$U_N(r, z) = A \frac{v_{0N}}{v_N(z)} \exp(-i[kz - \Phi_N(z)]) \exp \left[\left(\frac{ik}{2R_N(z)} - \frac{1}{v_N^2(z)} \right) r^2 \right] \\ \times \sum_{n=0}^n c_N L_n \left(\frac{2r^2}{v_N^2(z)} \right) \exp[-2iN\Phi_N(z)], \quad (5.7)$$

where

$$v_{0N} = w_0/\sqrt{N}. \quad (5.8)$$

The spot size has been rescaled accordingly, thereby re-defining the beam parameters such that,

$$(z_R)_N = \frac{z_R}{N}, \quad (5.9)$$

$$v_N = \frac{v_{0N}}{\sqrt{N}}, \quad (5.10)$$

$$v_N(z) = v_{0N} \sqrt{1 + \frac{z^2}{(z_R)_N^2}}, \quad (5.11)$$

$$\Phi_N(z) = \arctan \left[\frac{z}{(z_R)_N} \right], \quad (5.12)$$

$$R_N(z) = z + \frac{(z_R)_N^2}{z}. \quad (5.13)$$

By considering the intensity, $I = |U|^2$, of an FGB, it is clear that the beam does not hold its shape as a Gaussian beam does.

¹[62] provides the full derivation of the expression in eq. (5.7).

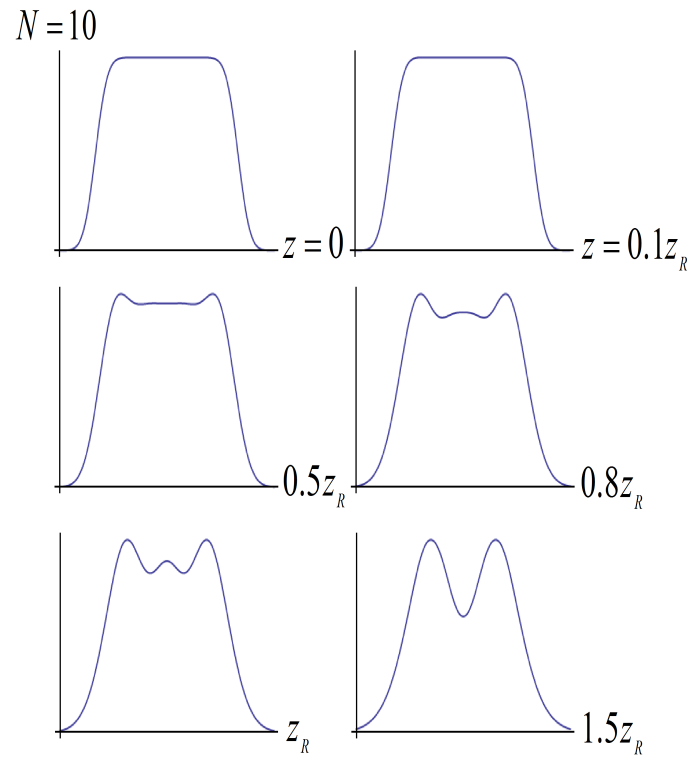


Figure 5.2: Intensity profile of a flattened Gaussian beam as it propagates along the z -axis.

Within the Rayleigh range, the flatness of the peak does deteriorate toward the end of the length, but even at $z = z_R$, it is still recognisable as a flat top. However, at $z = 1.5z_R$, there is no flattened section between the two peaks. Fig. 5.2 illustrates the propagation for a beam of order $N = 10$, if the order is increased, a more flattened profile is obtained, however the propagation for such a beam is much shorter than that for $N = 10$. That is, as the order of the beam increases, thereby producing a flatter beam, the shorter the distance in which the beam exists.

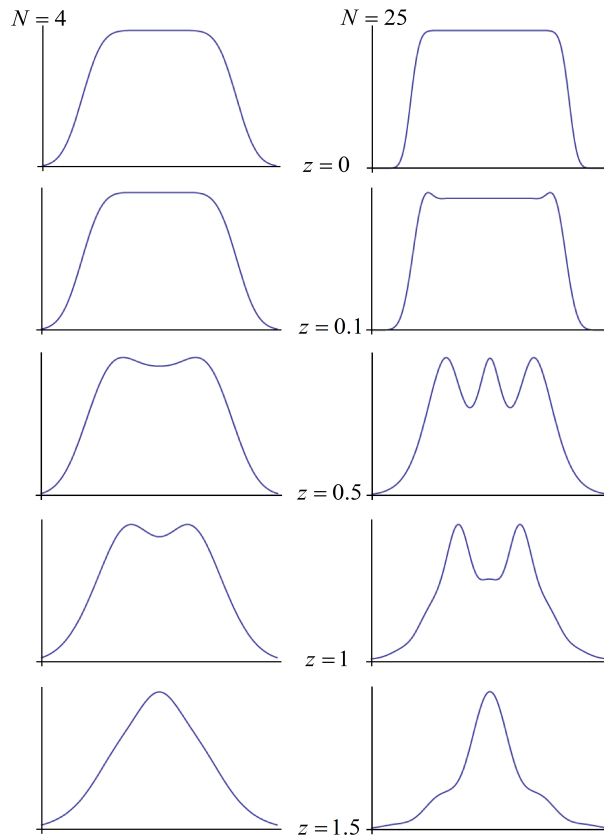


Figure 5.3: Comparison of the propagation of two flattened Gaussian beams of order $N = 4$ and $N = 25$. It is clear that at the same z -values, the beams differ quite drastically. The larger the order, the faster the beam loses its shape.

5.2 Intensity Gradient

According to eq. (3.7), the gradient force is proportional to the gradient of the intensity. Therefore the steeper the gradient of the beam intensity, the stronger the gradient force and ultimately, the stronger the trap. The intensity gradient in the lateral plane of super-Gaussian beam is much steeper than that of a Gaussian beam. Fig. 5.4 shows the intensity to be almost constant along the z -axis, resulting in no intensity gradient in that direction.

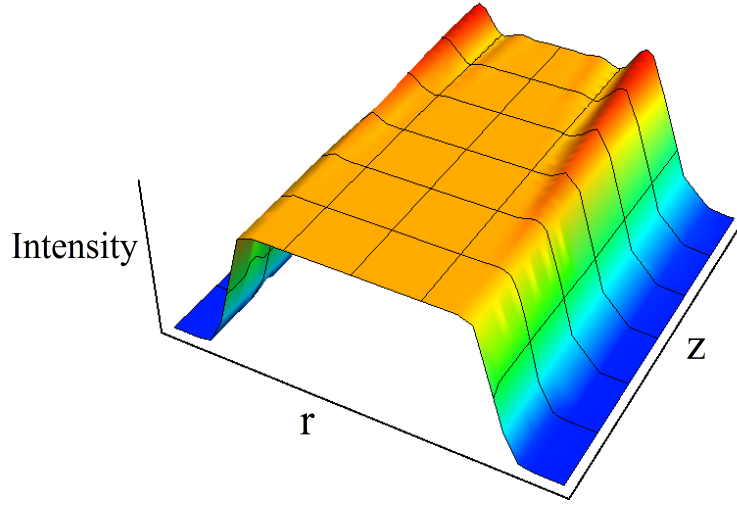


Figure 5.4: A three dimensional plot of the intensity distribution of a SG beam. The intensity gradient along r is very steep, while along the z -axis, there is little to no change in the intensity.

The higher the order of the SG beam, the flatter the peak intensity becomes. Fig. 5.5 illustrates that the intensity gradient and therefore the gradient force, increases with the order of the SG beam.

However, this gradient force acts only in the lateral direction and so a three dimensional trap is not obtained with a SG beam. Nonetheless, the lateral trapping strength of a super-Gaussian can simply be increased by increasing the order of the beam. This is a new concept and requires extensive experimental research to illustrate the idea effectively. A range of SG beams of varying orders must be shown to be able to trap a particle and subsequently, the strength of each SG trap must be determined. This can be achieved using the drag force method in section 3.3.2. Within this

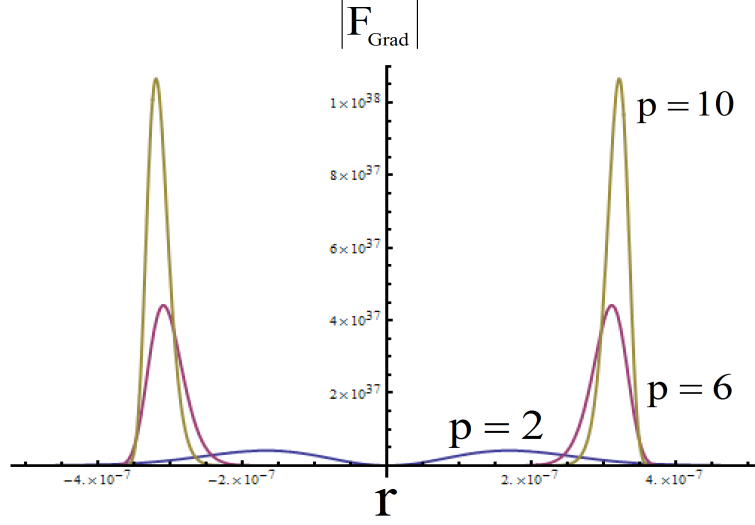


Figure 5.5: The lateral intensity gradient increases as the order of the super-Gaussian beam increases. For $p = 10$, the gradient force is stronger than the Gaussian beam ($p = 2$) by two orders of magnitude.

dissertation, a demonstration of particle trapping within a SG beam is provided, which is a crucial verification for future work.

5.3 Experimental Setup

The same experimental design used to produce a vortex trap was used to produce a super-Gaussian trap, where a spatial light modulator was used to generate the super-Gaussian beam. However, lenses L_1 and L_2 were not used as a 1:1 telescope, but rather as a focusing and imaging system. The Fourier transform of a super-Gaussian beam is a Sinc function. The phase pattern was therefore calculated for a Sinc function and lenses L_1 ($f_1 = 100$ mm) and L_2 ($f_2 = 100$ mm) formed a 1 : 1 telescope, allowing the SLM to be placed further from the objective lens. The objective lens acted as a Fourier lens, thereby producing a super-Gaussian at the focus of the objective lens.

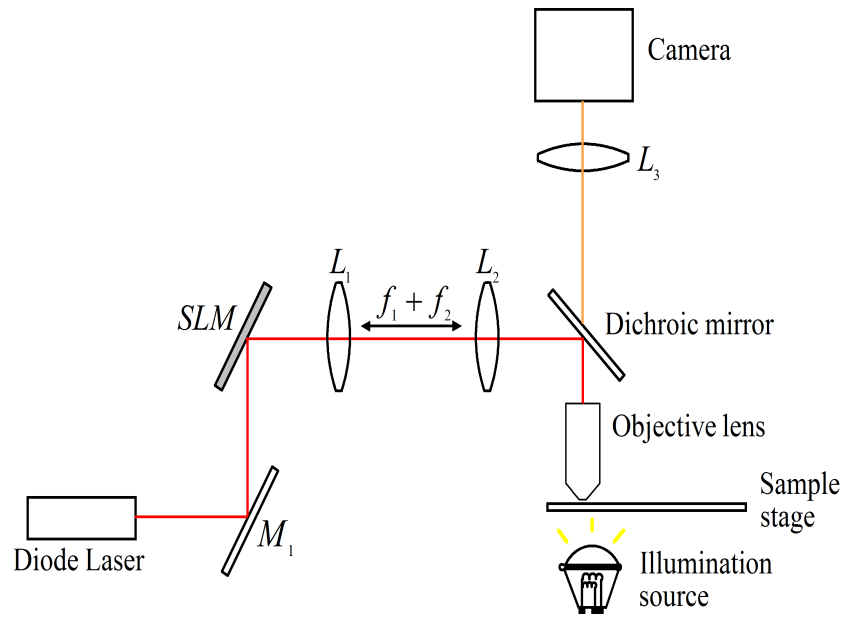


Figure 5.6: Experimental setup used for rotating particles about a point. A spatial light modulator (SLM) is used to generate a vortex beam, which carries orbital angular momentum. L_1 and L_2 act as a 1 : 1 telescope, allowing the SLM to be placed further from the objective lens. The objective lens acted as a Fourier lens, thereby producing a super-Gaussian at the focus of the objective lens.

The incoming Gaussian beam was expanded to a radius of $r = 2.1$ mm. As in section 4.2.1, the beam was centred on the SLM using the 3-axis translation stage. Fig. 5.7 shows the phase pattern calculated to generate a super-Gaussian beam. This particular phase pattern was calculated to produce a beam with a diameter of 1 mm at the focus of L_1 .

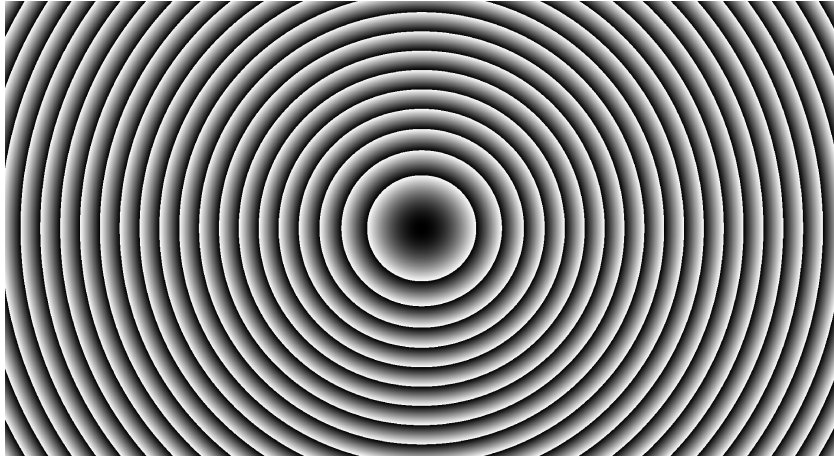


Figure 5.7: Calculated phase pattern for a super-Gaussian beam. The phase patterns were calculated and produced in Matlab.

5.4 Experimental Results

A circular flat-top beam was generated using the phase pattern in fig. 5.7. The peak intensity is roughly uniform and the steep intensity gradient is evident from the tight concentric colour changes seen in fig. 5.8 (a). A Gaussian intensity is shown alongside for comparison.

There is no axial gradient force in a super-Gaussian trap, only a lateral gradient force. A particle trapped in a SG beam, will remain fixed in the trap while the background particles in solution are re-positioned. Fig. 5.9 shows an untrapped particle being moved around a trapped particle in a square-like pattern. The untrapped particle is moved by moving the stage.

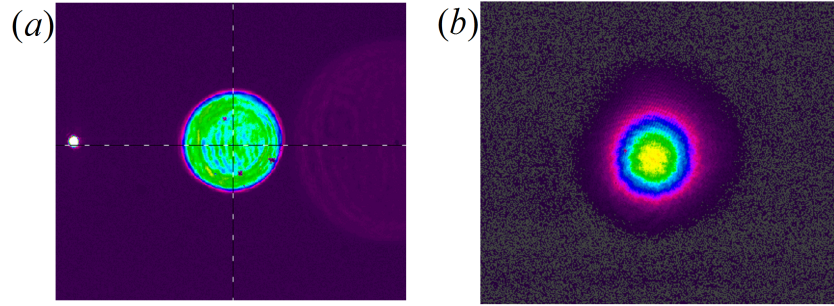


Figure 5.8: (a) Image of a circular flat-top beam of order 50. The intensity is uniform at the peak, and a steep gradient is evident from the immediate colour change, as compared to that of a Gaussian beam shown in (b).

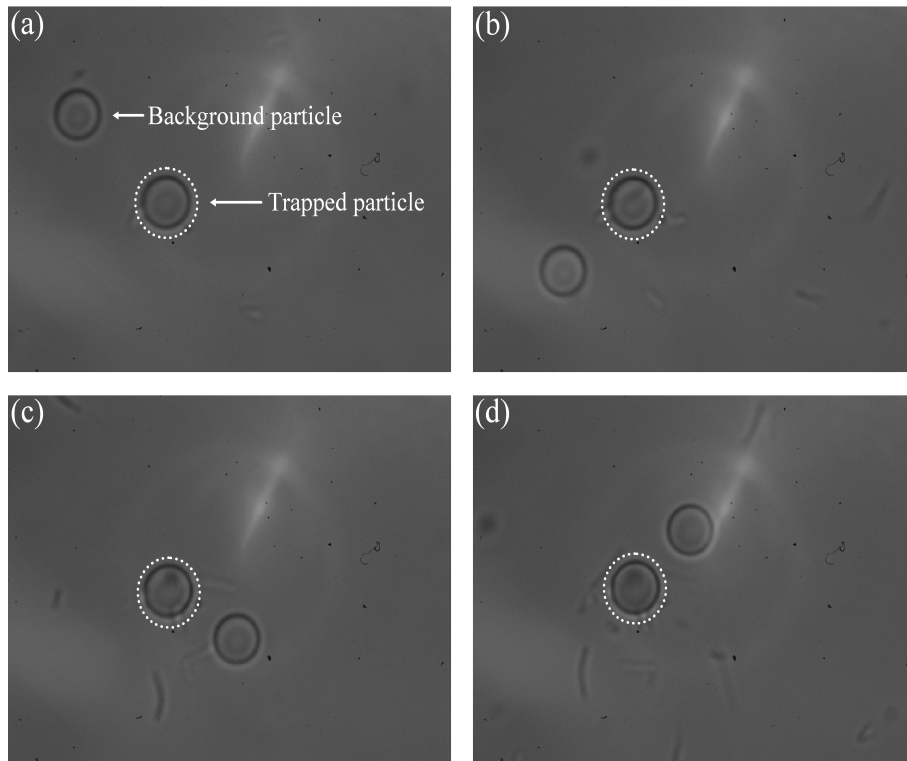


Figure 5.9: The gradient force only acts in the lateral plane, so the trapped particle does not change focus. The stage was moved in a square-like pattern, causing the untrapped particle to move in the same pattern.

The trapped particle was also shown to move with the re-positioning of the beam as shown in fig. 5.10. Movement of the laser was performed by moving the dichroic mirror and not a beam steering mirror as before. This prevented precise, defined movements of the particle and is not ideal as the beam translates across the back aperture of the objective lens. This can cause clipping of the beam, thereby reducing transmission through the objective. As a first attempt, the results are satisfactory, however future work will involve reconstructing the setup to include a steering mirror after the SLM.

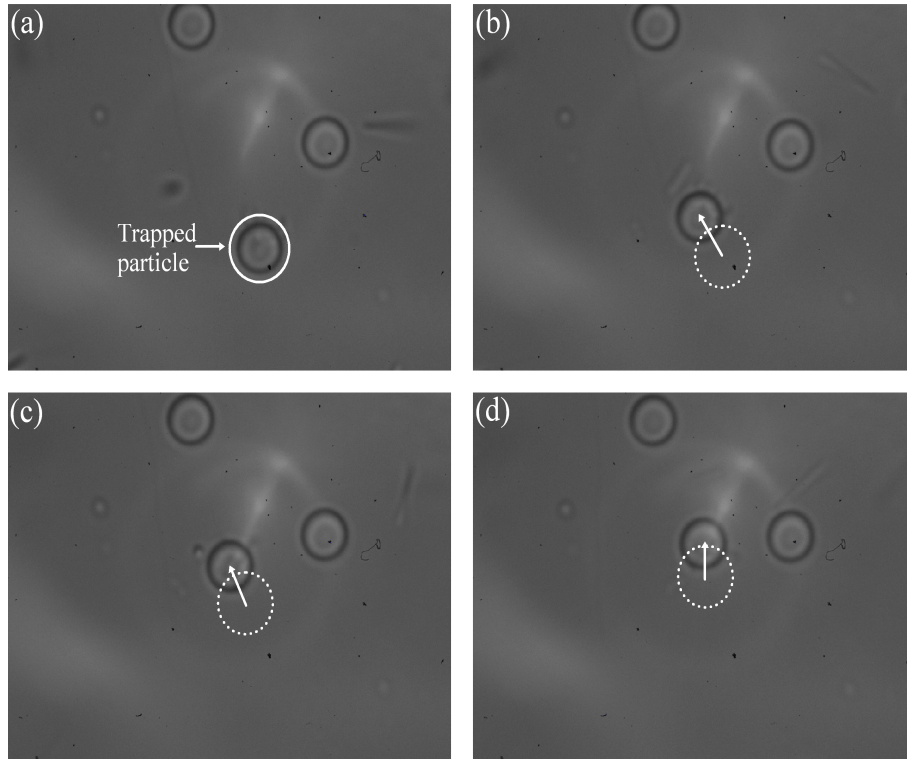


Figure 5.10: The trapped particle is moved towards the top of the image, while the untrapped particles remain fixed in position.

The super-Gaussian beam trapped particles in the lateral plane only, as expected. However, the next step is to determine the strength of these SG traps, and whether the trap strength increases as the order of the beam increases. The latter experiment will demonstrate a more efficient method of varying the trap strength than the current method of varying the power of the beam, especially as the exact power at the trap site is not accurately known.

Chapter 6

Conclusions and Outlooks

6.1 Summary

Optical tweezing began as a simple tool for measuring the forces of light on matter, but has now developed into an invaluable technique that benefits a spectrum of research fields, from studying the properties of DNA molecules [35] to examining the dynamics of aerosol droplets [40]. Any contribution towards these fields starts with building an optical tweezer and discovering both the theoretical and practical elements involved in this ever-growing technique.

Initially a Gaussian beam was used to demonstrate successful optical tweezing. The propagation of Gaussian beams was examined theoretically, while the trapping laser was characterised experimentally. Of course, the trapping laser and its propagation parameters are crucial in tweezing, however, just as important is the imaging system. As a commercial microscope was not used, a hand-made microscope was constructed. A number of optics principles had to be mastered, including how objective lenses function in terms of both imaging of the sample plane and the focusing of the laser light. While assembling the optical tweezer, various issues arose such as the vertical alignment of the optics that formed the microscope. The objective lens had to be carefully aligned with the collective lens and camera in order to image the trapping region correctly. Once each component had been optimally aligned, three dimensional optical tweezing was achieved using $4\text{ }\mu\text{m}$ silica beads. Not only were the beads repeatedly trapped and released, it was also shown that they could be manipulated by simply moving the laser beam. The main attraction of this manipulation method, is the non-invasive feature that it provides. Therefore, a human embryonic

kidney cell was also trapped and manipulated without killing the cell. The successful construction of the optical tweezer only partially fulfilled the aim of the project. The dimensions of the tweezer were then determined through a series of experimental measurements. This then produced a complete picture of the designed tweezer, on both the macro and microscopic levels.

Once the setup was permanently in place, the next goal was to demonstrate a series of techniques within optical tweezing that provide even more versatility. Bessel beams have two interesting properties that are exploited in optical traps. Their non-diffracting nature, together with their ability to reconstruct after encountering an obstacle, allows for multiple trapping and stacking of particles. This was shown by trapping a particle at various positions within the Bessel beam central maximum column. A new theoretical model for determining how a Bessel beam will reconstruct after any arbitrary obstacle was proven experimentally. It is an efficient method that can be used in conjunction with trapping of the various biological samples, such as DNA strands.

Particles have also been manipulated by simply changing the polarisation of the beam. Circularly polarised light was shown to rotate birefringent calcite particles at a constant speed, demonstrating the transfer of spin angular momentum from light to a material. Angular momentum of light consists of both spin and orbital angular momentum. Therefore it was also demonstrated that orbital angular momentum can be transferred to particles. By making use of a spatial light modulator, vortex beams of different orders were generated and directed onto a sample, where rotation was clearly observed.

Lastly, a novel beam shape was implemented into the tweezing setup, that of a super-Gaussian beam. This beam shape displays interesting properties that can be employed in optical tweezing. The steep intensity gradient offers an increased lateral trapping strength compared with a Gaussian beam. Experimental results showed the trapping ability of a SG beam, which will be followed with trap characterising experiments.

The experimental results obtained from each of the tweezing setups form a foundation for future work in the field, by either applying these techniques to other fields, or by generating novel beam shapes and further develop the technique.

6.2 Future Work

The construction of a functional optical tweezer, with known dimensions and trapping strength, allows the technique to be applied in any of the many fields that have already found it to be extremely useful. A basic, single trap can be readily used in biological studies. By focusing less on the physics involved in tweezing and more on the properties of a particular sample, biologists have a valuable tool for probing any of the many different cellular samples.

Microfluidics has already begun to make use of tweezers to rotate microgears. This is the beginning of using tweezers within this field and as new micro-structures are produced, tweezers will be involved in powering more complex micromachines. Of course, the solutions used in microfluidics can also be analysed to some extent. The flow rate of a fluid depends on the depth at which the measurement was taken. Optical tweezers will allow micro-systems to be constructed to exact specifications, producing more reliable results.

The combination of biology and microfluidics has also been shown by [39], where the effect of a stress solution on a yeast cell was efficiently demonstrated using the techniques of optical tweezing.

Naturally, novel beam shapes can be used within a tweezer, which may reveal even more applications. Super-Gaussian beams had not been used in tweezing, but their uniform peak intensity may offer new and interesting applications. Spatial light modulators allow a variety of beams to be generated easily and quickly, without changing much in the tweezing setup. It has also been used to generate multiple trapping sites with a single incoming beam. A series of phase patterns can then be combined and run to form a ‘movie’ of phase patterns. Each trap can then be manipulated individually from the others [65], allowing complex shapes to be made, including three dimensional structures. This versatility can be used in microfluidics to measure more than one parameter at a time, or flow rates at two different depths.

Although optical tweezing is over 30 years old, the applications of this technique continue to spread to more and more research fields.

Appendix A

Spatial Light Modulator

As the name suggests, a spatial light modulator or SLM places a spatially-varying modulation on a beam of light. This modulation can affect the beam's intensity or phase or both.

There are two distinct types of SLMs; an electrically addressed and an optically addressed SLM. The latter creates and changes an image by directing light encoded with an image onto the front or back surface of the SLM. The brightness of each pixel is detected by a photosensor and replicated using liquid crystals. The image on an electrically addressed SLM is created and altered electronically, where the input is typically received from a VGA or DVI input.

In section 4.2.2, fig. 4.9 (b) showed the different orders reflected off the SLM. These orders are not a property of a particular phase pattern and will be present without a phase pattern. They can be explained by first considering Young's double slit experiment, fig. A.1, where light is observed to form bright spots at points of constructive interference.

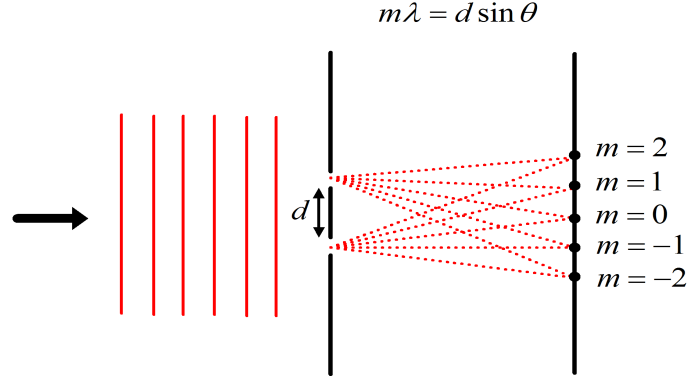


Figure A.1: After passing through the two slits, light interferes in a constructive and destructive manner, producing bright spots at points of constructive interference.

Similarly, the ‘dead spaces’ between the pixels of an SLM equate to those regions where light has been blocked in Young’s experiment. That is, each pixel acts as a slit, resulting in a grid of slits which produce a grid of diffraction orders reflecting off the SLM. This is an amplitude effect, and will therefore be observed even without a phase pattern on the SLM.

When a phase pattern is placed on the SLM, each pixel shifts the phase by a different amount. Therefore the phase on each pixel can be altered in such a way that the reflected light from each pixel interferes constructive at one particular point, forming a very bright spot. There is still constructive interference at other points, but ideally they are not as bright. To optimise this effect and obtain a single bright spot, an efficiency equation is used [66]:

$$\eta = \left(\frac{\sin(\pi m/N)}{\pi m/N} \right)^2, \quad (\text{A.1})$$

where p, n are any integers, $m = pN + 1$ is the m^{th} order, and $N = 2^n$. Here N is the number of levels used to approximate the changing phase of a function. Fig. A.2 demonstrates an approximation of a straight line using steps. The more steps used to approximate the function, the better the efficiency.

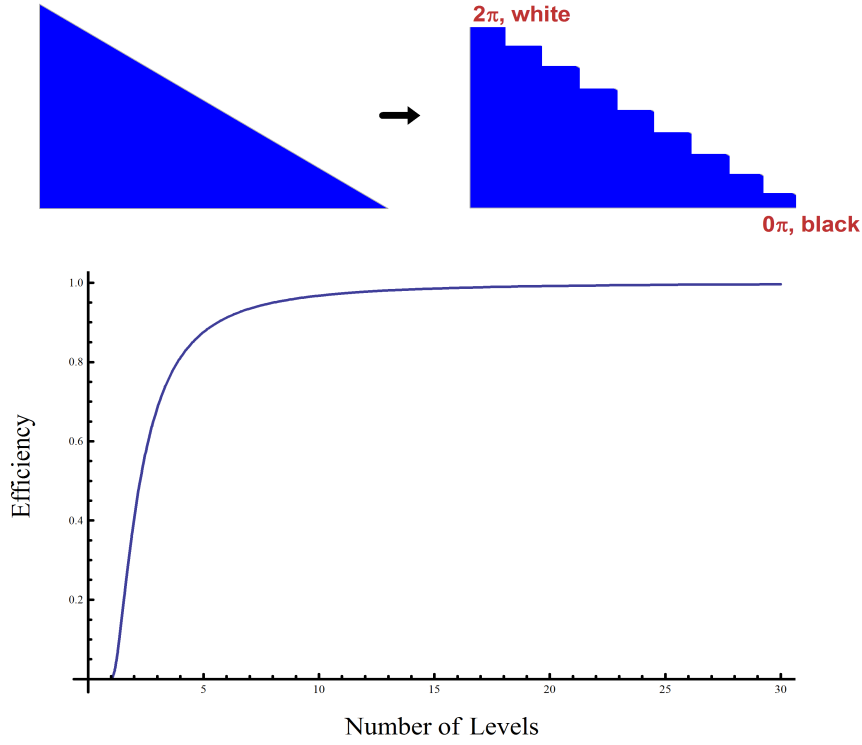


Figure A.2: SLMs cannot produce continuous phase patterns and therefore an approximation is made using a number of levels. The more levels used to approximate the function, the better the efficiency.

A high efficiency means that most of the light will be reflected to one particular spot. This spot is the $m = 1$ diffraction order, which corresponds to $p = 0$. Fig. A.3 illustrates the efficiency at the various orders for two different level systems, $N = 1$ and $N \rightarrow \infty$. For $N = 1$, there are at least two spots of equal brightness, where the spot brightness dims as the orders increase.

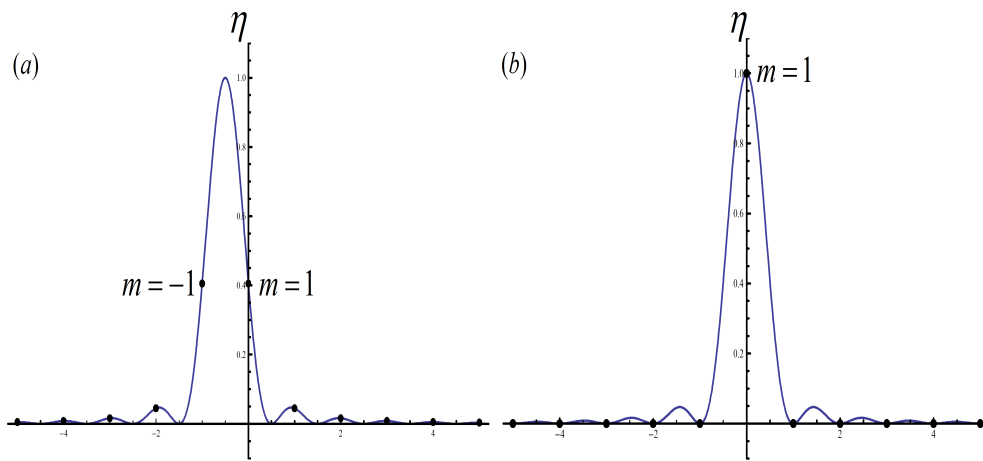


Figure A.3: (a) A two level system. The first order has only 41% of the energy. (b) An ideal system where the first order has all of the energy, while the other orders have zero.

Appendix B

List of Publications

Peer reviewed Journal Papers

1. I. A. Litvin, **M. G. McLaren** and A. Forbes, ‘A conical wave approach to calculating Bessel-Gauss beam reconstruction after complex obstacles’, *Opt. Commun.* (In press), 2008.
2. **M. G. McLaren**, A. Forbes and E. Sideras-Haddad, ‘Increased radial trapping strength with Super-Gaussian beams’, To be submitted to *J. of Mod. Opt.*, 2009.
3. **M. G. McLaren**, A. Forbes and E. Sideras-Haddad, ‘Precision measurements of microscopic forces and torques using optical tweezing techniques’, To be submitted to *S. Afr. J. Sci.*, 2009.

International Conference Papers

1. I. A. Litvin, **M. G. McLaren** and A. Forbes, ‘Propagation of obstructed Bessel and BesselGauss beams’, In *Laser Beam Shaping IX*, edited by Andrew Forbes & Todd E. Lizotte, Proceedings of SPIE Vol. 7062, SPIE Bellingham, WA, 706218-1, 2008.
2. **M. G. McLaren**, A. Forbes, R. Sparrow, and E. Sideras-Haddad, ‘Optical Tweezing and Micromanipulation’, In proceedings of *The US-African advanced Studies Institute of Environmental and Biological Applications of Lasers*, Cairo, 2008.

National Conference Papers

1. **M. G. McLaren**, A. Forbes, R. Sparrow, and E. Sideras-Haddad, ‘Optical Tweezing and Trapping’, In proceedings of *52nd South African Institute of Physics Conference, Lasers, Optics & Spectroscopy*, University of the Witwatersrand, Johannesburg, 2007.

2. **M. G. McLaren**, A. Forbes and E. Sideras-Haddad, ‘Optical Tweezing’, In proceedings of *53rd South African Institute of Physics Conference, Lasers, Optics & Spectroscopy*, University of Limpopo, 2008.
3. **M. G. McLaren**, A. Forbes and E. Sideras-Haddad, ‘Non-invasive Manipulation of cells’, In proceedings of *53rd South African Institute of Physics Conference, Applied & Industrial*, University of Limpopo, 2008.

Bibliography

- [1] A. Ashkin. Acceleration and trapping of particles by radiation pressure. *Phys. Rev. Lett.*, 24(4):156–159, 1970.
- [2] A. Chiou. Cellular mechanics and dna mechanics via optical manipulation. *Lasers and Electro-Optics*, 46:114–115, 2005.
- [3] R. A. Flynn, B. Shao, M. Chachisvilisa, M. Ozkanb, and S. C. Esenera. Counter-propagating optical trapping system for size and refractive index measurement of microparticles. *Biosens. Bioelectron.*, 21:1029–1036, 2006.
- [4] A. Ashkin and J. M. Dziedzic. Optical levitation by radiation pressure. *Appl. Phys. Lett.*, 19(8):283–285, 1971.
- [5] A. Ashkin and J.M. Dziedzic. Optical levitation of liquid drops by radiation pressure. *Science, New Series*, 187(4181):1073–1075, 1975.
- [6] A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm, and S. Chu. Observation of a single-beam gradient force optical trap for dielectric particles. *Opt. Lett.*, 11(5):288–290, 1986.
- [7] S. Chu. The manipulation of neutral particles. Nobel Lecture, 1997.
- [8] C. N. Cohen-Tannoudji. Manipulating atoms with photons. Nobel Lecture, 1997.
- [9] W. D. Phillips. Laser cooling and trapping of neutral atoms. Nobel Lecture, 1997.
- [10] B. E. A. Saleh and M. C. Teich. *Fundamentals of Photonics*. John Wiley & Sons, Inc., Hoboken, New Jersey, 2007.
- [11] J. Durnin. Exact solutions for nondiffracting beams. i. the scalar theory. *J. Opt. Soc. Am. A*, 4(4):651–654, 1987.

- [12] J. Durnin, J.J. Miceli, and J.H. Eberly. Diffraction-free beams. *Phys. Rev. Lett.*, 59(15):1499–1501, 1987.
- [13] F. Gori, G. Guattari, and C. Padovani. Bessel-gauss beams. *Opt. Commun.*, 64(6):491–495, 1987.
- [14] J. Arlt, V. Garces-Chavez, W. Sibbett, and K. Dholakia. Optical micro-manipulation using a bessel light beam. *Opt. Commun.*, 197:239–245, 2001.
- [15] J. Lu and J. F. Greenleaf. Diffraction-limited beams and their applications for ultrasonic imaging and tissue characterization. 1992. In *New Developments in Ultrasonic Transducers and Transducer Systems*, edited by Fredrick L. Lizzi. Proceedings of SPIE Vol. 1733 (SPIE, Bellingham, WA), 0-81 94-0906-5/92.
- [16] S. Sogomonian, S. Klewitz, and S. Herminghaus. Self-reconstruction of a bessel beam in a nonlinear medium. *Opt. Commun.*, 139:313–319, 1997.
- [17] Z. Bouchal and M. Bertolotti. Self-reconstruction of wave packets due to spatio-temporal couplings. *J. Mod. Opt.*, 47:1455–1467, 2000.
- [18] P. Fischer, H. Little, R. L. Smith, C. Lopez-Mariscal, C. T. A. Brown, W. Sibbett, and K. Dholakia. Wavelength dependent propagation and reconstruction of white light bessel beams. *J. Opt. A: Pure Appl. Opt.*, 8:477–482, 2006.
- [19] M. Anguiano-Morales, M. Mendez-Otero, and D. Iturbe-Castillo. Conical dynamics of bessel beams. *Opt. Eng.*, 46(7):078001–(1–9), 2007.
- [20] I. A. Litvin, M. G. McLaren, and A. Forbes. A conical wave approach to calculating bessel-gauss beam reconstruction after complex obstacles. *Opt. Commun.*, 282:1078–1082, 2009.
- [21] I. A. Litvin, M. G. McLaren, and A. Forbes. Propagation of obstructed bessel and bessel-gauss beams. 2008. In *Laser Beam Shaping IX*, edited by Andrew Forbes, Todd E. Lizotte, Proceedings of SPIE Vol. 7062 (SPIE, Bellingham, WA), 706218-1.
- [22] A. Ashkin. Forces of a single-beam gradient laser trap on a dielectric sphere in the ray optics regime. *Biophys. J.*, 61:569–582, 1992.
- [23] N.B. Simpson, D. McGloin, K. Dholakia, L. Allen, and M. J. Padgett. Optical tweezers with increased axial trapping efficiency. *J. Mod. Opt.*, 45(9):1943–1949, 1998.

-
- [24] K.C. Neuman and S.M. Block. Optical trapping. *Rev. Sci. Instrum.*, 75(9):2787–2809, 2004.
- [25] A. Rohrbach and E. H. K. Stelzer. Trapping forces, force constants, and potential depths for dielectric spheres in the presence of spherical aberrations. *Appl. Opt.*, 41(13):2494–2507, 2002.
- [26] S.P. Smith, S.R. Bhalotra, A.L. Brody, B.L. Brown, E.K. Boyda, and M. Prentiss. Inexpensive optical tweezers for undergraduate laboratories. *Am. J. Phys.*, 67(1):26–35, 1998.
- [27] K. Svoboda and S. M. Block. Biological applications of optical forces. *Annu. Rev. Biophys. Biomol. Struct.*, 23:247–285, 1994.
- [28] M. W. Berns, Y. Tadir, H. Liang, and B. Tromberg. Laser scissors and tweezers. *Methods in Cell Biology*, 55(50):71–98, 1998.
- [29] M. Parry-Hill, R. T. Sutter, and M. W. Davidson. Basic concepts and formulas in microscopy. Available: <http://www.microscopyu.com/tutorials/java/kohler/index.html>, 2000. Last accessed 24 October 2008.
- [30] M. Abramowitz and M. W. Davidson. Microscope illumination. Available: <http://www.olympusmicro.com/primer/anatomy/illumination.html>, 2000. Last accessed 24 October 2008.
- [31] K. Dholakia, P. Reece, and M. Gu. Optical micromanipulation. *Chem. Soc. Rev.*, 37:42–55, 2007.
- [32] J. Bechhoefer and S. Wilson. Faster, cheaper, safer optical tweezers for the undergraduate laboratory. *Am. J. Phys.*, 70(4):393–400, 2001.
- [33] D. McGloin and K. Dholakia. Bessel beams: diffraction in a new light. *J. Contemp. Phys.*, 46(1):15–28, 2005.
- [34] C. L. Kuyper and D. T. Chui. Optical trapping: A versatile technique for biomanipulation. *Appl. Spectrosc.*, 56(11):300A–312A, 2002.
- [35] Y. Arai, R. Yasuda, K. Akashi, Y. Harada, H. Miyata, K. Kinoshita, and H. Itoh. Tying a molecular knot with optical tweezers. *Nature (London)*, 399:446–448, 1999.
- [36] A. Ehrlicher, T. Betz, B. Stuhmann, D. Koch, V. Milner, M. G. Raizen, and J. Ka. Guiding neuronal growth with light. *Proc. Natl. Acad. Sci. USA*, 99(25):16024–16028, 2002.

-
- [37] D. J. Carnegie, D. J. Stevenson, M. Mazilu, F. Gunn-Moore, and K. Dholakia. Guided neuronal growth using optical line traps. *Opt. Express*, 16(14):10507–10517, 2008.
- [38] T. M. Squires and S. R. Quake. Microfluidics: Fluid physics at the nanoliter scale. *Rev. Mod. Phys.*, 77:977–1026, 2005.
- [39] E. Eriksson, J. Enger, B. Nordlander, N. Erjavec, K. Ramser, M. Goksor, S. Hohmann, T. Nystrom, and D. Hanstorp. A microfluidic system in combination with optical tweezers for analyzing rapid and reversible cytological alterations in single cells upon environmental changes. *The Royal Society of Chemistry*, 7:71–76, 2007.
- [40] R. J. Hopkins, L. Mitchem, A. D. Ward, and J. P. Reid. Control and characterisation of a single aerosol droplet in a single-beam gradient-force optical trap. *Phys. Chem. Chem. Phys.*, 6:4924–4927, 2004.
- [41] J. Buajarern, L. Mitchem, and J. P. Reid. Manipulation and characterization of aqueous sodium dodecyl sulfate/sodium chloride aerosol particles. *J. Phys. Chem. A*, 111:13038–13045, 2007.
- [42] M. D. Summers, D. R. Burnham, and D. McGloin. Trapping solid aerosols with optical tweezers: A comparison between gas and liquid phase optical traps. *Opt. Express*, 16(11):7739–7747, 2008.
- [43] L. Paterson, E. Papagiakoumou, G. Milne, V. Garcs-Chvez, S. A. Tatarkova, W. Sibbett, F. J. Gunn-Moore, P. E. Bryant, A. C. Riches, and K. Dholakia. Light-induced cell separation in a tailored optical landscape. *Appl. Phys. Lett.*, 87:123901–(1–3), 2005.
- [44] D. J. Griffiths. *Introduction to Electrodynamics*. Prentice-Hall International, Inc., Upper Saddle River, New Jersey, 1999.
- [45] S. M. Barnett and L. Allen. Orbital angular momentum and nonparaxial light beams. *Opt. Commun.*, 110:670–678, 1994.
- [46] H. He, M. E. J. Friese, N. R. Heckenberg, and H. Rubinsztein-Dunlop. Direct observation of transfer of angular momentum to absorptive particles from a laser beam with a phase singularity. *Phys. Rev. Lett.*, 75(5):826–829, 1995.
- [47] M. E. J. Friese, T. A. Nieminen, N. R. Heckenberg, and H. Rubinsztein-Dunlop. Optical alignment and spinning of laser-trapped microscopic particles. *Nature (London)*, 394:348–351, 1998.

-
- [48] N. B. Simpson, K. Dholakia, L. Allen, and M. J. Padgett. Mechanical equivalence of spin and orbital angular momentum of light: an optical spanner. *Opt. Lett.*, 22(1):52–54, 1997.
- [49] R. A. Beth. Mechanical detection and measurement of the angular momentum of light. *Phys. Rev.*, 50:115–125, 1936.
- [50] J. H. Poynting. The wave motion of a revolving shaft, and a suggestion as to the angular momentum in a beam of circularly polarised light. 1909. Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, Vol. 82, No. 557 (Jul. 31, 1909), pp. 560-567.
- [51] D. N. Moothoo, J. Arlt, R.S. Conroy, F. Akerboom, A. Voit, and K. Dholakia. Beths experiment using optical tweezers. *Am. J. Phys.*, 69(3):271–276, 2001.
- [52] T. A. Nieminen, N. R. Heckenberg, and H. Rubinsztein-Dunlop. Optical measurement of microscopic torques. *J. Mod. Opt.*, 48:405–413, 2001.
- [53] E. Hecht. *Optics*. Addison Wesley Longman, Inc., Reading, Massachusetts, 1998.
- [54] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman. Orbital angular momentum of light and the transformation of laguerre-gaussian laser modes. *Phys. Rev. A*, 45(11):8185–8189, 1992.
- [55] M. E. J. Friese, J. Enger, H. Rubinsztein-Dunlop, and N. R. Heckenberg. Optical angular-momentum transfer to trapped absorbing particles. *Phys. Rev. A: At. Mol. Opt. Phys.*, 54(2):1593–1596, 1996.
- [56] J. E. Curtis and D. G. Grier. Structure of optical vortices. *Phys. Rev. Lett.*, 90:133901, 2003.
- [57] A. Dudley, M. McLaren, and A. Forbes. Vortex beams. *Popular Mechanics (South Africa)*, 7(4):74–78, 2008.
- [58] G. Kroner, S. Parkin, N. R. Heckenberg, and H. Rubinsztein-Dunlop. Characterization of optically driven fluid stress fields with optical tweezers. *Phys. Rev. E*, 72:031507, 2005.
- [59] J. Leach, H. Mushfique, R. di Leonardo, M. Padgett, and J. Cooper. An optically driven pump for microfluidics. *The Royal Society of Chemistry*, 6:735739, 2006.

-
- [60] S. L. Neale, M. P. MacDonald, K. Dholakia, and T. F. Krauss. All-optical control of microfluidic components using form birefringence. *Nat. Mater.*, pages 1–4, 2005.
 - [61] S. De Silvestri, P. Laporta, V. Magni, and O. Svelto. Solid-state laser unstable resonators with tapered reflectivity mirrors: The super-gaussian approach. *IEEE J. Quantum Electron.*, 24(6):1172–1177, 1988.
 - [62] F. Gori. Flattened gaussian beams. *Opt. Commun.*, 107:335–341, 1994.
 - [63] C. J. R. Sheppard and S. Saghafi. Flattened light beams. *Opt. Commun.*, 132:144–152, 1996.
 - [64] M. Santarsiero and R. Borghi. Correspondence between super-gaussian and flattened gaussian beams. *J. Opt. Soc. Am. A*, 16(1):188–190, 1999.
 - [65] J. E. Curtis, B. A. Koss, and D. G. Grier. Dynamic holographic optical tweezers. *Opt. Commun.*, 207:169–175, 2002.
 - [66] L. Burger, I. A. Litvin, and A. Forbes. Simulating atmospheric turbulence using a phase-only spatial light modulator. *S. Afr. J. Sci.*, 104:129–134, 2008.



A conical wave approach to calculating Bessel–Gauss beam reconstruction after complex obstacles

Igor A. Litvin^{a,b}, Melanie G. McLaren^{a,c}, Andrew Forbes^{a,d,*}

^a CSIR National Laser Centre, P.O. Box 395, CSIR Campus Meiring Naude Road, Gauteng, Pretoria 0001, South Africa

^b Laser Research Institute, University of Stellenbosch, Stellenbosch 7602, South Africa

^c School of Physics, University of the Witwatersrand, Private Bag 3, P.O. Wits 2050, Johannesburg, South Africa

^d School of Physics, University of KwaZulu-Natal, Private Bag X54001, Durban 4000, South Africa

ARTICLE INFO

Article history:

Received 25 August 2008

Received in revised form 5 November 2008

Accepted 20 November 2008

Keywords:

Bessel–Gauss beams

Reconstruction

Ray optics

ABSTRACT

We present an efficient method to accurately calculate the reconstruction properties of a Bessel–Gauss beam after an arbitrary obstruction. Our method makes use of the well-known conical wave features of Bessel–Gauss beams, by considering the projection of the obstacle in space as a result of the travelling conical waves that produce the Bessel–Gauss beams. This approach, of projecting the obstacle boundaries rather than propagating the field itself, results in fast and accurate predictions of the field after any obstacle. We successfully predict the reconstruction properties of a Bessel–Gauss beam after three complex obstacles by calculating the boundaries of the various projected regions. This method has no restrictions regarding the geometry of the obstacle or its placement in the Bessel–Gauss field.

© 2008 Elsevier B.V. All rights reserved.

1. Introduction

The investigation into Bessel and Bessel–Gauss beams has been a topic of immense research during the past 20 years, due to the interesting properties they display. Bessel beams (BBs) represent a class of so-called diffraction free solutions to the Helmholtz equation, and have been studied extensively since the seminal work of Durnin et al. [1] in the late 1980s. One of the simplest solutions illustrated by Durnin [2], describes a monochromatic wave with an electric field envelope that is a zeroth-order Bessel function of the first kind, given by $J_0(k_r r)$, where k_r is the radial component of the wave vector k and r denotes the distance from the z -axis. This was the first mathematical concept of the Bessel beam, which ideally consists of an infinite number of rings of light, carrying an infinite amount of power over an infinite area. Experimentally, it is not possible to generate such a beam, and so an approximation is made in the form of a Bessel function enveloped by a Gaussian profile, thereby limiting the energy carried by the field to some finite value.

The electromagnetic field $u(r, z)$ of a Bessel–Gauss beam (BGB) propagating in free-space in the z -direction is given by [3]:

$$u(r, z) = A \frac{\omega_0}{\omega(z)} \exp \left[i \left(k - \frac{k_r^2}{2k} \right) z - i \Phi(z) \right] J_0[k_r r / (1 + iz/z_r)], \quad (1)$$

$$\times \exp \left[\left(\frac{-1}{\omega^2(z)} + \frac{ik}{2R(z)} \right) \left(r^2 + \frac{k_r^2 z^2}{k^2} \right) \right],$$

where A is an amplitude factor, which may be complex, and

$$z_r = \pi \omega_0^2 / \lambda$$

$$\omega(z) = \omega_0 \sqrt{1 + (z/z_r)^2}$$

$$R(z) = z[1 + (z_r/z)^2]$$

$$\Phi(z) = \tan^{-1}(z/z_r)$$

These equations represent the Rayleigh length, the beam width, the radius of curvature of the wavefronts and the Guoy phase of a Gaussian beam, respectively. The Rayleigh length determines the distance over which a Gaussian beam can propagate without diverging significantly and depends on the beam width at the waist, ω_0 , and the wavelength of the beam, λ . These “pseudo non-diffracting” beams maintain the properties associated with ideal Bessel beams, but only over a finite distance, $z_{\max}$ (see Fig. 1). Bessel beams not only exhibit diffraction free propagation, but also reconstruction of the amplitude and phase of the beam after encountering an obstruction, provided that the spatial extent of the Bessel beam is larger than that of the obstacle. For such beams, there is a minimum distance behind an obstacle of radius $\frac{1}{2} D$ before reconstruction occurs,

* Corresponding author. Address: CSIR National Laser Centre, P.O. Box 395, CSIR Campus Meiring Naude Road, Gauteng, Pretoria 0001, South Africa. Tel.: +27 12 8412368; fax: +27 12 841 3152.

E-mail address: aforbes1@csir.co.za (A. Forbes).

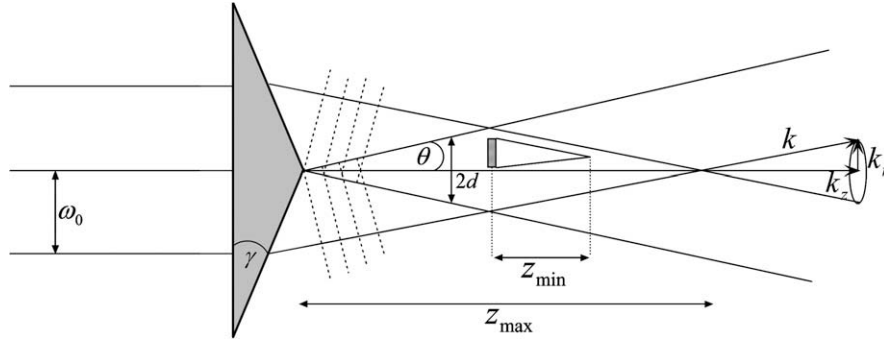


Fig. 1. Diagram illustrating the finite propagation distance of a Bessel beam generated by an axicon of opening angle γ . If an obstacle is placed in the Bessel field, there is a minimum distance, $z_{\min}$, before reconstruction of the Bessel beam occurs.

$$z_{\min} = D \frac{k_z}{2k_r} = \frac{D}{2 \tan \theta}, \quad (2)$$

where $\theta = (n-1)\gamma$ is the opening angle of the cone, n is the refractive index and γ is the opening angle of the axicon.

This property has been used in a variety of applications, including optical tweezing [4] and medical imaging [5], and is therefore an intriguing property worth investigating. Reconstruction has been well explained by firstly considering a Bessel beam as a superposition of plane waves, whose transverse amplitudes reform after encountering an obstruction [6]. Theoretical studies of this property describe reconstruction under varying conditions, including in a nonlinear medium [7] and by wave packets due to spatial-temporal links [8]. The reliability of the periodical self-reconstruction, as well as the wavelength dependence of white light Bessel beams have been examined [9].

Recently, the phenomenon of reconstruction was eloquently explained by considering the dynamics of the conical waves that form a Bessel beam [10]. Essentially, the cone of wave vectors previously mentioned, consists simultaneously of two conical waves, an incoming and an outgoing wave, which can be represented by Hankel functions [11] of the second, $H_0^{(2)}(k_r r)$, and first, $H_0^{(1)}(k_r r)$, kind, respectively:

$$H_0^{(1)}(k_r r) \exp(ik_z z) = [J_0(k_r r) + iN(k_r r)] \exp(ik_z z), \quad (3)$$

$$H_0^{(2)}(k_r r) \exp(ik_z z) = [J_0(k_r r) - iN(k_r r)] \exp(ik_z z), \quad (4)$$

where $N(k_r r)$ is the zero-order Newman function.

The incoming conical wave transforms into the outgoing conical wave and the superposition of these two conical waves is crucial in obtaining propagation invariant beams. Within a Bessel beam, although locally the light travels parallel to the axis of propagation, the energy flux follows the direction determined by the cone of wave vectors. For this reason, reconstruction after an obstruction is observed. That is, the reconstructed beam results from light that was not obscured by the obstacle. Despite this fluent explanation by Anguiano-Morales et al. [10], the numerical simulations demonstrated resorted back to solving the Helmholtz equation.

Although extensive work has been devoted to the property of reconstruction, the methods of modelling this characteristic are limited in accuracy and speed, and are yet to calculate Bessel fields after obstructions of arbitrary geometry. Conventionally, the Fresnel–Kirchoff diffraction integral together with Babinet's principle has been employed, which results in time consuming calculations. In this paper we present an efficient and accurate technique to predict Bessel and Bessel–Gauss beam propagation after encountering an obstruction of arbitrary geometry and complex orientation (i.e. no symmetry is required in the obstacle under study). The technique is a simple exploitation of the well-known conical wave formalism of Bessel and Bessel–Gauss beams. We consider the

projection of an obstacle in space due to the travelling conical waves that form the Bessel beams, rather than the propagation of the field itself. This is a departure from all previous methods of modelling the propagation after obstacles, and results in fast but accurate predictions, regardless of the complexity of the obstacle. We verify the concept experimentally for three obstacle geometries, and show that despite the method being based on a ray optics approximation, the method is resilient even when diffraction is evident.

2. Conical projection model

The model presented here is based on a geometrical approximation to the propagation of conical waves after obstacles. Consider the schematic in Fig. 1 where an arbitrary obstacle is placed off-centre in the path of a Bessel–Gauss beam, as bounded in the propagation direction by $z_{\max}$. The positive z -axis defines the direction of propagation of the Bessel–Gauss beam which we assume to have a spatial extend of $2d$ at the plane of the obstacle, as shown in Fig. 1. The surface of the obstruction, $S(r, \theta)$, results in a cross-section perpendicular to the z -axis as shown in Fig. 2, where we consider the cross-section passing through points a and b on the boundary of the obstacle. Since only a cross-section is considered, there are only two conical waves (labelled CW_1 and CW_2 , respectively) contributing to the Bessel–Gauss beam at any given point.

It is clear from Fig. 2 that the projection from the two conical waves results in distinct regions defined by (i) one conical wave contribution only, (ii) a shadow region where neither conical wave contributes, and (iii) the Bessel–Gauss beam where both waves contribute. The boundaries that define these regions are mapped by the projection of the obstacle's boundaries along a given z plane. Without any loss of generality, consider an obstacle with cross-section shown in Fig. 2, with full width D , and placed some distance $\rho = a$ from the central axis ($\rho = 0$) of the Bessel–Gauss beam, so that the two extreme edges of the obstacle are defined by $\rho = a$ and $\rho = b$, with $D = b - a$. From this geometry one can derive the projection of the obstruction boundaries due to each conical wave. If projections due to CW_1 are labelled a_1 and b_1 , and similarly a_2 and b_2 due to CW_2 , then one can show that at some distance z after the obstacle we have:

$$\begin{aligned} a_1 &= a + z \tan \theta \\ a_2 &= a - z \tan \theta \\ b_1 &= b + z \tan \theta \\ b_2 &= b - z \tan \theta \end{aligned} \quad (5)$$

Clearly the projection results in the creation of two zones defined by a single conical wave, with the boundaries of these zones moving farther apart at a rate of $2z \tan \theta$. The region behind the obstacle is

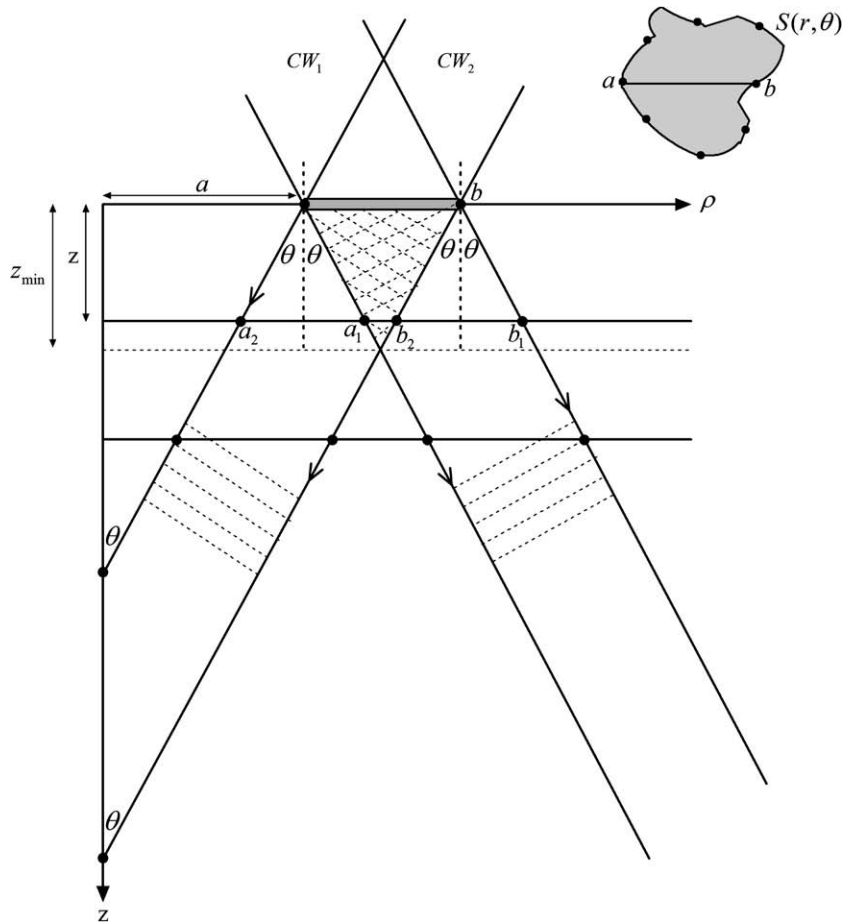


Fig. 2. Diagram illustrating the projection of the obstacle in space by two conical waves, CW_1 and CW_2 . The minimum distance before which the Bessel beam can reconstruct is given by $z_{\min}$, resulting in a shadow region where neither conical wave contributes.

then defined as follows (for a given z propagation and a chosen azimuthal angle ϕ):

$-d \leq \rho \leq a_2$ CW_1 and CW_2 add to form a complete Bessel–Gauss beam;

$a_2 < \rho < a_1$ CW_1 contribution only;

$a_1 < \rho < b_2$ no contribution from either CW_1 or CW_2 , resulting in a shadow region;

$b_2 < \rho < b_1$ CW_2 contribution only;

$b_1 \leq \rho \leq d$ CW_1 and CW_2 add to form a complete Bessel–Gauss beam.

If the process described above is carried out for all points along the boundary of an obstruction, then the complete movement of the boundaries of each region can easily be computed. Herein lies the power of this approach, for with full information of the boundary projections, the field at every point may be deduced. Thus with knowledge of the conical wave projections of the boundary of the obstacle, the complete intensity pattern at any position after an obstacle, regardless of the complexity of the geometry of the obstacle, may readily be found.

Note that this method does not require that the field reconstruct: full, partial or no reconstruction may with equal ease be computed. If reconstruction is to take place, then we require that at the distance $z = z_{\min}$ where $b_2 \leq a_1$ the shadow region vanishes

and the Bessel–Gauss beam is said to have reconstructed. The value for $z_{\min}$ can easily be found from Eq. (5) by solving for the equality $b_2 = a_1$:

$$\begin{aligned} b - z \tan \theta &= a + z \tan \theta \\ \Rightarrow b - a &= D = 2z \tan \theta \\ \Rightarrow z_{\min} &= \frac{D}{2 \tan \theta} \end{aligned}$$

which is consistent with the expression in Eq. (2), as expected.

It is worth pointing out some salient points in how these regions develop: firstly, the single conical wave regions move apart at a rate of $2z \tan \theta$. Thus in an off-centred geometry one expects the boundaries defining the Bessel–Gauss beam to move apart, in opposite directions, with one passing through the centre of the original Bessel–Gauss field. Secondly, any combinations of points on the boundary of an obstacle that find themselves equidistant from the central propagation axis of the Bessel–Gauss beam will result in boundaries that evolve in identical fashion. This latter fact can be exploited to simplify the calculation for arbitrary obstacles.

To illustrate the power of the approach, consider the propagation of a Bessel beam after an obstacle placed vertically in the field, as shown in Fig. 3. A cross-section of the field at the obstacle (Fig. 3a) shows only an immediate vertical shadow (black) and the Bessel field (white). Some distance later (Fig. 3b) three zones are defined: the reduced shadow (black), the Bessel field (white) and an enveloping area comprising of only one conical wave (grey). Propagation beyond $z = z_{\min}$ (Fig. 3c and d) results in the partial reconstruction of the original Bessel field (white lobes) surrounded

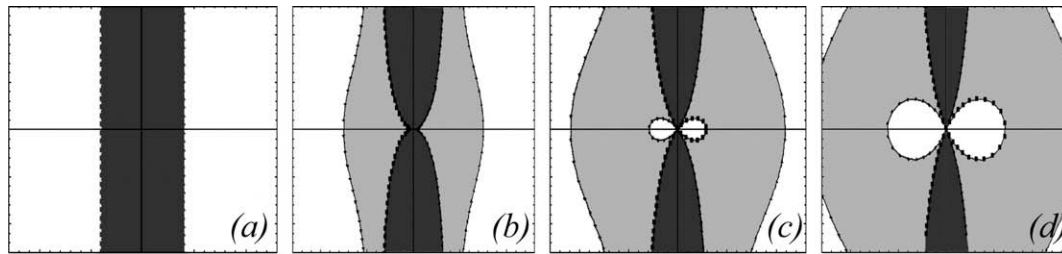


Fig. 3. Calculation of the propagation of a Bessel–Gauss beam after encountering an obstacle in the form of a vertical wire-like structure. (a) Initially the Bessel–Gauss beam (white) is obscured by the wire (black); (b) after propagating $z < z_{\min}$, a region with a single conical wave contribution forms (grey); (c) and (d) after propagating $z > z_{\min}$, the Bessel–Gauss beam shows partial reconstruction (white region) surrounded by a large single conical wave region (grey).

by a large field comprising of only one conical wave. Complete reconstruction does not take place in this scenario simply because the obstruction extends across the entire field in the vertical direction. We show in the section to follow that the calculated predictions made here are in excellent agreement with experiment.

3. Experimental methodology and results

Fig. 4 shows the experimental set-up used in generating the Bessel–Gauss beam. A 660 nm, 100 mW diode laser beam was expanded through a $5\times$ telescope before illuminating an axicon with an opening angle of $\gamma = 5^\circ$. The obstruction was placed on a thin transparent cover slip and positioned within the Bessel beam at a distance of $\frac{1}{2} z_{\max}$ (45 mm) from the axicon. The resulting obstructed beam was imaged onto a screen with a suitable magnification ($57\times$) to observe the intensity variations during propagation. The image plane could be varied continuously by means of a translation stage so as to allow measurement of the beam from fully obstructed to fully reconstructed. The geometries of the three test obstructions – a centred circle of diameter $D \sim 325 \mu\text{m}$ (approximately), an off centre oval $D \sim 325 \mu\text{m}$ and a vertical wire of width $D \sim 140 \mu\text{m}$ – are shown in Fig. 5. In each case the obstruction is opaque so that immediately after the obstruction a dark shadow region is evident, surrounded by an unperturbed Bessel–Gauss beam. We then use the method described earlier to calculate the boundary shape and sizes at various distances after the obstacles, from completely obstructed to partially or fully reconstructed.

Fig. 5 shows a sequence of measured Bessel–Gauss beam intensities during propagation after the three obstructions, with insets to the experimental images depicting the calculated boundaries of the conical waves from the source. Fig. 5a–d shows the change in the Bessel–Gauss beam intensity during reconstruction when the starting obstacle is an approximation to a centred circle, a scenario with radial symmetry. As the propagation distance from the obstruction was increased, so the shadow regions appeared to move precisely as predicted. Following the discussion in the previous section we noted that two boundaries appeared due to the projection of the obstacle by the two conical waves from the source. The intersection of the two boundaries defines the true shadow region, while the region between the two boundaries defines a single conical wave contribution. Elsewhere the original Bessel–Gauss beam is apparent due to the summation of both conical waves. This is evident in the sequence from (b) to (d). The final image at (d) was taken at a propagation distance of $z = 17 \text{ mm}$, exceeded $z_{\min} = 7.4 \text{ mm}$ so that complete reconstruction had taken place. This is predicted by the theory and has been confirmed experimentally. Verification of the reconstruction at the centre of the field, where the original obstacle was, is clear by the zoomed inset of the centre of beam in (d).

A similar sequence is shown in Fig. 5e–h and i–l. In these cases the radial symmetry is purposely broken. The sequence (e)–(h) illustrates the accuracy of the method for an arbitrary shaped obstacle that is placed off-centre in the Bessel–Gauss beam path. The prediction by the calculation method of the development of a “figure of eight” boundary, as is observed experimentally, is striking.

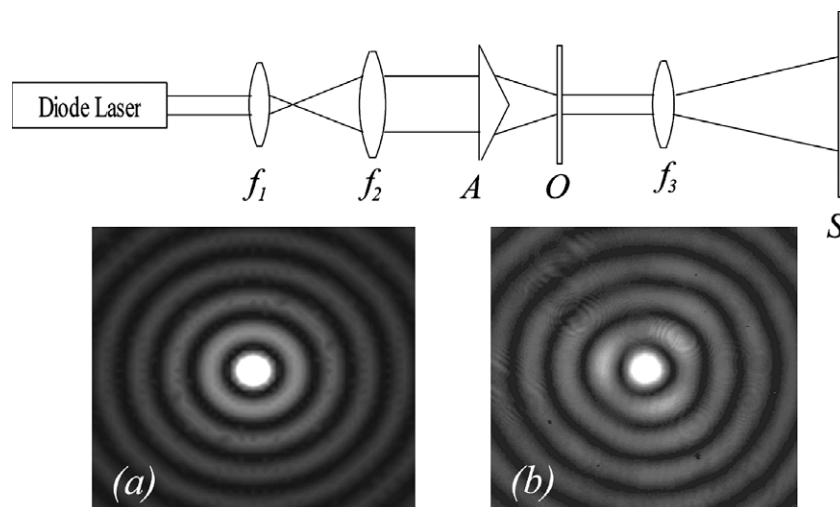


Fig. 4. Experimental design for generating a Bessel–Gauss beam: a telescope with lenses $f_1 = 50 \text{ mm}$ and $f_2 = 250 \text{ mm}$ expands the beam before illuminating the axicon (A). The obstacle, situated on a cover slip (O), was placed 45 mm from the axicon and was imaged onto a screen (S) by lens $f_3 = 25 \text{ mm}$. The inserts represent the theoretical prediction (left) and the experimentally measured (right) Bessel–Gauss beam.

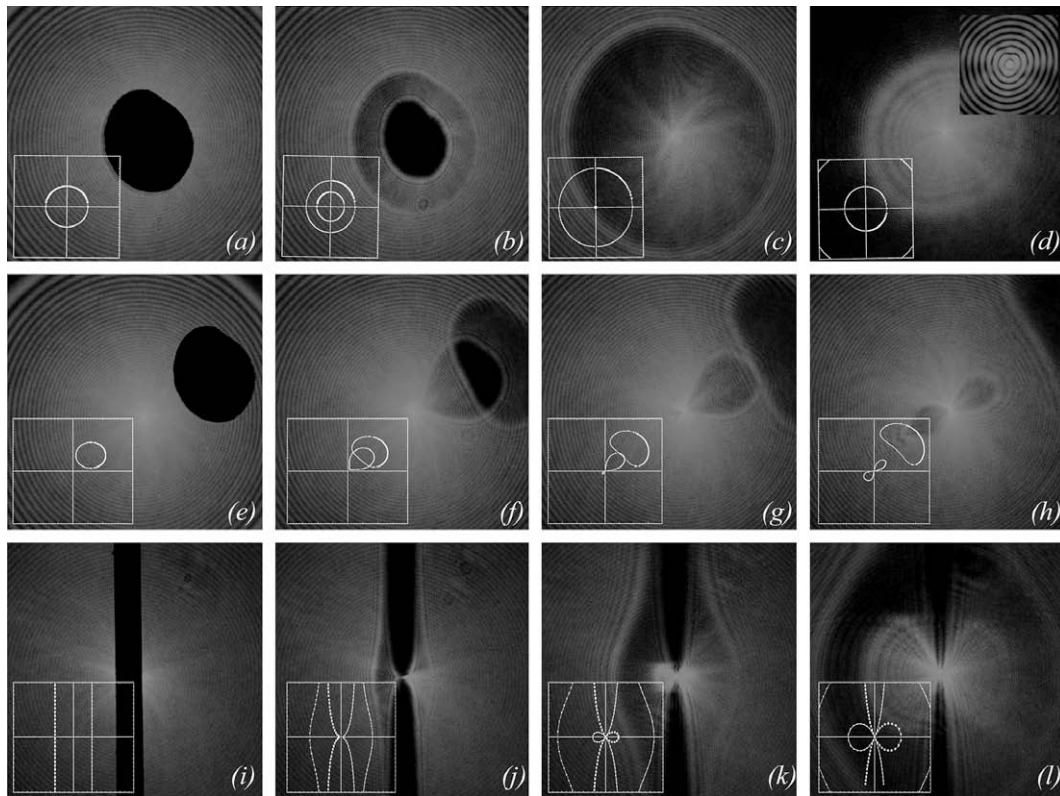


Fig. 5. Experimental images of three different obstacle geometries within a generated Bessel–Gauss beam, with the theoretical predictions of the shadow boundaries inset in the bottom left corners. The experimental images and calculated insets are in excellent agreement for all obstacle types considered. (a)–(d) propagation and reconstruction after a centred circle; (e)–(h) propagation and reconstruction after an off-centred oval; and (i)–(l) propagation and partial reconstruction after a narrow vertical obstruction.

ing. Likewise the vertical wire (Fig. 5i) is an interesting case, as here there is a region where no contribution from one of the conical waves is possible, as the obstruction extends across the entire Bessel–Gauss beam in the vertical plane following the wire. Again the agreement between calculation and experiment is very good.

From the experimental results, it is clear that our method correctly maps the boundaries of the reconstructed Bessel beam, regardless of the geometry or positioning of the obstacle. The model calculates only the boundaries of each region, where the intensity pattern at any position can then be appropriately added. The progression of the shadow regions is evident in each of the experimental images, with the theoretical predictions matching exactly. Such a simple yet fast and accurate method for predicting the regions of shadow, single conical wave, and Bessel–Gauss beam during propagation may be useful in applications where complex obstacles are encountered, for example, in the optical trapping of biological samples with Bessel beams, or the three dimensional optical sorting of particles with optical tweezers.

4. Conclusion

We have described an effective method to predict the propagation and self reconstruction of Bessel and Bessel–Gauss beams after arbitrary obstructions. The model predicts if the beam will reconstruct; where it will reconstruct; and also how the shadow and conical wave regions will change during propagation away from the obstacle. Note that while the method we present here is itself not dependent on the geometry or placement of the obstacle in the optical field (as we have already shown), the propagation of

the Bessel or Bessel–Gauss beam after the obstacle, and potential for reconstruction, is of course very dependent on these parameters. The advantage of the method is ease of computation and a more intuitive approach to the problem, by considering the projection of the obstacle as a means to determining the field, rather than calculating the field directly. We were able to verify our model experimentally using three different geometrical shapes, where the theoretical predictions and experimental results correspond especially well. We also illustrated the accuracy of our method by correctly predicting the reconstruction from an off-centred obstruction. We have therefore simplified the computations involved in evaluating the propagation and possible self reconstruction of Bessel–Gauss beams after obstacles, particularly in light of the fact that there are no restrictions on the geometry of the obstacle used, nor its placement in the Bessel–Gauss field.

References

- [1] J. Durnin, J.J. Miceli Jr., J.H. Eberly, *Phys. Rev. Lett.* 58 (1987) 1499.
- [2] J. Durnin, *J. Opt. Soc. Am. A* 4 (1986) 651.
- [3] F. Gori, G. Guattari, C. Padovani, *Opt. Commun.* 64 (1987) 491.
- [4] J. Arlt, V. Garcés-Chavez, W. Sibbett, K. Dholakia, *Opt. Commun.* 197 (2001) 239.
- [5] J. Lu, J.F. Greenleaf, *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* 37 (1990) 438.
- [6] Z. Bouchal, J. Wagner, M. Chlup, *Opt. Commun.* 151 (1998) 207.
- [7] S. Sogomonian, S. Klewitz, S. Herminghaus, *Opt. Commun.* 139 (1997) 313.
- [8] Z. Bouchal, M. Bertolotti, *J. Mod. Opt.* 47 (2000) 1455.
- [9] P. Fischer, H. Little, R.L. Smith, C. Lopez-Mariscal, C.T.A. Brown, W. Sibbett, K. Dholakia, *J. Opt. A: Pure Appl. Opt.* 8 (2006) 477.
- [10] M. Anguiano-Morales, M. Mendez-Otero, D. Iturbe-Castillo, *Opt. Eng.* 46 (2007) 078001-1.
- [11] S. Chavez-Cerda, *J. Mod. Opt.* 46 (1999) 923.

Propagation of obstructed Bessel and Bessel–Gauss beams

Igor A. Litvin^{a,b,†}, Melanie G. McLaren^{a,c} and Andrew Forbes^{a,d,‡}

^a CSIR National Laser Centre, PO Box 395, Pretoria 0001, South Africa

^b Laser Research Institute, University of Stellenbosch, Stellenbosch 7602, South Africa

^c School of Physics, University of the Witwatersrand, Private Bag 3, PO Wits 2050, Johannesburg, South Africa

^d School of Physics, University of KwaZulu–Natal, Private Bag X54001, Durban 4000, South Africa

ABSTRACT

The investigation into Bessel beams has been a topic of immense research during the past 20 years, due to the interesting properties they display. Bessel beams not only exhibit diffraction free propagation, but also reconstruction of the amplitude and phase of the beam after encountering an obstruction. Although this self reconstruction property has been previously modelled by numerous groups, the techniques involve rigorous, time-consuming computations. In this work we present an efficient method to accurately calculate the reconstruction of a Bessel beam after an arbitrary obstruction. Our method considers the well-known conical wave features of Bessel beams and looks at the projection of the obstruction in space as a result of the travelling conical waves that produce the Bessel beams.

Keywords: Bessel-Gauss beams; reconstruction; conical waves.

1. INTRODUCTION

Bessel beams (BBs) represent a class of so-called diffraction free solutions to the Helmholtz equation, and have been studied extensively since the seminal work of Durnin et al.¹ in the late 1980s. One of the simplest solutions illustrated by Durnin², describes a monochromatic wave with an electric field envelope that is a zeroth-order Bessel function of the first kind, given by $J_0(k_r r)$, where k_r is the radial component of the wave vector k and r denotes the distance from the z -axis. This was the first mathematical concept of the Bessel beam, which ideally consists of an infinite number of rings of light, carrying an infinite amount of power over an infinite area. Experimentally, it is not possible to generate such a beam, and so an approximation is made in the form of a Bessel function enveloped by a Gaussian profile, thereby limiting the energy carried by the field to some finite value.

The electromagnetic field $u(r, z)$ of a Bessel–Gauss beam propagating in free-space in the z -direction is given by³:

$$u(r, z) = A \frac{\omega_0}{\omega} \exp \left[i \left(k - \frac{k_r^2}{2k} \right) z - i \Phi(z) \right] J_0[k_r r / (1 + iz / z_r)] \times \exp \left[\left(\frac{-1}{\omega^2(z)} + \frac{ik}{2R(z)} \right) \left(r^2 + \frac{k_r^2 z^2}{k^2} \right) \right], \quad (1)$$

where A is an amplitude factor, which may be complex, and

$$z_r = \pi \omega_0^2 / \lambda$$

[†] Corresponding author: ilitvin@csir.co.za

[‡] Corresponding author: forbes1@csir.co.za

$$\omega(z) = \omega_0 \sqrt{1 + (z/z_r)^2}$$

$$R(z) = z[1 + (z_r/z)^2]$$

$$\Phi(z) = \tan^{-1}(z/z_r)$$

These equations represent the Rayleigh length, the beam width, the radius of curvature of the wavefronts and the Guoy phase of a Gaussian beam respectively. The Rayleigh length determines the distance over which a Gaussian beam can propagate without diverging significantly and depends on the beam width at the waist, ω_0 , and the wavelength of the beam, λ .

These “pseudo non-diffracting” beams maintain the properties associated with ideal Bessel beams, but only over a finite distance. In particular, they satisfy that the beam be propagation-invariant and also display the interesting property of reconstruction. The latter property has been used in a variety of applications, including optical tweezing⁴ and medical imaging⁵, and is therefore an intriguing property worth investigating. Reconstruction has been well explained by firstly considering a Bessel beam as a superposition of plane waves, whose transverse amplitudes reform after encountering an obstruction⁶. Theoretical studies of this property describe reconstruction under varying conditions, including in a nonlinear medium⁷ and by wave packets due to spatial-temporal links⁸. The reliability of the periodical self-reconstruction, as well as the wavelength dependence of white light Bessel beams have been examined⁹. Experimentally, reconstruction of Bessel beams has also been verified on numerous occasions by means of different methods.

Although extensive work has been devoted to this one property, the methods of modelling this characteristic are limited in accuracy and speed, and are yet to calculate Bessel fields after arbitrary obstructions. Conventionally, the Fresnel–Kirchoff diffraction integral together with Babinet’s principle has been employed, which results in time consuming calculations. Recently, the phenomenon of reconstruction was eloquently explained by considering the dynamics of the conical waves that form a Bessel beam¹⁰. Essentially, the cone of wave vectors previously mentioned consists simultaneously of two conical waves, an incoming and an outgoing wave, which can be represented by Hankel functions¹¹ of the second, $H_0^{(2)}(k_r r)$, and first, $H_0^{(1)}(k_r r)$, kind respectively:

$$H_0^{(1)}(k_r r) \exp(ik_z z) = [J_0(k_r r) + iN(k_r r)] \exp(ik_z z) \quad (2)$$

$$H_0^{(2)}(k_r r) \exp(ik_z z) = [J_0(k_r r) - iN(k_r r)] \exp(ik_z z), \quad (3)$$

where $N(k_r r)$ is the zero-order Newman function.

The incoming conical wave transforms into the outgoing conical wave and the superposition of these two conical waves is crucial in obtaining propagation invariant beams. Within a Bessel beam, although locally the light travels parallel to the axis of propagation, the energy flux follows the direction determined by the cone of wave vectors. For this reason, reconstruction after an obstruction is observed. That is, the reconstructed beam results from light that was not obscured by the obstacle.

Despite this fluent explanation by Anguiano-Morales *et al.*¹⁰, the numerical simulations demonstrated resorted back to solving the Helmholtz equation. In this paper we present an efficient and accurate technique to predict Bessel and Bessel–Gauss beam propagation after encountering an obstruction of arbitrary geometry and complex orientation (i.e., no symmetry is required in the obstacle under study). The technique is a simple exploitation of the well-known conical wave formalism of Bessel beams, and is based on a projection of the obstacle in space due to the travelling conical waves that form the Bessel beams. We verify the concept experimentally for two obstacle geometries, and show that despite the method being based on a ray optics approximation, the method is resilient even when diffraction is evident.

1.1. Fundamentals of Bessel beams

The propagation and generation of Bessel–Gauss (BG) beams have been well documented^{3,12,13}. Equation (1) can be simplified by considering the field at the plane $z = 0$, where the intensity profile is shown in Fig. 1 (a):

$$u(r,0) = AJ_0(k_r r) \exp[-(r/\omega_0)^2]. \quad (4)$$

Fig. 1 illustrates how the intensity profile of Eq. (1) changes along the axis of propagation (z -direction), from a Bessel beam (a) to an annular ring (b).

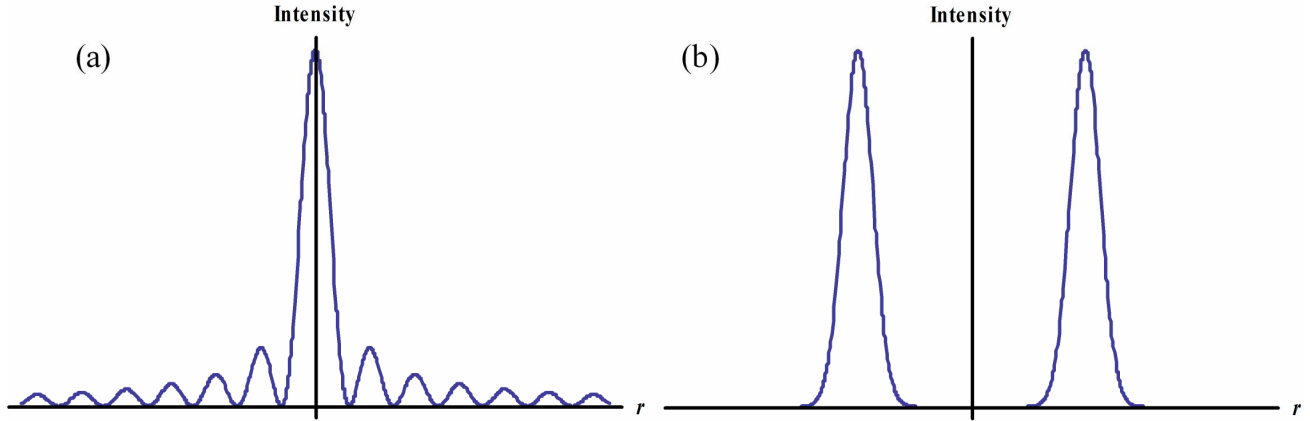


Fig. 1: Intensity profiles of a Bessel-Gauss beam. (a) A Bessel beam profile is produced near $z = 0$, (b) while an annular ring is produced at larger propagation distances.

An important property of the Bessel beam is its non-diffracting nature. That is, for propagation in the z direction, the intensity satisfies,

$$I(x, y, z \geq 0) = I(x, y), \quad (5)$$

where $I(x, y, z) \propto J_0^2(k_r r)$ for a zero-order Bessel beam.

Of course, for a BG beam this equality is true only for a finite distance of z . This distance can be calculated by analysing the experimental generation of a BG beam. There are two well established methods for Bessel beam formation, one of which makes use of an annular slit. A Bessel beam can be thought of as a superposition of plane waves whose wave vectors lie on a cone, which in k -space reveals itself as a ring. That is, the Fourier transform of a ring results in a Bessel beam, which allowed Durnin et al.¹ to produce an approximation to a Bessel beam by illuminating an annular slit onto the back focal plane of a focusing lens. A more efficient method of Bessel beam generation makes use of an axicon. Fig. 2 illustrates how a Bessel beam is created by illuminating a Gaussian beam onto an axicon. The opening angle of the cone is given by,

$$\theta = (n-1)\gamma, \quad (6)$$

where n is the refractive index of the axicon and γ is the opening angle of the axicon.

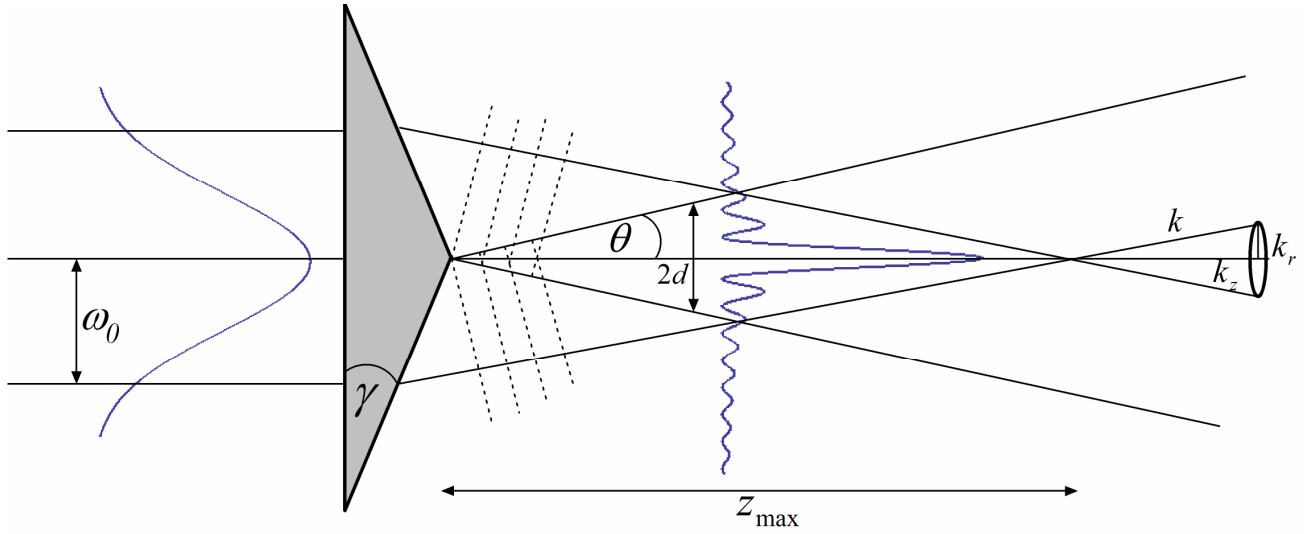


Fig. 2: Diagram illustrating the finite propagation distance of a Bessel beam generated by an axicon of opening angle γ . The superposition of plane waves has wave vectors, which lie on the surface of a cone with opening angle θ .

The finite distance after the axicon in which a BG beam is found is determined from the wave vectors of the Bessel beam that propagate on a cone. This maximum distance is given by,

$$z_{\max} = \frac{k}{k_r} \omega_0 \approx \frac{\omega_0}{\theta}, \quad (7)$$

Within this finite propagation distance, the properties associated with the ideal Bessel beam hold true for a BG beam. Using geometric optics, further information about the beam can be revealed. The maximum width of the BG beam, $r_{\max}$, is found at $z_{\max}/2$:

$$r_{\max} = z_{\max} \tan \theta, \quad (8)$$

These two parameters together with the Bessel function parameter, $k_r = k \sin \theta$, allow the number of rings, N , at $r_{\max}$ to be solved:

$$N = \frac{r_{\max}}{2} \frac{1}{\delta}, \quad (9)$$

where $\delta \approx \pi/k_r$ is the spacing between each ring.

For such beams, there is a minimum distance behind an obstacle of radius $\frac{1}{2} D$ before reconstruction occurs,

$$z_{\min} = D \frac{k_z}{2k_r} = \frac{D}{2 \tan \theta}, \quad (10)$$

We are able to verify this equation using our conical wave method. Although all the key aspects required to implement this technique are well known, previous models persisted with lengthy integral calculations.

2. GEOMETRICAL MODEL

The model presented here is based on a geometrical approximation to the propagation of conical waves after obstacles. As we will show later, this approximation is amazingly resilient even when the obstacle sizes are in the order of the wavelength of the illuminating light, and so while our method cannot be considered a complete substitute for a full vector diffraction based calculation, it offers a very accurate first approximation.

Consider the schematic in Fig. 3 where an arbitrary obstacle is placed off-centre in the path of a Bessel–Gauss beam. The obstruction cross-section due to this obstacle along one azimuthal direction is shown. The positive z -axis defines the direction of propagation of the Bessel–Gauss beam, which we assume to have a spatial extent of $2d$, as shown in Fig. 2. Since only a cross-section is shown, there are only two conical waves (labelled CW_1 and CW_2 respectively) contributing to the Bessel–Gauss beam at any given point.

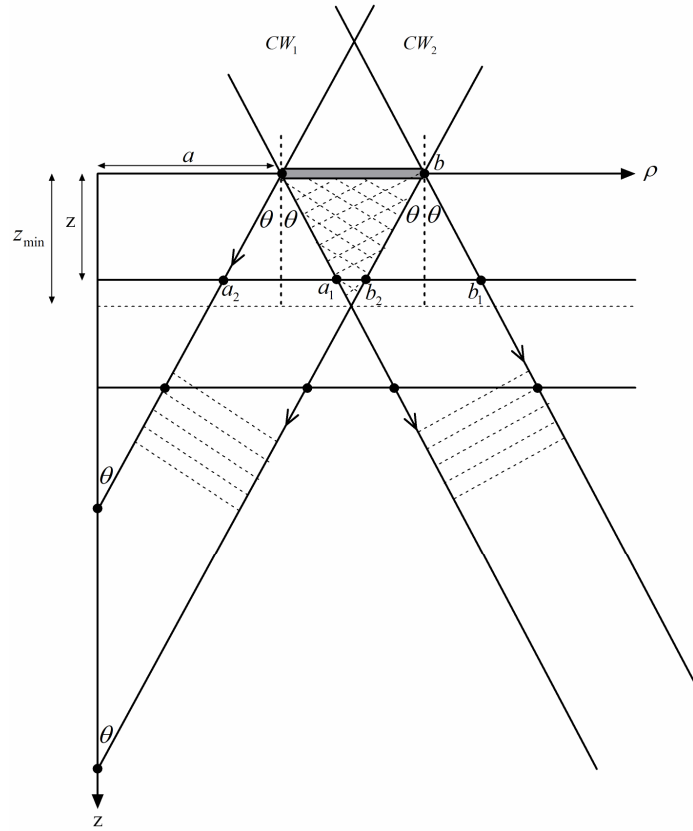


Fig. 3: Diagram illustrating the projection of the obstacle in space by two conical waves, CW_1 and CW_2 . The shadow region where neither conical wave contributes has a minimum distance given by $z_{\min}$.

It is clear from Fig. 3 that the projection from these two conical waves results in distinct regions defined by (i) one conical wave contribution only, (ii) a shadow region where neither conical wave contributes, and (iii) the Bessel–Gauss beam where both waves contribute. The boundaries that define these regions are mapped by the projection of the obstacle's boundaries along a given z plane. Without any loss of generality, consider an obstacle with cross-section shown in Fig. 3 with full width D and that is placed some distance $\rho = a$ from the central axis ($\rho = 0$) of the Bessel–Gauss beam, so that the two extreme edges of the obstacle are defined by $\rho = a$ and $\rho = b$, with $D = b - a$. From the geometry one can derive the projection of the obstruction due to each conical wave. If the new positions of these edges due to CW_1 are labelled a_1 and b_1 , and similarly a_2 and b_2 due to CW_2 , then one can show that at some distance z after the obstacle we have:

$$\begin{aligned}
a_1 &= a + z \tan \theta \\
a_2 &= a - z \tan \theta \\
b_1 &= b + z \tan \theta \\
b_2 &= b - z \tan \theta
\end{aligned} \tag{11}$$

Clearly the projection results in the creation of two zones defined by a single conical wave, with the boundaries of these zones moving farther apart at a rate of $2z \tan \theta$. The region behind the obstacle is then defined as follows (for a given z propagation and a chosen azimuthal angle ϕ):

$$\begin{aligned}
-d \leq \rho \leq a_2 & \quad \text{CW}_1 \text{ and CW}_2 \text{ add to form a complete Bessel–Gauss beam;} \\
a_2 < \rho < a_1 & \quad \text{CW}_1 \text{ contribution only;} \\
a_1 < \rho < b_2 & \quad \text{no contribution from either CW}_1 \text{ or CW}_2, \text{ resulting in a shadow region;} \\
b_2 < \rho < b_1 & \quad \text{CW}_2 \text{ contribution only;} \\
b_1 \leq \rho \leq d & \quad \text{CW}_1 \text{ and CW}_2 \text{ add to form a complete Bessel–Gauss beam;}
\end{aligned}$$

Clearly at the distance $z = z_{\min}$ where $b_2 \leq a_1$ the shadow region vanishes and the Bessel–Gauss beam is said to have reconstructed. The value for $z_{\min}$ can easily be found from Eq. (11) by solving for the equality $b_2 = a_1$:

$$\begin{aligned}
b - z \tan \theta &= a + z \tan \theta \\
\Rightarrow b - a &= D = 2z \tan \theta \\
\Rightarrow z_{\min} &= \frac{D}{2 \tan \theta}
\end{aligned}$$

which is consistent with the expression in Eq. (10), as expected. If the process described above is carried out for all points along the boundary of an obstruction, then the complete movement of the boundaries of each region can easily be computed. With this information the complete intensity pattern at any position after an obstacle, regardless of the complexity of the geometry of the obstacle, may readily be found.

It is worth pointing out some salient points in how these regions develop: firstly, the single conical wave regions move apart at a rate of $2z \tan \theta$. Thus in an off-centred geometry one expects the boundaries defining the Bessel–Gauss beam to move apart, in opposite directions, with one passing through the centre of the original Bessel–Gauss field. Secondly, any combination of points on the boundary of an obstacle that find themselves equidistant from the central propagation axis of the Bessel–Gauss beam will result in boundaries that evolve in identical fashion. This latter fact can be exploited to simplify the calculation for arbitrary obstacles.

3. EXPERIMENTAL METHODOLOGY AND RESULTS

Fig. 4 shows the experimental set-up used in generating the Bessel–Gauss beam. A 660 nm, 100 mW diode laser beam was expanded through a 5× telescope before illuminating an axicon with an opening angle of $\gamma = 5^\circ$.

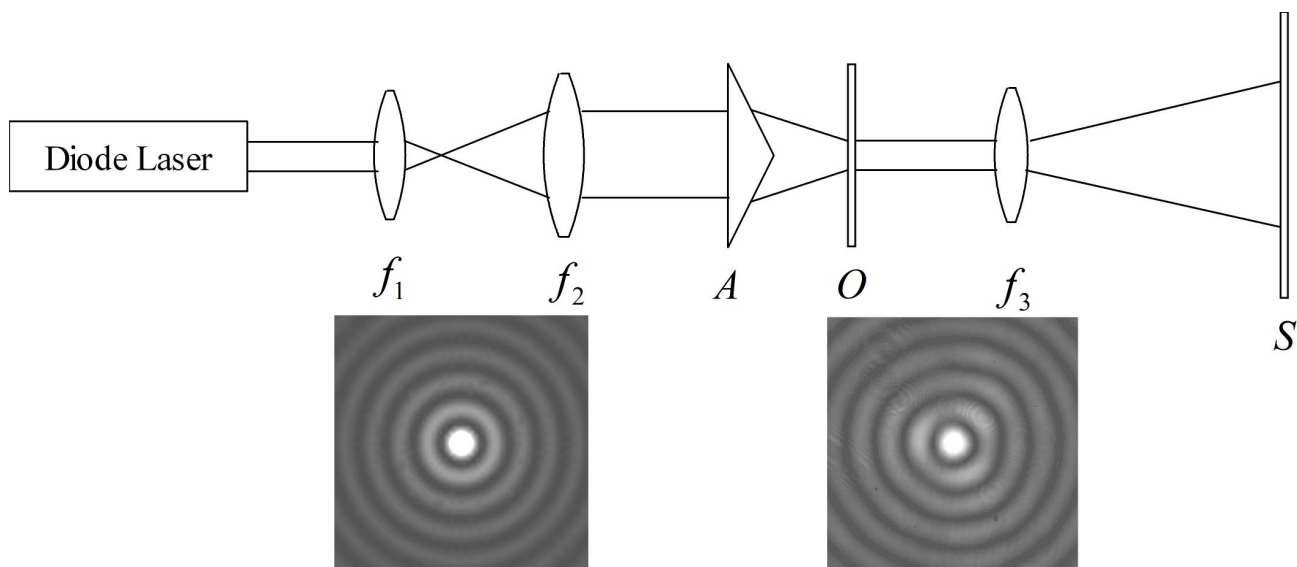


Fig. 4: Experimental design for generating a Bessel-Gauss beam: a telescope with lenses $f_1 = 50$ mm, $f_2 = 250$ mm expands the beam before illuminating the axicon (A). The obstacle (O) was placed 45 mm from the axicon and was imaged onto a screen (S) by lens $f_3 = 25$ mm. The inserts represent the theoretical prediction (left) and the experimentally measured (right) Bessel-Gauss beam.

The obstruction was placed on a thin transparent cover slip and positioned within the Bessel beam at a distance of $\frac{1}{2} z_{\max}$ from the axicon. The resulting obstructed beam was imaged onto a screen with a suitable magnification ($57\times$) to observe the intensity variations during propagation. The image plane could be varied continuously by means of a translation stage so as to allow measurement of the beam from fully obstructed to fully reconstructed. The geometries of the two test obstructions – a centred circle (approximately) and a vertical wire – are shown in Fig. 5. In each case the obstruction is opaque so that immediately after the obstruction a dark shadow region is evident, surrounded by an unperturbed Bessel-Gauss beam.

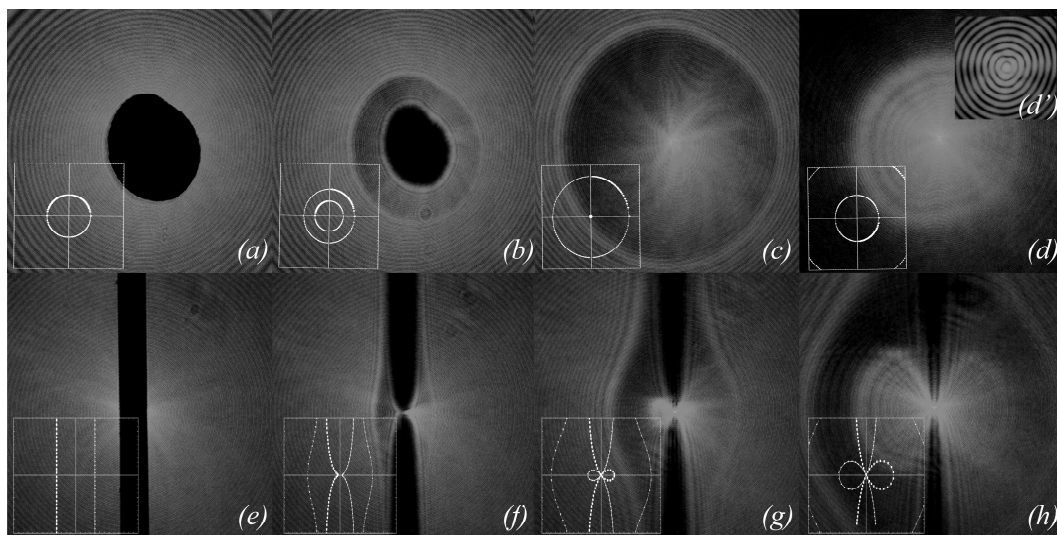


Fig. 5: Experimental images with the theoretical predictions inset in the bottom left corners. The images illustrate the excellent comparison for the self reconstruction of a Bessel beam.

Fig. 5 shows a sequence of measured Bessel–Gauss beam intensities during propagation after the two obstructions, with the theoretical predictions shown as insets on the experimental images depicting the calculated boundaries due to the conical waves from the source. Figs. 5 (a) – (d) show the change in the Bessel–Gauss beam intensity during reconstruction when the starting obstacle is an approximation to a centred circle, a scenario with radial symmetry. As the propagation distance from the obstruction was increased, so the shadow regions appear to move. Following the discussion in the previous section we note that, as expected, two boundaries appear due to the projection of the obstacle by the two conical waves from the source. The intersection of the two boundaries defines the true shadow region, while the region between the two boundaries defines a single conical wave contribution. Elsewhere, the original Bessel–Gauss beam is evident due to the summation of both conical waves. This is clear in the sequence from (b) to (d). The final image at (d) was taken at a propagation distance that exceeded $z_{\min}$, so that complete reconstruction has taken place. This is predicted by the calculation method and has been confirmed experimentally. Verification of the reconstruction at the centre of the field, where the original obstacle was, is indicated by the zoomed inset of the centre of the beam in (d). A similar sequence is shown in Figs. 5 (e) – (h). In this case the radial symmetry is purposely broken. The sequence illustrates an interesting case with the vertical wire, as here there is a region where no contribution from one of the conical waves is possible, since the obstruction extends across the entire Bessel–Gauss beam in the vertical plane following the wire. Again the agreement between calculation and experiment is excellent.

4. CONCLUSION

We have described an effective method to predict the self reconstruction of Bessel and Bessel–Gauss beams after arbitrary obstructions. The model predicts where the beam will reconstruct and also how the shadow regions will progress over the propagation distance. We were able to verify our model experimentally, where the theoretical predictions and experimental results correspond especially well. We have therefore simplified the computations involved in evaluating the self reconstruction of Bessel–Gauss beams, particularly in light of the fact that there are no restrictions on the geometry of the obstacle used, nor its placement in the Bessel field.

REFERENCES

1. Durnin J., Miceli Jr. J. J. and Eberly J. H., “Diffraction-Free Beams”, *Phys. Rev. Lett.*, **58**, 1499, (1987)
2. Durnin J., “Exact solutions for nondiffracting beams. I. The scalar theory”, *J. Opt. Soc. Am. A*, **4**, 651, (1986)
3. Gori F., Guattari G. and Padovani C., “Bessel-Gauss Beams”, *Opt. Commun.*, **64**, 491, (1987)
4. Arlt J., Garces-Chavez V., Sibbett W. and Dholakia K., “Optical micromanipulation using a Bessel light beam”, *Opt. Commun.*, **197**, 239, (2001)
5. Lu J. and Greenleaf J. F., “Diffraction-limited beams and their applications for ultrasonic imaging and tissue characterization”, *Proc SPIE*, **1733**, 92, (1992)
6. Bouchal Z., Wagner J. and Chlup M., “Self-reconstruction of a distorted nondiffracting beam”, *Opt. Commun.*, **151**, 207, (1998)
7. Sogomonian S., Klewitz S. and Herminghaus S., “Self-reconstruction of a Bessel beam in a nonlinear medium”, *Opt. Commun.*, **139**, 313, (1997)
8. Bouchal Z. and Bertolotti M., “Self-reconstruction of wave packets due to spatio-temporal couplings”, *J. Mod. Opt.*, **47**, 1455, (2000)
9. Fischer P., Little H., Smith R. L., Lopez-Mariscal C., Brown C. T. A., Sibbett W. and Dholakia K., “Wavelength dependent propagation and reconstruction of white light Bessel beams”, *J. Opt. A: Pure Appl. Opt.*, **8**, 477, (2006)
10. Anguiano-Morales M., Mendez-Otero M. and Iturbe-Castillo D., “Conical dynamics of Bessel beams”, *Opt. Eng.*, **46**(7), 078001-1, (2007)
11. McQueen C. A., Arlt J. and Dholakia K., “An experiment to study a “nondiffracting” light beam”, *Am. J. Phys.* **67**, 912, (1999)
12. Stockham A. and Smith J. G., “Optical Design for Generating Bessel Beams for Micromanipulation”, *Proc. SPIE*, **6326**, 63261B-1, (2006)
13. Chavez-Cerda S., “A new approach to Bessel beams”, *J. Mod. Opt.*, **46**, 923, (1999)

VORTEX BEAMS

South African scientists are putting a new twist on light, manipulating photons to create vortices with some very weird properties.

>> BY ANGELA DUDLEY,
MELANIE MCLAREN AND
ANDREW FORBES

EVIDENCE of angular momentum – that bodies can be made to spin and rotate – is very familiar to us from everyday life: twist a rubber band by applying a torque to it and hold it in tension; release it and it will unwind very fast, gaining angular momentum in the process.

Even our timekeeping is in some sense based on angular momentum: each new day arrives courtesy of the spin of the Earth about its axis (spin angular momentum), while the years are measured in the orbit of the Earth about our Sun (orbital angular momentum). Now scientists know that even photons can be twisted, forming so-called “twisted” light in the process. As with the released rubber band, this “twisted” light carries angular momentum.

For over a century now, scientists have known that light can be viewed as either a particle (photon) or a wave. If you could freeze a wave of ordinary light, you would see a series of crests and troughs, moving forward as a series of planes known as wavefronts. The energy transfer is in a straight line in the direction of the travelling planes.

The wavefronts of “twisted” light no longer look like those of ordinary light; but instead transform into a corkscrew-like pattern, with the direction of energy transfer spiralling around the axis of propagation. The

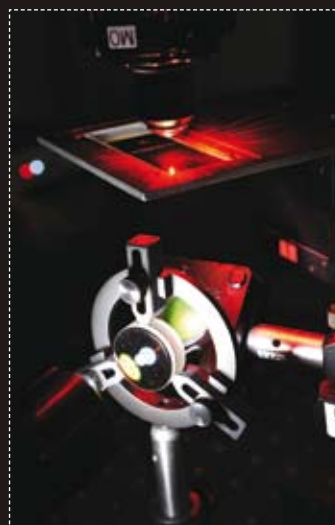
waves are still moving forward in a straight line, but twisting as they move, much like water spiralling down a drain.

Near the central propagation axis, the velocity of the spiralling waves becomes infinite, causing the intensity of the light in the centre to vanish, forming an optical vortex. This is one of the visual distinctions between “twisted” and ordinary light: if both are focused by a lens, “twisted” light appears as a ring of light, with the optical vortex manifesting itself as a dark centre; an ordinary laser beam appears as a bright spot with no dark centre.

There are an infinite number of possible twists that can be put on the light, for example, a simple single helix, a DNA-like double helix, or a triple helix akin to fusilli pasta. The more twists put on the light, the larger the dark centre of the beam becomes. When light is twisted in this manner, the twist is imparted to each photon, creating “twisted” photons. The result is that each photon carries a well-defined amount of orbital angular momentum.

As the twist is increased, so the amount of orbital angular momentum carried by each photon increases. Creating laser beams with

Dr Andrew Forbes and his group of mathematical optics researchers at the Council for Scientific and Industrial Research (CSIR) are studying various aspects of modern optics, including laser beam shaping, laser beam propagation and novel resonators. Their work encompasses the mathematical basis of these fields and applications in such diverse areas as optical tweezing in bio-photonics, high energy laser delivery through a turbulence atmosphere, novel gas lenses for variable focal lengths, and flat-top beams for studies in high pressure and high temperature physics.



▲ Main picture: Ordinary light appears as a series of plane wavefronts, with the direction of energy transfer along a straight line. Above left: Liquid crystal devices are used to create digital holograms.



Manipulating light at the CSIR National Laser Centre. Dr Andrew Forbes with Angela Dudley (far right) and Melanie McLaren.

multiple twists – so-called vortex beams – is the key ingredient in many emerging research fields, most notably optical tweezing, quantum computing and quantum information transfer, and is made possible in the laboratory by a fusion of the optical and digital worlds through digital holography.

Digital holography

When holography was first discovered in 1947 by Dennis Gabor (by serendipity; he was actually trying to improve the resolution of an electron microscope), he could not have imagined how all-pervasive the technique would become, nor how important it would become to modern optics in the 21st century. It was only after the invention of the laser in the early 1960s that holography became a serious tool in the armoury of optical scientists.

Waves can be described by both an amplitude and a position, and whereas photographs store information by recording the amplitude of a wave, holograms store information by recording the phase, or position of the wave. But light waves

travel very fast, so to store information on the position of the many waves requires that we slow them down – all the way to a standstill.

To do this, we create so-called “standing waves” – waves that don’t move, so that we can record the phase of the waves at every point using conventional photographic techniques. You can think of holography as taking a photograph of the phase of light and storing this information in some conventional medium, such as photographic film.

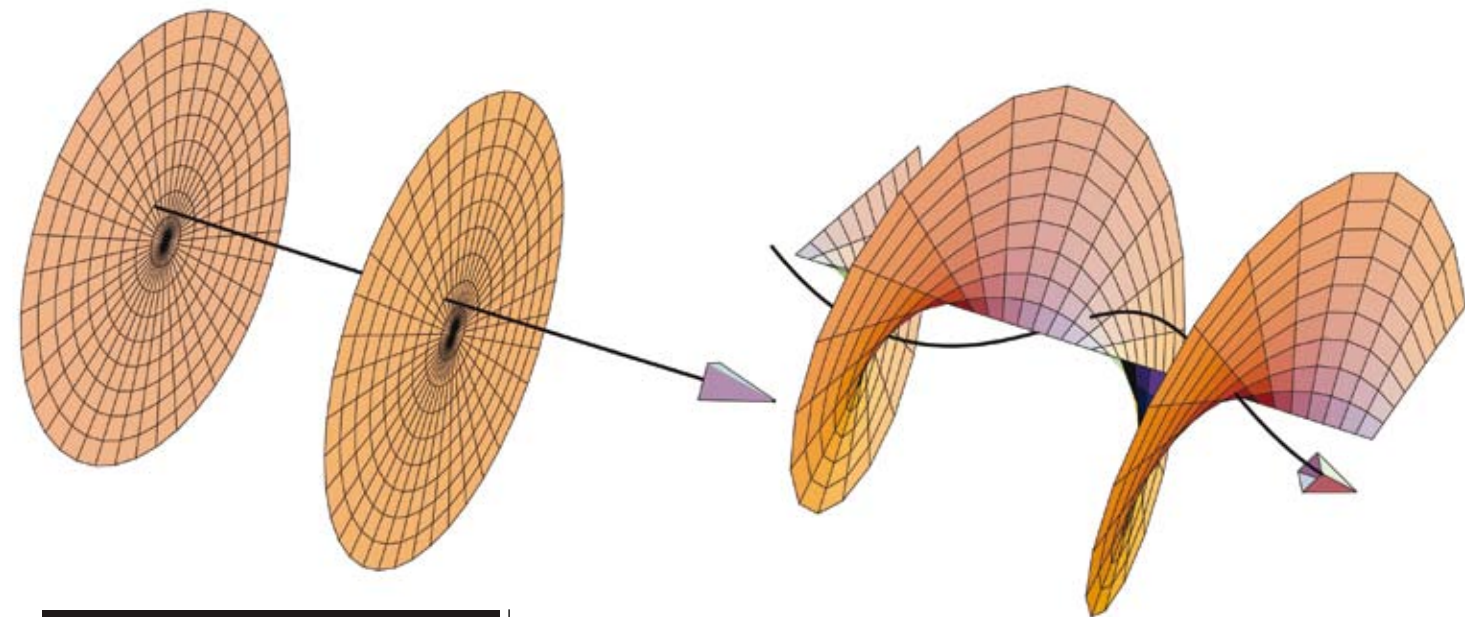
For many years, these standing waves were created by interference of two waves: a reference wave and an object wave that carries information about the object of which you intend to create a hologram. This is achieved in the laboratory by splitting a laser beam into two parts, reflecting one part off the object that one wishes to make a hologram of,

and then recombining the two waves on some photographic film.

The recombining creates standing waves by interference, and the resulting hologram stores information about the object using the phase of the waves. Once the phase information is stored in the hologram, it may be retrieved to create the original object by illuminating the hologram with the reference beam again. This may also be done with white light rather than a laser beam, and the result is the holograms we see on bank cards and some paper currencies.

But what if we want to make a hologram of something that does not exist as a physical object – for example, a vortex beam? In conventional holography, this would be impossible, as an object wave is crucial to the process. Today we get around this inconvenience by using a technique called Computer Generated Holograms (CGHs), otherwise known as digital holography.

Rather than let nature calculate the hologram using interference as described earlier, we use computers to do this, then



Above: Unlike plane waves (left), the helical wavefront of “twisted” light (right) results in the direction of energy transfer spiralling about the axis.

Below: Light diffracted off a liquid crystal splits into its many orders.

write the hologram to a liquid crystal display device. If this device is illuminated with a suitable laser beam to act as the reference beam, then the outgoing laser beam will carry the information from the hologram as before. The required hologram for creating a vortex beam takes the form of a fork-like pattern.

A non-vortex reference beam is used to illuminate a digital hologram of a fork-like dislocation; the number of forks in

the hologram determines the amount of twists put on to the out coming vortex beam: two forks means two twists, with photons carrying $2\hbar$ (pronounced h-bar) of orbital angular momentum, three forks means three twists, with $3\hbar$ of orbital angular momentum, and so on. This is how optical vortices, which are virtual objects, are created in the laboratory.

Quantum information and quantum computing

Previously in quantum communications systems, information was encoded in the spin angular momentum (polarisation) states of photons. In using the spin to encode information, one is restricted to only two options, “spin clockwise” and “spin anticlockwise”, and so is similar to

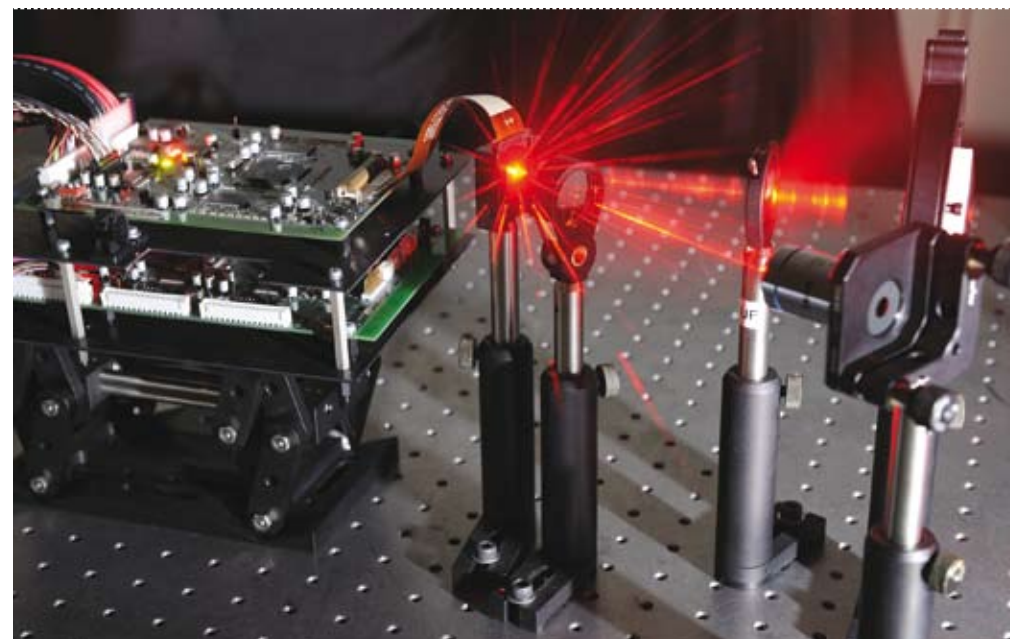
conventional communication methods of “on” and “off” modulated signals.

The orbital angular momentum state of light – its twisted nature – opens the way for an unlimited number of possible states to be used in a “twisted alphabet”. Rather than use many photons to create the letter “A” through an on-off sequence, “twisted” light can function as a form of communication by encoding the alphabet directly: a flash of single-helix light represents the letter “A”, a double-helix the letter “B”, and so on all the way to “Z”.

Vortex beams thus hold the key to dramatically increased bandwidth for the future. This is still some way off in practice, though, as unfortunately vortex beams with a large number of twists tend to spread out very quickly, and so for the moment cannot be used for long-distance communication.

Twisted photons are also making groundbreaking advances in the world of quantum computing. The growing interest in quantum computing stems from the fact that it outperforms conventional, everyday computing by exploiting the principle of quantum superposition (see “The Schizophrenic Atom”, POPULAR MECHANICS, February 2008).

In quantum computing, the number of calculations that can be performed in a given time depends on the number of superposition states a photon can possess. Considering the spin state of a photon, and taking clockwise to be denoted by 1 and anticlockwise by 0, we see that we have a superposition of four states: 00, 01, 10 and 11. Ideally, the number of superposition states needs to be increased, and this can be achieved by using the orbital angular momentum state of pho-



tons. Since there is no physical limit to the number of twists that can be put on the light, it's theoretically possible to create an infinite number of superposition states – the holy grail of quantum computing!

Optically driven motors

The ability to produce light carrying angular momentum may not seem to be an achievement of great importance, since the effects are visible only on the microscopic scale. However, for the past 30 years, there has existed a technique that specialises in the manipulation of microscopic particles – optical tweezers. In an optical tweezer, laser beams are used to produce pico-Newton forces to control microscopic samples in three dimensions: particles are “trapped” by the laser beam and then easily manipulated by steering the laser beam.

If the beam used in the tweezing is a vortex beam, the transfer of orbital angular momentum to the particle will cause it to rotate. Once rotating, optical tweezers can also control the rate of rotation by simply controlling the amount of angular momentum carried by the beam. By using

vortex beams to drive micro gears, it's possible to control fluid flow in micro-fluidic channels. By placing a number of these gears in series, a chain of rotating gears can be generated, which could make possible an optical pump by imparting angular momentum to one gear.

Today we are seeing the first demonstrations of integrated optical-mechanical

systems – photon-driven motors using twisted light!

● Angela Dudley and Melanie McLaren are PhD and MSc students respectively in the Mathematical Optics group at the CSIR National Laser Centre. Both are under the supervision of the group's leader, Dr Andrew Forbes. **PM**

