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First-year university students' algebraic thinking and its relationship to geometric conceptual understanding

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CHAPTER 1

INTRODUCTION AND OVERVIEW

1.1 Purpose and Background

This study investigated how first-year university students at Universidade Pedagógica (Pedagogical University) in Maputo-Mozambique bring their knowledge and thinking of algebra in understanding and working with geometry. The study explored how these students connected and used algebraic and geometric concepts and investigated whether this connection promoted students' conceptual understanding and problem solving performance in geometry.

According to the National Council of Teachers of Mathematics (NCTM) (1997) the *interplay* between geometry and algebra strengthens students' ability to formulate and analyse problems from situations both within and outside mathematics. This interplay is described by NCTM as follows:

1. translate between synthetic and coordinate representations;
2. deduce properties of figures using transformations and using coordinates;
3. identify congruent and similar figures using transformations;
4. analyse properties of Euclidean transformations and relate translations to vectors;
5. deduce properties of figures using vectors; and
6. apply transformations, coordinates, and vectors in problem solving (p. 29).

The NTCM´s contention about the connections between algebra and geometry points out a critical issue concerning connections in mathematics curriculum (Gascón, 2004). Gascón has considered those connections very important to avoid the so-called "thematic autism" which causes curricular disconnection and radical departmentalization of the disciplines and topics. This curricular disconnection hinders a holistic understanding of mathematics on students´s side. Therefore, I considered critical, in this study, to investigate the extent to which university students, who were on training to become secondary and high school mathematics teachers after a 3-year degree study (Bachelors) or 4-years study (Honours), would make

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connections and consequently, to see their ability to formulate and analyse problems from situations both within and outside mathematics in general and in the geometry in particular.

For this study I attempted to address these aspects of the interplay between algebra and geometry, although it is important to highlight that due to the type of data collected some of these aspects are addressed with less emphasis.

Ismael (2003, p. 25) has observed that students at Universidade Pedagógica performed well in advanced mathematics (e. g. calculus), while they experienced several gaps in school geometry. Moreover, Samo (2002, pp.22, 63) contended that the high school students faced a lot of difficulties in solving geometric problems that concerned the interplay between geometric and analytic approaches, possibly due to a weak grasp of geometric concepts from previous grades. Some research in Mozambique (Kilborn, 1994, p. 40) pointed out that poor performance by Grade 5 pupils in the national mathematics test, specifically in geometry, was primarily due to "the philosophy which underlies the mathematics syllabi in Mozambique". Kilborn contended that the school mathematics syllabi in Mozambique were a copy of those from some European countries, which were deprived of a concrete African country context like that of Mozambique. He was of the opinion that those syllabi were too ambitious. Many of the objectives set up in them were impossible to attain even in countries where the education system has been robust for many years. Some syllabi contents were taught before pupils had reached a sufficient cognitive development level or before they had consolidated previous concepts needed to understand new concepts. Kilborn acknowledged that the existence of a great number of untrained teachers could also influence the poor performance of pupils in mathematics.

In my own institution at Eduardo Mondlane University, I have observed that a significant number of newcomers pursuing Physics, Chemistry, Geology, Biology, Agronomy and Engineering seem to face a lot of difficulties when solving (Euclidean) geometry problems. When I asked them during a geometry lesson why they seemed to not be interested in geometry, they contended that this subject was difficult and they did not like it. Worldwide, it has also been recognized that teaching geometry, particularly producing geometry proofs, was more complex and often less successful than teaching numerical operations or elementary algebra (Duval, 1998 and Dindyal, 2003). This difficulty possibly arose due to the underlying cognitive complexity of geometrical activity.

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It seems clear that several factors have contributed to inadequate understanding and performance of students in geometry. These may be related to the nature of the discipline of geometry itself, the school mathematics syllabi, and the nature of the school geometry teaching in Mozambique. However, what is less clear, particularly in the context of mathematics education in Mozambique, is how students work with geometry problems, what algebraic knowledge, if any, they bring to the solving of these problems, and how knowledge of algebra and algebraic ways of thinking promote or hinder success in geometry problem-solving processes. Hence, this study proposed to focus on the former factor, the nature of the discipline of geometry related to algebraic thinking, that is, how students bring algebraic knowledge and thinking as they work with geometry at Universidade Pedagógica. The other factors were not taken into account at this level. On one hand it was assumed that the lecturers were well trained and on the other hand the mathematical syllabi have undergone reform in this institution (Ismael, 2003).

1.2 Research Questions

In this study, the aim is to explore how students work with geometry problems, what algebraic knowledge, if any, they bring to the solving of these problems, and how knowledge of algebra and algebraic ways of thinking promotes or hinders success (ie conceptual geometric understanding and performance) in geometry problem-solving processes. Accordingly, three research questions were considered:

1. How do first-year university students solve geometry problems? To what extent do they use algebraic knowledge and thinking in solving such problems? What kinds of meanings do students make of different algebraic and geometric concepts involved in problem solving situations?
2. To what extent does algebraic knowledge and thinking aid students' conceptual understanding and problem solving performance in geometry?
3. To what extent is geometric work linked to algebraic thinking in the first year university geometry course at the Universidade Pedagógica in Maputo?

The first question and sub questions attempt to find out whether first-year university students use algebraic symbols and algebraic relations, different representations, and patterns and generalisations (aspects of algebraic thinking considered in this study) while solving geometry problems. The aim was to find out the meanings held by the target students of

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different algebraic and geometric concepts involved in problem solving situations on (1) how they make relationships amongst those concepts; (2) how students use them and (3) how students account for the purpose of those concepts. The first type of meaning concerns the issue of context. The second and the third deal with method and validation aspects of meaning (epistemological meaning) (The Oxford English Reference Dictionary, 1995). To explore these three aspects of meaning, tests, interviews, and concept maps were designed according to the research methodology adopted by this study (Grounded Theory informed by the theoretical perspective Symbolic Interactionism).

The second question focuses on geometric conceptual understanding (visualization, construction and reasoning working in synergy) and performance (possibly and partially) triggered by algebraic thinking. Conceptual understanding should be understood as understanding that is rich in relationships (Even, 1988) and performance is considered as a result of conceptual understanding (Mousley, 2003 quoting Knapp *et al*, 1995).

The third question is concerned with algebraic and geometric concepts of the first year university geometry course which comprises Euclidean Geometry (first semester) and Analytic Geometry (second semester). In order to answer this question some lessons were video recorded together with the respective field notes of both courses to see whether a link of algebraic and geometric concepts took place during the teaching and learning process. The syllabi of those courses were also collected as an additional data source although they were not extensively analysed.

To inform my study about what is perceived as geometric conceptual understanding, three models of mathematical understanding were considered in this study, namely:

a) *Mathematical understanding* is portrayed as cumulative structuring (Mousley, 2003), that is, mathematics is a networked framework of ideas, where understanding of simpler ideas (axioms) forms a foundation or anchor point for higher-level ideas (propositions and theorems). This model accounts for the structural factor. In this study this model was used to examine whether the algebraic and geometric concepts in the Euclidean and Analytic Geometry courses at Universidade Pedagógica were constructed to facilitate the students' understanding of both disciplines. In broader terms, Gascón (2004) has explained the issue of the forms of structuring the mathematics topics to be learnt and the methods of organizing learning or teaching of those topics in schools in order to avoid the so-called "thematic

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autism". This notion characterises teaching which ignores the "why" and the "purpose" of each discipline in general and each topic in particular to be taught or learnt. Gascón argued that this phenomenon causes curricular disconnection and therefore, radical departmentalisation of the disciplines and topics.

b) *Mathematical understanding* is portrayed as a social process (ibid.), that is, one must be involved in social processes through discursive practice in particular settings in order to understand the "language" of mathematics. This model explains the contextual factor as one of the factors that may influence students' understanding and their use of algebraic knowledge and thinking in geometry. Kilborn (1994) pointed out some contextual factors that influenced poor performance by Grade 5 pupils in the national mathematics test in Mozambique, for instance the shortage of qualified teachers. In this study this model of mathematical understanding was not deeply used because of the nature of the data collected.

c) Once one is imbedded in such social processes, one can develop fluency of the "language" of mathematics at different levels depending on the different forms of meaning held and developed simultaneously but individually. In this way, *mathematical understanding* may be seen as a form of knowing (ibid.). Accordingly, the conceptual factor also plays an important role in students´ understanding and their use of algebraic knowledge and thinking in geometry. Making meaning in mathematics involves making connections among different concepts in mathematics, that is "… the forging of connections across domains [in mathematics]" (Noss and Hoyles, 1996, p. 130). Students' ability to make meaning and establish connections – "bridges" – between concepts is dependent on the nature of the subject matter[1] they are working with and the levels of understanding that is being demanded in the problem-solving context related to the subject[2]. This model was used to account for the meanings students held in relation to the connections they established during the entire research process.

Answers to these three research questions were intended to provide insights to the focus of this study: how students bring algebraic knowledge and thinking as they work with geometry at Universidade Pedagógica in Maputo – Mozambique. In other words, how they see geometry from an algebraic perspective and whether this perspective aids students' conceptual understanding and problem solving performance in geometry.

[1] This is an epistemological issue
[2] This is an contextual issue

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1.3 Overview of the Study

1.3.1 Methodology

This is a qualitative research study in which five target students were selected from a tertiary institution Universidade Pedagógica in Maputo - Mozambique. They were newcomers pursuing Bachelors and Honours of Mathematics Education to become teachers for lower and upper secondary school levels. The focus was on how these students bring their thinking and knowledge of algebra in understanding and working with geometry. To examine how students conceptually understood and performed in geometry, I observed them solving specific geometric tasks while thinking aloud, in other words to see them in action. Obviously, cognitive processes cannot be observed directly, but can only be inferred from the observations of some actions within social interaction. Even then, the correspondence between an action and the mental processes leading to that action is quite difficult to ascertain. In order to minimize this difficulty I found it worthwhile to take a theoretical perspective (symbolic interactionism) which helped me to understand social interaction. The main body of the research was done in three phases, which were analysed separately. The three phases were:

- The Pilot Study
- The Main Study – Euclidean Geometry Course
- The Main Study – Analytic Geometry Course

Data sources include students' artefacts (written responses to a test, elaboration task, and concept maps) and interview transcripts. The data were analysed using comparative method and grounded theory. Data collection was carried out during almost one academic year.

1.3.2 Theoretical Framework

Due to the context of the present study and the nature of the data collected, I drew on Prawat´s framework on learning and transfer (see Chapter 3) to analyse my data. As this framework allowed me to analyse how students with specific characteristics approached the tasks, rather than how students carried out the different tasks, which was the case in Dindyal's (2003) work. This framework comprises some perspectives similar to perspectives used by Dindyal (theory about thinking as communicating, socio-cultural theory or emergent perspective, and theory of knowledge connectedness).

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The theory about thinking as communicating in Dindyal´s work may be explained by the term *awareness* in the Prawat's framework. This awareness takes place when students articulate (communicate) their own thoughts in different ways and in different settings.

The theory of knowledge connectedness is accounted for the term *organisation*. According to Prawat organisation is equivalent to connectedness of key concepts and procedures which provide the glue that holds cognitive structures together. In addition, he affirmed that the adequacy of this structure determines the accessibility or availability of information later. Lawson and Chinnapan's (2000) study illuminated my research on "knowledge connectedness" from a concept-mapping perspective.

As to the socio-cultural theory or emergent perspective, Prawat termed it *disposition.* He distinguishes two types of disposition: performance and mastery dispositions. He suggested the need to develop a mastery orientation to learning in students rather than a performance orientation if the purpose "is to increase competence, to become more knowledgeable about or skillful at something" (p. 33). To attain this goal Prawat asserted, "They (students) must be strategically equipped to learn on their own" (p. 33). Meanwhile, the latter orientation, the intent of which is to get the job done as quickly and painlessly as possible, with learning serving as a means to an end and not an end in and of itself, he affirmed that it had its place. However, it was important for students to be able to access either one when appropriate. Moreover, in the dispositional category, he explained the relationship between dispositions and strategies (or cognitive skills). Prawat distinguishes two types of strategies: specific and general strategies. Specific strategies, when accessed, lead to a certain result, although they do not readily transfer to new, potentially relevant situations as general strategies do. However, general strategies are more difficult to access. He recommends, "Students should be aware of how factors such as the nature of the material to be learned or the kind of outcome they wish to achieve can influence the sort of strategy they select" (ibid. p. 33).

1.4 Significance of the Study

Research on students' thinking in geometry has mostly used the van Hiele's framework (Dindyal, 2003). Dindyal adds saying that this framework targets students' thinking in a purely Euclidean context and ignores the substantial interaction of geometry with algebra. However, my study investigated how first-year university students bring their knowledge and thinking of algebra in understanding and working with geometry. Besides this study dealt

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with geometry through use of coordinate geometry, transformations, and vectors, which involve algebra. Hence, this study appears to be significant as it used another framework in geometry thinking adapted from the work of Charbonneau (1996), Duval (1998), and Stillwell (1994, 1998). Certainly, this framework crosses somehow with van Hiele's (Chapter 3), meanwhile it has proper features. Hence, it is expected that the outcomes of this study will provide a basis for a more comprehensive framework for studying students' thinking in geometry and will also aid in the planning and implementation of instruction in geometry at the high school level and especially at the Universidade Pedagogica in Maputo – Mozambique.

1.5 Organisation of the Thesis

This chapter provides an overview of the research problem, the methodology, and the theoretical framework of the study. A rationale for the study was also given so as to state its significance. Chapter 2 provides the context where the study was carried out at Universidade Pedagogica. The context comprises the aim of this institution in relation to the syllabi of the courses of Euclidean Geometry and Analytic Geometry. Moreover it provides the philosophical aspects for designing these syllabi within curriculum reform which has taken place in this institution. Chapter 3 provides an overview of the related literature and theoretical framework for this study. Chapter 4 describes the methodology and Chapter 5 the data analysis and interpretation, finally Chapter 6 gives the conclusions of the study and a summary of the thesis, a discussion of the main findings, conclusions based on the findings, and recommendations for future research.

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