

Final Submission:
*Geometric and function analytic approaches to the
study of Sturm-Liouville theory.*
Correction report

Thomas T. Roth

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1 First examiner

Corrections according to numbering in 'suggested changes and corrections' section.

1) The history chapter has been merged with the Introduction chapter by adding history onto the end of introduction as a section.

2) Typographical and grammatical errors have been corrected as suggested in the hard copy of the draft. These corrections correspond to the page numberings and lines in the draft. Line numbering excludes lines written in the latex equation and section environments.

Chapter 3: Functional analysis

p.9 1.15 From: .. 'solution is unique solution'

To: ... 'solution is a unique solution'

p.10 1.2 From:... 'has a non-zero solutions f , called eigenfunctions;'

To: ... 'has a non-zero solution f , called an eigenfunction;'

p.10 1.10 From:... 'the study of the expansion of an arbitrary function from $\mathcal{L}^2[0, 1]$ in terms of the eigenfunctions of L .'

To: ... 'the study of expansion of function from $\mathcal{L}^2[0, 1]$ in terms of eigenfunctions of L .'

p.11 1.15 From:... 'infinite with only possible'...

To: ... 'infinite with the only possible'...

p.11 1.18 From:... 'sequence of eigenfunctions of associated with'...

To: ... 'sequence of eigenfunctions of T associated with'...

p.12 1.4 From:... we conclude that $\alpha_i(\lambda_{n+1} - \lambda_i) = 0, i = 1, \dots, n$. By assumption...

To: ... 'we conclude that

$$\alpha_i(\lambda_{n+1} - \lambda_i) = 0, \quad \text{for } i = 1, \dots, n.$$

By assumption' ...

p.12 1.8 moved footnote directly onto associated word.

p.14 1.17 From:... '(or mor precisely projectors)'

To: ... '(or more precisely projectors)'

p.15 1.1 From: 'In the is section'...

To: 'In this section'

p.15 1.3 From:... 'and its satisfies'...

To: ... 'and it satisfies'...

p.16 1.2 From:... 'adding $-\frac{1}{2}ih_-$ to both'...

To: ... 'adding $L_0^*(-\frac{1}{2}ih_-)$ to both sides'...

p.17 1.10 From:... 'using Lemma 3.16'...

To: ... 'using Lemma 3.6'...

p.18 1.9 From: 'Methods of linear algebra give that if N ' ...

To: 'From methods of linear algebra we have that if N ' ...

p.19 1.17 From:... 'maximal domain which to which L '...

To: ... 'maximal domain for which L '...

p.20 1.10 From:... $\bar{y}(x) = \bar{y}_0 + \int_{x_0}^x A(\xi)\bar{y}(\xi) + \bar{g}(\xi)d\xi$.

To: ... $\bar{y}(x) = \bar{y}_0 + \int_{x_0}^x (A(\xi)\bar{y}(\xi) + \bar{g}(\xi)) d\xi$.

p.21 1.6 From:... 'thus from the induction step we'...

To: ... 'thus by induction we' ...

p.22 1.4 From:... 'sufficiently close to α that '...

To: ... 'sufficiently close to α such that' ...

p.24 1.14 From:... ' g orthogonal to the all solutions' ...

To: ... 'if g is orthogonal to all solutions'...

p.25 1.16 From:... 'operator is has domain'...

To: ... 'operator has domain'...

Chapter 4: Eigenvalue Asymptotics

p.29 1.3 From:... 'we show through the Liouville transformation (1.1) can be transformed to this form which will be considered'.

To: ... 'the Liouville transformation that (1.1) can be transformed to the form under consideration. '

p.29 1.5 From:... 'A concrete representation of the resolvent operator given in terms of the Green's kernel.'

To: ... 'A concrete representation of the resolvent operator is given in terms of the Green's kernel.'

p.30 1.6 From: 'We leave to later to show that'...

To: 'We will show later that'...

p.31 1.14 Calculation changed as recommended.

p.35 1.7 From:... 'the left hand side is equal to'...

To: ... 'the left hand side of (3.22) is equal to'...

p.46 1.7 From: ... 'if $f(x)$ of bounded variation'...

To: ... 'if $f(x)$ is of bounded variation'...

p.46 1.9 From:... $\frac{1}{2\pi i} \int R(L : \lambda) i f d\lambda$

To: ... $\frac{1}{2\pi i} \int_{\Gamma} R(L : \lambda) i f d\lambda$

Chapter 5: Sturm theorems

p.49 1.7 From:... 'for the final final'...

To: ... 'for the final'...

p.50 1.4 From:... 'zero's of a solutions of a Sturm-Liouville equation to the distribution of zero's of a solutions'...

To: ... 'zeros of solutions of a Sturm-Liouville equation to the distribution of zeros of solutions'...

p.50 1.13 Term in calculation changed from ψ' to ψ as recommended.

p.52 1.10 From: 'All that is left is to show that'...

To: 'All that is left to show is that'...

p.53 1.10 From:... 'functions of λ on (Λ_1, Λ_2) bounds on them'...

To: ... 'functions of λ on (Λ_1, Λ_2) and bounds on them'...

p.55 1.18 From:... 'that as λ in increases'...

To: ... 'that as λ increases'...

p.56 1.2 From:... 'oscillation theorem to the'...

To: ... 'oscillation theorem to'...

Chapter 6: Variational approach

p.62 Subsection heading has been corrected.

p.64 1.11 Comma in equation has been moved to correct position.

p.65 1.7 From:... 'on in the compact interval'...

To: ... 'on the compact interval'...

p.66 1.3 From:... 'see that is is the'...

To: ... 'see that is the'...

p.75 1.8 From: 'We take the following lemmas'...

To: 'We state the following lemmas'...

Chapter 7: Classes of Self-Adjoint Operators.

p.95 1.9 From:...’where defined in’...

To: ...’are as defined in’...

p.98 1.18 From:...’ we must show there exists’...

To: ...’ we must show that there exists’...

Appendix

p.106 : Revision considered, the appendix was kept the same as it complies with AMS style standards.

p.107 1.3 From:...’ we and using’...

To: ...’ and using’...

3) **p.33** Notation has been changed to remove any ambiguity.

p.35 The equation directly after line 7 has been corrected by inserting a $-$.

p.45 The examiners concern is dealt with in the description of the problem (4.1)

p.47 The inequality following line 5 is valid.

4) **p.14** Removed unimportant details to make the ideas in the paragraph clearer.

p.59 This proof has been altered to include reference to other results used.

5) Page 65 has been inserted in the correct place.

6) Bibliography items [20] and [34] corrected.

2 Second Examiner

1) Literature reviews/introductions for chapters 3 and 6 in the first draft have been expanded. The conclusion at the end of the dissertation has been expanded

2) Typographical and grammatical errors have been corrected as suggested in examiner’s report. These corrections correspond to the page numberings and lines in the draft. Line numbering excludes lines written in the latex equation and section environments.

Abstract:

The abstract has been extended as recommended. Introduction:

p.2 1.9 From: Also considered are eigenvalue problem and the existence and uniqueness of solutions for the non-homogeneous problem.

To: Also considered are eigenvalue problems and the existence and uniqueness of solutions for their associated non-homogeneous problems.

Chapter 2: History
(section 1.3 in updated copy)

p.5 1.4 From:... 'set of real number'...

To: ... 'set of real numbers'...

p.5 1.19 From:... 'analytic approaches mention above'...

To: ... 'analytic approaches mentioned above'...

p.5 1.26 From:... 'self-adjoint (unbounded) operator in a'...

To: ... 'self-adjoint (unbounded) operators in a'...

p.6 1.7 From:... ' with a ordinary '...

To: ... ' with an ordinary '...

p.6 1.8 From:... 'which is represents'...

To: ... 'which represents'...

Chapter 4: Eigenvalue Asymptotics

p.29 1.3 This sentence has been rewritten under the recommendations of examiner 1.

p.30 1.6 This sentence has been rewritten under the recommendations of examiner 1.

p.30 1.11 From: The advantage of studying the problem (4.1) in this way is that it gives eigenfunction expansions, a form for the resolvent and the Green's function.

To: The advantage of studying the problem (4.1) in this functional analytic way is that it gives concrete representations of the eigenfunction expansion, the resolvent and the Green's function.

p.34 1.11 From: Towards our goal of obtaining the Sturm-Liouville expansion theorem we now apply the spectral theorem and integrate around the poles of the resolvent.

To: Towards our goal of obtaining the Sturm-Liouville expansion and its completeness theorem, we apply the spectral theorem and integrate around the poles of the resolvent.

p.40 1.8 From:... 'be transformed to '...

To: ... 'be transformed into '...

Chapter 5: The Sturm Theorems

p.49 1.1 From:... 'classical proof's of'...

To: ...'classical proofs of'...

p.49 1.4 From:... 'Ince[15]'...

To: ... 'Ince [15]'...

p.49 1.5 Corrected under recommendation of examiner 1.

p.49 1.11 From:... 'obtain that the'...

To: ... 'obtain the'...

p.50 1. 4 5 From:... 'zero's'...

To: ... 'zeros'...

p.56 1.7 From:... 'system into polar co-ordinates, in this way'...

To: ... 'system into polar co-ordinates. In this way'...

Chapter 6: Variational approach

p.71 1.9 'Lusternik-' not replaced as recommended.

p. From:... 'In this section we will give a variational proof of the Courant Nodal domain theorem, this theorem gives an upper bound for the number of zero's of each eigenfunction, and with the problem we are considering, it is the upper bound of the Sturm oscillation theorem.'...

To: ... 'In this section we will give a variational proof of the Courant Nodal domain theorem, this theorem gives an upper bound for the number of zero's of each eigenfunction. This is can be seen as the variational analogue of the upper bound of the Sturm oscillation theorem'...

Chapter 7: classes of self-adjoint operators

p.90 1.8 From:... 'Furthermore symplectic'...

To: ... 'Furthermore, a symplectic'...

p.94 1.5 Paragraphs seperated.

p.97 1.2 Remove the repeated 'the'

p.97 1.2 From:... 'To this end, we will consider'...

To: ... 'We will consider'...