Bull. Korean Math. Soc. **62** (2025), No. 3, pp. 631–639
https://doi.org/10.4134/BKMS.b210738
pISSN: 1015-8634 / eISSN: 2234-3016

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Reprinted from the
Bulletin of the Korean Mathematical Society
Vol. 62, No. 3, May 2025

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ON A CONJECTURE BY GRIGGS, KLEITMAN AND SHASTRI

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Abstract. Let G be a simple, connected graph. The *leaf number*, $L(G)$, is the maximum number of leaf vertices contained in a spanning tree of G. The cardinality of the smallest connected dominating set in G is the *connected domination number*, $\gamma_c(G)$. The *girth* $g(G)$ is the length of a shortest cycle in a graph. If G has girth $g(G) \geq 5$, minimum degree $\delta(G) \geq 3$, order $n(G)$, leaf number $L(G)$ and connected domination number $\gamma_c(G)$, we prove that $L(G) \geq \frac{2}{5}n(G)+\frac{4}{5}$. That is, $\gamma_c(G) \leq \frac{3}{5}n(G)-\frac{4}{5}$ and the bound is asymptotically sharp. This settles completely, a conjecture by Griggs, Kleitman and Shastri.

1. Introduction

Let $G = (V(G), E(G))$ be a simple, connected graph. The order of G denoted $n(G)$ is the number of vertices in G. The *degree* $\deg_G(v)$ of a vertex $v \in V(G)$ is the number of edges incident with v in G. The *minimum degree* of G, denoted $\delta(G)$, is defined as the smallest value of the degrees of vertices of G. The *leaf number* $L(G)$ of G is the maximum number of leaf vertices contained in a spanning tree of G, where a *leaf* is a vertex of degree one. The *girth* $g(G)$ of G is the length of a shortest cycle in G. A *cubic* graph is the one where every vertex is of degree 3. For some of the notation defined here, we drop the argument G where there is no danger of confusion.

A set $S \subseteq V(G)$ is *dominating* if every vertex in $V(G) \setminus S$ has a neighbour in S. S is a *connected dominating* set if it is dominating and the subgraph induced by S denoted $G[S]$ is connected. The cardinality of the smallest connected dominating set is the *connected domination number*, $\gamma_c(G)$. The leaf number and the connected domination number are related by formula $L(G) = n(G) - \gamma_c(G)$. We will find it most convenient to work with the leaf number of a graph in order to establish the results of this paper.

Received October 4, 2021; Revised May 3, 2022; Accepted February 28, 2025.
2020 *Mathematics Subject Classification.* 05C05.
Key words and phrases. Leaf number, connected domination number, minimum degree, order.

The leaf number of a graph, besides being a well-known and interesting graph theoretical parameter, is a crucial tool in the design and analysis of networks (see for instance [4, 5] for further details). The problem of finding a spanning tree with a prescribed number of leaves is known to be NP-complete, see for instance [5]. The question of determining spanning trees with many leaves in a graph was raised by Lovász and Saks[1], see also [8]. Payan, Tchuente and Xuong [18] proved that $L(G) \geq \frac{n(G)}{4}$ for cubic graphs. Storer [19] stated a similar result without proof that every cubic graph has leaf number at least $\frac{n(G)}{4} + 2$, see also a survey in [17]. Later, Linial[2], put forward the following conjecture, which attempts to generalize Storer's statement:

Conjecture 1. *Let G be a connected graph with minimum degree δ, leaf number $L(G)$ and order $n(G)$. Then $L(G) \geq \dfrac{\delta - 2}{\delta + 1} n(G) + c_\delta$, where c_δ is a positive constant that depends only on δ.*

Caro, West and Yuster [3] pointed out that a result by Alon [1] disproves Linial's conjecture for δ sufficiently large. For small $\delta \leq 5$, Conjecture 1 has been proved to be correct [6–8]. Mostly, the results follow due to brilliant ideas by Kleitman and West [8] known as the 'dead leaves method' (see for instance, [6, 10, 11]). For a brief introduction and understanding of the method, we refer the reader to [13]. For $\delta \geq 6$, Conjecture 1 is open to date. There is no doubt that Conjecture 1 opened a huge space for research on aspects of bounds on the leaf number (see for instance, [2, 6, 7, 10, 11, 13]), with some authors concentrating on graphs with some forbidden subgraphs. In addition, bounds of Linial's type proved to be a powerful tool in the establishment of sufficient conditions for the existence of spanning paths or cycles in a graph, see for example [12, 14–16].

Motivated by Conjecture 1, Griggs, Kleitman and Shastri [6] extended the study of lower bounds on the leaf number to graphs with some forbidden subgraphs. In the same paper [6], they posed the following conjecture, which has remained open since 1988 to date:

Conjecture 2 ([6])**.** *Let G be a simple connected, cubic graph with girth $g(G) \geq 5$, leaf number $L(G)$ and order $n(G)$. Then $L(G) \geq \dfrac{2}{5} n(G) + d$, where d is a positive constant.*

The purpose of this paper is to settle Conjecture 2 completely. To this end, we apply the dead leaves method in conjunction with some new techniques to establish the following theorem which is stronger than the conjecture:

Theorem 1. *Let G be a simple connected, graph with girth $g(G) \geq 5$, minimum degree $\delta \geq 3$, leaf number $L(G)$ and order $n(G)$. Then $L(G) \geq \dfrac{2}{5} n(G) + \dfrac{4}{5}$ and the bound is tight, apart from the additive constant.*

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[1] Private communication by L. Lovász and M. E. Saks

[2] Personal communication by N. Linial 1988

We mention here that attempts to settle Conjecture 2 were made in [9], where the proof to the bound $L(G) \geq \lceil \frac{3n(G)}{8} + \frac{9}{8} \rceil$ was presented. An infinite family of graphs which seemed at first to be a family attaining the bound $L(G) = \frac{6n(G)}{16} + 2$ was constructed in [9]. However, after some review, it was found that the said family meets the bound $L(G) = \frac{7n(G)}{16} + 2$. Hence, the aforementioned bound failed to be asymptotically sharp. Thus, the results in this paper also fill this gap. As part of further study, one could think to extend the results of this paper to graphs of given girth $g(G) \geq 6$.

We use the following notation, apart from those already defined: If H is a subgraph of a graph G, we write $H \leq G$. $V(G-H)$ denotes the set of vertices in G that are not in H.

2. Main results

Here, we apply the dead leaves method to settle Conjecture 2. We start off by describing the method: **The dead leaves approach method.** Generally, a spanning tree with many leaves is obtained for G by construction. We start off with a small tree T_0 of G which satisfies the hypothesis of the theorem. We then grow it through a series of iterations until we obtain a spanning tree T of G with many leaves. Let T_{i-1} be a current tree of G. We say a vertex $v \in V(T_{i-1})$ is 2-*split* if it has two neighbours in $V(G-T_{i-1})$. At each stage i, let T_i be a tree of G that is constructed by expanding the tree T_{i-1}. That is, we construct the tree T_i by adding n_i new vertices v to T_{i-1} and suitable edges uv. A leaf r of T_i is *dead* in T_i if r has no neighbour outside the tree T_i, otherwise it is *alive*. During the construction of T_i from T_{i-1}, we need to keep track of the number of leaves l_i gained (note $l_i = L(T_i) - L(T_{i-1})$, that is number of leaves in T_i minus number of leaves in T_{i-1}), the number of dead leaves d_i gained and the number of new vertices n_i added. The method therefore requires one to define a cost increasing function $C(l_i, d_i, n_i)$, where l_i, n_i and d_i are as defined previously. This is a function that requires one to find a nice initial tree, say T_0 of G with l_0 leaves, d_0 dead leaves and n_0 vertices such that $C(l_0, d_0, \eta_0)$ satisfies the theorem to be proved. We then expand the initial tree T_0 and subsequent partial trees to a spanning tree T of G. This is done by expanding the current tree, say T_{i-1} to T_i preserving $C(l_i, d_i, n_i) \geq 0$ at each stage i. Notice dead leaves of T_{i-1} are also dead in T_i and so the number of dead leaves gained at stage i are the dead leaves of the tree T_i created by expanding T_{i-1} to T_i. The final tree obtained after all the stages are exhausted becomes a spanning tree, say T, of G with all leaves dead and the result will follow.

Recall the statement of Theorem 1:

Theorem 1. *Let G be a simple, connected graph with girth $g(G) \geq 5$, minimum degree $\delta \geq 3$, order $n(G)$ and leaf number $L(G)$. Then $L(G) \geq \dfrac{2}{5}n(G) + \dfrac{4}{5}$ and the bound is sharp, apart from the additive constant.*

Proof. Assume G satisfies the hypothesis of the theorem. The following terminology is crucial in the construct of the proof.

Let $T_{i-1} \leq G$ be the current tree. If x and y are leaf vertices in T_{i-1} such that they share an interior neighbour, say z in T_{i-1}, we say x and y are *partners.* We call z a *child* for x and y. In this case, we also call x and y *guardians* for z. In addition, we call the set of elements x, y and z, a *family.* We call u a *leaf neighbour* of x and vice-versa if x and u are adjacent in G and both are leaf vertices in T_{i-1}. Let A be a family. We say A is a *strong family* if there exists a family B such that each of the partners in A has a leaf neighbour in B. In this case, if G is cubic and $B \neq A$, then B is also a strong family. In such a case where both A and B are strong families, we say A and B are *strongly related.* A family A that is not strong is called a *weak family.*

Define a cost increasing function $C(l_i, d_i, n_i)$ by

$$C(l_i, d_i, n_i) = 4l_i + d_i - 2n_i,$$

where at each stage i, l_i is the number of leaves gained, d_i is the number of dead leaves gained by adding n_i new vertices in expanding the current tree T_{i-1} to T_i. We start off with a small tree of G that satisfies the theorem. We then grow the tree by a series of steps, where at each stage $i \geq 1$, the addition of n_i new vertices, a gain of $d_i \geq 0$ dead leaves and a gain of $l_i \geq 0$ leaves preserve $C(l_i, d_i, n_i) \geq 0$. The argument is based on the fact that profit made at earlier stages by gaining l_i leaves compensates the loss accrued at a later stage where the addition of new vertices brings no gain in the number of leaves.

Clearly a star $K_{1,3}$ exists in G, since $\delta \geq 3$. Further, its leaf vertices are all 2-split with none of them sharing a neighbour, since $g(G) \geq 5$. Let $T_0 \leq G$ be a tree of order $n_0 = 10$ and leaf number 6 obtained from the aforementioned arguments, be our initial tree. That is, T_0 is the base case, since such a tree exists always in G. Then

$$C(l_0, d_0, n_0) \geq 4.$$

So, the initial tree satisfies the theorem. At each stage i, let T_{i-1} be the current tree which satisfies the theorem. Let l_i, d_i and n_i be the number of leaves gained, number of dead leaves gained and number of new vertices added, respectively, in expanding T_{i-1} to T_i. Then, the following operations guarantee that at each stage we can expand T_{i-1} to T_i at every stage i maintaining $C(l_i, d_i, n_i) \geq 0$:

(i) We first note that if there is a vertex not in T_{i-1} that is joined to an interior vertex of T_{i-1}, then $C(1, 0, 1) = 2$. Thus this operation yields a profit of 2 for every such vertex added. So at every stage, we shall assume that, no interior vertex of T_{i-1} has a neighbour outside T_{i-1}, otherwise we are done easily. Using the same argument, in what follows we assume that every vertex is of degree 3, otherwise we quickly gain leaves.

(ii) Assume first that there is a leaf vertex $x \in V(T_{i-1})$ that has two neighbours in $V(G - T_{i-1})$. Then we gain a leaf and two new vertices. So,

$$C(l_i, d_i, n_i) \geq C(1, 0, 2) \geq 0.$$

Thus, if there exists a vertex with two neighbours out, join it to all its neighbours. Do this until no leaf of the current tree has two neighbours outside it.

(iii) Assume there is a vertex v not in T_{i-1} that has three neighbours, say x, y and z in T_{i-1}. By the previous arguments, x, y and z are leaves. Let $T_i = T_{i-1} \cup \{xv\}$. Then y and z are two dead leaves gained and v is one new vertex added. Thus we have $C(0, 2, 1) = 0$. So, in what follows, we assume this operation does not hold, otherwise the proof is complete.

From the previous arguments, it suffices to say each leaf in T_{i-1} has at most one neighbour in $V(G - T_{i-1})$. Clearly, if all leaves of T_{i-1} are dead, then we are done by the previous arguments. Hence, we assume there exists at least one live leaf in the current tree. The next two operations contain a new technique of generating a gain in the leaf or dead leaves and is very crucial in the establishment of results for the remaining part of the proof.

(iv) Let A and B be strongly related families in the current tree T_{i-1}. If $A \cup B$ has at least one live leaf, then we can expand the current tree T_{i-1} to a tree T_i without incurring a loss, that is, preserving $C(l_i, d_i, n_i) \geq 0$. To show this, let $A = \{x, x', x''\}$ and $B = \{y, y', y''\}$ where u is a child for u' and u'' for $u = x$ or $u = y$. Further, u' and u'' are partners. Since A and B are strongly related, we can assume that x' and x'' are adjacent to y' and y'', respectively. In addition, since not all leaves are dead, we assume that y' has a neighbour $q' \in V(G - T_{i-1})$, other cases are treated likewise. Assume first that y'' is not dead. Since $g(G) \geq 5$, y' and y'' are neither adjacent nor share a neighbour, apart from y. So, if $q'' \in V(G - T_{i-1})$ is a neighbour of y'' then $q' \neq q''$. From T_{i-1}, delete edges xx', xx'' and add edges $x'y', y'q', x''y'', y''q''$. Then, with y' and y'' replaced by q' and q'', respectively, x is a leaf gained and x is also a dead leaf gained. Note here that q' and q'' are two new vertices gained. Thus, we build the tree T_i by this gain using the operation $C(l_i, d_i, n_i) \geq C(1, 1, 2) > 0$.

Assume now that y'' is dead. Then neither x' nor x'' is dead, otherwise, one of these and y'' are two dead leaves gained in expanding to $T_i = T_{i-1} \cup \{y'q'\}$ and we are done by $C(0, 2, 1)$. By similar arguments, we assume that neither x' nor x'' is adjacent to q', otherwise one of these and y'' are two dead leaves of the aforementioned tree and we are done. Let r' and r'' be neighbours of x' and x'', respectively, in $V(G - T_{i-1})$. Then by the aforementioned arguments q', r' and r'' are distinct. Let

$$T_i = (T_{i-1} \cup \{x'y', x'r', x''y'', x''r'', y'q'\}) - \{yy', yy''\}.$$

Then y is a leaf gained. Further, y and y'' are dead leaves in T_i. Thus, we are done by $C(1, 2, 3)$, since the cost function is increasing.

Therefore, this item is established.

(v) Let $A' = \{u, u', u''\}$ be a weak family, where u is a child for u' and u''. If for each of the partners in A', its leaf neighbour in T_{i-1} is alive then we can expand T_{i-1} to T_i maintaining $C(l_i, d_i, n_i) \geq 0$. To prove this, let y' and x'' be live leaf neighbours of u' and u'', respectively. Let r' and r'' be neighbours of y' and x'', respectively, outside T_{i-1}. Assume first that r' and r'' are distinct. Let

$$T_i = (T_{i-1} \cup \{y'u', y'r', x''u'', x''r''\}) - \{uu', uu''\}.$$

Then u is a leaf gained and u is a dead leaf gained as well. Thus, we have $C(l_i, d_i, n_i) \geq C(1, 1, 2) > 0$.

Now consider $r' = r''$. Neither u' nor u'' is dead, otherwise this together with y' are at least two dead leaves in $T_i = T_{i-1} \cup \{x''r''\}$ and

$$C(l_i, d_i, n_i) \geq C(0, 2, 1) = 0.$$

Also r' cannot have a third neighbour in T_{i-1}, otherwise we are done by Item (ii). Note that y' and x'' are not partners, since A' is a weak family or otherwise we would have been done by Item (iii). Let y'' be a partner of y' with y as their child. Further, let z be a leaf neighbour of y''. By properties of G and by following item (ii), $u, u', u'', x'', y, y', y''$ and z are distinct or else, we gain two dead leaves and we are done by $C(0, 2, 1)$. By a similar argument, we assume z is not dead, otherwise itself and y'' are two dead leaves gained by expanding to $T_i = T_{i-1} \cup \{x''r''\}$. Let s' and s'' be neighbours of u' and z, respectively, outside T_{i-1}. By the previous argument $r \notin \{s', s''\}$.

Consider first $s' \neq s''$. Let

$$T_i = (T_{i-1} \cup \{u''r'', u's', u'y', zy'', zs''\}) - \{yy', yy''\}.$$

Then, y is a leaf gained. Also y and y' are dead leaves gained. Therefore, we are done by $C(1, 2, 3)$.

Suppose $s' = s''$. If s' has a third neighbour in T_{i-1}, then y', s' and two neighbours of s are dead in $T_{i-1} \cup \{u's', x''r''\}$. Thus we are done by $C(0, 4, 2)$. Hence, we assume a third neighbour, say t' of s' is not in T_{i-1}. We further assume that $t' \neq r'' = r'$, otherwise we gain four dead leaves in $T_{i-1} \cup \{u's', s't'\}$ and the result follows. Let

$$T_i = (T_{i-1} \cup \{zs', s't', s'u', x''r'', x''u''\}) - \{uu', uu''\}.$$

Then, u and y' are dead leaves gained. In addition, u is leaf gained. Thus, we are done by $C(1, 2, 3)$.

Therefore all the sub-cases of this case are exhausted. To finish the proof, it is enough to consider a weak family such that one of its live leaf has a leaf neighbour which is dead in T_{i-1}.

(vi) Assume there is a weak family, satisfying the previously mentioned condition. We can consider it to be A' as defined in Item (iv). We also, consider its leaf neighbours as defined in the previous item. Assume that u' is alive and y' is dead, the other cases are treated likewise. Again, let s' be a neighbour of u' outside T_{i-1} and y'' be a partner of y', with z as a leaf neighbour of y''. We assume z is not dead in T_{i-1}, otherwise, y' and z are dead in $T_{i-1} \cup \{u's'\}$ and the result follows by $C(0, 2, 1)$. Note also that if z is adjacent to s' then y' and z are dead again in $T_{i-1} \cup \{u's'\}$ and we are done. So, we consider z not adjacent to s'. Let s'' be a neighbour of z outside T_{i-1}. Evidently, $s' \neq s''$. Let

$$T_i = (T_{i-1} \cup \{zy'', zs'', u'y', u's'\}) - \{yy', yy''\}.$$

Then, y is a leaf gained and y, y' are dead leaves gained. Finally,

$$C(l_i, d_i, n_i) \geq C(1, 2, 2) > 0$$

as required.

Therefore by summing up, we have grown T_0 by series of steps to a spanning tree $T \leq G$, where at each stage, we preserve the cost function positive. Finally if $L(T)$ is the number of leaves in T, then

$$5L(T) - 2n \geq 4.$$

Consequently, $L(G) \geq L(T)$ and the result is established.

To show that the bound is sharp, apart from the additive constant, we construct a family of cubic graphs, $\mathcal{G}$ as follows: For finite integers i and k with $1 \leq i \leq k$ and $k \geq 3$, let H_i be a component with vertex set

$$V(H_i) = \{x_0^i, x_1^i, x_2^i, x_3^i, x_4^i, x_5^i, x_6^i, x_7^i, x_8^i, x_9^i\}$$

and edge set

$$\begin{aligned} E(H_i) = \{&x_0^i x_1^i, x_0^i x_2^i, x_1^i x_3^i, x_1^i x_4^i, x_2^i x_5^i, x_2^i x_6^i, x_3^i x_6^i, x_4^i x_5^i, \\ &x_4^i x_7^i, x_6^i x_7^i, x_3^i x_8^i, x_5^i x_8^i, x_7^i x_9^i, x_8^i x_9^i\}. \end{aligned}$$

Take k components $H_1, H_2, H_3, \ldots, H_k$ and add edges $x_9^j x_0^{j+1}$, where $j \in \{1, 2, 3, \ldots, k\}$ is an integer and $x_0^{k+1} = x_0^1$. This forms a regular necklace of

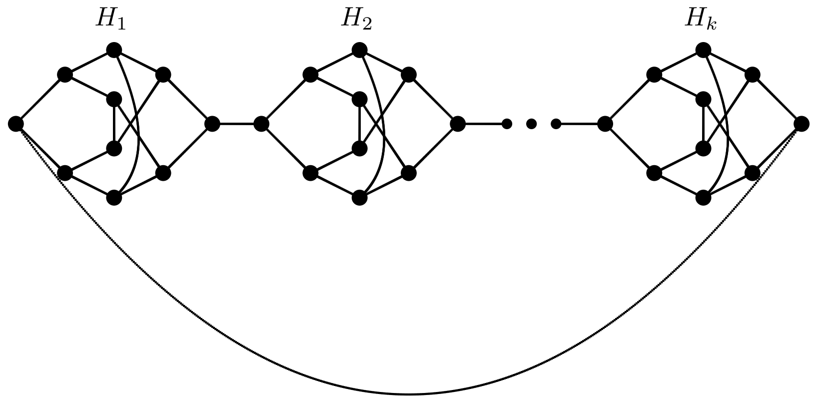

FIGURE 1. A regular necklace of beads, G_k

k beads which we denote by G_k, where each bead is the component isomorphic to H_i, see Figure 1. Let $\mathcal{G}$ be a collection of graphs G_k for all finite $k \geq 3$. Then, each graph G_k in the family $\mathcal{G}$ is a cubic graph with

$$L(G_k) = \frac{2n(G_k)}{5} + 2.$$

Hence the bound is sharp, apart from the additive constant. Thus Conjecture 2 is settled completely. This result imply the corresponding upper bound on the connected domination number. □

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