Number strategies of Grade 2 learners: Learning from performance on the Learning Framework in Number Test and the Grade 1 Annual National Assessments

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DECLARATION:

I declare that this Research Project is my own, unaided work. It is being submitted for the degree of Master of Science at the University of the Witwatersrand, Johannesburg. It was not been submitted for any degree or examination at any other University.

(Signature of candidate)

________________ day of____________________ 20___________ in _________________

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I dedicate this writing to my family: My late father, Georg Stegmann, my mother, Ria Stegmann,

husband Christo Weitz, son Tian Weitz, and two daughters

Marisa Weitz and Carina Weitz.

Table of Contents

List of Figures

List of Tables

List of Appendixes

Abstract

Several commentators describe the low performance of South African students in mathematics as ‘a crisis’. In the Foundation Phase specifically, there is evidence of a lack of shift from concrete counting-based strategies to more abstract calculation-based strategies (Ensor et al., 2009; Schollar, 2009). Concrete counting-based strategies refer to actions where the learner cannot find the answer to a mathematical problem without using concrete objects. In contrast, abstract calculation-based strategies involve strategies where the child does not need concrete objects to find the answer, but can instead use mental calculations in which numbers have been transformed into abstract objects upon which operations can then be carried out. Ensor et al argue that the poor mathematical results in South Africa are the result of inefficient moves made by learners from counting to calculating. In their study, many students failed to move their thinking sufficiently forward from concrete counting actions to abstract thinking.

The focus of this study is to investigate a sample of Grade 2 learners’ strategies on tasks drawn from the Learning Framework in Number (LFIN) test and responses on number related questions in the Annual National Assessment tests (ANA). I use the Learning Framework in Number to describe the stage of learners in their shift from concrete to a more abstract way of thinking about number. The theory of reification refers to the turning of processes into objects, and in this research, the origin of an abstract object in reification is explored. I also aim to understand the kinds of information I can get from children’s grasp of early number strategies, by looking at the responses of learners on the ANA and LFIN tests. My research question is: What do the two tests (ANA and LFIN) tell us about the strategies on early number used by a sample of Grade 2 learners in a township school in Gauteng? The two critical questions that follow from this are:

- How does learner performance on number problems compare across the two tests?
- What evidence in relation to concrete/abstract strategies is evident in the responses of learners in the two tests?

My findings showed that the learners in the school that I investigated still relied a great deal on concrete counting methods to answer questions. In spite of this, the mean ANA mark were much higher than the LFIN mean. The low number range of the ANA test, (1-34 for most of the number related questions), made it possible for the learners to use concrete counting (fingers or tallies) to answer the questions. The relatively low LFIN mark range indicated that children had difficulties in moving to more abstract ways of working with number. The implications of the reliance on concrete counting is potential difficulties when the learners move into higher grades where the number range is much higher, making the use of concrete methods time consuming and error prone.

Chapter 1 - Introduction

1.1 Defining the problem

The Bill of Rights, contained in **the Constitution, 1996** stipulates that, "... everyone has the right to a basic education, including adult basic education and further education, which the State, through reasonable measures, must progressively make available and accessible" (Government, 1996, p. 29). Implied in this statement is the intention to make mathematics, as a subject that forms a fundamental part of a basic education, accessible beyond an elite minority. This statement commits the government to provide a quality education to everyone in South Africa. Consequently, government has taken as imperative making mathematics education available and accessible to all in South Africa. In the last few years, the government has tried to accomplish this through several policy initiatives. The new National Curriculum and Assessment Policy Statement R-12 (CAPS) (DBE, 2011a) is one of these initiatives, and restates the commitment mentioned above in its principles. One of the principles upon which the CAPS documents is based is "social transformation; ensuring that the educational imbalances of the past are redressed, and that equal educational opportunities are provided for all sections of our populations "(DBE, 2011a, p. 3)

1.1.1 Poor performance

The focus in this research is on one of the most fundamental features of basic education. Mathematics needs to be accessible to everyone; however, the poor mathematics performance in South Africa means that access to the opportunities provided by high mathematics attainment at school remains limited. In the past, only a small minority of Grade 12 learners passed the Grade 12 Higher Grade Matriculation exam. Schollar (2009) states that only 1.5% of the 1995 grade 1 group managed to pass the HG Mathematics in 2006 when they were in grade 12. Prior to the 2006 reform of the Further Education and Training phase curriculum, many learners dropped Mathematics as a subject entirely in Grade 10. Mathematics is often described as a 'gateway' subject for further study, and is compulsory for a large number of those courses offered at university. Further, Mathematics is also described as a pragmatic subject – useful in real world situations, with mathematical literacy needed for everyday life contingent on mathematical thinking. This research underscores the importance of learner proficiency in mathematics since levels of performance at all stages of education indicate that this proficiency is not sufficiently attained in South Africa.

Several commentators describe the low performance of South African students in mathematics as ‘a crisis’ (Schollar, 2009; Fleisch, 2007; Van der Berg and Louw, 2006). Van der Berg and Louw (2006) highlight South Africa’s poor scores in mathematics tests compared to other countries in Africa. They also argue that tests scores obtained by South African students on international tests are much lower than those obtained by students of the same age in France, Hong Kong, Singapore and South Korea (Van der Berg & Louw, 2006). Ensor et al. (2009) and Fleisch (2007) note that international studies show South Africa’s performance for learners in numeracy is lower than that of eleven other African countries. Fleisch (2007) notes that a large nation-wide sample of Grade 4 learners scored an average of 30% in a numeracy test in 1999 and that in 2007, this average had only increased to 35 %. Ensor et al. (2009) found that the Western Cape Education Department (WCED) assessment documentation showed in 2006 that more than 60% of Grade 3 learners were performing below the expected level for numeracy, as defined by the National Curriculum statement. Some evidence shows that the performance in numeracy tests is actually worsening. Ensor et al. (2009) discover that the WCED’s documentation in 2006 showed that from 2002 and 2006, the average scores of Grade 3 learners dropped from 36, 6% to 32% in numeracy tests.

Jonathan Jansen, an education academic and public intellectual has stated[1] that “[in] the year 2000, about 1 035 192 pupils started school in Grade 1”, but that “only 496 090 showed up to write the finals in the Grade 12 class of 2011.” Of this number, Jansen writes, “only 41 586 of pupils who wrote mathematics passed, which in turn is 8,38 percent of all pupils who sat for the NSC examinations.” According to Jansen, a pupil that has written the 2011 NSC exams needs 40 percent in a home language, 40 percent in 2 other subjects, and 30 percent in 3 subjects in order to pass. This means that some of the 8,38% that passed mathematics, received only just more than 30%. He states that, “nothing demonstrates the systemic character of the schools crisis better than the mathematics results.” Thus, whilst evidence points to successes in South Africa with providing access to schooling at primary levels, the quality in terms of mathematical performance, even at primary level, remains problematic.

[1] Saturday Argus, 7 January, 2012.

The results of the international TIMSS (The Trends in International Mathematics and Science Study) tests, which were given to Grade 4 and 8 students in 34 countries in 1999, echo this bleak picture. In these tests, South Africa scored the lowest in the Grade 8 TIMSS tests. In 2003, the same test was conducted in 46 countries across Grades 4, 8 and 12 - and South Africa ranked the lowest again. The Netherlands scored the highest for the grade 12 tests, with a score of 560, where South Africa scored 356. Overall then, mathematical performance in South Africa, across all levels, is poor, and widespread problems with performance are in evidence in numeracy by the end of the Foundation Phase, as seen in learner performance on a range of standardised provincial, national, and regional tests.

1.1.2 Concrete/abstract strategies for number in Foundation Phase.

Concerning the Foundation Phase more specifically, there is evidence of a lack of shift from concrete counting-based strategies to more abstract calculation-based strategies (Ensor et al., 2009; Schollar, 2009). Concrete counting-based strategies refer to actions where the learner cannot find the answer to a mathematical problem without using concrete objects. In contrast, abstract calculation-based strategies involve strategies where the child does not need concrete objects to find the answer, but can instead use mental calculations in which numbers have been transformed into abstract objects upon which operations can then be carried out. Ensor et al. (ibid) argue that many South African learners stay dependant on concrete counting to solve problems, whilst noting that the curriculum requires children to have an abstract concept of numbers by the end of the Foundation Phase. This means that learners cannot solve problems without concrete counting, drawing tallies or using perceptual strategies such as feeling or seeing items.

Ensor et al. (2009) found that concrete methods to solve problems, such as tally counting, are dominant in the schools they investigated through classroom-based research. They also argue that the poor mathematical results in South Africa are the result of inefficient moves made by learners from counting to calculating. In their study, many students failed to move their thinking sufficiently forward from concrete counting actions to abstract thinking.

Ensor et al. (2009) state that at the end of children's primary school careers, learners need a strong understanding of counting, number and arithmetic. However, performance suggests that the majority of learners that leave primary school are not skilled in counting, number, and arithmetic, which they need for the secondary school. They argue that there are three aspects a child should master in order to achieve the requirements of Foundation Phase numeracy. These are: development in acquiring the number concept; the shift from concrete to abstract thinking and reasoning; and the move from counting to calculating, respectively (Ensor et al., 2009). Schollar (2009) supports the focus on number in his evidence showing that learners have specific problems with Learning Outcome One. According to the Revised National Curriculum Statement at that time, Learning Outcome One is about 'Numbers, Operations and relationships'. Learning Outcome One in Foundation Phase is achieved when "[a] learner is able to recognise, describe, and represent numbers in their relationships, and counts, estimates, calculates, and checks with confidence in solving problems" (DOE, 2004, p. 7). Schollar (2009) states from his evidence that many number problems are solved by Intermediate Phase learners through: "reducing the numbers involved to single unit marks and counting them one-by-one" (Schollar, 2009, p. 12). This is an example of a concrete counting based strategy.

This study concerns the extent to which various tests give us different information about learners' use of either more concrete or more abstract strategies for solving problems, and what these differences mean for the reporting of performance. This study is set in the context of increased standardised testing in primary schools, with the introduction of the Annual National Assessments (ANAs) in Language and Mathematics – national tests that are taken by all government school learners across the primary grades. One of the stated aims of these tests is to provide information about learner performance in key curriculum areas at the level of the class, grade, school, district, and province – information that can be used formatively to adapt teaching, by providing teachers with an further indication of where their own learners will need additional support throughout the course of the year (DBE, 2011).

The study is focused on two tests - the Annual National Assessment tests (ANA) and the research-based Learning Framework In Number tests (LFIN) (Wright, Martland, & Stafford, 2006). Whilst both of the tests will be described in detail later, my use of two tests is driven by wanting to understand and compare the kinds of information that tests can give teachers in relation to their learners' number competences, with a focus on a sample of Grade 2 learners in one school. In some of the ANA scripts that I analyzed within this study, I found evidence of the concrete counting that Schollar highlights as shown in the figures below:

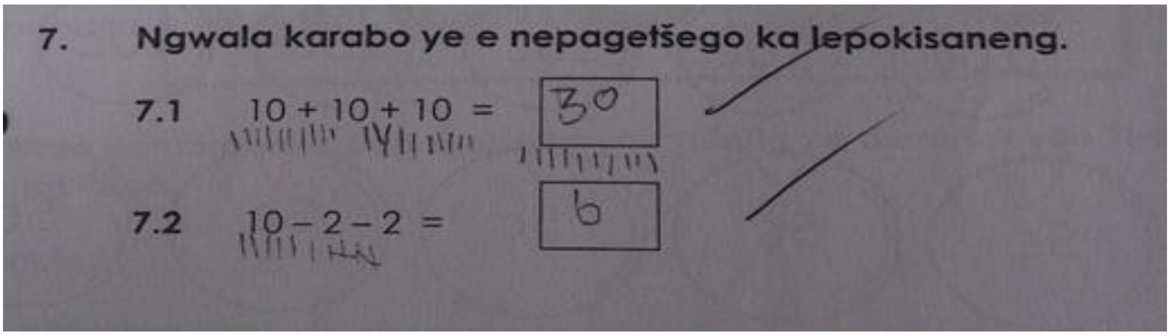

Figure1.1- An example of concrete counting in an ANA script

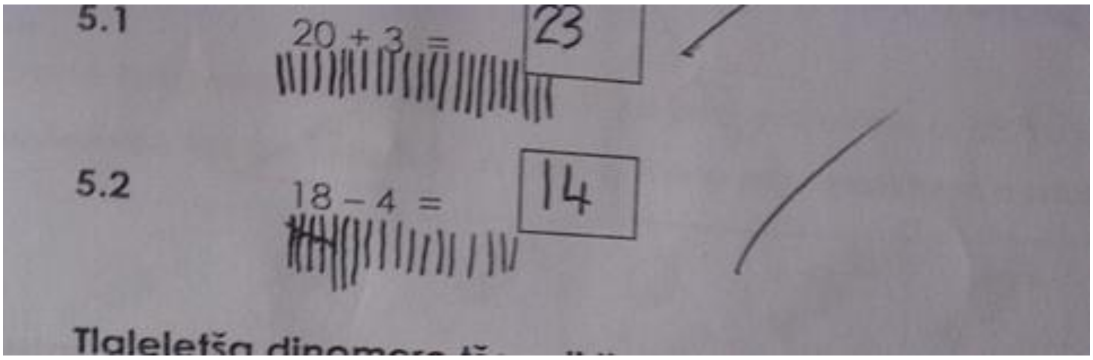

Figure 1.2 An example of concrete counting in an ANA script

In all of the problems above, the working shown indicates that these learners are using concrete conceptions of number. In 1.1b for example, the response suggests that the learner does not know that 20+3 is 23 as a recalled fact through mental calculation, or calculation based on ‘counting on’ rather than ‘counting all’; instead, there is a drawing of 20 tallies and 3 tallies to get the answer. In this working, neither 20 nor 3 exist as an abstract concept – both numbers only exist as outcomes of concrete unit counting processes.

Schollar’s (2009) findings, within the Primary Mathematics Research Project (RMRP) in 2000-2007 show that 79,5% of Grade Five learners and 60,3% of Grade Seven learners relied on unit counting to solve problems. He also found that 38,1% of Grade 5 learners and 11,5% of Grade seven learners relied exclusively on simple unit counting to solve problems. Schollar proposes that a key reason for the poor performance in South Africa is a failure to expand the ability of learners to move from unit counting to a higher level of working with number, which would demand abstract number-based calculations. He argues that learners are not given the opportunity to develop pathways to abstract calculations. Schollar also argues that the problem of being unable to do basic calculations and to see numeric relations, “...is caused by the application of ineffective learning practices in classrooms” (2009, p. 16).

Thus, problems of numeracy performance are linked to the nature of teaching, and what is taught in relation to numbers as abstract concepts.

1.2 Theory for understanding concrete/abstract strategies

Ensor et al. (2009) stated that learners are restricted from access to more abstract ways of working with number by classroom practices that privilege concrete models of representation. With regards to the Revised National Curriculum Statement (RNCS) they state that "...the shift from counting, to calculation which makes use of counting strategies, to calculation which does not rely on counting, takes place across Grades 1 to 3 and beyond" (2009, p. 11).

However, their empirical findings and broader literature show that in South Africa, performance in the Foundation Phase grades shows that for many learners, this shift is not achieved. They highlight the need for learners to shift their thinking towards a more abstract way of working with numbers. Sfard (2008) provides concepts relating to viewing mathematical learning in terms of shifts from discourses about concrete processes to abstract objects. She states that "the term abstracting is commonly used with reference to the activity of creating concepts that do not refer to tangible, concrete objects" (2008, p. 111). She also defines a concept as a"...symbol together with its uses" (2008, p. 111).

To have an abstract understanding of number, a child should be able to interpret the numerical symbol and its uses, without the need for concrete tallies or counters. Sfard (2008) argues that an *abstract object* originates in *reification*. The basic principle of reification is that the operational (process-orientated) conception emerges first and over time, then becomes reified into the mathematical object (structural conception) (A Sfard & Linchevski, 1994). The *theory of reification* refers to the turning of processes into objects. Sfard (1991) argues: "It seems therefore, that the structural approach should be regarded as a more advanced stage of concept development. In other words, we have good reason to expect that in the process of concept formation, operational conceptions would precede the structural" (p. 11).

Sfard (1992) also draws attention to the fact that process and object are inseparable; they are different facets of the same thing. If we look at number sense in the way Sfard's (1992) reification theory suggests, we can say that concrete counting is a way of understanding numbers as a process. When a learner understands number in a more abstract way, (i.e. the number exists without needing to be being associated with concrete counting actions), the learner has a structural understanding of numbers and reification has taken place. Sfard (2008) argues: "Such an act of reification - of discursively turning processes into object - is the beginning of *objectification*, which, if completed, will leave us convinced about the mind-independent, 'objective' existence of the object-like referent" (2008, p. 44).

Sfard (2008) states that objectification is the process which involves "... two tightly related, but not inseparable discursive moves: *reification*, which consist in substituting talk about actions with talk about objects, and *alienation*, which consists in presenting phenomena in a impersonal way, as if they were occurring of themselves, without the participation of human beings" (2008, p. 44). The author argues that objectification builds efficiency in mathematical communication. Because a learner/teacher can refer to mathematical objects and does not need to describe them in processes terms. A learner needs to objectify numbers to be able to participate effectively and efficiently in the communication in class. Ensor et al. showed in their study that most of their learners still used concrete methods to find answers to mathematical problems in Grade 3. When children use concrete methods to find the answer, this shows that they have not yet objectified numbers, and remain reliant instead on counting processes.

1.3 Implications of lack of shift to abstract strategies

The consequence of this lack of expansion in being able to deal with number in abstract ways when solving problems, is evident in the proportion of learners who are unable to access and meet the demands of the Intermediate Phase curriculum. Ensor et al. further state that, "[the] facts suggests that the majority of learners that leave the primary school are not competent in counting, number and arithmetic which they need for the secondary school" (Ensor et al., 2009). The two examples illustrated in Figure 1.1a and b earlier in this chapter provide indications of similar concrete counting, rather than calculations to solve problems in the context of my study. This situation suggests the need for extensive remediation - an issue dealt with in the next section.

Osana and Rayner (2010) note that research emphasizes that there is a strong relationship between early mathematical experiences and performance in mathematics later in a learner's life (Osana & Rayner, 2010). Wright et al. (2006) agree and argue that there is evidence in literature that children who are performing poorly in the lower grades will perform poorly in the higher grades. They also state that the gap between the low achievers and the high achievers grows from the lower grades to the higher grades. Wright et al. (2006) note that if gaps in low attainers' problem-solving strategies with numbers are not addressed in early years, the gap widens throughout their schooling. They quote the Cockcroft report (1982) that was produced in the UK, which states that a three-year difference in early years' numeracy can develop into a seven-year difference in mathematical attainment after ten years of school. The authors argue that low-attaining learners develop negative attitudes towards mathematics that are then difficult to change and that difficulties in number sense can affect performance in other aspects of mathematics (Wright et al., 2006).

This evidence points to the need to identify and address problems with numeracy early in the child's schooling. Leaving these issues unaddressed expands the problems encountered across mathematics in later phases. In the next section and in this study overall, I focus on responses to this need to identify and address problems with early numeracy at two levels. Firstly, I highlight the governmental policy responses to the problem with changes made in the curriculum, followed by the introduction of national standardised tests in the form of the ANA-test which is one the tests that I use to analyse learner performance on in this study. The second level that I want to highlight is a diagnostic test that was used in the ten primary schools participating in the Wits Maths Connect Primary Project, a 5 year research and development project. This individual, oral interview-based test was designed to identify and understand children's early number understandings and strategies. The items on this diagnostic test are drawn from the work of several researchers in mathematics education literature, and is based on the Learning Framework in Number (LFIN-test) (Wright et al., 2006) – a framework that identifies a network of aspects that feed into understanding and assessing a learner's level and stage of number sense. In this study, the focus is on one school within the WMC-P project, where 6 learners (selected from across the attainment range based on class teacher reports) in each of the six Grade 2 classes in 2011, took both the LFIN and ANA tests. These children's early number strategies, as indicated by their responses on the ANAs and LFIN, are investigated and compared. Since these two tests form the key instruments for this study, I provide brief details later in this chapter on the broader context of both of these tests, and the tests themselves.

1.4 SA Government responses to the problem – curriculum and assessment reform

In this section, I outline Government's curriculum and assessment responses in the post-apartheid years to both broadening access and trying to improve performance. The ANAs, which are a national assessment-level response to ongoing poor performance, form a part of this trajectory.

1.4.1 Curriculum responses

The South African government's own evaluations show that in ten years of democracy, there has been little development in educational outcomes, in spite of major policy changes (Carnoy et al., 2008). In 1997, the Department of Basic Education presented the "National Curriculum Statement" (NCS). The NCS (also called *Curriculum 2005*) aimed to develop the full potential of all learners as citizens of a democratic South Africa. It sought to create a learner who is "...confident and independent; literate, *numerate*, multiskilled; and compassionate, with a respect for the environment and the ability to participate in society as a critical and active citizen" (DOE, 1997).

Curriculum 2005 stressed integration across Learning Outcomes and across subjects. Critiques of Curriculum 2005 focused on the lack of explicit focus on content and progression (Chisholm, 2000); and as a consequence, in 2003 the Department for Basic Education introduced the Revised National Curriculum Statement (RNCS) (DOE, 2002). Following an ongoing lack of improvement in performance, the government published the Foundations Framework for Learning (FFL) in 2008. In the Foundations for Learning document, there was a significantly increased prescription of content coverage and sequencing, through the presentation of curriculum 'milestones' for each school term (DOE, 2008, p. iii). These Milestones were described as having interpreted curriculum content into units, on the basis of wanting to assist the teacher to develop the required assessment tasks per term. Rubrics for the last assessment task per term were also provided.

The introduction of this document followed evidence that teachers continued to feel the content was under-described, and were interpreting Assessment Standards at a lower level than the intended curriculum (WCED, 2003). The ANAs came in alongside the termly curriculum milestones and standardised termly assessments within the FFL, and are discussed in more detail below. The ANAs remained in place following the Government's introduction of the CAPS curriculum (DBE, 2011a) which was implemented in Foundation Phase in 2012. In this document the prescription that was built into the FFL curriculum was retained, with the presentation of a week by week outlining of content; with the argument that this was done 'to improve implementation' (DBE, 2011, p 2).

Thus, at the curriculum level, the Government aimed to address the problem of poor performance by exemplifying in much more detail, what should be taught, in what sequence and what pace. Over time, different standardised tests were also used to monitor performance in the context of these curriculum changes. These are discussed in the next section.

1.4.2 Assessment Level Responses

The Government of South Africa introduced The Systematic Evaluation Tests in Grades 3 in 2003 in order to determine the standard of implementation of the new curriculum (NCS). Early sittings of these tests showed that only half of the grade 3 learners passed the numeracy tests (Government-Gazette, 2010).

This Gazette of 2010 notes "...in South Africa there has been, for a long time, too much focus on the performance of the learners in the Grade 12 examinations" (2010, p. 10). The Gazette also stated that in recent years there had been more focus on how well young children in the lower grades understand numeracy and mathematics work. In order to want to improve the performance of the Grade 12 learners, an improvement in the performance in all grades, especially Grades 1 to 3 is required. As the research summarised above has noted, children with difficulties in Grade 1 to 3 often experience compounded difficulties in subsequent grades, because their foundation is not strong.

With this renewed focus on Foundation Phase, 27 goals were set out in the 2010 Gazette. The first goal amongst these is to "increase the number of learners in Grade 3 who by the end of the year have mastered the minimum numeracy competencies for Grade 3" (2010, p. 2). In this Gazette, the Government introduced the Annual National Assessment Tests (ANAs) as a standardised national test that would allow for the collection of performance data from schools, districts, and provinces on learner performance across the numeracy curriculum in each grade from 1-6. In this study, I look at Grade 2 learner performance on the Grade 1 ANA tests administered in Feb 2011, within my focus on number strategies as a central component of number sense. Whilst the general aim had been to administer this test to Grade 1-6 learners towards the end of each academic year, the modifications to school timetables during 2010 for the World Cup meant that the ANAs for 2010 were deferred to 2011, with grades taking the test for the previous year in February 2011

This test has importance on the ground in schools given that results are monitored at school, province, and national levels. My interest in this study was in examining the ANA (and LFIN) responses of a sample of learners in one school within the Wits Maths Connect-Primary project, with a view to understanding both their performance, as well as analysing the strategies that were used to solve number problems.

In the literature on mathematics education, there are other ways of identifying and assessing problems in early numeracy through diagnostic assessment. Wright et al. describe their LFIN-based assessment as a diagnostic assessment that "...aims to provide extensive and detailed information about the child's numerical knowledge" (2006, p. 30).

Diagnostic assessment involves a teacher or a mediator assessing a child's understanding of a concept, by looking at the strategies the child uses to find the answer. The teacher is not only interested in the answer of the child, but also in the methods that the child uses to get the answer. These tests often aim to provide more formative ways of addressing problems, in early numeracy. As stated already, the Learning Framework in Number (LFIN) test is a research-based early number test devised by Wright et al. (2006). The ANA and LFIN tests are introduced in more detail in the following section, given that they are the central instruments used in this study.

1.5 Study Focus

In this study, my focus is on understanding children's early number strategies from their ANA and LFIN test responses. I go on to compare findings from both tests. I aim to understand the kinds of information I can get from children's grasp of early number strategies, by looking at the responses of learners on the ANA and LFIN tests.

1.5.1 What is the ANA Test?

The ANA tests are the standardised national tests that are distributed to all government schools in the country. As stated already, the introduction of ANAs was part of the FFL policy, which was launched in 2009. There is an annual test for every grade from

Grades 1-6 and one for Grade 9. The teachers, according to the regulations of the Department (Government-Gazette, 2010), mark the tests internally using a rubric provided by the National Department, and this is followed by aggregate summaries that are sent out at district and provincial levels. There is also external moderation of a sample of scripts. Learners' achievement across district and provincial levels is compared through these summaries. The results are reported according to performance levels. Learner achievement is recorded on a four-point scale as follows:

Level 1, (0 to 34%) labelled as "Not achieved"
Level 2, (35% to 49%) labelled as "Partially achieved"
Level 3, (50% to 69%) labelled as "Achieved"
Level 4, (70% and above) labelled as "Outstanding"

In the beginning of 2010, the national department agreed to proceed with testing of all Grades 1 to 6 learners at the end of the 2010 year, as part of 'universal ANA'. The timing of the testing was consequently shifted to the beginning of 2011. The Department of Education states that "this meant that Grades 2 to 7 learners would be tested with respect to what they should have learnt by the end of the previous year. Thus, a learner in Grade 7 would be tested on what he or she should have learnt by the end of Grade 6. The tests were thus applicable to what had to be learnt in Grades 1 to 6" (DBE, 2011b, p. 11).

In Grades1 and 2, the ANA tests are orally administered, with the learners writing answers down on their scripts. In Grades 3-6, learners read and write answers to questions themselves.

The balance of items across the Learning Outcomes on the test reflects the allocations for each Learning Outcome in the curriculum documents. In the RNCS and FFL documents (these being the curricula in place at the time that the data used in this study was collected – 2011), LO1 is allocated approximately 70% (Based on my own analysis on the milestones per assessment tasks of the FFL). This was reflected in the large proportion of questions on LO 1 in the Grade 1 ANA, which involves number-based working.

1.5.2 What is the LEARNING FRAMEWORK IN NUMBER (LFIN) TEST?

Wright et al. (2006) devised the LFIN tests. These tests focus on early number topics and are broken down into a range of aspects that contribute to early number competence. The aspects the tests focus on are: Early Arithmetical Strategies; Forward and Backward number sequence; Numeral Identification, Base Ten strategies; Early Multiplication and Division and other Aspects of Early Arithmetical Learning like Combining and Partitioning, Patterns, Sequences and Base five Strategies. The LFIN is comprised of six sub-tests. The LFIN tests take the form of an orally conducted interview. The tests focus on understanding the strategies used by learners when solving number problems. Some of the questions are structured in such a way that if a learner answers the question correctly, the test leads on to a more advanced question, while a wrong answer leads on to a less advanced question. The aim of the test is to find out the early number sense level of each learner across the aspects, based on the strategies the learner uses to find the answer. These levels and the strategies underlying them are described in more detail in Chapters 2, 3 and 4. An explicit feature of the LFIN tests is that they are not only concerned with whether the answer is correct or not, but concerned with how the learner produces the answer. If we analyse the strategies used by the child across a range of tasks in the research-based framework, this allows us to diagnose the child to be at certain levels and stages according to the LFIN model (Wright et al., 2006).

In choosing to focus on what kinds of information can be obtained from children's number strategies across these two tests, my research questions were framed as follows.

1.5.3 Research question

What do the two tests (ANA and LFIN) tell us about the strategies on early number used by a sample of Grade 2 learners in a township school in Gauteng?

The two critical questions that follow from this are:

- How does learner performance on number problems compare across the two tests?
- What evidence in relation to concrete/abstract strategies is evident in the responses of learners in the two tests?

1.6 The structure of my study in terms of chapter sequencing

In Chapter 2, I present and analyse literature that is relevant for this report. The key bodies in my literature are related to the findings introduced in this chapter from Ensor et al. (2009) and Wright et al. (2006). Both of these pieces of writing relate to the transition from concrete to abstract ways of working with numbers. The work of Ensor et al. is central because they focus on the shift from concrete thinking to abstract understanding of numbers experienced by learners in South Africa. Ensor et al. draw attention to the concrete/abstract shift while Wright et al. describe this shift in fine nuance. Thus, the latter provides an analytical framework for this study. In Chapter 3, I elucidate the theoretical and analytical framework. First, I focus on the objectification and reification of number according to Sfard (2008). Sfard's notion of objectification describes what children need to accomplish in order to become capable of understanding and working with numbers. I then describe my analytical framework drawn from the LFIN framework of Wright et al. In Chapter 4 - dedicated to research design, I describe how I conducted this study, including explanations of the research design, samples, validity and reliability issues, and ethical concerns. In Chapter 5, I summarise my findings and then present an analysis of the data in Chapter 6. In Chapter 7, I discuss my conclusions and broader reflections on the process of conducting this research.

Chapter 2 - Literature Review

2.1 Introduction

My research is focused on the numeracy problem in South Africa. Specifically, in the previous chapter I noted the lack of a shift from concrete counting-based strategies to more abstract calculation-based strategies (Ensor et al., 2009; Schollar, 2009). To understand this problem in South Africa, I look at what international researchers have written about 'number sense' –the body of work that describes the early learning of number. The literature review is presented in four sections:

- Section one examines the problems in South Africa related to learning of numbers in the Foundation Phase (Ensor et al., 2009; Fleisch, 2007; Schollar, 2009).
- Section two focuses on what number sense involves as described in general in international literature. This part of my literature review allows me to highlight three key areas of number sense. These key areas are based on the framework that McIntosh et al. (1992) use to describe number sense. The first area is about knowledge and facility of numbers. Number sense firstly concerns number words and sequencing of these words. Secondly, it concerns knowledge and facility with operations on number. However, the one cannot go without the other. Therefore, the last area of number sense focuses on applying knowledge and facility with numbers and operations to solving mathematical problems.
- The third part is about progression and the shift from concrete methods of working with numbers to an abstract way of working with numbers (Ensor et al., 2009) . This part allows me to focus on how researchers describe the shift.
- Because I am looking at the problem that we have in South Africa, I will do a short analysis of what is said about concrete/abstract ways of working with numbers in the South African Curriculum in the fourth section, drawing attention to what is stipulated in the curriculum.

This research places emphasis on the fact that many children in South Africa have problems with number sense. Researchers' reports on this fact will be discussed in the next section.

2.2 South African evidence on early number learning

As pointed out in Chapter 1, underperformance in relation to the RNCS and the FFL curricula is widely evident by the end of Foundation Phase in South Africa. Certain authors have already investigated the detail of problems within teaching and learning in Foundation Phase. Ensor et al (2009), Schollar (2009) and Fleisch (2007) report on different aspects of the crisis in South Africa.

Fleisch (2007) points out from the results of the Systematic Evaluation in Grade 6 in December 2005, that 8 out of 10 learners do not 'achieve' in mathematics, with achieving taken to mean a score of 50% or better. In the cross-country Monitoring Learning Achievement (MLA) tests of Grade 4 in 1999, Fleisch finds that South Africa performed worse in numeracy tests than 11 other African countries (2007, p. 8). The study involved a nation-wide sampling of Grade 4 learners in literacy, numeracy, and life skills. South Africa took part together with other African countries. The mathematics study focused on four areas: numeracy; measurement; geometry; and everyday statistics. In the numeracy tests, learners were assessed on counting knowledge and abilities; writing the numbers in words; four operations; fractions and decimals; and word problems (Fleisch, 2008). Fleisch highlights, within his summary, a reliance on unit counting (using fingers or written tallies) without mathematical procedures relying on some abstraction of number.

The aspect of focus for Ensor et al. (2009) is the teaching of 'number' in the Foundation Phase. The main finding of the research is that they discern limited development across Grade 1 to Grade 3 in the growth from counting to an abstract way of understanding numbers. However, the learners in Ensor et al's study relied heavily on concrete objects for solving problems. The 'permissibility' of concrete counting methods across Foundation Phase is seen by these authors as a reason for the failure of many learners to think in a more abstract way (Ensor et al., 2009).

In the research that Schollar (2009) conducted in the Primary Mathematics Research project (PMRP), he found that data from national and international tests indicated the majority of learners in South Africa performing *well below* the accepted levels for their particular grades. Results from the National Systematic Grade 6 Cycle tests showed that 81% of learners have a score between 1% and 39%. The first phase findings of the PRMP were that many problems learners had were solved by unit counting - they reduced all numbers to single tallies and counted them one by one. He also found that multiplication and division problems were reduced to repeated addition and subtraction. As noted already, another finding is that 79,5% of Grade 5 and 60,3% of Grade 7 learners still rely on unit counting to solve problems and 38,1% of Grade 5 and 11,5% of Grade 7 relied entirely on unit counting. To illustrate, Schollar (2009, p. 13 and 34) presents the following examples:

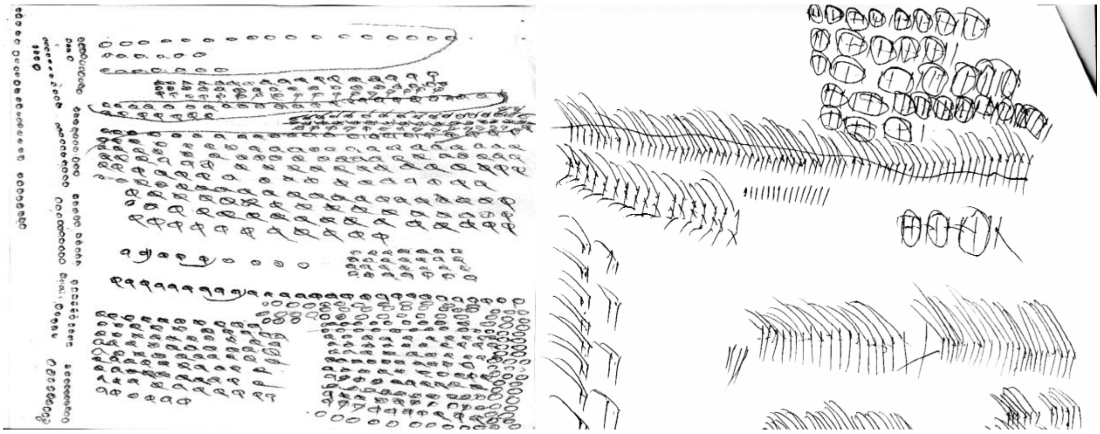

Fig 2.1-Example of concrete counting (Schollar, 2009)

According to these authors, it is clear that the overall mathematics performance of learners is poor and that for children, the shift from concrete counting to calculating is problematic in South Africa (Ensor et al., 2009; Schollar, 2009). There is therefore evidence over an extended period of time showing the prevalence of concrete actions across Foundation Phase and into the Intermediate Phase, with concrete strategies being used even with problems involving higher number ranges resulting in highly inefficient and error-prone methods. This suggests that looking for the ways in which tests can provide evidence of the use of more concrete/more abstract strategies used by learners in different test formats would be useful within this study in relation to the broader problem identified in South Africa. This rests on the view that teachers need to be able to see the shortcomings of concrete counting based strategies in order to support learners in moving towards more abstract ways of working with number.

I shift now to look at international literature foundation that describes what constitutes number sense. This is done in order to better understand the problems observed in the learning and progression of number sense in South Africa.

2.3 International Literature

2.3.1 How is number sense described in the literature?

From the literature it is clear that number sense is not easy to describe. McIntosh et al. (1992) highlight this fact and argue that number sense is "…nebulous and difficult to describe, although it is recognizable" (p. 8). It is not only difficult to describe, but also very difficult to define comprehensively. When researchers express their views about number sense, they focus on different issues and bring different characteristics together. Anghileri (2006, p. 1) states that children need "…an awareness of relationships that enable them to interpret new problems in terms of the results that they remember. Children who have this awareness and the ability to work flexibly to solve number problems are said to have a 'feel' for numbers or number sense."

The notion of connections and relationships between numbers is significant to Anghileri's (2006) view of number sense. She argues that learners with a good number sense exhibit an ability to use knowledge that they already have to work out the answer to subsequent problems. Children with an awareness of associations between problems that they have already seen and new problems are able to interpret new problems and solve them with greater ease. In other words, learners with 'number sense' have the ability to generalize patterns and processes and link new problems with the knowledge they already have. Cockcroft (1982) acknowledges the importance of familiarity with number within his broader notion of numeracy, where an 'at homeness' with number is described as one attribute of a numerate person: "...an 'at homeness' with numbers and the ability to make use of mathematical skills which enable an individual to cope with the practical mathematical demands of everyday life" (1982, p. 85).

McIntosh et al. (1992) bring the understanding of number, operations, strategies and the use of number symbols together:

Number sense refers to a person's general understanding of number and operations, along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations. It reflects an inclination and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information (1992, p. 2).

The definition above shows that number sense brings together concepts of number and operations on number, rather than separating concepts and procedures. The authors highlight the learner's preference and capability for using quantitative symbols and concepts within communication. McIntosh et al. set up a framework where three key areas of number sense play a role. The key areas are:

Knowledge of and facility with numbers – viz. the orderliness of number, multiple representations for numbers and a sense of relative and absolute magnitude of number.

Knowledge of and facility with operations – viz. the understanding of the effects of operations; and awareness of the rules of operations and the relationship between the operations

Applying knowledge and facility with numbers and operations to computational settings –viz. the understanding of the relationship between the problem in context and the necessary computation (1992, p. 4).

Several writers have written on one or more of these areas. My literature review will thus be presented within this framework.

2.3.2 Knowledge of and facility with numbers

This area includes a focus on symbolic and graphical representations of number and physical referents of number. According to Piaget (1964), these representations involve physical and social knowledge. 'Social knowledge' is based on conventions of language and culture. Knowledge of the number words is an example of social knowledge. 'Physical knowledge' is knowledge of an object's external existence. Physical knowledge can be attained by observation. In terms of number sense, physical knowledge develops from handling objects one by one and seeing the objects one by one while counting. This highlights the fact that concrete counting is an essential starting activity for children to understand numbers.

In relation to physical referents, Ensor et al. (2009) state that, the first aspect that children need to master, is counting. Gelman and Gallistel (1986) say that children have mastered counting when they have internalised the one-to-one principle. They argue that within this internalisation, there are two processes at work at the same time. The first process is that the child has to differentiate between 1) the items that have been counted; and 2) the ones that still need to be counted. The second process is presenting separate items one at a time. These two processes must happen simultaneously. Counting depends on connecting these processes with the number words. Torbeyns et al. (2002) highlight the importance of the use of number words - within the ability to count, viz. the ability to produce the forward and backward number words in the correct order, starting from any number other than one. Another important principle within the first aspect is understanding of the ordered number system. McIntosh et al. (1992) argue that there is a sense of orderliness in number, stating that, "number sense implies and understanding of how the Hindu Arabic number system is organised" (p. 5). They continue by arguing that "… an understanding of the number system helps the learner mentally organise, compare, and order numbers encountered in a mathematical environment" (1992, p. 5).

Haylock and Cockburn (2008), focusing on pre-school learners, bring out the difference between the cardinal and ordinal uses of numbers. The cardinal aspect is demonstrated when I can say there are 3 flowers, or children, or cars. If we count the flowers, the children, or the cars, there are 3. Anghileri (2006) defines cardinality as "[the] identification of a number word with quantity in a set is referred to as the cardinal (or quantity) aspect of number..." (2006, p. 20).

In contrast, there are situations where the use of numbers is not connected with cardinality e.g. the number 3 on a page. Here page 3 is located between page 2 and page 4, but may well not refer to a total of 3 pages in a book. This number principle is called the ordinality of numbers (Haylock & Cockburn, 2008). Anghileri states that, "the identification of a number with its position in a sequence is referred to as the ordinal (or location) aspect of number" (2006, p. 20).

McIntosh et al. (1992) state that counting beyond 20, children begin to understand the patterns in the number system and recognize pattern both orally and in written form and that once these counting patterns are identified, the child can use it as a powerful tool to extend the counting sequence (1992). Gelman and Gallistel (1986) emphasise the importance of the *stable order principle*. They argue that children need to know that counting proceeds in an organised, repeatable, and stable order (ibid).

Counting seen as both an organised system and as a system that is based on a one-to-one principle, come together within the counting of concrete objects. Haylock and Cockburn (2008) argued that the first thing that children need to do is to manipulate concrete materials because this helps them to understand that one object is represented by one number word in an organised sequence. Anghileri argues that children have to operate on some form of apparatus that represent the real objects, before they can operate with abstract numbers when they solve problems. It is important that for ‘number sense’ the number words as a ‘list’ need to be connected to an understanding of object ‘quantity’. Over time, the concrete objects are not needed as the connection to quantity comes to be internalised, and children can operate directly on numbers. Haylock and Cockburn (2008) argue that number as an abstraction is a major challenge in mathematics. When a child adds 3 and 4, the actual objects make no difference to the process. It can be 3+4 sweets, boys, or counters. “These are all represented by the same abstraction, 3+4=7” (2008, p. 42). This abstraction associates the word name with any set of that many objects. Gelman and Gallistel (1986) call this aspect the *abstraction principle*. This association represents a more abstract understanding of number.

The use of benchmarks is also an indication of working with number in an abstract way. Benchmarks are “...numerical values devoid of context, which have evolved from experience and/or instruction” (McIntosh et al., 1992, p. 6).

McIntosh et al. (1992) argue that benchmarks can be powers of 20 or multiples of ten and are mental referents for thinking about number. They argue that the difficulty of the benchmarks in decision-making is a good indication of number sense. For example, a child can say that he/she knows from experience that 5x20=100, and then say that 6x20 must be more than 100. In this case, 100 is the benchmark. A person that weighs 50kg can use this information to find the weight of other persons. Benchmarks bring together the relative size of numbers and the result of operations on numbers. Wright et al. (2010) also note the importance of 5 and 10 as benchmark numbers that support grouped counting. They state that “base-five strategies refers to those aspects of children’s early number knowledge that involve using the number five as a base” (2010, p. 13). Base-five strategies mean that “...combining and partitioning of numbers involving five is given special emphasis in children’s learning” (2010, p. 13).

From the above we note that number sense literature views the learning of number words and the order of the number words (counting) as important – which would mean that the ‘social knowledge’ (Piaget, 1964) ought to be a focus in this study. However, from the literature, it is also clear that children have to understand more about number than the number words and their sequence; a connection to quantity comes through early counting of concrete objects.

McIntosh et al. (1992) expand their investigation to more sophisticated concepts relating to ordering and representing number, with place; value; and equivalent forms through decomposing and recomposing and comparing as key amongst these. Being able to compare numbers is important within the selection of operations, and their second category is discussed below.

2.3.3 Knowledge of and facility of operations - Number operations into calculation procedures

McIntosh et al. (1992) argue that there are three things that a child has to understand when s/he does operations. Firstly, children need to understand the effect of operations. The authors include the understanding of operations on whole numbers and on fractions/decimal numbers. The second concern is that children need to understand the mathematical properties of operations, and the last concern is that children need to know the relationship between the operations. In my study, my focus is particularly on Grade 2 learners and on addition and subtraction, so I focus particularly on addition/subtraction operations in the writing below.

Wright et al. (2006) argue that children do operations on number, but use different strategies when they do so. If a child is asked to solve 4+5, she can 'count all' meaning from one to four and then from one to five and then from one to nine; alternatively she can count *on* from four to nine; or, she can answer it straight away because it is known to her as a fact. A triple count from one on her fingers, indicates a concrete understanding of number. If she counts-on from five to nine, the child has a more abstract understanding of the number five. If she can produce an answer straight away by doing the operation through mental calculations, she has an abstract understanding of the numbers five, four, and nine. To know what strategy the child uses, we have to analyse the child's way of doing the problem. We know now that there is a link between a child's number concept and his/her way of doing an operation. When calculating 4+5, if the child has an abstract concept of the number four, this creates possibilities for count on to 9 to get the answer; if the child does not have this abstract concept of the number four, she is likely to start from one (Wright et al., 2006). When a child starts to take shortcuts and does not count from one, she starts doing operations/procedures on *numbers*. These procedures start when the child counts on.

From the earliest stages, children should be encouraged to connect numbers in different ways (Anghileri, 2006). Such connections are not possible without an understanding of operations involved in more advanced counting (counting-on and breaking up numbers). Anghileri illustrates this point in relation to the number six. “The child has to see six, not only as a number of six objects, but also as the number after five and the number before seven, and six can be seen as a pattern of ‘2 and 2 and 2’ , or six is 4 and 2, or six is double 3” (2006, p. 3).

When children know the number six in such a connected way, they have a more abstract view of six that relies on relationships between numbers rather than between counted objects. If they can only count six objects one by one, their view of the number six is limited. Children develop, through experience, a sense of relative and absolute magnitude of number. McIntosh et al. argue that “The ability to recognise the relative value of a number or quantity in relation to another numbers and the ability to sense the relative size (or magnitude) of a given number or amount is a behaviour that develops with mathematical maturation and experience” (McIntosh et al., 1992, p. 6).

McIntosh et al. (1992) argue that much of school mathematics is dedicated to understanding operations. They argue that if a child understands an operation, he/she should know the effect of the operation, have an awareness of the mathematical properties of the operation, and have an awareness of the relationship between the operations. For example, initially understanding multiplication as repeated addition provides a concrete model to understanding multiplication. In this case, the child understands the effect of multiplication and addition on the numbers. If a child does the mental calculation of 36 x 4 as: 35 x 4 + 1 x 4 = 140 + 4, she uses the distributive property to recompose the numbers (McIntosh et al., 1992). This child understands the mathematical properties of the operations. If a child represents 36 with 35+1 in a calculation, he/she understands what 36 is, and has an abstract understanding of the number 36. McIntosh et al. argue that a child with a good number sense has an understanding of the relationship between operations. A child that knows that 8 x ? = 480 is the same as $\frac{480}{8}$ = ? understands the inverse operations of multiplication and division, and thus, the connections between the operations.

Operations that are more sophisticated like ‘grouping fives and tens’ and ‘grouping tens and ones’ are examples of more abstract thinking. When a child uses grouping to find the answer to 3 + 4, by partitioning 3 as 2 + 1 and then say 4 + 1 = 5 and 5 + 2 = 7, she understands what all the numbers represent and has an abstract view of these numbers (Wright et al., 2010). Counting in groups of ten, and on the decade, like 30, 40, 50,... and 44, 54, 64 ... to solve problems, are examples of more sophisticated operations. Mental strategies provide evidence of using operations that are more abstract. Beishuizen and Anghileri (1998) distinguish between two different mental ways of working. They discuss 'mental recall' and 'mental strategies'. 'Mental strategies' refer to informal mental processes of calculating. Learners that are strong in mental strategies have a well-developed base of memorised facts. If a child recognises that 2 + 2 + 2 + 2 is the same as 4 x 2, she a conceptual understanding of the relation between addition and multiplication, and the child can undertake operations flexibly. Decomposition and recomposition is when a child has to add 25 + 27 and she decomposes 27 as 25 + 2 and then adds 25 + 25 = 50, and says that 50 + 2 = 52 (McIntosh et al., 1992). Decomposition/recomposition is a verification that the child can work with number in an abstract way.

The relationship between number knowledge and operations will be discussed in the next section. Children have to move on and understand operations/procedures that they can make on numbers. According to Kilpatrick et al. (2001), algorithms and procedures are very important in mathematics. "Learning to use algorithms for computation with multidigit numbers is an important part of developing mathematical proficiency. Algorithms are procedures that can be executed in the same way to solve a variety of problems arising from different situations and involving different numbers" (2001, p. 116).

2.3.4 Applying knowledge and facility with numbers and operations to computational settings

The knowledge of procedures without understanding them fully proves ineffectual. McIntosh et al. argue that:

Solving real world problems which require reasoning with numbers and/or applying operations to numbers involves making a variety of decisions: deciding what type of answer is appropriate (exact or approximate), deciding which computational tool is efficient and/or accessible (calculator, mental computation), choosing a strategy, applying the strategy, reviewing the data and the result for reasonableness, and perhaps repeating the cycle utilising an alternative strategy (1992, p. 7).

This highlights the importance of understanding the problem as well as the relationship with the operation; and that the learner should have an awareness of a variety of potential strategies for performing the computation along with the ability to choose the most effective one. It also emphasises the learner's ability to review the result and to say if the answer make sense or not.

McIntosh et al. (1992) argue that the child should see a relation between the problem context and the operation. The problem context provides an indication of which computation to use and which numbers to use. Haylock and Cockburn (2008) argue that learning based on understanding is more enduring than are recipes for manipulating symbols. They argue that whilst mathematics involves the manipulation of symbols, this does not mean that focusing solely on learning rules or recipes will help with learners' understanding of mathematics. Their research shows that learning based on genuine understanding is more permanent. They state that learning with understanding is more useful and psychologically satisfying than learning that is based on the "...rehearsal of recipes and routines in low meaningfulness" (2008, p. 7).

Teachers have to see that children are involved in exploring relationships between mathematical symbols and other components of the child's experience of everyday life. Anghileri (2006) argues that the reform of mathematics has led to a shift from teaching procedures to facilitating the detection of patterns and relations, so that learners develop an insight into numbers. The realisation of patterns and counting in groups are the start of the operation of multiplication. When a child can see patterns and relations between numbers, she starts to develop an abstract understanding of number.

There are multiple strategies to solve mathematical problems. When a learner realises this and knows which strategy is the most efficient one, this constitutes good number sense (McIntosh et al., 1992). A child should have the ability to review the data and the results."When a solution is produced, people with number sense examine their answer in the light of the original problem (considering the numbers included as well as the questions asked) to determine if their answer 'make sense'"(McIntosh et al., 1992, p. 8).

Haylock and Cockburn (2008) draw attention to what is involved in young children's mathematical thinking. They argue that when children undertake mathematical thinking they are engaged in manipulating one or more of four components of mathematical knowledge. The four components of the framework are: actions on concrete materials; symbols;

language; and pictures. To explain children's understanding of number, they developed a framework to discuss number and number operations. The framework is based on the connections between these four components. An example of actions on concrete materials could be blocks, or any other material that children might use to construct mathematical concepts. They argue that learners can also manipulate symbols by selecting and arranging different symbols in the correct way. Thirdly, children manipulate language by reading instructions from cards or interpreting the teacher's instructions, and find the specific setting the numbers are in. Lastly, children manipulate pictures by drawing number strips and number lines (2008). Therefore, good number sense for these authors depends on building associations between these four components. From this framework, it is clear that concrete counting is not enough to understand number: children must be able to manipulate symbols; know how to communicate about number through words; and know how to interpret pictures (such as graphs) in mathematics. The words (or language) of the problem will help them to contextualise the problem. Moving between these four components with ease, and especially showing ability to work with symbolic representations, indicates a more abstract understanding of number than the mere handling of concrete materials.

It is important that a child knows mathematical procedures, however, having the procedures without the backup of understanding is regarded as limited. Thus, there are two concerns; the flexible use of procedures (which might indicate more abstract understanding) must be connected to and reified from the concrete actions in which they originate. Both are important. In this study, I will focus on assessing the number sense of a sample of learners based on the ANA and LFIN tests, and will explore what information the two tests can give me about the shift that learners have made from concrete thinking to abstract thinking about numbers; as a fundamental part of developing number sense.

If we say that children must move on from concrete thinking to abstract thinking, we are demanding development. In the next section, I summarise aspects of the available literature related to progression in number sense – as this is important in relation to more concrete/ more abstract strategies.

2.4 Progression in early number learning - The move from concrete to abstract

Some researchers see number sense in terms of progression, where the child needs to understand the principles and terminology of concrete counting and then move on to more abstract mental calculation. For these authors, a good understanding of number requires development in their knowledge and skills with numbers.

2.4.1 What are concrete counting strategies?

Given that so much of the work on early number rests on the work of Piaget, this work will be examined here. Piaget's *conservation of number* (Kamii & DeClark, 1985) refers to the ability to determine through reason that the quantity of a set can remain the same, even when the observed appearance is changed. However, some authors, like Donaldson (1978), have disputed Piaget's work on number conservation. The key number concepts of Piaget are social knowledge, physical knowledge, and logico-mathematical knowledge. Social and physical knowledge have been defined earlier in this chapter. Social knowledge is knowledge that is based on conventions of language and culture. Within the terrain of number sense, number words are an example of social knowledge. Moreover, physical knowledge is knowledge of an object's external existence. Physical knowledge can become known by observation. In terms of numbers, this can refer to one-by-one concrete counting of counters or objects. According to Piaget, "physical experience consist of acting upon objects and drawing some knowledge about the objects..." and "When one acts upon objects, the objects are indeed there..." (1964, p. 12). Thus, for a physical experience of number, the objects or counters must be available. According to Piaget, social knowledge is fundamental, but it is insufficient. A child who knows the number words (social knowledge) as a 'song' and does not develop a more advanced understanding of number as a concept, has not made the link with physical knowledge. I come to logico-mathematical knowledge later in this section

As mentioned before, Ensor et al. (2009) state that the first aspect that children need to master is counting. When a child thinks of number in a concrete way, she is able to count only visible and tactile objects (Wright et al., 2010). A child needs to understand the principles and terminology of concrete counting. In terms of Piaget's distinctions between different types of knowledge, the child needs the social knowledge of number words (1964), and to be able to link this knowledge to the physical world of perceived objects. When a child can count screened objects that she cannot see or touch, or can associate a number that is stated with a representation of this number on her fingers, some abstraction of number is evident. The degree of abstraction can vary, with some children needing to count out, say five, on their fingers in ones, with others able to show five fingers immediately. Concrete counting shifts can be more finely nuanced. Children count upwards from one or they count on from a number different from one, when they do mathematical problems that include addition and subtraction. They can also count down to a number or they can count down from a number (Wright et al., 2010). These fine nuances will be described later. In general, methods that are more concrete are associated with the child using his/her fingers, tallies, or counters in different ways.

2.4.2 What are abstract number strategies?

A vital challenge in mathematical learning is to abstract mathematical concepts (Haylock & Cockburn, 2008). When does a concept becomes abstract? Gray, Pitta and Tall (2000) say that the concept of a unit becomes abstract when a child does not need counters or tallies to solve problems. Through the eyes of Piaget, abstract thinking is *logico-mathematical knowledge*. Piaget describes logico-mathematical knowledge as "experience where the knowledge is not drawn from the objects, but it is drawn by the actions effected upon the objects" (Piaget, 1964, p. 12). An implication of this statement is that logico-mathematical knowledge:

"... is not the physical property of pebbles which the experience uncovered. It is the properties of the actions carried out on the pebbles, and this is quite another form of experience. It is the point of departure of mathematical deduction. The subsequent deduction will consist of interiorizing these actions and then combining them without needing the pebbles. The mathematician no longer needs his pebbles. He can combine his operations simply with symbols, the point of departure of his mathematical deduction is logical-mathematical experience, and this is not at all experience in the sense of the empiricists (1964, p. 12).

In this study, the physical knowledge and logico-mathematical knowledge of Piaget can therefore be linked broadly to concrete/abstract thinking. This point is related to progression, because logico-mathematical knowledge requires abstract thinking, where the child can undertake operations without objects. In order to be able to do that, the child has to develop/progress to and abstract understanding of number based on symbolic representations.

Ensor et al. argue that one aspect that a learner ought to accomplish is the ability to shift their understanding of number from a concrete understanding to an abstract understanding. They put forward that if learners cannot make this shift, they will not be able to "...understand that 10 is a concept, and will not be able to comprehend two digit numbers and place value" (2009, p. 11). They further link this shift with a move from counting to calculating when learners no longer rely on concrete counting strategies, but can operate directly on number as a symbolic object.

A range of important sub-skills is described in the literature during the shift to number as an abstract concept. The term 'subitize' refers to the ability to immediately ascribe a number to a collection of random dots correctly without counting them (Wright et al., 2006). If children can subitize a number, they do not deal with concrete counting, and so subitizing has been described as an important step to supporting the development of more abstract views of number. A child that can discern 5 dots on a domino *without counting them* has a more abstract understanding of number than one who has to count them one by one. Thus, tasks asking children to count objects in spatial arrangements can provide information on one way in which a child can show a more abstract view of number (Wright et al., 2006).

2.4.3 Steps in the concrete to abstract process

Carpenter et al. (1999) see the development of number sense as a process. They state that children need different ways to learn how to count and that children should be free to use their own ways to support simple mathematical problems. According to them, a child needs to progress by means of an evolution from counting concrete objects to doing mental calculation (Carpenter, Fennema, France, Levi, & Empson, 1999). Askew and Brown (2004) also see the development of number sense as a process, and note that for addition and subtraction that "there is a well-established sequence of development from counting into mental methods for addition and subtraction up to 20", whilst they add, "...children progress through a sequence of: count all, count on from the first number, count on from the larger number, use known facts and derive number facts" (2004, p. 6). The conclusion of this process is to develop mental methods, including recalled facts, for addition and subtraction up to 20. Askew and Brown (2004) argue that it is important that children develop mental methods to work in this low number range, in order to prepare for the kinds of methods that are essential for effective and efficient working with number in higher number ranges.

Torbeyns et al. (2002) highlight that researchers agree on ‘emerging numeracy’ of young children. McIntosh et al. state that the “...acquisition of number sense is a gradual, evolutionary process, beginning long before formal schooling” (1992, p. 3). What is clear from this is that concrete counting should develop into mental calculation. This literature points out that children can be at different stages in their abstract understanding of numbers. If we look at the strategies that children use, we can describe in more detail where a child is in this process. Carpenter et al. argue that in due course children will develop from “...direct modelling strategies...” to “...more sufficient counting strategies, which are generally more abstract...” (1999, p. 3). They use an example to explain their point. They ask the following question to two learners: “Eliz has 3 dollars to buy cookies. How many more does she need to earn to have 8 dollars?” (ibid). In their research (Carpenter et al., 1999), Jose solves the problem by counting on from three on his fingers to 4, 5, 6, 7 and 8. Then he counts the open fingers and states the answer as 5. Tanya solves the problem by counting out 3 counters and adding more until she has 8. She counted the ones added to find the answer as 5. The difference is that Jose has a more abstract view of number, because he knows he does not need to count 3 all over again. Jose understands the number 3 *more abstractly* than Tanya.

Carpenter et al. (1999) state that the process from concrete thinking to abstract understanding cannot simply be ‘given’ to a child. The direct modelling strategies and the more efficient counting strategies are strategies that children have to appropriate for themselves. Carpenter et al state that “...they do not have to be shown to count on or to be explicitly taught derived facts. In an environment that encourages children to use procedures that are meaningful to them, they will construct these strategies for themselves” (1999, p. 3).

For these authors, the shift is from direct modelling, to counting strategies, and then to flexible strategies. Of interest in relation to the trajectory of counting strategies noted by Askew and Brown (2004) earlier, Carpenter et al. start a step before ‘count all’, in that they begin in a situation, with counting viewed, to begin with, as ‘direct modelling’ of a situation.

Wright et al. (2006) describe this counting development from more concrete to more abstract strategies - for addition and subtraction specifically - in very fine nuances. Their model is described in the next section.

2.4.4 Progression through the eyes of Wright et al (2006)

Wright et al. (2006) state that the model that they developed has been adapted from research done by Steffe and colleagues (for example: Steffe, 1992; Steffe and Cobb, 1988; Steffe et al., 1983). They focus particularly on developing children's strategies for solving number problems. Like McIntosh et al. (1992), they also stress that strategies bring number concepts and operations together. Their central focus is counting in itself and in the context of addition and subtraction. Wright et al. (2006) see counting as a developmental process, which they break down into 6 stages: emergent; perceptual; figurative-counting; initial number sequence; intermediate number sequence; and facile number sequence. These stages are the most important part of the LFIN test according to Wright et al., and are referred to as the Stages of Early Arithmetical Learning (SEAL). The details of each stage below underscore that the focus is not only on whether the child can count, but also the strategies that the child uses to count. One example relates to whether a child could count onwards from a given number, or whether the child starts at one instead. Whilst the McIntosh et al. framework describes aspects of number sense, these stages point to the fact that progression within early number sense is central for Wright et al. (2006).

Table2.4.1 - The SEAL stages (Wright et al., 2006, p. 22)

	Name of the stage	**Explanation**
Stage 0	Emergent counting	Cannot count visible items
Stage 1	Perceptual counting	Cannot count perceived items, only seeing and feeling items
Stage 2	Figurative counting	Can count screened items, but counts from one - does not count on
Stage 3	Initial number sequence	Child uses counting on to solve addition/ count-down-from, but not counting down to solve subtraction
Stage 4	Intermediate number sequence	Child can count down to solve subtraction and missing subtrahend problems. Child can choose the more efficient of count down to or count-down from strategies.
Stage 5	Facile number sequence	Child uses procedures and not 'count by ones' method

These SEAL stages take the shift from concrete to abstract into account, and break it down to a more detailed set of development indicators. Perceptual counting, (stage 1 of

SEAL), refers to a child counting by seeing and feeling objects. This is a concrete way of dealing with number. If we look at figurative counting (stage 2), we see that a child can count unseen objects, but starts at one when she is asked to add 5 + 4 that is screened. Screened objects are objects that are not visible to the learner. In this stage, the child has the knowledge of what 4 and 5 are when associated to the same number of fingers, however, in order to add these two numbers, he/she has to start to count from 1 up to 9. When a child is at this stage, she has a more abstract understanding of number than a child who is still at stage 1. At Stage 3, a child uses counting-on strategies rather than count-all strategies. At this stage, the child can use 'count-down from' strategies to solve removed items tasks, but cannot use 'count-down to' strategies to solve missing subtrahend tasks. In the Intermediate stage (stage 4), the child can count down to, and from; can count on; and can choose the more efficient one to do particular addition and subtraction problems efficiently. If a child has to find the answer of a subtraction task like 10-3, it is better for her to look at it as a count-down-from task and remove 3; but 10 - 7 is better to take as a count down to task and the child must be able to count backwards to 7. If we set the following problem for 5 - ? = 3 (missing subtrahend) in a problem context, a child on stage 4 knows that she can count down from 5 and say 4, 3 ... and state the answer of 2. A child at stage 3 would not be able to answer this question, however, she would be able to count down from 5 to answer the question 5 - 3 (removed items task) and say 4, 3, 2 … also reaching the answer of 2. A child at stage 2 would not be able to do either the removed item task or the missing subtrahend task. The last stage is where a child can use a range of procedures flexibly to find the answer without counting by one. A child at this stage would be able to answer the problem: 5 - ? = 3 and 5 - 3 immediately, as recalled facts or strategies based on inverse operations, adding to 5, etcetera. A child at stage 5 is at the top of the SEAL 'ladder' concerning abstraction of number.

Wright et al. break down early number learning into an integrated framework which they refer to as the Learning Framework in Number (LFIN). The 11 aspects of the LFIN (all of which have been discussed within my literature review of number sense) are:

a) The SEAL stages
b) Base-ten Arithmetical Strategies
c) Forward Number Word Sequence and Number Word After
d) Backward Number Word Sequence and Number Word Before
e) Numeral Identification
f) Combining and Partitioning

g) Spatial Patterns and Subitizing
h) Temporal Sequences
i) Finger Patterns
j) Base Five Strategies

(Wright et al., 2006, p. 17).

The authors (Wright et al., 2006) provide levelled descriptions for some of these aspects as well. All the aspects feed into the SEAL stages. For example, a child who is able to subitize small numbers has effectively recognised the number shown as an abstract entity, which has been dissociated from a counting process. A more sophisticated strategy would lead a child to a higher stage of the SEAL stages. As noted already, higher SEAL stages mean that the child has a more abstract understanding of number.

Wright et al. (2006) view the SEAL stages as hierarchical, which would imply that in assessing a learner who achieved a higher SEAL level on some problems, but lower SEAL stages on others, the highest level consistently achieved would be counted. This is consistent with the point made earlier, that abstract manipulation of sums without a link to quantity and relationships, would indicate a lack of number sense. The framework presented by these authors provides a way of breaking down stages of the process from more concrete to more abstract strategies for working with number. I will use the LFIN framework to analyse strategies seen in learner responses across both tests, where appropriate, as an analytical tool. Both tests form the central instruments of my study and the number-based items on each, will be presented and discussed in the chapter dedicated to research design.

2.5 Analysing South Africa's Curriculum

Because the question of this research relate to the problem regarding the shift from concrete counting to abstract thinking in schools in South Africa, I found it useful to look for mention of concrete/abstract strategies in the curriculum documents in place at the time when my data was collected. I focused specifically on 'concrete counting' and 'abstract calculation' to see if either of the two (including the intermediate stages identified above) are visible in the curriculum.

I focus on the Revised National Curriculum Statement (RNCS) (DOE, 2004) and the Foundations for Learning (FFL) (DOE, 2008), because at the time that the data was collected, both of these curricula were in operation. The FFL was issued to strengthen the RNCS, and provided more description on content and sequencing. The ANAs are based on the RNCS/FFL, and I therefore provide a brief analysis of the ways in which the Foundation Phase curricula in these two documents provide advice to teachers on shifting towards more abstract working. I do this in order to understand (if there is advice supporting this), the extent to which curriculum may potentially offer support to teachers, or be a part of the problems with performance.

The curriculum balance on number is high, and numbers are used in all the other outcomes as well. Across both documents for Foundation Phase, there are five outcomes. The first outcome (Learning Outcome 1 - LO1) is: Numbers, Operations and Relations. Because my study is about number sense, I will only look at LO 1, described as "...the learner will be able to recognize, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems" (DOE, 2008, p. 70). In this description there is reference to concrete counting ('the learner must be able to … and to count') *as well as* abstract thinking (estimate, calculate, numbers and their relationships).

In the table below, I analyze assessment standards in the RNCS and 'milestone' statements in the FFL, for the extent to which they focus on encouraging more concrete/more abstract strategies. The FFL document is presented in 3 different sections. The first section is the Milestones per Term, then Milestones per Assessment Tasks, and the last section is Rubric for Assessment. This analysis will look across all of these sections. Given that the RNCS was associated with additional guide documents such as the Teacher's Guide, I make reference to this document in my commentary to ensure comparability. One of the FFL milestones in Grade 1 is: 'Is able to add and subtract 1 - 9 to any number up to 34.' This milestone can be accomplished by drawing tallies and finding the answer at the concrete extreme, or it can be done by making mental calculations at the abstract extreme. The relatively low number range of 1 - 34 makes both approaches feasible. This is the reason why some of the topics are in both columns. I looked at all the Grades in the Foundation Phase, to see how progression in number is described from Grade 1 to Grade 3.

Table 2.5.1 - Comparing concrete/abstract in the Curriculum

		Supporting more concrete thinking	Supporting some abstract thinking
RNCS-Grade 1		1. Counts to at least 34 everyday objects reliably 2. Using concrete apparatus 3. Counts forwards from 0-100	1. Count forwards and **backwards** in ones from **any** number between 0 and 100; and in tens from **any** multiples of 10 between 0 and 100 2. Order, describe and compare numbers 3. Performs **mental calculations** involving + and - up to 10 4. Building up and breaking down numbers
RNCS-Grade 2		1. Counts to at least 100 everyday objects reliably 2. Using concrete apparatus 3. Counts forwards from 0-200	1. Counts forwards and **backwards** in ones from **any** number between 0 and 200; tens from **any** multiple of 10 between 0 and 200; fives from **any** multiple of 5 between 0 and 200; twos from **any** multiple of 2 between 0 and 200. 2. Order, describe and compare numbers 3. Performs mental calculations involving +; -; x and ÷ up to 20 4. Building up and breaking down numbers
RNCS-Grade 3		1. Counts to at least 1000 everyday objects reliably 2. Using concrete apparatus 3. Counts forwards from 0-1000	1. Counts forwards and **backwards** in ones from **any** number between 0 and 1000; tens from **any** multiple of 10 between 0 and 1000; fives from **any** multiple of 5 between 0 and 1000; twos from **any** multiple of 2 between 0 and 1000 2. Order, describe and compare numbers 3. Performs **mental calculations** involving +; -; x and ÷ up to 50 4. Building up and breaking down numbers
FFL-Grade 1	**Milestones**	1. Orders numbers (1st -20th) 2. Counting out objects to 34 3. Counting to 100 on abacus and number line/number square 4. Counting in multiples of 2, 5, and 10 using concrete objects and number square 5. Is able to add and subtract 1-9 to any number up to 34 6. Solves problems, and explains solutions, using concrete objects and drawings with 7. Knows, reads and writes number names and symbols from 1-34	1. Knows, reads and writes number names and symbols from 1-34 and explores their relationship 2. Identifies numerosity (profile) of numbers 1 to 34 e.g. 20 is double 10, but 10 less than 30 3. Doubles and halves numbers 1-34 4. Completes repeated addition and subtraction of multiples of 2, 5, 10 5. Recognizes and designs own patterns using numbers to 34 6. Estimates up to 20 objects 7. Solves practical problems involving sharing and grouping with numbers to 34, including problems with remainders
FFL-Grade 1	**Milestones per Assessment**	1. One-to one correspondence 2. Counts out objects to 34 3. Counts to 100 on the number line 4. Solves problems using concrete objects 5. Counting in multiples of 2,5 ,10 using concrete objects	1.Identifies the numerosity of numbers(ex:16 is 1 more than 15 and 1 less than 17) 2. Estimate up to 10 objects

		6. Counts objects up to 100 7. Counts to 50 on the abacus 8. Count to 100 on the number line and number square	
FFL-Grade 1	**Rubric for Assessment**	1. Pack out objects next to the symbol 1-6 correctly. 2. Identify objects 1st to 6th in order 3. Count out objects to 20 4. Count to 20 on the number line 7. Solve problems using concrete objects and drawings with numbers up to 10 8. Count out objects to 20 9. Add two single digit numbers e.g. 2+6= 10. Subtract two single digit numbers e.g. 7-3=? 9. Solve problems involving sharing and grouping within the 1-10 number range, using concrete objects 10. Solve problems and explain solutions using concrete objects and drawings with numbers up to 34	1. Count in multiples of 2, 5, and 10 2. Identify the numerosity (profile) of numbers up to 34 e.g. 25 is 20+5 or 30-5 or 21+4 etc 3. Solve word problems involving any of the four operations (+, -, x, ÷) 4. Explain own thinking when solving word problems 5. Recognise numbers on a number chart to 50 6. Identify the numerosity (profile) of numbers 1 to 20 e.g. double 10 is 20 7. Complete repeated addition sums of 2 8. Recognise 'nearly doubles' e.g. 4+4=, 4+5=, 4+3= 9. Complete sums with two different operations correctly using numbers 1 to 10 10. Estimate up to 6 objects
FFL-Grade 2	**Milestones**	1. Counts out objects to 100 2. Solve problems, and explains solutions, using number charts and counters if needed with numbers up to 200 3. Is able to add and subtract two two-digit numbers e.g. 26+37=? 54-25=?(Can be done concretely with the 200 counters)	1. Counts forwards and backwards from any number in 1, 2, 5, 10 up to 20 2. Identifies the numerosity of numbers 1 to 100 3. Is able to add and subtract two two-digit numbers e.g. 26+37=? 54-25=? 4. Doubles and halves both odd and even numbers to 100 5. Calculates the multiplication of one digit numbers with one-digit numbers e.g. 6x5=? 9x5=? 6. Copies and extends number sequences to 500 7. Orders ½ and ¼ on a number line 8. Solves problems using grouping and sharing where the remainder is a fraction 9. Uses flow cards to decompose 3 -digit numbers as expanded notation e.g.241=200+40+1 10. Solve problems, and explains solutions, using number charts and counters if needed with numbers up to 200
FFL-Grade 2	**Milestones per Assessment**	Counts to 100 on abacus/number line/number square Counts out 100 objects	1. Counts forwards and backwards from any number in 1, 2, 5, and 10 up to 200 2. Identifies the numerosity of numbers(ex:16 is 1 more than 15 and 1 less than 17) from 1-100

			3. Decompose 3-digit numbers as expanded notation 4. Counts from any number in ones between 1-200 5. Expand 34 as 10+10+10+4 6. Calculates the multiplication of one-digit numbers 7. Identifies patterns in number work 8. Is able to subtract two digit numbers
FFL-Grade 2	**Rubric for Assessment**	1. Fill in a number chart correctly 2. Complete a given number line counting in 1's 3. Solve word problems involving all four operations (+, -, x, ÷) with numbers up to 100, using any or all of the following: concrete objects, drawings, flow cards, number squares, and number sentences. 4. Can do some of the +, -, x with 200 concrete objects	1. Use repeated addition leading to multiplication e.g. 5+5+5=3x5=15 2. Calculate multiplication of 2 single-digit numbers e.g. 6x5=? 9x2=? 3. Write two-digit numbers as expanded notation e.g. 47=40+7 or 47= 20+20+7 or 47=30+17 etc. 4. Add 2 two-digit numbers using expanded notation e.g. 27+35; 20+30; 50+7; 57+5; 62 or 27+35; 27+30;57+5=62 5. Subtract 2 two-digit numbers using expanded notation e.g. 72 – 36; 72-30; 42-6; 36 or 72-36; 72-10-10-10; 42-2; 40-4; 36 6. Build three-digit numbers using flow cards 7. Solve problems using sharing and grouping where the remainder is a fraction 8. Explain own thinking when solving problems
FFL-Grade 3	**Milestones**	1. Recognises and orders numerals and number names to 1000 2. Using counters up to 200, 500, 1000 if needed 3. Count out objects up to 100	1.Counts forwards and backwards in multiples 2, 3, 4, 5, 10, 20, 25, 50, 100, 1000 •2. Engages in using expanded notation of three-digit numbers in a variety of ways to 1000 e.g. 563=500+60+3 or 563=200+200+100+60+3 or563=300+100+50+50+40+23 or use flow cards 3. Calculates using addition and subtraction of two three-digit numbers e.g.300+259=? 4. Builds up multiplication tables of 2, 3, 4, 5, 9, 10, 11 up to 100 i.e. 5x20=100, 20x5=100 5. Calculates using division of two-digit by one-digit numbers e.g. 75÷5= 6. Is able to round off whole numbers to the nearest ten 7. Can extend number sequences to 1000 8. Solve problems, and explains solutions, using number charts if needed with
FFL-Grade 3	**Assessment Standards**	1 Count out objects up to 100	1. Counts forwards and backwards in multiples of 2, 3, 4, 5, 10, 20, 25, 50, 100, 1000 2. Develop expanded notation for numbers up to 1000

			3. Built up multiplication tables of 2, 3, 4, 5, 9, 10, 11, up to 100 can extend number sequences
FFL-Grade 3	**Rubric for Assessment**	1. None	1. Recognise and order numbers to 1000? 2. Decompose three-digit numbers to 999 in a variety of ways e.g. 498= 400+90+8 or 200+200+50+40+8 or 300+150+48 3. Correctly add and subtract a two-digit number and a three-digit number working over the ten/hundred? e.g. 597+28=? 326-48=? 4. Add two three-digit numbers correctly 5. Subtract two three-digit numbers correctly 6. Build multiplication tables of 2, 3, 4, 5, 9, 10, 11 up to 100? e.g. 20x3=60, 3x20=60 7. Calculate division of two-digit numbers by one-digit numbers e.g. 70÷5=? 8. Read and interpret data from a simple table? 9. Solve problems using grouping and sharing where the answer is a fraction or a remainder 10. Solve word problems using the four operations with numbers to 1000 (using number charts, number lines, etc. if needed

My analysis of the FFL suggests that there is a greater prominence of concrete counting and that processes that support concrete thinking are strongly visible in the FFL Grade 1 milestones. Across Grade 2 and 3 though, the FFL balance swings towards more emphasis on abstract thinking. In the Grade 1 RNCS there is clear mention of 'mental calculations', and a milestone that is visible in all four terms of Grade 1 is "knows, reads and writes number names and explore their relationship" (DOE, 2008, p. 3). According to the literature, to know and understand the relationship between numbers constitutes abstract thinking. In the FFL grade 3 milestones, the authors state that the children need to solve problems with counters with numbers up to 500. In the teacher's guide of the RNCS, it is stated that the assessment standards of LO 1 can be arranged into 3 groups viz. recognizing, classifying and representing numbers; application of numbers to problems (everyday problems are suggested); and calculation types – a categorization that seems to reflect McIntosh et al.'s three aspects of number sense. The last group requires that the child can perform calculations and performs mental calculations. This document also states that concrete counters "…should be available and learners must be allowed to use these learning aids as long *as they need them*" (DOE, 2003, p. 61).

From the above, it would appear that the curriculum statements communicate mixed messages. In Grade 1, the FFL milestones emphasize concrete understanding, but in the assessment rubric, they focus more on abstract thinking. At this point, it is worth noting that it is not possible to assess 'mental calculations' in the number range of 1-34 in a pen-and-paper test. This is due to the fact that it is not possible to tell *how* the child gets the answer, if we do not see how she does it. A diagnostic test like the LFIN test, with its oral interview format, provides access to this information. I use the LFIN test to help me to understand the concrete/abstract strategies of the child. I note this point because it means that the 'official' tests in the assessment landscape – the ANAs would therefore seem unable to assess mental calculations, and given the broad evidence of assessment driving teaching, there is a danger that mental calculation will not be emphasised in teaching. In the RNCS, the writers highlight key principles that guide the development of the NCS. One of the principles is outcome-based education. In addition, it is stated that: "consistent with an outcome-based approach..." this document does not "...specify a teaching method. How the outcomes and assessment standards will be taught is your responsibility as a teacher in the classroom" (DOE, 2002, p. 8).

In summary, my analysis suggests that there are mixed messages within and across the RNCS and FFL documents. However, my analysis of the FFL suggests that there is more prominence given to concrete counting. In Grade 1 particularly, there are some instances where concrete counting is emphasised, along with other elements where the need to allow learners to move to more abstract methods is left open. This entails that a concrete to abstract progression method is not specified; rather, only the outcome is specified. Statements like these tend to suggest that concrete counting with larger numbers is quite acceptable, and may contribute to the kinds of representational practices that Ensor et al. (2009) critique in their paper.

In the Teacher's Guide for the Development of Learning Programmes it is stated that:

> For the most part Numerical *concepts are abstract*. The use of *concrete objects* and apparatus in the early years — indeed in all years — can contribute to the development of understanding and must therefore be encouraged. The use of learning, teaching, and assessment contexts that are relevant to the lives of the learners can also contribute to understanding and should similarly be encouraged. However, it is important that the teacher also recognises that learners eventually need to *develop their understanding in the absence of concrete objects and contexts.* If learners are to develop rich numerical understanding then they need to be able to *visualise*

> *numerical concepts as objects* themselves. While the number 2 can be used to denote the number of bottle tops in a pile, in the statement 2 + 5, the number 2 is an object itself — i.e. it is independent of the situation that gave meaning to it (DOE, 2003, p. 63).

From this quote, it is clear that the aim is for learners to *shift* their thinking to more abstract ways of understanding number. This is despite the more mixed messages in both curriculum documents.

2.6 Conclusion

Across the literature reviewed here, a key similarity is that number sense begins in making sense of manipulating concrete objects. However, authors also state that children cannot stay at the simple level of manipulating objects. Another key point is that number sense is an intertwined competence, with many aspects. The child needs to see the connections between numbers and connections between number and operations in problem situations. The child has to explore and see patterns for herself, and the teaching of routine procedures without understanding is a futile exploit. Progression of number sense is important for this study, and the literature sees it as a development that needs to happen over time. The reason why I choose the LFIN Framework as my analytical framework is that it describes the progression from concrete to abstract strategies, across multiple aspects, in appropriately detailed stages. Wright et al. describe this shift or development systematically, drawing from an extensive literature base. Literature also brings to light that for a teacher to be able to help a child to progress in this development, the teacher needs to know where the child is on the road of progression. The LFIN test is a very useful tool to use to know where the child is in their numeracy evolution. In my study, I focus specifically on the road to abstraction, and what information the two tests used can give us on learner strategies.

In order to support this focus on the learner’s move from concrete to abstract strategies, my theoretical support is based on the work of Sfard’s ‘objectification’ of number and her reification theory. In the next chapter I will focus on the theoretical assumptions underlying the LFIN test and will then describe how reification and objectification theory can be applied to the number sense of learners.

Chapter 3 - Theoretical and Analytical Framework

3.1 Introduction

In the previous chapter, it was clear that there is a strong argument for focusing on the nature and extent of children's skills in working with less concrete/more abstract strategies across a range of number problems. I use Sfard's (2008) reification theory to explain the shift. However, I first draw attention to the broad constructivist framework of the authors of the LFIN test, based on the work of Piaget; and then consider Sfard's (2008) notion of Objectification as a way of understanding learners' dealing with mathematical ideas. I will depict how objectification manifests in number. Specifically, I will seek to bring to light how the theory of reification functions. I will describe how both reification and alienation function, so as to bring about objectification. In my study, I focused on reification specifically but I will describe objectification as a means to contextualise reification itself. The next section will focus on the relation of reification to understandings of mathematics. After linking reification with my study, I describe my analytical framework.

3.2 Theoretical framework of the LFIN test

The writers of the LFIN Test say that their view of "... knowledge and learning in early number is strongly constructivist" and they "...advocate a problem-based or inquiry-based approach to teaching" (Wright et al., 2010, p. 7). Within this constructivist orientation, whilst a teacher or an adult can give a child valuable information, the child can only receive the information if the child is in a state where she can understand the information. That means that the child "...must have a structure which enables him to assimilate this information" (Piaget, 1964, p. 8). Assimilation happens, according to Piaget, when a child perceives new objects in terms of existing structures. Another term that is important to Piaget is accommodation. 'Accommodation' describes a situation in which a child needs to change her existing structures to account for a new experience. For both assimilation and accommodation to occur, it is important to know the child's position in a progression between concrete and abstract thinking. If we know what structures the child currently displays, we know how to help her to a better understanding of number. The LFIN test, as a diagnostic test, is viewed as a tool for understanding this process.

Several authors, including Wright et al. (2006) working from the Piagetian framework, state that it is important that we are able to determine the logico-mathematical knowledge of the child, so that we can connect new knowledge with their extant knowledge.

If we know what knowledge and strategies the child uses and we understand the child's misunderstandings and misconceptions, we can intervene in ways that begin from 'where' the child is in order to take the child to more sophisticated sense of number. The administration of the LFIN test in oral interview format is premised on the view that learners' current understandings have to be 'diagnosed' in order to develop teaching sequences that link to current understandings. Whilst Piaget provides detail on the different types of knowledge that feed into understandings, Sfard (1992; 2008) provides detail on fundamental aspects of the process of moving from concrete to abstract understandings.

3.3 Objectification: Reification and Alienation

3.3.1 Reification

In Sfard's (1991) earlier work, she discusses "... the dual nature of mathematics: ... as different sides of the same coin" (1991, p. 1). The two sides are operational understanding and structural understanding of a notion. Sfard describes operational thinking in these terms: "...description speaks about processes, algorithms, and actions, rather than about objects. We shall say therefore, that it reflects an operational conception of a notion" (1991, p. 4). Structural thinking, in contrast, is described as: "Seeing a mathematical entity as an object means being capable of referring to it as if it was a real thing- a static structure, existing somewhere in space and time. It also means being able to recognize the idea 'at a glance' and to manipulate it as a whole, without going into details" (1991, p. 4).

Structural understanding is when one sees a mathematical concept as an abstract object (A Sfard, 1991, p. 1). If we look at numeracy and children's understanding of number, we can see operational understanding as the concrete counting of objects and structural understanding as the understanding of a number as an abstract concept that exists without the need for counting-based actions.

Sfard (1991, p. 3) argues that if we look carefully at any mathematical concept "... we shall find that it can be defined – thus conceived – both structurally and operationally." According to Sfard and Linchevski (1994): "The distinction between the two models of thinking, operational and structural, is delicate and not always easy to make" (1994, p. 89). Within number sense, the literature reveals that the aim is for learners to be supported to understand numerical objects structurally, and not just operationally.

The theory of reification describes the process of turning "...loose operational facts into meaningful manageable wholes" which are "object-like entities", where, Sfard says, structural understanding develops out of certain computational processes (1992, p. 60). This means that the learner needs to spend time repeatedly doing operations and computations on numbers in order to get to the point where reification takes place and the learner understands the concept of number structurally. To apply reification to number sense, a child has to count five objects repeatedly in different settings, and then realise that the last number word will always be five. Sfard (2008) explains reification in terms of the difference between someone carrying many loose objects in their hands, and then deciding to put all the objects in a bag. Even though reification is tricky to accomplish, when it happens, its benefits become immediately understandable (A Sfard & Linchevski, 1994). Therefore, reification takes place when a learner starts to think *structurally*.

Sfard notes also that operational and structural understandings are complementary. In problem solving, one can move from operational to structural and then back to operational thinking in order to solve a problem. The process of reification gives birth to abstract mathematical objects and this process is difficult (Sfard, 1992, Arcavi, 1995) and requires time and motivation on the side of the learner: "... insights, especially those at structural level, do not come easily" (Arcavi, 1995). Sfard (1992) argues that it is important for teachers to understand that it will sometimes take days, months, or even years before reification will take place, and can happen when it is least expected.

Sfard (2008) deals specifically with reification in the context of the learning of early number, and this theorisation is dealt with in the next section.

She argues that reification enhances the communicative efficiency of the discourse, saying that if someone speaks about a number, he/she imagines it as an object in its own right. In traditional definitions, the word 'number' can be used as a signifier and a signified. She distinguishes between the "number as such" and the words that refer to the numbers.

Amongst the first things, that children learn when their numerical education starts is how to count. If they count the same set repeatedly, they realise that they always stop at the same number-word. At this point, they realise that if they remember the last number-word, it is sufficient to say how many are in the set. It becomes “[a] shortcut for the action of counting” (2008, p. 47). Piaget states that the number-words that a child needs to learn are social knowledge (1964), and as noted already, social knowledge has to be taught and cannot be deduced. Dehaene cited in Sfard (2008, p. 73), states that our ability to use symbols for numbers, such as words or Arabic digits, distinguishes humans from animals. Sfard argues that numbers are “...mind-independent entities... whereas the words and symbols that people use in numerical discourses are mere ’avatars’ of the real thing” (2008, p. 47). The author argues that when a child says that there are five marbles in the box, the number word has a role of an additive action. Later, when the child says that 5 plus 3 is 8, these words have turned into nouns.

3.3.2 Alienation and Number Sense

Whilst I do not use ‘alienation’ in my study, I briefly describe it here in order to comprehensive. Sfard (2008) argues that once reified the supposed creation of the mind’s achievement “... undergoes the final objectification by being fully dissociated, or alienated, from the actor” giving the examples: “number is conserved as long as nothing is added to or taken away from a set” and “two plus three make five” (2008, p. 50). If we look at 2 + 3 make 5, we treat this mathematical object as ‘alienated’, and pre-existing or existing separately from any human activity, rather than as a product of humans. Participants of arithmetic discourses experience them as ‘happening to people’, not caused by them. She argues that the numbers “... appear to have a ‘life of their own.. [they]...communicate a message which is in the sovereign of the mind and subsist on their own” (2008, p. 50).

3.3.3 Objectification

According to Sfard (2008), objectification is a procedure in which a noun begins to be used for a process as if it signified an object. It consists of two sub processes: reification and alienation. It is important that the child objectify numbers, because according to Sfard (2008, p. 54), it increases the “communicative effectiveness” of the discourse. Objectification is a discursive process of double elimination, which frees people from both the extension in time, and from human agency. This double elimination consists of reification and alienation. We can communicate more economically by trading an extensive description of actions with a single sentence. This ‘short’ way to communicate also increases the flexibility and the applicability of our expressions. If a child has not yet reified the number five, it means that he/she cannot say that there are five marbles in the box. The child cannot use the numbers as adjectives yet. The only fact that the child knows, is that, “... if you count the marbles in this box, you end up with the word ‘five’.”(A Sfard, 2008, p. 53). If a child is at this level of understanding, the sentence 3 + 4 = 7 does not mean anything, because the number 3 and 4 and 7 do not function as nouns to them. Sfard argues that the equality 3 + 4 = 7 is then about the relation between counting to 3, and 4 and 7. Sfard (2008, p. 53) states that the lengthy description of 3 + 4 = 7 will be:

If I have a set so that whenever I count its elements, I stop at the word three,

and I have yet another set such that whenever I count its elements I stop at the word four,

and if I put these two sets together, then,

if I count the elements of the new set, I will always stop at seven

She argues that this example shows us how important it is for children to reify numbers. She states that the extent and intricacy of unreified objects speaks for itself. If the learners do not reify numbers, they will not be able to solve numbers problems with fluency and efficiency. Reification helps us to cope with new problems in terms of our experiences and “...gives us tools to plan for the future”. Sfard argues that “reifying sentences are not only concise, but also reassuring” (2008, p. 55). These reified tools are concise and easy to handle and thus supportive in the work of solving new problems.

3.4 Linking reification to the study

We can link the reification of Sfard with the SEAL stages of Wright et al. (2006) in the following ways. Stage 1 (when a child can count seen objects) refers to operational understanding, and the last Stage 5 (when a child uses procedures to find the answer) refers to structural understanding. We can link reification with the broad progression of number sense summarised by Askew & Brown (2004) as: count all; count on; and mentally calculate. Operational understanding refers to the ‘count all’ when the child will count from 1 if the question is to add 6 and 5. More structural understanding of 6 means that the child counts on her fingers from 6 and say: '7, 8, 9, 10, 11'. The child knows 6 in structural terms, and does not need to count from 1 to 6 (first part). The second number of the addition remains operational because the child counts from 6 one-by-one to 11 to get the answer. 'Mentally calculate' refers to mental deduction of what the answer is to the problem of 6 + 5. She might say that I know, as n known fact, that 6 + 4 is 10 and then I need one more, so the answer is 11 - without any counting.

Reification processes are also built into the progressive SEAL stages of Wright et al. Stage 1 (perceptual counting) refers to when a child can count only seen objects – reflecting an operational understanding. In Stage 2 (figurative counting) a child can count screened items, which means she can transfer a number of objects to her fingers or tallies, but then counts from one. This is still operational, but indicates a more abstract understanding than y, counting through seeing and feeling objects (Stage 1). In Stage 3, which is the 'initial number sequence' stage, the child uses counting on to solve addition, but does not use counting down to solve subtraction. In this stage, the child has some structural and some operational understanding of number. Stage 4 is the 'intermediate number sequence' stage and the child can count-down-to solve missing subtrahend problems. Here, the child can choose the more efficient of count-down to, or count down from strategies. This constitutes structural understanding, but counting by ones (operational understanding) is still present. The last Stage 5 (Facile number sequence) is when a child uses procedures to find the answer (structural understanding).

The SEAL stages therefore provide a framework in which degrees of operational and structural thinking can be analysed. As noted in the last chapter, Wright et al. (2006) note that several other aspects of the overall LFIN model feed into a child's SEAL stage. Given that the focus of this study is on Grade 2 learners, I used Wright et al.'s descriptions of five aspects of the LFIN framework in my analysis. I have discussed in the literature review chapter the ways in which more concrete and more abstract understandings relate to these aspects. These five aspects are now detailed.

3.5 Analytical Framework- The first 5 aspects of the LFIN Framework

The analytical framework that I use to help me to analyse my data is the LFIN Framework (Wright et al., 2006). Wolcott (1994) argues that to adopt any framework entails configuration of the descriptive account. He argues that, "by having the framework in mind during the fieldwork, the researcher, like a well prepared chef, is assured that when the various descriptive ingredients of the case are called for in an ensuing analysis, they will be at hand" (Wolcott, 1994, p. 20).

I used the first five aspects of the LFIN Framework that Wright et al. created to analyse the data. Within the videos, I articulated the stage or level of the child in question. Each child is coded according to the table below. All the responses on the questions of the LFIN test are analyzed thoroughly, and each child has a level or a stage for each of the five aspects based on Wright et al.'s descriptions.

Table 3.5.1 – LFIN Aspects

Learner	**Stage or Level**
MODEL	
1. Stage of Early Arithmetical Learning (SEAL STAGES)	Out of 5
2. Level of Forward Number Word Sequence (FNWS)	Out of 5
3. Level of Backward Number Word Sequence (BNWS)	Out of 5
4. Level of Numeral Identification	Out of 4
5. Level of tens and ones knowledge (Wright et al., 2006)	Out of 3

3.5.1 SEAL Stages: Early arithmetical strategies

The SEAL stage descriptions and link to the idea of reification have already been discussed extensively in this chapter. Below, I present brief summaries of how Wright et al. assign levels to the other four aspects. The 11 aspects of the LFIN should not be regarded as separate from each other. They are intertwined. The SEAL stages are the most important part of the 11 aspects of the LFIN (Wright et al., 2010). Wright et al. (2006) use the word 'stages' to describe the development of SEAL, and they use the word 'levels' to describe the development in 4 other aspects: FNWS, BNWS, Numeral Identification, Base Ten Arithmetical Strategies. All these aspects feed into the SEAL stages.

3.5.2 Forward Number Word Sequences (FNWS) and Number Word After

Level 0 – 5

0- Emergent FNWS (Child cannot produce the FNWS from ‘one’ to ‘ten’
1- Initial FNWS up to ten (Child can produce the FNWS from ‘one’ to ‘ten’, but cannot produce the number just after a given number)
2- Intermediate FNWS up to ten (Child can produce the FNWS from ‘one’ to ‘ten’ and can produce the number word just after a given number word, but drops back to ‘one’ by doing so)
3- Facile with FNWS up to ten (Child can produce the FNWS from ‘one’ to ‘ten’ and can produce the number word just after a given number word without dropping back to ‘one’)
4- Facile with FNWS up to thirty (Child can produce the FNWS from ‘one’ to ‘thirty’ and can produce the number word just after a given number word without dropping back to ‘one’.)
5- Facile with FNWS up to one hundred (Child can produce the FNWS from ‘one’ to ‘hundred’ and can produce the number word just after a given number word without dropping back to ‘one’) (Wright et al., 2010, p. 11)

3.5.3 Backward Number Word Sequences (BNWS) and Number Word Before

Level 0 – 5

0- Emergent BNWS (Child cannot produce the BNWS from ‘ten’ to ‘one’)
1- Initial BNWS up to ten (Child can produce the BNWS from ‘ten’ to ‘one’ but cannot produce the word just before a given number)
2- Intermediate BNWS up to ten (Child can produce the BNWS from ‘ten’ to ‘one’ and produce the word just before a given number word but drops back to ‘one’ by doing so)
3- Facile with BNWS up to ten (Child can produce the BNWS from ‘ten’ to ‘one’ and produce the word just before a given number word in the range of ‘ten’ to ‘one’ without dropping back to ‘one’)
4- Facile with BNWS up to thirty (Child can produce the BNWS from ‘thirty’ to ‘one’ and produce the word just before a given number word in the range of ‘thirty’ to ‘one’ without dropping back to ‘one’)
5- Facile with BNWS up to one hundred (Child can produce the BNWS from ‘hunderd’ to ‘one’ and produce the word just before a given number word in the

range of ‘hunderd’ to ‘one’ without dropping back to ‘one’) (Wright et al., 2010, p. 11).

3.5.4 Numeral Identification

When a child can identify numerals up to 100, he/she is on the highest level of the LFIN test.

Level 0 – 3

0- Emergent Numeral Identification (Cannot identify some or all numerals in the range of “1’ to ‘10’)
1- Numerals up to ‘10’(Can identify numerals in the range of ‘1’ to ‘10’)
2- Numerals up to ‘20’(Can identify numerals in the range of ‘1’ to ‘20’)
3- Numerals up to ‘100’(Can identify one- and two-digit numerals)(Wright et al., 2010, p. 12)

3.5.5 Levels: Base-Ten Arithmetical Strategies

When a child can see as a unit (see 10 as a structure or an object), s/he has an abstract view of 10.

Level 0 – 3

1- Initial concept of ten (Child does not see ten as a unit, but focus on the 10 items)
2- Intermediate concept of ten (Child see ten as a unit composed of ten ones)
3- Facile concept of ten (Child can solve addition and subtraction tasks involving tens and ones without concrete materials) (Wright et al., 2010, p. 9).

Wright et al argue that SEAL stage 3 is logically necessary for attainment of Level 1 within the base 10 aspect.

I now go to describe how I did my research in Chapter 4

Chapter 4 - Research Design

4.1 Introduction

In this chapter, I describe the ways in which data was gathered and analysed. The first part of the analysis is quantitative and the second part qualitative. Jackson (1995) defines qualitative research to be the use of theories and taxonomies to understand human actions in a way that reflects not only the researcher's view, but also the view of those whose behaviour is being researched. As stated earlier in this study I aim to ascertain what information can be gained on children's grasp of early number strategies, through looking at learners' responses on the ANA and LFIN tests. I will also describe the central data collection instruments used in this study – the ANA and LFIN tests. I will compare quantitatively as well as qualitatively, a sample of learner performance on the two tests. I am part of the Wits Maths Connect - Primary project (WMC-P). In the broader WMC-P project, working with 10 Gauteng primary schools identified as underperforming by the district, every Grade 2 class teacher in 2011 was asked to give names of 6 learners in each class, 2 low performers, 2 middle performers and 2 high performers. ANA scripts were collected for the selected learners and remarked. As part of the WMC-P team, I was involved in administrating the LFIN test in one of the 10 schools as part of the project's baseline data collection on learner performance. My sampling was therefore primarily a convenience sample, as I became interested in the responses recorded by learners in the township/informal settlement school I visited.

In my research, I wanted to ascertain whether some of the findings of an insufficient move from counting to calculating was prevalent in my selected school. I wanted to determine whether there were further findings in relation to the concrete to abstract shift required that could assist in understanding the number sense of Grade 2 learners in my sample in greater depth.

4.2 Data-gathering procedures/sample

I use a 'convenience sample' based on the data collection in which I was able to be involved. This school is in a township/informal settlement in Gauteng. Having met with the principals and the Grade 2 teachers of the schools before data collection, and having gained informed consent from principals, teachers, learners and parents, we started to collect the data. In this school, there were six Grade 2 classes and the teachers selected 6 learners in each class across the attainment range. As a result the tests were aimed at 36 learners. As stated earlier, the LFIN tests are oral interviews and all interviews were videotaped. The WMC Primary Project-team and I conducted the interviews on two of the LFIN tests (see Appendix A). I analysed the videos of the interviews of the 36 children and coded each one of them. All the Grade 2 learners that were present wrote the Grade 1 ANA tests in the beginning of 2011 and the scripts for learners in the LFIN sample were collected for the broader study. We ended up with matched data on both tests for 29 learners – who formed my overall sample for the initial quantitative analysis. For my study, I analysed both the LFIN test videos and the ANA test scripts using Wright et al.'s (2006) framework. I began by noting overall performance on the two tests quantitatively (based on levels and stages for the LFIN test, and overall mark based on my remarking according to the marking rubric for the ANAs). Levels and stages were allocated based on this analysis, according to the LFIN categories described. Then I compared the results of the two tests and looked for differences and similarities. I then formed a sub-sample based on particular groupings of learners that emerged from the quantitative analysis of the overall learner sample. This is discussed further in Chapter 5.

4.3 The research instruments

The results of the ANA test have implications for schools and teachers. The schools are judged on the results of these tests. In contrast, the LFIN tests used were 'low stakes' in that they did not have any consequences for the school, for learners or their teachers. In the Government Gazette where the ANA tests were introduced, the minister of Basic Education, Minister Angie Motshega, stated that she was interested in the input of Higher Education Institutions (Government-Gazette, 2010). In this study, I seek to understand some aspects of the potential and the limitations of the ANAs and LFIN tests, through examining the performance of a sample of learners. Particular focus is given to overlaps and contrasts in terms of the insights provided on learner understandings of number, through using the two different instruments. I focus also on the objectification of number within learners' test responses. The low number range of the ANA tests meant fewer openings to look for abstract strategies. I also need to acknowledge that internal marking carries some questions regarding accuracy which is acknowledged in the ANA report (DBE, 2011). As noted, all the ANA scripts gathered for this study, were re-marked. In the sections that follow, I detail and discuss the number items on both tests.

4.3.1 More about the LFIN Test

4.3.1.1 Reasons for use

The literature indicates that there should be a shift from a concrete understanding of number to a more abstract understanding of number over time. Associated strategies involve calculating procedures, rather than concrete counting. In the LFIN test, as noted in Chapter 3, we have a very fine description of the stages that lead into abstract thinking.

4.3.1.2 A closer look at LFIN

The LFIN Test consists of six assessment schedules that focus on all the aspects of number sense highlighted in the last chapter. However, given that the WMC-P team were working with Grade 2 learners early in the year, we administered only two of these tests, the two most basic tests, which focused on early counting, addition and subtraction tasks. The tests focus on understanding the strategies used by learners. Some of the questions are structured in such a way that if a learner answers the question correctly, the test leads to a more advanced question, with a wrong answer leading to a less advanced question. As I noted in the last chapter, higher SEAL stages are in particular associated with more abstract thinking. This means that a higher summary score on the LFIN is predicated on showing the use of more abstract strategies. This leads to an interest in what can be said similarly about high ANA scores, where marks are allocated for correct answers, rather than strategies used to derive them. As seen in Chapter 1, some correct answers were produced in these tests, with tallies on the answer sheet indicating that the child had used concrete counting. The LFIN questions are a fixed set of questions in a fixed order, and according to Breakwell (1995), this constitutes a structured interview. She also states that research interviews require a particularly systematic approach to data collection in order to maintain validity and reliability (Breakwell, 1995). Wright et al. (2006) advise that the interviewer does not change the questions or the order of the questions (Wright et al., 2006). In the LFIN tests there are a mix of situation based problems and more directly mathematical questions. There are also modelling action questions similar to those that Carpenter et al. (1999) describe. Because the LFIN test is videotaped, I could view the strategies that the learner used to find the answer.

By way of contrast, on the ANAs, I could usually only see that the child's answer was right or wrong as strategies were often not recorded. LFIN Test items

4.3.1.3 LFIN Test items

Table 4.3.1 – LFIN Test items

Question number and task in focus	**Specific sub-questions available**
Test 1.1	
Question1- Forward Number word Sequence (FNWS)	(1-32, 48-61, 76-84, and 93-112)
Question 2- Number Word After (NWA)	Say the number word after: Entry task- 14, 11, 19, 12, 23, 29, and 20. Less advanced task: 5, 9, 7, 3, 6. More advanced task: 59, 65, 32, 70, 99
Question 3- Numeral Identification	What numbers are these: Entry task: 10, 15, 47, 13, 21, 80, 12, 17, 99, 20, and 66. More advanced task: 100, 123, 206, 341, and 820. Less advanced task that is 8, 3, 5, 7, 9, 6, 2, 4, and 1
Question 4- Numeral recognition	In this task, the interviewer puts cards in a randomly order from 1-10 on the table and asks: which number is six, four, seven, nine, and eight? The child has to indicate the card with his finger.
Question 5- Backward Number Word Sequence (BNWS	Count backwards from: 10-1, 15-10, 23-16, 34-27, and 72-67.
Question 6-Number Word Before (NWB)	Say the number word before a certain number. The entry task: 24, 17, 20, 11, 13, 21, 14, and 30. The less advanced task: 7, 10 4, 8, and 3. The more advanced task: is 67, 50, 38, 100, 83, 41, and 99.
Question 7- Sequencing Numeral	The interviewer gives 10 cards, one by one, from 46-55 in a mixed order to the child. The child has to say the number word. Then the interviewer asks the child if he/she can put these cards in order, from the smallest to the biggest. Less advanced task: 1-10
Question 8 (a-e)- Additive tasks	Introductory task: 3+1 screened red (3) and yellow (1) counters. How many all together? Entry task: 5+4 and 9+6 (all counters screened) When the counters are screened, the child has to translate the counters to his/her fingers, and then add them, or do mental calculations. If a child can do this, the interviewer moved to the Supplementary additive task: 8+5, and 9+3 (counters screened and two colours).If a child can do this, the interviewer moe to Question 8f). If she cannot do the entry task, the interviewer move to a less advanced task, where only one of the group of counters are screened: 5+4, 7+3 and 9+4. If the child cannot answer the less advanced task correctly, then the interviewer move to the unscreened task were both groups of counters are unscreened: 5+2, 7+3 and 9+4. If a child cannot do this question, the interviewer place out 13 counters and asks how many are there. Then place out 18 counter.
Question 8 (f)- Missing Addend	Introductory task: The interviewer screens 4 red counters and ask the child to look away, and put two blue counters under the paper. The interviewer says, "Now there are 6 counters, how many more did I put under the paper?" Then '7+? =10 'and '12+?=15'
Question 9 (a)- Subtractive sentences task	Entry task: The interviewer presents a card with 16-12 on it and asks if the child knows what it means and if the child knows how to work out the answer. Supplementary task: 17-14 and asks the same question.
Question 9 (b)- Missing subtrahend task	Introductory task: The interviewer shows the child five counters, and removes two screened counters. Question: "There are 3 now, how many did I take away?" Entry task: 10-? =6; 12-? =9; More advanced task: 15-? =11

Question 9 (c) - Removed items.	Introductory task: The interviewer shows her 3 counters and takes away 1 counter. The total that is left is screened. The child has to tell how many are left. Entry task: 6-2, 9-4, 15-3. Advanced task: 27-4
Test 2.1	
Question 1- Subsitizing and Spatial Patterns	The interviewer shows cards with dots on it to the child briefly, for about half a second and asks the child how many dots they see. The child does not have time to count the dots. The cards have 4, 3, 2, 5, 6, 7, and 8 dots on it. Then the interviewer shows the child domino cards and asks the child to tell how many on each side and how many altogether. The interviewer has to show the cards briefly.(dominoes 5+3, 6+4, 4+4, 5+4)
Question 2- Finger Patterns 1-5	The first question is to show 3, 2, 5, 1 and 4 on their fingers. Then the child must show 3, 2, 5, and 4 fingers with two hands. The total of the fingers that the interviewer wants to see is 3, 2, 5, and 4 with two hands. The interviewer then asks the child to show 6 on their fingers and then show 6 in a different way.
Question 3- Finger Patterns 6-10	The next finger pattern question is to show 9 and then 10 on their fingers. The last finger pattern question is to show 8 on their fingers and then show 8 in a different way.
Question 4- Five Frame Pattern	Interviewer, shows the child a five-frame card with dots in it. The interviewer shows the card briefly. Not all the cells have a dots in it. The interviewer asks the child; how many dots do you see? The numbers that the interviewer shows to the child is 3, 2, 5, 1, and 4.
Question 5- Five-wise Patterns on a Ten Frame	The interviewer shows a ten-frame card with dots in and asks how many dots you can see. The cards that the interviewer shows are cards with 7, 10, 8, 6, and 9 dots in it.
Question 6- Pair-wise Patterns on a Ten Frame	The interviewer then shows the child Pair-wise Patterns on a ten-frame. The numbers that the interviewer shows to the child are 4, 2, 5, 1, 3, 7, 10, 8, 6, and 9.
Question 7- Combining to make Five	The interviewer says a number and the child has to give another number to make 5. The interesting question is to give a r number with 5 to make 5. Children of that age have difficulties with zero.
Question 8 - Combining to make ten.	Interviewer has to ask is to give three pairs of numbers that will give 10. Then the interviewer ask: I have 8 apples, how many do I need to get 10 apples? I have 4 apples, how many do I need to get 10? I have 7 apples, how many do I need to get 10?

In the LFIN test, the strategy the child uses demonstrates the level of development of her abstract understanding. The questions of the LFIN test aim to look across problem types and number range in order to ascertain the extent to which learners can apply more/ less abstract strategies.

4.3.1.4 How do the questions of the LFIN test relate to the LFIN Framework

A range of questions fed into each of the five aspects for which a summary stage/level is produced. Based on an analysis of the LFIN items, the following items were allocated to each aspect:

Table 4.3.2 – How does the LFIN items of test 1.1 relate to the LFIN Framework?

		Test 1.1											
		1.Count from/on	2. Number word after	3. Numeral Identification	4. Numeral recognition	5. Count down	6. Number word before	7. Sequencing numerals	8.(a-e) Additive task	8(f). Missing addend	9(a) Subtractive task	9(b) Missing subtrahend task	9(c) Removed items
1. SEAL Stages		x	x			x	x	x	x	x	x	x	x
2. Base ten arithmetical strategies													
3. FNWS		x	x				x	x	x	x	x	x	x
4. BNWS						x	x			x	x	x	x
5. Numeral Identification				x	x						x		

Most of the questions asked linked directly with the SEAL aspect of the LFIN Framework, as was the case with the ANA test. Two questions in test 1.1 did not link directly with the SEAL stages. They are questions 3 and 4 viz. numeral identification and numeral recognition. Nine questions helped the research team to understand a child's knowledge of the FNWS and six questions to understand BNWS. The oral interview format of the LFIN test allows counting tasks to be attempted without numerical identification. The written format of the ANA test, in contrast, results in a greater reliance on numerical identification.

Table 4.3.3 – How does the LFIN items of test 2.1 relate to the LFIN Framework?

	Test 2.1							
	1. Subsitizing and Spatial Patterns	2. Finger patterns 1-5	3. Finger patterns 6-10	4. Five frame Patterns	5. Five-wise pattern on a Ten Frame	6. Pair-wise pattern on a ten-frame	7. Combining to make five	8. Combining to make ten
1. SEAL Stages	x	x	x	x	x	x	x	x
2. Base ten arithmetical strategies					x	x		x
3. FNWS		x	x	x	x	x	x	x
4. BNWS				x	x	x		
5. Numeral Identification								

All the questions in test 2.1 linked with the SEAL stages. The difference between the two LFIN tests is that there are no questions in test 2.1 that can help to reveal the child's understanding of numeral identification. However, there are questions in test 2.1 that can help us to understand base ten strategies. In the next section, details are given of the Grade 1 ANA test that was written by the Grade 2 learners.

4.3.2 More about the Grade 1 ANA test of 2011

4.3.2.1 ANA test items

The ANA Test consists of 11 questions (Appendix B). Questions 4 and 8 concerned geometry; all the other questions contained a direct focus on number. I was looking at number sense and strategies of Grade 2 learners, and therefore omitted Questions 4 and 8. I now give details on the questions of the Grade 1 ANA test that the Grade 2 learners wrote in early 2011.

Table 4.3.4 – ANA test items

	Topic	Question	Commentary
1	Forward counting	Fill in the missing number in two tables: counting by one from 3 to 11 and counting in tens from 20-100.	In these questions, the child has to know which number is represented by the symbols and their sequence. The first part is straight forward, however to count in tens can indicate openings for more abstract conceptions of number than the first part.

2	Identify/produce number words	Write down the correct number words for 9 triangles. In the second part of the question, the child has to be able to draw 7 shapes to represent the word seven and the number symbol of 7.	In this question, the child can use concrete counting to answer this question; however, he/she has to know the symbols and the number words.
3	Ordinal nature of a number	A number line is given and shapes are drawn in the places of the number symbols. The child has to say what number is in the place of the scissor and what is in the place of the 6.	The child has to know the ordinal nature of a number.
5.1 and 5.2	Addition and subtraction	5.1 is about addition (20+3=?) and 5.2 is about subtraction (18-4=?).	In this question, the child has to know what the symbols stands for before they can get to the answer. A child can answer this question with a concrete understanding of number or with an abstract understanding of number. The marker of the answer sheet cannot tell what the child's understanding is, unless there are tallies as evidence of concrete counting on the paper. If tallies or any other written methods are absent, we cannot conclude on concrete/abstract understanding, because the child could use his/her fingers to get to the answer.
6.1 and 6.2	Doubling and halving	6.1-Double of 5, 6.2-Half of 20	This question can be answered by concrete or abstract understanding and we cannot tell what the child's understanding is by only looking at the final answer.
7.1 and 7.2	Addition and subtraction	In 7.1 they ask 10+10+10=? and 7.2 they ask 10-2-2=?	We cannot say if the child used concrete or abstract methods by looking at the answer. The child can use either concrete or abstract methods to get to the answer.
9.1 and 9.2	How many coins and which coins?	9.1- How many coins are the presented and 9.2-which 4 coins will you choose to add up to 25c The first part the child has to add all the coins to find the total of the money. The second part the child has to choose 4 coins that will add up to 25c.	This is an interesting question because it calls for abstract thinking. This question cannot be answered by concrete methods. The coins that the child can choose from are five 10c and five 5c. The child has to have an abstract understanding of what 25 is, and also how many 5c and 10c coins will add up to 25. The child has to have an abstract understanding of what 5 is and what 10 is and that two 5's makes 10. The important thing to remember is that he/she has to use 4 coins. I think there are many ways to find the answer on this question. One way of thinking can be trial and error. If he/she chooses two 10c coins, then he can only choose one 5c coin. That makes 3 coins, so this is not a possibility. The child has to restructure his/her thinking and argue that he/she is short of one coin. The way to have another coin is to break up one 10c coin into two 5c coins. Then the answer is three 5c coins and one 10c coin. There is a substantial abstract thinking in this question. I my analyses I will specifically focus on this question when I look at the ANA tests.
10	Division	You and two friends share 12 sweets. How many does each get?	
11	Concrete counting	The child has to count 7 apples and 5 bananas.	The child has to be able to write the number symbols down. These questions involve concrete counting and knowing the number symbols.

As noted, the ANA test is read out by the teacher with time give to the child to write his/her answers. The duration of the test is 60 minutes. It is important to note the relatively low number range allows most of the questions to be answered using concrete methods. It is only Question 9 that requires abstract understanding of number. In the analysis, I look at total performance score on number items based on the ANA test rubric and also look for evidence of concrete counting. Only two learners in my sample answered question 9 correctly, thereby indicating a more abstract understanding of number.

4.3.2.2 How do the ANA test and the LFIN Framework relate?

As with the LFIN tests, items on the ANAs that were relevant to the five selected aspects were identified – summary below:

Table 4.3.5– Comparing each Question of the ANA test with the LFIN Framework

Question in the ANA test	-	1.1	1.2	2a	2b	3.1	3.2	5.1	5.2	6.1	6.2	7.1	7.2	9.1	9.2	10	11.1 and 11.2
1. SEAL Stages		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
2. Base ten arithmetical strategies			x									x		x	x		
3. FNWS		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
4. BNWS									x				x				
5. Numeral Identification		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x

All the questions on the ANA test link with the SEAL stages in one way or another, corroborating the position that work across the curriculum rests to a large extent on a solid foundation in the understanding of number. However, forward counting is predominant (FNWS). Only three questions link with base ten arithmetical strategies. Two questions require the understanding of BNWS and as noted, the written format means that all the questions link with numeral identification.

4.3.3 Compatibility of test and research model

To understand number within these two tests it is necessary to know how the two tests differ. They vary in format and purpose. The format of the LFIN test is an oral interviewed-based diagnostic test, which focuses on strategies that learners use in answering number problem questions. Although the ANA test is orally administered, it is a written test that focused across all areas of the mathematics curriculum, not just number. The LFIN tests assess a range of aspects of early number knowledge. The ANA's stated purposes are at least partially summative, and reporting is focused on aggregate learner scores with the proportions meeting the different levels of performance. The diagnostic LFIN test is concerns on in-depth information on individual understanding, and helps in ascertaining the difficulties that a particular child experiences. In the LFIN test, the interviewer is not only concerned as to whether the answer is right or wrong, but also considers what strategies were used to find the answer. If we analyse the strategies, we can find which SEAL stage the child is at, and know the level is at which the child functions. The ANA tests require only the statement of answers and tell us little about how the answer was obtained. With all its advantages, the key disadvantage of the LFIN is that it is labour intensive and time consuming to administer. Most of the WMC-P LFIN interviews took approximately one and a half hours each. Thus, despite their pitfalls the ANAs allow breadth in terms of learner and curriculum coverage. A copy of the 2011 Grade 1 ANA test and LFIN tests 1.1 and 2.1 are included at the end of this report as Appendices A and B respectively.

4.3.4 How do the Questions of the ANA test and the LFIN test relate?

In the table below, I cross reference the overlaps between the different questions of the ANA test and the LFIN Framework. This allows me to compare learner responses on tasks related to the same aspects across both tests. Table 4.3.6 will give more information on the comparisons between the two tests.

Table 4.3.6– Comparing the LFIN items with the ANA items

					LFIN test 1.1														LFIN test 2.1						
	No of marks for the question	1. Count from…	2. Number word after	3. Numeral Identification	4. Numeral recognition	Count down from….	6. Number word before	7. sequencing numerals	8.(a-e) Additive task	8(f). Missing addend	9(a) Subtractive task	9(b) Missing subtra-hend task	9(c) Removed items		1(a,b) Subsitizing and Spatial Patterns	1(c) Domino-cards	2a) Finger patterns 1-5	Finger patterns on both hands	3. Finger patterns 6-10	4. Five frame Patterns	5. Five-wise pattern on a Ten Frame	6. Pair-wise pattern on a ten-frame	7. Combining to make five	8. Combining to make ten
Ana Test Question number	TEST 1.1	a)2-32 b)48-65 c)76-84 d)93-117	Up to 10, up to 30, up to 100	Up to 10, 10-99, more than 100	up to 10	10-1, 15-10, 23-16, 44-27, 72-67.	a)10-30, b)1-10, c)30-100.	a)46-55, b)1-10	5+4, 9+6, 8+5, 9+3	4+?=6, 7+?=10, 12+?=15	16-12, 17-14	5-?=3, 10-?=6, 12-?=9, 15-?=11	3-1, 6-2, 9-4, 15-3, 27-4	TEST 1.2	Flashed Regular/irregular	5+3, 6+4, 4+4, 5+4	Show me 3, 2, 5, 1, 4 on your fingers	Show me 3, 2, 5, 4 with two hands	Show me a number on your fingers, then different way. 6,9,10,8	3, 2, 5, 1, 4	7, 10, 8, 6, 9	4, 2, 1, 5, 3, 7, 10, 8, 6, 9	4, 2, 1, 3, 5	4+?=10, 7+?=10
1.1 (3,4,?,,6,7,?,9,?, 11)		x	x	x				x																
1.2 (20,?,40,50,60,? ,?,90,100)		x		x																				x
2.How many triangles.		x		x																				
3.1. Where is the scissor		x		x																				
3.2. indicate the 6		x			x																			
5.1. 20+3=		x		x					x							x								
5.2. 18-4=				x		x					x		x											
6.1. Double 7		x		x																				
6.2. Half 20				x		x																		
7.1. 10+10+10=		x		x					x							x								
7.2. 10-2-2=				x		x					x		x											
9.1. Money:5c+5c+1 0c+10c=		x		x					x							x								
9.2. 4 coins to make 25c				x					x															x
10. 3 share 12 sweets				x																				
11.1. How many apples?		x		x																				
11.2. How many bananas?		x		x																				

What is clear from the summary in table 4.3.6 is that the ANA test emphasizes certain aspects over others. This is not surprising, given that the LFIN test is based on the framework in use, whilst the ANA is not. Nonetheless, differences in emphasis allow increased understanding of the aspects of number sense in focus. An example to illustrate this claim is that FNWS is visible in most of the questions; however, BNWS is evident in only two questions. There are no Number Word After/Before questions in the ANA test. Also missing in the ANA tests are questions on missing addend and missing subtrahend. Flashcards showing numbers on fingers are not possible in a national test, however, questions on making five and ten in test 2.1 that are missing in the national test. An understanding of different ways of making five and ten is thus not directly tested in the ANA test. Overall then, there is a discrepancy between the demands/expectations of the national curriculum statements and their desired outcomes.

4.4 Recording and presenting the data

Based on learner performance across the relevant LFIN items, I was able to allocate each child a level or stage for the 5 aspects of the LFIN framework that are the focus of this study. The summary LFIN performance table below was completed for each learner.

Table 4.4.1 – Stages or levels of the LFIN Framework

	Learner	Stage or Level
	MODEL	
1	Stage of Early Arithmetical Learning	0-5
2	Level of Forward Number Word Sequence	0-5
3	Level of Backward Number Word Sequence	0-5
4	Level of Numeral Identification	0-4
5	Level of tens and ones knowledge (Wright et al., 2006)	0-3
	Maximum possible	22

I produced a quantitative overview for each learner by adding the total possible stages and levels according to Wright et al.'s (2006) model, which produced a total of 22. Each learner's actual score was translated to a percentage score based on this total. The ANA analysis began with a similar quantitative overview. The ANA marks total for the number items were calculated out of 18 and each learner's actual score here was also translated into a percentage. I used this quantitative summary of performance on both tests to select 6 learners for a more in-depth qualitative analysis. This is discussed in Chapter 5.

The analytical framework described in Chapter 3 was used to analyse each child's performance on the LFIN tests. The tables below were devised according **to** the LFIN framework, and helped me to record the responses of the learners. Wolcott (1994) argues that one of the most important lessons he learnt from Miles and Huberman (1984), is to 'think display' stating that "my impression is that, like me (Wolcott), most researchers make too little use of graphics and visualisation" (1994, p. 31).

Thus, the data and findings of this study are presented in tables and graphs. The videos of each LFIN interview were collated in summary tables. Tables 4.4.2 and 4.4.3 provide an extract of the qualitative descriptions that were written for each learner from videotaped observation. These descriptions formed the basis for the response summaries in the tables.

Table 4.4.2 – Example of a summarised table from observation of the video test 1.1

Test 1.1	**Lr:3FLS**										
FNWS 1	1-32/ fluent	48-61/ fluent	76-84/pause at 79 say 40 correct herself and say 80...84	93-112/fluent							
NWA 2a	14/	11/	23/	29/	19/	20/	12/				
2b	5	9	7	3	6						
2c	59/	65/	32/	70/	99/						
NI 3a	10/	15/	47/	13/	21/	80/	12/	17/	99/	20/	66/
3b	8	3	5	7	9	6	2	4	1		
3c	100/	123/	206wa (26hunderd)	341(34-1)	820/						
NR 4	6	4	7	9	8						
BNWS 5	10-1/fluent	15-10/fluent	23-16/fluent	34-27wa(34..32, 30...27) Skip 31	72-67wa(72waits, 71,70,...79, 78, 77)						
NWB 6a	24/	17/	20/	11/	13/	21/	14/	30/			
6b	7	10	4	8	3						
6c	67/	50/	38/	100/	83/	41/	99/				
SN (&NI) 7a	46-55/	7b	1-10								
Both scr 8a	5 + 4/NCI(immediately)	9 + 6/NCI(pause)	**Supp to 8a 8e**	8 + 5/NCI(pause)	9 + 3/ NCI(immediately)						
1st no scr 8b	5 + 2	7 + 3	9 + 4								
Both unscr 8c	5 + 2	7 + 3	9 + 4	**Perc count 8d**	13 counters	18 counters					
Missing addend 8f	7 + [ ] = 10/NCI	12 + [ ] = 15/NCI									
Subtrac sent 9a	16 – 12/	17 – 14WA of 4 but write on paper that the answer is 6	**Missing subt 9b**	5 – [ ] = 3/NCI	10 – [ ] = 6/NCI	12 – [ ] = 9/NCI					
Removed items 9c	i 3 – 1/NCI	Ii 6 – 2/NCI	9 – 4/ some counting on fingers	15 – 3/ Some counting on fingers	Iii 27 – 4/						

Table 4.4.3 – Example of the summarised table from observation of the video test 2.1

Test 2.1	Lr									
Int:										
Sub-Spat 1a	4/	3/	2/	5/	6/ All ok					
1b	6/NCI	7/NCI	4/NCI	5/	8/					
1c	5-3-8(used as a example)	6-4-10/NCI	4-4-8/NCI	5-4-9/NCI						
Fing Pat-5 2a	3/	2/	5/	1/	4/					
2b	3(2+1)	2(1+1)	5(3+2)	4(2+2)						
Fing Pat-10 3	6(3+3)	6(5+1)	9(5+4)	10/	8(4+4)	8(5+3)				
5 Frame 4	3/	2/	5/	1/	4/					
5-wise on 10 frame 5	7/	10wa 8	8wa 6	6/	9/					
Pair-wise on 10 frame 6	4/	2/	5	1/	3	7/	10/	8/	6/	9/
Comb to 5 7	4	2/	1/	3/	5					
Comb to 10 8	a [ ] + [ ] = 10/	b [ ] + [ ] = 10/	c [ ] + [ ] = 10/	d 8 + [ ] = 10/	e 4 + [ ] = 10/					
	f 7 + [ ] = 10/									

The following codes, related to SEAL Stage descriptions were used to record the strategies used:

/- correct answer

\CA - cannot answer

\WA-wrong answer

PC - Perceptual counting-concrete counting of seeing and feeling objects (count by ones)

CO - counting on

CD-counting down from;

NCI-Non count by one strategies (Use procedures to find the answer).

If I could see that a child used a counting on strategy, it is indicated by CO, and if I could see a child used a counting down from strategy, it is indicated by CD. None of the learners analyzed in this study used counting down to strategies. Wright et al. (2010) argue that it is possible that a learner uses strategies other than 'count by ones' to find the answer. These strategies are called "non-count-by-ones strategies" (NCI) by Wright et al. (2010). They argue that some of these strategies can be "...strategies such as compensation, using a known result, adding to ten, commutatively, subtraction as the inverse of addition, and the awareness of ten as a teen number" (2010, p 9).

In my analysis, I will use the term 'non-count-by-ones strategies' when a learner has been able to give an answer immediately, without showing any count-by-one strategies.

I began by compiling two summary tables of performance on the LFIN and the ANA tests. For convenience, I have placed them next to each other. This does not mean individual questions alongside each other are linked but that this linked focus on topics is built into the aspect related tables. For the sub-sample of 6 learners, I compared the answers of both tests using Table 4.4.4 and then focus on each aspect in turn (Tables 4.4.5-4.4.9).

Table 4.4.1 – Results of both tests

Results of LFIN			**Results of ANA**	
LFIN TEST 1.1	Comments		ANA test	Comments
1			1.1(3, 4, 5,......	
2			1.2 (20, 30, 40....	
3			2(a) Δ's(?)	
4			2(b) 7 objects	
5			3.1 Scissor?	
6			3.2 6?	
7			5.1 20+3	
8(a)			5.2 18-4=9	
8(f)			6.1 Double of 5	
9(a)			6.2 Half of 20	
9(b)			7.1 10+10+10	
9(c)			7.2 10-2-2	
LFIN TEST 2.1			9.1 coins =?	

Table 4.4.2 – SEAL Stages

SEAL Stages				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 5, 6, 7, 8(a) , 8(f), 9(a-c)		ANA test	1.1, 1.2, 2(a,b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 11.1, 11.2
LFIN TEST 2.1	1, 2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:			Overall comment on the ANA test:	
SEAL STAGE				
Overall comment on both tests				

Table 4.4.3 - FNWS

FNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 6, 7, 8(a-f), 9(a-c)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
LFIN TEST 2.1	2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:			Overall comment on the ANA test:	
FNWS Level				
Overall comment on both tests				

Table 4.4.4 - BNWS

BNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	5, 6, 8(f), 9(a-c),		ANA test	5.2, 7.2
LFIN TEST 2.1	4, 5, 6			
Overall comment on LFIN test:			Overall comment on ANA test:	
BNWS Level				
Overall comment				

Table 4.4.5 – Numeral Identification

Numeral Identification				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	3, 4, 9(a)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
Overall comment on LFIN test:			Overall comment on ANA test:	
Numeral Identification level:				
Overall comment				

Table 4.4.6 – Base ten Arithmetical strategies

Base ten Arithmetical strategies				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 2.1	5, 6, 8		ANA test	6.2, 7.1, 7.2, 9.1, 9.2,
Overall comment on LFIN test:			Overall comment on ANA test	
Base Ten Level:				
Overall comment				

4.5 Reliability and Validity

Reliability in research refers to the degree to which results are dependable over time. This means that if a study is repeated under the same methodology, and indicates the same results, then the instrument is considered to be reliable (Golafshani, 2003). Golafshani argues that reliability shows the degree to which the results of an instrument are replicable. In instances in the LFIN test administration where there was a sense that the child had made a random error, the question was repeated at a later time during the course of the interview (Wright et al., 2006), helping to ensure the reliability of findings in my study. Given that the project team did not administer the ANA, I cannot claim the same reliability for these tests. However as stated, I did re-mark each ANA script, and analysis was based on this score, rather than the score assigned by class teachers (these were mostly in agreement). Brinberg and McGrath (1985) argue that "...validity is not a commodity that can be purchased via techniques...rather, validity is like integrity, character and quality, to be assessed relative to purpose and circumstances" (p. 13).

Validity of a instrument indicates whether or not, the instrument measures what it supposes to measure (Golafshani, 2003). The LFIN test measures the level each child is on, and has been validated across a number of iterations in a range of countries. Part of the focus of my study is to assess the extent to which the Grade 1 ANA can be regarded as revealing useful information of learners' understandings of number.

4.6 Ethical considerations and limitations

In order to work ethically, the broader project ensured that all participants and parents, as well as class teachers and the principal, were aware of project aims. Only learners of parents who gave informed consent, were selected for our sample by the class teachers. Ethical clearance was gained from the University for data collection (Protocol number: 2011ECE012C). Learners were re-assured of anonymity in all reporting, and were told they could elect to stop the test at any point. Given that children may be scared and as a result not be able to give their best performance, the team took care to ensure that children were comfortable during the test process (Wright et al., 2006).

A further point of interest is that interpreters were employed to translate the test questions into the learner's home language on the LFIN tests, and to provided elaboration. This added to the time taken to run the test, and therefore, also needs to be acknowledged as a possible limitation. In all responses, learners responded with numbers given in English and actions that could be interpreted by me as researcher.

Chapter 5 - Findings

5.1 Introduction

I began my detailing of findings with a re-statement of the research question:

What do the two tests (ANA and LFIN) tell us about the strategies on early number used by a sample of Grade 2 learners in a township school in Gauteng?

The two critical questions within this focus are:

- How does learner performance on number problems compare across the two tests?
- What evidence in relation to concrete/abstract strategies is evident in the responses of learners in the two tests?

In answering these questions, I begin by looking at learner performance on number problems in both tests. I compared the performance of learners in the two tests quantitatively and qualitatively. The qualitative analysis provides the bulk of the analysis.

5.2 Quantitative analysis- Overview analysis

Initial marking and collating of learner performance on the two tests quickly revealed that performance on the tests was very different, with learners achieving higher marks in general on the ANA tests than on the LFIN test. The mean mark on the ANA test of the 29 learners was 61,1% and the mean mark on the LFIN test for the same learners was 42,2%. The median mark on the LFIN test is 45,5% and the median of the ANA test is 66,7%. This confirmed what was noted earlier in relation to the nature of the two tests namely that many students were using concrete 'count all' strategies to answer addition questions on the LFIN, and subsequently being allocated a SEAL Stage 2 based on their strategy, in spite of reaching the correct answer. In contrast, concrete counting as a strategy did not affect ANA marks if the correct answer was produced. Given that concrete/abstract strategies were of central interest in my study, I decided to explore the differences in performance in more depth.

I elected to use 60% as a cut off point to distinguish between high and low performance in the LFIN test and 65% in the ANA test. This meant that the cut off points were approximately 18% above mean on the LFIN test and approximately 4% above mean on the ANA test. I could not make the cut-off points of the different tests the same, because there would then have been too few learners that would achieve the ‘high’ criteria for the LFIN test. With this classification, most students fell into the ‘low’ category on LFIN. This will be explored further later in the chapter. In the larger Wits Maths Connect Primary Project (WMC-P) taking place in 10 schools, the focal school in my study was allocated a Code 3. I used the codes from the larger WMC-P project, where 3A1 for example, indicating that the data was collected from a learner in school 3, class A and learner 1. An overview quantitative summary of performance on the two tests is given in Table 5.1, including information on the SEAL stages and aspect levels achieved on the LFIN tests. Learners who were subsequently selected for the sub-sample have been given pseudonyms in this table.

Table 5.2.1 – Quantitative summary of performance on the two tests

	Ana Mark on number %	SEAL	FNWSs	BNWSs	Numeral Identification	Level of tens /ones	Total LFIN mark/22	LFIN mark %	Difference between the tests	High/low, high/high, low/high, low/low
3AL1	72.2	1	1	1	2	0	5	22.7	49.5	High ANA/Low LFIN
3AL3(3)-James	72.2	1	2	1	1	0	5	22.7	49.5	High ANA/Low LFIN
3AL4	72.2	1	2	2	3	0	8	36.4	35.9	High ANA/Low LFIN
3AL5	88.9	2	3	3	2	0	10	45.5	43.4	High ANA/Low LFIN
3AL6	88.9	2	5	4	3	0	14	63.6	25.3	High ANA/High LFIN
3BL1(5)-Jenny	33.3	0	1	0	1	0	2	9.1	24.2	Low ANA/Low LFIN
3BL3	77.8	2	4	1	3	0	10	45.5	32.3	High ANA/Low LFIN
3BL4	61.1	2	4	3	3	0	12	54.5	6.6	Low ANA/Low LFIN
3BL5	77.8	2	3	3	3	0	11	50.0	27.8	High ANA/Low LFIN
3BL6	66.7	2	3	3	4	0	12	54.5	12.1	High ANA/Low LFIN
3CL3	16.7	0	2	0	0	0	2	9.1	7.6	Low ANA/Low LFIN
3CL4	72.2	2	3	3	3	0	11	50.0	22.2	High ANA/Low LFIN
3CL5	83.3	1	3	3	3	0	10	45.5	37.9	High ANA/Low LFIN
3CL6	61.1	2	3	2	2	0	9	40.9	20.2	Low ANA/Low LFIN
3DL1	88.9	1	3	3	3	0	10	45.5	43.4	High ANA/Low LFIN
3DL2	83.3	2	3	3	3	0	11	50.0	33.3	High ANA/Low LFIN
3DL3	94.4	2	3	4	3	0	12	54.5	39.9	High ANA/Low LFIN
3DL5(4)-Happy	66.7	1	2	0	1	0	4	18.2	48.5	High ANA/Low LFIN
3DL6	66.7	2	2	2	2	0	8	36.4	30.3	High ANA/Low LFIN
3EL2	16.7	2	2	0	0	0	4	18.2	-1.5	Low ANA/Low LFIN
3EL3(6)-Buzi	27.8	1	3	1	1	0	6	27.3	0.5	Low ANA/Low LFIN

3EL4	11.1	1	3	0	1	0	5	22.7	-11.6	Low ANA/Low LFIN
3EL5(2)-Sipho	44.4	3	5	3	2	1	14	63.6	-19.2	Low ANA/High LFIN
3FL1	33.3	1	5	2	3	0	11	50.0	-16.7	Low ANA/Low LFIN
3FL2	61.1	2	4	3	3	0	12	54.5	6.6	Low ANA/Low LFIN
3FL3	44.4	2	4	3	3	0	12	54.5	-10.1	Low ANA/Low LFIN
3FL4	50.0	2	3	4	3	0	12	54.5	-4.5	Low ANA/Low LFIN
3FL5(1)-Cynthia	83.3	3	5	4	3	2	17	77.3	6.1	High ANA/High LFIN
3FL6	55.6	2	4	1	3	0	10	45.5	10.1	Low ANA/Low LFIN
Mean	61.1							42.2	0.3	

Table 5.2.1 shows that only a few learners have any 'base ten' awareness and that most of the learners are stronger in FNWS than in BNWS, as the literature indicates (Wright et al., 2006). In order to understand the different types of learner performance across the LFIN and ANA tests, I compared patterns of performance. In Figure 5.1 below, ANA marks, sorted from lowest to highest are plotted for each learner, with their corresponding LFIN mark.

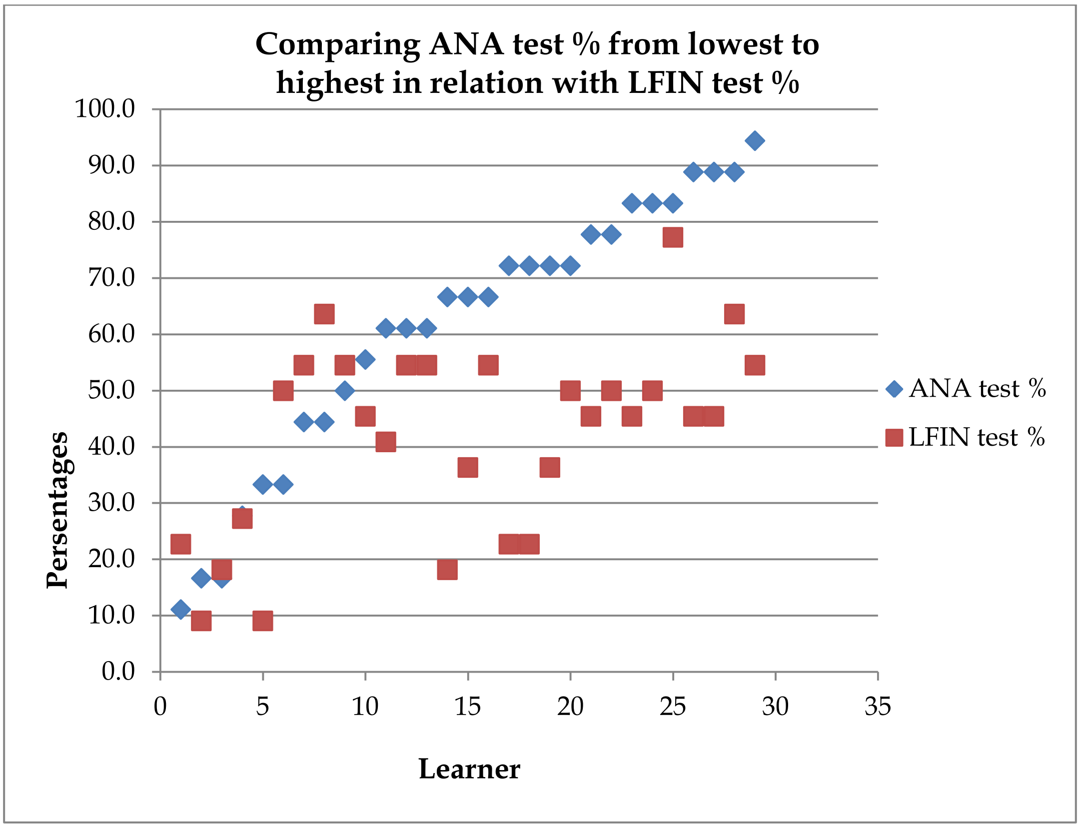

Figure 5.1– Comparing ANA and LFIN marks for each learner

Figure 5.1 confirms that LFIN test marks are generally much lower than the ANA test marks. It is also clear from graph 5.1 above, that high marks on the ANA test do not ensure high marks of the LFIN test. Only 3 learners manage to achieve 60% or more for the LFIN test, whilst 13 learners achieved 65% or more for the ANA test. Mostly, the 'high' performance in the ANA test links with a 'low' performance in the LFIN test. Only two learners scored 'high' in both tests. Fourteen learners scored a 'high' on the ANA test and 'low' on the LFIN test. This allowed me to think about my overall sample in terms of four groups, from which, in turn, I could select sub-sample learners for a more in-depth qualitative analysis.

Table 5.2.2 – Sub-sample learners

Groups	**No in each group**	**No selected for in-depth sub-sample**
Group 1: High ANA /High LFIN	2	1
Group 2: Low ANA/High LFIN	1	1
Group 3: High ANA/Low LFIN	14	2
Group 4: Low ANA/Low LFIN	12	2

In selecting learners within groups, I began by looking for the largest difference, but in one instance, I selected a smaller difference based on 'richer data' in terms of more visible strategies on the ANA script. My final sub-sample therefore consisted of the following learners (names are pseudonyms).

- **1 learners from the High ANA/ High LFIN-group**
  Learner 1 (Cynthia) She scored highest on the LFIN test in this group
- **1 learner from the Low ANA/High LFIN-group**
  Learner 2 (Sipho) Only one in this group
- **2 learners from the High ANA/ Low LFIN –group**
  Learner 3 (Happy) Highest difference between the two tests in this group (54)
  Learner 4 (James) Was chosen because of the rich data in the ANA test.
- **2 learners from the Low ANA/ Low LFIN-group**
  Learner 5 (Jenny) Highest difference between the two tests in this group (30)
  Learner 6-(Buzi)-Lowest difference between the two tests in this group

5.3 Findings from ‘stratified’ sub-sample (6 learners)

As noted in Chapter 4, I began with an overview of learner performance on each test, placed alongside each other. The aspect level tables for each sub-sample learner then follows with commentary on the strategies seen. These summaries then formed the basis of the analysis of similarities and differences across the two tests in the next chapter. The first learner (Cynthia) scored high on both tests.

5.3.1 Learner 1: High ANA/High LFIN (Cynthia)

Table 5.3.1 – Results of both tests (Cynthia)

Results on the LFIN tests	
LFIN TEST 1.1	Comments
1	Counts on from 1 and from other numbers fluently in the range 1-100
2	Gives the NWA immediately for numbers in the range of 1-100
3	Can identify numbers from 1-100
4	Can recognise numbers that was asked. Range 1-10
5	Counts backwards fluently for 1-30 range More hesitant counting backwards from 34 and 72, but gets answers correct
6	Can say the NWB for the numbers in the range of 1-100
7	Can sequence numerals correctly from 46-55.
8(a)	Answers 5+4, 9+6, 8+5 and 9+3 immediately as recalled facts. Did not use her fingers to count down

Results on the ANA test	
ANA test	Comments
1.1(3, 4, 5,......	✓ This links with the fact that she can count fluently from 1-100.
1.2 (20, 30, 40....	Can count in tens.
2(a) Δ's(?)	✓ Knows that there are 7 triangles.
2(b) 7 objects	✓
3.1 Scissor?	✓ Understands the ordinality of numbers.
3.2 6?	✓ Ordinality of numbers.
5.1 20+3	✓ Links with the LFIN test 1.1 question 8(a).
5.2 18-4=9	Wrong answer (9). Does not link with the LFIN test where she could

	or count forward.
8(f)	Using a ‘not count by ones’ strategy to answer ‘missing addend’ questions. Answered these questions as recalled facts, however, it takes more time to find the answer as the addition questions in 8(a).
9(a)	Wrote 16-14=4 on paper and got the 16-12 correct (Counting on fingers visible here), however, for 17-14 she got a wrong answer of 4. There is no tally counting visible on the paper. Links with question 5.2 where the answer 18-4=9 is given. No indication of how answers are deduced.
9(b)	Got all the questions for missing subtrahend correct, using ‘not count by ones’(use mental calculation or procedures) strategies. Some questions were answered as recalled facts and one of them were answered with the help of counting on fingers. It is not clear if this was counting down or counting forward.
9(c)	For numbers in the range 1-6 uses ‘not count by ones’ strategies; but for 9-4, 15-3, and 27-4, counts backwards to find the answer.
LFIN TEST 2.1	
1	Can ‘see’ (subitize) the numbers of dots on the cards without counting them one by one in the number range 1-10.

	count backwards and got question 9(c) correct in the LFIN test, however, links with 9(a) of the LFIN test where she answers 17-14=4. There is no tally counting visible on the ANA script.
6.1 Double of 5	✓
6.2 Half of 20	✓
7.1 10+10+10	✓ Links with LFIN test 1.1 questions 8(a) where she answers it correct.
7.2 10-2-2	✓ Links with question 9(c) where she answer the removed items question correctly.
9.1 coins =?	✓ Links with question 9(c) of the LFIN test.
9.2 ?4 coins=25c	✓ Evidence of the fact that she can uses procedures to answer the questions.

2 and 3	Can use her fingers to show numbers in the range of 1-10. She can also show numbers on two hands in the range 1-5. Can show numbers in different ways in the range 1-10.
4	Can say the number of dots on the five-frame flashing cards without counting them one by one.
5	Can say the number of dots on the ten-frame flashing cards without counting them one by one.
6	Can subitize the number of dots on the pair-wise ten-frame cards.
7	Can find the numbers that go with given numbers to make five as recalled facts. No fingers used.
8	Can find the numbers that goes with a given number to make ten in different ways. Has a awareness of 'ten' as a teen number.

10. 12 sweets/3	Wrong answer (66)
11.1- 7 Apples?	✓ Links with question 1 of the LFIN test.
11.2- 5 Bananas?	✓ Can count these objects.

Table 5.3.2 – SEAL stages (Cynthia)

SEAL Stages					
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 5, 6, 7, 8(a) , 8(f), 9(a-c)		ANA test	1.1, 1.2, 2(a,b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 11.1, 11.2
LFIN TEST 2.1	1, 2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:	Cynthia uses different strategies to find the answers in additive and subtractive situations. She uses non-count-by-one-strategies for 15-3 and 9-4, where some counting on her fingers were visible. She also uses known results to give the answers for additive questions like 5+4 and 9+3, which she answers immediately. However, she could		Overall comment on the ANA test:	She was one of two learners in the school that could answer question 9.2 correct She could not find the correct answer for 5.2 which is 18-4=9

	not answer 17-14 correctly. She can count-on from given numbers in the range 1-100. She can count down from numbers in the range 1-30. She cannot 'count-down' from a number bigger than 30			
SEAL STAGE	3 out of 5			
Overall comment	In terms of secure understanding, Cynthia is on stage 3 of SEAL. In some instances across both tests she appears to struggle with subtraction (of 17-14 in the LFIN test and 18-4 in the ANA test), and gives incorrect answers with no strategy apparent. However, she counts-on-from and counts–down- from any number in the number range 1=100. Whilst Cynthia is able to produce forward and backward number word sequences correctly in the 1-100 no range, she appears to bring this competence into play in addition contexts, but not in subtraction contexts. She can also use non-count-by-one strategies to find answers to problems like 5+4=? And give the answer immediately (stage 5). Within this work, she shows some abstract understanding of number because she uses procedures and mental calculations to get the answers. Backing this view, she can count on (range 1-100) and down (range 1-30) from any number.			

Table Error! No text of specified style in document.**.3 – FNWS (Cynthia)**

FNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 6, 7, 8(a-f), 9(a-c)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
LFIN TEST 2.1	2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:	Able to count fluently in the number range 1-100, but proceeds more hesitantly with number beyond this, using her fingers to keep track of her count. Extends her knowledge to being able to name the number word after a given number in the range 1-100. The number range for a grade 2 in the first term is 1-34. The number range of Grade 1 is 1-34 and for Grade 2 it is 1-100.		Overall comment on the ANA test:	Correct answers to most questions suggest competence in bringing knowledge of number words into operational problems.
FNWS Level	5 out of 5			
Overall comment	Cynthia is able to count fluently in the number range 1-100. She understands the 'orderliness of number' and she understands how the number sequence works.			

Table 5.3.4 – BNWS (Cynthia)

BNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	5, 6, 8(f), 9(a-c),		ANA test	5.2, 7.2
LFIN TEST 2.1	4, 5, 6			
Overall comment on LFIN test:	Gives the number of dots in the five and ten-frame cards with ease.		Overall comment on ANA test:	There are only two questions in the ANA test regarding backward counting. Could not find the answer for 18-4, however could find the answer of 9-4.
BNWS Level	4 out of 5			
Overall comment	Can count backwards, however, cannot use the ability to count backwards to solve subtraction tasks.			

Table 5.3.5 – Numeral Identification (Cynthia)

Numeral Identification				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	3, 4, 9(a)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
Overall comment on LFIN test:	She can Identify numbers in the range 1-100. She has difficulties to identify all the numbers beyond 100.		Overall comment on ANA test:	
Numeral Identification level:	3 out of 4			
Overall comment	She can identify the numbers in the ANA test, and in the LFIN tests with ease			

Table 5.3.6 – Base ten (Cynthia)

Base ten arithmetical strategies				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 2.1	5, 6, 8		ANA test	6.2, 7.1, 7.2, 9.1, 9.2,
Overall comment on LFIN test:	Has an abstract understanding of ten.		Overall comment on ANA test	She knows that 10 is the half of 20 and that 3 tens make 30. Also understands that 10-2-2=6.
Base Ten Level:	2 out of 3			
Overall comment	She has an abstract view of the number 10. She can give different combinations to make 10. This fact is clear in both tests.			

Overall comment on both tests for Cynthia

Cynthia's mark for the ANA test on the number items is 83,3%. She could answer question 9 in the ANA test, a question requiring more abstract thinking. No evidence of concrete counting is visible on the ANA script. She shows some abstract understanding of number in the LFIN tests because she uses procedures and mental calculations to produce answers to addition problems and she can 'subitize' the number of dots on a card in the range 1-10.

The second learner, Sipho, scored high on the LFIN tests, but low on the ANA test. He was the only one who scored higher on the LFIN test than the ANA test.

5.3.2 Learner 2: 3EL5-Low ANA/High LFIN (Sipho)

Table 5.3.7 – Results of both tests (Sipho)

Results on the LFIN tests	
LFIN TEST 1.1	Comments
1	Can counts from one and from other numbers fluently in the range 1-100.

Results on the ANA test	
ANA test	Comments
1.1(3, 4, 5,....	✓ This linked to the ability to count fluently from 1-

2	Can give the NWA for numbers in the range of 1-100.
3	He can identify numbers in the range from 1 to 100.
4	He can recognise numbers in the range of 1-10.
5	Can count backwards fluently from numbers in the range of 1-19. When counting from 23, he skips 20. Have problems with counting backwards from 34 and 72.
6	Can say the NWB for the numbers in the range of 1-100.
7	Cannot sequences numerals from 46-55. He arranges them: 52, 48, 54, 46...
8(a-e)	Use “non-count-by one strategies” to answer 5+4, 9+6, 8+5 and 9+3.
8(f)	Can do the 7+?=10, but not the 12+?=15
9(a)	Can answer 16-12. Cannot give an answer for 17-14.
9(b)	Cannot answer the missing subtrahend tasks correctly. Only 10-?=6 he answers correctly.
9(c)	Can do the removed items tasks in the number range of 1 to 20. He

	100.
1.2 (20, 30, 40...	✓ Can count in tens.
2(a) Δ's(?)	✓ He knows that there are 7 triangles
2(b) 7 objects	Wrong answer (present 9 objects)
3.1 Scissor?	✓ Does not understands the ordinality of numbers (Position of number).
3.2 6?	Wrong answer.
20+3	✓ Links with the LFIN test 1.1 question 8(a)
18-4	Wrong answer (8). Does not links with the LFIN test where he could count backwards and got question 9(c) correct in the LFIN test, however, links with 9(a) of the LFIN test where he could not give a correct answer for 17-14=?
6.1 Double of 7	Wrong answer (11)
6.2 Half of 20	Wrong answer (9)
7.1 10+10+10	✓ Links with LFIN test 1.1 question 8(a) where he answer it

	cannot answer 27-4 correctly.
LFIN TEST 2.1	
1	Can 'see' all the number of dots 1-5 without counting them one-by-one. Cannot 'see' number of dots when there are more than 5 dots. Can see both sides of the domino cards and give the sum correctly, but uses fingers to help to get the answers.
2	Can show a number, using both hands.
3	Can use fingers to show numbers in the range of 1-10. Can also show numbers on both hands in the range 1-5. Can show numbers in different ways in the range 1-10.
4	Can say the number of dots on the five-frame flashcards without counting them one by one.
5	Can say the number of dots on the ten-frame flashing cards without counting them one by one, except 6.
6	Can say the number of dots on the pair-wise ten-frame flashcards without counting them one by one, except 8.
7	Can find the numbers that goes with a given number to make five.
8	Can find the numbers that goes with a given number to make ten in different ways. He has a awareness of 'ten' as a teen number.

	correct.
7.2 10-2-2	Wrong answer(10). Cannot use his knowledge of backward counting to find the answer of subtraction tasks.
9.1 coins =?	Wrong answer (20)
9.2 ?4 coins=25c	Wrong answer (2x10c+2x5c)
10 12 sweets/3	✓
11.1 (counting of 7 and 5 objects)	✓ Can count 7 apples
11.2	✓ Can count 5 bananas

Table 5.3.8 – SEAL Stages (Sipho)

SEAL Stages				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 5, 6, 7, 8(a) , 8(f), 9(a-c)		ANA test	1.1, 1.2, 2(a,b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 11.1, 11.2
LFIN TEST 2.1	1, 2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:	Sipho use sometime non-count-by-one procedures to find the answer to addition tasks, however, he also use count by ones strategies. He uses counting-on strategies.		Overall comment on the ANA test:	He makes mistakes on questions involving the ordinality, but not cardinality of numbers. He cannot use his knowledge of backward counting to find the answer of subtraction questions.
SEAL STAGE	3 out of 5			
Overall comment	Sipho uses counting-on strategies rather than counting from one to solve addition or missing addend tasks; however, he can answer some questions without using his fingers and give some answers immediately, which show us that he knows some answers as recalled facts. It does not links with his bad score in the ANA test.			

Table 5.3.9 – FNWS (Sipho)

FNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 6, 7, 8(a-f), 9(a-c)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
LFIN TEST 2.1	2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:	Can produce the FNWS in the range of 1-100, he can also produce the NWB for a given number in the range 1-100		Overall comment on the ANA test:	He answered the addition questions correctly.
FNWS Level	5 out of 5			
Overall comment	Can produce the FNWS in the range 1-100; Can produce the number word after for this range without dropping back to one			

Table 5.3.10 – BNWS (Sipho)

BNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	5, 6, 8(f), 9(a-c),		ANA test	5.2, 7.2
LFIN TEST 2.1	4, 5, 6			
Overall comment on LFIN test:	He can produce the BNWS from 19 to 1, however he can produce the NWB for numbers in the range of 1 to 100.		Overall comment on ANA test:	He answered the subtraction questions wrongly
BNWS Level	3 out of 5			
Overall comment	BNWS is not as well developed as FNWS. In the ANA test both the subtraction question were answered wrongly. Cannot bring backward counting (LFIN test) and subtraction (ANA test) together. Can count backwards in the number range 1-19 and all the questions about subtracting in the ANA test were in that number range.			

Table 5.3.11 – Numeral Identification (Sipho)

Numeral Identification				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	3, 4, 9(a)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
Overall comment on LFIN test:	Can identify numbers in the range of 1-20		Overall comment on ANA test:	He scored very badly on the ANA test.
Numeral Identification level:	2 out of 4			
Overall comment	Can identify numbers in the range of 1-20. His bad score on the ANA test does not link with his good score on the LFIN test. The reason could be because he is not so good on numeral identification that the ANA test gives high emphasis.			

Table 5.3.12 – Base ten (Sipho)

Base ten arithmetical strategies				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 2.1	5, 6, 8		ANA test	6.2, 7.1, 7.2, 9.1, 9.2,
Overall comment on LFIN test:	Can see 10 as a unit. Can find the numbers that goes with a given number to make ten in different ways. Has a awareness of 'ten' as a teen number.		Overall comment on ANA test	Answers all the questions that link with base-ten strategies wrongly, except 7.1 in the ANA test.
Base Ten Level:	1 out of 3			
Overall comment	He cannot see 10 as a unit, however, he can count-on and count down when tasks involving tens are asked.			

Overall comment on both tests for Sipho

Sipho scored 44,4% on the ANA test on the number items. He could not answer the question 9.2 in the ANA test, which requires abstract thinking. No evidence of concrete counting is visible on his ANA script. In the LFIN test, he shows some abstract understanding of number because he uses counting-on strategies to get the answers. Sipho used counting-on strategies rather than counting-from-one to solve addition or missing addend tasks; however, he was able to answer some questions without using his fingers and give some answers immediately, which show us that he knows several answers as recalled facts. He could count-on, however, he did not use this ability to answer the questions in the ANA test. He could 'see' the number of dots in the number range 1-5, indicating that he has a partially abstract understanding of numbers 1-5. The reasons for these contradicting patterns of performance are unclear in this instance.

5.3.3 Learner 3: 3AL3 –High ANA/ Low LFIN (Happy)

Table 5.3.13 – Results for both tests (Happy)

Results on the LFIN tests	
Relevant LFIN Questions	Comments
LFIN TEST 1.1	
1 and 2	Can produce the FNWS for numbers from 1-32. After 49, says "forty-ten, forty-eleven... Says "...after 29 comes 'twenty-ten' ". This idea used more times. Can say the NWA for numbers in the range 1-10,
3 and 4	He can identify numbers in the number range 1-20. He recognised numbers in the number range 1-10
5	Can produce the BNWS for numbers from 10 to 1. He cannot produce the BNWS from numbers bigger than 10.
6	Cannot produce NWB for all the given numbers in the range 1-10. Numbers bigger than 10 is also problematic.
7	Cannot order the numbers 46-55
8(a-e)	Can count perceived items.
8(f)	No answer.
9(a)	Interviewer offered counters and gets the answer by concrete counting.
9(b)	Cannot answer all the questions on 'missing subtrahend' correctly.
9(c)	The question on removed items is problematic.
LFIN TEST 2.1	

Results on the ANA test	
Relevant ANA Question	Comments
1.1(3, 4, 5,......	✓ Writes 10 as '01'
1.2 (20, 30, 40....	✓ Can count in tens
2(a) Δ's(?)	✓ Cannot write the number word for 9.
2(b) 7 objects	✓ Links with the LFIN test where he can produce the FNWS for numbers from 1-32. He appears to know the cardinality of numbers.
3.1 Scissor?	✓ He knows the ordinality of the number 4
3.2 6?	✓ He knows the ordinality of the number 6.
5.1 20+3	Wrong answer (1)
5.2 18-4	Wrong answer (7). Links with the LFIN test where he could not count backwards from numbers bigger than 10 and links with question 9(c) where he cannot produce the correct answer for the removed items tasks.
6.1 Double of 7	Wrong answer(7)
6.2 Half of 20	✓
7.1 10+10+10	✓ .
7.2 10-2-2	✓ Does not link with question 9(c); answered the removed items

1	Can see the numbers of dots on a card in the number range of 1-6
2 and 3	Can show numbers 1-5 on his fingers. Cannot show numbers smaller than 5 on both hands. Cannot show bigger than 5 numbers in different ways. There is evidence of concrete counting.
4	Can see numbers on a five-frame card.
5	Cannot see all the numbers on a ten-frame card.
6	Cannot see the number of dots on a pair-wise ten-frame card.
7	Cannot give numbers to make five.
8	Cannot give numbers to make numbers bigger than 5 in different ways.

	question wrongly.
9.1 coins =?	✓ Does not link question 9(c) of the LFIN test 1.1; could not answer the removed items correctly.
9.2 ?4 coins=25c	Wrong answer (2x10+5)
10 12 sweets/3	✓
11 (counting of 7 and 5 objects)	✓ Links with question 1 of the LFIN test.

Table 5.3.14 – SEAL Stages (Happy)

SEAL Stages				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 5, 6, 7, 8(a) , 8(f), 9(a-c)		ANA test	1.1, 1.2, 2(a,b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 11.1, 11.2
LFIN TEST 2.1	1, 2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:	Can count feeling and seeing objects. Uses concrete counting. When counting, always dropping back to 'one'		Overall comment on the ANA test:	Scored 75% for the ANA test, which is a very high score
SEAL STAGE	1 out of 5			
Overall comment	Happy can count perceived items, which involved feeling and seeing objects in the range of 1-32. He uses concrete counting. He always drops back to 'one' when he counts. He cannot count on or down when doing addition or subtraction tasks. Given that the number range of the ANA test is from 1-30 (except for the question on counting in tens) it is understandable that he could do so well in the ANA test. 75% for the ANA does not correspond with his bad performance in the LFIN test. Happy does not have an abstract			

	view of numbers because he is always falling back to one to do the tasks. He has a sound understanding of concrete counting.

Table 5.3.15 – FNWS (Happy)

FNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 6, 7, 8(a-f), 9(a-c)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
LFIN TEST 2.1	2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:	Can produce the FNWS and say the NWA for numbers 1-10. After ten counting is problematic to say the NWA.		Overall comment on the ANA test:	Good score in the ANA test does not link with the bad performance in the LFIN test. Writes 10 as 01 and when writing the numbers, flips them. Not sure of number symbols.
FNWS Level	2 out of 5			
Overall comment	Happy can produce the number sequence from 1-32. The ANA test focused on numbers that is in a number range of 1-32. Concrete counting strategies seen in the LFIN test suggest that these may have been used to produce correct answers on ANA addition and subtraction problems. He does not know what comes after 11 without going back to 1.			

Table 5.3.16 – BNWS (Happy)

BNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	5, 6, 8(f), 9(a-c),		ANA test	5.2, 7.2
LFIN TEST 2.1	4, 5, 6			
Overall comment on LFIN test:	Can produce the BNWS for 10 to 1. Cannot say the NWB a give number in the range 1-10		Overall comment on ANA test:	Could not answer both subtraction tasks correctly in the ANA test.
BNWS Level	1 out of 5			
Overall comment	Happy can produce the BNWS from 10-1. He cannot produce the number word before a number. There is evidence from both tests that he cannot do subtraction tasks.			

Table 5.3.17 – Numeral Identification (Happy)

Numeral identification				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	3, 4, 9(a)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
Overall comment on LFIN test:	Can identify numbers in the number range 1-10.		Overall comment on ANA test:	Low level of numeral identification can be the reason why he could not answer question 5, due to the fact the numbers were bigger than 10.
Numeral Identification level:	1 out of 4			
Overall comment	He cannot identify numbers that are bigger than 10. This does not link with his good score in the ANA test were the range is 1-32. The fact that the teacher conducted the test orally could be the reason why he knew which numbers were used.			

Table 5.3.18 – Base Ten (Happy)

Base ten Arithmetical Strategies				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 2.1	5, 6, 8		ANA test	6.2, 7.1, 7.2, 9.1, 9.2,
Overall comment on LFIN test:	Does not see 10 as a unit, and focus on the 10 units. One ten and ten ones do not exist at the same time. Cannot count on or down from a certain number when doing subtraction and addition tasks.		Overall comment on ANA test	Could do 4 out of 5 correct of the questions that had to do with 10 as a unit.
Base Ten Level:	0 out of 3			
Overall comment	The results of the two tests does not compare.			

Overall comments on both tests for Happy

Happy scored 72,2% in the ANA test on the number items but 18,2% for the LFIN test. He could not answer the question 9.2 in the ANA test that requires abstract thinking. No evidence of concrete counting is visible on his ANA script. In the LFIN test, he did not show much abstract understanding of number, because he counted from one when he needs to add 4+5=?; however, he could subitize the numbers 1-6, which show some abstract understanding for those numbers.

5.3.4 Learner 4: 3DL5 –High ANA/Low LFIN (James)

Table 5.3.19 – Results of both tests (James)

Results on the LFIN tests	
Relevant LFIN Questions	Comments
LFIN TEST 1.1	
1	He cannot count pass 29. He cannot count on from a number without dropping back to one.
2	He can say the NWA a given number in the number range 1-10, he has difficulties with NWA beyond 10.
3	He could identify most numbers in the range 1-100 except 21 and 80
4	He could recognise the numbers 1-10
5	He cannot count backwards
6	He cannot say the NWB for any number.
7	46-55 was not asked. He can order and identify numbers from1 to 10.
8(a-e)	No answer for (a), however can give the answer when it was not screened and he could count them one by one.
8(f)	Can not answer.
9(a)	Wrong answers
9(b)	Wrong answers

Results on the ANA test	
Relevant ANA Question	Comments
1.1(3, 4, 5,......	✓ This links with the fact that he can count fluently from 1-100
1.2 (20, 30, 40....	✓ He can count in tens
2(a) Δ's(?)	✓ He knows that there are 7 triangles
2(b) 7 objects	✓
3.1 Scissor?	✓ He understands the ordinality of numbers
3.2 6?	✓ Ordinality of numbers
5.1 20+3	✓ Links with the LFIN test 1.1 question 8(a).Evidence of concrete counting (tallies)
5.2 18-4	Wrong answer (41). Evidence of concrete counting
6.1 Double of 7	No answer
6.2 Half of 20	No answer
7.1 10+10+10	✓ There is evidence of concrete counting. See some tallies on the script
7.2 10-2-2	Wrong answer (9).He had the correct number of tallies, but could not present the right answer.

9(c)	Wrong answers
LFIN TEST 2.1	
1	Cannot see the total number of dots on the cards without counting them
2 and 3	He can show the numbers 1-10 on his fingers. He cannot show the numbers on two hands and cannot show numbers in different ways.
4	Can see numbers on a 5 frame card
5	Can see numbers on a ten frame card
6	Can see numbers in the range of 1-5 in a pair-wise 10 frame card, but not bigger than 5
7	Cannot give a number to make 5
8	Cannot give a number to make 10

9.1 coins =?	Wrong answer.
9.2 ?4 coins=25c	Wrong answer (5x 10c)
10 12 sweets/3	✓
11 (counting of 7 and 5 objects)	✓

Table 5.3 20 – SEAL Stages (James)

SEAL Stages			
	Relevant LFIN Questions		Relevant ANA Question
LFIN TEST 1.1	1, 2, 5, 6, 7, 8(a) , 8(f), 9(a-c)	ANA test	1.1, 1.2, 2(a,b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 11.1, 11.2
LFIN TEST 2.1	1, 2, 3, 4, 5, 6, 7, 8		
Overall comment on the LFIN test:	He cannot count screened objects. He can count perceived items that he can see and feel.	Overall comment on the ANA test:	He scored 70% on the ANA test. Tallies is visible in questions 5.2, 5.2, 7.1, 7.2. He used concrete counting to find the answers.
SEAL STAGE	1 out of 5		
Overall comment	Evidence from the LFIN tests shows that he cannot counts screened items. That links with the fact that he used tallies to find the answers in the ANA test. The fact that the number range is only from 1-32 in the ANA test is the reason why he could manage to get a high score of 70% for the ANA test		

Table Error! No text of specified style in document. **– FNWS (James)**

FNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 6, 7, 8(a-f), 9(a-c)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
LFIN TEST 2.1	2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:	Can produce the FNWS and NWA for numbers 1 to 10, but drops back to 'one' to do so. Cannot count on or down from any given number.		Overall comment on the ANA test:	Knows the number word sequence from 1-29 and uses tallies to find the answers.
FNWS Level	2 out of 5			
<u>Overall comment</u>	He has a concrete understanding of number. He cannot count on or down from (LFIN test) and he used tallies in the ANA test to find his answers.			

Table 5.3.22 – BNWS (James)

BNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	5, 6, 8(f), 9(a-c),		ANA test	5.2, 7.2
LFIN TEST 2.1	4, 5, 6			
Overall comment on LFIN test:	Cannot produce the BNWS from 10 to 1.		Overall comment on ANA test:	He has both subtraction questions wrong
BNWS Level	0			
<u>Overall comment</u>	The results of both tests show a limited proficiency with subtraction. BNWS is also limited.			

Table 5.3.23 – Numeral Identification (James)

Numeral Identification				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	3, 4, 9(a)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
Overall comment on LFIN test:	Can identify numerals in the range 1-10		Overall comment on ANA test:	Can identify the numbers in the ANA test in the range of 1-32
Numeral Identification level:	1			
Overall comment	Does not make sense that he could identify numbers in the range of 1-32 in the ANA test, but not numbers bigger than 10 in the LFIN test.			

Table 5.3.24 – Base ten (James)

Base ten arithmetical strategies				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 2.1	5, 6, 8		ANA test	6.2, 7.1, 7.2, 9.1, 9.2,
Overall comment on LFIN test:	Does not see ten as a unit of any kind. Cannot count-on or count-down from a number.		Overall comment on ANA test	Uses tallies to find his answers. See 10 only as 10 tallies
Base Ten Level:	0			
Overall comment	He does not see ten as a unit that exists out of 10.			

Overall comments on both tests for James

James scored 66,7% for the ANA test on the number items but only scored 18,2% for the LFIN test. He could not answer the question 9.2 in the ANA. There is evidence of concrete counting on his ANA script, which might explain his higher mark for the ANA test. In the LFIN test, he did not show much abstract understanding of number because he cannot count screened objects. That means he is dependent on concrete objects to answer addition and subtraction questions. He could not subitize the numbers which show that he does not have much abstract understanding of numbers.

5.3.5 Learner 5: 3BL1- Low ANA/Low LFIN (Jenny)

Table 5.3.25 – Results of both tests (Jenny)

Results on the LFIN tests	
Relevant LFIN Questions	Comments
LFIN TEST 1.1	
1 and 2	Can count from 1-29. After 29, he says 40, 41, 42... He cannot count from any other number than 1. She can say the NWA from numbers in the range from 1 to 10.
3 and 4	Can identify numbers in the number range 1-10, however, when asked to arrange them from the smallest to largest, was unable. Put the numbers upside down and arrange them as follows: 8, 7, 6, 4, 9, ...
5	She cannot count backwards
6	Cannot find the NWB for 10, 4, 8, but he can find the NWB for 7 and 3.
7	Cannot identify the numbers from 46-55 and cannot order them. Puts the numbers upside down. Can identify the numbers 1to 10, but he cannot order them. (8, 7, 6, 4, 9...) . Numbers are upside down.
8(a-e)	Cannot do 8(a) or 8(b) where the counters are screened. Can do 8(c) by counting the counters one by one. Can count the 13 feeling and seeing items of question 8(d) but not the 18 counters. When counting the 18 counters, there is evidence that the difference between the counters that are already counted and the ones that still need to be counted is not perceived.
8(f)	Cannot do this question
9(a)	Cannot do this question

Results on the ANA test	
Relevant ANA Question	Comments
1.1(3, 4, 5,......	Wrong answer
1.2 (20, 30, 40....	Wrong answer
2(a) Δ's(?)	Wrong answer
2(b) 7 objects	✓
3.1 Scissor?	Wrong answer
3.2 6?	Wrong answer
5.1 20+3	✓
5.2 18-4	✓
6.1 Double of 7	✓

9(b)	Cannot do this question
9(c)	Cannot do this question
LFIN TEST 2.1	
1	Cannot see the numbers of dots.
2 and 3	Can show 3, 5, 1, 4 on fingers but not 2. Cannot show numbers on two hands. Cannot show 6, 9, 10 and 8 on his fingers.
4	Can see numbers on the five frame cards, but not 5.
5	Cannot see numbers on a 10 frame card.
6	Cannot see numbers on a pair wise 10 frame card.
7	Cannot give a number to make five.
8	Cannot give a number to make 10.

6.2 Half of 20	Wrong answer
7.1 10+10+10	Wrong answer. However she answered 20+3 correctly
7.2 10-2-2	Wrong answer. She could answer 18-4 correctly
9.1 coins =?	Wrong answer
9.2 ?4 coins=25c	Wrong answer
10. 12 sweets/3	✓
11 (counting of 7 and 5 objects)	Can count the 7 apples, but not the 5 bananas.

Table 5.3.26 – SEAL Stages (Jenny)

SEAL Stages				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 5, 6, 7, 8(a) , 8(f), 9(a-c)		ANA test	1.1, 1.2, 2(a,b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 11.1, 11.2
LFIN TEST 2.1	1, 2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:	Cannot count visible items. Cannot coordinate the number words with the items.		Overall comment on the ANA test:	The questions in the ANA test that were correct were difficult questions, e.g. the division question. Does not link with the fact that there are problems in counting up to 12
SEAL STAGE	0			
Overall comment	She cannot count visible items with a number bigger than 10, however, she can say the sequence from 1-29. In the ANA test, she could not count the 5 bananas in question 11.			

Table 5.3.27 – FNWS (Jenny)

FNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 6, 7, 8(a-f), 9(a-c)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
LFIN TEST 2.1	2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:	Can produce the FNWS for number 1-10. Cannot produce the NWA for numbers 1-10.		Overall comment on the ANA test:	Can add 20+3, but she cannot count 5 bananas. Unclear.
FNWS Level	1			
Overall comment	She can say the number words up to 29. She can say 'the number word after' for numbers between 1 and 10.			

Table 5.3.28 – BNWS (Jenny)

BNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	5, 6, 8(f), 9(a-c),		ANA test	5.2, 7.2
LFIN TEST 2.1	4, 5, 6			
Overall comment on LFIN test:	Cannot produce the BNWS from any number		Overall comment on ANA test:	Does not make sence that she could do 18-4 correct
BNWS Level	0			
Overall comment	She does not understands what backward counting means and does not know what the 'number word before' means.			

Table 5.3.29 – Numeral Identification (Jenny)

Numeral Identification				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	3, 4, 9(a)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
Overall comment on LFIN test:	On the Initial level of numeral Identification. Can identify numbers from 1-10.		Overall comment on ANA test:	Cannot identify numbers.
Numeral Identification level:	1			
Overall comment	She can identify the numbers 1to 10, but she cannot order them. (8, 7, 6, 4, 9...). Numbers are upside down.			

Table 5.3.30 – Base ten (Jenny)

Base ten arithmetical strategies				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 2.1	5, 6, 8		ANA test	6.2, 7.1, 7.2, 9.1, 9.2,
Overall comment on LFIN test:	Does not see ten as a unit of any kind. Cannot count-on or count-down from a number.		Overall comment on ANA test	Has all the questions that links with 10 as a unit wrong.
Base Ten Level:	0			
Overall comment	Does not see 10 as a unit and cannot counts-down			

Overall comments on both tests for Jenny.

Jenny scored 33,3% for the ANA test on the number items but 9,1% for the LFIN test. She could not answer Question 9.2 in the ANA test, which requires abstract thinking. No evidence of concrete counting is visible on the ANA script. In the LFIN test, she cannot count visible items with a number bigger than 10, however, she can enunciate the sequence from 1-

29. In the ANA test, she could not count the 5 bananas in Question 11. She could not subitize the numbers 1-6, which show that she has no abstract understanding of numbers.

5.3.6 Learner 6: - Low ANA/Low LFIN (Buzi)

Table 5.3.31 – Results of both tests (Buzi)

Results on the LFIN tests	
LFIN TEST 1.1	Comments
1	Can count from 1-32. She cannot count from a number that is not one.
2	Can say the number word after for numbers 1-14. Does not drop back to 1 when doing so. Gives her answers immediately.
3	Can identify numbers in the range of 1-13.
4	Can recognise numbers in the range 1-10.
5	Can count backwards from 10-1.
6	Can say the NWB in the number range 1-7.
7	Can put numbers from 1-10 in the correct order and say the number words for each one of them.
8(a and b)	Could do only 5+4 when both were screened. Could do all the questions were one of the numbers were screened.
8(f)	Cannot do.
9(a)	Cannot do.
9(b)	Could answer 5-?=3of the missing subtrahend questions, but not the others.
9(c)	Could do 3-1 of the removed items questions, but not the others.
LFIN TEST 2.1	
1	Can 'see' the numbers of dots on a card in the number range 1-4, sometimes she can 'see' 5 dots.
2 and 3	Can show numbers 1-8 on her fingers, but not in different ways.

Results on the ANA test	
ANA test	Comments
1.1(3, 4, 5,......	✓
1.2 (20, 30, 40....	Wrong answer
2(a) Δ's(?)	Wrong answer
2(b) 7 objects	✓ Can present 7 objects correctly
3.1 Scissor?	✓ She knows the ordinality of the number 4
3.2 6?	Wrong answer
5.1 20+3	Wrong answer
5.2 18-4=9	Wrong answer
6.1 Double of 7	Wrong answer
6.2 Half of 20	Wrong answer
7.1 10+10+10	Wrong answer
7.2 10-2-2	Wrong answer
9.1 coins =?	Wrong answer
9.2 ?4 coins=25c	Wrong answer
10. 12 sweets/3	Wrong answer

4	Can 'see' numbers 1-5 on a five frame card.		11.1- 7 Apples?	✓
5	Can 'see' numbers 1-7 on a ten-frame card.		11.2- 5 Bananas?	✓
6	Interviewer did not ask this question.			
7	Cannot find another number to make five.			
8	Cannot find two numbers that will make ten.			

Table 5.3.32 – SEAL Stages (Buzi)

SEAL Stages				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 5, 6, 7, 8(a) , 8(f), 9(a-c)		ANA test	1.1, 1.2, 2(a,b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 11.1, 11.2
LFIN TEST 2.1	1, 2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:	Can count from 1-32, however cannot count-on from any other number than 1. Can add 5+4 when both are screened and give an answer without showing how it is obtained. Does not use her fingers. Is at stage 1 because she cannot count on.		Overall comment on the ANA test:	Can present 7 objects and can count 5 and 7 objects. Understands the ordinality of the number 9. Uses concrete counting methods, however, cannot use concrete methods for numbers bigger than 13.
SEAL STAGE	1			
Overall comment	Even though she uses concrete methods for numbers in the range 1-13, she has a 'partially' abstract view of numbers 1-4, because she can 'see' the number of dots on a card without counting them one by one.			

Table 5.3.33 – FNWS (Buzi)

FNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	1, 2, 6, 7, 8(a-f), 9(a-c)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
LFIN TEST 2.1	2, 3, 4, 5, 6, 7, 8			
Overall comment on the LFIN test:	Can produce the FNWS from 1-32, and can produce the NWA for numbers in the range 1-10 without dropping back to ten when doing so. Has difficulties		Overall comment on the ANA test:	Can fill in the missing numbers in the range 3-11 of question 1.1, but cannot fill in the missing numbers when counting in tens are required. Can present 7 objects and can count 5 and 7 objects.

	to produce the NWA for numbers bigger than 10.			
FNWS Level	3			
Overall comment	From the LFIN test, we can see that she can produce the FNWS from 1-32, however, she does not use this ability to answer the questions in the ANA test correctly.			

Table 5.3.31 – BNWS (Buzi)

BNWS				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	5, 6, 8(f), 9(a-c),		ANA test	5.2, 7.2
LFIN TEST 2.1	4, 5, 6			
Overall comment on LFIN test:	Can produce the BNWS in a number range 10-1. She cannot produce the NWB for number in the range 1-10. Can only produce the NWB for numbers 1-6.		Overall comment on ANA test:	Cannot answer 18-4 or 10-2-2 correctly.
BNWS Level	1			
Overall comment	Cannot answer 18-4 because she cannot produce the BNWS in that range, however, she could say the BNWS for numbers 10-1 and did not use that ability to answer 10-2-2 in the ANA test correctly.			

Table 5.3.35 – Numeral Identification (Buzi)

Numeral Identification				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 1.1	3, 4, 9(a)		ANA test	1.1, 1.2, 2(a, b), 3.1, 3.2, 5.1, 5.2, 6.1, 6.2, 7.1, 7.2, 9.1, 9.2, 10, 11.1, 11.2
Overall comment on LFIN test:	Can identify numerals in the range 1-10.		Overall comment on ANA test:	The range of the ANA test is 1-34.
Numeral Identification level:	1			
Overall comment	A reason for her poor performance in the ANA test could be that she cannot identify numbers bigger than 10. The range of most of the questions in the ANA test is 1-34.			

Table 5.3.32 – Base ten (Buzi)

Base ten Arithmetical strategies				
	Relevant LFIN Questions			Relevant ANA Question
LFIN TEST 2.1	5, 6, 8		ANA test	6.2, 7.1, 7.2, 9.1, 9.2,
Overall comment on LFIN test:	Does not see ten as a unit of any kind. Cannot count-on or count-down from a number.		Overall comment on ANA test	Has all these questions wrong in the ANA test.
Base Ten Level:	0			
Overall comment	She does not see ten as a unit and focuses on the 10 individual items. One ten and ten ones does not exists at the same time for her. She cannot count down or count on sufficiently, that is why she is on level 0.			

Overall comments on both tests for Buzi

Buzi scored 27,8% for the ANA test on the number items but 27,3% for the LFIN test. She could not answer Question 9.2 in the ANA test that requires abstract thinking. No evidence of concrete counting is visible on the ANA script. In the LFIN test, she showed a 'partially' abstract view of numbers 1-4, because she can 'see' the number of dots on a card without counting them one by one, but used concrete methods of counting in number situations.

In the next chapter, I discuss the differences and similarities between these 6 learners and compare the responses of the learners with the literature review. I also comment on responses in relation to reification theory.

Chapter 6 - Analysis of the data

6.1 Analysis

The literature shows that children usually learn to count first (Ensor et al., 2009) and that they then learn to connect the word names to quantity through the 1-1 principle (Gelman & Gallistel, 1986) before they can reify number (Sfard, 2008). In the next section, I show where each one of the 6 children analysed in this research are on the path to abstraction. A summary of the learner performance analysed based on the LFIN aspects is provided in Table 6.1. The SEAL Stages vary from 0 to 3 for these 6 learners.

Table 6.1.1 – Stages/Levels of sub-sample Learners

Learner	SEAL Stage	FNWS	BNWS	Numeral identification	Base ten knowledge	Total
1 Cynthia	3	5	4	3	2	17
2 Sipho	3	5	3	2	1	14
3 Happy	1	2	1	1	0	5
4 James	1	2	0	1	0	4
5 Jenny	0	1	0	1	0	2
6 Buzi	1	3	1	1	0	6

I begin with the learner with the lowest identified SEAL stage within this commentary. Jenny (Learner 5) was at the Emergent counting stage (stage 0). She knew the number words from 1-29, however, she could not count visible items, as seen on items 7(b) of the LFIN test. The 1-1 principle includes the two processes of differentiating between the items that were counted and ones that still need to be counted; and presenting separate items one at a time (Gelman & Gallistel, 1986). Jenny knew how to count the items one by one, but LFIN data shows that she could not differentiate between the objects that had been counted and those that still needed to be counted. She could identify numbers and say the numbers words for numbers 1-10; however, she could not arrange them in the correct order. She did not have an understanding of ten as a unit of any kind (Wright et al., 2010) and was not able to count 10 objects correctly. Across all these aspects, I note the recurrence of a strongly concrete counting action conception of number, with 1-1 correspondence still not consistently in place.

She managed to get 33,3% for her ANA test. To answer the first question in the ANA test, (fill in the missing number: 20, ?, 40, 50, 60, ?, ?, 90, ?) she responded with 20, <u>15, 16</u>, 40, 50, 60 , <u>31, 14, 15</u>, however she could answer 18 - 4 correctly, but could not solve 10 -2 - 2. Jenny did not reify numbers as Sfard (2008) describes reification, and she depended on concrete counting when doing problems in mathematics.

Buzi (learner 6) could count objects correctly, even when they were screened. Buzi was on stage 1 of the SEAL, because she could not answer all the addition questions when two collections were screened. She could find the answer of 5 + 4 when both were screened, but not the others. However, she could answer all addition questions in the number range 1 - 10 when one of the collections was screened. Even though she used concrete methods for numbers in the range 1 - 13, she had a ‘partially’ abstract view of numbers 1 - 4, because she could subitize the number of dots on a card without counting them one by one. From the evidence in the LFIN tests, she reified numbers in the range 1 - 4, because she could state the number of dots on the cards without counting them one by one. A similarity between Jenny (Learner 5) and Buzi (Leaner 6) is that they can say the forward number words from 1 to 29. The curriculum stipulates that a child in the first term of Grade 2, need be able to count from 1 - 34. Both learners were not on standard regarding counting. The difference between Buzi and Jenny is that Buzi could count visible and feeling items, but Jenny could not always counts perceived items correctly.

Buzi scored 27,8% in the ANA test. She could count the 7 apples and the 5 bananas and she understood the cardinality of the number 9, but she wrote 07 and 08 when she means 70 and 80, and used 000 for 100.

Learner 4 (James) was at the Perceptual counting stage. He could count perceived items, which involved seeing and feeling objects, but he could not count on or down from a given number. In James’ (Learner 4) ANA script, I observed evidence of concrete counting in the question: 20 + 3 = ? James drew 20 and then another 3 tallies and presented the correct answer. In Question 7.1, which is 10 + 10 + 10 = ? we also see 30 tallies, which shows us that he is dependent on concrete counting (Ensor et al., 2009) to find this answer – which he gets correct. If we look at the answers on the additive tasks of the LFIN test, we conclude that

James cannot add 5 + 4 if objects are screened. He appears unable to work out for himself how to obtain the answer if he does not have the privilege of tallies. Therefore, he could not count unseen objects. In the LFIN test, James could not count past 29. He also had difficulties in counting on from a number that is bigger than 30. When the interviewer asked him, what number comes after 23, he started to count from one to get to 23, and then was able to answers 24. The interviewer then asked him what comes after 19. James counted from one and arrives at 20; but then the interviewer asked what comes after 20 and James started counting from one again. He needs to count from one every time the interviewer asks him what comes after a number, reverting to one to solve all problems (Wright et al., 2006). On the question ‘What comes after 59?’ he started counting from one, got lost, and gave the answer of 10. When the interviewer asked him what comes after 99, he started counting from one and never got to the answer. Wright et al describe reverting to ‘one’ this way in terms of ‘dropping back’ and note that low attainers show a prevalence of this (2006). The fact that he has to drop back to ‘one’ when he worked with number means that he did not reify numbers yet (Sfard, 2008). For backward counting, he is on level 0. He cannot count from 10 to 1. He cannot produce the BNWS from any given number. The subtractive tasks in the LFIN show us that James did not know what 16-12 means. He could not answer the question 9 - 4. This correlates with his answers in T he ANA tests. He could not answer both subtractive tasks in the ANA correctly. He was on the level 1 for numeral identification; meaning that he only knew numbers 1 - 10. He did not see ‘one ten’ as the same as ‘ten ones’ at the same time. He could not count on from a given number and he could not count down from a given number. He could not subitize numbers 1-4, which means that he needs to count the dots on the cards one by one.

James scored a high 66,6% for the ANA test. Evidence of concrete counting is visible on his ANA script. This and the results above show that he did not reify numbers and is still depending on concrete methods to find his answers (Sfard, 2008).

Learner 3 (Happy) showed a lack of awareness of the repeating structure of the counting sequence in the 1-100 range. When counting on from 48, he says: ‘forty-eight, forty-nine forty-ten, forty-eleven...’ He could count from 1-32, but then he did not know how to proceed. He appeared to be familiar with early number as a list: one, two, three, four, five, six... He could say the NWA in the range of 1-10. When asked what comes after 23, he answered 22; after 19, he first offered 8; and then answered 18. After 29 he counted twenty ten; after 65 he moved to 75; after 32 he counted 31; and after 70, 80. Here, there could be confusion between NWA and NWB (Wright et al., 2006). The mix of answers indicates a lack of solid awareness of the forward counting sequence. He arranged the number cards in the LFIN test as follows: 47, 49, 54, 48, 53, 46, 50, 55, 52, 51. He had no idea of how to count with bigger numbers. However, he arranged the numbers 1 to 10 in the correct way. This correlates with the high score he has in the ANA test, where there is a low number range. He can count backwards form 10 to 1, but when trying to count downward from any number bigger than 10, he had difficulties in doing that. He could not count down from 15, 23, 34 and 72. The 'number word before' was very difficult for him, because he had to count from one to find the answer. He attained some correct answers, but when he counted, he 'sang the song' and lost track of which number he was at, and counted 'over' the number. When the interviewer asked him, what comes before 67, he started to count from one, and answered 33. The fact that he started to count from one showed us that he did not have an idea of what the number 67 meant. He did not have a good understanding of 67 and he did not know how long it will take to get to 67. After the interviewer asked him what came before 50, he started counting from one again. He did not get to the answer. He did not use any abstract calculations to answer the questions. It can be observed that on the video that he used his fingers to find answers to the questions. When the interviewer asked him what comes before 7, he started counting on his fingers from one and answered 6. In the missing addend task, Happy can produce the correct answer for 4 + ? = 6, however, 7 + ? = 10 and 12 + ? = 15, he could not answer correctly. He answered the missing subtrahend task incorrectly, without thinking. The same happened with the removed item task. He answered the question immediately, however, the answers was wrong. He could identify of some of the numbers bigger than 30, however for 80 he said 28. He could not identify 21; he said 22. Numbers between 1 and 10 were easy for him to identify. He did not see 'one ten' as the same as 'ten ones' at the same time. He could not count on from a given number and he could not count down from a given number. Ensor et al. (2009) argue that when this happens, children are using concrete methods to find answers and do not move on to abstract methods to find the answers.

For the ANA test, Happy scored 72,2% for the number items in the ANA test. He could produce the sequence from 3 to 11, however, he wrote 01 for 10. He did not produce the number word for nine, but he was able to represent 7 objects. He produced a nine for the scissor on the number line, which indicated that he understands the cardinality of 9. His performance on the items in the ANA test indicates that he is familiar with numbers in the number range 1-30, which is the number range of the ANA test. Question 1.2 is in the range of 20-100, but it is counted in 10's. 20, 30, 40, 50. In the ANA test, question 5.2 asks 18 - 4 = ? Happy's answer was 7. This correlates with the LFIN tests results. However, in question 7.2, which asks 10 – 2 - 2 = ?, Happy gave the correct answer of 6. The number range of this question is 10 and under, and it is confirmed that Happy was more sure of the numbers from 1-10. He could identify some of the numbers bigger than 30, however for 80 he said 28. He could not identify 21; he said 22. Numbers between 1 and 10 are easy for him to identify. Like some of the learners above, he did not see 'one ten' as the same as 'ten ones' at the same time. He could not count on from a given number and he could not count down from a given number. Ensor et al. (2009) argue that children are using concrete methods to find answers and do not move on to abstract methods to find the answers. Happy stands as an exemplary of this. Happy's ANA results prove sufficient, and no one would think that he has a problem with number sense. The fact that Happy did so well in the ANA test, even though he does not have abstract tools to help him, is because the number range of the ANA tests were 1-34. Concrete methods can be used because the numbers are small.

Sipho (Learner 2) scored high in the LFIN test and low in the ANA test. He could count from 1-32 and he could count on from any number between 1 and 100. He used sometimes non-count by one procedures to find the answer to addition tasks, however, he also used count by ones strategies (Wright et al., 2006). Sipho could produce the FNWS in the range of 1-100, he could also produce the NWA for a given number in the range 1-100. He could produce the BNWS for numbers from 19 to 1 but not for numbers higher than 19, however he can produce the NWB for numbers in the range between 1 and 100. He could identify numbers in the range of 1-20. He could answer 3-1=? and 6-2=? in the LFIN, and he used his fingers to find the answer to 9-4. He answered the question 15-3=?, immediately, which showed us that he could answer the question without concrete counting (Wright et al., 2006). This indicated a more abstract understanding. However, he could not answer 16-12=? and 17-14=? correctly. He had an awareness of 'ten' as a object, because he does see 10 as a unit. This is noted because he could find the numbers that goes with a given number to make ten in different ways. He has a partially abstract view of number and he has reified numbers in the range of 1-6 (Sfard, 2008). Evidence for this is that he subitized numbers 1-6 by saying the number words of the number of dots on the cards in the second LFIN test (Wright et al., 2006). Whilst he seems to be able to deal with cardinality of number, ordinality appears to be a problem (Gelman & Gallistel, 1986). The reason why he scored so low in the ANA test is not clear. He does not use the knowledge and understanding of count-on and backward counting and his partially abstract understanding of numbers to his answers in the ANA test.

Sipho scored 63,6% for the number items in the LFIN test and scored 44,4% on the ANA test. He could not use his understanding of number in the ANA test. He could not produce a nine for the scissor in question 3 of the ANA test, which indicates that he does not understand cardinality and position. He produces 9 objects to represent the symbol 7, which indicates that he does not have a good awareness of number identification.

The learner with the most abstract understanding of number in this study is Cynthia (Learner 1). Cynthia, in many instances, appears to produce answers as recalled facts. This is because she showed no concrete working with her fingers or counters, and can produce answers immediately. It was not possible to determine whether she used mental facts or mental strategies (Beishuizen & Anghileri, 1998). It appears on the video only that she could answer some additive tasks immediately. This suggests the fact that she used mental procedures or that she knows the answers as facts. She is at SEAL stage 3, because she could not find the answer for the supplementary task 17-14 correctly in the LFIN test. Cynthia was able to count forwards fluently in the 1-100 number range, but proceeded more hesitantly with numbers beyond this, using her fingers to keep track of her count for numbers bigger than 100. This fluency extends to being able to name the number word after a given number in the 1-100 range, and is further backed up by her written answers to questions 1, 2, 3, 6, 7 and 9 of the ANAs – all of which require awareness of the forward number sequence. She could not count backwards from 72, however she could say the ‘number word before’ 24, 17, 20, 11, 13, 21, 14, and 30; and she could give the ‘number word before’ (NWB) 67, 50, 38, 100, 83, 41, and 99 correctly. This indicates that Cynthia has constructed methods or strategies to deduce a given NWB. Cynthia could count backwards; however, she did not use this ability to answer subtraction questions. She did not know that she could use backward counting to do subtraction questions. From evidence in the LFIN test, it is possible to observe some fragility with subtraction. Because Cynthia gave some answers after a while without using her fingers, it is difficult to find evidence of how she arrived at her answers. She could not answer 17-14=? correctly. She could answer the questions on NWB of a given number correctly in the number range 100 to 1; however, counting backwards shows some gaps. She identifies numbers up to 100. She could say the number words for the symbols, however, could not identify numbers bigger than 100. She could order and identify the cards from 46-55. The fact that she could see the number of dots on the domino cards is also an indication that she has an abstract understanding of numbers in the range 1-10.

Cynthia scored 83,3% on the number items in the ANA test and 77,3% on the LFIN test. There are no questions that focus explicitly on the backward counting or the counting-down sequence in the ANA test, but producing the correct answer to 10 - 2 - 2 =? suggested that she was able to work with BNWS in the 1-10 range. Despite this, Cynthia could not answer Question 5.2 in the ANA test where they ask 18-4=? correctly. Question 1 and 2 of the ANA test relate well with this question in the LFIN test, where ordering and identifying are at issue. Only two learners out of 29 learners answered Question 9.2 correctly. Cynthia is one of them. The results in the ANA and LFIN tests show that Cynthia has some abstract understanding of number for addition; however, she did not have an abstract understanding for subtraction. I say this because in the LFIN test, she uses most of the times mental calculations to find the answers to addition questions, but she cannot answer all subtraction questions correctly.

The difference between Learner 3 (Happy), Learner 4 (James) and Learner 5 (Jenny) is that Happy could produce the FNWS past 29 to 32. A similarity regarding counting between Learner 3, Learner 4 and Learner 6, is that all of them are at SEAL Stage 1. Learner 5 is at Stage 0. Learners 3 and 6 can count from 1-32, but Learner 4 and Learner 5 can only say the FNWS in the number range 1-29. Learner 4 could find the NWA for numbers between 1-24 but counted from one to find the answer, and Learner 3 gave answers that made no sense. The curriculum stipulates that children that is in Grade 2 first term, need to be able to count from 1-34. Jenny and James, Learners 4 and 5 did not fulfil the requirements of the curriculum, however Happy and Buzi did as they can count past 29.

The next chapter offers conclusions to this study.

Chapter 7 - Conclusions

Many researchers highlight the fact that there is a general problem with number sense in South Africa (Ensor et al., 2009; Fleisch, 2008; Schollar, 2009). My sample shows that the the results in the Grade 1 ANA test are good overall, but the LFIN results indicate that the learners do not have a reified understanding of number. Ensor et al. (2009) argue that children are dependent on concrete counting to find answers to tasks that involve numbers in the Foundation Phase. The findings of this study corroborate this, with my data also highlighting that some of the Grade 2 children in my sample cannot yet count and do not understand the 1-1 principle (Gelman & Gallistel, 1986). Jenny (Learner 5) can say the number words from 1-29, but cannot count concrete objects. All the other 5 learners can count concrete objects. Learner 3 (Happy) scored well in the ANA test, but did not score well in the LFIN test. He does not have an abstract view of numbers because he always falls back to concrete counting. He does not use mental calculations or calculation procedures to find his answers. He always starts to count from one to find the answers to questions in the LFIN test. He uses tallies in the ANA test to find answers to the questions. All 6 learners use their fingers at some points to find answers in the LFIN test, which indicates the presence of some concrete counting amongst the stronger, as well as weaker learners. This would suggest that more detailed guidance on supporting stronger learners towards more abstract number concepts would be useful.

My findings also point to the fact that children can do well in the Grade 1 ANA test without having a good number sense. Because the number range of the Grade 1 ANA test is 1-34, reflecting the range stipulated in the curriculum documents, all the questions can be answered with the help of tallies or fingers. The high performance on the Grade 1 ANA tends to obscure the prevalence of concrete counting strategies. Children can answer most of the questions with a concrete understanding of number. The literature notes that it is important to start developing abstract view of number in Grade 1 and 2 and answer the questions with mental calculations and operations. If they do not have an abstract view of number and have procedures in place for small numbers, they are going to have problems when the number range gets bigger. Then it would not make sense to try to count 500 counters. My data shows that children lose track of their counting when the numbers are bigger than 30. Learner 3

(Happy) is a good example of losing track when he starts to count from one to find the NWA 99.

The above highlights the limited nature of ANA information in a concluding commentary across all learners. It is possible that the results of each child can show a false picture of the number sense of that child. The ANA test is a summative test, where only the last part of the response to the question is important, and that is the answer. If a teacher looks only at the answer, she will therefore not have insights into the child's conception of number and thus the teacher will not know if a child has an abstract or a concrete understanding. The process to the answer is important, because then, the teacher knows what the child understands, and the level of reification that they are able to achieve. Children have to reify numbers in the early years, because this is the start of reification of bigger numbers, which are important for the higher grades. When the numbers are bigger, concrete methods are impossible.

Teachers can help children to reify numbers by avoiding the ongoing provision of resources for concrete counting (Ensor et al., 2009) and building connections across tasks in ways that encourage avoiding recounting (Askew & Brown, 2004). Sfard (2008) argues that children reify numbers when they can use a number without counting it again. When children keep on counting numbers from one repeatedly, they have not been able to reify numbers. They can teach themselves to count-on from a number to find the answer to addition questions and to count down to find the answer to subtraction questions. The first thing teachers need to know is what the child knows and understands. The child must move forward to an abstract understanding and the teacher must not persist in taking the child back to concrete counting if the child is competent in concrete counting. The teacher must be able to take the child to higher level of understanding. I believe that we need to perceive the child's misunderstandings and misconceptions in order to find a process to intervene and develop the child to a place where he has a better sense of number (Wright et al., 2010). It would be useful to do the LFIN test with individual learners with problems in numeracy to try to find out more about the understanding of that learner.

In the Gazette where the ANA tests were introduced, the minister of Basic Education stated that she is interested in the input of Higher Education Institutions (Government-Gazette, 2010). As seen in the analysis of the ANA test against the LFIN test, it is clear that there is significant emphasis on the FNWS, but little sign of the BNWS in the ANA.

Counting backward is very important for subtracting, and more focus on BNWS can help students to do subtraction. More focus on BNWS in the curriculum is important. Learner 1 (Cynthia) has difficulties with backward counting and the results were fragility with subtraction. Another finding suggest that the 'mental calculations' section that we see in the curriculum's foundations of learning (DOE, 2008) should be described in more detail.

The ANA results seem to be good in terms of the performance of my learner sample, but looking at the results of the LFIN tests, we can see that most of the children in this study have difficulties with reifying numbers - and through this, with achieving an abstract understanding of numbers. International literature states that we need to identify children that fall behind with early numbers, but looking at my data, it seems that the problems with numbers is not only a few, but it is the norm.

I started my research report in chapter 1, with the claim that everybody is entitled to a basic education, which includes 'number sense'. Concrete number sense is not enough. A person with a concrete understanding of number will be limited in life and will not be able to do abstract algebra, for instance in higher grades. South Africa has to focus more on the lower grades, for progression in the higher grades to be assured.

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