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Application of Webb's depth of knowledge on learners' responses to probability questions

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APPLICATION OF WEBB'S DEPTH OF KNOWLEDGE ON LEARNERS' RESPONSES TO PROBABILITY QUESTIONS

A Research Report submitted in partial fulfilment of the requirements for the degree of Master of Education

Faculty of Humanities

University of the Witwatersrand

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30 April 2021

DECLARATION

I Deepa Gopal declare that this Research Report is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of references. It is being submitted for the partial fulfilment for the requirements for the degree of Master of Education at the University of the Witwatersrand, Johannesburg. I have not previously submitted this work or part of it for any degree or examination at any other University.

Signature: _____  ____ Date: 30 April 2021

Deepa Gopal

ABSTRACT

This study focuses on Probability Questions from a selection of National November examinations. Probability was selected as a focus for this study because it is a topic that has been made compulsory across grades 4 to 12 in recent years. This study is two-fold as it investigated the Depth of Knowledge in selected Grade 12 Probability questions, and it looks at a sample of learners' responses to those probability questions. Given the unpredictable Pandemic we are experiencing, a total of 29 learners and one teacher participated in this study. A mixed method research methodology was employed in the study. Webb's Depth of Knowledge (DOK) Model was used as a framework of analysis. Probability questions from November examinations (2017 to 2019) were analysed to determine the DOK level for each question. Probability questions from a selection of November examinations were then reorganised in order to design the Probability Revision Task (PRT). Learner performance was examined by analysing their responses to the PRT. The study found that DOK level 1, 2 and 3 were present in all selected November examination Probability questions. However, there was no consistency with the weighing and spread of DOK levels from those examinations. The results revealed that learners in this study performed better than the cohorts of learners whose scripts were sampled and used to analyse the November examinations. Learner performance for DOK level 1 questions exceeded the performance in DOK level 2 and DOK level 3 questions by almost 31%. In an interview with the teacher, the Pandemic motivated some learners and provided relief for others from various pressures experienced in school.

DEDICATION

This study is dedicated to my family for their support during this journey. I dedicate this study to my mother Lata Sooka who motivated and encouraged me and in soul my father Yogesh Gopal and grandfather Harilal Daya who have inspired me throughout this journey.

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ACRONYMS

C2005 – Curriculum 2005

CAPS – Curriculum and Assessment Policy Statement

DBE – Department of Basic Education

DOK – Depth of Knowledge

FET – Further Education and Training

GDE – Gauteng Department of Education

NCS – National Curriculum Statement

NSC – National Senior Certificate

OBE – Outcomes-based Education

RNCS – Revised National Curriculum Statement

PRT – Probability Revision Tasks

NDR – National Diagnostic Report

CHAPTER 1: CONCEPTUALISATION OF THE STUDY

1.1. INTRODUCTION

Since the inception of democracy in 1994 there has been noticeable reorganisation and development in the South African mathematics curriculum. The introduction of a new curriculum aims to provide education for all learners in South Africa, irrespective of previous discriminatory policies. There have been changes in the curriculum that took place after the first democratic election in 1994. At first, education was based upon the Outcomes Based Education (OBE) curriculum (implemented in the late 1990s), in an attempt to overcome past inequalities. OBE was then revised and curriculum 2005 (C2005) was introduced and implemented in 1998 with the expectation of it being fully implemented across all grades by the year 2005. The Revised National Curriculum Statement (RNCS) was then implemented in 2002. RNCS was then replaced by the Curriculum and Assessment Policy Statement (CAPS) in 2012. The plan was a phased in implementation of the CAPS as follows: Grades R; 1; 2; 3 and 10 in 2012, Grades 4; 5; 6 and 11 in 2013 and finally Grades 7; 8; 9 and 12 in 2014. To date, CAPS is being implemented and followed in South African schools. This study focused on selected Probability questions from Grade 12 November examinations. Probability is a branch in mathematics which measures the chance or likelihood of an event occurring. The value of probability ranges from 0 to 1; these values can also be represented as percentages. Probability is a topic that has been made compulsory for all learners in school and is an examinable topic in the final high-stake Grade 12 examination.

1.1.1. Curriculum changes in South Africa

These series of curricular changes aimed to revise the past segregated curriculum and created a united curriculum for all South African learners. This means that teachers now must adapt their methodologies to ensure that all learners are included. Jojo (2019) claims that the curriculum reform has been generally politically driven, and the redress is to ensure that all learners have some form of mathematics which they have engaged with by the time they would have completed their matric (year 12 of schooling in South Africa). At present mathematics is compulsory for learners from Grade 1 to Grade 9 (aged 14 years - 15 years). In the FET phase from Grade 10 to Grade 12 (aged 16 years - 18 years) learners must choose between Mathematics, Mathematical Literacy or Technical Mathematics.

1.1.2. Mathematics education in a South African context

"The teaching of mathematics in South African schools has been pronounced to be among the worst in the world. Unacknowledged poor teaching of mathematics in most public schools deprive many learners access to both higher education and modern, knowledge-intensive work skills" (Jojo, 2019, p. 1). According to Kanjee (2009) several factors impact the effectiveness of assessment practice in classrooms; in a South African context some factors include; large class sizes, scarcity and lack of access to teaching and learning resources, high teacher workloads, inadequate teacher training with a poor understanding of assessment and the new curriculum, the inability or unwillingness of teachers to adjust assessments to address the needs of their learners in the new education system and a continued dependence on traditional teaching and assessment practices.

1.2. RATIONALE

As a novice mathematics teacher, it is vital to know and understand what mathematics content has to be taught. Not only should I know the content for the current grade I teach, but knowing content across the grades will be an advantage. Knowledge and awareness about the topics and contents that are covered across the various grades, other than the grades that an individual teaches, will provide the educator with an understanding of the content that learners should know and will give them the foresight for how the content of the current year is going to be utilised during future years. During my experience of teaching mathematics, I have noticed that learners struggle with certain topics especially probability, chosen for this study.

Probability as a topic was chosen because the questions that are assessed are contextualised. This links close to specific aim number 2 in the mathematics CAPS document.

> Mathematical modelling is an important focal point of the curriculum. Real life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible. (Department of Basic Education, 2011, p. 8)

Mathematical modelling is important in learning probability as real-life problems help learners in developing a wider range of strategies and methods (Gage, 2012). The incorporation of real-life problems has been emphasised by the Department of Basic

Education (DBE) in such a way that they are incorporated in various sections whenever possible.

A reason for choosing probability is that it is a relatively new compulsory topic in the curriculum. In the previous curriculum probability was not a compulsory topic. Learners who were assessed on probability wrote an optional examination (Mathematics Paper 3) in which the topic was assessed. According to Memnun, Ozbilen and Dinc (2019) Turkey has also made several changes regarding the inclusion of probability into the curriculum so that the key skill of developing probabilistic thinking can be achieved. At present, probability is a compulsory topic and incorporated into the mathematics syllabus from Grade 4 to Grade 12.

There are numerous reasons for including probability in school curricula. As a result, there has been an increase in the number of countries that are including probability in the curriculum (Gage, 2012; Batanero & Diaz, 2012; Vargas, Benincasa, Cian, & Martignon, 2020), as global recognition of the importance of probabilty is noted (Pratt, 2005). These reasons include: the helpfulness of probability in daily life as well as professional life; the instrumental role of probability in other disciplines; the essential role of probability reasoning in decision making (Batanero & Diaz, 2012); the importance of probability in data management; risk evaluation and decision theory (Vargas et al., 2020). Consequently, an early introduction to probability has been incorporated in school curricula in several countries (Batanero, 2009; Batenero, Chernoff, Engel, & Lee, 2016).

In a modern society people need to understand probability as they will be faced with having to make decisions that involve uncertainty (Paul & Hlanganipai, 2014). These reasoning skills and decision-making skills are important (Batenero et al., 2016) and should be developed in school. This decision-making skill will then assist individuals so that they are able to make informed decisions in their adulthood. These types of skills supports specific aim number 8 in the CAPS mathematics document (Department of Basic Education, 2011). Teachers also avoid having to explain why an experiment "does not work" (Gage, 2012) due to their own misconceptions or lack of experience. This will impact on the teaching and learning of probability at school level and has the ability to filter through into out of school experiences.

In this study, I investigated the Depth of Knowledge (DOK) levels that are present in a sample of Probability questions from November examinations (2017 to 2019) set by the DBE. The focus of this research is on learners' responses to those questions that were analysed using Webb's Depth of Knowledge framework (Webb, 1997). DOK model consists of four levels that can be used as a tool for creating, maintaining and analysing learning events or resources. The DOK model incorporates the content assessed and the intended cognitive demand (thinking and depth of understanding that is required in order to perform tasks).

1.3. MOTIVATION OF STUDY

The motivation for this study stems from my curiosity and desire to want to know more about mathematics in general. The focus of this study is probability for a variety of reasons. The weighting of specific skills and concepts that are present in assessment are of particular interest to me. In this study I am curious about the distribution of questions that are at different Depth of Knowledge Levels. By investigating further, I am hoping to gain a better understanding toward perceptions of Probability as a topic and possible reasons for poor performance in Probability as a topic within the Mathematics curriculum. The research process as well as analysis of results will contribute towards a better understanding for me as a novice mathematics teacher. This process will hopefully positively impact my methodologies and outlook towards mathematics, probability and assessment.

1.4. PROBLEM STATEMENT

Mathematic results in South Africa are poor. These poor results can be attributed to several factors including content-based factors, didactic factors, systemic factors, student personal factors and social-cultural factors (Mbungua, Kibet, & Nkonke, 2017; Luneta, 2014). Mathematics teachers should be aware of factors that contribute to learner performance, especially those factors over which they have control. The poor results might be due to the inclusion of difficult topics like probability in the CAPS syllabus. Probability (regarded as a main topic in the FET mathematics curriculum) is a topic that was not compulsory in the previous curriculum. With the previous NCS and RNCS curriculum, Mathematics Paper 1 and Paper 2 were compulsory and Mathematics Paper 3 (where probability was assessed) was optional.

Table 1.1: Mathematics examinations and the topics assessed

	NCS	CAPS	DURATION & MARKS
PAPER 1	Patterns Annuities & finance Functions & graphs Algebra Equations & inequalities Calculus Linear programming	Patterns & sequences Finance, growth & decay Functions & graphs Algebra, equations & inequalities Differential Calculus Probability	150 marks 3 hours
PAPER 2	Coordinate geometry Transformations Trigonometry Data handling	Euclidean Geometry Analytical Geometry Statistics & regression Trigonometry	150 marks 3 hours
PAPER 3 (OPTIONAL)	Sequences defined recursively Geometry Descriptive stats & interpretations Probability Bivariate data.	No longer written	100 marks 2 hours

Table 1.1 displays the examinable mathematics topics obtained from the FET mathematics examination guidelines of 2009 (Department of Basic Education, 2009) and 2017 (Department of Basic Education, 2017). Major restructuring in examination occurred when curriculum changes were enforced. People who have been in school during the time of the NCS curriculum and previous curricula and who are now teaching, are required to teach content to their learners without them necessarily having engaged with the specific school level content.

According to the CAPS document (Department of Basic Education, 2011), for Grade 10, probability constitutes about 15 ± 3 marks out of a 100 mark examination, which amounts to 12-18% in a Grade 10 examination. In Grade 11 probability is 20 ± 3 marks out of a 150 mark examination, which is 11-17% in a Grade 11 examination. For Grade 12 probability the mark allocation is 15 ± 3 marks from a 150 mark examination which amounts to 8-12% in a Grade 12 examination. In this study, Grade 12 mathematics examination papers were chosen as a focus because it is a written national assessment. It is also the last official school leaving assessment that is written before learners embark on their tertiary education journey.

Annually a National Diagnostic Report (NDR) is composed based on the performance in each subject from the final Grade 12 examination. In the NDR a question-by-question analysis, challenges and trends noticed; possible reasons for the results and suggestions for improvement strategies are presented.

Table 1.2: Average for Probability questions (NDR for 2017 to 2019)

	2017	2018	2019
PROBABILITY AND COUNTING PRINCIPLES QUESTIONS	Question 10 (probability) 41% Question 11 (counting principle) 25%	Question 11(counting principle) 34% Question 12 (probability) 31%	Question 10 (probability) 21% Question 11 (probability and counting principle) 26%
AVERAGE	33%	32.5%	23.5%

Table 1.2 displays the average for questions on probability and counting principles over a three-year period. From 2017 to 2018 there was only a slight decrease of **0.5%** for the probability and counting principle questions however, from 2018 to 2019 there was a decrease in performance of **9%**. For the 2019 examination the average was below the required **30%** for promotion. This means that most candidates did not achieve a pass result in these questions.

Table 1.3: Grade 12 learner performance in Mathematics Examinations (2014 to 2019)

YEAR	NUMBER WROTE	NUMBER ACHIEVED 30% AND ABOVE	% PASSED (ACHIEVED 30% AND ABOVE)	NUMBER ACHIEVED 40% AND ABOVE	% ACHIEVED AT 40% AND ABOVE	% FAILED
2014	225 458	120 523	53.5	79 050	35.1	46.5
2015	263 903	129 481	49.1	84 297	31.9	50.9
2016	265 912	136 011	51.1	89 119	33.5	48.9
2017	245 103	127 197	51.9	86 096	35.1	48.1
2018	233 858	135 638	58.0	86 874	37.1	42.0
2019	222 034	121 179	54.6	77 751	35.0	45.4

Table 1.3 provides a summary of learner performance from 2014 to 2019. Data from Table 1.3 is taken from the diagnostic reports from 2017 to 2019. The number of candidates who wrote the mathematics examination from 2014 to 2016 increased; sadly from 2016 to 2019 there has been an annual decrease in the number of candidates who wrote mathematics (Department of Basic Education, 2020). The percentage of learners who passed at 30% and above seems to fluctuate over the given time period, with a noticeable decrease of **4.4%** from 2014 to 2015 and a noticeable increase of **6.1%** from 2017 to 2018. The percentage of learners who have failed the examination is just less than half of the total who wrote. These results have been consistent over the period of 2014 to 2019 and have just shown slight fluctuation. Possible factors and reasons for poor performance remain important to be identified. The implementation of strategies for improvement and maintenance of results is crucial.

South Africa has repeatedly been on the bottom ranks of Trends in International Mathematics and Science Study (TIMSS) for both fourth and eighth grades and there are correspondingly poor results from the NSC Grade 12 high-stake examination (Jojo, 2019), evident in Figure 1.1. Poor performance in mathematics is present throughout the phases and grades; it is not specific to grade 12 only.

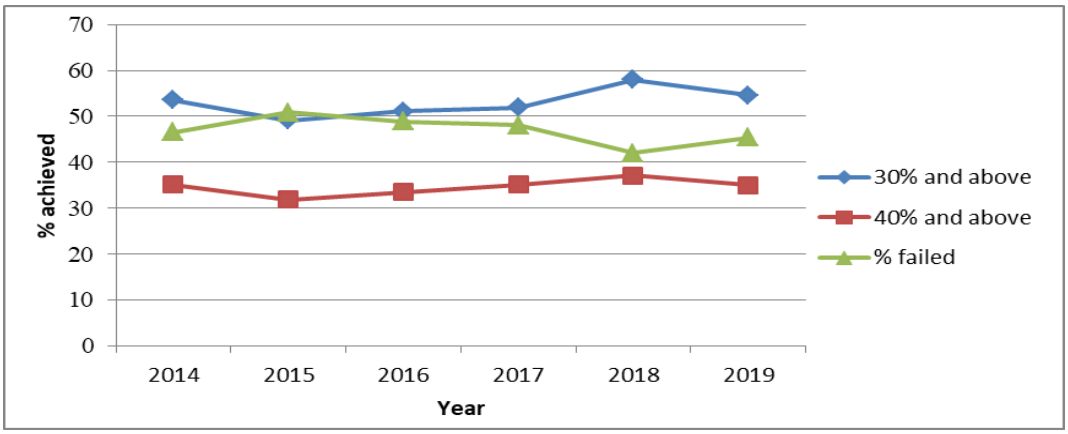


Figure 1.1: Percentage achievement in Mathematics from 2014 to 2019

Figure 1.1 shows the percentage of achievement in mathematics from 2014 to 2019. The results show fluctuations between the years. The results provided in Table 1.2 and Figure 1.1 displays steady increase in pass rates from 2015 to 2018 (learners achieving 30% and above), with a larger increase from 2017 to 2018. Learners who have achieved 40% and above has also steadily increased from 2015 to 2018 with a slight decrease from 2018 to 2019.

1.5. RESEARCH AIMS AND OBJECTIVES

This study is aimed at investigating the Depth of Knowledge levels that are present in probability questions from selected November examinations. An analysis of learners written responses will be based on Webb's Depth of Knowledge (DOK) model. In doing so, clarity regarding the dominant DOK level that is utilised for probability questions and the DOK level that learners are either excelling or struggling with will become evident. The DOK levels in probability questions will be classified in terms of being conceptual and/or procedural in nature. The manner in which learners respond to questions that test procedural and/or conceptual knowledge will be investigated. Thereafter a link between Depth of Knowledge levels and procedural and/or conceptual knowledge may become apparent. The DOK model is discussed in detail in Chapter 3, Section 3.3.

1.6. RESEARCH QUESTIONS

The two main research questions for this study are:

Main research question 1: What Depth of Knowledge is evident from the Probability questions in selected November examinations?

Main research question 2: How do learners respond to Probability Questions that are at different Depth of Knowledge Levels?

The sub-research questions for this study are as follows:

These two questions are sub-questions for main research question 1:

1.1. Which Depth of Knowledge Level is most prominent from the selected Probability Questions?

1.2. From the selected Probability Questions which Depth of Knowledge Level has the highest weighting?

These five questions are sub-questions for main research question 2:

2.1. From the selected Probability Questions, which Depth of Knowledge Level questions do learners seem to be performing better?

2.2. From the selected Probability Questions, which Depth of Knowledge Level questions are learners struggling to respond to/ answer?

2.3. How do learners respond to Probability questions that require Procedural Knowledge?

2.4. How do learners respond to Probability questions that require Conceptual Knowledge?

2.5. How have learners responded to Probability questions at Depth of Knowledge Level 1, 2 and 3?

1.7. SIGNIFICANCE OF THE STUDY

The outcomes of this study can provide teachers and examiners with insight into the questioning techniques and how their requirements impact learner performance. Results from this study may assist with the improvement and development of training material and study material in probability for both teachers and learners in the FET phase. The development and improvement of material can be overseen by DBE officials, tertiary institutions, teacher development units or tuition providing institutions. Results from this study may guide teachers when they are planning lessons so that they can anticipate possible outcomes; the

report may also provide them with solutions to challenges that they may experience when teaching probability.

1.8. DEFINITION OF TERMS

Grade 12

Grade 12 is the exit grade in the FET phase, it is the school leaving grade in the South African schooling system.

Further Education and Training (FET)

Refers to grades 10, 11 and 12 in the South African schooling system

General Education and Training (GET)

Refers to grades 0 -9 in the South African schooling system

Teachers

Teachers referred to in the study are currently teaching mathematics to learners in the FET phase. These teachers are teaching in schools that are located in Johannesburg South District.

Learners

Learners referred to in this study are in Grade 12 (2020) from specifically selected schools in the Johannesburg South District, who are taught by the teachers participating in the study.

Curriculum Assessment Policy Statement (CAPS)

CAPS refers to the official document that outlines all assessment topics and concepts that have to be taught and assessed in school level in South Africa.

Depth of Knowledge (DOK)

The levels are based on Webb's Depth of Knowledge taxonomy, which is utilised to rate questions and the thinking that is required to answer the questions correctly. These levels include recall and reproduce; applications of skills/ concepts; strategic thinking and extended thinking.

Final Examination

The final examination is a high-stake National Examination that is written at the end of the schooling year, usually in October and November, and this examination follows the preliminary examination. Probability questions from a selection of final examinations are used as the data collection instrument in this study.

National Diagnostic Report (NDR)

The Diagnostic Report is a National publication in which an analysis of each subject and the examination papers written for the subject are provided. The report is done annually and is based on the final examination that is written in that year. The report provides the reader with an overview of performance in the paper, the average performance per topic, average for each question, common errors and misconceptions and suggestions for improvement.

Probability Revision Task (PRT)

Is the revision task that was administered to learner participants in this study. The PRT consists of eight probability questions which were sourced from previous years (2017 to 2019) final examinations. The probability questions from the final examination were rearranged and grouped in order to design the PRT.

Procedural Knowledge

Rittle-Johnson and Alibali (1999) describe procedural knowledge as the action sequences for solving problems. This means that procedural knowledge consists of rules and procedures that are required in order to perform a calculation. This type of knowledge is often memorised and used in isolation.

Conceptual Knowledge

Rittle-Johnson and Alibali (1999) describe conceptual knowledge as the explicit or implicit understanding of the principles that govern a specific domain and the interrelations between pieces of knowledge in a domain. This means that conceptual knowledge can be seen as a connected and integrated web of knowledge that is rich in relations. This type of knowledge links pieces of information, skills and concepts to create a bigger and better understanding of the content at hand.

1.9. Outline and Organisation of the Report

Chapter 1 provides a background for this study. The problem statement, rationale, the research aims and objectives, research questions, possible contributions of the study, are provided. Chapter 2 discusses literature in relation to the topic of this study. The literature forms a foundation for the study and its guiding principles. Chapter 3 discusses the theoretical framework, based on Webb's Depth of Knowledge (DOK) levels which will be utilised to analyse probability questions from past examination papers and learners' responses to those questions. This chapter includes a discussion and comparison between Bloom's taxonomy, the CAPS taxonomy and Webb's DOK model. Chapter 4 engages with the research design, sample, methodologies, data collection and data analysis of this study. The validity and reliability of the instruments and ethical issues that were applicable for this study are also described. Chapter 5 includes a detailed analysis of the research instruments. Chapter 6 presents the results obtained and an analysis of the data. Qualitative and quantitative results are presented as this study utilises mixed methods research strategies. In Chapter 7, the interpretations of the analysed data are presented alongside their possible implications.

1.10. Summary and Conclusion

This chapter presented a background for this study. The importance and applicability of probability to school and everyday life have been highlighted. This contributed to developing the research questions that govern this study. It is necessary that an investigation be made to determine the DOK levels that are present in probability questions from national examinations. The findings can hopefully inform teacher planning, teacher development and assessment requirements regarding active approaches that can be utilised for the future teaching and learning of probability for learners, teachers and examiners.

CHAPTER 2: LITERATURE REVIEW

2.1. INTRODUCTION

In this Chapter, I discuss the literature that relates to the research questions of this study. The literature surveyed looks at probability as a topic in mathematics, assessment in mathematics and learners' responses to probability questions. The concept of knowledge that can be classified as procedural or conceptual or both will be discussed and linked to probability and Webb's DOK model in Chapter 3. The literature review allows the researcher to determine what has already been done and to discover the gaps from previous research done. It allows the researcher to identify central issues and provides a framework for the importance of the study (Ary, Jacobs, & Sorensen, 2010; Alzahrani, 2020; Boote & Beile, 2015; Creswell, 2012; Creswell, 2014). Section 2.2 discusses mathematics education and probability. Section 2.3 discusses different types of learner knowledge focusing on procedural and conceptual knowledge. Section 2.4 discusses assessments and focuses on mathematical assessments in school. Section 2.5 provides a conclusion for the literature review.

2.2 MATHEMATICS EDUCATION AND PROBABILITY

2.2.1. School Mathematics Curriculum

School mathematics curricula changes have placed an increase on the importance of probability in the past few decades (Bezzina, 2004; Papaieronymou, 2009; Gage, 2012; Dvorakova et al., 2017; Koparan & Koparan, 2019) and learners have to engage with probability at school level. Probability represents real-life mathematics as well as applications to physics, weather predictions, music and predicting the risks of new medical treatments (Taylor, 2011) amongst other applications, which is essential to daily life.

Dvorakova et al. (2017) claim that the primary reason for including probability in school curricula is to provide learners with understanding and to develop critical thinking around the role of probability in their daily lives and in mathematics. Probability is a useful concept and should be introduced during the early stages of child development when children are aged 5-7 years (Taylor, 2011).

It is suggested that critical thinking skills should be developed from learners' early years in school and not only later in their school lives as it is suggested that critical thinking is necessary for contributing to mathematically proficient citizens of the world (Aizikovitsh-Udi

& Cheng, 2015). Probability has been made a compulsory topic in the South African curriculum, and learners engage with this topic from Grade 4 (aged 9-10 years old). At times traditional rote learning dominates the mathematics classroom. However, probability is a topic in which learners' everyday lives and interests can be incorporated to develop critical thinking and deep understanding. This study focuses on Grade 12 probability questions and how learners respond to them. Learners' responses may provide insight into their understanding of the topic as well as their misconceptions or errors within this topic.

Table 2.1.: Concept development in Probability from Grade 4 to 12

GRADE	TOPICS IN PROBABILTY
4	Perform simple repeated events and list possible outcomes for experiments such as: -tossing a coin -rolling a die
5	• Perform simple repeated events and list possible outcomes for experiments such as: - tossing a coin - rolling a die - spinning a spinner • Count and compare the frequency of actual outcomes for a series of trials up to 20 trials
6	• Perform simple repeated events and list possible outcomes for experiments such as: - tossing a coin - rolling a die - spinning a spinner • Count and compare the frequency of actual outcomes for a series of trials up to 50 trials
7	Perform simple experiments where the possible outcomes are equally likely and • List the possible outcomes based on the conditions of the activity • Determine the probability of each possible outcome, using the definition of probability
8	• Consider a simple situation (with equally likely outcomes) that can be described using probability and**:** - List all the possible outcomes - Determine the probability of each possible outcome using the definition of probability - Predict, with reasons, the relative frequency of the possible outcomes for a series of trials based on probability - Compare relative frequency with probability and explain possible differences
9	• Consider situations with equally probable outcomes, and: - Determine probabilities for compound events using two-way tables and tree diagrams - Determine the probabilities for outcomes of events and predict their relative frequency in simple experiments - Compare relative frequency with probability and explains possible differences
10	1. The use of probability models to compare the relative frequency of events with the theoretical probability. 2. The use of Venn diagrams to solve probability problems, deriving and applying the following for any two events A and B in a sample space S: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$; A and B are mutually exclusive if $P(A \text{ and } B) = 0$; A and B are complementary if they are mutually exclusive; and $P(A) + P(B) = 1$. Then $P(B) = P(\text{not } A) = 1 - P(A)$.
11	1. Revise the addition rule for mutually exclusive events: $P(A \text{ or } B)=P(A)+P(B)$, the complementary rule: $P(\text{not } A)=1-P(A)$ and the identity P(A or B) = P(A) + P(B) - P(A and B) 2. Identify dependent and independent events and the product rule for independent events: $(A \text{ and } B) = (A)\times P(B)$. 3. The use of Venn diagrams to solve probability problems, deriving and applying formulae for any three events A, B and C in a sample space S. 4. Use tree diagrams for the probability of consecutive or simultaneous events which are not necessarily independent.
12	1. Revise: • Dependent and independent events; • The product rule for independent events: P(A and B) = P(A) × P(B). • The sum rule for mutually exclusive events A and B: $P(A \text{ or } B) = P(A) + P(B)$

	• The identity: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ • The complementary rule: $P(\text{ not A}) = 1 - P(A)$ 2. Probability problems using Venn diagrams, trees, two-way contingency tables and other techniques (like the fundamental counting principle) to solve probability problems (where events are not necessarily independent). 3. Apply the fundamental counting principle to solve probability problems.

Table 2.1 provides details on the topic of Probability according to the CAPS document from Grade 4 to 12. This table allows one to see how the topic of probability is developed over eight years during the schooling process. Probability is introduced to learners in Grade 4 and is assessed every year till Grade 12.

According to Baternero and Diaz (2012) and Dvorakova et al. (2017) to change the teaching methods of probability at various school levels, teachers will have to be convinced of its importance; the extent to which teachers are convinced will then impact on the change in their teaching. Along with the convincing of teachers on the importance of probability, teachers will have to be provided with opportunities to receive adequate teacher training for teaching the topic of probability. Teacher participants in this study may have different educational backgrounds and experience; this may impact on their teaching methodology and outlook towards the subject and specifically the topic of probability.

2.2.2. Teacher Understanding and Training

Over the past two decades an increase in attention has been given to the teaching and learning of statistics education. However, probability remains a section that both teachers and learners regard as being difficult. The view of probability being a topic that is challenging is supported by Sahoo (2013) who claims that probability theory is challenging for learners to grasp and for teachers to explain.

Koparan (2015) conducted a study where teachers (n=10) were interviewed to gain a better understanding about the difficulties in teaching statistics subjects in middle school. Difficulties for probability included, amongst others, recognising the fact that probability ranges between 0 and 1, learners' attitudes towards probability, certain concepts or definitions that might be incomplete or incorrect, calculations incorporating knowledge of permutation and combination, the assumption that all events have equal probability and ignoring sample when performing calculations. It is important to know the possible difficulties that teachers and learners face. Knowledge of these difficulties may provide better

insight when analysing learners' responses and from teacher interview responses. These difficulties may also be common with those that are presented in the annual National Diagnostic Report (NDR) in South Africa.

Batenero and Diaz (2012) analysed possible reasons as to why teachers find probability difficult to teach. Research revealed that textbooks and curriculum documents for teachers may not adequately support teachers. As sometimes the resources provide a narrow and limiting view of the concepts, application of probability is limited to games of chance and the definitions are incomplete or incorrect.

Kristanto and Santoso (2020) argue that textbooks are an important resource as it often impacts the sequencing and materials used in teaching. More importantly, textbooks are the main resources that learners use outside the classroom. A study conducted in Indonesia across 10 public and private universities via an electronic survey through Google Forms, investigated preservice teachers' perceptions on mathematics textbooks. Huge gaps between preservice teacher expectations and their perceptions were identified in terms of the mathematical content, presentation, language, graphic properties and incorporation of technology in mathematics textbooks.

Teacher training and development is crucial. However due to time constraints and pressure, it may not always be possible to adequately prepare teachers for teaching probability. In South Africa there are several mathematics textbooks that are available for use in the FET phase; the DBE does not have a prescribed textbook. This means that schools have the choice of selecting one or more textbooks that teachers will use when teaching. Textbooks should be aligned with CAPS requirements as a requirement by DBE. Teachers can utilise a variety of resources, as long as they are CAPS aligned, during their lessons. This allows teachers the freedom to even develop their own tasks that would be best suited for their learners. Most teachers feel that answering pass examination papers is the best preparation for examinations as it provides the learner with possible examination type questions. Some teachers incorporate pass examination questions in their lessons, if any participant teachers do incorporate this, results obtained will be of interest.

2.2.3. Studies on Probability in School

In a study conducted in Maltese schools, several errors in probabilistic thinking of learners were identified (Bezzina, 2004). Reading information from tables and statistical diagrams were classified as sophisticated skills. It was recommended that teachers should not assume that learners are comfortable with such skills. The lack of reading skills was evident in poor performance in probability. Learners should be provided with opportunities whereby they are required to utilise and develop their interpretation skills to effectively extract important information from tables and statistical diagrams. Probability questions often require learners to interpret given information. The given information may have to be represented graphically in some instances and in other cases a graphical representation may assist learners with answering questions that follow. Bezzina (2014) has warned that correct answers from learners do not necessarily indicate that the concept has been understood or grasped by the learner. The struggles alluded to by Bezzina (2014) is of importance in this study because learners will be expected to read information and interpret the information and perform various tasks, such as drawing Venn diagram and performing calculations. Also, the information presented in probability questions are not always in paragraphs, sometimes information in provided in tables or in bullet points.

Awuah (2018) studied the problem-solving skills (in probability) of Grade 12 learners, (n=490) from seven secondary schools. Socioeconomic factors were considered when schools were approached for participation. In the study Bloom's taxonomy as well as the CAPS taxonomy were utilised as frameworks for analysis of instruments. Learners were more successful when answering questions at the knowledge level compared to questions at the evaluation level. The results from this study shows that learners displayed a greater proficiency when utilising Venn diagrams as an aid as opposed to tree diagrams or contingency tables as aids in solving probability problems. Low levels of success were recorded for questions requiring higher cognitive demand. Learners also performed poorly in questions that required the fundamental counting principle, tree diagrams and contingency tables when compared to other aspects in probability. The errors identified that misconceptions emerged from poor understanding of concepts from previous grades, when solving problems. In my study, learners responded to probability questions that vary in difficulty. The questions required learners to engage with several concepts in probability such as; Venn diagrams, tree diagrams, independent and mutually exclusive events.

Lister (2019) investigates the relationship between mathematics discourse and the enacted levels of cognitive demand when solving mathematical tasks that are set at four different DOK levels with Grade 5 learners. Learners (n=97) had to complete two task-sets, each consisted of questions having four DOK levels. The tasks utilised in the project were all pregenerated mathematics task-sets from NextLesson (Lister, 2019, p. 124). Reasons for the selection of these specific tasks include; tasks are based on real-world problems, tasks are pre-coded using Webb's DOK model and tasks are organised in four levels which utilised a mixture of DOK levels. For the first task-set learners worked individually and for the second task-set learners collaborated in small groups. The relationships within and between groups were analysed. The results from this study showed that most learners' responses, both written and verbal were high and there were no significant differences with the quality when learners engaged in different types of discourse. There were significant differences when learners engaged with different organisational practices with the discussion and learner responsibilities within the small groups. Results in this study have shown that learners who were struggling were supported better with reflective discourse compared to exploratory discourse; from this study it was noted that learners who are struggling were either ignored or rejected during exploratory discourse. It was suggested that teachers should make use of both reflective and exploratory small-group discourse in order to increase the enacted levels of cognitive demand to promote deeper understanding of mathematics. Teachers can support learners who are struggling with mathematics by allowing them time to think about their own ideas before group or class reflection on a task. This study was however limited in terms of generalisability with regard to the Grade 5 level, mathematics content and the task length. Unlike Lister (2019) who utilised tasks set at predetermined DOK levels, in this study the DOK levels of examination questions will first be established before a presentation on learners' responses to those examination questions.

Tasks that challenge learners often results in dialogue, as learners engage with the task. According to Bezzina (2004) and Al-Mutawah et al., (2019) found that learning involves struggling, grappling and engaging with concepts that learners find challenging. Questions that incorporate real-life situations help develop an understanding of concepts in probability and problem-solving strategies. By engaging learners with real-life scenarios that incorporate elements of probability learners are made to engage in dialogue. This encourages the formation of solution methods as opposed to applying a procedure. This type of learning with

understanding develops problem solving skills required throughout life. In this study a comparison between learners' responses to probability questions that require procedural and conceptual knowledge is done and presented in Chapter 6. It is of interest to notice how learners respond to questions that challenge their thinking and those that do not.

Teacher knowledge can impact the learners' knowledge of probability hence teachers should have sufficient probability content and teaching knowledge (Papaieronymou, 2009). This is particularly true in a South African context, as many mathematics teachers have not studied probability during their own years of schooling (Gage, 2012). De Kock (2015) found that a sample of teachers, from the Tshwane South district in Gauteng, with a diploma qualification performed better when compared with teachers with degree qualifications in a probability test. Teachers with degrees were able to answer the test but did not feel comfortable with the topic of probability. Teachers with diploma qualifications had more years of teaching experience which they feel contributed to their abilities in answering probability questions.

In her study on teacher knowledge, De Kock (2015) looked at the relationship between Grade 11 and Grade 12 learner performance and the mathematical content knowledge (MCK) of teachers' while focusing on probability. Probability questions were analyzed using the CAPS taxonomy (refer to Chapter 3, Section 3.2.2). Teacher participants (n=8) had varying levels of qualifications. The results revealed a low level of MCK in probability for teachers. Teachers also struggled to answer questions that required knowledge and problem solving because there was no formula sheet provided in the written test, higher levels of competency were shown for questions based on routine and complex procedures. The average score for Grade 11 (n=89) learners was 47% and Grade 12 (n=75) learners was 30% for probability. Learners struggled with questions that required knowledge, the recalling of an identity and rules on probability. Learner performance at the levels of routine procedures, complex procedures and problem solving were expected as questions that are challenging. In my study Grade 12 probability questions are analyzed using Webb's DOK model, questions from November examinations over a three-year period are used. Teachers' qualifications will be discussed in an interview but no tests will be conducted with teachers.

Kodisang (2015) focused on Grade 6 teachers' probability knowledge. Teachers in this study have been able to obtain an average of greater than 60% for learner performance over a three-

year period (2012 to 2014). The results from this study show that cooperation, discussions and problem-based approaches were utilized for the teaching and learning of probability. Essential aspects of the identified teaching strategies were enforced during lessons however, limited knowledge on how to successfully implement these strategies were evident. Teachers (n=3) in this study displayed a great reliance on conducting experiments and recording the results. Teachers noted that procedural understanding and conceptual understand are equally important. It was recommended in this study that teachers allocate some time for a reflection of their lesson as well incorporation and attendance of teacher development training. Both studies by De Kock (2015) and Kodisang (2015) focused on mathematics teachers and probability as a topic. In my study the focus is on learners' responses to probability questions and the DOK levels that are present in those questions.

2.3 PROCEDURAL AND CONCEPTUAL KNOWLEDGE

2.3.1. Procedural Knowledge or Conceptual Knowledge OR IS IT Procedural Knowledge and Conceptual Knowledge

Procedural knowledge is viewed in terms of two kinds of knowledge: The first kind of procedural knowledge is a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols. The second kind of procedural knowledge consists of rules or procedures for solving mathematical problems. Many of the procedures that learners possess probably are chains of prescriptions for manipulating symbols (Hiebert & Lefevre, 1986).

Conceptual knowledge is typically viewed as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network. (Hiebert & Lefevre, 1986).

According to Star (2000), the endpoint for the acquisition of procedural knowledge is when routines and skills are executed with fluency and has become automatized. Compared to the endpoint for the acquisition of conceptual knowledge is when such knowledge is understood. However, according to current research the situation is reversed. Knowing that is knowledge of concepts which is more complex, it is the acquisition of knowledge which can either be

understood or memorized. Knowing how is knowledge of procedures which is straightforward as procedures and skills are either known or not known. With this change or shift in classifying knowing that and knowing how we need to examine different characteristics of knowledge, knowledge type and knowledge quality.

A distinction between knowledge type and knowledge quality needs to be established. Knowledge type refers to what is known and knowledge quality refers to the way in which something is known (Star & Stylianides 2013). Knowledge type can be either conceptual or procedural knowledge that is known. Knowledge quality can be between different levels – deep level, superficial level or anything in between. Holmes (2012) states that the depth of content difficulty relates to the test item and not the responses provided from teacher or learner.

The distinction between conceptual and procedural knowledge can be a complex task. Research done in the early 2010s describe that the dichotomy between the two types of knowledge are not as distinct as previously believed (Kadijevich, 2018) There is a claim that each type of knowledge, procedural knowledge and conceptual knowledge, predict gains in the other type of knowledge (Rittle-Johnson, Schneider, & Star, 2015), and develop in an iterative way. This means that there is a bidirectional relation (Rittle-Johnson et al 2015) with a complex interdependence (Engelbrecht, Bergsten, & Kagesten, 2009) between procedural and conceptual knowledge. This implies that both procedural and conceptual knowledge are important in the teaching and learning of mathematics. Teachers should expose learners to the skills required for attainment of both procedural and conceptual knowledge. Haapasalo and Kadijevich (2000) claim that procedural knowledge often calls for automated and unconscious steps whereas conceptual knowledge requires conscious thinking and conceptual understanding may be poorly developed due to a lack of conceptual understanding.

A study conducted by Chappell and Kilpatrick (2007) show that students in the concept-based learning environment scored higher than students in the procedure-based learning environment in both the mid-year examination and the final examination. This impacts the teaching methodologies that are utilised in classrooms. Teaching methodologies are factors that the teacher can control in order to ensure an improved learner attainment. "Cognitive type of knowledge matter: teachers' knowledge of facts and procedures (type 1 knowledge)

has a relatively smaller positive effect on the quality of instruction and on students' achievement than teachers' knowledge of concepts and connections (type 2 knowledge) (Tchoshanov, 2011) as citied in (Fauskanger, 2015).

Skemp (1976) refers to mathematical understanding in instrumental understanding (rote memorisation) and relational understanding (deep conceptual knowledge). Skemp suggests that a teacher needs relational understanding in order to help students gain deep understanding of the content taught. Recent studies confirm that having conceptual and connected mathematical knowledge is a premise for teaching that helps students develop relational understanding (Fauskanger, 2015).

2.3.2. Studies on Procedural and Conceptual Knowledge in Schools

Manandhar (2018) investigated the level of procedural and conceptual knowledge of algebra that learners are acquiring as well as the relationship between the two types of knowledge. Grade 8 learners (n=360) responded to a survey across public schools (n=9). To verify the results obtained from learners, interviews with teachers (n=4) and learners (n=6) were conducted. The focus of this study was learners' abilities to define concepts in algebra, the comparison of algebraic expressions, representation of concepts with figures and diagrams, the ability to express verbal problems into mathematical sentences and critical thinking. Learners had to respond to four questions that were based on procedural knowledge and 20 multiple choice questions that were based on conceptual knowledge. It was found that there was a low level of conceptual knowledge in algebra and learners had higher procedural knowledge. However, there was a positive correlation between procedural and conceptual knowledge. Interviews with teachers revealed that learners were weak in reasoning and critical thinking and displayed difficulties when having to compare algebraic quantities and graphically representing knowledge with diagrams and pictures. These difficulties stem from poor interest from learners to learn algebra, the lack of incorporation of effective teaching and learning methodologies, lack of linking daily life to contextual problems during lesson presentation and a lack of practical work. Teachers in this study wanted their learners to develop both procedural and conceptual knowledge in mathematics. It was noted that the construction and development of conceptual knowledge should be prioritised without neglecting procedural knowledge. A suggestion was for the development of a curriculum that nurtures conceptual knowledge.

The predominance of developing learners' procedural skills through initial demonstrations and the rigorous practice for similar examples is worrisome because mathematical understanding is just as important as procedural competencies (Hatisaru, 2020). As highlighted in the study by Manandhar (2018) procedural and conceptual knowledge is important.

In a study conducted in Bahrain, a positive correlation between problem-solving skills and conceptual understanding was found Al-Mutawah et al. (2019). In their study learners (n=350) were assessed on five mathematical domains (numbers and operations; algebra; geometry; measurements and data handing and probability) within each of the domains procedural knowledge and conceptual understanding were tested. Results from this study show that learners scored an average of 66% in questions that tested procedural knowledge, as the problems are familiar to learners from their previous years of schooling and the problems can be easily solved once the procedure is known. For questions that required an understanding of the concepts and application to different types of problems, the results were significantly lower; an average of 33% was obtained. Reasoning of procedures proves to be having a significant impact on learner performance. A positive correlation shown between problem-solving skills and conceptual understanding was noted. Participants also struggled with the translation of word problems into mathematical statements. The translation of word problems to mathematical statements is pertinent for probability questions especially when learners are required to draw Venn or tree diagrams.

In this study a comparison of learners' responses to probability questions that require procedural and conceptual knowledge is presented in Chapter 6, Section 6.5. Interviews with Grade 12 mathematic teachers are presented in Chapter 6, Section 6.7 the results may provide a different perspective on learners approaches to probability questions. This study differs from studies conducted by Manandhar (2018) and Al-Mutawah et al. (2019) as probability is the focal topic and the study is conducted on a smaller scale. This study does not only focus on the presence of procedural and conceptual but incorporates the cognitive demand that is required when answering questions.

Pratama and Retnawati (2018) claim that higher order thinking skills can be developed and require practice as it is not automated. These higher order thinking skills go beyond recalling facts and procedures. It requires something to be done to the facts such as understanding it, linking them and making conclusions. Usually when learners display higher order thinking skills they tend to steer way from memorisation and recall and utilise logic and in this process master concepts that can be used in order to solve more challenging problems. Higher order thinking skills are present in the top six levels of Bloom's revised taxonomy (understanding, applying, analysing, evaluating and creating).

Barber (2018) advocates for the use of computer simulations when teaching because online simulations can lead to the mastery of concepts as it may engage learners in higher-level thinking and challenge them to actively engage with new ideas. Online simulations are an effective means to promote DOK and conceptual understanding. Aizikovitsh-Udi and Amit (2011) claim that by creating learning environments that foster critical thinking help in developing and improving learners' conceptual thinking and their critical thinking skills. Engaging with mostly procedural tasks is limiting and boring for learners (Sullivan, 2011). In order to attract learners' attention and keep them interested in a topic, conceptual questions proved a good means to engage learners with content as they are no longer applying rules but are actively engaged and challenged. According to Al-Mutawah et al. (2019) problem solving incorporates four processes: reasoning, communicating, connecting, and representing. Through the problem-solving process learners are able to apply content knowledge in the various mathematical domains. Problem solving affords the opportunity for one to better understand a learner's mathematical thinking and is a major contributor to assessment. Without conceptual understanding learners memorise facts and cling onto previous understanding because they are unable to understand the concepts behind them (Konicek-Moran & Keeley, 2015). Learners' responses to probability questions will be analyzed and their problem solving will be displayed, especially in questions that do not funnel and guide thinking but instead allow the learner freedom with solution strategies.

In a study conducted in Turkey by Hatisaru (2020), drawings of learners' perceptions of the teaching and learning practices in mathematics classrooms were analysed. It was found that learners did not experience a variety of learning experiences in the classroom. Drawings revealed that procedural tasks dominated over representational, open-ended and contextual

tasks. The repetition of routine problems requiring the application of facts and procedures were emphasised (Hatisaru, 2020). Task utilised for teaching and assessment purposes can require procedural and conceptual understanding. In this study an analysis of learners' responses to questions requiring procedural and conceptual knowledge is done. Saiman, Wahyuningsih, and Hamdani (2017) claim that students frequently do not see how problem solving strategies and skills that are learnt in one topic or subject apply to other activities, topics and subjects. This links to the study done by Hatisaru (2020) as learners are not aware of the importance of conceptual knowledge.

Focussed, quality instruction requires teachers to make judgements, and one key judgement is the consideration of the place of procedural knowledge and conceptual knowledge in the teaching of mathematics (Hurrell, 2021, p. 60). Although conceptual and procedural knowledge cannot always be separated, it is useful to distinguish between the two types of knowledge to better understand knowledge development (Saiman, Wahyuningsih, & Hamdani, 2017, p. 3). According to Star and Stylianides (2013) the manner in which one classifies knowledge as being either conceptual, procedural, neither or both is subjective and depends on their perspective; this depends on their exposure to literature, their knowledge about the subject and their personal experiences. Brodie (2004) claims that the distinction between procedural and conceptual knowledge is limited and limiting and that a useful formation and integration of both should be used for mathematical knowledge and teaching practices.

In a study conducted in South Africa and Sweden, it was found that a small sample of senior engineering students from South Africa recognised the importance of conceptual questions outside the mathematics curriculum and a larger portion saw the importance of procedural questions outside the mathematics curriculum whereas engineering students from Sweden views were the opposite (Bergsten, Engelbrecht, & Kagesten, 2017). Research is unambiguous in accepting the important of both procedural and conceptual knowledge (Rittle-Johnson, Schneider, & Star, 2015). The recognition of the importance of both forms of knowledge allows debates to determine the relationship between the two forms of knowledge (Hurrell, 2021).

2.4 ASSESSMENTS IN SCHOOL

My study incorporates probability questions from National Assessments that have been written. The results in this study are compared to the National average recorded for that assessment. The research instrument has been administered as a test under controlled conditions that most school assessment follow. "Assessment is closely linked to education; it is inherent to it because any educational process must also be assessed" (Rincon, 2019, p. 1318). Assessments have been used for various purposes; providing learner grades to individuals and their families, placements in schools or tertiary education institutions, for national accountability, improvement of teaching and learning and even resource allocation within a specific area or district (Suurtamm et al., 2016). Assessment does not only take place at the end of the term, it occurs constantly and is an integral part of daily classroom practice. Assessment is of utmost importance to the teacher as it focuses on paying attention to learners' thinking and will inform the teacher's behaviour accordingly. An important aspect of assessment is the purpose of the assessment (Suurtamm et al., 2016). Some assessments are used in order to grade and promote learners. While responses provided in assessments are used to provide feedback, training or guide future lessons. Assessment can also be used to monitor the system such as the TIMMS and NSC final examination in Grade 12. Assessment results can be used in order to accept or deny placement to educational institutions.

Assessments in South Africa must align with the CAPS requirements. 'The four cognitive levels used to guide all assessment tasks are based on those suggested in the TIMSS study of 1999 (Department of Basic Education, 2011, p. 53). There is a guideline for the approximate percentage for cognitive level that should be present in all assessments. Table 2.1 shows the suggested allocation for the cognitive levels. From this guideline, routine procedures are allocated with the greatest weighting for any mathematical assessment.

Table 2.2: Cognitive Levels and Approximate % for Assessment Tasks according to CAPS

COGNITIVE LEVEL	APPROXIMATE PERCENTAGE IN TASKS
Knowledge	20%
Routine procedures	35%
Complex procedures	30%
Problem solving	15%

The CAPS framework is discussed in further detail in Chapter 3 Section 3.2.2. In this study my focus is on the weighing of the DOK levels in probability questions that are selected from 2017 to 2019 final November examinations that have been written.

The final National Grade 12 examination is used to gauge the mathematical knowledge of South African learners and it can be used to do a comparison of learner knowledge between countries. According to Suurtamm et al. (2016) large scale assessments are often used for system monitoring, evaluation of programmes or to make student placements. The NCS examination is a form of a large-scale assessment. These examinations are written country wide in South Africa under controlled conditions and have to comply with strict measures which are regularly monitored by representatives from the DBE. The results from this examination is used to grant learners access into tertiary educational institutions and used for placements in various courses, internships and other programmes. High status is attached to university degrees and hence high school and university examinations are highly competitive. As a result high importance is given to mathematics as a subject at school level as it contributes to the high school or university that one can attend (Hatisaru, 2020). In this study the variation of the DOK levels in probability questions from selected National examinations are analysed as well as learners responses to those questions.

Educational assessment affects the content (what) is taught and the methodology (how) learning occurs (Baird & Hopfenbeck, 2017). The purpose of assessments is to supply proof which can be utilised in order to make choices about or to enhance mathematics education, this then raises concerns about mathematical elements of competence that need to be assessed, the methodology as well as the intention of the assessment (Nortvedt & Buchholtz, 2018). According to Suurtamm et al. (2014) attention must be paid to the purpose of the assessment and then the results of the assessment can be interpreted and utilised appropriately for the intended purpose. Classroom assessment usually uses a variety of teacher-selected or teacher-made assessments which will be most effective when closely aligned with the needs of the learners and what they have been learning (Baird et al., 2014). In this study the purpose of the assessment administered is to gain a better understanding of how learners respond to questions that are at different DOK levels as well as how learners respond to questions that require procedural or conceptual knowledge. This task was specially designed for this study and comprises of questions selected by the researcher.

The assessment process needs to be considered and accounted for when considering how to improve the quality of education (Cordoba, 2006). Assessment is an important factor in determining how to improve the quality of education and this is important for every country,

as each country wants to improve their education system. External examinations; such as the Grade 12 National Senior Certificate Examination often influences the instructional practice of teachers and importance is given to assessments which makes assessments a topic of interest to educators (Suurtamm et al., 2016). According to Kanjee and Bhana (2020) assessments are used to gain evidence; evidence on what learners know, what they understand, what they can do and this information is used to then adjust teaching so that learners are supported on their journey of mastering content. South Africa has repeatedly been on the bottom ranks of TIMSS and there are equally poor outcomes from the high-stake NSC Grade 12 examination results (Jojo, 2019). In this study the results documented in the National Diagnostic Reports (NDR) will be compared to the results obtained from the Probability Revision Task (PRT) that was compiled from the three examination papers. This comparison will be presented in Chapter 6 of the report, specifically in Section 6.6. According to the CAPS document, knowledge should only consist of about 20% in any assessment. In this study, different questioning levels will be analysed and a weighting of each level will be provided.

2.5 Summary and Conclusion

The literature in this Chapter was positioned around previous studies that focused on mathematics. In these studies the importance of procedural and conceptual knowledge and different types of tasks was the focus. It was found that tasks used for teaching and learning should be stimulating and captivating for learners, one way of doing this is to incorporate everyday experiences into tasks. The importance of assessment has been presented and will significantly contribute to analyses provided later in the report. The interdependence of both procedural and conceptual knowledge and the importance in mathematics has been discussed. This will provide a good foundation for the researcher when analysing November examination question papers and the memoranda.

CHAPTER 3: THEORETICAL FRAMEWORK

3.1. INTRODUCTION

After having discussed the background, motivation, defining research questions and reviewing literature concerning this study, it is important that a theoretical framework is established. This is done to link the concepts that are crucial in this study. In this chapter, I will discuss Bloom's Taxonomy; the Revised Taxonomy; CAPS Taxonomy and Webb's DOK Model. I will provide justification for utilising Webb's DOK model as a theoretical framework for this study.

3.2. TAXONOMIES / MODELS FOR TEACHING AND LEARNING

Taxonomy is a classification system that organises and categorises systems. In education there are several taxonomies for teaching and learning. This section provides a brief discussion on Bloom's taxonomy, Bloom's revised taxonomy, CAPS framework and Webb's DOK model. This is done so that one is able to have an overview of the taxonomy or framework before the justification for the chosen teaching and learning model is provided. The CAPS framework is discussed because it is the policy that is prescribed by the DBE in South Africa and currently being followed. Webb's DOK model has been selected because it incorporates the content and cognitive demand of tasks unlike other educational models.

3.2.1. Bloom's taxonomy and Bloom's revised taxonomy

Bloom's taxonomy (1956) is a hierarchical model with six levels. The levels are: knowledge, comprehension, application, analysis, synthesis and evaluation. Knowledge occupies the lowest level and evaluation occupies the highest level (Hess, Jones, Carlock, & Walkup, 2009; Krathwohl 2002). Limitations to this model were identified, such as several verbs appearing at various levels of the hierarchy and it does not indicate the specified complexity. This taxonomy accounts for the cognitive dimension only. With these shortfalls amongst others a revised taxonomy was introduced.

The revised Bloom's Taxonomy focuses on the process dimension and not only the cognitive dimension. The revised Taxonomy (2001) consists of six levels: remember, understand, apply, analyse, evaluate and create as shown in Figure 3.1 (Krathwohl 2002; Anderson & Krathwohl 2001). Remember occupies the lowest level and create occupies the highest level.

The two topmost levels have been rearranged and there are now expanded and clarified descriptors. The revision of level descriptors has incorporated both processes and the knowledge required. The revision has acknowledged the importance of the interaction between content and thought processes.

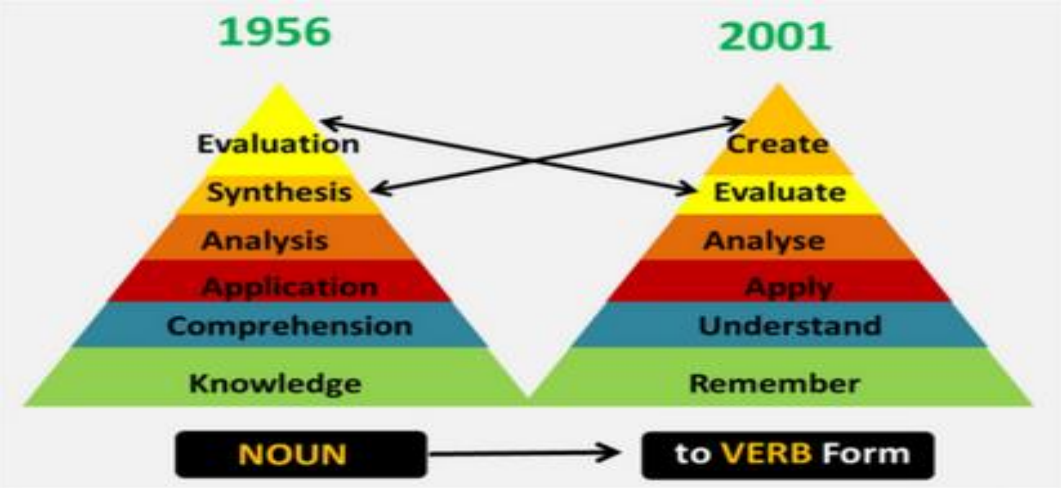


Figure 3.1: Bloom's Taxonomy and Bloom's Revised Taxonomy

3.2.2. CAPS framework

The CAPS framework consists of four cognitive levels as suggested in the TIMSS study of 1999 and are clearly stated in the CAPS document (DoE, 2011, p. 53). These cognitive levels are: knowledge; routine procedures; complex procedures and problem solving. The framework must be utilised when designing assessment tasks. The description of skills and complexity of questions at each level are shown in figure 3.2.

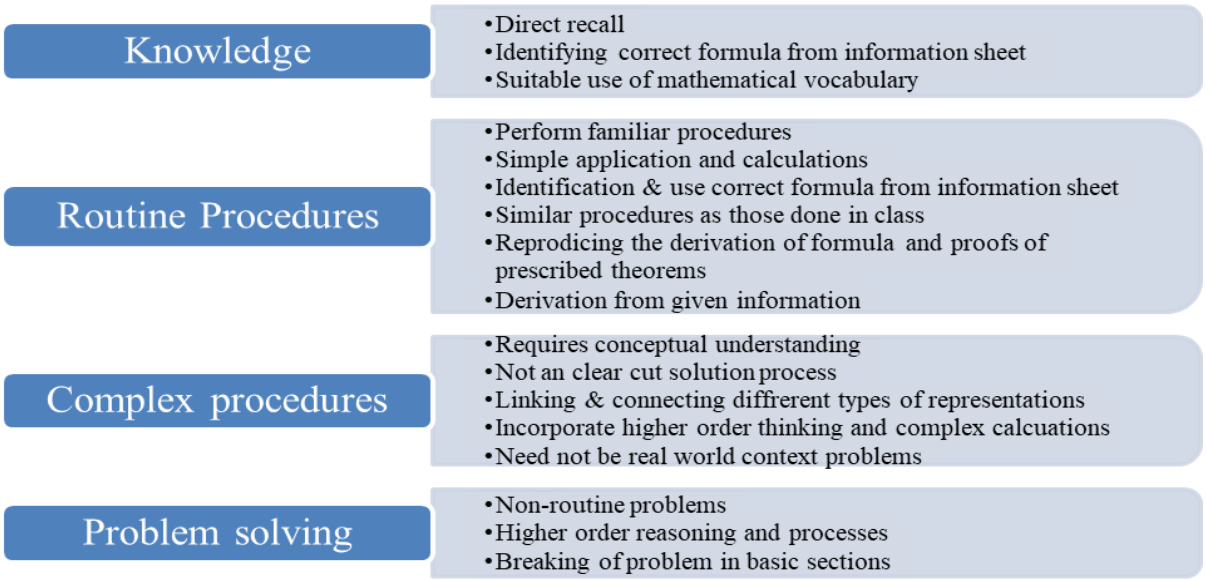


Figure 3.2: Cognitive levels according to CAPS and a description thereof (DoE, 2011)

3.2.3. Webb's DOK model

Webb's DOK model was designed by Norman Webb and it refers to the comprehension and interpretation that is mandatory in order to perform an activity or reach a solution.

> Standards and assessments can be aligned not only on the category of content covered by each, but also on the basis of the complexity of knowledge required by each. *Depth-of-knowledge consistency between standards and assessment indicates alignment if what is elicited from students on the assessment is as demanding cognitively as what students are expected to know and do as stated in the standards*. (Webb, 2002, p. 4)

The DOK model consists of four levels and can be used as a tool that can assist teachers in creating rich environments in which learners learn at a high level, as it can be used during task design in order to improve the rigor of instruction and implementation. A detailed description of the DOK model is presented in Section 3.3.

3.3. WEBB'S DOK MODEL

DOK differs on several dimensions; the cognitive complexity, ability to transfer knowledge, formation of generalisations and the prerequisite knowledge required (Webb, 1997). The strength and number of mental connections that learners have to make describes the DOK. Understanding in mathematics, is among the aspects the ability to make connections between ideas, facts, or procedures (Hiebert & Carpenter, 1992, p. 67). Figure 3.3 shows the four DOK levels.

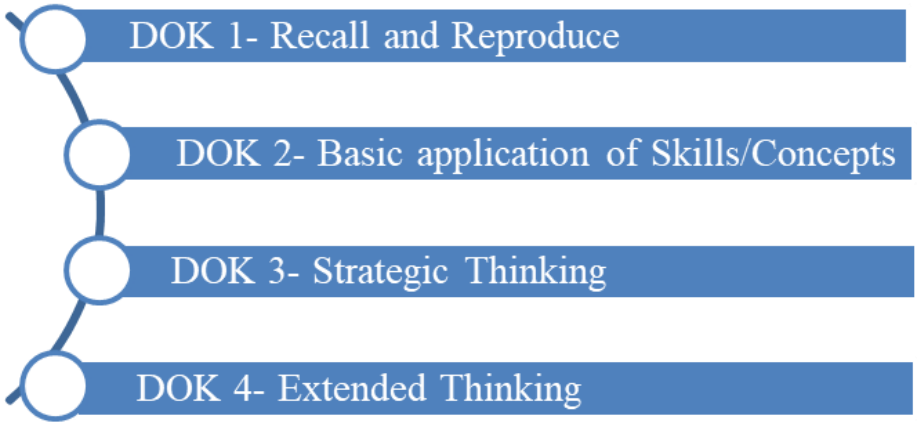


Figure 3.3: Webb's Depth of Knowledge (DOK) model

DOK allows for the inclusion of both content being assessed and the depth to which understanding is going to be assessed. The depth to which understanding is going to be assessed and the content assessed is incorporated in the DOK model. A shift from the end

results to the cognitive and thinking processes is noted. The DOK model goes beyond the "what" and seeks to go deeper into the "how".

The classifying of questions into DOK levels will be determined by the number of connections required to reach a solution according to the intricacy of thinking required. This highlights the way learners have to interact with content (Nappi, 2017). A description from Hess et al. (2009) of each DOK level is given in figure 3.4.

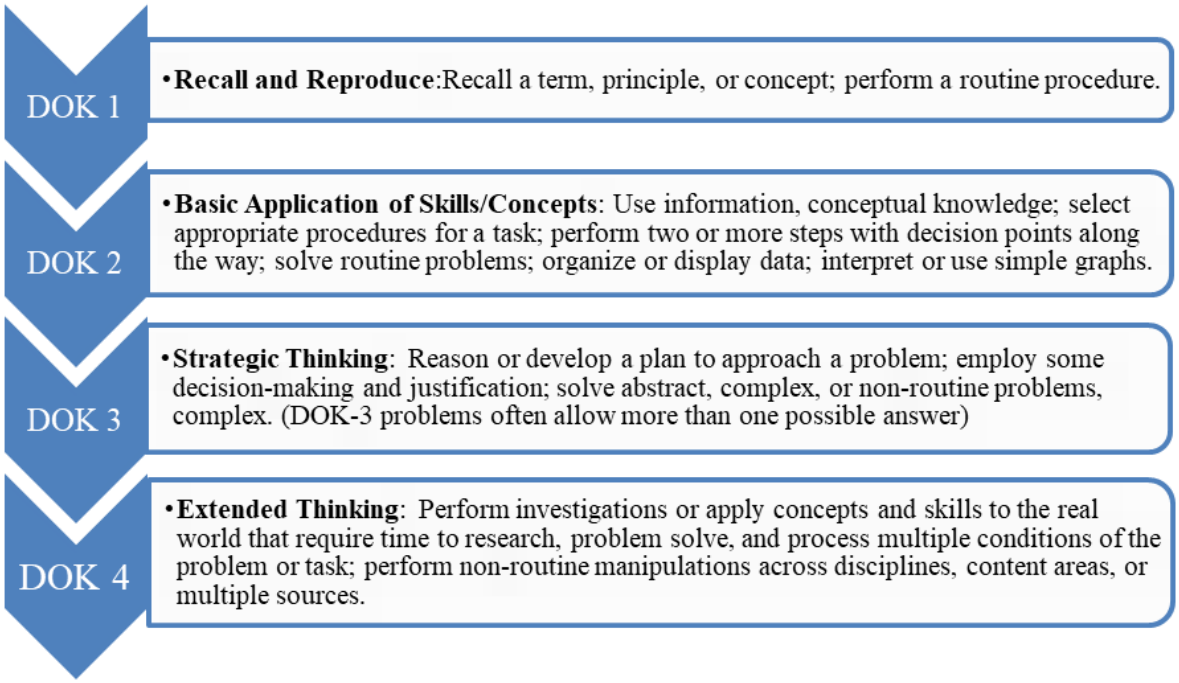


Figure 3.4: DOK levels as described in Hess et al. (2009)

DOK 1: Recall and Reproduce

DOK 1 represents the rote memorisation and basic knowledge acquisition which makes higher level tasks possible. This is essential as the mastery of concepts at this level contributes to a solid foundation for future learning. Learners are expected to acquire information that they require to develop deeper knowledge thinking. This can include calculating the probability of a single event occurring (for example, tossing a coin results in two predictable outcomes; heads or tails).

DOK 2: Basic Application of Skills/ Concepts

This level includes limited application of skills and concepts to perform a task. It is common to find multistep problems and this requires learners to make decisions at various stages

during the solution process. This can include instances where learners are required to draw a Venn diagram or Tree diagram based on a specific scenario. Demonstration and communication of both conceptual and procedural knowledge is required. Personal understanding of learning will be displayed and connection between pieces of information, concepts and skills are required.

DOK 3: Strategic thinking

Questions at this level usually involve analysis and evaluation of information. Engagement with real-world problems that may have predictable outcomes (for example, having the choice to pick out two cards from a set, with or without replacing the first card) occurring frequently. Various problem-solving strategies and skills that they have acquired both in and out of the mathematics classroom can be incorporated into learning.

DOK 4: Extended Thinking

This level requires an extension of thinking to solve complex problems with unpredictable outcomes. Strategic thinking, analyses, investigation and reflection during the problem-solving process are important. Tasks at this level are usually project-based research or investigations which are usually open-ended, and learners can explore several creative and feasible options. Exploration and extension of thinking beyond the mathematics classroom is encouraged. An example would be for learners to design a product or application that can make mathematics' learners lives easier. This type of thinking is often developed over a period of time, sometimes weeks or even months.

For the scope of this study investigations or research investigations will not be done hence no level 4 analyses will be included. Only DOK 1, 2 and 3 are considered. Figure 3.5 provides examples of possible probability questions at DOK 1, 2 and 3 respectively. Detailed examples of probability questions at DOK 1, 2 and 3 are explored in Chapter 5 when the question papers are analysed.

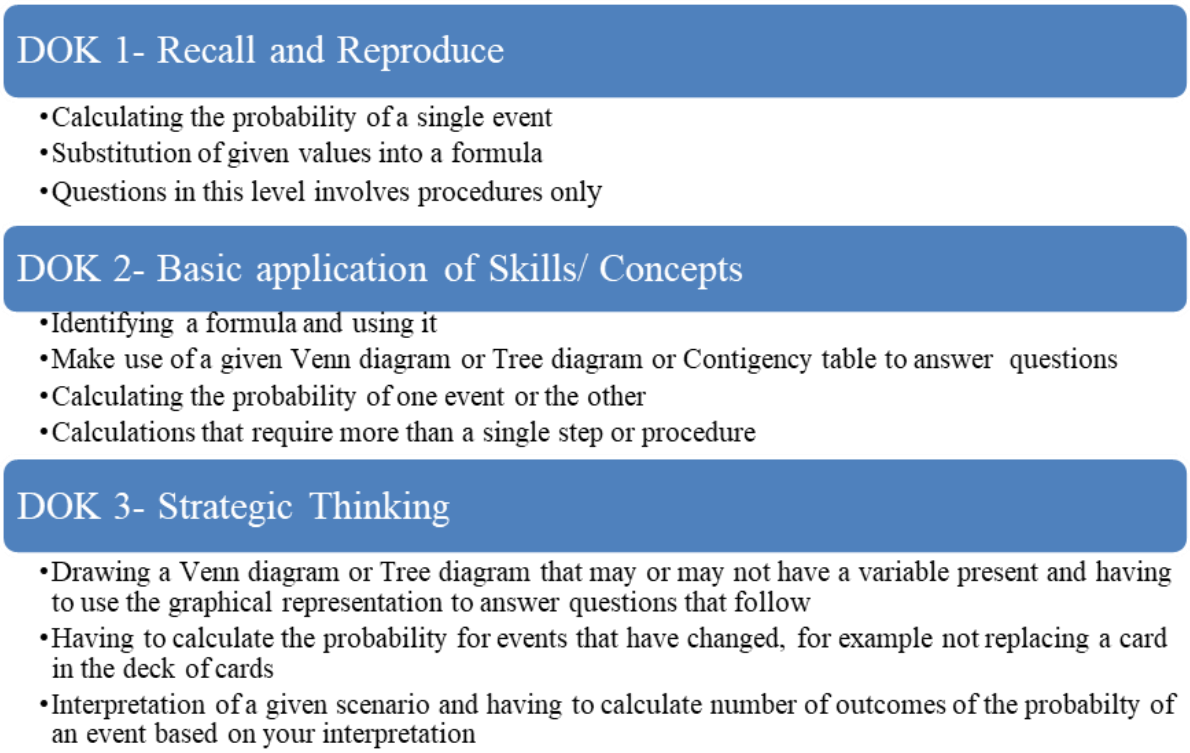


Figure 3.5: Possible examples for DOK levels in probability

3.4. A COMPARISON OF THE TAXONOMIES/ MODELS

Each taxonomy or models discussed in previous sections have advantages and pitfalls when it involves teaching and learning of mathematics. Teachers should be well informed of the various taxonomies and models that are available to enhance teaching and learning. Figure 3.6 provides a side-by-side comparison of Bloom's revised taxonomy, CAPS taxonomy and Webb's DOK model. The comparison highlights the overlapping and unclear boundaries amongst the various models. The goal for a given task or lesson will determine which model is utilised as they all provide the user with a grading or scaling system suitable for teaching and learning.

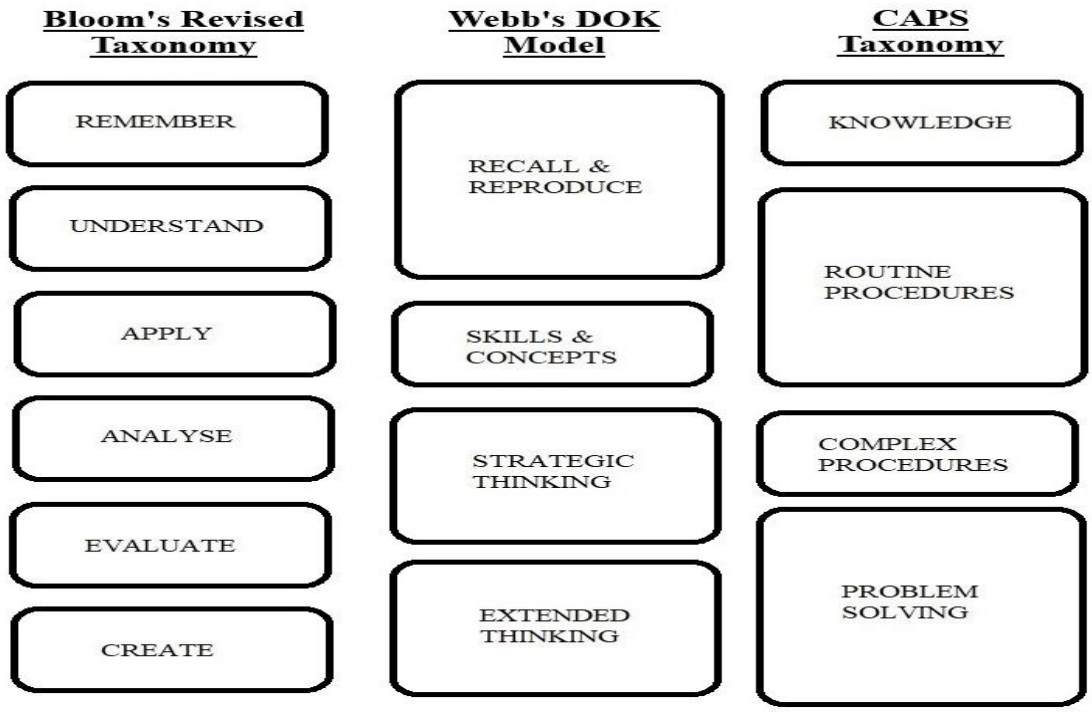


Figure 3.6: A comparison of Bloom's Revised Taxonomy, Webb's DOK Model and the CAPS Taxonomy

3.5. THE LINK BETWEEN WEBB'S DOK MODEL WITH PROCEDURAL AND CONCEPTUAL KNOWLEDGE

Learners' responses to questions will display some form of understanding. This understanding of knowledge can be categorised into two, namely; Procedural Knowledge or Conceptual Knowledge. A link between the DOK model and procedural or conceptual knowledge is made and displayed in Table 3.1. Rittle-Johnson, Schneider and Star, (2015) claim there is bidirectional relation between procedural and conceptual knowledge. Both procedural and conceptual understanding are required for mathematical competence as they reinforce each other (Rittle-Johnson, Siegler, & Alibali, 2001) as gains in procedural knowledge enhance conceptual knowledge and vice versa. Conceptual knowledge may influence or even become procedural with repeated exposure, while procedural knowledge may support the development of conceptual knowledge (Artigue, 2007). If one repeatedly performs a task that requires conceptual knowledge, the conceptual nature of the task will eventually become procedural.

Table 3.1: Link between Webb's DOK with Procedural and Conceptual Knowledge

DOK LEVEL	WEBB'S DOK	PROCEDURAL / CONCEPTUAL
1	Recall and remember	Procedural
2	Basic application of skill/concepts	Conceptual
3	Strategic thinking	Conceptual
4	Extended thinking	Conceptual

Holmes (2012) reviews various frameworks that are utilised in the teaching fraternity. A comparison of Webb's DOK with the Diagnostic Teacher Assessment of Mathematics and Science (DTAMS) as well as a comparison with Porter's Cognitive Complexities were done (Holmes, 2012). Figure 3.7 shows the DTAMS and Webb's DOK comparison; from this image we can see that DOK 3 and 4 are both classified as problem-solving, DOK 2 represents understanding and DOK 1 is memorisation. This figure further helps us when it comes to analysing responses to mathematics' probability questions. Figure 3.8 shows DOK 2 consists of two levels of Porter's cognitive complexities (performance procedures and communicate understanding). DOK 1, like with the comparison with DTAMS, represents memorisation; DOK 3 represented solving non-routine problems and DOK 4 is prove.

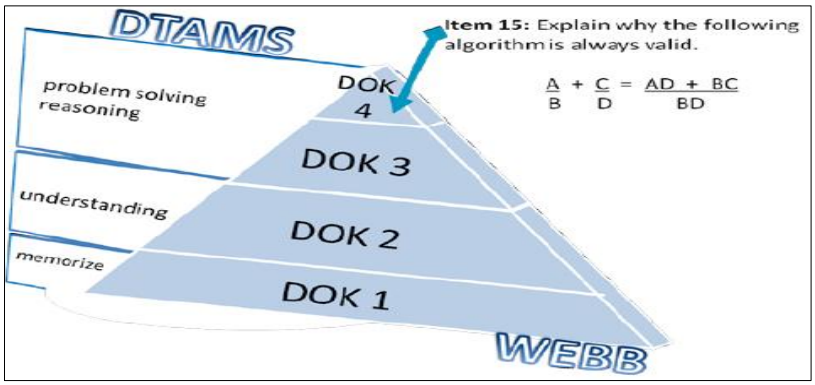


Figure 3.7: Comparison of DTAMS and Webb's DOK levels (Holmes, 2012, p. 62)

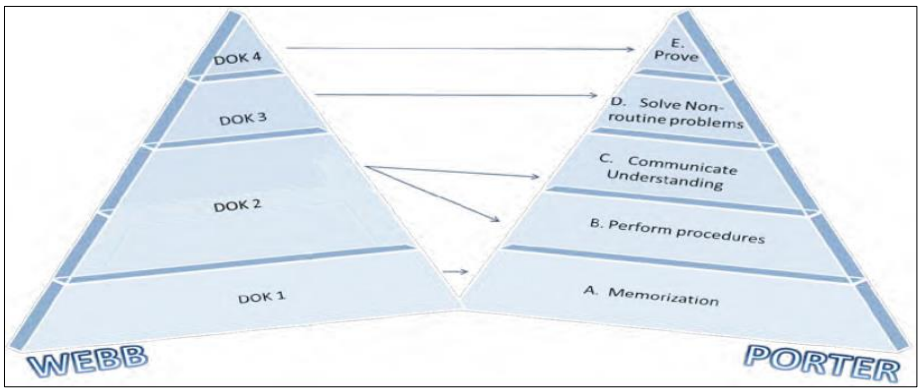


Figure 3.8: Comparison of Webb's DOK with Porters Cognitive Complexity Framework (Holmes, 2012, p. 63)

According to Hiebert and Lefevre (1986) one form of procedural knowledge involves symbols of the system and syntactic norms; the second form of procedural knowledge involves the rules and processes for solving mathematical problems. Rittle-Johnson and Alibali (1999) define procedural knowledge as the action sequences for problem solving. From the above descriptions, procedural knowledge is being able to identify and utilise a correct procedure in order to solve a problem, without much thought about the actual process and the reasoning behind why you are using a specific process.

Hiebert and Lefevre (1986) claim that conceptual knowledge is rich in relationships that form connected networks of knowledge where the linking relations are as prominent as individual pieces of information so that all information is eventually connected to some network. Rittle-Johnson and Alibali (1999) define conceptual knowledge as the understanding of principles and interrelations between various pieces of knowledge. From the above descriptions, conceptual knowledge is rich in relations between various knowledge networks and requires deep understanding of concepts and their connections.

According to Rittle-Johnson et al., (2015) there is a bidirectional relation between procedural and conceptual knowledge. This means that development of procedural and conceptual knowledge is an iterative process (Rittle-Johnson et al., 2001), such that development of one leads to gains in the other as well, this displays the complex interdependence (Engelbrecht, Bergsten, & Kagesten, 2009) between knowledge. Haapasalo and Kadijevich (2000) claim that procedural knowledge requires involuntary recall whereas conceptual knowledge requires purposeful thinking. This highlights the importance of both types of knowledge for the teaching and learning of mathematics (Rittle-Johnson & Alibali, 1999; Rittle-Johnson et al., 2015; Rittle-Johnson et al., 2001). Teachers should expose learners to the skills required for attainment of both procedural and conceptual knowledge.

3.6. JUSTIFICATION FOR WEBB'S DEPTH OF KNOWLEDGE MODEL

Bloom's revised taxonomy provides inadequate guidance in articulating instruction and assessment strategies however, the thinking processes required to answer questions are described (Hess et al., 2009). There is a gap between the taxonomy levels and the depth of understanding which is necessary for responding to problems. The DOK model bridges this

gap. Webb's DOK model links the depth of understanding for a task from beginning to the end; this includes the planning, research and conclusions (Hess et al., 2009).

Bloom's taxonomy and revised taxonomy are hierarchical unlike Webb's DOK model. With DOK it means that achievement and accomplishment of one level is not required to progress to the next. Kuzmich (2015) claims DOK model is not a linear model as all levels are necessary over time, the order only matters for the sequence of instruction and learning in relation to the curriculum. Providing learners with DOK level 3 tasks can engage and motivate learning that might take place at level 1 or 2. Learners of all ages can engage with task at all DOK levels because the DOK levels are not age-related. The classification of tasks will differ; for example, a task at Grade 9 level may be classified as DOK 3 but the same task will be classified as DOK 1 for Grade 12.

The thinking process is the focus when identifying the DOK level. In comparison to Bloom's revised taxonomy which focuses on the verbs to classify thinking. Within DOK the focus is not on verbs; the focus is on learners' demonstration of learning content and the depth of thinking required, so any verb must be considered in the context of learning content (Kuzmich, 2015).

CAPS taxonomy consists of four cognitive levels. The highest level, or the most challenging level is problem solving. However, there may be instances in which the skills of problem solving can be used to answer lower levels of questions. Problem solving can be used in reaching a solution for complex procedures. This categorization of cognitive levels may not always be applicable; at times more than one cognitive level may be present in a question.

In summary, the DOK or the cognitive demands relates to the number of mental connections that have to be made. The DOK of a task depends on the number of connections that learners have to make; their level of reasoning and problem-solving skills not forgetting other factors such as; representations, and the transferability and generalisability of concepts and skills from one problem to others (Webb, 1997). The DOK model classifies content knowledge in terms of the cognitive difficulty as well as the refining of pedagogy whilst driving instruction and most importantly it provides a means to plan for and promote higher order thinking behaviour, for both the teacher and learner (Holmes, 2012). An analysis of probability

questions will be presented in Chapter 5. This analysis will be based on Webb's DOK model and the depth of thinking that is required to answer probability questions.

3.7. SUMMARY AND CONCLUSION

In this study three taxonomies/ models were discussed in terms of their characteristics and application. Different levels of DOK showed how questions at those levels may look. In this study DOK 1, 2 and 3 questions will be considered. Justification for the DOK model was given, Webb's DOK model is going to be utilised further in the study. This was done because probability questions as well as learners' responses to probability questions are investigated further in this study. Chapter 4 discusses the research methodology used in this study. Chapter 5 provides a detailed analysis of the research instrument incorporating the DOK model and Chapter 6 proves an analysis of learners' responses to probability questions.

CHAPTER 4: RESEARCH DESIGN AND METHODOLOGY

4.1. INTRODUCTION

In this chapter, the research design and methodology of this study will be discussed. The discussion begins with the research strategy which involves the research methods and the research sample. The data collection instruments and processes as well as the data analysis methods are discussed in detail. The discussion includes necessary information about quantitative and qualitative methods for data analysis. Thereafter the reliability, validity and ethical considerations of this are discussed. The limitations and a summary of the data collection strategy are then provided.

4.2. RESEARCH STRATEGY

4.2.1. Research Methods

In this study mixed methods of research are utilised. Mixed methods research encourages us to view the world through various lenses and to share these views and values hence, mixed methods recognises that research is neither exclusively quantitative nor qualitative (Cohen, Manion, & Morrison, 2018).

A post-positivist paradigm guides this study, assumptions in this paradigm that have been used in traditional forms of research and which hold true for quantitative and qualitative research. "Post-positivism places emphasis on the identification of causality and its effect, focus on variables and their manipulation, careful observation and measurement" (Cohen et al., 2018, p. 34). Creswell and Creswell (2018) claim the accepted approach to research by post-positivists follows a process whereby a researcher selects a specific theory; which is then followed by data collection. The results from the data collection can either support or refute the theory. Finally, the researchers should then be able to make appropriate alterations and conduct tests.

According to Creswell and Creswell (2018) mixed methods research takes a position in between the middle of the research spectrum as it incorporates features of both qualitative and quantitative research methods. Mixed methods paradigm rest on the ontology which recognises that phenomena are complex; that a single method approach may only results in partial or incomplete understanding and on an epistemology that requires a sequential pragmatic combination of methods. This combination of methods run in parallel or are

synthesised in order to fully comprehend and embrace phenomena in order to do justice. (Cohen, Manion, & Morrison, 2018). Mixed methods utilises a model or conceptual framework in to guide the questions in the study. That conceptual framework also informs both the quantitative and qualitative components in the study (Creswell & Creswell, Research Design: Qualitative, Quantitative, and Mixed Methods Approaches, 2018). Quantitative research results are usually presented as quantities or numbers whereas qualitative research results are usually presented through words by reviewing transcripts or field notes (Patten & Newhart, 2018). Generalisability is a goal for quantitative research but it cannot always reach all members of a group and hence the findings are bound to a specific sample. Qualitative approaches emphasise interpretive processes and quantitative approaches seek to generalise methods with goals of objectivity and standardisation. Strengths of qualitative research are the ability to provide insights on interpretation, context or the meaning of phenomena and results are often presented in narrative formats.

Qualitative analysis relies on interpretation and there is no single correct way to analyse and present the data (Cohen, Manion, & Morrison, 2018). "A basic interpretive study provides descriptive accounts targeted to understanding a phenomenon using data that might be collected in a variety of ways, such as interviews, observations, and document reviews. The purpose is to understand the world or experience of another" (Ary et al., 2010, p. 29). This research is interpretive as it focuses on interpreting learners' responses to probability questions and teachers' responses in the interview. It is also descriptive as it records and represented information based on the frequency an extent that something occurs. Qualitative methods used in this study include preparing and organises data into useful formats; describing and presenting that data in manners that are complimentary to the research; analysing that data by finding linkages and relationships between and among the results obtained; interpretation of the data and finally drawing conclusions and reporting on the findings of the data (Cohen, Manion, & Morrison, 2018). The purpose of this methodology is to satisfy the research plan and to target the devised by the researcher.

4.2.2. The Research Sample

This study utilises a purposeful sample as there are specific criteria (Patten & Newhart, 2018) that should be met by the participants. Participating schools have to offer mathematics as an examinable subject in Grade 12 and be registered for NSC examinations. For convenience,

schools sampled are located in the surrounding area of the researcher's residence, to reduce logistical issues for the data collection processes. Participation in this study was voluntary and withdrawal would not have any implications on participants.

Six schools were approached and informed about the study, one school agreed to participate. Due to the Corona Virus pandemic several schools opted not to participate and the school that agreed to participate did so with extremely strict rules and regulations. One teacher consented to be actively involved and be interviewed. The initial sample was 40 learners however, 11 learners withdrew which resulted in a sample of 29 learners. Several learners had also opted not to participate in the study due to the extreme circumstances caused by the pandemic. Since one school was participating, making the sample smaller, the research was broadened by increasing the number of data collection questions.

4.3. DATA COLLECTION

Data collected from this study was used to identify and explain phenomena that are guided by the research question based on probability.

> When qualitative inquirers interpret or explain the meaning of events, actions, and so forth, they generally use one of the following types of interpretation: (1) construction of patterns through analysis and resynthesis of constituent parts, (2) interpretation of the social meaning of events, or (3) analysis of relationships between events and external factors. These interpretations may lead to the generation of theories, be guided by existing theories or concept maps, or seek to further explicate or expound upon a theory. (Ary, Jacobs, & Sorensen, 2010, p. 421)

A discussion of the data collection instruments and the data collection process is done in order to address the two main research questions guiding this study.

The two main research questions for this study are:

Main research question 1: **What depth of knowledge is evident, from Probability questions in selected November examinations?**

Main research question 2: **How do learners respond to Probability questions that are at different Depth of Knowledge levels?**

4.3.1. Data Collection Instruments

4.3.1.1. November Examinations - Probability Questions and Answers

The November final examination is a National Examination and is therefore set and moderated by Department of Basic Education (DBE). The examination is written in all schools, which are registered with the Department of Education for the National Senior Certificate Examination, across the country on the same day at the same time. Schools follow a national timetable for the November examinations and have to collect examination papers and writing material from a nodal collection point on the day of the examination. All learners writing this examination write at the same time, for the same duration and under standardised conditions set out by the GDE. Once the examination is over, answer scripts are collected and delivered to the nodal points after which they are distributed to the relevant examination authorities for storage until national marking of examination scripts commence. Marking of examination scripts are an external process and the school or teachers in the school have no control over this marking process. That means that the school does not have access to learners' scripts. In this study data was collected from November 2017 to 2019 examinations, the probability questions from those examination papers have been used.

4.3.1.2. Probability Revision Task (PRT) - Questions for Learners to Answer

November examinations are not kept by the school and hence scripts could not be accessed via the school. To collect data, probability questions from three November examinations were reorganised and given to learners to answer. The questions were reorganised and designed into the PRT that was completed once the topic of probability has been taught in class. This task would be beneficial for learners as they are now able to apply what they have learnt to a variety of questions for the topic of probability.

Probability questions were used from November 2017 to 2019 examinations to form the second data collection instrument – the Probability Revision Task (PRT). The PRT was used to collect the necessary data. A selection of three years probability questions were used in the project so as to broaden the scope of questions that have been answered. This also then allows the research to compare results obtained in this study with three cohorts of learners who have written the examinations.

A summary of the PRT and the probability questions and concepts are discussed below. This provides the reader with a better understanding of the PRT. Probability questions were reorganised in order to form the PRT. The original November examination, mark allocation and PRT number (1 to 8) are presented in Table 4.1.

Table 4.1: Original source and mark allocation for PRT

ORIGINAL SOURCE	QUESTION NUMBER	MARK ALLOCATION	PRT NUMBER
NOVEMBER 2017 EXAMINATION	10	8	5
	11	6	1
NOVEMBER 2018 EXAMINATION	11	7	2
	12.1	3	3
	12.2	6	6
NOVEMBER 2019 EXAMINATION	10	5	7
	11.1	5	4
	11.2	6	8

Probability questions from three sets of November Paper 1 examinations (2017 to 2019) were reorganised to compile the PRT. The PRT consists of eight tasks (T_1-T_8), in which some tasks have sub-questions. PRT has a grand total of 46 marks and learners were given an hour to complete it. Learners wrote this Revision Task before their final November examinations, this Revision Task formed part of their revision programme in getting prepared for their final school leaving high-stake examination. The task was reorganised so that learners can engage with the various questions within the probability topic by answering questions that require certain skills or concepts. Table 4.2 provides a summary of the probability concepts that are present in each task. Through the reorganisation process of the PRT, the questions remained the same, only the sequences of the questions have been rearranged.

Table 4.2: Probability concepts assessed in PRT

PROBABILITY REVISION TASK	PROBABILITY CONCEPTS
Task 1 (T_1)	Codes and Fundamental Counting Principle
Task 2 (T_2)	Codes and Fundamental Counting Principle
Task 3 (T_3)	Mutually Exclusive Events
Task 4 (T_4)	Independent Events and Venn Diagram
Task 5 (T_5)	Venn Diagram
Task 6 (T_6)	Interpretation and Tree Diagram
Task 7 (T_7)	Interpretation
Task 8 (T_8)	Interpretation and Fundamental Counting Principle

T_1 and T_2 in the PRT involve specific codes that are being generated; these questions also require the counting principle to be incorporated into reaching a solution. T_3 and T_4 require learners to recall properties of mutually exclusive and independent events. T_4 and T_5 require

learners to utilise the graphical representation of a Venn diagram as a means of expressing their solutions. T_6 requires learners to interpret the given information and a learner may find it helpful by drawing a tree diagram in order to reach a solution. T_4, T_5 and T_6 incorporate a graphical representation in problem solving strategy. T_7 and T_8 require the learners to interpret the given scenario and answer questions that follow. T_1, T_2 and T_8 incorporate the Fundamental Counting Principle as a concept that should be utilised in order to reach a solution.

The researcher followed necessary protocols for conducting research in schools. Permission for conducting research at a school only took place after permission had been granted from the University of the Witwatersrand, school of education ethics committee (Protocol number 2019ECE046M), and the GDE (2019/346- 8/4/4/1/2).

The data collection process took place once the Principal (refer to appendix C and D) had granted the researcher permission. Learners (refer to appendix I and J) submitted their permission to participate in research consent form as well as forms of parental/ guardian acknowledgment and consent if the learner is younger than 18 years (refer to appendix E and F).

4.3.1.3. Interviews

Interviews with Grade 12 mathematics teachers were conducted. The aim of this interview is to gain insight into teachers' outlook, challenges, suggestions and opinion on how they feel learners approach probability. Mostly open-ended questions were used in the interview; this allows the teacher to give valuable input without having to settle for a predetermined response to a specific question. The interview would allow for the triangulation of data. Data triangulation allows the researcher to explore whether data collected with one procedure, for example; document analysis; confirms data collected using a different procedure; for example, interviews (Ary et al., 2010; Cohen et al., 2018; Patten & Newhart, 2018). Interviews with teachers only took place after teachers agreed to participate (refer to appendix G and H) for the permission and consent forms. Interviews took place after school hours; this ensured that teaching and learning was not disrupted for the teacher, learners or researcher. The researcher had an estimated duration for the teacher interview of 30 minutes

however; the duration of the interview was dependent on the responses provided by the teacher.

4.3.2. Data Collection Process

In this study the data collection process occurred in sequential phases. Each phase supports the researcher when answering the research questions. To begin the data collection process, the researcher sourced November Mathematics Paper 1 Examinations from 2017 to 2019. Probability questions from those examinations were then retrieved. The researcher answered all the probability questions before marking the responses. Once the responses were marked the researcher then analysed the questions according to Webb's DOK model. The CAPS document and the NDR reports for mathematics paper 1 for 2017 to 2019 were then analysed Thereafter data collection from learners and teachers began.

Learners participating in the study wrote the PRT, during the month of August. This ensured that it is completed after their Preliminary examination but before their final November examination. PRT formed part of their revision programme before their final examination. Due to the Corona Virus pandemic, several learners opted not to participate in the study and hence the sample space was reduced significantly. PRT was written during school hours as part of a revision programme. Figure 4.1 provides a detailed summary of the data collection process that occurred and guided the researcher in this study.

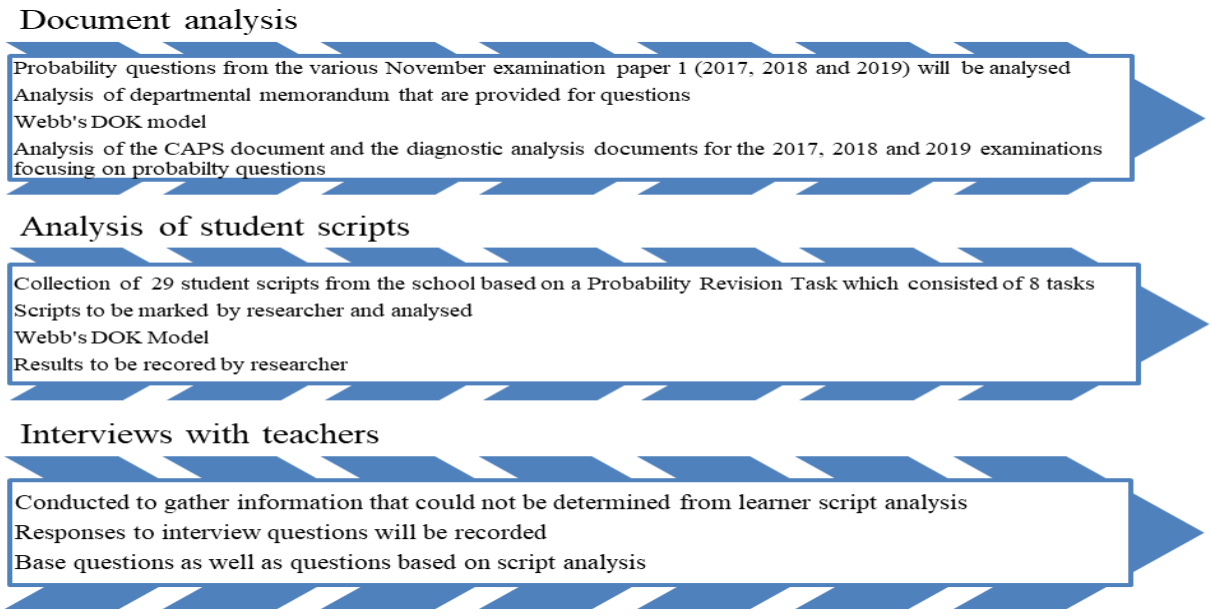


Figure 4.1: Data collection process

4.4. DATA ANALYSIS

In this study data was analysed using descriptive methods, the descriptive statistics involved calculating the frequencies, measures of central tendencies (averages), measures of dispersion (standard deviations, coefficient of variation, minimum and maximum). Descriptive statistics were used to compare the achievements of the learners according to Webb's DOK model and also according to certain categories and concepts of probability taught. This informed the researcher which of the DOK levels learners are struggling with and which of the DOK levels that learners are excelling in. A comparison between results from this study and results that have been recorded in the National Diagnostic Reports are presented. A comparison between probability questions that require procedural and conceptual knowledge is provided.

Bar charts are useful for presenting discrete data whereas histograms are useful for continuous data; pie charts are useful for showing proportions; box plots are useful for displaying the distribution of values for several variables together in a single chart with their range and medians (Cohen et al., 2018). Highest and lowest values can be easily seen from a bar chart and the data from bar charts can be easily compared. A histogram can provide a brief overview of the frequency distribution from the data that was collected. Proportions can be easily seen from a pie chart. A box plot contains several variables that are used in order to describe the distribution of data, using minimum value, quartile 1, median, quartile 3, the maximum value and any possible outliers. The sample box plot (Figure 4.2) has been used as it provides a good graphical representation of the spread of data and displays any outliers if present. Comments regarding skewness can also be made from the box plot. In my study I have displayed results obtained as various representations, such as a bar chart; histogram; pie chart and box plot where necessary. Representations used would be dependent on the analysis and interpretation required.

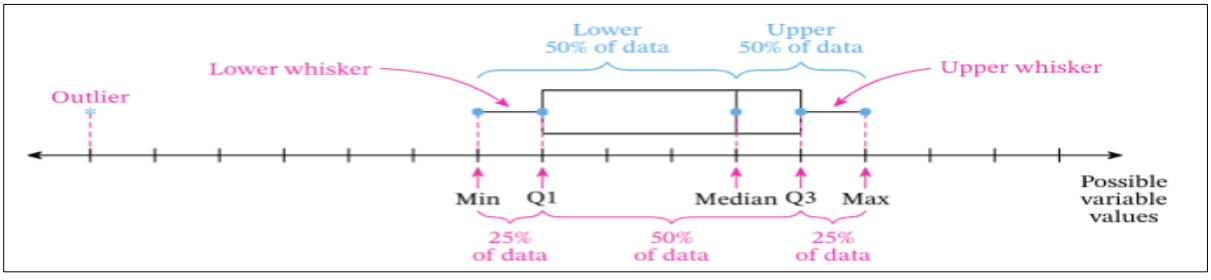


Figure 4.2: Box Plot display

Content analysis can utilise a variety of sources, the analysis methods are determined by the researcher's goals. The focuses of the analysis are based on the interpretation of the data that has been collected. The selection of documents was determined by the research questions and what the researcher believes is the best possible documents that can provide answers and reasoning to their questions. Based on the analysis provided above Denscombe (2010) provides a possible sequence for content analysis. The content analysis process begins by preparation of data followed by the initial exploration of data and then an analysis of data follows. Thereafter the data is presented and displayed in appropriate means and finally the data is validated. A content analysis provides a detailed qualitative analysis of the probability questions (Chapter 5) and learners' responses to probability questions (Chapter 6).

4.4.1. November Examinations - Probability Questions and Answers

Probability questions from November 2017 to 2019 examinations were used to compile the PRT. Due to a smaller than anticipated sample space the research was broadened by in-depth documentary analysis of three examination question papers (in terms of DOK) as opposed to the initial one examination paper. Probability questions from November examinations were first answered by the researcher and then compared to the memorandum that is available. Thereafter the researcher identified the various DOK levels that are evident, whether procedural or conceptual knowledge was required in order to answer the question.

The documentary analysis allowed for comparison of learners' responses amongst three different examination papers that were intended for three different cohorts of learners. The focus was on how the papers compared and how learners responded to different questions testing concepts from three different examination papers. The concepts that are tested in the Probability Revision Task (PRT) include: fundamental counting principle, different types of events, graphical representations of events and interpretive questions.

Data from the November examination probability questions will be analysed both quantitatively and qualitatively. Qualitative analysis involved the researcher answering all of the probability questions and then comparing the answers to those that are provided in the memoranda. There after a more quantitative analysis consisted of categorising the questions into the three DOK levels and identifying whether procedural or conceptual knowledge was required when answering. All probability questions that are analysed have been presented in

Chapter 5. In this chapter, the results in tables include the question, solution, mark allocation and DOK level. This is followed by a brief discussion of each sub-question. Finally, a table is used to summarise the questions from each November examination showing the DOK levels; the questions numbers, mark allocation, percentage from total and whether procedural or conceptual knowledge is required.

4.4.2. Probability Revision Task (PRT) -Learner Scripts

Data analysis is presented with various graphical and statistical representations in Chapter 6. Once the researcher had access to the learners' responses to the PRT, it had then be stored in envelopes, kept in a locked cupboard in a secured building during and after analysis has taken place. Pseudonyms were assigned to each learner's response to the PRT in order to protect learner's identity. Learner's scripts were marked and results tabulated and interpreted by the researcher. The identity of the school and the learners must remain confidential so as to ensure that it is not brought to public attention. All scripts have then been analysed qualitatively and quantitatively in terms of the DOK model.

The analysis of learners scripts were conducted systematically in order to meet the requirements outlined in the report. Once scripts were collected from the school and in the researchers' possession, the analysis process began. Initially the research had to go over each script and cover up any names if learners had written their name. Thereafter the researcher assigned a letter (A to CC) to each of the scripts. The process of renaming scripts had to be done in order to protect the identity of all participants when referring to their work in this project. After this process was done the analysis process began.

To being the qualitative processes, scripts were marked and a total for each task was recorded, then the total mark and percentage for the PRT was also recorded. The results obtained in each task from the PRT can be seen in Appendix U and V. Section 6.2 discusses the overview of results from the PRT. In this section a bar graph, histogram, descriptive statistics, box and whisker plot and tables are used. Learner performance was then grouped using achievement descriptions used by the DBE. Table 4.3 displays the description of performance and the percentage required for each level.

Table 4.3: Achievement description (adapted from DBE , 2011, p. 56)

ACHIEVEMENT DESCRIPTION	PERCENTAGE
Outstanding	80-100%
Meritorious	70-79%
Substantial	60-69%
Adequate	50-59%
Moderate	40-49%
Elementary	30-39%
Not Achieved	0-29%

A histogram was used to display the spread of results obtained from the PRT according to the achievement descriptions that are provided in table 4.3 a box plot was used to display the scores obtained in the PRT.

The researcher then began the quantitative process whereby responses were categorised according to the response provided. Table 4.4 provides the different classification for learners' responses, it is important to note that the category of 'partial' consists of two sections- 'partly correct calculations and procedures' as well as 'nearly correct calculations and procedures'. This process was done for all eight tasks in the PRT. For each task in the PRT learners marks were displayed in a bar chart. Alongside the bar chart is a table with the categories, as shown in table 4.4, which summarises learner performance for each of the sub-question in the task. This can be seen in section 6.3

Table 4.4: Categories of learners' responses

CATEGORY	CLASSIFICATION OF LEARNERS' RESPONSES
NO ATTEMPT	Did not answer
ZERO	Incorrect calculations and procedures
PARTIAL	Partly correct calculations and procedures
	Nearly correct calculations and procedures
MAXIMUM	Correct calculations and procedures

Learner performance was also grouped according to DOK level, this is presented in Chapter 6 and a summary is provided in section 6.3.9. Once this was done the results were further analysed in order to see any trends and relations from learners' responses. Thereafter scripts were scanned per task and according to each category. The scanned responses are discussed in section 6.4.

Section 6.5 compares the spread of procedural and conceptual knowledge from the PRT. In this section a breakdown of the questions, mark allocation and the average obtained in this

study are tabulated for procedural and conceptual knowledge. The distribution of procedural and conceptual knowledge has been presented as a pie chart; this gives the reader a graphical representation of the spread. This is then followed by a discussion of the descriptive statistics obtained for procedural and conceptual knowledge. A comparison of results obtained in the PRT and results recoded in the NDR are tabulated in section 6.6 of this report.

The document analysis enables the reader to anticipate which DOK level the learner should be utilising. The analysis of the memorandum contributed to answering sub research questions that focus on procedural and conceptual knowledge; the analysis would determine if procedural or conceptual knowledge or both are required when answering the question. The classification of knowledge required to answer the question has been used when analysing learner scripts. Finally from the learners' scripts the researcher may get a better understanding of how learners interpret questions that test procedural or conceptual knowledge (sub-research questions). The researcher collected qualitative data in the form of learner written responses to probability questions and analysed the responses in a sequential manner.

4.4.3. Interviews

After the data from learner scripts were analysed I went back to the school for an interview with the mathematics teacher (refer to appendix K). For this study qualitative interviewing (Mason, 2002) questioning was used. The interview consisted of a few open and close response questions. These questions were aimed at teachers' perceptions on the topic, on learner attainment and teacher practices in classes. Caper and Leask (2009) provide detailed information on how to design and conduct interviews. The interview for this study was designed after data analysis from learner scripts and related to the DOK model. Even if the interview did not directly answer research questions in this study the aim of the interview was to gain insight and a have a better understanding of an FET mathematics teacher with a focus on probability as a Grade 12 topic. Responses from the teacher interview have been interpreted and linked to DOK levels. The responses from the teacher interview were typed out verbatim and a short interpretation was done, the results are presented in section 6.7.

4.5. AN OVERVIEW OF RESEARCH QUESTIONS, DATA COLLECTION AND DATA ANALYSIS

Table 4.5 provides an overview of the instruments and methodology utilised in order to approach and answer each research question.

Table 4.5: Research questions, instruments and methodology used

RESEARCH QUESTIONS	INSTRUMENTS & METHODOLOGY
MAIN RESEARCH QUESTIONS	
What Depth of Knowledge is evident, from the Probability Questions in selected November Examinations?	Analysis of Probability questions from November examinations questions papers.
How do learners respond to Probability Questions that are at different Depth of Knowledge levels?	Analysis of learners' responses to PRT and interview
SUB- RESEARCH QUESTIONS	
Which Depth of Knowledge Level is most prominent from the selected Probability Questions?	Identified after having analysed the probability question from selected November examinations.
From the selected Probability Questions which Depth of Knowledge Level has the highest weighting?	
From the selected Probability Questions, which Depth of Knowledge Level questions do learners seem to be performing better in?	After having analysed learners' responses to the Probability task, the DOK level(s) can be identified.
From the selected Probability Questions, which Depth of Knowledge Level questions are learners struggling to respond to/ answer?	
How do learners respond to Probability Questions that require Procedural Knowledge?	After having analysed learners' responses to the Probability task, the researcher can focus on questions that require procedural knowledge and how learners responded to those questions.
How do learners respond to Probability Questions that require Conceptual Knowledge?	
How have learners responded to Probability Questions at Depth of Knowledge Level 1, 2 and 3?	After having analysed learners' responses to the Probability task, the performance of learners at the different DOK levels can be identified.

The document analysis of the probability questions from November examination enables the researcher to ascertain the different DOK levels that are present in the selected probability questions as well as determine whether procedural or conceptual knowledge is required when answering questions. The document analysis of learners responses to the probability questions are used in order to establish which DOK level questions learners are excelling in and which they are struggling with. The analysis of probability questions from November 2017 to 2019 examinations contributed towards answering main research Question 1. The analysis of learners' responses to PRT contributed to answering main research Question 2.

4.6. RELIABILITY AND VALIDITY

Validity concerns the extent to which the data collection instrument measures what it was intended to measure (Maree, 2016) , the extent to which theory and evidence support the interpretation of data (Oluwatayo, 2012). The questions that are taken from the November

examinations (2017 to 2019) utilised in this study have been set and moderated by the Department of Education, this ensures validity as both the questions and answers for the probability questions have been approved by the DBE and have met the required standards for assessments in the country. Utilising these questions as revision material would be beneficial to the learners as they would be answering past examination questions and can familiarise themselves with questioning styles and techniques.

Reliability refers to the repeatability of the data collection instrument (Maree, 2016). Does the instrument yield the same results if administered to other members of the population? The National examination results are presented and reported in a diagnostic report by the Department of Education. However in this study the focus was on a small group of learners from the same geographical area. Results obtained from the participating learners are then compared to the results from the diagnostic reports; this has been done in Chapter 6 of the report.

4.7. ETHICAL CONSIDERATIONS

Trust and transparency are important for any research. In a school setting various levels of power within the organisation should allow access to the institution itself (Morrison, 2005).
Ethics Clearance and permission from the University of the Witwatersrand, school of education ethics committee (Protocol number 2019ECE046M), and the GDE (2019/346-8/4/4/1/2) was granted before the researcher proceeded with the study. Documents for clearance can be referred to in Appendix L1 and L2. Permission from the University of the Witwatersrand and GDE was granted before the researcher went to schools.

According to Gregory (2003, pp. 49) "Confidentiality is best assured on the basis of anonymizing the collection of data" In this study, both teachers and learners were assured that their names would not be used at any time. They were given pseudonyms for this research study and their results have not been shared with other individuals. The protection of the identity of the school and the learners has remained a priority for the researcher throughout the process; appropriate measures have been taken to ensure that the identities remain anonymous.

Consent forms were issued to seek permission for research to be conducted. Consent forms were issued to the principal, teachers, learners and parents. Data collection began after permission from all stakeholders was received and permission was granted. Information letters and consent forms contained detailed information about the study Informed consent, anonymity, confidentiality and withdrawal (Vithal & Jansen, 2010) were explained and contact details for the researcher has been provided for any further information that was required. All participants were made aware that participation is purely voluntary and that they can withdraw from the process at any given time. Learners were also made aware that the results of this task shall not impact their academic grades for the year and that they should answer questions to the best of their ability. Non-participation in this study does not affect their mark or grading for the year. Learners did not have to write their names on answer scripts so as to protect their identities.

4.8. LIMITATIONS OF THE STUDY

There are shortcomings and limitations to any study. This study encountered several limitations that the researcher could not control. Given the unpredictable nature of the worldwide Corona Virus pandemic, data collection for this study was affected. Several schools opted not to participate in this study, some due to time constraints regarding curriculum coverage, newly implemented operating protocols and to prevent the spread of the virus among other reasons. When permission for participation in this project was declined by the principal the sample space of the study was reduced. Learner participation depended on parental or guardian permission if the learner was less than 18 years old. Participation was voluntary and non-participation remained an option for teacher and learners. Time constraints were crucial in this study, as schools were closed for an extended period of time in order to curb the spread of the corona virus. This meant that interviews with learners would not be feasible and a deeper understanding of learner reasoning was not possible. Teacher interviews had to ensure that teachers were not distracted from their other commitments and would have to take place during a convenient time. When analysing learner scripts, the omission of an answer may distort results obtained and not provide a true reflection of their potential. Probability having a relatively low weighing in examinations may result in the topic being rushed to complete syllabus.

4.9. SUMMARY AND CONCLUSION

This Chapter discussed the guiding research methodology for this study. The data analysis techniques were selected so that the researcher can respond to the research questions that guide this study. In order to achieve this, probability questions and learners responses to the probability questions have to be analysed. A teacher interview enables triangulation the data collected. Specific quantitative and qualitative research methods were incorporated into a mixed methods research strategy in order to suit the needs of this study. Learners and teachers are participants in this study as their responses are analysed and interpreted. It is from the analyses and interpretations of results that the researcher aims to respond to the guiding research questions for this study.

CHAPTER 5: ANALYSIS OF PROBABILTY QUESTIONS FROM NOVEMBER 2017, 2018 AND 2019 EXAMINATIONS

5.1. INTRODUCTION

In this chapter a discussion about the probability questions is presented which will include a detailed analysis of each question and sub-question using the DOK framework. The discussion elaborates on the requirements, possible solution strategy, DOK level and type of knowledge necessary for the specific question. The analysis of the question papers is used to address the first research question: **What depth of knowledge is evident from probability questions in selected November examinations?** This research questions will be supported by two sub-questions: **Which Depth of Knowledge Level is most prominent from the selected Probability Questions?** And **From the selected Probability Questions which Depth of Knowledge Level has the highest weighting?** A documentary analysis of the question paper and departmental memorandum (Chapter 5 Section 5.2) is presented, followed by an analysis of learners' responses to the probability questions (Chapter 6). Section 5.3 provides a summary of the three November examination question papers. This is followed by the composition of the PRT in Section 5.4.

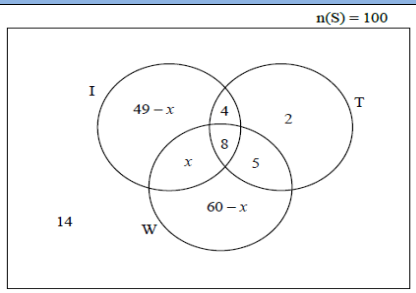
5.2. ANALYSIS OF PROBABILITY QUESTIONS IN NOVEMBER EXAMINATIONS

Probability questions from three previous November examinations (2017 to 2019) that were set by the Department of Basic Education (DBE) are presented with questions, solution, mark allocation and then analysed in terms of the DOK level (DOK 1- recall and reproduce; DOK 2- basic application of skills/ concepts and DOK 3- strategic thinking) and requirement of procedural or conceptual knowledge.

5.2.1. November 2017 Examination

Table 5.1: Question 10 from November 2017 examination

QUESTION 10
A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following: • 8 used all three. • 12 used Instagram and Twitter • 5 use Twitter and WhatsApp, but not Instagram • x use Instagram and WhatsApp, but not Twitter • 61 use Instagram • 19 use Twitter • 73 use WhatsApp • 14 use none of these applications

QUESTION NUMBER AND QUESTION	SOLUTION	MARK ALLOCATED	DOK LEVEL
10.1. Draw a Venn diagram to illustrate the information above		4	3
10.2. Calculate the value of x	$(49 - x)+x+8+4+5+2+(60 - x) + 14 = 100$ $- x + 142 = 100$ $x = 42$	2	2
10.3. Calculate the probability that a learner, chosen randomly, uses only ONE of these applications	P (use only one application) $= \frac{7+2+18}{100}$ $= \frac{27}{100}$ or 27 %	2	1

Question 10 8 marks

A Venn diagram consisting of three interlocking circles must be drawn and used to answer subsequent questions. Information, given in point form, must be interpreted in order to fill in the Venn diagram. The variable x affects three values in the Venn diagram. Errors in the Venn diagram will negatively impact subsequent questions. Both procedural and conceptual knowledge is required when answering the questions.

Question 10.1 can be classified as **DOK 3**, as procedures with multiple steps and multiple decision points are required. Conceptual understanding of the relations between variables and events will contribute towards correct values placed in the Venn diagram. The selection of an appropriate Venn diagram is crucial in this question, as the values placed in the various sections of the Venn diagram ultimately impact the subsequent questions. Conceptual understanding is necessary in order to perform the multiple calculations that are required so that the Venn diagram is filled in correctly.

In question 10.2 the value for variable x has to be found, this process can be classified as **DOK 2**. Information has to be extracted from the Venn diagram in order to reach a solution. Conceptual understanding is necessary to respond to this question, as the relations between the social media platforms and the sample must be understood, all values from the Venn diagram should be used. An equation has to be set up in order to reach a solution, only then can the value of x be found.

A simpler calculation is required in question 10.3 and can be classified as **DOK 1**. The sum of values (7 + 2 +18 = 27) is then divided by the sample space (100); the quotient should then be represented as an appropriate probability value (percentage, fraction or decimal). Procedural understanding is displayed as selected values should be utilised in order to calculate the probability of a specific event.

Overall, this question requires a strong understanding of the relations that govern the given scenario. Interpretation of information is crucial. Errors made early in the question, will filter through the rest of the questions and result in incorrect answers. A comprehensive conceptual understanding of the information is required to answer subsequent questions. Questions of this nature are introduced in grade 11 (refer to Appendix P) and skills learnt in Grade 11 have to be applied to a new scenario.

Table 5.2: Question 11 from November 2017 examination

<table>
<tr><th colspan="4">QUESTION 11</th></tr>
<tr><td colspan="4">A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR45789

The letters are chosen from A; D; R; S and U. Letters may be repeated in the code.

The digits 0 to 9 are used, but NO digit may be repeated in the code</td></tr>
<tr><th>QUESTION NUMBER AND QUESTION</th><th>SOLUTION</th><th>MARK ALLOCATED</th><th>DOK LEVEL</th></tr>
<tr><td>11.1. How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits?</td><td>Number of codes = (letter) × (digits)
= (5 × 5) × (10 ×9)
= 2250
2250 clients can be identified</td><td>3</td><td>1</td></tr>
<tr><td>11.2. Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system.</td><td><table><tr><th>Digits used</th><th>Letters</th><th>Digits</th><th>Total</th></tr><tr><td>1</td><td>5 × 5</td><td>10</td><td>250</td></tr><tr><td>2</td><td>5 × 5</td><td>10×9</td><td>2250</td></tr><tr><td>3</td><td>5 × 5</td><td>10×9×8</td><td>18000</td></tr><tr><td>4</td><td>5 × 5</td><td>10×9×8×7</td><td>126000</td></tr><tr><td>5</td><td>5 × 5</td><td>10×9×8×7 ×6</td><td>756000</td></tr><tr><td>6</td><td>5 × 5</td><td>10×9×8×7 ×6×5</td><td>3780000</td></tr></table>A code consisting of 2 letter and 5 digits will ensure unique codes for 700000 clients</td><td>3</td><td>2</td></tr>
</table>

Question 11 6 marks

An alphanumeric code (code utilising alphabets and digits) of varying lengths are the focus for this question. A selection of alphabets (which can be repeated) and all digits (0-9, with no repetition) can be utilised to generate codes. Graphical representations are not necessary for this question.

Question 11.1 can be classified as **DOK 1**, as a simple calculation is required. The recalling of information based on repetition and non-repetition of variables is necessary. In this question, there are five alphabets (repetition allowed) and nine digits (no repetition), which mean 5×5 and 10×9 should be multiplied to reach a solution. Procedural understanding of probability concepts in a basic calculation is displayed.

Question 11.2 can be classified as **DOK 2** as an extension of a given pattern is required under certain conditions. A target of 700000 codes is desired. Due to the fixed number (2) of alphabets only the number of digits can vary. The exact number of 700000 may not be possible, but a value over that is acceptable. Conceptual understanding of the constraints of the scenario is necessary and inclusion of alphabets is crucial. Solution approaches for this question is a personal choice as there is no guidance provided and the question is free to individual interpretation.

Overall, this question required one simple calculation and slightly longer calculation. Although the questions are at different DOK levels, the intensity/ difficulty of the responses required does not change that much. DOK 2 question requires a few calculations and reasoning. The fundamental counting principle, which is required for this question, is introduced in Grade 12 (refer to appendix P).

Table 5.3: Summary of probability questions from November 2017 examination.

DOK LEVEL	QUESTION	MARK ALLOCATED	% OF TOTAL (FOR PROBABILITY QUESTIONS)	PROCEDURAL OR CONCEPTUAL
1: RECALL AND REPRODUCE	10.3 & 11.1	2 + 3 = 5	36%	Procedural
2: BASIC APPLICATION OF SKILLS/ CONCEPTS	10.2 & 11.2	2 + 3 = 5	36%	Conceptual
3: STRATEGIC THINKING	10.1	4	28%	Conceptual

2017 **14 marks**

5.2.2. November 2018 Examination

Table 5.4: Question 11 from November 2018 examination

QUESTION 11			
Given the digits: 3 ; 4 ; 5 ; 6 ; 7 ; 8 and 9			
QUESTION 11.1			
Calculate how many unique 5-digit codes can be formed using the digits above, if:			
QUESTION NUMBER AND QUESTION	**SOLUTION**	**MARK ALLOCATED**	**DOK LEVEL**
11.1.1. The digits may be repeated	$7^5 = 16\,807$ Or $7 \times 7 \times 7 \times 7 \times 7 = 16\,807$	2	1
11.1.2. The digits may not be repeated	$7 \times 6 \times 5 \times 4 \times 3 = 2520$ Or $= \frac{7!}{2!} = 2520$	2	1
QUESTION 11.2			
How many unique 3-digit codes can be formed using the digits above, if: • Digits may be repeated • The code is greater than 400 but less than 600 • The code is divisible by 5			
QUESTION NUMBER AND QUESTION	**SOLUTION**	**MARK ALLOCATED**	**DOK LEVEL**
11.2	$2 \times 7 \times 1 = 14$	3	2

Question 11 7 marks

A selection of digits is provided in order to generate specific 5-digit and 3-digit codes. There are specific restrictions given for each sub-question, such as the repetition of digits. Graphical representations are not necessary for these questions. Both procedural and conceptual knowledge is required when answering the questions.

Question 11.1.1 can be classified as **DOK 1** as it involves recalling a method and performing a routine procedure. Properties for the repetition of digits are necessary. Two possible methods which result in the same answer are options. Solutions as $7 \times 7 \times 7 \times 7 \times 7$ or 7^5 are acceptable. Procedural knowledge is required to reach a solution.

Question 11.1.2 can be classified as **DOK 1** as it requires learners to recall and reproduce what they have already learnt. Two possible methods which result in the correct solution are possible. Options as $7 \times 6 \times 5 \times 4 \times 3$ or using the formula $\frac{n!}{(n-r)!}$ will result in correct solution. The property of non-repetition is very important for reaching a solution. Procedural knowledge is emphasised in this question.

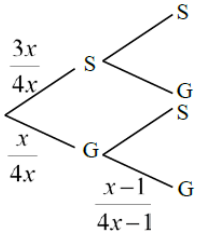
Question 11.2 has more than one restriction that has to be adhered to in order to generate codes. There is no set formula for this type of question, thus the interpretation and

understanding of conditions is crucial. This question can be classified as **DOK 2** and required conceptual understanding of the given situation. This question requires analysis and incorporation of given restrictions.

Overall these questions utilise similar solution strategies where conditions for repetition and non-repetition are crucial. Questions of this nature that require the fundamental counting principle are discussed in Grade 12 (refer to appendix P). Question 11.2 requires a more complex type of thinking compared to question 11.1.

Table 5.5: Question 12 from November 2018 examination

<table>
<tr><th colspan="4">QUESTION 12</th></tr>
<tr><th colspan="4">QUESTION 12.1</th></tr>
<tr><td colspan="4">Given: P(A) = 0,45 ; P(B) = y and P(A or B) = 0,74</td></tr>
<tr><th>QUESTION NUMBER AND QUESTION</th><th>SOLUTION</th><th>MARK ALLOCATED</th><th>DOK LEVEL</th></tr>
<tr><td>12.1. Determine the value(s) of y if A and B are mutually exclusive.</td><td>P(A or B) = P(A) + P(B)
0,74 = 0,45 + y
y = 0,29</td><td>3</td><td>1</td></tr>
<tr><th colspan="4">QUESTION 12.2</th></tr>
<tr><td colspan="4">An organisation decided to distribute gift bags of sweets to a Grade R class at a certain school. There is a mystery gift in exactly $\frac{1}{4}$ of the total number of bags.

Each learner in the class may randomly select two gift bags of sweets, one after the other. The probability that a learner selects two bags of sweets with a mystery gift is $\frac{7}{118}$.</td></tr>
<tr><th>QUESTION NUMBER AND QUESTION</th><th>SOLUTION</th><th>MARK ALLOCATED</th><th>DOK LEVEL</th></tr>
<tr><td>12.2. Calculate the number of gift bags of sweets with a mystery gift inside.</td><td>

Let the number of mystery gift bags = x
The total number of bags = $4x$
$\left(\frac{x}{4x}\right)\times\left(\frac{x-1}{4x-1}\right)=\frac{7}{118}$
$\frac{1}{4}\times\frac{x-1}{4x-1}=\frac{7}{118}$
$\frac{x-1}{4x-1}=\frac{28}{118}$
$118x-118=112x-28$
$x=15$</td><td>6</td><td>3</td></tr>
</table>

Question 12 9 marks

In these questions, different types of calculations have to be performed; the complexity of the responses depends on the approach and method selected. Both procedural and conceptual knowledge is required to reach solutions.

Question 12.1 is based on a specific mutually exclusive event, and the probability of event B occurring has to be computed. Properties of mutually exclusive events are taught in grade 11 (refer to appendix P). This question is classified as **DOK 1** as it requires recalling of knowledge of mutually exclusive events and the appropriate formula. Remembering that P(A and B) is not possible because the events are mutually exclusive one can then proceed to using P(A or B) = P(A) + P(B) in order to reach a solution. Procedural knowledge is required, as a correct formula must be selected and values substituted in order to reach a solution.

Question 12.2 can be classified as **DOK 3** and requires conceptual understanding of the situation at hand. Information for this question is presented in a paragraph and had to be read with understanding and interpreted for one to design a solution strategy. Several decisions about a possible solution strategy must be made and strategic thinking is important for approaching this question. Possible solution methods could be a tree diagram or an equation.

Overall, the questions require using different formulae and performing various calculations to reach a solution. Visual representations, such as a Venn diagram for question 12.1 and tree diagram for question 12.2, may greatly assist in reaching a solution. Tree diagrams are introduced in Grade 9 (refer to appendix O) and the concept is further developed in FET phase (refer to appendix P). Venn diagrams are introduced in Grade 10 and developed till Grade 12 (refer to appendix P). Graphical representations should assist to better understand the given scenario.

Table 5.6: Summary of probability questions from November 2018 examination

DOK LEVEL	QUESTION	MARK ALLOCATED	% OF TOTAL (FOR PROBABILITY QUESTIONS)	PROCEDURAL OR CONCEPTUAL
1: RECALL AND REPRODUCE	11.1.1 & 11.1.2 & 12.1	2 + 2 + 3 = 7	44%	Procedural
2: BASIC APPLICATION OF SKILLS/ CONCEPTS	11.2	3	19%	Conceptual
3: STRATEGIC THINKING	12.2	6	37%	Conceptual

2018 **16 marks**

5.2.3 November 2019 Examination

Table 5.7: Question 10 from November 2019 examination

QUESTION 10			
The school library is open from Monday to Thursday. Anna and Ben both studies in the school library one day this week. If the chance of studying any day in the week is equally likely, calculate the probability that Anna and Ben studied on:			
QUESTION NUMBER AND QUESTION	**SOLUTION**	**MARK ALLOCATED**	**DOK LEVEL**
10.1. The same day	P(same day) = $\frac{4}{16}$ or $\frac{1}{4}$ or 0,25 or 25%	2	1
10.2. Consecutive days	P(2 consecutive days) = $\frac{3 \times 2}{16} = \frac{3}{8}$	3	2

Question 10 5 marks

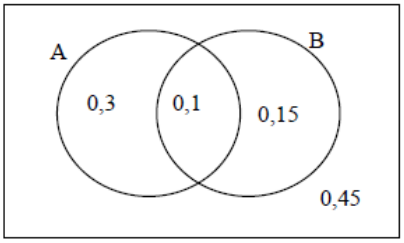
This question presents information in a short paragraph. Terminology and understanding the conditions are important in order to answer the questions that follow. A combination of procedural and conceptual knowledge is required.

Question 10.1 can be classified as **DOK 1** that requires procedural knowledge because a calculation of a simple procedure must be reproduced. This question is based on routine procedures that are introduced in Grade 7 (refer to appendix O).

Question 10.2 can be classified as **DOK 2** as reasoning about conditions has to be considered when reaching a solution. Conceptual understanding of the given scenario and understanding terminology is important in this question. Incorporating basic probability skills, like listing possible outcomes (refer to appendix O, Grade7) will make understanding the given constraint easier.

Overall this question required good reading, comprehending and understanding of the given scenario. Probability skills that are introduced and developed in Grade 7 are incorporated into these questions. Fundamental counting principle, Grade 12 content, is tested in these questions.

Table 5.8: Question 11 from November 2019 examination

<table>
<tr><th colspan="4">QUESTION 11</th></tr>
<tr><th colspan="4">QUESTION 11.1</th></tr>
<tr><td colspan="4">Events A and B are independent. P(A) = 0,4 and P(B) = 0,25</td></tr>
<tr><th>QUESTION NUMBER AND QUESTION</th><th>SOLUTION</th><th>MARK ALLOCATED</th><th>DOK LEVEL</th></tr>
<tr><td>11.1.1. Represent the given information on a Venn diagram. Indicate on the Venn diagram the probabilities associated with each region.</td><td>P(A) ×P(B) independent events
= 0,40 × 0.25 = 0,1

</td><td>3</td><td>3</td></tr>
<tr><td>11.1.2 Determine P(A or NOT B)</td><td>P(A or NOT B) = 1 – P(only B)
= 1 – 0,15
= 0.85</td><td>2</td><td>2</td></tr>
<tr><th colspan="4">QUESTION 11.2</th></tr>
<tr><td colspan="4">Motors Incorporated manufacture cars with 5 different body styles, 4 different interior colours and 6 different exterior colours, as indicated in the table below.

<table>
<tr><th>BODY STYLES</th><th>INTERIOR COLOURS</th><th>EXTERIOR COLOURS</th></tr>
<tr><td>Five body styles</td><td>Blue
Grey
Black
Red</td><td>Silver
Blue
White
Green
Red
Gold</td></tr>
</table>

The interior colour of the car must NOT be the same as the exterior colour.

Motors Incorporated wants to display one of each possible variation of its car in their showroom. The showroom has floor space of 500 m^2 and each car requires a floor space of 5 m^2.</td></tr>
<tr><th>QUESTION NUMBER AND QUESTION</th><th>SOLUTION</th><th>MARK ALLOCATED</th><th>DOK LEVEL</th></tr>
<tr><td>11.2. Is this display possible? Justify your answer with the necessary calculations.</td><td>(5× 1× 5) + (5× 1× 6) + (5× 1× 5) + (5× 1× 6) = 110
There are 110 possible variations of cars
110× 5 m^2 =550 m^2
550 m^2 > 500 m^2
This display is not possible because there is not enough floor space</td><td>6</td><td>3</td></tr>
</table>

Question 11 11 marks

This question requires the interpretation and understanding of given information in order to design a possible solution strategy. Various calculations are utilised when responding to the questions. Questions require conceptual and procedural knowledge in order to answer.

Question 11.1.1 can be classified as **DOK 3** as given information must be interpreted and calculations performed to present the given information in a Venn diagram. Conceptual

understanding of independent events and Venn diagrams are required when responding to this question. Independent events and the conditions are covered in Grade 11 (refer to appendix P). A sequence of calculations is required to populate the Venn diagram; an error here will result in errors in the question that follows.

Question 11.1.2 can be classified as **DOK 2** as there are various solution methods and the information has to be understood and properly interpreted in order to reach a correct solution. It is important that event 'only B' is omitted in this calculation. Conceptual understanding of the scenario and solution method is required for this question.

Question 11.2 can be classified as **DOK 3** and conceptual understanding is required. Given information is summarized in a table and a paragraph with certain constraints are provided. A good understanding of the constraints will ensure a realistic solution strategy is incorporated. Limitations of floor space and repetition of colours affect the possible number of outcomes in this scenario, such concepts are covered in the senior phase (refer to appendix O) and further developed in the FET phase (refer to appendix P).

Overall, this question promotes strategic thinking and critical thinking by ensuring the given information is understood and correctly interpreted. These questions incorporate conceptual understanding of the various relations and conditions for each of the given scenarios. The questions require a good understanding and application of various probabilistic concepts and skills that have been covered in various phases of schooling (refer to appendix O and P).

Table 5.9: Summary of probability questions from November 2019 examination

DOK LEVEL	QUESTION NUMBER	MARK ALLOCATED	% OF TOTAL (FOR PROBABILITY QUESTIONS)	PROCEDURAL OR CONCEPTUAL
1: RECALL AND REPRODUCE	10.1	2	13%	Procedural
2: BASIC APPLICATION OF SKILLS/ CONCEPTS	10.2 & 11.1.2	3 + 2 = 5	31%	Conceptual
3: STRATEGIC THINKING	11.1.1 & 11.2	3 + 6 = 9	56%	Conceptual
2019				**16 marks**

5.3. COMPARING THREE PAPERS IN TERMS OF DOK AND COMPLEXITY

Summary of all the probability questions from the three November examinations papers (2017 to 2019) are presented in Table 5.10 in terms of the DOK level, marks allocation and whether procedural or conceptual knowledge is required to reach a solution.

Table 5.10: Overview of 3 November examinations Probability Questions

	2017	**2018**	**2019**	**PROCEDURAL / CONCEPTUAL**
DOK 1	Q 10.3& 11.1 5 Marks = 36%	Q 11.1.1 & 11.1.2 & 12.1 7 Marks = 44%	Q 10.1 2 Marks = 13%	Procedural
DOK 2	Q 10.2 & 11.2 5 Marks = 36%	Q 11.2 3 Marks = 19%	Q 10.2 & 11.1.2 5 Marks = 31%	Conceptual
DOK 3	Q 10.1 4 Marks = 28%	Q 12.2 6 Marks = 37%	Q 11.1.1 & 11.2 9 Marks = 56%	Conceptual
TOTAL MARKS	14 Marks	16 Marks	16 Marks	46 Marks

The results based on the six sub-questions over a three-year period reveal that, DOK 1, 2 and 3 questions were present in each of the November examination papers. The sub-total for probability questions in the three examinations differs by two marks and all adhere to the required mark allocations according to CAPS requirements. From the distribution there is no evident trend for the DOK level that is most prominent in November examinations in the probability questions. In 2017 there is an equal distribution between **DOK 1** and **2** questions (**36%**) whereas in 2018 **DOK 1** questions (**44%**) were most prominent. In the November 2019 questions **DOK 3** questions (**56%**) dominated. The weighting of questions at the three DOK levels varies for each examination over the three-year period. In all questions that were analysed DOK 1 questions required procedural knowledge and DOK 2 and 3 questions required conceptual knowledge.

5.4. PROBABILITY REVISION TASK

The questions from the three examination papers were reorganised and designed into a revision task, Probability Revision Tasks (PRT). The PRT was completed by learners after they had covered the topic of probability in school. The tasks are beneficial for learners as they are able to apply what they have learnt to a variety of questions for the topic of probability as a whole and use them to revise for the upcoming November examinations.

PRT has a grand total of **46 marks**. The tasks were given to learners to complete in a maximum of 1 hour duration. This revision task allows learners to utilise various skills that

have been taught to them in a single task. A summary of the source for the probability questions in the PRT can be seen in Table 4.1.and the concepts assessed in Table 4.2.

5.5. SUMMARY AND CONCLUSION

The analysis of the probability questions from three examination papers reveals that DOK 1, 2 and 3 questions are present in all however, the weighing of DOK levels differed substantially in each examination. The DOK levels, marks allocated and type of knowledge were presented alongside the question and possible solution strategy. An overview of the probability questions that have been reorganised for the PRT has been discussed. The PRT is used as the data collection instrument in this study.

CHAPTER 6: PRESENTATION AND ANALYSIS OF DATA

6.1. INTRODUCTION

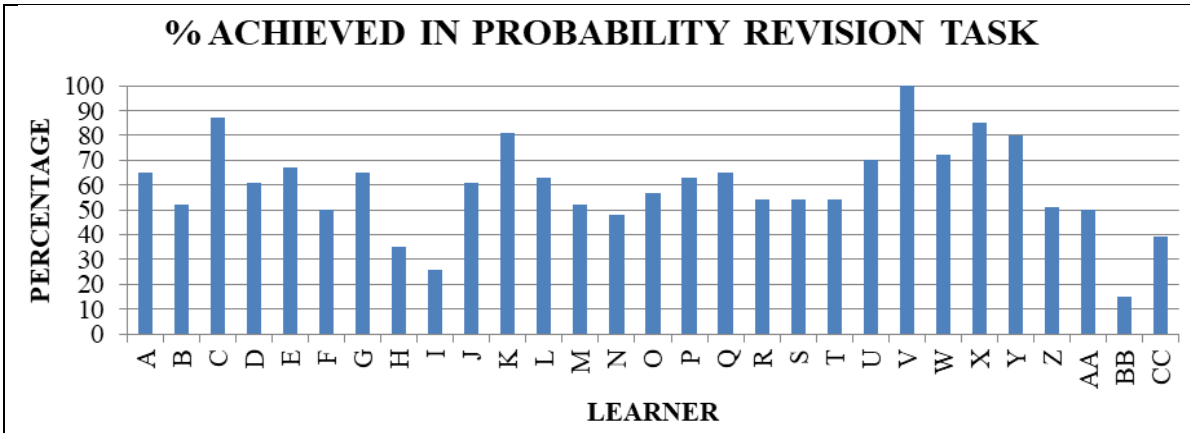
This chapter provides a quantitative and qualitative analysis of the data collected in this study; with the Probability Revision Tasks (PRT), in order to address the second research question: **How do learners respond to Probability Questions that are at different Depth of Knowledge Levels?** The analysis of learners' written responses and teacher interview responses will be utilised in order to answer the research question. The research question is supported by five sub-questions:

- From the selected Probability Questions, which Depth of Knowledge Level questions do learners seem to be performing better in?
- From the selected Probability Questions, which Depth of Knowledge Level questions are learners struggling to respond to/ answer?
- How do learners respond to Probability questions that require Procedural Knowledge?
- How do learners respond to Probability questions that require Conceptual Knowledge?
- How have learners responded to Probability questions at Depth of Knowledge Level 1, 2 and 3?

The quantitative data discussion utilises descriptive methods and is done on learners' written responses in the PRT. Descriptive statistics including measures of central tendency and variation are used to describe the data. The qualitative data includes images of learners' responses in the PRT and a teacher response to a semi-structured interview is presented. In the sections that follow, L_A refers to learner A, T_2 refers to Task 2 in the PRT and in the instant where data values are referred to, the percentage is displayed and in parenthesis the frequency is given. For example, 17% (n=5) means that 5 learners from the 29 learner sample make up the 17% that is being referred to.

6.2. OVERVIEW OF PROBABILITY REVISION TASK (PRT)

Results from the PRT obtained from learners (**n=29**) is discussed. Learner achievement is described using descriptive statistics. The spread of data is the focus in this section, as it provides the basis for the sections that follow.



ACHIEVEMENT DESCRIPTION	LEARNERS ACHIEVING IN THIS LEVEL		LEARNER
Outstanding (80-100%)	5		C; K; V; X; & Y
Meritorious (70-79%)	2		U & W
Substantial (60-69%)	8	17	A; D; E; G; J; L; P; & Q
Adequate (50-59%)	9		B; F; M;O; R; S; T; Z; & AA
Moderate (40-49%)	1		N
Elementary (30-39%)	2		H & CC
Not Achieved (0-29%)	2		I & BB

Figure 6.1: Results from PRT

Probability questions from November 2017 to 2019 examinations were used in the PRT. Figure 6.1 displays the percentage obtained by each learner as well as a comprehensive overview of the achievement description for the results obtained. It is evident that only two learners (L_I & L_{BB}) fall below the 30% threshold, this means that only **7%** (n=2) have not achieved (failed) the PRT. Which results in a **93%** (n=27) pass rate, according to the requirements in the CAPS document, which is extremely high. L_V has achieved **100%** and L_{BB} achieved **15%** in this task. In this study **83%** (n=24) of the learners have scored **50%** or more in the probability task. An outstanding performance was achieved by **17%** (n=5) of the participants, this qualifies as a distinction at school level. Overall learners performed very well in these tasks. This could be attributed to several factors. Learners may have studied for the PRT and utilised this task to their advantage while studying for assessments that were going to contribute to their promotion mark. Probability was already taught in school, so learners would have the necessary skills and knowledge that was required in order to answer questions. According to the interview that was conducted with the teacher, the teacher ensured that all exercises for probability was done and when marking those exercises full explanations were provided to learners. Also the teacher tried to relate probability to the real-world and often used career interests of the learners in order to maintain and grab their attention. These can be a few possible reasons for the outstanding performance amongst others.

The values related to the descriptive statistics for the PRT is summarized in Table 6.1. Descriptive statistics allows the researcher and reader to better understand the data set that is being considered.

Table 6.1: Descriptive statistics and distribution of data for PRT

MEAN	60%	**STANDARD DEVIATION**	17.89%
MEDIAN	61%	**KURTOSIS**	0.876
MODE	65%	**COEFFICIENT OF VARIATION**	29.82%
SKEWNESS	-0.2086		
MAXIMUM	100.0	**FIRST QUARTILE**	52%
MINIMUM	15.22	**THIRD QUARTILE**	67%
RANGE	84.78	**INTERQUARTILE RANGE**	15%
DISTRIBUTION	Negatively skewed		

The mean value (**60%**) for PRT indicates substantial performance by learners. The standard deviation (**17.89%**) is not too high, meaning that the data items are moderately spread out. This is supported by a coefficient of variance of **29.82%**, which implies that data values are moderately dispersed. The mode of this sample is **65%.** When considering the spread of the data we see that the mean and median differ by one unit and therefore the data values are spread slightly more to the left of the median. This is supported by the negative skewness value (**-0.2086**) which implies that the data is fairly symmetrical and approximately skewed. The kurtosis value (**0.876**) supports the data being negatively skewed, having a right tail and being a platykurtic distribution. The probability scores in this data set are spread over a large range of **85%** as the minimum score obtained was **15.22%** and the maximum was **100%**. The middle **50%** of data in this sample is between **52%** and **67%**; resulting in an interquartile range of **15%**. This is regarded as a significant finding of this study. As **52%** (n=15) of the data values are concentrated over a **15%** range. When looking at the achievement descriptions in Figure 6.1, we see that **59%** (n=17) of achievement is categorised as adequate or substantial performance. In order to see the spread of the data a histogram (Figure 6.2) and a box plot (Figure 6.3) is used.

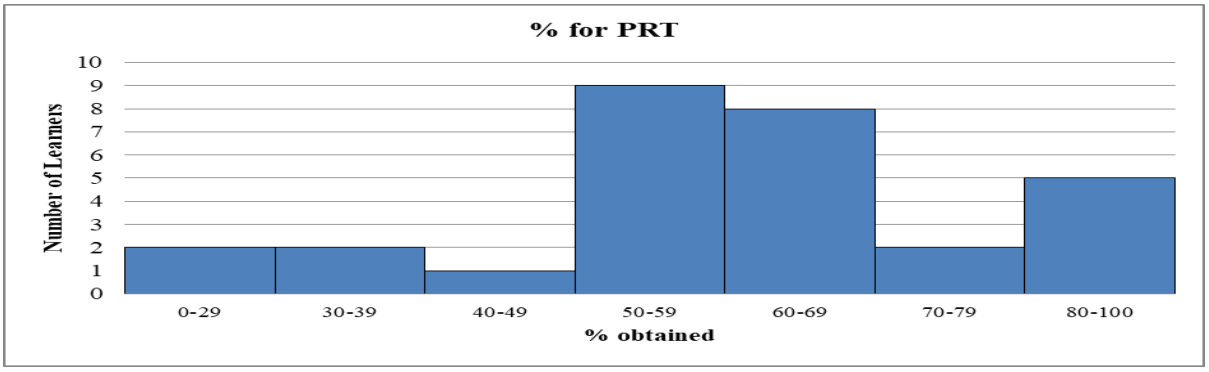


Figure 6.2: Histogram for the scores obtained in the PRT

The histogram provides a graphical representation of the PRT results. It is evident that majority of the learners **59%** (n=17) obtained between **50%** and **69%**. The frequency of results that are less than **50%** ranges between one (1) and two (2) for the given categories. The frequency of results that is greater than **69%** range from two (2) to five (5). This shows that the data has a longer tail on the left hand side which means that the data is skewed to the left.

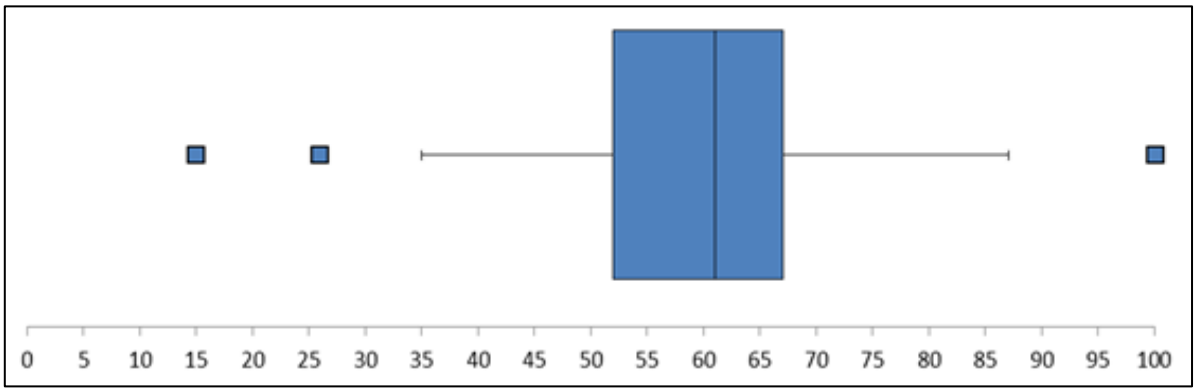


Figure 6.3: Box Plot representing the scores obtained in the PRT

The box plot (Figure 6.3) confirms the skewedness of the data, as there is a bigger spread of data on the left-hand side of the mean compared to the spread on the right hand side of the mean. The Box plot show us three (3) outliers for this set of data which mean that those values are extreme (very low or very high) in relation to the other scores obtained. One can see that the data values are more spread on the left had side of the mean, and hence the data is negatively skewed. Meaning that most scored obtained were greater than **50%**.

In table 6.2 a summary for learners (n=29) responses for the Tasks (n=8) in the PRT are categorised, showing the Tasks that they excelled in and those that they did not.

Table 6.2: Categories of learners' responses in the PRT

	T1	T2	T3	T4	T5	T6	T7	T8
NO ATTEMPT	0	0	0	2	0	**12**	2	2
ZERO	2	3	0	**12**	0	5	7	4
PARTIAL	**20**	9	2	11	11	8	14	13
MAXIMUM	7	17	**27**	4	18	4	6	10
TOTAL	**29**	**29**	**29**	**29**	**29**	**29**	**29**	**29**

No attempt to answer the task was the greatest for T_6 where **41%** (n=12) did not respond to the task, in most cases an empty space was left and nothing was written next to the question number. Almost all learners **93%** (n=27) were able to score the maximum mark for T_3. T_3

was a relatively short task that tested learners' ability to recall a formula for mutually exclusive events. Whereas in T_4 and T_6 only **14%** (n=4) of learners were able to score maximum marks for the tasks. T_4 was a task in which **41%** (n=12) of learners received **zero** for the solution, that means learners attempted to answer the questions but did not receive any marks for their work presented. T_4 required learners to recall how a Venn diagram for independent events looks like and how to calculate the values that belong in each region. Overall T_4 and T_6 seem to have either been poorly answered or not answered at all by learners; as these tasks have the highest count for not being attempted or learners having scored zero for their responses.

Table 6.3: Average Percentage for Tasks in PRT

TASK NUMBER	1	2	3	4	5	6	7	8
AVERAGE % FOR TASK	59	82	98	26	85	31	47	49
PRT AVERAGE%	59.63							

The average for each of the eight Tasks is displayed in Table 6.3; with the average percentage for the PRT being almost **60%**. The results show that learners excelled in T_3 which was based on mutually exclusive events, with a **98%** average. T_4 (with the lowest average of **26%)** required learners to draw a Venn diagram for independent events and perform a calculation based on their Venn diagram. T_5 (**85%** average) required to draw a Venn diagram with appropriate values as well as perform a calculation based on their Venn diagram. These results indicate that learners have the necessary skills for drawing Venn diagrams and reading information off the diagram. Both T_3 and T_5 incorporated Venn diagrams, results obtained for the tasks show a difference. Revision of these concepts can improve learner performance. This indicates that learners have the necessary skills for drawing Venn diagrams and reading information off the diagram. However, revision of these concepts will improve results. T_6 did not provide any guidance for a solution and required a deep level of understanding the given scenario, an average of **31%** was obtained. For this task a tree diagram would have been an appropriate graphical representation and would have assisted learners in reaching a solution. A more extensive investigation may reveal reasons as to why performance in this task is elementary. Learners have responded better to T_8 (**49%** average) compared to that of T_6; it should be noted that both tasks did not have any sub-questions that may have possibly guided learners thinking and solution strategy. A possible reason for T_8 being answered better than the other interpretive questions could be attributed to the displaying of information. A table may have been easy to read as information was explicitly listed for the given scenario. T_1 and

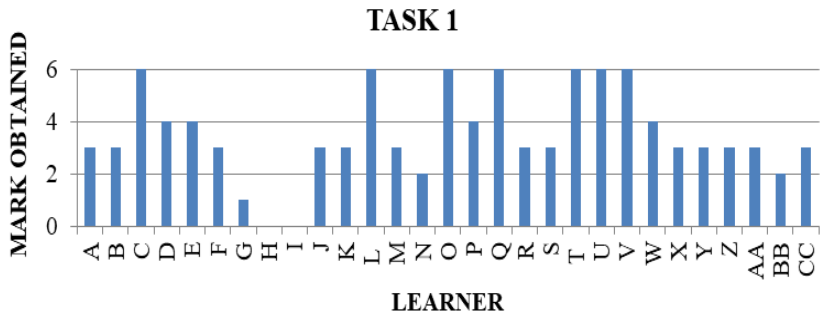
T_2 both involved a specific type of code that was to be generated and required learners to utilise the fundamental counting principle. Learners performed better in T_2 (**82%** average) compared to T_1 (**59%** average) there is a substantial difference in the performance for both tasks that incorporated codes and the fundamental counting principle. This result can be associated with the DOK levels in questions. T_2 had a greater weighing for DOK questions compared to T_1.

The above analysis all pertains to eight tasks that make up the PRT. The section that follows provides an analysis and discussion for each task and the sub-questions that made up the task.

6.3. Categories Of Learners' Responses To Probability Revision Task

In this section a discussion for individual tasks and the corresponding sub-questions are presented. Results obtained in each task from the PRT are presented in this section. Scores obtained in each task are graphically represented; the graphical representation allows the reader to easily gain an idea of the overall achievement in each task. Learners' responses are categorised as "**no attempt"** meaning that an answer was not provided, **"zero"** meaning that zero marks were awarded for the solution presented; **"partial"** means that some marks were awarded for the solution presented and **"maximum"** means that the learner obtained the highest possible mark for that question. The DOK level is given in parenthesis below the sub-question or task number. For the scores obtained refer to Appendix U and V.

6.3.1. Task 1



	T1	
QUESTION & DOK LEVEL	**1.1 (1)**	**1.2 (2)**
NO ATTEMPT	0	8
ZERO	3	9
PARTIAL	2	5
MAXIMUM	24	7
TOTAL	**29**	**29**

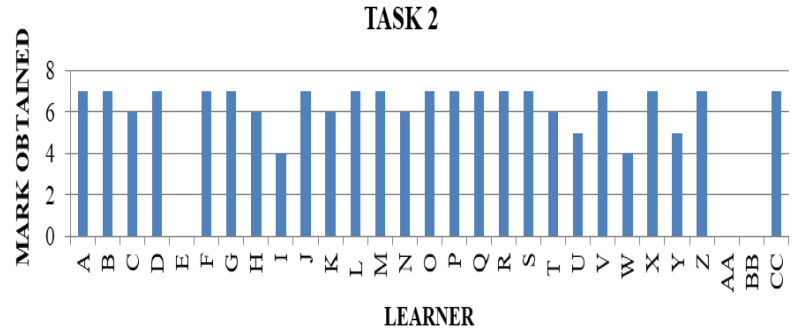
Figure 6.4: Results for Task 1 in PRT

In this task the specifications of a coding system used by a company is described. The only information given is that the code is made up of two digits and a sequence of digits is provided, no other information regarding the company s provided. Two sub-questions, each

for three marks, make up the six marks. Learners results (Figure 6.4) show that only two learners (L_H & L_I) obtained **zero** for the task. All learners (n=29) made an attempt to answer this task. L_G received **1** mark for the task and L_N and L_{BB} scored **2** marks for the task. The remainder of the learners all received three marks or more for the task.

Question 1.1 (DOK 1) was answered by all learners whereas in question 1.2 (DOK 2) eight learners did not answer the question. **83%** (n=24) learners were able to get full marks for question 1.1 compared to only **24%** (n=7) learners obtaining full marks for question 1.2. The results from question 1.1 and 1.2 show that learners were able to answer a question that required procedural knowledge and that was DOK 1 better than a question that required conceptual knowledge and at DOK 2.

6.3.2. Task 2



	T2		
QUESTION & DOK LEVEL	2.1.1 (1)	2.1.2 (1)	2.2 (2)
NO ATTEMPT	0	0	0
ZERO	4	4	4
PARTIAL	0	0	7
MAXIMUM	25	25	18
TOTAL	**29**	**29**	**29**

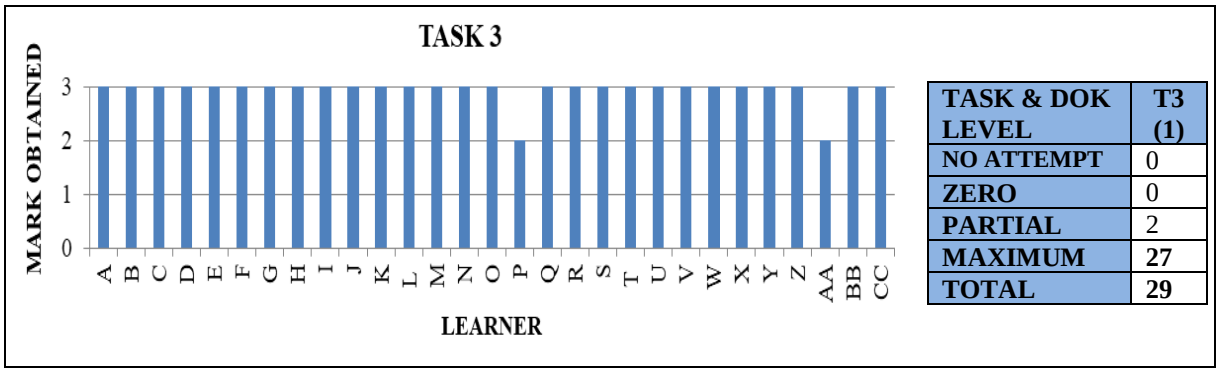
Figure 6.5: Results for Task 2 in PRT

Figure 6.5 presents data obtained from T_2. T_2 was based on a numeric code that can be generated from a certain selection of digits. This task consists of three sub questions for a total of seven marks. Three learners (L_E, L_{AA} & L_{BB}) obtained **zero** and 17 learners were able to score seven marks for this task. All learners attempted to answer this task and **90%** (n=26) of learners were able to get more than **30%** (4 marks or more) for the task.

Question 2.1 consisted of two questions, each for two marks. Both questions required procedural knowledge to be answered and both were classified as **DOK1**. Question 2.2 (**DOK 2**) required conceptual understanding to be answered. Looking at question 2.1 and 2.2, more learners were able to score full marks for question 2.1 than 2.2. Results obtained show that learners seem to have answered procedural knowledge questions that are at DOK 1 better

than the conceptual question that was classified as DOK 2. A very small group of learners (n=3) were unable to score any marks for the question.

6.3.3. Task 3

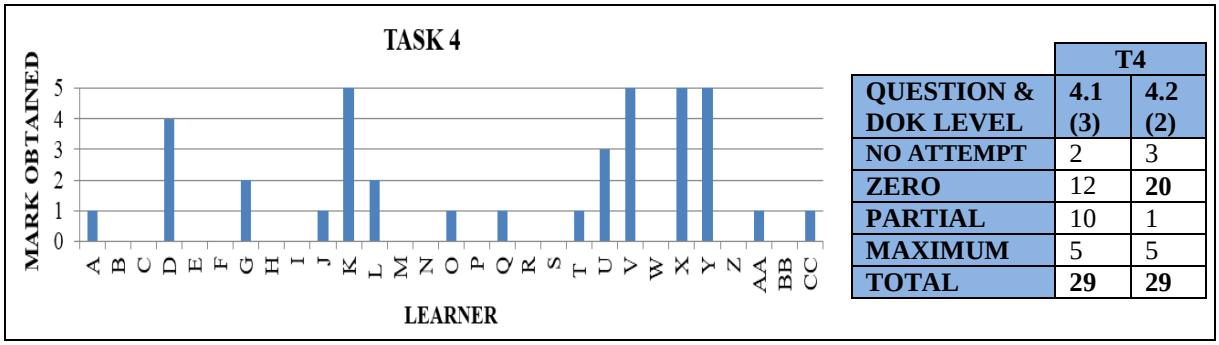


TASK & DOK LEVEL	T3 (1)
NO ATTEMPT	0
ZERO	0
PARTIAL	2
MAXIMUM	27
TOTAL	29

Figure 6.6: Results for Task 3 in PRT

This was a relatively short task for three marks. Learners had to recall the conditions for mutually exclusive events. From figure 6.6 we see that, **93%** (n=27) were able to get full **three** marks for this task. L_P & L_{AA} scored **two out of three** for the task. This task was classified as **DOK 1** that required procedural understanding. Most learners were able to correctly identify and utilise the formula for mutually exclusive events.

6.3.4. Task 4



	T4	
QUESTION & DOK LEVEL	4.1 (3)	4.2 (2)
NO ATTEMPT	2	3
ZERO	12	**20**
PARTIAL	10	1
MAXIMUM	5	5
TOTAL	**29**	**29**

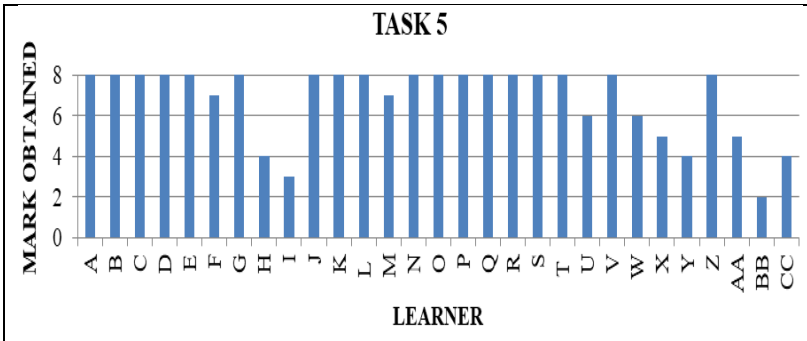
Figure 6.7: Results for Task 4 in PRT

A Venn diagram had to be drawn from given information and questions that followed were based on the diagram. There are two sub-questions worth five marks. Figure 6.7 shows that this task was not answered well by learners. Only four learners (L_K; L_V; L_X & L_Y) were able to score five marks for this task. Seven learners (L_A; L_J; L_O; L_Q; L_T; L_{AA} & L_{CC}) managed to score 20% (1 mark) for this task. Almost half of the sample (n= 14) did not score any marks.

Question 4.1 (**DOK 3)** required learners to draw a Venn diagram, and question 4.2 (**DOK 2**) was based on the Venn diagram. Both questions require conceptual understanding. Five

learners received full marks for question 4.1 and question 4.2. Question 4.1 was not answered by two learners and 12 learners answered the question but obtained **zero** marks for their response. Question 4.2 was not answered by three learners and twenty learners answered the question but did not score any marks. Question 4.2 seems to have been poorly answered by learners when compared to question 4.1, as nine more learners obtained **zero** for the question. For this Task, the DOK 3 question was answered slightly better than the question at DOK 2. This finding is of importance in this study as DOK 3 questions require slightly more complex thinking processes than DOK 2 questions, even though both questions have been categorised as requiring conceptual knowledge. Overall there was a poor performance for questions that required conceptual knowledge.

6.3.5. Task 5



	T5		
QUESTION & DOK LEVEL	**5.1 (3)**	**5.2 (2)**	**5.3 (1)**
NO ATTEMPT	0	1	2
ZERO	0	3	6
PARTIAL	7	2	0
MAXIMUM	22	23	21
TOTAL	**29**	**29**	**29**

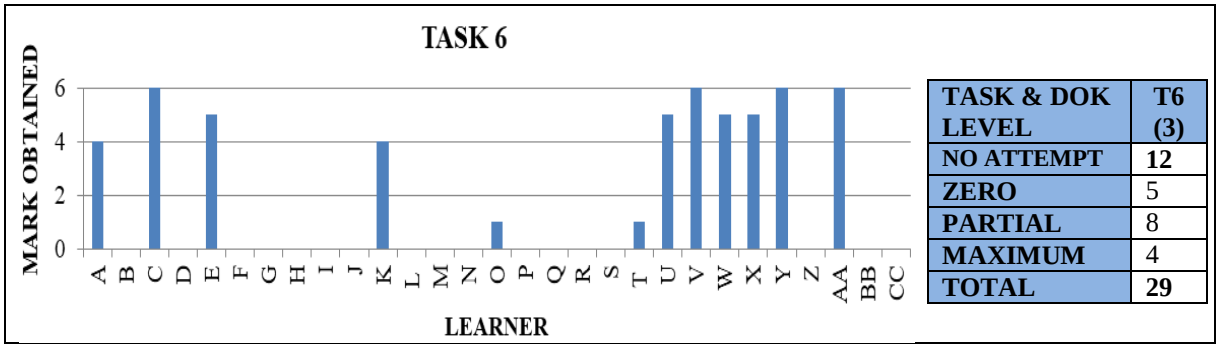
Figure 6.8: Results for Task 5 in PRT

A Venn diagram from the given information has to be drawn; calculations that follow are based on the diagram drawn by the learner. Three sub-questions for eight marks made up this task. Figure 6.8 show that all learners (n=29) answered this task and received a minimum of two marks.

Question 5.1 (**DOK 3**) required learners to draw the Venn diagram from the given information and required conceptual understanding. **76%** (n=22) of learners scored full marks. Question 5.2 (**DOK 2**) requiring conceptual understanding was based on the response presented in question 5.1. Three learners obtained **zero** marks after having attempted to answer the question and **79%** (n=23) of learners were able to score full **2** marks for this sub-question. Question 5.3 (**DOK 1**) was not answered by two learners; six learners got **zero** after answering the sub-question and twenty-one learners scored full **2** marks for this sub-question. The Venn diagram drawn in question 5.1 is required to answer the subsequent questions. Errors in the Venn diagram would filter into question 5.2 and question 5.3. The reliance of

previous answers could be a possible reason why an increasing number of learners either did not answer or got zero for question 5.2 and 5.3.

6.3.6. Task 6

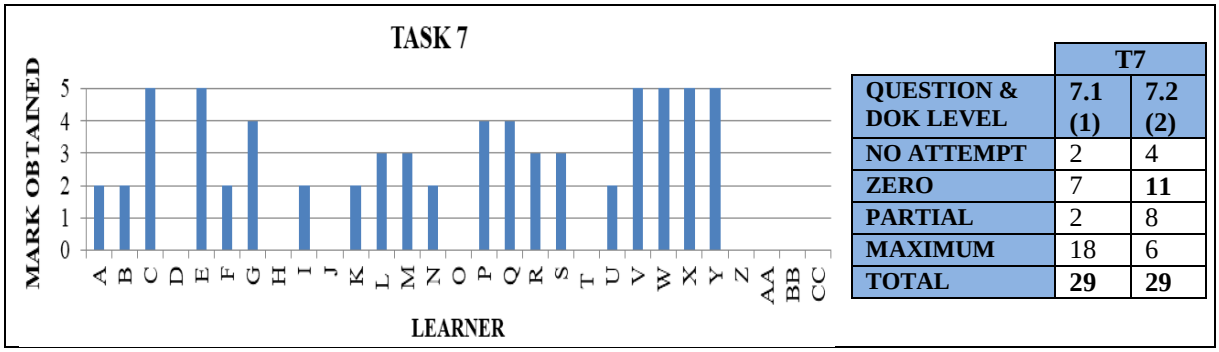


TASK & DOK LEVEL	T6 (3)
NO ATTEMPT	12
ZERO	5
PARTIAL	8
MAXIMUM	4
TOTAL	29

Figure 6.9: Results for Task 6 in PRT

In T_6 there are no guiding questions or sub-questions. The task requires conceptual knowledge and has been classified as **DOK 3**. Interpretation of given information is crucial to answer the question. The scenarios with the necessary conditions are described in four short and explicit sentences. Figure 6.9 shows that only few learners managed to score marks. **41%** (n=12) did not answer T_6 at all. **17%** (n=5) obtaining **zero**. L_C; L_V; L_Y and L_{AA} (**14%**) were able to score full **six** marks for T_6. One possible explanation for the poor performance could be that information was presented as statements and not graphically. Marks were only allocated to the equation that learners set up and solved, a tree diagram would have assisted in expressing the equation. Interpretation and representation of given information was vital in the answering T_6.

6.3.7. Task 7



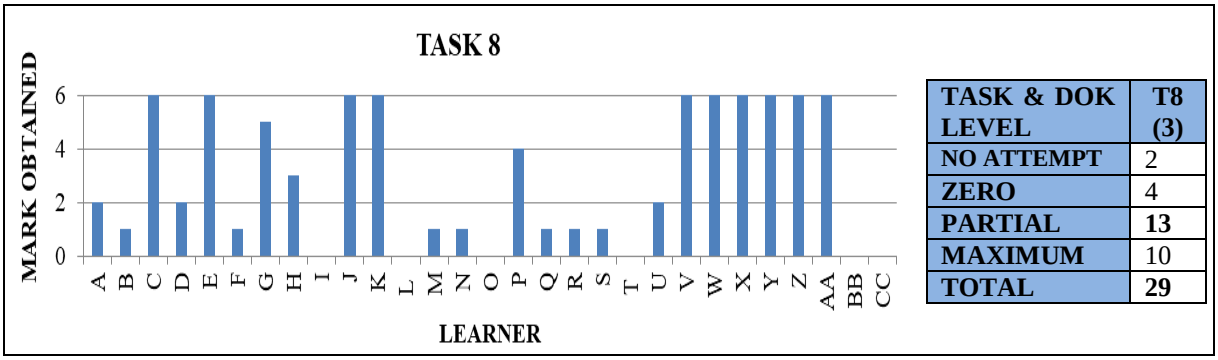
	T7	
QUESTION & DOK LEVEL	7.1 (1)	7.2 (2)
NO ATTEMPT	2	4
ZERO	7	11
PARTIAL	2	8
MAXIMUM	18	6
TOTAL	29	29

Figure 6.10: Results for Task 7 in PRT

Questions are based on two learners studying in a library. Two sub-questions make up the task for five marks. From figure 6.10 one can see that nine learners (**31%**) received zero for the task. Six learners (L_C; L_E; L_V; L_W; L_X & L_Y) were able to get five marks for this task.

Question 7.1 (**DOK 1**) requires procedural knowledge. Looking at question 7.1; two learners did not answer the question, seven learners scored **zero** for the sub-question and eighteen learners were able to get the maximum of **two** marks. Question 7.2 (**DOK 2**) required conceptual knowledge to answer. This question was not answered by four learners, 11 learners obtained **zero** for this sub-question and six learners managed to score **three** marks. From the results obtained it is evident that learners struggled to answer question 7.2; **38%** (n=11) of learners scored **zero** while four learners did not answer the question.

6.3.8. Task 8



TASK & DOK LEVEL	T8 (3)
NO ATTEMPT	2
ZERO	4
PARTIAL	**13**
MAXIMUM	10
TOTAL	**29**

Figure 6.11: Results for Task 8 in PRT

Information about a car company is provided in paragraph form as well as in a table. No sub-questions were present, and the question had a total of six marks. Figure 6.11 shows that **34%** (n=10) of learners were able to score **six** marks. Two learners did not answer the task and four learners obtained **zero** marks for their response. This task was classified as **DOK 3** that required conceptual knowledge in order to answer it.

6.3.9. Summary of the PRT results for DOK levels

The DOK levels present in the PRT had different mark allocations and weightings. Table 6.4 shows the average for DOK 1, 2 and 3.

Table 6.4: Average for DOK levels in PRT

	PRT	AVERAGE
DOK 1	Q 1.1. (86.2%); Q 2.1.1. (86.2%); Q2.1.2. (86.2%); T 3 (98%); Q 5.3. (72%); Q 7.1. (65.5%)	82.35%
DOK 2	Q 1.2. (31%); Q 2.2. (77%); Q 4.2. (19%); Q 5.2. (82.8%); Q 7.2. (34.5%)	48.86%
DOK 3	Q 4.1. (31%); Q 5.1. (92%); T 6 (31%); T 8 (49%)	50.75%

The average for each DOK level differs, with the smallest difference between DOK 2 and 3 questions. A substantial difference between DOK 1 with DOK 2 and 3 questions have been noted. Such results are expected as DOK 2 and 3 questions are more challenging than DOK 1 questions.

6.4. PRESENTATION OF LEARNERS' RESPONSES TO PRT

In this section a detailed discussion of learners' (n=29) responses to each Task is provided. Each category ('no attempt'; 'zero'; 'partial' and 'maximum') that was discussed earlier, are further analysed. Table 6.5 displays how each category is reported on in terms of the calculations and procedures that were required when answering the task. A selection of responses that best describe the categories of '**did not answer'**; '**incorrect calculations and procedures**'; '**partly correct calculations and procedures**'; '**nearly correct calculations and procedures**' and '**correct calculations and procedures**' are shown. The mark allocation for each response is discussed as well. In cases where there were no responses that matched with the criteria '**none**' is written under that category. For the category of '**did not answer**', only a small sample of responses are shown; as majority of responses that were classified as '**did not answer**' just has empty space on the answer sheets.

Table 6.5: Categories of how learners' responses are presented

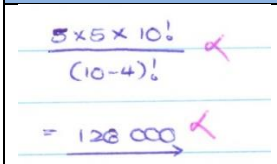
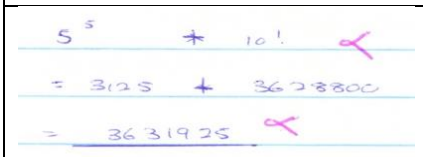
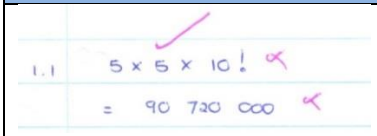
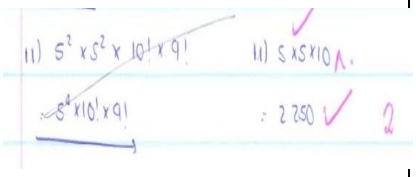
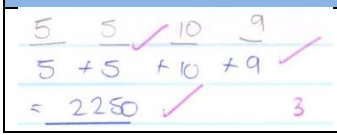
CATEGORY	CLASSIFICATION OF LEARNERS' RESPONSES
NO ATTEMPT	Did not answer
ZERO	Incorrect calculations and procedures
PARTIAL	Partly correct calculations and procedures
	Nearly correct calculations and procedures
MAXIMUM	Correct calculations and procedures

Table 6.5 provides the different classification for learners' responses, it is important to note that the category of 'partial' consists of two sections- 'partly correct calculations and procedures' as well as 'nearly correct calculations and procedures'. In the section that follows the eight tasks will be presented according to the categories in Table 6.5. The question from the PRT is presented thereafter learners' responses are discussed.

6.4.1. Task 1

Table 6.6: Responses for Question 1.1

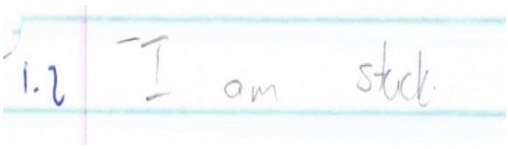
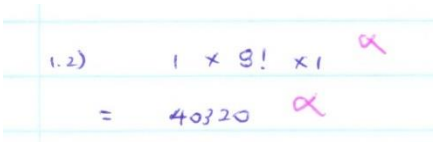
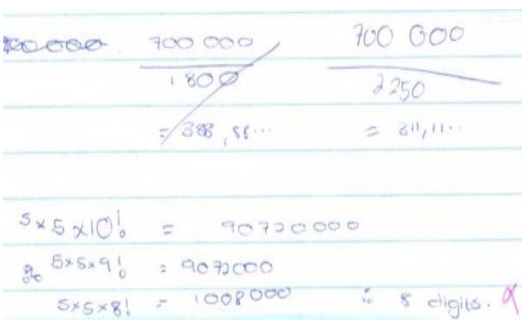
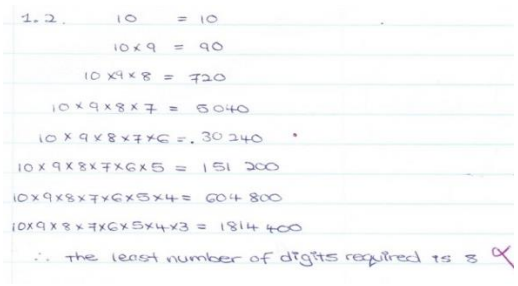
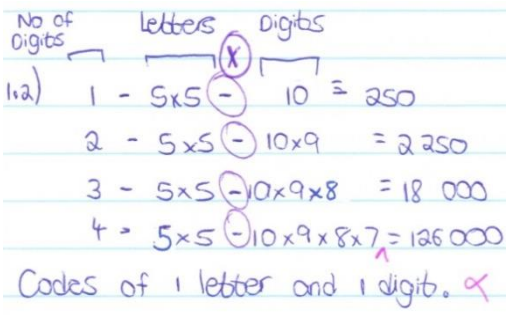
Question 1.1 (DOK 1): A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR45789. The letters are chosen from A; D; R; S and U. Letters may be repeated in the code. The digits 0 to 9 are used, but NO digit may be repeated in the code. **How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits?** **(3)**

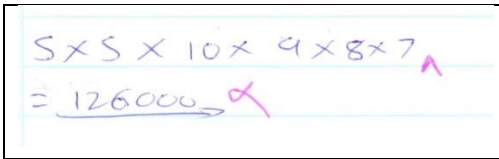
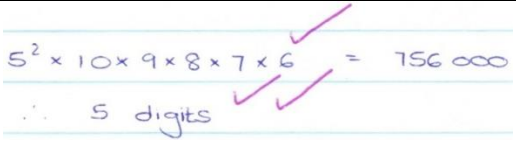
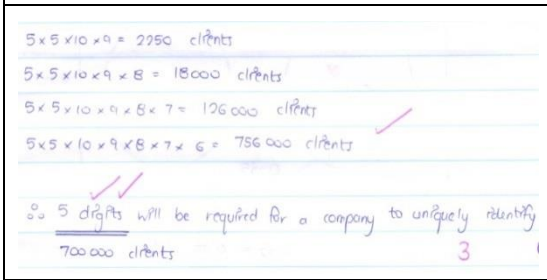
LEARNER RESPONSE	DISCUSSION
INCORRECT CALCULATIONS AND PROCEDURES	
	Factorial notation is used. However, this is not necessary for reaching a solution and hence there are no marks awarded for this response. The numerator of 5 × 5 ×10 is present; however the addition of factorial to the 10 is not required. Response was also presented in a fraction form, but this was unnecessary. A mark was not awarded to the multiplication of five as the entire expression in the fraction in incorrect.
	Factorial notation is used. There is another response that has been cancelled out. In both responses the methods utilised are similar. Values from the original response were changed. This method does not allow marks to be awarded for their solution process.
	Factorial notation is used. No marks are awarded for this solution process and this method yields an incorrect final answer.
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	A portion of the expression is correct and was awarded one mark. Unlike the first example for NCP. The first part of the response was correct however, the remainder of the expression was not correct and hence an incorrect solution is presented.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
	The answer is correct as well as the first part of the solution. An omission of × 9 in the expression is evident however the answer is correct. The response marked was the second response that was provided as the first attempt was cancelled out.
CORRECT CALCULATIONS AND PROCEDURES	
	Four spaces have been allocated for the requirements of the code. Thereafter the spaces have been filled out with digits and the product of the digits is calculated. The maximum of three marks were awarded for this response.

In the first four responses factorial notation was utilised when in fact it was not necessary. In some instances, exponential notation and factorial notation were incorporated.

Table 6.7: Responses for Question 1.2

Question 1.2 (DOK 2): A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR45789. The letters are chosen from A; D; R; S and U. Letters may be repeated in the code. The digits 0 to 9 are used, but NO digit may be repeated in the code. **Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system.** **(3)**

LEARNER RESPONSE	DISCUSSION
DID NOT ANSWER	
	Question was not answered but a response of 'I am stuck' was provided. No marks awarded.
INCORRECT CALCULATIONS AND PROCEDURES	
	No marks have been awarded. Factorial notation is used. Both before and after the factorial there is multiplication by one. Handwriting on the script is challenging to read, responses will have to be double checked and confirmed before awarding marks.
	In the second attempt for this question. A fraction in which the numerator is 700000, the required amount of codes and 2250 as the denominator is set up and simplified. Thereafter a partially correct expression for computing the answer is written however, this expression is incorrect and does not yield a correct answer.
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	No marks awarded for this response as the learner has forgotten to include the 5 ×5 for the two alphabets that are still to be used in the codes. In the response the correct expression of 10×9×8×7×6 is presented. However, due to the omission of the alphabets the incorrect number of digits that should be used is presented as a final answer.
	A very similar method to the one which is provided in the memorandum is presented. However, subtraction of the values for the digit from the alphabets instead of multiplying is shown. Unfortunately, no marks were awarded for this response as the expression, totals and answer is incorrect. Also, multiplication until 6 is not present and this is required in order to reach an answer. A conclusion about the number of alphabets and digits for the code is presented, but in the context of this question it does not make sense.

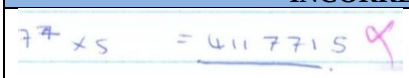
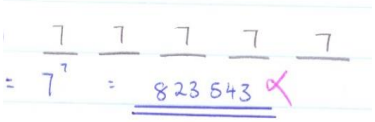
	There is no sequence of calculations shown, with different numbers of digits that can be used. Thus no conclusion regarding the number of alphabets and digits is made, in order to make the required number of unique codes for the company. No marks are awarded for this response.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
None	
CORRECT CALCULATIONS AND PROCEDURES	
	Three marks are awarded for this response. Correct calculation and conclusion is present. Although there is no process of how an answer of five digits is obtained. The memorandum allocates one mark for the calculation and two marks for the conclusion.
	Trial and error method for the number of alphabets and digits and that are used in the code is shown. This response is very similar to the answer in the memorandum. Three marks are awarded for this response. As the calculation and the conclusion made from the calculations are correct.

Some learners continued to use factorial notation in this response. Most learners have presented a set of calculations in order to answer the question but a conclusion regarding the number of digits required was not always made explicit.

6.4.2. Task 2

Table 6.8: Responses for question 2.1.1

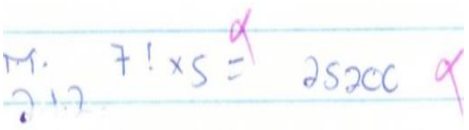
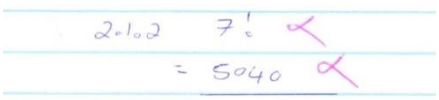
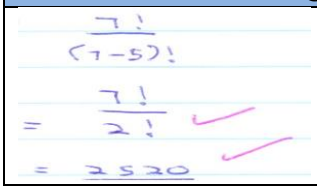
Question 2.1.1 (DOK 1): Given the digits: 3; 4; 5; 6; 7; 8 and 9. Calculate how many unique 5-digit codes can be formed using the digits above, if **The digits may be repeated.** **(2)**

LEARNER RESPONSE	DISCUSSION
INCORRECT CALCULATIONS AND PROCEDURES	
	The digit 7 is used as a base but with an incorrect exponent, this is then multiplied by 5. Incorrect answer and no marks awarded.
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	A graphical representation of the possible number of digits that may be utilised is given. The digit 7 is placed on each space allocation. However, when expressing this as scientific notation, with a base and exponent, a mistake is made. The correct base of 7 with an incorrect exponent of 7 is used. The graphical representation of the answer is correct but the mathematics thereafter is incorrect and no marks can be awarded for this response.
	The digit 7 is correctly used as a base correctly but with an incorrect exponent. No marks were awarded for this incorrect response.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
None	
CORRECT CALCULATIONS AND PROCEDURES	
	Two marks are allocated for the answer. The correct calculation and answer is presented.

In all these responses, a base of 7 was used however the value of the exponent differed. This affected the final answer of this question.

Table 6.9: Responses for question 2.1.2

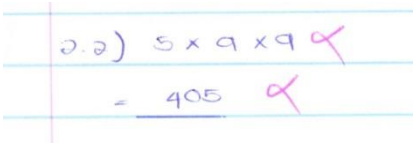
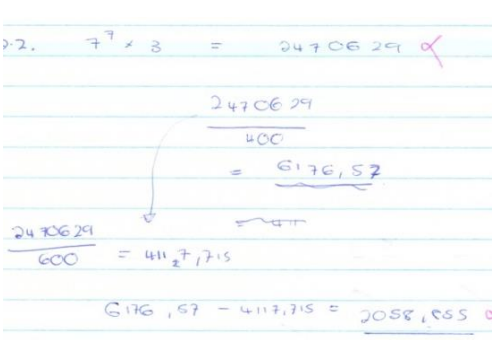
Question 2.1.2 (DOK 1): Given the digits: 3; 4; 5; 6; 7; 8 and 9. Calculate how many unique 5-digit codes can be formed using the digits above, if **The digits may not be repeated.** **(2)**

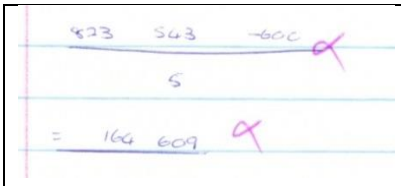
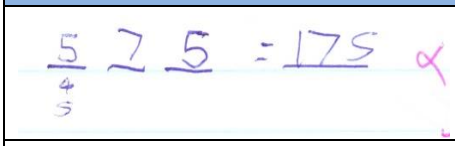
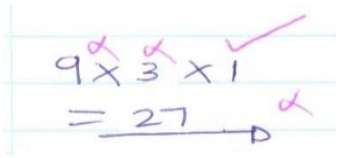
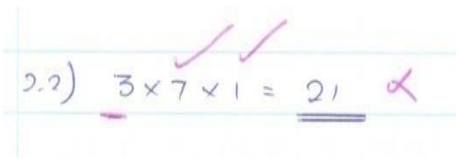
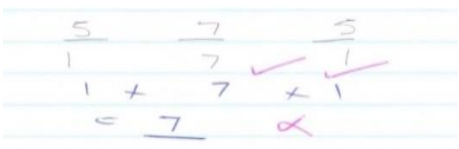
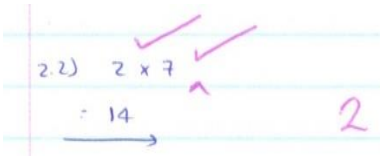
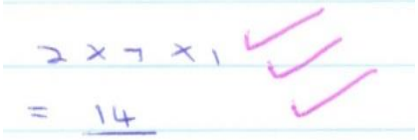
LEARNER RESPONSE	DISCUSSION
INCORRECT CALCULATIONS AND PROCEDURES	
	Factorial notation for 7 is correctly utilised but the condition that letters cannot be repeated is not incorporated. An incorrect answer is presented, no marks awarded.
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	Some correct procedure with factorial notation is present. Repetition of digits is not taken into account resulting in an incorrect and no marks are awarded.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
None	
CORRECT CALCULATIONS AND PROCEDURES	
	Two marks awarded for response. A fraction utilising factorial notation is correctly formed. One mark is awarded for the correct fraction. Another mark is awarded to the correct answer of 2520.

Learners have correctly used factorial notation (7!) however the expression or lack thereof resulted in incorrect answers.

Table 6.10: Responses for question 2.2

Question 2.2 (DOK 2): Given the digits: 3; 4; 5; 6; 7; 8 and 9. **How many unique 3-digit codes can be formed using the digits above, if:** • **Digits may be repeated** • **The code is greater than 400 but less than 600** • **The code is divisible by 5** **(3)**

LEARNER RESPONSE	DISCUSSION
INCORRECT CALCULATIONS AND PROCEDURES	
	There are three digits that are utilised however; all digits that are used are incorrect. No marks are awarded for this response.
	No correct procedures or calculations for the required question presented. The difference between two quotients is presented as the final answer. No marks awarded for this response.

	An incorrect value from a previous question is used in this solution. The solution method is incorrect. No marks are awarded for this response.
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	Three place holders are drawn on which values are placed. An answer is given after these values on the placeholder. It is not made explicit what was done with the values in order to reach a solution. No marks awarded for the response.
	Three digits are multiplied, only one of the digits is correct. Some correct procedure shown, by multiplying three digits. One mark awarded for the one correct value that was used
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
	Multiplication of three digits where two of the three digits are correct. Two marks were awarded for this response.
	Correct identification of two out of the required three digits. Multiplication between digits shown and two marks awarded for the response.
	Correct identification of two out of the three digits. One digit is not present in the solution. Two marks awarded for the response.
CORRECT CALCULATIONS AND PROCEDURES	
	Correct product of the digits two, seven and one is presented. Three marks awarded. There is no mark allocated to the actual answer. Instead all the marks are allocated to the correct digits that are used.

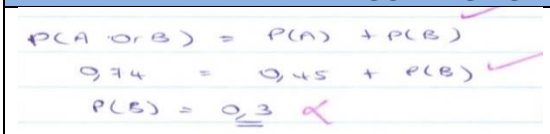
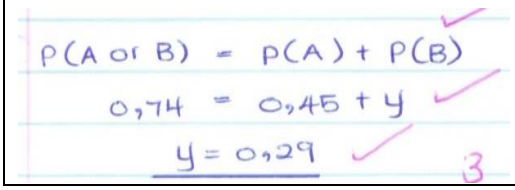
For this question most learners made use of the correct number of spaces (3) however, the conditions that were stipulated were not taken into account, or learners did not know how to express these conditions mathematically.

6.4.3. Task 3

Table 6.11: Responses for question 3.1

Question 3.1 (DOK 1): Given: P(A) = 0,45 ; P(B) = y and P(A or B) = 0,74. **Determine the value(s) of y if A and B are mutually exclusive.** **(3)**

LEARNER RESPONSE	DISCUSSION
INCORRECT CALCULATIONS AND PROCEDURES	
None	
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
None	

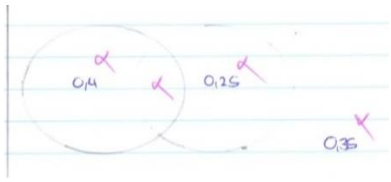
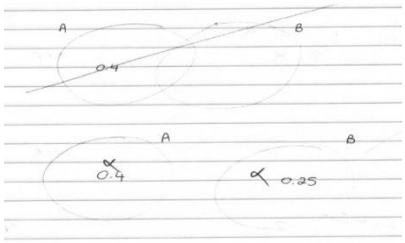
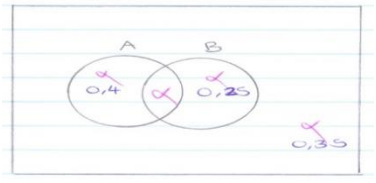
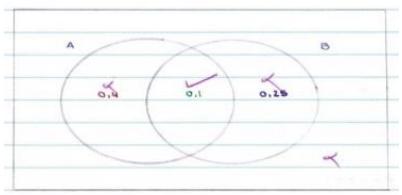
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
	Two out of the possible three marks awarded. A mark for the correct formula written is given as well as for correct substitution. The answer is incorrect and hence mark for final answer is forfeited. Possible rounding off for the answer (0.29 to 0.3) present, according to the memorandum this is incorrect.
CORRECT CALCULATIONS AND PROCEDURES	
	Correct identification of a formula for mutually exclusive events. Correct values are substituted. This response was awarded the maximum of three marks.

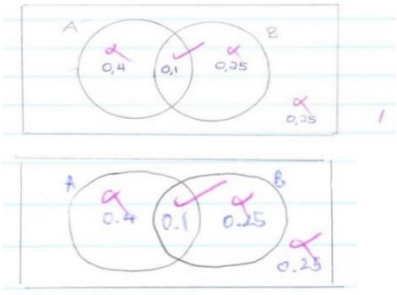
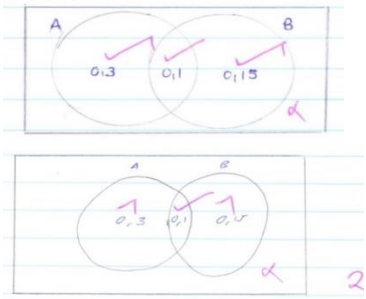
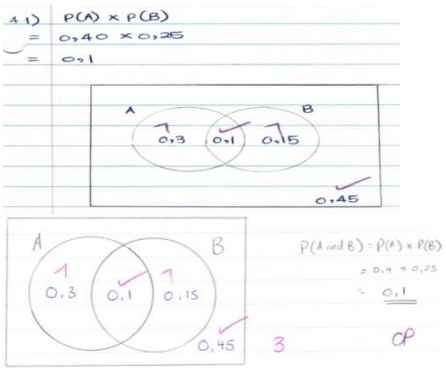
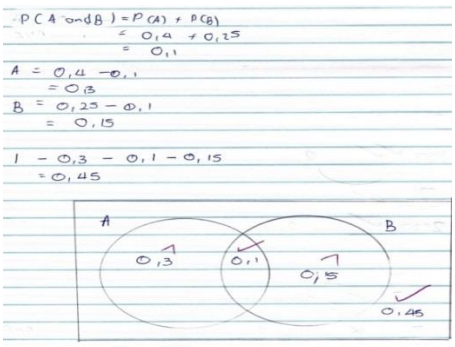
Correct formula is used and correct substitution. Arithmetic errors resulted in the final answer being wrong and the learners not scoring full marks.

6.4.4. Task 4

Table 6.12: Responses for question 4.1

Question 4.1 (DOK 3): Events **A** and **B** are independent. P(A) = 0,4 and P(B) = 0,25. **Represent the given information on a Venn diagram. Indicate on the Venn diagram the probabilities associated with each region**. **(3)**

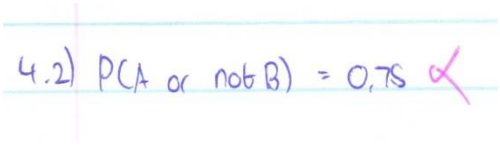
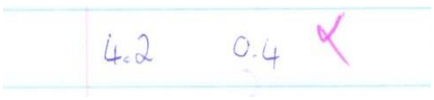
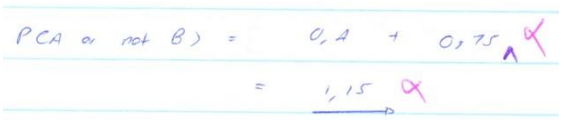
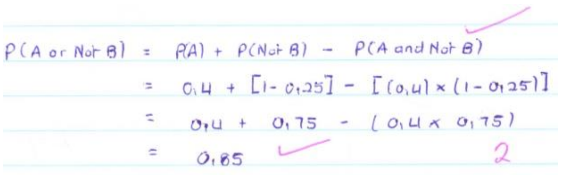
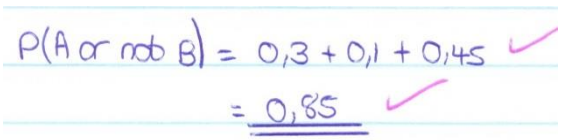
LEARNER RESPONSE	DISCUSSION
INCORRECT CALCULATIONS AND PROCEDURES	
	Correct Venn diagram drawn. Given values are directly placed into the Venn diagram without computing the value that belongs in the intersection. The events A and Bare not labelled in the Venn diagram. No marks were awarded for this response.
	Initially the correct Venn diagram for this question is drawn. This option was cancelled out and another attempt was made. The second attempt represents mutually exclusive events. Also, the Venn diagram is not drawn in a block which represents the entire sample. In the given diagram there is no value for anything that is not part of event A or B. given vales have simply been placed in the diagram and events have not been labelled. No marks have been awarded for this response .
	Correct Venn diagram drawn. No value for the intersection written or calculated. Substitution of given values are made into the Venn diagram. No marks were awarded for this response.
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	Correct Venn diagram drawn correct value for the intersection for A and B. however, the value in the intersection is not subtracted from the given values for P (A) and P (B). No value that is provided for neither event A or B provided. Only one mark has been awarded for this response.

	Correct Venn diagrams drawn in responses. No value for probability of an event not part of event A or event B. The value for the intersection of A and B has been correctly calculated. However, due to the intersection value P (A) and P (B) values have not been correctly calculated. Although there are some correct procedures and calculations that have been displayed, only one mark has been awarded for each of these responses.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
	Venn diagram and calculation of the intersection value correct. Correct value for both event A and event B. however, no value for the sample that does not belong in either A or B. For these responses, a maximum of two marks is awarded out of a possible three marks.
CORRECT CALCULATIONS AND PROCEDURES	
	Three marks awarded for each response. The calculation for the intersection value is shown in both responses. In both responses, no calculations for the value that is neither part of event A or B is shown, but this value is correctly calculated. One mark is awarded for the value in the intersection of A and B; one mark for the value that is neither part of event A or B, finally one mark is allocated for the correct value for A and B.
	Several calculations are shown, each of the calculations are necessary for populating the Venn diagram. First the intersection value is calculated. Thereafter the learner calculates the values for event A and B. Finally, the third calculation is for the value that is neither part of event A or event B. The Venn diagram is then drawn with all values correctly placed. This response is awarded a maximum of three marks.

In most cases the correct Venn diagram is drawn (two intersecting circles), some learners have correctly calculated the intersection value. Several learners do not subtract the intersection value from the given values for P(A) and P(B). Some learners omit the value that does not form part of event A or B.

Table 6.13: Responses for question 4.2

Question 4.2 (DOK 2): Events **A** and **B** are independent. P(A) = 0,4 and P(B) = 0,25. **Determine P(A or NOT B)** **(2)**

LEARNER RESPONSE	DISCUSSION
INCORRECT CALCULATIONS AND PROCEDURES	
	No formula written. Only the required probability expression was written down and an answered. No marks are awarded here as no substitution is shown and the answer is incorrect.
	No written expression or formula. Only a value of 0.4 is presented as an answer. This is an incorrect answer and no marks have been awarded.
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	No formula is written but instead values are substituted next to the expression P (A or NOT B). The Venn diagram presented this script is partially correct, and this may have impacted on this answer. The probability of A and not B is not subtracted in the solution. The answer is greater than one, basics of probability is that probability of an event ranges from zero to one. Having an answer that is greater than one does not make sense.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
None	
CORRECT CALCULATIONS AND PROCEDURES	
	Correct identification of a formula and correct substitution into the equation. This response has been awarded two marks.
	No formula is made explicit. However, correct values for reaching the answer are utilised. It is highly likely that values from the Venn diagram in question 4.1 are used in this response. Two marks awarded for correct substitution and answer.

Learners' response depended on the values in their Venn diagram for question 4.1. Errors in question 4.1 would have impacted this response. Some learners are unable to identify which values contribute to P(A or not B) and hence the incorrect substitution of values results in an incorrect final answer.

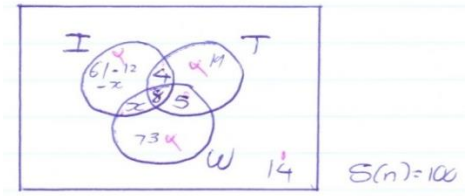
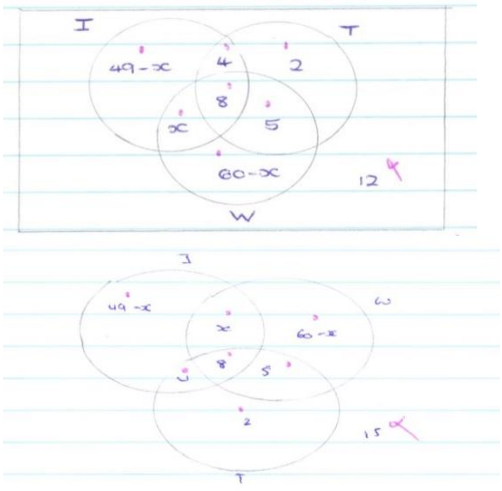
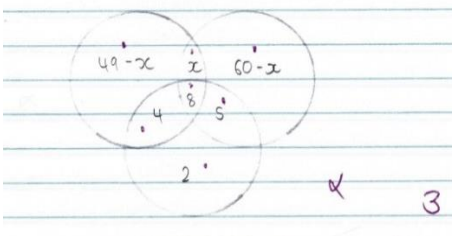
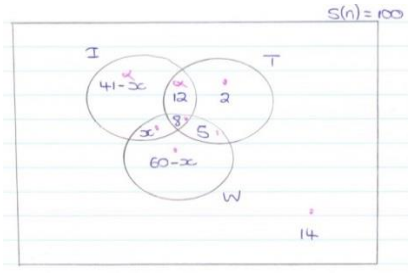
6.4.5. Task 5

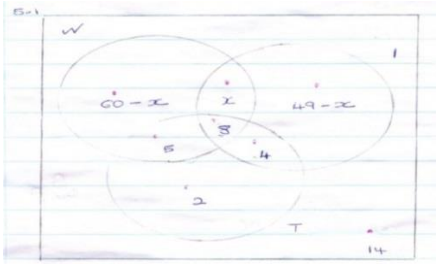
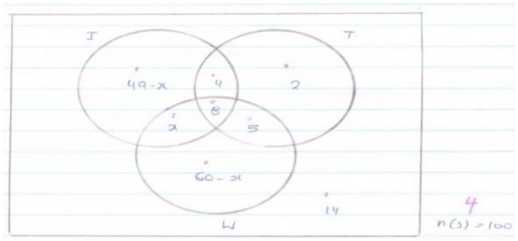
Table 6.14: Responses for question 5.1

Question 5.1 (DOK 3) A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 used all three.
- 12 used Instagram and Twitter
- 5 use Twitter and WhatsApp, but not Instagram
- x use Instagram and WhatsApp, but not Twitter
- 61 use Instagram
- 19 use Twitter
- 73 use WhatsApp
- 14 use none of these applications

Draw a Venn diagram to illustrate the information above. **(4)**

LEARNER RESPONSE	DISCUSSION
INCORRECT CALCULATIONS AND PROCEDURES	
None	
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	Correct Venn diagram is drawn, three interlocking circles. Correct placement of variable x and three other values belonging in the intersections and the number of pupils who do not use any of the social media applications. Incorrect values for Instagram, Twitter and WhatsApp alone are presented. Resulting in three incorrect values for the Venn diagram. Two marks awarded for this response.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
	In these responses three marks out of four marks are awarded. The only value that has been calculated incorrectly is the number of grade 12s who do not use any of the social media applications. The remaining values that belong in the Venn diagram and intersections are all presented correctly with the variable x.
	For this response, seven out of the eight values in the Venn diagram are correctly placed. The total for the sample and the number of grade 12s who do not use any of the social media applications are not shown. Three marks awarded for this response.
	Six out of the eight required values are correctly placed. Including the correct placement of variable x in the Venn diagram. An error is made when calculating the probability for Instagram only and for the intersection of Instagram and Twitter. The remaining values on the Venn diagram all are correct. Three marks awarded for this response.

CORRECT CALCULATIONS AND PROCEDURES	
	The maximum of four marks awarded. All eight values have been correctly calculated and placed in the Venn diagram.
	Correct calculation and placement of all eight values. Sample size is provided on the left of the Venn diagram. Four marks awarded for this response.

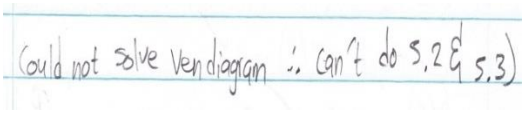
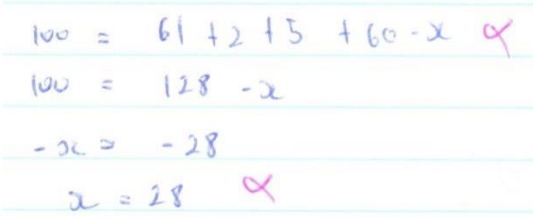
Correct Venn diagrams (3 intersecting circles) are draw by learners. The correct placement of variable x is presented. Thereafter mathematical calculation errors are made which results in certain values being expressed incorrectly.

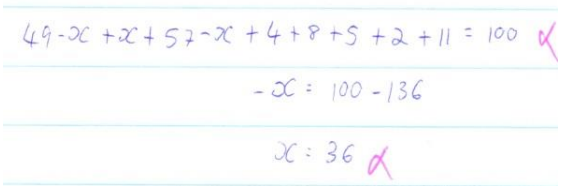
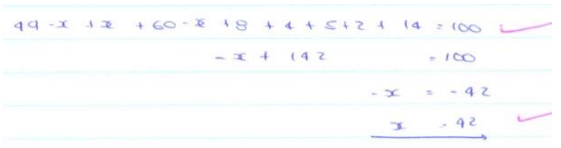
Table 6.15: Responses for question 5.2

Question 5.2 (DOK 2) A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 used all three.
- 12 used Instagram and Twitter
- 5 use Twitter and WhatsApp, but not Instagram
- x use Instagram and WhatsApp, but not Twitter
- 61 use Instagram
- 19 use Twitter
- 73 use WhatsApp
- 14 use none of these applications

Calculate the value of x. **(2)**

LEARNER RESPONSE	DISCUSSION
DID NOT ANSWER	
	A note stating, "Could not solve ven diagram ∴ can't do 5.2 & 5.3" is provided. This learner was able to correctly calculate and place all eight values for the Venn diagram however, did not know how to solve for the unknown and hence was not able to answer the questions that followed.
INCORRECT CALCULATIONS AND PROCEDURES	
None	
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	An equation is set up in order to reach a solution. Not all values from the Venn diagram are used and hence an incorrect answer is presented. Unfortunately no marks were awarded for this response. The left hand side of the equation is correct as well as three of the values on the right hand side, but this does not qualify for any marks.

NEARLY CORRECT CALCULATIONS AND PROCEDURES	
	An equation is set up in order to reach solution. There are no marks awarded for this response as one value in the equation is incorrect. This one incorrect value then impacts the final answer, which is also incorrect. Although all the values from the Venn diagram are used, one value is incorrect the final answer is then affected.
CORRECT CALCULATIONS AND PROCEDURES	
	Correct values utilised when setting up an equation. Two marks awarded for this response. One mark for having set up the correct equation and another mark for the final answer of 42.

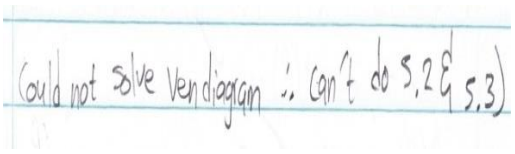
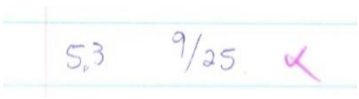
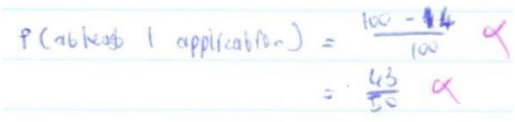
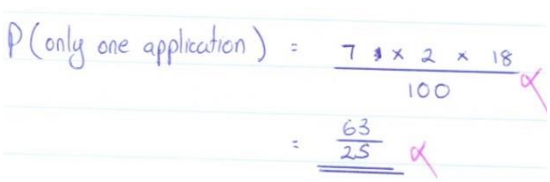
This question relied on the Venn diagram that was drawn in question 5.1. Some learners did not include all values in their expression and others used incorrect values in their expressions. This resulted in incorrect answers for variable *x*.

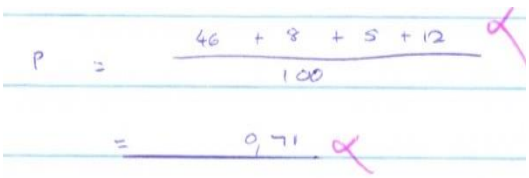
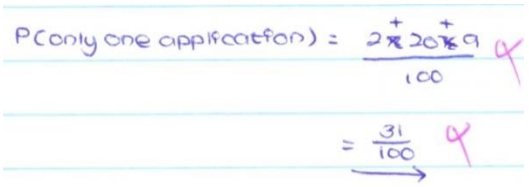
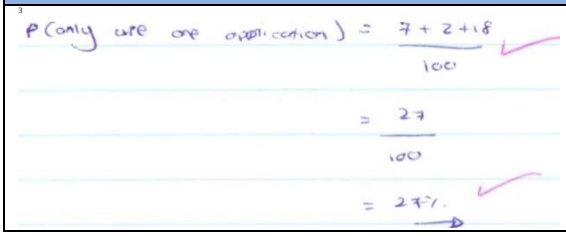
Table 6.16: Responses for question 5.3

Question 5.3 (DOK 1) A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 used all three.
- 12 used Instagram and Twitter
- 5 use Twitter and WhatsApp, but not Instagram
- *x* use Instagram and WhatsApp, but not Twitter
- 61 use Instagram
- 19 use Twitter
- 73 use WhatsApp
- 14 use none of these applications

Calculate the probability that a learner, chosen randomly, uses only ONE of these applications (2)

LEARNER RESPONSE	DISCUSSION
DID NOT ANSWER	
	A note stating, "Could not solve ven diagram ∴ can't do 5.2 & 5.3" is provided. This learner was able to correctly calculate and place all eight values for the Venn diagram however, did not know how to solve for the unknown and hence was not able to answer the questions that followed
INCORRECT CALCULATIONS AND PROCEDURES	
	No formula that may have been used or substitution is shown. The fraction that is provided is incorrect and no marks have been awarded for this response.
	A fraction is set in order to answer the question. However, incorrect values are used in the numerator of the fraction. No marks have been allocated for this response.
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	A fraction is set in order to answer the question All three values in the numerator are correct and so is denominator. The error occurs when the values in the numerator are multiplied instead of addition. No marks have been awarded for this response.

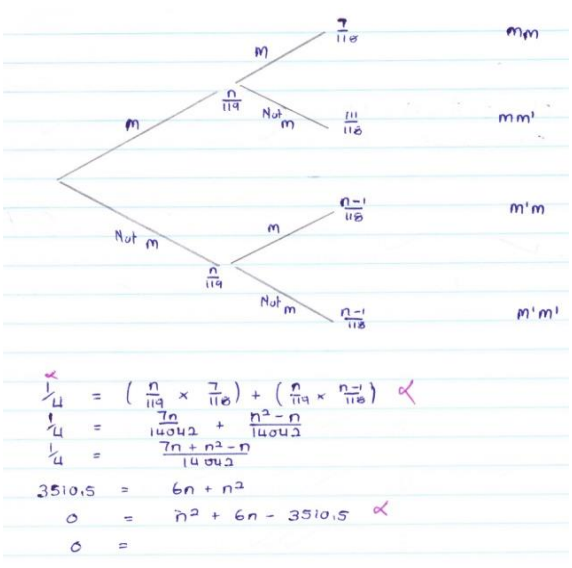
	A fraction with the correct denominator is presented. Values used in the numerator are incorrect. No marks are awarded for this response.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
	A fraction in which three values are used in the numerator and 100 as the denominator is presented. Incorrect values in the numerator are used. Initially multiplication between the values in the numerator are present however, it was then correctly changed it to addition. No marks were awarded for this question.
CORRECT CALCULATIONS AND PROCEDURES	
	Fraction presented consists of three correct values in the numerator which is added. Denominator used is correct. Two marks awarded for this response.

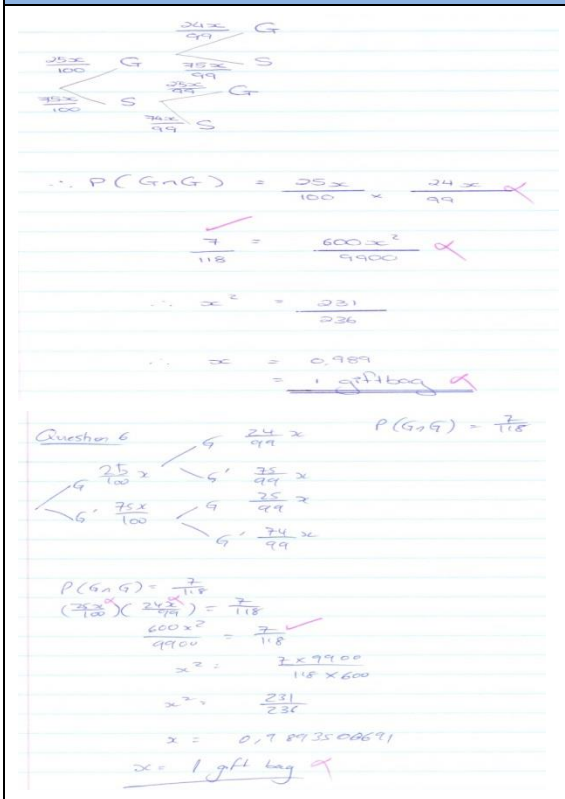
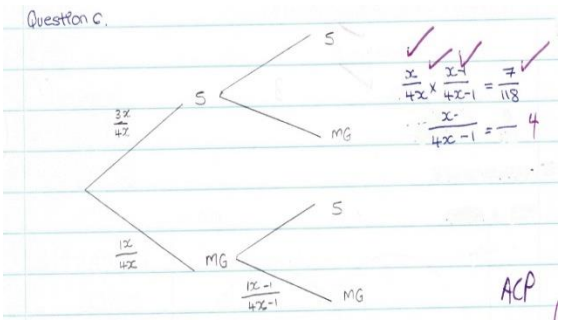
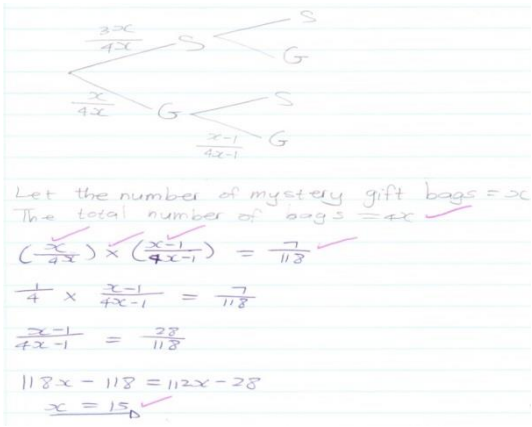
Most learners had three values which represented each of the applications but incorrect mathematical procedures results in the wrong answer.

6.4.6. Task 6

Table 6.17: Responses for question 6.1

Question 6.1 (DOK 3): An organisation decided to distribute gift bags of sweets to a Grade R class at a certain school. There is a mystery gift in exactly $\frac{1}{4}$ of the total number of bags.
Each learner in the class may randomly select two gift bags of sweets, one after the other. The probability that a learner selects two bags of sweets with a mystery gift is $\frac{7}{118}$.
Calculate the number of gift bags of sweets with a mystery gift inside. **(6)**

LEARNER RESPONSE	DISCUSSION
INCORRECT CALCULATIONS AND PROCEDURES	
	Tree diagram drawn in order to represent the given scenario. Below the tree diagram an equation is formulated in order to reach a solution. Figures that are used in the tree diagram are incorrect and hence the expression written is also incorrect. Both left and right hand side of the equation is incorrect. Eventually a quadratic expression is obtained and no solution is provided for this. No marks have been awarded for this response.

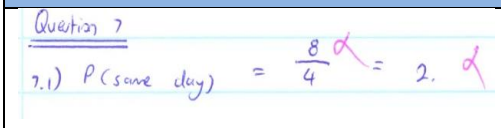
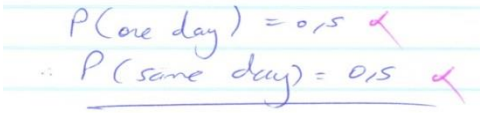
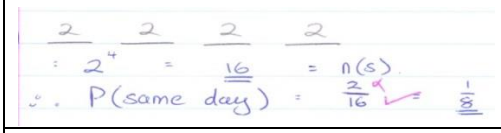
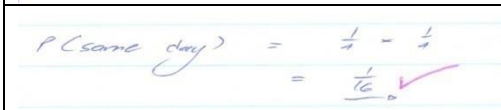
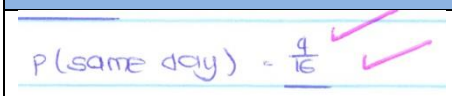
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	Similar responses to the question are provided in these responses. It should be noted that one of these responses is the third attempt at answering the question. Initially tree diagrams are presented and thereafter an equation is written. The tree diagram that has been presented has the correct number of branches and options. When setting up the mathematical equation in order to reach a solution one side of the equation is correct and not the other. Both equations have correctly placed $\frac{7}{118}$ as one side of the equation however; because the values in their tree diagram are incorrect the other side of their equation is also incorrect. In both responses the figures used can be simplified. Even with simplification correct answer would not be obtained. This is because the variable *x* has not been incorporated into the denominator. One mark awarded for these responses.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
	In this response almost all the procedures and calculations are correct. The solution begins with a tree diagram in which the number of branches, options and values are all correct. Thereafter there is an equation which is written, the equation is correct and marks are awarded. Thereafter a portion of the solution is covered correction fluid and remains incomplete. There is no final answer presented. Four marks awarded for this response.
CORRECT CALCULATIONS AND PROCEDURES	
	Correct tree diagram drawn with the correct number of branches and values. Thereafter a statement about the number of mystery gift bags and total bags is presented. This is followed by an equation from which the final answer is obtained. Six marks awarded for this response.

For this task some learners struggled with expressing the conditions mathematically. Some made mistakes on their tree diagram and hence wrote an incorrect equation.

6.4.7. Task 7

Table 6.18: Responses for question 7.1

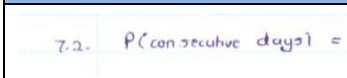
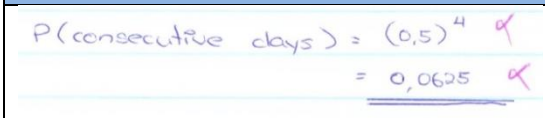
Question 7.1 (DOK 1): The school library is open from Monday to Thursday. Anna and Ben both studies in the school library one day this week. If the chance of studying any day in the week is equally likely, calculate the probability that Anna and Ben studied on: **The same day.** **(2)**

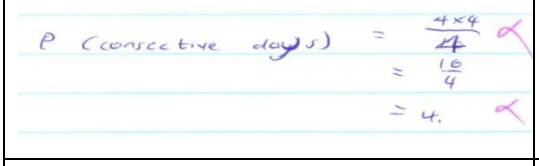
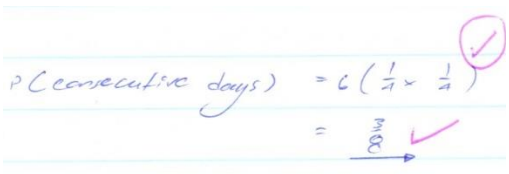
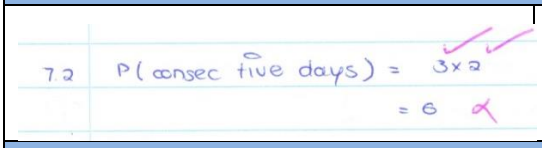
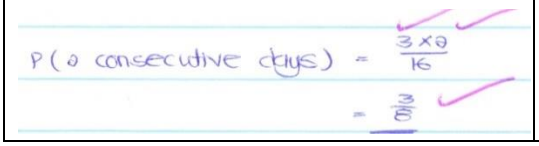
LEARNER RESPONSE	DISCUSSION
INCORRECT CALCULATIONS AND PROCEDURES	
	Answer presented in a fraction form, and then simplified. The learner did not check to see if their answer makes sense, as probability values ranges from zero to one. No marks awarded for response.
	Answer represented as a decimal. The answer is incorrect and no marks have been awarded.
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	One mark awarded for having the correct denominator. The solution begins with four place holder and 2 as the base. The numerator that is used is incorrect and hence an incorrect answer is presented.
	One mark awarded, for the correct denominator value of 16. Multiplication of fractions present, but the numerator values were incorrect and this resulted in an incorrect answer.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
None	
CORRECT CALCULATIONS AND PROCEDURES	
	Correct numerator and denominator identified. Two marks awarded for this response.

The formation of a fraction with correct numerator and denominator values proved challenging for some learners. Identifying the number of possible days for the given duration was important.

Table 6.19: Responses for question 7.2

Question 7.2 (DOK 2): The school library is open from Monday to Thursday. Anna and Ben both studies in the school library one day this week. If the chance of studying any day in the week is equally likely, calculate the probability that Anna and Ben studied on: **Consecutive days.** **(3)**

LEARNER RESPONSE	DISCUSSION
DID NOT ANSWER	
	P (consecutive days) is written but no answer is presented. No marks were awarded for this response.
INCORRECT CALCULATIONS AND PROCEDURES	
	Fraction not presented from which an answer can be reach. The previous incorrect answer is used and is raised to the power of four. No marks awarded for this response.

PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	A fraction is set up however, incorrect values are used. No marks are awarded this response.
	Some correct procedures and calculations, as their final answer is correct. The calculation that is provided is not what is on the memorandum and hence full marks were not awarded to the learner. Partial marks were awarded for the presented solution process. A portion of the previous answer is used in this response. Two marks awarded for the response.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
	Two marks for multiplying 2 with 3 are awarded. However, the answer was not part of a fraction and the denominator value is not present. This results in an incorrect final answer.
CORRECT CALCULATIONS AND PROCEDURES	
	Correct product for the numerator is calculated and the denominator is correct. Three marks awarded for this response.

Terminology used in this question may have affected learners understanding and performance. Calculating the number of consecutive days that are possible is key for answering this question.

6.4.8. Task 8

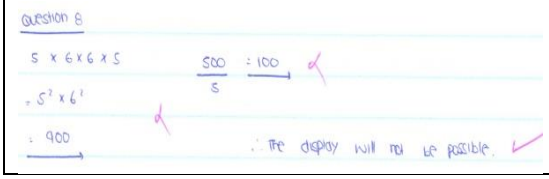
Table 6.20: Responses for question 8.1

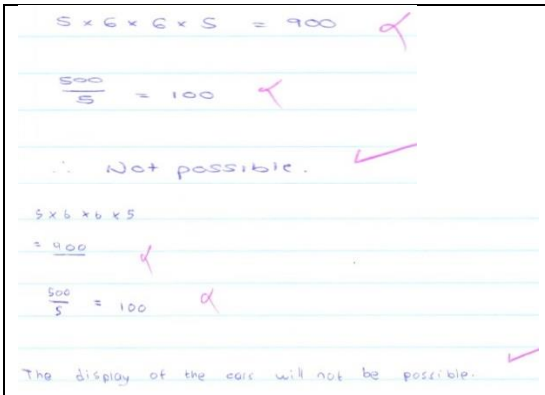
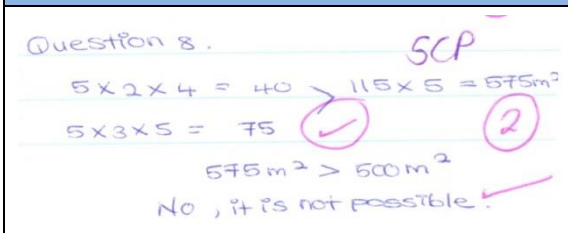
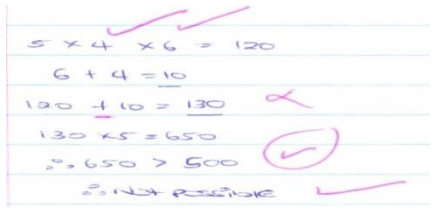
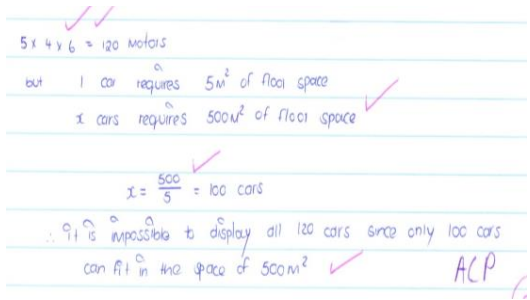
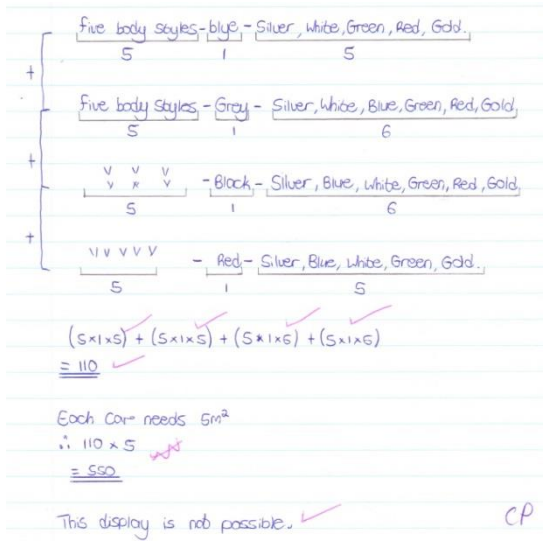
Question 8.1 (DOK 3): Motors Incorporated manufacture cars with 5 different body styles, 4 different interior colours and 6 different exterior colours, as indicated in the table below.

BODY STYLES	INTERIOR COLOURS	EXTERIOR COLOURS
Five body styles	Blue Grey Black Red	Silver Blue White Green Red Gold

The interior colour of the car must NOT be the same as the exterior colour.
Motors Incorporated wants to display one of each possible variation of its car in their showroom. The showroom has floor space of 500 m^2 and each car requires a floor space of 5 m^2.

Is this display possible? Justify your answer with the necessary calculations. (6)

LEARNER RESPONSE	DISCUSSION
INCORRECT CALCULATIONS AND PROCEDURES	
	In these responses no correct calculations or procedures are presented. However correct conclusion for the question is written. From the calculations that are presented it is evident that the number of possibilities for the different types of cars are calculated. Thereafter the number of cars that can fit in the allocated space was

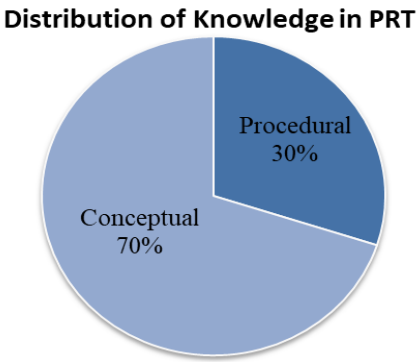
	calculated. Finally they wrote a correct conclusion is given. One mark is awarded for the correct conclusion that is given.
PARTLY CORRECT CALCULATIONS AND PROCEDURES	
	A very short and compressed answer is presented. The solution presented shows that one possibility is greater than another and a correct conclusion is made. Two marks awarded for this response. No detailed calculations shown in the limited space in which solution was written.
	Various calculations presented and a correct conclusion is made. Four marks were awarded for the procedure that was followed. In this response there was no mention of the repetition that is not allowed.
NEARLY CORRECT CALCULATIONS AND PROCEDURES	
	A sequence of the thinking process followed is evident to the marker. First the amount of floor space occupied by one car is calculated, then the amount of floor space *x* cars requires. The amount of cars that can fit in the given space is then calculated. Thereafter a correct conclusion is made. Five marks awarded for this response as the conditions for repetition of colours that are not allowed were not considered.
CORRECT CALCULATIONS AND PROCEDURES	
	Possible choices based on the number of body styles, interior colours and exterior colours are written out. This display is then translated into the form of a mathematical expression. The number of variations for cars that are possible with the given requirements is then calculated. Thereafter the floor space for the number of cars is required is calculated. Finally the conclusion that the display is not possible is given. This response is awarded six marks.

For this task many learners failed in incorporating the given restrictions. Some learners struggled to link the idea of a car occupying a certain floor area with the total number of cars and floor space that is available.

This section presented examples of learners' responses to each sub-question from the PRT. Along with samples of learners' responses a description of each response and the marks that were allocated to the response was provided. In the section that follows a comparison of learners' response that required procedural and conceptual knowledge is presented.

6.5. Comparison of Procedural and Conceptual Knowledge Tasks from the Probability Revision Task Responses

In this section a comparison between tasks that required procedural knowledge and conceptual knowledge is presented. Looking at the spread of tasks and sub-questions that required procedural and conceptual knowledge in order to answer, one can clearly see that majority of the questions required conceptual knowledge in order to be answered. This does not mean that procedural knowledge is not important or that is should be ignored.



PROCEDURAL KNOWLEDGE			CONCEPTUAL KNOWLEDGE		
SUB-QUESTION NUMBER	MARK ALLOCATION	AVERAGE % IN STUDY	SUB-QUESTION NUMBER	MARK ALLOCATION	AVERAGE % IN STUDY
1.1	3	86%	1.2	3	31%
2.1.1 & 2.1.2	4	86%	2.2	3	77%
3	3	98%	4.1 & 4.2	5	25%
5.3	2	72%	5.1	4	92%
7.1	2	66%	5.2	2	83%
			6	6	31%
			7.2	3	34%
			8	6	49%

Figure 6.12: Distribution of Procedural and Conceptual Knowledge in PRT

A pie chart is useful for showing proportions (Cohen et al., 2018); Figure 6.12 represents the composition of the PRT in terms of procedural and conceptual knowledge with the mark allocation and percentage obtained in this study. Figure 6.12 displays that bulk of the PRT **70%** required learners to incorporate conceptual knowledge in their solution processes. Sub-questions that required procedural knowledge seem to all have a higher average percentage than sub-questions that required conceptual knowledge. The lowest percentage obtained for a question that required procedural knowledge was **66%** whereas the lowest percentage for a question that required conceptual knowledge was **25%**. There is a noticeable difference between the minimum averages achieved for the PRT. Questions that require procedural knowledge have an average that is **41%** higher than questions that require conceptual knowledge. The difference is higher than the national pass requirement. This value is of importance because it highlights that even though learners are able to answer questions that require both procedural and conceptual knowledge; the sample in this study performed better for questions that required procedural knowledge but the task consisted of more questions that required conceptual knowledge. The highest average for procedural knowledge was **98%** and that for conceptual knowledge was **92%**. The difference between the highest average values obtained for the PRT is much lower, and only differs by **6%**.

Descriptive statistics for questions that require procedural and conceptual knowledge are presented in order to better understand the data set.

Table 6.21: Descriptive statistics for Procedural and Conceptual knowledge in PRT

KNOWLEDGE / STATISTICS	PROCEDURAL KNOWLEDGE	CONCEPTUAL KNOWLEDGE
MINIMUM	66%	25%
MAXIMUM	98%	92%
RANGE	32%	67%
SUM	408	422
COUNT	5	8
MEAN	81.60%	52.75%
STANDARD DEVIATION	12.68%	27.06%
COEFFICIENT OF VARIATION	15.54%	51.30%
SKEWNESS	-0.0204	0.5173
DISTRIBUTION	Negatively skewed	Positively skewed

Table 6.21 provides a summary of the basic statistics for the questions that required procedural and conceptual knowledge. The range for procedural knowledge questions is **32%** and the range for conceptual knowledge questions is **67%**. The higher range for conceptual knowledge questions highlight that there is more variation in the results. Questions that require procedural knowledge are fairly symmetrical (**-0.0204**) and questions that require

conceptual knowledge are moderately skewed (**0.5173**), this is determined by the skewness value. The coefficient of variation shows that questions that required conceptual knowledge has a great variation relative to the mean (**15.54%** for procedural knowledge & **51.30%** for conceptual knowledge). A lower standard deviation for questions that required procedural knowledge (**12.68%**) suggests that the data is more clustered around the mean. A much higher standard deviation for questions that require conceptual knowledge (**27.06%**) suggests that the data values are more spread around the mean. In this study it implies that there is a large amount of variation from the scores obtained for probability questions that required conceptual knowledge. This is supported by a high coefficient of variation value. Both Figure 6.12 and Table 6.21 provide evidence for better achievement in questions that require procedural knowledge when compared to questions that required conceptual knowledge.

This section discussed how learners responded to Tasks that required procedural or conceptual knowledge. In the section that follows, results from this study will be compared to the National Diagnostic Report results for the appropriate questions.

6.6. Comparison of Probability Revision Task with Results from the Diagnostic Reports

In this section a comparison between results obtained in this study and the results from the National Diagnostic Report (NDR) is presented. The NDR is an annual report that is presented based on the achievements for the NCS November Examination. The NDR reports a detailed analysis is done per question and sub-question for the examination question papers. The NDR provides recommendations and suggestions for the reader. Diagnostic Reports for the 2017 to 2019 examinations will be refereed to.

It has to be noted that the conditions for when the PRT and national examination were written, are not the same. Participants in this study are not being assessed in a high stake examination compared to learners who wrote the national examination and whose results were used to in the formulation of the NDR. An important aspect to note is that the sample size of this study is smaller than the sample of scripts used in the NDR.

Table 6.22: Learner performance from NDR (2017 to 2019)

YEAR	QUESTION AND TOPIC	AVERAGE PER SUB-QUESTION	AVERAGE FOR THE QUESTION	NATIONAL AVERAGE
2017	Question 10: Probability	10.1: 58% 10.2: 36% 10.3: 29%	41%	33%
	Question 11: Counting Principles	11.1: 34% 11.2: 16%	25%	
2018	Question 11: Counting Principles	11.1: 41% 11.2: 26%	34%	32.5%
	Question 12: Probability	12.1: 69% 12.2: 13%	31%	
2019	Question 10: Probability	10.1: 34% 10.2: 12%	21%	23.5%
	Question 11: Probability and Counting Principles	11.1: 26% 11.2: 64%	26%	

Table 6.22 summarises the results for 2017 to 2019 for probability questions and counting principles only. A definite decline in the average for questions from 2017 to 2019 is noted. The largest decline is from 2018 to 2019 with a decreased of **9%**. The 2019 average is the lowest for the three year period. From the comparison of probability questions (question 10 - November 2017, question 12 -November 2018 and question 10 -November 2019) a constant decrease of **10%** per annum is evident. These results highlight a serious concern for probability questions because not only has performance decreased but there is a decline in performance over the three year period. For questions based on the fundamental counting principle, there has been an increase in performance from 2017 to 2018. The improvement must be noted as the difference is **9%**. A comparison between 2019 (question 11) with 2017 and 2018 will not be done because question 11 (2019) incorporated both probability and counting principles in the same question. Unlike previous years when questions on probability and counting principles were separated. Information in Table 6.22 was based on the figures reported in the NDR and were obtained for three different cohorts of learners who wrote the examinations. In the table that follows, results obtained in this study is linked to the NDR results for the three year period.

Table 6.23: Comparison of NDR % and PRT %

NOVEMBER EXAMINATION SOURCE	QUESTION NUMBER & TOPIC	SUB-QUESTION & AVERAGE % ACHIEVED	PRT NUMBER	SUB-QUESTION NUMBER	AVERAGE % ACHIEVED 2020 PRT
2017	Question 11:	11.1: 34%	1	1.1	86%
		11.2: 16%	1	1.2	31%
2018	Question 11:	11.1: 41%	2	2.1	86%
		11.2: 26%	2	2.2	77%
2018	Question 12:	12.1: 69%	3	3	98%
2019	Question 11:	11.1: 26%	4	4.1 & 4.2	25%
2017	Question 10:	10.1: 58%	5	5.1	92%
		10.2: 36%	5	5.2	83%
		10.3: 29%	5	5.3	72%
2018	Question 12:	12.2: 13%	6	6	31%
2019	Question 10:	10.1: 34%	7	7.1	66%
		10.2: 12%	7	7.2	34%
2019	Question 11:	11.2: 64%	8	8	49%

Table 6.23: displays the results from the PRT in this study alongside the results published in the NDR. Results presented in Table 6.23 show that participants in this study performed exceptionally better when compared to the average results that were recorded in the NDR. This table has been arranged in order of the PRT number and not according to the year in which questions were used.

Participants in this study obtained averages for T_4 (**25%**) and T_8 (**49%**) that are lower when compared to the national averages (**26%** and **64%** respectively). Both T_4 and T_8 were originally from the November 2019 examination. One possible reason for a decrease in performance for the tasks can be attributed to having less time to engage with the 2019 questions compared to questions from 2017 or 2018.

T_4 was based on a specific independent event and learners had to draw an appropriate Venn diagram as well as perform a calculation based on their Venn diagram. For this question the national average was **26%** (Department of Basic Education, 2020). Participants in this study were able to score an average of **25%**, this is a negligible difference. Common errors and misconceptions that were recorded in the NDR (refer to Appendix T, question 11.1.1) have also been noted in this study. It is evident from Table 6.11 that learner's confused mutually exclusive events with independent events, as some drew Venn diagrams without any intersection or they were unable to calculate probabilities correctly. Table 6.13 displays responses for question 4.2 from the PRT, in this question it should be noted that learners may have not considered the intersection between sets A and B and also some may have not

calculated P (A or not B), these errors have been recorded in the NDR (refer to Appendix T, question 11.1.2). T_8 incorporated the fundamental counting principle and required learners to really understand and interpret the given information before designing a solution strategy. The national average recorded is **64%** (Department of Basic Education, 2020) and the average obtained in this study is **49%**. A greater decline is displayed when compared to T_4. Average obtained in this study is **15%** lower than the national average. Learners in this study displayed incorrect understanding about the variables and concepts involved in this question. Learners struggled with total floor area and the area occupied by one car. They also struggled in expressing their thinking mathematically and writing out their solutions. Errors that were made can be seen in Table 6.20.

The NDR (Department of Basic Education, 2020) has noted five suggestions for improvement (refer to Appendix T, question 11.2); suggestions can be read in Appendix S. It is emphasised in these suggestions that there should be an understanding of concepts such as mutually exclusive events, independent events and complementary events in probability. It is also noted that the concept of repetition being allowed and repetition not being allowed should be explained by using practical examples in order for learners to relate these concepts. Factorial notation should only be introduced to learners after they have a good understanding of the fundamental counting principle. Implementation of these suggestions would improve the results for learners who attempt answering probability questions.

Question 1.1 from T_1 and question 2.2 from T_2 stand out with the highest average obtained when compared to the national average. Question 1.1 (from November 2017 examination) was a question which required learners to calculate the number of codes that could be generated given certain restrictions for a company. The National average for this question was **34%** (Department of Basic Education, 2018) and the average obtained in this study was **86%**, a difference of **52%**. Appendix R (question 11) includes an extract from the NDR, from the comments made in the report we can see that most learners in this question did not confuse the number of alphabets that could be used in this code however; some of them did confuse 5×5 with $5! \times 5!$, examples of these errors can be seen in Table 6.6. Question 2.2 (from November 2018 examination) required learners to determine the number of three-digit codes that can be generated whilst adhering to certain restrictions. The National average for this question was **26%** (Department of Basic Education, 2019) and the average obtained in

this study was **77%**, a difference of **51%**. Learners' responses to this question can be seen in Table 6.10; evidently learners did not understand the difference between the total number of options and the number of different arrangements. Other conditions for the code such as it being divisible by 5 and only consisting of three digits may have been misinterpreted or completely ignored by learners. Detailed examples of errors, misconceptions and suggestions for improvement can be seen in Appendix S (question 11).

Other Tasks from the PRT have all recorded a higher average than that in the NDR. Question 5.2 from the PRT recorded an average (**83%**) which is **47%** higher than the national average (**36%**). Question 2.1, is not far behind, with an increase of **45%** in this study (**86%** average in study and national average of **41%**). Question 5.3 in the PRT (**72%**) also displayed an average that is **43%** higher than the recorded national average (**29%**). Tasks that recorded the lowest percentage improvement are question 1.2 with a **15%** increase (**31%** average in this study and a **16%** national average) and Task 6 with an **18%** increase (**31%** average in this study and a **13%** national average). The remainder of the sub-questions in the PRT have averages that are between **22%** and **34%** higher than the recorded national averages.

Overall participants have responded extremely well in the PRT when compared to the three cohorts that wrote the examinations. This is supported by the values presented in Table 6.23 which indicate the participants in this study performed better than the classes of 2017, 2018 and 2019.

6.7. Results from Teacher Interview

In this section the results that were obtained from the interview conducted with one Grade 12 mathematics teacher is presented. The results from the interview were transcribed and are presented below. The aim of the interview was to gain a better understanding of the teacher's views, opinions, challenges and strategies used for the teaching and learning of mathematics. The teacher was assured that there are no incorrect responses. As a researcher I feel that the teacher interview was necessary for this research project as the sample of learner scripts were relatively small. By including an interview with the mathematics teacher some insight into the challenges, strategies, methodologies and successes of teaching probability can be brought to light. Given the unpredictable circumstances of the pandemic, the teacher's voice is extremely important and can provide the researcher with valuable information. Due to the

current conditions, questions about Corona Virus were included in the interview in order to better understand the teaching and learning situation of participants in this study. In the excerpts below **R** refers to the researcher and **T** refers to the teacher being interviewed.

In order to begin the interview the teacher was asked about her qualifications and experiences about teaching in general. The discussion shows that the teacher is aware of the demanding nature of teaching and can adapt to teaching different grades.

> R: Please provide a background about yourself with regards to your qualifications and teaching career.
> T: Ok… I initially completed my BSc degree and I majored in computer science and mathematics. Then after some career and personal decisions I decided to do my BEd Honours… I am very excited, nervous and anxious because this is my first year teaching grade 12 after being in the FET phase for 9 years. I am looking forward to the new challenges that await me for teaching Grade 12.

The teacher's qualifications show that she has extensive knowledge of mathematics. This can be one contributing factor to the results obtained in PRT.

> R: What do you feel probability entails?
> T: Basically it is a branch of Mathematics that involves numerical representations and calculations that uhm… determine the likelihood of an event occurring or not.
>
> R: From your personal experience, how do you feel learners respond to probability questions?
> T: I've noticed that the average or weaker learner often struggle with the counting principle questions even though probability is usually regarded as one of the sections that they can score (good marks) in.
>
> R: Do you think learners make use of probability in an everyday context, out of school?
> T: I think they do
>
> R: Can you explain why you think that?
> T: I think they do…but they are not directly aware of it. For example, what is the probability of it snowing today or what is the probability of getting accepted into a University of their choice… are examples of probability in everyday life but I do not think that learners are always able to make this connection.
>
> R: Do you feel that there should be more teacher training on probability?
> T: Yes…I would like attend training sessions. Training is not only beneficial for teachers…learners benefit from it as well. Training equips teachers with various teaching methodologies so they can use the best possible methods suited for their learners.

This teacher displays a good understanding of the topic of probability, what it entails and recognises the importance of the topic in the school curriculum and the relevance in daily life.

> R: In all your teaching experience, how would you describe performance in probability?

T: A little more than average. I feel that the time allocated to this topic is short. What usually happens is that teachers do not spend too much time on this topic… then learners don't tend to spend much time practicing it. Sometimes I feel that learners perceive it to be an easy topic to score marks in…. but then the struggle with exam type questions that are not so basics… the higher level questions.
R: I am sure that you are familiar with the CAPS taxonomy…
T: The four different levels?

R: Yes; so… which questions are learners doing well with? Or, at which level questions are they performing better.
T: Ok… level 1 and 2 questions can be mastered with practice from answering different questions. … learners struggle with level 3 and 4 questions. Because they are expected to reflect…display analytical and critical thinking skills. Not all learners are able to think differently for these kind questions … especially when they writing tests and are stressed.

R: What challenges have you experienced when teaching probability to your learners over the years?
T: Wow, there were so many challenges… but I will summarise the main issues for me. Getting learners to remember the rules… for inclusivity and mutually exclusive events for example. In my experience learners really struggle to understand and make sense of the questions especially with the fundamental counting principles. Sometimes questions are ambiguous and led to a lot of confusion.

R: What strategies or methodologies do you use when teaching? Do you change any of them or incorporate any others when having to teach probability?
T: Revision is important, so I spend a considerable amount of time revising previous years' content. With this topic I make sure that all the exercises in the chapter are covered from end to end. Unlike some of the other longer topics like functions and graphs. I make sure I give homework. When I mark… I provide full explanations. I always try and relate probability to the real-world context. With Grade 12 learners it's easy to link probability to their careers they have chosen or show interest in.

The teacher displays a deep understanding of how learners respond to and perceive probability. She has familiarised herself with the CAPS taxonomy and is aware of how learners respond to questions at different levels. She is aware of the challenges when teaching probability and has adapted her methodologies to align with the needs for the topic and her learners.

R: During the Corona Virus Pandemic, how have you changed or improvised on your approaches to teaching and learning?
T: A lot has changed recently… I've had to adapt and review processes continuously in order to ensure I am able to reach my learners and provide them with the best possible outcome given these scary times. With online classes, I made my lessons very interactive so that I got maximum participations from all the learners. I started making videos of myself … used videos from other maths experts. So that learners can replay the video at their leisure and as many times as they wanted. Eventually when schools reopened, when learners were at school I conducted short informal tests. This really helped me to check if learners were on track with the work that I had done online. I became very strict with the online class. Non-responsiveness was logged and parents were notified if this occurred frequently.

R: How did your school enhance or support teaching and learning during these uncertain times?
T: The school invested in Microsoft Teams for online lessons for learners and teachers. This together with the WhatsApp groups became the standardised platforms. Priority was given to Grade 12 learners for face-to-face classes. This means that I saw the 2 Grade 12 classes almost every day for about a month and after that at least two to three times a week. This helped greatly. Parents of underachievers were contacted and the necessary support was provided by the school.

R: Do you feel that this uncertainty will impact on your learner's achievements? Please explain in detail.
T: The uncertainty was unsettling… and learners had to rely on themselves much more than they would have previously. I felt that they actually worked harder last year than they did in Grade 10 and 11… Please take note, I taught these Grade 12 learners in Grade 10 and 11 as well). Some learners were disadvantaged because the heavily relied on having a teacher in front of them and struggled to work independently. And then there were learners who flourished while being at home.

R: Please can you elaborate further on the learners who were struggling and those that excelled while working from home?
T: Some learners really need a teacher in front of them to monitor and motivate them. Then there were some learners who preferred being at home because they escaped social pressures like bullying in school. I found that the weaker learners ended up collaborating with each other through the tough times and managed to develop the skill of being self-reliant.

The teacher was required to improvise and adapt her lesson due to the corona virus, the impact on teaching and learning and the consequences that followed. After final results were announced the researcher contacted the teacher to get a final statement about her first-year teaching Grade 12 through a pandemic.

R: How do you feel having taught Grade 12 mathematics for your first year and having to do so during the uncertain times given the pandemic?
T: I was prepared for a challenge, to teach Grade 12 but the challenges I faced surpassed my expectations and I've had to adapt in order to survive.

R: What are your views with the pass rate for mathematics?
T: I am very impressed! I must say…. Learners worked under tremendous pressure and they came out victorious. For the trial exams there were about 7 failures. So I was expecting about 85% pass rate. I was shocked when I saw a 100% pass for mathematics… I am still so excited when I saw this…. I still cannot believe it! All 40 learners pass mathematics in 2020, Corona and all. This raises the bar for me as a teacher as I have to live up to a 100% pass for the years to come.

It is evident that this teacher has not only a good understanding of mathematical content but is also able to adapt and revise according to the learners needs. This teacher has provided insight into the positive and negative impact which was experienced by learners and herself when forced to move to an online platform for learning.

6.8. SUMMARY AND CONCLUSION

In this Chapter it is evident that learners are able to answer and score marks in probability questions at DOK 1, 2 and 3. Learners have answered DOK 1 questions much better than DOK 2 and 3 questions in PRT. Results for this study were above the national averages obtained for three cohorts of learners. DOK 3 questions from the PRT were answered slightly better than DOK 2 questions. Performance in questions requiring procedural knowledge exceeded performance in questions requiring conceptual knowledge. This chapter provided a detailed analysis of the Probability Revision Task as well as responses from the interview conducted. In general learners struggled with questions that required conceptual knowledge; this can be beneficial to teachers and curriculum designers when planning for the future. Utilising three sets of probability examinations from past papers provides the reader with greater insight not only for determining how learners may respond, but to make changes where necessary to improve performance.

CHAPTER 7 INTERPRETATIONS AND CONCLUSION

7.1. INTRODUCTION

In this chapter, the results for the study which was centred around Probability Questions and Learners' responses to Probability questions are interpreted. This study consisted of two main research questions: **What Depth of Knowledge is evident, from the Probability Questions in selected November Examinations?** And **How do learners respond to Probability Questions that are at different Depth of Knowledge levels?** These research questions are answered by looking at the different Depth of Knowledge Levels that is evident in Probability Questions as well as Learners' responses to those Probability Questions. This chapter begins with an overview of the research questions. Thereafter an attempt to answer the sub-research questions and the main research questions are presented. Recommendations for further studies are discussed which is followed by the limitations of this study and suggestions for future studies.

7.2. OVERVIEW OF THE RESEARCH

This study focused on Probability Questions from a selection of Grade 12 National November Examinations. From the motivation for this study, which was discussed in Chapter 1, the topic of Probability and performance for this topic was emphasised. In Chapter 2 ideas and previous studies that relate to this study were presented. Webb's Depth of Knowledge Levels, procedural and conceptual knowledge; guided by literature from authors such as Hiebert and Lefevre (1986); Holmes (2012); Rittle-Johnson et al., (2015); Engelbrecht, Bergsten, and Kagesten (2009) and Haapasalo and Kadijevich (2000) amongst other that are utilised in the study were discussed in Chapter 3. In this chapter a brief description of Bloom's Taxonomy and Revised Taxonomy as well the CAPS Taxonomy was provided. These descriptions contribute to the justification of utilising Webb's Depth of Knowledge Model for this study and its intended purposes. Chapter 4 included the research methodology for this study, which incorporated qualitative methods as well as some quantitative methods. In this study a selection of Probability Questions from November Examinations were discussed in detail in Chapter 5. The selected Probability Questions were then reorganised in order to develop the PRT. The PRT was the instrument that was utilised in order to collect data for this study. Chapter 6 provided a detailed discussion and analysis on the learners' responses that were obtained in this study.

The data collection process for this study was done in stages. The first stage of data collection required the researcher to source Probability Questions that would be suitable for the purposes of this project. The second stage required the researcher to analyse the Probability Questions that would be utilised for data collection. The researcher had to ensure that the necessary permission from Ethics committee, GDE research unit, Schools, Parents/ Guardians and learners who would be participating in the study were obtained before going ahead and collecting data. The third stage involved the participating learners who wrote the PRT which was compiled by the researcher. Finally, an interview was conducted with the Mathematics teacher who taught the learners who participated in this study, in order to get a teachers perspective on the topic of probability.

The purpose of this study was to investigate the Depth of Knowledge Levels that are present in standardised Probability Questions from the National November Examination database as well as to determine how learners are responding to Probability questions that are at various DOK levels. This led to the main research questions for the study:

- **Main Research Question 1: What Depth of Knowledge is evident, from the Probability Questions in selected November Examinations?**
- **Main Research Question 2: How do learners respond to Probability Questions that are at different Depth of Knowledge levels?**

The sub-questions will be used in the section below in order to address the research questions for this study.

7.3. ANSWERING RESEARCH QUESTION 1 USING SUB-QUESTIONS

The results from the analysis from November 2017 to 2019 Probability questions, as seen in Chapter 5, are utilised in this discussion.

7.3.1. Which Depth of Knowledge Level is most prominent from the selected Probability Questions?

The Probability questions from November 2017 to 2019 examinations incorporated three DOK levels in each of the questions. The distribution of DOK levels across the three years examination papers showed a drastic difference. November 2017 probability question had an equal spread of **DOK 1** and **2** of **36%** each (refer to Table 5.3). The probability questions

from November 2018 consisted of **44%** for **DOK 1** and **DOK 2** had the lowest weighting of **19%** (refer to Table 5.6). Analysis of the November 2019 probability questions share no similarities with November 2017 or 2018. The November 2019 probability questions had a **56%** allocation of **DOK 3** questioning and **DOK 1** was only present in **13%** of questions (refer to Table 5.9). From the analysis presented in Chapter 5, Section 5.2; we see that there is no definite pattern with regards to the spread of DOK level question for the probability section. This study focused on the probability section questions only, the spread of DOK level questions for the rest of the question paper is not known. However, the questions in all November examinations should align to the CAPS framework as it is the governing framework for the South African curriculum.

7.3.2. From the selected Probability Questions which Depth of Knowledge Level has the highest weighting?

The weighting of DOK level questions differed from the Probability questions that were used in this study. For two out of the three November examinations (2017 and 2018), **DOK 1** had the highest weighting however from this; November 2017 had both **DOK 1** and **2** with an equal allocation. For the November 2019 examination **DOK 3** has the highest weighting. It should be noted that for each of the November examination probability questions the DOK level with the lowest weighting differs each year. For the November 2017 examination, **DOK 3** had the lowest weighting of **28%**, November 2018 examination **DOK 2** had the lowest weighting of **19%** and November 2019 examination **DOK 1** had the lowest weighting of **13%**. From the results obtained in this study there was no consistency with the weighting and spread of DOK levels for probability questions. The inconsistency of the spread of DOK levels is interesting and further probing may provide insight as to why this is the case.

7.3.3. Relating results to literature

The incorporation of real-life experiences into mathematics content has been encouraged and emphasized. From the selection of probability questions from selected November examinations, the incorporation of real-life mathematics is evident. Almost all questions from the November examinations incorporated real-life scenarios that learners should be able to relate to, with the exception of question 12.1 from November 2018 and question 11.1 from November 2019. These two questions were more theoretical questions that did not provide any explicit real-life situation. Question 11 from November 2017 and 2018 deal with coding

systems which are used, each with their own specifications. Learners should be able to relate to codes, as most electronic devices and even bank card make use of some sort of code in order to grant access. Question 10 from November 2017 incorporated social media and how a certain sample of learners used social media applications. Question 10 from November 2019 was about two learners using a library. Both these questions should be extremely relatable to learners everyday lives as social media applications is a part of most learners day-to-day experiences and visiting a library should be common to learners. Question 12.2 from November 2018 was based on a selection of gift bags where a certain number of bags were mystery gifts. This scenario is a little more challenging to understand but is still a real-life scenario. Lastly, question 11.2 from November 2019 was about a car sales room and the amount of floor space that is available for a specific car display.

It is evident from the probability questions that real-life mathematics (Taylor, 2011) is being incorporated in assessments. These types of questions can definitely develop critical thinking skills (Dvorakova, et al., 2017) that are required for answering questions that do not only rely on memory (Pratama & Retnawati, 2018). Such examples can be seen in questions which are identified as DOK 2 and 3, here learners are expected to engage with the given facts by understanding and linking them to the given scenario. A study conducted by Lister (2019) also made use of questions at different DOK levels that were based on real-world problems. One major difference between this study and Lister (2015) is that DOK levels in this study were identified by the researcher and not predetermined.

Questions that learners are expected to answer are set at different levels of difficulty. The level of difficulty can be determined and analysed with several models or taxonomies. Popular taxonomies such as Bloom's Taxonomy, Bloom's Revised Taxonomy and the CAPS Taxonomy were discussed in Chapter 3 Section 3.2. In this study, Webb's DOK model was used as a tool that guided the analysis. Studies by De Kock (2015) used the CAPS taxonomy and that by Lister (2019) used the DOK model. For purposes of this study the DOK model is better suited and justification for DOK model is provided in Chapter 3, Section 3.6.

Overall, the selection of November examination probability questions encompassed a variety of DOK 1, 2 and 3 questions without any fix pattern that was noticed in the spread of DOK levels.

7.4. ANSWERING RESEARCH QUESTION 2 USING SUB-QUESTIONS

The results from the analysis of the data collected for this study, as seen in Chapter 6, are utilised in this discussion.

7.4.1. From the selected Probability Questions, which Depth of Knowledge Level questions do learners seem to be performing better in?

In order to respond to this question the responses to the PRT were analysed and an average calculated for each sub-question. The averages for each sub-question can be seen in Table 6.4. When considering the percentage achievement for this study, **DOK 1** questions have an average of **82%**; **DOK 2** questions have a percentage of **49%** and **DOK 3** questions have an average of **51%**. These results clearly show that participants in this study have performed better with questions that are at **DOK 1**. It has to be noted that the average learner performance for **DOK 2** and **3** questions only have a marginal difference of **2%**. Question 5.1 had an unusually high average of **92%;** this may have contributed to a higher average for DOK 3. Results for DOK 2 and 3 were unexpected as those questions are usually challenging for learners. When considering **DOK 1** probability questions the lowest average obtained was **72%** and the highest was **98%**. Task 3 yielded an average of **98%**, in this task learners were required to perform a calculation by recalling the conditions for mutually exclusive events.

The results from this study imply that learners performed considerably well for probability questions at different DOK levels. One should take note that the PRT consisted of more questions at DOK 1 compared to the number of questions at DOK 2 or 3. Learners have provided the most correct solutions to DOK 1 questions. Responses for questions at DOK 2 or 3 have been attempted relatively well. These samples of learners have performed the best at DOK 1 probability questions.

7.4.2. From the selected Probability Questions, which Depth of Knowledge Level questions are learners struggling to respond to/ answer?

From this study it is evident that learners seem to be struggling with questions that are at **DOK 2** more than they are struggling with questions that are at **DOK 3**. The difference in performance for the two DOK levels is just **2%**. When performance is compared for questions that are at **DOK 1**, there is a difference. **DOK 2** and **3** questions will require a greater cognitive demand from any person responding to those questions when compared to

questions at **DOK 1**. The reasons for certain performance outcomes can be impacted from a variety of reasons. It is not in the scope of this study to identify specific reasons for performance at certain DOK levels as learners' views were not accounted for.

For the questions at **DOK 2** question 4.2 had the lowest average of **19%**. In this question learners had to use information from their Venn diagram in the previous question to calculate P (A and NOT B). For this question, an alarming 20 learners received zero for their responses after having attempted to answer the question and 3 learners did not answer the question at all.

For questions at **DOK 3** both question 4.1 and question 6.1 has an average of **31%**. For sub-question 4.1 learners were required to represent given information in a Venn diagram. Question 6.1 required learners to interpret a given situation with specific criteria and then perform a calculation for marks. This question has no guiding questions and learner performance was based purely on their interpretation and solution methods.

For **DOK 1** questions, the question with the lowest average was sub-question 5.3, in this question learners were required to use information from their Venn diagram to answer the question. A possible reason for poor performance in this question can be attributed to errors that were made on the Venn diagram and previous questions that would have negatively impacted on the average performance for this question.

From these results it is evident that learners may not necessarily struggle with graphical representations of probability concepts but they definitely have barriers with the interpretation and application of information from graphical representations.

7.4.3. How do learners respond to Probability Questions that require Procedural Knowledge?

The descriptive statistics for learner performance in probability questions that require procedural knowledge is presented in Table 6.21. The percentage for each sub-question can be viewed in Figure 6.12 which also shows that procedural knowledge questions accounted for **30%** of the PRT, there were 14 marks out of the 46 that required procedural knowledge in order to present and appropriate solution and answer the question. The mean score for

questions that required procedural knowledge in the PRT is 81.6% (refer to Table 6.21). This is a noteworthy difference when compared to the averages reported in the NDR over the years. Evidence from this study shows that learners' have responded exceptionally well to questions that require knowledge.

7.4.4. How do learners respond to Probability Questions that require Conceptual Knowledge?

The descriptive statistics for learner performance in probability questions that require conceptual knowledge is presented in Table 6.21. The percentage for each sub-question can be viewed in Figure 6.12 which also shows us that conceptual knowledge questions accounted for **70%** of the PRT, there were 32 marks out of the 46 that required conceptual knowledge to provide an answer. The mean score for questions that require conceptual knowledge was **52.75%** (refer to Table 6.21). This achievement is well above the pass requirements for schooling in South Africa. Participants in this study have displayed an adequate performance when answering questions that require conceptual knowledge in order to reach a solution.

7.4.5. How have learners responded to Probability Questions at Depth of Knowledge Level 1, 2 and 3?

In this study it is evident that learners performed exceptionally well for the overall PRT. The learners surpassed the researcher's expectations for probability questions. The performance for **DOK 1** (**82%** average) questions was extraordinary and the results for questions that were identified as **DOK 2** (**49%** average) or **DOK 3** (**51%** average) were attempted with a great sense of understanding. The performance of learners in this study exceeded the results that were recorded in the NDR. Learners have provided the most correct solutions for DOK 1 questions. Their responses to DOK 2 and DOK 3 questions are slightly more flawed that DOK 1 questions however, the results obtained are above the national pass rate and higher than results reported in NDR over the years. Responses from the teacher interview reveal that learners struggle with higher level exam type questions even though the learners perceive probability to be an easy topic. The teacher also mentioned that learners struggled with questions that are CAPS level 3 and 4. This means that they will find DOK level 3 and 4 questions challenging.

7.4.6. Relating results to literature

According to Suurtamm et al. (2016) an important aspect of assessment is the purpose of the assessment. There are several reasons for utilising assessment; the purpose of the PRT was to gain insight as to how learners respond to probability questions that are at different DOK levels. The PRT was compiled using teacher (researcher) selected material in order to align with the requirements of this study (Baird et al., 2014). The PRT assisted the researcher in responding to research questions in this study.

The results obtained in this study align with Awuah (2018) study, where it was found that learners were more successful when answering questions at the knowledge level compared to questions at the evaluation level. Similarly, DOK 1 questions required learners to recall and reproduce knowledge. DOK 3 required strategic thinking and in doing such, the learner will be required to evaluate and analyse given information in order to devise a suitable solution strategy. Pratama and Retnawati (2018) claim that learners tend to steer away from memorisation and recall and use logic in order to solve more challenging problems, for example higher order thinking skills that are present in the top levels of Bloom's taxonomy. If such a claim is true, then it is evident in this study. When looking as learners' responses to DOK 2 and 3 questions one can notice how they have struggled in some cases, changed their answers and used different approaches to respond to questions. This is evident for learners displaying their thinking processes, showing how they planned, at times their initial plan did not allow them to reach a solution or they omitted something, hence they reverted to planning again and attempted to find a solution with a different strategy.

In response to questions that required higher-order thinking and critical thinking skills (such as question 4.1; 5.1; 6.1 and 8.1), mathematical understanding (Hiebert & Carpenter, 1992) is displayed as connections between facts, procedures and ideas are necessary in order to reach a solution. The problem-solving process as described by Al-Mutawah et al. (2019) is evident. As learners had to make connections, use different representations and reason with what was provided to them. By exposing learners to questions that encourage critical thinking their conceptual thinking is also enhanced (Aizikovitsh-Udi & Amit, 2011).

The results for Venn diagram question and tree diagram question in PRT agree with correlate found in the study conducted by Awuah (2018). In that study Awuah (2018) concluded that

learners displayed greater proficiency when using Venn diagrams as opposed to other aids when solving probability questions. However, when comparing responses for question 5.1, where learners had to draw a Venn diagram, this question had a **92%** average and question 6.1 where learners best option was to draw a tree diagram in order to reach a solution, this question had a **31%** average. The results from the Venn diagram and tree diagram questions concur with Awuah (2018) as learners displayed a much greater proficiency with Venn diagrams compared to tree diagrams.

De Kock (2015) found that learners struggled with questions that required knowledge and recalling rules on probability. Results from Task 3 contrast with those findings, as learners excelled in recalling the formula for mutually exclusive events and the substituting appropriately. It should be noted that in the study conducted by De Kock (2015) data was collected from learners who wrote their November examination that was set either by the school or GDE and other data was collected from the preliminary examination that learners wrote. The condition in which the assessments were administered differs in this study and those conducted by De Kock (2015). The difference in examination conditions are possible contributing factors to learner performance.

The mean score for conceptual knowledge (**53%**) and procedural knowledge (**82%**) concur with results from a study by Manandhar (2018) and Al-Mutawah et al. (2019)in which it was found that learners had a low level of conceptual knowledge and a higer procedural knowledge in selected mathematics topics. The mean score for procedural knowledge (**82%**) is exceptionally well and is regarded as a distinction for schooling in South Africa. Learners in this study displayed outstanding procedural knowledge (Hiebert & Lefevre, 1986) as they were able to make use of symbol and syntactic norms as well as apply rules and procedures. Conceptual knowledge as described by Hiebert and Lefevre (1986) was adequate in this study as learners displayed a decent understanding of relations that form part of a network of knowledge in certain questions.

In PRT average for procedural knowledge questions (**82%**) is higher than the average for conceptual knowledge (**53%**); these results do align with Konicek-Moran and Keeley (2015) who claim that without conceptual understanding learners hang onto their previous understanding and memorise facts. Simultaneously a bidirectional relation between

procedural and conceptual knowledge (Rittle-Johnson et al., 2015) is noted; as they enforce and support the development of each other. Based on the way learners responded to questions it is evident that they are able to draw upon procedural and conceptual knowledge in a manner that supports each other, these results are similar to Manandhar (2018) who found a positive correlation between procedural and conceptual knowledge for algebra questions.

The PRT had an unequal spread of procedural and conceptual tasks, where conceptual knowledge dominated. In contrast, a study conducted by Hatisaru (2020) revealed that procedural tasks dominated over conceptual tasks when drawing of learners' perceptions of their mathematics classroom was analysed.

From the teacher interview it is evident that the teacher is aware of the importance of probability and this has impacted her teaching, this is made explicit by Batenero and Diaz (2012) as well as Dvorakova et al. (2017). This teacher is aware of the importance of a topic like probability and acknowledges this. As a result the teacher recognises the importance of training and is willing to attend sessions; this is with a long term goal that the learners will actually benefit from her having attended training. Unlike results obtained by Kodisang (2015) this teacher was able to successfully implement teaching strategies that are beneficial to her learners for the topic of probability. Papaieronymou (2009) asserts that teachers should have sufficient probability content and teaching knowledge, this was evident in learners' responses as the teacher's knowledge definitely impacted learners grasp and understanding of probability.

Although textbooks are an important resource (Kristanto & Santoso, 2020) they may not provide the necessary support (Batanero & Diaz, 2012) expected by teachers. A textbook may not prove all the necessary support required by a teacher, but it does provide some support. A variety of textbooks and other resources should be used in order for teachers to have the necessary support. The incorporation of videos into lessons shows that the teacher does not only rely on the textbook. In order to ensure that learners do not find probability challenging (Sahoo, 2013) the teacher ensures that all activities for this section is completed with full explanations.

7.5. CONSOLIDATION OF RESEARCH QUESTIONS

Discussions of sub-research questions significantly contribute towards answering the main research questions for this study. Results presented in Chapter 5 and Chapter 6 were reflected upon when answering sub-research questions. Those results can also support the discussion of the two main research questions.

Main research Question 1: What Depth of Knowledge is evident, from the Probability Questions in selected November Examinations?

From the results of this study it is evident that Probability Questions from selected November examinations (2017 to 2019) contain a variety of questions that are at different DOK levels. The detailed analysis presented in Chapter 5, analysed each of the selected examination papers and the probability questions. From the analyses no patterns or trends in the allocation of DOK levels are evident. Each of the primary sources of probability questions had a different DOK level that was most prominent. In the November 2017 examination there was an equal allocation for **DOK 1** and **2** at **36%** each, and **DOK 3**, was slightly less, at **28%** (refer to Table 5.3). In the November 2018 examination **DOK 1** had the highest allocation of **44%**, followed by **DOK 3** with **37%** and finally **DOK 2** accounted for only **19%** of the probability questions (refer to Table 5.6). In the November 2019 examination, **DOK 3** accounted for a staggering **56%** of the probability questions, **DOK 2** was **31%** and **DOK 1** only accounted for **13%** (refer to Table 5.6). DOK 4 was not present in any of the probability questions from November examinations. Questions that are categorised at DOK 4 require extended thinking and in order to do, an investigation or research processes would have to occur. With these processes, multiple problem-solving skills for non- routine tasks will have to be conducted. Probability questions at DOK 4 can be implemented at school level. A task at DOK 4 can be a take home project that learners have to complete over a certain period of time. As this will allow them to refer to multiple sources and incorporate necessary information and skills from a variety of content areas and subjects.

Main research Question 2: How do learners respond to Probability Questions that are at different Depth of Knowledge levels?

From the results obtained in this study, learners have performed extremely well in the PRT. When considering the different DOK levels, it is evident that learners have performed substantially better in questions at **DOK 1**, which was expected. The performance in

probability questions at **DOK 2** and **3** do not differ by much. When comparing the average performance for **DOK 1** and **DOK 2**, there is a shocking difference of **33%**, and when comparing the average performance for **DOK 1** and **DOK 3**, there is a difference of **31%**. The average performance for all DOK levels in this PRT has all exceeded the national pass rate of 30%.

The overall performance by learners fluctuated; learners performed the best for DOK 1 questions. Performance for DOK 2 and 3 was not bad either. The overall performance in the PRT exceeded the researcher's expectations. These results may have been very different if the sample space was larger; if assessments were conducted over a longer period of time or even if results were taken from actual examinations that learners wrote and not a revision task.

7.6. RECOMMENDATIONS OF THE STUDY

This study has enlightened and provided me with great insight not only for mathematics but specifically for the topic of Probability. Like with any other that is conducted, there is always room for improvement. The recommendation for this study is guided by the data collected and the methodologies utilised.

In order to gain a better understanding of learners' understanding of the topic, interviews with learners can be conducted. Also, if learners are given more questions that require interpretation and have few guiding questions, a more detailed analysis of learner understand could have been produced. This study only utilised questions that were designed for Grade 12 mathematics learners. If a larger focus group is utilised, such as FET learners, or if learners and their progress is tracked for their FET journey, results obtained may offer teachers and curriculum developers a better understanding about the views and perceptions on probability as well as the strengths and challenges that learners and teachers are faced with. The monitoring of types of dialogue that learner engage with when posed with unfamiliar questions can add value and insight to a study like this.

Results from past, present and future studies should also be considered when designing curricula, teaching and learning outcomes and resources. The development of a curriculum that nurtures conceptual knowledge should become a priority. One major trend is the need for

more regular and adequate teacher training that should be made available for both novice and experienced teachers. Training should start from as early as pre-service teacher training and continue with in-service teachers. Training can be provided by the GDE or any other educational training institution. Mathematics facilitators and mathematics learning communities can arrange and conduct training in order to help develop teachers based on their personal experiences as well.

7.7. LIMITATIONS OF THE STUDY

This specific study was extremely limited with the participants. Due to the Corona Virus Pandemic, permission and consent for participation in this voluntary research project was limited. This study had a small sample from which data was collected, that means that results from this study cannot be generalised and applicable for a larger population.

Not only did the Corona Virus Pandemic affect the number of participants, but it also impacted the data collection for the study. One major factor that was influenced by the pandemic was time. Due to the pandemic, there was valuable teaching and learning time lost, but teachers were under severe pressure to complete the syllabus with extreme time limitations. Given the extraordinary circumstances, the researcher was able to collect data in a manner that did not interrupt on teaching and learning time of the learners in school.

The participant's option of withdrawing from the study at any given time is a reality that impacts the sample and data collected. This was a significant factor in this study as the sample space was reduced from 40 to 29 participants. As a researcher it was my duty to comply with the decision of participants and their wanting to continue or to withdraw from the study. This impacted the quantitative component of this study. Due to small sample size in this study a proper quantitative analysis of results obtained was not feasible.

This study focused on Probability Question that was assessed in the final school leaving examination only. The study did not look at the Probability topic for the entire FET phase or high school journey. This contributes as a limitation of this study and results from this study cannot be generalised.

7.8. REFLECTIONS OF THE STUDY

From the inception to the completion of this research report I have gained valuable knowledge regarding Depth of Knowledge in probability questions. Although probability can be considered as a relatively new topic in the South African curriculum, which has a relatively small weighting in the syllabus, there is room for improvement. This study allowed me to explore the complex nature of probability questions and view various responses to probability questions that motivates and encourages me to want to investigate even further. The research process has allowed me to improve on my teaching and learning practices and the urge to improve learner understanding by incorporating different teaching strategies and resources into my planned lessons.

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APPENDIX A – PROBABILITY QUESTIONS

An Extract of the probability questions from the National November Examination 2017, 2018 and 2019 mathematics paper 1.

2017 **(14/150 = 9%)**

QUESTION 10

A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 use all three.
- 12 use Instagram and Twitter.
- 5 use Twitter and WhatsApp, but not Instagram.
- x use Instagram and WhatsApp, but not Twitter.
- 61 use Instagram.
- 19 use Twitter.
- 73 use WhatsApp.
- 14 use none of these applications.

10.1 Draw a Venn diagram to illustrate the information above. (4)

10.2 Calculate the value of x. (2)

10.3 Calculate the probability that a learner, chosen randomly, uses only ONE of these applications. (2)
[8]

QUESTION 11

A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR 45789.

The letters are chosen from A; D; R; S and U. Letters may be repeated in the code.

The digits 0 to 9 are used, but NO digit may be repeated in the code.

11.1 How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits? (3)

11.2 Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system. (3)
[6]

2018 (16/150= 11%)

QUESTION 11

Given the digits: 3 ; 4 ; 5 ; 6 ; 7 ; 8 and 9

11.1 Calculate how many unique 5-digit codes can be formed using the digits above, if:

11.1.1 The digits may be repeated (2)

11.1.2 The digits may not be repeated (2)

11.2 How many unique 3-digit codes can be formed using the above digits, if:

- Digits may be repeated
- The code is greater than 400 but less than 600
- The code is divisible by 5 (3)

[7]

QUESTION 12

12.1 Given: $P(A) = 0,45$; $P(B) = y$ and $P(A \text{ or } B) = 0,74$

Determine the value(s) of y if A and B are mutually exclusive. (3)

12.2 An organisation decided to distribute gift bags of sweets to a Grade R class at a certain school. There is a mystery gift in exactly $\frac{1}{4}$ of the total number of bags.

Each learner in the class may randomly select two gift bags of sweets, one after the other. The probability that a learner selects two bags of sweets with a mystery gift is $\frac{7}{118}$. Calculate the number of gift bags of sweets with a mystery gift inside. (6)

[9]

2019 (16/150 = 11%)

QUESTION 10

The school library is open from Monday to Thursday. Anna and Ben both studied in the school library one day this week. If the chance of studying any day in the week is equally likely, calculate the probability that Anna and Ben studied on:

10.1 The same day (2)

10.2 Consecutive days (3)

[5]

QUESTION 11

11.1 Events **A** and **B** are independent. P(A) = 0,4 and P(B) = 0,25.

11.1.1 Represent the given information on a Venn diagram. Indicate on the Venn diagram the probabilities associated with each region. (3)

11.1.2 Determine P(**A** or NOT **B**). (2)

11.2 Motors Incorporated manufacture cars with 5 different body styles, 4 different interior colours and 6 different exterior colours, as indicated in the table below.

BODY STYLES	INTERIOR COLOURS	EXTERIOR COLOURS
Five body styles	Blue	Silver
		Blue
	Grey	White
	Black	Green
		Red
	Red	Gold

The interior colour of the car must NOT be the same as the exterior colour.

Motors Incorporated wants to display one of each possible variation of its car in their showroom. The showroom has a floor space of 500 m^2 and each car requires a floor space of 5 m^2.

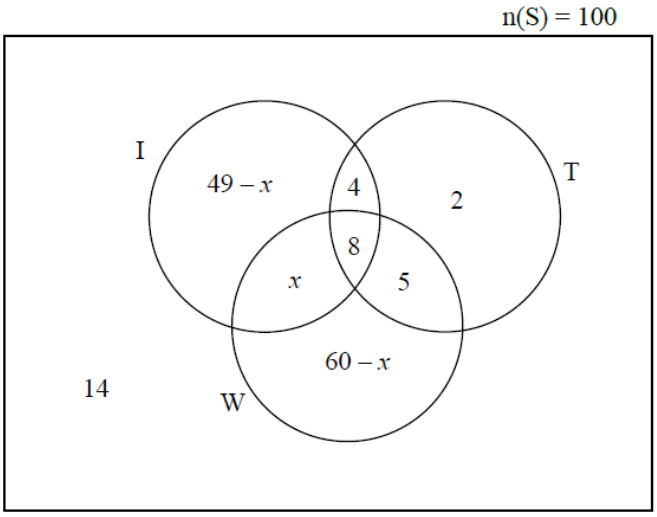
Is this display possible? Justify your answer with the necessary calculations. (6)
[11]

APPENDIX B – PROBABILITY MEMORANDUM

An Extract of the probability answers from the National November Examination 2017, 2018 and 2019 mathematics paper 1.

2017

QUESTION/*VRAAG* 10

10.1		8 values need to be placed in correct position: 2 or 3 correct: 1 mark 4 or 5 correct: 2 marks 6 or 7 correct: 3 marks 8 correct: 4 marks (4)
10.2	$(49-x)+x+8+4+5+2+(60-x)+14=100$ $-x+142=100$ $x=42$	✓ setting up equation ✓ answer (2)
10.3	P (use only one application) $=\frac{7+2+18}{100}$ $=\frac{27}{100}$ or 27%	✓ $\frac{7+2+18}{100}$ ✓answer (2) **[8]**

QUESTION/*VRAAG* 11

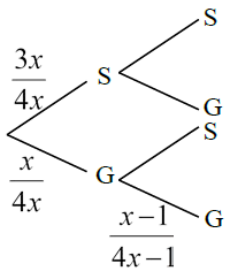
<table>
<tr>
<td>11.1</td>
<td>5 x 5 x 10 x 9
= 2250</td>
<td>✓ 5 x 5
✓ 10 x 9
✓ 2250 (3)</td>
</tr>
<tr>
<td>11.2</td>
<td>
<table>
<tr><th>No of digits used</th><th>Letters</th><th>Digits</th><th>Total</th></tr>
<tr><td>1</td><td>5 x 5</td><td>10</td><td>250</td></tr>
<tr><td>2</td><td>5 x 5</td><td>10 x 9</td><td>2 250</td></tr>
<tr><td>3</td><td>5 x 5</td><td>10 x 9 x 8</td><td>18 000</td></tr>
<tr><td>4</td><td>5 x 5</td><td>10 x 9 x 8 x7</td><td>126 000</td></tr>
<tr><td>5</td><td>5 x 5</td><td>10 x 9 x 8 x7 x 6</td><td>756 000</td></tr>
</table>
Codes of two letters and five digits will ensure unique numbers for 700 000 clients.
</td>
<td>✓ 5 x 5 x 10 x 9 x 8 x 7 x 6
✓✓ five digits (3)
[6]</td>
</tr>
</table>

2018

QUESTION/*VRAAG* 11

11.1.1	$7^5 = 16\,807$	✓✓ answer (2)
11.1.2	$7\times6\times5\times4\times3$ $= \frac{7!}{2!} = 2520$	✓ $7\times6\times5\times4\times3$ or $\frac{7!}{2!}$ ✓answer (2)
11.2	$2\times7\times1 = 14$	✓✓✓ $2\times7\times1$ (3)
		[7]

QUESTION/*VRAAG* 12

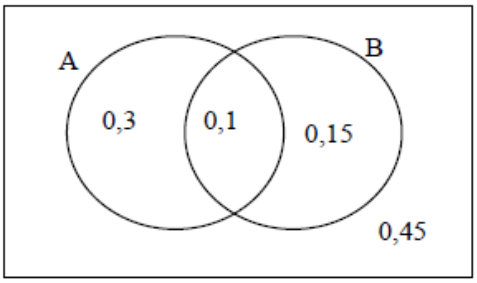
12.1	P(A or B) = P(A) + P(B) 0,74 = 0,45 + y y = 0,29	✓ P(A or B) = P(A) + P(B) ✓substitution ✓answer (3)
12.2	 Let the number of mystery gift bags = x The total number of bags = $4x$ $\left(\frac{x}{4x}\right)\times\left(\frac{x-1}{4x-1}\right)=\frac{7}{118}$ $\frac{1}{4}\times\frac{x-1}{4x-1}=\frac{7}{118}$ $\frac{x-1}{4x-1}=\frac{28}{118}$ $118x - 118 = 112x - 28$ $x = 15$	✓ $4x$ ✓$\left(\frac{x}{4x}\right)$ or $\left(\frac{1}{4}\right)$ ✓$\left(\frac{x-1}{4x-1}\right)$ ✓$\frac{1}{4}\times\frac{x-1}{4x-1}$ ✓ equating to $\frac{7}{118}$ ✓answer (6)
	OR/OF P(gift and gift) = P(gift at first draw) × P(gift at second draw) $\frac{7}{118}=\frac{1}{4}\times$ P(gift at second draw) P(gift at second draw) $=\frac{7}{118}\div\frac{1}{4}$ $=\frac{14}{59}$ Therefore: P(gift at first draw) $=\frac{15}{60}$ And: 15 bags had mystery gifts inside	**OR/OF** ✓$\frac{1}{4}$ ✓$\frac{1}{4}\times$ P(gift at 2nd draw) ✓ $\frac{7}{118}=\frac{1}{4}\times$ P(gift at 2nd draw) ✓$\frac{14}{59}$ ✓ $\frac{15}{60}$ ✓answer (6)
		[9]

2019

QUESTION/*VRAAG* 10

10.1	P(same day) $= \frac{4}{16}$ or $\frac{1}{4}$ or 0,25 or 25%	✓4 numerator ✓16 denominator (2)
10.2	P(2 consecutive days) $= \frac{3 \times 2}{16} = \frac{3}{8}$	✓3 ✓×2 ✓ answer (3)
		[5]

QUESTION/*VRAAG* 11

11.1.1	$P(A) \times P(B)$ independent events $= 0,40 \times 0,25 \quad = 0,1$  	✓0,1 ✓0,15 and 0,3 ✓0,45 (3)
11.1.2	$P(\text{A or not B}) = P(\text{A}) + P(\text{not B}) - P(\text{A and not B})$ $= 0,4 + 0,75 - 0,3$ $= 0,85$ **OR/OF** $P(\text{A or not B}) = 1 - P(\text{only B})$ $= 1 - 0,15$ $= 0,85$ **OR/OF** From Venn diagram: 0,3 + 0,1 + 0,45 = 0,85	✓ substitution ✓answer (2) **OR/OF** ✓ 1 – 0,15 ✓answer (2) **OR/OF** ✓ substitution ✓ answer (2)
11.2	$(5 \times 1 \times 5) + (5 \times 1 \times 6) + (5 \times 1 \times 6) + (5 \times 1 \times 5) = 110$ $110 \times 5 = 550 > 500$ Not possible, because not enough space **OR/OF** $(5 \times 2 \times 5) + (5 \times 2 \times 6) = 110$ $110 \times 5 = 550 > 500$ Not possible because not enough space **OR/OF**	✓5 × 1 × 5 ✓5 × 1 × 6 ✓5 × 1 × 6 ✓5 × 1 × 5 ✓110 ✓ conclusion (6) **OR/OF** ✓✓5 × 2 × 5 ✓✓5 × 2 × 6 ✓ 110 ✓ conclusion (6) **OR/OF**

APPENDIX C – INFORMATION SHEET FOR PRINCIPAL

INFORMATION SHEET FOR PRINCIPAL

Date _____

Dear Principal

Request to participate in a research project

My name is Deepa Gopal and I am a mathematics teacher. I am a student in the School of Education at the University of the Witwatersrand. I am currently doing my Master's Degree in Mathematics Education under the supervision of Dr B. Mofolo-Mbokane.

I am doing research on the topic of Probability in mathematics paper 1 examination papers. The aim of this study is to identify possible factors that may result in poor performance achieved by learners. The outcome of this study may lead to some solutions to the current performance of learners in mathematics. My research project involves analyzing answer scripts of learners as well as interviewing teachers. The interview with teacher(s) aims to gather more information on the teaching and learning of probability.

I hereby request permission to carry out my research at your school. The information data collection process will take place after school hours so as not to cause any disruptions to the school. The research will require the following:

- Learners to write a Probability Revision Task.
- Interview with Grade 12 Mathematics teacher(s) for 30 minutes.

Research participants will not be advantaged or disadvantaged in any way. They will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and no participants will be paid for this study. The identity of all participants and the school will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project. Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,
Deepa Gopal

If you have any questions, please feel free to contact either myself or my supervisor.

Student	Supervisor …. Mbokane …….
Deepa Gopal	Dr. Mofolo-Mbokane, B
Cell-phone: 084 370 9722	Tel: 011 717 3411
e-mail: deepa23gopal@gmail.com	e-mail: batseba.mbokane@wits.ac.za

APPENDIX D – CONSENT FORM FOR PRINCIPAL

CONSENT FORM FOR PRINCIPAL

Date _____

Please fill in and return the reply slip below if you agree to allow the school to participate in my study.

I _____ the principal of _____ (schools name) agree that a research study conducted by Deepa Gopal (student number 560530) from the University of the Witwatersrand may be undertaken in this school.

Informed consent

I understand that:

- Learner participants will write a Probability Revision Task.
- Interview with Grade 12 Mathematics teacher(s) shall be conducted and audiotaped
- My name and information, as well as the names and information of learners and teacher(s) that participate will be kept confidential and safe and that my name and the name of the school will not be revealed.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Signed: _____ Date: _____

APPENDIX E – INFORMATION SHEET FOR PARENT(S)/GUARDIAN(S)

INFORMATION SHEET FOR PARENTS(S)/ GUARDIAN(S)

Date _____

Dear Parent(s)/ Guardian(s)

Request to participate in a research project

My name is Deepa Gopal and I am a mathematics teacher. I am a student in the School of Education at the University of the Witwatersrand. I am currently doing my Master's Degree in Mathematics Education under the supervision of Dr B. Mofolo-Mbokane.

I am doing research on the topic of Probability in mathematics paper 1 examination papers. The aim of this study is to identify possible factors that may result in poor performance achieved by learners. The outcome of this study may lead to some solutions to the current performance of learners in mathematics. My research involves analyzing the answer scripts of learners. The reason why I have chosen your child/ward class is because this class has mathematics as a subject that is examinable. Learners are only expected to complete a Probability Revision Task that will be written after their preliminary examination as part of revision for their national November final examination. Teachers will be interviewed so as to gain more information on the challenges that they may experience when teaching probability.

Please note that the research will require the following:

- Learner participants will write a Probability Revision Task.
- Interview with Grade 12 Mathematics teacher(s) for 30 minutes.

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study. Your child's name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project. Please let me know if you require any further information.

Thank you very much for your help.

Please feel free to contact me if you have any questions.

Yours sincerely,
Deepa Gopal

Student 	Supervisor …Mbokane …….
Deepa Gopal	Dr. Mofolo-Mbokane, B

Cell-phone: 084 370 9722	Tel: 011 717 3411
e-mail: deepa23gopal@gmail.com	e-mail: batseba.mbokane@wits.ac.za

APPENDIX F – CONSENT FORM FOR PARENT(S)/GUARDIAN(S)

CONSENT FORM FOR PARENT(S)/ GUARDIAN(S)

Date _____

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in my research project.

Informed Consent

I understand that:

- My child's/ward's name and information will be kept confidential and safe and that my name, and the name of my school will not be revealed.
- He /she does not have to answer every question and can withdraw from the study at any time.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

I _____ give permission for my child _____ to participate in this research.

I _____ give permission that my child's results from the mathematics preliminary examination may be used in this study. I am aware that the findings of this research will be used to promote teaching and learning and will be published. I am aware that my child will remain anonymous.

Signed: _____ Date: _____

APPENDIX G – INFORMATION SHEET FOR TEACHERS

INFORMATION SHEET FOR TEACHERS

Date _____

Dear Teacher

Request to participate in a research project

My name is Deepa Gopal and I am a mathematics teacher. I am a student in the School of Education at the University of the Witwatersrand. I am currently doing my Master's Degree in Mathematics Education under the supervision of Dr B. Mofolo-Mbokane.

I am doing research on the topic of Probability in mathematics paper 1 examination papers. The aim of this study is to identify possible factors that may result in poor performance achieved by learners. The outcome of this study may lead to some solutions to the current performance of learners in mathematics

My research involves analyzing the answer scripts of learners. The reason why I have chosen your school is because this school offers mathematics as a subject that is examinable and the location of the school is in a community that I want to make a difference in. Learners are only expected to complete a Probability Revision Task that will be written after their preliminary examination as part of revision for their national November final examination. Teachers will be interviewed so as to gain more information on the challenges that they may experience when teaching probability.

Please note that the research will require the following:

- Learner participants will write a Probability Revision Task.
- Interview with Grade 12 Mathematics teacher(s) for 30 minutes.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project. You will not be advantaged or disadvantaged in any way. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.

Please let me know if you require any further information.

Yours sincerely,
Deepa Gopal

Student	Supervisor …. Mbokane …….
Deepa Gopal	Dr. Mofolo-Mbokane, B
Cell-phone: 084 370 9722	Tel: 011 717 3411
e-mail: deepa23gopal@gmail.com	e-mail: batseba.mbokane@wits.ac.za

APPENDIX H – CONSENT FORM FOR TEACHERS

CONSENT FORM FOR TEACHERS

Date _____

Please fill in and return the reply slip below indicating your willingness to be a participant in my research project.

I _____ volunteer to participate in this research by answering question based on teaching probability and giving you access to learners scripts, who gave given consent to take in this project. I am aware that the findings of this research will be used to promote teaching and learning and will be published. I am aware that my identity will be protected and I will remain anonymous.

Permission to review/collect documents/artifacts	**Circle one**
I agree that preliminary examination answer scripts can be used for this study only.	YES/NO

Permission to be audiotaped	**Circle one**
I agree to be audiotaped during the interview or observation lesson	YES/NO
I know that the audiotapes will be used for this project only	YES/NO

Informed Consent

I understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Signed: _____ Date: _____

APPENDIX I – INFORMATION SHEET FOR LEARNERS

INFORMATION SHEET FOR LEARNERS

Date _____

Dear Grade 12 learner

Request to participate in a research project

My name is Deepa Gopal and I am a mathematics teacher. I am a student in the School of Education at the University of the Witwatersrand. I am currently doing my Master's Degree in Mathematics Education under the supervision of Dr B. Mofolo-Mbokane.

I am doing research on the topic of Probability in mathematics paper 1 examination papers. The aim of this study is to identify possible factors that may result in poor performance achieved by learners. The outcome of this study may lead to some solutions to the current performance of learners in mathematics. My research involves analyzing the answer scripts of learners. The reason why I have chosen your class is because this class has mathematics as a subject that is examinable. As a learner you are only expected to complete a Probability Revision Task after your preliminary examination as part of revision for you national November final examination. Your teacher(s) will be interviewed so as to gain more information on the challenges that they may experience when teaching probability.

Please note that the research will require the following:

- Learner participants will write a Probability Revision Task.
- Interview with Grade 12 Mathematics teacher(s).

You will not be advantaged or disadvantaged in any way. You can withdraw your permission at any time during this project without penalty. There are no foreseeable risks in participating and you will not be paid for this study. I will not be using your own name but I will make one up so no one can identify you. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join in the study.

Thank you very much for your help.

Please feel free to contact me if you have any questions.

Yours sincerely,
Deepa Gopal

Student	Supervisor …. Mbokane …….
Deepa Gopal	Dr. Mofolo-Mbokane, B
Cell-phone: 084 370 9722	Tel: 011 717 3411
e-mail: deepa23gopal@gmail.com	e-mail: batseba.mbokane@wits.ac.za

APPENDIX J – CONSENT FORM FOR LEARNERS

CONSENT FORMS FOR LEARNERS

Date_____

Please fill in and return the reply slip below if you agree to participate in my study.

I _____ agree that my answer script for the Probability Revision Task can be used for this study.

I volunteer to participate in this research. I am aware that the findings of this research will be used to promote teaching and learning and will be published. I am aware that my identity will be protected and I will remain anonymous.

Informed Consent

I understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Signed: _____ Date: _____

APPENDIX K – SAMPLE INTERVIEW QUESTIONS

SAMPLE INTERVIEW QUESTIONS

Date _____

Good afternoon Madam/Sir

1. How many years have you been teaching Grade 12 mathematics?
2. How many years have you taught FET mathematics?
3. What do you feel probability entails?
4. What are the challenges when teaching probability?
5. How do you feel learners respond to probability questions?
6. Are learners able to answer conceptual or procedural questions better? Why do you think this is the case?
7. In terms of the Caps Taxonomy, which questions are learners doing well with? (Please motivate why)
8. Do you think learners make use of probability in an everyday context, out of school? (Please motivate)
9. Do you feel that there should be more teacher training on probability? (Why?)
10. Explain any intervention strategies that you have in place in order to support learners?
11. How would you describe performance in probability? Elaborate.
12. What strategies or methodologies do you use when teaching? Do you change any of them or incorporate any others when having to teach probability?
13. During the Corona Virus Pandemic, how have you changed or improvised on your approaches to teaching and learning?

Thank you very much for your time

APPENDIX L1 – ETHICS CLEARANCE LETTER: SCHOOL OF EDUCATION

WITS SCHOOL OF EDUCATION

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

SCHOOL OF EDUCATION ETHICS COMMITTEE
CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	PROTOCOL NUMBER: 2019ECE046M
PROJECT TITLE	Application of Webb's Depth of Knowledge on learners' responses to Probability questions
INVESTIGATOR	Deepa Gopal
SCHOOL/DEPARTMENT OF INVESTIGATOR	WITS SCHOOL OF EDUCATION
DATE CONSIDERED	15 October 2019
DECISION OF THE COMMITTEE	Approved unconditionally
EXPIRY DATE	Date of submission of the project report

ISSUE DATE OF CERTIFICATE 22 October 2019 CHAIRPERSON (Dr. Paul Goldschagg)

cc: Supervisor: Dr. Batseba Mofolo-Mbokane

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** emailed to the Ethics Office: Matsie.Mabeta@wits.ac.za .

I fully understand the conditions under which I am authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

Signature

23 / 10 / 2019
Date

APPENDIX L2 – GDE RESEARCH APPROVAL LETTER



GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**04 December 2019**
Validity of Research Approval:	**10 February 2020 – 30 September 2020** **2019/346**
Name of Researcher:	**Gopal D**
Address of Researcher:	**6 Eosin Close, Extension 13**
	Lenasia
	Johannesburg, 1820
Telephone Number:	**011 852 5266/ 084 370 9722**
Email address:	**deepa23gopal@gmail.com**
Research Topic:	**Application of Webb's Depth of Knowledge on Learners Responses to Probability questions.**
Type of qualification	**Masters' in Education**
Number and type of schools:	**Four Secondary Schools**
District/s/HO	**Johannesburg South**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

04/12/2019

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

1. *Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
4. *A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

.....

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: ...04/12/2019.....

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

APPENDIX M – PROBABILITY TASK FOR LEARNERS TO ANSWER

Probability

Probability involves the logic of uncertainty and randomness. Uncertainty and randomness occur in just about every field of application and in daily life, so it is extremely useful and interesting to understand probability. Probability trains you to make decisions in situations which there are observable patterns, but a degree of uncertainty.

The following sets of questions allow you to apply knowledge that you have learnt to situations that may be unfamiliar to you.

Task 1

A company uses a coding system to identify its clients. Each **code is made up of two letters and a sequence of digits**, for example AD108 or RR45789

The letters are chosen from **A; D; R; S** and **U**. Letters may be repeated in the code.

The digits **0 to 9** are used, but NO digit may be repeated in the code

1.1. How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits? (3)

1.2. Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system. (3)

Task 2

Given the digits: **3; 4; 5; 6; 7; 8** and **9**

2.1. Calculate how many unique 5-digit codes can be formed using the digits above, if:

2.1.1. The digits may be repeated (2)

2.1.2. The digits may not be repeated (2)

2.2. How many unique 3-digit codes can be formed using the digits above, if:

- Digits may be repeated
- The code is greater than 400 but less than 600
- The code is divisible by 5 (3)

Task 3

Given: **P(A) = 0,45** ; **P(B) = y** and **P(A or B) = 0,74**

Determine the value(s) of y if A and B are mutually exclusive. (3)

Task 4

Events **A** and **B** are independent. **P(A) = 0,4** and **P(B) = 0,25**

4.1. Represent the given information on a Venn diagram. Indicate on the Venn diagram the probabilities associated with each region. (3)

4.2. Determine P(**A** or NOT **B**). (2)

Task 5

A survey was conducted among 100 Grade 12 learners about their use of **Instagram (I)**, **Twitter (T)** and **WhatsApp (W)** on their cell phones. The survey revealed the following:

- 8 used all three.
- 12 used Instagram and Twitter
- 5 use Twitter and WhatsApp, but not Instagram
- x use Instagram and WhatsApp, but not Twitter
- 61 use Instagram
- 19 use Twitter
- 73 use WhatsApp
- 14 use none of these applications

1.1. Draw a Venn diagram to illustrate the information above. (4)

1.2. Calculate the value of x. (2)

1.3. Calculate the probability that a learner, chosen randomly, uses only ONE of these applications. (2)

Task 6

An organisation decided to distribute gift bags of sweets to a Grade R class at a certain school. There is a **mystery gift in exactly $\frac{1}{4}$ of the total number of bags**.

Each learner in the class may randomly select two gift bags of sweets, one after the other. The probability that a learner selects **two bags of sweets with a mystery gift is $\frac{7}{118}$.**
Calculate the number of gift bags of sweets with a mystery gift inside. (6)

Task 7

The school library is open from **Monday to Thursday**. Anna and Ben both studied in the school library **one day this week**. If the chance of studying any day in the week is **equally likely**, calculate the probability that Anna and Ben studied on:

7.1. The same day. (2)

7.2.Consecutive days. (3)

Task 8

Motors Incorporated manufacture cars with **5 different body styles, 4 different interior colours and 6 different exterior colours**, as indicated in the table below.

BODY STYLES	INTERIOR COLOURS	EXTERIOR COLOURS
Five body styles	Blue Grey Black Red	Silver Blue White Green Red Gold

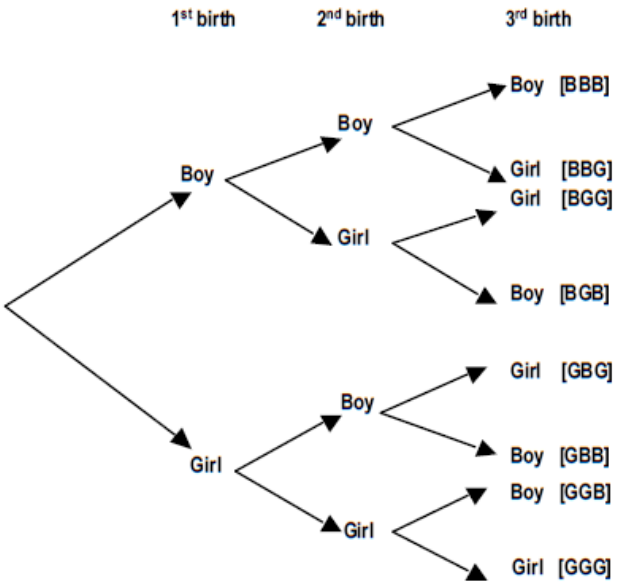
The interior colour of the car must NOT be the same as the exterior colour.
Motors Incorporated wants to display one of each possible variation of its car in their showroom. The showroom has floor space of 500 m^2 and each car requires a floor space of 5 m^2.
Is this display possible? Justify your answer with the necessary calculations. (6)

APPENDIX N – INTERMEDIATE PHASE PROBABILITY (GRADES 4; 5 AND 6)

	Concepts and skills	Clarification and teaching guidelines
Grade 4 (2 hours)	**Probability experiments** • Perform simple repeated events and list possible outcomes for experiments such as: -tossing a coin -rolling a die	**What is different from Grade 3?** Learners do not work with probability in Grade 3. Everything about probability is new in Grade 4. **Performing simple repeated events** Learners need to perform experiments by tossing a coin or rolling a die. Doing experiments with a coin is easier than with a die because the coin can only have two outcomes (heads or tails), while rolling the die can have 6 outcomes (numbers1-6). Learners should first list the possible outcomes before doing the experiments. They should learn how to record the results of their experiments in a table using tally marks.
Grade 5 (2 hours)	**Probability experiments** • Perform simple repeated events and list possible outcomes for experiments such as: - tossing a coin - rolling a die - spinning a spinner • Count and compare the frequency of actual outcomes for a series of trials up to 20 trials	**Performing simple repeated events** Learners need to perform experiments by tossing a coin, rolling a die, or spinning a spinner. Doing experiments with a coin is easier than with a die because the coin can only have two outcomes (heads or tails), while rolling the die can have 6 outcomes (numbers1 - 6). The spinner can have any number of outcomes, depending on number of divisions made on the spinner. Learners must first list the possible outcomes before doing the experiments. They should learn how to record the results of their experiments in a table using tally marks. They then count how many times heads or tails, or each number, or colour on a spinner, occurs in 20 trials. If learners do this in groups, the results from all the groups can be collated. They can then compare the number of outcomes that occur as the number of trials increase.
Grade 6 (2 hours)	**Probability experiments** • Perform simple repeated events and list possible outcomes for experiments such as: - tossing a coin - rolling a die - spinning a spinner • Count and compare the frequency of actual outcomes for a series of trials up to 50 trials	**Performing simple repeated events** Learners need to perform experiments by tossing a coin, rolling a die or spinning a spinner. Doing experiments with a coin is easier than with a die because the coin can only have two outcomes (heads or tails), while rolling the die can have 6 outcomes (numbers 1 - 6). The spinner can have any number of outcomes, depending on the number of divisions made on the spinner. Learners must first list the possible outcomes before doing the experiments. They should learn how to record the results of their experiments in a table using tally marks. Learners then count how many times heads or tails, or each number, or colour on a spinner, occurs in 20 trials. If learners do this in groups, the results from all the groups can be collated. They can then compare the number of outcomes that occur as the number of trials increase.

APPENDIX O – SENIOR PHASE PROBABILITY (GRADES 7; 8 AND 9)

	Concepts and skills	Clarifications and teaching guidelines
Grade 7 (4.5 hours)	**Probability** Perform simple experiments where the possible outcomes are equally likely and • list the possible outcomes based on the conditions of the activity • determine the probability of each possible outcome, using the definition of probability	**Probability experiments** In the Intermediate Phase learners did experiments with coins, dice and spinners. In this grade experiments can be done with other objects, like, different coloured buttons in a bag; choosing specific cards from a deck of cards, etc. **Example** If you toss a coin there are two possible outcomes (head or tail). The probability of a head is 1/2 which is equivalent to 50% (since it is one out of two possible outcomes)
Grade 8 (4.5 hours)	**Probability** • Consider a simple situation (with equally likely outcomes) that can be described using probability and**:** - list all the possible outcomes - determine the probability of each possible outcome using the definition of probability - predict, with reasons, the relative frequency of the possible outcomes for a series of trials based on probability - compare relative frequency with probability and explain possible differences	**Probability experiments** In the Intermediate Phase and Grade 7 learners did probability experiments with coins, dice and spinners. In Grade 8 doing actual trials of experiments become less important, and learners should consider probability for hypothetical events e.g. the probability of white as a successful outcome on a roulette table, or the probability of getting a Coca Cola at the shop if you know what the total number of drinks is that they stock and how many cans of Coca Cola they have. **Comparing relative frequency and probability** • The relative frequency is the observed number of successful outcomes for a finite sample of trials. • For example, if you toss a coin 50 times, the results are 27 heads and 23 tails. Define a head as a successful outcome. The relative frequency of heads is: 27 /50 = 54% • The probability of a head is 50% (one of two likely outcomes). The difference between the relative frequency of 54% and the probability of 50% is due to small sample size. • The more trials you do, the closer the relative frequency gets to the probability. This can be compared in class by combining results from trials done in groups or pairs.
Grade 9 (4.5 hours)	**Probability** • Consider situations with equally probable outcomes, and: - determine probabilities for compound events using two-way tables and tree diagrams - determine the probabilities for outcomes of events and predict their relative frequency in simple experiments - compare relative frequency with probability and explains possible differences	**Probability experiments** In Grades 8 and 9 probability experiments are less important, and learners should consider probability for hypothetical events. In Grade 9, learners have to consider outcomes of compound events and use two-way tables and tree diagrams to work out the probability of an outcome. **Probability of compound events** For example, what is the probability of a woman giving birth to two boys after each other? A two-way table can be used:

		<table><tr><td rowspan="2">1st birth</td><td colspan="2">2nd birth</td></tr><tr><td>Boy</td><td>Girl</td></tr><tr><td>Boy</td><td>BB</td><td>BG</td></tr><tr><td>Girl</td><td>GB</td><td>GG</td></tr></table> Two boys after each other (BB) is 1 of 4 possible outcomes, so the probability of two boys is 1 out of 4, or 25%. How does this probability change if you ask what is the probability of giving birth to three boys after each other? A tree diagram can then be used:  Total outcomes 8 Three boys after each other, [BBB], is 1 out 8 of possible outcomes, hence the probability of three boys after each other is 1/8 or 12,5%.

APPENDIX P – FET PHASE PROBABILITY (GRADES 10, 11 AND 12)

	Concepts and skills/ curriculum statement	Clarification
Grade 10 (2 weeks)	1. The use of probability models to compare the relative frequency of events with the theoretical probability. 2. The use of Venn diagrams to solve probability problems, deriving and applying the following for any two events A and B in a sample space S: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$; A and B are mutually exclusive if $P(A \text{ and } B) = 0$; A and B are complementary if they are mutually exclusive ; and $P(A) + P(B) = 1$. Then $P(B) = P(\text{ not } (A)) = 1 - P(A)$.	It generally takes a very large number of trials before the relative frequency of a coin falling heads up when tossed approaches 0,5. **Example**: In a survey 80 people were questioned to find out how many read newspaper S or D or both. The survey revealed that 45 read D, 30 read S and 10 read neither. Use a Venn diagram to find how many read 1. S only; (C) 2. D only; and (C) 3. both D and S. (C)
Grade 11 (2 weeks)	1. Revise the addition rule for mutually exclusive events: $P(A \text{ or } B)=P(A)+P(B)$, the complementary rule: $P(\text{not } A)=1-P(A)$ and the identity P(A or B) = P(A) + P(B) - P(A and B) 2. Identify dependent and independent events and the product rule for independent events: $P(A \text{ and } B)=P(A)\times P(B)$. 3. The use of Venn diagrams to solve probability problems, deriving and applying formulae for any three events A, B and C in a sample space S. 4. Use tree diagrams for the probability of consecutive or simultaneous events which are not necessarily independent.	**Comment**: • Venn Diagrams or Contingency tables can be used to study dependent and independent events. **Examples**: 1. P(A) = 0,45, P(B) = 0,3 and P(A or B) = 0,615. Are the events A and B mutually exclusive, independent or neither mutually exclusive nor independent? (R) 2. What is the probability of throwing at least one six in four rolls of a regular six sided die? (C) 3. In a group of 50 learners, 35 take Mathematics and 30 take History, while 12 take neither of the two. If a learner is chosen at random from this group, what is the probability that he/she takes both Mathematics and History? (C) 4. A study was done to test how effective three different drugs, A, B and C were in relieving headaches. Over the period covered by the study, 80 patients were given the opportunity to use all two drugs. The following results were obtained: from at least one of the drugs? (R) 40 reported relief from drug A 35 reported relief from drug B 40 reported relief from drug C 21 reported relief from both drugs A and C 18 reported relief from drugs B and C 68 reported relief from at least one of the drugs 7 reported relief from all three drugs. 4.1 Record this information in a Venn diagram. (C) 4.2 How many subjects got no relief from any of the drugs? (K) 4.3 How many subjects got relief from drugs A and B, but not C? (R) 4.4 What is the probability that a randomly chosen subject got relief from at least one of the drugs? (R)
Grade 12 (2	1. Revise:	**Examples:**

weeks)	• dependent and independent events; • the product rule for independent events: P(A and B) = P(A) × P(B). • the sum rule for mutually exclusive events A and B: $P(A \text{ or } B) = P(A) + P(B)$ • the identity: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ • the complementary rule: $P(\text{ not A}) = 1 - P(A)$ 2. Probability problems using Venn diagrams, trees, two-way contingency tables and other techniques (like the fundamental counting principle) to solve probability problems (where events are not necessarily independent). 3. Apply the fundamental counting principle to solve probability problems.	1. How many three-character codes can be formed if the first character must be a letter and the second two characters must be digits? (K) 2. What is the probability that a random arrangement of the letters BAFANA starts and ends with an 'A'? (R) 3. A drawer contains twenty envelopes. Eight of the envelopes each contain five blue and three red sheets of paper. The other twelve envelopes each contain six blue and two red sheets of paper. One envelope is chosen at random. A sheet of paper is chosen at random from it. What is the probability that this sheet of paper is red? (C) 4. Assuming that it is equally likely to be born in any of the 12 months of the year, what is the probability that in a group of six, at least two people have birthdays in the same month? (P)

Concepts and Skills and Clarification that are provided for FET phase in the CAPS document (Department of Basic Education, 2011)

APPENDIX Q – ELABORATION OF PROBABILITY TOPIC FOR FET PHASE

Probability		
Grade 10	**Grade 11**	**Grade 12**
(a) Compare the relative frequency of an experimental outcome with the theoretical probability of the outcome. (b) Venn diagrams as an aid to solving probability problems. (c) Mutually exclusive events and complementary events. (d) The identity for any two events A and B: *P(A or B) = P(A) + (B) - P(A and B)*	(a) Dependent and independent events. (b) Venn diagrams or contingency tables and tree diagrams as aids to solving probability problems (where events are not necessarily independent).	(a) Generalisation of the fundamental counting principle. (b) Probability problems using the fundamental counting principle.

Elaboration of topics in the FET Phase from the CAPS document (Department of Basic Education, 2011, p. 14)**.**

APPENDIX R – ANALYSIS OF PROBABILITY QUESTIONS FROM THE NDR FOR 2017

QUESTION 10: PROBABILITY

Common errors and misconceptions

a. Some candidates could not interpret the given information correctly. For example, they did not realise that the 12 learners using Instagram and Twitter should be split up between two parts of the diagram, with 8 in the one area and 4 in the other. Also amongst the 61 learners using Instagram, there were some who used some of the other applications as well.
b. Some candidates took the intersection of the three events to be *x*.
c. In Q10.2, many candidates could not set up the correct equation because they omitted the 14 learners who used none of the applications. Some candidates obtained answers that were negative numbers or fractions.
d. Candidates included parts of the intersection, even though 'ONE of the applications' was emphasised.

Suggestions for improvement

An in-depth explanation of the Venn diagram will clarify the number of events that occur in the various sections, in particular where exactly 1 event occurs, where two events occur simultaneously and where three events occur simultaneously.

QUESTION 11: COUNTING PRINCIPLE

Common errors and misconceptions

a. The language in this question could have been a barrier to the candidates. Candidates did not read the question carefully. Some used all 26 letters in the alphabet while others excluded 0 as a digit.
b. Candidates did not know the difference between 5 x 5 and 5! x 5!
c. Many candidates did not have any idea of how to solve Q11.2. They had difficulty in performing the reverse calculations.

Suggestions for improvement

a. The section on the fundamental counting principle needs to be taught as clearly and simply as possible. Steer away from formulae and reasoning out scenarios, using diagrams where needed.
b. The difference between *repeating* and *not repeating* should be explained to learners.

APPENDIX S – ANALYSIS OF PROBABILITY QUESTIONS FROM THE NDR FOR 2018

QUESTION 11: COUNTING PRINCIPLES

Common errors and misconceptions

a. In Q11.1, some candidates could not understand the process of selecting digits for a code and the implication of digits being repeated in the code. Some gave the answer as 57 instead of 75. They confused the number of options with the number of digits. Some candidates did not count the number of digits correctly, i.e. from 3 to 9 is 7 digits. They used the number of digits as 9 in their calculations.
b. In Q11.1.2, some candidates applied the rule without real understanding. As a result, they gave an answer of 7! instead of 7 x 6 x 5 x 4 x 3.
c. In Q11.2, some candidates gave the answer as 2! x 7! X 1!. These candidates did not know the difference between the number of options and the number of different arrangements of the number of options. Some candidates misinterpreted 'the code is divisible by 5' to mean that the number of codes was divided by 5. Some ignored the fact that this code could only comprise 3 digits.

Suggestions for improvement

a. The section on the fundamental counting principle needs to be taught as clearly and simply as possible, using diagrams to explain scenarios. Choose practical scenarios to demonstrate the concepts of 'repetition is allowed' and 'repetition is not allowed'. Learners will then be able to relate to these concepts.
b. Teachers should refrain from teaching rules to different situations, rather aim at reasoning out the calculations are required to answer the question.
c. Teachers should only introduce factorial notation once learners have a good understanding of the fundamental counting principle.

QUESTION 12: PROBABILITY

Common errors and misconceptions

a. In Q12.1, some candidates confused mutually exclusive events with independent events. Some candidates had little idea of the notation and substituted P(A) with P(0,45). Some candidates used the formula P(A or B) = P(A) + P(B) – P(A and B) but did not realise that P(A and B) = 0. Consequently, they were unable to calculate the value of *y*. Some candidates rounded off the answer of 0,29 to 0,3 although there was no need to round off the answer.
b. In Q12.2, very few candidates were able to draw the tree diagram for dependent events. Some candidates attempted to use a formula without really understanding how

it applied to the situation, e.g. P(A) x P(B) = P(A and B). Language was a barrier to answering this question.

Suggestions for improvement

a. When teaching Probability, emphasis should be placed on the understanding of the concepts like mutually exclusive events, independent events and complementary events.
b. The formulae in this section should not be an abstract idea. Teachers should explain these formulae in the context of Venn diagrams.
c. Tree diagrams are a useful tool to visualise compound events. Tree diagrams also assist learners in understanding the sequence of events. Teachers should make greater use of tree diagrams when dealing with questions that involve compound events. These should include independent events and dependent events.

APPENDIX T – ANALYSIS OF PROBABILITY QUESTIONS FROM THE NDR FOR 2019

QUESTION 10: PROBABILITY

Common Errors and Misconceptions

a. In Q10.1 some candidates did not read the question and used 7 days of a week in their calculations. Many candidates did not know how to answer this question. The most common response to this question was 1 /4 × 1/4 = 1/16.
b. Many candidates did not answer Q10.2. Some candidates attempted to draw a tree diagram but could not draw it correctly.

Suggestions for Improvement

Tree diagrams are a useful tool to visualise compound events. Tree diagrams also assist learners in understanding the sequence of events. Teachers should make greater use of tree diagrams when dealing with questions that involve compound events. These should include independent events and dependent events.

QUESTION 11: PROBABILITY AND COUNTING PRINCIPLES

Common Errors and Misconceptions

a. In Q11.1.1 some candidates confused mutually exclusive events with independent events. They drew a Venn diagram having no intersection of A and B. Some candidates were able to calculate that P(A and B) = 0,1. However, they were unable to calculate the other probabilities correctly. On the Venn diagram they indicated P(A only) as 0,4 instead of 0,3.
b. In Q11.1.2 many candidates calculated P(A or B) instead of P(A or not B). A few took into account that there was an intersection of the two events: A and not B.
c. Many candidates used factorial notation when writing down the various options, e.g. 5! ×1 × 6! instead of 5 ×1 ×6.

Suggestions for Improvement

a. When teaching Probability, emphasis should be placed on the understanding of the concepts like mutually exclusive events, independent events and complementary events.
b. The formulae in this section should not be an abstract idea. Teachers should explain these formulae in the context of Venn diagrams.
c. The section on the fundamental counting principle needs to be taught as clearly and simply as possible, using diagrams to explain scenarios. Choose practical scenarios to demonstrate the concepts of 'repetition is allowed' and 'repetition is not allowed'. Learners will then be able to relate to these concepts.

d. Teachers should refrain from teaching rules to different situations, rather aim at reasoning out the calculations that are required to answer the question.
e. Teachers should only introduce factorial notation once learners have a good understanding of the fundamental counting principle.

APPENDIX U – RESULTS OBTAINED FOR EACH TASK IN THE PRT

LEARNER	TASKS								TOTAL	%
	1	2	3	4	5	6	7	8		
A	3	7	3	1	8	4	2	2	***30***	***65***
B	3	7	3	0	8	0*	2	1	***24***	***52***
C	6	6	3	0	8	6	5	6	***40***	***87***
D	4	7	3	4	8	0	0	2	***28***	***61***
E	4	0	3	0*	8	5	5	6	***31***	***67***
F	3	7	3	0	7	0*	2	1	***23***	***50***
G	1	7	3	2	8	0	4	5	***30***	***65***
H	0	6	3	0	4	0	0	3	***16***	***35***
I	0	4	3	0	3	0*	2	0	***12***	***26***
J	3	7	3	1	8	0	0	6	***28***	***61***
K	3	6	3	5	8	4	2	6	***37***	***80***
L	6	7	3	2	8	0*	3	0	***29***	***63***
M	3	7	3	0	7	0*	3	1	***24***	***52***
N	2	6	3	0	8	0*	2	1	***22***	***48***
O	6	7	3	1	8	1	0	0	***26***	***57***
P	4	7	2	0	8	0	4	4	***29***	***63***
Q	6	7	3	1	8	0*	4	1	***30***	***65***
R	3	7	3	0	8	0*	3	1	***25***	***54***
S	3	7	3	0	8	0*	3	1	***25***	***54***
T	6	6	3	1	8	1	0	0	***25***	***54***
U	6	5	3	3	6	5	2	2	***32***	***70***
V	6	7	3	5	8	6	5	6	***46***	***100***
W	4	4	3	0	6	5	5	6	***33***	***72***
X	3	7	3	5	5	5	5	6	***39***	***85***
Y	3	5	3	5	4	6	5	6	***37***	***80***
Z	3	7	3	0*	8	0*	0*	6	***27***	***59***
AA	3	0	2	1	5	6	0	6	***23***	***50***
BB	2	0	3	0	2	0*	0	0*	***7***	***15***
CC	3	7	3	1	4	0*	0*	0*	***18***	***39***

0* represent questions that learners did not answer.

x represents questions in which learners scored full marks

APPENDIX V – RESULTS OBTAINED FOR EACH SUB-QUESTION IN THE PRT

LEARNER	TASKS & SUB-QUESTIONS															TOTAL
	1		2			3	4		5			6	7		8	
	1.1 (3)	1.2 (3)	2.1.1 . (2)	2.1.2 . (2)	2.2 (3)	3 (3)	4.1 (3)	4.2 (2)	5.1 (4)	5.2 (2)	5.3 (2)	6 (6)	7.1 (2)	7.2 (3)	8 (6)	
A	3	0	2	2	3	3	1	0	4	2	2	4	2	0	2	**30**
B	3	0*	2	2	3	3	0	0	4	2	2	0*	2	0	1	**24**
C	3	3	2	2	2	3	0	0	4	2	2	6	2	3	6	**40**
D	3	1	2	2	3	3	2	2	4	2	2	0	0	0	2	**28**
E	3	1	0	0	0	3	0*	0*	4	2	2	5	2	3	6	**31**
F	3	0*	2	2	3	3	0	0	3	2	2	0*	2	0	1	**23**
G	1	0	2	2	3	3	1	1	4	2	2	0	2	2	5	**30**
H	0	0	2	2	2	3	0	0	3	1	0	0	0	0	3	**16**
I	0	0*	2	0	2	3	0	0	3	0	0	0*	2	0	0	**12**
J	3	0	2	2	3	3	1	0	4	2	2	0	0	0	6	**28**
K	3	0*	2	2	2	3	3	2	4	2	2	4	2	0*	6	**37**
L	3	3	2	2	3	3	2	0	4	2	2	0*	1	2	0	**29**
M	3	0*	2	2	3	3	0	0	3	2	2	0*	2	1	1	**24**
N	2	0*	2	2	2	3	0	0	4	2	2	0*	2	0	1	**22**
O	3	3	2	2	3	3	1	0	4	2	2	1	0	0	0	**26**
P	3	1	2	2	3	2	0	0	4	2	2	0	2	2	4	**29**
Q	3	3	2	2	3	3	1	0	4	2	2	0*	2	2	1	**30**
R	3	0*	2	2	3	3	0	0	4	2	2	0*	2	1	1	**25**
S	3	0*	2	2	3	3	0	0	4	2	2	0*	2	1	1	**25**
T	3	3	2	2	2	3	1	0	4	2	2	1	0	0	0	**25**
U	3	3	0	2	3	3	3	0	4	2	0	5	1	1	2	**32**
V	3	3	2	2	3	3	3	2	4	2	2	6	2	3	6	**46**
W	3	1	2	2	0	3	0	0	2	2	2	5	2	3	6	**33**
X	3	0	2	2	3	3	3	2	3	2	0*	5	2	3	6	**39**
Y	3	0	2	2	1	3	3	2	4	0*	0*	6	2	3	6	**37**
Z	3	0	2	2	3	3	0*	0*	4	2	2	0*	0*	0*	6	**27**
AA	3	0	0	0	0	2	1	0	4	1	0	6	0	0	6	**23**
BB	0	2	0	0	0	3	0	0	2	0	0	0*	0	0	0*	**7**
CC	3	0	2	2	3	3	1	0*	4	0	0	0*	0*	0*	0*	**18**

0* represent questions that learners did not answer.

x represents questions in which learners scored full marks

APPENDIX W – TURN IT IN REPORT

560530:Gopal_Final_TurnItIn_2021.docx

ORIGINALITY REPORT

4%	4%	1%	0%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	manualzz.com Internet Source	1%
2	uir.unisa.ac.za Internet Source	1%
3	scholarship.shu.edu Internet Source	<1%
4	www.amesa.org.za Internet Source	<1%
5	link.springer.com Internet Source	<1%
6	hdl.handle.net Internet Source	<1%
7	www.pmena.org Internet Source	<1%
8	files.eric.ed.gov Internet Source	<1%
9	eprints.utas.edu.au Internet Source	<1%

10	www.edwardsmaths.com Internet Source	<1 %
11	www.coursehero.com Internet Source	<1 %
12	www.mathematik.uni-dortmund.de Internet Source	<1 %
13	moam.info Internet Source	<1 %
14	ecexams.co.za Internet Source	<1 %
15	my.unisa.ac.za Internet Source	<1 %
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