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An investigation on the impact of low-grade material in the selection of a Global Mean for Simple Kriging on an economic reef horizon at South Deep Gold Mine

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AN INVESTIGATION ON THE IMPACT OF LOW-GRADE MATERIAL IN THE SELECTION OF A GLOBAL MEAN FOR SIMPLE KRIGING ON AN ECONOMIC REEF HORIZON AT SOUTH DEEP GOLD MINE

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A research report is submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Mining Engineering.

2022

DECLARATION

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ABSTRACT

In this research study, the impact of using low-grade samples, here also referred to as low-grade material, in the estimation of the global mean for an economic reef horizon at South Deep Gold Mine is investigated, where, due to sparse data density, Ordinary Kriging would not be optimal. Simple Kriging, which is reliant on a global mean, is thus the estimation method of choice. The global mean, therefore, plays an important role and due to the positively skewed nature of the probability distribution of the gold grades, the estimation of a global mean poses a problem. Three grade zones have been identified on the Modderfontein B bottom band auriferous reef, namely a high-, a medium-, and a low-grade zone. From each of these zones one grade domain is selected for the investigation. For each of these domains three bottom cut values were then defined relative to the average grade of each domain. The bottom cut defined the threshold at which low-grade material was removed before declustering at an optimal block size. The research study showed that low-grade material does impact the estimate of the global mean of a domain, inclusion of low-grades results in low mean grade estimates. As expected, the global mean estimate increased as the bottom cut-off grades increased for all the domains. The research study also showed specifically for the low-grade domain investigated, that the low-grade samples should be removed before declustering, additional research into other low-grade domains would be required to verify if this holds true. It was found that the low-grades have a greater impact on the local mean estimate for individual declustered blocks than on the declustered average grade estimate of the domain. Where the number of samples within the individual declustered blocks is small, it is recommended that a Sichel-*t* estimator for the mean grade be considered. The grade estimates produced in this research study were dissimilar to estimates used by South Deep, and although the grade differences were not excessive it produced significantly higher gold content estimates as the bottom cut-off grade increased. It follows that, if the low-grade material are indeed low-grade outliers, then the Mineral Resources could be underestimated. Two-parameter lognormal estimates of the global mean produced, for the three domains investigated, were found to be high compared to the global mean estimates obtained using the bottom cut-off grades. Mineral Resource estimation was therefore not conducted using the lognormal estimates. It is further recommended that an investigation into other domains be conducted to potentially develop a structured methodology for determining a global mean estimate at South Deep Gold Mine. Different methodologies may be required for different domains.

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1 INTRODUCTION

South Deep Gold Mine is a trackless mechanised low-grade bulk mining operation. The mine exploits sixteen reef horizons that form part of one of the world's largest ore bodies located in the Witwatersrand Basin. Resource estimation of the reef horizons are done using Simple Kriging (SK) which requires an estimate for the global mean as an input, the SK estimates are then post-processed using localised direct conditioning (LDC) to produce local estimates.

1.1 Purpose of the Study

The purpose of this research study is to improve the current estimation process and introduce more robustness to the process of determining the global mean required in SK estimation rather than selecting it. This will be done by investigating the impact of low-grade sample values on the estimation of a global mean value for specific estimation domains (low, medium, and high-grade domains) of the economic Modderfontein B bottom band (MBB) auriferous reef at South Deep Gold Mine.

1.2 Geology Background and Location

The Witwatersrand Basin was formed approximately 3074 million years ago over a period of 360 million years (Robb & Meyer, 1995). The basin attains a maximum thickness of approximately 7000 m and comprises of predominantly clastic sedimentary rocks deposited in a fluvio-deltaic environment (Robb & Meyer, 1995). The auriferous reefs of the Upper Elsburg Formation exploited at South Deep are part of the West Rand Goldfield, located on the north-western rim of the Witwatersrand Basin. Koglin, et al., (2010) provides a description of the sedimentation of the Elsburg Formation.

South Deep Gold Mine, located in the West Rand area, near the town of Westonaria is approximately 45 km south-west of Johannesburg as shown in Figure 1.1. The local geology consists of outliers of the Karoo Supergroup shales and sandstones which are underlain by Pretoria Group sediments, Chuniespoort Group dolomites, and Klipriviersberg Group volcanic rocks. These Groups are underlain by the Central Rand Group that hosts the gold-bearing conglomerates exploited at South Deep (Osborn, et al., 2014). The reefs exploited at South Deep form part of the Upper Elsburg reefs of the Mondeor Formation and the Ventersdorp Contact Reef (VCR) of the Ventersdorp Supergroup. The Upper Elsburg reefs, an easterly divergent clastic wedge, comprises 15 alternating bands of conglomerates and quartzites, that subcrops against the VCR towards the west, shown in Figure 1.2 (Osborn, et al., 2014).

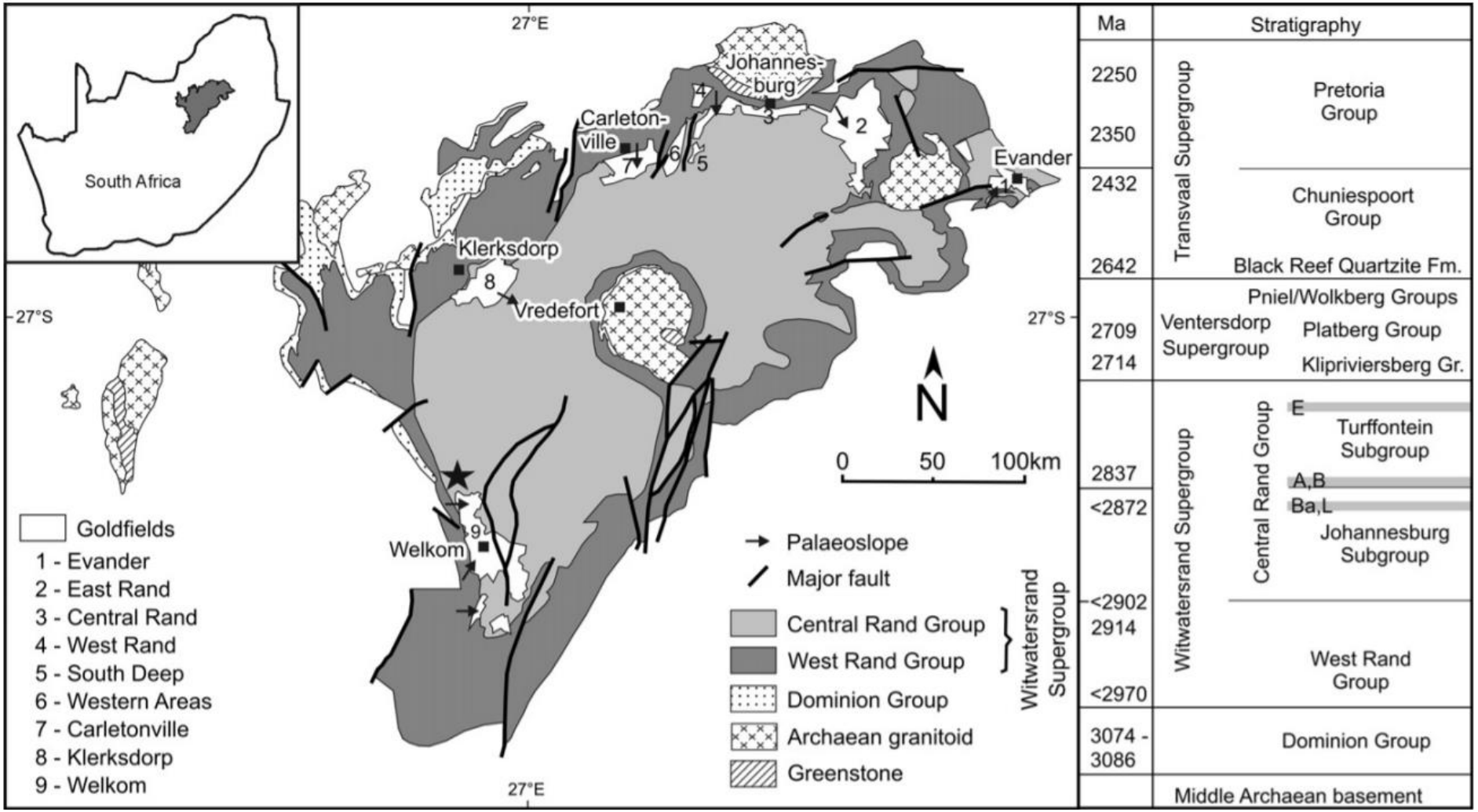


Figure 1.1: Locality map showing the location of South Deep Gold Mine and stratigraphic column of the Witwatersrand Basin (Koglin, et al., 2010)

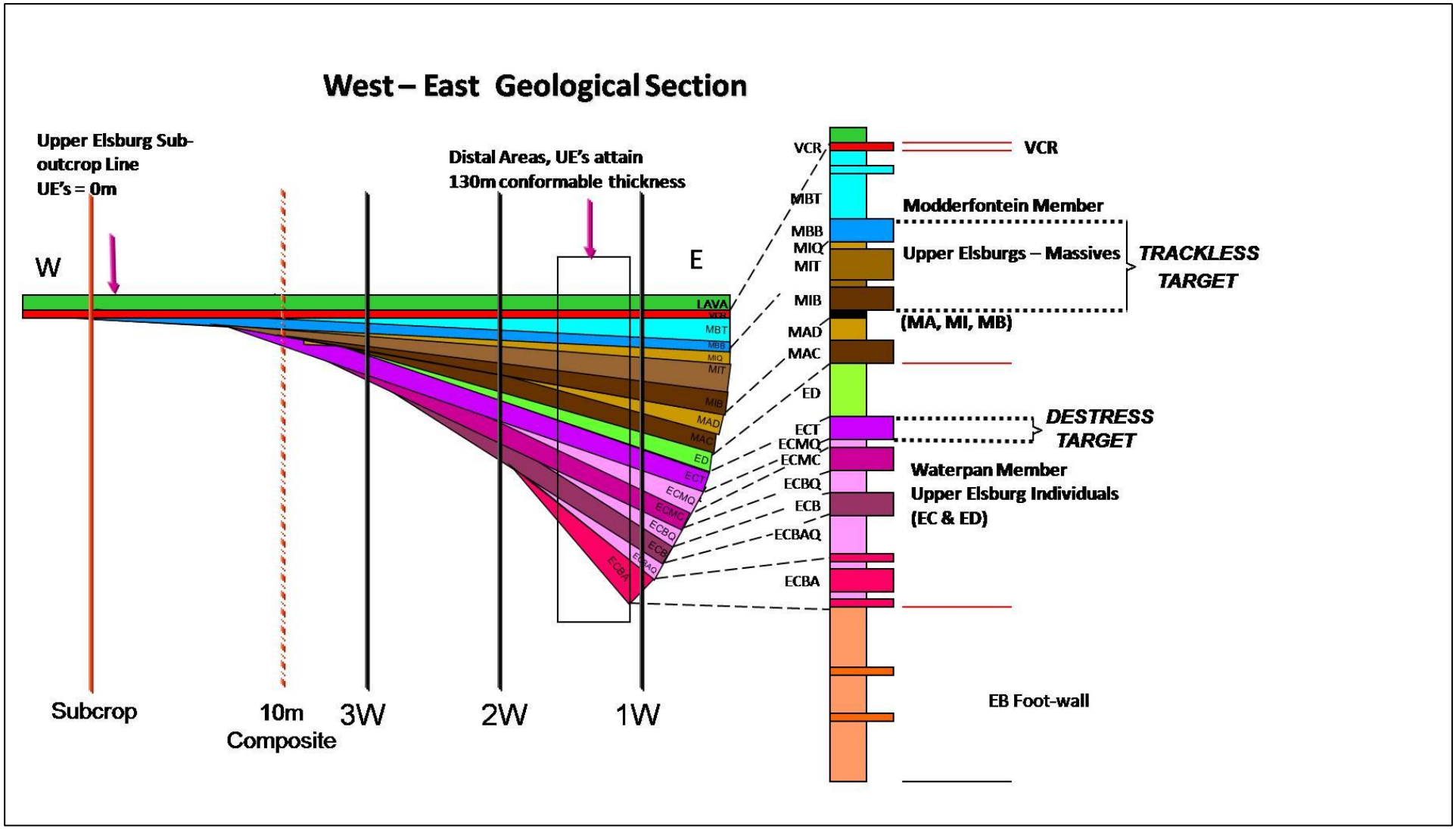


Figure 1.2: West - East section showing geology at South Deep Gold Mine (Osborn, et al., 2014)

1.3 Research Background

The geological model for the orebody exploited at South Deep is based on drilling data, mapping information and expert interpretation and knowledge on the geological formation of the orebody. Borehole core is obtained from three drilling programmes, that is, surface exploration drilling, long inclined borehole (LIB) drilling, and grade control (within the stope) drilling. The surface exploration drilling programme, which was decommissioned in 2013, yielded 81 borehole intersections on a drilling grid of approximately 550 m x 550 m. A single surface rig could yield 12 borehole intersections. Data from these boreholes were used in conjunction with seismic surveys to define the VCR top contact position and to provide geological structural definition. The surface exploration boreholes were used primarily for identifying future mining areas, more than ten years ahead of the current mining face. The LIB drilling programme focuses on drilling long flat boreholes far ahead of the mining face on a drilling grid of 300 m x 300 m to provide information on a two-to-ten-year window ahead of the current mining face. Grade control boreholes are drilled on a drill spacing aimed at an optimal 30 m x 30 m grid, but in general, only achieving a 60 m x 60 m grid. These boreholes are drilled either upwards or downwards within development stopes and are used to support current mining activities by providing grade information and logging data for reef identification.

Borehole logging is used for structural definition, facies determination, and stratigraphic modelling. The borehole core is sent for assaying to measure the gold (Au) grade (g/t) at that specific location in the orebody. For LIB boreholes and grade control boreholes, full core samples are assayed, while half or split cores were assayed for surface exploration boreholes. The orebody resources are estimated using the Au assay grades of the core samples.

The borehole intersections are composited at intervals of one metre per reef unit. Estimation is done on a block size of 30 m x 30 m x 1 m, which is further sub-celled into 3 m x 2 m x 1 m blocks if required. This is done to facilitate improved wireframe filling for each reef being estimated. The resource estimation methodology applied at South Deep Gold Mine is SK, and is done per reef, per domain. SK has been identified as the preferred methodology, based on operational research and experience which has taken into consideration the geology, sedimentary deposition of the reefs and a wider than optimal drilling grid leading to a paucity

of borehole data. Any Ordinary kriging (OK) being carried out is done only for comparative purposes to the SK estimate.

The existing procedure for the estimation of the global mean for the MBB reef used in SK is described next. Cell declustering on a block size of 90 m x 90 m x 50 m is used to avoid bias in the estimation of the global mean for a domain. The global mean for each domain in the MBB reef is calculated as the weighted average of the 90 m x 90 m x 50 m declustered local means in the domain, where the weights are the number of blocks containing samples. A “local” block global mean for SK in more densely sampled areas, that include grade control samples, is based on a block size of 90 m x 90 m x reef thickness (m). In the case of the MBB, the reef thickness is 5 m as the average thickness of the MBB reef is 5 m. In cases where, in a 90 m x 90 m x 5 m block, there are sufficient samples available the average grade of that specific block will be used as a global mean in the SK estimation.

The minimum number of samples for each reef was established from an internal study conducted by Gold Fields. That study considered the thickness of each reef mined at South Deep Gold Mine and found, that for a 90 m x 90 m panel, at least four boreholes are required for the mean of that block to override the global mean of the domain wherein the block is located. In the case of the MBB reef, which has an average thickness of 5 m, one borehole can yield approximately five one-metre composite samples, therefore 20 (4 x 5) one-metre composite samples are required for that local 90 m x 90 m x 5 m block mean to be used as the global Domain mean. The use of this approach in areas where sufficient sample data is available achieves an SK estimate that is similar to the OK estimate.

For variography purposes, a top cut value is determined using the log-transformed (ln) gold (Au) g/t data distribution and percentile-percentile (PP) plots of the ln(Au g/t) distribution and a normal probability distribution for each domain. If the sample data set matches the specified probability distribution, the points will plot along a 45-degree line (Dohm, 1995). In this research study, the cumulative relative frequency of the ln(Au g/t) data is compared to the cumulative Gaussian distribution. The PP plot is here used only to determine if there are outliers present within the dataset. The outlying ln(Au g/t) values identified for each domain are then back transformed to the real space Au (g/t) top cut values. For the variography analysis, the top cut thresholds identifies that grade value beyond which assay values are excluded; as a guide it is the point where the cumulative plot deviates from the 45˚ line at the

upper end. This is done to reduce spatial variability and emphasise grade continuity per domain. Outliers are removed from the composited dataset before the experimental variogram calculations and variogram modelling are done for every domain for each reef. These models define the direction and search parameters for sample composites to be used during SK.

SK is conducted using composited Au assay data that has been 'top capped'. The top cap value is determined using the data distribution and PP plots described above for the top cut. The top cap does not remove the outliers; however, the outlying assay values are restricted to that of the determined top cap value. This is done to avoid erroneous overestimation of the kriged resource model.

The SK estimated model produced is then post-processed using a LDC process. The LDC process, developed for South Deep Gold Mine is detailed in Assibey-Bonsu, et al., (2014), and provides for support corrections and addresses the inherent smoothing effects created during the kriging process.

1.4 Justification for Research

The global mean used in SK plays a major role in the grade estimation at South Deep Gold Mine. Most mining areas at South Deep Gold Mine have limited data, these areas are informed by the global mean in the SK estimation model. The output of the SK model is the primary input for the LDC post-processing, therefore making the global mean selection crucial.

Resource models are the primary input for mine planning processes and hence estimate the eventual gold to be extracted. Mineral Resource models have major financial implications for the business. The resource model is the primary input for mine planning processes. Mineable stopes are selectively planned within the resource block model according to the specific economic cut-off grades and mining methods. The resource within these mineable stopes represents the Mineral Reserves. The Mineral Resources and Mineral Reserves publicly reported as tonnage and grade estimates above the cut-off grade, have a significant influence on the financial statements of the company. It is therefore critical that the estimation methodology used for the global mean be optimal for all reefs being exploited.

The effect of low-grade or isolated high-grade samples can impact cell declustering and thus impact the global mean estimate of a domain. The determination of a global mean estimate

is critical in that it informs the grade of most reef domains at South Deep Gold Mine. In quartzites, high-grade outlying samples from pebble lags can skew the declustering calculation and the selected global mean can lead to overestimation. Conversely in the high-grade gold-bearing reef, nonrepresentative low-grade samples could conceivably also skew the declustered mean calculation, again affecting the global mean selection and possibly causing underestimation during SK.

While the impacts of top capping and top cutting are understood and applied to high-grade outliers, no tangible research on the impact of low-grade outliers has been performed. The effect of very high-grade outliers is controlled by top capping during kriging to not overestimate the grade. Low-grade outliers are not considered and are used for estimation purposes. These low-grade outliers thus have a double impact, first on the global mean estimate of a domain and second on the grade estimation in the domain. The average grade above cut-off could be underestimated which could impact the mine's grade control and mine call factor (MCF).

This research intends to only assess the impact of low-grade samples on the estimate of a global domain mean to be used in SK. Low-grade material will, however, still be included during grade estimation to produce mining selectivity on a local scale.

1.5 Assumptions

Outliers can originate from different processes and mechanisms that include, data errors, sampling errors, and incorrect distribution assumptions (Osborne & Overbay, 2004). Declustering is only effective if the data is not spatially biased, that is, borehole sampling is representative of the true distribution of the orebody (Pyrcz & Deutsch, 2007).

The assumptions made in this research study are:

- The Quality Assurance Quality Control (QA/QC) processes followed at South Deep Gold Mine are effective, leading to the assumption that the Au (g/t) assay of the MBB reef data used for this study is precise and accurate.
- That outliers within the data originate from the inherent variability of the orebody.
- The data to be used is spatially representative of each domain of the MBB reef.

1.6 Research Methods

The research conducted was experimental and quantitative in nature. Original Au (g/t) assay and thickness data collected and stored on the Datamine Fusion® database were used for this research study. Borehole data along with underground mapping data was used to model the MBB reef in Leapfrog Geo® software. The MBB reef is currently divided into 16 domains at South Deep Gold Mine. For comparative purposes the domains delineated by South Deep Gold Mine were used in this research study. Borehole data, within their respective domains, were composited and compiled for statistical use, and statistical analyses were conducted on all 16 reef horizons to select three domains for this research study. A high-grade zone, a medium-grade zone and a low-grade zone were identified. One domain from each of these three grade zones was selected based on the area of the domain and the spatial representativity of the available data within the domain to investigate the different impacts low-grade samples have on domains of different average grades.

Three bottom cut-off values were defined for each of the three domains depending on the average grade of the domain. The high-grade domain was assigned higher bottom cut-off values than the medium-grade and low-grade domains. Cell declustering, to determine a global mean, was done for all composited data within their respective domains. Cell declustering was then repeated for each domain for each bottom cut-off grade. This was done by removing the assay values below each defined bottom cut-off threshold and repeating the cell declustering process. Each domain was therefore declustered four times, each with a different data set to produce four global mean estimate for each domain. The different global means produced, within their respective domains, were compared to each other as well as to the selected global mean used by South Deep Gold Mine. Cell declustering was also conducted using two methods, for comparative purposes, one with a fixed origin for the grid of cells and the second with a moving origin for the grid of cells. Different methodologies for estimating a global mean were explored and compared.

In order to not introduce additional variability that might affect the outcomes of the estimation, the variograms modelled by South Deep Gold Mine were used to define sample search directions and search ranges for grade estimation. The standard SK grade estimation methodology followed at South Deep Gold Mine was then carried out using each of the three bottom cut-off global means for each domain of the MBB reef. The resulting three SK models

produced per domain were compared on an estimated grade and gold content basis to assess the influence of the low-grade values. Tonnage estimates would be unaffected as the block model volumes did not change. The estimated grades produced using the different global means were compared to the grade reported by South Deep Gold Mine to reach a conclusion about the impact of low-grade samples on the estimate of a global mean of three of the 16 estimation domains.

1.7 Questions Addressed by this Research

The following research questions on the effects of low-grade material on the estimation of the global mean for SK are addressed by this study:

- What are the implications of low-grade outliers on the selection of the global mean?
- Should low-grade outliers be included in the declustering process?
- Are resources underestimated as a result of including low-grade outliers in the global mean estimate?
- What is the difference on the Mineral Resource estimate if a lognormal estimate for the global mean is used, in comparison to using a global mean that is estimated without including low-grade material?

1.8 Research Objectives

Declustering methods rely on the weighting of sample data to be spatially representative, only the influence of each sample is changed, the sample value itself does not change (Pyrcz & Deutsch, 2007). A low-grade sample therefore will have some influence on the resultant declustered mean. The main objective of this research study is to optimise the estimation of the global mean to be used in SK. This will be done by investigating the method of determining the global mean for three grade domains of the MBB reef and specifically the influence of low grades on this global mean estimate.

1.9 Report Layout

The research report is divided into nine chapters and a brief description of each will be given in this section. Background information, data, theories, statistical and geostatistical analyses, and comparisons pertinent to this research study will be presented here.

Chapter 1 Introduction: This chapter is dedicated to describing the purpose and objectives for this research study. The background information associated with the problem, the assumptions that are made for the research to be valid, and the research methodologies used to carry out the investigation are defined and described.

Chapter 2 Literature Review: In this chapter, definitions, theories, and concepts are researched. Previous literature and studies conducted, and statistical and geostatistical methods are explored to gain an understanding on the estimation methods implemented for this research study.

Chapter 3 Logging and Assay Data Validation: This chapter describes the sources of the data used, the process of data validation and the methods implemented to compile the data for use for the research study analyses.

Chapter 4 Modelling and Data Analysis: This chapter provides a description of the 3D geological modelling process of the MBB reef, the domaining and domain selection process, and statistical analysis of the borehole data within selected domains.

Chapter 5 Declustering and Global Mean Estimation Options: This chapter describes the cell declustering methodology and presents the results obtained from declustering for the various domain data sets to assess the impact of low-grade sample material. Results from other estimation methods for the global mean including a 2-parameter lognormal estimate are also described and compared.

Chapter 6 Variography: In this chapter the spatial distribution of the sample data is described and search parameters for SK estimation are defined. The application and justification of applying top-cutting and top-capping of the Au (g/t) data in variography and estimation are discussed.

Chapter 7 Grade Estimation and Results: This chapter describes the SK estimation conducted on each domain and the results are presented.

Chapter 8 Discussion of Results: The results from the analyses presented in Chapter 5 and Chapter 7 are interpreted and presented in this chapter with the focus on answering the

fundamental research question, that is, what is the impact of low-grade material on the global mean estimate?

Chapter 9 Conclusion and Recommendations: Concluding remarks from the research findings are made and recommendations are proposed for future studies in this final chapter.

In summary this research is conducted to investigate the impact of low-grade samples on the estimation of the global mean for SK. The MBB reef, one of the 16 reefs exploited at South Deep Gold Mine, will be considered for this investigation. Research will be done using validated borehole and mapping data supplied by South Deep Gold Mine. The analysis will be confined to three domains of different average grades, that is, a high-grade zone, a medium-grade zone, and a low-grade zone. The analysis and findings from this research study will be compared to the results obtained by South Deep Gold Mine.

2 LITERATURE REVIEW

This Chapter is dedicated to research on published literature and previous studies relevant to this research study. Statistically and spatially representative grade distributions are important in resource estimation more so in the case of SK, where this statistical and spatial grade distribution forms part of the estimation process. Methodologies to conduct an exploratory data analysis were therefore researched to gain insight on the statistical probability distributions of the data. Au values typically have a positively skewed or lognormal shaped distribution, which causes difficulty in estimating the mean grade. The main aim of this research is to estimate the Au mean of the positively skewed grade distribution.

The mean grade or global mean of a domain is an important input parameter for the SK estimation that is conducted at South Deep. Different methodologies and considerations around this global mean estimate were therefore considered for this research. Data transformation is considered, where the Au grades are transformed to the log space to make the distribution more normally distributed to estimate the mean, and subsequently back transformed to the normal space. Cell declustering, which is a widely used technique to determine a declustered global mean, is also examined in this chapter.

The result or impact of using a global mean for estimation is seen in the final output, which is the resource estimation model, therefore, literature on estimation methodologies were researched. The literature review includes the calculation of an experimental variogram, modelling of a variogram and the SK estimation technique. OK estimation theory is also reviewed for comparison to SK estimation, and outlier studies are done to define an outlier and methods of outlier identification.

An in-depth understanding of the geology of the orebody is fundamental to the development of any statistical processes to be conducted. The orebody is unique and therefore dictates the methodologies and processes to follow. There are no two orebodies that formed in the same way, nature has no prescribed method for the formation of orebodies and therefore there cannot be a prescribed method for estimation of the orebody. The spatial and statistical grade distribution depends on the mode of formation and the provenance of the material amongst other factors. A review of the orebody geology is done in Section 1.2. Geological modelling is an essential aspect that needs to be conducted before any type of geostatistical analysis or

estimation can take place (Emery & Ortiz, 2005). Reef horizons need to be delineated into homogenous zones or geological domains based on mineralisation, alteration, and lithological aspects such as the percentage conglomerate, the channel width, and the average clast size. Geostatistical domains can then be further delineated based on accumulation or grade information on the variable of interest. Geostatistical analysis is then conducted within these homogeneous geostatistical domains.

Exploratory data analyses are fundamental in Mineral Resource estimation to improve the quality of the estimate through a comprehensive understanding of the statistical and spatial distribution of the variable being estimated (Sinclair, 1998). Sinclair (as cited in Sinclair, 1998) describes specific aims of an exploratory data analysis to include: recognition and elimination of errors and anomalies; providing a comprehensive understanding of the characteristics of the variable being estimated; distinguishing and documenting interrelationships among variables of interest; identifying systematic spatial variation of the variable of interest such as grade and thickness of the mineralised zone; identifying and delineating homogenous geological domains to be evaluated independently during estimation; outlier identification and understanding; and evaluation of similarities/dissimilarities between different types of raw data including those of different sample support.

The data analysis procedure followed by South Deep Gold Mine, described in Section 1.3, will be followed for this research along with the assumptions outlined in Section 1.5. Descriptive statistics defines information such as the mean, standard deviation, coefficient of variation, maximum, and minimum amongst other statistics, and are useful in summarising data and provides a platform for comparison of different data subgroups (Sinclair, 1998). Scatter plots, histograms, probability plots and contour plots provide a graphical representation of the data from which outliers, anomalies and homogeneity can be identified. Histograms provide a simple representation of the numerical data and are useful in determining the type of data distribution, the spread of data, and the range of greatest concentration of data (Sinclair, 1998). According to Sinclair (1998), scatter plots can be utilised for outlier detection, while probability plots can be used to identify multiple sub-populations within a dataset which may represent different domains and allows for the detection of outliers. Any type of estimation will come with its errors, this is unavoidable, a comprehensive exploratory data analysis will

provide a better understanding of the data distributions at play and will assist to minimise this error.

To determine what constitutes an outlier, research was done on previously published outlier and trend analyses studies. Estimates can be skewed by extreme values known as outliers. An outlier is a data point that lies so far outside the norm of the data distribution that it can cause suspicion as to its origin (Hawkins, 1980). Sinclair (1998) describes an outlier as an observation which seems to be inconsistent with most of the data values. It is important to distinguish between an outlier and a data point that is representative of the population.

Extreme values that keep recurring may be an indication that the sample is representative and removing these outliers would not be justified. Removing high-grade values, for example, that are not outliers would result in a global negative bias in the estimates and the MCF would appear to improve because of it (Krige, 1999). The author of this research study suspects that the same could be true for removing recurring low-grade values that are not outliers resulting in a global positive bias in the estimates.

Before outliers can be identified, it is recommended that data are divided into domains, and that those domains are based on sound geological data (Krige, 1999). Outliers can arise from several causes or events and can be classified into two major categories according to how they were formed: those created from erroneous data and those formed by the inherent variability of the orebody (Anscombe, 1960). This research study is based on outliers created by the inherent variability of the MBB orebody at South Deep Gold Mine.

Osborne and Overbay (2004) give a broad definition of an outlier as a data point that is three or more standard deviations away from the mean They highlight that outliers can have disastrous consequences on statistical tests by increasing the error variance and reducing its statistical power. Outliers can decrease the normality of data with a non-parametric distribution and introduce bias that inaccurately influences the parameter being estimated.

Krige and Magri (1982) studied the effects of outliers on variogram estimates and concluded that data outliers can seriously distort or mask actual variogram patterns. They concluded that the best method to identify outliers, particularly when data are limited and outliers are a result of inherent variability, is to transform the data to a logarithmic scale. This research study will

examine if the same holds true for outlying low-grade material in what is known to be a high-grade gold horizon such as the MBB reef to be investigated.

There are many researchers with different views as to what should be done with outlying data, Barnett & Lewis, and Judd & McClelland (as cited in Osborne & Overbay, 2004) for example, make strong arguments for the removal of outliers to get the best estimate of a parameter. Srivastava (as cited in Costa, 2003), conducted comprehensive studies and provides detailed guidance on the procedure to follow in how to identify and manage outliers. However, in the end, it comes down to the researcher doing what is best suited to the situation at hand, with the data at hand. No two data sets are the same, hence no two data compilation methods need to be the same.

All the research papers reviewed thus far, have been related to concerns about high-grade outliers or on how to manage outlying data on the upper end of the spectrum. McKinstry (1984, cited in Sinclair, 1998), for example, raised concerns directed toward high-grade outliers within datasets. Costa (2003), for example, proposed a methodology based on robust kriging (RoK) to mitigate problems associated with high-grade outliers. In this methodology, the impact caused by the presence of outliers in grade estimation is minimised by applying weights to each sample such that each original sample value is estimated using the surrounding samples and the robust estimator for the variogram (Costa, 2003). Fourie, et al., (2019), discusses a post-processing methodology to minimise the influence of high-grade outliers smearing grade onto otherwise low-grade areas. The post-processing involves the calculation and application of restriction factors on the OK weights of the samples that restricts the range of influence proportionately to the probability of their occurrence (Fourie, 2019).

While there is concern about overestimating a parameter, and justly so, for it can come with major financial implications, the outliers on the lower end of the spectrum are given an automatic pass-through to estimation. In the case of SK, where the global mean plays an important role in areas of limited data, these low-grade outliers have a double effect, that is, first in the estimation of the global mean and again during the estimation process.

A global mean estimate for positively skewed data can be substantially increased by a few high-grade samples with a low probability. Sichel (1947, cited in Sichel, 1966) showed that the 2-parameter log-transformed data distributions for positively skewed Au grades are more

symmetrical and less sensitive to outliers and may therefore give a more reliable global mean estimate.

The global mean estimate using a 2-parameter lognormal distribution is calculated using the equation below (Dohm, 1995).

$$\mu^* = \tau = e^{\xi + 0.5\sigma_L^2} \quad where,$$

$\mu^* is\ the\ lognormal\ estimate\ of\ the\ mean,$

$\tau\ represents\ the\ back\ transformed\ mean\ (g/t),$

$\xi\ is\ the\ mean\ of\ the\ log-transformed\ data,$

$\sigma_L^2\ is\ the\ variance\ of\ the\ log-transformed\ data.$

Krige (1960) discussed a regular pattern of departure from the 2-parameter lognormal Au distributions for certain mines in areas, which included the Witwatersrand, and noted that these departures can lead to biased and inefficient statistical estimates of the population. Krige (1960) suggested the addition of a constant value, known as the additive constant, to all the Au values which lead to the 3-parameter lognormal distribution.

Sichel (1966), discusses the effects of outliers on the estimation of the mean for a small number of samples, and shows that the Sichel-t estimator is unbiased, has a minimum variance and in many cases is a more precise method to determine the unknown true average grade than the arithmetic average. Although the 2-parameter lognormal will be considered in this research study for comparative purposes, the researcher is more interested in an estimate of the mean grade that considers the spatial positioning of the Au grades.

A literature review was also conducted on cell declustering methodologies which gave insight to choosing an optimal cell declustering method. Harding and Deutsch (2021) discuss the concept of stationarity and modelling data with trends. Deutsch (2015) emphasise the importance of representative global distributions and the principle of cell declustering. The cell declustering methodology produces a declustered or global mean that can be used in SK. Deutsch (2015) states that the cell declustering cell size should be approximately the spacing of the sample data in areas of low data density. Deutsch (2015) further describes the declustering process as a grid of equal-volume cells placed over the domain, the number of occupied cells are counted, and each cell is given an equal weighting with respect to the

number of occupied cells. If there is a single data point within a cell, that data point gets the full weight of the cell, if there are multiple data points within one cell the weighting of the cell is shared equally between all data points, this is demonstrated in Figure 2.1.

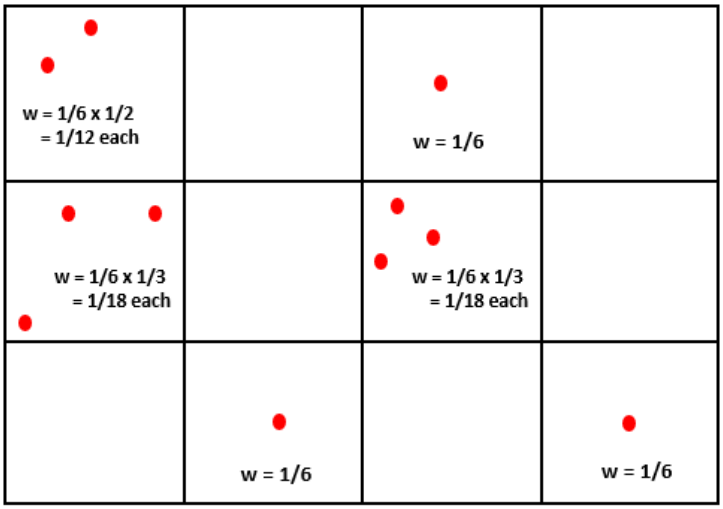


Figure 2.1 Cell Declustering grid with weighting assigned to the samples in the six occupied cells (adapted from Deutsch, 2015)

In Figure 2.1 above, there are six occupied cells, and each cell is given an equal weighting of one sixth. Multiple samples within a cell share the weighting of one sixth, whereas cells with a single sample are given the entire one sixth of the weighting.

Au grade can be referred to as a regionalised variable because the value of the grade depends on its spatial position. An experimental variogram is calculated to capture an approximation of the spatial continuity of sample grade values that are a certain distance apart and in a certain direction.

The experimental variogram is calculated using the following equation (Armstrong, 1998):

$$2\gamma(h) = Var[Z(i) - Z(i+h)] = E\{[Z(i) - Z(i+h)]^2\}$$

$where, 2\gamma(h)\ is\ the\ experimental\ variogram\ value\ in\ a\ certain\ direction,$
$i\ \ and\ i+h\ are\ the\ locations\ of\ the\ sample\ pairs,$

$Z(i)\ and\ Z(i+h)\ are\ the\ value\ \ of\ the\ sample\ pairs, and$

$h\ is\ the\ lag\ distance\ between\ sample\ pairs.$

The experimental semi-variogram value for lag distance *h* is calculated by dividing the above equation by two times the number of sample pairs. The experimental semi-variogram is then modelled in a certain direction to define the search ranges to be used in kriging.

A variogram model provides an estimate of the spatial continuity of samples values at a certain distance apart and in a certain direction. Single or double structured spherical variograms are modelled for each reef domain at South Deep. A typical single structured spherical variogram model is shown in Figure 2.2.

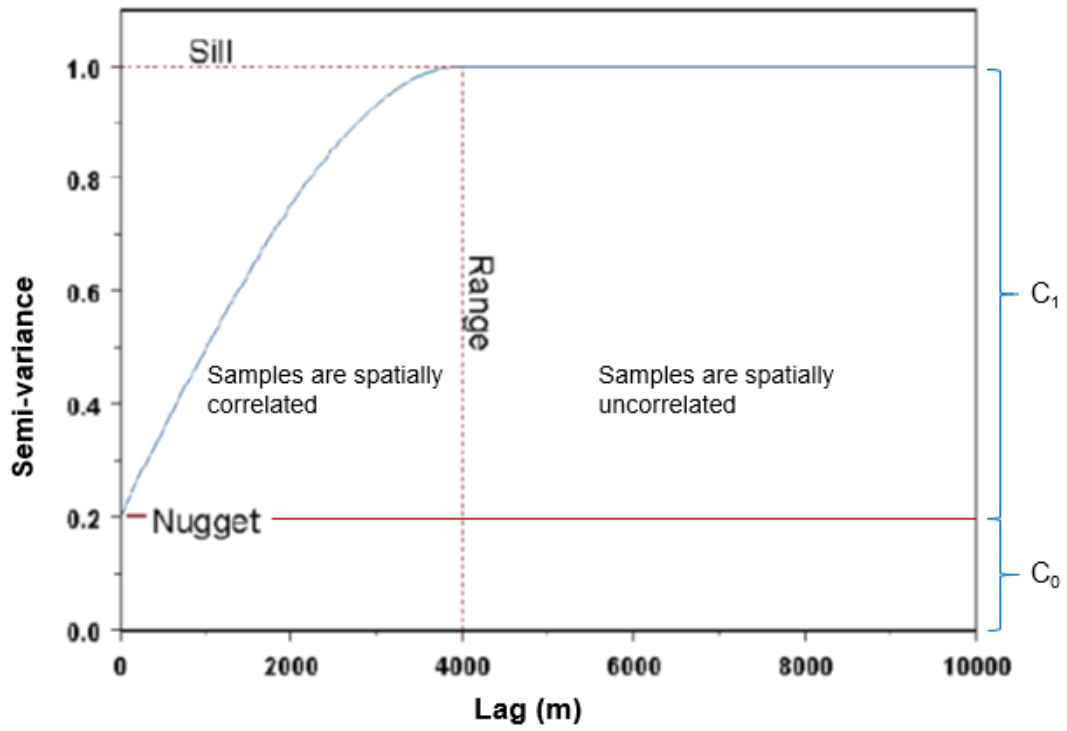


Figure 2.2 Spherical semi-variogram model (modified after Armstrong, 1998)

The nugget effect is the inherent variability within an orebody, which influences the variability between one sample and another sample immediately next to it (Armstrong, 1998). A simplistic example of a semi variogram model is presented in Figure 2.2, the nugget C_0 represents the random component of the semi-variogram while C_1 represents the structured component. The sill is defined as the total spatial variance, that is the sum of the nugget and structure C_1, and it is reached when the semi-variance becomes constant or levels out. The range is the lag distance at which total variance is reached. Beyond the range of influence, sample data are no longer spatially correlated. A double structured spherical semi-variogram model is defined by the following equations (Armstrong, 1998):

$$\gamma(h) = 0, \qquad if\ h = 0$$

$$\gamma(h) = C_0 + C_1\left[1.5\ {}^{h}/_{a_1} - 0.5\left({}^{h}/_{a_1}\right)^3\right] + C_2\left[1.5\ {}^{h}/_{a_2} - 0.5\left({}^{h}/_{a_2}\right)^3\right],$$
$$if\ 0 < h \leq a_1$$

$$\gamma(h) = C_0 + C_1 + C_2\left[1.5\ {}^{h}/_{a_2} - 0.5\left({}^{h}/_{a_2}\right)^3\right], \qquad if\ a_1 < h \leq a_2$$

$$\gamma(h) = C_0 + C_1 + C_2\,, \qquad if\ h > a_2$$

$where, h\ is\ the\ lag\ distance,$

$\gamma(h)\ is\ the\ semivariance\ value\ at\ distance\ h,$

$C_0\ is\ the\ nugget\ effect\ or\ random\ component.$

$C_1\ and\ C_2\ are\ the\ first\ and\ second\ structured\ components\ respectively,$

$a_1\ and\ a_2\ are\ the\ first\ and\ second\ ranges\ of\ influence\ respectively.$

According to Krige (1994), SK is basically a classical statistical technique that considers the population mean and the grades of regularised data blocks in the vicinity of each ore block that is being estimated. SK, when properly implemented, on South African gold mines, proved to be the only technique that consistently ensured the elimination of conditional biases; provided sufficient data were available (Krige, 1994).

Kriging considers the number of samples, the spatial positions that is, the spatial clustering or non-clustering of the samples; the distance between the known sample values and the unknown point/block to be estimated; and the spatial continuity of the variable being estimated by applying a variogram model (Armstrong, 1998). Thus, kriging is an estimation method that produces unbiased linear estimates of point values or of block averages with the least variance (Armstrong, 1998).

The two types of kriging techniques that are commonly used for gold estimation are OK and SK. During OK estimation the mean is unknown and assumed to fluctuate over the area; the sum of the weights assigned to each sample equates to one; and there needs to be stationarity, i.e., no trend in the data (Armstrong, 1998). The equation for an OK block estimate is given by the equation below (Armstrong, 1998):

$$Z_{ok}^{*} = \sum_{i=1}^{n} \lambda w_i \, v_i, \qquad where \sum \lambda_i = 1 \ and \ there \ is \ stationarity$$

$Z_{ok}^{*} \ is \ the \ OK \ \ estimate \ of \ block \ V \ , and$

$w\lambda_i \ is \ the \ optimum \ weight \ assigned \ to \ sample \ v_i$

The OK kriging variance for the block, which is the minimum variance, is given by the following equation (Armstrong, 1998):

$$\sigma_K^2 = \sum_{j=1}^{N} \lambda_i \, \bar{\gamma} \, (x_i, V) - \, \bar{\gamma} \, (V, V) + \mu$$

$where, \sigma_K^2 \ is \ the \ kriging \ variance \ of \ block \ V,$

$\bar{\gamma} \, (x_i, V) \ is \ the \ average \ of \ the \ variogram \ between \ x_i \ and \ the \ volume \ V,$

$\bar{\gamma} \, (V, V) \ is \ the \ average \ of \ the \ variogram \ between \ any \ two \ points \ in \ the \ volume \ V,$

$\lambda_i \ is \ the \ optimal \ weight \ assigned \ to \ sample \ x_i,$

$\mu \ is \ the \ Lagrange \ multiplier,$

The OK kriging variance is given by the equation below (Armstrong, 1998):

$$\sum_{j=1}^{N} \lambda_j \, \gamma\left(x_i, x_j\right) + \mu = \bar{\gamma}\left(x_i, V\right), where \; i = 1,2, \ldots N \; and \; \sum_{i}^{N} \lambda_i = 1$$

$where, \gamma\left(x_i, x_j\right) \; is \; the \; average \; of \; the \; variogram \; between \; x_i \; and \; x_j,$

$\bar{\gamma}\left(x_i, V\right) \; is \; the \; average \; of \; the \; variogram \; between \; x_i \; and \; the \; volume \; V,$

$\lambda_j \; is \; the \; optimal \; weight \; assigned \; to \; sample \; x_j, \mu \; is \; the \; Lagrange \; multiplier$

On the other hand SK estimation requires the mean to be known, and the sum of the sample weights is not required to equate to one because the mean of the samples is introduced to the estimation equation (Armstrong, 1998). The equation for the SK block estimate is given below (Armstrong, 1998):

$$Z_{sk}^{*} = \sum_{i=1}^{n} \lambda_i \, v_i + \left(1 - \sum_{i=1}^{n} \lambda_i\right) \times m$$

$Z_{sk}^{*} \; is \; the$ SK $\; estimate \; for \; block \; V \; block \; estimate,$

$\lambda_i \; is \; the \; optimum \; weight \; assigned \; to \; sample \; v_i, and \; m \; is \; the \; selected \; SK \; mean$.

In summary, this chapter presented the literature review of the technical aspects to be considered in the estimation of a mean. The chapter highlighted the importance of geology and geological domaining; the benefits of a comprehensive exploratory data analysis and the methods that can be employed; descriptions and discussions around outlying data; methods for estimating a global mean; calculating and modelling spatial distributions; and estimation techniques. Outliers have a significant effect on the estimation of the mean and increase the variability during the calculation of a variogram that is not necessarily true. Cell declustering is seen to be an appropriate technique to use to obtain a globally representative mean because the spatial positioning of the data is considered. From this literature review the gap in knowledge regarding the estimate of a global mean for SK estimation has been identified, that is, the impact of low-grade outliers on the global mean estimate has not been assessed and will be investigated. The methods and techniques described in this chapter will be used to conduct the research study.

3 LOGGING AND ASSAY DATA VALIDATION

"Without data you're just another person with an opinion" – W. Edwards Deming

This chapter addresses the sources of data used for both geological modelling and resource estimation, the data validation, and the data compilation process. The borehole information provided by South Deep Gold Mine forms the basis for this research. This database is considered and validated for further use in geological, statistical, and geostatistical analysis in the research study analyses.

3.1 Reef Thickness and Grade Units

Boreholes with collar coordinates, survey data, lithology/reef logs with thicknesses, and Au assay grades stored on the Datamine Fusion® database were used for this research study. Underground mapping data was also used to facilitate geological modelling.

Au assay grades are measured as gram per tonne (g/t). Due to the type of deposition and massive nature of the orebody, with a thickness of up to 130 m, there is no obvious correlation between the channel width and the grade of the reef. The reason for this is the occurrence of multiple depositional events for a single reef horizon, hence centimetre gram per tonne (cm.g/t) units are not relevant to this research study

3.2 Quality Assurance and Quality Control

Data validation was performed to ensure that the data used for this research study was clean and error free. Original Au (g/t) assay and thickness data collected and stored on the Datamine Fusion® database were used for this study. A full extraction of all borehole sample data with a status of "passed" were extracted from the database for the MBB reef. The "passed" status means that these samples passed the QA/QC process of South Deep Gold Mine. Certified reference materials (CRMs) and blanks were inserted between borehole samples before being sent to the laboratory for fire assay testing. The CRMs are of a known Au-grade and are used to test for accuracy of the fire assay methodology. The blanks are quartzitic material devoid of Au-grade and are used to test for contamination during the sample preparation process. All material, that is, samples, CRMs, and blanks were sent to the laboratory for fire assay analysis to determine the Au grade. The assaying results obtained were uploaded to the Fusion®

database and samples either pass or fail in batches of ten depending on the results obtained for the CRMs and blanks.

The detection limit for a CRM is three standard deviations (3SD) above and 3SD below the expected Au-grade of the CRM, as shown in Figure 3.1. The CRM, AMIS0729, in Figure 3.1 failed once, this failure is represented by the red dot on the graph which is above the 3SD line.

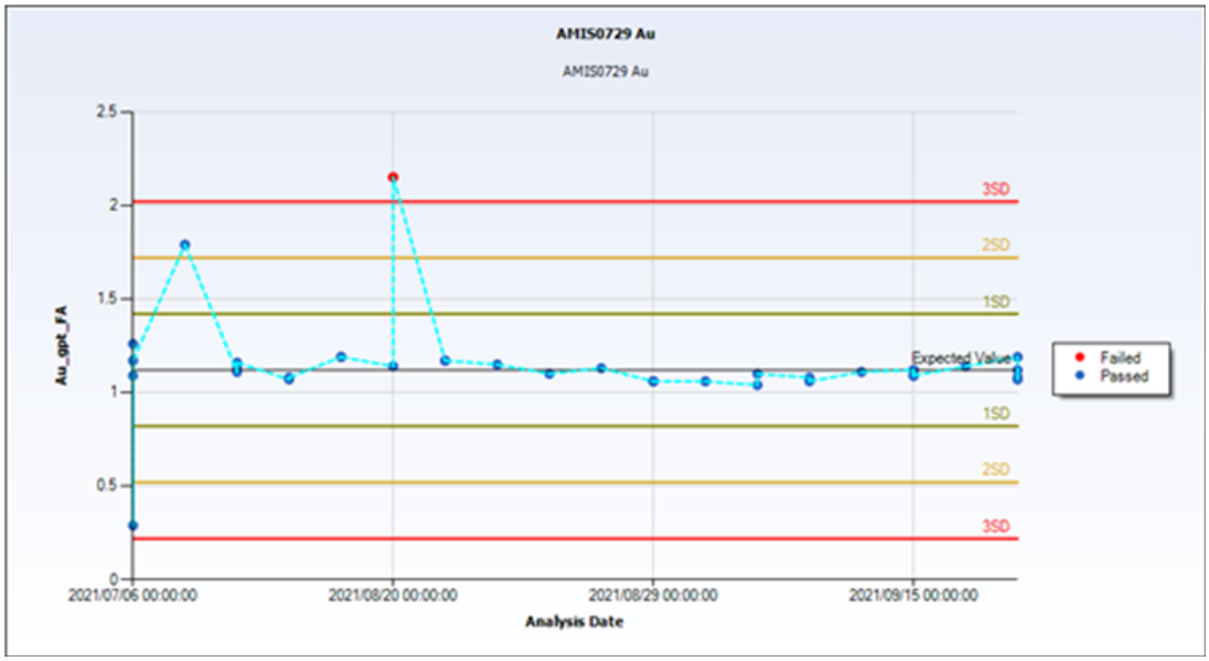


Figure 3.1 Au performance of CRM (AMIS0729) showing 3SD limits above and below the expected value of 1.2 g/t (output from database)

If the assay results of a CRM fall outside of the specified limits, as in Figure 3.1 the CRM fails on the system, along with nine samples assayed before, and nine samples assayed after that CRM in the laboratory. This batch of samples including the CRM are re-assayed at the laboratory. In some cases, if the amount of CRM available for the re-assay is insufficient, a new CRM is inserted.

The detection limit for blank material is 0.06 g/t, blank material will fail the QA/QC process if a value of 0.06 g/t or above is the result from fire assay testing. Figure 3.2 below shows an example of the performance of blank material over a period.

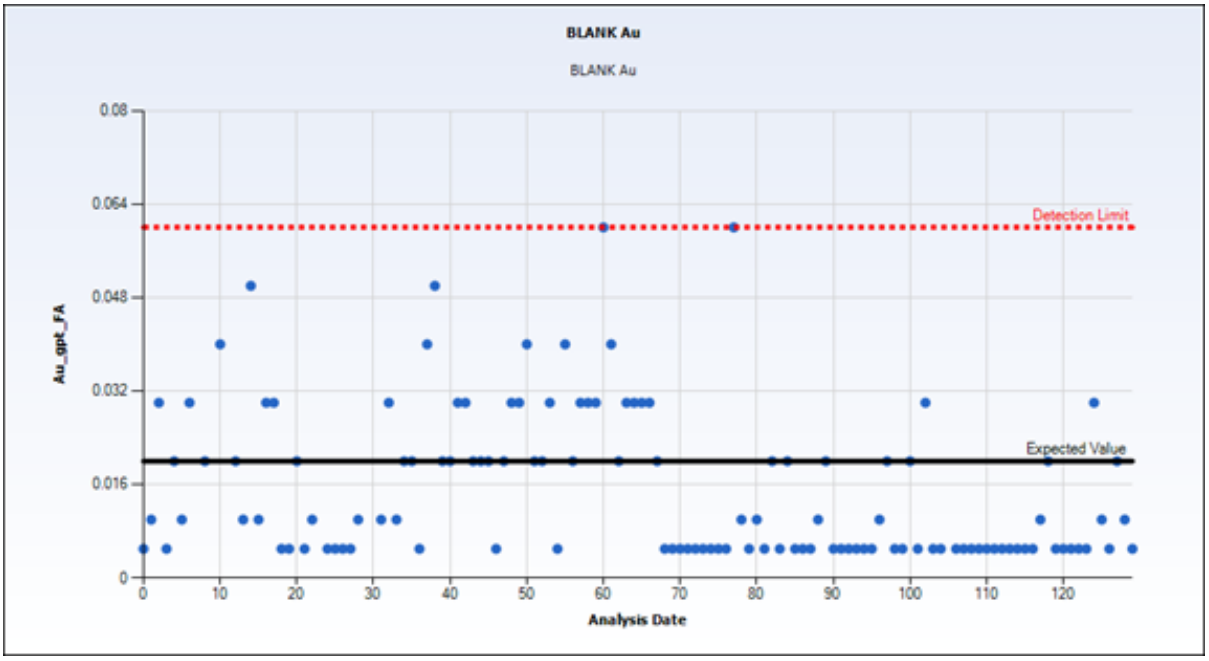


Figure 3.2: Graph showing the performance of blank material at a detection limit of 0.06 g/t (output from database)

The failure of blank material indicates contamination during sample preparation at the laboratory and results in nine samples failing before the blank and nine samples failing after the blank in a sequence. The samples failing the QA/QC process, at the time of the data extraction from the database, does not form part of the estimation process. This includes samples that are still at the laboratory for reassaying.

The QA/QC process also includes QA/QC on the laboratory that conducts the fire assay testing. Replicate or duplicate samples are submitted to the laboratory to test the laboratory's precision. The duplicate samples are of two forms, that is, a course duplicate, which is taken after a sample is crushed, and a pulp duplicate, which is taken after a sample is pulverised. Accuracy of the laboratory is also tested by sending sample pulps, tested at the primary laboratory, to an umpire laboratory also for fire assay testing.

The borehole data passing QA/QC were also visually checked to ensure that collar position coordinates were correct and appropriate and spatially plot within the mine lease area. The downhole survey was checked to ensure that the orientation and inclination of the boreholes were captured correctly, a positive dip means the borehole was drilled in a downward direction and a negative dip means the borehole was drilled in an upward direction. Boreholes

with errors that could not be rectified or reconciled were removed from the final dataset and were not used for the estimation process in this research study.

3.3 Sample Compositing

Borehole data that passed the QA/QC process were extracted from the database. The MBB reef data, relevant to this study, was filtered out from the Datamine Fusion® database and represent the validated raw data used for this research study.

Borehole sample sizes vary from 10 cm up to 1 m, to achieve a common sample support, the samples were then composited in Datamine®. A composite support of 1 m was used, to align with the selective mining unit (SMU) support of 5 m x 5 m x 1 m. A minimum composite length of 0.2 m and maximum gap length of 0.5 m was used. Full reef composites were also done for statistical comparison. To verify that the 1 m sample compositing was successful, the histogram of the composite length was plotted as shown in Figure 3.3.

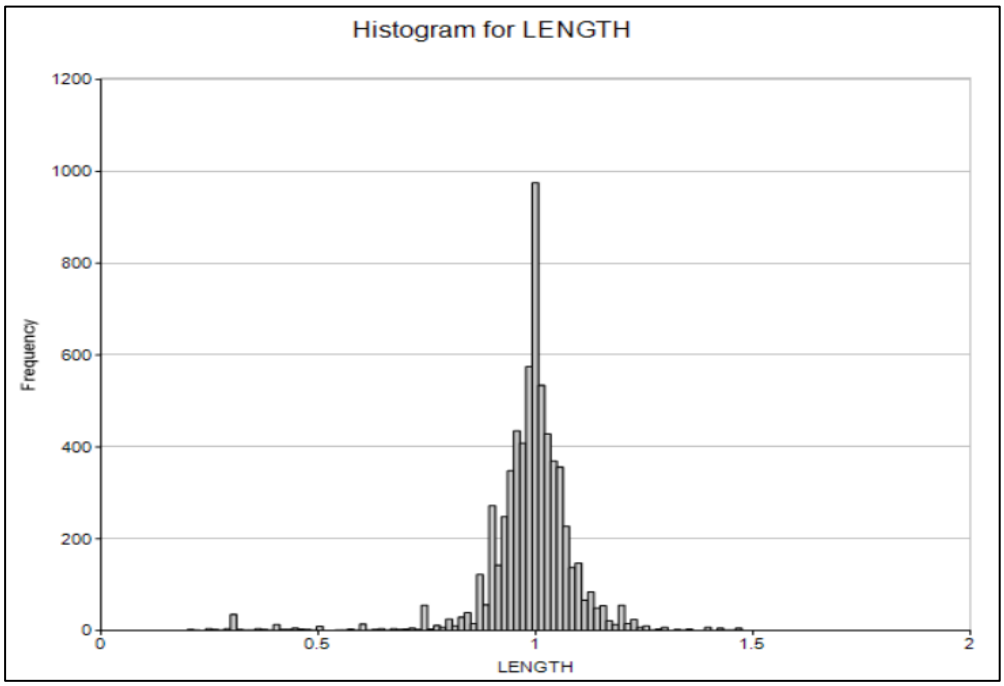


Figure 3.3: Histogram of composite sample lengths

The composited samples were flagged according to each domain, to constrain composites to their respective domains. Thereafter data was rotated to the horizontal plane to counteract the 15˚ south-easterly dip of the ore body. The data was then projected to a flat datum plane

to counteract or remove the fault displacement. The MBB borehole data in the real space, rotated space and flattened space are depicted in Figure 3.4.

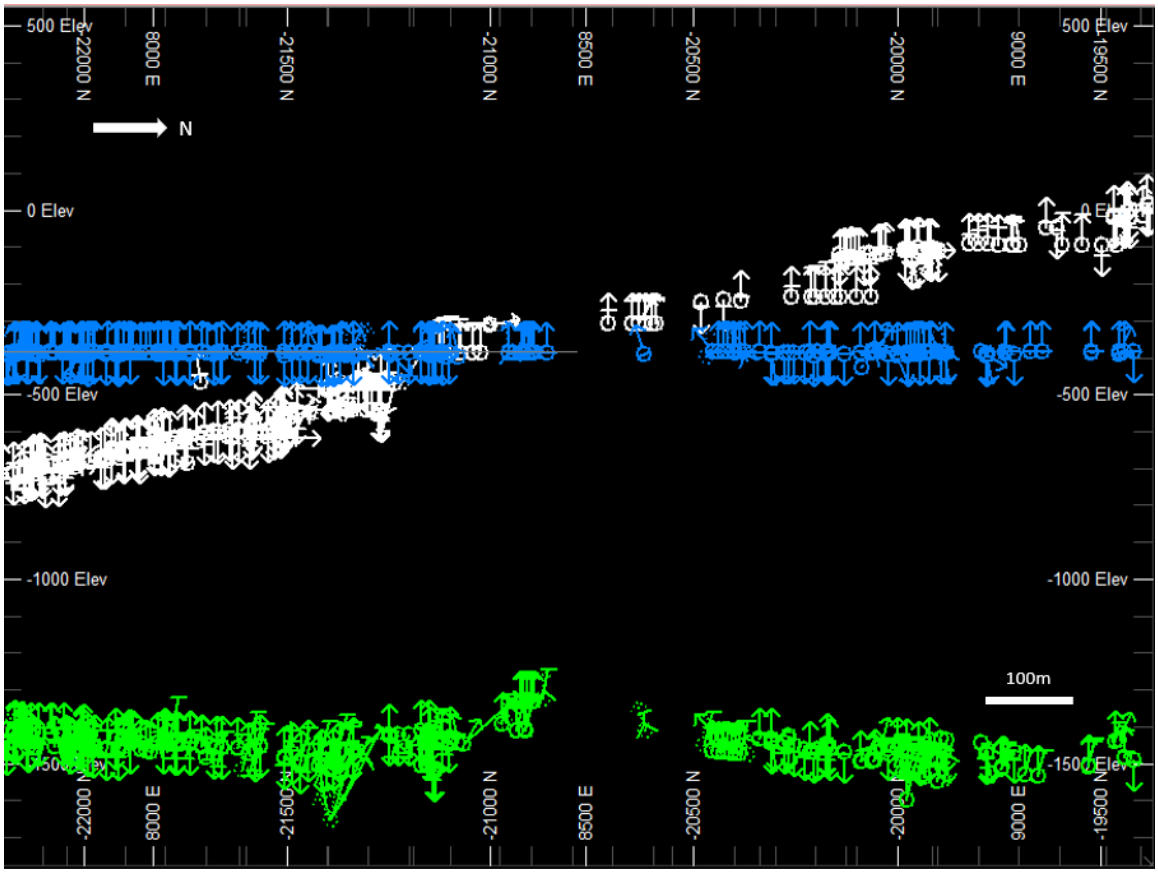


Figure 3.4 MBB borehole data in real space (white), rotated space (green), and flattened space (blue)

The rotation was done to facilitate better wireframe filling where the reef volume is filled with cells that will be estimated into. The flattening of the data enables a greater data selection within the specified search ellipse during kriging.

In summary, this chapter addressed the sourced, validated, and compiled data that will be used going forward for statistical and geostatistical analyses to investigate the impact of low-grade material on the global mean estimate in the chapters to follow. The borehole data in conjunction with mapping data will be used to model the MBB reef in Leapfrog Geo®.

4 MODELLING AND DATA ANALYSIS

This chapter describes the geological modelling process, the exploratory data analysis (EDA) and the domain selection process followed in this research study. Borehole data, mapping data and domain boundaries are the primary data inputs considered in this chapter.

4.1 Geological Modelling

A sound geological model is the foundation on which geostatistical analyses are built. Modelling of the MBB reef was done using Leapfrog Geo® 3D software. Fault blocks were created in Leapfrog Geo® by establishing fault relationships, and reef surfaces were modelled within each of these fault blocks. The MBB reef contacts mapped underground by production geologists along with borehole data were used to create the top and bottom surfaces confining the MBB reef within fault block structures. Volumes for the MBB reef was created, using these surfaces and boreholes, for each fault block as shown in Figure 4.1. The MBB reef has a relatively constant channel width thickness of 5 m.

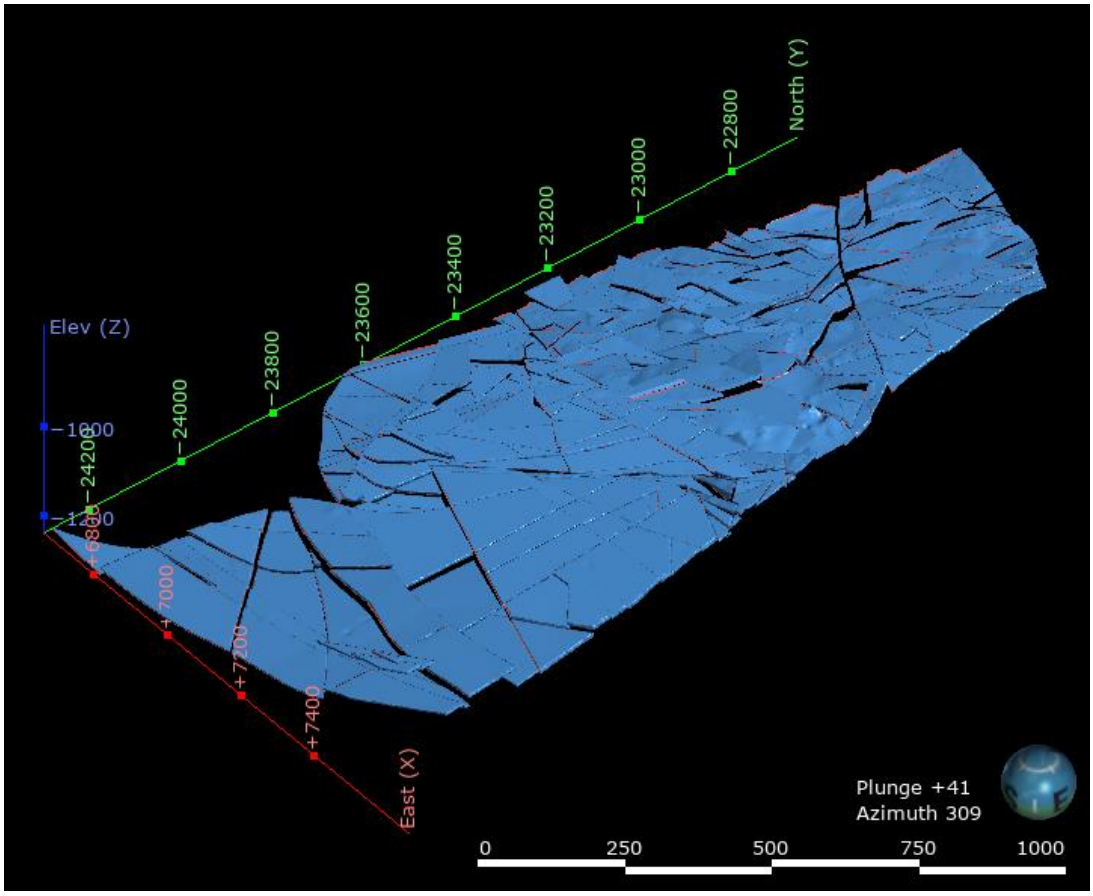


Figure 4.1: Showing MBB reef modelled in Leapfrog Geo® software

4.2 Geological Domaining and Domain Selection

Domains are homogenous zones within which statistical and geostatistical analyses are constrained. Sedimentological facies parameters such as channel width, percentage conglomerate and average clast size were considerations for the geological domaining of the MBB reef at South Deep Gold Mine. These geological domains formed the foundation from which the geostatistical domains were constructed by taking into consideration the Au grade. Figure 4.2 shows the geological domains on the left and the geostatistical domains on the right as they are currently delineated at South Deep Gold Mine.



Figure 4.2: MBB reef with geological domains (left) and geostatistical domains (right)

For comparative reasons, the geostatistical domains were kept as they are currently at South Deep Gold Mine. Statistical analyses were conducted on the Au (g/t) grade distributions for all 16 domains in a process to determine the three domains that will be used for this research analysis. Descriptive statistics were calculated for all 16 domains, the detailed results of which can be found in Table A 1 in the Appendix.

The average Au grade (g/t) and number of samples in each domain were noted, then the domains grouped or zoned into three broad grade zones (low- medium- and high- Au grade zones) according to their average grade and are shown in Table 4.1 below together with the currently selected South Deep global means to be applied in SK.

Table 4.1 Domains of the MBB reef zoned by average grade

Grade Zone	Domain	Global means (South Deep)	Average	No. of data points	Research Domains Chosen
Low-grade	14	3.05	2.46	176	
	10	3.09	3.27	120	
	15	3.37	3.18	381	Dom 15
	11	5.80	5.37	260	
	6	6.21	6.43	128	
	16	6.23	5.49	496	
Medium-grade	9	8.00	8.84	480	Dom 9
	12	7.83	7.54	468	
	1	8.31	7.14	31	
	13	9.46	10.15	531	
	7	10.16	10.42	114	
	8	12.33	16.38	156	
	5	12.92	13.83	393	
High-grade	3	15.44	16.44	157	
	4	19.74	17.85	341	Dom 4
	2	20.66	20.20	258	

From Table 4.1, the cells highlighted in blue were zoned as low-grade domains, the domains highlighted in orange were zoned as medium-grade domains, and the domains highlighted in red were zoned as high-grade domains. It was decided to identify one representative domain per grade zone for further analysis in this research study. Domains with sufficient samples and with a favourable spatial distribution of the sample locations see Figure 4.3 below, were considered as possible representative domains in the research study investigation. Domains with a high number of data points are highlighted in green in Table 4.1.

For the low-grade zone, the options were either Domain 16 (496 samples) or Domain 15 (381 samples). Domain 15 was chosen despite having less samples than Domain 16, because the average grade of the domain was lower, and this choice would ensure a wider average grade range or more variation in grade for the domains being investigated in this research study.

The three options identified for the medium grade zone were Domain 9 (480 samples), Domain 12 (486 samples), and Domain 13 (531 samples). Domain 9 was chosen over both Domain 13 and Domain 12 even though these domains had more data, the areal extent of these domains and the spatial distribution of the data within them as shown in Figure 4.3 also were considered. Domain 9 is larger than both these two domains, and the SK global mean would have more influence in a zone with a scattered sample location distribution, an aspect that is relevant to this research study. For the high-grade zone, Domain 4 was selected with 341 samples, the other two domains have relatively small numbers of data points.

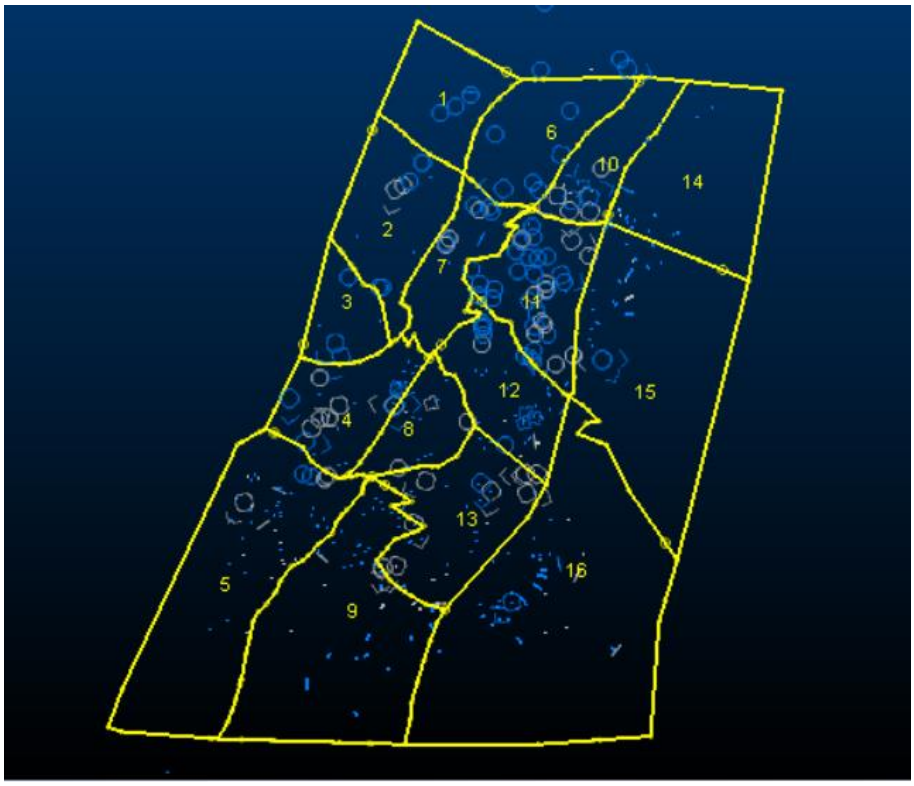


Figure 4.3 MBB domains showing the spatial distribution of the data

Cumulative probability distributions (CPDs) of the Au grade were constructed for all 16 MBB reef domains and grouped according to the low- medium- and high- Au grade zones as highlighted in Table 4.1. The CPD curves for the three grade zones are respectively presented in Figure 4.4, Figure 4.5, and Figure 4.6 below. The shapes of the curves within each figure are similar to each other, however, the shapes of the curves between figures are different, confirming that the grade zoning applied is plausible and acceptable. From Figure 4.4 it is seen that Domains 10, 14, and 15 and Domains 6, 11, and 16, all of which make up the low-grade zone defined in Table 4.1, have similar grade distributions that respectively cluster together.

Figure 4.5 shows similar behaviours in the CPD curves for the medium-grade domains, and Figure 4.6 shows similar CPD curves for the high-grade domains. From the interpretation of these figures, it is concluded that the individual domains chosen from each grade zone is representative of that grade zones and that the grade zones are uniquely different which is desirable for this research study.

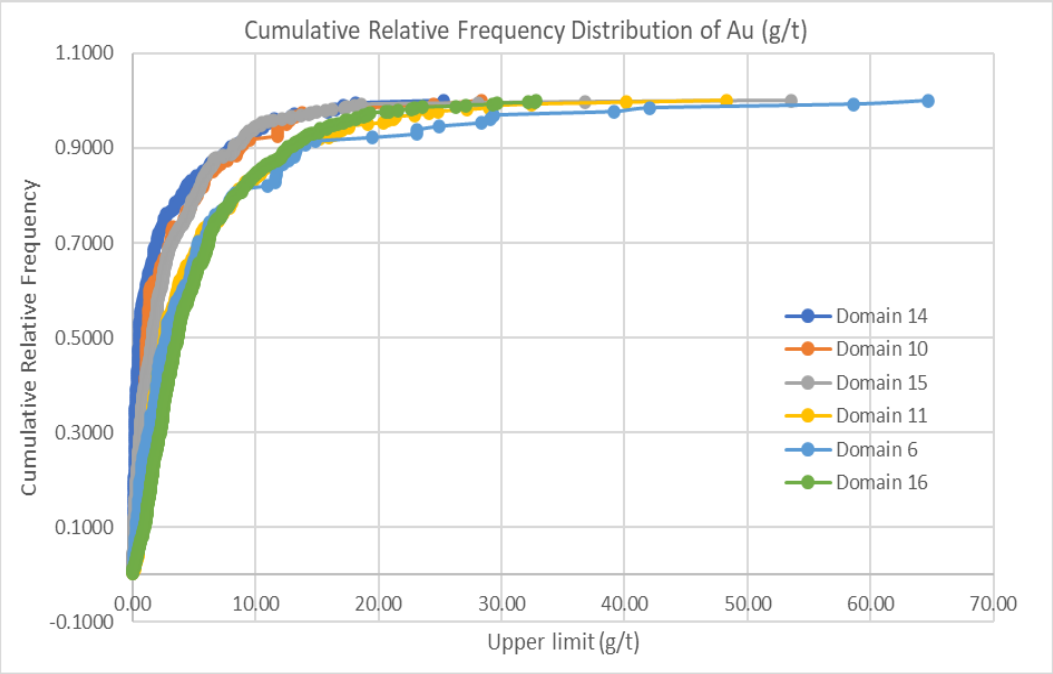


Figure 4.4 Cumulative Frequency Distribution Curves for Domains 6, 10, 11, 14, 15, and 16

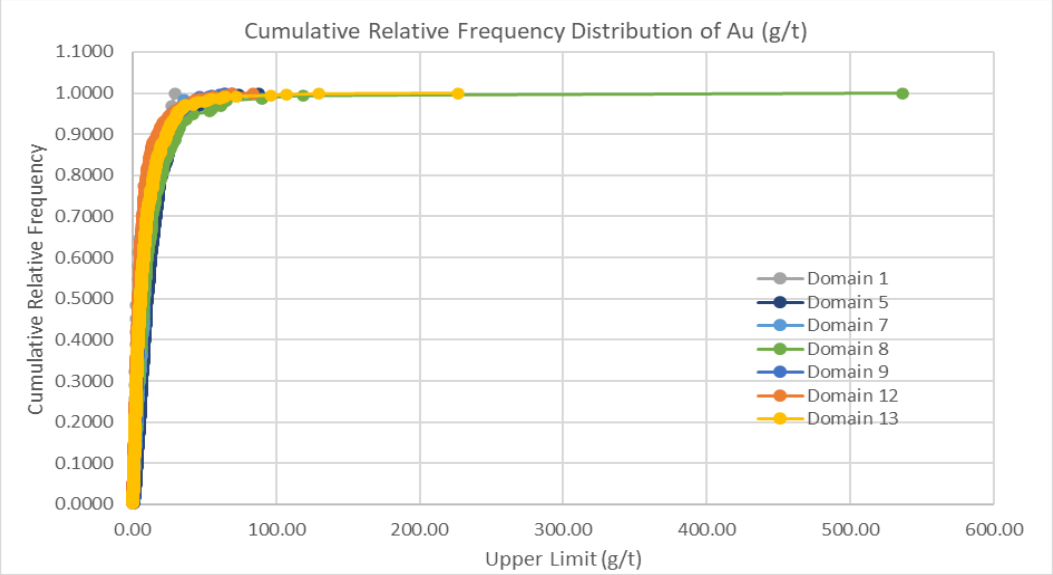


Figure 4.5 Cumulative Frequency Distribution Curves for Domains 1, 5, 7, 8, 9, 12, and 13

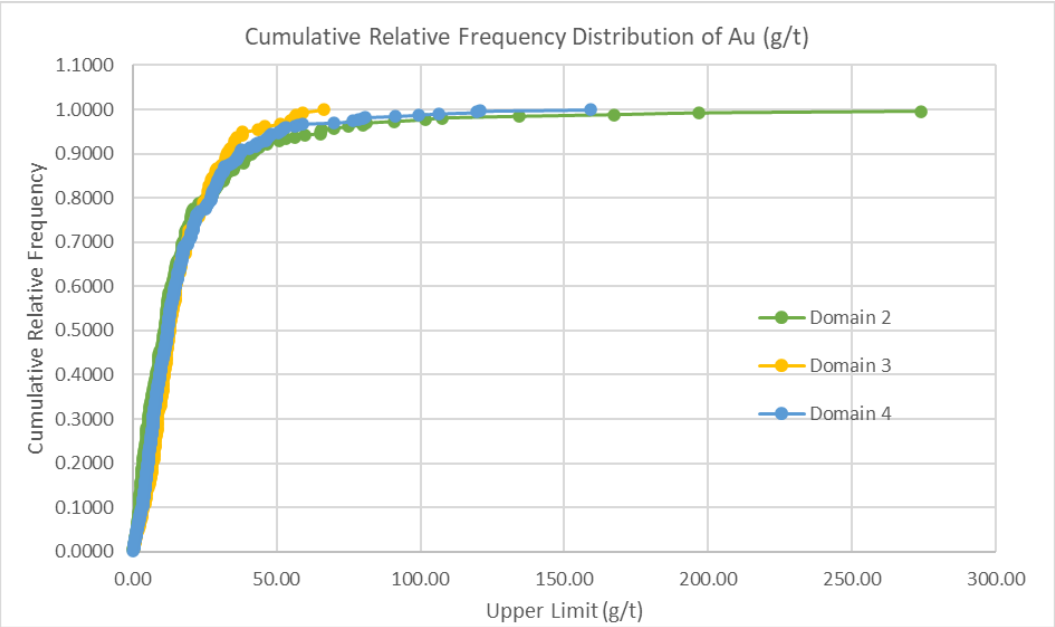


Figure 4.6 Cumulative Frequency Distribution Curves for Domains 2, 3, 4, 5, 7, and 8

The delineation of the domains selected for this study is shown along with the MBB reef wireframe on the left in Figure 4.7. Inverse distance estimation was conducted in Datamine® software with the MBB sample data and the MBB reef volume filled with cells of size 30 m x 30 m x 1 m. A regularized Au grade grid, as depicted on the right in Figure 4.7, was created showing the average grades of the three selected domains.

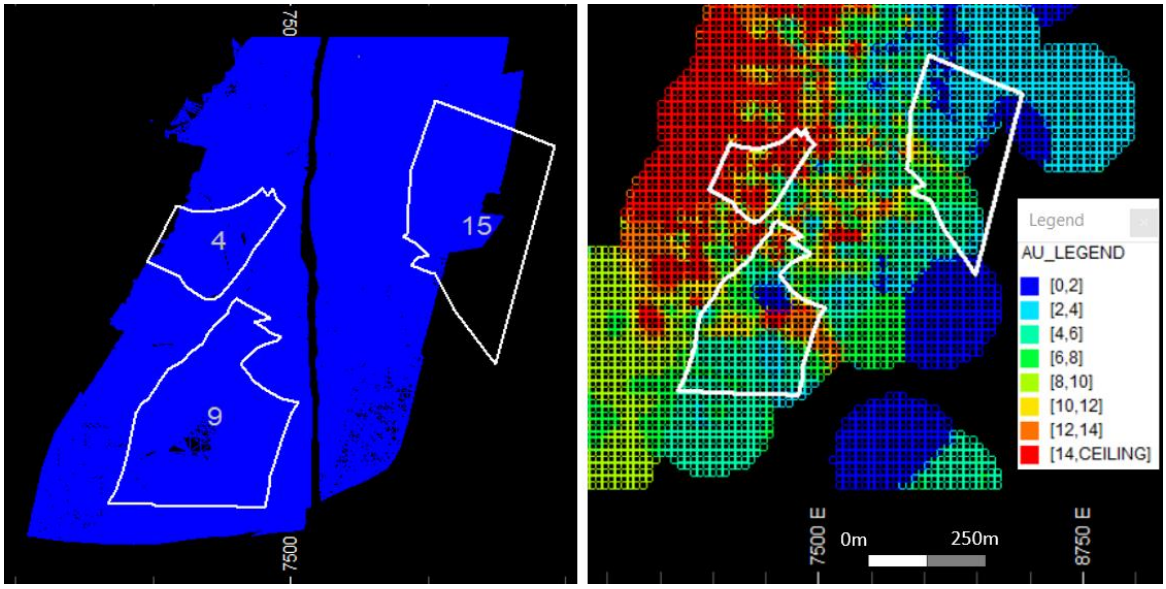


Figure 4.7: MBB reef highlighting selected Domains 4, 9 and 15 (left) and regularised 30 m x 30 m Au grades (right)

4.3 Exploratory Data Analysis

An exploratory data analysis was conducted on the three chosen domains to develop an understanding of the sample data and the underlying distributions. Descriptive statistics were calculated on one meter sample composites, the compilation of which was described in Chapter 3. Table 4.2 below gives a summary of the numerical descriptive statistics for each of the selected domains. Full reef composite statistics are given in Table A 2 of the Appendix.

Table 4.2: Descriptive Statistics of Domains 4, 9 and 15 for 1 m composites of the MBB reef

Domain	No. of samples	Minimum (g/t)	Maximum (g/t)	Mean (g/t)	Variance (g/t)^2	Standard deviation (g/t)	CoV
4	341	0.215	159.157	17.855	388.697	19.715	1.104
9	480	0.102	69.158	8.835	106.036	10.297	1.165
15	381	0.019	53.539	3.18	23.75	4.873	1.532

It is seen from Table 4.2 above that the high-grade Domain 4 has an average grade of 17.86 g/t, the medium-grade Domain 9 has an average of 8.84 g/t, and low-grade Domain 15 has an average of 3.18 g/t. The standard deviation can be viewed as the typical difference between an actual sample value and the mean of the samples (Sinclair, 1998). The coefficient of variation (CoV) is a relative measure of variability, it measures the variability that remains after the influence of the average has been removed (Sinclair, 1998). The CoV is a ratio of the standard deviation to the mean and therefore independent of units which allows different data sets to be compared directly with each other. It is observed that the standard deviation and CoV for each of the domains increases from the low-grade zone to the high-grade zone. A CoV value of less than 0.7 typically represents low variance within a dataset. The CoV for the domains in this study range from 1.1 to 1.5 indicating a wider dispersion in the data sets. The CoV values of the full reef composites are less than those for the 1 m composites (Table A 2 in the Appendix) as is expected for longer composites lengths.

Graphical summaries of the Au grades are given in Figure 4.8 and Figure 4.9. The histograms and PP plots provide a graphical presentation of the data distribution and were used to assist in identifying outliers to determine “top-cut” and “top-cap” thresholds.

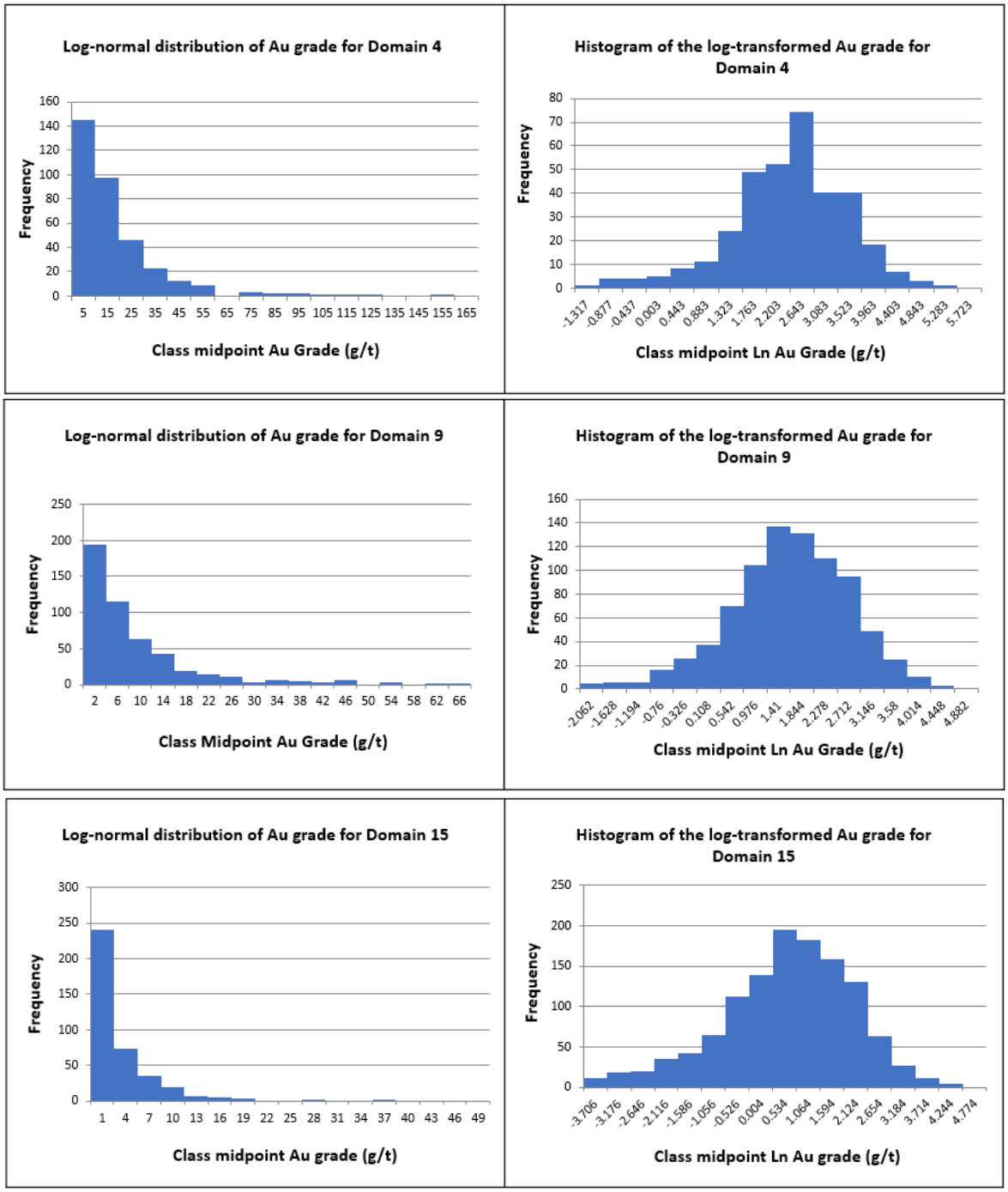


Figure 4.8: Histograms for Domain 4, 9 and 15 of the MBB reef

The Au grade histograms shown in Figure 4.8 on the left are positively skewed, as expected for a gold deposit with more low-grade samples and few occurrences of high-grade samples. It is also observed that the histograms of the log-transformed grades shown on the right in

Figure 4.8 are more symmetrical and similar to a normal distribution. The CFDs were plotted for each domain and are shown in Figure 4.9. The CFD can be used to estimate the median, mean, and interquartile range (IQR). The median can be read off from the CFD graph at a cumulative frequency of 0.5.

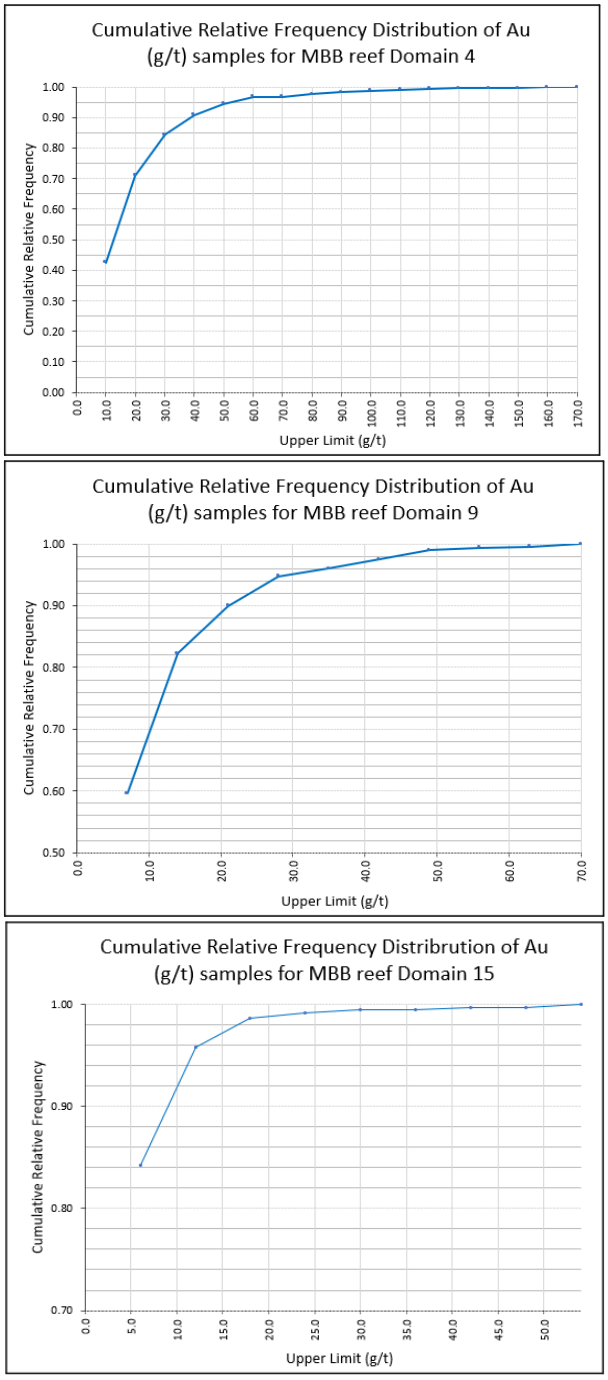


Figure 4.9 Cumulative relative frequency distribution curves for Domains 4, 9 and 15 for the MBB reef

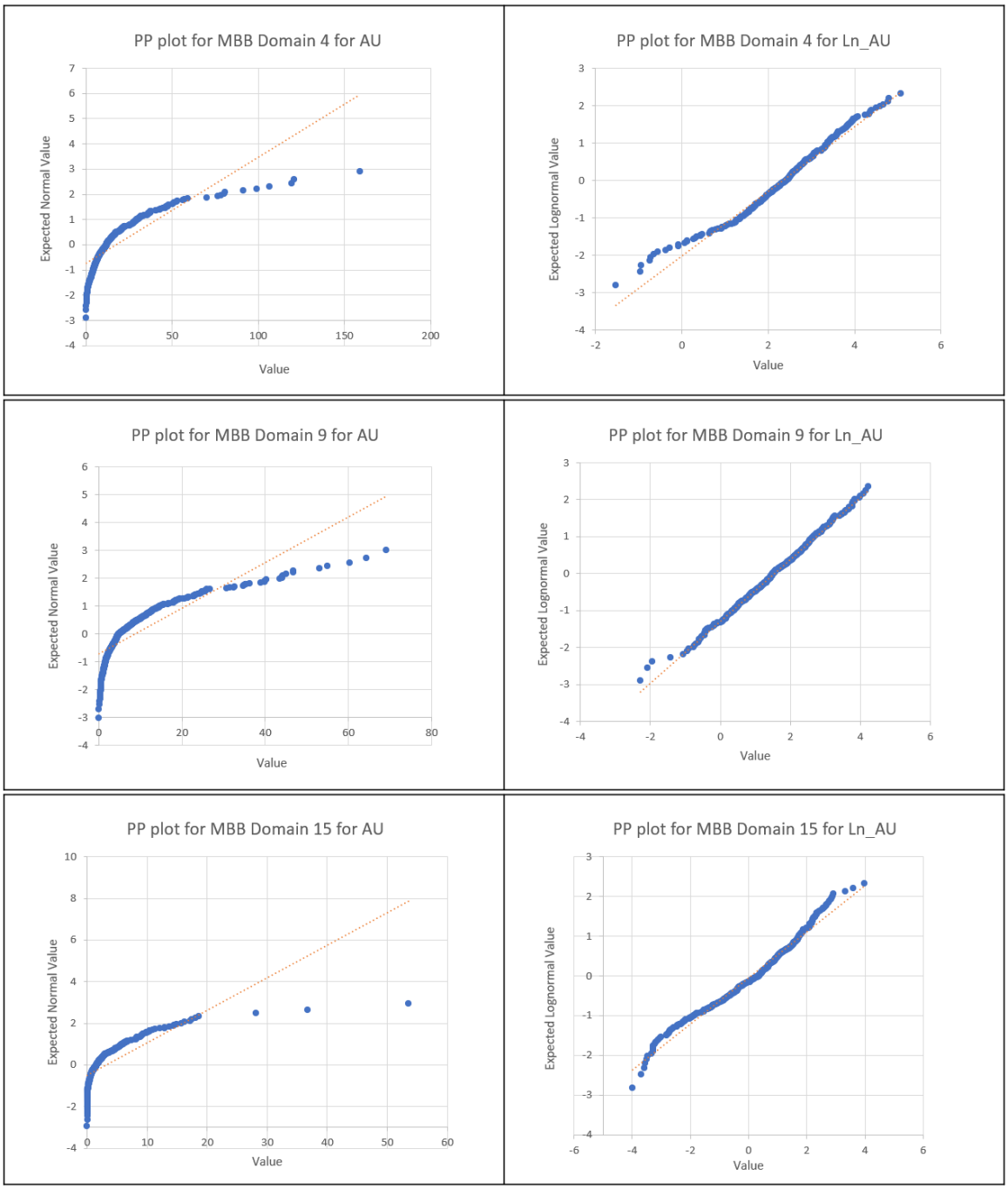


Figure 4.10 PP plots for the MBB reef Domains 4, 9 and 15

The kriging top caps and variogram top cuts were chosen using the histograms in Figure 4.8 and the PP plots in Figure 4.10. The top cut and top cap values were validated against the CFD curves in Figure 4.9. The values are tabulated below in Table 4.3 for each domain.

Table 4.3 Kriging "top caps" and variogram "top cuts" for each domain

Domain	Maximum value (g/t)	Variogram top cut (g/t)	Kriging top cap (g/t)
4	159.16	52	130
9	69.16	28	50
15	53.54	15	20

The kriging top cap for each domain was chosen to remove outlying high-grade values by replacing the high-grade value with that of the chosen top cap value. The data is top capped without removing any samples. Grades higher than the kriging top cap threshold are given the value of the kriging top cap and the sample number remains the same. The kriging top cap is more inclusive of samples so that those values can still inform the local grade during kriging. The variogram top cut values were chosen to remove unwanted or erratic variability within the data to determine the direction and distance of continuity of the sample data within each domain. The variogram top cut removes the outlying samples from the data set and the number of samples decrease.

In summary, borehole and mapping data provided by South Deep Gold Mine was utilised for the 3D geological modelling and domain selection analysis. Spatial data distributions and statistical analyses were conducted on all 16 domains delineated by South Deep Gold Mine. The rationale for selecting the specific high-, medium- and low-grade domains was presented. The data statistics and grade distributions aided the selection of three domains, one form each grade zone, to be used for this research analysis. Top cut and top cap values were defined, to be used for variography and grade estimation.

5 DECLUSTERING AND GLOBAL MEAN ESTIMATION OPTIONS

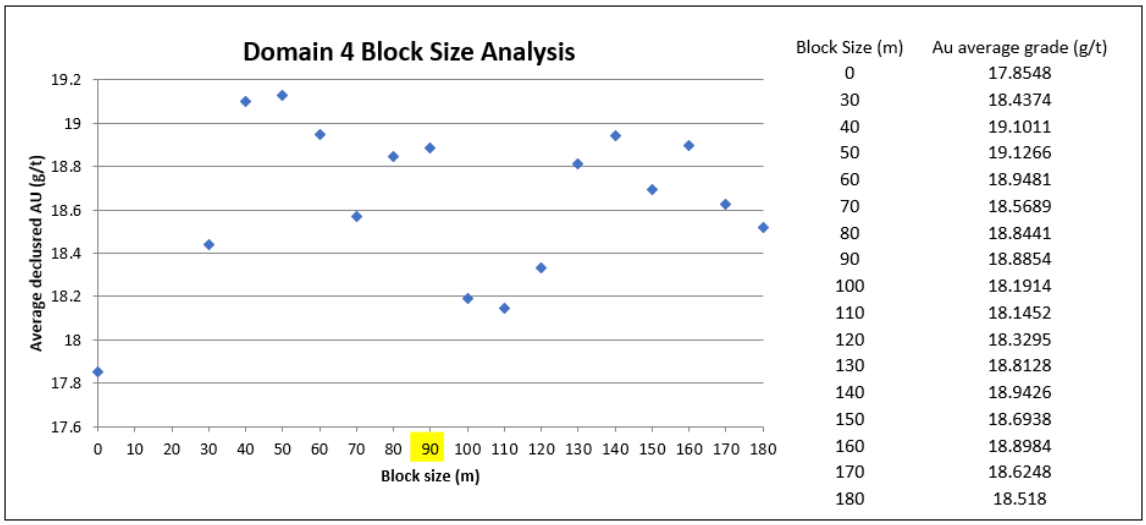
THIS chapter details the declustering process from obtaining the optimum block size to determining the declustered average of an estimation domain. At South Deep Gold Mine a global mean is chosen by taking into account a number of considerations, the most important being the declustered average. In this research study a declustered average is also calculated after removing low-grade material at different grade thresholds for comparison in the selected domains. In this chapter different methods of estimating a global mean are investigated and compared to the determined SK mean currently in use per domain.

5.1 Cell Declustering

Clustering of data is caused by irregular drilling especially in high-grade areas where higher confidence is required resulting in more boreholes being drilled in a cluster. Declustering is done for statistical purposes to obtain a representative global mean for each domain. The global mean plays an important role during SK in areas where there is insufficient data as more weight will be assigned to this value in SK in these instances.

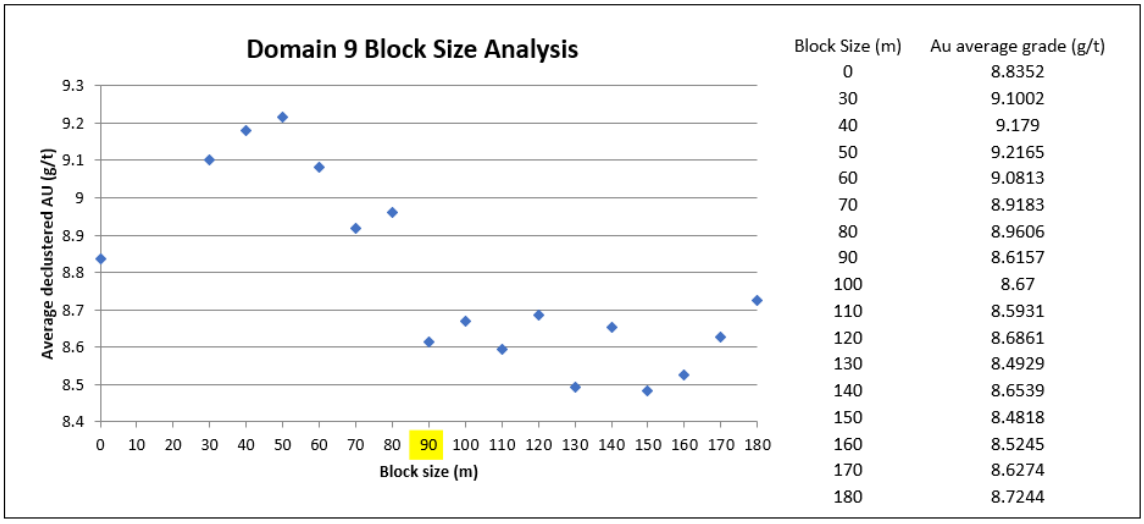
Determining the local mean involves declustering of the sample data. Declustering is a process of placing a regular grid of cells over the data and calculating the local mean for each cell in the specific grid chosen and then calculating a global mean from those local means. The declustering methodology was described in Chapter 2.

To determine the optimal grid size, the weighted average grade was calculated for a range of cell sizes using the GRIDDC function in Datamine® software per domain. The cell size that minimised the weighted average grade was selected as the optimal declustering cell size. It is recommended to not choose a block size greater than 120 m x 120 m x 50 m as the domains are relatively small and not to choose a block size that is too small as data is sparse. From the graphs in Figure 5.1, Figure 5.2, and Figure 5.3 below, a cell size of 90 m x 90 m x 50 m was chosen as the optimal declustering cell size.



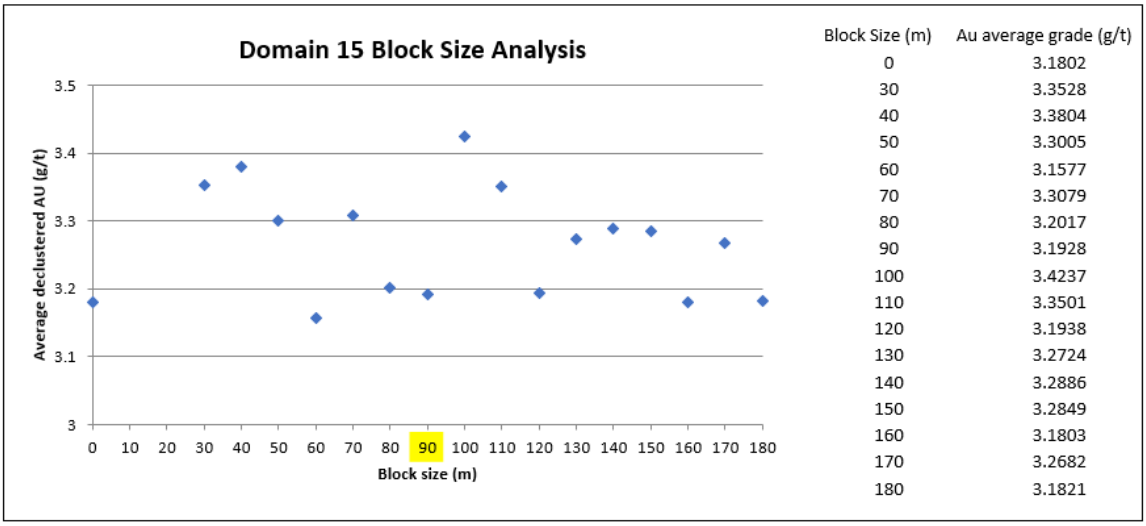
Block Size (m)	Au average grade (g/t)
0	17.8548
30	18.4374
40	19.1011
50	19.1266
60	18.9481
70	18.5689
80	18.8441
90	18.8854
100	18.1914
110	18.1452
120	18.3295
130	18.8128
140	18.9426
150	18.6938
160	18.8984
170	18.6248
180	18.518

Figure 5.1 Optimal block size analysis for Domain 4



Block Size (m)	Au average grade (g/t)
0	8.8352
30	9.1002
40	9.179
50	9.2165
60	9.0813
70	8.9183
80	8.9606
90	8.6157
100	8.67
110	8.5931
120	8.6861
130	8.4929
140	8.6539
150	8.4818
160	8.5245
170	8.6274
180	8.7244

Figure 5.2 Optimal block size analysis for Domain 9



Block Size (m)	Au average grade (g/t)
0	3.1802
30	3.3528
40	3.3804
50	3.3005
60	3.1577
70	3.3079
80	3.2017
90	3.1928
100	3.4237
110	3.3501
120	3.1938
130	3.2724
140	3.2886
150	3.2849
160	3.1803
170	3.2682
180	3.1821

Figure 5.3 Optimal block size analysis for Domain15

In this research study declustering was done using a block size of 90 m x 90 m x 50 m for the full data set of each domain for a grid with a fixed origin and repeated for a grid with a moving origin. Using an optimal block size of 90 m x 90 m x 50 m made the results obtained comparable with those obtained by South Deep Gold Mine where the same optimal block size is implemented. The results are reflected in Table 5.1 along with the global means that are currently in place at South Deep Gold Mine using the same data and data support for all domains.

Table 5.1 Declustered global mean results

Domain	Average (g/t)	Fixed origin declustered mean (g/t)	Moving origin declustered mean (g/t)	Global mean used at South Deep Gold (g/t)
4	17.86	19.74	18.89	19.74
9	8.84	7.96	8.62	8.00
15	3.18	2.67	3.19	3.37

From Table 5.1 for the high-grade Domain 4, it is seen that the fixed origin declustered mean is used as the global mean by South Deep. The declustered mean for both the fixed and moving origin are higher than the average for Domain 4. The moving origin declustered mean for both Domain 9 and Domain 15 are higher than the fixed origin declustered mean. In Domain 9 the global mean used by South Deep is lower than the moving origin declustered mean and the domain average but is higher than the fixed origin declustered mean. The global mean for Domain 15 used by South Deep is higher than both the declustered means and the average of the domain.

The data for each domain was then bottom cut at various grades to gradually remove low-grade material and the declustering process was reiterated. A bottom cut is similar to the top cut, except here, samples below the specified threshold are removed for the purposes of determining a global mean for SK. The bottom cut-off grades for each domain differ due to difference in average grade in each domain. A high-grade domain cannot be dealt with in the same manner as a low-grade domain. High-grade Domain 4 will have higher bottom cut-off thresholds than the low-grade Domain 15. This was done because removing low-grade samples (less than 1 g/t) in a high-grade zone does not have much of an impact on the overall average. The average of the high-grade zone is not sensitive to removing a few low-grade

samples. When removing low-grade samples from a low-grade domain a more visible impact on the average would be observed. It is for this reason that different bottom cut-off grades were chosen for each domain depending on the average grade of the domain.

The bottom cut-off grades chosen for the high-grade Domain 4 were 1 g/t, 2 g/t and 3 g/t. The sample data below each of these bottom grade cut-offs were removed and declustering was again performed on the remaining data above the bottom grade cut-off grades. The declustered mean results for Domain 4 appear in Table 5.2. At a bottom grade cut-off of 1 g/t the declustered mean increased from 19.74 g/t when no cut-off was applied to 20.01 g/t. A slight increase is seen as the bottom cut-off grade was increased from 1 g/t to 3 g/t, but it is not significant. The fact that the declustered mean does not change significantly could imply that these low-grade samples could be low-grade outliers that should be removed.

Table 5.2 Domain 4 declustered mean grades at different bottom cut-off grades

All Data g/t	**> 1 g/t**	**> 2 g/t**	**> 3 g/t**
19.74	20.01	20.57	20.78
Number of samples removed	11	23	33

The results tabulated in Table 5.3 below are that of the local means for each 90 m x 90 m x 50 m declustered block. The declustering of Domain 4 produced 17 declustered blocks shown in Table 5.3 along with the number of samples present in each declustered block.

Table 5.3 Domain 4 declustered block mean grades per block at various bottom cut-off grades

Block No.	**Block Average Au g/t all data**		**Au > 1 g/t**		**Au > 2 g/t**		**Au > 3 g/t**	
	Au (g/t)	No. of samples	Au (g/t)	No. of samples	Au (g/t)	No. of samples	Au (g/t)	No. of samples
1	11.37	18	11.37	18	11.96	17	13.24	15
2	28.79	12	28.79	12	28.79	12	28.79	12
3	17.26	10	17.26	10	17.26	10	17.26	10
4	18.64	10	18.64	10	18.64	10	18.64	10
5	14.62	51	14.62	51	15.43	48	16.61	44
6	15.17	66	15.40	65	16.07	62	16.75	59
7	21.13	21	21.13	21	21.13	21	21.13	21
8	17.65	3	17.65	3	17.65	3	17.65	3
9	17.16	1	17.16	1	17.16	1	17.16	1
10	15.37	23	16.03	22	16.74	21	16.74	21
11	11.38	5	11.38	5	11.38	5	11.38	5
12	24.47	10	24.47	10	30.29	8	30.29	8
13	15.94	5	15.94	5	15.94	5	15.94	5
14	57.90	12	57.90	12	57.90	12	57.90	12
15	15.29	54	17.85	46	18.21	45	18.56	44
16	18.22	18	19.25	17	19.25	17	19.25	17
17	15.24	22	15.24	22	15.88	21	15.88	21

It is interesting to note in block numbers 5, 10, 12, and 15, highlighted in Table 5.3, that the average grade of Domain 4 increased by at least 2 g/t with the removal of a few low-grade samples. Of specific interest is block number 12, where two low-grade samples out of ten samples were removed, and the average grade increased drastically from 24.47 g/t to 30.29 g/t. Removing low-grade samples does have an impact, albeit small, on the declustered mean, but there appears to be more of an impact on a local scale as seen when comparing Table 5.2 to Table 5.3. Also, to note is that there is only one sample informing the high-grade of 17.16 g/t in block 9.

The same methodology described for Domain 4 was applied to the medium-grade Domain 9 and low-grade Domain 15. The bottom cut-off grades chosen for Domain 9 were 0.5 g/t, 1 g/t, and 1.2 g/t. The declustered means and the declustered block means at the different bottom cut-off grades are presented in Table 5.4 and Table 5.5 respectively. Bottom cut-off grades chosen for low-grade Domain 15 were 0.3 g/t, 0.5 g/t, and 0.7 g/t. Declustered means, and block means are shown in
Table 5.6 and Table 5.7 respectively.

Table 5.4 Domain 9 declustered mean grades at different bottom cut-off grades

All Data g/t	>0.5 g/t	>1 g/t	>1.2 g/t
7.96	8.04	8.76	8.92
Number of samples removed	9	35	47

From Table 5.4 we see an increase of almost 1 g/t in grade from declustering with all data compared with a bottom cut-off grade of 1.2 g/t. It is a considerable grade increase taking into consideration that the bottom cut-off values for Domain 9 are low. It is also worth noting that the declustered mean at a bottom cut-off grade of 0.5 g/t is 8.04 g/t, a mere 0.04 g/t difference to the global mean of 8.00 g/t currently in place at South Deep.

Looking locally at each declustered block in Table 5.5, there are some interesting results for highlighted blocks 5, 8, 10,14, and 24. The mean grades increase from between 1 g/t to over 4.5 g/t as the bottom cut-off grades were increased. Again, although there is an impact on the declustered mean when removing low-grade samples, we see more of the impact on a local scale within the individual 90 m x 90 m x 50 m declustered block for this medium-grade domain when comparing Table 5.4 to Table 5.5.

Table 5.5 Domain 9 declustered block mean grades at various bottom cut-off grades for each 90 m x 90 m x 50 m declustered block in the domain

Block No	All Data		> 0.5 g/t		> 1 g/t		> 1.2 g/t	
	Au (g/t)	No. of samples	Au (g/t)	No. of samples	Au (g/t)	No. of samples	Au (g/t)	No. of samples
1	1.10	5	1.63	3	3.44	1	3.44	1
2	12.62	8	12.62	8	12.62	8	12.62	8
3	0.60	1	0.60	1				
4	10.04	7	10.04	7	10.04	7	10.04	7
5	6.56	13	6.56	13	8.32	10	8.32	10
6	12.64	7	12.64	7	12.64	7	12.64	7
7	6.95	21	6.95	21	6.95	21	7.57	19
8	8.11	17	8.11	17	9.09	15	9.09	15
9	4.57	1	4.57	1	4.57	1	4.57	1
10	12.95	26	12.95	26	13.95	24	13.95	24
11	15.49	19	15.49	19	15.49	19	16.30	18
12	3.66	16	3.66	16	3.66	16	3.66	16
13	5.42	2	5.42	2	5.42	2	5.42	2
14	10.53	49	11.70	44	13.79	37	14.91	34
15	10.02	49	10.02	49	10.21	48	10.21	48
16	5.27	7	5.27	7	5.27	7	5.27	7
17	9.21	11	9.21	11	9.21	11	9.21	11
18	5.42	44	5.53	43	5.53	43	5.86	40
19	11.25	27	11.25	27	11.25	27	11.25	27
20	10.96	23	10.96	23	10.96	23	10.96	23
21	9.28	38	9.28	38	9.28	38	9.28	38
22	12.12	17	12.12	17	12.12	17	12.12	17
23	5.46	43	5.46	43	5.82	40	5.93	39
24	6.56	24	6.82	23	8.15	19	8.53	18
25	2.16	5	2.16	5	2.49	4	2.97	3

Table 5.6 Domain 15 declustered means at different bottom cut-off grades

All Data g/t	> 0.3 g/t	> 0.5 g/t	> 0.7 g/t
2.67	3.11	3.36	3.97
Number of samples removed	74	100	127

The declustered mean for low-grade Domain 15 increases with an increase in the bottom cut-off grade shown in Table 5.6. An increase of almost 1 g/t is seen for a bottom cut-off grade of 0.5 g/t which is substantial for a low-grade area. It could mean the difference between a block being considered or not considered during mine planning, that is, the block is either above the cut-off grade and is planned for or below the cut-off grade and not planned to mine. Also, very interesting to note is that the global mean used by South Deep for this domain is 3.37 g/t (Table 5.1), which is significantly higher than the declustered mean of 2.67 g/t considering all domain data. The chosen 3.37 g/t amounts to a 0.01 g/t difference to the declustered mean with a bottom cut of 0.5 g/t. It is almost as if the low-grade material was being accounted for by the intuition and experience of the South Deep team.

Table 5.7 Domain 15 declustered block mean grades at various bottom cut-off grades for each 90 m x 90 m x 50 m declustered block

	All Data		> 0.3g/t		> 0.5g/t		> 0.7g/t	
Block No.	Au (g/t)	No. of samples	Au (g/t)	No. of samples	Au (g/t)	No. of samples	Au (g/t)	No. of samples
1	3.13	15	3.13	15	3.34	14	3.79	12
2	2.35	3	3.38	2	3.38	2	3.38	2
3	6.05	63	6.05	63	6.14	62	6.32	60
4	3.65	59	4.04	53	4.11	52	4.11	52
5	5.77	22	5.77	22	6.30	20	6.30	20
6	0.41	2	0.54	1	0.54	1		
7	0.57	5	1.06	2	1.06	2	1.45	1
8	1.93	56	2.66	40	2.99	35	3.29	31
9	2.35	63	3.73	39	3.91	37	4.31	33
10	1.67	41	2.66	25	3.21	20	3.85	16
11	3.20	9	3.20	9	3.20	9	3.52	8
12	1.91	20	2.10	18	2.96	12	4.13	8
13	1.71	23	2.16	18	2.50	15	3.19	11

There are also some substantial mean grade increases within individual declustered blocks, shown in Table 5.7, as the bottom cut-off grade is increased, but these increases in grade comes with a substantial loss of samples in most cases. This is as expected because Domain 15 is a low-grade domain.

5.2 Global Mean Estimation Methods

While declustering is the method used to determine the global mean at South Deep Gold Mine, there are other possible ways to estimate the global mean. In this section different methods of estimating a global mean are investigated and compared. A global mean estimate for positively skewed data can be substantially increased by a few high-grade samples with a low probability. The log-transformed data distributions are more symmetrical and less sensitive to outliers and may therefore give a more reliable global mean estimate (Dohm, 1995). A 2 -parameter lognormal estimate, the median and the average were used to estimate the global mean. The global mean estimate using a 2 -parameter lognormal distribution was calculated using the equation presented in the literature review in Chapter 2. The results of which are presented in Table 5.8.

Table 5.8 Different estimates of the global mean

Domain	Average (g/t)	Median (g/t)	Declustered mean (g/t)	2-parameter lognormal estimate (g/t)	Global mean used by South Deep (g/t)
4	17.85	12.07	19.74	19.43	19.74
9	8.84	4.91	7.96	9.42	8.00
15	3.18	1.60	2.67	4.35	3.37

From Table 5.8 it is seen that no two estimation methods for the mean result in the same grade value. Using the median as an estimate of the global mean is biased and this estimate significantly lower than all other estimates. This is attributed to the positively skewed nature of the Au grade distribution of the data for which the mode<median<mean. The median as an estimate for the global mean would not be optimal for positively skewed distributions. A few high-grade samples with a low probability would increase the estimate of the mean, making the average sub-optimal also, especially for a few samples. The 2 -parameter lognormal mean estimates are higher than the estimated means.

In summary this chapter described the cell declustering conducted for different data sets for each domain. Each data set was manipulated by removing low-grade sample values below bottom cut-off threshold values. Declustering results from all data sets were compared to each other and to the global mean used by South Deep Gold Mine. The resultant global mean estimates produced from applying the various bottom cut-off grades will be used in the SK grade estimation analysis presented in Chapter 7.

6 VARIOGRAPHY

In this chapter the variography process and parameters used for kriging is described. In this research, the semi-variograms modelled and in place at South Deep were used for the sake of a grade estimation comparison to be made. Grade estimation results from global means used by South Deep Gold Mine and global means produced after removing low-grade samples will be compared in the next chapter.

The equation for calculating the experimental variogram was outlined in Chapter 2. Experimental downhole and planar variograms were calculated for the one metre Au (g/t) composites. The downhole experimental variograms were used to estimate the nugget effect for each domain. The planar experimental variograms were used to plot variogram contour fans to determine the directions of longest and shortest continuity of Au mineralisation within each domain. Semi-variograms were then modelled to the experimental variograms for each domain using the determined nugget to sill ratio in the various directions of continuity.

Horizontal contour fans were constructed for each domain using Supervisor® software. The horizontal contour fan for Domain 4 is shown below in Figure 6.1.

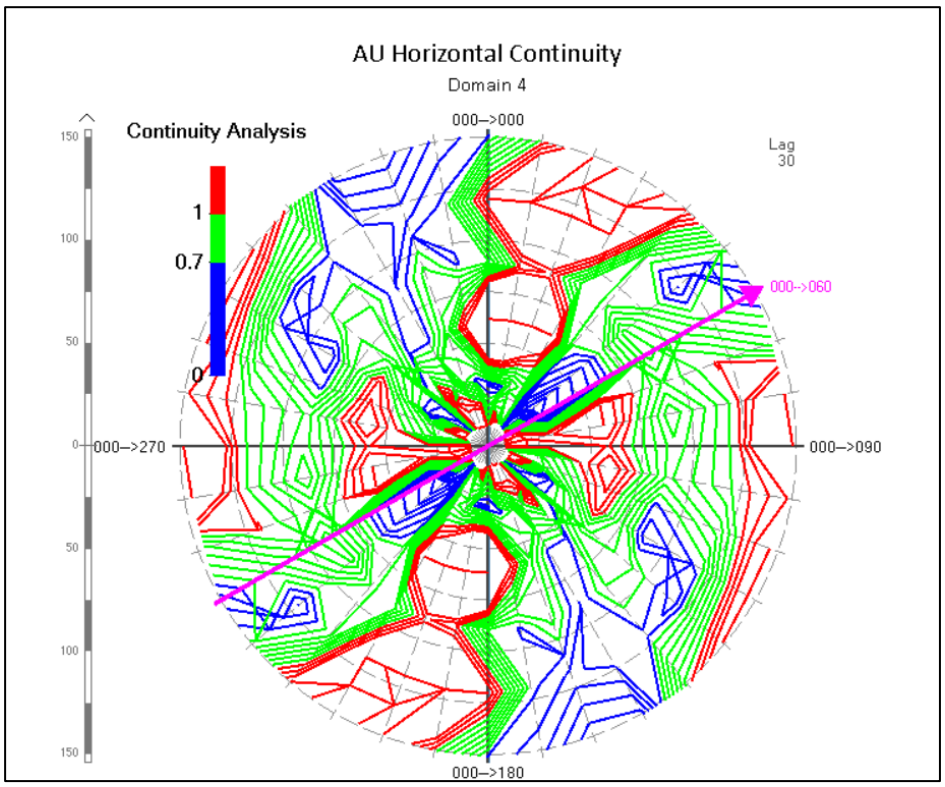


Figure 6.1 Horizontal continuity fan for Domain 4

The contours were constructed for sample pairs at a lag distance of 30 m (to mimic the desired borehole drilling grid). The direction of longest continuity used by South Deep is depicted by the pink arrow shown in Figure 6.1. Contour fans constructed for Domains 9 and 15 appear in Figure A 1 and Figure A 2 respectively in the Appendix.

Double structured semi-variograms were modelled for Domains 4, 9 and 15 by the South Deep team. The modelled semi-variogram using Datamine® software for Domain 4 and used by South Deep is shown in Figure 6.2. The nugget was modelled at a ratio of 0.345 of the total variance with a range of 47 m for the first structure and 113 m for the second structure. The modelled semi-variograms for Domain 9 and Domain 15 appear in the Appendix as Figure A 3 and Figure A 4 respectively. The double structured spherical variogram model parameters for all three domains are tabulated in Table 6.1.

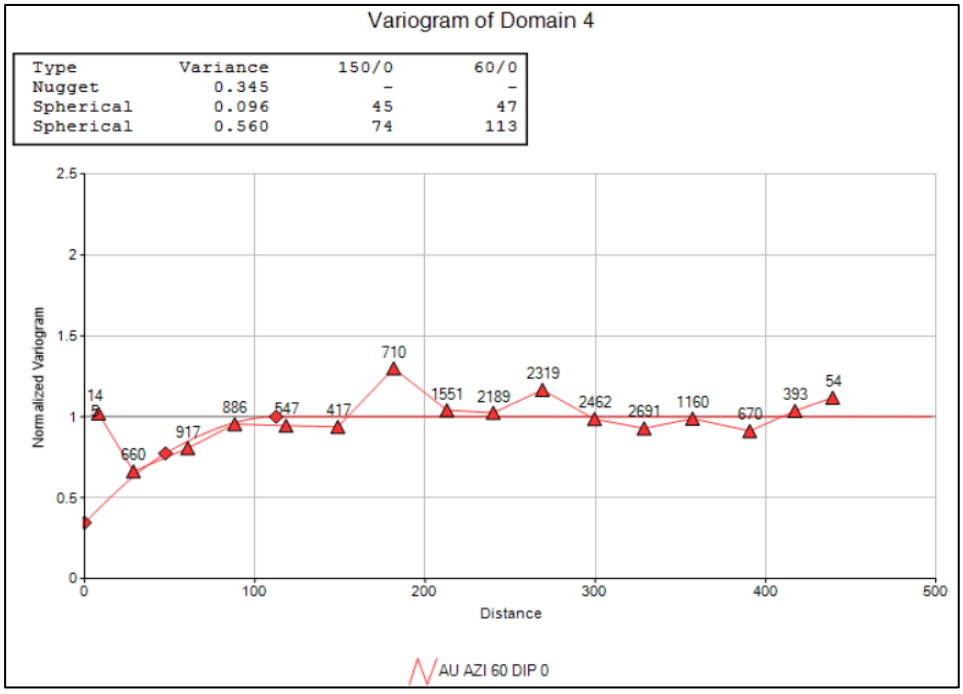


Figure 6.2 Variogram model for Domain 4 modelled in Datamine

Table 6.1 Variogram Model Parameters for Domains 4, 9 and 15

Domain	Direction of longest continuity (˚)	Nugget	Sill 1	Sill 2	Range 1 (m)	Range 2 (m)
4	60˚	0.345	0.096	0.560	47	113
9	60˚	0.403	0.349	0.248	62	209
15	30˚	0.347	0.066	0.588	67	255

The variograms modelled for each domain have nugget values ranging from approximately 35 % to 40 % of the total variance, shown in Table 6.1, is typical of a gold deposit. The general direction of continuity for each of the domains are North-North-East which is subparallel to the VCR subcrop position.

In summary, the variograms modelled in this chapter for Domains 4, 9, and 15 are all double-structured spherical models. The variogram models produced here will be used to define the sample search parameters in SK estimation in the next chapter.

7 GRADE ESTIMATION AND RESULTS

The objective of this research study is to assess the impact of low-grade material on the global mean used in SK. In this chapter the SK grade estimation is carried out for the three domains considered based the global means derived using the bottom cut-off grades selected in Chapter 5.1. The resultant SK grade estimates are compared to the grades based on the SK methodology as applied at South Deep Gold Mine.

7.1 Estimation Process

Estimation is conducted within the modelled geological wireframe volumes using all available composited data. Estimation is constrained within domains but allows a 30 m skin or "*soft boundary*" outside the domain from which composites can be used for estimates within the domain. This is done to reduce the edge effect around the boundary of a domain. The MBB wireframe volume was rotated to the horizontal plane by removing the dip and plunge of the reef using the CDTRAN command in Datamine® and then flattened. Thereafter blocks or cells of 30 m x 30 m x 1 m were created within the flat reef volume, using the TRIFIL command in Datamine® shown on the right in Figure 7.1. This flat model filled with empty cells is referred to as a start model in flat space and Au grade will be estimated into these cells. Estimation is performed in the flat horizontal space with the flattened composited data, the process of which was described in Chapter 3.3.

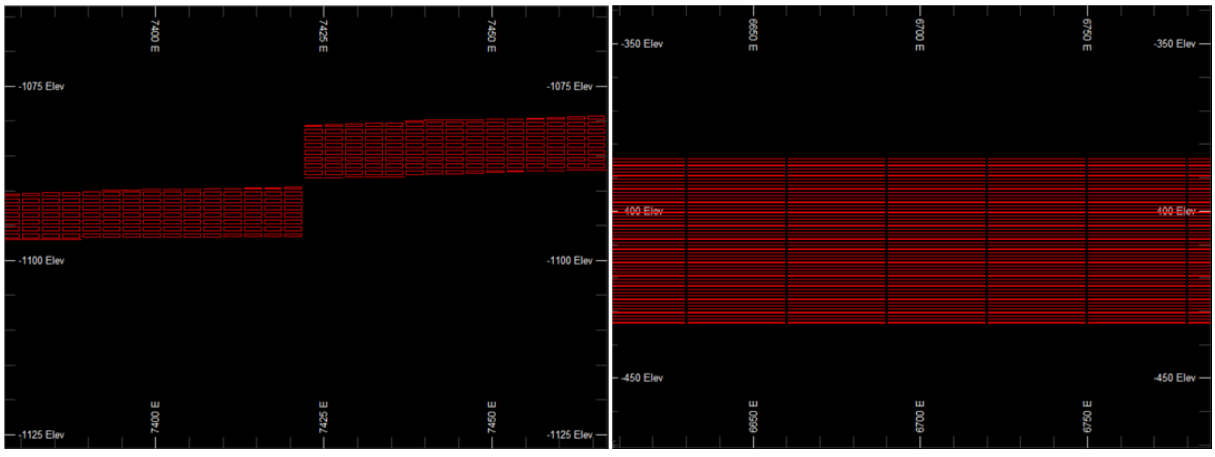


Figure 7.1 MBB wireframe volume filled with cells in the original space (left) and flat space (right)

The wireframe volume in the original space is also filled with cells using the same method described above but using a cell size of 15 m x 10 m x 1 m, with an option to sub-cell on 5 m x 5 m x 1 m if required. The sub-cell option creates smaller blocks of size3 m x 2 m x 1 m within the wireframe volume and ensures that the volume is entirely filled with cells. This start model in the original space, shown on the left in Figure 7.1, is used later in the process when the estimates from the flat space are projected and rotated back to the original space.

The variogram models discussed in Chapter 6 defined the search ranges for samples within each domain. SK estimation was carried out into 30 m x 30 m x 1 m blocks which were divided into 5 m x 5 m x 1 m blocks for discretisation. Discretisation is done to determine the spatial relationship between each sample point and the block being estimated, facilitating better block estimates from point data and to recognise the change in support between sample points and blocks. The global mean of the domain was used for the SK estimate if the number of samples within the search ellipse was less than 20; the minimum number of samples required for the local mean to be used as established in Chapter 1.3.

7.2 Estimation Results

Once SK estimation was concluded for every bottom cut global mean estimate for each domain, the block estimates were projected and rotated back to the original space. The resultant tonnage and grade estimates of block models were summarised for comparison, the results of which are given in Table 7.1.

Table 7.1 Domain estimation results of South Deep SK means and bottom cut-off grade means

In-situ SK estimation results				
Domain 4	**South Deep**	**> 1 g/t**	**> 2 g/t**	**> 3 g/t**
Grade (g/t)	17.69	17.70	17.79	17.83
Tonnes (t)	904 091.77	904 091.77	904 091.77	904 091.77
Content (kg)	15 993.383	16 002.424	16 083.793	16 119.956
Domain 9	**South Deep**	**> 0.5 g/t**	**> 1 g/t**	**> 1.2 g/t**
Grade (g/t)	8.27	8.34	8.81	8.92
Tonnes (t)	4 469 695.14	4 469 695.14	4 469 695.14	4 469 695.14
Content (kg)	36 964.379	37 277.257	39 378.014	39 869.680
Domain 15	**South Deep**	**> 0.3 g/t**	**> 0.5 g/t**	**> 0.7 g/t**
Grade (g/t)	3.33	3.37	3.48	3.75
Tonnes (t)	2 074 431.19	2 074 431.19	2 074 431.19	2 074 431.19
Content (kg)	6 907.856	6 990.833	7 219.021	7 779.117

The following are observed from Table 7.1, the South Deep in-situ SK grade estimate is less than the estimates where the respective bottom cut-off grades have been applied in the calculation of a mean for the SK process. This is to be expected as an increasing number of low-grade samples are excluded as the bottom cut-off grades increases. Also, the tonnage estimates for the different domains are the same because the tonnage is a function of the volume and density and is unaffected by the SK mean used in the different estimation scenarios per domain.

Domain 4: Whilst the South Deep in-situ grade estimate is very similar to the various bottom cut estimates for this high-grade domain, it is slightly less than that of the bottom cut-off SK estimated in all cases. The differences are 0.01 g/t, 0.10 g/t and 0.14 g/t for the respective increasing bottom cut-off grades. This very small increase in grade as the bottom cut-off increases results in increases in the Au content estimates of 9.04 kg, 90.4 kg, and 126.57 kg respectively. This is substantial when considering the price of Au per kilogram, while this fluctuates, it can cost in the ballpark of R 900,000 for a single kilogram. These miniscule differences in the grade estimate brought about from excluding low-grade material from the global mean estimate can therefore amount to millions in the resource and reserve reporting.

Domain 9: In this domain the differences the in-situ grade estimate for South Deep compared to the grade estimates for the various bottom cut-offs are more noticeable but still small. The South Deep grade estimate is less in all cases by 0.07 g/t, 0.54 g/t and 0.65 g/t for the respective increase in bottom cut-off grades. The increase in grade results in Au content increases of 312.88 kg, 2,413.64 kg, and 2,905.30 kg respectively. Again, the small differences in the Au grade estimate are seen to result in large Au content differences, more so in this medium-grade domain than the high-grade domain. These differences amount to major financial implications in the resource reporting for the company.

Domain 15: While the South Deep in-situ grade estimate for this domain is very similar to the resultant grade estimates for the various bottom cut-off grades, it is still less than all the bottom cut-off grade estimates. The differences are 0.04 g/t, 0.15 g/t, and 0.42 g/t as the bottom cut-off grades are increased from 0.3 g/t to 0.5 g/t to 0.7 g/t respectively. The increase in grade results in Au content increases that are less drastic but still noticeable at 82.98 kg, 311.17 kg, and 871.26 kg respectively

The grade estimate results for the different bottom cut-off grades for Domain 9 and Domain 15 were comparable to each other, whereas the grade estimates results for Domain 4 was very marginal. In all three domains though for very small increases in Au grade the Au content increase was considerable as the different bottom cut-off grades were increased.

In summary, from the results in Table 7.1, the grade estimates produced for each of the domains for the various bottom cut-off grades were all less than those produced by South Deep, albeit marginally so in some instances. The grade estimates for the different domains at the various bottom cut-off grades resulted in Au content amounts that were considerable.

8 DISCUSSION OF RESULTS

This study investigated the impact that low-grade material has on the global mean estimate for SK. The global mean estimates generated from declustering samples and using bottom grade cut-offs showed interesting estimation results for the different domains. The global mean estimate increased as the bottom cut-off grades increased for all the domains. The resultant Au grade estimates and Au content also increased as the bottom cut-off grades were increased. The lognormal estimates of the global mean were found to be high when compared to other global mean estimates and Mineral Resource estimation was not considered for the lognormal estimates.

Domain 4 The impact of excluding the low-grade material from the estimation of the declustered mean for the high-grade Domain 4 was very small. It is possible for low-grade material to be excluded in the declustering of high-grade domains; however, more research would need to be conducted to confirm this opinion. For a bottom cut of 1 g/t, 11 samples were removed. The question that arises is: are these 11 samples out of 341 samples part of the population (domain) or are the samples low-grade outliers? The declustered mean increased from 19.74 g/t to 20.01 g/t, as this is a relatively small increase of 0.27 g/t, the researcher is inclined to consider these 11 samples as low-grade outliers, however, more research work would be needed to confirm this opinion.

While there was a small increase in the global mean estimate for Domain 4 (Table 5.2), it was insignificant compared to the impact the low-grade samples had on a local scale in the individual declustered blocks (Table 5.3) which is mostly due to the few samples in the individual 90 m x 90 m x 50 m blocks of Domain 4. The average grade estimates resulting from the use of the global means at the different bottom cut-off grades in SK produced results that increased with each increase in the bottom cut-off. The average grade estimate produced by South Deep was less than those produced from applying each of the bottom cut-off grades. The resulting Au content also increased by 9.04 kg to 90.4 kg to 126.57 kg (Table 7.1) as the bottom cut-off grades increased when compared to the Au content estimated by South Deep Gold Mine.

For **Domain 9** the declustered mean increased by almost 1 g/t with a bottom cut of 1.2 g/t which resulted from the exclusion of 47 samples out of a total of 480 samples (Table 5.4) about

10 percent of the data, which is substantial, thus more research would be required. Removal of the low-grade material had more of an impact on the global mean estimate for this medium-grade domain compared to the high-grade Domain 4. The individual declustered blocks of 90 m x 90 m x 50 m also showed interesting results, with the removal of a mere 2 to 3 low-grade samples the local mean of those block estimates increased between 0.98 g/t to 1.76 g/t (refer to blocks 5, 8, and 10 in Table 5.5). For **block 14** of the individual declustered blocks, the removal of 15 low-grade samples out of a total of 49 samples resulted in an increase in the average grade from 10.53 g/t to 14.91 g/t, that is a massive 4.38 g/t increase in average grade for the 90 m x 90 m x 50 m block (Table 5.5). While this is substantial, it comes with the removal of 30 percent of the sample data which is also substantial. However when considering that during SK, a local mean for a block size of 30 m x 30 m x 1 m is used as a global mean, if the minimum number of sample criteria is satisfied (refer to Chapter 1.3). This then questions the necessity of the low-grades in the global mean estimate for the domain as a whole. It is also worth noting that the nugget effect for this domain is the highest of the three domains considered at 40 percent. Which indicates high variability at short distances in the dataset.

The resulting average Au grade estimates after SK for Domain 9 increased as the bottom cut-off grades increased. The South Deep average grade estimate of 8.00 g/t, when compared to the bottom cut-off grade estimates, was less in all three instances (Table 7.1). This domain showed the most drastic increases in the Au average grade estimates and Au content for the three bottom cut-offs when compared to the other domains. The difference in Au content when compared to the South Deep Au content, increased by 312.88 kg, 2,413.64 kg, and 2,905.30 kg for each of the bottom cut-off grades respectively (Table 7.1).

For the low-grade **Domain 15** the low-grade sample removal had the most impact on the declustered mean, however, this came with the removal of a large amount of sample data. For a bottom cut-off grade of 0.3 g/t, there was an increase of almost 0.5 g/t in the declustered mean from 2.67 g/t to 3.11 g/t (Table 5.6). This, however, came with the removal of 74 samples out of a total of 381 samples, that is almost 20 percent of the sample data, which is significant. That being said, the global mean used by South Deep (3.37 g/t) for this domain is very close to the declustered mean derived by using the 0.5 g/t bottom cut-off (3.36 g/t). Interestingly enough, this 3.37 g/t global mean in place at South Deep is significantly higher

than the 2.67 g/t declustered mean produced when no bottom cut-off grade was used. Could this imply that the low-grade material was intuitively accounted for? The researcher considers it as likely. The average Au grade and the Au content estimates for this domain also increased as the bottom cut-off grade increased. The Au content increased by differences of 82.98 kg, 311.17 kg, and 871.26 kg for the different bottom cut-off grades respectively when compared to the South Deep Au content

Looking at the declustered means when no bottom-cut-off grades were applied compared to the global means in place at South Deep (refer to Table 5.1 in Chapter 5.1), it is only in the high-grade Domain 4 that the fixed origin declustered mean was used as a global mean estimate for South Deep. For Domains 9 and 15, the global mean estimate used by South Deep is higher than the declustered mean. As noted earlier (and in Chapter 5.1) for Domain 15, the declustered mean with a bottom cut of 0.5 g/t had a 0.01 g/t difference with the global mean estimate chosen by South Deep. The methodology for removing low-grade samples before declustering, with more research, could potentially provide a clear method for determining a global mean estimate for different domains.

In summary, the impact of low-grade samples on the various domains investigated produced different results. It was observed that these low-grade samples had a greater impact on the local mean of the individual declustered blocks of size 90 m x 90 m x 50 m. With the removal of a few low-grade samples there were increases in the local mean grade of up to 4 g/t. The increase was evident in all domains investigated at varying amounts. For these individual blocks of 90 m x 90 m x 50 m, because of the small number of samples, it may be worth considering a Sichel-*t* estimator, or a 2 -parameter lognormal estimator or a 3 -parameter lognormal estimator of the local mean. The impact of removing low-grade samples for the determination of a global mean estimate and subsequentially an SK average grade estimate was also noticeably different for the different domains investigated, the researcher therefore recommends additional research into this methodology. The high-grade Domain 4 was least impacted by the removal of the low-grade sample removal for the global mean estimate, while the low-grade Domain 15 was the most impacted. The in-situ SK estimation results at the various bottom cut-off grades for each domain were similar in Au grade but more noticeably different in Au content when compared to the estimates of South Deep.

9 CONCLUSION AND RECOMMENDATIONS

This chapter summarises the conclusions of the research study on the impact of low-grade samples on the global mean to be used in SK estimation prior to the post processing step of the final estimation. Low-grade samples do have an impact on the global mean estimate. The significance of the impact differs depending on the average grade of the domain. In high-grade domains the impact of low-grade samples on the global mean is small, while in low-grade zones there is a significant impact. It could point to a difference of being above or below the economic cut-off grade. The number of samples within individual declustered blocks were, in some instances, very low as demonstrated in Table 5.3, Table 5.5, and Table 5.7 in Section 5.1. These few samples represent the grade of the individual declustered block and carries an equal weight with respect to other individual declustered blocks in the calculation of the declustered mean. If one low-grade sample represents an individual block in a high-grade zone this greatly affects the calculation of the declustered mean, the number of samples within an individual declustered block therefore affects the calculation of the declustered mean

It is further concluded that the local low-grade samples have a significant impact on individual declustered blocks. In cases where only a local impact was seen, it is still significant because these outliers affect the grade estimate produced locally. If the minimum number of samples (20) are available for a block being estimated, the local mean is used instead of the global mean in SK. The local impact that low-grade samples have on the estimate should therefore not be taken lightly. For each domain SK at the various bottom cut-off grades produced similar Au grade estimates. However, the resulting Au content estimates were remarkably different compared to South Deep the estimates per domain. This makes a significant financial difference when considering resources reported by the company. It also follows that, if the low-grade material are indeed low-grade outliers, the Mineral Resources could be underestimated.

From the findings of the research study, the researcher recommends removing low-grade sample data from known high-grade zones only, where low-grade samples are few and noticeable outliers for the estimation of the global mean of these zones. All sample grades are to be used in the SK resource modelling. For the low-grade Domain 15, a global mean estimate higher than that of the declustered mean was chosen by South Deep. More research would need to be conducted on other low-grade zones of different reef horizons to investigate if this

is usual practice. If it is commonly practiced for low-grade domains, then it would be wise to remove the low-grade samples before declustering. Mining is a selective process, if estimation can be improved on a local scale, mine planning processes could be optimised.

The researcher recommends an analysis into the determination of the local mean for the 90 m x 90 m x 50 m individual declustered blocks using a Sichel-*t* estimator, 2-parameter, and 3-parameter lognormal estimator of the mean. These estimates would be appropriate because there are, in some instances, very few samples within individual declustered blocks, and better estimating these individual blocks could lead to a better estimate of the global mean of the domain.

It is further recommended that a similar study be carried out on all domains to develop the general methodology for South Deep Mine to reduce the impact of the low-grade outlying material from influencing the local mean during SK especially as new borehole information becomes available. Further studies including more domains could give insight to the approach to be taken when determining a global mean estimate. The global mean identification process could potentially be simplified, and the method of determination could possibly be different for different domains. It is concluded that the research questions regarding the effects of low-grade material on the estimation of the global mean for SK posed at the beginning (Chapter 1.7) of this study were addressed by conducting this research.

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APPENDIX

Table A 1 Descriptive statistics of all 16 Domains of the MBB reef

Domain	14			Domain	10			Domain	15		
Number	176	176	176	Number	120	120	120	Number	381	381	381
Min	0.01	-4.31		Min	0.10	-2.3026		Min	0.02	-3.9705	
Max	25.27	3.23		Max	28.36	3.34484		Max	53.54	3.98	
Range	25.26	7.54		Range	28.26	5.64743		Range	53.52	7.95089	
Median	0.52	-0.65		Median	1.16	0.1518		Median	1.60	0.4673	
Average	2.46	-0.40		Average	3.27	0.42395		Average	3.18	0.2098	
Variance	16.78	2.95		Variance	22.25	1.52007		Variance	23.75	2.52141	
Std Dev	4.10	1.72		Std Dev	4.72	1.23291		Std Dev	4.87	1.58789	
Cov	1.66			Cov	1.44	2.90815		Cov	1.53	7.56985	
	LN Est	2.94			LN Est	3.27			LN Est	4.35128	
	Med est	0.52			Med est	1.16393			Med est	1.59568	

Domain	11			Domain	6			Domain	16		
Number	260	260	260	Number	128	128	128	Number	496	496	496
Min	0.01	-4.61		Min	0.05	-3.09		Min	0.01	-4.61	
Max	48.32	3.88		Max	64.71	4.17		Max	32.84	3.49	
Range	48.31	8.48		Range	64.67	7.26		Range	32.83	8.10	
Median	2.63	0.97		Median	2.75	1.01		Median	3.68	1.30	
Average	5.37	1.05		Average	6.43	0.91		Average	5.49	1.20	
Variance	46.61	1.41		Variance	106.96	2.27		Variance	29.95	1.35	
Std Dev	6.83	1.19		Std Dev	10.34	1.51		Std Dev	5.47	1.16	
Cov	1.27	1.13		Cov	1.61	1.66		Cov	1.00	0.97	
	LN Est	5.75			LN Est	7.72			LN Est	6.56	
	Med est	2.63			Med est	2.75			Med est	3.68	

Domain	9			Domain	12			Domain	1			Domain	13		
Number	480	480	480	Number	468	468	468	Number	31	31	31	Number	531	531	531
Min	0.10	-2.28		Min	0.11	-2.21		Min	0.56	-0.57		Min	0.10	-2.30	
Max	69.16	4.24		Max	83.89	4.43		Max	29.47	3.38		Max	226.48	5.42	
Range	69.06	6.52		Range	83.78	6.64		Range	28.90	3.96		Range	226.38	7.73	
Median	4.91	1.59		Median	4.15	1.42		Median	3.62	1.29		Median	5.50	1.70	
Average	8.84	1.62		Average	7.54	1.36		Average	7.14	1.35		Average	10.15	1.63	
Variance	106.04	1.25		Variance	110.01	1.50		Variance	72.62	1.24		Variance	259.46	1.53	
Std Dev	10.30	1.12		Std Dev	10.49	1.22		Std Dev	8.52	1.12		Std Dev	16.11	1.24	
Cov	1.17	0.69		Cov	1.39	0.90		Cov	1.19	0.82		Cov	1.59	0.76	
	LN Est	9.42	5.05598		LN Est	8.20			LN Est	7.22			LN Est	10.95	
	Med est	4.91			Med est	4.15			Med est	3.62			Med est	5.50	

Domain	7			Domain	8			Domain	5		
Number	114	114	114	Number	156	156	156	Number	393	393	393
Min	0.51	-0.67		Min	0.29	-1.25		Min	0.49	-0.71	
Max	63.97	4.16		Max	536.66	6.29		Max	87.75	4.47	
Range	63.46	4.83		Range	536.37	7.54		Range	87.26	5.19	
Median	8.23	2.11		Median	7.64	2.03		Median	10.07	2.31	
Average	10.42	1.92		Average	16.38	1.95		Average	13.83	2.28	
Variance	95.99	1.04		Variance	2031.95	1.63		Variance	148.30	0.77	
Std Dev	9.80	1.02		Std Dev	45.08	1.28		Std Dev	12.18	0.88	
Cov	0.94	0.53		Cov	2.75	0.66		Cov	0.88	0.39	
	LN Est	11.46			LN Est	15.79			LN Est	14.35	
	Med est	8.23			Med est	7.64			Med est	10.07	

Domain	3			Domain	4			Domain	2		
Number	157	157	157	Number	341	341	341	Number	258	258	258
Min	0.33	-1.09		Min	0.22	-1.54		Min	0.15	-1.92	
Max	66.25	4.19		Max	159.16	5.07		Max	378.52	5.94	
Range	65.92	5.29		Range	158.94	6.61		Range	378.38	7.86	
Median	12.94	2.56		Median	12.07	2.49		Median	11.15	2.41	
Average	16.44	2.47		Average	17.85	2.40		Average	20.20	2.29	
Variance	161.10	0.85		Variance	388.70	1.13		Variance	0.49	0.53	
Std Dev	12.69	0.92		Std Dev	19.72	1.06		Std Dev	0.70	0.73	
Cov	0.77	0.37		Cov	1.10	0.44		Cov	0.03	0.32	
	LN Est	18.14			LN Est	19.43			LN Est	12.83	
	Med est	12.94			Med est	12.07			Med est	11.15	

Table A 2 Descriptive statistics for Domains 4, 9 and 15 for full reef composites of the MBB reef

Domain	No. of samples	Minimum (g/t)	Maximum (g/t)	Mean (g/t)	Variance (g/t)^2	Standard deviation (g/t)	CoV
4	93	1.715	120.158	19.012	294.029	17.147	0.902
9	56	0.549	29.281	9.496	38.064	6.170	0.650
15	51	0.407	17.685	4.345	12.215	3.495	0.804

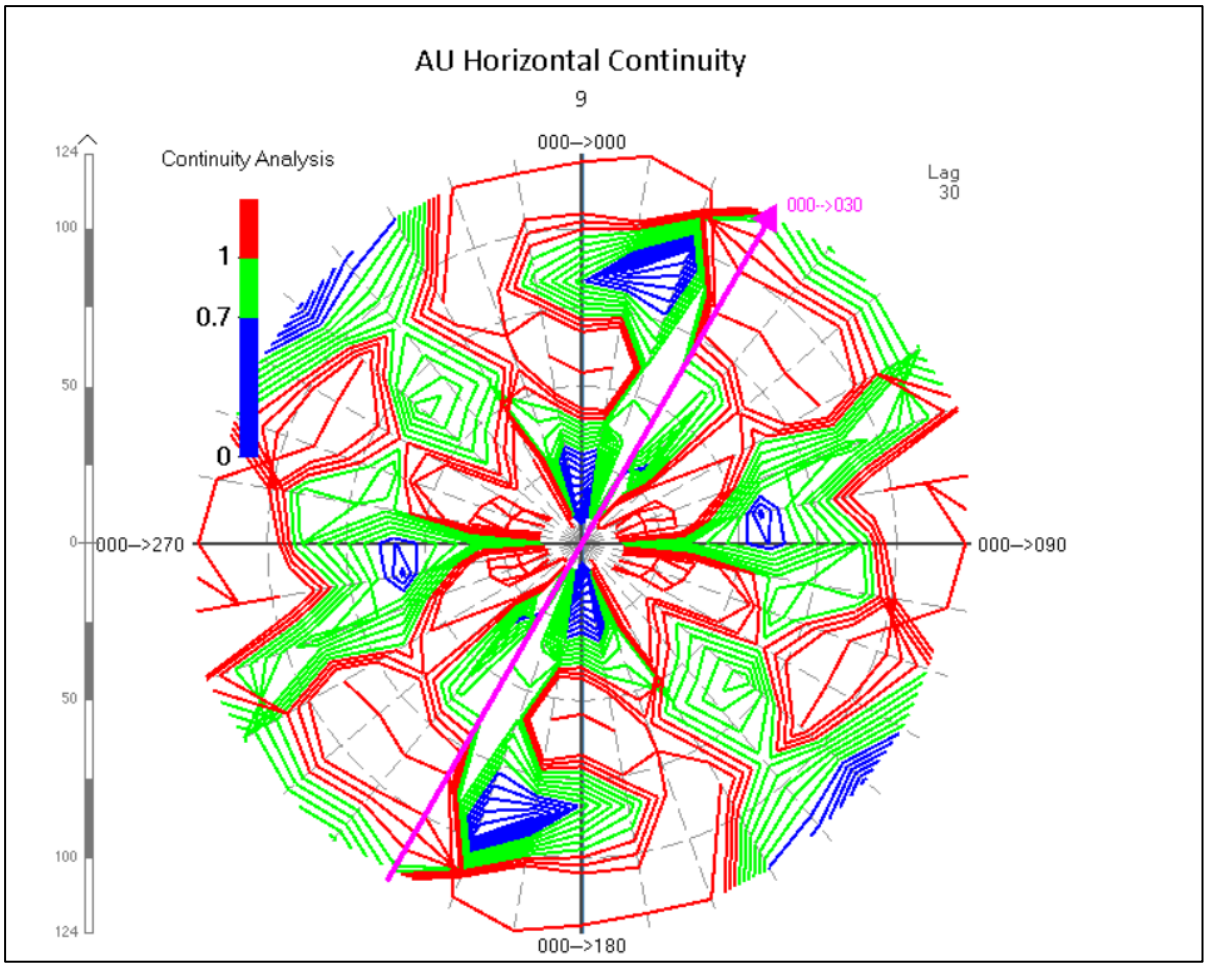

Figure A 1 Horizontal continuity fan for Domain 9

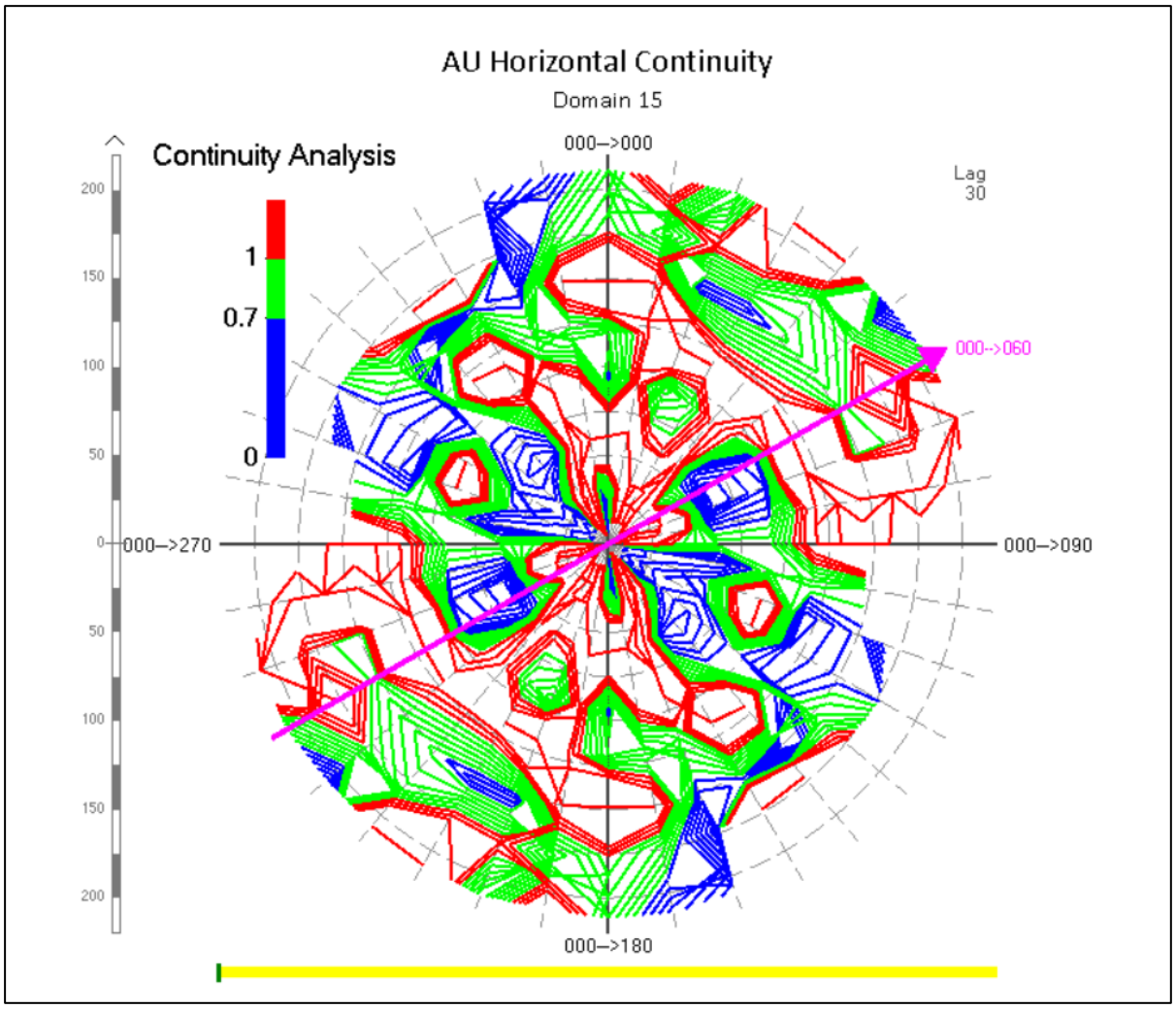

Figure A 2 Horizontal continuity fan for Domain 15

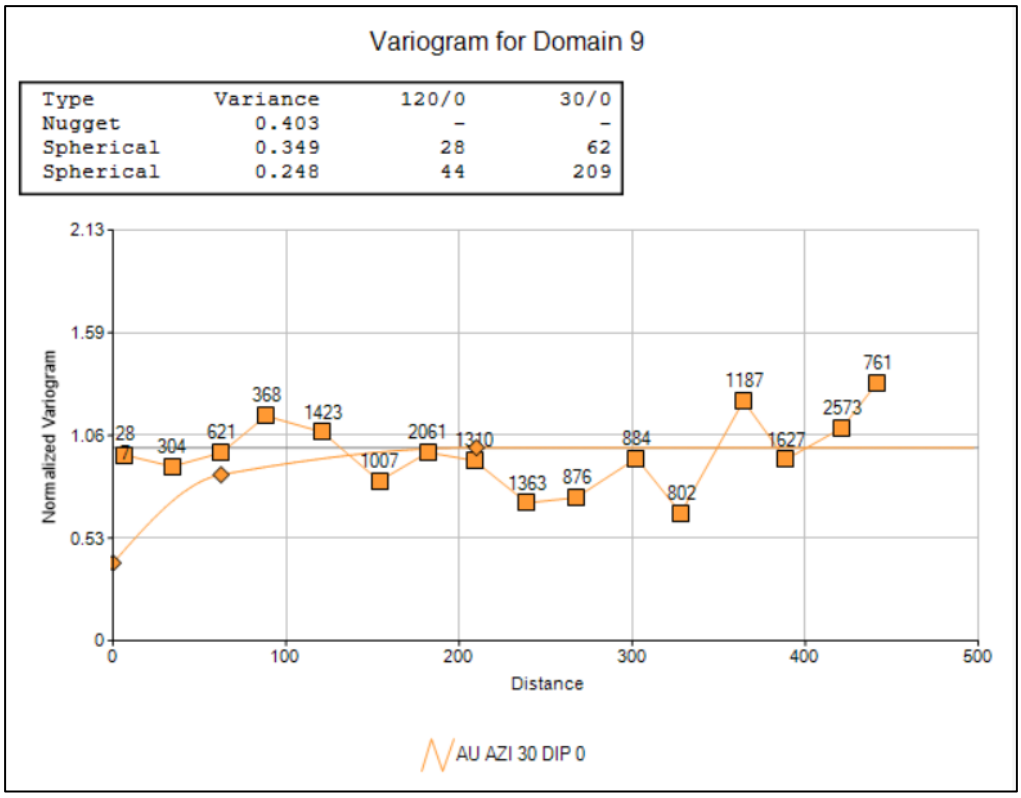

Figure A 3 Variogram model for Domain 9

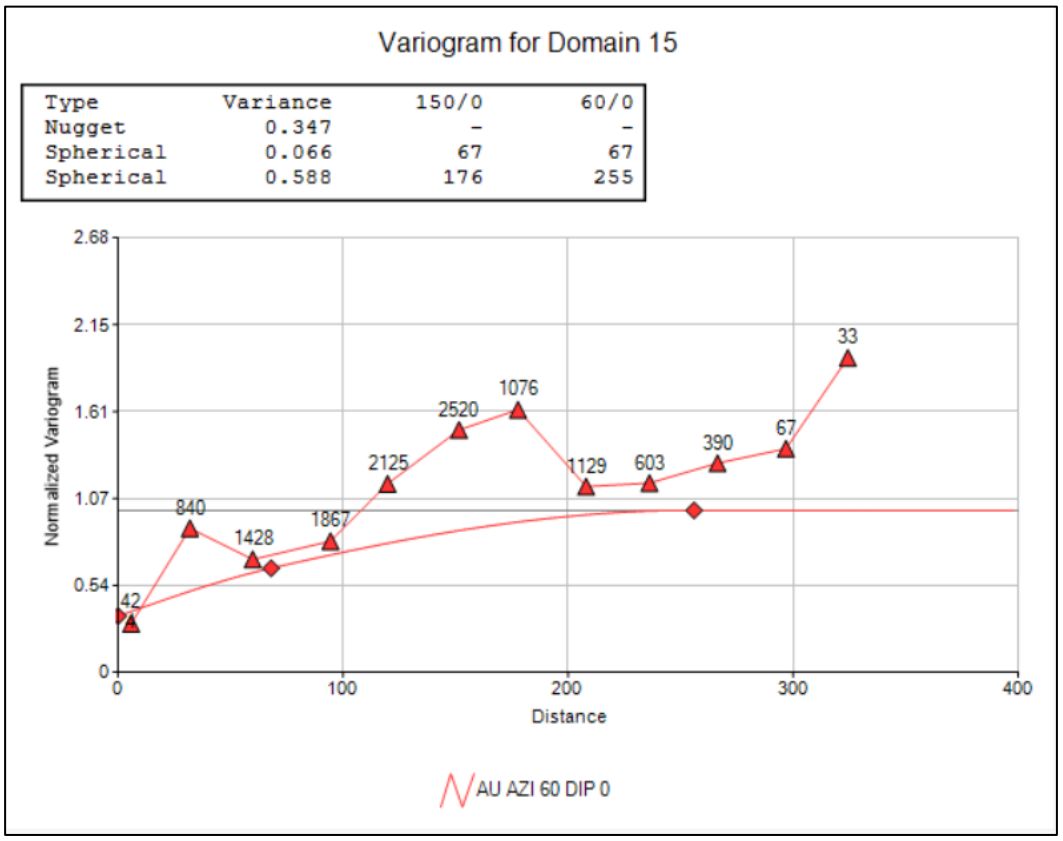

Figure A 4 Variogram model for Domain 15