EXPLORING MATHEMATIZING PROCESSES OF SOUTH AFRICAN GRADE 2 LEARNERS FOR SOLVING ADDITIVE RELATION PROBLEMS

A thesis submitted to the Wits School of Education, Faculty of Humanities, University of the Witwatersrand in fulfilment of the requirements for the degree of Doctor of Philosophy

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Declaration

I declare that this thesis is my own unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

Thulelah B. Takane

March 2021

Dedication

To every child who has been marginalised and abused and to the Takane family.

Abstract

Small scale intervention studies have been conducted in South Africa in order to find ways of mitigating low performance in primary mathematics (Dlamini, 2014; Tshesane, 2014). The results in these studies suggest that improvements are possible in the South African context. In this study I embarked on a small-scale intervention in order to explore mathematizing processes of South African grade 2 learners for solving additive relation problems. This exploration occurred in the context of a single school and two classrooms with isiZulu speaking learners, in which one class constituted an intervention group and the other a control group for comparative purposes. A total of 13 lessons across one year were undertaken with the intervention group. Data sources within this study consisted of pre- and post-tests with both classes, and task-based follow-up pre- and post-intervention interviews with the intervention group. In the background I also had learners' written work from the intervention lessons, the researcher's journal, and lesson plans for the work.

The intervention made use of the themes and approaches proposed in Askew's (2004) *Big Book of Word Problems* classroom materials developed in a British context. The problems in these materials are drawn from the categories of word problems developed in Cognitively Guided Instruction (CGI) (Carpenter, Fennema, Franke, & Levi, 1999). The pedagogic approach adopted for the intervention drew on the guided re-invention approach, developed within Realistic Mathematics Education (RME) theory (Freudenthal, 1973), the main theory underpinning this study.

Key findings of the study reflected high gains in performance for the intervention group, which significantly outperformed the control group.

Further findings indicate that these gains were underpinned by improvement in sense making of additive situations by the intervention group which were reflected in the mathematizing processes of these learners. The intervention group also reflected willingness to use more appropriate models to calculate answers resulting in correct answers, which was not the case for the control group.

Beyond the evidence that this intervention approach can support learning gains for African learners in a Home language medium context, this study also contributes theoretically through identifying 'extent of structuring'- evident in the distinguishing of quantities in learners' work – as a factor within the higher gains made by the intervention group and similarly, progress towards use of a more formal mathematical register as also associated with higher gains.

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List of Abbreviations

ANA	Annual National Assessment
CA	Count All
CAPS	Curriculum and Assessment Policy Statement
CbC	Calculating-by-counting
CbS	Calculating-by-structure
CD	Count Down
CFMR-C	Complete form of mathematical register-Correct
CFMR-I	Complete form of mathematical register - Incorrect
CGI	Cognitively Guided Instruction
COF	Count On From
COL	Count on From Larger
COT	Count on To
CU	Change Unknown
DF	Derived Facts
DM	Direct Modelling
DQ-C	Distinguished quantities – Correct
DQ-I	Distinguished quantities – Incorrect
ENL	Empty Number Line
FP	Foundation Phase
HL	Home Language
HM	Horizontal Mathematization
JA	Joining All
JT	Joining To
LiEP	Language in Education Policy

LoLT	Language of Learning and Teaching
M	Matching
NoM-C	No Overt Model - Correct
NoM-I	No Overt Model – Incorrect
PiC	Pictorial
PPW	Part-part whole
RFI	Recalled Fact Immediate
RME	Realistic Mathematics Education
RU	Result Unknown
SA	South Africa
SANC	South African Numeracy Chair
SF	Separating From
SFMR-C	Some inclusion of Formal Mathematical Register - Correct
SFMR-I	Some inclusion of Formal Mathematical Register – Incorrect
SN-NL	Symbolic Number-Based Number Line
SS-HNS	Symbolic Syntactic-Horizontal Number Sentence
Sp	Splitting
St	Stringing
ST	Separate To
SU	Start Unknown
TE	Trial & Error
V	Varying
VM	Vertical Mathematization
WMC-P	Wits Maths Connect Primary

CHAPTER 1

INTRODUCTION

1.1 Background to the Study

This study focuses on exploring the mathematizing processes of Grade 2 South African learners when solving additive relation problems prior to, and after, an intervention that aimed to support this mathematizing. Mathematizing refers to the activity of organizing matter from reality or organizing of mathematical matter (Freudenthal, 1968 as cited in van den Heuvel-Panhuizen, 2003), with the emphasis being on mathematics as an 'activity' rather than just a body of knowledge. Attention to mathematizing processes in this study incorporates attention to models and the extent of structuring within them, to the calculation strategies used in the context of various models, and to the mathematical register that learners used in their problem-solving. The study was located within one school in the broader longitudinal Wits Maths Connect Primary (WMC-P) Project in which the focal school was involved, with the broader work focused on supporting teachers to work on building number sense.

A substantial body of research reflects poor performance in South African primary schools (Ensor et al., 2009; Fleisch, 2008; Schollar, 2008). Fleisch (2008) refers to poor learning outcomes as a 'crisis' and Schollar (2008) refers to the 'enduring persistence of poor outcomes'. When annual testing existed in South Africa, the Annual National Assessment (ANA) results illustrated patterns of performance in the school system. A mean performance of 56%

in Grade 3 in 2014 contrasted with a mean of 37% in Grade 4. In Grade 9, the national average for mathematics in the same year was 11% (DBE, 2014). What is seen in these results is that the challenges in mathematics in South Africa begin in the early years where performance needs to be bolstered if better learning outcomes in the later grades are to be achieved. Poor performance in Grade 9 concurs with what Pritchett and Beatty (2012) theorised in international developing country contexts about weaker students falling so far behind the curriculum that virtually no learning can subsequently take place. The causes of such poor outcomes can be attributed to various factors and these are introduced later in this chapter.

Two important aspects of working successfully in mathematics are highlighted in the international literature. The first involves helping learners to make sense of situations, everyday and mathematical, in order to choose sensible models and calculation approaches that support problem-solving. A second important aspect for making progress in mathematics involves moving to increasingly efficient calculation strategies. In South African middle grades contexts, a problem linked to sense-making relates to instruction happening in a language (most commonly English), that is not the primary language of learners and teachers. Learners' work on word problems has been particularly important in the research related to all three of these aspects. In the next section, I summarise issues highlighted in the international literature and in South African writing related to these aspects.

1.2 Research base on sense-making

A wide variety of international research has reflected a predominance of children who tend to answer school mathematics word problems with no sense of the situations described in these problems (eg. Verschaffel, De

Corte, & Lasure, 1994). These authors further assert that evidence that mathematics is seen as not making sense goes further than word problems, but word problems provide a particularly fertile site for investigating learner sense-making given that they are often presented in everyday language, rather than in symbolic language. Evidence of lack of sense-making when presented with word problems has been gathered in a range of ways. Some studies have intentionally presented problems that do not make sense and investigated learner responses to them. For example, Greer (1997), using the problem: 'There are 125 sheep and 5 dogs in a flock. How old is the shepherd?' – noted that many learners simply gave the answer as 130, reflecting that they were not making sense of the problem.

Other difficulties have to do with how learners tend to ignore the relevance of the context when asked to solve a problem such as:

> *'An army bus holds 36 soldiers. If 1 128 soldiers are being bussed to their training site, how many buses are needed?'*

Greer (1997) reports from various studies of learners solving this problem that of the 70% of the children who correctly carried out the division of 1128 by 36 to get a quotient of 31 and remainder of 12, only 23% of the total gave the answer as 32 buses, while 19% gave the answer as 31 buses and 29% gave the answer as 31 remainder 12.

Several writers, including Sepeng (2014) in South Africa, have argued that the difficulties with word problems stem from the manner in which word problems are approached in the classroom by teachers. Sepeng further asserts that the mechanical way in which students solve problems reflects the textbooks used by mathematics teachers for teaching and assessment. He states that teachers therefore must pay greater attention to sense-making of situations in their presentations of word problems from textbooks. Xin, Jitendra, and Deatline-Buchman (2005) refer to two commonly used ineffective instructional approaches. The first approach is the "key word" approach, in which students are encouraged to use key words that cue them as to what operation to use in solving problems. In this case, students are taught that *altogether* indicates addition, and *left* indicates subtraction. Similarly, the word *times* calls for multiplication, and *among* indicates division. Parmar, Cawley and Frazita (1996) argue that "the outcome of such training is that the student reacts to the cue word at a surface level of analysis and fails to perform a deep-structure analysis of the interrelationships among the word and the context in which it is embedded" (p. 427). The focus is usually on which operation to use rather than whether the problem makes sense. The second approach according to Parmar et al. (1996) is the four-step (read, plan, solve, and check) general heuristic procedure which may not facilitate sense making of the problem for the students.

A substantial number of international researchers has investigated questions pertaining to the role of language interpretation in making sense of word problems. Some researchers have suggested that the layer of interpretation required to make sense of language makes word problems harder when compared to 'bald' number calculation problems (Cummins, Kintsch, Reusser & Weimer, 1988; Martiniello, 2009). Others though, including authors in the Research in Mathematics Education (RME) tradition that framed the intervention and analysis adopted in this study, have argued that word problems that present situations in everyday language that are real, realistic, or imaginable by learners, can help children to see mathematical sense-making as linked to sense-making in everyday life, and through this, support mathematical problem-solving (Barnes & Venter, 2010).

Concerns particular to language issues in South Africa relate to low literacy levels (Fleisch, 2008; Spaull, 2013), and there is evidence from middle grades and above, where the language of learning and teaching (LoLT) is commonly English, of teachers and learners having to make sense of situations in a language that is not their main language (Sepeng, 2014; Setati & Barwell, 2006). Further concerns relate to limited research on the role of language in early grade mathematics which is pointed out by Essien (2018) as surprising since it is during the early grades that a substantial number of learners globally learn in a language which is not their first language.

1.3 Research base on efficient models and strategies

Research relating to indicators of sense-making focuses on learners' ability to set-up and in addition work with models of situations. These models are described in various literature (Carpenter, Moser & Bebout, 1988; Ensor et al., 2009; Gravemeijer & Stephan, 2002; Worthington, 2014) as involving verbal re-descriptions of situations, acted out replays of stories, drawn iconic images, indexical diagrams and mathematical models such as number sentences and number lines. Across this sequence of models that can indicate that a child has made sense of the situation, there is increasing abstraction. According to Carpenter et al. (1999), varying problem types determine how children model the situation in an increasingly sophisticated manner.

As number ranges increase, an important aspect of early number working is a move to more efficient models (as described in the previous paragraph) and problem-solving strategies. In additive situations (which are the focus of this study), the trajectory of strategies can be overviewed in terms of direct modelling strategies that move into counting strategies, using manipulatives that move into increasingly efficient counting and calculating (Carpenter et al., 1999). While the detail of each of these strategy groups is given in Chapter 2, the broad trajectory is from modelling with concrete objects in direct modelling strategies, to modelling situations with counted quantities using fingers or tallies in counting strategies, to making use of known results within recalled/derived facts. van den Heuvel-Panhuizen (2008) refers to calculating-by-structuring strategies as stringing, splitting and varying and these strategies will be discussed in detail in Chapter 2. Increasingly efficient strategies are important because there has to be a progression in the way children understand and solve word problem situations. The use of more sophisticated strategies allows children to solve problems more quickly with the most appropriate strategy. This is particularly important as the number range gets bigger, where it is the efficiency of the strategies used that allows children to effectively and efficiently solve word problems.

1.4 South African evidence on learners' difficulties with sense-making and efficiency

Two key areas that are important to highlight in Foundation Phase (FP) (Grades R-3) numeracy classes in SA are the evidence of lack of sense making and the evidence of inefficient calculation strategies. The latter has received more attention than the former but in research sense-making is noted as problematic among older learners (Sepeng & Sigola, 2013), and evidence of problems with coherence in instruction in the early grades that tends to disrupt the possibilities for supporting sense-making (Mathews, 2014; Venkat & Askew, 2012). The need to focus on learner participation and sense making has been highlighted by Graven (2015) as rich opportunities to repair negative relations with mathematics. In the author's study carried through the South African Numeracy Chair projects Graven (2015) reports that in one learner's response there was lack of awareness of the importance of sense making.

Ensor et al. (2009) have identified the widespread use of inefficient models and strategies in solving addition and subtraction problems as a key problem in South African FP classrooms (Grades R-3), age appropriate learners (between 6 and 8). They specifically found that learners were geared towards concrete unit counting strategies for solving problems at FP level. They linked this form of learner working with teaching that privileged concrete models and strategies which limit more abstract ways of working with number.

Other studies point to inefficient strategies continuing to feature well beyond the FP grades (e.g. Schollar, 2008), as is evident in the example in Figure 1.1 below:

Figure 1. 1 First image: unit counting in the context of multiplication; second image: grade 5 script reflecting 1420 ÷ 20 (Extracted from Schollar, 2008, p.6)

The first image reflects a Grade 5 script where unit counting is used to solve a multiplication problem. The second image reflects a Grade 7 script where skip counting in multiples of 20 is used to solve 1420 ÷ 20. These strategies reflect extended models which tend to be associated with more inefficient strategies and this has been identified as a key problem in South African primary mathematics, and among the most widely written about issues nationally (Spaull, 2013). The use of inefficient mathematizing processes to model addition and subtraction situations and failure of students to progress to calculating strategies has been presented as a significant contributing factor to poor mathematics achievement of students in South African schools (Ensor et al., 2009).

It was this background of problems seen in the South African context and suggestions for productive working in ways that promoted sense-making through everyday language and working with increasingly structured and abstract models of situations, that could support moves to more efficient calculation that provided the motivation for this study. The literature base also provided the rationales for the theoretical and methodological approaches underpinning the study-described next.

1.5 Rationale for the study

A significant body of research highlights to the need to focus on problem solving through word problems since this approach promotes modelling from reality which helps learners to make sense of mathematics and to connect the informal world to formal mathematics (see Gravemeijer, 1997; Greer, 1997; Peled & Balacheff, 2011). Gravemeijer (1994) points to word problems representing real or realistic situations as very useful for supporting the sense making and modelling processes that figure within effective and, over time, increasingly efficient problem-solving (Gravemeijer, 1994; Peled & Balacheff, 2011), and particularly so in the context of early number operations. Carpenter et al. (1999) argue that before learners receive formal instruction in addition and subtraction word problems, they are able to represent situations with physical objects and to use object counting strategies that reflect the action or relationships described in the problem. I decided therefore, to work with word problems in this study as a tool for emphasising mathematical sense-making.

The broader international literature suggests that strategies are usually associated with models that act as precursors of problem-solving strategies (Carpenter et al., 1999; Clements & Sarama, 2009; Thompson, 1999a, 1999b). Within RME, models are viewed as representations of problem situations which then function as models for calculating. This view led to the decision that encouraging the setting-up and discussion of models of situations would be an important part of an intervention lesson sequence in which the lessons aimed to promote sense-making and efficient problem-solving.

The widespread evidence of one-by-one counting in early grade South African classrooms and beyond led to a focus on addition and subtraction in Grade 2. Evidence of problems in the area of addition and subtraction has been presented in various studies carried out in South African contexts, including my own prior work (Ensor et al., 2009; Takane, 2013). While this evidence tends to emphasize the need for moving on from concrete counting strategies, the international literature on early addition and subtraction also suggests the importance of presenting children with a range of additive relation situations (Carpenter & Moser, 1984). These authors argue that this range provides a way to support children to make sense of the common underlying structure of all additive problems beyond providing access to direct modelling of particular situations (Askew, 2012; Carpenter et al., 1999; Clements & Sarama, 2009). While initially, solving word problems usually involves counting strategies which work well in small number ranges, these strategies become problematic as the number range increases. Thus, increasing the number range provides an important motivation for working with number more structurally. This means, as Venkat, Askew, Watson, and Mason (2019) have noted, working with number relationally, rather than as one-by-one counted quantities. Models that support attention to this relational structure in additive situations, like part-part-whole bar models and/or number lines, are particularly important here and are discussed in more detail in Chapter 2.

The choice to work in Grade 2 was also intentional. Given the added complexities pointed out in the literature in supporting sense-making in an additional language, I chose to focus the intervention in the South African FP (Grades R-3) as classes in township schools serving disadvantaged learners usually use home languages as the LoLT in these early grades, although in some schools this is difficult to achieve, given the range of languages spoken by learners in any one class. The intervention was located in an isiZulu classroom, where I, as a fluent isiZulu speaker, acted as teacher/researcher. Through the intervention lessons (which I taught), I set out to give learners opportunities to do, talk and record their mathematical thinking through word problem-solving activities in whole class discussions, in their acting out of word problem situations, pair work and individual activities. Pre-test, pre-interviews, post-test, post-interviews were included to determine mathematizing processes of children before, during and at the end of the intervention.

The intervention approach, focused on sense-making via mathematizing and using models of situations for increasingly efficient calculation, is one that has support in terms of both RME theory and the South African Curriculum and Assessment Policy Statement (CAPS). According to DBE (2011, p.10):

> *'Foundation Phase Mathematics forges the link between the child's pre-school life and life outside school on the one hand, and the abstract Mathematics of the later grades on the other hand. In the early foundational grades children should be exposed to mathematical experiences that give them many opportunities "to do, talk and record" their mathematical thinking'*

This advocacy once again points to RME as a useful theoretical framing for the pedagogical approach adopted in the intervention. Furthermore according to the CAPS document children in Grade 2 are expected to 'solve word problems in context and explain own solution to problems involving addition and subtraction with answers up to 99' (p.21). Solution pathways are described as involving a range of possible approaches: drawings or concrete apparatus such as counters; building up and breaking down of numbers; doubling and halving; and using number lines. While progressions towards efficient working are not made explicit across these approaches, the inclusion of more concrete and more abstract models, and of more and less structured models does suggest some expectation that children at this level are expected to progress from using concrete apparatus to using more sophisticated models and strategies to solve word problems. This progression is referred to in RME, which will be discussed in detail in the next sections, as 'progressive mathematization' which incorporates 'horizontal mathematization- (HM)' and 'vertical mathematization-(VM)'.

I go on now to introduce the research questions that I sought to answer in this study. Then I go on to to introduce the RME theoretical framing and the literature on additive relations models that were the key reference bases for my intervention and data analysis. More detail on the research design is also provided.

1.6 Research Questions

The questions that I sought to answer in this study are:

- What mathematizing processes related to additive relation problems were reflected by learners across the intervention and control groups prior to the intervention?
- What mathematizing processes related to additive relation problems were reflected by learners across the intervention and control groups post the intervention?
- How did the shifts (if any) made by the intervention group compare with the shifts (if any) made by the control group in overall performance and mathematizing processes at the end of the intervention?

1.7 Theoretical underpinnings of the study

The theory that underpins this study is Realistic Mathematics Education which is founded on the premise that mathematics is of human value and therefore should make sense and should initially be close to children's experiences. This study is focused on the mathematizing processes of learners in two South African Grade 2 classrooms in the context of working on word problems. Freudenthal's (1968) critique of traditional mathematics teaching was that teaching abstract mathematics first, and afterwards showing learners how to apply this abstract mathematics meant that learners did not need to organize or to solve problems. Freudenthal (1971, p.413-414) viewed mathematics as a human activity which he described as:

> *"…an activity of solving problems, of looking for problems, but it is also an activity of organizing a subject matter. This can be a matter from reality which has to be organized according to mathematical patterns if problems from reality have to be solved. It can also be a mathematical matter, new or old results, of your own or others, which have to be organized according to new ideas, to be better understood, in a broader context, or by an axiomatic approach".*

This organising activity which involves turning 'matter from reality' into 'mathematical matter' is termed 'mathematizing'. It was the organising itself rather than translation into a ready-made system of symbols that was central to Freudenthal's (1971) approach to teaching mathematics. Treffers, 1968 cited in Gravemeijer and Terwel (2000), later distinguished between horizontal mathematization (HM) and vertical mathematization (VM) within their RME-based approach. Freudenthal (1971) characterised HM as leading from the world of life to the world of symbols, and VM as a world where symbols are manipulated mechanically, with comprehension and reflection. Treffers (ibid.) later termed this combined process 'progressive mathematization' and argued that learners should start by mathematizing subject matter from reality and then progress to analysing their own mathematical activity. According to Barnes and Venter (2010), the traditional approach to teaching mathematics that has dominated in South African classrooms for a long time has resulted in learners' lack of use of horizontal mathematization. The research conducted by Barnes (ibid.) focused on grade 8 learners and the findings were that learners did not really know which mathematical algorithm to use and that sometimes, if by chance they chose a correct one, they could not justify their decisions. In a traditional approach, during lessons, learners are required to put the algorithms they have been taught into practice in order to solve a problem. According to RME this type of approach positions learners immediately in the more formal vertical mathematization process. This becomes concerning in that that when learners have by-passed the process of HM there is a strong possibility of forgetting the algorithms they were taught, which will result in not having a strategy at their disposal to assist them in solving the problem. This study explored RME approaches with younger children in an isiZulu classroom, with the aim of addressing the concerns with expanded models produced through HM, and the issues of lack of progression beyond inefficient strategies while using those models through VM that the literature has highlighted. This theoretical base meant that in the intervention it was important to elicit and discuss models described as particularly useful for working with additive relations. Key models and strategies for additive working are described in the next section.

1.8 Literature underpinning the study

Initial models for additive working are described as iconic/indexical (pictorial). Subsequently, key models for additive working are described as number tracks, part-part whole models, and structured/semi-structured/empty number lines. These are expected to follow after early iconic/indexical working. Models are usually associated with calculation strategies described differently across a range of literature. The progression trajectory referred incorporating Carpenter et al. (1999) and van den Heuvel-Panhuizen (2008) is direct modelling, calculating by counting, calculating by structuring and formal calculating. Calculating by structuring can make use of some recalled facts within working which van den Heuvel-Panhuizen (2008) refer to as stringing, splitting and varying. RME incorporates models and strategies in horizontal and vertical mathematization. These mathematizing processes will be unpacked in detail in Chapter 2. For the intervention in this study, initially, the empty number line was my starting point as a useful and widely advocated emergent model for additive situations, with the expectation that this model might be employed across all problem types. However, for a range of reasons described in the study, and related to evidence of a lack of sense-making, the intervention approach was adapted to pay increasing attention to the setting up of the concrete and iconic/indexical models that were used across the problem types.

I foreground these models in this introduction because previously conducted intervention studies have noted explicit instruction on models as particularly salient in improving performance on additive items. Findings from Askew, Bibby, and Brown (2001) study, which focused on year 4 in England, reflected limited gains between the pre-test and post-test where learners were not explicitly instructed to use a specific model. Most learners chose unit tally models in the pre-test and column models in the post-test. However, results of a delayed post-test where learners were explicitly instructed to use the number line reflected more substantial gains when compared to the pre-test. These findings point to the efficacy of using a number line model when solving additive relation situations – a model that is widely advocated in literature (Gravemeijer, 2009; Klein, Beishuizen, & Treffers, 1998; van den Heuvel-Panhuizen, 2008) and a model that was initially central to the conducted intervention.

1.9 Research design

This study focused on an intervention which aimed at exploring and developing the mathematizing processes children employ in solving additive relation situations located within word problems. I also aimed to explore whether an extended intervention based on a sequence of 13 lessons, taught twice a week across the year, could produce gains in overall performance and in levels of mathematizing in the Foundation Phase. The intervals in between the lessons were to allow the children to process the learning as well as to allow the reflection on the lessons for possible considerations by the teacher for the next lesson. The intervention lessons consisted of instruction based on contextual problems set in situations that were likely to be familiar to learners and involving number ranges initially dictated by accessibility to learners, and working towards the number ranges specified in the curriculum statement (addition and subtraction of whole numbers in the 1-100 range by the end of Gr2). The classroom approach involved learners attempting to solve a set of linked problems through initial whole class discussion in pairs/groups, followed by discussion in pairs/groups, and then teacher-led whole class attention to common structural features. This collective working was then followed by time for individual learner working on solving similar problems using their own mathematizing processes. More detail regarding the intervention is provided in the methodology chapter.

Askew (2008) argues that learners are natural problem solvers and therefore they can also solve mathematical problems for which they do not have formal techniques on condition that the problems are presented in a meaningful way. Askew's (2004) resources consist of word problems which are aimed at eliciting mathematizing processes. These word problems have been formulated within real world themes that learners can relate to and make sense of and they provide learners with opportunities to: “do, talk and record their mathematical thinking” as the CAPS document suggests. By using a linked set of problems in each lesson I aimed to focus learners’ attention on comparison and contrast their underlying structural similarities, and to elicit and discuss less efficient and more efficient mathematizing processes. This way of working, according to Gravemeijer and Stephan (2002), allows learners to reinvent their own mathematics which supports learners to take ownership of their own learning. Learners must be given opportunities to explain and justify their solutions to mathematical problems in order to develop general mathematical reasoning (Clements & Sarama, 2009).

The principles of RME provided guidance for both teaching and learning during the intervention. Learning was seen as happening through a process of progression through HM and VM mathematizing processes where learners begin by modelling a situation and end by solving the problem. This process foregrounds key concepts and principles such as ‘mathematics as a human activity, mathematizing, guided re-invention, didactical phenomenology and emergent models’ (Freudenthal, 1973; Gravemeijer & Stephan, 2002), discussed in detail later in the study. One of RME’s principles is emergent models (Gravemeijer, 2009; Gravemeijer & Stephan, 2002) which could be either less or more sophisticated. Since, as discussed earlier, there is evidence of a tendency for children to solve word problems using concrete and less sophisticated mathematizing processes, and there is consensus on the need to move children to more sophisticated mathematizing processes. RME suggests the number line as a more sophisticated emergent model because of its flexibility in that it allows children to model the situation based on their own cognitive understandings of number and number relationships.

1.10 Outline of chapters in the study

In **Chapter 1** I have outlined the global picture of what the present study aims to explore. In doing so, I have dealt with the background to the study, the research base for sense making, research base on efficient models and strategies, the rationale for my focus on mathematizing processes, emanating from the particularities related to primary mathematics in South Africa, and further motivations for the study drawn from the international literature. Also detailed in this chapter are the research questions and some limited information on early grades intervention studies in Home Language (HL) contexts.

In **Chapter 2** I discuss literature on the following bodies of writing: word problems, additive relations, modelling, models and progression, all of which are central to the present study. Due to the specificity of this study, I discuss each of these topics in two parts: the international literature and the South African literature. The chapter identifies knowledge gaps in evidence about moves toward vertical mathematization in South African contexts, which are addressed in this study.

In **Chapter 3** I discuss the RME theory which underpins this study. I deal specifically with the philosophical and epistemological stance taken in the theoretical underpinning of the study and the rationale for my choice of this theoretical lens.

In **Chapter 4** I outline the methodology employed in this study. I delve into the research design, context of the study, selection of participants, research instruments, data sources and procedures, data analysis, trustworthiness of the research and ethical considerations.

Chapter 5 outlines an analysis of the shifts in performance across the intervention and control groups and then follow this with an analysis of the mathematizing processes (models and strategies) related to additive relation problems that were evident in learners' responses to tasks, across the intervention and control groups both prior to and after the intervention.

Chapter 6 reports on the key contribution of this study – the development of a language of description to identify and develop important 'stages of progression' towards vertical mathematization in South African classrooms. In developing this language, I formulated two constructs - the 'extent of structuring' and 'progression towards formal mathematical register', with each construct consisting of sub-categories and codes.

In **Chapter 7** I present and discuss data from the interviews with learners which were conducted in order to establish the kinds of mathematizing processes that they used. In this chapter the focus is on calculation strategies since the mathematizing process incorporates both models and strategies**.**

In **Chapter 8** I discuss the outcomes of the intervention and what the results suggest for the research field, for policy, and to practitioners in relation to mathematizing approaches in low attainment contexts such as found in many South African classrooms.

CHAPTER 2

SITUATING THE STUDY IN EXISTING LITERATURE

2.1 Introduction

This study is located within the content specific domain of additive relations. In simple terms 'additive relations' problems refer to situations underpinned by an additive structure. Additive relations word problems involve addition or subtraction situations expressed in narrative forms. Early additive work is described in the CAPS as being fundamental to the foundational knowledge that should minimize learners' difficulties in handling additive work in higher grades (DBE, 2011a). This study thus focused on an intervention that used familiar additive word problems to support the following:

- sense-making of situations, including the incorporation of language as a resource for learning
- focusing on emergent models as a pedagogic approach
- progression, seen across a range of aspects of early additive working including working beyond rudimentary counting-based strategies

The focus on sense-making was included for a range of reasons, some based on the international literature and some based on particular features of the South African landscape. Problems located in familiar situations have been widely described as useful for promoting mathematical sense-making and employing language as a resource. The use of familiar situations and familiar language, in turn, supports the focus on sharing and discussing emergent models of these situations, which can then be used as a basis for solving a range of additive problems in increasingly efficient ways. In this chapter, I use the above headings to focus on the key bodies of literature that I drew on to understand what has been advocated for the teaching of additive relations. While the first two categories of literature were useful to me in understanding the field of additive relations teaching, and for devising the intervention content and approach, it is the section on progression, and the range of sub-aspects covered there, that was particularly useful for interpreting the differences seen in learner performance over time in this study. Thus, this section forms the largest part of this chapter.

2.2 Sense making in mathematics

2.2.1 Why is sense-making is seen as important?

Researchers' increasing interest in additive word problems suggests that these problems comprise an important part of the mathematics curriculum at primary school level. A number of authors have discussed word problem solving as affording learner opportunities to link their school knowledge with their everyday knowledge. Sense making is a core idea within the RME approach that underpins this study. This sense making can be seen in children's constructions of initial models of problem situations.

There is evidence that sense-making, sometimes described as comprehension, is an important first step in problem-modelling and problem-solving. In order for mathematics to make sense, comprehension through language is very critical in the teaching and learning of mathematics. This makes the multilingual language context in many South African classrooms important to understand, as it expands to the complexity of working with language as a resource in classrooms. In 1997 a new language in education policy (LiEP) that recognized 11 South African official languages was adopted. With this policy, the nine African languages - Sesotho, Sepedi, Setswana, Tshivenda, Xitsonga, IsiNdebele, IsiXhosa, IsiSwati and IsiZulu received official status. This policy afforded schools the option of choosing their favoured language of learning and teaching (LoLT) and also to adopt multilingual approaches such as code-switching and translanguaging. In urban contexts of substantial inward migration such as the one that this study is located in, this policy results in urban township schools constituting classes in terms of a range of different languages within the foundation phase grades. For example, there might be two IsiZulu classes, one Xitsonga and one Tshivenda class in a grade. This leads to complexities for policy makers when thinking about policy, materials and intervention roll-outs in schools, but also affords possibilities for incorporating ways of working with languages as resources for sense-making, seen in initial informal models of situations.

'Progressive mathematization' is a process through which informal models of situations function as base inventions for formalization and generalization. This process depends on initial sense making, or comprehension of the word problem. Lastly, according to CAPS (DBE, 2011a), in order to develop mathematical knowledge and skills a learner must 'build an awareness of the important role that mathematics plays in real-life situations, including the personal development of the learner' (pg. 9). This awareness is evident in the construction of representations of situations by learners in mathematical language, pictures and symbols (Haylock & Cockburn, 2008), which, in turn, provide evidence of learners' sense making of situations.

Evidence that learners do not make sense, in and of mathematical problems, in non-first language of learning settings feeds into language being seen as compounding problems of comprehension in the SA context (Sepeng, 2014), but problems with sense-making extend beyond non-first language settings. According to Verschaffel and De Corte (1997) word problems were initially used to train learners to apply the formal mathematical knowledge and skills learned at school to real-world situations. At a later stage, they were given other attributes such as being thought of as vehicles for developing students' general problem-solving capacity or for making the mathematics lessons more pleasant and motivating. In spite of this advocated practice, the international literature presents evidence that word problems still do not fulfil these functions well. This aspect is also alluded by Verschaffel and De Corte (1997) and is related to how the models used by children reflect a guessing game or riddle approach to word problems. An example is where children were asked to solve:

> *"Pete has 3 apples. Ann also has some apples. Pete and Ann have 9 apples altogether. How many apples does Ann have?" (p. 80).*

The children's answers were 'some apples, a couple, a few'. The authors note that such answers are inappropriate in the context of word problems in a school context, although they might not be totally incorrect, they are inappropriate in the context of word problems in a school context. Additionally, random guessing of answers is also confirmed by Hoadley (2012) in her work located within the SA terrain. In this study there is also evidence of children plucking numbers out of word problems and guessing at operations on the given numbers, which also suggest widespread lack of sense-making in word problem situations.

2.2.2 Pedagogic approaches for supporting sense-making

Word problems based on familiar situations are suggested by RME researchers as useful because mathematics is viewed as a human activity that must make sense and must initially be close to the children. Halai (2009), working in two additional language contexts, concurs that word problems can be used to promote sense making.

Beyond the use of word problems located in familiar situations, classroom discussion of the problem situation - according to RME - aids comprehension. In a study conducted by Verschaffel et al. (1994) 'telling' and 're-telling' were used to gather data on children's sense making strategies. 'Telling' and 'retelling' refer to social acting or performance of the given situation by the children, including attempting to tell their own similar stories and paraphrasing existing stories in a way that makes sense to them. These kinds of physical or verbal replays of situations provide indications of how children have understood situations. This kind of 'telling' and 'retelling' can also be seen in children's informal inscribed or written models of the situation. In this study children's informal models of situations came to be a particularly important focus in the face of evidence of lack of sense-making. These informal models were supported through encouragement to model situations through performance and through inscribed representations.

In the influential writing of Carpenter et al. (1999) within their extensive work on Cognitively Guided Instruction, the evidence points to children naturally engaging with informal modelling of the additive relation problems presented to them. This, however, was not always the case in the context where this study is located. Such evidence led to the need to explore the literature for avenues for supporting children to construct informal models of situations.

Verschaffel and De Corte (1997) argue that the first kind of knowledge involved in understanding and working with additive word problems is schemata of the problem situations which they refer to as structures of the basic classes of situations that can be modeled by addition and subtraction. According to Savard, Polotskaia, Freiman, and Gervais (2013), sense-making is seen in learner's understanding of the relationship between the quantities in the situation in context-based additive problems. These latter authors further argue for a 'relational paradigm' where the focus is on enabling learners to make sense of additive relation situations instead of promoting knowledge of addition and subtraction for solving additive relation word problems.

A further aspect relevant to supporting sense-making of additive situations relates to literature that points to the range of underlying situation types underpinned by an additive structure. This range of situation types is important because the idea of making sense of additive situations therefore involves being able to make sense of situations across this typology. A first important overview distinction is between what Carpenter et al., (1999) describe as 'dynamic' and 'static' situations. An example of a dynamic situation is:

> *'Thulelah has 3 sweets. Herman gives her 5 more sweets. How many sweets does Thulelah have now?'*

This situation involves the dynamic 'action' of 'giving'. By contrast, static situations do not involve any action. For example:

'Thulelah has 3 strawberry sweets. Herman has 5 chocolate sweets. How many sweets do they have altogether?'

In static situations, both given quantities are present concurrently. The relevant point for sense-making is that additive relations sense-making thus involves sense-making of different types of situations that need to be included in classroom teaching. Further sub-types, with literature suggesting different difficulty levels, and therefore progression, are detailed in section 2.4 on progression.

Instruction happening in a language that learners are familiar with in order to support sense making is advocated as important (Essien, 2018; Halai, 2004). Whilst the language policy states that children can learn in their mother tongue in the Foundation Phase (Grades 1-3), studies on the learning of mathematics in multilingual contexts argue that teachers' and researchers' focus should not be on LoLT use per se, but rather on language as a resource for the learning of mathematical concepts (Adler, 2001; Moschkovich, 1999; Setati, 1998). Given the evidence across phases in South African mathematics classrooms of instruction with gaps in mathematical coherence (Adler & Venkat, 2012), this evidence also points to the need to pay more extensive attention to the context of familiar situations and everyday (HL) for making mathematical sense of the situations. Halai (2009) explored sense making in her study and concluded that in a multilingual classroom in which learners are using an additional language as the LoLT, their sense-making of the mathematics is determined by (a) how they understand the particular usage and structure of the language, (b) how the use of everyday language shapes mathematics learning, (c) how learners express mathematical thinking in their own language(s), and finally, (d) how language is used in the textbooks as compared with how the teacher uses language.

A further pedagogical approach suggested for supporting sense making is language code-switching. Linguistic code-switching refers to instances where, in the face of evidence of learner difficulties in understanding some ideas or concepts, the teacher switches to a language in which the learner is more proficient to reiterate the main idea (Brock-Utne, 2005). Code-switching is especially useful in classrooms where learners learn in a language that is not their first language and also in classrooms where the languages of their homes have not yet been fully developed in terms of vocabulary for mathematical concepts. According to Setati (2008) code-switching assists with mediating textual meanings for learners who have limited control over the language of those texts. Code switching can also be used to qualify the key components of a concept (Halai, 2009), or to reformulate the teacher's instructions or learners' utterances (Setati, 1998, 2005).

While the literature broadly supports extensive working in home language as a resource for making sense of both unfamiliar language and mathematical language, the attention to language in this study also includes attention to moves into the mathematics register. According to Verschaffel and De Corte (1997): 'The mathematics register is a set of meanings that belong to the language of mathematics (the mathematical use of natural language) and that a language must express if it is used for mathematical purposes'(p.76). Before learners can proceed to using a mathematical register in a language, they need communicative proficiency in the 'everyday' registers of that language as a starting point for processing or interpreting the mathematics presented to them. Learners therefore have to have a language that they understand and can communicate with. Therefore, in order to learn mathematics, there has to be some fluency in a language that will be of assistance in understanding the more technical register of mathematics. According to Setati and Adler (2000), the mathematics register includes words; phrases; symbols; abbreviations; and ways of speaking, reading, writing and arguing that are specific to mathematics. These authors further distinguish between informal and formal mathematical registers and argue for the importance of being able to move from one form to the other. Informal language is the kind that learners use in their everyday life to express their mathematical understanding. Formal mathematical language refers to the standard terminology that is usually developed within formal settings like schools. Progression towards the formal register is one of the aspects dealt with in the 'progression' section.

2.2.3 Difficulties with incorporating word problems of real or realistic situations in instruction

While there is widespread support for the use of word problems depicting familiar situations to support sense-making in early mathematics, an extensive literature base points to challenges for teachers and learners with incorporating this approach. In this section, I overview key aspects of these challenges as they helped me to think about the approaches to incorporate in my intervention lesson sequence.

Traditional instruction, focused on teaching particular strategies and memorization, has been described as a widespread pedagogic approach in mathematics education (Verschaffel & De Corte, 1997), and an approach that is particularly common in South Africa (Sepeng & Sigola, 2013). However, approaches based on particular strategies and memorization do not work consistently across word problems. In South Africa, some of these memorization-orientated approaches are entrenched in the policy space, for example, the 2014 Annual National Assessment diagnostic report (DBE, 2014, p.32) includes a comment that:

'Learners should be exposed to techniques of dealing with word problems by firstly underlining the key words in the written sentence; and, learners need to be taught how to translate the key words into correct mathematical operations (i.e. addition, subtraction, multiplication and division)'

For example:

There are 10 children in a class and 20 more joined them. Half of them are boys. How many learners are boys?

(10 + 20) ÷ 2

Learners need to know that "more" means addition, hence the "20" is added to the original 10. They must also understand that half means dividing by 2, hence the total of 10 + 20 must be divided by 2.

The literature base has pointed out extensively the problems with this keyword approach, given that what are described as 'inconsistent' word problems can have the word 'more' related to subtraction rather than addition as in this example extracted from *Roberts (2016),*

> *'Mahlodi has 15 sweets. Moeketsi has 6 sweets. How many more sweets must Moeketsi get to have as many as Mahlodi?*

While drilling leads to more attention to memorizing strategies and algorithms and, whilst children may become better at calculation using algorithms, there is a danger that the need to make sense of the context of word problems is sidelined. Verschaffel and De Corte (1997) note that by the end of primary school many learners do not know how to apply apply their formal mathematical knowledge to real-world situations. These authors also argue traditional instruction is likely to result in limited development of students' general problem-solving skills, and limited building of conceptual understanding of mathematical procedures. In support of Verschaffel and De Corte's argument, some South African studies have found that the manner in which students approach problem solving reveals a lack of sense making of situations (Sepeng & Sigola, 2013).

A compounding factor in South Africa is that in spite of the language policy outlined earlier being adopted, several challenges related to language have been reported by researchers (Fleisch, 2008). One of these challenges is that large proportions of school learners complete their primary education without being able to read fluently in their school's LoLT (Spaull, 2013). Problems with learner reading and writing add to the challenges teachers face in their work with word problems.

2.2.4 Implications of the sense-making literature for this study

The implications of the sense making literature for this study pointed to the usefulness of using word problems set in familiar situations. Aspects such as contexts which the children were familiar with had to be considered in the design of the intervention.

Another advocated approach used during the intervention was the discussion of the word problems (Pimm & Keynes, 1994). At the beginning of each of the intervention lessons, the context of the word problems was discussed with the whole class and then individual word problems were also discussed in order to make sense of the actual situations. In some instances, children were asked to formulate their own word problems in order to reveal their awareness of the additive structure of word problems and the range of problems they could devise. The aim of a problem-solving approach involving extensive discussion of word problems in this study was to emphasize the importance of understanding the situation, rather than delving straight into bald calculations, with little understanding of why particular numbers and particular operations have been selected.

Evidence suggests the need for more extensive oral working in home languages (Essien, 2018) in the context of work with word problems in order to support children with making sense of problem-situations, while providing access to these situations in written format. Since word problems are contextual and require comprehension, the use of learners' home languages for the intervention thus became an important aspect to consider. The intervention was located within an isiZulu medium classroom. Word problems in the tests, interviews, and lessons used in the intervention lesson sequence were also all in isiZulu. As mentioned earlier, the aim of using isiZulu was to enable learners to engage with the word problems and with learning in the everyday language that they were more fluent in. Next, I discuss emergent models as a pedagogic approach.

2.3 Focusing on emergent models as a pedagogic approach

2.3.1 Models of situations

Following attention to initial comprehension of word problems located in familiar situations, a stage described as important in order to both see evidence of learners' comprehension of the problem, and to pull towards problem solution is 'emergent modelling'. For modelling to be possible, children must comprehend the situation. Modelling is described by various authors as a process through which a real situation manipulated by the 'problem solver' according to their interests (Nesher, Hershkovitz, &

Novotna, 2003). This leads to a model of the original situation which, on the one hand, still contains essential features of the structure of the original situation, but is, on the other hand, already schematized in that it potentially allows for a solution approach with mathematical means. In referring to the 'structure' of the original situation, I refer to the relationship between the quantities in the problem, following the description of structure presented in Venkat et al. (2019) writing.

Initially, models of situations are often context-specific in their reference to a meaningful problem situation that is experientially real for the student; it is a 'model of' that situation. Then, through working with this model, the model gradually acquires a more generic character and develops into a 'model for' mathematical reasoning. This is possible because of the development of new mathematical objects in a more abstract framework of mathematical relations to which the model starts to refer. In this study, the 'models of' a situation were viewed as the models that children created from additive word problems. While learners' written responses did not tell me whether these models were seen as 'context-specific' or as 'generic', the task-based interview data collected from a sample of learners in the intervention class did provide some insights into the extent to which learner talk as they worked on the task was more context-specific or more generic. This outlining of models of situations leads into a discussion of what RME theorists describe as emergent models.

2.3.2 Emergent models

In RME approaches, the position taken is that a model should not be formed from the intended mathematics, instead models should be grounded in the way contextual situations are understood and structured by children. It is this view that has influenced the notion of emergent models. Emergent models are informal models that function as base inventions for formal models, thereby allowing the process of 'progressive mathematization' (Gravemeijer, 1994), a process which promotes progression from informal to formal models. Emergent modelling refers to the model which emerges in the process of structuring the problem situation as part of "organizing" activity (Gravemeijer & Stephan, 2002). A model of the situation emerges through this activity. Gravemeijer and Stephan (2002) and Worthington (2014) refer to 'emergent models' as 'children's mathematical graphics' and describe a range of concrete representations that children use. These emergent models may be less sophisticated and more informal than what are referred to as 'didactical' models – models such as the bar model and double number line (see for example Klein et al., 1998). Some examples of less sophisticated emergent models as described by Carpenter et al. (1999) and Worthington (2014) are physical counter arrangements, scribbles, drawings, iconic marks, and a combination of invented and standard symbols. These emergent models often do not reflect more constricted mathematical structure, reasoning and generalization because of their concrete nature. Carpenter et al. (1999) use the phrase 'direct models' to describe the kinds of models that learners initially naturally devise. These direct models involve reconstructions of the situation using concrete manipulatives such as counters.

2.3.3 Pedagogic approaches for supporting emergent modelling

Pedagogic approaches for supporting emergent modelling have been described as involving presenting learners with well-chosen contextual problems that allow them opportunities to develop informal solution models and strategies (Barnes, 2004; Uzel & Uyangor, 2006). This approach is important as it affords learners an opportunity to use their informal descriptions to create informal or more formal mathematical forms (Barnes, 2004; Doorman, 2001). The invention process by learners, advocated by RME, is thus stimulated. According to Uzel and Uyangor (2006), "the process of progressive mathematization mentioned earlier is possible through guided reinvention where 'invention' refers to steps in the learning processes, and the meaning of 'guided' is the instructional environment of the learning process"' (Uzel & Uyangor, 2006 p.1952).

A further pedagogical approach is to promote the use of models such as the bead string where '5' and '10' are incorporated as benchmarks as advocated by RME. According to Gravemeijer and Stephan (2002) the bead string acts as a precursor for the use of the number line viewed by RME as an emergent model which the students can adjust to their thinking. They further argue that incorporating a bead string is aligned to the number line which can play a role in elaborating students' informal models and strategies. According to Klein et al. (1998) the empty number line is appropriate for linking up with informal solution procedures because its linear character can reflect a useful representation of numerical magnitude. The empty number line further provides the opportunity to raise the level of the students' activity (Gravemeijer, 1994), by allowing for more efficient working, for example by using 5 and 10 as benchmarks or other quantity relationships for calculating, rather than counting in 1s.

2.3.4 Implications of focusing on emergent models in pedagogy for this study

The implications of the emergent models' literature for this study pointed to initially encouraging learner input on setting up direct or informal models of situations as a way of observing their sense-making of situations. This setting up was done by acting out the problems using manipulatives, and then by drawing a replay of this acting out to create a record of the actions.

Quantitative relationships were then discussed through the models displayed by the children. As mentioned earlier, structural similarities between situations were built into the instructional design in order to encourage attention to 're-using' or revisiting models devised for previous problems. However, in the enactment of the intervention lessons, for reasons that are detailed in the Methodology chapter, more extensive attention was given to sense making and emergent models in the lessons than on linking between emergent models across examples.

While sense-making and emergent modelling fed into the design of the intervention, it was ideas about progression in early additive working that provided me with a range of developmental trajectories that allowed for analysis and interpretation of learning outcomes from the pre- and post-tests.

2.4 Progression in early additive working, including working beyond rudimentary counting-based strategies

Attention to progression in early number learning was particularly important to me in the intervention given the evidence from studies of South African classrooms, introduced in Chapter 1, of ongoing use of highly inefficient one-by-one counting strategies well beyond the early grades. Progression into more efficient ways of calculating has been described extensively in the literature and this progression formed one of the ways of thinking about progression in this study. However, the literature also pointed to a number of other ways of exploring progression in the context of early additive working. In this section, I introduce the key progression routes that were considered in the design of the intervention and the analysis of the data emanating from the data collection. The progression routes that featured in this study are:

- additive problem categories
- position of the unknown
- extent of formality of mode of representation
- extent of awareness of structure
- calculation strategies

2.4.1 Additive problem categories

Verschaffel and De Corte (1997) report that classification of additive word problems goes as far back as 1978 when Greeno and his associates introduced a classification schema for addition and subtraction problems which distinguished between three basic categories of problem situations: Change, Combine and Compare problems. Change situations involve action where there is a decrease and increase in quantities over time. Combine and Compare situations are static in nature with parts and wholes present simultaneously. Combine problems can involve either a missing part or a missing whole. Compare problems involve situations where comparison of quantities is involved, and often involve 'how many more/less' or 'how much more/less' phrasings. Carpenter, Hiebert, and Moser (1981) used a different terminology where 'change' became 'Joining' (increase) and 'Separate' (decrease) and 'combine' became 'part-part whole'. They introduced ‘equalizing’ as a particular kind of compare problem involving making one set or quantity equal to another and retained ‘compare’ as a separate category.

Various authors’ refinements and changes of terminology over time to the classification framework are reflected in Table 2.1 below. In this study I decided to use Carpenter et al.’s (1999) terminology because their finding that children had more difficulties with subtraction than addition, suggested that distinguishing the join and separate problems would be empirically useful. Given the evidence of limited exposure to word problems in South Africa, I decided that that it would be useful in the context of the present study to introduce join and separate problems in sequence rather than together, so their terminology was particularly useful. Further, the ‘part-part whole’ terminology used by Carpenter et al. (1999) also explicitly describes the quantities in the situations and their relation, and was therefore regarded as being useful within teaching.

Table 2.1 Additive Relation Word Problems and change of terminology over time

Example	Carpenter, 1981	Riley et al. (1983)	Verschaffel & DeCorte (1994)	Carpenter et al. (1999)	Clements & Sarama (2009)	Askew (2012)	This Study
There were 23 people on the bus. 5 more people got on. How many people are now on the bus?	Joining	Change	Change	Join	Change plus	Change Increase/	Join
There were 12 bottles on the table. Mom removed 5. How many bottles are now left?	Separating			Separate	Change Minus	Change Decrease	Separate
12 girls and 8 boys are playing outside. How many children are playing outside altogether?	Part-part Whole	Combine	Combine	Part-part Whole	Collection	Combine	Part-part Whole
Sam has 5 marbles. Lawan has 8 marbles. What could Sam do to have as many marbles as Lawan?	Equalizing	Equalise	Equalise				
Thulelah has 8 sweets. Herman has 12 sweets. How many more sweets does Herman have than Thulelah?	Comparison	Compar e	Compare	Compare	Compare	Compare	Compare

Carpenter et al. (1999), in providing detail on the additive problem types, suggest more broadly that join and separate problems, due to their dynamic nature involving change over time, tend to be easier for children to solve than the other two categories. They found that compare type problems were usually the hardest of all for young children to solve.

Riley, Greeno and Heller (1983), describe a progression hierarchy in additive problem categories slightly differently by distinguishing between action and static situations (see Table 2.2). One of the ways the problems differ is in the semantic relations used to describe the problem situation. By semantic relations I refer to conceptual knowledge about increases, decreases, combinations, and comparisons involving sets of objects. The first two categories: ‘change’ and ‘equalizing’ - describe addition and subtraction as actions that cause increases or decreases in some quantity. For example, in ‘change increase’ (1) the initial quantity or start set of 23 people in the bus is increased by the action of 5 more people getting onto the bus (the change set). The resulting quantity or result set is 28. In ‘change decrease’ (1) the initial or start set of 28 people in the bus is decreased by the action of 5 people getting off the bus (change set). ‘Equalizing’ problems involve two separate quantities, one of which is changed to be the same as the other quantity. In ‘Equalizing’ problem (1) the problem solver is asked to change the size of Thulelah’s set to be the same as the size of Sam’s set.

The remaining categories ‘combine’ and ‘compare’ involve static relations between quantities. In ‘combine’ (1) there are two distinct quantities that do not change - 12 girls and 8 boys and the problem solver is asked to consider them in combination: How many children are playing outside altogether? ‘Compare’ (1) also describes two quantities that do not change, but this time the problem solver is asked to determine the difference between them: How many more sweets does Herman have than Thulelah? Since in this case Herman's sweets are being compared to Thulelah's, Herman’s sweets are called the compared set and Thulelah’s sweets are called the referent set.

While described differently, taken together, Carpenter et al.,’s (1999) work and Riley, Greeno & Heller’s (1983) work reveal that empirical evidence indicates a hierarchy of progression for word problem situations for this study consisting of: join, separate, part-part whole, compare problem situations. ‘Equalize’ falls away in the hierarchy of progression in this study as Carpenter et al.’s (1999) framework does not include it. The progression mentioned above is intercut by another feature – the position of the unknown – that I consider below.

2.4.2 Position of the unknown

Each situation type introduced above includes several distinct types of problems depending upon which quantity is unknown. In each kind of problem: join, separate, part-part whole, equalizing, combine, and compare there are three items of information. Different problems can be formed by varying the items of information given and those to be found by the problem solver. In change problems, the three items of information are the start, change, and result sets. Any of these can be found if the other two are given, yielding three different cases: The unknown may be the start, the change, or the result. Furthermore, the direction of change can either be an increase or a decrease, so there are a total of six kinds of change problems. As noted above, in this study I refer to change problems involving increases as join problems, and change problems involving subtraction as separate problems. A similar set of variations exists for compare problems, where the direction of difference may be more or less and the unknown quantity may be the amount of difference between the referent set and the compared set, or either of the two sets themselves. Equalizing problems usually restrict the unknown to the difference between the given quantity and the desired quantity, although a total of six variations is possible. In combine (part-part whole) problems there are fewer possible variations: The unknown is either the combined set or one of the subsets.

Before 1980 research on additive word problem solving focused mostly on the effects of the number words in the problem, and the structure of the number sentence hidden in the problem (Verschaffel & De Corte, 1997). In recent years, the focus has shifted from superficial mathematical task variables towards the problem type of a situation. More recent research (Nunes, Bryant, Evans, Bell & Barros, 2011; Pape, 2003; Thevenot, 2010) reveals that some word problems are especially difficult for children because they require a rigorous analysis of their mathematical structure. Nunes et al., (2011) found that children had more difficulty with solving start unknown (? + a = b) word problems than solving result unknown (a + b = ?) word problems because in result unknown word problems the relations between quantities are operationally transparent, while in start unknown word problems an ability to work out the relations between the quantities involved in the word problem is required prior to selecting the numbers that need operating on and the operation required.

While 'Join' additive situations are usually easier for children because of their action-oriented nature, children use different strategies to solve problems based on varying the position of the unknown. This means that such problems vary significantly in difficulty, with change unknown and start unknown problems found to be harder than result unknown problems (Carpenter et al., 1999)

For the static part-part whole situations there are fewer possible variations. The unknown is either the combined set or one of the subsets. In 'Compare' situations, a set of variations exists where the direction of the difference may be more or less and the unknown quantity may be the amount of difference between the referent set and the compared set, or either of the two sets themselves.

In Table 2.2, each problem type is exemplified in term of situation and varying position of the unknown.

The distinction between whole unknown and part (including start) unknown turned out to be important in this study, with very different learner performance based on this feature.

Table 2.2 Differentiating between action and static situations (Riley, Greeno & Heller, 1983)

Action	Static
Change Increase (Join) 1. Result Unknown: There were 23 people on the bus. 5 more people got on. How many people are now on the bus? 2. Change Unknown: There were 23 people in the bus. More people got on. There are now 28 people on the bus. How many more people got on the bus? 3. Start Unknown: There were some people in the bus. 5 more people got on. There are now 28 people. How many people were in the bus to start with? Change Decrease (Separate) 1. Result Unknown: There were 23 people on the bus. 5 people got off. How many people are now on the bus? 2. Change Unknown: There were 28 people in the bus. Some people got off. There are now 23 people on the bus. How many people got off the bus? 3. Start Unknown: There were some people in the bus. 5 people got off. There are now 23 people. How many people were in the bus to start with?	Combine (Part-part whole) 1. Combine Value Unknown: 12 girls and 8 boys are playing outside. How many children are playing outside altogether? 2. Subset Unknown: 12 girls and some boys are playing outside. There are 20 children altogether. How many boys are playing outside?
Equalizing (The action in equalizing problems is in the future) 1. Thulelah has 3 marbles. Sam has 8 marbles. What could Thulelah do to have as many marbles as Sam? (How many marbles does Thulelah need to have as many as Sam?) 2. Thulelah has 8 marbles. Sam has 3 marbles. What could Thulelah do to have as many marbles as Sam?	Compare 1. Difference Unknown: Thulelah has 8 sweets. Herman has 12 sweets. How many more sweets does Herman have than Thulelah? 2. Compared Set/Quantity Unknown: Thulelah has 8 sweets. Herman has 4 more sweets than Thulelah. How many sweets does Herman have? 3. Referent Set Unknown: Thulelah has 7 sweets. She has 4 more sweets than Herman. How many sweets does Herman have?

Next, I discuss the implications for focusing on additive problems classification and position of the unknown.

2.4.3 Implications of focusing on additive problem classification and the position of the unknown in this study

The intervention lessons were initially designed with the aim of promoting progression within and across all problem classes and types. In the pre-test all problem classes and types were therefore included. Two assumptions influenced the initial decision to focus on all problem classes and types. Firstly, there was the assumption based on Carpenter at al. (1999) that children would naturally directly model additive relation situations. Secondly, following from the first assumption, it was hoped, drawing from RME, that the empty number line would be used by at least some children in the intervention class.

In addition, the lack of exposure to word problems which was evident during the pilot lesson became a strong motivation for attempting to expose the children to all problem classes and types during the intervention so that they would become more familiar with all of them, even if they did not become fluent in solving all of them. As discussed, the literature suggests progressive difficulty in solving word problems across word problem classes and types. Carpenter et al.'s (1999) framework provides a suggested path of progression across and within problem classes. For example, 'Join' problems are usually the easiest for the children to solve followed by 'Separate'. The action in these situations makes the situations easier for the children. On the other hand, 'Part-part whole' and 'compare' problems tend to be more difficult for the children because of their static nature with 'Compare' being the most difficult. However, this progression is intercut with the position of the unknown. 'Part-part whole' whole unknown situations tend to be more similar to 'Join Result Unknown' situations and both of these are seen as easier than 'Part-part whole -Part Unknown' problems if the number range remains similar. This implies that the progression process is not linear but depends on the situation to be dealt with at a given time. In this study progression was explored by offering learners word problems with some range in difficulty in order to understand progression empirically.

In the intervention lessons, join (addition) problems were thus used to introduce additive relation word problems and were later combined with Separate (subtraction) additive relation problems in order to show the inverse and in order to incorporate progression. As the intervention unfolded in the early lessons, I found that the children had extensive difficulties with the Join and Separate word problems, suggesting that they were not familiar with them in spite of the requirement from the CAPS document that children should solve word problems and explain own solutions to problems involving addition and subtraction (DBE, 2011). These difficulties resulted in the intervention limiting its focus to Join and Separate problems only, incorporating all problem types also referred to as variations within these two categories. Part-part whole 'Whole Unknown' problems are solved similarly to the 'Join' Result Unknown problem types and these were therefore included in the worksheets during the intervention in order to consolidate sense making and were also included in the post-test. The part-part whole - part unknown problem type, although not addressed during the intervention, was also included in the post-test in order to establish whether the promotion of sense making had been effective in assisting the children to solve this category of so called 'more complex' problems.

Below in table 2.3 I provide examples of additive relation situations as per each class and problem type that were addressed during the intervention and during assessments.

Table 2.3 below reflects problem types that had common items in the pre- and post-test:

Table 2.3 Problem Classes covered during the intervention and in the assessments

Problem Classes included in Pre-Test	**Problem Classes covered During Intervention**	**Problem Classes covered in the Post-Test**
Action and Static	**Action and Static**	**Action and Static**
Join (Action) • Result Unknown • Change Unknown • Start Unknown **Separate (Action)** • Result Unknown • Change Unknown • Start Unknown **Part-Part Whole (Static)** • Whole Unknown • Part Unknown	**Join** • Result Unknown • Change Unknown • Start Unknown **Separate** • Result Unknown • Change Unknown • Start Unknown	**Join (Action)** • Result Unknown • Change Unknown • Start Unknown **Separate (Action)** • Result Unknown • Change Unknown • Start Unknown **Part-part Whole (Static)** • Whole Unknown • Part Unknown

Next, I discuss the extent of formality of modes of representations.

2.4.4 Extent of formality of modes of representations

Ensor et al.'s (2009) framework, alluded to earlier, provides a trajectory of progression from concrete to abstract modes of representation. This framework incorporates the following: concrete, pictorial (indexical/iconic), symbolic number-based, symbolic syntactic. These authors' indicators for recognizing each of these modes of representations are detailed below with the counting/calculating strategies they can be associated with:

- Concrete – entails manipulation of physical objects. e.g. counters
- Iconic - Images of everyday context-realistic depictions. Used the same as concrete but manipulated differently e.g. pictures
- Indexical - Indexes everyday context – generic rather than realistic depictions of everyday context
- Symbolic Number-based - Use of numerals to represent numbers
- Symbolic Syntactic - Use of mathematical notation to produce mathematical statements. (Explicates some kind of relationship between items)

This framework provides a language useful for describing progression of models in this study. This progression can be referred to as increasingly abstract models as observed in the data collected for this study.

In table 2.4 I exemplify each of these model forms and the increasing abstraction across them that forms the basis of progression across these models, drawing from examples of models seen in learner responses in this study that were associated with the task: There were 12 bottles on the table. Mom removed 7. How many bottles are left now? I include comments on some of the different ways in which each of the modes of representation was evident in the dataset.

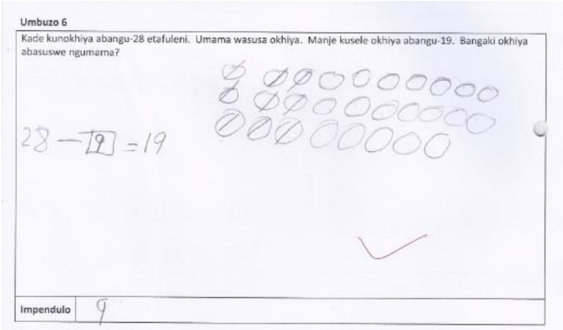
Table 2.4 Exemplification of model forms and increasing abstraction:

Mode of representation	Emergent model illustration	Comment
Concrete	There were 12 bottles on the table. Mom removed 7. How many bottles are now left? 	Associated with 'direct models' as described by Carpenter et al., (1999) and with concrete models as described by Ensor et al. (2009) and seen in learner working with counters.
Iconic (Pictorial)	'There were 12 bottles on the table. Mom removed 7. How many bottles are now left?' 	Associated with 'counting models' as described by Carpenter et al. (1999) and with 'iconic models' as described by Ensor et al. (2009) and seen in learner starting with bottle figures.
Indexical (Pictorial)	'There were 12 bottles on the table. Mom removed 7. How many bottles are now left?' 	Associated with 'counting models' as described by Carpenter et al., 1999 and with 'indexical models' as described by Ensor et al. (2009) and seen in tally marks.
Symbolic Number based (Abstract – NL)	'There were 12 bottles on the table. Mom removed 7. How many bottles are now left?' 	Associated with 'number fact models' as described by Carpenter et al., (1999) and with 'symbolic number based models' as described by Ensor et al. (2009) seen in using an empty number line.
Symbolic Syntactic (Abstract – HNS)	'There were 12 bottles on the table. Mom removed 7. How many bottles are now left?' 	Associated with 'number fact models' as described by Carpenter et al., 1999 and with 'symbolic syntactic' models as described by Ensor et al. (2009) seen in a horizontal number sentence.

The iconic model in Table 2.4 shows iconic bottle images initially, moving towards more generic indexical images mid-way. This 'mid-way' from more iconic towards increasingly indexical figures was relatively common in my dataset. Given this, I found it more useful to collect iconic and indexical images under the heading of 'pictorial' models. I therefore ended up with progression in abstraction of models marked by the following sequence: Concrete, Pictorial, Symbolic-Number based-(Abstract NL), Symbolic Syntactic (Abstract-HNS). In my analysis of data I looked for instances of progression in models across children's work, noting the prevalence of each of the above model types in responses across the pre- and post-tests. Alongside this coding, the models were also coded for whether they indicated: correct/incorrect answer, appropriate/inappropriate model quantities distinguished/not distinguished (with the latter arising as an emergent feature of difference that was important within shifts in performance).

A further point of interest in the learner written test response dataset was the presence of more than one model within a response to a particular item. This meant that there were sometimes examples of more concrete and more abstract models *within* a problem, as well as examples of progression from more concrete to more abstract models across problems, when looking across the pre- and post-tests.

Below there are examples of the progression from a concrete to an abstract model within a problem and across problems. The first illustration below is drawn from a response to the problem: 'There were 28 padlocks on the table. Mom removed some padlocks. Now there are 19 padlocks left. How many padlocks did mom remove?':



Umbuzo 6

Kade kunokhiya abangu-28 etafuleni. Umama wasusa okhiya. Manje kusele okhiya abangu-19. Bangaki okhiya abasuswe ngumama?

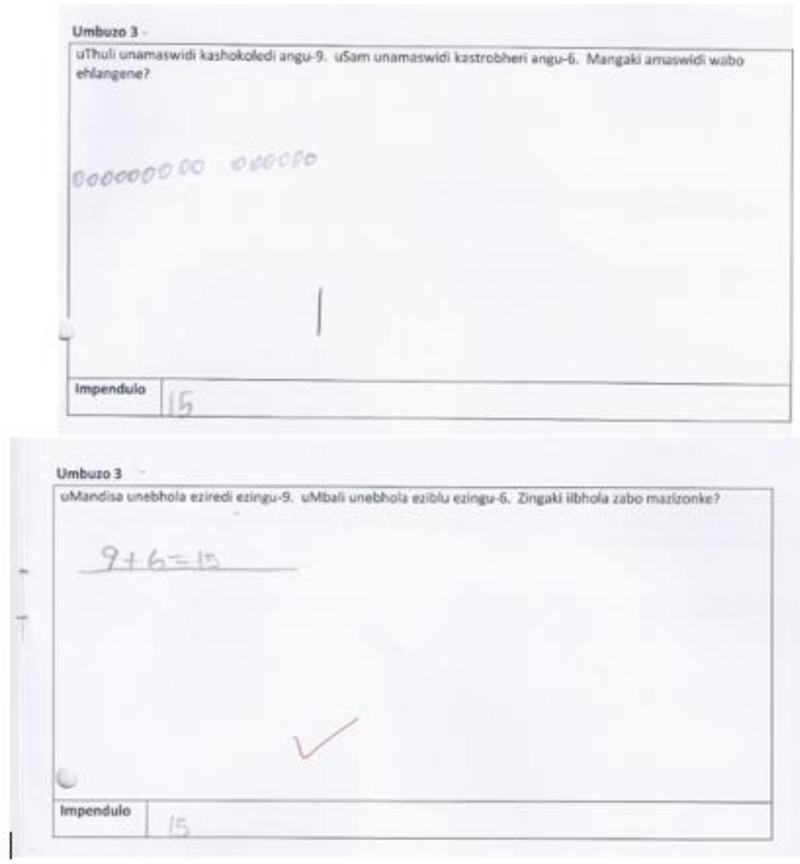
$28 - \boxed{9} = 19$

Impendulo 9

Figure 2.1 Evidence of pictorial and horizontal number sentence within a problem.

In this problem there is evidence of a pictorial model. In the same problem there is evidence of a number sentence.

Since this is written work, it is not clear which model the learner started with in order to work out the answer, but there is evidence of familiarity with both of these model types.


Umbuzo 3

uThuli unamaswidi kashokoledi angu-9. uSam unamaswidi kastrobheri angu-6. Mangaki amaswidi wabo ehlangene?

Impendulo 15

Umbuzo 3

uMandisa unebhola eziredi ezingu-9. uMbali unebhola eziblu ezingu-6. Zingaki iibhola zabo mazizonke?

9+6=15

Impendulo 15

Figure 2.2 Pictorial to horizontal number sentence models across pre and post-tests

In the problems above, the learner has used an informal pictorial model in the pre-test '*Thuli had 9 chocolate sweets. Sam had 6 strawberry sweets. How many sweets do they have altogether?'* and has used a formal number sentence in the post-test *'Mandisa has 9 red balls. Mbali has 6 blue balls. How many balls do they have altogether?*

There is wide agreement in mathematics education that moves between representations are a key indicator of conceptual understanding (Greer, 1997). Written responses and interview responses provided evidence of the extent to which learners were able to construct models and move between representations, thus providing indicators of both sense-making and conceptual understanding. Figure 2.1 also reflects a consideration of multiple models which Greer (1997) argues enhances the activity of modelling. This author does not provide details of how this is reflected empirically, but the evidence of multiple representations resonates with the empirical data in my study where children used multiple models to solve a word problem. For example, in some instances as in Figure 2.1 above, children sometimes used tally mark models to calculate their answer and then translated this model into a number sentence.

Given this evidence, the broader use of modes of representation was seen in this study as one indicator of increasing facility with more sophisticated models, especially in the post-test which was conducted after the intervention lessons.

2.4.5 Implications for focusing on the extent of abstraction of modes of representation.

In South African context scholars argue strongly for progression from the construction of more concrete models towards increasingly abstract models. Whilst RME promotes emergent modelling and is a pedagogic approach that was adopted in this study, Ensor et al. (2009) found through their study that students depended mostly on emergent models involving concrete unit counting strategies to solve additive problems at FP level. This finding has been widely reported in a number of other studies as well and well beyond FP level (Hoadley, 2012; Schollar, 2008). Increasingly formal models involve models that do not lend themselves to counting in ones can involve structured bead strings, number tracks, number sentences and empty number lines. The SA evidence presented earlier indicates that learners use mostly concrete and indexical models, the latter involving children using tally marks to solve additive word problems. This evidence called for attention to be given to increasing formality due to limited evidence of this kind of progression. For example progression towards the employment of the empty number line (ENL) begins with a bead string, moves to a structured number line, to an empty number line and finally to a number sentence. This evidence suggested that there was also a need to go beyond children's initial emergent models, to enable moves towards increasingly efficient and more formal models.

In learners' written responses seen on the pre- and post-test scripts, it was usually not possible to ascertain the order of working with multiple modes of representation. This presented difficulties for coding of 'emergent models' and subsequent translations of these initial models, which led to a decision to record all the models as noted above. However, the in-depth task-based interviews conducted with a cross-attainment sample of 12 learners provided evidence of lack of linear progression towards increasingly formal models in the pre-tests, while backing up the evidence of construction of multiple models, and of improvement towards the extent of awareness of structure in the post-test.

Models were in evidence in the empirical data, where children provided a model in a test response, or through interview responses. In instances where children used artefacts such as counters or bead strings (available in the interview setting), these models were categorised as 'concrete models'. In instances where they used written realistic images, tally marks or circles, these 'iconic' and 'indexical' models were collapsed into 'pictorial' models. In instances where they used number lines or number sentences, these were referred to as 'Symbolic number based - NL' and 'Symbolic – syntactic - NS' respectively.

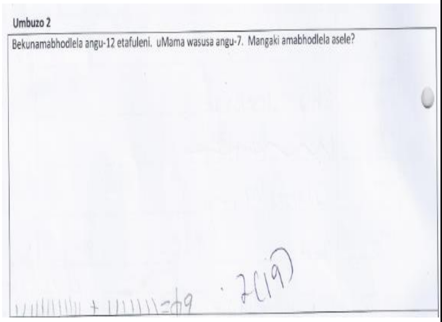
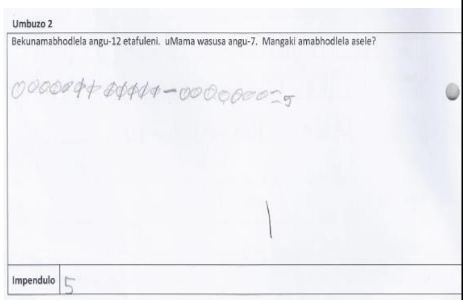
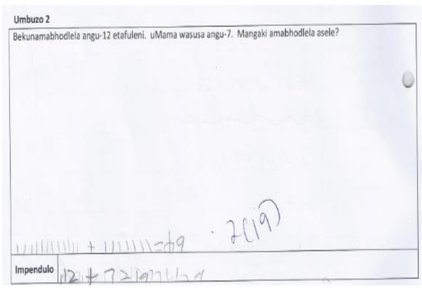
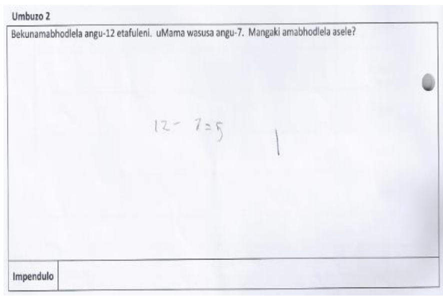
2.4.6 Extent of awareness of structure

An additional indicator that proved useful in this study was based on Mulligan and Mitchelmore's (2009) research whose results reflected that mathematics achievement in early schooling was strongly correlated with the child's development and perception of mathematical structure. Awareness of structure is related to being able to perceive and represent the ways in which given quantities in a problem are related to each other. Awareness of structure is the awareness of the way that quantities in a problem are related to each other and is seen in the construction of an appropriate model as in the work of Venkat et al.'s (2019) work. Schöner and Benz (2017) showed that perceiving quantity in terms of separate sets is more advanced than perceiving in ones. In Mulligan and Mitchelmore's formulation, the first level of awareness of structure is evident in the absence of construction of an appropriate model. The next level is the construction of an appropriate model. The third level becomes the demarcation of given quantities within an appropriate model, with the inclusion of a mathematical symbolic operation. Mulligan and Mitchelmore argued that the extent of awareness of structure in emergent models appeared to be an important marker for children who calculate more efficiently. Awareness of structure can be seen in all modes of representation (Ensor et al., 2009). These modes of representation, as previously mentioned, are concrete, iconic/indexical (pictorial), symbolic number based/symbolic syntactic (abstract) and they are important because teachers and researchers need to know how they can see awareness of structure.

2.4.7 Inclusion of some formal mathematical register

Within the empirical data that reflected children's demarcation of quantities, there was also evidence that demarcation sometimes included use of a more formal mathematical register. In this study, moves towards the use of more formal mathematical registers involved inclusion of operational notation such as '+' and/or ' – 'by the learners in their calculations. There were instances with and without this inclusion in the dataset. Readings of moves to more formal register in this study was similar with moves in Setati and Adler's (2000) work in that they also looked at moves between spoken and written responses. In this study spoken responses were in the form of discussion and acting out of contexts where children described the action taking place within a situation. In written responses learners modelled the situation by including either a plus or minus sign in order to describe the action in the situation. Further similarities include some instances where moves from informal to formal register was promoted through use of word problems. Differences point to use of formal register as a starting point in Setati and Adler's (2000) work, which was not the case in this study. Another difference is that in most cases in these authors' writing these moves were interpreted mostly by pointing to discussions that a teacher is having with learners in trying to solve a mathematical problem. In this study however interpretations of these moves were based on the learner's written work in pre and post-tests. Table 2.5 below reflects learners' distinguishing of quantities used together with the inclusion of formal mathematical register. Two categories of coding were used to distinguish responses: some form of mathematical register (SFMR) was used to denote pictorial models that include a plus or minus sign, and complete form of mathematical register (CFMR) indicated including of number sentences and/or number lines. All illustrations, once again refer to the problem: ‘There were 12 bottles on the table. Mom removed 7. How many bottles are now left?’

Table 2.5 Empirical data displaying including of some formal mathematical register

Cluster 1 (SEP-RU 12 - 7 =?)			
Including of mathematical register	**Description**	**Incorrect Answers**	**Correct Answers**
DQ-SFMR	Overt distinguishing of quantities within an appropriate model (DQ)		
DQ-CFMR-NS	Overt distinguishing of quantities within an appropriate model (DQ)		

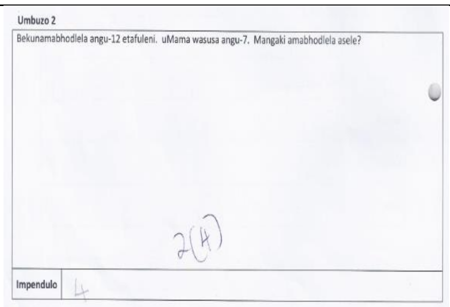
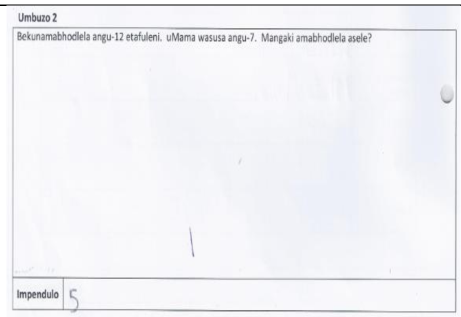
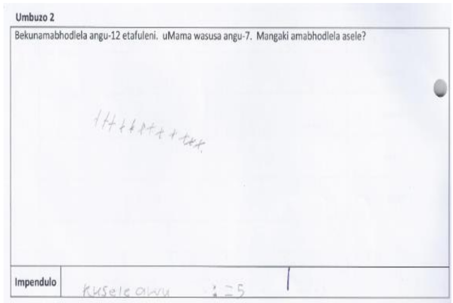
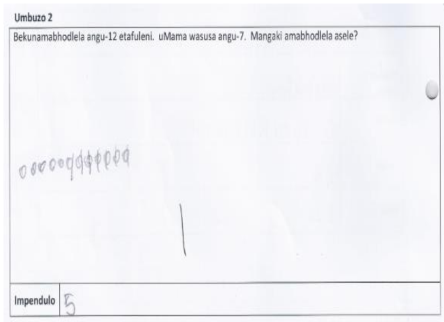
2.4.8 Implications of focusing on the extent of structure/some formal mathematical register in this study

Differences in the models that were visible in the empirical data resulted in the construct referred to as ‘the extent of awareness of structure’. In this study, these differences led to coding for ‘distinguished’ and ‘not distinguished’ quantities (English, 2006). In this study, distinguished quantities refer to evidence of explicitly demarcated quantities depicting an appropriate model of the situation in the problem, while quantities not distinguished entailed lack of evidence of quantities..

The instances of some distinguishing of quantities and some inclusion of a formal mathematical register meant that it was useful to adapt the progression of modes of representation detailed above to include interim stages related to pictorial modes. The empirical data reflected evidence of not distinguishing of quantities, distinguishing of quantities, some form of mathematical register, and complete form of mathematical register in relation to the modes of representations. This progression was then adapted to include 'Not distinguished quantities and Distinguished quantities-NDQ/DQ'; 'Some form of mathematical register – SFMR' and 'Complete form of mathematical register – CFMR'. Details of the adapted progression follow the table.

As presented in Table 2.5 below, learners' distinguishing of quantities was used together with the extent of their progression towards using a more abstract mathematical register, thus incorporating Ensor et al's (2009) modes of representation and informal and formal register.

Table 2. 6 Empirical data displaying extent of awareness of structure

Cluster 1 (SEP-RU 12 - 7 =?)			
Extent of Awareness of Structure	**Description**	**Incorrect Answers**	**Correct Answers**
NoM	Response does not include any evidence of a model	 Umbuzo 2 Bekunamabhodlela angu-12 etafuleni. uMama wasusa angu-7. Mangaki amabhodlela asele? 2(4) Impendulo 4	 Umbuzo 2 Bekunamabhodlela angu-12 etafuleni. uMama wasusa angu-7. Mangaki amabhodlela asele? Impendulo 5
NDQ/PiC	No overt distinguishing of quantities within an appropriate model (NDQ)		 Umbuzo 2 Bekunamabhodlela angu-12 etafuleni. uMama wasusa angu-7. Mangaki amabhodlela asele? Impendulo
DQ/PiC	Overt distinguishing of quantities within an appropriate model (DQ)		 Umbuzo 2 Bekunamabhodlela angu-12 etafuleni. uMama wasusa angu-7. Mangaki amabhodlela asele? Impendulo 5

The explanation of the table above and the different levels is as follows:

- An inappropriate model, or no model with an incorrect answer, suggests lack of sense making in HM of the problem (NoM-I)

- Moves to an appropriate model represent the beginnings of sense making of the situation. At the first level, an appropriate model would not distinguish quantities (so NDQ-I/C are both better than NoM-I).
- Models where the quantities are distinguished appropriately point to explicit marking of the relationship between the quantities, even if the subsequent work with the model suggests that the VM element has not played through. (So DQ-I/C are both better than NDQ-I). DQ-C is interpreted, following Mulligan and Mitchelmore's (2009) evidence on explicit structure awareness, as better in terms of explicit marking of the relationships between quantities than NDQ-C, acknowledging that it could be the case that the relationship between quantities may have been internalized but not explicitly marked in NDQ-C cases).
- In responses with No overt model with a correct answer, the assumption being made is that a model that distinguishes quantities with awareness of the relationship between them has been internalized (so NoM-C is better than both DQ-I/C). The demarcation of these quantities ranged from No Overt Model Incorrect (NoM-I) where children did not take up any model for solving problems and the answer was incorrect, to No distinguished quantities-Incorrect/Correct (NDQ-I/C) where children used a model but without distinguishing quantities to Distinguishing Quantities-Incorrect/Correct (DQ-I/C) where children used models where quantities were demarcated. No Overt Model Correct (NoM-C) indicates children did not taking up a model but giving a correct answer.

2.4.9 Calculation Strategies

In the early stages of learning, children tend to use informal solution strategies to directly model problems (Verschaffel & De Corte, 1997). According to Carpenter et al. (1999) direct modelling usually requires concrete representations of the problem.

A wide range of literature presents evidence on additive relation strategies (Anghileri, 2006, 2007; Carpenter et al., 1999). According to Carpenter et al., (1999) children should be able to progress from the use of concrete strategies to more abstract strategies. Concrete strategies refer to 'direct modeling' and 'counting strategies' whilst abstract strategies refer to 'number fact strategies' and these are outlined in detail in the sections that follow. In South African classrooms, as mentioned earlier, many children remain at the level of concrete use of strategies (Ensor et al., 2009).

Other strategies are outlined in table 2.8 below and adapted from Carpenter et al.,'s (1999) framework of progression of strategies in relation to problem classes and problem types. For the *Join* class and problem types (*Result Unknown*, *Change Unknown*, and *Start Unknown*) the progression in use of strategies is across *Direct Modeling Strategies*, *Counting Strategies* and *Number Fact Strategies.* Number fact strategies also incorporate calculating by structuring strategies which have been termed 'stringing', 'splitting' and 'varying' (van den Heuvel-Panhuizen, 2008). Carpenter et al., (1999) also describe progression within *Direct Modeling Strategies* – detailed in table 2, with 'joining all' being the least sophisticated strategy and 'matching' the most sophisticated. Within *Counting Strategies*, Counting On from First at the top of the table is the least sophisticated and Counting Down To the most sophisticated, halfway through the table. Strategies reflected as 'less likely to be used by children' in Table 2.8 were reported by Carpenter et al., (1999) from their observations of children. They found that the complexity of part-part whole and compare problems resulted in children not attempting to solve them at all.

Strategies reflected through Carpenter et al.'s (1999) and van den Heuvel-Panhuizen's (2008) work are grouped into the following categories:

- **Direct Modeling Strategies (DMS)** - *Joining All (JA), Joining To (JT), Separating From (SF), Separating To (ST), Matching (M) and Trial and Error (TE)*
- **Counting Strategies (CS)-** *Counting On From First (COF), Counting On From Larger (COL), Counting On To (COT), Counting Down (CD)* and *Counting Down To (CDT)*
- **Calculating-by-structuring strategies** – *Stringing, Splitting, Varying*
- **Number Fact Strategies (NFS) –** *Recalled Facts (RF) and Derived Facts (DF)*

As mentioned above, there is progression within and across the strategies: *Direct Modeling* strategies are less sophisticated because children tend to directly model or represent the action or relationship in the situation by using physical objects; *Counting Strategies* are more sophisticated than *direct modeling* strategies because in the latter case, physical objects are needed to keep track of the counting sequence rather than working with a representation of the situation. *Calculating-by-structuring strategies* include evidence of some moves beyond one-by-one counting through the use of number relations, often based on the base ten structure, but may involve some one-by-one counting as well. *Number Facts Strategies* involve mental reasoning and are therefore more sophisticated than counting strategies and structuring strategies. In these cases, learners show evidence that they have internalized the outcomes of working with structured representations, and can state outcomes as either recalled or derived facts without the need for number line or other diagrammatic representations for support. Below is a table which reflects strategies in relations to problem classes and types. Following the table is a description of these strategies.

Table 2.7 Strategies in relation to additive classes and problem types (Adapted from Carpenter et al., 1999 and van den Heuvel-Panhuizen, 2008)

Additive Word Problems			Strategies			
Classes of problems	**Problem Types**	**Examples**	**Direct Modelling Strategies**	**Counting Strategies**	**Calculating by structuring**	**Number Fact Strategies**
Join	**Result Unknown**	Thulelah had 2 toy cars. Her parents gave her 5 more cars for her birthday. How many toy cars did she have then?	Joining All	Counting On from First/Larger	**Stringing, splitting, and varying**	**Recalled Facts and Derived Facts**
	Change Unknown	Thulelah had 5 toy cars. Her parents gave her some more cars for her birthday. Then she had 7 toy cars. How many toy cars did Thulelah's parents give her for her birthday?	Joining To	Counting on To		
	Start Unknown	Thulelah had some toy cars. Her parents gave her 2 more cars for her birthday. Then she had 7 toy cars. How many toy cars did Thulelah have before her birthday?	Trial and Error	Less likely to be used by children		
Separate	**Result Unknown**	Herman had 8 sweets. He gave 3 to Thulelah. How many sweets did Herman have left?	Separating From	Counting Down		
	Change Unknown	Herman had 8 sweets. He gave some to Thulelah. Then he had 5 sweets left. How many sweets did Herman give to Thulelah?	Separating To	Counting Down To		
	Start Unknown	Herman had some sweets. He gave 3 to Thulelah. Then he had 5 sweets left. How many sweets did Herman have to start with?	Trial and Error	Less likely to be used by children		
Part-part Whole	**Whole Unknown**	6 boys and 4 girls were playing soccer. How many children were playing soccer?	No commonly used strategy corresponding to the action or relationship described in the problem.	No commonly used strategy corresponding to the action or relationship described in the problem.		
	Part Unknown	10 children were playing soccer. 6 were boys. How many girls were playing soccer?	No commonly used strategy.	No commonly used strategy.		

2.4.8.1 Direct Modelling strategies

a) Joining All

Additive relation situation: *Thuli had 2 toy cars. Her parents gave her 5 more cars for her birthday. How many toy cars did she have then?*

A set of 2 objects and a set of 5 objects are constructed. The sets are joined and the union of the two sets is counted in ones.

b) Counting All:

Thuli had 2 toy cars. Her parents gave her 5 more cars for her birthday. How many toy cars did she have then?

A set of two objects and a set of 5 objects are drawn. Both sets are counted in ones.

c) Joining To

Additive relation situation: *Thuli had 5 toy cars. Her parents give her some more toy cars for her birthday. Then she had 7 toy cars. How many toy cars did Thuli's parents give her for her birthday?*

A set of 5 objects is constructed. Objects are added to this set until there is a total of 7 objects. The answer is found by counting the number of objects added.

d) Separating From

Additive relation situation: *Thuli had 8 puppies. She gave 3 to Thabo. How many puppies does Thuli have left?*

A set of 8 objects is constructed. Three objects are removed. The answer is the number of remaining objects which are counted. This strategy is also referred to as 'take away' (Beckmann, 2005)

e) Separating To

Additive relation situation: *Thuli had 8 puppies. She gave some puppies to Thabo. Then she had 5 puppies left. How many puppies did Thuli give to Thabo?*

A set of 8 objects is counted out. Objects are removed from it until the number of objects remaining is equal to 5 (strategy). The answer is the number of objects removed which are counted. This strategy is also referred to as 'difference' (Beckmann, 2005)

f) Matching

Additive relation situation: *Mark has 3 mice. Joy has 7 mice. Joy has how many more mice than Mark?*

A set of 3 objects and a set of 7 objects are matched 1-to-1 until one set is used up. The answer is the number of unmatched objects remaining in the larger set which are counted (strategy). Anghileri (2006) argues that a skill required by this strategy is for a child to be able to check that both objects have a 'starting point' from which the quantities can be viewed. She cautions that if children use objects with different sizes in the different rows, they might be misled into thinking that the row with shorter or smaller objects has a smaller quantity.

g) Trial and Error

Additive relation situation: *Colleen had some guppies. She gave 3 guppies to Roger. Then she had 5 guppies left. How many guppies did Colleen have to start with?*

A set of objects is constructed. A set of 3 objects is added to the set, and the resulting set is counted. If the final count is 5, then the number of objects in the initial set is the answer. If it is not 5, a different initial set is tried.

2.4.8.2 Counting Strategies

Counting strategies are aligned to both concrete and pictorial models. According to Carpenter et al.,'s (1999) hierarchy, counting strategies are more sophisticated than direct modelling strategies. When using a counting strategy, children fundamentally recognize that they do not have to actually construct and count sets, rather they can find the answer by focusing on the counting sequence itself. According to Carpenter et al., 1999 'Counting strategies generally involve some sort of simultaneous double counting, and the physical objects a child may use (fingers, counters, tally marks) function as a model of the situation, that are then used as a 'model for' keeping track of counts rather than to represent objects in the problem' (p. 23). Carpenter et al. (1999) have identified counting models as:

a) **Counting On From First**

Additive relation situation: *Thuli had 2 toy cars. Her parents gave her 5 more cars for her birthday. How many toy cars did she have then?*

The counting begins with 2 and continues on for 5 more counts. The answer is the last number in the counting sequence. For example, a child may count, '2'(pause), 3,4,5,6,7'. In this counting a child may use their fingers or counters with each count, when he/she has extended 5 fingers or counters he / she stops counting and gives the answer. The model in the case of 'counting strategies' becomes slightly more sophisticated than the model in 'direct modelling' above since there is no construction of the first number as the strategy begins with counting on.

b) **Counting On From Larger**

Additive relation situation: *Thuli had 2 toy cars. Her parents gave her 5 more cars for her birthday. How many toy cars did she have then?*

The counting sequence begins with 5 and continues on 2 more counts. The answer is the last number in the counting sequence. The model here could be that children keep 5 in their head and use fingers or counters to count on. This model becomes more sophisticated than the 'counting on model' since children require an awareness of commutativity for them to know that they can start counting from the larger number.

c) **Counting On To**

Additive relation situation: *Thuli had 5 toy cars. Her parents gave her some more cars for her birthday. Now Thuli has 7 toy cars. How many toy cars did her parents give her?*

A forward counting sequence starts from 5 and continues until 7 is reached. The answer is the number of counting words in the sequence. The model allows for a more sophisticated strategy than the 'counting on from larger strategy' because a child needs to keep track of the number of the counting words in the sequence and not confuse them with the verbal number 7 which they are counting to.

d) **Counting Down (Take Away)**

Additive relation situation: Thuli had 8 puppies. She gave 3 to Roger. How many puppies does Thuli have left?

A backward counting sequence is initiated from 8. The sequence continues for 3 more counts. The last number in the counting sequence is the answer. According to Carpenter et al., (1999) children generally find it difficult to count backwards, so this model allows for more sophisticated strategies.

e) **Counting Down To (Difference)**

***Additive relation situation**: Thuli had 8 puppies. She gave some puppies to Roger. Then she had 5 puppies left. How many puppies did Thuli give to Roger?*

A backward counting sequence starts from 8 and continues until 5 is reached. Since the answer is the number of words in the counting sequence, this model becomes more sophisticated than the 'count down' model above. Furthermore, similarly to the *count on to strategy,* children need to keep track of the number of words in the sequence.

2.4.8.3 Calculating by structuring

van den Heuvel-Panhuizen (2008) refers to calculating-by-structuring strategies as stringing, splitting and varying. With the *stringing strategy* in addition and subtraction of two-digit numbers, the first number is kept in-tact and the second number is split into tens and ones. This strategy can be associated with the NT strategy discussed earlier. With regards to the second strategy, *splitting*, the numbers are split into tens and ones and processed separately when the operations are carried out. This strategy can be associated with the TT strategy also discussed earlier. The third and last strategy is *varying* which is related to the children's knowledge of number relationships and properties of operations involving what van den Heuvel-Panhuizen (2008) refers to as 'smart calculation strategies' such as 5 + 5 for solving 5 + 6.

2.4.8.4 Number Facts

Carpenter et al., (1999) suggest that children learn number facts both in and out of school and apply the knowledge in problem solving. They maintain that children often use a small set of memorized facts to derive solutions for problems involving other number combinations. Children use *Derived Facts* solutions which are based on understanding relations between numbers. For example, if a child is asked to solve 5 + 7 =? - they may say immediately that the answer is 12 from knowing that 5 + 5 is 10 and adding 2 to arrive at 12. In this case the child has used a derived fact. Carpenter et al., (1999) suggest that counting strategies and derived facts are relatively efficient strategies for solving additive problems.

2.5 Concluding remarks

In this chapter I have looked at sense making, emergent models and progression across a range of aspects of early additive working. There is a consensus both nationally and internationally of the need to promote sense making and the use of learners' languages as a resource where word problems are concerned. Whilst the focus on emergent models was useful for understanding the field of additive relations teaching and for devising the intervention content and approach, the section on progression was particularly useful for interpreting the differences observed in this study of learner performance over time. The extent of formality and extent of structure were particularly interesting nuances in the study data especially for the SA context.

CHAPTER 3

THEORETICAL FRAMEWORK

3.1 Introduction

I ended the previous chapter with a summary and discussion of a range of aspects that constituted progression for this study. This range of aspects allowed for multiple ways of understanding the differences seen in learner working across the pre-test, post-test and interviews administered in the course of the intervention focused on additive relations word problems, and for making claims that these differences constituted improvement. The aspects of progression listed emerged from two primary sources: CGI and RME. CGI approaches indicate that incorporating a range of additive problem types is important, and that compare situations, in particular, are seen by children as more difficult, with some evidence also, that action-oriented change situations are the easiest for children. CGI approaches also indicate moves towards increasingly abstract calculating strategies over time, starting from what have been described as 'direct models' where situations are replayed in actions or by using concrete manipulatives which lead directly to answers.

I began this study with the intention of focusing on all the different additive relations problem types in the intervention lesson sequence, and of supporting moves towards increasingly efficient calculating strategies that moved beyond unit counting. However, learner difficulties with sense-making of situations led to more focus in the intervention lessons on 'join' and 'separate' word problems than I had anticipated. This led me to view Realistic Mathematics Education as the central theory that frames my study. In this chapter I justify the use of this theory in addition to CGI and give a brief history of development of the theory. Then I explain key concepts and principles of the theory and use some of the data from the study to support the explanation. I found that the work of *Realistic Mathematics Education* (RME) researchers provided a useful framework for thinking further about concepts and principles important for teaching and learning during the intervention that is central to this study.

Realistic Mathematics Education (RME) was first developed at the Freudenthal Institute in the Netherlands. This theory was birthed because Freudenthal critiqued traditional instruction and proposed an alternative incorporating a philosophy, or theory, of mathematics education, its elaboration in developmental research, and an understanding of the overarching process of educational development in which this is embedded (Gravemeijer & Terwel, 2000). Freudenthal (1973) argued that curriculum development was not to be conducted using a ready-made recipe from academics, but rather should be based in schools, in collaboration with teachers and students. Freudenthal opposed general education theories arguing that they do not fit the situation of mathematics education. He proposed that the activity of 'reality-structuring' should be considered and further argued that it is via structuring that students get a grip on reality. What this structuring involves is detailed below. The key concepts of RME detailed by Freudenthal (1971; 1973) are *mathematics as a human activity*, as *mathematizing* and as guided *re-invention* (Gravemeijer, 1994). Each of these incorporates guiding principles which are '*guided re-invention through progressive mathematization, didactical phenomenology, and self-developed or emergent models'.* The mind-map below provides an overview of the concepts and principles and is followed by details of each.

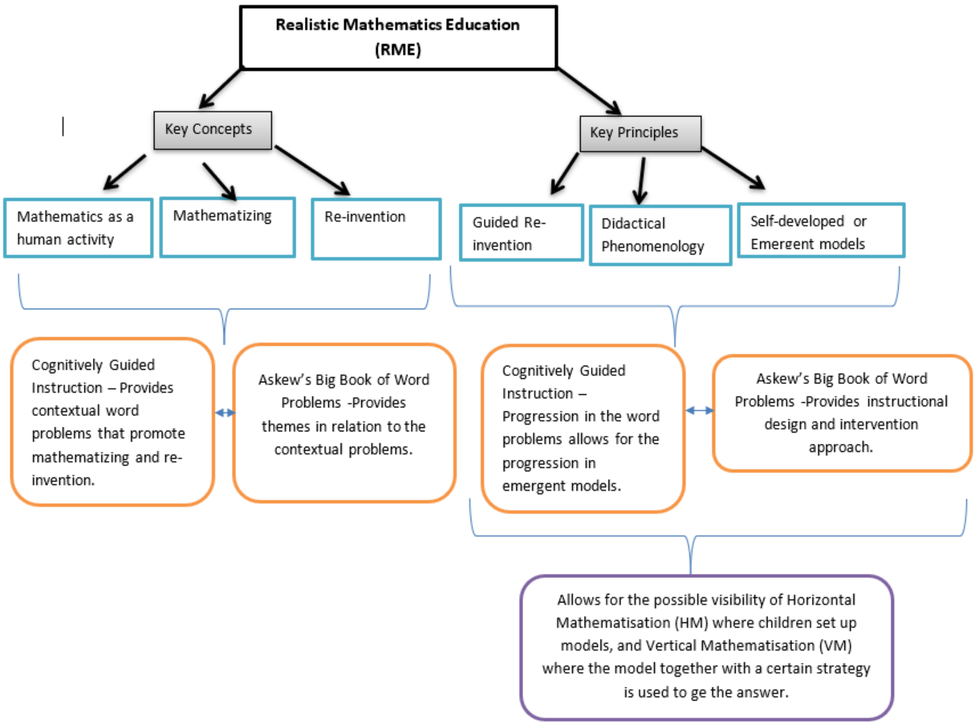


Figure 3. 1 Mind-map of RME Concepts and Principles adapted from Freudenthal (1971; 1973*)*

3.2 RME Key Concepts

3.2.1 Mathematics as a human activity

> *"[Mathematics as a human activity] is an activity of solving problems, oflooking for problems, but it is also an activity of organizing a subject matter. This can be a matter from reality which has to be organized according to mathematical patterns if problems from reality have to be solved. It can also be a mathematical matter, new or old results, of your own or others, which have to be organized according to new ideas, to be better understood, in a broader context, or by an axiomatic approach." (Freudenthal, 1971, p.413-414)*

The essence of the quotation above is Freudenthal's idea of organizing matter from reality into mathematical matter. He termed this organizing 'mathematizing' which will be discussed later in this chapter. Freudenthal (1983) believed that mathematical structures emerge from organizing reality and expand continuously in individual and collective learning processes. He argued that in order for mathematics to be of human value it should be connected to reality, stay close to children and be relevant to society (Freudenthal, 1997, cited in van den Heuvel-Panhuizen, 2003). Following from these ideas, the use of realistic contexts has become one of the determining characteristics of the RME approach to mathematics education. Askew (2004), in his 'Big Book of word problems' text and teaching approach that I used in this study, centres tasks on word problems contextualised in situations that are close to children in a real or imaginable sense. This idea of imagination implies that students can be presented with problems that they will be able to 'imagine'. For example, a context problem such as: *Thulelah has 5 dresses, her mother buys her 3 more dresses. How many dresses does Thulelah have now?*' – while not necessarily authentic, can be imagined by children with its use of objects (dresses) and numbers that children are familiar with. During the intervention in this study children were presented with problems such as '*There were 3 people on the bus. 8 more people got on. How many people are now on the bus?'* The children were expected to 'picture' the situation through approaches such as acting it out or imagining it in their minds. These kinds of problems were close to the children in that they were all familiar with the situation of a bus and people getting in and out of it.

3.2.2 Mathematics as mathematizing

Mathematizing, according to Freudenthal (1971), is an organizing activity that involves both 'matter from reality' and 'mathematical matter'. With the aim of elaborating the concept of mathematizing, Treffers (1987) distinguished between horizontal mathematization (HM) and vertical mathematization (VM). HM is concerned with converting a contextual problem into a mathematical problem and VM with taking mathematical matter onto a higher plane. (Freudenthal, 1991, p.41-42) characterized this distinction as follows:

> “Horizontal mathematization leads from the world of life to the world of symbols. In the world of life one lives, acts (and suffers); in the other one symbols are shaped, reshaped, and manipulated, mechanically, comprehendingly, reflectingly: this is vertical mathematization. The world of life is what is experienced as reality (in the sense I used the word before), as is a symbol world with regard to abstraction. To be sure the frontiers of these worlds are vaguely marked. The worlds can expand and shrink - also at one another’s expense.”

It is evident from the quote above that the boundary between HM and VM is not clear-cut which led to a conclusion that the differentiation lies with the actor. For example, the starting point of one learner may be that they can model a situation using a number sentence while another learner models the same situation with a ‘replay’ involving concrete objects. The number sentence in the first case, and the concrete object arrangement in the second case can both be described as HM, while the strategy children use to work with their model to calculate the answer can be described as VM.

Freudenthal (1971) viewed mathematizing as a process of discovery through organising matter that would promote re-invention. According to Gravemeijer and Terwel (2000) mathematizing means ‘making more mathematical’ which may incorporate characteristics such as generality, certainty, exactness, brevity.

Various researchers have expanded on the concepts of HM and VM. For example, Deguara and Nutbrown (2017) and Uzel and Uyangor (2006) adhere to the same definitions and provide examples of the indicators of HM as: identifying or describing the specific mathematics in a general context, schematizing, formulating and visualizing a problem in different ways, discovering relations, discovering regularities, recognizing isomorphic aspects in different problems, and transferring a real world problem to a mathematical problem. According to these authors, representing a relation in a formula, proving regularities, refining and adjusting models, using different models, combining and integrating models, formulating a mathematical model, and generalizing can all be involved. Barnes (2005) summarises the definitions of HM and VM by describing HM as 'when learners use their informal strategies to describe and solve contextual problems' and VM as 'when the learners' informal strategies lead them to solve the problem using mathematical language or to find a suitable algorithm' (p.50).

Although at definition level there are overlaps between Barnes' summary and the earlier writing, there are also some differing emphases. For example, while Freudenthal, 1971; Treffers, 1987 and Uzel & Uyangor (2006) consider HM to involve organising and solving a problem situated in daily life, for them this organising can also involve formal models. For Barnes on the other hand, HM involves describing and solving a contextual problem through use of informal strategies. This overlaps with the 'direct modelling' strategies described by Carpenter et al. (1999) where in HM physical action or concrete replays of the situation lead directly to the answer. In this kind of HM, there is a possibility that there may be no separate VM step in some learner solutions. In this study, children's written responses in the pre- and post-tests often showed HM of some sort, but this dataset did not clearly show how children worked with the model. Fuller VM detail was available in the interview data. In interactional situations like task-based interviews, it is often possible to probe for learners' approaches to both HM and VM. In written responses, while HM can often be seen on paper, there can sometimes be gaps in terms of evidence of VM, with an answer providing little information on how a HM model was used (or, in some cases, not used), to solve the problem. In this study, I gathered both written response data from written test data and task-based interview data from a sample of learners in the intervention group in order to verify which aspects of the responses could be categorized as HM or VM. Figure 3.2 below reflects an example of how HM and VM were interpreted in written response data and I then explain additional detail gathered from the intervention response. The English version of the problem in the example below is *'Mandisa has 9 red balls. Mbali has 6 blue balls. How many balls do they have altogether?'*

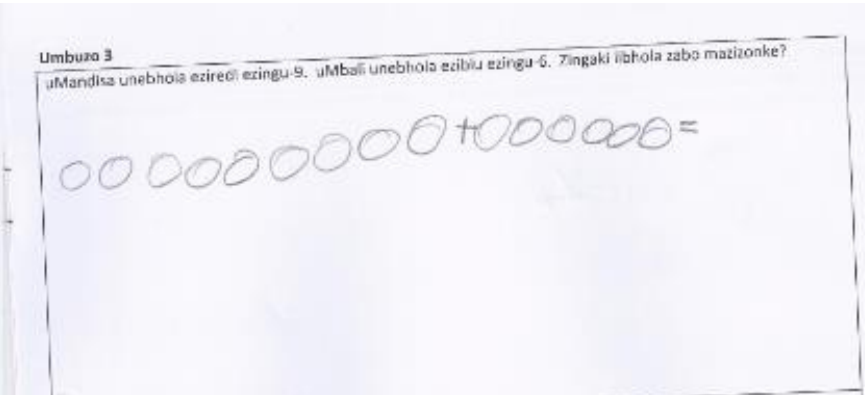


Figure 3. 2 Example of HM and VM from written test data

The child drew 9 circles and then a plus sign followed by 6 circles, thus setting up a pictorial model of the situation and exemplifying HM in doing this. There are various possibilities for VM, but we do not know which route is taken. We can infer that since the answer is correct an appropriate VM option was taken. I provide an example from the interview dataset in order to illustrate the difference between HM and VM:

The teacher (myself) reads the word problem for the child:

> *"uMandisa uneebhola eziredi ezingu-9. uMbali uneebhola eziblu ezingu-6. Zingaphi iibhola zabo mazizonke?"*
>
> *("Mandisa has 9 red balls. Mbali has 6 blue balls. How many balls do they have altogether?")*
>
> T: Uzoyithola kanjani impendulo? (How will you work out the answer?)
>
> (The learner drew 9 circles followed by a '+' sign followed by 6 circles followed by an equal sign. Then 'counted on' verbally pointing to the 6 circles…10,11,12,13,14,15 and wrote answer as 15)
>
> M: Impendulo ngu-15 (The answer is 15)

In this example the child sets up the pictorial model first (HM), then uses a 'count on' strategy (VM) to get the answer.

RME-based theory suggests that children are able to progress from the use of informal models and strategies to more sophisticated models and strategies. Levels of progression within mathematizing are described by Gravemeijer and Stephan (2002) as: level 1 (activity setting), level 2 (referential), level 3 (general); level 4 (formal). The authors state that these four levels indicate a developmental progression, but are not strictly hierarchical as students may move back and forth between various levels. Levels 1 and 2 fit with horizontal mathematization since students are still dependent on the situation to construct a model. Levels 3 and 4 fit with vertical mathematization since students are no longer dependent on situation-specific imagery and, in some cases, may not need the support of any external model for more formal mathematical reasoning. These levels have been useful in thinking about progression in this study although the constructs are different. Next, I discuss mathematics as re-invention.

3.2.3 Mathematics as re-invention

According to Freudenthal (1973), re-invention is often known as discovery or re-discovery of mathematics by learners. Gravemeijer (1994) submits that mathematizing enables learners to reinvent mathematics. The re-invention principle is constituted by learners' informal solution procedures which often anticipate more formal procedures (Uzel & Uyangor, 2006). This re-invention of mathematical insights, knowledge and procedures takes place through the processes of *progressive mathematization* described above in the stages and levels referred to as horizontal and vertical mathematization.

3.3 Guiding principles of RME instructional design

Three guiding principles of RME instructional design discussed by various authors (Barnes, 2004; Doorman, 2001; Gravemeijer, 1994; Uzel & Uyangor, 2006) are *didactical phenomenology', 'self-developed or emergent models' and* '*guided re-invention through progressive mathematization'.* These principles need to be considered by teachers in designing tasks and conducting lessons, and were considered for designing the intervention lessons in this study.

3.3.1 Didactical phenomenology

Freudenthal (1983) proposed the use of phenomenologically rich situations: situations that are 'begging' to be organized. He emphasized that situations should be selected so they can be organized by the mathematical objects which the teacher wants students to construct. Didactical phenomenology is aimed at making phenomena accessible for modelling and calculating activity through organizing. Situations in which a given mathematical topic is to be applied need to be suitable as points of introduction to a process of progressive mathematization. In other words, situations must have the potential to evoke solution procedures that can be a basis for construction of formal mathematics. This construction is referred to as re-invention.

Where guided re-invention is concerned, the instructor is required to play an important role. According to van den Heuvel-Panhuizen (2003), this requirement puts the onus on the teacher to develop materials that allow an instructional approach which uses problem situations that make sense to the learners. These materials further need to be suitable for model building, and should fit with the trajectory that elicits the further evolution of the model into a didactic model that opens up a path to higher levels of understanding for learners. Two demands that need to be met are that the problem situation can be readily schematized and that there should be a necessity for model building. Therefore the situation has to include model-eliciting activities, such as planning and executing solution steps, generating explanations, identifying similarities and differences and making predictions (van den Heuvel-Panhuizen, 2003). This process takes place within and across situations. In order to discover which problems and activities can bring students to identify mathematical structures and concepts, 'didactical analyses' (Freudenthal, 1983) are needed. According to van den Heuvel-Panhuizen (2003), these analyses focus on how mathematical knowledge and concepts can manifest themselves to students and how they can be constituted. She further suggests that some means of thought experiments and inter-colleague deliberation are vital in this analysis. Inclusion of discussions with teachers in which both in which both knowledge about students and ideas about the desired mathematical concepts function as a guiding pre-image further need to be considered.

In my study, colleagues were involved in both the design and the observation of the intervention lessons and in giving feedback afterwards. The teachers were also involved in discussing an appropriate isiZulu language that could be used during the intervention lessons. The class teacher for the intervention group was present in the classroom for the duration of the intervention lessons both to observe and to give input.

A second part of didactical analysis is described as occurring while working with students and analysing students' work. In this way what is important for constituting the model is what has to be in the problem situation, so that situation-specific solutions can be promoted, which can be schematized, and which can result in vertical moves (van den Heuvel-Panhuizen, 2003).

Didactical phenomenology is therefore viewed as a heuristic design since it suggests ways of identifying possible instructional activities that might support individual activity and whole class discussions in which the students engage in progressive mathematization. In RME the instructional aim of individual activity is to enable progressive mathematization through which re-invention of the mathematics takes place. HM and VM can only be evaluated from the individual's response data. The aim of whole class discussion is so that through interaction with the teacher and other children, an individual child can make sense of the context of the word problems, allowing him or her to model the situation in a way that makes sense and then use the model for calculation.

3.3.2 Self-developed or emergent models

English (2006) posits that children are required to make sense of a situation in order to model it and mathematize it in ways that are meaningful to them. Within RME the term 'emergent models' is adopted and is described as playing an important role in bridging the gap between informal and formal knowledge (Gravemeijer, 2009; Gravemeijer & Stephan, 2002; Peled & Balacheff, 2011). Gravemeijer and Stephan (2002) and Worthington (2014) refer to 'emergent models' as 'children's mathematical graphics' to describe a range of concrete representations children use. These emergent models may be less sophisticated and more informal than 'didactical' models (see for example Klein et al., 1998).

In RME, models play a role in bridging the gap between informal knowledge connected to the 'real' or imagined reality on the one hand, and the understanding of formal systems on the other (Sembiring & Hadi, 2008; van den Heuvel-Panhuizen, 2003). Van den Heuvel-Panhuizen (2003) further elaborates that within RME models are viewed as representations of problem situations, which reflect critical aspects of mathematical concepts and structures that are relevant for the problem situation, but that can have different expressions. The emergent modelling approach aims to build on the informal solution methods of the students and assist them to develop these to more formal solution methods (Gravemeijer & Stephan, 2002).

In additive situations, the empty number line[1] has been identified as an important didactical or emergent model in RME for teaching calculations with numbers up to 100 and above (Gravemeijer & Stephan, 2002; van den Heuvel-Panhuizen, 2008). As mentioned earlier in the literature review chapter, the empty number line, from RME's point of view, is to be used as a flexible mental model to support addition and subtraction rather than for steering the students' thinking. Instead, it is meant to be adapted by the students to fit their thinking (Gravemeijer & Stephan, 2002). The expectation is that students decide what steps to make in marking the number line in order to create a model of the situation and which strategies to use on the empty number line. Key features of many South African teaching and learning contexts have led to a more 'guided'

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[1] An empty number line is a blank number line with no numbers or markers' (van den Heuvel-Panhuizen, 2008, p.6)

approach, developed through classroom discussion. In particular the highly authoritarian cultures of many South African classrooms have been widely noted (Jansen, 1990) as have those in sub-Saharan Africa more generally (Tabulawa, 2004). In these cultures, there is a strong expectation from children, teachers, parents and the broader system, that they will be 'instructed' on what to do. In the study, learners' lack of exposure to word problems and to the use of the number line also fed into a more guided approach to classroom discussions on emergent models to fit the dominant cultural context, with this more guided approach linked to the emphasis in RME on 'guided re-invention' that I go on to next .

3.3.3. Guided re-invention through progressive mathematization

When it comes to reinvention, learners are not expected to reinvent everything by themselves, rather they are guided through the process because according to Freudenthal (1991) the emphasis in re-inventing is on the character of the learning process rather than on the invention as such. Since re-invention is often known as discovery or re-discovery, the principle of guided reinvention necessitates learners being presented with well-chosen contextual problems that allow them opportunities to develop informal solution mathematizing processes (Barnes, 2004; Uzel & Uyangor, 2006). As discussed earlier, the invention process is stimulated when learners use their informal descriptions of informal or more formal mathematical forms. Gravemeijer and Stephan (2002) refer to the shift from informal models to more formal models as *model of* and *model for*. In RME terms 'model of' refers to the move in activity from creating a model of the situations towards using that model for thinking about how to solve the problem. It is hard to see the model of/mode for shift in written work but there may be evidence of it in the interview data. The use of models should be incorporated into progressive mathematization. Within RME the term 'model' is implicated within the progression from the 'model-of' a situated activity to a 'model-for' more sophisticated (more abstract or more general) reasoning (Gravemeijer & Stephan, 2002). To offer the intended support to learning processes, models must have important characteristics such as being rooted in realistic, imaginable contexts, being sufficiently flexible to be applied also on a more advanced or more general level and being able to be re-invented by students on their own. In my study what emerged from the data were models which reflected realistic backgrounds for example, in pictorial images. Learners tended to draw circles, tallies or lines or pictures related to the specific situation described in the word problem. As discussed previously, these models are referred to by Ensor et al. (2009) as indexical models and iconic models.

3.4 Intervention materials and their alignment with RME materials.

Askew's (2004) 'Big Book of Word problems' is a materials text that follows several aspects of the RME approach described above. For the purposes of the intervention, some of these word problems were adapted in order to elicit model building. These problems were chosen because the themes of the problem situations and the structure of the word problems could be adapted to be close to the experiences of the learners. Initially the empty number line promoted by RME as an emergent model was adopted in this study with the assumption that its use by the learners would evolve into formal mathematics or vertical mathematization.

In the Big Book's approach, part of the classroom discussion at the end of the three initial problems focuses on discussing different approaches and on pointing out efficiencies, as part of an explicit attempt to build towards more efficient models in the classroom community. This approach had to be adapted in order to fit the context and the level of the children in my study. From the start of the intervention it was evident that the learners had not been exposed sufficiently to word problems and to the use of the number line. Therefore, the focus came to be on an orientation to word problems, to making sense of them and to modelling the use of a number line. The emphasis on initial sense making of situations led to a working with individual problems rather than the three problems collectively. In line with RME principles, the sense making focused on individual problems rather than the three problems collectively, as in the Big Book's approach which focuses on the interface between individual emergent models and progression towards the collective level through this mechanism.

The instructional design involved adjusting the approach taken in the Big Book of Word Problems by compiling a set of problems that could lead to a series of processes which together were aimed at the reinvention of the intended formal mathematics (Barnes, 2004; Doorman, 2001). In this study, an environment was provided in which learners engaged in organizing activities and were guided through the exploration of the empty number line as a model of number which they could utilize to solve additive relation situations. These organising activities began with a whole class discussion which promoted the understanding of the context. This was followed by the solving of the word problems individually in order for each learner to engage in the process of re-invention through mathematization. The models produced by the children suggested that they had not made sense of the situation and this became evident in the use of the number line which fell apart. The focus of the intervention then shifted in order to attend more to extended discussion of situations and to acting out the situations to promote greater success in producing appropriate emergent models in the first instance.

The Big Book of word problem's approach followed in this study aligned with and deviated from the RME principles underpinning the study in various ways. Firstly, didactical phenomenology is concerned with providing phenomena or situations that would be useful for organising and which can be easily schematized. The Big Book of word problem's approach provided themes accessible to children which were carefully chosen and adapted to suit a South African (SA) context. For example, themes such as buses, sweets and pencils were used. This principle of accessibility also promotes 'analysis' which entails thought experiments and inter-colleague deliberation, discussion with teachers, and analysing students' work while working with them. During the intervention different colleagues observed the lessons and gave feedback based on the observations of how the children were responding and the kinds of the organising activities that were emerging from them. The children's class teacher was constantly involved during the intervention lessons and also gave feedback based on her observations. The children's books were collected and analysed after every lesson. Although the aim of incorporating this principle was that the outcomes would be progression from situation concrete focused models to mathematical models, it was not a linear process as there was evidence that the children had not been exposed to word problems and the translation of situations to models which could then be used for calculating.

3.5 Conclusion

In this chapter I have discussed both the theoretical underpinning of this study, and the theoretical assumptions upon which it is based. I have also explained the rationale for this choice of theoretical framing. In Chapters 4 and 5 I offer a detailed discussion of the interaction between theory and data in relation to the study's methodology and research design, and to the development of a language of description for the data analysis.

CHAPTER 4

RESEARCH DESIGN AND METHODOLOGY

4.1 Introduction

As stated previously this study aimed to explore mathematizing processes employed by South African Grade 2 isiZulu speaking learners in solving additive relation word problems. This chapter includes a description of both the broader research design and the specific intervention design within which this exploration occurred. It also includes details of the data sources used to answer the research questions guiding this study, and the analytical methods employed in this process.

4.2 Research Design

This study adopts a broadly qualitative approach with some quantitative analysis incorporated in it. It also stems from an interpretivist view as I aim to "interpret the meaning of social action and thereby give a causal explanation of the way the action proceeds and the effects which it produces" (Weber 1922, pg.7 cited in Bertram & Christiansen, 2014). Within the interpretivist paradigm, researchers attempt to describe and understand how people make sense of their world, and how they make meaning of their particular actions. In this study I analysed how children make sense of word problems through the ways in which they solve them. The assumption here is that there is a set of realities or truths which are historical, local, specific and non-generalisable. The study took a case study approach, focusing on a single school and two classrooms with isiZulu speaking learners, and therefore conclusions can only be drawn for these participants. Many researchers seem to be in agreement about the definition of a case study. For example, Creswell (2009) defines a case study as a strategy of inquiry in which the researcher explores in depth a program, or event, an activity or a process, or one or more individual A case study is also described as 'an opportunity for one aspect of a problem to be studied in some depth within a limited timescale' (Bell, 1999, p.10). For Creswell, case studies are bounded by time and activity, and researchers collect detailed information using a variety of data collection procedures over a sustained period of time. Yin (2009) supports the idea that case study designs can include both qualitative and quantitative data. A critique of case studies is that they cannot be generalised. In order to mitigate this limitation various researchers propose 'relatability' (Bell, 2005; Opie, 2004) of the approach rather than generalisation as a key goal, noting that:

> *'an important criterion for judging the merit of a case study is the extent to which the details are sufficient and appropriate for a teacher working in a similar situation to relate his decision making to that described in the case study'*

(Bassey, 1984, p.85)

In this study I explored, in depth, the mathematizing processes of South African grade 2 isiZulu speaking learners in two classrooms while they were solving additive relation word problems. The quantitative aspects of this study focused initially on overall performance before the intervention. This assisted with choosing 12 learners for interview -4 high attaining, 4 middle, and 4 low attaining learners for more in-depth understanding of their mathematizing processes at the beginning and end of the intervention. These 12 learners were interviewed immediately after the pre-test. However, after the post-test, two of the learners in the interview sample had stopped coming to school and could not be interviewed. Therefore, the analysis came to be based on the 10 learners that were retained as part of the interview sample for whom I had pre- and post-interview data. The quantitative outcomes also enabled identification of overall performance gains over time.

4.2.1 Empirical Setting

The research was carried out at a quintile 1[2] government primary school in Gauteng in a township area with a low economic status. This school was one of the ten schools participating in the WMC-P project and has adhered to the broad language in education policy (Taylor & Vinjevold, 1999) that stipulates that FP children have a right to receive a basic education in the language of their choice. The township school divided foundation classes into African languages linked to the children's home languages. In the study school, there were two isiZulu Grade 2 classes which eliminated the challenges of sampling the classes. With pre-test evidence indicating very similar patterns for the two classes, I chose one class as the intervention group and the other as the control group. In the intervention class (n=38 registered class) I used the 'Word Problems resource' which consisted of the Big Book and Teacher's notes described earlier. Learners in the control class continued with their normal lessons and received no intervention but were included in writing the pre-test, post-test and delayed post-test. Thirteen lessons from the 'Big Book of word problems' resource were taught by myself across 3 months twice a week.

4.2.2 Participants

Merriam (1998) differentiates between two levels of sampling in a case study:

- The selection of the case to be studied
- Sampling of participants within the case

[2] Quantile 1 schools designate the poorest institutions and the categorisation is based on rates of income, unemployment and illiteracy around the school's catchment area.

The phenomena of interest in this study are the mathematizing processes of the learners in solving additive relation word problems, across the intervention and control groups prior to, and after, the intervention lesson sequence. The intervention class was composed of 38 isiZulu speaking learners, but absences meant that matched pre-and post-test data was available for n=28 matched learners for analysis against n=38 registered learners, and n=27 for the control class matched learners for analysis against 37 registered learners. The interview sub-sample were selected on the basis of 4 high attaining, 4 middle attaining, and 4 low attaining learners. This sub-sample was selected based on their performance in the pre-test.

4.3 Intended and enacted intervention plan & consequences for data gathering

4.3.1 Pre-Test and Pilot

The structure and design of the questions for the pre-and post-tests were drawn from Askew's (2004) *Big Book of word problems* and were adapted to suit a South African context. During the writing of both the pre and post-tests the learners were seated one at a table to avoid any copying of each other's work. One script was handed out to each learner so that individual working out could be done. As the teacher and researcher, I read out the questions to the children and those that could read and understand for themselves were given the option to read for themselves and continue with the working out. The first instrument used was a pre-test, administered to both classes in September 2014 in order to ascertain performance and the kinds of mathematizing processes of both the intervention and control groups in solving additive relation word problems. The purpose of the pre-test in this study was to answer research question one which is "What mathematizing processes related to additive relation problems were reflected by these learners across the intervention and control groups prior to the intervention?" The questions in the pre-test consisted of additive relation situations which were adapted and translated from the *Big Book of word problems* and themes into isiZulu and were relevant to a South African context. In addition, the questions consisted of additive relation situations that would be likely to make sense to the learners. The number of questions in the pre-test was determined by the problem types identified in Carpenter et al.'s (1999) framework. Initially, there were 14 questions in total across all the problem types (Join, Separate, Part-part whole, Compare). All tests were administered by myself, as researcher and teacher in this study, with the questions read out to the class, and children provided with response scripts with all questions and space to write down their answers. The choice to read out questions was based on widespread evidence of poor reading skills (Fleisch, 2008; Fleisch & Schöer, 2014) and was therefore aimed at ensuring that poor reading skills did not hamper learners from accessing questions and solving them.

Immediately after the pre-test was administered a pilot lesson was introduced in order to gain insight into the following three issues: the relevance of the IsiZulu language into which the contextual problems were translated; learners' familiarity with the themes of the word problems; how the learners translated the situations into a mathematical world.

Among the findings from the pilot lesson was that while the language of isiZulu was appropriate for the learners, different dialects needed to be included in order to accommodate children from different contexts and geographic backgrounds (Nkambule, 2013). Although this study focused on South African learners, the concept of migration became relevant as South Africans move from villages or rural areas into urban areas there is a need for researchers to accommodate dialects different from those that learners have been used to. Nkambule (2013) investigated how teachers created opportunities for learning for immigrant learners and found that learners were successful in their learning when teachers used code switching to support this learning. In South African contexts code switching may incorporate switching between dialects of a language. This form of codeswitching was taken into consideration during the intervention lessons, with support from the teacher and isiZulu speaking friends and colleagues.

In the pilot lesson, during the discussion of the context of the word problems children showed familiarity with the bus context, with most children agreeing that they had been on a bus when travelling back and forth between the township and their families' hometowns. After the discussion of the context the children were asked to work in pairs in order to solve a word problem and write answers in the exercise books provided for the intervention lessons. The learners were encouraged to do all their working out in the book as some were spotted writing on pieces of papers, also it was evident that learners were not familiar with explaining their answers. While the learners indicated familiarity with the context, they seemed unable or unwilling to move into mathematizing without further input. During the pilot lesson the children were asked to solve the word problem presented to them, using their own mathematizing processes. This was to determine how they would translate the informal situations into formal mathematics and to ascertain their mathematizing processes.

The empty number line, which had been initially proposed for the intervention, did not feature at all in the working of the children. When it was introduced some children showed some familiarity with structured number lines, but none indicated familiarity with an empty number line. Important to mention is that lack of sense making and natural modelling of situations in drawings or diagrams by the learners was evident in the pilot lesson. Quaglia, Longobardi, Lotti and Prino (2015) consider drawing a developmental product, whilst Hopperstad

(2008) views drawings as ‘a meaning-making process in which children draw signs to express their understanding and ideas in a visual-graphic form’ (p.78*)*.

Each of the findings outlined above necessitated some adaptations to the planned intervention, with its intended focus on supporting moves towards more sophisticated calculation strategies. The pilot lesson findings were taken into consideration during the design of the intervention lessons, with attention given to how to meet the needs of learners while remaining aligned to the design principles of RME.

A record of my observations and reflections after the pilot lesson and learner working from this lesson were informally analysed before the commencement of the intervention lessons and focused on language and theme observations.

4.3.2 Intervention

The theoretical underpinning of the intervention was influenced by RME coupled with CGI and the Big Books approach. All of these approaches provided a framework for designing the intervention and for developing insights into the processes, guided by the concepts and principles of RME discussed in the theory chapter. RME provided the concepts and principles for the instructional design of the intervention. CGI provided the framework for the progression of the word problems and the *Big Book of word problems* provided the outline for and the pedagogic approach adopted for the intervention lessons. As mentioned earlier, the focus of the intervention and analysis shifted to giving in-depth attention to sense making and modelling of situations. Next, I discuss in detail how the concepts and principles were incorporated in the design and implementation of the intervention programme.

4.3.2.1 Mathematics as a human activity

As discussed earlier, mathematics must make sense and must initially be close to children and achieving this is possible through the employment of word problems. In this study the *Big Book of word problems* provided a structure of word problems categorised according to themes and in a context that makes sense and is close to the students (van den Heuvel-Panhuizen, 2003). Relevant themes such as buses, sweets, pencils and bottles were incorporated into the design of the word problems since they are familiar to children in South African contexts. The structure of the word problems was simple, and the wording was straight forward in order to make sure that they were as accessible to the children as possible. In order to deal with the children's different dialects, during the discussions of the word problems in the intervention lessons, words were explained in all the possible different dialects. Below in table 4.1 I present an intervention lesson schedule that guided the intervention lessons.

Table 4. 1 Intervention Lesson Schedule

Intervention Lesson Schedule	
Activity	**Date**
Join (Result Unknown)	Tuesday 14 October 2014
Join • Result Unknown • Change Unknown • Start Unknown	Thursday 16 October 2014 Tuesday 21 October 2014 Thursday 23 October 2014
Separate • Result Unknown • Change Unknown • Start Unknown	Tuesday 28 October 2014 Thursday 30 October 2014 Tuesday 4 November 2014 Thursday 6 November 2014
• Mixed (Join, Separate) • (PPW-Whole Unknown in Worksheets)	Monday 10 November 2014 Tuesday 11 November 2014
• Mixed (Join, Separate) • (PPW-Whole Unknown in Worksheets)	Thursday 13 November 2014 Monday 17 November 2014

4.3.2.2 Mathematics as mathematizing

The word problems presented to the learners enabled modelling through the progressive mathematization process of HM and VM. As mentioned earlier, progressive mathematization is shifting or progressing from HM and VM, with HM forming a base for progressing to the desired outcome of VM. Progressive mathematization was promoted through direct modelling of situations after learners had made sense of them. The aim was to promote progressive mathematization towards VM since it had already been established that the learners were operating at HM. During the mathematizing processes it was evident that learners were leaning towards concrete models such as tallies and circles to solve additive relation situations. RME is not explicit about the progressive stages of mathematization from HM to VM. It is through the data analysis to be discussed later in this chapter and in the analysis chapter that this finding emerged.

4.3.2.3 Guided re-invention

As explained in the theory chapter, the principle of guided reinvention necessitates learners being presented with well-chosen contextual problems that enable them to develop informal solution mathematizing processes (Barnes, 2004; Uzel & Uyangor, 2006). For example, one of the themes in the *Big Book of word problems* is ‘Cats and Dogs’ and the word problem entails the learners solving a problem of how many cats and dogs are playing in a field. Another important aspect to be taken into account in relation to themes is culture and context (Bussi, Sun, and Ramploud (2013). For the isiZulu speaking learners from disadvantaged socio-economic backgrounds, some of the themes in the *Big Book of word problems* were adapted to suit both the culture and context of the participants. For example, a theme of ‘buses’ and the context of travelling from one place to another was used since the children were familiar with it. One of RME's key drivers is Model-Eliciting Activities (MEA) which Bansilal (2013) describes as '*open ended mathematical tasks in which students develop a mathematical model to solve a real world problem*' (p.261). Proponents of RME postulate that the open-ended nature of the tasks often results in students developing and articulating a diversity of mathematical ideas. In my study the tasks that the learners were presented with allowed them to model the situations in different ways. For example, some modelled the situation using tally marks while others used pictures, number sentences and number lines. Learners were then guided to solve the problem in a more mathematically meaningful and sophisticated way. Sembiring et al. (2008, p. 73) suggest the following task design principles in relation to an RME approach:

- Problem solving tasks should start with an analysis of the given 'real world' situation involving its socio-cultural context.
- The model must compel students to take into consideration the relationships involved between the quantities.
- Mathematical operations must be used to find the result.
- The result should be evaluated and interpreted within the initial 'real-world' context.
- The task should use a socio-cultural context in which students can identify themselves as active agents.
- The task should not contain any explicit and immediate questions that could be answered by finding one particular number with the aim to prevent students from immediately calculating the answer.

All of these principles were taken into consideration during the intervention. For example, a word problem that was presented in the test and intervention lesson was: ‘There were 8 people on the bus. 5 more people got on. How many people are on the bus now?’. At the beginning of the intervention lessons a situation was first discussed in order to ascertain learners’ familiarity with it. Discussions of different mathematizing processes were aimed at encouraging take up of more sophisticated mathematizing processes. Learners were presented with contextual problems in an environment that encouraged them to reinvent their own mathematics in ways that would later allow generalisations in the employment of mathematizing processes. Thus, the structure of the lessons based on the adapted version of the *Big Books of word problems* resource was as follows:

Table 4. 2 GeneraL lesson structure

<table>
<tr><td colspan="2"><u>General Lesson Structure</u>
<u>CONTEXT - BUS</u></td></tr>
<tr><td colspan="2"><u>OBJECTIVES:</u>
• Understand addition problems involving joining two sets
• Understand addition problems involving problem types
• Model addition using the number line</td></tr>
<tr><td>Activities</td><td>Resources</td></tr>
<tr><td><u>ACTIVITY 1 – (PAIR/WHOLE CLASS DISCUSSION)</u>
• Paste picture of the bus on the board. Discuss it with the children. What do they see etc. (More guidelines on Pg 14 and 15 of the Teacher's guide)
• Paste word problem on the board underneath the bus.
Read through the problem with the children. Encourage them to use whatever means they like to find an answer. (Refer to Pg 14 and 15 of the teacher's guide)
Problem 1: Result Unknown
Bekunabantu abangu-7 ebhasini. Bese kugibela futhi abantu abangu-4. Bangaki abantu ebhasini manje? (There were 7 people on the bus. Four more people got on. How many people are now on the bus?)
• Walk around the class, select 3 children using different methods and invite them to come and share their methods with the class. (Take their book to make sure they do the same thing)
• If anyone has not done so, <u>model</u> how to find the answer by using a structured number line, then go back to the story and emphasize starting from the number already given (7 in this case), then <u>model</u> using the empty number line.
<u>Problem 2 : Change Unknown</u>
Bekunabantu abangu-7 ebhasini. Bese kugibela futhi abanye abantu. Manje kunabantu abangu-12 ebhasini. Bangaki abantu abaze bagibela futhi ebhasini? (There were 7 people in the bus. More people got onto the bus. There are now 12 people in the bus. How many people got onto the bus?)</td><td></td></tr>
<tr><td><u>ACTIVITY 2 (INDIVIDUAL ACTIVITY)</u>
Repeat same steps 2, 3, and 4.
Change Numbers on the word problem and get learners to work out the problem individually.
<u>Problem 3: Start Unknown</u>
Bekunabantu ebhasini. Bese kugibela futhi abantu abangu-7. Manje kunabantu abangu-13 ebhasini. Bangaki abantu kade besebhasini kuqala?
(There were people in the bus. 7 more people got onto the bus. There are now 13 people in the bus. How many people got onto the bus?)
Repeat same steps 2, 3, and 4.
Change Numbers and get learners to work out the problem individually.</td><td></td></tr>
<tr><td><u>ACTIVITY 3: WRAP UP</u>
Ask learners: 'What have we learnt today?'</td><td></td></tr>
</table>

4.3.2.4 Didactical phenomenology

Didactical Phenomenology suggests different ways of identifying possible activities to support individual activity and whole class discussions in which the students engage in progressive mathematization. The structure of the lessons adapted from Askew's (2004) *Big Books of word problems* teaching approach was such that the lesson started with a whole class discussion where a picture pasted on the board was discussed to establish the context, then a word problem in relation to the picture was pasted on the board and discussed, taking into consideration the mathematics in the situation. In some instances, in addition to the whole class discussion, the situation was acted out by the learners in front of the classroom. Van den Heuvel Panhizen's (2003) notion of the 'didactical use of models' that facilitate the process of mathematizing (HM), played an important role in guiding how the lessons were conducted. The pre-test and pilot lesson analysis led to more extended discussion and horizontal mathematizing of situations and a restriction of focus to join and separate problems. This restriction resulted in a narrow focus in the post-test in comparison to the pre-test which incorporated all problem types. Below is the synopsis of the intervention lesson programme:

Table 4. 3 Intervention Lesson Programme

Lesson No	**Context**	**Lesson Objectives**
1 (Pilot)	Bus	• Solve Join 'Result Unknown Word Problems'
2	Bus	• Understand addition problems involving joining two sets • Understand addition problems involving problem types • Model addition using the number line
3	Pencils	• Understand addition problems involving joining two sets • Understand addition problems involving Change Unknown Problems • Model addition using the number line
4	Bus Pencils Sweets	• Understand addition problems involving joining two sets • Understand addition problems involving Result Unknown, Change Unknown and Start Unknown Problems • Understand translating a situation into a number sentence • Model addition using the number line
5	Bus Pencils Sweets	• Continue with Activity 2 of lesson 4 • Understand and solve addition problems involving Result Unknown, Change Unknown and Start Unknown Problems • Model addition using the number line
6	Frogs	• Use context that will translate to model advocated-eg. frog jumps • Understand how dots translate to number line • Understand 'count on strategy' starting from any number and adding '10' • Model addition using the number line
7	Bus Sweets	• Use word problems to engage children in discussing the problems and the mathematics in them • Use word problems for children to act and directly model • Use word problems of different contexts to elicit learners' understanding
8	Bus	• Use word problems to engage children in discussing the problems and the mathematics in them • Use word problems for children to act and directly model • Use word problems of different contexts to elicit learners' understanding
9	Bus	• Use word problems to engage children in discussing the problems and the mathematics in them • Use word problems for children to act and directly model • Use word problems of different contexts to elicit learners' understanding
10	Sweets	• Use Part-part whole word problems to engage children in discussing the problems and the mathematics in them • Use word problems for children to act and directly model • Use word problems in different contexts to elicit learners' understanding
11	Sweets	• Continue with Lesson 10 so learners can work on more word problems within the context of sweets.
12	Sweets	• Explore 'Change Unknown Problem' Types • Explore acting out of the problems • Elicit learners' understanding through drawings or any other direct model
13	Sweets Bus Pencils	• Explore 'Start Unknown Problem' Type • Learners work on mixed word (Join, Separate, Part-part whole) problems individually

Background information was captured in my reflective journal, teacher's lesson plans and video observations of lessons (aimed at capturing the nature of task presentation within the guided re-invention frame, and teacher-learner interaction). The notebooks in which learners wrote and attached their worksheets were also looked at after each lesson, and informed the forward focus in the following lesson. As noted already, the lack of evidence of spontaneous sense-making led to a shift during the intervention lessons away from the intended model approach focused on the use of empty number lines to greater emphasis on children's own mathematizing processes. In addition, there were disturbances in the school. These factors contributed to a delay in the intervention lessons and as a result part-part whole problem types were briefly introduced and no compare problems were included. The pilot, early lesson outcomes and other factors discussed above led to a narrower focus in the intervention lessons and a narrower set of items in the post-test. Details of the narrower focus are further discussed in the next section.

4.4 Data sources and procedures

4.4.1 Pre and post-tests

Data was collected using multiple sources and a range of instruments as informed by the design of the study. The main instruments were a pre-test, pre-test interviews, post-test and post-test interviews. As already noted, the intervention lessons and post-test came to a focus on a narrower range of items. Details of the common items that featured in the central analyses of this study are included in Appendix B. The aim of the pre and post-tests was to answer all research questions.

While there were additional items in the pre-test across all categories, and additional Join, Separate and PPW items in the post test, the analysis focuses on the 7 common items, with performance and models in the additional items scanned for whether they indicated support for the patterns seen on the common items. The patterns and claims remained valid across the additional items, allowing for illustrations of responses to be drawn from the broader item set where this was useful for providing strong exemplars. Analysis of models created in HM phase and overall patterns of performance on items was possible to see from written test responses. Next, I detail pre-and post-follow up interviews which provided access in particular to VM, which was less easy to access in written responses.

4.4.2 Pre and post test follow-up interviews

Interviews were conducted in order to gain insights into learners' thinking and modelling processes. The aim of the interviews was to follow up on the test responses in order to obtain clarification and deeper understanding of the mathematizing processes for solving additive relation situations used by the learners. Interviews form part of qualitative data collection methods and are 'useful when participants cannot be directly observed and allow a researcher control over the line of questioning' (Creswell, 2009 p.179). Two of the disadvantages of interviews are that they are time consuming, and sometimes can pose a danger of bias because of their more subjective techniques (Bell, 2005). However, the advantage of interviews is that they can provide richer information than is possible from other data sources. Semi-structured interviews, according to Opie (2004 p.119)

> *'allow for a depth of feeling to be ascertained by providing opportunities to probe and expand the interviewee's responses, also they allow for deviation from a prearranged text and to change the wording of questions for the order in which they are asked'*

In this study the interviews were task-based, semi-structured and were video recorded. The written tests of the 10 learners were re-visited and participants were given the opportunity:

- to expand their ideas, explain their answers and way of thinking as shown in their responses.
- to discuss items that they did not answer (if any) or items in which directions were not followed correctly.
- to discuss ideas that were not expressed in writing. This was particularly useful in this study for gaining insights into learners' vertical mathematizing: the ways learners used models for calculating answers to the problems – as this was frequently not visible in their written test responses.

Interviews allowed for preservation of natural language, capturing of information that might be missed from note taking, re-analysis of data and capturing of the interviewer's contribution. In this study, the interviews took place during school hours. Children were called out of the class individually for the interviews which lasted between 30 and 60 minutes. Questions were all in isiZulu, with numbers read in English as learners were familiar with this format. In the next section I discuss the process of data analysis.

4.5 Data analysis

4.5.1 Analytical framework

After all data had been collected, I attempted to use RME principles and the CGI framework of mathematizing processes for organisation and analysis. Since Creswell (2012) describes the process of analysing qualitative data as being inductive in form, going from the detailed data to general codes and themes, I decided to take an inductive approach in which I developed codes and themes from the emerging data. Important to note is that ‘Extent of structuring’ was a category that emerged from noticing a phenomenon that occurred in some scripts. Next I discuss the coding process.

4.5.2 Coding of written responses for quantitative analysis

The tests were designed so that one mark was allocated for a correct answer and 0 (incorrect answer) for an incorrect answer. This scoring enabled me to see both the overall total for each child and patterns of common errors. The marks were captured onto separate spread sheets for each class. Below there is an example of a marked script with questions (translated as): ‘There were 3 people on the bus. 5 more people got on. How many people are now on the bus?’ and ‘There were 12 padlocks on the table. Mom removed 5. How many padlocks are now on the table?’:

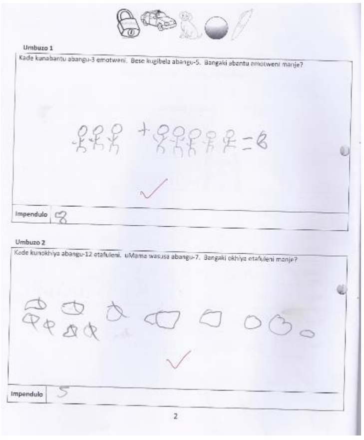


Figure 4. 1 Example of a marked script

4.5.3 Coding of data for qualitative analysis

In order to clarify the mathematising processes that emerged from the data analysis, below I provide a brief description and some exemplification from the children's written work.

- Iconic (pictorial): drawings (realistic depictions) of everyday context. According to Ensor et al. (2009) iconic models are more sophisticated examples of concrete models. This example also shows the 'distinguished quantities ' (DQ) discussed earlier and later on in the analysis chapters.

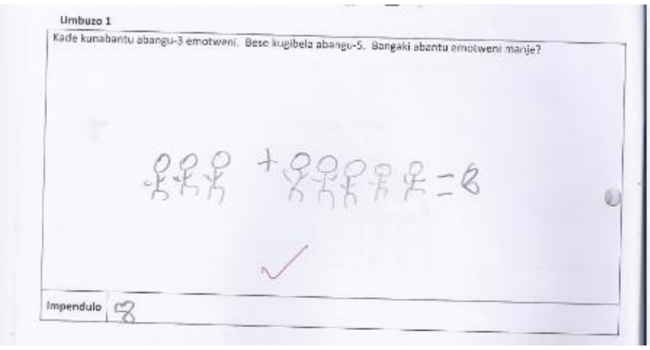


Figure 4. 2 Example of an Iconic/Pictorial Model

- Indexical (pictorial): generic depictions of everyday context – drawings of sticks, tallies, dots, circles and other shapes (blocks) -, also less sophisticated model

Figure 4.3 Example of Indexical Models-Circles/Pictorial and Tally Marks (Pictorial)

- Symbolic-Number based (SNB): Number Lines: Symbolic Number-based (Use of numerals to represent numbers). Supports calculation without counting but could also be used for calculation-by-counting strategies.

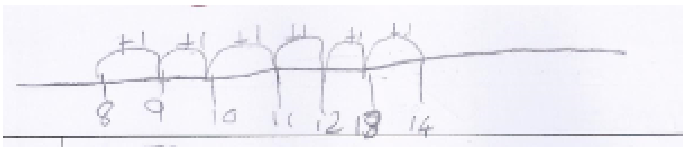


Figure 4. 4 Example of a Symbolic Number-based model-ENL

- Horizontal Number Sentence (HNS) - Use of mathematical notation to produce mathematical statements. Relies on known number facts and facts which can be derived from*)* counting (explicates some kind of relationship between items)

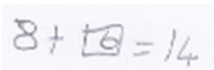


Figure 4. 5 Example of a Horizontal Number Sentence:

- No Overt Model-Correct (NOM-C): This code represents instances where there is no evidence of learners’ working at all, but the learners have only provided a correct answer.

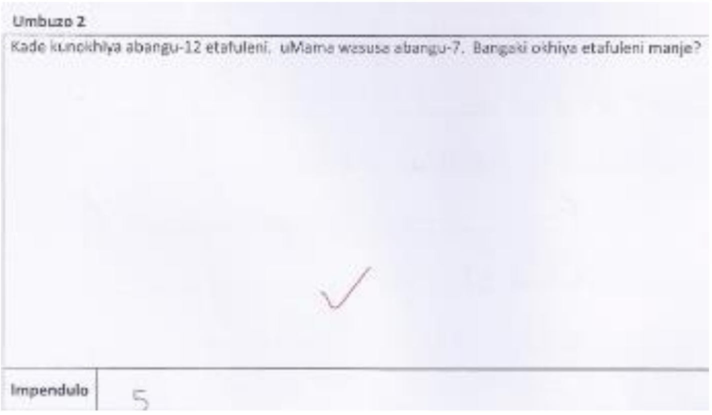


Figure 4. 6 Example of a 'No Model' Correct

- No Overt Model-Incorrect (NOM-I): This code represents instances where there is no evidence of children's working at all, but the children have only provided an incorrect answer. It is inferred that the internal model used to work out the answer is incorrect and is explicated in the incorrect answer.

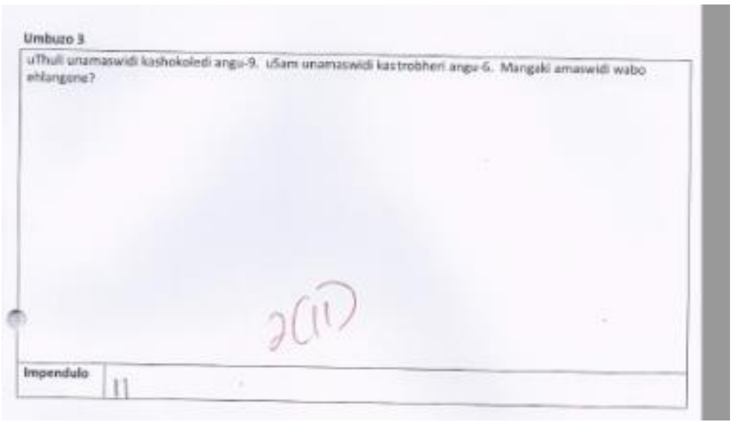


Figure 4. 7 Example of a 'No Model' Incorrect

The models above, extracted from learners' work, allowed for the categorisation in table 4.4 below. All 'models' were categorised into themes and presented together with summaries of the most prevalent mathematizing processes and commentaries on these.

In the next section I explain how the tests were analysed and turn to the interviews. The tests were analysed and categories were formed using the mathematizing processes that emerged from the learners' working out. In table

4.3 below, I present a brief framework that was formulated in order to code the mathematizing processes that emerged from the tests. The themes were coded based on what was explicitly visible from what the learners were presenting with regards to the number of models presented by the learners, whether they were correct or incorrect and whether they were appropriate or inappropriate. In the table In table 4.3 below I briefly exemplify by using a few number of learners as a complete framework is presented in table 4.4.

Table 4. 4 A brief framework for the coding of mathematising processes

Learner Coded Numbers	Total Number of Models per number of learners	Correct Answer (1)	Incorrect 0 (Answer)	Model	Appropriate Model	Inappropriate Model
L17						
L10,L15,L16 L 22 L 23						

In table 4.4 below I provide a detailed example of coding for mathematising processes emerging from the tests as already alluded above:

Table 4.5 Detailed data coding for mathematising processes emerging from tests

Name	Total Number of Models	Correct Answer (1)	Incorrect 0 (Answer)	Model	Appropriate Model	Inappropriate Model
L17	1	1		Pictorial	A(10 sweets + 5 sweets =15 in one row	
L 10 L 15 L16 L 22 L 23	5		0(13) 0(12) 0(16) 0(6) 0(18)	No Model	None overtly seen but incorrect answer	n/a
L2 L4 L 5 L7 L8 L9 L19 L21 L 24 L 27 L 14	11	1		No Model	None overtly seen but correct answer	
L 26	1		0(14)	Pictorial		I (2 sweets)
L 6 L 18 L 20 L 28	4	1		Pictorial	A (9 tally marks distinct to 6 tally marks all in one row.	
L 1 L 3 L4 L11 L 28	4	1		A [Symbolic Syntactic (Horizontal Number Sentence)]	A (9 + 6=15)	
L 12 L 13	2	1		Pictorial	A (15 circles in one row)	
L25	1		0(14)	Pictorial		I (9 tally marks distinct to 7 tally marks all in one row.

For the description of the models emerging from the data I used Ensor et al.,'s (2009) forms of representations described earlier in the literature review as Concrete, Pictorial, Symbolic Number-based-Number line, Symbolic syntactic-Horizontal Number Sentence. Where learners did not show any mathematizing processes, a 'No Model' code was used. In the first column are the learners' names (L=Learner). Learners with similar models or mathematizing processes were grouped together in order to give a summarized number of responses exhibiting specific mathematizing processes. In the second column is the total number of models and in the third and fourth columns are the correct and incorrect answers respectively. A ‘1’ is indicated for correct answers and 0 for incorrect answers. The ‘model’ is in the fifth column. Ensor et al. (2009) forms of representations discussed in the literature review chapter were used for categorisation of models as these fitted well with what was emerging from the data. In the sixth and seventh column are ‘appropriate’ and ’inappropriate’ models respectively. This category was included as the appropriateness of the models appeared to be associated with correct answers. In most cases it was assumed that an appropriate model should lead to a correct answer unless there was an error such as a miscount. After analysing the data that emerged from the learners’ written responses, the analysis was revised into the summary table presented in Table 4.5 below:

Table 4.6 Example of coded categories collapsed into themes for PPW-WU (9 + 6 =?)

Model	**Total number of attempts for this problem type**	**Number of attempts of the particular model**	**Number of appropriate models**	**Number of inappropriate models**	**Correct Answers**	**Incorrect answer**
No Model	29	16	n/a	n/a	11	5
Indexical (Pictorial)	29	7	6	1	6	0
Iconic (Pictorial)	29	2	1	0	1	1
Symbolic-syntactic	29	4	4	0	4	0

Looking at the learner performance in the way reflected above led to the identification of clusters of items with different patterns of gains from pre- to post test. These cluster are Result unknown high initial; Part unknown high gains; Part unknown lower gains. These categories are introduced and described in Chapter 5.

4.5.4 Emergence of 'The extent of awareness of structure' in relation to progression into use of more formal mathematical register.

The results of the post-test for the intervention group reflect an improvement in the visibility of 'mathematical means to solve the problem'. According to various literature modelling takes place when learners are solving word problems. According to English (2006) mathematical modelling involves three shifts in the approach to the teaching and learning of mathematics. These shifts are in 1) the nature of quantities and operations that are useful, 2) the use of contexts that will elicit the creation of useful systems or models, 3) the development and refinement of such models in ways that are generalizable. English (2006) further posits that when solving typical school word problems learners tend engage in a one or two step process of mapping the problem information onto arithmetic quantities and operations. In addition, she refers to a reflection tool which encourages children to focus on structural elements of modelling. The study data indicated that learners distinguished between quantities in various ways and this observation influenced the decision to code for 'extent of structuring'. As already mentioned in Chapter 2, Schöner and Benz (2017) showed that perceiving quantity in terms of separate sets is more advanced than perceiving in ones, and the data in my study indicated moves towards distinguishing quantities in models which was linked with higher facility overall.

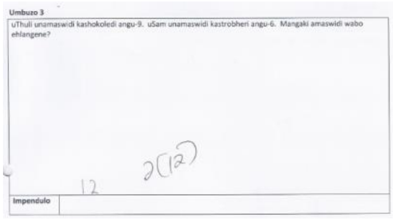
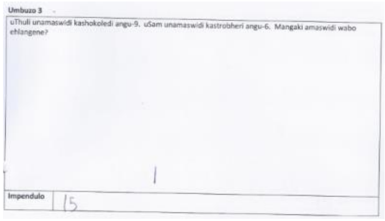
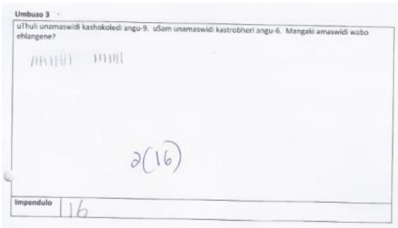
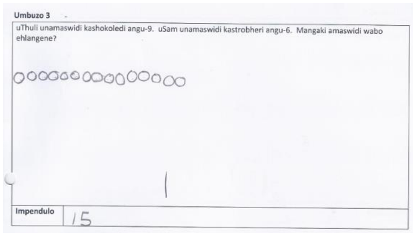
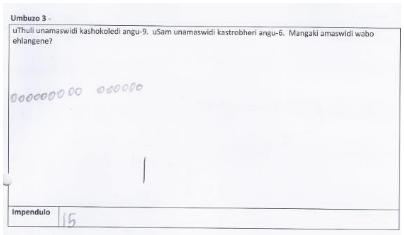
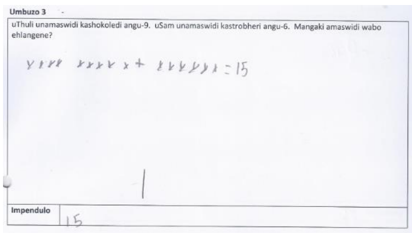
The process described by English (2006) provided guidelines for thinking about stages in the mathematizing processes that were visible in the data and resulted in the construct referred to as 'the extent of structure' - an improvement in the visibility of 'mathematical means to solve the problem'. I therefore coded for HM and VM in terms of 'the awareness of extent of structure' in relation to the progression towards use of more formal mathematical register.

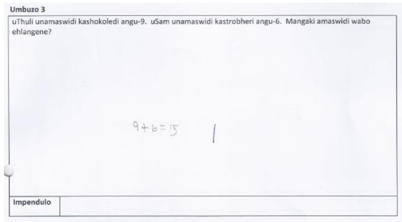
During coding for HM and VM I realised that was what emerging quite clearly from the data was learners' differentiation of quantities in ways that either yielded correct or incorrect answers, depending on how they were applied. This realisation, in addition to awareness of the constructs discussed above contributed to the formulation of 'the extent of awareness of structure'. What also emerged were the different representations the learners used for these quantities and this led to the formulation of the code ‘progression into use of more formal mathematical register’. Sub-categories under each construct were then formulated with levels that are progressive but are not hierarchical and with the formulation based on the constructs mentioned earlier. Below I present the framework, describe the codes used in this analysis which are accompanied by examples from learners’ work, and finally I present a summary of the analysis.

Operational definitions and a refined coding system were formulated from the learners’ responses to the tests and interviews. In order to illustrate development levels of structure I discuss representative examples of learners’ responses to the tasks presented in the tests. The analysis focuses on how representations conform to structural features such as numerical quantity and use of formal notation.

Table 4.7 illustrates categorisation for ‘extent of structure’ and progression towards use of a more formal register. More examples of the coding are included in Appendix C.

Table 4.7 Coded categories for the extent of structuring in relation to progression towards use of a more formal mathematical register

Cluster 1 (PPW-WU 9 + 6 =?)			
Extent of Awareness of Structure	**Description**	**Incorrect**	**Correct**
NoM	Response does not include any evidence of a model	**10(16),15(12),16(13),22(6),23(18)** 	**2,4,5,7,8,9,14,19,21,24,27** 
NDQ/PiC	No overt distinguishing of quantities within an appropriate model (NDQ)	**25(16),26(14)** 	**13** 
DQ/PiC	Overt distinguishing of quantities within an appropriate model (DQ)	**n/a**	**6,18,20** 
DQ-SFMR	Overt distinguishing of quantities within an appropriate model (DQ)		**12,17,28** 

DQ-CFMR-NS	Overt distinguishing of quantities within an appropriate model (DQ)		**1,3,11,28**  **Umbuzo 3** uThuli unamaswidi kashokoledi angu-9. uSam unamaswidi kastrobheri angu-6. Mangaki amaswidi wabo ehlangene? 9+6=15 1 **Impendulo**

4.6 Coding of interview data

Given the lack of evidence in many cases, in the written data, of how children used their constructed models for calculation, the interview data was used specifically to focus on the vertical mathematization processes. Table 4.7 below summarises the progression of calculation strategies discussed in Chapter 2 and these strategies are adapted based on the empirical data. The table includes the categories and indicators used for analysing the interview data. I begin with a brief explanation of the categories. The ‘Level’ represents the progression of strategies. For example, learners at level 0 use Recalled Fact-Immediate-Incorrect strategies (RFI-I), at level 1 they use Direct-*Modelling based Strategies (DM)*, at level 2 they use Calculating by-*Counting Strategie*s (CbC), level 3 Calculating by structuring strategies (CbS), and level 4 *Recalled Fact-Immediate-Correct* strategies (RFI-C). ‘Problem Type’ indicates that learners could either be presented with word or abstract problems to solve. ‘Strategies’ represent strategies which learners might use to solve either word or abstract problems. ‘Description’ depicts what learners tended to do when solving a problem. For example, at level 1 learners tended to construct the first set and then the second set when asked to solve a join problem. Lastly, ‘Physical Representations’ represents the nature of objects learners used at a specific level for a specific strategy in order to solve a problem.

Table 4. 8 Categories and Indicators for analysing interview data

Level	Strategy		Description
0	**Recalled Fact Immediate – Incorrect (RFI-I)**	Recalled Fact Immediate – Incorrect (RFI-I)	Immediate answer is given without counting or calculating, and the answer is incorrect.
1	**Direct Modelling:**	• Joining All (JA)	Learners use physical counters to work out the answer. The sets are joined and the unstructured union of the two sets is counted in ones.
		• Counting All (CA)	Learners write their answers in pictorial form which sometimes include formal operations. The structured union of the two sets is counted in ones.
		• Joining To (JT)	Objects are added to a set until the total is reached. The answer is found by counting the number of objects added.
		• Separating From (SF)	Objects are removed from a set. The answer is the answer of remaining objects which are counted.
		• Separating To (ST)	Objects are removed from a set until the number of remaining objects remaining is equal to the total left. The answer is the number of objects removed which are counted.
		• Matching (M)	A set of objects are matched 1-to-1 until one set is used up. The answer is the number of unmatched objects remaining in the larger set which are counted.
		• Trial & Error (TE)	The answer is found by playing around with the given quantities.
2	**Counting Strategies:**	• Count On From First (COF)	The answer is found by counting on from the first given number. The answer is the last number in the counting sequence.
		• Count On from Larger (COL)	The answer is found by counting from the larger number. The answer is the last number in the counting sequence.
		• Counting On To (COT)	A forward counting sequence starts from the first given number and continues until the given total. The answer is the number of counting words in the sequence.
		• Counting Down (CD)	A backward counting sequence is initiated from the total number. The counting continues for the number of second set. The last number in the counting sequence is the answer.
		• Counting Down To (CDT) (Difference) = (Diff)	A backward counting sequence starts from the initial set, and continues until the total set left. The answer is the number of words in the counting sequence.

3	**Calculating-by-structuring strategies**	• Stringing (St)-(NT) •	The first number is kept intact, and the second number is split into tens and ones.
		• Splitting (Sp) - (TT)	The numbers are split into tens and ones and processed separately when the operations are carried out.
		• Varying (V)	Related to the children's knowledge of number relationships and properties of operations involving strategies such as '5 + 5' for solving '5 + 6'
4	**Recalled Fact Immediate-Correct (RF(-C)**	• Recalled Fact Immediate (RFI)	Immediate answer is given without counting or calculating, and the answer is correct.

Similarly to the analyses in the previous chapters, the analyses of the interview data focuses on the 7 common items aligned to clusters. The rationale for this option was to explore similar patterns with regards to progression between the models and the strategies.

4.7 Reliability

According to Opie (2004) reliability of findings often serves as an indicator of 'goodness' or quality in research. Cited in Opie, (2004), Wellington (2000, p.200) describes reliability as 'the extent to which a test, a method or a tool gives consistent results across a range of settings, and if used by a range of researchers. Two of my colleagues in the WMC-P project have conducted similar studies to mine and their findings have helped me to test the reliability of those in my own study. For example Tshesane (2014) reported that children in his study struggled with the uptake of the ENL. He also reported that children used more than one model for a problem which is a finding replicated in my study. Lastly, he found that unstructured pictorial models were dominant and were mostly error prone. The findings of Tshesane (2014) and others point to the reliability of the patterns of results in this study.

Triangulation, which is the process of corroborating evidence from different sources of data, was an important part of the design of this case study in which data was collected from pre- and post-tests, videotaped pre- and post-interviews, learners' worksheets, learners' written work, classroom video observations and my personal journal.

4.7 Validity

Mouton (1996) argues that validity should be viewed as a synonym for 'best approximation of truth' (p.109). In his view the goal or destination of all social inquiry is to produce knowledge that is as close as possible to the truth. Wellington (2000, p. 201, cited in Opie, 2004)) defines validity as 'referring to the degree to which a method, a test or a research tool actually measures what it is supposed to measure'. Opie (2004) suggests that the best way to think of validity is to focus on the relationship between a claim and the result of a data-gathering process. Cohen, Manion, and Morrison (1994) distinguish between internal and external validity. Internal validity has to do with a claim being considered to be valid if it is a valid claim about the content of the research study. In my study all research content is reported in order for any claims made to be valid. External validity has to do with the extent to which the results of the study can be generalized to other people and settings. I noted earlier that the results of my study are not generalizable to other people and settings since this case study involved only two classrooms (Intervention and control classes) located in one school.

4.8 Credibility

Several strategies that can enhance the credibility of case study research are listed in Opie (2004) as follows:

- Data-gathering procedures are explained.
- Data is presented transparently and in ways that enable ready re-analysis.
- ‘Negative instances’ are reported.
- Biases are acknowledged.
- Fieldwork analyses are explained.
- The relationships between claims and supporting evidence are clearly expressed.
- Interpretation is distinguished from description.
- Procedures are used to check the quality of the data.

In my study, as outlined in this chapter, I have taken all these aspects into consideration. In order to deal with biases, colleagues from the WMC-P project were involved in the observation of the lessons and in offering feedback based on their observations. ‘Negative instances’ such as learners not being familiar with word problems and empty number line which resulted in a narrow focus of the study in some instances, are outlined. Data is initially described as per the empirical evidence then followed by an interpretation of the analyses. Learners’ written and video-taped responses were used to check the quality of data. Examples based on empirical evidence are provided in order to explain fieldwork analyses.

4.9 Research Ethics

I have complied with the University of the Witwatersrand’s ethical codes for research which required seeking approval from the Ethics Committee of the School of Education and the Gauteng Education Department to conduct the proposed study at a government school. Informed consent was obtained from all the relevant participants in the study. It was explained to other classes that the intervention could not take place in more than one class, but once a term discussion and feedback were scheduled with all the teachers involved in Grade 2 teaching in the school, as part of the broader WMC-P project remit. The teacher and learners were told that only my supervisor, the author of the 'word problems resource' (who is an advisor to the WMC-P project) and myself would have access to the pre and post-tests, field-notes, workbooks, and recorded information. I also communicated that I would be using pseudonyms in all reporting of the study so that no participants could be identified. All collected information has been stored safely and will be destroyed five years after the completion of the research. Lastly, participants were told that their participation was voluntary and without consequence for their broader school performance. The same information was communicated to the parents concerning their children. Parents were informed that their children were already familiar with video-taping as a result of their participation in the WMC-P project.

4.10 Conclusion

In this chapter, I have provided a description of the research design and given reasons for the choices made in the case study design and choice of participants for the study. I have provided reasons for the use of the particular theory that underpinned the analysis of the data collected and explained how this theory was used in my study. I have also dealt with issues of ethics, validity and reliability.

In the next chapter, I report on the key findings emanating from the analysis that led to the descriptive performance and models of the intervention and control group. Further in Chapter 6 I report on the emergence of the 'extend of structuring’ in relation to the ‘progression towards use of a more formal mathematical register’, consisting of codes and categories that represent stages of implementation towards a more sophisticated way of modelling and structuring in the context of South African resource-constrained primary schools.

CHAPTER 5

ANALYSIS OF OVERALL PERFORMANCE AND MODELS OF THE INTERVENTION AND CONTROL GROUPS

5.1 Introduction

As an introduction to the first of the data analysis chapters, the research questions addressed in the study are re-presented:

- What mathematizing processes related to additive relation problems were reflected by the learners across the intervention and control groups prior to the intervention?
- What mathematizing processes related to additive relation problems were reflected by the learners across the intervention and control groups post the intervention?
- How did the shifts (if any) made by the intervention group compare with the shifts (if any) made by the control group in overall performance and mathematizing processes at the end of the intervention?

In this chapter I begin by providing an analysis of the shifts in performance across the intervention and control groups and then follow this with an analysis of the mathematizing processes (models and strategies) related to additive relation problems that were evident in learners' responses to tasks and to interview questions, across the intervention and control groups both prior to and after the intervention.

In this chapter, the detailed analysis focuses on the seven common items, with examples sometimes drawn from the 'non-common' items in instances where these provided particularly telling examples of working, in order to illustrate a pattern of facility levels where these were similar across the non-common and common item set. Where facility levels were substantially different, likely reasons for this are discussed. The written test responses only provided access to the end results of the children's working rather than the process of assembling models and calculating answers. More of this processing evidence was available in the interview data, analysis of which follows in Chapter 6.

In section 5.2 I present a descriptive analysis of the performance of the Intervention and Control groups across the pre and post-tests. I go on to present an analysis of the kinds of models the children used to solve additive problems in the context of different word problem situations, distinguished on the basis of position of the unknown, which – as noted earlier in this study – emerged as a particularly strong demarcator of facility. The position of the unknown indicator was also linked to the formality of modes of representation. Changes in this formality between the pre and post-tests across different items are also discussed. The mathematization process involves both the formation of models and use of these formulated models and (strategies) to calculate answers (discussed in later sections of this chapter). Therefore, models and strategies as reflected in the empirical data are analysed and discussed.

As discussed in the methodology chapter these models are coded from the data as:

- Concrete – Entails manipulation of physical objects. Used for counting and calculation by counting strategies, e.g. Counters
- Pictorial – Iconic Images of everyday context-realistic depictions. Used the same as concrete but manipulated differently, (e.g.

Pictures). Also Indexical - Indexes everyday context using generic rather than realistic depictions of everyday context. Used for counting and for calculating-by-counting strategies, e.g. Tallies and Circles

- Symbolic Number-based (Number Line) - Use of numerals to represent numbers. Supports calculation without counting but could also be used for calculation-by-counting strategies.

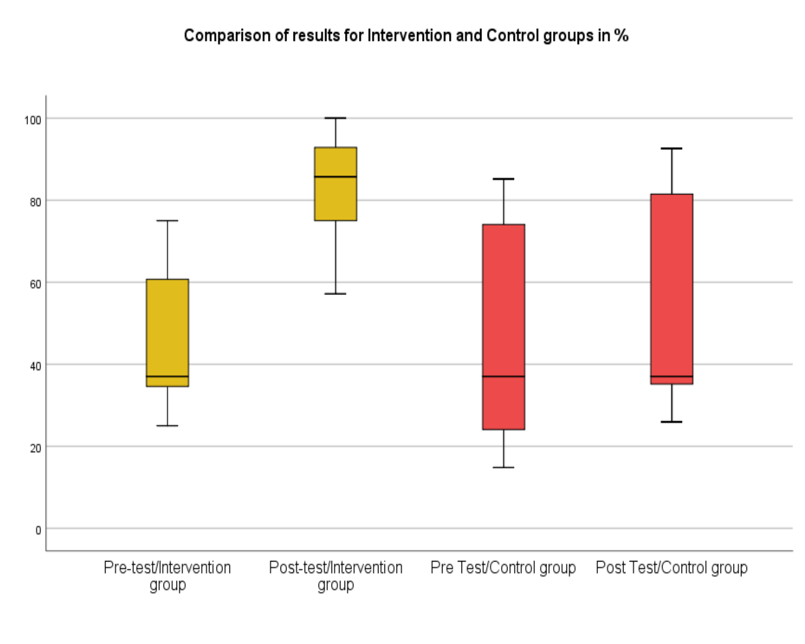
- Symbolic Syntactic (Number Sentence) - Use of mathematical notation to produce mathematical statements. Relies on known number facts and facts which can be derived with counting (Explicates some kind of relationship between quantities).

The processing of 'concrete' models will be discussed in detail in the interview-based analysis as these refer to manipulatives, which were not available during the written-test administration.

5.2 Descriptive analysis of overall performance for the Intervention and Control groups

I begin the presentation of findings with an overview of the performance of the intervention and control groups across the pre- and post-tests on the (n= 7) items that were common across both tests. I do this because the statistically significant differences between the intervention and control groups in post-test performance provides a useful point of entry into the other research questions' focus on mathematizing processes pre- and post- the intervention.

In Figure 5.1, the box and whisker plots show the profile of performance in the pre- and post-tests for the n=27 matched learners in the intervention group and the n=28 matched learners in the control group. Matched learners were drawn from the 37 and 36 intervention group learners who wrote the pre- and post-test, and the 38 and 35 control group learners who wrote both tests. The five-point summary figures are presented alongside the box and whisker plots



	Intervention Group		Control Group	
	Pre	Post	Pre	Post
Max	75	100	84	95
Q3	60	95	75	81
Median	38	86	38	36
Q1	35	75	23	35
Min	25	58	15	25

Figure 5. 1 Comparison performance results in the pre- and post- tests for the Intervention and Control Groups

The key finding indicated by the box and whisker plot is a substantial difference in the median post-test outcomes between the Intervention and Control groups (86% for the intervention group and 36% for the control group), contrasting with a pre-test median of 38% for both groups. An independent samples t-test confirmed the statistical significance of this result: the pre-test means of the two groups indicated no statistically significant difference, while the post-test means showed highly statistically significant differences at the 95% level. The difference in performance between the intervention group (M=5.79 [82.7%], SD=1.50 [21.4%]) and control group (M=3.89 [55.6%]; SD=1.98 [28.2]) was significant: t (53) = -4.01, p = 0.000.

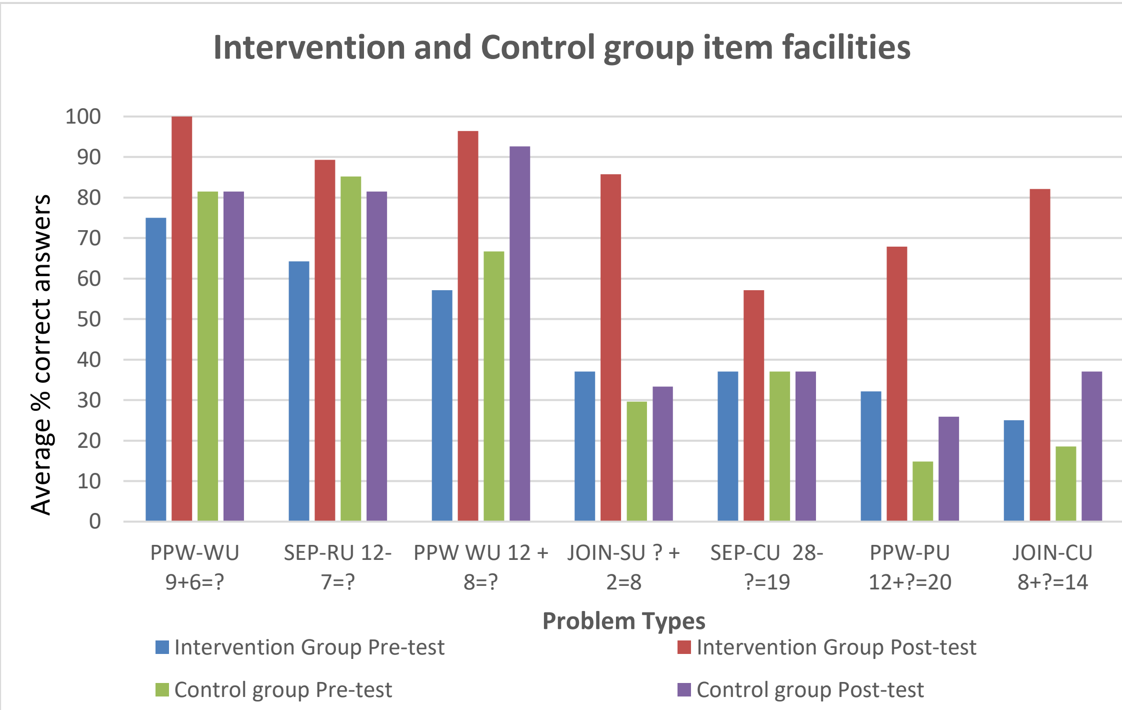
This result strongly suggested the value of trying to understand the detail of mathematizing processes underlying the differences in outcomes between the two groups. Before getting to this detail though, a few further comments about the similarities and differences in the performance profiles of the two groups are necessary in order to note differences in the pre-test make-up of the two classes that may also have featured in the differences in their profiles. This commentary is also used to point to ways in which the profiles of performance of the two groups changed in very different ways between the pre- and post-test, i.e. before and after the intervention lesson sequence.

Figure 5.1 points to important similarities and differences between the intervention and control groups' performance profiles in the pre- and post-tests. The range and interquartile range were greater for the control group in the pre-test than for the intervention group. The median for the intervention group from pre- to post-test increased by 48% whilst for the control group it decreased by 2%. The range for the intervention group from pre- to post-test decreased by 8% whilst for the control group it increased by 1%. The IQ range for both the intervention and the control groups from pre- to post-test decreased by similar amounts: 5% for the intervention group and 6% for the control group. What this suggests is that the intervention, as well as raising the average levels of attainment in the intervention class substantively between the pre- and post-test, achieved this with a closing of the attainment gap between the highest and lowest attainers in the intervention class at the range and IQ levels. By contrast, in the control class, the attainment gap between the highest and lowest attainers remained largely unchanged across the same period. This means that the intervention appears to have supported learning across the attainment range, rather than in more localized ways for particular sub-groups of learners.

The highly statistically significant difference in favour of the intervention group in the post-test led to an interest in understanding whether these differences were localised to a particular cluster of items within the seven common items set. This is the focus of the next section.

5.2.1 Common item level differences in performance between pre and post-tests

For this analysis, Intervention and Control group performance on the seven common pre- and post-test items was recorded on the basis of correct/incorrect answers. This produced facility levels for each item based on the matched learner responses. This data is presented in Figure 5.2.



Int. Mean facility:

Pre - (n=28) 46,8%

Post - (n=28) 82.7%

Con. Mean facility:

Pre- (n=27) 47.6%

Post- (n=27) 55.6%

Figure 5. 2 Intervention and Control Group Test Comparison Results

The outcomes in the overview above suggested that it would be useful to group the items into three key clusters, described and summarised in Table 5.1, with additional commentary below.

Table 5. 1 Overview of the three key clusters and item descriptions

	Cluster 1: ‘Result unknown high initial’	Cluster 2: ‘Part unknown high gains’	Cluster 3: ‘Part unknown lower gains’
Description	Items with ‘high’ facility levels in the pre-test (above 50% facility – which was also above the mean facility of around 47% across the Int & Con groups).	Items with ‘low’ (<50%) pre-test facility but high gains for the Int group (>40% point gains)	Items with ‘low’ (<50%) pre-test facility and lower gains for the Int group (<40% point gains)
Items	PPW-WU (9+6=?) SEP-RU (12-7=?) PPW-WU (12+8=?)	JOIN-SU (? +2=8) JOIN-CU (8+?=14)	SEP-CU (28-?=19) PPW-PU (12+?=20)

In looking for what could connect the items in Cluster 1, it was useful to refer to Carpenter et al. (1999)’s view that although PPW-WU problems are static problems where there is no action in the situation, when directly modelling these problems, children solve them similarly to Join- and Separate- Result Unknown (J-RU/S-RU) problems because the structure of the problem provides known numbers that can be operated with to find the missing ‘result’. These items all had initial high facility so the terminology used for this set of problems is ‘***Result Unknown High Initial***’.

Clusters 2 and 3 both involved missing numbers on the ‘operational’, rather than the ‘result’, side of the equals sign. Both Join items (described by Carpenter et al. (1999) as easier than Separate and PPW category items) fell in Cluster 2, while the Separate and PPW operational part unknown items fell in Cluster 3. However, it also needs to be noted that the number range in Cluster 2 items was a little lower than the number range in Cluster 3 items. I therefore tentatively refer to the second cluster as ***‘Part Unknown High Gains’*** and the third cluster as ‘***Part Unknown Lower Gains***’**.** These clusters form the frame for the analytical sections that follow. For each cluster I begin the section by presenting a tabular overview of performance and gains for that cluster for the intervention and control groups and follow this with a brief summary commentary. This commentary is followed by a tabulated overview of mathematizing processes for each cluster for the intervention group in order to understand changes in these processes related to the intervention.

5.3 Result Unknown Cluster high initial

Table 5. 2 Performance and gains of the Intervention and Control Groups

Groups	Test	PPW-WU (9 + 6 = ?)	PPW-WU (12 + 8=?)	SEP-RU (12 – 7=?)
Intervention	Pre-Test	75%	57.1%	64.3%
	Post-Test	100%	96.4%	89.3%
Gains		+25%	+39.3	+25%
Control	Pre-Test	81.5%	66.7%	85.2%
	Post-Test	81.5%	92.6%	81.5%
Gains		+0%	+25.9%	+3.7%

While there was high performance in this cluster for both groups in the pre-test, the intervention group gains far outstripped the gains for the control group, with a mean gain of 29,8% points for the former compared with a mean gain of 9,9% points for the latter. A further contrast was that, for the intervention group, substantial gains were seen across all three items in the cluster whereas for the control group, large gains were only seen on the 12+8=? item. While the intervention group began with a lower facility profile in this cluster in the pre-test, by the post-test, matched learners were doing better than the control group on all three items.

In the next section I present the mathematizing processes for each problem in this cluster for the pre- and post-tests for the intervention and control groups. I focus the discussion on the results of the intervention group in order to probe for what produced higher gains than were achieved by the control group.

Focusing on mathematization involved attending to the models constructed within the learners' problem-solving. I recorded the category of model evident in learners' pre- and post-test responses using a typology adapted from the work of Ensor et al. (2009), summarised earlier in this chapter. I then coded for the extent to which the model seen was 'appropriate' or 'inappropriate' in terms of match with the problem context. In a small number of cases, responses indicated formulations that included the correct answer but in a mathematically inappropriate syntax – for example, in one response to the PPW-WU (9+6=?) item, '15' was given as the answer with working that showed ten pictorial pictures of sweets and 6 pictorial picture of sweets'. The correct answer here suggested an implicit awareness of an appropriate model but this was not clearly illustrated in the response, and so these responses were coded as an 'unclear' model. The percentage of models in each of the appropriate/ inappropriate/ unclear categories linked to a correct answer was then recorded. In some learner scripts, a response included two models as shown in Figure 5.3 in an example that includes a pictorial and a horizontal number sentence model. In these cases, both models presented were counted. This meant that thenumberof models counted for each group did not necessarily tally with the number of children in the matched sample.

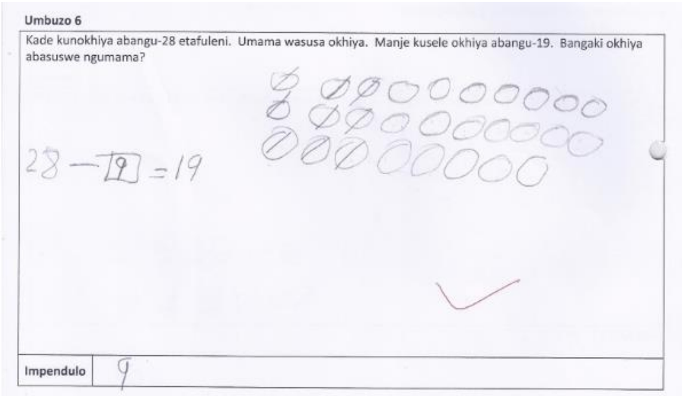


Figure 5. 3 Example of PiC and SS-HNS models

Summary Tables 5.3 to 5.5 show the model categories and prevalence, appropriateness of model and extent of correct answers for the items in the 'result unknown' cluster. The most prevalent model for the group within the test is highlighted. Each table includes a summary of key comments. These tables are followed by analysis that identifies key patterns in the nature of mathematization in this cluster of items. Differences between the intervention and control groups are noted, as well as differences between the two groups in patterns of shifts in mathematization between the pre- and post-tests. In all the tables, blank spaces reflect that there was no uptake for that model.

Table 5. 3

Pre-Test Part-Part Whole-Whole Unknown(9 + 6 =)

Group	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group (28 lrns)	**NoM 16**	**55.2 16/29**		**68.8 11/16**				
	PiC 9	31 9/29	77.8 7/9	85.7 6/7	11.1 1/9	0 0/1	11.1 1/9	100 1/1
	SN-NL 0	0 0/29						
	SS-HNS 4	13.8 4/29	100 4/4	100 4/4	0		0	
	N = 29 models							
C Group (27 lrns)	NoM 4	14.8 4/27		50				
	PiC 22	**81.5 22/27**	**91 20/22**	**100 20/20**	**4.5 1/22**	**n/a**	**4.5 1/22**	**n/a**
	SN-NL 0	0 0/27						
	SS-HNS 1	3.7 1/27	100 1/1	100 1/1	0		0	
	N = 27 models							

Post-Test Part-Part Whole-Whole Unknown (9 + 6 = ?)

Group	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group (28 lrns)	NoM 4	11.1 4/36		100 4/4				
	PiC 25	**69.4 25/36**	**96 24/25**	**100 24/24**	**4 1/25**	**0 0/1**	**0**	
	SN-NL 0	0						
	SS-HNS 7	19.4 7/36	100 7/7	100 7/7	0	n/a	0	
	N = 36 models							
C Group (27 lrns)	NoM 8	29.7 8/27						
	PiC 18	66.7 18/27	83.3 15/18	100 15/15	11.1 2/18	0 0/2	5.6 1/18	100 1/1
	SN-NL 0	0 0/27						
	SS-HNS 1	3.7 1/27	100 1/1	100 1/1	0		0	n/a
	N = 27 models							

Key: NoM: No Overt Model; PiC – Pictorial; SN-NL – Symbolic Number based (Number Line); SS-HNS – Symbolic Syntactic (Horiz Number Sentence)

Key comments

- C group showed a higher prevalence of Pic models than I group in the pre-test (81.5% in comparison to 31% of all models), while the I group showed higher 'NoM' prevalence (55.2% in comparison to 14.8% of all their pre-test models). I group showed slightly higher pre-test proportions of SS-HNS than the C group (13.8% vs 3.7%).
- For both groups, where models were attempted, they were largely appropriate and associated with correct answers
- I group showed substantial shift in the model category profile from pre- to post-test, with the percentage of NoM offers decreasing from 55.2% to 11.1% and the percentage of Pic models increasing from 31% to 69.4%. This contrasted with C group where there was a reduction in the prevalence of Pic models and an increase in the prevalence of NoM responses. Proportions of appropriate models and correct answers also improved for I group.

Table 5. 4

Group	Pre-Test Part - Part Whole-Whole Unknown (12 + 8 =?) Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group (28 lrns)	**NoM 18**	**64.3 18/28**		**39 7/18**				
	PiC 7	25 7/28	85.7 6/7	100 6/6	14.3 1/7	0 0/1		
	SN-NL 0	0 0/28	0					
	SS-HNS 3	10.7 3/28	100 3/3	100 3/3	0			
	N=28 models							
C Group (27 lrns)	NoM 8	29.6 8/27		75 6/8				
	PiC 19	**70,4 19/27**	**47.4 9/19**	**88.9 8/9**	**36.8 7/19**	**100 7/7**	**15.8 3/19**	**100 3/3**
	SN-NL	0	0					
	SS-HNS 0	0	0					
	N= 27 models							

Group	Post-Test Part-Part Whole-Whole Unknown (12 +8 = ?) Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group	NoM 5	17.2 5/29		100 5/5				
	PiC 19	**65,5 19/29**	**94.7 18/19**	**100 18/18**	**0 0/19**		**5.3 1/19**	**100 1/1**
	SN-NL 0	0	0					
	SS-HNS 5	17.2 5/29	80 4/5	100 4/4	20 1/5	0 0/1	0 0/5	
	N= 29 models							
C Group	NoM 8	29.6 8/27		75 8/8				
	PiC 18	**66,7 18/27**	**77.8 14/18**	**100 14/14**	**0 0/18**		**22.2 4/18**	**100 4/4**
	SN-NL	0	0					
	SS-HNS 1	3.7 1/27	100 1/1	100 1/1	0 0/1		0 0/5	
	N= 27 models							

Key: **NoM: No Overt Model; PiC – Pictorial; SN-NL – Symbolic Number based (Number Line); SS-HNS – Symbolic Syntactic (Horiz Number Sentence)**

Key comments	
	- C group continued to show a higher prevalence of Pic models than I group in the pre-test. I group continued to predominantly show NoM responses, Again, more SS-HNS responses in I group than C group. - I group showed substantial shift in the model category profile from pre- to post-test, with the percentage of NoM offers again decreasing and the percentage of Pic models increasing. For C group there was a reduction in the prevalence of Pic models. Proportions of appropriate models are above 80% and correct answers close to 100% for I group.

Table 5. 5

Group	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
Pre-Test Separate-Result Unknown (12 – 7 =?)								
I Group (28 lnrs)	**NoM 14**	**46.7 14/30**		**77.8**				
	PiC 12	40 12/30	75 9/12	77.8 7/9	25 3/12	0 0/3		
	SN-NL 0	0 0/30	0					
	SS-HNS 4	13.3 4/30	75 3/4	66.7 2/3	25 1/4	0 0/1	0	
	N=30 models							
C Group (27 lrns)	NoM 4	14.8 4/27		75 3/4				
	PiC 22	**74.1 22/27**	**68.2 15/22**	**100 15/15**	**18.2 4/22**	**0 0/4**	**13.6 3/22**	**33.3 1/3**
	SN-NL 0	0 0/27	0					
	SS-HNS 1	3.7 1/27	100 1/1	100 1/1	0		0	
	N= 27 models							

Group	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
Post-Test Separate Result Unknown (12 – 7 = ?)								
I Group (28 lnrs)	NoM 3	8.6 3/35		100 3/3				
	PiC 24	**68,6 24/35**	**83.3 20/24**	**100 20/20**	**12.5 3/24**	**0 0/3**	**4.2 1/35**	**100 3/3**
	SN-NL 1	3 1/35	100 1/1	100 1/1	0		0	
	SS-HNS 7	20 7/35	85.7 6/7	100 6/6	14.3 1/7	0	0	
	N= 35 models							
C Group (27 lrns)	NoM 9	33.3 9/27		55.6 5/9				
	PiC 17	**63 17/27**	**76.5 13/17**	**100 13/13**	**11.8 2/17**	**0 0/2**	**11.8 2/17**	**100 2/2**
	SN-NL 0	0	0					
	SS-HNS 1	3.7 1/27	100 1/1	100 1/1	0		0	
	N= 27 models							

Key: **NoM: No Overt Model; PiC – Pictorial; SN-NL – Symbolic Number based (Number Line); SS-HNS – Symbolic Syntactic (Horiz Number Sentence)**

Key comment	
	- C group continued to show a higher prevalence of Pic models than I group in the pre-test. I group continued to show NoM as main response, Again, more SS-HNS responses in I group that C group. - I group showed substantial shift in the model category profile from pre- to post-test, with the percentage of NoM offers again decreasing and the percentage of Pic models increasing. C group pattern as in first item – decrease in Pic model prevalence; increase in NoM prevalence. Proportions of appropriate models and correct answers close to 100% for both groups in post-test, but higher for I group. I group, again, increased the number of models offered from pre- to post-, contrasting with C group where number of models stayed constant.

5.3.1 Mathematization Analysis: Result unknown cluster

Several patterns of interest emerge in the above analysis of mathematization processes underlying the findings of both high initial facility in this cluster, and the larger gains seen for the intervention group across these three items. Firstly, while the overview analysis of performance across all the items indicated that the intervention and control groups began with very similar means (46,8% and 47,6% respectively), performance in this cluster points to higher pre-test facilities for the control group than for the intervention group. In the post-test though, the intervention group outperformed the control group with higher facilities on all three items.

The mathematization analysis also indicates pre-test differences between the intervention and control groups with the control group showing a greater prevalence of Pictorial models across all three items, contrasting with the prevalence of 'No overt model' responses for the intervention group. Across all three items, the most prevalent model for the intervention group shifted from No overt model to Pictorial models between the pre- and post-test, with more than double the number of Pictorial models seen in the post-test compared to the pre-test on two items and a percentage shift from 40% to 68% on the third item. The proportions of appropriate models and correct answers from these models for the intervention group exceeded the proportions of appropriate models and correct answers in the responses from the control group. The intervention group responses indicate a slightly higher prevalence of Symbolic Syntactic-Horizontal Number Sentence models in the pre-test in comparison to the responses of the control group. The prevalence of this model increased in the post-test responses for all three items for the intervention group, while remaining near to absent in the control group across the pre- and post-tests.

What the mathematization analysis in combination with the overview of facility for the 'result unknown' cluster indicates is that competence with setting up appropriate pictorial models broadened substantially within the intervention group between the pre- and post-test. By contrast, within the control group, competence with setting up appropriate Pictorial models expanded on only one of the three items (PPW-WU: 12+8=?). Additionally, within the intervention group, 7/28, 4/28 and 6/28 learners respectively were able to correctly answer the three items using horizontal number sentences in the post-test. In the control group, only 1/27 matched learners was able to correctly answer each of the three items using this model.

For the intervention group, expanding their capacity to set up a model of the situation - horizontal mathematization in RME terms - focused on making initial sense of a situation (Gravemeijer and Stephan, 2002). This expanded capacity appears to have been critical to improving outcomes in this cluster. The lowest facility item in the cluster for the intervention group in the pre-test was the (PPW-WU: 12+8=?) item with a 57,1% facility, compared to a 66,7% lowest level facility on this item for the control group. In the post-test, the lowest facility item for the intervention group was the (SEP-RU: 12-7=?) item with a 89,3% facility, compared with a 81,5% facility on this item (and the PPW-WU: 9+6=?) item for the control group. This may reflect Carpenter et al.'s (1999) finding of SEP-RU problems being somewhat harder to solve because they involve subtraction and counting backwards rather than counting forwards.

While the South African literature has emphasised the need to support moves into symbolic working (Ensor et al., 2009), the mathematization findings in this cluster suggest that this move may need to be preceded by attention to helping children to make sense of the situation and set up an appropriate model using their most accessible mode of representation. This finding, in what Carpenter et al., (1999) have acknowledged to be the easiest class of problems, provides warning signs of the need for more attention to horizontal mathematization in the harder problem classes, discussed in subsequent sections.

I now turn to the performance results for the next cluster, which is the Part Unknown Cluster High Gains.

5.4 Part Unknown Cluster High Gains

Table 5. 6 Performance and gains of the Intervention and Control Groups

Groups	Test	JOIN-SU ?+2=8	JOIN-CU 8+?=14
Intervention	Pre-Test	37%	25%
	Post-Test	85.7%	82.1%
Gains		+48.7%	+57.1%
Control	Pre-Test	29.6%	18.5%
	Post-Test	40.7%	37%
Gains		+11.1%	+18.5%

The intervention group gains far outstripped the gains for the control group, with a mean gain of 52.9% points for the former compared with a mean gain of 14.8% points for the latter. A further contrast is that although the gap between the performance of the two groups in the pre-test is not vast, and although there is a slight improvement in performance by the control group, by the post-test, the intervention group reflected a much higher performance than the control group, where facility remained at less than 50% on both items.

Summary Tables 5.7 and 5.8 show the model categories and prevalence, appropriateness of model and extent of correct answers for the items in the 'part-unknown high gains' cluster. As for the previous tables, the most prevalent model for the group is highlighted, followed by a summary of key comments and an analysis of key patterns in the nature of mathematization in this cluster of items. In all the tables, blank spaces reflect that there was no uptake for that model.

Table 5. 7

Group	Pre-Test –Join-Start Unknown (? + 2 = 8)							
	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group (28 lrns)	**NoM 22**	**76 22/29**		**36.4 8/22**				
	PiC 5	17.2 5/29	40 2/5	100 2/2	60 3/5	0 0/1	0 0/5	n/a
	SN-NL 0	0 0/29						
	SS-HNS 2	7 2/29	50 1/2	100 1/1	50 1/2	0 0/1	0 0/5	
	N = 29 models							
C Group (27 lrns)	NoM 12	44.4 12/27		33.3 4/12				
	PiC 15	**55.6 15/27**	**13.3 2/15**	**100 2/2**	**73.3 11/15**	**0 0/11**	**13.3 2/15**	**100 2/2**
	SN-NL 0	0 0/27						
	SS-HNS 0	0 0/27						
	N = 27 models							

Group	Post-Test Join-Start Unknown (? + 2 = 8)							
	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group (28 lrns)	NoM 6	18.2 6/33		83.3 5/6				
	PiC 14	**42.4 14/33**	**50 7/14**	**100 7/7**	**21.4 3/14**	**0 0/3**	**28.6 4/14**	**100 4/4**
	SN-NL 4	12.1 4/33	75 3/4	100 3/3	25 1/4	0 0/1	0 0/4	
	SS-HNS 9	27.3 9/33	88.9 8/9	100 8/8	11.1 1/9	0 0/1	0 0/9	
	N = 33 models							
C Group (27 lrns)	NoM 9	32.1 9/28		0				
	PiC 17	**60.7 17/28**	**52.9 9/17**	**55.6 5/9**	**41.2 7/17**	**0 0/7**	**5.9 1/17**	**100 1/1**
	SN-NL 0	0 0/28	0 0/0					
	SS-HNS 2	7.1 2/28	50 1/2	100 1/1	50 1/2	0 0/1	0 0/2	
	N = 28 models							

Key: NoM: No Overt Model; PiC – Pictorial; SN-NL – Symbolic Number based (Number Line); SS-HNS – Symbolic Syntactic (Horiz Number Sentence)

Key comment s

- C group showed a higher prevalence of Pic models than I group in the pre-test (55.6% in comparison to 17.2% of all models), while the I group showed higher 'NoM' prevalence (76% in comparison to 44.4% of all their pre-test models). I group again showed slightly higher pre-test proportions of SS-HNS than the C group (7% vs 0%).
- For both groups, where models were attempted, they were largely inappropriate and associated with incorrect answers
- I group showed substantial shifts in the model category profile from pre- to post-test, with the percentage of NoM for I offers decreasing from 76% to 18.2% , the percentage of PiC models increasing from 17.2% to 42.4%, percentage of SN-NL increasing from 0% to 12.1% and percentage of SS-HNS increasing from 7% to 27.3%. C group showed similar shifts to I group with slightly fewer percentage points and no proportions of SN-NL. Proportions of appropriate models and correct answers close to/at 100% for I group.For C group approp PiC models only produced correct answers in 50% of cases, while for I group, this proportion went up to 100% in post-test. In I group, 9/33 (27%) models involved SS-HNS all linked with correct answers, while in C group this proportion was only 2/28 (7%). Also greater prevalence of 'unclear' models for I group always with correct answers suggesting that these children have understood the idea but not the representational format.

Table 5. 8

	Pre-Test –Join-Change Unknown (8 + ? =14)							
	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group (28 lrns)	**NoM 16**	**55.2 16/29**		**31.3 5/16**				
	PiC 10	34.5 10/29	0 0/10	0 0/0	90 9/10	0 0/9	10 1/10	100 1/1
	SN-NL 0	0 0/29						
	SS-HNS 3	10.3 3/29	33.3 1/3	100 1/1	66.7 2/3	0 0/2	0 0/3	
	N = 29 models							
C Group (27 lrns)	NoM 7	25.9 7/27		14.3 1/7				
	PiC 20	**74.1 20/27**	**0 0/20**		**80 16/20**	**0 0/16**	**20 4/20**	**100 4/4**
	SN-NL 0	0 0/27						
	SS-HNS 0	0/27						
	N = 27 models							

	Post-Test Join-Change Unknown (8 + ? =14)							
	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group (28 lrns)	NoM 5	15.6 5/32		80 4/5				
	PiC 17	**53.1 17/32**	**58.8 10/17**	**100 10/10**	**23.5 4/17**	**0 0/4**	**17.6 3/17**	**100 3/3**
	SN-NL 1	3.1 1/32	100 1/1	100 1/1	0 0/1		0 0/1	
	SS-HNS 9	28.1 9/32	88.9 8/9	100 8/8	11.1 1/9	0 0/1	0 0/9	
	N = 32 models							
C Group (27 lrns)	NoM 9	33.3 9/27		11.1 1/9				
	PiC 18	**66.7 18/27**	**22.2 4/18**	**75 3/4**	**55.6 10/18**	**0 0/10**	**22.2 4/18**	**100 4/4**
	SN-NL 0	0 0/27						
	SS-HNS 1	3.7 1/27	100 1/1	100 1/1	0 0/1		0 0/1	
	N = 27 models							

Key: NoM: No Overt Model; PiC – Pictorial; SN-NL – Symbolic Number based (Number Line); SS-HNS – Symbolic Syntactic (Horiz Number Sentence)

Key comme nts	- C group showed a higher prevalence of PiC models than I group in the pre-test (74.1% in comparison to 34.5% of all models), while the I group showed higher 'NoM' prevalence (55.2% in comparison to 25.9% of all their pre-test models). I group showed proportions of SS-HNS and none for C group (10.3% vs 0%). - For both groups, where models were attempted, they were largely inappropriate and associated with incorrect answers. - I group showed substantial shifts in the model category profile from pre- to post-test, with the percentage of NoM offers decreasing from 55.2% to 15.6% and the percentage of PiC models increasing from 34.5% to 53.1%. This contrasted with C group where there was a reduction in the prevalence of PiC models from 74.1% to 66.7% and an increase in the prevalence of NoM responses from 25.9% to 33.3%. Proportions of appropriate models and correct answers also improved for I group close to/at 100%, contrasting with C group where most models were inappropriate and associated with incorrect answers. Nearly 1/3 models are SS-HNS for I group – a high increase from pre- to post- test contrasting with C group where the increase is not seen.

5.4.1 Mathematization Analysis: Part Unknown high gains cluster

Performance in this cluster points to marginally higher pre-test mean facility for the intervention group than for control group (Intervention group mean: 46.8%; Control group mean: 47.6%), and therefore low facility for both groups. In the post-test however, the intervention group substantially outperformed the control group on both items, with mean post-test facility reflecting this starkly (Intervention group mean: 82.7%; Control group mean: 55.6%).

The mathematizing analysis indicated pre-test differences between the intervention and control groups, with the control group once again showing a greater prevalence of Pictorial models across both items, contrasting with the prevalence of 'No overt model' responses for the intervention group. Across both items, the most prevalent model for the intervention group shifted, as in the last cluster, from 'No overt model' to Pictorial models between pre- and post-test, with more than double the number of Pictorial models seen in the post-test compared to the pre-test (5/29 models to 14/33 models) for the first item (Join-Start Uknown ? + 2 = 8) and with a high increase (10/29 to 17/32) for the second item (Join-Change Unknown 8 + ? =14). Of interest, there was not a single appropriate Pictorial model in either group on the pre-test for the second item. In the post-test, in contrast to pre-test mathematization, the intervention group responses showed that more than half of all the Pictorial models were appropriate and that all of the appropriate models were linked with correct answers. For the control group, appropriate Pictorial model proportions improved to above 50% for the first item but only to 22% for the second item. Once again, the proportion of appropriate and correct models for the intervention group exceed the proportions of appropriate models and correct answers in the responses from the control group.

As in the first cluster, the intervention group responses indicated a slightly higher prevalence of Symbolic Syntactic-Horizontal Number Sentence models in the pre-test in comparison to the control group. The prevalence of this model increased in the post-test responses for both items for the intervention group, while remaining near absent in the control group across the pre- and post-tests.

Similarly to findings for the first cluster above, the mathematization analysis, in combination with the overview of facility for the 'part-unknown' cluster, indicates that competence with setting up appropriate pictorial models broadened substantially within the intervention group between the pre- and post-test. By contrast, within the control group, competence with setting up appropriate Pictorial models improved on both items. As in the first cluster, the prevalence of appropriate models and correct answers across symbolic number line and number sentence models improved for the intervention group (1 correct answer to 11 correct answers in this category on first item and 1 correct answer to 9 correct answers on the second item). As before, the proportion of appropriate models linked to correct answers remained low and unchanged in the control group (0 to 1 correct answer across both items). Additionally, within the intervention group, 1/28, 3/28 matched learners respectively were able to correctly answer the two items using a number line in the post-test and none of the control group matched learners were able to correctly answer either of these two items using this model. Also within the intervention group, 8/28, 8/28 matched learners respectively were able to correctly answer both items using horizontal number sentences in the post-test. In the control group, only 1/27 and 1/27 matched learners respectively were able to correctly answer each of the three items using this model.

For the intervention group, expanding their capacity to set up a model of the situation - horizontal mathematization in RME terms - focused on making initial sense of a situation (Gravemeijer & Stephan, 2002). This expanded capacity appears to have been critical to improving outcomes in this cluster. The lowest facility item in the cluster for the intervention group in the pre-test was the (JOIN-Change Unknown: 8 + ?=14) item with a 25% facility, compared to a 18.5% lowest level facility on this item for the control group. In the post-test, the intervention group improved to 82.1% contrasted with an insignificant increase to 37% for the control group. This finding contrasts with some of the findings of Carpenter et al. (1999) which reflect that children find Join-Change Unknown problems easier to solve Join-Start Unknown because of the position of the unknown.

Similar to the findings for the previous cluster, the mathematization findings in this cluster suggest that attention to helping children to make sense of a situation and to set up an appropriate model, using their most accessible mode of representation, need to precede any other move.

I now turn to the performance results for the next cluster, which is the Part Unknown Cluster – Lower Gains.

5.5 Part Unknown Cluster - Lower Gains

Table 5. 9 Performance results and gains of the intervention and control groups

Groups	**Test**	**Separate-Change Unknown (28-?=19)**	**Part Part Whole -Part Unknown (12+?=20)**
Intervention	Pre-Test	37%	32.1%
	Post-Test	57.1%	67.9%
Gains		+20.1%	+35.8%
Control	Pre-Test	37%	14.8%
	Post-Test	37%	26%
Gains		+0%	+11.2%

The intervention group gains far outstripped the gains for the control group, with a mean gain of 28% points for the former compared with a mean gain of 5.1% points for the latter. While pre-test facilities were identical for the first item in this cluster, the intervention group began with stronger facility on the second item. However, the intervention group also reflected a much higher gain than the control group which reflected no gains at all.

Below, the same format as for the previous clusters is used to present summary Table 5.10 and Table 5.11 and to present and analyse patterns of shifts in mathematization between the pre- and post-tests. In all the tables, blank spaces reflect that there was no uptake for that model.

Table 5. 10

Group	Pre-Test –Separate Change Unknown (28 - ? =19)							
	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group (28 lrns)	**NoM 15**	**51.7 15/29**		**26.7 4/15**				
	PiC 12	41.4 12/29	58.3 7/12	85.7 6/7	41.7 5/12	0 0/5		
	SN-NL 0	0/29						
	SS-HNS 2	7 2/29	50 1/2	100 1/1	50 1/2	0 0/1		
	N = 29 models							
C Group (27 lrns)	NoM 6	22.2 6/27	n/a	0				
	PiC 21	**77.8 21/27**	**23.8 5/21**	**100 5/5**	**52.4 11/21**	**0 0/11**	**23.8 5/21**	**100 5/5**
	SN-NL 0	0 0/27						
	SS-HNS 0	0 0/27						
	N = 27 models							

Group	Post-Test Separate-Change Unknown (28 - ? =19)							
	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group (28 lrns)	NoM 3	8.6 3/35		66.7 2/3				
	PiC 24	**68,6 24/35**	**62.5 15/24**	**100 15/15**	**37.5 9/24**	**0 0/9**	**0 0/24**	
	SN-NL 1	2.9 1/35	0 0/1	0 0/0				
	SS-HNS 7	20 7/35	42.9 3/7	100 3/3				
	N = 35 models							
C Group (27 lrns)	NoM 8	29.6 8/27		12.5 1/8				
	PiC 19	**70.4 19/27**	**36.8 7/19**	**71.4 5/7**	**36.8 7/19**	**0 0/7**	**26.3 5/19**	**100 5/5**
	SN-NL 0	0 0/27						
	SS-HNS 0	0 0/27						
	N = 27 models							

Key: NoM: No Overt Model; PiC – Pictorial; SN-NL – Symbolic Number based (Number Line); SS-HNS – Symbolic Syntactic (Horiz Number Sentence)

Key comments

- C group showed a higher prevalence of Pic models than I group in the pre-test (77.8% in comparison to 41.4% of all models), while the I group showed higher ‘NoM’ prevalence (51.7% in comparison to 22.2% of all their pre-test models). I group showed slightly higher pre-test proportions of SS-HNS than the C group (7% vs 0%).
- For I group where in the post-test models were attempted, they were largely appropriate and associated with correct answers, in contrast with C group where models were largely inappropriate with incorrect answers. .
- I group showed substantial shift in the model category profile from pre- to post-test, with the percentage of NoM offers decreasing from 51.7% to 8.6% and the percentage of Pic models increasing from 41.4% to 68.6%. This contrasted with C group where there was a reduction in the prevalence of Pic models and an increase in the prevalence of NoM responses. Proportions of appropriate models and correct answers also improved for I group close to/at 100%

Table 5. 11

Group	Pre-Test –Part-Part Whole-Part Unknown (12 + ? =20)							
	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group (28 lrns)	**NoM 19**	**63.3 19/30**		**32 6/19**				
	PiC 8	26.7 8/30	37.5 3/8	100 3/3	50 4/8	0 0/4	12.5 1/8	100 1/1
	SN-NL 0	0 0/30						
	SS-HNS 3	10 3/30	66.7 2/3	100 2/2	33.3 1/3	0 0/1		
	N = 30 models							
C Group (27 lrns)	NoM 8	29.6 8/27		0 0/8				
	PiC 19	**70.4 19/27**	**15.8 3/19**	**100 3/3**	**84.2 16/19**	**0 0/16**	**0 0/19**	
	SN-NL 0	0 0/27						
	SS-HNS 0	0 0/27						
	N = 27 models							

Group	Post-Test Part-Part Whole-Part Unknown (12 + ? =20)							
	Model	% prevalence of model	% appropriate models	% correct answer with approp model	% inappropriate models	% correct answer with inapprop model	% unclear model	% correct answer with unclear model
I Group (28 lrns)	NoM 5	16.1 5/31		80 4/5				
	PiC 18	**58.1 18/31**	**61.1 11/18**	**100 11/11**	**33.3 6/18**	**0 0/6**	**5.6 1/18**	**100 1/1**
	SN-NL 1	3.2 1/31	100 1/1	100 1/1				
	SS-HNS 7	22.6 7/31	57.1 4/7	100 4/4	42.3 3/7	0 0/3	0 0/7	
	N = 31 models							
C Group (27 lrns)	NoM 10	35.7 10/28		0 0/10				
	PiC 17	**60.7 17/28**	**29.4 5/17**	**80 4/5**	**58.8 10/17**	**0 0/10**	**11.8 2/17**	**100 2/2**
	SN-NL 0	0 0/28						
	SS-HNS 1	3.6 1/28	0 0/1		100 1/1	0 0/1	0 0/1	
	N = 28 models							

Key: NoM: No Overt Model; PiC – Pictorial; SN-NL – Symbolic Number based (Number Line); SS-HNS – Symbolic Syntactic (Horiz Number Sentence)

Key comments

- C group showed a higher prevalence of Pic models than I group in the pre-test 70.4% in comparison to 26.7% of all models), while the I group showed higher 'NoM' prevalence (63.3% in comparison to 29.6% of all their pre-test mode. I group showed higher pre-test proportions of SS-HNS whilst for the C group it was non-existent (10% vs 0.
- For both groups, where models were attempted, they were largely appropriate and associated with correct answers.
- I group showed substantial shift in the model category profile from pre- to post-test, with the percentage of NoM offers decreasing from 63.3% to 16.1% and the percentage of Pic models increasing from 26.7% to 58.1%. This contrasted with C group where there was a reduction in the prevalence of Pic models and an increase in the prevalence of NoM responses. Proportions of appropriate models and correct answers also improved for I group.

5.5.1 Mathematizing analysis: Part unknown lower gains cluster

In the post-test, the intervention group substantially outperformed the control group on both items, with there being no improvement on one item for the control group.

The mathematizing analysis indicates pre-test differences between the intervention and control groups with the control group showing a greater prevalence of Pictorial models across both items. By contrast, 'No overt model' responses were more prevalent for the intervention group. Across both items, the most prevalent model for the intervention group shifted from 'No overt model' to Pictorial models between pre- and post-test, with more than double the number of Pictorial models seen in the post-test compared to the pre-test for the second item (Part-Part-Whole-Part Unknown 12 + ? =20 = 8) and with a high increase for the first item (Separate-Change Unknown 28 - ? =19). The proportion of appropriate and correct models for the intervention group exceeded the proportion of appropriate models and correct answers in the responses from the control group.

The intervention group responses indicate a higher prevalence of Symbolic Syntactic-Horizontal Number Sentence models in the pre-test in comparison to the control group. The prevalence of this model and small increase in prevalence of 'Symbolic Number-based-Number Line' in the post-test responses for both items for the intervention group, while remaining near absent in the control group across the pre- and post-tests.

The mathematization analysis, in combination with the overview of facility for the 'part-unknown lower gains' cluster, indicates that competence with setting up appropriate pictorial models broadened substantially within the intervention group between the pre- and post-test. By contrast, within the control group, there was a decrease in the prevalence of Pictorial (PiC) models and an increase in 'No Overt Model' competence. Additionally, within the intervention group, 3/28, matched learners was able to correctly answer the two items using a number sentence in the post-test and none of the control group matched learners were able to correctly answer either of these two items using this model.

For the intervention group, expanding their capacity to set up a model of the situation - horizontal mathematization in RME terms - focused on making initial sense of a situation (Gravemeijer & Stephan, 2002). This expanded capacity appears to have been critical to improving outcomes in this cluster. The lowest facility item in the cluster for the intervention group in the pre-test was the (Part Part Whole-Part Unknown (PPW-PU): 12+?=20) item with a 32.1% facility, compared to a 14.8% lowest level facility on this item for the control group. In the post-test, the lowest facility item for the intervention group was the (Separate-Change Unknown: 28 - ?=19) item with a 57.1% facility, compared with a 37% facility on this item for the control group. This contrasts with Carpenter et al.'s (1999) finding of PPW-PU problems being somewhat harder to solve because they are static in nature. Lastly, there is a smaller increase in this cluster to setting up appropriate PiC models for the intervention group in the post-test. Appropriate PiC models were always linked with correct answers. There was less prevalence of symbolic models in this cluster.

5.6 Summary comment on mathematization across all three clusters

The key shifts for the intervention group, in contrast to the control group, in all three clusters are:

- The increased prevalence of pictorial models which are largely appropriate and associated with correct answers. The prevalence of these models indicates learners' increased willingness to set up models in the post-test as opposed to the pre-test. This willingness suggests, in turn, an increased proficiency with making sense of the situations.

- An increase in the uptake of more formal models, such as the Horizontal number sentence (HNS), which is largely appropriate and associated with correct answers. Some of the limitations that need to be noted are that although there is some evidence of use of the number line, it is very limited in spite of the attention to number lines in the intervention.
- Evidence of take up of pictorial models being lower in the 'Part unknown lower gains' cluster. This suggests that horizontal mathematization remains at an early stage, with learners more able to set up appropriate pictorial models on the problems that Carpenter et al. (1999) describe as easier.

CHAPTER 6

ANALYSIS AND FINDINGS FOR SHIFTS IN THE EXTENT OF AWARENESS OF STRUCTURE

6.1 Introduction

The analysis in Chapter 5 pointed to some key points of interest where substantial gains in performance for the intervention group were found to be underpinned by an increasing prevalence, in particular, of appropriate pictorial models as well as an increased incidence of symbolic models. Both these findings suggest some evidence of gains being related to moves BETWEEN models occurring in the time between the pre- and post-test. Closer inspection of the models evident in learners' working across the pre-and post-tests also indicated moves WITHIN models as well as BETWEEN them, with literature pointing to it being possible to view both of these as progression moves.

In the first part of this chapter, moves within models for each of the clusters identified are analysed in terms of 'extent of structuring'. To recap briefly on the literature, extent of structuring refers to awareness of the way that quantities in a problem are related to each other as seen in the construction of an appropriate model (Venkat et al., 2019). This extent of structuring is cross-referenced with the moves between models for each of the clusters presented in the previous chapter, analysed in terms of the progression towards increasingly abstract models of representation presented in Ensor et al.'s (2009) work linked with the writing on moves towards more formal mathematics. This work is linked with writing on moves towards more formal mathematical registers seen in the work of Setati and Adler (2000) which connects with Ensor et al.'s (2009) work on symbolic modes.

Operational definitions and a refined coding system were formulated from the learners' responses to the tests (pre-test and post-test). In order to illustrate development levels of structure I discuss representative examples of learners' responses to the tasks presented in the tests. The analysis centres on how representations conform to structural features such as quantitative relationships and use of formal notation.

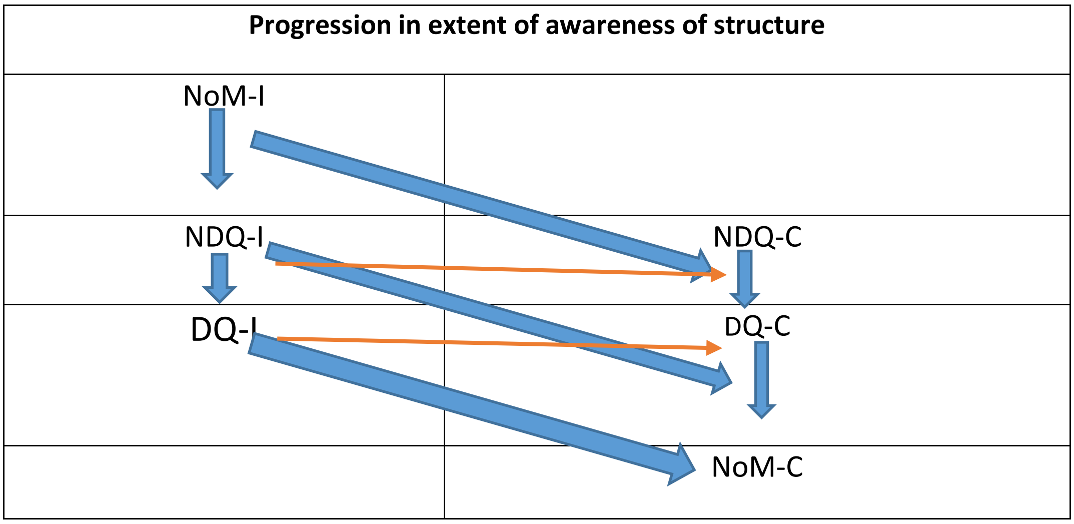
Operationalizing the framework

In the development of Tables 6.1 and 6.2 below, I have distinguished between instances where the quantities reflected the quantities given in the problem, and those where this was not the case, because this is important within the emphasis in the definition of HM on making sense of the situation. Next I present some theoretical positions that underpin the progression in extent of awareness of structure. I have decided to combine the modes of representation discussed in Chapter 2 with the categories for progression in the 'extent of awareness of structure' presented in the four bullet points. In Table 6.1, all of the arrows represent potential progression indicators when looking at learner responses across the pre- and post-tests. Given that this progression was studied in relation to progression of modes of representation, Table 6.2 includes examples of these. The categories of progression are as follows:

- An inappropriate model or no model with an incorrect answer suggests lack of sense making in horizontal mathematization terms, and thus, indicates no evidence of being able to start working on the problem (No Overt Model- Incorrect : NoM-I)

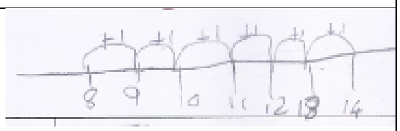
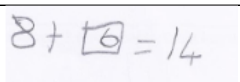
- Moves to an appropriate model represents the beginnings of sense-making of the situation. At the first level, an appropriate model would not distinguish quantities. Thus, ‘no distinguishing of quantities’ (NDQ) with either an incorrect (I) or a correct (C) model, abbreviated as NDQ-I and NDQ-C respectively, are both better than ‘No overt Model’ (NoM)-I
- Models where the quantities are distinguished appropriately point to explicit marking of the relationship between the quantities, even if the subsequent work with the model suggests that the vertical mathematization (VM) element has not played through. (So ‘distinguished quantities-incorrect/correct’ (DQ-I/C) are both better than ‘no distinguished quantities-Incorrect (NDQ-I). ‘Distinguished quantities-correct’ (DQ-C) is interpreted, following Mulligan and Mitchelmore’s (2009) work on explicit structure awareness, as better in terms of explicit marking of the relationship between quantities than ‘no distinguished quantities’ NDQ-C, acknowledging that it could be the case that the relationship between quantities may have been internalised but not explicitly marked in ‘no distinguished quantities’ (NDQ-C) cases.
- In responses with No model with a correct answer, the assumption being made is that a model that distinguishes the quantities with awareness of the relationship between them has been internalised (so ‘no overt model-correct’ (NoM-C) is better than both ‘distinguished quantities-incorrect’ - DQ-I/C)

Table 6. 1 Progression in extent of awareness of structure



This move to including attention to whether, or not, quantities were distinguished in model leads to a more differentiated model than the earlier Concrete-Pictorial-Symbolic Number Based-Symbolic Syntactic mode of representation towards increasingly formal modes because learner responses suggested finer-grained steps towards more formal working. Specifically, given the prevalence of pictorial models, the pictorial category was broken up into two sub-categories: completely pictorial and pictorial with some inclusion of formal mathematical register, for example, '+' and '-' used with pictorial representations of quantities. The progression is thus interwoven with Ensor et al. (2009) progression in terms of modes of representation, presented in Chapter 2. This means that, for example, on the one hand that distinguished quantities in the pictorial register were interpreted as more abstract that 'non-distinguished' quantities in the pictorial register, and therefore, at a higher progression level, and on the other hand, more concrete and therefore, at a lower progression level, than distinguished quantities in more increasingly symbolic, or formal registers. In table 6.2 there are descriptions and examples of the progression into a more formal register, cross-referenced with the progression between lack of distinguishing of quantities (NDQ) and distinguished quantities (DQ). Only categories present in the empirical data are illustrated. In the next table, table 6.3 I present examples of progression into more formal mathematical register.

Table 6. 2 Progression into more formal mathematical register

Progression into use of more formal mathematical register	**Description**	**Example**
Completely pictorial representations-PiC (Papic et al.)	Children use representations such as tally marks, circles or pictures. Iconic and indexical are grouped together because if a child is presented with a 'pie' problem and they draw circles as a representation, it is difficult to tell whether it is indexical or iconic.	 
Some inclusion of formal mathematical register (SFMR)	Children use pictorial representations that include some symbolic representation of mathematical operations.	
Complete use of formal mathematical register (CFMR-NL):	Children use number lines etc.	
Complete use of formal mathematical register (CFMR-NS):	Children use number sentences to solve word problems.	

This trajectory expands the modes of representation explained in Chapter 5. The distinguishing of quantities of quantities was, in some modes, intertwined with moves to the inclusion of some elements of more formal mathematical registers but not always. Comparing the second example in the completely pictorial category in table 6.2 with 'Some Form of Mathematical Register' (SFMR) example, we can see that inclusion of symbolic operation symbols incorporated the distinguishing of quantities, but the distinguishing of quantities also occurred, in some cases, without any inclusion of more formal registers. Thus, the distinguishing of quantities and the inclusion of more formal registers were maintained as distinct categories in the analysis.

It is important to note that the modes of representation present in each cluster were based on the existence of the category in the empirical data also in order to keep the tables in their simplest form, hence in some instances such as in cluster 1 'complete form of mathematical register-Number sentence' (CFMFR-NS) does not have an 'no distinguished quantities' (NDQ) option. Next I present tabular summaries, per cluster, of the pre-and post-test analysis for the extent of the awareness of structure for the intervention and control groups.

As in the previous chapter, the items are grouped into the three key item clusters in order to establish whether there is a difference in the extent of structure in relation to the clusters or not, with the clusters re-iterated in table 6.3.

Table 6.3 Items grouped into clusters

	Cluster 1: 'Result unknown high initial'	Cluster 2: 'Part unknown high gains'	Cluster 3: 'Part unknown lower gains'
Description	Items with 'high' facility levels in the pre-test (above 50% facility – which was also above the mean facility of around 47% across the Int & Con groups).	Items with 'low' (<50%) pre-test facility but high gains for the Int group (>40% point gains)	Items with 'low' (<50%) pre-test facility and lower gains for the Int group (<40% point gains)
Items	PPW-WU (9+6=?) SEP-RU (12-7=?) PPW-WU (12+8=?)	JOIN-SU (?+2=8) JOIN-CU (8+?=14)	SEP-CU (28-?=19) PPW-PU (12+?=20)

6.2 Summary of the clusters in table form

In this section, I present, with summary tables 6.4, 6.5 and 6.6, an analysis of the categories of the 'extent of structuring' against 'progression into more formal mathematical register' and for each category in Cluster 1 – 'Result Unknown high initial'; Cluster 2 – Part unknown high gains; Cluster 3 – Part unknown lower gains. In all three tables (6.4; 6.5; 6.6) the most prevalent category within the test for the group is highlighted. Each table includes a summary of key comments. These tables are followed by analysis that identifies key patterns in the extent of structure in this cluster of items. Differences between the intervention and control groups are noted, as well as differences between the two groups in patterns of shifts in mathematization between the pre- and post-tests.

It should be noted that the total number of learners does not necessarily match the total number of models as some children used more than one model within a problem. For example, in some instances the total number of learners is 28, while the total number of models is 29.

6.2.1 Cluster 1: Result Unknown high initial

Table 6. 3

	Pre-Test Part-Part Whole-Whole Unknown 9 + 6 =?									
	% NoM-I	%NDQ-I/PiC	%NDQ-C/PiC	%DQ-I/PiC	%DQ-C/PiC	%DQ-I/SFMR	%DQ-C/SFMR	%DQ-I/CMFR-NS	%DQ-C/CFMR-NS	%NoM-C
Int Group (28 lnrs)	17.2 5/29	7 2/29	3.4 1/29	0 0/29	10.3 3/29	0 0/29	10.3 3/29	0 0/29	13.8 4/29	**38 11/29**
	N=29 Models									
Cont Group (27 lnrs)	7.4 2/27	7.4 2/27	7.4 2/27	3.7 1/27	**55.6 15/27**	0 0/27	7.4 2/27	3.7 1/27	0 0/27	7.4 2/27
	N=27 Models									

	Post-Test Part-Part Whole-Whole Unknown 9 + 6 = ?									
	% NoM-I	%NDQ-I/PiC	%NDQ-C/PiC	%DQ-I/PiC	%DQ-C/PiC	%DQ-I/SFMR	%DQ-C/SFMR	%DQ-I/CMFR-NS	%DQ-C/CFMR-NS	%NoM-C
Int Group (28 lnrs)	0 0/33	0 0/33	3 1/33	0 0/33	**33.3 11/33**	0 0/33	**33.3 11/33**	0 0/33	18.2 6/33	12.1 4/33
	N=33 Models									
Cont Group (27 lnrs)	11.1 3/27	7.4 2/27	0 0/27	0 0/27	**59.3 16/27**	0 0/27	0 0/27	0 0/27	3.7 1/27	18.5 5/27
	N=27									

Key: NoM – I (No overt Model-Incorrect) ; NDQ-I/PiC (No overt distinguished quantities-Incorrect)/Pictorial); NDQ-C/PiC (No overt distinguished quantities-Correct/Pictorial); DQ-I/PiC (Overt distinguished quantities-Incorrect/Pictorial); DQ-C/PiC (Overt distinguished quantities/Pictorial); DQ-I/SFMR (Overt distinguished quantities-I/Some inclusion of Formal mathematical register); DQ-C/SFMR (Overt distinguished quantities-C/Some inclusion of formal mathematical register); DQ-I/CFMR-NS (Overt distinguished quantities-Incorrect/Complete use of formal mathematical register-Number Sentence); DQ-C/CFMR-NS (Overt distinguished quantities-Correct/Complete use of formal mathematical register-Number Sentence); NoM-C (No overt Model-C)

Key Comments	
	- I group showed a greater prevalence of models in which quantities are distinguished, with correct answers: DQ-C/Pic rose from 10.3% to 33.3% and DQ-C/SFMR rose in exactly the same way. The large decreased in % of NoM-C from 38% to 12.1% and the % of NoM-I from a little under 20% to 0%, the two decreases reflect a greater willingness to produce the models of situations pointed to earlier, coupled with increases in extent of distinguishing of quantities in the responses. - C group showed no substantial shift in terms of distinguishing of quantities in comparison to the I group. Instead, there was an increase of % NoM-C from 7.4% to 18.5% which reflects an increase in prevalence of working without producing models of situations.

Table 6. 4

	Pre-Test Part-Part Whole-Whole Unknown 12 + 8 = ?									
	% NoM-I	%NDQ-I/PiC	%NDQ-C/PiC	%DQ-I/PiC	%DQ-C/PiC	%DQ-I/SFMR	%DQ-C/SFMR	%DQ-I/CMFR-NS	%DQ-C/CFMR-NS	%NoM-C
Int Group (28 lnrs)	**41.4 12/29**	3.4 1/29	0 0/29	10.3 3/29	0 0/29	0 0/29	10.3 3/29	0 0/29	10.3 3/29	24.1 7/29
	N=29 Models									
Cont Group (27 lnrs)	7.4 2/27	7.4 2/27	7.4 2/27	3.7 1/27	55.6 15/27	0 0/27	7.4 2/27	3.7 1/27	0 0/27	7.4 2/27
	N=27 Models									

	Post-Test Part-Part Whole-Whole Unknown 12 + 8 = ?									
	% NoM-I	%NDQ-I/PiC	NDQ-C/PiC	DQ-I/PiC	DQ-C/PiC	DQ-I/SFMR	DQ-C/SFMR	DQ-I/CMFR-NS	DQ-C/CFMR-NS	NoM-C
Int Group (28 lnrs)	3.3 1/30	0 0/30	3 1/30	3.3 0/30	**33.3 11/30**	0 0/30	26.7 8/30	0 0/30	16.7 5/30	12.1 4/30
	N=30 Models									
Cont Group (27 lnrs)	7.4 2/27	0 0/27	18.5 5/27	3.7 1/27	44.4 12/27	0 0/27	0 0/27	0 0/27	3.7 1/27	22.2 6/27
	N=27 Models									

Key: NoM – I (No Overt Model-Incorrect) ; NDQ-I/PiC (No overt distinguished quantities-Incorrect)/Pictorial); NDQ-C/PiC (No overt distinguished quantities-Correct/Pictorial); DQ-I/PiC (Overt distinguished quantities-Incorrect/Pictorial); DQ-C/PiC (Overt distinguished quantities/Pictorial); DQ-I/SFMR (Overt distinguished quantities-I/Some inclusion of Formal mathematical register); DQ-C/SFMR (Overt distinguished quantities-C/Some inclusion of formal mathematical register); DQ-I/CFMR-NS (Overt distinguished quantities-Incorrect/Complete use of formal mathematical register-Number Sentence); DQ-C/CFMR-NS (Overt distinguished quantities-Correct/Complete use of formal mathematical register-Number Sentence); NoM-C (No overt Model-C)

Key Comments	- I group showed a greater prevalence of models in which quantities are distinguished, with correct answers: DQ-C/Pic rose from 0% to 33.3%. There is a large decrease in % NoM-C from 24% to 12.1% and large drop in NoM-I by a little under 40%. These two latter results reflect the greater willingness to produce models of situations pointed to earlier, but importantly what is seen here is the extent of distinguishing of quantities in the responses. - DQ-C/SFMR also showed a large increase for I group in the post-test. - C group showed no substantial shift in terms of distinguishing of quantities in comparison to the I group. Instead, there is a large decrease of DQ-C/PiC from 55.6% to 44.4%, and an increase of % NoM-C from 7.4% to 22.2% which reflects an increase in prevalence of working without producing models of situations.

Table 6. 5

	Pre-Test Separe-Result Unknown 12 - 7 = ?										Post-Test Separate Result Unknown 12 – 7= ?									
	% NoM-I	%NDQ-I/PiC	NDQ-C/PiC	DQ-I/PiC	DQ-C/PiC	DQ-I/SFMR	DQ-C/SFMR	DQ-I/CMFR-NS	DQ-C/CFMR-NS	NoM-C	% NoM-I	%NDQ-I/PiC	NDQ-C/PiC	DQ-I/PiC	DQ-C/PiC	DQ-I/SFMR	DQ-C/SFMR	DQ-I/CMFR-NS	DQ-C/CFMR-NS	NoM-C
Int Group (28 lnrs)	**25 7/28**	3.6 1/28	0 0/28	3.6 1/28	21.4 6/28	7.1 2/28	7.1 2/28	3.6 1/28	7.1 2/28	25 7/28	0 0/31	6.5 2/31	3.2 1/31	0 0/31	**61.3 19/31**	0 0/31	6.5 2/31	0 0/31	12.9 4/31	9.7 3/31
	N=28 Models										**N=31**									
Cont Group (27 lnrs)	3.7 1/27	14.8 4/27	11.1 3/27	3.7 1/27	52 14/27	0 0/27	0 0/27	0 0/27	3.7 1/27	11.1 3/27	14.8 4/27	7.4 2/27	3.7 1/27	0 0/27	55.6 14/27	0 0/27	0 0/27	0 0/27	3.7 1/27	18.5 5/27
	N=27										**N=27**									

Key: NoM – I (No Overt Model – Incorrect); NDQ-I/PiC (No overt distinguished quantities-Incorrect/Pictorial); NDQ-C/PiC (No overt distinguished quantities-Correct/Pictorial); DQ-I/PiC (Overt distinguished quantities-Incorrect/Pictorial); DQ-C/PiC (Overt distinguished quantities-Correct/Pictorial); DQ-I/SFMR (Overt distinguished quantities-Incorrect/Some inclusion of Formal mathematical register); DQ-C/SFMR (Overt distinguished quantities-Correct/Some inclusion of formal mathematical register); DQ-I/CFMR-NS (Overt distinguished quantities-Incorrect/Complete use of formal mathematical register-Number Sentence); DQ-C/CFMR-NS (Overt distinguished quantities-Correct/Complete use of formal mathematical register-Number Sentence); NoM-C (No overt Model-Correct)

Key Comments	- I group showed a greater prevalence of models in which quantities are distinguished, with correct answers: DQ-C/Pic rose from 21.4% in pre to 61.3% in post. There was a large decrease in % NoM-C from 25% to 9.7% and large drop in NoM-I by 25%. These two latter results reflect learners' greater willingness to produce the models of situations pointed to earlier, but importantly what is seen here is the extent of distinguishing of quantities in the responses. C group showed no substantial shift in terms of distinguishing of quantities in comparison to the I group. Instead, there is a large increase of NoM-I from 3.7% to 14.8% and an increase of NoM-C from 11.1 to 18.5 reflection lack of willingness to produce models.

6.2.1.1 Analysis of the extent of structure for Cluster 1

In the post-test, the intervention group substantially outperforms the control group on all 3 items in terms of the extent of awareness of structuring.

The mathematizing analysis indicated pre-test differences between the intervention and control groups, with the control group showing a greater prevalence of Pictorial models across both items and the intervention group a prevalence of 'No overt model' responses. Across all items, the most prevalent extent of awareness of structuring response for the intervention group shifted from 'No overt model' to Distinguished quantities-Pictorial (DQ-PiC) by more than 20% and was associated with correct answers for all items. The proportion of DQ-PiC associated with correct answers for the intervention group exceeds the proportion of DQ-PiC associated with correct answers in the responses from the control group.

In the post-test the intervention group responses indicated a slightly higher increase of 'Distinguished quantities-some inclusion of formal mathematical Register' (DQ-SFMR) in PPW-WU 9 + 6 =? and PPW-WU 12 + 8 =? with none of these increases evident for the control group. The intervention group further showed a decrease in both 'No overt model' – Incorrect (NoM-I) and 'No overt model-Correct' (NoM-C) in all three items which suggests willingness to produce models which make it easy to see the extent of the awareness of structure. This shift contrasted with the responses of the control group where there was an increase in NoM-C for 2 items and increase in NoM-I in 1 item.

The mathematization analysis for this cluster indicates substantial moves in the ability to distinguish quantities on the part of the intervention group in the post-test, that could be interpreted using the theoretical trajectory as an important aspect within the already identified increase in prevalence of pictorial models.

By contrast, there are no substantial shifts in comparison to the pre-test in the responses of the control group in which there a prevalence of ‘No overt model’ (NoM) and no distinguishing of quantities was retained in the post-tests. For the intervention in the post-test there is a substantial increase in inclusion of ‘Some inclusion of formal mathematical register’ (SFMR) for the first two items and ‘Complete formal mathematical register’ (CFMR) for all 3 items. This is contrasted by the control group where there were no substantial shifts in the post-test.

6.2.2 Cluster 2: Part Unknown high gains

Table 6. 6	Pre-Test Join-Start Unknown ? + 2 = 8										Post-Test Join-Start Unknown ? + 2 = 8									
	% NoM-I	%NDQ-I/PiC	NDQ-C/PiC	DQ-I/PiC	DQ-C/PiC	DQ-I/SFMR	DQ-C/SFMR	DQ-I/CMFR-NS	DQ-C/CFMR-NS	NoM-C	% NoM-I	%NDQ-I/PiC	NDQ-C/PiC	DQ-I/PiC	DQ-C/PiC	DQ-I/SFMR	DQ-C/SFMR	DQ-I/CMFR-NS	DQ-C/CFMR-NS	NoM-C
Int Group (28 lnrs)	**48.3 14/29**	10.3 3/29	0 0/28	0 0/29	3.4 1/29	0 0/29	3.4 1/29	3.4 1/29	3.4 1/29	27.6 8/29	3.3 1/30	6.7 2/30	13.3 4/30	0 0/30	**20 6/30**	0 0/30	6.7 2/30	0 0/30	23.3 7/30	16.7 5/30
	N=29 Models										**N=30**									
Cont Group (27 lnrs)	29.6 8/27	**40.7 11/27**	3.7 1/27	0 0/27	11.1 3/27	0 0/27	3.7 1/27	0 0/27	0 0/27	11.1 3/27	33.3 9/27	33.3 9/27	3.7 1/27	3.7 1/27	25.9 7/27	0 0/27	0 0/27	0 0/27	0 0/27	0 0/27
	N=27 Models										**N=27 Models**									

Key: NoM – I (No Overt Model – Incorrect); NDQ-I/PiC (No overt distinguished quantities-Incorrect)/Pictorial); NDQ-C/PiC (No overt distinguished quantities-Correct/Pictorial); DQ-I/PiC (Overt distinguished quantities-Incorrect/Pictorial); DQ-C/PiC (Overt distinguished quantities/Pictorial); DQ-I/SFMR (Overt distinguished quantities-I/Some inclusion of Formal mathematical register); DQ-C/SFMR (Overt distinguished quantities-C/Some inclusion of formal mathematical register); DQ-I/CFMR-NS (Overt distinguished quantities-Incorrect/Complete use of formal mathematical register); DQ-C/CFMR-NS (Overt distinguished quantities-Correct/Complete use of formal mathematical register); DQ-NoM-C/SFMR (No overt Model-C/Some inclusion of formal mathematical register)

Key Comments	- I group showed a greater prevalence of models for which quantities are distinguished, with correct answers: DQ-C/Pic rose from 3.4% to 20% . There was a relatively large decrease in % NoM-C from 27% to 16.7% and a large drop in NoM-I by 45%. These two latter results reflect learners' greater willingness to produce models of situations pointed to earlier, but importantly what is seen here is the extent of distinguishing of quantities in the responses. - C group showed no substantial shift in terms of distinguishing of quantities in comparison to the I group. Instead, there was an increase of NoM-I - from 29.6% to 33.3% and a decrease of NoM-C from 11.1% to 0%.

Table 6. 7

	Pre-Test Join-Change Unknown 8 + ? = 14									
	% NoM-I	%NDQ-I/PiC	NDQ-C/PiC	DQ-I/PiC	DQ-C/PiC	DQ-I/SFMR	DQ-C/SFMR	DQ-I/CMFR-NS	DQ-C/CFMR-NS	NoM-C
Int Group (28 lnrs)	**38** **11/29**	20.7 6/29	3.4 1/29	0 0/29	0 0/29	10.3 3/29	0 0/29	7 2/29	3.4 1/29	17.2 5/29
	N=29 Models									
Cont Group (27 lnrs)	22.2 6/27	59.3 16/27	7.4 2/27	0 0/27	7.4 2/27	0 0/27	0 0/27	0 0/27	0 0/27	3.7 1/27
	N=27 Models									

	Post-Test Join-Change Unknown 8 + ? = 14									
	% NoM-I	%NDQ-I/PiC	NDQ-C/PiC	DQ-I/PiC	DQ-C/PiC	DQ-I/SFMR	DQ-C/SFMR	DQ-I/CMFR-NS	DQ-C/CFMR-NS	NoM-C
Int Group (28 lnrs)	3.2 1/31	9.7 3/31	9.7 3/31	3.2 1/31	9.7 3/31	0 0/31	**22.6** **7/31**	0 0/31	19.4 6/31	13 4/31
	N=31 Models									
Cont Group (27 lnrs)	25.9 8/27	33.3 9/27	7.4 2/27	0 0/27	22.2 6/27	0 0/27	0 0/27	0 0/27	3.7 1/27	3.7 1/27
	N=27 Models									

Key: NoM – I (No Overt Model-Incorrect); NDQ-I/PiC (No overt distinguished quantities-Incorrect)/Pictorial); NDQ-C/PiC (No overt distinguished quantities-Correct/Pictorial); DQ-I/PiC (Overt distinguished quantities-Incorrect/Pictorial); DQ-C/PiC (Overt distinguished quantities/Pictorial); DQ-I/SFMR (Overt distinguished quantities-I/Some inclusion of Formal mathematical register); DQ-C/SFMR (Overt distinguished quantities-C/Some inclusion of formal mathematical register); DQ-I/CFMR-NS (Overt distinguished quantities-Incorrect/Complete use of formal mathematical register); DQ-C/CFMR-NS (Overt distinguished quantities-Correct/Complete use of formal mathematical register); DQ-NoM-C/SFMR (No overt Model-C/Some inclusion of formal mathematical register)

Key Comments	
	- I group showed a greater prevalence of models for which quantities were distinguished, with correct answers: **DQ-C/SFMR** rose from 0% to 22.6% . There was a large decrease of NDQ-I/PiC from 20.7% to 9.7%. There was a small decrease in %NoM-C from 17.2% to 13% and a large drop in NoM-I by 35%. There was a large increase in DQ-C/CFMR-NS from 3.4% to 19.4%. These three latter results reflect learners' greater willingness to produce models of situations pointed to earlier, but importantly what is seen here is the extent of distinguishing of quantities in the responses. - C group showed no substantial shift in terms of distinguishing of quantities in comparison to the I group. Instead, there was an increase of NoM-I - from 22.2% to 25.9%. There was a slight shift with a decrease of NDQ-I/PiC from 59.3% to 33.3%. and an increase of DQ-C/Pic from 7.4% to 22.2%

6.2.2.1 Analysis of the extent of structure for Cluster 2

The earlier analysis for the intervention group showed that performance improved from an already high base for this cluster and that there was a drop in incidences of No overt Model (NoM). This analysis shows that responses shifted in different ways for both questions. For example, in Join-Start Unknown (J-SU? + 2 =8) there is an increase in the extent of distinguishing of quantities seen in DQ-PiC for the intervention group. However, in Join- Change Unknown (JOIN-CU) there is an increase in the extent of awareness of structure seen in Distinguished quatities-Some include of formal mathematical register (DQ-SFMRR) for this group.

In the pre-test for both items, the intervention group reflects a high prevalence of NoM-I whilst the control group reflects a high prevalence of No distinguished quantities – Pictorial (NDQ-I/PiC). In the post-test, however, the intervention group responses show that these learners outperform the control group in terms of the extent of distinguishing quantities with correct answers for both items. The control group reflects no improvement between pre-test and post-test in the extent of distinguishing of quantities for both items.

The mathematizing analysis further indicates that for the intervention group, across both items, the prevalence of No overt Model (NoM-I) shifted to Distinguished quantity (DQ-PiC in J-SU ? + 2 =8) and to Distinguished quantity-Some inclusion of formal mathematical register (DQ-SFMR) in Join-Start Unknown (J-CU 8 + ? = 14). Lastly, for both items, the prevalence of Distinguished quantities-Complete formal mathematical register – Number sentence (DQ-CFMR-NS) increased substantially. These results contrast with those of the control group where in response to both items there was an increase in No overt Model – Incorrect (NoM-I) in the post-test, although there was an increase in Distinguished quantities – Pictorial (DQ-C/PiC).

The mathematization analysis for this cluster indicates the ability of the intervention group to distinguish quantities substantially, especially in the post-test. By contrast for the control group, there were no substantial shifts in comparison to the pre-test in which there was a prevalence of NoM and no distinguishing of quantities. There was also evidence of substantial increases of 'some inclusion of mathematical register' (SFMR) and 'complete form of mathematical register' (CFMR) for both items.

6.2.3 Cluster 3: Part Unknown lower gains

Table 6. 8

Pre-Test Separate-Change Unknown 28 - ? = 19	% NoM-I	%NDQ-I/PiC	NDQ-C/PiC	DQ-I/PiC	DQ-C/PiC	DQ-I/SFMR	DQ-C/SFMR	DQ-I/CMFR-NS	DQ-C/CFMR-NS	NoM-C
Int Group (28 lnrs)	**38 11/29**	17.2 5/29	3.4 1/29	0 0/29	20.7 6/29	0 0/29	0 0/29	7 2/29	0 0/29	13.8 4/29
	N=29									
Cont Group (27 lnrs)	22.2 6/27	**40.7 11/27**	18.5 5/27	0 0/27	18.5 5/27	0 0/27	0 0/27	0 0/27	0 0/27	0 0/27
	N=27 Models									

Post-Test Separate – Change Unknown 28 - ? = 19	% NoM-I	%NDQ-I/PiC	NDQ-C/PiC	DQ-I/PiC	DQ-C/PiC	DQ-I/SFMR	DQ-C/SFMR	DQ-I/CMFR-NS	DQ-C/CFMR-NS	NoM-C
Int Group (28 lnrs)	3.2 1/30	26.7 8/30	0 0/30	0 0/30	**46.7 14/30**	0 0/30	**0 0/30**	10 3/30	6.7 2/30	3.3 1/30
	N=30									
Cont Group (27 lnrs)	**25.9 7/27**	**25.9 7/27**	18.5 5/27	7.4 2/27	18.5 5/27	0 0/27	0 0/27	0 0/27	0 0/27	3.7 1/27
	N=27 Models									

Key: NoM – I (No overt distinguished quantities-Incorrect); NDQ-I/PiC (No overt distinguished quantities-Incorrect)/Pictorial); NDQ-C/PiC (No overt distinguished quantities-Correct/Pictorial); DQ-I/PiC (Overt distinguished quantities-Incorrect/Pictorial); DQ-C/PiC (Overt distinguished quantities-Correct/Pictorial); DQ-I/SFMR (Overt distinguished quantities-Incorrect/Some inclusion of Formal mathematical register); DQ-C/SFMR (Overt distinguished quantities-Correct/Some inclusion of formal mathematical register);DQ-I/CFMR-NS (Overt distinguished quantities-Incorrect/Complete use of formal mathematical register-Number Sentence); DQ-C/CFMR-NS (Overt distinguished quantities-Correct/Complete use of formal mathematical register-Number Sentence); NoM-C (No overt Model-Correct)

Key Comments	
	- I group showed a greater prevalence of models for which quantities were distinguished, with correct answers: DQ-C/PiC rose from 20.7% to 46.7% There was a large decrease in %NoM-C from 13.8% to 3.3% and large drop in NoM-I by 35%. These two latter results reflect the learners' greater willingness to produce models of situations pointed to earlier, but importantly what is seen here is the extent of distinguishing of quantities in the responses. - C group showed no substantial shift in terms of distinguishing of quantities in comparison to the I group. Instead, there was an increase of NoM-I - from 22.2% to 25.9%. There was a slight shift with a decrease of NDQ-I/PiC from 40.7% to 25.9%.

Table 6.9

	Pre-Test Part-Part Whole-Part Unknown 12 + ? =20												
	% NoM-I	%NDQ-I/PiC	NDQ-C/PiC	NDQ-I/SFMR	NDQ-I/CFMR-NS	DQ-I/PiC	DQ-C/PiC	DQ-I/SFMR	DQ-C/SFMR	DQ-I/CMFR-NS	DQ-C/CFMR-NL	DQ-C/CFMR-NS	NoM-C
Int Group (28 lnrs)	3.4 1/29	0 0/29	3.4 1/29	3.4 1/29	20.7 6/29	0 0/29	**24.1 7/29**	0 0/29	17.2 5/29	0 0/29	3.4 1/29	10.3 3/29	13.8 4/29
	N=29												
Cont Group (27 lnrs)	**29.6 8/27**	**48.2 13/27**	**0 0/27**	**7.4 2/27**	**0 0/27**	**3.7 1/27**	**11.1 3/27**	**0 0/27**	**0 0/27**	**3.7 1/27**	**11.1 3/27**	**0 0/27**	**0 0/27**
	N=27 Models												

	Post-Test Part-Part Whole-Part Unknown 12 + ? =20										
	% NoM-I	%NDQ-I/PiC	NDQ-C/PiC	NDQ-I/CFMR-NS	DQ-I/PiC	DQ-C/PiC	DQ-I/SFMR	DQ-C/SFMR	DQ-I/CMFR-NS	DQ-C/CFMR-NS	NoM-C
Int Group (28 lnrs)	3.2 1/30	26.7 8/30	0 0/30	0 0/27	0 0/30	**46.7 14/30**	0 0/30	0 0/30	10 3/30	6.7 2/30	3.3 1/30
		N=30									
Cont Group (27 lnrs)	**37 10/27**	**25.9 7/27**	**7.4 2/27**	**3.7 1/27**	**11.1 3/27**	**14.8 4/27**	**0 0/27**	**0 0/27**	**0 0/27**	**0 0/27**	**0 0/27**
	N=27 Models										

Key: NoM – I ; NDQ-I/PiC (No overt distinguished quantities-Incorrect)/Pictorial); NDQ-C/PiC (No overt distinguished quantities-Correct/Pictorial); NDQ-I/CFMR-NS (No overt distinguished quantities-Incorrect/Complete use of formal mathematical register) ;DQ-I/PiC (Overt distinguished quantities-Incorrect/Pictorial); DQ-C/PiC (Overt distinguished quantities-Correct/Pictorial); DQ-I/SFMR (Overt distinguished quantities-I/Some inclusion of Formal mathematical register); DQ-C/SFMR (Overt distinguished quantities-C/Some inclusion of formal mathematical register); DQ-I/CFMR-NS (Overt distinguished quantities-Incorrect/Complete use of formal mathematical register-number sentence); DQ-C/CFMR-NL (Overt distinguishing of quantities-Correct/Complete use of formal mathematical register-Number Line) ; DQ-C/CFMR-NS (Overt distinguished quantities-Correct/Complete use of formal mathematical register); DQ-NoM-Correct (No overt Model-Correct)

Key Comments	- I group showed a greater prevalence of models for which quantities were distinguished, with correct answers: DQ-C/PiC rose from 24.1% to 46.7% . There was a large decrease of NDQ-I/CFMR from 20.7% to 0%. There is decrease in %NoM-C from 13..8% to 3.3%.. These results reflect willingness to produce models of situations pointed to earlier, but importantly what is seen here is the extent of distinguishing of quantities in the responses. - C group showed no substantial shift in terms of distinguishing of quantities in comparison to the I group. Instead, there was an increase of NoM-I - from 29.6% to 37%. There is a slight decrease of NDQ-I/PiC from 48.2% to 25.9%.

6.2.3.1 Analysis of the extent of structure for Cluster 3

The analysis showed a mix in the shifts in both items. For example, in Separate – Change Unknown (SEP-CU) 28 -? = 19 for the intervention group there was a prevalence of No overt Model-Incorrect (NoM-I) in the pre-test which decreased substantially in the post-test, resulting in the prevalence of Distinguished quantities-Pictorial (DQ-PiC). By contrast, in the control group, there was a prevalence of No distinguished quantities-Pictorial (NDQ-I/PiC) in the pre-test, with a slight decrease in the post-test but a slight increase in post-test NoM-I. In Part-Part-Whole-Part Unknown (PPW-PU) 12 +? =20 Distinguished quantities-pictorial (DQ-C/PiC) was prevalent for the intervention group and increased greatly in the post-test. By contrast, in the control group there was a high prevalence of NDQ-I/PiC in the pre-test, with a slight decrease in the post-test but an increase in NoM-I.

However, in the post-test, the intervention group outperformed the control group in terms of the extent of distinguishing quantities with correct answers for both items. The control group's responses reflected no improvement in the extent of distinguishing of quantities for either item. Important to note here is the evidence of lack of Distinguished quantities-Some inclusion of formal mathematical register (DQ-SFMR) for I group in contrast to earlier clusters.

The mathematizing analysis further indicated that, across both items, for the intervention group the prevalence of NoM-I shifted to DQ-PiC in SEP-CU 28 - ? =19 by 20% and PPW-PU 12 + ? =20 by 22.6%. The intervention group's responses showed a decrease of No distinguuished quantities-Incorrect/Complete form of mathematical register-Number sentence (NDQ-I/CFMR-NS) from 20.7% in the pre-test to 0% in the post-test. None of these changes was evident in the control group's responses.

For this cluster, the mathematization analysis indicates the ability of learners in the intervention group to distinguish quantities more substantially, with this ability especially evident in the post-test. By contrast, for the control group, there were no substantial shifts in this feature in comparison to the pre-test in which a prevalence of NoM and no distinguishing of quantities were evident.

CHAPTER 7

ANALYSIS OF INTERVIEW DATA

7.1 Introduction

In this section I present an analysis of interview data in order to establish the mathematizing processes seen within learners' calculation strategies. In the previous analysis chapters I have looked at shifts in the nature of models and extent of structuring across the clusters of items identified earlier. In this chapter I look at calculation strategies across these clusters since mathematizing processes incorporate both the models set up from situations and the strategies enacted, using the models, for calculating answers. Calculation strategies can be considered in terms of vertical mathematization processes within this study's RME framing. The interview data was analysed as it provided evidence of the kinds of strategies the intervention group children produced while solving the problems, which was often not possible to see in their written responses. By way of reminder, interviews were conducted with a sub-sample of 12 learners following the written pre- and post-tests. The interview sub-sample were selected on the basis of 4 high attaining, 4 middle attaining, and 4 low attaining. This sub-sample was selected based on the performance in the pre-test. After the post-test (administered in November 2014) two of the learners (both low attaining) in the pre-interview sample stopped attending school and were therefore not interviewed as part of the post-interview sample. This resulted in the data from ten learners being analysed. In the interviews, unlike the written tests, counters and bead strings were made available to learners if they wanted to use them but this was not directed.

In Table 7.1 I present examples for strategies that were observed in this analysis. As mentioned in the Methodology Chapter, there is a progression in the strategies: Level 0 represents incorrect immediate Recalled Fact strategies, Level 1 represents the least sophisticated ‘Direct Modelling Strategies’, Level 2 represents ‘Counting Strategies’ and is more sophisticated than level 1, Level 3 represents ‘Calculating by structuring’, and lastly Level 4 represents ‘Recalled Fact’ strategies with correct answers which can be interpreted as the most sophisticated strategy for a specific task. Within each of these levels, the literature identifies a selection of strategy options that fall within the scope of the main level. In the second column are the strategies as per Carpenter et al., (1999). In analysing the responses that children offered for each item, I recorded the total number of attempts of each of these strategies among the matched interview sample, and noted the number of these attempts that had incorrect and correct answers.

Summarising the responses in this way allowed an exploration of shifts in strategies within and across levels. In the literature review chapter, I have detailed the sub-categories that provided indicators for each level, and these helped me to code the data.

Table 7. 1 Outline of initial coding approach for each interview task

Progression	Level	Strategy	Total Number of Attempts	Correct	Incorrect
	0 Recalled Fact Immediate Incorrect (RFI-I)				
	1 Direct modelling-based strategies (DM)	Joining All (JA)			
		Joining To (JT)			
	2 Calculating-by-counting strategies (CbC)	Count On From First (COF)			
	3 Calculating – by – Structuring strategies (CbS)	Stringing, Splitting, Varying			
	4 Recalled Fact Immediate (Correct)	Recalled Fact Immediate (RFI)			

As per the previous sections the analysis is presented within the format of the clusters identified in Chapter 5, **cluster 1 – 'Result Unknown high initial'; cluster 2 – Part unknown high gains; cluster 3 – Part unknown lower gains.** I did this, as before, in order to explore whether patterns of calculation strategy use were similar or different across the three clusters.

7.2 Result Unknown high initial

In the next section, I provide summary tables 7.2 – 7.4 showing the progression in strategy use from the pre- to the post-interviews for the Intervention group in this cluster. As in the written test analyses, the most prevalent category – now in terms of calculation strategy - within the item for the group is highlighted. Each table includes a summary of key comments. These tables are followed by analysis that picks up key patterns in the progression of strategies in this cluster of items.

Differences between the pre- and post-tests are noted for the n=10 learners. In column 1 I present the progressive strategy levels detailed above, with column 2 detailing the sub-categories of strategies that I looked for in learner responses from the interviews. In summarising information from my initial recording of responses, I include, in columns 3 and 4, the pre- and post-interview prevalence of each strategy type and extent of correct answers linked with the strategy, in fractional and percentages format respectively. In column 4 I also list learners that used each strategy in order to track individual-level shifts from pre to post-test.

Table 7.2 Summary of strategies from the interview data for PPW-WU 9 + 6 = ?

PPW-WU 9 + 6 = ?						
Progression	**Level**	**Strategy**	**Pre-Interview**		**Post-Interview**	
			Prevalence of strategy	**% Correct Answers**	**Prevalence of strategy**	**% Correct Answers**
	0 Recalled Fact Immediate - Incorrect (RFI-I)	**Recalled Fact Immediate - Incorrect (RFI-I)**	3/10 (30%) (L2, L4, L6)	0/3 (0%)		
	1 Direct modelling-based strategies (DM)	**Joining All (JA)**	2/10 (20%) (L5, L10)	2/2 (100%)		
		Count All (CA)			3/10 (30) (L2, L3, L6)	3/3 (100%)
		Joining To (JT)	1/10 (10%) (L1)	1/1 (100%)		
	2 Calculating-by-counting strategies (CbC)	**Count On From First (COF)**	**4/10 (40%) (L3, L7, L8, L9)**	**4/4 (100%)**	**6/10 (60%) (L4, L5, L7, L8, L9, L10)**	**6/6 (100%)**
	3 Calculating-by-Structuring strategies (CbS)	**Stringing, Splitting, Varying**				
	4 Recalled Fact Immediate – Correct (RFI-C)	**Recalled Fact Immediate – Correct (RFI-C)**			1/10 (10%) (L1)	1/1 (10%)

Key 1 (Learners): L1 – Learner 1; L2 – Learner 2; L3 – Learner 3 (up to L10)
L 1-L4 (High Attaining); L5 – L8 (Middle Attaining); L9 – L10 (Low Attaining)

Comments -Level 2-COF was the most prevalent strategy in pre- and post-interviews with prevalence increasing from 40% to 60%.
-All three learners at level 0 in the pre-interview shifted to either Level 1 or Level 2 in the post-interview.
-L1 shifted from Level 1 in pre-interview to Level 4 in post-interview.
-In one case L3 shifted from Level 2 in pre-interview to Level 1 in post-interview.

Table 7. 3 Summary of strategies from the interview data for SEP-RU 12 – 7 =?

SEP-RU 12 – 7 =?						
Progression	**Level**	**Strategy**	**Pre-Interview**		**Post-Interview**	
			Prevalence of strategy	**% Correct Answers**	**Prevalence of strategy**	**% Correct Answers**
	0 Recalled Fact Immediate – Incorrect (RFI-I)	**Recalled Fact Immediate – Incorrect (RFI-I)**	1/10 (10%) (L10)	0/1 (0%)		
	Level 1 Direct Modelling-based Strategies (DM)	**Separate From (SF)**	3/10 (30%) (L1, L4, L8)	3/3 (100%)	1/10 (L10)	1/1 (10%)
	2 Calculating-by-counting strategies (CbC)	**Count On From First (COF)**				
		Count On To (Copper, Nye, Charlton, Lindsay, & Greathouse)	**4/10 (40%) (L2, L5, L6, L7)**	**1/4 (25%)**		
		Count Down (CD)	2/10 (20%) (L3, L9)	0/2 (20%)		
		Count Down To (CDT) /Difference (Diff)			**8/10 (90%) (L2, L3, L4, L5, L6, L7, L8, L9)**	**8/8 (100%)**
	3 Calculating-by-Structuring strategies (CbS)	**Stringing, Splitting, Varying**				
	4 Recalled Fact Immediate – Correct (RFI-C)	**Recalled Fact Immediate – Correct (RFI-C)**			1/10 (10%) (L1)	1/1 (100%)

Key 1 (Learners): L1 – Learner 1; L2 – Learner 2; L3 – Learner 3 (up to L10)
L 1-L4 (High Attaining); L5 – L8 (Middle Attaining); L9 – L10 (Low Attaining)

Comments -Level 2-COT and Level 2-CDT were the most prevalent strategies in pre- and post-interviews respectively increasing to 40% in pre-and to 80% in post-interview. The interesting shift here is Level 2-CDT because it is a strategy that is not linked to a 'take-away' situation.

-L1 shifted from Level 1 in the pre-interview to Level 4 in post-interview.

- All 3 learners in Level 1 shifted to either Level 2 or Level 4 in post-interview and 2 learners remained in Level 2.

Table 7. 4 Summary of strategies from the interview data for PPW-WU 12 + 8 =?

PPW-WU 12 + 8 =?						
Progression	Level	Strategy	Pre-Interview		Post-Interview	
			Prevalence of strategy	% Correct Answers	Prevalence of strategy	% Correct Answers
	0 Recalled Fact Immediate -Incorrect (RFI-I)	**Recalled Fact Immediate - Incorrect (RFI-I)**				
	1 Direct modelling-based strategies (DM)	**Joining All (JA)**	4/10 (40%) (L3, L7, L8, L10)	3/4 (75%)	1/10 (10%) (L3)	
		Separate From (SF)	1/10 (10%) (L5)	0/1 (0%)		
	2 Calculating-by-counting strategies (CbC)	**Count On From First (COF)**	**5/10 (50%) (L1, L2, L4, L6, L9)**	**5/5 (100%)**	**8/10 (80%) (L2, L4, L5, L6, L7, L8 , L9, L10)**	
	3 Calculating-by-Structuring strategies (CbS)	**Stringing, Splitting, Varying**				
	4 Recalled Fact Immediate – Correct (RFI-C)	**Recalled Fact Immediate – Correct (RFI-C)**			1/10 (10%) (L1)	

Key 1 (Learners): L1 – Learner 1; L2 – Learner 2; L3 – Learner 3 (up to L10)
L 1-L4 (High Attaining); L5 – L8 (Middle Attaining); L9 – L10 (Low Attaining)

Comments -Level 2-COF was the most prevalent in pre- and post-interviews with prevalence increasing from 50% to 80%.
-Three out of four learners shifted from Level 1 in pre-interview to Level 2 in post-interview. The fourth learner - L3 remained in Level 2.
-L1 shifted from Level 2 in pre-interview to Level 4 in post-interview.

7.2.1 Analysis of the strategies for Cluster 1

The analysis for this cluster reflects that from learners' pre-interview responses, Level 2 showed greater prevalence with learners identifying with COF strategies for PPW-WU 9 + 6 and PPW-WU 12 + 8 with the exception of Level 2-COT for SEP-CU 12 – 7 =? The COF strategies were associated with correct answers and COT with incorrect answers. The COF strategy consisted, in learner responses in the pre-interview, of use of fingers and counters where learners mostly started counting from the first given number in the problem and counting on to get the answer. Using fingers to calculate an answer for a number range that is bigger than 10 can result in errors for children. In addition, counters were mostly used in an un-structured manner where quantities and answers were in heaps which made it difficult for the learners to see distinguished quantities. The Count on to (COT) strategy in the pre-interview consisted of learners using fingers to count on from 12 adding 7 resulting in mostly incorrect answers of '19' instead of '5'.

The learners' post-interview responses reflected a maintained prevalence of Count on from (COF) for Part-Part-Whole-Whole Unknown 9 + 6 =? and PPW-WU 12 + 8 =? but a shift from Count on to (COT) to Count down to (CDT) for Separate-Result Unknown 12 – 7 =? Both the COF and CDT strategies consisted of learners drawing pictorial representations with distinguished quantities some with an inclusion of some formal mathematical register such as a plus sign.

In this cluster, low attainer L9 did not show shifts between pre- and post-interviews, remaining in Level 2 strategies, but the other low attainer L10 moved from Level 0 to Level 1 strategies in PPW-WU 9 + 6 =? and Level 1 to Level 2 strategies in Separate-Result Unknown (SEP-RU) 12 – 7 =? and PPW-WU 12 + 8 =? In the middle attainer group (L5-8), the predominant move was into Level 2 strategies, sometimes from a Level 1 start, and other times from a Level 0 start. And in the high attainer group (L1-4), most learners moved from Levels 0/1 strategies to Level 2 strategies, but L1 was able to move consistently into Level 4 strategies, indicating awareness of recalled facts. While there was little distinction between high, middle and low attainers in the level of strategy use in pre-interview, in the post-interview, middle and high attainers showed more progress to higher level strategies. In some instances, there is evidence of a tendency for high attainers to start with low level strategies, however, they were able to progress into higher level strategies.

Lastly, there are two key results in this cluster. Firstly, the first shift is into increased move into Level 2 calculation by counting category-seen across all three items. Secondly, the item that is interesting within this move is the Separate Result (SEP-RU) 12-7=? question, where 8/10 learners, were able to use Count down to (CDT) strategies using their fingers. In separate problems, direct models would be reflected in Count down from (CDF) strategies in which 7 is 'taken away' from 12. This suggests a growing awareness of number relationships and the ability to decide on the more efficient way of calculating-by-counting.

7.3 Cluster 2: Part Unknown high gains

In this section, summary Table 7.5 -7.6 show the progression of strategies described by learners from the intervention group to the pre- and post-interview questions on items in Cluster 2. This is followed by a discussion of findings from the analysis.

Table 7. 5 Summary of strategies from the interview data for J-SU ? + 2 = 8

J-SU ? + 2 =8						
Progression	**Level**	**Strategy**	**Pre-Interview**		**Post-Interview**	
			% of Strategies	**% of Correct Answers**	**% of Strategies**	**% of Correct Answers**
	0 Recalled Fact Immediate – Incorrect (RCI-I)	Recalled Fact-Immediate – Incorrect (RCI-I)	**5/10 (50%) (L2, L4, L7, L9, L10)**	**0/5 (0%)**	1/10 (10%) (L10)	0/1 (0%)
	1 Direct modelling-based strategies	**Joining All (CA)**	1/10 (10%) (L5)	1/1 (100%)		
		Trial & Error (TE)	1/10 (10%) (L8)	1/1 (100%)		
	2 Calculating-by-counting strategies	**Count On From First (COF)**			2/10 (30%) L6, L7	0/2 (0%)
		Count On from Larger (COL)	1/10 (10%) (L6)	0/1 (0%)		
		Counting On To (COT)			**4/10 (40% (L2, L3, L4, L5,)**	**4/4 (100%)**
		Counting Down (CD)			1/10 (10%) (L8)	0/1 (0%)
	3 Calculating-by-structuring strategies	**Stringing, Splitting, Varying**				
	4 Recalled Fact Immediate – Correct (RFI-C)	**Recalled Fact Immediate (RFI-C)**	2/10 (20%) (L1, L3)	2/2 (100%)	2/10 (20%) (L1, L9)	2/2 (100%)

Key 1 (Learners): L1 – Learner 1; L2 – Learners 2; L3 – Learner 3 (…up to L10)
L 1-L4 (High Attaining); L5 – L8 (Middle Attaining); L9 – L10 (Low Attaining)

Comments -Level 0-Recalled fact – Incorrect (RFI/I) was the most prevalent strategy in pre-interview with prevalence of 50%, whilst Level 2-Count on to (COT) was the most prevalent strategy in post-interview with prevalence of 50%.

- in some instances, learners counted on from 2 and stopped at 10 Count on from (COF) – indicating an incorrect model.

-Four learners in Level 0 in pre-interview shifted to either Level 2 or Level 4 in post-interview.

-Both learners in Level 1 in pre-interview shifted to Level 2 in post-interview.

-L6 shifted from Level 2-Count on from larger (COL), in this strategy the learner counted from 8 and stopped at 10.

- In one case L3 shifted from L4 in pre-interview to Level 2 in post-interview.

Table 7. 6 Summary of strategies from the interview data for J-CU (8 + ? =14)

J-CU (8 + ? = 14)						
Progression	Level	Strategy	Pre-Interview		Post-Interview	
			% of Strategies	% of Correct Answers	% of Strategies	% of Correct Answers
	0 Recalled Fact Immediate – Incorrect (RCI-I)	Recalled Fact-Immediate – Incorrect (RCI-I)	2/10 (20%) (L7, L10)			
	1 Direct modelling-based strategies	**Join All (JA)**	1/10 (10%) (L5)	0/1 (0%)		
		Trial & Error (TE)	2/10 (20%) (L8, L9)	2/2 (100%)		
	2 Calculating-by-counting strategies	**Count On From First (COF)**	2/10 (20%) (L4, L6)	0/2 (0%)		
		Count On from Larger (COL)				
		Counting On To (COT)	2/10 (20%) (L1, L2)	2/2 (100%)	**8/10 (80%) (L2, L4, L5, L6, L7, L8, L9, L10),**	**8/8 (100)**
	3 Calculating-by-structuring strategies	**Stringing, Splitting, Varying**				
	4 Recalled Fact Immediate – Correct (RFI-C)	**Recalled Fact Immediate (RFI-C)**	1/10 (10%) (L3)	1/1 (100%)	2/10 (10%) (L1, L3)	2/2 (100%)

Key 1 (Learners): L1 – Learner 1; L2 – Learner 2; L3 – Learner 3 (up to L10)
L 1-L4 (High Attaining); L5 – L8 (Middle Attaining); L9 – L10 (Low Attaining)

Comments	
	-Level 2-Count on from/Count on to (COF/COT) were both the most prevalent strategies in the pre-interview and post-interview with prevalence increasing from 40% to 80%. The natural strategy option here is Count on to (COT) where learners started counting from 8 and end at 14. In instances where learners started counting from 8 and stopped at 22 (adding 14 and indicating an incorrect model), this move was coded as Count on from (COF). -Both learners at Level 0 in pre-interview shifted to Level 2 in post-interview. -All learners in Level 1 in the pre-interview shifted to Level 2 in the post-interview. -L1 shifted from Level 2 in the pre-interview to level 4 in the post-interview.

7.3.1 Analysis of strategies for Cluster 2

The key result in this cluster is a decreasing prevalence of what looks like random guessing that was frequently incorrect in the pre-interview. As mentioned in Chapter 2, Hoadley (2012) reported evidence of the random guessing phenomenon in SA FP classrooms. The findings of this phenomenon coming down in this study back up the increase in sense-making through model setup that has been noted. The Count on from (COF) strategy in Join-Change Unknown (J-CU) consisted, in learner responses in the pre-interview, of use of fingers and counters where, in some instances, learners added the given quantities by counting on. In addition, counters were mostly used in an un-structured manner where quantities and answers were in heaps which made it difficult for the learners to see distinguished quantities. The Count on to (COT) strategy in the pre-interview consisted of learners using fingers to count on from 8 to 14 when calculating the answer for 'J-CU'.

The learners' post-interview responses reflected a shift to a greater prevalence of Level 2-COT strategies for both items. The COT strategy consisted of learners drawing pictorial representation with distinguished quantities some with an inclusion of some formal mathematical register such as a plus sign. In some instances, learners would add 2 and 8 indicated an incorrect model for calculating the answer for J-SU.

In this cluster, in the low attainer group (L1-L2), the predominant move was into Level 2 strategies, sometimes from Level 0/1 start. In the middle attainer group (L5-L8) overall, most learners moved from Level 0/1 strategies to Level 2 strategies. In high attainer group most learners moved from Level 0/1 strategies to Level 2 strategies, but L1 was able to move consistently into Level 4 strategies.

7.4 Cluster 3: Part unknown lower gains

In this section summary Tables 7.7 and 7.8 show the progression of strategies evident in intervention group learners' answers to questions in the pre- and post-interviews about their responses to questions in Cluster 3. The tables are followed by a discussion of these findings.

Table 7. 7 Summary of strategies from the interview data for PPW-PU 12 + ? = 20

PPW-PU 12 + ? =20						
Progression	Level	Strategy	Pre-Interview		Post-Interview	
			% of Strategies	% of Correct Answers	% of Strategies	% of Correct Answers
	0 Recalled Fact Immediate – Incorrect (RFI-I)	**Recalled Fact Immediate – Incorrect (RFI-I)**	1/10 (10%) (L10)	0/1 (0%)		
	1 Direct modelling-based strategies (DM)	**Count All (CA)**	1/10 (10%) (L8)	1/1 (100%	2/10 (20%) (L5, L7)	0/2 (0%)
	2 Calculating-by-counting based strategies (CbC)	**Count On From First (COF)**	1/10 (10%) (L7)	0/1 (0%)		
		Count On from Larger (COL)	1/10 (10%) (L6)	0/1 (0%)		
		Counting On To (COT)	**5/10 (50%) (L1, L2, L4, L5, L9)**	**4/5 (80%)**	**7/10 (70%) (L2, L3, L4, L6, L8, L9, L10)**	**6/7 (86%)**
	3 Calculating-by-Structuring strategies (CbS)					
	4 Recalled Fact Immediate-Correct (RFI-C)	**Recalled Fact Immediate answer (RFI)**	1/10 (10%) (L3)	1/1 (100%)	1/10 (10%) (L1)	1/1 (100%)

Key 1 (Learners): L1 – Learner 1; L2 – Learners 2; L3 – Learner 3 (up to L10)

L 1-L4 (High Attaining); L5 – L8 (Middle Attaining); L9 – L10 (Low Attaining)

Comments - Level 2-Count on to (COT)was the most prevalent strategy in the pre-interview and in the post-interview with the prevalence increasing from 50% to 70%.

-L10 and L8 shifted from Level 0 and Level 1 respectively in pre-interview to Level 2

-L1 shifted from Level 2 in pre-interview Level 4in post-interview.

-In one case L3 shifted from Level 4 in pre-interview to Level 2 in post-interview.

Table 7. 8 Summary of strategies from the interview data for SEP-CU 28 - ? =19

SEP-CU 28 - ? =19						
Progression	Level	Strategy	Pre-Interview		Post-Interview	
			% of Strategies	% of Correct Answers	% of Strategies	% of Correct Answers
	0 Recalled Fact Immediate – Incorrect (RFI-I)	Recalled Fact Immediate – Incorrect (RFI-I)	**3/10 (30%) (L4, L7, L10)**	**0/3 (0%)**	1/10 (10%) (L10)	0/1 (10%)
	1 Direct modelling-based strategies (DM)	Separating From (SF)	**3/10 (30%) (L5, L6, L8)**	**3/3 (100%)**	1/10 (10%) (L6)	1/1 (100%)
	2 Calculating-by-counting based strategies (CbC)	Counting Down (CDT) – Difference (Diff)	2/10 (20%) (L2, L9)	1/ 2 (50%)	**7/10 (70%) (L2, L3, L4, L5, L7, L8, L9)**	**3/7 (43%)**
	3 Calculating-by-Structuring strategies (CbS)	Stringing Splitting Varying				
	4 Recalled Fact Immediate-Correct (RFI-C)	Recalled Fact Immediate answer (RFI)	2/10 (20%) (L1, L3)	2/2 (40%)	1/10 (10%) (L1)	1/10 (10%)

Key 1 (Learners): L1 – Learner 1; L2 – Learners 2; L3 – Learner 3 (up to L10)
L 1-L4 (High Attaining); L5 – L8 (Middle Attaining); L9 – L10 (Low Attaining)

Comments	-Level 0-Recalled Fact Immediate – Incorrect (RFI-I) and Level 1-Separating from (SF) were the most prevalent strategies in the pre-interview with the prevalence of 30% for each strategy. -In post-interview Level 2-Count down to (CDT) was the most prevalence strategy by 70%. Most of the incorrect answers were from miscounting since learners used counters in an unstructured manner. -Two of the three learners in Level 0 in the pre-interview shifted to Level 2 in the post-interview. -In one case, L3 shifted from L4 in pre-interview to L2 in post-interview.

7.4.1 Analysis of strategies for Cluster 3

The analysis for this cluster reflects that from learners' pre-interview responses, Level 2-Count on to (COT) showed a greater prevalence of 50% in Part Part Whole-Part Unknown (PPW-PU) 12 + ? = 20, and Level 0 and Level 1 showed a shared a greater prevalence of 30% associated with incorrect and correct answers, respectively. The Count on to (COT) strategy consisted, in learner responses in the pre-interview, of use of fingers and counters where, in some instances, learners (incorrectly) added the given quantities either by counting all or by counting on. In addition, counters were mostly used in an un-structured manner where quantities and answers were in heaps which made it difficult for the learners to see distinguished quantities.

The learners' post-interview responses reflected a shift to a greater prevalence of Level 2-COT and Count down to (CDT) strategies. The COT strategy consisted of learners drawing pictorial representations with distinguished quantities, some with an inclusion of some formal mathematical register such as a plus sign. The CDT strategy consisted of learners drawing pictorial representations mostly with no distinguished quantities and with some calculations resulting in miscounts.

In this cluster, in the low attainer group (L9 and L10), there were no substantive shifts except for one learner's shift in one item. In the middle attainer group (L5-L8), most learners moved to Level 2 strategies, sometimes from Level 0/1 start. In the high attainer group (L1-L4), most learners showed some moves although not substantively.

7.5 Summary comment on strategies of the interview data

Analysis of the interview data reflects findings across all the clusters that are similar to those from the analysis of the test data. In some instances there was evidence of taking up of more models reflecting a move towards sense making. Overall, the results show less random guessing in the post-interviews. What is also more prevalent is calculating-by-counting strategies associated with correct answers. More use of this strategy involves awareness of number relations. In the pre-test and in the pre-interview there was usually a prevalence of limited sophisticated models and structuring, with progression evident for the intervention group in the post-test and post-interview.

An observation important to note is that during the pre-interviews learners were very slow in solving problems and were mostly able to offer appropriate answer after intervention. This observation however changed during the post-interviews where learners offered answers quicker and without intervention. During the post-interviews there was also evidence of structuring where children were demarcating quantities with the inclusion of formal mathematical registers in some instance which was limited in the pre-interviews.

CHAPTER 8

FINDINGS, RECOMMENDATIONS AND CONTRIBUTIONS

8.1 Introduction

In this final chapter I reflect on the research process and highlight the key findings emanating from this study by situating these within the current literature base on mathematizing processes grade 2 learners in solving additive relation situations. I relate findings from this study to those of national and international studies that are also based on mathematizing processes and discuss how these are both similar and different. This reflection allows for careful consideration of the outcomes of the study that I consider to be useful for teachers and/or policy makers and the theoretical and methodological choices that enabled these findings to emerge. I further note the limitations of the study and offer recommendations for future research relating to the results that I have highlighted as interesting or surprising in relation to existing South African research and the directions of its policy recommendations.

8.2 What were the foci of this research?

This research aimed to explore the efficacy of a short-term intervention, based on Cognitively Guided Instruction (CGI) and Realistic Mathematics Education (RME) principles, for exploring the mathematizing processes of Grade 2 learners for solving additive relation situations in a South African primary school. In addition to these principles the *Big Book of Word Problems* was used as a resource to provide a teaching approach for the intervention. I considered additive relation situations to be an important focus because of the broad research evidence on South African learners' poor numeracy levels at the completion of their primary school education (Ensor et al., 2009; Schollar, 2008; Spaull, 2013), and the specific evidence of the importance of sense making in mathematics at an early age in order for learners to be successful in later grades.

Using a 'teaching experiment' approach I chose to work with isiZulu speaking learners from the isiZulu 2 classes in one township school, using one as an intervention group and the other as a control group. The changes recorded across the learners' pre and post-tests allowed for a determination of the learning gains made across the two groups. Comparative analysis between the gains made by the intervention and control groups determined whether the intervention based on CGI, RME and the Big Books improved the learners' sense making of the problems and the extent of structuring in relation to more formal mathematical register. In another layer of analysis, interview data from 10 learners was analysed was used in order to understand more comprehensively their VM vertical mathematizing processes.

The analysis focused on progression in early additive word problems and incorporated additive problem categories, position of the unknown, extent of formality and extent of awareness of structure.

In this chapter I present a summary of the study's findings and provide recommendations for practice based on these. I then present what I believe are the contributions to knowledge made by this work in the field of additive relation situations. I also discuss limitations to this study as well as the steps I took to mitigate these. Lastly I share some biographical reflections.

8.3 Key findings

High level gains made by the intervention group in comparison to the control group

When I compared the nature of performance gains made by the intervention group and control groups I found that the intervention group outperformed the control group on all the items tested. For Cluster 1 across all three items, the most prevalent model for the intervention group shifted from No overt model to Pictorial models between the pre- and post-test, with more than double the number of Pictorial models seen in the post-test compared to the pre-test on all three items. The proportion of appropriate models and correct answers from these models for the intervention group exceeded the proportion of appropriate models and correct answers in the responses from the control group. The intervention group's responses indicated a slightly higher prevalence of Symbolic Syntactic-Horizontal Number Sentence models in the pre-test than did the control group. The prevalence of this model increased in the post-test responses to all three items for the intervention group, while remaining near absent in the responses of the control group across the pre- and post-tests.

Significantly higher level gains for Cluster 1 'Result unknown high gains' with take-up of models to solve problems.

The higher initial performance seen in Cluster 1 reflects the finding of Carpenter et al. (1999) that children find it easier to solve 'result unknown' problems than 'part unknown'. However, in the post-test, the intervention group reflected higher gains and a take-up of models in comparison to the pre-test and to the pre- and post-tests of the control group. In addition to take-up of models, the intervention group showed an increase in the extent of structuring where there was demarcation of quantities associated with appropriate models and correct answers. For example, the most prevalent category was Distinguished quantities-Pictorial (DQ-PiC) in all clusters with some increases in Distinguished quantities – some inclusion of formal mathematical register (DQ-SFMR) and Distinguished quantities-Complete form of mathematical register/Number Sentence (DQ-CFMR-NS) in some items. Also for the intervention group there was evidence of take-up of a more formal mathematical register such as Number Sentences. Interesting shifts were seen in calculating strategies in that the intervention group's responses to the post-test indicated that the most prevalent strategies were located within Level 2 which is more sophisticated than Level 1 but less sophisticated than Level 3. Also important to note is that shifts occurred mostly within Level 2 rather than across levels. What this suggests for practice is that teachers need to capitalise on result unknown problems in order to promote emergent modelling with appropriate models for the learners. In addition, teachers need to pay attention to the different stages that learners need to go through whilst working towards more sophisticated strategies.

8.4 Empirical Contributions

Sense making of situations and extent of structuring correlating with attainment

The South African literature (Ensor et al., 2009; Schollar, 2008) has tended to argue predominantly that early grades' teachers need to equip learners to use a formal mathematics register and move beyond counting into more sophisticated calculation strategies coupled with more abstract modes of representation. Findings from this study suggest, instead, that attention to initial sense making and setting up of models with learners is required in order to provide a base for calculating answers to problems. Teachers need to pay more extensive attention to discussing, acting out and setting up models of situations in order to place emphasis on sense making, before moving to try and push attention to more sophisticated calculation strategies. This discussion needs to incorporate re-telling of situations in word problems, telling of learner's own word problems. Once sense-making has been established teachers then need to help children set up appropriate models which can be linked to the learners' extent of structuring via attention to distinguishing quantities and early inclusion of some aspects of formal mathematical register.

A further finding of importance for teaching is that word problems that are close to learners' contexts and experiences should be used in order to promote sense making so that learners are able to direct model appropriately and attain high levels of performance.

8.4.1 Principles to consider for sense making

Another finding of this study is that the emphasis on sense making in the international literature and RME theory proved important and relevant. Such sense making was enabled by the following:

- Use of language with which learners are familiar
- Selection of themes with which learners are familiar
- Promotion of discussion during lesson (telling and re-telling)
- Promotion of acting out of situations

A further empirical contribution relates to the finding from this study that there was a move towards employment of more sophisticated mathematizing processes on the part of learners in the intervention group. Analysis of data related to progression indicate that progressive stages need to be considered in a particular order through guided re-invention (Freudenthal, 1971) as learners are moving towards more sophisticated mathematizing processes. Lastly, in their research on structuring Mulligan and Mitchelmore (2009) found that the extent of structuring correlated with the attainment of learners in that learners who were able to use the relationship of quantities in solving word problems usually obtained correct answers. This was the case in this study in the post-test after the intervention.

8.5 Theoretical contributions

The extent of structuring (distinguishing of quantities in a situation in relation to the extent of progression towards use of more formal mathematical register) is the theoretical contribution my study makes to solving of additive relation situations.

The study has involved the development of a theoretically informed, and empirically grounded, expanded progression model for moving towards increasingly abstract modes of representation. The model incorporates attention to 'distinguished quantities' (DQ) and 'formal mathematical register' (FMR) as shown in Table 8.1 below.

Table 8. 1 Indicators for the extent of structuring

Moving from left to right is regarded as 'Progression'													
NoM-I	**NDQ-I/CO**	**NDQ-I/PiC**	**NDQ-I/SFMR**	**NDQ-I/CFMR**	**DQ-C/CO**	**DQ-C/PiC**	**DQ-I/SFMR**	**DQ-C/SFMR**	**DQ-I/CFMR-NL**	**DQ-C/CFMR-NL**	**DQ-I/CFMR-NS**	**DQ-C/CFMR-NS**	**NoM-C**
Key: NoM-I (No Overt Model-Incorrect); NDQ-I/CO (No distinguished quantities-Incorrect/Concrete); NDQ-I/PiC (No distinguished quantities-Incorrect/Pictorial); NDQ-I/SFMR (No distinguished qualities-I/Some form of mathematical register); NDQ-I/CFMR (No distinguished quantities-Incorrect/Complete form of mathematical register); DQ-C/CO (Distinguished quantities – Correct/Concrete) DQ- C/PiC (Distinguished quantities-Correct/Pictorial); DQ-I/SFMR (Distinguished quantities—Incorrect/Some form of mathematical register); DQ-C/SFMR (Distinguished quantities – Correct/Some form of mathematical register); DQ-I/CFMR-NL (Distinguished quantities-Incorrect/Complete form of mathematical register-Number line); DQ-C/CFMR-NL (Distinguished quantities-Correct/Complete form of mathematical register-Number line); DQ-I/CFMR-NS (Distinguished quantities-Incorrect/Complete form of mathematical register-Number sentence); DQ-C/CFMR-NS (Distinguished quantities-Correct/Complete form of mathematical register-Number sentence); NoM-C (No overt model – Correct)													

Examples pertaining to these constructs are included in the appendices. In this study DQ-PiC appeared to be most frequently associated with more correct answers which suggests that with more interventions such as the one in this study, learners can be assisted to move towards a more formal mathematics register.

8.6 Limitations of the study

I began this study with aim of promoting the use of the empty number line which, according to RME, is an emergent model that can support moves towards the development of more sophisticated calculation strategies. This aim was based on my initial agreement with the view that sense-making was something that young children would engage with 'naturally' if presented with familiar situations set in a home language. However, a few lessons into the intervention I realised that focusing on the ENL was not appropriate, given the evidence that children were not taking up openings to make sense of situations in any natural way. Therefore, the primary focus became sense making, followed by direct modelling of situations and then solving of problems. This shift – during the intervention lesson sequence – had consequences for the range of common items across the pre- and post-tests and interviews in this study, leading to a more limited base of matched items than I had originally envisaged. Pilot interviews prior to administering the pre-test may have helped to draw attention to the need for a more restricted focus on join/separate items on the assessment instruments and in the intervention ahead of time.

A second limitation was a language issue as it became evident during the pilot lesson that some of the children in the intervention were immigrants from various rural areas and were used to different dialects of isiZulu. In order to mitigate the possible negative effects of this limitation I conducted a workshop with teachers who teach isiZulu in which they were asked to translate word problems from English to isiZulu. During the workshop we had to reach consensus on which term would be appropriate for the learners and I was advised to also use different dialects to try and explain certain concepts.

The interventions had to be reshuffled and shortened as I realised during the lessons that learners were not familiar with addition relation problems although, according to the CAPS curriculum, they should have been exposed to such word problems from grade R. Weaknesses in sense-making related to simpler classes of problems became clearer after the pilot lesson and early in the intervention sequence. This led to the need for more focus on join and separate problems and to a more narrowly focused post-test. Ideally, pre- and post-tests would have had more questions on simpler categories.

8.7 Self Reflection

During this doctoral journey I had to adapt some of my assumptions about what learners are expected to know at a specific grade level. For example, I had assumed that since it is stipulated in the curriculum the children should be familiar with word problems and models such as a number line, they would have acquired the knowledge and skills expected of learners in a particular grade. The finding that this was not the case has implications for the progress of learners in mathematics, especially in FP.

The CGI approach and RME principles have made me appreciate the role that teachers should play in promoting guided re-invention with learners so that these learners are able to reinvent their mathematics and thereby improve their performance.

8.8 Implications

For Materials Development

Teachers need to pay attention to the kinds of materials that are relevant for learners in FP classes when teaching and when designing interventions. For example, contexts need to be considered given that in some contexts teachers might need to improvise depending on the number of learners they have in their classroom. For example, for the intervention in this study, I had to print out A3 pages with pictures in order for learners to have access to problems from the *Big Book of word problems.* Language and relevant themes also need to be taken into consideration. For example, in this study I used items and objects such as buses, sweets with which learners were familiar.

For Teacher Education

In order for teachers to make mathematics accessible to FP children, RME principles and concepts such as sense making, and guided re-invention need to be taken into consideration. Graven (2015) pointed to a reality that ‘fear’ appears to be common in the early grades among FP learners. Learners fear to make mistakes and to try things out which might be an underlying part of why children failed to ‘naturally’ try and make sense of the problems that were presented to them. Teachers need to promote sense making by allowing learners to make mistakes and to try things out. Lastly, teachers need to consider that progression towards more a formal mathematical register takes places in stages. Therefore, teachers need to be aware of these stages and explore ways of moving learners forward within and across these stages.

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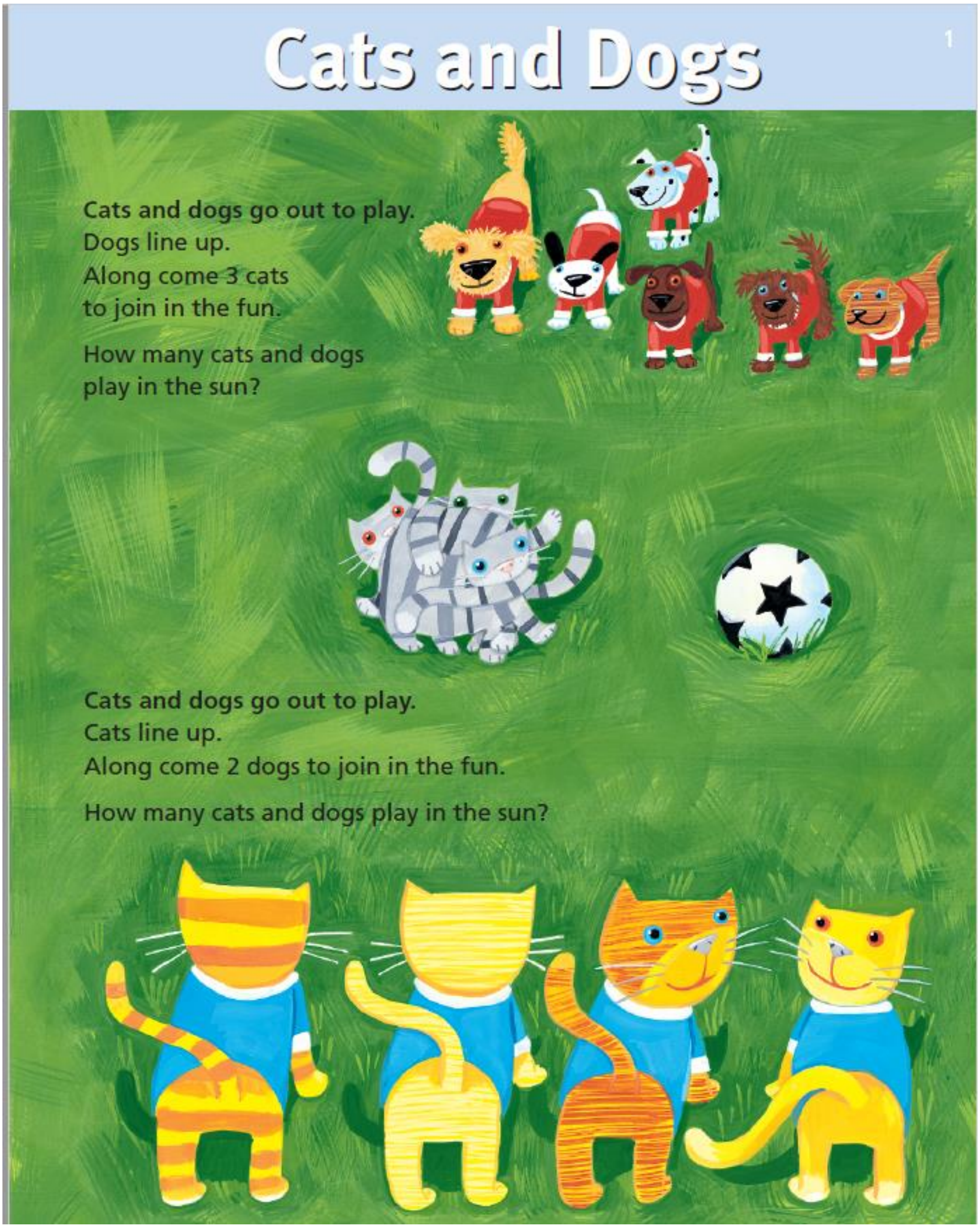
APPENDIX A

Big Book Example of word problems and themes (Askew, 2004)



Intervention Lesson Schedule	
Activity	**Date**
Join (Result Unknown)	Tuesday 14 October 2014
Join • Result Unknown • Change Unknown • Start Unknown	Thursday 16 October 2014 Tuesday 21 October 2014 Thursday 23 October 2014
Separate • Result Unknown • Change Unknown • Start Unknown	Tuesday 28 October 2014 Thursday 30 October 2014 Tuesday 4 November 2014 Thursday 6 November 2014
• Mixed Problems	Monday 10 November 2014 Tuesday 11 November 2014
• Mixed Problems	Thursday 13 November 2014 Monday 17 November 2014

<table>
<tr><td colspan="2"><u>PILOT LESSON</u></td></tr>
<tr><td>Activities</td><td>Resources</td></tr>
<tr><td><u>ACTIVITY 1 – 10 minutes</u>
<u>More and less than 1</u>

Which number is 1 more than 21? (Yiliphi inani elidlula u-21 ngo-1)
Which number 1 is less than 21? (Yiliphi inani elincane kuno-21 ngo-1?)
Which number is 1 more than 6? (Yiliphi inani elidlula u-6 ngo-1)
Which number is 1 less than 6? (Yiliphi inani elincane kuno-6 ngo-1?)

<u>More and less than 2</u>
Which number is 2 more than 21? (Yiliphi inani elidlula u-21 ngo-2)
Which number is 2 less than 21? (Yiliphi inani elincane kuno-21 ngo-2?)
Which number is 2 more than 6? (Yiliphi inani elidlula u-6 ngo-2)
Which number is 2 less than 6? (Yiliphi inani elincane kuno-6 ngo-2?)

<u>Ordering of Numbers (Writing Exercise)</u>
<table>
<tr><td>17</td><td>16</td><td>23</td><td>22</td><td>11</td><td>10</td></tr>
<tr><td>15</td><td>14</td><td>21</td><td>20</td><td>9</td><td>8</td></tr>
<tr><td>13</td><td>12</td><td>19</td><td>18</td><td>7</td><td>6</td></tr>
</table></td><td>‘Washing Line’
(Wool)

Number Cards</td></tr>
<tr><td><u>ACTIVITY 2 – 20 minutes (PAIR/WHOLE CLASS DISCUSSION)</u>

<u>Problem 1 (Whole Class) 10 min</u>

Show the picture of the Bus to the children and discuss it to establish the context for the problem. Where do people normally take a bus to? (More guidelines on Pg 14 and 15 of the Teacher’s guide)

Read the through the problem with the children. Encourage them to use whatever means they like to find an answer. (Refer to Pg 14 and 15 of the teacher’s guide)

Join (Result Unknown): Bekunabantu abangu-21 ebhasini. Bese kugibela futhi abantu abangu-6. Bangaki abantu ebhasini manje?

Ask for volunteers to show/tell the class how they have solved the problem.

<u>Problem 2 10 min</u>

Bekunabantu abangu-9 ebhasini. Bese kugibela futhi abantu abangu-4. Bangaki abantu ebhasini manje? (There were 9 people on the bus. 4 more people got on. How many people are on the bus now?)</td><td>Printed A3 page with picture of a Bus</td></tr>
<tr><td><u>ACTIVITY 3-Linking up the problems - 10 min</u>

Refer to Pg 15 of TG (Big Books of WP)
<u>ACTIVITY 4- Making up own problems - 10 min</u>
Give out handout with 5 pencils, ask learners to add their own then tell a story.
<u>ACTIVITY 5 – Wrap up – 10 minutes</u>
Refer to Pg 15 TG (Big Books of WP)</td><td>Worksheets

Pencils</td></tr>
</table>

<table>
<tr><td colspan="2"><u>LESSON 1</u>
<u>CONTEXT - BUS</u></td></tr>
<tr><td>Activities</td><td>Resources</td></tr>
<tr><td><u>ACTIVITY 1: FILL IN MISSING WORDS ON THE ‘ROPE’ (10 min)</u>

1) Ask for two volunteers to hold the rope on either sides
2) Hang 1, 4, 7, 10, 13, 16, 19
Q1: Which number comes after or is 1 more than 7, 13, 19, 1, 4, 10, 16?
Q2: Which number comes before 7, 13, 19, etc…
Is there any number missing?</td><td>‘Rope” (Wool)

Number Cards</td></tr>
<tr><td><u>ACTIVITY 2 – JUMPING ON THE NUMBER TRACK (10 min)</u>

PUT NUMBER TRACK ON THE FLOOR FROM ANY NUMBER, eg. 8 (NOT 1) to 12. TELL THE STORY AND ASK FOR A VOLUNTEER TO COME AND DEMONSTRATE THE STORY IN THE WAY THE UNDERSTAND IT.

STORY: The bus picks up people at house no8 then stops at house no12. How many houses will the bus jump before it reaches house number 12? Ibhasi igibelisa abantu kwanumber 8 bese igcina kwanumber 12. Idlula izindlu ezingaki phambi kokokuthi ifike kwanumber 12?

<table><tr><td><u>8</u></td><td><u>9</u></td><td><u>10</u></td><td><u>11</u></td><td><u>12</u></td></tr></table>

Ask another learner to describe the demonstration-look for keywords such as ‘jumps’ etc

ASK IF THERE IS ANYONE WHO CAN SHOW ON THE BOARD WHAT HAS BEEN DEMONSTRATED</td><td>Number Track</td></tr>
<tr><td><u>ACTIVITY 3 – 30 minutes (PAIR/WHOLE CLASS DISCUSSION)</u>

<u>Problem 1 (Whole Class)</u>

Show the picture of the Bus to the children and discuss it to establish the context for the problem. Where do people normally take a bus to? (More guidelines on Pg 14 and 15 of the Teacher’s guide)

Read the through the problem with the children. Encourage them to use whatever means they like to find an answer. (Refer to Pg 14 and 15 of the teacher’s guide)

Join (Result Unknown): Bekunabantu abangu-5 ebhasini. Bese kugibela futhi abantu abangu-4. Bangaki abantu ebhasini manje?

Ask for volunteers to show/tell the class how they have solved the problem.

<u>Problem 2</u>

Use same context but change numbers. Establish if learners are able to make sense of the problem and how the numbers are connected to the situation.

Eg Bekunabantu abangu-6 ebhasini. Bese kugibela futhi abantu abangu-8. Bangaki abantu ebhasini manje?</td><td>A3 printed page with Bus picture</td></tr>
<tr><td><u>ACTIVITY 4 - WRAP UP (10 MIN)</u>

Remind learners about worksheets, tell them about stickers for those who do homework.</td><td>Worksheets</td></tr>
</table>

LESSON 2 **CONTEXT – PENCILS/AMAPENSELA** **OBJECTIVES:** • Understand addition problems involving joining two sets • Understand addition problems involving Change Unknown Problems • Model addition using the number line	
Activity	**Resources**
Activity 1: WHOLE CLASS DISCUSSION (20 min) • ***Put picture of the pencils on the board. Discuss it with the children. What do they see etc.*** *(More guidelines on Pg 14 and 15 of the Teacher's guide)* • *Read the through the problem with the children. Encourage them to use whatever means they like to find an answer. (Refer to Pg 14 and 15 of the teacher's guide)* **Change Unknown Problems** **Problem 1:** uThando kade enamapensela angu-5. uZola wamupha amanye amapensela. Manje uThando unamapensela angu-9. uZola umuphe amapensela amangaki uThando? (Thando had 5 pencils. Zola gave her some more pencils. Now Thnado has 9 pencils. How many more pencils did Zola give to Thando? Ask the children: • What is the story about? • Who can show us how they can find the answer? **Problem 2:** uNandi kade enamapensela angu-8. Wathenga amanye amapensela. Manje uNandi unamapensela angu14. uNandi uthenge amapensela amangaki? (Nandi had 8 pencils. She bought some more pencils. Now Nandi has 14 pencils. How many more pencils did Nandi buy?) Ask the children: • What is the story about? • Who can show us how they find the answer? Discuss the link between the two problems (Refer to Pg 15 of TG)	A3 printed page with picture of pencils
Activity 2: INDIVIDUAL (30 min) **Hand out Worksheets with Problems**	Worksheets
Activity 3: Wrap Up (Whole Class) 10 Minutes Discussions of observations	

<table>
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<u>**LESSON 3**</u>
<u>**CONTEXT – PENCILS/AMAPENSELA**</u>

<u>**OBJECTIVES:**</u>

- Understand addition problems involving joining two sets
- Understand addition problems involving Change Unknown Problems
- Model addition using the number line

</td></tr>
<tr><td><u>**Activities**</u></td><td>**Resources**</td></tr>
<tr><td>

<u>**Activity 1: WHOLE CLASS DISCUSSION (20 min)**</u>

- <u>***Put picture of the pencils on the board. Discuss it with the children. What do they see etc.***</u> *(More guidelines on Pg 14 and 15 of the Teacher's guide)*
- *Read the through the problem with the children. Encourage them to use whatever means they like to find an answer. (Refer to Pg 14 and 15 of the teacher's guide)*

Change Unknown Problems
Problem 1:
uThando kade enamapensela angu-5. uZola wamupha amanye amapensela. Manje uThando unamapensela angu-9. uZola umuphe amapensela amangaki uThando? (Thando had 5 pencils. Zola gave her more pencils. Now Thando has 9 pencils. How many pencils did Zola give to Thando?)

Ask the children:

- What is the story about?
- Who can show us how they can find the answer?

Problem 2:
uNandi kade enamapensela angu-8. Wathenga amanye amapensela. Manje uNandi unamapensela angu14. uNandi uthenge amapensela amangaki? (Nandi had 8 pencils. She bought more pencils. Now Nandi has 14 pencils. How many pencils did Nandi buy?)
Ask the children:

- What is the story about?
- Who can show us how they find the answer?

Discuss the link between the two problems (Refer to Pg 15 of TG)

</td><td>A3 printed page with picture of pencils.</td></tr>
<tr><td>**Activity 2: INDIVIDUAL (30 min)**
Hand out Worksheets with Problems</td><td>Worksheets</td></tr>
<tr><td><u>**Wrap Up (Whole Class) 10 Minutes**</u>
Discussions of observations</td><td></td></tr>
</table>

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<tr><td colspan="2">

LESSON 4

CONTEXT – BUSES/AMABHASI, PENCILS/AMAPENSELA and SWEETS/AMASWIDI

OBJECTIVES:

- Understand addition problems involving joining two sets
- Understand addition problems involving Result Unknown, Change Unknown and Start Unknown Problems
- Understand translating a situation into a number sentence
- Model addition using the number line

</td></tr>
<tr><td>

Activities

</td><td>

Resources

</td></tr>
<tr><td>

Activity 1: WHOLE CLASS DISCUSSION (20 min)

a) RECAP of JOIN RESULT UNKNOWN PROBLEMS and CHANGE UNKNOWN PROBLEMS

- ***Paste picture of the Pencils on the board. Underneath put Join Result Unknown Word Problem.***

uThando kade enamapensela angu-9. uZola wamupha amapensela angu-12. Mangaki amapensela kaThando manje? (Thando had 9 pencils. Zola gave her 12 more. How many pencils does Thando now have?)

- ***Paste picture of Bus on the board. Underneath put Join Change Unknown Word Problem.***

Bekunabantu abangu-9 ebhasini. Bese kugibela futhi abanye abantu. Manje kunabantu abangu-21 ebhasini. Bangaki abantu abaze bagibela futhi? (There were 9 people in the bus. More people got on. Now there are 21 people in the bus. How many people got on?)

b) INTRODUCTION OF START UNKNOWN PROBLEMS

- ***Put picture of sweets on the board. Underneath put Join Start Unknown Word Problem***

uLungi kade enamaswidi. Wathenga amaswidi angu-7. Manje unamaswidi angu-21. Mangaki amaswidi kade uLungi enawo ekuqaleni? (Lungi had some sweets. She bought 7 more sweets. Now she has 21 sweets. How many sweets did Lungi have to start with?)

Ask for 3 learners to show how they would solve the problem.

Once the learners are done, Discuss the Word problems and how they have been solved.

</td><td>

A3 printed page with picture of pencils.

A3 printed page with picture of a bus.

A3 printed page with picture of sweets.

</td></tr>
<tr><td>

Activity 2: Writing Exercise (Individual)

Hand out Worksheets with Word Problems

</td><td>

Worksheets

</td></tr>
<tr><td>

Activity 3: Wrap Up (Whole Class) 10 Minutes

Choose 3 Word problems with different problem types: Result Unknown, Change Unknown, and Start Unknown for children to show on the board how they worked them out.

</td><td></td></tr>
</table>

LESSON 5

CONTEXT – BUSES/AMABHASI, PENCILS/AMAPENSELA and SWEETS/AMASWIDI

OBJECTIVES:

- Continue with Activity 2 of lesson 4
- Understand and solve addition problems involving Result Unknown, Change Unknown and Start Unknown Problems
- Model addition using the PPW diagram
- Model addition using the number line

Activities	**Resources**
Activity 1: GROUP WORK ***a) Call first group of learners (low attaining) to the front with their books.*** • Write two (Join RU) word problems (writing one first) on the board and guide learners to draw either circles or tally marks to solve the problems • Then guide them to convert drawings to numbers, then solve the problem • Depending on progress send them to their desks to do worksheet 1 using the discussed strategies ***b) Call second group of learners (middle attaining) to the front with their books.*** • Write example on the board (Join RU), use PPW to explain the relationships between the numbers. Then show them how to find answer using the empty number line, incorporating the 'count on' strategy. • Take 2 examples from worksheet 2-repeat same process • Depending on progress send them to their desks to complete the worksheets ***c) Call third group of learners (high attaining) to the front with their books.*** • Repeat process as per group two • Shift to jumps of 10 and units	**Learners' Exercise books**
Activity 2: Wrap Up (Whole Class) 10 Minutes Choose 3 Word problems with different problem types: Result Unknown, Change Unknown, and Start Unknown for children to show on the board how they worked them out.	

LESSON 6

CONTEXT – FROGS/AMAXOXO

OBJECTIVES:

- Use context that will translate to model advocated-eg. frog jumps
- Understand how dots translate to number line
- Understand 'count on strategy' starting from any number and adding '10'
- Fluencies
- Model addition using the dots/circles
- Model addition using the number line

Activities	Resources
Activity 1: • **Count from 1 to 30 using the 100 square (In English and IsiZulu)** • **FWNS from 8 to 10…Use the frog to jump from 8 making 2 jumps to 10, 18 making 2 jumps to 20, 28…etc.** • **BWNS from 10 to 8…etc…** • **Ask learners what they notice about the columns 8, 9, 10 (look for 'we're counting in 10's, or we're jumping in 10's etc)**	**100 Square** **Frogs cut out papers**
Activity 2: Writing exercise Write sums on the board 8 + 2 =? ; 20-2=?; 28 + 2 =? ; 30-2 =? ; 48 + 2 =? ; 50-2 = ? ; 68 + 2 =? ; 70-2 =? ; 88 + 2 =? ; 98 + 2 =? When learners are finished, write answers on the board.	
Activity 3: Whole Class Solve word problems with frog jumps. Hand out No 1 first, do it together with learners. Then hand out No 2 and do it together with learners to demonstrate adding 10 without counting in ones-use dots in alignment with number line.	
Activity 4: Wrap Up Discuss Observations	

<table>
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<u>LESSON 7</u>

<u>CONTEXT – BUSES (AMABHASI), SWEETS (AMASWIDI)</u>

<u>OBJECTIVES:</u>

Use word problems to engage children in discussing the problems and the mathematics in them
Use word problems for children to act and directly model
Use word problems of different contexts to elicit understanding of learners

</td></tr>
<tr><td>Activities</td><td>Resources</td></tr>
<tr><td>

Activity 1:

Count from 1 to 100 using the 100 square, learners must point to the numbers as they count (In English and IsiZulu)
Number Identification…Show me 13…which number comes after 13, which number comes before 13 etc
Turn 100 square upside down…ask learners individually, which number comes after, and before…etc

</td><td>100 Square</td></tr>
<tr><td>

<u>Activity 2: Word Problems</u>

Problem 1: Bus: Separate Result Unknown

Paste Picture of Bus on the board
Paste Word Problem underneath:

Bekunabantu abangu-6 ebhasini. Abangu-3 behla. Bangaki abantu ebhasini manje? (There were 6 people in the bus. 3 people got off. How many people are in the bus now?)

Problem 2: Bus: Join Result Unknown

Paste Picture of the bus on the board
Paste Word Problem underneath:

Bekunabantu abangu-6 ebhasini. Kwagibela futhi abantu abangu-3. Bangaki abantu ebhasini manje? (There were 6 people in the bus. 3 more people got onto the bus. How many people are in the bus now?)

Problem 1

Choose 6 learners to form a line before coming to the front of the classroom. Then learners pretend to be moving in a bus towards the front of the classroom. The purpose of this is that learners need to realise that there were already 6 people in the bus. People didn’t get on one by one.

Once learners are in front of the classroom talk about what happens next based on the story. 3 learners must ‘get off’ (sit down). Now ask learners. So how many people are now left in the bus?

Write the answer on the board as : There are now 3 people in the bus, or Manje kunabantu abangu-3 ebhasini.

Problem 2

Repeat same process as per problem 1. Now 3 more people must get onto the bus. Ask learners how many people are now in the bus?

</td><td>A3 printed page with picture of a Bus</td></tr>
</table>

Write answer on the board as: There are now 9 people in the bus. • Talk about the numbers 6 and 3 in both the problems and then talk about the different answer. Ask learners why is the answer 3 in the first problem and 9 in the second problem**.**	
<u>Activity 3</u> • Now ask learners to attempt drawing the first story in their books. Walk around to see what they are doing. • Those who are finish can attempt drawing the second story in their books. • Those who manage to finish quickly get to work on fluency worksheets	Learners' Exercise books
Activity 4: Wrap Up	

<u>LESSON 8</u>

<u>CONTEXT – BUSES (AMABHASI)</u>

<u>OBJECTIVES:</u>

- Use word problems to engage children in discussing the problems and the mathematics in them
- Use word problems for children to act and directly model
- Use word problems of different contexts to elicit understanding of learners

Activities	**Resources**
Activity 1: • Count forward in 1's from any number 5…15…25… • Count backwards in 1's from any number 6…16…26… • Start counting from any number and select individual learners to say number after and before etc. • Ask learner to write a number on the board, ask which number comes after and before.	
<u>Activity 2: Word Problems</u> **Recap lesson 7** **Continue with Activity 3 of Lesson 7** • Select children noted on reflective journal to draw the stories on the board starting with the Separate-RU problem. • Also emphasizing writing their answers as: .There are now …people on the bus'. • Learners to make corrections, finish off WP1 to WP4 • Word problem 3 and 4 were added during the lesson. • WP3 was: There were 7 people on the bus. 4 more people got. How many people are now on the bus? • WP4 was: There were 7people on the bus. 4 people got off. How many people are now on the bus? • For those who are finished all 4 problems from Lesson 7-hand out more problems with numbers varied. Give learners sitting next to each other different problems to avoid copying.	**Learners' Exercise books**
<u>Activity 3: Wrap Up</u> Discuss problem 4 with '7 and 4'. Establish if the learners understand the difference between the two problems. Ask if there are any questions.	

LESSON 9 CONTEXT – BUSES (AMABHASI) OBJECTIVES: • Use word problems to engage children in discussing the problems and the mathematics in them • Use word problems for children to act and directly model • Use word problems of different contexts to elicit understanding of learners • Help children improve counting fluencies	
Activities	**Resources**
Activity 1: Written Activity: Mental Hando out matrix sheets for learners to complete. Give 10 minutes then collect.	**Matrix sheets**
Activity 2: Word Problems Hand out random Word Problems to those who have finished the 4 word problems from Lesson 7 and 8, and walk around marking.	
Activity 3: Wrap Up Fluencies – use hundred square Count forward in 2's starting from 3 to 30 Count backward in 2's starting from 29 Show me 13…jump twice going forward (siya phambili) Show me 7…jump twice going backwards (sibuyela emuva)	100 Square

<table>
<tr><td colspan="2">

<u>LESSON 10</u>
<u>CONTEXT – SWEETS (AMASWIDI)</u>

<u>OBJECTIVES:</u>

- Use word problems to engage children in discussing the problems and the mathematics in them
- Use word problems for children to act and directly model
- Use word problems of different contexts to elicit understanding of learners
- Help children improve counting fluencies

</td></tr>
<tr><td><u>Activity</u></td><td>Resources</td></tr>
<tr><td><u>Activity 1: Written Activity: Mental</u>
Hundred square-talk about the pattern 1 to 10…11 to 20 (horizontally) and vertically. Use hundred square for horizontal explanation.
Write on the board for vertical explanation 1 to 10…11 to 20
Use one of the matrix sheets as an example.
Learners to fill in the numbers on the board.</td><td>100 Square</td></tr>
<tr><td><u>Activity 2: Word Problems (Whole Class Discussion)</u>
Put up a picture of sweets on the board-Have a discuss about buying/getting or selling/giving.
Put up a problem on the board for learners to ACT OUT:
Join-Result Unknown
uMary kade enamaswidi angu-5. uMpilo wamupha amaswidi angu-2. Mangaki amaswidi kaMary manje?
Separate – Result Unknown
uZama kade emanaswidi angu-5. Wapha uWilliam amaswidi angu-2. Mangaki amaswidi kaZama manje?
Ask learners to pose a story and get other learners to act it out.</td><td>A3 printed page with picture of sweets.</td></tr>
<tr><td><u>Activity 3: Individual Writing Activity</u>
Hand out word problems on slips. Walk around to see what the learners are doing.</td><td>Slips</td></tr>
<tr><td><u>Activity 4: Wrap Up</u>
Discuss similarities between bus problems and sweet problems</td><td></td></tr>
</table>

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<u>LESSON 11 - MONDAY 17 NOVEMBER 2014</u>
<u>CONTEXT – SWEETS (AMASWIDI)</u>

<u>OBJECTIVES:</u>

- Conduct activities that will help children improve counting fluencies
- Continue with Lesson 10 so learners can work on more word problems within the context of sweets

</td></tr>
<tr><td><u>Activities</u></td><td>Resources</td></tr>
<tr><td><u>Activity 1: Written Activity: Mental</u>
Use hundred square to count from 1 to 100
Learners to show on the fingers 2,4,3,5,7,6,8,10,9
Show me 2 + 3 on fingers…what is the answer?
Show 4 + 5…what is the answer?
What is 6 + 2?
What is 7 + 3?
What is 8 + 1?
Learners to fill in the matrix sheet.</td><td>100 Square</td></tr>
<tr><td><u>Activity 2: Individual Writing Activity</u>
Learners pose and write own problems using the context of sweets and any numbers. (Write on clean white paper)</td><td>Blank paper and Exercise books</td></tr>
<tr><td><u>Activity 3: Individual writing Activity</u>
Learners continue with solving more ‘sweets’ word problems.</td><td></td></tr>
<tr><td><u>Activity 4: Wrap Up</u>
Discuss links between bus problems and sweet problems.</td><td></td></tr>
</table>

LESSON 12

CONTEXT – SWEETS (AMASWIDI)

OBJECTIVES:

- Conduct activities that will help children improve counting fluencies
- Use number track
- Use number line
- Introduce 'Change Unknown Problem' Type
- Explore acting out of the problems
- Elicit understanding of learners through drawings or any other direct model

Activity	Resources
Activity 1: Counting Fluencies FNWS from 1 to 100 BNWS from 100 to 1 Identify numbers where is 5…16…31…58…70…84…99 Writing activity: Hand outs Write 2 in the first block…jump 3 times Demonstrate on the board…should make first jump from 2 to 3, to 4 to 5 Now go to number line… start at 2…make 3 jumps	**Learners' Exercise books**
Activity 2: Whole Class: Word Problems Put up 2 Word problems on the board: a) uJabu kade enamaswidi angu-4. uSipho wamupha amaswidi. Manje uJabu unamaswidi angu-7. uSipho umuphe amaswidi amangaki uJabu? (Jabu had 4 sweets. Sipho gave him more sweets. Now Jabu has 7 sweets. How many sweets did Sipho give to Jabu?) b) Let learners act it out c) uSipho kade enamaswidi angu-4. Wapha uJabu amaswidi. Manje uSipho uneswidi engu-1. uSipho umuphe amaswi amangaki uJabu? (Sipho had 4 sweets. He gave some sweets to Jabu. Now Sipho has 1 sweet. How many sweets did Sipho give to Jabu? d) Learners act it out Learners to pose own problems	**A3 printed sheets with word problems.**
Activity 3: Individual writing Activity Learners solve change unknown word problems individually by using pictures.	
Activity 4: Wrap Up Ask 2 to 3 learners to show on the board how they have solved their problems.	

LESSON 13

CONTEXT – SWEETS (AMASWIDI)/MIXED

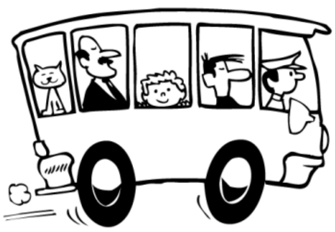
OBJECTIVES:

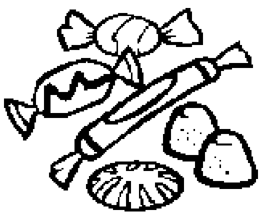
- Conduct activities that will help children improve counting fluencies
- Learners show working on number line
- Explore 'Start Unknown Problem' Type

Learners work on mixed word problems individually

Activities	Resources
Activity 1: Addition and Subtraction Solve: 2 + 3=? 12 + 3=? 22 +3 =? 27-5=? 17-5=? 7-5=?	**Learner's exercise books**
Activity 2: Whole Class: Word Problems Put up Word problem on the board: a) uJabu kade enamaswidi. uSipho wamupha amaswidi angu-4. Manje uJabu unamaswidi angu-7. uJabu kade enamaswidi amangaki ? b) uJabu kade enamaswidi. Wapha uSipho amaswidi angu-4. Manje uJabu unamaswidi angu-3. Kade emangaki amaswidi kaJabu?	**A3 printed sheets with word problems.**
Activity 3: Individual writing Activity Learners solve mixed word problems.	
Activity 4: Wrap Up Ask a few learners to show on the board how they have solved their problems.	

Examples of pictures that were pasted on the board for discussion





Example of a Worksheet

Mixed Word Problems

1. Bekunabantu abangu-6 ebhasini. Kwehla abantu abangu-2. Bangaki abantu ebhasini manje? (There were 6 people on the bus. Two people got off. How many people are on bus now?)

2. uThuli kade enamaswidi angu-7. uJabu wamupha amanye amaswidi. Manje uThuli unamaswidi angu-12. uJabu umuphe amaswidi amangaki uThuli? (Thuli had 7 sweets. Jabu gave her some more sweets. Now Thuli has 12 sweets. How many sweets did Jabu give Thuli?)

3. Bekunabantu abangu-14 ebhasini. Kwagibela futhi abantu abangu-12. Bangaki abantu ebhasini manje? (There were 14 people on the bus. Twelve more people got on. How many people are now on the bus?)

4. Bekunabantu abangu-18 ebhasini. Kwehla abantu abangu-13. Bangaki abantu ebhasini manje? (There were 18 people on the bus. Thirteen people got off. How many are on the bus now?)

5. ULizo kade enamaswidi. Wathenga amanye amaswidi angu-9. Manje unamaswidi angu-17. Mangaki amaswidi kade enawo ekuqaleni uLizo? (Lizo had some sweets. He bought 9 more sweets. Now he has 17 sweets. How many sweets did Lizo have to start with?)

6. uSizwe kade enamapensela angu-21. uSifundo wamupha amanye amapensela angu-14. Mangaki amaswidi kaSizwe manje? (Sizwe had 21 pencils. Sifundo gave him 14 more pencils. How many pencils does Sizwe have now?)

7. Bekunamantombazane angu-19 eklasini. Amantombazane angu-8 aphuma phandle. Mangaki amantombazane asele ekilasini? (There were 19 girls in the classroom. Eight girls went outside. How many girls are left in the classroom?)

8. Bekunabantu abangu-15 ebhasini. Kwagibela abanye futhi. Manje kunabantu abangu-24. Bangaki abantu abagibele futhi ebhasini? (There were 15 people on the bus. Some more people got on. Now there are 24 people. How many more people got onto the bus?)

9. uMbali unamaswidi kashokoledi angu-18. uMpilo unamaswidi kastrobheri angu-6. Mangaki amaswidi wabo uma ephelele? (Mbali has 18 chocolate sweets. Mpilo has 6 strawberry sweets. How many sweets do they have altogether?)

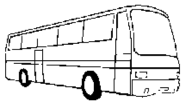
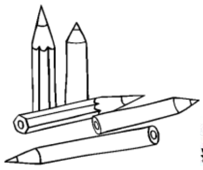
10. uThoko kade enamapensela. uNosi uwamupha namapensela angu-4. Manje uThoko unamapensela angu-9. uThoko kade enamapensela amangaki? (Thoko had some pencils. Nosi gave him 4 more pencils. Now Thoko has 9 pencils. How many pencils did Nosi give to Thoko?)

APPENDIX B

Wits Maths Connect – Primary Project

Learner booklet

Name	
Teacher's name	
School	
Grade	
Date	

Umbuzo 1 (Question 1)

Bekunabantu abangu-23 ebhasini. Bese kugibela abangu-5. Bangaki abantu ebhasini manje? (There were 23 people on the bus. Five more people got on. How many people are on the bus now?)	
Impendulo (Answer)	

Umbuzo 2 (Question 2)

Bekunamabhodlela angu-12 etafuleni. uMama wasusa angu-7. Mangaki amabhodlela asele? (There were 12 bottles on the table. Mom removed 7. How many bottles are left?)	
Impendulo (Answer)	

Umbuzo 3 (Question 3)

uThuli unamaswidi kashokoledi angu-9. uSam unamaswidi kastrobheri angu-6. Mangaki amaswidi wabo ehlangene? (Thuli has 9 chocoloate sweets. Sam has 6 strawberry sweets. How many sweets do they have altogether?)	
Impendulo (Answer)	

Umbuzo 4 (Question 4)

uSandile unamapensile angu-3. uThabo unamapensile angu-21. Amapensile kaThabo maningi kangaki kunakaSandile? (Sandile has 3 pencils. Thabo has 21 pencils. How many pencils does Thabo have more than Sandile?)

Impendulo (Answer)	

Umbuzo 5 (Question 5)

Bekunabantu abangu-8 ebhasini. Kwagibela abanye futhi. Manje kunabantu abangu-14. Bangaki abantu abaze bagibela ebhasini? (There were 8 people on the bus. More people got on. Now there are 14 people. How many more people got on the bus)

Impendulo (Answer)	

Umbuzo 6 (Question 6)

Bekunamabhodlela angu-28 etafuleni. Umama wasusa amabhodlela athize kwaze kwasala amabhodlela angu-19. Mangaki amabhodlela asuswe ngumama? (There were 28 bottles on the table. Mom removed some bottles and there were 19 bottles were left. How many bottles did mom remove?)

Impendulo (Answer)	

Umbuzo 7 (Question 7)

uThuli unamaswidi kashokoledi angu-12. uSam unamaswidi kastrobheri athize. Banamaswidi angu-20 behlangene. Mangaki amaswidi kaSam? (Thuli has 12 chocolate sweets. Sam has some strawberry sweets. They have 20 sweets altogether. How many sweets does Sam have?)	
Impendulo (Answer)	

Umbuzo 8

uSandile unamapensile angu-14. uThabo unamapensile amaningi kunakaSandile nga-4. Mangaki amapensile kaThabo? (Sandile has 14 pencils. Thabo has 4 pencils more than Sandile. How many pencils does Thabo have?)	
Impendulo	

Umbuzo 9

Kade kunabantu ebhasini. Kwaze kwagibela abantu abangu-2. Manje kunabantu abangu-8. Bangaki abantu kade besebhasini? (There were people on the bus. Two more people got on. Now there are 8 people. How many people were on the bus to start with?)	
Impendulo	

Umbuzo 10 (Question 10)

Kade kunamabhodlela etafuleni. Umama wasusa amabhodlela angu-5 kwaze kwasala angu-13. Mangaki amabhodlela kade esetafuleni? (There were bottles on the table. Mom removed 5 bottles and there were 13 bottles left. How many bottles were on the table to start with?)	
Impendulo	

Umbuzo 11 (Questiion 11)

Izingane ezingu-29 zidlala ngaphandle. Amantombazane angu-14 nabafana abathize. Bangaki abafana? (Twenty nine children are playing outside. There are 14 girls and some boys. How many boys are there?**)**	
Impendulo (Answer)	

Umbuzo 12 (Question 12)

uSizwe unamapensile angu-10. Unamapensile amancane kunakaThabo nga-2. Mangaki amapensile kaThabo? (Sizwe has 10 pencils. He has 2 less pencils than Thabo. How many pencils does Thabo have?)	
Impendulo (Answer)	

Umbuzo 13 (Question 13)

Abafana abangu-12 namantombazane angu-8 adla amaswidi. Zingaki izingane ezidla amaswidi? (Twelve boys and 8 girls are eating sweets. How many children are eating sweets?)	
Impendulo (Answer)	

Umbuzo 14 (Question 14)

uSizwe unamapensile angu-17. uNosi unamapensile angu-13. uNosi unamapensile amancane kangaki kunakaSizwe? (Sizwe has 17 pencils. Nosi has 13 pencils. How many less pencils does Nosi have than Sandile?)	
Impendulo (Answer)	

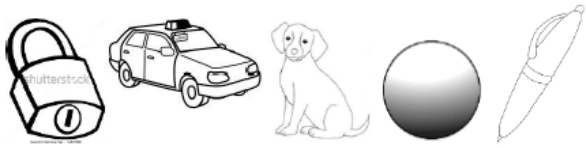
UNIVERSITY OF THE WITWATERSRAND JOHANNESBURG

Post-test

Wits Maths Connect – Primary Project

Learner booklet

Name	
Teacher's name	
School	
Grade	
Date	



Key: Umbuzo (Question); Impendulo (Answer)

Umbuzo 1

Bekunabantu abangu-3 emotweni. Bese kugibela abangu-5. Bangaki abantu emotweni manje? **(There were 3 people in the car. Five more people got on. How many people are in car now?)**	
Impendulo	

Umbuzo 2

Bekunokhiya abangu-12 etafuleni. uMama wasusa abangu-7. Bangaki okhiya etafuleni manje? **(There were 12 padlocks on the table. Mom removed 7. How many padlocks are left now?)**	
Impendulo	

Umbuzo 3

uMandisa unebhola eziredi ezingu-9. uMbali unebhola eziblu ezingu-6. Zingaki iibhola zabo mazizonke? **(Mandisa has 9 red ball. Mbali has 6 blue balls. How many balls do they have altogether?)**	
Impendulo	

Umbuzo 4

uManqoba kade enamapeni angu-13. uSifundo wamupha amanye amapeni. Manje uManqoba unamapeni angu-21. uSifundo umuphe ampeni amangaki uManqoba? (Manqoba had 13 pens. Sifundo gave him more pens. Now Manqoba has 21 pens. How many pens did Sifundo give to Manqoba?)	
Impendulo	

Umbuzo 5

Bekunabantu abangu-8 emotweni. Kwagibela abanye futhi. Manje kunabantu abangu-14. Bangaki abantu abaze bagibela futhi emotweni? **(There were 8 people in the car. More people got on. Now there are 14 people. How many more people got onto the car?)**	
Impendulo	

Umbuzo 6

Bekunokhiya abangu-28 etafuleni. Umama wasusa okhiya. Manje kusele okhiya abangu-19. Bangaki okhiya abasuswe ngumama? **(There were 28 padlocks on the table. Mom removed some keys. Now there are 19 keys left. How many keys did mom remove?)**	
Impendulo	

Umbuzo 7

uLizo unebhola eziredi ezingu-12. uZama uneebhola eziblu. Banebhola ezingu-20 mazihlangene. Zingaki iibhola zikaZama? **(Lizo has 12 red ball. Zama has blue balls. They have 20 balls altogether. How many balls does Zama have?)**	
Impendulo	

Umbuzo 8

uSibu kade enamapeni angu-24. Wapha uThabo amapeni angu-14. Mangaki amapeni kaSibu manje? **(Sibu had 24 pens. She gave 14 pens to Thabo. How many pens does Sibu have now?)**	
Impendulo	

Umbuzo 9

Kade kunabantu emotweni. Kwaze kwagibela abantu abangu-2. Manje kunabantu abangu-8. Bangaki abantu kade besemotweni kuqala? **(There ware people in the car. Two more people got on. Now there are 8 people. How many people were in the car to start with?)**	
Impendulo	

Umbuzo 10

Kade kunokhiya abangu-13 etafuleni. Umama wasusa okhiya abangu-5. Bangaki okhiya abasetafuleni manje? **(There were 13 padlocks on the table. Mom removed 5 padlocks. How many padlocks on the table now?)**	
Impendulo	

Umbuzo 11

Izinja ezingu-29 zidlala ngaphandle. Ezi-brown zingu-14 kanye nezimhlophe. Zingaki izinja ezimhlophe? **(29 dogs are playing outside. There are 14 brown and some white. How many dogs are white?)**

Impendulo	

Umbuzo 12

uJohn kade enamapeni angu-15. Wapha uThemba amanye amapeni. Manje uJohn unamapeni angu-10. uJohn uphe uThemba amapeni amangaki? **(John had 15 pens. He gave some pens to Themba. Now John has 10 pens. How many pens did John give to Themba?)**

Impendulo	

Umbuzo 13

Izinja ezibrown ezingu-12 zidlala nezinja ezimhlophe ezingu-8. Zingaki izinja mazizonke? **(12 brown dogs are playing with 8 white dogs. How many dogs are there altogether?)**

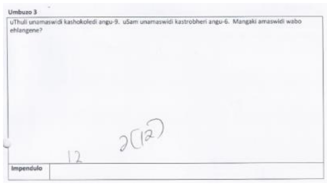
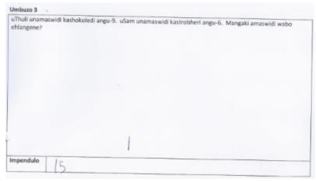
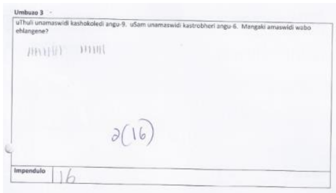
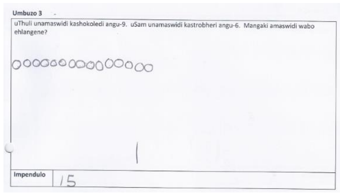
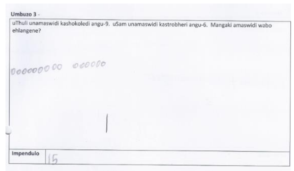
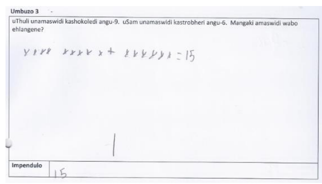
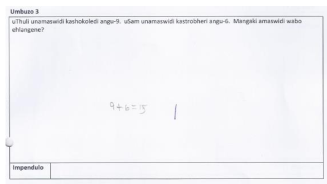
Impendulo	

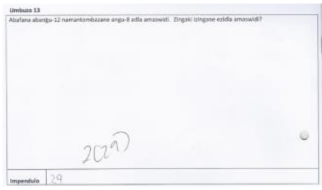
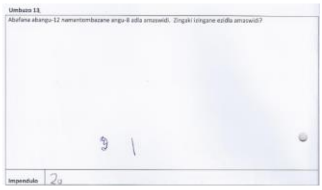
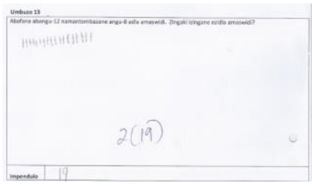
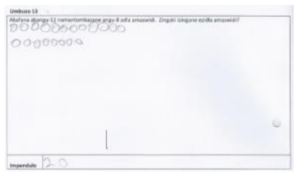
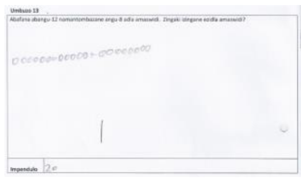
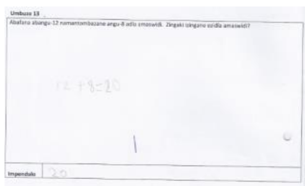
Umbuzo 14

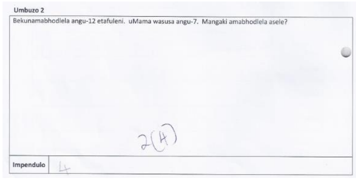
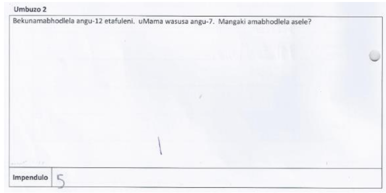
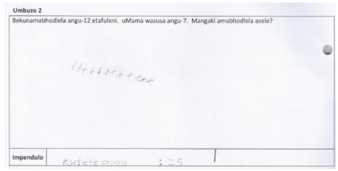
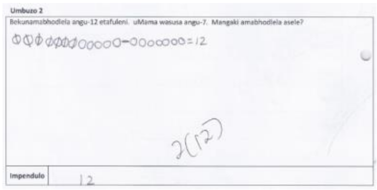
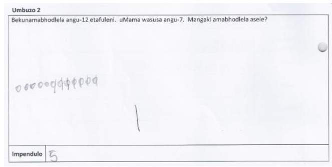
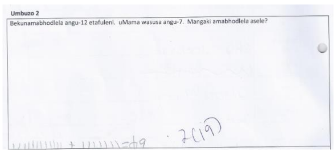
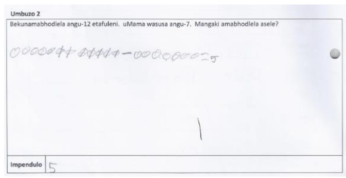
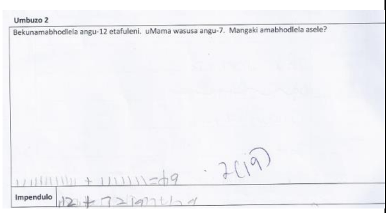
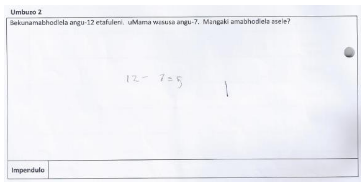
uBusi kade enebhola ezingu-24. Wathenga ibhola ezingu-13. Zingaki ibhola zikaBusi manje? (Busi had 24 balls. She bought 13 more balls. How many balls does Busi have?)

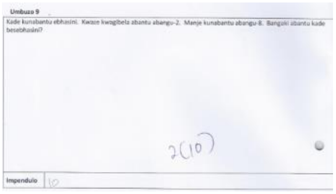
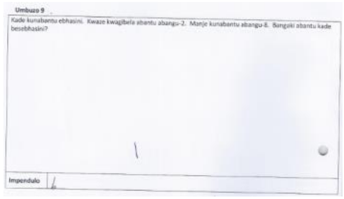
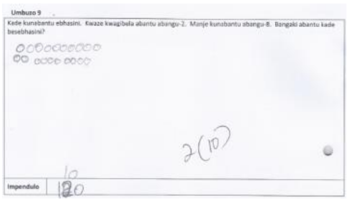
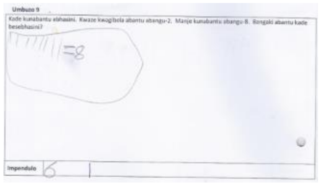
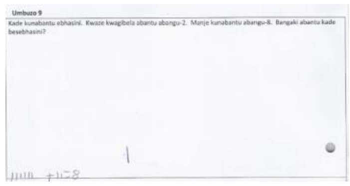
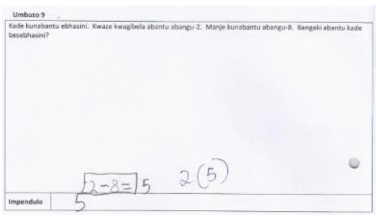
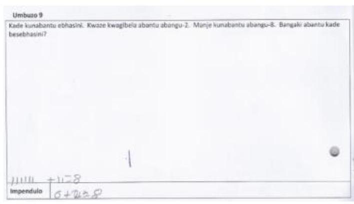
Impendulo	

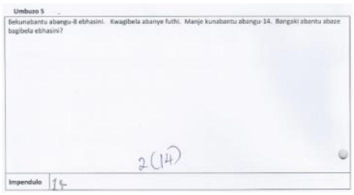
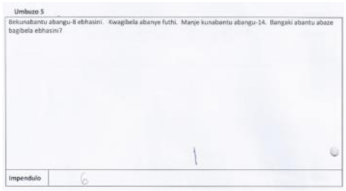
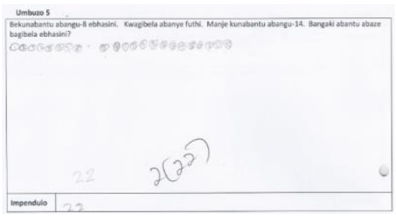
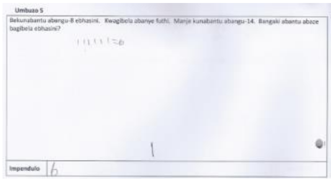
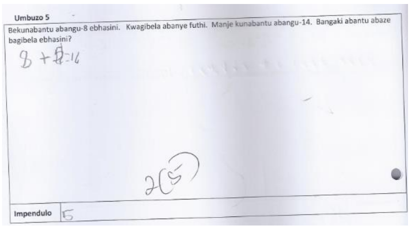
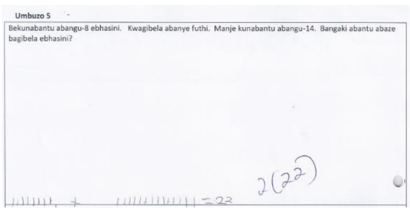
APPENDIX C

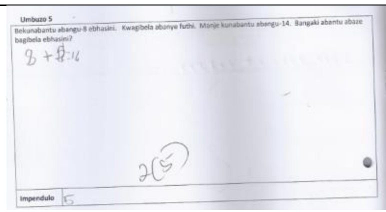
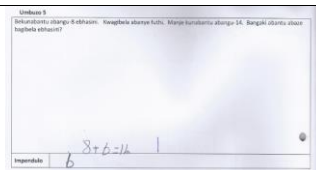
Cluster 1 (PPW-WU 9 + 6 =?)			
Extent of Awareness of Structure	**Description**	**Incorrect**	**Correct**
NoM	Response does not include any evidence of a model	**10(16),15(12),16(13),22(6),23(18)** 	**2,4,5,7,8,9,14,19,21,24,27** 
NDQ/PiC	No overt distinguishing of quantities within an appropriate model (NDQ)	**25(16),26(14)** 	**13** 
DQ/PiC	Overt distinguishing of quantities within an appropriate model (DQ)	**n/a**	**6,18,20** 
DQ-SFMR	Overt distinguishing of quantities within an appropriate model (DQ)		**12,17,28** 
DQ-CFMR-NS	Overt distinguishing of quantities within an appropriate model (DQ)		**1,3,11,28** 

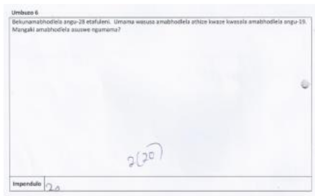
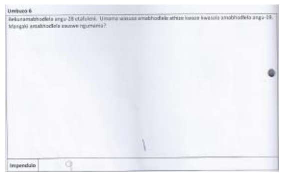
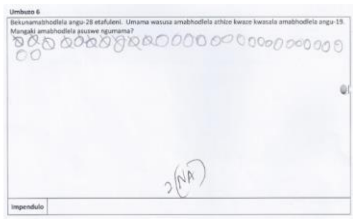
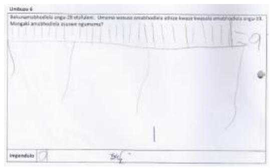
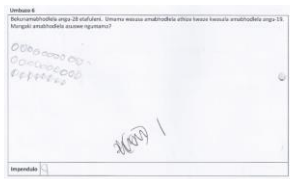
Cluster 1 (PPW-WU 12 + 8 =?)			
Extent of Awareness of Structure	**Description**	**Incorrect**	**Correct**
NoM	Response does not include any evidence of a model	**8(29),9(4),12(8),13(4),14(8),15(18),19(8),20(8),21(8),26(21),27(21)** 	**2,4,5,11,18,23,24** 
NDQ/PiC	No overt distinguishing of quantities (NDQ)	**25(19)** 	
DQ/PiC	Overt distinguishing of quantities within an appropriate model (DQ)		**7,10,22** 
DQ-SFMR	Overt distinguishing of quantities within an appropriate model (DQ)		**6,16,17** 
DQ-CFMR-NS	Overt distinguishing of quantities within an appropriate model (DQ)		**1,3,28** 

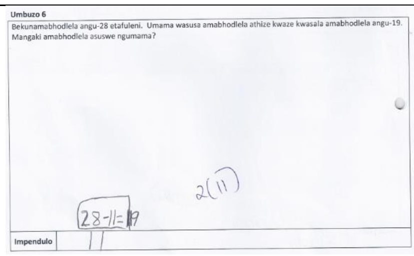
Cluster 1 (SEP-RU 12 - 7 =?)			
Extent of Awareness of Structure	**Description**	**Incorrect**	**Correct**
NoM	Response does not include any evidence of a model	**4(4),8(3),10(3),15(19),16(19),20(12),27** 	**2,5,7,9,21,23,24** 
NDQ/PiC	No overt distinguishing of quantities within an appropriate model (NDQ)		**18** 
DQ/PiC	Overt distinguishing of quantities within an appropriate model (DQ)	**22(19)** 	**6,12,14,17,25,26** 
DQ-SFMR	Overt distinguishing of quantities within an appropriate model (DQ)	**13(12),28(19)** 	**11,19** 
DQ-CFMR-NS	Overt distinguishing of quantities within an appropriate model (DQ)	**28(19)** 	**1,3** 

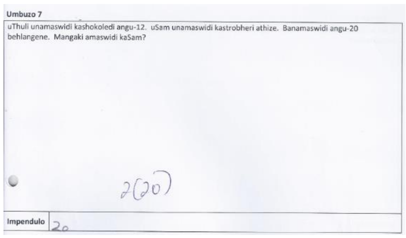
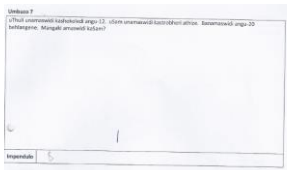
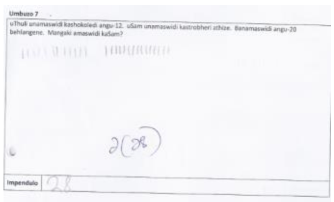
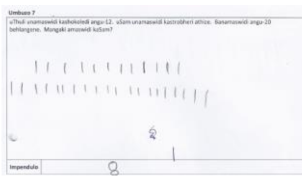
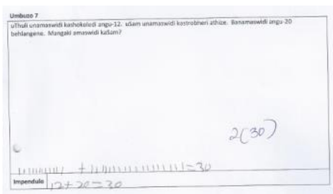
Cluster 2 (JOIN-SU ? + 2 = 8) Done			
Extent of Awareness of Structure	**Description**	**Incorrect**	**Correct**
NoM	Response does not include any evidence of a model	**1(10),4(4),6(10),7(18),8,10(10), 11(14),13,14,16(10),18(10),20,23(4),27** 	**2,5,9,12,19,21,24,26** 
NDQ/PiC	No overt distinguishing of quantities (NDQ)	**15(10),22(10),25(9)** 	
DQ/PiC	Overt distinguishing of quantities within an appropriate model (DQ)		**17** 
DQ-SFMR	Overt distinguishing of quantities within an appropriate model (DQ)		**28** 
DQ-CFMR-NS	Overt distinguishing of quantities within an appropriate model (DQ)	**3** 	**28** 

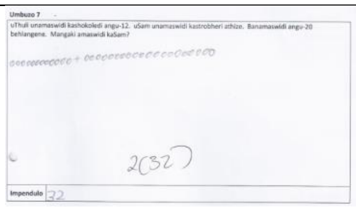
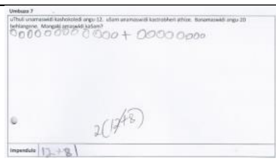
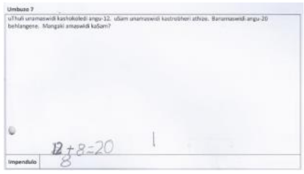
Cluster 2 (JOIN-CU 8 + ? =14) - Done			
Extent of Awareness of Structure	**Description**	**Incorrect**	**Correct**
NoM	Response does not include any evidence of a model	**2(14),4(28),8(8),10(22),11(22),13(8),14(14),16(22),19(22),23(22),27(23)**  Umbuzo 5 Bekunabantu abangu-8 ebhasini. Kwagibela abanye futhi. Manje kunabantu abangu-14. Bangaki abantu abaze bagibela ebhasini? 2(14) Impendulo 14	**1,5,21,24,26**  Umbuzo 5 Bekunabantu abangu-8 ebhasini. Kwagibela abanye futhi. Manje kunabantu abangu-14. Bangaki abantu abaze bagibela ebhasini? 1 Impendulo 6
NDQ/PiC	No overt distinguishing of quantities (NDQ)	**15(22),17(22),18(22),20(22),22(22),25(22)**  Umbuzo 5 Bekunabantu abangu-8 ebhasini. Kwagibela abanye futhi. Manje kunabantu abangu-14. Bangaki abantu abaze bagibela ebhasini? 22 2(22) Impendulo 22	**9**  Umbuzo 5 Bekunabantu abangu-8 ebhasini. Kwagibela abanye futhi. Manje kunabantu abangu-14. Bangaki abantu abaze bagibela ebhasini? 1 Impendulo 6
NDQ/CFMR	No overt distinguishing of quantities (NDQ)	**28(22)**  Umbuzo 5 Bekunabantu abangu-8 ebhasini. Kwagibela abanye futhi. Manje kunabantu abangu-14. Bangaki abantu abaze bagibela ebhasini? 8 + ?=14 2(5) Impendulo 5	
DQ/PiC	No evidence of this category in this cluster		
DQ-SFMR	Overt distinguishing of quantities within an appropriate model (DQ)	**6(22),7(22),28(22)**  Umbuzo 5 Bekunabantu abangu-8 ebhasini. Kwagibela abanye futhi. Manje kunabantu abangu-14. Bangaki abantu abaze bagibela ebhasini? 2(22) = 22	
DQ-CFMR-NS	Overt distinguishing of quantities within	**12(Brock-Utne),28(22)**	**3**

	an appropriate model (DQ)		

Cluster 3 (SEP-CU 28 - ? =19) Done			
Extent of Awareness of Structure	**Description**	**Incorrect**	**Correct**
NoM	Response does not include any evidence of a model	**2(10) 8(17),9 (10) 10(37),14(28),16(47),19(11),20(19) ,23(28),26(47),27(19)** 	**1,4,5,24** 
NDQ/PiC	No overt distinguishing of quantities (NDQ)	**7(19)),12(19),15(8),22(47),25(19)** 	**17** 
DQ/PiC	Overt distinguishing of quantities within an appropriate model (DQ)		**6,11,13,18,21,28** 
DQ-SFMR	**No evidence of this category in this cluster**		
DQ-CFMR-NS	Overt distinguishing of quantities within an appropriate model (DQ)	**3(11),28(19)**	

Cluster 3 (PPW-PU 12 + ? =20)			
Extent of Awareness of Structure	**Description**	**Incorrect**	**Correct**
NoM	Response does not include any evidence of a model	**2(32),4(12),5(23),10(33),12(6),13(32),14(32),15(23),16(32),18(32),20(22),26(22),27(6)** 	**1,9,19,21,23,24** 
NDQ/PiC	No overt distinguishing of quantities (NDQ)	**25(28),28(30)** 	**22** 
NDQ/CFMR-NS	No overt distinguishing of quantities within an appropriate model (NDQ)	**28(30)** 	
DQ/PiC	No evidence of this category in this cluster		
DQ-SFMR	Overt distinguishing of quantities within	**6(32),8(29)**	**7,11,17**

	an appropriate model (DQ)	 Umbuzo 7 uThuli unamaswidi kashokoledi angu-12. uSam unamaswidi kastrobheri athize. Banamaswidi angu-20 behlangene. Mangaki amaswidi kaSam? 2(32) Impendulo 32	 Umbuzo 7 uThuli unamaswidi kashokoledi angu-12. uSam unamaswidi kastrobheri athize. Banamaswidi angu-20 behlangene. Mangaki amaswidi kaSam? Impendulo
DQ-CFMR-NS	Overt distinguishing of quantities within an appropriate model (DQ)		**3,7**  Umbuzo 7 uThuli unamaswidi kashokoledi angu-12. uSam unamaswidi kastrobheri athize. Banamaswidi angu-20 behlangene. Mangaki amaswidi kaSam? 12+8=20 Impendulo 8

APPENDIX D

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

Student Number:
696188
Protocol Number:
2013ECE109D

Date:18 November 2013

Dear Blessing Takane

Application for Ethics Clearance: Doctor of Philosophy

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

Exploring how grade 2 South African learners solve addition and subtraction word problems.

The committee recently met and I am pleased to inform you that clearance was granted.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely

MMabeta

Matsie Mabeta
Wits School of Education

011 717 3416

CC Supervisor: Prof. H Venkatakrishnan

LETTER TO THE PRINCIPAL

DATE

Dear NAME

My name is Thulelah Takane. I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing research **on exploring how grade 2 South African learners solve addition and subtraction word problems.**

My research involves a pre-test and a post-test, video-taping and audio taping of the interviews of learners in one Grade 2 class in your school. I will be conducting an intervention in 1 grade 2 class in your school and interviewing 12 learners as individuals as well ask keeping field-notes and analysing samples of their work. This investigation will be done throughout the course of the school term over 20 lessons altogether.

Since permission has already been granted for your school to participate in the Wits Maths Connect Primary project, I was wondering if you would grant me permission to do my research in one of your grade 2 classes in your school.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.

I look forward to your response as soon as is convenient.

Yours sincerely,

SIGNATURE:

NAME : Thulelah Blessing Takane

ADDRESS :24-313 Wits Junction, Cnr Rich and Boundary Rd, Parktown, 2193

EMAIL: thulelahblessing@gmail.com

TELEPHONE NUMBERS : 011-717 3088(office), 0769183569 (cell)

LETTER TO THE CLASS TEACHER

DATE

Dear NAME

My name is Thulelah Takane. I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing research on exploring how grade 2 South African learners solve addition and subtraction word problems.

My research involves a pre-test and a post-test, video-taping and audio taping of the interviews of learners in one Grade 2 class in your school. I will be conducting an intervention in 1 grade 2 class in your school and interviewing 12 learners as individuals as well ask keeping field-notes and analysing samples of their work. This investigation will be done throughout the course of the school term over 20 lessons altogether.

Since permission has already been granted for your school to participate in the Wits Maths Connect Primary project, I was wondering if you would grant me permission to do my research in your grade 2 class in your school.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.

I look forward to your response as soon as is convenient.

Yours sincerely,

SIGNATURE:

NAME : Thulelah Blessing Takane

ADDRESS :24-313 Wits Junction, Cnr Rich and Boundary Rd, Parktown, 2193

EMAIL : thulelahblessing@gmail.com

TELEPHONE NUMBERS : 011-717 3088(office), 0769183569 (cell)

INFORMATION SHEET PARENTS

DATE

Dear Parent

My name is Thulelah Blessing Takane and I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing research through an intervention approach on how grade 2 South African learners solve addition and subtraction word problems.

My research involves a pre-test and a post test, video-taping and audio taping. I will also be interviewing your child as an individual as well as keeping field-notes and analysing samples of his/her work. This investigation will be done through-out the course of the school terms and will consist of 20 lessons altogether. Your child is amongst a group of 1 class on which I intend focusing my research.

The reason why I have chosen your child's class is because the school is participating in the Wits Maths Connect Primary project in which my study is located and both the HOD and the teacher have agreed for the class to participate.

I was wondering whether you would mind if I invited your child to be a participant in my research and use the information mentioned in the paragraph above for the purposes of my research.

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

Your child's name, school name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

SIGNATURE:

NAME : Thulelah Blessing Takane

ADDRESS : 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193

EMAIL: thulelahblessing@gmail.com

TELEPHONE NUMBERS : 011- 717 3319 (Office) or 0769183569

TELEPHONE NUMBERS : 011- 717 3319 (Office) or 0769183569

Consent Form Parents: Learner Documents

Please fill in and return the reply slip below indicating your willingness to allow me to use certain documents (workbooks, worksheets, and any other written work) of your child for my voluntary research project called:

Exploring how grade 2 South African learners solve addition and subtraction word problems.

Permission for the use of (workbooks, worksheets, and any other written work)

I, ________________________ the parent of ______________________

Give/do not give* my consent for the use of the following documents (workbooks, worksheets, and any other written work)

[ ] I know that my child may withdraw from the study at any time and that s/he will not be advantaged or disadvantaged in any way.

[ ] I know that my child's documents (workbooks, worksheets, and any other written work) will be used for this study only.

Parent Signature: ________________________ Date: ____________________

Contact person:
NAME: Thulelah Blessing Takane
ADDRESS: 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193
TEL NUMBER: 011- 717 3319 (Office) or 0769183569

Consent Form Parents: Learner Interview

Please fill in and return the reply slips below indicating your willingness to allow your child to be interviewed in my voluntary research project called:

Exploring how grade 2 South African learners solve addition and subtraction word problems.

Permission to be interviewed

I, ________________________ the parent of ______________________

Give/do not give* my consent for my child to be interviewed.

[ ] I know that my child may withdraw from the study at any time and that s/he will not be advantaged or disadvantaged in any way.

[ ] I am aware that the researcher will keep all information confidential in all academic writing.

[ ] I am aware that my child's interview will be destroyed between 3—5 years after completion of the project.

Parent Signature: ________________________ Date: ____________________

Contact person:

NAME: Thulelah Blessing Takane
ADDRESS: 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193
TEL NUMBER: 011- 717 3319 (Office) or 0769183569

Consent Form Parents: Learner Observation

Please fill in and return the reply slip below and indicate your willingness to allow your child to be observed in my voluntary research project called:

Exploring how grade 2 South African learners solve addition and subtraction word problems.

Permission to be observed

I, __________________________ the parent of ________________________

Give/do not give* my consent for my child to be observed for this project.

[ ] I know that I may withdraw from the study at any time and that I will not be advantaged or disadvantaged in any way.

[ ] I am aware that the researcher will keep all information confidential in all academic writing.

[ ] I know that the observations will only be used for this project.

Parent Signature: ___________________ Date:

Contact person:

NAME: Thulelah Blessing Takane
ADDRESS: 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193
TEL NUMBER: 011- 717 3319 (Office) or 0769183569

Consent Form Parents Audiotaping

Please fill and return the reply slip below and indicate your willingness to have your child's interview audiotaped for my voluntary research project called:

Exploring how grade 2 South African learners solve addition and subtraction word problems.

Permission to have my child's interview audio-taped

I, ______________________ the parent of ______________________

give/do not give* my consent to have the interview recorded.

[ ] I know that I may withdraw from the study at any time and that my child will not be advantaged or disadvantaged in any way.

[ ] I know that the tapes will be destroyed between 3-5 years after completion of the project.

Parent Signature: ______________________ Date: ______________________

Contact person:
NAME: Thulelah Blessing Takane
ADDRESS: 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193
TEL NUMBER: 011- 717 3319 (Office) or 0769183569

Consent Form Parents Videotaping

Please fill and return the reply slip below and indicate your willingness to have your child's interview audiotaped for my voluntary research project called:

Exploring how grade 2 South African learners solve addition and subtraction word problems.

Permission to have my child's interview videotaped.

I, ________________________ the parent of ______________________

give/do not give* my consent to have the interview recorded.

[ ] I know that I may withdraw from the study at any time and that my child will not be advantaged or disadvantaged in any way.

[ ] I know that the tapes will be destroyed between 3-5 years after completion of the project.

Parent Signature: _____________________ Date: ____________________

Contact person:
NAME: Thulelah Blessing Takane
ADDRESS: 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193
TEL NUMBER: 011- 717 3319 (Office) or 0769183569

INFORMATION SHEET LEARNERS

DATE

Dear Learner

My name is Thulelah Blessing Takane and I am a PhD Student in the School of Education at the University of the Witwatersrand.

I am doing research on how grade 2 South African learners solve addition and subtraction word problems.

My investigation involves a pre-test, post-test, video-taping and audio taping you together with others. I will also be interviewing you as an individual as well as keeping field-notes and analysing samples of your work. Only my Supervisors, Prof H. Venkatakrishnan-the leader of the Wits Maths Connect-Primary Project, Prof Mike Askew, the author of the 'Big Book' model which I will be using in my study, and myself will have access to the pre and post-tests, field-notes, workbooks, and recorded information. I will not be using your name, but I will make one up so that no one can identify you. Also, all collected information I will store safely and destroy between 3-5 years after I have completed my research. This investigation will be done through-out the course of the school terms which consists of 20 lessons across the year.

I was wondering whether you would mind if I asked for your permission to allow me to go ahead with my investigation for the project. I need your help with participating in my study.

Remember, the test is not for marks, and your participation is voluntary, which means that you don't have to do it. Also, if you decide halfway through that you prefer to stop, this is completely your choice and will not affect you negatively in any way.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join us in the study.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you.

SIGNATURE:

NAME : Thulelah Blessing Takane

ADDRESS : 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193

EMAIL : thulelahblessing@gmail.com

TELEPHONE NUMBERS : 011-717 3319 (Office) or 0769183569

Consent Form Learners Documents

Please fill in the reply slip below if you agree to have the following documents: Work books, worksheets, and any other written work used for my study called:

Exploring how grade 2 South African learners solve addition and subtraction word problems.

Permission for documents (Workbooks, Worksheets, and any other written work)

My name is: ________________________

I agree that (workbooks, worksheets, and any other written work) can be used for this study only YES/NO

I know that Thulelah B. Takane will keep my information confidential YES/NO

Sign_____________________________ Date___________________________

Contact person:

NAME: Thulelah Blessing Takane
ADDRESS: 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193
TEL NUMBER: 011-7173319 (Office) or 0769183569

Consent Form Learners Observation

Please fill in the reply slip below if you agree to let me observe you in the Maths club.
I will use my notes for my study called:

Exploring how grade 2 South African learners solve addition and subtraction word problems.

Permission for observations

My name is: ______________________

I would like to be part of this project	YES/NO
I agree to be observed in Maths Club	YES/NO
I know that Thulelah B. Takane will keep my information confidential	YES/NO
I know that I can leave the study any time I want	YES/NO

Sign____________________________ Date__________________________

Contact person:

NAME: Thulelah Blessing Takane
ADDRESS: 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193
TEL NUMBER: 011-717 3319 (Office) or 0769183569

Consent Form Learners Interview

Please fill in the reply slip below if you agree to be interviewed. I will use your answers to my questions for my study called:

Exploring how grade 2 South African learners solve addition and subtraction word problems.

Permission for interview

My name is: ______________________

I would like to be interviewed for this study	YES/NO
I know that Thulelah B. Takane will keep my information confidential	YES/NO
I know that I can stop the interview at any time and don't have to answer all the questions asked	YES/NO

Sign____________________________ Date__________________________

Contact person:

NAME: Thulelah Blessing Takane
ADDRESS: 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193
TEL NUMBER: 011-717 3319 (Office) or 0769183569

Ifomu Yemvume

Ngicela ugcwalise lencwadi uma uvuma ukuthi ngisebenzisane nawe kungeno nocaningo lokuhlola abafundi bebanga lesibili lokuhlanganisa nokususa izibalo zamazwi.

Imvume:

Igama lami ngingu: ________________________

Ngiyavuma ukuthi ngirekhodwe ngeteyipu, ngevidiyo teyipu.
YEBO/CHA

Ngiyavuma ukuthi ngibenezingxoxo mgomsebenzi wami
YEBO/CHA

Ngiyavuma ukuthi ngihlolwe ngokubhala
YEBO/CHA

Ngiyavuma ukuthi umsebeni wam womngeno uhlolwe
YEBO/CHA

Ngiyazi ukuthi ngingayeka nasiphi isikhathi
YEBO/CHA

Ngiyazi ukuthi ledatha izosetshenziswa kulolucaningo kuphela
YEBO/CHA

Ngiyazi ukuthi ledatha izolahIwa phakathi kwamashumi amathathu nemihlanu ma ucaningo seluphelile.
YEBO/CHA

Isignesha_________________________Usuku___________________________

Umuntu wokuthintwa:

NAME: Thulelah Blessing Takane
ADDRESS: 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193
TEL NUMBER: 011-717 3319(Ihofisi) or 0769183569

Consent Form Learners Videotaping

Please fill in the reply slip below if you agree to be videotaped. I will use these videotapes for my study called:

Exploring how grade 2 South African learners solve addition and subtraction word problems.

Permission to be videotaped.

My name is: ______________________

I agree to be video-taped during individual interviews	YES/NO
I know that I can stop the videotaping at any time	YES/NO
I know that the videotapes will be used for this project only	YES/NO
I know that the videotapes will be destroyed between 3-5 years after completion of this study.	YES/NO

Sign___________________________ Date_________________________

Contact person:

NAME: Thulelah Blessing Takane
ADDRESS: 24-313 Wits Junction, Cnr Rich and Boundary Road, Parktown, 2193
TEL NUMBER: 011-717 3319(Office) or 0769183569