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EXPLORING GRADE 7 LEARNERS' PERFORMANCE IN MULTIPLICATIVE AND ADDITIVE REASONING WORD AND PURE NUMERICAL PROBLEMS FROM WRITTEN TESTS

A Research Report submitted in partial fulfilment of the requirements for the degree of Master of Education

By

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DECLARATION

I declare that the work of this Research Report is my own work except where indicated otherwise. I have followed the required conventions in referencing the thoughts and ideas of others.

……..
Chemedzai Mufara
November, 2019

ABSTRACT

This study aimed at exploring learners' performance in multiplicative and additive reasoning word and pure numerical problems from their written scripts. To achieve this, a content analysis of the grade 7 learners' scripts of the pre- and post- tests administered by the Wits Maths Connect Primary (WMCP) as part of their broader project- Multiplicative Reasoning Intermediate Phase (MRIP) project in the Gauteng province of South Africa was done. This was done by comparing differences in learners' performance across parallel word and pure numerical problems that demand multiplicative and additive reasoning, solution strategies and detectable errors in written responses.

Scripts from 143 learners who took part in the main project were used as sources of data for this research. The 143 learners took part in two tests each with 8 problem items divided between word and pure numerical problems. This meant that 2288 learners' responses became available for analysis. The analysis and coding were done per test, and within each test, a comparison was made between mathematical reasoning (i.e. additive and multiplicative reasoning), problem format (word or pure numeric problems) and the correctness of learners' responses coded as correct or incorrect. In addition, the study established the solution strategies employed by the learners and the detectable errors that they committed as they responded to the mathematical problems presented in the tests.

Findings revealed no significant difference in learners' performance in word and pure numerical problems. However, for both problem formats, multiplicative reasoning was more challenging for the learners than additive reasoning. The predominant strategies used by learners across additive and multiplicative reasoning were 'writing the answer only without showing any visible working' and standard algorithm. It was further established that the most prevalent detectable error in the learners' scripts was of a procedural nature rather than operational. Based on these findings, the study recommends that teachers' emphasis should be on the development of learners' multiplicative reasoning and encourage learners to show all their working when solving mathematical problems.

DEDICATION

This research is dedicated to my husband and five children. The entire family is a special gift that God gave me in life. They are all special to me in their individual capacities.

ACKNOWLEDGEMENTS

It is with gratitude that I acknowledge the unwavering support that I received from my supervisors Dr Lawan Abdulhamid and Professor Mike Askew. The two supervisors were very supportive for me during the entire journey of this research study. Without their expert guidance and motivation, it would not have been possible to timeously complete the work and with such rigour and quality.

I also extend my thankfulness to members of the Wits Maths Connect team who were always available for me whenever I needed research material from the archives of the main project. Special acknowledgements go to Dr Craig Pounara for facilitating financial support for my studies, Professor Hamsa Venkat for allowing me to use data from the project she leads, and Yvonne Saunders for ratifying the reliability of my coding framework.

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CHAPTER 1: INTRODUCTION

1.1 Introduction

This section provides the background to the study, problem statement, research questions to which the research sought to get answers, the rationale of the study and the organisation of the entire report.

1.2 Background to the study

Results of Annual National Assessments (ANAs) administered to grades 1-6 and 9 in 2012, 2013 and 2014 showed that solving word problems is an area of weakness across the three Intermediate Phase grades 4-6 in South Africa (Department of Basic Education, 2012; DBE, 2013; DBE, 2014). Despite specific interventions directed to the teaching and learning of solving word problems by the Department of Basic Education (DBE) in 2013 and 2014, solving word problems continued to be an area of weakness (DBE, 2015). This situation reflects my teaching experience with Grade 8 learners who rarely respond correctly to word problems on baseline assessments administered to them on entry into high school. Though the South African curriculum emphasises solving word problems across all phases and grades, learners are still challenged by solving word problems (DBE, 2011). In response to this, Wits Maths Connect-Primary project (WMC-P) set up a series of interventions, in previously disadvantaged primary schools in Johannesburg, aimed at supporting learners' work on word problems, the Multiplicative Reasoning Intermediate Phase (MRIP) project.

Koedinger and Nathan (2004) posited that it is a belief held by practitioners and researchers in mathematics education, and the general public that word problems are difficult for learners. Theoretically, reasons for the difficulty lie in the two phases involved in solving word problems: the comprehension phase that requires linguistic processing and the solution phase that comprises different computational strategies (Koedinger & Nathan, 2004). However, in South Africa, the diagnostic report on ANA results for 2012 also highlighted that learners' performance in number problems, where they had to perform basic mathematical operations without any distracting language components was no different from performance where they had to interrogate textual information and translate it into mathematical expressions and computations (DBE, 2013). This suggests difficulties for South African learners in both of Koedinger and Nathan's phases.

With awareness of South African learners' problems across both the comprehension and solution phases in solving mathematics problems, the WMCP-MRIP project designed and implemented a middle grades intervention focused on multiplicative reasoning. Their design included pre- and post-tests that incorporated parallel pure numeric and word problems across the four operations (8 items) as the researchers were interested in whether learners could distinguish between additive and multiplicative situations.

Literature has shown that multiplicative and additive reasoning are basically different (Askew, 2018; Bryant & Nunes, 2009; Caddle &Brizuela, 2011; Clark & Kamii, 1996; Jacob & Willies, 2003). Multiplicative reasoning involves multiplication and division operations while additive reasoning comprises addition and subtraction operations. Additive reasoning is employed in one-variable problems, where measures of the same kind are put together, separated or compared, whereas multiplicative reasoning involves two variables linked by a fixed-ratio (Askew, 2018; Bryant & Nunes, 2009; Venkat & Mathews, 2018). Structural understanding of multiplication and division is more complex than for addition and subtraction and is also important for the development of learners' foundations in different mathematical domains (Askew, 2018; Brown, Küchemann & Hodge, 2010; Bryant & Nunes, 2009; Hurst & Hurrell, 2017; Jacob & Mulligan, 2014; Venkat & Mathews, 2018). An intervention focusing on multiplicative reasoning was therefore seen as worthwhile.

1.3 Problem statement

Learners' challenges in solving word problems, across grades, prompted my interest to get greater insight into how grade seven learners respond to problems that demand multiplicative and additive reasoning and presented in parallel word and pure numerical formats. The intention of this work was to get an insight of learners' performance and performance differences in multiplicative and additive reasoning in the parallel problem formats.

In related research, Askew and Venkat (2018) made an analysis that provides an overview of differences in learners' performance based on item format across grades 5-7 in pre-intervention tests administered by the WMCP-MRIP project. Askew and Venkat (2018) found that learners' performance on arithmetic problems set either as word problems or purely numerical problems was similar. Neither problem format was associated with better performance than the other and this concurred with the findings of the 2012 ANA diagnostic report. Askew and Venkat's (2018) analysis left out data obtained by the WMCP-MRIP during post-test. In addition to

Askew and Venkat's (2018) analysis, the research presented below went further, to explore the effects of word and pure numerical format on performance and learners' performance differences in additive and multiplicative reasoning problems on the post-intervention test. The present work also extended Askew and Venkat's work by exploring the solution strategies that learners used in solving multiplicative and additive reasoning problems presented as word and pure numerical formats on the pre- and post- tests and the detectable errors they were committing.

Further afield, Hickendorff (2013) compared the performance and solution strategies of grade 6 learners' in word and pure numerical problem formats in the four mathematical operations in the domain of arithmetic, in Netherlands. Cummins, Kintsch, Reusser and Weimer (1988) investigated the effect of problem format on grade 1 learners' performance in the United States of America. Koedinger and Nathan (2004) explored the effects of problem representation on performance, solution strategies and errors on grade nine learners in the United States of America while Koedinger, Alibali and Nathan (2008) did a similar study with college students. All these studies contribute to the debate of effect of question item format on learner performance.

The current study, therefore focused, in-depth, on grade 7 learners' performance on eight parallel items across problem format and mathematical reasoning in the pre- and post-tests administered by the WMCP-MRIP project. The WMCP-MRIP project investigated the impact of a targeted intervention across 10 Gauteng primary schools. The schools that were used for the research are in previously disadvantaged settings in Johannesburg. Furthermore, the present study identified the solution strategies employed by learners during the tests as well as the errors that the learners committed during the two tests.

Grade 7 is the last grade of primary education, so analysing learners' written responses provides an understanding of their performance on multiplicative and additive reasoning word and pure numerical problems at the exit point of primary education. This present study compared the learners' performance differences in the pre- and post- test. In Koedinger & Nathan's (2004) problem solving framework performance is underpinned by comprehension and solution phases, so this work also intended to get an insight of learners' performance, solution strategies and detectable errors to multiplicative and additive word problems compared to multiplicative and additive pure numerical problems in the pre and post-tests.

1. 4 Research questions

The study sought to find answers to the following questions:

1. What are the differences in grade 7 learners' performance on multiplicative and additive reasoning word and pure numerical problems in the pre- and post- tests?
2. What solution strategies do grade 7 learners employ when solving multiplicative and additive reasoning word and pure numerical problems in the pre- and post- tests?
3. What errors are detectable in the grade 7 learners' written responses to multiplicative and additive reasoning word and pure numerical problems in the pre- and post- tests?

Taken together the answers to these questions will also indicate what, if any, shifts the intervention implemented by the WMCP-MRIP project appeared to have influenced on learners' performance, solution strategies and detectable errors.

1.5 Rationale of the study

As a high school teacher who encounters frequent problems with learner performance in mathematics, I wanted to closely analyse learners' written responses which reflect both the comprehension and solution phases of solving mathematics problems in order to gain insights into what might underlie the performance issues seen in high school mathematics. The current study opted to use data from the pre- and post-tests to get insight to learners' performance differences with or without any influence of other learning activities that are not part of their scheduled learning programmes.

This study contributes to the existing debate on the effect of problem format on learner performance in mathematics by making public the results of the research. The study also contributes to the literature on learners' performance across multiplicative and additive reasoning problems. Identifying the strategies used by learners and detectable errors they committed will give a picture of the multiplicative and additive reasoning of the grade 7 learners at the exit of primary schooling.

1.6 Organisation of this report

This report comprise of a literature review, research design and methodology, data analysis, findings and discussions, conclusions, limitations and recommendations. The literature review has six sections. The first section deals with the theory and concepts that guide this research.

This is followed by literature on mathematical reasoning, the effect of question item format on performance, solution strategies and errors and lastly the summary of the literature review. The third part of the report elaborates on the research design and methodology that was followed. Presentation of the data analysis, findings and discussions of findings precedes the summary, conclusions arrived at and recommendations arising from the research.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This section offers the theoretical framework that underpins the study as well as a review of literature on mathematical reasoning (additive and multiplicative reasoning), the effect of question item format on performance, written solution strategies used by learners as they respond to test items and detectable errors on learner written work. A summary of literature review including a conceptual framework of the research is also provided.

2.2 Theoretical framework

As my research focused on learner mathematical reasoning, an Information – Processing theory provided the lens through which a suitable methodology and a conceptual framework was constructed. The major tenet of Information – Processing theory centres on the notion of Input-Processing-Output relationship as in a computer (Brian, 2012). In this view, the learner's mind captures input data brought to it through sensory organs. The data, here, are the mathematical objects in their various forms; symbols, words, diagrams, formula, etc. (Koedinger & Nathan, 2004). Once captured, the learner tries to comprehend the data and creates mental models and schemas through which the data is processed (Koedinger & Nathan, 2004).

Solving mathematical problems involves cognitive processes that culminate in written or verbal solutions (Koedinger & Nathan, 2004). The cognitive processes that learners undergo when solving mathematical problems involve some knowledge constructions whereby the problem will be the new idea that will be fitted to what the learner already knows (Piaget, 1972). For the learner to come up with the solution, what the learner already knows will be restructured to fit the presented mathematical problem (Piaget, 1972). This is in line with what was highlighted by Skemp (1976) that a concept that was previously learnt is triggered in the mind when an example of it is encountered. This applies to mathematical problems learners come across in tests they activate what was previously learnt that is linked to the problem so that the learner can start the constructions of the answer to the problem.

Cognitive processing of the data is referred to as the comprehension of the mathematical problem. Comprehension according to Koedinger and Nathan (2004) refers to the cyclic processing of the problem be it verbal or symbolic problem by the problem solver when solving the problem to get the answer. When comprehension is taking place, at first there is production

of corresponding external representations like a mathematical operation or expression followed by a solution strategy (Koedinger & Nathan, 2004).

The internal Processing provides cognitive solutions through the various constructions in the schemas and models (Koedinger & Nathan, 2004). These mental representations are then presented as a solution through the external written or verbal responses (Koedinger & Nathan, 2004). Since I worked with learners' written scripts, I am interpreting what the learners wrote on their scripts as the product of what their minds captured, processed and returned as solution notations.

These external written responses when the learner works on his/her own, are an indication of the learners' comprehension and solution phases. A correct response indicates correct comprehension and correct solving of the problem, resulting in the solution strategy used being reliable. An incorrect response is an indication of a fault in comprehension phase or wrong solution phase or both resulting in errors and unreliable solution strategies or approaches.

This information processing theory can be illustrated schematically using Koedinger and Nathan, (2004) model/framework for analysing solving of mathematical problems. Figure 1 shows the Koedinger and Nathan framework.

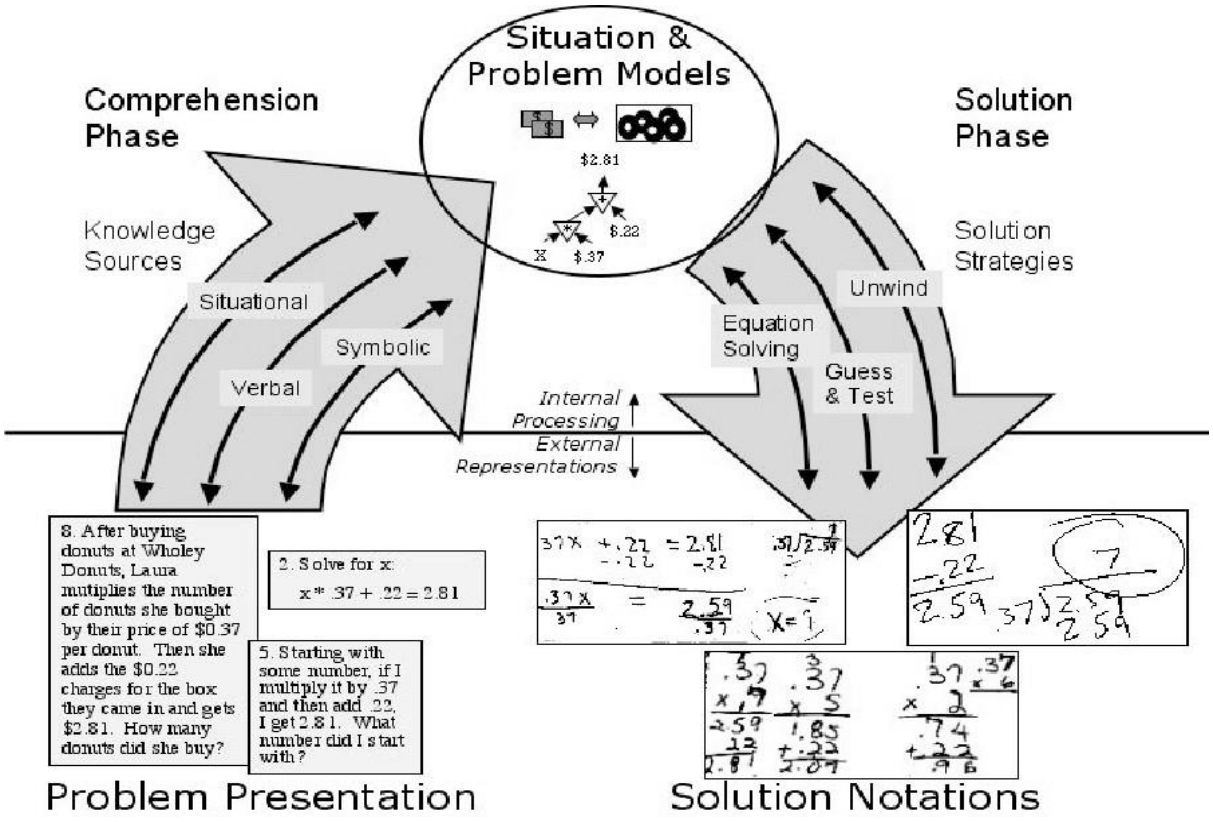


Figure 1: Framework for analysing solving of mathematical problems (Koedinger & Nathan, 2004, p.4)

As illustrated in the Koedinger and Nathan (2004) model, the three phases of the Information Processing theory take place during task execution. The first and last of the three phases are externally observable but the second one is comprised of processes that take place in the learner's mind as internal processes. When a mathematical problem is presented to a learner, this problem is accessed by the learner through the sensory organs as an input. Once the problem has been received, some internal cognitive processes start to take place (not externally observable). These processes constitute the comprehension phase, situation and problem models of the Koedinger and Nathan model. This is analogous to the processing phase of the information processing theory and it entails incorporation of new constructs into existing mental schemas and models (assimilation and accommodation). The resultant constructs lead to a solution phase which eventually produce a solution notation as an output (eternally observable). I adopted the Koedinger and Nathan (2004) framework to inform my coding framework that will be detailed in the next chapter.

2.3 Literature on Mathematical Reasoning

According to Lithner (2008), mathematical problem solving requires mathematical reasoning which is 'no less than a basic skill' (Ball & Bass 2003, p.28). Reasoning is the line of thought adopted to produce claims and reach inferences in task solving (Lithner, 2008). It does not necessarily need to lead to a correct answer so long as it is backed by some kinds of sensible (to the reasoner) reasons (Lithner, 2008).

Lithner (2008) proposed a reasoning structure that comprises four steps. Firstly, a problem solver encounters the task. The task may be simple or problematic depending on how easy it is for the solver to get a clue on how to proceed. After encountering the task, the solver chooses a strategy to use for solving the problem. The strategy may be a known procedure or any general approach. For some problem solvers, strategy choice is supported by predictive argumentations which are justifications of why the chosen strategy will solve the task. Following this, the strategy is implemented. Lastly, a conclusion is achieved which is end of the reasoning structure.

Lithner (2008) identified two major reasoning types, *Imitative* and *Creative* reasoning, and then proceeded to divide these into sub-categories. Imitative reasoning refers to reasoning where a problem solver emulates a solution procedure memorised from the textbook. Empirical studies have identified two main types of imitative reasoning: Memorised and Algorithmic reasoning.

With Memorised reasoning, the strategy choice is founded on recalling a complete answer so the strategy implementation only consists of writing down the answer. Memorised reasoning is used by learners when answering questions that require definitions, for example, knowing sub units in a bigger unit like millilitres in a litre and so forth. Learners may also memorise a proof from a textbook and reproduce it even at times without understanding it.

Usually, many school tasks ask for calculations where it is more appropriate to recall not the answer but an algorithm. "An algorithm is a finite sequence of executable instructions which allows one to find a definite result for a given class of problems" (Brousseau 1997, p. 129). Such cases require algorithmic reasoning where the problem solver just needs to recall a solution algorithm. On implementing the strategy (algorithm), the solver is guaranteed of a correct conclusion unless s/he commits a careless mistake in the process.

As already stated, the second type of reasoning is Creative reasoning which is reasoning that comprises both novelty and mathematically founded arguments. Novelty involves creating a new (to the reasoner) reasoning sequence or re-creating a forgotten sequence. The problem

solver develops arguments to support the strategy choice and/or strategy implementation by motivating why the conclusions are true or credible. All the arguments developed for this purpose are based on fundamental mathematical properties of the components involved in the reasoning.

Inferring from the above theories, research has suggested that mathematical reasoning comprises of the understanding of the meanings of numbers in two perspectives (Ching & Nunes, 2017; Nunes & Bryant, 2015; Nunes, Bryant, Barros & Sylva, 2012; Vergnaud, 2009). Firstly, meaning of numbers when used as symbols in the context of mathematical language and being able to manipulate the numbers sensibly. Second, meaning of numbers when representing and operating with quantities and relations between the quantities. Narrowing down from the broad reasoning types discussed above, this current research focused on learners' performance in mathematical problems that demand multiplicative and additive reasoning. Though literature provides many different definitions for multiplicative reasoning (MR) and additive reasoning (AR), for the purpose of this present research definitions have been chosen that are more direct and easy to connect with the work done during this study as detailed below.

Additive reasoning (AR) consists of the two mathematical operations: addition and subtraction, that both contribute to the key concept of the additive composition of numbers that a whole can be additively composed of two parts (Canobi, 2004; Resnick, 1992). AR is based on quantities connected by part-whole relations employed in one-variable problems, where measures of the same kind are put together, separated or compared (Askew, 2018; Bryant & Nunes, 2009; Ching & Nunes, 2017). The calculations that involve the AR include two central properties of part-whole relations that are commutative property of addition ('a + b = c' is the same as 'b + a=c') and the inverse relation between addition and subtraction ('a + b = c' can be expressed as 'c – a = b' and also that 'c –b = a') (Ching & Nunes, 2017).

Multiplicative reasoning (MR) involves multiplication and division operations, contributing to the key fact of relating three quantities in the form of 'c = a × b' (Venkat & Mathews, 2018). MR involves two variables linked by a fixed-ratio (Bryant & Nunes, 2009; Askew, 2018; Venkat & Mathews, 2018). The key concepts in play in MR are the many–to-one correspondence among the three quantities that sets the composite unite and the compositions of the composite units (Bryant & Nunes, 2009; Clark & Kamii, 1996; Venkat & Mathews, 2018).

Clark and Kamii (1996) highlighted that MR can also be considered as a more complex reasoning constructed out of AR at a higher level of abstraction. Siemon, Breed and Virgona (2005) inferred that transition from AR to MR is one of the major barriers to learning mathematics in the middle years. For many learners, working with multiplication before they have an adequate understanding and access to efficient strategies for addition and subtraction is terrifying and undermines all their subsequent school mathematics experience. This result is noted in studies done exploring the development of learners AR to MR citing differences between the two (Degrande, Verschaffel &Van Dooren, 2018; Dooren, Bock & Verschaffel, 2010; Jacob & Willis, 2001; 2003; Siemon, Breed & Virgona, 2005)

Research has established that MR is a difficult topic for it takes different forms and deals with many different situations (Breed, 2011). It is because of this complex nature of the MR that, though it is introduced to learners in early years of primary schooling, many learners struggle to understand it (Breed, 2011; Clark & Kamii, 1996). Despite of all the difficulties associated with MR, it is important in the learning of mathematics for it provides the fundamental base for many mathematics content areas at primary and secondary school levels and beyond (Askew, 2018; Bryant & Nunes, 2009; Hurst & Hurrell, 2017; Venkat & Mathews, 2018).

2.3.1 Studies on Multiplicative and Additive reasoning

Ching and Nunes (2017) carried out a longitudinal study that examined the relative importance of counting ability, additive reasoning, and working memory in children's mathematical achievement (calculation and story problem solving). The study was done with one hundred and fifteen 6-year-old Chinese children in Hong Kong who participated in two assessments. AR resulted in explaining a substantial and significant amount of variance in success on calculation and story problem items in both assessments after the effects of age, working memory, and counting ability were controlled. Further, both knowledge of the commutativity and complement principles were unique predictors. This finding highlights the importance of additive reasoning in children's mathematical development.

In a research on multiplicative reasoning and mathematics achievement, Bryant and Nunes (2009) carried out two studies. In the first study, they investigated the conditions under which learners were relatively more successful in solving MR problems. The study was conducted with 81 children from two state schools in their third year of schooling with minimum instruction on multiplication and none on division. It was concluded that the learners were able to solve MR problems with relatively high levels of success with best performance when

materials that facilitated the representational resources of both variables were available. Where that access was reduced there were lower levels of success (Bryant & Nunes, 2009).

The second study investigated whether 52 learners' multiplicative reasoning had a noteworthy influence on the prediction of their mathematics achievement in their first year of schooling. (Bryant & Nunes, 2009). The study concluded that the learners' performance in the multiplicative reasoning task contributed significantly to the prediction of their mathematics achievement.

Ching and Nunes (2017) highlighted the importance of additive reasoning in learners' mathematical achievement and Bryant and Nunes (2009) put forward the importance of multiplicative reasoning in learners' mathematical achievement. My study therefore explored learners' performance on multiplicative and additive reasoning abilities as presented in their written responses from parallel word and pure numerical problems in a pre-test and a post-test administered by the Wits Maths Connect project in the Gauteng province. In the process, the solution strategies that learners used and errors they committed were also analysed.

2.4 Literature on effect of question item format on performance.

In Askew & Venkat's (2018) analysis, three competing views about the effect of question item format on performance are summarised. The first view regards the word problem format as more difficult than a pure symbolic problem format. The second view claims that a word problem format is easier than pure symbolic problem format. Lastly, the two formats are considered to result in similar performance. For the purpose of this work, word problem format refers to descriptions of situations in words, assumed to be comprehensible to the reader, and within which mathematical questions can be contextualised (Sepeng, 2014). In this study pure numerical format refers to a problem presented in a numerical form or as an equation. Below I review previous studies that provide evidence in relation to the different views.

2.4.1 Word problem format as more difficult than pure numerical format

Koedinger & Nathan (2004) reported that the general public, textbook authors, teachers, mathematics education researchers and learning science researchers often voice the belief that word problem formats are difficult for learners. A survey done by Nathan and Koedinger (2000) of 67 high school mathematics teachers revealed that most teachers predicted that word problems would be more difficult than matched symbolic equations for algebra students. Another survey of 35 mathematics education researchers showed that most of them predicted

that word problems would be more difficult than matched equations for algebra students (Nathan & Koedinger, 2000)

The belief in the difficulty of word problems stems from the word problem solving processes that can be theoretically described, as noted above, as involving two phases: the comprehension and solution phases (Koedinger & Nathan, 2004). The comprehension phase involves the linguistic processing of the word problem into a corresponding model of the relationship present in the problem (Askew & Venkat, 2018; Cummins, Kintsch, Reusser & Weimer, 1988; Hickendorff, 2013; Koedinger & Nathan, 2004). For the purpose of this study linguistic processing is the interpretation of language in the question, to understand and develop a meaningful problem model (Koedinger & Nathan, 2004). From the problem model then the learner will have to create an appropriate mathematical model from which the answer can be calculated (Askew & Venkat, 2018; Hickendorff, 2013; Koedinger & Nathan, 2004). The solution phase involves the use of the mathematical model as the base for computational strategies.

Symbolic problem solving also involves the two phases but the comprehension phase focuses on an appropriate mathematical model (Koedinger & Nathan, 2004). The comprehension for symbolic problem does not involve the linguistic processing, it is just comprehending the given expression. The solution phase is the same as in word problem format (Koedinger & Nathan, 2004). The difficulty of the word problem lies in the linguistic processes which differentiate the word problem solving process from the symbolic problem solving process. According to Sepeng (2014), a lack of competence in processing language when solving word problems is likely to result in incorrect models and wrong solutions.

Difficulty of word problems was studied in the United States of America by Cummins, Kintsch, Reusser and Weimer (1988) who concluded that "word problems are notoriously difficult to solve" (p. 405). This was highlighted after carrying out a study on grade one learners' performance on matched arithmetic word and numeric problems. The study found that performance on word problems was poorer than performance on matched numeric problems due to difficulty in text comprehension processes (Cummins, Kintsch, Reusser & Weimer, 1988).This suggests that learners' understandings of language could have been their main challenge.

In the same way, Hickendorff (2013) carried out a study with grade six learners to assess the effects of arithmetic word problem format and bare numerical format that involve all the four

operations of mathematics on performance, in Netherlands. The study revealed that word problems involving division were more difficult than their numerical counterparts. However, for the other operations, differences in learner performances were insignificant. This can be an indication that the operation involved in the word problem, particularly if it is division, may contribute to the difficulty of the problem.

In the domain of algebra, Koedinger, Alibali and Nathan (2008) on their study with college students in the United State of America discovered that in more complex problems, word problems pose a disadvantage over pure symbolic problems. Symbolic problems have the advantage of placing fewer demands since they are concise and demand the computation of the unknown only (Koedinger, Alibali & Nathan, 2008).

2.4.2 Word problem format can be easier than pure symbolic format.

Word problems activate two knowledge systems; knowledge that is developed in the mathematics classroom and knowledge that is developed through real world experience (Sepeng, 2014). Sepeng argues that the real world experience bridges the gap that exists between the abstraction of pure mathematics and its application to the real world situation. The activation of real-world knowledge is an advantage to learners in getting many different and effective strategies in arriving at a correct solution (Hickendorff, 2013; Koedinger & Nathan, 2004). For the purpose of this work real-world knowledge is culturally constituted systems of quantification (Sepeng, 2014). This type of knowledge is not needed in pure numeric problems.

Considering the studies done with respect to algebraic problems Koedinger and Nathan (2004) explored the effects of problem representation on performance of grade nine learners in the United States of America. They found that learners performed better in word problems than in symbolic problems (equations) because they used unplanned, informal strategies as they solved word problems (Koedinger & Nathan, 2004). This result agrees with that obtained by Baranes, Perry and Stigler (1989) in a study with Brazilian children. In their, work, Baranes et al., established that though there are certain conditions under which word problems are easier than their pure numerical counterparts, (money contexts and numbers involving multiples of 25), word problems activate real–world knowledge that help learners to arrive at correct solutions. Word problems elicit different but more effective solution strategies than pure numerical problems (Koedinger & Nathan, 2004). For instance, Nunes, Schliemann and Carraher (1993),

concluded that word problems evoke oral solution strategies as opposed to pure numerical problems which evoke less effective written arithmetic strategies.

Sepeng (2014) highlighted that word problems give learners an intuitive way of connecting with practical issues of the world they live in. That being the case, word problems can arouse interest in learners of doing the mathematics and in the process give them an impetus to succeed. Consequently, they come up with innovative ways of solving the problem at hand resulting in unplanned and informal strategies (Koedinger & Nathan, 2004). Mostly, the strategies are peculiar to individual students so they make them take ownership of their learning without having to memorise specific textbook procedures. This makes their learning easy and enjoyable. As such they find word problems more interesting and easy to deal with.

2.4.3 The two formats are considered to result in similar performance

In the arithmetic domain, Hickendorff (2013) carried out a study on grade six learners that assessed the effects of arithmetic word problem format and pure numerical format that involve all the four operations of mathematics on performance in Netherlands. The study revealed that the word and pure numerical problems that involve the operations of addition, subtraction and multiplication were equally difficult, with little difference in performance (Hickendorff, 2013).

In South Africa, Annual National Assessments (ANAs) were administered by Department of Basic Education (DBE) to grades 1-6 and 9 in 2012, 2013 and 2014 (DBE, 2012; 2013; 2014). The purpose of the ANAs was to determine what learners can and cannot do with regards to the skills and knowledge that they have acquired as a result of teaching and learning experiences in school. After the administration of each ANA test, the DBE compiled a diagnostic report in which an analysis was made to provide evidence to inform and direct appropriate interventions (DBE, 2012; 2013; 2014).

The 2012 diagnostic report stated that there was no difference in learners' performance on word and symbolic problems; they were just difficult (DBE, 2012). This indicated that format of questions, be it word problem or numerical was not of any advantage to the cohort of 2012 learners.

ANA tests were testing mathematics covered in the content areas: Numbers, Operations and Relations; Patterns Functions and Algebra; Space and Shape (Geometry); measurement and Data handling. My work focused on problems that involve additive and multiplicative relations in the domain of arithmetic tested to grade 7 learners from four schools. The intention was to get insight of the learners' performances in multiplicative and additive reasoning problems as

well as explore the effect of problem format on performance with respect to a specific mathematics content area unlike in the ANAs where all content areas were examined.

Likewise, Askew and Venkat (2018) carried out a study that examined South African middle grade (grades 5-7) students' performance on problems set either within a word problem format or purely numeric format. The study involved 910 learners and was part of the WMCP-MRIP that includes pre –and post-tests. Learners' pre-test scripts from 10 public primary schools in the Johannesburg area were explored. The results showed little difference on performance across the two formats. The difference was negligible to an extent that it couldn't support any claim that states that word problems are easier or harder for learners than purely numerical problems. The study also revealed that learners performed better on additive reasoning problems as compared to multiplicative reasoning ones.

This present study used data from the WMC- MRIP, where Askew and Venkat are team members. Learners' scripts from the WMC-MRIP project schools were used. Unlike Askew and Venkat who analysed grades 5-7 pre-test learners' scripts, I analysed grade 7 pre-test and post-test learners' scripts of 4 schools. In Askew and Venkat's case, they compared performance differences between word and pure numerical problems as well as between additive and multiplicative relations of the pre-test. In addition to such a comparison, I went on further to explore the effects of word and pure numerical format on performance and learners' performance differences in additive and multiplicative reasoning problems on the post-test. The work was also extended by exploring the solution strategies that learners used in solving multiplicative and additive reasoning problems presented as word and pure numerical formats on the pre- and post- tests and the detectable errors they were committing.

2.5 Literature on solution strategies

A solution strategy for solving a mathematical problem is an option that an individual makes out of existing options related to the problem structure (Ferreira & Serrazina, 2011). The choice is made with the intention to lead the individual to the execution of computational steps related to the numbers in the problem.

Literature contains a number of studies carried out that analysed and characterised solution strategies. For instance Kouba (1989) carried out an empirical study in New York with 43 first graders, 35 second graders, and 50 third graders from one small mid-western school to characterize their solution strategies for equivalent set of multiplication and division word

problems in an examination and identified two dimensions. The first dimension pertains to the degree of abstractness involved in the solution strategy that include the categories of direct representation, double counting, transitional counting (counting sequence based on multiples of a factor in the problem), additive o subtractive (use of repeated addition or subtraction to calculate an answer) and recalled number fact .The second dimension relates to the way in which physical objects were used that include the following classification of objects used as representations of the unique elements in each set, objects used as tallies or repeated references for the involved numbers , and no objects used.

Anghileri (1989) reported the results of her observations of the behaviours and successful solution strategies of 152 children aged from 4- 12 years as they carried out multiplication tasks. The results suggested a progression of children's strategies from unitary counting or counting by ones, skip counting and repeated addition through to the use of a multiplication fact (Anghileri, 1989).

A considerable number of studies have characterised the solution strategies according to education level and the mathematical topic being studied. Anghileri, Beishuizen and Putten (2002), for example, explored written calculation methods for division used by 276 learners in England and 259 learners in the Netherlands at two points in the same school year. The research concluded that, as a response to different teaching approaches used by teachers, learners' solution strategies shift from informal to more structured procedures. In addition, the research categorised the learners' solution strategies as follows: using tally marks or some symbol for each unit, operating with the digits independently (The main idea involved in this category is breaking down numbers using ideas of place value.), low level 'chunking', high level 'chunking', the traditional algorithm involving formal layout, mental calculation showing an answer but no working, unclear strategies, no attempt.

Koedinger and Nathan (2004), analysed the solution strategies used by learners as they responded to algebraic problems and categorized the solution strategies as: unwind, guess and test, symbolic manipulation, answer only, unknown, No response.

Hickendorff (2013) worked with grade 6 learners to explore strategies they used to solve multidigit mathematics problems that involve the four mathematical operations. Hickendorff realised that learners used the following strategies: traditional algorithm, Realistic Mathematics Education (RME) approach, partitioning operands, repeated addition/subtraction, indirect addition, other written strategy, no written work, unclear strategy and skipped problem.

In a study of solution strategies and achievement, Hickendorff, Heiser and Van Putten (2009) analysed the assessments of grade 6, the end of primary school, investigating the solution strategies learners used in solving the division problems, in the Netherlands. They classified the solution strategies into four main categories of traditional long division algorithm, RME strategies, writing the answer only with no written working and unclear strategies (Hickendorff, Heiser & Van Putten, 2009).

In the South African context, solution strategies are characterised as concrete or abstract acknowledging progression from concrete to abstract (Ensor, Hoadley, Jacklin, Kuhne, Schmitt, Lombard, & Van den Heuvel, 2009; Hoadley, 2007; Schollar, 2008). Concrete strategies involve the use of physical objects or drawing of solid objects, tallies, counters to aid tangible counting (Weitz, 2013). On the other hand, abstract strategies involve working with mathematical symbols like numbers without the need for physical counting (Weitz, 2013).

Venkat and Mathews (2018) conducted a study in which they acknowledged the progression in solution strategies from the inefficient counting-based solution strategies to more efficient abstract solution strategies. In their analysis, they highlighted that learners made use of unit/multiple counting, repeated addition/subtraction, T-tables/double number lines and the traditional algorithms of multiplication and division (Venkat & Mathews, 2018).

Like the above mentioned researches, where the researchers explored strategies employed by learners, this research used learners' scripts to find out the types of solution strategies that were used to solve multiplicative and additive word and pure numerical problems. The above sets of strategies identified by other studies have a close resemblance with the strategies that were employed by learners in this research. Besides being concrete and abstract, the strategies can be structured or unstructured making them either simple or difficult to use by learners. What is common between the above sets is that learners use manipulatives as a natural solution strategy. Manipulatives are physical objects used to engage students in hands-on learning of mathematics (Smith, 2009). Besides using manipulatives, learners have a tendency to write the answer only without showing any working. Learners also leave the problem unanswered. This present research explored learners' written scripts with a view of identifying the solution strategies that the learners used in answering multiplicative and additive word and pure numerical problems in pre – and post-tests.

2.6 Literature on errors

Research has shown that regardless of the nature of mathematical content area, learners commit errors as they solve mathematical problems. In one study, Luneta and Makonye (2010) regarded an error as a mistake, slip, blunder or inaccuracy and a deviation from accuracy. Prior to this, Luneta (2008) had defined errors as signs of the challenges experienced by learners in a learning process. Such challenges in the learning process may hinder or manifest as problems of conceptual understanding, and, as such, Swan (2001) regarded an error as a mistake or lack of understanding of the information given. Nesher (1987) argued that errors are systemic, persistent and pervasive mistakes performed by problem solvers: in her view, it is difficult for the person who commits the errors to realize them.

Another dimension to errors was given by Legutko (2008) by stating that an error occurs when a person takes the false as the truth or vice versa. This happens without the person realising that the truth has been exchanged with falsehood. In mathematics, errors also involve the exclusion of essential characteristics in a given class of objects. This might also entail inclusion of unnecessary characteristics into the definition when defining mathematical concepts and its application (Legutko, 2008). When essential characteristics of a mathematical object are excluded from its description or discussion, chances are that a different object might end up being discussed.

According to Riccomini (2005) errors can be differentiated as systematic and non-systematic errors. Non-systematic errors (also referred to as slips, lapses or unintended mistakes) are mistakes which learners make un-intentionally and are easy to correct with no assistance (Riccomini, 2005). Systematic errors were described as symptomatic of a faulty line of thinking, they are often repeated and thus symbolise an underlying incorrect hypothesis. Systematic errors are procedural and often feature as an incorrect routine in an otherwise correct method. Herholdt and Sapire (2014) considered systematic errors as bugs of a conceptual or procedural understanding involving an incorrect routine. Riccomini (2005) believes that these types of errors occur when a learner is faced with a difficult or unfamiliar feature of a task. Due to the unfamiliarity of the task, the learner gets to an impasse that results in him/her modifying a known procedure and incorrectly apply it to the task.

It is crucial for teachers to understand the nature of errors and why they are made as this helps to inform best practices for remediation (McGuire, 2013). Best practices for remediation include knowledge of the mathematical concepts that learners struggle with and the underlying

levels of cognitive processes involved (Ashlock, 2006; Shulman, 1986; Sousa, 2008). When learners commit errors, it is a demonstration of faulty comprehension and this eventually impacts on solutions to mathematical tasks (Davis, 1984). Usually, error analysis enables teachers to get a better insight of the mathematical reasoning of the learners (Herholdt & Sapire, 2014).

Luneta & Makonye (2010) highlighted that errors are visible in learners' artefacts such as written text or speech. There are a number of studies that analysed and classified learners' errors. These classifications are much dependent on the education level and topic area concerned. In the study presented here, the errors will be those detectable in grade 7 learners' scripts as they answered word and pure numerical problems that required multiplicative and additive reasoning.

Watson (1980) conducted a study with children in third year at a Melbourne primary school to determine why they had made errors, on sixteen arithmetical tasks. The errors were analysed using the Newman model which stated that all errors can be categorised and placed into eight different categories. The error analysis led to the identification of types of errors that include reading, comprehension, transformation, processing, encoding and carelessness.

In another work, Elbrink (2008) also devised her own classification of errors, consisting of four categories namely mechanical errors, application errors, careless errors and order of operation errors. Sarwadi and Shahril (2014) also analysed errors on scripts of repeating 'O' level students that wrote a test that comprised of 13 questions. The test items were designed to cover the topics on numbers (fractions and decimals), Algebra, Geometry, measures including conversions of units of measurement and area, Statistics and probability. The errors that were identified were whole number to fraction error, conversion error, inverting the wrong fraction error, cancelling error and incorrect operation- cross multiplication.

In a study of algebra Koedinger and Nathan (2004) analysed the scripts of eighth grade learners that participated in a test that involved 3 levels of problem presentation, namely story, word equation and symbol equation in all the four mathematical operations. Students' errors were put into three broad categories which are no response, arithmetic error and conceptual error. No response category was coded when nothing was written for that problem while arithmetic error was for a mistake in performing arithmetic operations. The category of conceptual errors

include order of operation error, algebra manipulation error, argument error, inverse operator error, incomplete guess- and- test error and decimal alignment arithmetic error.

Koedinger, Alibali and Nathan (2008) also conducted research on error analysis with university students on two college-level algebra courses. They came up with 3 broad categories of errors in the study. The categories are; no response, conceptual and arithmetic errors. Conceptual errors involve giving up on the task, challenges with order of operations, comprehension, missing operator, bad algebra, inversion and writing the answer only without showing working where it is expected to do so. On the other hand, arithmetic errors entail wrong additions, subtractions, multiplication and division as well as copy slips. In this current study, operation choice, procedural and computational errors were regarded as the main types of errors detectable in learners' written scripts all of which are in a way related to the errors highlighted above.

Herholdt and Sapire (2014) carried out error analysis research using learner responses to tests that were administered as part of the evaluation of an intervention project that aimed at teaching mathematical problem solving skills to grades 1-4 learners in South Africa. The errors that were identified were conceptual and procedural across the grades and mathematical topics.

Luneta 2015 analysed errors students made when solving coordinate geometry problems in the final Grade 12 examination in South Africa. The analysis was of 1000 scripts from the 2008 Mathematics examination. The study revealed that the errors that were identified were conceptual errors and procedural errors.

Elbrink (2008) and Luneta (2015) defined procedural errors as errors that occur when a learner applies a procedure incorrectly whereas computational errors are mistakes in addition, subtraction, multiplication and division of numbers.

Conceptual errors are structural errors, which originate from a failure to realize the relationship involved in the problem or to grasp some principle essential to the solution and are due to lack of understanding of the concept (Luneta, 2015; Orton ,1983). As already alluded to earlier, the term conceptual error is a broad category that encompasses a wide range of learner challenges as they execute mathematical tasks. For the sake of this study the conceptual errors are the errors that include the wrong choice for the operation and operating with it with the numbers in the problem.

Though the literature has recorded a considerable number of types of errors as already discussed, in this research, I chose to focus much on those that are perceived to be strongly

linked to comprehension and solution phases. Without downplaying the effect of other types of errors on learners' comprehension and/or quality of solution, my work places emphasis on the following conceptual (operation choice error), procedural, computational and others errors. The errors that are referred to as "others" in this study are the group of errors that could not be characterised as operation choice nor procedural nor computational. For example, copying incorrectly the numbers that are involved in the test item.

2.7 Summary of Literature review

The literature that was reviewed for the purpose of this research focused on mathematical reasoning and also on the effect of problem format on learners' performance in mathematics.

To this effect, there are three competing views about the effect of problem format on learners' performance. There is a view that word problems are more challenging to learners than pure numerical problems (Cummins, Kintsch, Reusser & Weimer, 1988; Hickendorff, 2013). Another view is that word problems are easier for learners than pure numerical problems (Koedinger & Nathan, 2004). The third view, is a middle ground view which states that the two problem formats are equally challenging to learners (Askew & Venkat, 2018, DBE, 2012, 2013, 2014; Hickendorff, 2013).

This current research adds a voice to the existing debates about effect of problem format on learners' performance and performance differences in problems that involve multiplicative and additive reasoning. As already shared earlier, the research which is close to what was done by this present research is that of Askew and Venkat, 2018. They made an analysis of grade 5-7 learners' scripts taken from the main project as mine but their analysis focused on the pre-test and left out the post-test scripts. My research fills the gap left by the work of Askew and Venkat (2018) by exploring grade 7 learners' multiplicative and additive reasoning in pre and post-test during a WMCP-MRIP project. The reason for choosing grade 7 in this research was that this is the exit grade for primary education so understanding grade 7 learners' mathematical reasoning sheds light into underlying problems experienced by high school learners in mathematics.

The conceptual framework that guided the exploration of the learners' performance differences, solution strategies and detectable errors is diagrammatically represented in Figure 2 below.

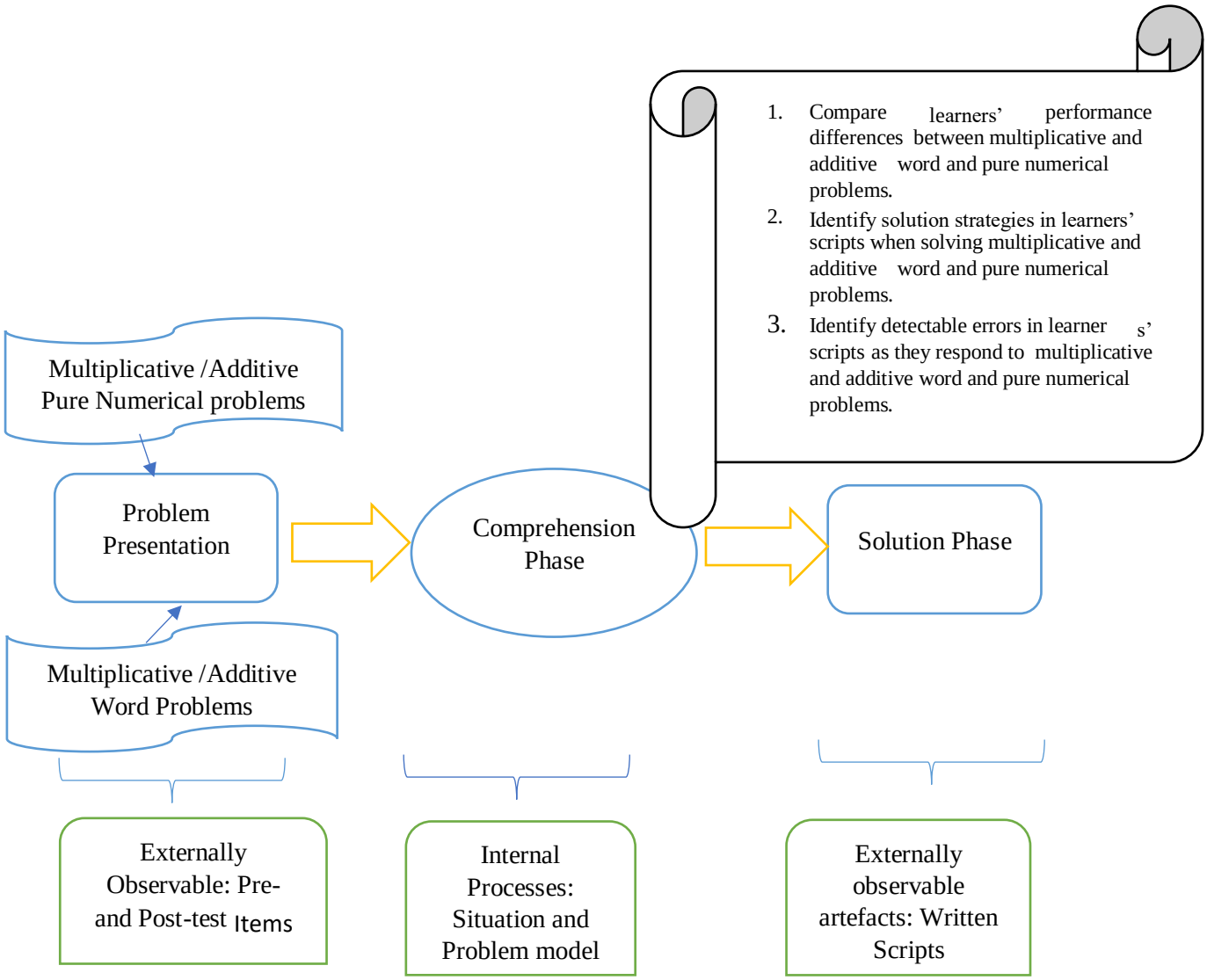


Figure 2: Conceptual framework for this research

1. Problem presentation:

   In this research, the mathematical problems are presented to learners in two formats; Multiplicative /Additive word and Multiplicative /Additive pure numerical formats.

2. Internal processes: These processes are not visible to the researcher but are assessed through learners' solution notations. The internal processes cause comprehension or lack of it as learners work with mathematical problems presented to them. Such comprehension will manifest through learners' external representations. Correct comprehension will enable learners to operate correctly with numbers and quantities presented in the problem statement.

3. Solution notations: The solution notations will show the:
    - Written answers to the mathematical problems.
    - Solution strategies used to arrive to an answer.
    - Errors that are identifiable on the solution notations that lead to the wrong answer.

The solution notations allow classification of responses. The responses will be classified as correct or incorrect and they also allow evaluation of solution strategies used and detection of errors committed by the learners. By way of analysing the artefacts of the solution phase, answers to the three research questions outlined earlier in this research will be sought.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter provides a description of the research design, data sources, sampling technique, development of coding framework for data analysis and details of how the research addresses issues of ethics, reliability and validity.

3.2 Research design

In this research, a quantitative content analysis was done on grade 7 learners' responses to the pre- and post-tests conducted by Multiplicative Reasoning–Intermediate Phase (MRIP) project. Quantitative content analysis is a research approach that involve analysing the appearance, counting and quantifying of a textual element (Hsieh & Shannon, 2005, Zhang & Wildemuth, 2009). Content analysis is used to develop systematic and objective interpretations about a theme of interest in any type of communication through the process of coding of raw messages (Elo & Kynga¨ s 2007; Hsieh & Shannon, 2005). Quantitative content analysis also involves classification of parts of the written response through the application of a coding framework from which conclusions can be deduced through descriptive statistics like frequency counts (Rose, Spinks & Canhoto, 2015). The analysis was done to explore learners' performance in multiplicative and additive reasoning word and pure numerical problems on written responses. The content analysis is suitable for this study because of its perceived advantages that the design has for a research of this nature. Firstly, according to Allen (2017), content analysis examines messages that are close to the communicator because a message is either created by or recorded from the communicator. The examined message is directly from the communicator with no risk of losing its originality through transcribing. That being the case, by collecting and analysing actual communicative messages, content analysis provides researchers with richer data than what other designs would do.

Since content analysis is done on existing texts, the work of collecting data is made minimal. This way, the researcher only needs the data, and not the source, to conduct the analysis. By so doing, content analysis can bypass human subjects' boards because the research neither involves nor affects actual participants (Allen, 2017). Furthermore, as a process, content analysis is safe, not time consuming and relatively cheap. In the event of an error occurring during the research process, the error is easy to fix in content analysis than in other designs. With other designs, a process error might cause a revamp of the research whereas with content analysis, the researcher only traces the point of the error and fix it (Allen, 2017).

Scott and Morrison (2006) stated that the selection of research methods to use is guided by the nature of the research question(s) of the study. This study used quantitative content analysis to respond to the research questions. The analysis of the appearance, counting and quantifying of the categories answer, comprehension, solution strategies and errors was done on the written responses of the pre and post-tests. The data was analysed using statistical methods such as T-tests and presented in the form of tables and graphs

3.3 Data Sources

This research used the data that was collected by the Wits Maths Connect Primary project-Multiplicative Reasoning (M-RIP) project intermediate phase (grades 4-7) that investigated the impact of a targeted intervention across 10 Gauteng primary schools. The M-RIP project involved work with Intermediate phase Mathematics teachers on developing and implementing 4-6 intervention lessons, which took place between pre- and post-tests. Data was collected through lesson observations, teacher reflection meetings and learner performance in the pre- and post-tests for the study of the impact of the intervention. This research does not focus on the impact of the intervention, but on the data that was collected from the pre- and post-tests. From all the data that was collected by the M-RIP project, this work only used grade 7 learners' written responses in the pre and post-tests.

3.4 Sampling

Sampling is the practice of choosing a subgroup or a sample from a bigger group or population (Alvi, 2016; Scott & Morrison, 2006). Scott and Morrison (2006) postulated that the approaches that one uses in selecting the sample would determine the quality and validity of the findings that are brought about from the study of the selected sample.

The well-known types of sampling are probability sampling and non-probability sampling. Probability sampling is in place when every element of the bigger group has an equal chance of being selected to be part of the sample (Scott & Morrison, 2006). Non-probability sampling occurs when the selection of the sample is done basing on subjective judgment of the researcher such that the elements from the bigger group do not have an equal chance of being selected (Scott & Morrison, 2006).

This study used purposive sampling which is a non-probability sampling technique by selecting grade sevens' pre and post-tests learners' scripts. Purposive sampling transpires when the sample is selected by taking into consideration a certain purpose and that the criteria of

selecting the elements to be included in the study is explicitly defined (Alvi, 2016). My research was aimed to explore performance in multiplicative and additive reasoning word and pure numerical problems of grade 7 learners from written tests. M-RIP project included some grade seven learners in their project hence their data (pre and post- tests) was targeted by this research. The selection of grade 7 was because they are at the exit point of primary education, which research has established that the standards of teaching and learning of mathematics in South African Primary schools are poor (Venkat & Naidoo, 2012). Focusing on grade 7 gave room for insights into learners' performance as they finish the primary education. The research questions of this study influenced the selection of all the samples used for this study because these questions focused on grade 7 learners' performance that can be revealed from the written tests. Further, two levels of sampling were carried out: sampling of learners' scripts, and sampling of the questions for the content analysis.

3.4.1 Sampling of Scripts

Six grade 7 classes from six different schools participated in the MRIP project. Four out of the six grade 7 classes participated in the second phase trial hence a broader interest in making a content analysis of these four specific classes. In these four grade 7 classes, 143 learners wrote both pre and post-test. Hence, a total of 286 scripts were analysed in this study; 143 scripts for pre-test and 143 scripts for post-test.

3.4.2 Sampling of Questions

The pre- and post- tests comprised of 19 items: 15 word problems; 12 involving multiplicative relations (MR), three additive relations (AR) and 4 pure numeric problems (See Appendix 1 for the test). This study focused on eight matched parallel test items, 4 word problems with 2 that involve MR and 2 that involve AR and four pure numeric problems with two that involve MR and two that involve AR. The selection of these eight test items was on purpose so as to be able to get insight into learners' performance differences with respect to problem format with identical mathematical operation, solution strategies and errors. The eight parallel problems are shown in Table 1.

Table 1: Sampled Questions for the study

Word problem	Pure numeric problem
4. Corin puts out 81 pies on one tray and put 19 pies on another tray. How many pies does Corin put out all together?	16. 81 + 19 =
5. Hamsa is counting how many tiles cover the bathroom floor. She counts 12 rows of tiles. There are 11 tiles in each row. How many tiles cover the floor?	18. 12 x 11 =
11. Sam cycles 112 km. Sameera cycles 99 km. How much further does Sam cycle than Sameera?	17. 112 – 99 =
12. A company packs pencils into boxes. Each box contains 15 pencils. HOW many boxes are needed to pack 195 pencils	19. 195 ÷ 15 =

Table 2 outlines the five stages that were carried out during the content analysis of the learner scripts.

Table 2: Five Stages of the content and data analysis carried out in the study

Activity	**Purpose and Description**
Developing a Coding framework	To work with a selected number of learners' scripts (16) so as to identify emerging codes from the data. Data was already available from the main project- WMCP-MRIP. To develop a coding framework for the research using emergent codes from the piloting exercise. Codes were developed for the three research questions.
Coding	Coding of 286 learners' scripts using the developed coding framework.
Coding reliability check	To check reliability of the coding framework. For this purpose a peer was asked to recode 32 scripts.
Data capturing	To capture 8 questions on two excel spread sheets for processing; one for pre-test and another for post-test. The capturing was done per test item with each test item having 4 aspects to be captured on it, which were the answer, comprehension, solution strategy and errors
Data analysis	Presenting the data in tables, graphs and narratives.

In the next section, the process of the development of the coding framework is provided

3.5 Development of the Coding Framework for my study

Coding is the process of organizing communication content in a way that allows easy identification or retrieval of content relevant to research questions (Kondracki, Wellman & Amundson, 2002). The process gives room for data to be converted into electronic form; when the codes are numerical they can be statistically processed using computer software (Rose, Spinks & Canhoto, 2015). The frameworks developed by Hickendorff (2013) and Koedinger and Nathan (2004) were used at the initial stage for the coding of the learner's responses to the test items. 16 scripts (from 8 learners; comprising of pre-and post-test) were used at this stage. The aim was to understand how these two frameworks could help me to read the data as well as to answer my research questions. Following detailed scrutiny of this initial coding, with the help of my supervisors, a refined coding framework that speaks to my data and the research questions emerged. Table 3 presents the coding framework that was used in my analysis.

Table 3 : Coding framework for my analysis of learner scripts

S/N	Categories	Codes and their descriptions
1	Answer	0 Blank answering space 1 Correct answer 2 Incorrect answer
2	Comprehension	3 Correct comprehension 4 Incorrect comprehension
3	Solution strategy/ approach	10 drawing objects, tallies or counters 11 Repeated addition or repeated subtraction 12 T- table or double number line 13 Standard algorithm 14 Answer only with no visible working 15 Others
4	Errors	21 Operation choice error 22 Computational error 23 Procedural error 24 Others

It is important to note that the order of the strategies and errors presented in Table 3 are not hierarchical. Below I provide illustrative examples of how I conceptualized each of the four categories and their accompanying codes as provided in Table 3.

3.5.1 Conceptualization of the 'Answer' category

The answers for the word and pure numeric problems were coded in the same way. During the coding process, each and every learner script was looked over from beginning to end in search for any notes, workings and answers written by the learner. This was done to ensure that every learner's attempts were inspected. It is common practice that, despite the fact that answering spaces are provided for learners to use, they do calculations to questions on the additional space and latter only transfer the answers to the answering space. As the focus of the research was on how mathematics was done by learners, instances where correct calculations were done in the additional space but wrong transfers of the answers (answer only cases) were made to the answering space, the coding was done with respect to what was in the additional space where

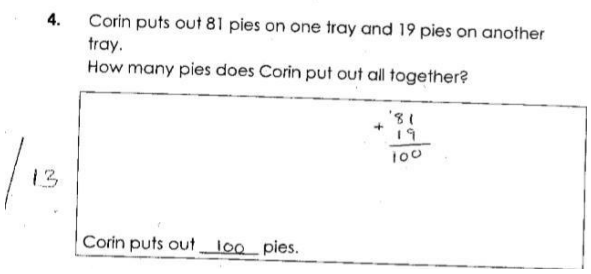
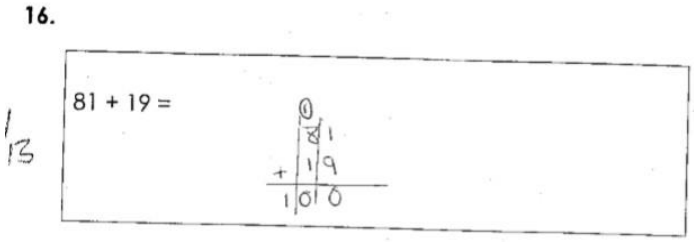
there was complete working and the work in the answering space was ignored. Code 0 was given for blank answering space, where nothing was written, no working nor an answer was shown anywhere on the script with respect to the question. The correct answer was assigned code 1. An incorrect answer was coded 2.

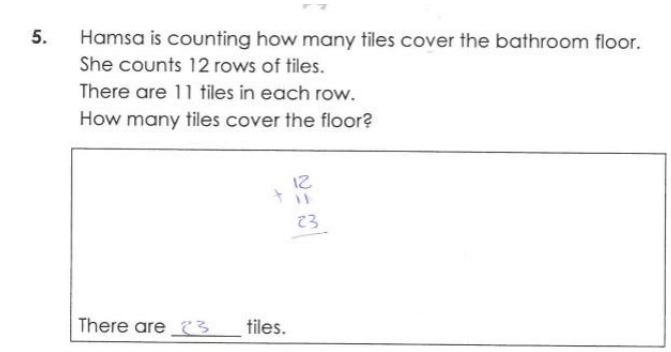
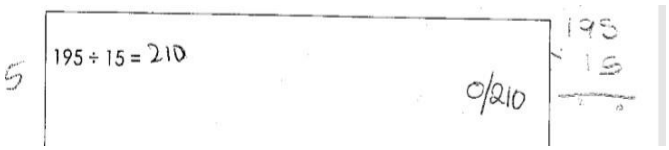
3.5.2 Conceptualization of the 'Comprehension' category

According to Koedinger and Nathan (2004), comprehension refers to the cyclic processing of the problem, be it verbal or symbolic, by the problem solver when solving the problem to get the answer. The comprehension of the word problem involved, representing the word problem with pictures, diagrams or symbolic form (Koedinger &Nathan, 2004). On the other hand, the comprehension of the pure numerical problem was signified by solution workings that indicated any relevant strategy and procedure for the mathematical operation concerned in setting up of the initial model of the problem depending on the problem format and assessed by means of following learners' solutions.

Koedinger and Nathan (2004) highlighted that most researchers considered comprehension as present in solving word problems only and not in pure numerical problems but this is not the case as comprehension is indispensable when solving any mathematical problem regardless of its format of presentation. In this study I considered comprehension as present in both problem formats. However, the nature of comprehension required in word problems cannot be equated to that required for pure numerical problems. That being the case, using the framework that was developed by Koedinger and Nathan (2004), comparison of comprehension was done on problems of the same format not on problems of different formats. It was important to analyse comprehension because an incorrect comprehension would result to incorrect answer or learners not attempting to respond leaving the question blank. Incidences of blanks per problem format per operation were also analysed for each test. The analysis of comprehension and blanks informed how the responses that were needed to be used as data for solution strategies' analysis were obtained. Table 4 presents the summary of the two types of comprehension, and code identification descriptors (details of the development of the coding framework for comprehension are in Appendix 2).

Table 4: *Comprehension codes and identification descriptors*.

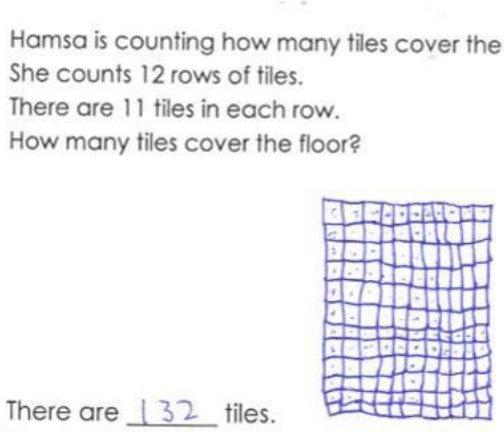
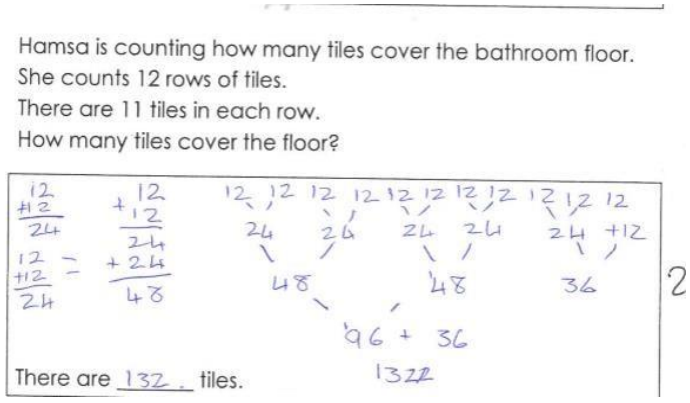
Comprehension Types	Code	Identification descriptors
Correct comprehension	3	**Word problems:** Modelling of word problems into correct pictorial, symbolic or any other semiotic representation. This is irrespective of whether the answer is correct or not. Example of code 3  From the example, there is correct comprehension since the solution workings showed the correct operation of addition and adding of the numbers involved in the question. **Pure numerical problems:** For pure numerical problems, correct comprehension was signified by solution workings that indicated any strategy and procedure for the mathematical operation concerned.  In question 16 above the learner showed the column algorithm of addition which corresponds to the question to be solved. For both problem formats any response with correct answer but without any visible solution workings was also considered as having a correct comprehension and was also given code 3.

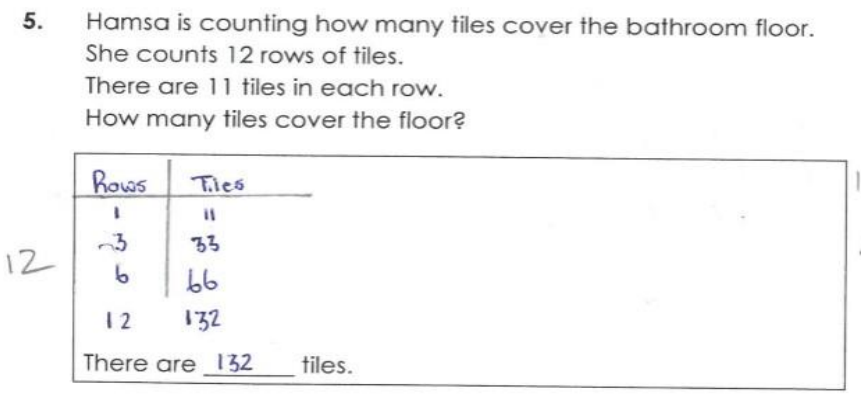
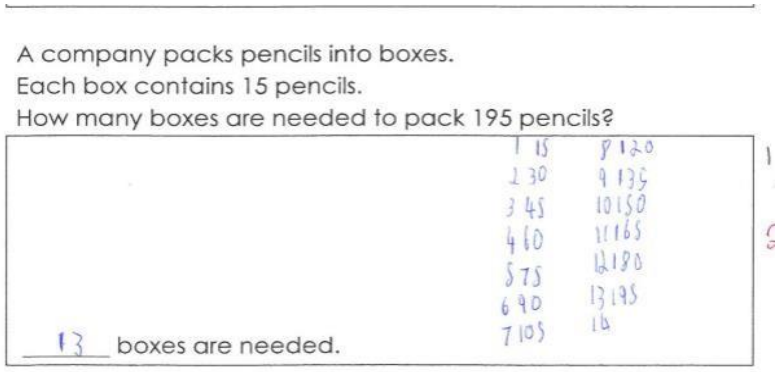
Incorrect comprehension	4	**Word problems:** Incorrect modelling of word problems into pictorial, symbolic or any other semiotic representations. This could also take the form of incorrect choice of operations Example of code 4  5. Hamsa is counting how many tiles cover the bathroom floor. She counts 12 rows of tiles. There are 11 tiles in each row. How many tiles cover the floor? 12 11 23 There are 23 tiles. In question 5 above the learner added the numbers involved instead of multiplying. **Pure numerical problems:** In pure numerical problem format solution workings indicating any strategy and procedure apart from the one of the operation illustrated in the question was coded 4.  5 195 ÷ 15 = 210 0/210 195 15 In the above case the learner wrote the solution strategy of column algorithm for addition and added the two numbers involved instead of division algorithm and dividing.

3.5.3 Conceptualization of the 'Solution strategy' category

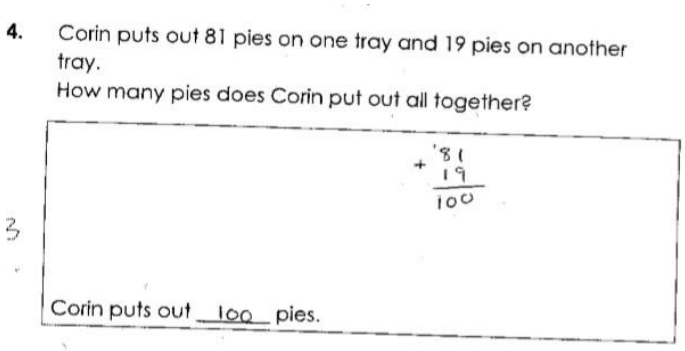
As presented in Table 5, there are six different solution strategies that were used in coding the data. The solution strategy was coded based on how the answers were obtained. Table 5 provides different types of solution strategies, and code identification descriptors. (See Appendix 3 for complete development of coding framework for solution strategies)

Table 5: Solution strategies' codes and identification descriptors

Solution strategies	Codes	Identification Descriptors
Drawing objects, tallies or counters	10	Denotes the strategy of working out the answer by drawing solid objects, tallies or counters. Example of drawing solid objects: 
Repeated addition or repeated subtraction	11	Represents the repetition of adding and subtracting the same number till the final answer is obtained. Repeated addition In the case of multiplication, the multiplicand was added the number of times that was equivalent to the multiplier's value.  In question 5 above, 12 was repeatedly added 11 times in both the word and pure numerical problems.

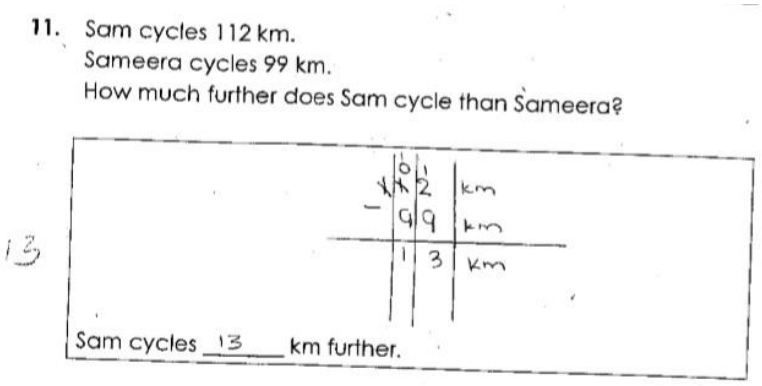
T-table or double number line	12	Signifies a solution strategy that used a T-table or double number line as solution workings. This solution strategy is comprised of establishing a relationship of numbers in the two columns of the T-Table or rows of the double number line by establishing the pattern of getting the multiples of the multiplier or divisor. T-table:  5. Hamsa is counting how many tiles cover the bathroom floor. She counts 12 rows of tiles. There are 11 tiles in each row. How many tiles cover the floor? 12 Rows | Tiles 1 | 11 3 | 33 6 | 66 12 | 132 There are 132 tiles. Double number line:  12. A company packs pencils into boxes. Each box contains 15 pencils. How many boxes are needed to pack 195 pencils? 1 15 | 8 120 2 30 | 9 135 3 45 | 10 150 4 60 | 11 165 5 75 | 12 180 6 90 | 13 195 7 105 | 14 13 boxes are needed.
Standard algorithm	13	Denotes solution strategies that showed the standard algorithms for all the operations. A standard algorithm is a step by step way to solve a problem. This standard algorithm strategy includes the column algorithms for addition, subtraction and multiplication in which the numbers had to be aligned vertically so that the digits in the same position represented the same place value. The place values are read from right to left as units, tens and hundreds. Sub-additions/subtractions/multiplications are carried out from the right to the left starting with the units followed by

the tens then hundreds. In the case where the sub-additions were more than 10 or sub-subtractions where a bigger number is subtracted from a smaller number, regrouping was expected to have taken place. Addition



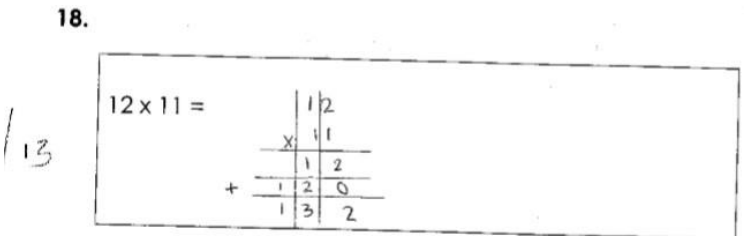
Question 4 above showed the column algorithm for addition

Subtraction



Question 11 above was an example of column algorithm for subtraction

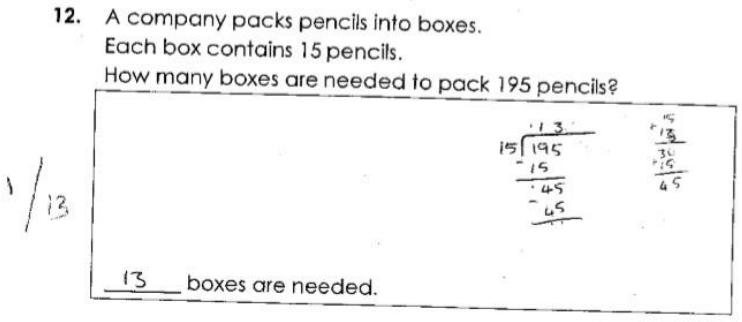
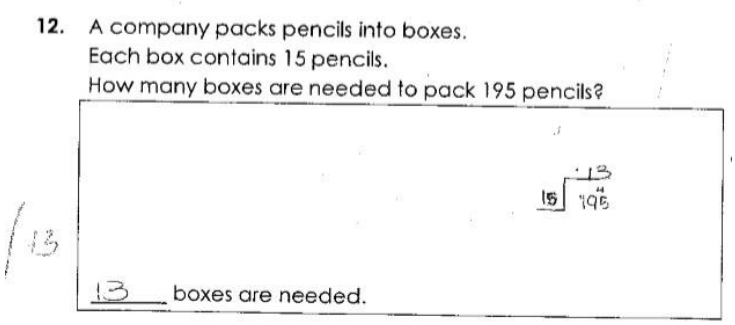
Multiplication

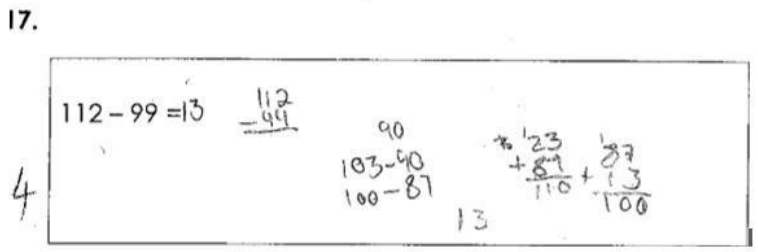


In the response for question 18, the column algorithm of multiplication was illustrated

Division

For division, the standard algorithm strategy includes either the long division or short division algorithm in which the numbers had to be aligned horizontally with a vertical line in between the divisor and the

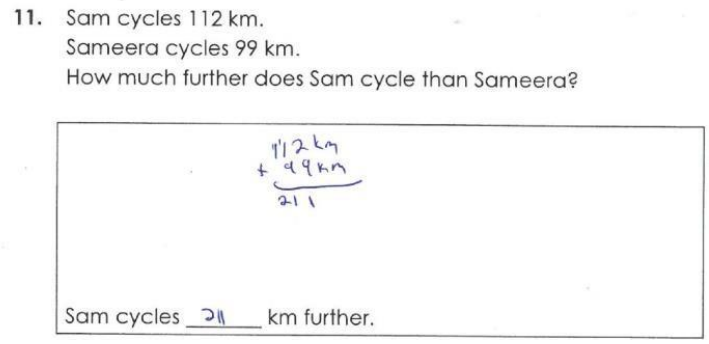
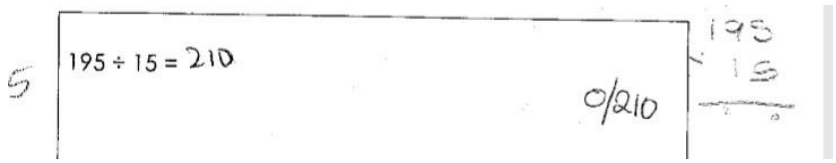
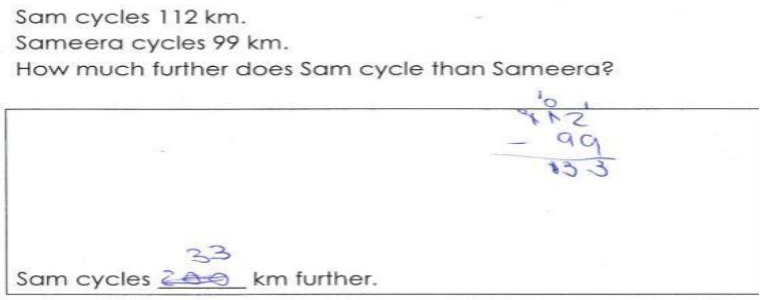
		dividend. A horizontal line was underneath the divisor and another horizontal line above the dividend or the dividend could be written above the divisor as in the long division. The working was done from the left to the right. Sub-divisions were done by dividing the divisor into the two highest values of the dividend since it was a division of a three digit number by a two digit number. Long division:  12. A company packs pencils into boxes. Each box contains 15 pencils. How many boxes are needed to pack 195 pencils? 13 boxes are needed. The long division algorithm was shown in the response for question 12 above. Short division:  12. A company packs pencils into boxes. Each box contains 15 pencils. How many boxes are needed to pack 195 pencils? 13 boxes are needed. Another response for question 12 showing the short division algorithm is shown above.
Answer only with no visible working	14	Represents a strategy of presenting an answer only without any visible working.

Others	15	All other solution strategies that were not able to be coded as code 10, 11, 12, 13 or 14 but which could be followed to get to know how the answer was obtained were assigned code 15.  The solution above can be followed to see all the steps involved in the solution strategy as to how the answer was arrived at but the solution strategy cannot be codded as 10, 11, 12, 13 or 14.

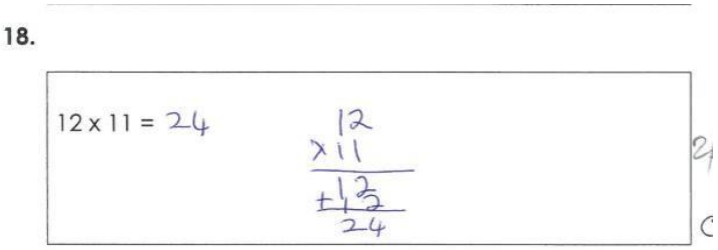
3.5.4. Conceptualization of the 'Errors' category

Four types of errors were detected in the learners' responses as shown in Table 6. The coding of errors from the data was done as shown by the examples for each error type. In the event that a learner committed more than one error, the first error to be detected was the one which was considered for that response. Subsequent errors committed were not taken into consideration. The four different types of errors, and code identification descriptors are shown in Table 6 (See Appendix 4 for complete development of coding framework for errors)

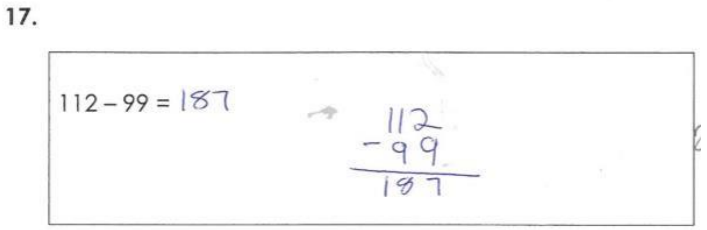
Table 6: Errors' codes and identification descriptors

Errors	Codes	Identification Descriptors
Operation choice error	21	Manifested when learners carried out an operation different from the one required by the context. Word problem:  In question 11 above learner presented the column algorithm for addition and chose to add the two numbers in place of subtracting. Pure numerical problem:  The response shows the column algorithm for addition and the learner added the two numbers instead of dividing them.
Computational error	22	These involve wrong additions/subtractions/multiplications/divisions of numbers in the question.  In question 11, the learner did not compute the sub-subtraction of 10-9 correctly resulting in the computational error.
Procedural error	23	Procedural errors were identifiable through wrong steps of executing a procedure of a defined operation or incomplete workings with respect to a

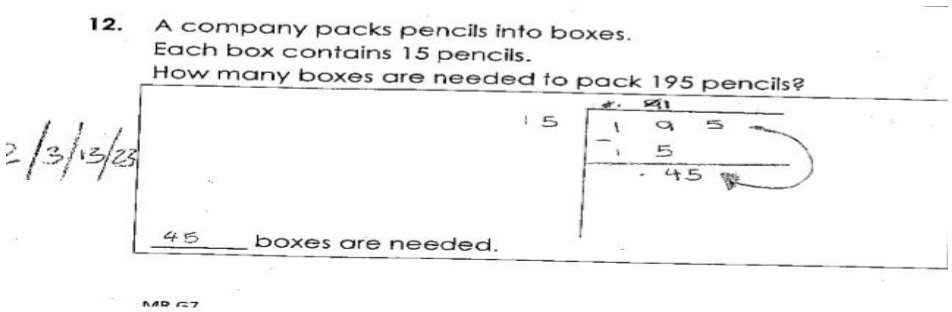
specific strategy. Wrong steps included failing to regroup when there is a need to, or writing answers of the sub-additions/subtractions/ multiplications/divisions on the wrong place or operating from the left to the right or vice versa. Examples of these are presented below.



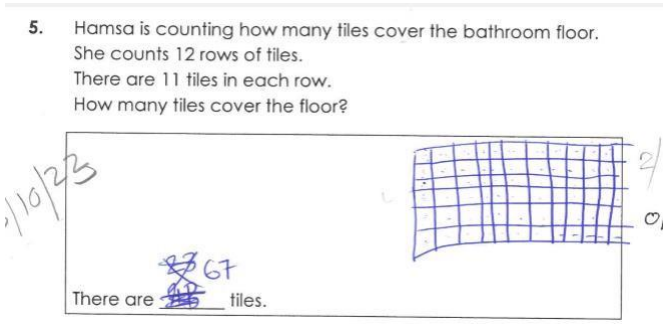
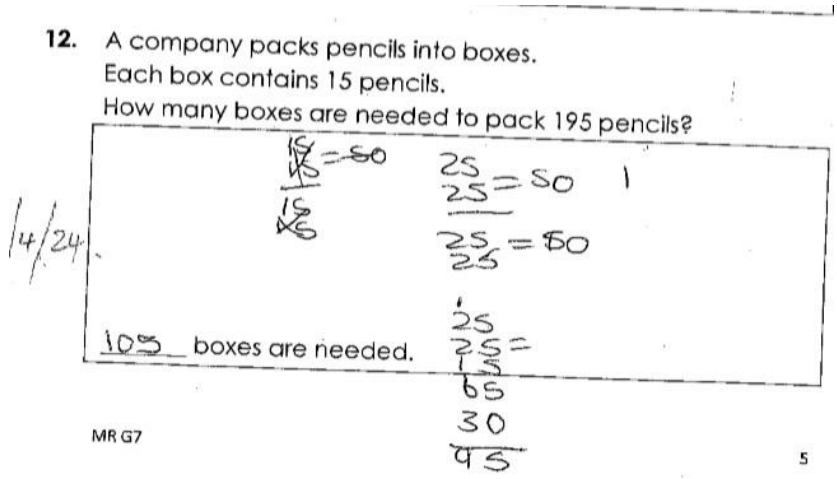
In the response to question 18 above, the solution workings showed that the learner did not write the answers to sub-multiplications at the correct places according to place values resulting in the wrong procedure of multiplying.



In the response to question 17 above, the solution workings showed that the learner did not do the step of regrouping when subtracting a bigger number from a smaller number. The learner just subtracted the smaller number from a bigger number without considering which one is the subtrahend or the minuend.

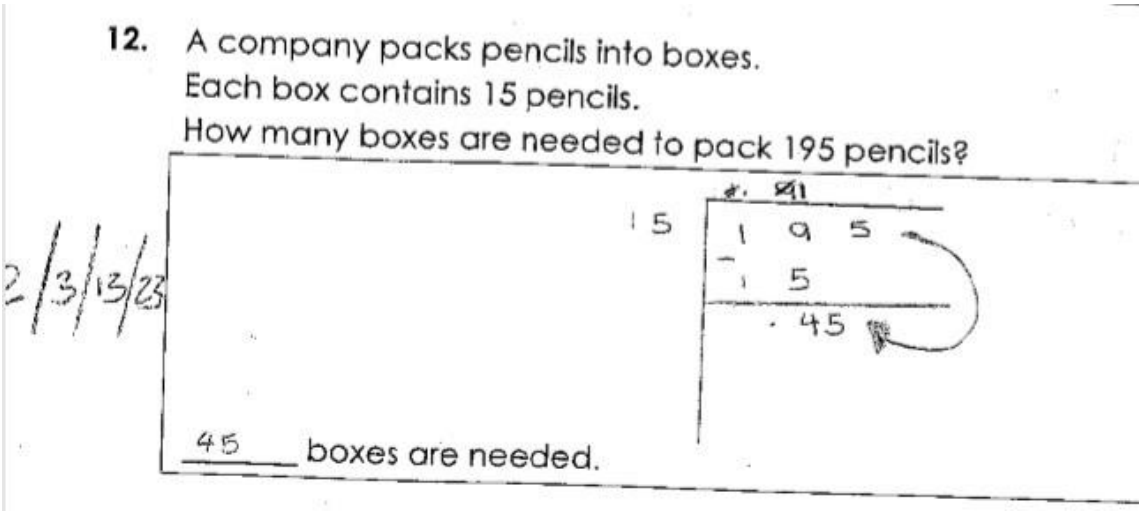


The solution workings to the response of question 12 above showed incomplete steps of long division, instead of continuing dividing until the dividend of the sub-divisions is zero, the learner opted to write the sub-dividend as the answer.

		 In question 5 above, the response presented incomplete drawings that resulted in a wrong answer.
Others	24	All other errors that were presented on solution workings that resulted in an incorrect answer, but could not be coded as 21, 22 or 23. Example of a responses that was coded 24 is shown below.  In the response above, numbers that are not involved in the question were used in the solution strategy.

Exemplification of the coding of scripts:

Each learner script was coded using the coding framework explained above for the 8 selected questions. The coding was in the order: answer/comprehension/solution strategy/error. This is illustrated in the following learner script.



The codes for the above response was: 2 / 3 /13 / 23 meaning the answer is wrong with correct comprehension where the solution strategy used was a standard algorithm of division with a procedural error.

After the coding process, the information of the 8 questions was captured on two excel spread sheets for processing; one for pre-test and another for post-test. The data was captured with reference to the question, with each question having 4 aspects to be captured on it, which was the answer, comprehension, solution strategy and errors. All the codes were numerical to allow implementation of a quantitative analysis.

3.6 Ethical issues

The participants to this research were the grade 7 learners who participated in the Wits Maths Connect Primary project- Multiplicative Reasoning (M-RIP) project intermediate phase (grades 4-7). These grade 7 learners had previously given assent (as had their parents given consent) under the auspices of the larger M-RIP project (protocol Number H17/07/30) for their anonymized test scripts to be used for research. No new participants were involved in this research. The research reported on here did a quantitative content analysis of learners' written responses of pre and post assessment tests of M-RIP. For this research clearance to use the scripts was sought and granted by Wits University under Protocol Number 2018ECE013M.

3.7 Reliability

Scott and Morrison (2006) stated that reliability has to do with the quality and trustworthiness of the data and findings of the study. Mayring (2014) highlighted that for content analysis, coding done by more than one researcher can increase the reliability which is of particular significance. To ensure reliability of the methods and procedures used in the research, a pilot

study was done and all the codes were discussed with the research supervisors until a consensus was reached. According to Joppe (2000) reliability can be considered as the extent to which the results of a study can be reproduced using the same methodology. A peer was invited to recode some already coded scripts using the coding framework to check whether the solution workings were given the same codes with the researcher to ensure the reliability of the coding process of the scripts (Rose, Spinks & Canhoto, 2015).

The reliability of the coding process is a common methodological requirement that is arrived at with the duplication of the coding process (Krippendorff, 2011). In the case of this research, reliability was the extent to which the invited peer, managed to recode the already coded scripts in the same way as the researcher using the designed coding framework.

Randomly 32 scripts which constitute 11,18 percent of the whole data set were selected and recoded, 16 scripts for pre-test and 16 scripts for post-test. The following results were obtained.

A total of 32 variables were analysed of which 23 resulted in 100 percent agreement (Scott's PI =1, Cohen's Kappa =1, and Krippendorff's Alpha =1) implying that in 72% of the cases there was 100% agreement of results by the recoding exercise as were obtained in the actual research coding practice. However, there were 6 variables where the results of the recoding differed slightly with the initial coding. For each of these 6 scripts, there was 1 instance where the codes could not concur. For theses 6 scripts, the percent agreement was 96, 87 percent (Scott's PI =0.932131, Cohen's Kappa =0.932203, and Krippendorff's Alpha =0.933192). For the remaining 3 variables, there was a 93, 75 percent agreement (Scott's PI =0.775044, Cohen's Kappa =0.775439, and Krippendorff's Alpha =0.778559) caused by the fact that for each of these 3 scripts there were 2 codes on each script where coding disagreed. (See Appendix 5 for the calculations).A discussion was held concerning the different coding and the scripts were later coded according to the outcome of the discussion. Largely, basing on the three different reliability coefficients shown above (all of which are high), there was a considerably high percentage agreement between my coding and that of my peer implying that the results of the coding process are reliable.

3.8 Validity

Scott and Morrison (2006) highlight that validity has to do with the accuracy of descriptions to be made and the applicability of the study to other situations. Considering Henning, Rensburg and Smit (2004) data itself when it is available is neither valid nor invalid. What matters most is the conclusion that is established from it that determines its validity. With respect to validity,

scripts used in the analysis are available for confirmation and validation of all the answers, comprehension, solution strategies and errors identified. A coding framework was developed to ensure systematic coding of the data, codes in the coding framework were also explicitly specified such that the coding can be reproduced (Rose, Spinks & Canhoto, 2015). Henning et al. (2004) postulated that a researcher should guard against bias and be extra cautious with both procedures and decisions in a manner that is critical in order to achieve validity. In order for me to stay focused and consistent with the coding framework, I kept on consulting my supervisors with regard to some learners' responses that were not clear with respect to the coding framework and sought clarifications on certain assumptions.

CHAPTER 4: DATA ANALYSIS, FINDINGS AND DISCUSSIONS

4.1 Introduction

This study set out to explore learners' performance in multiplicative and additive reasoning word and pure numerical problems of grade 7 learners from written tests (pre- and post-test). This chapter therefore presents the data analysis, findings and discussions. The data is mainly of a quantitative nature and was analysed using statistical methods, and presented in the form of tables and graphs. Table 7 represents the research questions and how each was addressed.

Table 7: *Research questions and how each was answered*

Research question	How the research question was answered
What are the differences in grade 7 learners' performance in multiplicative and additive reasoning word and pure numerical problems in the pre- and post- tests?	The answers were classified and quantified as blank spaces, correct answers and incorrect answers. The total frequencies were illustrated in a table per test item, problem format and per test. In some cases graphs were used to display the proportions of correct answers. T-tests were used to compare differences in performance with reference to problem format and mathematical reasoning.
What solution strategies do grade 7 learners employ when solving multiplicative and additive reasoning word and pure numerical problems in the pre- and post- tests?	The solution strategies were classified and quantified as: 1. Working out drawing solid objects tallies or counters. 2. Repeated addition or repeated subtraction. 3. T- Table or double number line. 4. Standard algorithm. 5. Answer only with no visible working. 6. Other. Tables were used to present total frequencies per test item, mathematical reasoning, and problem format and per test. Graphs were used to show the proportions of solution strategy use.

What errors are detectable in the grade 7 learners' written responses for multiplicative and additive reasoning word and pure numerical problems in the pre- and post- test?	The incorrect answers were further analysed with respect to errors that made the response to be incorrect. The errors were classified and quantified with respect to the errors: 1. Operation choice error. 2. Computational error 3. Procedural error 4. Other. Total frequencies per test item, mathematical reasoning, problem format and test were shown in tables. Graphs were used to display the proportions of error detection.

As mentioned in the previous chapter, there were 8 test items that were considered in each test that were responded to by 143 grade 7 learners resulting in a total of 1144 responses in each test. All the learners' responses were analysed and coded using the coding framework detailed in Chapter 3. The 1144 responses per test consisted of 572 responses to multiplicative and additive word problems and 572 responses to multiplicative and additive pure numerical problems. Since there were two tests considered for the research, this resulted in 2288 responses that were coded and analysed in total. For the purpose of this study when referring to the pre-test and post-test, it is only the 8 test items (parallel word and pure numerical problems that demand multiplicative and additive reasoning) that will be referred to not the whole test.

The analysis and coding were done per test, and within each test, a comparison was made between mathematical reasoning (i.e. additive and multiplicative reasoning), problem format (word or pure numeric problems) and the correctness of learners' responses coded as correct or incorrect. In addition, the study established the solution strategies employed by the learners and the detectable errors that they committed as they responded to the mathematical problems presented in the tests. The presentation of the analysis is organized according to the three research questions relating to: Learner performance; solution strategies; and detectable errors.

4.2 Learners' performance

To be able to answer the first research question, an in-depth analysis of learner performance in both the pre- and post-tests was done. A comparison was made in their performances across the two problem formats, and the two mathematical reasoning as evident in the test. T-test analysis was carried out to test the level of significance across the variables in the test, and between the pre- and post-test.

4.2.1 Learners' performance in Pre-test

The analysis on performance focuses on coding of correct and incorrect responses, and is presented across the problem format and mathematical reasoning. Table 8 presents the summary of proportions (in parenthesis) and percentages of correct answers based on mathematical reasoning and problem format, and detail of the analysis by each test item in the pre-test is presented in Appendix 6.

Table 8: *Correct answers based on mathematical reasoning and problem format in pre-test.*

Problem format	**Additive reasoning**	**Multiplicative reasoning**	**Total**
Word problems	(195/286) 68%	(67/286) 23%	(262/572) 46%
Pure numerical problems	(189/286) 66%	(84/286) 29%	(273/572) 48%
Total	(384/572) 67%	(151/572) 26%	(535/1144) 47%

Table 8 shows that the overall performance of learners in the pre-test was 47% on the 8 items considered. On the aspect of problem format, the learners' performance in word and pure numerical problems was 46% and 48% respectively. Learners' performance in the 4 test items that respectively demand additive and multiplicative reasoning was 67% and 26% respectively. Finding indicates that the overall learners' performance was slightly below 50% and there was similar pattern of learners' performance across the problem format. However there was wide difference (41%) with higher learners' performance in additive reasoning problems in comparison with multiplicative reasoning problems. Overall low learners' performance in the test was attributed to poor learners' performance in multiplicative reasoning items in the test.

To test the significance of the difference between the learners' performance in word and pure numeric problems, t-test analysis was computed at 95% confidence interval. The mean

performance of the learners in the word problems was 1.83 with standard deviation of 1.14 and the mean performance of the learners in the pure numerical problems was 1.91 with standard deviation of 1.23. Using Social Sciences Statistics computations software, the means difference of the results obtained in the learners' performances in two problem formats were computed and the results were, the *t*-value was -1.23508 and the *p*-value was .108911. The result was not significant at $p < .05$. It was concluded that there was no significant difference between the mean performance of the learners in word and pure numerical problems in the pre-test. Learners' performance in word and pure numerical problems was similar in this test. An extract of the detail statistical computations is presented in Appendix 7.

The significance of the numerical difference amid the learners' performance in problems that demand additive and multiplicative reasoning was determined using a t-test analysis at 95% confidence level. The mean performance of the learners in the additive reasoning problems was 2.69 with standard deviation of 1.20 while the mean performance of the learners in the multiplicative reasoning problems was 1.06 with standard deviation of 1.31. The significance of the mean difference of the learners' performance in problems that demand the different mathematical reasoning were computed. The *t*-value was 10.91628 and the *p*-value was < .00001. The result was significant at $p < .05$. It was established that the difference between the mean performance in additive and multiplicative reasoning problems was significant. An extract of the detail statistical computations is shown in Appendix 8. During the pre-test, learners performed better in problems that demand additive reasoning than in those that demand multiplicative reasoning.

4.2.2 Learners' performance in Post-test

The analysis on learners' performance focuses on coding of correct and incorrect responses, and is presented across the problem format and mathematical reasoning. Table 9 presents the summary of proportions (in parenthesis) and percentages of correct answers in post-test based on mathematical reasoning and problem format, and detail of the analysis by each item in the test is presented in Appendix 9.

Table 9: Correct answers based on mathematical reasoning and problem format in post-test.

Problem format	Additive reasoning	Multiplicative reasoning	Total
Word problems	(209/286) 73%	(113/286) 40%	(322572) 56%
Pure numerical problems	(215/286) 75%	(133/286) 47%	(348/286) 61%
Total	(429/572) 74%	(246/572) 43%	(670/1144) 59%

The overall performance of the learners in the 8 items in post-test was 59%. The learners' performance in problems that demand additive and multiplicative reasoning was 74% and 43% respectively while it was 56% and 61% respectively for word and pure numerical problems. Findings therefore indicate learners' performance that is above 50% in the post-test.

On comparison between problem formats, there was higher learners' performance (61%) in pure numeric than in word problems (56%) with difference of 5%. Albeit the numerical difference, there was need to check the statistical significance of the mean difference before a conclusion could be reached. A t-test analysis was done. A mean of 2.25 with standard deviation of 1.24 of the learners' performance in word problems was compared against a mean of 2.43 with standard deviation 1.25 of learners' performance in pure numerical problems. The t-value was -1.23508 and the p-value was .108911. The result was not significant at $p < .05$. It was concluded that the difference between the means was not statistically significant. This is an indication that learners' performance in the word and pure numerical problems were similar. An extract of the Social Sciences Statistical computations in this regard is presented in Appendix 10.

While there was noticeable improvement in the post-test, there exist wide differences (31%) between learners' performance in additive reasoning and multiplicative reasoning. A t-test analysis was done to check the statistical significance of the difference of the means of learners' performance in multiplicative and additive reasoning problems. Additive reasoning problems had a mean of learners' performance of 2.97 with a standard deviation of 1.20 was compared to multiplicative reasoning problems that had a mean of learners' performance of 1.72 with standard deviation of 1.43. The *t*-value was 8.0656 and the *p*-value was < .00001. The result was significant at $p < .05$. The t-test revealed that there was a statistically significant difference between the mean of learners' performance in additive and multiplicative reasoning problems

in post-test. Learners' performance in additive reasoning problems was better than that in multiplicative reasoning problems. An extract of the Social Sciences Statistical computations in this regard is presented in Appendix 11.

4.2.3 Comparison of learners' performance in the pre- and post-test.

A comparison was made between the pre- and post-tests across additive and multiplicative reasoning, and word and pure numeric problem formats. This comparison was not to comment on the effectiveness of the intervention done by the WMC-P MR project as it was not the aim of this research. However, since the pre- and post-tests were considered as variables in this study, it was vital for the comprehensive analysis of the tests to carry out this comparison and get insights of any shifts the intervention appeared to have influenced on learners' performance.

Table 10 represents the summary of proportions and percentages of learners' performance in the pre- and post-tests on additive and multiplicative reasoning.

Table 10: *Learners' performance in the pre- and post-tests based on additive and multiplicative reasoning.*

	Additive reasoning	**Multiplicative reasoning**	**Total**
Pre-test	(384/572) 67%	(151/572) 26%	(535/1144) 47%
Post-test	(424/572) 74%	(246/572) 43%	(670/1144) 59%
Differences	(40/572) 7%	(95/572) 17%	(135/1144) 12%

The overall learners' performance indicated a difference of 12%, with 7% difference in additive reasoning problems and 17% difference in multiplicative reasoning problems. Higher difference is recorded in multiplicative reasoning, meaning a greater improvement in learners' performance in multiplicative reasoning problems than in problems that demand additive reasoning. A t-test was done to check the statistical significance of the difference in the overall learners' performance. The mean of the learners' performance in pre-test was 3.74 with standard deviation 2.18 and for the post-test was 4.69 with standard deviation of 2.29. The t value was -3.57216 and the p-value is .000208. The result was significant at $p < .05$. It was concluded that the difference between the means was statistically significant. Learners performed significantly better in the post-test than in the pre-test. An extract of the Social Sciences Statistics computations is presented in Appendix 12. The improved learners'

performance in the post-test was credited to the improved learners' performance in problems that demand multiplicative reasoning.

The data revealed that generally learners performed better in problems that demand additive reasoning than multiplicative reasoning. Furthermore, learners performed significantly better in the post-test than in the pre-test with greatest improvements in problems that demand multiplicative reasoning. It was also revealed that there was no significant difference between learners' performance in word and pure numerical problems.

4.3 Learners' Solution Strategies

The learners' responses were coded for the solution strategies that the learners used as they solved word and pure numerical problems that demand multiplicative and additive reasoning in the pre- and post-tests. A quantitative analysis of the number of times a specific solution strategy was established per each test based on problem formats, mathematical reasoning and strategy effectiveness. Analysis of strategies with respect to problem formats was done to get an insight of what strategy is aligned most with word problems or pure numerical problems. The analysis with respect to mathematical reasoning was done so as to get a better understanding of strategies that learners use when they encounter mathematical problems that demand multiplicative or additive reasoning. Strategy effectiveness was established by means of analysing the percentage of correct answers associated with each strategy. Finally, a comparison of strategies that were used in the pre- and post-test was done to check if there was any shift in the use of the strategies.

As detailed in Chapter 3, the following solution strategies were coded in this analysis on problems that involve additive and multiplicative reasoning in word problems and pure numerical formats as described in Table 11.

Table 11: *Summary of codes of solution strategies and their descriptions*

Code	**Description**
10	Drawings of solid objects tallies or counters
11	Repeated addition or repeated subtraction.
12	T- Table or double number line.
13	Standard algorithm
14	Answer only with no visible working
15	Other.

4.3.1 Pre-test solution strategies

Out of 1144 responses in the pre-test 170 responses were discarded. Responses were discarded because they were either blank spaces or they displayed incorrect comprehension. The frequencies of the correct and incorrect comprehension for each test item in the pre-test are detailed in Appendix 13. Hence, 974 responses were analysed in terms of the strategies used by learners. The frequencies of the solution strategies used per test item, problem format and mathematical reasoning in the pre-test are detailed in Appendix 14 for each test item. Table 12 displays how the proportions of the responses targeted for solution strategy analysis was established based on problem format in pre-test.

Table 12: *Responses targeted for solution strategy analysis based on problem format in pre-test.*

Problem format	**Total**	**Blanks**	**Incorrect comprehension**	**Responses targeted for solution strategy analysis.**
Word problems	(572/1144) 50%	(17/572) 3%	(86/572) 15%	(469/572) 82%
Pure numerical problems	(572/1144) 50%	(44/572) 8%	(23/572) 4%	(505/572) 88%
Total	(1144) 100%	(61/1144) 5%	(109/1144) 10%	(974/1144) 85%

The 170 responses that were discarded were distributed as follows, 61 were no responses (Blanks) and 109 showed incorrect comprehension. This leaves 85% of the responses worth analysing for solution strategies. The usage of the identified solution strategies in word problems was 82% and 88% in pure numerical problems. There was slightly more appearance of the solution strategies in the pure numerical problems than in word problems.

Table13 displays the summary of proportions and percentages of the grade 7 learners' solution strategies usage based on problem format in the pre-test.

Table 13: Grade 7 learners' solution strategies based on problem format in the pre-test.

Solution strategies	Word problems	Pure numerical problems	Grand total
Drawings of solid objects, counters and tallies	(23/469) 5%	(8/505) 2%	(31/974) 3%
Repeated addition/ subtraction	(31/469) 7%	(10/505) 2%	(41/974) 4%
T-table/ Double number line	(0/469) 0%	(0/505) 0%	(0/974) 0%
Standard algorithm	(180/469) 38%	(151/505) 30%	(331/974) 34%
Answer only with no visible working	(229/469) 49%	(330/505) 65%	(559/974) 57%
Other	(6/469) 1%	(6/505) 1%	(12/974) 1%
Total	(469/974) 48%	(505/974) 52%	(974) 100%

Table 13 showed that the most used solution strategy during the pre-test was the 'writing of the answer only with no visible working' with 57% usage, followed by standard algorithm with 34% usage. The T-table/double number line was not used at all in the pre-test.

The performance differences were analysed across problem format, Figure 3 presents the solution strategies used by grade 7 learners based on problem format in pre-test

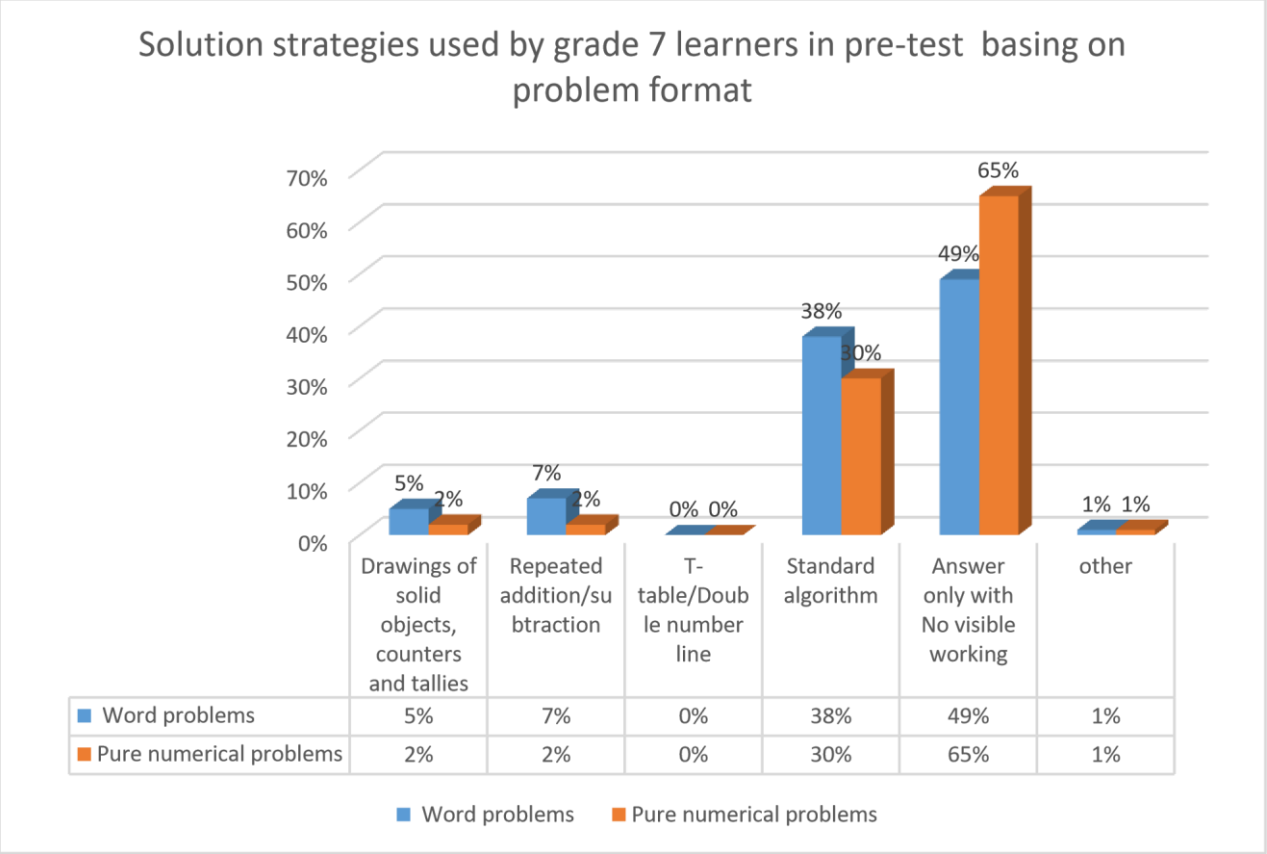


Figure 3: Solution strategies used by grade 7 learners in pre-test basing on problem format

Figure 3 displays that the most used solution strategy in both problem formats was writing the answer only with no visible working, with 65% and 49% usages in pure numerical and word problems respectively. The second most used solution strategy was the standard algorithm with 30% and 38% usages in pure numerical and word problems.

As the research sought to explore learners' performance basing on mathematical reasoning, Table 14 presents the solution strategies used by grade 7 learners based on mathematical reasoning in pre-test.

Table 14: Grade 7 learners' solution strategies based on mathematical reasoning in pre-test.

Solution strategies	Additive reasoning	Multiplicative reasoning
Drawings of solid objects, counters and tallies	(19/523) 4%	(12/451) 3%
Repeated addition/ subtraction	N/A	(41/451) 9%
T-table/ Double number line	N/A	(0/451) 0%
Standard algorithm	(215/523) 41%	(116/451) 26%
Answer only with no visible working	(283/523) 54%	(276/451) 61%
Other	(6/523) 1%	(6/451) 1%
Total	(523/974) 54%	(451/974) 46%

Figure 4 presents solution strategies used by grade 7 learners in word and pure numerical problems that demand additive reasoning

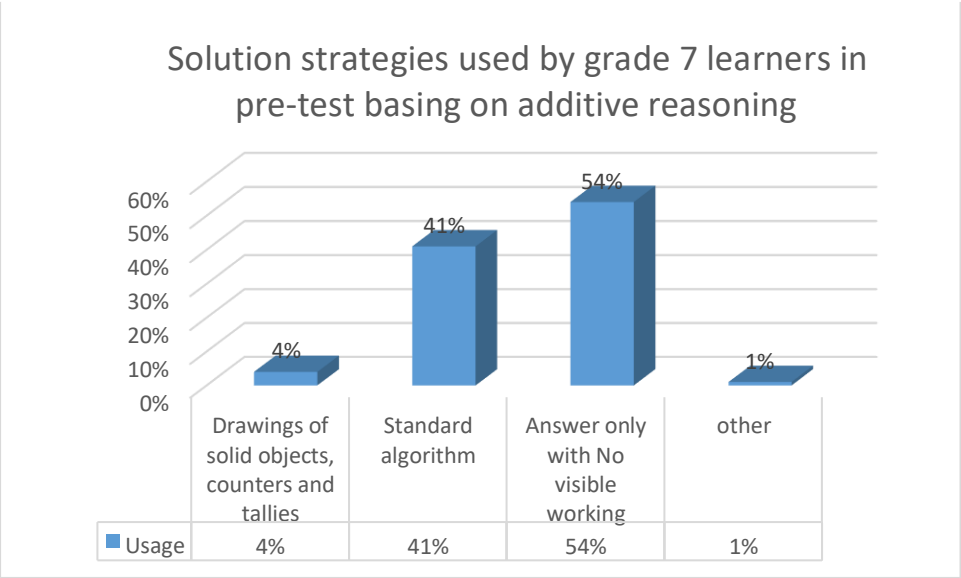


Figure 4: Solution strategies used by grade 7 learners in pre-test basing on additive reasoning.

The most used solution strategy by grade 7 learners on additive word and pure numerical problems was writing the answer only with no visible working with 54% followed by standard algorithm with 41%.

Figure 5 presents solution strategies used by grade 7 learners in word and pure numerical problems that demand multiplicative reasoning.

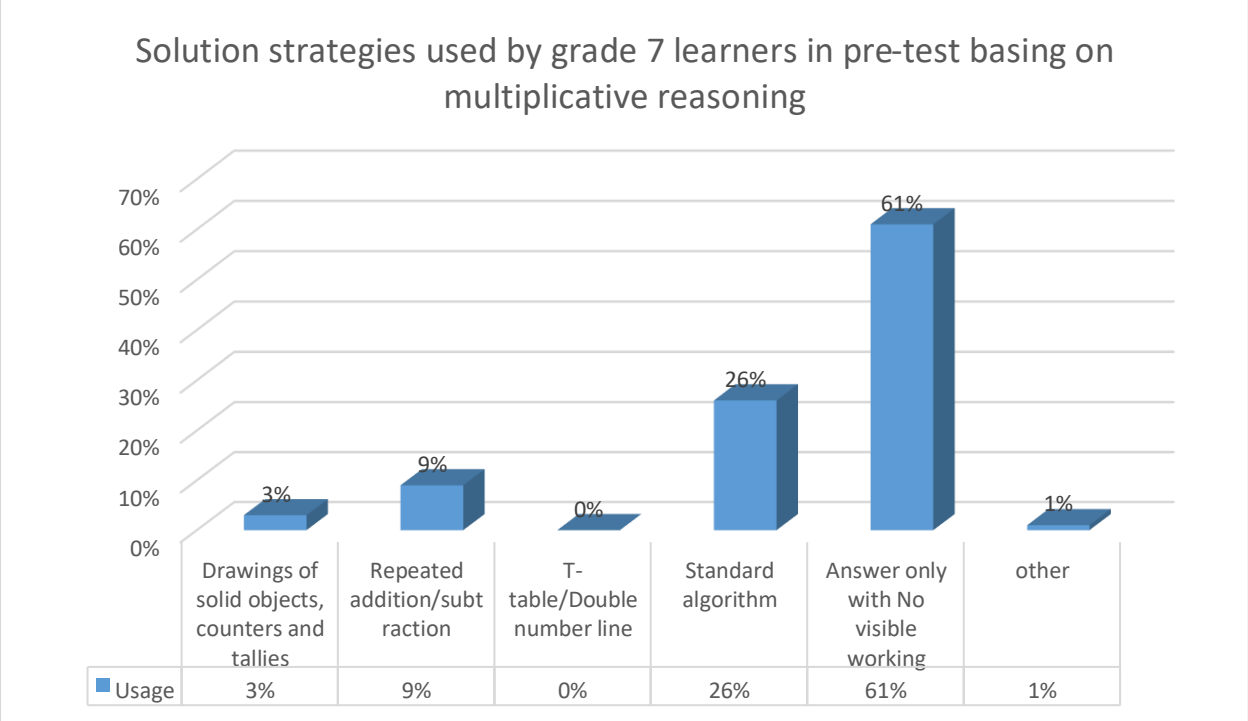


Figure 5: Solution strategies used by grade 7 learners in pre-test basing on multiplicative reasoning.

Writing the answer only with no visible working with 61% usage was the most used solution strategy in word and pure numerical problems that demand multiplicative reasoning followed by standard algorithm with 26% usage.

The trend of the solution strategies usage across the mathematical reasoning is the same as across problem format as well with "writing the answer only with no visible working" as the most used solution strategy followed by the standard algorithm.

A further analysis was done to compare how each solution strategy was associated with correct answers in the pre-test. Table 15 presents the summary of grade 7 learners' correct responses across solution strategies used in the pre-test with reference to the required mathematical reasoning.

Table 15: Grade 7 learners' correct responses across solution strategies used in the pre-test with reference to the required mathematical reasoning.

Solution strategies	Additive reasoning	Multiplicative reasoning	Total
Drawings of solid objects, counters and tallies	(13/19) 68%	(3/12) 25%	(16/31) 52%
Repeated addition/ subtraction	N/A	(15/41) 37%	(15/41) 37%
T-table/ Double number line	N/A	0 (0%)	0 (0%)
Standard algorithm	(170/215) 79%	(66/116) 57%	(236/331) 71%
Answer only with no visible working	(198/283) 70%	(65/276) 24%	(263/559) 47%
Other	(3/6) 50%	(2/6) 33%	(5/12) 42%
Total	(384/523) 73%	(151/451) 33%	(535/974) 55%

Table 15 revealed that of the 974 solution strategies employed by the learners in pre-test, 535 solution strategies resulted in correct responses with the use of standard algorithm generating higher number of correct responses 236/331 constituting 71%. This is followed by using drawings (52%), and answer only with no visible working (47%). Surprisingly, the least was repeated addition (37%). The learners' correct usage of solution strategies in problems that demand additive reasoning was well above 50% with 73% whereas for multiplicative was below 50% with 33% having a difference of 40%. This analysis revealed that for each solution strategy that was used in both additive and multiplicative reasoning problems, the percentage of correct answers achieved using the strategy in problems that require multiplicative reasoning was consistently lower than the one of problems that require additive reasoning. This trend might suggest that the use of a specific solution strategy does not necessarily contribute to performance instead it is the type of the mathematical reasoning involved.

4.3.2 Post-test solution strategies

Out of 1144 expected responses in the post-test there were 137 that were excluded in the solution strategies analysis. The excluded were based on either blank spaces or showed incorrect comprehension. The frequencies of correct and incorrect comprehension for each test item in the post test are illustrated in Appendix 15. The results of incidences of the solution strategies used per test item, problem format and mathematical reasoning in the post-test are shown in Appendix 16.

Table 16 shows the establishment of proportions of the responses targeted for solution strategy analysis based on problem format in post-test.

Table 16: *Responses targeted for solution strategy analysis based on problem format in post-test.*

Problem format	Total	Blanks	Incorrect comprehension	**Responses targeted for solution strategy analysis.**
Word problems	(572/1144) 50%	(7/572) 1%	(78/572) 14%	(487/572) 85%
Pure numerical problems	(572/1144) 50%	(39/572) 7%	(13/572) 2%	(520/572) 91%
Total	(1144) 100%	(46/1144) 4%	(91/1144) 8%	(1007/1144) 88%

Table 16 showed that there were total of 137 no response or incorrect comprehension that were discarded in this analysis. Hence, a total of 1007 responses with evidence of strategy used were analysed. This constituted a total of 88% of the total expected responses, which is good enough representation that one can make conclusion from the analysis. Out of the 1007 responses, 520 were for pure numeric problems, while 485 were for word problems.

Table 17 provides the summary of proportions and percentages of the grade 7 learners' solution strategies usage based on problem format in post-test.

Table 17: Grade 7 learners' solution strategies based on problem format in post-test.

Solution strategies	Word problems	Pure numerical problems	Grand total
Drawings of solid objects, counters and tallies	(7/487) 1%	(2/520) 0%	(9/1007) 1%
Repeated addition/ subtraction	(25/487) 5%	(7/520) 1%	(32/1007) 3%
T-table/ Double number line	(37/487) 8%	(2/520) 0%	(39/1007) 4%
Standard algorithm	(218/487) 45%	(138/520) 27%	(356/1007) 35%
Answer only with no visible working	(189/487) 39%	(356/520) 68%	(545/1007) 54%
Other	(11/487) 2%	(15/520) 3%	(26/1007) 3%
Total	(487/1007) 48%	(520/1007) 52%	(1007) 100%

The summary in Table 17 showed that the most used solution strategy during the post-test was the 'writing of the answer only with no visible working' with 54% usage, followed by standard algorithm with 35% usage. This is followed by T table/Double number line with 39 incidents of usage constituting 4%.

Figure 6 presents the solution strategies used by grade 7 learners across word and pure numerical problems in post-test.

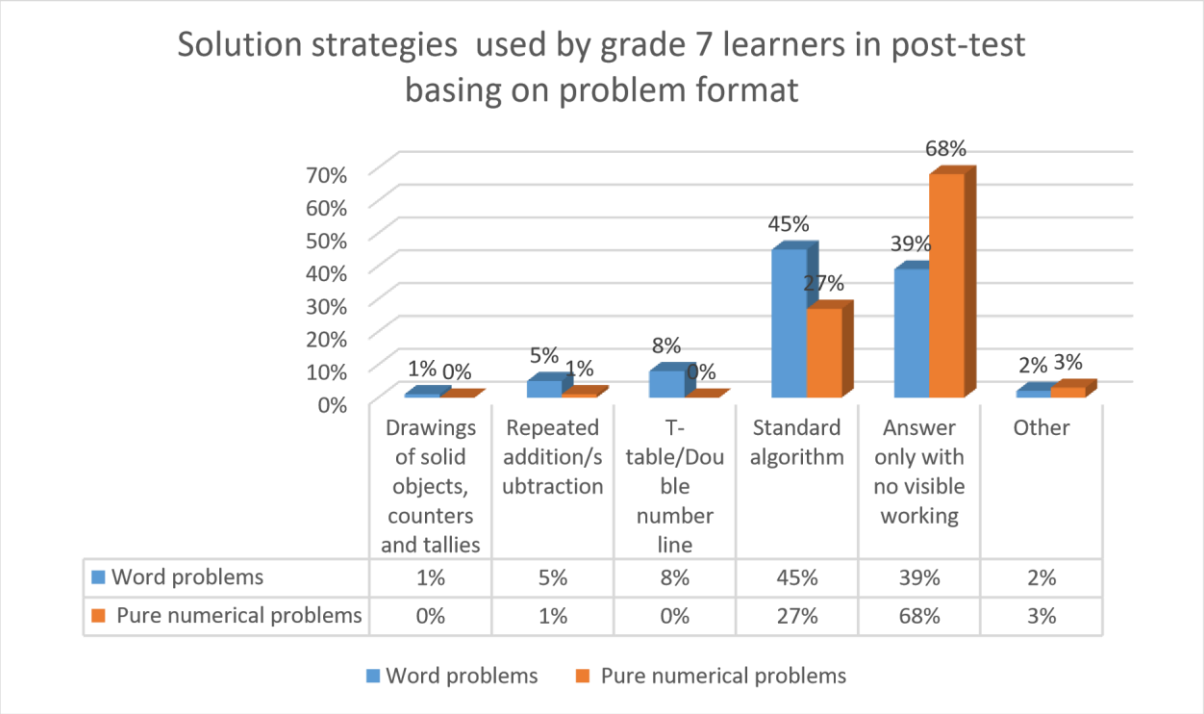


Figure 6: Solution strategies used by grade 7 learners basing on problem format in post-test.

The most used solution strategy across the problem format in the post-test is different for word and pure numerical problems. In pure numerical problems, answer only with no visible working was the most used solution strategy with 68% usage followed by standard algorithm with 27% usage. In word problems, the most used solution strategy was standard algorithm with 45% usage trailed by answer only with no visible working with 39% usage and third most used solution strategy was T-table /double number line with 8% usage.

Table 18 illustrates the solution strategies used by grade 7 learners across the mathematical reasoning.

Table 18: Grade 7 learners' solution strategies based on mathematical reasoning in post-test

Solution strategies	Additive reasoning	Multiplicative reasoning
Drawings of solid objects, counters and tallies	(3/522) 1%	(6/485) 1%
Repeated addition/ subtraction	N/A	(32/485) 7%
T-table/ Double number line	N/A	(39/485) 8%
Standard algorithm	(232/522) 44%	(124/485) 26%
Answer only with no visible working	(269/522) 52%	(276/485) 57%
Other	(18/522) 3%	(8/485) 2%
Total	(522/1007) 51%	(485/1007) 48%

Figure 7 presents the solution strategies used by grade 7 learners based on additive reasoning in post-test.

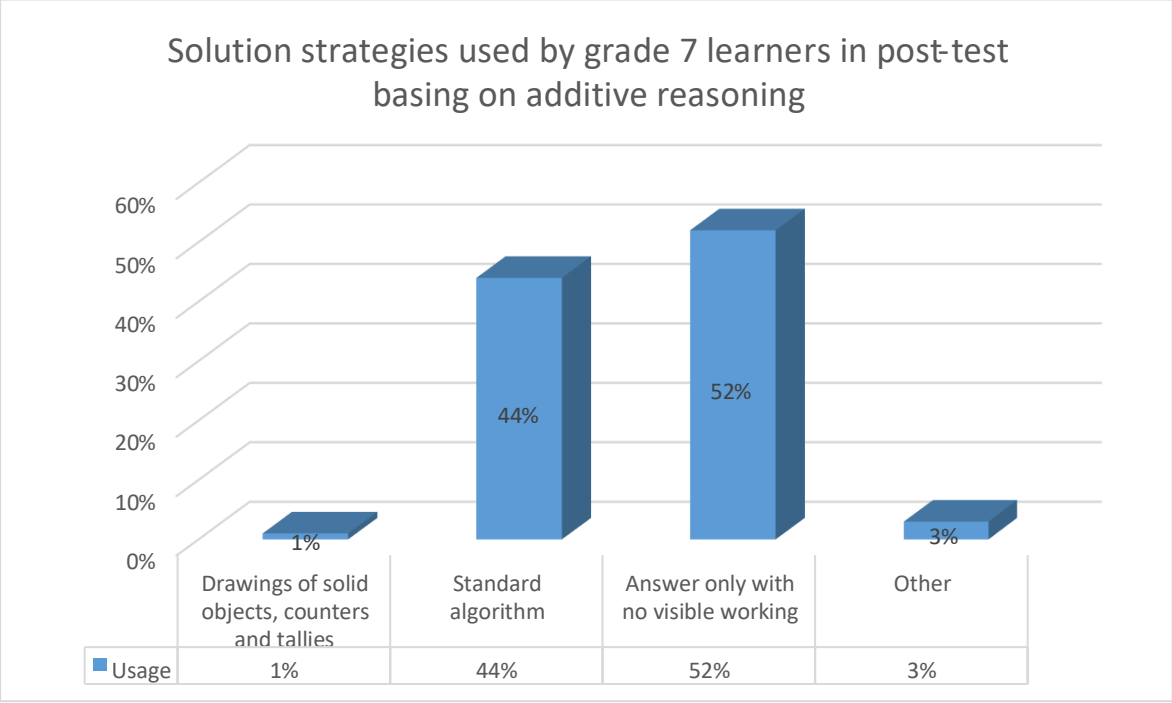


Figure 7: Solution strategies used by grade 7 learners basing on additive reasoning in post-test.

Figure 8 illustrates the solution strategies used by grade 7 learners based on multiplicative reasoning in post-test

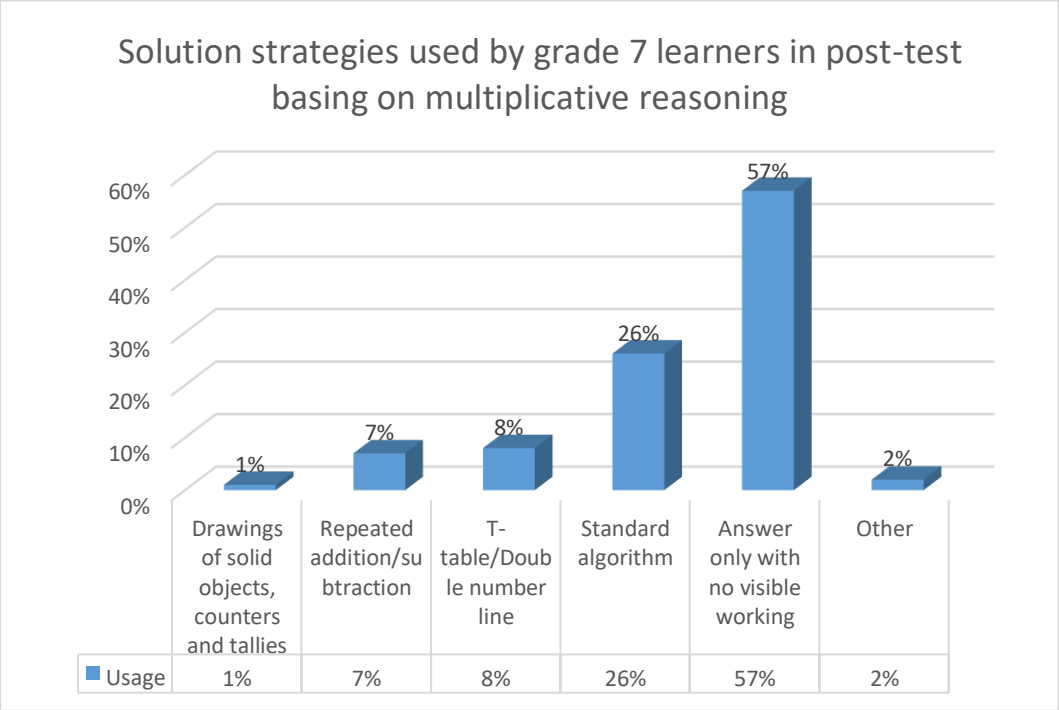


Figure 8: Solution strategies used by grade 7 learners basing on multiplicative reasoning in post-test.

The most used solution strategy basing on mathematical reasoning was answer only with no visible working with 57% usage in multiplicative reasoning problems and 52% in additive reasoning problems followed by standard algorithm with 44% in additive reasoning problems and 26% in multiplicative reasoning problems. The third most used solution strategy in multiplicative reasoning problems was the T-table /double number line with 8% usage.

A further analysis was done to compare how each solution strategy was associated with correct answers in the pre-test. Table 19 presents the summary of grade 7 learners' correct responses across solution strategies used in the post-test with reference to the required mathematical reasoning.

Table 19: Grade 7 learners' correct responses across solution strategies used in post-test with reference to mathematical reasoning.

Solution strategies	Additive reasoning	Multiplicative reasoning	Total
Drawings of solid objects, counters and tallies	(3/3) 100%	(1/6) 17%	(4/9) 44%
Repeated addition/ subtraction	N/A	(12/32) 38%	(12/32) 38%
T-table/ Double number line	N/A	(22/39) 56%	(22/39) 56%
Standard algorithm	(196/232) 84%	(83/124) 67%	(279/356) 78%
Answer only with no visible working	(210/269) 78%	(125/276) 45%	(335/545) 61%
Other	(15/18) 83%	(3/8) 38%	(18/28) 69%
Total	(424/522) 81%	(246/485) 51%	(670/1007) 67%

Table 19 revealed that the use of standard algorithm generated higher number of correct responses 236/331 constituting 78%. This is followed by answer only with no visible working (61%), and T-table/double number line (56%). The least was repeated addition (38%). The learners' correct use of strategies in problems that demand additive reasoning was very high stands at 81% whereas for multiplicative stands at 51% having a difference of 30%. This analysis also revealed that for each solution strategy that was used in both mathematical reasoning problems, the percentage of correct answers achieved using the strategy in problems that require additive reasoning was always higher than the one of problems that require multiplicative reasoning. This trend might also suggest the contribution of the mathematical reasoning to performance.

4.3.3 Comparison of strategy usage during the pre and post-tests.

Although this research sought to identify strategies employed by learners in solving problems that involve multiplicative and additive reasoning in both word and pure numerical problems, the use of a pre-test and a post-test motivated me to compare strategies used between the two tests. It is not the intention of this analysis to make argument about the effectiveness of the intervention conducted by WMC-P MR project but to get an insight of solution strategy shifts if any that were influenced by the intervention. The proportions of each solution strategy usage in each test is shown in Table 20.

Table 20: *Usage of strategies in pre- and post-test.*

Solution strategies	Pre-test	Post-test
Drawings of solid objects, counters and tallies	(31/974) 3%	(9/1007) 1%
Repeated addition/ subtraction	(41/974) 4%	(32/1007) 3%
T-table/ Double number line	(0/974) 0%	(39/1007) 4%
Standard algorithm	(331/974) 34%	(356/1007) 35%
Answer only with no visible working	(559/974) 57%	(545/1007) 54%
Other	(12/974) 1%	(26/1007) 3%
Total	(974/1144) 85%	(1007/1144) 88%

A comparison of the strategies used during the pre-test to those used during the post- test shows a rise in the use of the T-table/ double number line from no incident of its use in pre-test to 39 incidents of usage in the post-test. Of the 39 learners, who used the T-table strategy during the post-test, 56% of them used it effectively to get correct answers. This substantial increase suggest increase level of awareness in the use of t-table/double number line as a strategy for multiplicative situations. The presence of this new strategy (T-table), also caused a drop in the use of other strategies as follows: Drawings of solid objects, counters and tallies dropped from 3% during pre-test to 1% during post-test, Repeated additions/subtractions from 4% to 3% and answer only with no visible working from 57% to 54%.

Overall the most used solution strategy by grade 7 learners as they answer multiplicative and additive problems in word and pure numerical formats in the pre- and post- test was answer only with no visible working followed by the standard algorithm.

The results revealed that for each solution strategy that was used in both mathematical reasoning problems, the percentage of correct answers achieved using the strategy in problems that require multiplicative reasoning was always lower than the one of problems that require additive reasoning. This trend might suggest that the use of a specific solution strategy does not necessarily contribute to performance instead it is the type of the mathematical reasoning involved.

There was a rise in the use of the T-table/ double number line from 0% in pre-test to 4% in post-test and a drop in the use of other strategies such as Drawings of solid objects, counters and tallies, Repeated additions/subtractions and Answer only with no visible working.

4.4 Learners' Detectable Errors

The analysis of learners' responses was done with the aim of exploring the detectable errors in the grade 7 learners' written responses as they performed multiplicative and additive reasoning in word pure numerical problems. This analysis was carried out on incorrect responses, as no errors can be found in the incidences of correct responses. A quantitative analysis of the number of times a specific detectable error appeared was established per test item, problem formats, mathematical reasoning and per test. A comparison of errors that were committed in the pre- and post-test was also done to check if there was any shift in the detectable errors across pre- and post-test. As detailed in Chapter 3, Table 21 provides the summary of the kinds of possible detectable errors committed by learners as they performed multiplicative and additive reasoning in word and pure numerical problems. Table 21 presents the summary of codes of detectable errors and their descriptions.

Table 21: summary of codes of detectable errors and their descriptions

Code	Error
21	Operation choice error
22	Computational error
23	Procedural error
24	Other error

4.4.1 Pre-test detectable errors

There were 548 learner responses with incorrect answers (48% in pre-test) that were subjected to detectable error analysis. As noted in the previous sections, the majority of learners' strategies in the pre-test were in the category of 'answer only with no visible working', and therefore not subjected to error analysis because there was no detailed information that could be coded for learner detectable error. Frequencies of incorrect answers with no visible working per test item in pre-test are displayed in appendix 13. The establishment of targeted responses for error analysis is presented in Table 22.

Table 22: Responses targeted for detectable error analysis in pre-test.

Mathematical reasoning	Incorrect answers	Incorrect answers with no visible working	**Incorrect answers with visible working (Responses targeted for detectable error analysis)**
Additive reasoning	(165/548) 30%)	(86/297) 29%	(79/251) 31%
Multiplicative reasoning	383/548) 70%	(211/297) 71%	(172/251) 69%
Total	(548) 100%	(297/548) 54%	(251/548) 46%

Table 22 shows a total of 548 learners' responses that had incorrect answers, of which 31% were for additive reasoning problems and 69% for multiplicative reasoning problems. 54% of the incorrect answers were obtained through the use of solution strategy answer only with no

visible working, which were not considered in error analysis. Only 251 (46%) of the incorrect responses in the pre-test were subjected for the error analysis. Details of the analysis per each test-item is presented in Appendix 17. The summary of this analysis is presented and interpreted in this section.

Table 23 presents the summary of proportions and percentages of the grade 7 learners' detectable errors based on problem format in the pre-test.

Table 23: *Grade 7 learners' detectable errors based on problem format in pre-test.*

Errors	Word problems	Pure numerical problems	Grand total
Operation choice error	(62/161) 39%	(7/90) 8%	(69/251) 27%
Computational error	(22/161) 14%	(12/90) 13%	(34/251) 14%
Procedural error	(40/161) 25%	(46/90) 51%	(86/251) 34%
Other	(37/161) 23%	(25/90) 28%	(62/251) 25%
Total	(161/251) 64%	(90/251) 36%	(251) 100%

Across problem format, word problems were associated more with detectable errors with the incidence rate of 64% as compared to pure numerical problems with 36%. The most evident error in word problems was the operation choice error with an error incident of 39% while for pure numerical problems was the procedural error with 51%

Table 24 illustrates the summary of proportions and percentages of the grade 7 learners' detectable errors across the mathematical reasoning in the pre-test.

Table 24: *Grade 7 learners' detectable errors across additive and multiplicative reasoning problems in pre-test*

Errors	Additive reasoning problems	Multiplicative reasoning problems	Grand total
Operation choice error	(15/79) 19%	(54/172) 31%	(69/251) 27%
Computational error	(7/79) 9%	(27/172) 16%	(34/251) 14%
Procedural error	(36/79) 46%	(50/172) 29%	(86/251) 34%
Other	(21/79) 27%	(41/172) 24%	(62/251) 25%
Total	(79/251) 31%	(172/251) 69%	(251) 100%

The learners had an incidence of 31% of the detectable errors in problems that require additive reasoning and 69% in problems that require multiplicative reasoning. This revealed that learners' multiplicative reasoning is more associated with detectable errors as compared to additive reasoning with a numerical difference of 37%. The most detectable error in additive reasoning problems was procedural error with an error detection of 46% .On the other hand for multiplicative reasoning the most detectable error was operation choice error with an error detection of 31%.

Table 25 illustrates the summary of detectable errors basing on problem format and mathematical reasoning in pre-test.

Table 25: *Detectable errors basing on problem format and mathematical reasoning in pre-test*

Errors	**Additive reasoning problems**			**Multiplicative reasoning problems**			**Grand Total**
	Word problems	Pure numerical problems	Total	Word problems	Pure numerical problems	Total	
Operation choice error	(14/44) 32%	(1/35) 3%	(15/79) 19%	(48/117) 41%	(6/55) 11%	(54/172) 31%	(69/251) 27%
Computational error	(4/44) 9%	(3/35) 9%	(7/79) 9%	(18/117) 15%	(9/55) 16%	(27/172) 16%	(34/251) 14%
Procedural error	(12/44) 27%	(24/35) 69%	(36/79) 46%	(28/117) 24%	(22/55) 40%	(50/172) 29%	(86/251) 34%
Other	(14/44) 32%	(7/35) 20%	(21/79) 27%	(23/117) 20%	(18/55) 33%	(41/172) 24%	(62/251) 25%
Total	(44/79) 56%	(35/79) 44%	(79/251) 31%	(117/172) 68%	(55/172) 32%	(172/251) 69%	(251) 100%

As shown in table 25, the procedural error was visible most frequently during the pre-test with an incidence rate of 34% whilst the computational error had the least prevalence of 14%. The operation choice error and the other error appeared 27% and 25% respectively. The results revealed that word problems that demand multiplicative reasoning were associated with more detectable errors than any other problems with 68% incident rate as shown in Table 23.

4.4.2 Post-test detectable errors

Altogether there were 428 (37% in post-test) responses with incorrect answers, of these 206 responses were targeted for error analysis in the entire post-test. The detectable error analysis focused on responses that presented incorrect answer with visible working. The total frequencies of incorrect answers with no visible working is shown in appendix 15. The total frequencies of the errors that were detectable in the post-test per test item, problem format and mathematical reasoning is presented in appendix 18.

Table 26 presents how the 206 responses were identified for analysis of detectable errors per mathematical reasoning during the post-test.

Table 26: *Responses targeted for detectable error analysis in post-test.*

Problem format	**Incorrect answer**	**Incorrect answer with no visible working**	**Incorrect answer with visible working (Responses targeted for error analysis)**
Additive reasoning	(132/428) 31%	(58/132) 44%	(74/132) 56%
Multiplicative reasoning	(296/428) 69%	(164/296) 55%	(132/296) 45%
Total	(428) 100%	(222/428) 52%	(206/428) 48%

The responses with incorrect answers in the post test were distributed as follows, 31% were in additive reasoning problems whereas 69% were for multiplicative reasoning. Additive reasoning problems resulted in 56% targeted for detectable error analysis while 45% was for multiplicative reasoning. This process of identifying the responses for error analysis resulted in 48% of the incorrect responses targeted for the detectable error analysis in post-test.

Table 27 shows the detectable errors based on problem format in post-test.

Table 27: *Grade 7 learners' detectable errors based on problem format in post-test.*

Errors	Word problems	Pure numerical problem	Grand total
Operation choice error	(53/146) 36%	(3/60) 5%	(56/206) 27%
Computational error	(37/146) 25%	(10/60) 17%	(47/206) 23%
Procedural error	(32/146) 22%	(34/60) 57%	(66/206) 32%
Other	(24/146) 16%	(13/60) 22%	(37/206) 18%
Total	(146/206) 71%	(60/206) 29%	(206) 100%

In the post-test, of the 206 detectable errors across the problem format, 71% of the detectable errors were in word problems and 29% in pure numerical problems. The trend of detectable errors in the post-test with respect to problem format was the same as the one of pre-test of having more detectable errors present in word problems as compared to pure numerical problems.

There was need of detectable error analysis across mathematical reasoning. Table 28 presents summary of proportions and percentages of the grade 7 learners' detectable errors based on additive and multiplicative reasoning.

Table 28: Grade 7 learners' detectable errors based on additive and multiplicative reasoning in post-test.

Errors	Additive reasoning problems	Multiplicative reasoning problems	Grand Total
Operation choice error	(23/74) 31%	(33/132) 25%	(56/206) 27%
Computational error	(10/74) 14%	(37/132) 28%	(47/206) 23%
Procedural error	(24/74) 32%	(42/132) 32%	(66/206) 32%
Other	(17/74) 23%	(20/132) 15%	(37/206) 18%
Total	(74/206) 36%	(132/206) 64%	(206) 100%

The frequency of detectable errors in the post test was 206 with 64% in problems that demand multiplicative reasoning and 36% in problems that demand additive reasoning. The multiplicative reasoning problems presented more detectable errors than the additive reasoning problems with a numerical difference 28%. The results on detectable error analysis in the post test with respect to the demanded mathematical reasoning revealed a trend similar to one in the pre-test where it was noticed that detectable errors were more noticeable when learners solved problems that demand multiplicative reasoning (64%) than when solving problems that demand additive reasoning (36%).

Table 29 displays the summary of proportions and percentages of grade 7 learners' detectable errors across problem formats and mathematical reasoning in the post-test.

Table 29: *Grade 7 learners' detectable errors across problem format and mathematical reasoning in post-test.*

Errors	**Additive reasoning**			**Multiplicative reasoning**			**Grand Total**
	Word problems	Pure numerical problems	Total	Word problems	Pure numerical problems	Total	
Operation choice error	(22/44) 49%	(1/29) 3%	(23/74) 31%	(31/101) 31%	(2/31) 6%	(33/132) 25%	(56/206) 27%
Computational error	(7/44) 16%	(3/29) 10%	(10/74) 14%	(30/101) 30%	(7/31) 23%	(37/132) 28%	(47/206) 23%
Procedural error	(3/44) 7%	(21/29) 72%	(24/74) 32%	(29/101) 29%	(13/31) 42%	(42/132) 32%	(66/206) 32%
Other	(13/44) 29%	(4/29) 14%	(17/74) 23%	(11/101) 11%	(9/31) 29%	(20/132) 15%	(37/206) 18%
Total	(44/74) 61%	(29/74) 39%	(74/206) 36%	(101/132) 77%	(31/132) 23%	(132/206) 64%	(206) 100%

Likewise in the post-test, the word problems that demand multiplicative reasoning had the highest rate of detectable error incidences of 77%. The detectable error that resulted to have the highest frequency was the procedural error with 32% and the other error had the least frequency of 18%. Operation choice error was the second highest with 27% and the computational error was the third highest with 23%.

4.4.3 Comparison between pre- and post-tests patterns of learner detectable errors

As already noted, there was a statistically significant difference between learners' performance during the two tests. For this reason, I developed an interest to analyse the differences in error detection between additive and the multiplicative reasoning problems.

Table 30 presents the comparison of proportions and percentages of detectable errors in pre and post-tests basing on mathematical reasoning.

Table 30: *Detectable errors in pre- and post-tests basing on additive and multiplicative reasoning*

	Additive reasoning problems	Multiplicative reasoning problems	Total errors in test
Pre-test	(79/572) 14%	(172/572) 30%	(251/1144) 22%
Post-test	(74/572) 13%	(132/572) 23%	(206/1144) 18%
Differences	(5/572) 1%	(40/572) 7%	(45/1144) 4%

There was reduction of detectable errors across the additive and multiplicative reasoning problems with greater difference shown in the multiplicative reasoning problems where the detectable errors were reduced from 30% to 23% resulting in 7% difference in error detection.

Table 31 displays the proportions of detectable errors in each test.

Table 31: *Detectable errors in pre- and post- tests*

	Pre-test	**Post-test**
1. Operation choice error	(69/251) 27%	(56/206) 27%
2. Computational error	(34/251) 14%	(47/206) 23%
3. Procedural error	(86/251) 34%	(66/206) 32%
4. Other	(62/251) 25%	(37/206) 18%
TOTAL	(251/1144) 22%	(206/1144) 18%

There was a noticeable shift in detectable error incidence between the pre-test and the post test. Overall there were more times that detectable errors were evident during the pre-test than during the post test, 22% during pre-test to 18% during post-test. For each detectable error type there was a reduction in the incidences except for computational error that rose from 14% to 23%.

This study revealed that the most detectable error on the grade 7 learners' responses during the two tests was the procedural error. On the mathematical reasoning strand, the problems that demand multiplicative reasoning were associated with more detectable errors than the problems that demand additive reasoning. It was further revealed that there were more detectable errors visible in word problems than in pure numerical problems. Lastly, there was a noticeable shift in error incidences between the pre-test and the post-test as there were more times that errors were committed by learners during the pre-test than during the post test. The error shift between the two tests is attributed to the reduction of errors in multiplicative reasoning problems

4.5 Discussions of findings

The discussion of findings is organised into three sections in line with the research questions. These are: learner performance; solution strategies, and detectable errors. In each case, the results of the data analysis is given and also a comparison is made based on literature.

4.5.1 Learner Performance

Two major findings were revealed as the study attempted to find out the differences in grade 7 learners' performance in multiplicative and additive reasoning word and pure numerical problems in the pre- and post- test.

Findings revealed no significant difference in learners' performance in word problems as compared to their performance in pure numerical problems. This finding is consistent with the report released by the South African Department of Basic Education which stated that the learners' performances in the two problem formats was equally poor (DBE, 2012.). Similarly, Askew and Venkat (2018) conducted research with middle grade learners in suburban and township schools of South Africa. Their findings also revealed that learners' performance in word problems was similar to their performance in pure numerical problems.

Hickendorff (2013), conducted research in arithmetic word and pure numerical problem formats with grade 6 learners, in the Netherlands, and concluded that there was a same ability dimension in solving problems in both problem formats. My research has also shown that learners attempted to answer word problems more than they did with pure numerical problems. This was shown by more blank spaces in pure numerical problems as compared to the blank spaces in word problems. The more attempts in answering the word problems might be a sign of having interest in answering the problems presented in words.

On the learner performance based on mathematical reasoning strands, findings indicated low performance in multiplicative reasoning questions as compared to additive reasoning with 41% difference point in pre-test and 30% difference in post-test. This finding may be attributed to the structural understanding of multiplication and division that is more complex than that of addition and subtraction (Askew, 2018; Brown; Venkat & Mathews, 2018 Küchemann & Hodge, 2010; Jacob &Mulligan, 2014; Bryant & Nunes, 2009).

4.5.2 Learner Solution Strategies

Answering the research question about the solution strategies that grade 7 learners employ when solving multiplicative and additive reasoning word and pure numerical problems in the pre- and post- test, findings were revealed. The most used solution strategy was answer only with no visible working in pre-test constituting of 57% usage in pre-test and 54% in post-test. This high frequency of the usage of this solution strategy could be attributed to the South

African curriculum advocating the mental mathematics without requiring learners to record their working process (DBE, 2011). The second most used strategy was the standard algorithm that had 34% usage in pre-test and 35% in post-test.

On comparing the most and the second most used strategies and their achievements on correct answers it was established that standard algorithm was associated with more correct answers than writing answer only with no visible working. Standard algorithm had an achievement of 71% in pre-test and 78% in post-test compared to the achievement of writing the answer only with no visible working that had 47% in pre-test and 61% in post-test. This suggests that though the most used strategy was not the most effective strategy. Hickendorff, Heiser, Putten and Verhelst (2009) conducted a study of analysing solution strategies and achievement on grade 6 learners in the content area of arithmetic. Their findings were similar to the one in this study that the use of standard algorithm was more successful than writing answer only with no visible working.

Findings also indicated that the choice of strategy by learners was dependent on problem formats and mathematical reasoning tasks. In solving of pure numeric problems, learners opted more for the use of the strategy of answer only without any visible working. The most probable explanation as to why learners employed this strategy the most might be found in Koedinger and Nathan (2004) problem solving model. The model assumes a cyclical pattern in problem solving with most of the cognitive processes (internal processes), done without need for any external representation. As learners solve a mathematical problem, they experience a comprehension phase, they situate and model the problem before they get to the solution phase. All these cognitive processes can be done without writing anything on paper. This leads to arriving at the answer without putting anything on paper. Consequently, by the time the learner thinks of writing, he/she writes the answer only without having to go over all that was done cognitively.

As this was true for pure numeric problems, it was also true for word problems but with a slightly lower incidence. The drop in incidence of use of this strategy in word problems might be due to the need to comprehend the word problem. Comprehension of a word problem involves a linguistic processing of the word problem into a corresponding problem model that illustrates the relationship in the problem (Askew & Venkat, 2018; Cummins, Kintsch, Reusser & Weimer, 1988; Hickendorff, 2013; Koedinger & Nathan, 2004). The learner then translates

the problem model into a mathematical model. For the purpose of getting a solution, the learner most likely writes down the mathematical model before answering it (Askew & Venkat, 2018; Hickendorff, 2013; Koedinger & Nathan, 2004). Once the model has been written down, in this case it means answer only has already been avoided. However, some learners would go on to write the answer without showing the mathematical model on paper.

The other noticeable trend on the use of solution strategies by the grade 7 learners in the comparison of usage of strategies during the pre- and post-test, in different problem formats, mathematical reasoning and test items was the shift from concrete strategies to more abstract strategies. This observed trend shows a progression from concrete strategies to abstract strategies (Ensor et al., 2009; Hoadley, 2007; Schollar, 2008). T-table/ double number line in this case was not used at all meaning 0% usage in the pre-test but was the third most used in the post-test with 4% usage with 56% achievement of correct answers. A concrete solution strategy, drawings of solid objects, counters and tallies had 3% usage in pre-test had 0% usage in post-test

4.5.3 Learner Detectable Errors

The data analysis suggested that the most detectable error on the grade 7 learners' responses was the procedural error that had the highest frequency in both tests of 34% detection in pre-test and 32% in post-test. The operation choice error had the second highest frequency in both tests of 27% in pre and post-tests. This finding is in line with what was found by Herholdt and Sapire (2014) after conducting an error analysis on grades 1-4 learners' responses to tests that identified most conceptual and procedural errors across the grades and mathematical topics. Luneta (2015) also arrived at the same finding after analysing students' solutions on coordinate geometry problems in the final Grade 12 examination in South Africa that identified most conceptual and procedural errors. This is worrying because a procedural error symbolises a bug of a procedural understanding involving an incorrect routine thus having a symptom of a faulty line of thinking (Herholdt & Sapire, 2014; Riccomini, 2005).

Regarding the type of mathematical reasoning, multiplicative reasoning problems were associated with more errors as compared to their additive reasoning counterparts. However, it is this multiplicative reasoning that is important to the development of learners' foundations in mathematical domain that include proportional relations and linear functions (Askew, 2018; Brown, Küchemann & Hodge, 2010; Bryant & Nunes, 2009; Jacob &Mulligan, 2014). Having

more detectable errors in the multiplicative reasoning at the end of primary education imply problems at high school on the related content areas.

The statistics in the detectable error analysis show that there were more detectable errors visible in word problems than in pure numerical problems. There are two probable reasons behind this trend. The first reason behind this might be the lack of competence in processing language and that likely resulted in incorrect models and wrong solutions (Sepeng, 2014). The language of instruction in the schools from which the data was collected is English, which is a second or third additional language to learners and this might have contributed to the incorrect comprehension of the problems. The second reason behind this trend might be due to the high usage of the strategy of answer only with no visible working in pure numerical problems. Due to use of this strategy, learners' work had less written components to be analysed for detectable errors in pure numerical problems.

The detectable error analysis revealed that some learners had an incorrect comprehension of symbolic expressions (pure numerical problem). This failure to comprehend the given mathematical operation resulted in them having an operation choice error. It must not be taken for granted that comprehension is only challenging in word problems where the learners have to cross-examine textual information and decode it into mathematical expressions and computations (DBE, 2012; Koedinger, Alibali & Nathan, 2008; Koedinger &Nathan, 2004). Operation choice error was not supposed to be there in pure numerical problems because the operation sign was explicitly given but some learners were choosing not to use the given operation and operated with the numbers using another operation. This might have been caused by the incorrect comprehension of symbolic expressions (Koedinger & Nathan, 2004). The presence of operation choice error in pure numerical problems suggests that due diligence is needed on the part of both teachers and learners when approaching problems of this format.

There was a noticeable shift in error incidence between the pre-test and the post test. There were more times that errors were committed by learners during the pre-test than during the post test, 22% detection during pre-test as compared to 18% detection during post-test. There was a reduction in the total frequency of each error type committed by learners in pre-test as compared to post-test except for the computational error. However, there was an observation that was made about the commitment of the computational error. During the pre-test, there was 14% detection of computational errors committed by learners but during the post-test the

detection of computational errors rose to 23%. After looking at the learners' scripts, it was discovered that the introduction of a new strategy, T-table, in between the two tests, was partly the cause of the increase in computational errors during the post test. The learners were not yet conversant with the new strategy. The T-table is an abstract strategy so like all other abstract strategies, learners take a while to master it before they can start to use it with a minimal number of errors.

4.5.4 Conclusion

This research has established that learners' performance in word problems compares the same as their performance in pure numerical problems. This suggests that mathematical reasoning and not problem format might be the reason for performance differences. The analysis of the data also revealed that the problems that demand multiplicative reasoning were performed poorly as compared to problems that demand additive reasoning.

Regarding the solution strategies employed by learners during both tests, this study identified that grade 7 learners used: Drawings of solid objects tallies or counters, Repeated addition or repeated subtraction, T- Table or double number line, Standard algorithm, Answer only with no visible working and other less frequent strategies just bunched as Other in this research. Of these strategies, the most used strategy was answer only with no visible working. Writing answer only with no visible working was most used but resulted to be less effective when compared to some other solution strategies. Writing the answer only with no visible working can be used but it hinders teacher- learner communication that is much needed on activities like error analysis.

Lastly, in an endeavour to find out the detectable errors on learners responses, this study established that these detectable errors were of four types: Operation choice error, Computational error, Procedural error and other error (bunch of less prevalent errors). The most detectable error on grade 7 learners' responses was the procedural error followed by operation choice error. These two errors were also detectable in lower grades (grade 1- 4) across all chapters (Herholdt & Sapire, 2014) and in higher grades (grade 12) in geometry (Luneta, 2015).

CHAPTER 5: SUMMARY, CONCLUSION, AND RECOMMENDATIONS

5.1 Introduction

This chapter presents the summary of the study, conclusion derived from the research, limitations of the research and recommendations.

5.2 Summary of the Study

This research explored grade 7 learners' performance in multiplicative and additive reasoning word and pure numerical problems by means of analysing their written pre -test and a post-test scripts on a specific set of sub-items. As already alluded to earlier, the reason for selecting grade 7 was informed by my position in the research which doubles as a researcher and as a high school educator. Over the years, I have noted with concern that learners' perform badly in baseline assessment mathematical tasks during their induction into high school. No attempt has been made by neither me nor my school to find out the underlying cause of the problem.

This research therefore focused on learner performance in problems that demand multiplicative and additive reasoning. The problems were presented in word and pure numerical formats. In addition to learner performance, the research investigated the solution strategies used by the learners as they grappled with mathematical reasoning in problem solving. In the process of solving the mathematical problems, learners committed some errors which made them to get wrong solutions to the problems. This research also identified detectable errors and analysed them.

My study is part of a bigger project (WMCP-MRIP) done by Wit Maths Connect. The project (WMCP-MRIP) involves a series of interventions in previously disadvantaged primary schools in Johannesburg aimed at supporting learner work on word problems. This is in response to poor performance by learners in problem solving. The interventions involved pre-test, an intervention and a post test. The pre and post-tests consist of mathematical problems presented in word and pure numerical formats. These problems entail multiplicative and additive reasoning. As for my research, I examined grade 7 learners' scripts to explore their performance in multiplicative and additive reasoning word and pure numerical problems as presented in their written scripts. Scripts from 143 learners were used for my research. This resulted in 246 scripts being examined as all the 143 learners participated in both tests.

The tests consisted of 4 word and 4 pure numerical problems both formats of which were equally divided between multiplicative and additive reasoning problems. The 8 problems per test

multiplied by the 143 learners meant that 1144 responses were examined per test. Each script had to be coded for performance, solution strategy and detectable errors with the results of the coding captured in an excel spreadsheet. This being the case, it implies that at the end of the coding, each learner script had codes that explained correctness of the learner's answer, the strategy used as well as any error detectable in the answer.

When all coding was done, the work had to be checked for reliability by means of asking for recoding of 32 scripts by a peer who is familiar with the main project. The recording confirmed accuracy of my coding framework with an agreement rate of 96%. After this confirmation by the recoding exercise, the data from the spreadsheet was then analysed and processed leading to establishment of findings presented earlier.

5.3 Conclusion

Through this research I have established that learners are more challenged by problems associated with multiplicative reasoning than additive reasoning. That being the case, teaching and learning need to focus on developing learners' skills in multiplicative reasoning. Research (Askew, 2018; Brown, Küchemann & Hodge, 2010; Bryant, Nunes & Tzekaki, 2009; Jacob & Mulligan, 2014; Venkat & Mathews, 2018) has already stated the importance of multiplicative reasoning by pointing out that it develops learners' skills needed in other mathematical domains. As a result, emphasising on multiplicative reasoning throughout learners' primary school life, but more so at grade 7, prepares them for high school mathematical domains that rely on multiplicative reasoning competence.

Furthermore, this study has established that there is no performance difference between mathematics problems presented as either word or pure numerical problems. This conclusion agrees with conclusions reached at by other earlier studies in the same field (Askew &Venkat 2018; DBE 2012; Hickendorff, 2013). In light of this, there is need to change the way teachers, textbook writers and curriculum planners regard these two problem formats. Most textbooks are written in such a way that word problems are always placed at the end of chapters as closing exercises. The current position was influenced by the thinking that word problems are more challenging than their pure numerical counterparts. Since this research has concluded that the ability dimension is the same for the two problem formats, there is no more need to start with teaching and presenting of mathematical concepts in a pure numerical format all the times. Introducing mathematical concepts by teaching and presenting them in context based problems might help to make the subject more appealing to learners.

Of the six strategy categories that were examined in the research, it resulted that learners preferred to write the answer without showing any visible working. However though this was their preferred strategy, it was not as effective as using a standard algorithm. Also, though writing answer only is a preferred answering strategy in primary mathematics, showing all the working is important for it enables the identification and rectification of errors rather than focusing on the answer. The answer is an essential component of learners' work, but teachers should identify learners' errors in their work.

Lastly, it was noted that the error that was most detectable in learners' scripts was the procedural error. This error entails wrong steps of executing a procedure of a defined operation or incomplete workings by the learners. It was also established that learners committed an operation choice error in pure numerical problems where the operation was already given. The operation choice error was the second highest detectable error in the research. The implication is that teachers should not take it for granted that if an operation is given in a mathematical problem, learners will use it. Due diligence and caution need to be emphasised with the learners in this respect.

5.4 Limitations

The fact that this research was conducted through the analysis of learners' scripts only, the analysis was done on what was made available by learners on paper of which when the learner opted to write the answer only nothing was analysed for the solution strategy and errors in case of an incorrect answer. An interview with learners who presented errors or who opted to write the answer only would have given further insight into the mental processes they went through to find out the causes of the errors and preferences to different solution strategies.

Despite the fact that there were 548 incorrect responses in the pre-test and 428 in the post test, the learners' responses targeted for detectable error analysis in the pre-test was 46% and in post-test was 48% which is less than half of the total number of incorrect responses in both tests. The results of the error analysis could have been different if all the incorrect responses were eligible for the error analysis.

The research was done focusing on grade 7 learners, in the mathematical content area number, operations and relations (with respect to the South African curriculum). It is not known what the results could have been had the analysis been done to scripts of learners in a different grade or different content area or both.

5.5 Recommendations

Basing on the findings of the study I recommend the following:

1. Introducing mathematical concepts presenting them alternating the word and pure numerical formats might help to make the subject more appealing to learners unlike following the sequence presented in textbooks for the learner performance is similar in the different problem formats.
2. Teachers' emphasis be on the development of learners' multiplicative reasoning throughout primary school mathematics. Multiplicative reasoning provides the base to many content areas in secondary education.
3. Learners be encouraged to show all their working starting from the lower grades in their primary school mathematics. As much as writing answer only is common in primary school mathematics, in high school, the emphasis changes to showing all working with more marks allocated to the working than to the answer.
4. Learners be encouraged to use solution strategies that are effective in terms of producing the correct answer.

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Appendices

Appendix 1: Sample of the pre- and post-test administered to grade 7 learners by the (WMCP) - (MRIP) project

MR Assessment Grade 7

First name: _____

Surname: _____

Boy or girl: _____

Your date of birth: _____Day/month/year

Instructions

1. **Show all your working in this book**
2. **Do not rub out. If you make a mistake, put a line through it.**
3. **No calculators allowed.**

1. Learners sell bags of mangoes during market day. They put 6 mangoes in each bag. How many mangoes will they need to fill 9 bags?

They need _____ mangoes.

2. A class of 56 children go out to play games. They get into teams of 8. How many teams do they make?

They make _____ teams.

3. The total weight of 7 same-sized bags of potatoes is 63 kg. What is the weight of one bag of potatoes?

The weight of one bag is _____ kg.

4. Corin puts out 81 pies on one tray and 19 pies on another tray. How many pies does Corin put out all together?

Corin puts out _____ pies.

5. Hamsa is counting how many tiles cover the bathroom floor.
She counts 12 rows of tiles.
There are 11 tiles in each row.
How many tiles cover the floor?

There are _____ tiles.

6. Zwi collected 8 bottles for recycling.
Her friend collected 6 times as many bottles as Zwi. How many bottles did the friend collect?

Her friend collected _____ bottles.

7. 105 people are going on a car trip. Each car can carry 7 people.
How many cars will the people need?

They need _____ cars.

8. One bar of chocolate costs R12.
How much will 19 bars of this chocolate cost?

The cost is R_____ .

9. A baker puts pies in rows on a tray.
He puts the same number of pies in each of 8 rows.
The baker has 128 pies altogether.
How many pies must he put in each row?

There are _____ pies in each row.

10. Joey bought 108 metres of wire to put up a fence.
This is 6 times more wire than Peter bought.
How much wire did Peter buy?

Peter bought _____ metres of wire.

11. Sam cycles 112 km.
Sameera cycles 99 km.
How much further does Sam cycle than Sameera?

Sam cycles _____ km further.

12. A company packs pencils into boxes. Each box contains 15 pencils.
How many boxes are needed to pack 195 pencils?

_____ boxes are needed.

13. Viren has some pencils.
His friend gave him 12 pencils.
Now Viren has 144 pencils.
How many pencils did Viren start with?

Viren started with _____ pencils.

14. Denozo cycles at 14 km per hour.
How many hours will it take for Denozo to travel 84 km?

It takes _____ hours.

15. A farmer is planting rows of apple trees. He has 144 apple trees. He plants 12 trees in each row. How many rows will there be?

There are _____ rows of apple trees.

16.

81 + 19 =

17.

112 – 99 =

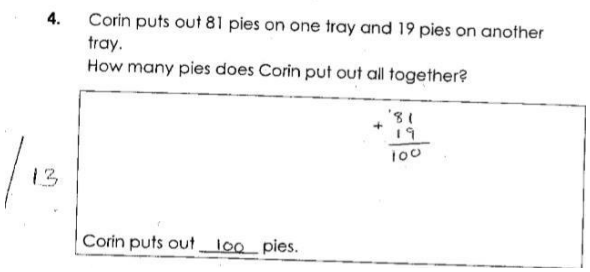
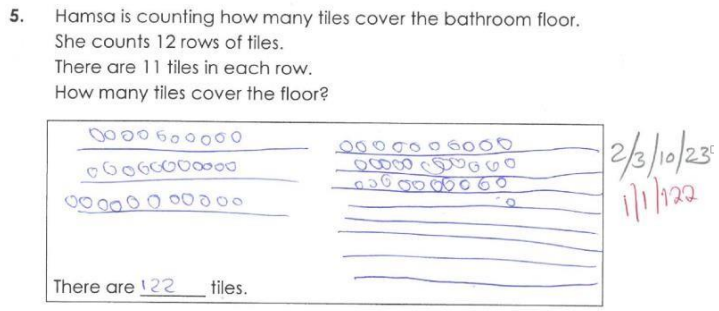
18.

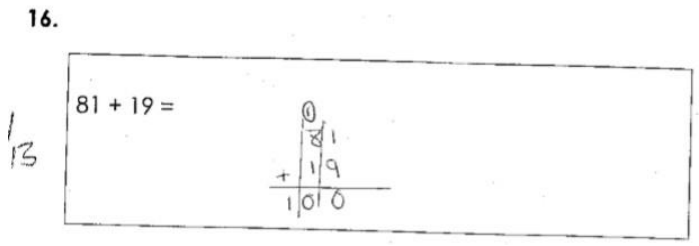
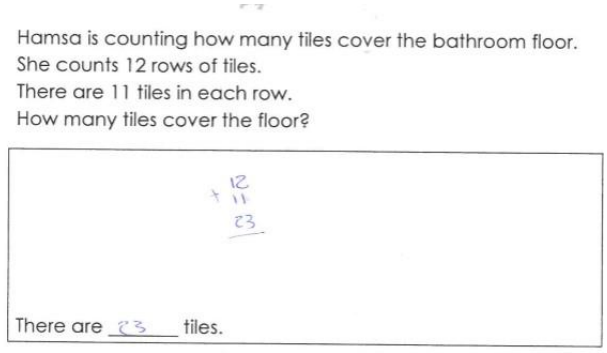
12 x 11 =

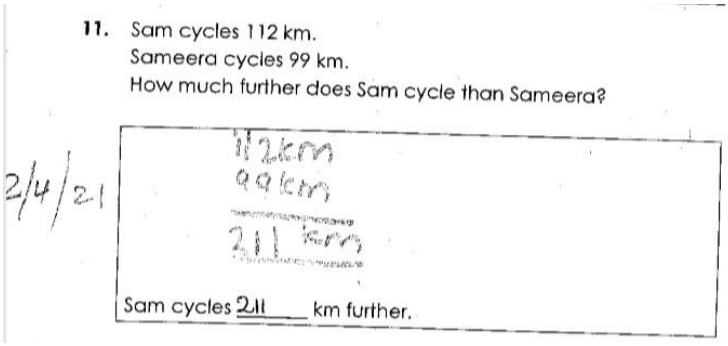
19.

195 ÷ 15 =

Appendix 2: Conceptualisation of the category Comprehension.

Comprehension Types	Code	Identification descriptors
Correct comprehension	3	**Word problems:** Modelling of word problems into correct pictorial, symbolic or any other semiotic representation. This is irrespective of whether the answer is correct or not. Examples of code 3 Example1:  From the example, there is correct comprehension since the solution workings showed the correct operation of addition and added the numbers involved in the question. Example 2:  Code 3 also entails another scenario that involves writing a correct operation but operates wrongly with that operation. The solution workings in question 5 above showed drawings representing the tiles in each raw but are not complete, only 2 rows had 11drawings drawn to it, the other 4 rows had 10 drawings drawn instead of 11 and this resulted in a wrong answer though the comprehension was correct.

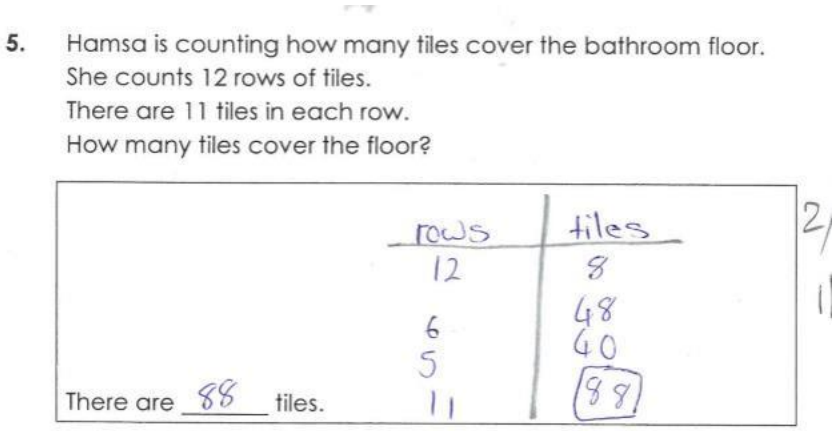
		Pure numerical problems: For pure numeric problems, correct comprehension was signified by solution workings that indicated any strategy and procedure for the mathematical operation concerned.  In the above example the learner showed the column algorithm of addition which corresponds to the question to be solved. For both problem formats any response with correct answer but without any visible solution workings was also considered as having a correct comprehension and was also given code 3.
Incorrect comprehension	4	**Word problems:** Incorrect modelling of word problems into pictorial, symbolic or any other semiotic representations. This could also take the form of incorrect choice of operations Examples of code 4 Example 1  In the case above the learner added the numbers involved instead of multiplying. Example 2



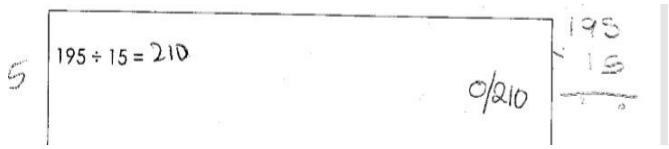
In the case above the learner did not explicitly show the operation but the answer shows that the learner added not subtracting the two numbers in context. Though the operation was not visible the solution workings showed that addition had been carried out.

Example 3:

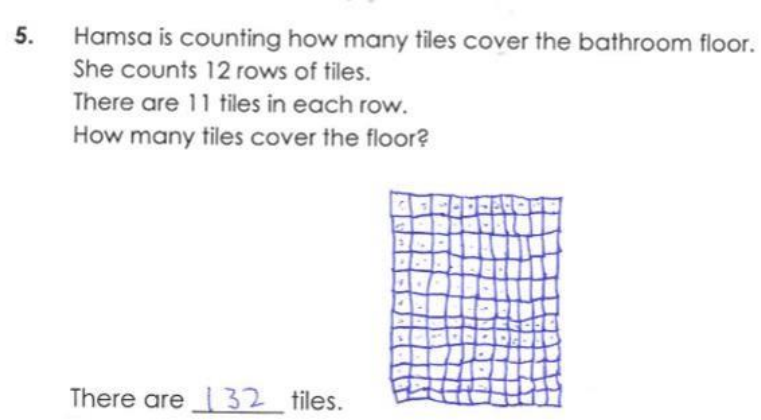
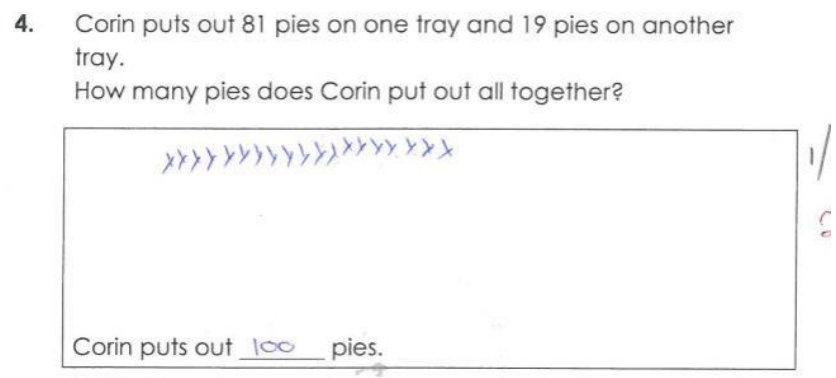
Another form of incorrect comprehension is when the solution strategy showed numbers that were not according to the context of the question.

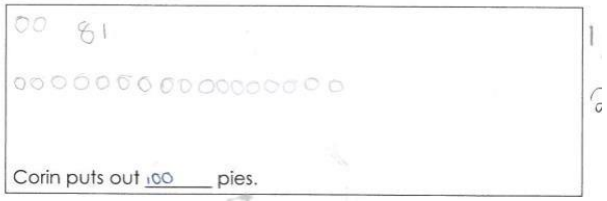
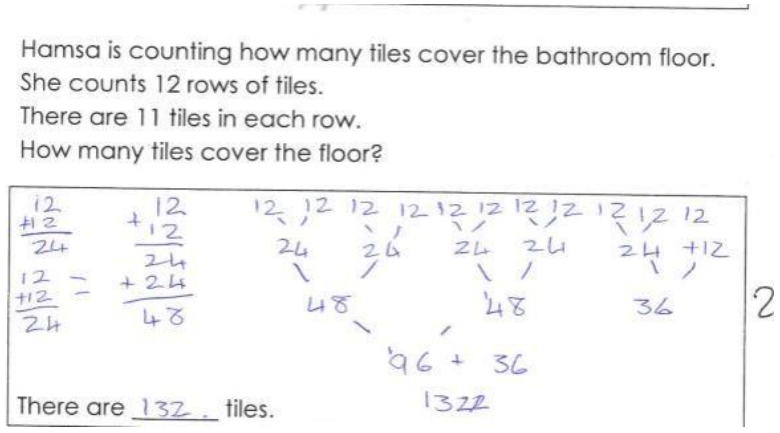
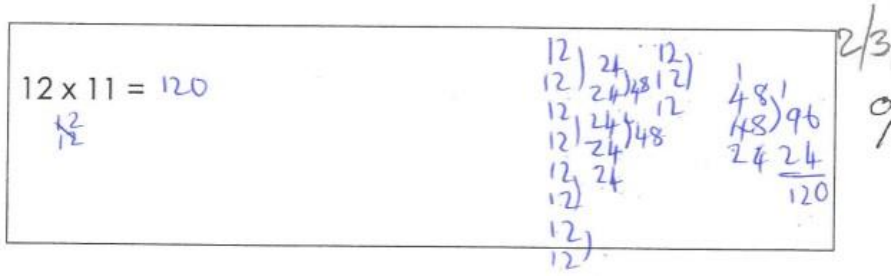


In the example above the learner opted to use multiples of 8 instead of multiples of 11 on the number of tiles in specific row.

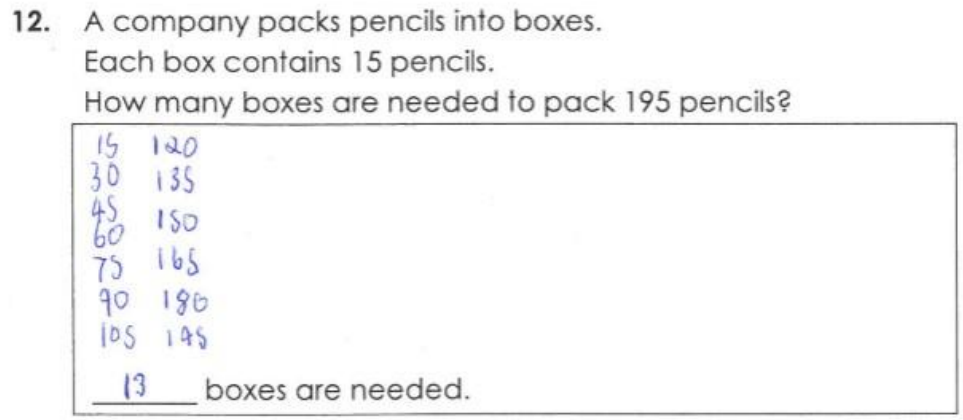
		Pure numerical problems: In pure the numeric problem format solution workings indicating any strategy and procedure apart from the one of the operation illustrated in the question was coded 4.  In the above case the learner wrote the solution strategy of column algorithm for addition and added the two numbers involved instead of division algorithm and dividing.

Appendix 3 Conceptualisation of the category Solution strategies

Solution strategies	Codes	Identification Descriptors
Drawing objects, tallies or counters	10	Denotes the strategy of working out the answer by drawing solid objects, tallies or counters. Example of drawing solid objects:  5. Hamsa is counting how many tiles cover the bathroom floor. She counts 12 rows of tiles. There are 11 tiles in each row. How many tiles cover the floor? There are 132 tiles. Example of drawing tallies:  4. Corin puts out 81 pies on one tray and 19 pies on another tray. How many pies does Corin put out all together? Corin puts out 100 pies. Example of drawing counters:

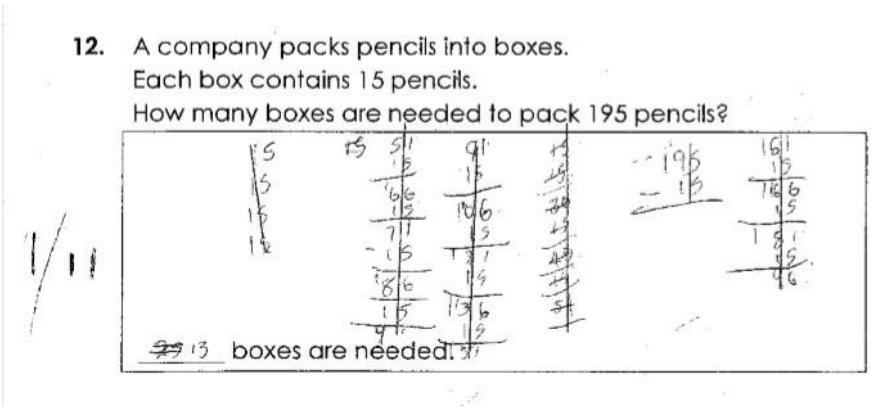
		4. Corin puts out 81 pies on one tray and 19 pies on another tray. How many pies does Corin put out all together?  Corin puts out 100 pies.
Repeated addition or repeated subtraction	11	Represents the repetition of adding and subtracting the same number till the final answer is obtained. Repeated addition In the case of multiplication, the multiplicand was added the number of times that was equivalent to the multiplier's value. Word:  5. Hamsa is counting how many tiles cover the bathroom floor. She counts 12 rows of tiles. There are 11 tiles in each row. How many tiles cover the floor? There are 132 tiles. Pure numeric  18. 12 x 11 = 120 In question 5 and 18 above, 12 was repeatedly added 11 times in both the word and pure numeric problems.

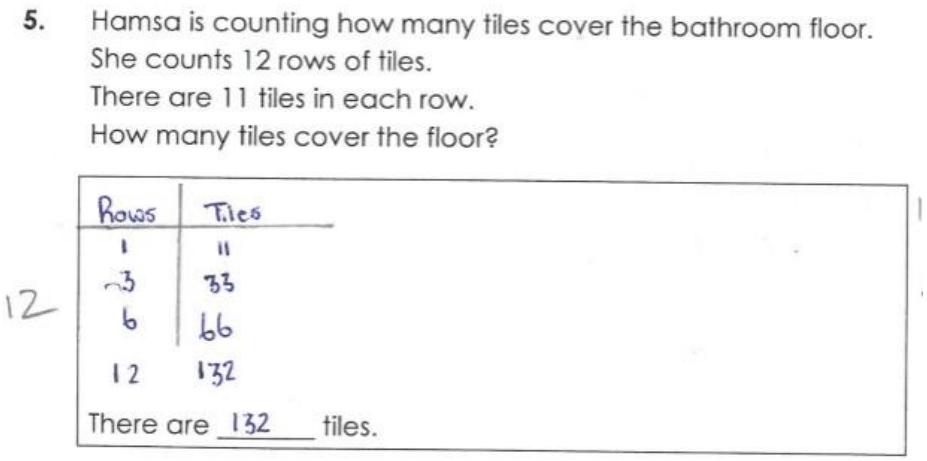
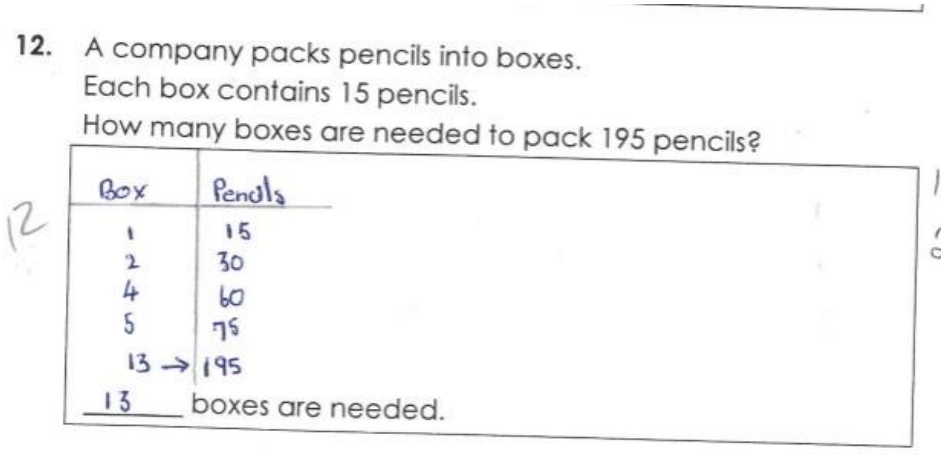
If the repeated addition is used in division, the divisor was repeatedly added until the answer was equal to the dividend. In question 12 above, 15 was repeatedly added until 195 until was reached. The answer was the number of times 15 was added to get 195.

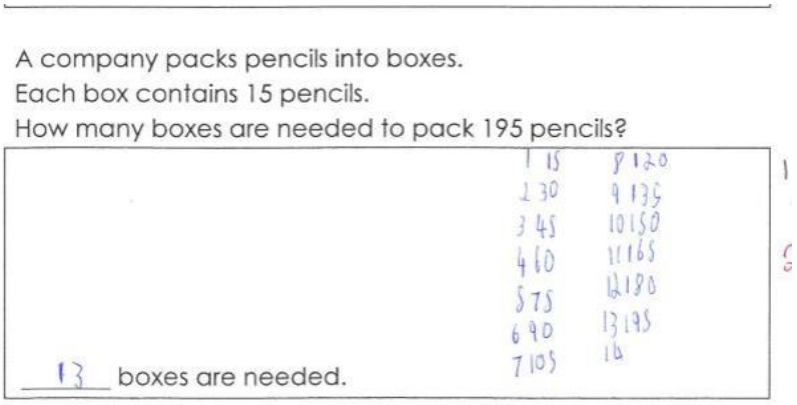
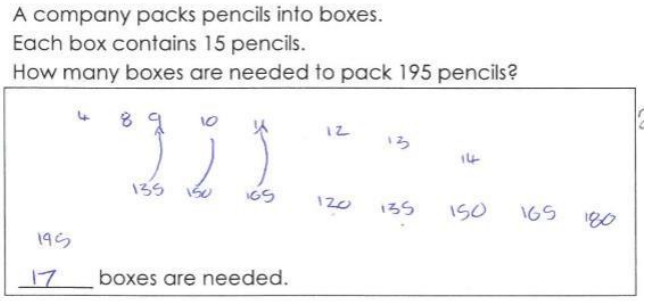


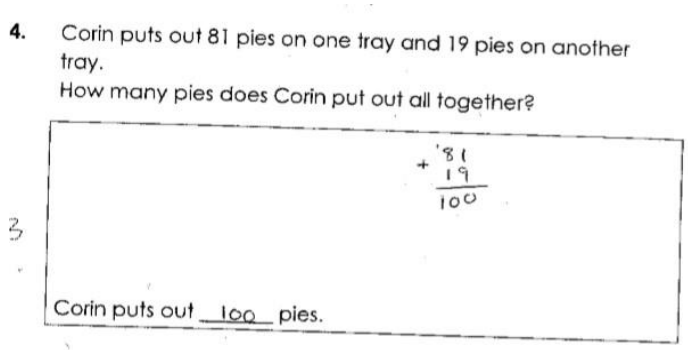
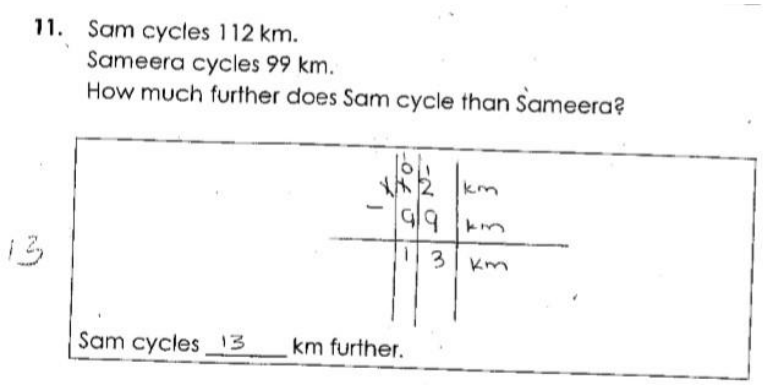
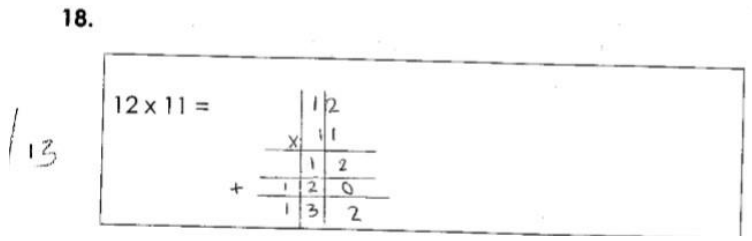
Repeated subtraction:

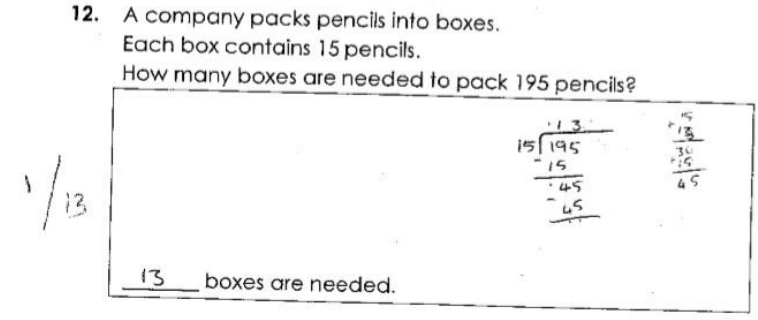
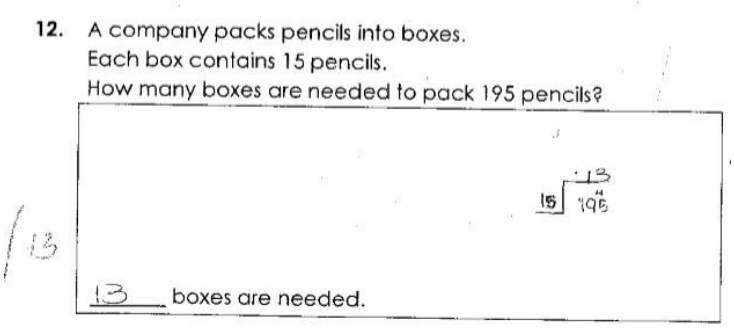
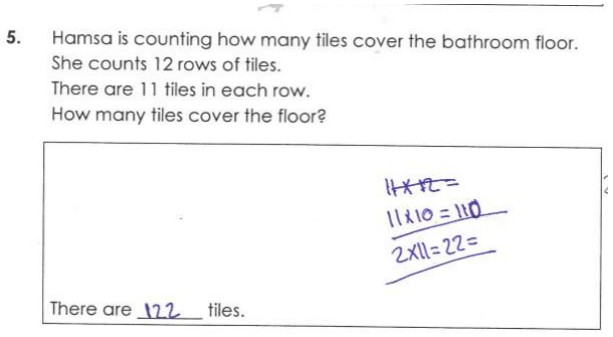
If repeated subtraction was used on the division operation, the divisor was repeatedly subtracted from the dividend until it was a zero then the answer was the number of times the divisor was subtracted.

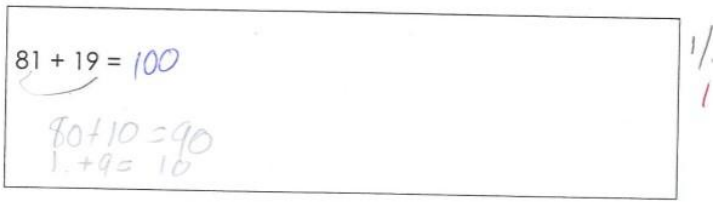
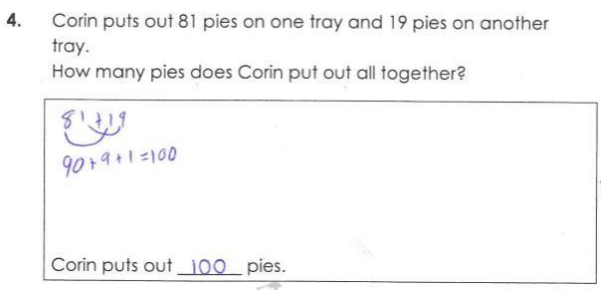
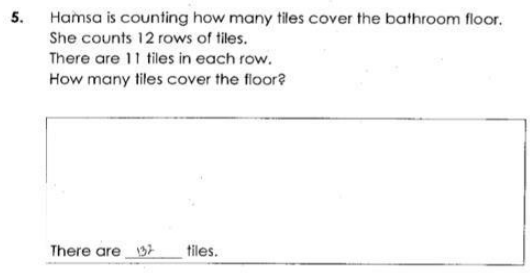


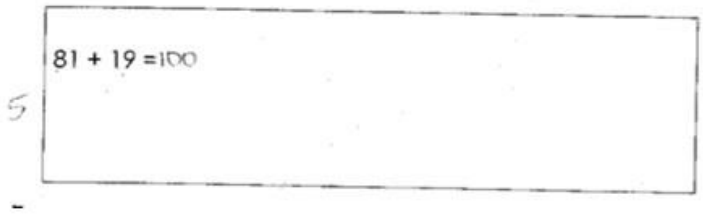
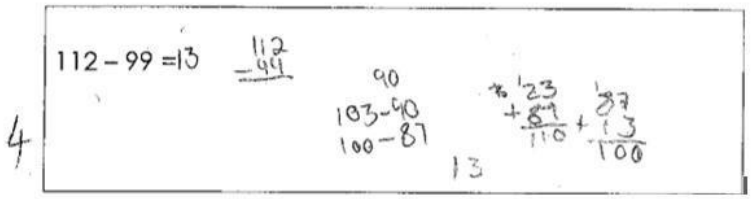
T- table or double number line	12	Signifies a solution strategy that used a T-table or double number line as solution workings. This solution strategy is comprised of establishing a relationship of numbers in the two columns of the T-Table or rows of the double number line by establishing the pattern of getting the multiples of the multiplier or divisor. T-table: In multiplication:  5. Hamsa is counting how many tiles cover the bathroom floor. She counts 12 rows of tiles. There are 11 tiles in each row. How many tiles cover the floor? 12 Rows | Tiles 1 | 11 3 | 33 6 | 66 12 | 132 There are 132 tiles. In division  12. A company packs pencils into boxes. Each box contains 15 pencils. How many boxes are needed to pack 195 pencils? 12 Box | Pencils 1 | 15 2 | 30 4 | 60 5 | 75 13 → | 195 13 boxes are needed. Double number line: Vertical double number line:

		 12. A company packs pencils into boxes. Each box contains 15 pencils. How many boxes are needed to pack 195 pencils? 1 15 8 120 2 30 9 135 3 45 10 150 4 60 11 165 5 75 12 180 6 90 13 195 7 105 14 13 boxes are needed. Horizontal double number line:  12. A company packs pencils into boxes. Each box contains 15 pencils. How many boxes are needed to pack 195 pencils? 4 8 9 10 11 12 13 14 135 150 165 120 135 150 165 180 195 17 boxes are needed.
Standard algorithm	13	Denotes solution strategies that showed the standard algorithms for all the operations. A standard algorithm is a step by step way to solve a problem. This standard algorithm strategy includes the column algorithms for addition, subtraction and multiplication in which the numbers had to be aligned vertically so that the digits on the same position represented the same place value. The place values are read from right to left as units, tens and hundreds. Sub-additions/subtractions/multiplications are carried out from the right to the left starting with the units followed by the tens then hundreds. In the case where the sub-additions was more than 10 or sub-subtractions where a bigger number

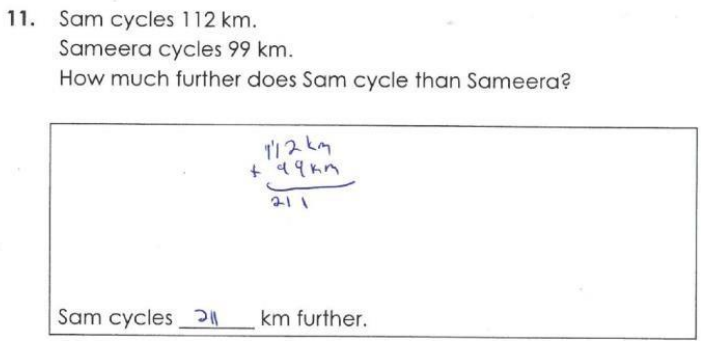
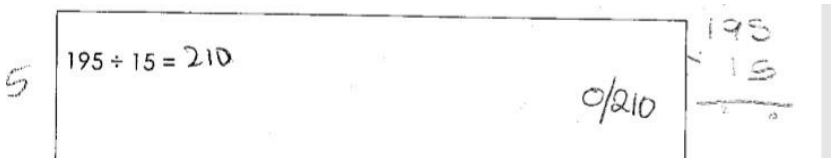
		is subtracted from a smaller number, regrouping was expected to have taken place Addition  4. Corin puts out 81 pies on one tray and 19 pies on another tray. How many pies does Corin put out all together? 3 + 81 / 19 / 100 Corin puts out 100 pies. Question 4 above showed the column algorithm for addition Subtraction  11. Sam cycles 112 km. Sameera cycles 99 km. How much further does Sam cycle than Sameera? 13 1 1 2 km − 9 9 km 1 3 km Sam cycles 13 km further. The question 11 above was an example of column algorithm for subtraction Multiplication  18. 13 12 x 11 = 1 2 x 1 1 1 2 + 1 2 0 1 3 2 In the response for question 18, the column algorithm of multiplication was illustrated Division For division the standard algorithm strategy includes either the long division or short division algorithm in which the numbers had to be aligned horizontally with a vertical line in between the divisor and the dividend. A horizontal line was underneath the divisor and another horizontal line above the dividend or

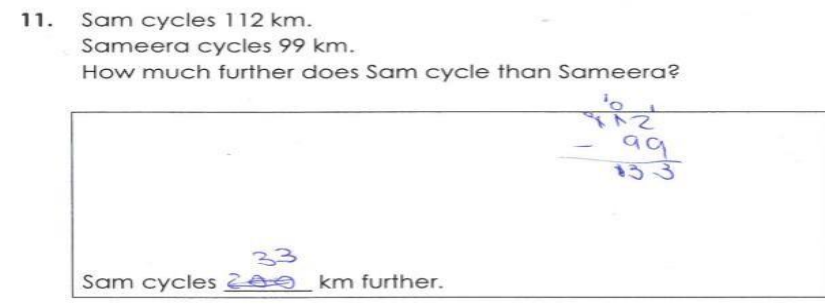
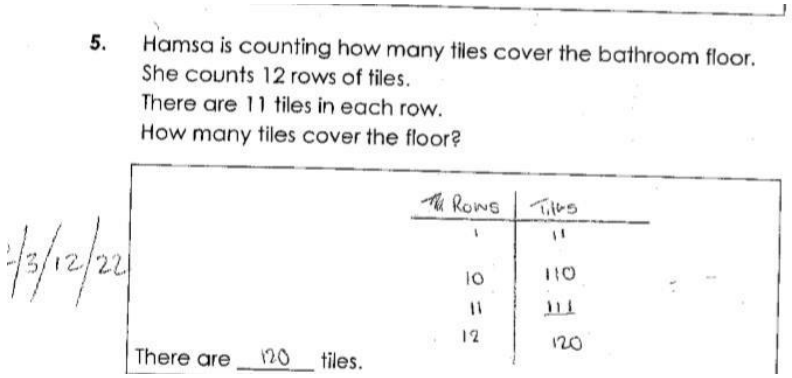
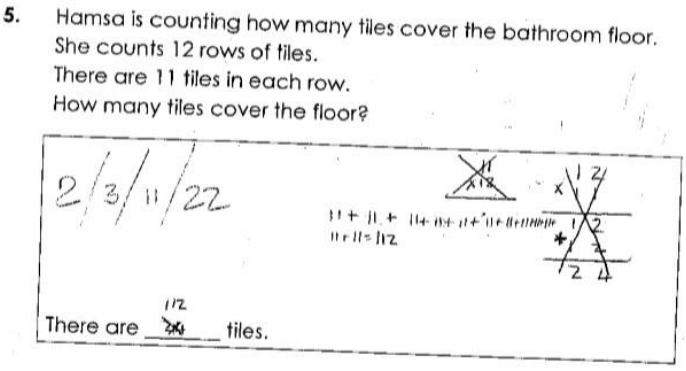
		the dividend could be written above the divisor as in the long division. The working was done from the left to the right. Sub-divisions were done by dividing the divisor into the two highest values of the dividend since it was a division of a three digit number by a two digit number. Long division:  12. A company packs pencils into boxes. Each box contains 15 pencils. How many boxes are needed to pack 195 pencils? 13 boxes are needed. The long division algorithm was shown in the response for question 12 above. Short division:  12. A company packs pencils into boxes. Each box contains 15 pencils. How many boxes are needed to pack 195 pencils? 13 boxes are needed. Another response for question 12 showing the short division algorithm is shown above. Another standard algorithm involves the split of the numbers according to place value of units, tens and hundreds. In word problem:  5. Hamsa is counting how many tiles cover the bathroom floor. She counts 12 rows of tiles. There are 11 tiles in each row. How many tiles cover the floor? There are 122 tiles. Pure numeric

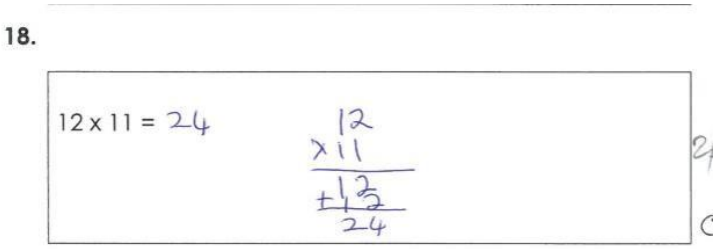
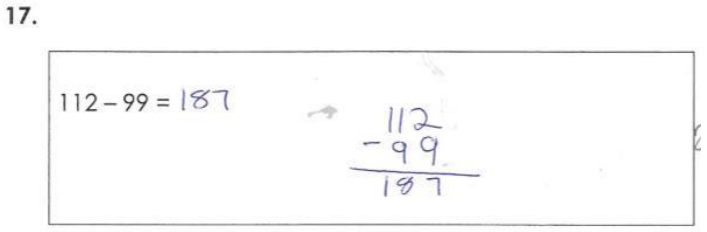
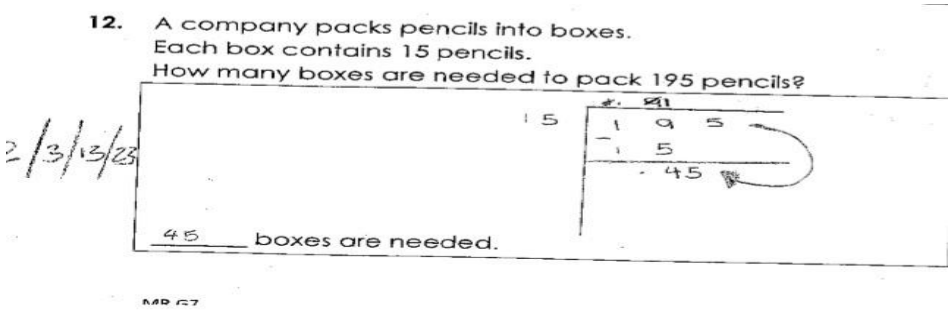
		16.  1/ 1 Though there are literature disagreements on the relationship between a number sentence and a standard algorithm, in this research number sentences were considered as standard algorithms. Since a number sentence involves construction of a mathematical structure, cognitively it should entail a step by step way of constructing the structure to solve the problem at hand as shown in question 4 below. 
Answer only with no visible working	14	Represents a strategy of presenting an answer only without any visible working.  Or

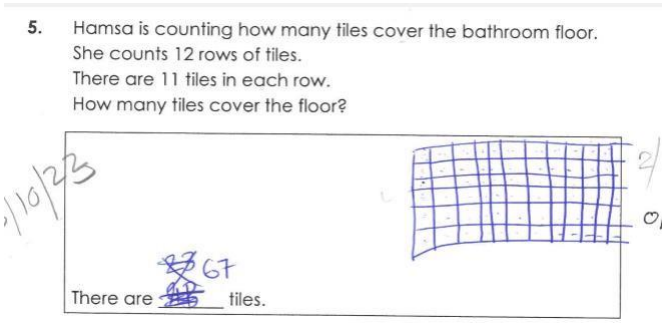
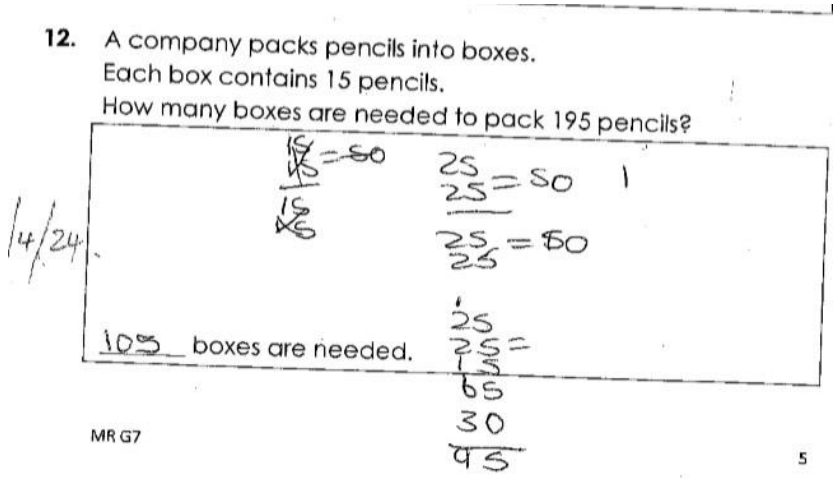
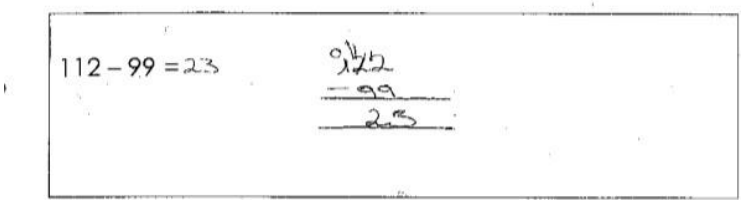
		16. 
Others	15	All other solution strategies that were not able to be coded as code 10, 11, 12, 13 or 14 but which could be followed to get to know how the answer was obtained were assigned code 15. 17.  The solution above can be followed and see all the steps involved in the solution strategy as to how the answer was arrived at but the solution strategy cannot be codded as 10, 11, 12, 13 or 14.

Appendix 4: Conceptualisation of the category Errors.

Errors	Codes	Identification Descriptors
Operation choice error	21	Manifests when learners carried out an operation different from the one required by the context. Word problem:  In question 11 above learner presented the column algorithm for addition and chose to add the two numbers in place of subtracting. Pure numerical problem:  The response shows the column algorithm for addition and the learner added the two numbers instead of dividing them.

Computational error	22	These involve wrong additions/subtractions/multiplications/divisions of numbers in the question.  11. Sam cycles 112 km. Sameera cycles 99 km. How much further does Sam cycle than Sameera? Sam cycles 33 km further. In question 11, the learner did not compute the sub-subtraction of 10-9 correctly resulting in the computational error.  5. Hamsa is counting how many tiles cover the bathroom floor. She counts 12 rows of tiles. There are 11 tiles in each row. How many tiles cover the floor? 2/3/12/22 There are 120 tiles. In the response for question 5 above, the multiplication of 11 and 11 was wrongly done and gave a result of 111 in place of 121.  5. Hamsa is counting how many tiles cover the bathroom floor. She counts 12 rows of tiles. There are 11 tiles in each row. How many tiles cover the floor? 2/3/11/22 There are 112 tiles. 1n the response above, the repeated addition of 11 was wrongly done and the result was 112 instead of 132.

Procedural error	23	Procedural errors were identifiable through wrong steps of executing a procedure of a defined operation or incomplete workings with respect to a specific strategy. Wrong steps included failing to regroup when there is a need to or writing answers of the sub-additions/subtractions/ multiplications/divisions on the wrong place or operating from the left to the right or vice versa. Examples of these are presented below.  In the response to question 18 above, the solution workings showed that the learner did not write the answers to sub-multiplications at the correct places according to place values resulting in the wrong procedure of multiplying.  In the response to question 17 above, the solution workings showed that the learner did not do the step of regrouping when subtracting a bigger number from a smaller number. The learner just subtracted the smaller number from a bigger number without considering which one is the subtrahend or the minuend.  The solution workings to the response of question 12 above showed incomplete steps of long division, instead of continuing dividing until the dividend of the sub-divisions is zero the learner opted to write the sub dividend as the answer.

		 5. Hamsa is counting how many tiles cover the bathroom floor. She counts 12 rows of tiles. There are 11 tiles in each row. How many tiles cover the floor? There are 67 tiles. In question 5 above, the response presented incomplete drawings that resulted in a wrong answer.
Others	24	All other errors that were presented on solution workings that resulted in an incorrect answer, but could not be coded as 21, 22 or 23. Examples of responses that were coded 24 are shown below.  12. A company packs pencils into boxes. Each box contains 15 pencils. How many boxes are needed to pack 195 pencils? 105 boxes are needed. MR G7 5 In the response above, numbers that are not involved in the question were used in the solution strategy. 17.  112 – 99 = 23 In question 17 above, the response presented a copying error of 112 as 122 resulting in a wrong answer.

Appendix 5: Extract of the computation of the coding reliability.

FILENAME	ChemeVSYvonne.csv
file size	5152 bytes
n columns	64
n variables	32
n coders per var	2

	Percent Agreement	Scott's Pi	Cohen's Kappa	Krippendorff's Alpha	N Agreements	N Disagreements	N Cases	N Decisions
Variable 1 (cols 1 & 2)	100	1	1	1	32	0	32	64
Variable 2 (cols 3 & 4)	100	1	1	1	32	0	32	64
Variable 3 (cols 5 & 6)	100	1	1	1	32	0	32	64
Variable 4 (cols 7 & 8)	100	1	1	1	32	0	32	64
Variable 5 (cols 9 & 10)	96.875	0.932131	0.932203	0.933192	31	1	32	64
Variable 6 (cols 11 & 12)	100	1	1	1	32	0	32	64
Variable 7 (cols 13 & 14)	96.875	0.957475	0.957503	0.95814	31	1	32	64
Variable 8 (cols 15 & 16)	100	1	1	1	32	0	32	64
Variable 9 (cols 17 & 18)	100	1	1	1	32	0	32	64
Variable 10 (cols 19 & 20)	96.875	0.944493	0.944541	0.94536	31	1	32	64
Variable 11 (cols 21 & 22)	93.75	0.893333	0.893511	0.895	30	2	32	64
Variable 12 (cols 23 & 24)	93.75	0.775044	0.775439	0.778559	30	2	32	64
Variable 13 (cols 25 & 26)	100	1	1	1	32	0	32	64
Variable 14 (cols 27 & 28)	100	1	1	1	32	0	32	64
Variable 15 (cols 29 & 30)	100	1	1	1	32	0	32	64
Variable 16 (cols 31 & 32)	96.875	0.941659	0.941712	0.942571	31	1	32	64
Variable 17 (cols 33 & 34)	100	1	1	1	32	0	32	64
Variable 18 (cols 35 & 36)	100	1	1	1	32	0	32	64
Variable 19 (cols 37 & 38)	100	1	1	1	32	0	32	64
Variable 20 (cols 39 & 40)	100	1	1	1	32	0	32	64
Variable 21 (cols 41 & 42)	100	1	1	1	32	0	32	64
Variable 22 (cols 43 & 44)	96.875	0.937561	0.937622	0.938537	31	1	32	64
Variable 23 (cols 45 & 46)	96.875	0.936946	0.937008	0.937931	31	1	32	64
Variable 24 (cols 47 & 48)	93.75	0.848161	0.848341	0.850534	30	2	32	64
Variable 25 (cols 49 & 50)	100	1	1	1	32	0	32	64
Variable 26 (cols 51 & 52)	100	1	1	1	32	0	32	64
Variable 27 (cols 53 & 54)	100	1	1	1	32	0	32	64
Variable 28 (cols 55 & 56)	100	1	1	1	32	0	32	64
Variable 29 (cols 57 & 58)	100	1	1	1	32	0	32	64
Variable 30 (cols 59 & 60)	100	1	1	1	32	0	32	64
Variable 31 (cols 61 & 62)	100	1	1	1	32	0	32	64
Variable 32 (cols 63 & 64)	100	1	1	1	32	0	32	64

Appendix 6: Summary of Results of the performance of the learners per test item, problem format and mathematical reasoning in the pre-test.

Pre-test	**Additive reasoning**							**Multiplicative reasoning**									**GRAND TOTAL**
	Word problems			**Pure numerical problems**				**Word problems**			**Pure numerical problems**				**Total word problems**	**Total pure numerical problems**	
	Q4	Q11	Total	Q16	Q17	Total	Total	Q5	Q12	Total	Q18	Q19	Total	Total			
Correct	(118) 83 %	(77) 54 %	(195) 68%	(119) 83 %	(70) 49 %	(189) 66%	(384) 67%	(45) 31%	(22) 15 %	(67) 23%	(53) 37 %	(31) 22 %	(84) 29%	(151) 26%	(262) 46 %	(273) 48 %	(535) 47 %
Incorrect	(24) 17 %	(60) 42 %	(84) 29%	(16) 11%	(65) 45%	(81) 28%	(165) 29%	(96) 67 %	(113) 79 %	(209) 73%	(82) 57 %	(92) 64 %	(174) 61%	(383) 67%	(293) 51 %	(255) 45 %	(548) 48 %
blank	(1) 0 %	(6) 4 %	(7) 2%	(8) 6 %	(8) 6 %	(16) 6%	(23) 4%	(2) 1 %	(8) 6 %	(10) 3%	(8) 6 %	(20) 14 %	(28) 10%	(38) 7%	(17) 3 %	(44) 8 %	(61) 5 %
total	143	143	286	143	143	286	572	143	143	286	143	143	286	572	572	572	1144

Appendix 7: Extract of statistical computations to examine difference between performances across problem format in pre-test

Difference Scores Calculations

Treatment 1

N_1: 143
$df_1 = N - 1$ =9 143 - 1 = 142
M_1: 1.91 SS_1: 213.82 $s^2_1 = SS_1/(N - 1)$ =
213.82/(143-1) = 1.51

Treatment 2

N_2: 143
$df_2 = N - 1 = 143 - 1 = 142$
M_2: 1.83 SS_2: 185.97 $s^2_2 = SS_2/(N - 1)$ =
185.97/(143-1) = 1.31

T-value Calculation

$s^2_p = ((df_1/(df_1 + df_2)) * s^2_1) + ((df_2/(df_2 + df_2)) * s^2_2) = ((142/284) * 1.51) + ((142/284) * 1.31) = 1.41$

$s^2_{M1} = s^2_p/N_1 = 1.41/143 = 0.01$ s^2_{M2}
$= s^2_p/N_2 = 1.41/143 = 0.01$

$t = (M_1 - M_2)/\sqrt{(s^2_{M1} + s^2_{M2})} = 0.08/\sqrt{0.02} = 0.55$

Significance Level: 0.05

The *t*-value is 0.54822. The *p*-value is .291987. The result is *not* significant at $p < .05$.

Appendix 8: Extract of statistical computations to examine difference between performances across mathematical reasoning format in pre-test

Difference Scores Calculations

Treatment 1

N_1: 143
$df_1 = N - 1 = 143 - 1 = 142$
M_1: 2.69 SS_1:
208.84
$s^2_1 = SS_1/(N - 1) = 208.84/(143-1) = 1.47$

Treatment 2

N_2: 143
$df_2 = N - 1 = 143 - 1 = 142$
M_2: 1.06 SS_2:
243.55
$s^2_2 = SS_2/(N - 1) = 243.55/(143-1) = 1.72$

T-value Calculation

$s^2_p = ((df_1/(df_1 + df_2)) * s^2_1) + ((df_2/(df_2 + df_2)) * s^2_2) = ((142/284) * 1.47) + ((142/284) * 1.72) = 1.59$

$s^2_{M1} = s^2_p/N_1 = 1.59/143 = 0.01$ s^2_{M2}
$= s^2_p/N_2 = 1.59/143 = 0.01$

$t = (M_1 - M_2)/\sqrt{(s^2_{M1} + s^2_{M2})} = 1.63/\sqrt{0.02} = 10.92$

The *t*-value is 10.91628. The *p*-value is < .00001. The result is significant at $p < .05$.

Appendix 9: Summary of Results of the performance of the learners per test item, problem format and mathematical reasoning in the post-test

Post-test	**Additive reasoning**							**Multiplicative reasoning**									
	Word problems			**Pure numerical problems**				**Word problems**			**Pure numerical problems**				**Total word problems**	**Total pure numerical problems**	
	Q4	Q11	total	Q16	Q17	Total	Total	Q5	Q12	total	Q18	Q19	total	Total			
Correct	(126) 88%	(83) 58%	(209) 73%	(125) 87%	(90) 63%	(215) 75%	(424) 74%	(74) 52%	(39) 27%	(113) 40%	(81) 57%	(52) 36%	(133) 47%	(246) 43%	(322) 56%	(348) 61%	(670) 59%
Incorrect	(16) 11%	(58) 41%	(74) 26%	(14) 10%	(44) 31%	(58) 20%	(132) 23%	(68) 48%	(101) 71%	(169) 59%	(52) 36%	(75) 52%	(127) 44%	(296) 52%	(243) 42%	(185) 32%	(428) 37%
Blank	(1) 0%	(2) 1%	(3) 1%	(4) 3%	(9) 6%	(13) 5%	(16) 3%	(1) 0%	(3) 2%	(4) 1%	(10) 7%	(16) 11%	(26) 9%	(30) 5%	(7) 1%	(39) 7%	(46) 4%
Total	143	143	286	143	143	286	572	143	143	286	143	143	286	572	572	572	1144

Appendix 10: Extract of statistical computations to prove difference between performances across problem format in post-test across problem format

Difference Scores Calculations

Treatment 1

N_1: 143
$df_1 = N - 1 = 143 - 1 = 142$
M_1: 2.25 SS_1: 216.94 $s^2_1 = SS_1/(N - 1) =$
216.94/(143-1) = 1.53

Treatment 2

N_2: 143
$df_2 = N - 1 = 143 - 1 = 142$
M_2: 2.43 SS_2: 223.12 $s^2_2 = SS_2/(N - 1) =$
223.12/(143-1) = 1.57

T-value Calculation

$s^2_p = ((df_1/(df_1 + df_2)) * s^2_1) + ((df_2/(df_2 + df_2)) * s^2_2) = ((142/284) * 1.53) + ((142/284) * 1.57) = 1.55$

$s^2_{M1} = s^2_p/N_1 = 1.55/143 = 0.01$ s^2_{M2}
$= s^2_p/N_2 = 1.55/143 = 0.01$

$t = (M_1 - M_2)/\sqrt{(s^2_{M1} + s^2_{M2})} = -0.18/\sqrt{0.02} = -1.24$

Significance Level: 0.05

The *t*-value is -1.23508. The *p*-value is .108911. The result is *not* significant at $p < .05$.

Appendix 11: Extract of statistical computations to examine difference between performances across mathematical reasoning format in post-test

Difference Scores Calculations

Treatment 1

N_1: 143
$df_1 = N - 1 = 143 - 1 = 142$
M_1: 2.97 SS_1:
194.83
$s^2_1 = SS_1/(N - 1) = 194.83/(143-1) = 1.37$

Treatment 2

N_2: 143
$df_2 = N - 1 = 143 - 1 = 142$
M_2: 1.72 SS_2:
288.81
$s^2_2 = SS_2/(N - 1) = 288.81/(143-1) = 2.03$

T-value Calculation

$s^2_p = ((df_1/(df_1 + df_2)) * s^2_1) + ((df_2/(df_2 + df_2)) * s^2_2) = ((142/284) * 1.37) + ((142/284) * 2.03) = 1.7$

$s^2_{M1} = s^2_p/N_1 = 1.7/143 = 0.01$ s^2_{M2}
$= s^2_p/N_2 = 1.7/143 = 0.01$

$t = (M_1 - M_2)/\sqrt{(s^2_{M1} + s^2_{M2})} = 1.24/\sqrt{0.02} = 8.07$

The *t*-value is 8.0656. The *p*-value is < .00001. The result is significant at $p < .05$

Appendix 12: Extract of statistical computations to examine difference between performances in pre-test and post test

Difference Scores Calculations

Treatment 1

N_1: 143
$df_1 = N - 1 = 143 - 1 = 142$
M_1: 3.74 SS_1: 673.43 $s^2_1 = SS_1/(N - 1) =$
673.43/(143-1) = 4.74

Treatment 2

N_2: 143
$df_2 = N - 1 = 143 - 1 = 142$
M_2: 4.69 SS_2: 744.84 $s^2_2 = SS_2/(N - 1) =$
744.84/(143-1) = 5.25

T-value Calculation

$s^2_p = ((df_1/(df_1 + df_2)) * s^2_1) + ((df_2/(df_2 + df_2)) * s^2_2) = ((142/284) * 4.74) + ((142/284) * 5.25) = 4.99$

$s^2_{M1} = s^2_p/N_1 = 4.99/143 = 0.03$ s^2_{M2}
$= s^2_p/N_2 = 4.99/143 = 0.03$

$t = (M_1 - M_2)/\sqrt{(s^2_{M1} + s^2_{M2})} = -0.94/\sqrt{0.07} = -3.57$ Significance

Level:

The *t*-value is -3.57216. The *p*-value is .000208. The result is significant at $p < .05$.

Appendix 13: Summary of pre-test comprehension and incorrect answers with no visible working.

Pre-test	Q4	Q5	Q11	Q12	Total	Q16	Q17	Q18	Q19	Total	Grand total
Correct comprehension	(129) 90%	(76) 53%	(90) 63%	(43) 30%	(338) 59%	(123) 88%	(94) 66%	(77) 54%	(43) 30%	(338) 59%	(676) 59%
Incorrect comprehension	(5) 3%	(25) 17%	(17) 12%	(39) 27%	(86) 15%	(0) 0%	(5) 3%	(3) 2%	(15) 10%	(23) 4%	(110) 10%
Wrong with no visible working	(6) 4%	(40) 28%	(30) 21%	(53) 37%	(131) 23%	(11) 8%	(36) 25%	(55) 38%	(64) 45%	(166) 29%	(297) 26%

Appendix 14: Summary of frequencies and percentages of the solution strategies used per test item, problem format and mathematical reasoning in the pre-test

Pre -test		**Additive reasoning problems**							**Multiplicative reasoning**									**Grand Total**
		Word problems			**Pure numerical problems**				**Word problems**			**Pure numerical problems**				**Total Word problems**	**Total Pure numerical problems**	
		Q4	Q11	Total	Q16	Q17	Total	Total	Q5	Q12	Total	Q18	Q19	Total	Total			
Drawings of solid objects, counters and tallies	Extent	(5) 4%	(9) 8%	(14) 5%	(3) 2%	(2) 2%	(5) 2%	(19) 4%	(6) 5%	(3) 3%	(9) 4%	(2) 2%	(1) 0%	(3) 1%	(12) 3%	(23) 5%	(8) 2%	(31) 3%
	Correct answers	(4) 80%	(4) 44%	(8) 57%	(3) 100%	(2) 100%	(5) 100%	(13) 68%	(1) 17%	(1) 33%	(2) 22%	(1) 50%	(0) 0%	(1) 33%	(3) 25%	(10) 43%	(6) 86%	(16) 52%
Repeated addition/subtraction	Extent	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(16) 14%	(15) 16%	(31) 14%	(5) 4%	(5) 5%	(10) 4%	(41) 9%	(31) 7%	(10) 2%	(41) 4%
	Correct answers	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(6) 38%	(6) 40%	(12) 39%	(1) 20%	(2) 40%	(3) 30%	(15) 37%	(12) 39%	(3) 27%	(15) 37%
T-table/Double number line	Extent	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%
	Correct answers	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%
Standard algorithm	Extent	(82) 59%	(40) 33%	(122) 47%	(46) 34%	(47) 36%	(93) 35%	(215) 41%	(39) 34%	(19) 20%	(58) 27%	(39) 30%	(19) 18%	(58) 24%	(116) 26%	(180) 38%	(151) 30%	(331) 34%
	Correct answers	(72) 88%	(32 80%	(104) 85%	(41) 89%	(25) 53%	(66) 71%	(170) 79%	(24) 62%	(11) 58%	(35) 60%	(20) 51%	(11) 58%	(31) 53%	(66) 57%	(139) 30%	(97) 64%	(236) 71%
Answer only-No visible working	Extent	(51) 37%	(70) 58%	(121) 47%	(85) 63%	(77) 59%	(162) 61%	(283) 54%	(52) 45%	(56) 59%	(108) 51%	(86) 65%	(82) 76%	(168) 70%	(276) 61%	(229) 49%	(330) 65%	(559) 57%
	Correct answers	(42) 82%	(40) 57%	(82) 67%	(74) 87%	(42) 55%	(116) 72%	(198) 70%	(12) 23%	(4) 7%	(16) 15%	(31) 36%	(18) 22%	(49) 29%	(65) 24%	(98) 43%	(165) 50%	(263) 47%
other	Extent	(0) 0%	(1) 0%	(1) 0%	(1) 0%	(4) 3%	(5) 2%	(6) 1%	(3) 3%	(2) 2%	(5) 2%	(0) 0%	(1) 0%	(1) 0%	(6) 1%	(6) 1%	(6) 1%	(12) 1%
	Correct answers	(0) 0%	(1) 100%	(1) 100%	(1) 100%	(1) 25%	(2) 40%	(3) 50%	(2) 67%	(0) 0%	(2) 40%	(0) 0%	(0) 0%	(0) 0%	(2) 33%	(3) 50%	(2) 33%	(5) 42%
Total	Extent	138	120	258	135	130	265	523	116	95	211	132	108	240	451	469	505	974
Total	Correct answers	118	77	195	119	70	189	384	45	22	67	53	31	84	151	262	273	535

Appendix 15: Summary of post-test comprehension and incorrect answers with no visible working

Post-test	Word problems					Pure numerical problems					Grand total
	Q4	Q11	Q5	Q12	totals	Q16	Q17	Q18	Q19	totals	
Correct comprehension	(132) 92%	(88) 62%	(106) 74%	(72) 50%	(398) 70%	(128) 90%	(114) 80%	(92) 64%	(64) 45%	(398) 70%	(796) 70%
Incorrect comprehension	(5) 3%	(25) 17%	(17) 12%	(31) 22%	(78) 14%	(1) 0%	(2) 1%	(1) 0%	(9) 6%	(13) 2%	(91) 8%
Wrong answer with no visible working	(6) 4%	(30) 21%	(20) 14%	(40) 28%	(96) 17%	(14) 10%	(27) 19%	(50) 35%	(72) 50%	(163) 28%	(259) 23%

Appendix 16: Summary of frequencies and percentages of the solution strategies used per test item, problem format and mathematical reasoning in the post-test

Post-test		Additive Reasoning problems							Multiplicative Reasoning problems									Grand total
		Word Problems			Pure Numerical problems				Word Problems			Pure Numerical problems				Total word problems	Total pure numerical problems	
Solution strategies		Q4	Q11	Total	Q16	Q17	Total	Total	Q5	Q12	Total	Q18	Q19	Total	Total			
Drawings of solid objects, counters and tallies	Extent	(1) 1%	(1) 1%	(2) 1%	(1) 1%	(0) 0%	(1) 0%	(3) 1%	(5) 4%	(0) 0%	(5) 2%	(1) 0%	(0) 0%	(1) 0%	(6) 1%	(7) 1%	(2) 0%	(9) 1%
	Correct answers	(1) 100%	(1) 100%	(2) 100%	(1) 100%	(0) 0%	(1) 100%	(3) 100%	(1) 20%	(0) 0%	(1) 20%	(0) 0%	(0) 0%	(0) 0%	(1) 17%	(3) 43%	(1) 50%	(4) 44%
Repeated addition/subtraction	Extent	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(8) 6%	(17 16%	(25) 11%	(2) 2%	(5) 4%	(7) 3%	(32) 7%	(25) 5%	(7) 1%	(32) 3%
	Correct answers	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(6) 75%	(4) 24%	(10) 40%	(1) 50%	(1) 20%	(2) 29%	(12) 38%	(10) 40%	(2) 29%	(12) 38%
T-table/Double number line	Extent	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(18) 14%	(19) 18%	(37) 16%	(1) 0%	(1) 0%	(2) 0%	(39) 8%	(37 8%	(2) 0%	(39) 4%
	Correct answers	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(0) 0%	(11) 61%	(10) 53%	(21) 67%	(0) 0%	(1) 100%	(1) 50%	(22) 56%	(21) 57%	(1) 50%	(22) 56%
Standard algorithm	Extent	(89) 65%	(55) 47%	(144) 57%	(39) 28%	(49) 38%	(88) 33%	(232) 44%	(47) 38%	(27) 25%	(74) 32%	(27 21%	(23) 19%	(50) 20%	(124) 26%	(218) 45%	(138) 27%	(356) 35%
	Correct answers	(83) 93%	(49) 89%	(132) 92%	(37) 95%	(27) 55%	(64) 73%	(196) 84%	(30 64%	(17) 65%	(47) 64%	(19) 70%	(17) 74%	(36) 72%	(83) 67%	(179) 82%	(100) 72%	(279) 78%
Answer only with no visible working	Extent	(43) 31%	(56) 48%	(99) 39%	(95) 69%	(75) 58%	(170) 63%	(269) 52%	(45) 36%	(45) 42%	(90) 39%	(99) 76%	(87) 73%	(186) 74%	(276) 57%	(187) 39%	(356) 68%	(545) 54%
	Correct answers	(38) 88%	(29) 52%	(67) 68%	(85) 89%	(58) 77%	(143) 84%	(210) 78%	(26) 58%	(8) 18%	(34) 38%	(60) 61%	(31) 36%	(91) 49%	(125) 45%	(101) 54%	(234) 66%	(335) 61%
Other	Extent	(4) 3%	(4) 3%	(8) 3%	(3) 2%	(7) 5%	(10) 4%	(18) 3%	(2) 2%	(1) 1%	(3) 9%	(1) 0%	(4) 3%	(5) 2%	(8) 2%	(11) 2%	(15) 3%	(26) 3%
	Correct answers	(4) 100%	(4) 100%	(8) 100%	(2) 67%	(5) 71%	(7) 70%	(15) 83%	(0) 0%	(0) 0%	(0) 0%	(1) 100%	(2) 50%	(3) 60%	(3) 38%	(8) 73%	(10) 67%	(18) 69%
Total	Extent	137	116	253	138	131	269	522	125	109	234	131	120	251	485	487	520	1007
Total	Correct answers	126	83	209	125	90	215	424	74	39	113	81	52	133	246	322	348	670

Appendix 17: Summary of frequencies and percentages of detectable errors per test item, problem format, mathematical reasoning in the pre-test.

Pre-test	**Additive reasoning**							**Multiplicative reasoning**									**GRAND TOTAL**
	Word problems			**Pure numerical problems**				**Word problems**			**Pure numerical problems**				**Total word problems**	**Total pure numerical problems**	
	Q4	Q11	total	Q16	Q17	Total	Total	Q5	Q12	Total	Q18	Q19	Total	Total			
Operation choice error	(0) 0%	(14) 48%	(14) 32%	(0) 0%	(1) 3%	(1) 3%	(15) 19%	(21) 38%	(27) 44%	(48) 41%	(2) 7%	(4) 14%	(6) 11%	(54) 31%	(62) 39%	(7) 8%	(69) 27%
Computational error	(2) 13%	(2) 7%	(4) 9%	(2) 40%	(1) 3%	(3) 9%	(7) 9%	(11) 20%	(7) 11%	(18) 15%	(4) 15%	(5) 18%	(9) 16%	(27) 16%	(22) 14%	(12) 13%	(34) 14%
Procedural error	(3) 20%	(9) 31%	(12) 27%	(3) 60%	(21) 70%	(24) 69%	(36) 46%	(17) 30%	(11) 18%	(28) 24%	(19) 70%	(3) 11%	(22) 40%	(50) 29%	(40) 25%	(46) 51%	(86) 34%
Other	(10) 67%	(4) 14%	(14) 32%	(0) 0%	(7) 23%	(7) 20%	(21) 27%	(7) 13%	(16) 26%	(23) 20%	(2) 7%	(16) 57%	(18) 33%	(41) 24%	(37) 23%	(25) 28%	(62) 25%
TOTAL	15	29	44	5	30	35	79	56	61	117	27	28	55	172	161	90	251

Appendix 18: Summary of frequencies and percentages of detectable errors per test item, problem format, mathematical reasoning in the post-test.

Post-test	**Additive reasoning**							**Multiplicative reasoning**									**GRAND TOTAL**
	Word problems			**Pure numerical problems**				**Word problems**			**Pure numerical problems**				**Total Word problems**	**Total Pure numerical problems**	
	Q4	Q11	Total	Q16	Q17	Total	Total	Q5	Q12	Total	Q18	Q19	Total	Total			
Operation choice error	(3) 23%	(19) 59%	(22) 49%	(0) 0%	(1) 4%	(1) 3%	(23) 31%	(7) 18%	(24) 38%	(31) 31%	(0) 0%	(2) 11%	(2) 6%	(33) 25%	(53) 36%	(3) 5%	(56) 27%
Computational error	(3) 23%	(4) 13%	(7) 16%	(0) 0%	(3) 12%	(3) 10%	(10) 14%	(16) 42%	(14) 22%	(30) 30%	(1) 8%	(6) 33%	(7) 23%	(37) 28%	(37) 25%	(10) 17%	(47) 23%
Procedural error	(2) 15%	(1) 3%	(3) 7%	(2) 67%	(19) 73%	(21) 72%	(24) 32%	(15) 39%	(14) 22%	(29) 29%	(9) 69%	(4) 22%	(13) 42%	(42) 32%	(32) 22%	(34) 57%	(66) 32%
Other	(5) 38%	(8) 25%	(13) 29%	(1) 33%	(3) 12%	(4) 14%	(17) 23%	(0) 0%	(11) 17%	(11) 11%	(3) 23%	(6) 33%	(9) 29%	(20) 15%	(24) 16%	(13) 22%	(37) 18%
TOTAL	13	32	45	3	26	29	74	38	63	101	13	18	31	132	146	60	206