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Elastic properties of glasses and glass ceramics at high pressures and high temperatures

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have been published using simultaneously combined high pressure and high temperature. According to the measurements made on fused quartz by Levitt (1987) and compared to Kondo et al (1981), this ultrasonic technique is accurate within 0.03%. Thus it could be a step forward in this field of extreme conditions.

1.2 Glasses

Vitreous silica and certain other glasses are anomalous in many respects, including some of their mechanical properties (Uhlmann and Kreidl, 1980). These anomalies can in most cases be plausibly explained as consequences of the existence of "free volume" within the structure of the material. This free volume is associated with the inherent inefficiency of random packing, but more especially with the inability of the bulky tetrahedral SiO_4 groups to pack as closely together as do the spherical atoms of metals and alloys. The attainment of optimum packing is particularly hindered by the extremely high viscosity that is typical of glasses.

Uhlmann and Kreidl (op. cit.) reported that silica and borosilicate glasses increase in compressibility with increasing pressure, contrary to the behaviour of other materials. The higher the silica content, the more pronounced this anomalous behaviour; an aspect that has also been reported by Manghnani (1970). At a certain critical pressure the compressibility was observed to become quite large, and a portion of the compression was found to be irreversible; that is, a permanent densification occurred. Both the anomalous compressibility and the ultimate structural collapse relate basically to the same cause, namely, inefficient packing. The unoccupied space remains potentially available and the application of hydrostatic pressure forces the small shifts of atomic position and flexures of bonds that permit an overall decrease in the volume of the system.

Work has been published which illustrates the effect of the open glass structure on the elastic behaviour of a glass. Sato and Anderson (1980) observed a positive $(\partial B/\partial P)_T$ but a negative $(\partial C_{44}/\partial P)_T$ for a soda-lime - silica glass. They pointed out that the presence of alkali metal ions in the interstices of the silica network inhibited the soft

transverse vibrations. Anomalous pressure or temperature dependences of elastic moduli are observed in crystalline solids which have soft acoustic phonon modes, or, in crystals with low coordination numbers so that bending vibrations are allowed (Brassington, et al., 1981). Brassington et al. investigated an iron phosphate glass and also observed a strongly positive $(\partial B/\partial P)_{P=0}$ (+4.73) and a just negative $(\partial C_{44}/\partial P)_{P=0}$ (-0.16). This means that the longitudinal modes stiffen as usual but the shear modes soften. The elastic behaviour of this glass is similar in kind to that of the soda-lime - silica glass where the transverse vibrations are inhibited by a network modifier. The small but negative value found for $(\partial C_{44}/\partial P)$ implied that for the iron phosphate glass the structure is open enough to allow a degree of bond-bending vibration. For samarium phosphate glasses $\partial B/\partial P$ and $\partial G/\partial P$ are negative and $\partial B/\partial T$ and $\partial G/\partial T$ are positive (Sidek et al., 1988). Although uncommon, such behaviour is known in silica-based glasses and has been attributed to the open fourfold-coordinated structure which enables bending vibrations of the bridging oxygen ions, corresponding to transverse motion against small force constants.

Zachariasen (1932) proposed that the energy content of a substance in the glassy state must not be greatly different from that of the corresponding crystal network. It follows from this that for a glass forming oxide the coordination number of the cation must be closely similar in a glass to that observed in a crystal. This means that the units of structure in a glass and in a crystal will be practically identical. In the crystal these structural units are built up to give a regular lattice but in the glass there is sufficient distortion of bond angles to permit the structural units to be arranged in a non-periodic fashion giving a random network (McMillan, op. cit.). Figure 1.1 shows the differences between the regular crystalline lattice and the random network for an oxide having the formula M_2O_3. In both cases the structural unit is the MO_3 triangle. Thus glasses possess short-range order since the oxygens are arranged in fairly regular polyhedra but long-range order is absent. This view of glass structure is entirely consistent with the liquid-like nature of glass.

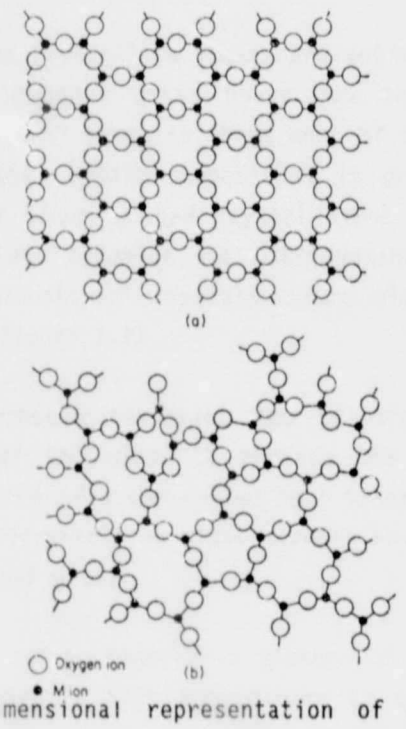


Fig. 1.1 Two-dimensional representation of an oxide M_2O_3 in (a) the crystalline form (b) the glassy form (after McMillan, 1979)

The original use of glass was for decorative purposes. Next glass was used for containers, and this use is still the most important today. Flat glass for windows in buildings and vehicles is now the second largest item of glass manufacture. There are many special applications of glass, namely glass ceramics, fibre glass, mirror blanks of fused silica, glass electrodes for alkali ions, and glass lasers (Doremus, 1973).

1.3 Glass ceramics

Glass ceramics are polycrystalline solids prepared by the controlled crystallisation of glasses (McMillan, 1979). Crystallisation is accomplished by subjecting suitable glasses to a carefully regulated heat-treatment schedule which results in the nucleation and growth of crystal phases within the glass. In many cases, the crystallisation process can be taken almost to completion but a small proportion of residual glass-phase is often present.

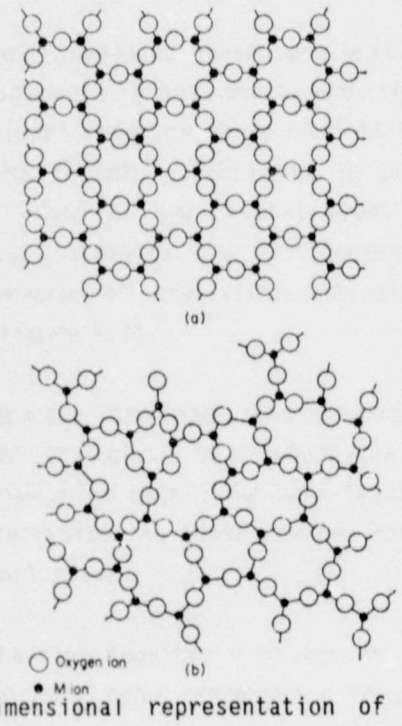


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In glass ceramics, the crystalline phases are entirely produced by crystal growth from a homogeneous glass phase and this distinguishes these materials from traditional ceramics where most of the crystalline material is introduced when the ceramic composition is prepared. However, some recrystallisation may occur or new crystal types may arise due to solid state reactions. Glass ceramics are distinguished from glasses by the presence of major amounts of crystals since glasses are amorphous or non-crystalline (see figure 1.1).

The glass ceramic process comprises the preparation of a homogeneous glass, the shaping of the glass to produce the required articles and, finally, the application of a controlled heat-treatment process to convert the glass into a microcrystalline glass ceramic having properties superior to those of the original glass.

Glasses are made by heating together a mixture of raw materials (known as "batch") at a sufficiently high temperature to permit the materials to react with one another and to permit the refining of the glass. Upon completion of the refining process, the glass is cooled from the melting temperature to the working temperature where the glass has a higher viscosity. Various shaping can then be applied to the glass to produce articles of the required form.

In selecting raw materials the most important aspect to be taken into account is the purity. Economic considerations will also play an important part in the choice of materials. In the case of glass ceramics, materials of quite high purity will be used since some types of impurity, even in quite small concentrations, could affect the crystallisation characteristics of the glass. Undoubtedly, the most important constituent of the glass is silica, since it is the principal glass-forming oxide present in most glasses. Small amounts of alumina maybe present and iron oxide, Fe_2O_3, will certainly be present. CaO and MgO are often introduced in the form of their naturally occurring carbonates, limestone in the case of CaO and magnesite in the case of MgO. The raw materials are accurately weighed out and after thorough mixing are charged into a furnace maintained at the melting temperature which can range from 1 250 to 1 600 °C, depending upon the glass composition.

It is especially important to achieve a high mechanical strength and since this is favoured by a fine-grained microstructure, the aim is to produce a glass ceramic containing crystals of small dimensions which are closely interlocked.

An important feature of the glass ceramic process is that it is applicable to a wide range of compositions and this together with the variations which can be applied in the heat-treatment process, means that various crystal types can be developed in controlled proportions. As a result, the physical characteristics of glass ceramics can be varied in a controlled manner and this fact has an important bearing upon the practical applications of glass ceramics.

Glass ceramics have become established as commercially important materials in fields such as consumer products, vacuum tube envelopes, telescope mirror blanks, radomes for the aerospace industry and protective coatings for metals. Glass ceramics can be regarded as a most valuable addition to the materials available to the design engineer. Being inorganic and non-metallic they combine useful high temperature capabilities with a high degree of chemical stability and corrosion resistance. Their unique combination of properties is likely to make them attractive for a number of specialised engineering applications. The high mechanical strengths of glass ceramics render them suitable for applications in mechanical engineering as special purpose bearings, in pumps, valves and pipes, heat exchangers and furnace construction as well as applications in nuclear engineering and medical fields.

1.4 High pressures and high temperatures

Fujisawa and Ito (1983) developed a new method for measuring the ultrasonic wave velocities in a solid under very high pressures up to 14 GPa. For pressures below 4 GPa, a piston-cylinder apparatus was employed. For pressures above 4 GPa a double stage split-sphere type high pressure apparatus was used. The pulse echo overlap method was used for the wave velocity measurement together with a piston-cylinder apparatus. Earlier, a simple pulse transmission method has been the only applicable method in the higher pressure range. Examples of data for simpler materials, such as NaCl and baked pyrophyllite, were presented.

Other published work is the elastic constants for silver, sodium and cesium chlorides which have been obtained for pressures from 0 to 10 GPa at 298 K (Voronov and Grigor'ev, 1976). An analysis of the results for silver chloride showed the occurrence of a phase transition at 7.1 ± 0.3 GPa in all the elastic properties. All the elastic properties of sodium cesium chlorides rose monotonically with pressure and their pressure coefficients decreased. On comparing the rise of the shear modulus of sodium chloride and of cesium chloride, the former was anomalously small. This was due to the occurrence of a phase transition in sodium chloride at higher pressure.

High pressure and high temperature studies on more complex materials were also produced. Bayuk and Tedeyev (1974) studied the effect of temperature on the longitudinal wave propagation velocity in rock samples at high triaxial pressures up to 0.4 GPa. The apparatus was an arrangement for producing high hydrostatic pressures up to 0.5 GPa as described by Volarovich and his co-workers (1957, 1960). The pressure in the chamber was created by the compression of nitrogen utilizing a system of compressors. The pressure was measured by a spring manometer. A furnace was designed and placed inside the high-pressure chamber to heat the samples to a temperature of 450 °C.

The ultrasonics technique was used to determine elastic constants of steels. Young's modulus, shear modulus and Poisson's ratio for 17-4 PH stainless steel and uranium-2 molybdenum were determined through the temperature range -40 °C to 800 °C by Gieske (1980). Gerlich and Hart (1984b) determined the first pressure derivatives of the elastic moduli for three austenitic stainless steels, AISI 304, 310 and 316.

The pulse-echo-overlap (PEO) method was also used by Rotter and Smith (1966) to measure the single crystal elastic constants of iron and their pressure variation to 3.6 kbar at room temperature.

High pressure and high temperature studies were also done on glasses and glass ceramics. Manghnani (1970) measured the pressure and temperature dependence of the elastic parameters of silicate and aluminate glasses. The pressure system used had been described by Sokolowski and Manghnani

(1969). Nitrogen was used as the pressure medium. The temperature of the specimen was maintained at 25 ± 0.1 °C during pressure measurement up to 0.7 GPa. Temperature dependence of the elastic parameters of the glasses was investigated to 300 °C by employing a well thermally-insulated furnace powered by a solid-state control device.

Gerlich and Wolf (1978) measured the elastic moduli and their temperature and pressure derivatives for Zerodur glass ceramic also using two separate techniques. For measuring the pressure variation between 0 MPa and 70 MPa the sample and transducer were placed in a pressure bomb connected to a gas pressure generator. For the temperature dependence the sample and transducer were placed in a dry gas cell where the temperature was varied between room temperature and 0 °C.

Sigalas, et al (1985) determined the volume static compression of Duran 50 glass (a high silica content, borosilicate glass) by monitoring the piston displacement up to 4 GPa in a static high pressure piston cylinder apparatus. Duran 50 possesses an open crystal strucure based on SiO_4 tetrahedra. This feature as had already been observed for Pyrex (Bridgman, 1924), gives rise sometimes to a decreasing bulk modulus with pressure. However, Kondo, et al. (op. cit.) measured the elastic constants of fused quartz up to 3 GPa and found that the bulk modulus reached a minimum at 2.12 GPa. It is thus surprising that up to 3.6 GPa the bulk modulus of the silica-rich Duran 50 did not show such an effect. Sigalas, et al. (op. cit.) stated that the decrease of the bulk modulus with pressure is attributed to the bending of the bonds rather than their becoming shorter in length. The addition of network modifiers in Duran 50 opens up the bond network, therefore causing the bond-bending effect.

Hart (1983) measured the elastic moduli of two telluride glasses as functions of both low temperature and high pressure. He found normal elastic behaviour in that the pressure derivatives are positive and their temperature dependences are negative.

Kandil, et al. (1984) determined the elastic constants of an yttria-stabilized zirconia ceramic through the temperature range 20 °C to 700 °C using the ultrasonics technique. Here, Kandil and his co-workers

looked at the effect of increasing Y_2O_3 content on C_{11}, C_{12} and C_{44}.

Thorp and Bushell (1985) examined a series of partially nitrided reaction-bonded silicon-nitride (RBSN) ceramics whose weight gains varied from 22% to 64% representing full nitridation. The PEO method was used to measure the longitudinal and shear velocities at 15 MHz and to derive the Young's and bulk moduli at room temperature.

Gerlich and co-workers (1976) used the ultrasonic pulse superposition technique to measure the elastic moduli, and their temperature and pressure derivatives for ULE titanium silicate glass. Anomalous positive temperature and negative pressure derivatives have been found.

2. THEORY

2.1 Introduction

The elastic properties of a material are of great importance in determining its behaviour when subjected to deformation (McMillan, 1979). For example, the modulus of elasticity fixes the levels of stress which are generated if a glass ceramic is strained by the application of a temperature gradient. This can occur when the material is suddenly heated or cooled. In this case a low modulus of elasticity is desirable. Since solids resist both volume change and shape change, they have at least two independent elastic constants, the shear modulus and the bulk modulus.

2.2 Elasticity theory

2.2.1 Strain components

The local elastic strain of a body may be specified by six parameters (Kittel, 1957). If α, β, γ are the angles between the unit cell axes $\underline{a}$, $\underline{b}$, $\underline{c}$, the strain may be specified by the changes $\Delta\alpha$, $\Delta\beta$, $\Delta\gamma$; Δa, Δb, Δc resulting from the deformation. Consider three orthogonal axes $\underline{f}$, $\underline{g}$ and $\underline{h}$ of unit length, imbedded in the unstrained solid shown in figure 2.1. After a small deformation has taken place the axes, which are now labelled $\underline{f}'$, $\underline{g}'$, $\underline{h}'$, are distorted in orientation and length. With the same atom as origin:

$$\begin{aligned} \underline{f}' &= (1 + \epsilon_{xx})\underline{f} + \epsilon_{xy}\underline{g} + \epsilon_{xz}\underline{h} \\ \underline{g}' &= \epsilon_{yx}\underline{f} + (1 + \epsilon_{yy})\underline{g} + \epsilon_{yz}\underline{h} \\ \underline{h}' &= \epsilon_{zx}\underline{f} + \epsilon_{zy}\underline{g} + (1 + \epsilon_{zz})\underline{h} \end{aligned} \qquad (2.1)$$

The fractional changes of length of the $\underline{f}$, $\underline{g}$ and $\underline{h}$ axes are ϵ_{xx}, ϵ_{yy} and ϵ_{zz}, respectively, to the first order. The strain components e_{xx}, e_{yy} and e_{zz} are defined by the relations:

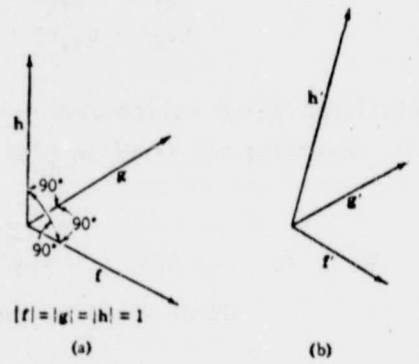


Fig. 2.1 Coordinate axes for the description of the state of strain; the orthogonal unit axes in the unstrained state (a) are deformed in the strained state (b)

$$e_{xx} = \epsilon_{xx};\quad e_{yy} = \epsilon_{yy};\quad e_{zz} = \epsilon_{zz} \qquad (2.2)$$

The strain components e_{xy}, e_{yz} and e_{zx} to the first order may be defined as the changes in angle between the axes:

$$\begin{aligned} e_{xy} &= \underline{f}'.\underline{g}' = \epsilon_{yx} + \epsilon_{xy} \\ e_{yz} &= \underline{g}'.\underline{h}' = \epsilon_{zy} + \epsilon_{yz} \\ e_{zx} &= \underline{h}'.\underline{f}' = \epsilon_{zx} + \epsilon_{xz} \end{aligned} \qquad (2.3)$$

This completes the definition of the six strain components. A deformation is called uniform if the values of the strain components are independent of the location of the origin.

Note that merely rotating the axes does not change the angle between them. Using equation (2.3) we see that for a pure rotation $\epsilon_{yx} = -\epsilon_{xy}$; $\epsilon_{zy} = -\epsilon_{yz}$; $\epsilon_{zx} = -\epsilon_{xz}$. If we exclude pure rotations as not being of interest here, we may without further loss of generality take $\epsilon_{yx} = \epsilon_{xy}$; $\epsilon_{zy} = \epsilon_{yz}$; $\epsilon_{zx} = \epsilon_{xz}$, so that

in terms of the strain components we have:

$$\begin{aligned} \underline{f}' - \underline{f} &= e_{xx}\underline{f} + \tfrac{1}{2}e_{xy}\underline{g} + \tfrac{1}{2}e_{zx}\underline{h} \\ \underline{g}' - \underline{g} &= \tfrac{1}{2}e_{xy}\underline{f} + e_{yy}\underline{g} + \tfrac{1}{2}e_{yz}\underline{h} \\ \underline{h}' - \underline{h} &= \tfrac{1}{2}e_{zx}\underline{f} + \tfrac{1}{2}e_{yz}\underline{g} + e_{zz}\underline{h} \end{aligned} \qquad (2.4)$$

We consider, under a deformation being substantially uniform near the origin, a particle originally at the position:

$$\underline{r} = x\underline{f} + y\underline{g} + z\underline{h} \qquad (2.5)$$

After deformation the particle is at:

$$\underline{r}' = x\underline{f}' + y\underline{g}' + z\underline{h}' \qquad (2.6)$$

so that the displacement is given by:

$$\underline{\chi} = \underline{r}' - \underline{r} = x(\underline{f}' - \underline{f}) + y(\underline{g}' - \underline{g}) + z(\underline{h}' - \underline{h}) \qquad (2.7)$$

If we write the displacement as:

$$\underline{\chi} = u\underline{f} + v\underline{g} + w\underline{h} \qquad (2.8)$$

we have from equations (2.4) and (2.7) the following expression for the strain components:

$$\begin{aligned} &e_{xx} = \partial u/\partial x; \quad e_{yy} = \partial v/\partial y; \quad e_{zz} = \partial w/\partial z; \\ &e_{xy} = \partial v/\partial x + \partial u/\partial y; \quad e_{yz} = \partial w/\partial y + \partial v/\partial z; \\ &e_{zx} = \partial u/\partial x + \partial w/\partial z \end{aligned} \qquad (2.9)$$

Derivatives have been written for application to non-uniform strain. The expressions (2.9) are frequently used in the literature to define strain components. For a uniform deformation the displacement $\underline{\chi}$ has the components:

$$\begin{aligned} u &= e_{xx}x + \tfrac{1}{2}e_{xy}y + \tfrac{1}{2}e_{zx}z; \\ v &= \tfrac{1}{2}e_{xy}x + e_{yy}y + \tfrac{1}{2}e_{yz}z; \\ w &= \tfrac{1}{2}e_{zx}x + \tfrac{1}{2}e_{yz}y + e_{zz}z \end{aligned}$$

2.2.2 Stress components

The force acting on a unit area in the solid is defined as the stress. There are nine stress components: X_x, X_y, X_z, Y_x, Y_y, Y_z, Z_x, Z_y, Z_z. The capital letter indicates the direction of the force, and the subscript indicates the normal to the plane to which the force is applied. Thus the stress component X_y represents a force applied in the x direction to a unit area of a plane whose normal lies in the y direction, etc. (See figure 2.2). The number of independent stress components is reduced to six by applying to an elementary cube (figure 2.2) the condition that the angular acceleration vanishes, and hence that the total torque must be zero. It follows that:

$$Y_z = Z_y; \quad Z_x = X_z; \quad X_y = Y_x$$

and the independent stress components may be taken as X_x, Y_y, Z_z, Y_z, Z_x, X_y.

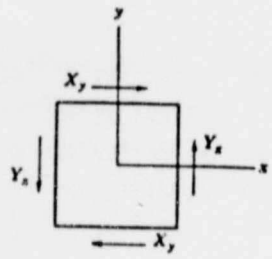


Fig. 2.2 Demonstration that $Y_x = X_y$ in order that the body may be in equilibrium

The elastic coefficients C_{ijkl} and s_{ijkl} which are symmetrical in the first pair and last pair of indices makes it possible to simplify the notation by changing from the three-dimensional fourth-rank tensor to a six dimensional matrix, whose elements (as for any matrix) are governed by two subscripts. This is the Voigt notation. This transition is performed by replacing pairs of subscripts with the values 1, 2, 3 by one taking the values 1, 2, 3, 4, 5, 6 as follows, with allowance for the symmetry of the pairs:

tensor notation	11	22	33	23 32	13 31	12 21
matrix notation	1	2	3	4	5	6

Therefore Hooke's law, which states that for small deformations the strain is proportional to the stress, so that the strain components are linear functions of the stress components, can be written as follows:

$$
\begin{aligned}
e_{xx} &= s_{11}X_x+s_{12}Y_y+s_{13}Z_z+s_{14}Y_z+s_{15}Z_x+s_{16}X_y\\
e_{yy} &= s_{21}X_x+s_{22}Y_y+s_{23}Z_z+s_{24}Y_z+s_{25}Z_x+s_{26}X_y\\
e_{zz} &= s_{31}X_x+s_{32}Y_y+s_{33}Z_z+s_{34}Y_z+s_{35}Z_x+s_{36}X_y\\
e_{yz} &= s_{41}X_x+s_{42}Y_y+s_{43}Z_z+s_{44}Y_z+s_{45}Z_x+s_{46}X_y\\
e_{zx} &= s_{51}X_x+s_{52}Y_y+s_{53}Z_z+s_{54}Y_z+s_{55}Z_x+s_{56}X_y\\
e_{xy} &= s_{61}X_x+s_{62}Y_y+s_{63}Z_z+s_{64}Y_z+s_{65}Z_x+s_{66}X_y
\end{aligned}
\qquad (2.10)
$$

Conversely, the stress components are linear functions of the strain components:

$$
\begin{aligned}
X_x &= C_{11}e_{xx}+C_{12}e_{yy}+C_{13}e_{zz}+C_{14}e_{yz}+C_{15}e_{zx}+C_{16}e_{xy}\\
Y_y &= C_{21}e_{xx}+C_{22}e_{yy}+C_{23}e_{zz}+C_{24}e_{yz}+C_{25}e_{zx}+C_{26}e_{xy}\\
Z_z &= C_{31}e_{xx}+C_{32}e_{yy}+C_{33}e_{zz}+C_{34}e_{yz}+C_{35}e_{zx}+C_{36}e_{xy}\\
Y_z &= C_{41}e_{xx}+C_{42}e_{yy}+C_{43}e_{zz}+C_{44}e_{yz}+C_{45}e_{zx}+C_{46}e_{xy}\\
Z_x &= C_{51}e_{xx}+C_{52}e_{yy}+C_{53}e_{zz}+C_{54}e_{yz}+C_{55}e_{zx}+C_{56}e_{xy}\\
X_y &= C_{61}e_{xx}+C_{62}e_{yy}+C_{63}e_{zz}+C_{64}e_{yz}+C_{65}e_{zx}+C_{66}e_{xy}
\end{aligned}
\qquad (2.11)
$$

The coefficients s_{11}, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}, ... are called the elastic compliances or elastic moduli; the coefficients C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, ... are called the elastic stiffnesses or elastic constants. Other names are also current.

The definition of energy density can be used to find the general relation:

$$C_{ij} = C_{ji}$$

between the elastic stiffnesses in equation (2.11). The thirty-six elastic stiffnesses are in this way reduced to twenty-one. Similar relations hold among the elastic compliances. The matrix of the C's or s's is therefore symmetrical.

2.2.3 Elastic constants

a) The C°_{11} and C°_{44} elastic constants (superscript zero indicates quasi-isotropic) follow from single crystal notation for the Voigt elastic-stiffness constants, the C_{ij}'s. Cubic crystals, for example, have three independent C_{ij}'s: C_{11}, C_{12} and C_{44}. Here the relations for the elastic constants are:

$$\begin{aligned} &C_{11} = C_{22} = C_{33} \\ &C_{44} = C_{55} = C_{66} \text{ and} \\ &C_{12} = C_{13} = C_{23} = C_{32} = C_{31} = C_{21} \\ &C_{14} = C_{25} = C_{36} = C_{63} = C_{52} = C_{41} = 0 \\ &C_{45} = C_{56} = C_{64} = C_{46} = C_{65} = C_{54} = 0 \\ &C_{15} = C_{26} = C_{34} = C_{43} = C_{62} = C_{51} = C_{16} \\ &\quad = C_{24} = C_{35} = C_{53} = C_{42} = C_{61} = 0 \end{aligned} \qquad (2.12)$$

It is possible to convert the notation for stresses and strains to a more convenient one for isotropic materials. When equations (2.10) and (2.11) expressing Hooke's law are written in the Voigt notation, they can each be contracted to a single equation:

$$S_i = s_{ij}T_j \qquad (2.13a)$$

and

$$T_i = C_{ij}S_j \ (i,j = 1, 2, 3, \ldots \text{ or } 6) \qquad (2.13b)$$

Here S_i and T_i represent the six strain and the six stress components, respectively, and i,j = 1, 2, 3, ... or 6).

Single crystals are not elastically isotropic. The only strictly isotropic materials are well-annealed glasses and similar non-crystalline materials; many polycrystalline materials with random orientation of individual grains are found to be effectively isotropic in samples of adequate size. Among such quasi-isotropic materials are most metals, sintered ceramics and even rocks.

In an isotropic aggregate the elastic constants are identical to those in the cubic system, together with an additional relation between them, i.e.

$$\tfrac{1}{2}(C_{11} - C_{12}) = C_{44} \qquad (2.14)$$

This means that two independent constants will be sufficient for specifying the elastic properties. Usually either the Lamé constant C_{12}, or Young's modulus E and the shear modulus G, are used (Hearmon, 1961). From the isotropic equations involving the stiffnesses it can be derived that

$$L = C_{11} = C_{12} + 2G; \quad \tfrac{1}{2}(C_{11} - C_{12}) = C_{44} = G \qquad (2.15)$$

where L is the longitudinal modulus.

b) From the equations involving the compliances follows the relation

$$2(s_{11} - s_{12}) = 2(1 + \sigma)/E = 1/G$$

(σ is Poisson's ratio) that is,

$$G = \tfrac{1}{2}E/(1 + \sigma) \qquad (2.16)$$

which is a familiar relation between the elastic parameters of an isotropic solid.

c) The relation of elastic constants to the bulk modulus (B) is useful to bridge the gap between the independent elastic constants determined from ultrasonic velocities and B which is used to define the equation of state:

$$B_T = -V(\partial P/\partial V)_T \qquad (2.17)$$

Subscript T denotes a specific temperature, V is volume and P is pressure. This condition of hydrostatic pressure is satisfied in the case of ultrasonic measurements, since the pressure is truly hydrostatic (fluid pressure media) or at least quasi-hydrostatic (solid pressure media). The application of hydrostatic stress conditions to the generalized Hooke's law leads to considerable simplification.

By definition, the stress system corresponding to hydrostatic pressure is

$$T_1 = T_2 = T_3 = -P$$
$$T_4 = T_5 = T_6 = 0$$

Hooke's law then becomes

$$\begin{aligned} S_1 &= -(s_{11} + s_{21} + s_{31})P \\ S_2 &= -(s_{12} + s_{22} + s_{32})P \\ S_3 &= -(s_{13} + s_{23} + s_{33})P \\ S_4 &= -(s_{14} + s_{24} + s_{34})P \\ S_5 &= -(s_{15} + s_{25} + s_{35})P \\ S_6 &= -(s_{16} + s_{26} + s_{36})P \end{aligned} \qquad (2.18)$$

A general change in angles as well as volume of the body is effected by applying hydrostatic pressure. Compressibility (K) is defined by

$$\begin{aligned} K &= -(S_1 + S_2 + S_3)/P \\ &= s_{11} + s_{22} + s_{33} + 2(s_{12} + s_{23} + s_{13}) \end{aligned} \qquad (2.19)$$

By relating the compliances and stiffnesses and by establishing the relations between the elastic constants, K can be calculated. For the cubic system it follows,

$C_{11} = (s_{11} + s_{12})/X_3$; $C_{12} = -s_{12}/X_3$, and

$C_{44} = 1/s_{44}$,

where $X_3 = (s_{11} - s_{12})(s_{11} + 2s_{12})$

and in addition for the isotropic system

$$C_{44} = \tfrac{1}{2}(C_{11} - C_{12}) = 1/(2(s_{11} - s_{12})). \qquad (2.20)$$

In the case of a cubic system

$$\begin{aligned} s_{13} &= s_{12} \qquad & s_{44} &= 2(s_{11} - s_{12}) \\ s_{22} &= s_{11} \qquad & s_{55} &= 2(s_{11} - s_{12}) \\ s_{23} &= s_{12} \qquad & s_{66} &= 2(s_{11} - s_{12}) \end{aligned} \qquad (2.21)$$

By substituting equations (2.21) in equation (2.19) it follows

$$K = 3(s_{11} + 2s_{12}) \qquad (2.22)$$

Thus from equation (2.20) for the cubic system,

$$C_{11} + 2C_{12} = 1/(s_{11} + 2s_{12})$$

Thus

$$B = 1/K = (1/3)(C_{11} + 2C_{12}) \qquad (2.23)$$

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