

Investigating concrete and abstract strategies Grade 2 learners use when working with early number concepts.

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Abstract:

This study focuses on understanding the strategies used by a sample of high ability and low ability Grade 2 learners drawn from two government primary schools in Gauteng, with emphasis on more concrete or more abstract strategies learners use to solve early number problems. This study takes place against the backdrop of poor performance in South African schools more especially across the foundation phase and also amidst claims that learners remain largely dependent on concrete strategies for solving problems. The theoretical background for this study is drawn from Sfard's (1992) "Dual Nature of Mathematical Conceptions" and also Sfard's (1992) theory of reification. I used on a wide range of literature on strategies for counting, addition, subtraction within my analysis of nine videos of high ability learners and 9 videos of low ability learners with the aim of examining the strategies these learners use when dealing with early number concepts.

My findings pointed to the limited use of higher levels of abstraction in solving early number problems. Whilst there is progression from the concrete to the abstract levels of conception this is not happening at a pace and depth that is required for Foundation Phase learners in order for them to effectively engage with more challenging and complicated arithmetic in the Intermediate Phase.

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Chapter 1: Introduction

The Trends in International Mathematics and Science Studies (TIMSS) for 1995, 1999 and 2003 indicated poor learner performance in mathematics across the Intermediate Phase and the Senior Phase in South Africa. This has turned the spotlight on the learning and teaching of mathematics in South African schools. The Western Cape Education Department (WCED, 2006) produced documentation stating that more than 60% of grade 3 learners in 2008 were performing below the levels expected by the curriculum for literacy and numeracy. These documents also revealed that numeracy levels dropped from 36.6% in 2002 to 32% in 2006. Whilst 2012 Annual National Assessment analyses indicate a small upturn in performance in Foundation Phase Numeracy, the overall performance levels remain low with 41% being the national mean score.

Ensor, Hoadley, Jacklin, Kuhne, Schmitt, Lombard, and van den Heuvel-Panhuizen, (2009) have concluded after their research that there is a crisis that is being faced in mathematics teaching and learning, especially in the teaching of number in the Foundation Phase. They focused particularly on the transition from more concrete to more abstract strategies for working with number, noting that abstract conceptions of number are required by the end of the Foundation Phase in terms of the curriculum. Ensor et al (2009) found through their research that there were shifts from the concrete to abstract notions of number evident in all the classrooms they observed but that this did not happen at a pace or depth which would allow learners to engage with more complicated arithmetic operations in the Intermediate Phase. Their findings raise the question as to whether our learners in South African schools are adequately transitioning from concrete to abstract concepts of number. Ensor et al (2009) note the lack of South African research on learners' understanding of number in South Africa and their transition from concrete to abstract notions of number. This study aims to focus on this gap by examining the strategies learners use when they deal with early number problems.

Ensor et al's (2009) key finding was that whilst there was a trajectory of development across grade 1 to grade 3 with respect to counting towards more abstract ways of working with number, learners remained largely dependent on concrete strategies for

solving problems. Concrete methods relate to learners' reliance on the use of counters, fingers or something tangible to count in order to perform simple numerical operations. According to Ensor et al (2009) the conception of a number becomes wholly abstract when the learner no longer requires any tangible objects to count or perform simple operations. There are intermediate stages in this shift, where we get partial abstract conceptions where one number in a sum, maybe the first number, is treated in abstract concept terms, but the second number or 'operation number' still requires perceptual objects for the calculations to be carried out. This can be seen in the 'count-on' strategies that can be used within early addition problems which are detailed in Chapter 2 where, I provide a literature review which details the sequence of shifts that constitute the gradual move over from more concrete to more abstract conception of numbers.

This report seeks to focus on the problems associated with poor levels of performance of learners through exploring the strategies from the more concrete to the more abstract used to solve early number problems by a sample of weaker and stronger learners drawn from two schools in the greater Johannesburg area. This study compares the strategies a sample of grade 2 learners use with respect to early number processing, in order to understand in more detail the range of understandings of concrete and abstract strategies, and in order to identify routes to supporting more learners to move towards more abstract number strategies. This is done in order to fill the gap noted by Ensor et al (2009) of the lack of empirical studies on the trajectory of strategies used by South African learners, leading to the need to borrow from literature drawn largely from Europe and the United States.

The theoretical background underlying this study is based on Sfard's (1991) claim that structural objects in mathematics develop out of operational processes. In other words abstract mathematical objects emerge from certain operational processes. Thus for Sfard (1991), mathematical concepts like 'number', 'algebraic expressions', etc. can be viewed both as objects and as processes. Sfard (1991) refers to this as the dual nature of mathematics and it is the existence of the dual nature of mathematics that allows for flexible mathematical working; Sfard (1991) notes also that reification of counting processes into objects is necessary for effective and efficient mathematical working as it is the progression to the abstract levels which ensures effective and efficient learning of mathematics. Sfard (1991) also states that

the absence of abstract conceptions of mathematical objects may hinder further development. Reification increases problem solving and learning abilities and Sfard (1991) further states that the more structural or abstract the approach, the greater the learner's confidence in what they are doing. I provide more detail on Sfard's theoretical concepts in Chapter 3 (Theoretical Framework).

In this study I observe the responses of a higher attaining and lower attaining sample of learners to number problems drawn from a research based test which is part of "The Mathematics Recovery Programme" as drawn from the work of Wright et al (2010). These assessment instruments aim at assessing "Early Arithmetic Strategies and Numerical Knowledge". The instrument contains items which allow the interviewer to focus on the counting strategies used by responding learners across counting, addition and subtraction tasks. Test based interviews were conducted with these learners within the broader Wits Maths Connect – Primary project towards the end of 2011, following some intervention work with the teachers during the year. My focus within this study, in analysing learner responses, is on seeing whether and if so, to what extent reification is taking place in the context of early number learner, and to understand how this differs between stronger and weaker learners. A concept becomes reified when the learner is able to see the mathematical object as an entity that exists without needing to stem from operational activities; it exists as an object to which new processes can then be applied. Sfard (1991) describes this shift as a quantum leap when the learner is able to recognise the concept as an object i.e. moving beyond the concrete phase, and this is when we can say that the concept is reified.

Given that Ensor et al's (2009) evidence has pointed out the prevalence of more concrete approaches to number problems (i.e. non-reified number conceptions), I hope to explore differences in strategies used by learners who have been identified by class teachers in the two schools as mathematically 'weak' or 'strong'. This will help determine the progression from concrete to abstract for 'weak' or 'strong' learners. This focus leads to my research questions.

Research Questions

1. What strategies do a sample of weaker/stronger grade 2 learners from two Johannesburg schools use with respect to early number problem-solving?

2. To what extent is the concrete/abstract strategies distinction useful to characterise the strategies used by weaker/stronger learners? How do these strategies link with literature calling for development from more concrete strategies towards more abstract strategies in early number learning?
3. What do my findings suggest for the implementation of the Foundation Phase Curriculum Assessment Policy Statement (CAPS) curriculum?

The purpose of this study is to examine and compare the strategies that weaker/stronger Foundation Phase learners use when working with numbers. I will also track the learners transitioning from the concrete to the abstract across a range of problems drawn from Wright et al's (2010) tests. Ensor et al (2009), state that South African Foundation Phase learners remain highly dependent on concrete strategies rather than following the trajectory of development from counting to more abstract ways of working with numbers. Schollar's (2008), report from his primary mathematics research project backs up this finding with evidence indicating that learners are reliant on concrete methods such as tally counting to solve problems in mathematics in the early grades and into the Intermediate Phase. In this study I aim to explore the extent to which learners identified as weaker/stronger rely on concrete modes, whilst also looking for differences in their intermediate transitions towards more abstract modes. Theoretically, as stated above, I draw on the writings of Sfard (1992), especially the notion of the dual nature of mathematics and also the theory of reification.

The remainder of this report is structured as follows: In Chapter 2, I review literature on early number learning more especially on counting, addition, subtraction and also the progression from abstract conceptions to concrete conceptions. Chapter 3 consists of details of the theoretical frameworks that used to assess learners who are involved in this study. In Chapter 4, I explain the research methodology and instruments that were used to gather data related to the research questions. In Chapter 5, I analyse the learner responses to the "The Mathematics Recovery Programme" assessment instrument. In Chapter 6, I discuss the findings and provide concluding remarks on this study.

Chapter 2: Literature Review

In this chapter I review some of the literature related to early number learning. Subsequently, I make reference to this literature in the analysis chapter to discuss the findings from my South African learner sample in relation to the broader literature base. I deal with a broad range of literature on early number learning and as this literature focuses centrally on counting, addition and subtraction, I also deal with the issues of counting, addition and subtraction. With respect to counting I focus on:

- What does counting consist of?
- Why is counting important in early number learning?
- What does progression in counting consist of and how this progression can be related to concrete and abstract concepts?

With respect to addition I focus on: count all, count on, count on from the first, count on from the larger, flexible group strategies and recall strategies. With regard to subtraction I focus on; counting-out, counting-back-from, counting-back-to and counting-up strategies. Thereafter, I explore the progression from more concrete to more abstract strategies. I end this chapter by examining Learning Framework in Number (Wright, Martland and Stafford, 2006). Within this focus on counting, I refer to literature by Steffe (1983), Wright et al's (2010), Gelman and Gallistel (1978), Ensor et al (2009), Gray (1997), Thompson (1995), Ellemor-Collins and Wright (2007) and Wright (2000).

2.1 Counting

Counting is an important exercise for learners since it helps them explore the relationships between numbers. In this aspect of the task I focus on; what counting consists of, why counting is considered important in early number learning, what progression in counting consists of and how this literature can be linked to the progression from the concrete to abstract.

2.1.1 What does counting consist of?

Meaningful counting activities begin in Grade R and counting at this level involves at least two separate skills. Firstly, a learner must be able to produce the standard list

of counting words in order;” one, two, three, four ...” Secondly, a learner must be able to connect this sequence in a one-to-one manner with the items being counted where each item must get one and only one count. Steffe (1983) defines counting as the ability to produce a sequence of number words, to the extent where each number word is accompanied by the production of a unit item. Learners are required to pay attention to cardinality and ordinality when counting. Learners are required to provide the cardinal number of items being counted, in other words the total number of items being counted. Whilst paying attention to cardinality the learner has to also pay attention to ordinality i.e. the learner has to ensure that sequence or order of numbers are correct. Slips in ordinality will result in the production of an incorrect cardinal number.

According to CAPS (DBE, 2011) a grade 2 learner should be able to:

- Count to at least 100 everyday objects reliably.
- Count forwards and backwards in 1s, 2s, 3s, 4s, 5s and 10s between 0 and 200.
- Recognise number symbols and names between 0 and 200.
- Order, describe and compare 2-digits whole numbers.

This specification suggests that learners at Grade 2 level should exhibit fluency with the counting sequence to 100 in forward and backward directions. Learners find it easier to count by pointing to one object at a time synchronised with uttering the number words and in so doing keep track of the objects being counted. The learners used in this research task were assessed using Wright et al's (2006) “The Mathematics Recovery Programme” LFIN tests. Within this test, some items asked learners to forward count from a certain number, state the word number after, identify numerals, backward counting from a certain number, say the number word before and sequence numerals.

Gelman and Gallistel (1978) state that there are five principles that guides a learner's development of counting procedures. These principles are:

- One-to-one correspondence, which implies that one and only one number word, can be assigned to each object that is being counted. The learner is required to master a link between two processes namely partitioning and

tagging or marking. Partitioning entails differentiating within the set of items between which has been counted and those which have yet to be counted. Gelman and Gallistel (1978) claim that partitioning can be achieved mentally or physically. Tagging or marking on the other hand entails using distinct markers or tags, one at a time, for example an item in a set that has been assigned “3” cannot also be assigned “5”. Learners who count using their fingers normally display partitioning and tagging simultaneously by holding up finger that have already been counted and folding back fingers that are yet to be counted. Three kinds of errors can arise; errors in partitioning, such as tagging an item more than once, errors in the use of tags i.e. using a tag more than once and failure to co-ordinate the two processes adequately.

- The stable order principle refers to the fact that the number words are always used in the same order for example counting in the order “1, 2, 3, 5” is incorrect. Gelman and Gallistel (1978) claim that counting involves more than assigning tags to items in a collection. Learners need to recognise that the tags themselves are arranged in a repeatable, stable order. They also argue that much of the learner’s engagement with early number learning is based on rote learning the first 12 or so number words.
- Cardinality refers to the fact that the value of the last number word used when counting indicates the quantity of items in the set for example, counting “1, 2, 3, 4” means that there are four items in the set. This understanding is crucial to the learner’s later number reasoning since the number 4 encapsulates numerosity and represents the total number of items.
- Abstraction means that any set of items can be subject to counting procedures. However, Steffe et al (1983) argue that there are five different types of countable items which present increasing difficulty for learners in the early stages of number learning: perceptual units (can be seen), figural items (not present but recallable - for example numbers of people dwelling in a home), motor units (counting movements like steps and handclaps) verbal units (uttering number words) and abstract items as discussed above.

- Order irrelevance means that items can be counted in any order (e.g., counting from right to left, left to right, or in no particular sequence at all will result in the same total number of items). The order in which the items are counted does not affect its numerosity.

Gelman and Gallistel (1978) state further that the first three principles are the basic “how to count” rules and these three principles set the structure for developing knowledge and understanding of counting. Anghileri (2006) argues that oral work is vital in the early years because it encourages learners to work with numbers mentally. In the process these learners develop rich connections and strategies which serve as a platform for more difficult procedures. She states that forward and backward counting in 1s, 2s, 3s, 4s, 5s, etc. develops an understanding of patterns that assist learners in early addition and subtraction. Oral counting enables learners to memorise number sequences which they should be able to reorganise and to reproduce in written form. Moreover, mental strategies are more about the application of known or quickly calculated number facts in combination with specific properties of the number system to find the solution of a calculation whose answer is not known. They also incorporate the idea that, given a collection of numbers to work with, children will select the strategy that is the most appropriate for the specific numbers involved.

Thompson (1995) states that there is a worry, especially in the case of lower attaining learners, that over-dependence on counting may lead to these learners not committing number facts to memory. On other hand learners who know many number facts and have developed a range of sophisticated calculations strategies are in a position to combine these number facts and strategies with counting techniques in order to derive unknown facts. He also states that learners need to learn how to compress counting procedures in order for the learners to be in a position to make choices between strategies. The issue of compressing counting procedures will be explained further under the next sub-heading viz. 2.1.2 Why is counting important in early number learning?

Thompson (1995) claims that if learners are left to their own devices they appear able to develop sophisticated mental calculations strategies. However, he states that most learners need to be taught a range of mental methods and these mental methods can be taught. He further states that learners should be taught specific skills, developing recall of facts and build awareness of important aspects of the number system and number relationships. These factors contribute to construction by the learner of mental strategies that is appropriate to a given problem situation.

Karpov (2003), states that learners need to be presented with opportunities to solve problems and to develop a range of strategies. Learners need problems and activities that will allow them to make connections, develop strategies and commit facts to memory. Karpov (2003) also states that learners must be taught a range of possible strategies for adding, subtracting, multiplying and dividing numbers.

2.1.2 Why is counting important in early number learning?

The meaning attached to counting is the key conceptual idea on which all other number concepts are developed. Counting is critical to learner's understanding of numbers and it is through counting that they are able to understand numbers represent quantities. Gelman and Gallistel (1986) state that "counting provides the representations of reality upon which the numerical reasoning principles operate. That is, counting serves to connect a set of reasoning principles to reality" (p.161). They underscore the importance of moving from counting to calculating strategies and eventually to calculation that does not rely on counting. A number is derived from the counting process. Gelman and Gallistel (1986) state that arithmetic reasoning grasps that "the laws of arithmetic govern an abstraction called number" (p.181).

As learners learn to count they come to understand that these symbols represent different quantities (i.e. different numbers of objects). Counting lays the basis for understanding the relative magnitude of numbers for example the learner they compare numbers to a benchmark value such as 5, 10, 25, etc. and further to benchmark values learners are able to identify inequalities such as "1 more", "1

less", "10 more", "10 less", etc. and these inequalities help learners realise the distances between numbers.

McIntosh, Reys and Reys (1992) note the importance of benchmarks within their framework for basic number sense, and they state that the system of benchmarks establishes essential mental referents for thinking about numbers. According to McIntosh et al (1992) "numerical benchmarks are generally powers of 20, multiples of powers of 10, or midpoints such as $\frac{1}{2}$ or 50%, although any value for which the learner has a confident understanding can serve in this capacity" (p. 6). Benchmarks are often used to estimate or judge the size of an answer or to round a number so that it is easier to mentally process. Benchmarks for these authors are numerical values devoid of context, which have evolved from experience and/or instruction:

"Benchmarks may also evolve from personal attributes or encounters. For example, a person weighing 50 kg may use this information in estimating the weight of another person. Similarly, a child who attends a baseball game where the attendance is 50,000 may at a later time use this as a referent for judging the size of other crowds. The variety and complexity of the benchmarks in making decisions about numbers and numerical contexts is a valuable indicator of number sense." (p. 6)

As much as learners are aware of order irrelevance (Gelman and Gallistel, 1978) i.e. at items can be counted in any order and the order in which the items are counted does not affect its numerosity (as discussed earlier), they also come to the understanding that a number represents a specific quantity and therefore 4 is smaller than 5 and will appear before 5 if these numbers are ordered. Therefore, counting helps the learner to connect number-words and numerals to positions and the quantities they represent.

Ensor et al (2009) concur with Gelman and Gallistel (1978), stating that in order to enhance understanding significantly the learner has to move away from seeing numbers as being useful for counting towards viewing numbers as objects which are manipulated according to certain laws. The learner must be able to see the shift from objects of the real world to objects of the arithmetical world. If learners do not make

the shift from process to concept they will not be able to understand that the number 10 is a concept and will find it difficult to comprehend two digit numbers and place value.

According to Ensor et al (2009) learners move from the perceptual world i.e. the use of counters and fingers to using representations of it i.e. the use of tallies and later number words. The concept of unit becomes fully abstract when the learner no longer needs material to create countable items or any counting process. The understanding of number requires the teacher to assist learners to deepen the notions of counting, develop flexible and powerful means of representing number using apparatus such as beads, number lines, empty lines and so on, so that learners gain confidence in using counting as means of calculating. Counting strategies form the basis for learning addition and subtraction, initially using strategies like counting all, then counting on or counting back. The movement from the concrete to abstract helps the learner to develop confidence to compute without the reliance on counting strategies and with experience learners develop a repertoire of number facts and number skills. Carpenter et al (1982) state that counting strategies lay the foundation for learning addition and subtraction after using strategies like counting all, counting on or counting back.

Drawing from this literature, this study therefore also explore strategies learners use in order to track their shift from viewing number as process to number as concept i.e. move from counting with their fingers to understanding the concept of a particular number without relying on concrete aids.

Gray (1997) states that the compression of counting procedures will help develop a powerful tool for success in arithmetic. Thurston (1990) in Gray and Tall (2007), states that compression is the mechanism by which information is held in an economical manner:

Mathematics is amazingly compressible: you may struggle a long time, step by step, to work through some process or idea from several approaches. But once you really understand it and have the mental perspectives to see it as a whole, there is often a tremendous mental compression. You can file it away, recall it quickly and completely when you need it, and use it as just one step in some other mental process.

The insight that goes with this compression is one of the real joys of mathematics (Thurston, 1990, p.847)

At this stage I need to explain what compressions of counting procedures are about by using the following example: a count all procedure is seen as a triple counting procedure where the learner counts one set, counts another set and then puts them together and counts them together. For example, to solve $4 + 5$ a learner will count 4 counters and then counts out 5 counters and then puts them together and counts all. These three counting procedures may be compressed into a count on procedure. Gray (1997) states that if counting procedures are not compressed this will not easily lead to the development of flexible known facts. A skilful counting and understanding of the numerical system improves the learner's number sense. Learners who have a keen number sense are able to decompose numbers, partition numbers and regroup numbers and this plays a significant role in simplifying their problem solving strategies. Practice and success in arithmetical operations relies largely on the storage of basic facts about numbers. For example a learner should be able to, at a relatively early stage in their scholastic life be able to solve " $5 + 3$ " without having to count.

Thompson (1997) states that a sound grasp of counting involves procedural competence (example: the learner is able to correctly determine that there are seven items in a collection), conceptual competence (understanding why a procedure works) and utilization competence (using procedures in appropriate contexts and in ways that are most adaptive to achieving the arithmetic goal). He states that counting is understood by some learners as an enactive process that is carried out on concrete objects whilst other learners will implicitly recognise that counting can take place in the absence of available countable objects.

2.1.3 What does progression in counting consist of?

Ensor et al (2009), state that in order for learners to master Foundation Phase numeracy there must be progression in acquiring the number concept, the movement from concrete to abstract reasoning and relatedly, the shift from counting

to calculating. According to Ensor et al (2009) recent classroom-based research shows that many foundation phase learners in South Africa rely on concrete methods such as tally counting to solve problems. This method is ineffective and it results in failure of many learners to abstract from concrete representations as the number range increases and this is seen as a major contributor to poor mathematics achievement in South African schools. They further state that there are limited opportunities to grasp the symbolic system of mathematics since concrete modes of representation are favoured by Foundation Phase teachers. This together with inefficient use of class time inhibits abstract ways of working with number.

According to Gray et al (2000) a cognitive shift from concrete to abstract involves a qualitative change through which the concept of number can be conceived as a construct that can be manipulated in the mind. In investigating this shift, they reported instances of children seeing full picture images of fingers, in others it was 'finger like'. The essential thing is that the object of thought was the 'finger' and the mental use of finger invoked a double counting procedure or "counting all". Low achievers - found it even more difficult to mentally hold the different initial inputs, appearing to place much greater reliance on a visual stimulus and creating and manipulating images associated with this. These learners have a much greater tendency to talk about things that may be captured by the senses and their imagery tends to be strongly associated with real concrete objects.

In summary, according to Gelman and Gallistel (1978) learners master the skill of counting when they:

- Can mark off an item in a collection with clear markers or tags where only one marker is used for each item.
- Realise that the tags are organised in a repeatable, stable order.
- Understand that a number, such as 7, represents the total number of items.
- Realise that a number can be manipulated.
- Understand that counting procedures can be applied to any collection of items.

Gray, Pitta and Tall (2000) in Ensor et al (2009) state that as learners move from the perceptual world (the use of counters, fingers and so forth) to using representations

of it (the use of tallies, and later number words) they are engaged in processes which are basically similar in that all are comparable to the process of counting.

McGuire, Kinzie and Berch (2011) suggests that the first three principles of counting by Gelman and Gallistel (1978) form the core for learner's emerging counting and that these principle alone indicate how learners count. These principles are: one-to-one correspondence (each item should be tagged with one and only one numeric tag, the stable-order (knowing that the number-name list i.e., one, two, three, etc. consists of a fixed order) and the cardinal principle (the counting tag for the last object in the count indicates the total number of objects in the set). According to McGuire et al (2011) the mastery of these three essential counting principles does not occur simultaneously but rather that "knowledge of the stable-order principle appears to develop first, followed by one-to-one correspondence, with a mastery of the cardinality principle developing the slowest" (p. 217). McGuire et al (2011) encourage the use of five-frames to help learners to use subitization to support early counting t. The following figure is an example of a five-frame showing three objects.



McGuire et al (2011), state that five-frames are useful tools for the development of number sense for the following reasons:

- It provides a basic organizational structure around the number five which is regarded as a benchmark number.
- Working with a small set of frames reduces the cognitive load and also minimizes errors with respect to counting.
- It presents opportunities for learners to establish connections with different numerical representations.
- Five-frames are similar to ten-frames, so early exposure to five-frames will help in their understanding of ten-frames.

This literature points to the usefulness of looking at the contexts and representations in which children are able to work with early counting.

In the following section, I deal with specific aspects of progression that relate to addition and subtraction – which were key topics in focus within the Wright et al (2010) tests administered within this study. The strategies that have been introduced in the previous sections are described and discussed in more detail within the addition and subtraction sections. Addition and subtraction strategies are linked since they are inverse operations. Given that some literature deals with addition and subtraction strategies separately I will also deal with addition and subtraction strategies separately in this literature review.

2.2 Addition

Progression across the stages involves the learner using counting in gradually more sophisticated ways to solve additive and subtractive tasks. For example; when asked to provide an answer for $3 + 4$, some learners showed evidence of the concept of 3 being reified into an object since they were able to count on from 3 to provide an answer i.e. 4, 5, 6 and 7. Some learners were able to provide an answer without using a count all or count on strategy; it is evident that this learner has reified both the 3 and 4 into objects. Askew et al (2001) summarises the trajectory of processes used for addition as: count all, count on and count from larger, and working with recalled and derived facts.

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2.2.1 Count All

According to Gray (2008) the most elementary method used to add is to 'count all' which involves the triple count strategy explained earlier. For example the addition of 3 and 4, could be executed by counting three objects (1, 2, 3), then to count four objects (1, 2, 3, 4) and then to put all the objects together and count the total (1, 2, 3, 4, 5, 6, 7).

2.2.2 Count on

The next stage occurs when the learner realises that it is not necessary to count a set of four objects and then a set of three objects. Gray (2008) states that one of the numbers i.e. the 4 or the 3 can be seen as a number object and the learner can simply count-on. The sum of $4 + 3$ becomes 4...5, 6, 7. The count-on can be seen as another procedure to carry out addition. Gray and Tall (1991) used the term “procept” to describe symbols such as $3 + 2$ which could evoke both a process (addition) and a concept (sum). Gray (2008) states that learners can indicate a growing sophistication in the objects they use to count on from. Learners may change the numbers around, and start with the larger number because it is closer to the answer and count-on from the first. Thus, there are two sub-categories within the count on strategy:

2.2.2.1 Counting on from the first number

The learner starts counting by repeating the first number and then continues the counting by starting from that number. When faced with $3 + 4$ the learner might say 34, 5, 6, 7. The 3 is the reified concept. Learners may use their fingers or any concrete material to count on, for example the learner may hold up 4 fingers and then count on from the first i.e. 4, 5, 6, 7 pointing to each of the 4 fingers as they count on.

2.2.2.2 Counting on from the larger number

In this case the learner may realise the efficiencies that can be gained by changing the numbers around so that counting on starts with the larger number. In the case of $3 + 4$, the numbers will be changed around so that the larger number appears first i.e. $4 + 3$. The learner will say 4 ...5, 6, 7. The fact that the learner realises that they can swop the numbers suggests a grasp of the concept that addition is commutative Gray (2008). The commutative property refers to the fact that you can change the order in which you add or multiply two numbers without changing the answer. For example, a child who understands the commutative property of arithmetic knows that the sum of $4 + 3$ is the same as the sum of $3 + 4$. In an example such as $4 + 15$, the larger number, 15 is closer to the answer or the answer is 4 fingers or objects away

from the answer, with commutativity providing a significantly quicker way of counting on. Therefore awareness of the commutative law for addition is reflected in learners' ability to count on from the larger.

2.2.3 Recalled Facts:

Learners normally select their strategies according to the size and familiarity of the numbers involved. Thompson (1995) states that the first number facts learners learn are the doubles. For example when presented with $4 + 5$ the learner will recall that $5 + 5 = 10$ therefore $4 + 5$ is one less and therefore the answer is 9. Thompson (1995) claims that this might be so because learners meet many situations in real life where things come in pairs or there might be less of a load on the memory.

The learners may also resort to “using fives” (Thompson, 1999), for example when the learner is presented with $6 + 7$ he/she sees 6 as 5 and 1 and 7 as 5 and 2 and therefore $6 + 7 = 5 + 5 + 1 + 2 = 13$. The bridging by ten strategy involves knowing compliments of 10. For example, to solve $8 + 6$ the learner recalls that $8 + 2 = 10$, so the learner removes 2 from 6 and is left with 4 and as a result $10 + 4 = 14$. To solve $9 + 5$ the learner may resort to compensation i.e. $10 + 5 = 15$ therefore $9 + 5 = 14$. Thompson (1999), claims that the concept of adding more than is required and then compensating is quite sophisticated and it needs to be carefully taught if it is to be used successfully used.

When children have automatic recall of facts, they can quickly retrieve answers from memory without having to rely on counting procedures, such as counting on fingers. Lack of automatic recall is a problem as children advance into the intermediate and senior phases, because the need to rely on laborious counting procedures creates a drain on mental resources needed for learning more advanced mathematics. Here the focus will be on automatic recall of addition and subtraction facts, which develops in normally-achieving youngsters by about the end of second grade. Poor automatic recall of facts has been identified as very common among struggling students Thompson (1995).

2.2.4 Flexible grouping strategies

Also known as derived number facts – this refers to instances where learners use a fact that they know by heart to calculate one that they do not know. In the initial stages there is a tendency to use the doubles, so that $6 + 5$ might be found by saying:

“ $5 + 5$ is 10 and 1 more makes 11”, or “ $6 + 6$ is 12, but because it is one less so it must be 11”.

Learners may study how numbers are related to 5 and 10 so they can apply these relationships to their strategies for knowing $5 + 4$ or $8 + 7$. Students might picture $5 + 4$ on a ten-frame to mentally see 9 as the answer. For remembering $8 + 7$, students might think —since 8 is 2 away from 10, take 2 away from 7 to make $10 + 5 = 15$.

Askew (2008) claims that those learners who can make links between known and deduced facts make good progress, because the known and the derived support each other. These two aspects of mental mathematics – known facts and derived facts – are complimentary. Higher attaining pupils are able to use known facts to figure out other number facts.

Learners may also engage in skip counting to add numerals. Thompson (1995) describes skip counting as counting in two, three or any number. For example when calculating $13 + 15$ the learner recognises that fifteen is a multiple of five and that thirteen comprised of two fives and a three. The learner is expected to say 15, 20, 25, 28. The difficulty would be to keep track of the number of multiples to add on.

2.3 Subtraction

Thompson (1997) identifies six particular strategies, which he suggests are increasingly complex, for dealing with subtraction situations: counting out, counting-back-from, counting-back-to, counting-up, inverse properties and bridging-down-through-ten. I will discuss each of these strategies using the example $7 - 3$.

2.3.1 Counting-out Strategies

The learner starts by counting 7 fingers and these fingers are held in a raised position and then the learner lowers 3 fingers one at a time. The learner then counts the remaining fingers. This strategy can involve learners in three separate forward counts and thus, this strategy parallels the triple count that is part of the count all approach to addition. Mastery of this strategy may result in learners setting up or removing a given number very quickly and sometimes this could be achieved in one movement and then read off from their fingers how many remain. This strategy is normally very successful when dealing with numbers up to ten and is less useful for larger numbers because of the finger shortage problem. Brighter learners may tend to modify this strategy if the numbers are larger than 10. For example: $11 - 6$ a learner may use their 10 fingers and an object in the vicinity of the learner such as a pencil or any object within the learners range of sight. This extra object is usually counted out first before returning to the familiar territory of their ten fingers.

2.3.2 Counting-back-from

The learner counts out 7 fingers and then counts back saying ...6, 5, 4 and lowering a finger as each number is counted backwards. The fingers that remain raised are then counted, giving the answer four. This strategy is quite sophisticated since the learner needs to execute a range of skills successfully. The learner must be able to forward count, backward count and also find some system of keeping track. This strategy involves two simultaneous processes which in effect go in opposite directions. Common errors in the count back from process that have been identified in the literature include learners counting back from the wrong number; the learner may say 7, 6, 5 instead of 6, 5, 4. (Thompson, 1997)

2.3.3 Counting-back-to

This strategy involves counting down to the subtrahend (in this case 3); 7.....6, 5, 4, 3 and arrives at the answer of 4. The learner raises a finger for each backward count and stops and realises the answer when the subtrahend is mentioned, so the answer is the number of fingers raised rather than the last number spoken. Unlike count

back from approaches, this strategy involves seeing subtraction as difference rather than take away.

Learners do not make connections with counting in larger steps or known addition facts, often because they do not understand subtraction as difference. According to Haylock and Cockburn (1989), it is important that learners experience and understand subtraction as both take away and difference. This is from a very early age and relates to real experiences they will have had. They will often be looking to see if others have more. Children also experience take away very early on. As learners are first coming to grips with these two aspects of subtraction it is important the language, the situation and the numbers involved all reflect the understanding of subtraction that is the most efficient way of solving the problem.

For example; if Adam collected 25 cans and George collected 22 cans. How many more cans did Adam collect than George? How many fewer did George have than Adam? This relates to “What is the difference between 25 and 22” and can be solved by counting up from 22 to 25 or down to 22 from 25 or using a known fact ($2 + 3 = 5$). The situation, the language and the numbers indicate difference. In the same way talking about a box of chocolates with 30 in and you want to give your friend 4 chocolates relates to taking 4 away from 30 and can be solved by counting back or using a known fact ($6 + 4 = 10$). The situation, the language and the numbers indicate take away.

2.3.4 Counting-up

With respect to the counting up strategy the learner starts counting up from the subtrahend i.e. 3.....4,5,6,7.... it's 4. Carpenter and Moses (1984) suggest that learners prefer to use the ‘counting-up’ rather the ‘counting-down’ strategy in missing addend and subtrahend problems, but Thompson (1995) states that in subtraction situations learners are more likely to do the countdown to strategy rather than the counting up strategy, and thus argues that counting up represents a more sophisticated approach and understanding.

2.3.5 Flexible Group Strategies:

The learner could engage in doubles fact (Thompson, 1995), to solve $18 - 9$ the learner uses the inverse argument that $9 + 9 = 18$ therefore, $18 - 9 = 9$. With respect to near doubles subtraction for example $9 - 5$ the learner states that $10 - 5$ is 5 therefore $9 - 5$ is one less that is 4. The learners may also view subtraction as the inverse of addition for example, $7 - 3$ the learner recalls the addition fact that $3 + 4 = 7$ therefore, $7 - 3 = 4$. Lastly, the learner engages in bridging through ten. For $12 - 4$ the learner will reason that $12 - 2$ is 10 and $10 - 2$ is 8.

2.3.6 Progression from concrete to abstract

According to Ellemor-Collins and Wright (2007) research and curriculum reforms has placed emphasis on mental computation. They state that if mental (abstract) strategies are emphasised earlier in the life of the learner rather than later, there may be better support for number sense and conceptual understanding of multi-digit numbers. The mental strategies also support the development of important connections to related knowledge. They differentiate between mental strategies that are 'sequence-based' and 'collection-based'.

With respect to sequence based or the 'jump' strategy, the learner keeps the first number whole and adds or subtracts the second number using a series of jumps. For example, when adding 37 and 26 a learner using jump may reason as follows: 37 and ten is 47 and ten more is 57; three more is 58, 59, 60 and three more makes 63. Sequence-based strategies depend on sequential structures to jump by ten and to make steps and hops in the number sequence. Learners may rely on a number line where the decades are highlighted.

A typical collections-based strategy is the 'split' strategy. This strategy involves partitioning both numbers into tens and ones, and then adding or subtracting the tens and ones separately, and finally combining the tens and the ones. For example, when adding 37 and 26 using split reasoning, the learner will add as follows: 30 and 20 is 50, 7 and 6 are 13, 50 and 13 makes 63. According to Ellemor-Collins and Wright (2007) low-attaining learners tend to use split strategies more than sequence-based strategies, and this indicates the development of knowledge of collections-based structure. They also state that research suggests that many low-attaining

learners do not develop the strategy of jumping by tens and therefore may not develop sequence-based structures. It is therefore unlikely that low-attaining learners can advance to integrated sequence-collections-based strategies which, they argue, are important for number sense and mental computation. Subtraction tasks can be potentially confusing when attempting to use the split strategy. Success with the split strategy requires a strong number sense and subtle insight into the procedure itself if errors are to be avoided – which is frequently not the case.

These indicators of the various increasingly sophisticated strategies for number conception, addition and subtraction flesh out the framework that has been used to analyse data in this study. I draw on a framework that has been developed by Wright et al (2006). The LFIN framework is explained in greater detail in 2.4 of this chapter and as noted earlier, some aspects of the LFIN framework are incorporated in my observation schedule.

Wright (2000) states that young learners find it easier to learn about things that are tangible and directly accessible to their visual, auditory, tactile, and kinaesthetic senses. As these learners become familiar with the concrete, they grow in their ability to understand abstract concepts, manipulate symbols, reason logically, and generalize. Wright (2000), states that these skills develop slowly, and he affirms that the dependence of most learners on concrete examples of new ideas persists throughout life. Concrete experiences are most effective in learning when they occur in the context of some relevant conceptual understanding. Transitions towards greater use of abstract number concepts run across the progressions identified in counting, addition and subtraction. These transitions are underlain by the gradual shift from more concrete to more abstract number understandings, making Sfard's (1991) notion of reification appropriate as the overarching theoretical frame for this study, with the specific progression trajectories identified in the literature across these three areas, forming the specific sub-frameworks used within the data analysis.

2.4 Learning Framework in Number (LFIN)

In this aspect of the report I provide a broad overview of the LFIN framework using a flow chart. Thereafter, I discuss the Stages of Early Arithmetic Learning (SEAL) in greater detail followed by Base-Ten Arithmetic Strategies.

2.4.1 Wright et al's (2010) "Learning Framework in Number" (LFIN)

The LFIN framework comprises of four parts. Figure 1 provides a breakdown of the complete LFIN:

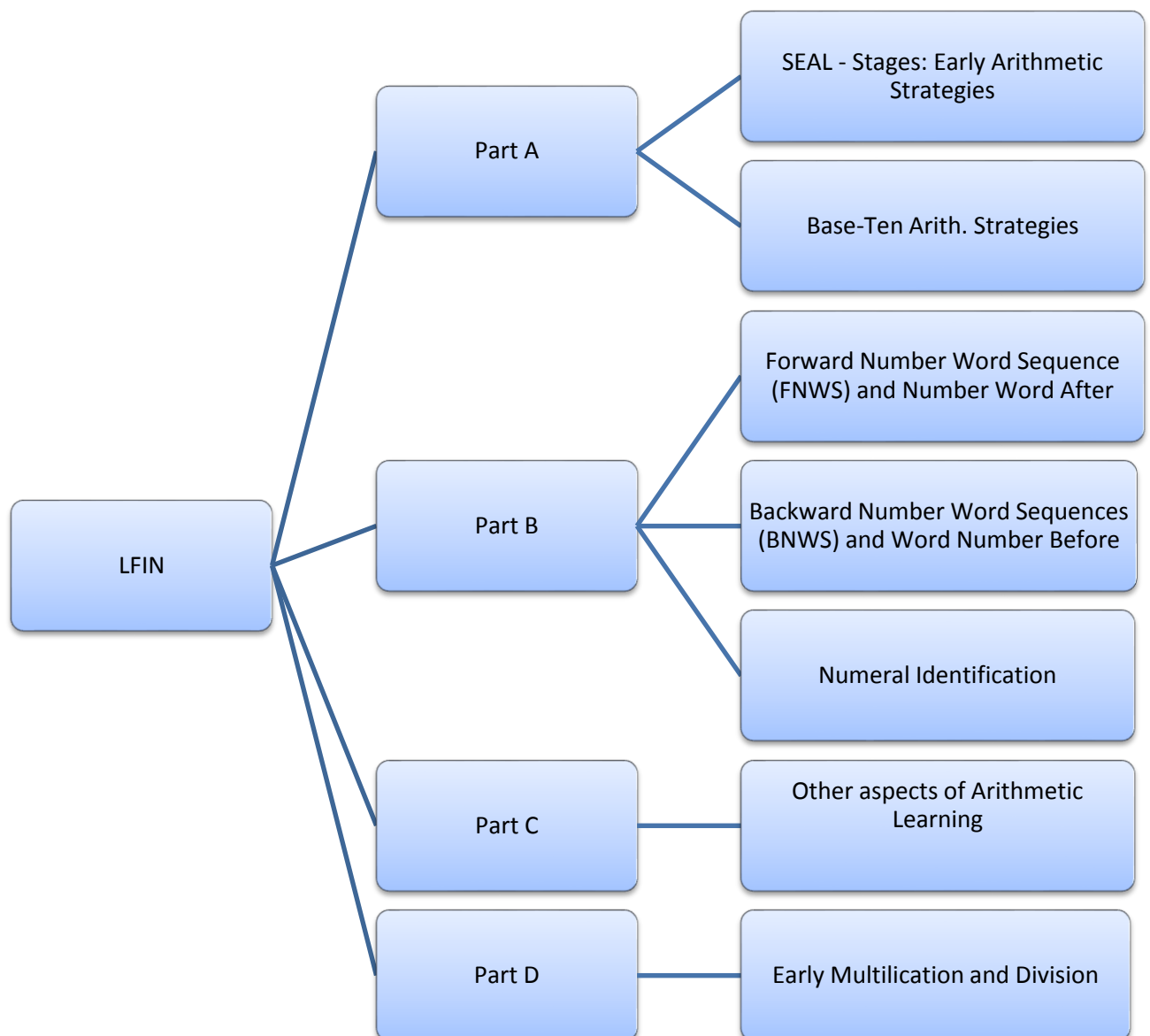


Figure 1: The Learning Framework in Number (Wright et al, 2002)

For the purposes of my research I will be focussing on Part A and Part B of LFIN. Part A of LFIN comprises of “The Stages of Early Arithmetic Learning (SEAL)” and “Base- Ten Arithmetic Strategies” as drawn up by Wright et al (2000). Part B contains a range of aspects that are important within the development of more sophisticated counting strategies.

2.4.2 The Stages of Early Arithmetic Learning (SEAL)

“The Stages of Early Arithmetic Learning (SEAL)” provides a clear framework for analysing early number strategies used by learners primarily for counting, addition and subtraction. It moves gradually from more concrete strategies to more abstract strategies. An important focus of this research will be on what strategies emerge from learner’s responses when answering the questions posed to them. Each stage has a clear description of what the learner should be able to achieve at that particular stage. The SEAL stage descriptions provide some useful intermediate steps in the process of moving from more concrete to more abstract strategies.

Stage 0 is called **emergent counting** and in this stage the learner experiences difficulty in counting visible items. The learner either does not know the number or cannot coordinate the number words with items.

Stage 1 is called **perceptual counting**, here the learners can count perceived items but cannot count concealed collections. This stage may involve seeing, hearing or feeling items.

Stage 2 is called **figurative counting** and in this stage the learner can count screened items in a screened collection for example when presented with two screened collections and asked how many counters in all, the child will count from one instead of counting on.

Stage 3 is called the **initial number sequence** and in this stage the learner uses counting-on rather than counting from one to solve addition tasks or missing addend tasks (eg. $6 + \Delta = 9$). The learner may use a count-down-from strategy (eg. $17 - 3$ as 16, 15, 14 – answer 14) but not count-down-to strategies (eg. $17 - 14$ as 16, 15, 14 – answer 3)

Stage 4 is called the **intermediate number sequence** and here the learner counts-down-to solve missing subtrahend tasks. The learner can choose the more efficient strategy of count-down-from and count-down-to (eg. $17 - 14$ as 16, 15, 14 – answer 3).

Stage 5 is called the **facile number sequence** and in this stage the learner uses a range of what are referred to as non-count-by ones strategies. These strategies involve procedures other than counting in 1's. It may include some counting by 1's alongside other strategies. Learners may count in 5's or in 10's. Strategies learners could use:

- Compensation – eg. If one number in an addition problem gets bigger by one and the other gets smaller by one then the answer will be the same.
- Commutativity – reversing the sum.
- Seeing subtraction as the inverse or opposite of addition.
- Awareness of using 10 as a block for addition or subtraction.

The model of SEAL provides a means of understanding the progression of learner's early number learning from perceptual counting strategies in which learners rely on concrete materials to a point where learners have relatively sophisticated and flexible counting strategies for addition and subtraction (abstract). Feeding into the SEAL stages, Wright et al (2000) provide a range of sub aspects or significant tasks for each stage.

2.4.3 Base-Ten Arithmetic Strategies

Wright et al (2010) claim that around the time the learners attain stage 3, 4 or 5 of SEAL, learners begin to understand or develop knowledge of the tens and ones structure of the numeration system. For learners who have attained stage 5, development of knowledge of the ten's and one's structure is absolutely vital. The following is a Model for Developing of Base-Ten Arithmetic Strategies:

Level 1: Initial Concept of Ten. The learner does not see ten as a unit. The learner focuses on the individual items that make up the ten. With respect to addition and subtraction tasks involving tens, learners count forward or backwards by ones.

Level 2: Intermediate Concept of Ten. Ten is seen or understood as a unit comprising of ten ones. The learners are dependent on representations of units of ten such as hidden ten-strip or open hands of ten fingers. The learner can perform addition and subtraction tasks involving tens when presented with material such as covered strips of tens and ones. The learner is not able to solve addition and subtraction tasks involving tens and ones when presented in a number sentence.

Level 3: Facile Concept of Ten. The learner can solve addition and subtraction tasks involving tens and ones without using materials or re-representations of materials. The learner can at this level solve number sentences with tens and ones by adding and subtracting units of tens and ones. The learner utilises reference numbers involving five-wise (5-plus) and pair-wise (doubles) structures to combine and partition numbers in the range 1 to 10 without counting.

The main idea underlying this strategy is complements in ten. For example, 6 is the complement in ten of 4 and 2 is the complement in ten of 8. This concept according to Thompson (1995) is not usually treated as an important contributor to mental calculations but argues that it should be. For example when adding $8 + 5$ a learner will resort to making the eight a ten by adding the two that is removed from the five i.e. $8 + 2 + 3 = 13$. A learner may also opt to make eight a ten and then count 11, 12, 13.

According to Thompson (1995), to use this strategy effectively learners need to follow the following steps:

- Determine what is needed to build one of the numbers up to ten;
- Partition the other number into two appropriate parts,
- Adding these two parts separately by counting-on or by using their knowledge of adding a single digit numeral to ten.

2.4.4 Part B of LFIN

This part of LFIN consists of Forward Number Word Sequence (FNWS) and Number Word After, Backward Number Word Sequence (BNWS) and Number Word Before and Numeral Identification. According Wright et al (2006), a learner at this stage is

expected to be facile with FNWSs to 100 and beyond and in the case of BNWSs the learner is expected to be facile beyond 30 but not necessarily to 100. A range of examples can be found in the observation schedule (ANNEXURE A).

I conclude this chapter with a summary of the key aspects of the literature that was focussed upon. The four key aspects of this chapter were: counting, addition, subtraction and the LFIN framework. The key focus of counting was the requirements for CAPS with respect to counting and also Gelman and Gallistel (1978) five principles of counting. The main counting strategies for addition are count-all, count-on (from the first and from the larger), recall strategies and flexible group strategies or derived strategies. The key areas of focus for subtraction are counting out, counting back (from and to), counting up and lastly flexible group strategies. With respect to the LFIN framework I focussed on the Stages of Early Arithmetic Learning (SEAL) and the Base Ten Arithmetic Strategies.

In the next chapter I deal with the theoretical framework for this study.

Chapter 3 Theoretical Framework

In this chapter, I detail the theoretical framework that features in this study. I use Sfard (1991) notion of the “Dual Nature of Mathematical Conceptions” as the theoretical framework for this study. I also draw on Sfard’s theory of reification to explain the progression to abstract conceptions of number.

Sfard’s (1991) “Dual Nature of Mathematical Conceptions”

Sfard (1991), states that abstract notions can be conceived in two different ways namely structurally as objects and operationally as processes. She refers to these notions as the “dual nature of mathematical conceptions”. Although these notions are incompatible they are complementary, there is interplay between the operational and the structural conceptions with respect to the same notions. Sfard (1991) claims that the operational conception is the first step in acquiring new mathematical notions. She also states that the movement from the computational operations to abstract objects is a long and difficult process but it can be accomplished through three stages in concept development: interiorization, condensation and reification. At this point I think it is necessary to elaborate on the above mentioned stages namely interiorization, condensation and reification.

Sfard (1991) describes interiorization in the following terms: “the learner gets acquainted with the processes which will eventually give rise to a new concept” (p.18) – for example, the process of ‘adding one more’ to a given set of counters leads to ‘new numbers’ arising in the early stages of counting. This stage involves lower-level mathematical processes which eventually become interiorized.

At the condensation stage there is a compacting of lengthy sequences of operation into more manageable units. Sfard (1991) describes this process in terms of: “squeezing lengthy sequences of operations into more manageable units. At this stage a person becomes more and more capable of thinking about a given process as a whole, without the feeling an urge to go into details” (p. 19). Sfard (1991) also states that, “Thanks to the condensation, combining the process with other processes, making comparisons, and generalizing becomes easier” (p. 19).

Condensation may be assessed through the learner's proficiency in combining processes with some reified objects, for examples when the learner is required to add $7 + 2$, the learner may resort to count-on (7, 8, 9). When the learner is able to conceive the notion as a fully-fledged object then the concept is reified. Sfard (1991) claims that reification is inherently difficult since it is not always known how much can be done to move the learner from the operational to the structural conception. Sfard's (1991) theory of reification relates to Gray's (2008) compression of the counting as discussed in Chapter 2.

Figure 2 below is a "General model of concept formation" (Sfard, 1991, pp. 22). The figure shows a three- phase schema which is to be understood as a hierarchy. This implies that one stage cannot be attained until all the former steps are taken. The learner must be able to apply processes on concrete objects viz. interiorization, condensation and reification before moving on to Concept B. Each stage in the schema has a trajectory from processes to objects.

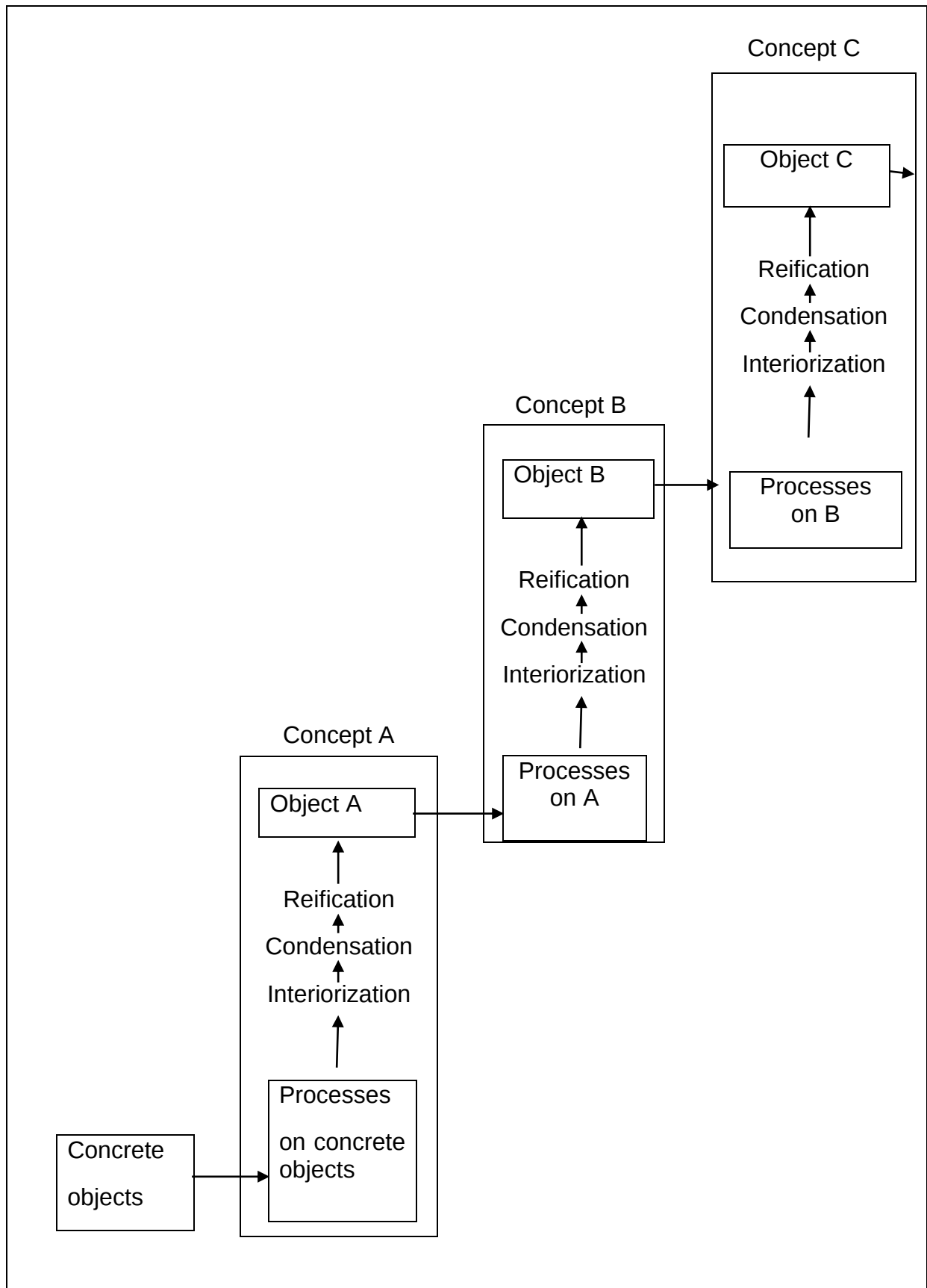


Figure 2: General model of concept formation (Sfard, 1991, pp. 22)

Sfard argues that reification can have beneficial effects on learning. The formation of structural conception implies that the cognitive schema is reorganised by the inclusion of new layers. Sfard (1991) states that the absence of structural conceptions may hinder further development. In the audio visual data I examine the extent to which structural conceptions of number can be seen in the responses of my sample of learners. Reification increases problem solving and learning abilities.

In the literature review, various interim steps in the transition from concrete to abstract counting are discussed. I summarise them here as they provide me with an analytical framework that operationalises the theory. Detail on where they are drawn from is given in the next section. In early number learning: if the learner is presented with $3 + 4$ the learner could respond with a concrete count all strategy. The learner could count out three fingers on one hand and four on the other hand and then count out the total of 7 fingers one by one. In the case where the concept of 3 is reified the learner may 'count on' using his or her fingers i.e. 4, 5, 6, 7. The learner who has reified 3 and has a reified understanding of the notion of 5 as a 'benchmark' may do the sum as '3 add 2 makes 5, and two more makes 6, 7'. Here the 3 has been reified and the 4 has been partially reified through being broken down into $2 + 1 + 1$. The learner who has reified both 3 and 4 may be able to provide an answer without relying on any concrete counting – here the answer is stated as a recalled fact, 3, 4 and 7 all existing as abstract objects disassociated from the process of counting. Counting from the larger number may also be a strategy learner's use when adding two numbers. For example when adding $5 + 13$, the learner will count on from 13.

Sfard (1991) also states that the development of the notion of number goes through three phases:

1. The pre-conceptual stage, at which the learner becomes familiar with operations on the already known numbers or in the case of counting it will rely on concrete objects. Routine manipulations are treated as processes and nothing else.
2. After a long period of the operational approach new numbers will begin to emerge out of the familiar processes, this is what is referred to as a new abstract construct.

3. The structural phase, when the number in question has eventually seen as a mathematical object. Upon this new number different processes could be performed and thus producing more advanced kinds of numbers.

Chapter 4 Methodology

To accomplish this task, video-based test interview data from the Wits Maths Connect Primary (WMC-P) programme was analysed to observe what strategies learners used when working with early number. The broader Wits Maths Connect Primary project is aimed at “Improving teaching and learning of primary school numeracy/mathematics”. Within this project, a sample of Grade 2 learners across all Grade 2 classes in two schools were assessed at the end of 2011 using “The Mathematics Recovery Programme” assessment instruments drawn from the work of Wright et al. (2010)) in order to explore strategies being used.. The tests were administered using an individual oral interview method. These interviews were captured on video for analysis. Within the broader project, learner responses were assessed at an overview level using Wright et al’s (2010) LFIN framework comprising of the following aspects and stages/levels:

- Stages of Early Arithmetic Learning (6 stages, numbered 0-5),
- Base-Ten Arithmetic Strategies (3 levels, numbered 1-3),
- Forward Number Word Sequences and Number Word After (6 levels, numbered 0-5),
- Backward Number Word Sequences and Number Word Before (6 levels, numbered 0-5)
- Numeral identification (5 levels, numbered 0-4).

The following table provides a summary of the LFIN framework. Part A and Part B of the framework are relevant to my research:

Part A Early Arithmetic Strategies Base-Ten Arithmetical Strategies	Part B Forward Number Word Sequence and Number Word After Backward Number Word Sequence and Number Word Before Numerals
Stages: Early Arithmetic Strategies 0 – Emergent Counting 1 – Perceptual Counting 2 – Figurative Counting 3 – Initial Number Sequence 4 – Intermediate Number Sequence 5 – Facile Number Sequence Levels: Base-Ten Arithmetic Strategies 1 – Initial Concept of Ten 2 – Intermediate Concept of Ten 3 – Facile Concept of Ten	Levels: Forward Number Word Sequence (FNWSs) and Number Word After 0 – Emergent FNWS 1 – Initial FNWS up to 10 2 – Intermediate FNWSs up to 10 3 – Facile with FNWSs up to 10 4 – Facile with FNWSs up to 30 5 – Facile with FNWSs up to 100 Levels: Backward Number Word Sequence (BNWSs) and Number Word Before Numerals 0 – Emergent BNWS 1 – Initial BNWS up to 10 2 – Intermediate BNWSs up to 10 3 – Facile with BNWSs up to 10 4 – Facile with BNWSs up to 30 5 – Facile with BNWSs up to 100 Levels: Numeral Identification 0 – Emergent Number Identification 1 – Numerals to 10 2 – Numerals to 20 3 – Numerals to 100

Table 1: Summary of LFIN Framework

The learners were assigned points within the WMC-P project team, based on the stages and levels detailed in Wright et al's (2010) work, in order to quantify this qualitative data. Summing these points created an overall summary score for each learner with 20 as the overall total possible score. These scores provided the basis of my selection of a sub-sample of learners in nine learners in each of two groups, namely weaker (scores from 6 to 13) and stronger learners (scores above 16). These learners were drawn from the two schools in which these tests were repeat conducted at the end of 2011, following their administration in all ten project schools at the start of 2011. Analysis of baseline data using the same tests in an earlier sitting in 2011 within the WMC-P project with a learner sample drawn from all ten participating schools (n = 231) revealed that over three quarters of the learners were using 'count all' strategies (Venkat, 2011). Within the theoretical framework I am using in this study, this strategy represents a highly concrete approach to counting, addition and subtraction.

Data gathering instruments – The tests

The assessment interview schedule that is used to assess the learners involved in the study is attached as ANNEXURE A. Drawn from Wright et al's work, these tests focus on:

- Early Arithmetical Strategies and Numerical Knowledge,
- Base-Ten and Advanced Arithmetical Strategies,
- Early Grouping: Structuring Numbers 1 to 10,
- Advanced Grouping: Structuring Numbers 1 to 20,
- Early Multiplication and Division and
- Advanced Multiplication and Division

For the purposes of this study I focus on the first two test items viz. Early Arithmetical Strategies and Numerical Knowledge and Base-Ten and Advanced Arithmetical Strategies.

My research methodology will be based on observing and analysing videos of a sample of grade 2 learners in two schools answering questions on a test that was administered orally to individual learners. The test is therefore the key research instrument in this study, and it is built on the idea of examining less and more

sophisticated strategies learners use to display their early number sense, understood in terms of a trajectory from more concrete towards more abstract number concepts identified from the literature.

The broader project administered part A of the test to all learners in the early 2011 sample, and part B to learners who appeared able to deal with the higher level Base 10 and addition and subtraction in the higher number ranges Part C was omitted entirely given the levels of learner performance seen in the earlier 2011 sitting. Given that my study is focused on counting, addition and subtraction, I work with the data gathered from Parts A and B only from the end 2011 sitting that was carried out with the learner sample in two of the project schools.

Part A focuses on the early strategies used by learners in arithmetical situations which are problematic for them, and typically involve counting, addition or subtraction. The assessment tasks involve activities such as counting the items in a collection given the numbers of counters, addition and missing addend tasks involving working out how many counters in visible screened collections, and various kinds of subtractive tasks presented with visible and screened collections of counters (Wright et al, 2006). Counters were screened for the addition and subtraction tasks. The following is an extract from the Assessment Interview Schedule that illustrates how counter were screened within a particular test item.

“Additive Tasks (screened, use counters of two colours)

Introductory task:

There are three red counters under here, and one yellow counter under here.

How many counters are there altogether?” (Wright et al, 2000, pp.161)

The Assessment Interview Schedule provided the interviewer instructions as to how the screening should take place for each of the test items that required screening. In the WMC-P the interviewers used A4 paper to screen counters.

For “Early Arithmetical Strategies and Numerical Knowledge” the test items were grouped according to the following categories; entry task, less advanced task and

more advanced task. For example the items for number word after were presented as follows in the Assessment Interview Schedule:

Number word after:

Say the word that comes straight after..... . Example: Say the word that comes after one.

- a) Entry task: 14; 11; 19; 12; 23; 29; 20
- b) Less advanced task: 5; 9; 7; 3; 6
- c) More advanced task: 59; 65; 32; 70; 99

In the sections below, I present examples of items from the test focused on the three key areas of interest within my study – counting, addition and subtraction. The following extract contains some examples for each category. The observation schedule (ANNEXURE B) contains all the examples used in the test.

Topic: Early arithmetic and numerical knowledge.

1. Forward number word sequence – count from

- a) 1 (to 32)
- b) 48 (to 61)

2. Number word after – say the word after

- a) 14;11;19;12;23;29;20
- b) 5;9;7;3;6

3. Numeral identification

- a)10; 15; 47; 13; 21; 80; 12; 17; 99; 20;66
- b) 8;3;5;7;9;6;2;4;1

4. Numeral recognition – arrange cards from 1 to 10.

5. Backward number word sequences – count backwards

- a) 10 (down to 1)
- b) 15 (down to 10)

6. Number word before – say the number word that comes before.

- a) 24;17;20;11;13;21;14;30
- b) 7;10;4;8;3

7. Sequencing numerals – say the number and then place the cards in sequence.

- a) cards from 46 to 55

b) cards from 1 to 10

8. Additive tasks (screened use of counters of two colours)

3 + 1 (3 green and 1 yellow concealed – how many altogether)

a) 5 + 4; 9 + 6 (both concealed)

b) 5 + 2; 7 + 3; 9 + 4 (both concealed)

c) 5 + 2; 7 + 3; 9 + 4 (unscreened)

d) Perceptual counting 13; 18 counters

e) Supplementary additive tasks: 8 + 5; 9 + 3

f) Missing addend – here are four red counters. Now look away. How many yellow counters were added if I now have six counters altogether? $4 + [] = 6$;
 $7 + [] = 10$; $12 + [] = 15$

9. Subtractive tasks –

a) Subtraction sentences

flash a card with $16 - 12$ (no counters involved); $17 - 14$

b) Missing subtrahend –

i) $5 - [] = 3$

ii) $10 - [] = 6$; $12 - [] = 9$

c) Removed items – Here are 6 counters (briefly display then screen). If I take away 2 counters (remove 2 counters display briefly and then re-screen), how many are left under here?

i) $3 - 1$

ii) $6 - 2$; $9 - 4$; $15 - 3$

Topic: Base ten and advanced arithmetic strategies.

a) Ten and one's task – count by tens with strips

i) Put down a ten strip. How many dots are there?

ii) How many altogether? Put down 8 strips one at a time. How many dots are there altogether?

iii) Pick up all the strips. How many dots do we have? How many strips are there?

b) Incrementing by ten -

- i) Place one 'four dot' strip out. How many dots are there?
- ii) Place a ten strip on the right of the four strip. How many dots are there altogether?
- iii) Continue placing ten strips to the four strip. How many dots are there altogether? 24; 34; 44; 54; 64
- iv) If necessary, repeat the task with either the 3 strip or the 7 strip.

c) Place the following strips in the following order:

10; 3; 10; 10; 4; 3; 10; 2; 10. How many dots altogether?

d) Place the following strips in the following order:

4; 10; 10; 2×10 ; 10; 3; 10×2 ; 5. How many dots altogether?

3. Horizontal Sentences:

Do you have a way to figure out what is?

$16 + 10$; $42 + 23$; $38 + 24$; $39 + 53$; $56 - 23$; $43 - 15$; $73 - 48$

In this report I analyse 18 videos of test interviews – nine videos of high performing grade 2 learners and nine videos of poor performing grade 2 learners. The rationale for selecting nine high performing learners was to compare the extent to which the high performers were relying on concrete strategies to provide answers in relation to the low performing learners or whether they have transitioned to more abstract strategies. These learners were classified as high performing according to the Wits Maths Connect Project team's composite scores for them based on Wright et al's Learning Framework in Number's (LFIN) levels and stages. Whilst I have used this tool for selecting the learners I focus on (and whilst this score is partially based on a framework that is built on progression towards more abstract strategies), my study is exploratory, with a specific focus on examining the extent of use of more abstract strategies, rather than whether the learners get answers correct or incorrect. The learners were selected from two schools; one from the suburbs of Gauteng and the

other from a township in Gauteng. As stated above these two schools were the only schools in the broader project where the LFIN test was administered at the end of 2011. The suburban school used English as the language of learning and teaching, whilst the township/informal settlement school used classes based on a range of home languages. Speakers of each of the home languages were available in the township school to provide translation and clarification of all the test items. Almost all learners in this school gave numerical answers in English. Interview allows for clarification of understandings and visibility of counting strategies – which were necessary for the focus on this study adding to depth and validity. . For this study, I undertook a secondary in depth analysis using the video data as my data source.

Justification for data collection methods:

The advantage of audio visual data is that it provides visual evidence of the strategy the learner is using to arrive at a particular answer whether right or wrong, including the capture of data relating to finger count or physical object count based strategies – which audio based data would not have been able to capture. Opie (2004) states that the advantage of the observation method of research is that information about the physical environment and about human behaviour or response can be recorded directly by the researcher. Creswell (2012) states that observation as a research method is very direct method for collecting data or information and affirms that it is best used for the study of human behaviour. In this study observing human response to counting, addition and subtraction tasks facilitates the process of identifying the strategy the learner uses. For example, the count on strategy is easiest to identify by observation i.e. seeing and hearing the learner count on using their fingers.

Video analysis also provides data that is observable and thus makes the analysis open to validation. As standards of academic rigour, both validity and reliability are rooted in the assumption that the information contained in documentary evidence is embedded in data that is objectively identifiable. The use of video data is also versatile since the data can be repeatedly analysed if the need arises.

The data will be quantitatively and qualitatively analysed with the aim being to understand and determine the ways in which more concrete and more abstract

strategies play out in the approaches used by weaker and stronger learners, and the extent to which the subjects being studied transition from the concrete to the abstract with respect to the strategies they use in early number learning. From my initial observation of some of the videos it is apparent that some learners rely wholly on concrete strategies using their fingers for forward counting and backward counting. One of the limitations of the study would be the fact that some learners may be intimidated by the presence of cameras and they may not respond the way they would in a classroom setting, but in the WMC-P data collection, data collection was terminated in all instances where learners appeared uncomfortable. Informed consent for participation in the test based interviews was obtained from all learners and their parents/carers. Ethical clearance was obtained for the secondary data analysis from the University (Protocol Number: 2012 ECE119).

An observation schedule linked to my overview of the literature and Sfard's theoretical framework was used to record the responses of each of the learners in the sample. With respect to counting learners responses were categorised under the following counting strategies: pre-conceptual, operational and structural. For addition learner responses were recorded under the following strategies: count all, count on from the first, count on from the larger, recall and flexible group strategies or derived strategies. With regard to subtraction learners' responses were recorded according to the following strategies; counting out, counting back from, counting back to, counting up and flexible group strategies. An extract of the observation schedule is attached, refer to ANNEXURE A.

Chapter 5: Data Analysis

In this chapter I examine the strategies high ability and low ability learners use when working with early number. I will at the outset consider the strategies used by the high ability level learners and then I will deal with the low ability level learners. For each group I provide a table summarising the strategies used by that group of learners. Thereafter, for each of the above mentioned sub groups I will assess individual performance under the following aspects; counting, addition, subtraction. Based on the responses seen by some learners in the high attaining group who were able to access problems in the higher number ranges, I also analyse their work with base ten strategies. Thereafter, I compare learner performance and strategies within and across the two ability groups. This approach allowed me to note common approaches that characterised elements of the ways in which lower attaining learners worked with early number problems in contrast with the ways in which higher attaining learners worked with the same problems. I was also able to comment on similarities between the two groups, and ways in which my findings compared both with prior findings in the South African and international contexts.

I used a code for each learner allowing for anonymity. The schools were assigned a code of either “1” or “8” (based on the numbering of the broader WMC-P project) and then “HA” or “LA” representing higher ability or lower ability learners respectively, followed by a number representing the learner. The LFIN score is indicated in brackets after the code. In the case of each learner, I begin with a qualitative description of their strategies on counting/ addition/ subtraction tasks, before going onto examining the whether the strategy used is concrete or abstract.

5.1 High Ability Level Learners

Table 2 that follows provides a summary of the strategies used by the high ability level learners:

	Counting			Addition Strategies					Subtractive Strategies				
	Pre-conceptual	operational	structural	Count all	Count on		Flexible group str	Recall	Counting out	Counting back		Counting up	Recall
					From 1 st	From larger				from	to		
1 HA 1		√	√		√			√		√			
1 HA 2		√	√		√			√		√			√
1 HA 3		√	√		√	√		√		√			√
1 HA 4			√		√			√		√			√
1 HA 5		√	√		√			√		√			√
1 HA 6		√	√		√			√		√			√
8 HA 7			√			√		√					√
8 HA 8			√					√					√
8 HA 9			√		√			√		√	√		√

Table 2: Summary of Strategies used by High Ability Learners.

1 HA 1 (16/20):

Counting Tasks:

This learner was able to forward count from 1 to 112 without any difficulty. He was able to say the number word for numbers up to 100. The learner was able to identify both the 2- and 3-digit numbers that he was shown. He was able to count backwards from 10 to 0, 15 to 10, 23 to 16, 34 to 27 and 72 to 67, and was also able to say the number word before for numbers up to 99. With respect to sequencing numerals he was able to say the number and then was able to place the number cards in sequence for numbers from 1 to 10 and then for numbers from 46 to 55 fluently.

He used his fingers to track his counting and he had mastered the technique of using his fingers. He was able to count on from any number given to him in the range (refer to items 1a, b, c, d in the test) for example; when he had to count from 76 to 84, he held up six fingers and counted 76, 77, 78, etc.. The learner held up six fingers to help him keep track of the unit's digit of the counting range 76 to 84. Whenever he reached 10 or a multiple thereof he clapped his hands. Sfard (2008) states that learners realise that repetitive counting of the same set of numbers always end with the same number word as in the case of the above scenario where the learner clapped his hands when he reached 10 or a multiple thereof. Sfard (2008) refers to this as "ritualized chanting of number-words" (p47) and the number words become reified and turns into a metaphorical object. He also used his fingers to count backwards. So with respect to counting this learner performed at the operational level and structural level. He used his fingers to keep track of his counting.

Addition Tasks:

He was able to add concealed and unscreened items from the addition tasks of "The Mathematics Recovery Programme". The learner resorted to counting on from the first number for example when given $9 + 6$ he held up nine fingers and then held up six fingers and then counted 10, 11, 12, 13, 14, 15 and stated the answer 15. The first number i.e. 9 was reified and he counted on from the reified number. However, he was not able to solve for the missing addend (8f) for example $4 + [] = 6$. For these examples he did not use his fingers and provided incorrect answers.

Subtractive Tasks:

He was able to provide correct answers for cards with examples $16 - 12$ and $17 - 14$. For $17 - 14$ he counted out 17 on his fingers and then counted back 14 from 17 until he got to three fingers in a raised position and he gave the answer 3. He resorted to counting back from, to arrive at an answer. He was not able to state the missing subtrahend (11b) for example, $5 - [] = 3$; $10 - [] = 6$; $12 - [] = 9$ and $15 - [] = 11$. He was also not able to solve removed items (11c) for example, '3 – 1, here are 3 counters (briefly display then screened) take away 1 counter (display briefly and then re-screened), how many are there left under?' Of interest here, is the fact that he was able to solve by taking away in the context of the symbolic sums (e.g. 17

– 14), but unable to do this in the physical context described here. This may suggest unfamiliarity with the direct modelling style tasks used here (Carpenter, Moser and Rebout, 1988).

Base Ten and Advanced Arithmetic Strategies:

This learner was able to provide correct answers for base ten strategies problems such as counting by tens, incrementing by ten and adding the number of dots on a strip for example $10 + 4 + 10 + 10 + 10 + 10 + 10$. However he was slow in processing because he started all over again instead of counting on.

1 HA 2 (18/20):

Counting:

This learner was able to count up to 112 and was able to say the number word after up to 99. With respect to numeral identification she was not able to identify 206 and identified 820 on the second attempt. She was able to count backwards from 72 and she was able to say the number word before up to 99. With regard to sequencing numerals she was able to say the number correctly and was also able to place the cards in the correct sequence.

This learner operated at the structural level for counting since the use of her fingers for counting were merely to keep track of her counting and to ensure accuracy in her counting.

Addition tasks:

She was able to add a one digit numeral to another one digit numeral without any difficulty. She was able to solve for the missing addend easily. She resorted to counting on from the first when presented with $5 + 4$. She held up 5 fingers and the on the other hand she raised one finger at a time as she counted i.e. 5.... 6, 7, 8, 9. For $9 + 6$ she was able to give an answer of 15 after counting on using her fingers 9..... 10, 11, 12, 13, 14, 15. The first number, in both of the examples quoted above, is reified and therefore the learner operated at an abstract level for addition of one digit numbers.

Subtractive Tasks:

She was able to correctly subtract a two digit number from a two digit number and she was able to determine the missing subtrahend without any difficulty. With respect to $16 - 12$ and $17 - 14$ the learner was able to provide the correct answer by using recall strategies, she showed no reliance on concrete objects. With regard to solving for the missing subtrahend she was able to use recall strategies to solve: $5 - [] = 3$, $10 - [] = 6$ and $15 - [] = 11$. For $12 - [] = 9$. She resorted to counting on from 9 (9....10, 11, 12) and she raised a finger as she counted and as a result she had three fingers raised when she reached 12 and gave an answer of three. Thus, she was able to use the idea of addition as the inverse of subtraction, and worked through this problem using a count-up-to strategy (Thompson, 1999).

Base Ten and Advanced Arithmetic Strategies:

She was able to provide correct answers for all of the base ten and advanced arithmetic strategies. She was able to use recall strategies for all of the examples in this aspect of the test except for example 'c' where she resorted to counting on. She used a sequenced based strategy fluently.

1 HA 3 (17/20):

Counting Strategies:

This learner used her fingers to count from 1 to 32. She placed both hands on her head when she got to 10 or a multiple thereof and she placed one hand on her head when she counted to five or a multiple thereof. Thereafter, I could not tell whether she continued to use her fingers to count since her fingers were under the desk. Although her hands were moving under the desk I could not tell for sure if she used her fingers to count or not. She was not able to count beyond 107 and was hesitant when counting from 93 to 112. She was able to say the number word after without any reliance on any tangible objects and she was able to say the number word after without any difficulty. With regards to numeral identification she was able to identify all numerals except for 123 and 820. She was able to count backwards without any difficulty and without any reliance on any tangible object. She was also able to provide the correct answers for number word before. However, she was very

hesitant and took a while before the answers were provided and she took long to say the number word before 20.

Although this learner used her fingers to count from 1 to 32 she showed no reliance on tangible objects for aspects of counting thereafter. This learner operated at a structural level for counting.

Addition Tasks:

When presented with $5 + 4$, she started by tapping one hand on her head (denoting five) and then she counted on from the first i.e. 6, 7, 8, 9. When presented with $9 + 6$ she counted on from the larger that is 9, and said 10, 11, 12, 13, 14, 15. The learner consistently applied this addition strategy to all addition examples presented to her. With respect to examples with missing addends for example; $4 + \square = 6$, she was able to count on i.e. counted on her fingers 5, 6 and gave the answer of two. Thereafter she was able to use recall strategies to provide answers for $7 + \square = 10$ and $12 + \square = 15$.

This learner operated at an abstract level for addition she was able to use count on from first, count on from larger and therefore showed evidence of reification of at least one number in the sum. Note the existence of the dual nature of mathematics when the learner engaged in recall strategies to provide answers for $7 + \square = 10$ and $12 + \square = 15$ and according to Sfard (1992) it is the dual nature of mathematics that allows for flexible mathematical working.

Subtraction Tasks

She was able to use the counting up strategy to solve $16 - 12$ and $17 - 14$. She counted and raised a finger as she counted that is 13, 14, 15, 16 and for $17 - 4$ she counted 15, 16, 17. With respect to the items with missing subtrahends she was able to provide answers from recall and she was also able to count up from. For example $5 - \square = 3$ she was able to answer without using any counting strategy but for $10 - \square = 6$, she counted up i.e. 7, 8, 9, 10 and gave an answer of 4. She however, gave the wrong answer of 6 for $15 - \square = 11$.

This learner used abstract strategies for subtraction; she did not count out or engage in treble counting but rather counted up or she engaged in recall strategies.

1 HA 4 (18/20):

Counting Tasks:

Although this learner was able to count without reliance on any tangible objects to count, he was not able to count from 76 to 84. He was able to count from 1 to 32 and 48 to 61 without any difficulty but when he had to count from 76 to 84 he counted..... 68, 69, 80, 81, and when he had to count from 93 to 112 he counted..... 108, 109, 110, 114,..... . He was able to say the number word after without any reliance on his fingers and was successful with this task. With regard to numeral identification he was able to successfully identify numerals in the task except for 206 where he fumbled at first and got it right at the second attempt and with 341 he read this number as 343. He experienced no problems with counting backwards, saying the number word before and the sequencing tasks and with respect to these tasks he did not rely on tangible objects.

This learner showed structural conception for the counting tasks however, he did not meet the CAPS requirements for a Grade 2 learner.

Addition Tasks:

With respect to $5 + 4$ he was able to engage in recall for this example in the task. This learner was able to count on from the first number, when presented with $9 + 6$ he gave an answer of 15 without any evidence of using his fingers. When asked how he got the answer, he explained that he counted 9.... 10, 11, 12, 13, 14, 15. However, he adopted a different strategy when presented with $8 + 5$ he gave an answer of 13 and was asked to explain how he arrived at 13. He got his answer by adding 2 to 8 and then adding 3. He used the same strategy when presented with $9 + 3$. He was able to successfully use flexible group strategies. With respect to the tasks with missing addends he was able to resort to recall when providing answers. When he was asked to explain how he got his answer to $12 + [] = 15$ he explained; 12.... 13, 14, 15 therefore the answer is 3.

This learner used a range of strategies from recall to counting on from the first to flexible group strategies. He displays greater progression to abstract strategies than the learners discussed thus far. The use of flexible group strategies shows the progression from process to object.

Subtraction Tasks:

When presented with $16 - 12$ he resorted to a counting down from strategy i.e. 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4. However, when he was presented with $17 - 14$, he gave an answer of 4 after a while because he kept on losing track of his counting when using the counting back from strategy. With respect to the missing subtrahend tasks he was able to resort to the recall strategy. For the rest of the subtraction tasks that is $3 - 1$, $6 - 2$, $9 - 4$, $15 - 3$ and $27 - 4$ he resorted using a counting back to strategy for example $27 - 4$ he said 26, 25, 24, 23.

Whilst this learner has shown evidence that he has progressed from the concrete to abstract conception for subtraction he has not resorted to counting up and this indicates that there is room for further progression into abstract conception.

Base Ten Strategy Tasks:

This learner was able to count in multiples of ten and was able to add the units and tens separately to arrive at an answer for example when asked for the total number of dots 10; 3; 10; 10; 4; 3; 10; 2; 10. He added $3 + 4 + 3 + 2$ and counted 22, 32, 42, 52, 62. He showed evidence of using a sequenced based strategy.

1 HA 5 (16/20):

Counting Tasks:

He used his fingers to help him to count from 1 to 32. When counting from 48 to 61 he jumped from 59 to 70 and I feel that this error was a slip since he was able to count from 76 to 84 without any problems. However, he was not able to count beyond 109. With respect to number word after he found it difficult to understand the instruction but was able to correctly say the number word after for all the examples in the task except for the number word after 29. He was not able to identify the following numerals; 47 which he read as 74, 206 which he read as 126, 341 which he read as 34 and 820 which he read as 180. He was able to count backwards from

72 down to 1 without any reliance on concrete objects. He was able to correctly say the number word before for all of the examples in the task.

This learner operated at an operational and structural level for counting since there were instances when he used his fingers to count and there were instances when he did not rely on his fingers or any tangible objects.

Addition Tasks:

This learner was able to operate at a recall level for some of the addition test items that is for $5 + 4$, $5 + 2$, $7 + 3$. With respect to $9 + 6$ he counted on from the first number using his fingers, he said 10, 11, 12, 13, 14, 15. For $9 + 4$ he gave an answer of 11 without using his finger to count on. When asked to think about his answer he then used his fingers and counted on; 10, 11, 12, 13 and gave an answer of 13. He was able to use the recall strategy when asked to determine the missing addend for example; $4 + \square = 6$ and $7 + \square = 10$. With respect to $12 + \square = 15$ he provided two incorrect answers and the instruction had to be repeated to him in his home language. When he understood the instruction he then gave the correct answer.

This learner operated at an abstract level for addition since he was able to count on and also use recall strategies.

Subtraction Tasks:

When presented with $16 - 12$ he struggled to understand the instruction and he was not able to provide a correct answer. He was able to solve $17 - 4$ using counters and he engaged in treble counting or the counting out strategy i.e. he counted out 17 counters and then of the 17 he counted out 14 counters and then he was left with 3 counters. With respect to the missing subtrahend tasks he was able to provide answers using the recall strategy.

For subtraction this learner operated at a concrete level for the take away examples and at an abstract level for the examples with missing subtrahends.

1 HA 6 (17/20):

Counting Tasks:

This learner was able to count from 1 to 112 and she used her fingers to keep track of the unit digit as she counted. She was able to say the number word after without any problems (up to 99). With regards to numeral identification, she was able to successfully identify all the numerals in the task without reliance on any tangible objects. She was able to count backwards from 72 down to 1. She was able sequence numerals without any problems.

This learner operated at an abstract level with respect to numeral identification, she performed at an operational and also structural level for counting.

Addition Tasks:

She used the count on from the first strategy for all of the examples in the addition tasks. For example $5 + 4$, she counted on her fingers 5.... 6, 7, 8, 9. With regards to the tasks with the missing addends she also used the count on from the first strategy. In the case of $4 - \square = 6$, she held up four fingers and then counted on until she got to six and gave the answer 2.

Therefore this learner operated at an abstract level for addition. Although she resorted to an abstract strategy of counting on from the first she did not attain higher levels of abstraction since she did not use recall strategies or flexible group strategies.

Subtraction Tasks:

This learner resorted to counting out or triple counting. With respect to $16 - 12$ she counted 16 on her fingers and toes and then counted out 12 from the 16 and was left with 4 which she also counted. With regards to the missing subtrahend tasks she used her fingers and toes, for $5 - \square = 3$ she held up five fingers and then dropped 3 fingers and gave said 2. She used the counting down to strategy. However, she struggled to keep track when there double digit numerals and as a result she gave incorrect answers for $12 - \square = 9$ and $15 - \square = 11$.

This learner used concrete strategies when solving $16 - 12$; she used the strategy of counting out or triple counting. She used abstract strategies when finding the missing

subtrahend. Although she used concrete strategies she was able to use the strategy effectively and this is evident in her score of 17 out of a total of 20.

Base ten strategies

She was able to count in tens and she was able to count by adding ten each without any problems. For example, $4 + 10 + 10 + 10 + 10 + 10 + 10 + 10$ she was able to correctly count 14, 24, 34, 44, 54, 64, 74. She used the count on strategy when determining the number of dots in 10, 3, 10, 10, 4, 3, 10, 2, 10. She was able to use the sequence based strategy or the jump strategy effectively.

8 Ha 7 (18/20):

Counting Tasks

This learner was able to counting up to 112 without any reliance on tangible objects. He was able to say the number word after up to 99 and was able to identify 3 digit numerals without relying on any concrete objects. However, with respect to counting backwards he fumbled when counting down from 34 to 27. His response was as follows: 34, 33, 32, 31, 30, 39, 38, He was able to count backwards from 72 to 67 without any problems. He was able to say the number word before and he was able to sequence numerals without any problems.

This learner therefore operated at the structural level for counting since he did not make use of any concrete objects to count.

Addition Tasks:

This learner operated at a recall level for most of the addition tasks. For $9 + 6$ there was some finger movement and also a little lip movement which suggested that he counted on. With regard solving for the missing subtrahend he was able to provide answers without the use of his fingers or any other concrete object.

This learner operated at an abstract level and he was able to see the object in the examples presented to him i.e. he has progressed beyond the process stage.

Subtraction Tasks:

For $16 - 12$ he was able to provide an answer without showing any reliance on any tangible object or doing any counting and when asked how he got the answer he explained that he put 16 into his head and took away 12 from it. With regards to $17 - 14$ he counted with fingers but was not able to produce a correct answer at the first attempt. He then counted up because after pointing to three fingers he was able to produce the answer. With regard to solving for the missing subtrahend he was also able to provide answers without any reliance on tangible objects.

He therefore operated at an abstract level for the subtraction tasks since he was able to use recall strategies and count up strategies.

8 HA 8 (17/20):

Counting Tasks:

This learner was able provide correct answers to forward number counting, number word after, numeral identification, backward counting and sequencing numerals without reliance on tangible objects. However, the only item he got wrong for counting was identifying the numeral 341.

He operated at a structural level for counting and this implies that he performed at an abstract level for counting.

Addition Tasks:

He was able to use the recall strategy to answer task 8a to 8e for addition. With regard to the task with missing addends he was also able to resort to using the recall strategy.

This learner was able to operate at an abstract level for all items of addition since he resorted to using the recall strategy.

Subtraction Tasks:

For the subtraction tasks he showed no evidence of reliance on tangible object and was able to provide answers purely by recall. However, for $17 - 14$ his first attempt was 2 which was incorrect and on the second attempt he was able answer correctly.

There were times when he looked at his fingers but did not physically use his fingers to count.

He operated at an abstract level for subtraction he resorted to using the recall strategy.

8 HA 9 (18/20):

Counting Tasks:

This learner was able to correctly answer all examples in the counting tasks (i.e. forward number counting, number word before, numeral identification, backward number counting and numeral sequencing) without reliance on any tangible objects.

This learner was able to operate at a structural level for all counting tasks.

Addition Tasks:

For most of the examples in the addition tasks (8a to 8e) she was able to resort to recall strategies except for $9 + 6$, where resorted to using her fingers and where she used a count on from the first strategy. With respect to the items with the missing addend examples she was able to recall the answers without the reliance any tangible object.

This learner used abstract strategies i.e. counting on from the first and recall.

Subtraction Tasks:

The learner was given a pencil and paper to work out $16 - 12$. The learner drew 16 little circles and then counted out 12 circles, striking out a circle as she counted and was left with 4 circles in other words the counting back strategy. The following is a simulation of the strategy she used to solve the problem:

o o o o

o o o o

o o o o

o o o o

She used the same strategy for 17 – 14. The following is a simulation of the strategy she used to solve the problem:

Step 1:

o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

(forward counting)

Step 2:

o	o	o	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
			14	13	12	11	10	9	8	7	6	5	4	3	2	1

← (counting back from)

Step 3: Answer: 3

Note the sophistication in the understanding of number, when subtracting from 16 items she saw 16 as a multiple of 4 and grouped the circles in fours and this helped her keep track of her counting and also it has helped to attain greater levels of accuracy. With respect to the problem with 17 items she could not regroup 17 and therefore we see the linear arrangement of circles and a concrete counting strategy.

With regard to the problems with missing subtrahends she was able to recall answers to the items of the test with smaller digits and counting back strategies for items with larger numerals. For example $10 - \square = 6$ she resorted to counting back to i.e. 10, 9, 8, 7, 6, and gave the answer 4. With $12 - \square = 9$ she provided the answer straight away and when she was asked how she got to the answer said $12 - 3 = 9$ and $9 + 3 = 12$.

Therefore, the learner operated at an abstract level for subtraction since she was able to resort to using the counting back from and also recall strategies - both of which entail the use of at least some reified number elements.

The shaded columns in summary Table 2 represent the concrete level for each category i.e. counting, addition and subtraction. The table shows that the high ability learners are all operating at some level of abstraction and none of them relied on

concrete modes of learning at this level in their understanding of early number. However, the summary also indicates that most of these higher attaining learners, whilst able to work in more abstract ways in counting situations, seem unable to use this skill consistently in addition and subtraction situations, where the predominance of count on and count back from strategies indicates only partial and relatively low level progress into abstraction.

5.2 Low Ability (LA) Level Learners

The following table provides a summary of the strategies used by the low ability level learners:

	Counting			Addition Strategies					Subtractive Strategies				
	Pre-conceptual	operational	structural	Count all	Count on		Flexible group	Recall	Counting out	Counting back		Counting up	Recall
					From 1 st	From larger				from	to		
8 LA 1		✓	✓	✓	✓			✓					✓
1 LA 2			✓				✓	✓	✓	✓			✓
1 LA 3		✓	✓					✓					
1 LA 4		✓	✓					✓		✓			
1 LA 5		✓	✓	✓		✓			✓	✓			
1 LA 6		✓						✓					✓
1 LA 7			✓		✓								✓
8 LA 8			✓		✓					✓			
8 LA 9		✓	✓	✓						✓			

Table 3: Summary of Strategies used by Low Ability Learners.

8 LA 1 (13):

Counting Tasks:

This learner operated at a structural level for counting, she did not rely on any tangible objects when asked to count, and she could not count beyond 110. After 110 she counted 111 as “ten hundred and one” and for 112 she said “ten hundred and two” and so on. She was not able to identify the numerals 206 and 341. She identified 206 as 260 and 341 as 314. This learner was only able to count backwards from 10 to 1 and thereafter was not able to accurately count backwards. For example; when asked to count backwards from 15 down to 10 she hesitantly responded as follows 15, 14, 13, 20 and counting down from 23 to 16 her response was as follows: 23, 21, 30, 32. She consistently switched to forward number counting. When asked to state the number word before the learner stated the number word after for example; for the number word before 17 the learner gave an answer of 18. She also failed to place cards in sequence from 46 to 55, her response was as follows: 51; 49; 52; 48; 47; 54; 46; 50; 53; 55.

This learner performed at a structural level for counting but did not meet the requirements of the syllabus as stipulated in the CAPS document.

Addition Tasks:

With regards to addition examples she resorted to counting out or triple counting. For example when adding 5 and 4, the learner counted out 5 fingers and then counted out 4 fingers and then counted the fingers that were held up and gave the answer of 9. She was able to successfully execute this strategy for all of the addition examples. However, with respect to addition examples with missing addends, she did not possess the skill to use the above mentioned concrete strategy to solve the problem and as a result she resorted to guessing. Here are some of her responses: for $4 + \square = 6$ her answer was 3, for $12 + \square = 15$ her answer was 4. She provided these answers without any activity.

This learner performed at a concrete level since she resorted to counting out or triple counting.

Subtraction Tasks:

This learner was not able to subtract two digit numbers from two digit numbers, her response to $16 - 12$ was 2 and her response to $17 - 14$ was 4, without any activity. She was able to provide correct answers to $5 - \square = 3$, $10 - \square = 6$ and $12 - \square = 9$ by counting back from. With respect to $15 - \square = 11$ she provided incorrect answers in her first two attempts and got the answer on the third attempt and these answers were provided without any activity. It seemed as if the learner resorted to guessing since she provided answers that did not seem reasonable for example: for $6 - 2$ her answer was 3, $9 - 4$ her answer was 1, for $15 - 3$ her answer was 5 on the first attempt and 8 on the second attempt and for $24 - 4$ her answer was 20.

Although this learner was able to resort to abstract strategies for subtraction these strategies were not consistently applied and as a result she got many of the test items for subtraction incorrect.

1 LA 2 (11/20):

Counting Tasks:

This learner was able to count at a structural but this was only up to 61. When asked to count from 76 to 84, her response was, "76, 78, 79, 61, 62" and when asked to count from 93 to 112 her response was "93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 200". This learner has not attained the required level of competency for counting that is expected of grade 2 learners. However, she showed competence with respect to number word after and numerical identification. With respect to backward counting she was able to count backward from 34 to 1, she struggled to count backward from 34 to 27. Her response was 33, 31, 30,.... . With respect to sequencing numerals, she was not able to place numerals in the correct sequence. When asked to place cards from 46 to 55 in sequence, her response was, "51, 52, 53, 54, 55, 50, 46, 47, 48, 49". Although this learner was able to use abstract strategies at times, she was not consistent in applying these counting strategies through all the items in the test.

Addition Tasks:

With respect to addition task the learner was able to resort to flexible group strategies for example when presented with $5 + 4$ the learner said $5 + 5 = 10$ therefore $5 + 4 = 9$ that is 1 less. She was able solve $9 + 6$ without any activity that is without reliance on tangible objects. With respect to items with missing addends she gave an incorrect response to $4 + \square = 6$ and was able to answer $7 - \square = 10$ and $12 + \square = 15$ correctly without any reliance on tangible objects. This learner took a while to provide these answers, so assumedly this learner was processing her answers mentally. Whilst this learner operated at higher abstract levels i.e. she was able to use flexible group strategies, she lacked consistency in applying these strategies to all of the items in the test.

Subtraction Tasks:

She was not able to provide answers for $16 - 12$ and $17 - 14$ and with respect to items with missing subtrahends she was able to correctly solve $5 - \square = 3$ and $10 - \square = 6$ without any activity. She was able to apply recall strategy to $6 - 2$ and for $9 - 4$ she used her fingers to count back from. With respect to $15 - 3$ she was not able to provide an answer in spite of using her fingers. She engaged in triple counting for $27 - 4$, she wrote 1, 2,22, 23, 24 25, 26, 27 and struck off 27, 26, 25, 24 and counted the numbers all over again the number that were not struck off. This learner used both concrete and abstract strategies to solve subtraction examples.

1 LA 3 (7/20):

Counting Tasks:

This learner could not understand the instructions which were presented in English. His teacher had to translate the instructions to Zulu for him. He was able count from 1 to 32 without any reliance on any concrete objects. He was not able to count from 48 to 61; he jumped from 48 to 70. The interviewer moved on to the next aspect without him answering all items in that category. He was able state the number word after for values less than 100 but could not state the number word after for 123, 206, 341 and 820. He could not count backwards nor could he state the number word before. He could not comprehend the instruction for number word before and as a

result he stated the number after instead the number word before. With respect to sequencing numerals he could arrange the cards from 46 to 50 correctly and 51 to 55 was incorrectly arranged. This learner did not use any tangible objects to help him through the problems. I feel that he has not acquired the necessary skills for using tangible objects and he needs a firmer grounding in the use of concrete strategies in order for him to effectively progress to the use of abstract strategies.

Addition Tasks:

This learner could not add a one digit numeral to a one digit numeral, for example for $5 + 4$ he gave answer of 6 and for $9 + 6$ he gave an answer of 11. There was no evidence of the learner making use of concrete strategies. With respect to missing addends he was able to answer $4 + [] = 6$ correctly but supplied incorrect answers to $7 + [] = 10$ and $12 + [] = 15$. With respect to missing addends he answered all items incorrectly and made no attempt to use concrete strategies. He lacked competence in the use of concrete skills to solve addition problems.

Subtraction Tasks:

He lacked competence with all the subtraction tasks and seemed very distracted at this stage of the test. With regards to missing subtrahend for example $5 - [] = 3$ he was not able to supply the correct answers even after using his fingers to count.

1 LA 4 (9/20):

Counting Tasks:

This learner was able to provide correct answers to forward number counting without any reliance on her fingers or any other concrete items. She got into a counting rhythm, clapping her hands every time she reached 10 or a multiple thereof, Sfard (2008) describes this way of counting as ritualized chanting of number-words. She was able to correctly supply answers for number word after and she looked at her fingers all the time as if her fingers gave her a hint towards the answer. With respect to numeral identification she was not able to identify 341 and 820. She struggled with number word before, where she was not able to provide the number word before for the first three items i.e. 24, 17 and 20.

Addition Tasks:

She was able to provide correct answers for $5 + 4$, $9 + 6$, $5 + 2$ and $7 + 3$ without showing any reliance on tangible objects. At this stage her hands were under the desk and she gave no indication that she was using her fingers to count under the desk. However, with $9 + 4$ she counted on from the first using her fingers. There were times when it seemed as if she resorted to guessing because she gave incorrect answers to $8 + 5$ and $9 + 3$ without using her fingers to count. With regards to items with missing addends she was able only get $7 + [] = 10$ correct without the use of any tangible objects. She was not consistent in applying addition strategies; there were times when she chose not to count on and resorted to a recall strategy with little success.

Subtraction Tasks:

She displayed a lack of competence in using subtraction strategies; with regards to $16 - 12$ she used her finger and gave an answer of 2. With regards to missing addends she again resorted to guessing since she could not use any of the strategies. However, she was able to use the counting back from strategy when finding the missing subtrahend. For example $10 - [] = 6$, she held up 10 fingers and then counted out 6 fingers and was left with 4 but gave an answer of 6. She was unsuccessful with $12 - [] = 9$ and $15 - [] = 11$, even though she resorted to the counting back strategy using her fingers.

1 LA 5 (9/20):

Counting Tasks:

This learner was able to count up to 109, she used her fingers to help her count. She was not able to count beyond 109, when reached 109 she jumped to 200. She was able to correctly provide the number word after. She was not able to identify numerals 206 and 341, she read 341 as 314. She experienced some difficulty with counting backwards especially when asked to count backwards from 34 and 72. When counting backwards from 72 her response was 72, 77, 78, With regard to number word before 17 and 20 she gave answers of 71 and 21 respectively. She showed no reliance on concrete strategies for number word after, counting

backwards and number word before. She was not able to place cards from 45 to 55 in sequence. This learner performed at operational and structural level for counting.

Addition Tasks:

With respect to addition she was able to provide correct answers but took a while to provide answers because she resorted to counting all. For example for $5 + 4$, she counted out 5 fingers and then counted out 4 fingers and then counted all her fingers. With respect to missing addends she was able to successfully use the count on from the larger number strategy. This learner operated at a concrete level for addition and at an abstract level for solving for the missing addend.

Subtraction Tasks:

When presented with $16 - 12$ she gave an answer of 12 without engaging with any tangible objects. With regard to missing subtrahend she resorted to counting back strategy for solving $5 - [] = 3$ and $10 - [] = 6$ and was able to provide correct answers for these examples but did not get the answer correct for $12 - [] = 9$. She struggled with numerals that were larger than 10. For $15 - 3$, she counted 15 on her fingers and toes (she pointed to her toes when counting), then counted out 3 and then counted out 12. This is what is referred to as a triple counting or counting out strategy. This learner resorted to using a concrete strategy (triple counting) and an abstract strategy (counting back from).

1LA 6 (6/20):

Counting Tasks:

This learner was able to count from 1 to 32 by using his fingers to help him. However, he was not able to count from 48 to 84. He looked at his fingers in vain since starting at 48 posed a problem for him. He also struggled to provide the number word after for 14, 59 and 65. He gave an answer of 41 for 14 instead of the number word after. He was not able to identify the following numbers; 47, 21, 80 and 123. With regards to counting backwards he used his fingers to count backwards. He struggled to provide the number word before for most of the items in the test.

Addition Tasks:

With regards to addition he provided answers without reliance on concrete objects. With respect to $5 + 4$ his response was 8 and with respect to $5 + 2$ his response was 6. He was not able to provide the missing addends. He would have stood a better chance of providing correct answers if he was encouraged to use his fingers or any tangible object. This learner needs a firm grounding in the use of concrete objects in order for him to progress to the use of abstract strategies.

Subtraction Tasks:

He was not able to solve $16 - 2$ and $17 - 14$ and showed no intent of using any concrete aids. The interviewer moved on to the next item whenever he struggled with any problem. Whilst, this learner used only recall strategies he was not very successful at it. He should be encouraged to use concrete strategies before moving on to abstract strategies.

1 LA 7 (9/20):

Counting Tasks:

This learner was able to count up to 84 and there were times when she looked at her fingers to ensure that she is on track with her counting. Her response was 80 for the number word after 70 and this was her only error for this aspect of counting. For numeral identification her incorrect responses were as follows; for 123 she said 131, for 206 she said 162, for 341 she said 133 and for 820 she said 128. She was able to say the number word after for numerals for up to 100. With regard to backward counting she was able to count backward from 10 to 1 and her response to counting backwards from 15 to 10 was; 15, 59, 58. She provided the number word after when she was asked to provide the number word before for 20, 14, 30 and 7. With respect to sequencing numerals she was not able to sequence numbers from 46 to 55, her response was as follows; 50, 47, 48, 52, 51, 49, 46, 55, 54, 53. She was however able to sequence numerals from 1 to 10. This learner did not make use of any tangible objects in the counting process and therefore she operated at an abstract level for counting.

Addition Tasks:

For the addition items of the test she resorted to counting on from the first but she was not always successful in providing the correct answers. Here are some of her responses; $5 + 4 = 9$, $9 + 6 = 19$, $7 + 3 = 10$ and $8 + 5 = 16$. She was not presented with the missing addends task. This learner operated at an abstract level for the addition.

Subtraction Tasks:

This learner was not able to solve $16 - 14$, she was able to provide correct answers to $3 - 1$, $9 - 4$. For $15 - 3$ her answer was 5 and she did not use any tangible objects. This learner operated at an abstract level.

8 LA 8 (13/20):

Counting Tasks:

This learner was able to count up to 32 without using any tangible objects. When he asked to count from: 48 to 61 he counted, 58, 59, 70

76 to 84 he counted, 78, 60, 61, 62

93 to 112 he counted, 99, 91, 92.

He was able to say the number word after with a few intermittent problems and these were the items that were problematic for this learner: he said 41 for the number word after 14, 28 for the number word after 29, 52 for the number word after 59 and 64 for the number word after 65. As seen here he has given the number word before instead of the number word after. With respect to numeral identification he was not able to identify the following numbers and his response is written in brackets: 21(12), 123(103), 341(no answer) and 820(180). With regard to number word before; he said 15 for the number word before 13, 23 for the number word before 21, 43 for the number word before 14 and 34 for the number word before 30. He was not able to sequence numerals from 46 to 55 and could not proceed beyond the following; 52, 53, 54, 55, 50 and he was stopped. This learner did not make use of any tangible

objects to help him count and this lack of grounding in the concrete affected his progression to the abstract.

Addition Tasks:

With regard to addition of two single digit numerals he answered two of the four items presented to him correctly. He produced the following answers after counting on from the first; $5 + 4 = 10$ and $8 + 5 = 14$. With respect to missing addends he was able to answer one out of the three items presented to him. He was able to count on from the larger number. He was not able to consistently apply abstract strategies.

Subtraction Tasks:

His response to subtraction of two digits from two digits numbers is as follows: $16 - 12 = 13$. He also struggled with items where he had to solve for the missing subtrahend; here he was able to answer one out of four items correctly. He was able to count back from 10 to produce an answer of 4 for $10 - [] = 6$. Although he was able to use an abstract strategy he was not able to consistently apply this strategy.

8 LA 9 (12/20):

Counting Tasks:

This learner was able to count up to 32 and thereafter he could count accurately at each level of forward number counting. When counting from 48 to 61 he counted 54, 55, 56, 57, 58, 59, 40, 41. He used his fingers to help him keep track of his counting and was not very successful with this strategy for numbers beyond 48. With respect to number word after he gave an answer of 61 for the number word after 59 and this was his only incorrect answer for this aspect of counting. He failed to identify numerals 123, 341 and 820 and his response to these items were 101, don't know and 80 zero respectively. With regard to backward counting he was able to count back from 15 to 1. When he was asked to count backwards from 23 his response was as follows: 23, 22, 21, 20, 10, 9, 8, 7, 6. He switched to number word after when he was asked to state the number word before. He failed at the first attempt to place the number cards in order. He performed at an operational and structural level for counting. However, he lacked in competence to operate consistently at these levels.

Addition Tasks:

With regards to adding of single digit numerals he was successful with $3 + 1$, $5 + 4$, $7 + 3$ and $9 + 3$. When asked how he got the answers he explained that he counted the first and then counted the second and then counted all. There was no physical evidence of how he counted. He was not able to solve for the missing addend. This learner operated at a concrete level for addition since he engaged in a count all strategy.

Subtraction Tasks:

For $16 - 12$ the learner counted out 16 counters and then the same 16 counters and went on up to 20 and gave an answer of 20. He used the counting out strategy but used it incorrectly. With regard to missing subtrahends he was able to produce correct answers for $5 - [] = 3$, $10 - [] = 6$ and $12 - [] = 9$. He was able to use the counting down from strategy to find the missing subtrahends. This learner operated at an abstract level for subtraction.

With respect to counting learners operated at an operational and structural level. Even though these learners used the same strategies as the high ability level learners these learners were not able count up to required level for the category as stated in the test and therefore not meeting the required level as stated by the CAPS document. Three learners operated at a concrete level for addition. These learners generally lacked the competence to apply the abstract strategies they used consistently to arrive at the correct answers. Learners resorted to using abstract strategies without a solid grounding in the concrete levels. The same holds true for subtraction, there was no solid grounding at the concrete level and this level is necessary for the progression to abstract levels.

I this stage I look at key overlaps and key contrasts between the higher ability and the lower ability learners. With respect to the counting tasks both groups performed at an operational level and at a structural level with more of the higher ability learners

performing at a structural level. None of the learners performed at a pre-conceptual level for counting which implies that none of the learners performed at a concrete level for counting.

With respect to the addition tasks none of the high ability learners used the count all strategy i.e. none of these learners operated at a concrete level for addition. With regard to the low ability learners 3 of them used the count all strategy (concrete strategy). With respect to the counting on from the first strategy 7 of the learners from the high ability were able to use this strategy and all of the learners in this group were able to use the recall strategy. However, with the low ability group only 3 of the learners were able to use the count on from the first strategy and 5 learners from this group were able to use the recall strategy. None of the high ability learners were able to use the flexible group strategies and 1 of the low ability learners used the flexible group strategies. There is therefore a clear indication that there is a lack of progression into higher levels of abstraction with respect to addition.

With regard to the subtraction tasks none of the high ability learners used counting out strategies i.e. one of these learners used concrete strategies whilst 2 learners from the low ability group used the counting out strategy. The high ability learners operated on occasions at low levels of abstraction since 7 out of 9 learners used the counting back from strategy. With respect to using recall strategies 8 out of 9 high ability learners were able to use this strategy. On the other hand 5 of the low ability learners were able to use the counting back from strategy and 4 of the low ability learners were able to use recall strategies.

Chapter 6: Concluding Remarks

In my sample, the higher attaining learner group were able to work with more abstract strategies in the context of counting, addition, subtraction and base ten strategies. However, as much as the high ability learners are operating at an abstract level these learners were largely not able to adopt flexible or derived strategies and recall strategies. The high ability learners largely resorted to abstract counting strategies i.e. counting on, counting on from first and counting on from the larger. The limited progression to higher levels of abstraction could be one of the reasons for the poor performance in mathematics at all levels of schooling. Gray (2008), states that compression of counting is vital for the progression to higher levels of abstraction.

Predictably, there was greater dependence on concrete strategies among the lower ability learners. There were lower ability learners who lacked competence in using concrete strategies and these learners resorted to guessing since they provided answers that were incorrect and there was no reliance on concrete objects. It seemed as if some of these learners lacked the experience of using concrete objects in the addition and subtraction of numerals. With respect to the counting items in the test these learners were largely able to use abstract strategies but some of these learners were not able meet the requirement of the CAPS for Grade 2 learners i.e. in many cases learners were not able to count up to 100 nor were they able to recognise numerals up to 200. This could have serious repercussions on learner competence in mathematics especially in the later years.

A point to note is that whilst the higher ability learners showed greater progression to abstract conception of early number than the lower ability learners, the higher ability learners are still not able to work consistently with abstract strategies in higher number ranges or in missing addend or subtrahends and missing start problems. Their use of more abstract strategies is therefore relatively limited. Thus, whilst literature says that more concrete strategies are retained by lower attainers at the level of 8 year olds with consequent need for focused remediation (Wright et al, 2000), my data shows that some counting based strategies are being used by several of the higher attainers in my sample. This suggests in turn, that whilst the

problems seen in South Africa are also seen in the international landscape, these problems appear to be working across a much larger proportion of learners in South Africa. Given that I selected learners identified as higher attaining by their teachers, the problems associated with a lack of comprehensive shift to more abstract number conceptions would seem to be prevalent amongst the majority, rather than within a minority, of learners.

This limited progression into more abstract levels pose a problem for learners especially when they are faced with more complicated operations in the intermediate phase. Learners need to be provided with greater opportunities to progress to abstract levels of conception. Ensor et al (2009) argue that learners' opportunities to progress to abstract levels of conception is limited by classroom practices that favour concrete modes of representations and this inhibits the access to more abstract ways of working with number and they also highlight that class time is inefficiently used.

One of the limitations of this research task is the fact that learners from two schools were used. A cross spectrum of schools should have been used i.e. schools in different settings; schools in the rural areas, townships schools, sub-urban schools etc. should have been used. The study was also limited because only 18 learners were used in the study. The use of a larger cohort of learners would have made the research more credible. Another limitation is the fact that the learners could have been intimidated by the presence of cameras. Another limitation was the language of instruction; many learners did not understand English and the interviewer or the teacher on standby had to translate. In these instances the test took longer to administer.

However, the research was able to show case the strategies used by the low ability learners and the high ability learners. There is a need to progress to more abstract strategies and literature used in this research highlight the call for development with respect to the progression from the concrete to abstract levels.

Sfard (1991) claims that the operational conception is the first step in acquiring new mathematical notions. Some of the weaker ability learners, in this research, were not competent enough to use concrete strategies. Sfard (1991) also states that the movement from the computational operations to abstract objects is a long and

difficult process but this transition from concrete strategies to abstract strategies must take place in order for learners to be successful with complicated arithmetic that will be encountered at a later stage. According to Ellemor-Collins and Wright (2007) research and curriculum reforms has placed emphasis on mental computation. They state that if mental (abstract) strategies are emphasised earlier in the life of the learner rather than later, there may be better support for number sense and conceptual understanding of multi-digit numbers. The mental strategies also support the development of important connections to related knowledge.

Critique of the interview process: Two or three interviews (from my observation) ran simultaneously in the same room and this posed a problem at times, more especially when the audio from other interviews were louder than the interview being observed. On the other hand, the volume on some of the videos were very low and it was difficult to make sense of the answers given by the learners. I had to observe what the interviewer was recording to assess whether the learner answered correctly or not. Learners being interviewed were also distracted by their peers who were engaged in their interviews in the same room. The interviewers should have read through the Assessment Interview Schedule before the interview and they should have rehearsed the procedure for test items that involved screening. I felt that the learners lost focus when the interviewers spent time silently reading through the Assessment Interview Schedule in the presence of the learner being interviewed. Learners should have been encouraged to place their hands on the desk so that the strategies used were clearly observable.

The conclusions of this study should provoke a study into classroom practices with a view of examining whether learners are given enough opportunity to progress to abstract levels of conception. Ensor et al (2009), state that the dominance of concrete methods such as tally counting in the early grades results in the failure of learners to abstract from concrete representations.

It must also be noted that the high ability learners in spite of not attaining high levels of abstraction were able to score high marks in the oral test administered to them. This begs the question as to whether educators are satisfied with learners getting by with lower levels of abstraction. The school should be diligent in bridging the gap

between known and derived facts for children, gradually weaning them away from “counting on” at the earliest opportunity. Children should be actively encouraged to see the connection between known facts and derived facts and not treat each calculation as if coming to a blank canvas.

ANEXURE: A

Observation Schedule:

Code for learner: _____ Ability level: high/low

Counting	Pre-conceptual	Operational	Structural
Topic: Early arithmetic and numerical knowledge. 1. Forward number word sequence – count from a) 1 (to 32) b) 48 (to 61) c) 76 (to 84) d) 93 (to 112) 2. Number word after – say the word after a) 14;11;19;12;23;29;20 b) 5;9;7;3;6 c) 59;65;32;70;99 3. Numeral identification a) 10;15;47;13;21;80;12;17;99;20;66 b) 8;3;5;7;9;6;2;4;1 c) 100;123;206;341;820 4. Numeral recognition – arrange cards from 1 to 10. 5. Backward number word sequences – count backwards a) 10 (down to 1) b) 15 (down to 10) c) 23 (down to 16) d) 34 (down to 27) e) 72 (down to 67) 6. Number word before – say the number word that comes before. a) 24;17;20;11;13;21;14;30 b) 7;10;4;8;3 c) 67;50;38;100;83;41;99 7. Sequencing numerals – say the number and then place the cards in sequence. a) cards from 46 to 55 b) cards from 1 to 10			

Addition	Count all	Count on		Flexible group strategies	Recall
		From 1 st	From larger		
8. Additive tasks (screened use of counters of two colours) 3 + 1 (3 green and 1 yellow concealed – how many altogether) a) 5 + 4; 9 + 6 (both concealed) b) 5 + 2; 7 + 3; 9 + 4 (both concealed) c) 5 + 2; 7 +3; 9 + 4 (unscreen) d) perceptual counting 13 ; 18 counters e) Supplementary additive tasks: 8 +5 ; 9 + 3 f) Missing addend – here are four red counters. Now look away. How many yellow counters were added if I now have six counters altogether? $4 + [] = 6$; $7 + [] = 10$ $12 + [] = 15$					

Subtraction	Counting out	Counting back		Counting up	Flexible group strategies
		From	to		
9. Subtractive tasks – a) flash a card with $16 - 12$ (no counters involved); $17 - 14$ 11. Missing subtrahend – b) i) $5 - () = 3$ ii) $10 - [] = 6$; $12 - [] = 9$ iii) $15 - [] = 11$ c) Removed items – Here are 6 counters. (briefly display then screen). If I take away 2 counters (remove 2 counters display briefly and then re-screen), how many are left under here? i) $3 - 1$ ii) $6 - 2$; $9 - 4$; $15 - 3$ iii) $27 - 4$					

Topic: Base ten and advanced arithmetic strategies.	Count all	Count on		Flexible group strategies	Recall
		From 1 st	From larger		
a) Ten and one's task – count by tens with strips i) How many dots are there? ii) How many altogether? Put down 8 strips one at a time. How many dots are there altogether? b) Incrementing by ten - i) Place one 'four dot' strip out. How many dots are there? ii) Place a ten strip on the right of the four strip. How many dots are there altogether? iii) Continue placing ten strips to the four strip. How many dots are there altogether? 24;34;44;54;64 c) Place the following strips in the following order: 10; 3; 10; 10; 4; 3; 10; 2; 10. How many dots altogether? d) Place the following strips in the following order: 4; 10; 10; 2x10; 10; 3; 10x2; 5. How many dots altogether? 3. Horizontal Sentences: Do you have a way to figure out what is? $16 + 10$ $42 + 23$ $38 + 24$ $39 + 53$ $56 - 23$ $43 - 15$ $73 - 48$					

ANNEXURE B

Assessment Interview Schedule:

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5 Backward Number Word Sequence

Example: Count backwards from 3. *Three, two, one.*
Now count backwards from ** and keep going until I say stop.

- (a) 10 (down to 1) (b) 15 (down to 10)
(c) 23 (down to 16) (d) 34 (down to 27)
(e) 72 (down to 67)

6 Number Word Before

Say the number word that comes just before ... Example: Say the number just before 2.

(a) Entry task
24 17 20 11
13 21 14 30

(b) Less advanced task
7 10 4 3

(c) More advanced task
67 50 38 100
83 41 99

7 Sequencing Numerals

Show the ten numbered cards face up in random order, asking the child to identify each number as you put it out. Then say: Can you place the cards in order? Start by putting the smallest down here.

(a) Entry task
Cards from 46 to 55

(b) More advanced task
Cards from 1 to 10

8 Additive Tasks (screened, use counters of two colours)

Introductory task:

There are three red counter under here, and one yellow counter under here. How many counters are there altogether?

$$\begin{array}{|c|} \hline 3 \\ \hline \end{array} + \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

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1 Forward Number Word Sequence

Start counting from ** and I'll tell you when to stop.

- (a) 1 (to 32) (b) 48 (to 61)
(c) 76 (to 84) (d) 93 (to 112)

2 Number Word After

Say the word that comes straight after ... Example: Say the word that comes straight after one.

(a) Entry task
14 11 19 12
23 29 20

(b) Less advanced task
5 9 7 3 6

(c) More advanced task
59 65 32 70 99

3 Numeral Identification

Show each card in turn, saying: What number is this?

(a) Entry task
10 15 47 13 21
80 12 17 99 20 66

(b) Less advanced task	(c) More advanced task
8 3 5 7 9	100 123 206
6 2 4 1	341 820

4 Numeral Recognition

Arrange the cards from 1 to 10 randomly. Which number is ...?

6 4 7 9 8

(a) Entry tasks (both collections screened)

5	+	4	=	9	+	6	=
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If one or more incorrect continue below to 8(b). If both correct go to 8(e).

(b) Less advanced task (first collection screened)

5	+	2	=	7	+	3	=	9	+	4	=
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If (b) is too difficult go to (c)

(c) Unscreened collections

5 + 2	7 + 3	9 + 4
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(d) Perceptual counting

Would you count to see how many counters there are altogether in this group?

Place out 13 counters.

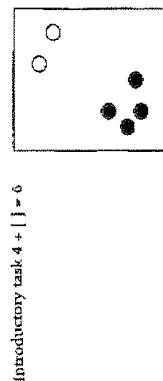
(e) Supplementary additive tasks (screened, use counters of two colours)

Supplementary tasks to (a) if further clarification is needed. The tasks are presented totally screened.

8	+	5	=	9	+	3	=
---	---	---	---	---	---	---	---

(f) Missing addend

Here are four red counters. Now look away. While you were looking away I put some more yellow counters under here. Now there are 6 counters altogether. How many yellow counters did I put under here?



Introductory task $4 + \square = 6$

Tasks $7 + \square = 10$

$12 + \square = 15$

9 Subtractive Tasks

(a) Subtraction sentences

Present the tasks as a written number sentence on card. Say to the child, What does this say? Do you have a way to work out what the answer is? [Note: using counters is not an option]

Entry task	Supplementary task
$16 - 12$	$17 - 14$

(b) Missing subtrahend

[Note: use counters of one colour only]

Here are five counters. (Ask the child to look away. Remove and screen two counters.) There were five counters. While you were looking away I took some away. Now there are only three. How many did I take away?

- (i) Introductory task $5 - \square = 3$
- (ii) Entry tasks $10 - \square = 6$
 $12 - \square = 9$
- (iii) More advanced task $15 - \square = 11$

(c) Removed items

Here are three counters. (briefly display, then screen). If I take away one, (remove one counter, display briefly, then re-screen) how many are left under here? (Indicate the first screen.)

(i) Introductory task	3 - 1
(ii) Entry tasks	6 - 2 9 - 4 15 - 3
(iii) More advanced task	27 - 4

1 Tens and Ones Tasks

(a) Counting by tens with 'strips' - informal familiarization with material	
(i) Put down a ten strip. How many dots are there? if the child says 'one', ask, How many dots are there?	
(ii) 'How many altogether?' Put down one ten strip at a time to 8 strips.	
10 20 30 40 50 60 70 80	
(iii) Pick up all the strips. How many dots do we have? How many strips are there?	

(b) Incrementing by ten	
(i) Place out the 'four dot' strip. How many dots are there?	
(ii) Place out a ten strip to the right of the four strip. How many dots are there altogether?	
(iii) Continue placing ten strips to the right of the four strip. How many dots are there altogether?	24 34 44 54 64 74
(iv) If necessary, repeat the whole task with either the 3 strip or the 7 strip.	

(c) Uncovering tasks: Board One	
Upon each uncovering ask, How many dots are there now?	

(d) Uncovering tasks: Board Two	
Upon each uncovering ask, How many are there now?	

Example of Uncovering tasks: Board One, fourth move



2 Horizontal Sentences	
(a) Do you have a way to figure out what is?	16 + 10 =
So what is 16 + 9?	
(b) Do you have a way to figure out what is?	42 + 23
If correct ask, Do you have another way to work it out or check?	
(c) Do you have a way to figure out what is?	38 + 24
If correct ask, Do you have another way to work it out or check?	
(d) Repeat the above questions for:	39 + 53
(e)	56 + 23
(f)	43 + 15
(g)	73 + 48

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