

DEVELOPMENT AND APPLICATION OF GENERAL CIRCUIT THEORY TO SUPPORT CAPACITIVE COUPLING

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Declaration

I declare that this dissertation is my own unaided work. It is being submitted to the degree of Master of Science in Engineering at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other university.

Signed on: ____ day of _____ 20____

Abstract

In textbook literature, the phenomenon of mutual inductance has been described in rigorous detail from the magnetic field-level behaviour all the way to equivalent circuit models, and these are valid for circuits consisting of any number of coils where there may be magnetic flux linkage. Unlike mutual inductance, the description of multi-body systems which exhibit electric flux coupling has not been carried through from a field level to equivalent circuit models in the same way. Most circuit models used to describe capacitive coupling are therefore different and cannot be easily compared.

In this dissertation, a general circuit model describing capacitive coupling is developed from field-level theory. This model is based on a four-body physical structure, and forms a restricted dual to the well-known two-body inductive coupling circuit model. A quantity representing coupling capacitance was defined and given the symbol S , and this quantity is the dual to the mutual inductance term commonly referred to by the symbol M in textbook literature. An in-depth analysis is documented into the coupling capacitance term S , showing that it is possible to obtain a system which exhibits positive, negative or zero coupling. Experimental verification was done for systems exhibiting zero coupling, 100 % coupling and arbitrary coupling. For all cases, the experimental results had very good agreement to the values predicted using the capacitive coupling circuit model.

The circuit properties of the capacitive coupling model hold in the same way as it does for inductive coupling, as expected of a dual model. In general, interconnections of capacitive coupled networks can also be made as long as specific conditions are met.

A concise discussion into different possible practical applications of the circuit model is then provided, together with circuit diagrams. This is followed by a detailed discussion into a condition-monitoring application for inductive (power) transformers. It is shown theoretically and experimentally that the capacitive coupling circuit model can be used in condition-monitoring of power transformers to detect mechanical movements of coils.

The dissertation is concluded with a discussion on possible future work.

Opsomming

Die verskynsel van onderlingeinduktansie is al breedvoerig in die literatuur beskryf vanuit beide die veldteorie sowel as ekwivalente stroombaanmodelle. Hierdie stroombaanmodelle is geldig vir enige aantal spoel waar daar gemeenskaplike magnetiese vloedkoppeling voorkom. In teenstelling hiermee is daar geen soortgelyke beskrywing van die elektriese vloedkoppeling van veelgigaamstelsels waar die veldteorie deurgetrek word na die stroombaan model nie. Die meeste stroombaanmodelle wat gebruik word om kapasitiewe koppeling te beskryf verskil en dit is dus moeilik om verskillende stelsels te vergelyk.

In hierdie verhandeling word 'n ekwivalente stroombaanmodel wat kapasitiewe koppeling beskryf vanuit veldteorie afgelei. Hierdie model is gebaseer op 'n vierliggaam-fisiese struktuur. Vanuit 'n terminale perspektief vorm dit 'n sogenaamde beperkte dualis van die welbekende onderlingeinduktansie stroombaan model met twee spoel (liggame). 'n Grootheid wat die koppelingkapasitasie verteenwoordig is gedefinieer en word die simbool S gegee. Hierdie grootheid is tweeledig tot die onderlingeinduktansie term wat in teksboekliteratuur algemeen deur die simbool M voorgestel word. 'n Diepgaande ontleding van die koppelingkapasitasie term S , is gedokumenteer en toon aan dat dit moontlik is om 'n stelsel wat positiewe, negatiewe of nul-koppeling toon te verkry. Eksperimentele verifikasie is gedoen vir stelsels wat nul-koppeling, 100 %-koppeling en arbitrêre koppeling vertoon. Deur gebruik te maak van die kapasitiewekoppeling-stroombaanmodel het die eksperimentele uitslae in al die gevalle baie goed ooreengestem met die voorspelde waardes .

Die stroombaaneienskappe van die kapasitiewekoppelingmodel is soortgelyk aan die vir induktiewekoppeling, soos verwag sou word van 'n dualis. Interbindings van kapasitief-gekoppelde netwerke is ook moontlik solank bepaalde toestande geld.

'n Bondige bespreking van verskillende moontlike werklike toepassings van die stroombaanmodel word vervolgens gegee, tesame met stroombaandiagramme. Dit word gevolg deur 'n dieper bespreking van 'n kondisie-monitering toepassing vir induktiewe (krag-) transformators. Die verhandeling sluit af met 'n bespreking van moontlike toekomstige werk.

ملخص

ان ظاهرة الحث المتبادل مفهومة على نطاق واسع وموثقة في الكتب المنهجية بشكل جيد. فقد وصفت بتفصيل دقيق من سلوك مستوى المجال المغناطيسي الى النماذج المعادلة للدوائر وهي دقيقة للدوائر التي تتكون من أي كمية من الملفات التي من الممكن ان تكون في توصيل التدفق المغناطيسي.

ومشابهها للحث المتبادل، فان نظم الاجسام المتعددة التي تعرض ربط التدفق الكهربائي قد وصفت بشكل دقيق على مستوى المجال الا ان هذا الوصف لم يتم من خلال وصف مستوى المجال المغناطيسي مروراً بالنماذج المعادلة للدوائر كما هي الحالة التي وصفت بها ظاهرة الحث المتبادل. هذا يعني: اولاً، ان معظم النماذج المعادلة للدوائر الموظفة من اجل وصف الربط السعوي مختلفة، وبالتالي فانه من الصعب مقارنة انظمة مختلفة معتمداً على نفس النموذج. ثانياً، ان العلاقة ما بين نموذج الدائرة ومستوى نشاط المجال غير واضح أحياناً.

في هذه الأطروحة، ان النموذج المعادل للدائرة والذي يصف الربط السعوي سوف يستخلص من نظرية مستوى المجال. ويستند هذا النموذج على هيكل فيزيائي رباعي، ويشكل ثنائية معروفة للجسمين الحثيين المرتبطين بنموذج الدائرة من منظور الربط النهائي. وقد عرفت كمية تمثل الربط السعوي واعطيت الرمز S ، وهذه الكمية هي ثنائية لمصطلح الحث المتبادل ويشار اليها عادة بالرمز M في الكتب المنهجية.

وقد تم توثيق تحليل معمق لمصطلح الربط السعوي S ، مبيناً أنه من الممكن الحصول على نظام يمكن ان يعرض ربط ايجابي او سلبي او منعدم. كذلك تم اظهار انه وبسبب العزل الكهربائي الواضح بين روابط الإدخال والإخراج لا يوجد أهمية للإشارة بمفهوم المصطلح S . وقد تم التحقق من تجريبية الأنظمة التي تعرض ربط منعدم، او ربط 100%، او ربط متراوح. وفي جميع الحالات، فإن النتائج التجريبية متفقة بشكل جيد للغاية مع القيم المتنبأ بها عند استخدام نموذج الربط السعوي للدائرة.

وقد تم استكشاف خصائص دائرة الربط السعوي ووجدت بانها تعمل بنفس الطريقة التي تعمل في الربط الحثي كما هو متوقع من النموذج الثنائي. وبشكل عام تم الكشف عن ان عملية الربط السعوي للشبكات ممكنة بتوافر شروط معينة.

كما تم التزويد بمناقشة موجزة عن مختلف التطبيقات التجريبية الممكنة لنموذج الدائرة وبمخططات نموذج الدائرة. واعقب هذا مناقشة تفصيلية تدور حول شروط تطبيق رصد المحولات (القوى) الحثية. وثبت نظرياً ومخبرياً ان نموذج الربط السعوي للدائرة يمكن ان يستخدم في شروط رصد محولات القوى لتحري حركات ميكانيكية للملفات. وختمت الأطروحة بمناقشة حول الاعمال المحتملة في المستقبل.

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Chapter 1: Background and Problem Statement

1. Introduction and Problem Statement

The content of this chapter begins with a description of the problem statement and a literature review of current work in this field. Details regarding the research methodology are then provided, followed by the conclusion.

In textbook theory, the subject of magnetostatics has been described in rigorous detail from the field level behaviour all the way to electric circuit models. The field level behaviour is represented by a lumped element called an inductor at a circuit level. This component represents a quantity called inductance, which has a direct link back to the physical properties of the system. This theory is very general in that it remains valid for any arbitrary number of coils to make up the system. For a single coil system, the inductance is usually termed as the self-inductance of that coil. However, in a multiple-coil system where there may be a certain amount of magnetic coupling between the coils, each inductor interacts with the field. This interaction between coils leads to an added component of inductance seen at the terminals of each inductor, which is commonly referred to as mutual inductance. As with self-inductance, mutual inductance is well described and the link has been made from the field-level theory all the way to its circuit representation.

Using this general theory, many different application-specific circuit models have been developed. The limitations of these models have been explored and experimentally verified using multiple approaches; the results of which are generally consistent. This theory has been used in many physical systems, including high-power applications (examples include electric power transformers and electric motors and drives) as well as low-power applications (examples include wireless charging and communication).

Similarly, the subject of electrostatics has also been well documented in textbook theory. The behaviour of electrostatics at a field level has been rigorously described and modelled. This was done for both single and multiple body (a conductive structure in space) systems, and analogies can be drawn between the electrostatic and magnetostatic systems in this regard. In order to represent the electrostatic behaviour at a circuit level, a lumped element called a capacitor was defined. Capacitors represent a quantity called capacitance, which has a direct link back to the physical properties of the system. Unlike with inductance, however, no consistent definition for “mutual capacitance” can be found for a multi-body system with electric field coupling using an approach similar to mutual inductance.

The lack of a simple, consistent method for dealing with multi-body capacitive coupling at a circuit level has meant that designers generally have had to develop individual application-specific circuit models in every instance. This is unlike inductive coupling where the same circuit model is used for every application. While the former approach presents a more challenging design procedure, it also has the disadvantage of producing different circuit models for different applications. This means that it is difficult to compare different physical systems in the same way that it can be done for systems based on inductive coupling. Many of the application-specific models give good agreement with experimental data, but very few have a direct link from the field level all the way to the circuit level. These attempts are also generally concerned with the constraints of the specific application rather than supporting general circuit theory.

There is therefore a missing link in current circuit theory for systems with multiple bodies which exhibit capacitive coupling behaviour. For this link to be made, a general circuit model must be developed similar to the inductive coupling model, and this model must have its basis in field theory. It is this general model which is developed in this dissertation, and the boundaries of its validity are explored.

The theory developed in this dissertation is restricted to a four body system, i.e. a system with two sets of terminals as shown in Figure 1. This is in order to compare to a two-body inductively coupled system (a system also with two sets of terminals), as shown in Figure 2.

The detailed reasoning behind the choice of four bodies for the capacitive coupling circuit model will become clearer in later chapters.

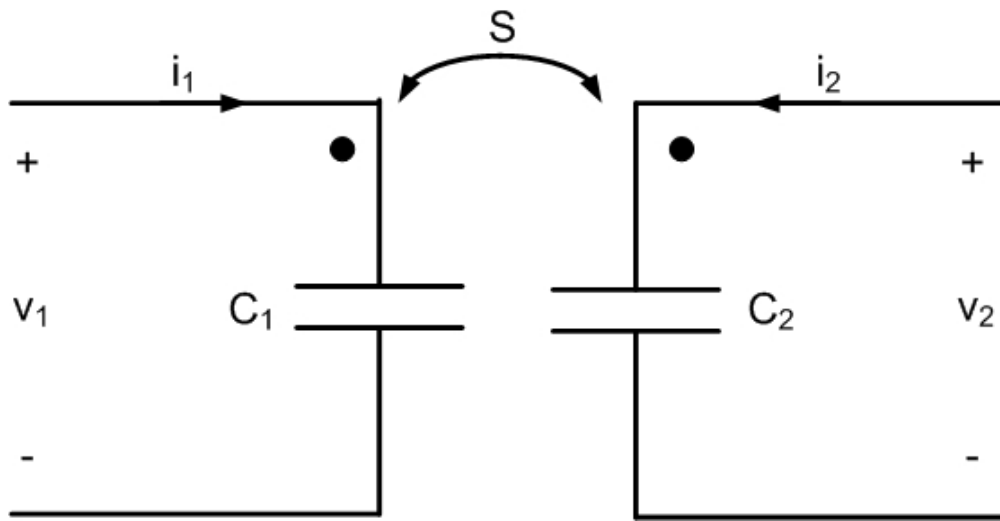


Figure 1: Capacitive Coupling

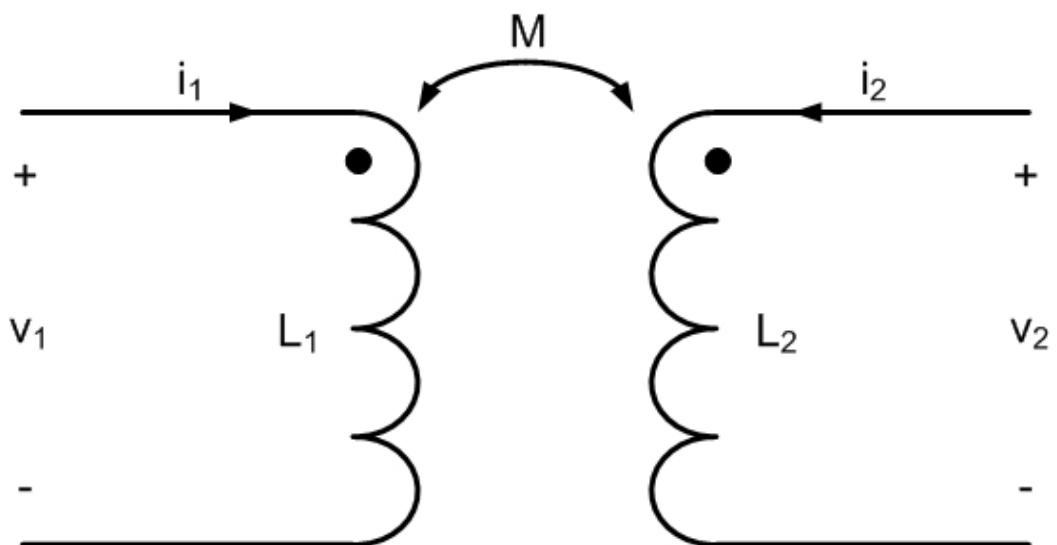


Figure 2: Inductive Coupling

It must be noted that a mixed physical-circuit notation is often used in this dissertation. This is a relaxed notation in that it mixes the field level behaviour and circuit level symbols, and is simply used to help show the link from the field level to the circuits domain. The final model lies entirely in the circuits domain.

In the following section, a review of the various traditional methods of dealing with the problem of capacitive coupling is provided.

2. Literature Review

In order to describe the electric fields present in a system, it is helpful to model it in some way at a circuit level. This model allows the designer to both predict an output to a given input, as well as vary parameters within the model to try and attain a specific outcome. There have been various circuit models proposed in the past to deal with electric-field coupling, and these models are usually concerned with the constraints of the particular application at hand. In this section, a review of some of the proposed models is provided, and the limitations of these models are highlighted.

Method 1: Elementary Model

Capacitive coupling is commonly a parasitic effect; and one of the typical cases in which it arises is that of inter-winding capacitance in high frequency inductors. A physical application of this is in a high frequency 1:1 transformer as shown in Figure 3.

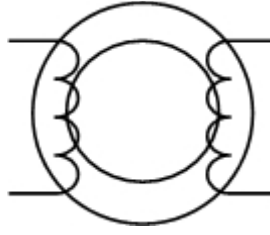


Figure 3: High Frequency Isolation Transformer

It is common in such a system to model the capacitance in a very elementary way, i.e. to lump the distributed interwinding capacitance into a single capacitor which is connected across the terminals of the inductor at a circuit level, as shown in Figure 4.

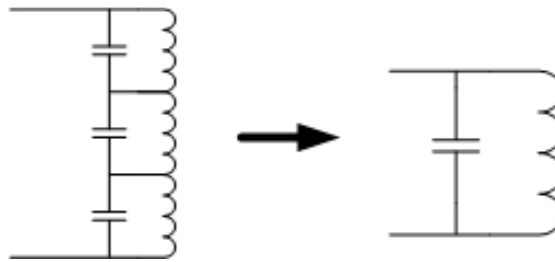


Figure 4: Lumped Capacitor Model

This type of modelling is often done in order to represent inductive coupling losses arising from the displacement current flowing through the capacitors; and is generally a measured value (as opposed to a prediction based on geometry). The elementary model is solely used to quantify the coupling so that it can be reduced by some other mechanism. Since it is a measured value, it cannot be reduced/maximised by design, so a circuit is designed to cancel its effect, i.e. it is a re-active approach to the problem of capacitive coupling. One such case was documented by Wang and Lee [1] where a model similar to Figure 4 was used for a high frequency inductor.

Using such an elementary model, very little insight is achieved into the cause of the parasitic capacitance. If this information was available, it may be possible for

the designer to design the inductor such that its inductive properties remain, but the parasitic capacitance effect is minimised. This could possibly alleviate the need for a cancellation circuit altogether, but would require a more comprehensive circuit model.

This form of elementary modelling is therefore extremely limited in its usefulness, and more comprehensive modelling of capacitively coupled systems is required.

Method 2: Trend-based Optimisation

It was shown above that lumped capacitor modelling does not provide much insight into the cause of coupling capacitance. It would be more useful to use an approach where the coupling capacitance can be related to the physical geometry of the structure. It is well known that capacitance is a quantity dependent on the geometrical parameters of a structure. Therefore, with even the simplest of capacitive coupling models, changes in the geometrical parameters should affect the coupling which occurs.

Using this basis, Liu et al. developed a circuit model for a capacitive power transfer system [2] as shown in Figure 5.

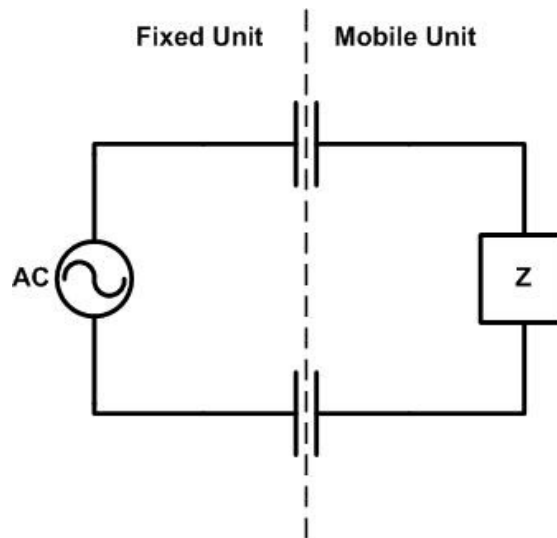


Figure 5: Typical Capacitive Power Transfer Device

The premise of their argument was that a capacitive power transfer system consists of an input circuit with a power source, which is open circuited using two plates (with zero coupling between the plates). An output circuit with a load is also open circuited with two plates that have zero coupling between the plates. When the plates of the input and output circuits are placed on top of each other, coupling occurs between the input and output circuits, forming the circuit shown in Figure 6.

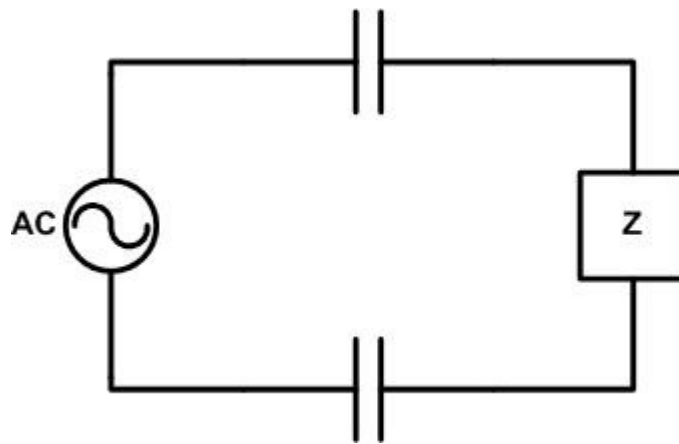


Figure 6: Typical Power Transfer Circuit

Using this circuit model, the authors developed expressions for the capacitors in Figure 6 which were functions of physical alignment parameters that were variable in two dimensions. The authors had the aim of maximising power transfer to the load, and found that in order to use this type of circuit effectively, the plates (one in the input circuit and the other in the output circuit), had to be aligned perfectly, or less power transfer will occur due to less coupling.

The method presented by the authors to develop a capacitive power transfer circuit model offers a simple circuit-level description of the capacitive coupling behaviour that occurs, as well as the way it changes as the alignment is varied. Based on their experimental results, it seems to tie in well with a physical system and therefore the model can be said to be valid for a specific region of application.

It is perhaps important to consider the disadvantages and implications of such a model to highlight its limits of operation. One of the most fundamental assumptions implicitly made by the authors is that the only coupling which occurs is between the plates connecting the input and output circuits, and there is zero coupling between each plate to every other plate. This assumption may be reasonable if there is excellent coupling between the plates connecting the input and output circuits, however, as this coupling is reduced by varying the geometry, this assumption may no longer be valid. The implication of such an assumption is at the very least a discrepancy in terms of what the model predicts and actual results; in the extreme case the model will not be applicable at all.

A second drawback of the model is that it is based on a 2-D alignment analysis of the plates making up the system. It can be expected that most practical power transfer systems will have a 3-D alignment variability and in this case the model cannot be used at all. The model presented by the authors cannot be easily extended to allow for 3-D analysis.

The method therefore has an area of application, however, it is inefficient in its design and is based on certain fundamental assumptions of which the designer must be aware. While it is more comprehensive than the Method 1 discussed earlier, it is not representative of all the coupling which occurs, and therefore cannot be said to be a general representation of an arbitrary four body system.

Method 3: Generalised Model

It was shown above that there is a need for a general model which can represent arbitrary geometries in a capacitively coupled system. There have been attempts in the past to develop a general model, each with different levels of success. One such model was listed as a curiosity in the Problems section of [9], with no further discussion. In its form, however, it appears to be similar to those discussed in the papers below.

In [3], Yang proposed the method of using the duality principle in order to develop a general capacitive coupling circuit model as a dual to the inductive coupling model. The duality principle is discussed in detail in Chapter 2. Since the mutual inductance in the inductive coupling circuit is an inductance, he postulates that there should exist a dual quantity to this, which he calls a “mutual capacitance”. Also, the duality dictates that the capacitive coupling circuit model should take the form of the dual to the inductive coupling circuit, i.e. the equations take the same form, and the “mutual capacitance” refers the currents/voltages from the one side of the circuit to other; and there is full electrical isolation between the two sides.

Using this premise, he developed a circuit model to represent the capacitive coupling which occurs in a four-body system. His circuit model is shown in Figure 7 with the “mutual capacitance” term given the symbol M_C . To support the premise of duality, he shows that certain properties of the defined “mutual capacitance” quantity are similar to those of mutual inductance.

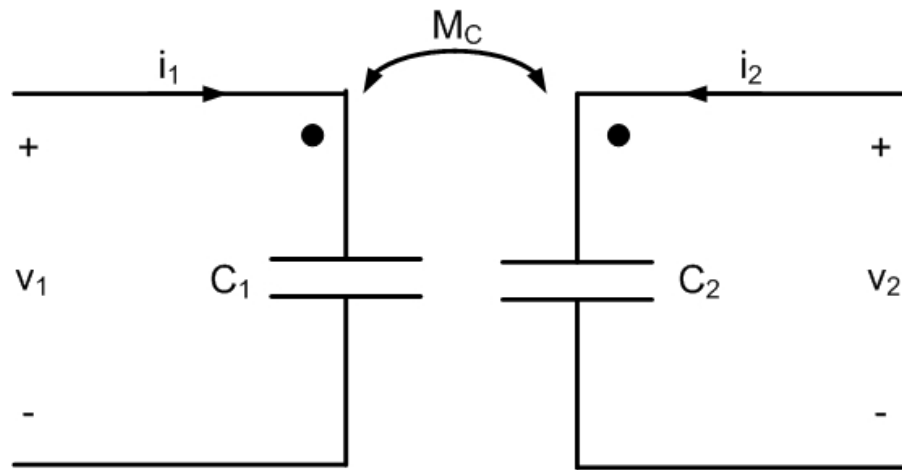


Figure 7: Dual Capacitive Coupling Model

Although Yang appears to have a promising basis for his work, the method is incoherent, making it difficult to engage with his results. It is perhaps notable

however, that his paper appears to be one of the first pieces of work suggesting a dual circuit to inductive coupling. The duality approach has since been widely used by researchers attempting to develop general circuit theory to support capacitive coupling.

In attempting to derive a general capacitive coupling model, Liu et al [4] used a similar approach to Yang [3]. Also based on the duality principle, the authors took a general four-body system and neglected the self-capacitances of the bodies, which gives rise to a network of six capacitors between the four bodies. This is shown in Figure 8.

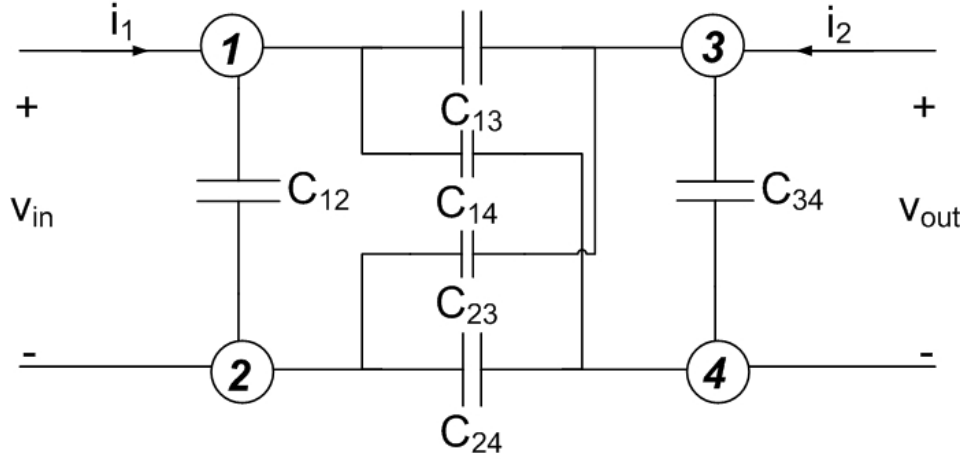


Figure 8: Six Capacitors between four bodies

The authors then simplified the circuit in Figure 8 to obtain the layout of the circuit in Figure 7, To do this, they developed a “mutual capacitance” term as a function of the capacitors joining the input to the output terminals, namely C_{14} , C_{13} , C_{23} and C_{24} . Using this mutual term, they developed a circuit of the form shown in Figure 7, with the terms shown in (1) to (3).

$$C_1 = C_{12} \tag{1}$$

$$C_2 = C_{34} \tag{2}$$

$$M_C = f(C_{13}, C_{24}, C_{23}, C_{14}) \quad (3)$$

In the expressions above, C_1 is analogous to the self-inductance of coil 1 in the inductive coupling circuit, C_2 to the self-inductance of coil 2 and M_C to the mutual inductance between the coils. This representation takes the form of the dual to the inductive coupling model, and it is this model which is presented as a generalised coupling representation. This circuit was identified as a two-port network, and using network theory a simplified π circuit model was developed with a common reference between the input and output stages.

In developing the two port circuit model in Figure 7 with expressions (1) to (3), the authors assumed that expressions (1) and (2) are true, without doing further circuit analysis. It is shown by papers reviewed in the following paragraphs that this is in fact not true. This has significant consequences in that voltages and currents cannot be predicted correctly using their model. The assumption therefore presents a fundamental flaw in the validity of the model.

A second assumption made by the authors was that any arbitrary interconnections can be made between the input and output stages of the capacitive coupling model. Using the basis that the developed circuit is a true dual to the inductive coupling circuit, it is assumed that all properties which hold for the latter also hold for the former. It can clearly be seen from Figure 8 that the connection of a common reference between the input and the output stages will modify the “mutual capacitance” term by shorting out certain capacitors. Further investigation therefore is required into the extent of validity of a proposed dual circuit, and this is discussed in detail in Chapter 4.

Similar to the paper discussed above, the authors in [6] also developed a two-port representation of the coupling for use in planar microwave filters. A quantity for mutual capacitance was defined to represent the coupling which takes place in a case similar to Figure 7. The authors noted that the circuit takes the form of the

standard conductance parameter (Y-parameter) layout; however, similar to the paper discussed above, the authors assumed that the connection of a common reference can be made between the input and output according to standard network theory. It was highlighted above that this type of connection cannot be made as it modifies the mutual term. This is discussed in detail in Chapter 4.

Mahomed and Hofsajer proposed a general capacitive coupling circuit model also based on duality [5]. In order to derive the model, a general four-body system was used as in Figure 8, with the self-capacitances of each body neglected. By explicitly stating the conditions for which the approach is valid, the authors simplified the six-capacitor network fully using standard two-port network theory. This approach gives a capacitive coupling circuit of the form shown in Figure 7, but where the expressions for the terms are different to those in [4]. It is shown to give excellent agreement with experimental results. Since this is an analytic model, it is general in nature and can be used in an arbitrary four-body system. The limits of the duality of this circuit were not explored, and no conclusion can be drawn as to the types of interconnections which can be made for which the model is still valid. The mathematical derivation is consistent, but the approach is entirely a circuit level simplification. The link from the physics all the way to the circuit model is therefore missing, and as such there may be conditions required for the model to be valid, which may not be apparent.

In an earlier paper [7], Hofsajer proposed a capacitive coupling circuit similar to that shown in Figure 7, however, the approach used to develop the expressions for the terms in the circuit was slightly different. He referred to the “mutual capacitance” term as a coupling capacitance, and derived expressions for the term from a field as well as a circuit point of view. From a circuit perspective, the expressions presented in this paper correspond to those developed using the two-port network simplification approach discussed in the paper above. Using a test structure, the circuit model was verified and found to be in good agreement to the predicted values.

Following an identical approach to Hofsajer in [7], Wang presented the same circuit model as discussed above for use in an application for EMI noise suppression [8]. This model was also verified experimentally and found to be in good agreement.

All the papers discussed above present the concept of duality as the basis for the development of a capacitive coupling model. The developed circuit, in general, is presented as a true dual to mutual inductance, and has good agreement with experimental data. However, it is perhaps important to note that in general, textbook theory states that a dual to mutual inductance does not exist. In order to investigate these two conflicting views, a detailed discussion on duality is provided in Chapter 2.

3. Research Methodology

The literature review reveals that the duality approach to the problem appears to give good results [5][6][7][8]. It was also highlighted that textbook theory in general shows that a dual to mutual inductance does not exist. In order to develop this link, it is necessary to investigate the concept of duality in detail, and then develop the circuit model from a field level instead of doing a circuit-level simplification to produce a dual to the inductive model.

In order to solve this problem, the duality between the fields of electrostatics and magnetostatics is rigorously investigated in Chapter 2 to determine the boundaries for which the duality holds. The duality is investigated at different levels, from field-level behaviour to circuit-component duality. This is done in order to develop a meaningful definition of duality for the mutual component of capacitance as compared to mutual inductance.

Using this definition, a capacitive coupling circuit model is developed. Unlike previous work, the development of this model is approached from the field level

theory, and it forms a dual circuit to the inductive coupling circuit model under certain conditions. Since the duality does not fully carry through to the field level, there are limitations in its operation as compared to the inductive model, and these are investigated.

With the capacitive coupling model developed, a concise review of typical applications is provided to highlight the possible widespread use of this circuit theory. As a detailed design application, a diagnosis methodology based on capacitive coupling is proposed for use in a condition monitoring application for inductive power transformers.

The detailed application is followed by a concluding chapter summarising the contents of the dissertation.

4. Conclusion

In summary, the literature review has highlighted a missing link in contemporary circuit theory in the field of multi-body capacitor systems. It was observed that mutual inductance has been extensively described and modelled; however, this has not been done in a similar manner for a coupled capacitor system. The methodology proposed in this chapter to find a solution to this problem is to investigate the duality of inductance and capacitance extensively, and thereby define a capacitive coupling circuit model which is a dual to the mutual inductance circuit. With this circuit as a goal, its description would be developed from the field level in order to close the missing link in circuit theory.

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Chapter 2: Duality between inductance and capacitance and its application to capacitive coupling

1. Introduction

It was highlighted in Chapter 1 that there is a missing link in contemporary circuit theory for dealing with coupled capacitance systems. Where there exists a simple universal circuit model for mutual inductance, a similar circuit model does not exist for “mutual capacitance”. It was also shown that certain authors have presented a circuit model to close this missing link, and these models are generally based on the concept of duality. However, these models are generally a circuit level simplification of a coupled system, and as such lack a clear basis in field theory.

In this chapter, the concept of duality is discussed in detail to determine the extent to which it can be used as a basis for the development of a circuit model for capacitive coupling.

2. Duality

The principle of duality as a circuit level construct describes a symmetry between two networks. At a very basic level, two electrical networks are said to be duals of each other if the loop equations of one network have the same form as the mesh equations of another [4]. It is this definition that is used throughout this dissertation, for reasons that will become clearer in the following paragraphs. In the mathematical representation, the equations of two dual networks differ only by the symbols used to describe specific quantities. In literature, dual networks are sometimes referred to as reciprocal or inverse networks, or in the case where an electrical system is described by a similar set of equations to a physical system, these systems are said to be analogs of each other [5].

If the investigation of duality is restricted to a circuit level, there are differences of opinion in literature as to the exact conditions required for a network to have a dual. The authors of [5] and [6] have treated this subject in detail, and [5] classifies the different definitions into two categories, namely completely dual networks and restricted dual networks. A short summary of each category is provided below to contextualise the content of this chapter.

The complete dual is the strictest definition of duality. This definition enforces an exact structurally symmetrical duality, where there are dual elements for every element in each network, i.e. there are an equal number of elements in both networks [5]. In this definition, only planar and connected networks can have duals [6].

The planarity requirement is shown in Figure 1 and Figure 2, and basically states that a non-planar graph cannot be redrawn on a flat plane such that the lines do not cross, while this is possible for a planar graph.

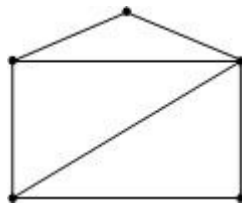


Figure 1: Planar Network

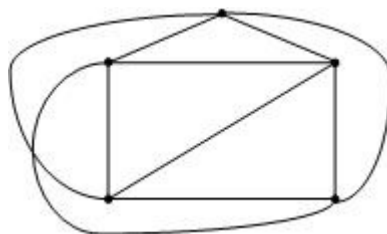


Figure 2: Non-Planar Network

This requirement arises from the fact that a non-planar network must have at least one element common to more than two meshes. This means that the dual element must be in contact with more than two nodes, and this is not physically realisable [5].

A connected network implies that there is at least one node in common between different sections of a network as shown in Figure 3 and Figure 4.

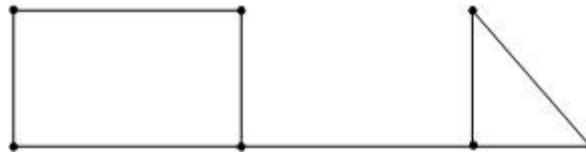


Figure 3: Connected Network

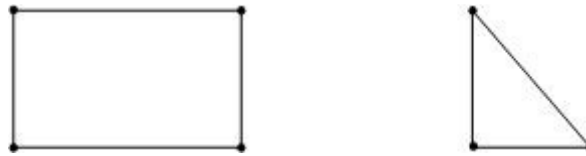


Figure 4: Unconnected Network

Based on the definition for complete duality, it is clear that mutual inductance cannot have a dual element since it presents an unconnected network in the fundamental case where there is full electrical isolation between the two inductors.

The second definition for duality describes the restricted dual network. In this category, two networks form a restricted dual if dual behaviour can be observed at specified terminal pairs, i.e. it is not required to observe dual behaviour from all terminal pairs as required by two completely dual networks. A schematic is shown in Figure 5.

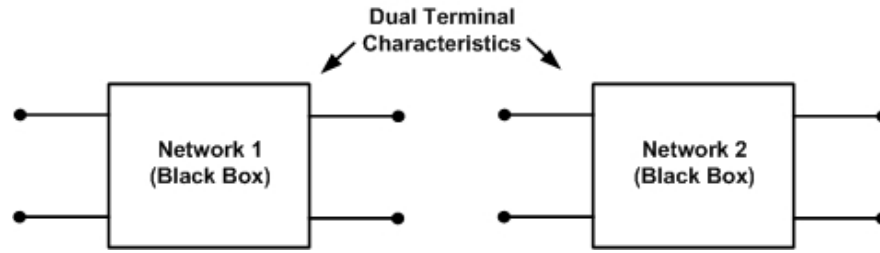


Figure 5: Restricted Dual Networks

A consequence of this more relaxed definition is that dual networks need not be structurally similar, allowing any network, including one with mutual inductance, to possess a dual [5]. Since this is a restricted definition, it is clear that limitations will arise in the operation of these dual networks. The two most obvious conditions are listed below.

- Since duality is declared at specified terminal pairs, the behaviour of the two networks can only be compared when operated from those terminals, i.e. the dual network cannot necessarily be used to solve for the original network if another set of terminals is used.
- Duality may fail if using a different set of terminals, and as such, arbitrary interconnections at the terminals of either network may not yield similar results as expected of a true dual.

Since the aim of this dissertation is to develop a capacitive coupling circuit model which forms the dual of the inductive coupling model, it is clear from the above that the developed network must form a restricted dual to mutual inductance, and the consequences of this must be fully described. In order to develop the model, it is useful to investigate the duality between electrostatics and magnetostatics from a very fundamental level, as this highlights the physical differences between the two fields.

In the following sections, the duality between the two fields is investigated. This is not a circuit level duality, but rather a mathematical duality where the equations take the same form. As is shown, duality can be observed on many levels; each

with its own limitations. A circuit model describing capacitive coupling is then presented using the duality approach and is shown to be the restricted dual of the two-body inductive coupling model. In the following chapters, the term “dual” will be used to refer to the restricted dual unless specified otherwise.

In order to aid with the discussion of the duality between magnetostatics and electrostatics, it is perhaps useful to first review the concept of mutual inductance as well as that of absolute potentials. The description of mutual inductance is referred from Chapter 1. Both concepts are discussed in the following subsections.

3. Topic Review

The following two subsections provide information to assist the reader to engage with the contents of the remainder of this chapter.

3.1 Mutual Inductance

For a single coil system, its inductance is usually termed as the self-inductance of that coil. However, in a multiple-coil system where there may be a certain amount of magnetic coupling between the coils, each inductor interacts with the field. This interaction between coils leads to an added component of inductance seen at the terminals of each inductor, which is commonly known as mutual inductance. For a two-inductor system, the circuit-level representation is shown in Figure 6 with the definitions for each term shown in (1).

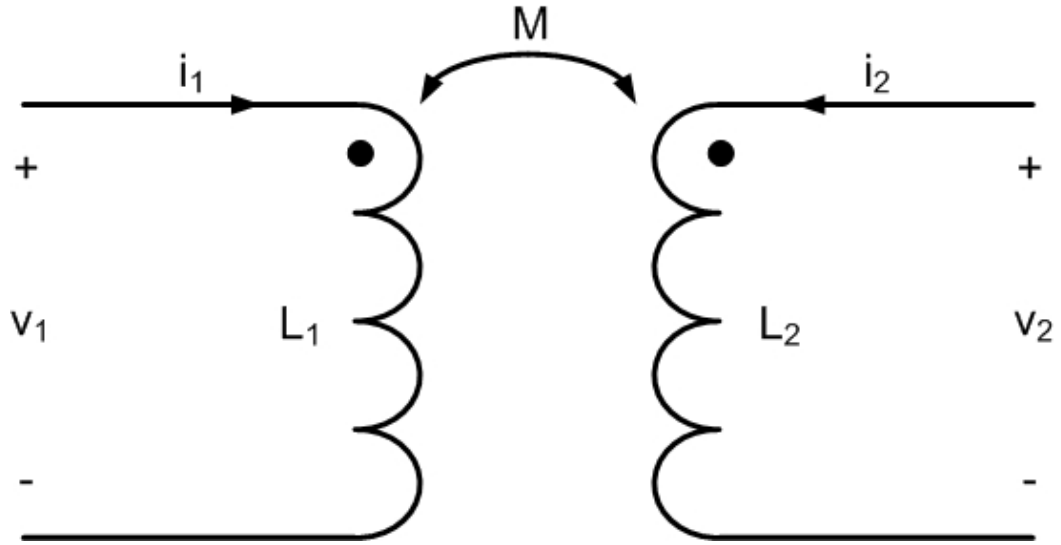


Figure 6: Inductive Coupling

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} L_1 & M \\ M & L_2 \end{pmatrix} \begin{pmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{pmatrix} \quad (1)$$

3.2 The Concept of an Absolute Potential

At a circuit level, all voltages are described as differential voltages. At a field level, however, all components also have a voltage with respect to infinity (or ground/earth). This zero reference does not exist on a circuit level. A schematic of this type of situation is shown in Figure 7 with infinity shown as the earth node. In this case it is shown that there exists a capacitance between each body and the zero reference, as well as a capacitance between the two bodies. These capacitances give rise to potentials on each body. The capacitances between the bodies are often referred to as differential capacitors or inter-body capacitors, while the capacitances to the zero reference are referred to as the self-capacitances of each body. Since circuit level theory is only concerned with differential voltages, the self-capacitances are neglected and a capacitor element is defined for the capacitance C_{12} , i.e. bodies 1-2 form a single capacitor at a circuit level.

Although the absolute reference is neglected in the circuit modelling, the existence of it can present limitations in certain applications. This will be shown later in the chapter.

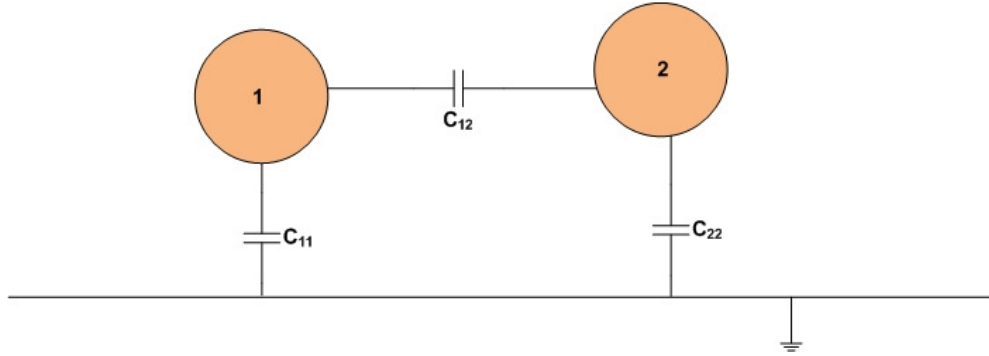


Figure 7: Absolute Voltages

4. Duality in the fundamental case

It is widely known that inductance is based on magnetic fields and capacitance on electric fields. In order to present the duality of the two concepts, then, it is useful to look at the duality of the fields as well. To do this, one needs to consider the equations that define the fields and see if a duality can be observed. This is shown below.

Magnetostatics

In textbook theory, the topics of electrostatics and magnetostatics are introduced using a basic property of each field which is that it is able to exert a force on something; and this force is proportional to the field. One of the governing equations of magnetic fields is the expression of the Lorentz magnetic force on a moving charge, as shown in equation (2) [1].

$$F_M = qv \times B \quad (2)$$

In (2) F_M is the Lorentz magnetic force acting on a charge q which is moving at a velocity v in a magnetic field with flux density B .

Electrostatics

Similar to the magnetic field case, a fundamental equation for electric fields is that given in equation (3) [1].

$$F_E = qE \tag{3}$$

In (3) F_E is the electric force acting on a charge q which is in an electric field E .

Discussion of duality

From the expressions given in equations (2) and (3), it can be seen that they are of a similar form in that both B and E are “sources of field” which act on a charge to produce a force. These expressions therefore appear to exhibit a mathematical duality, with the only difference being that in the magnetic case, the charge must be moving for a force to be exerted on it due to the magnetic field. This is not a requirement for electric fields, however.

If the duality presented above is accepted, this leads to the conclusion that the magnetic flux density, B , is the dual of the electric field, E .

5. Duality at a circuit component level

At a circuit component level, an inductor is typically a coil of wire wound around a magnetic core which concentrates the magnetic flux. A capacitor is defined to be a two-body parallel plate device with a dielectric material between the two plates which concentrates the electric flux in the gap. The reason for this definition ties

into the theory presented earlier regarding absolute voltages, and is explored in more detail later in this chapter.

For inductors and capacitors, the relationships in equations (4) and (5) hold [1].

$$B = \mu H \quad (4)$$

$$D = \epsilon E \quad (5)$$

In (4), B is the magnetic flux density, μ is the magnetic permeability of the magnetic core and H is defined as the magnetic field intensity vector. In (5), D is the electric flux density, ϵ is the electric permittivity of the dielectric material and E is the electric field intensity. Although it appeared in the preceding section that B is the dual of E , this will not be considered in this analysis, for reasons which become apparent later.

Magnetostatics

Equation (4) can be simplified further to obtain a component-level description of inductance based on geometric parameters. This is by using equations (6) and (7).

$$B = \frac{\phi}{A} \quad (6)$$

$$H = \frac{Ni}{l} \quad (7)$$

In (6), B is described by a flux ϕ per unit area A . In (7), H is described as N times a current i per unit of magnetic path length l . Since the inductors are ideal and all flux is confined to the core, the flux density is uniform throughout the core and the magnetic path length is clearly defined by the geometric parameters of the core. The current i is the current flowing through the coil which makes up the

inductor, and the factor N arises since more turns on the coil results in more flux in the core. Combining (4), (6) and (7) gives (8).

$$\phi = Ni \frac{\mu A}{l} \quad (8)$$

Using Faraday's law to get an expression relating voltage to current in (8), expression (9) is obtained.

$$v = N \frac{d\phi}{dt} = N^2 \frac{\mu A}{l} \frac{di}{dt} \quad (9)$$

In (9) there is a component-level description of terminal voltage due to a current flowing in a coil. The quantity called inductance can be used to describe the geometric constant relating current to voltage in the component, and this is given by (10).

$$L = N^2 \frac{\mu A}{l} \quad (10)$$

Electrostatics

Using a similar procedure as above, equation (5) can be simplified using expressions (11) and (12) which hold for a parallel plate system with ideal behaviour.

$$D = \frac{Q}{A} \quad (11)$$

$$E = \frac{v}{l} \quad (12)$$

In (11) the electric flux density D is expressed as an electric flux Q per unit area A . It is notable that in the electric field case, the electric flux has the dimensions of charge. In (12), the electric field in the gap E is given by the potential difference v

between the two plates divided by the distance across which the electric flux “is”, i.e. the electric path length l . Combining (5), (11) and (12) and then differentiating with respect to time yields (13).

$$\frac{dQ}{dt} = i = \frac{\varepsilon A}{l} \frac{dv}{dt} \quad (13)$$

In (13), there is a component-level description of the current flow due to a potential difference across two plates. The quantity called capacitance can be used to describe the geometric constant relating current to voltage in the component, and this is given by (14).

$$C = \frac{\varepsilon A}{l} \quad (14)$$

Discussion of duality

The information presented in the preceding sections leads to significant conclusions. Given that duality holds for capacitance and inductance at a circuit level, i.e. equations (9) and (13) are valid; it means that the generally accepted circuit-level duality actually exists between a single body inductor and a two-body capacitor. In other words, the duality exists between self-inductance and mutual-capacitance, rather than the intuitive case in which self-inductance is the dual to self-capacitance. Going further back into the physics domain, it is evident that the magnetic permeability μ is the dual for the electric permittivity ε , and consequently a duality between the B and the D fields is declared. This is contrary to the duality shown between the B and the E fields shown in the preceding section.

It is important to note that the analysis presented in this section is heavily dependent on geometric parameters. Because of this, it is extremely difficult to extend it to a system of N conductors which is required to present a more general approach. This is also required to investigate coupled circuit components. If it is

accepted that the terms inductance and capacitance are purely geometry-dependent terms describing the voltage-current relationships for the components, it may be possible to present the duality in a more general manner. This is shown in the following section.

6. Duality for a multibody system

This section presents the case for multibody systems in order to investigate the general duality between the magnetostatic and the electrostatic systems.

Magnetostatics

For a system of N inductors, the flux-current relationship shown in equation (15) holds [2].

$$\begin{pmatrix} N_1 \phi_1 \\ N_2 \phi_2 \\ \vdots \\ N_N \phi_N \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} & \cdots & L_{1N} \\ L_{21} & L_{22} & \cdots & L_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ L_{N1} & L_{N2} & \cdots & L_{NN} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{pmatrix} \quad (15)$$

The matrix of L quantities (the inductance matrix) in (15) is a matrix consisting of geometrical quantities relating the magnetic flux to the current. The elements on the main diagonal are the self-inductances of the system, and the remaining elements are the mutual inductances. As is evident, a mutual inductance gives rise to additional flux existing in a coil. The term ϕ_j is the flux in the j -th coil, and the term i_j is the current in the j -th coil. In the absence of more than one coil, the matrix reduces to the case presented in the section dealing with single component duality. To show the inductances, a schematic of a three-body case is shown in Figure 8.

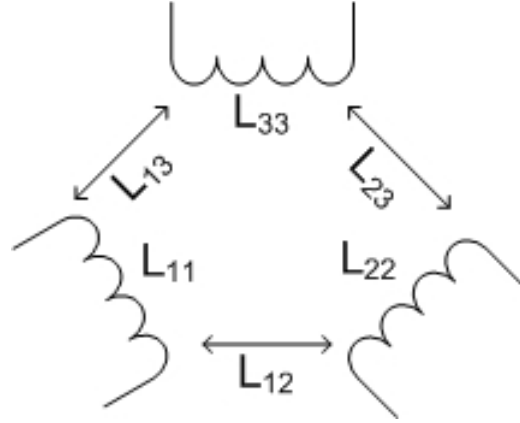


Figure 8: Three-Body inductive system

Electrostatics

For a system of N conductors, the electric flux-voltage relationship shown in equation (16) holds [3].

$$\begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_N \end{pmatrix} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \cdots & \epsilon_{1N} \\ \epsilon_{21} & \epsilon_{22} & \cdots & \epsilon_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{N1} & \epsilon_{N2} & \cdots & \epsilon_{NN} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{pmatrix} \quad (16)$$

In (16), the charges on each body, Q_j , are described in terms of some constant $\mathcal{C}$ multiplied by the potential of a body v_j . The $\mathcal{C}$ constants populate a matrix of quantities called coefficients of capacitance, and are purely geometric constants. The potentials are absolute potentials with reference to infinity (a reference which does not appear on a circuit, as described earlier in the chapter). To show the coefficients of capacitance, a schematic of a three-body case is shown in Figure 9, together with the potentials of each body V_x with reference to infinity (shown as the earth node in the schematic).

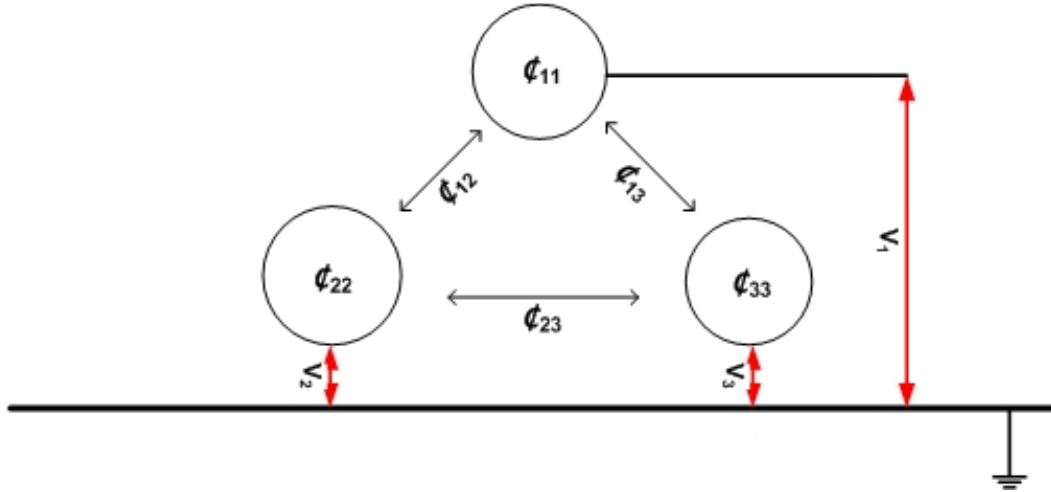


Figure 9: Three-body system with co-efficients of capacitance

Discussion of duality

If it is accepted that current is the dual of voltage, then the expression in (15) must be the dual of that in (16). This means that self-inductance must be the dual of the self co-efficient of capacitance, and similarly with the mutual terms. If the analysis is restricted to two body systems, then a two-body capacitor must form the dual of a two body coupled inductor. This is however not the case at a circuit level, as shown earlier in the chapter. This is because at a circuit level, a two body capacitor is regarded as a single circuit component. It was mentioned earlier that the reason it is considered a single component is that this definition is required to enable the sole use of voltage differences (as opposed to absolute voltages) in the circuits domain. This is a mathematical duality then, in that the expressions take the same form. It is not a circuit level duality and cannot be extended to a circuit level.

7. Duality for inductive and capacitive coupling

It was shown in preceding section that a general duality between inductance and capacitance cannot be traced from the governing field equations all the way to a circuit level. This is mainly due to the fact that at a circuit level, a two-body

capacitor is the dual of a single-body inductor. In this section, the two-body inductive coupling model is presented and an expression is derived for a capacitive coupling model, for which the duality between the two systems holds at a circuit level. As before, an analysis of this duality is provided.

Inductance

For this investigation, the analysis is restricted to a two-body system for the simplest case of inductive coupling. Equation (15) therefore reduces to the well-known expression shown in (1), after applying Faraday's law. A circuit diagram for the system is given in Figure 6.

In (1), L_1 and L_2 represent the two self-inductances of the inductively coupled system. The term M is the mutual inductance which arises due to the inductive coupling. The mutual inductance is equal regardless of which port is used as the input. The method of determining the values for L_1 , L_2 and M is widely documented and will not be discussed here.

Capacitance

If a circuit model representing coupled capacitors is to be a dual to the inductive model shown in (1), the model must have two "self-capacitances" and a coupling capacitance. It was shown earlier in the chapter that capacitors form two-body components at a circuit level. This means that the coupling model in the capacitive case must consist of a four body system, unlike the two body system in the inductive case. The model should be as shown in Figure 10 with its governing equation given in (17), where C_1 is the input capacitance, C_2 the output capacitance, and S the coupling capacitance of the system. The derivation of this model is presented in the following section.

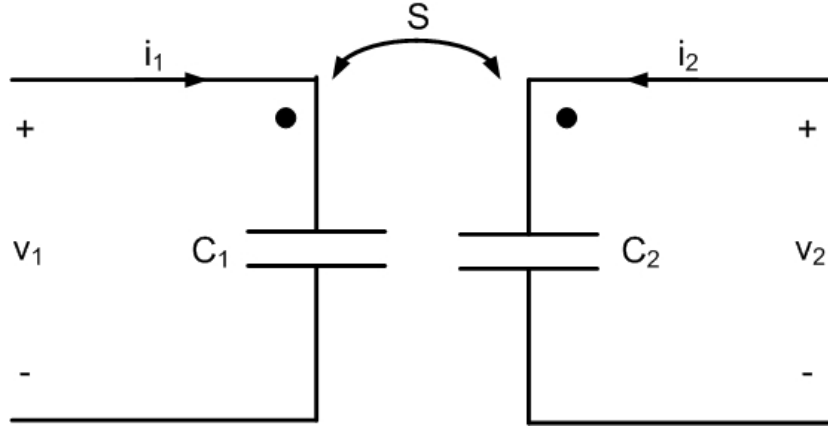


Figure 10: Capacitive Coupling Model

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} C_1 & S \\ S & C_2 \end{pmatrix} \begin{pmatrix} \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{pmatrix} \quad (17)$$

The capacitive model in Figure 10 as well as the inductive model Figure 6 form two-port networks, and may be referred to as such in later chapters.

7.1 Derivation of Model

Equation (16) describes the electric flux-voltage relationship for a general multi-conductor system. Since the voltages are absolute, it is not a useful representation for circuit-level analysis. Through mathematical manipulation, it is possible to convert Equation (16) to Equation (18), where the C quantities are circuit level capacitors which are based on the coefficients of capacitance. All primed voltages are absolute voltages. The terms C_{ii} are the self-capacitances of the system and C_{ij} ($i \neq j$) are the mutual capacitances. Also, a symmetry exists whereby $C_{ij} = C_{ji}$. The matrix of C values is often referred to as the capacitance matrix, and the relationship provides a component-level description of the electrostatic behaviour.

$$\begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_N \end{pmatrix} = \begin{pmatrix} C_{11}V_1' + C_{12}(V_1' - V_2') + \dots + & C_{1N}(V_1' - V_N') \\ C_{21}(V_2' - V_1') + C_{22}V_2' + \dots + & C_{2N}(V_2' - V_N') \\ \vdots & \\ C_{N1}(V_N' - V_1') + C_{N2}(V_N' - V_2') + \dots + & C_{NN}V_N' \end{pmatrix} \quad (18)$$

Further analysis is restricted to a four body system to fit the form shown in Figure 10. The expression in (18) therefore has the circuit representation shown in Figure 11.

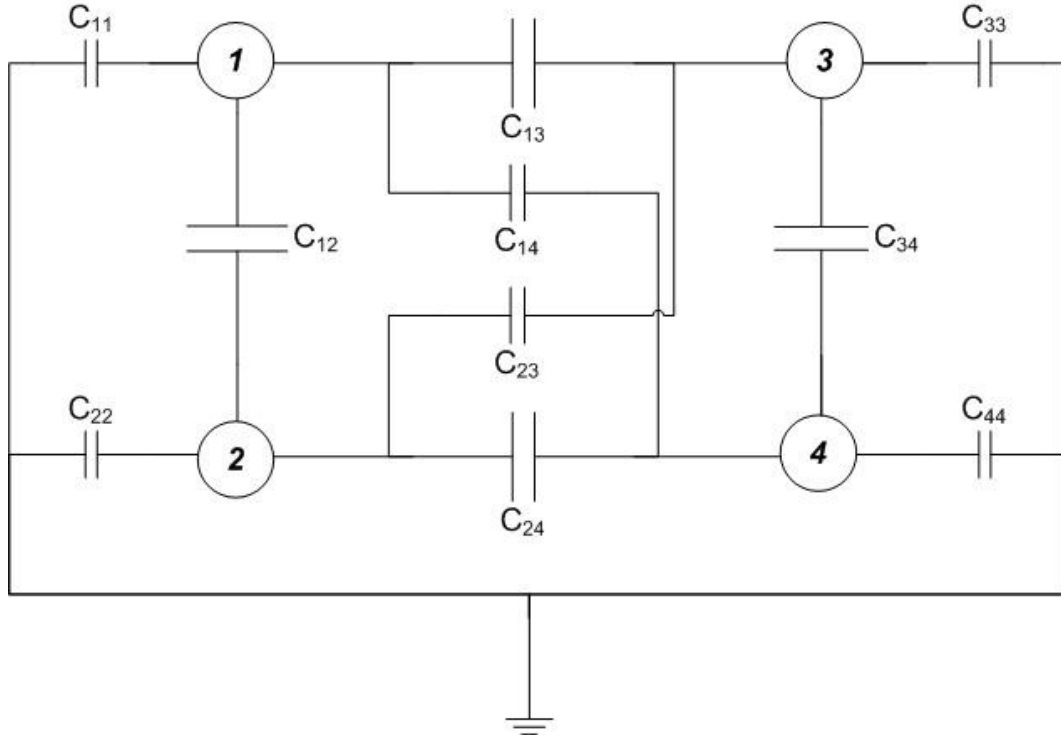


Figure 11: Ten capacitors for a four body system

For the four body system described above, expression (18) can re-written as shown in (19).

$$\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{pmatrix} = \begin{pmatrix} C_{11}V_{11}' + C_{12}V_{12} + C_{13}V_{13} + C_{14}V_{14} \\ C_{21}V_{21} + C_{22}V_{22}' + C_{23}V_{23} + C_{24}V_{24} \\ C_{31}V_{31} + C_{32}V_{32} + C_{33}V_{33}' + C_{34}V_{34} \\ C_{41}V_{41} + C_{42}V_{42} + C_{43}V_{43} + C_{44}V_{44}' \end{pmatrix} \quad (19)$$

The expression in (19) is a fourth order expression. This eventually needs to reduce to a second order mathematical representation in order to form the expression in (17), with the circuit-level representation in Figure 10.

As was discussed earlier, a circuit capacitor is a two body system, so two bodies need to be used as the input terminals and two bodies as the output terminals of the network. Terminals 1-2 are therefore arbitrarily chosen to form the input terminals with terminals 3-4 to form the output terminals. Clearly it is useful to express as many voltages as possible in terms of the input and output voltages as these form the terminals of the device. For the system to remain fully described, however, it must remain dependent on at least one absolute voltage. Using this approach, a voltage transformation expression can be developed as shown in (20).

$$\begin{pmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \\ V_{12} \\ V_{13} \\ V_{14} \\ V_{23} \\ V_{24} \\ V_{34} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & 0 & 1 & -1 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_{12} \\ V_{34} \\ V_2' \\ V_4' \end{pmatrix} \quad (20)$$

It is now possible to rewrite the expression describing the arbitrary four body system shown in (19), using only the four voltages chosen above. This is shown in (21).

$$\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{pmatrix} = \begin{pmatrix} C_{11} + C_{12} + C_{13} + C_{14} & -C_{13} & C_{11} + C_{14} + C_{13} & -C_{13} - C_{14} \\ -C_{12} & -C_{23} & C_{22} + C_{23} + C_{24} & -C_{23} - C_{24} \\ -C_{13} & C_{13} + C_{23} + C_{33} + C_{34} & -C_{13} - C_{23} & C_{13} + C_{23} + C_{33} \\ -C_{14} & -C_{34} & -C_{14} - C_{24} & C_{14} + C_{24} + C_{44} \end{pmatrix} \begin{pmatrix} V_{12} \\ V_{34} \\ V_2' \\ V_4' \end{pmatrix} \quad (21)$$

It is important to recall that absolute voltages are not defined at a circuit level, therefore some condition needs to be placed such that they are not apparent from a

terminal perspective. Recall from the introduction that the aim is to obtain a duality from a terminal perspective as the network by definition cannot be a complete dual to mutual inductance.

By inspection of the expression in (21), it is clearly possible to have a different charge on each of the four bodies. However, it was stated earlier that the proposed circuit model has one input (circuit-level) capacitor and one output (circuit-level) capacitor. At a circuit level then, the current which flows through the two input bodies (the input circuit-level capacitor) must be equal, and similarly at the output. This is evident from the circuit described in (17). Mathematically this condition can be applied in terms of charge by the relationships in (22) and (23).

$$Q_1 + Q_2 = 0 \quad (22)$$

$$Q_3 + Q_4 = 0 \quad (23)$$

By expressing the charges in the above expression using their definitions from the four-body system in (19), the conditions given in (24) and (25) can be obtained. At this stage the four-body system now comprises of two sets of bodies possessing the same charge. At a circuit level, this translates to differential currents at the input and output terminals.

$$\begin{aligned} &_1 + C_{13} + C_{14})V_{12} + (C_{11} + C_{13} + C_{14} + C_{22} + C_{23} + C_{24})V_2' + (-C_{13} - C_{23})V_{34} \\ &+ (-C_{13} - C_{14} - C_{23} - C_{24})V_4' = 0 \end{aligned} \quad (24)$$

$$\begin{aligned} &(-C_{13} - C_{14})V_{12} + (-C_{13} - C_{14} - C_{23} - C_{24})V_2' + (C_{13} + C_{23} + C_{33})V_{34} \\ &+ (C_{33} + C_{13} + C_{14} + C_{44} + C_{23} + C_{24})V_4' = 0 \end{aligned} \quad (25)$$

The conditions in (24) and (25) provide two equations as functions of four voltages. It is therefore possible to solve for two voltages in terms of another two.

It was mentioned earlier that the absolute voltages may not present themselves from a terminal perspective, since the absolute reference does not exist at a circuit level. In order to remove the absolute voltages from the circuit definition, voltages V_2' and V_4' can be solved for in terms of V_{12} and V_{34} , based on the expressions in (24) and (25). For the sake of clarity, a short review the procedure used to develop the model is provided below:

- i) A system with four bodies was defined in terms of a mixture of differential and absolute voltages.
- ii) Terminals were chosen for input and output, and all voltages from above were expressed in terms of input/output voltages and two absolute voltages, i.e. a total of four voltages.
- iii) The condition of differential currents at the input and output was highlighted, and using this condition it was possible to express the two remaining absolute voltages in terms of the input/output voltages, resulting in the system being fully described using only two voltages: input and output.

Every voltage is now fully described in terms of the input and output voltages: V_{12} and V_{34} . The expressions are too expansive to print in this chapter, and are therefore provided in Appendix A. It is clear that Q_2 is simply the negative of Q_1 , and Q_4 the negative of Q_3 . The terms Q_2 and Q_4 can therefore be dropped from the system as they are no longer independent. At this step it is clear that the order of the system has been reduced from a fourth order system to a second order system. It immediately follows that the resulting expression fits the form given in (17). The expressions for the terms C_1 , C_2 and S are given in Appendix A.

Discussion of duality

At this point a capacitive coupling circuit model has been developed which is of similar form to the inductive coupling model. This model holds as long as the condition of differential currents at the input and output terminals is externally

enforced. In this way the net current flow from the input stage to the output stage through the self-capacitances is forced to be equal to zero. The absolute voltages are therefore fully defined, but are not apparent from a terminal perspective. This had consequences in terms of electrical isolation and is discussed in detail in the next subsection. From a terminal perspective, the system appears to be a dual to the inductive coupling model. This duality does not hold at the electromagnetic level, however, for the simple reason that the one system has two bodies while the other system has four.

In reducing the fourth order capacitive system to a second order “dual” system, the input and output capacitances became a complicated conglomeration of the individual capacitors which make up the system. This is unlike the inductive model which is physically a second order system where the input and output inductances are equivalent to the self-inductances of the system. However, similar to the inductive model, the capacitive model is reciprocal in that the mutual term is the same no matter which port is used as the input.

Since the developed dual network forms a restricted dual to inductive coupling, there are certain boundaries in its operation. Deeper insight into the coupling term and the network as a whole are presented in Chapters 3 and 4.

Electrical Isolation

The capacitive coupling model developed in the above section forms a dual to the inductive coupling model from a terminal perspective, i.e. the equations describing the two systems from a terminal perspective take the same form and differ only in their mathematical symbols. However, it is evident from the derivation of the capacitive coupling model that the absolute voltages are fully defined in terms of the differential voltages. The consequence of this is that every absolute voltage in the circuit, both in the input and output stages, can be solved for, i.e. there is always a common reference between the input and output, and this common reference is at a zero potential at infinity. This concept may be better

understood by reviewing the circuit diagram in Figure 11, with the zero potential shown as the earth node.

This means that there cannot be true electrical isolation between the input and the output of the capacitive coupled circuit model in Figure 10, unlike in the inductive case in Figure 6. This is despite the fact that it tends to behave as an electrically isolated system when balanced currents are enforced at the input and output. The circuit therefore presents one of the cases where despite the fact that an absolute reference is missing from the model, its existence may have consequences on the operation of the circuit. This will become evident in Chapter 4.

Simplification of Coupling Model: Level 1

Although the developed capacitive coupling network presents a simple circuit level description of the electric field coupling, the expressions for each term in the model are expansive and therefore could complicate its implementation. The following subsections describe two levels of simplification of the model to ease its implementation in certain cases.

In most physical four body capacitive systems it may be reasonable to assume that the bodies will be arranged in proximity to each other such that the capacitors to infinity (the self-capacitances) will be of similar magnitudes. In such a case, the model described above can be simplified further by making the self-capacitances C_{22} , C_{33} and C_{44} equal to C_{11} . This gives simplified expressions for the terms C_1 , C_2 and S . The terms remain too large to print in this chapter and are shown in Appendix A.

This level of simplification provides slightly less complicated expressions for each term in the model, and has the benefit of requiring only a single self-capacitance measurement as compared to the measurement of four self-capacitances in the full model. It is therefore easier to implement.

Simplification of Coupling Model: Level 2

If a physical four body system is set up in such a way that the magnitudes of the capacitors between the bodies are much larger than the self-capacitances of the bodies, then it may be possible to neglect the effect of the self-capacitances altogether. This entails making the values for C_{11} , C_{22} , C_{33} and C_{44} equal to zero in the detailed two port model derived earlier. This leads to a further simplified two port network description of capacitive coupling, where the expressions for C_1 , C_2 and S are given by (26) to (28).

$$C_1 = \frac{(C_{13}+C_{14})(C_{24}+C_{23})}{(C_{13}+C_{24}+C_{23}+C_{14})} + C_{12} \quad (26)$$

$$C_2 = \frac{(C_{13}+C_{23})(C_{24}+C_{14})}{(C_{13}+C_{24}+C_{23}+C_{14})} + C_{34} \quad (27)$$

$$S = \frac{(C_{23}C_{14}-C_{24}C_{13})}{(C_{13}+C_{24}+C_{23}+C_{14})} \quad (28)$$

It can be noted that the expressions in (26) to (28) correspond to those presented by other authors referenced in the literature review of this dissertation. In this dissertation, however, the expressions were developed using a very rigorous approach with a direct link to field theory. This is as opposed to the papers discussed in the literature review, where the expressions were developed by neglecting all the self-capacitors at the outset and then performing a circuit-level simplification. The fact that different methods were used in literature to obtain the same result gives confidence in the method presented in this dissertation.

8. Conclusion

A summary of duality was provided to contextualise the contents of the chapter. It was shown that mutual inductance cannot have a dual in the complete sense, but it

is possible in a restricted sense. If a restricted dual is developed, it will have some operational limitations and these must be fully defined.

Duality between electrostatics and magnetostatics was explored in order to highlight the link between field level duality and circuit level duality. It was shown that there are certain inconsistencies in the duality when approached from different perspectives, and no duality extends from the field level to a circuit level. This is primarily due to the fact that circuit theory only deals with differential voltages, whereas any conductor has a self-capacitance with an absolute voltage; the latter which does not exist on a circuit level. The typical circuit capacitor is therefore a “mutual capacitor”, i.e. a capacitor between two physical bodies, unlike an inductor which is inherently a single bodied system.

A capacitive coupling circuit model was developed which forms a restricted dual to the inductive coupling model. This is physically made up of four conductive bodies. It is restricted in the sense that it appears to be a dual from a terminal perspective, but does not form a full structural dual. The absolute voltage of each body is fully defined internally, but these do not appear on a circuit level as long as the network is operated with differential currents enforced at the input and output terminals. Since these voltages are defined, there is no full electrical isolation between the input and output of the network.

The full capacitive coupling model was simplified to two levels, taking into account typical possible operating conditions of the model. It was highlighted that the full simplified model corresponds to those presented in earlier literature, giving confidence to the method used to develop the model in this dissertation.

9. List of References

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Chapter 3: A deeper analysis of the coupling capacitance

1. Introduction

In Chapter 2, a term called coupling capacitance was derived and given the symbol S . This coupling term was shown to be the dual of the mutual inductance term (usually known as M) to a large extent. This is arguably the most significant term of the three terms describing the coupled circuit, namely C_1 , C_2 and S , primarily because it is a quantity that describes the extent of coupling between two separate circuits. In this chapter, a detailed analysis of the coupling term S is documented to explore the properties of this term and to compare it with the mutual inductance term M where appropriate. Experimental verification of specific scenarios is also provided.

2. Coupling Terms

In Chapter 2, the initial derived expression for the coupling capacitance term S takes the form shown in (1). Two levels of simplification of this expression were developed based on typical operating conditions, with the most simplified version fully independent of self-capacitances. Every version however, takes the form shown in (1), with f_4 , f_5 and f_6 each being a sum of various capacitors, i.e. f_4 , f_5 and f_6 are all positive terms and their definitions can be found in Appendix A. In order to analyse the coupling term in this chapter, the most complicated form (the original form) of the coupling term S will be used, so as not to neglect any information.

$$S = f_3(\text{self capacitance}, \text{interbody capacitance}) = \frac{f_4(C) - f_5(C)}{f_6(C)} \quad (1)$$

3. Analysis

The denominator of the S term is a sum of both self-capacitors and inter-body-capacitors, and thus cannot have a negative value. It can therefore either assume a positive non-zero value or a value equal to zero, and this depends on the physical construction of the four-body system. For the denominator to be zero, the four body system tends to the case where there is zero coupling between the input and output stages. Physically this means placing the two bodies comprising the output terminals far from the two bodies comprising the input terminals. We are primarily concerned with the coupled case in this dissertation, i.e. where the bodies are in proximity to each other such that the capacitances are not negligible, so we will only focus on the situation where the denominator is positive and greater than zero. In this case, the denominator can be seen purely as a scaling factor.

The numerator, however, contains a negative sign. Theoretically then, it is possible to design systems that exhibit one of three types of coupling: positive, negative and zero. These are discussed below.

3.1 Positive/Negative Coupling

Before designing a system to have either positive or negative coupling, it is important to know exactly what the sign between the terms in the numerator of (1) represents in a physical system. In order to do this, it is perhaps useful to consider the well-known inductive coupled system given by (2).

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} L_1 & M \\ M & L_2 \end{pmatrix} \begin{pmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{pmatrix} \quad (2)$$

In (2), the M term is the mutual inductance in the system, and shown in Chapter 2 to be a dual to the S term in (1). However, it is not shown in textbooks to have a mathematical form like S in (1). Despite this, it can also have a positive or negative polarity and this arises from the physical winding direction of the coils comprising the system.

The sign of the M term defines the polarity of the induced voltage with reference to the applied voltage, whilst maintaining positive current flow, i.e. if M is positive, then the induced voltage will have the same polarity as the applied voltage, whereas in the negative case the voltages have the opposite polarity. This is shown in Figure 1 and Figure 2. In the figures the red arrows show the defined positive current direction, and the red polarity markings show the defined positive voltage polarity. The blue polarity markings show the polarity of the induced voltage with reference to the applied voltage, and this corresponds to the sign of M . The placement of the dots is often used in circuit theory to denote the sign of M .

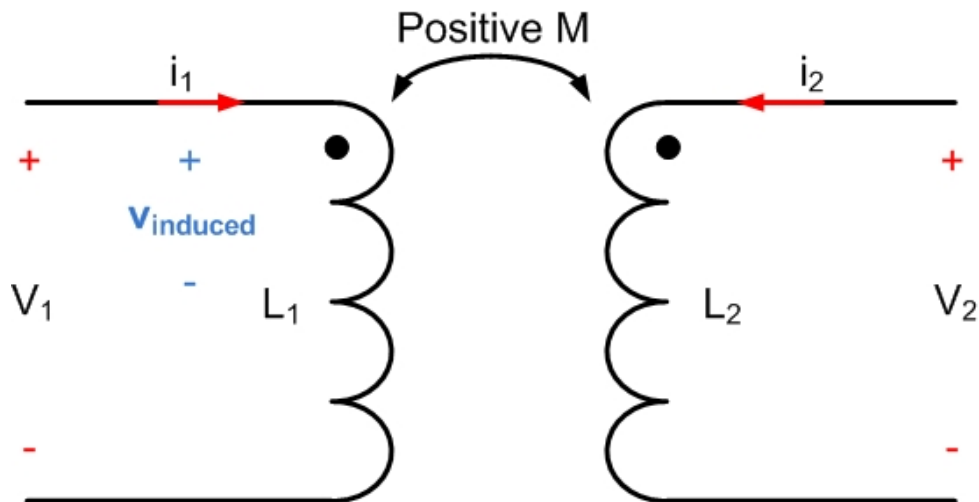


Figure 1: Positive mutual inductance

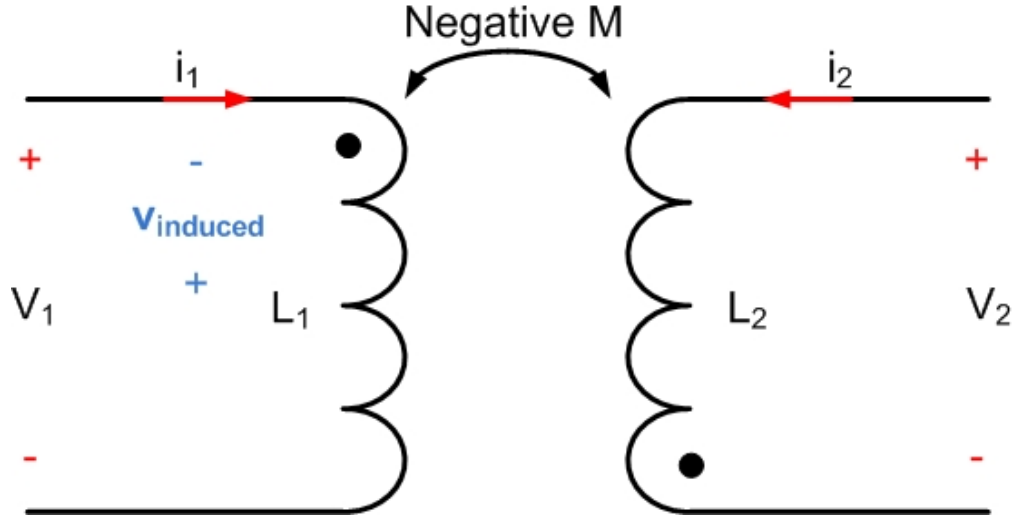


Figure 2: Negative mutual inductance

In order to physically achieve a negative value for M , it entails the winding of the secondary coil in a different direction to that of the primary coil, consequently changing the direction of flux coupling. However, due to the electrical isolation between the input and output, if an oppositely polarised voltage is required at the output, the terminals of the output can simply be interchanged without having any effect on the coupled circuit. The only case where the sign of the M term matters therefore is the case of where there is no electrical isolation between the input and output, for example in the autotransformer. In this case both the load and supply have a common reference, and the terminals of the output cannot be as easily swapped around to give an oppositely polarised voltage whilst maintaining the common reference in the supply and load circuits.

Returning to the capacitive case, the coupled system is given by (3).

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} C_1 & S \\ S & C_2 \end{pmatrix} \begin{pmatrix} \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{pmatrix} \quad (3)$$

As a dual to the inductive case, in (3), the sign of the S term defines the direction of the induced current with reference to the applied current, whilst maintaining

positive voltage polarity i.e. if S is positive, then the induced current will have the same direction as the applied current, whereas in the negative case the currents have opposite directions. This is shown in Figure 3 and Figure 4. Similar to before, the red polarity markings show the defined positive voltage polarity, and the red arrows show the defined positive current direction. The blue arrows show the direction of the induced current with reference to the applied current, and this corresponds to the sign of S . The placement of the dots corresponds to the sign of S .

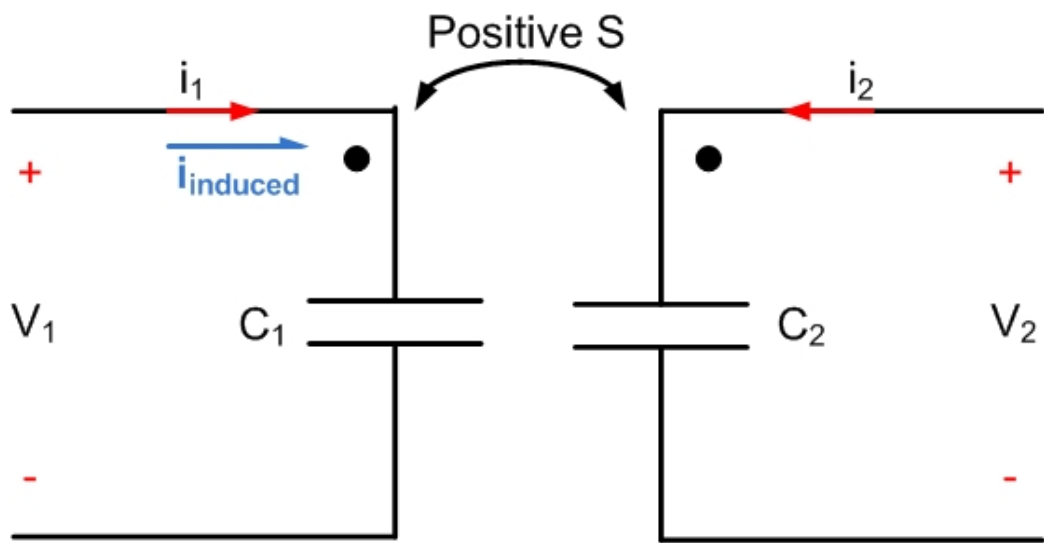


Figure 3: Capacitive Coupling Positive

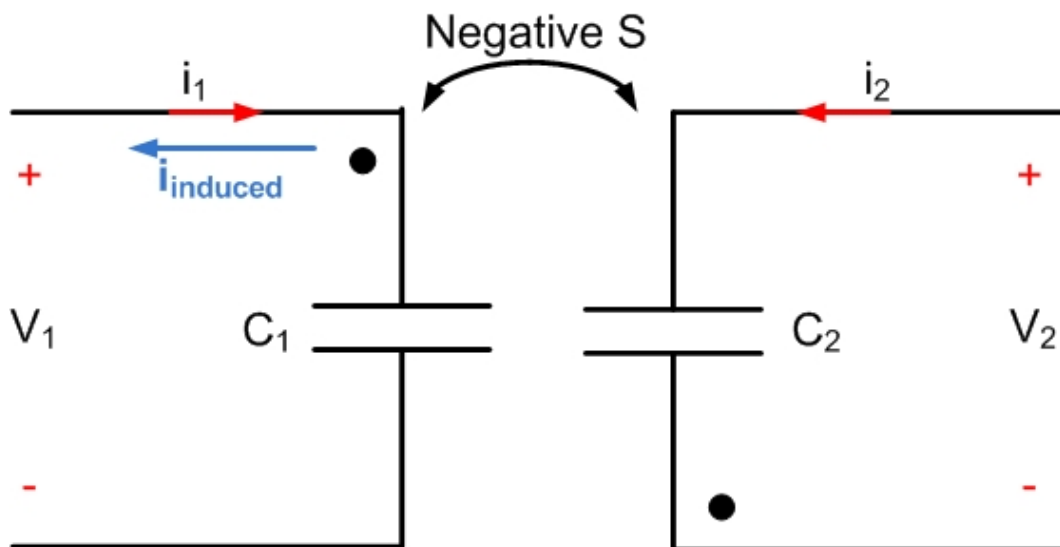


Figure 4: Capacitive Coupling Negative

The dot polarity described for the inductive case can therefore be extended to the capacitive case. Based on the expression for S , it is evident that a negative value for S can indeed be obtained. The sign of S can therefore be likened to the winding direction in an inductively coupled system. By modifying the sign of S , a designer can change the “direction” of the electric flux coupling, consequently changing the direction of induced current flow, assuming constant voltage polarity.

It was shown in the inductive case that the winding direction was irrelevant in the case of full electrical isolation between the input and output, due to the lack of a common reference point between the two stages. This is different to the capacitive case, however, as it was shown in Chapter 2 that the reference at infinity forms a common reference between the input and the output of the network, i.e. the capacitive coupling network always has a common reference.

Taking the above into account, it is the fact that the system behaves like an electrically isolated system that is of great importance. This is because even though current can flow through the self-capacitors to infinity, the differential current requirement placed at the terminals means that the same current will have to flow back from infinity into the circuit. This is shown by the red arrows in Figure 5.

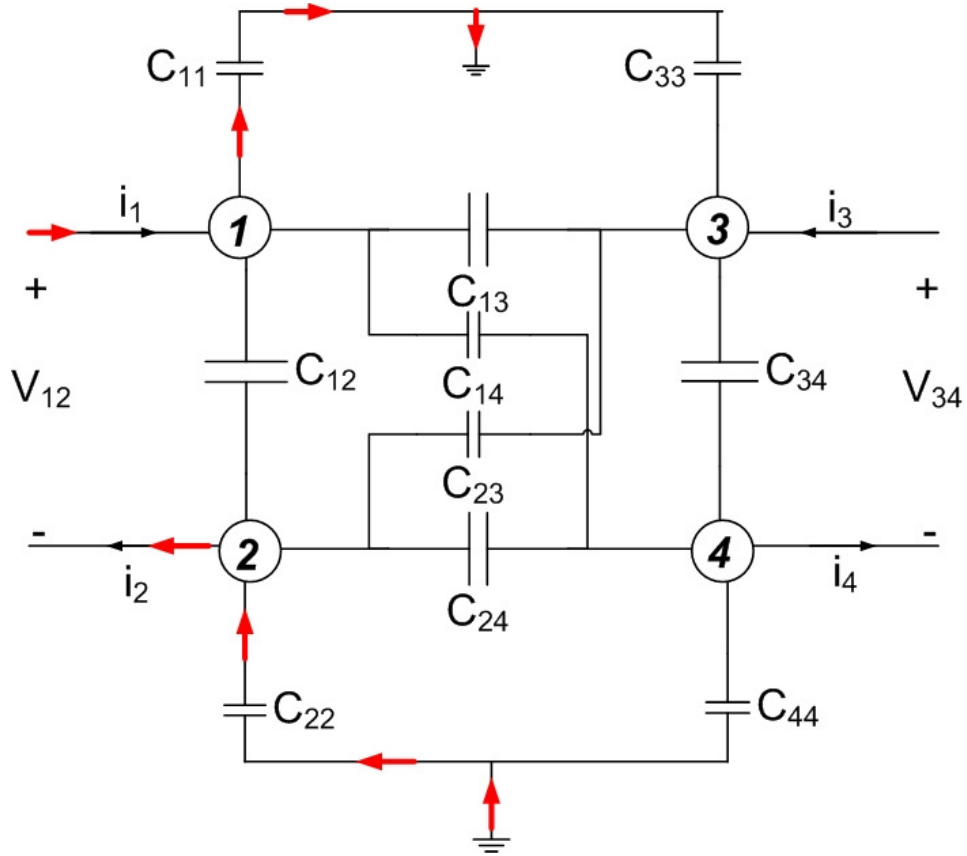


Figure 5: Current flow through the self-capacitors

From a terminal perspective, then, the input and output terminals of the network appear to be electrically isolated from each other. We will refer to this type of electrical isolation as a pseudo-electrical isolation from this point forward. The output terminals can therefore easily be switched around to give a different polarity at the output if required, because at a circuit level, the supply and load circuits do not have a common reference. The sign of the S term is therefore irrelevant in the capacitively coupled system just as it is in the inductive system, despite the fact that the capacitive system is not electrically isolated in the true sense. In terms of design, the magnitude of the term is the only item of significance.

It was shown earlier that the sign of the mutual inductance term M becomes relevant in the case of a common reference between the input and output circuits. It is shown in Chapter 4 that the physical connection of a common reference

between the input and output is not allowed by the capacitively coupled circuit model as it causes a violation of the balanced currents condition at the terminals of the network. Since this connection is not allowed, i.e. the model is not defined in this case; no further conclusion can be made as to the significance of the sign of S .

Based on the above analysis, it can be concluded that the sign of S holds no real significance, but instead it is only the magnitude that is of any importance. This conclusion is valid for as long as the circuit is operated in a region for which the model is valid, i.e. with balanced currents at the input and output.

The following experiments were done in order to validate the capacitive coupling model derived in Chapter 2. The first experiment is to test the model's validity for an arbitrary setup, followed by experiments with coupling parameters which were designed to achieve a specific output. These include a system that exhibits 100 % coupling followed by a system that exhibits zero coupling.

4. Experimental Validation – Arbitrary Coupling

In order to verify the validity of the model for a system with non-zero coupling, a parallel-plate system consisting of four plates was constructed. A parallel-plate geometry was chosen because the mathematical expression for capacitance between parallel plates is very well defined in textbook theory. The setup comprised of a four-layer PCB with arbitrary plate sizes and 0.1 mm thick FR₄ as the dielectric between the plates. The setup is shown in Figure 6 with a schematic in Figure 7.



Figure 6: The test structure

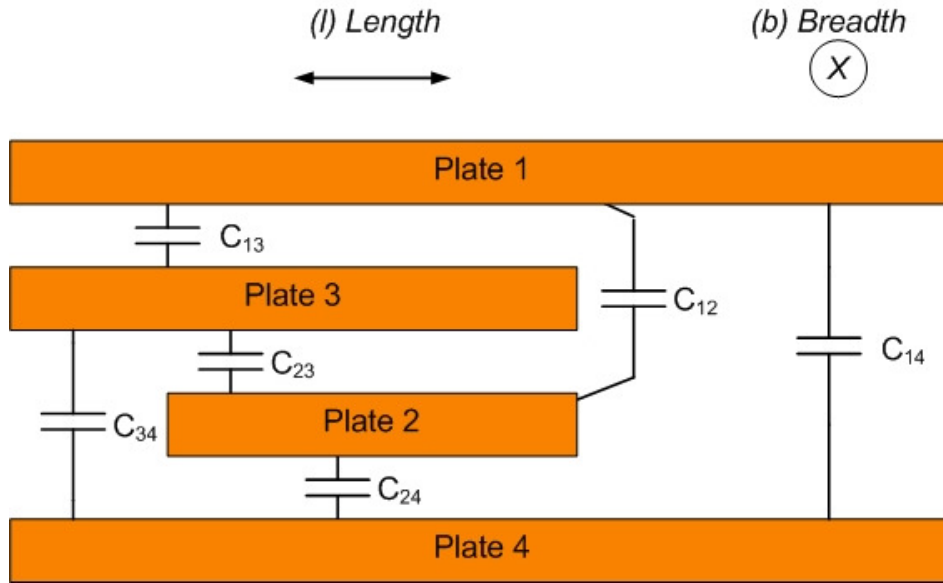


Figure 7: The test structure schematic

The geometrical properties of the structure are given in Table 1. As shown in the schematic, length is defined as the horizontal distance and breadth defined as the length into the page. The quantity d is the vertical distance between plates. The subscripts are used to show which geometrical dimensions correspond to specific capacitors, e.g. l_{12} is the horizontal length of the overlap between plates 1 and 2, b_{12} is the distance into the page of overlap between plates 1 and 2, and d_{xx} is the vertical distance between any two plates.

Table 1: Parameters for test structure

Parameter	Magnitude
l_{12}	0 cm (no overlap)
l_{13}	3.9 cm
l_{14}	3.9 cm
l_{23}	3.6 cm
l_{24}	6 cm
l_{34}	2.1 cm
b_{12}	0 cm
b_{13}	3.8 cm
b_{14}	4.3 cm
b_{23}	3.8 cm
b_{24}	4.8 cm
b_{34}	3.8 cm
d_{xx}	0.1 mm
ϵ_0	8.854 pF/m
ϵ_D	~4

In order to use the full model, the self-capacitances of the individual bodies, namely C_{11} , C_{22} , C_{33} and C_{44} need to be measurable. To achieve this, the parallel-plate setup was put into a Faraday cage – the body of the cage forming the “earth” terminal.

Using the physical dimensions in Table 1, it was possible to theoretically determine the six capacitances definable between the conductors. This was achieved by using standard parallel plate capacitor theory and the capacitors are shown in Table 2. The self-capacitance of each of the bodies cannot be easily predicted using geometrical parameters and thus no prediction was made. All the capacitor values were then verified experimentally using the measurement routine in Appendix B and this is shown in Table 2.

Table 2: Capacitor sizes for test layout

Capacitor Name	Capacitance Measured	Capacitance Predicted (nF)
C_{12}	0.003 nF	0.000
C_{13}	1.278 nF	1.262
C_{14}	1.524 nF	1.427
C_{23}	0.264 nF	0.410
C_{24}	2.494 nF	2.390
C_{34}	0.341 nF	0.324
C_{11}	0.000 pF	No Prediction
C_{22}	12.00 pF	No Prediction
C_{33}	12.00 pF	No Prediction
C_{44}	1.225 pF	No Prediction

It is clear from Table 2 that there is good agreement between the predicted and measured capacitances. The differences can be attributed to fringing flux and inaccuracies in the construction process.

Putting the measured capacitors from Table 2 into the expressions in Appendix A, the values for C_1 , C_2 and S in the capacitive coupling model were calculated. The values for C_1 and C_2 were then measured directly using the routine in Appendix B. Using two separate load resistors, the value for S was inferred according to the routine detailed in Appendix B. All calculated and measured values are shown in Table 3.

Measurements were taken using an Agilent precision LCR-meter for increased accuracy, with excitation at 1V 1kHz, to minimise inductive effects.

Table 3: Circuit Parameters for Arbitrary Coupling

Parameter	Measured Capacitance (nF)	Calculated Capacitance (nF)	Percentage Discrepancy
C_1	1.394	1.394	0 %
C_2	1.458	1.459	0.062 %
S	-0.499 (inferred using 50 k Ω load)	-0.499	0 %
	-0.499 (inferred using 100 k Ω load)		

It is clear from Table 3 that the capacitive coupling model is an extremely accurate model of the electric field coupling which occurs in a four-body system. There is almost no discrepancy between the predicted and measured values within the resolution of the experiment.

This level of accuracy was aided by the controlled environment in which the test structure was placed, as well as the fact that all electric field effects were taken into account, i.e. the self-capacitances of the various bodies were not neglected. An experiment where the structure was not placed in a Faraday Cage and the self-capacitors were neglected is discussed in a later section.

5. Experimental Validation – 100 % Coupling

In Chapter 4, an expression describing the extent of coupling between the input and output of a coupled circuit is developed. This expression is given in (4).

$$S = k' \sqrt{C_1 C_2}; \quad 0 \leq k' \leq 1 \quad (4)$$

Given this expression, it appears to be theoretically possible to construct a system such that $k' = 1$, i.e. a system which exhibits 100 % coupling. In order to do this, the process shown in Figure 8 was used, whereby a structure was designed such that only capacitors C_{13} and C_{24} would be significant. In order to do this, a circuit was set up using two capacitor elements in the layout of the bottom diagram in Figure 8. The magnitudes of these capacitors are shown in Table 4.

Table 4: Capacitor Values

Capacitor Name	Capacitance (nF)
C_{13}	10
C_{24}	10

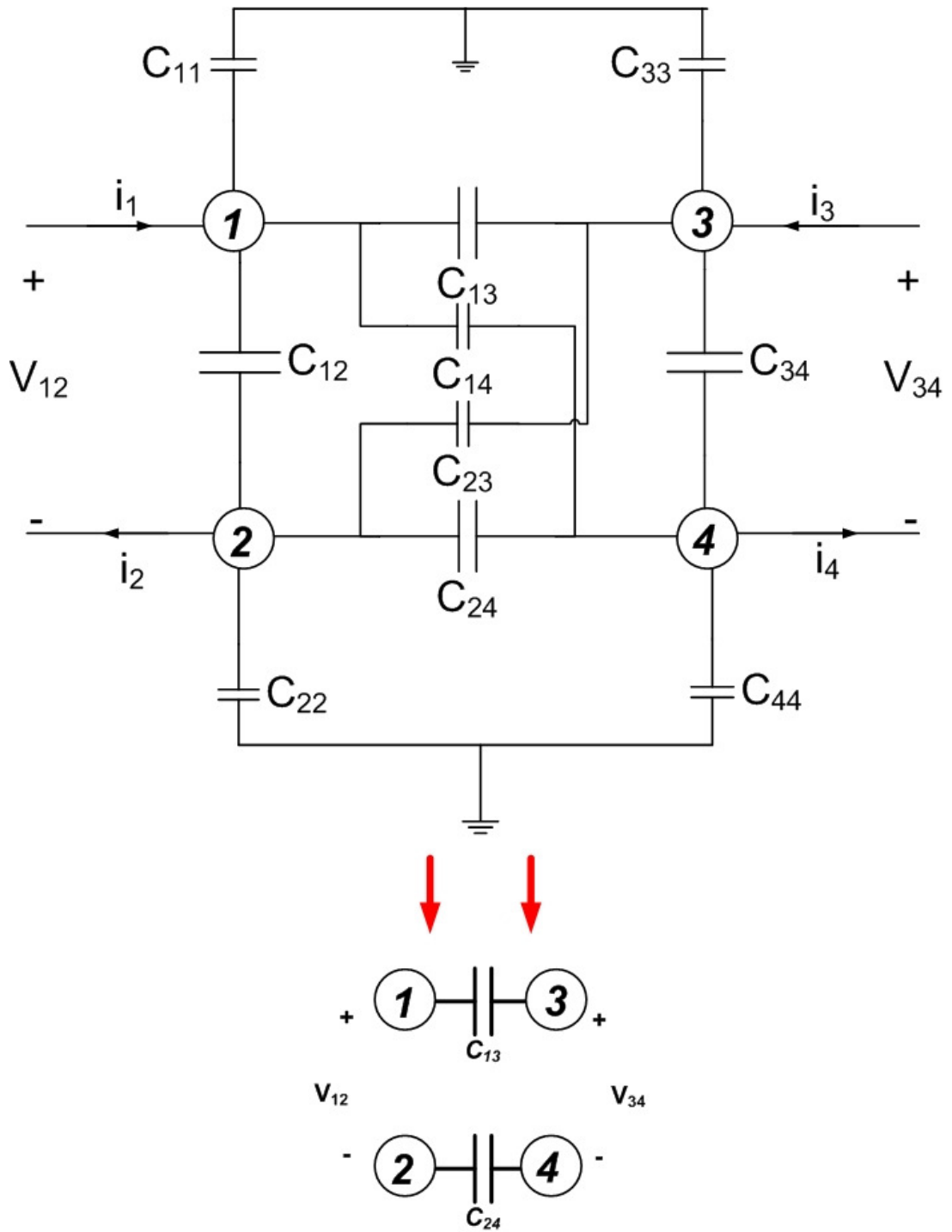


Figure 8: Circuit with 100 % Coupling

For increased accuracy, the circuit was placed in a Faraday cage to allow the self-capacitors to be easily measured. Following the measurement procedure in Appendix B, all capacitances were measured and this shown in Table 5.

Table 5: Capacitor values for circuit with 100 % Coupling

Capacitor Name	Capacitance Measured
C_{12}	2.500 pF
C_{13}	10.02 nF
C_{14}	0.890 pF
C_{23}	0.890 pF
C_{24}	9.991 nF
C_{34}	1.500 pF
C_{11}	3.240 pF
C_{22}	1.500 pF
C_{33}	3.240 pF
C_{44}	1.500 pF

Putting the measured capacitors from Table 5 into the expressions in Appendix A, the values for C_1 , C_2 and S in the capacitive coupling model were calculated. The values for C_1 and C_2 were then measured directly using the routine in Appendix B. Using two separate load resistors, the value for S was inferred according to the routine detailed in Appendix B. All calculated and measured values are shown in Table 6.

Table 6: Circuit Parameters for 100 % Coupling

Parameter	Measured Capacitance (nF)	Calculated Capacitance (nF)	Percentage Discrepancy
C_1	5.005 nF	5.007 nF	0.039 %
C_2	5.004 nF	5.006 nF	0.039 %
S	5.002 nF (inferred using 10 M Ω load)	5.002 nF	0 %
	5.002 nF (inferred using 560 k Ω load)		

In Table 6 we see very good agreement between predicted and measured results. Using the parameters in Table 6 we can calculate the extent of coupling given by k' in the expression in (4). This is shown in (5).

$$k' = \frac{S}{\sqrt{C_1 C_2}} = \frac{5.002nF}{\sqrt{5.005nF * 5.004nF}} = 0.999 \quad (5)$$

From (5) it is clear that the designed system exhibits close to 100 % coupling.

6. Zero Coupling

The coupling term S in a capacitive system can be made to have a zero magnitude. This is a system with bodies arranged such that the voltages and currents sum in such a way that despite the capacitance between the bodies, there is no apparent coupling between the input and the output. In order to achieve this, the numerator of the S term needs to be made equal to zero, i.e. based on (1), the relationship in (6) must hold.

$$f_4(C) - f_5(C) = 0 \quad (6)$$

The mathematics does not pose any restriction on a design which allows for (6) to hold, so the only possible limiting factor is the geometrical structure. A coupled system was designed to show that it is indeed possible, and this is discussed in the following section.

6.1 Experimental Validation – Zero Coupling

The aim of this design was a proof of concept rather than optimisation for a specific application. There are many applications for a zero-coupled capacitive system, and some are discussed concisely in Chapter 5.

Similar to the previous structure with arbitrary coupling, a parallel plate layout was chosen for simplicity. A schematic for the structure is shown in Figure 9.

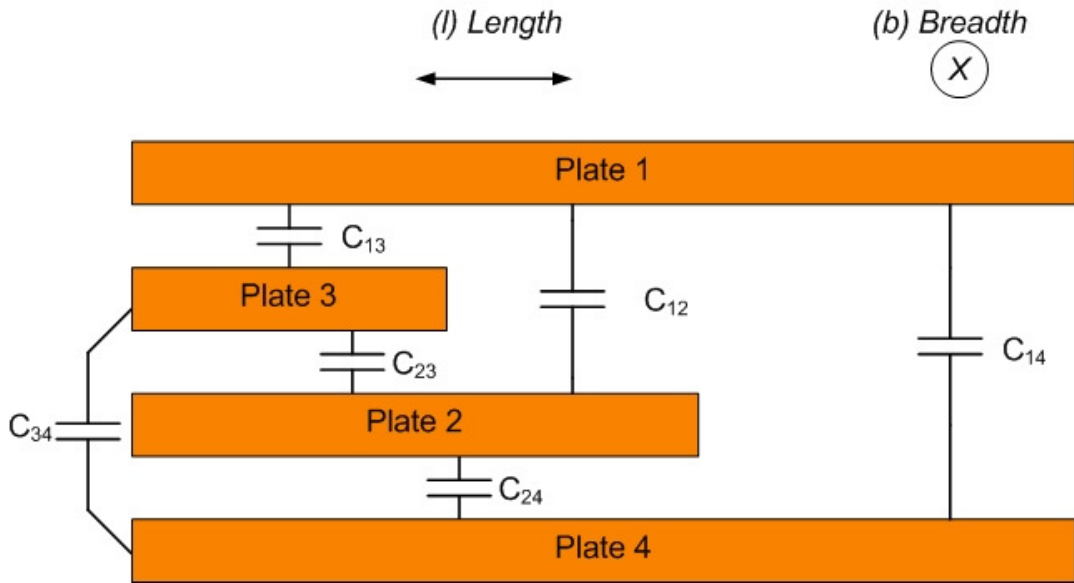


Figure 9: Schematic for structure with zero coupling

Using an iterative design procedure, the parameters shown in Table 7 were obtained to give a system with zero coupling. The subscripts are used to show which geometrical dimensions correspond to specific capacitors, e.g. l_{12} is the horizontal length of the overlap between plates 1 and 2, b_{xx} is the distance into the page of overlap between any two plates, and d_{xx} is the vertical distance between any two plates.

Table 7: Parameters for system with zero coupling

Parameter	Magnitude
l_{12}	1 cm
l_{13}	2 cm
l_{14}	15 cm
l_{23}	2 cm
l_{24}	3 cm
l_{34}	0 cm (no overlap)
b_{xx}	10 cm
d_{xx}	0.01 mm
ϵ_0	8.854 pF/m
ϵ_D	~ 4

In this experiment, the test structure was not placed in a Faraday cage, and thus the self-capacitances of the bodies could not be easily measured. It is shown later that in this particular case, the neglect of the self-capacitors does not have a major effect on the results.

Using the routine in Appendix B which neglects self-capacitances, the magnitudes of the six inter-body capacitors were measured and these are shown in Table 8.

Table 8: Capacitor sizes for zero-coupled layout

Capacitor Name	Measured Capacitance (nF)
C_{12}	0.094
C_{13}	0.457
C_{14}	0.933
C_{23}	0.403
C_{24}	0.741
C_{34}	0.006

Based on the measured capacitances, the expression in (6) is very close to zero as shown in (7).

$$f_4(C) - f_5(C) = 37.36 \times 10^{-21} \text{ F} \quad (7)$$

Using the measured capacitors, the values for C_1 , C_2 and S were determined, and these are shown in Table 9. These parameters were also experimentally determined using the routine described in Appendix B. The experimental values are also provided in Table 9.

Table 9: Circuit Parameters for zero-coupled layout

Parameter	Measured Capacitance (nF)	Calculated Capacitance	Percentage Discrepancy
C_1	0.720 nF	0.721 nF	0.139 %
C_2	0.571 nF	0.574 nF	0.525 %
S	15.62 pF (inferred using 220 kΩ load)	14.52 pF	7 %

It is clear from Table 9 that there is very good agreement between the values predicted using the model and the values which were measured directly. Although there is a 7 % discrepancy in the S parameter, this is considered acceptable as the order of magnitude of S is much smaller than that of the parameters C_1 and C_2 . This slightly higher error level can be attributed to the difficulty of the inferred measurement – since S is so small (designed to be zero), the input impedance barely changes when a load is applied to the output. At this resolution, parasitic effects such as temperature drift become apparent and may have affected the measurement slightly.

Since the capacitive coupling model used in this experiment was the simplified model which neglects self-capacitors, the high level of agreement between predicted and measured results show that in this case the neglect of self-capacitors

is a valid assumption. Evidently the relative magnitudes of the self-capacitors are much smaller than the inter-body capacitors, and this is achieved by the construction of the structure where the bodies had large overlap area and small inter-body distance.

Using expression (4) to quantify the coupling of this system, we get (8).

$$k' = \frac{S}{\sqrt{C_1 C_2}} = \frac{15.62pF}{\sqrt{0.720nF * 0.571nF}} = 0.024 \quad (8)$$

Based on the results above, it is clear that it is indeed possible to construct a system which exhibits close to zero coupling as predicted by the model.

7. Conclusion

An in-depth analysis was done into the properties of the coupling capacitance S which forms part of the capacitive coupling network developed in Chapter 2. It was shown that although a designer can obtain positive, negative and zero coupling with a four-body system, there is no real significance in the sign of S as the output terminals can easily be switched to obtain a different voltage polarity. It was also shown that this holds regardless of the fact that the system is not fully electrically isolated, since it behaves like an electrically isolated system when the condition of balanced currents is enforced at the input and output terminals.

Experimental verification of three capacitively coupled systems were documented. These included a case with arbitrary coupling between the input and output, a case with close to 100 % coupling and a case with close to zero coupling. All experimental evidence showed excellent agreement with the results predicted by the capacitive coupling circuit model. This shows that the model can indeed be used to describe the electrostatic behaviour of an arbitrary four body system.

Chapter 4: Properties of the Capacitive Coupling Model

1. Introduction

In the previous chapters, the need was highlighted for general circuit theory to support capacitive coupling in a similar way that inductive coupling is described. A circuit model for a four body system was then developed from the field level theory which formed a dual to the inductive coupling model. A term called “coupling capacitance” was defined to be the dual quantity to mutual inductance, and this quantity was explored in detail in Chapter 3. Experimental verification showed good agreement between the theory and a physical system.

At this point, a “new” circuit component has been developed. In this chapter, certain circuit properties of this component are explored in order to gain insight into the operation and limitations of the component. For reference purposes, the capacitive coupling model is shown in Figure 1 with the governing equation in (1). The inductive coupling model is shown in Figure 2 with its governing equation in (2).

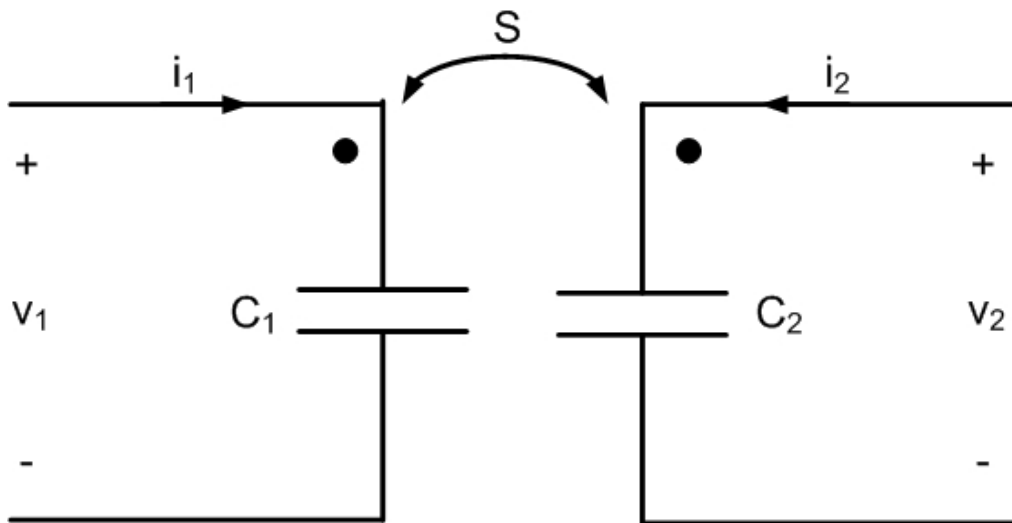


Figure 1: Capacitive Coupling Model

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} C_1 & S \\ S & C_2 \end{pmatrix} \begin{pmatrix} \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{pmatrix} \quad (1)$$

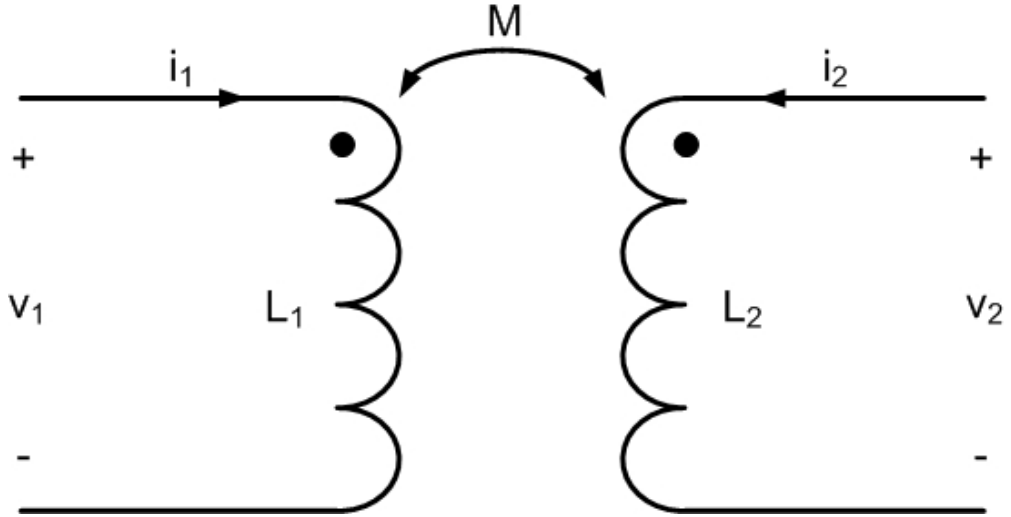


Figure 2: Inductive Coupling Model

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} L_1 & M \\ M & L_2 \end{pmatrix} \begin{pmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{pmatrix} \quad (2)$$

2. Circuit Properties

Typical circuit properties are discussed in the following subsections.

2.1 Linearity, Memory and Causality

The system of equations which describe the capacitive coupling model is easily shown to be linear under the condition that the coefficients C_1 , C_2 and S are linear. Since these quantities translate to physical capacitors, the condition imposed for

the system to be linear is that the dielectrics of the capacitors be linear. In the inductive model, the requirement for linearity is that the permeability of the magnetic core is linear. The linearity property thus appears to impose similar requirements in both the capacitive and inductive coupling models.

Since the voltage-current relationships of a capacitor are based on values at previous time intervals, the capacitive coupling model can be said to have memory. There is no future dependence, so the system is causal. These properties also hold in the inductive coupling model.

2.2 Energy Storage

In order to develop the capacitive coupling circuit model, the only condition which was placed on the general four-body coupled system is that there are differential currents at the input and output ports. This condition implies that there is zero net current flow from the input side of the network to the output side. By enforcing this condition, the system is fully described by the parameters as described in Appendix A. The energy storage of the system can therefore be calculated as follows.

Short circuit the output port V_2 . Let V_1 increase from 0 V up until V_1' . The total energy stored in the network is given by (3), where P represents power.

$$E_1 = \int_{t_0}^t P dt = \int_{t_0}^t V_1 I_1 dt = C_1 \int_0^{V_1'} V_1 \frac{dV_1}{dt} dt = \frac{1}{2} C_1 V_1'^2 \quad (3)$$

Let V_1 remain constant, i.e. $\frac{dV_1}{dt} = 0$. Now let V_2 increase from 0 V up until V_2' .

The total energy stored in the network is given by (4).

$$\begin{aligned}
E_2 &= \int_{t_0}^t P dt = E_1 + C_2 \int_0^{V_2'} V_2 \frac{dV_2}{dt} dt + S \int_0^{V_2'} V_1' \frac{dV_2}{dt} dt \\
&= \frac{1}{2} C_1 V_1'^2 + \frac{1}{2} C_2 V_2'^2 + S V_1' V_2
\end{aligned} \tag{4}$$

Since the system is linear, the expression for total energy stored is the same if the procedure is reversed. The expression in (4) is the dual to the expression in (5) which represents the energy storage in a two-body inductively coupled system.

$$E_L = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2 \tag{5}$$

In order to verify the expression in (4), an arbitrary four body system was simulated using the circuit model and the total energy was calculated using (4). A similar four body system was then set up in a FEM package and the field energy was computed numerically. The energy calculated using these two methods were found to be identical.

2.3 Limit on Coupling Capacitance

In the inductive model in (2), the inductances on the off diagonal of its co-efficient matrix L_{ii} correspond to the self-inductances of the system, and the term M on the main diagonal is the mutual inductance of the system. Both quantities have a direct link to the field level theory.

If it is stated that the capacitive model in (1) is the circuit level dual to the inductive model in (2), then at a circuit level the self-inductances must be the duals of the C_{ii} terms on the off diagonal of the capacitive co-efficient matrix in (1). The same relationship applies to the mutual terms on the main diagonal. It is important to recall, however, that these relationships do not extend to the field level.

In an inductively coupled circuit, the mutual inductance can be expressed in terms of the self-inductances of the system, as shown in (6). The term k is a ratio defined to indicate the “extent” of the coupling between the two coils.

$$M = k\sqrt{L_1 L_2}; \quad 0 \leq k \leq 1 \quad (6)$$

The question arises as to whether or not a similar expression to (6) can be found for the capacitively coupled system. In order to do this, boundary conditions must be applied to the system to determine the maximum value for the mutual term S .

Recall the expression for energy storage in (4). To find the maximum value for S , a boundary condition that can be enforced is that the energy storage in the capacitive coupling system must be minimised. This condition is physically realised when the coupling term S is negative, giving an expression for energy storage is given (7).

$$E = \frac{1}{2}C_1 V_1^2 + \frac{1}{2}C_2 V_2^2 - S V_1 V_2 \quad (7)$$

Since the energy storage of the system is always greater than or equal to zero, this reduces to the expression in (8).

$$\frac{1}{2}C_1 V_1^2 + \frac{1}{2}C_2 V_2^2 \geq S V_1 V_2 \quad (8)$$

By completing the square and solving for S , the expression in (9) can be derived.

$$S \leq \sqrt{C_1 C_2} \xrightarrow{\text{yields}} S = k' \sqrt{C_1 C_2}; \quad 0 \leq k' \leq 1 \quad (9)$$

Based on the analysis above, it is shown that in a capacitively coupled system, the coupling capacitance S can be expressed in terms of the two capacitances C_1 and C_2 , similar to the inductive coupling case. A coupling term k' can also be defined

to describe the “extent” of the coupling in the network. This property ties in with the dual nature between the inductive and capacitive coupling networks.

3. Applications of model

This section describes some important applications of the capacitive coupling model with comparisons to the inductive coupling model.

3.1 Theoretical Application: Interconnections between networks

One of the main benefits of two-port modelling is the ability to analyse complicated networks in terms of combinations of different two-port network circuits. The converse of this is to construct a complicated network using two-port network components in various combinations. Two-port networks can be interconnected in a few basic ways, namely: series, parallel and cascade. These can be split into two groups and are discussed separately below.

Group 1: Series and Parallel Connections

For a two port network interconnection in this group, the two port parameters describing the interconnected network is simply the sum of the component network parameters, as long as a set of validity tests known as the Brune tests hold [1]. The Brune tests verify that each component network behaves the same way when it is interconnected as it does in isolation, by verifying that the input and output ports have differential currents even when interconnected. These interconnections are shown in Figure 3 and Figure 4.

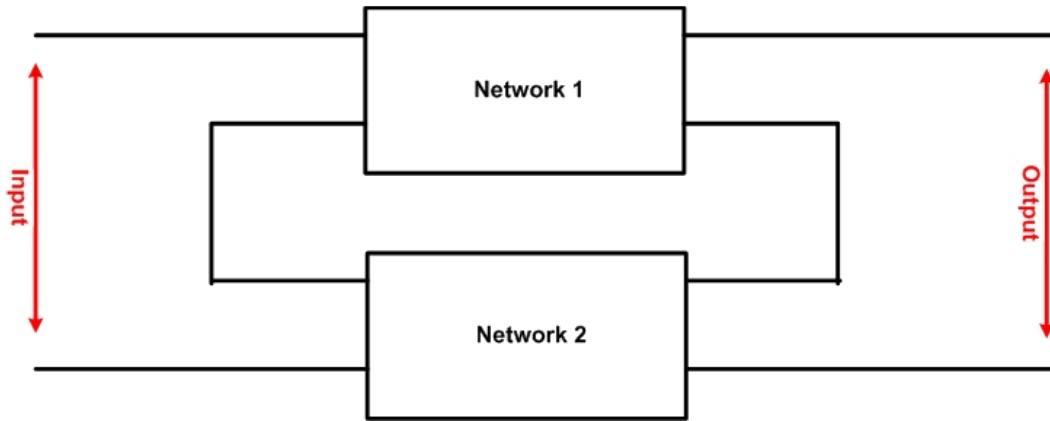


Figure 3: Two Port Series Connection

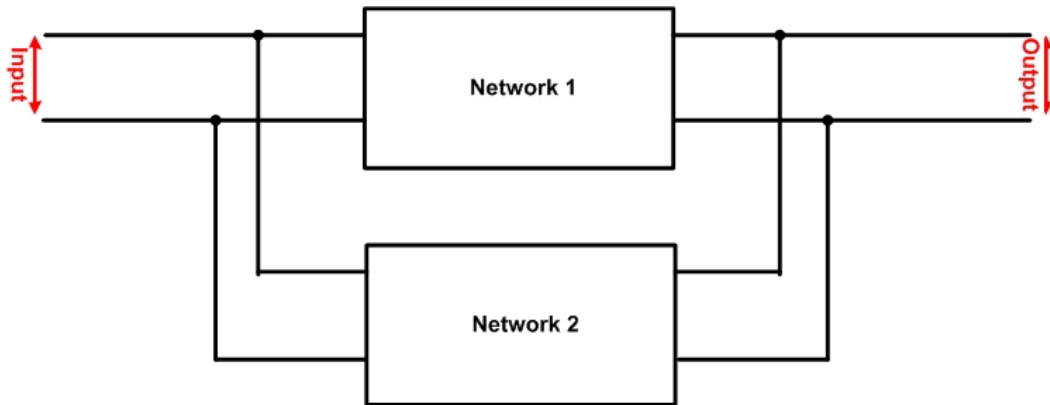


Figure 4: Two Port Parallel Connection

Recall from Chapter 2 that in the inductive coupling model, the two-port network representation has a direct link to the physics upon which it is based. Since the model and the physics show no electrical connection from the input to the output stages of the network, the condition for differential currents at both ports is internally enforced by the physical system. Because of this, the Brune tests are inherently satisfied, allowing any combination of the above interconnections to be made. The two-port representation of the interconnected network is therefore the sum of the component two-port parameters.

In contrast, the capacitive coupling two port model requires that the condition for differential currents should be externally enforced for the model to hold. This means that the Brune tests do not necessarily hold in any arbitrary

interconnection, and thus the two-port parameters describing the interconnected network is not necessarily the sum of the component network parameters. Further analysis needs to be done on an interconnected system to develop an input-output model. Alternatively, a 1:1 transformer can be used between each interconnection to ensure differential currents at the ports and satisfy the Brune tests; after which the two-port representation of the interconnected network will be the sum of the component two-port parameters.

Although a difference in the operation of the inductive and capacitive models is highlighted here, it does not show a breakdown in the duality theory at a circuit level. A condition for the duality was that differential currents must be enforced, and as long as this condition is satisfied the models can be interconnected in the same way.

Group 2: Cascade interconnection

This type of connection can be made between arbitrary component networks, and the two port parameters of the interconnected network is simply the product of the parameters of the component networks. It is shown in Figure 5.

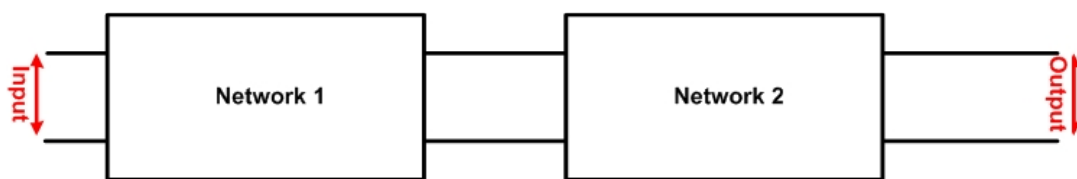


Figure 5: Two Port Cascade Connection

The cascade connection can be made for both the inductive and capacitive coupling two-port networks, and the condition for differential currents is inherently satisfied due to the terminations of the interconnected network on both ends.

3.2 Theoretical Application: Arbitrary configurations

Due to the physics upon which the inductively coupled circuit is based, the output stage may be connected to the input stage of the circuit in either a series or parallel configuration whilst maintaining the coupling; however the electrical isolation will be lost. A common case of this is in the autotransformer, where a series connection is made between the input and output stages, as shown in Figure 6.

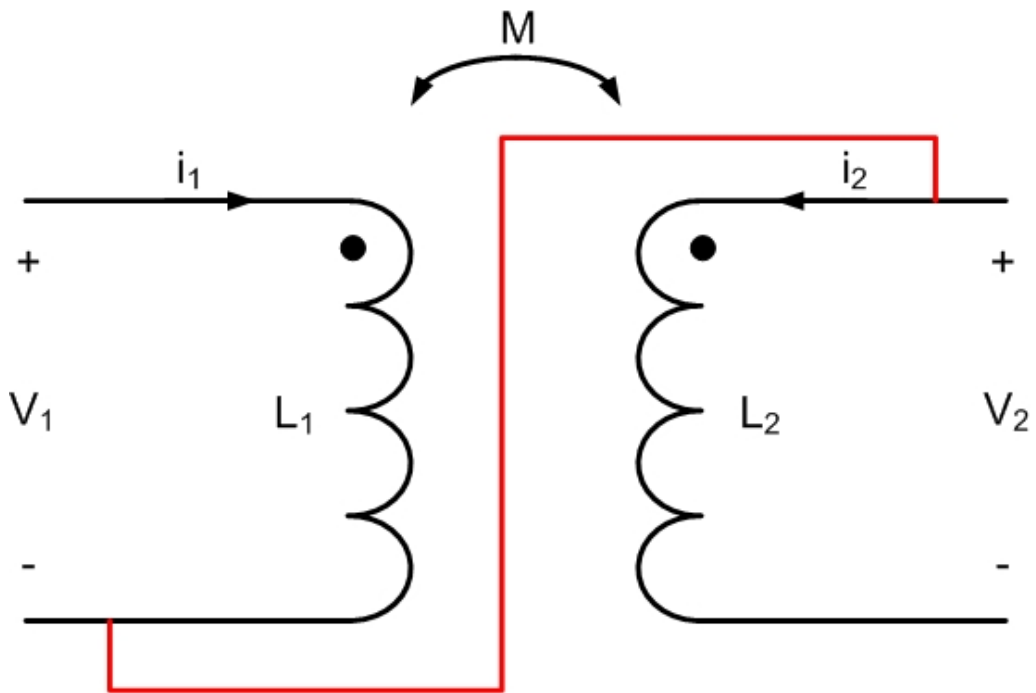


Figure 6: Autotransformer configuration

The inductance looking into the input terminals is given by (10).

$$L_{input} = L_1 + L_2 \pm 2M \quad (10)$$

Based on the duality between the two port representation of capacitive coupling and inductive coupling, a similar expression can be found for the capacitance

looking into the input terminals when a series connection is made. However, this expression does not carry through to a field level.

The reason for the breakdown in the duality in this particular application is that the condition upon which the capacitive coupling model is based is not satisfied. This condition is that the input of the circuit must be applied between two bodies, and the output between the other two bodies; and there must be differential current at both the output and the input ports. However, in the case of the autotransformer, the output is tapped off between one of the original output ports and one of the input ports, i.e. a common reference is used for both the output and input. The two-port capacitive coupling model therefore cannot be used in any configuration that entails the use of a common terminal for both the input and output. A different model needs to be developed based on the physics of the system to allow for the two cases of 1) a single common terminal between the input and output (the series connection); and 2) two common terminals between the input and output (the parallel connection).

Although it was stated above that the model fails when a common reference is implemented between the input and output, a significant difference of the capacitive coupling model can be highlighted. In an inductively coupled circuit, there is only a single configuration that can be used such that there will be electrical isolation between the input and output circuit, and this is the configuration as shown in (2). However, in the capacitive coupling model, it is simply a condition that the input be applied between two bodies, and the output between two bodies. This means that as long as this condition is satisfied, any configuration of input/output ports will achieve pseudo-electrical isolation, as shown in Figure 7. Pseudo-electrical isolation is discussed in detail in Chapter 3. All of the combinations of input/output ports for a four-body system are shown in Table 1. To aid with this concept, the similarity between the networks in row 1 and row 2 of Table 1, using the same four bodies is shown in Figure 7.

Table 1: Configurations of capacitive coupling system

Input port terminals	Output port terminals
1-2	3-4
1-3	2-4
1-4	2-3

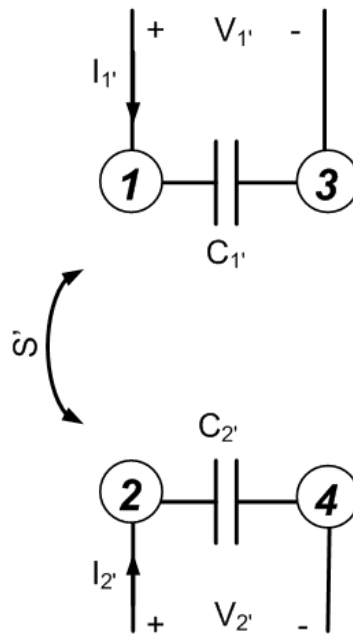
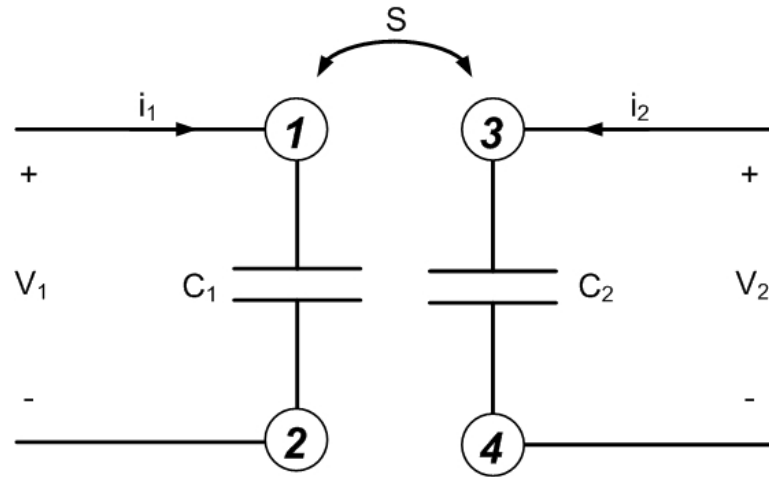


Figure 7: Capacitive Coupling showing alternate excitation

In order to use the various configurations shown in Table 1 in a circuit, it is simply a matter of substituting for the individual capacitors in the same two-port model described in this dissertation. This difference arises from the physical basis of the capacitive circuit, due to the fact that the capacitive coupling circuit is a four-body system as compared to the inductive circuit which physically consists of two bodies.

4. Use of the circuit model in circuit analysis

With the properties of the capacitive coupling circuit model described in the preceding sections, it is perhaps useful to show how the model can be used in circuit analysis. In this regard, a few examples are provided in Appendix C.

5. Conclusion

In this chapter, certain circuit level properties of the capacitive coupling model were explored to gain insight into the operation and limitations of the model. It was shown that the properties of linearity, memory and causality hold in a similar way that it does for inductive coupling. An expression for total energy storage in a capacitively coupled network was developed, and this expression also appears to exhibit a duality with the expression for energy storage in an inductively coupled network. Using the expression for energy storage, an expression for the maximum value of the coupling capacitance of the network was developed. This is also shown to be of similar form to the inductively coupled network.

Interconnections between multiple capacitively coupled networks were then explored, and it was found that in general these connections can be made without the duality breaking down, as long as the condition of differential currents at the input and output of the network is enforced. It was then mentioned that it is not possible to enforce this condition when there is a connection of a common

reference between the input and output of the network - hence this type of connected is not allowed by the model.

A difference of the capacitive coupling model was then highlighted in that the model can be used to describe a system with any set of ports chosen as the input and the other set as the output. This is not the case with an inductively coupled system.

Lastly, general examples are provided detailing how to use the model in standard circuit analysis.

6. List of References

- [1] Ruston H, Bordogna J. *Electric Networks: Functions, Filters and Analysis*. McGraw-Hill Inc, USA, 1966.

Chapter 5: A Discussion on Typical Applications for the Capacitive Coupling Circuit Model

1. Introduction

In the previous chapters, a circuit model was developed to deal with capacitive coupling between four conductive bodies. In this model, the input is applied across two bodies and the output is tapped off the other two bodies. The aim of the model was to further develop general circuit theory, as there was no consistent, rigorously defined capacitive coupling circuit model similar to the well-known inductive coupling model. This meant that many designers used different circuit models in different applications and it was difficult to make any comparisons between different systems. Since the model derived is general in nature, it has the potential to be used in a variety of applications and allow comparisons to be made between different systems.

In this chapter, typical practical applications of the capacitive coupling model are documented. For each type of application, a discussion is provided as to whether or not the use of the capacitive coupling model would be advantageous. The list of applications provided is by no means an exhaustive list of all possible applications, but rather a concise group of different applications to give the reader an indication of the possible widespread benefits of such a model.

2. Applications

Typical applications are discussed in the following subsections.

2.1 Tapping power from overhead transmission lines

The presence of AC voltage on floating conductors in proximity to high voltage overhead AC transmission lines has received much attention in literature. This voltage can exist due to three mechanisms: conductive coupling, inductive coupling and capacitive coupling [1]. The voltage due to the coupling is often undesirable as it can result in electric shock when humans or animals come into contact with it. In these cases, it is important to take the electric fields into account, and implement measures to mitigate its effect. In order to do this, some sort of modelling of the coupling effect must be done. With this in mind, Al-Badi and Salam [1] developed a finite-element-modelling based system to predict the voltage on a gas pipeline in the vicinity of an overhead transmission due to capacitive coupling. Although this model gave good agreement to an experimental setup, it is limited in that it is not a circuit model, and as such cannot easily be used to describe the interaction between the capacitive system and other circuit systems which may be present.

While the case described in the preceding paragraph had undesirable coupling, there are also applications where capacitive coupling can be used for beneficial purposes. One such application is the Sishen-Saldana railway link in South Africa, where the locomotive is powered by the high voltage overhead line [2]. In this application, there is no low-voltage supply easily available alongside the train for track-side services, and other options such as step down inductive transformers and PV arrays are not feasible or viable solutions. In his paper [2], Nicolae presents a capacitive coupling model that allows for sufficient energy tapping off the transmission line for powering up of these devices. This model is an analytic model, and corresponds to the simplified model (neglecting self-capacitance) presented in Chapter 2 of this dissertation. It is shown experimentally that the model does in fact give good agreement with experimental data, and can be used in such a situation.

Another use for capacitive power tapping off transmission lines is for powering cellular masts in remote locations. In recent years, mobile phone usage has become very widespread, to the extent that there are more mobile phones in use than traditional landline phones. Because of this, cellular network operators are continuously increasing their network coverage to support the increasing number of subscribers. In urban areas, this is not too difficult as there is generally a power source easily available. However, this is not necessarily the case in outlying areas where there is no electrical infrastructure available. Blais and Berthiaume [3] proposed a method of tapping power from the overhead ground cable of an electric power transmission line. The mechanism upon which it is based is capacitive coupling. This is widely regarded as the more economical method of capacitive power tapping as compared to the use of a capacitive voltage divider [3]. The basis of their work is that capacitive coupling gives rise to a voltage on the overhead ground cable due to the group of electrified 3-phase cables in its vicinity. This voltage can be used as a voltage source to power a device. This setup is shown in Figure 1. Using this information, they derived a Thevenin circuit model of the setup; the output of which provides the power to the cellular mast as shown in Figure 1. In a test case of a 735 kV transmission line, they were able to tap off 2 kW of power for every kilometre of wire length.

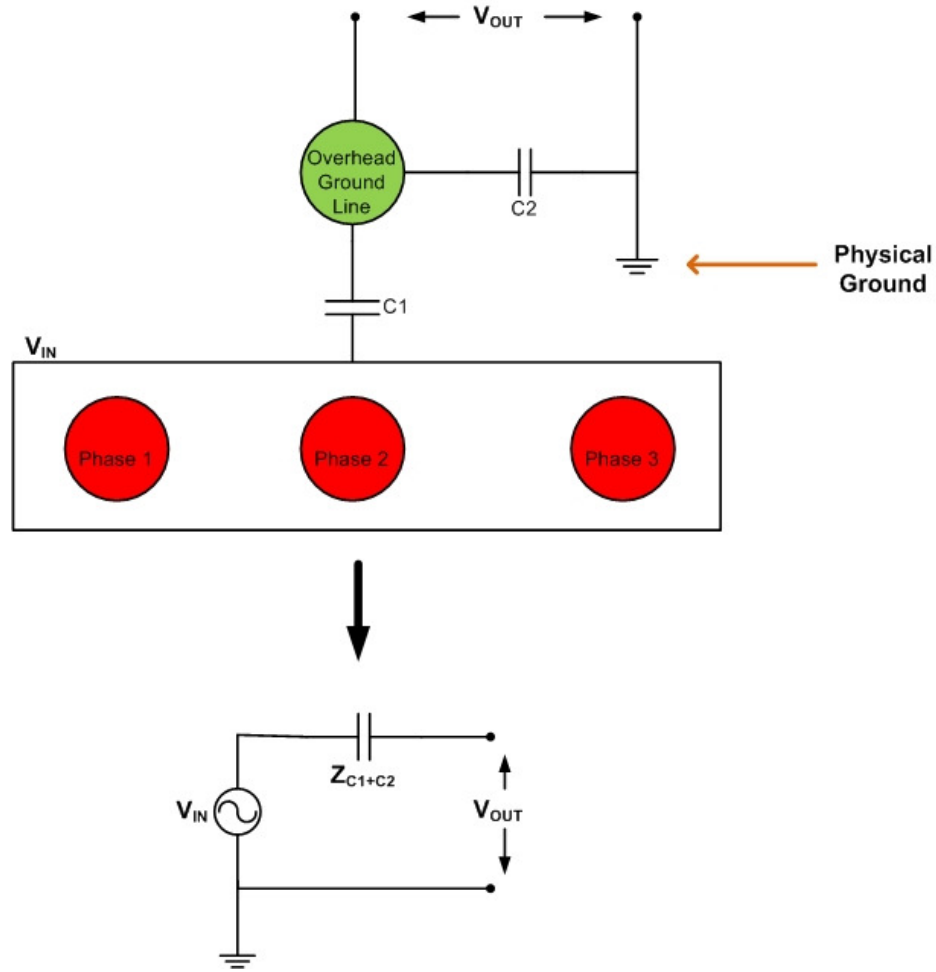


Figure 1: Power tap off from ground wire

In terms of their design process, although they proposed that the mechanism of power transfer was capacitive coupling, the capacitive effects were not modelled, and assumed to be uniform for the length of the line. There is no measurement of these capacitances with a link back to the proposed circuit model. Instead, the voltage between the overhead ground and earth was simply used as the two terminals providing power to the cellular mast. Because of this, the only method to optimising power transfer is to increase the length of the line.

In the case where the capacitances are uniform and no modification is made to the overhead lines, this simplified approach is fully valid and gives good experimental results as shown by the authors. However, it is extremely limited in that the output cannot be predicted theoretically as the capacitances are unknown.

In order to have a more general approach to capacitive tap-off from overhead transmission lines, it will be useful to use a capacitive coupling circuit model such as the one derived in Chapter 2. In order to use the model, it is required that there are differential currents at the input and the output terminals, as shown in Figure 2. Similar to the paper discussed above, the overhead ground cable and earth can be used as the two output terminals since the condition of differential currents at the output will be enforced by the powering a passive load.

At the input, the three 3-phase cables can again be modelled as a single body, and this forms one of the input terminals. This is possible since the three 3-phase cables are at different potentials, and their electric fields add to give a resultant field in space. Due to phasing, this resultant field will only be non-zero within close proximity to the group of cables, and further investigation is required to quantify this exact distance. It is not possible to use earth as the second input terminal because a common reference between the input and output is not allowed as discussed in Chapter 4. To get around this, a test conductor can be inserted into the field to form the second terminal at the input. In this way, the condition of differential currents at the input can be enforced. The test conductor can have any geometry and placement, such that the system gives the desired output. This gives the designer the benefit of a level of variability in the design, i.e. the designer is not entirely restricted by the geometry of the lines.

Investigation must be done into the consequences on the surrounding fields due to the placement of a test conductor, as well as the maximum distance from the group of three-phase lines at which the resultant electric field will be of a magnitude such that it can be modelled as a single body.

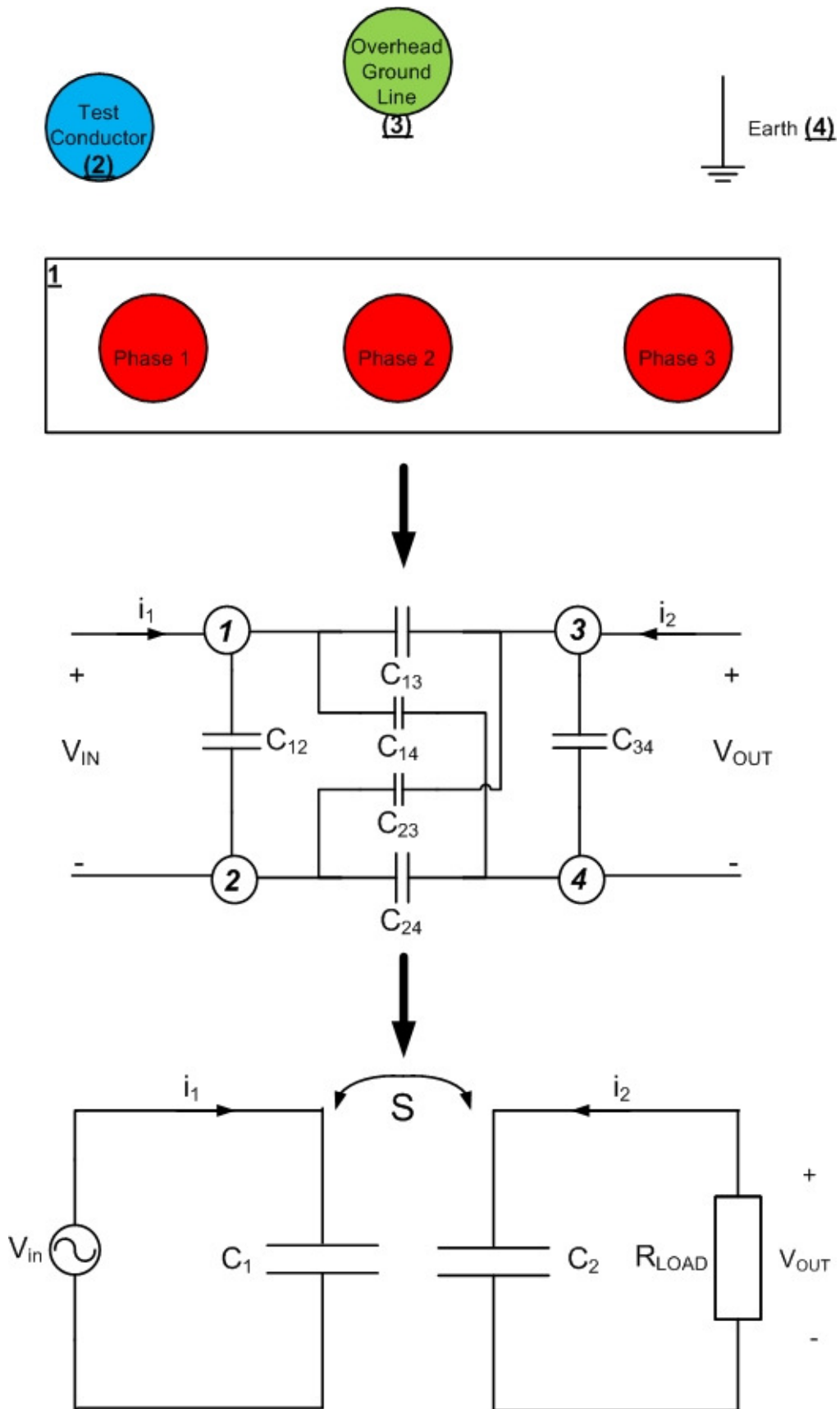


Figure 2: Power tap off from ground wire using model

The advantage of using this type of system is that the fourth body (the test conductor) can be designed and placed specifically to optimise power output, since the other three bodies have no variability. Also, the output can be determined theoretically for any given geometry of transmission lines at any voltage. Therefore this is not only a more comprehensive model than the traditional one, but it also offers designers more flexibility in a design of such a system.

2.2 Human Proximity Sensors

There are many applications where some sort of human proximity sensing is required. Generally these sensors are used to preserve the safety of human beings who are in proximity to some sort of machinery. One of these applications is presented in [5], where the authors describe a proximity sensor which detects a human close to the blade of a chainsaw, and implements electricity cut-off. The mechanism of operation is simple as shown in Figure 3 – a transmitter plate is placed on the one side of the blade and a receiver is placed on the other. A switching waveform is applied to the transmitter plate and the voltage at the receiver plate is measured. Capacitors are defined between the various bodies as shown in Figure 4. Because of the grounded blade placed between the transmitter and receiver, there is a weak capacitance between the two plates and it relies on fringing flux. When a human body part enters that field, it disturbs the flux and less voltage appears at the receiver. It is found that by using this mechanism, a human hand can be detected at a distance of 15 cm, and false alarms are not triggered by the presence of wood.

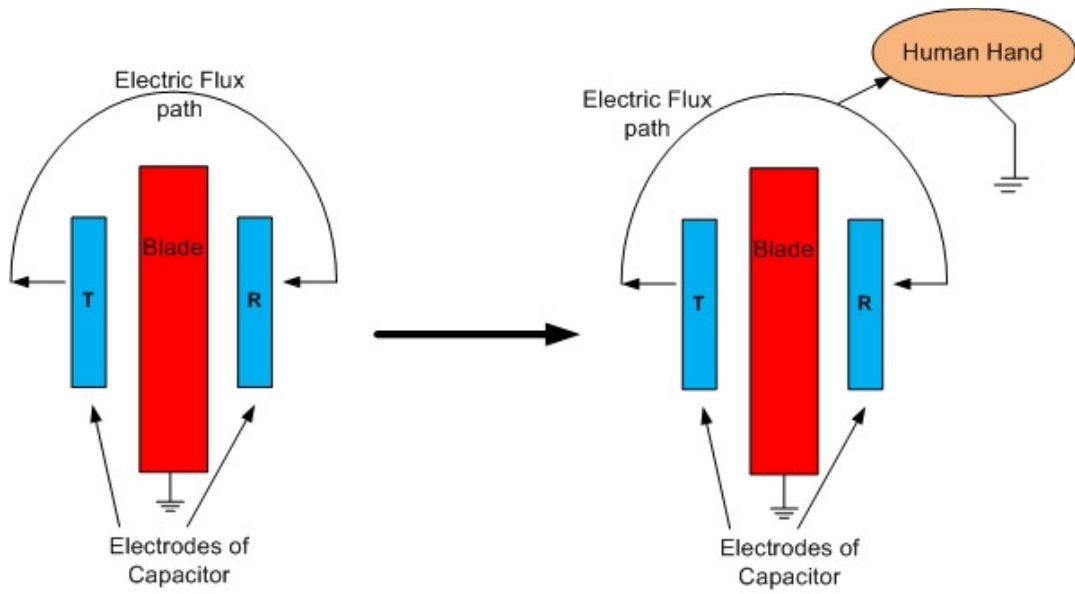


Figure 3: Proximity Sensor for Chainsaw

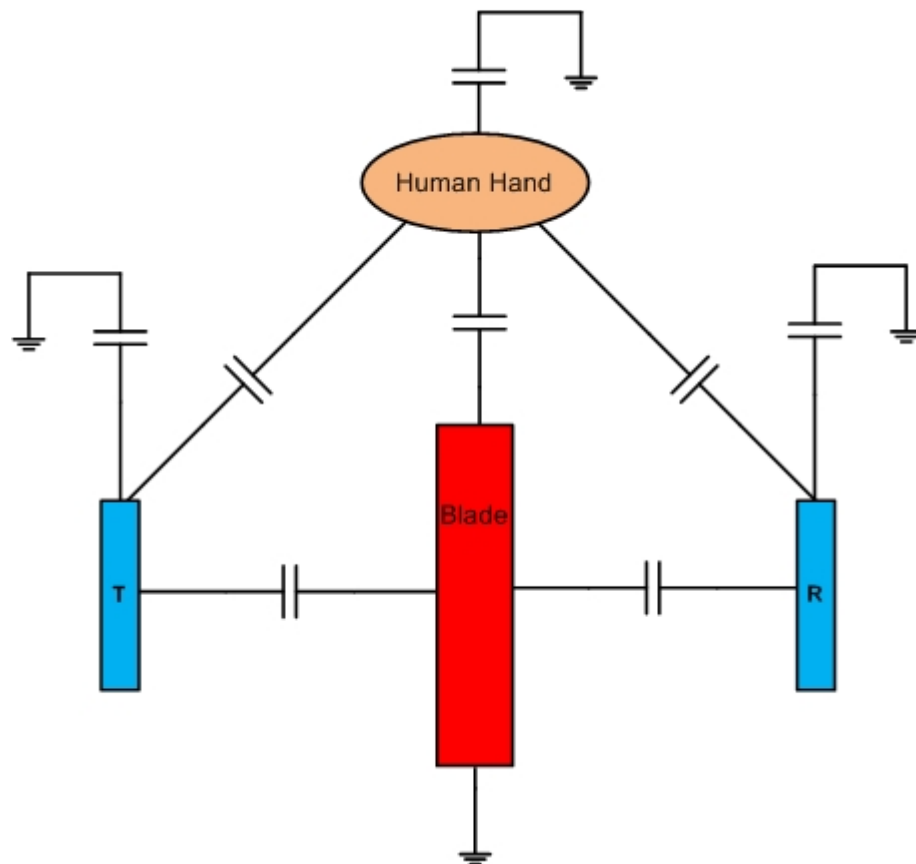


Figure 4: Chain saw proximity sensor circuit

It is clear from Figure 4 that the circuit is made up of four bodies can be simplified using the model presented in this dissertation. An alternative circuit model for this application is shown in Figure 5. By using the setup in Figure 5, it is not a requirement that the blade be connected to physical ground, unlike the circuit used in [5]. The ground symbols used in Figure 5 therefore refer to the absolute zero reference, or infinity, to which all bodies have a self-capacitance.

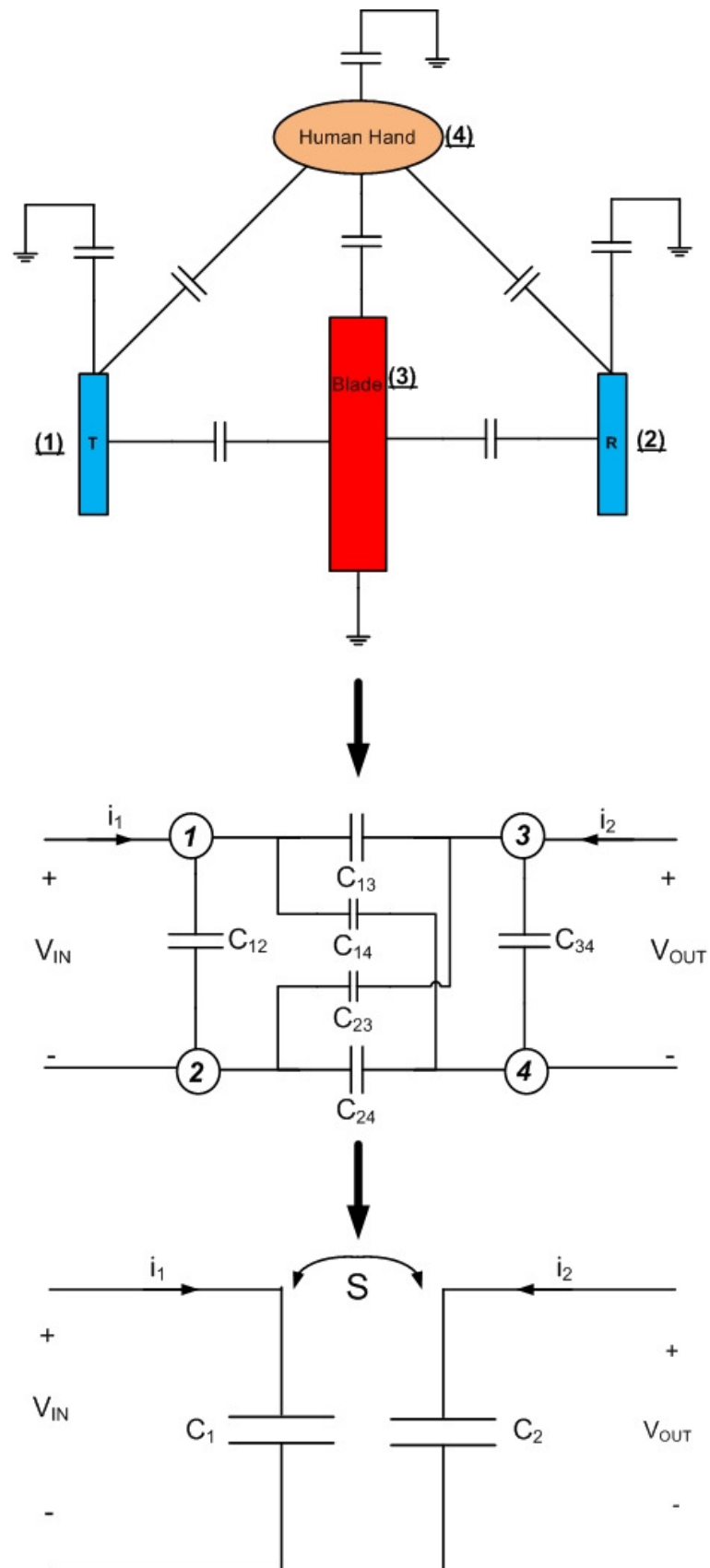


Figure 5: Using Capacitive Coupling Model for Chainsaw

Other applications of human proximity sensing can be found in buildings where existing electrical infrastructure is utilised to detect human presence and implement measures such as powering on lights or activating an alarm. In a higher voltage environment such as an electrical substation, human proximity sensors can be used to raise an alarm if a person steps too close to a live installation.

Buller and Wilson [6] described a method of measurement and modelling the coupling capacitance between electrical wiring in buildings and humans. The technique employed to measure the coupling capacitance between the human and the electrical wiring was to make use of a musical instrument called a theremin. This device operates on the principle that the human acts as a conductor with a capacitance to ground, and modifies the capacitance controlling the frequency of a variable frequency oscillator. The signal is compared to a fixed frequency oscillator, and the change in capacitance observed due to a human interacting with the field can be found as a function of the difference between these two signals. A series of experiments were performed with different humans and a numerical consistency was found in the results obtained for specific distances away from the electrical wiring. It was found that this type of sensor could predictably detect the proximity of a human up to a distance of 1 m away.

An analytic first order model was then proposed to represent the coupling effect, and the model results agree with the measurements taken using the theremin only in terms of order of magnitude.

Although the analytical model presented by the authors of the paper does have a level of agreement with experimental results, it is a very crude estimation of the capacitive effects which are manifest in such a setup. The model is not detailed enough to use as a basis for a design of such a system, but merely provides a step in the right direction. The authors mainly focused on the experimental setup to measure the capacitances and show that there is indeed a capacitive coupling that

occurs between humans and wiring, and that this coupling is large enough to be measureable.

In order to fully represent the electric field effects that are manifest, a much more comprehensive analytical model is required. The capacitive coupling model presented in this paper takes into account all electric field effects in a four body system, and this can be beneficial in such an application. A proposed setup is shown in Figure 6.

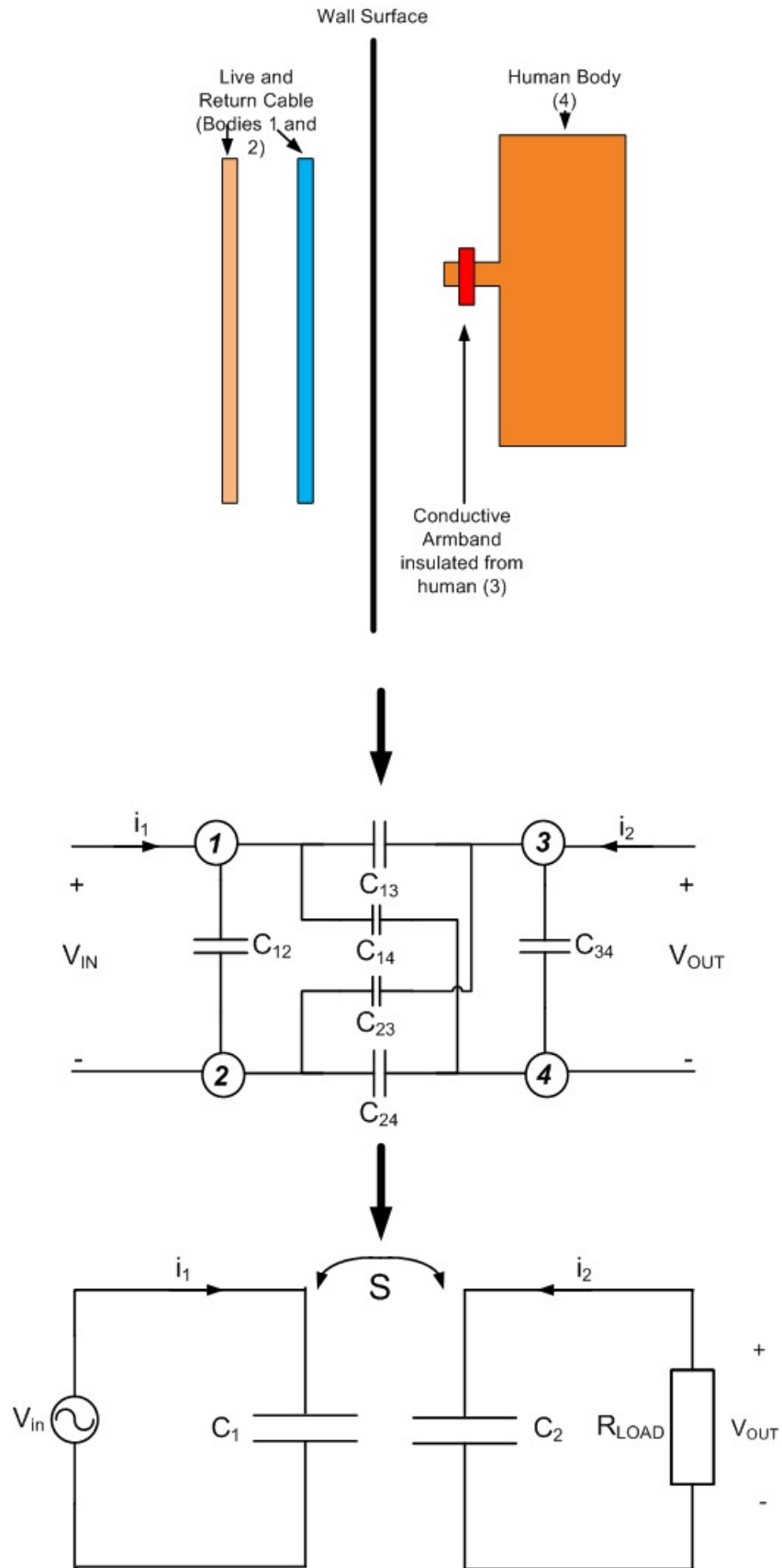


Figure 6: Human Proximity to Wall Wiring

In this particular application, there is electric wiring in the walls of the building. It is reasonable to assume that there is an outgoing and return cable in the walls for the electric supply, and these form two bodies. Outside the wall is the person. It is proposed that the person wears a conductive body on their person, which is not electrically connected to their body, e.g. an armband. In order to enforce the condition of differential currents at the output port, a load can be used with a connection to the band and the human. Depending on the coupling, a certain amount of power can be dissipated in the load, and this gives an indication of the proximity of the human to the electric cable. This kind of setup can also be used in high voltage installations, where a light can be made to glow on a person's armband to notify him when he moves his hand close to an electrified line.

The advantages of using the proposed model instead of the one detailed in the paper include the fact that all capacitive effects are fully modelled. The model also allows for calculation of outputs for any arbitrary configuration of wiring and human proximity.

2.3 The Electric Transformer

3-D integrated circuits have emerged in recent years as one of the technologies necessary for use in high end processing chips. The benefit of the extra dimension is that it allows for parallel processing power with short interconnects [7]. One of the main challenges in manufacturing 3-D chips is that of transferring data and power across the various layers, as galvanic connections restrict the layout and design of the chip significantly. While there have been methods proposed for data transfer between layers such as optical and inductive vias, these do not compete with the interconnect density of galvanic connections; and the power transfer across different dies is still an issue [7].

Culurciello and Andreou proposed a method of using capacitive coupling between different dies on a 3-D chip to allow communication as well as power transfer between different layers [7]. This is shown in Figure 7.

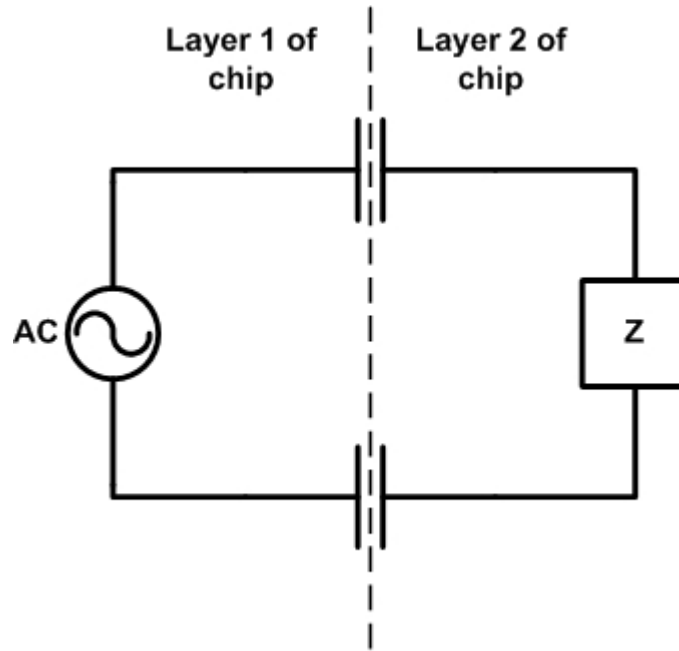


Figure 7: Capacitive Power Transfer for 3-D chips

The concept behind their approach is to have metal pads on different layers. These pads are aligned accurately to overlap each other, and these form parallel-plate capacitors across which displacement current can flow. In this way, power and data can be transmitted across layers, with each power transfer circuit requiring a minimum of two capacitors to complete the circuit. Using this concept, a power supply circuit was developed and an expression was developed to show the relationship between output current and the physical geometry of the capacitors. The setup was tested experimentally and found to yield good results. For their application, a reasonable amount of power was transferred using capacitive coupling, and it is shown that capacitive power transfer is a very viable technology for use in 3-D circuits.

Although the authors attempted to transmit power across layers using capacitors, a very fundamental assumption was made in their approach, namely that each two-body capacitor does not interact with the bodies alongside them. In other words it is assumed that the capacitors are very well defined in terms of their construction, and there is no coupling between the two capacitors which make up the power transfer circuit. There is a lot of precision in the design of 3-D integrated circuits, so this assumption is a reasonable assumption to make. However, in the case where the capacitors are brought very close to each other due to space constraints, this may pose a problem. For a more general approach then, it is beneficial to make use of the capacitive coupling model derived in Chapter 2. This takes all the effects of capacitive coupling between four bodies into consideration, and will provide a much more accurate model for the circuit.

A second application of capacitive power transfer is that of battery charging. Due to technological developments in recent years, most consumer devices have some sort of battery built in to the device to allow for mobility. Some of these include mobile phones, mobile computers, electric toothbrushes and electric cars. One of the biggest inconveniences of these devices is that they need to be plugged in to an electrical supply to get charged, and this often entails making a direct contact between the supply and the device using a cable. There have been many methods proposed to wirelessly charge devices in the past as it offers many benefits, including convenience and electrical isolation [8]. The most common technique used for wireless power transfer is inductive coupling, however this has its limitations. Another method to achieve wireless power transfer is to use capacitive coupling, and this has been used in niche applications. A typical example was investigated by Hu, Liu and Li in [9], where the authors developed a contactless battery charging system for a soccer-playing robot.

In this application, the authors developed a stationary power supply circuit with two flat plates, and a mobile power receiving circuit with another two flat plates, similar to the circuit shown in Figure 7. The method of operation is that when the battery charge on the robot falls to a certain level, it would return to the charge

station, and the plates would line up. When these plates are aligned, there is a well-defined capacitance between the plates on the stationary and mobile circuits, and due to the high frequency of operation, displacement current flows across the plates, completing the circuit.

This device is seen to give good experimental results, however, it is exceptionally limited in its design. Firstly, the authors rely on good coupling of plates between the stationary and mobile units, and zero coupling between the two plates on the stationary unit or the two plates on the mobile unit. The immediate consequence of this is that when the mobile unit is not charging, there will still be coupling in the stationary circuit, resulting in unnecessary power draw. Also, the mobile unit must align itself perfectly with the stationary unit when charging. If this does not happen, the coupling is not predictable. Another consequence is that the capacitances of their circuit must be very well defined if the other coupling capacitors are to be ignored (the capacitor between the stationary plate 1 and the mobile plate 2 and vice versa). Also, the self-capacitance of each plate is ignored entirely.

The factors mentioned in the previous paragraphs can potentially lead to many limitations in the general applicability of the approach used by the authors. If the capacitive coupling model discussed in this dissertation is used instead, many of these limitations can be accounted for and a much more general device can be designed. It is notable that this type of charging is not restricted to the niche application of soccer-playing robots, but rather it can be extended to wireless mobile phone chargers and the like.

2.4 Minimisation of parasitic capacitance in inductor windings

In the design of high frequency inductors, there is much emphasis placed on the electromagnetic coupling, where unwanted capacitive coupling takes place between windings in the inductor. This is especially true for filter inductors like

the one shown in Figure 8, where the capacitance between the windings leads to displacement current flow across the windings rather than conduction current through them. This effect is exacerbated by the high frequency of operation. There have been a number of measures proposed to reduce this effect by varying the placement and orientation of components, or by modelling it using finite element analysis [10], however these measures are non-general in nature. As such, it is difficult to use this theory in any arbitrary configuration.

Hofsajer [11] and Wang [13] proposed a capacitive coupling circuit model to deal with the inter-winding capacitance for minimisation of the parasitic effect in a high frequency inductor.

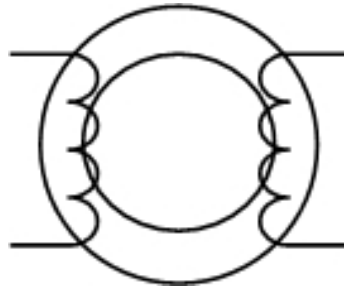


Figure 8: High Frequency Inductor

This capacitive coupling model presented in [11] and [13] to represent the parasitic effect is identical to the simplified model derived in Chapter 2, where the self-capacitances were ignored. This is shown in Figure 9.

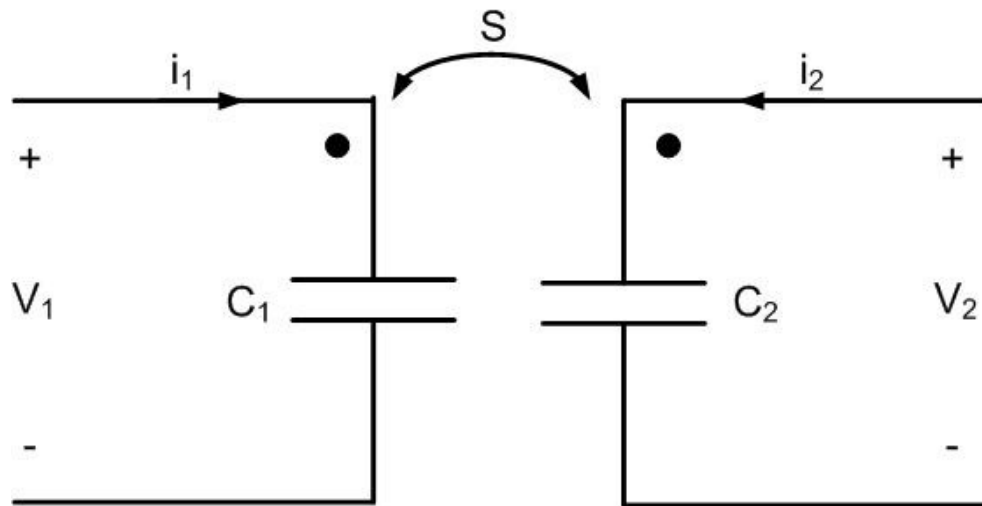


Figure 9: Capacitive Coupling Model

Using this model, an experiment was set up to arrange conductors in a layout to minimise the coupling term. A planar inductor, which usually gives problems at higher frequencies due to inter-winding capacitance, was built with the sizing of the plates accurately determined for coupling minimisation.

This was compared to a similar planar inductor which was not optimised for coupling minimisation. The results obtained show that the model indeed represents the coupling behaviour in such a configuration, and can be used to design inductors such that the coupling can be minimised.

Although the analysis performed by Hofsaier [11] was done on a planar inductor, the approach can easily be extended to the more common wound inductor. This means that the model can be used to reduce parasitic capacitance between transformer windings as well, by making use of a negative or zero coupling term as discussed in Chapter 3.

2.5 Capacitive Crosstalk

Ribbon cables are commonly used in electronics because they offer numerous benefits over other types of cable in certain applications. A typical commercially available application is that of medical ultrasound probes [12]. In this application it offers the benefits of improved ergonomics, decreased capacitance and lower cost than the conventional co-axial cable [12].

The typical configuration of four-wire ribbon cable and its electrical model are shown in Figure 10 [12]. This configuration represents a section of the ribbon cable, i.e. the terminations of the individual cables are not included in the model. The purpose of the multiple ground wires is to electrically shield each of the signal wires from each other, i.e. instead of signal wires coupling with other signal wires, they now couple with adjacent ground wires, thereby reducing the coupling between different circuits. Although the coupling between signal wires is now reduced, it cannot be assumed to be zero. However, it is clear from Figure 10 that the typical circuit model for the cable neglects the coupling which still occurs between the signal wires, i.e. it only takes into account the coupling between adjacent wires.

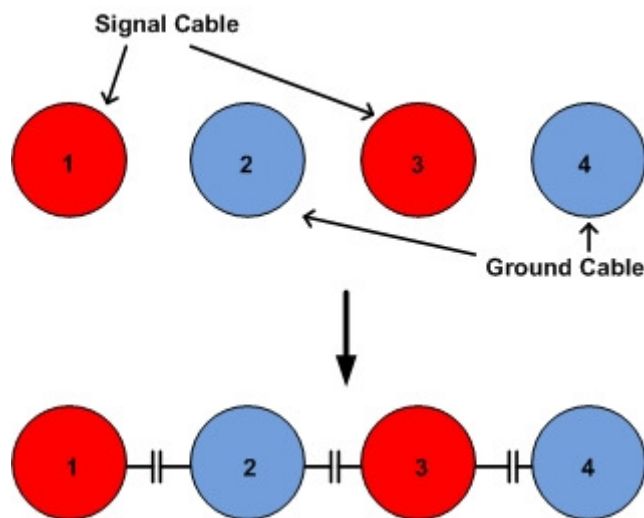


Figure 10: Ribbon Cable

The authors of [12] have highlighted that despite the enhanced crosstalk-reducing properties of ribbon cable, the crosstalk remains a problem in the design of the probes. This crosstalk arises due to imperfect shielding of the signal cables by the ground cables over a certain length.

To reduce the effect, the authors proposed that instead of the constant spacial separation used in conventional ribbon cable, a variable geometry ribbon cable should be used. Since the cable has a variable geometry over its length, the lumped model in Figure 10 can no longer be used. As an alternative, they proposed a model using Multiconductor Transmission Line (MTL) theory to compute the per unit capacitances along the line. These distributed quantities were lumped for certain sections, and these sections were cascaded to form the full model for the cable.

Using this model together with a variable geometry cable, it was found in simulation that the capacitive coupling could be reduced as compared to a standard ribbon cable.

It is evident from the above discussion that in the simplified case of Figure 10, the capacitive coupling between signal wires is not modelled at all. To solve this problem the authors of the paper developed a second model which is much more complicated than the one in Figure 10. As an alternative to the model presented in the paper, the capacitive coupling model in this dissertation can also be used, as shown in Figure 11.

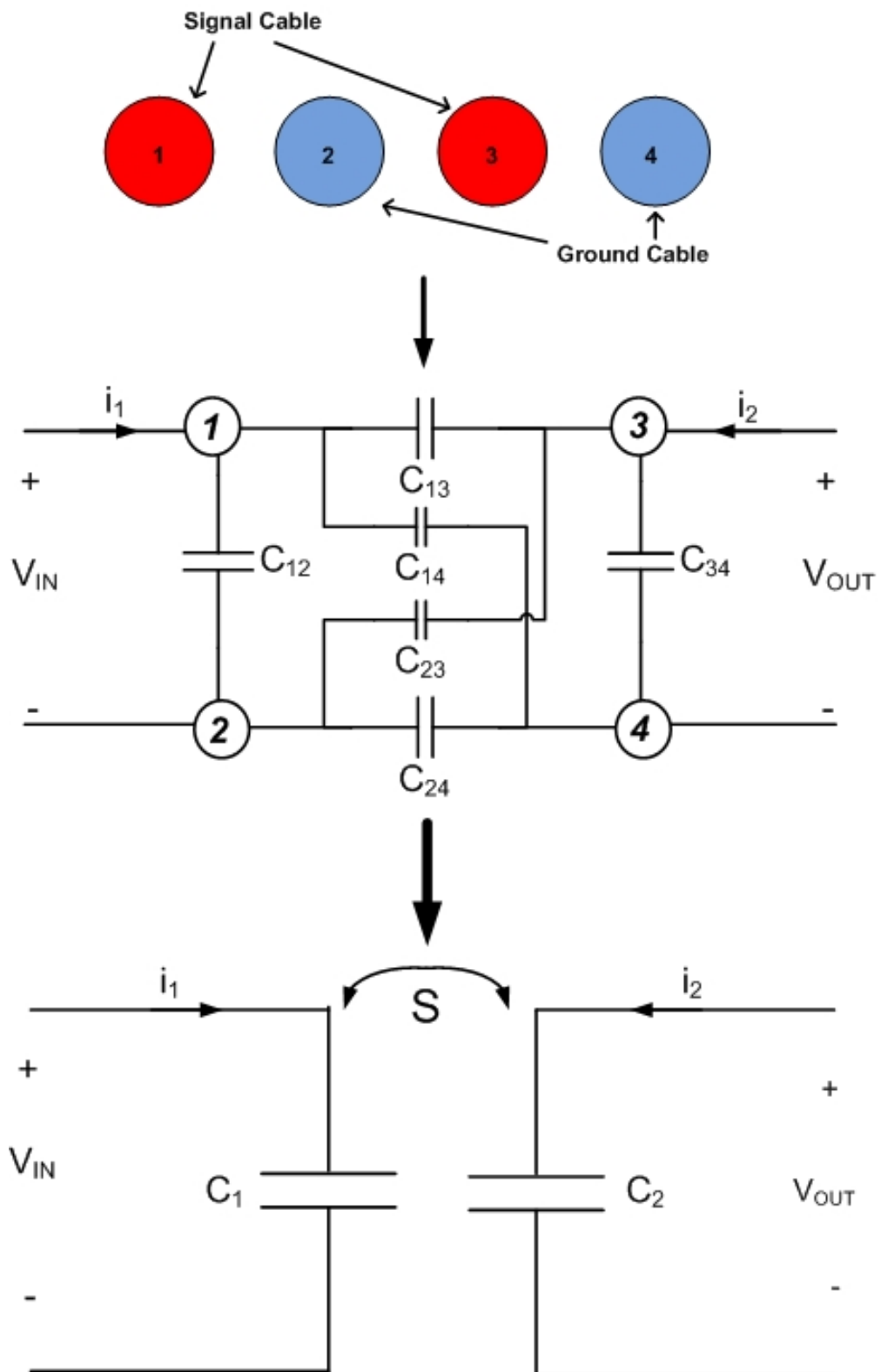


Figure 11: Ribbon Cable using Capacitive Coupling Model

Based on the modelling in Figure 11 it may appear that a common reference is created between the input and output in the capacitive coupling model, due to the ground cables being short circuited at the cable terminations. Recall from Chapter 4 that the capacitive coupling circuit model is not valid when the connection of a common reference is made. This issue is addressed by the discussion in the following paragraphs.

For the circuit in Figure 11, the return current of cable 1 flows only along cable 2, and the return current of cable 3 flows only along cable 4 since these are the paths of least inductance. However, at the termination of the cable, cables 2 and 4 are connected to each other as they both form the same digital ground.

However, due to the high frequency switching signals in the cable, inductive effects along its length are no longer insignificant, and this gives rise to voltage drops along the length of cable. Thus, despite the fact that cables 2 and 4 may be connected to each other at the termination of the cable, the inductive voltage drops along the length of the individual lines result in the two cables being at different potentials along their lengths. This means that if a small section of cable is considered, cables 2 and 4 will be at different potentials and will not appear to be connected to each other. As a result, the capacitive coupling model can be used to model the electric field coupling manifest in a ribbon cable used for high frequency switching signals.

Figure 11 therefore shows a simple alternate circuit model to the one presented in the paper, and takes into account the capacitive coupling between signal wires in a ribbon cable. In the case where a variable geometry ribbon cable is used to reduce the coupling effect, this model can still be used by using the same method as the authors, i.e. split the cable into small sections such that the lumped parameter approach is valid and use the model for each section. The model can be cascaded with itself as shown in Chapter 4 of this dissertation.

3. Conclusion

Based on the discussion presented in the preceding section, it is evident that the capacitive coupling model has a very widespread area of application due its general nature. In situations where the model is not directly applicable, minor modifications can be done to the experimental setup to allow the model to be used. The numerous applications of the model emphasise the fact that as a new circuit “component”, the coupled capacitor network is a much needed tool in the electric circuits domain. The list of applications discussed is a non-exhaustive one, and more detailed research may be required to implement the model in some of these applications.

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Chapter 6: A different perspective on condition monitoring for inductive (power) transformers

1. Introduction

Various applications for the capacitive coupling circuit model were discussed in the previous chapter. These applications were discussed in a very concise manner, i.e. without any detailed design considerations taken into account. In this chapter, an in-depth look into a condition monitoring application for power transformers using the capacitive coupling model is provided. In order to contextualise the application, some background information is provided. This is followed by experimental results, analysis and a conclusion.

2. Background

Power transformers are widely used in industry for various applications. These transformers are designed such that they are able to withstand a certain amount of mechanical and electrical stress, however, due to aging or electrical faults, mechanical deformations can occur within the transformer and this may affect the performance and the health of the device. It is therefore necessary to have some form of diagnosis routine in place to detect these mechanical deformations so that transformer failure can be prevented. There are many techniques currently available for diagnosis of mechanical faults, including short-circuit impedance measurement, vibration measurement, and frequency response analysis [1]. Each method has a range of application, in that some methods can detect only certain types of faults, whereas other methods may be able to detect more faults but their implementation in the field may not be easy.

Frequency Response Analysis (FRA) is one of the more recently developed diagnosis techniques, proposed by Dick and Erven [2]. This technique entails the

use of a frequency sweep generator to apply sinusoidal voltages at certain terminals of the transformer, with the measurement of the voltage and phase at specific terminals. Plots of the voltage and phase can then be made as a function of frequency. The concept behind this is that when a mechanical deformation occurs, the voltage/phase plots will change; and depending on the change and the frequency that it appears at, a specific fault can be diagnosed. The benefits of this method include a high sensitivity and good repeatability.

Much research has gone into the development of diagnosis procedures based on FRA; and a comprehensive review of these methods can be found in [1]. In terms of the measurement procedure, most methods entail the use of a frequency sweep from low to high frequencies, but the exact boundaries of these frequencies differ from method to method. The methods also differ in the quantities which are measured, as well as the terminal conditions.

Once the measurements have been taken, there is also a variety of analysis tools used to interpret the data [1]. A common method is to compare the voltage/phase plots of a damaged transformer with the voltage/phase plots of a fully working similar transformer using a statistical approach. Another method entails the use of circuit models with a predefined topology, and finding numerical values for each component such that the response of the circuit model matches the response of the physical transformer. The circuit model of the undamaged transformer is compared to the model of a damaged transformer, and depending on which component changes in the model, specific faults can be diagnosed.

There is therefore an extensive amount of measurement and analysis techniques for fault diagnosis using FRA, and there are no industry standards currently in place for a general FRA routine to detect specific faults. In this chapter, an alternative approach to fault diagnosis related to FRA is proposed, which is based on the capacitive coupling model developed in Chapter 2. This method is discussed in the following section.

3. Method

If an arbitrary four-body system exhibits capacitive behaviour internally, it can be represented by the capacitive coupling model regardless of the physical makeup of the internal structure. This means that a conventional inductively coupled transformer may be modelled as a capacitively coupled transformer at a frequency such that the internal impedances are capacitive. Since capacitors are fully geometry dependent, it can be expected that the capacitances will change if the coils have been mechanically deformed due to electrical or mechanical stress. Therefore, a condition-monitoring diagnosis routine can be proposed which entails modelling a healthy power transformer using the capacitive coupling model at a specific frequency. This circuit model forms the “reference signature” for that specific transformer, and on a periodic basis the transformer can be re-measured and verified against the reference signature. Any mechanical deformations will translate to a difference between the reference signature and the current signature, and further action can be implemented if this difference is highlighted.

As an example, a typical impedance/angle plot verses frequency for a 1:1 50 Hz power transformer is shown in Figure 1. The data for this plot was measured using a precision LCR meter with a logarithmic frequency sweep from 20 Hz to 2 MHz. It is evident that at lower frequencies, the transformer is primarily inductive as expected. However, as the frequency is swept to high orders of magnitude, resonant points can be seen and the transformer exhibits capacitive behaviour. Using the standard expression for capacitor impedance with sinusoidal excitation, the “Capacitor Match” line can be drawn to intersect with the impedance plot of the transformer by varying the capacitance. At around 300 kHz, the input impedance of the transformer is purely capacitive, and the capacitance can be determined from the “Capacitor Match” line. If all impedances making up the four-terminal device can be shown to be capacitive, then the capacitive coupling model can be fully defined.

The exact diagnosis routine is detailed in the following section.

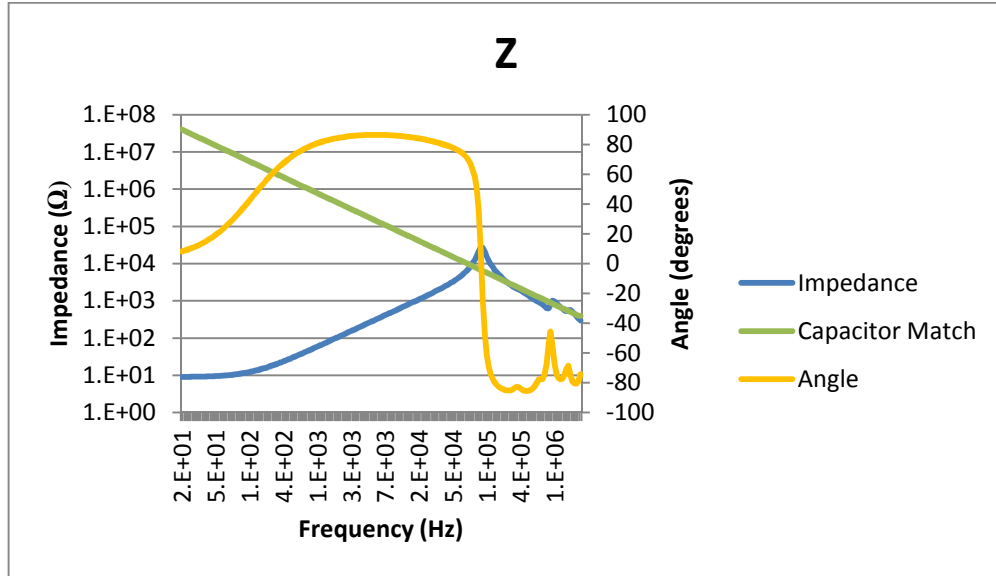


Figure 1: Impedance plot

3.1 Diagnosis routine

There are a few steps to follow in the routine and these are detailed below.

Step 1: Obtain a Signature

As mentioned in the previous section, the first step in the diagnosis routine is the formation of a reference signature for the power transformer. In practice, this will be done before the transformer leaves the factory or on any transformer which is certified to be in optimal working condition. To do this, either a frequency sweep LCR meter or a frequency sweep function generator with an oscilloscope is required. The purpose of this step in the routine is to find a frequency such that the impedances are capacitive in nature.

There are two ways to formulate the reference signature for the transformer. The first is based on the measurement of the individual capacitors upon which the parameters of the capacitive coupling model is based, and the second is to measure the parameters of the coupling model directly. Both are discussed in the next few paragraphs.

Six Capacitor Measurement for calculation of model parameters

Although the six-capacitor measurement routine is discussed here (neglecting self-capacitances), a similar procedure can be followed in the case where all ten capacitors need to be accounted for (the case where self-capacitances are significant).

There are six measurements that need to be taken in order to determine the six capacitors upon which the parameters of the capacitive coupling model is based. The exact routine for the six measurements is similar to the one detailed in Appendix B, with a slight variation. Instead of measuring a capacitance directly for each of the six measurements, it is required that a frequency sweep is performed from low to high frequencies to measure the impedance and angle throughout the chosen frequency range. This is because the impedance will not be capacitive for the entire range of frequencies; hence the impedance is measured to allow for an appropriate frequency to be chosen later.

After taking the measurements, plot the impedances as functions of frequency and choose a frequency at which all impedances are mostly capacitive. At this frequency, the six capacitors can be determined from the impedance plot using the line matching technique discussed above. This is the frequency at which all future measurements are taken.

Using the capacitors as calculated above, the capacitors C_1 , C_2 and S of the capacitive coupled circuit model can be determined.

Direct Measurement of Model Parameters

The second method to formulate the circuit model as discussed above is to measure the parameters of the coupling model directly. Parameters C_1 , C_2 and S of the capacitive coupled circuit model can be measured directly using the routine in Appendix B, by doing a frequency sweep similar to the one discussed above. It is now possible to model the power transformer using the capacitive coupling model at the frequency chosen in this routine. The model is shown in Figure 2.

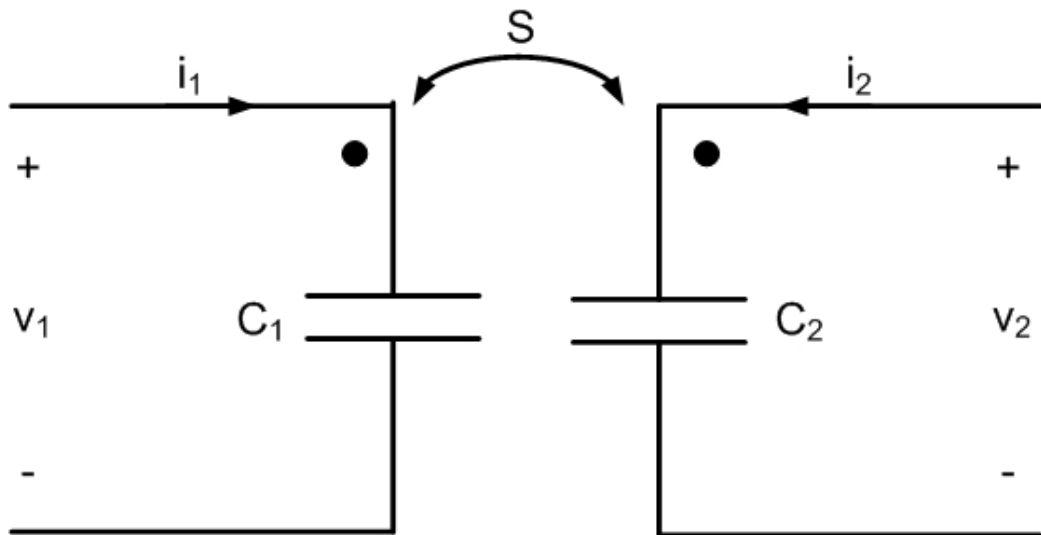


Figure 2: Capacitive Coupling Model describing Power Transformer

Step 2: Measure at periodic intervals or after a suspected occurrence

In order to verify the geometrical integrity of the transformer, the capacitive signature can be verified periodically or after an occurrence which could have affected it. Since the model for the transformer in Figure 2 is valid for a specific frequency, all that is required is to excite the circuit with a function generator at

the correct frequency, place a load on the output, and measure the voltages and currents. A typical circuit configuration is shown in Figure 3.

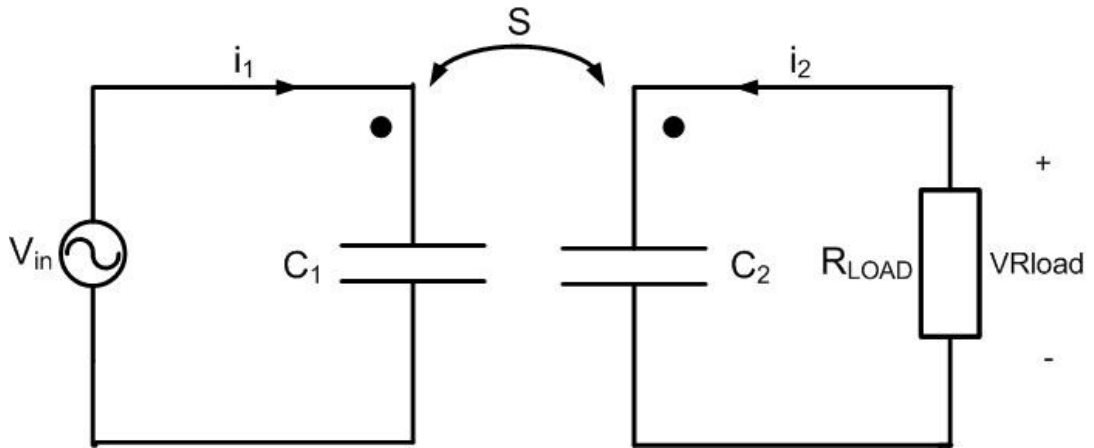


Figure 3: Test configuration

Step 3: Analyse

This step entails the comparison of the voltages and currents predicted by the model to those measured at a later stage. If there are any significant differences between the two, it can be suspected that some sort of mechanical change has occurred.

4. Experimental Verification

In order to experimentally verify the diagnosis routine discussed in the previous section, a 50 Hz 300 VA 1:1 power transformer was used. This is shown in Figure 4.



Figure 4: Power transformer under test

The transformer core was removed to allow easy access to the coils for physical modification to emulate a deformed coil structure. The mechanical deformation chosen to be implemented was axial displacement of the secondary coil as shown by the arrow in Figure 5. This was chosen purely for ease of implementation, as the theory shows that any form of mechanical deformation will result in a change in geometry and hence a change in the capacitive signature. The dismantled power transformer is shown in Figure 5.

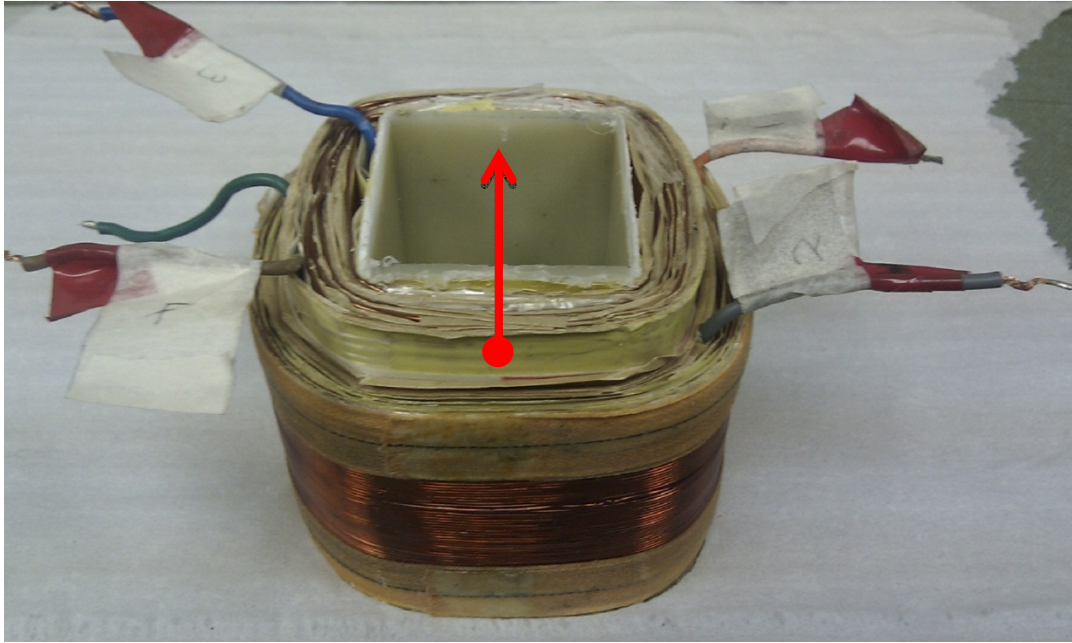


Figure 5: Dismantled transformer

Using the dismantled transformer without coil displacement, impedance plots were obtained. These are shown in Figure 6 to Figure 11. The capacitor match lines have also been drawn in. For this specific transformer the frequency chosen for development of the capacitive coupling circuit is 300 kHz.

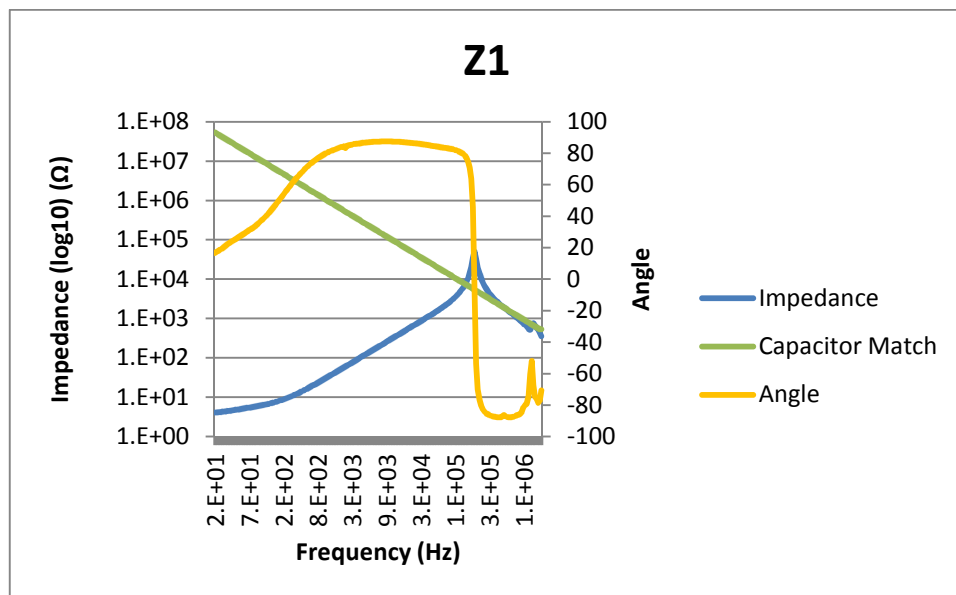


Figure 6: Impedance plot 1 of 6

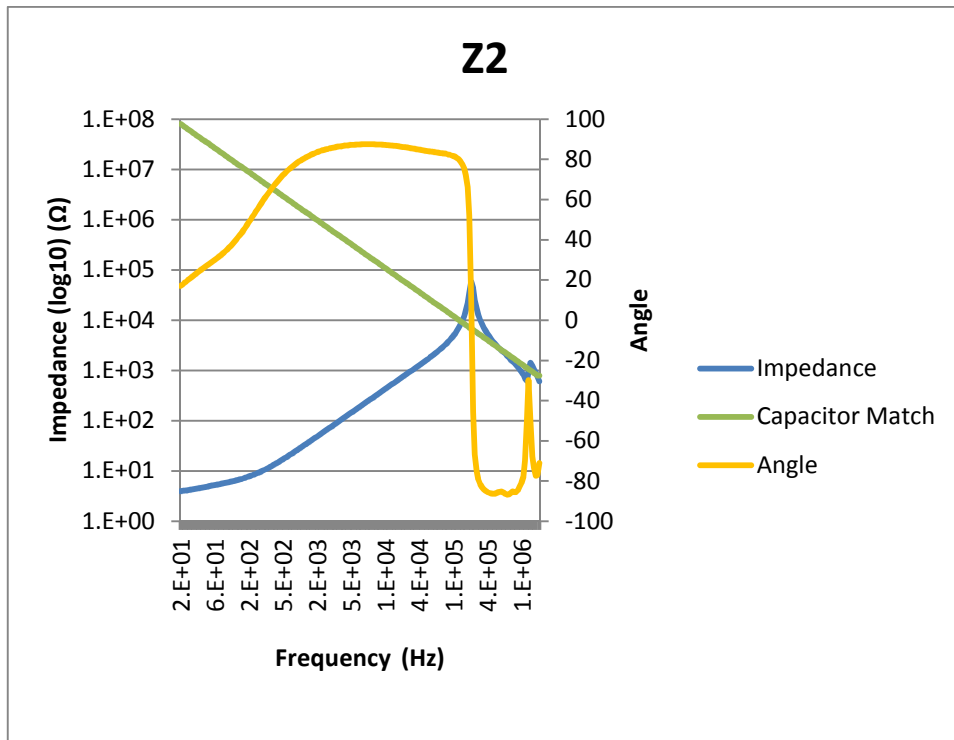


Figure 7: Impedance plot 2 of 6

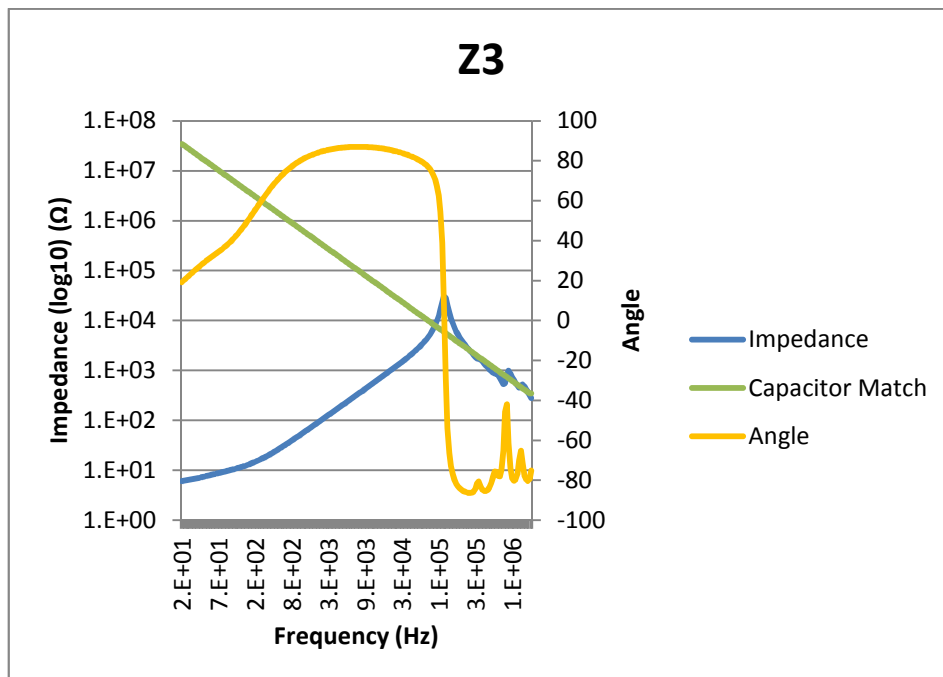


Figure 8: Impedance plot 3 of 6

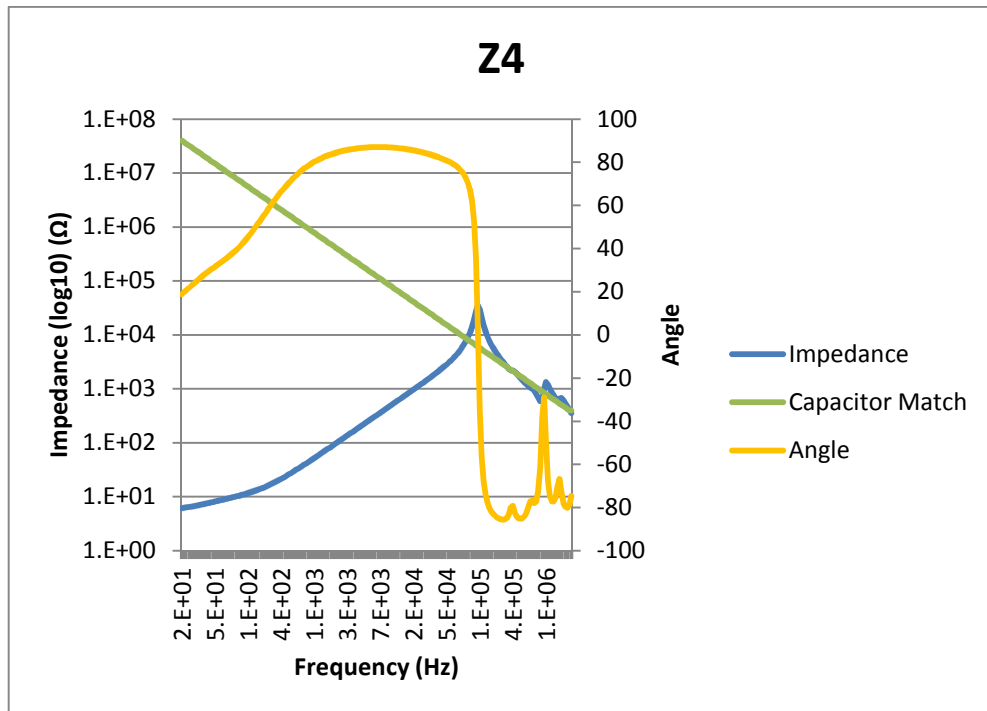


Figure 9: Impedance plot 4 of 6

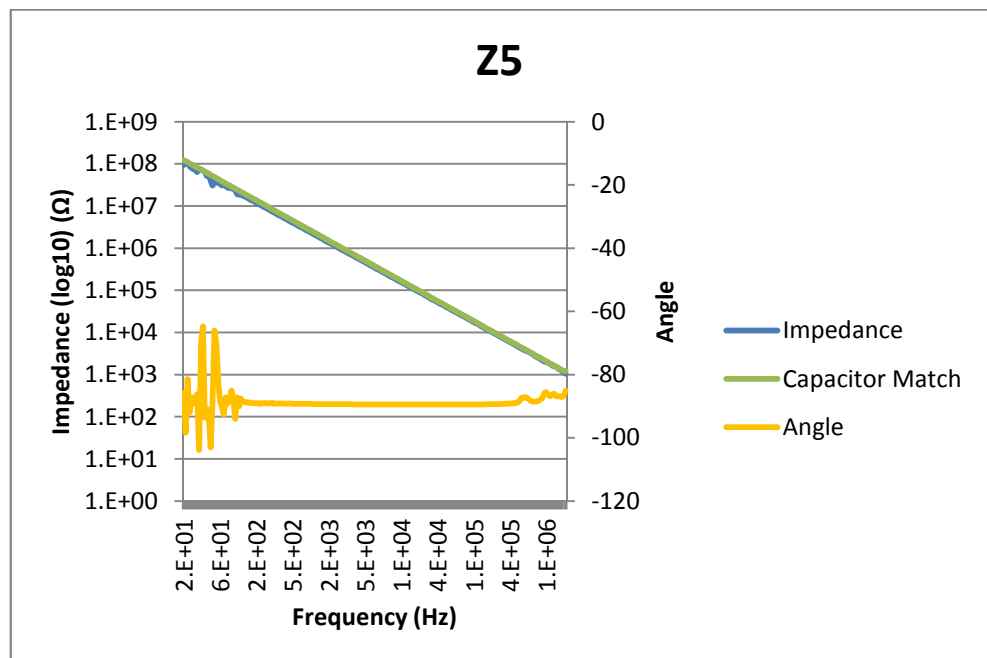


Figure 10: Impedance plot 5 of 6

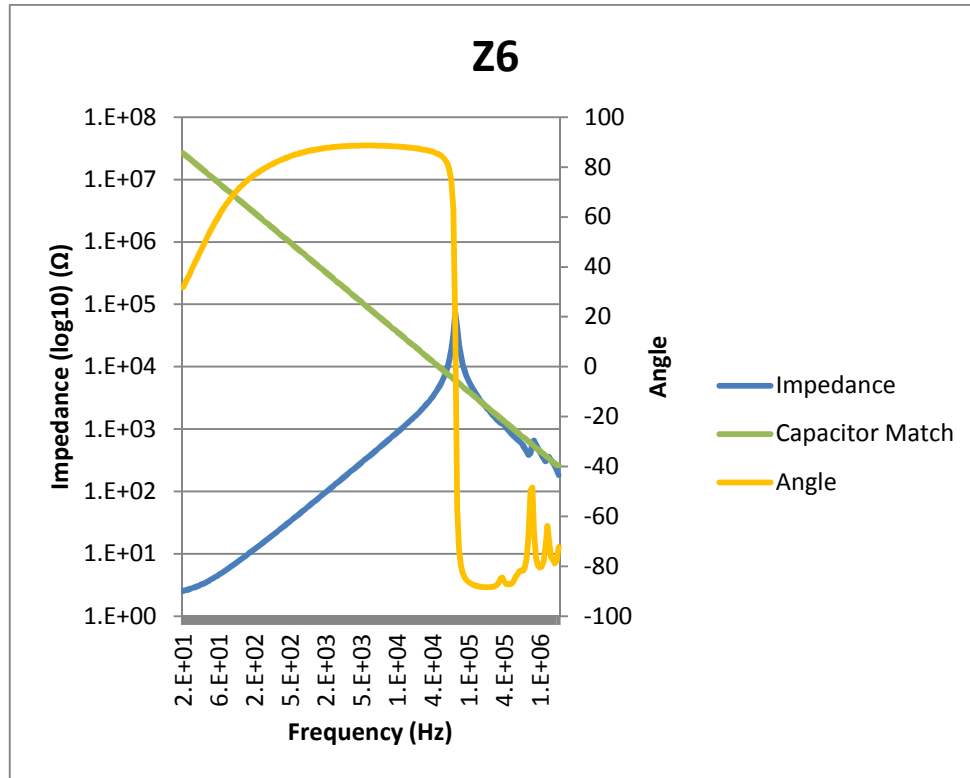


Figure 11: Impedance plot 6 of 6

The impedance plots above are used to obtain the capacitive signature for the transformer. Using the routine in Appendix B, the six capacitors obtained at a frequency of 300 kHz are shown in Table 1. Based on these capacitors, the values for C_1 , C_2 and S in the coupled circuit were obtained and are shown in Table 2.

The secondary set of coils was then displaced in two increments and new capacitive signatures were obtained. The first displacement increment was a 25% axial movement of the secondary coil. At the same frequency of 300 kHz, the capacitances were re-measured. The process was repeated for a 50% axial movement of the secondary coil and the capacitances recorded. All values are shown in Table 1 and Table 2. Although it is very unlikely that such big coil displacement will occur in practice, these increments were chosen to show a proof of concept that coil displacement can indeed be detected. Based on the large

percentage changes recorded in Table 2, it is likely that smaller coil displacements will also be detectable.

Table 1: Six capacitors

Capacitor Name	Value for reference signature (nF)	Value for signature with 25% coil displacement (nF)	Value for signature with 50% coil displacement (nF)
C_{12}	0.1825	0.16	0.1765
C_{13}	0	-0.015	-0.0050
C_{14}	0.0175	0.0550	0.0285
C_{23}	0.0075	0.03	0.0085
C_{24}	0.04	-0.01	0.0150
C_{34}	0.0925	0.0550	0.0865

Table 2: Capacitors in Capacitive Coupling Circuit

Capacitor in Coupling Circuit	Value for reference signature (nF)	Value for signature with 25% coil displacement (nF)	Value for signature with 50% coil displacement (nF)
C_1	0.1953	0.1733 (11.26 %)*	0.1883 (3.58 %)*
C_2	0.0991	0.0663 (33.10 %)*	0.0897 (9.49 %)*
S	0.0020	0.0025 (25.00 %)*	0.0068 (240 %)*

* Values in brackets represent the percentage difference with respect to the reference signature

4.1 Analysis of experimental data

Based on Table 1, changes can be observed in all six capacitors as the secondary coil is displaced. This is expected, since the coupling between the various conductors is modified. There are also negative values in the table. These are not “negative capacitors” in the true sense of the word, but instead they result from the measurement process, where it is assumed that all measurements are purely capacitive (i.e. the phase is exactly -90°) in the reference signature measurement

as well as in all subsequent measurements. This is not necessarily the case, as a change in geometry often results in a movement of the resonant points; therefore the use of a single frequency for all measurements is prone to a level of error. While this does invalidate the model to an extent, it does not invalidate the diagnosis routine as the routine is centred on the detection of a change in its parameters as opposed to its absolute values.

Based on Table 2, a change is also observed in all parameters describing the coupled circuit. This immediately shows that the capacitive coupled circuit can be used for a condition monitoring application, as a physical deformation in the coil structure results in an observable change in the coupled circuit model.

It is notable that there are many different types of physical deformations which can occur, and each can occur to different extents. In this verification procedure, only a single fault was emulated, namely axial displacement of coils, and this was done in only two increments. The experimental verification described above is therefore a proof of concept; however, further analysis is required to show the relationship between each capacitor value and specific faults.

5. General Discussion

It was shown above that the capacitive approach to condition monitoring shows a change in capacitor values when there is a change on geometry. This means that if the transformer is tested after having been deformed, the currents and voltages will not be as predicted and the deformation can therefore be highlighted.

There are however a few interesting points to note: Based on the experimental data, there does not appear to be any obvious relationship between each capacitor and a specific fault/extent of fault. If there was, it may have been possible to simplify the routine to the measurement of a single/few capacitors to detect a specific fault.

Furthermore, each capacitance in the coupled network (C_1 , C_2 and S) is a function of the six capacitors, and it may be possible that a certain change in geometry will change certain of the six capacitors, while inversely changing other capacitors, resulting in an unchanged value for C_1 , C_2 or S . This geometry change will therefore not be detectable using the model, and presents an exception to this approach. However, in this case, the tester can simply revert to observing changes in the individual six capacitors, which will highlight changes regardless.

Lastly, the approach discussed does not strictly fall into the domain of FRA as the frequency is swept only once from low to high frequencies to determine the optimal frequency for the capacitive model. Every subsequent measurement is done at a single frequency. While this differs from the general method of FRA where the frequency is swept on every iteration [1], it does offer the benefit of an easy, quick measurement procedure. One of the limitations which this single-frequency approach introduces is the possibility that certain faults may show at different frequencies to the test frequency, and as a result become undetectable. One of the main benefits of FRA is that different faults can be diagnosed based on the frequency at which a difference is observable in the FRA plots.

In the case where the preceding requirements have been met, i.e. the capacitive model is used to detect a fault which presents itself at the operating frequency of the model, it may be possible to detect a change in the capacitive model on a fully operational healthy power transformer. This may arise due to external interference, different wire-loop areas in the measurement equipment, etc. Research is therefore required into developing tolerance limits for measurements to cater for these factors.

6. Conclusion

A diagnosis routine for condition monitoring of power transformers was proposed. This method is based on the capacitive coupling model developed in Chapter 2 of this dissertation, and is related to Frequency Response Analysis. It requires an initial frequency sweep measurement to choose an appropriate frequency at which a reference signature is formed; and every subsequent measurement is taken only at this frequency. The routine offers the benefit of a very simple measurement process, in that a single frequency input is used for all validation measurements using inexpensive equipment.

The routine was verified experimentally using a 300 VA 1:1 power transformer with an axial coil displacement fault. It was shown that the model does indeed show a change in parameters as the coil is displaced. The benefits and drawbacks of this diagnosis approach were discussed.

7. List of References

- [1] J.R. Secue, E. Mombello, *Sweep frequency response analysis (SFRA) for the assessment of winding displacements and deformation in power transformers*, Electric Power Systems Research, Volume 78, Issue 6, June 2008, Pages 1119-1128.
- [2] E.P. Dick, C.C. Erven, *Transformer diagnostic testing by frequency response analysis*, IEEE Trans. Power Apparatus Syst. PAS-97 (November/December (6)) (1978) 2144-2153.

Chapter 7: Conclusion

1. Overall Summary

The literature review highlighted a missing link in contemporary circuit theory in the field of multi-body capacitor systems. Although mutual inductance has been extensively described and modelled, this has not been done in a similar manner for a coupled capacitor system. It was proposed that a coupled capacitor circuit model should be developed from a field level, and should form a dual to mutual inductance.

Duality theory showed that mutual inductance cannot have a complete dual but instead only a restricted dual, and this gives rise to certain operational limitations. The duality between electrostatics and magnetostatics was then explored in order to highlight the link between field level duality and circuit level duality. This highlighted certain inconsistencies in the duality when approached from different perspectives, and no duality extends from the field level to a circuit level. One of the reasons for this is that a circuit capacitor physically consists of two bodies while an inductor is a one-body system.

A capacitive coupling circuit model was then developed which forms a restricted dual to the inductive coupling model. This is physically made up of four conductive bodies. It is restricted in the sense that it appears to be a dual from a terminal perspective, but does not form a full structural dual. The absolute voltage of each body is fully defined internally, but these do not appear on a circuit level as long as the network is operated with differential currents enforced at the input and output terminals. Since these voltages are defined, there is no full electrical isolation between the input and output of the network, unlike in the inductive coupling circuit model.

The full capacitive coupling model was simplified to two levels, taking into account typical possible operating conditions of the model. The full simplified model corresponds to those presented by other authors in earlier literature, giving confidence to the method used to develop the model in this dissertation.

An in-depth analysis was done into the properties of the coupling capacitance term S which forms part of the capacitive coupling circuit model. Although it is possible to obtain positive, negative and zero coupling with a four-body system, there is no real significance in the sign of S as the output terminals can easily be switched to obtain a different voltage polarity. This holds regardless of the fact that the system is not fully electrically isolated, since it behaves like an electrically isolated system when the condition of balanced currents is enforced at the input and output terminals.

Experimental verification of three capacitively coupled systems were documented. These included a case with arbitrary coupling between the input and output, a case with close to 100 % coupling and a case with close to zero coupling. All experimental evidence showed excellent agreement with the results predicted by the capacitive coupling circuit model.

Certain circuit level properties of the capacitive coupling model were explored to gain insight into the operation and limitations of the model. The properties of linearity, memory, causality, energy storage and coupling factor hold in a similar way that it does for inductive coupling as expected of a dual.

Interconnections between multiple capacitively coupled networks were explored. In general these connections can be made without the duality breaking down, as long as the condition of differential currents at the input and output of the network is enforced. It is not possible to enforce this condition when there is a connection of a common reference between the input and output of the network, and hence this type of connected is not allowed by the model.

A difference of the capacitive coupling model was highlighted in that the model can be used to describe a system with any set of ports chosen as the input with the other set as the output. This is not the case with an inductively coupled system. General examples are provided detailing how to use the model in standard circuit analysis.

A discussion on numerous practical applications of the circuit model is provided. The capacitive coupling model has a very widespread area of application due its general nature. In situations where the model is not directly applicable, minor modifications can be done to the experimental setup to allow the model to be used. The list of applications discussed is a non-exhaustive one, and more detailed research can be done to implement the model in some of these applications.

An in-depth discussion of a diagnosis routine for condition monitoring of power transformers was proposed. This method is based on the capacitive coupling model, and is related to Frequency Response Analysis. It requires an initial frequency sweep measurement to choose an appropriate frequency at which a reference signature is formed; and every subsequent measurement is taken only at this frequency. The routine offers the benefit of a very simple measurement process, in that a single frequency input is used for all validation measurements using inexpensive equipment.

The routine was verified experimentally using a 300 VA 1:1 power transformer with an axial coil displacement fault. It was shown that the model does indeed show a change in parameters as the coil is displaced. The benefits and drawbacks of this diagnosis approach were discussed.

2. Future work

Although the capacitive coupling circuit model was rigorously developed and its properties, limitations and possible applications discussed, there is yet much

research that can be done to broaden the scope of this content. In the following paragraphs, some ideas are proposed.

At a very basic level, the capacitive coupling circuit model in this dissertation consists of four bodies, and there does not appear to be any clear method to extend its applicability to a system consisting of a different number of bodies. This is unlike the inductive case where the theory of mutual inductance is easily extended to a system consisting of any number of coils. Research is therefore required into development of the capacitive coupling circuit model for a system consisting of an arbitrary number of bodies.

At an operational level, further research can be done into the coupling capacitance term S and how it compares to the M term describing mutual inductance. In particular, it is well known that an inductive coil can link magnetic flux more than once, hence giving a transformer the ability to step voltages or currents up. It is suspected that the capacitive system will not be able to step voltages or currents up, however, this has not yet been conclusively shown.

Also at an operational level, research can be done into the behaviour of the capacitive coupling system when operated in a background uniform and non-uniform electric field. In addition to this, the model can be extended to include dielectric and conductive losses.

At a production level, routines need to be designed that easily allow a designer to physically realise a system given its circuit model. These routines need to account for construction and material limitations, as well as operational environments.

APPENDIX A

Capacitive Coupling Model Parameters

This document contains the expressions for the parameters used to describe the capacitive coupling circuit model.

1. Parameters for full model

The expressions in this section do not neglect any capacitors. Equations (1) to (3) describe C_1 , C_2 and S .

$$C_1 = f_1(\text{self capacitance}, \text{interbody capacitance}) = \frac{\left(\begin{aligned} &C_{12}(C_{22}C_{23}+C_{22}C_{24}+C_{22}C_{33}+C_{23}C_{33}+C_{24}C_{33}+C_{22}C_{44}+C_{23}C_{44}+C_{24}C_{44}+C_{13}(C_{22}+C_{33}+C_{44})+C_{14}(C_{22}+C_{33}+C_{44})) \\ &+(C_{13}+C_{14})((C_{23}+C_{24})(C_{33}+C_{44})+C_{22}(C_{23}+C_{24}+C_{33}+C_{44}))+C_{11}(C_{14}C_{22}+C_{14}C_{23}+C_{22}C_{23}+C_{14}C_{24}+C_{22}C_{24}+ \\ &C_{13}(C_{22}+C_{23}+C_{24})+ \\ &C_{22}C_{33}+C_{23}C_{33}+C_{24}C_{33}+C_{22}C_{44}+C_{23}C_{44}+C_{24}C_{44}+C_{12}(C_{13}+C_{14}+C_{23}+C_{24}+C_{33}+C_{44})) \end{aligned} \right)}{\left(\begin{aligned} &C_{14}C_{22}+C_{22}C_{23}+C_{22}C_{24}+C_{14}C_{33}+C_{22}C_{33}+C_{23}C_{33}+C_{24}C_{33}+C_{14}C_{44}+C_{22}C_{44}+C_{23}C_{44}+C_{24}C_{44}+C_{13}(C_{22}+C_{33}+C_{44})+ \\ &C_{11}(C_{13}+C_{14}+C_{23}+C_{24}+C_{33}+C_{44}) \end{aligned} \right)} \quad (1)$$

$$C_2 = f_2(\text{self capacitance}, \text{interbody capacitance}) =$$

$$\frac{\left((C_{14}C_{22}C_{23} + C_{22}C_{23}C_{24} + C_{14}C_{22}C_{33} + C_{14}C_{23}C_{33} + C_{22}C_{24}C_{33} + C_{23}C_{24}C_{33} + C_{14}C_{22}C_{34} + C_{22}C_{23}C_{34} + C_{22}C_{24}C_{34} + C_{14}C_{33}C_{34} + C_{22}C_{33}C_{34} + C_{23}C_{33}C_{34} + C_{24}C_{33}C_{34} + C_{14}C_{23}C_{44} + C_{22}C_{23}C_{44} + C_{23}C_{24}C_{44} + C_{14}C_{33}C_{44} + C_{22}C_{33}C_{44} + C_{23}C_{33}C_{44} + C_{24}C_{33}C_{44} + C_{14}C_{34}C_{44} + C_{22}C_{34}C_{44} + C_{23}C_{34}C_{44} + C_{24}C_{34}C_{44} + C_{13}(C_{24}C_{33} + C_{33}C_{34} + C_{24}C_{44} + C_{33}C_{44} + C_{34}C_{44} + C_{14}(C_{22} + C_{33} + C_{44}) + C_{22}(C_{24} + C_{34} + C_{44})) + C_{11}(C_{23}C_{24} + C_{24}C_{33} + C_{23}C_{34} + C_{24}C_{34} + C_{33}C_{34} + C_{14}(C_{23} + C_{33} + C_{34}) + C_{23}C_{44} + C_{33}C_{44} + C_{34}C_{44} + C_{13}(C_{14} + C_{24} + C_{34} + C_{44})) \right)}{\left((C_{14}C_{22} + C_{22}C_{23} + C_{22}C_{24} + C_{14}C_{33} + C_{22}C_{33} + C_{23}C_{33} + C_{24}C_{33} + C_{14}C_{44} + C_{22}C_{44} + C_{23}C_{44} + C_{24}C_{44} + C_{13}(C_{22} + C_{33} + C_{44}) + C_{11}(C_{13} + C_{14} + C_{23} + C_{24} + C_{33} + C_{44})) \right)} \quad (2)$$

$$S = f_3(\text{self capacitance}, \text{interbody capacitance}) = \frac{f_4(C) - f_5(C)}{f_6(C)} =$$

$$\frac{\left((C_{14}C_{22}C_{23} + C_{14}C_{22}C_{33} + C_{14}C_{23}C_{33} + C_{14}C_{23}C_{44} + C_{11}C_{14}C_{23} + C_{11}C_{23}C_{44}) - (C_{11}C_{13}C_{24} + C_{11}C_{24}C_{33} + C_{13}C_{22}C_{24} + C_{13}C_{22}C_{44} + C_{13}C_{24}C_{33} + C_{13}C_{24}C_{44}) \right)}{\left(C_{14}C_{22} + C_{22}C_{23} + C_{22}C_{24} + C_{14}C_{33} + C_{22}C_{33} + C_{23}C_{33} + C_{24}C_{33} + C_{14}C_{44} + C_{22}C_{44} + C_{23}C_{44} + C_{24}C_{44} + C_{13}(C_{22} + C_{33} + C_{44}) + C_{11}(C_{13} + C_{14} + C_{23} + C_{24} + C_{33} + C_{44}) \right)} \quad (3)$$

2. Parameters for Level 1 simplification of model

This section assumes that all self-capacitors are of the same magnitude and equal to C_{II} . Equations (4) to (6) describes C_I , C_2 and S .

$$C_1 = f_7(\text{self capacitance}, \text{interbody capacitance}) =$$

$$\frac{(2C_{11}^2 + 4C_{11}C_{12} + 3C_{11}C_{13} + 3C_{11}C_{14} + 3C_{11}C_{23} + 4C_{13}C_{23} + 4C_{14}C_{23} + 3C_{11}C_{24} + 4C_{13}C_{24} + 4C_{14}C_{24} + 4C_{12}(C_{13} + C_{14} + C_{23} + C_{24}))}{(4(C_{11} + C_{13} + C_{14} + C_{23} + C_{24}))} \quad (4)$$

$$C_2 = f_8(\text{self capacitance}, \text{interbody capacitance}) = \frac{(2C_{11}^2 + 3C_{11}C_{13} + 3C_{11}C_{14} + 3C_{11}C_{23} + 4C_{14}C_{23} + 3C_{11}C_{24} + 4C_{13}C_{24} + 4C_{11}C_{34} + 4C_{13}C_{34} + 4C_{14}(C_{13} + C_{34}) + 4(C_{24}C_{34} + C_{23}(C_{24} + C_{34})))}{(4(C_{11} + C_{13} + C_{14} + C_{23} + C_{24}))} \quad (5)$$

$$S = f_9(\text{self capacitance}, \text{interbody capacitance}) = \frac{f_{10}(C) - f_{11}(C)}{f_{12}(C)} = \frac{((C_{11}C_{14} + C_{11}C_{23} + 4C_{14}C_{23}) - (C_{11}C_{13} + C_{11}C_{24} + 4C_{13}C_{24}))}{(4(C_{11} + C_{13} + C_{14} + C_{23} + C_{24}))} \quad (6)$$

APPENDIX B

Capacitance Measurements

1. Introduction

In this document the measurement routines related to the capacitive coupling model are provided.

2. Measurement Procedures

2.1 Measurement of all capacitors present in four body system

In a four-body system, there are ten capacitors between the various bodies and infinity/earth that describe the electric field coupling as shown in Figure 1.

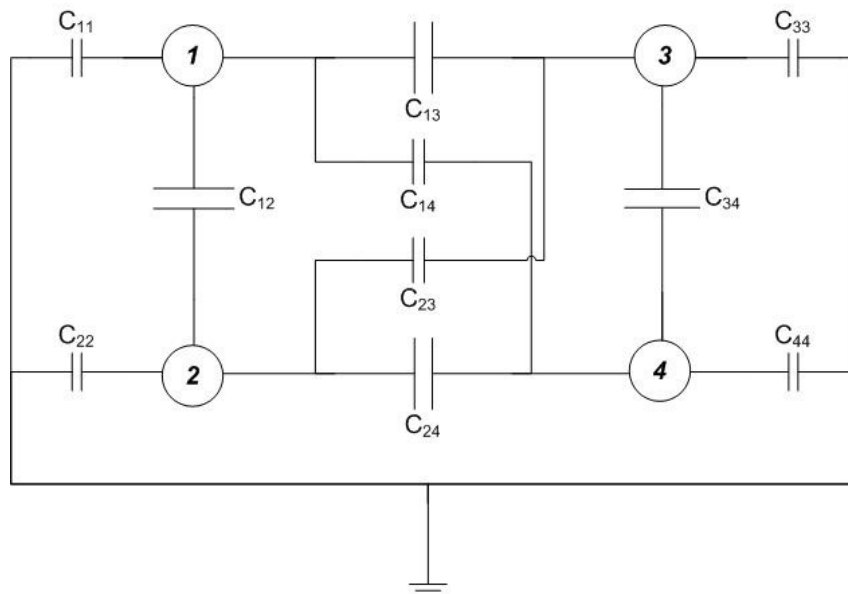


Figure 1: Ten capacitors between four bodies

There are ten measurements that need to be taken in order to determine the ten capacitors making up the circuit in Figure 1. Each measurement entails connecting certain terminals together, and then using simple circuit analysis to formulate an equation for capacitances seen at the input terminals of the circuit. Clearly, this routine can only be used in the case where the self-capacitors of each body can be measured, i.e. where an earth terminal can be defined. We will refer to the earth terminal as terminal 5 in this routine.

The LCR meter-based routine is detailed in Table 1. The same routine can be followed if a function generator and oscilloscope are used instead of an LCR meter; simple use of Ohm's law and the expression for impedance of a capacitor with sinusoidal excitation, will allow for the determination of capacitance from current and voltage readings.

Table 1: Measurement Routine for Ten Capacitors

Measurement Number	Terminal 1 of test device	Terminal 2 of test device	Measurement taken
1	1	2-3-4-5	C_i
2	2	1-3-4-5*	C_{ii}
3	3	1-2-4-5*	C_{iii}
4	4	1-2-3-5*	C_{iv}
5	5	1-2-3-4*	C_v
6	1-2*	3-4-5*	C_{vi}
7	2-3*	1-4-5*	C_{vii}
8	3-4*	1-2-5*	C_{viii}
9	4-5*	1-2-3*	C_{ix}
10	1-5*	2-3-4*	C_x

* Connect these bodies to each other when using the set of bodies as a single terminal

The ten capacitors in Figure 1 can then be calculated using the expression shown in (1).

$$\begin{pmatrix} C_{12} \\ C_{13} \\ C_{14} \\ C_{11} \\ C_{23} \\ C_{24} \\ C_{22} \\ C_{34} \\ C_{33} \\ C_{44} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} C_i \\ C_{ii} \\ C_{iii} \\ C_{iv} \\ C_v \\ C_{vi} \\ C_{vii} \\ C_{vii} \\ C_{ix} \\ C_x \end{pmatrix} \quad (1)$$

Using the capacitors as calculated above, the capacitors C_1 , C_2 and S of the capacitive coupled circuit model in Figure 3 can be calculated. The expressions for C_1 , C_2 and S in terms of the ten capacitors are shown in Appendix A.

2.2 Measurement of six capacitors between four bodies neglecting self-capacitors

In the case where the self-capacitances of a four-body system can be neglected, there are six capacitors between the bodies that describe the electric field coupling as shown in Figure 2.

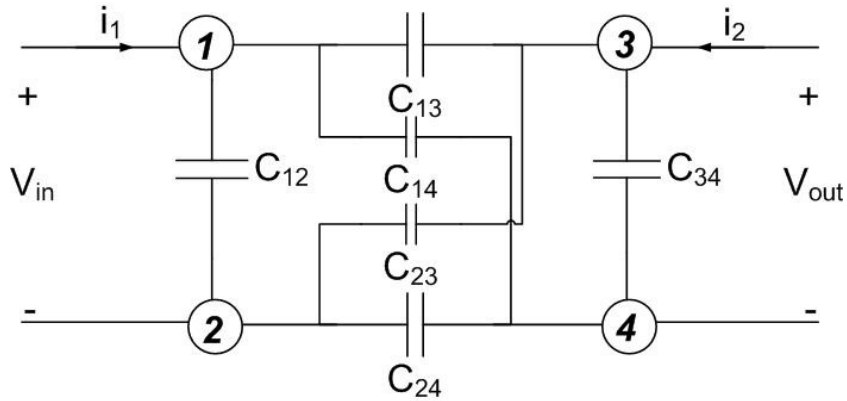


Figure 2: Six capacitors between four bodies

There are six measurements that need to be taken in order to determine the six capacitors making up the circuit in Figure 2. Similar to before, each measurement entails connecting certain terminals together, and then using simple circuit analysis to formulate an equation for capacitances seen at the input terminals of the circuit. The LCR meter-based routine is detailed in Table 2. The same routine can be followed if a function generator and oscilloscope are used instead of an LCR meter; simple use of Ohm's law and the expression for impedance of a capacitor with sinusoidal excitation, will allow for the determination of capacitance from current and voltage readings.

Table 2: Measurement Routine for Six Capacitors

Measurement Number	Terminal 1 of test device	Terminal 2 of test device	Measurement taken
1	1-2-3*	4	C_A
2	1-2-4*	3	C_B
3	1-3-4*	2	C_C
4	2-3-4*	1	C_D
5	1-2*	3-4*	C_E
6	1-3*	2-4*	C_F

* Connect these bodies to each other when using the set of bodies as a single terminal

The six capacitors in Figure 2 can then be calculated using the expression shown in (2).

$$\begin{pmatrix} C_{12} \\ C_{13} \\ C_{14} \\ C_{23} \\ C_{24} \\ C_{34} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} C_A \\ C_B \\ C_C \\ C_D \\ C_E \\ C_F \end{pmatrix} \quad (2)$$

Using the capacitors as calculated above, the capacitors C_1 , C_2 and S of the simplified capacitive coupled circuit model in Figure 3 can be calculated. The expressions for C_1 , C_2 and S in terms of the six capacitors are shown in a chapter in the dissertation.

2.3 Direct Measurement of Capacitive Coupling Model Parameters

In the case where the three parameters of the coupling model, namely C_1 , C_2 and S need to be measured directly, i.e. not as functions of interbody capacitors as in the routines above, the following routine can be used. The coupling model is shown in Figure 3.

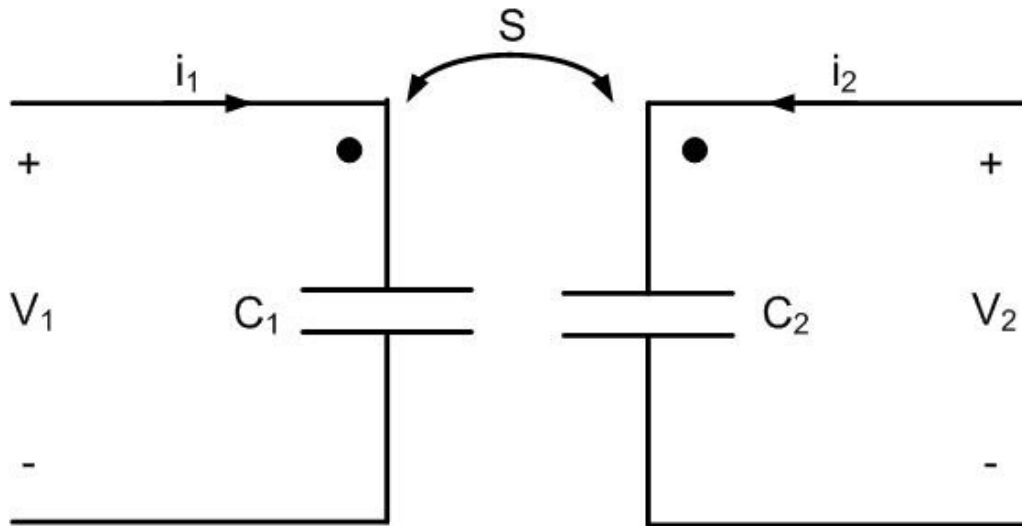


Figure 3: Capacitive Coupling Model

If the circuit in Figure 3 is excited sinusoidally, it is trivial to show that it can be represented by Figure 4. It is interesting to note that this corresponds to the standard admittance parameter (Y-Parameter) two-port network model found in textbook theory. In Figure 4 ω is the angular frequency of operation and j is the imaginary number in the complex domain.

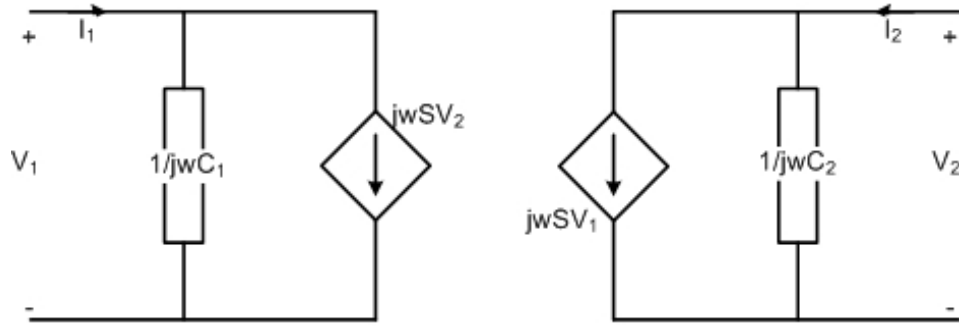


Figure 4: Impedance description of Capacitive Coupling Model

It is easy to see from Figure 4 that the parameters C_1 , C_2 and S can be measured experimentally using the following steps:

Measurement 1:

Short circuit V_2 .

Excite the circuit sinusoidally at V_1 , and record the voltage V_1 , the current I_1 and the frequency of the sinusoidal input signal. C_1 can then be calculated using (3).

$$C_1 = \frac{I_1}{j\omega V_1} = \frac{1}{y_{11}} \quad (3)$$

Alternatively, a LCR meter can be used to measure the impedance looking into terminals at V_1 . C_1 can then be calculated using (4), where Z_1 is the impedance looking into the terminals.

$$C_1 = \frac{1}{j\omega Z_1} = \frac{1}{y_{22}} \quad (4)$$

Measurement 2:

Short circuit V_1 .

Excite the circuit sinusoidally at V_2 , and record the voltage V_2 , the current I_2 and the frequency of the sinusoidal input signal. C_2 can then be calculated using (5).

$$C_2 = \frac{I_2}{j\omega V_2} \quad (5)$$

Alternatively, a LCR meter can be used to measure the impedance looking into terminals at V_2 . C_2 can then be calculated using (6), where Z_2 is the impedance looking into the terminals.

$$C_1 = \frac{1}{j\omega Z_2} \quad (6)$$

Measurement 3:

Short circuit V_2 .

Excite the circuit sinusoidally at V_1 , and record the voltage V_1 , the current I_2 and the frequency of the sinusoidal input signal. S can then be calculated using (7).

$$S = \frac{I_2}{j\omega V_1} \quad (7)$$

It is clear from (7) that the S parameter is a function of the current on one side of the network and the voltage on the other side. In order to use a LCR meter in this case, it is therefore required that the meter is capable of measuring the current and voltage without the use of a common reference between the two measurement circuits. As this is not easily available, an alternative method for Measurement 3 is detailed below.

Alternative Method for Measurement 3 (Inferred Measurement)

It is clear that if a load is placed at the output terminals of the network (i.e. at V_2), the impedance looking into the input terminals of the network (i.e. at V_1) will

change. Using simple circuit analysis, an expression for S can be found as a function of the load impedance, the input impedance and the two port parameters. This is given in (8).

$$S = \frac{\sqrt{1 - y_{11}Z_{in} + y_{22}Z_L - y_{11}Z_{in}y_{22}Z_L}}{j\omega\sqrt{Z_{in}}\sqrt{Z_L}} \quad (8)$$

In (8), Z_L is the load impedance measured before inserting the load into the circuit. Z_{IN} is the impedance looking into the input terminals of the network after placing the load at the output terminals.

Therefore, to measure S , a load is placed across the output terminals. The magnitude of the load must be such that when placed at the terminals, a change in the input impedance of the network can be observed. Using the expression in (8), the parameter S can be determined. This is an inferred measurement of S as compared to the direct measurement in the routine above.

3. Conclusion

In this document, procedures were discussed to allow for experimental measurement of the various capacitors that exist between four bodies. This is to allow for calculation of the parameters which make up the capacitive coupling model. A procedure was also provided to directly measure the parameters which make up the capacitive coupling model.

APPENDIX C

Typical Examples

1. Introduction

This document has a set of examples to illustrate how to use the capacitive coupling model in circuit analysis.

2. Examples

2.1 Determine an expression for the voltage across a load impedance in terms of the input voltage and capacitive coupling model parameters.

Solution:

We recall the capacitive coupling circuit model in Figure 1 with its governing equation in (1).

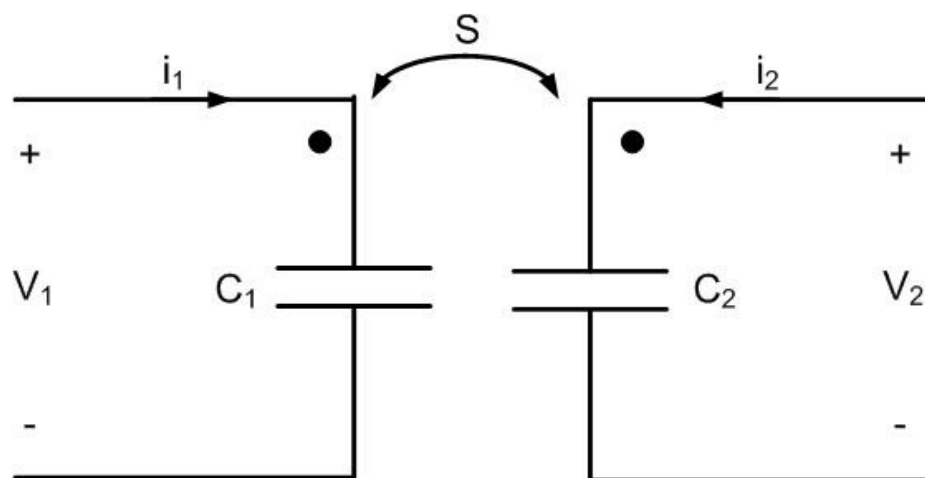


Figure 1: Capacitive Coupling Circuit Model

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} C_1 & S \\ S & C_2 \end{pmatrix} \begin{pmatrix} \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{pmatrix} \quad (1)$$

Writing (1) in the frequency domain for sinusoidal excitation we get the circuit in Figure 2.

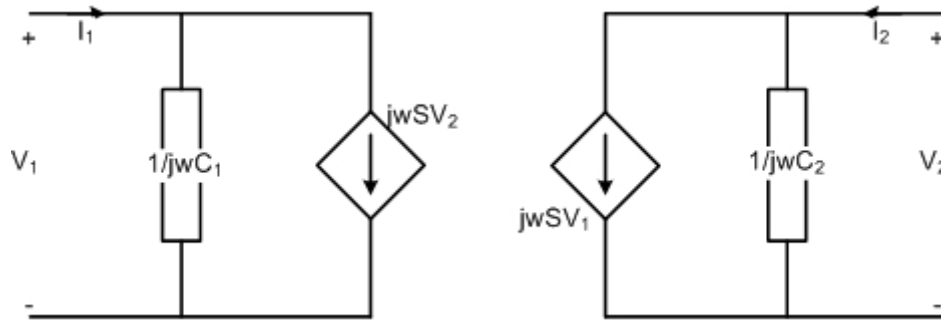


Figure 2: Frequency domain representation of Capacitive Coupling Model

Taking the input voltage to be V_1 , and the output at V_2 , we place a load at V_2 . The output stage of the circuit then looks like Figure 3.

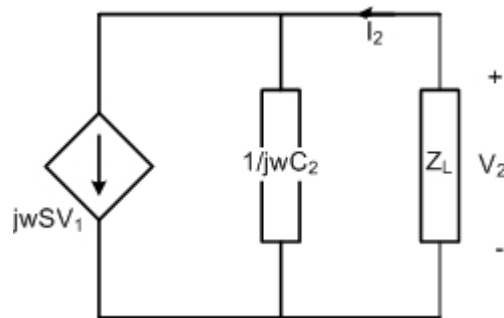


Figure 3: Output Stage with Load

We can now simplify the two parallel impedances into a single impedance as shown in (2).

$$Z_{TOTAL} = \frac{Z_L}{Z_L j\omega C_2 + 1} \quad (2)$$

We now have the circuit in Figure 4.

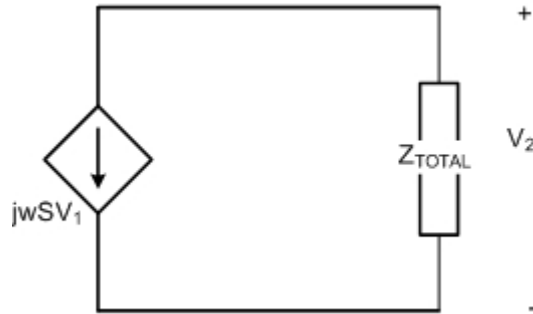


Figure 4: Output stage with total load

By using Ohm's Law, we get (3).

$$V_2 = IZ = -j\omega S V_1 Z_{TOTAL} = -j\omega S V_1 \frac{Z_L}{Z_L j\omega C_2 + 1} \quad (3)$$

2.2 Calculate impedance looking into the input terminals of the network in the above question.

Solution:

In the above question we developed an expression for the voltage V_2 across the load. We now look at the input stage of the network as shown in Figure 5.

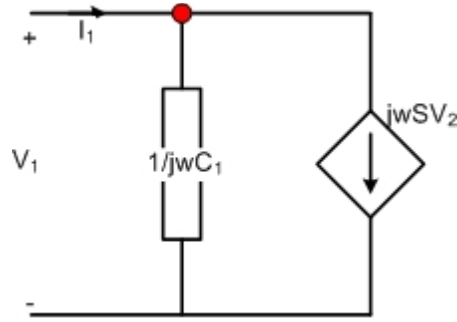


Figure 5: Input Stage of Network

We can now use Kirchoff's Current Law to sum the currents at the node marked in red, as shown in (4).

$$I_1 = V_1 j\omega C_1 + j\omega S V_2 \quad (4)$$

The input impedance is given by (5).

$$Z_{IN} = \frac{V_{IN}}{I_{IN}} = \frac{V_1}{I_1} = \frac{V_1}{V_1 j\omega C_1 + j\omega S V_2} \quad (5)$$

But we already have an expression for V_2 from the previous example in (3). Therefore substituting (3) into (5) we get (6).

$$Z_{IN} = \frac{1}{j\omega C_1 + \frac{Z_L \omega^2 S^2}{Z_L j\omega C_2 + 1}} \quad (6)$$

2.3 Determine the current through a load impedance in terms of C_I and S given a system with 50 % coupling.

Solution:

We recall the expression relating the extent of coupling to circuit parameters as shown in (7), where $k = 0.5$ for 50 % coupling.

$$k = \frac{s}{\sqrt{C_1 C_2}} \quad (7)$$

From (7) we can determine the capacitance C_2 in terms of the given parameters as shown in (8).

$$C_2 = \frac{s^2}{k^2 C_1} \quad (8)$$

By applying Ohm's law at the load we can develop an expression for the current flowing through the load as shown in (9).

$$I_{LOAD} = \frac{V_{LOAD}}{Z_{LOAD}} \quad (9)$$

From Figure 3 we know that the voltage across the load is V_2 , and the expression for the voltage is given in (3). Therefore by substituting (3) into (9), we get (10).

$$I_{LOAD} = \frac{V_{LOAD}}{Z_{LOAD}} = \frac{V_2}{Z_{LOAD}} = \frac{-j\omega S V_1 \frac{Z_{LOAD}}{Z_{LOAD} j\omega C_2 + 1}}{Z_{LOAD}} = \frac{-j\omega S V_1}{(Z_{LOAD} j\omega C_2 + 1)} \quad (10)$$

Lastly we have an expression for C_2 from (8), so we substitute it into (10) to get the final expression in (11).

$$I_{LOAD} = \frac{-j\omega S V_1}{(Z_{LOAD} j\omega \frac{s^2}{k^2 C_1} + 1)} = \frac{-j\omega S V_1 k^2 C_1}{(Z_{LOAD} j\omega S^2 + k^2 C_1)} \quad (11)$$