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Mathematics Teachers' Noticing Practices in Their Classrooms

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A RESEARCH REPORT SUBMITTED TO THE FACULTY OF EDUCATION, UNIVERSITY OF WITWATERSRAND, IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF EDUCATION

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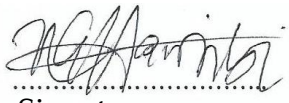
TITLE OF THE RESEARCH: MATHEMATICS TEACHERS NOTICING PRACTICES IN THEIR CLASSROOMS.

Declaration

I declare that this research study is my own work and no part of it has been copied from another source (unless indicated as a quote). All phrases, sentences and paragraphs taken directly from other works have been cited and the reference recorded in full in the reference list.



Signature

N.G. Hantibi

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ABSTRACT

Classrooms must be communities in which mathematical sense making is developed. Most mathematical lessons consist of teacher presentation of mathematical problems along with the algorithm of solving it, followed by the assignment of a similar set of problems for students to work through individually. The above characteristics do not provide the conditions necessary for the development of the students' capacity to think and to reason mathematically (Stein, Grove & Henningsen 1996). Teachers must come up with practices and skills that will bring about mathematical sense making in the classroom. Emphasis on teacher accountability for student learning outcomes necessitates the development of effective instructional tactics among all teachers including those just beginning their professional practice (Thomas, Jong & Fisher, 2017). One of the strategies that can be used by the teachers is 'noticing'. Noticing is the teacher's capacity to attend to the students' mathematics conceptions and practices, secondly to interpret these conceptions and practices and thirdly to decide upon a productive instructional course of action based on this interpretation (Jacobs, Lamb & Phillip, 2010). This study explored the noticing practices that are used by the teachers in their classrooms. Qualitative mathematics learning and sense making of mathematics by the learners was also explored. This was done by observing three educators in their classrooms. A method of interviewing was also used. These methods helped the researcher to realise what is taking place in the mathematics classrooms.

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Chapter one

Background of the study

1.1 Introduction

The main purpose of the teacher is to bring about learning. It is to develop knowledge in learners. The teachers need to come up with the strategies and approaches that can be used to bring about knowledge in a mathematics classroom. Calvert (2001) argues that to understand how new knowledge is generated and accepted by the discipline, it is essential to know how to acquire mathematical knowledge. There are different perspectives that enhance our understanding of mathematical knowledge. These include cognitive, socio cultural and situated perspectives. These help us to come up with the teaching approaches that are used by the teachers in a classroom.

Looking at cognitive perspective of understanding mathematics classrooms (Stein, Grove & Henningsen, 1996) argue that complete understanding includes the capacity to engage in the process of mathematical thinking, that is, framing and solving problems, looking for patterns, making conjectures etc. They argue that most mathematics lessons consist of teacher presentation of a mathematical problem along with the algorithm of solving it, followed by the assignment of a similar set of problems for students to work through individually. Students' work consists almost entirely of memorising the procedures, algorithms or applying formula. They further state that these do not provide the conditions necessary for the development of students' capacity to think and to reason mathematically. Conceptual understanding has been emphasized in the curriculum and principles and standards across nations as it is the key in mathematical learning. However, mathematics instruction in classrooms often relies on rote memorization of mathematical rules and formulae without conceptual connections (Kim, 2020). As a result of this (Stein, Grove & Henningsen, 1996, p.457) call for the establishment of instructional environments characterised by an increased emphasis on problem solving. The instruction will then be a means of building student capacity for mathematical thinking and reasoning.

They state that one model that could be used is the student mediation model. This model specifies that teaching does not directly influence student learning but rather, that teaching influences students' cognitive processes or thinking which in turn influences their learning. A mediating variable is therefore the nature of students' thinking processes in the classroom and how these processes are altered when teachers attempt to create enhanced instructional environment. This enhanced instructional model is a means of building student capacity for mathematical thinking and reasoning. In this the students must first be provided with opportunities, encouragement and assistance to engage in thinking, reasoning and sense making in the classroom (Stein et al 1996, p. 457). Students need to be encouraged to share their mathematical thinking and ideas in their classrooms. Having students share strategies enables them to learn from each other and to be exposed to different strategies, thus creating flexible mathematical problem solvers.

Another perspective that enhances the learners' understanding of mathematical knowledge is socio cultural perspective. Socio cultural theorists are of the idea that children's learning begins long before they attend school. They argue that any learning a child encounters at school always has a previous history. They further state that children begin to study arithmetic in school, but long before hand they have had some experience with quantity (Vygotsky, 1978, p.84).

Mediation is seen as essential in Vygotsky's account of social formation. Mediators serve as the means by which the individual acts upon and is acted upon by social, cultural and historical factors (Daniels, 2009). Like socio-cultural theorists, cognitive theorists also see mediation as of importance in the building of the students' mathematical thinking and reasoning. They see mediation as the enhanced instructional model which is used to alter students' thinking processes in the classroom. The teachers are therefore regarded as the mediators in the classrooms because they are the ones responsible for providing mathematical knowledge and

building student capacity and thus altering mathematical processes and thinking in learners.

In addition to the two perspectives discussed above, there is also a situated perspective. Lave (2001) states that all learning is situated. Situated learning is the way in which learning is linked to the situation or context in which it takes place (Lave 2001). She further argues that learning is an integral and inseparable aspect of social practice. Learning is viewed as situated activity whose central defining characteristic is legitimate peripheral participation (LPP). LPP is where learners participate in communities of practitioners and that the mastery of knowledge and skill requires newcomers to move toward full participation in the socio cultural practices of society (Lave, 2001, p. 29). In order for this to happen, there are three types of participation that need to be observed. Central, complete and full participation. Central participation can be seen as the school or the classroom where mathematical teaching and learning takes place. Complete participation is the collective practice for which there might be measurable degrees of acquisition by newcomers. This is also known as closed domain of knowledge (Lave, 2001, p. 36). I can compare complete participation to the process of mediation. This is where there is a teacher whose main aim is to build mathematical knowledge of the learners. This is also where there are learners who completely participate in what the teacher is teaching them. This can also be evident when the learners are able to share knowledge with each other. Full participation places emphasis on what partial participation is not. This is when a learner moves from periphery, that is, from being a newcomer to be able to participate fully in the practice. This is also evident when the learners are able to explain to each other whatever was taught to them. This shows they are no longer on the side lines, they are no longer newcomers to the concept being taught but they can fully participate. This is when learners are able to perform duties on their own. This is when the learners move from actual developmental level towards the level of potential development.

Mathematics is one of the learning areas (subjects) that are taught in a school environment. Mathematics can be seen as a special type of discourse, made distinct, among others, by its objects, mediators and rules (Sfard, 2009). A discourse counts as mathematical if it features mathematical words, such as those related to quantities and shapes (Sfard,2009). Learning mathematics may be defined as individualizing mathematical discourse, that is as a process of becoming able to have mathematical communication not only with others, but also with oneself (Sfard, 2009). Sfard (2009) further argues that acquisitionists view learning as resulting from the learners' direct efforts to arrive at a coherent vision of the world, whilst participationists see learners as arising mainly from one's attempt to make sense of other people's vision of this world. When mathematics learning takes place, a learner will therefore have understanding of mathematics. Mathematics understanding brings about mathematical proficiency in learners.

Mathematical proficiency has five strands or components. The five strands are interwoven or interdependent in the development of proficiency in mathematics (Kilpatrick, Swafford & Findell, 2001). The five strands are: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (Kilpatrick et al, 2001). Proficiency should enable learners to cope with the mathematical challenges of daily life and enable them to continue their study of mathematics in high school and beyond (Kilpatrick et al, 2001). Kilpatrick et al (2001) argues that mathematics proficiency is not a one-dimensional trait, and it cannot be achieved by focusing on just one or two of these strands. Therefore, in order for learners to acquire the mathematics proficiency, instructional programmes and practices by the teachers must be developed. These programmes and practices must be used efficiently in the classroom.

One of the practices used by the teacher in a classroom is 'noticing'. To notice is to be conscious / aware of someone or something by seeing, hearing or feeling them (Fowler & Fowler, 1995). Teachers become aware of the learning of the learners in their

classrooms. This is mostly done by the people of the same profession who have similar goals and outcomes. This study therefore explored the noticing practices used by the teachers in their classrooms, and how these noticing practices impact student learning of mathematics.

"The classroom is a complex environment with multiple interactions occurring at the same time. A teacher cannot possibly pay attention to all that is happening, instead teachers must be selective in determining where to focus their attention (Sherin &Van Es, 2005, p. 477). This is where the skill of noticing comes. Noticing is the teachers' capacity to firstly attend to the students' practices, and thirdly to decide upon a productive instructional course of action based on this interpretation (Jacobs et al, 2010). Mason (2020) argues that it is not sufficient merely to 'notice' some aspect of a situation, it is important to have an action available as a result of noticing. One of the techniques or skills used in the noticing process is listening. Zaslavsky (2020) points to the need to listen carefully to what the learner says. Listening to what a learner is saying. Or perhaps trying to express, is quite different from listening-for what is already in the teacher's head (Davis,1996). Noticing is a situation in which it would be preferable to listen-to what is being expressed rather than listening-for what is expected. This could trigger an action of supressing what is in the teacher's head and focussing on what the learner is expressing, or trying to express (Mason, 2020). Listening carefully to learners can take time and may call upon mathematical experience and sophistication on the part of the teacher beyond what appears to be the mathematical sophistication of the topic (Mason, 2020). The study therefore explored the noticing practices employed and used by the mathematics teachers in their classrooms and, how these noticing practices impact mathematics learning.

1.2 Problem statement

Teachers are expected to teach and produce good results. Good results are possible when the learners understand what is taught by the teacher. The teachers therefore need to notice the talk and action of the learners in their classrooms. Teachers need to

be aware of the problems their learners have. Teachers need to acknowledge when their learners have problems in understanding mathematics. Information based on national mathematics matric results show that learners do not perform very well in mathematics. Some say teachers do not teach, some say teachers do not have content knowledge, and some say learners do not have proper mathematics foundation. Several studies show teachers' lack of understanding of students' thinking and reasoning in mathematics, and difficulties in attending to the details of students' strategies (Wallach & Even, 2005). Teaching students so they learn and know mathematics conceptually along with understanding of how to move students' mathematical knowledge forward is a complex enterprise (Neumann, 2014). Neumann (2014) further states that most teachers learned mathematics through direct instruction that emphasized fact-based, low-level questions, and rote memorization. As a result of this many teachers now use this as their model of how to teach and therefore struggle to understand what it means to teach mathematics in a way which would bring about desired results. Reform in mathematics instruction (i.e., curriculum, teaching and assessment) is implemented with little or no support for teachers, which leaves them to interpret reform measures on their own (Ball & Cohen, 1996). If teachers cannot practice the type of instruction needed to produce desired results because they have never experienced it, then how can they be expected to apply instruction effectively? Complete mathematics understanding include the capacity to engage in the process of mathematics thinking, that is, framing and solving problems, looking for patterns and making conjectures (Stein, Grove & Henningsen, 1996).

Classrooms must be communities in which mathematical sense making is developed (Stein et al, 1996). Students need much more than abstract concepts and self -contained examples, they need to be exposed to the use of domain's conceptual tools in authentic activity (Brown, Collins & Duguid, 1989). "Mathematics instruction advocated by NCTM asks teachers to: use mathematical activities that help students develop a conceptual understanding of the mathematics involved, to encourage students to use a variety of mathematics strategies to solve

problems effectively and flexibly, to discuss and examine mathematical strategies through classroom discourse for the underlying mathematical concepts, and to determine what students understand by listening to and by probing their students' thinking (Neumann, 2014,p. 2). Therefore, my study will explore how noticing practices used by the teachers in their classrooms impact mathematics learning.

1.3 Purpose of the study

The purpose of the study is to explore the noticing practices used by different teachers in their classrooms (for example, how do the teachers attend to their learners' mathematics conceptions, how do they interpret these conceptions, what listening and hearing skills do the teachers use, and how do the teachers reflect on the ideas and conceptions given by the learners). Another purpose is to examine what impact do these noticing practices by the teachers have on mathematics learning.

1.4 Research questions

1. How can observed mathematics teachers' noticing practices on learner mathematical thinking be explained in terms of noticing frameworks?
2. How can professional noticing be a tool for teachers to include learners in qualitative mathematical conversations?

1.5 Significance of the study

The study enabled us: to have an idea of what is taking place inside mathematics classrooms, to understand the interactions (talk and action) between the teachers and their learners, and to understand the practices that the teachers use in their classrooms. Moreover, this study led to the formulation of intervention strategies needed in order for mathematics learning to take place OR the formulation of the improvement plans that can be used for learners to improve. When noticing, the teachers need to interpret the specific mathematics thinking and reasoning each learner has engaged. The teacher will also need to figure out how to engage these interpretations in the classroom – how to mediate between them and the mathematical notions (Thomas et al., 2017). When teachers engage the thinking and reasoning of the learners in the

classroom, they will be bringing about responsive instruction which will benefit the learners.

1.5.1. Techniques and strategies that were used by the teachers

Whole class discussion strategy was used by the teachers in their classrooms. This was used to promote and extend the students' mathematical understanding and their proficiency (Ball & Bass, 2000). Listening techniques were also used by the teachers in their classrooms. These listening techniques are part of the grounded theory of the study. They ae discussed more in the Theoretical Framework chapter of the study. The four types of interpretation analysed by Wallach & Even (2005) were also of importance in the teaching and learning of the learners in the study.

1.5.2. How the learners benefited from the techniques and strategies employed by the teachers during the lessons

The strategies and techniques used by the teachers in the classrooms helped the learners to be able to voice out their ideas and thoughts with each other. These helped them to share their understanding of a mathematics concept and generated shared meanings amongst themselves. Learners were encouraged to engage in dialogues and to entertain the points of views of their peers. This was an indication that there was transformation of experience and that the process of learning took place.

1.5.3. The significance of study to policy makers

Department of Basic Education (DBE) policy makers would benefit from the study if they would be given the findings of the study. They would be able to explore different teaching methods and practices used by the teachers in their classrooms. Policy makers need to pay attention to ways that the teachers can use to help them shift from the practice of 'telling' only, but also to focus on how to link mathematics to real life events and prepare learners to be investigators and problem solvers. In addition to this workshops and seminars should be introduced in order to equip teachers with more skills and MKT especially in the less disadvantaged areas. The reality in south

African mathematics education context is that the quality of mathematics teaching in disadvantaged areas remain poor (Taylor & Reddi 2013). In order to support teachers and learners, some mathematics textbooks and need to be revised and a variety of contextually mathematics problems have to be introduced

1.6 Definition of terms

Noticing: To take note of something happening or taking note of an event. To observe, to pay attention and to act upon something.

Professional noticing: it is a skill used to identify and act upon salient mathematical options of children, it has three components: attending, interpreting and deciding (Thomas et al, 2017).

Attending: to focus one's attention on something. The teacher focusing his/her attention towards the learner's mathematics actions, ideas, conceptions and practices. Attending: Refers to the noting of the mathematical details in children's strategies (Thomas et al, 2017).

Interpreting: to analyse and to evaluate these ideas, conceptions and practices of the learners. Interpreting is when a teacher constructs a complete picture of a child's mathematical understanding (Jacobs et al, 2010).

Deciding: to act upon these actions and ideas of the learners by coming up with the productive instructional method based on the teacher's analysis and evaluation of the learner's ideas and conceptions (Jacobs et al, 2010).

NCTM: National Council of Teachers of Mathematics

Sampling: A selection of a portion of a population to be researched

Ethics: A set of moral principles used to protect the rights of the research participants.

Validity: An extent to which an instrument measures what it claims to measure (Ary, Jacobs & Sorensen, 2010).

Reliability: Degree to which a research finding remains the same when data are collected and analysed several times (Bloor & Wood, 2006)

Chapter two

Literature review and theoretical framework

2.1. Introduction

In this chapter I discussed the literature review and the theoretical framework of my research project. Literature review helped me to analyse what other researchers found out or say about the professional noticing skills and practices used by the teachers in their classrooms. Literature review also helped me to note the errors and misconceptions that the teachers notice in their learners during the teaching and learning process, for example Makonye (2015) states that some types of learner errors in elementary algebra can be reduced as a result of pedagogical intervention Therefore it is of importance for the teachers to take note of whatever is taking place in their classrooms. When teacher notice learners' errors they need to consciously attend, interpret and respond to them, rather than just saying it's wrong. They need to be curious of hat the learner is thinking and interrogate that (Makonye, 2015). Theoretical framework looked at the how the teachers listen to their learners during the teaching and learning period. This was done by using Davis' three forms of listening.

2.2. Literature review

2.2.1. Definition of Professional noticing

Professional noticing is defined as an expertise of interrelated skills used by a group of individuals who have the same goals. In this instance the group of individuals is teachers and their goal is for the learners to understand what is taught to them in the classrooms. The skills used by teachers include attending to children's strategies, interpreting children's understandings and deciding how to respond on the basis of children's understanding (Jacobs et al., 2010, p.169). Mason (2011) referred to the practice of noticing as a collection of techniques for 'pre-paring' to notice in the moment….. and 'post-paring' by reflecting. Mason (2011) described these techniques according to 'accounts of' and 'accounting for'. 'Accounts of' provide brief descriptions of the learners' strategies, while 'accounting for' not only what was observed but why it struck the observer sufficiently to be identified. Teaching students to know mathematics conceptually along with understanding is not a simple exercise.

This can be due to the fact that teachers themselves learned mathematics through direct instruction that emphasised fact-based, low level questions and rote memorization (Raymond, 1997). Many teachers therefore use this as their model of how to teach and therefore struggle to understand what it means to teach mathematics with conceptual understanding.

2.2.2. An outline of professional noticing in South African schools. Discussion of teaching and learning practices in South Africa using Socio-economic and political issues in the South African system of education

In South Africa the poor outcomes in mathematics has received much attention in recent times (Umugiraneza & Bansilal, 2017). For example, in the Grade 12 mathematics examination in 2015, the percentage of learners who achieved 50% and above was only 20% (Department of Basic Education (DBE), Republic of South Africa, 2016, p. 151). This means that 80% of the learners who wrote were only able to achieve a mark below 50%. To avoid or to deal with poor mathematics results, mathematics teaching approaches should be made more effective. The crucial issue in this regard is that of the actual methods of teaching employed by teachers to facilitate mathematics (Umugiraneza & Bansilal, 2017). Innovative teaching approaches can enable learners to link mathematics to real life and prepare learners to be investigators and problem solvers. Based on their research study conducted in Kwa-Zulu Natal South Africa, Umugiraneza & Bansilal, (2017) found that teachers mostly focus on teacher-led instructional methods. In the study conducted by Umugiraneza & Bansilal (2017), it was found that teachers in that Kwa-Zulu Natal, South African School did try to engage in progressive methods such as classroom discussion, group work and practical examples in their classrooms, however teacher-led instruction methods were still their first choice.

Mathematics teaching practices in South Africa are also affected by the areas in which the schools are situated. Mhakure (2019) conducted a study on the school-based Continuous Professional Development (CPD) of mathematics teachers in schools

located in disadvantaged areas in the South African mathematics education context. Schools located in these areas usually do not have good educational and mathematical resources. Research in the past three decades reports that, on the one hand, teachers who are exposed to frequent, rich learning opportunities have in turn been helped to teach in more ambitious and effective ways (Little, 2012, p. 1), while on the other hand, teachers who experience sporadic, superficial and disconnected CPD activities are left disappointed by limitations in learning opportunities (DeMonte, 2013). Recent lines of research concerning high school mathematics teachers in South Africa have shown that these teachers possess inadequate common content knowledge (CCK) and specialised content knowledge (SPK), and that frequent and quality professional development activities are lacking (Fleish, 2008). While the notion is true that teachers can learn from social gatherings such as workshops and seminars outside the school, the reality in the South African mathematics education context is that the quality of mathematics teaching in disadvantaged areas remain poor (Taylor & Reddi, 2013).

2.2.3. Relationship between professional noticing and Mathematical Knowledge for Teaching

CCK is the knowledge of the curriculum or the knowledge of the content that must be taught. SPK refers to the mathematical knowledge and skill unique to teaching, and is not typically needed for purposes other than teaching, (Ball, Thames & Phelps, 2008, p. 400). Mathematical knowledge for Teaching (MKT) refers to the amalgamated knowledge required to effectively teach mathematics (Thomas et al, 2017). There is an important and a possible relationship between professional noticing and MKT. the formulation of MKT is an attempt to define and describe such knowledge in the area of mathematics teaching, conversely, professional noticing has been linked to descriptions of outer and inner attention (Erickson, 2011). Knowledge of mathematics content is a prerequisite for effective teaching method, and such knowledge has been linked to improved student learning outcomes. Professional noticing allows for theoretically locating and analysing responsive instructional practices while MKT

provides framework for considering and investigating the varied knowledge-types required for rich mathematics teaching (Thomas et al, 2017). Adler and Davis (2006) wrote about applications of MKT. they termed these applications as unpacking and decompressing. This is where the teacher would need to interpret the specific mathematical thinking and reasoning in which the learner has engaged……the teacher will also need to figure out how to engage these interpretations in the classroom – how to mediate between them and the intended mathematical notions (p. 274). The idea of interpreting and mediating students' mathematical thinking, lead to how teachers choose and use examples in the classroom. This should provide insights into their knowledge of mathematics teaching (Chick, 2009). From Chick's perspective, the use of mathematical examples signifies both pedagogical unpacking and decompressing as well as some knowledge of the general mathematical idea embodied within the example. MKT is of importance in the teaching of mathematics in the classroom.

Mathematics instruction advocated by NCTM asks teachers to: use mathematical activities that help students develop a conceptual understanding of the mathematics involved, encourage students to use a variety of mathematics strategies to solve problems effectively and flexibly, discuss and examine mathematical strategies through classroom discourse for the underlying mathematical concepts, and determine what students understand by listening to and probing their students' thinking (McLaughlin & Mitra, 2001, p. 307). This request by NCTM leads teachers to come up with strategies that can help them to meet these requirements. Barrows (1986, p. 1) suggests that the integration of a variety of teaching methods and assessment strategies would be the most helpful factor to improve the effectiveness of teaching and learning practices. Furthermore, it may be the case that learners' preferred learning styles may not be their most effective learning styles. Therefore, the use of different teaching approaches has the advantage of challenging learners to think more laterally. One of the strategies that can be used is the approach of noticing by teachers.

The capacity of the teacher to attend to the students' mathematical conceptions and practices (Jacobs et al., 2010) is in line with Mason's technique of 'accounts of'- the noticing of the mathematical details in children's strategies. These are in line with Thomas et al skill of identifying children's mathematical actions (2017). Jacobs et al (2010) further state that, after attending to the students' mathematical conceptions and practices, the teacher interprets these conceptions. Interpreting involves the coordination of the observed strategies with current theory of mathematical development. This skill / practice is similar to Mason's technique of 'accounting for' (Thomas et al., 2017). When noticing, the teacher attends or identifies mathematics actions of children, interprets these actions and lastly decide upon a productive instructional course of action based on this interpretation (Jacobs et al., 2010).

Professional noticing has also been linked to the descriptions of outer and inner attention (Thomas et al., 2017). When the teacher attends to the learners' understandings, the teacher must attend to the inner and the outer ways of students' understanding. Outer attentions are the easily observed behaviour cues (for example, sitting still, talking to a neighbour, etc.), while inner attention refers to internal interests and thoughts of the student (Thomas et al, 2017), these are not easily observed by the teacher. When noticing, the teacher needs to interpret the specific mathematical thinking and reasoning in which each learner has engaged. The teacher will also need to figure out how to engage these interpretations in the classroom – how to mediate between them and the mathematical notions (Thomas et al., 2017). Professional noticing is to bring about responsive instruction tool in the classroom. It has to bring about mathematical development in learners. During this process the teacher's reasoning has to be consistent with both the details of the specific child's strategies and the children's mathematical development (Jacobs et al., 2010). Professional noticing determines the extent to which teachers use what they have learned about the children's understandings from the specific situation and whether

the teachers' reasoning is consistent with children's mathematical development (Jacobs et al, 2010).

2.2.4. The analysis of four different types of teacher interpretation (describing, explaining, assessing and justifying)

The other noticing techniques are associated with attempts to understand what students are saying, showing, feeling and doing – 'to hear students' (Wallach & Even, 2005). The nature of hearing and interpreting what students say, show, feel and do is of importance in the classroom. Wallach & Even (2005) analyse four different types of teacher interpretation: describing, explaining, assessing and justifying. Describing is when the teacher describes students' talk, thought, feelings and actions by direct quotation. Explaining is when the teacher explains the students' talk and action. Justifying – the teacher justifies her assessment of or the meaning she attributes to students' talk and action. Justifying differs from the other three because it involves the teachers' reflection on her own interpretation rather than on the students' talk and action (Wallach & Even, 2005). These types of interpretation determine how the teacher hears her students. One way teachers learn about their students' mathematical understanding or strategies is by asking them to explain their mathematical thinking through high level probing, and/ or pressing questions, and mathematical discourse (Kazemi & Stipek, 2001). When learners share their strategies and ideas for mathematics problem solving, they engage in mathematics discourse with their peers and their teacher. Kazemi and Stipek (2001) advocated that children should share and discuss reasoning that justifies the answer, rather than just give the answer itself or a summary of procedural steps. This social interaction of sharing and justifying their mathematical reasoning and strategies with their peers is what Vygotsky calls socially

constructing new knowledge-they are developing the cognitive skills needed to learn new information (Wertsch, 1984).

When learners are given the opportunity to share their thinking with their peers and their teacher, they begin to have conceptual understanding. The teachers should therefore encourage sharing of the ideas by their learners. One way of encouraging this process is the use of collaborative classroom practices. This process analyses two conceptual models: one of the teacher's role, along with specific instructional strategies the teacher uses to organise a collaborative learning environment, and another of the process by which the class's capacity to participate in collaborative inquiry practices developed over time (Staples, 2007). Classrooms with these collaborative practices are seen by NCTM as places where students have opportunities to reason and construct their understandings as part of a community of learners (Staples, 2007). The value of these reform practices has been demonstrated with respect to student learning, particularly students' conceptual understanding (Boaler, 1997).

Staples (2007) states two conceptual models of a teacher's instructional strategies that supported students' participation in the whole class discussions. Whole class discussions are used by the teachers to promote and extend students' mathematical understandings and develop their proficiency with a range of mathematical practices such as conjecturing, justifying and reconciling (Ball & Bass, 2000). Collaborative implies a joint production of ideas, where students offer their thoughts, and attend and respond to each other's ideas, and generate shared meaning or understanding through their joint efforts. This is process of noticing which is used by the learners on other learners. This is where the learners notice each other's conceptions and ideas of mathematics. The teacher in this practice acts as a facilitator of the process. In facilitating, the teacher uses practices such as eliciting student ideas. In doing this, many teachers use generic questions such as why, how do you know, who else has an idea? These requests open space for student ideas (Kazemi & Stipek, 2001). Requests

assume that a student has an idea that is reasonably well formulated, and given the opportunity, can and would be willing to share the idea with others (Staples, 2007). The role of the teacher in this practice is a more experienced or a more capable other who assists students in a performance they would not have achieved on their own. This is where the teacher uses the practice of scaffolding. This is a strategy used to elicit students' ideas and to uncover the students' thinking and support them in expressing their ideas (Staples, 2007).

Examination of describing and explaining types of interpretation indicates five different characteristics of teacher hearing students: over-hearing, compatible hearing, under-hearing, non-hearing and biased hearing (Wallach & Even, 2005). Over-hearing is when the teacher hears things that were not said by the learner. Under-hearing is when the teacher ignores some of the things said or done by the students. It denotes hearing where some parts are missing, whereas non-hearing implies that the whole thing said by the learner is missing (Wallach & Even, 2005). This is when the teacher does not hear anything said by the learner. Biased hearing is when the teacher does not hear what the learner says, but the attention of the teacher looks at who the learner is. The required type of hearing is compatible hearing. This is where the teacher hears exactly what the learner says.

Another method used by the teachers to 'listen' to their learners is the analysis of student work and student thinking. Jacobs et al (2010) conceptualised the term 'professional noticing' of children's mathematical thinking as: (a) attending to children's strategies using meaningful ways to discern patterns and chunk information in complex situations. When children share their strategies for solving problems they participate in mathematical discourse with their peers and teacher (Neumann, 2014). Kazemi & Stipek (2001) advocated that children should share and discuss reasoning that justifies the answer, rather than just give the answer itself or a summary of the procedural steps. Neumann (2014) argues that the social interaction of sharing and justifying their mathematical reasoning and strategies with their peers

and teachers is what Vygotsky calls socially constructing new knowledge. She further states that they are developing the cognitive skills needed to learn new information. When teachers focus on understanding students' thinking, teachers are more likely to change their beliefs and instructional strategies to cater for children's thinking.

Listening is essential for teachers to understand students' perspectives. There may be some ideas hidden behind the students' problem solving process. It is necessary therefore to have a tool to understand what the students are doing, a reason behind their ideas, and to be able to adjust their procedures or ways of thinking to become acceptable (Wetbunpot & Inprasitha, 2015). This I shall link with Jacobs et al (2010) second concept of professional noticing of children's mathematical thinking. They argue that after attending to children's strategies, the teacher then interprets children's mathematics understanding based on the details of the children's strategies. Jacobs et al's (2010) third concept of professional noticing of children's mathematical thinking states that the teacher decides on how to respond based on the interpretation of the children's strategy and understanding. Neumann (2014) argues that there are several ways to move a child's thinking forward, but any direction should be grounded in the child's way of thinking. Wilson and Berne (1999) found that when teachers focused on understanding students' thinking during professional development experiences, teachers were more likely to change their beliefs and instructional strategies to listen to children's thinking. It was also documented that when teachers focused on student's work and understanding students' mathematical thinking, they were better able to recognise their students' mathematical competencies and developed appropriate learning benchmarks for moving students forward mathematically (Franke & Kazemi, 2004). A response by the teacher should be an instruction which is based or led by the child's thinking and understanding. The teacher needs to ensure that his / her instruction based on the child's thinking leads to the mathematical and conceptual understanding for the learner.

One of the methods used by the teacher in constructivism is a method of group work. Group work promotes higher order thinking among students, and it helps both the low and high achievers gain from each other. However, for these groups to work more effectively requires that the group members share some common social and cultural orientations that help them to critically reflect, and find possible solutions to the problem in question (Ampadu & Danso, 2018). So cooperative learning is one of the principles used in constructivism. This is where students interact effectively with one another to develop new ideas and knowledge.

This sometimes is not always the case. This is according to the research study conducted in Ghana by Ampadu & Danso in 2018. They argued that most students prefer to work alone as they will not be allowed to do cooperative work during their final examination. Also, based on their study, all the students indicated their willingness to participate in the teaching and learning process, but they prefer not to participate since they will be mocked at by their peers if they provide wrong answers. Wrong answers are the errors that the learners make when answering mathematics questions in class. Errors that the learners make are mostly the results of the misconceptions that the learners have Misconceptions often occur because of assimilation through over-generalisation. When a learner meets a new mathematical object, he/she might think that the mathematical object belongs to a class which he/she already has. This could be a misconception of inclusion (Luneta & Makonye, 2013). It is necessary that the learner becomes aware that the mathematical object does not belong to the class he/she puts it (Luneta & Makonye, 2013). In order fo this to happen, a teacher should be able to take note and to understand where the misconception comes from.

There are different types of errors that the learners make when answering questions in mathematics. These include systematic errors, constant errors, reading errors, comprehension errors, transformation errors, process skills errors, encoding errors and careless errors ((Makonye, Cox, 1975; White, 2005). symptoms of the underlying

conceptual structures that are the causes of the errors (2015). It is these underlying beliefs and principles in the cognitive structure that are the cause of the systematic conceptual errors, and these are called misconceptions (Olivier, 1989, p.4). A misconception is the misapplication of a rule, an over or under generalisation or an alternative conception of a situation (Drews, 2005). The errors made by learners are therefore caused by their misconceptions of a particular mathematics concept or situation. Misconceptions that the learners have include misconception of interference and misconception of meaning. Misconception of interference is when the previously learned skills interfere with new skills (Olivier, 1989). "Addition schema is constructed first and is well developed, thus when a question is asked about multiplication, which is a more recent piece of learning, the pupil replaces it with a question dealing with earlier material' (Olivier,1989, p.11). Misconception of meaning occurs when learners conceive open expressions as incomplete and tend to finish them (Booth, 1988). For example, when a learner sees 3a + 5, a learner with a misconception of meaning does not consider 3a + 5 as a final answer, he sees this as incomplete and therefore has to complete it (by adding these two terms together) and get the answer as 8a.

On the other hand, the culture of acknowledging only correct answers on the part of the teacher is another factor affecting students' participation (Ampadu & Danso, 2018). This is where the teacher uses the evaluative form of listening. Coles 2008) argues that if a teacher is listening in an evaluative manner, students' contributions are judged as either right or wrong. This form of listening leads the learners to hesitate in participating in the classroom. The teacher needs to use the interpretive form of listening which is characterised by an awareness of the fallibility of the sense being made (Davis, 1997). This is where a response by the teacher might offer feedback to the speaker (Coles, 2008). These forms of listening will be clearly explained in the following chapter of Theoretical Framework.

2.2.5. Conclusion of literature review

In this chapter professional noticing has been discussed in detail. Firstly, this was done by expanding on its definition. The definition of professional noticing involves attending to children's mathematical strategies in the classroom, interpreting these strategies, and deciding on how to respond as a teacher to these strategies. Secondly, South African professional noticing practices were also outlined. This was done by discussing the teaching and learning practices used in South Africa using Socioeconomic and political issues in the South African system of education. This was also done by discussing Mathematical knowledge for Teaching (MKT) possessed by teachers in different South African areas. The analysis of four different types of teacher interpretation (describing, explaining, assessing and justifying) were also discussed.

2.3. Theoretical Framework

Noticing can help teachers to develop new ways of 'seeing' whatever is happening in the classrooms. For this to happen teachers are supposed to pay close attention to the ideas that students raise and use these ideas as the basis for the lesson in progress (Sherin &Van Es, 2005). This study has used Davis' enactment theory as a theoretical framework. The theory is about Davis' three forms of listening.

Noticing is a three-part practice: firstly, it identifies noteworthy aspects of a classroom situation, secondly it uses knowledge about the context to reason about the classroom interactions, and lastly, it makes connections between the specific classroom events and broader principles of teaching and learning (Sherin & an Es, 2005

One of the ways of improving noticing is the practice of listening by the teachers. The act of listening is to hear attentively, to give ear to, to pay attention, to make an effort to hear something (Coles, 2008). It involves an act of will or decision on the part of the listener to make an effort. Each teaching episode is characterised by a particular manner of listening. Davis (1997) uses the notion of listening in this way, arguing that an attentiveness to how mathematics teachers listen may be a worthwhile route to

pursue to help better understand their practice. Listening is both an effective means of interpreting classroom phenomena and as useful starting place for transforming mathematics teaching practice (Davis, 1997). When a student shares mathematical thinking and a teacher actively listens to the strategies the child shares, the teacher can then determine what the student knows and understands. She or he can then ask questions of the child that further push the students' thinking forward, or probes the child's thinking so that a better diagnosis of the students' gaps can be made. These three aspects of teaching (sharing, listening and probing students' mathematical thinking) also help students build conceptual understanding). (Jacob, et al, 2010).

Davis (1997) argues that there are three forms of listening used by the teachers in their classrooms. These include evaluative, interpretive and hermeneutic listening. Coles (2008) argues that if a teacher is listening in an evaluative manner, students' contributions are judged as either right or wrong. Learners in this situation do not get any tangible feedback or interpretation from the teacher. The teacher will only say yes or no. there is no justification whatsoever from the teacher (Coles, 2005). Another quality of evaluative listening is revealed in the sorts of questions posed by the teacher. In every case, the teacher already had a 'correct' answer in mind. The teacher in this situation is not really interested in what the learner is saying, rather, the teacher's actions and questions are generally pre-determined and allow little room for deviation (Davis, 1997).

Interpretive listening is characterised by an awareness of the fallibility of the sense being made (Davis, 1997). This is where a response might offer feedback to the speaker not by evaluating what was said, but by offering an interpretation and asking for clarification (Coles, 2008). In interpretive listening there is an attempt to interpret and make sense of the speaker says, but always from the point of view of the listener (Coles, 2008, p.24). Interpretive listening brings about an action of probing and of eliciting by the teacher. The teacher wants to uncover the ideas of the learner, or the thinking behind the learner's question or answer. In this form of listening the listener

is aware that whatever is spoken may not be the connection, idea or meaning the speaker intended (Coles, 2008). That is why interpretation or clarification is necessary.

Transformative (hermeneutic) listening is defined as an imaginative participation in the formation and the transformation of experience through an ongoing interrogation of the taken for granted and the prejudices that frame these perceptions and actions (Davis, 1997). Evidence of transformative listening in a classroom includes a willingness to alter ideas in a discussion, to engage in dialogue, to entertain other points of view and hold them as valid, independent of whether they are accepted or not (Coles, 2008). Coles (2008) further states that if a student makes a connection to a previous piece of work or links something that has been said before, this would indicate the transformation of experience, the restructuring of categories. This sense of restructuring of previous categories or ideas is indicative of learning (Coles, 2008).

2.4. Conceptual framework

Conceptual framework is used to refer to at least three different things: First, it is seen as purely visual representation of a study's organisation or major theoretical tenets. This representation is usually included within one's literature review, as a standalone figure (Rowitch & Riggan, 2012). A second perspective treats conceptual and theoretical frameworks as essentially the same thing. A third view sees the conceptual framework as a way of linking all the elements of the research process: researcher disposition, interest, and positon, literature, theory and methods (Rowitch & Riggan, 2012). This study does not define conceptual framework in terms of the second perspective of Rowitch & Riggan (2012). In their second perspective, they argue that conceptual framework and theoretical framework are basically the same thing. This study looks at the conceptual framework as a standalone thing. The contents of this conceptual framework are also included in the literature review of the study. The conceptual framework of this study will basically explore the three key elements or definitions of the noticing process: attending, analysing (interpreting) and responding

(deciding) (Jacobs, et al, 2010). These elements or definitions of the noticing process together with the theoretical framework will be used in the analysis of the data of the study.

Levels of sophistication for noticing skills.

Skill	Low sophistication	Medium sophistication	High sophistication
Attending	Highlights classroom events, teacher pedagogy, student behavior, and/or classroom climate. No attention to student thinking.	Highlights student thinking with respect to the collection of data from a scientific inquiry (science procedural focus).	Highlights student thinking with respect to the collection, analysis, and interpretation of data from a scientific inquiry (science conceptual focus)
Analyzing	Little or no sense-making of highlighted events; mostly descriptions. No elaboration or analysis of interactions and classroom events; little or no use of evidence to support claims.	Begins to make sense of highlighted events. Some use of evidence to support claims.	Consistently makes sense of highlighted events. Consistent use of evidence to support claims.
Responding	Does not identify or describe acting on specific student ideas as topics of discussion; offers disconnected or vague ideas of what to do differently next time.	Identifies and describes acting on a specific student idea during the lesson; offers ideas about what to do differently next time.	Identifies and describes acting on a specific student idea during the lesson *and* offers specific ideas of what to do differently next time in response to evidence; makes logical connections between teaching and learning.

Figure 1. Levels of sophistication for noticing skills

Adapted from: Barnhart, T., & van Es, E. (2015).

Figure 1 shows the levels of sophistication of noticing skills. I now discuss these levels of sophistication in terms of the types or forms of listening by Davis (1997). Davis states three forms of listening: evaluative, interpretive and transformative forms of listening. I will link low sophistication with the evaluative form of listening. In evaluative listening the learners' responses are judged as either right or wrong (Coles, 2005). Learners do not get any tangible feedback from the teacher. Low level of sophistication there is little or no sense making of highlighted events. When responding, the teacher offers disconnected or vague ideas of what to do differently next time. Medium sophistication is linked to the interpretive form of listening. Interpretive listening attempts to interpret and make sense of the speaker but always on the point of view of the listener (Coles, 2005). It brings about an action of probing and to uncover the ideas of the learner. The medium level of sophistication highlights students' thinking and begins to make sense of highlighted events. It identifies and describes students' ideas during the lesson and offers ideas on what to do differently next time. Transformative listening is now linked to high sophistication level of noticing. Transformative listening is an imaginative participation and a willingness to alter ideas in a discussion. It is to engage in a dialogue and to entertain other points of views and hold them as valid (Coles, 2005). High level of sophistication highlights students' thinking with respect to the collection, analysis and interpretation of data. It consistently makes sense of the highlighted events. It identifies and describes acting on a specific student ideas during the lesson and offers specific ideas of what to do differently next time. It makes logical connections between teaching and learning.

When noticing, teachers attend, interpret (analyse) and respond to their learners' strategies and ideas. When attending to learners' ideas, teachers identify key aspects

in learners' ideas and strategies. After identifying these key aspects, teachers need to analyse these aspects. In analysing these ideas, teachers interpret these ideas in a way that can be understandable to the rest of the learners. After analysing and interpreting teachers need to respond to the learners based on the interpretations of the ideas. Teachers' responses can be to come up with the best instructional strategy which will bring about mathematical understanding to the learners. As indicated in figure 1 there are levels of sophistication for these noticing skills. There are low, medium and high levels of sophistication.

	What Teachers Notice	How Teachers Notice
Level 1 *Baseline*	Attend to whole class environment, behaviour, and learning and to teacher pedagogy.	Form general impressions of what occurred; Provide descriptive and evaluative comments; Provide little or no evidence to support analysis.
Level 2 *Mixed*	Primarily attend to teacher pedagogy; Begin to attend to particular students' mathematical thinking and behaviours.	Form general impressions and highlight noteworthy events; Provide primarily evaluative with some interpretive comments; Begin to refer to specific events and interactions as evidence.
Level 3 *Focused*	Attend to particular students' mathematical thinking.	Provide interpretive comments; Refer to specific events and interactions as evidence; Elaborate on events and interactions.
Level 4 *Extended*	Attend to the relationship between particular students' mathematical thinking and between teaching strategies and student mathematical thinking.	Provide interpretive comments; Refer to specific events and interactions as evidence; Elaborate on events and interactions; Make connections between events and principles of teaching and learning; On the basis of interpretations, propose alternative pedagogical solutions.

Figure 2: Snapshots of mathematics noticing levels

Adapted from

Choy, B. H. (2015).

2.4. Conclusion of theoretical and conceptual framework

Different forms of listening were discussed in this chapter. These include evaluative, interpretive and transformative (hermeneutic) listening (Davis, 1997). These were discussed as part of the theoretical framework. Contents of the conceptual framework of the study are mostly based on the key elements of the definition of professional

noticing: attending, interpreting and deciding. These were outlined and discussed in detail in the literature review of the study.

Chapter three

Methodology

3.1. Introduction

The study is of qualitative nature. It focused on the exploration of noticing practises that the teachers use in their classrooms. Qualitative research takes place in the natural setting (Creswell, 2003). The study was conducted in a school environment in three different classrooms. The qualitative researcher often goes to the site of the participants to conduct the research. This enables the researcher to develop a level of detail about the individual or place and to be highly involved in actual experiences of the participants (Creswell, 2003, p.181). The study was conducted in the teachers' classrooms. I went to the classrooms so that I can identify the noticing practices that the teachers use in their classrooms. How these noticing practices impact mathematics learning were also explored.

Qualitative research is descriptive and interpretive. This means that the researcher makes an interpretation of the data (Creswell, 2003). Interpreting refers to the individual and collective efforts to give meaning to symbols, talk, action, and environment to explain what is going on (Miller & Dingwal, 1997). They further argue that it is an unending process that makes research scripting, staging and performing possible.

This study was conducted in three classrooms with three different teachers. Interpretation of the data collected was done. Contrast, classification and integration of all findings were also performed. These were done by taking the findings from one teacher and compare them to the findings of another teacher. The purposes of the descriptive designs seek to acquire evidence concerning a situation or population and

to identify norms or baseline information which can be used for comparative purposes (Adams & Schcaneveldt, 1991).

3.2 Research Design - Ethnography

Ethnography is a type of qualitative research that gathers observations, interviews and documentary data to produce detailed and comprehensive accounts of different social phenomena (Reeves, Peller, Goldman & Kitto, 2013). I refer to my study as ethnographic research. The data of the study were gathered using methods of observation and interviews. Ethnography is the study of social interaction and culture groups, whether these groups are designed as societies, communities, organisations or teams (Reeves et al, 2013). My research is a study of social interaction within a school community. My research explored the practices that are used by the mathematics teachers in their classrooms.

The task of ethnography is to document the culture, the perspectives and practices of the people in the settings that they inhabit (Hammersley, 1992). I gathered the information of the study by going to the classrooms of the teachers. I inhabited the settings of the mathematics teachers. I wanted to document the culture, the perspectives and practices of the teachers in their classrooms. The ethnographic approach means that the ethnographer goes into the field to explore a cultural group and / or explore certain social interactions (Reeves et al, 2013). In my study the exploration of certain classroom interactions between a teacher and the learners was administered.

3.2 Sampling

Without data, it is impossible to do research. Data are obtainable through some type of sampling process. The type, size, composition and nature of the sample are depended on the purpose of the study, the population available, research design, and the resources available to the researcher (Adams & Schvaneveldt, 1991, p. 172). In this research study a sample was selected. This was done because the intention of the

researcher was not to study a relatively large population, but to identify a small group of people.

The discussion on survey was removed as per the recommendation by the examiner

The sample of the study is the purposive sample. This sampling procedure Cresswell, (2003) refers to it as convenience sampling. This is where respondents are chosen based on their convenience and availability (Cresswell, 2003).

The study used this kind of sampling. The sample used in the study was chosen in relation to the purpose of the study. The purpose of the study was to explore the noticing practices used by the teachers in their classrooms. This kind of sampling was therefore chosen based on the purpose of the study. There are also limitations in this sampling procedure. Conclusions drawn are strictly limited to the population actually studied. The findings of this study were only limited to the teachers that were interviewed and observed. These findings can therefore not be generalised.

Sample of the study

In the following I discuss the characteristics of the participants.

Pulane

Pulane is a male teacher who is 27 years of age. He graduated from university of Pretoria in South Africa in 2016. He has honours degree in Bachelor of Education. His major subjects are Mathematics and Life Sciences. He has less than three years of experience of teaching. He is currently teaching grades 8, 9 and 10 Mathematics. I observed Pulane teaching his two grade 8 classes, his grade 8A class has forty learners and his grade 8B class has 38 learners.

Themba

Themba is a male teacher who is 34 years of age. He is a product of the University of Limpopo in South Africa. He graduated in the year 2010. His qualification is Bachelor of Science with Mathematics and Physical Science majors. He has less than 10 years of

teaching experience. He is presently teaching grade 11 and 12 Mathematics. He was observed teaching his grade 11 mathematics class. His grade 11A class has 24 learners.

Mary

Mary is a female teacher who is 54 years of age. She graduated in Johannesburg College of Education in 1990. She also graduated in B. Ed (Honours) from the University of Witwatersrand South Africa in 2011. Her majors are Mathematics and Life Sciences. She is currently teaching grade 8 and 9. She has about 30 years of teaching experience. She was observes teaching her grade 8 classes. Her grade 8C class has 40 learners and her grade 8D class has 42 learners.

Eddie

Eddie is a male teacher who is 50 years of age. He graduated in Bloemfontein College of Education. His major subjects are Physical Science and Mathematics. He has 25 years of teaching experience. He graduated in Bachelor of Science. He is currently teaching grade 8 and 10 Mathematics. He was not observed teaching. He was only interviewed based on his noticing practices in his classroom.

Eric

Eric is a male teacher who is 45 years of age. He has 20 years of teaching experience. He graduated in the University of Limpopo. He is teaching Mathematics grade 8, and Mathematical literacy grade 10 and 11. He graduated in Bachelor of Arts and later qualified in Post Graduate Certificate in Education. He was not observed in the classroom. He was also interviewed based on his noticing practices in her classroom.

Gift

Gift is a male teacher who is 31 years of age. He has less than 10 years of teaching experience. He graduated in the University of Witwatersrand in South Africa. His qualification is Bachelor of Education. He is teaching grade 9 Mathematics and grade

10 and 11 Physical Science. He was also not observed in the classroom. He was only interviewed based on his noticing practices in the classroom.

The empirical site

The sampling procedure of this study was a non-random sampling procedure. The key features of the sample of this study are convenience, availability and accessibility. The data were collected at my own school. My school is a school in the East of Johannesburg. There are about 1200 learners and 40 educators. There are five different departments of learning areas. These include, Mathematics and Science department, Languages department, Life Orientation department, Life Sciences department and Creative Arts department. Sampling is the link between population and its generalisation to the wider population. The units of a sample may be individuals, institutions and communities (Bloor & Wood, 2006). The sample of the study is six teachers who were teaching in their classrooms. Three of these six teachers were observed while teaching in their classrooms. The other three teachers were interviewed. The interview was mostly about the noticing practices that they use when teaching in their classrooms.

Conclusion of Sampling

In concluding this sampling section, I argue that this study used convenience sample. This is where respondents are chosen based on their convenience and availability (Creswell, 2003). This sample is of convenience to me because these are the teachers that I teach with in my school. We are in the same mathematics department.

The data collected in this research study cannot be generalised in all mathematics teachers of the province. They cannot be generalised even for the teachers in the same district. Conclusions drawn on the study are strictly limited to the population that is actually studied. This procedure is appropriate if you do not intend to draw conclusions about a larger group on the basis of the group you study (Bouma et al, 2009). This procedure is appropriate and suitable for this research study because the

data collected or the research findings of this study are only intended for research report purposes only.

3.3 Data collection methods

Introduction

This section outlined the research methods that were used to collect the data of the study. Data were collected through classroom observations and interviews with teachers. Three teachers were observed while teaching mathematics in their classrooms. In addition to that, three more teachers were interviewed about their professional noticing practices that they use when teaching in their classrooms. I did not want to observe or interview all the teachers of my sample. I observed three teachers so that I could compare and contrast the practices that they each use in their classrooms. I interviewed only three of the teachers because I wanted to see whether the conclusions and findings from both methods would yield the same results.

Doing research involves more than just data collection. The research process does not begin nor does it end with data collection. Before data collection can be done, the researcher must: focus on the problem, identify and define basic concepts involved, select a research design that will provide the desired information, decide on a sampling procedure and draw the sample (Bouma et al, 2009). Unless each of these essential first steps is completed, data collection will often be done in a wasteful, haphazard and unproductive way. If the above steps are completed, data collection can proceed smoothly, efficiently and with little wasted time or effort on the part of either the researcher or the participants in the research (Bouma et al, 2009). The problem statement of the study focuses on how the teachers teach or the ways and methods that the teachers use when teaching mathematics. It also focuses on how these ways and methods are used by the teachers in order to produce the desired results. So, before data were collected the researcher focused more on the problem statement of the study, and how the study would identify it. That is what guided the researcher to select the identified sample of the study. After the sample was selected data collection methods were identified. The identified methods were believed to be able to provide the desired information of the study.

Qualitative researchers rely on four primary methods for gathering information. The research methods that were used in this study are observation and interviews. Research methods are used in describing for the reader how the data will inform the research questions (Marshall & Rossman, 2011). In answering the research questions, observation of the three educators in their classrooms were conducted. This was followed by in-depth interviews of these three educators. In addition to interviews of these three previously observed teachers, two additional teachers were interviewed.

3.3.1 Observation method

Observation is the systematic viewing of a specific phenomenon in its proper setting for the specific purpose of gathering data (Bell, 2005). Observation is a basic qualitative research technique that requires discipline, planning and alertness. There are two basic forms of observation: non-participant and participant observation (Bouma et al, 2009).

Participant observation has some disadvantages. Firstly, the researcher's presence may affect the situation and cause other participants to feel self-conscious or even 'tress-passed' upon. Secondly, the researcher might be unable to observe much of the situation while participating, for an example, if the researcher is much older than the participants, it may be difficult to blend in with the youth (Bouma et al, 2009). The other disadvantage is that, when a stranger enters that situation, the social interactions will inevitably change. People may talk to the newcomer, trying to place them in the social context. They might try to make sense of their presence, and locate them in the social network of the situation (Oliver, 2010). The study did not use participant observation. Participating in the study was going to defeat the purpose of the research. The researcher wanted to observe and take notes of what the teacher was doing and saying. If the researcher participated she was not going to be able to thoroughly observe and take notes. The study used non-participant observation method.

In non-participant observation, the researcher stands back from the situation and observes – for instance, when a researcher sits in a corner at a committee meeting to find out how the committee arrives at decisions (Bouma et al, 2009). This is where the researcher does not take part in the research process. The only thing that he / she is doing is to observe and record the information that he is looking for.

Observation was chosen in this study because first-hand information will be obtained. The research study used non-participant and undisguised observation. The researcher only observed and did not participate in any events / activities of the classrooms. Undisguised observation is when the respondent is aware of being observed (Bell, 2005). The research study used undisguised observation method. The researcher did not disguise herself. Both the classroom teacher and the learners were well aware of her presence and also of the reason why the researcher was there. The advantages of using observation method include non-biased responses, accurate and realistic data based on first-hand information, and observing events as they occur (Bell, 2005). Field notes were taken during and after the process of observation.

The disadvantages of observation technique include subjective interpretations. This is where researchers rely only on their own perceptions of observation. The second disadvantage is: when respondents are aware of observation they tend to act or behave in a different way (Bell, 2005). The respondents were ensured that the data collected in their classrooms is only for research report purposes only, and therefore there will not be any reason for the respondents to behave in an 'un-normal' way.

The researcher explored the noticing practices used by the teachers in their classrooms when teaching. Observation of how these teachers attend to their learners' mathematics strategies and conceptions were conducted, how the teachers interpret the learners' strategies were also looked at. The researcher also analysed the responses of the teachers based on the interpretation of the leaners' strategies. The purpose for observing a system being used is to obtain in a limited amount of time relevant data

needed to change and / or improve the system. Field notes were used to collect data during the observation process.

Field notes

Field notes refer to qualitative notes recorded by scientists or researchers in the course in the course of field research, during or after their observation of a specific phenomenon they are studying (Gibbs, 1949). These are actually notes taken by a researcher during the process of the research. The purpose of field notes is to help the researcher to remember the activities / actions that were taken during the research process. Gibbs (1949) states that the notes are intended to be read as evidence that gives meaning and aids in the understanding of the phenomenon. They allow the researcher to access the subject and record what they observe in an unobtrusive manner (Gibbs, 1949).

According to Bogdan & Biklen (1982), field noted usually consist of two broad kinds: descriptive and reflective. One major disadvantage of taking field notes is that they are recorded by an observer and thus subject to memory and possibly the conscious or unconscious bias of the observer (Gibbs, 1949). It is therefore best to record field notes immediately after leaving the site to avoid forgetting important details Most observers will try to develop systematic ways in which they can record their field notes. Ultimately, the purpose of the research will be to develop a theory concerning the social setting, and hence one purpose of the field notes will be to try to note the roles and relationships in that setting (Oliver, 2010). During observations the study used descriptive field notes. This is where the researcher described what she saw, heard and what she experienced during the observation process. The field notes were taken during the observation process. Some additions were done by the researcher immediately after leaving the classroom, as the events were still fresh in the mind of the researcher at the time.

3.3.2 Interviews

Interviews are categorised into three general types: the informal conversational interview, the interview guide or topical approach, and the standardised open ended approach (Marshall & Rossman, 2011).

The three teachers that were observed in their classrooms were also interviewed by the researcher. A list of questions was prepared for them. The questions were based on what has been observed. Questions like, what prompted you to say/do that when learner A said this…. Did you pre plan to ask/ do this, or you did that because learner B said that….

Interviews provide the best opportunity to find out what someone else thinks or feels. The idea of in-depth interviewing is to get a 'window' on reality from the point of view of a participant and to allow them to tell their story as they wish and identifying the issues that are important to them (Bouma et al, 2009). The interview schedule usually contains more information than simply the questions to be asked. It may include a reminder to the interviewer of the information to be given to potential interviewees before the commencement of the interview. This may be the reminder about one or two formalities, such as thanking the interviewees for their time, and it might also include a brief restatement of the purpose of the research (Oliver, 2010).

One of the main advantages of the interview is that the interviewer can explain the purpose of the study, discuss the interview and respond to any questions a respondent might have.

Two problems often occur during interviews: First, because participants have a lot of freedom in how they respond to general questions, they often drift to topics unrelated to the research (Bouma et al, 2009). Listening to this idle talk is polite and can also be productive, particularly when it generates rapport. The strategy here is to show interest in the participant's discussion before redirecting them back to the research topic. The researcher needs to tell the participants that he respects them enough to

engage in friendly conversations at a later stage, and then gently restate the research question. The interviewer comes to the situation with goals in mind, objectives to be attained, and questions to be used in accomplishing these purpose.

Three additional teachers were also interviewed. These teachers were not observed. They were just interviewed based on their experiences as Mathematics teachers. The standardised type of interview was used in interviewing these teachers. The questions that were asked were based on their own noticing practices in their classrooms. They were asked how they attend to their learners' strategies or mathematical thinking, how they interpret these strategies and how they respond to the interpretations. The teachers were also asked to provide some examples of how they perform these things.

The interview process of the study took place in the Mathematics department office in our school. This is the office that the mathematics teachers of our school share whenever there is a mathematics departmental meetings. This is a comfortable setting for both the interviewer and the interviewees. Three educators were interviewed. The interview processes did not take place at the same time nor on the same date. The only convenient time for the interviews was during break and after school. One interviewee suggested to be interviewed during his non-teaching period (free period), but that time was not convenient for the interviewer because the interviewer was teaching at that time. Break time and after school were the convenient times for both the interviewer and the interviewees. The interviewees preferred to be interviewed during break. They said after school they are tired and the only thing that they think about is to go home. We therefore agreed to conduct the interviews during the break.

The average duration for each interview was 15 – 20 minutes. There was no need to introduce myself. We know each other we work in the same place. I thanked them deeply for their time and for the opportunity they gave me to interview them. I did not have the problems of the interviewees drifting away from the research question. I think it was because break is only 40 minutes, and in this 40 minutes the teaches had to eat and to respond to the question posed on them. Therefore, there was no time for

small talk. The interviewees were motivated and willing to participate. We even discussed the interview time together. The interview instrument with questions was well prepared, and the interviewees were informed again of the purpose of the study. The interview instrument with questions is attached as an appendix of the study. Reflective field notes were used during the interview processes.

In concluding this chapter, observational data refer to the raw material an observer collects from observations, interviews, and materials such as reports that others have created. Data may be recorded in several ways, written notes, sketches, tape recordings, photographs and videotapes (Bailey, 2008). This research study gathered information using interviews and observations. Data were recorded using mainly field notes.

3.4 Data analysis

Qualitative data analysis involves organising, accounting for and explaining the data: in short, making sense of data in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities (Cohen, Manion & Morrison, 2007, p.479). The analysis of data in a qualitative study is mostly influenced by the number of data sets and by the people from which the data is collected.

Having organised my data, how do I analyse or 'make sense of it' so that I can write about 'what is going on' with the benefit of an informed understanding? Most importantly I have to develop my sense of having 'been there' to the situation so that I can look at the data with the sensitivity of someone who knows the situation personally (Bouma et al, 2009). Analysis can achieve a variety of objectives. It can compare the data from similar or different situations, it can draw general themes out of the data, and enable us to make more general statements which will have wider applicability, it can take a range of isolated fragments of data and infer broader or underlying trends, which can be useful for making predictions, and it can start us on the process of developing theories which can be useful to try to explain the reasons for the data taking the form they do (Oliver, 2010).

Data analysis process involves an attempt to identify any themes or concepts which will appear to emerge from the data. These themes and concepts are then analysed and interpreted using the grounded theory and / or the conceptual framework. The cycle of identifying themes and concepts continues with a progressive attempt to either identify new themes or amend existing ones. Gradually these themes will be fitted together into a social theory which will slowly develop more coherence and clarity (Oliver, 2010). In some forms of qualitative data analysis, an attempt was made to identify the most significant concepts in a text, or piece of qualitative data, and perhaps also to identify the number of times a concept is used. This process can reveal the extent to which the responded or writer is relying upon a particular concept in transmitting ideas. It is part of an overall process known as content analysis (Oliver, 2010, p.120).

The data that were collected in the study were analysed and interpreted using the grounded theory and the conceptual framework of the study. The grounded theory of the study focusses on Davis' three forms of listening. The conceptual framework of the study explores the three key elements or definitions of the noticing processes: attending, analysing (interpreting) and responding (deciding) (Jacobs et al, 2010). Themes and concepts were identified from the data collected and these were linked and fitted together into the grounded theory and into the conceptual framework of the study. The data were presented and analysed for each participant first. Later, in the discussion stage, themes and issues emerging across the participants were amalgamated.

Data analysis involves making sense out of text and image data. The data collected in the study was analysed in order to reflect on the information supplied by the participants. Data in this study were analysed following Creswell's (2003) three steps.

Creswell (2003) argues that data analysis involves three generic steps. The first step is to organise and prepare the data. This involves transcribing interviews, optically scanning material and typing up field notes. This step is to obtain the general sense of

the information and to reflect on its overall meaning. The second step is to read through all the data. This is to identify the general impression of the overall depth, credibility and the use of the information. The third step is to begin a detailed analysis with a coding process. This involves taking text data or images into categories and labelling those categories with a term (Creswell, 2003).

Cohen et al (2007) argue that researchers decide whether to present the data individually or to combine key issues emerging across the individuals. Data in this study is presented firstly individually. Combination of key issues from all participants is also considered. Data in this study were analysed using the conceptual framework and the grounded theory. Grounded theory research is designed to develop a theory of social phenomena based on the field data collected in the study. From an inductive analysis of the study, the researcher constructs concepts, after constructing concepts the researcher then forms a theory by plausible relationships among the concepts (Ary et al, 2010, p.30). Inductive method is making observations and then draw conclusions from these observations. The theory that was mostly used to interpret the data is Davis's enactment theory. This theory evaluates and explains the three forms of listening used by the teachers and learners in the classrooms. These forms of listening are: evaluative, interpretive and hermeneutic listening.

3.5 Validity and reliability

Validity is an extent to which research produces an accurate version of the world (Bloor & Wood, 2006). This study improved validity of data by reviewing the literature of the study and by analysing the data using the theory. Bloor & Wood (2006) argue that validity can be improved by thorough data analysis in which the researcher searches for deviant cases, thereby revising the theory in the light of data. They further state that validity reflects a need to provide an improved understanding of the research subject rather than improved accuracy (2006, p. 148). Internal validity of the study was portrayed by the study. The researcher's conclusions correctly portray the data collected.

Reliability is the extent to which research produces the same results when replicated. It is concerned with the extent to which research findings are reproducible, that is whether a different researcher who replicated the study would come to the same or similar conclusions (Bloor & Wood, 2006). It refers to the consistency and dependability of the fact itself (Adams & Schvaneveldt, 1991). Reliability is the degree to which a research finding remains the same when data are collected and analysed several times (Bloor & Wood, 2006). To increase reliability of the study, the process of triangulation was used. The study employed multiple methods in collecting the data. Observation of teachers teaching mathematics in their classrooms was used. Some teachers were interviewed to collect data. The use of multiple methods in this study was used to increase reliability of the study. Data triangulation was also used in the study. Theory triangulation was also used in the study. This is the use of multiple perspectives to analyse data (Denzin, 1978). The study used multiple perspectives to analyse data. Theoretical framework, conceptual framework and literature review were used to analyse the data of the study. Reliability of the study has been realised. The findings of the three different classroom observations have been observed and these findings are replicated.

3.6 Ethics

The purpose of this chapter is to outline ethical considerations that were followed before the data were collected. The research process will usually be more important to the researcher, but much less so to the participants. It is easy for researchers in all subjects and areas to become so preoccupied with collecting their data that they forget about the dignity of the persons providing those. One way in which researchers can give something back to the respondents is to be aware of the ways in which that research might help them or other people (Oliver, 2010)

Qualitative research often brings researchers and participants into close contact and creates a need for the interests of both to be balanced. Researchers can improve the lives of participants if they have access to such data and they need support in their

efforts (Bouma, Ling & Wilkinson, 2009). During the research process ethical considerations should be followed. Researchers must treat participants in their research with dignity and respect.

Participants must not be forced into taking part in the research process, they must be fully informed about the purpose and the intended results of the research project. Therefore, it is in their right to refuse participating in any research project. The decision to participate must be based on knowledge of what will be involved, what will be demanded in terms of time, activity and topics covered, what risks are likely, and where to lodge complaint should that become necessary. This information is usually provided by means of a consent form which is also referred to as a 'plain language statement (Bouma et al, 2009).

Participants in this study were first verbally requested to participate in the study. It was discussed with them of what the study is about. They were told that the study is only for research purposes. The consent form was then given to them. They were given some time to read through, and sign it when they feel comfortable to do so. The copy of the consent form of this study is attached as an appendix of this report. The consent form was endorsed by me and my supervisor. The form described who I am. It also stated the reason why I am doing the research (for my Master's degree purposes), it also stated what the research will require from the people I was seeking permission from. A consent form usually concludes with the information about what the participant should do if they have any questions or concerns about the research (Bouma et al, 2009).

When conducting a research project, the rights of people are greater than the researcher's 'need to know'.People who take part in a research need to be referred to as 'participants' rather than 'subjects', this shows that as researchers we remember that we are dealing with people and not objects. This helps to retain the levels of respect and concern appropriate to humans (Bouma et al, 2009).

In concluding this chapter, ethics are guidelines or sets of principles for good professional practice, which serve to advise and steer researchers as they conduct their work. It is concerned with thinking about morality, integrity and distinction between right and wrong (Bloor & Wood, 2006). This study has followed all the ethics considerations. Permission from the teachers was requested. Letters requesting permission from Gauteng Department of Education, the school principal and the School Governing Body were prepared and submitted to them. These bodies granted me the opportunity to conduct the research in their school.

Chapter four

Data analysis

4.1. Introduction

Qualitative data analysis involves the identification, examination and interpretation of patterns and themes in textual data and determines how these patterns and themes help answer the research questions (Patton, 1999). Qualitative data analysis transforms data into findings (Patton, 1999). This chapter considered the observations and interviews of my six teacher participants. Pulane, Mary and Themba were observed teaching in their classrooms. Eddie, Gift and Eric were interviewed. These names of the teachers that were observed are pseudonyms for the purpose of the research. The lesson observation processes took place in the teachers' classrooms. The lesson observations took place over several days.

There were three lessons observed for Pulane. These were three in different grade 8 classes that he teaches. The first lesson was about the angles that add up to 180°. It took place on Wednesday the 11th of September 2019. It was on period 4. The duration of each period was 40 minutes. It was a class of 38 learners.

The second lesson that was observed took place on the Thursday the 12th of September 2019. It was the continuation of the previous day' lesson. It was also about the angles that add up to 180°, but in this lesson parallel lines and a transversal were given. It was a grade 8 class as well even though it was a not the same class that I observed the previous day.

The third and the last lesson that was observed for Pulane took place the following week. It happened on the 18th of September 2019. The lesson was about the algebraic fractions. It was a grade 8 class.

There were two lesson observations conducted in Mary's classrooms. The first lesson took place on Friday 13th of September 2019. The lesson was about revision of multiplying and dividing expressions. It was a grade 8 class of 35 learners. The second lesson that was observed was about equations. It took place on Monday the 16th of September 2019. It was the same grade 8 class of 35 learners that I observed previously.

The third teacher that was observed was Themba. Two lessons were observed for Themba. The first lesson observation was conducted on Tuesday the 17th of September 2019. The topic of the lesson was depreciation. The lesson was conducted when the teacher was teaching 30 grade 11 learners. The second lesson observation was conducted the following day the 18th of September 2019. It was the continuation of the depreciation lesson with the same class of the grade 11s.

The whole process of lesson observations process took about two weeks.

4.2. Data analysis of observed teachers

I begin my data analysis by discussing each teacher's professional noticing in relation to learners' mathematical thinking during the lesson observations

Analysis of observation on Pulane's teaching with respect to professional noticing of learners mathematical thinking.

Pulane is referred to as Teacher 1 in the sampling chapter. He is a 27 years old. He is a teacher with less than 3 years of teaching experience.

Lesson observation 1: Grade 8 classroom:

In the following I write a transcript of what transpired in Pulane's classroom. I write about my observation of his lesson. This lesson was mostly about the angles that add up to 180°. It took place on Wednesday the 11th of September 2019.

I discuss in detail my observation in Pulane's lesson 1.

The teacher greeted the learners. The teacher used a data projector. The first thing that he did was to put an exercise on the slide. The exercise was:



Calculate the value of x, give a reason for your statement.

What type of quadrilateral is ABCD? Justify your answer.

Learners were told to copy down the exercise and not to answer it. After copying it down the learners waited for the teacher to give them an instruction.

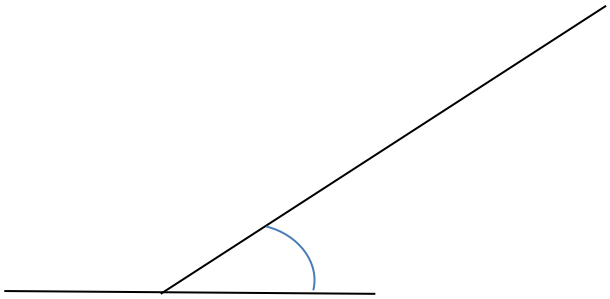
The teacher then initiated the conversation. He said "yesterday we spoke about the angles on the same straight line". The learners agreed with him saying yes sir we said angles on the straight line add up to 180˚.

Teacher: Yes, good. We said they add up to 180˚. Which other angles add up to 180? (Learners raised up their hands).

Sibusiso: Angles in a triangle.

Teacher: Angles in a triangle add up to 180˚.

The teacher drew a straight line on the board. I want you to look at this. This is a straight line. I want us to talk about the angles on a straight line. He drew a slanted line on top of the straight line to make two angles



Teacher: What is the value of angle a. I don't want any answers from the calculator. (learners raised their hands. The teacher pointed 1girl)

Lebo: Angle a is 132˚.

Teacher: Explain to us how you got that answer.

Lebo: We said that angles on a straight line add up to 180˚. Now we are given a straight line. One angle is 48˚ so I said 180˚-48˚=132˚. So when I add 132 and 48 I get 180˚.

Teacher: Good. You explained it very well. Angles on a straight line add up to 180˚. It does not matter how many angles are there on a particular straight line. They will all add up to 180˚. Just like the angles in a triangle.

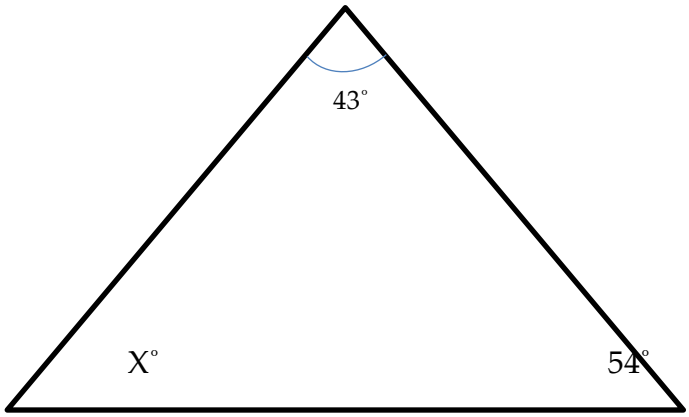
Teacher: How many angles does a triangle have?

Learners: 3 angles

Teacher: Good. There are 3 angles in a triangle, and the value of these angles add up to….?

Learners: 180˚

The teacher drew a triangle on the board with angles 43, 54 and x



Teacher: Find the value of x. Do not use the calculator.

Learners worked on their books for about 3-5 minutes. When they finished they raised up their hands. The teacher called Bonga to come to the board and show the answer. Bonga stood up and walked towards the board and wrote:

$43^\circ + 54^\circ = 97^\circ$ $180^\circ - 97^\circ = 83^\circ$ $x = 83^\circ$

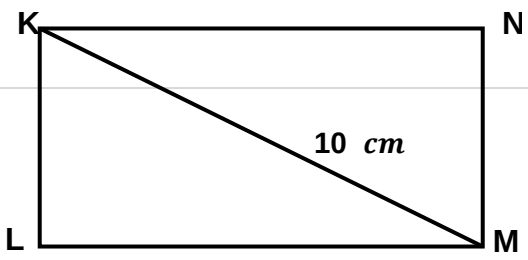
Teacher: Correct. Thanks my boy.

This leads us to talking about quadrilateral.

Teacher: What do you think a quadrilateral is? (The teacher looked at all the learners whose hands were up and asked one boy)

Lelethu: A quadrilateral is a shape that has 4 sides and 4 angles.

The teacher then added another triangle to the triangle that he drew previously.



Teacher: If this is the case, how do we get a quadrilateral?

Agnes: Angles on a quadrilateral add up to 360˚.

Teacher: Why do you think that is the case?

Agnes: Eehh because

Teacher: no. I am not asking you alone. I am asking everyone… if learner E is correct when saying angles of a quadrilateral add up to 360˚, why is that the case… Yes learner B, what do you think?

Brian: I think eehh… in the board you added another triangle on top of the triangle that was there already. So there are 2 triangles now, so the value of the angles of 2 triangles is 360˚.

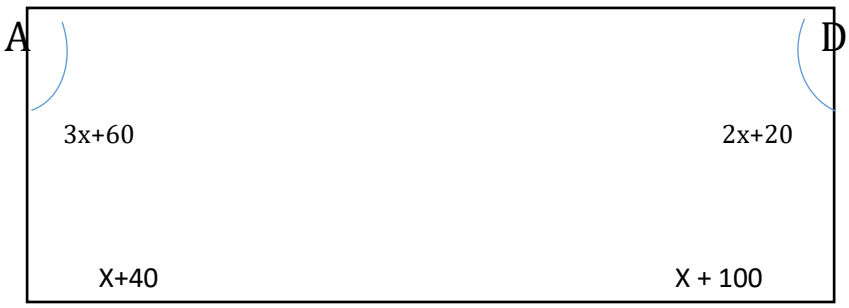
Teacher: Did you all hear what he said?

Learners: Yes

Teacher: I want anyone of you to repeat what he said.

Betty: Sir, there are two triangles. One triangle has 3 angles adding up to 180˚. Also the second triangle has 3 angles adding up to 180˚. So those two triangles have angles that add up to 360˚.

Teacher: Ok. Now let us look at the examples that I gave you beginning of the lesson. (He said this pointing at the first diagram that he gave them).



I gave you a quadrilateral with four angles. What are the four angles that you see in the quadrilateral?

Learners: 3x+60, 2x+20, x+40 and x+100

Teacher: ok Angle 1 is 3x+60, Angle 2 is 2x+20, angle 3 is x+40 and angle 4 is x+100 right?

Learners: Yes

Teacher: What I want you to do now is add all these angles and equal them to 360˚, like this Angles 1+2+3+4 =360˚. Before you do that I want you to look at this example of triangle. He wrote on the board x˚ + 2x˚ + 30˚ = 180˚. Let's find x. To find x we know we have to collect the like terms. How do we collect like terms class?

A girl responded and said, "we group the ones that are the same".

Teacher: Let's group them, which ones are the same?

Grace: x and 2x are the same.

Teacher: Good. Which others are the same?

Betty: 30˚ and 180˚ are the same as well.

Teacher: That's very good. But why do we say that they are the same?

Betty: Because x and 2x have x variable. 30˚and 180˚ are just numbers without any variable.

Teacher: Ok. What do we need to do next? I want someone to come and write the answer on the board.

Lerato: x + 2x = 180˚- 30˚ *3x = 150˚* *x = 30˚*

Teacher: That's good. Now let's get back to our previous question. I said Angles 1+2+3+4=360˚, you said in the beginning Angle 1 is 3x+60, angle 2 is 2x+ 20 and so on….. So I want you to add these angles. In the place of angle 1 put 3x+60 in angle 2's place put 2x+20 etc. then after doing that I want you to find the value of x. I am giving you about 5 minutes to do it. I don't want you to use a calculator. I want you to list and group the like terms like we have done with this (pointing at what Learner F wrote) example.

Learners started writing. Others talking to each other about what they wrote. The other one was saying to the other I found 7x = 140˚. I also got that, the other one said. The teacher gave them 5 minutes to complete the task.

Teacher: We have about 2 minutes towards the end of the period. Make fast.

Teacher: Ok. What is the answer? What is the value of x?

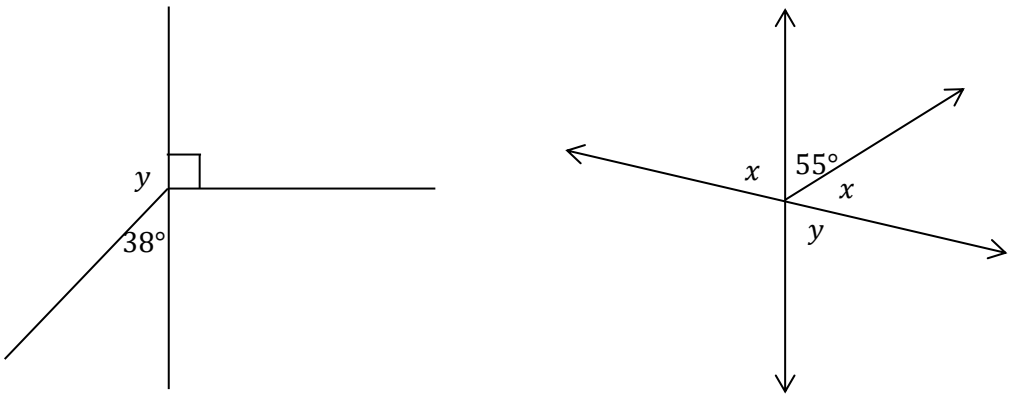
Learners: x=20

Lesson observation 2: Pulane's grade 8 classroom

I now write a transcript of the second lesson observation of Pulane's lesson. This lesson was about angles. The teacher warned me in advance, that this class is a bit slow. The teacher gave them the following task:

Question 1

Find the values of x and y. Give reasons for your answer.

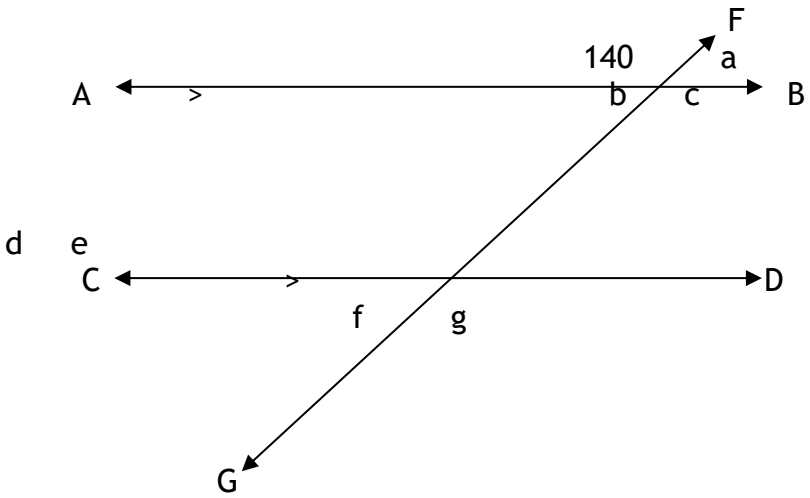


The teacher told them to quickly copy it down. They quickly copied it down and waited for him.

Teacher: This is what I'd like us to do today. In order for us to answer these questions, I want us to look at this:

Question 2

If AB // CD



We are given parallel lines AB and CD and a transversal FG. Angle ACF=140˚. We are required to find angles a to g, with reasons.

Teacher: What are parallel lines? What are they?

Thando: They are equal and they will never meet.

Teacher: Ok. I will take that answer.

Tumi: There is a certain distance between them.

Teacher: The distance between them will never change, this leads them never to meet.

Teacher: I want us to find angles a to g. You can start with the easiest that you think you can get. You don't have to start with angle a.

Cindy: 140˚+ a = 180˚ a = 180˚- 140˚ a = 40˚

Teacher: Ok, Priscilla started with a, yes anyone else

Chloe: c = 140˚…… vertically opposite angles

Teacher: ok, another one?

Tshepi: b is also 40˚…… vertically opposite angles

Teacher: Ok, but vertically opposite with which angle, angle c?

Tshepi: No sir with angle a

Teacher: Ok. We got angles a, b and c. How about d now?

Cindy: d = 140˚……. Corresponding angles

Tumi: g is also 140˚…… vertically opposite with d

Teacher: Good. We are now left with e and f.

Grace: e =40˚. Alternate angles with b. They form a z

Teacher: Good, how about f?

Tumi: f = 40˚ vertically opposite with e

Teacher: That's very good, now let's go back to the question. Let's try and answer now number 1.

Tumi: y = 180˚- 38˚ y = 142˚

Teacher: Do you all agree with what Sam is saying?

Learners: Yes sir we agree.

Teacher: Why?

Yonela: Because they are the angles in a straight line.

Teacher: ok, that's good. Now let's go to number 2. Let's do number 2 now class.

Anna: Sir can I come and show the answer on the board:

Teacher: Yes come, you are most welcome

Anna went to the board. She wrote

y + 55˚ = 180˚……. Angles on a straight line

y = 180˚ - 55˚ y = 125˚

x = 55˚

x = 180˚- 55˚ x = 125˚

After writing all this, she stood aside.

Teacher: ok. Can you explain to us what you wrote there?

Anna: No sir, I can't explain.

Teacher: Is there anyone who did it the same way?

Teacher: Do you guys agree with her solution?

Tshepi: No sir, I don't agree. The answer is x = 0. Sir, can I go to the board and show how I got zero. He went to the board and wrote

X = 55˚ + 125˚ x = 180˚ x = 180˚ - 180˚ = 0

Teacher: Ok. Your answer is 0, so let's substitute your answer on x + x + 55˚, does it add up to 180˚?

Teacher: Is he correct class?

Learners: No. He is not correct.

Yonela: Sir can I go to the board and solve it?

Teacher: Sure. Go.

Teacher: Guys you must know that I won't do it for you. You have to get the answer yourselves. You need to apply the rule when solving this.

Teacher: How do you solve the angles on a straight line. That's the only clue I can give you, only.

The teacher overheard one girl whispering to her friend saying x = 62,5

Teacher: Sarah, say that loud for all of us to hear. We want to hear what you are saying.

Somi: I'm saying x = 62,5

Teacher: Why do you say that?

Somi: Because 62,5 + 62,5 + 55 = 180

Lesson observation 3: Pulane's grade 8 class

This is day three of my observation of Pulane's classroom. The lesson of this day about algebraic expressions. Pulane said to his learners, "yesterday we did Pythagoras theorem, and I believe you did your part at home with the homework I gave you".

Teacher: Let's do algebraic expressions before we attempt to do the exercise I gave you about scientific notation.

The teacher wrote down 3 algebraic expressions:

$10xy^5$ 2. $(2x^3 - 3x + 5x^2) - (4x^3 - x - x^2)$ 3. $(x - 2)(x + 2)$

Teacher: We need to simplify before we learn to find the unknown variables and solve for x. We must know how to simplify.

Teacher: Let's look at number 1. We are dividing two parts, what do you think happens here?

Lundi: (Raising his hand) Sir, can I do it on the board?

Teacher: ok. Come and do it.

Lundi wrote $2x2y^5$

Teacher: Explain to us how you got that answer

Lundi couldn't explain

Teacher: Is it how we do it class?

Learners: Some said yes, some said no

Teacher: ok class listen. 10 and 5 are both co- efficients because they are in front of variables. So let's divide now the co-efficients without variables. 10 ÷ 5 is…..(waiting for the learners to finish off the sentence)

Learners: 2

Teacher: What do we do when dividing exponents of the same base?

Learners: (Mumbling)

Sethu:: The answer is $2xy^4$ *sir.*

Teacher: (looking at his watch, said, ok let's go to number 2

$(2x^3 - 3x + 5x^2) - (4x^3 - x + x^2)$

Teacher: Here we have terms separated by a positive or negative signs. How many terms do we have?

Learners: 3

Teacher: Ok. What is it that we need to do now?

Learners: We will use BODMAS to remove brackets.

Teacher: How do we remove the second bracket:

Learners: We multiply by a negative sign, then we add the like terms

Teacher: Lelethu identify the like terms for us.

Sethu identified them by pointing at all the like terms.

Teacher: Thanks Lelethu. Then what's the next thing to do

Learners: We add them, then we are done.

Teacher: You identified 3 sets of like terms, add them for me. Choose the ones you like then the arrangement will be seen later.

Learners: $-2x^3 + 4x^2 - 2x$

Example 3. $(x - 2)(x + 2)$

Teacher: What is the method that we use in this example class?

Learners: Quiet

Teacher: Method is FOIL

Teacher: I want someone to use FOIL to solve the problem. FOIL is First, Outer, Inner and Last

Lungi: First x^2 Outer $+2x$ Inner $-2x$ Last -4, then the answer is $x^2 - 4$

Teacher: Why is the answer $x^2 - 4$. What happened to the other numbers?

Lungi: $-2x$ and $2x$ are like terms with different signs, so their final answer is 0. That is why the answer is $x^2 - 4$

Teacher: now I will explain scientific notation that I gave you yesterday.

Teacher: when you press in a calculator then you get $2{,}5 \times 10^{-5}$ or $2{,}5 \times 10^{5}$ what is that?

Lethu: it's a scientific notation.

Teacher: ok. It is good to use scientific notation because it makes long numbers like 0.00007 short.

(the bell rang for the end of the period. The teacher promised the learners to see them and to finish the lesson the following day).

Analysis of Pulane's lesson observations with respect to professional noticing of their learners' mathematical thinking

Research Question 1: How can observed mathematics teachers' noticing practices on learner mathematical thinking be explained in terms of noticing practices?

Research Question 2: How can professional noticing be a tool for teachers to include learners in qualitative mathematical conversations?

Analysis of observation on Pulane's teaching

Mason (2011) referred to the practice of noticing as a collection of techniques for 'pre-paring' to notice in the moment…. And 'post-paring' by reflecting. The first technique that the teacher Pulane used in lesson 1 was to give his learners an exercise and tell them to just copy it down and not to write any answers. This was to prepare learners for what he was going to teach them. He was pre-paring them to notice in the moment (Mason, 2011). His instruction became a means of building their capacity for mathematical thinking and reasoning (Stein, Grove & Henningsen, 1996). The teacher wanted to engage them in the process of mathematical thinking for the topic that was going to follow the exercise. This is the process that influences the students' cognitive processes which in turn influence their learning. Stein et al (1996) term this process the student mediation model. Mediators serve as the means by which an individual act upon and is acted upon by social, cultural and historical factors (Daniels, 2011). The learners copied down the exercise and waited for their teacher to give them another instruction. Learners at this stage used social norms. Social norms are general ways that students participate in classroom activities (Kazemi & Stipek, 2001). Situated type of learning took place in this instance. Situated learning is the way in which learning is linked to the situation or context in which it takes place (Lave, 1993). When the teacher saw that the learners were done copying down the exercise, he reminded them about what was done the previous day and said, yesterday we spoke about angles on the same straight line. Professional noticing has also been linked to the descriptions of inner and outer attention (Thomas et al, 2017). Outer attentions are the easily observed behaviour cues, while inner attention refers to internal interests and thoughts of the students (Thomas et al, 2017). The teacher attended to the outer attentions of the learners when he saw that the learners finished writing down the exercise. The teacher in this instance initiated the conversation. The response by learners to his initiation was positive. The learners agreed with him, they said yes sir, we said that the angles on a straight line add up to 180˚. The teacher then built from what the learners were saying, and said yes, good, which other angles add up to 180˚? In this instance the teacher also attended to the inner attentions of the learners. Inner attentions are the internal interests and

thoughts of the students (Thomas et al, 2017). In his initiation, the teacher only said, yesterday we spoke about the angles on a straight line. The students said angles on a straight line add up to 180°. They said this even before the teacher ask. This shows the internal interest of the learners in this particular topic. The teacher then interpreted this reasoning / interest of the learners by asking them a question: which other angles add up to 180°. Interpreting involves the coordination of the observed strategies with current theory of mathematical development (Thomas et al, 2017). Learning that took place here can be viewed as situated activity whose central defining characteristic is legitimate peripheral participation. This is where learners move from being newcomers toward full participation (Lave, 1993). In this lesson there is complete participation by learners. Complete participation is the collective practice by which there might be a measurable degrees of acquisition by newcomers (Lave, 1993).

The teacher then gave examples of the straight line and of a triangle. He told them to find the values of the missing angles. The teacher asked one girl who gave the correct answer of 132°. The teacher asked her to explain to everyone how he got 132°. The teacher here wanted to know whether the learner can justify her correct answer. The teacher in this instance urged the learner to 'account for' her mathematical reasoning. This was through the interpretive type of listening by the teacher. In interpretive listening there is an attempt to interpret and to make sense of what the speaker says, but always from the point of view of the listener (Coles, 2008). Interpretive listening brings about an action of probing and of eliciting by the teacher. The teacher wants to uncover the ideas of the learner, or the thinking behind the learner's answer. In this form of listening the listener is aware that whatever is spoken may not be the connection, idea or meaning the speaker intended (Coles, 2008). It can happen that the learner copied the 132° from another learner, or she just overhead someone saying the answer is 132°, that is why justification, interpretation and clarification is necessary.

Another form of listening that the teacher used is transformative (hermeneutic) listening. This is defined as imaginative participation in the formation and transformation of experience through an ongoing interrogation of the taken for granted and the prejudices that frame these perceptions and actions (Davis, 1997). When the teacher was dealing with quadrilateral example, he asked the learners what they think a quadrilateral is. One learner responded and said a quadrilateral is a shape with four sides. The teacher then drew another triangle on top of the triangle that was already on the board. After doing that he asked the learners how we get the quadrilateral. One boy said angles of a quadrilateral add up to 360˚. The teacher asked, why that is the case. Another boy responded and said; "Sir, you have added another triangle on top of the other triangle, so there are two triangles now. These two triangles make a quadrilateral. Previously we said that angles of a triangle add up to 180˚. Now we have two triangles so their angles add up to 360˚. Coles (2008) states that if a student makes a connection to a previous piece of work or link something that has been said before, this would indicate the transformation of experience, the restructuring of categories or ideas is indicative of learning. This shows that the skill of the teacher has included the learners to be able to come up with qualitative mathematics conversations.

In lesson observation 2 the teacher warned me in advance that the class that I was going to observe is a very slow class. That caught my attention and wanted to see what different strategy will he use when teaching them. The teacher gave them the examples to copy down. They quickly copied down everything and waited. The teacher wrote down another example on the board, he drew two parallel lines AB and CD, and transversal FG. Angle FCA = 140˚was given. He said in order for us to answer these examples (pointing at the 2 examples) I want us to look at this (pointing at the parallel lines with transversal). The teacher again here was preparing the learners cognitively. He was engaging them in the process of mathematical thinking for the topic / examples they were going to look at in few minutes. The teacher used student mediation model, the process that influences the students' cognitive processes which in turn influence their learning (Stein et al, 1996).

The teacher asked them what parallel lines are. One boy said, they are equal and they will never meet. Another boy said there is a certain distance between them. The teacher said this distance between them will never change, this leads them never to meet. The teacher revoiced their answers. The teacher's reasoning in this instance is consistent with both the details of the specific child's strategies and the children's mathematics development (Jacobs et al, 2010). After attending to these students' mathematical conceptions (answers) the teacher interpreted these by combining their answers and telling them that, that is the reason why the parallel lines will never meet. In this example of parallel lines learners were required to find the values of angles a to g with reasoning. The teacher told them they can start even with the easiest one. The teacher was facilitating the process, allowing the learners even to come to the board and show their calculations. The whole class discussion was taking place. The teacher got students to share their thinking and this gave learners the chance to engage in sustained reasoning (Chapin, O'connor and Anderson, 2003). After this process of giving answers for parallel line questions, the teacher then said let's go back now to the main examples of the day. He said let's try and answer number 1. One boy quickly raised his hand, the teacher noted him and asked him to answer. The boy said y = 180° - 38° = 142°. The teacher asked the class if they all agree with him. They said yes. The teacher asked why. The learners said they are the angles on a straight line. The learners here were asked to justify their answer. This shows that interpretive and transformative listening was used by the teacher.

Example 2: Anna asked to come and show the answer on the board. She wrote:

Y = 55° = 180°…….. angles on a straight line

Y - 180° - 55° = 125°

X = 55°

X = 180° - 55° = 125°

When she finished writing the answer she stood aside and waited for the teacher to comment. The learner used social and socio mathematical norms of the classroom. Social norms are general ways that students participate in the classroom (Kazemi & Stipek, 2001). The learner also used socio mathematical norms. These are based on mathematical explanation of activities. She went to the board and presented her work. The teacher in this instance used reform-minded mathematics instruction. This includes the teacher asking the learners to present their strategies and solutions to the class and they try to make mathematics activities interesting (Kazemi & Stipek, 2001, p.80). the teacher asked her to explain what she wrote. She said she can't explain. The learner in this situation made a transformation error. Transformation errors occur when the children understand what the question asks but they are unable to identify the operation, or sequence of operations, needed to solve the problem (White, 2005). The learner understood what the question asked, because she even asked the teacher if she can go to the board and demonstrate to other learners on how to answer the question. The learner's error was caused by the misconception that she had. The definition of misconception includes: (a) a misunderstanding or misinterpretation or to have a wrong idea of a topic or a concept, (b) misapplication of a rule, an overgeneralisation or an alternative conception of a situation (Drews, 2005). The learner in this instance made an error due to the misconception of meaning. Lerman (1994) claims that mathematics is a matter of internal mental operations, and meaning is an association of mental operations with mathematical symbols. He further states that this is a private process which cannot be witnessed by anyone else, all one has to go on is the visible results of those mental operations, the writing speaking or other behaviour of students. In this situation the learner wrote an incorrect answer. The other learners saw the answer but were unable to explain how she got that answer. The teacher asked if there is anyone in class who did it the same way. Learners did not respond. The teacher then asked the class if they agree with her solution. A boy said no I don't agree, the answer is $x = 0$. The teacher asked him to go to the board to show his solution. He went to the board and wrote:

X = 55° + 125° x = 180 ° x = 180° - 180° = 0

The teacher said ok, your answer is x = 0, so let's substitute your answer in x + x+ 55°. These are the angles in a straight line. Let us see if they will add up to 180°. Remember, the angles in a straight line must add up to 180°. Does it add up to 180°, the teacher asked? The teacher here was justifying why the learner's answer is incorrect. Justifying is when the teacher justifies his/ her assessment of or the meaning she / he attributes to students' talk and action. It involves the teacher's reflection on his / her own interpretation rather than on student's talk and action (Wallach & Even, 2005).

Analysis of observations on Mary's teaching with respect to professional noticing of learners mathematical thinking

Mary is female teacher who is 54 years old. She has more than 20 years of teaching experience. The lesson that she had with her grade 8 learners was on multiplying of expressions.

Lesson observation 1: Mary's grade 8 class

In the following, I write a transcript of what transpired in Mary's classroom. Mary and I entered the classroom together. She was from another class. I was already waiting outside the classroom. After greeting the children, she told the learners that Mrs Hantibi (referring to me) will be here today to observe our lesson. She then proceeded with the lesson. She said that today we will look at multiplying the expressions.

Teacher: We added and subtracted expressions, now we will multiply and divide the expressions.

Teacher: I will give you 3 examples, after that I will give you the classwork.

The teacher wrote the examples on the board

EXAMPLES

$25a^3X\ 4a^2$ 2. $24a^3b \div 4a^2b$ 3. $4(2b-3c)-2(b-6c)$

She gave the learners 3 minutes to copy down the examples

Teacher: let us look at example 1 now…. When we multiply and divide expressions, we go back to the rule of exponents. What does the rule of exponents say, class?

Learners kept quiet.

Teacher: If u multiply powers of the same base, what do you do with the exponents?

Learners: We add them.

Teacher: What do we call 25 and 4 in that example?

Queeny: co-efficients

Teacher: Good. So what must we do with the co-efficients

Learners: We must multiply them

Teacher: So what is 25 X 4

Learners: 100

Teacher: So it will be 100(a^3 X a^2). What is the final answer class?

Learners: $100a^5$

Teacher: Good. But why to the power 5

Learners: We add powers of the same base. 3 +2 =5

Teacher: Anyone who doesn't understand how we got 5

One boy raised up his hand. The teacher asked one learner to explain to him.

Example 2

$24a^3b \div 4a^2b$

Teacher: What is the second law or rule of exponents?

Queeny: When we divide powers of the same base we subtract them

Teacher: ok, what is 24÷4

Learners: 6

Teacher: what must we do now class?

Learners: $a^3 \div a^2 = a$

Teacher: Do we all know why we say a

Learners: yes

Teacher: Then what next class

Gladys: The answer is 6a

Teacher: Why, what happened to b?

Kagiso: Any base to the exponent of 0 is 1. So b doesn't exist anymore, that is why we have 6a

Buntu: I don't understand ma'am

Teacher: What is the exponent of b?

Learners: 1

Teacher: Any number to the exponent of 1 is 0. We said when we divide powers of the same base we subtract them, therefore b – b =0. So we cannot say 6a0 that is why we say the answer is 6a.

Example 3

4(2b – 3c) – 2(b -6c)

Teacher: in this situation we distribute 4 inside the bracket and we keep the sign. We distribute the outside numbers inside the bracket

Teacher: Let's do it class

Learners:8b – 12c – 2b + 12c

Teacher: Good. Then what do we need to do next?

Thandi: 12c -12c = 0

Teacher: There is nothing like 0c. c is gone as well

Learners: we collect like terms

Teacher: that's correct. We put the bs and cs together. We take them with their signs. We can't leave their signs. So it's

Learners (with the teacher): 8b + 2b – 12c +12c = 6b + 0

The teacher gave them classwork after this exercise.

Analysis of Mary's lesson observations with respect to professional noticing of their learners' mathematical thinking

Research Question 1: How can observed mathematics teachers' noticing practices on learner mathematical thinking be explained in terms of noticing practices?

Research Question 2: How can professional noticing be a tool for teachers to include learners in qualitative mathematical conversations?

Analysis of observation on Mary's teaching

After greeting the learners, the teacher initiated the lesson by saying, yesterday we added and subtracted expressions, and today we will multiply and divide the expressions. She continued and said, I will give you 3 examples after that I will give you the classwork. Then she wrote the examples on the board. She gave the learners 3 minutes to copy down the examples. After the learners have finished copying down the examples. The teacher said, when we multiply and divide expressions we go back to the rule of exponents. What does the rule of exponent say, class. First, the teacher was 'telling' the learners. She said when we multiply or divide expressions we go back to the rule of exponents. 'Telling' is the providing of intellectual resources, steering, an offering of something intended both to contribute to and to shape the discussion (Chazan &Ball, 1999). The teacher here was contributing and shaping the discussion of the day.

After telling them that we go back to the rule of exponent, she asked them what the rule says. She was engaging them. She wanted them to think, to reason and to come up with mathematical ideas. The learners did not respond to her question. The teacher rephrased her question by trying to make it easy for them. She said if you multiply powers of the same base, what you do with the exponents. Then the learners said, we add them the teacher in this situation identified mathematical actions of her learners (keeping quiet). She realised that the learners did not know the answer (interpreting their actions). She then decided upon a productive instructional course of action based on her interpretation (she lowered the cognitive demand of the question). Professional noticing is a skill that the teachers use to identify and act upon salient mathematical actions of children (Thomas et al, 2017).

The teacher worked on the 3 examples that she gave them. She was asking them questions and the learners would answer. It was a question and answer session until the learners were given the classwork. Example no. 1 was $25a^3 \times 4a^2$. There was a boy who did not understand how they got 5 in a^5. The teacher asked one learner to explain to him. Davis (1997) argues that there are three forms of listening used by the teachers in their classrooms. These include evaluative, interpretive and hermeneutic listening. One quality of evaluative listening is revealed in the sorts of questions posed by the teacher (Coles, 2008). In every case, the teacher already had the correct answer in mind. The teacher is not really interested in what the learner is saying, rather the teacher's actions and questions are generally predetermined and allow little room for deviation (Davis, 1997).

Example 2 $24a^3b \div 4a^2b$

In this example interpretive listening was used by the teacher.

Teacher: what is the second law of exponents

Queeny: When we divide powers of the same base we subtract them.

Teacher: Ok. What is 24 divide by 4

Learners: 6

Teacher: What must we do now, class?

Learners: $a^3 \div a^2 = a$

Teacher: ……

Teacher: Do we all know why we say yes

Learners: Yes ma'm

Teacher: Then what next now?

Gladys: the answer is 6a

The teacher asked why, what happened to b.

Kagiso: Any base to the exponent of 0 is 1. So b doesn't exist anymore, that is why we have 6a. Interpretive listening brings about an action of probing and eliciting by the teacher. The teacher wants to uncover the ideas of the learner, or the thinking behind the learner's answer.

Analysis of observations on Themba's teaching with respect to professional noticing of learners' mathematical thinking

Themba is a male teacher who is 34 years old. He has less than 10 years of teaching experience. The lesson that I observed in his class was on depreciation.

Lesson observation 1 of Themba's grade 11 class

In the following I write a transcript of what happened in Themba's classroom. I observed him teaching depreciation to his grade 11 class.

Teacher: we are going to look at depreciation today. Yesterday we looked at the formula $A = P(1+i)^n$

Teacher: What does depreciation mean?

Nick: Decay

Teacher: What is decay?

Nick: Decrease in quality

Teacher: Basically loss of value.

Teacher: If you move from 35 years to 40 years, will you decrease or increase?

Bongi: You won't lose value, financially you increase, but physically you decrease.

Teacher: Ok, good. Now let's talk about a car. When the car is 30 years old, does it decrease or increase. A Mercedes benz made in 1993, will it go up or down?

Thami: It will increase sir because it's a vintage

Teacher: Ok. If the car was made in 1965, it can be bought for less. There are 2 values. The book value and the scrap value. Book value is when the value is lost for a period of time, for example, at the end of 5 years the value of the car is Book value. Scrap value is when the car has reached the end of its usefulness. It is a car made long time ago. It might not even reach 5km.

Imagine a granny of 105 years of age. How useful is she at home?

Kwena: She is useful because of pension money

Everyone laughed

Teacher: There are two methods that we use to calculate depreciation: Linear Value and reducing balance.

Linear value is Simple Interest and the formula is $A = P(1 - in)$

Reducing balance is Compound interest and the formula is $A = P(1 - i)^n$

Teacher: A in both formulae represent the Book value or scrap value.

Teacher: Let's look at this example class

Joshua wants to sell his car. Every year his car loses value of 14%. Calculate the book value in 5 years' time

Straight line method

Reducing balance method

Teacher: Are these As representing the same thing?

Thami: In a way, they are the same, because they represent Future Value.

Teacher: Not necessarily future amount. They represent the worth in the future. The other is investment (increase), the other is depreciation (loss).

Thami: it's still the future amount still, because you are looking for the value in the future

Teacher: The other is increase, the other is loss…… let's go back to the example.

$A = R60\,000\,(1 - 0{,}14)^5$ $A = R60\,000\,(1 - 0{,}14 \times 5)$

$=R28\,225$ $=R18\,000$

Teacher: Which method would you choose when dealing with the dealer

Learners: straight line method

Teacher: ok, let's look at the following example.

A laptop costs R9 000. After 4 years it is worth R2 000. Calculate the rate of depreciation

Sam: i stands for interest rate

Teacher: How can you explain interest?

Sam: Money added on

Teacher: i stands for depreciation rate here because you are losing in this example, you are not adding.

Straight line method *reducing balance method*

$R2000 = R9\ 000\ (1 - i4)$ $R2000 = R9000\ (1 - i)^4$

$2/9 = 1 - i4$ $2/9 = (1 - i)^4$

$i.4 = 1 - 2/9$

$i = 7/9\ x1/4$

$i = 0,19 = 19\%$

Sam: Why 2/9sit?

Teacher: Because I divided both sides b 9 000

Sam: Why can't we expand the bracket into (1-i) (1-i) (1-i) (1-i)

Teacher: (In answering) solve the following, for x

$X^2 = 2$ $x^3 = 7$ $x^5 = 8$ $x^{10} = 20$

Sam: $x^2 = 2$ we solve it by square root

$X^3 = 7$ we solve by cube root

Teacher: Then how do you solve $x^5 = 8$ and $x^{10} = 20$

Sam: Kept quiet

Teacher: you must use the nth root. Nth root makes it to the power of 1/n. So that's also exactly what you do as well in this example

$2/9 = (1 - i)^4$

$\sqrt[4]{2/9} = \sqrt[4]{(1- i)^4}$

$\sqrt[4]{2/9} = 1 - i$

$i = 1 - \sqrt[4]{2/9}$

$i = 0{,}3134 = 31{,}34\%$

Lesson observation 2 of Themba's grade 10 class

Teacher: Today we are going to finish off the financial Maths topic. Yesterday we spoke about the inflation rate. What is inflation rate?

Sipho: Inflation is the growth of prices overtime.

Teacher: Yes correct. Yesterday we said inflation is the rate of change in the prices of goods and services over a period of time. For example how much was a loaf of bread in 1986? In 2009 -2010 the price of a loaf of bread was R10.

Senzo: Today is R15.

(there was a discussion about the prices and the difference in the brands of prices)

Teacher: Previously we said Hire Purchase is calculated based on Simple Interest. Teacher: What interest is used to calculate inflation?

Learners kept quiet

Teacher: Which people are responsible for inflation?

Busi: Reserve bank

Teacher: reserve bank borrows banks money at repo rate. If the repo rate is increased or decreased by reserve bank it affects inflation.

Teacher: Who owns South African money? The president of the reserve bank signs the money. He signs one note, then the machine makes copies.

Bethuel: How about the code numbers in these notes. Sir you say the president of the reserve bank signs only one copy then the machine makes copies, so what about the code numbers in different notes, because all notes have different codes?

Teacher: The machine will focus on that signature only not on anything else. We call that technology my boy.

Teacher: Going back to our question. Compound interest will be used for inflation.

Ben: Why not use Simple Interest.

Teacher: we only use Simple interest in Hire Purchase.

The teacher wrote the formula on the board $A = P(1 + i)^n$ *where A is a new price, P is the current price, i is the rate of inflation and n is the number of years.*

Teacher: Let us look at this, how much did the loaf of bread cost 25 years ago if the rate of inflation was 10,8%? The price of the bread now is R9,40.

Teacher: the price that we are looking for, will it be smaller or bigger than R9,40?

Learners: It will be smaller.

Teacher: If it is not smaller then you must know you have done something wrong.

The teacher wrote $P = R9,40 / (1,108)^{25}$. *After writing he told the learners to punch their calculators and get the answer.*

Teacher: Do not forget that your answer must be smaller than R9,40. What is the answer?

Learners: R0,72

Teacher: That's correct.

Teacher: When solving this kind of problem articulate all the necessary information before you can substitute. You need to read these problems more than once so that you can articulate correctly.

Analysis of observations on Themba's lessons with respect to professional noticing of their learners' mathematical thinking

Research Question 1: How can observed mathematics teachers' noticing practices on learner mathematical thinking be explained in terms of noticing practices?

Research Question 2: How can professional noticing be a tool for teachers to include learners in qualitative mathematical conversations?

Analysis of observation on Themba's teaching The teacher initiated the lesson by saying today we are going to look at depreciation. He said yesterday we looked at the formula $A = P(1 + i)^n$. the teacher still initiated the discussion by asking, what does depreciation mean. One boy responded and said decay. The teacher asked, what does decay mean. The same boy said, it is decrease in quality. The teacher revoiced the response of the learner and said basically it's a loss in value. The teacher in this situation gave feedback to the responses of the learner. IRF format was used. Giving feedback to the responses of the learners meant that the teacher was listening to the learners. The teacher did not listen in an evaluative manner, giving yes or no answers (Coles, 2008), but the teacher listened interpretively to the responses of the learners. Interpretive listening is characterised by an awareness of the fallibility of the sense being made. This is where a response might offer feedback to the speaker not by evaluating what was said, but by offering an interpretation and asking for clarification (Coles, 2008)

Wallach & Even (2005) analyse four different types of teacher interpretation: describing, explaining, assessing and justifying. The teacher explained the meanings of book value and scrap value using examples. He used the examples of a car made in 1965, and of a granny of 105 years of age. These examples helped the learners to understand the meanings of these values. The teacher used real life mathematical situations. This made the students to make sense of the school mathematics.

The teacher gave the learners two methods that must be used to calculate these values: linear value and reducing balance value.

Linear value: Simple interest $A = P(1 - in)$

Reducing balance: Compound interest $A = P(1 - i)^n$

The teacher gave them examples where these formulae can be used. He gave them real world examples. These examples helped the learners in making sense of these mathematics situations. The teacher taught relational understanding to the learners. Skemp (1976) argues that relational understanding is about knowing what to do and why. When mathematics is learned relationally the connected information, or the network of connected ideas is simply more likely to be retained over time than disconnected information. Relational understanding therefore has the welcome potential to motivate the learner to new insights and ideas, and the creation of new inventions and discoveries in mathematics (SAIDE, 2007).

The teacher used different forms of listening to his learners in these examples. Interpretive and transformative listening were used by the teacher. When the teacher was explaining book value and scrap value he asked the learners if the car is 30 years old, does it decrease or increase. Before the learners responded, he said a Mercedes Benz made in 1993, will it go up or down. One boy said, it will increase sir because it's vintage. The teacher interpreted this response of the learner by saying, if the car was made in 1965, it can be bought at a higher price by businesses, but for individual use it can be bought for less.

There was also another example given by the teacher. An example of a laptop which costed R9000. After 4 years it was worth R2000. When the teacher was calculating depreciation of this laptop using the reducing balance method, the teacher wrote:

$R2000 = R9000\ (1 - i)^4$

$2/9 = (1 - i)^4$

One boy asked why 2/9 sir. The teacher replied I divided both sides by R9000. The boy asked why can't we expand the bracket into (1-i) (1-i) (1-i) (1-i). The teacher in answering the boy said, solve the following for x:

$X^2 = 2$ $x^3 = 7$ $x^5 = 8$ $x^{10} = 20$

The boy replied $x^2 = 2$ can be solved by square root, $x^3 = 7$ can be solved by cube root The teacher asked, how then can you solve $x^5 = 8$ and $x^{10} = 20$. The boy could not answer. The teacher said you must use the nth root. Nth root makes it to the power 1/n. so that's also what you need to do in this example. The teacher in this situation used transformative listening. Transformative listening is defined as an imaginative participation in the formation and transformation of experience through an ongoing interrogation of the taken for granted and the prejudices that fame these perceptions and actions (Davis, 1997). Evidence of this form of listening in a classroom includes a willingness to alter ideas in a discussion, to engage in a dialogue, to entertain other points of view and hold them as valid, whether they are accepted or not (Coles, 2008). The teacher also used Mathematical Knowledge for Teaching (MKT) in this instance. MKT refers to the amalgamated knowledge required to effectively teach mathematics. It provides a structured perspective of subject matter and pedagogical content knowledge related to mathematics teaching (Thomas et al, 2017). The teacher noticed that the learner was using a different approach in solving the given problem. The approach that the learner wanted to use to solve the given problem was not entirely wrong, but it was not suitable for the problem at hand. The teacher therefore used his specialised knowledge of mathematics content to help the learner to realise that there is another method that can be used to solve the problem. The teacher figured out how to engage the mathematical thinking and reasoning of the learner by applying his MKT. this application of MKT is referred by Adler and Davis (2006) as unpacking and decompressing. In responding or in explaining to the learner the teacher used another example to provide insight to the learner. The use of mathematical examples signifies both pedagogical unpacking and decompressing as well as some knowledge of the general mathematical idea embodied within the example (Thomas et al, 2017).

The merging of MKT and professional noticing practice produces Mathematics for Teaching (M_4T) (Davis & Renert, 2014). They define M_4T as "a way of being with mathematics knowledge that enables a teacher to structure learning situations, interpret student actions mindfully and respond flexibly in ways that enable learners to extend their mathematical understandings' and expand the range of their interpretive possibilities through access to powerful connections and appropriate practice" (p.4). In lesson observation 2, when the learners were calculating the price of bread in the past 25 years, the teacher asked tem whether the price would be smaller or bigger than R9,40 (the current price). The learners responded that it will be smaller. The teacher further told them that if they get a number bigger than R9,40, they must just know that something is wrong. The teacher in this situation used M_4T. The teacher enabled the learners to structure the learning situation. The learners understood that the price of bread 25 years ago cannot be the same or cannot be more than R9,40. The teacher also extended the mathematical understanding of his learners in this situation. When he told them that if they get a number bigger than R9,40 they must just know that they have done something wrong in their calculations. The teacher enabled them to be able to correct themselves (To check where they went wrong in their calculations) even before they go to the teacher.

The learner here made a connection of previous piece of work, that we need to expand when there are powers. This sense of restructuring of previous categories or ideas is indicative of learning (Coles, 2008). In addition to this, the learner had misconception of interference about this part of the topic. The misconception of interference is when previously learned skills interfere with new skills (Olivier, 1989).

Phase two :

4.3. Analysis of interviews

Three teachers were interviewed, Eddie, Eric and Gift. These are not their real names. These are just pseudonyms. The interview processes did not take place in one day. It took about a week for the interview process to be completed.

Interview question 1

How do you attend to the learners' mathematics conceptions (thinking, reasoning, ideas)in your classroom?

Eddie: Learners' conceptions help me teach a certain mathematics concept from their point of view. Hence, I scaffold them to figure out solutions using their own thinking and ideas. I also use their thinking to help me make them understand the concepts.

Eric: Mathematical conception requires strong theoretical base of the concept, for example, solving quadratic equations, mathematical fact states that whenever numbers are multiplied with zero the answer is zero. X (x + 2) = 0 therefore x = 0 or x + 2 = 0. I teach mathematical theories and rules.

Gift: By paying attention to whether they can explain which skills of mathematics have they used for every step leading to the solution.

Interview question 2

How do you interpret these conceptions?

Eddie: I understand that conception of learners is at times built from the previous knowledge in previous grades they have attended. I believe learners are not empty containers waiting to be filled with knowledge, however, they have knowledge which needs to be properly moulded by me.

Eric: I always use arithmetic to develop conceptual understanding. Arithmetic is prior knowledge. I listen to the learners' explanations, and interrupt at any point they make an error. I listen carefully to hear whether an error is a misconception or a mistake.

Gift: Before we start with a topic, I use a mathematics dictionary to give them terminology and notation related to the topic.

Interview question 3

Do you decide upon a productive instructional course of action based on your intepretations:

Eddie: Yes. Based on my interpretations I make my classroom to be learner centred. Thus, instilling learners with a mind-set that they are responsible and accountable for their own learning.

Eric: By listening to their presentations, I can decide how to help restructure his / her understanding.

Gift: Yes. When I give them terminology from the Maths dictionary, I am preparing them for the instruction that will be given.

Data analysis for interviews

Eric's noticing practices that he uses in his classroom are: the use of content knowledge. He argues that the mathematics conception requires strong theoretical base of the concept. In order for his learners to think and reason mathematically, he teaches them mathematical theories and rules. He also states that he uses arithmetic to develop conceptual understanding of the learners. In order for him to interpret their thinking and reasoning, he listens carefully to heir explanations and interrupt them at any point they make an error. The teacher here does not just use evaluative listening but he uses interpretive listening. He listens carefully to the learners' explanations and try and make sense of what the learner is saying. He then checks if the errors the students make are due to misconceptions or are just mistakes. Based on this interpretation the teacher therefore decides on how to help the child restructure his / her understanding.

Eddie and Gift also listen very carefully to their learners' mathematics conceptions. Eddie states that his learners' mathematics conceptions help him to teach them in their point of view. In other words, he decides upon a productive instructional course of action based on the learners' mathematical conceptions. He argues that he scaffolds them to figure out solutions using their own thinking and ideas. Eddie uses a constructivist view / theory towards learning. Constructivists believe that learners construct their own understanding. Gift uses both interpretive and transformative listening. He states that he pays attention to whether the learners can explain which skills of mathematics they have used for every step leading to the solution. He therefore probes the learners to explain and to justify their work.

Both Eddie and Alice see prior and previous knowledge of learners as of importance in understanding the learners' conceptions. Eddie states that learners' conceptions are at times built from previous knowledge in the previous grades they have attended. Alice argues that he uses arithmetic to develop conceptual understanding. He further states that arithmetic is prior knowledge. Errors and misconceptions of learners are sometimes a result of previous knowledge. White (2005) wrote about two types of errors: transformation errors and process skills errors. He argues that transformation errors take place when the child understands what the question asks but is unable to identify the operation needed to solve the problem. Process skills errors is when the child identifies appropriate operation, but do not know the procedures of how to carry out these operations accurately (White, 2005). Based on these definitions we can therefore conclude that errors are sometimes the result of the previous knowledge. Another practice that teacher 3Gift uses in his classroom is the process of students' mediation model. This is the process that influences the students' cognitive processes which in turn influence their learning. The teacher argues that before he starts with a new topic he uses a mathematics dictionary to give them terminology and notation related to the topic. By so doing, he is preparing the learners cognitively for the topic at hand.

Chapter five

5.1 Discussion of findings

In this chapter I discussed in detail the different teaching practices and skills that were used by the teachers in their classrooms. Teachers utilise different teaching practices and skills that will bring about mathematics sense making in the classroom. One of the practices that the teachers use in their classrooms is 'noticing'. Classrooms must be communities in which mathematics sense making is developed (Stein et al, 1996). It is also argued that students need much more than abstract concepts and self-contained examples, they need to be exposed to the use of domain's conceptual tools in authentic activity (Brown et al, 1989). Professional noticing is defined as an expertise of interrelated skills used by a group of individuals who have the same goals (Jacobs et al, 2010). In this instance the group referred to is the teachers who have the same goal of helping the learners to make sense of mathematics concepts taught to them. This study explored the noticing practices used by teachers in their classrooms. Qualitative mathematics learning and sense making of mathematics was also explored. The skills that the teachers use include attending to children's strategies, interpreting children's understandings, and deciding on how to respond on the basis of children's understanding (Jacobs et al, 2010). The practice or the skill of noticing by the teachers helps the teacher to attend or to identify the mathematics thinking and reasoning of the learners. After identifying these ideas by the learners, the teachers interpret these ideas and later come up with responsive instructional strategies which will benefit the learners.

There are a lot of instructional strategies that were used by the teachers in this study. These include whole class discussion, explaining, interpreting, justifying and describing. Whole class discussions are used by the teachers to promote and extend the students' mathematical understandings and their proficiency with a range of mathematical practices such as conjecturing, justifying and reconciling (Ball & Bass,2000). This helped the learners to voice out their ideas and thoughts with each other. They shared their understanding of a mathematics concept and generated shared meanings and understanding of a particular mathematical concept. Examination of describing and explaining types of interpretation indicates five different characteristics of teacher hearing students: over-hearing, compatible hearing, under-hearing, non-hearing and biased hearing (Wallach & Even, 2005). These different characteristics of interpretation were also used by the teachers in the observed classrooms of the study. A listening technique was also used by the teachers in this study. Noticing helps teachers to develop new ways of 'seeing' whatever is happening in the classrooms. For this to happen teachers are supposed to pay close attention to the ideas that students raise, and use those ideas for the lesson in progress (Sherin & Van Es, 2005). The grounded theory used in the study is Davis' enactment theory. This theory is about Davis' three forms of listening: evaluative listening, interpretive listening and transformative (hermeneutic) listening. All these forms of listening were used by the teachers in the study.

The first key issue that was observed from the teachers was that, when initiating the lessons, the teachers greeted their learners and told them of what was done the previous day. They reminded the learners of what transpired previously in order to prepare them for the topic of the day. After reminding the learners, the teachers asked the learners questions based on the work that was done previously. The learners' responses would then lead the teachers to the lesson of the day. The second practice that was evident on all classes observed was that, the teachers asked the learners to write some work or copy down the work from the board. After this activity the teachers would interpret he work to their learners. In doing this, the teachers used the four types of interpretation: describing, explaining, assessing and justifying (Wallach & Even, 2005). The third common practice used by the observed teachers was the practice of listening to the learners. They used evaluative, interpretive and transformative forms of listening.

All three of the teachers that were observed, when initiating their lessons, they greet their learners and tell them of what was done the previous day. This, I saw as a way of reminding the learners of what transpired previously in order to prepare them for the topic of the day. After reminding them about the work of the previous day, they ask them some questions based on the work that was done previously. The responses that the learners give would lead the teachers to the lesson of the day. In their introduction and initiating of the learners of the lesson, the teachers use the 'telling' approach. Chazan & Ball (1999) defined telling by including the following actions: introducing conventional terminology, reminding students of a conclusion on which they have already agreed, rephrasing students' comments for the whole class, telling students that an idea is unclear, challenging students' ideas when they reach a mathematically incorrect consensus. Telling therefore is a necessary approach in conveying knowledge to the learners. It is an instruction tool that is used to introduce the concept, to control and to manage the behaviours of learners, to correct the learners and to share the information.

The second practice that was evident in all the learners observed was to write some work on the board and asking the learners to copy down the work. The learners did not question their teachers, they did not even complain. They just copied the work in their books. After copying it down, they waited for their teachers to give hem instruction. This practice by the teacher and by the learners in the classroom can be seen as the use of social norms and mathematical norms. Social norms are general ways that students participate in the classroom activities (Kazemi & Stipek, 2001). These activities were common in all three classroom observations. Learners followed the instructions given by their teachers they copied down the examples and waited for another instruction from their teachers. The teachers then interpreted the work to their learners. Thy used the four types of interpretation analysed by Wallach & Even (2005). These four different types of interpretation are: describing, explaining, assessing and justifying. The teachers described and explained the student talk, thought, feelings and actions by direct quotation. They justify their explanations to the learners, they also assessed their learners by asking them questions based on what was taught.

Listening to students' thinking, ideas and responses is vital for mathematics instruction to be effective. The three observed teachers used this practice carefully. Davis (2007) states three forms of listening: evaluative, interpretive and transformative listening. Evaluative listening which only evaluates the correctness and incorrectness of the learners' answers was not used much at all. The forms of listening that were used mostly in these observed classrooms were interpretive and transformative listening approaches. Learners were asked to explain their thinking. The teachers used interpretive listening to uncover the ideas of the learners or the thinking behind the learners' answers or responses. The learners were asked to explain why their answers are like that. The teachers probed their learners to justify their answers. Transformative listening was also used by the teachers. This is where the teachers would allow a dialogue and discussion by the learners. Transformative listening includes willingness to alter ideas in a discussion, to engage in dialogue and to entertain other points of view (Coles, 2008). Transformative listening was also evident in these classrooms when the learners linked or connected the work they are doing with the previous work. Coles (2008) states that if a student makes a connection to a previous piece of work or links something that has been said before, this would indicate the transformation of experience and the restructuring of categories. This sense of restructuring previous categories or ideas is indicative of learning (Coles, 2008).

Chapter six

6.1. Conclusion and recommendations

The study took place in a school environment. It was conducted in three different classrooms. The sampling procedure of the study is a non-random sampling procedure. The key features of the sample of the study are convenience, availability, and accessibility. The data were collected at the researcher's own school with the teachers from the same department as that of the researcher. The sample of the study is six teachers. Ethics considerations were also followed in this study. Participants were first verbally requested to participate in the study. They were told what the study was about and were also told that the study is only for Masters' research purposes. The consent form was given to them. They were given some time to read through, and sign it when they feel comfortable to do so.

Three teachers were observed while teaching in their classrooms and three more teachers were interviewed based on their noticing practices or experiences in their classrooms. Non-participant and undisguised observation was used. The researcher was not participating in any way in the study and everyone in class was aware of her presence. The observed teachers were also interviewed based on what was observed. Guided (topical) interview was used in this regard. With the guided interview the researcher explores a few general topics to help uncover the participant's views, but otherwise respects the way the participant frames and structures the responses (Marshall & Rossman, 2011). The standardised type of interview was used to interview the teachers that were not observed. Standardised interviews are carefully 'scripted', asking specific questions in a specific sequence, sometimes without follow-up (Marshall & Rossman, 2011). The interview questions were written down by the researcher in a specific sequence. The teachers were asked how they attend or identify their learners' mathematics ideas and strategies. After this question they were asked how they interpret these ideas and strategies by their learners. Lastly they were asked how they responded or what is the responsive instruction that they used based on their interpretations of the learners' ideas. The teachers were also asked to provide examples of how these things were performed. These three teachers were interviewed during break and after school hours. The average duration of interview process was 15 to 20 minutes. Teachers were interviewed based on the noticing skills that they use in their classrooms.

Data were analysed using Creswell's (2003) three generic steps: the first step is to prepare and organise the data, secondly is to read through all the data, and lastly to begin a detailed analysis with coding process (Creswell, 2003). Combination of key issues emerging from all participants was considered. Conceptual framework and grounded theory were used in the analysis of the data. Inductive method of analysis was also considered. Inductive method is about making observations and then draw conclusions from these observations. The study also used Davis' enactment theory. This theory evaluated and explained three forms of listening that were used by the teachers in their classrooms.

The first key issue that was observed from the teachers was that, when initiating the lessons, the teachers greeted their learners and told them of what was done the previous day. They reminded the learners of what transpired previously in order to prepare them for the topic of the day. After reminding the learners, the teachers asked the learners questions based on the work that was done previously. The learners' responses would then lead the teachers to the lesson of the day. The second practice that was evident on all classes observed was that, the teachers asked the learners to write some work or copy down the work from the board. After this activity the teachers would interpret he work to their learners. In doing this, the teachers used the four types of interpretation: describing, explaining, assessing and justifying (Wallach & Even, 2005). The third common practice used by the observed teachers was the practice of listening to the learners. They used evaluative, interpretive and transformative forms of listening.

6.2. Findings

In this chapter I present my findings with respect to the research questions.

1. **How can observed mathematics teachers' noticing practices on learner mathematical thinking be explained in terms of noticing frameworks**

   The teachers that were observed used different teaching strategies and practices in their classrooms. These practices include the practice of noticing. This is where the teachers attend to the learner's mathematics ideas, interpret these ideas, and decide on how to act upon them in order to bring about understanding to the learners. The strategies that were used in doing this included those of telling, writing, the use of whole class discussions where learners were encouraged to share ideas. Four different types of interpretation were also utilised by the teachers. These types of interpretation are describing, explaining, assessing and justifying (Wallach & Even, 2005).

2. **How can professional noticing be a tool for teachers to include learners in qualitative mathematical conversations?**

   One of the skills that the teachers used was a skill of listening to the learners. Three forms of listening were effectively used. These include evaluative, interpretive and transformative listening (Davis, 2007). Interpretive and transformative listening were mostly used by the observed teachers. Learners were asked to explain their thinking and to share their thinking with their peers. In so doing, learners were all included in the mathematical conversations of the classrooms. Learners were encouraged to engage in dialogues and to entertain the points of view of others. Learners were able to make connections to previous pieces of work. This was an indication that there was transformation of experience. Tis also indicated that learning took place.

6.2. Recommendations

There are different findings which were observed in the study. These findings cannot be generalised on all mathematics teachers. It is argued that teachers in the less disadvantaged areas in South Africa have less MKT and they possess inadequate common content knowledge. It is also argued that in the grade 12 mathematics examination in 2015, the percentage of learners who achieved 50% and above was only 20% (Deparment of Education, DBE, Republic of South Africa. 2016, p.151). to avoid or to deal with poor mathematics results, mathematics teaching approaches should be made more effective, the crucial issue in his regard is that of the actual methods of teaching employed by teachers to facilitate mathematics (Umugiraneza & Bansilal, 2017). I therefore recommend that the DBE policy makers to use the findings of the different research studies to explore different teaching methods and practices used by teachers in their classrooms. Policy makers also need to pay attention to ways that teachers can use to help them shift from the practice of 'telling' only, but also to focus on how to link mathematics to real life and prepare learners to be investigators and problem solvers. In addition to this, workshops and seminars should be introduced in order to equip teachers with more skills and MKT especially in the less disadvantaged areas. The reality in the South African mathematics education context is that the quality of mathematics teaching in disadvantaged areas remain poor (Taylor & Reddi, 2013).

Recommendation for the practice of teaching would also be the improvement of the teaching methods and practices which will bring about the improvement in the mathematics results if learners. Mathematics teachers need to promote the use of more sophisticated teaching strategies. Teachers can improve their noticing by changing what they notice (for example, by moving from a focus on teacher's actions to students' conceptions). This is where the teacher would need to interpret the specific mathematics thinking in which the learner has engaged. This idea of interpreting and mediating students' mathematical thinking, leads to how teachers choose and use examples in the classroom (Adler & Davis, 2006).

6.4. Limitations of the study

The study only observed three teachers and two teachers of the same school. One of the findings of the study was that the teachers observed used the four types of interpretation analysed by Wallach & Even (20005). These are describing, explaining, assessing and justifying. The three observed teachers used these forms when teaching mathematics in their classrooms. It was also found out that they used the three forms of listening by Davis (1997). These forms are: evaluative, interpretive and transformative.

The conducting of the study had some limitations. The first limitation was that, the time allocated for the study was not enough to go further with the study. This is because the study was only for the purpose of the Masters research project. As a researcher, I was only able to observe and to interview teachers from only one school. It was going to be very interesting to conduct research in more than one school. Different schools have different policies and different ways of doing things. For example, in my school (where research was conducted) the duration of 1 lesson period is 40 minutes. In one of the neighbouring schools the duration of 1 lesson period is 30 minutes. Other schools have 1- hour period depending on the school's policy. If I had more time to conduct the research in other schools, I would be able to compare and contrast the noticing practises used by other educators in their schools.

Another limitation that I encountered in conducting the research was that, I was only supposed to be a non-participant observer in the study. I was not supposed to comment or to ask questions during the lesson even though I had some questions. For example, when teacher Themba was teaching his grade 11 class about 'interest' I wanted to ask a question based on what he was

teaching. The question was about the book value and the scrap value. I could not ask the question because asking questions and commenting was not part of the agreement we had. During the research process ethical considerations must be followed. Even though the participants were verbally requested first to participate in the study, the consent forms were given to them. The consent was that, the researcher will be a non-participant observer. It does not matter how you think your research is, as a researcher you must put participants' well-being first (Bouma et al, 2009).

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APPENDIX A

LETTER TO THE PRINCIPAL, SGB Chair, etc. DATE: 01 July 2019

Dear Mr. Principal

My name is Nwabisa Gloria Hantibi. I am a Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on MATHEMATICS TEACHERS NOTICING PRACTICES IN THEIR CLASSROOMS.

My research involves: (a) OBSERVATION OF THREE MATHEMATICS TEACHERS WHEN THEY TEACH MATHEMATICS IN THEIR CLASSROOMS. A TEACHER WITH 3 YEARS OF EXPERIENCE IN MATHEMATICS TEACHING, A TEACHER WITH MORE THAN 3 YEARS EXPERIENCE IN TEACHING MATHEMATICS AND A TEACHER WITH MORE THAN 10 YEARS EXPERIENCE IN MATHEMATICS TEACHING WILL BE OBSERVED.

(b)INTERVIEWING OF THREE MORE MATHEMATICS TEACHERS ABOUT THE PROFESSIONAL NOTICING PRACTICES THAT THEY USE IN THEIR CLASSROOMS.

THE PURPOSE OF THE RESEARCH IS TO EXPLORE THE NOTICING PRACTICES THAT THE TEACHERS USE WHEN TEACHING MATHEMATICS IN THEIR CLASSROOMS, ALSO TO EXAMINE THE IMPACT THAT THESE NOTICING PRACTICES HAVE IN MATHEMATICS LEARNING. AUDIO RECORDING WILL ALSO BE USED AS AN ADDITIONAL AID FOR THE RESEARCHER.

I therefore invite your school teachers to participate in this research.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely

NAME OF THE STUDENT: NWABISA GLORIA HANTIBI

NAME OF THE SUPERVISOR: DR. JUDAH MAKONYE

EMAIL OF THE STUDENT:nwabisahantibi8@gmail.com

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TELEPHONE NUMBERS: 0814776188 / 0849197430

TELEPHONE NUMBERS OF THE SUPERVISOR: 078 689 4572

APPENDIX B

A LETTER TO THE TEACHERS

DATE: 01 July 2019

Dear teachers

My name is Nwabisa Gloria Hantibi. I am a Masters Student in the School of Education at the University of the Witwatersrand.

I am doing research on MATHEMATICS TEACHERS NOTICING PRACTICES IN THEIR CLASSROOMS

My research involves OBSERVATION OF THREE TEACHERS TEACHING MATHEMATICS IN THEIR CLASSROOMS. A TEACHER WITH 3 YEARS OF TEACHING EXPERIENCE IN MATHEMATICS, A TEACHER WITH MORE THAN 3 YEARS OF TEACHING EXPERIENCE, AND A TEACHER WITH MORE THAN 10 YEARS OF MATHEMATICS TEACHING EXPERIENCE. THE PURPOSE OF THE STUDY IS TO EXPLORE THE NOTICING PRACTICES THAT THE TEACHERS USE IN THEIR CLASSROOMS, AND ALSO TO EXAMINE HOW THESE NOTICING PRACTICES IMPACT LEARNERS' MATHEMATICS LEARNING. AUDIO RECORDING WILL ALSO BE USED AS AN ADDITIONAL AID FOR THE RESEARCHER.

I therefore invite you to be one of the three teachers who will be participation in my study.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

You will not be advantaged or disadvantaged in any way. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

SIGNATURE

NAME OF THE STUDENT: Nwabisa Gloria hantibi

EMAIL ADDRESS OF THE STUDENT: nwabisahantibi8@gmail.com

NAME OF THE SUPERVISOR: Doctor Judah Makonye

EMAIL ADDRESS OF THE SUPERVISOR: Judah.makonye@wits.ac.za

Telephone number of the student: 081 477 6188

Telephone numbers of the supervisor: 011 717 0000



APPENDIX C

Teacher's Consent Form

Please fill in and return the reply slip below indicating your willingness to be a participant in my voluntary research project called:

I, _____ give my consent for the following:

Circle one

Permission to observe you in class

I agree to be observed in class. YES/NO

Permission to be audiotaped

I agree to be audiotaped during the observation of lesson YES/NO

I know that the audiotapes will be used for this project only YES/NO

Informed Consent

I understand that:

my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.

I do not have to answer every question and can withdraw from the study at any time.

I can ask not to be audiotaped, photographed and/or videotaped

all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____ Date_____

Appendix D

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Department: Education

REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

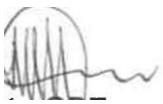
GDE RESEARCH APPROVAL LETTER

Date:	**31 July 2019**
Validity of Research Approval:	**04 February 2019 - 30 September 2019** **2019/179**
Name of Researcher:	**Hantibi N.G**
Address of Researcher:	**146 Bertha Street**
	Kenilworth
	Johannesburg,2090
Telephone Number:	**084 919 7430/ 081 477 6188**
Email address:	**814553@students.wits.ac.za**
Research Topic:	**Mathematics teachers noticing practices in their classrooms.**
Type of qualification	**Masters' in Education**
Number and type of schools:	**One Secondary School**
District/s/HO	**Johannesburg East**

Re: ***Approvalin Respect of Request to Conduct Research***

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Making education a societal priority
Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355-0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za



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The following conditions apply to E research! The researcher may proceed with the

above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.

The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.

A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcherls have been granted permission from the Gauteng Department of Education to conduct the research study.

A letter I document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs *and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*

The Researcher will make every effort obtain the goodwill and co-operation of all the GOE officials, principals, and chairpersons of the SGBs, *teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*

Research may only be conducted after school hours so *that the normal school programme is not interrupted. The Principal (if at* a *school) and/or Director (if at* a *district/head office) must be consulted about an appropriate time when the researcherls may carry out their research at the sites that they manage.*

Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.

Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.

It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.

The researcher is responsible for supplying and utilising his/her own research resources, such as *stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*

The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.

On completion of the study the researcher ls must supply the Director: Knowledge Management

& Research with one Hard Cover bound and an electronic copy of the research.

The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GOE officials and theschools concerned.

Should the researcher have been involved with research at a *school and/or* a *district/head office level, the Director concerned must also be supplied with* a *brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the fmdings of your research study.

.....

Mr Gumani Mukatuni

Acting CES: Education Research and Knowledge Management DATE: ... .\\9..\?:: \1.....

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Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix E

WITS SCHOOL OF EDUCATION



SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE **PROTOCOL NUMBER: 2019ECE035M**

PROJECT TITLE Mathematics teachers noticing practices or habits in their classrooms

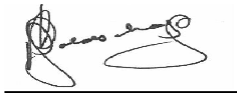
INVESTIGATOR Nwabisa Gloria Hantibi

SCHOOL/DEPARTMENT OF INVESTIGATOR WITS SCHOOL OF EDUCATION

DATE CONSIDERED 07 September 2019

DECISION OF THE COMMITTEE Approved unconditionally

EXPIRY DATE Date of submission of the project report

ISSUE DATE OF CERTIFICATE 12 September 2019 **CHAIRPERSON** 

(Dr. Paul Goldschagg)

cc: Supervisor: Dr. Judah Makonye

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** emailed to the Ethics Office: Matsie.Mabeta@wits.ac.za .

I fully understand the conditions under which I am authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

_____

_____/_____/_____

Signature **Date**

PLEASE QUOTE THE PROTOCOL NUMBER ON ALL ENQUIRIES