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A novel acoustic forced vibration study for application in high temperature gas reactors

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A NOVEL ACOUSTIC FORCED VIBRATION STUDY FOR APPLICATION IN HIGH TEMPERATURE GAS REACTORS

Yerishca Mudaly

A Research Report submitted to the Faculty of Science, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science.

Johannesburg, 2010

DECLARATION

I declare that this Research Report is my own unaided work. The mathematical formulation of the various acoustic wave solutions in this Research Report has been with the guidance of my technical supervisor, Mr M Cepkauskas.
It is submitted for the Degree of Master of Science in the University of Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

_____

Yerishca Mudaly

_____ day of _____ 2010

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To my Parents and Neo, my heart is forever indebted to you for always pushing me to the extreme limits where knowledge is involved and always reminding me to be the best I can be. I am proud to make this Report as manifestation of this and appreciate all the support I have received. For my love, Lyndon, I would like to thank you for all the patience, love and support through my Masters and pushing me when there was not much left.

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ABSTRACT

Excessive vibration of pipes, structures or components has been determined as one of the main causes of Nuclear Power Plant degradation. These vibrations can lead to potential damage of plant systems, structures and components, which can negatively impact the plant performance and safety integrity of an operational unit. Should resonance conditions be experienced due to the vibrations, the vibration can be further amplified and when this exceeds a permissible limit, potential failure of the structure can occur. In the nuclear environment being able to predict such phenomena is highly important. Specialised analysis provides a proactive risk management process to predict such phenomena before they occur. This approach is becoming more necessary and important during the design of new generation Nuclear Power Plants. This Research Report taps into this requirement and aims to provide a method in determining the acoustic pressure distribution for predicting high fluid vibrational areas or possible resonance conditions. Various methods have been employed by specialists to produce adequate acoustic solutions. In various papers by Cepkauskas, he introduces a transformation technique used to change the form of the problem to a non-homogenous differential equation with homogenous boundary conditions by utilising an auxiliary function. Cepkauskas also demonstrated that, unlike other solutions produced, an auxiliary function defined on the interior of the media is unnecessary.

In this Research Report, we investigate the Cepkauskas methodology and adapt it further by using a one dimensional wave equation and non-homogenous boundary conditions and through the transformation technique to produce four forced-vibration acoustic solutions with different boundary conditions existing in a pipe-loop configuration. Specific Jolley series have been selected that ensure a proper representation of each of the four forced-vibration acoustic solutions. The Jolley series have been applied to determine the acoustic pressure distribution within a pipe over a series of incremental lengths and time. It is demonstrated that these acoustic forced-vibration solutions can be used to properly couple various

individual pipes, while still maintaining the physical acoustic behaviour within the pipe-loop or a pipe system. A general acoustic subroutine is developed using the selected Jolley series and applied to specific conditions in two pipe-loop systems (general pipe-loop and a simplistic HTGR pipe-loop). For the HTGR pipe-loop, pipe geometries and fluid temperatures from a Computation Fluid Dynamic (CFD) software computer code, Flownex is used to calculated these conditions and provides the input for the acoustic loop subroutine model for steady state conditions. The series of unknown constants required at the pipe to pipe interface that are necessary to maintain pressure distribution and pressure gradient continuity, are solved via matrix operations and applying Kramers rule. In order to verify accuracy and gain confidence in the mathematics of this methodology, the subroutine is applied to two case studies, a general pipe-loop model and a model representing a simplified HTGR environment. This methodology can also be used to determine the natural and forced frequencies in a system to predict potential flow-induced vibrations or resonant conditions. It can also be used for other various applications that will be further elaborated on in this Research Report.

The results of this study has led to the publication of this work at the 20th International Conference on Structural Mechanics in Reactor Technology, (SMiRT-20) in Finland on August 2009, Division V, Paper 1577, where it was open for judgement and no significant findings on this methodology were found, but received well by the conference.

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CHAPTER 1

INTRODUCTION AND CONTEXT

In an operational nuclear environment, processes such as the selection of process control, expert supervision, optimisation and maintenance methodologies are gaining fast in importance. It is beneficial that these areas should operate together to provide the best technical solution to ensure safe plant operation. If any of these processes are found to be ineffective, it can negatively influence the performance and safety integrity of the plant, caused by sudden failures of key plant components like turbines, pipe structures etc. These failures can culminate in unscheduled outages of electricity producing power plants, manifesting in negative economic, social and environmental implications. The operation of a Nuclear Power Plant (NPP) is rigidly regulated to ensure public and worker safety, therefore, maintaining the integrity of components is vital in preventing any accident condition caused by structural degradation. Excessive flow induced-vibration of key plant components is one type of phenomenon that has caused significant plant degradation. Over the years, NPPs have experienced vibration induced degradation of plant equipment during operation at the original licensed power and under up-scaled conditions. “Up-scaled” is an operational mode that is beyond its original operational condition that has resulted from optimisation or upgrading to achieve an increased output. In such environments, it may cause vibrations in the plant due to components operating beyond its designed capability.

Vibration studies are being conducted internationally in an attempt to predict these occurrences within a plant system as accurately as possible. There are three major approaches when examining vibration and acoustic phenomena, consisting of analytical, experimental, and numerical methods. However, a total understanding of some characteristics of fluid-structure interaction problems has not been attained. The significance of the analytical approach is important to assess the effect of these phenomena on the system response and is the foundation of this Research Report.

In this section, vibration and its applicability in the engineering industry are explored. Several industrial cases are presented to reinforce the importance of being able to predict the pressure behaviour of fluids that produce vibrations. Internationally, there have been several instances of valve failures and system inoperability occurring as a result of normal and transient operational vibration. It will be mentioned that the consequences of such damage (arising from excessive vibrations) can cost companies investment, capital and production loss and in some cases hamper safety.

1.1 Vibration in Nuclear Power Plant Environments

1.1.1 Vibrations at Loviisa NPP under up-scaled conditions

This case study is obtained from the operational experience of a NPP in Finland, the Loviisa Power Plant. It is classified as a Pressurised Water Reactor (PWR) and started its operation in February 1977. The present electrical power capacity of the Loviisa NPP is approximately 10% larger than what it was originally designed for. The power upgrading of these units has been achieved by increasing the reactor, steam generator and other plant systems capacities in steam and feedwater mass-flow generation. This resulted in an increase in the flow velocity within the feed and steam piping, causing flow-induced vibrations resulting in extensive vibrations of the pipe lines. Figures 1.1 and 1.2 are illustrations of the type of damage caused by the vibration fatigue experienced at the Loviisa NPP.

The engineers, in some cases, attempted redesigning the piping support system consisting of strengthening and installation of additional elastic supports, as such done traditionally. All the measures that were employed did not provide positive effects in shifting the system's vibration and did not influence the vibration level very much. At the same time the transferring of vibration to environmental structures had been increased. NPP operational experience shows a correlation between piping operation reliability and service life limit from one side and the level of piping operational vibration from another [KO02].

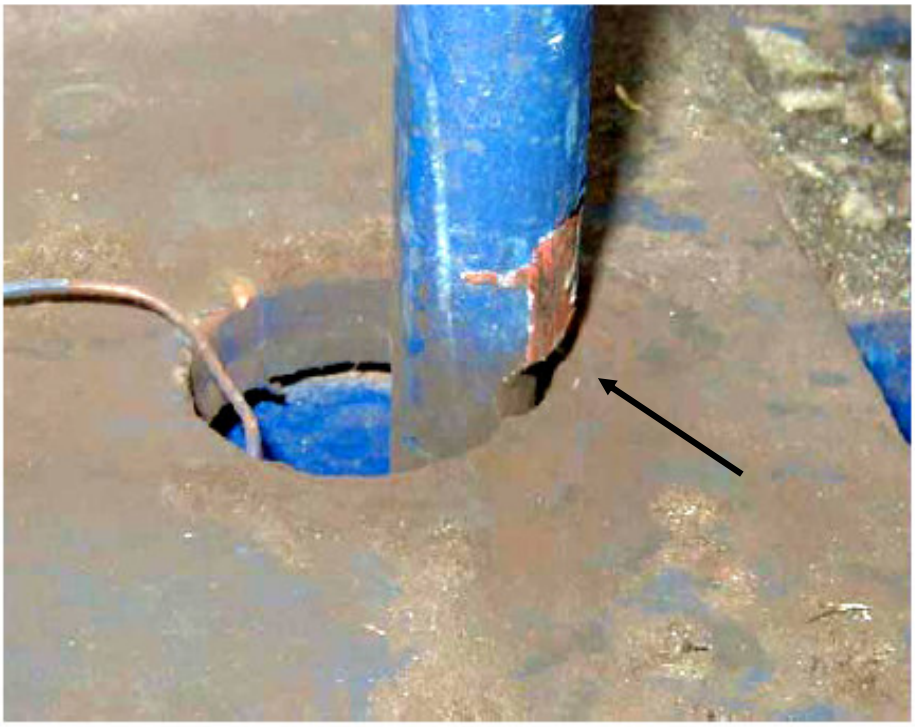


Figure 1.1: Wear of rod hanger at Loviisa NPP.

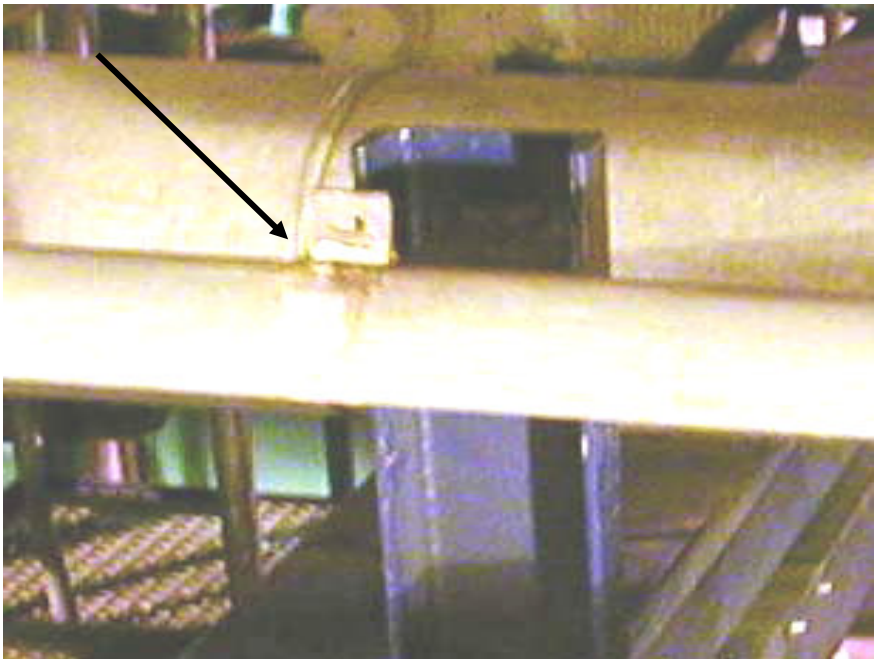


Figure 1.2: Fatigue collapse of elastic vibration support.

1.1.2 Failure of a steam dryer cover plate after a recent power up-scale

In March 2002, Quad Cities Unit 2, a Boiling Water Reactor (BWR), completed a refuelling outage that included a modification to add baffle plates to the steam dryer. It was done to reduce excessive moisture carry over expected as a result of an extended power up-scale increase of 17.8%. In June 2002, the unit began experiencing fluctuations in steam flow, reactor pressure and level, and carry over in the main steam lines. An engineering evaluation of one of the fluctuations, determined that the possible cause of the steam flow irregularities could be from loose parts in the steam line impairing the proper functioning of the safety systems. A small scale test was developed and produced results indicating that the dryer cover plate had failed due to high cycle fatigue. On the basis of the test results, it was concluded that the fatigue resulted from excessive vibration caused by the synchronisation of the cover plate resonance frequency, the nozzle chambers' standing wave frequency, and the vortex shedding frequency. All three of these frequencies synchronised in a very narrow band of steam flow at or near the steam flow required to reach full power under up-scaled conditions. The lessons learnt from this operational experience stated that resonance frequencies in upgrades may cause degradation as a result of high cycle fatigue [DO03].

1.2 Vibration in commercial engineering industries

Vibration problems are also common in commercial engineering industries. In one example, a damaged cooler nozzle in a production plant in the United States of America was found to be cracked due to excessive vibrations caused by pressure pulsations. The crack resulted in a shut down of the failed unit and the back-up unit. The loss of production amounted to over US$10 000 per day. In this case, ad-hoc approaches such as orifice plates, braces, added pipe supports were used to attempt to control the vibrations in the field, but the end result was that the unit could only be safely run at one speed in the range of 700 to 1200 rpm. Computational analysis of this system indicated very high shaking forces present throughout the system, providing a clear indication that the system was not designed to handle the conditions it endured. Had the necessary analysis been

conducted prior to implementing any changes for the increase in production, the problem would have been identified and possibly prevented [HO01].

In another example, a broken compressor frame was caused by excessive vibrations. The unit where this compressor was located experienced no problems while it was operating at lower speeds. The demand for gas increased and the decision was made to increase the speed of the compressor to handle the additional load. This caused the mechanical natural frequency of the pipe to be excited by the motion of the cylinder assembly. The large vibrations resulted in stresses that cracked the frame of the compressor. Although this was not an acoustic pulsation problem, the gas forces inside the cylinder created an environment which made the cylinder assembly get longer and shorter. With proper analysis during the design phase that could predict the mechanical natural frequency of the free standing elbows, this problem could have been avoided [HO01].

Problems that are created by excessive vibration in machinery can have serious economic impact. Frequently these problems are caused by large pressure fluid pulsation in the associated piping. An increasing concern in the engineering society regarding high-cycle piping vibration fatigue has been explained in the trend to revise the existing American Society of Mechanical Engineers (ASME) fatigue curves extending the cycle range together with considering of negative environmental effects [KO02]. As the engineering field advances, innovating engineering turbo-machinery designs are developed to achieve higher plant thermal efficiencies and hence to have a higher electrical output. Such an example is high-speed and high temperature turbines that are becoming more advanced. However, together with great engineering advances, problems have arisen which require more sophisticated analytical methods to predict them.

Only a few national recommendations and guidelines were developed based on operational experience of safety related piping subjected to vibration loads, this is also shown in NRC Regulation Guide 1.20 [US07]. This is why a practical and traditional solution for the resolving piping vibration often consists of a piping support system upgrade, such as tuning or changing parameters of the existing

system, using new supports or special devices [KO02], but as technology advances these types of solutions will become limited.

1.3 Fluid pulsation and dynamic modelling

Fluid pulsations in piping and vessels are a very common source of excessive vibration. Pumps, compressors and turbines are common sources of pressure pulsations that can cause fluctuations in the flow medium. This leads to metering errors, increased system pressure drops, and distortion of the pressure-volume curve of the system (pump, compressors etc), adding to operating costs and lost production. With high rotational-speed machines as an example, the increased speeds create shorter pressure-pulsation wave-lengths in the fluid that propagate through it. The mechanical natural frequencies of the machine need to be significantly high to avoid the higher frequencies of the unbalanced forces and moments inherent in reciprocating machines [HO01]. In particular, if the half wave-length coincides with a length of pipe, induced forces can be enormous. Also, fluid pulsation can lead to very high forces in the vertical direction on cylinders, which may be damaging. Vibration damage from rotating equipment caused by resonance is also a recognised phenomenon. However, the resonant conditions leading to equipment failure are frequently difficult to identify [LE07]. The phenomenon that drives this can be complex and is at present little understood by engineers.

Performing specialised analysis techniques can be used to highlight potentially at risk locations of a plant and provide adequate information in advance to establish the mitigating engineering actions for reducing the risk of the vibration related problem. Making proper use of specialised dynamic modelling in the design phase of a project is a proactive risk management approach. Of particular concern in the present investigation is the licensing of a new generation Nuclear Power Plants, where it is essential that measures are taken to predict the behaviour of all piping and vessel components under any postulated plant condition to ensure that the consequences of pulsations do not lead to significant damage that would

compromise nuclear safety. If a standard or an experienced-based design is implemented it may work well, but there will be a substantial risk that there will be problems that can impact on reliability, performance, safety and profitability. In some cases, the cost of correcting the problems is frequently large. Design optimisation will reduce risk but this should be done through analysis of various factors such as: criticality of the component, cost of loss of production, safety risks, newness of design, location and accessibility. Dynamic modelling and analysis as a component of design will contribute to the above benefits. During start-up, field analysis of vibration, pulsation and performance will further contribute to assuring economic success.

In the nuclear environment being able to predict flow induced vibrations is very important. For example, the existence of coolant flow in Pressurised Water Reactors (PWRs) may cause vibrations of the core barrel. Indeed, the effect of protracted vibration may cause fatigue of the barrel, wear away the connections, and even bring about an accident scenario or plant shutdown [YA02]. Flow induced vibrations have led to many problems in reactor internal components. Here, the pressure fluctuations of the reactor coolant generated at the pump discharge can interact with the internal components producing damaging vibrations [LE91]. This particular scenario has led to many specialists studying these phenomena to predict these conditions, which are investigated further on in this Research Report.

Having pipe breaks or boundary breeches could potentially lead to radiological exposures that exceed National Nuclear Regulator (NNR) limits for the public and workers. The most common cause for pipe breaks or system boundary breeches is excessive stresses in the system that occur from vibrations of the structures produced by excessive fluid pulsations impacting on them. In order to assess a plant design before construction, it is essential to model these phenomena and provide a confident prediction of how a structure will behave under various conditions and justify that should any breaks or damage occur, no compromise of

nuclear safety will result. Generally, vibration analysis attempts to optimise engineering design in two ways:

1) To ensure that the dynamic excitation forces are acceptably low, such an example is the forces produced from pulsations in piping systems.
2) To limit the response from vibration and dynamic stress of the mechanical components to prevent resonance by eliminating the coincidence between the natural frequencies of the system and the excitation frequencies.

There are various ways in the engineering field where engineers design structures, systems and components to suppress vibration. Passive damping is presently the major mechanisms of suppressing unwanted vibrations. The theory of damping will be discussed in Section 2.3. The primary effect of having increased damping in a structure is a reduction of vibration of amplitudes at resonances, with corresponding decreases in stresses, displacements, fatigue and sound radiation. Passive damping is separated into two classes: inherent and designed-in. Inherent damping is damping that exists in a structure due to the friction in joints, material damping, rubbing of cables, etc. Designed-in damping refers to passive damping that is added to a structure by design. This built-in damping supplements the structures' inherent damping, and it can increase the passive damping of a structure by substantial amounts. All passive damping treatments share a common goal: to absorb significant amounts of strain energy in the modes of interest and dissipate this energy through some energy-dissipation mechanism. The effectiveness of all passive damping methods varies with frequency and temperature [JO09].

In flow systems, wide band flow-induced excitation as well as structural interaction form part of excitation forces. Research is on going to understand the mechanisms of flow induced vibration. For example, the core barrel of a PWR is one of the key components of the reactor internals. The coolant flow may cause vibration of the barrel; the effect of protracted vibration may cause fatigue of the barrel, wear away the connection, and even bring about an accident or a shut down. Theoretically, it is very difficult to obtain the flow-induced pulse pressure

acting on barrel structure because of the complexity of fluid flow in the reactor. In reactor structural mechanics, problems are encountered with flow-induced vibration and the engineering relies entirely on a theoretical base to solve pulse pressure loading on the barrel by the fluid [YA02]. In new generation High Temperature Gas Reactors, (HTGRs), performing these analyses is of paramount importance, due to the fluid medium being a high temperature gas, and indeed necessary to demonstrate that flow-induced vibrations, resonance from standing pressure waves or structural will have no adverse effect on the safety of the plant when in operation, start-up and importantly during plant transients. In addition, flow analysis provides an opportunity to expertly understand the fluid behaviour of the system by predicting a plants' behaviour under different operational conditions.

1.4 Research methodology and scope of the investigation

Many experts in the field have attempted to derive adequate forced-vibration acoustic solutions that can properly estimate or predict the pressure distribution of a fluid medium in vessels and piping. Simulation tools are available; however, each method has its own limitations in ranges of applicability and with the mathematical modelling itself. This Research Report will discuss the simple one-dimensional acoustic wave-equation, the foundation for acoustic pulsation modelling performed. The investigation will explore different specialists' solutions and approaches to the modelling of acoustic waves and select the most appropriate method to expand on. The results of this study will provide a mechanism to assist others by providing a methodology that can aid the determination of the behaviour of the piping structures subjected to acoustic waves in a system, by accentuating possible areas of stress and vibrational fatigue within power conversion units of HTGRs. An earlier paper published by Cepkauskas [CE79], introduced a simple one-dimensional acoustic wave equation for a Pressurized Water Reactor (PWR) inlet pipe and pump, will be re-evaluated and used to establish new analytical models for different scenarios. This will be done by covering the following areas in this Research Report:

- Investigating different acoustic wave solution approaches of three specialists in the field and selecting the most appropriate approach.
- The selected methodology will be expanded through the determination of a simple one-dimensional analytical pipe solution consisting of an acoustic wave equation with non-homogeneous boundary conditions.
- Different acoustic forced-vibration solutions will be formulated by considering the various boundary conditions existing for pipes in a pipe-loop model.
- It will be shown that each case is solved using a transformation technique that transforms the homogeneous differential equations with time-dependent boundary conditions to non-homogenous differential equations with homogenous boundary conditions.
- The formulated solutions will be analytically proven to be adequate for application to real case scenario modelling and will further substantiate that the selected methodology is analytically correct.
- It will also be demonstrated by applying the cases to a pipe-loop model, that these acoustic solutions can be used to properly couple various individual pipes, while still maintaining the correct physical acoustic behaviour within the system.

Areas for future work and application of these analytical models will also be discussed. The methodology of this study can also be adapted further to be coupled with CFD and Finite Element Models (FEM) to determine the fluid behaviour during transients and the stress loads on components where significant acoustic or vibration effects are present.

1.5 Research Report structure and chapter outline

This Research Report will cover the basics of vibrations, waves and acoustics to provide a better foundation to understand the derivation of the one-dimensional acoustic wave equation. Acoustic forced-vibration solutions, CASES 1 to 4, will be derived for application in a pipe model consisting of pipes with different

boundary conditions. The present study will provide a method to determine the acoustic pressure waves and pressure distribution caused by compressor or pump operations for application in a High Temperature Gas Reactor (HTGR) environment. The Research Report is structured as follows:

- **Chapter 2** provides a comprehensive background into the fundamentals of vibration, acoustics and waves. Information will be provided to describe how these phenomena interact to produce flow induced vibrations and acoustic waves in fluids. Other specialist authors' solutions for acoustic waves are introduced and the basis for selecting the most appropriate analytical approach will be discussed.
- **Chapter 3** provides the approach adopted to derive an analytical model that would represent the pressure distribution in a pipe. It is demonstrated this approach will be applied to four scenarios with different boundary conditions to formulate different forced-vibration acoustic solutions (CASES 1 to 4). These solutions will be verified analytically and tested with applications to a general pipe-loop system and a simple HTGR pipe-loop. The results of the methodology adopted and its application will be presented in this section.
- **Chapter 4** presents the conclusions of the investigation for the formulation and verification of the four acoustic forced-vibration wave-solutions. It will also present the results of the application to a general pipe-loop model and to a simplified HTGR pipe model and will also discuss potential future areas of work to improve the methodologies used.

CHAPTER 2

THEORY OF VIBRATION, WAVES AND ACOUSTICS

2.1 Introduction

To understand the phenomena of fluid pulsations, acoustic vibrations, flow induced vibrations, resonance etc. it will require insight into the fundamental physics that govern these phenomena. In classical physics and mechanics, the acoustic wave-equation governs the propagation of acoustic waves through a material medium. Vibration of structures and components often give rise or generate pressure waves in the fluid medium, which propagate in the systems impacting on its interfacing structures, this process of impact is termed as fluid-structure interaction. Among the sources of vibration are pump-induced pulsations, flow turbulence, cavitation, vortex shedding, etc. In consideration that fluid damping is typically very low, a large amplification is usually observed at resonance frequencies [VA03]. There are many types of acoustic resonances and most are difficult to predict, the most common type results from longitudinal standing wave patterns [OE89]. Vibrations, waves and acoustics are specific phenomena and are discussed in detail in this section to elucidate how they are integrated to produce acoustic waves, flow induced vibrations and resonance. After establish these concepts, the one-dimensional wave-equation will be discussed. It will also be shown how various technical specialists applied the one-dimensional wave equation to produce different forced-vibration acoustic solutions

2.2 Principles of waves

2.2.1 Theory of waves

One of the properties of matter is that it can support the transfer of mechanical energy, whether the matter is solid, gas or liquid, without any net movement of the molecules involved. Such transfers are referred to as wave motion or transitory

displacement of atoms within the matter that passes on kinetic energy. Wave motion is formulated by disturbances or variations (vibrations) in a fluid medium thereby progressively transferring energy from one point to another. This occurrence may take the form of an elastic deformation or of a variation in pressure, electric or magnetic intensity, electric potential, or temperature. As mentioned previously, waves travel and transfer energy from one point to another, often with little or no permanent displacement of the particles of the medium (i.e with little or no associated mass transport). Instead, they oscillate around almost fixed positions in the form of vibrations. The energy of a vibration is moving away from the source in the form of a disturbance within the surrounding medium [HA80] in the form of waves. The disturbance may take a number of shapes in the manner it propagates, from a finite width pulse to an infinitely-long sine wave. The magnitude of a waves' velocity depends on the properties of the medium (and for certain waves, the shape of the wave). Once a wave is produced, the only reason for its speed changing is if it enters a different medium or if the properties of the medium changes. Putting more energy into the wave makes it more intense, not faster [CR09].

Waves can also spread out in all directions from every point on the disturbance that has created them. Infinitely many patterns are possible, but linear or plane waves are the simplest to analyse. In the physics of wave propagation, a plane wave it is at a constant frequency whose wave-fronts (surfaces of constant waves) are infinite parallel plane on constant amplitude normal to the phase velocity vector. A plane wave satisfies the one-dimensional wave equation in Cartesian coordinates [CR09].

There are two kinds of waves: transverse, and compression or longitudinal waves. The difference in these types of waves is explained in Section 2.2.1.1. Waves have a specific speed of propagation that depends on the matter transmitting the wave and the environmental conditions. In cases of high temperature, the molecules are moving more quickly and can transmit energy faster. Depending on the origin of the waves, it can introduce certain wave characteristics particular to the properties

of the medium involved. For example, in solids the disturbance is carried by mechanical waves, in various forms including longitudinal, transverse and surface waves. Waves with different amplitudes are variations in magnitude between the peaks and troughs of the wave, with the power of the wave increasing with amplitude (*A*). The amplitude is thus a measure of the magnitude of the wave or vibration [CR09]. Waves have different wavelengths, λ, or spacings between successive regions of maximum density (for a compression wave) or peaks (for a transverse wave). The distance spanned by one repetition is referred to as one wavelength. The wavelength is related to the frequency, *f*, of the wave, or number of times it oscillates in a second while passing through a fixed location (number of vibrations). The rapidity of a vibration, in terms of vibrations per second is called the frequency. The period of the waves, *T*, is the time between the pulses of the waves i.e. the time between passages of consecutive crest troughs of the wave. Waves have a phase, φ, in that the peaks of one wave may not coincide in time with the peaks of another wave of the same frequency. When two waves are in phase, they have a phase difference of 0 degrees. In summary waves are characterised by:

- Frequency (*f*): rate or number of cycles per unit time (also is taken as the inverse of the period);
- Phase (φ)**:** starting position of the wave;
- Amplitude (*A*): magnitude of the vibration or the amount of vibration. In general, amplitude can be expressed equivalently in terms of maximum displacement, velocity, or pressure relative to a reference value [VA01].

Any wave that is periodic will also display a repeating pattern when graphed as a function of position. Sinusoidal waves are the most important case of periodic waves. Fourier showed that any periodic wave with a frequency can be constructed as a superposition of sine waves with different frequencies. In this sense, sine waves are the basic, pure building blocks of all waves [CR09]. Therefore, the most recognisable and for purposes of mathematical analysis fundamental, form of a wave is a sine wave, which is produced by the trigonometric sine function. Refer

to Section 2.3.4, Fig. 2.9 for a graphical representation of these wave characteristics.

2.2.1.1 Transverse and Longitudinal waves

Transverse waves are formed from vibrations that are perpendicular to the direction of the propagation of the wave, as illustrated in Fig 2.1. Transverse waves can only arise in a solid medium or at the interface between a liquid and gaseous medium. Longitudinal waves are similar, but the vibrations are parallel to the direction of the propagation of the waves, as illustrated in Fig 2.2. Such an example includes most sound waves.

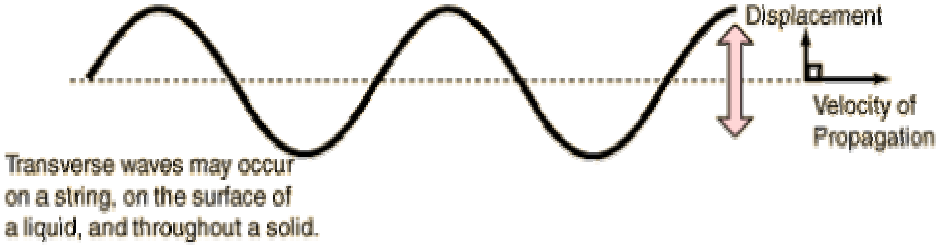


Figure 2.1: Illustration of a transverse wave.



Figure 2.2: Illustration of a longitudinal wave.

2.2.1.2 Standing waves

Normal modes in an oscillating system are special solutions where all the parts of the system are oscillating with the same frequency; standing waves are a continuous form of a normal mode. This phenomenon can arise in a stationery medium as a result of interference between two waves travelling in opposite direction. If two waves of the same wave length and amplitude are travelling through the same medium at the same velocity but in opposite directions. When

these two waves collide, they create a standing wave. In a standing wave, all the space elements (x, y, z coordinates) are oscillating with the same frequency and are in phase with different amplitudes, as illustrated in Fig 2.3.

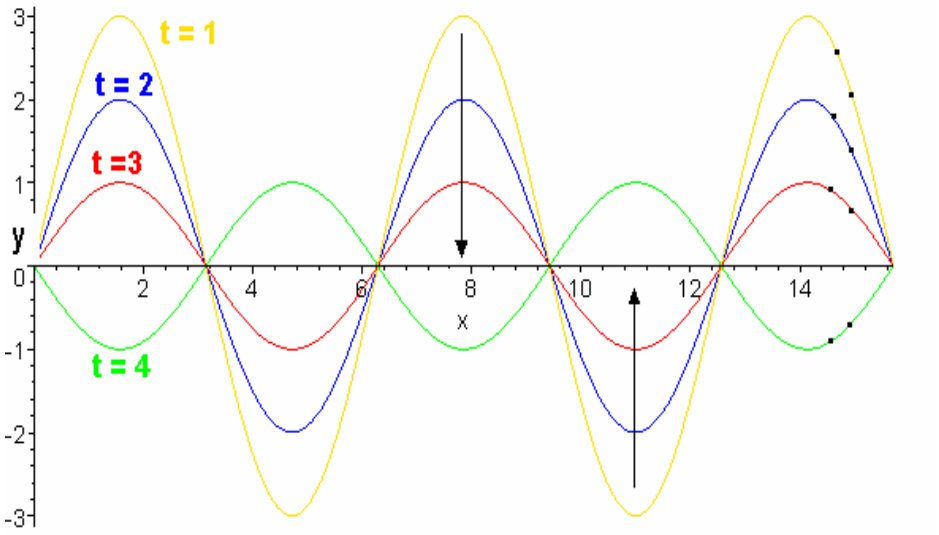


Figure 2.3: Characteristics of a Standing Wave Pattern.

As displayed in Fig 2.3, each wave reaches the equilibrium point together, the stationery points on the wave are referred to as nodes. Unlike travelling waves, standing waves appear to vibrate in place [CR09, RA45]. One form of standing wave is resonance. The term standing wave is often applied to a resonant mode of an extended vibrating object. Normally, if an object is excited to vibrate, the vibration will fade away due to damping. However, all objects have a preferred natural vibration frequency, ω_n called the natural resonance frequency. These vibrations are reinforced as standing waves within the object. If not excited continuously, an object vibrating at resonance will eventually calm down, due to damping, but over a relatively longer period of time [CR09].

2.2.1.3 Acoustic pressure waves

Acoustic or sound pressure is the local deviation from ambient (average or equilibrium) atmospheric pressure caused by a sound wave. Sound or acoustic refers to small amplitude, propagating pressure perturbations in a compressible

medium. These pressure disturbances are related to the corresponding density perturbation via material equation of state. The manner in which these disturbances propagate, are governed by a wave equation. This is further discussed in Section 2.4. Acoustic pressure waves consist of compressions and expansions, rather than side ways vibrations. They are typically considered longitudinal waves, since the molecules transmitting the waves move back and forth along the direction of propagation of the motion, as described in Section 2.3.1.1 [CR09].

Acoustic pressure waves are produced from acoustic pressures, particle displacements, density changes etc., having common phases and amplitudes at all points on any given plane perpendicular to the direction of the wave propagation. The energy involved in the propagation of acoustic waves through a fluid medium is of two forms, the kinetic energy of the moving particles and the potential energy inherent in a compressed fluid. They are readily produced within a rigid pipe, through the action of a vibrating piston located at the end of the pipe. In actual situations acoustic energy within a system is always irreversibly transmitted into the walls and, therefore, lost from the fluid [FR62]. The acoustic resonance of the fluid medium (steam or water) is considered the most probable source of flow-induced vibration [VA01].

Acoustic waves differ from other types of waves, such as in optics, since sound waves are related to a mechanical rather than an electromagnetic wave-like transfer or transformation of vibratory energy [VA01]. Three dimensional acoustic waves are more complicated than waves that are travelling in one or two dimensions. Based on the mechanical origin of the acoustic wave there can be a moving disturbance in space-time if and only if the medium involved is neither infinitely stiff nor infinitely pliable. If all the parts making up a medium were rigidly bound, then they would all vibrate as one, with no delay in the transmission of the vibration resulting in no wave motion (or rather infinitely-fast wave motion). On the other hand, if all the parts were independent then there would not be any transmission of the vibration and again, no wave motion (or rather infinitely slow wave motion). Acoustic wave characteristics are strongly

dependent on the properties of the medium involved, therefore, concepts such as mass, density, temperature, momentum, inertia, or elasticity become crucial in describing acoustic wave processes [VA01].

2.3 Fundamentals of vibration

Vibration is more narrowly used to describe a mechanical oscillation, where an oscillation is the repetitive variation, typically in time, of some measure about a central value, often a point of equilibrium, or between two or more different states [CA87]. For a mechanical oscillation to occur a system must posses two quantities, elasticity and inertia and when this system is displaced from its equilibrium position, the elasticity provides a restoring force such that the system tries to return to its equilibrium position. Vibration is usually divided into two characteristic types:

- steady-state and
- dynamic transient vibration.

2.3.1 Simplified vibration theory

The simplest mechanical oscillating system is a spring-mass system, as illustrated in Fig. 2.4. In Fig. 2.4, a mass, *m*, is attached to a linear spring with a stiffness, *k*, and is subjected to no other external forces. The system is in an equilibrium state when the spring is static. If force is applied to the spring, in the vertical direction, the system is displaced by a distance of *x*. There is a net restoring force on the mass, tending to bring it back to equilibrium. If a constant force, such as gravity, acts on the system the point of equilibrium will be shifted [CR09]. In the spring-mass system, oscillations occur as a result of the static equilibrium displacement, the mass has kinetic energy that is converted into potential energy that is stored in the spring at the extremes of its path. The time taken for an oscillation to occur is referred to as the oscillatory period (*T*). Mechanical oscillations are, therefore, periodic conversions of energy from potential energy to kinetic energy to potential energy, causing continuous mechanical oscillations about an equilibrium point, giving rise to vibration. In the spring-mass system, when the motion is measured

accurately, its *x-t* graph is nearly a perfect sine-wave shape. A sinusoidal vibration is known as simple harmonic motion [CR09].

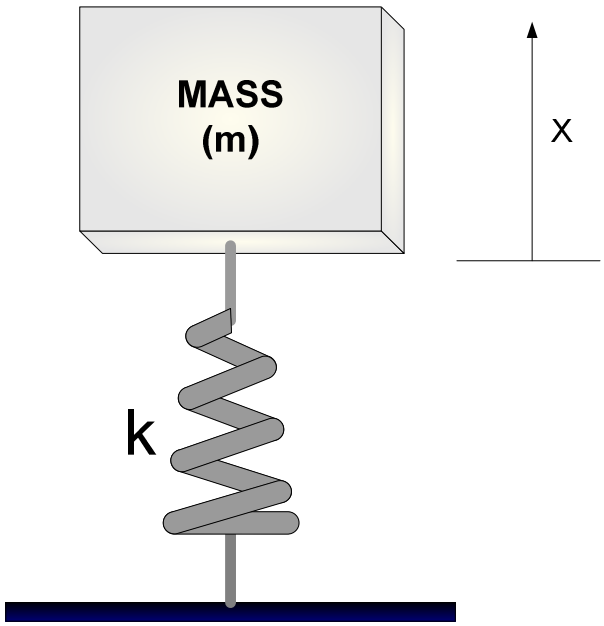


Figure 2.4: Basic components of a Spring-mass System.

Vibration has two common modes of operation, free and forced-vibration. **Free vibration** occurs when a mechanical system is subjected to an initial input and then allowed to vibrate freely. 'Free vibration' is used for the study of natural vibration modes in the absence of external loading [CR09]. The mechanical system will then vibrate at one or more of its natural frequencies and then damp down to zero, as depicted in Fig 2.5. If there is force consistently applied to the system, it will continue to vibrate or oscillate. The oscillation of the simple pendulum is an example of free vibration [RA09]. **Forced-vibration** occurs when an alternating force or motion is applied to a mechanical system. The frequency of the vibration is dependent on the frequency of the force applied, but the magnitude of the vibration is strongly dependent on the properties of the mechanical system itself, such as inertia, mass etc. [CR09]. This is depicted in Fig. 2.6. The vibrations caused by the movement of a compressor (compressor driven vibration) and/or discharge gas pulsations (pulsation driven vibration) emitted from the compressor,

are perfect examples of forced-vibration systems. Gas- pulsation driven vibration is the most common cause of forced-vibration. The term 'forced-vibration' excludes vibration due to any piping system resonances [OE89].

All reciprocating compressors emit discharge pulsations (a reciprocating compressor generates a constant stream of pulsating flow). When discharge gas pulsations react with the piping system geometry in such a way that an oscillatory force is created, discharge vibration may occur. An oscillation in the gas, when it hits an elbow in the piping geometry, may increase in the oscillation creating a significant amount of line vibration. The pressure pulsations can occur at multiples of the pump or compressor rotor and blade passing frequencies. Although all these exciting frequencies are deterministic, the magnitude of the pressure fluctuations in the pumps become a very complicated problem and usually can only be solved by an experimental determination [PE73]. The magnitude and spatial distribution of these pressures are dependent on the pump/compressor, geometry of the flow path and temperature of the fluid medium [YA02].

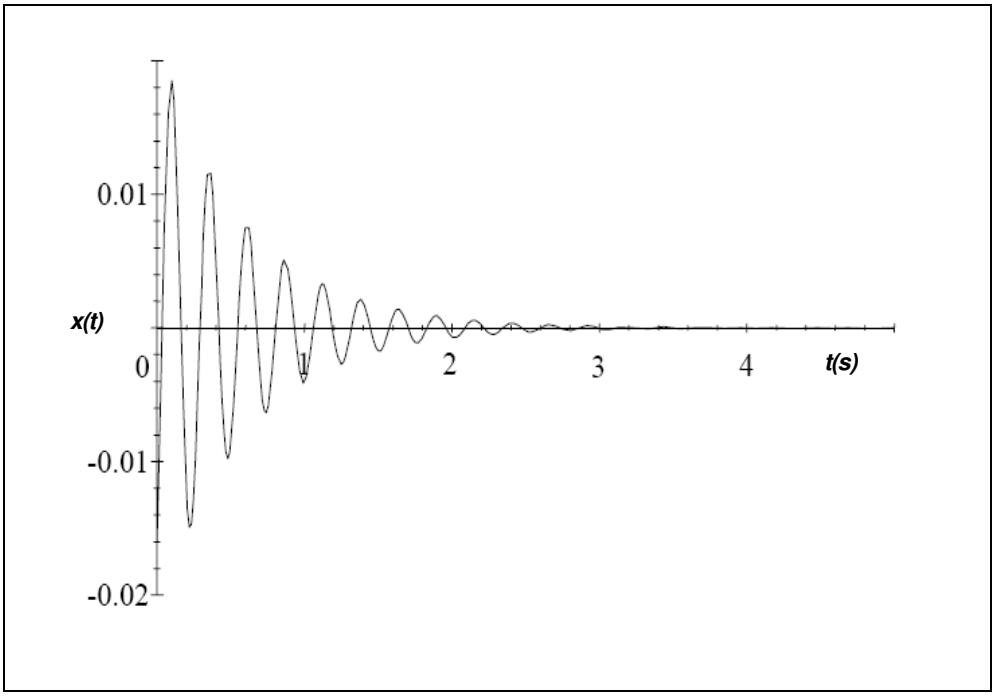


Figure 2.5: Free vibration response signal.

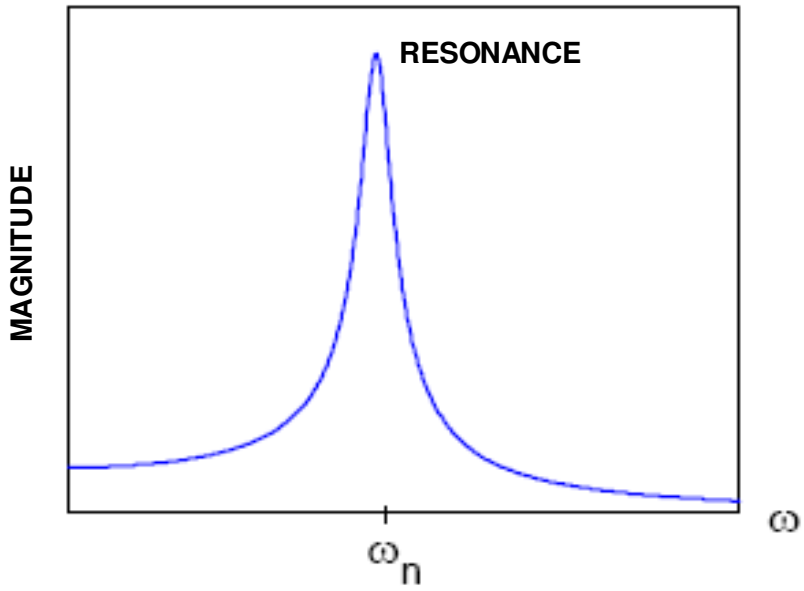


Figure 2.6: Forced vibration response signal.

2.3.2 Vibrational resonances

Energy is lost from a vibrating system for various reasons such as the conversion of heat via friction or emission of sound. This effect is called damping and will cause the vibrations to decay exponentially unless energy is applied to the system to replace the loss [CR09]. If energy is applied to a spring-mass system, it is set into oscillation and with no external friction acting on the system to dissipate its energy it will vibrate continuously at its natural frequency. As dissipative (frictional) forces arise (damping), it decreases the amplitude of the free oscillations with time [FR62]. A driving force that pumps energy into the system may drive the system at its own natural frequency or at some other frequency. A lightly damped system exists when the forcing frequency nears the natural frequency of the system. In such a system the amplitude of the vibration can get extremely high. This phenomenon is called resonance (the natural frequency of a system is also often referred to as the resonant frequency) [CR09]. More information for resonance can be found in Section 2.2.1.2.

Resonance is the tendency of a system to oscillate at larger amplitude at some frequencies than at others, termed as resonant frequencies. It can create excessive vibrations when the natural frequency of a structure or fluid coincides with excitation frequency [LE07]. At these frequencies, even small periodic driving forces can produce large amplitude vibration, because the system stores vibrational energy. When the damping is small the resonant frequency is approximately equated to the natural frequency of the system, which is the frequency of free vibrations. Figure 2.7 illustrates the increasing amplitude of damped simple harmonic oscillator; it shows that as damping decreases the input (excitation) frequency, ω_A or ω_p, approaches resonance or natural frequency, ω_n or ω_0. Figure 2.7 displays the damping coefficient, δ, which is a representation of the amount of damping existing in the system, refer to Section 2.3.4.2. The coincidence of these frequencies defines resonance. A physical system can have as many resonance frequencies as it has degrees of freedom; each degree of freedom can vibrate as a simple harmonic oscillator. As the number of coupled harmonic oscillators grows, the time it takes to transfer energy from one to the next becomes significant. The vibrations in them begin to travel through the coupled harmonic oscillators in waves, from one oscillator to the next [CR09]. This process allows one to construct complex acoustic models, based on the coupling of simple harmonic oscillators.

In a fluid-structure system, the forced-vibration of the system depends on the magnitude and frequency of the exciting forces, and in the dynamic characteristics of the structure. The vibration can become larger if the frequency on an important force component coincides with a main natural frequency of the structure. The system would then resonate [YA02].

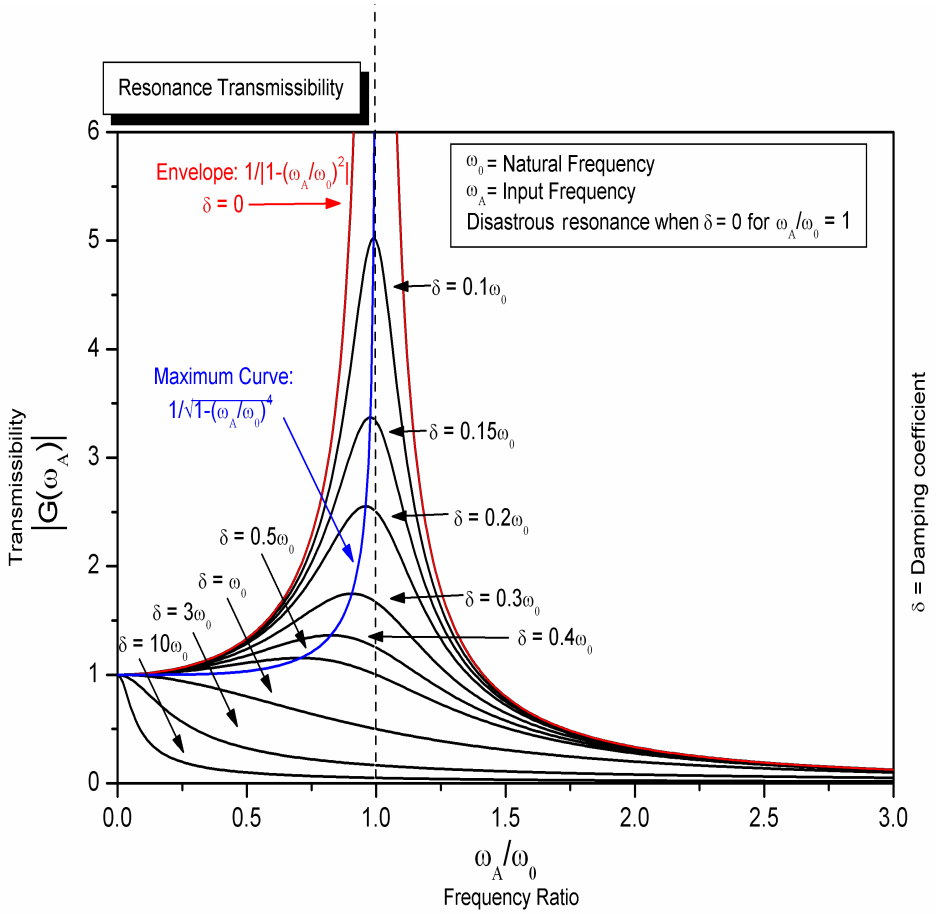


Figure 2.7: Resonance in a simple damped harmonic system [CR09].

2.3.2.1 Acoustic and Structural resonances

Pressure waves propagate at the speed of sound in the medium and the length of pipe and pipe design, (i.e. narrowing or bends in the line that may define reflection points) as well as the speed of the pressure wave in the medium (determined by the temperature and density of the medium) determine whether a standing wave will exist. Acoustic resonances may occur as a result of the propagation of longitudinal standing-wave patterns [OE89]. There are many types of acoustic resonance but most are very difficult to predict. Acoustical resonances amplify the gas pulsations at the specific locations in such a magnitude as to cause significant vibration, also known as flow induced vibrations. The acoustic resonance of the fluid medium (steam or water) is considered as most probable source of flow-induced vibration [VA03].

This pressure pulsation affects structures by causing unbalanced forces at locations where direction of flow or velocity is changed (elbows, tees, local orifices etc.). Every structure having a mass and elasticity has the capacity to vibrate. Structures will tend to oscillate at certain specific frequencies when subjected to forces. When the amplitude of these vibrations exceeds the permissible limit, resonance occurs which may lead to possible failure of the structure [RA09]. One mechanism that can produce structural resonance is having an excitation force in its system with a frequency component that coincides with the structures' natural frequency. For example, in piping systems, the vibration of the piping can become greatly amplified at the point where the exciting frequency matches the frequency of the piping. The resonant frequency of this system is a function of the mass and stiffness of the pipe line [OE89]. Structural resonances are very hard to predict and extensively testing a pipe design in a laboratory environment, is at present the best method of avoiding structural resonances. Another important method of determining or predicting if these resonance factors will occur, is by performing specialised acoustic, vibrational analysis coupled with stress analysis with the use of appropriate analytical models.

2.3.3 Vibration analysis

In analysing vibrations, the key to understanding how an object will vibrate is to know how the force acting on the object depends on the objects position [CR09]. The simplest example is the mass on the spring (simple-harmonic oscillator). Once the mass is set into motion, a mathematical model can be produced that can takes into account known laws of physics. Certain assumptions are taken to simplify the problem to such an extent that the known laws of physics can be applied. In a general case, the motion of a vibrating system is due to both the initial conditions and the exciting forces. The mathematical model of this system would be a linear non-homogenous differential equation of the second order. To produce the particular solution of a non-homogenous equation, it is assumed that the excitation can be approximated by a harmonic function. Such a case can be referred to as the harmonic excitation, and is best described by a simple-harmonic oscillator system.

2.3.4 Simple-harmonic oscillator

A simple-harmonic oscillator is commonly used to describe the mechanics of vibration. The simple harmonic oscillator consists of an ordinary spring that is suspended vertically from a fixed support, as displayed in Fig. 2.8. At the lower end of the spring, a body with mass, *m*, is attached. If the body is pulled down a certain distance and then released, it will undergo motion. This motion, assuming the motion is in the vertical direction is the displacement which will be determined as a function of time. If this system behaves linearly, the vibration is known as linear vibration, if not, it is considered non-linear vibration [RA09].

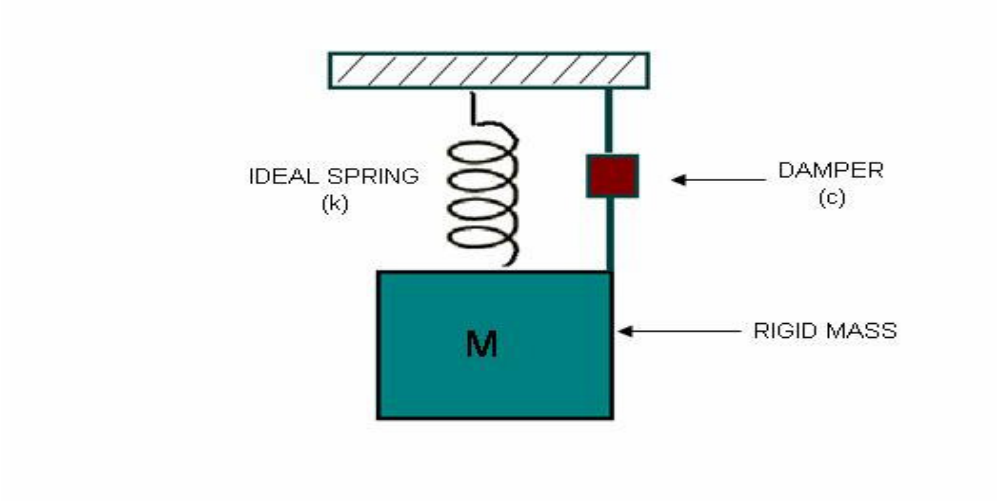


Figure 2.8: **Simple-harmonic oscillator diagram**

The simple-harmonic oscillator is neither driven nor damped. This solution will oscillate with simple-harmonic motion that has amplitude of, *A* (a constant) and a frequency, *f*. The frequency (*f*) is one of the most important quantities in vibration analysis and is called the undamped natural frequency. Measurement shows that the frequency of the vibration is constant and that the displacement of the mass from its rest position is a sinusoidal function of time (*t*), with constant amplitude, *A*. Sinusoidal vibrations of this type are called simple-harmonic vibrations. Any single-frequency travelling wave will take the form of a sine wave:

$$f(t) = A\sin\omega t \tag{2.1}$$

The angular frequency, ω, is a scalar measure of rotation rate or frequency with which phase changes and is given by:

$$\omega = 2\pi f \ . \tag{2.2}$$

In addition to its amplitude, the motion is characterised by its period, *T*, the time for a single oscillation, its frequency, *f*, the reciprocal of the period $f = 1/T$ (which is the number of cycles per unit time), and its phase, ϕ, which determines the starting point on the sine wave. The period and frequency are constants which are determined by the overall system, while the amplitude and the phase are determined by the initial conditions (position and velocity) of that system. This is illustrated in Fig. 2.9.

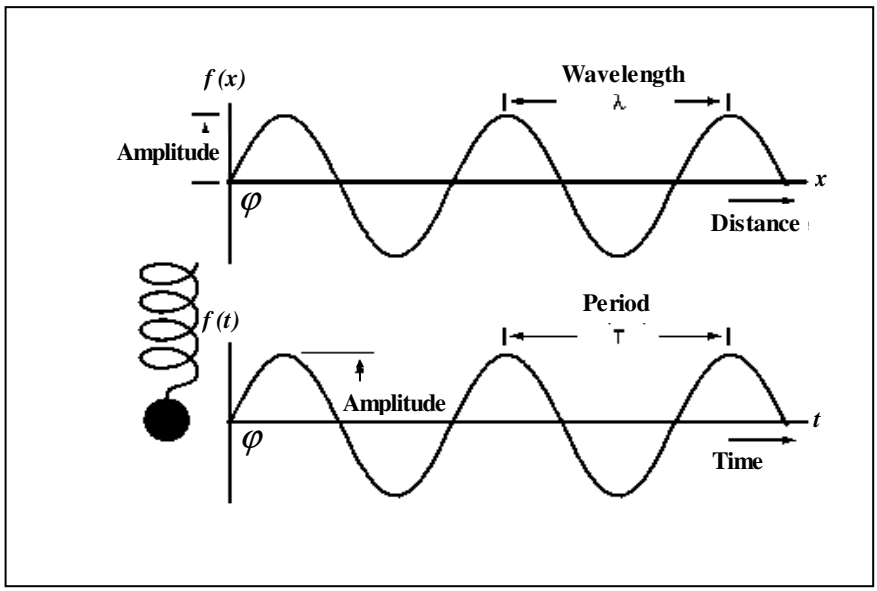


Figure 2.9: Simple-harmonic function as a sine-wave

It can be shown, both experimentally and theoretically, that the mass will vibrate with simple-harmonic motion whenever the restoring force resulting from the stiffness of the spring is directly proportional to the displacement of the mass from its rest position [FR62]. In deriving the solution of the vibration, all forces must be accounted for. The main forces acting on the mass are:

- attraction of gravity,

- spring force exerted by spring in the system and
- Newtons' second law of motion, where force is equal to mass times acceleration.

2.3.4.1 Undamped harmonic oscillator, free vibration without damping

If no energy is lost or dissipated in friction or other resistance during an oscillation in the system, the vibration is known as undamped vibration. In one dimension, the direction of the force can be represented by a positive or negative sign from the equilibrium point, where the force is zero (at rest). The force on the mass is given by Hooke's law, which states that the stiffness of the spring, k is directly proportional to the displacement of the mass from its central position by a stretching force, F:

$$F = kx, \tag{2.3}$$

where F is the force of the spring, k is the stiffness of the spring (spring-modulus) and x is the displacement of mass, m. If the spring force, F is the only force acting on the system, the system is called a simple-harmonic oscillator. The equation of the motion for this contains a complete description of the motion and other parameters that can be calculated from it. As mentioned previously, the position of an object vibrating will trace out a sine wave as a function of time (or if a mass on a spring is carried at constant speed, it will trace out a sine wave), and is represented by Eq. (2.1), where $\omega = \sqrt{(k/m)}$. This is represented in Fig. 2.10.

In an undamped harmonic oscillator, the damping is assumed to be negligible and no external force is applied to the mass, i.e no forced-vibration is induced. Utilising all the forces acting on the mass, the system attains a linear equation with constant coefficients that depict the motion of the mechanical system. As such,

- from Hooke's Law, Eq. (2.3), the restoring force tends to restore the mass to the equilibrium position and is then expressed as $F = -kx$ and,
- from Newtons second law of motion, the force generated by the mass is proportional to the acceleration of the mass, the general equation of linear motion is used as follows: $\sum F = ma = m\ddot{x} = mdx^2 / dt^2$.

Combining all forces on the mass produces:

$$m\frac{\partial^2 x}{\partial t^2} = -kx$$
$$m\ddot{x} + kx = 0. \tag{2.4}$$

The solution to this differential equation is sinusoidal, as described in Section 2.4.2 and the acceleration, velocity and displacement is also depicted in Fig. 2.11 as function of time. It is also assumed that the spring is stretched by a distance of *A*, and is released, the solution that expresses the motion of the mass becomes:

$$x(t) = a\sin\left(2\pi ft + \phi\right) \tag{2.5}$$

where ϕ is the phase shift. Equation (2.5) indicates that the mass will oscillate with simple-harmonic motion, having an amplitude of *A* and a frequency, ω , also called the undamped natural frequency. Here, *A*, ω, and ϕ are constants and each represents an important physical property of the motion. The velocity, *v*, and the acceleration, *a*, of the mass are given by:

$$v(t) = \frac{dx}{dt} = -A\omega\sin\left(\omega t + \phi\right)$$
$$a = \frac{d^2 x}{dt^2} = -A\omega^2\cos\left(\omega t + \phi\right). \tag{2.6}$$

The acceleration can also be expressed as a function of displacement, $a(x) = -\omega^2 x$ and considering, $ma = -m\omega^2 x = -kx$, therefore $\omega^2 = k/m$, producing $\omega = \sqrt{(k/m)}$. In addition $\omega = 2\pi f$, thus the frequency of vibration can be expressed as:

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k}{m}\right)}. \tag{2.7}$$

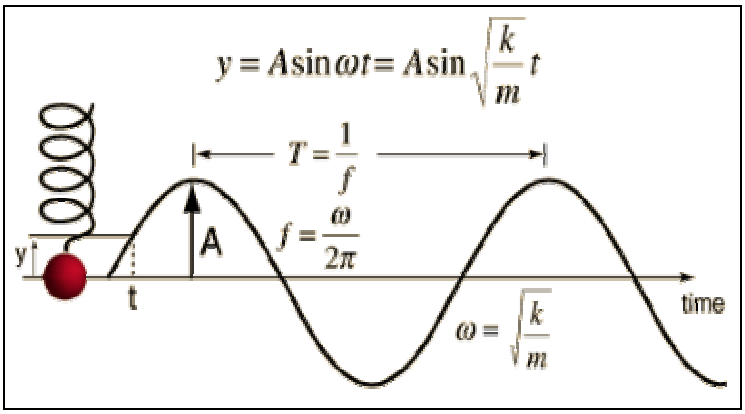


Figure 2.10: Simple-harmonic motion (undamped).

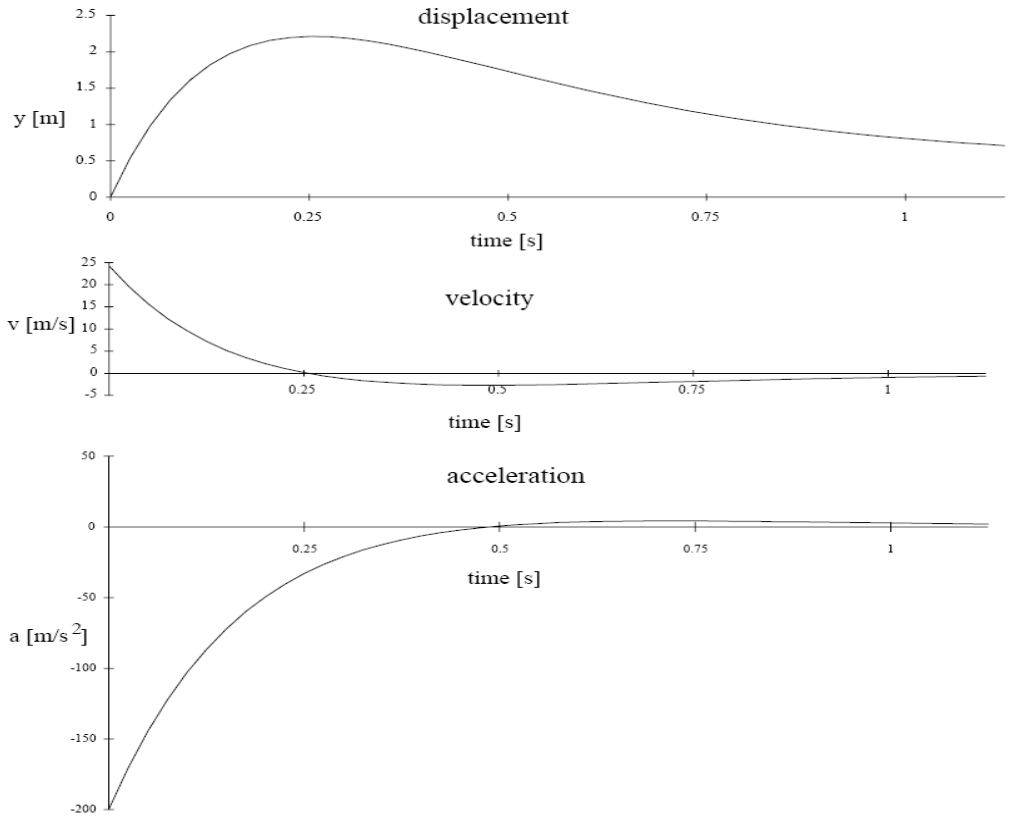


Figure 2.11: Displacement, velocity and acceleration *vs* time for a typical vibrating system.

2.3.4.2 Damped harmonic oscillator, free oscillations with damping

If the system has energy lost or dissipated by friction or from other resistances during oscillation, it is called damped vibration. In many physical systems, the amount of damping is so small that it is considered negligible for most engineering purposes. However, the consideration of damping becomes extremely important in analysing vibratory systems near resonance [RA09]. A damper added to the model (as seen in Fig. 2.8) produces a force, F_{d}, that is proportional to the velocity of the mass. It is called a viscous damper since it models the effect of an object within a fluid. The proportionality constant, *c*, is the damping coefficient, also termed as δ in Section 2.3.2 with units Ns/m and F_{d} is given by,

$$F_{\mathrm{d}} = -cv - c\dot{v} = -c\frac{dx}{dt}. \tag{2.8}$$

The damping coefficient represents the inefficiencies of the material due to energy loss at a molecular level or of the system due to component interaction. As done previously, the forces around the mass are summed to obtain the following ordinary differential equation, $m\ddot{x} + c\dot{x} + kx = 0$. The damping coefficient significantly affects the motion of the mass. If the damping is small enough, the system will vibrate but eventually over time it with cease to vibrate and is called ***underdamping***. If the damping is large enough, to the point where the system does not oscillate, this is referred to as ***critical damping***. If the damping exceeds this point, the system becomes ***overdamped***.

The amount of damping in a system, is expressed as the damping ratio/factor, ξ. The formula to calculated this ratio/factor is $\xi = c\Big/\left(2\sqrt{km}\right)$. The damping ratio is introduced as a measure of the severity of the damping. In summary the three distinct ratio values that characterise system behaviour can be categorised as follows:

- ζ **> 1: Overdamped.** The system is so well damped that it will return to its equilibrium point without a single oscillation (see right hand side Fig. 2.12).
- ζ **= 1: Critically damped**. The system is on the verge of oscillating. It will return to its equilibrium point the fastest (see right hand side of Fig. 2.12).
- ζ **< 1: Underdamped**. The system exhibits a decaying oscillatory motion (see left hand side of Fig. 2.12).

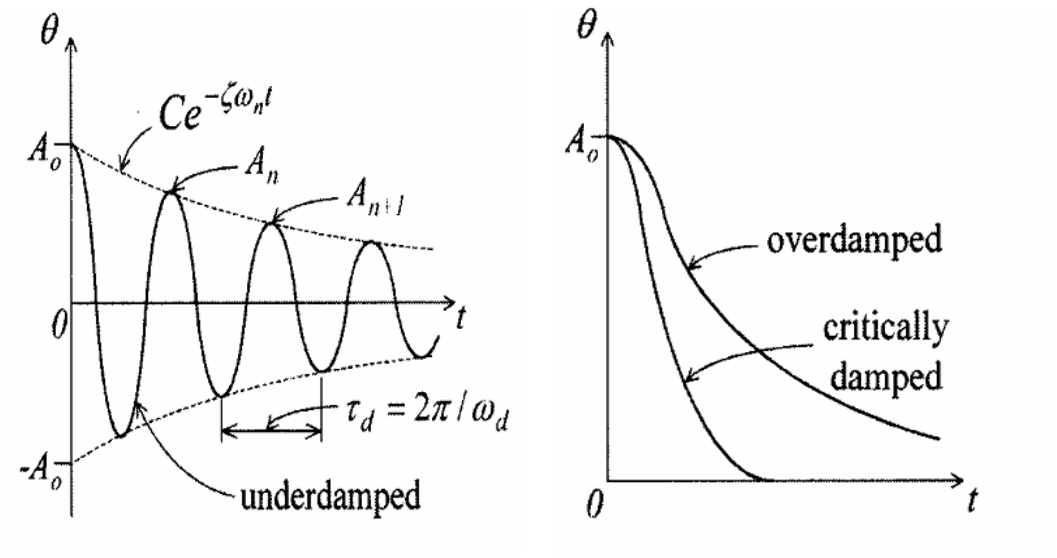


Figure 2.12: Left hand side: underdamped and right hand side: overdamped and critically damped systems.

For the mass-spring damper, the solution for an underdamped system is the following:

$$x(t) = Xe^{-\xi\omega_n t}\cos\left(\sqrt{1-\xi^2}\omega_n t - \phi\right). \tag{2.9}$$

The value of the initial magnitude (X) and the phase shift (ϕ) are determined by the amount the spring is stretched and the exponential term defines how quickly the system damps down to zero. The larger the damping ratio, the quicker it damps

down. The cosine is the oscillatory part, but the frequencies are different from the undamped case. The damped natural frequency, f_n, is related to the undamped natural frequency by $f_d = \sqrt{1-\xi^2 f_n}$. This is illustrated in Fig. 2.13 for values of $\xi = 0.1$.

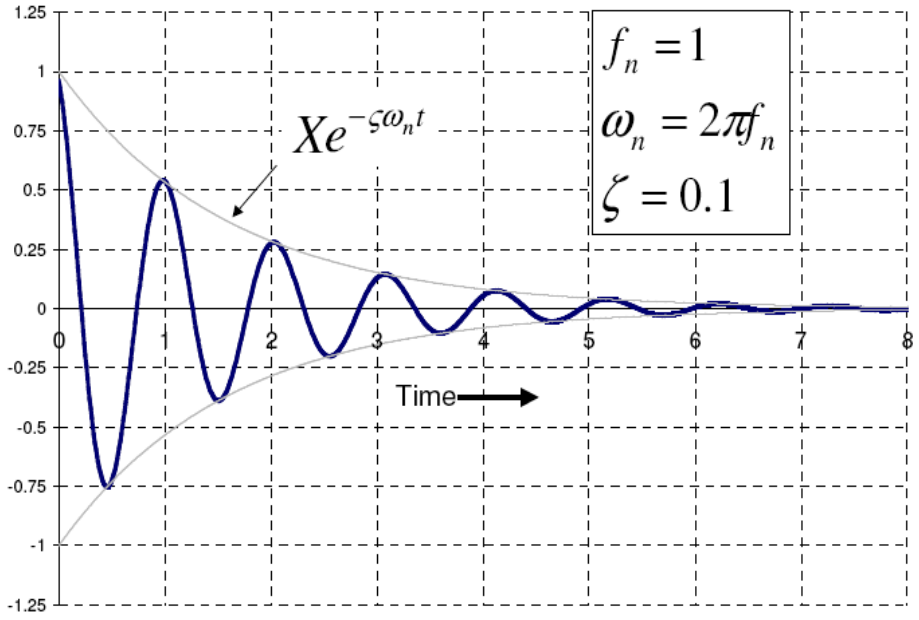


Figure 2.13: Free vibration illustration.

2.3.4.3 Forced oscillations with damping

In this system, a harmonic force is added to the spring-mass damper. It is often maintained in a condition of vibration by the application of a sinusoidal driving force. The source of this force, described in Section 2.2.1, is also referred to as a body force in later parts of this Report. By summation of the all the forces on the body, the differential equation for the motion of the damped oscillator becomes:

$$m\frac{\partial^2 x}{\partial t^2} + c\frac{dx}{dt} + kx = F_0 \cos(2\pi f t). \tag{2.10}$$

where the driving force is expressed as $F = F_0 \cos(2\pi f t)$.

The steady-state solution becomes $x(t) = X\cos(2\pi ft - \phi)$ and the amplitude of the vibration is defined as,

$$X = F_0 / k\left[1/\sqrt{(1-r^2)^2 + (2\zeta r)^2}\right]. \tag{2.11}$$

where $\phi = \arctan\left(2\zeta r/1-r^2\right)$, and the ratio, r, of the harmonic force frequency over the damped natural frequency of the mass-spring damper model and the phase shift is defined as $r = f / f_n$.

2.4 One dimensional acoustic wave equation

Most vibrating systems in nature are not as simple as explained above, the systems discussed were of single degree-of-freedom models and it was assumed that the mass moves up and down only. In the case of more complex systems we need to discretize the system into more masses and allow them to move in more than one direction, thereby adding more degrees of freedom. It, therefore, becomes necessary when specifying the motion completely to give the displacement, *x*, of each point as a function of time, *t*. This is illustrated in Fig 2.14. Fluids exhibit fewer types of constraints relative to possible deformations than with solids. As a result, the restoring force responsible for propagating a wave is the non-directional elastic opposition that arises when a fluid is compressed. The fundamental wave, as explained earlier, is a sinusoidal (harmonic) wave form.

The wave-equation is a differential equation that describes the evolution of a harmonic wave over time. In physics, the general wave-equation governs the propagation of acoustic waves through a material medium. This equation has slightly different forms depending on how the wave is transmitted and the type of fluid medium it is travelling through. To properly utilize the wave-equation, it is important to understand its derivation. This equation will be the foundation for deriving the various acoustic equations later in Chapter 3.

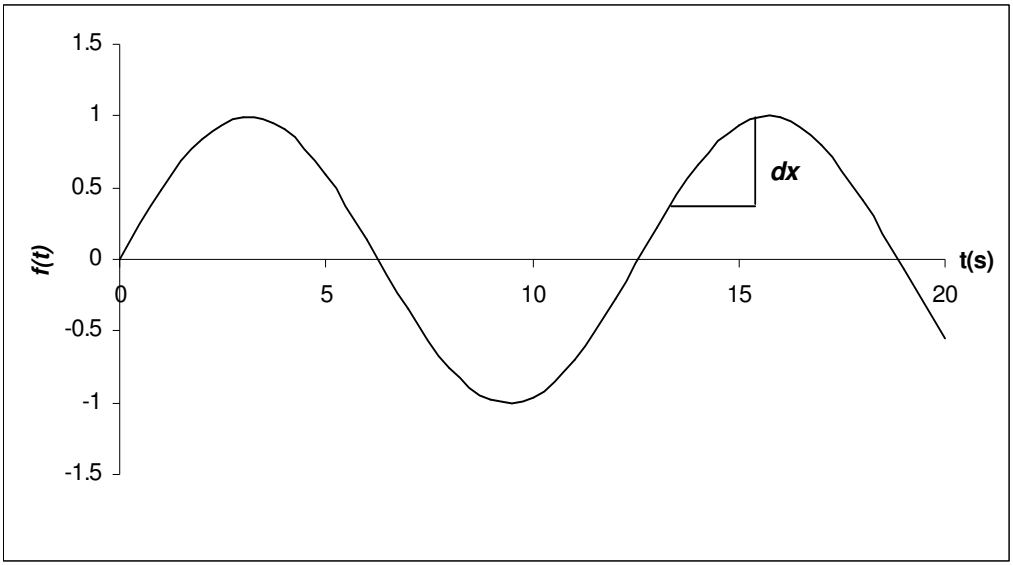


Figure 2.14: Frequency of fluid *versus* displacement.

The derivation of the general wave-equation is based on incorporating the following into the formulation of the solution [RA45]:

- *Eulers Force equation* (conservation of momentum) for a fluid medium: $\rho d\vec{V} / dt + \nabla P = 0$ and $\rho dv / dt + \partial P / \partial x = 0$,
- *Continuity equation (conservation of mass)* of the particles of the fluid in one dimension: $\partial\rho / \partial t + \nabla\left(\rho\vec{V}\right) = 0$ and
- *Equation of state* (ideal gas law): $PV = nRT$.

It is to be noted that in an adiabatic process, pressure is a function of density, ρ, which can be linearised to $P = C\rho = f(\rho)$. For incompressible fluids, the pressure of the fluid does not vary significantly over time, thus $dP / dt = 0$ and $(PVA)_1 = (PVA)_2$ remains constant as the wave propagates through the fluid. However, for compressible fluids, the pressure differs significantly and the movement and the conservation of momentum can be expressed as:

$$\partial P / dt + v\partial P / dx + \rho\partial v / dx = 0. \qquad (2.12)$$

The following assumptions are used:

$$P = P_0 + \tilde{P},$$
$$\rho = \rho_0 + \tilde{\rho},$$
$$v = v_0 + \tilde{v}, \tag{2.13}$$

where, P is the local deviation from the ambient pressure, P_o is the ambient pressure, $\tilde{P}$ is the fluctuating pressure around P_o as illustrated in Fig 2.15, $\tilde{\rho}$ is the fluctuating density, $\tilde{v}$ is the fluctuating velocity and v_o, is the minimum velocity.

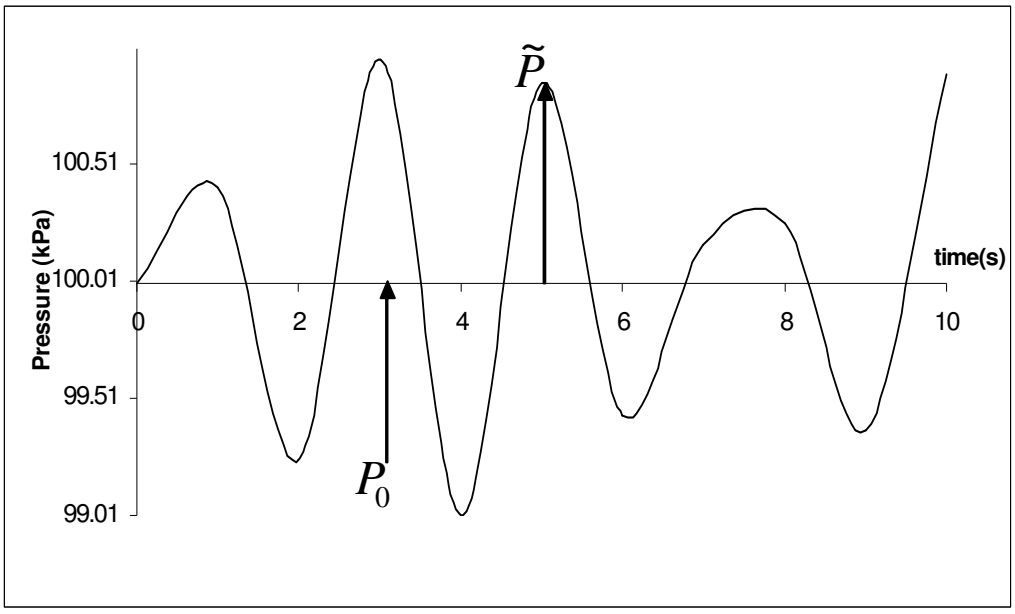


Figure 2.15: Schematic of fluctuating pressure in a pipe length.

Since the velocity is small and the fluctuating velocity is much less, the fluctuating velocity can be assumed to be negligible. Perturbation Theory is used to find an approximate solution to the problem [SC82]. It is applied using the assumptions made in Eq. (2.13) and substituted into Eq. (2.12). This is reduced further by neglecting the velocity terms, resulting in $d(\tilde{P})/dx + \rho_o d\tilde{v}/dt = 0$. A Taylor series is applied to the equation in the form of:

$$P(\rho_0 + \rho) = P(\rho_0) + \tilde{\rho}\left(dP/d\rho\right). \tag{2.14}$$

In the case of the fluid medium being a gas, as in this Research Report, the second order of speed of sound with respect to the fluids' density and pressure is used as the representation of the fluid's properties, instead of its density, as such:

$$dP/d\rho = c_0^2 \,, \tag{2.15}$$

where c_0 is the speed of sound in the fluid (later on to be applied to helium). The ideal gas law is used to calculate the speed of sound in the medium or gas (helium) using:

$$c_0 = \sqrt{\gamma_{\text{He}} RT / M}\,, \tag{2.16}$$

where, γ is the expansion coefficient for helium, R is the universal constant for gases, M is the Mach number, and T is the temperature of the gas in degrees Kelvin [SC82]. Equation (2.12), Eq. (2.14) and the assumptions made, are combined and the time derivative of the continuity equation and the spatial derivative of the force equation are taken to yield:

$$\frac{\partial^2 \tilde{P}}{\partial x^2} + \frac{\partial^2 \tilde{\rho}}{\partial t^2} = 0. \tag{2.17}$$

The fluctuating symbols are omitted, Eq. (2.15) and (2.16) are incorporated and the higher- order terms are ignored to produce a homogeneous second-order partial differential equation, the one-dimensional general wave-equation:

$$\frac{\partial^2 P(x,t)}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 P(x,t)}{\partial t^2} = 0. \tag{2.18}$$

Equation (2.18) describes an acoustic pressure wave as a function of space and time. The actual pressure in Eq. (2.18) is a fluctuating pressure deviating from ambient pressure, if one has a mathematical solution which produces large values of $\tilde{P}$, it will violate the original perturbation assumptions thereby providing inaccurate results. Thus some of the non-linear terms discarded in the development

of the wave equation needs to be further examined. The one-dimensional wave-equation can be readapted to suite various types of applications. Below are examples of the type of variations that can be produced [SC82]:

- Navier Stokes equation leads to a viscous damping term.
- Non-zero velocity leads to a Mach number term in the wave-equation.
- If the fluid is replaced with an elastic material longitudinal and shear wave equations are produced.
- Incorporating electromagnetism produces a vector wave-equation.

2.5 Acoustic wave prediction solutions

In this section, it is explored how the general acoustic wave-equation has been utilised by various experts in this field to derive their own solutions for cases of flow-induced vibrations. It has mostly been investigated and applied in Pressurised Water Reactors (PWRs) as part of solving flow-induced vibrations experienced in industry, especially in reactor core internals. It is also found that the most probable and widely observed mechanism of piping steady-state vibrations is from flow-induced vibration. Refer to Sections 2.2.1.2, 2.2.1.3 and 2.3.2.1 for more information.

2.5.1 Historical account

Pressure pulsations in PWRs were first studied by Penzes [PE73] and later by Bowers and Horvay [BO75] and Cepkauskas [CE79]. The acoustic response of reactor piping and reactor core support structures has also been a subject at the International Conference in Structural Mechanics in Reactor Technology (SMiRT) conferences since Penzes introduced his Cleaver model in 1973 [PE73].

Most of the derived acoustic wave solutions by specialists use the Penzes mathematical approach as the basis. Three different specialists' methodologies were selected for this Research Report for investigation. All three specialists have formulated their forced-vibration acoustic solutions in accordance with the Linearised Navier-Stokes equations by assuming a compressible, inviscid liquid.

Penzes and Lee and Im [LE92] have utilised an equivalent body force to represent the source of the pump pulsations. Lee and Im in their publications stated:

"Penzes, Bowers and Horvay and Cepkauskas, treated the problem of pulsations by introducing a body force concept, but they neglected to recognise the restrictions on the body force to make the boundary conditions homogenous"[LE92].

Cepkauskas, sought to resolve this by demonstrating that the auxiliary terms need not be defined to determine the pressure distribution of the acoustic wave. This theory made by Cepkauskas will also be investigated or proved in Chapter3. Many authors have published detailed accounts of their solutions, but as with most mathematical solutions, it has its limitations on how it is applied. Three main specialists were selected to investigate their approaches, the reasons for these are:

1) **Penzes and Horvay**

   Penzes was one of the first known published authors to produce acoustic wave solutions in forced-vibration systems for LWRs. Much work subsequently produced has been based on expansion of the work conducted by Penzes and Horvay.

2) **Kye Bock Lee and In Young Im**

   Kye Bock Lee and In Young Im stated in their publications [LE92, LE94] that they have developed an improved analytical model which satisfies all boundary conditions having also considered all these missing constraints on the auxiliary functions. In addition, it was mentioned that other specialists (in this field), neglected to recognise the restrictions on the body force necessary to make the boundary conditions homogenous while determining their acoustic solutions. Also stated was that Lee and Chandra (other specialists in the field) analysed the pump induced pulsating pressure in a reactor coolant pipe but missed the constraints on the auxiliary function to make the boundary conditions homogenous

[LE80]. Thus, the solution cannot satisfy the boundary conditions at the pipe end for the piston-spring supported end case.

3) **M. Cepkauskas**

Cepkauskas utilised a technique that transformed the homogeneous differential equation with time-dependent boundary conditions to one of a non-homogeneous differential equation with homogeneous boundary conditions. This technique was unique in that the previous authors chose auxiliary functions to make the boundary conditions homogeneous, and in his methodology, this was not needed [CE79, CE81 and CE08].

2.5.2 Penzes methodology

Penzes utilized his methodology to calculate the pressure distribution that predicts the dynamic responses of the reactor internal components. The mathematical formulation used by Penzes was in accordance with the linearised Navier-Stokes equation and assumed a compressible, inviscid liquid and the concept time-dependent "body forces" as forcing functions. In Penzes case, the time-dependent mixed valued boundary value problem is replaced by a forced-vibration problem approximation with homogenous boundary conditions. The boundary conditions selected are two concentric rigid walls in the radial direction. Penzes formulation determines a time-dependent, mixed boundary-value problem, where for this case the separation of variables technique cannot be applied and a solution of this problem would require an extremely lengthy application of numerical analysis techniques. Penzes extended his research from the one-dimensional application to the three-dimensional one to incorporate the geometry of a PWR annulus. Only the approach Penzes utilised for the one-dimensional application will be covered.

2.5.2.1 Penzes and Horvay mathematical approach

The time-dependent mixed boundary-value problem is solved approximately by using the concept of forced-vibration. Penzes solution for obtaining the unknown pressure distribution was developed by using two steps in his method:

1) determining the free vibration in terms of the liquid frequencies and pressure mode shapes and,

2) obtaining the pressure distribution using the free vibration solution as the solution of the forced-vibration equation.

Both Penzes' and Horvay's solution are based on the assumption that the pressure pulsations are due to excitations at the inlet nozzles of a PWR reactor core annulus. The pressures in the annulus are calculated based on prescribing the pressure at the inlet nozzles and on the concept of the time-dependent forcing function (also referred to in his literature as a body force) in the governing differential equations [PE10]. Penzes expresses the *equation of compressibility* in terms of fluctuating dynamic pressure as:

$$q = P - P_0 = c_0^2 \left(\rho - \rho_0 \right),$$

where P_0 is the reference pressure, c_0 is the reference sound velocity and q is the dynamic fluctuating pressure. The equation of continuity follows from Berry and Reissners' equation:

$$\frac{\partial P}{\partial t} + \rho_0 \mathrm{div} \bar{V} = 0. \tag{2.19}$$

Equation (2.19) is differentiated with respect to time and substituted with the density, (from equation of compressibility) and expressions for velocity components, resulting in the governing differential equation (expressed in terms of the unknown fluctuation pressure),

$$\nabla^2 q = \frac{1}{c_0^2} \frac{\partial^2 q}{\partial t^2} + \mathrm{div} \bar{P} = 0, \tag{2.20}$$

where $\bar{P}$ is the forcing function. Equation (2.20) is also a representation of the induced forced-vibration in the liquid. When the forcing function is eliminated, the governing differential equation is reduced to the well-known damped wave-equation in terms of the fluctuating pressure,

$$\nabla^2 q = \frac{1}{c_0^2}\frac{\partial^2 q}{\partial t^2} \quad . \tag{2.21}$$

The above is an idealised hydrodynamic model that provides a basic understanding of the relationships between the pulsating forcing function and the periodic pressure fluctuations at any location of the liquid, including the inner and outer surfaces with the pump inlet of the reactor [FR73]. It is also assumed that the well-known damped wave-equation is represented by the phenomenon of standing waves. Penzes makes use of this assumption by applying the separation of variables technique to produce:

$$q_1 = Ae^{\omega t}$$
$$q_2 = B_1 \sin \alpha z + B^2 \sin \alpha z,$$

where A and B are arbitrary constants, z is the axial co-ordinates and ω, α are separation constants. It is mentioned that the separation and integration constants can be determined up to a single set of arbitrary constants by specifying the boundary conditions.

The following types of boundary conditions are considered: open, closed and piston-spring supported for the axial direction. They are mathematically expressed as:

- **Open ended**: $q_2\big|_{z=z_1} = 0$
- **Closed ended**: $q_2 / dz\big|_{z=z_1} = 0$
- **Piston-spring supported**: $q_2\big|_{z=z_1} = k/\left(\rho_0 \omega^2 A\right) \partial q_2 / \partial z_{z=z_1}$,

where ω is the natural frequency of the liquid, A is the area of the piston and k is the spring constant. The forced-vibration system is created by the pulsating forcing function placed near to the location of the pulsating surface pressure. A periodic pump pulsation is assumed and the driving force of the liquid in the radial component P_r is described as:

$$P_r = P_0 \cos \omega_p t, \tag{2.22}$$

where ω_p is the driving frequency and P_0 is the constant pressure at volume 1. The pressure is also known as the reference pressure. The driving force, Eq. (2.22), is incorporated into Eq. (2.21) yielding Penzes solution for a forced-vibration system as:

$$\nabla^2 q = \left(1/c^2\right)\frac{\partial^2 q}{\partial t^2} + \left(P/r\right)\cos\omega_p t. \tag{2.23}$$

2.5.3 Kye Bock Lee and In Young Im

Lee and Im derived a method of estimating the amplitude of the acoustic pressures induced by the external coolant pump in the inlet pipe and the annulus of a PWR. Only the methodology employed at the inlet pipe is explained in this section, and not that of the annulus. The separation of variables technique is not used as it is stated that it would prove too complex to solve using numerical analysis techniques [LE92, LE94].

2.5.3.1 Kye Bock Lee and In Young Im Mathematical Approach

The Lee *et al.* [LE92] hydrodynamic differential equations of the analytical model are applied to a fluid that is assumed to be compressible and inviscid. The mathematical analysis is formulated in accordance with the linearised Navier Stokes equations. The pressure pulsations considered are smaller than the static pressure and the small pressure perturbation method is used to reduce the Navier-Stokes equations to the following wave-equation as such,

$$\nabla^2 - \left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]P = 0.$$

The variation of the pressure pulsations in the radial and circumferential direction is ignored and the equation reduces to,

$$\left[\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]P_1 = 0. \tag{2.24}$$

The excitation, or forcing function of the system, is taken to occur at the pump discharge at $x = 0$, making the boundary condition:

$$P_1\big|_{x=L} = P_D \cos \omega_p t, \tag{2.25}$$

where P_D is the pressure wave at the pump discharge, ω_p is the pump frequency and x is the pipe length. The pressure wave at the pump discharge can be obtained through analytical means. The boundary conditions at $x = L$ (total length of the pipe) considered are:

- **open end**: $P\big|_{x=L} = 0$
- **closed end:** $\partial P / \partial x\big|_{x=L} = 0$
- **piston-spring supported end:** $\left[(\partial P / \partial x) - A\rho / k\partial^2 P / \partial t^2\right]_{x=L} = 0$.

The boundary conditions along with Eq. (2.24) and Eq. (2.25) describe the boundary-value problem with time-dependent non-homogeneous boundary conditions. The boundary condition is rendered homogeneous through transformation technique, in assuming the form of:

$$P_1(x,t) = Q_1(x,t) + g_1(x) f_1(t) + g_2(x) f_2(t),$$

where $g_1(x)$ and $g_2(x)$ are auxiliary functions and Q is the transform equation. Restrictions are placed on the auxiliary functions, they are arbitrary within $0 < x < L$. The frequency equation and normal modes were determined to be:

$$\tan\left(\omega_n L_1 / C_1\right) + K / \left(\rho A C_1 \omega_n\right) = 0,$$
$$X_n = \sin\left(\omega_n / C_1\right) x.$$

More details on the formulation of the solution can be found in [LE93]. The pressure distribution in the pipe region is then determined to be:

$$P_1(x,t)=\sum_{n=1}^{\infty}\left(\frac{P_D I_{2n}+\alpha I_{3n}}{I_{1n}}\right)\cos\omega_p t\sin\left(\frac{\omega_n}{C_1}\right)x+g_1(x)f_1(t)+g_2(x)f_2(t). \qquad (2.26)$$

The resultant solutions for open-end and closed-end boundary conditions are displayed in Ref. [LE93].

2.5.4 Cepkauskas methodology

Cepkauskas utilized a technique that transformed a homogeneous differential equation with time dependent boundary conditions to one of a non-homogeneous differential equation with homogeneous boundary conditions. This technique was unique, in that the previous authors chose auxiliary functions to make the boundary conditions homogeneous [CE79, CE08, CE82]. For the simple pipe acoustics, Lee and Chandra [LE80] choose unique auxiliary functions to formulate the problem. This was shown to be unnecessary and was expounded upon in a paper with Fisher and Chandra [FI79]. Cepkauskas in his studies aimed at demonstrating that the use of a body force to derive the axial pressure distribution is not required. He derived a technique based on the above methodology, and published it at the 5th International conference on Structural Mechanics in Reactor Technology (SMiRT), where he termed his solution as SMiRT5 [CE79].

2.5.4.1 Cepkauskas mathematical approach

Cepkauskas formulation consists of applying a transformation technique that changed the form of the problem to a non-homogenous differential equation with homogenous boundary conditions by utilising an auxiliary function. Cepkauskas also demonstrated that unlike other solutions an auxiliary function defined on the interior of the media is unnecessary. Cepkauskas demonstrated that SMiRT5 is a valid solution and can be used to determine the acoustics for a pipe with a steady-state driving frequency, time-dependent boundary conditions. It is also noted that this solution may have an advantage in that it can be applied to a loop configuration since the frequencies do not require a transcendental equation. Cepkauskas studied other previously published solutions involving a coolant annulus that were formulated by the use of an equivalent forcing function to

circumvent the complexities of a time-dependent mixed-value boundary condition problem. Cepkauskas found that due to physical arguments concerning the correct form of the body force, different solutions were obtained. Cepkauskas mathematical approach in deriving SMiRT5 will be explored in Section 3.3.2, where SMiRT5 is used as the basis and is termed as the Forced Vibration Acoustic CASE 2 in Chapter 3.

Both Penzes and Bowers treated the problem of pulsations in the coolant annulus by introducing a time-dependent boundary condition at the inlet of the pipe annulus interface. The correct form of this boundary was questioned by Bowers and Horvay, which resulted in an alternate solution. Two problems were identified from the other methodologies (see Sections 2.5.2 and 2.5.3), these are:

1) derivation of a theoretical solution that resolves discrepancies between varying physical arguments, and
2) analytical determination of the inlet pressure.

The Cepkauskas approach attempts to resolves these issues, by utilising a transformation technique to reduce each region of study to non-homogenous differential equations with homogenous boundary conditions. Standard normal mode superposition method is used to derive the transformed forced-vibration response of both regions. The inverse transformation solutions are derived and the complete description of the system is obtained by mathematically requiring pressure continuity at the inlet pipe interface. The time-dependent boundary condition concept is demonstrated to be analogous to the transformation used by Cepkauskas, both Penzes and Bowers did not recognise the restrictions necessary to make the boundary conditions homogenous [CE81].

2.5.5 Rationale for the approach selected

Lee *et al.* [LE92] stated that Lee and Chandra did not meet the boundary conditions with their chosen auxiliary functions and proceeded to formulate solutions using different auxiliary functions; therefore, their solution cannot satisfy the boundary conditions for the piston-spring supported end case. They further

stated that they neglected the constraints on the auxiliary functions to make the boundary conditions homogenous. Cepkauskas addressed the same simple pipe but coupled the solution with the reactor-core annulus solution, as mentioned earlier, using a transformation technique that changed the form of the problem to a non-homogenous differential equation with homogenous boundary conditions by utilising an auxiliary function. Lee *et al.* [LE92, LE93] and Cheong *et al.* [CH99] questioned Cepkauskas' methodology and stated:

"Lee and Chandra, Bowers and Horvay and Cepkauskas "missed the constraints on the auxiliary functions to make the boundary conditions homogeneous" [LE92, LE93].

Cepkauskus produced SMIRT5 where he demonstrated his solution to resolve any discrepancies regarding the proper acoustic models used in these scenarios. The work of Cepkauskas has thus been selected to re-emphasise the methodology employed and verify its applicability. In addition, the methodology is found to have an advantage in loop configuration due to the fact that the frequencies do not require a transcendental equation, this statement will be further tested in Chapter 3. In doing this, SMiRT5 is used as the basis and is applied in a simple acoustic vibration model that can be used for an HTGR application. The method of solving this type of problem is also found in the work of Fisher *et al.* [FI79] and is illustrated in several papers by Cepkauskas [CE79, CE81 and CE08].

CHAPTER 3

ACOUSTIC WAVE MATHEMATICAL FORMULATIONS

The approach to produce complex acoustic models, is first to understand the acoustic wave phenomena by applying simple models and then to build more complex models to quantify the results. The basic spring-mass model approach is used to construct the complex and sophisticated models that can represent a fluid system having a pressure force at one boundary. The mathematical formulation used is in accordance with the linearised Navier-Stokes equation and assumes helium as the fluid medium as compressible and inviscid.

The analytical model presented in this chapter consists essentially of a one dimensional wave-equation with time dependent non-homogenous boundary conditions. Different boundary conditions are selected depending on the conditions present in a system. SMiRT5 [CE79] is the closed-form solution for a problem that has been derived using a linear transformation technique that reduced the problem to one involving a non-homogenous differential equation with homogenous boundary conditions. This same approach is used to produce four acoustic forced-vibration solutions for integration in a pipe-loop model.

The SMiRT5 pressure distribution acoustic model will be referred to as Acoustic Forced Vibration CASE 2 due its specific boundary conditions and application. The same derivation process is applied to other pipes within the pipe-loop configuration with different boundary conditions. The formulation of these solutions plays a major part in this Research Report, the foundation for the derivation is physics based, and information presented in Chapter 2 will be used to construct the basic physical expression of the solutions. To complete the derivations, it will be predominately mathematical and in some cases, quite complex. All four acoustic forced-vibration solution cases, along with their derivations and verifications of accuracy will be presented. The solutions will be applied to a simple pipe-loop model to obtain the proper response and to test the

solutions. It will then be applied to a typical HTGR pipe-loop model to determine the applicability for those operating conditions.

3.1 Delineation of the acoustic simple pipe model

The source of pressure pulsations or time-dependent boundary occurs at pipe one or at the first entry of the system. It has been assumed that the inlet pipe of length L_1 can be represented as being straight and rigid with the wave propagation in the x-direction at the speed of sound in the fluid, c_o. Four different cases with corresponding boundary conditions are considered:

- **CASE 1**: Represents the entry in a pipe of length, L_1, with a pump at the left hand side and is specified to be open end at the right, this is considered a more general case of demonstrating the methodology and is not applied to the pipe-loop model.
- **CASE 2**: Represents the entry into a pipe with an arbitrary pressure gradient at the right.
- **CASE 3**: Represents the entry into a pipe with an arbitrary pressure gradient at the left with an open end at the right.
- **CASE 4**: Represents the entry into a pipe, also is the more general case, with a pressure gradient at both sides.

A basic illustration of the four pipes in the loop system with the applicable boundary conditions in represented in Fig. 3.1. These four models consist of the interior being described by a simple one-dimensional wave equation with the boundary conditions as indicated in Fig. 3.1. The applicable boundary conditions for each acoustic case can also be found in Table 3.1.

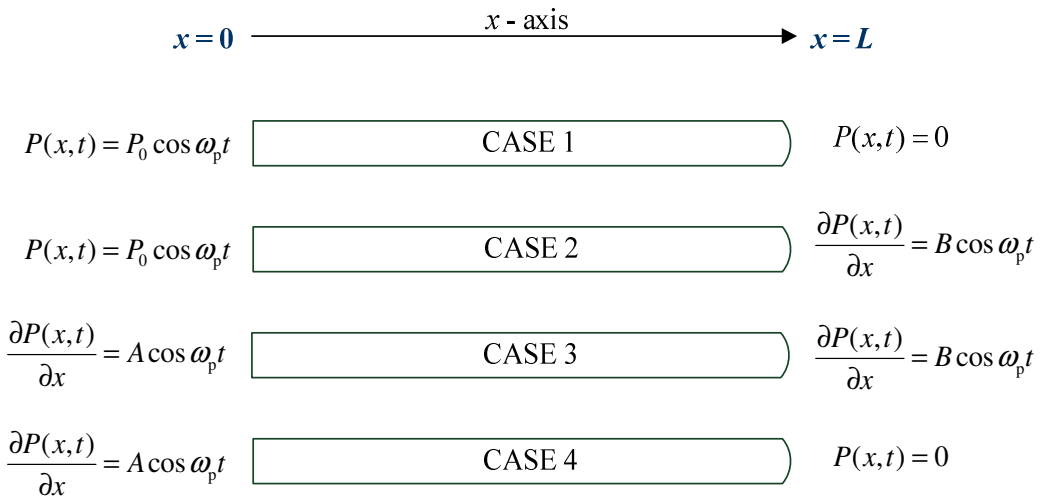


Figure 3.1: Four different acoustic cases with applicable boundary conditions.

3.1.1 Mathematical formulation for different acoustic solutions

The solution outline for the four models in order to develop the acoustic loading is dependent on applying mathematical models. These solutions are termed as forced-vibration acoustic solutions with different boundary conditions unique to the application. The solution outline for the four models consists of:

1) Utilising Eq. (2.18) as the basis of the acoustic formulations.

2) The boundary conditions selected with a choice of two of the following four:
   - At $x = 0$: $P(0,t) = f(t)$ and $\partial P(x,t)/\partial x\rangle_{x=0} = f(t)$.
   - At $x = L$: $P(L,t) = g(t)$ and $\partial P(x,t)/\partial x\rangle_{x=L} = g(t)$.

3) The transformation equation is assumed in the form of:

$$P(x,t) = Q(x,t) + h(x)f(t) + i(x)g(t) \tag{3.1}$$

4) The boundary conditions thus become:

- $P(0,t) = f(t) - [h(0)f(t) + i(0)g(t)]$
- $\left.\frac{\partial P(x,t)}{\partial x}\right\rangle_{x=0} = f(t) - \left\{\frac{\partial}{\partial x}[h(x)f(t) + i(x)g(t)]\right\}_{x=0}$
- $P(L,t) = g(t) - [h(L)f(t) + i(L)g(t)]$
- $\left.\frac{\partial P(x,t)}{\partial x}\right\rangle_{x=L} = g(t) - \left\{\frac{\partial}{\partial x}[h(x)f(t) + i(x)g(t)]\right\}_{x=L}$

5) Phase shift, ϕ, is assumed as zero.

6) The forcing frequency, ω_p, is calculated using Eq. (2.2).

3.1.1.1 Acoustic wave CASE 1 formulation

The acoustic wave CASE 1 formulation is based on a pipe model where the pipe has an open ended boundary. This is considered to be the general acoustic solution case. As mentioned in Section 2.4.1, the frequency of the vibration is constant and the displacement of the mass from its rest position is a sinusoidal function of time.

The corresponding boundary conditions for CASE 1, as presented in Fig. 3.1 are taken as:

- non-homogenous boundary conditions at x =0: $P(0,t) = P_0 \cos \omega_p t$

  This boundary represents a pump (time dependent function) as its acoustic excitation source. This was selected to represent a body force which produces a periodic frequency.
- homogenous boundary conditions at $x = L$: $P(L,t) = 0$.

  This boundary is open ended with no restrictions in the pressure at its exit. This condition is selected to determine the internal pipe flow conditions with a pipe open to ambient pressure.

For CASE 1, Eq. (2.18) is tranformed by adapting Eq. (3.1) to be suitable for this case in the form of:

$$P(x,t) = Q(x,t) + g(x)P_0 \cos \omega_p t. \tag{3.2}$$

The boundary conditions are transformed by applying Eq. (3.2)

- at $x = 0$:

  $P(0,t) = P_0 \cos \omega_p t,$

  $Q(0,t) = P_0 \cos \omega_p t - g(0) P_0 \cos \omega_p t.$

- at $x = L$:

  $P(L,t) = 0,$

  $Q(L,t) = -g(L) P_0 \cos \omega_p t = 0.$

To make the above set of equations homogenous, restrictions are imposed on the auxiliary functions. The transformed equations are placed in the form of a non-homogenous equation with homogenous boundary conditions by requiring $g(0) = 1$ and $g(L) = 0$ and applying to Eq. (3.2), then making $Q(0,t)$ and $Q(L,t) = 0$. The free vibration solution is used to guess the initial solution of $Q(x)$, in the form of:

$$Q_i(x) = C_i \sin\left(i\pi x/L\right), \tag{3.3}$$

and the forced vibration solution is assumed to be of the form:

$$Q(x,t) = \cos \omega_p t \sum_{i=1,..}^{\infty} C_i \sin\left(i\pi x/L\right). \tag{3.4}$$

Here, the angular natural frequency, ω_i of the wave is described with $\omega_i = i\pi c_0 / L$ (rad/s) and the mode shape by $\sin\left(i\pi x/L\right)$.

The wave Eq. (2.18) is transformed using Eq. (3.2) as follows:

$$\begin{aligned} &\frac{\partial^2}{\partial x^2}\left[Q(x,t) + P_0 g(x) \cos \omega_p t\right] \\ &-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\left[Q(x,t) + P_0 g(x) \cos \omega_p t\right] = 0. \end{aligned} \tag{3.5}$$

The forced vibration solution, Eq. (3.4) and Eq. (3.2), is incorporated into Eq. (3.5) and is differentiated as follows:

$$\frac{\partial^2}{\partial x^2}\left[\cos\omega_p t\sum_{i=1,..}^{\infty}C_i\sin\left(i\pi x/L\right)+P_0 g(x)\cos\omega_p t\right]$$
$$-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\left[\cos\omega_p t\sum_{i=1,..}^{\infty}C_i\sin\left(i\pi x/L\right)+P_0 g(x)\cos\omega_p t\right]=0,$$

$$P_0\cos\left(\omega_p t\right)g''(x)+g(x)\frac{P_0\omega_p^2}{c_0^2}\cos\omega_p t=-\frac{\omega_p^2}{c_0^2}\cos\omega_p t\sum_{i=1,2}^{\infty}C_i\sin\left(i\pi x/L\right)$$
$$+\cos\omega_p t\sum_{i=1,2}^{\infty}C_i\left(i\pi/L\right)^2\sin\left(i\pi x/L\right).$$

The factor of $\cos\omega_p t$ is eliminated and is multiplied by $\sin\left(j\pi x/L\right)$:

$$P_0 g''(x)\sin\left(j\pi x/L\right)+\frac{P_0\omega_p^2}{c_0^2}g(x)\sin\left(j\pi x/L\right)=$$
$$\sum_{i=1,2}^{\infty}C_i\left[\left(i\pi/L\right)^2-\frac{\omega_p^2}{c_0^2}\right]\sin\left(i\pi x/L\right)\sin\left(j\pi x/L\right).$$

The solution is integrated over the entire length of the pipe from $x = 0$ to $x = L$:

$$P_0\int_0^L g''(x)\sin\left(j\pi x/L\right)dx+\frac{P_0\omega_p^2}{c_0^2}\int_0^L g(x)\sin\left(j\pi x/L\right)dx=$$
$$\sum_{i=1,2}^{\infty}C_i\left[\left(i\pi/L\right)^2-\frac{\omega_p^2}{c_0^2}\right]\int_0^L\sin\left(i\pi x/L\right)\sin\left(j\pi x/L\right)dx. \qquad (3.6)$$

Equation (3.6) is too large to be integrated in one step, therefore Integral 1, I_1 is created and treated separately by applying integration by parts [KR67]:

$$I_1=P_0\int_0^L g''(x)\sin\left(j\pi x/L\right)dx=P_0\sin\left(j\pi x/L\right)g'(x)\Big|_0^L-P_0\left(j\pi x/L\right)\int_0^L g'(x)\cos\left(j\pi x/L\right)dx,$$
$$I_1=P_0\left[\sin\left(j\pi x/L\right)g'(x)\Big|_0^L-\left(j\pi/L\right)g(x)\cos\left(j\pi x/L\right)\Big|_0^L-\left(j\pi/L\right)^2\int_0^L g(x)\sin\left(j\pi x/L\right)dx\right].$$

Auxiliary function restrictions, $g(0) = 1$ and $g(L) = 0$, and $Q(L,t) = 0$ are applied on I_1. This further yields:

$$I_1 = P_0 \begin{Bmatrix} \left[g'(L)\sin\left(j\pi L/L\right) - g'(0)\sin\left(j\pi 0/L\right)\right] - \left(j\pi/L\right)\begin{bmatrix} g(L)\cos\left(j\pi L/L\right) \\ +g(0)\cos\left(j\pi 0/L\right)\end{bmatrix} \\ -\left(j\pi/L\right)^2 \int_0^L g(x)\sin\left(j\pi x/L\right)dx, \end{Bmatrix}$$

$$I_1 = P_0\left(j\pi/L\right) - P_0\left(j\pi/L\right)^2 \int_0^L g(x)\sin\left(j\pi x/L\right)dx.$$

It is to be noted that the orthagonality for the differential equation on the right-hand side of Eq. (3.6) is applied; thereby eliminating the summation sign from the solution [KR67], as displayed below:

$$\int_0^L \sin\left(i\pi x/L\right)\sin\left(j\pi x/L\right)dx = \begin{cases} 0 & if \quad i \neq j \\ L/2 & if \quad i = j \geq 1 \end{cases}.$$

The completed I_1 is incoporated back into Eq. (3.6) and the orthogonality is employed. The solution is further rearranged to obtain C_i as follows:

$$P_0\left[\left(j\pi/L\right) - \left(j\pi/L\right)^2 - \frac{\omega_p^2}{c_0^2}\right]\int_0^L g(x)\sin\left(j\pi x/L\right)dx = C_i\left[\left(i\pi/L\right)^2 - \frac{\omega_p^2}{c_0^2}\right]\frac{L}{2},$$

$$C_i = 2P_0\left\{\left[\left(j\pi/L\right) - \left(j\pi/L\right)^2 - \frac{\omega_p^2}{c_0^2}\right]\int_0^L g(x)\sin\left(j\pi x/L\right)dx \Big/ L\left[\left(i\pi/L\right)^2 - \frac{\omega_p^2}{c_0^2}\right]\right\}. \qquad (3.7)$$

Equation (3.7) is multiplied by c_0^2 and is simplified:

$$C_i = 2P_0\left[c_0^2 j\pi \Big/ L^2\left(\omega_i^2 - \omega_{\mathrm{p}}^2\right)\right] - \left(2P_0/L\right)\int_0^L g(x)\sin\left(j\pi x/L\right)dx.$$

The unknown constant, C_i, from the above equation is placed back into Eq. (3.4),

$$Q(x,t) = \cos\omega_{\mathrm{p}} t \sum_{j=1,2,}^{\infty} \left\{ \begin{array}{l} 2P_0 \left[c_0^2 \, j\pi / L^2 \left(\omega_i^2 - \omega_{\mathrm{p}}^2 \right) \right] \\ -\left(2P_0 / L \right) \int_0^L g(x) \sin\left(j\pi x / L \right) dx \end{array} \right\} \sin\left(i\pi x / L \right).$$

The solution of $Q(x,t)$ is then placed back into Eq. (3.2) to further yield:

$$\begin{aligned} P(x,t) = {} & \frac{2 j\pi P_0 c_0^2}{L^2 \left(\omega_i^2 - \omega_{\mathrm{p}}^2 \right)} \cos\omega_{\mathrm{p}} t \sum_{i=1,2}^{\infty} \sin\left(i\pi x / L \right) - \left(2P_0 / L \right) \cos\omega_{\mathrm{p}} t \\ & \sum_{i=1,..}^{\infty} \left\{ \int_0^L g(x) \sin\left(i\pi x / L \right) dx \right\} \sin\left(i\pi x / L \right) + P_0 g(x) \cos\omega_{\mathrm{p}} t. \end{aligned} \tag{3.8}$$

The second and third terms of Eq. (3.8) cancel since the second term is the negative Fourier series of the third term. This is proved by:

$$\frac{2P_0}{L} \cos\omega_{\mathrm{p}} t \sum_{i=1,2..}^{\infty} \left\{ \int_0^L g(x) \sin\left(i\pi x / L \right) dx \right\} \sin\left(i\pi x / L \right) = P_0 g(x) \cos\omega_{\mathrm{p}} t. \tag{3.9}$$

The term $P_0 \cos\omega_p t$ is cancelled from Eq. (3.9) and is multiplied by $\sin\left(j\pi x / L \right)$ and is integrated over the entire length of the pipe from $x = 0$ to $x = L$ as follows:

$$\sum_{\mathrm{i}=1,2,3}^{\infty} \frac{2}{L} \left[\int_0^L g(x) \sin\left(\frac{j\pi x}{L} \right) dx \right] \sin\left(\frac{i\pi x}{L} \right) \sin\left(\frac{j\pi x}{L} \right) = \sum_{\mathrm{i}=1,2,3}^{\infty} g(x) \sin\left(\frac{j\pi x}{L} \right)$$

$$\frac{2}{L} \sum_{\mathrm{i}=1,2,3}^{\infty} \left[\int_0^L g(x) \sin\left(\frac{j\pi x}{L} \right) dx \right] \int_0^L \sin\left(\frac{i\pi x}{L} \right) \sin\left(\frac{j\pi x}{L} \right) = \sum_{\mathrm{i}=1,2,3}^{\infty} \int_0^L g(x) \sin\left(\frac{j\pi x}{L} \right) dx$$

$$\frac{2}{L} \sum_{\mathrm{i}=1,2,3}^{\infty} \left[\int_0^L g(x) \sin\left(\frac{j\pi x}{L} \right) dx \right] \frac{L}{2} = \sum_{\mathrm{i}=1,2,3}^{\infty} \int_0^L g(x) \sin\left(\frac{j\pi x}{L} \right) dx$$

$$1 = 1.$$

Equation (3.8) is then reduced and arranged to formulate the final solution of the forced vibration acoustic solution of CASE 1:

$$P(x,t) = \sum_{j=1,2,3}^{\infty} \frac{2P_0 c_0 \omega_i}{L\left(\omega_i^2 - \omega_p^2\right)} \sin\left(j\pi x/L\right)\cos\omega_p t. \tag{3.10}$$

The initial conditions represented by Eq. (3.10) are:

- The natural frequency, ω_i, of the fluid medium is represented by $\omega_i = j\pi c_0 / L$ (measured in radians/sec).
- The mode shape is characterised by $\sin\left(j\pi x / L\right)$.

3.1.1.2 Acoustic wave CASE 2 formulation

The CASE 2 approach is aimed at producing a homogenous one dimensional acoustic wave-equation with non-homogenous time-dependent boundary conditions. The details of this case can be found in references [CE08, CE80] and will not be reproduced in detail in this section. Only the important aspects of the solution will be provided. The applicable boundary conditions are illustrated in Fig. 3.1. It is also strongly emphasised that the same mathematical methodology as used for CASE 1 is applied to CASE 2 to determine $P(x,t)$.

For CASE 2, Eq. (2.18) is tranformed by adapting Eq. (3.1) to be suitable for this case in the form of:

$$P(x,t) = Q(x,t) + P_0 f_1(x,t)\cos\omega_p t + f_2(x,t)\cos\omega_p t. \tag{3.11}$$

Restrictions are imposed on the auxiliary functions, $f_1(x)$ and $f_2(x)$, in order to make the above set of equations homogenous. These restrictions are:

$$f_1(0) = 1$$
$$\left.\frac{\partial f_1(x)}{\partial x}\right|_{x=L} = 0.$$

The description of CASE 2 becomes a non-homogenous differential equation, with boundary conditions becoming homogenous by requiring the restrictions given above. This is applied to the auxiliary functions f_1 and f_2. Following the same mathematical procedure as for CASE 1, this will result in the final forced vibration acoustic solution for CASE 2:

$$P(x,t) = -\sum_{j=1,3,5}^{\infty} \frac{2c_0\left(P_0\omega_j + Bc_0 \sin\left(j\pi/2\right)\right)}{L\left(\omega_j^2 - \omega_{\mathrm{p}}^2\right)} \sin\left(j\pi x/2L\right)\cos\omega_{\mathrm{p}}t. \tag{3.12}$$

The following conditions are represented by Eq. (3.12):

- The natural frequency, ω_1, of the fluid medium is represented by $\omega_1 = j\pi c_0 / 2L$ (measured in radians/sec).
- The mode shape of the wave is described by: $Q(x) = \sin\left(j\pi x/2L\right)$.

3.1.1.3 Acoustic wave CASE 3 formulation

Acoustic wave CASE 3 is commonly applicable for pipes that are not located adjacent to a time-dependent boundary condition. This case is applied, for example to interconnecting pipes. The boundary conditions contain two constants that can represent some resistance and amplitude at that boundary. The same mathematical methodology used in CASES 1 and 2 are applied to this case. The formulation basis of the solution uses Eq. (2.18). The boundary conditions applicable are illustrated in Fig. 3.1. For CASE 3, Eq. (2.18) is transformed by adapting Eq. (3.1) and thus is assumed in the form of:

$$P(x,t) = Q(x,t) + Ag(x)\cos\omega_{\mathrm{p}}t + Bh(x)\cos\omega_{\mathrm{p}}t. \tag{3.13}$$

The corresponding boundary conditions for CASE 3, as presented in Fig. 3.1 are taken as:

- non-homogenous boundary conditions at x =0: $\dfrac{\partial P(x,t)}{\partial x} = A\cos\omega_{\mathrm{p}}t.$

This boundary represents a resistance at its' interface where is connected with another pipe, with no time-dependent forcing function present. It is reasonable to assume the pressure gradient at this interface is an unknown constant times the same time dependency from the pressure exiting its connecting pipe.

- homogenous boundary conditions at $x = L$: $\dfrac{\partial P(x,t)}{\partial x} = B\cos\omega_{\mathrm{p}}t.$

  This boundary represents a resistance at its' interface where is connected with another pipe, with no time-dependent forcing function present. It is reasonable to assume the pressure gradient at this interface is an unknown constant times the same time dependency from the pressure exiting its connecting pipe.

The boundary conditions specified above are incorporated into Eq. (3.13) as such:

- at $x = 0$:

$$\left.\frac{\partial P}{\partial x}\right|_{x=0} = A\cos\omega_{\mathrm{p}}t - \Psi\left[Ag(x)\cos\omega_{\mathrm{p}}t + B\frac{\partial}{\partial x}h(x)\cos\omega_{\mathrm{p}}t\right]_{x=0},$$

- at $x = L$:

$$\left.\frac{\partial Q}{\partial x}\right|_{x=0} = B\cos\omega_{\mathrm{p}}t - \Psi\left[A\frac{\partial}{\partial x}g(x)\cos\omega_{\mathrm{p}}t + B\frac{\partial}{\partial x}h(x)\cos\omega_{\mathrm{p}}t\right]_{x=L},$$

where the operator is given by $\Psi \equiv \dfrac{\partial^2}{\partial x^2} - \dfrac{1}{c_0^2}\dfrac{\partial^2}{\partial t^2}$. The auxiliary functions for CASE 3 are defined as:

- at $x = 0$: $dg(x)/dx\big|_{x=0} = 1$ and $dh(x)/dx\big|_{x=0} = 0$.
- at $x = L$: $dg(x)/dx_{x=L} = 0$ and $dh(x)/dx_{x=L} = 1$.

The free vibration solution used to guess the initial solution is in the form of:

$$Q(x) = C\cos\left(\omega/c_0\right)x + D\sin\left(\omega/c_0\right). \tag{3.14}$$

This is differentiated to $Q(x)/dx = -(\omega/c)C\sin(\omega/c)x + (\omega/c)D\cos(\omega/c)x$ and the following restrictions are imposed on the auxiliary functions to make the solution homogenous:

- at $x = 0,\ \ Q(x)/dx = -(\omega/c)C\sin(\omega/c)(0) + (\omega/c)D\cos(\omega/c)(0) = 0$
- at $x = L,\ Q(x) = C\cos(i\pi c_0/L)$.

The forced vibration solution, from Eq. (3.4), is assumed to be of the form:

$$Q(x,t) = \sum_{j=1,2,3}^{\infty} \cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t. \tag{3.15}$$

The wave Eq. (2.18) is combined with Eq. (3.13) and transformed accordingly:

$$\frac{\partial^2}{\partial x^2}\left[Q(x,t) + Ag(x)\cos\omega_{\mathrm{p}}t + Bh(x)\right] - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\left[\begin{pmatrix} Q(x,t) + Ag(x)\cos\omega_{\mathrm{p}}t \\ +Bh(x)\cos\omega_{\mathrm{p}}t \end{pmatrix}\right] = 0.$$

The forced vibration solution, Eq. (3.15) is incorporated into the solution above and is differentiated as follows:

$$\frac{\partial^2}{\partial x^2}\left[\sum_{j=1,3,5}^{\infty} C_j\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t + Ag(x)\cos\omega_{\mathrm{p}}t + Bh(x)\cos\omega_{\mathrm{p}}t\right]$$
$$-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\left[\left(\sum_{j=1,3,5}^{\infty} C_j\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t + Ag(x)\cos\omega_{\mathrm{p}}t + Bh(x)\cos\omega_{\mathrm{p}}t\right)\right] = 0,$$

$$\frac{\partial^2}{\partial x^2}\sum_{j=1,3,5}^{\infty} C_j\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\sum_{j=1,3,5}^{\infty} C_j\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t =$$
$$+\frac{A}{c_0^2}\frac{\partial^2}{\partial t^2}g(x)\cos\omega_{\mathrm{p}}t + \frac{B}{c_0^2}\frac{\partial^2}{\partial t^2}h(x)\cos\omega_{\mathrm{p}}t - A\frac{\partial^2}{\partial x^2}g(x)\cos\omega_{\mathrm{p}}t - B\frac{\partial^2}{\partial x^2}h(x)\cos\omega_{\mathrm{p}}t,$$

$$-\left(j\pi/L\right)^2 \cos\omega_p t \sum_{j=1,3,5}^{\infty} C_j g(x)\cos\left(j\pi x/L\right) + \frac{\omega_p^2}{c_0^2}\cos\omega_p t \sum_{j=1,3,5}^{\infty} C_j \cos\left(j\pi x/L\right) =$$

$$-A\cos\omega_p t\; g''(x) - B\cos\omega_p t\; h''(x) - A\frac{\omega_p^2}{c_0^2} g(x)\cos\omega_p t - B\frac{\omega_p^2}{c_0^2} h(x)\cos\omega_p t.$$

The factor of $\cos\omega_p t$ is eliminated (it is constant on both sides of the solution), is multiplied by $\cos\left(j\pi x/L\right)$ and integrated over the entire length of the pipe from $x = 0$ to $x = L$:

$$\sum_{j=1,3,5}^{\infty} C_j \left[\frac{\omega_p^2}{c_0^2} - \left(j\pi/L\right)^2\right] \int_0^L \cos\left(j\pi x/L\right)\cos\left(j\pi x/L\right)dx = -A\int_0^L g''(x)\cos\left(j\pi x/L\right)dx$$

$$-B\int_0^L h''(x)\cos\left(j\pi x/L\right)dx - \frac{\omega_p^2}{c_0^2} A\int_0^L g(x)\cos\left(j\pi x/L\right)dx - \frac{\omega_p^2}{c_0^2} B\int_0^L h(x)\cos\left(j\pi x/L\right)dx. \qquad (3.16)$$

Two integrals, I_1 and I_2 are formed in order to perform the integration:

$$I_1 = A\int_0^L g''(x)\cos\left(j\pi x/L\right)dx, \text{ and } I_2 = B\int_0^L h''(x)\cos\left(j\pi x/L\right)dx.$$

The solutions of the integrals are obtained and the auxiliary function restrictions are applied:

- at $x = 0$, $h(0) = 0$ and $g(0) = 1$,
- at $x = L$, $h(L) = 1$ and $g(L) = 0$.

This reduces the integrals solutions to:

$$I_1 = -1 - \left(j\pi/L\right)^2 \int_0^L g(x)\cos\left(j\pi x/L\right)dx.$$

$$I_2 = \cos j\pi - \left(j\pi/L\right)^2 \int_0^L h(x)\cos\left(j\pi x/L\right)dx.$$

The completed I_1 and I_2 are incoporated back into Eq. (3.16) and the orthagonality for the differential equation on the right hand side of equation is applied, this

eliminates the summation sign from the solution. The solution is further rearranged to obtain, C_j as follows:

$$-\sum_{i=1,3,5}^{\infty} C_j \left[\left(j\pi/L \right)^2 - \frac{\omega_{\mathrm{p}}^2}{c_0^2} \right] \frac{L}{2} = A + A\left(j\pi/L \right)^2 \int_0^L g(x) \cos\left(j\pi x/L \right) dx - B\cos j\pi$$

$$+B\left(j\pi/L \right)^2 \int_0^L h(x) \cos\left(j\pi x/L \right) dx - \frac{\omega_{\mathrm{p}}^2}{c_0^2} A \int_0^L g(x) \cos\left(j\pi x/L \right) dx$$

$$-\frac{\omega_{\mathrm{p}}^2}{c_0^2} B \int_0^L h(x) \cos\left(j\pi x/L \right) dx,$$

$$-C_j \left[\left(j\pi/L \right)^2 - \frac{\omega_{\mathrm{p}}^2}{c_0^2} \right] \frac{L}{2} = A \left[\left(j\pi/L \right)^2 - \frac{\omega_{\mathrm{p}}^2}{c_0^2} \right] \int_0^L g(x) \cos\left(j\pi x/L \right) dx$$

$$+B \left[\left(j\pi/L \right)^2 - \frac{\omega_{\mathrm{p}}^2}{c_0^2} \right] \int_0^L h(x) \cos\left(j\pi x/L \right) dx + A - B\cos j\pi,$$

$$C_j = -\frac{2A}{L} \int_0^L g(x) \cos\left(j\pi x/L \right) dx - \frac{2B}{L} \int_0^L h(x) \cos\left(j\pi x/L \right) dx - \left(\frac{2\left(A - B\cos j\pi \right)}{L\left[\left(j\pi/L \right)^2 - \frac{\omega_{\mathrm{p}}^2}{c_0^2} \right]} \right). \tag{3.17}$$

Equation (3.17) is multiplied by c_0^2 / c_0^2 and is simplified to:

$$C_j = -\frac{2A}{L} \int_0^L g(x) \cos\left(j\pi x/L \right) dx - \frac{2B}{L} \int_0^L h(x) \cos\left(j\pi x/L \right) dx - \left(\frac{2c_0^2\left(A - B\cos j\pi \right)}{L\left[\omega_j^2 - \omega_{\mathrm{p}}^2 \right]} \right).$$

The unknown constant, C_{j}, from the above equation is placed back into Eq. (3.15) as such,

$$Q(x,t) = \sum_{i=1,3,5}^{\infty} \left[\begin{array}{l} -\frac{2A}{L} \int_0^L g(x) \cos\left(j\pi x/L \right) dx - \frac{2B}{L} \int_0^L h(x) \cos\left(j\pi x/L \right) dx \\ -\left(\frac{2c_0^2\left(A - B\cos j\pi \right)}{L\left[\omega_i^2 - \omega_{\mathrm{p}}^2 \right]} \right) \end{array} \right] \cos\left(i\pi x/L \right) \cos \omega_{\mathrm{p}} t.$$

The solution of $Q(x,t)$ is placed back into Eq. (3.13) to further yield:

$$P(x,t)=\cos\omega_{\mathrm{p}}t\sum_{i=0,1,2}^{\infty}\left[\begin{array}{l}-\dfrac{2A}{L}\displaystyle\int_0^L g(x)\cos\left(j\pi x/L\right)dx-\int_0^L h(x)\cos\left(j\pi x/L\right)dx\\ -\left(\dfrac{2c_0^2\left(A-B\cos j\pi\right)}{L\left[\omega_j^2-\omega_{\mathrm{p}}^2\right]}\right)\end{array}\right]\cos\left(j\pi x/L\right)$$

$$+g(x)A\cos\omega_{\mathrm{p}}t+h(x)\cos\omega_{\mathrm{p}}t.$$

Thus,

$$\begin{aligned}P(x,t)&=\sum_{i=0,1,2}^{\infty}\left(\frac{2c_0^2\left(A-B\cos j\pi\right)}{L\left[\omega_{\mathrm{p}}^2-\omega_i^2\right]}\right)\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t\\&-\frac{2A}{L}\sum_{j=0,1,2}^{\infty}\left[\int_0^L g(x)\cos\left(j\pi x/L\right)dx\right]\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t+Ag(x)\cos\omega_{\mathrm{p}}t\\&+\frac{2B}{L}\sum_{j=0,1,2}^{\infty}\int_0^L h(x)\cos\left(j\pi x/L\right)dx\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t+Bh(x)\cos\omega_{\mathrm{p}}t.\end{aligned}\tag{3.18}$$

The second and third terms of Eq. (3.18) cancel since the second term is the negative Fourier series of the third term; the same will apply to the third and fourth terms. Refer to CASE 1 and CASE 4 for the justification. This reduces $P(x,t)$ to:

$$P(x,t)=\sum_{j=0,1,2}^{\infty}\left(\frac{2c_0^2\left(A-B\cos j\pi\right)}{L\left[\omega_{\mathrm{p}}^2-\omega_i^2\right]}\right)\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t.\tag{3.19}$$

Equation (3.19) is then arranged to formulate the final solution, Eq. (3.20) for the forced vibration acoustic solution of CASE 3.

$$\begin{aligned}P(x,t)&=A\sum_{j=0,1,2}^{\infty}\left(\frac{2c_0^2}{L\left[\omega_{\mathrm{p}}^2-\omega_j^2\right]}\right)\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t\\&-B\sum_{j=0,1,2}^{\infty}\frac{2c_0^2}{L\left[\omega_{\mathrm{p}}^2-\omega_j^2\right]}\cos j\pi\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t.\end{aligned}\tag{3.20}$$

The following conditions are represented by Eq. (3.20):

- The natural frequency, ω_1, of the fluid medium is represented by $\omega_1 = j\pi c_0 / L$ (measured in radians/sec).
- The mode shape of the wave is described by: $\cos(j\pi x / L)$.

3.1.1.4 Acoustic wave CASE 4 formulation

Acoustic wave CASE 4 has been derived to represent a pipe that is entering a system such as a pump or vessel. The left boundary condition for this case contains a constant that is representative of a resistance or amplitude at that boundary and the right boundary condition is open ended. An example of this is a pipe that enters the compressor/pump. The boundary conditions applicable are further illustrated in Fig. 3.1. For CASE 4, the wave Eq. (2.18) is tranformed by adapting Eq. (3.1) to be suitable in the form of:

$$P(x,t) = Q(x,t) + Ag(x)\cos\omega_p t. \tag{3.21}$$

Boundary conditions for CASE 4 are:

- at $x = 0$, $\dfrac{\partial P(x,t)}{\partial x} = A\cos\omega_p t.$

  This boundary represents a resistance at its' interface where is connected with another pipe, with no time-dependent forcing function present. It is reasonable to assume the pressure gradient at this interface is an unknown constant times the same time dependency from the pressure exiting its connecting pipe.

- at $x=L$, $P(x,t) = 0.$

  This boundary is open ended with no restrictions in the pressure at its exit. This condition is selected to determine the internal pipe flow conditions with a pipe exiting into a vessel or opening.

The auxiliary functions defined for this case are:

- at $x = 0$: $dg(x)/dx\big|_{x=0} = 1$ and $dh(x)/dx\big|_{x=0} = 0$

- at $x=L$: $dg(x)/dx_{x=L} = 0$ and $dh(x)/dx_{x=L} = 1$

Restrictions are imposed on the auxiliary functions and applied to the free-vibration solution, Eq. (3.14), and by applying the boundary conditions gives:

- **boundary condition where $x = 0$:**

  $\partial Q / dx|_{x=0}^{L} = (\omega_i / c_0)[-A\sin(\omega_i / c_0)x + B\cos(\omega_i / c_0)x] = 0$,

  where constant B equates to 0, reducing the equation to:

  $Q(x) = A\cos(\omega_i / c_0)x$

- **boundary condition where $x = L$:**

  $Q(x) = A\cos(\omega_i / c_0)L = 0$,

  but $A \neq 0$, therefore the solution becomes:

  $\cos(\omega_i / c_0)L = 0 = j\pi/2$, where j =1, 3, 5.

The formulated free-vibration solution is incorporated for the non-homogeneous solution and the forced-vibration for CASE 4 is assumed to be in the form of:

$$Q(x,t) = \sum_{j=1,3,5}^{\infty} C_j \cos\left(j\pi x/2L\right)\cos\omega_{\mathrm{p}}t, \tag{3.22}$$

where,

- The natural frequency, ω_{i}, of the fluid medium is represented by $\omega_j = j\pi c_0/2L$. (measured in radians/sec).
- The mode shape is described by $Q(x) = A\cos\left(j\pi x/2L\right)$.

The wave Eq. (2.18) is transformed by Eq. (3.21) as follows:

$$\frac{\partial^2}{\partial x^2}\left[Q(x,t) + Ag(x)\cos\omega_{\mathrm{p}}t\right] - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\left[Q(x,t) + Ag(x)\cos\omega_{\mathrm{p}}t\right] = 0. \tag{3.23}$$

Equation (3.22) is incorporated into Eq. (3.23), rearranged and differentiated, yielding:

$$\frac{\partial^2}{\partial x^2}\left[\sum_{j=1,2,3}^{\infty}\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t + Ag(x)\cos\omega_{\mathrm{p}}t\right]$$
$$-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\left[\sum_{j=1,2,3}^{\infty}\cos\left(j\pi x/L\right)\cos\omega_{\mathrm{p}}t + Ag(x)\cos\omega_{\mathrm{p}}t\right] = 0,$$

which further yields,

$$-\sum_{i=1,3..}^{\infty} C_{\mathrm{i}}\cos\omega_{\mathrm{p}}t\left(\frac{j\pi}{2L}\right)^2\cos\left[j\pi x/2L\right] + Ag''(x)\cos\omega_{\mathrm{p}}t$$
$$+\frac{1}{c_0^2}\sum_{i=1,3..}^{\infty} C_{\mathrm{i}}\omega_{\mathrm{p}}^2\cos\omega_{\mathrm{p}}t\cos\left[j\pi x/2L\right] + \frac{A\omega_{\mathrm{p}}^2}{c_0^2}g(x)\cos\omega_{\mathrm{p}}t.$$

The factor of $\cos\omega_p t$ can be eliminated and the solution rearranged:

$$Ag''(x) + \frac{A\omega_{\mathrm{p}}^2}{c_0^2}g(x) = \sum_{j=1,2,3}^{\infty} C_j\left(\frac{j\pi}{2L}\right)^2\cos\left[j\pi x/2L\right] + \frac{\omega_{\mathrm{p}}^2}{c_0^2}\sum_{j=1,2,3}^{\infty} C_j\cos\left[j\pi x/2L\right].$$

The solution is multiplied by $\cos\left(j\pi x/2L\right)$ and integrated over the entire length from $x = 0$ to $x = L$, producing:

$$A\int_0^L g''(x)\cos\left(j\pi x/2L\right)dx + \frac{A\omega_{\mathrm{p}}^2}{c_0^2}\int_0^L g(x)\cos\left(j\pi x/2L\right)dx =$$
$$\sum_{j=1,3..}^{\infty} C_j\left[\left(\frac{j\pi}{2L}\right)^2 - \frac{\omega_{\mathrm{p}}^2}{c_0^2}\right]\int_0^L\cos\left[j\pi x/2L\right]\cos\left(j\pi x/2L\right)dx. \qquad (3.24)$$

Integral 1 (I_1) is extracted from Eq. (3.24) and treated separately by applying the integration by parts technique [KR67], where I_1 is:

$$I_1 = A\int_0^L g''(x)\cos\left(j\pi x/2L\right)dx,$$

$$I_1 = A\begin{bmatrix} g'(x)\cos\left(j\pi x/2L\right)\big|_0^L + \left(j\pi/2L\right)g'(x)\sin\left(j\pi x/2L\right)\big|_0^L \\ -\left(j\pi/2L\right)^2\int_0^L g(x)\cos\left(j\pi x/2L\right)dx. \end{bmatrix}.$$

Auxiliary function restrictions, $g\,(0) = 1$ and $g\,(L) = 0$, are applied on I_1, yielding:

$$I_1 = A\left\{\left[g'(L)\cos\left(j\pi L/2L\right) - g'(0)\cos\left(j\pi 0/2L\right)\right] + \left(j\pi/2L\right)\begin{bmatrix} g'(L)\sin\left(j\pi L/2L\right) \\ -g'(0)\sin\left(j\pi 0/2L\right)\end{bmatrix}\right\}$$

$$-\left(j\pi/2L\right)^2\int_0^L g(x)\cos\left(j\pi x/2L\right)dx,$$

after further simplification gives, $I_1 = A - A\left(j\pi/2L\right)^2\int_0^L g(x)\cos\left(j\pi x/2L\right)dx.$

The orthagonality for the integration of equation $\int_0^L \cos\left[j\pi x/2L\right]\cos\left(j\pi x/2L\right)dx$ is applied [KR67], where: $\int_0^L \cos\left(i\pi x/L\right)\cos\left(j\pi x/L\right)dx = \begin{cases} 0 & if \quad i\neq j \\ L/2 & if \quad i=j\geq 1 \end{cases}$.

Integral I_1 is incorporated into Eq. (3.24)and rearranged to produce:

$$-A - A\left[\left(j\pi/2L\right)^2 - \frac{\omega_p^2}{c_0^2}\right]\int_0^L g(x)\cos\left(j\pi x/2L\right)dx = C_j\left[\left(\frac{j\pi}{2L}\right)^2 - \frac{\omega_p^2}{c_0^2}\right]\frac{L}{2},$$

$$C_j = \frac{-2A}{L\left[\left(\frac{j\pi}{2L}\right)^2 - \frac{\omega_p^2}{c_0^2}\right]} - \frac{2A\int_0^L g(x)\cos\left(j\pi x/2L\right)dx}{L}. \qquad (3.25)$$

The unknown constant, C_i from Eq. (3.25) is placed back into Eq. (3.22) and is substituted into Eq. (3.21):

$$Q(x,t)=\sum_{j=1,3,5.}^{\infty}\left\{\frac{-2A}{L\left[\left(\frac{j\pi}{2L}\right)^2-\frac{\omega_p^2}{c_0^2}\right]}-\frac{2A\int_0^L g(x)\cos\left(j\pi x/2L\right)dx}{L}\right\}\cos\left(j\pi x/2L\right)\cos\omega_p t,$$

$$P(x,t)=\sum_{i=1,..}^{\infty}\left\{\begin{array}{l}\dfrac{-2A}{L\left[\left(\dfrac{i\pi}{2L}\right)^2-\dfrac{\omega_p^2}{c_0^2}\right]}\cos\left(i\pi x/2L\right)\cos\omega_p t\\ -\dfrac{2A}{L}\cos\omega_p t\int_0^L g(x)\cos\left(i\pi x/2L\right)\cos\left(i\pi x/2L\right)dx\end{array}\right\}+Ag(x)\cos\omega_p t. \quad (3.26)$$

The second and third terms of Eq. (3.26) cancel since the second term is the negative Fourier series of the third term. This is proved by:

$$\sum_{i=1,3}^{\infty}\frac{2A}{L}\cos\omega_p t\int_0^L g(x)\cos\left(i\pi x/2L\right)\cos\left(i\pi x/2L\right)dx=\sum_{i=1,3}^{\infty}Ag(x)\cos\omega_p t.$$

The term $P_0\cos\omega_p t$ is cancelled from the above solution, multiplied by $\cos\left(j\pi x/2L\right)$ and integrated over the entire length of the pipe from $x = 0$ to $x = L$, yielding:

$$\sum_{i=1,3..}^{\infty}\frac{2A}{L}\int_0^L g(x)\cos\left(i\pi x/2L\right)dx\int_0^L\cos\left(i\pi x/2L\right)\cos\left(j\pi x/2L\right)dx=\sum_{i=1,3..}^{\infty}Ag(x)\cos\left(j\pi x/2L\right),$$

$$\sum_{i=1,3..}^{\infty}\frac{2A}{L}\left(\frac{L}{2}\right)\int_0^L g(x)\cos\left(j\pi x/2L\right)dx=\sum_{i=1,3..}^{\infty}A\int_0^L g(x)\cos\left(j\pi x/2L\right)dx,$$

$$1 \quad = \quad 1.$$

Eq. (3.26) is reduced and rearranged to formulate the final forced vibration acoustic solution for CASE 4:

$$P(x,t)=\sum_{j=1,3,5}^{\infty}\frac{2Ac_0^2}{L\left[\omega_j^2-\omega_p^2\right]}\cos\left(j\pi x/2L\right)\cos\omega_p t. \quad (3.27)$$

The following conditions are represented by Eq. (3.27):

- The natural frequency, ω_1, of the fluid medium is represented by $\omega_1 = j\pi c_0 / 2L$ (measured in radians/sec).
- The mode shape of the wave is described by: $\cos\left(j\pi x/2L \right)$.

A summary of CASES 1 to 4 can be found in Table 3.1.

Table 3.1: Summary of forced vibration responses and Boundary conditions

Case	Forced Response	Left Boundary Condition @ $x = 0$	Right Boundary Condition @ $x = L$	Natural Frequencies	Mode Shapes	Series "j" [1]
1	$P(x,t) = \sum_{j=1,2,3}^{\infty} \frac{2P_0 c_0 \omega_i}{L\left(\omega_i^2 - \omega_p^2\right)} \sin\left(j\pi x/L\right)\cos\omega_p t.$	$P(x,t) = P_0 \cos\omega_p t$	$P(x,t) = 0$	$\frac{j\pi c_0}{L}$	$\sin\left(\frac{j\pi x}{L}\right)$	1,2,3...
2	$P(x,t) = -\sum_{j=1,3,5}^{\infty} \frac{2c_0\left(P_0\omega_j + Bc_0 \sin\left(j\pi/2\right)\right)}{L\left(\omega_j^2 - \omega_p^2\right)} \sin\left(j\pi x/2L\right)\cos\omega_p t.$	$P(x,t) = P_0 \cos\omega_p t$	$\frac{\partial P(x,t)}{\partial x} = B\cos\omega_p t$	$\frac{j\pi c_0}{2L}$	$\sin\left(\frac{j\pi x}{2L}\right)$	1,3,5…
3	$P(x,t) = A\sum_{j=0,1,2}^{\infty}\left(\frac{2c_0^2}{L\left[\omega_p^2 - \omega_j^2\right]}\right)\cos\left(j\pi x/L\right)\cos\omega_p t - B\sum_{j=0,1,2}^{\infty}\frac{2c_0^2}{L\left[\omega_p^2 - \omega_j^2\right]}\cos j\pi\cos\left(j\pi x/L\right)\cos\omega_p t.$	$\frac{\partial P(x,t)}{\partial x} = A\cos\omega_p t$	$\frac{\partial P(x,t)}{\partial x} = B\cos\omega_p t$	$\frac{j\pi c_0}{L}$	$\cos\left(\frac{j\pi x}{L}\right)$	0,1,2,3..
4	$P(x,t) = \sum_{j=1,3,5}^{\infty}\frac{2Ac_0^2}{L\left[\omega_j^2 - \omega_p^2\right]}\cos\left(j\pi x/2L\right)\cos\omega_p t.$	$\frac{\partial P(x,t)}{\partial x} = A\cos\omega_p t$	$P(x,t) = 0$	$\frac{j\pi c_0}{2L}$	$\cos\left(\frac{j\pi x}{2L}\right)$	1,3,5…

3.2 Mathematical evaluation of the acoustic wave solutions

The acoustic solutions are tested to ensure that they are mathematically correct and can be applied to ensure pressure continuity at the interfaces. Each solution for the different cases is applied in a series of matrix operators to determine the constants. These values are to be determined in order to solve the pressure distribution continuously from interface 1 to the end (entry back to interface 1). Specific Jolley series are selected and applied for the prediction of the forced response within the pipes and to demonstrate the continuity of the pressure gradient at the interfaces.

3.2.1 Jolley series equivalent to the forced response solutions

The Jolley series [JO05] is investigated to determine if it can be used to simplify the complexity of the solutions produced by reducing them to simple equations. Specific Jolley series that is applicable for each case is selected and applied to a arbitrary case study to test for the accuracy of the acoustic solutions produced and the suitability of the Jolley series to represent the acoustic solutions. The four solutions and the corresponding Jolley series are plotted and compared by using arbitrary values for P_o, ω_p, c_o. The correlations between the results of these two equations are observed to determine the accuracy of the Jolley series in representing the acoustic solutions. The corresponding Jolley series found for each forced vibration solution determined for CASES 1, 2, 3 and 4 are as follows:

- **CASE 1 forced vibration response solution corresponds to Jolley series # 560 [JO05]:**

$$\sum_{j=1,2,3}^{\infty} \frac{2P_0 c_0 \omega_i}{L\left(\omega_i^2 - \omega_p^2\right)} \sin\left(j\pi x/L\right)\cos\omega_p t = P_0 \frac{\sin\alpha\left(1-x^*\right)}{\sin\alpha}, \tag{3.28}$$

where $x^* = x/L$ (the incremental distance of *x* over the total pipe length of *L*) and $\alpha = L\omega_p/c_0$. For comparison purposes both these functions are plotted in Fig. 3.2.

- **CASE 2 forced vibration response solution corresponds to Jolley series # 556 and 558 [JO05]:**

$$-\sum_{j=1,3,5}^{\infty} \frac{2c_0\left(P_0\omega_j + Bc_0\sin\left(j\pi/2\right)\right)}{L\left(\omega_j^2 - \omega_p^2\right)}\sin\left(j\pi x/2L\right)\cos\omega_p t = \frac{P_0\cos[\alpha(1-x^*)] + (\frac{Bc_0}{\omega_p})\sin(\alpha x^*)}{\cos\alpha}, \tag{3.29}$$

where $x^* = x/L$ (the incremental distance of *x* over the total pipe length of *L*) and $\alpha = L\omega_p/c_0$. For comparison purposes the first term, $P_0\cos\alpha\left(1-x^*\right)/\cos\alpha$ is plotted in the upper part of Fig. 3.3 and the second term, $Bc_0\sin\alpha(x^*)/\omega_p\cos\alpha$ in the lower part of Fig. 3.3 for both these functions since the second term of the forced response solution of CASE 2 is multiplied by the constant *B*.

- **CASE 3 forced vibration response solution corresponds to Jolley series # 559 and 558 [JO05]:**

$$P(x,t) = A\sum_{j=0,1,2}^{\infty}\left(\frac{2c_0^2}{L\left[\omega_p^2 - \omega_j^2\right]}\right)\cos\left(j\pi x/L\right)\cos\omega_p t - B\sum_{j=0,1,2}^{\infty}\frac{2c_0^2}{L\left[\omega_p^2 - \omega_j^2\right]}\cos j\pi\cos\left(j\pi x/L\right)\cos\omega_p t = A\frac{c_0}{\omega_p}\left\{\frac{\cos[\alpha(1-x^*)]}{\sin\alpha}\right\} - B\frac{c_0}{\omega_p}\left\{\frac{\cos[\alpha(x^*)]}{\sin\alpha}\right\}, \tag{3.30}$$

where $x^* = x/L$ (the incremental distance of *x* over the total pipe length of *L*) and $\alpha = L\omega_p / c_0$. For comparison purposes both these functions are plotted in Fig. 3.4.

- **CASE 4 forced vibration response solution corresponds to Jolley series # 555 [JO05]:**

$$\sum_{j=1,3,5}^{\infty} \frac{2Ac_0^2}{L\left[\omega_j^2 - \omega_p^2\right]} \cos\left(j\pi x/2L\right) \cos\omega_p t = -\frac{Ac_0 \sin\alpha\left(1-x^*\right)}{\omega_p \cos\alpha} \ , \qquad (3.31)$$

where $x^* = x/L$ (the incremental distance of *x* over the total pipe length of *L*) and $\alpha = L\omega_p / c_0$. For comparison purposes both these functions are plotted in Fig. 3.5.

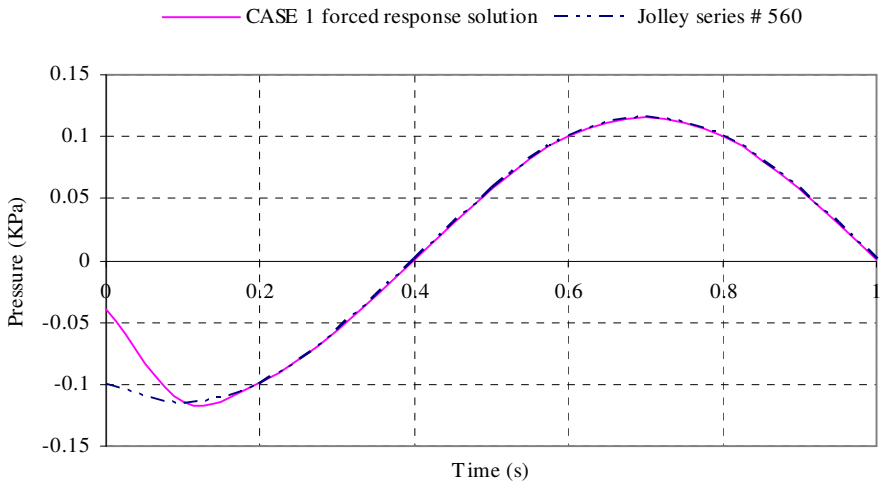


Figure 3.2: Acoustic forced response CASE 1 and corresponding Jolley series # 560.

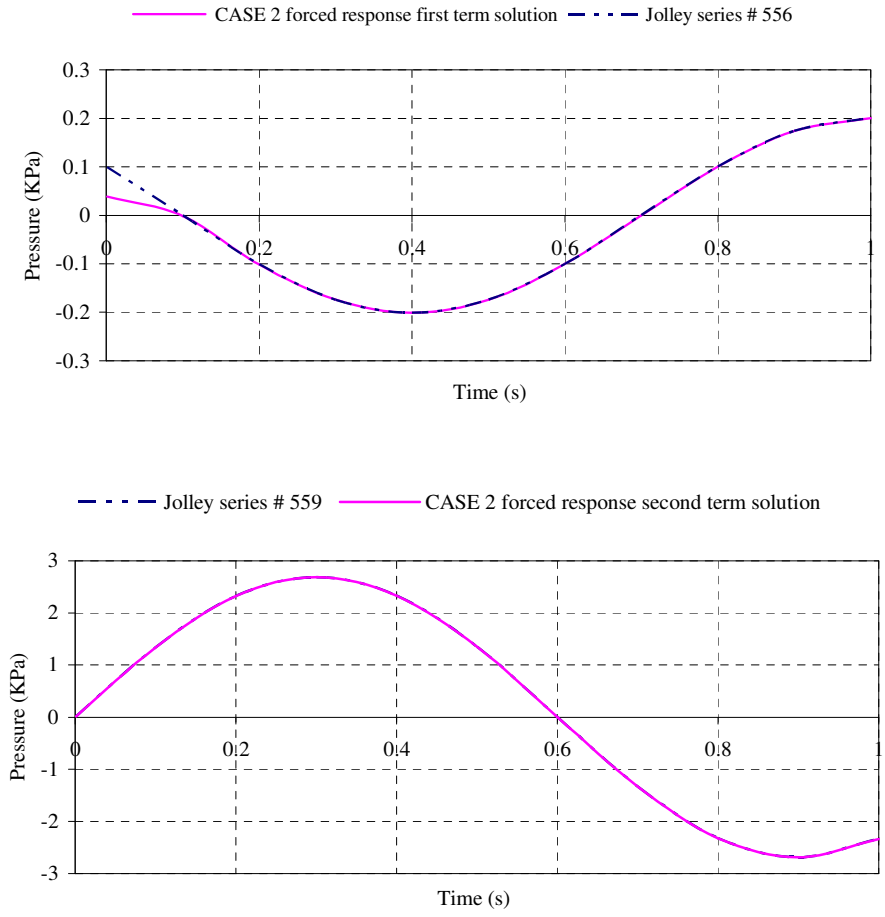


Figure 3.3: Acoustic wave CASE 2 forced response solution and the corresponding Jolley series # 556 & 559 for the first term (upper part) and the second term (lower part).

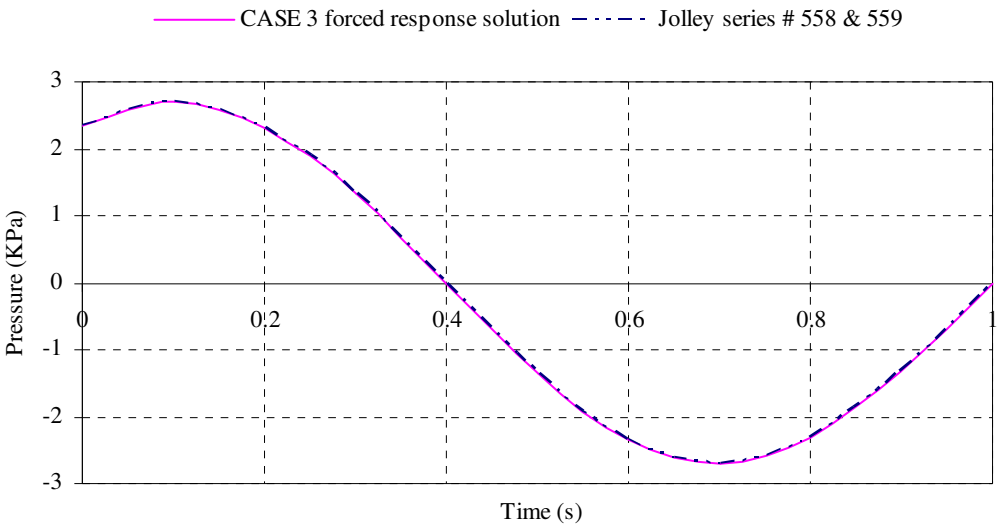


Figure 3.4: Acoustic wave CASE 3 forced response solution and the corresponding Jolley series # 558 & 559.

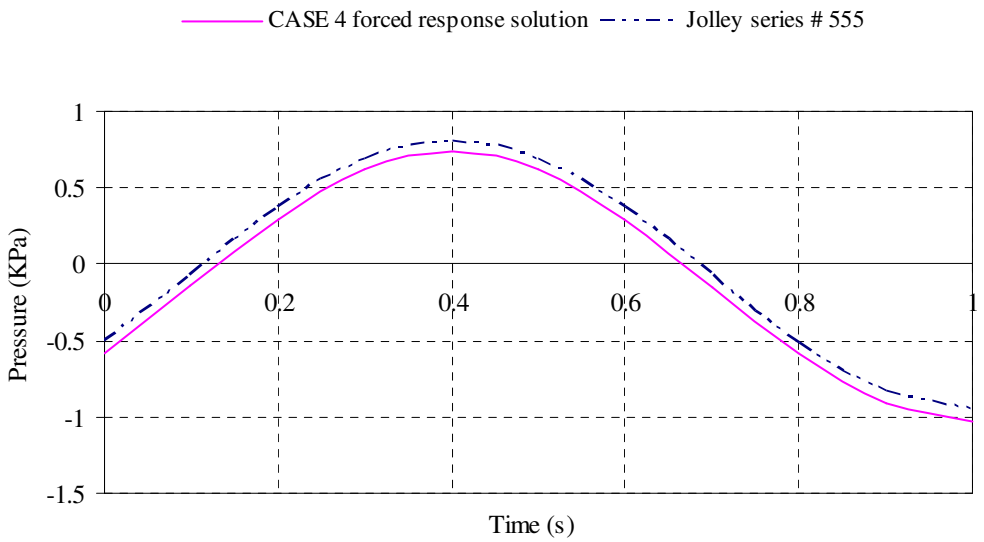


Figure 3.5: Acoustic wave CASE 4 forced response solution and the corresponding Jolley series # 555.

As seen in all the cases (Fig 3.2 to 3.5) the acoustic wave solutions and the corresponding Jolley series are in excellent agreement with each other. This provides confidence that the Jolley series selected is a good representation of its respective force vibration acoustic case. The Jolley series can thus be used as a simpler equation for further applications.

3.2.2 Satisfying the boundary conditions of the differential equations

The boundary conditions of the differential equations have to be satisfied to ensure that the assumptions made for those conditions are justified. MathCad was selected to perform a solution check. The Jolley series is substituted into the wave equation, Eq. (2.18) to ensure that it is homogenous. In all cases, it tends to zero proving that the wave equation is satisfied [KR67]. The solutions are then solved by taking the first derivatives of the solution for each case with respect to x and the second derivatives with respect to time and x. This results in:

- **CASE 1 at $x = 0$ and $x = L$**

  at $x = 0$

  $$P_0\left[\sin\alpha\left(1-0/L\right)\cos\left(\omega_p t\right)\right]/\sin\alpha = P_0\cos\left(\omega_p t\right)$$

  at $x = L$

  $$P_0\left[\sin\alpha\left(1-L/L\right)\cos\left(\omega_p t\right)\right]/\sin\alpha = 0$$

- **CASE 2 at x = 0 and $x = L$**

  at $x = 0$

  $$\left\{P_0\cos\left[\alpha\left(1-0/L\right)\cos\left(\omega_p t\right)\right]+\left(Ac/\omega_p\right)\sin A(0/L)\cos\left(\omega_p t\right)\right\}/\cos\alpha =$$

  $$P_0\cos\omega_p t$$

  at $x = L$

  $$\frac{\left\{A\cos\left(L\omega_p/c_0\right)\cos\left(\omega_p t\right)-\left(\frac{P_0\omega_p}{c_0}\sin\left[\left(L(L/L-1)\omega_p\right)/c_0\right]\cos\left(\omega_p t\right)\right)\right\}}{\cos\left(L\omega_p/c_0\right)} =$$

  $$A\cos\omega_p t$$

- **CASE 3 at $x = 0$ and $x = L$:**

  at $x = 0$

  $$\left\{A\sin\left[L\left(0/L-1\right)\omega_{\mathrm{p}}/c_0\right]\cos\left(\omega_{\mathrm{p}}t\right)-B\sin\left(\omega_{\mathrm{p}}/c_0\right)\cos\left(\omega_{\mathrm{p}}t\right)\right\}/\cos\left(L\omega_{\mathrm{p}}/c_0\right).$$

  $$= A\cos\omega_p t$$

  at $x = L$

  $$\left\{A\sin\left(L((L/L)\omega_{\mathrm{p}}/c\right)\cos\left(\omega_{\mathrm{p}}t\right)-\left(B\sin\left[L\omega_p/c\right]\cos\left(\omega_{\mathrm{p}}t\right)\right)\right\}\Big/\sin\left(L\omega_{\mathrm{p}}/c\right)$$

  $$\rightarrow B\cos\omega_p t$$

- **CASE 4 pump at $x = 0$ and $x = L$:**

  at $x = 0$

  $$Ac_0\sin\left[L\left(0/L-1\right)\omega_{\mathrm{p}}/c_0\right]\cos\left(\omega_{\mathrm{p}}t\right)/\cos\left(L\omega_{\mathrm{p}}/c_0\right) = A\cos\omega_{\mathrm{p}}t$$

  at $x = L$

  $$Ac_0\sin\left[L(L/L-1)\omega_{\mathrm{p}}/c_0\right]\cos\left(\omega_{\mathrm{p}}t\right)\omega_{\mathrm{p}}\Big/\cos\left(L\omega_{\mathrm{p}}/c_0\right) = 0$$

In all the acoustic solution cases, when the differentiation is performed, the results indicate that the assumptions made on the initial selection of the boundary conditions are met. This proves that the formulated acoustic-wave solution differential equations are indeed satisfied and further reaffirms that the solutions are mathematically correct and are valid for further application.

3.2.3 Methodology for determining the constants

The conditions at the interfaces of the pipes in the loop are important in determining the constants of the acoustic forced vibration solutions. It is assumed that pressure gradient is the same at each interface, as such:

$$\begin{aligned} P_1(x = L) &= P_2(x = 0) \\ P_2(x = L) &= P_3(x = 0) \\ P_3(x = L) &= P_4(x = 0). \end{aligned} \tag{3.32}$$

The first step is to apply this pressure continuity assumption at the pipe-pipe interfaces. Figure 3.6 displays the location of these interfaces. This results in the

constants existing at the pipe-pipe interfaces being equal (due to the continuity of the pressure gradients). Thus, only three unknown constants are to be determined. These bounding conditions are applied to each case applicable for the four pipes at the interfaces, producing a linear set of three equations with constants of the form:

$$a_{11}B_1 + a_{12}B_2 + a_{13}B_3 = b_1$$
$$a_{21}B_1 + a_{22}B_2 + a_{23}B_3 = b_2$$
$$a_{311}B_1 + a_{32}B_2 + a_{33}B_3 = b_1,$$

where a_n, B_n and b_n are the parameters that form the matrix. Using the selected Jolley series for CASES 1 to 4, the applicable case for each pipe is utilised. This creates unknown constants in each equation that exist at the boundaries of the pipes. The *a*–values in the matrix can be determined with known values of α, c_0 and x^* of the Jolley series and assuming the pressure gradients are the same at the interfaces. The a_n values are applied in a matrix and solved utilising Kramer's Rule [KR67] for determining the values of the constants. Two computer codes, Matlab and MathCad, were used to model this approach. Two independent codes are used as a benchmarking exercise to ensure the methodology employed is consistent and accurate and verify that the results are correct.

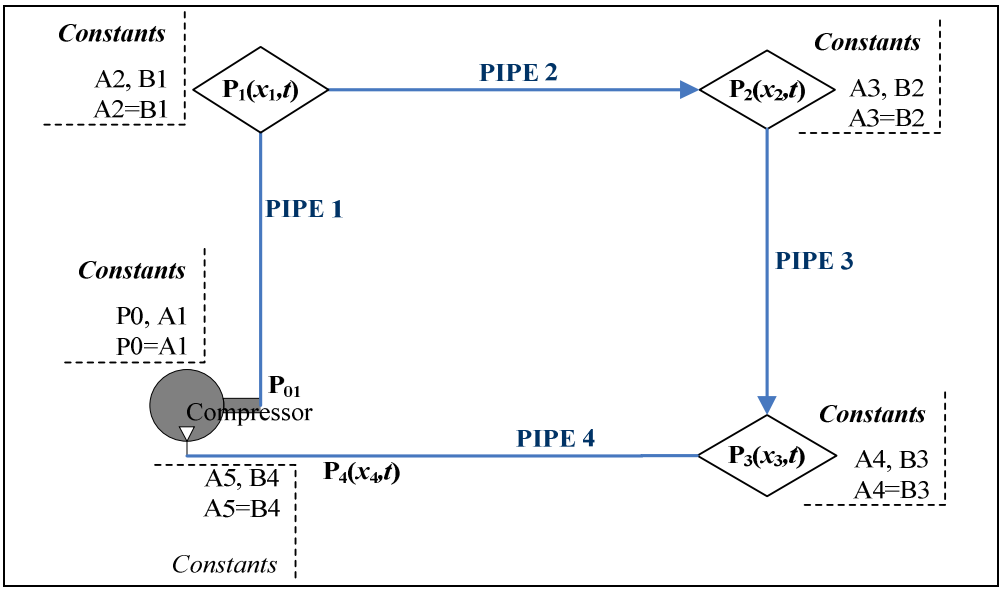


Figure 3.6: Location of constants existing at the pipe-pipe interfaces.

3.2.4 Pressure gradient at the interfaces

To ensure pressure continuity at the interfaces, the gradients of pressure distribution at the interfaces need to be determined. The first order derivative of the acoustic wave solutions is taken and applied to the acoustic cases applicable for the pipe-loop model. The following set of equations with the unknown constants, B_1, B_2 and B_3 are produced:

- **Pipe 1:** $\left(\left\{\alpha\left[P_0 \sin\alpha\left(1-x^*\right)\right]+\left(B_1 c_0/\omega_p\right)\cos\alpha x^*\right\}/\cos\alpha\right)/L$
- **Pipe 2:** $\left\{\begin{array}{l}\left[B_1(c_0/\omega_p)[\sin(\alpha(1-x^*))/\sin\alpha]\right]\\ +B_2\left[(c_0/\omega_p)[\sin(\alpha x^*)/\sin\alpha]\right]\end{array}\right\}/L$
- **Pipe 3:** $\left\{\begin{array}{l}\left[B_2(c_0/\omega_p)[\sin(\alpha(1-x^*))/\sin\alpha]\right]+\\ B_3\left[(c_0/\omega_p)[\sin(\alpha x^*)/\sin\alpha]\right]\end{array}\right\}/L$
- **Pipe 4:** $(\alpha/L)\left[B_3(c_0/\omega_p)\cos\alpha\left(1-x^*\right)\right]/\cos\alpha)$.

If the pressure is continuous from pipe to pipe and no significant variations in pressure exist, it can be concluded that the interface conditions (boundary conditions) is consistent from pipe to pipe and no numerical problems are present

with the mathematical modelling used. This will be tested by applying this methodology to a general pipe-loop model, as done in the Section 3.3.

3.3 Application for verification of the acoustic solutions

An important objective for selecting Cepkauskas methodology is the possibility that the acoustic forced vibration solutions can be coupled in a loop configuration. To prove this possibility two case studies are looked at. The first study is a general pipe-loop system, with a compressor as the source of the forced pulsations and the fluid medium being helium gas. This case will test the suitability of the solutions to be coupled. The second case study emulates a typical HTGR environment. This case study is selected to test the suitability of the solutions for a known HTGR scenario.

3.3.1 Application to general pipe loop system

A general case study is used to test the methodology employed from Sections 3.1 and 3.2. Arbitrary pressures and temperatures that are averaged over the pipe length were selected for the four pipes in the general pipe-loop model in Fig 3.6.

The inputs used were:

P_0 = 0.01 kPa

f = 100 Hz (taken as general frequency of a pump/compressor)

T = 273.15 K.

Pipes 1 to 4 temperatures are taken as averaged over the pipe length which is displayed in Table 3.2. Using the temperatures, the respective values of c_0 are calculated using Eq. (2.16).

Using Eq. (2.2), the angular forcing frequency is calculated to be:

ω_p = 628.32 radians s^{-1}.

Table 3.2: Input values for the pipes in the general pipe-loop system.

Pipe Number	Pipe Length (m)	Temperature (°C)
1	10	100
2	15	200
3	20	300
4	25	400

The applicable acoustics solutions for each pipe are determined as:

- **Pipe #1** is represented by forced-vibration acoustic **CASE 2**, and incorporating the applicable conditions in Eq. (3.29), results in the pressure distribution being:

$$P_1(x_1^*,t)=\frac{P_0\cos\left[\alpha\left(1-x_1^*\right)\right]+\dfrac{B_1c_{0,1}}{\omega_p}\sin\left(\alpha x_1^*\right)}{\cos\alpha_1}.$$

- **Pipe #2 and Pipe #3** can be described with forced-vibration acoustic **CASE 3**, Eq. (3.30) is used and the pressure distribution for both pipes becomes:

$$P_2(x_2^*,t)=A_2\frac{c_{0,2}}{\omega_p}\left\{\frac{\cos\left[\alpha_2\left(1-x_2^*\right)\right]}{\sin\alpha_2}\right\}-\frac{B_2c_{0,2}}{\omega_p}\left\{\frac{\cos\left[\alpha_2\left(x_2^*\right)\right]}{\sin\alpha_2}\right\}.$$

$$P_3(x_3^*,t)=A_3\frac{c_{0,3}}{\omega_p}\left\{\frac{\cos\left[\alpha_3\left(1-x_3^*\right)\right]}{\sin\alpha_3}\right\}-\frac{B_3c_{0,3}}{\omega_p}\left\{\frac{\cos\left[\alpha_3\left(x_3^*\right)\right]}{\sin\alpha_3}\right\}.$$

- **Pipe #4** is represented by forced vibration acoustic **CASE 4**, Eq. (3.31) is used and the pressure distribution becomes:

$$P_4\left(x_4^*,t\right)=-\frac{A_4c_{0,4}}{\omega_p}\frac{\sin\left[\alpha_4\left(1-x_4^*\right)\right]}{\cos\alpha_4}.$$

From Section 3.2.3, the constants at the interfaces are determined by programming this methodology to produce an acoustic subroutine. It is to be noted that $B_1 = A_2$, $B_2 = A_3$ and $B_3 = A_1$, due to the continuity of the pressure gradients at the pipe-pipe interfaces. These constants are determined by adapting Eq. (3.32) to produce:

$$P_1(x_1^* = 1, t) = P_2(x_2^* = 0_1, t)$$
$$P_2(x_1^* = 1, t) = P_3(x_2^* = 0_1, t)$$
$$P_3(x_1^* = 1, t) = P_4(x_2^* = 0_1, t).$$

The constants determined are given in Table 3.3. A reverse calculation was performed by inserting these constants back into the original equations of the matrix to determine if the matrix conditions are met. It was found that the matrix equations equated zero, with the exception of matrix indicator b_1 of $a_0P_0 = 0.0137$, which should be the result. This indicates that the matrix conditions are indeed satisfied and the process selected to determine the constants are accurate for this application. To ensure that the pressure gradient is continuous at the interfaces, the first order derivation to produce the pressure gradient of the acoustic wave solution for each pipe, as explained in Section 3.2.4. This is calculated from the acoustic subroutine and plotted, as illustrated in Fig. 3.7.

The entire acoustic pressure distribution in the pipe-loop is calculated using the acoustic sub routine and is displayed in Fig. 3.7. The individual pipe pressure distribution for each of the four pipes can be found in Fig. 3.8. The code for general acoustic subroutine and results can be found in Appendix B1-1.

Table 3.3: Values of the constants determined for general pipe-loop system.

Constants	Matlab	MsExcel
B1	-0.00046	-0.00042
B2	-0.00651	-0.00645
B3	0.00438	0.00434

Figure 3. 7: Continuous acoustic pressure distribution and pressure gradient for general pipe-loop system

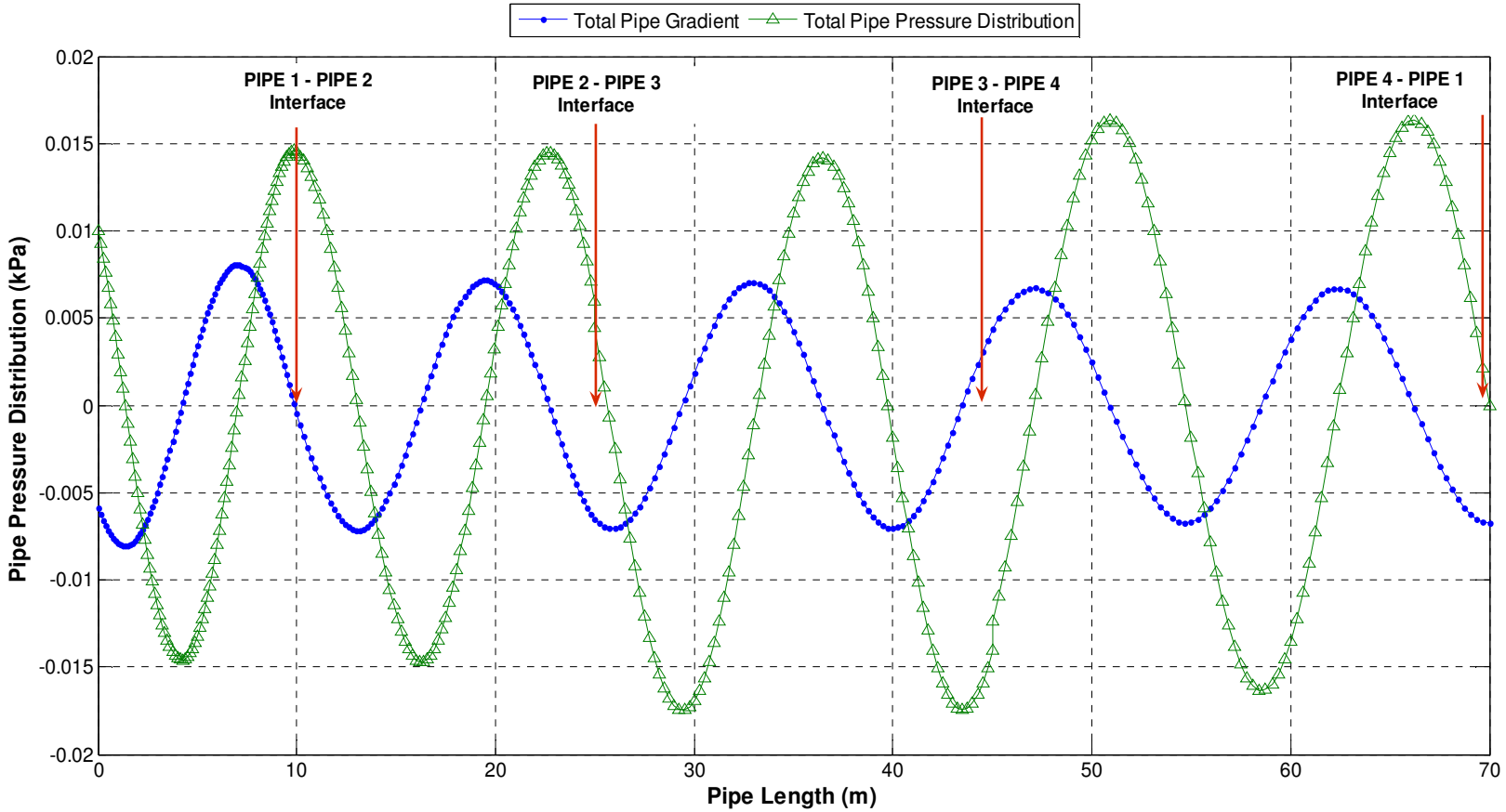
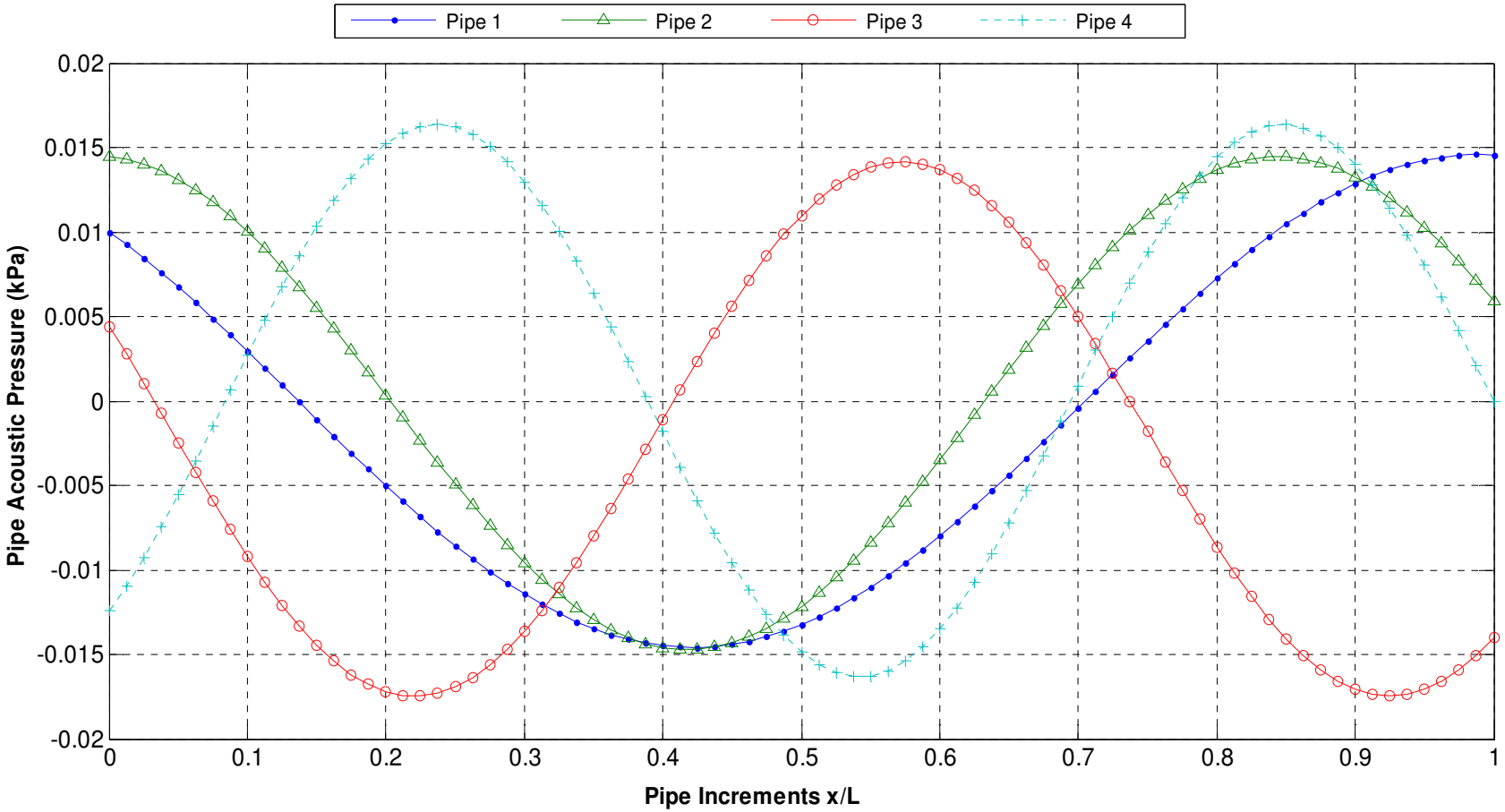


Figure 3. 8: General acoustic pressure distribution for individual pipes in the general pipe system



3.3.1.1 Simulation of HTGR case study

To determine whether the acoustic solutions can be applied to a real system, a HTGR design is selected. The HTGR contains helium as the working fluid and consists of turbomachinery for extracting energy from the fluid and maintaining the necessary flow in the loop. The acoustic solutions cannot at this stage be applied to the flow in the complex geometries of the turbomachinery and heat exchangers due to the geometrical difference in these components. Although this can be achieved, the present study is only aimed at demonstrating generally that these acoustic equations can be applied to an HTGR pipe network for testing the pipe-loop configuration compatibility.

3.3.1.2 HTGR plant model configuration

A simplified model of and HTGR was formulated in a computation fluid dynamic code Flownex® [FL06]. The Flownex model consists of a simplified reactor and primary loop comprising of the main turbomachinery components, such as a compressor to emulate the time dependent boundary, turbine, and heat exchangers to produce the temperature variations in the pipe-loop. This model can be found in Appendix A1-1, Fig. A.1. The Flownex HTGR model is used to simulate the steady state conditions such as temperatures, pressures and pipe lengths at each pipe interface. The pressure and temperature results of the model are represented in Appendix A1-1, Fig. A.2.

3.3.1.3 Inputs for simplified HTGR pipe loop model

The Flowex HTGR model is further reduced to a simplified four pipe-loop model by averaging the pipe lengths and temperatures. This simplifies the application of the acoustics solution and restricts it only to four pipes, as done in Section 3.3.1. This simplified pipe-loop is represented in Fig 3.9 and the averaged values of pipe length and temperatures are given in Table 3.4. An acoustics HTGR subroutine was produced, similar to the general pipe-loop acoustic subroutine. These values are used in this code and run to observe the pressure distributions in the primary loop.

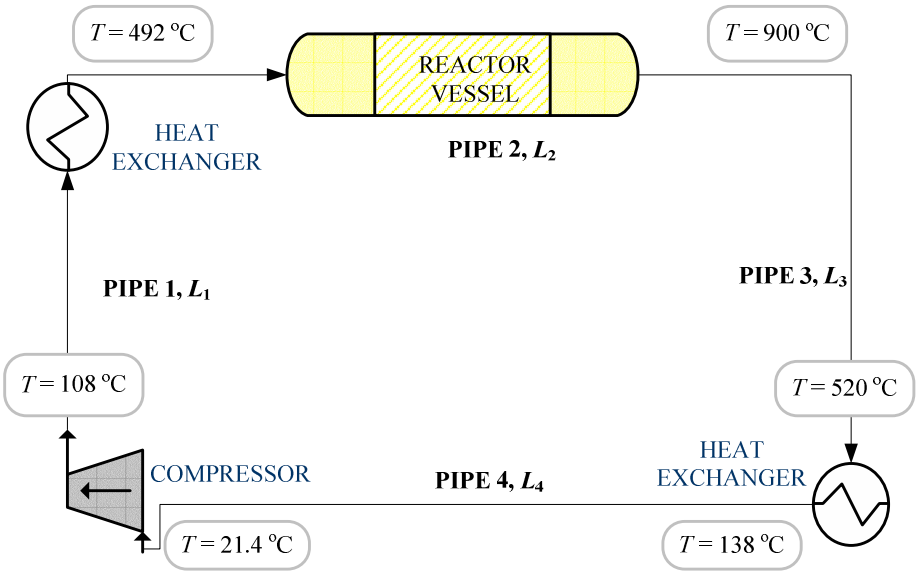


Figure 3.9: Conditions applicable to the HTGR simplified pipe-loop.

Table 3.4: Input values of the HTGR simplified pipe model.

Pipe No	Pipe Length (m)	Average Temperature (°C)
1	6.74	108
2	33.4	(492 + 900) / 2 = 696
3	25	(900 + 520) / 2 = 710
4	37	(138 + 21.4) / 2 = 79.7

As per the general pipe-loop study the acoustic solution cases applied to the four pipes are:

- **Pipe #1** is represented by forced-vibration acoustic **CASE 2**, and incorporating the applicable conditions in Eq. (3.29), results in the pressure distribution being**:**

$$P_1(x_1^*,t)=\frac{P_0\cos\left[\alpha\left(1-x_1^*\right)\right]+\dfrac{B_1 c_{0,1}}{\omega_p}\sin\left(\alpha x_1^*\right)}{\cos\alpha_1}.$$

- **Pipe #2 and Pipe #3** can be described with forced-vibration acoustic **CASE 3**, Eq. (3.30) is used and the pressure distribution for both pipes becomes:

$$P_2(x_2^*,t) = A_2 \frac{c_{0,2}}{\omega_p}\left\{\frac{\cos\left[\alpha_2\left(1-x_2^*\right)\right]}{\sin\alpha_2}\right\} - \frac{B_2 c_{0,2}}{\omega_p}\left\{\frac{\cos\left[\alpha_2\left(x_2^*\right)\right]}{\sin\alpha_2}\right\},$$

$$P_3(x_3^*,t) = A_3 \frac{c_{0,3}}{\omega_p}\left\{\frac{\cos\left[\alpha_3\left(1-x_3^*\right)\right]}{\sin\alpha_3}\right\} - \frac{B_3 c_{0,3}}{\omega_p}\left\{\frac{\cos\left[\alpha_3\left(x_3^*\right)\right]}{\sin\alpha_3}\right\}.$$

- **Pipe #4** is represented by forced vibration acoustic **CASE 4**, Eq. (3.31) is used and the pressure distribution becomes:

$$P_4\left(x_4^*,t\right) = -\frac{A_4 c_{0,4}}{\omega_p}\frac{\sin\left[\alpha_4\left(1-x_4^*\right)\right]}{\cos\alpha_4}.$$

The subroutine and the results can be seen in Appendix B1-2.

Figure 3. 10: Simplified HTGR individual pipe-loop acoustic pressure distribution.

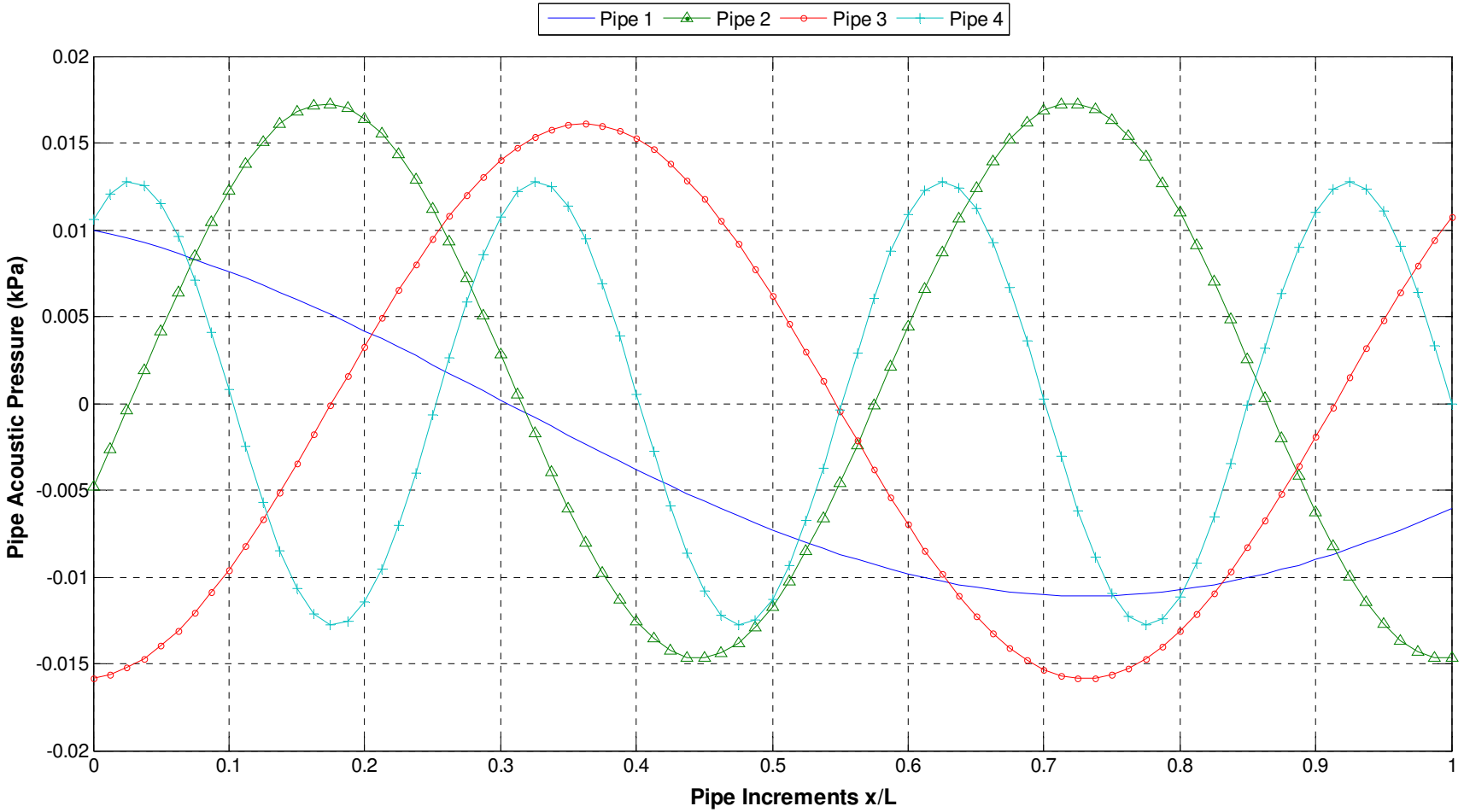
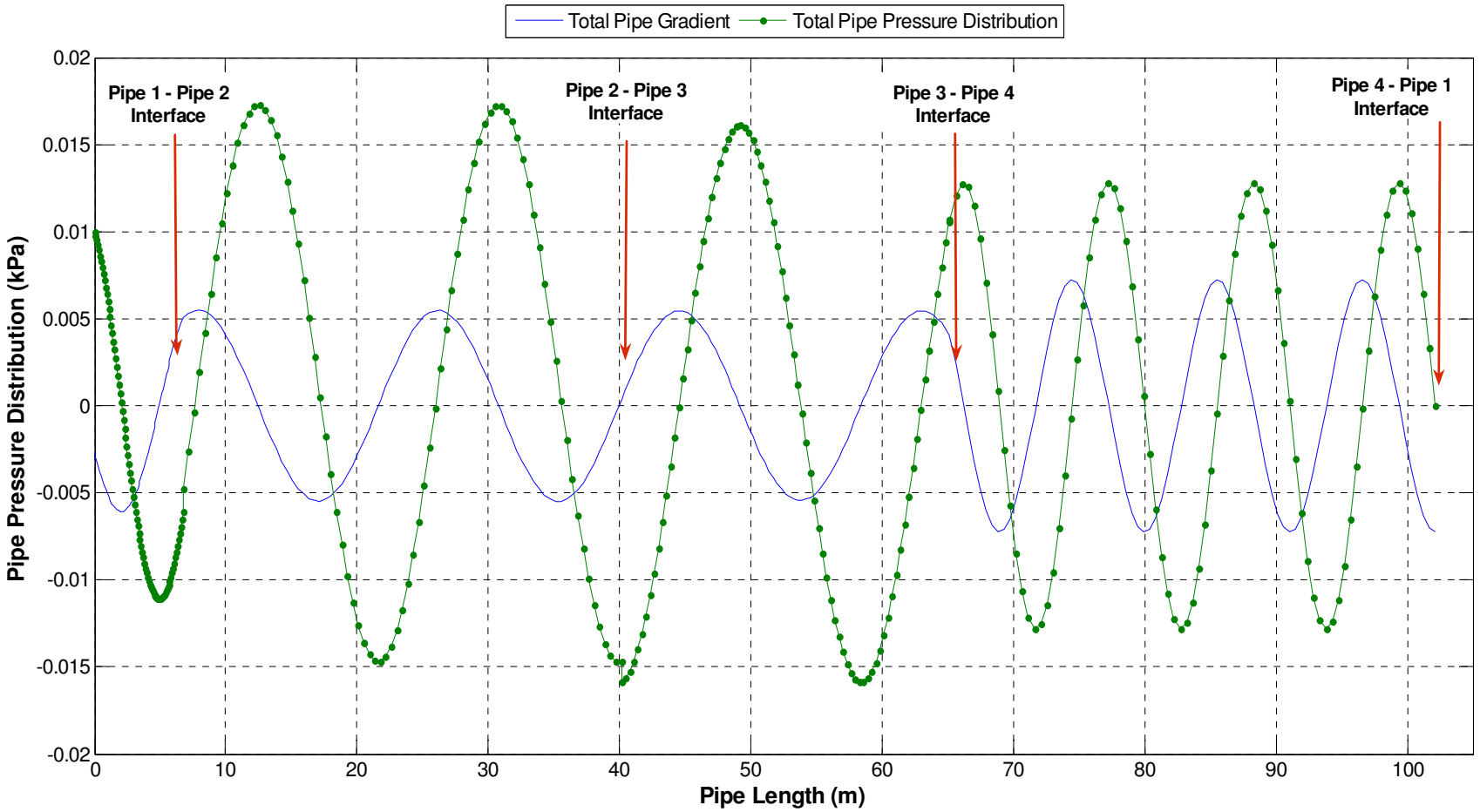


Figure 3.11: Continuous acoustic pressure gradient and pressure distribution for HTGR pipe-loop system.



3.4 General acoustic results and discussion

3.4.1 Acoustic solution formulations

The Cepkauskas SMiRT 5 approach has been successfully adapted to produce four acoustic cases with different boundary conditions. The boundary conditions existing for each pipe in the pipe network were determined as:

- **CASE 1** considered a general acoustic case, depicted a pipe with boundary conditions of pump or compressor at the one end and the other end as open ended. This is considered a more general case (refer to Eq. (3.10)).
- **CASE 2** depicted a pipe representing a pump/compressor at one end and a general boundary condition at the other end (refer to Eq. (3.12)).
- **CASE 3** depicts a pipe with the boundary conditions containing constants that represented some resistance or amplitude at both ends (refer to Eq. (3.19)).
- **CASE 4** depicted a pipe entering a vessel or compressor, with boundary condition containing a constant (representing a resistance or amplitude) at one end and open ended at the other end (refer to Eq. (3.27)).

It was shown in Section 3.1 and in Refs. [CE79, CE08] that when restrictions are placed on the auxiliary functions and their derivatives at the boundaries, the boundary conditions are made homogeneous. Utilising the transformation technique enables a more manageable method of solving of the acoustic equations that is less complicated than it otherwise would be. Therefore, there is no need to define the auxiliary functions in the interior for the specific problem, thereby reaffirming the Cepkauskas methodology as correct. This has been achieved by solving the problem in the transformed space, and after substituting back into the untransformed equation, terms appear that are negative Fourier series expansions of the auxiliary functions and thus the auxiliary functions terms cancel. It thus shows that the auxiliary terms need not be defined in order to determine the pressure distribution within the pipe. This approach was successfully adopted and applied to the four pipe cases. Using the same mathematical approach of SMiRT5, it is shown in Section 3.1 that four solutions are produced that are representative

of the pressure distribution in a pipe length; these results are found in Table 3.1. The series term in the solutions allows the solution to be applied for many time steps therefore the solution can be applied in transient and steady state situations. This in turn indicates a possibility of coupling to CFD codes that can run plant transients to determine the acoustic behaviour of the fluid.

3.4.2 Jolley series application

From Section 3.2.1, it is found that the plots of the Jolley series and acoustic CASES 1 to 4, Fig. 3.2 to 3.5, and the selection of specific Jolley series for each forced vibration acoustic solution correlated with each other and no significant deviations were found. This proves that the Jolley series selected is valid in representing its respective acoustic solutions and enables one to reduce the acoustic solutions to a more convenient and simpler form. It is then appropriate to conclude that the Jolley series selected can be used to represent the more complex form of the four acoustic solutions. It should be noted that Fisher et al., showed a numerical convergence difficulty at the boundaries. The use of the Jolley Series provides an exact solution at the interior and at the boundaries.

3.4.3 Validity of the acoustic solutions

The results of the differentiation for each case with respect to x and the second derivatives with respect to time and x performed in MathCad, Section 3.2.2 shows that the assumptions made on the initial selection of the boundary conditions are met and that the assumptions made on the formulated acoustic-wave solution differential equations are indeed satisfied. This reaffirms that the solutions are mathematically correct and are valid for application.

To test the validity of the derived forced-vibration acoustic solutions, each applicable solution were applied to the four pipes in a general pipe-loop model, with helium as a fluid medium and a compressor as the time-dependent boundary condition. Kramer's Rule was applied to determine the constants. This methodology and acoustic case were programmed in a Matlab acoustic subroutine for the general pipe-loop, Appendix B1-1. The generalised input parameters

representing the system conditions were inputted into the subroutine. The results provided values for the solution constants' and satisfied the conditions of the matrix (refer to Sections 3.2.2 and 3.2.3). The acoustic pressure distribution as described in Fig. 3.7 in the pipe-loop, shows smooth pressure wave transitions at all pipe interfaces. The pressure gradients at the interfaces are continuous and in agreement with the pressure distribution curves. This affirms that the acoustic cases produced can be applied to individual pipes and can be coupled together for a pipe-loop to adequately represent to total pressure distribution in a system. Figure 3.8 represents the individual acoustics in a pipe. From this plot, one can isolate the localised fluid behaviour in an area of a pipe-loop.

The acoustic subroutine generated values for c_0, ω_i and ω_p, (the acoustic behaviour and the frequencies of the fluid). In flow systems, wide band flow-induced excitation as well as structural interaction form part of excitation forces. Thereby knowing these values will enable one to simulate if acoustic resonances or wide band flow-induced excitation are present. Should one know the systems' structural natural frequencies transporting this fluid, it can be predicted if the fluids' pressure wave excitation frequency produced from an excitation force (e.g. compressor or pump) approaches the structures' natural frequency.

Since the solutions contain time dependent functions, it can be applied to a system transient. Hence, it can be applied to a system consisting of compressors and pumps starting up from zero conditions or deviating from a steady state plant condition. During the plant transients, the fluid properties such as temperatures, densities and pressures are changing continuously. The solutions should be able to calculate the acoustic forcing and natural frequencies of the fluid and the pressure distribution as the fluid medium properties changes at different time steps. If one has known values for the natural frequencies of the structural components supporting the flow of the fluid, it can be monitored if the fluids' natural frequencies coincide with the structural natural frequencies. In knowing when these conditions exist, it can avoid a potential resonant condition or excessive structural vibration. Ideally, one would prevent the system from being at this

condition for prolonged periods of time, and one would rather want to transitional quickly through these regions.

The acoustic cases can also be applied to systems used during optimisation of process flow, since the flow parameters (pressures, temperatures, fluid mediums) can be changed and the natural and forcing frequencies can be observed and possibly used to predict potential resonant conditions or forced vibration problems. As explained in Section 2, this is a condition that is to be avoided due to the possibility of mechanical failure.

3.4.4 HTGR pipe loop application

To test the suitability of the acoustic force-vibration solutions in a simplified HTGR environment, the general acoustic subroutine is used. The input values are changed according to the HTGR conditions to produce an HTGR acoustic subroutine, Appendix B1-2. The input to the subroutine requires only temperature and pressures, which was obtained from results of a simplified HTGR Flownex model produced (refer to Appendix A1-1 for the Flownex HTGR model and Appendix A1-2 for the steady state parameters). The unknown constants determined for the acoustic solutions used for this steady state simplified model can be found Appendix B1-2. The acoustic behaviour of the helium in each pipe and acoustic pressure distributions and pressure gradients for each pipe and in the overall loop in the simplified network were thus found.

The HTGR acoustic subroutine produced good acoustic responses in each individual pipe as seen in Fig. 3.10. No significant inconsistencies were found thereby concluding that the solutions are compatible to the conditions of the pipes.

From the pressure gradient results (Fig 3.10), the graph is in general a continuous acoustic wave, demonstrating that the interface or boundary conditions are largely consistent from pipe to pipe. However, the pressure gradient results revealed slight deviations in the pressure continuity when compared to the results of the general pipe model. This is due to the large temperature gradients existing in the pipe-loop model. The temperature gradients, seen in the HTGR Flownex

simulation (Appendices A1-1 and A1-2) indicate large temperature gradients located at the heat exchangers and turbomachinery (interfaces of pipe 4 to pipe 1, pipe 1 to pipe 2, pipe 3 to pipe 4). In the acoustic subroutine, these temperatures are averaged for input into the code. The approach taken is quite simplistic, but in reality, this is not a true representation of these components. Having said this, the continuous pressure distribution and gradient in the entire loop, especially at the pipe interfaces where complex machinery exists, cannot be realistically demonstrated in this case, as it is not a true representation of its conditions. For this reason, these components have to be discretised into smaller sections (to represent many channels or pipes) to take into account the influences of the geometry on the wave propagation, such as reflections. The acoustic force-vibration solutions are applied with the appropriate boundary conditions. It then can be acoustically modelled and coupled to the main pipe network. Having achieved this, it will definitely be possible to demonstrate a continuous pressure-distribution and gradient along the entire HTGR pipe-network. The process is simple in principle but complex analytically due to the geometries of these components. It is an area of future work and investigation that is currently underway.

Since the inputs to both acoustic subroutines only require pipe geometries, fluid temperatures at various points in the system, and the acoustic solutions is solvable for many time steps. It was found that a code such as Flownex can readily produce these parameters at different time steps. This supported the possibility of integrating these two codes for transient application. This has been proved for a steady state run and provides a high probability that the acoustic solutions can be coupled to a CFD code to produce complete acoustic simulations for transients of any pipe network system with time dependent boundary conditions. Although there are no experimental data from the an operating HTGR plant that can validate the results of this HTGR case study, the main objective was to verify the mathematics for the acoustic solutions and to demonstrate it can be coupled for a pipe-loop system. This objective has thus been demonstrated by the justification

provided. The next step in validating these results should be through experimental means.

CHAPTER 4

SUMMARY AND CONCLUSIONS

Pressure disturbances produced by longitudinal waves propagate through the compressible fluids in some cases as acoustic waves. These waves always produce excitation (wide band) and when these waves resonate (i.e. when standing pressure waves are produced). When these resonances coincide with any natural frequencies in the system, the resulting vibration of the components is amplified. Such vibrations can impact on the piping structures, known as fluid-structure interaction, and over time cause failure from material fatigue. If the vibrations are excessive it can be very damaging having financial, economic and productive impact. This condition must be avoided and therefore, being able to predict these conditions is extremely important. At present, one of the most useful tools available is to perform specialised analyses to predict this phenomenon and as such is a proactive risk management process. This is commonly known as acoustic vibrational analysis.

The fundamentals of vibration and waves have been explored. This has been expanded to the simplest mechanical system, a spring-mass system, which is used to describe a typical vibrating system. The system is mathematically described as a linear non-homogeneous differential equation of the second order. This mathematical model incorporated the laws of physics that govern vibration. For complex systems, modelling such as system is complex, breaking down the complex system into smaller systems is the best approach to use. When a simple-harmonic system is coupled to form a series of similar systems (smaller harmonic oscillators) as a complex acoustic model, it can be used to determining the overall vibration behaviour of that system. This Research Report investigated a forced vibration system. The forced vibration is produced by an excitation force, approximated by a harmonic function arising from forces produced by the compressor pulsating in the fluid medium (helium gas).

The well-known one-dimensional wave equation, described as a linear homogeneous differential equation, has been explored and selected to best describe the system of study. The wave-equation provides an estimation of the displacement of mass at each point as a function of time in order to completely describe the motion of an acoustic wave. The derivation of the wave equation has been explored and broken down into its fundamental properties that define it. It is discussed how it can be adapted to describe waves in a compressible and inviscid (with no phase change) fluid and can be modified to various other types of applications.

An extensive study has been performed of the results of specialists who investigated flow induced vibrations and acoustic responses of reactor piping and reactor core support in PWRs. Notably, the work of Penzes and Horvay, Lee and Im and Cepkauskas were looked at more closely. Each investigation produced acoustic solutions that aimed at predicting the acoustic behaviour of fluids in a system. An assessment of each solution resulted in selecting the Cepkauskas SMiRT5 approach because of its unique applications. The Cepkauskas method consists of transforming a homogeneous differential equation with time dependent boundary conditions to one of a non-homogeneous differential equation with homogeneous boundary conditions. The Cepkauskas approach is also unique in comparison to the other solutions in that it did not require the use of defined auxiliary functions on the interior of the medium and proved to be more manageable and mathematically rigorous. In addition, this transformation technique was found to be a simpler method of solving these complex differential equations in the transformed space. The Cepkauskas approach also in turn had the possibility to be integrated into a pipe-loop configuration due to the frequencies not requiring a transcendental equation, therefore the possibility of applying it to a plant transient makes it more appropriate to use.

The Cepkauskas approach was questioned by Lee and Im, when they stated [LE92, LE94]:

"Penzes, Bowers and Horvay and Cepkauskas, treated the problem of pulsations by introducing a body force concept, but they neglected to recognise the restrictions on the body force to make the boundary conditions homogenous".

However when investigating the Cepkauskas methodology, it was found that the auxiliary terms do indeed cancel thus affirming that the auxiliary terms need not be defined to determine the pressure distribution of the acoustic wave. This has been proved by solving the problem in transformed space and, upon substituting back into the untransformed equation, terms appear that are the negative Fourier series expansions of the auxiliary functions and thus the auxiliary function terms cancel. The results produced by the preliminary investigation provided confidence that this approach can be applied as the basis for deriving the other acoustic solution of the pipe-loop model. This approach has been demonstrated in several references [FI79, CE82 and CE08], and published by Cepkauskas as SMiRT5 [CE79].

In the present study, SMiRT5 has been analysed and the methodology applied to four forced-vibration acoustic cases with different boundary conditions. This was done to prove that the methodology is correct and to adapt it to various conditions existing in a pipe-loop system. Four acoustic cases, CASES 1 to 4, were successfully produced based on applying the linear transformation technique, as used in Penzes, Horvay and Bowers and Cepkauskas and using a time dependent boundary condition as the source of the pressure pulsations. In all four cases, the results demonstrated that when restrictions were placed on the auxiliary functions the solutions were made homogenous and that the auxiliary terms do cancel, reaffirming that the auxiliary terms need not be defined to determine the pressure distribution of the acoustic wave.

All four forced-vibration acoustic solutions were investigated to determine if they can be coupled to represent a pipe-loop configuration, containing four pipes and a compressor as the source of the pressure pulsations. The boundary conditions were found to be satisfied by taking the first and second derivatives of the

acoustic solution with respect to distance and time. This confirmed that the original assumptions made on the interior of the functions were met. To simplify the complexity of the acoustic solution, Jolley series [JO05] were selected that can adequately represent the acoustic solutions. It should be noted that Fisher et al., showed a numerical convergence difficulty at the boundaries. The use of the Jolley Series provided an exact solution at the interior and at the boundaries.

An acoustic subroutine consisting of the acoustic solutions was created and applied to a general pipe-loop model for determining the pressure distribution in the system. Specific forced vibration solutions are applied to each pipe dependent on its boundaries conditions; this generated a set of linear equations containing unknown constants. Kramers Rule [KR67] has been successfully applied to generate values of these constants in the solutions. Reverse calculations were performed to ensure the matrix conditions have been met. This demonstrated that the correct approach has been used for determining unknown constants in the acoustic solutions for a pipe network.

In the general pipe-loop model test case, arbitrary values for temperatures and geometries were used to determine the overall acoustic pressure distribution. The pressure distribution was found to be continuous from pipe to pipe and no significant numerical problems were found with the methodology used. The overall pressure distribution and gradient results showed a continuous gradient through the interfaces, from the exit to the pump/compressor, through the pipes and back into the entry of the pipe, affirming that the acoustic solutions are mathematically correct and indeed can be coupled into a pipe network or pipe-loop model to determine the fluid behaviour within a system. The acoustic subroutine was applied to an HTGR Flownex model at steady state conditions. The temperature and pipe geometries were extracted from the CFD code as inputs to the HTGR acoustic subroutine. The results showed that there was good representation of the acoustic behaviour of the helium in the individual pipes. The pressure gradient and overall pressure distribution in the entire pipe-loop was successfully produced, but indicated slight irregularities in its distribution. These

irregularities were produced by large temperature gradients existing at the heat exchanger and turbomachinery locations. These irregularities can be eliminated by discretising these components into many smaller sections, thereby reducing the temperature gradient, and applying the appropriate acoustic cases to them.

The application of the acoustic solutions has generated values for forcing (excitation) and natural frequencies of the fluid pressure waves within the pipes. It also produced information of the resonances due to standing waves. This information can be used for identifying various problems such as flow-induced vibration or structural resonances. With knowing the excitation and natural frequencies of the fluid medium, one can predict if acoustical resonances will occur. Acoustical resonances are known to amplify the gas pulsations at the specific locations in such a magnitude as to cause significant vibration, flow induced vibrations. Should one know the systems' structural natural frequencies transporting this fluid, it can be predicted if the fluids' pressure wave excitation frequency produced from an excitation force (e.g. compressor or pump) approaches the structures' natural frequency. This situation you will want to avoid due to resonance conditions it will generate. The search for resonance in the piping is to determine pressure that causes the pipes to vibrate. A coupling of structural, acoustic and pump frequencies along with a contributing modal participation factor is needed for large displacements.

The series term in the acoustic solutions will also enable it to be applied for dynamic plant systems, such as plant transients (heat-up, shut down, start-up). In a start-up or heat-up plant mode, the parameters such pressures, temperatures and densities are continuously changing. The pressure of the fluid plays an important role as it influences the speed of sound and hence the speed of pressure waves in the fluid medium. These parameters are inputs into the one-dimensional wave equation and the four forced-vibration acoustic solutions. It can be obtained from a thermal hydraulic code simulating a particular plant transient, generating the results of these parameters at different time steps. The acoustic sub-routine can be coupled to this CFD code and generate the resonant frequencies, acoustic pressure

wave natural frequencies at each time step of the transient. This information can be used to predict the stages (or time period) where resonance will occur in the plants' transient and to avoid extended operation at these conditions. The resonance can be produced by presence of possible standing waves in the fluid and by the excitation natural frequencies of the fluid impacting on structural natural frequencies, thereby causing structural resonance. These regions need to be determined and either the acoustic energy dissipated through design changes (such as designed- in damping) or start up rates accelerated to avoid fatigue damage (from excessive vibration). Once you accelerate the transitioning through these stages, one can pass quickly through the resonant phases with minimal impact. It is only when one is operating for an extended period of time in these phases then significant impact is experienced. A reactor coolant pump for a Pressurised Water Reactor comes to full speed very rapidly, thus the driving frequency starts at zero and arrives at its final frequency, ω_p. This sweeps through any acoustic resonance quick enough to avoid high structural response. However, in reality the damping of structures help dissipate the acoustic energy.

The future work that has been identified from this study includes:

1) **Transient application**

   The acoustic subroutine is to be adapted to allow for a time-step change within transients. The resulting acoustic pressures can be then imported to a finite element pipe model to determine pipe vibration response (fluid-structure interaction).

2) **Discretising of complex components**

   Complex components, such as heat exchangers and turbomachinery, produce large temperature gradients in a system. These components should be discretised to smaller increments to reduce the temperature gradients. The acoustic solutions can then be applied and coupled with a pipe network model to determine a complete system response.

3) **Coupling of simulation packages**

The original acoustic code produced in this Research Report is a subroutine. This subroutine can easily be coupled to a CFD code to produce an integrated forced vibration acoustic analytical tool. This is feasible since the subroutine requires only temperature, pressure and geometries for inputs. In the case of transients, the CFD code is ideal to produce values that are changing with respect to time, therefore coupling it to this subroutine is best. For determining fluid-structure interaction (pipe vibration response) and/or stress analysis, the subroutine can be coupled to a Finite Element Model (FEM) code.

4) **Validation through experimental methods**

The final validation of the four forced vibration acoustic solutions produced would be needed to be done through experimental methods. A real physical model of an acoustic problem should be established and comparing the results gathered would assess if the numerical analysis used for the acoustic solutions to predict the acoustic pressure distribution is accurate.

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APPENDIX A1-1.

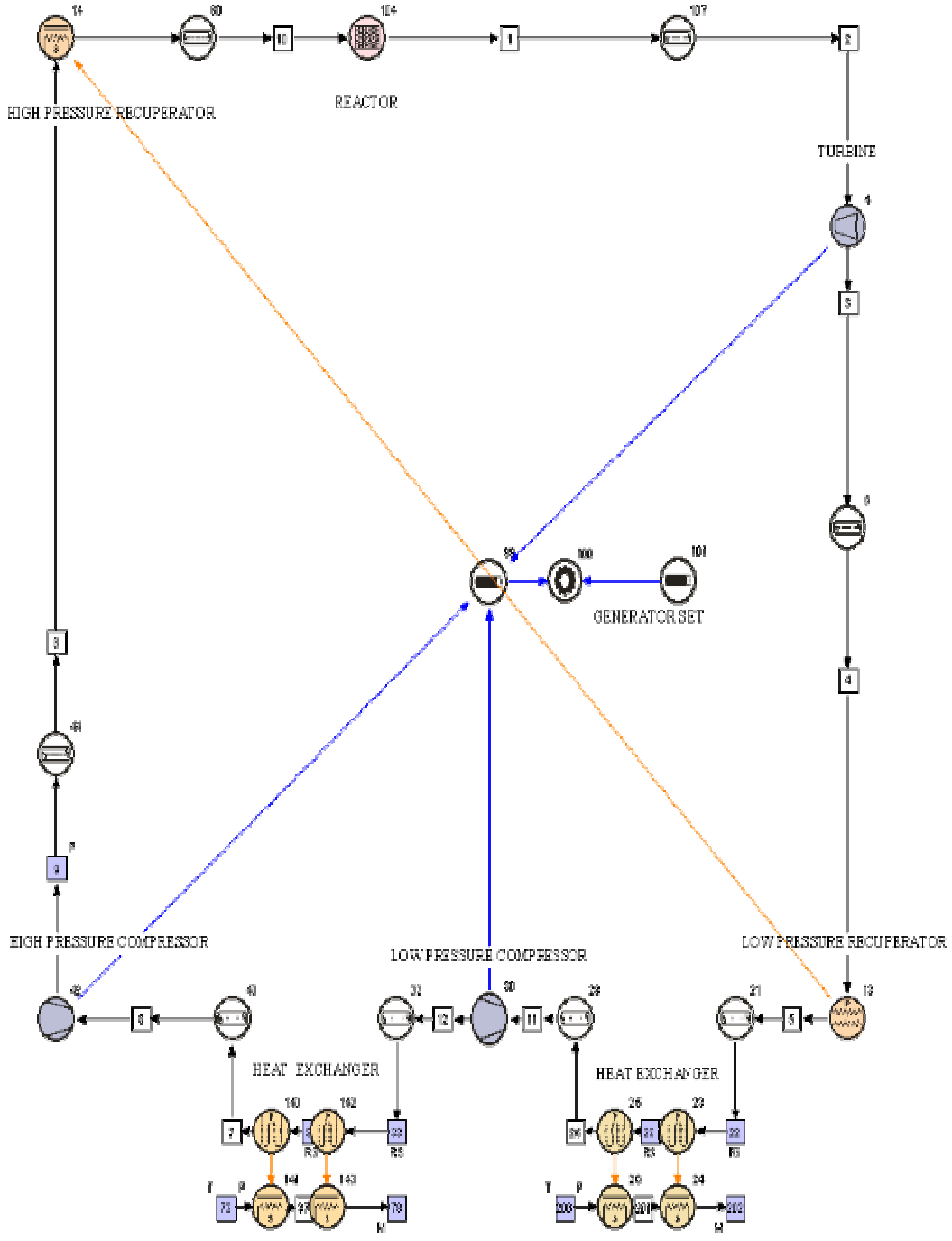


Figure A.1: Simplified HTGR Flownex Model

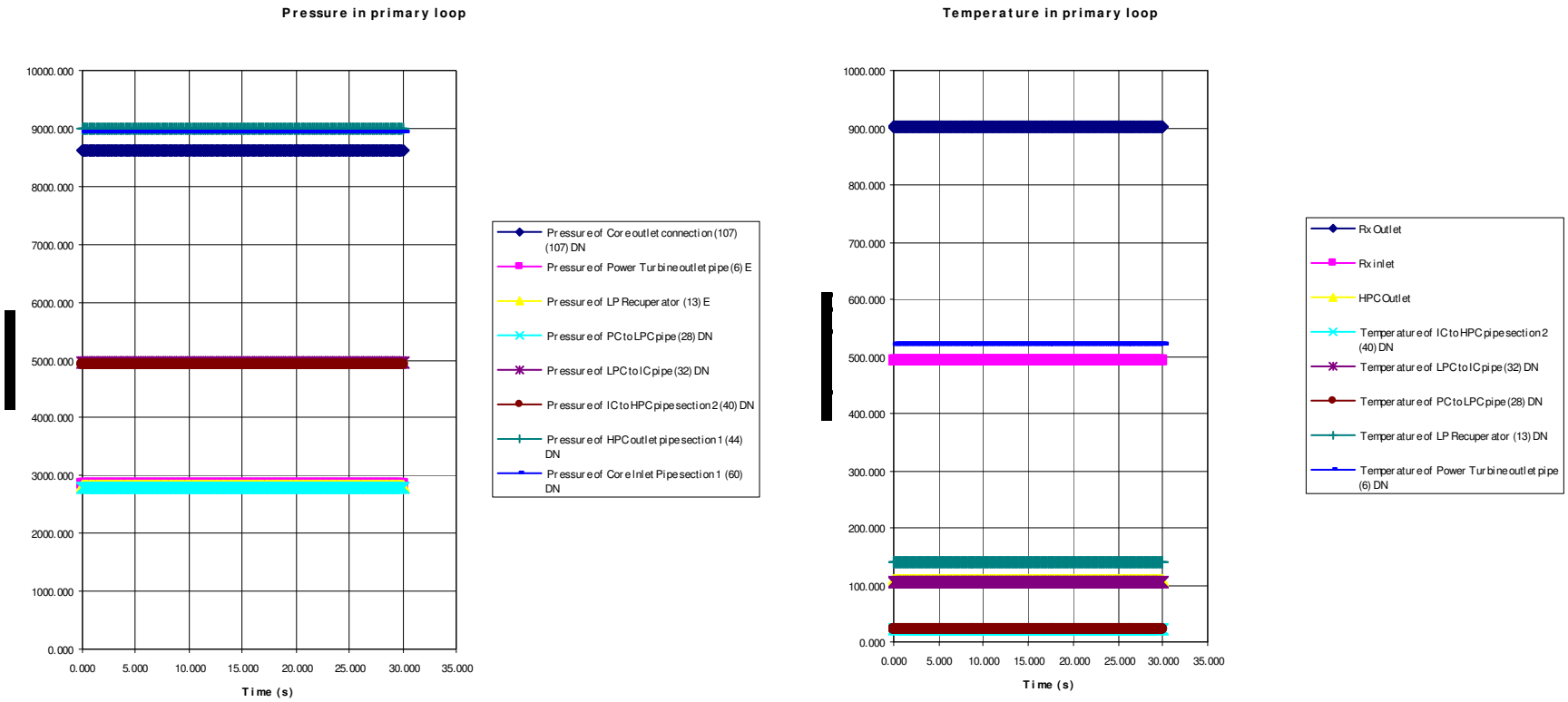


Figure A.2: HTGR Flownex steady state conditions: **Left Handside**-Pressure in primary loop, **Right Handside**-Temperature in primary loop

APPENDIX B1-1.

General case study acoustic subroutine.

```
P = [10; 2; 3; 4]
Lp = [10; 15; 20; 25]
T = [100;200;300;400]
</pre><pre class="codeoutput">
P =

     1
     2
     3
     4


Lp =

    10
    15
    20
    25


T =

   100
   200
   300
   400
```

Constant Variables

```
kelvin = 273.15;
Po = 0.01;
f = 100;
R = 8.314;
G = 1.667;
M = 0.004;
```

Determination of Constants

```
Co = sqrt(G*R*(T+kelvin)/M);
Wp = 2*pi*f;
A = (Wp*Lp)./(Co);
```

First Interface boundary at the exit of the compressor to the first pipe

Application of Summation of Series(Jolley

```
% P1(x=l) = P2(x=0)

x11=1;
```

```
x21 = 0;
K0 = cos((A(1,1))*(1-x11))/ cos (A(1,1));
K1 = (Co(1,1)/Wp)*(sin(A(1,1)*x11)/cos(A(1,1)));
K2 = (Co(2,1)/Wp)*((cos(A(2,1)*(1-x21))/sin(A(2,1))));
K3= (Co(2,1)/Wp)*((cos(A(2,1)*x21)/sin(A(2,1))));
```

Second Interface Boundary
Application of Summation of Jolley Series

```
P2(x=l) = P3(x=0)</span>
x22 = 1;
x32 = 0;
K4 = (Co(2,1)/Wp)*(((cos(A(2,1)*(1-x22)))/sin(A(2,1))));
K5 = (Co(2,1)/Wp)*((cos(A(2,1)*x22)/sin(A(2,1))));
K6 = (Co(3,1)/Wp)*(((cos(A(3,1)*(1-x32)))/sin(A(3,1))));
K7 = (Co(3,1)/Wp)*((cos(A(3,1)*x32)/sin(A(3,1))));
</pre><h2>End Interface Boundary<a name="7"></a></h2><pre
class="codeinput"> <span class="comment">% Application of
Summation of Jolley Series</span>
        <span class="comment">% P3(x=l) = P4(x=0)</span>
        <span class="comment">% At interface Aj = B(j-1) : A4 =
B3</span>
x33 = 1;
x43= 0;

K8 = (Co(3,1)/Wp)*(((cos(A(3,1)*(1-x33)))/sin(A(3,1))));
K9 = (Co(3,1)/Wp)*((cos(A(3,1)*x33)/sin(A(3,1))));
K10 = (Co(4,1)/Wp)*(sin(A(4,1)*(1-x43))/cos (A(4,1)));

KValues=[K0;K1;K2;K3;K4;K5;K6;K7;K8;K9;K10];
```

Constructing the Matrix

```
BM1 = [(K2-K1); -K4; 0];
BM2 = [(-K3); (K5+K6); -K8];
BM3 = [ 0; -K7; (K9-K10)];
C = Po*K0;
Mtx = [BM1,BM2,BM3]
Con = [C; 0; 0];
DeterMatrix = det(Mtx);
w = Mtx;
w(1:3,1)=[Con];
B1 = det(w)/det(Mtx)
m = Mtx;
m(1:3,2) = [Con];
B2 = det(m)/det(Mtx)
n = Mtx;
n(1:3,3) = [Con];
B3 = det (n)/det(Mtx)

Mtx =

   2.80614336231391  -2.31341974181624                   0
  -2.31341974181624  -2.93829166237818  -4.61504440706338
                   0  -4.61504440706338  -6.86026247558943


B1 =   -4.686216505486100e-004
```

B2 = -0.00651721854074
B3 =   0.00438427145945

Checking of Matrix

```
a= B1*(K2-K1) + B2*(-K3)+ B3*(0);
b= B1*(-K4)+ B2*(K5+K6) + B3*(-K7);
c = B1*(0) + B2*(-K8)+ B3*(K9-K10);
```

Application of Forced Responses- Plotting

```
xl = 0:0.0125:1;
xL = [xl'];
```

All Pipes Pressure Distribution

```
PipePressDist = [(Po*cos(A(1,1)*(1-
xL))+(B1*Co(1,1)/Wp)*sin(A(1,1)*xL))/cos(A(1,1)),(B1*(Co(2,1)/Wp)*
((1/A(2,1))+<span class="keyword">...</span>
    ((cos(A(2,1)*(1-xL)))/sin(A(2,1)))))-
(B2*(Co(2,1)/Wp)*(((cos(A(2,1)*xL))/sin(A(2,1)))),(B2*(Co(3,1)/Wp
)*((1/A(3,1))+<span class="keyword">...</span>
    ((cos(A(3,1)*(1-xL)))/sin(A(3,1)))))-
(B3*(Co(3,1)/Wp)*(((cos(A(3,1)*xL))/sin(A(3,1)))),<span
class="keyword">...</span>
    (-B3*Co(4,1)/Wp)* (sin(A(4,1)*(1-xL))/cos(A(4,1)))]

plot(xL,PipePressDist)


    a1= Lp(1,1)*xl;
    b1= Lp(1,1)+ Lp(2,1)*xl;
    c1= Lp(1,1)+Lp(2,1)+(Lp(3,1)*xl);
    d1=Lp(1,1)+Lp(2,1)+Lp(3,1)+Lp(4,1)*xl;
PipePressDist =

   0.01000000000000   0.01443413503533   0.00441377520306  -
0.01239372387376
   0.00924305377638   0.01428478708848   0.00275027826137  -
0.01092511979585
   0.00844202634427   0.01401349199929   0.00103230008040  -
0.00927617649231
   0.00760073789752   0.01362254492964  -0.00071883607206  -
0.00747411280960
   0.00672320063949   0.01311525329957  -0.00248139537683  -
0.00554867512810
   0.00581359964839   0.01249590880644  -0.00423350123241  -
0.00353164634330
   0.00487627291810   0.01176975111708  -0.00595340678382  -
0.00145632123051
   0.00391569066985   0.01094292354009  -0.00761976484105
0.00064304314663
   0.00293643403313   0.01002242105333  -0.00921189283697
0.00273179291220
   0.00194317319780   0.00901603112630  -0.01071002953595
0.00477544940395
```

```
   0.00094064514138   0.00793226783793  -0.01209558030718
0.00674027830720
  -0.00006636896213   0.00678029984736  -0.01335134791820
0.00859384650181
  -0.00107306654428   0.00556987282688  -0.01446174598419
0.01030555743056
  -0.00207464654611   0.00431122701334  -0.01541299242365
0.01184715615170
  -0.00306633231502   0.00301501057554  -0.01619328051935
0.01319319573882
  -0.00404339438516   0.00169218953051  -0.01679292546144
0.01432145732920
  -0.00500117303283   0.00035395497080  -0.01720448455372
0.01521331688710
  -0.00593510049923  -0.00098837161238  -0.01742284959119
0.01585405262777
  -0.00684072277463  -0.00232343410927  -0.01744531026227
0.01623308802761
  -0.00771372084008  -0.00363993786437  -0.01727158778872
0.01634416640919
  -0.00854993126535  -0.00492674522954  -0.01690383838585
0.01618545421923
  -0.00934536606473  -0.00617296978868  -0.01634662649986
0.01575957129474
  -0.01009623171632  -0.00736806845706  -0.01560686815470
0.01507354761786
  -0.01079894725365  -0.00850193067599  -0.01469374511153
0.01413870727299
  -0.01145016134386  -0.00956496394838  -0.01361859090611
0.01297048152193
  -0.01204676827049  -0.01054817499145  -0.01239475017884
0.01158815408251
  -0.01258592274507  -0.01144324582005  -0.01103741304322
0.01001454281528
  -0.01306505347664  -0.01224260411707  -0.00956342654862
0.00827562307270
  -0.01348187543448  -0.01293948729527  -0.00799108557742
0.00640009892811
  -0.01383440074574  -0.01352799970901  -0.00633990577187
0.00441892936211
  -0.01412094817584  -0.01400316253148  -0.00463038130912
0.00236481722760
  -0.01434015114644  -0.01436095587567  -0.00288373053076
0.00027166942889
  -0.01449096425284  -0.01459835280274  -0.00112163258424  -
0.00182596277414
  -0.01457266824964  -0.01471334492990   0.00063404165523  -
0.00389345409827
  -0.01458487348089  -0.01470495942148   0.00236150104249  -
0.00589667679101
  -0.01452752173839  -0.01457326721903   0.00403930463054  -
0.00780256397198
  -0.01440088653935  -0.01431938244122   0.00564662779214  -
0.00957965546275
  -0.01420557182190  -0.01394545295837   0.00716352069175  -
0.01119861709496
  -0.01394250906484  -0.01345464222139   0.00857115589960  -
0.01263272492484
```

```
  -0.01361295284536  -0.01285110249896   0.00985206207480  -
0.01385830636112
  -0.01321847485574  -0.01213993974921   0.01099034081670  -
0.01485513092475
  -0.01276095640783  -0.01132717042326   0.01197186399335  -
0.01560674419021
  -0.01224257946081  -0.01041967056590   0.01278444909758  -
0.01610073939614
  -0.01166581621520  -0.00942511764409   0.01341801045426  -
0.01632896224174
  -0.01103341732261  -0.00835192559550   0.01386468440213  -
0.01628764548858
  -0.01034839876761  -0.00720917364636   0.01411892689628  -
0.01597747114578
  -0.00961402748409  -0.00600652950104   0.01417758231992  -
0.01540355921218
  -0.00883380577492  -0.00475416755313   0.01403992265139  -
0.01457538316136
  -0.00801145460902  -0.00346268280980   0.01370765650029  -
0.01350661356449
  -0.00715089587563  -0.00214300125785   0.01318490790043  -
0.01221489243237
  -0.00625623368037  -0.00080628742965   0.01247816512302  -
0.01072154200157
  -0.00533173477227   0.00053615004902   0.01159620014530  -
0.00905121277159
  -0.00438180819515   0.00187295413018   0.01054995977418  -
0.00723147660303
  -0.00341098426038   0.00319281542454   0.00935242977621  -
0.00529237159320
  -0.00242389294125   0.00448456787909   0.00801847370036  -
0.00326590624199
  -0.00142524179215   0.00573728324218   0.00656464839407  -
0.00118553109266
  -0.00041979349764   0.00694036351674   0.00500899850230
0.00091441343103
   0.00058765684135   0.00808363061953   0.00337083250031
0.00299926387675
   0.00159230457587   0.00915741248796   0.00167048304003
0.00503460594772
   0.00258935842295   0.01015262490591  -0.00007094541559
0.00698684257409
   0.00357406331575   0.01106084835638  -0.00183183853774
0.00882374849442
   0.00454172308104   0.01187439925077  -0.00359034040530
0.01051500219305
   0.00548772283581   0.01258639493222  -0.00532462477703
0.01203268641248
   0.00640755099618   0.01319081190300  -0.00701316599534
0.01335174897920
   0.00729682079366   0.01368253678342  -0.00863500615925
0.01445041633590
   0.00815129119622   0.01405740957115  -0.01017001525052
0.01531055295418
   0.00896688713418   0.01431225883487  -0.01159914098416
0.01591796069484
   0.00973971893479   0.01444492854473  -0.01290464528231
0.01626261317431
```

```
   0.01046610087246   0.01445429631231  -0.01407032443643
0.01633882126859
   0.01114256874646   0.01434028288611  -0.01508171022505
0.01614532702272
   0.01176589640209   0.01410385282200  -0.01592624949109
0.01568532441561
   0.01233311111659   0.01374700632307  -0.01659345994952
0.01496640663761
   0.01284150777640   0.01327276231784  -0.01707506029187
0.01400044075084
   0.01328866177817   0.01268513292010  -0.01736507297244
0.01280337180151
   0.01367244059197   0.01198908948633  -0.01745989840069
0.01139495961757
   0.01399101393160   0.01119052055793  -0.01735835961869
0.00979845263630
   0.01424286248339   0.01029618204403  -0.01706171690943
0.00804020414612
   0.01442678515204   0.00931364006639  -0.01657365215430
0.00614923727694
   0.01454190478875   0.00825120694982  -0.01590022313425
0.00415676591998
   0.01458767237446   0.00711787089977  -0.01504978834153
0.00209567948500
   0.01456386963818   0.00592321996194  -0.01403290323549
0
```

<h2>**Continuous Pressure Distribution in Loop**<a

```
TotalPipeLength=[a1,b1,c1,d1]';
PipeDist1=[(Po*cos(A(1,1)*(1-
xL))+(B1*Co(1,1)/Wp)*sin(A(1,1)*xL))/cos(A(1,1));(B1*(Co(2,1)/Wp)*
((1/A(2,1))+...
    ((cos(A(2,1)*(1-xL)))/sin(A(2,1)))))-
(B2*(Co(2,1)/Wp)*(((cos(A(2,1)*xL))/sin(A(2,1)))));(B2*(Co(3,1)/Wp
)*((1/A(3,1))+...
    ((cos(A(3,1)*(1-xL)))/sin(A(3,1)))))-
(B3*(Co(3,1)/Wp)*(((cos(A(3,1)*xL))/sin(A(3,1)))));...
    (-B3*Co(4,1)/Wp)* (sin(A(4,1)*(1-xL))/cos(A(4,1)))];
Pipegrad1= (A(1,1)*((Po*sin(A(1,1)*(1-
xl))+(B1*Co(1,1)/Wp)*cos(A(1,1)*xl))/cos(A(1,1)))/Lp(1,1))';
Pipegrad2=(A(2,1)*(((Co(2,1)*B1/Wp)*(sin(A(2,1)*(1-
xl))/sin(A(2,1))))+((B2*Co(2,1)/Wp)*(sin(A(2,1)*xl)/sin(A(2,1)))))
/Lp(2,1))';
Pipegrad3=(A(3,1)*(((Co(3,1)*B2/Wp)*(sin(A(3,1)*(1-
xl))/sin(A(3,1))))+((B3*Co(3,1)/Wp)*(sin(A(3,1)*xl)/sin(A(3,1)))))
/Lp(3,1))';
Pipegrad4=((A(4,1)/Lp(4,1)*B3*Co(4,1)/Wp)*(cos(A(4,1)*(1-
xl))/cos(A(4,1))))';
TotalPipeGrad = [Pipegrad1;Pipegrad2;Pipegrad3;Pipegrad4]
</pre><pre class="codeoutput">
TotalPipeGrad =

       -0.0059                            -.0068       0.0044
          -0.0062      -0.0005       -0.0065       0.0050
          -0.0066      -0.0011       -0.0062       0.0055
          -0.0069      -0.0018       -0.0058       0.0060
```

```
-0.0072     -0.0024    -0.0022         0.0063
-0.0074     -0.0030      -0.0028       0.0066
-0.0076     -0.0036      -0.0034       0.0067
-0.0078     -0.0041      -0.0040       0.0067
-0.0079     -0.0047      -0.0045   -0.0059
-0.0080     -0.0051      -0.0050
-0.0080     -0.0056      -0.0055       0.0066
-0.0081     -0.0060   -0.0053          0.0064
-0.0080     -0.0063      -0.0047       0.0061
-0.0080     -0.0066      -0.0041       0.0057
-0.0079     -0.0068      -0.0035       0.0052
-0.0077     -0.0070      -0.0028       0.0046
-0.0076     -0.0071      -0.0020       0.0040
-0.0074     -0.0072      -0.0013       0.0032
-0.0071     -0.0071      -0.0005       0.0025
-0.0068     -0.0071       0.0003       0.0016
-0.0065     -0.0070       0.0011       0.0008
-0.0062     -0.0068       0.0019      -0.0001
-0.0058     -0.0065       0.0026      -0.0009
-0.0054     -0.0062       0.0033      -0.0018
-0.0050     -0.0059       0.0040      -0.0026
-0.0045     -0.0055       0.0046      -0.0034
-0.0041     -0.0050       0.0052      -0.0041
-0.0036     -0.0045       0.0057      -0.0047
-0.0031     -0.0040       0.0061      -0.0053
-0.0026     -0.0034       0.0065      -0.0058
-0.0020     -0.0028       0.0067      -0.0062
-0.0015     -0.0022       0.0069      -0.0065
-0.0009     -0.0016       0.0070      -0.0067
-0.0004     -0.0009       0.0071      -0.0067
 0.0002     -0.0003       0.0070      -0.0067
 0.0007      0.0004       0.0068      -0.0065
 0.0013   0.0010          0.0066      -0.0063
 0.0018      0.0017       0.0063      -0.0059
 0.0024      0.0023       0.0059      -0.0054
 0.0029      0.0029       0.0054      -0.0049
 0.0034      0.0035       0.0048      -0.0043
 0.0039      0.0041       0.0042      -0.0036
 0.0044      0.0046       0.0036      -0.0028
 0.0048      0.0051       0.0029      -0.0020
 0.0053      0.0055       0.0022      -0.0012
 0.0057      0.0059       0.0014      -0.0003
 0.0061      0.0063       0.0006       0.0006
 0.0064      0.0066      -0.0002       0.0014
 0.0067      0.0068      -0.0009       0.0022
 0.0070      0.0070      -0.0017       0.0030
 0.0073      0.0071      -0.0025       0.0038
 0.0075      0.0072      -0.0032       0.0045
 0.0077      0.0072      -0.0039       0.0051
 0.0078      0.0071      -0.0045       0.0056
 0.0079      0.0070      -0.0051       0.0060
 0.0080      0.0068      -0.0056       0.0064
 0.0081      0.0066      -0.0060       0.0066
 0.0081      0.0063      -0.0064       0.0067
0.0080       0.0059      -0.0067       0.0067
 0.0079      0.0055      -0.0069       0.0066
 0.0078      0.0051      -0.0070       0.0064
 0.0077      0.0046      -0.0071       0.0061
 0.0075      0.0041      -0.0070       0.0057
```

0.0072	0.0035	−0.0069	0.0051
0.0070	0.0029	−0.0066	0.0046
0.0067	0.0023	−0.0063	0.0039
0.0064	0.0017	−0.0055	0.0031
0.0060	0.0010	−0.0050	0.0024
0.0056	0.0004	−0.0044	0.0015
0.0052	−0.0003	−0.0037	0.0007
0.0048	−0.0009	−0.0030	−0.0002
0.0043	−0.0016	−0.0023	−0.0010
0.0038	−0.0059	−0.0015	−0.0019
0.0033	−0.0062	−0.0008	−0.0027
0.0028	−0.0065	0.0000	−0.0035
0.0023	−0.0065	0.0008	−0.0042
0.0017	−0.0068	0.0016	−0.0048
0.0012	−0.0070	0.0023	−0.0054
0.0006	−0.0070	0.0031	−0.0059
0.0001	−0.0070	0.0037	−0.0062
−0.0005	−0.0070	0.0044	−0.0065
			−0.0067
			−0.0067

APPENDIX B1-2

Simplified HTGR Model Acoutic Subroutine

Creation of M-file to be linked with Flownex for the Acoustics Application Input Variables Constant Variables Determination of Constants First Interface boundary at the exit of the compressor to the first pipe Second Interface Boundary End Interface Boundary Constructing the Matrix Checking of Matrix Application of Forced Responses- Plotting Continuous Pressure Distribution in Loop

Creation of M-file to be linked with Flownex for the Acoustics Application

Defining of the Global Variables
global kelvin Po f R G M
format long

Input Variables

Piping, Temperature Inputs to coding from Flownex model This would need more detail for a proper representation of the PBMR plant
P = [1;2;3;4]
Lp = [6.74;33.4;25;37]
T = [108;696;710;80]

P =

1
2
3
4

Lp =

6.74000000000000
33.40000000000000
25.00000000000000
37.00000000000000

T =

108
696
710
80

Constant Variables

kelvin = 273.15;
Po = 0.01;

```
f = 100;
R = 8.314;
G = 1.667;
M = 0.004;
```

Determination of Constants

```
Co = sqrt(G*R*(T+kelvin)/M);
Wp = 2*pi*f;
A = (Wp*Lp)./(Co);
```

First Interface boundary at the exit of the compressor to the first pip

Application of Summation of Series(Jolley)

```
        % P1(x=l) = P2(x=0)
x11=1;
x21 = 0;
```

Second Interface Boundary

Application of Summation of Jolley Series

```
        % P2(x=l) = P3(x=0)
x22 = 1;
x32 = 0;
```

End Interface Boundary

```
 % Application of Summation of Jolley Series
      % P3(x=l) = P4(x=0)
      % At interface Aj = B(j-1) : A4 = B3
x33 = 1;
x43= 0;

KValues=[K0;K1;K2;K3;K4;K5;K6;K7;K8;K9;K10];
```

Constructing the Matrix

```
BM1 = [(K2-K1); -K4; 0];
BM2 = [(-K3); (K5+K6); -K8];
BM3 = [ 0; -K7; (K9-K10)];
C = Po*K0;
Mtx = [BM1,BM2,BM3]
Con = [C; 0; 0];
DeterMatrix = det(Mtx);
w = Mtx;
w(1:3,1)=[Con];
B1 = det(w)/det(Mtx)
m = Mtx;
m(1:3,2) = [Con];
B2 = det(m)/det(Mtx)
n = Mtx;
n(1:3,3) = [Con];
B3 = det (n)/det(Mtx)
```

```
Mtx =

  -2.53772394030833   3.24935333328282                   0
   3.24935333328282  -3.69719029408620  -3.70902746586376
                   0  -3.70902746586376   0.33052833115732


B1 =

   0.00506936943576


B2 =

    3.634790087039232e-004


B3 =

   0.00407878387256
```

Checking of Matrix

```
a= B1*(K2-K1) + B2*(-K3)+ B3*(0)
b= B1*(-K4)+ B2*(K5+K6) + B3*(-K7)
c = B1*(0) + B2*(-K8)+ B3*(K9-K10)

a =

  -0.01168358845088

b =

    1.734723475976807e-018

c =

   -2.168404344971009e-019
```

Application of Forced Responses- Plotting

```
xl = 0:0.0125:1;
xL = [xl'];

% All Pipes
PipePressDist = [(Po*cos(A(1,1)*(1-
xL))+(B1*Co(1,1)/Wp)*sin(A(1,1)*xL))/cos(A(1,1)),(B1*(Co(2,1)/Wp)*((1/A(2,1))+...
    ((cos(A(2,1)*(1-xL)))/sin(A(2,1)))))-
(B2*(Co(2,1)/Wp)*(((cos(A(2,1)*xL))/sin(A(2,1)))),(B2*(Co(3,1)/Wp)*((1/A(3,1))+...
    ((cos(A(3,1)*(1-xL)))/sin(A(3,1)))))-(B3*(Co(3,1)/Wp)*(((cos(A(3,1)*xL))/sin(A(3,1)))),...
    (-B3*Co(4,1)/Wp)* (sin(A(4,1)*(1-xL))/cos(A(4,1)))]

plot(xL,PipePressDist)
```

```
grid on
xlabel('Pipe Increments x/L')
ylabel('Pipe Acoustic Pressure (kPa)')
legend('Pipe 1','Pipe 2','Pipe 3','Pipe 4');
legend('Orientation','Vertical','Location','NorthEastOutside')
title('Pipe Acoustic Pressures')

format short

   a1= Lp(1,1)*xl;
   b1= Lp(1,1)+ Lp(2,1)*xl;
   c1= Lp(1,1)+Lp(2,1)+(Lp(3,1)*xl);
   d1=Lp(1,1)+Lp(2,1)+Lp(3,1)+Lp(4,1)*xl;


PipePressDist =

   0.01000000000000  -0.00479052575579  -0.01582598301391   0.01058475641071
   0.00976878989147  -0.00261907863326  -0.01562242906074   0.01208641682495
   0.00951685538239  -0.00036764087232  -0.01524081488304   0.01275872925276
   0.00924473094957   0.00191772855548  -0.01468545536588   0.01255556083554
   0.00895299390256   0.00419027651873  -0.01396262992087   0.01149085262134
   0.00864226315890   0.00640351218219  -0.01308051148523   0.00963766295545
   0.00831319793109   0.00851215809684  -0.01204907411102   0.00712315434932
   0.00796649632809   0.01047307646696  -0.01087998018917   0.00411986781981
   0.00760289387423   0.01224615164510  -0.00958644858369   0.00083388343781
   0.00722316194888   0.01379511080126  -0.00818310516683  -0.00250932051224
   0.00682810614994   0.01508826597722  -0.00668581744539  -0.00568033944301
   0.00641856458475   0.01609916234552  -0.00511151514794  -0.00846158379066
   0.00599540609209   0.01680711941129  -0.00347799880149  -0.01066220961584
   0.00555952839892   0.01719765408514  -0.00180373846223  -0.01213121396842
   0.00511185621588   0.01726277697214  -0.00010766487583  -0.01276779643589
   0.00465333927551   0.01700115581552   0.00159104457114  -0.01252827588468
   0.00418495031740   0.01641814275142   0.00327318268864  -0.01142908778030
   0.00370768302455   0.01552566481713   0.00491972965750  -0.00954565641552
   0.00322254991526   0.01434197995276   0.00651206808519  -0.00700721943228
   0.00273058019509   0.01289130348793   0.00803219351140  -0.00398795976926
   0.00223281757341   0.01120331275469   0.00946291798325  -0.00069505354477
   0.00173031804919   0.00931253996109   0.01078806439823   0.00264554599248
   0.00122414767066   0.00725766574571   0.01199264941763   0.00580461296523
   0.00071538027378   0.00508072786534   0.01306305288206   0.00856537793132
   0.00020509520403   0.00282626120417   0.01398717181370   0.01073840220938
  -0.00030562497336   0.00054038669773   0.01475455726380   0.01217457681737
  -0.00081569677010  -0.00173013218992   0.01535653245824   0.01277535406024
  -0.00132403807342  -0.00393884613349   0.01578629090518   0.01249950969397
  -0.00182957044179  -0.00604057018741   0.01603897335564   0.01136597165834
  -0.00233122139280  -0.00799230816051   0.01611172274666   0.00945252127603
  -0.00282792667843  -0.00975413221405   0.01600371650591   0.00689045603950
  -0.00331863254286  -0.01128999968883   0.01571617585247   0.00385558021529
  -0.00380229795797  -0.01256849045076   0.01525235198859   0.00055614147434
  -0.00427789683190  -0.01356344967084   0.01461748933857  -0.00278145868521
  -0.00474442018589  -0.01425452288971   0.01381876625042  -0.00592820019796
  -0.00520087829481  -0.01462757242064   0.01286521383078  -0.00866815937244
  -0.00564630278685  -0.01467496657245   0.01176761383079  -0.01081332518295
  -0.00607974869792  -0.01439573577540   0.01053837673764  -0.01221650024492
  -0.00650029647641  -0.01379559241630   0.00919140144997  -0.01278140123226
  -0.00690705393395  -0.01288681397685   0.00774191812408  -0.01246926566450
  -0.00729915813822  -0.01168799186606   0.00620631596751  -0.01130151171777
```

```
-0.00767577724365  -0.01022365108500   0.00460195792749  -0.00935826854851
-0.00803611225618  -0.00852374850460   0.00294698436924  -0.00677287797612
-0.00837939872829  -0.00662306002048   0.00126010796414  -0.00372274480937
-0.00870490838080  -0.00456046912227  -0.00043959789313  -0.00041716365034
-0.00901195064788  -0.00237817143145  -0.00213291474620   0.00291704252125
-0.00929987414210  -0.00012081148094  -0.00380069637869   0.00605108652928
-0.00956806803635   0.00216543060403  -0.00542408529921   0.00876991596201
-0.00981596335966   0.00443378383960  -0.00698472596169   0.01088696967828
-0.01004303420433   0.00663784320452  -0.00846497231025   0.01225697929440
-0.01024879884159   0.00873251897462  -0.00984808730199   0.01278593723699
-0.01043282074358   0.01067495914957  -0.01111843215148   0.01243754737205
-0.01059470950950   0.01242542610111  -0.01226164315751   0.01123571557982
-0.01073412169376   0.01394810950886  -0.01326479411242   0.00926290937662
-0.01085076153467   0.01521185895287  -0.01411654245786   0.00665449914360
-0.01094438158183   0.01619082117585  -0.01480725753428   0.00358946925685
-0.01101478322117   0.01686496897831  -0.01532912947415   0.00027813650435
-0.01106181709623   0.01722051092662  -0.01567625750761  -0.00305228147027
-0.01108538342506   0.01725017349238  -0.01584471668203  -0.00617325743011
-0.01108543221190   0.01695334985123  -0.01583260224118  -0.00887063566916
-0.01106196335326   0.01633611229701  -0.01564005116221  -0.01095932698825
-0.01101502663811   0.01541108801731  -0.01526924060684  -0.01229600917990
-0.01094472164227   0.01419720077176  -0.01472436330432  -0.01278896153812
-0.01085119751716   0.01271928375764  -0.01401158014458  -0.01240435856675
-0.01073465267338   0.01100757158277  -0.01313895051742  -0.01116859102367
-0.01059533435978   0.00909708173852  -0.01211634118560  -0.00916645503485
-0.01043353813891   0.00702689822665  -0.01095531472200  -0.00653533353807
-0.01024960726002   0.00483937199514  -0.00966899877241  -0.00345576931514
-0.01004393193081   0.00257925454020  -0.00827193762218  -0.00013907647380
-0.00981694848967   0.00029278239896  -0.00677992774495   0.00318715954275
-0.00956913847993  -0.00197326873834  -0.00520983919307   0.00629469845597
-0.00930102762833  -0.00417254094582  -0.00357942484903   0.00897030658563
-0.00901318472964  -0.00626004243297  -0.00190711969488   0.01103038855793
-0.00870622044001  -0.00819306796793  -0.00021183236916   0.01233358528685
-0.00838078598142  -0.00993207252322   0.00148726863177   0.01279047377808
-0.00803757176018  -0.01144148027135   0.00317097169059   0.01236970317256
-0.00767730590221  -0.01269041238027   0.00482023929344   0.01110014598559
-0.00730075270829  -0.01365331871953   0.00641642328577   0.00906891692715
-0.00690871103267  -0.01431050055418   0.00794147572575   0.00641539524869
-0.00650201258826  -0.01464851353327   0.00937815295109   0.00332166079181
-0.00608152018219  -0.01466044272871   0.01071021055190   0
```

Continuous Pressure Distribution in Loop

```
TotalPipeLength=[a1,b1,c1,d1]';
PipeDist1=[(Po*cos(A(1,1)*(1-
xL))+(B1*Co(1,1)/Wp)*sin(A(1,1)*xL))/cos(A(1,1));(B1*(Co(2,1)/Wp)*((1/A(2,1))+...
   ((cos(A(2,1)*(1-xL)))/sin(A(2,1)))))-
(B2*(Co(2,1)/Wp)*(((cos(A(2,1)*xL))/sin(A(2,1)))));(B2*(Co(3,1)/Wp)*((1/A(3,1))+...
   ((cos(A(3,1)*(1-xL)))/sin(A(3,1)))))-(B3*(Co(3,1)/Wp)*(((cos(A(3,1)*xL))/sin(A(3,1)))));...
   (-B3*Co(4,1)/Wp)* (sin(A(4,1)*(1-xL))/cos(A(4,1)))];
Pipegrad1= (A(1,1)*((Po*sin(A(1,1)*(1-
xl))+(B1*Co(1,1)/Wp)*cos(A(1,1)*xl))/cos(A(1,1)))/Lp(1,1))';
Pipegrad2=(A(2,1)*(((Co(2,1)*B1/Wp)*(sin(A(2,1)*(1-
xl))/sin(A(2,1))))+((B2*Co(2,1)/Wp)*(sin(A(2,1)*xl)/sin(A(2,1)))))/Lp(2,1))';
Pipegrad3=(A(3,1)*(((Co(3,1)*B2/Wp)*(sin(A(3,1)*(1-
xl))/sin(A(3,1))))+((B3*Co(3,1)/Wp)*(sin(A(3,1)*xl)/sin(A(3,1)))))/Lp(3,1))';
```

```
Pipegrad4=((A(4,1)/Lp(4,1)*B3*Co(4,1)/Wp)*(cos(A(4,1)*(1-xl))/cos(A(4,1))))';
TotalPipeGrad = [Pipegrad1;Pipegrad2;Pipegrad3;Pipegrad4]
```

TotalPipeGrad =

-0.0026	0.0055	0.0041	
-0.0029	0.0055	0.0041	
-0.0031	0.0054	0.0024	
-0.0033	0.0052	0.0005	
-0.0036	0.0049	-0.0014	
-0.0038	0.0044	-0.0032	
-0.0040	0.0039	-0.0048	
-0.0042	0.0033	-0.0060	
-0.0044	0.0027	-0.0069	
-0.0046	0.0020	-0.0072	
-0.0048	0.0012	-0.0071	
-0.0049	0.0005	-0.0065	
-0.0051	-0.0003	-0.0054	
-0.0052	-0.0011	-0.0040	
-0.0054	-0.0019	-0.0023	
-0.0055	-0.0026	-0.0004	
-0.0056	-0.0032	0.0015	
-0.0057	-0.0038	0.0033	
-0.0058	-0.0044	0.0048	
-0.0059	-0.0048	0.0061	
-0.0059	-0.0051	0.0069	
-0.0060	-0.0053	0.0073	
-0.0060	-0.0055	0.0071	
-0.0060	-0.0055	0.0065	
-0.0061	-0.0054	0.0054	
-0.0061	-0.0052	0.0039	
-0.0060	-0.0048	0.0022	
-0.0060	-0.0044	0.0004	
-0.0060	-0.0039	-0.0015	
-0.0059	-0.0033	-0.0033	
-0.0059	-0.0027	-0.0049	
-0.0058	-0.0019	-0.0061	
-0.0057	-0.0012	-0.0069	
-0.0056	-0.0004	-0.0073	
-0.0055	0.0004	-0.0071	
-0.0054	0.0004	-0.0064	
-0.0052	0.0009	-0.0053	
-0.0051	0.0015	-0.0039	
-0.0049	0.0020	-0.0022	
-0.0047	0.0026	-0.0003	
-0.0046	0.0031	0.0016	
-0.0044	0.0035	0.0034	
-0.0042	0.0039	0.0050	
-0.0040	0.0043	0.0062	
-0.0038	0.0046	0.0070	
-0.0035	0.0049	0.0073	
-0.0033	0.0051	0.0071	
-0.0031	0.0053	0.0064	
-0.0028	0.0054	0.0053	
-0.0026	0.0054	0.0038	
-0.0023	0.0054	0.0021	

-0.0021	0.0053	0.0002	
-0.0018	0.0052	-0.0017	
-0.0015	0.0050	-0.0035	
-0.0012	0.0047	-0.0050	
-0.0010	0.0044	-0.0062	
-0.0007	0.0041	-0.0070	
-0.0004	0.0036	-0.0073	
-0.0001	0.0032	-0.0071	
0.0001	0.0027	-0.0064	
0.0004	0.0022	-0.0052	
0.0007	0.0017	-0.0037	
0.0010	0.0011	-0.0020	
0.0012	0.0005	-0.0001	
0.0015	-0.0001	0.0018	
0.0018	-0.0006	0.0035	
0.0021	-0.0012	0.0051	
0.0023	-0.0018	0.0062	
0.0026	-0.0023	0.0070	
0.0028	-0.0028	0.0073	
0.0031	-0.0033	0.0070	
0.0033	-0.0037	0.0063	
0.0035	-0.0041	0.0052	
0.0038	-0.0045	0.0037	
0.0040	-0.0048	0.0019	
0.0042	-0.0050	0.0000	
0.0044	-0.0052	-0.0018	
0.0046	-0.0054	-0.0036	
0.0047	-0.0054	-0.0051	
0.0049	-0.0054	-0.0063	
0.0051	-0.0054	-0.0070	
0.0051	-0.0053	-0.0073	
0.0053	-0.0051		
0.0055	-0.0049		
0.0055	-0.0046		
0.0054	-0.0043		
0.0052	-0.0039		
0.0049	-0.0034		
0.0045	-0.0030		
0.0040	-0.0025		
0.0034	-0.0019		
0.0028	-0.0014		
0.0021	-0.0008		
0.0013	-0.0003		
0.0005	0.0003		
-0.0002	0.0009		
-0.0010	0.0015		
-0.0018	0.0020		
-0.0025	0.0025		
-0.0032	0.0030		
-0.0038	0.0035		
-0.0043	0.0039		
-0.0047	0.0043		
-0.0051	0.0046		
-0.0053	0.0049		
-0.0055	0.0051		
-0.0055	0.0053		
-0.0054	0.0054		
-0.0052	0.0054		

-0.0049 -0.0045 -0.0040 -0.0034 -0.0027 -0.0020 -0.0013 -0.0005 0.0003 0.0011 0.0018 0.0025 0.0032 0.0038 0.0043 0.0048 0.0051 0.0053	0.0054 0.0053 0.0052 0.0050 0.0047 0.0044		

```
plot(TotalPipeLength,TotalPipeGrad,TotalPipeLength,PipeDist1)
xlabel('Pipe Length (m)');
ylabel('Pipe Pressure Distribution (kPa)');
grid on
title('Pipe Acoustic Pressures')
h=legend('Total Pipe Gradient','Total Pipe Pressure Distribution');
legend('Orientation','Horizontal','Location','NorthEast')
title('Total Pipe Acoustic Pressure Distribution'
```

