

UNIVERSITY OF THE WITWATERSRAND
SCHOOL OF MECHANICAL ENGINEERING

SWIRLING FLOW IN HELICALLY WELDED PIPES AND
ITS EFFECT ON ORIFICE FLOW METERS

BY

CHRISTOPHER ANDREW AUSTIN HIGMAN

A dissertation presented to the Faculty of Engineering,
University of the Witwatersrand, in fulfilment of the
requirements for the degree of Master of Science in Eng-
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Johannesburg, 1977

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DECLARATION

I, Christopher Andrew Austin Higman, declare that this dissertation

- (a) is my own unaided work,
- (b) has not been submitted previously, in whole or in part for a degree at this or any other university, and
- (c) that information contained therein has not been obtained in the course of employment or under the auspices of any person or organisation other than the university.

CAH Higman

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	<u>PAGE</u>
<u>CHAPTER 4:</u> EXPERIMENTAL INVESTIGATIONS	49
4.1 Preliminary Investigations	49
4.2 Description of Equipment	50
4.3 Conduct of Experiments	52
 <u>CHAPTER 5:</u> DISCUSSION OF RESULTS	 54
5.1 Swirl Growth	54
5.2 Effects on Meter	58
 <u>CHAPTER 6:</u> CONCLUSIONS	 60
 <u>APPENDICES</u>	
A. Stability Analysis for Tangential Equation .	61
B. Numerical Technique for Axial Equation . .	63
C. List and Explanation of Variables for Comp- uter Programme	69
D. Computer Programme	72
 REFERENCES	 81

LIST OF FIGURES

	<u>PAGE</u>
FIGURE 1: Helical Weld Pipe Mill	3
FIGURE 2: Geometry of Helically Welded Pipe .	3
FIGURE 3: Mixing Length Vector	15
FIGURE 4: Von Kármán's similarity hypothesis .	26
FIGURE 5: Stresses on a Fluid Element in Swirling Flow	33
FIGURE 6: Grid Mesh for Computer Programme . .	40
FIGURE 7: Overall Logic Diagram for Computer Programme	46
FIGURE 8: Logic Diagram of Computer Solution of Tangential Equation	47
FIGURE 9: Logic Diagram of Computer Solution of Axial Equation	48
FIGURE 10: Apparatus for Preliminary Investigation	50
FIGURE 11: Apparatus for Main Experimental Work	51
(a) Diagrammatic layout of test rig	
(b) Detail of typical test section with instrumentation	
FIGURE 12: Axial Swirl Growth: $Re = 3.47 \times 10^5$. .	55
FIGURE 13: Radial Distribution of Swirl for Varying Downstream Positions: $Re = 3.47 \times 10^5$	55
FIGURE 14: Radial Distribution of Swirl for Varying Reynolds Number: $l/d=62.3$. .	56
FIGURE 15: Velocity Profiles for Varying Reynolds Number: $l/d=62.3$	57
TABLE 1: Orifice Metering Error due to Presence of Wire Helix	59

NOMENCLATURE

As far as possible, the nomenclature conventional in literature on the subject has been adhered to. A summary of the most important symbols used in this paper is included for reference.

A, A'	defined on page 40
C_s	similarity constant (page 30)
F	body force per unit mass on a fluid element
$F(r, z)$	function defined on page 36
j	numbering of grid mesh in radial direction
J	maximum value of j
K_s	similarity constant (page 29)
ℓ	length scale in turbulent flow field
$\underline{L}$	mixing length vector
n	numbering of grid mesh in axial direction
p	hydrostatic pressure
$P(r, z)$	function defined on page 36
Q	$-\frac{p}{\rho} + \frac{z^2}{2}$ (page 14)
$Q(r, z)$	function defined on page 36
$\underline{q} = (u, v, w)$	velocity vector
Re	Reynolds number
u, v, w	velocity components in rectangular co-ordinates
v_r, v_θ, v_z	velocity components in polar co-ordinates

v_f	friction velocity (page 25)
α_i	defined on page 31
$\beta(r, z)$	angular momentum for unit mass
δ_{ij}	= 1 for $i = j$ and = 0 for $i \neq j$
$\Delta r, \Delta z$	grid mesh widths
ϵ	eddy viscosity
κ	constant (page 31)
λ	defined on page 32
μ	absolute viscosity
ξ, η, ζ	vorticity components
ρ	density (mass per unit volume)
τ_{ij}	stress tensor
$\underline{\Omega} = (\xi, \eta, \zeta)$	vorticity vector
∇	differential operator

SUPERSCRIPTS

$\bar{}$	mean value
$\prime$	turbulent component
$*$	similarity parameters
n	value at $z = n$ in grid mesh

SUBSCRIPTS

$\circ$	reference value
x, y, z	components in rectangular co-ordinates
r, θ, z	components in cylindrical polar co-ordinates

i, j, k

components in a generalised co-ordinate system

j

values at $r = j$ in grid mesh.

The nomenclature used in the computer programme for the solution of the swirl equations is included in Appendix C.

CHAPTER 1

INTRODUCTION

1.1 HELICALLY WELDED STEEL PIPE

Although the first patents for the manufacture of helically welded steel pipes date back to early in the present century, the slow development of welding techniques and a number of problems associated with forming methods held back commercial exploitation of the process till the mid-1950's. During the last 15 years or so, however, this type of pipe has gained increasingly wide acceptance under the most stringent specification requirements because of the advantages which helically welded pipe offers to both manufacturer and user.

Production costs on a helical-weld pipe mill can be considerably less than on a longitudinal weld mill for large bore pipes, partly through reduced capital cost, partly through reduced labour costs and also through reduced stock costs since the same coiled strip can be used for varying pipe sizes merely by alteration of the helix angle. The mill's great inherent flexibility also allows shorter production runs at economical cost. The principal components of such a mill are shown in Fig 1.

Further economic advantages of the helically welded pipe are that it is manufactured in a continuous process and so pipe lengths can (in theory) be as long as is desired, thus reducing substantially the number of line repairs. In practice, lengths of up to 100 feet have been made.

Helical weld pipe mills are now designed to be transportable so that they can be transported from site to site without difficulty. When these mobile machines are operated in situ, substantial savings can be effected due to the difference in shipping costs of pipes and steel in coils.

From the strength aspect, helically welded pipe has an inherently greater factor of safety than longitudinally welded pipe. Since, when the pipe is subject to internal pressure, the hoop stress is twice the longitudinal stress, the weld in a longitudinally welded pipe is subjected to the maximum stress. The weld seam in a helically welded pipe is generally at about 45° to the axis of the pipe and so the stresses in the seam (and the heat affected zone) are reduced by 30 to 40%. The geometry of the pipe is shown in Fig 2. Additional technical advantages of helically welded pipe are

- (a) its greater resistance to tearing in the event of failure, and
- (b) its improved tolerances on the straightness, roundness, wall thickness and diameter of the pipe.

A helically welded pipe also has disadvantages, however. Despite the statement made by Eckhardt and Robson (1) it is the author's experience that great difficulty is encountered when attempting to hotbend helically welded piping because the weld reinforcement tends to encourage creasing on the inside of the bend (28). It would appear that Eckhardt and Robson's comment refers to cold field bending of line pipe, where bend angles of over 7° are rarely exceeded.

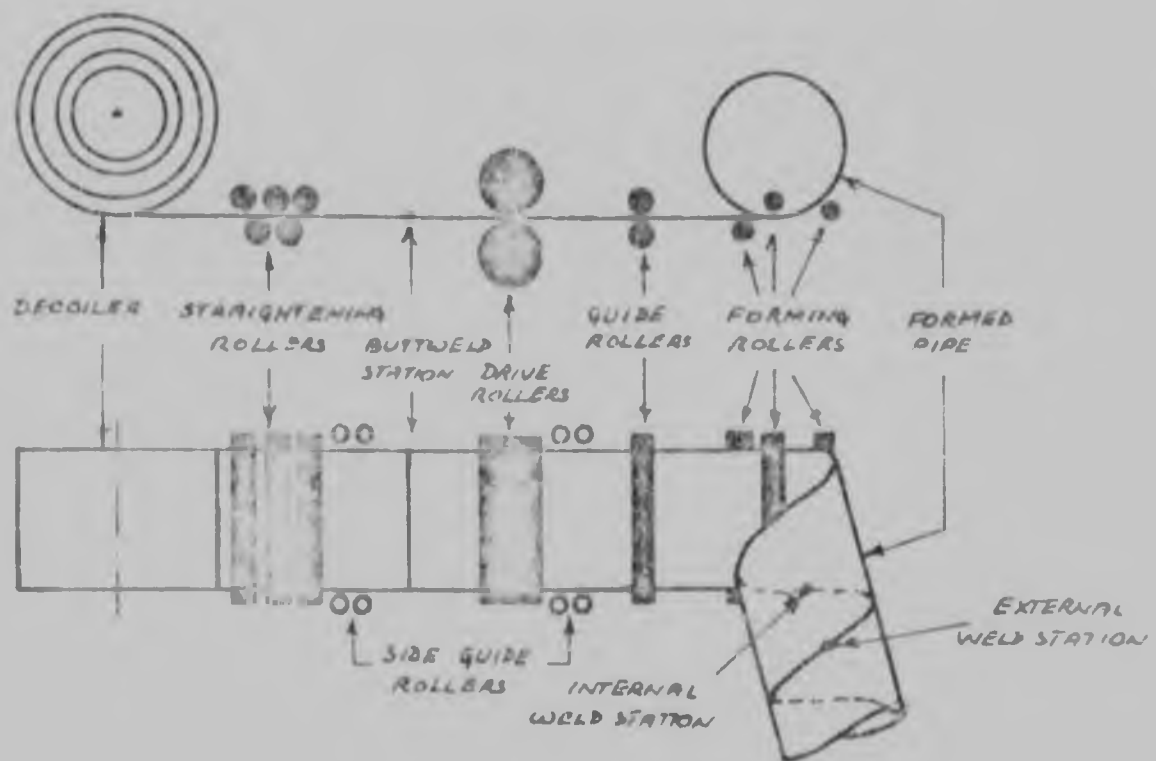


Figure 1: SPIRAL WELD PIPE MILL (adapted from Ref 10)

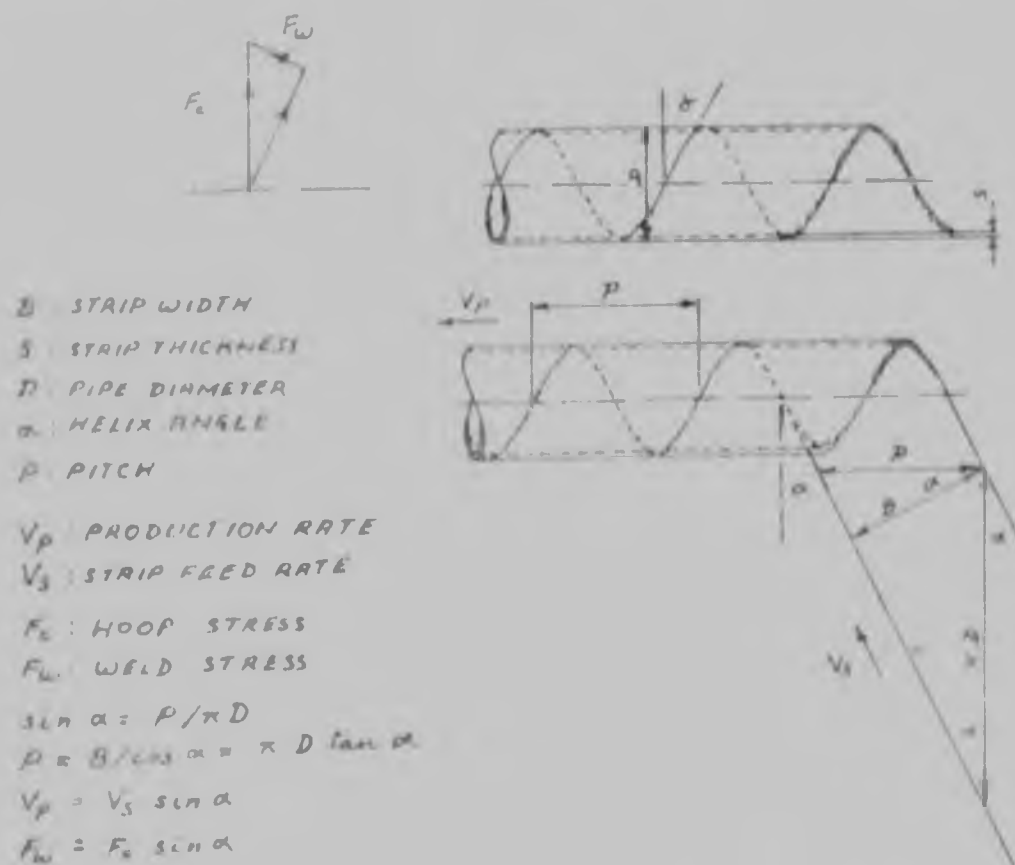


Figure 2: GEOMETRY OF HELICALLY WELDED PIPE (adapted from Refs 10 and 16)

Another possible disadvantage of helically welded pipe is that the hydraulic characteristics may be adversely affected by the weld bead and the deformation caused by anticlastic curvature of the strip. Few of the international specifications place any limit on the size of this weld bead but it is limited in SABS 719 (27) to 5 mm. Conrad and Brandts (6) have shown that the effect of the weld bead in helically welded pipes has less influence on the friction losses than the general roughness of the pipe.

It has been suggested (7), however, that a swirling flow may be generated by a helical weld bead although no work appears to have been done to investigate this aspect. This swirling flow might be sufficient to affect the results of pressure differential flow meters (4, 15, 32) but no guidance is given in the British Standards Flow Measurement Code (4) with regard to the use of helically welded pipe.

In view of the above the author has felt it desirable to investigate the development of flow patterns in helically welded pipes under controlled conditions and their effect on the discharge coefficient in the light of a theoretical analysis of swirling flows.

1.2 WORK ON SWIRLING FLOW ORIFICE APPROACH LENGTHS

The British Standard for flow measurement with pressure differential devices (BS 1042:1964) (4) excludes their application where there is any swirling motion upstream of the device. Guidelines are given for the choice of straightening lengths downstream of certain flow disturbing features such as bends, valves, etc. Part 1 of the

Standard provides a guide to the magnitude of the error to be expected when deviating from the standard approach conditions but no reference is made to swirling flows. Appendix I of Part 1 of the Standard gives some advice on the damping of swirling motion. It also states that strong swirls may persist for 100 or more pipe diameters downstream of the disturbance.

The ASME Guide to Fluid Meters (1) also makes reference to the desirability of excluding swirl in the approach flow and gives guidelines similar to those in BS 1042. It also specifies close tolerances on the pipe bore, roundness and surface roughness. This is, however, limited to the immediate vicinity of the orifice (4 diameters upstream and 2 diameters downstream).

In his work on the development of a slow straightener for use with orifice plate flow meters in disturbed flows, Zanker (32) investigated the effect of different types of swirling flows on the discharge coefficients of an orifice meter. He found that forced vortex swirls tended to give a high value of the discharge coefficient and that a free vortex swirl gave a low value. A reduction of area ratio tended to reduce these effects but not consistently or within acceptable limits. He also found that the discharge coefficient was dependent on the momentum flux in the outer third of the pipe cross-sectional area.

Lugt (15) investigated the effect of swirling flow on various types of pressure differential flow metering devices using varying area ratios. Using a free vortex type of swirl he found that with a swirl angle of 200° the error on the meter discharge coefficient was about 10% depending on the type and area ratio of the device.

In view of the large variation of error depending on swirl angle and the difficulty of determining the amount of swirl in practical installations, Lugt concluded that there was no point in attempting to incorporate correction factors into the standard specifications.

1.1 ANALYTICAL INVESTIGATIONS OF TURBULENT SWIRLING PIPE FLOWS

In recent years considerable interest has been shown in swirling flows because of their wide applications among which are (Gartshore) (8):

- Heat Exchangers (Kumar and Judd) (15)
- Vortex Reactors (Ragsdale) (19)
- Vortex decay in pipe flows (Talbot) (29), (Collatz and Görtler) (5) (Baker) (2)
- Meteorological Disturbances
- Ranque-Hilsch effect (Hilsch) (11).

Some of this work (eg, Gartshore, Talbot, and Collatz and Görtler) has been devoted to the study of laminar flows but since the flow of water in pipelines is almost always in the turbulent region, this investigation will be limited to that regime.

Although statistical methods have been used to study turbulence effects, the fact that these have not yet been fully developed resulted in a decision to use the simpler and more commonly used phenomenological theories based on the concepts of eddy viscosity and mixing length.

Early work on the mathematical formulation of a theory for turbulent flow was conducted by Reynolds (20) who

introduced the concept of the "Reynolds stresses" and by Boussinesq (3) who in 1877 conceived the idea of "eddy viscosity" to account for the Reynolds stresses in an analogy with molecular viscosity for shear stresses. Boussinesq's "eddy viscosity", however, had the disadvantage that unlike the molecular viscosity, the eddy viscosity is not a property of the fluid but depends on the flow itself.

This difficulty held up development of the theory till 1925 when Prandtl (18) proposed the concept of a mixing length. He developed this concept using a momentum transport theory thus arriving at an expression for the eddy viscosity in terms of his mixing length. Although this involved replacing one unknown with another, the fact that it is simpler to make plausible assumptions about the mixing length than about the eddy viscosity represented a considerable advance. By making assumption about the mixing length, Prandtl's theory has been successfully applied to the study of turbulent motion along fixed solid boundaries.

Taylor (30) introduced a modified version of the theory by assuming a vorticity transport theory which has proved more successful in problems of free turbulence. The drawback of both Prandtl's and Taylor's hypotheses is their dependence on assumptions about the nature of the mixing length. Von Kármán's similarity hypothesis (31) has enabled us to develop a mathematical expression for the mixing length (and hence the eddy viscosity) by assuming that turbulent fluctuations are similar at all points in the flow field. This theory was initially applied only to rectangular channel flow, but Kinney (13) developed it for rotational flows and Rochino and Lavan (23) have extended it to swirling pipe flows. (6) (11)

investigation Rochino and Lavan compared the result of applying the above approach with published experimental results of others and generally found a good correlation except for those portions of the flow field where certain of their assumptions were not valid - principally in the area of the pipe close to the axis.

Roberts (22) has suggested that the scalar model used for the eddy viscosity is not adequate and has used a vector model. However, in view of the fact that his component for the radial transport of angular momentum is the same as the scalar model of Rochino and Lavan and bearing in mind the latter's general success with their model, it has been decided to adopt a similar approach in this work.

CHAPTER 2

DEVELOPMENT OF THE SWIRL EQUATION

2.1 THE EQUATION OF TURBULENT INCOMPRESSIBLE FLOW AND REYNOLDS STRESSES

For the development of an equation to represent the swirling flow in a pipe, it is proposed to use as the starting point the Navier-Stokes equations for a generalised fluid motion. The derivation of these equations can be found in most standard works (eg, (26)) and in vectorial form, assuming constant viscosity, they can be written as follows:

$$\frac{d}{dt}(\rho \underline{q}) = \rho \underline{F} - \nabla p + \mu \nabla^2 \underline{q} + \frac{1}{3} \mu \nabla (\nabla \cdot \underline{q}) \quad . \quad . \quad (2.1.1)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{q}) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.1.2)$$

If we neglect external body forces (which in the case of pipe flow is allowable) and also assume that the fluid is incompressible then these equations reduce to

$$\rho \frac{d \underline{q}}{dt} + \rho \underline{q} \cdot \nabla \underline{q} = - \nabla p + \mu \nabla^2 \underline{q} \quad . \quad . \quad . \quad . \quad . \quad (2.1.3)$$

$$\text{and} \quad \nabla \cdot \underline{q} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.1.4)$$

These equations apply to the full flow variables ρ and $\underline{q}$ but to conduct an analysis of turbulent flow it is convenient to split up these variables into their time mean value $\bar{\rho}$ and $\bar{\underline{q}}$ and fluctuating components ρ' and $\underline{q}'$ to represent the superimposed turbulent motion. The time

averages of ρ' and q' are of course zero and

$$\rho = \bar{\rho} + \rho' \text{ and } q = \bar{q} + q' \quad (2.1.5)$$

By substituting these values of ρ and q into equation (2.1.3), we find the following

$$\frac{\partial}{\partial t}(\bar{q} + q') + ((\bar{q} + q') \cdot \nabla)(\bar{q} + q') = \frac{1}{\rho} \left[-\nabla(\bar{\rho} + \rho') + \mu \nabla^2(\bar{q} + q') \right] \quad (2.1.6)$$

If we now take mean values term by term (the validity of this procedure is discussed by Hinze in [12]) the following terms drop out since their means are zero:

$$\overline{\frac{\partial q'}{\partial t}}, \quad \overline{\bar{q} \cdot \nabla q'}, \quad \overline{q' \cdot \nabla \bar{q}}, \quad \overline{\frac{1}{\rho} \nabla \rho'} \quad \text{and} \quad \overline{\frac{\mu}{\rho} \nabla^2 q'}$$

and we are left with

$$\begin{aligned} \frac{\partial \bar{q}}{\partial t} + \bar{q} \cdot \nabla \bar{q} + \overline{q' \cdot \nabla q'} &= \frac{1}{\rho} (-\nabla \bar{\rho} + \mu \nabla^2 \bar{q}) \\ \text{or } \frac{\partial \bar{q}}{\partial t} + \bar{q} \cdot \nabla \bar{q} &= \frac{1}{\rho} (-\nabla \bar{\rho} + \mu \nabla^2 \bar{q}) - \overline{q' \cdot \nabla q'} \end{aligned} \quad (2.1.7)$$

By the same process equation (2.1.3) produces

$$\nabla \cdot \bar{q} = 0 \quad (2.1.8)$$

It is apparent that with the exception of the last term on the right hand side of equation (2.1.7), equations (2.1.7) and (2.1.8) for the mean flow variables are formally identical to equation (2.1.2) and (2.1.4) for the full flow variables.

This additional term $\overline{\underline{z}' \cdot \nabla \underline{z}'}$ can be expressed as $\sum_j \overline{\frac{\partial u'_i u'_j}{\partial x_j}}$. In his development of the Navier-Stokes equations Schlichting (26) shows that the surface force per unit area can be expressed by

$$\sum_i \frac{\partial \tau_{ij}}{\partial x_i}, \text{ where } \tau_{ij} = -p \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

is a stress tensor. It can thus be seen that the effect of the additional term $-\underline{z}' \cdot \nabla \underline{z}'$ in (2.1.7) is to increase the stress tensor by $-\rho \overline{u'_i u'_j}$. Thus the effect of the turbulence is to make the mean flow depart from the laminar flow by the imposition of these additional stresses known variously as "apparent" or "Reynolds stresses" after Osborne Reynolds who first developed this model for turbulent flow (20).

By considering only the motion after a steady state has been reached, we can further simplify the equation to

$$\underline{\bar{z}} \cdot \nabla \underline{\bar{z}} = -\frac{1}{\rho} \nabla \bar{p} + \frac{\mu}{\rho} \nabla^2 \underline{\bar{z}} - \underline{\bar{z}}' \cdot \nabla \underline{\bar{z}}' \quad . \quad . \quad . \quad (2.1.9)$$

The equations (2.1.8) and (2.1.9) have more dependent variables than available equations and in order to overcome this difficulty, Boussinesq (3) assumed that the turbulence stresses behave like the viscous stresses ie, that they are directly proportional to the velocity gradient. This assumption would imply the existence of an "eddy viscosity" such that

$$-\overline{u'_i u'_j} = \epsilon \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \quad . \quad . \quad . \quad . \quad (2.1.10)$$

According to Boussinesq's original idea ϵ was a scalar quantity but subsequently there has been considerable discussion about the mathematical nature of eddy viscosity (eg, Roberts (22) and Rochino and Lavan (23)). Hinze

even goes as far as to suggest the introduction of "the eddy diffusion coefficient as a fourth-order tensor", although he goes on to say that "this would complicate the whole matter exceedingly". Since eddy viscosity is anyway only a formal concept introduced to simplify the analysis of turbulent flows and does not represent any true physical property of the fluid, it would appear self-defeating to allow discussion of its mathematical nature to introduce further complications of this sort.

In this study it is proposed to adopt a vector scheme for the eddy viscosity on the grounds that it can account for different values in different directions without involving unnecessary complications. This scheme, however, precludes the use of a relationship such as (2.1.10) for the product of a first order tensor (vector), ϵ_{ij} and a second order tensor $(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i})$ is a first order tensor which clearly cannot be equated with the second order tensor $\overline{u'_i u'_j}$.

A formulation for a vector eddy viscosity can however be developed using phenomenological models which are discussed in the next two sections.

2.2 MIXING LENGTH THEORIES

The phenomenological theories which will be used in this work are those of Prandtl, Taylor and von Kármán, all of which are based on the concept of a "mixing length". All these theories assume that in turbulent flow lumps of fluid transfer certain of their transferable properties from one layer to another just as molecular agitation transfers properties like heat and momentum in a non-turbulent fluid. The lumps conserve these transferable

properties as they traverse a length, l , characteristic of the flow and known as the mixing length. Having gone this distance the lumps then merge with their new surroundings and assume the average properties of the fluid in that region.

In certain respects mixing length theories show an analogy with the kinetic theory of gases, the mixing length playing the part of the mean free path. The analogy breaks down because of the difference of scale between molecules, which are discrete elements moving in vacuo, and the fluid lumps, which even if assumed to be discrete elements for the purposes of the theory, are in reality part of a continuum which is continuously exerting forces on the lumps. Part of the failure of mixing length theories to account for all the phenomena of turbulence must be due to their non-recognition of this fact.

The general description of mixing length theories given above omits detail of

- (a) the property which is considered to be conserved in the fluid lumps, or
- (b) any hypothesis for determining the size of the mixing length.

Prandtl assumed that momentum would be preserved where Goldstein proposed a vorticity transport theory. Goldstein (9) says that though "there is nothing to choose between the two theories for flow between parallel planes the vorticity transfer theory gives results in better agreement with observation than does the momentum transfer theory for pipe flow". Taylor's theory has therefore been adopted.

Neither Prandtl or Taylor discusses in detail the mechanism according to which the lumps of fluid adapt their transferable property to their new surroundings. von Kármán's similarity theory fills this gap and provides the hypothesis required for determining the mixing length. This theory is discussed in the next section.

In order to use the vorticity transport theory we must first express the equation of motion (2.1.9) in a form which gives us access to the turbulent vorticity component $\underline{\Omega}'$ thus:

$$\underline{\underline{z}} \cdot \nabla \underline{\underline{z}} = -\nabla Q + \frac{\mu}{\rho} \nabla^2 \underline{\underline{z}} + \overline{\underline{\underline{z}}' \times \underline{\underline{\Omega}}'}. \quad (2.2.1)$$

where $Q = \left(\frac{\bar{u}^2}{2} + \frac{\underline{\underline{z}}'^2}{2} \right)$

If we consider figure 3 which shows the spacial relationships between a point A (a, b, c,) where the vorticity at time t_0 is $\underline{\underline{\Omega}}_0$ and a point X to which the fluid lump moves before mixing with the new environment, the vector $\underline{\underline{AX}}$ or $\underline{\underline{L}}$ is the mixing length. Let the vorticity at X have mean value $\underline{\underline{\Omega}}$ and turbulent component $\underline{\underline{\Omega}}'$. By the vorticity transport theory

$$\underline{\underline{\Omega}}_0 = \underline{\underline{\Omega}} + \underline{\underline{\Omega}}' \quad (2.2.2)$$

However, if we assume that the mixing length is small then a Taylor expansion around A for the mean vorticity (which at A is of course $\underline{\underline{\Omega}}_0$) yields

$$\underline{\underline{\Omega}}_0 = \underline{\underline{\Omega}} - (\underline{\underline{L}} \cdot \nabla) \underline{\underline{\Omega}} + \dots \text{ to first order} \quad (2.2.3)$$

$$\text{Thus to first order} \quad \underline{\underline{\Omega}}' = -(\underline{\underline{L}} \cdot \nabla) \underline{\underline{\Omega}} \quad (2.2.4)$$

so that the equation of motion becomes

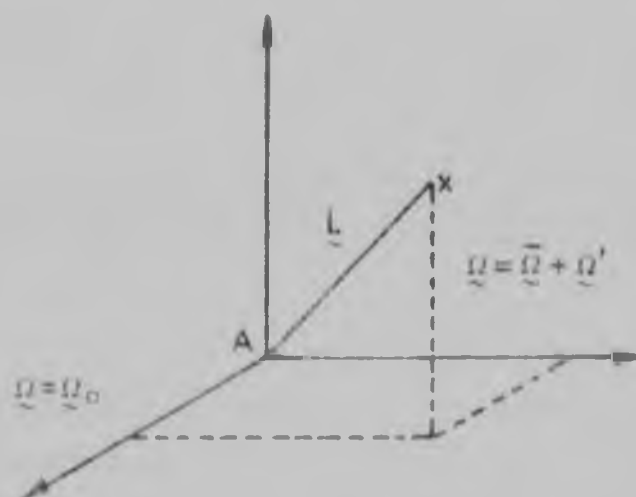


Figure 3: MIXING LENGTH VECTOR

$$\bar{\underline{z}} \cdot \nabla \bar{\underline{z}} = -\nabla Q + \frac{\mu}{\rho} \nabla^2 \bar{\underline{z}} - \overline{\underline{z}' \times (\underline{L} \cdot \nabla \bar{\underline{z}})} \quad (2.2.5)$$

In the discussion of pipe flows it will be convenient to express equation (2.2.5) in cylindrical co-ordinates as follows, with the bars on mean flow velocities and viscosity dropped for convenience

Radial Component

$$v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} = -\frac{\partial Q}{\partial r} + \frac{\mu}{\rho} \nabla^2 v_r$$

$$- v_\theta' \left[L_r \frac{\partial \gamma}{\partial r} + L_\theta \frac{\partial \gamma}{\partial \theta} + L_z \frac{\partial \gamma}{\partial z} \right] + v_z' \left[L_r \frac{\partial \eta}{\partial r} + L_\theta \frac{\partial \eta}{\partial \theta} + L_z \frac{\partial \eta}{\partial z} \right] + \frac{L_\theta}{r} \left[\frac{\partial \gamma}{\partial \theta} + \frac{\partial \eta}{\partial r} \right]$$

(2.2.6a)

Tangential Component

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r} = -\frac{1}{r} \frac{\partial Q}{\partial \theta} + \frac{\mu}{\rho} \nabla^2 v_\theta$$

$$- v_z' \left[L_r \frac{\partial \gamma}{\partial r} + L_\theta \frac{\partial \gamma}{\partial \theta} + L_z \frac{\partial \gamma}{\partial z} \right] + v_r' \left[L_r \frac{\partial \eta}{\partial r} + L_\theta \frac{\partial \eta}{\partial \theta} + L_z \frac{\partial \eta}{\partial z} \right] + \frac{L_\theta}{r} \left[\frac{\partial \gamma}{\partial \theta} + \frac{\partial \eta}{\partial r} \right]$$

(2.2.6b)

Axial Component

$$v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} = - \frac{\partial Q}{\partial z} + \frac{\mu}{\rho} \nabla^2 v_z$$

$$- v_r' \left[L_r \frac{\partial \eta}{\partial r} + \frac{L_\theta}{r} \frac{\partial \eta}{\partial \theta} + L_z \frac{\partial \eta}{\partial z} + \frac{L_\theta \xi}{r} \right] + v_\theta' \left[L_r \frac{\partial \xi}{\partial r} + \frac{L_\theta}{r} \frac{\partial \xi}{\partial \theta} + L_z \frac{\partial \xi}{\partial z} - \frac{L_\theta \eta}{r} \right]$$

$$- v_r' \left[L_r \frac{\partial \eta}{\partial r} + \frac{L_\theta}{r} \frac{\partial \eta}{\partial \theta} + L_z \frac{\partial \eta}{\partial z} + \frac{L_\theta \xi}{r} \right] + v_\theta' \left[L_r \frac{\partial \xi}{\partial r} + \frac{L_\theta}{r} \frac{\partial \xi}{\partial \theta} + L_z \frac{\partial \xi}{\partial z} - \frac{L_\theta \eta}{r} \right] \quad (2.2.7c)$$

In the swirling pipe flow under consideration we shall assume axisymmetry. This implies that there are no body forces (which we have already ignored) and that there will be no cavitating core which may take up a helical shape. Although these two conditions will be good approximations this will still not assure axisymmetrical flow. The above equations are sufficiently intractable, however, that for further development of the theory, axisymmetry must be assumed. This assumption leads to the following simplification

Radial Component

$$v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} = - \frac{\partial Q}{\partial r} + \frac{\mu}{\rho} \left[\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{\partial^2 v_r}{\partial z^2} \right]$$

$$- v_\theta' \left[L_r \frac{\partial \xi}{\partial r} + \frac{L_\theta}{r} \frac{\partial \xi}{\partial \theta} \right] + v_z' \left[L_r \frac{\partial \eta}{\partial r} + L_z \frac{\partial \eta}{\partial z} + \frac{L_\theta \xi}{r} \right]$$

(2.2.7a)

Tangential Component

$$v_r \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_\theta}{\partial z} + \frac{v_\theta v_r}{r} = \frac{\mu}{\rho} \left[\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} + \frac{\partial^2 v_\theta}{\partial z^2} \right]$$

$$- v_r' \left[L_r \frac{\partial \xi}{\partial r} + L_z \frac{\partial \xi}{\partial z} - \frac{L_\theta \eta}{r} \right] + v_z' \left[L_r \frac{\partial \xi}{\partial r} + L_z \frac{\partial \xi}{\partial z} \right]$$

(2.2.7b)

Axial Component

$$\begin{aligned} v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = - \frac{\partial Q}{\partial r} + \frac{\mu}{\rho} \left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2} \right] \\ - v_r' \left[L_r \frac{\partial \eta}{\partial r} + L_z \frac{\partial \eta}{\partial z} + \frac{L_\theta \xi}{r} \right] + v_\theta' \left[L_r \frac{\partial \xi}{\partial r} + L_z \frac{\partial \xi}{\partial z} - \frac{L_\theta \eta}{r} \right] \end{aligned} \quad (2.2.7c)$$

In order to simplify the equation further Taylor assumed that the anisotropy of the turbulence was sufficiently small that the turbulence transport coefficients $\overline{v_i L_j}$ could be considered nearly isotropic, ie, $\overline{v_i L_j} = 0$ if

$$j \neq i \quad \text{and} \quad \overline{v_i L_j} = \epsilon_{(i)} \quad \text{if} \quad i = j$$

where i and j take on the values r, θ, z . If we now write the vorticity components in terms of the mean flow velocities remembering that the flow is axisymmetrical we have

$$\xi = - \frac{\partial v_\theta}{\partial z} ; \quad \eta = - \frac{\partial v_z}{\partial r} ; \quad \zeta = \frac{1}{r} \frac{\partial (r v_\theta)}{\partial r} \quad (2.2.8)$$

Equations (2.2.7) now become

Radial Component

$$\begin{aligned} v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} = - \frac{\partial Q}{\partial r} + \frac{\mu}{\rho} \left[\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{\partial^2 v_r}{\partial z^2} \right] \\ + \epsilon(r) \left[\frac{\partial^2 v_r}{\partial z^2} - \frac{\partial^2 v_z}{\partial r \partial z} \right] \end{aligned} \quad (2.2.9a)$$

Tangential Component

$$\begin{aligned}
 v_r \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_\theta}{\partial z} + \frac{v_\theta v_r}{r} &= \frac{\mu}{\rho} \left[\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} + \frac{\partial^2 v_\theta}{\partial z^2} \right] \\
 (\epsilon) \quad (1) \quad (1) \quad (1) &\quad (1) \quad (1) \quad (1) \quad (1) \\
 &+ \epsilon_{(2)} \left[\frac{\partial^2 v_\theta}{\partial z^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} + \frac{\partial^2 v_\theta}{\partial r^2} \right] \\
 &\quad (1) \quad (1) \quad (1) \quad (1) \\
 &\quad \dots \dots \dots (2.2.9b)
 \end{aligned}$$

Axial Component

$$\begin{aligned}
 v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} &= - \frac{\partial Q}{\partial z} + \frac{\mu}{\rho} \left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2} \right] \\
 (\epsilon) \quad (1) \quad (1) \quad (\epsilon) &\quad (1) \quad (1) \quad (1) \quad (\epsilon) \\
 &- \epsilon_{(2)} \left[\frac{\partial^2 v_r}{\partial r \partial z} - \frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial z} - \frac{1}{r} \frac{\partial v_\theta}{\partial r} \right] \\
 &\quad (\epsilon) \quad (1) \quad (\epsilon) \quad (1) \\
 &\quad \dots \dots \dots (2.2.9c)
 \end{aligned}$$

We have now made what simplifications to the equations we are able to by global assumptions about the flow and we must now investigate them term by term in an order of magnitude analysis to determine what further simplification can be made.

In the swirling flow under consideration axial and tangential velocities are large compared with radial velocities so that if we regard small quantities as of order

$$v_\theta = O(1), v_z = O(1) \text{ and } v_r = O(\epsilon). \quad (2.2.10)$$

Furthermore, changes in the axial velocity over axial distances are small compared with changes in the radial direction so that

$$\frac{\partial v_z}{\partial r} = O(1) \text{ and } \frac{\partial v_z}{\partial z} = O(\epsilon) \quad (2.2.11)$$

By omitting terms of first or higher order in ϵ we can simplify the equations to the following

Radial Component

$$\frac{v_\theta^2}{r} = \frac{\partial Q}{\partial r} \quad (2.2.12a)$$

Tangential Component

$$v_z \frac{\partial v_\theta}{\partial z} = \left[\epsilon(0) + \frac{\mu}{\rho} \right] \left[\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} + \frac{\partial^2 v_\theta}{\partial z^2} \right] \quad (2.2.12b)$$

Axial Component

$$\frac{\partial Q}{\partial z} = \left[\epsilon(z) + \frac{\mu}{\rho} \right] \left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} \right] \quad (2.2.12c)$$

It is interesting to note that to the order of accuracy of these equations eddy viscosity and molecular viscosity are purely additive in their effect on the mean flow.

By differentiating equations (2.2.12a) and (2.2.12c) with respect to z and r respectively we can eliminate Q from this system as follows

$$\frac{\partial}{\partial z} \left(\frac{v_\theta^2}{r} \right) = \frac{\partial^2 Q}{\partial r \partial z} = \frac{\partial}{\partial r} \left[\left(\epsilon(z) + \frac{\mu}{\rho} \right) \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} \right) \right] \quad (2.2.13)$$

We now have, in equations (2.2.12b) and (2.2.13), two simultaneous partial differential equations for v_θ and v_z and provided satisfactory models can be provided for $\xi(z)$ and $\xi(\theta)$, a numerical solution should be obtainable.

However, before performing any numerical work it is desirable to non-dimensionalise the equations using the following reference scales:

V_{z0} = Maximum axial velocity of the non swirling flow profile

r_w = Pipe radius

ν = Kinematic molecular viscosity

If we denote dimensionless values by asterisks

$$v_\theta = V_{z0} v_\theta^* \quad ; \quad r = r_w r^* \quad ; \quad \xi(\theta) = \nu \xi(\theta)^*$$

$$v_z = V_{z0} v_z^* \quad ; \quad z = r_w z^* \quad ; \quad \xi(z) = \nu \xi(z)^*$$

so that the equations become

$$v_z^* \frac{\partial v_\theta^*}{\partial z^*} \cdot \frac{V_{z0}^2}{r_w^2} = \frac{\nu V_{z0}}{r_w^2} \cdot [\xi(\theta)^* + 1] \left[\frac{\partial^2 v_\theta^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial v_\theta^*}{\partial r^*} - \frac{v_\theta^*}{r^{*2}} + \frac{\partial^2 v_\theta^*}{\partial z^{*2}} \right]$$

. (2.2.14a)

and

$$\frac{\partial}{\partial z^*} \left(\frac{v_\theta^{*2}}{r^*} \right) \cdot \frac{V_{z0}^2}{r_w^2} = \frac{\nu V_{z0}}{r_w^2} \cdot \frac{\partial}{\partial r^*} \left[(\xi(z)^* + 1) \left(\frac{\partial^2 v_z^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial v_z^*}{\partial r^*} \right) \right]$$

. (2.2.14b)

As a convention we will now drop the asterisks since from this point on all equations will be dimensionless. The above equation therefore becomes

$$v_z \frac{\partial v_\theta}{\partial z} = \frac{(\epsilon(z)+1)}{Re} \left[\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} + \frac{\partial^2 v_\theta}{\partial z^2} \right] \quad (2.2.15a)$$

$$\frac{\partial}{\partial z} \left(\frac{v_\theta^2}{r} \right) = \frac{\partial}{\partial r} \left[\frac{(\epsilon(z)+1)}{Re} \left(\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} \right) \right] \quad (2.2.15b)$$

2.3 THE EDDY VISCOSITY MODEL AND VON KÁRMÁN'S SIMILARITY HYPOTHESIS

In the search for a satisfactory model for eddy viscosity various approaches have been tried. In Prandtl's momentum transfer theory the eddy viscosity is a scalar which may be written as the product of the square of the mixing length and the lateral derivative of the mean velocity. For free turbulence the mixing length was assumed proportional to the width of the mixing zone and in wall turbulence to the distance from the wall.

Von Kármán proposed that the mixing length was determined by the local flow conditions and that the flow pattern would show geometrical similarity throughout the flow field. Rochino and Lavan (23) followed this approach using a vector formulation with the components in the radial and tangential shear planes being equal. Roberts (22) went one stage further suggesting different values for the two components. For the transport of angular momentum he has adopted the same method as Rochino and Lavan and for axial transport phenomena he has assumed the formulation

$$\epsilon(z) = \frac{C_\mu \kappa}{\omega v_z}$$

where C_μ is the constant in the $\kappa-\omega$ turbulence model, κ is the turbulence kinetic energy and ω is the square of vorticity fluctuations.

$$(\bar{\xi}, \bar{\eta}, \bar{\zeta}) = \left(-\frac{1}{r} \frac{\partial}{\partial z} (r \bar{v}_\theta), 0, \frac{1}{r} \frac{\partial}{\partial r} (r \bar{v}_\theta) \right) \quad . \quad . \quad . \quad (2.3.5)$$

since the flow is axisymmetric.

If ξ', η', ζ' are the fluctuating vorticity components we can then express the instantaneous vorticity as the sum of mean and fluctuating values thus

$$(\xi, \eta, \zeta) = \left(\xi' - \frac{\partial \bar{v}_\theta}{\partial z}, \eta', \zeta' + \frac{1}{r} \frac{\partial}{\partial r} (r \bar{v}_\theta) \right) \quad . \quad . \quad . \quad (2.3.6)$$

The instantaneous velocities are

$$(v_r, v_\theta, v_z) = (v_r', \bar{v}_\theta + v_\theta', \bar{v}_z + v_z') \quad . \quad . \quad . \quad (2.3.7)$$

Substituting (2.3.6) and (2.3.7) into equation (2.3.4) we find the following, assuming axisymmetry.

Radial Component

$$\begin{aligned} \frac{\partial \xi'}{\partial t} + v_r' \frac{\partial}{\partial r} \left(\xi' - \frac{\partial \bar{v}_\theta}{\partial z} \right) + (v_z' + \bar{v}_z) \frac{\partial}{\partial z} \left(\xi' - \frac{\partial \bar{v}_\theta}{\partial z} \right) - (v_\theta' + \bar{v}_\theta) \frac{\eta'}{r} \\ - \left(\xi' - \frac{\partial \bar{v}_\theta}{\partial z} \right) \frac{\partial v_r'}{\partial r} - \left(\zeta' + \frac{1}{r} \frac{\partial}{\partial r} (r \bar{v}_\theta) \right) \frac{\partial v_r'}{\partial z} + \frac{\eta'}{r} (v_\theta' + \bar{v}_\theta) = 0 \end{aligned}$$

. (2.3.8a)

Tangential Component

$$\begin{aligned} \frac{\partial \eta'}{\partial t} + v_r' \frac{\partial \eta'}{\partial r} + (v_z' + \bar{v}_z) \frac{\partial \eta'}{\partial z} + \frac{1}{r} (v_\theta' + \bar{v}_\theta) \left(\xi' - \frac{\partial \bar{v}_\theta}{\partial z} \right) \\ - \left(\xi' - \frac{\partial \bar{v}_\theta}{\partial z} \right) \frac{\partial (v_\theta' + \bar{v}_\theta)}{\partial r} - \left(\zeta' + \frac{1}{r} \frac{\partial}{\partial r} (r \bar{v}_\theta) \right) \frac{\partial (v_\theta' + \bar{v}_\theta)}{\partial z} - \frac{\eta' v_r'}{r} = 0 \end{aligned}$$

. (2.3.8b)

Axial Component

$$\begin{aligned} \frac{\partial \xi'}{\partial t} + v_r' \frac{\partial}{\partial r} \left(\xi' + \frac{1}{r} \frac{\partial (r \bar{u}_\theta')}{\partial r} \right) + (\bar{u}_\theta' + \bar{u}_\theta) \frac{\partial}{\partial z} \left(\xi' + \frac{1}{r} \frac{\partial (r \bar{u}_\theta')}{\partial r} \right) \\ - \left(\xi' - \frac{\partial \bar{u}_\theta}{\partial z} \right) \frac{\partial}{\partial r} (\bar{u}_2' + \bar{u}_2) - \left(\xi' + \frac{1}{r} \frac{\partial (r \bar{u}_\theta')}{\partial r} \right) \frac{\partial}{\partial z} (\bar{u}_2' + \bar{u}_2) = 0 \end{aligned}$$

. (2.3.8c)

If we further assume the axial and radial change in $\frac{\partial \bar{u}_\theta}{\partial z}$ to be negligible, then the equations simplify to the following:

Radial Component

$$\begin{aligned} \frac{\partial \xi'}{\partial t} + v_r' \frac{\partial \xi'}{\partial r} + \bar{u}_2' \frac{\partial \xi'}{\partial z} + \bar{u}_2 \frac{\partial \xi'}{\partial z} - \xi' \frac{\partial v_r'}{\partial r} + \frac{\partial \bar{u}_\theta}{\partial z} \cdot \frac{\partial v_r'}{\partial r} - \xi' \frac{\partial v_r'}{\partial z} \\ - \frac{1}{r} \frac{\partial}{\partial r} (r \bar{u}_\theta') \frac{\partial v_r'}{\partial z} = 0 \end{aligned}$$

. (2.3.9a)

Tangential Component

$$\begin{aligned} \frac{\partial \eta'}{\partial t} + v_r' \frac{\partial \eta'}{\partial r} + \bar{u}_2' \frac{\partial \eta'}{\partial z} + \bar{u}_2 \frac{\partial \eta'}{\partial z} + \frac{\bar{u}_\theta' \xi'}{r} - \frac{v_r'}{r} \cdot \frac{\partial \bar{u}_\theta}{\partial z} + \frac{\bar{u}_\theta \xi'}{r} \\ - \frac{\bar{u}_\theta}{r} \frac{\partial \bar{u}_\theta}{\partial z} - \xi' \frac{\partial \bar{u}_\theta'}{\partial r} - \xi' \frac{\partial \bar{u}_\theta}{\partial r} + \frac{\partial \bar{u}_\theta}{\partial r} \cdot \frac{\partial v_r'}{\partial r} + \frac{\partial \bar{u}_\theta}{\partial z} \cdot \frac{\partial \bar{u}_\theta}{\partial r} \\ - \xi' \frac{\partial v_r'}{\partial z} - \xi' \frac{\partial \bar{u}_\theta}{\partial z} - \frac{1}{r} \frac{\partial (r \bar{u}_\theta')}{\partial r} \frac{\partial \bar{u}_\theta'}{\partial z} - \frac{1}{r} \frac{\partial (r \bar{u}_\theta')}{\partial r} \frac{\partial \bar{u}_\theta}{\partial z} - \frac{\xi' v_r'}{r} = 0 \end{aligned}$$

. (2.3.9b)

Axial Component

$$\begin{aligned} \frac{\partial \bar{z}'}{\partial t} + \bar{v}_r' \frac{\partial \bar{z}'}{\partial r} + \bar{v}_r' \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (r \bar{v}_\theta')}{\partial r} \right) + \bar{v}_z' \frac{\partial \bar{z}'}{\partial z} + \bar{v}_z' \frac{\partial}{\partial z} \left(\frac{1}{r} \frac{\partial (r \bar{v}_\theta')}{\partial r} \right) \\ + \bar{v}_z' \frac{\partial \bar{z}'}{\partial z} + \bar{v}_z' \frac{\partial}{\partial z} \left(\frac{1}{r} \frac{\partial (r \bar{v}_\theta')}{\partial r} \right) - \bar{z}' \frac{\partial \bar{v}_z'}{\partial r} + \frac{\partial \bar{v}_\theta'}{\partial z} \cdot \frac{\partial \bar{v}_z'}{\partial r} \\ - \bar{z}' \frac{\partial \bar{v}_z'}{\partial z} - \frac{1}{r} \frac{\partial (r \bar{v}_\theta')}{\partial r} \cdot \frac{\partial \bar{v}_z'}{\partial z} = 0 \end{aligned}$$

. (2.3.9c)

Von Kármán's similarity hypothesis postulates that the turbulent fluctuations are similar at all points of the field of flow and that they differ only by time and length scale factors. In more concrete terms, if we take the flow pattern of the turbulent eddies at any arbitrary point P_0 in the flow field, we only have to stretch (or shorten) the length and time scales in this pattern by suitable factors to be able to obtain the pattern at any other desired point in the field.

While this assumption would appear to have little theoretical basis, if the application of the theory is restricted to a small region near P_0 , the results obtained by Rochino and Lavan would indicate that within limitations it can provide a useful tool for analysing turbulent flows.

Since we are considering only the immediate neighbourhood of P_0 (see fig 4) it will be convenient to take for our length scale ℓ , the mixing length at P_0 and for our time scale, ℓ/v_f where v_f is the friction velocity given by

$$v_f = \left(\frac{\tau_{w,f}}{\rho} \right)^{1/2} = \left(\overline{u'_i u'_j} \right)^{1/2} (2.3.10)$$

(see Reference 26, page 485).

Then for any point $P(r, \theta, z)$ in the neighbourhood of $P_0(r_0, \theta_0, z_0)$

$$r = r_0 + \ell r^* ; \quad r\theta = r_0 \theta_0 + \ell \theta^* ; \quad z = z_0 + \ell z^* ; \quad t = \ell / v_f \cdot t^*$$

where the asterisked variables are of order unity in the newly scaled field. Furthermore we can state that the turbulence velocity and vorticity components are

$$v_r' = v_f v_r'^* ; \quad v_\theta' = v_f v_\theta'^* ; \quad v_z' = v_f v_z'^*$$

$$\xi' = \frac{v_f}{\ell} \xi'^* ; \quad \eta' = \frac{v_f}{\ell} \eta'^* ; \quad \zeta' = \frac{v_f}{\ell} \zeta'^*$$

$$\frac{\partial v^*}{\partial r} = \frac{1}{\ell} ; \quad \frac{\partial \theta^*}{\partial \theta} = \frac{r_0}{\ell} ; \quad \frac{\partial z^*}{\partial z} = \frac{1}{\ell} ; \quad \frac{\partial t^*}{\partial t} = \frac{v_f}{\ell}$$

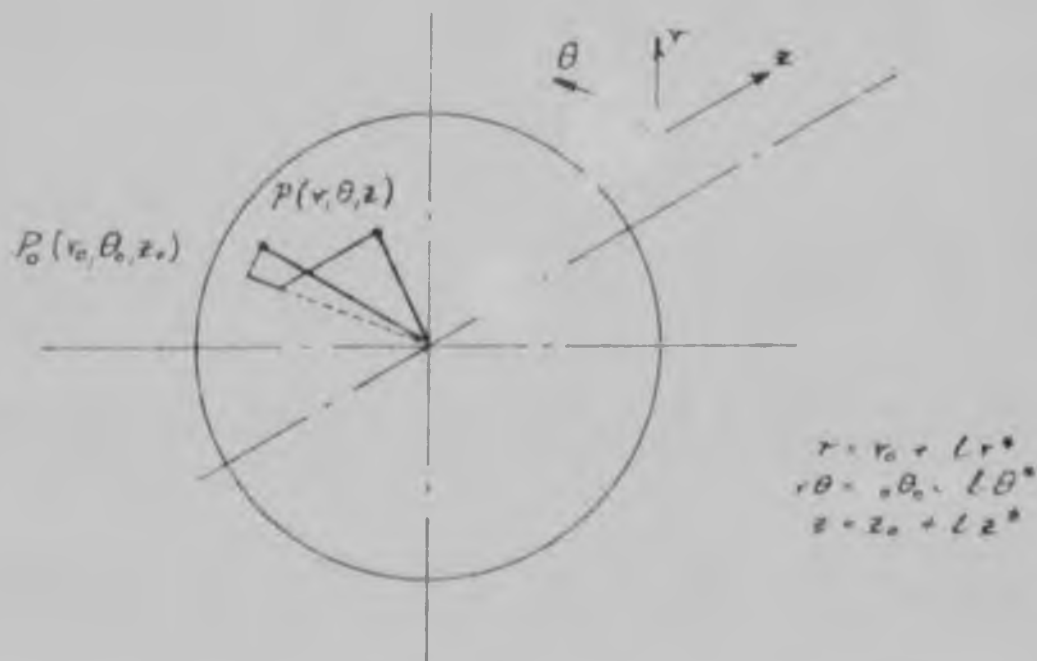


Figure 4) VON KÁRMÁN'S SIMILARITY HYPOTHESIS

To first order of accuracy we shall also assume that the value of $\frac{\partial}{\partial r}(r \bar{v}_\theta)$ at the point P near P_0 is the same as it is at P_0 .

For the term $\frac{\bar{v}_\theta}{r}$ we shall take the first term in the Taylor expansion about P_0 . ie, at P

$$\frac{\bar{v}_\theta}{r} = (r - r_0) \left[\frac{\partial}{\partial r} \left(\frac{\bar{v}_\theta}{r} \right) \right]_0 \quad \dots \quad (2.3.11)$$

Similar treatment will be used for other terms of equation (2.3.9).

If we substitute all the above relationships into (2.3.9) we find

Radial Component

$$\begin{aligned} & \frac{v_f}{l^2} \cdot \frac{\partial \xi'^{**}}{\partial t^{**}} + \frac{v_f^2}{l^2} \cdot v_r'^{**} \cdot \frac{\partial \eta'^{**}}{\partial r^{**}} + \frac{v_f^2}{l} \cdot v_z'^{**} \cdot \frac{\partial \xi'^{**}}{\partial z^{**}} + \bar{v}_{z_0} \frac{v_f}{l^2} \cdot \frac{\partial \xi'^{**}}{\partial z^{**}} - \frac{v_f^2}{l^2} \cdot \xi'^{**} \frac{\partial v_r'^{**}}{\partial r^{**}} \\ & + \left(\frac{\partial \bar{v}_\theta}{\partial z} \right)_0 \frac{v_f}{l} \frac{\partial v_r'^{**}}{\partial r^{**}} - \frac{v_f^2}{l^2} \cdot \xi'^{**} \frac{\partial v_z'^{**}}{\partial z^{**}} - \left(\frac{1}{r} \frac{\partial}{\partial r} (r \bar{v}_\theta) \right)_0 \cdot \frac{v_f}{l} \cdot \frac{\partial v_r'^{**}}{\partial z^{**}} = 0 \\ & \dots \dots \dots (2.3.12a) \end{aligned}$$

Tangential Component

$$\begin{aligned} & \frac{v_f^2}{l^2} \cdot \frac{\partial \xi'^{**}}{\partial t^{**}} + \frac{v_f^2}{l} \cdot v_r'^{**} \cdot \frac{\partial \eta'^{**}}{\partial r^{**}} + \frac{v_f^2}{l} \cdot v_z'^{**} \cdot \frac{\partial \eta'^{**}}{\partial z^{**}} + \bar{v}_{z_0} \frac{v_f}{l^2} \cdot \frac{\partial \eta'^{**}}{\partial z^{**}} + \frac{v_f^2}{l} \cdot \frac{\eta'^{**}}{r_0} \\ & - v_f \left(\frac{\partial \bar{v}_\theta}{\partial z} \right)_0 \frac{v_r'^{**}}{r_0} + \frac{v_f}{l} \cdot \frac{\bar{v}_{\theta_0}}{r_0} \xi'^{**} + \left(\frac{\bar{v}_\theta}{r} \cdot \frac{\partial \bar{v}_\theta}{\partial z} \right)_0 - \frac{v_f^2}{l^2} \cdot \xi'^{**} \frac{\partial v_\theta'^{**}}{\partial r^{**}} - \frac{v_f}{l} \cdot \xi'^{**} \left(\frac{\partial \bar{v}_\theta}{\partial r} \right)_0 \\ & + \left(\frac{\partial \bar{v}_\theta}{\partial z} \right)_0 \frac{v_f}{l} \cdot \frac{\partial v_\theta'^{**}}{\partial r^{**}} + \left(\frac{\partial \bar{v}_\theta}{\partial z} \cdot \frac{\partial \bar{v}_\theta}{\partial r} \right)_0 - \frac{v_f^2}{l^2} \cdot \xi'^{**} \frac{\partial v_\theta'^{**}}{\partial z^{**}} - \frac{v_f}{l} \cdot \xi'^{**} \left(\frac{\partial \bar{v}_\theta}{\partial z} \right)_0 \\ & - \left(\frac{1}{r} \frac{\partial}{\partial r} (r \bar{v}_\theta) \right)_0 \cdot \frac{v_f}{l} \cdot \frac{\partial v_\theta'^{**}}{\partial z^{**}} - \left(\frac{1}{r} \frac{\partial}{\partial r} (r \bar{v}_\theta) \cdot \frac{\partial \bar{v}_\theta}{\partial z} \right)_0 - \frac{v_f^2}{l} \cdot \frac{\eta'^{**}}{r_0} v_r'^{**} = 0 \\ & \dots \dots \dots (2.3.12b) \end{aligned}$$

Axial Component

$$\begin{aligned}
 & \frac{v_f^2}{\ell^2} \frac{\partial \xi'^*}{\partial t^*} + \frac{v_f^2}{\ell^2} v_r'^* \frac{\partial \xi'^*}{\partial r^*} + v_f v_r'^* \left(\frac{\partial}{\partial r} \cdot \frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right)_0 + \frac{v_f^2}{\ell^2} v_z'^* \frac{\partial \xi'^*}{\partial z^*} \\
 & + v_f v_z'^* \left(\frac{\partial}{\partial z} \cdot \frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right)_0 + \bar{u}_{z0} \frac{v_f}{\ell^2} \frac{\partial \xi'^*}{\partial z^*} + \left(\bar{u}_z \frac{\partial}{\partial z} \cdot \frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right)_0 \\
 & - \frac{v_f^2}{\ell^2} \xi'^* \frac{\partial v_z'^*}{\partial r^*} + \left(\frac{\partial \bar{u}_\theta}{\partial z} \right)_0 \frac{v_f}{\ell} \frac{\partial v_z'^*}{\partial r^*} - \frac{v_f^2}{\ell^2} \xi'^* \frac{\partial v_z'^*}{\partial z^*} \\
 & - \left(\frac{1}{r} \frac{\partial}{\partial r} (r \bar{u}_\theta) \right)_0 \frac{v_f}{\ell} \frac{\partial v_z'^*}{\partial z^*} = 0
 \end{aligned}$$

(2.3.12c)

Rearranging this equation in terms of v_f and ℓ for the sake of convenience

Radial Component

$$\begin{aligned}
 & \frac{v_f^2}{\ell^2} \left[\frac{\partial \xi'^*}{\partial t^*} + v_r'^* \frac{\partial \xi'^*}{\partial r^*} + v_z'^* \frac{\partial \xi'^*}{\partial z^*} - \xi'^* \frac{\partial v_r'^*}{\partial r^*} - \xi'^* \frac{\partial v_z'^*}{\partial z^*} \right] \\
 & + \frac{v_f}{\ell^2} \bar{u}_{z0} \frac{\partial \xi'^*}{\partial z^*} + \frac{v_f}{\ell} \left(\frac{\partial \bar{u}_\theta}{\partial z} \right)_0 \frac{\partial v_r'^*}{\partial r^*} + \frac{v_f}{\ell} \left[\frac{1}{r} \frac{\partial}{\partial r} (r \bar{u}_\theta) \right]_0 \frac{\partial v_r'^*}{\partial z^*} = 0
 \end{aligned}$$

(2.3.13a)

Tangential Component

$$\begin{aligned}
 & \frac{v_f^2}{\ell^2} \left[\frac{\partial \eta'^*}{\partial t^*} + v_r'^* \frac{\partial \eta'^*}{\partial r^*} + v_z'^* \frac{\partial \eta'^*}{\partial z^*} - \xi'^* \frac{\partial v_\theta'^*}{\partial r^*} - \xi'^* \frac{\partial v_\theta'^*}{\partial z^*} \right] + \frac{v_f}{\ell^2} \bar{u}_{z0} \frac{\partial \eta'^*}{\partial z^*} \\
 & + \frac{v_f^2}{\ell} \frac{1}{r_0} (v_\theta'^* \xi'^* - \eta'^* v_r'^*) - \frac{v_f}{r_0} \left(\frac{\partial \bar{u}_\theta}{\partial z} \right)_0 v_\theta'^* + \frac{v_f}{\ell} \left(\frac{\bar{u}_\theta}{r} \right)_0 \xi'^* - \frac{v_f}{\ell} \left(\frac{\partial \bar{u}_\theta}{\partial r} \right)_0 \xi'^* \\
 & + \frac{v_f}{\ell} \left(\frac{\partial \bar{u}_\theta}{\partial z} \right)_0 \left(\frac{\partial v_\theta'^*}{\partial r^*} - \xi'^* \right) - \frac{v_f}{\ell} \left(\frac{1}{r} \frac{\partial}{\partial r} (r \bar{u}_\theta) \right)_0 \frac{\partial v_\theta'^*}{\partial z^*} + \left(\frac{\bar{u}_\theta}{r} \frac{\partial \bar{u}_\theta}{\partial z} \right)_0 + \left(\frac{\partial \bar{u}_\theta}{\partial z} \cdot \frac{\partial \bar{u}_\theta}{\partial r} \right)_0 \\
 & - \left(\frac{1}{r} \frac{\partial}{\partial r} (r \bar{u}_\theta) \frac{\partial \bar{u}_\theta}{\partial z} \right)_0 = 0
 \end{aligned}$$

(2.3.13b)

Axial Component

$$\begin{aligned} & \frac{v_f^2}{\ell^2} \left[\frac{\partial \bar{z}^{1*}}{\partial t^*} + v_r^{1*} \frac{\partial \bar{z}^{1*}}{\partial r^*} + v_z^{1*} \frac{\partial \bar{z}^{1*}}{\partial z^*} - \bar{z}^{1*} \frac{\partial v_r^{1*}}{\partial r^*} - \bar{z}^{1*} \frac{\partial v_z^{1*}}{\partial z^*} \right] \\ & + v_f \left(\frac{\partial}{\partial r} \frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right)_0 \cdot v_r^{1*} + v_f \left(\frac{\partial}{\partial z} \frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right)_0 \cdot v_z^{1*} + \frac{v_f}{\ell^2} \bar{u}_{z0} \frac{\partial \bar{z}^{1*}}{\partial z^*} \\ & + \frac{v_f}{\ell} \left(\frac{\partial \bar{u}_\theta}{\partial z} \right)_0 \cdot \frac{\partial v_z^{1*}}{\partial r^*} - \frac{v_f}{\ell} \left(\frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right)_0 \frac{\partial v_z^{1*}}{\partial z^*} + \left(\bar{u}_z \frac{\partial}{\partial z} \frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right)_0 = 0 \end{aligned}$$

(2.3.13c)

If the turbulent flow properties are similar then the above equations must be independent of ρ_0 since it was chosen arbitrarily. Without loss of generality we can divide these equations by v_f^2/ℓ^2 and deduce the following relationships.

$\frac{1}{v_f} \bar{u}_{z0} = \kappa_1$	$\frac{\ell}{v_f} \left(\frac{\bar{u}_\theta}{r} \cdot \frac{\partial \bar{u}_\theta}{\partial z} \right)_0 = \kappa_8$
$\frac{\ell}{v_f} \left(\frac{\partial \bar{u}_\theta}{\partial z} \right)_0 = \kappa_2$	$\frac{\ell^2}{v_f^2} \left(\frac{\partial \bar{u}_\theta}{\partial z} \cdot \frac{\partial \bar{u}_\theta}{\partial r} \right)_0 = \kappa_9$
$\frac{\ell}{v_f} \left(\frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right)_0 = \kappa_3$	$\frac{\ell^2}{v_f^2} \left[\frac{1}{r} \cdot \frac{\partial (r \bar{u}_\theta)}{\partial r} \cdot \frac{\partial \bar{u}_\theta}{\partial z} \right]_0 = \kappa_{10}$
$\frac{\ell}{r_0} = \kappa_4$	$\frac{\ell^2}{v_f^2} \left[\frac{\partial}{\partial r} \frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right]_0 = \kappa_{11}$
$\frac{\ell^2}{v_f} \cdot \frac{1}{r_0} \left(\frac{\partial \bar{u}_\theta}{\partial z} \right)_0 = \kappa_5$	$\frac{\ell^2}{v_f^2} \left[\frac{\partial}{\partial z} \frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right]_0 = \kappa_{12}$
$\frac{\ell}{v_f} \left(\frac{\bar{u}_\theta}{r} \right)_0 = \kappa_6$	$\frac{\ell^2}{v_f^2} \left[\bar{u}_z \frac{\partial}{\partial z} \frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right]_0 = \kappa_{13}$
$\frac{\ell}{v_f} \left(\frac{\partial \bar{u}_\theta}{\partial r} \right)_0 = \kappa_7$	

Many of these constants are not independent, eg,

$$\kappa_{13} = \kappa_1 \times \kappa_{12}$$

$$\kappa_8 = \kappa_2 \times \kappa_6$$

$$\kappa_{10} = \kappa_3 \times \kappa_2$$

$$\kappa_5 = \kappa_4 \times \kappa_2$$

$$\kappa_9 = \kappa_2 \times \kappa_7$$

$$\kappa_7 = \kappa_3 - \kappa_6$$

And if we neglect $\frac{\partial^2 \bar{v}_\theta}{\partial z \partial r}$

$$\kappa_{12} = \kappa_4 \times \kappa_1$$

After a small amount of further rearrangement these relationships result in the following similarity conditions

$$l = C_1 r \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.3.14a)$$

$$v_f = C_2 l \frac{\partial \bar{v}_\theta}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.3.14b)$$

$$v_f = C_3 l \left(\frac{\partial \bar{v}_\theta}{\partial r} + \frac{\bar{v}_\theta}{r} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.3.14c)$$

$$v_f = C_4 l \left(\frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.3.14d)$$

$$v_f = C_5 \bar{v}_z \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.3.14e)$$

$$v_f = C_6 l^2 \frac{\partial}{\partial r} \left(\frac{\partial \bar{v}_\theta}{\partial r} + \frac{\bar{v}_\theta}{r} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.3.14f)$$

There has been considerable discussion on the significance of these similarity conditions and good accounts are given by Kinney (13) and Rochino and Lavan (23). The latter derive seven conditions but two of these are not independent. The additional one given here is (2.3.14e).

Now from a dimensional argument by Goldstein (9) we can see that $\epsilon = \pm \alpha \left| \frac{\tau_{ij}}{\rho} \right|^{1/2} l$ where α is a constant and the $\pm$ sign ensures that ϵ is positive. But from page 11 we know that the turbulence stress tensor $\tau_{ij} = -\rho \overline{u_i u_j}$

$$\mathcal{E}(\theta) = \pm r^2 \left(\frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r} \right) \times \text{constant} \quad (2.3.17a)$$

$$\text{and } \mathbf{E}_{(2)} = \pm r^2 \left(\frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r} \right) \times \text{constant} \quad (2.3.17b)$$

Now these expressions for eddy viscosity only hold in the free stream where the similarity theory represents a close approximation to the situation. Close to the wall Prandtl's hypothesis that the mixing length is proportional to the distance from the wall has been shown to be accurate and the above expressions need to be modified. We should now use the expression

[illegible]

with (2.3.16) to obtain the modified expressions

$$\varepsilon_{(\theta)} = \pm \text{constant} \times (1-\gamma)^2 \left(\frac{\partial \bar{v}_\theta}{\partial \gamma} - \frac{\bar{v}_\theta}{\gamma} \right) \quad (2.3.19a)$$

$$\text{and } \mathcal{E}(z) = \pm \text{constant} \times (1-\gamma)^2 \left(\frac{\partial \bar{v}_0}{\partial \gamma} - \frac{\bar{v}_0}{\gamma} \right) \quad (2.3.19b)$$

These latter expressions will be used for $0.9 < r \leq 1.0$ or the outer 10% of the radial distance.

Thus we have finally the following eddy viscosity expression

$$\mathcal{E}(\theta) = \mathcal{E}(\mathbf{z}) = \rho_e \kappa^2 \lambda^2 \left(\frac{\partial \bar{u}_\theta}{\partial r} - \frac{\bar{u}_\theta}{r} \right) \quad (2.3.20)$$

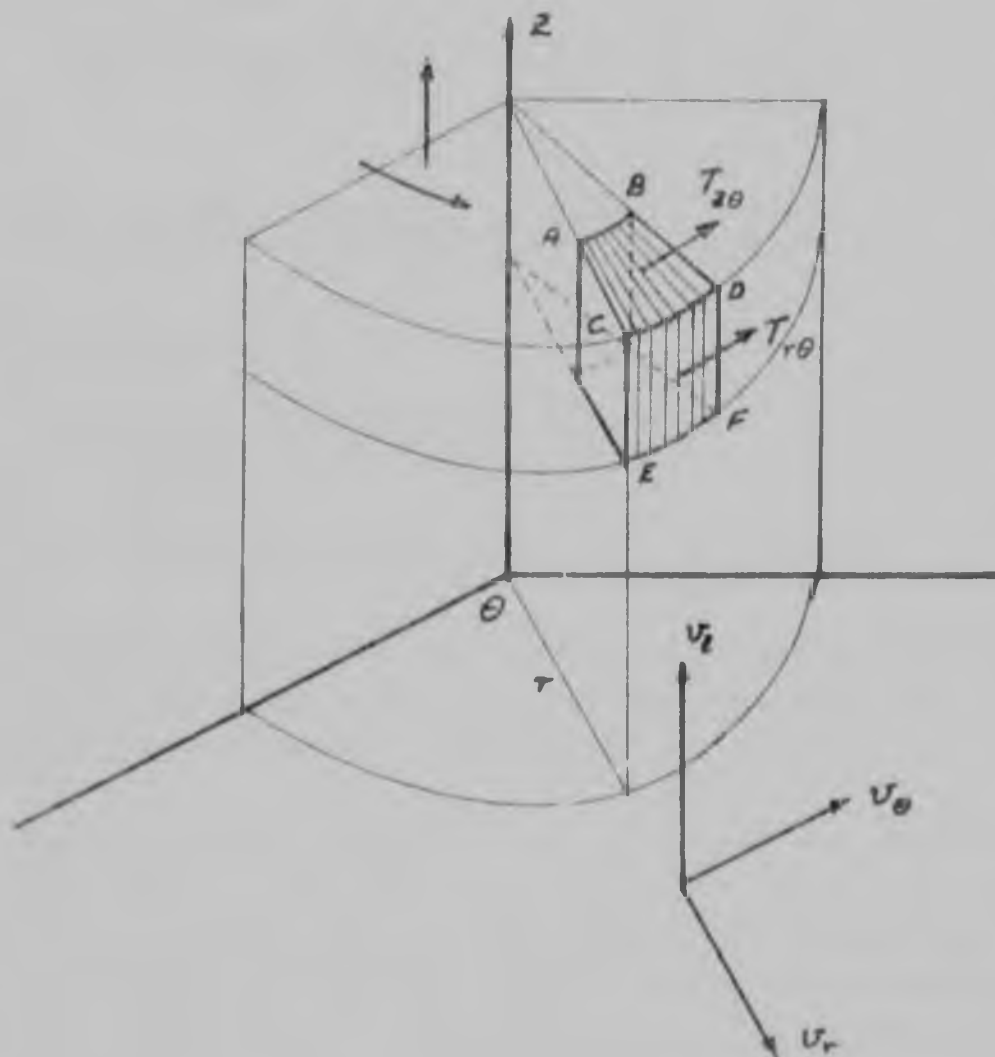
where

$$\lambda = r \quad 0 \leq r \leq 0.9$$

$$= 1-r \quad 0.9 \leq r \leq 1.0$$

and where κ is a constant for the type of flow which has to be determined experimentally. Rechin and Lavan found

its value to be 0,0333. The Reynolds number is based on the pipe radius and the maximum mean flow velocity.



= stress on face perpendicular to i in direction of j .

Figure 5: STRESSES ON A FLUID ELEMENT IN SWIRLING FLOW

2.4 THE SWIRL EQUATIONS

The governing equations for swirling flow (2.2.15a and b) are as follows

$$\bar{v}_z \frac{\partial \bar{v}_\theta}{\partial z} = \frac{(\epsilon(\theta) + 1)}{Re} \left(\frac{\partial^2 \bar{v}_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r^2} + \frac{\partial^2 \bar{v}_\theta}{\partial z^2} \right) \quad (2.4.1a)$$

$$\text{and } \frac{\partial}{\partial z} \left(\frac{\bar{v}_\theta^2}{r} \right) = \frac{\partial}{\partial r} \left[\frac{(\epsilon(\theta) + 1)}{Re} \left(\frac{\partial^2 \bar{v}_z}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{v}_z}{\partial r} \right) \right] \quad (2.4.1b)$$

From (2.3.20) we have

$$\epsilon(\theta) = \epsilon(\lambda) = Re \kappa^2 \lambda^2 \left(\frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r} \right) \quad (2.4.2)$$

where $\lambda = r$ $0 \leq r \leq 0.9$; $\lambda = 1-r$ $0.9 < r \leq 1.0$

These equations are too complex to permit analytical solution but a small amount of further analysis will enable the numerical work to be simplified.

The form of the equations suggests that the solution will best be obtained by solving (2.4.1a) for $\bar{v}_\theta$ on the assumption that $\bar{v}_z$ is independent of z . Using this solution for $\bar{v}_\theta$ we can now obtain a better approximation for $\bar{v}_z$ from (2.4.1b). Provided there are no problems over convergence, further iteration between these two equations should provide an accurate solution. Bearing this in mind as our ultimate numerical system, the two equations (2.4.1a) and (2.4.1b) can be simplified as follows.

In order to simplify (2.4.1a) we will introduce a new variable $\beta(r, z) = r \bar{v}_\theta$.

Differentials of β are

$$\frac{\partial \beta}{\partial z} = r \frac{\partial \bar{v}_\theta}{\partial z} ; \quad \frac{\partial \beta}{\partial r} = \bar{v}_\theta + r \frac{\partial \bar{v}_\theta}{\partial r} = \frac{\beta}{r} + r \frac{\partial \bar{v}_\theta}{\partial r}$$

$$\text{so } \frac{\partial \bar{v}_0}{\partial r} = \frac{1}{r} \left(\frac{\partial \beta}{\partial r} - \frac{\beta}{r} \right)$$

$$\text{and } \frac{\partial^2 \bar{v}_0}{\partial r^2} = \frac{1}{r} \frac{\partial^2 \beta}{\partial r^2} - \frac{1}{r} \frac{\partial \beta}{\partial r} - \frac{1}{r^2} \frac{\partial \beta}{\partial r} + \frac{2}{r^3} \beta$$

so inserting (2.4.2) and the above into (2.4.1a) we have

$$\frac{\partial \beta}{\partial z} = \frac{1}{\bar{v}_z} \left(\frac{1}{Re} + \frac{\kappa^2 \lambda^2}{r} \left(\frac{\partial \beta}{\partial r} - \frac{\beta}{r} \right) \right) \left(\frac{\partial^2 \beta}{\partial r^2} - \frac{1}{r} \frac{\partial \beta}{\partial r} \right) \quad (2.4.3)$$

Expansion of (2.4.1b) gives

$$\begin{aligned} 2 \frac{\bar{v}_0}{r} \frac{\partial \bar{v}_0}{\partial z} &= \frac{(\epsilon(z)+1)}{Re} \left(\frac{\partial^3 \bar{v}_z}{\partial r^3} + \frac{1}{r} \frac{\partial^2 \bar{v}_z}{\partial r^2} - \frac{1}{r^2} \frac{\partial \bar{v}_z}{\partial r} \right) \\ &+ \frac{1}{Re} \frac{\partial}{\partial r} \epsilon(z) \left(\frac{\partial^2 \bar{v}_z}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{v}_z}{\partial r} \right) \quad (2.4.4) \end{aligned}$$

But for $0 \leq r \leq 0.9$ $\lambda = r$ so

$$\begin{aligned} \frac{1}{Re \kappa^2} \frac{\partial (\epsilon(z))}{\partial r} &= r^2 \left(\frac{\partial^2 \bar{v}_0}{\partial r^2} - \frac{1}{r} \frac{\partial \bar{v}_0}{\partial r} + \frac{\bar{v}_0}{r} \right) + 2r \left(\frac{\partial \bar{v}_0}{\partial r} - \frac{\bar{v}_0}{r} \right) \\ &= r^2 \frac{\partial^2 \bar{v}_0}{\partial r^2} + r \frac{\partial \bar{v}_0}{\partial r} - \bar{v}_0 \quad (2.4.5) \end{aligned}$$

When $0.9 < r \leq 1.0$ $\lambda = 1-r$ so

$$\begin{aligned} \frac{1}{Re \kappa^2} \frac{\partial (\epsilon(z))}{\partial r} &= (1-r)^2 \left(\frac{\partial^2 \bar{v}_0}{\partial r^2} - \frac{1}{r} \frac{\partial \bar{v}_0}{\partial r} + \frac{\bar{v}_0}{r^2} \right) - 2(1-r) \left(\frac{\partial \bar{v}_0}{\partial r} - \frac{\bar{v}_0}{r} \right) \\ &= \frac{\partial^2 \bar{v}_0}{\partial r^2} (1-r)^2 + \frac{\partial \bar{v}_0}{\partial r} \left(r - \frac{1}{r} \right) + \bar{v}_0 \left(\frac{1}{r^2} - 1 \right) \quad (2.4.6) \end{aligned}$$

So for both values of λ we have

$$\frac{\partial}{\partial r} (\xi(z)) = f(\bar{v}_\theta, \frac{\partial \bar{v}_\theta}{\partial r}, \frac{\partial^2 \bar{v}_\theta}{\partial r^2}, r) \text{ or } f(r, z)$$

We can therefore write (2.4.4) as

$$\frac{\partial^3 \bar{v}_z}{\partial r^3} - P(r, z) \frac{\partial^2 \bar{v}_z}{\partial r^2} - Q(r, z) \frac{\partial \bar{v}_z}{\partial r} - F(r, z) = 0 \quad (2.4.7)$$

where for $0 \leq r \leq 0.9$

$$P(r, z) = \frac{-1}{r} - \kappa^2 \left(r^2 \frac{\partial^2 \bar{v}_\theta}{\partial r^2} + r \frac{\partial \bar{v}_\theta}{\partial r} - \bar{v}_\theta \right) / \left[\frac{1}{Re} + \kappa^2 r^2 \left(\frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r} \right) \right] \quad (2.4.8a)$$

$$Q(r, z) = \frac{1}{r^2} - \kappa^2 \left(r \frac{\partial^2 \bar{v}_\theta}{\partial r^2} + \frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r} \right) / \left[\frac{1}{Re} + \kappa^2 r^2 \left(\frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r} \right) \right] \quad (2.4.8b)$$

$$F(r, z) = 2 \frac{\bar{v}_\theta}{r} \frac{\partial \bar{v}_r}{\partial z} / \left[\frac{1}{Re} + \kappa^2 r^2 \left(\frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r} \right) \right] \quad (2.4.8c)$$

$$\quad (2.4.8c)$$

For $0.9 \leq r \leq 1.0$

$$P_1(r, z) = \frac{-1}{r} + \frac{\kappa^2 \left(\frac{\partial^2 \bar{v}_\theta}{\partial r^2} (1-r)^2 + \frac{\partial \bar{v}_\theta}{\partial r} \left(r - \frac{1}{r} \right) + \bar{v}_\theta \left(\frac{1}{r^2} - 1 \right) \right)}{\frac{1}{Re} + \kappa^2 (1-r)^2 \left(\frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r} \right)} \quad (2.4.9a)$$

$$Q_1(r, z) = \frac{1}{r^2} + \frac{\frac{\kappa^2}{r} \left(\frac{\partial^2 \bar{v}_\theta}{\partial r^2} (1-r)^2 + \frac{\partial \bar{v}_\theta}{\partial r} \left(r - \frac{1}{r} \right) + \bar{v}_\theta \left(\frac{1}{r^2} - 1 \right) \right)}{\frac{1}{Re} + \kappa^2 (1-r)^2 \left(\frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r} \right)} \quad (2.4.9b)$$

$$F(r, z) = 2 \frac{\bar{v}_\theta}{r} \frac{\partial \bar{v}_r}{\partial z} / \left[\frac{1}{Re} + \kappa^2 (1-r)^2 \left(\frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r} \right) \right] \quad (2.4.9c)$$

It can now be seen that given the value of β and hence $\bar{v}_0$ from (2.4.3), the solution of (2.4.7) is nearly that of a linear third order differential equation.

Although (2.4.3) cannot be linearised, it has been simplified to a form where it can be solved numerically.

2.5 BOUNDARY CONDITIONS

The determination of boundary conditions for the swirl equations, while simple in principle, presents a number of difficulties.

If one assumes that the fluid stream enters a helically welded pipe from a straight length of seamless or longitudinally welded pipe then the initial flow profile would be that due to normal turbulent pipe flow.

$$\text{ie, for } z=0 \quad \bar{v}_z(r,0) = \bar{v}_z(0,0) \left(\frac{1-r}{r} \right)^{1/7} \quad \dots (2.5.1a)$$

$$\bar{v}_\theta(r,0) = 0 \quad \text{for all } r. \quad \dots (2.5.1b)$$

5.1a) is an empirical equation given by Schlichting (6).

At the pipe wall will be zero but the algebraic representation of the shape of the wall is so complicated that this boundary condition can only be determined by physical measurement of the flow a small distance from the pipe wall.

Since we have already made the assumption that the flow is axisymmetrical then the tangential velocity component will be zero on the pipe centre line. The axial velocity

component will vary from its initial value at $z = 0$, but this variation can only be determined as we solve the equation along the pipe. The procedure for overcoming this difficulty is described in Section 3.2 and Appendix B.

CHAPTER 3

SOLUTION OF THE SWIRL EQUATIONS

3.1 NUMERICAL SOLUTION OF THE TANGENTIAL FLOW EQUATION

The tangential flow equation is

$$\frac{\partial \beta}{\partial z} = \frac{1}{U_2} \left[\frac{1}{Ra} + \frac{\kappa^2 \lambda^2}{r} \left(\frac{\partial \beta}{\partial r} - 2 \frac{\beta}{r} \right) \right] \left[\frac{\partial^2 \beta}{\partial r^2} - \frac{1}{r} \frac{\partial \beta}{\partial r} \right]$$

. (3.1.1)

where $\lambda = r$ for $0 \leq r \leq 0.9$; $\lambda = 1-r$ for $0.9 < r \leq 1.0$

To solve this equation it will be converted into a difference equation which is capable of being solved numerically using a digital electronic computer. Using the grid mesh shown in figure 6, β and its derivatives are replaced by the finite difference approximations:

$\beta(r, z)$ is replaced by $\beta(j, k)$ which is denoted by β_j^k

$$\frac{\partial \beta}{\partial z} = \frac{\beta_j^{k+1} - \beta_j^k}{h} ; \quad \frac{\partial \beta}{\partial r} = \frac{\beta_{j+1}^k - \beta_j^k}{\kappa} ; \quad \frac{\partial^2 \beta}{\partial r^2} = \frac{\beta_{j+1}^k - 2\beta_j^k + \beta_{j-1}^k}{\kappa^2}$$

. (3.1.2)

Substituting into (3.1.1) we have

$$\frac{\beta_j^{k+1} - \beta_j^k}{h} = \frac{1}{U_2} \left[\frac{1}{Ra} + \frac{\kappa^2 \lambda^2}{r} \left(\frac{\beta_{j+1}^k - \beta_j^k}{\kappa} - 2 \frac{\beta_j^k}{r} \right) \right] \left[\frac{\beta_{j+1}^k - 2\beta_j^k + \beta_{j-1}^k}{\kappa^2} \right]$$

$$- \frac{1}{r} \left(\frac{\beta_{j+1}^k - \beta_j^k}{\kappa} \right)$$

. (3.1.3)

If we let $A = \frac{1}{\bar{u}_2} \left[\frac{1}{Re} + \frac{\kappa^2 \lambda^2}{r} \left(\frac{\beta_{j+1}^n - \beta_j^n}{\kappa} - 2 \frac{\beta_j^n}{r} \right) \right]$. (3.1.4)

equation (3.1.3) can be rearranged as an explicit equation for β_j^{n+1} thus:

$$\beta_j^{n+1} = \beta_j^n \left(1 - \kappa A \left(\frac{2}{\kappa^2} - \frac{1}{r\kappa} \right) \right) + \beta_{j+1}^n \kappa A \left(\frac{1}{\kappa^2} - \frac{1}{r\kappa} \right) + \beta_{j+1}^n \frac{\kappa A}{\kappa^2}$$

. (3.1.5)

For computation we use the following form of the equation:

$$\beta_j^n = A' \left[\beta_j^{n-1} \left(\frac{1}{A'} - 2 - \frac{\kappa}{r} \right) + \beta_{j+1}^{n-1} \left(1 - \frac{\kappa}{r} \right) + \beta_{j+1}^{n-1} \right]$$

. (3.1.6a)

where for $0 \leq r \leq 0.9$

$$A' = \frac{\kappa}{\kappa^2 \bar{u}_2(r)} \left[\frac{1}{Re} + \frac{\kappa^2 r}{\kappa} \left(\beta_{j+1}^{n-1} - \beta_j^{n-1} \left(1 + \frac{2\kappa}{r} \right) \right) \right]$$

. (3.1.6b)

and for $0.9 < r \leq 1.0$

$$A' = \frac{\kappa}{\kappa^2 \bar{u}_2(r)} \left[\frac{1}{Re} + \frac{\kappa^2 \kappa}{r} \left(\frac{1}{\kappa} - \frac{r}{\kappa} \right)^2 \left(\beta_{j+1}^{n-1} - \beta_j^{n-1} \left(1 + \frac{2\kappa}{r} \right) \right) \right]$$

. (3.1.6c)

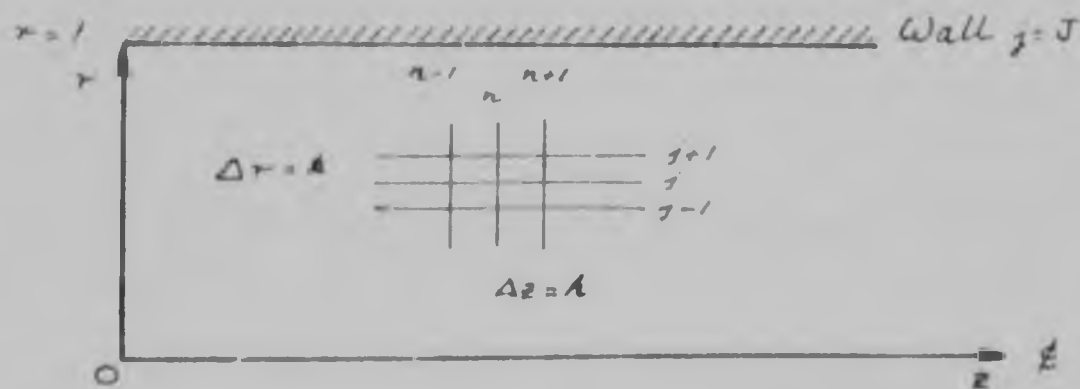


Figure 6: GRID MESH FOR COMPUTER PROGRAMME

The boundary conditions are $\beta(r, 0) = 0$; $\beta(0, z) = 0$;
 $\beta(0.95, z) = f(z)$ which transforms to

$$\beta_j^0 = 0 \quad \text{for all values of } j \quad . \quad . \quad . \quad . \quad . \quad (3.1.7a)$$

$$\beta_n^0 = 0 \quad \text{for all values of } n \quad . \quad . \quad . \quad . \quad . \quad (3.1.7b)$$

$$\beta_{0.95}^n = F(n) \quad . \quad . \quad . \quad . \quad . \quad (3.1.7c)$$

$F(n)$ will be determined from the results measured under experimental conditions.

The numerical technique for the solution of equations (3.1.6) together with boundary conditions (3.1.7) will be to evaluate each β_j^n by starting at β_1^0 to β_j^0 and repeat this procedure for each value of n .

A brief investigation into the stability of this equation is given in Appendix A.

3.2 NUMERICAL SOLUTION FOR THE AXIAL SWIRL EQUATION

The axial swirl equation is given by

$$\frac{\partial^3 \bar{v}_z}{\partial r^3} - P(r, z) \cdot \frac{\partial^2 \bar{v}_z}{\partial r^2} - Q(r, z) \cdot \frac{\partial \bar{v}_z}{\partial r} - F(r, z) = 0$$

. (3.2.1)

where $P(r, z)$, $Q(r, z)$ and $F(r, z)$ are given by equations (2.4.8) and (2.4.9).

P , Q and F are only implicitly dependent on z and their explicit dependency is in terms of r , $\bar{v}_0$, $\frac{\partial \bar{v}_0}{\partial r}$ and $\frac{\partial^2 \bar{v}_0}{\partial r^2}$. Given the (temporary) solution of the tangential swirl

equation for $\bar{v}_\theta$, by the means of Section 3.1 and provided the same grid notation is used for this section, there should be no problem about calculating these functions at each node point of the grid. The problem would appear then to be to find a method of numerical solution for a third order differential equation. This can be done by a predictor - corrector method such as is outlined in McCracken and Dorn (17). We have a problem with the boundary conditions, however, for in order to use such a method we would need to know the value of $\bar{v}_z$ on the pipe centre line all along the pipe. For the initial solution of the tangential equation, the assumption that $\bar{v}_z(0)$ was constant may be sufficient. For the axial equation this is, however, not good enough for the variation in $\bar{v}_z$ may lead to a violation of the continuity equation.

It is proposed, therefore, after each pass in the radial direction to check for the continuity and if necessary to adjust the value of $\bar{v}_z(0)$ to preserve it.

Further details on the technique of solution are given in Appendix B.

3.3 A COMPUTER PROGRAMME FOR THE SWIRL EQUATIONS

Sections 3.1 and 3.2 illustrate the basic numerical techniques for solving each of the two swirl equations independently. The purpose of this section is to illustrate the method of translating these techniques into an integrated computer programme for solving the simultaneous partial differential equations.

Fig 7 shows the logic network for the programme as a

whole while figures 8 and 9 show the detailed logic for solving each of the equations separately. Following the logic of figure 7, the first step is to define adequately the boundary conditions. Although this can in part be done theoretically, as outlined in Section 2.5 this work uses values determined from the test rig. Step C determines the total flow in the pipe by numerical integration of the initial flow profile. This serves as the standard of comparison when checking the results of the axial equation. In order to begin the main calculation portion of the programme it is necessary to assume that the velocity profile is unchanged over the axial distance being considered. This is only true to first order and is corrected later in the programme (at Step L). Steps D and E are also required to start the programme. The two main calculation steps are G (the tangential equation) and L (the axial equation). Closer solutions are obtained by solving these equations alternately in an iterative process (loop A). The iteration counters for the iteration loops are set in step F. At step K the solutions are checked for convergence and if the solutions have converged successfully then the results are printed out. Otherwise the iteration counter is increased and another iteration cycle started at step G. If more than five cycles are required the programme automatically aborts and prints out the most recent values of $\bar{v}_\theta$ and $\bar{v}_z$ obtained.

Figure 8 shows the detailed logic sequence of solving the tangential equation which is step G of the figure 7. The explicit nature of equation (3.1.6a) means that no iterative procedures are needed. Values of β are calculated passing in the radial direction and then in the axial direction. The branch step in the logic is required to cope with the alternate value of λ . After the full

β -array is calculated, the $\bar{v}_\theta$ -array is deduced from the relation

$$\beta(r,z) = r \bar{v}_\theta(r,z) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.3.1)$$

The programme then proceeds to step H of figure 7.

Figure 9 shows the detailed logic sequence for solving the axial equation which is step L of figure 7. The axial flow velocities for $n=1$ are part of the boundary conditions so that the calculation starts at $n=2$. At step L/A an iteration counter ITNB is set to 1 before calculating $\bar{v}_z(2,2)$ by the modified Euler method described in Appendix B. Since we now have the value of $\bar{v}_z(1,2)$ and $\bar{v}_z(2,2)$ we can use the predictor-corrector method for $j=3$. The predicted value is calculated in step L/B. In step F the iteration counter ITNC for the corrector loop was set to 1. The corrector loop is contained in stage L/C. The mathematics of the predictor-corrector method is detailed in Appendix B. Step L/D is the error correction required after the final corrector step. The DO loop 4302 repeats the predictor-corrector stages for increased values of j (ie, it completes the radial pass for $n=2$). In order to compensate for any error due to the assumption of a value for $\bar{v}_z(\infty,1)$ the total flow at $n=2$ is calculated in step L/E and compared with the initial total flow (calculated at step C of figure 7) in step L/F. If the error is outside acceptable tolerances then the assumed value of $\bar{v}_z(\infty,1)$ is corrected and the process repeated from step L/A. This iteration is controlled by ITNB. When flow continuity is preserved, further radial passes are made for increased values of n until the complete $\bar{v}_z$ -array is calculated. The programme then proceeds to step M of figure 7.

Work on the solution of the swirl equation was still in progress when the author was transferred to Germany. At this point solutions to the tangential equations were available (as indicated in the graphs of Section 5) but the influence of the axial equation had not yet been included. Due to a lack of suitable computer facilities a demonstration that the influence of the latter is only slight has not yet been possible. As and when computer facilities should become available, it is hoped that this can be adequately confirmed.

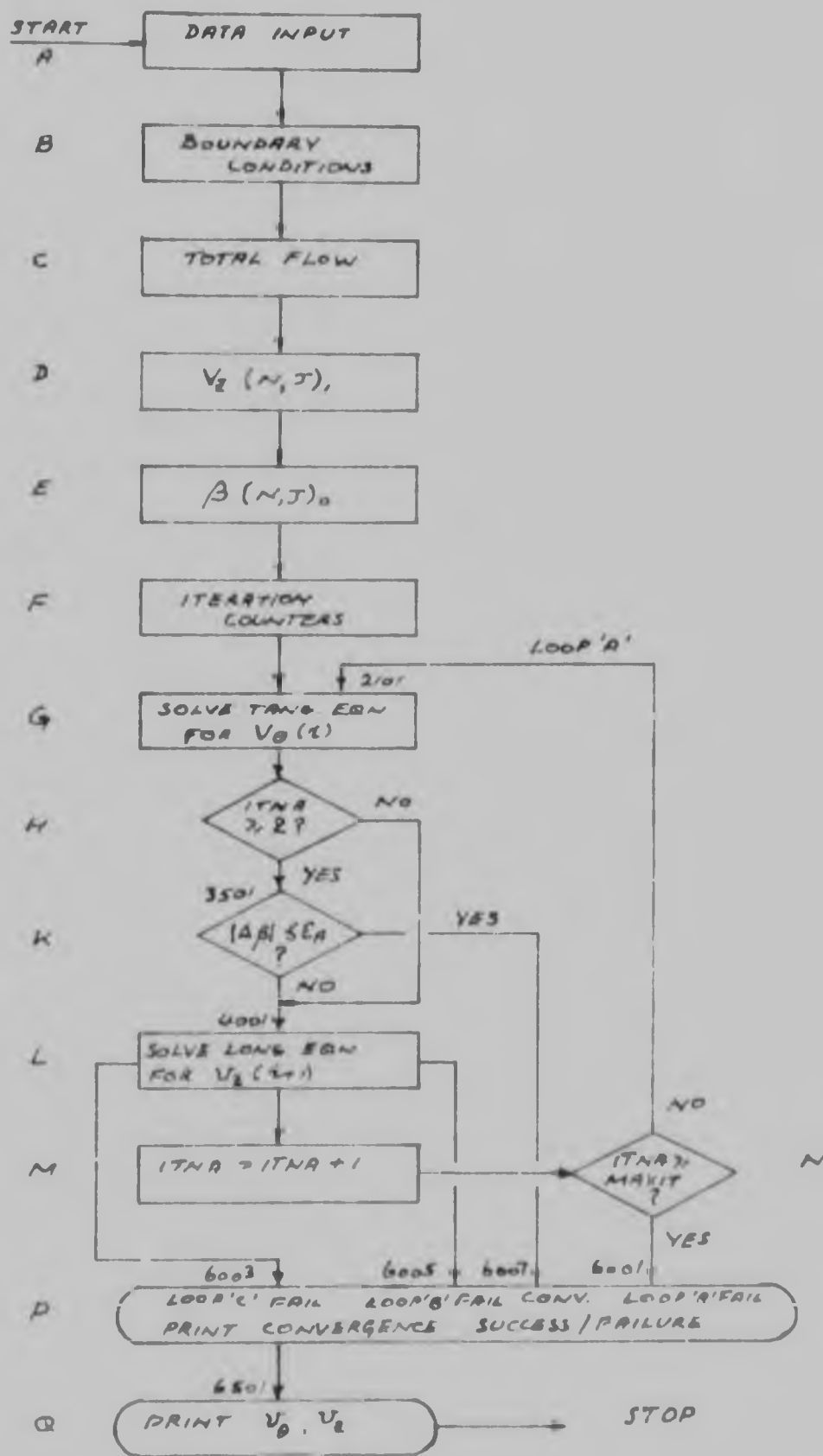


Figure 7: COMPUTER PROGRAMME FOR SOLUTION OF SWIRL EQUATIONS: OVERALL PROGRAMME LOGIC

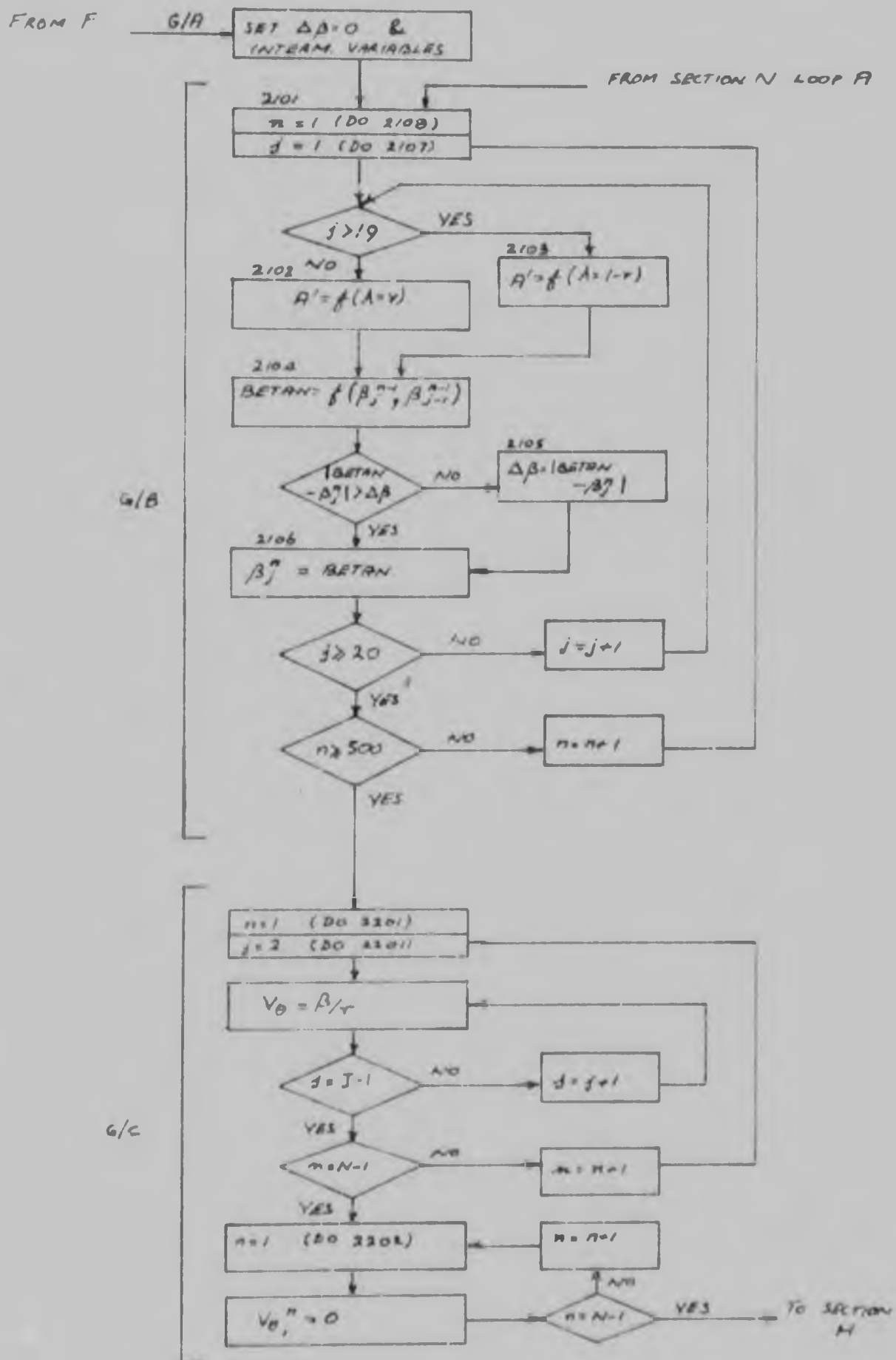


Figure 8: COMPUTER PROGRAMME FOR SOLUTION OF SWIRL EQUATIONS: TANGENTIAL EQUATION (SECTION G)

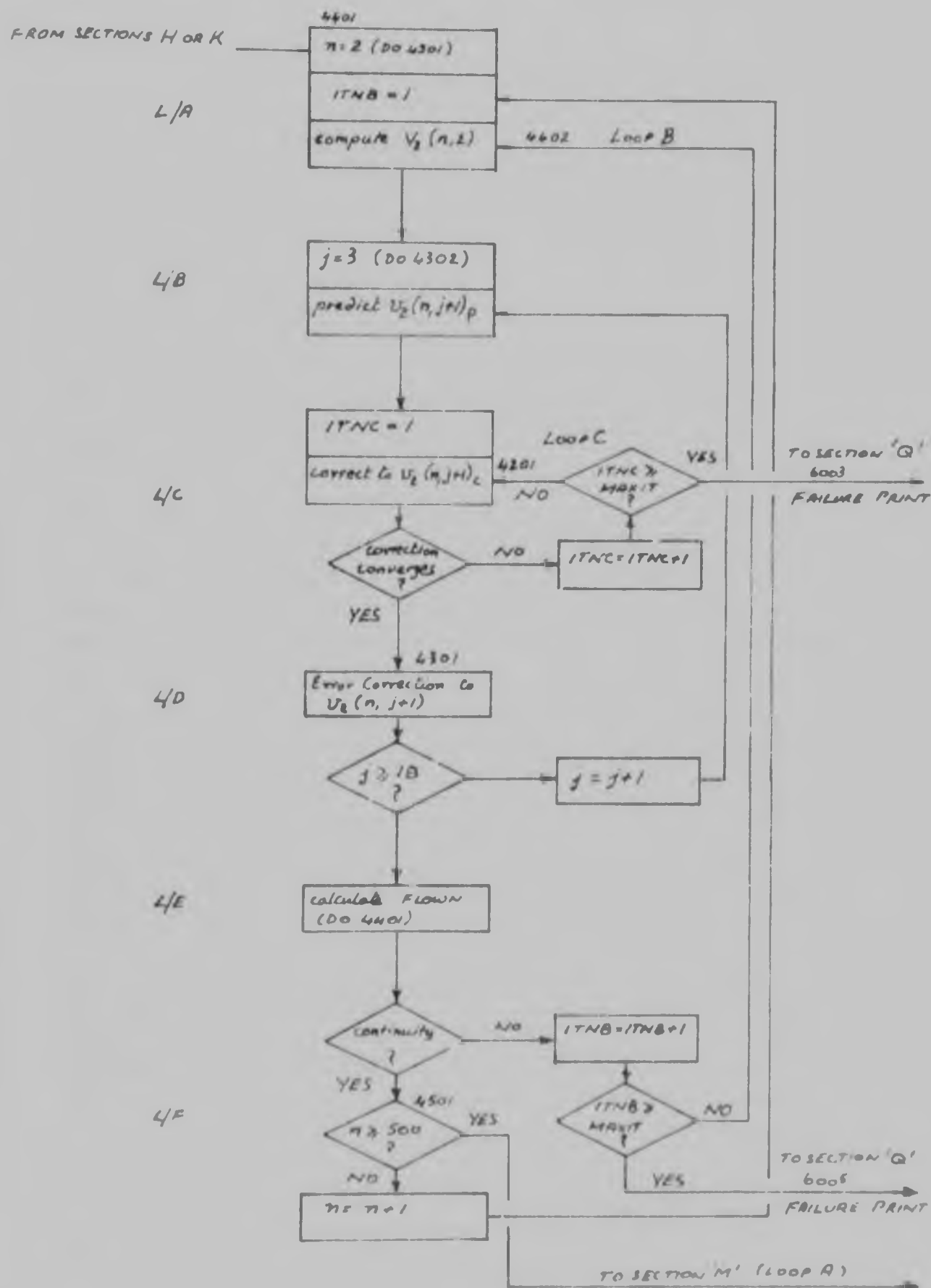


Figure 9: COMPUTER PROGRAMME FOR SOLUTION OF SWIRL EQUATIONS: AXIAL EQUATION (SECTION L)

CHAPTER 4

EXPERIMENTAL INVESTIGATIONS

4.1 PRELIMINARY INVESTIGATION

Since there is a lack of published work in the generation of swirl in helically welded pipes, a small pilot experiment was carried out to determine the likelihood of sufficient swirl being generated to disturb the operation of orifice meters.

The apparatus (shown in figure 10) consisted simply of a length of clear perspex pipe into which a wire insert was placed to simulate the disturbance of the weld seam in a helically welded steel pipe. This method of simulating the weld seam was necessary partly because helically welded pipe is not commercially available in sizes below about 250 mm in diameter and partly because perspex was required to permit the flow to be seen. The pipe length was connected up to a length of garden hose which had a sufficiently long straight length upstream to settle out any flow disturbances.

Dye was injected into the pipe through a hypodermic syringe and was observed to acquire a flow pattern showing considerable swirl after a length of test pipe equivalent to 70 diameters. The degree of swirl was such that, based on Lugt's results (15), a meter could be expected to show as much as 7-10% error. Zanker (32) has shown that the degree and the direction of the error are very much dependent on the nature of the swirl which could not be determined in this case because of the scale and crudity of the apparatus.

Despite the limitations of the apparatus for this preliminary investigation, however, the results show that swirl can certainly be generated in pipes with a protruding helix of the order of magnitude of that of the weld bead in a helically welded pipe and that this effect could be large enough to create serious errors in orifice plate meters. Further and more detailed investigation of the phenomenon at a larger scale was therefore justified.

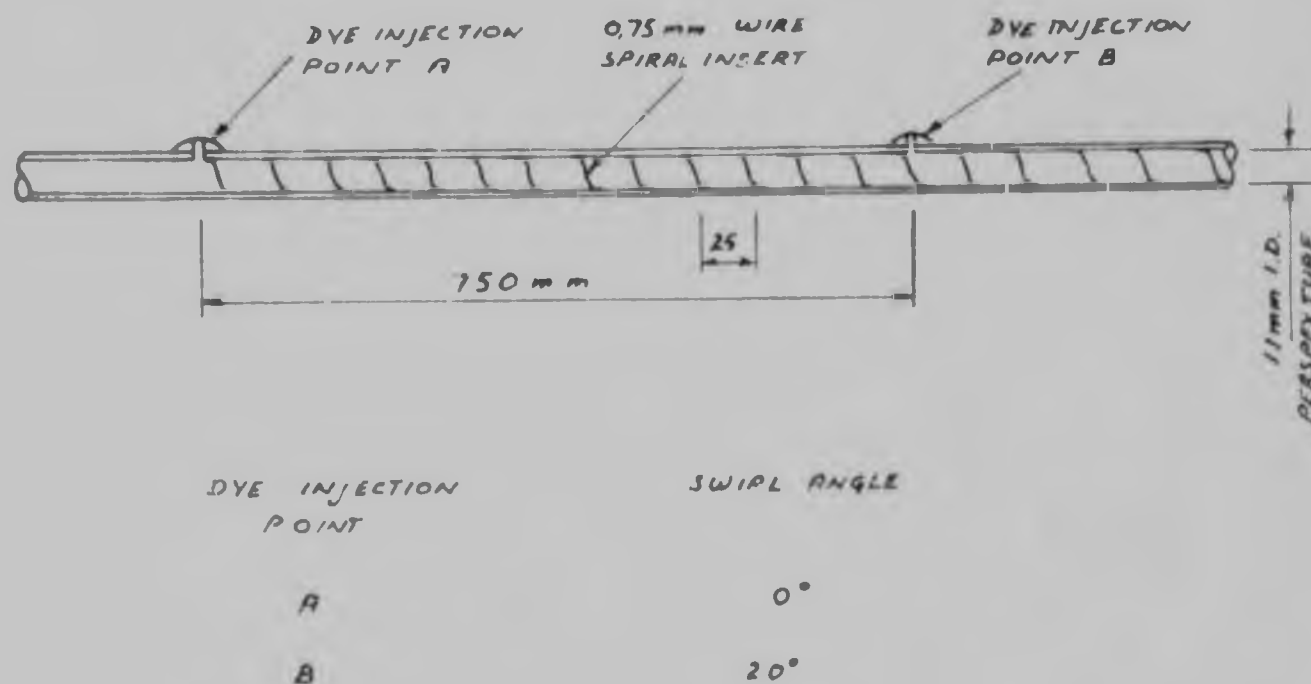


Figure 10: APPARATUS FOR PRELIMINARY INVESTIGATION

4.2 DESCRIPTION OF APPARATUS

Figure 11 illustrates the main portions of the apparatus used in the subsequent experimental investigations. Pumping limitations restricted the diameter of pipe used to 100 mm. Commercial helically welded pipe was not

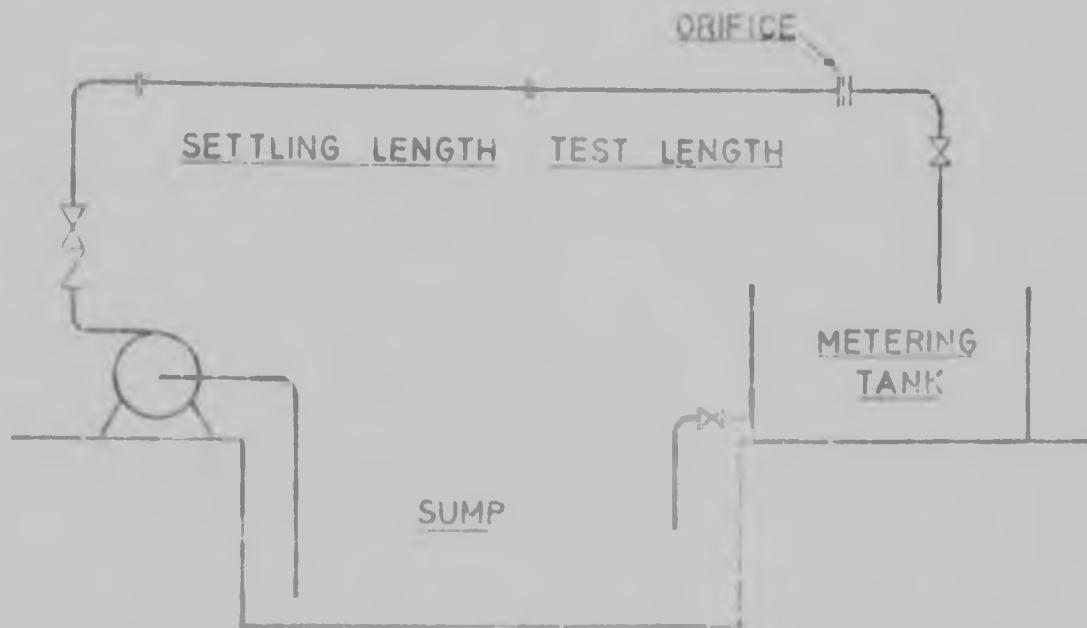


Figure 11(a): DIAGRAMMATIC LAYOUT OF TEST RIG

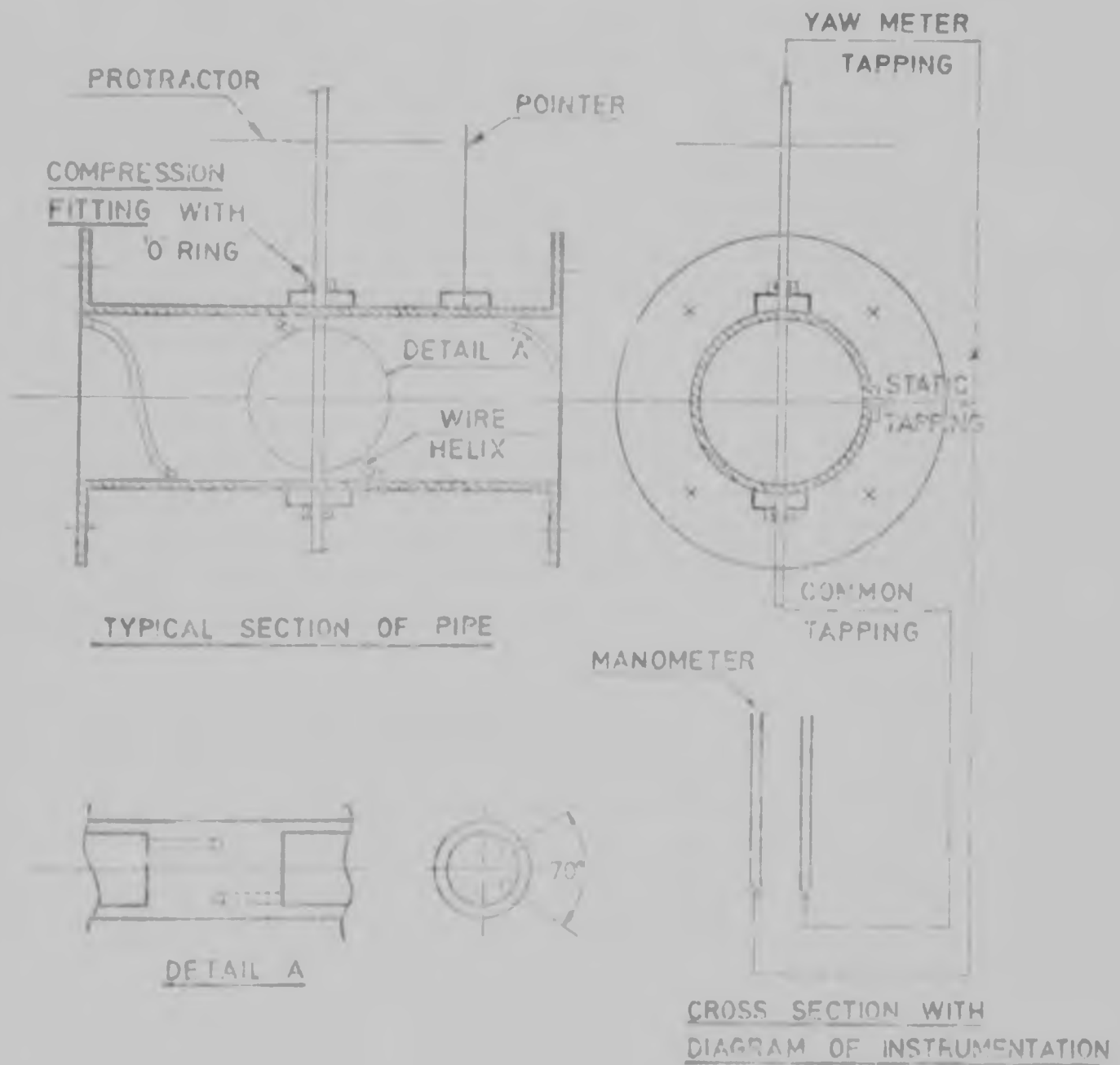


Figure 11(b): DETAIL OF TYPICAL TEST SECTION WITH INSTRUMENTATION

used because of the same two primary considerations which prevailed in the preliminary work, viz:

1. Commercial pipe is not made by the helical weld process under about 250 mm, and
2. Transparent perspex windows were required to show up the flow pattern.

Water was pumped into the test section through a flow settling length of resistance welded longitudinally seamed pipe. The test length was primarily seamless steel pipe with a helical wire insert to simulate the disturbance caused by the weld seam. Short lengths of perspex tube were inserted in the test length to provide windows to observe the build-up of any swirl which might occur. The test orifice installation immediately after the test section complied fully with BS 1042 except for the helical insert in the upstream pipe. The water was finally discharged to a metering tank to provide a positive means of calibration for the orifice.

Instrumentation was kept as simple as possible by using a yaw-meter/pitot tube arrangement to provide data on speed and direction of flow at cross sections corresponding to each of the perspex windows on the test section. Differential pressures for this and the orifice were measured with mercury manometer.

4.3 CONDUCT OF EXPERIMENTS

Two series of experiments were carried out, the first to determine the growth of swirl in the pipe and the second to examine the possible deviation of the meter reading from that in standard conditions.

In the first set of experiments the work consisted of determining the flow profiles at the different windows in the test length using the following procedure. First the direction of flow was found using the diametrical probe as a yaw meter (ie, with the static tapping blanked off). The probe was then rotated through 35° so as to present the hole corresponding to the common tapping to the now established direction of flow. The other hole was blanked off and the manometer leg formerly connected to it was connected to the static pressure tapping - thus converting the probe into a pitot tube to measure the flow velocity.

This procedure enabled the superior direction finding characteristics of the yaw-meter to be used to assist in the determination of the resultant flow velocities.

The second series of experiments consisted simply of a calibration of the orifice installation at various flow rates using the metering tanks.

CHAPTER 5

DISCUSSION OF RESULTS

5.1 SWIRL GROWTH

The results of a series of test runs conducted to examine the growth of swirl in the flow at different Reynolds Numbers is displayed in the accompanying four graphs (figures 12 - 15). The growth of swirl in the longitudinal and radial directions is shown in figures 12 and 13. Figure 14 shows the radial swirl distributions for a variety of Reynolds Numbers. Over the restricted range investigated, swirl appears to be independent of Reynolds Number.

In Reference 31, flow meters are shown to be affected by the axial flow profile as well as swirl. Figure 15 was plotted to show this profile. The skew pattern exhibited was found to increase along the test length and, as can be seen in the figure, was more pronounced at lower Reynolds Number.

Figure 13 shows the radial distribution of swirl at various cross sections of the pipe. It exhibits a comparatively slow growth of swirl in the centre of the pipe despite the rapid rise in the outer annulus. The actual growth of swirl was considerably more than that predicted by equation (2.4.1) using the results displayed for $r/R=0.95$ in figure 12 as a boundary condition. The equation was found to be very sensitive to changes in κ and the plot shown was based on the value of 0,0333 of Rochino

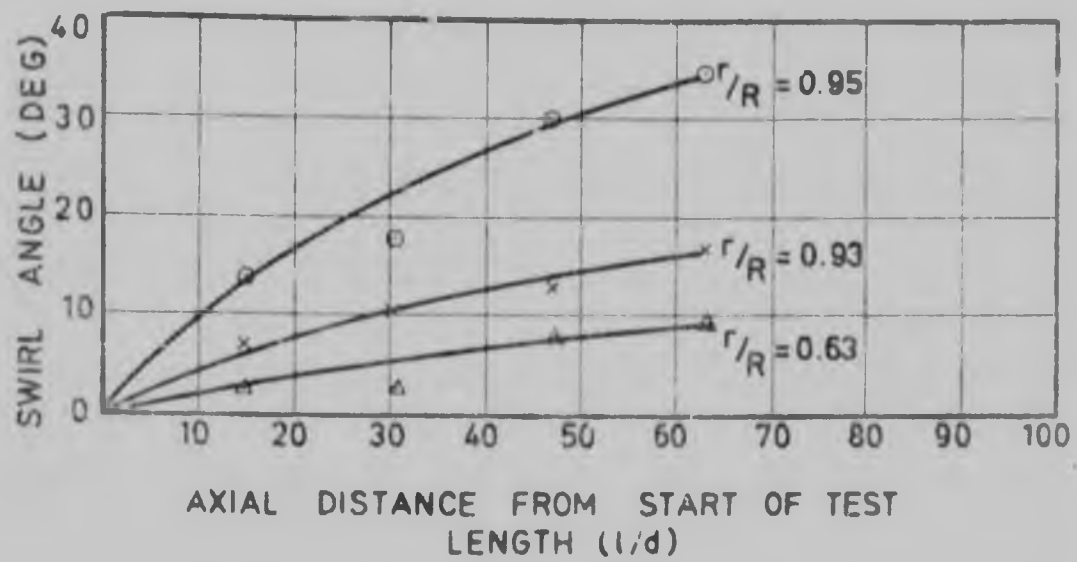


Figure 12: AXIAL SWIRL GROWTH: $Re = 3.47 \times 10^5$

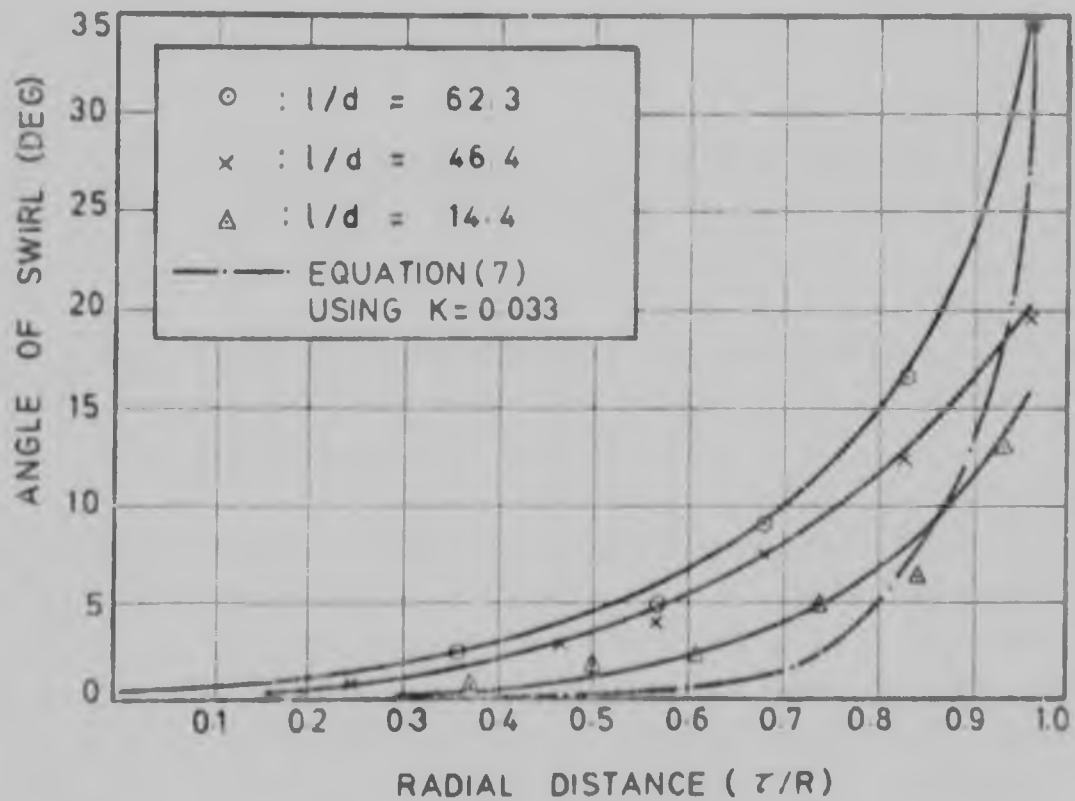


Figure 13: RADIAL DISTRIBUTION OF SWIRL FOR VARYING DOWNSTREAM POSITIONS: $Re = 3.47 \times 10^5$

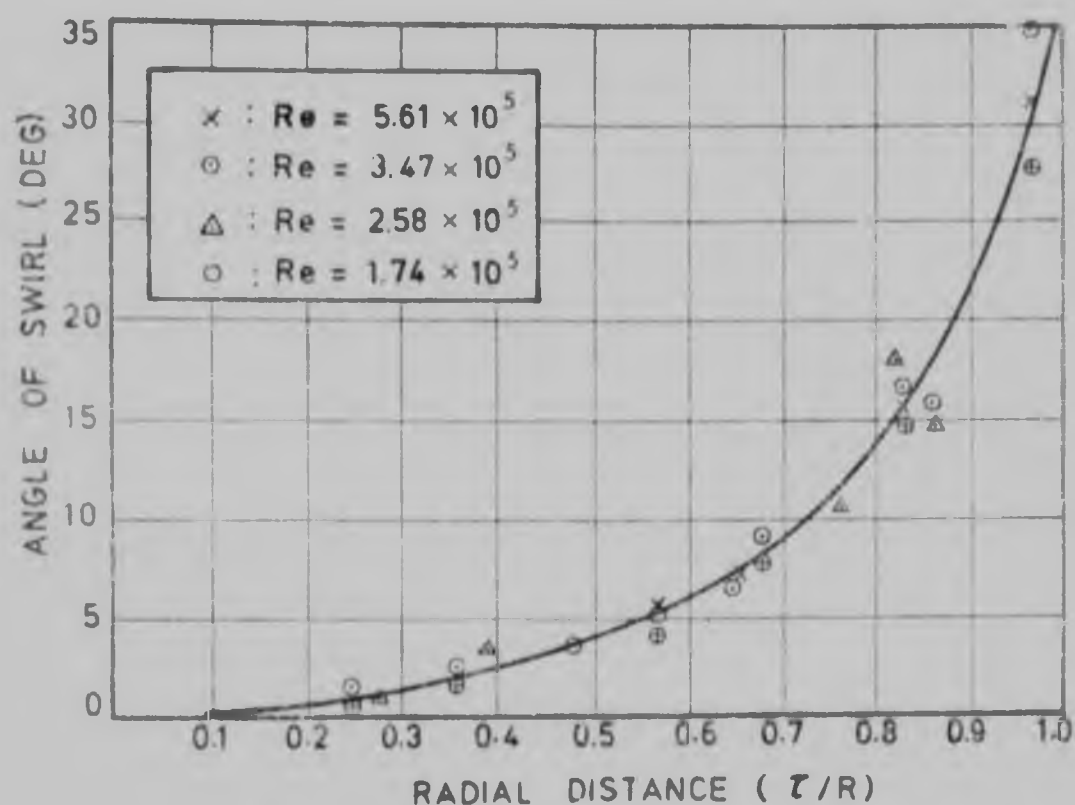


Figure 14: RADIAL DISTRIBUTION OF SWIRL FOR VARYING REYNOLDS NUMBER! $\tau/d = 62.3$

and Lavan (23). It would appear therefore that additional work is required to confirm the accuracy of the value for this flow pattern.

The axial plot of swirl growth in figure 12 demonstrates that the swirl builds up slowly over a considerable length of pipe and the 60 pipe diameters available on the test rig were insufficient to allow maximum swirl to develop. Extrapolation of the curves would imply that 100 or more diameters are required before the axial swirl gradient vanishes.

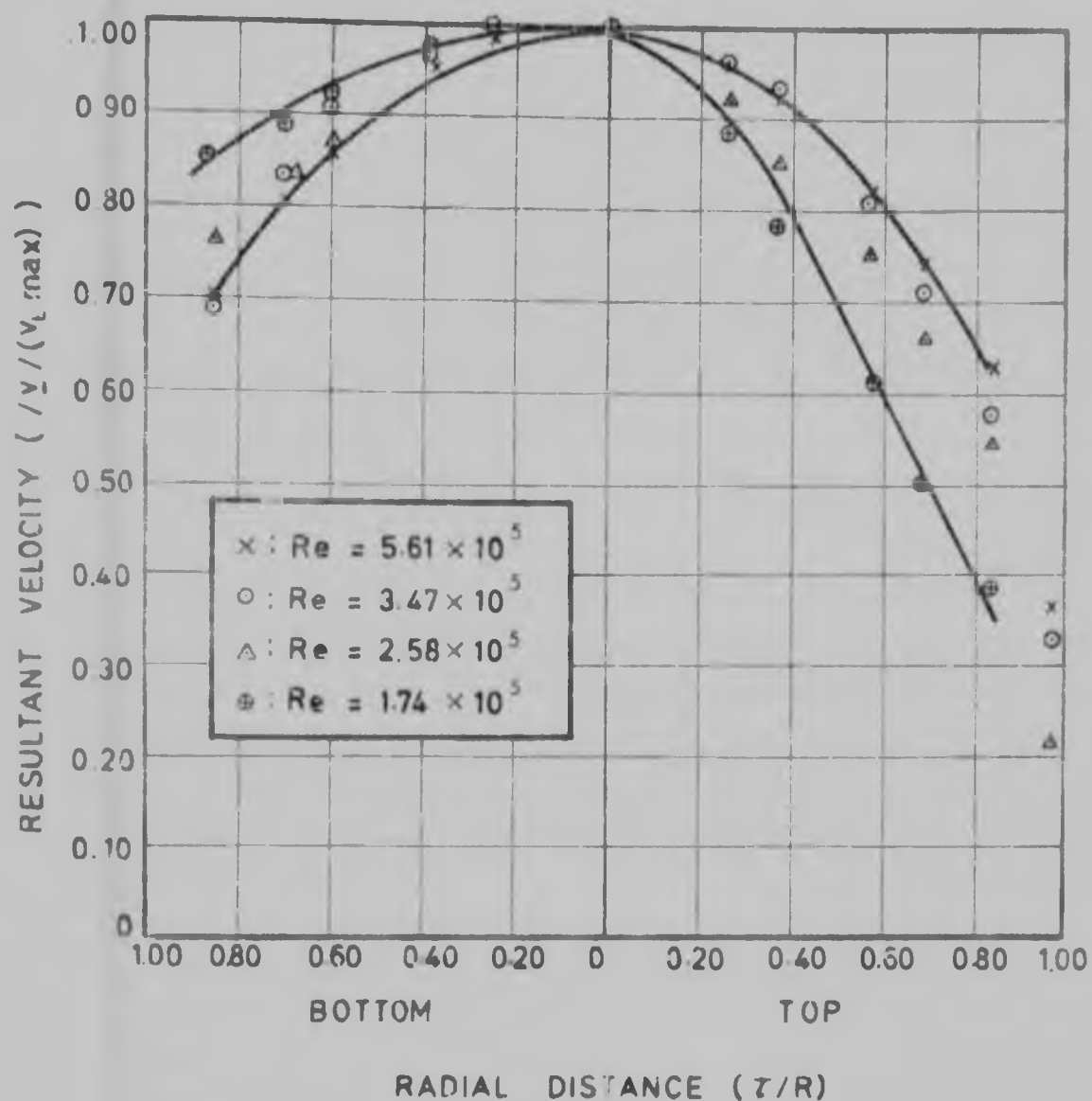


Figure 13: VELOCITY PROFILES FOR VARYING REYNOLDS NUMBER:
 $r/d = 62.3$

The apparent independence of swirl growth and Reynolds Number depicted in figure 14 confirms one aspect of equations (2.4.1) and (2.4.2). From (2.4.2) it can be seen that $\mathcal{E}$ is of the order of magnitude of Re so that at high Reynolds Numbers omission of the term $\frac{1}{Re}$ in (2.4.1) would not materially affect its solution.

The skew velocity profile in figure 15 has no immediately obvious explanation. The profile became more skew along the test length and was also more pronounced at lower Reynolds Number. Since the skewness was found in the vertical plane it could be caused by some gravitational effect but further investigation is required to examine this aspect. The skew velocity profile must, however, be a secondary source of error in the orifice meter.

5.2 EFFECTS ON METER

After an initial calibration of the meter in its installed position using straight seamless pipe in the test length, a comparison was made between the metered flowrate as measured at the orifice and the actual flowrate as determined by use of the metering tank. Table 1 presents this comparison for the range of flowrates measured.

The error, amounting to an average of -12.3% with a spread of +1.8% and -2.1% which is introduced into the meter is clearly considerable. Although the swirl was independent of Reynolds Number the skewness and variation in velocity profile must be partly responsible for the size of spread. Errors of similar magnitude were reported by Lugt (15). He examined orifices of varying area ratios and found that the area ratio had considerable influence on the error. The meter used for the experimental work had an area ratio of $m = 0.404$. It would therefore seem that the above work might usefully be extended to find the effect of the flow pattern under consideration on orifice plates of other area ratios. Of more importance, however, is the need to determine a suitable settling length for the induced swirl since it introduces error of unacceptable magnitude.

Table 1: ORIFICE METERING ERROR DUE TO PRESENCE OF WIRE
HELIX

Actual Flowrate l/s	Linear Flowrate m/s	Reynolds Number $\times 10^5$	Metered Flowrate l/s	Meter Error %
12.25	1.77	2.38	10.71	-12.4
15.45	2.23	3.00	13.31	-13.7
16.80	2.42	3.26	14.98	-10.9
18.65	2.69	3.61	16.20	-13.1
20.30	2.93	3.94	17.41	-14.1
21.90	3.16	4.25	19.22	-12.1
26.05	3.76	5.05	23.00	-11.9
28.90	4.16	5.61	25.95	-10.2

CHAPTER 6

CONCLUSIONS

Although work has been conducted on the use of helical swirl inducers (14) designed specifically to promote a swirling flow, little work has been done to investigate the effects of the small helical surface deformations such as in a helically welded pipe. The present work investigates some aspects of this phenomenon. It demonstrates that even on this scale a significant amount of swirl can be generated and that this can affect the accurate performance of an orifice plate flow meter. Further work is required to determine suitable settling lengths for damping this type of swirl but in the absence of such information it is recommended that the use of helically welded pipe in connection with metering installations be avoided.

APPENDIX A

STABILITY ANALYSIS OF SWIRL EQUATION

For the purpose of this section we express the equation as

$$\beta_j^n = \beta_j^{n-1} \left(1 - B \lambda \left(\frac{2}{R^2} - \frac{1}{rR} \right) \right) + \beta_{j+1}^{n-1} B \lambda \left(\frac{1}{R^2} - \frac{1}{rR} \right) + \beta_{j-1}^{n-1} \cdot \frac{B \lambda}{R^2} \quad (A.1)$$

where $B = \frac{1}{\bar{u}_2(r)} \left[\frac{1}{R^2} + \frac{\kappa^2 \lambda^2}{r} \left(\frac{\beta_{j+1}^{n-1}}{\kappa} - \beta_j^{n-1} \left(\frac{1}{R} - \frac{2}{r} \right) \right) \right]$

let $\tau = B \lambda \left(\frac{2}{R^2} - \frac{1}{rR} \right)$, $\gamma = B \lambda \left(\frac{1}{R^2} - \frac{1}{rR} \right)$, $\phi = \frac{B \lambda}{R^2}$

Then $\beta_j^n = \beta_j^{n-1} (1 - \tau) + \beta_{j+1}^{n-1} \gamma + \beta_{j-1}^{n-1} \phi \quad (A.2)$

Let $\beta_j^n = \sum_j \psi^n e^{ij\kappa}$ be a Fourier Series.

Then (A.2) reduces to

$$\sum_j \psi^n e^{ij\kappa} = (1 - \tau) \sum_j \psi^{n-1} e^{ij\kappa} + \gamma \sum_j \psi^{n-1} e^{i\kappa(j+1)} + \phi \sum_j \psi^{n-1} e^{i\kappa(j-1)}$$

$$\text{or } \sum_j \psi^{n-1} e^{ij\kappa} (\psi - (1 - \tau) - \gamma e^{i\kappa} - \phi e^{-i\kappa}) = 0$$

$$\text{or } \psi = 1 - \tau + \gamma e^{i\kappa} + \phi e^{-i\kappa}$$

$$= 1 - \tau + (\gamma + \phi) \cos \kappa + (\gamma - \phi) \sin \kappa \quad (A.3)$$

Von Neumann's necessary (but not sufficient) condition for stability is $|\psi| \leq 1$. Taking the worst case of (A.3)

$$R = \pi ; \cos R = -1 , \sin R = 0$$

$$\psi = 1 - (\tau + \gamma + \phi) = 1 - 2\tau$$

$$\text{so } |1 - 2\tau| \leq 1$$

$$\text{or } 0 \leq 2\tau \leq 2 \quad \text{ie. } 0 \leq \tau \leq 1$$

Hence a necessary condition for stability is

$$0 \leq Bk \left(\frac{2}{R^2} - \frac{1}{rR} \right) \leq 1$$

This expression is always the same sign as B , the sign of which is negative only in areas of flow reversals in the mean flow.

APPENDIX R

SOLUTION OF AXIAL SWIRL EQUATION

The axial swirl equation is as follows

$$\frac{1}{r} \cdot 2 \bar{v}_\theta \cdot \frac{\partial \bar{v}_\theta}{\partial z} = \frac{(\varepsilon+1)}{Re} \left[\frac{\partial^3 \bar{v}_z}{\partial r^3} + \frac{1}{r} \frac{\partial^2 \bar{v}_z}{\partial r^2} - \frac{1}{r^2} \cdot \frac{\partial \bar{v}_z}{\partial r} \right] + \frac{1}{Re} \cdot \frac{\partial \varepsilon}{\partial r} \left(\frac{\partial^2 \bar{v}_z}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \bar{v}_z}{\partial r} \right) \quad . . . (B.1a)$$

where for $0 \leq r \leq 0.9$ $\lambda = r$ so that

$$\frac{1}{Re \kappa^2} \cdot \frac{\partial \varepsilon}{\partial r} = r^2 \cdot \frac{\partial^2 \bar{v}_\theta}{\partial r^2} + r \frac{\partial \bar{v}_\theta}{\partial r} - \bar{v}_\theta \quad . . . (B.1b)$$

and for $0.9 < r \leq 1.0$ $\lambda = 1-r$ so that

$$\frac{1}{Re \kappa^2} \cdot \frac{\partial \varepsilon}{\partial r} = \frac{\partial^2 \bar{v}_\theta}{\partial r^2} (1-r)^2 + \frac{\partial \bar{v}_\theta}{\partial r} (r - \frac{1}{r}) + \bar{v}_\theta \left(\frac{1}{r^2} - 1 \right) \quad . . (B.1c)$$

Hence for both values of λ we have expressions for $\frac{1}{Re} \cdot \frac{\partial \varepsilon}{\partial r}$ which are functions of r and z though the z -dependency is only via $\bar{v}_\theta(r, z)$.

The solution of the axial equation uses the same grid format as that of the tangential equation.

The notation for the (i)th iterative value of $\frac{\partial \bar{v}_z}{\partial r}$ at the grid point n, j is $\bar{v}_z'{}^{(i)}$ where additional primes denote further differentiation w.r.t. r .

As far as boundary conditions are concerned, we can assume derivatives of $\bar{v}_z$ w.r.t. z are small, so for short inter-

vals along the pipe axis $\bar{v}_z$ and its derivatives w.r.t. r will be constant. By making this assumption along the centre line for one step, we can then make a pass in the radial direction obtaining the full velocity profile shape. An integration of the velocity profile will determine any violation of the continuity of the flow and a readjustment to the value of $\bar{v}_z$ on the centre line can be made before a final radial pass.

We know the initial velocity profile; the flow at the outside of the pipe is determined from experiment; and the velocity at the centre line is determined by the above procedure. The situation for derivatives is the same so we have adequate boundary conditions for the equation.

From the tangential equation we have a complete array for $\bar{v}_\theta$ so that values of all derivatives of $\bar{v}_\theta$ and hence of $F(r, z)$, $P(r, z)$ and $Q(r, z)$ are known. Rather than store large arrays for these variables, they will be calculated as and when required.

Equation (B.1) can be written as

$$\frac{\partial^3 \bar{v}_z}{\partial r^3} - P(r, z) \frac{\partial^2 \bar{v}_z}{\partial r^2} - Q(r, z) \frac{\partial \bar{v}_z}{\partial r} - F(r, z) = 0 \quad . \quad . \quad (B.2)$$

where P , Q and F are functions known and defined by 2.4.8 and 2.4.9.

The method of solution follows that given by McCracken and Dorn (17) (on page 390) since we can regard (B.2) as an ordinary differential equation for $\bar{v}_z$ w.r.t. r . Each new step in the axial direction will be treated as a new O.D.E. The major logic steps are shown in figure 9. The basic logic is as follows:

1. V_θ array determined from Tangential Equation.
2. $V_z(N,1)$ (ie, on $\frac{1}{2}$) is boundary condition.
3. $V_z(N,2)$ is calculated by Modified Euler Method from $V_z(N,1)$.
4. $V_z(N,j)$ $j=3 \dots 21$ is predicted using $V_z(N,j-1)$ and $V_z(N,j-2)$.
5. $V_z(N,j)$ is corrected iteratively until there is satisfactory convergence.
6. $V_z(N,j)$ has its error corrected.
7. When the full velocity profile is determined this is integrated and compared with the total flow.
8. If necessary $V_z(N,1)$ is corrected and we return to step 2.
9. This procedure is then repeated for the next step in the axial direction.

Steps 3, 4, 5 and 6 require to be examined in more detail before actual programming.

MODIFIED EULER METHOD

For a third order equation the modified Euler Method provides the following equations

$${}^n \bar{v}_2 = {}^n \bar{v}_2 + \Delta v {}^n \bar{v}_2' + \frac{(\Delta v)^2}{2} {}^n \bar{v}_2'' \quad \dots \quad (B.3a)$$

$${}^n \bar{v}_2' = {}^n \bar{v}_2' + \Delta v {}^n \bar{v}_2'' + \frac{(\Delta v)^2}{2} g(0, {}^n \bar{v}_2, {}^n \bar{v}_2''). \quad \dots \quad (B.3b)$$

$$\begin{aligned} \frac{1}{2} \bar{v}_2'' = \frac{1}{2} \bar{v}_2'' + \frac{\Delta r}{2} [g(0, \frac{1}{2} \bar{v}_2', \frac{1}{2} \bar{v}_2'') \\ + g(\Delta r, \frac{1}{2} \bar{v}_2' + \Delta r \frac{1}{2} \bar{v}_2'', \frac{1}{2} \bar{v}_2'' + \Delta r g(0, \frac{1}{2} \bar{v}_2', \frac{1}{2} \bar{v}_2'')))] \end{aligned} \quad (B.3c)$$

where $g(r, \bar{v}_2', \bar{v}_2'') = P(r, z) \bar{v}_2'' + Q(r, z) \bar{v}_2' + F(r, z)$

In the computer programme equation (B.3c) has been represented as $\frac{1}{2} \bar{v}_2'' = \frac{1}{2} \bar{v}_2'' + \frac{\Delta r}{2} (g + G)$ and for ease of programming we have

$$X = \kappa^2 \left(\frac{\partial \bar{v}_0}{\partial r} - \frac{\bar{v}_0}{r} \right) \text{ and } XA = X + \kappa^2 r \frac{\partial^2 \bar{v}_0}{\partial r^2} \quad (B.4)$$

which enables us to write

$$g(r, \bar{v}_2', \bar{v}_2'') = \frac{\left(\frac{-1}{r} - rXA \right) \bar{v}_2'' + \left(\frac{1}{r^2} - XA \right) \bar{v}_2' + 2 \frac{\bar{v}_0}{r} \frac{\partial \bar{v}_0}{\partial z}}{\left(\frac{1}{r^2} + r^2 X \right)} \quad (B.5)$$

PREDICTOR STAGE

Once we have $\nabla z(N,1)$ and $\nabla z(N,2)$ we can use the predictor-corrector method. The predictor stage provides first approximations to $\tilde{\bar{v}}_2''^{(0)}$, $\tilde{\bar{v}}_2'^{(0)}$ and $\tilde{\bar{v}}_2^{(0)}$

For the third order equation we have:

$$\tilde{\bar{v}}_2''^{(0)} = \tilde{\bar{v}}_2'' + 2 \Delta r g((j-2)\Delta r, \tilde{\bar{v}}_2', \tilde{\bar{v}}_2'') \quad (B.6a)$$

$$\tilde{\bar{v}}_2'^{(0)} = \tilde{\bar{v}}_2' + 2 \Delta r \tilde{\bar{v}}_2'' \quad (B.6b)$$

$$\tilde{\bar{v}}_2^{(0)} = \tilde{\bar{v}}_2 + 2 \Delta r \tilde{\bar{v}}_2' \quad (B.6c)$$

Again for ease of programming we have introduced the following notation

$$\tilde{v}_2^{(0)} = VZD2RP \quad \tilde{v}_2^{(1)} = VZD2RA$$

$\tilde{v}_2^{(2)} = VZD2RB$ etc as well as intermediate variables XP and XAP analogous to X and XA above.

CORRECTOR STAGE

The corrector stage can be repeated as often as necessary as an iterative process to improve the values $\tilde{v}_2^{(0)}$ etc obtained from the predictor stage.

For the third order equation we have

$$\begin{aligned} \tilde{v}_2^{(i+1)} = \tilde{v}_2^{(i)} + \frac{\Delta v}{2} \left[g((j-2)\Delta v, \tilde{v}_2^{(i)}, \tilde{v}_2^{(i)}) \right. \\ \left. + g((j-1)\Delta v, \tilde{v}_2^{(i)}, \tilde{v}_2^{(i)}) \right] \end{aligned} \quad (B.7a)$$

$$\tilde{v}_2^{(i+1)} = \tilde{v}_2^{(i)} + \frac{\Delta v}{2} \left[\tilde{v}_2^{(i)} + \tilde{v}_2^{(i)} \right] \quad (B.7b)$$

$$\tilde{v}_2^{(i+1)} = \tilde{v}_2^{(i)} + \frac{\Delta v}{2} \left[\tilde{v}_2^{(i)} + \tilde{v}_2^{(i)} \right] \quad (B.7c)$$

For programming purposes

$$\tilde{v}_2^{(i+1)} = VZD2RC \quad \text{and} \quad \tilde{v}_2^{(i)} = VZD2RX \quad \text{etc}$$

ERROR CORRECTION

After convergence of $\tilde{v}_2^{(i+1)}$ has been obtained we can

then add the truncation error back onto it and its derivatives by the following equations

$$\tilde{v}_2 = \tilde{v}_2^{(i+1)} + \frac{1}{5} (\tilde{v}_2^{(i)} - \tilde{v}_2^{(i+1)}) \quad . \quad . \quad . \quad (B.8)$$

and two similar equations for the two derivatives.

APPENDIX C

LIST AND EXPLANATION OF VARIABLES USED IN THE COMPUTER PROGRAMME

The variables are listed alphabetically within the main sections of the programme for which they are used.

1. Main Variables

DELAR	= Radial Mesh width Δr
DELZED	= Axial Mesh width Δz
J	= Radial Mesh co-ordinate
N	= Axial Mesh co-ordinate
VEZED (501, 21)	= $\bar{v}_z$
VTHETA (501, 21)	= $\bar{v}_\theta$
VZD1R	= $\partial \bar{v}_z / \partial r$
VZD2R	= $\partial^2 \bar{v}_z / \partial r^2$
Z(501)	= z

2. Tangential Equation

ADASH	= Intermediate variable
BETA (501, 21)	= β
BOUND (501)	= Value of β for $r/R=0.95$
F	= Intermediate variable
P	= Intermediate variable
Q	= Intermediate variable
Y	= Intermediate variable

3. Axial Equation - Modified Euler Method Section

BIGG	= Intermediate variable
LITG	= Intermediate variable
VZD1RA	=
VZD1RB	= $\frac{\partial \bar{v}_z}{\partial r}$
VZD1RN	=
VZD2RA	=
VZD2RB	= $\frac{\partial^2 \bar{v}_z}{\partial r^2}$
VZD2RN	=
X	= Intermediate variable
XA	= Intermediate variable

4. Axial Equation - Predictor Section

VZD1RP	= $\frac{\partial \bar{v}_z}{\partial r}$
VZD1RX	=
VZD2RP	= $\frac{\partial^2 \bar{v}_z}{\partial r^2}$
VZD2RX	=
VZP	= $\bar{v}_z$
VZX	=
XAP	= Intermediate variable
XP	= Intermediate variable

5. Axial Equation - Corrector Section

VZD1RC	= $\frac{\partial \bar{v}_z}{\partial r}$
VZD2RC	= $\frac{\partial^2 \bar{v}_z}{\partial r^2}$
VZC	= $\bar{v}_z$
XAC	= Intermediate variable
XC	= Intermediate variable

6. Continuity Equation

FLOW	= Pipe flow (actual)
FLOWN	= Pipe flow (uncorrected)
FLOWR	= Flow in annular ring of width Δr

7. Control Variables

DIFBTA	= Max change in β between iterations
EPSA	= ϵ_A Convergence test
EPSB	= ϵ_B Convergence test
EPSC	= ϵ_C Convergence test
ITNA	= Iteration counter
ITNB	= Iteration counter
ITNC	= Iteration counter
MAXIT	= Maximum number of iterations

8. Sundry

KAPPA	= κ , Kinney's constant
PITCH	= Helix pitch
REYNO	= Re, Reynolds number
SWIRL	= Angular velocity as $\omega \rightarrow \infty$
WIRED	= Wire insert diameter

APPENDIX D

COMPUTER PROGRAMME


```

0041      WRITE(10,*) Y, X, Z
0042      IF (X.EQ.0.0) THEN
0043          X = 1.0
0044          Y = 1.0
0045          Z = 1.0
0046      ELSE
0047          X = 1.0
0048          Y = 1.0
0049          Z = 1.0
0050      END IF
0051      IF (X.EQ.0.0) THEN
0052          X = 1.0
0053          Y = 1.0
0054          Z = 1.0
0055      ELSE
0056          X = 1.0
0057          Y = 1.0
0058          Z = 1.0
0059      END IF
0060      IF (X.EQ.0.0) THEN
0061          X = 1.0
0062          Y = 1.0
0063          Z = 1.0
0064      ELSE
0065          X = 1.0
0066          Y = 1.0
0067          Z = 1.0
0068      END IF
0069      IF (X.EQ.0.0) THEN
0070          X = 1.0
0071          Y = 1.0
0072          Z = 1.0
0073      ELSE
0074          X = 1.0
0075          Y = 1.0
0076          Z = 1.0
0077      END IF
0078      IF (X.EQ.0.0) THEN
0079          X = 1.0
0080          Y = 1.0
0081          Z = 1.0
0082      ELSE
0083          X = 1.0
0084          Y = 1.0
0085          Z = 1.0
0086      END IF
0087      IF (X.EQ.0.0) THEN
0088          X = 1.0
0089          Y = 1.0
0090          Z = 1.0
0091      ELSE
0092          X = 1.0
0093          Y = 1.0
0094          Z = 1.0
0095      END IF
0096      IF (X.EQ.0.0) THEN
0097          X = 1.0
0098          Y = 1.0
0099          Z = 1.0
0100      ELSE
0101          X = 1.0
0102          Y = 1.0
0103          Z = 1.0
0104      END IF
0105      IF (X.EQ.0.0) THEN
0106          X = 1.0
0107          Y = 1.0
0108          Z = 1.0
0109      ELSE
0110          X = 1.0
0111          Y = 1.0
0112          Z = 1.0
0113      END IF
0114      IF (X.EQ.0.0) THEN
0115          X = 1.0
0116          Y = 1.0
0117          Z = 1.0
0118      ELSE
0119          X = 1.0
0120          Y = 1.0
0121          Z = 1.0
0122      END IF
0123      IF (X.EQ.0.0) THEN
0124          X = 1.0
0125          Y = 1.0
0126          Z = 1.0
0127      ELSE
0128          X = 1.0
0129          Y = 1.0
0130          Z = 1.0
0131      END IF
0132      IF (X.EQ.0.0) THEN
0133          X = 1.0
0134          Y = 1.0
0135          Z = 1.0
0136      ELSE
0137          X = 1.0
0138          Y = 1.0
0139          Z = 1.0
0140      END IF
0141      IF (X.EQ.0.0) THEN
0142          X = 1.0
0143          Y = 1.0
0144          Z = 1.0
0145      ELSE
0146          X = 1.0
0147          Y = 1.0
0148          Z = 1.0
0149      END IF
0150      IF (X.EQ.0.0) THEN
0151          X = 1.0
0152          Y = 1.0
0153          Z = 1.0
0154      ELSE
0155          X = 1.0
0156          Y = 1.0
0157          Z = 1.0
0158      END IF
0159      IF (X.EQ.0.0) THEN
0160          X = 1.0
0161          Y = 1.0
0162          Z = 1.0
0163      ELSE
0164          X = 1.0
0165          Y = 1.0
0166          Z = 1.0
0167      END IF
0168      IF (X.EQ.0.0) THEN
0169          X = 1.0
0170          Y = 1.0
0171          Z = 1.0
0172      ELSE
0173          X = 1.0
0174          Y = 1.0
0175          Z = 1.0
0176      END IF
0177      IF (X.EQ.0.0) THEN
0178          X = 1.0
0179          Y = 1.0
0180          Z = 1.0
0181      ELSE
0182          X = 1.0
0183          Y = 1.0
0184          Z = 1.0
0185      END IF
0186      IF (X.EQ.0.0) THEN
0187          X = 1.0
0188          Y = 1.0
0189          Z = 1.0
0190      ELSE
0191          X = 1.0
0192          Y = 1.0
0193          Z = 1.0
0194      END IF
0195      IF (X.EQ.0.0) THEN
0196          X = 1.0
0197          Y = 1.0
0198          Z = 1.0
0199      ELSE
0200          X = 1.0
0201          Y = 1.0
0202          Z = 1.0
0203      END IF
0204      IF (X.EQ.0.0) THEN
0205          X = 1.0
0206          Y = 1.0
0207          Z = 1.0
0208      ELSE
0209          X = 1.0
0210          Y = 1.0
0211          Z = 1.0
0212      END IF
0213      IF (X.EQ.0.0) THEN
0214          X = 1.0
0215          Y = 1.0
0216          Z = 1.0
0217      ELSE
0218          X = 1.0
0219          Y = 1.0
0220          Z = 1.0
0221      END IF
0222      IF (X.EQ.0.0) THEN
0223          X = 1.0
0224          Y = 1.0
0225          Z = 1.0
0226      ELSE
0227          X = 1.0
0228          Y = 1.0
0229          Z = 1.0
0230      END IF
0231      IF (X.EQ.0.0) THEN
0232          X = 1.0
0233          Y = 1.0
0234          Z = 1.0
0235      ELSE
0236          X = 1.0
0237          Y = 1.0
0238          Z = 1.0
0239      END IF
0240      IF (X.EQ.0.0) THEN
0241          X = 1.0
0242          Y = 1.0
0243          Z = 1.0
0244      ELSE
0245          X = 1.0
0246          Y = 1.0
0247          Z = 1.0
0248      END IF
0249      IF (X.EQ.0.0) THEN
0250          X = 1.0
0251          Y = 1.0
0252          Z = 1.0
0253      ELSE
0254          X = 1.0
0255          Y = 1.0
0256          Z = 1.0
0257      END IF
0258      IF (X.EQ.0.0) THEN
0259          X = 1.0
0260          Y = 1.0
0261          Z = 1.0
0262      ELSE
0263          X = 1.0
0264          Y = 1.0
0265          Z = 1.0
0266      END IF
0267      IF (X.EQ.0.0) THEN
0268          X = 1.0
0269          Y = 1.0
0270          Z = 1.0
0271      ELSE
0272          X = 1.0
0273          Y = 1.0
0274          Z = 1.0
0275      END IF
0276      IF (X.EQ.0.0) THEN
0277          X = 1.0
0278          Y = 1.0
0279          Z = 1.0
0280      ELSE
0281          X = 1.0
0282          Y = 1.0
0283          Z = 1.0
0284      END IF
0285      IF (X.EQ.0.0) THEN
0286          X = 1.0
0287          Y = 1.0
0288          Z = 1.0
0289      ELSE
0290          X = 1.0
0291          Y = 1.0
0292          Z = 1.0
0293      END IF
0294      IF (X.EQ.0.0) THEN
0295          X = 1.0
0296          Y = 1.0
0297          Z = 1.0
0298      ELSE
0299          X = 1.0
0300          Y = 1.0
0301          Z = 1.0
0302      END IF
0303      IF (X.EQ.0.0) THEN
0304          X = 1.0
0305          Y = 1.0
0306          Z = 1.0
0307      ELSE
0308          X = 1.0
0309          Y = 1.0
0310          Z = 1.0
0311      END IF
0312      IF (X.EQ.0.0) THEN
0313          X = 1.0
0314          Y = 1.0
0315          Z = 1.0
0316      ELSE
0317          X = 1.0
0318          Y = 1.0
0319          Z = 1.0
0320      END IF
0321      IF (X.EQ.0.0) THEN
0322          X = 1.0
0323          Y = 1.0
0324          Z = 1.0
0325      ELSE
0326          X = 1.0
0327          Y = 1.0
0328          Z = 1.0
0329      END IF
0330      IF (X.EQ.0.0) THEN
0331          X = 1.0
0332          Y = 1.0
0333          Z = 1.0
0334      ELSE
0335          X = 1.0
0336          Y = 1.0
0337          Z = 1.0
0338      END IF
0339      IF (X.EQ.0.0) THEN
0340          X = 1.0
0341          Y = 1.0
0342          Z = 1.0
0343      ELSE
0344          X = 1.0
0345          Y = 1.0
0346          Z = 1.0
0347      END IF
0348      IF (X.EQ.0.0) THEN
0349          X = 1.0
0350          Y = 1.0
0351          Z = 1.0
0352      ELSE
0353          X = 1.0
0354          Y = 1.0
0355          Z = 1.0
0356      END IF
0357      IF (X.EQ.0.0) THEN
0358          X = 1.0
0359          Y = 1.0
0360          Z = 1.0
0361      ELSE
0362          X = 1.0
0363          Y = 1.0
0364          Z = 1.0
0365      END IF
0366      IF (X.EQ.0.0) THEN
0367          X = 1.0
0368          Y = 1.0
0369          Z = 1.0
0370      ELSE
0371          X = 1.0
0372          Y = 1.0
0373          Z = 1.0
0374      END IF
0375      IF (X.EQ.0.0) THEN
0376          X = 1.0
0377          Y = 1.0
0378          Z = 1.0
0379      ELSE
0380          X = 1.0
0381          Y = 1.0
0382          Z = 1.0
0383      END IF
0384      IF (X.EQ.0.0) THEN
0385          X = 1.0
0386          Y = 1.0
0387          Z = 1.0
0388      ELSE
0389          X = 1.0
0390          Y = 1.0
0391          Z = 1.0
0392      END IF
0393      IF (X.EQ.0.0) THEN
0394          X = 1.0
0395          Y = 1.0
0396          Z = 1.0
0397      ELSE
0398          X = 1.0
0399          Y = 1.0
0400          Z = 1.0
0401      END IF
0402      IF (X.EQ.0.0) THEN
0403          X = 1.0
0404          Y = 1.0
0405          Z = 1.0
0406      ELSE
0407          X = 1.0
0408          Y = 1.0
0409          Z = 1.0
0410      END IF
0411      IF (X.EQ.0.0) THEN
0412          X = 1.0
0413          Y = 1.0
0414          Z = 1.0
0415      ELSE
0416          X = 1.0
0417          Y = 1.0
0418          Z = 1.0
0419      END IF
0420      IF (X.EQ.0.0) THEN
0421          X = 1.0
0422          Y = 1.0
0423          Z = 1.0
0424      ELSE
0425          X = 1.0
0426          Y = 1.0
0427          Z = 1.0
0428      END IF
0429      IF (X.EQ.0.0) THEN
0430          X = 1.0
0431          Y = 1.0
0432          Z = 1.0
0433      ELSE
0434          X = 1.0
0435          Y = 1.0
0436          Z = 1.0
0437      END IF
0438      IF (X.EQ.0.0) THEN
0439          X = 1.0
0440          Y = 1.0
0441          Z = 1.0
0442      ELSE
0443          X = 1.0
0444          Y = 1.0
0445          Z = 1.0
0446      END IF
0447      IF (X.EQ.0.0) THEN
0448          X = 1.0
0449          Y = 1.0
0450          Z = 1.0
0451      ELSE
0452          X = 1.0
0453          Y = 1.0
0454          Z = 1.0
0455      END IF
0456      IF (X.EQ.0.0) THEN
0457          X = 1.0
0458          Y = 1.0
0459          Z = 1.0
0460      ELSE
0461          X = 1.0
0462          Y = 1.0
0463          Z = 1.0
0464      END IF
0465      IF (X.EQ.0.0) THEN
0466          X = 1.0
0467          Y = 1.0
0468          Z = 1.0
0469      ELSE
0470          X = 1.0
0471          Y = 1.0
0472          Z = 1.0
0473      END IF
0474      IF (X.EQ.0.0) THEN
0475          X = 1.0
0476          Y = 1.0
0477          Z = 1.0
0478      ELSE
0479          X = 1.0
0480          Y = 1.0
0481          Z = 1.0
0482      END IF
0483      IF (X.EQ.0.0) THEN
0484          X = 1.0
0485          Y = 1.0
0486          Z = 1.0
0487      ELSE
0488          X = 1.0
0489          Y = 1.0
0490          Z = 1.0
0491      END IF
0492      IF (X.EQ.0.0) THEN
0493          X = 1.0
0494          Y = 1.0
0495          Z = 1.0
0496      ELSE
0497          X = 1.0
0498          Y = 1.0
0499          Z = 1.0
0500      END IF
0501      IF (X.EQ.0.0) THEN
0502          X = 1.0
0503          Y = 1.0
0504          Z = 1.0
0505      ELSE
0506          X = 1.0
0507          Y = 1.0
0508          Z = 1.0
0509      END IF
0510      IF (X.EQ.0.0) THEN
0511          X = 1.0
0512          Y = 1.0
0513          Z = 1.0
0514      ELSE
0515          X = 1.0
0516          Y = 1.0
0517          Z = 1.0
0518      END IF
0519      IF (X.EQ.0.0) THEN
0520          X = 1.0
0521          Y = 1.0
0522          Z = 1.0
0523      ELSE
0524          X = 1.0
0525          Y = 1.0
0526          Z = 1.0
0527      END IF
0528      IF (X.EQ.0.0) THEN
0529          X = 1.0
0530          Y = 1.0
0531          Z = 1.0
0532      ELSE
0533          X = 1.0
0534          Y = 1.0
0535          Z = 1.0
0536      END IF
0537      IF (X.EQ.0.0) THEN
0538          X = 1.0
0539          Y = 1.0
0540          Z = 1.0
0541      ELSE
0542          X = 1.0
0543          Y = 1.0
0544          Z = 1.0
0545      END IF
0546      IF (X.EQ.0.0) THEN
0547          X = 1.0
0548          Y = 1.0
0549          Z = 1.0
0550      ELSE
0551          X = 1.0
0552          Y = 1.0
0553          Z = 1.0
0554      END IF
0555      IF (X.EQ.0.0) THEN
0556          X = 1.0
0557          Y = 1.0
0558          Z = 1.0
0559      ELSE
0560          X = 1.0
0561          Y = 1.0
0562          Z = 1.0
0563      END IF
0564      IF (X.EQ.0.0) THEN
0565          X = 1.0
0566          Y = 1.0
0567          Z = 1.0
0568      ELSE
0569          X = 1.0
0570          Y = 1.0
0571          Z = 1.0
0572      END IF
0573      IF (X.EQ.0.0) THEN
0574          X = 1.0
0575          Y = 1.0
0576          Z = 1.0
0577      ELSE
0578          X = 1.0
0579          Y = 1.0
0580          Z = 1.0
0581      END IF
0582      IF (X.EQ.0.0) THEN
0583          X = 1.0
0584          Y = 1.0
0585          Z = 1.0
0586      ELSE
0587          X = 1.0
0588          Y = 1.0
0589          Z = 1.0
0590      END IF
0591      IF (X.EQ.0.0) THEN
0592          X = 1.0
0593          Y = 1.0
0594          Z = 1.0
0595      ELSE
0596          X = 1.0
0597          Y = 1.0
0598          Z = 1.0
0599      END IF
0600      IF (X.EQ.0.0) THEN
0601          X = 1.0
0602          Y = 1.0
0603          Z = 1.0
0604      ELSE
0605          X = 1.0
0606          Y = 1.0
0607          Z = 1.0
0608      END IF
0609      IF (X.EQ.0.0) THEN
0610          X = 1.0
0611          Y = 1.0
0612          Z = 1.0
0613      ELSE
0614          X = 1.0
0615          Y = 1.0
0616          Z = 1.0
0617      END IF
0618      IF (X.EQ.0.0) THEN
0619          X = 1.0
0620          Y = 1.0
0621          Z = 1.0
0622      ELSE
0623          X = 1.0
0624          Y = 1.0
0625          Z = 1.0
0626      END IF
0627      IF (X.EQ.0.0) THEN
0628          X = 1.0
0629          Y = 1.0
0630          Z = 1.0
0631      ELSE
0632          X = 1.0
0633          Y = 1.0
0634          Z = 1.0
0635      END IF
0636      IF (X.EQ.0.0) THEN
0637          X = 1.0
0638          Y = 1.0
0639          Z = 1.0
0640      ELSE
0641          X = 1.0
0642          Y = 1.0
0643          Z = 1.0
0644      END IF
0645      IF (X.EQ.0.0) THEN
0646          X = 1.0
0647          Y = 1.0
0648          Z = 1.0
0649      ELSE
0650          X = 1.0
0651          Y = 1.0
0652          Z = 1.0
0653      END IF
0654      IF (X.EQ.0.0) THEN
0655          X = 1.0
0656          Y = 1.0
0657          Z = 1.0
0658      ELSE
0659          X = 1.0
0660          Y = 1.0
0661          Z = 1.0
0662      END IF
0663      IF (X.EQ.0.0) THEN
0664          X = 1.0
0665          Y = 1.0
0666          Z = 1.0
0667      ELSE
0668          X = 1.0
0669          Y = 1.0
0670          Z = 1.0
0671      END IF
0672      IF (X.EQ.0.0) THEN
0673          X = 1.0
0674          Y = 1.0
0675          Z = 1.0
0676      ELSE
0677          X = 1.0
0678          Y = 1.0
0679          Z = 1.0
0680      END IF
0681      IF (X.EQ.0.0) THEN
0682          X = 1.0
0683          Y = 1.0
0684          Z = 1.0
0685      ELSE
0686          X = 1.0
0687          Y = 1.0
0688          Z = 1.0
0689      END IF
0690      IF (X.EQ.0.0) THEN
0691          X = 1.0
0692          Y = 1.0
0693          Z = 1.0
0694      ELSE
0695          X = 1.0
0696          Y = 1.0
0697          Z = 1.0
0698      END IF
0699      IF (X.EQ.0.0) THEN
0700          X = 1.0
0701          Y = 1.0
0702          Z = 1.0
0703      ELSE
0704          X = 1.0
0705          Y = 1.0
0706          Z = 1.0
0707      END IF
0708      IF (X.EQ.0.0) THEN
0709          X = 1.0
0710          Y = 1.0
0711          Z = 1.0
0712      ELSE
0713          X = 1.0
0714          Y = 1.0
0715          Z = 1.0
0716      END IF
0717      IF (X.EQ.0.0) THEN
0718          X = 1.0
0719          Y = 1.0
0720          Z = 1.0
0721      ELSE
0722          X = 1.0
0723          Y = 1.0
0724          Z = 1.0
0725      END IF
0726      IF (X.EQ.0.0) THEN
0727          X = 1.0
0728          Y = 1.0
0729          Z = 1.0
0730      ELSE
0731          X = 1.0
0732          Y = 1.0
0733          Z = 1.0
0734      END IF
0735      IF (X.EQ.0.0) THEN
0736          X = 1.0
0737          Y = 1.0
0738          Z = 1.0
0739      ELSE
0740          X = 1.0
0741          Y = 1.0
0742          Z = 1.0
0743      END IF
0744      IF (X.EQ.0.0) THEN
0745          X = 1.0
0746          Y = 1.0
0747          Z = 1.0
0748      ELSE
0749          X = 1.0
0750          Y = 1.0
0751          Z = 1.0
0752      END IF
0753      IF (X.EQ.0.0) THEN
0754          X = 1.0
0755          Y = 1.0
0756          Z = 1.0
0757      ELSE
0758          X = 1.0
0759          Y = 1.0
0760          Z = 1.0
0761      END IF
0762      IF (X.EQ.0.0) THEN
0763          X = 1.0
0764          Y = 1.0
0765          Z = 1.0
0766      ELSE
0767          X = 1.0
0768          Y = 1.0
0769          Z = 1.0
0770      END IF
0771      IF (X.EQ.0.0) THEN
0772          X = 1.0
0773          Y = 1.0
0774          Z = 1.0
0775      ELSE
0776          X = 1.0
0777          Y = 1.0
0778          Z = 1.0
0779      END IF
0780      IF (X.EQ.0.0) THEN
0781          X = 1.0
0782          Y = 1.0
0783          Z = 1.0
0784      ELSE
0785          X = 1.0
0786          Y = 1.0
0787          Z = 1.0
0788      END IF
0789      IF (X.EQ.0.0) THEN
0790          X = 1.0
0791          Y = 1.0
0792          Z = 1.0
0793      ELSE
0794          X = 1.0
0795          Y = 1.0
0796          Z = 1.0
0797      END IF
0798      IF (X.EQ.0.0) THEN
0799          X = 1.0
0800          Y = 1.0
0801          Z = 1.0
0802      ELSE
0803          X = 1.0
0804          Y = 1.0
0805          Z = 1.0
0806      END IF
0807      IF (X.EQ.0.0) THEN
0808          X = 1.0
0809          Y = 1.0
0810          Z = 1.0
0811      ELSE
0812          X = 1.0
0813          Y = 1.0
0814          Z = 1.0
0815      END IF
0816      IF (X.EQ.0.0) THEN
0817          X = 1.0
0818          Y = 1.0
0819          Z = 1.0
0820      ELSE
0821          X = 1.0
0822          Y = 1.0
0823          Z = 1.0
0824      END IF
0825      IF (X.EQ.0.0) THEN
0826          X = 1.0
0827          Y = 1.0
0828          Z = 1.0
0829      ELSE
0830          X = 1.0
0831          Y = 1.0
0832          Z = 1.0
0833      END IF
0834      IF (X.EQ.0.0) THEN
0835          X = 1.0
0836          Y = 1.0
0837          Z = 1.0
0838      ELSE
0839          X = 1.0
0840          Y = 1.0
0841          Z = 1.0
0842      END IF
0843      IF (X.EQ.0.0) THEN
0844          X = 1.0
0845          Y = 1.0
0846          Z = 1.0
0847      ELSE
0848          X = 1.0
0849          Y = 1.0
0850          Z = 1.0
0851      END IF
0852      IF (X.EQ.0.0) THEN
0853          X = 1.0
0854          Y = 1.0
0855          Z = 1.0
0856      ELSE
0857          X = 1.0
0858          Y = 1.0
0859          Z = 1.0
0860      END IF
0861      IF (X.EQ.0.0) THEN
0862          X = 1.0
0863          Y = 1.0
0864          Z = 1.0
0865      ELSE
0866          X = 1.0
0867          Y = 1.0
0868          Z = 1.0
0869      END IF
0870      IF (X.EQ.0.0) THEN
0871          X = 1.0
0872          Y = 1.0
0873          Z = 1.0
0874      ELSE
0875          X = 1.0
0876          Y = 1.0
0877          Z = 1.0
0878      END IF
0879      IF (X.EQ.0.0) THEN
0880          X = 1.0
0881          Y = 1.0
0882          Z = 1.0
0883      ELSE
0884          X = 1.0
0885          Y = 1.0
0886          Z = 1.0
0887      END IF
0888      IF (X.EQ.0.0) THEN
0889          X = 1.0
0890          Y = 1.0
0891          Z = 1.0
0892      ELSE
0893          X = 1.0
0894          Y = 1.0
0895          Z = 1.0
0896      END IF
0897      IF (X.EQ.0.0) THEN
0898          X = 1.0
0899          Y = 1.0
0900          Z = 1.0
0901      ELSE
0902          X = 1.0
0903          Y = 1.0
0904          Z = 1.0
0905      END IF
0906      IF (X.EQ.0.0) THEN
0907          X = 1.0
0908          Y = 1.0
0909          Z = 1.0
0910      ELSE
0911          X = 1.0
0912          Y = 1.0
0913          Z = 1.0
0914      END IF
0915      IF (X.EQ.0.0) THEN
0916          X = 1.0
0917          Y = 1.0
0918          Z = 1.0
0919      ELSE
0920          X = 1.0
0921          Y = 1.0
0922          Z = 1.0
0923      END IF
0924      IF (X.EQ.0.0) THEN
0925          X = 1.0
0926          Y = 1.0
0927          Z = 1.0
0928      ELSE
0929          X = 1.0
0930          Y = 1.0
0931          Z = 1.0
0932      END IF
0933      IF (X.EQ.0.0) THEN
0934          X = 1.0
0935          Y = 1.0
0936          Z = 1.0
0937      ELSE
0938          X = 1.0
0939          Y = 1.0
0940          Z = 1.0
0941      END IF
0942      IF (X.EQ.0.0) THEN
0943          X = 1.0
0944          Y = 1.0
0945          Z = 1.0
0946      ELSE
0947          X = 1.0
0948          Y = 1.0
0949          Z = 1.0
0950      END IF
0951      IF (X.EQ.0.0) THEN
0952          X = 1.0
0953          Y = 1.0
0954          Z = 1.0
0955      ELSE
0956          X = 1.0
0957          Y = 1.0
0958          Z = 1.0
0959      END IF
0960      IF (X.EQ.0.0) THEN
0961          X = 1.0
0962          Y = 1.0
0963          Z = 1.0
0964      ELSE
0965          X = 1.0
0966          Y = 1.0
0967          Z = 1.0
0968      END IF
0969      IF (X.EQ.0.0) THEN
0970          X = 1.0
0971          Y = 1.0
0972          Z = 1.0
0973      ELSE
0974          X = 1.0
0975          Y = 1.0
0976          Z = 1.0
0977      END IF
0978      IF (X.EQ.0.0) THEN
0979          X = 1.0
0980          Y = 1.0
0981          Z = 1.0
0982      ELSE
0983          X = 1.0
0984          Y = 1.0
0985          Z = 1.0
0986      END IF
0987      IF (X.EQ.0.0) THEN
0988          X = 1.0
0989          Y = 1.0
0990          Z = 1.0
0991      ELSE
0992          X = 1.0
0993          Y = 1.0
0994          Z = 1.0
0995      END IF
0996      IF (X.EQ.0.0) THEN
0997          X = 1.0
0998          Y = 1.0
0999          Z = 1.0
1000      ELSE
1001          X = 1.0
1002          Y = 1.0
1003          Z = 1.0
1004      END IF
1005      IF (X.EQ.0.0) THEN
1006          X = 1.0
1007          Y = 1.0
1008          Z = 1.0
1009      ELSE
1010          X = 1.0
1011          Y = 1.0
1012          Z = 1.0
1013      END IF
1014      IF (X.EQ.0.0) THEN
1015          X = 1.0
1016          Y = 1.0
1017          Z = 1.0
1018      ELSE
1019          X = 1.0
1020          Y = 1.0
1021          Z = 1.0
1022      END IF
1023      IF (X.EQ.0.0) THEN
1024          X = 1.0
1025          Y = 1.0
1026          Z = 1.0
1027      ELSE
1028          X = 1.0
1029          Y = 1.0
1030          Z = 1.0
1031      END IF
1032      IF (X.EQ.0.0) THEN
1033          X = 1.0
1034          Y = 1.0
1035          Z = 1.0
1036      ELSE
1037          X = 1.0
1038          Y = 1.0
1039          Z = 1.0
1040      END IF
1041      IF (X.EQ.0.0) THEN
1042          X = 1.0
1043          Y = 1.0
1044          Z = 1.0
1045      ELSE
1046          X = 1.0
1047          Y = 1.0
1048          Z = 1.0
1049      END IF
1050      IF (X.EQ.0.0) THEN
1051          X = 1.0
1052          Y = 1.0
1053          Z = 1.0
1054      ELSE
1055          X = 1.0
1056          Y = 1.0
1057          Z = 1.0
1058      END IF
1059      IF (X.EQ.0.0) THEN
1060          X = 1.0
1061          Y = 1.0
1062          Z = 1.0
1063      ELSE
1064          X = 1.0
1065          Y = 1.0
1066          Z = 1.0
1067      END IF
1068      IF (X.EQ.0.0) THEN
1069          X = 1.0
1070          Y = 1.0
1071          Z = 1.0
1072      ELSE
1073          X = 1.0
1074          Y = 1.0
1075          Z = 1.0
1076      END IF
1077      IF (X.EQ.0.0) THEN
1078          X = 1.0
1079          Y = 1.0
1080          Z = 1.0
1081      ELSE
1082          X = 1.0
1083          Y = 1.0
1084          Z = 1.0
1085      END IF
1086      IF (X.EQ.0.0) THEN
1087          X = 1.0
1088          Y = 1.0
1089          Z = 1.0
1090      ELSE
1091          X = 1.0
1092          Y = 1.0
1093          Z = 1.0
1094      END IF
1095      IF (X.EQ.0.0) THEN
1096          X = 1.0
1097          Y = 1.0
1098          Z = 1.0
1099      ELSE
1100          X = 1.0
1101          Y = 1.0
1102          Z = 1.0
110
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MAIN

FORTRAN IV 01 RELEASE 2.0

141 WRITE(6,70141) ITHA,IINH,IINC)
142 FORMAT(13,15STATE=16 ITHA=1,12,1,IINH=1,12,1,IINC=1,12)
143 IF(IAMSFLOW=FLOWN) LEA FPMH GO TO 4501
144 ITHA=IINH+1
145 IF(IINH.GE. MAXIT) GO TO 4005
146 VEZFC(1,1)=V-ZED(N,1)*FLOW/FLOWN
147 C(1)=XZC(1)-C
148 V7-PH=VZC2/C
149 GO TO 4012

450 CONTINUE

141 WRITE(6,7017) (ITHA,IINH,IINC)
142 FORMAT(13,15STATE=17 ITHA=1,12,1,IINH=1,12,1,IINC=1,12)
143 IF(IAMSFLOW=FLOWN) LEA FPMH GO TO 4501

SECTION 144

ITHA=IINH+1

SECTION 145 LOOP 144 CONVERGENCE TEST

IF (ITHA.GE. MAXIT) GO TO 4001
GO TO 4101

SECTION 146 CONVERGENCE FAILURE DIAGNOSTICS

4001 WRITE(6,6002) ITHA
4002 FORMAT(12,15OVERALL CONVERGENCE FAILURE, ITHA=1,12)
GO TO 4501

4003 WRITE(6,6004) IINC
4004 FORMAT(12,15CONVERGENCE FAILS TO CONVERGE, IINC=1,12)
GO TO 4501

4005 WRITE(6,6006) ITHA
4006 FORMAT(12,15CONTINUITY FAILS TO CONVERGE, ITHA=1,12)
GO TO 4501

4007 WRITE(6,6008)
4008 FORMAT(12,15C CONVERGENCE FAILURES)

C SECTION 147 PRINT OUT
C SECTION 148 TANDEM TAIL EQUATION

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DEPT. OF THE ARMY, WASHINGTON, D. C.

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INDEXES FOR CRYSTALLINITY COUNTING

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Some implications of the generation of swirl by helically welded steel pipes

C A A HIGMAN*

Synopsis

The paper examines the extent to which helically welded steel pipes will impart swirl to a pipe flow and the consequent effects, particularly on the accuracy of differential pressure flow metering orifices. An investigation of these effects is described. On the basis of the results obtained a recommendation is made regarding the need to avoid the use of helically welded pipe upstream of differential pressure flow metering devices.

Opsomming

In hierdie verhandeling word ondersoek ingestel na die mate waarin helies gesweiste staalpyp versneling aan 'n pyp se vloei sal verleen en die uitwerking daarvan, veral op die akkuraatheid van differensiaaldrukstuskmeters. Die ondersoek na hierdie uitwerking word beskryf. Op die grondslag van die verkree resultate word 'n aanbeveling gedoen betreffende die noodsaaklikheid om die gebruik van helies gesweiste pyp stroomop van differensiaaldrukvloei-meettoestelle te voorkom.

Notation

d	Pipe internal diameter
$\epsilon_{m\theta}$	Tangential component of eddy viscosity
$\mathbf{l}$	Mixing length vector
l	Axial distance along pipe from start of helical deformation
m	Orifice area ratio
p	Hydrostatic pressure
Q	As defined ($Q = p \cdot q = 2$)
$\mathbf{q} = (q_r, q_\theta, q_z)$	Velocity vector
R	Pipe internal radius
Re	Reynolds number
r, θ	Cylindrical polar co-ordinates
k	Constant in Equation (6)
z	As defined for Equation (6)
μ	Absolute viscosity
ρ	Density
Ω	Vorticity vector
$\bar{}$	Mean value of variable
τ	Turbulence value of variable

Helically welded steel pipe

Although the first patents for the manufacture of helically welded steel pipes date back to early in the present century, the slow development of welding techniques and a number of problems associated with forming methods held back commercial exploitation of the process till the mid 1950s (1). During the last 15 years or so, however,

this type of pipe has gained increasingly wide acceptance because of the advantages which helically welded pipe offers to both manufacturer and user.

Production costs on a spiral-weld pipe can be considerably less than on a longitudinal weld mill for large bore pipes, partly through reduced capital cost, partly through reduced labour costs and also through reduced stock costs since the same coiled strip can be used for varying pipe sizes merely by alteration of the helix angle. The mill's great inherent flexibility also allows shorter production runs at economical cost.

Further economic advantages of the helically welded pipe are that since it is manufactured by a continuous process, pipe lengths can (in theory) be as long as is desired, thus reducing substantially the number of line joints. In practice lengths of up to 30 m have been made. Spiral weld pipe mills are designed to be transportable so that they can be transported from site to site without difficulty. When these mobile machines are operated *in situ*, substantial savings can be effected from the difference in shipping costs of pipes compared with steel in coils.

From the strength aspect helically welded pipe has an inherently greater factor of safety than longitudinally welded pipe. Since, when the pipe is subject to internal pressure, the hoop stress is twice the longitudinal stress, the weld in a longitudinally welded pipe is subjected to the maximum stress. The weld seam in a helically welded pipe is generally at about 45° to the axis of the pipe and so the stresses in the seam are reduced by 50 to 40 per cent. Additional real advantages of helically welded pipe are (a) its greater resistance to tearing in the event

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of failure, and (b) its improved tolerances on the straightness, roundness, wall thickness and diameter of the pipe.

Despite the assertion made by Eckardt and Robson that helically welded pipe presents no problems in bending (1), it is the author's experience that great difficulty is encountered when attempting to hot-bend helically welded piping because the weld reinforcement tends to encourage creasing on the inside of the bend. Eckardt and Robson's statement therefore presumably refers to cold field bending where offsets are generally limited to 7.5%. Another possible disadvantage of helically welded pipe is that the hydraulic characteristics may be adversely affected by the weld bead and the deformation caused by anti-elastic curvature of the strip (bamboo effect). Few of the international specifications place any limit on the size of this weld bead but it is limited to S A B S 719 (2) to 5 mm. Conrad and Brandts (3) have shown that the effect of the weld bead in helically welded pipes has less influence on the friction losses than the general roughness of the pipe.

It has been suggested, however, that a swirling flow may be generated, although no work appears to have been done to investigate this aspect despite the possible detrimental effects which could arise. Two such possibilities which spring immediately to mind are (i) the accuracy of pressure differential flow meters could be impaired, and (ii) adverse prerotation in the suction pipes of centrifugal pumps could lower the suction performance characteristics.

Swirling flow in orifice approach lengths

The British standard for flow measurement with pressure differential devices (B S 1042:1964) (4) excludes their application where there is any swirling motion upstream of the device. Guidelines are given for straightening lengths downstream of certain flow disturbing features such as bends, valves, etc. but no mention is made of helically welded pipe. Part 3 of the standard provides a guide to the magnitude of the error to be expected when deviating from the standard approach conditions but no reference is made to swirling flows. Appendix F of Part 1 gives some advice on the damping of swirling motion. It also states that strong swirls may persist for 100 or more pipe diameters from the disturbance.

The A S M E guide to fluid meters (5) also makes reference to the desirability of excluding swirl in the approach flow and give guidelines similar to those in B S 1042. It also specifies close tolerances on the pipe bore roundness and surface roughness. This is, however, limited to the immediate vicinity of the orifice (four diameters upstream and two diameters downstream).

In his work on the development of a flow straightener for use with orifice plate flow meters in disturbed flows, Zanker (6) investigated the effect of different types of swirling flows on the discharge coefficients of an orifice meter. He found that forced vortex swirls tended to give a high value of the discharge coefficient and that a free vortex swirl gave a low value. A reduction of area ratio tended to reduce these effects but not consistently or within acceptable limits. He also found that the discharge

coefficient was dependent on the momentum flux in the outer third of the pipe cross-sectional area.

Lugt (7) investigated the effect of swirling flow on various types of pressure differential flow metering devices and using varying area ratios. Using a free vortex type of swirl he found that with a swirl angle of 20° the error on the meter discharge coefficient was ± 10 per cent depending on the type and area ratio of the device. In view of the large variation of error depending on swirl angle and the difficulty of determining the amount of swirl in practical installations, Lugt concluded that there was no point in attempting to incorporate correction factors into the standard specifications.

Theoretical considerations

The geometry of the interior surface of a helically welded pipe is sufficiently complex that it effectively precludes any purely mathematical approach to the problem. It is possible, however, to determine the inward growth of a swirl pattern from an empirically determined swirl growth in the outer annulus of the pipe. The fundamentals of the method are well set out by Rochino and Laven (8) who used it to examine a number of different rotating and swirling flows.

In vector form the equation for a steady state flow, neglecting body forces and assuming constant viscosity is:

$$\mathbf{q} \cdot \nabla \mathbf{q} = -\frac{1}{\rho} \nabla p - \frac{\mu}{\rho} \nabla^2 \mathbf{q} \quad (1)$$

If the velocity vector $\mathbf{q}$ is broken up into its mean component $\bar{\mathbf{q}}$ and its time dependent or turbulence component $\mathbf{q}'$ then Equation (1) can be rewritten

$$\mathbf{q} \cdot \nabla \mathbf{q} = -\frac{1}{\rho} \nabla p - \frac{\mu}{\rho} \nabla^2 \mathbf{q} + \mathbf{q}' \cdot \nabla \mathbf{q}' \quad (2)$$

The derivation of this equation can be found in any standard work such as Schlichting (9). It will be observed that this equation for the mean flow variables differ only from Equation (1) for the full flow variables by the additional term $\mathbf{q}' \cdot \nabla \mathbf{q}'$. This term has been considered to be due to the action of a purely notional but nonetheless useful concept, eddy (or turbulence) viscosity. This concept is useful mainly because the mathematical effect of the additional term is that we can simply add the eddy and molecular viscosities. To demonstrate this it is convenient to express Equation (2) in the form

$$\mathbf{q} \cdot \nabla \mathbf{q} = -\frac{1}{\rho} \nabla p - \frac{\mu}{\rho} \nabla^2 \mathbf{q} + \Omega \quad (3)$$

where $\Omega = (\rho/\mu) \mathbf{q}' \cdot \nabla \mathbf{q}'$

The turbulence component of vorticity, Ω , can be expressed in terms of the mean vorticity component by use of Prandtl's mixing length as described in the appendix. This enables Equation (3) to be expressed as

$$\mathbf{q} \cdot \nabla \mathbf{q} = -\frac{1}{\rho} \nabla p - \frac{\mu}{\rho} \nabla^2 \mathbf{q} + \frac{1}{L} \nabla \Omega \quad (4)$$

Assuming that the turbulence is isotropic, simplified by neglecting the following cylindrical

Radial component:
Tangential component:

Axial component:

The tangential Eq. can be used to determine assuming that the undisturbed by the. Although this cannot be so to first order.

von Karman's Similarity Law is provided by the following equation:

where $z = r$ for $n = 1$ and k is a constant determined from experimental data.

Lavan (based on work to dimensionless form

This is a non-linear equation which has been solved using the method of successive approximations. The results are discussed later in the paper.

Description of apparatus: Fig. 1 describes the apparatus used in the experiments. The test section consisted of a commercial helically welded pipe of diameter 100 mm under about 250 mm. Windows were required for observation. Water was pumped through the flow settling length into the test section. The test pipe was a helical weld.

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Assuming that the flow is axisymmetrical and that the turbulence is isotropic Equation (4) can be further simplified by neglecting terms of second order yielding the following cylindrical polar equations.

Radial component:

$$\frac{v_{\theta}^2}{r} = \frac{cQ}{cr} \quad (5a)$$

Tangential component:

$$\frac{1}{r} \frac{\partial v_{\theta}}{\partial z} = \left(\frac{v_{\theta}}{r} \right) \frac{\partial v_{\theta}}{\partial r} + \frac{\mu}{\rho} \left(\frac{\partial^2 v_{\theta}}{\partial r^2} + \frac{1}{r} \frac{\partial v_{\theta}}{\partial r} - \frac{v_{\theta}}{r^2} - \frac{\partial^2 v_{\theta}}{\partial z^2} \right) \quad (5b)$$

Axial component:

$$\frac{cQ}{cz} = \left(\frac{v_z}{r} \right) \frac{\partial v_z}{\partial r} + \frac{\mu}{\rho} \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} \right) \quad (5c)$$

The tangential Equation (5b) (the swirl equation) can be used to determine the tangential flow in the pipe assuming that the axial velocity, v_z , is essentially undisturbed by the swirl velocity superimposed on it. Although this cannot be strictly true, it is assumed to be so to first order. An expression for the eddy viscosity $\nu_{\theta} = c_{\theta} \mu$ is provided by Rochino and Lavan (8) using von Kärman's Similarity Hypothesis. This is:

$$c_{\theta} = Re k^2 \lambda^2 \left(\frac{1}{r} \frac{\partial v_{\theta}}{\partial r} - \frac{v_{\theta}}{r^2} \right) \quad (6)$$

where $\lambda = r$ for $0 < r < 0.9$
 $\lambda = 1 - r$ for $0.9 < r < 1.0$

and k is a constant which has to be determined from experimental data. The value used by Rochino and Lavan (based on work by Kinney) was 0.0333. Reduced to dimensionless form, the swirl equation therefore is:

$$\frac{1}{r} \frac{\partial v_{\theta}}{\partial z} = \left[\frac{1}{Re} + k^2 \lambda^2 \left(\frac{1}{r} \frac{\partial v_{\theta}}{\partial r} - \frac{v_{\theta}}{r^2} \right) \right] \left[\frac{\partial^2 v_{\theta}}{\partial r^2} + \frac{1}{r} \frac{\partial v_{\theta}}{\partial r} - \frac{v_{\theta}}{r^2} - \frac{\partial^2 v_{\theta}}{\partial z^2} \right] \quad (7)$$

This is a non-linear partial differential equation which has been solved using a digital computer. The solutions are discussed later together with the results from the practical experiments.

Description of apparatus

Fig 1 describes the main portions of the apparatus used in the experimental investigation. Pumping limitations restricted the size of pipe used to 100 mm. Commercial helically welded pipe was not used because (i) commercial pipe is not made by the helical weld process under about 250 mm and (ii) transparent perspex windows were required to show up the flow pattern.

Water was pumped into the test section through a flow settling length of resistance welded longitudinally seamed pipe. The test length was primarily seamless steel pipe with a helical wire insert to simulate the disturbance

caused by the weld seam. Short lengths of perspex tube were furnished in the test length to provide windows to observe the build up of any swirl which might occur. The test orifice installation immediately after the test section complies fully with BS 1042 except for the helical insert in the upstream pipe. The water was finally discharged to a metering tank to provide a positive calibration facility for the orifice.

Instrumentation was kept as simple as possible by using a yaw-meter pitot tube arrangement (see Fig 2) to provide data on speed and direction of flow at cross sections corresponding to each of the perspex windows on the test section. Differential pressures for this and the orifice were measured with mercury water and water air manometers.

Experimental work

The purpose of the experiments was to examine the build-up of the swirl component of velocity and to study its effect on an orifice flow meter. In addition other data were collected which might throw some light on the mathematical model of swirling flow outlined above, particularly regarding the effect of swirl buildup on the axial velocity profile.

The procedure used to find the direction and velocity

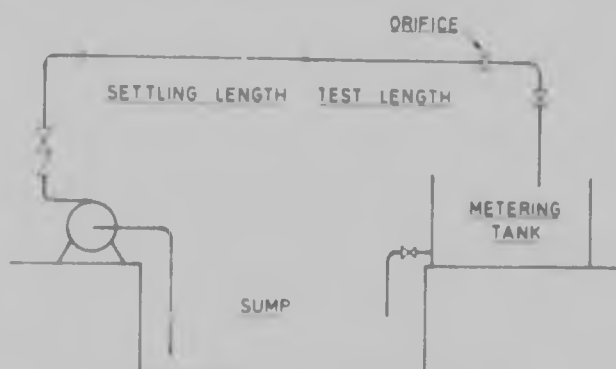


Fig 1 Diagrammatic layout of test rig

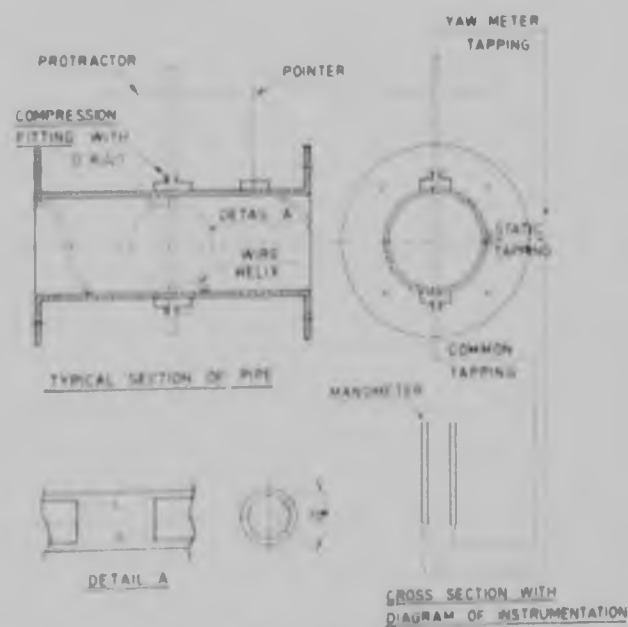


Fig 2 Detail of typical test section with instrumentation

of flow at each point was as follows. First the direction of flow was as found using the vertical diametrical probe as a yaw-meter and then the probe was rotated through 35° so as to present the hole corresponding to the common tapping to the direction of flow. The other hole was blanked off and the manometer leg formerly connected to it was connected to the static pressure tapping thus converting the probe into a pitot tube to measure the flow velocity. This procedure enabled the superior directional characteristics of the yaw-meter to be used to assist in determination of resultant flow velocities.

Typical results are displayed in Figs 3 to 6 and Table 1.

TABLE 1
Orifice metering error due to presence of wire helix

Actual flowrate l/s	Linear flowrate m/s	Reynolds' number	Metered flowrate l/s	Meter error %
12.25	1.77	2.38 × 10 ⁵	10.71	12.4
15.45	2.23	3.00	13.31	13.7
16.80	2.42	3.26	14.98	10.9
18.65	2.69	3.61	16.20	13.1
20.30	2.93	3.94	17.41	14.1
21.90	3.16	4.25	19.22	12.1
26.05	3.76	5.05	23.00	11.9
28.90	4.16	5.61	25.95	10.2

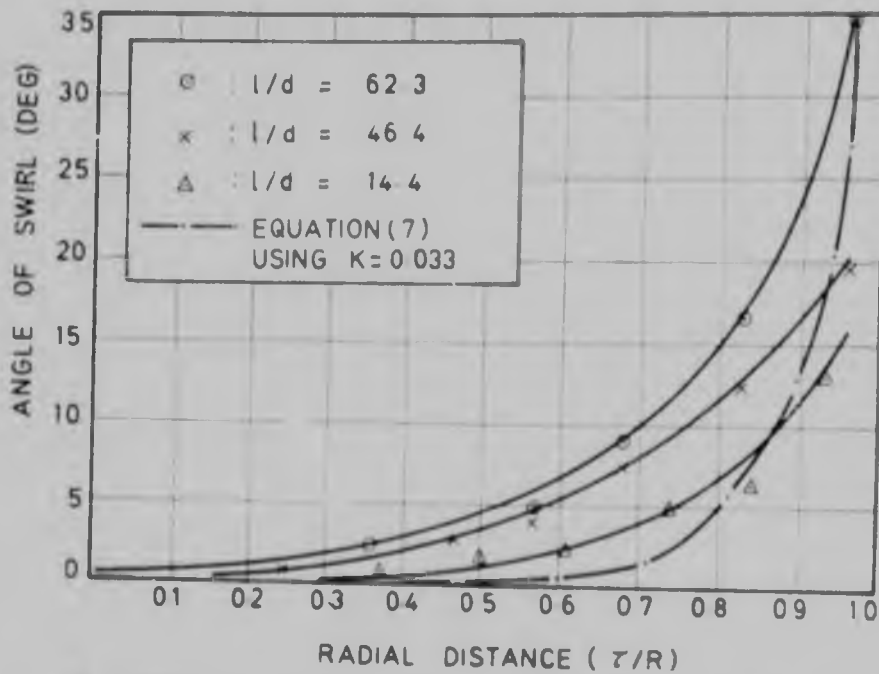


Fig 3 Radial distribution of swirl at various downstream positions.
 $Re = 3.47 \times 10^5$

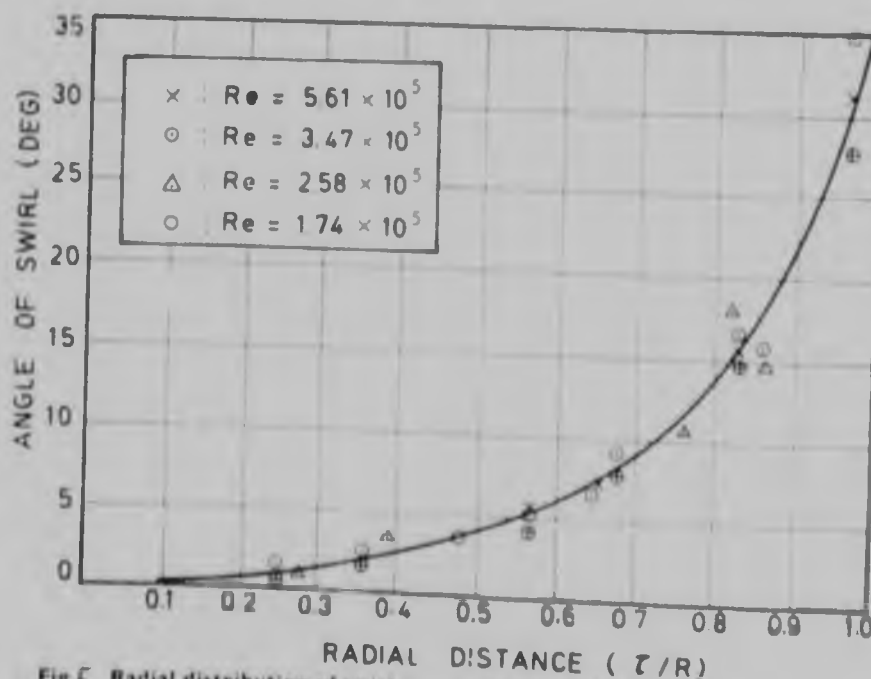


Fig 5 Radial distribution of swirl for various Reynolds numbers. $l/d = 62.3$

The growth of swirl direction is shown in radial swirl distribution numbers. Over the range appears to be independent of the effect of axial orifice meters (6). The skew pattern of the test length and at lower Reynolds numbers Table 1 indicate number for the orifice of $m = 0.44$. The error is 12.3 per cent.

Discussion

Fig 3 shows the radial cross sections of the slow growth of swirl. The rapid rise in the swirl was considered in Equation (7) using $K = 0.033$ in Fig 4 as a boundary condition. It was found to be very close to the experimental data shown was based on Lavan. It would appear that the swirl builds up of pipe and the 60°

TABLE 1
Effect of presence of wire helix

Orifice number	Metered flowrate l/s	Meter error %
10 ¹	10.71	12.4
	13.31	13.7
	14.98	10.9
	16.20	13.1
	17.41	14.1
	19.22	12.1
	23.00	11.9
	25.95	10.1

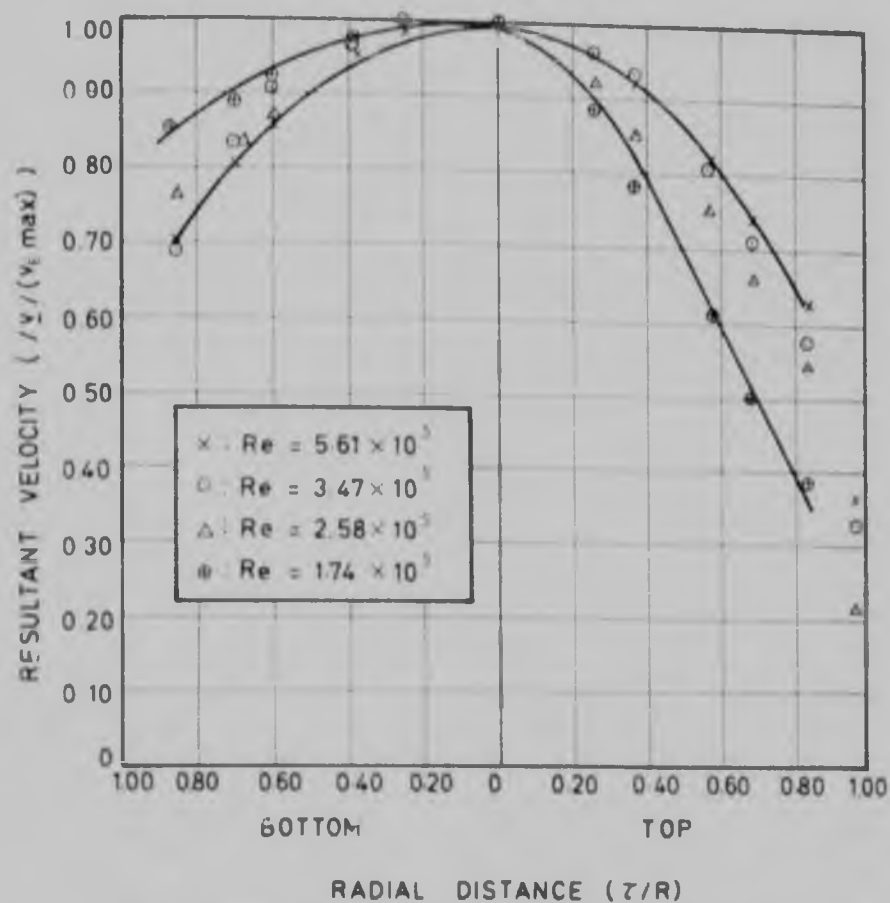


Fig 6. Velocity profiles for various Reynolds numbers. $Id = 62.3$

The growth of swirl in the longitudinal and radial direction is shown in Figs 3 and 4. Fig 5 shows the radial swirl distribution for a variety of Reynolds numbers. Over the restricted range investigated, swirl appears to be independent of Reynolds Number. Because of the effect of axial velocity profile as well as swirl on orifice meters (6), Fig 6 was plotted to show this profile. The skew pattern exhibited was found to increase along the test length and as can be seen was more pronounced at lower Reynolds numbers. The results tabulated in Table 1 indicate the meter error at different Reynolds number for the orifice used which had an area ratio $a/m = 0.404$. The results display an average error of 12.3 per cent.

Discussion

Fig 3 shows the radial distribution of swirl at various cross sections of the pipe. It exhibits a comparatively slow growth of swirl in the centre of the pipe despite the rapid rise in the outer annulus. The actual growth of swirl was considerably more than that predicted by Equation (7) using the results displayed for $r/R = 0.95$ in Fig 4 as a boundary condition. The equation was found to be very sensitive to changes in α and the plot shown was based on the value of 0.0333 of Rochino and Lavan. It would appear, therefore, that additional work is required to confirm the correctness of their value.

The axial plot of swirl growth in Fig 4 demonstrates that the swirl builds up slowly over a considerable length of pipe and the 60 pipe diameters available on the test

rig was insufficient to allow maximum swirl to develop. Extrapolation of the curves would imply that 100 or more diameters are required before the axial swirl gradient vanishes.

The apparent independence of swirl growth and Reynolds number depicted in Fig 5 confirmed one aspect of Equation (7). It can be seen by inspection that at high Reynolds numbers omission of the term would not materially affect the solution.

The skew velocity profile in Fig 6 has no immediately obvious explanation. The profile became more skew along the test length and was also more pronounced at lower Reynolds number. Since the skewness is in the vertical plane it could be caused by some gravitational effect but further investigation is required to examine this aspect. The skew velocity profile must, however, be a secondary source of error in the orifice meter and also upsets one of the major assumptions of the theoretical investigation.

The error introduced into the meter is very clear from Table 1. The error is 12.3 per cent with a spread of 1.8 per cent to 2.1 per cent. Although the swirl was independent of Reynolds number the variation in velocity profile must be partly responsible for the size of spread. Errors of similar magnitude were reported by Lugt (7). He examined orifices of varying area ratios and found that the area ratio had considerable influence on the error. It would therefore seem that the above work should be extended to find the effect of the flow pattern under consideration on orifice plates of other area ratios. Of



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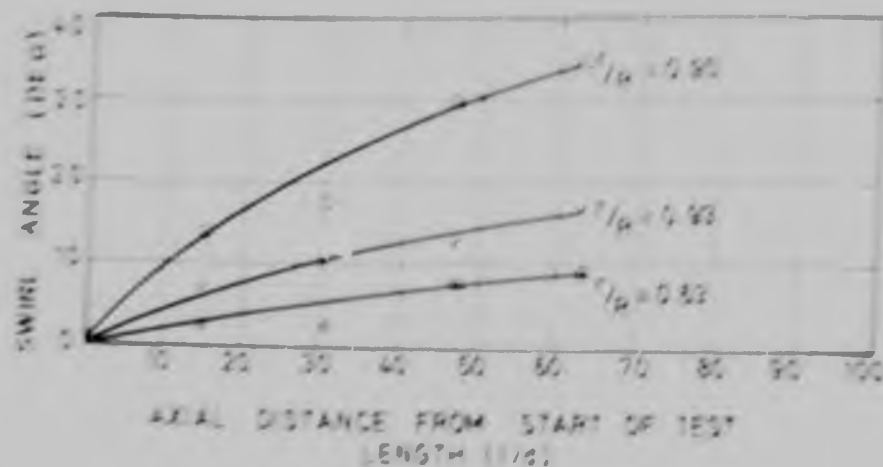


Fig 6 Axial swirl growth $K = 1.07 \times 10^{-4}$

more important, however, is the need to determine a suitable settling length for the straight section since it involves some of the most difficult problems.

Conclusion

Although work has been done on the use of helical vanes in industrial mixers, specifically in the case of a swirling flow, this work has not done to investigate the effects of the small helical surface deformations such as in a helically twisted pipe. The present work demonstrates some aspects of this phenomenon. It demonstrates that even in the case of a significant amount of swirl can be generated and that this can affect the accurate performance of an orifice plate flow meter. Further work is required to determine suitable settling lengths for damping this type of swirl but in the absence of such information it is suggested that the use of helically twisted pipe in connection with metering installations be avoided.

Appendix

The concepts of mixing length developed by Prandtl and Taylor are all discussed by Schlichting (9). All the theories based on this concept assume that in turbulent flow lumps of fluid transfer certain of their transferable properties from one fluid layer to another just as molecular agitation transfers properties like heat and momentum in a non-turbulent flow. The lumps conserve their transferable properties as they traverse a length, L , characteristic of the flow and known as the mixing length. Having gone this distance they merge with their new surroundings and assume the average properties of the fluid in that region. Prandtl assumed that transferable property to be momentum whereas Taylor assumed a vorticity transport phenomenon. Both theories have proved to be of value for different types of flow, but for a pipe flow such as that under consideration the vorticity transport theory gives better results.

If we consider the transport of vorticity between the points A and X in Fig 7 we can deduce the following. A fluid lump starts at A with vorticity Ω_0 at time t_0 and

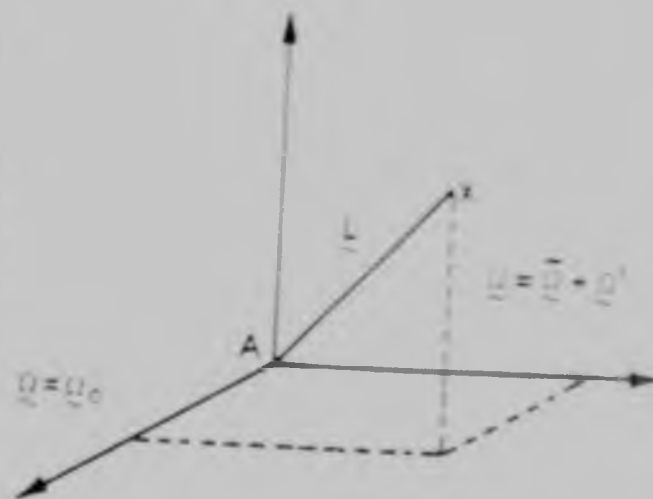


Fig 7

traverses the mixing length L to a point X before it mixes with the new environment. If the vorticity at X has mean value $\bar{\Omega}$ and turbulence component Ω' then at time t_1 when the fluid lump arrives at X

$$\bar{\Omega} = \Omega' = \Omega_0 \quad (2.1)$$

However, if we assume that the mixing length is small then a Taylor expansion around t_0 for the mean vorticity (which is Ω_0 at t_0) yields

$$\bar{\Omega} = \Omega_0 + L \frac{d\Omega}{dt} \quad (2.2)$$

Thus to first order

$$\Omega = \bar{\Omega} = \Omega_0 + L \frac{d\Omega}{dt} \quad (2.3)$$

which enables us to express the turbulence component of vorticity in terms of the mean vorticity and the mixing length.

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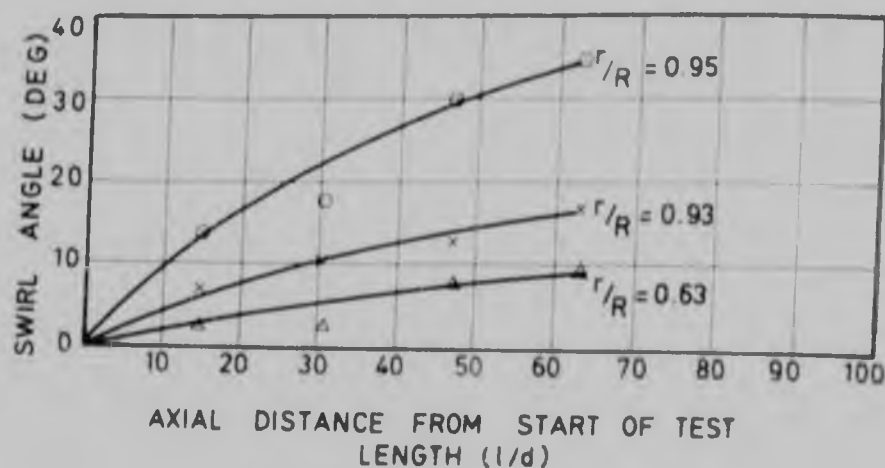


Fig 6 Axial swirl growth. $R_e = 3.47 \times 10^5$

more importance, however, is the need to determine a suitable settling length for the induced swirl since it introduces errors of unacceptable magnitude.

Conclusion

Although work has been conducted on the use of helical swirl inducers (10) designed specifically to promote a swirling flow, little work has been done to investigate the effects of the small helical surface deformities such as in a helically welded pipe. The present work investigates some aspects of this phenomenon. It demonstrates that even on this scale a significant amount of swirl can be generated and that this can affect the accurate performance of an orifice plate flow meter. Further work is required to determine suitable settling lengths for damping this type of swirl but in the absence of such information it is suggested that the use of helically welded pipe in connection with metering installations be avoided.

Appendix

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If we consider the transport of vorticity between the points A and X in Fig 7 we can deduce the following. A fluid lump starts at A with vorticity Ω_0 at time t_0 and

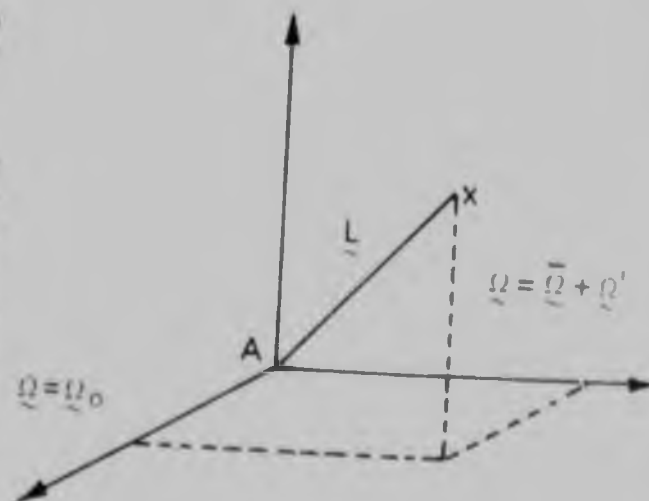


Fig 7

traverses the mixing length L to a point X before it mixes with the new environment. If the vorticity at X has mean value $\bar{\Omega}$ and turbulence component Ω' then at time t_1 when the fluid lump arrives at X

$$\bar{\Omega} = \bar{\Omega} + \Omega' \quad (2.1)$$

However, if we assume that the mixing length is small then a Taylor expansion around A for the mean vorticity (which is $\bar{\Omega}_0$ at A) yields

$$\bar{\Omega} = \bar{\Omega}_0 + \frac{d\bar{\Omega}}{dx} L \quad \text{to first order in } L \quad (2.2)$$

Thus to first order

$$\bar{\Omega} = \bar{\Omega}_0 + \frac{d\bar{\Omega}}{dx} L \quad (2.3)$$

which enables us to express the turbulence component of vorticity in terms of the mean vorticity and the mixing length.

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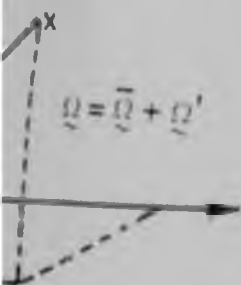


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CHRISTOPHER HIGMAN came to South Africa from the U.K. after obtaining a BA(Hons) in Applied Mathematics at Oxford. He received his basic engineering training with the Rand Water Board. In 1969 he joined the Pipework Engineering Department of Stewarts and Lloyds as a contract engineer where he was responsible for the design and installation of piping systems for power stations and chemical plants. In 1974 he moved to Frankfurt, West Germany, since when he has been involved in the design and construction of two large ammonia plants. He is currently employed by Lurgi Mineraloltechnik GmbH.



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Proceedings of Annual General Meeting

The 25th Annual General Meeting of the Institution was held in the Main Hall, Kelvin House, Johannesburg on Thursday the 10th June, 1976 at 18h00.

The President, Prof E A Bunt, extended a warm welcome to those present and expressed his appreciation of the support of the Presidents of so many sister engineering institutions.

The minutes of the Annual General Meeting which was held on 12th June, 1975, having been published in the October, 1975 Journal were taken as read and confirmed.

Annual Report and Balance Sheet

Copies of the Report and Balance Sheet for the 1975/76 Session had been previously circulated and the President commented briefly on the salient points of the report. He proposed the adoption of the Annual Report and Accounts and called on the Honorary Treasurer, Mr W M Kinghorn to second his proposal.

Revenue and Expenditure Account

In seconding the report, Mr Kinghorn drew attention to the increase in administration, stationery and printing costs.

Further items of expenditure are the annual John Orr Memorial Lecture, branch grants, travelling expenses and following the usual practice bursaries to the amount of R1 800,00.

The situation which existed in previous years of expenses being offset by revenue received from subscriptions and investments no longer exists as the income rate has dropped below the rate of expenditure and an increase in subscriptions has only been avoided by profit received from the Journal and symposia. As these

are not a reliable source of income it will necessitate keeping a closer control over income and expenditure in the future.

The Annual Report and Accounts were adopted.

Awards

The Institution's Awards for papers published during the calendar year 1975 were then announced and were presented to the following persons:

Silver Medal: N G W Cook as co-author of the paper entitled: *Work softening behaviour of brittle materials.*

Certificates: J P M Hojem and C Heins as co-authors of the above paper.

Rand Mines Award: N G W Cook, J P M Hojem and C Heins as co-authors of the paper entitled: *Work softening behaviour of brittle materials.*

Jubilee Award: P B Power, for his paper entitled: *Ventilating freeway tunnels for public safety.*

Thomas Price Award: J van der Walt and D G Kroger as co-authors of the paper entitled: *Hydrodynamics of liquid film down vertical surface.*

Visitors Award: J de Kock, for his paper entitled: *South African coal industry.*

Dr N G W Cook responded suitably on behalf of the recipients.

Honorary Fellowship

Honorary Fellowship of the Institution was conferred upon Professor C J Rallis, Mr C J Hopewell and Mr J G H Loubser, who expressed their appreciation and gratitude of this honour.



Mr A R Adams speaking at the Annual General Meeting

Author Higman C A A

Name of thesis Swirling Flow in Helically Welded Pipes and its Effect on Orifice Flow Meters 1977

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