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An exploration of first year university students' constructions of proofs in elementary set theory

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ABSTRACT

In this thesis I explored the development of first year university students' proof construction abilities in the context of consultative group sessions. In order to do this I investigated students' difficulties in proof construction in the area of elementary set theory and the forms of guidance offered as they participated in consultative group sessions. Vygotsky's (1987) socio-cultural theory is the theoretical framework for the study. His premise that all higher mental functions which include the activity of mathematical proof construction, develop as a result of mediated activity in the context of more knowing others, motivated my exploration. Ten students purposefully chosen from a first year mathematics major class at the University of Limpopo (a historically disadvantaged university) participated in weekly consultative sessions. Students were encouraged to share their thoughts and ideas and critique other students as they attempted proof construction exercises. The lecturer (myself) was present to offer guidance whenever necessary. By establishing the sociomathematical norms pertinent to successful proof construction, my aim was to support students in becoming intellectually autonomous and to empower those with the potential to become more knowing peers to develop their capabilities. With this in mind I investigated the nature of the interactions of the students and lecturer in the consultative sessions. I also traced the journeys of two case study students as they progressed in the first two sessions.

Two complementary analytical frameworks incorporating social and cognitive aspects of students' development enabled me to obtain a holistic picture of the development and scaffolding of proof construction abilities in consultative group sessions.

Students' difficulties were found to be similar to those reported in the literature and included difficulties within meanings of mathematical terms, symbols, signs and definitions, logical reasoning and proof methods and deductive reasoning processes and justification. The most persistent of these difficulties seemed to be the challenge of knowing how to use the knowledge of the definitions of relevant mathematical objects, proof methods, deductive reasoning processes and justification. This is also referred to as strategic knowledge (Weber, 2001).

The two case study students showed great improvement in all aspects of their proof construction abilities as they progressed from the first to the second session. This highlighted the effectiveness of the consultative sessions in facilitating access to the observed students' zones of proximal development and in allowing students to make functional use of the various mathematical objects and processes needed in successful proof construction. This functional use together with the scaffolding received from their peers and the lecturer enabled students to develop and internalise proof construction skills and abilities.

Investigation of the nature of the interactions in the consultative sessions examined the lecturer's use of requests for clarification, reflection on proof construction strategy, critique and justification, while eliciting elaboration of contributions which could drive the proof construction process forward. The importance of the correct interpretation of definitions and their role in providing the logical structure and the justification of each step of the proof construction was emphasized. As the sessions progressed more knowing peers emerged from the group who took over the role and responsibilities of the lecturer and provided most of the scaffolding to their peers. I often called upon these more knowing peers to explain and elaborate on completed proof constructions. Their presentations were observed to be effective learning opportunities for other students.