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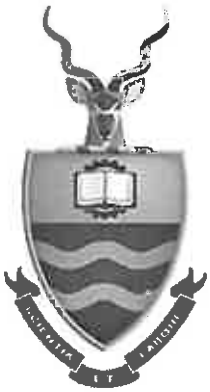
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Research project 2017



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A qualitative study exploring South African high school teachers' beliefs about teaching critical thinking in mathematics.

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Abstract

This study focused on Mathematics teachers' beliefs about teaching critical thinking and how these beliefs appear to inform teaching Mathematics in practice. Two sets of interviews were conducted that investigated what conceptions of critical thinking, teaching strategies and tasks, and assessment methods, teachers believed promoted critical thinking in their Mathematics classroom. Qualitative methods of inductive analysis were used to analyse teachers' responses based on grounded theory where categories were constructed and re- organised into themes. The study found that teachers had a limited understanding of critical thinking and their beliefs about teaching critical thinking skills in Mathematics did not appear to be reflected in practice. These newly qualified teachers demonstrated that they need assistance in developing and using teaching strategies and tasks that promote critical thinking skills in Mathematics.

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Chapter 1

Introduction

Critical thinking skills which are a central component of higher order thinking have been recognised by policy makers and researchers as one of the most important educational goals in modern societies (Lewis and Smith, 1993; Pithers and Soden, 2000; Ab Kadir, 2007). Educational reforms over the last decade identify critical thinking skills as crucial and necessary to develop in learners who are destined to be part of the ever increasingly complex information rich societies. Critical thinking skills have long been regarded by researchers, educationalists, employers and policy makers as core life skills which are essential for learners in a post school reality that demands individuals to be competitive. They are useful because they help learners adapt in an ever evolving job market which requires them to make important life choices for their benefit and the societies they live in (Pithers and Soden, 2000; Alwadai, 2014; Lai, 2011). For these reasons it is important that a modern educational system like we have in South Africa that aligns itself with similar objectives should ensure that the goals that are consistent with critical thinking are met or taught in the classroom.

Despite widespread consensus on the importance of critical thinking, there is still a lack of agreement among researchers in defining critical thinking (Innabi and El Sheikh, 2007; Kuhn, 1999). Critical thinking is often taken to include components of skills that can be used in making decisions and judgements such as in deciding what to do or to believe (Lai, 2011). One example of the many definitions of critical thinking is that it is a component of skills "that enables seeing different sides of arguments, being open to new knowledge that challenges beliefs and ideas, reasoning dispassionately, demanding that claims are backed up by factual evidence, deducing and inferring conclusions based on facts, solving problems and a host of other thinking skills that encourages good thinking" (Willingham, 2008, p. 21). Critical thinking as argued by researchers involves cognitive skills as well as dispositions. These dispositions can be seen as habits of minds like fair mindedness, propensity to seek reason, flexibility, and reserving judgements by first looking at diverse view points as well as other attitudes that constitute good thinking (Lai, 2011). Despite the lack of a single definition in the literature, which will be discussed in the following chapter, critical thinking is often referred to and also held in high regards with higher order thinking skills because of the important benefits in learning these skills.

The objective of this study is to find the connection between Mathematics teachers' beliefs about teaching critical thinking and how those beliefs appear to inform their teaching and assessment of Mathematics. This is because learning Mathematics is seen by Schoenfeld (1992) as empowering. Schoenfeld (1992) argues that learners who have a deep understanding of Mathematics are then, among other skills, capable of interpreting the large amount of quantitative data that is the daily experience of an information rich society and making balanced informed judgements based on those interpretations. They are capable of using Mathematics in many practical ways like budget management and statistical analyses. They are also flexible thinkers who are analytical and through a repertoire of skills, perspectives as well as techniques can think for themselves in examining arguments from different sources with careful and thoughtful consideration of different issues and make informed decisions and judgements about what is acceptable or not. These abilities and skills among many are consistent with critical thinking and educational goals of societies which desires citizens who are informed and good thinkers. For this reason Mathematics teachers who are tasked with teaching mathematical skills which are very similar and constitute important elements of thinking, are primarily responsible with overseeing that learners are taught these thinking skills as they are of critical importance to society.

However despite the recognised importance of critical thinking as an educational goal, evidence in the literature suggest that teachers are not aware and do not pay enough attention to critical thinking and do not know how to appropriately teach or assess the required associated thinking skills to learners (Halpern, 2001; Innabi and El sheikh 2007). Critical thinking is understood by researchers and educationalists to be the product of intentional teaching requiring specialised classroom instruction, task design and assessment (Kuhn, 1999; Halpern, 2001; Beineke, 1985). According to Snyder and Snyder (2008) most of the current barriers to teaching critical thinking are due to the educational trend of standardising curricula. This trend Snyder and Snyder (2008) argue promotes a conception of teaching that focuses on test scores and detracts the teacher's ability to address critical thinking skills as often the focus is on the content away from learner centred instruction. The focus or emphasis on content distracts the learning process and teaching of critical thinking through pressure on teachers and learners from strict accountability measures such as the amount of time a certain area of content should be finished and assessment mark levels to evaluate competence of understanding content. Often, as South African Mathematics teachers would attest, the amount of content and the time given to teach, promote and importantly properly assess thinking skills is limited because there is not enough time to allow

for learners to freely explore the content, analyse resources, make reasoned judgements about their grasp and understanding of content, apply their knowledge and other important thinking skills that are critical as well as important to learn. What is now often the case is the undesirable conception of teaching and learning called 'teaching to the test' which is not consistent with the desired outcomes of the curriculum as it misses the point of what learning and teaching should be about, to promote and develop well-adjusted citizens who are critical thinkers who are knowledgeable and problem solvers among other skills. In this study I will be looking at Mathematics teachers' beliefs about teaching Mathematics and how they are connected with their beliefs about teaching critical thinking. In my analyses I will also look at whether such beliefs and conceptions of teaching mathematics appear to influence practice and whether these are consistent with the tenants of teaching critical thinking in literature and to what extent teachers are disposed to apply such beliefs about teaching and assessing critical thinking skills.

1.2 Context and background

The widespread acceptance of critical thinking is testament to its value. Modern societies in the 20th century through to the 21st century unanimously recognise it as a critical educational goal in education systems. This recognition is aligned with the aims and objectives of developing citizens who are thoughtful about their participation in academic, social and employment aspects of society (Pithers and Soden, 2000; Halpern, 2001). Critical thinkers possess intellectual autonomy which equips them with the abilities to make sound and well informed decisions by critically examining information and knowledge as well overcoming inclinations to any bias or fallacious thinking, among other negative influence (Ab Kadir, 2007; Alwardai, 2014; Halpern, 2001). It is this important component of critical thinking that I believe makes it valuable in Mathematics education in South Africa, particularly given the central role Mathematics holds in the country and the current worrying state of teaching. Within the current and last decade, Mathematics literacy of learners in South Africa is rated as second last in the countries that participates in the Trends in International Mathematics and Science Study 2015(Reddy, Visser, Winnraar, Arends, Juan, Prinsloo, and Isdale 2015). Looking at the status Mathematics holds in many countries because of its usefulness and importance in many institutions of society especially those that influence economic development like science and technology, it is a worrying state of affairs for the future of this country.

In recognising the importance of an educational system that is consistent with global trends and developments South Africa has in the last few decades' implemented curricular reformations that recognise the value of critical thinking as in the South African Curriculum Assessment

Policy Statements (CAPS). One of the primary directives of CAPS amongst promoting a globally competitive school curriculum is to remedy social injustice and to ensure that the highest basic education qualification holders, the National Senior Certificate (NSC) are critical thinkers. The CAPS main aims and objectives are to endow the country's future citizens with practical and thinking skills which will allow them to meet the demands of the ever evolving economy in the age of globalisation and technology (Pithers and Soden, 2000). It lists in its aims and objectives that teaching and learning should promote "Active and critical learning: encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths" (CAPS: Mathematics Grade 8-12:4 2011). Active and critical learning are key in teaching that encourages critical thinking in learners and in ensuring rational future citizens who value and can critically examine knowledge and information (Barak, Ben-Chaim and Zoller 2007).

Key to meeting the objectives of the curriculum in South Africa was to make Mathematics or Mathematics literacy compulsory for a school leaving certificate. The primary reason for this might be recognition of the importance of mathematical skills which can be applied in a variety of situations and are widely used in many disciplines (Kranz, 2007). Mathematics as a subject or discipline is the most consistently precise science through its logical and reasoning structure which allows it to be a very powerful system of investigating and fact-finding if done critically. Mathematics and mathematical skills are required for employment, in tertiary education and even in everyday life as shopping or buying a car and any activity that requires intricate use of numbers, relationships, and analyses of patterns or many skills rooted in mathematical principles. Mathematics in CAPS (Mathematics Grade 8-12:8 2011) is defined as "...a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem-solving that will contribute in decision-making." This definition and the status of the discipline in promoting thinking skills places Mathematics as a core subject in meeting the aims and objectives of the curriculum because it recognizes the personal and social aspects of the subject as an important to tool or system of principles that can be used to develop good or useful thinking habits.

Teaching Mathematics as it pertains to this study and whether it meets the standards of what can be considered as appropriate in teaching or even assessing learners as critical thinkers will be drawn from literature on the subject together with similar studies exploring the same topic of

this study. The country's performance has been consistently low despite a lot of effort being directed to improving learners' performance as well as the recommendation that teaching of Mathematics ought to encompass critical thinking skills. Because of the definition of Mathematics in the CAPS document as well as the nature and structure of Mathematics, the expectation is that if teaching were to be consistent with what is stipulated in the curriculum aims and objectives, there would be more improvement in achievement. It is in this context that this study will be looking at a small sample of Mathematics teachers and their beliefs about both the teaching critical thinking and the teaching of Mathematics as a subject to understand the extent in which teacher's beliefs appears to inform their teaching in South African classrooms in light of findings sighted above.

1.3 Justification of study

Learning to think critically does not occur spontaneously. Empirical studies show that critical thinking skills are acquired through purposeful instruction and task design (Barnett, Jerrold and Francis, 2012; Innabi and El Sheikh, 2007;Tsui, 1999; Sezer, 2008; Ballew, 1967) . One such study by Lampert (2007) compared general critical thinking skills between college discipline groups, and found that fine arts and humanities students performed better than other disciplines when critical thinking skills were assessed. Problem-based inquiry with an emphasis on critical thinking skills like critical analysis and understanding multiple perspectives were found to be the biggest factor for this, which was how fine arts was taught and assessed. A study conducted by Price (1967) assessed critical thinking skills in Mathematics through designed discovery activities. Findings of the study showed a significant gain in the critical thinking skills of those taught the discovery method from a control group learning through traditional instruction. Another study looking at Islamic teachers' perception about teaching critical thinking in elementary schools found that there is a need to design teaching or assessment tasks and teaching strategies that help learners develop critical thinking skills as traditional methods were either less effective or not at all (Alwadai, 2014). What this, and many other studies, suggests is that critical thinking can and should be intentionally taught and assessed in the classroom to develop in learners (Barnett, Jerrold and Francis, 2012; Innabi and El Sheikh, 2007;Tsui, 1999; Sezer, 2008; Ballew, 1967). Because critical thinking coupled with critical learning and teaching are at the centre of educational goals in CAPS, it is important that mathematical teachers should meet knowledge requirements to teach and assess critical thinking since these skills are clearly needed in our country.

1.4 Research question

The study has one major aim which is to look at the connection between Mathematics teachers' beliefs about teaching critical thinking and how those beliefs appear to inform views of the teaching of Mathematics. This will be done through studying what teachers say about teaching critical thinking in Mathematics through conducting semi-structured interviews and the consistency in which they appear willing to apply their conception and beliefs about critical thinking in practice. The research question it will address is

What are Mathematics teachers' beliefs about teaching and fostering critical thinking skills in high school Mathematics and how do their beliefs appear to inform their practice?

1.4.1 Sub-questions

- What are high school Mathematics teacher's awareness of critical thinking and the pedagogical implications?

- How do teacher beliefs for instructional and tasks design in their practice indicate an effort to foster critical thinking in Mathematics?

- What are the challenges teachers identify as limiting or impeding their effort to teach and assess critical thinking in the Mathematics classroom?

1.5 Purpose of the study

The overall purpose of the study is to study teacher beliefs about teaching and enforcing/developing critical thinking in Mathematics based on what they say about those beliefs and how they appear to inform their practice. This will be done through looking at;

1. What conceptions of critical thinking do teachers hold and how do they relate to Mathematics. Teachers' conceptions of teaching Mathematics and critical thinking refers to the ideas or principles teachers speak about using in their practice and the teacher interviews will be analysed based on their consistency with critical thinking and related to conceptions of critical thinking based on literature and what the study recognises as critical thinking. Conceptions will also include how teachers define critical thinking and their awareness of important elements that are part of critical thinking in teaching Mathematics.

2. Teaching strategies and tasks that teachers identify as promoting critical thinking in Mathematics. The strategies and tasks that teachers speak about using will be analysed based on the extent they were explained or identified as encapsulating a broader or limited understanding of critical thinking strategies as identified in the literature. The analysis will also include

teaching techniques teachers speak about using in assessing critical thinking skills and interpreted in the context of the teaching strategies they use. A focus will also be placed on challenges that teachers identify as impeding their beliefs of enforcing critical thinking in Mathematics

3. How teachers assess learners in making judgements about the effectiveness of their teaching in promoting critical thinking in Mathematics. The focus of analysis will be on what forms of assessments teachers use in assessing their teaching as well as learners. This will be done drawing on teachers' beliefs, conceptions and awareness of intellectual criteria and standards and how this informs their assessment and teaching decisions.

Chapter 2 Review of related literature and theoretical framework

2.1 review of related literature

In this chapter I will present a review of literature on critical thinking, why it is important in education and especially Mathematics education, issues related to teaching of critical thinking, how critical thinking may be taught, obstacles teachers encounter in teaching critical thinking and a contextual summary of important literature as it relates to this study. I will then outline the theoretical framework which will inform the analysis of the study and discuss how it links with the topic of the research questions as well as literature on critical thinking and teacher beliefs.

2.1.1 Importance of critical thinking in education

Researchers and educationalists have in recent decades recognised the importance and value of teaching learners the ability to think about what they learn in their curricular subjects. It is recognised that learning to understand and apply the skills and knowledge that are being taught in school subjects requires learners to think about the meaning embedded in the subject content. Kuhn (1999) points out that critical thinking has been the major driving force which has shaped most curriculum reforms in response to the need for learners to think about what is learnt in schools. What is believed by both researchers and educationalist is that thinking is a skill that can be taught (Facione, 2000; Lewis and Smith, 1993; Lai, 2011; Alwadai, 2014; Innabi and El Sheik, 2007; Halpern, 2001).

Teaching higher order thinking skills has been a central objective of many educational reforms of democratic society (Pithers and Soden 2012). The need for learners who are rigorous, careful thinkers as an outcome of educational objectives is a widely accepted notion about schooling (Ennis, 2012; Lai, 2011; Alwardai, 2014). Educational institutions are tasked with the responsibility of developing such thinking skills. This is in the interest of safeguarding and promoting the rapid advancements of knowledge and technology as well as meeting challenges of modern societies (Abrami, Bernard, Borokhovski, Waddington, Wade, Persson, 2015; Pithers and Soden, 2012; Willingham, 2008; Vieira, Tenreiro-Vieira, Martins, 2011). Reformation of curricula in democracies have been consistent with the principles stated above at the basic schooling level in prioritizing efforts towards realizing and achieving these educational goals.

The need for critical thinking skills which are a subset of higher order thinking skills as part of learners' skills is based in the recognition that good thinking is an important core life skill for

the individual and for society. Higher order thinking skills are believed to be essential in developing those cognitive and behavioural skills in school learners who go on to be useful in both personal and public life as citizens of a democratic society (Alwadai, 2014). Facione (2000) argues that if the educational goals are for an intellectually virtuous society, it is critical that educational goals are aligned to higher order thinking skills such as critical thinking and dispositions, as simply focusing on specific skills and knowledge alone is not enough. Critical thinking is of particular importance in educational goals in recent years because it is central to higher order thinking skills in encouraging sound judgements in what to do and believe. What is now argued is that as educational systems and goals are aligned with encouraging critical thinking learners, this should be reflected in instructional assignment as well as educational outcomes in assessments to ensure that learners meet the set goal (Innabi and El Sheik, 2007; Ab Kadir, 2007).

2.1.2 Critical thinking and other thinking skills

Lewis and Smith (1993) discuss some of the skills teachers have often been asked to develop in the classroom and that are identified as important in achieving educational goals. The skills which are often identified as important to promote include critical thinking, higher order thinking, problem solving, rational thought, and a host of other thinking skills. These authors note that there is a lot of difficulty in defining thinking skills, reasoning, critical thinking, creative thinking and problem solving. An important distinction between the mental skills which Lewis and Smith (1993) discuss is that critical thinking, creative and problem solving are not the same but rather they are interdependent in that critical thinking incorporates both creative thinking and problem solving amongst their skill set.

Critical thinking because it encourages purposeful and deep thinking as discussed above in some literature is often conflated or confused with thinking skills such as higher order thinking, creative thinking, and problem solving (Lewis and Smith, 1993; Alwadai, 2014). Such terms are often thought of and used synonymously simply because they all refer to some mental skills or thinking processes. Critical thinking is a rich concept which has many different overlapping definitions which are multi-disciplinary and multi-contextual encompassing definitions of different thinking skills of intellectually disciplined symbolizing higher order thinking. In its most basic form, critical thinking is about making judgments or decisions which are based on universal intellectual standards and criteria of thinking about any subject, problem or content in which one engages in self-directed, skillful analysis, self-disciplined, self-correcting and self-monitoring as a guides to belief and action. Critical thinking is about developing thinking skills

that would allow learners to deal with social, ethical, scientific and many everyday practical problems in a meaningful and thoughtful way (Snyder and Snyder 2008).

Like higher order thinking skills, the use of critical thinking encompasses universal intellectual standards and criteria these intellectual standards include such as accuracy, reasoning, precision, consistency and reliance on good evidence among other intellectual standards and criteria. Which much like Mathematics transcends subject matter in how they are used as the basis of being a critical thinker. Mathematical thinking has been found to be effective decision making and/or judgment, solving problems (Facione, 2000; Alwadai, 2013; Lai, 2011). An example of a definition of critical thinking that demonstrate similar applicability of mathematical skills given by Ennis (1985, cited in Innabi, El Sheik, 2007 p. 46-47) is that "1) critical thinking starts as problem-solving process in the context of interacting with people and the world; 2) it continues as a reasoning process, informed by background knowledge and previously accepted conclusions, and it results in drawing a number of inferences through induction, deduction, and value judging; and 3) the critical thinking process ends with a decision of what to do or believe." The use of critical thinking skills and meaning of the above definition may be universal in applicability like mathematical skills, however learning and applying such thinking skills differs in many contexts and how they are used in many disciplines dependent on domain-specific knowledge such as methodological principles or competences. The use of critical thinking skills and mathematical skills by a physicist is not necessarily the same in his discipline as those of an economist or any person who engages in any everyday task like shopping in which they may use critical thinking or mathematical skills. Critical thinking can thus be said to be encapsulated or interwoven by different modes of interdependent thinking among them: higher order thinking skills, mathematical thinking, scientific thinking and moral thinking to mention just a few, which are self-guided and self-monitoring thinking skills whose primary goal is to develop and encourage reasoning at the highest level or quality while rooted in fairmindedness and the willingness to exercise critical thought in everyday experiences (Lai 2011; Alwadai, 2013).

The definition above by Ennis is one of many other definitions which identify a myriad of processes or skills thought to be central to critical thinking but also other thinking skills like problem solving. As argued above critical thinking encompasses intellectual standards of reasoning and thinking skills making it an important and necessary element of quality thinking that promotes rigorous standards of intellectual excellence and mindful command of their use in

making judgements on any subject or problem (Alwadai, 2013; Lewis and smith, 1993; Lai, 2011). Logical thinking, rational thinking, reflective thinking, evaluative thinking, analytical thinking and a whole many other different types of thinking are all essential parts of critical thinking which when learnt predisposes one to be a good thinker. Good thinking encourages effective problem solving, honest communication, and the commitment to seek and overcome our predisposed native misbeliefs, misinformation, irrationality, biases, prejudice, and other self-delusions which negatively affect our thinking, knowledge and way of life due to the lack of coherent intellectual standards of thinking to which the accuracy and adequacy of our thinking can be assessed. This is what makes critical thinking very important to develop in schooling as well as a subject like Mathematics because such thinking skills and intellectual standards promote and encourage the development of many elements of what could be considered good thinking which will be discussed in sections to follow. Good thinking the way it is defined in this study refers to the ability and disposition to using appropriate kinds of thinking in deciding what to do or believe through using critical thinking skills and intellectual standards of thinking.

2.1.3 Perspectives of critical thinking

There are two perspectives on critical thinking that are relevant to this study. They are the psychological and the philosophical perspectives which both define critical thinking skills and abilities. I will describe and discuss their differences in defining critical thinking for this study.

2.1.3.1 Psychological perspective

Psychologists focus on the processes of cognition that include the components of thought and cognitive operations used to address academic and practical issues like problem solving and decision making (Ab Kadir, 2007). Cognitive and developmental psychologists in defining critical thinking tend to focus on how people actually think instead of how they should or could think. These include symbolic activities that are geared towards productive ideas and analysing and synthesising arguments for cognitive biases. Rather than focusing on characteristics or standards and criteria to define critical thinking, psychologists identify a number of thinking skills and procedures of good thinkers based on action and behaviour that

one can do through a list of skills and procedures (Lewis and Smith, 1993; Lai, 2011). Their major definitions are (Lai, 2011 p. 8):

- "the mental processes, strategies, and representations people use to solve problems, make decisions, and learn new concepts (Sternberg, 1986, p. 3)

- the use of those cognitive skills or strategies that increase the probability of a desirable outcome (Halpern, 1998, p. 450)

- seeing both sides of an issue, being open to new evidence that disconfirms your ideas, reasoning dispassionately, demanding that claims be backed by evidence, deducing and inferring conclusions from available facts, solving problems, and so forth (Willingham, 2007, p. 8)"

Psychologists' main concern is the process of good thinking and how it can help people make sense of their experiences through constructing meaning and imposing structure. A critical thinking disposition in psychology emphasises problem solving rather than reflective thinking where decisions and judgements are made rationally (Lewis and Smith, 1993). If a learner uses critical thinking skills it provides evidence that they are disposed to use the skills. Bloom's hierarchical taxonomy of cognitive processes structures is an example of a psychological approach that informs task design and problem solving with some levels with direct relevance to the psychological definition of critical thinking (Ab Kadir, 2007). One major criticism of this perspective is that it is reductionist in its simplification of the complex interaction of knowledge and skill by reducing it to a collection of procedures and steps (Lai, 2011).

2.1.3.2 Philosophical perspective

Philosophers emphasise good thinking as logical reasoning and perfection of thought to decide what to do or believe (Lewis and smith, 1993; Ab Kadir, 2007). Their conception dates back to the time of Socrates and Aristotle who promoted an approach to discipline thinking to guard against accepting false claims as well as unquestioned truths (Innabi, El Sheik, 2007). This approach idealises a critical thinker through defining the characteristics of behaviour or action of a critical thinker. Examples of behaviours that idealise what it means to be a critical thinker include being open minded, a desire to be well informed, willingness to suspend judgement while considering other perspectives and other dispositions that emphasise quality and standard of thought (Lai, 2011; Lewis and smith 1993). The philosophical perspective emphasises the use of intellectual standards and criteria of adequacy and accuracy of thought as a means to assess or judge critical thinking. The major definitions from the main proponents of this perspective define critical thinking as (Lai, 2011 p. 6)

- "the propensity and skill to engage in an activity with reflective scepticism (McPeck, 1981, p. 8)

- reflective and reasonable thinking that is focused on deciding what to believe or do (Ennis, 1985, p. 45);

- skilful, responsible thinking that facilitates good judgment because it 1) relies upon criteria, 2) is self-correcting, and 3) is sensitive to context (Lipman, 1988, p. 39)

- purposeful, self-regulatory judgment which results in interpretation, analysis, evaluation, and inference, as well as explanation of the evidential, conceptual, methodological, criteriological, or conceptual considerations upon which that judgment is based (Facione, 1990, p. 3);

- disciplined, self-directed thinking that exemplifies the perfections of thinking appropriate to a particular mode or domain of thought (Paul, 1992, p. 9);

- thinking that is goal-directed and purposive, "thinking aimed at forming a judgment," where the thinking itself meets standards of adequacy and accuracy (Bailin et al. 1999b, p. 287)

- judging in a reflective way what to do or what to believe (Facione, 2000)".

The philosophical perspective's major concern is disciplined thinking which commits to impartiality and objectivity, relevance, consistency, and searching for defensible reasons for behaviour (Lewis and Smith, 1993). One of the major criticisms of this perspective is that its use of standards and criteria often do not correspond to reality. This, according to Lai (2011), is because of the idealising of a critical thinker through actions and behaviour with the capacity of meeting standards and criteria is limited in revealing how people actually think.

2.1.4 A unifying conception of critical thinking

As the preceding indicates, there is still no accepted universal conception among researchers of what constitutes critical thinking. However, although there are differences in conception, both psychological and philosophical perspectives have a large area of agreement of what is a critical thinker. Both perspectives agree upon a mutual goal, to help learners develop skills of reasoning and logic to become good thinkers in making decisions of what to do or believe. To fully encompass the goal in an educational sense both perspectives are essential to complement one another in a number of important aspects and both can lead to similar qualities of thinking (Lewis and Smith, 1993). There are varying ideas on both perspectives but there are evident overlaps in basic emphasis of what intellectual standards and criteria of adequacy and accuracy of thought and thinking skills that are of common importance (Ab Kadir, 2007; Bailin, 2002). The overlaps, to name just a few, include (Ab Kadir, 2007 p. 5)

- "a sensitivity to value- or ideology-laden assumptions;

- an insistence on appropriate supporting grounds before accepting disputable claims;
- an appreciation of the various criteria applicable to good reasoning and argument (whether general or subject dependent);
- skill and judgment in the analysis and evaluation of claims and arguments; and
- a disposition to be self-reflective, sensitive to one's own possible biases or assumption"

As argued by Schoenfeld (1992) and the CAPS document, learners who know and understand Mathematics should be able to develop the intellectual standards and criteria of thought outlined above. This is because unlike most disciplines as discussed earlier Mathematics knowledge and skills are based on logical induction or deductions which are based on empirically verifiable principles and evidence among others. This makes them ideal in establishing extractable intellectual standards through the discipline's system of logical tools which are very consistent and reliable. Mathematical tools are specifically useful for establishing intellectual standards as they are abstraction as well as symbolic representation and manipulation through a coherent system of logic and reasoning which means learning Mathematics is learning to think. In how thinking and learning Mathematics are intertwined, mathematical principles and valuing the processes through which it establishes its logical system are at the core of the subject. These are transferable to any discipline and would still be as coherent. No other principles of a discipline are as prevalent as the mathematical principles which makes the discipline very useful in teaching and learning of important intellectual standards and skills of thinking. This would include thinking that avoids idea-laden assumption, that bases its claims on logic and empirical evidence that can abstract and find patterns, and making reasoned judgements after analysing and evaluating claims which are all part of high school Mathematics and are included in the definition of Mathematics in the CAPS document.

There is further agreement between proponents of both perspectives regarding critical thinking skills and abilities with researchers in both fields agreeing that there is some difference between possessing such skills and one's ability to use them (Facione, Sanchez, Facione, Gainen, 1995; Bailin, 2002; Lai, 2011). Lai (2002) and Facione et al (1995) argue that empirical studies have shown that critical thinking abilities and dispositions do not necessarily cause one another. To develop the component thinking skills associated with critical thinking is not the same as using them. To make an example from the context of this study, to learn the principles and logical

system of Mathematics does not mean people can be logical and systematic thinkers in all contexts or activities they undertake. What both perspectives agree on is that there needs to be some internal motivation of a critical thinker to encourage an action or response to different situations in ways that are consistent and reflect critical thinking abilities. To focus on skills when talking about critical thinking in schooling or teaching contexts runs the risk of trivializing and characterizing critical thinking as just as some combination of techniques rather than the complex, purposeful and thoughtful process of formulating judgement through reasoning and evidence (Facione, 2000). What is called for by recent proponents of critical thinking in education is that emphasis is placed on the dispositional side of thinking as teaching the skills alone is not adequate to ensure their use, learners must demonstrate abilities to use thinking skills to show that they have learnt them. These dispositions which researchers have identified include (Lai, 2011 p. 11-12);

- "open-mindedness (Bailin et al., 1999; Ennis, 1985; Facione 1990, 2000; Halpern, 1998);
- fair-mindedness (Bailin et al., 1999; Facione, 1990);
- the propensity to seek reason (Bailin et al., 1999; Ennis, 1985; Paul, 1992);
- inquisitiveness (Bailin et al., 1999; Facione, 1990, 2000);
- the desire to be well-informed (Ennis, 1985; Facione, 1990);
- flexibility (Facione, 1990; Halpern, 1998); and
- respect for, and willingness to entertain, others' viewpoints (Bailin et al., 1999; Facione, 1990)."

Facione (2000, p. 64) describes dispositions as "a person's consistent internal motivation to act toward, or to respond to, persons, events, or circumstances in habitual, and yet malleable, ways." This means that knowing a person's disposition allows for discernment of how the person is more likely to act or react in a variety of situations. This is especially useful in teaching and learning since learners are at the center of the learning process, dispositions allow discernment of critical thinking abilities and skills by the teacher making them more assessable to examine the extent of their use in both the subject and in general. This is because the link between skills and disposition is that a person who is skilful tends to manifest those skills in performing a variety of tasks with fewer mistakes than a person who is less skilled (Facione, 2000). Learners

who are critical thinkers in Mathematics will be more disposed and inclined to using critical thinking skills in demonstrable ways such as making fewer conceptual errors, misconceptions, and many other mathematical errors that are created by inability to formulate complex and thoughtful processes of thinking about the principles and logic of Mathematics. Making critical thinking skills more actionable gives ways which can be assessed in a variety of learning situations which is important in assessment. Learners can be assessed and would be disposed to use mathematical skills like deductive or inductive reasoning, self-regulation, analysis, inference, interpretation and many other skills at the core of critical thinking and Mathematics in varied practical multi-contextual situations (Facione, 2000; Lai, 2011).

2.1.5 Challenges of unifying conception

As argued above recent curricular reforms have been driven by the need for educational goals to be aligned to encourage critical thinking within schools. Educationalists and educators have, in many aspects of the curriculum, reached a consensus of ensuring that critical thinking skills are taught through subject knowledge like Mathematics. However due to the lack of a unifying consensus on what constitutes critical thinking, much of the consensus gets eroded when these goals have to be concretely interpreted into the curriculum (Bailin, 2002; Ab Kadir, 2007). The lack of a unifying definition has led to problems in how critical thinking skills are described and interpreted as educational goals in the curriculum. An example of this is in the definition given in the mathematical CAPS document where logical thinking, problem solving, accuracy and decision making are listed as separate from critical thinking and not as part of it. This lack of a coherent description for critical thinking, as argued above, has been a major cause of breakdown in teaching critical thinking in the classroom.

There are often two main causes of this breakdown especially in mathematical education. The first can be attributed to the difference in emphasis between the psychological and philosophical conception of critical thinking as argued above. This has inadvertently led to the second cause of a breakdown which is about how critical thinking can be encouraged and taught in the classroom. Most work and literature on curriculum planning in Mathematics and science education regarding critical thinking tends to focus on particular skills, processes and procedures of critical thinking. These include argumentation, problem solving, scientific method and other similar skills, processes and procedures without any reconciliation of the skills and procedures as part of critical thinking (Bailin, 2002). Because of the different lists of skills, processes and procedures associated with critical thinking which are often treated separately, the task of teaching these skills is made that much greater than any Mathematics teacher can do on their

own (Pingry, 1951; Innabi and El sheikh, 2007). It is too much of a task for a teacher to be expected to focus on teaching and to reconcile the different skills, processes and procedure especially if there is no consensus from educational experts.

However conceptions of critical thinking which are used in work done for the purpose of aligning educational goals with critical thinking have, according to Bailin (2002), value in bringing useful insights, ideas, and informative pedagogical strategies together. But as argued above working on the basis of varied definitions, lack of clarity, vagueness and varied conceptions causes potential problems with educational implications in the long run. Critical thinking as argued by many researchers can be taught through explicit instruction that focuses on developing the ability to think critically as a goal (Innabi and El Sheikh, 2007; Lai, 2011; Alwadai, 2014). If, in contrast, teaching is grounded in different or vague conceptions of critical thinking, such teaching runs the risk of only focusing on certain thinking skills and not critical thinking. This is particularly a problem in Mathematics due to the nature of subject as it is important that the knowledge, skills and understanding are built in the area where learners are supposed to do the thinking (Pingry, 1951). It is important that learners have a rich background of knowledge, experiences and skills of Mathematics if they are to have any success of learning mathematical skill like problem solving. Skills, knowledge and understanding of Mathematics to develop and promote critical thinking, should be cultivated by teachers as part of the subject in the teaching and learning process. If learners are supposed to use critical thinking skills in social situations then they should be taught mathematical knowledge using social contexts. It should not be the case that in teaching a topic like geometry for example logical thinking is confined to critical thinking within Mathematics as a subject or a section of a topic, it should also be taught how logical thinking can be used in everyday situations effectively.

2.1.6 Adopted conception of critical thinking

Bailin (2002) argues from a philosophical perspective that in looking at the educational implications arising from the major definitions, as well as assumptions from the perspectives in how most researchers define and conceptualize critical thinking, means that there are problems with how critical thinking should be used as an educational goal in teaching and learning. Her major criticism of major definitions by all sides is that the definitions and conceptions such as the ones discussed above view critical thinking as cognitive and thinking skills which leads to the limiting of teaching critical thinking as focusing primarily on skills, processes and procedures. This limiting view of equating critical thinking with mental processes and procedures which must be practiced and performed is evident in educational research literature

and curriculums goals. Examples where critical thinking is conceptualised as skills and processes include instances where it is described as specific aspects such as formulating questions, problem solving, interpreting, analysing, seeking answers and communicating. Conceptualising critical thinking as either a mental process or procedure is problematic in that it fails to see critical thinking for the standard of norms of behavior it is. As argued above critical thinking is based on universal intellectual standards and disposition of thinking about any subject, problem or content in which one engages in self-directed, skillful analysis, self-disciplined, self-correcting and self-monitoring. A critical thinker is a person who can demonstrate the kind of dispositional behaviors in action which meet norms which are considered as adequate for critical thinking by their adherence to intellectual standards and criteria. Equating critical thinking with skills, processes and procedures is to fail to see that critical thinking is a normative concept that refers to good thinking (Bailin, 2002; Ab Kadir 2007). The normative nature of critical thinking according to Bailin (2002 p. 363-364) is based on "...the quality of the thinking which distinguishes critical from uncritical thinking, and this quality is determined by the degree to which the thinking meets the relevant standards and criteria. It is then adherence to certain criteria which is the defining characteristic of critical thinking." This is because for a person to be a critical thinker they must demonstrate the thinking skills that reflect the standards or behavior associated with critical thinking and not demonstrate thinking skills, behavior or standards that can be performed uncritically. There are qualities which thinking that a critical thinker should demonstrate that are distinguishable from thinking that is not critical such asking prompting or deep questions to get deeper understanding of issues or problems, an inclination to collect thorough and unbiased evidence when addressing issues, approach problems with an open mind by avoiding biases or fallacious presumption either personal or from outside sources and other normative qualities of good thinking.

Specifically in Mathematics the failure to see the normative nature of critical thinking can cause problems especially through viewing critical thinking as mental skill or procedure. To clarify, the term mental skill is based on mental activities which cannot be observed but can only be inferred from action. Learners' mental skills for example in doing Mathematics cannot be directly attributed to mental skills even if they are able to solve mathematical problems. An example of this is when learners are given a mathematical assignment in statistics where the purpose is that they have to go through the process of investigating a certain issue, collect data, analyse and evaluate the data, synthesise the information they get to solve a problem or come to a decision. To use terms like analyse, evaluate and synthesis in doing Mathematics does not

necessarily constitute mental activities but rather tasks that require thinking where the process can be inferred to have taken place after its completion. In other words it is possible to do the above mathematical activities procedurally without any thinking and fail to meet the standards of thinking that adhere to critical thinking as it is possible to do all these processes uncritically through superficially engaging with the process. An example of uncritical thinking is collecting insufficient or biased data, evaluating the data in a closed minded procedural manner, failing to recognise the biasness of methods used in collecting or analysing data, failing to recognise untruthful or untrustworthy sources, and making invalid or unwarranted inferences when reaching a conclusion (Bailin, 2002).

Evidence of and a case for the normative conception of critical thinking can be seen through its synonymous use to refer to good thinking by most researchers. Researchers have argued that critical thinking and subsequently critical thought can only be exhibited by people who have both the skills and disposition to think critically (Lai, 2011; Bailin, 2002). A person who has learnt critical thinking skills but unable to utilize them in thoughtfully or intentionally in a demonstrable way is not considered as a critical thinker. This is why attributing critical thinking to procedures and activities especially those which are routine and requiring no deeper thought is problematic as it is possible for learners to do Mathematics uncritically. Conceptions attributing critical thinking to procedures often define a set of behaviours or activities which are given as steps to be followed such as problem solving which can be systemized into steps that may not reflect critical thinking because they would lack the necessary intellectual thought processes considered as thinking critically. This would also lack the dispositional aspect of critical thinking that demonstrates good thought as a norm through making self-guided or self-monitoring thinking skills that encourage making good decisions and judgements (Lai, 2011; Bailin, 2002; Ab Kadir 2007). What is required is a coherent and consistent system that would distinguish in assessable ways what is a critical thinker and what a non-critical thinker is, especially in an educational sense.

Bailin, (2002) proposes the idea of defining critical thinking through intellectual standards and criteria based on the quality of thinking. The argument is that critical thinking should then be based on the quality that determines and demonstrates whether the standards and criteria meet the degree of being critical. Although proponents of the psychological aspect of critical thinking in education often don't emphasize the use of criteria or even standards to determine the quality of thinking the philosophical perspective emphasizes its importance in education. Bailin (2002) and proponents of the philosophical perspective maintain that there should be some intellectual

standard and criteria to judge or evaluate the adequacy and accuracy of thinking by the teacher and for the learners to evaluate their own thinking. Both maintain that some criteria is needed to evaluate arguments from other people, evaluating one's thoughts as well as beliefs, evaluating evidence, evaluating problems and how to solve them and other aspects that require some level of judgement or decision about adequacy or accuracy towards critical thinking. These criteria may be formulated into some intellectual standards of thinking on judging adequacy of thinking such as the strength of inductive arguments, the adequacy and consideration of moral and ethical issues in how one solves or approaches solution to complex problems. Criteria can even be made into some regulations or laws regarding thinking (Lai, 2011). These intellectual standards and criteria are argued for by Bailin (2002) and other proponents of the philosophical perspective in education because they communicate to learners and teacher what qualities of thought they should strive for such as relevancy, clarity, accuracy, precision, logic, consistency, fairness, significance and adequacy among many skills of good thinking. The proponents of the philosophical and psychological perspective can be reconciled in their belief that it is important that teachers inform learners on what intellectual standard is used to evaluate their work which is a very important and critical part of teaching them to become critical thinkers.

In reconciling the different persepectives of critical thinking, it is important tpo first recognise current problems with how it is defined in literature. According to Bailin (2002) the problems of defining critical thinking as skills, process or procedures, as just discussed earlier, is that these can often be applied without employing critical thought. Analysing, problem solving, gathering data, interpreting data and any aspect of critical thinking based on defining mental skills and procedures through inference can be done without meeting standards that would be considered as thinking critically. What would be considered as critical in such instances would be thinking that adhered to a certain standards and criteria to judge the intellectual performance. With respect to Mathematics this would mean that teaching would help learners build standards and norms of good reasoning or logic to help them realise that mathematical reason and logic is not limited to subject knowledge but can be applied to real life non-mathematical situations (Pingry, 2005). An example where this could apply in a real life context is when buying groceries where they can use Mathematics skills such as focusing on rands per unit quantity rather than size in calculating value for money or even considering healthy options of food by critically analyzing the ingredients of the food they buy. Mathematical and critical thinking skills and abilities can also be used in assessing statistical information as argued above for any bias, misuse and

misinterpretation through proper assessment and investigation of any inherent assumptions or reporting of data.

Paul (1997) cited in Thomas (1999) also argues for the importance of the use of universal standards and criteria like clarity, precision, depth, breadth, accuracy, logic, and relevance which can be applied when assessing reasoning about any problem, issue or situation in many subjects including Mathematics to promote good thinking rather than certain skills and processes. He states that one can be said to be a critical thinker by the degree to which they adhere to those standards especially in an educational context. He argues that this should be done in a classroom context through the teacher probing learners' reasoning through asking questions that would hold learners accountable for their thinking as well as help them become good and reflective thinkers. Such questions should be consistent in use by the teacher in instruction and assessment that they become internalised by the learner and become part of their thinking. Paul (1997) identifies logic, relevance, clarity, accuracy, precision, depth and breath as among the most important universal standards of assessing learners thinking and reasoning which can be applied and are part of core principles in teaching Mathematics and promoting critical thinking.

Bailin (2002) discusses well founded criticism of the conception of critical thinking through standards and criteria which can be made especially in instances where procedures and characterization are vaguely defined as it is possible in an educational context. She argues that it is possible to state clearly the procedures and characterization of the normative nature of critical thinking for specific activities or topics. This she argues can be done through using criterion in defining specific procedures and characterization of outcomes when deciding teaching and learning goals. The criterion and standards can then be judged as focused on critical thinking based on outcomes. This is of particular importance because no amount of description of critical thinking procedures and characterisation alone in Mathematics can ensure a normative dimension that can encapsulate the thinking skills as it is current practice in curriculum documents. Criteria cannot be simplified to a list of steps or actions but by standards of adherence. Mathematics teachers when deciding lesson outcomes need to draw emphasis on what is important and should be the focus of learning based on already established intellectual standards and criteria. This is useful in teaching Mathematics where thinking and concepts are abstract due to the nature of the subject because, it makes it possible to contextualize or tailor-make lesson goals to emphasize or promote certain standards of thinking based on how activities and problems are structured. Intellectual standards and criteria can then be described using the basis of mathematical concepts because the thinking skills are often embedded in the concepts.

An example of this would be using algebra or algebraic functions and even statistics to solve mathematical word problems and relationships encountered by learners on a daily basis which strengthen logical and reasoning skills.

Based on efforts towards reconciling the different perspectives and problems in current conceptions, it is important that in defining a conception of critical thinking for this study it must be noted that Mathematics and Science education are often about encouraging the development of critical thinking skills (Alwadai, 2014; Bailin, 2002). The content which is being taught in Mathematics serves in most cases to acquire and develop certain skills with disposition to use them. These skills like deductive and inductive reasoning, self-regulation, analysis, inference, interpretation and other similar skills are not learned through following processes and procedures but through deeper or critical thinking about knowledge in the process of teaching and learning Mathematics. Mathematical knowledge which is acquired in the classroom is emphasised through the use of certain criteria or standards of what learners should be able to do which are set by the policy makers in the curriculum and are interpreted and used by the teacher in the classroom to evaluate learners work. Since this study focuses on teacher beliefs about teaching critical thinking in Mathematics as well as the disposition for promoting these skills in their practice, critical thinking as defined in this study will refer to a standard or criteria of a person's ability and disposition to use appropriate thinking based on mathematical skills and principles in judging and making decision.

This study, based on the definition above, adopted a conception and definition of critical thinking that accepts both intellectual standards and criteria. Both this definition and its associated conceptions are based on the common agreements or overlaps between the philosophical perspective and psychological perspective with regards to critical thinking skills and disposition. Aspects of both perspectives addressing the quality and standards of thought from the philosophical perspective and thinking skill and procedures from the psychological procedures are taken into consideration because both are based on actions as well as behavioural attributes which can be assessed through inferring thinking skills to draw focus on what can be judged by the teacher as relating to evidence of learning and the consequence of their teaching as this study is focused on education. The study will focus on aspects of teacher beliefs as it relates to qualities and characteristics of teaching Mathematics through drawing inference or connections to what teachers say about their beliefs on teaching critical thinking in Mathematics.

2.1.7 Teaching critical thinking in Mathematics

In reflecting on the arguments above in how the contribution of the different perspectives on critical thinking are useful in the teaching and learning of Mathematics one has to look at what is it about Mathematics that is so important. Kranz (2007 p. 1) states that "A mathematician is a master of critical thinking, of analysis, and of deductive reasoning." He bases that statement in the argument made earlier that skills learnt in Mathematics can be applied in a variety of situation and disciplines. Mathematical skills and principles transverse many disciplines whether in humanities, medicine, commerce and almost every branch of human knowledge. Teaching and assessing mathematical skills in the classroom should therefore be synonymous with teaching critical thinking. This is because appropriate knowledge of Mathematics if it should be meaningful must dispose the learner to use the skills in their school work and in multiple situations they encounter in everyday life. Mathematical skills like self-regulation, analysis, inference, interpretation, deductive and inductive reasoning which are also critical thinking skills should be promoted in the classroom in a wider context as they can be applied as part or actual life skills. This can be done through focusing on how teachers can help develop the necessary cognitive skills of good thinking in learners who are then competent in self-formulating purposeful and thoughtful processes of judgements and decision making through using reason inferred from understanding and using mathematical skills critically.

My study is concerned with beliefs of Mathematics teachers in South Africa in how their beliefs about teaching critical thinking skills are connected to teaching Mathematics based on what appears to do in classrooms. As their conception cannot be divorced from what is prescribed and recommended in the curriculum the study will look at the elements that influence teachers' beliefs about teaching critical thinking and teaching Mathematics based on conceptions from themselves and other forces as they relate to their beliefs and how these are reconciled. A gap in teacher beliefs in practice is discussed in a study in Jordan by Innabi and El sheikh (2007) looking at the educational curriculum and how it was interpreted by teachers in a Mathematics classroom in teaching critical thinking skills. The researchers compared findings in 1988 from 12 schools and 47 teachers in teaching critical thinking with the other teachers in 2004 after reforms that required teaching critical thinking. They found that teachers in 2004 demonstrated little knowledge of what critical thinking was and how to teach it similar to their 1988 counterparts. About half of them couldn't identify one teaching approach for critical thinking and how it related to learning Mathematics. Their findings show that there was no significant

difference between the teaching in 1988 and 2004 despite educational reforms which required that critical thinking skills must be taught in Mathematics classrooms.

Studies discussing the teaching of critical thinking agree that it can only be addressed through the intentional pedagogical strategies of the teacher (Kuhn, 1999; Halpern, 2001; Beineke, 1985). Instructional and content design of a lesson are said to be the most effective ways to address intellectual standards and criteria of critical thinking as non-specific methods have proven less effective (Halpern, 2001, Alwadai, 2014, Lai, 2011). Halpern (2001, p.276) suggests a few but general guidelines which are essential to teach critical thinking:

- "Teaching strategies used by the teacher that require active involvement by the learner,
- practice of thinking skills in multiple settings whether through inquiry or argumentative, use of examples that are similar to the situations in which the skills will be used which invokes the notion of transfer,"
- "students' monitoring of their comprehension and progress toward an identifiable goal which is metacognition and dispositional stance,
- a rationale for learning the skills which are pre-set as well as outcomes of instruction,
- and use of intrinsic motivational techniques and multiple learning strategies are most likely to yield optimal results"

The significance of teaching strategies that have been effective based on similar models are discussed by Innabi and El Sheikh (2007). Onosko (1992) found that teachers who reflect on their own practices, value thinking, and emphasize depth over breadth of coverage of content tend to have classrooms with a measurable climate of thoughtfulness. He stated that teacher's beliefs in critical thinking and how they perceive thinking inform instructional theory. Zhang (2001) expanding on the influence of beliefs found a relationship between teaching approaches and thinking styles in teaching. Teachers who value and commit themselves to critical thinking in their teaching are more likely to demonstrate a reflective style of teaching. Lai (2011) discusses numerous empirical studies done to explore the effectiveness of critical thinking instruction and notes that they all show significant improvement whenever instruction or assessment is directed towards critical thinking and disposition. A common thread that is present in many studies and those discussed earlier is that critical thinking is a product of teaching and can be taught.

2.1.8 Instructional design towards teaching critical thinking

Instruction which is geared towards critical thinking requires special consideration for effectiveness. As argued, learning critical thinking skills alone does not dispose the learner to use them. Instructional design must therefore give learners opportunities to apply the thinking skills and abilities in a variety of context and subject areas with appropriate assessment methods to judge the use of those skills and abilities. Learners should be desensitised to only dealing with shallow procedural problems and should be exposed to deeper problems as often their thinking is disposed to finding quick surface solutions through shorthand mathematical procedures and methods without understanding (Lai, 2011; Alwadai, 2014). For example, in Mathematics simply giving learners problems which can be solved by following certain procedure with no connection to complex problems requiring different insight and decision making cannot be considered as promoting critical thinking. There should be instructional consideration emphasising thinking in setting lesson goals, planning and assessing progress of the learning of critical thinking skills and abilities. Learners should be taught how to recognise problem structures in order to learn to deploy different strategies through the use of authentic real life activities to promote transfer of critical thinking skills. This instructional consideration of encouraging focus on processes of thinking and not content alone is especially useful in Mathematics because of the subjects' structural connections and systematical connections across different areas allowing for structural learning to be practiced in different contexts or settings. As argued earlier Mathematics is a discipline that is known for emphasizing rationality and logical thought through using clear and concise language and thoughtfulness in all aspects of the discipline. The subject also prioritizes and pays attention to all assumptions and decision making techniques used to make conclusions in its methodology which are consistent with intellectual standards and criteria of critical thinking (Marcut, 2005).

On the question of how critical thinking skills and abilities can be defined to supplement learning outcomes of actions and behaviours as dispositions in Mathematics, Pingry (1951) described some important aspects that high school Mathematics teachers should develop as part of mathematical instruction. The guidelines specify how critical thinking strategies can be directed towards mathematical instruction. Pingry (1950 p. 470) states teachers can;

- "help the students have a rich background of experiences in Mathematics as related to other fields and life needs;
- Invent situations that will expose the student to question provoking experiences;

- Help the learner develop attitudes that will lead to thoroughness of study, organization of materials, and suspension of judgement until sufficient data is present;
- Set a climate in the classroom sympathetic to questions and questioning;
- Help the students gain ability in "if then" thinking and knowledge of some principles of logic; and
- Help the students build a set of standards or norms of good reasoning against which they can compare their own reasoning of others. The Mathematics teacher can help the students realize that the method of postulational thinking is not dependent upon the subject matter of Mathematics but is applicable to reasoning in non-mathematical areas as well"

One critical aspect of instruction especially as it relates to teaching strategies is the importance of knowledge or content in critical thinking teaching. This aspect of teaching as it relates to teaching is acknowledged by both proponents of the philosophic and psychology perspectives. It is argued that for learners to be taught to think critically, they need something to think critically about and thus teaching should be aware of how knowledge and thinking are interlinked (Bailin, 2002; Lai, 2011, Halpern, 1998). It is argued by researchers that aspects like domain-specific knowledge are important to critical thinking teaching because they demand certain explanations, evidence, arguments and evaluations that vary across different domains which can show transferability of critical thinking skills. One example of this in teaching and why Mathematics is closely linked to critical thinking is that Mathematics is the only disciple that concerns itself with proof. Knowledge about proof requires understanding the logical structure of Mathematics as well as methodological principles which are consistent and can be used in everyday reasoning and many contextual situations. It is this reason that critical thinking skills and teaching Mathematics should be taught in ways that are compatible with as well as transcend specific subject or discipline knowledge as the teaching strategies above suggest. This is because, although teaching subjects requires domain specific knowledge, as argued earlier, mathematical knowledge is not limited to a list of logical operations and procedures. Mathematical knowledge includes decision making techniques like problem solving and creative thinking which if taught for transference can be applied on other situations the learner may encounter. The value of domain specific knowledge in Mathematics and critical thinking is that it allows learners to be aware of the varied explanations, evidence and evaluation that are valued in different domains of knowledge and subjects which requires learners to use different thinking skills and

abilities to be applied in different context (Lai, 2011). This is because mathematical knowledge and principles can be found in all disciplines and spheres of life.

2.1.9 Recommendations of assessment strategies

Assessing critical thinking skills and disposition is a complex process. This is because as argued above, critical thinking skills and disposition are not the same and assessing thinking through inference is very problematic. The process of assessing critical thinking is made trickier if the focus is on content specific subject like Mathematics as it requires the consideration of thinking skills of the subject with general critical thinking skills (Abrami et al, 2015). There are however measures that can be used in a school context that can to some extent allow teachers to assess critical thinking skills and disposition in very reasonable and practical ways.

In matters of assessing critical thinking skills and disposition, there are obvious problems with the reliability and validity of methods which can be used to measure critical thinking. The current measures that are used often fall short because they rely on performance which has been shown to be prone to be subjective and thus can be prone to error. It is not easy to measure the development of thinking skills especially over a long period of time as different task and domain specific knowledge can communicate different messages from what is actually happening in the mind of the learner. Disposition and ability especially in assessment are distinct and assessing them can prove difficult to pick up their different effects especially in a performance assessment. To use standardised instruments to assess critical thought may lead to measuring extraneous constructs of biasness in assessing by the test maker. Standardising may lead to assessing already held beliefs like political, religious or any ideological stance the learners hold (Lai, 2011). However it is important to find assessment methods that can measure specific intellectual criteria and standards which are practical and can be used in the classroom.

Researchers have proposed some methods of assessment which can be used to measure the cognitive and dispositional aspects of critical thinking within practical implication in the classroom. One way is to use open-ended types of problems as they are more sensitive to dispositional aspects of critical thinking. There should be a mixture of text items of open- ended questions with multiple choice answers which will assess both the use of skill and disposition of critical thinking. Another method is for teachers to use activities which would allow self-constructed answers by learners or those which would encourage practicing thinking skills in an everyday context with follow up probing questions. An example of this in Mathematics are modelled problems which can model any real life problem like statistics problems which can be

found everywhere and can have a variety of solution if chosen well. What researchers recommend is the use of structured or ill-structured problems which simulate real life problems with more than one plausible solution which require learners to go beyond given information about the problem (Lai, 2011). One example that can be structured or unstructured, in Mathematics is a broad concept like functions which examples are found in almost all aspects of life in which problems that require learners to find, define or even use them can be easily conceived to allow learners to think deeper as well as critically about the mathematical problems.

Assessment tasks should also be used carefully as certain task will assess different skills and dispositions in different levels. What is suggested is that tasks should be directed at assessing the arguments underlying a position rather than on the correctness of the solution. Tasks which seek to measure thinking should not focus on recalled information if they are to assess critical thinking skills and disposition but should require the use of already learned knowledge in different context in innovative ways (Lewis and Smith, 1993; Lai, 2011). Assessment modes or tasks should focus on identifying and measuring thinking skills in ways that are visible through following learner's thinking process through requiring them to justify or rationalise their work. With regard to Mathematics this can be done much more effectively because of the logically coherent structure of the subject by requiring justification for any mathematical argument they make. It can be easily acknowledged that most of the assessment strategies above are not very practical for South African teachers due to curriculum demands and time. However, it is still possible for teachers to infuse some of the elements in their own assessment methods through assigning a few critical thinking problems or emphasising some intellectual standards in lesson goals.

2.2 Theoretical framework

The need to address critical thinking skills in Mathematics is recognised as important both in literature and the CAPS (2001). Stols Ono and Rogan (2015) discuss some studies in Mathematics done in South Africa which identify a gap in teacher beliefs and practice (Webb and Webb, 2004; Van Putten, Stols & Howie, 2014; Morar, 2000). What was generally found was that teachers embraced certain beliefs in theory but they did not seem to be effective in applying them to their own practice. As it is recognised that teaching critical thinking is a product of intentional teaching and that teachers' beliefs shape if and how it is taught. This study will look to understand the connection between teachers' conception and beliefs about

developing critical thinking and beliefs about teaching Mathematics and what theese beliefs shape their views of teaching emphasised by the CAPS document.

Mathematics teachers' knowledge, beliefs and attitude towards the subject are important in their practice (Ernest, 1989). Knowledge of pedagogy and curriculum informs teachers' use of mathematical content in the classroom. Beliefs are teachers'conception of how mathematical knowledge can be used for teaching and learning and as stated above there is often a disparity between what teachers believe and what they practice (Ernest, 1989, Stols et al, 2015). Conceptions provide explanation of teachers thought process about how their beliefs translate to their practice. They explain the relationships between teachers' intuitive perception about how they perceive their own actions and their consequences. Attitudes shape teaching by defining the atmosphere and values that permeate in the classroom. All three elements – knowledge, beliefs and attitudes – link theoretical aspects with the practical aspect of teaching practice.

Beliefs are important as they inform the actions and interactions of the teacher and provide the lenses to how teachers make sense as well as create meaning in the classroom. They also shape planning and curricular decisions which determines the path of instruction and what is taught. They inform how the teacher thinks and are found to be more influential than knowledge in the teacher plans and organises tasks and activities and the behaviour of the teacher (Xu, 2012). The objective of this study is to find the connection between Mathematics teachers' beliefs about teaching critical thinking and how those beliefs inform their teaching of Mathematics. This will be done by looking at teacher interpretations and reflection on their teaching of Mathematics through eliciting their conception of practice. The study will not focus on attitudes as they tend to be variable construct within a single person and may be affected and influenced by factors other than what the study is trying to understand. Beliefs relate more to disposition and have a powerful impact on teaching process like selection of content and what that content emphasises, teaching style and modes of learning. These elements are closely linked to elements important to consider in teaching critical thinking. Attitudes although important are behaviourist through emphasis on action and thus do not encapsulate the concrete aspect the study is interested in which does not take the directly literal relationship dimension of teachers and learners.

As identified above in the literature review, beliefs inform the teaching of critical thinking and it is generally found that there are disparities between what teachers do and what they believe. Three factors are attributed to this disparity according to Ernest (1989). The first is the depth of the adopted beliefs and how they are integrated with pedagogical and content knowledge. If

teachers' espoused beliefs are disconnected from other beliefs and knowledge, then they are likely to be disconnected in practice. Second is the teacher's awareness of their beliefs, how they reflect on their beliefs in their practice and how it leads to the integration into practice. Teaching requires awareness of one's views, assumptions and the ability to justify and articulate why one holds the beliefs he or she has. It also requires the disposition to reconcile beliefs with conflicting beliefs and with classroom practice. The third is the influences of the external professional and social context. This encompasses what the teacher is expected to do by the curriculum and superiors which exerts a powerful influence on conception and practice. These disparities can have a big influence in teaching and promoting critical thinking as they influence how knowledge is utilised in the classroom.

Such disparities will be used to inform how teaching critical thinking links to how teachers view Mathematics and how they teach it. Based on beliefs, teachers' instrumental views of Mathematics influence their instructional model of teaching and how they implement teaching tasks and activities in lessons. Innabi and Elsheikh (2007) and Halpern (2001) argue that teachers should demonstrate conceptions or rather and in the context of this study, beliefs about critical thinking if they are to teach it in practice. The main argument is that beliefs guide thoughts and actions which influence teaching attitude, method and policy. These elements are critical factors of the disparities that exist between beliefs and practice and this study will try to look at how they could influence teaching critical thinking through Mathematics. The teaching model by Halpern (2001) exemplifies an idealized model that teachers should commit themselves to teach critical thinking in Mathematics which will be used to examine and assess the extent of influences exerted by disparities as they apply to the connection to teacher beliefs about teaching critical thinking and teaching Mathematics.

2.3 Conclusion

The focus of this study is centred on teacher beliefs about teaching Mathematics and critical thinking as based on the connections and nature of these connections between teacher beliefs and how they appear to influence practice. The fact that teachers play a crucial role in teaching and learning, as well as implementing changes in schools, is the main reason why many educationalists and researchers believe that the complex nature and role of teacher beliefs and practice is an important area of study in how educational goals are achieved (Savaşcı Açıkalın, 2009). Beliefs as they relate to teaching and learning are said to be concerned with manipulation of knowledge or information processing for a particular purpose based on psychologically held understandings, premises, or propositions that one holds with some conviction as true (Savaşcı

Açıkalın, 2009; Xu, 2012). Educationalists and researchers argue that beliefs are dispositional and actionable in that they determine behaviour. Mathematics teachers' beliefs about teaching Mathematics and critical thinking can determine practice in how the teacher designs or implement their teaching strategies or task in the classroom (Ernest, 1989; Stols et al, 2015; Savaşcı Açıkalın, 2009, Xu, 2012, Kuhn, 1999; Halpern, 2001; Beineke, 1985). As argued earlier by Lai (2011) and Alwadai (2014) it is important that teachers are critical thinkers or possess appropriate beliefs about teaching critical thinking in order that they can develop critical thinking skills in learners through designing and/or selecting appropriate teaching strategies, mathematical tasks and assessment methods that can promote good thinking skills.

Beliefs, especially the ones that shape lifelong convictions, are said to often be formulated or enculturated early on in one's life and tend to self-perpetuate themselves against later contradictions of time and even schooling (Savaşcı Açıkalın, 2009). This makes teaching learners critical thinking skills in school an important educational goal as is the recommendation of the CAPS document (2011) because it is the best possible time to enculturate critical examination of beliefs through promoting intellectual standards of thinking which develop good thinking. Learners should be encouraged to be critical thinkers and be tentative about what they believe by always developing their thinking habits and being scrupulous as well as logical about the information or knowledge they are using in whatever context as much as mathematicians would be. To develop critical thinking requires that Mathematics teachers and learners develop the complex, purposeful and thoughtful process of formulating judgement through reasoning, logic and evidence as this process is often carried out in Mathematics (Facione, 2000). It is the hope of this study then to investigate the nature of beliefs of newly appointed Mathematics teachers to both find out what is the nature of their beliefs between teaching Mathematics and teaching critical thinking and Mathematics. This is in hoping to promote or reinvigorate a culture of critical thinkers in learners which through appropriate teaching will develop future South African citizens who can solve many of the complex problems in all of the country's institution and equally important, to have people who hold well examined and reasoned beliefs about the world.

Chapter 3 Research methodology

3.1 Study design

This is a qualitative case study of teachers' beliefs and understandings of teaching critical thinking in Mathematics. It is descriptive in nature and looks at teachers' conception of what they believe about teaching critical thinking in Mathematics. One primary focus was to examine whether, based on their beliefs, teachers expressed a connection between teaching critical thinking and teaching Mathematics in their classrooms. The primary focus is consistent with the aims as set out as an important objective in the South African curriculum. The data was collected by conducting two sets of interviews in a space of a month with six different Bachelor of Education graduates teachers working in six different schools around Gauteng. All the selected Mathematics teachers are graduates of one of the top universities in South Africa who have been teaching for more than a year. No observational data was collected in the study due to the scope of the research as well as time constrains.

The main question of the study is, what are Mathematics teachers' beliefs about teaching and fostering critical thinking skills in high school Mathematics and how their beliefs dispose or appear to inform their practice? The first interview was designed to explore the teachers' initial conceptions of critical thinking and their general understanding of critical thinking through reflecting on their current practices. The interview questions were taken and adapted from a study which investigated the extent and conditions of high school teachers in articulating and demonstrating elements of critical thinking in their instruction by Thomas (1999). Questions were adapted for the purpose of looking at Mathematics teachers' beliefs and piloted before being used to collect data. The second interview was designed to explore further the teachers' beliefs through reflections of their practices together with their responses to video lesson vignette that they were asked to analyse. The video lesson depicted a primary school lesson promoting critical thinking Mathematics with learners working on their own with direction from the teacher in making judgements and decision about a problem they were given. The video was retrieved from YouTube with the link of the three parts in references. The lesson was chosen among others because of its focus on school Mathematics, a characteristic not shared by other videos. It is acknowledged that there were limitations in extrapolation as the culture of teaching and learning in a primary school are not necessarily similar to high school teaching, a characteristic very visible in the video. Teachers were made aware of this limitation and encouraged to only focus on elements they were familiar with like teaching strategy, kind of tasks used, learner involvement and assessment. The teachers were shown a three part video

lesson where they were asked to reflect on elements of the lesson based on their beliefs and conception of teaching critical thinking.

The initial intention of the data collection process was to conduct the two semi-structured interviews with standard sets of questions which were adjustable based on the responses by teachers. Because of time constrains and scheduling issues there were a few changes in the study design. The first change was on the first interview which some questions had to be reduced because of time issues as other teachers could only spare less than 30 minutes of time when around 45 minutes was more appropriate. Some questions were altered during the course of the interview because of the type of responses teachers were giving which were judged as not consistent with the purpose of the interview. Other changes to the design were also made to the interviews as issues of time and scheduling meant two out of the six teachers could not schedule and the second of the two interviews was incomplete. Two were completed without complication while the last was rushed because of time. In total all six first interviews were carried out and only four were carried out for the second interview. However due to the complications in conducting the second interview as well as the nature of such data, it was decided that analysis would focus mostly on the first interview. The inconsistency in the data collection process in the second interview rendered the data unreliable in drawing appropriate categories and themes. Some portion of the data from two completed second interviews was used for the teachers who did the interviews to support ideas and themes from the first interview.

3.2 *Setting*

This study focuses on high school Mathematics teachers' beliefs about teaching Mathematics. There were six teachers in total in the study: two female and four male Mathematics teachers. They were all new to the profession, with experience ranging from two to three years. They were teaching Mathematics or Mathematics literacy as their primary subject. Four of the teachers were teaching core Mathematics while two were teaching Mathematics literacy. All interviews were conducted in locations the teachers identified which all opted to hold them in their respective schools. The selection criteria for teachers was based on being newly employed sharing the same graduation year and university. Each of the teachers completed their studies or degrees in the same year from the University of the Witwatersrand class of 2014.

All six teachers were teaching in different schools mostly in township schools all located in Gauteng province. All schools were comparably similar in resources with no reported problems

in terms of major disadvantages regarding with teaching and learning. Interviews were conducted in each teacher's classrooms or office. There were a few noise disturbances as interviews were carried out in school days. All interviews were conducted out of teaching time in order not to interfere with teacher's responsibilities and recorded using a digital recorder and tablet.

3.3 Research instrument

The first semi-structured interview (Appendix A) consisted of six main questions and follow up questions. The designs of the questions were originally from a study titled; *California Teacher Preparation for Instruction in Critical Thinking: Research Findings and Policy Recommendations California Commission on Teacher Credentialing, Sacramento California, 1997.* They were adapted from this other study which investigated high school teacher's perception of critical thinking in general. Based on consultation with literature, I used the same questions and adapted and added some questions to highlight and focus on the teaching of Mathematics. I then gave out a separate information sheet for teachers (Appendix C) which highlighted the purpose of the interview which was the teacher's belief about teaching Mathematics.

The reason for giving teachers the information sheet was to avoid confusion over the common conflation of critical and critique as well as allow focus teachers on the topic of the interviews. Some teachers confessed to have not gone over the documents because of the busy schedule and some did go over it. The information sheet mainly focused on the topic and what is required of the teacher and effort on the wording was applied and checked that it did not allude to any questions in the interview. In any case it is acknowledged that teachers could have prepared for the interview by preparing answers or research on the topic and in such instances all teachers were elaborately probed in their responses for example as well as nuanced explanations to avoid superficial answers.

I piloted the interview with two different Mathematics teachers who were also new to the profession. The purpose of the pilot was to check for clarity in questions as well as language issues as the participants were not first language English speakers. Feedback from the teachers in the pilot like unclear questions, phrasing and unclear words was taken and appropriate steps to resolve them were taken prior to the official interview. The pilot also provided some data which was studied for appropriateness in meeting the purpose of this study. This was done by grounding this study on findings from other studies where the questions were taken as well as

other similar studies which investigated similar topics. Foresight on potential problems as well methods of collecting and analyzing more meaningful data were adapted to suit this study. Methods like coding and how to link and integrate similar categories were among the extrapolations from other studies.

The second interview involved another semi-structured interview which was conducted as a discussion between me and the interviewee. Each teacher was asked to comment on a vignette video lesson depicting a classroom lesson geared towards promoting critical thinking. The purpose of using a video to direct the interview was to ground teacher beliefs in practice through identifying elements they could pick up from the lesson which they could identify with. The video consisted of three sections with each approximately eight minutes in length. In the video a teacher asks learners to resolve a problem of inconsistent answers of a Mathematics paper they submitted as there were likely more incorrect responses than correct. He urges them to come up with a system the whole class can use in order to get to a common solution on their paper much quicker. The learners work on their own to set up incentives to motivate or punish slackers in order that others work hard in getting solutions. They end up agreeing on a system and start implementing it by helping each other with their Mathematics and discussing which solutions make sense and which are incorrect.

Teachers were asked to comment on elements of the videos they could identify from their own teaching of Mathematics and the elements they believed promoted critical thinking. Three questions were asked which were modelled from a study that used videos to investigate teacher beliefs about a teaching programme (Van Putten, Stols & Howie, 2014). The three questions teachers were asked were;

- Can you identify segments in the lesson that you regard to be the most useful in promoting critical thinking and explain your choice?
- Can you identify segments in the lesson that you regard to be the least useful in promoting critical thinking and explain your choice?
- Can you identify segments in the lesson that resemble your own teaching of Mathematics and explain what that has in common with your conception of critical thinking?

Qualitative data was collected from the two semi-structured interviews per teacher through a recording device. The purpose of using semi-structured interviews is to collect descriptive information data that describe teachers' beliefs about teaching as well of teaching critical thing

in Mathematics (Opie, 2004). The semi-structured interviews were used to enable teachers to provide explanations of their thought process about how their beliefs dispose them to their practice in teaching critical thinking. The use of the video lesson after the first interview was expected to help them to explain the relationships between their conceptions of critical thinking and how they perceived their own practice by focusing on specific elements like their idea of what critical thinking is, what tasks or teaching strategies they believe should be used, how they would assess if their teaching is effective in promoting critical thinking and other educational aspects of teaching.

3.4 Data collection

Qualitative data was collected only from the two semi-structured interviews. Interviews were chosen based on recommendations from Opie (2004) and the scope of this study. The use of interviews according to Opie (2004) is that they give detailed and descriptive information given limits and time of a master's dissertation. Only six teachers were chosen to participate which made interviews the most preferable data collection instruments to gather in-depth data.

Verbatim recordings of the each set of interviews were made as part of the data collection process. Notes from the interview were also made to assist with the analysis process but are not part of analysis. The interviews were conducted by me as the main researcher and lasted between 30-45 minutes each session. They were paced according to the interviewee's responses and convenience. Two recording were made of each interview for maximum clarity in transcribing.

3.5 Analysis

The interviews were transcribed verbatim from which an inductive analysis was carried out. I used qualitative data analysis through using open coding. Huq Khandkar (2009) as "includes labelling concepts, defining and developing categories based on their properties and meaning." I then broke down the data into similar chunks and used both in vivo and constructed codes based on properties. After coding my data I analysed for patterns, themes, categories through a repeated reading of the data to confirm and establish the accuracy of my findings. The process proceeded using grounded theory and Hitch's (2002, p. 162) adapted steps of inductive analysis which proceed as;

1. "Reading the data and identifying frames of referencing
2. Create domains based on semantic relationship discovered within frames of analysis

3. Identify salient domains, assign them a code, and put others aside

4. Reread the data; refine salient domains and keeping a record of where relationships are found in the domains

5. Deciding if the domains are supported by the data and search data for examples that do not fit with or run counter to the relationship in the identified domains

6. Complete an analysis within domains

7. Search for themes across domains

8. Create a master outline expressing relationships within and among domains

9. Select data excerpts to support the elements of the outline"

The identified domains and outlines that emerged are discussed in detail as part of the findings and in the light of my theoretical framework as well as related research in the literature review. This was done in line with the research question and purpose of this study which is to investigate South African mathematical teachers' conception about teaching critical thinking as required by the curriculum. This is done for the purpose of revealing themes, categories and possibly new ways of looking at the issue of teaching critical thinking in the discussion and interpretation section.

3.6 Rigour, validity and reliability

To ensure issues of trustworthiness and validity the study I adapted and piloted questions from an interview protocol that was used in a similar study (Thomas, 1999) provided as appendix A. That study was investigating the extent and conditions of 40 American high school teacher articulations and demonstration of critical thinking elements in their classroom. It relied on classroom observation and interviews in its methodology. My study however is different in that it focuses on beliefs about critical thinking and how mathematics teachers appear to interpret and dispose themselves in their practice. Descriptive qualitative data was obtained from what teachers said in the interviews which is acknowledged may not be the full range of phenomenon that can be analysed as no classroom observations was conducted. The vignette lesson responses which were used to aid teacher reflection from a classroom that is modelled from critical thinking elements were used to assess the reliability of what was said in the interviews. The analytical tool used is grounded theory and to ensure non-superfluous themes guidelines in figure 1 were followed. To ensure that the standards of evidence were met results and themes

were compared to two similar peer reviewed studies of the topic done in other countries. One of these studies is where the interview protocol was adapted (Thomas, 1999) and the other is Innabi & El Sheikh (2007) investigating the change in Mathematics teachers' perceptions of critical thinking after 15 years of educational reform in Jordan.

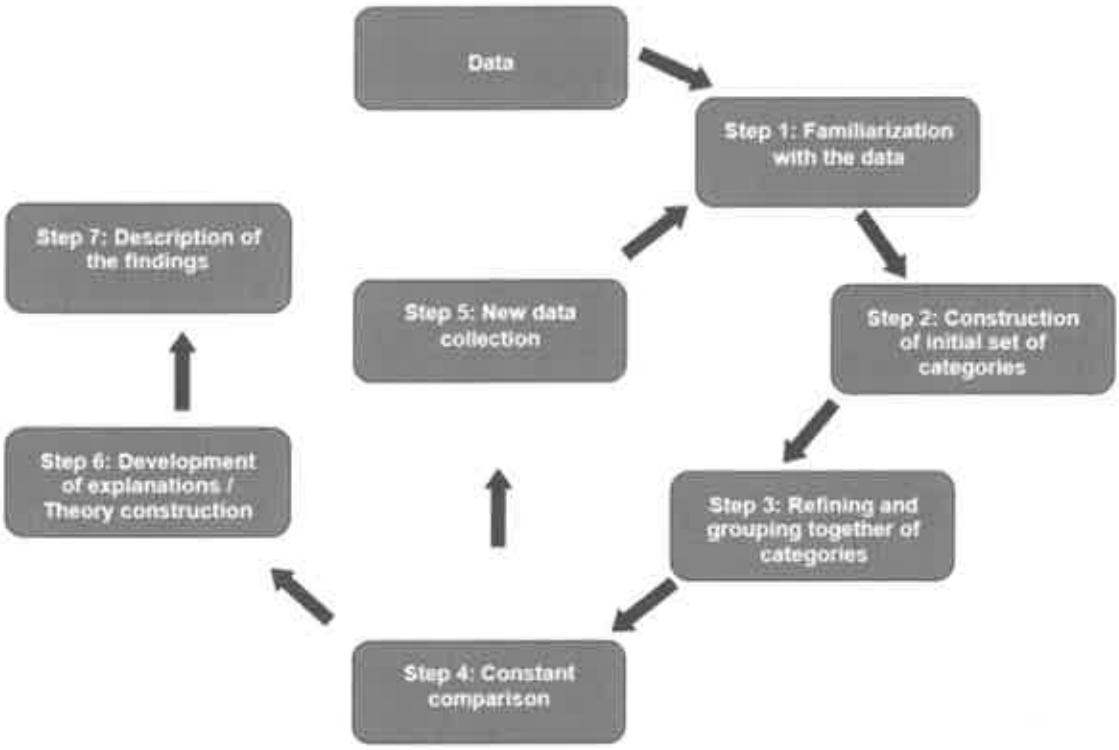


Figure 1: *Model adapted from Kohn and Christiaens (2016)*

3.7 Limitation of study

The study makes no claim of generalisation in its findings as there were only six participants. All teachers who were interviewed were all newly appointed teachers meaning they do not necessarily reflect the teaching of Mathematics in general. They are a product of the same university and therefore do not represent the full spectrum of teachers who were trained from other institutions. Teachers were not observed and all interpretation or analysis was based on what they say only. Teachers were also provided with an information sheet about the interview as stated above and it is acknowledged that they might have prepared some of the responses they gave beforehand. My study however tries to draw attention to the issues around Mathematics teachers' attention towards promoting critical thinking as a very important educational goal.

3.8 Ethical considerations

This study focuses exclusively on teachers and does not directly include any learner, principal and parent. The interviews were not conducted during school times to avoid infringing on the teachers' responsibilities in the classroom and rights of learners to learn. Anonymity of teachers and the school are prioritised in all instances of data collection. All writings in findings, discussion and interpretation do not link to any names of teachers directly. Teachers were told that they have the freedom to choose to opt out of the interviews at any time as it was voluntary. If the teacher chose to give any practical examples it was welcomed and encouraged with no specific learner names to be given and where it is it will not be mentioned in the transcript.

4 Chapter Results, findings and discussion

4.1 Introduction of results

In this chapter I will present and discuss the results of the study. The objective of this study was to find the connection between Mathematics teachers' beliefs about teaching critical thinking and how those beliefs informed their views of teaching of Mathematics. This was done in line with literature in Mathematics that teaching critical thinking is a product of teacher's intentional instructional goal and strategies (Ab kadir, 2007; Alwadai, 2014; Facione, 2000). The primary purpose of the analysis process was to answer the main question of this study as well as its sub-questions from teacher incite. The results are divided into three sections. The first section focuses on teacher conception of critical thinking, the second on teaching strategies and task, and the last section on assessment of critical thinking.

4.2 Teachers' conception of critical thinking.

One of the main areas of focus in this study is teachers' conception of critical thinking concerning teaching Mathematics. The focus was both on general or specific beliefs and conceptions of teaching critical thinking in Mathematics. Three themes emerged from analysis of teacher responses which were;

- Critical thinking as meeting certain condition and requirements
- Critical thinking as abilities to apply what was taught
- Critical thinking as processes and skills

Beliefs and conceptions of critical thinking by each of the six teachers shifted in the course of the two interviews with more emphasis on practical questions about teaching. Some teachers' responses shifted as more specific questions were asked which required more mathematical examples of conception of critical thinking and how they could apply in practice. Some teacher responses remained the same when talking about particular beliefs about critical thinking but developed in what thinking skills or processes they associated with teaching critical thinking skills. Shifts encapsulated broader or sometimes newer descriptions of conceptions or beliefs of critical thinking when talking about teaching Mathematics. I will discuss the main conceptions and beliefs that emerged from teachers in both interviews and relate them to literature on teaching critical thinking and Mathematics to highlight features that are useful or less useful.

4.2.1 Critical thinking as meeting certain condition and requirements

One conception of critical thinking the teachers expressed was meeting certain conditions of thinking or achievement. These conditions or achievements they described were in fulfilling certain requirements of thinking within teaching and learning situations. Teacher expressed their conception by identifying certain situations where they believed critical thinking was fostered or that the outcome was critical thinking. There was variation in conditions and requirements to be fulfilled which shifted between focusing on learners and focusing on teachers or even both. More emphasis was on what teachers needed to do with learners in teaching critical thinking. This variation of teachers' ideas regarding teaching Mathematics suggested that teachers either hold different beliefs about teaching critical thinking or the level of awareness of elements of critical thinking is not the same. Two categories of conditions and/or requirement emerged from analysing responses. The first was more emphasis what was expected of learners; the second was what is expected from the teacher.

4.2.2 Conditions and requirement for critical thinking with emphasis on learners

Regarding the conditions of critical thinking focusing on the learners, teacher talked about a variety of situations which formulated their conception and beliefs of critical thinking. Teacher 6 in describing his conception of critical thinking spoke about how it applies to teaching in a mathematical classroom. The teacher stated that "*first as a teacher, I will say you need to reserve your judgement and allow learners to use their understanding in a classroom.*" He was talking about discovery learning which the teacher said was part of his mathematical teaching that promotes critical thinking in his classroom. He elaborated on this idea by giving a mathematical example that;

> *"You need to give them, they need to find ways , especially Mathematics there's one thing our learners don't get it but we use ehhr...equations or what we use methods, it should not be like they should think about, method is a way of finding an answer. A method is the simplest way to find even if you are in a situation whereby you don't know the answer but you can be able to find it using the similar methods that you know they are known. So, what is important is that as a teacher you need to allow kids to think."*

Critical thinking skills in Mathematics for teacher 6 are based on the condition that learners are allowed opportunities to think and find different ways to solve problems and not rely on equations or methods. Another conception was based on the requirement that learners know conceptual links in Mathematics to be critical thinkers. This view was held by teacher 4 who

stated that "*If you don't know there is a relationship between the concepts then there is no critical thinking there, you are just doing a procedure without linking*". In elaborating teacher 4 argued that an important requirement for critical thinking is the ability to think of the "*why's*" and "*how's*" of mathematical concepts and considering questions of where and how concepts came about. Another teacher who also spoke about mathematical concepts and critical thinking said "*in relation to Mathematics I understand critical thinking as something that tells me conceptual understanding, so you need to know the concepts behind everything*" (Teacher 3). The teacher saw conceptual understanding as a condition of critical thinking. Teacher 3 also spoke of the importance of prior knowledge as an important condition and requirement of critical thinking by stating that "*Critical thinking only applies when a learner has a clear prior knowledge or something like that so that they can have a starting point*". The teacher believed that learners need some mathematical knowledge first before they can think critically. Teacher 1 gave a mathematical example that explained meeting certain learning conditions and requirements of knowledge and said;

> "*there was a question that was given to learners, they've all been to the pool so it's on volume. So what it asked 'given all the dimensions you calculate volume' so the shape of the pool was a rectangle, it was a compound shape a rectangle and two semicircles. So they were given all the measurements and they were asked, 'what is the volume of that?' So the learners actually failed to understand the question, because they failed to answer the question. So they were unable to actually visualise what it is they need to calculate and what they need to work out*"

The teacher defined visualisation as part of her conception of critical thinking and the examples she gives suggests that learners have to possess critical thinking skills to do their Mathematics. Teacher 4 spoke about co-operation and comprehension environment in the classroom as necessary for critical thinking. She stated that;

> "*coming to critical thinking one must be cooperative, there has to be thick comprehension, you can't critically think if you don't comprehend...ah its cooperating and comprehend because if you understand you can critically think, but if you can also work with people you can critically think with the ideas obviously coming from other members*"

The teacher believed that working together and understanding other learners' ideas then there is critical thinking. Other teachers identified other conditions and requirements as part of their

conception of critical thinking. These included the need for instrumental and conceptual understanding which, learners using appropriate mathematical methods and procedures which, and the ability to reflect on ones' own ideas among other situations teachers thought necessary for critical thinking (teacher 4, 5 and 6).

4.2.3 Conditions and requirements for critical thinking with emphasis on teacher

Teacher's conception and beliefs of critical thinking also focused on conditions and requirements of teaching in Mathematics. These conditions were either general to teaching Mathematics or specifically to how teachers believed Mathematics should be taught. One condition or requirement was "*asking high level questions*" (teacher 5). This conception was based on the levels of questioning from Bloom's taxonomy either in school work or classroom discussions which the teacher believed conceptualises a hierarchy of thinking skills. Teacher 3 spoke about teaching critical thinking in Mathematics by using probing question by saying "*sometimes the question that you ask, if you ask higher order questions; it requires critical thinking. If you ask lower order questions, it does not require it because sometime you have to recall now and then*". Teacher 3 also believed that questioning is necessary in promoting critical thinking.

Teacher 1 stated that "*my understanding of critical thinking is that, ok let me put it in the classroom, when you teach a learner it's not just about ahh let me recall the concept it's about making part of the learners' life so in essence, you should be able to apply it in any situation that you're given*". The teacher was arguing that it was necessary that teaching critical thinking in Mathematics means showing learners how the subject holds practical everyday relevance. This she elaborated by saying "*So for me it's like whenever you can visualise what's going on, so you see the concept so outside the classroom so if you can see the concept outside the classroom as well it's a good thing*". The rest of the teachers made reference to the requirement that mathematical knowledge be contextualised in real life. Teacher 6 spoke of the necessity of encouraging an enquiry climate in the classroom saying "*in a classroom, to help them to understand, to help them to emphasize thinking, in my class I always give them the opportunity to explore, discover on their own before I even tell them what the real solution...and especially I don't give them problems that I know answers to.*" The teacher recognised as important allowing learners to openly explore mathematical problems without being influenced by the teacher which he believed encourages thinking them to think. Teachers generally agreed that it was important for a mathematical teacher to encourage a learning environment based in questioning

or enquiry. This was more prevalent when teachers discussed teaching strategies and tasks they believed fostered critical thinking which will be discussed later in the second section of this chapter.

4.2.4 Discussion

Intentional teaching of critical thinking is argued in literature as one of the fundamental requirement for fostering critical thinking skills. Pingry (1951) argues that critical thinking would be of little use if it is not characterised with specific learning outcomes of actual behaviour and characteristics of thinking abilities. This means there could be some specialised conditions and requirements of teaching that could best promote critical thinking. This is because encouraging abilities, skills and disposition of critical thinking can be characterised into teaching situations where thinking can be promoted from subject matter more effectively (Pingry, 1952; Facione, 1990; Lai 2011). Examples of some conditions and requirements that can be found in literature include; domain-specific knowledge, consistently using questions to probe learner reasoning and understanding until they internalise the question and use them on their own, and identifying some standards of thinking that learners have to adhere to in order to promote critical thinking within a particular subject (Thomas, 1999). In teaching Mathematics this includes understanding specialised mathematical methods and procedures as well as mathematical principles and what situations would enable learners to participating or engage in any norm related principle related to intellectual standards and criteria of critical thinking effectively. This norm related principle includes thinking skills that can be used in making decisions and reasonable judgements in a variety of situations which adhere to intellectual standards like accuracy, fairmindedness, valuing good evidence among other standards (Facione, 1990; Lai 2011). Teachers in this study identified elements they believed exemplify conditions and requirements required that can help learners think critically from learning or knowing Mathematics. Teachers' conceptions of conditions and requirements in this study demonstrated some awareness to important teaching and learning situations that could generally promote critical thinking, however there were some important elements that were not recognised. This means that at the least and based on the teachers interviewed, there is some potential for teachers to teach critical thinking skills especially if they are aware of what it is and can identify situations which can promote the thinking skills.

Most of the teacher conceptions of critical thinking through conditions and requirements focused on understanding Mathematics, thinking about Mathematics and then applying the knowledge in the subject or real life. Conceptions recognised some important elements of critical thinking and

Mathematics through encouraging the development of some critical thinking skills and abilities like using higher order questions, understanding mathematical concepts and principles, making Mathematics relevant to everyday life situation among those mentioned. This demonstrates that there is a level of awareness of the decisional and judgmental aspects of thinking in teachers' beliefs and conceptions of critical thinking. To some extent teachers could talk about skills like inference, interpretation, deductive and inductive reasoning and also recognised that these thinking skills can be developed through encouraging understanding, thinking and application of mathematical skills in a critical thinking manner. A conception of critical thinking based on conditions and/or requirements in the classroom as the teachers discussed hold the advantage that learning outcomes can be linked to certain situations making them easier to assess. Intellectual criteria and standards can be constructed around the learning situations where critical thinking skills and disposition can be promoted in Mathematics because they will be based on performance which can be assessed. However one of the critical elements that teachers (except teacher 6) seem to have ignored is that critical thinking skills and abilities involve making well informed or reasonable self-directed judgements not in a manner that the outcomes have been predetermined or not open to other possible solutions.

Teachers in this interview gave examples that were seemed rigid where actions or performance was defined mostly by the teacher. Teaching critical thinking in Mathematics should promote learner centred and deep understanding of mathematical principles by learners so they can see how they can be used to think and deal with everyday life situation. There is a danger that creating generic situations of teaching and learning can defeat the function or purpose of critical thinking skills by limiting elements such as transfer and self-exploration. For example, if a teacher takes too much authority in how the course of learning is progressing and how learners should use or make decisions about what they are learning in Mathematics, it can create dependence by learners on the teacher thus limiting their own intellectual growth. One important element of critical thinking is that it is based on universal intellectual standards of thinking about any subject, problem or content in which one engages in self-directed, skillful analysis, self-disciplined, self-correcting and self-monitoring. It should then be reflected with classroom interactions instead of focusing simply on mastering mathematical content as the teacher dictates. Learning situations that promote critical thinking should inform thinking in making judgements even in non-mathematical situations

4.2.5 Critical thinking as ability to apply what was taught

Another prevailing conception of critical thinking that emerged from teachers encapsulates elements from the two main themes in this chapter focused on teachers and learners abilities. Teachers in explaining their conceptions of critical thinking conceived of some quality or character of both teacher and learner which demonstrates critical thinking. Emphasis in this conception of critical thinking was the belief that a learner's or teacher's abilities or inabilities of demonstrating certain actions or behaviours could be conceived as critical thinking or lack of. There were two categories of responses teachers gave as part of their conception of what demonstrates critical thinking through characteristics and qualities. These categories were; what abilities should teachers or learners demonstrate and what should be the roles of teachers and learner in the classroom.

4.2.6 What abilities should teachers or learners demonstrate?

Explanations given by teachers were mostly about what they do and conceive to be appropriate critical thinking qualities, behaviour or characteristics for specific teaching and learning situations in Mathematics. Teacher 1 speaking about the requirements by the department of education in the curriculum as influencing how she characterises learning and teaching critical thinking in a classroom said that "*you should be able to apply it in any situation that you're given. That's what I call critical thinking, because the department keeps on saying, whenever you giving learners a test, even though you've taught them like things they need to know in order to understand that concept, when you give them a test, it's supposed to test, do they understand and can they apply what they've learned. So when they can't do that then they have failed*". This explanation by teacher 1 characterises a critical thinker as the ability to apply what is taught and non-critical thinker as one who cannot apply what is taught. According to the teacher the curriculum and the department of education are characterises a critical thinker as a learner who can apply what they learn in the classroom based on the instructions they communicate. Teacher 5 talking about qualities and characteristics of a critically thinker in Mathematics said he judges their ability to use or select appropriate formulas and theorems like Pythagoras without the help of the teacher. This according to the teacher demonstrates thinking because the learner can understands Mathematics and can use Mathematics skills. Teacher 4 explained an example of calculating dimensions and how learners not thinking critically about things like measurement area and perimeter could lead to real life problems. She said "*It catches up with you later when people are selling a yard to you and they then tell you 420 square meters, most people up to this stage are very confused as to what is meant by a square meter, but this traces back to the*

classroom, a learner didn't understand what a square meter is actually." According to the teacher understanding of mathematical concepts means the ability to use them in real life situation. Teacher 2 and 3 spoke about how learners' ability to answer higher order questions is one of the defining characteristic of critical thinking. Both teacher spoke of the value of asking engaging and deep questions to encourage thinking in Mathematics.

4.2.7 What should be the roles of teachers and learner in the classroom?

Regarding what ought to happen in the classroom based on roles of both teachers and learners, teachers also defined critical thinking as based on designated duties and actions of both teachers and learners in the teaching and learning process. For example, teachers described certain responsibilities that either they or the learners have to fulfil in order that teaching and learning can be said to be promoting critical thinking. These included actionable responsibilities such as creating a conversational dialog between learners and teachers in a classroom and also duties like teachers should explain the mathematical content or explanations which they believe are appropriate for learning, and learners should engage as well as ask questions. Teacher 3 on how he introduces topics through using scenarios spoke about the importance of questioning teacher's ability to motivate and engage learners. The teacher stated that "*So if you give them that thing that requires them now to think and sometimes they can even question and say ok sir, why are you saying this? If they start questioning you, it means critical thinking, if they start questioning you but if they don't it means you are doing something wrong.*" This explanation by the teacher places emphasis on responsibility as well as roles of teachers and learners between mathematical content and critical thinking in the teaching and learning. The teacher describes a situation where critical thinking is embedded in learners engaging in dialog through asking questions about what the teachers gave them. The teacher suggests that it should be the teacher that plays a leading role in helping learner understand their Mathematics through giving explanations. This he based on how the learners perceive or engage with the explanation he would give through asking questions which informs him on whether their thinking is critical with regards to what they are learning. Teacher 4 spoke about the importance of questioning as characteristic of a critical thinker in that "*What I mean is that if you are a critical thinker you must be able to come up with different questions for every problem, you must think of what are the possible problems that I could get to solve a certain concept*". The teacher here speaks of a specific quality and ability of a learner in that she can judge critical thinking skills of the learner based on their question formulating skills. Teachers 1, 2, 3 and 6 talked about the role teachers should play in monitoring and directing mathematical lesson to ensure that learners can think

critically and not go astray with wrong ideas they come up with in doing Mathematics. Teacher 3 explained this by saying "*if you put a learner to lead, as a teacher you must be involved because a learner doesn't have that grade of a teacher, you (a teacher) can be there but helping that learner*". A view which seems to characterises learners' thinking process as dependent on the teacher's directives to lead learners so they can understand their Mathematics. Other roles and abilities teachers identified include learners helping each other, researching and collecting their own information in some tasks and the principal idea that teachers should maintain some authority in the classroom.

4.2.8 Discussion

The philosophical and psychological perspectives conceptualise critical thinking through actions and behaviours. Researchers working on the subject of critical thinking agree that demonstrating critical thinking abilities and dispositions are crucial elements of defining what a critical thinker is (Lai, 2011). Researchers in both perspectives agree that to conceive of critical thinking skills and abilities without disposition cannot be considered as representing critical thinking. The reason for this is that only consistent characteristics of good thinking based on actions and behavioural of a critical thinker can be attributed to critical thinking. Teachers in this study identified some of the characteristics and behaviour they believed crucial in learning critical thinking skills in Mathematics. Some of the behavioural qualities could be said to encompass elements of critical thinking skills and dispositions while some others were not. Teachers spoke about the abilities of applying and questioning as the most important dispositions of critical thinking in mathematical. Teachers specifically identified dispositions like inquisitiveness, flexibility and open mindedness as important in promoting critical thinking. This were discussed by teachers as normative aspects of actions and behaviours in learning Mathematics like questioning and posing question which teachers identified as part of learning and teaching Mathematics and promoting critical thinking. This showed that teachers were aware of some fundamental principles of encouraging thinking in their teaching of Mathematics.

Each teacher could generally identify at least one aspect that characterises a critical thinking disposition. Looking closely at the responses, teachers demonstrated a mixture of awareness of certain aspects of critical thinking but a limited awareness of broader conception of critical thinking abilities and dispositions. Teachers focused mostly on critical thinking skills as they apply to learning Mathematics or as processes or skills related to understanding Mathematics. Although both of the aforementioned aspects are important in teaching and learning Mathematics, they neglect to promote the crucial aspect of critical thinking which is about

making sound decisions and judgments through encouraging, among other abilities, self-directed, skillful analysis, self-disciplined, self-correcting and self-monitoring through mathematical principle in any situation. Although teachers could recognise the importance of encouraging learners to scrutinise and apply their knowledge, most of the explanation focused on classroom Mathematics and not entirely on mathematical skills. The focus was more on what should happen in a mathematical classroom in terms of responsibilities and not on what or how mathematical skills can help learners self-formulate sound decisions or judgment in the process of learning Mathematics. Teachers were speaking about dispositions as they applied to teaching and learning of Mathematics and in instances where they referred to transferring the skills to a broader contexts did so by making the assumption that it was automatically embedded in the subject and should not be taught as part of the skill set.

4.2.9 Critical thinking as skills and processes

All six teachers through the two interviews talked about and identified some skill and/or process that formulated their conception of critical thinking. Teachers identified either a skill or process or a combination of the two with main emphasis on either one or both as part of their conception of critical thinking. The responses fell into two categories of answers, either identifying a skill or identifying a process. There were many instances where teachers spoke of skills and processes as the same thing. This was when teacher were speaking about analysing, synthesising, evaluating and applying. For purposes of clarity when referring to skills we mean the ability to carry out a particular task with predetermined result and when we are talking about a process we are talking about a series of actions and steps towards a particular end. There was no clear indication in some instances whether teachers were speaking about processes or skills.

4.2.10 Identifying a specific skill

Teachers' answers included naming a skill they believed necessary for a critical thinker. Most of the initial answers given by teachers hardly mentioned some critical thinking skills and abilities teachers identified as critical thinking. Teacher 3 and 4 identified analysing and problem solving as skills. An example is teacher 4 who spoke about problem solving skills and critical thinking in Mathematics by saying;

> *"financial maths, let's look at financial maths, one reason these kids aren't getting their financial maths properly is because; one the problem solving skill is not there, there was a problem where the question said John got a lump sum of this much, you see the word lump sum on its own confused the learners so much then you realise that these kids then don't see the application behind the maths because even if you don't know what lump*

sum is, there won't be that application, I don't know if you see what I mean, so instead of me just saying that application, understanding of the terms is important. I don't know how you would put it down, but terms in Mathematics are important to be understood, this was a seven marks question by the way but most learners struggle to answer and when we asked why they simply said they didn't know what the lump sum is."

Teacher 5 talked about application as a skill in that if a learner cannot see how a mathematical process applies in real life then they are not critically thinking and are doing it procedurally. On those instances where teachers were asked question that required identifying specific sets of skills and practical examples of their conceptions, teacher could identify new skills or extended from skills in their earlier conception. An example of this is teacher 3 who initially identified analysis as a skill and problem solving as a skill later included synthesising and reasoning as thinking skill as part of the higher levels of Blooms taxonomy. When asked he responded by saying "*Critical thinking for me goes from level, level what... if they are six, it's from 3 upwards, the first 2 or 3 but after that...you evaluate*" (teacher 3). This was part of a broader definition of thinking skills as part of the upper higher levels of Bloom's taxonomy. Another teacher who initially did not identify any skill associated with critical thinking identified the skills of co-operation and comprehension as part of her conception of critical thinking. The teacher stated that"*...its cooperating and comprehend because if you understand you can critically think, but if you can also work with people you can critically think with the ideas obviously coming from other members ...*" (teacher 4). Another teacher also talked about co-operation among learners as part of critical thinking and understanding Mathematics by saying "*learners understand more if they speak to one another than speak to a teacher*" (teacher 5). Teacher 5 also identified self/group evaluation as a thinking skill in solving mathematical problems and to overcoming language barriers in multilingual classroom as it requires learners to self-monitor, self-direct and reflection each other or themselves.

4.2.11 Identifying a process

There were conceptions of critical thinking that referred to certain processes which were either mathematical or about thinking. These conceptions would identify a sequence of actions or steps learners had to follow. These were either expressed directly from the teachers' conceptions of critical thinking or formulated from explaining classroom proceedings that teachers believed promoted critical thinking. Teacher 1 spoke about two processes in her teaching. The first is a process where one has to "*actually extract what you've taught them, the concept from the new situation*" this was part of a problem solving process she encouraged her learners to use by

focusing on key mathematical terms. She gave an elaborated example of how she applies this in her classroom and said;

> *"I prefer, instead of giving them assignments, I prefer to give them investigations. If they go out and do an investigation, regardless of what the end result is, if their process of them going through trying to answer the question, they go through a particular process, collecting the information putting it together and then actually putting it together and go ok this is what I get from my information. Then I feel that actually enforces critical thinking because it is not looking at specific answer it is looking for a process, a thinking process is it on the right path or not, can you support what you have found if you can convince me that that is, what do you call that... if you can convince me logically that what you found is the truth and makes sense".*

The second process the teacher spoke about was using Cornel note taking. She explained that this note taking process involves learners taking their own notes, asking themselves questions and reflecting on what they have learned over a lesson or topic with the teacher using a rubric to assess understanding. Another teaching process was self-discovery/explorative learning. Teacher 3 gave an example about trigonometric functions and how changing certain variables can help learners discover the different transformation by themselves.

> *"you want a task, okay lets look at trig functions. You give them trig functions in grade 10, you concentrate on y=a sin x +q so you look at the effect of a and q so that effect of a and q learners must by themselves discover that if a is 1, q is 0. Theta is an element of...give them a domain maybe from 0° to 160° so they discover for themselves. If a is 1, what happens when a is 2 and a is -2, what will happen to that graph? Okay look at when a is 1 and now we are gonna make q=1, q=-1 so they going to see that there is a shift there is a stretch so those ones you don't have to teach, they can discover themselves...the effects of a and q."*

The teacher explained that this was one of his teaching processes which he believed promoted critical thinking in Mathematics. Some teachers also talked about critical thinking as a process in itself. Their explanation conceived of it as a tool within a process or procedure used to teach

Mathematics. Examples of these included responses like "*the problem with that thing is critical thinking if you are using it too much, you will end up not finishing ATP*" (Teacher 3). Teacher 1 conceived of critical thinking as synonymous with independent thinking and visualisation. She described her conception by saying "*I think because you must have a picture in your head of what this question wants me to do. When you have the picture then you know what tools I am supposed to use*". This was describing how visualising formulated her concept of critical thinking.

4.2.12 Discussion

The conception of critical thinking as skills and processes is common because of the difficulty in defining critical thinking. The psychological and philosophical perspectives both identify critical thinking through identifying skills and process that would constitute a critical thinker (Alwadai, 2003; Lewis and smith, 1993; Lai, 2011). Both perspectives agree that critical thinking means possessing the ability to use some important skills and processes that determine quality of thinking (Lewis and Smith, 1993). This lack of coherence is what Bailin (2002) identified as a challenge in education and teaching as this usually leads to the failure to recognise the normative nature of critical thinking. Critical thinking means different things depending on the context it is used and this is why it is often described as skills and processes. However definitions which should be used for education should encapsulate all the necessary elements of both decision making and judgement for practical used and teaching purposes.

The skills and processes teachers gave in the interviews demonstrated limited understanding of critical thinking, as it was argued in chapter 2 and in literature that teaching critical thinking is a product of intentional instruction and cannot occur spontaneously (Facione, 2000; Lewis and Smith, 1993; Lai, 2011; Alwadai, 2014; Innabi and El Sheik, 2007; Halpern, 2001). A limited conception of critical thinking that only focuses on specific skills and processes may also limit the extent in which critical thinking can be encouraged in lessons or educational outcomes. This is because critical thinking is associated with many thinking skills like evaluative thinking, analytical thinking and logical thinking. This means teaching that focuses only on encouraging some of the skills separately without considering the link with other different kinds of thinking could limit critical thinking. This is especially a problem in Mathematics as it is easy for teachers to fall into the trap of focusing only on thinking skills or processes that are only necessary for certain sections of Mathematics without linking them to other kinds of thinking.

According to Pingry (1951) a Mathematics teacher's main responsibility is to help learners grow in all areas of Mathematics. This includes developing their abilities in all aspects of problem solving, helping them gain understanding of skills in Mathematics, and a great number of dispositions as well as intellectual standards of thought which can be overwhelming because of the need to that the curriculum imposes on teachers. Teachers, as it will later be discussed in this chapter, feel that they have to focus on the sections they are to teach without paying too much attention to promoting a culture of critical thinking. Pingry (1951) argues however that because of the nature of Mathematics, the Mathematics teacher is in the best position to develop every aspect of thought processes associated with critical thinking. If teachers do not hold a normative conception involving intellectual standards or criteria that focus on describing a range, list of activities, or behaviors as part of thinking skills and processes then teaching thinking skills may be compartmentalized, fragmented or limited to thinking skills based on certain mathematical sections or topics resulting in failure to teach critical thinking skills.

4.2.13 Summary of section

Most of the teachers in how they defined their conception of critical thinking skills suggest awareness of some components of critical thinking. However teachers' conceptions and beliefs were not completely coherent to be described as encouraging critical thinking in Mathematics. The reasons for this were that even though they could identify some elements, they could not provide a consistent definition which can be used for teaching. Teaching based on an incomplete conception of critical thinking could result in teachers only teaching other thinking skills and not critical thinking. This is because for Mathematics teachers to promote critical thinking in their teaching they first have to be critical thinkers themselves so that they can be able to use their mathematical knowledge, pedagogical strategies, and intellectual standards and criteria as well as dispositional aspects to help learners become critical thinkers.

4.3 Teaching strategies and tasks that foster critical thinking

The second focus of the analysis is on the teaching strategies and teaching task that teachers used in their Mathematics classroom. As argued in chapter 2, literature states that critical thinking can be encouraged through intentional instruction by the teacher. Teachers were asked about the teaching strategies they used in their teaching which they believed fostered critical thinking. Analysis in this section was done with a focus on teaching strategies and tasks as suggested in literature. Following the analysis there were prevailing themes that emerged

- Teaching strategies and tasks that demonstrates a broad understanding of critical thinking
- Teaching strategies and tasks that demonstrates a limited understanding of critical thinking

The analysis focused on the teaching strategies and task that teachers associated with their teaching of Mathematics and critical thinking from statements of belief made by the teachers. The main issue in this section was to find out both; what the teachers said they were doing to foster critical thinking, and what they believed fostered critical thinking. There was often some conflation between strategies and task in teacher responses where teachers spoke of both as the same or one being part of the other, for instance describing asking open questions as a teaching strategy as well as teaching task and in those instances the focus was drawn on what was emphasised in the context of what was said by the teacher.

4.3.1 Teaching strategies and tasks that demonstrates a broad understanding of critical thinking

On the question of strategies which teachers believed should be used in classroom to promote critical thinking, teachers responded by identifying different methods and processes in teaching Mathematics. Explanation of strategies and tasks focused on what methods teachers would use in their classroom to promote critical thinking. One of the prevalent examples teachers gave is how they would introduce and administer work in teaching critical thinking to ensure that learners think about Mathematics as a subject and how it can apply in everyday life. What was common in their explanation of teaching strategies and tasks was the need to allow learners to work on their own or be inquisitive as most of their conceptions of critical thinking were based on that premise.

Most of the responses by teachers that can be said to promote critical thinking envisioned an interactive, participatory classroom of teaching Mathematics. An example of this is the teacher who stated that "*in my classroom I try to get it to be learner centered, instead of giving them what the solutions are, I want to get their understanding from them. So it's more discussion based, you tell me what you think, then you must back up what you think with a reason and you mustn't just be any reason, it must be logical, it must make sense to the situation that we are dealing with*" (Teacher 1). Teacher 3 said about his teaching that "*you give them that thing that requires them now to think and sometimes they can even question and say ok sir, why are you*

saying this?" all teachers emphasised the importance of a participatory and learner centered classroom as a central teaching strategy to promote critical thinking.

On practical strategies that teachers identified or believed promote critical thinking in Mathematics, there were varied elements that teachers identified. One of the most common and useful strategies teachers admitted to using was to asking higher order questions. Teachers believed that this would encourage learners to think critically. Teacher 2 stated that "*in terms of the level of questions that you give them it should not be level one questions that allows them to remember. But if the level of question maybe you provide them with level four questions where they have to think outside the box and analyse the situation*". This was part of an explanation of teaching strategies cantered on questioning or rather level of questions as the teacher talked about using Bloom's taxonomy. Other examples of this are from teacher 3 and 4 who both talk about questioning as also central in their teaching and both teachers together with teacher 6 believed that asking appropriately leveled mathematical questions that are practical to learner's everyday experiences is essential to helping learners think. The teachers stated that they generally use the six step Bloom's taxonomy and believed that the 3 higher levels promote critical thinking. Another example is teacher 6 who stated that "*you can ask deep questions like the one that require reasoning. Ya, they need to use reasoning like... reasoning is one of the things that you can try to know their thinking in a way that they can be critical at the end of the lesson about certain situation*". The teacher (6) gave an example to elaborate on how he uses probability and deep questions to encourage critical thinking and said;

> *"if I should make a practical of it, uhhmm... maybe uhhmm, to analyse the weather news...or not the weather news or...for instance why; oh okay. You can analyse why at casino they choose 7 to be a winning number and why actually those, whatever that you have to choose, they choose it to be the winning thing like if they roll a ball in that machine; why they choose to say if you it ended on this point its uhhmm...what you call, you get a jackpot. Understand, or why when you play maybe poker; I don't know much about poker but if it's you get this certain cards, it's why is it a win you understand. So, there of course the concept is that the, actually the main idea is that uhmm...the number that seems to appear have lighter chances for you to win is the one that they uses. So, meaning the person who is responsible for or for whatever ehh...maybe casino whatever, they know that now if one you win, how many will lose like nothing for instance; if one person win, how many person lost billions over the money that they advertised that you are going to win 1.5 million but if you were to analyse what are the chances of winning*

lotto, they are very slim but the chances for personal responsible for lotto which is ithuba (chance) neh, they will win but the people are losing".

The teacher gave this as an example using higher order questions that also encourages learners to be inquisitive because it is a real life problem. Teacher 1 explaining her process in her teaching said that she uses "*open ended discussion right. Ok I start with an open ended question and then they give whatever they give me, so when they give me something, I throw back another question again, right sub-questions right try to get them to think about the concepts*" All teachers spoke of the importance of using a lot of questions in their teaching especially higher order questions

Another practical strategy teachers identified was self-directed or exploratory learning as important in their beliefs about teaching Mathematics and critical thinking. Teacher 2 spoke of demonstrating through practical examples mathematical ideas like volume to encourage learners' self-exploration through extracting the concepts on their own as part of learning Mathematics and critical thinking. Teacher 3 said that he employs the strategy of "*discovery learning, learners they discover like before I explain, I explain the concept behind this thing they are going to do*", teacher 1 stated that "*ok what I do is, you ask a question 'why is it that you are answering this way?', so if you can think about your thinking when you answer that question, you are being critical of what you have done*". As stated earlier all teachers spoke about some ideas of learners taking responsibility for their learning. This teacher explained that critical thinking in Mathematics can be achieved through exposing learners to challenging questions during lessons or any mathematical problem or task chosen by the teacher in the process of teaching and developing learners' critical thinking skills and abilities.

There was also reference made of the importance of critical thinking in helping learners to understand Mathematics. Teacher 1 used a different approach of combining concepts in teaching. Speaking about teaching perimeter, area and volume she stated that "*using critical thinking, I can cover those three concepts faster, like I can go back and forth in the concept because they understand what they are. So instead of me doing treating everything separately, all concepts are linked somehow...*" which was part of the idea that learners can understand Mathematics better if they learn to see connections on their own with guidance from the teacher. Teacher 4 talked about critical thinking as questioning which is her method of teaching. She spoke of giving learners different questions on the same topic to familiarise themselves with the

ideas and then using complex questions from the tasks she gives to encourage them to see relationships and think deeper about the concepts.

Another strategy that teachers spoke about in teaching critical thinking in Mathematics was utilizing learner's prior knowledge. Teachers talked about the importance of prior knowledge as a requirement of teaching learners to critically think. This includes knowledge of Mathematics to understand problems as well as to know general ideas underpinning mathematical problems. Teacher 3 argued that where there are gaps in knowledge, critical thinking cannot be taught. He argued that there needs to be prior knowledge of Mathematics from which critical thinking can be developed. Teacher 3's argument was that without some knowledge of Mathematics to develop critical thinking as a starting point of teaching, it may not be possible to develop critical thinking because the thinking is based on understanding mathematical principles and engaging in mathematical problems. He said that "*Critical thinking only applies when a learner has a clear prior knowledge or something like that so that they can have a starting point. If you don't have a starting point, there's no premise and if there is no premise it's going to be very difficult for me to start. They need to start from somewhere.*" This he said referring to his own teaching and learners. Teacher 2 and teacher 6 also spoke about using learner's prior knowledge to set up mathematical concepts and encourage critical thinking. Teacher 4 talked about the need of prior knowledge of Mathematics to encourage critical thinking as learners in her school were failing. She said "*these kids are failing everything because of the lack of something called critical thinking, they can't relate to the things we teach them*". She argued the point that most of the difficulty of understanding Mathematics and promoting critical thinking is that learner's prior/everyday knowledge is often not related to Mathematics in how teaching takes place in the classroom from her experiences.

4.3.2 Teaching task that promote a broad understanding of critical thinking in Mathematics.

Examples of teaching and learning tasks that teachers gave which they believed promotes critical thinking were consistent with their teaching strategies. An example of this consistency between teaching strategies and task can be demonstrated by teacher 4 who emphasised the importance of cooperativeness and questioning between learners as well as the teacher. Asked about an example of a task that she uses to promote critical thinking the teacher said she asks her learners to "*Formulate a problem and give it to your friends, they do and then swap. Ok if one's problem is able to be solved, then the learner can critically think, that means that learner is able to apply a situation in real life*". This she believed promotes teaching critical thinking in

Mathematics. The teacher elaborated on how she incorporates self-directed and discovery learning in her classroom through using complex questions and a lot of examples that allows learners to see mathematical relationships. She gave the example;

> *"remember for you to critically think, you must also be able to identify relationships right, so if I have given these learners a lot and loads of work on different sections of geometry and then at the end I combine them, I give them one complex thing to deal with and they are able to solve it with the use of the ideas and concepts they have learnt on those small bits I have been giving them, these learners are applying their critical thinking. They are applying what they have learnt; they know why they have been learning it. Because if they see a complex structure they can't do because they are used to doing the easier ones, they don't understand anything, they just have the procedure then, but if they can apply what they have learnt on the small one and the big ones, they are show their critical thinking."*

Teachers 2, 5 and 4 spoke about giving tasks with questions like finding the unknown, open ended tasks and asking questions to foster critical thinking in Mathematics. Teacher explained that "*if you have to help them to think critically, it's a matter of...for example you can provide them with a task where they can [come up] with their own conclusion, then after that you can use or analyse on a certain criteria and use it as a reference point to correct mistakes and misconceptions*". Teacher 1 spoke about discovery learning tasks that are based on questioning, she gave an example of a task she used which was;

> *"yes because I can't go into a classroom and teach concept and not ask them a question that will take them out of the particular situation that they are in, or a known scenario. So out of the known scenario give them something that probably delivers an experience but try to explain what the scenario is and then see are they able to input the concept that they have just learned now"*

Other teachers as discussed earlier in their conceptions of critical thinking believed that discovery learning task where learners figure things out for themselves are more effective in promoting critical thinking in Mathematics. Teacher 1 also suggested that her teaching involved learners taking their own notes and critically reflecting on what they have learnt and wrote during lesson to assess the extent of their understanding of the concepts being discussed.

Teacher 6 expressed the importance of giving tasks that rely on learners' everyday experiences and current affairs to encourage thinking because they relate more to learner thinking and makes it easier for them to apply mathematical skills.

Teachers also gave some practical examples of how they believed critical thinking should be taught in mathematical content. Teacher 3 when asked how critical thinking can be encouraged or developed in teaching gave the following example;

> *"Like now, when you are talking about Euclidian geometry midpoint theorem, you want them to show proof of the midpoint theorem using the reverse of it. That one requires learners to see okay, if this is the reverse of the midpoint theorem...you can ask them now ok, why is it this one equal to this one? Can you see, why when two parallel lines in a triangle if those two lines are supposed to join the two points (about the midpoint). This requires some learner to identify ok, this one no...they are just trying to want us to analyse to evaluate so those kind of things"*

Teacher 4 when also asked the same question and how to ensure effectiveness in teaching critical thinking in Mathematics talked about an example of geometry and proof. She explained that she prefers building understanding by using smaller elements of concepts and then increasing the complexity to challenge learners to think deeper. She gave the example of

> *"ok I could go to geometry in proofs, different proofs. You give them similar proofs to prove, prove it's a parallelogram, prove that the two triangles are congruent, prove that prove that prove... when you then bring a complex structure, a circle that has triangles and everything in it within it, you give the learner and say now prove that this is a quadrilateral given that it has maybe different triangles, ... learners can then collect all the ideas from the familiar proofs that they've been doing or from the number of proofs they have been doing all along, they connect them and they do the final thing that you want them to do."*

Teacher 1 speaks about using teaching task based on applying mathematical knowledge to real life situations as part of her conception and belief about critical thinking gave the example that;

> *"Alright, I'm going back to perimeter and areas and stuff. So we were dealing with the concept of perimeters. So I started off with, because I knew that they have been doing this since grade ten and this particular maths lit, so I asked them that what is your*

understanding of one perimeter is? And then like then I bring a real life object like this table here and ask is that 'will this have perimeter and why do you say it has perimeter and in what scenario would you be looking for the perimeter, what are you doing when using the perimeter measurement of this table?'. So these are the kinds of questions that I will be asking, we don't go deep but then I try to"

Other teachers gave examples about tasks that were based on their teaching strategies and belief as well as conceptions of teaching critical thinking. Teacher 6 similar to teacher 1 gave examples of tasks drawn from real life Mathematics situations which they believed promoted critical thinking, teacher 5 gave examples on using mathematical strategies and formulas to solve mathematical problems which were consistent with teaching strategies that he said he used in his teaching. Teacher 5 stated that if learners understand mathematical formula and where they can be applied, this showed that learners were critical thinkers. Teacher 2 gave examples that encouraged mathematical problems solving and self-discovery as teaching tasks that promoted and developed learners' critical thinkers.

4.3.3 Discussion

Researchers overwhelmingly agree that critical thinking can only be taught through intentional strategies and content design (Kuhn, 1999; Halpern, 2001; Beineke, 1985). Teaching strategies are one of the most important components of teaching critical thinking in Mathematics. Halpern (2001) and Pingry (1951) speak about teaching strategies which can be used to encourage critical thinking skills, abilities and disposition from mathematical skills. This skills include; an active involvement of learners, practicing thinking skills in multiple settings through inquiry or argumentative, encouraging a mathematical classroom climate that is inquisitive, help learners have some background on Mathematics and how it relates to other subjects and disciplines, learners monitoring their own learning towards a particular identifiable goal, setting a rational for the skills to be learnt before teaching and other learning and teaching strategies that incorporate tasks which enforces critical thinking. It is argued in literature that the only way to encourage critical thinking is that teachers help learners develop critical thinking skills and abilities and develop attitudes in learners so they are disposition to using such skills for their learning and in other areas of their lives.

On the teaching strategies that teachers talked about using in their teaching, each teacher could identify some teaching strategies or tasks that adheres to strategies and task purposes discussed by Halpern (2001) and Pingry (1950) as essential in promoting critical thinking in Mathematics

and in general. Teachers spoke about teaching strategies like learners participating and regulating their learning with the help of the teacher, learners practicing different skills through participation in a question and discussion classroom which encouraged argumentation, and teachers requiring that their learners reason and justifying their mathematical arguments. Teachers could identify teaching strategies that could promote thinking skills in Mathematics and what thinking skills could be cultivated. However in explaining their teaching strategies and tasks, teachers demonstrated to a certain extent a lack of understanding crucial aspects of teaching critical thinking abilities. One of these aspects was helping the learners to learn to make judgements in applying their knowledge of the subject and how mathematical skills could be used in non-mathematical situations. Strategies were mostly focused on using mathematical skills to make decision about appropriate strategies to use. Strategies teachers discussed did not suggest any encouragement of learners evaluating the value of the mathematical processes for their own understanding.

On teaching tasks that teachers use, there was no clear indication on literature about what specific tasks can be used to encourage critical thinking. In cases that tasks are mentioned there are usually suggestions that task should be closely linked to teaching strategies. As discussed above teachers' responses in this study tended to resemble their teaching strategies. Almost all teachers spoke about teaching tasks that focused on explorative and discovery forms of learning. Teaching tasks that teachers talked about using in teaching Mathematics were mostly focused on developing mathematical skills with critical thinking skills and abilities. Explanations of teaching tasks developed through the interview as the explanation either placed more focus on the teacher or learners' initiative when engaging deeper with the content of the tasks however some tasks and strategies placed limited emphasis on learners' decision making or judgement. Overall the teaching tasks suffered from the same defects as the strategies as they do not suggest any focus on learners formulating their own self-directed decisions and judgement in working through them.

4.3.4 Teaching strategies and tasks that demonstrates a limited understanding of critical thinking in Mathematics

There were some teaching strategies that were limited in how they conceived of teaching and learning critical thinking skills in Mathematics. Those tasks and strategies were limited based on two aspects; how they relate to strategies that are recognised in literature (Halpern, 2001; Pingry, 1951) and teaching issues that were identified to be limiting by the teacher.

4.3.5 Teaching strategies that promoted a limited understanding of critical thinking in Mathematics

There were teaching strategies that were limited in their conception of teaching critical thinking in Mathematics. Those strategies were the ones that ignored important aspect which are identified in literature as effective in encouraging thinking critically within Mathematics. One of the common teaching strategies that promoted a limited conception of critical thinking by teachers was how they conceptualised the role of mathematical content in teaching learners critical thinking. The majority of teachers generally agreed that critical thinking should be integrated in the mathematical content with only two teachers stating that the two should be treated separately. Teacher 4 was one of the teachers and expressed the view that there should be some separation of critical thinking and Mathematics. The reason given was that in her experience learners tended to have difficulties in understanding Mathematics because they lack critical thinking and if they were to be taught critical thinking separately they would help in understanding their Mathematics. She expressed this by saying

> *"You know based on the school I teach in, I would say it needs to also come in as separate entity. By separate entity I would say now come...for maths it will be applied maths, do you realise that for applied maths its more about critical thinking do you see what I mean. Because of the maths we teach here and the learners we have here, you'd be mostly focused on procedures I would have to be honest with you. We cut off I mean we have short time, CAPS is on our backs you know basically what you need to do when exams are approaching to catch up you need to finish and simply show these kids how to do it. You will never show them the why but if the was a separate section of teaching these learners the why's and what...ja so I think critical thinking should be taught as a different subject. Ja I would say that it should be even a subject because really these kids are failing everything because of the lack of something called critical thinking, they can't relate to the things we teach them."*

The justification of the position of not integrating mathematical content and critical thinking were based on teaching responsibilities and demands of the curriculum on what has to be taught and when or how long particular topics have to be taught. Other examples of teaching strategies that demonstrated a limited view of critical thinking were like those expressed by teacher 3 when he stated that "*yes yes..but ya the problem with that thing is critical thinking if you are using it too much, you will end up not finishing ATP (Annual Teaching Plan)*". This demonstrates a limited understanding critical thinking. The teacher sees critical thinking as

something to be used in teaching Mathematics and not part of the discipline. Teacher 4 stated that "*there is and honestly it happens, this happens to me many times, when I, in science you are forced to teach critical thinking there and there, when you do that you are bound to be a week or three weeks behind honestly there is lot of content.*" The view shared by teacher 5 also demonstrates a limited understanding as it takes the view that teaching critical thinking means using long and elaborate strategies that take up time. This view fails to see that well planned lesson can encourage critical thinking without requiring intervention by the teacher or using too much time.

Another teaching issue that demonstrated a limited view developing critical thinking skills in Mathematics was the influence of the curriculum document (CAPS). According to teachers the pacing of the topics in CAPS is imposing restrictions on time and methods of teaching. Teachers expressed concerns that the curriculum requires them to cover too much mathematical content in a short amount of time to effectively teach critical thinking skills in Mathematics. Teacher 4 expressed the concern that the CAPS document and district officials were imposing restriction on what the teachers can do in the classroom by making it hard for them to be creative in their teaching. One way teachers expressed this imposition was that teachers were sometimes forced to teach for the purpose of learners passing tests and exams rather than for them to learn to think critically about the subject because of deadlines. Teacher 4 stated that "*Because of the maths we teach here and the learners we have here, you'd be mostly focused on procedures I would have to be honest with you. We cut off I mean we have short time, CAPS is on our backs you know basically what you need to do when exams are approaching to catch up you need to finish and simply show these kids how to do it.*" Teacher 6 expressing a similar view that "*to be honest for me as a teacher right now neh, its the time is very limited in terms of if you want to achieve the objectives in terms of you want learners to be critical and stuff. Sometimes you end up teaching for exams.*" Another teacher who made reference to outside forces and how they impose on his teaching stated that "*Critical thinking not just critical thinking even conceptual understanding, it requires and needs time and for most of us we don't have time. We need to talk about ATP we need to finish it they come, people from the district they ask you why are you not teaching this, like this people who are facilitators are impossible.*" All teachers expressed the same point that their teaching of Mathematics was severely compromised by the pacing and deadlines in the curriculum or enforced by officials which often mean they have to work on an inflexible schedule. They openly admitted that they often did what was required which means they do not

teach in a way that promotes critical thinking in their classroom despite believing that it is important.

4.3.6 Teaching tasks that demonstrated a limited understanding of critical thinking in Mathematics

Teaching tasks which demonstrated a limited understanding of critical thinking were also mostly linked with teaching strategies and challenges the teachers faced in teaching. There were not too many examples from teachers that demonstrated tasks that could be fully interpreted as not containing elements of critical thinking on their own. However there was one recurring limitation in almost all the examples that demonstrated that teachers may not fully understand why it is important to teach critical thinking in Mathematics. The limit was on the outcomes and goals of using or acquiring critical thinking within the Mathematics. Some of the examples of tasks teachers discussed lacked one fundamental principle of critical thinking which is to help learners explore their beliefs and decide what to do using their Mathematics. One example of a tasks that showed a limited understanding of critical thinking is;

> *"in maths? I would say with the grade 8s I would go down to grade 8s now, after having taught simplifying then going to factorise, and you say lets factorise, then you say to the learners now show that your answers are right. They must now remember that for them to get it to be factorised it was in its simpler form, and if they are able to take it back to its simplest form by simplifying then they critically thinking. That is what I do a lot with grade 8s, actually even when we are solving for x I say now take your answer, show to me that your answer is right, the learner must simply take the answer and substitute back. It shows they critically thinking, they understand what x is about, its not just something that comes out of the blue, because you can actually give learners a simple sum like X + 2 = 5 solve for x, then the learner will say X = X do you see that the learner doesn't even understand what you are talking about, what's this x is about they don't know but if a learner is able to work procedurally with it and then you say take your answer put it back, show me that this learner understands what they are doing"*

This example demonstrates a limited view of learning critical thinking as the teacher conflates performing mathematical procedures as critical thinking. Another example of a task one teacher talked about using in the classroom that they stated as promoting critical thinking is from teacher 2 who said "*critical thinking when it comes to maths, differentiates between a person who can do maths a person who understand maths.*" The teacher was talking about abilities relating to using mathematical methods, formula or procedures as indicating critical thinking.

When the teacher was asked elaborating about this idea he responded by saying "*we can refer to critical thinking when someone is able to use a specific formula with understanding, but if there is a lack of critical thinking they just use it for the sake of using it because they know how to use it.*" This view exemplifies the limited perspective of critical thinking by the teacher as he also conflates doing Mathematics as thinking critically.

Almost all teachers when asked to make reference to the major decisions about teaching and giving work referred to the curriculum document as the primary source of directions. The curriculum documents was also identified as the primary reason for teachers not focusing on teaching or using tasks that promote critical thinking as discussed above. The justification of using the curriculum document by teachers was that it was the official policy that dictated teaching responsibilities or requirements. Teacher 1 elaborating on this point stated that "*because we are chasing the learners to pass, so I don't spend as much time as I would like on activities that fosters critical thinking because at the end of the day, you need to have evidence that you have taught.*" Teacher 3 said "*We need to talk about ATP we need to finish it, they come people from the district they ask you why are you not teaching this, like this people who are facilitators are impossible.*" Teacher 4 stated that "*We cut off I mean we have short time, CAPS is on our backs you know basically what you need to do when exams are approaching to catch up you need to finish and simply show these kids how to do it*". All teachers referred to the curriculum document as an obstacle to their teaching because of the timing and the emphasis on keeping time from the district officials which they argued imposes limitation on the tasks or any work they give to learners. A limiting factor in the context of the analyses of what teachers were saying was not recognising that teaching critical thinking can be integrated into their current model of teaching Mathematics through encouraging good thinking elements such as self-exploration, self-monitoring, open minded and unbiased habits of thinking to make decision or judgements. As discussed in Chapter 2, planning or creating teaching tasks, strategies and the assessment of critical thinking skills takes a lot of time especially if the teacher does the planning or creating. Creating teaching conditions and tasks that promote critical thinking requires careful considerations about potential pitfalls especially in Mathematics as there is always the likelihood of shorthand uncritical methods of completing activities. This requires a lot of time in planning assessments that would identify thinking that is critical or uncritical. A limiting factor in the context of the analyses of what teachers were saying was not recognising that teaching critical thinking can be integrated into their current model of teaching mathematics through encouraging good thinking elements such as self-exploration, self-monitoring, open

minded and unbiased habits of thinking to make decision or judgements. As discussed in Chapter 2, planning or creating teaching tasks, strategies and the assessment of critical thinking skills takes a lot of time especially if the teacher does the planning or creating. Creating teaching conditions and tasks that promote critical thinking requires careful considerations about potential pitfalls especially in mathematics as there is always the likelihood of shorthand uncritical methods of completing activities. This requires a lot of time in planning assessments that would identify thinking that is critical or uncritical.

4.3.7 Discussion

As stated above teaching strategies and task design are important to critical thinking. Teachers could recognise the important aspect of setting goals for teaching and rationalising the teaching of critical thinking skills as argued by Halpern (2001) and Pingry (1950) as discussed above. Considering the teaching strategies and tasks that are discussed above teachers generally viewed teaching critical thinking as requiring more time and responsibility for the teacher. Teachers viewed critical thinking as an impediment to normal teaching of Mathematics and could conceive it as part of their everyday teaching responsibility. One of the reasons for this view which was cited again was the curriculum which they stated forces them to teach procedurally or teaching for tests or exams. Similar sentiments about teaching tasks could be said about teaching strategies. Teachers talked about the pressure of focusing on tests or exams because of the demands of the curriculum. Some teachers conflated mathematical content with critical thinking through speaking about the act of doing a mathematical task as learning to critically think through the content. This view goes against literature which suggests that critical thinking cannot spontaneously occur from mathematical content on its own. This indicated that the teachers were unsure of what roles teaching strategies and tasks play in teaching mathematical skills and critical thinking and what kind of planning is required to ensure that both teaching tasks and strategies were effective. Teachers emphasised the issue of time as it applied to teaching time and did not directly express being inconvenienced by the planning process of their lessons which given the complains about packed curriculum meant less time for focusing on focusing on promoting critical thinking skills.

4.3.8 Summary of section

Teachers demonstrated knowledge of the importance of using teaching strategies and task design in promoting critical thinking. They also could identify how critical thinking and Mathematics can be taught in the classroom to encourage deeper understanding of content as well as learning thinking skills. However one striking issue that immerged is that there seem to be a gap between

what teachers believe about teaching and what they appear to do in the classroom. Teachers honestly admitted that although they believe teaching critical thinking is important, they did not enforce such a belief in their teaching of Mathematics because it was limited by teaching loads and time constraints of the curriculum.

4.4 Teacher assessment of critical thinking

In this section the focus is on teachers' understanding and beliefs of how critical thinking could be used as a tool for understanding mathematical content in teaching and learning rather than to view critical thinking as a distraction or separate entity/area of teaching and learning in assessment of Mathematics. This is done through looking at the extent to which teachers have a clear, elaborate and substantive conception of how to assess critical thinking as well as what their conception constitute for teaching and learning of Mathematics. One theme emerged from analysing teacher responses to questions which was;

- Use of standard forms of assessments of critical thinking

Teachers in responding to questions on assessing critical thinking gave responses on what intellectual criteria and standards they used to judge their teaching or learners' responses of mathematical work. Among the responses teachers mentioned common procedures they would use and of challenges they faced in using assessments in terms of promoting critical thinking in Mathematics. The views and beliefs they expressed about assessment of their learners and themselves on being critical thinkers suggested that they focused on standardized forms of assessments like classroom activities, assignments, projects and tests given to be marked for a certain mark.

4.4.1 Use of standard forms of assessments of critical thinking

Teachers were explicitly asked about their conception of intellectual standards and criteria and how they would use these to assess critical thinking in Mathematics. Responses suggested that teachers could only identify a few intellectual criteria and standards, if any. When teachers were asked how they assessed critical thinking in their classrooms in teaching and learning, the responses given suggested that teachers often relied on assessment criteria and standards that were recommended in official documents like the curriculum. Formal forms of assessment as referred to what teachers said mean the more common classroom assessment which are either summative or formative which include giving questions to be marked for correct or incorrect answer, responding to learners' questions during teaching, and any common form of teacher assessment that are normal in a classroom. In referring to how they conduct assessments in their

teaching, learner's effectiveness in solving mathematical problems was often the measure identified as indicating critical thinking skills, ability and disposition. These responses were characterised by three categories of response which were their conception of intellectual criteria and standards and when or how intellectual criteria and standard may apply.

4.4.2 Conceptions of intellectual criteria and standard

On conceptions of intellectual criteria and standards for the purpose of assessing and teaching critical thinking in Mathematics most of the teachers could not readily identify a clear example or knowledge of what constitute intellectual standards and criteria. Some teachers could identify some of the intellectual standards and criteria like logic and reasoning through discussing elements of assessing performance of learner and their Mathematics skills. Teacher 1 spoke at length of the importance of learners' use of logic and reasoning as the core of being critical thinkers. She spoke about the importance of using intellectual criteria and standard saying "*logic, I look at logic because the question that... alright so if they give a solution... so how did you go about doing this and then with an extension I ask them, as they explain you go how come you used this and not this process and why did you add and not subtracted so things like that*". Teacher 6 spoke about a similar sentiment in that logical thinking and reasoning are and should be the focus in assessing learner responses with regards to their ability to think critically about their mathematical work:

> *"I will say because of...the thing is now I've been experienced in mathematical literacy. Mathematical literacy our criteria is you looking at the context, the context should be the one that judges, so did you consider about the context. If your answer is only based on like number 1. I will say that err...on a personal view, if it's a personal view I will say that uhhmm no we will reject it if it's a personal view but if your reasoning is universally so then it can be accepted because universal can apply, it does not depend on a condition, it is acceptable"*

The teacher is talking about elements of using logic through rejecting responses that contain biasness which she believed that reasoning to reflect critical or intellectual standard and criteria should focus on objective facts rather than individual experiences. Expanding on this Teacher 6 talked about

> *"As I mentioned above about all the consequences, when you consider about all the consequences. When you justify what you are saying with evidence, facts then we can say your reasoning was good but if you just reason because of you are answering a question*

it becomes a challenge because of number 1. For instance if I were to say, there is a scale there are two scale and one is to hundred and one is to two hundred, you want to draw a scoop map , which scale are you going to use and why? You need to provide that, because I have a certain book I will be using A4 and I will be drawing my map scoop on my A4 so I can see 1 is to 100 it's a much better scale than 1 is to 200 even if 1 is to 200 it's so small it can fit on my paper but the problem is people won't be able to visualise my map but if I were to use 1 is to 100 people are going to be able to see my map as well as it will fit on my paper."

Other teachers mentioned or alluded to other elements of logic and reasoning which they believed are important intellectual standards and criteria of a critical thinker, for example teacher 3 who stated that "*no when you are looking at accuracy you are looking at the answer, no you are not looking at the answer you are looking at how they can do the answer. The accuracy is always there because maths got the one...always correct answer*" when speaking using criteria and standards in assessing critical thinking skills and abilities

On the use of other intellectual criteria and standards teachers were seemingly unaware or unsure of how they can apply these in teaching or assessing critical thinking in their classroom. Some teachers could mostly speak about intellectual criteria and standards like accuracy, relevance and depth in a limited sense or conversational level and other teachers used their own definition of some of the intellectual criteria and standards. An example of a different conception is teacher 1 who spoke about an example of relevance as an intellectual criteria and standard by saying "*when I look at what relevance is, is it something that will happen in their lives is it something that, is it something relevant in their circumstances as well*". Teacher 4 also speaking about accuracy responded by referring to it in a general sense saying "*yes, are the concepts applied correct yes. And it comes back to the idea that it doesn't have to be precise, because by precise we mean if the thing follows when the answer is right.*" Teacher 3 talking about how accuracy may apply in teaching critical thinking spoke seemingly against by saying "*I don't think accuracy for me applies in this one because if you are accurate procedurally then you have an understanding*". Teachers 2, 4, 5 and 6 confessed to not being aware of intellectual criteria and standard even though they were explained to a certain extent during the interview. This indicates that teacher do not have a clear grasp of how to assess critical thinking skills.

4.4.3 Nature of assessment tasks

One other aspect of assessment that some teachers identified as promoting critical thinking in assessments was using open questions. Teachers as discussed in the section of teaching strategies and tasks recognised the importance of using questions that encouraged exploration of mathematical problems with no certain answers to assess and promote learners' critical thinking abilities. Teacher 1 stated that "*the questions have to be open ended, so they can have an input. In terms of me deciding whether it's critical or not...because critical thinking to me is independent thinking, from a situation you extract and then look at it in like is this going where I like?*" Teacher 6 also recognised some element of this aspect assessing and said "*so what I do, what I emphasis is that they should acquire skills so that they can be able to solve unfamiliar situations because you know in mathematical literacy specific the questions are forever changing with the current affairs*". Although some of the teachers spoke about using discovery or exploratory teaching strategies, they did not identify any specialised assessment methods besides teacher 1 and 2 identifying open ended questions as their preferred method.

4.4.4 How intellectual standards may apply

When it comes to formal forms of assessment that teachers used in assessing critical thinking abilities and dispositions in their classrooms, teachers could identify some understanding of how the criteria and standards applied to their practice. Teachers could identify the importance of using intellectually challenging questions to help learners think more deeply about mathematical problems. Teacher 3, for example, in talking about questions used to assess critical thinking in Mathematics spoke about the importance of using higher order questions that test synthesising and analysing. One method the teachers confessed to using was Bloom's taxonomy in setting and assessing learning objectives. Teacher 3 gave an example on setting higher order questions to assess learner by saying "*if a learner is able to see that if ok this questions sometimes they just answer...let's say higher order questions they can ask you a...even the marks they, sometimes the mark determines the level of thinking*". Teacher 4 talked about "*the district actually wants us to use the Bloom's taxonomy when setting tests, remember the Bloom's taxonomy a learner must remember, understand, be able to apply, be creative, analyse, evaluate and apply.*" This according to teacher 3 and 4 means a learner who scores higher in a test designed through questions in a hierarchy of Bloom's taxonomy means a learner who can critically think.

On the question of when intellectual criteria and standards would influence formal assessment of critical thinking in Mathematics, different assessment conception emerged. Teachers expressed

different approaches in how to assessing knowledge of the subject and thinking. Examples of this are when some teachers spoke about doing certain task as inferring critical thinking as discussed above in the section regarding tasks and strategies. Other teachers gave different methods of assess like "*if you give them a problem...these kids have been using different methods to solve the problem I mean mathematically correct methods and they all busy leading to the same answer, it shows that these learners have some form of critical thinking*" (Teacher 4), which indicates that teachers assessed thinking based on dispositions of learners following logical mathematical steps in formal assessment: "*logic, if it follows logically right then if its mathematically correct, because I must check if the maths makes sense, but for me its logic if one argument follows from the next then I feel its...they are using critical thinking*" (Teacher 1), and other such statements about assessing knowledge and thinking about Mathematics discussed earlier.

4.4.5 Teacher challenges in assessment

On the challenges teachers faced in assessing for critical thinking using formal forms of assessments, there were a few obstacles identified. There was again the pressure of getting the stipulated amount of work required by most teachers as directed by the curriculum. Teacher 1 spoke about "*I have to look at because I need to go according to...curriculum like this week I must teach this topic and all that and stuff*" which meant following the stipulation with the only time to deal and assess critical thinking coming only if there is extra time left. Teachers again identified time as a factor as it was discussed in the section of teaching tasks and strategies as another challenge to assessment. Teacher 6 in talking about issues of time and assessment said that "*to be honest for me as a teacher right now neh, it's the time is very limited in terms of if you want to achieve the objectives in terms of you want learners to be critical and stuff. Sometimes you end up teaching for exams*". This was another challenge for teachers as some felt there was a lack of resources that they can use to teach and assess critical thinking

4.4.6 Discussion

The use of criteria and standards to assess critical thinking disposition is mostly used in the philosophical perspective (Lewis and Smith, 1993). To get a balance of the elements of critical thinking Lai (2011) argues for using both the cognitive and dispositional to identify intellectual criteria and standards to assess critical thinking. One way this can be used in assessment is to use open-ended types of problems as they are more sensitive to dispositional aspects of critical thinking. Another method of assessing using criteria and standards is for teachers to use activities which would allow self-constructed answers by learners or those which would

encourage practicing thinking skills in an everyday context with follow up probing questions. These methods of assessments can be used abundantly in Mathematics as mathematical problems exist in all aspects of everyday life and the internet. One exemplary aspect of critical thinking is that it encourages thinking about any subject or discipline, problem or content in which one engages in self-directed, skilful analysis, self-disciplined, self-correcting and self-monitoring. Mathematics skills and principles can be found and used in many subjects or disciplines and emphasizes many aspects of problem solving which develop autonomous decision making and judgement which are all central aspects of critical thinking.

Teachers' responses suggested that they rely mostly on the standard forms of assessment inherent in Mathematics. Teachers used logic and reason to assess thinking skills and formal school work. Most examples they gave showed evidence of being taken from teaching sources like textbooks to which their standards and criteria of completion were often not adjusted to serve instructional goals which can be considered as promoting critical thinking. The lack of knowledge and methods of assessing intellectual criteria and standards demonstrated limited mathematical teaching which meant not enough attention would be given to cognitive and dispositional aspects of critical thinking within Mathematics. Teachers seemed to be unaware of the critical role of defining intellectual criteria and standards as lesson outcomes to ensure that the normative aspects of critical thinking are achieved. According to Lai (2011) one can only be said to be a critical thinker through dispositions which demonstrate critical thinking skills abilities. Looking at teacher responses there is a strong sense that teachers are aware of some assessment requirements learners should demonstrate to be considered critical thinkers like logic and reason but are either unaware or do not pay careful attention of other intellectual thinking skills that can be learnt through Mathematics. All of this reifies that the teachers do not have a coherent or complete conception of critical thinking and that they do not appear to teach in a way that would promote the complete elements of critical thinking in their classroom despite their beliefs that it is important.

4.4.7 Summary of section

Teachers showed a fair inclination and loyalty to what can be considered as standard forms of assessment that are generally used in the classroom as they believed that the demands of the curriculum does not allow for more open forms of assessment as recommended for promoting critical thinking. There was indication that should they get more freedom to explore thinking skills, they would take up the task with enthusiasm as current conditions do not allow for any time to assess critical thinking skills outside those that are valued in what is perceived as more

demonstrating successful learning in Mathematics like marks obtained or test scores. The inability or lack of time to use appropriate methods to assess critical thinking skills in Mathematics by the teacher due to the curriculum by the teacher could mean that they would be less likely to promote the skills even though they believe that the skills are important in Mathematics and educational outcomes. These statements by the teachers lead to the implication that teachers may not be adequately focusing on teaching and assessing critical thinking skills as some teachers confessed to not doing which would deprive learners of important and necessary skills that would allow them to be very competent citizens. There is a great need to focus on helping teachers meet the challenges of planning, teaching and assessing mathematical skills especially in areas of encouraging good thinking as proposed by proponents of critical thinking. There should be a lot of effort by important stakeholders like curriculum planners and textbook authors and distributors to assist teachers by providing useful resources for teaching and learning.

Chapter 5 Conclusion

5.1 Summary of findings

Finding in this study are consistent with key finding made by Thomas (1999) and Innabi & El Sheikh (2007) in their respective studies. One of these key findings that is common with this study is that mathematical teachers could not express a comprehensive view of critical thinking. Each teacher expressed and identified different critical thinking skills and abilities and dispositions when they were asked about them. Generally teachers differed in identifying what critical thinking skills they believed are important in promoting critical thinking in Mathematics and what it means for those thinking skills to demonstrate critical thinking in teaching, learning and assessing Mathematics. Teachers expressed different beliefs on the role of mathematical skills and mathematical content in relation to devoloping critical thinking skills. Teachers spoke of requirements, conditions and the role of Mathematics interchangeably in relation to critical thinking without properly focusing on the qualities of mathematical skills and principles. Only three of the teacher identified the important quality of logic and reason in Mathematics. Others spoke about Mathematics and questioning as important in promoting critical thinking by referring to Bloom's taxonomy.

Teachers identified different teaching strategies and tasks that are similar to those identified by Halpern (2001) as among the most effective in promoting critical thinking. However the teachers mostly agreed on questioning and learner participation as the centre of critical thinking and varied in their views on other teaching strategies. There was further variation in responses on how effectively thinking skills and mathematical skills can be assessed from tasks and strategies for their quality and relation to critical thinking. Teachers could only speak about what strategies or tasks they believed promoted critical thinking and how they can develop learners thinking skills. Even the tasks that teachers identified as promoting critical thinking varied, mostly because they were closely related to teaching strategies. Again teachers, with the exception of one or two, mostly focused on conditions, requirements and procedures in their understanding of critical thinking and could not demonstrate any clear examples of how they assess the quality of thinking that is related to mathematical skills and critical thinking.

Another aspect in the findings was the role of dispositions as well as intellectual criteria and standards of critical thinking in Mathematics teaching. Teachers were mostly unaware of some intellectual criteria and standards like clarity, precision, depth, breadth, accuracy, and relevance as well as how to assess the disposition to use them. Some teachers did identify reasoning and

logic and how they can be assessed to judge learners' thinking abilities. Teachers could identify how they could assess critical thinking abilities in learners but the assessment were mostly focused on correctness of answers and not disposition of using critical thinking skills in Mathematics in an open minded context. Teachers were not really sure of what important role intellectual standards and criteria played in teaching critical thinking within Mathematics and how they could plan their teaching using them as outcomes.

One major finding of this study is that Mathematics teachers' beliefs about teaching critical thinking do not reflect their teaching of the subject. All teachers agreed that critical thinking is an important educational goal and should be promoted in teaching, however all the teachers confessed that they did not focus their teaching on promoting critical thinking. They identified factors like the CAPS document, district officials, time constrains as the main impediments to teaching critical thinking in Mathematics. They believed that the curriculum is too packed with content to cover, the district officials impose too much authority on their teaching responsibilities and that they do not have enough time because of this. Time was also a problem in that teachers believed that if they were to teach in a way that promotes critical thinking, they would fall behind on their teaching schedule or that strategies that promote critical thinking skills require more than the usual time to teach.

5.2 *Implications and recommendations*

Teachers openly admitted that their teaching often focused on teaching in order to fulfil the Annual Teaching Plan (ATP). They stated that due to the amount of work that is recommended by the curriculum, they often had to resort to procedural teaching or teaching towards tests and exams. For that reason they believed that trying to promote critical thinking within teaching Mathematics would take up time which they believed they did not have. This is a problem to a country already getting a bad reputation of low standards of mathematical literacy in the world. If newly appointed teachers who are still young reflect beliefs that suggest frustration regarding teaching critical thinking skills to learners which is one of the fundamental goals of our education system the situation could lead to producing learners who are not appropriately prepared for the demands complexities of the 21st century citizenry. The teaching of critical thinking skills and abilities and Mathematics are the cornerstone of our education system and should not be regarded as a policy rhetoric that is meant to represent ideals with no purposeful intentions to be achieved. The teachers in this study expressed some resignation in trying to effect some change to their future teaching because of the lack of support and direction of how to teach critical important skills thinking. This implies that their teaching of Mathematics will

continue to be about learning concepts and procedures with limited effort in trying to uphold the important objectives and aims of the CAPS document. What is more concerning is that they often point to CAPS document as an impeding force in their desire to teach critical thinking skills.

There is some need in supporting newly qualified teachers in meeting these challenges which they identified as limiting teaching critical thinking in Mathematics. One way teachers can be assisted is to provide specific programs that would focus on teaching critical thinking in Mathematics. Teachers should be assisted in understanding what critical thinking is and how it relates to other forms of thinking. They need support in developing teaching materials and teaching strategies in Mathematics to lessen the burden they seem to be feeling in their everyday responsibilities. Teachers should also be shown how critical thinking skills and abilities can be encouraged and promoted within normal teaching by adjusting and reinventing teaching strategies teachers already use. They should know how to help their learners learn critical thinking skills and what kinds of assessments they should use that would encourage and motivate learners to see the value of Mathematics.

5.3 Conclusion

It is both argued in this paper and a well-established goal of education to produce and encourage learners who are critical thinkers. Mathematics and mathematical skills and principles are some of the most important skills to learn in society (Schoenfeld, 1992). Learners who are critical thinkers in Mathematics are more disposed and inclined to use critical thinking skills. Innabi & El Sheikh (2007) argue that learners will not learn to be critical thinkers unless their teachers can think critically, have comprehensive knowledge about critical thinking and the content knowledge of the subject as well as how to teach it, and more importantly they should believe that they are able to teach it for all students. Based on the finding of this study, newly appointed professional teachers demonstrate some abilities, willingness and expertise that disposes them to teaching critical thinking within Mathematics. However there are evident gaps in their beliefs of teaching critical thinking and their practice which are worrying as they mean that critical thinking skills are not being taught in schools. More awareness and encouragement of focus must be given to critical thinking as it is an important educational goal. New Mathematics teachers feel teaching schedules are too congested for teaching responsibilities to focus on encouraging thinking and as long as that is the prevailing sentiment, teaching critical thinking will be severely compromised in Mathematics and schools to the detriment of the future of the country and its citizens. The teachers expressed a willingness, some capabilities and beliefs

about teaching Mathematics that could encourage teaching critical thinking in Mathematics. Teachers therefore need to be assisted in developing teaching strategies and tasks that can promote critical thinking skills especially in a subject like Mathematics where the skills can be useful core life skills.

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Appendix A

Interview questions
Candidate Number:

1. Would you explain to me your concept of critical thinking?
Perhaps you could begin by completing the following sentence: *"To* me, critical thinking *is "*

Follow-up:
• *Could you elaborate further on your conception?*
• *Could you give me an example of your use of critical thinking in Mathematics outside the classroom that illustrates your concept of it? (e.g., as a consumer, as a parent, as a citizen, or in a personal relationship)*
• *In your answer you've mentioned processes such as analysis, synthesis, evaluation, and application.*
• *Do you think critical thinking is important to society?*
• *What skills or processes would you relate to critical thinking?*
• *What intellectual standards in Mathematics would you use to distinguish whether or not these processes are being done critically vs. un-critically?*
• *Does your conception of critical thinking involve any traits of mind?*
• *An example of a trait that some might identify is open-mindedness.*

If you do prime by mentioning this trait and value, and you receive an affirmative answer, ask for an elaboration *"How do you teach for this trait in the classroom?"*, to determine whether the traits or values mentioned have actually been thought through or are simply being rhetorically expressed.
(Remember, you are looking here to determine the extent to which an interviewee has developed a clear, elaborated, and substantive conception of critical thinking.)

2. Is there anything you do whether teaching strategy or mathematical task in the classroom that you believe fosters critical thinking in your learners?

Follow-up Questions:
• *Do you have any other classroom structures that you find particularly effective in teaching your learners to think critically about Mathematics?*
• *Can you give an example of a task for learners you would use to engage them in critical thinking in Mathematics?*
• *How do you emphasize critical thinking within the task?*
• *Can you give an example of a strategy you would use to teach critical thinking in Mathematics?*
• *How do you emphasize critical thinking within the strategy?*

• *Do you think critical thinking should be taught explicitly within mathematical content or integrated in the content?*

3. Some teachers may feel they have too much mathematical content to cover to have much time left for fostering critical thinking. What is your view of this position?

Follow-up Questions:

• Do *you teach or have you taught any mathematical topics that require a great deal of coverage, and if so, how would you foster critical thinking in those mathematical topics?*
• *Could you please give me an example from the design of your classes?*
• *How would you involve learners in the topic?*
(Remember, here we are seeking to see if the interviewee understands how critical thinking can be used as a tool for the deeper understanding of content rather than to see the learning of critical thinking as a distraction from the learning of content. In other words, reconciliation occurs when a teacher discovers that learners truly learn content when they think that content through critically).

4. What particular critical thinking skills do you believe are most important for your learners to develop in your teaching of Mathematics?

List of skills to share?

If they hesitate for 5-10 seconds, say *"Is the question clear or do you want some clarification?"*

Follow-up Questions:
• *In your answer you've mentioned processes such as analysis, synthesis, evaluation, and application. What intellectual standards in learner engagement and responses in the Mathematics would you use* to *distinguish whether or not these processes are being done critically vs. un-critically?*
•

• *Could you give me an example of the use of one of those skills in some everyday context, outside of the classroom? (e.g., as a consumer, as a parent, citizen, in a personal relationship)*
• *would you rather design your own tasks/material or do you prefer to have them designed for you?*
If the interviewee asks what you mean by a particular or specific critical thinking skill, answer *"Well, some would identify the ability* to *assess information for its relevance* to *an issue* or *the ability* to *clarify an issue or problem as important component critical thinking skills." "In your views which are the most important component critical thinking skills?"*
(Remember that we are seeking to determine here whether the interviewee has thought through distinguishable component critical thinking skills and can explain what those skills constitute).

5. If you had the task of assessing the extent to which your current teaching was or was not emphasizing or fostering critical thinking through your mathematical instruction, how would you go about making that assessment?

Or looking at tasks you've used and choosing tasks that would engage learners in critical

What intellectual standards would you use to distinguish whether or *not these processes are being done critically vs. un-critically?*

Prime if necessary by asking *"For example, would you use classroom task? What exactly would you be looking for in the task that would inform you that learners are thinking critically?"* If they say, "I would look at their tests and answers," then ask *"What criteria would you use* to *evaluate your learners' tests and response to questions?"* If they say, "I would look to see if learners use their own and ideas," then say *"How would you assess the quality of the learners' thinking and ideas in doing Mathematics?"*

(Remember, you are looking here to determine the extent to which a teacher has developed a clear, elaborated, and substantive conception of how to assess their teaching and learners to determine the extent to which that teacher was or was not fostering critical thinking in his/her learners.)

6. What is your personal conception of intellectual criteria and standards as they apply to critical thinking in Mathematics?
Or what specific skills would you look for

Is *the question clear to you?* I *can give you further clarification if you want.*

Follow up Questions:
• *"What qualities do you look for in your learners' reasoning that tells you whether or not they are reasoning well or poorly?" For example, if you emphasize the importance of being accurate in their thinking, then "accuracy" is a general criteria or standard you value.*
• *In your answer you've mentioned processes such as analysis, synthesis, evaluation, and application.*
What intellectual standards would you use to distinguish whether or not these processes are being done critically vs. un-critically?

• *If a student said to you, "what criteria should* I *use in deciding when to accept or reject some mathematical argument or some position that someone defends?" what advice would you give them?*
• *What intellectual standards do you use to evaluate learners' mathematical reasoning?*
• *Could you name some and elaborate on them?*
Here is another example to illustrate what we mean by "intellectual standards":

A study was conducted in which elementary learners were asked, "There are 26 *sheep and* 10 *goats on a ship. How old is the captain?"* 76 *of the* 97 *learners "solved" the problem by adding, subtracting, multiplying or dividing. This example illustrates the intellectual standard of relevance. That is, the learners did not recognize that the information they were given was not relevant to the question that they were asked.*

(If an interviewee asks what we mean by "intellectual standards" the common answer should be: *"By intellectual standards we mean general criteria that one uses to decide what to accept as true or false, reasonable or unreasonable."* You could also say, *"Well, a concern for accuracy is an intellectual criterion or standard essential to Mathematics. That is the sort of thing we have in mind, general intellectual criteria that a thinker might use to assess what people assert or claim."*

Appendix B

Interview protocol questions for interview 2

- Can you identify segments in the lesson that you regard to be the most useful in promoting critical thinking and explain your choice?

- Can you identify segments in the lesson that you regard to be the least useful in promoting critical thinking and explain your choice?

- Can you identify segments in the lesson that resemble your own teaching of Mathematics and explain what that has in common with your conception of critical thinking?

Appendix C

Interview information for teachers

Most modern democratic societies have structured their educational goals around critical thinking as one of the main core skill that must be learned and taught (Halpern, 2001). The Curriculum Assessment Policy Statements (CAPS) is an example of such educational systems as in its aims and principles lists critical thinking skills and dispositions as one of the main educational objectives to be taught in school. In referring to Mathematics it states in the subject and general objectives that teaching must promote "Active and critical learning: encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths" (CAPS, 2011 pg. 4). Critical learning and active learning are some of the disposition associated with critical thinking. The CAPS document defines Mathematics as

"...a language that makes use of symbols and notations to describe numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem-solving that will contribute in decision-making." (CAPS, 2011 pg. 4).

The definition illustrates the significance of the development of thinking skills in Mathematics that can be useful to learners academically as well in their social lives. Critical thinking is taken as important because it encourages thinking in learners "that enables seeing different sides of arguments, being open to new knowledge that challenges beliefs and ideas, reasoning dispassionately, demanding that claims are backed up by factual evidence, deducing and inferring conclusion based on facts, solving problems and a host of other thinking skills that encourages good thinking" (Willingham, 2008, p. 21).

You are asked as the teacher to think about your teaching and how it reflects elements that encourage learners to think critically in their lessons. This will include the teaching strategies as well as the tasks you use in the classroom that you believe encourages critical thinking. You are encouraged to reflect on specific examples and instances that you believe reflect your ideal conception of the necessary conditions of developing critical thinking in your classroom. This will be the subject of the first interview and you are encouraged to be as open and honest as you can be as the subject of the study is your beliefs, not assessing your practice.

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Willingham, D. T. (2008). Critical thinking: Why is it so hard to teach?. *Arts Education Policy Review*, *109*(4), 21-32.

Appendix D

LETTER TO THE PRINCIPAL, SGB Chair, etc. DATE: 09/09/2016

Dear sir/ma'am

My name is Trust Simelane and I am a Master of science (M.Sc) student in the School of Education at the University of the Witwatersrand.

I am doing research on teacher beliefs on teaching critical thinking skills in their Mathematics classroom. Teaching critical thinking in Mathematics is one of the important objectives set by the CAPS document and critical thinking is recognized as one of the most important objectives of any educational system of a democratic country. This is because critical thinking has been shown to be important in cultivating learners who can think for themselves in academics as well as their personal lives. Research has shown that teachers do not pay enough attention to cultivating critical thinking skills in learners which is recognized as important to teach.

My research involves interviewing one of your teachers who is a Wits graduate as part of my inquiry teacher beliefs about critical thinking skills. Two separate interviews over a space of a month will be carried out outside of school time, at the teachers chosen location to minimize work disruption. I will also work with the teacher to adapt some of their usual teaching tasks to incorporate critical thinking in line with CAPS expectations. This will be adapted from their lesson plan as well as tasks they were planning to use in their classes.

The reason why I have chosen your school is because as mentioned above, the chosen teacher is a Wits graduate in Bachelor of Education (B.Ed) and thus has had some education in the teaching critical thinking skills.

I am inviting your school to participate in this research as this project aims at helping both teacher and learners to learn a vital thinking skill.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

SIGNATURE 

Trust Simelane
5495 Kearona street
Rooderkop ext 22
Germiston
simelanetrust@gmail.com or 480428@students.wits.ac.za
0734977783

I have read (or have been read) the above information regarding this research study on teacher beliefs about teaching critical thinking, and consent to participate in this study. I also provide consent for the selected teacher to participate in this study.

_____ (Printed Name)
_____ (Signature)
_____ (Date)

University of the Witwatersrand Johannesburg

Declaration

I ***Trust wiseman Simelane*** Student Number ***480428*** am a

Student registered for ***MSc in mathematics education*** in the year 2017.

I herby declare the following:

- I am aware that plagiarism (the use of someone else's work without their permission

And/or without acknowledging the original source) is wrong.

- I confirm that the work submitted for assessment is my own unaided work except where I have explicitly indicated otherwise.
- I have followed the required convention in referencing the thoughts and ideas of others.
- I understand that the University of the Witwatersrand may take disciplinary actions against me if there is a belief that this is not my own unaided work or that I have failed to acknowledge the source of the ideas or words in my writing.

Signed:  date: 31 - August - 2017

INFORMATION SHEET TEACHERS

DATE:

Dear NAME

My name is Trust Simelane and I am a Master of science (M.Sc) student in the School of Education at the University of the Witwatersrand.

I am doing research on teacher beliefs on teaching critical thinking skills in their mathematics classroom. Teaching critical thinking in mathematics is one of the important objectives set by the CAPS document as well as being recognized as one of the most important objectives of any educational system of a democratic country. This is because critical thinking has been shown to be important in cultivating learners who can think for themselves in academic study as well as their personal lives.

My research will be about your beliefs about the teaching of critical thinking skills in mathematics lessons. I would like to ask you to take part in two separate interviews about a month apart, each lasting no more that 45 minutes. The interview will be carried out outside of school time, at your chosen location to minimize work disruption Between the interviews I would like you to adapt one of your normal lessons to incorporate critical thinking as specified in CAPS. This will be adapted from your existing lesson plan as well as tasks you were planning to use in your class and we will work together to adapt the lesson. The lesson will be conducted at your discretion after the first interview. You may opt not to conduct the lesson. The reasoning, however, for using the lesson is for your reflections in the second interview on how critical thinking can be incorporated into normal teaching.

The reason why I have chosen you is because you have graduated from the University of the Witwatersrand with the Bachelor of Education during which you will have been introduced and trained in teaching critical thinking skills.

I invite you to participate in this study, which is voluntary and anonymous to participate in. The interviews will be recorded and information concealed and changed to protect your identity. When it is complete, you may, on request, receive a copy of the research.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be securely stored and destroyed between 3-5 years after completion of the project.

You will not be advantaged or disadvantaged in any way. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

SIGNATURE 

Trust Simelane
5495 Kearona street
Rooderkop ext 22
Germiston
simelanetrust@gmail.com or 480428@students.wits.ac.za
0734977783

I have read (or have been read) the above information regarding this research study on teacher beliefs about teaching critical thinking, and consent to participate in this study.

_____ (Printed Name)
_____ (Signature)
_____ (Date)

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064
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26 April 2016

Student Number:

Protocol Number: 2016ECE017M

Dear Trust Simelane

Application for Ethics Clearance:

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

To evaluate if there is any emphasis on critical thinking skills in the mathematics curriculum as emphasised by the objectives of the CAPS documents and if the resources can be adapted to promote critical thinking

The committee recently met and I am pleased to inform you that **clearance was granted.** However, there were a few small issues which the committee would appreciate you attending to before embarking on your research.

The following comments were made:

- In Section 6.1 there is a typing error on the word pseudonyms. Please state the Grade that the 6 teachers will be responsible for;
- The section of project information needs to contain an answer to the question, "Where will the research be carried out?";
- The documents have to be proofread again with specific attention paid to the capitalization of words;
- Section 2.5 on the GDE form is filled out in grey and not in black.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

M Mokoena

Wits School of Education

011 717-3416