

School of Mining Engineering



UNIVERSITY OF THE
WITWATERSRAND,
JOHANNESBURG

**DETERMINING AN OPTIMAL SMALL MINING
UNIT FOR THE TWANGIZA MINE USING THE
ANALYTIC HIERARCHY PROCESS**

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A research report submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

Johannesburg, July 2022

DECLARATION

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ABSTRACT

Optimising the mineral extraction process for a more profitable operation is crucial for the wealth of a mining company. Mine planners define the smallest mining unit (SMU) during the optimisation process to serve many different factors. These factors include the main orebody indicators such as the size of the mining equipment, the direction of mining, and the depositional environment of the orebody. In this study, the selection of the SMU size for an open-pit mine is discussed. The Twangiza gold deposit in the Democratic Republic of Congo is used as the case study where the drilling database is used to construct five in-situ resource models at the block size of 5x5x2.5m, 7.5x7.5x2.5m, 10x10x5m, 12.5x12.5x5m and 15x15x7.5m. The pit optimisation results coupled with the mining operations factors such as the equipment size and productivity are used to determine the SMU size that best suits the Twangiza north deposit. The average mining grade, the dilution factor, the ore tonnage, the stripping ratio, the project NPV, the environmental impact, the productivity and the life of mine are the primary criteria used to select the best SMU size using the analytical hierarchy process (AHP). The grade-tonnages curves performed on the in-situ resource models demonstrate the impact of the SMU on selectivity, which is that as the block dimension gets larger, the average mining grade, the tonnage above the cut-off, and the metal content drop. During pit optimisation, the 5x5x2.5m block provides higher NPV and a larger pit shell. However, the sensitivity study shows that the smaller block size yields an

increased dilution factor and cost of selectivity. When these two parameters are also considered in the pit optimisation, the NPV of the smaller SMU size drops at the advantage of the 10x10x5m. When adding the mining parameters to the pit optimisation results to construct an AHP model to select the best alternative, the 10x10x5 remains the SMU that best suits the deposit. Although the study confirms that the SMU of 10x10x5m used at Twangiza is the optimal SMU, it is recommended that the Twangiza mine planners should use a scientific method to determine such vital factors in the mine optimisation.

DEDICATION

This research is dedicated to my wife “Neema Mugoneke Nelly”, my daughter “Faraja Rudahya Anaelle”, my sister “Dr Antho Rudahya” and my brother “Brazos Abraham”.

ACKNOWLEDGMENTS

I return all the glory to God Almighty, Jesus Christ, who by His grace paved the way to the completion of this study.

My profound acknowledgement to all staff of the School of Mining Engineering at the University of the Witwatersrand, in particular, to Mr. Clinton Birch for his supervision and Mrs. Thobile Sedumedi for her constant administrative support.

I wish to acknowledge the valuable contributions made by Banro Corporation and particularly Twangiza mine that partially sponsored the study financially and provided me with the required data and software for the study. Please find my gratitude here: Dan Bansah, Chris Bawah, Peter Kersi, Philip Muteba, Magloire Kashiba, Pete Flindell.

My profound acknowledgement also goes to my late Father Tegere Rudahya, many thanks for your encouragement and passion for education. My gratitude to my mother, Suzane M'Kahamagarwa, for encouragement and prayers. To all my brothers and sisters, Rudahya's family, thanks for your encouragement and support during this study.

Lastly, I would like to thank my colleagues within the Banro Group who have contributed to the study by answering the research questionnaire and for their technical advice.

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ACRONYMS AND ABBREVIATIONS

\$:	United States dollar
\$ mill	Million United States dollars
%:	Percentage
\$/t:	United States dollar per tonne
°:	Degree
Ag:	Silver
As:	Arsenic
Au:	Gold
Bi:	Bismuth
CaO:	Calcium oxide
cm:	Centimetre
Co:	Cobalt
Cr:	Chromium
Cu:	Copper
E-W:	East-West
g/t:	Grams per tonne
K tonnes:	Kilo tonnes
kg/t:	Kilogram per tonne
kg:	Kilograms
ktpa:	Kilo tonne per annum
m:	Metre
Ma:	Million annum

Mo:	Molybdenum
MPa:	Mega Pascal
MTPA:	Million tonnes per annum
NaCN:	Sodium cyanide
Nb:	Niobium
Ni:	Nickel
NNE-SSW:	North northeast - South southwest
NPV:	Net present value
N-S:	North-South
NW-SE:	Northwest-Southeast
Oz:	Troy Ounce
Pb:	Lead
Se:	Selenium
SMU:	Small mining unit
Ta:	Tantalum
V:	Vanadium
W:	Wolfram
Zn:	Zinc

1 INTRODUCTION

1.1 Chapter 1 Overview

This chapter one introduces the background of the research, the problem addressed, the objectives as well as the motivation for the research. A brief description for the Twangiza mine, used as case study for this research study, is presented. Lastly the chapter gives an outline of the next chapters of the research report.

1.2 Research Background

Mining operations consist of extracting valuable minerals from an orebody. These minerals are finite natural resources that require a maximum extraction at a minimum cost. Therefore, operations research (OR) techniques have been adopted in mine planning and optimisation to develop algorithms that will optimally define the ultimate pit limit (ULP) of a three-dimensional block model. Each block representing ore or waste in the geological block model is identified by x,y and z coordinates along the x, y and z directions. Block with the mineral content above the cut-off is classified as ore; otherwise, it is treated as waste (Nieto & Zhang, 2013). The block size is defined by the Small Mining Unit (SMU) representing the size of the block that can be mined selectively. The selection of the SMU is based on a combination of the critical orebody indicators (Neito, 2010) and the size of the extraction equipment.

Several authors; Birch, 2014, Jara, *et al.*, 2006, and Rocca, *et al.*, 2007 among others, have discussed the impact of the SMU on the geological block model. It was shown that any variation in the SMU influences the block boundary dilution (tonnes of ore and tonnes of waste) and the diluted head grade.

Lerchs and Grossman (LG) is a three-dimension algorithm used to optimise open pit limits based on the graph theory approach, by converting the three-dimensional grid of blocks in the orebody model into a directed graph (Lerchs & Grossman, 1965). During optimisation, with the LG algorithm, the economic value of the block is used to add or reject the block in the final optimal pit. The head grade is one of the most determinant aspects in economic valuation of a block model; hence, the project economic value will be influenced by the SMU used for the pit optimisation.

Twangiza Mining SA is a subsidiary company of Bayin International Investment, a Chinese based company operating in the eastern part of the Democratic Republic of Congo (DRC). The Twangiza North deposit, located within the Twangiza Concession, has 4.58 million tonnes of probable mineral reserves at an average grade of Au 2.84 g/t (Twangiza Mining, 2015). Twangiza North deposit was used as a case study for this research study because the orebody is awaited for extraction, hence the research findings may be used to review the mine planning process.

The mine design and layout for the Twangiza North orebody was completed by the Twangiza mine engineers using 10x10x5m SMU. It was essential to understand whether the value of the Twangiza final pit can be increased by applying a different SMU size. This assessment was done by conducting a comparative study whereby the 10x10x5m SMU was used as the base case, and multiplier factors of +/-25% and +/-50% were applied to gauge the degree of sensitivity of the final pit shell results to the SMU size. Knowing that Twangiza North orebody was drilled at 40m spacing and the minimum modelled domain thickness is 10m (Fall, 2008), the 25% and +/-50% SMU variation allowed to keep the SMU in a range between the one-half the average drillhole spacing and one-half the size of the smallest geological feature to be modelled according to the Society for Mining, Metallurgy, and Exploration (SME) Mining Engineering Handbook cited by Birch, 2019.

1.3 Research Motivation

Determining the optimum pit is the basis of mine planning. During optimisation process, mine planners define the SMU to suit several different factors, including the size of the mining equipment, the critical ore deposit indicators, the direction of mining, and the depositional environment of the orebody. Knowing that SMU influences the block boundary dilution and the diluted head grade, the difference in the size of the SMU will lead to different block economic values. It is imperative during pit optimisation process to select the SMU that best fits the mining operation, yielding a higher net present value (NPV). This statement motivated the current research

whereby five different SMUs were applied on a real gold deposit to optimise different pit limits and determine which SMU is appropriate to maximise the NPV.

1.4 Problem Statement

The purpose of optimising the extraction process is to maximise economic value realised from a mining project. Twangiza mine uses an SMU of 10x10x5m for pit optimisation using the LG algorithm (Twangiza Mining, 2015). Mine engineer gut-feeling and experience largely influenced the selection of this SMU. There was a lack of a scientific evidence in the determination of such vital factor in the mine optimisation at Twangiza Mine. It was essential to understand whether this SMU of 10x10x5m is optimal or not, hence the importance of testing various combinations of SMU to select an appropriate SMU to maximise the economic value realised from the orebody.

1.5 Research Hypothesis/Questions

Block models are used for the planning of mining operations and designs. Knowing that the final pit limit results depend to some extent on the SMU size, the question is, "Does the SMU used at the Twangiza mine provide the optimal pit net present value?" To answer the question, the 10x10x5m SMU opted by Twangiza Mine is used as the base case and sensitivity analysis at +/-25% and +/-50% SMU variation is done, ascertaining whether the deposit can provide a better NPV by changing the SMU size.

1.6 Research Objectives

The aim of this research is to select the optimal SMU for maximising the economic value realised from the Twangiza North orebody. The specific objectives of the research are:

- To analyse the geological setting of Twangiza Mine to ascertain its impacts on extraction activities;
- To analyse modelling process of the orebody used for economic valuation of the Twangiza mine;
- To Analyse algorithms used to optimise open pit limits in determining the ultimate pit limits; and
- To analyse multi-criteria decision analysis (MCDA) methods and select an appropriate technique to rank different SMU scenarios.

1.7 Description for the Twangiza Project

1.7.1 Location of Twangiza project

The Twangiza mining concession is centred at 2°52' South and 28°45' East in the province of South Kivu, approximately 45 kilometres south-southwest of Bukavu town. Six different exploitation permits cover 1,164 square kilometres. Figure 1.1 is a location map showing the Twangiza mine concession.

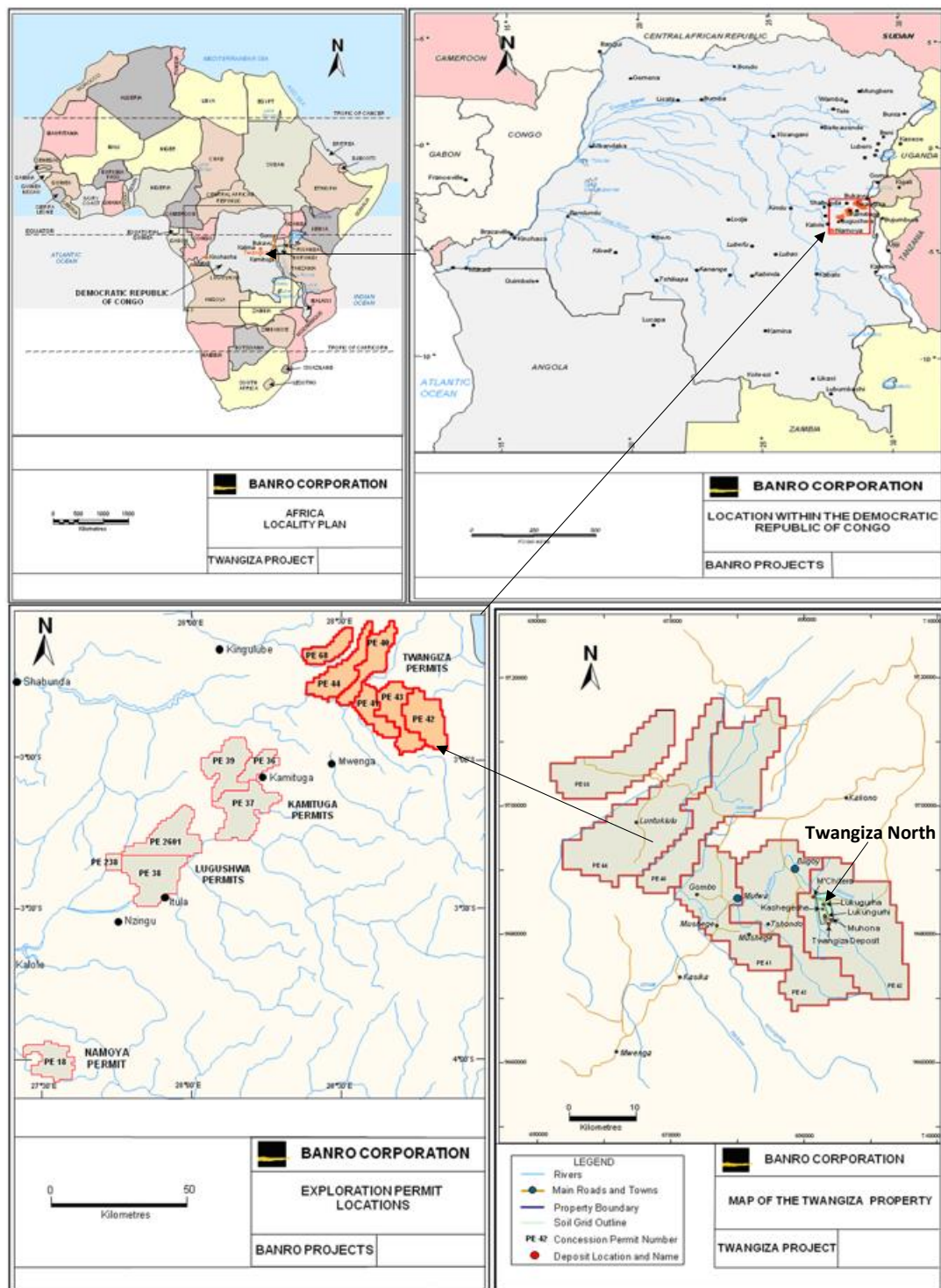


Figure 1.1 Map locating the study area within DRC (Twangiza Mining, 2015).

1.7.2 Regional Geology

The area of Twangiza is in the northern half of the Great Lakes sub-province of High Africa, one of the world's major significant Precambrian orogenic-metallogenic provinces (see Figure 1.2). The region is part of Kibara, a Proterozoic intracontinental mobile belt that extends from Uganda through Rwanda, Burundi and eastern DRC. The Kibara belt is located between the Tanzanian Craton in the east and the Congo Craton in the west (Büttner, *et al.*, 2016). The belt has a major structural trend in an NNE-SSW direction from Katanga to Lake Victoria for over 1,700 km, and attains a maximum width of approximately 500 km northward the study area. Almost 66% of the Kibara belt is located in the DRC (Walembe, 2001).

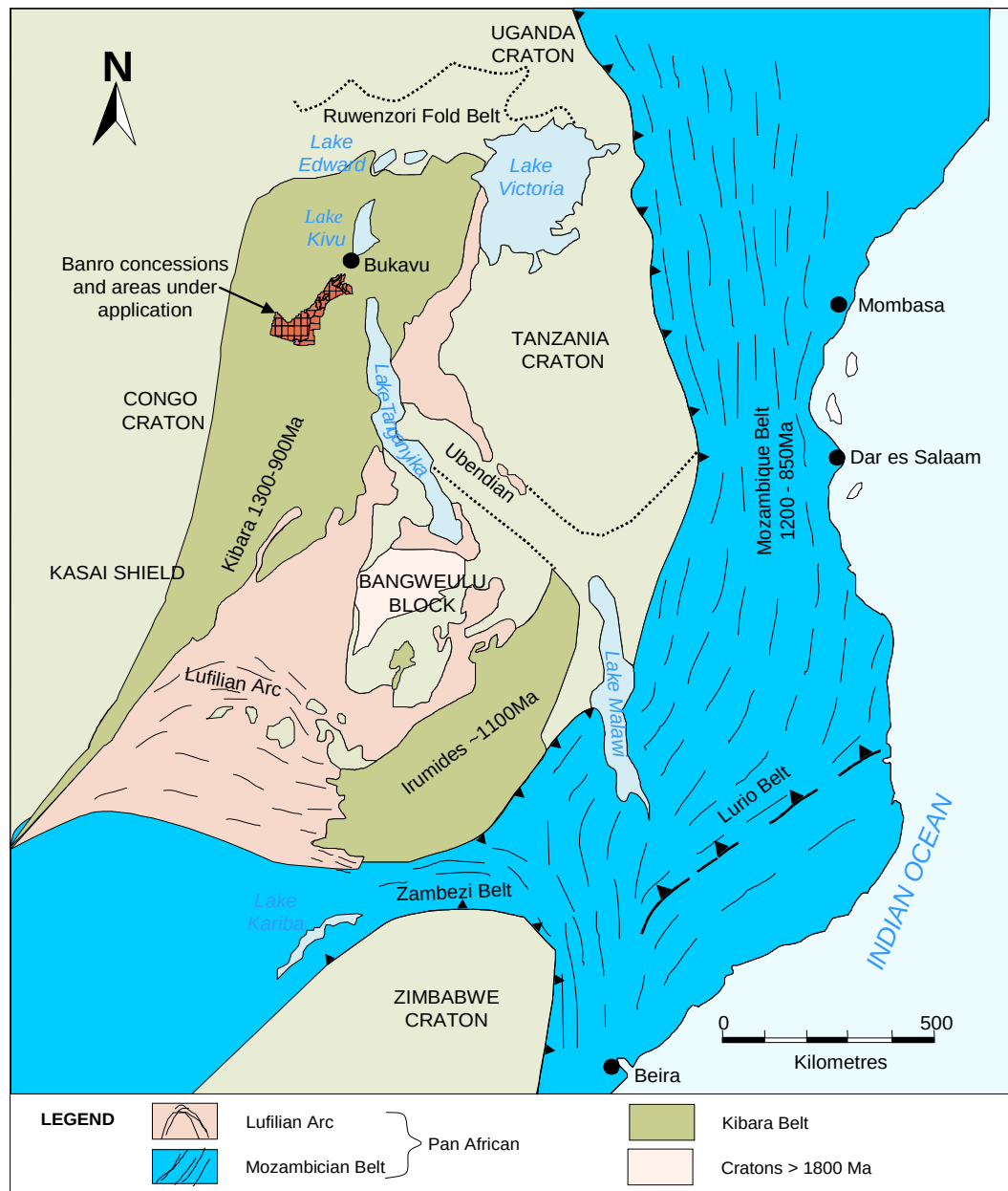


Figure 1.2 Regional Setting of the Kibara Belt (Twangiza Mining, 2015).

1.7.3 Local Geology

Three litho-structural terrains were mapped within the concession of Twangiza mine (see Figure 1.3). The Neoproterozoic folded sediment, broadly trending N-S, characterises the eastern terrain, forming part of the

Itombwe synclinorium (see Figure 1.4). This regional-scale fold expands for about 150 km southwards from the Twangiza area (Walemba, 2001). The Paleoproterozoic terrains with a distinct NW-SE tectonic grain characterise the western domain. In contrast, the third domain, which is Mesoproterozoic, appears in the north where recent basalts cover the Proterozoic rocks (Twangiza Mining, 2015).

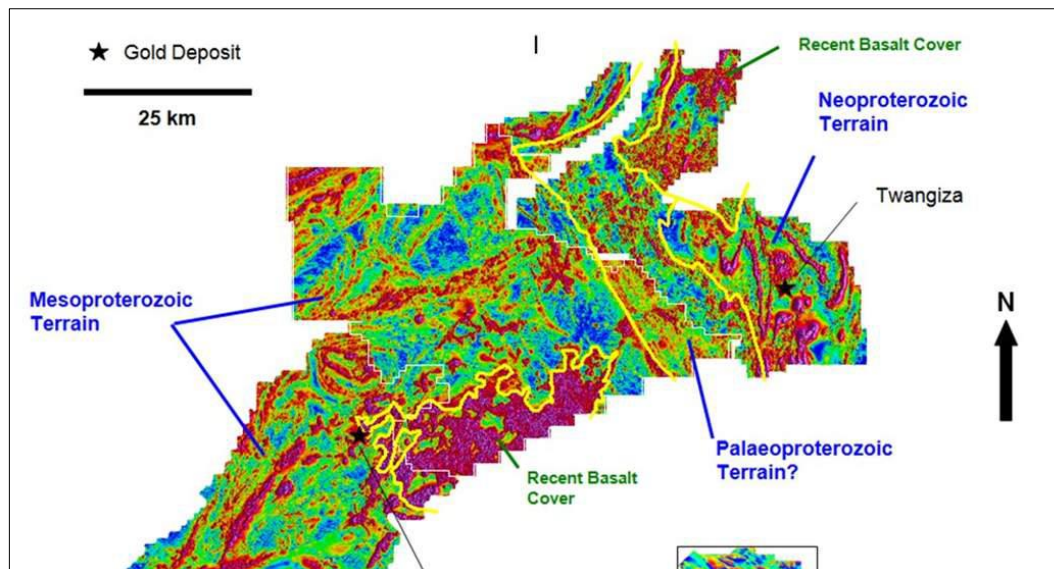


Figure 1.3 Magnetic signal displaying Litho-Structural Domains in the Twangiza mining boundary (Twangiza Mining, 2015).

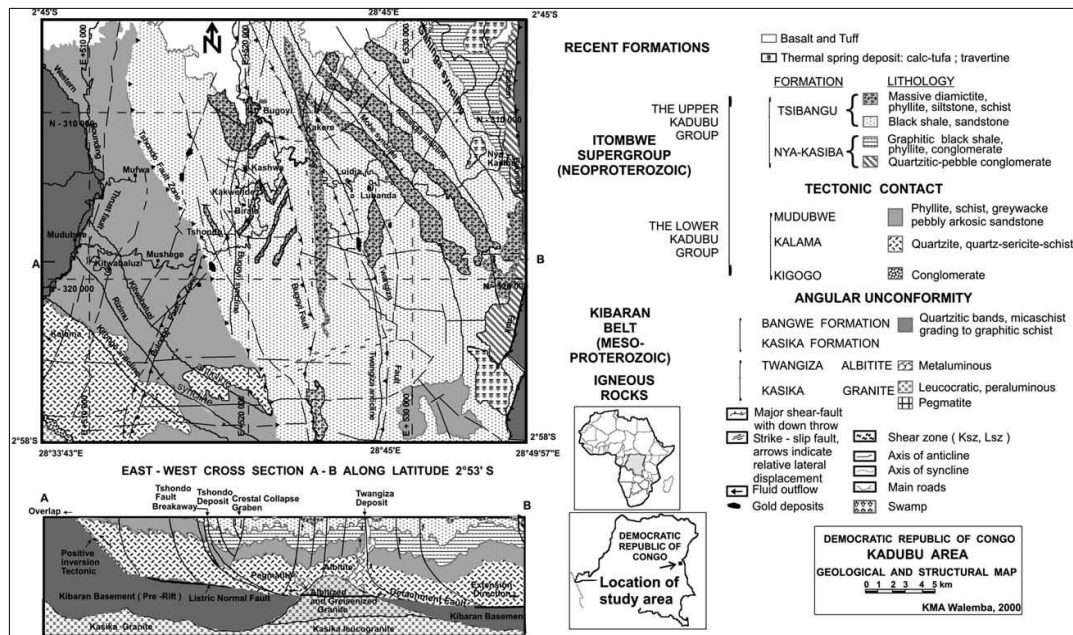


Figure 1.4 Geology of the Itombwe Synclinorium in the Kadubu area (Walemba & Master, 2005).

The lithology of the Neoproterozoic is weakly metamorphosed. It is mainly made up of mudstone, habitually with a considerable amount of carbonaceous material, intercalated with siltstone and thin layers of sandstone and quartz wacke (Walemba, 2001). A characteristic texture of the Twangiza terrain is the occurrence of conglomerate with a significant amount of detrital magnetite forming a relatively high magnetite unit defining the geometry of the folds at the concession scale. This conglomerate unit is clasts of granite, siltstone, and mudstone within dark grey silty mud (Twangiza Mining, 2015).

Porphyritic sills ranging from less than 1 metre to 50 metres have intruded the Neoproterozoic sediments of Twangiza (Twangiza Mining, 2015). These sills are part of alkaline intrusives emplaced in the region at about 750 Ma,

and have undergone widespread hydrothermal alteration (Büttner, *et al.*, 2016).

A series of folds trending N-S on concession-scale characterise the Neoproterozoic terrains at Twangiza mine and result from the E-W compression in the Pan African orogeny at about 550 Ma (Walembe, 2001). Geometrically, the folds at Twangiza are open to almost isoclinal, with the limbs dipping between 50° and 80°. On a prospect scale, smaller-scale folds plunging to the North and South are found. Some of them are doubly-plunging as the ones hosting the Twangiza deposits (Twangiza Mining, 2015).

1.7.4 Deposit Type

The Kibara Belt contains gold deposits and tin-wolfram mineralisation. This spatial association has led scientists to think that bearing fluids were associated with the intrusion of the tin bearing granites (G4 granites) dated 975 Ma and responsible for tin mineralisation (Büttner, *et al.*, 2016). This assumption does not stand because the rocks hosting the Twangiza deposit are Neoproterozoic in age (post-dating the G4 granites). It is suggested that the Twangiza mineralisation is somewhat related to the Pan African orogeny at about 550-520 Ma with gold-bearing fluids resulting from the devolatilisation of the lower crust (Twangiza Mining, 2015).

The Pan African orogeny involved E-W compression that deformed the Neoproterozoic sediments and sills into a series of folds trending N-S. Gold-

bearing fluids were focused into these structural traps, and the most significant traps being the low-pressure hinge zones of anticlinal folds (Twangiza Mining, 2015). Figure 1.5 depicts a model of the Twangiza deposit geology.

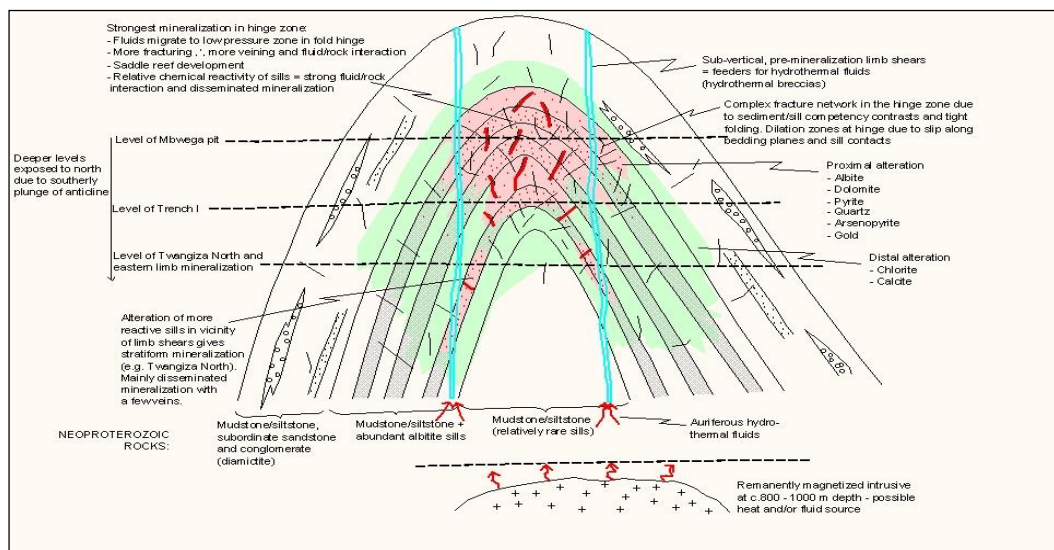


Figure 1.5 Twangiza Deposit Geology (Twangiza Mining, 2015).

The feeder structures for the bearing fluids are considered sub-vertical, limb-parallel structures or limb shears (Twangiza Mining, 2015). Hence, the mineralisation should decrease down the fold limbs distal the fold hinge and from the porphyritic zones where the stratigraphy is nearly homogeneous. The Twangiza main deposit located at the fold closure zone has the mineralisation crosscutting sedimentary rock and porphyritic sills due to high fracturing in the anticlinal hinge zones. The Twangiza North deposit situated in the limb of the fold hosts strata bound mineralisation confined within the internally highly fractured porphyritic sills that have constituted the traps for

auriferous fluids (Twangiza Mining, 2015). Figure 1.6 represents typical drill hole sections of the Twangiza Main deposit and Twangiza north deposit.

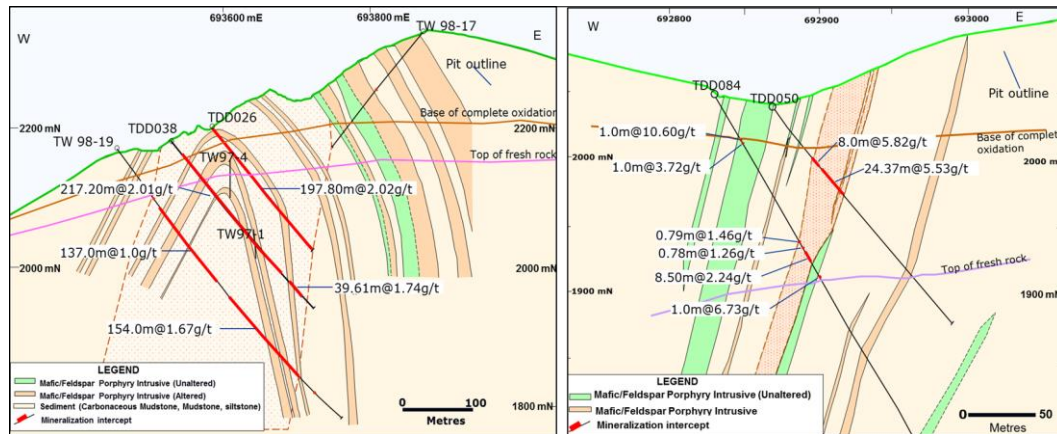


Figure 1.6 Drillhole section, Twangiza Main (left) and Twangiza North (Right) (Fall, 2008).

1.7.5 Mineralisation

The Twangiza mineralisation is characterised by a very high gold (Au) Silver (Ag) ratio (10:1 to 35:1) and abundant sulphide mineralisation (pyrite, arsenopyrite, pyrrhotite). The metal association comprises Au, Ag, As, Bi, Se with low concentrations of Zn, Cu, Ni, V, Nb, Pb, Cr, Co, Ta, W, Mo (Walemba, 2001).

The sulphides occur as disseminated crystals, stringers, coarse crystalline veins up to 10 cm in width, often intergrowing with quartz and carbonate veins. Gold grade and sulphide content show a positive correlation (Twangiza Mining, 2015).

The mineralised sills contain pyrite and arsenopyrite (about 65% pyrite: 35% arsenopyrite), with total sulphide content averaging 3% of the rock. Still, in places, a 1m sample can contain up to about 30% of sulphide. The mineralised sediments contain generally lower quantities of fine-grained disseminated sulphides. Quartz veins intergrowing with pyrite and arsenopyrite are prevalent in the sediments (Twangiza Mining, 2015).

1.7.6 Mining Method

A conventional open-pit shovel and truck method is applied at Twangiza mine to produce 1.7MTPA of run of mine ore which is supplied to the plant. Currently, there are three active pits out of five optimised pits in the run of mine production is realised from (see Figure 1.8)

The Twangiza North and the Alluvial pits mining are on hold pending community resettlement. A total of 91,468,000 tonnes of material, including 22,380,000 tonnes of ore averaging 2.28 g/t diluted grade, were estimated to be mined at a rate of 7,622,000 tonnes per year for 12 years from all the pits (Twangiza Mining, 2015). Based on the recoverable gold grade, a cut-off grade of 0.84g/t is applied to demarcate waste from the run of mine ore. Furthermore, the ore is categorised into high and low grades using an economic cut-off grade of 1 g/t recoverable. If the recoverable grade is above 1g/t it is classified as high grade, otherwise it is classified as low grade.

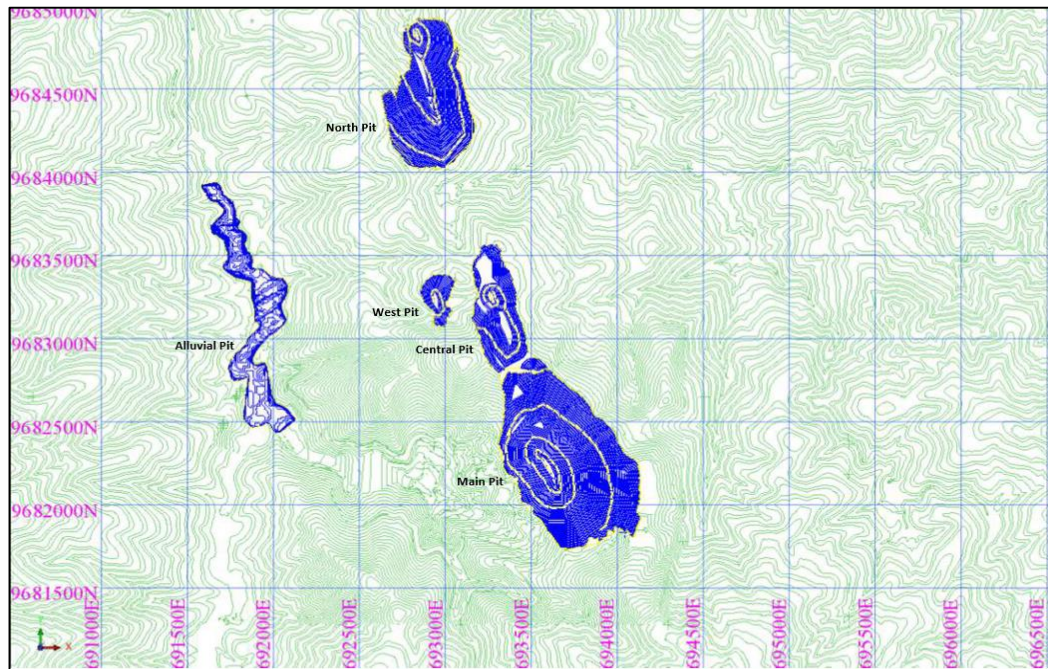


Figure 1.7 Twangiza final open pits – plan view (Twangiza Mining, 2015).

1.7.7 Processing

Twangiza Mine has a carbon-in-leach (CIL) process plant with major components such as single-stage crushing, ball milling, detoxification of tailings, elution, gold smelting, and tailings disposal. Figure 1.9 shows the process flow used at the Twangiza plant.

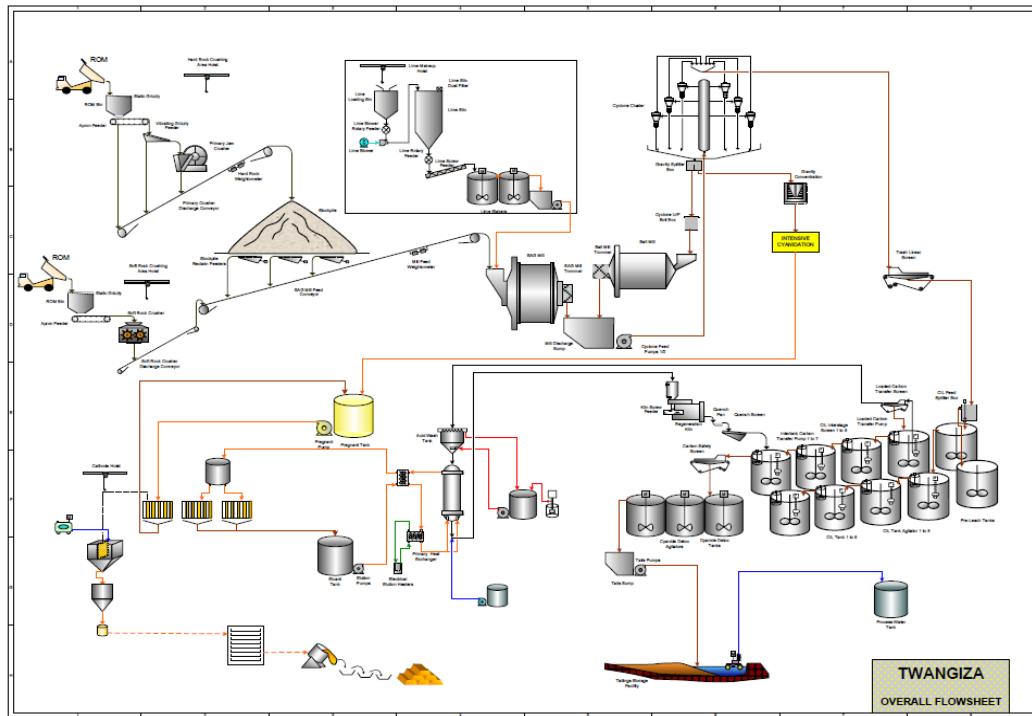


Figure 1.8 Twangiza Plant overall flow diagram (Twangiza Mining, 2015).

The plant has been designed to process 1.7 million tonnes per annum of oxide ore and soft transition ore at a maximum hardness of 60 MPa. Hence, the hard transition, beyond 60 MPa and fresh sulphidic ore are stockpiled for future treatment with an appropriate plant.

The variability leach test was conducted on oxide ore where minimum recovery of 83.66% and maximum of 88.51% was obtained from samples with head grade varying from 1.99 g/t to 5.04 g/t. The cyanidation of oxide variability samples with grades from 0.74 g/t to 1.97 g/t showed gold dissolution varying from 70.12% to 89.77%. This range in the dissolution expressed high variability of ore, which could signify some refractory material in the oxide zone (Twangiza Mining, 2015).

1.8 Research Structure

The report is structured as below:

Chapter 1 is the Introduction. - This chapter presents the motivation of the research, the problem statement, the objectives as well as its structure. The case study location and its geological settings are also discussed.

Chapter 2 is the Literature review. This chapter discusses the previous works conducted in the field of open-pit optimisation techniques. This includes orebody block modelling, SMU size selection, algorithms for open pit design and ultimate pit optimisation steps.

Chapter 3 discusses the methodology used to conduct the research.

The case study and result interpretation is found in Chapter 4. Open pit optimisation is done using the orebody model with different SMU sizes, pit shell produced, and Mineral Reserve tabulated. Thereafter, they are compared using a multi-criteria decision-making analysis to rank the best SMU size from the obtained results.

Chapter 5 gives the conclusion and recommendations. This chapter summarises the research outcomes by answering the research questions. It assesses the importance of the research project as well based on the results obtained. Recommendations for future research are also provided.

1.9 Chapter 1 Summary

In this chapter, the background of the research, the problem addressed, the objectives as well as the motivation for the research were discussed. The description of Twangiza mine, selected as case study for this research study was presented with an emphasis in geology and mineralisation styles, mining and processing methods. The next chapter, namely literature review, will provide the relevant literature for tackling the research question.

2 LITERATURE REVIEW

2.1 Chapter 2 Overview

The relevant literature for tackling the research question are summarised in this chapter. This included the required data for building an economic block models and definition of the ultimate pit for mining planning. Figure 2.1 depicts the input data and flow chart of mine planning.

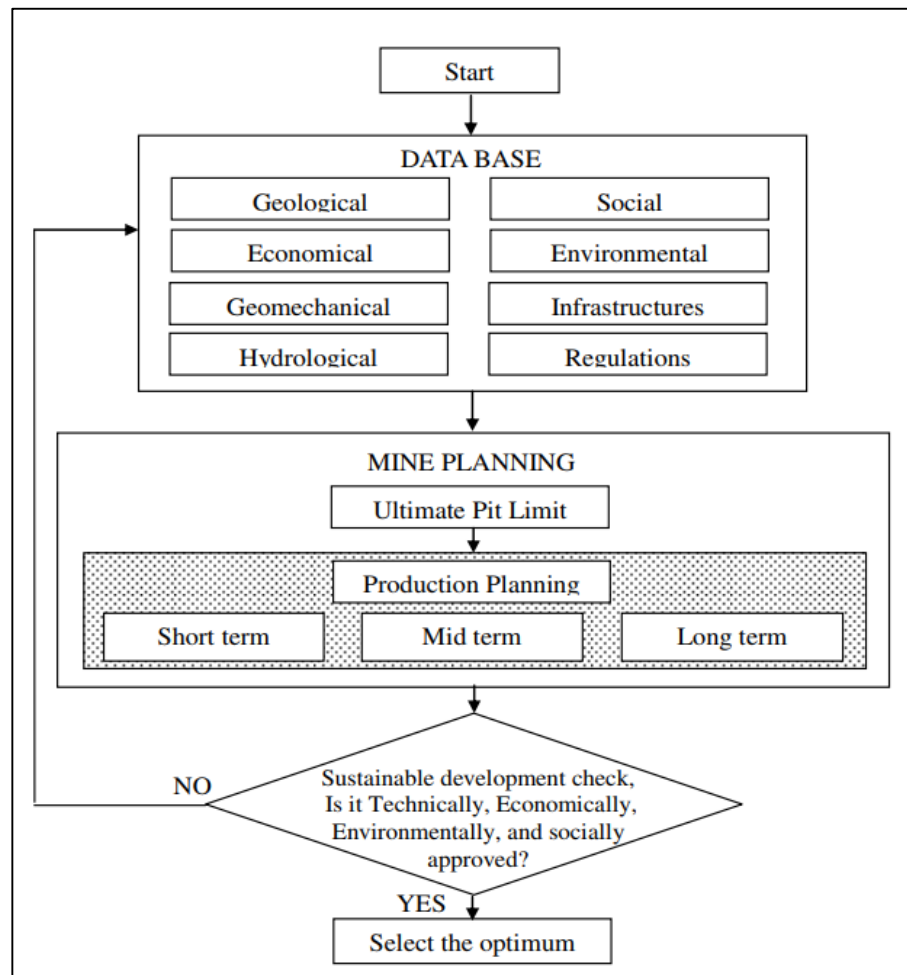


Figure 2.1 Required data of mine planning process (Mahdi & Morteza, 2014).

The key themes reviewed in the following sections included:

- Orebody modelling principles, and its impact on the selection of the appropriate SMU to be used during extraction activities and the impact of SMU on dilution effects;
- Various algorithms with their limitation in application open pit optimisation; and
- Multi-criteria Decision Analysis Methods with an emphasis on the analytic hierarchy process (AHP) that was deemed appropriate for the ranking of the SMU scenarios.

2.2 Orebody Block Modelling

Predicting the tonnage and grade of a deposit to be mined is often a challenging exercise. The geology model is a computerised illustration of geochemical, lithological, geophysical and structural, and drill hole information on and below the topography (Fallara, *et al.*, 2006). It is not easy to reproduce the exact representation of the geological body. According to Ronald (2017), cited by Deacon (2017), the aim is not to produce exactly the orebody but rather a helpful model for the purpose.

2.2.1 Geological database

The drill hole data is the most crucial and critical information required to determine the location, shape, quality, and quantity of an orebody. Geological modelling commences with the establishment of a database.

This database often contains compulsory files; namely collar, survey, lithology and assay.

Collar data contains easting, northing and elevation (x,y,z) coordinates of the drillhole header; survey includes coordinates of drill holes, azimuth and bearings at different depths. Stratigraphy database comprises the information of geological formations intercepted by each hole and the assay database contains the quality values of the ore deposit.

Since drill holes intercept different lithological units with different thickness and assay values, classical reserve estimation methods such as triangulation and polygon will require a drill hole compositing to obtain total thickness and unique grade. After a cut-off grade has been used to demarcate ore from waste, the composited grade is the thickness-weighted average of the composited zone (Hustrulid, *et al.*, 1995). Equations 1 and 2 express the total thickness of the ore and the resulting composited grade, respectively.

$$t_{total} = \sum_{i=1}^n t_i \quad (1)$$

$$g_{comp} = \frac{\sum_{i=1}^n g_i t_i}{\sum_{i=1}^n t_i} \quad (2)$$

Where t_{total} is the total thickness of ore, g_{comp} is the composite grade, t_i is the thickness, and g_i is the grade of i^{th} sampled core piece within n sampled core pieces.

Triangulation or polygons methods are then used after drill holes have been composited to build the mineralised ore solids.

2.2.2 Solid three-dimensional modelling

A geological cut-off grade is used to demarcate ore zones in the drill hole cross-sections during ore modelling. Due to the mineral commodity price cycles, the definition of the optimum cut-off grade of ore at different periods throughout the life of mine to maximise the NPV of the mine is one of the most challenging problems in modelling (Nieto & Bascetin, 2007). A cut-off grade is the lowest value at which a specified mining unit may be mined profitably; hence, it is used to separate ore and waste in the deposit. The material with the mineral content above the cut-off is classified as ore; otherwise, it is treated as waste (Nieto & Zhang, 2013).

Once the geological cut-off grade is defined, closed segments are digitised to form the outlines of the mineralised zones interpreted on each section that are then combined together to form a solid model (Appianing & Mireku-Gyimah, 2015). This manual creation of an orebody solid called explicit modelling presents several shortcomings in the orebody modelling process. Apart from being a time-consuming process (Birch, 2014), the following

additional disadvantages have been highlighted by Cowen (2003) cited by Deacon (2017):

- The generated model largely depends on the modeller's intuition and gut feeling during the interpretation process. This creates the lack of reproducibility due to different approaches and methods used in mining industries; and
- It is difficult to update the model within a reasonable time due to the complexity of the update exercise; hence, models are usually not constantly updated when new results are available.

Implicit modelling has been introduced in geological modelling to overcome these disadvantages related to traditional explicit modelling. In the implicit modelling with the radial basis function, three-dimensional models are built or updated efficiently and quickly from different data sources such as borehole data, outcrop data, hand-interpreted sections, and structural data (Birch, 2014).

2.2.3 Mineral resource estimation

Once an orebody model is created, and all the required attributes are determined, a grade is assigned to each block to calculate the deposit's mineral content. Different estimation methods have been developed for mineral reserve estimation. The ones commonly applied in the mining industry are the nearest neighbour, inverse distance, geostatistics and artificial neural networks (Erarslan, 2012).

Nearest neighbour method

The nearest neighbour method called point sampling or proximal interpolation applies a multivariate interpolation in one or more dimensions. The extrapolation is done by selecting the value of the nearest sample in the predefined space. The shortcoming is that the values of other neighbouring points are not considered in the interpolation (Erarslan, 2012).

Inverse distance method

The inverse distance method is simply a weighted average method whereby weights are calculated inversely (Erarslan, 2012). All the samples within a predefined space influence the estimated value of the assignment point. This influence, however, is a function of the distance. The larger the distance from the assignment point, the less is the contribution to the average results of the estimate. In other words, the weighted average result is estimated by considering the weighted average of parametric values by distances mathematically expressed in Equation 3 (Erarslan, 2012).

$$P_i = \frac{\sum_{j=1}^G \frac{P_j}{D_{ij}^n}}{\sum_{j=1}^G \frac{1}{D_{ij}^n}} \quad (2)$$

Where:

P_i = the value at sampled station i ;

P_j = the value at sampled station j ;

D_{ij} = the distance between i to j ;

G = the number of sampled stations; and

n = the inverse-distance weighting power.

The value of n , in practice, defines the area of influence of each of the sample stations.

The major drawback of the inverse distance method is that estimates are restricted by the extreme in the sampled values (Watson & Philip, 1985). According to Erarslan (2012), the method produces useful results in an environment of geological structure with a linear behaviour. Sudden changes in thickness or grade may result in having biased estimations.

Geostatistical methods

According to Matheron (1997), the regional variability of parameters within an area is a basic concept of the geostatistical approach. When sample values of a deposit present random behaviour, hence the variability between the samples values cannot be defined by a mathematical model, thus, probabilistic methods of statistics can be helpful.

In the geostatistics, apart from the distance relation between sample points and assignment points, the location and direction of the sample pairs are also considered (Webster & Oliver, 2007). The two major stages in the estimation process are variogram modelling and kriging.

Variogram modelling. The geostatistical process starts by creating a variogram model that displays the relation between distance and parametric variance in a particular direction of sample values (Deutsch & Journel, 1998). The variance is expressed by Equation 4:

$$\gamma(h) = \frac{\sum_{n=1}^n (z - z')^2}{2n} \quad (3)$$

Where,

$\gamma(h)$ = variance sample pairs positioned h distance (lag) apart.

z = value at location z .

z' = value at location, h distance (lag) apart from z .

n = total number of pairs located h distance (lag) apart.

Sample pairs are found for a different direction θ° and lag distances. A tolerance distance $\pm \Delta h$ and angle $\pm \Delta\theta^\circ$ is always considered to facilitate the process as it is difficult to find direction and distance conditions absolutely (Erarslan, 2012). When variogram models produced from different directions are almost identical, the ore body is characterised as isotropic, otherwise it is called anisotropic (Erarslan, 2012).

For each different h distance (lag), a variance $\gamma(h)$ is calculated and graphically plotted to produce an experimental variogram (see Figure 2.2).

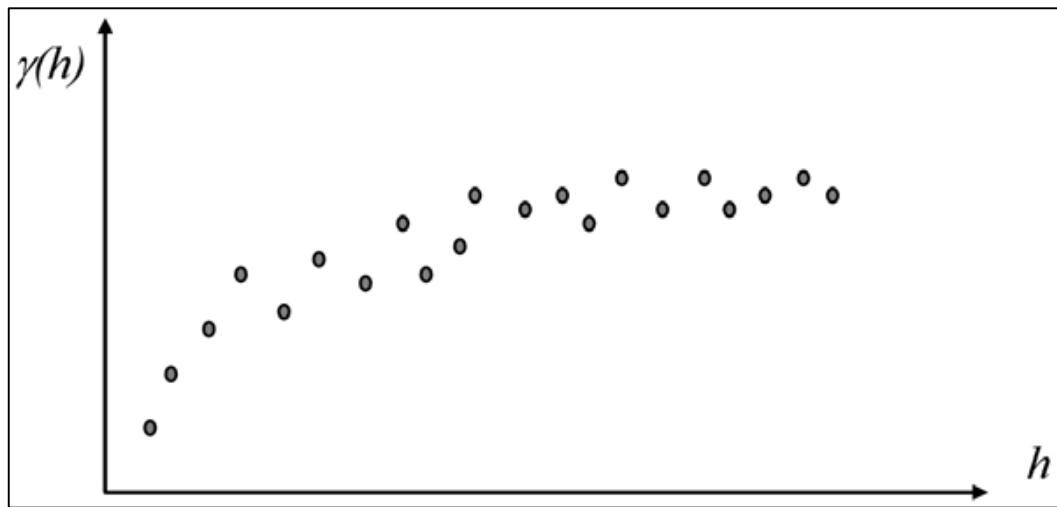


Figure 2.2 Values of variance marked for h distanced (lag) sample pairs (Deutsch & Journel, 1998).

The values of variances on an experimental variogram show a trend from which the final variogram model will be generated. The variogram model is the one that best fits the spotted points and has a mathematical expression in the form of an equation (Figure 2.3). It is decided by visual interpretation in respect to the experimental models being a spherical model, exponential model or linear model (Erarslan, 2012)

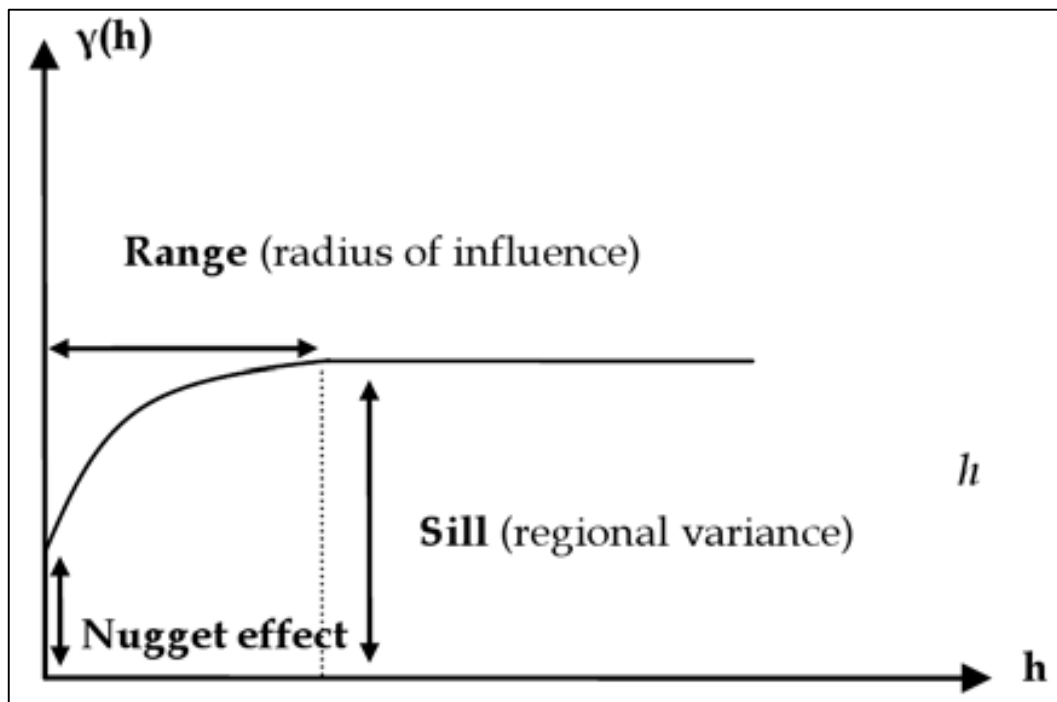


Figure 2.3 Variogram model showing its different parameters (Deutsch & Journel, 1998).

The nugget (c_0) is expressed as the variance value at zero lag distance (h). In theory, the nugget value should be zero; however, in practice, the nugget value is superior to zero due to sampling selection errors, tolerances in pairing, and structural discontinuity. Sill (c) is the constant value where variances get parallel to the lag axis while the range (a) is the distance (h) at which the sill value is obtained (Deutsch & Journel, 1998).

The variogram model is expressed by Equations 5, 6 and 7.

Spherical Model

$$\gamma(h) = \begin{cases} c \left(\frac{3h}{2a} - \frac{h^3}{a^3} \right) + c_0 & h \leq a \\ c + c_0 & h > a \end{cases} \quad (4)$$

Linear Model

$$\gamma(h) = a . h \quad (5)$$

Exponential Model

$$\gamma(h) = c \left(1 - e^{-h/a} \right) \quad (6)$$

Kriging. Developed by Krige in 1966, the kriging process is an estimation mainly applied in the mining industry. The method considers covariances between assignment sample points and within sample points mutually (Erarslan, 2012). The purpose is to extend the sample data to estimate unknown values by calculating the weight for each sample point.

After derivations, kriging is expressed by Equation 8.

$$z^*(i, j) = \sum_{k=1}^n \omega_k \cdot z(x_k) \quad (7)$$

$$\sum_{k=1}^n \omega_k = 1$$

Where:

$z^*(i, j)$ = assigned value to (i, j) node point respect to surrounding n sample points (drill holes)

$z(x_k)$ = parametric value of x_k sample point

ω_k = weight of sample point k .

Equation 9 is a simple representation of matrices.

$$[\sigma_{xx}] \cdot [\omega_x] = [\sigma_{vx}] \quad (8)$$

Where:

$[\sigma_{xx}]$ = matrix comprising mutual variances between samples points

$[\sigma_{vx}]$ = matrix comprising variances between sample points and assignment point.

$[\omega_x]$ = weight matrix

The weight matrix is calculated, and weights are used to predict unknown parameters at the assignment point. σ variance values in matrices is defined by putting distance amounts between sample points mutually (σ_{xx}) and sample to assignment points (σ_{vx}) into h term in the variogram model (Erarslan, 2012).

The main shortcomings of the geostatistics method are the visual assessment and interpretation of experimental variogram, which may be arguable and user intuitive. An acceptable amount of samples is required. Sen (2009) suggested that mostly over thirty samples are suitable. However, the estimation error can be gauged, which is part of the advantages of the geostatistics method. The estimation error is estimated using Equation 10.

$$e^2 = (z^* - z)^2 \quad (9)$$

Where:

e^2 = estimation error (variance);

z^* = estimated value; and

z = actual value.

Artificial neural networks

An Artificial neural network is a computational model that based on artificial intelligence techniques mimicking the neuron cell structures of the biological nervous system. Application of the artificial neural networks to grade estimation consist of mapping the relationship between the sample location and its corresponding grade as a complex function, where the system models the spatial variability of the ore grade to determine ore grades in unknown points. The locations of the known samples are used as input data while the ore grades are used as an output result (Kapageridis & Denby, 1998)

Artificial neural networks can be seen as an interesting alternative to the conventional geostatistical ore reserve estimation, mainly in the case where, within the orebody, the second order stationary assumption about the spatial distribution of ore grade is not satisfied. Another advantage is that in the conventional method, a linear weighting is used to estimate the grade of a specified location from the nearby sample points, while the artificial neural the nonlinear input-output mapping via a set of connection weights is used (Nezamolhosseini, *et al.*, 2017)

2.2.4 Dilution consideration for Mine Planning

Dilution refers to the amount of unwanted material (material with grade below the cut-off grade or waste material) that is not segregated from ore during mining operation and sent to the process plant (Jara, *et al.*, 2006).

Mining dilution affects the mining projects. On a short-term plan, dilution increases the tonnages of material to be milled, increasing the processing cost. In the long term, the dilution affects the overall NPV considerably because it (Ebrahimi, 2013):

- Prolongs the life of mine by reducing the mill's effective capacity;
- Reduces the plant feed grade;
- Lowers the mill recovery by lowering the feed grade; and
- Increases the cut-off grade hence reducing the ore utilisation of the mine.

According to Ebrahimi (2013), internal dilution and external dilution are the two main types of dilution in a mining operation (Figure 2.4). Internal dilution occurs when a mining block contains inclusions of unwanted material and is sent to the plant with ore. External dilution or contact dilution refers to the waste outside of the orebody extracted with ore within the mining block. A proper mining practice and the use of appropriate mining equipment can reduce the effect of contact dilution.

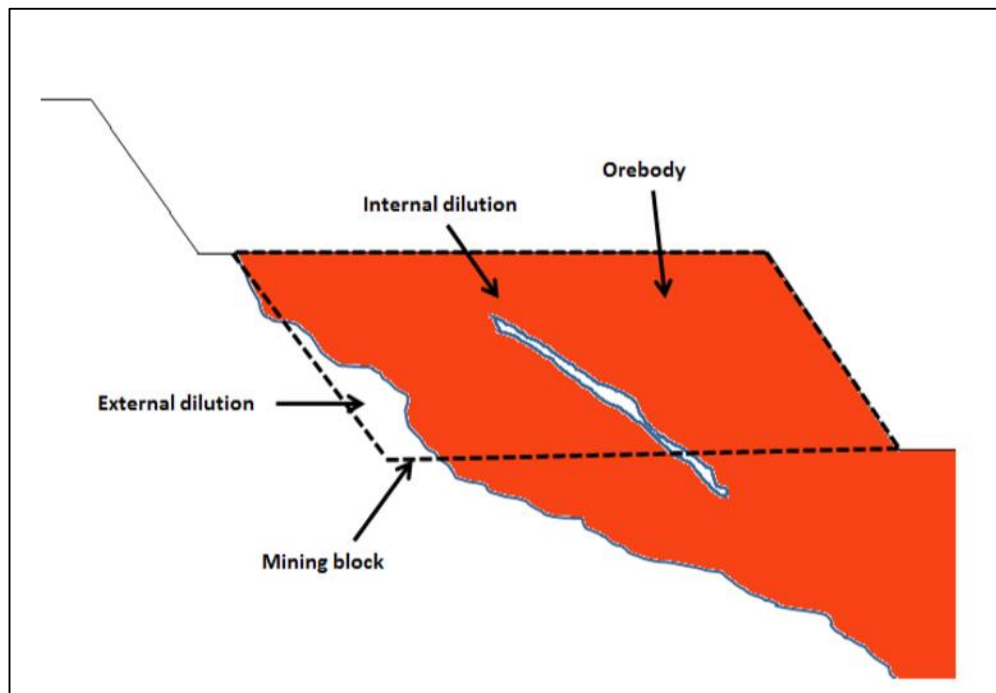


Figure 2.4 Mining block in an open pit showing different types of dilution (Ebrahimi, 2013).

Depending on the cut-off grade applied, the material previously considered ore can be reclassified as waste or low grade due to dilution impact. This is explained by a case study by Ibrahim (2013) where 10% dilution was used with a cut-off grade of 0.2g/t, as shown in Table 2.1.

Table 2.1 Effect of Dilution on Grade in a Gold Mine, Case Study (Ebrahimi, 2013)

Original Ore (g/t)	Dilution (%)	Waste Au (g/t)	Mill Feed (g/t)
0.50	10%	0.05	0.46
0.40	10%	0.05	0.37
0.30	10%	0.05	0.28
0.20	10%	0.05	0.19

During mine planning and design, engineers often make general assumptions about dilution in place of measuring it due to insufficient budget, time for studies, and lack of an appropriate methodology to quantify the dilution (Ebrahimi, 2013). Commonly, a general dilution is assumed, such as 5% for massive deposits and 10% for tabular shape deposits. Even though these assumptions may be tolerated in the early stages of mining studies, it does not consider the complexity of the problem (Ebrahimi, 2013).

Defining the contact surface between ore and unwanted material is necessary to determine the dilution factor to incorporate in the mine planning (Amir, *et al.*, 2018). The size of the SMU has an impact on the contact dilution; the smaller is an SMU, the larger the contact surface, hence higher contact dilution. The particular configuration of the ore and waste blocks also play a significant influence on the contact zone dilution, as shown in Figure 2.5, where case(b) has a higher contact dilution factor than case (a), with the same number of blocks (Amir, *et al.*, 2018).

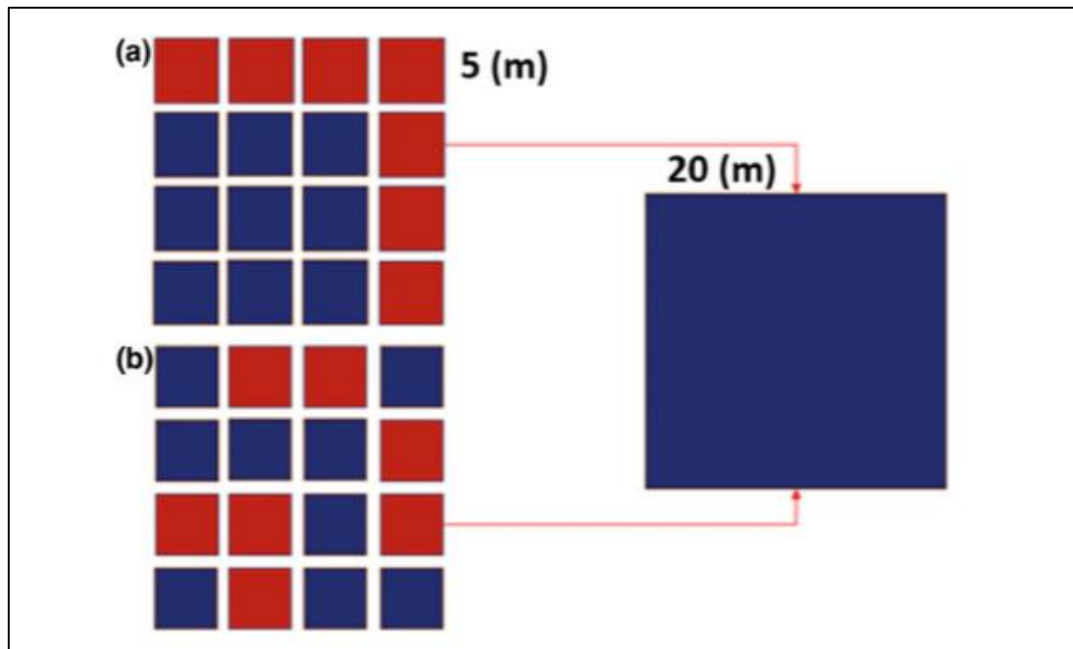


Figure 2.5 Effect of ore block (in blue) distribution on dilution (Amir, *et al.*, 2018)

Ibrahim (2018) further defined two methods to apply dilution factors into a mine plan; fixed dilution factor and variable dilution factor. The fixed dilution factor is the method widely used by mining planning practitioners, which consists of applying a fixed dilution factor to all or a selected domain of the grade model. Instead, the variable dilution factor determines the contact surface between the ore and waste and then calculates the diluted grade of this contact zone. A case study on a small copper deposit showed that the result from designs with fixed dilution factors was not optimum (see Figure 2.6) (Ebrahimi, 2018).

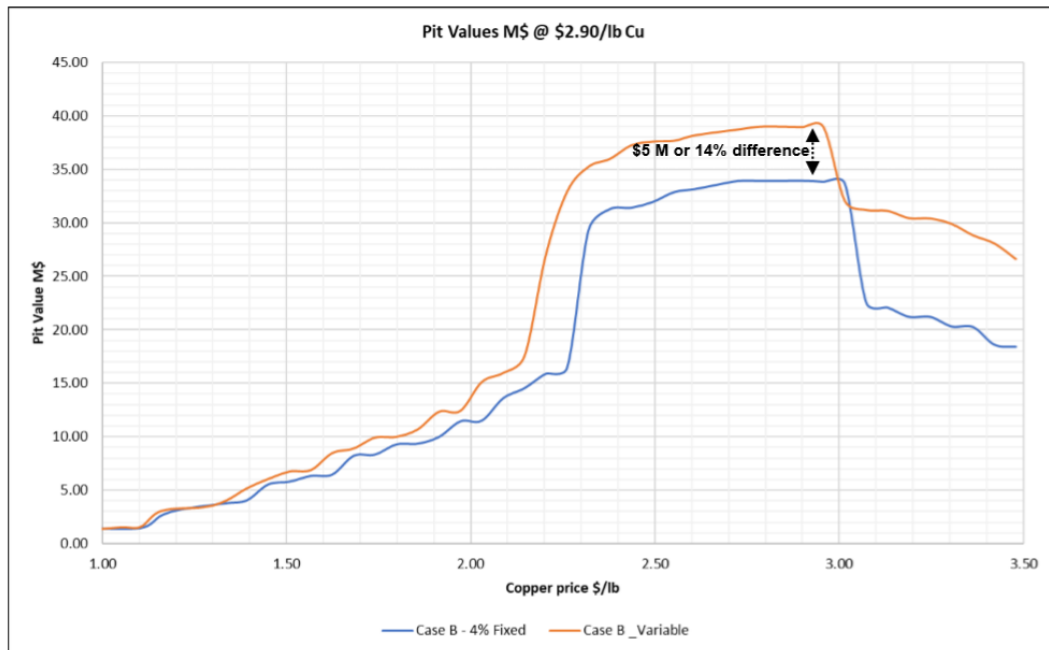


Figure 2.6 Comparison of pit values from fixed dilution and variable dilution (Ebrahimi, 2018)

2.2.5 Geotechnical consideration for ultimate pit design

One of the critical factors in the planning and optimisation of the surface mines is determining the correct slope angle. There is always a trade-off between a steeper slope angle that reduces the amount of waste to be moved and the flattened slope that gives more stability. A slight change in angle can provide a sizeable economic benefit by reducing waste and determining the limit between the stable and unstable conditions (Steffen, *et al.*, 2008). The main risks associated with slopes failure are (Steffen, *et al.*, 2008):

- Safety of personnel;
- Equipment loss or damage;

- Loss of production impacting on the economics of the project;
- Force majeure;
- Industrial action; and
- Effect on public relations, comprising stakeholder reluctance to support company's actions due to social or environmental impact, among others.

Determining the acceptable slope angle is difficult due to the multitude of factors within the rock mass and interaction between them (Steffen, *et al.*, 2008). Figure 2.7 depicts some of those factors affecting slope instability in open-pit mines.

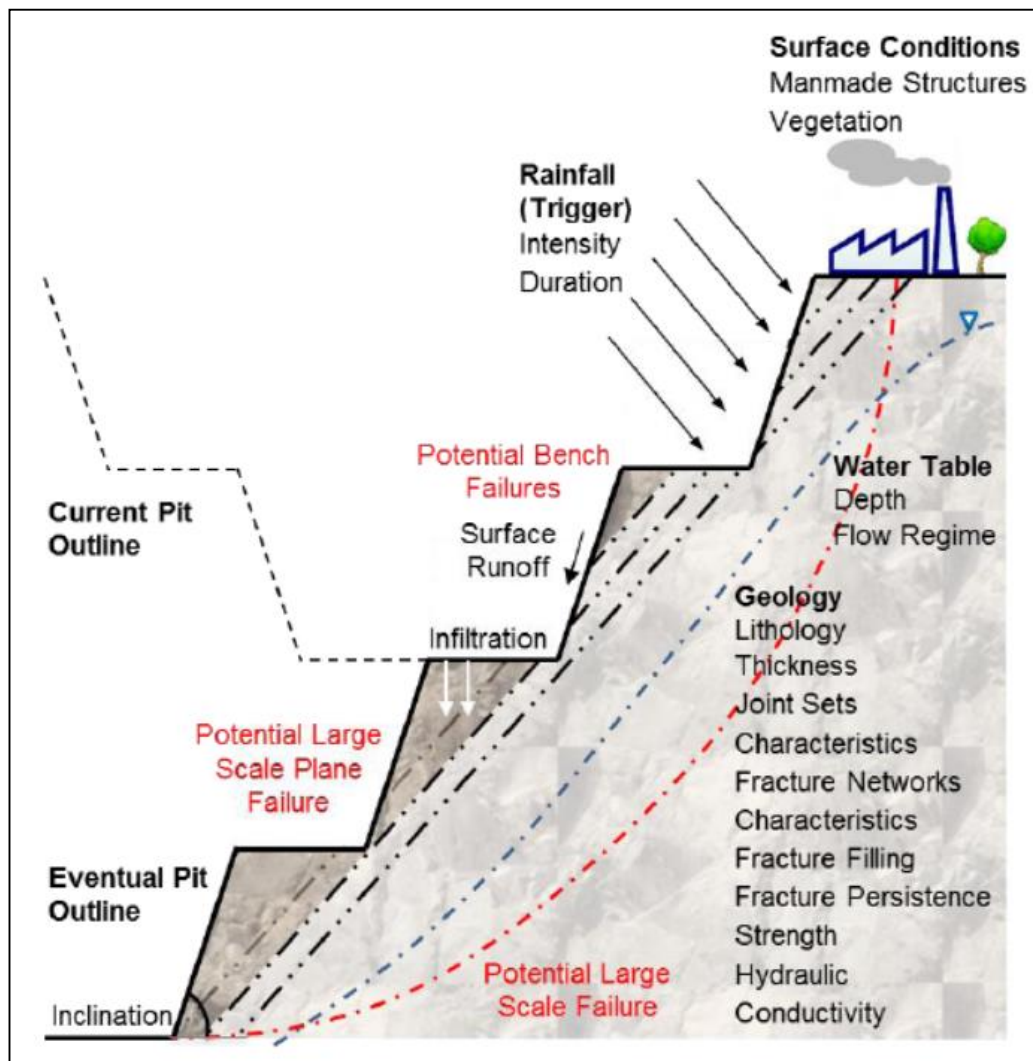


Figure 2.7 Some factors affecting slope instability in open pit mines (Steffen, *et al.*, 2008).

The main elements considered when designing a pit slope are shown in Figure 2.8 (Sullivan, 2006). These comprise the bench and berm geometry, the inter-ramp slope, and the overall slope.

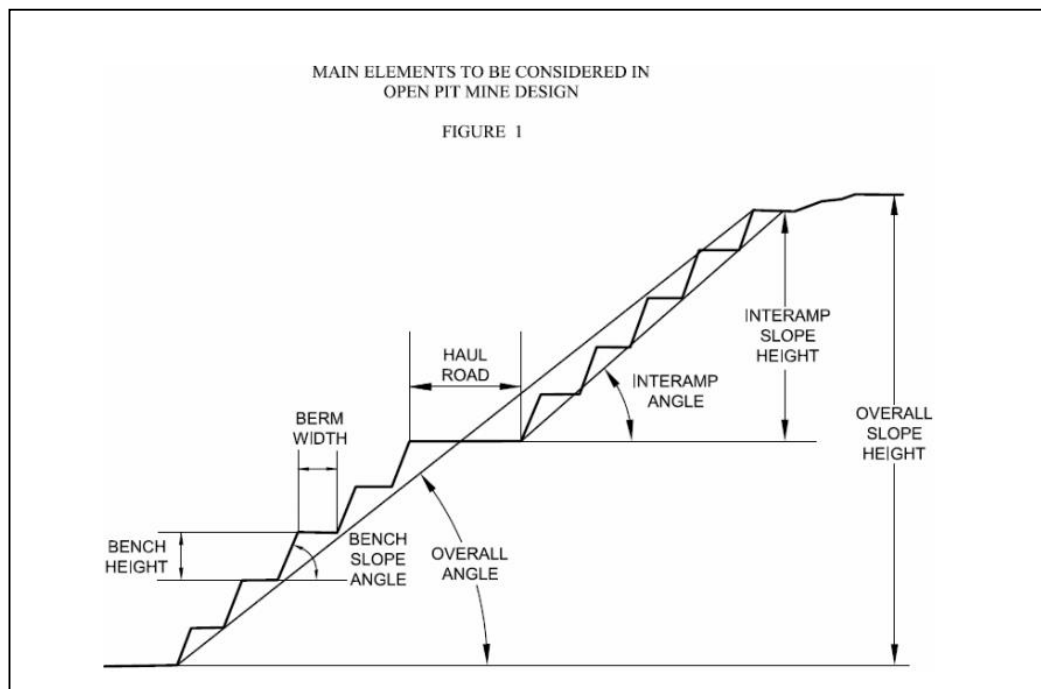


Figure 2.8 Main elements to be considered in open pit design (Sullivan, 2006).

At Twangiza mine, the slope angles are deducted from the estimation of the factor of safety. The probability of failure is not determined. The used slope angles in the final pit design are as follow, assuming dry mining conditions (Twangiza Mining, 2015):

- The overall slope angle of 27.5 to 30° in upper oxide zone;
- The overall slope angle of 34 to 38° in oxide material;
- The overall slope angle of 41 to 42° in transitional material; and,
- The overall slope of 53° angle in fresh material.

2.3 Small Mining Unit Size Selection

Design and production scheduling in a mining project is developed based on a block model. The orebody is subdivided into regular blocks that require essential parameters to estimate a value for each block. The selection of the parameters requires a thorough understanding of their impact on mining operations.

In most cases, a block value is the undiscounted net revenue estimated from elements such as grade, block volume, density, and other geological and geotechnical aspects. This value is the revenue expected by extracting and refining the block and selling its metal content (Jara, *et al.*, 2006).

The small mining unit (SMU) size is part of the critical fundamental parameters of an orebody model. Its impact on the operation and overall mining costs are considerable (Khorram, *et al.*, 2017). Conventionally, the definition of SMU is the smallest volume of material on which ore or waste classification is done (Sinclair & Blackwell, 2002).

Key ore deposit indicators, deducted from the strength of the ore and host rock, the shape, dip, and size of the deposit, and the grade and uniformity of ore (Neito, 2010), will primarily affect the choice of selecting the SMU size. In addition, the constraints imposed by the size of the extraction equipment are also considered. Some authors suggested that the SMU size should be a function of bucket width and truck capacity. Referring to a mine study at Buzwari (Rocca, *et al.*, 2007), the bucket width should be less than

75% of the SMU block dimension and a minimum of two truckloads per block.

Most open pit optimisation algorithms are applied to a regular, fixed, three-dimensional block model of the orebody. Some of these algorithms, such as the Lerchs-Grossmann use block economic value in the optimisation process (Lerchs & Grossman, 1965).

For production scheduling purposes and mine layout optimisation, it is important to express each block by its economic value to indicate its net worth, such as its dollar values. The main reason is that blocks with the same grade value may have a different dollar value due to their location, geotechnical, geometallurgical or other engineering properties (Ataee-pour, 2005).

During optimisation, the larger SMU has advantages in reducing computing time, while the disadvantage is the loss of resolution of grade variances within the orebody (consequently the net revenue). This means, the smaller the SMU size, the greater the grade estimation error. Thus, the greater the unreliability of the revenue block model used in the pit optimisation. Loss of selectivity, dilution of the head grade and cost of extraction are the main parameters affected by the SMU (Jara, *et al.*, 2006). That why tests are often required to gauge the impact of the SMU to the final pit shell.

2.4 Open Pit Limits Optimisation.

Since 1965, the determination of ultimate pit limits (UPL) has been part of the mining industry's operational research projects, and many algorithms have been developed. Musingwini (2016) stated that open pit mine planning optimisation is well advanced and extensively applied. For open pit, the mining direction is principally down and outward towards the pit limits. This two-directional mining has facilitated the easy development of algorithms for surface mining planning and optimisation.

An orebody represented in three dimensions defines the ultimate pit limit by implementing an appropriate algorithm. Each block in the block model is identified by an x,y, and z, indicating the corresponding axis value in x,y, and z coordinates.

Figure 2.9 illustrates an example of a three-dimensional block model for UPL determination.

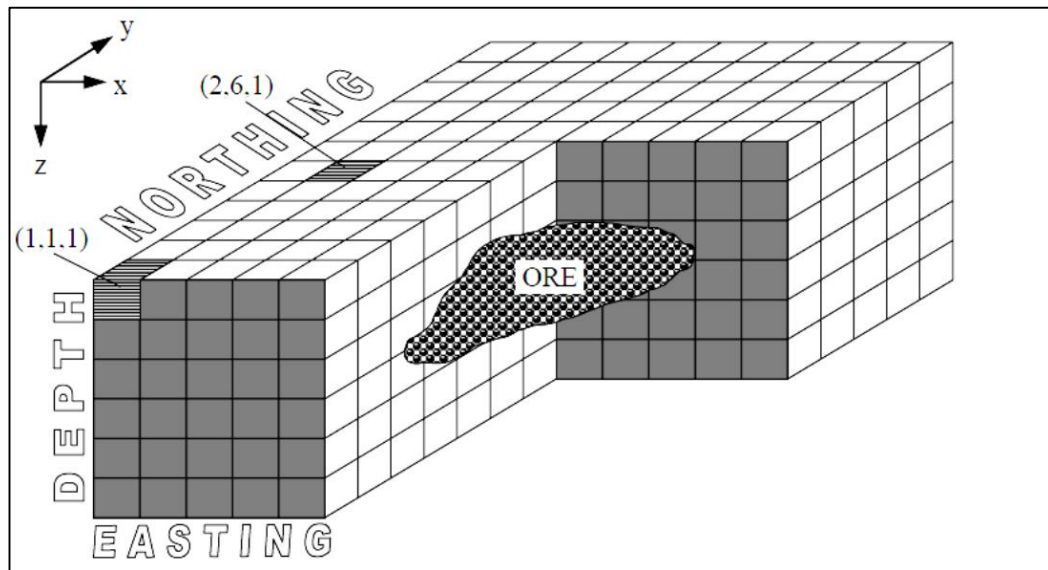


Figure 2.9 An example of a three-dimensional block model for ultimate pit limits optimisation (Gholamnejad & Mojahedfar, 2010).

The algorithms for ultimate pit design can be classified into rigorous and heuristic categories depending on the optimality yield. The rigorous algorithms can find optimum solution while the heuristic algorithms mostly fail to provide true optimal designs (Shahriar, *et al.*, 2007). Figure 2.10 depicts the typical surface mining limit optimisation algorithms.

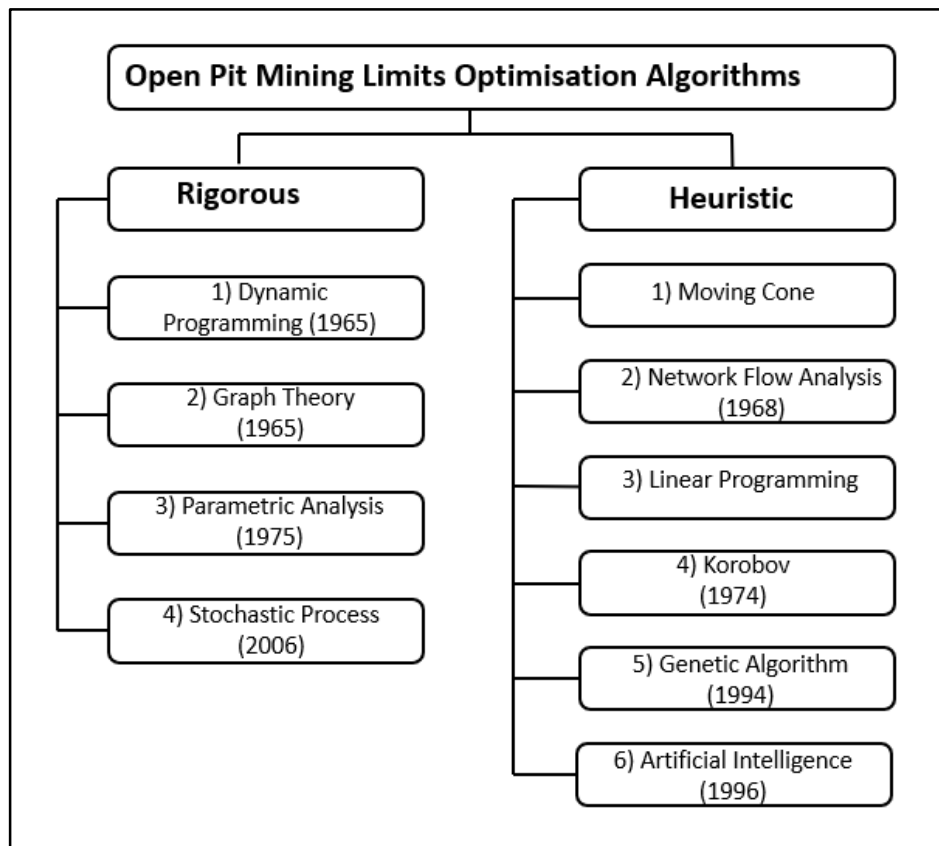


Figure 2.10 Common open pit mining limits optimisation algorithms (Adapted from Shahriar, *et al.*, 2007).

2.4.1 Lerchs and Grossmann algorithm

The first algorithm developed from operations research to optimise pit limit was the Lerchs-Grossman (LG) algorithm formulated by Lerchs and Grossmann (1965). Dynamic programming is used in the two-dimension LG algorithm, while graph theory is applied in the three-dimension pit design LG algorithm (Lerchs & Grossman, 1965).

Dynamic programming algorithm by Lerchs and Grossmann

The problem is solved using a forward pass and backward pass algorithm. In this approach, the mining block with monetary value sequencing problem

is split into a series of simpler sub-problems called stages and each sub-problem is then optimised (Shahriar, *et al.*, 2007)

The algorithm approach is as follows:

- Step 1: Add a row to the top of the mine. Set the values of these blocks to zero.
- Step 2: For each column, starting from the top, change the block value to the cumulative total of all block values above, expressed by Equation 11.

$$M_{ij} = \sum_{k=1}^i m_{kj} \quad (10)$$

Where M_{ij} is the cumulative block value at row k and column j , while m_{kj} is the economic value of the block located in row k and column j .

- Step 3: Start from the leftmost column. For each block in the column, estimate the overall cumulative value P_{ij} , which is the sum of M_{ij} and the highest value among (Equation 12):
 - Block on the upper left;
 - Block on the left; and
 - Block on the lower left.

$$P_{ij} = M_{ij} + \max\{P_{i-1,j-1}, P_{i,j-1}, P_{i+1,j-1}\} \quad (11)$$

- Step 4: Take the block with maximum positive value in the first row, follow the arrows from and to the left of the block. This forms the optimal pit. If there is no positive block in the first row, the pit is not profitable (Shahriar, *et al.*, 2007).

In 1971, Johnson and Sharp, cited by Shariar *et al.* (2007), formulated a direct extension of the Lerchs-Grossman algorithm that considers the three dimensions of the ore body. The procedure consists of two main basic steps, firstly, the algorithm is run along a cross-section and lastly, along the strike of the orebody in a longitudinal direction.

The dynamic programming algorithm obtains good cross-sectional results, but the extension to three-dimensional (3D) space yields erratic results. The reasonable computing times and storage when running the algorithm in 3D are also constraints. Many modifications were developed to overcome this shortcoming, and to provide optimal results (Khalokakaie, *et al.*, 2000).

Graph theory algorithm by Lerchs and Grossmann

Lerchs and Grossmann (1965) developed an algorithm to optimise open pit limits in 3D based on the graph theory approach, which converts the three-dimensional grid of blocks in the orebody model into a directed graph.

In this algorithm, the assumption is that:

- Each block in the grid is denoted by a vertex which is allocated a mass equal to the net revenue value of the corresponding block; and
- Arcs link the vertices so that the connections leading from a particular vertex to the surface define the set of vertices that must be detached if that vertex is to be extracted.

The algorithm approach is as the following (Nogholi, 2015):

- Step 1: Construction of an initial tree and dummy root is added;
- Step 2: Labelling of arcs;
- Step 3: Normalising a tree; and
- Step 4: Select the strong nodes which define a set of blocks with maximum value to form the ultimate pit.

2.4.2 Moving cone

Pana (1965) cited by Shahriar *et al.* (2007), described the moving cone algorithm to look for cones that satisfy the criterion that the total weight of the blocks in the cone should be positive. The moving cone algorithm is among the easiest and fastest algorithms applied to define the final pit limits (Elahizeyni, *et al.*, 2011). The Korobov algorithm is an example of a cone-based approach. The Korobov algorithm performs from top to bottom by evaluating the positive net values of blocks at each level.

The algorithm can be briefly described as follows:

- Step 1: Start from the first level, for each block, construct an extraction cone;
- Step 2: Allocate positive values in each cone against negative value until no negative block remains;
- Step 3: If the value of the cone is positive, the cone is in the optimum solution set, remove this cone; and
- Step 4: If any positive block remains, return to Step 1

Moving cone algorithm is easy to code, delivers the result in a short time frame, and its concept is simple. The principal limit is that the algorithm does not providing an optimum solution (Shahriar, *et al.*, 2007).

2.4.3 Simulated annealing

The simulated annealing (SA) algorithm is a stochastic optimisation method that replicates the iterative improvement philosophy of the Metropolis algorithm consisting of simulating the collection of particles in thermal equilibrium at a given temperature (Consuegra & Dimitrakopoulos, 2009).

The application of simulated annealing method to the open-pit mine production scheduling is accomplished by the following steps (Kumral & Dowd, 2005):

- Step 1: Start with a non-optimal feasible solution;
- Step 2: Select a portion of the blocks;

- Step 3: Option 1. change ore-waste discrimination. Change ore blocks to waste and waste to ore by some probability. Option 2. change period of the given block to a previous or following period by some probability;
- Step 4: Re-estimate NPV for the newly found solution;
- Step 5: Use Metropolis criterion as the acceptance criterion; and
- Step 6: If NPV has not improved for the last n steps, end the process or go to Step 2.

The simulating annealing algorithm does not guarantee the optimality of the final result (Kumral & Dowd, 2005).

2.4.4 Ant colony optimisation

The ant colony optimisation (ACO) algorithm mimics the behaviour of ants in detecting an optimal path in graph theory. Gilani and Sattarvand (2016), cited by Musingwini (2016), have used the ACO for pit optimisation and mine production scheduling.

To simulate the ACO process for the mine production scheduling problem, pheromone trails were used for the blocks as programming variables equal to the number of production periods. The approach was combined with a traditional method because Lerchs-Grossmann was used to generate an initial image. The mine schedules are developed through ACO iterations based on pheromone trails (Sari, 2014).

The algorithm does not guarantee the optimality of the final result (Khan & Niemann-Delius, 2014).

2.4.5 Parametric analysis algorithm

The parametric analysis approach to pit optimisation is to parameterise the pit design as a function of several variables. The algorithm divides the problem into two different components, namely, the practical part and cost-effective part. The assumptions are that the final selected pit limit is the one that provides the maximum tonnage of the mineral of interest. Parametric analysis algorithm is incorporated into the Whittle 4-D software, which produces incremental pit limits by increasing the economic parameters (Shahriar, *et al.*, 2007).

The main disadvantage of the parametric analysis algorithm is that the time and other variables such as exploitation rate, the types of ore, blending ratio are only secondarily incorporated in the optimisation through modifying the cost function. In addition, the algorithm does not as well guarantee optimality (Caccetta & Hill, 2003).

2.4.6 Linear programming techniques

Linear programming techniques were introduced in surface mining planning by Meyer in 1969 cited by Shahriar *et al.* (2007). Network algorithm is an example of a linear programming approach. The logic in the network

algorithm is that each positive block that can pay for its overlaying negative blocks will be part of the final pit (Khodayari, 2013).

The main limitation of the linear programming techniques to ultimate pit limit optimisation is that the computing time and computer memory do not allow better processing when dealing with vast amounts of data (Shahriar, *et al.*, 2007). Therefore, this can make the optimisation problem intractable.

2.4.7 Genetic algorithm

John Holland describes the genetic algorithm (GA) as a method to imitate the processes of biological evolution through natural selection or the principle of 'survival of the fittest' for resolving global optimisation problems (Musingwini, 2016). Application of this technique to the optimal pit problem is accomplished by Denby and Schofield (1995) cited by Sari (2014) where random pits are coded as genes. Fitness function included maximising NPV, minimising early stripping and for multiple mineral cases, balancing ore production for production blending in a central processing facility to maximise NPV.

The algorithm does not guarantee the optimality of the final result (Khan & Niemann-Delius, 2014).

From the seven ultimate pit optimisation algorithms discussed, the Lerchs-Grossman (LG) algorithm was selected to be used in this study. This preference is since the LG algorithm is commonly used in the mining

industry for defining the ultimate pit limits optimisation (Khan & Niemann-Delius, 2014).

2.5 Mineral Reserve Conversion

At Twangiza mine, the definition and mineral resource classification are taken from NI 43-101, the Canadian Institute of Mining, Metallurgy and Petroleum (CIM) Standards on Mineral Resources and Reserves.

Figure 2.11 shows the conversion process from exploration results to Mineral Resources and Mineral Reserves per NI 43-101.

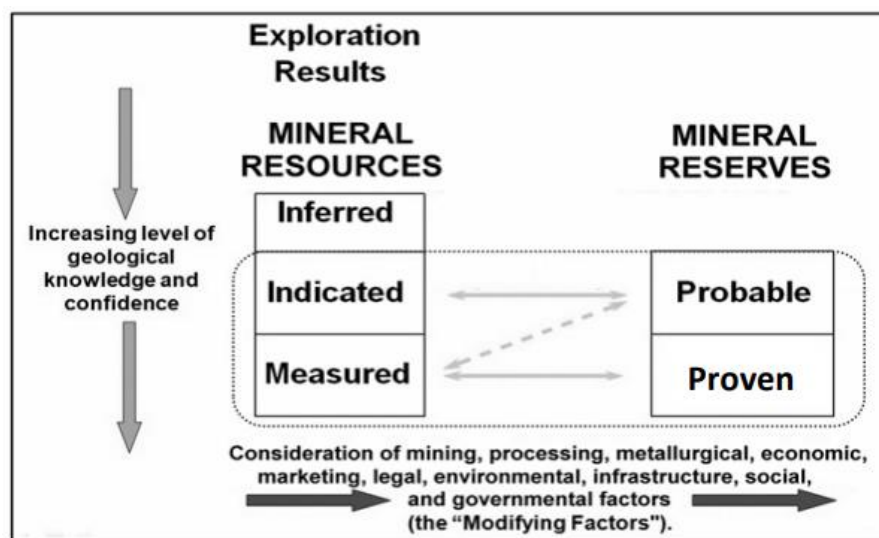


Figure 2.11 Relationship between Mineral Resources and Mineral Reserves (CIM Standing Committee on Reserve Definitions, 2014).

2.5.1 Classification method

The mineral resource classification is based on increasing levels of geological knowledge and confidence (Twangiza Mining, 2015). At Twangiza mine, this classification stands on a combination of the drill hole

spacing (as depicted in figure 2.12), the confidence in the geological wireframe, and the slopes of regression, where:

- Inferred Mineral Resources have been restricted to 50m–100m drill hole spacing.
- Indicated Resources within 40 metres spacing, and
- Measured Resources are within 20 metres drill holes spacing;

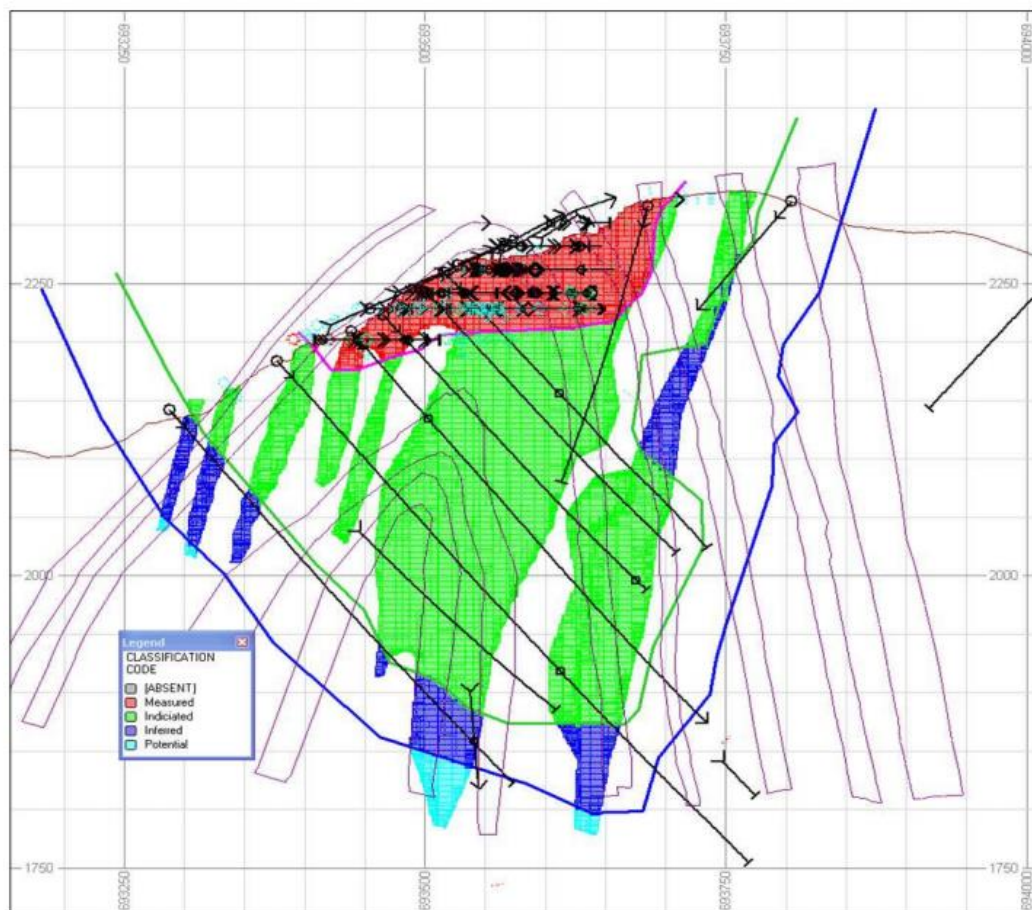


Figure 2.12 Twangiza Typical section displaying the mineral resources classification (Twangiza Mining, 2015).

2.6 Mine Production and Scheduling

Mine scheduling is one of the post-design techniques used in mining operations. The purpose is to determine an optimal sequence in which ore blocks will be mined to generate higher NPV subject to constraints of mining, economics and processing (Caccetta & Hill, 1987). Currently, a stochastic approach is being used in mine scheduling to integrate the uncertainties into the optimisation of the life of mine. It was demonstrated that apart from understanding risks and their consideration in the planning process, the stochastic approach offers a higher NPV than the conventional (deterministic) method. A case study by Consuegra & Dimitrakopoulos (2009) on a copper mine showed that the stochastic approach using simulated annealing algorithm provided 10% additional NPV and increased the life of mine by one year compared to the conventional approach (Consuegra & Dimitrakopoulos, 2009).

2.7 Multi-Criteria Decision Analysis

Multi-criteria decision analysis (MCDA) is a tool used in operations research when several contradictory criteria have to be evaluated simultaneously in decision-making (Musingwini, 2010).

The MCDA methodology is based on a generic structure whereby a decision taker or a team of decision takers must select an alternative, A_i , from a set of other alternatives, $A = \{A_1, A_2, \dots, A_m\}$, such that A_i gives the best trade-off among decision criteria defined by a set, $C = \{C_1, C_2, \dots, C_n\}$. O_{ij} is the

outcome that expresses the efficiency of alternative i compared to criterion j . Generic MCDA procedure is summarised as follows (Musingwini, 2010):

- Define objectives and record them into criteria;
- Assign weights to criteria;
- Aggregate the O_{ij} values using the weights; and
- Take the decision.

There are several MCDA techniques used in mining industries, among them the preference ranking organisation method for enrichment evaluation (PROMETHEE), Elimination et choix traduisant la réalité (ELECTRE), multiple-attribute utility (MAUT) and analytic hierarchy process (AHP) and its generalisation, the analytical network process (Musingwini, 2010).

In this study, the Analytical Hierarchical Process method (AHP) was used as the MCDA technique due to its simplicity in the calculation and ability to detect inconsistent judgements and estimate the degree of inconsistency when ranking the criteria.

2.7.1 Analytic hierarchy process

The analytic hierarchy process (AHP) methodology developed by Saaty in 1990 is an MCDA tool used to merge qualitative and quantitative factors in the electing process (Musingwini & Minnit, 2008). Its basis is a mathematical framework formed by a matrix and vector algebra that can efficiently be

executed in Microsoft Excel (Karel, 2015). Figure 2.13 illustrates the AHP solution methodology.

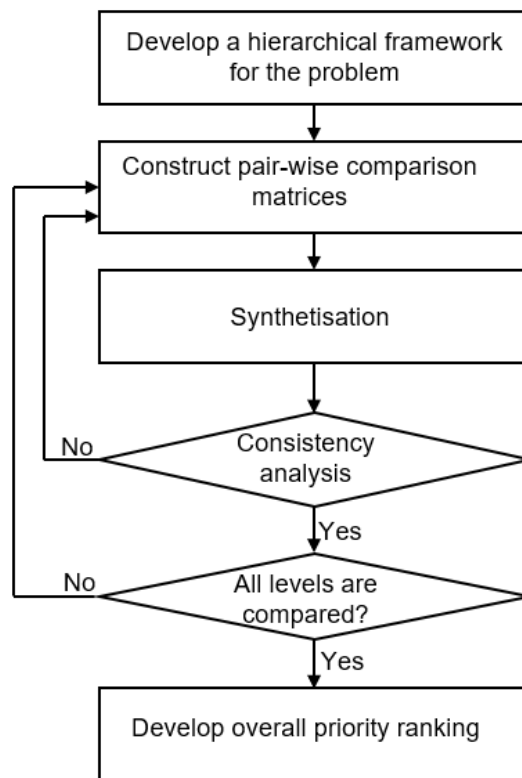


Figure 2.13 Solution methodology for AHP (Sameer, *et al.*, 2013).

Pairwise Comparison

The mathematical framework starts with a pairwise comparison of each criterion's relative weight or ranks over another. The relative weight of criterion C_i over criterion C_j is represented by w_{ij} such that (see Equation 13):

$$w_{ij} = \frac{1}{w_{ji}} \quad (12)$$

for $i \neq j$ and $w_{ii} = 1$, for all i since a criterion is as important as itself.

Where w_{ij} represents the relative weight of criterion C_i over C_j while w_{ji} represents the relative weight of criterion C_j over C_i .

The weights form a square matrix $W = (w_{ij})$, of order n . This order corresponds to the number of criteria. The fact that the inverse of one criterion over another is equal to the weight of the second criterion over the first one, the matrix, W , is classified as a reciprocal matrix. In this case, if the life of mine is twice as important as the grade in the open pit optimisation process, then logically, grade will be half as important as the life of mine.

Transitive axiom

Another important property of the matrix, W , is the transitivity. If the life of mine is twice as important as the grade and NPV is four times as important as the life of mine, then it follows that criterion NPV should be eight times as important as the grade. When a matrix satisfies the transitive axiom, the judgements are referred to as consistent. Here is where the AHP is very important in gauging the degree of inconstancy in typical human decisions.

The mathematical expression of the transitivity between weights is shown in Equation 14.

$$w_{ik} = w_{ij}w_{jk} \quad \text{for all } i, j, \text{ and } k \quad (13)$$

A vector, w , of order n can be determined. $Ww = \lambda w$, whereby the vector, w , is an eigenvector of the matrix W , and the constant λ represents its corresponding eigenvalue. The matrix W is consistent if the eigenvalue (λ) is equivalent to the order of the matrix (n).

When dealing with inconsistent human judgements, the eigenvector, w , will only satisfy the condition Ww equals $\lambda_{max}w$ such that $\lambda_{max} \geq n$. The decisions are consistent if λ_{max} is equivalent to n . Otherwise, the discrepancy between λ_{max} and n shows that the decisions were inconsistent.

To estimate the eigenvector, different methods are used. Among these is the geometric mean that provides a simple and close approximation of the eigenvector. According to Costa (2007), cited by Musingwini and Minnitt (2008), the advantage of the geometric mean is that it tends to reduce the effect of very high or low values, which could bias the mean in case an arithmetic mean was estimated.

Consistency index and consistency ratio

A consistency index, CI , is calculated from λ_{max} and n using the relationship expressed mathematically as follows (Equation 15):

$$CI = \frac{(\lambda_{max} - n)}{(n - 1)} \quad (14)$$

To test whether judgments made are reasonably consistent, a consistency ratio is determined by evaluating the defined consistency index (CI) against

judgements performed entirely at random. Saaty (1980) cited by Musingwini and Minnit (2008) simulated large samples of random matrices of increasing order and defined their equivalent consistency index, which are random indices (RI). For matrices of order n ranging from 1 to 15, Saaty (1987) set the corresponding RIs as shown in Table 2.2. The CR is then a ratio between the CI and its corresponding RI (see Equation 16).

Table 2.2 Random index for matrix of order n (Saaty, 1987)

Matrix Order	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

$$CR = \frac{CI}{RI} \quad (15)$$

According to Saaty (1980), the judgments are reliable if the C_r does not exceed 0.1. Otherwise, the decisions are inconsistently made, leading to biased conclusions. In other words, a CR of 0.1 means that the decisions are approximately 10% random, and a ratio of 1.0 would therefore imply that the judgements are completely too arbitrary, hence untrusted. However, in some circumstances, CR of more than 0.1 can be accepted if the reasonable justification for their acceptance is provided (Coyle, 2004).

Criterion normalisation

Suppose the judgements are considered consistent or tolerable. In that case, the efficiencies of all alternatives on a criterion, E_{ij} , are then normalised to remove the effect of different units of measure for each criterion as shown in the following Equation 17.

$$E^N = \frac{E_{ij}}{\sum_{i=1}^m E_{ij}} \quad (16)$$

Where:

- m = number of alternatives;
- E_{ij} = alternative on a criterion; and
- E^N = normalised criterion

Score

Finally, the matrix of normalised performance outcomes is multiplied by the eigenvector to determine the consolidated AHP performance score. Logically, the preferable alternative is the one with the highest AHP performance score (Musingwini & Minnit, 2008).

2.7.2 AHP limitations

There are three main limitations of the AHP methodology which are that (Coyle, 2004):

- The AHP works only if the matrix for the criteria weights is a positive reciprocal matrix;

- The weight vector varies if the scale for estimating the relative importance of criteria with respect to each other is changed. This may affect the final decision; and
- Increase in the number of criteria may render the calculations more complex.

2.8 Chapter 2 Summary

Previous research that has been conducted concerning selection of an appropriate SMU size to be used during extraction were discussed in this chapter. It was noticed that the orebody characteristics deduced from the strength of the ore and host rock, the shape, dip, and size of the deposit, and the grade and uniformity of ore largely influence the selection of SMU size. In addition to the orebody characteristics, the SMU size is also influenced by the capacity of the machinery and extraction parameters.

Several algorithms for surface mine planning and optimisation have been discussed and their limitation in open pit optimisation, among which, the Lerchs-Grossman (LG) algorithm was selected to be used in this study.

Lastly, Multi-criteria decision analysis (MCDA) was discussed as a tool used in operations research when several contradictory criteria have to be evaluated simultaneously in decision-making. The analytic hierarchy process (AHP) was deemed appropriate for the ranking of the SMU scenarios in this study due to its simplicity in the calculation and ability to

detect inconsistent judgements and estimate the degree of inconsistency when raking the criteria.

The next chapter describes the research methodology used in this study.

3 RESEARCH METHODOLOGY

3.1 Chapter 3 Overview

The methodology used in this research report is described in this chapter. The methodology aimed to respond the research question “does the SMU used at the Twangiza mine provide the optimal pit discounted value?” The chapter summaries the research approach, the type of data that was needed in order to answer the research question, the instruments used to collect data and how the data was analysed. Lastly the chapter described the ethical considerations in the development of this research.

3.2 Research Approach

A quantitative approach was used in this study. According to Creswell 2009, when an objective is tested by examining the relationships between variables, the research approach is qualified of quantitative. Mathematical techniques are used to measure and examine the variables.

3.3 Research Setting

Twangiza Mining SA is a subsidiary company of Bayin International Investment, a Chinese based company operating in the eastern part of the Democratic Republic of Congo (DRC). The Twangiza North orebody, located within the Twangiza Concession, has 4.58 million tonnes at an average grade of Au 2.84 g/t as a probable reserve (Twangiza Mining, 2015) and was used as a case study for this research study. This orebody was selected because the orebody is awaited for extraction; the expectation

being that the research findings may be used to review the mine planning process.

3.4 Data Collection

Data used in the research has two sources; Twangiza Mine database and a questionnaire survey.

3.4.1 Twangiza Mine Database

The student accessed the Twangiza Mine drilling database in order to reproduce geological block models. The drill hole database for the deposit contains 87 diamond drill holes totaling 15,850.53 metres and High-resolution LIDAR data in CSV format for topographic modelling.

In order to reproduce ore models the following software were used:

- Micromine to create Implicit grade shell to be used to constrain the grade interpolation;
- Datamine resource module (RM) was used for geostatitics analysis and grade interpolation within the grade shell created in Micromine;
- Geovia Surpac was used to create parameters files for pit shell optimisation using the models exported from Datamine RM; and finally
- Geovia Whittle was used to perform pit shell optimisation and the NPV of the pits using the parameters models created in Geovia Surpac.

3.4.2 Questionnaire

The yield average mining grade, the dilution factor, the ore tonnage, the stripping ratio, the project NPV, the amount of waste, the productivity and the life of mine obtained from pit optimisation in Whittle coupled with the mining operations factors were used simultaneously as criteria to select the best SMU size from the five alternatives of SMU size using a multi-criteria decision analysis (MCDA).

In this study, AHP was used as the MCDA technique due to its simplicity in the calculation and ability to detect inconsistent judgements and estimate the degree of inconsistency when ranking the criteria. The MCDA techniques can be applied to solve problems that have both quantitative and qualitative data (Musingwini, 2010).

The information was collected through close-ended questionnaires distributed personally to the selected participants by the researcher. The questionnaire with a five-point scale for each of the eight factors relating to the SMU selection was designed to rank their respective importance (Table 3.1).

Table 3.1 Research questionnaire to assess the importance of each factor

Criterion		Importance				
		1 None	2 Minor	3 Substantial	4 Fundamental	5 Highest
1	Grade					
2	Dilution					
3	Ore tonnes					
4	Strip ratio					
5	LOM					
6	NPV					
7	Environmental Impact					
8	Productivity					

The Study Population and Sample

Burns and Grove (1993) defined a population as the individuals, objects or events that meet the sample criteria for inclusion in a study. In this research, mine engineers and geologists involved in the mining planning process within the Banro Mining Group constituted the study population. A total of 16 participants were contacted.

3.5 Ethical Considerations

The University of Witwatersrand in general and the school of mining engineering in particular, requires that all research should be conducted following the university ethical standards. Any research that may impact directly or indirectly on people, animals or the environment need to seek ethics clearance before any research is conducted. An ethical clearance was obtained from the university ethics committee and the rights to self-

determination, anonymity, confidentiality and informed consent were observed throughout the collection of information from the participants.

A permission to conduct a study was also granted by Banro Corporation in order to use their data in the study.

3.6 Data Analysis

Data was analysed using resource modelling and pit optimisation software to produce economics models used in this study. For analysis of closed-ended questions, data was analysed by using descriptive statistics in Microsoft Excel. Lastly, the optimisation data and the closed-ended questionnaire results were combined to build an AHP model in Microsoft excel.

3.7 Chapter 3 Summary

The data used in this research has two main sources; the Twangiza Mine database and the questionnaire. The Twangiza Mine drilling database was used to reproduce geological block models and optimal pit shells using micromine, datamine RM, Geovia Surpac and Geovia Whittle software.

The MCDA techniques was applied to determine the SMU size that best suits the Twangiza North orebody. A quantitative approach was used to rank the alternatives. A questionnaire with a five-point scale for each of the selected factors relating to the SMU selection was designed to rank their respective importance. The participants on the survey were geologists and

mining engineers involved in mining planning process within the Banro Mining Group.

An ethical clearance was obtained from the university ethics committee for ethics compliance throughout the research. A permission to conduct a study was also granted by the company in order to use their data in the study.

The next chapter namely Case study is the main body of this study comprising data selection, analysis and interpretation.

4 CASE STUDY

4.1 Chapter 4 Overview

The previous chapter presented the methodology used to answer the research question. The source of data and the instruments used to analyse the data was discussed. Chapter 4, namely Case Study, is the main body of the research. The data collected is being analysed and interpreted.

The chapter has two components. Firstly, it focuses on investigating the degree of sensitivity of the final pit result to the changes of SMU sizes. Orebody models at five different SMU sizes were created and pit shells optimised. The yield NPV was used as criteria of selecting the best SMU size. In the second component, AHP is used as a multi-criteria decision making tool. An AHP models is constructed in order to rank the five different SMU sizes.

4.2 Database

The drill hole database for the Twangiza North deposit contains 87 diamond drill holes totalling 15,850.53 metres. In addition to the collar and survey files, the drill hole data includes:

- Assay; containing 15,046 samples collected at a maximum of 1metre length interval;
- Lithology;
- Oxidation;

- Structure;
- Density; and
- High-resolution LIDAR data in Microsoft Excel csv format for topographic modelling.

Figure 4.1 shows the location of the Twangiza North deposits within the Twangiza mine footprint.

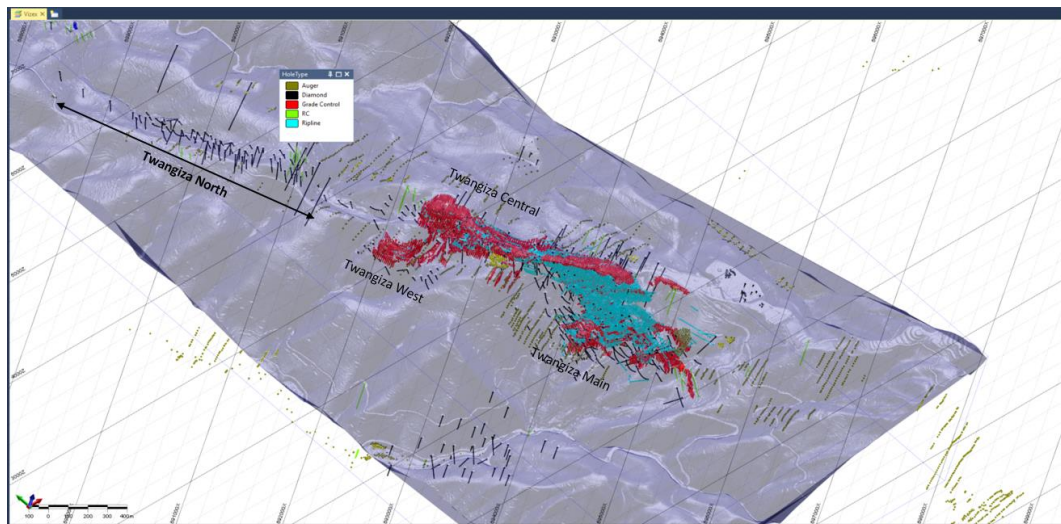


Figure 4.1 Location of the Twangiza North deposit within the Twangiza mine footprint

4.3 Grade Shell and Resource Modelling.

The traditional explicit modelling method was used by the Twangiza Mine to generate the grade shell of the Twangiza North deposit. This method consists of digitising closed segments from sections and then stitched together to form the mineralisation solid. The shortcoming of this method was discussed in the literature review under section 2.2.2. In this research,

the student has then preferred to use an implicit method to generate the grade shell in Micromine software. An exploration cut-off grade of 0.3 g/t was used to demarcate the ore from waste. The sampling thickness has composited the grade.

Histograms of raw data showed that most samples were around 1m in length (see Figure 4.2a). Given that the minimum modelled thickness of a domain at Twangiza Mine is four metres, a nominal downhole composite of two metres was adopted (Figure 4.2b) to ensure no bias exists when performing the descriptive statistics and geostatistics.

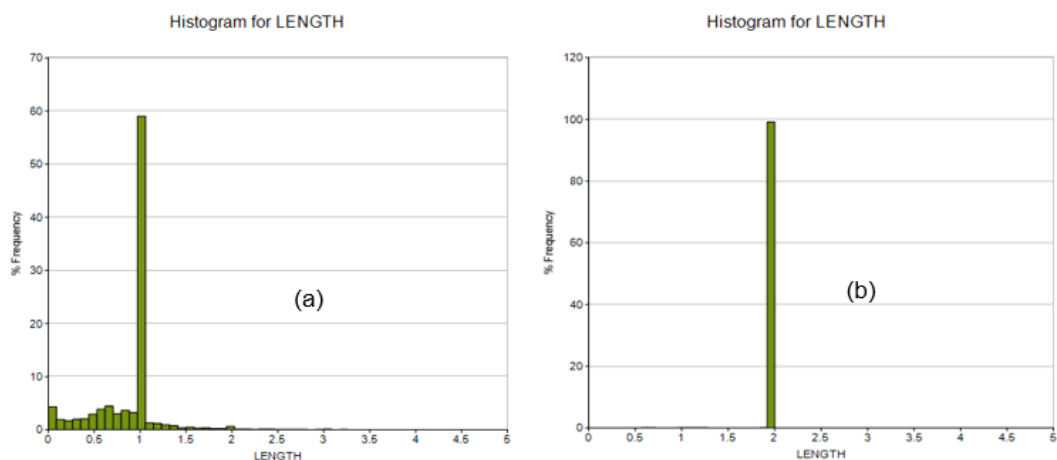


Figure 4.2 Histogram for sample length of raw data (a) and composite data (b)

The drill holes were drilled at a grid of 40m x 40m, with a sparse grid towards the north (Figure 4.3).

In most cases, the feldspar porphyry units host the gold mineralisation, as depicted by Figure 4.4. During modelling, lithological domains, and oxidation surfaces were generated.

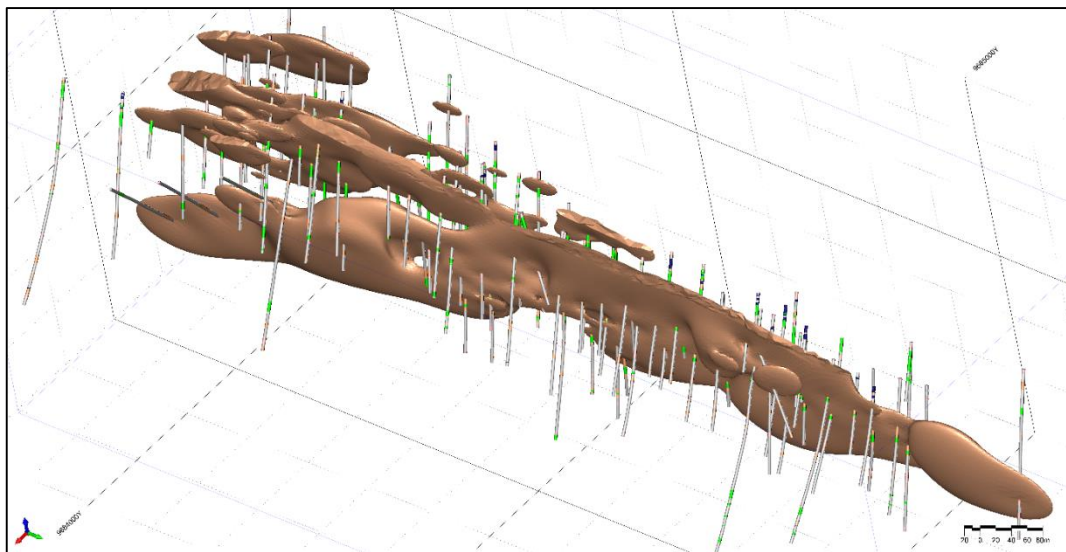


Figure 4.3 Grade shell of Twangiza North deposit

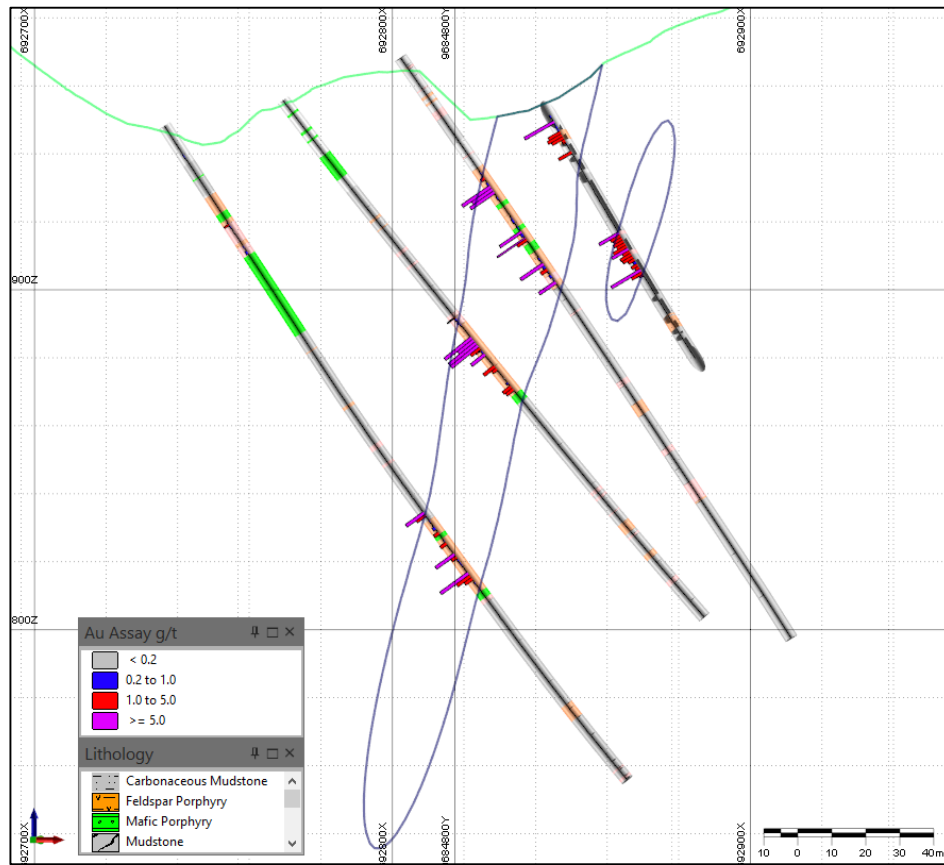


Figure 4.4 Typical cross-section of Twangiza North deposit showing the lithology and the gold grade.

4.4 Block Modelling

At Twangiza North, the minimum modelled domain thickness is 4 metres, while the flitch height of 2.5 metres is generally used. The “wireframe fill” interface in Resource Model (RM) package of Datamine commercial software was used to create the ore block within the grade shell for estimation.

Five different scenarios were created based on block size, which are: 5x5x2.5m, 7.5x7.5x3.75m, 10x10x5m, 12.5x12.5x6.25m and 15x15x7.5m.

Twangiza Mine uses a mining flitch height of 2.5 metres, thus, the block height along the z-axis was kept at an integer multiplier of the mining flitch height. Hence, the final block created were 5x5x2.5m, 7.5x7.5x2.5m, 10x10x5m, 12.5x12.5x5m and 15x15x7.5m for scenarios 1, 2, 3, 4, and 5, respectively. Tables 5.1 to 5.5 show the block model setups for the different scenarios.

Table 4.1 Block model setup for scenario 1; 5x5x2.5m SMU

Extents	Minimum Centre	Block size	Maximum Centre
East	692,503	5	693,298
North	9,683,795	5	9,685,440
Z	1,670	2.5	2,285
Total Blocks			7,921,655

Table 4.2 Block model setup for scenario 2; 7.5x7.5x2.5m SMU

Extents	Minimum Centre	Block size	Maximum Centre
East	692,503	7.5	693,298
North	9,683,795	7.5	9,685,437
Z	1,670	2.5	2,285
Total Blocks			5,757,072

Table 4.3 Block model setup for scenario 3; 10x10x5m SMU

Extents	Minimum Centre	Block size	Maximum Centre
East	692,503	10	693,293
North	9,683,795	10	9,685,435
Z	1,670	5	2,285
Total Blocks		Total Blocks	1,593,588

Table 4.4 Block model setup for scenario 4; 12.5x12.5x5m SMU

Extents	Minimum Centre	Block size	Maximum Centre
East	692,503	12.5	693,290
North	9,683,795	12.5	9,685,432
Z	1,670	5	2,285
Total Blocks			1,015,119

Table 4.5 Block model setup for scenario 5; 15x15x7.5m SMU

Extents	Minimum Centre	Block size	Maximum Centre
East	692,503	15	693,298
North	9,683,795	15	9,685,430
Z	1,670	7.5	2,285
Total Blocks			473,714

After grade modelling was done, the waste model outside the grade shell was created and combined with the ore model. All blocks were coded by oxidation state and lithology. The topography was used to code all the blocks above the topography.

4.5 Grade Modelling

4.5.1 Statistical analysis

Statistical analysis of the 2m composite assay data within the ore wireframe shows an average grade of 2.57 g/t, as depicted in Figure 4.5.

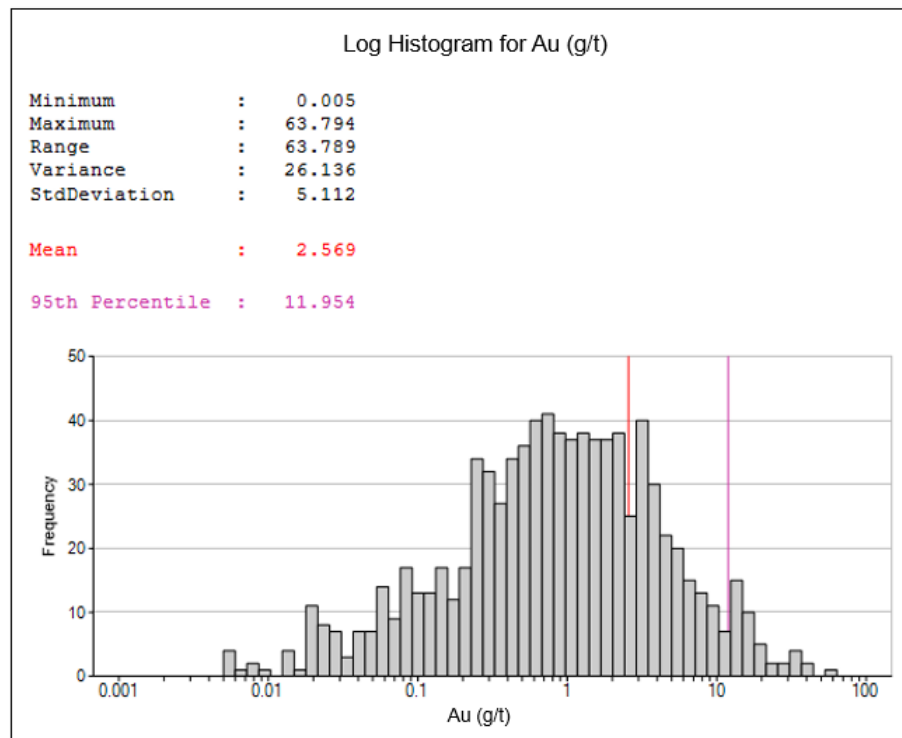


Figure 4.5 Log-histogram plot of 2m composite Au data for Twangiza North.

The histogram shows a unimodal structure. This allows keeping the ore within one domain. Based principally on the histogram plot of 2m composite grades (Figure 4.6), a common top-cut grade of 20 g/t Au was used to deal with outlier grades during estimation. Additionally, a visual inspection of the extreme top-cut values spatial occurrence was done to determine if they constituted distinct zones.

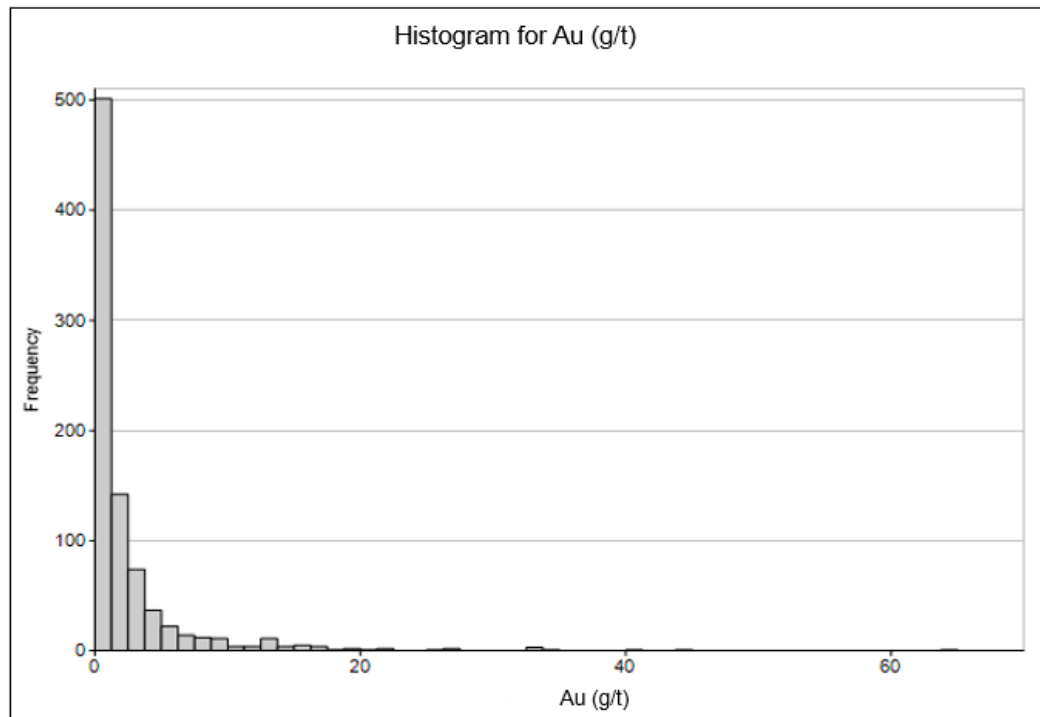


Figure 4.6 Histogram plot of 2m composite Au data for Twangiza North

4.5.2 Variography

Geostatistical analysis was performed on the 2m composite samples. Visualising the plans and sections, it was noticed that the dominant orientation of the data sets was 350° , dipping at 75° with a plunge of 18° towards 175° . The variogram maps have been produced accordingly to investigate the directional anisotropy as shown in Figure 4.7, Figure 4.8 and Figure 4.9.

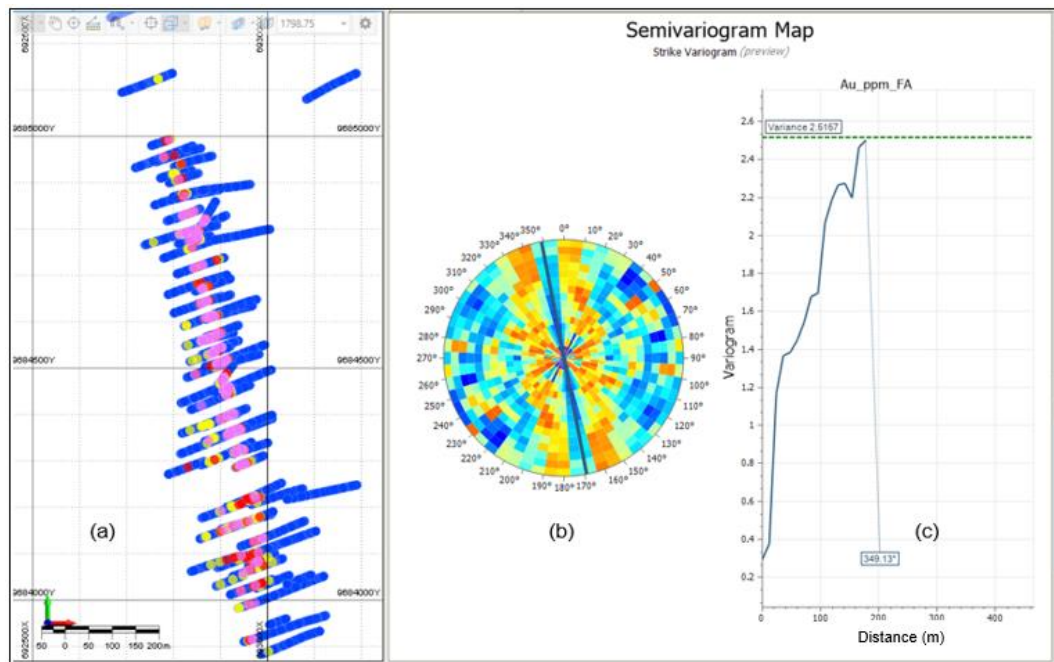


Figure 4.7 Drill hole plan view (a) strike variogram map (b) and strike variogram (c) of Au data for Twangiza North

The strike variogram map (b) shows a maximum directional continuity of 355° which is almost parallel to the strike of the orebody as per the drill hole plan view.

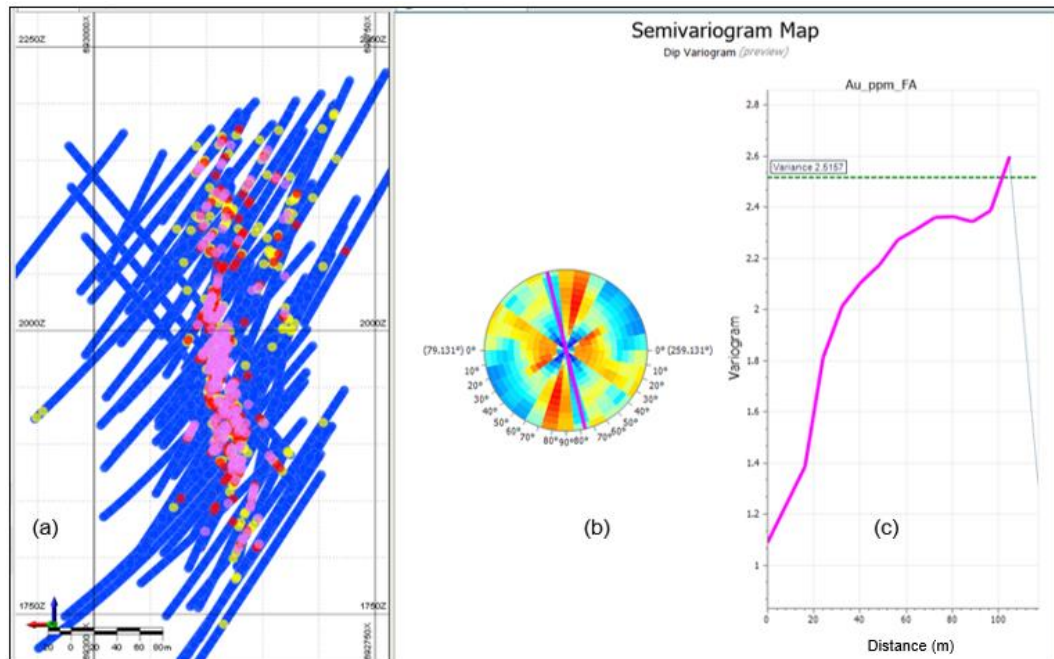


Figure 4.8 Drillhole cross-sectional view (a), dip variogram map (b) and dip variogram (c) of Au data for Twangiza North

The variogram shows a maximum continuity dipping at 75°, coinciding with the light blue continuity on the variogram map. Although the red zone in the variogram map shows as well a strong continuity, when aligned with the drillhole cross-sectional view, this red zone continuity coincides with the downhole directional variance.

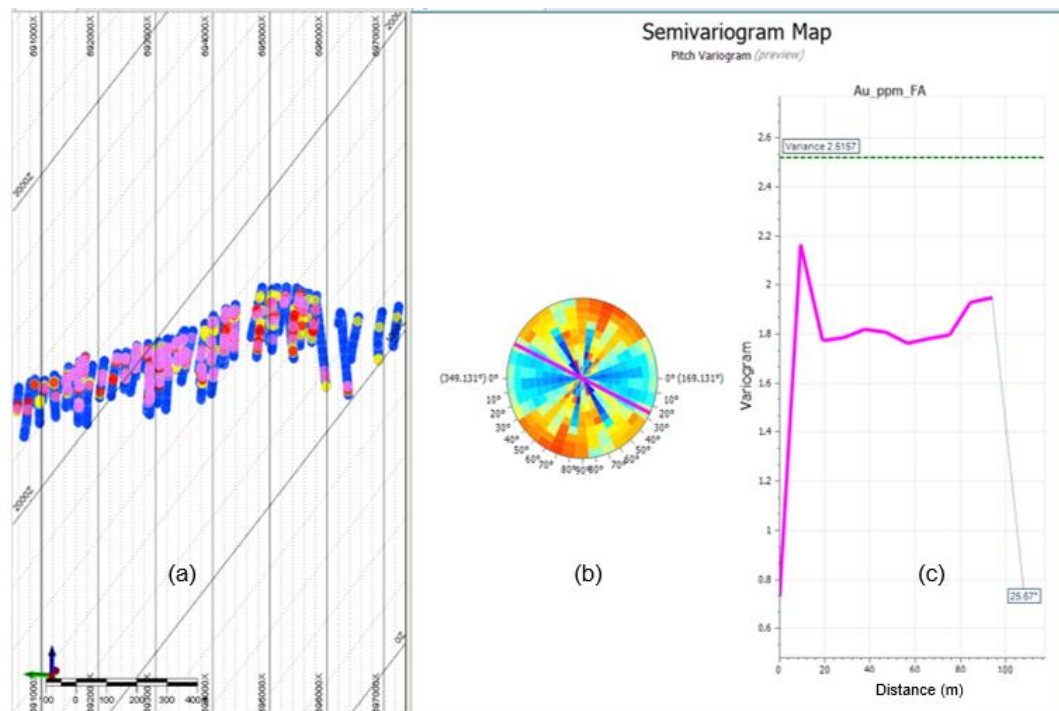


Figure 4.9 Drillhole longitudinal-sectional view (a), pitch variogram map and pitch variogram of Au data for Twangiza North

The variogram shows four different directions of continuity. Drillhole longitudinal-sectional view aligned with the pitch variogram map helped in determining the maximum direction of continuity. This direction plunged 22° towards 170°.

4.5.3 Semi-variograms

Based on the anisotropy directions, variograms have been computed, and the kriging parameters optimised. Firstly, an omnidirectional down-hole variogram was calculated to estimate the nugget variance to apply to the directional variograms. This shows a nugget of 30%, indicating a relatively high degree of variability (see Figure 4.10). The directional variograms are shown in Figure 4.11.

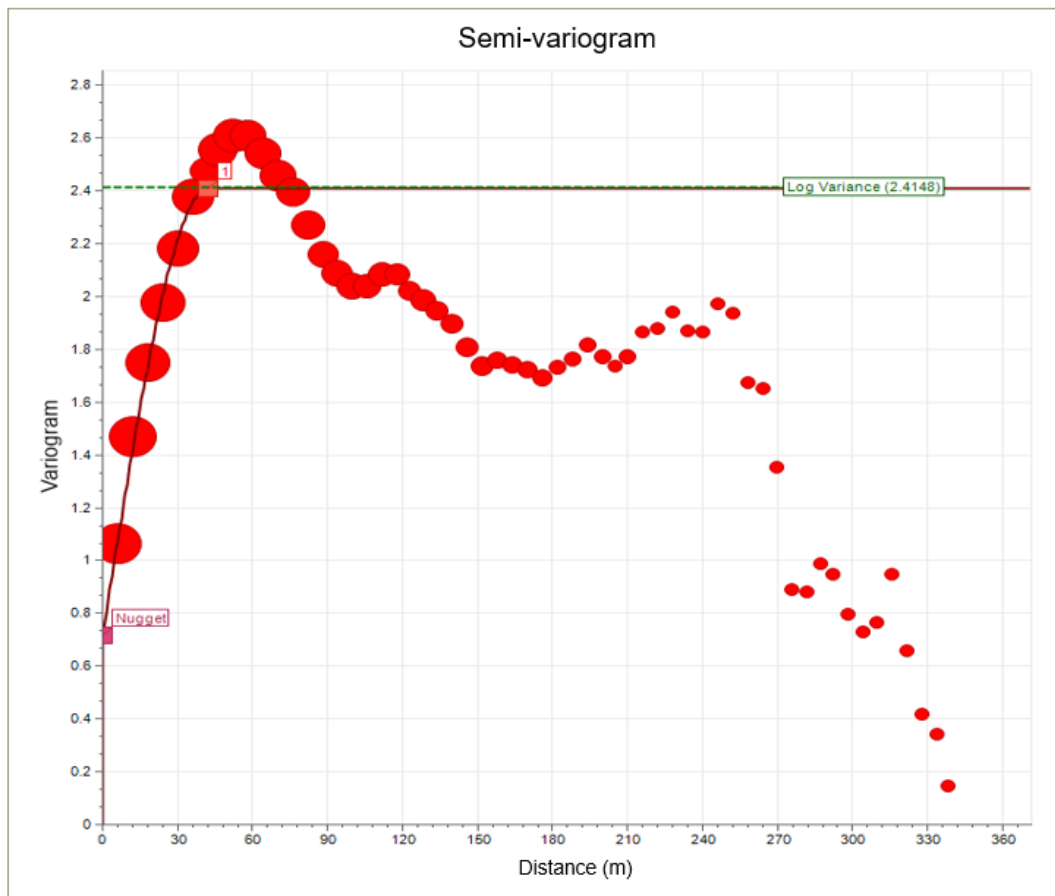


Figure 4.10 Downhole semi-variogram for the Twangiza North Au data

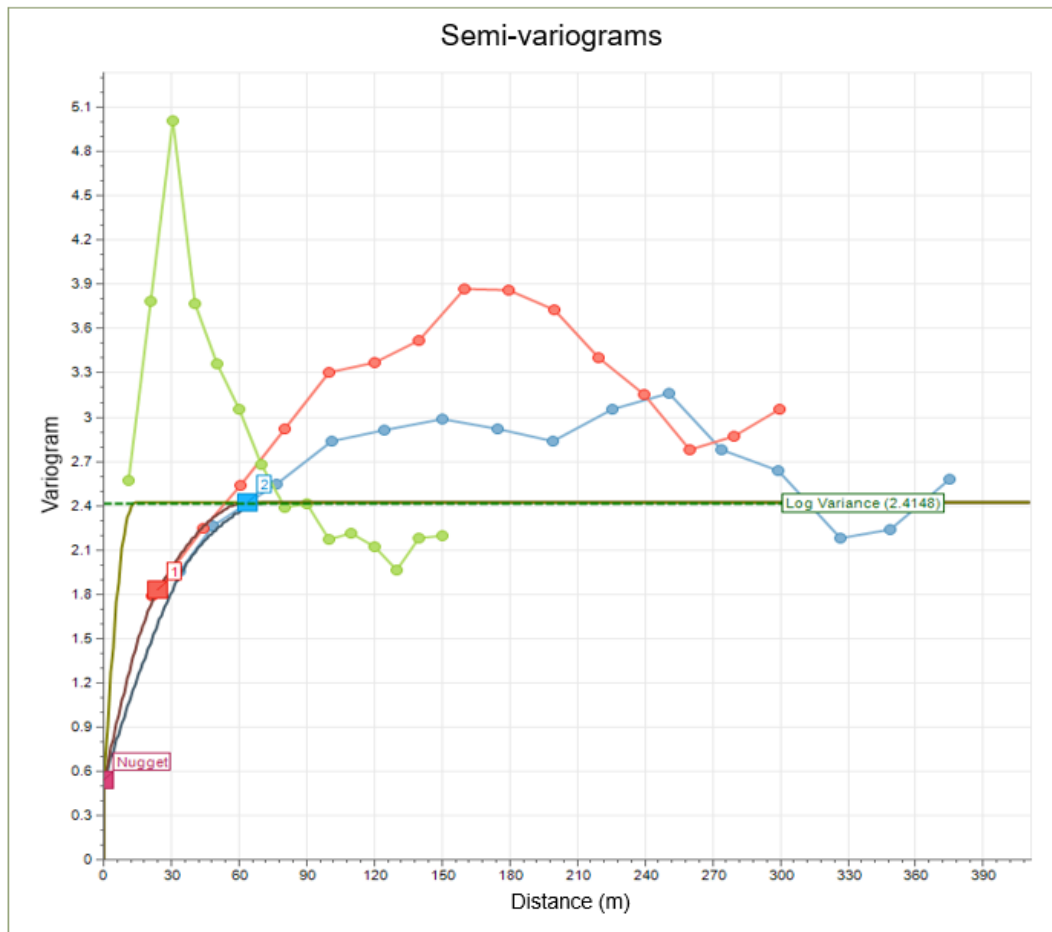


Figure 4.11 Directional semi-variograms for the Twangiza North Au data

The Figure 4.11 showed that the three-directional variogram are not identical, hence the Twangiza orebody is anisotropic. The data was firstly transformed using the Discrete Gaussian transformation algorithm embedded in RM package in Datamine commercial software for experimental variograms. Later, these models were back transformed for use in the grade estimation using kriging.

4.5.4 Grade estimation

Grade estimation of the five block models (5x5x2.5m, 7.5x7.5x2.5m, 10x10x5m, 12.5x12.5x5m and 15x15x7.5m) was conducted using ordinary kriging. A kriging neighbourhood analysis (KNA) exercise within the RM package in Datamine commercial software was performed to optimise the kriging parameters before the final estimation. This shows a minimum of five composite sample per sector (see Figure 4.12) with an optimum of 25 samples as shown in Figure 4.13. Sample discretisation of 4x4x4 was also defined as optimum as presented in Figure 4.14.

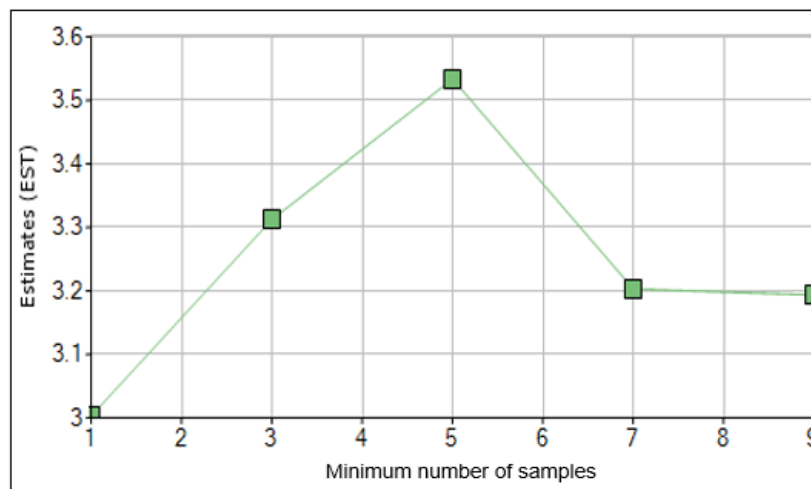


Figure 4.12 Kriging Neighbourhood Analysis for minimum sample per sector

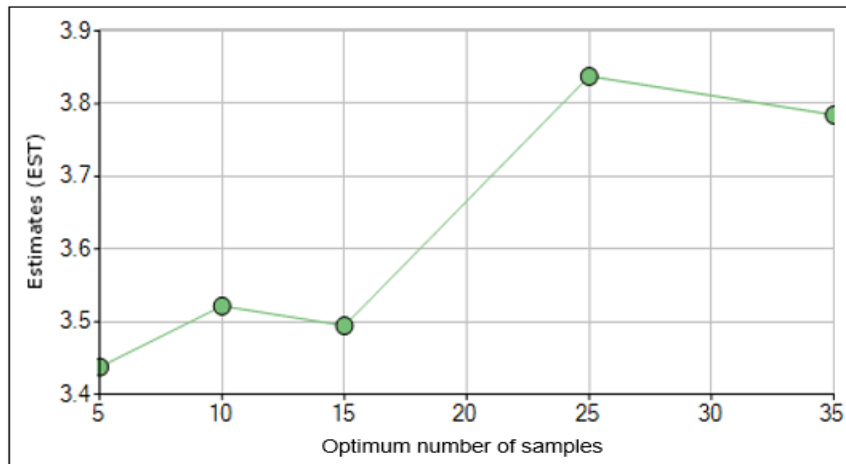


Figure 4.13 Kriging Neighbourhood Analysis for optimum sample per sector

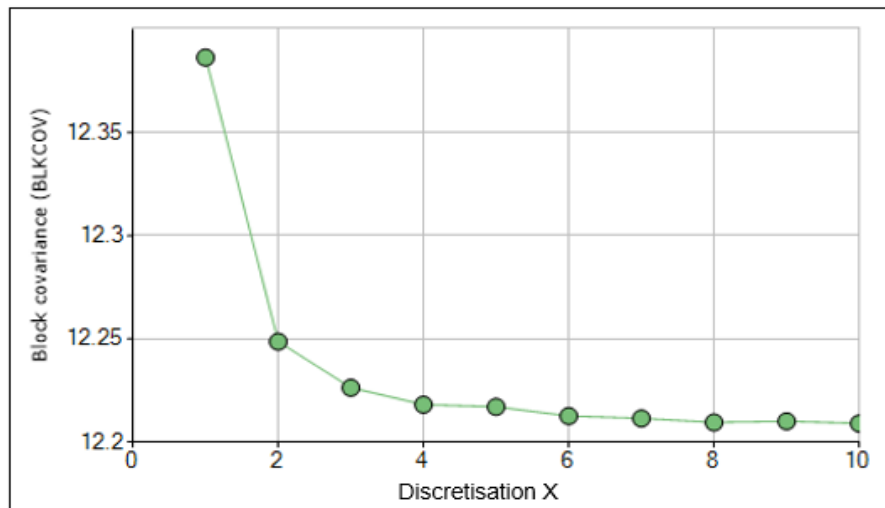


Figure 4.14 Kriging Neighbourhood Analysis for sample discretisation.

4.6 Mineral Resource Estimate

4.6.1 Break-even grade

As stated in the Section 4.4, an exploration cut-off grade of 0.2g/t was applied to construct the grade shell used in constraining the grade interpolation. A breakeven grade is calculated to determine the blocks that may return a profit if they are extracted. At break-even, the total unit cost

(Equation 18) equals the unit revenue (Equation 19). The break-even grade is then simply expressed by Equation 20 (Birch, 2014):

$$UTC = \left(\frac{TFC}{x} \right) + UVC \quad (17)$$

$$UR = Grade * MRF * Price \quad (18)$$

Knowing that the revenue equals the total unit cost;

$$\begin{aligned} Grade * MRF * Price & \quad (19) \\ &= \left(\frac{TFC}{x} \right) + UVC \\ & Grade \\ &= \frac{\left(\left(\frac{TFC}{x} \right) + UVC \right)}{(MRF * Price)} \end{aligned}$$

Where:

- UTC = Unit total cost (\$/tonne);
- TFC = Total fixed cost (\$/tonne);
- UVC = Unit variable cost (\$/tonne);
- X = Volume (tonnes);
- UR = Unit revenue (\$/g);
- Grade = Break-even grade (g/t);
- MRF = the mine recovery factor (%), which is the mine call factor (MCF), multiplied by the plant recovery factor (PRF); and

- Price = commodity price quoted in US\$ per troy ounce of gold.

At Twangiza North, with a mill capacity of 1,7 million tonnes, the total unit cost is estimated at \$34.91 (Twangiza Mining, 2015). Using a long term gold price of \$1400/oz, mining recovery of 95% and plant recovery of 88%, the break-even grade for the Twangiza North deposit is estimated at 0.93 g/t. When including the royalties and selling costs of \$63.19/oz, the break-even grade is calculated at 0.97 g/t.

4.6.2 Effect of the block size on the grade model

The review of the grade-tonnage curves associated with each SMU size shows that the block size impacts the average grade and the tonnage above the cut-offs. The study shows a considerable loss of ore from 5x5x2.5m block model to 7.5x7.5x 2.5 m block model than 7.5x7.5x2.5m to 10x10x5m, as depicted by the metal versus tonnage curve shown in Figure 4.15. This suggests that the effect of the SMU is more critical in the mining selectivity for small blocks than for big blocks.

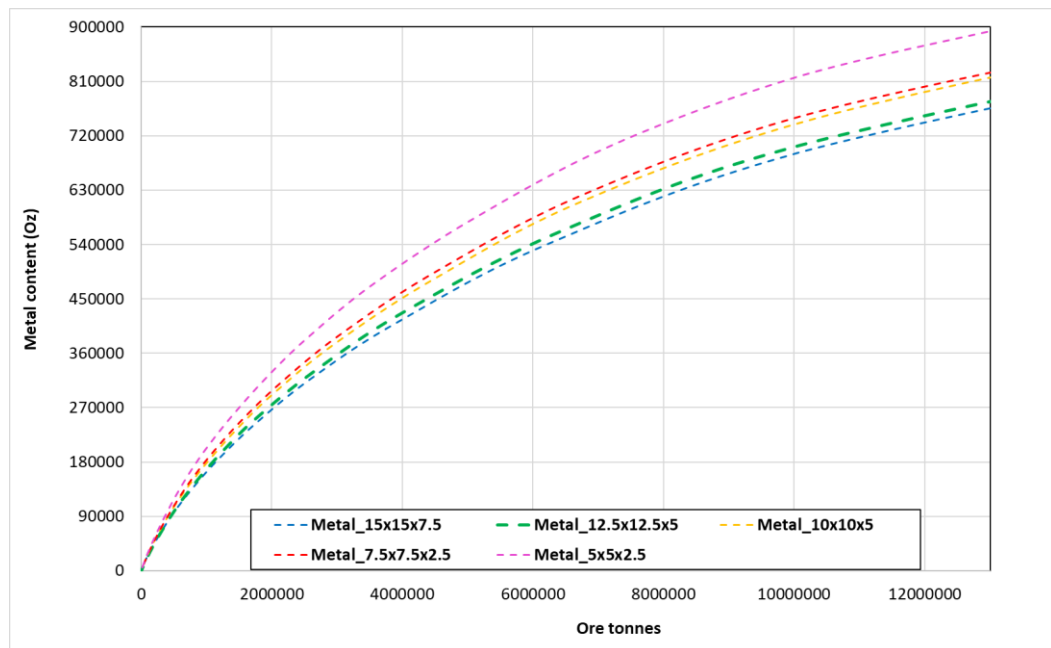


Figure 4.15 Metal vs. tonnage curves associated with the mineral resources for the different block sizes

The comparison of the tonnage vs cut-off grade curves also reveals that as the block gets larger, the average mining grade and the tonnage above the cut-off grade drop (Figure 4.16). The difference in the tonnages is pronounced when the cut-off increases. This shows that the effect of the SMU is higher on blocks of high grades than for the low grades.

On the low-grade portion (above the break-even grade) however, as the block gets larger, the average mining grade decreases but the tonnage above the cut-off increases. This shows that when mining the smaller blocks of low grade, a high degree of selectivity will be required.

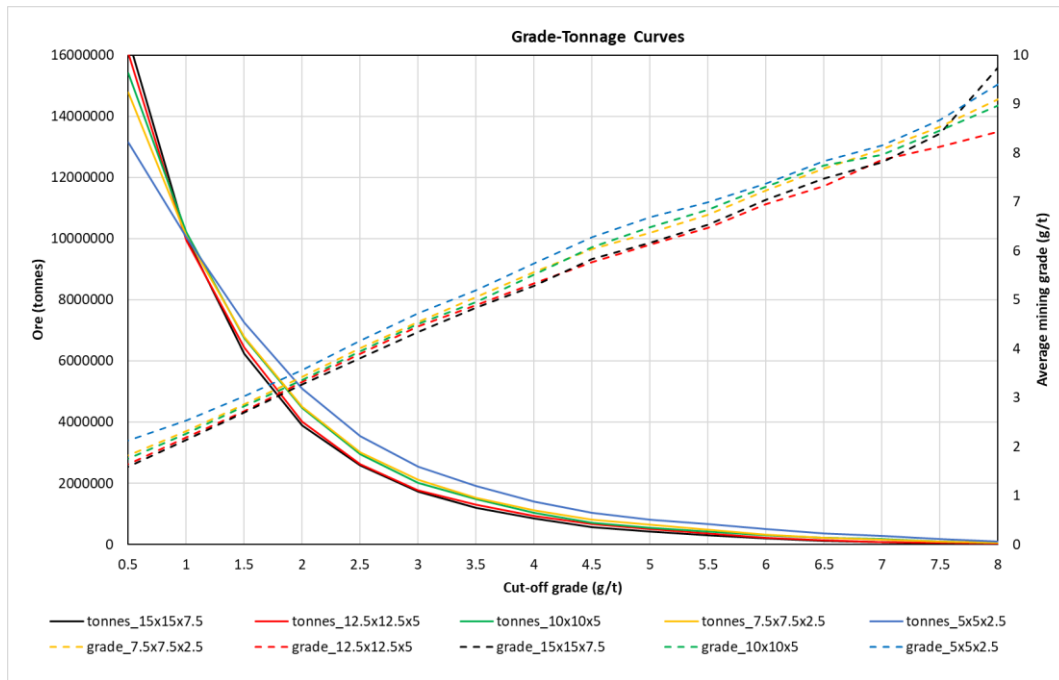


Figure 4.16 Grade vs tonnage curves associated with the mineral resources for the different block sizes

4.6.3 Resource tabulation

The breakeven grade of 1 g/t defined for the Twangiza North deposit is used as the cut-off grade to demarcate the deposit portion that may be converted into reserve and mined economically (Figure 4.17). In the next section, Whittle was used to determine the mineable material considering the different conversion factors. The tonnage, average grade and metal contents for the five different models are shown in Table 4.6 and Figure 4.18. Figure 4.18 shows that the metal content decreases with the increase of the block size.

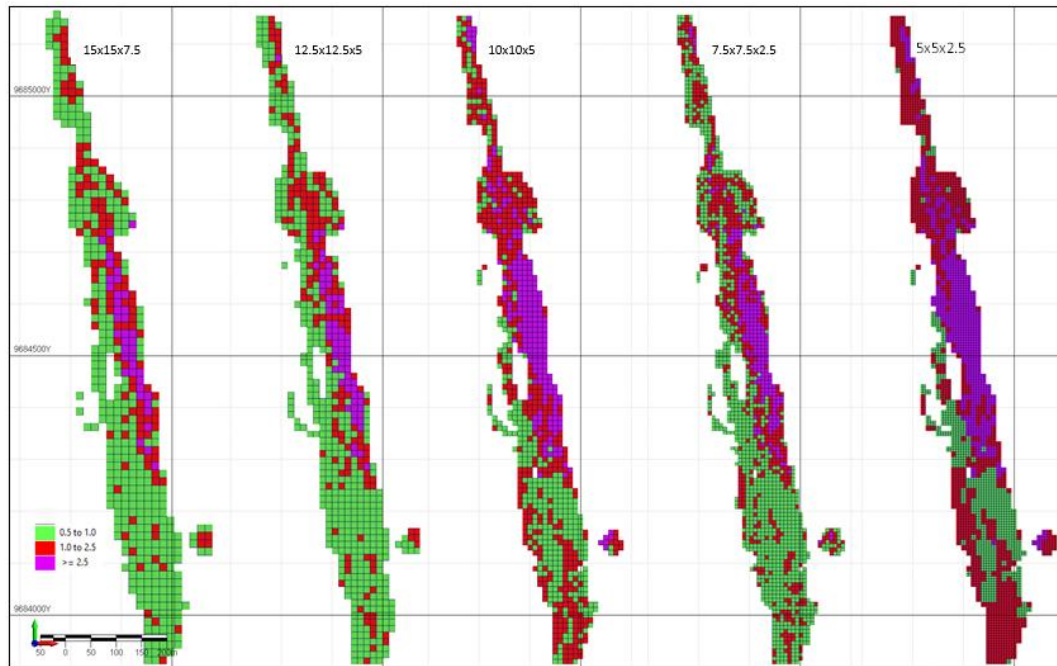


Figure 4.17 Resource block models for the different five sizes

Table 4.6 The resource estimate for the different five sizes

Scenario	Size (m)	Tonnage (tonnes)	Au g/(t)	Metal content (oz)
1	5x5x2.5	6,881,003	3.05	674,760
2	7.5x7.5x2.5	6,422,428	2.87	592,624
3	10x10x5	6,344,275	2.81	573,173
4	12.5x12.5x5	6,073,125	2.72	531,103
5	15x15x7.5	5,970,713	2.68	514,468

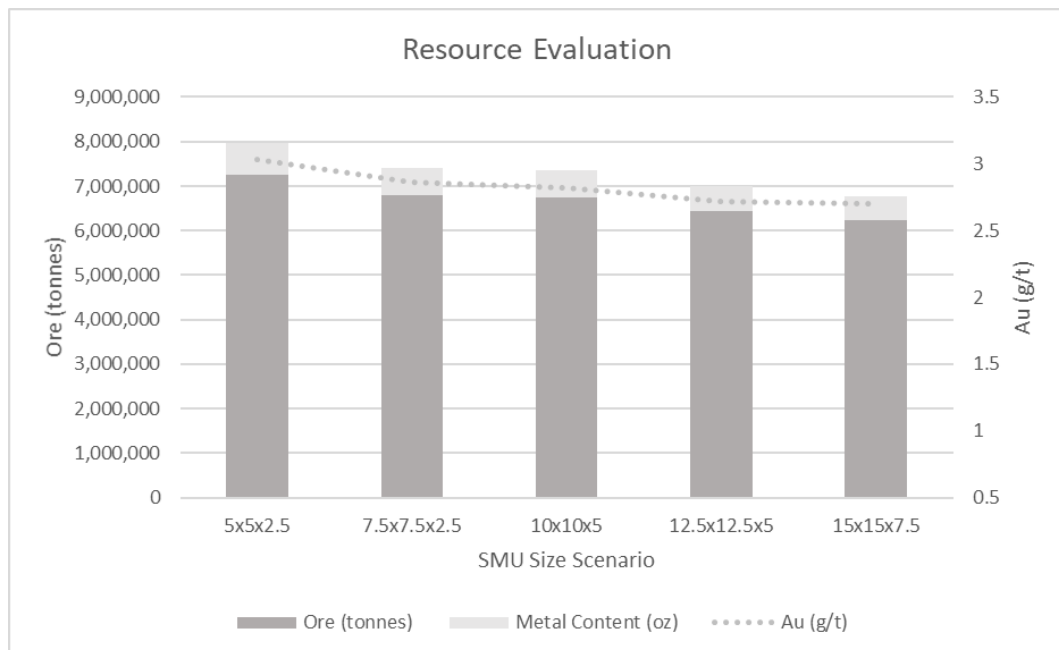


Figure 4.18 Graph showing the tonnage, grade and metal versus block size

4.7 Pit Optimisation

Pit shell optimisation was performed using the Lerchs-Grossman algorithm imbedded in GEOVIA Whittle software. The input data used to produce a set of nested pits included the resource block model, the economic and technical parameters. Figure 4.19 is a flow chart summarising the pit optimisation process.

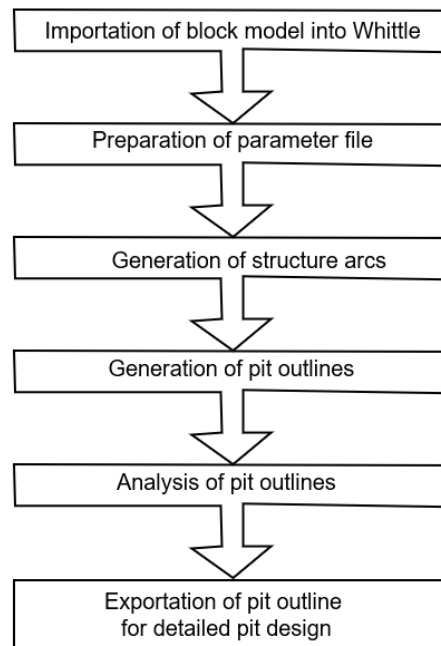


Figure 4.19 Flow chart for optimisation in Whittle software (Appianing & Mireku-Gyimah, 2015)

4.7.1 Importation of block model into Whittle

The block model was exported from GEOVIA Surpac software to Whittle using the “block model to Whittle ” interface in the Surpac software for pit optimisation. The “rock” attributes are based on the weathering status. They comprise OXID (oxide zone), TRAN (Transition) and FRSH (Fresh sulphide zone) in addition to the “AIR” zone. Weathering status was selected due to the process recovery factor varying with the oxidation status.

Intensive cyanidation tests performed on the Twangiza North orebody showed a high recovery variability (Twangiza Mining, 2015). This is due to the inclusion of refractory material in the orebody. The results are summarised in Table 4.7.

Table 4.7 Variability Tests of the Twangiza North ore (Twangiza Mining, 2015)

Ore Type	Sample ID	Grade (g/t)	NaCN Consumpmtion (Kg/t)	CaO consupmtion (kg/t)	Overall gold recovery (%)
Oxide	#78	0.50	0.56	3.11	94.40
	#79	4.26	0.85	5.37	92.00
	#80	3.06	0.48	5.10	87.60
	#81	2.78	0.83	5.88	87.80
	#82	1.35	0.54	4.80	94.10
	#83	1.01	0.48	3.28	69.30
	#84	2.24	0.45	5.80	80.80
	#85	8.76	0.83	6.24	93.70
	#86	2.76	0.65	6.07	82.80
	#87	9.67	0.62	5.44	87.70
Transition	#38	1.51	0.62	1.39	92.10
	#39	3.62	0.71	1.09	79.50
	#40	10.35	0.27	1.19	96.60
	#41	3.56	0.71	0.79	92.10
	#42	0.85	0.71	0.72	98.00
	#60	0.73	0.65	3.11	89.10
	#61	1.29	0.56	1.91	91.00
	#62	2.72	0.68	3.15	81.80
Fresh	#63	1.12	0.61	1.38	78.90
	#64	1.50	0.67	1.35	78.40
	#65	2.81	0.78	1.85	86.70
	#66	1.36	0.58	1.35	79.40
	#67	2.74	0.72	1.51	92.20

The average recovery factor used in the pit optimisation at Twangiza Mine is 89%, 91% and 79% for Oxide, transition and fresh ore, respectively (Twangiza Mining, 2015). The processing and mining cost adjustment factors were incorporated in the model and set at 1, assuming the costs do not vary with the type of ore.

4.7.2 Preparation of parameter file

The essential economic parameters for pit optimisation were set up in Whittle software. These include mining and plant recovery factors, cost values, slope angles, mining dilution factors and types of the run-of-mine. Table 5.8 summarises the cost figures and parameters used in the pit optimisation for Twangiza North (Twangiza Mining, 2015). A long-term gold price of US\$1400/Oz was used in this study instead of the \$1200 used in the Twangiza 2014 reserve estimate in order to conform with the recent reserve review for Twangiza Mine, done in 2019 (unpublished). Table 4.8 shows the costs and parameters used in the pit optimisation process in this study.

Table 4.8 Costs and Parameters used for Pit Optimisation

Parameters	Unit	Value
Mining cost	\$/tonne mined	3.51
Processing cost	\$/tonne processed	19.11
General and administration cost	\$/tonne processed	12.29
Royalties and selling costs	\$/Oz	63.19
Gold price	\$/Oz	1400
Discount rate	%	10
Mining recovery	%	100
Mining dilution	%	5
Mining limit	Tonnes/year	12,775,000
Processing limit	Tonnes/year	1,700,000

4.7.3 Slope angles for pit optimisation

Slope angles are defined in Whittle by structure arcs. The structure arcs are generated using the FXST programme in Whittle software. The aim is to convert the defined slope angles into a suitable format for pit optimisation (Thobaven & Willoughby, 2012).

The SMU of a block model dictates the number of structure arcs. The smaller the SMU, the greater the number of benches and structure arcs, and the slower the optimisation process. In other hand, block model with less number of benches for arc generation, provides great slope error between desired and achieved slope (Thobaven & Willoughby, 2012). The number of benches for slope generation produced during structure arc generation is estimated using Equation 21.

$$\text{Number of Benches} = \frac{\text{Maximum } (x, y) * 8}{z} \quad (20)$$

Where: x , y and z are the block dimensions in the x , y , and z directions

Hence, for a model with an SMU of 5x5x2.5m, the number of benches for slope generation is 16.

Whittle provides the slope error after running the slope node. In case the slope error is above 2 degrees, then the number of benches should be increased. A significant slope error in the optimal shell means that the actual

pit design cannot achieve the tonnes and grade reported in the optimisation result (Thobaven & Willoughby, 2012).

The slope angles used in the study were set based on the geotechnical properties captured in geological model. At Twangiza, the following overall slope angles are used for different geotechnical zones based on the oxidation status (Twangiza Mining, 2015).

- The overall slope angle of 38° in oxide material;
- The overall slope angle of 42° in transitional material; and,
- The overall slope angle of 53° in fresh material.

Table 4.9 shows the number of benches and slope error reports after running the slope node.

Table 4.9 Number of benches and slope error for the five different block sizes

Scenario	SMU size (m)	Number of benches	Average slope error	Number of arcs
1	5x5x2.5	256	0.4	1,375,825,216
2	7.5x7.5x3.75	170	0.4	408,443,005
3	10x10x5	128	0.4	183,288,857
4	12.5x12.5x6.25	102	0.4	96,500,202
5	15x15x7.5	85	0.4	57,354,119

4.7.4 Generation of pit outlines

Lerchs-Grossman algorithm imbedded into GEOVIA Whittle software was used to perform the pit outline optimisation. The optimisation carried out on the five different models produced 50 different nested pit outlines according to the set revenue factor in the range of 0.2 to 2.

Many analyses were carried out to select the optimal pit. This was done based on the worst and best-case scenario graphs of yield NPV against the pits. In Whittle, the best-case scenario refers to mining out the inner shell and then mining out subsequently each pit in a top-down approach before starting the new pit shell, while in the worst-case scenario, each bench is fully mined before moving to the next bench (Thobaven & Willoughby, 2012).

The ultimate pit limit is the one that provides the highest NPV. Considering an example from the 5x15x2.5m SMU model (see Appendix 1 and Figure 5.20), it is clear that the NPV rises gradually to Pit 41 and starts dropping from Pit 42 to Pit 50. Hence, the optimal pit for Scenario 1 is pit 41. Figure 4.20 to Figure 4.24 shows the NPV versus Pit number for the remaining SMU sizes under investigation.

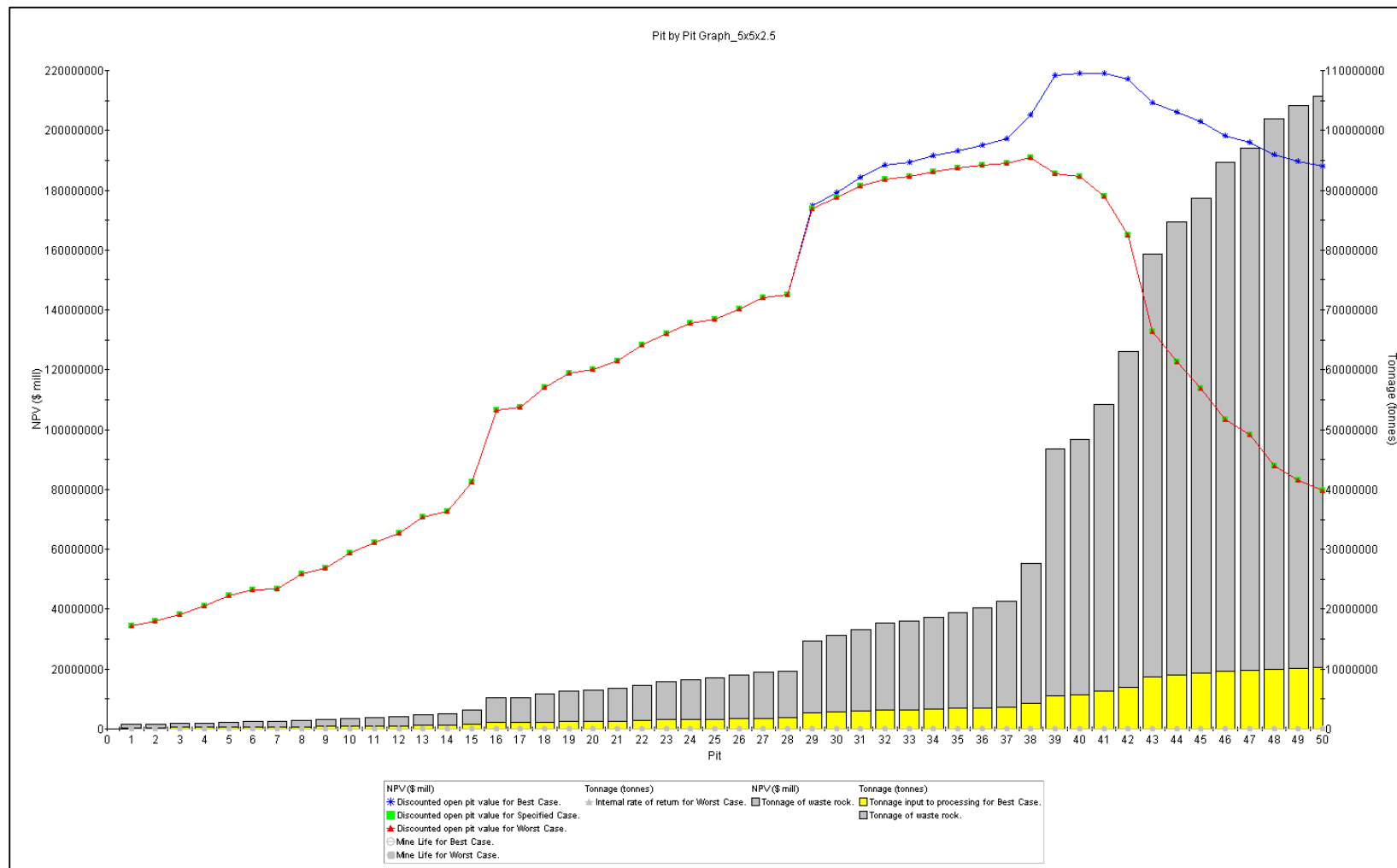


Figure 4.20 NPV versus pit shells for 5x5x2.5 SMU

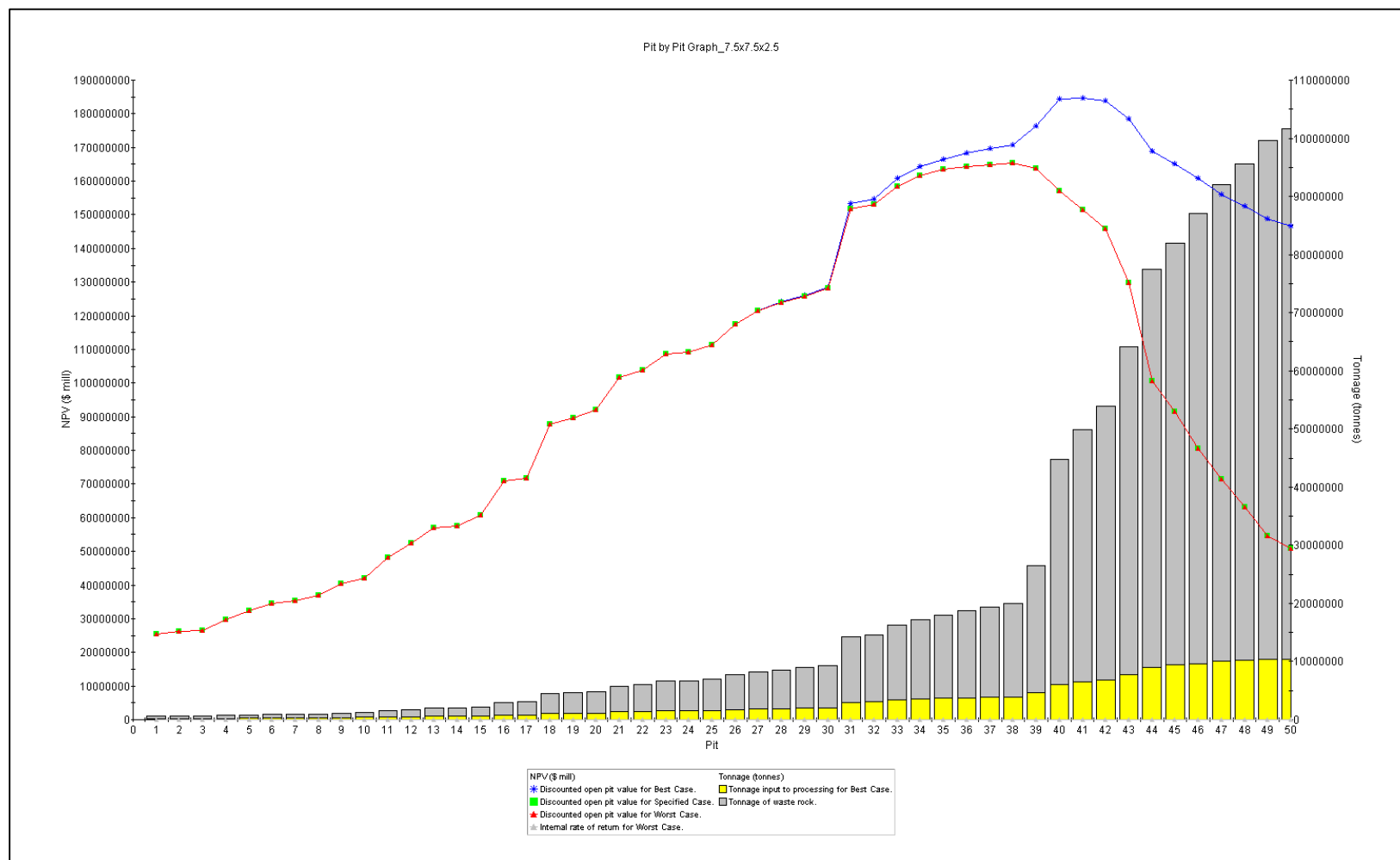


Figure 4.21 NPV versus pit shells for 7.5x7.5x2.5 SMU

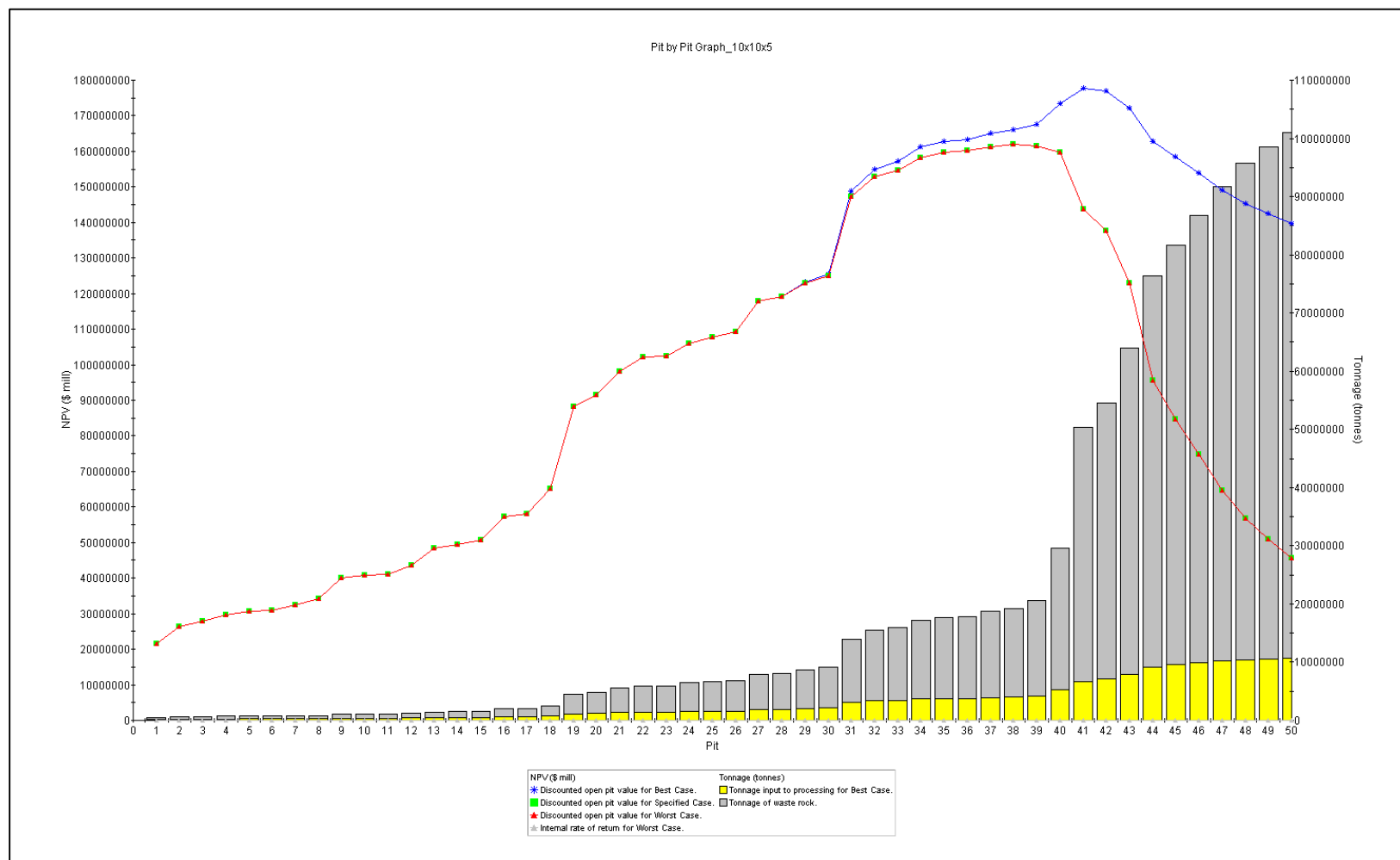


Figure 4.22 NPV versus pit shells for 10x10x5 SMU

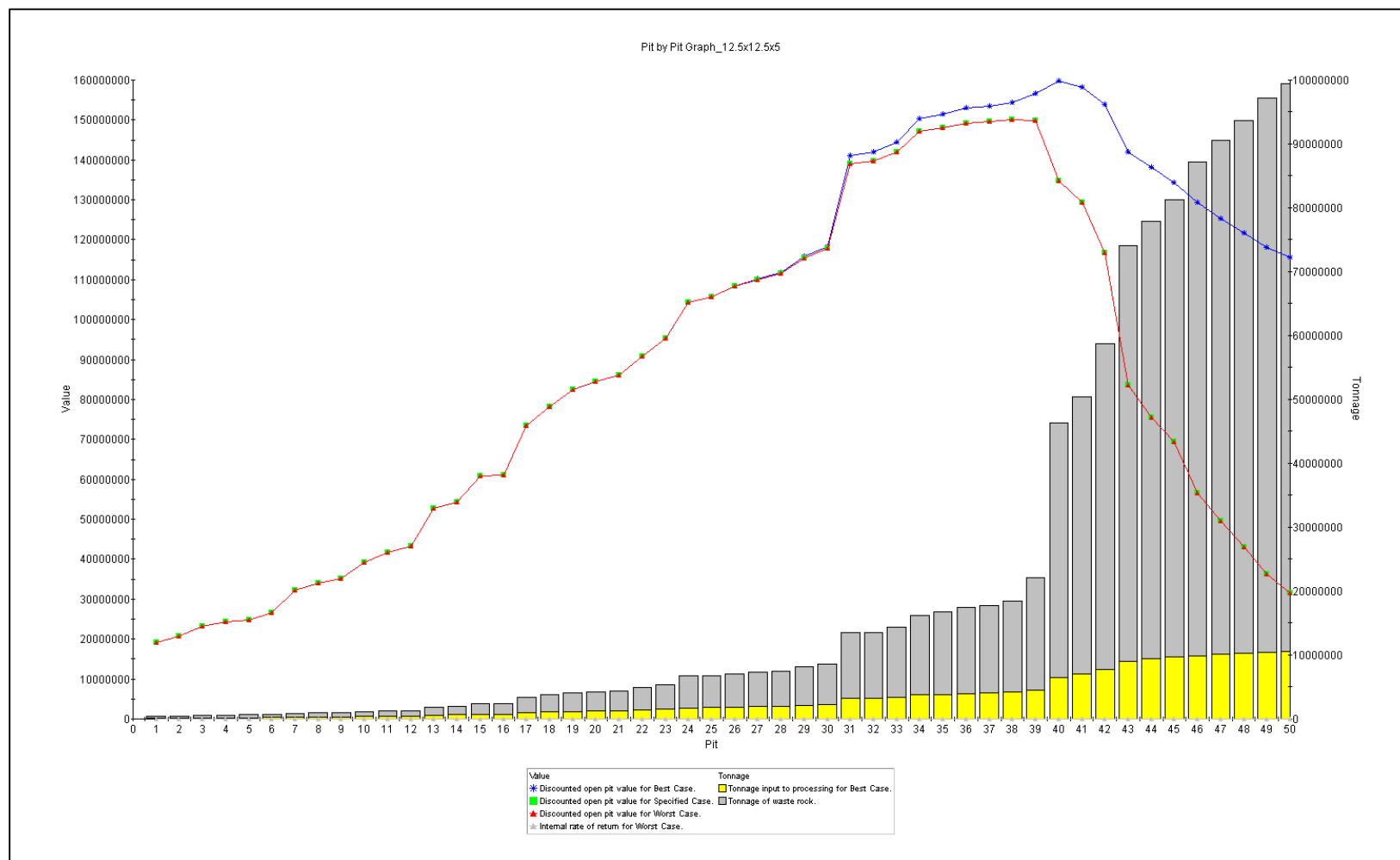


Figure 4.23 NPV versus pit shells for 12.5x12.5x5 SMU

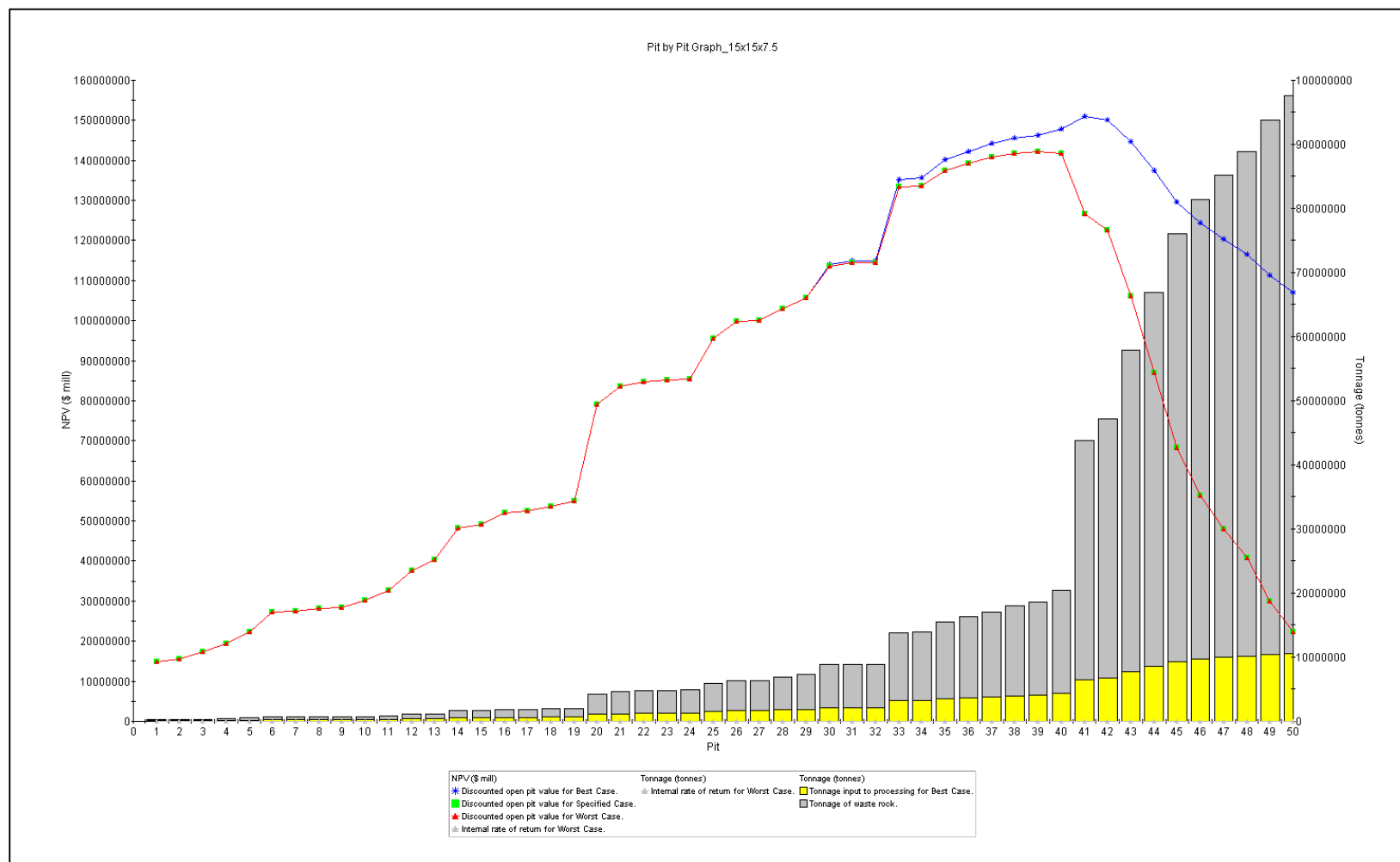


Figure 4.24 NPV versus pit shells for 15x15x7.5 SMU

Summary of optimal pit results for the five scenarios are presented in Table 4.10. Scenario 1 generated the highest recoverable gold, NPV, and the longest life of mine. Scenario 1 produced a pit shell and NPV that are 29% and 19% larger than the largest SMU, which is scenario 5. This is alluded to occurrence of dilution when reblocking to larger blocks. Figure 4.25 represents a comparison between the size of the optimised pit shells for the five scenarios.

Table 4.10 Whittle optimisation results for the five scenarios

		Scenario 1 5x5x2.5	Scenario 2 7.5x7.5x2.5	Scenario 3 10x10x5	Scenario 4 12.5x12.5x5	Scenario 5 15x15x7.5
Ore mined to Process	Kt	6,306	6,493	6,700	6,534	6,443
Insitu recoverable grade	g/t	2.80	2.50	2.42	2.31	2.25
Recoverable gold	Oz	486,166	447,631	447,524	416,721	399,962
Waste mined	Kt	47,850	43,409	43,670	39,765	37,356
Total material	Kt	54,156	49,901	50,370	46,299	43,800
Strip ratio	t/t	7.59	6.69	6.52	6.09	5.80
NPV @ 10%	\$ mill	177.98	151.37	143.83	134.83	126.61
Life of mine	Year	5.21	5.02	5.13	4.61	4.64

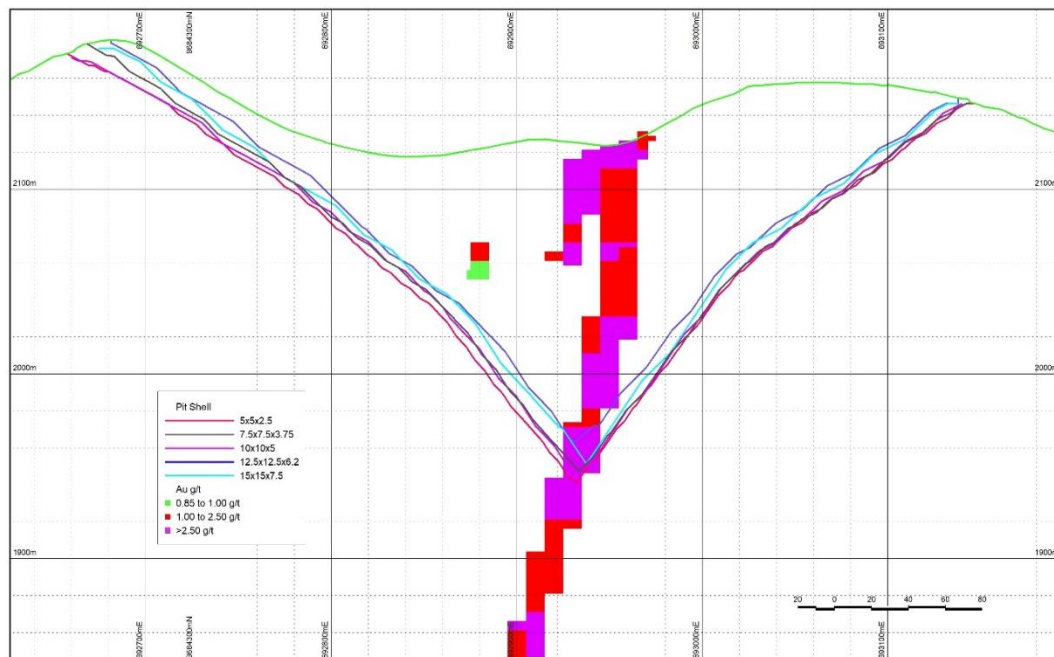


Figure 4.25 Comparison of the pit shells generated from the five scenarios

4.7.5 Sensitivity analysis

A set of sensitivity studies were carried out because the changes in the SMU size affect the tonnage and the average mining grade above the cut-off, the dilution factor, and the scheduling profile.

Effect of the Cut-off Grade

Different cut-off grades were applied to gauge the effect of the cut-off grade on the average mining grade, tonnage above the cut-off grade and NPV for the different SMU sizes. Fixed cut-off grades varying from 0.5g/t to 4g/t were used in Whittle, and the five scenarios were re-optimised. The results are presented in Appendix 2 and graphically represented in Figure 4.26 to Figure 4.28.

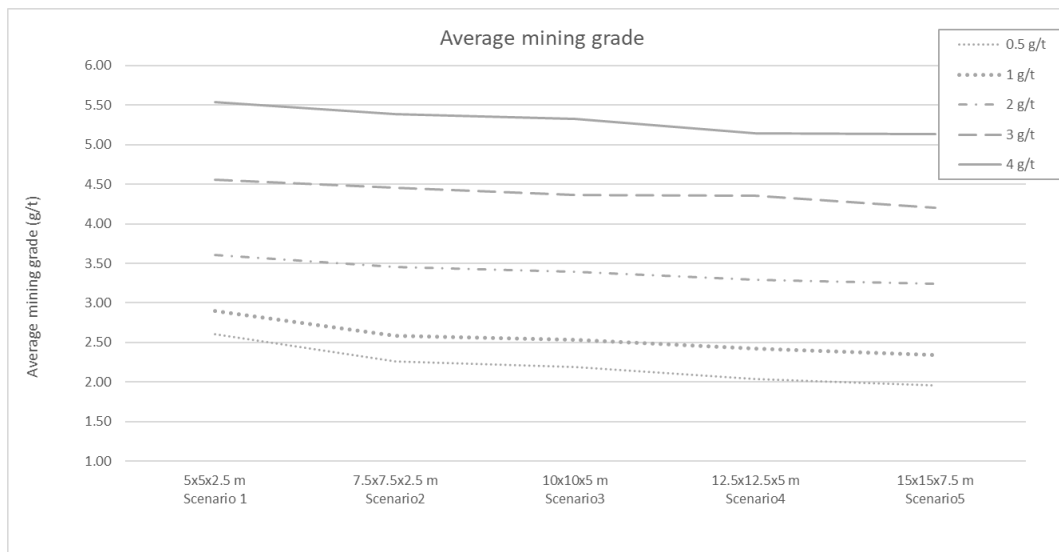


Figure 4.26 Average mining grade for each cut-off grade related to the SMU

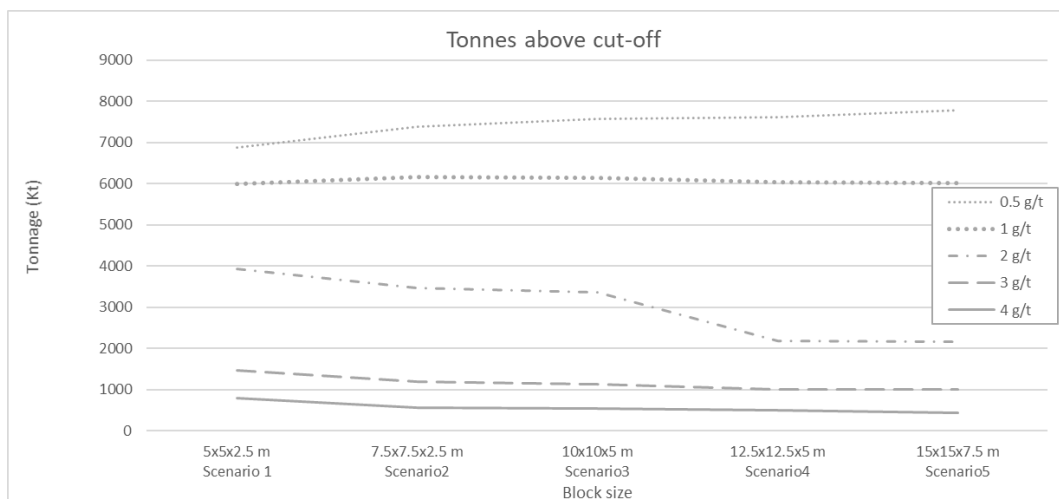


Figure 4.27 Ore tonnes for each cut-off grade related to the SMU

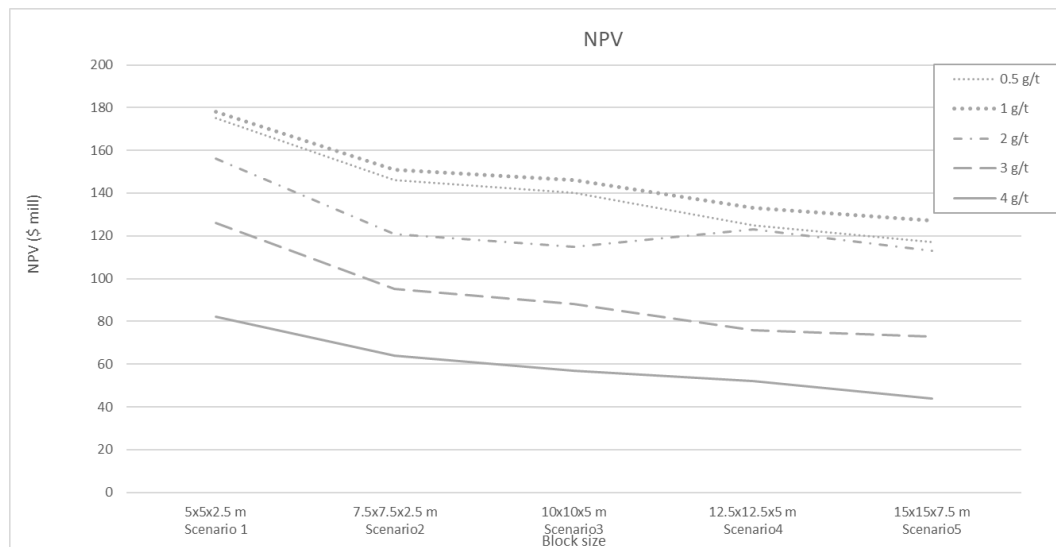


Figure 4.28 NPV for each cut-off grade related to the SMU

The result shows that at the different cut-off grades applied, the average mining grade (see Figure 4.26), the tonnage above the cut-off grade (see Figure 4.27) and the NPV (figure 4.28) decrease as the block gets larger. However, below the cut-off grade, the tonnage increases as the block get larger. This implies that when mining low-grade ore, larger blocks are required. Oppositely, when mining the smaller blocks of low grade, a high degree of selectivity will be needed to minimise ore dilution.

The following section investigates the cost of selectivity and ore dilution associated with mining smaller blocks.

Effect of dilution

At Twangiza Mine, the mine planners use a fixed dilution factor of 5%, assumed commonly for massive deposits. In the Twangiza North a different dilution factor should be applied because the deposit is strata bound. In

addition, as discussed in section 3.2.4, when the fixed dilution is used, the result of the design is not optimum because in real mining environment, the grade is inhomogeneous. Therefore, in the current study, variable dilution was adopted.

A contact surface of one metre between ore and waste material is considered in the study due to the dip of the orebody (75 degree), the size of the equipment used in extraction and the disposition of the block (unrotated) in respect to the strike of the mineralisation. The “dilution interface” in RM module of Datamine commercial software was used to calculate the diluted grade of the dilution skin.

Figure 4.29 shows how dilution factor changes with change in SMU size, for example, the dilution factor decreases at the SMU size increase.

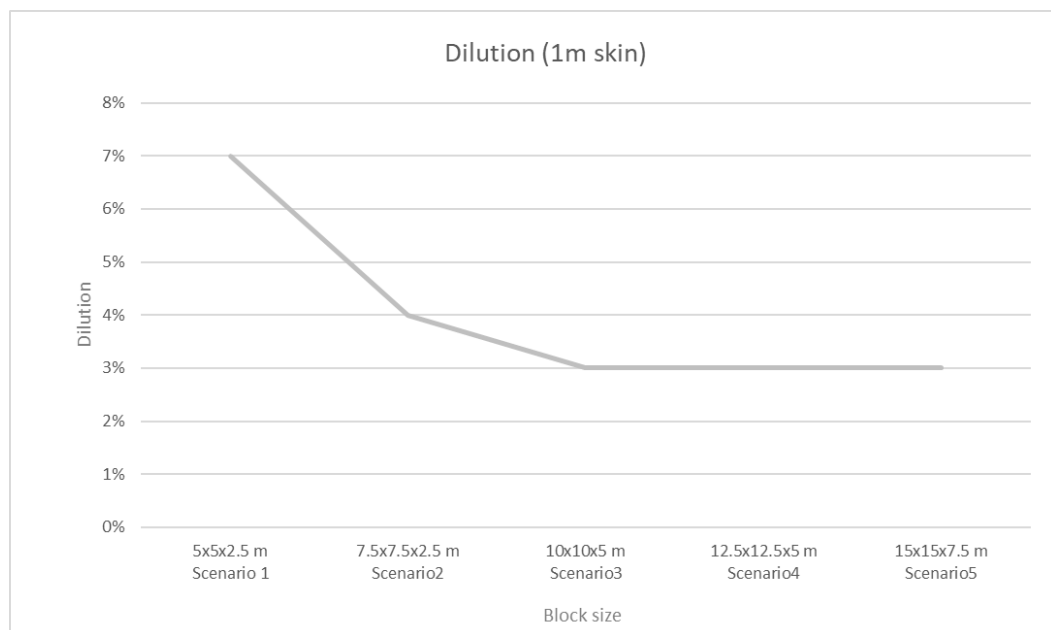


Figure 4.29 Dilution factor in relation to the SMU size at Twangiza North

By adopting the variable dilution, during ultimate pit optimisation in Whittle, the dilution factor is set to 1 since the grade model already contains the diluted grade on the contact surface. The Whittle optimisation results are shown in Table 4.11.

Effect of Selectivity

The fact that extracting smaller blocks requires a certain degree of selectivity, it is crucial to incorporate the cost of selectivity effect in the optimisation process.

In the optimisation of mining block size study by Birch (2019), to investigate the effect of selectivity on profit, the mining rate was adjusted to capture the loss of production due to selectivity. The same context is applied in the current study.

In the second semester of 2018, the orebody mined at Twangiza Mine from the northern end of the central pit varied from 2 m to plus 10 m in width. During this period, when mining the narrow ore zones (in the range of 5 metres), a high degree of selectivity was needed, and the daily production dropped to 27,000 tonnes compared to a planned production of 32,000 tonnes. While mining a zone of total waste where selectivity was not required, the production was more than 35,000 tonnes a day (see Figure 4.30).

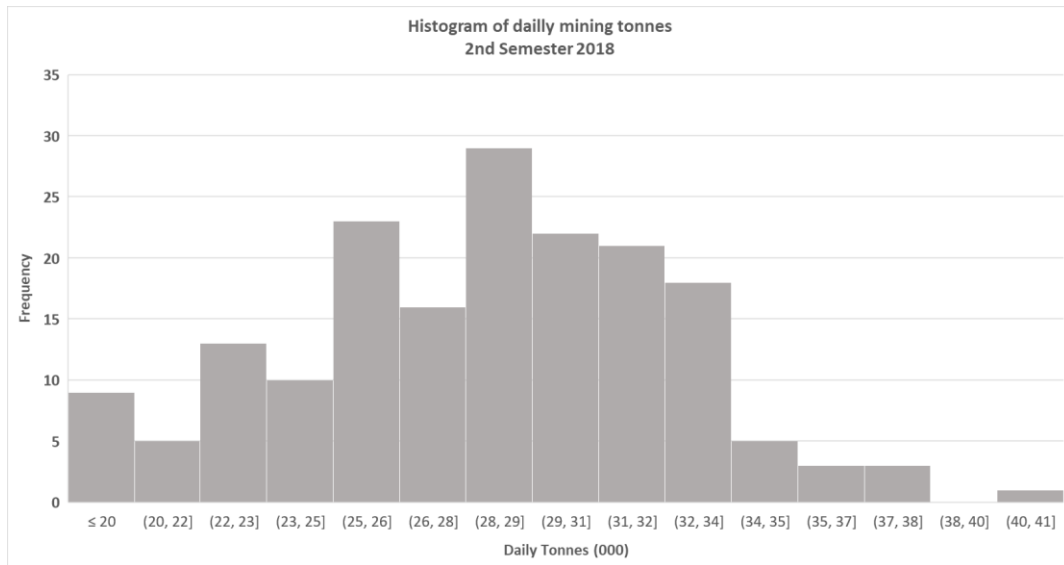


Figure 4.30 Daily mining tonnes during the second semester of 2018 at Twangiza Mine, Central pit.

This was factored as the effect of selectivity that comes with the reblocking as follows:

- For Scenario 1: the mining rate was reduced to 26,000 tonnes per day;
- For Scenario 2: the mining rate was reduced to 29,000 tonnes per day;
- For Scenario 3: the mining rate was kept at the planned 32,000 tonnes per day;
- For Scenario 4: the mining rate was increased to 33,500 tonnes per day; and
- For Scenario 5: the mining rate was increased to 35,000 tonnes per day.

Mining cost is not a fixed cost but varies with the volume excavated. It is crucial to consider the varying cost that comes with the production volume to gauge the effect of selectivity on the final pit values. Table 4.11 shows the estimated mining cost for the different block sizes when considering the cost of selectivity. The benchmark for the estimation is based on scenario 3 (10x10x5m SMU size) for the mining cost and mining volume which is used during mining planning at Twangiza mine.

Table 4.11 Estimated mining cost taking into account the cost of selectivity.

Scenario	SMU size (m)	Daily production (t)	Cost (\$/t)
1	5x5x2.5	26,000	4.32
2	7.5x7.5x2.5	29,000	3.87
3	10x10x5	32,000	3.51
4	12.5x12.5x5	33,500	3.35
5	15x15x7.5	35,000	3.21

Considering the selectivity and the variable dilution factor, the Whittle optimisation results are as shown in Table 4.12 and Figure 4.31.

Table 4.12 Whittle optimisation results for the five scenarios taking into account the cost of selectivity and variable dilution

		Scenario 1 5x5x2.5	Scenario 2 7.5x7.5x2.5	Scenario 3 10x10x5	Scenario 4 12.5x12.5x5	Scenario 5 15x15x7.5
Ore mined to Process	kt	5,788	6,283	6,630	6,477	6,595
In-situ recoverable grade	g/t	2.83	2.54	2.46	2.35	2.27
Recoverable gold	Oz	452,762	440,099	448,939	418,936	413,398
Waste mined	kt	42,035	42,141	43,762	40,061	39,896
Total material	kt	47,823	48,424	50,392	46,538	46,491
Strip ratio	t/t	7.26	6.71	6.60	6.19	6.05
NPV @ 10%	\$ mill	142.95	137.72	146.85	142.52	139.65
Life of mine	Year	5.43	5.14	5.11	4.67	4.67

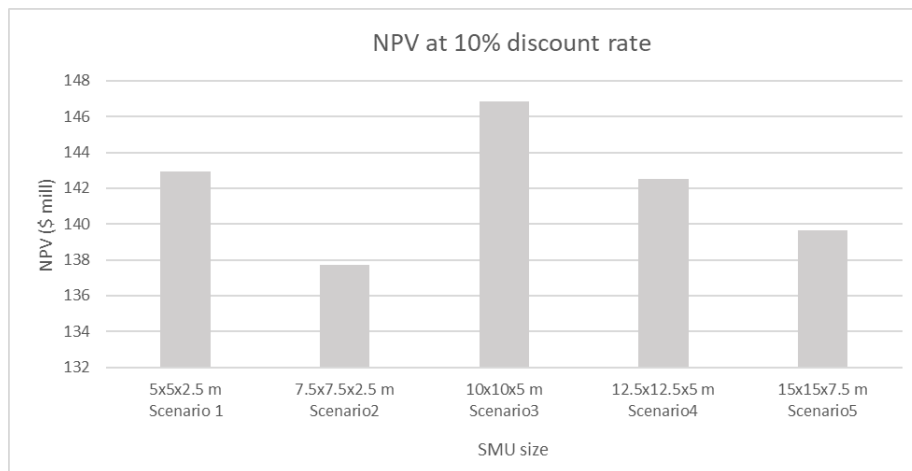


Figure 4.31 NPV for each SMU size when the cost of selectivity and variable dilution are considered

In the Figure 4.31, it is clear that when considering the cost of selectivity and dilution factor, the optimal SMU size is no longer the 5x5x2.5m but the 10x10x5m. Despite the grade and the produced ounces are 13% and 1% respectively lower than that of the scenario 1 (5x5x2.5m), the associated

mining efficiencies and the variable dilution influence the increase of the NPV with the scenario 3 (10x10x5m size).

4.8 Defining the Optimal Smallest Mining Unit Using the AHP

Selection of the optimal SMU size is one of the most critical activities in mining planning. The fact that the final economic result of the project is to some extent influenced by the SMU size, it is imperative to select an SMU that best fits the deposit characteristics. The primary purpose is to maximise the NPV while providing a safe environment for workers and the surrounding community.

4.8.1 Identification of alternatives and hierarchical structure

Mine planners consider numerous factors when selecting the appropriate SMU block size. In addition to the pit optimisation result from the block economic model, the capacity of the machinery and extraction parameters are also taken into account. In section 4.7.5, scenario 1 (5x5x2.5m block size) yield the largest NPV, however, when extraction related parameters are considered, scenario 1 yields less tonnage compared to larger SMU block size. The reduced production output is alluded to the effect of selectivity when mining smaller blocks. This shows that some of the factors used in the selection of the SMU size are conflictual. Hence, a scientific and efficient approach is required to weight the available options. In this study, the analytical hierarchy process (AHP) method was used to analyse several criteria by considering their different relative weights.

Identification of Decision Criterion.

There are many criteria that influences selection of the SMU size. In this study, eight main criteria were selected to determine the optimal SMU size for the Twangiza North deposit; these criteria are:

- Dilution factor (C1);
- Equipment productivity due to selectivity (C2);
- Stripping ratio (C3);
- Environmental impact (C4);
- Average mining grade (C5);
- Ore tonnes above the cut-off grade (C6);
- Life of mine (C7); and
- NPV (C8).

Other criteria that influence the selection of SMU size relating to the geostatistical parameters (such as kriging efficiency and slope of regression) and the size of the size of the extraction equipment (such as the capital expenditure, fleet maintenance cost, fume and dust generation) are not used in the AHP model due to time constraint for this study and the complexity in building an AHP model with an increased number of criterion.

Parameters C3 to C8 are the outcomes of pit optimisation obtained from Whittle software. Parameters C4, the environmental criterion, is more defined from the extensiveness of the waste dump, thus dependent on the quantity of waste generated. The parameter C2, the equipment productivity,

is estimated from the mine data record. Parameters C1, C3, and C4 are criteria with negative impact; their preference in the selection decreases when their values increase.

To determine the optimal block size, SMU sizes of 5x5x2.5m (A1), 7.5x7.5x2.5 m(A2), 10x10x5m (A3), 12.5x12.5x5m (A4) and 15x15x7.5m (A5) were used as alternatives in the construction of the AHP models. Table 5.13 shows the decision matrix for the AHP. The problem of selecting the SMU size is then constructed in a hierarchical structure is shown in Figure 4.32, while the flowchart for the process is depicted by the Figure 4.33.

Table 4.13 Decision matrix of selection of optimal SMU for Twangiza by AHP approach

	C1: Dilution (%)	C2: Productivity (t/day)	C3: Stripping ratio (t/t)	C4: Environnemental Impact (Mt)	C5: grade (g/t)	C6: Run of Mine (Mt)	C7: Life of mine (Year)	C8: NPV (M\$)
5x5x2.5 (A1)	7	26.0	7.59	47.85	2.8	63.06	5.21	177.98
7.5x7.5x3.75 (A2)	4	29.0	6.69	43.40	2.5	64.93	5.02	151.37
10x10x5 (A3)	3	32.0	6.52	43.67	2.42	67.00	5.13	143.83
12.5x12.5x6.25 (A4)	3	33.5	6.09	39.76	2.31	65.34	4.61	134.83
15x15x7.5 (A5)	3	35.0	5.80	37.36	2.25	64.43	4.64	126.61

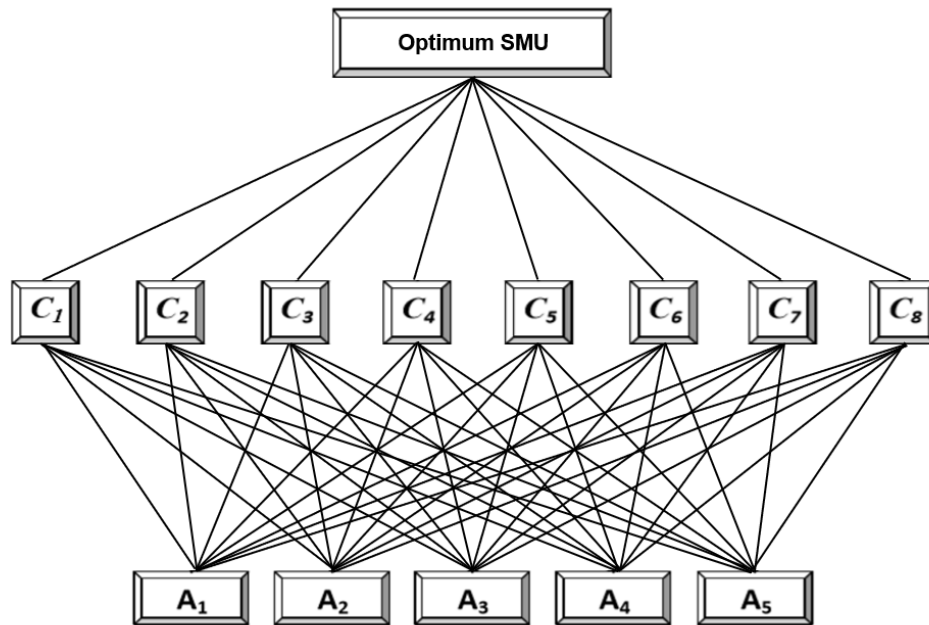


Figure 4.32 Hierarchy structure of selection of optimal SMU for Twangiza Mine using AHP approach (Adapted from Hayati, *et al.*, 2015)

The Figure 4.32 shows that the hierarchy of SMU size selection can be divided into three levels: The main goal of the hierarchy is presented in level 1 while level 2 and level 3 contains respectively the main criteria and the alternatives representing the five different SMU sizes.

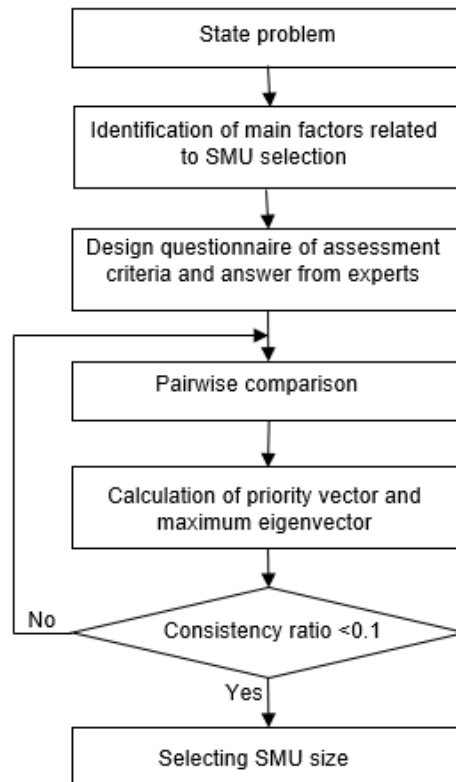


Figure 4.33 Flowchart of selection of optimal SMU using AHP approach

4.8.2 Questionnaire design and results of the survey

Questionnaire design, study population and Sample were discussed in section 3.4.2. A total of 10 out of 16 participants responded to the questionnaire and are all involved in the mining planning process within the Banro Mining Group. The Results of the survey is given in Table 4.14.

Table 4.14 The results of the survey

Criteria		Average	Standard deviation
1	Dilution	2.2	0.79
2	Productivity	3.9	0.99
3	Strip ratio	2.9	0.88
4	Environmental Impact	2.1	0.99
5	Grade	3.7	0.82
6	Ore tonnes	3.1	0.88
7	Life of Mine	3.1	0.88
8	NPV	4.5	0.71

4.8.3 Pairwise comparison

After the hierarchical structure is defined, the comparison of factors is made. Each factor is evaluated against another to measure their influence on the main objective at the same level. A comparison matrix constructed by comparing pairs of the selected criterion to define the SMU is presented in Table 4.15. The discounted net present value of the pit was rated as a significant factor, while dilution and environment impact were rated as least critical.

Table 4.15 Pairwise comparison matrix for SMU selection

	C1	C2	C3	C4	C5	C6	C7	C8
C1	1.00	0.50	0.50	1.00	0.50	0.50	0.50	0.33
C2	2.00	1.00	2.00	2.00	1.00	2.00	2.00	0.50
C3	2.00	0.50	1.00	2.00	0.50	1.00	1.00	0.50
C4	1.00	0.50	0.50	1.00	0.50	0.50	0.50	0.33
C5	2.00	1.00	2.00	2.00	1.00	2.00	2.00	0.50
C6	2.00	0.50	1.00	2.00	0.50	1.00	1.00	0.50
C7	2.00	0.50	1.00	2.00	0.50	1.00	1.00	0.50
C8	3.00	2.00	2.00	3.00	2.00	2.00	2.00	1.00

4.8.4 Priority vector and the principal eigenvalue

To determine the relative importance of the criterion, the priority vector (eigenvector) and the principal eigenvalue, $\lambda_{\max}$ of the comparison matrix were calculated in Microsoft Excel (see Table 4.16). The approximation method was used as follows:

- Step 1: Sum the values in each column of the matrix;
- Step 2: Normalise the judgements by dividing each element of the matrix by the sum of values in each column to normalise the judgements;
- Step 3: Create a new normalised matrix from the normalised values;
- Step 4: Determine the priority vector that is the average of the values in each row of the normalised matrix; and lastly
- Step 5: Calculate the principal eigenvalue by summing the products between each element of the priority vector and the sum of the matrix columns.

Table 4.16 The normalisation of the matrix

	C1	C2	C3	C4	C5	C6	C7	C8	eigenvector	λ
C1	0.07	0.08	0.05	0.07	0.08	0.05	0.05	0.08	0.06	8.14
C2	0.13	0.15	0.20	0.13	0.15	0.20	0.20	0.12	0.16	8.21
C3	0.13	0.08	0.10	0.13	0.08	0.10	0.10	0.12	0.11	8.10
C4	0.07	0.08	0.05	0.07	0.08	0.05	0.05	0.08	0.06	8.14
C5	0.13	0.15	0.20	0.13	0.15	0.20	0.20	0.12	0.16	8.21
C6	0.13	0.08	0.10	0.13	0.08	0.10	0.10	0.12	0.11	8.10
C7	0.13	0.08	0.10	0.13	0.08	0.10	0.10	0.12	0.11	8.10
C8	0.20	0.31	0.20	0.20	0.31	0.20	0.20	0.24	0.23	8.18
Sum/ Average	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	$\lambda_{\max} =$ 8.15

4.8.5 Consistency measurement

The principal eigenvalue $\lambda_{\max}$ of the matrix is used to measure the consistency of the judgement. The principal eigenvalue $\lambda_{\max}$ of 8.15 obtained implies that the decision is not consistent because $\lambda_{\max}$ is greater than the number of criteria ($\lambda_{\max} > n$). The consistency ratio (CR) is then calculated to determine whether the degree of inconsistency is tolerable. The CR is derived from the consistency index (CI) and the random consistency index (RI). CI is obtained according to Equation 21.

$$CI = \frac{(\lambda_{\max} - n)}{(n - 1)} \quad (21)$$

$$CI = \frac{(8.15 - 8)}{(8 - 1)} = 0.02$$

With a random consistency index of 1.41 for a matrix of order 8 (RI=1.41), the consistency ratio of 0.01 was calculated using Equation 22 and it is less than the recommended 0.1. Therefore, the pair-wise judgement is acceptable.

$$CR = \frac{CI}{RI} \quad \text{Equation 22}$$

$$CR = \frac{0.02}{1.41} = 0.01$$

4.8.6 Information synthesis

The synthesis of the information is done after the priority vector has been determined. The method is as follows:

- Step 1: Normalise the outcomes of all the alternatives (A) for each criterion (see Table 4.17). Note that parameters C2, C4, and C7 are criteria with negative impact, their values have been multiplied by 10^{-1} to keep all the alternatives with positive impact;
- Step 2: Multiply the normalised alternative-criterion matrix with the eigenvalue to produce the priority scores (see Table 4.18); and
- Step 3: Rank the alternatives based on the priority score (Figure 4.34).

Table 4.17 Normalised decision matrix

SMU size (m)	C1: Dilution (%)	C2: Productivity (t/day)	C3: Stripping ratio (t/t)	C4: Environmental Impact (Mt)	C5: grade (g/t)	C6: Run of Mine (Mt)	C7: Life of mine (Year)	C8: NPV (M\$)
5x5x2.5	0.10	0.17	0.17	0.18	0.23	0.19	0.21	0.24
7.5x7.5x3.75	0.18	0.19	0.19	0.19	0.20	0.20	0.20	0.21
10x10x5	0.24	0.21	0.20	0.19	0.20	0.21	0.20	0.20
12.5x12.5x6.25	0.24	0.22	0.21	0.21	0.19	0.20	0.20	0.18
15x15x7.5	0.24	0.23	0.22	0.23	0.18	0.20	0.19	0.17
Sum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 4.18 Raking of the SMU sizes based on the priority score

	SMU size (m)	Priority score (%)	Rank
Scenario 1	5x5x2.5	19.86	4
Scenario 2	7.5x7.5x2.5	19.78	5
Scenario 3	10x10x5	20.28	1
Scenario 4	12.5x12.5x5	20.04	2
Scenario 5	15x15x7.5	20.04	3

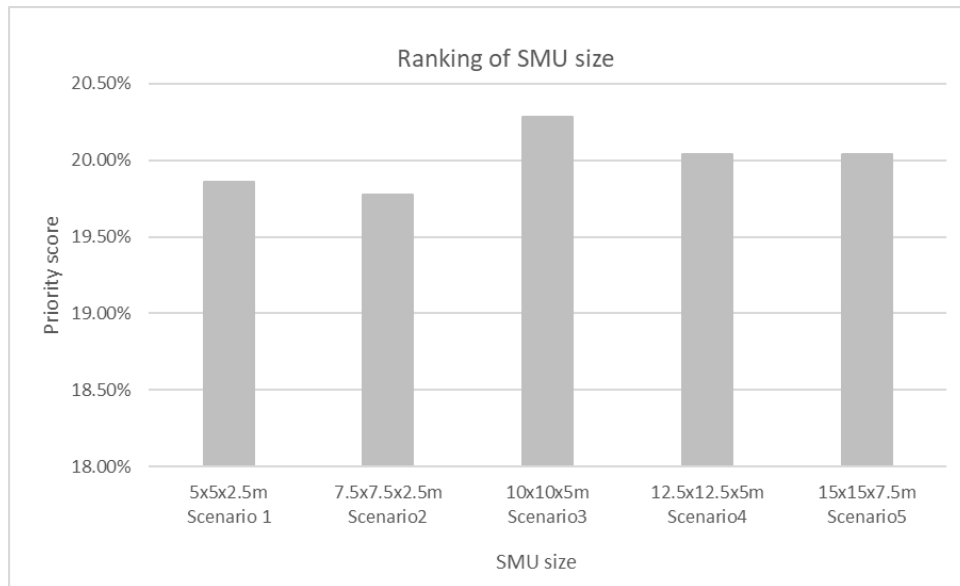


Figure 4.34 Ranking of SMU size for the five scenarios

From Figure 4.34, scenario 3 (i.e. 10x10x5m SMU size) with a rating of 20.28% is most preferred and is followed by 12.5x12.5x5, 15x15x7.5, 5x5x2.5 and 7.5x7.5x2.5, respectively, in order of preference.

4.9 Chapter 5 Summary

In the fourth chapter the Twangiza orebody was used as case study to assess the impact of the SMU size in ultimate pit design and then select the SMU that best suit the Twangiza orebody. It was noticed that pit shell and yield NPV of the smaller SMU (5x5x2.5m) were 29% and 19% larger than the largest SMU (15x15x7.5m). The main reason was that when larger blocks are used, the dilution of the grade increases, hence the monetary value drops.

A set of sensitivity studies was carried on cut-off grades, dilution factors and cost of selectivity. It was observed that when extracting low-grade ore, larger

SMU are suitable. Otherwise when the smaller blocks of low grade are extracted, a high degree of mining selectivity is required to minimise ore dilution.

Considering the cost of selectivity and the mining factors associated with the different SMU sizes, the NPV of 5x5x2.5m dropped and the suitable SMU was the 10x10x5m.

Lastly, AHP was used as a tool to select the best SMU size. Eight main criteria were selected to determine the optimal SMU size for the Twangiza North deposit. A questionnaire with a five-point scale for each of the eight factors relating to the SMU selection was designed to rank their respective importance. A principal eigenvalue $\lambda_{\max}$ of 8.15 was obtained implying that the decision was not consistent ($\lambda_{\max} > n$). The consistency ratio (CR) was then calculated to determine whether the degree of inconsistency was tolerable. This was below the recommended 0.1, showing that the pair-wise judgement was acceptable. The 10x10x5m SMU came with a rating of 20.28% and was the most preferred SMU size.

The final chapter five will provide the final conclusion of the entire research report and some major findings with regards to the SMU size selection. Recommendations, limitations of the study and further research works will be also presented.

5 CONCLUSIONS AND RECOMENDATIONS

The selection of the ultimate SMU size during surface mine planning is a complex process because there are multiple variables that influence the choice of the SMU size. The following conclusions can be made from the selection of ultimate SMU in this study.

A. During resource modelling:

- The geostatistics tools help to define the suitable block size for resource modelling. This size is often the half of the drilling space but should also consider other geological parameters such as the thickness of the smallest modelled unit;
- The mining parameters are used to determine the final SMU for mining planning. Often, this SMU size is different from the one used for modelling; hence, reblocking is done;
- The reblocking has an impact on the average grade and the tonnage above the cut-off grade;
- As the block gets larger, the average mining grade, the tonnage above the cut-off and the metal content drop;
- When the cut-off grade increases, the discrepancy in tonnages for the different SMU sizes is pronounced, showing that the effect of SMU is higher on blocks of a high grade than for the blocks of low grade;
- The mining grade decreases on the low-grade portion as the block gets larger but the tonnage above the cut-off increases. This shows

that when mining blocks of low grade, a high degree of selectivity is required; and

- The dilution factor varies with the SMU sizes. As the SMU get larger, the contact surface between ore and waste reduced, hence the variable dilution decreases.

B. During the optimisation of the pit limit:

- The optimised pit shell shows that the in situ recoverable grade drops when the SMU increases. The tonnage above cut-off increases gradually to a peak and starts dropping with the increase of the SMU;
- The small SMU size (5x5x2.5m) yields a larger pit shell with a higher strip ratio of 7.59;
- The NPV decreases with the increase of SMU size;
- The dilution and selectivity vary with the changes in the SMU size. As the block gets larger, the dilution factor and the selectivity decrease; and
- When the effect of selectivity and dilution are factored in the optimisation process of the pit shell, the 10x10x5m yields the maximum NPV of US\$145.85 million in place of the 5x5x2.5m (US\$142.95 million).

C. During the optimisation of mine operations:

- When the mining operation parameters are considered, the choice of the optimal SMU size becomes complex;

- A multi-criteria decision-making tool is required. In this study, the AHP method was selected; and
- Eight criteria were selected to determine the optimal SMU size for Twangiza Mine using the AHP method. The 10x10x5m SMU had the highest rating of 20.28 and it was selected as the preferred SMU.

5.1 Recommendations

The purpose of the study was to determine the effect of the SMU on mine planning and optimisation, and to investigate whether the SMU size used at Twangiza Mine for mine planning and optimisation was optimal. Although the study confirms that the SMU of 10x10x5m used at Twangiza is the optimal SMU, the mine planners should use a scientific method to determine such vital factors in the mine optimisation. Considering the multiple criteria that influence the choice of the SMU size, the AHP is a perfect tool to assist in decision making. Twangiza mine should also use the implicit modelling to overcome the shortfalls of the traditional explicit modelling, and adopt the variable dilution instead of the fixed dilution which may provide a suboptimum result during the mining planning process.

5.2 Limitations of the Study and Further Research

The research was unable to embrace all the parameters that influence the selection of SMU due to the time constraint and the availability of data. Therefore, further research is required to embrace those aspects that the student was not able to integrate in this study. These are:

- The geostatistical parameters such as kriging efficiency and slope of regression during resource modelling;
- The mining schedule profile from the different SMU sizes;
- The capital expenditure: The size of the SMU will dictate the size of equipment to be used for extraction, hence, the capital expenditure will be affected by SMU size employed;
- The operating cost: It is plausible that the operating and maintenance costs will be related to the size of the mining equipment; and
- The environmental impacts: The study was limited to the waste dump foot-print as environmental impact. However, a more detailed aspect can be considered knowing that the air pollution from the engine fume and dust generation may vary with the size of the equipment used.

REFERENCE

- Amir, R., Morales, N. and Cceres, A., 2018. Analysis of the Impact of the Dilution on the Planning of Open-Pit Mines for Highly Structural Veined-Shaped Bodies. *Proceedings of the 27th International Symposium on Mine Planning and Equipment Selection - MPES, 2018*, pp. 187-196.
- Appianing, E. J. A. and Mireku-Gyimah, D., 2015. Open Pit Optimisation and Design: A Stepwise Approach. *Ghana Mining Journal*, Vol. 15, No.2, pp. 27-35.
- Ataee-pour, M., 2005. A Linear Model for Determination of Block Economic Values. *The 19th International Mining Congress and Fair of Turkey, IMCET,2005, Izmir, Turkey*, pp. 289-294.
- Balt, K., 2015. *A Methodolgy for Implementing the Analytical Hierarchy Process to Decision-Making in Mining*, MSc report, School of Mining Engineering, University of the Witwatersrand.
- Birch, C., 2014. New Systems for Geological Modelling–Blackbox or Best Practice?, *The Journal of The Southern African Institute of Mining and Metallurgy*, Vol. 114, No.12, pp. 993-1000.
- Birch, C., 2019. Optimisation of Mining Block Size for Narrow Tabular Gold Deposits Tabular Gold Deposits, *Proceedings of the 27th International Symposium on Mine Planning and Equipment Selection - MPES 2018*, pp 121–141

- Burns, N. and Grove, S.K., 1993. The practice of nursing research: Conduct, critique and utilisation, 2nd ed. W.B. Saunders, Philadelphia, IL.
- Büttner, S. H., Reid, W.K., Glodny, J., and Wiednbeck, M., 2016. Fluid Sources in the Twangiza–Namoya Gold Belt (Democratic Republic of Congo): Evidence from Tourmaline and Fluid Compositions, and from Boron and Rb–Sr Isotope Systematics. *Precambrian Research*, Vol. 280, pp. 161-178.
- Caccetta, L. (2007). Application of Optimisation Techniques in Open Pit Mining. In: Weintraub, A., Romero, C., Bjørndal, T., Epstein, R., Miranda, J. (eds) *Handbook Of Operations Research In Natural Resources*. International Series In Operations Research amp; Mana, vol 99, pp 547–559, Springer, Boston, MA.
https://doi.org/10.1007/978-0-387-71815-6_29
- Caccetta, L. & Hill, S.P., 2003. An Application of Branch and Cut to Open Pit Mine Scheduling. *Journal of Global Optimisation*, Vol. 27, No. 2-3, pp. 349–365 (2003). <https://doi.org/10.1023/A:1024835022186>
- Consuegra, F. and Dimitrakopoulos, R., 2009. Stochastic Mine Design Optimisation Based on Simulated Annealing: Pit Limits, Production Schedules, Multiple Ore Body Scenarios and Sensitivity Analysis. *Institute of Materials, Minerals and Mining*, Vol. 118, No.2, pp. 79-90.
- Coyle, G., 2004. *The Analytic Hierarchy Process, Practical Strategy, Open Access Material*. London: Pearson Education Ltd.

- Creswell, J. W., 2009. *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches*. 3rd ed. Thousand Oaks: CA: Sage Publications.
- Deacon, J., 2017. Explicit and Implicit Geological Modelling Methods on Resource Definition and Resource Utilisation–Sishen Iron Deposit Case Study, *MSc report*, School of Mining Engineering, University of the Witwatersrand.
- Deutsch, C.V., and Journel, A.G., 1997. *GSLIB: Geostatistical Software Library and User's Guide*. 2nd ed. New York: Oxford University Press.
- Ebrahimi, A., 2013. The Importance of Dilution Factor for Open Pit Mining Projects, Available from:
<https://www.srk.com/en/publications/importance-of-dilution-factor-for-open-pit-mining-projects>
- Ebrahimi, A., 2018. *Mining Dilution: Variable versus Fixed Factor*. Minneapolis, SME Annual Conference and Expo, Available from:
<https://www.srk.com/en/publications/mining-dilution-variable-versus-fixed-factor>
- Elahizeyni, E., Kakaie, R. and Yousefi, A., 2011. A new algorithm for optimum open-pit design: Floating cone method III. *Journal of Mining & Environment*, Vol. 2, No.2, pp. 118-125
- Fallara, F., Legault, M. and Rabeau, O., 2006. 3-D Integrated Geological Modeling in the Abitibi Subprovince (Québec, Canada): Techniques

and Applications. *Exploration and Mining Geology*, Vol. 15, No.1, pp. 27-43. <https://doi.org/10.2113/gsemg.15.1-2.27>

Fall, H., 2008. *Overview of the Geology, Gold mineralisation and Resource Potential*. RD Congo: Banro Congo Mining SARL.

Gholamnejad, J. and Mojahedfar, A.R., 2010. Determination of the largest pit with the non-negative net profit in the open-pit mines. *Journal of Mining and Environment*, Vol. 1, No.2, pp. 45-52.

Hayati, M., Rajabzadeh, R. and Darabi, M., 2015. Determination of Optimal Block Size in Angouran Mine Using VIKOR Method. *Journal of Materials and Environmental Science*, Vol. 6, No. 6, pp. 3236-3244.

Hustrulid, W.A., Kuchta, M. and Martin, R.K., 1995. *Open pit mine planning & design- Vol. 1*, 3rd ed. Florida: CRC Press.

Jara, R.M., Coubelt, A., Emery, X., Magri, E.J., and Ortiz, J.M., 2006. Block size selection and Its impact on open-pit design and mine planning. *The Journal of The Southern African Institute of Mining and Metallurgy*, Vol. 106, No. 3, pp. 205-212.

Kaan Erarslan (2012). *Computer Aided Ore Body Modelling and Mine Valuation*, In: Earth Sciences, Dr. Imran Ahmad Dar (Ed.), ISBN: 978-953-307-861-8, InTech, pp. 197-207, Available from: <http://www.intechopen.com/books/earth-sciences/computer-aided-ore-body-modelling-and-mine-valuation>

- Kapageridis, I. & Denby, B., 1998. Neural network modelling of ore grade spatial variability. Neural network modelling of ore grade spatial variability, In: Proceedings of the International Conference for Artificial Neural Networks (ICANN), Vol. 1, Springer-Verlag, Skovde.
- Khalokakaie, R., Dowd, P.A., and Forwell, R.J., 2000. Lerchs-Grossman algorithm with variable slope angles. *Mining Technology*, vol. 109, No.2, pp. 77-85.
- Khan, A. and Niemann-Delius, C., 2014. Production Scheduling of Open Pit Mines Using Particle Swarm Optimisation Algorithm. *Advances in Operations Research*, Vol. 2014.
- Khodayari, A.A., 2013. A New Algorithm for Determining Ultimate Pit Limits Based on Network Optimisation. *International Journal of Mining and Geological Engineering*, Vol. 47, pp. 129-137.
- Khorram, F., Asghari, O. and Memarian, H., 2017. Change of support based on modified localised uniform conditioning and direct block multivariate simulation on the Sungun porphyry deposit. *Iran, Bollettino di Geofisica Teorica ed Applicata* Vol. 58(2), pp. 115-136.
- Kumral, M. and Dowd, P., 2005. A simulated annealing approach to mine production scheduling. *Journal of the Operational Research Society*, Vol. 56, No. 8, pp. 922-930.
- Lerchs, H. and Grossman, L., 1965. Optimum Design of Open-Pit Mines. *Canadian Mining and Metallurgy Bulletin*, Vol. 58, pp. 47-54.

Mahdi, R. and Morteza, O., 2014. Determining the Most Effective Factors on Open Pit Mine Plans and Their Interactions. In: *Mine Planning and Equipment Selection*. Switzerland : Springer International Publishing, pp. 197-207.

Matheron, G., 1971. The theory of regionalised variables and its applications, Les cahiers du Centre de Morphologie Mathematique, Ecole superieure nationale des mines, Fontainebleau, Paris.
Available from:
http://cg.ensmp.fr/bibliotheque/public/MATHERON_Ouvrage_00167.pdf

Musingwini, C. 2010. A review of the theory and application of multi-criteria decision analysis techniques in mine planning. *Proceedings of Mine Planning and Equipment Selection (MPES) 2010, Australasian Institute of Mining and Metallurgy*, Melbourne. pp. 129–140.

Musingwini, C., 2016. Presidential Address: Optimisation in Underground Mine Planning-Developments and Opportunities. *The Journal of the Southern African Institute of Mining and Metallurgy*, Vol. 116, No. 9, pp. 809-820.

Musingwini, C. and Minnit, R.C.A., 2008. Ranking the efficiency of selected platinum mining methods using the analytic hierarchy process (AHP). *Third International Platinum Conference 'Platinum in Transformation'*. The Southern African Institute of Mining and Metallurgy, pp.319-326.

Neito, A., 2010. Key Deposit Indicators (KDI) and Key Mining Method Indicators (KMI) in Underground Mining Method Selection.

Transactions of the Institution of Section A: Mining Technology, Inc, Littleton, Co, Vol. 328, pp. 381-396.

Nezamolhosseini, S., Mojtahedzadeh, S. & Gholamnejad, J., 2017. The application of artificial neural networks to ore reserve estimation at Choghart iron ore deposit. *Analytical and Numerical Methods in Mining Engineering*, Vol.6, pp. 73-83.

CIM Standing Committee on Reserve Definitions, 2015. Definition Standards for Mineral Resources & Mineral Reserves, Available from: https://mrmr.cim.org/media/1128/cim-definition-standards_2014.pdf

Nieto, A. and Bascetin, A., 2007. Determination of optimal cutoff grade policy to optimize NPV using a new approach with optimisation factor. *The Journal of The Southern African Institute of Mining and Metallurgy*, Vol. 107, No.2, pp. 87-94.

Nieto, A. and Zhang, K.Y., 2013. A Cutoff grade economic strategy for a byproduct mineral commodity operation: A rare Earth case study. *Mining Technology Journal*, 122(03), pp. 166-171.

Nogholi, M., and Alah. A., 2015. *Optimisation of open pit mine block sequencing, PhD Thesis*, Australia: Queensland University of Technology.

Rocca, F., Sebbag, M. and Taimre, T., 2007. *Buzwagi Open Pit Study. Open Pit Mining Conference 2007*. Perth, The Australian Institute for Mining and Metalurgy, pp. 119-128.

- Saaty, R., 1987. The analytic hierarchy process—what it is and how it is used. *Mathematical Modelling*, vol. 9(3-5), pp. 161-176.
- Sameer, A. A., Nandagopan, O.R., and Dhiwakar, V. M., 2013. Application of Analytic Hierarchy Process for Optimal Design Decisions in Product Development. *International Journal on Theoretical and Applied Research in Mechanical Engineering (IJTARME)*, Vol. 2, No. 3, pp. 47-54.
- Sari, Y.A., 2014. *Mine production scheduling through heuristic memory based, improved simulated annealing*, MEng degree, Canada: McGill university, Montreal, Quebec.
- Sen, Z., 2009. *Spatial Modeling Principles in Earth Sciences*. New York: Springer.
- Shahriar, K., Oraee, K. and Bakhtavar, E., 2007. A Study of the Optimisation Algorithms for Determining Open-Pit and Underground Mining Limits. Bulgaria, VII-th International Scientific Conference, SGEM.
- Sinclair, A.J. and Blackwell, G., 2002. *Applied Mineral Inventory Estimation*. United Kingdom: Cambridge University Press.
- Steffen, O. K. H., Contreras, L. F., Terbrugge, P.J. and Venter, J., 2008. A Risk Evaluation Approach for Pit Slope Design. *42nd U.S. Rock Mechanics - 2nd U.S Canada Rock Mechanics Stmposium*.

- Sullivan, T. D., 2006. Pit Slope Design and Risk - A view of the Current State of the Art. *The International Symposium on Stability of Rock Slopes in Open Pit Mining and civil Engineering*, pp. 51-80.
- Tack, L., Wingate, M.T.D, De Waeles, B., Meert, J., Belonsova, B., Griffin, A., Tahon, M., Fernandez-Alonso, 2010. The 1375 Ma “Kibaran Event” in Central Africa: Prominent emplacement of bimodal magmatism under extensional regime, Vol. 180, No. 1-2, pp.63-84.
- Thobaven, D. and Willoughby, J., 2012. *Strategic Mine Planning in Whittle 4.5 and Surpac v6.3*. Perth: Gemcom Software Australia Pty Ltd.
- Twangiza Mining, 2015. *Twangiza Reserve Technical Report*, Ontario.
Available from: <https://minedocs.com/22/Twangiza-TR-07292015.pdf>
- Walemba, K.M.A, 2001. Geology, Geochemistry and Tectono-Metalogenic Evolution Neoproterozoic Gold Deposits in the Kadubo Area. *Kivu, Democratic Republic of Congo, PhD*, Johannesburg: University of Witwatersand.
- Walemba, K.M.A and Master, S., 2005. Neoproterozoic Diamictites from the Itombwe Synclinorium, Kivu Province, Democratic Republic of Congo: Palaeoclimatic significance and regional correlations. *Journal of African Earth Sciences*, Vol. 42 (2005), pp. 200-210.
- Watson, D.F. and Philip, G.M., 1985. A Refinement of Inverse Distance Weighted Interpolation. *Geoprocessing*, Vol. 2, No. 4, pp. 315-327.
- Webster, R. and Oliver, M.A., 2007. *Geostatistics for Environmental Scientists*. 2nd ed. England: John Wiley & Sons Inc.

Appendix 1 Incremental pit values for 5x5x2.5 SMU

Pit Number	Best Case NPV (\$)	Increase in NPV
1	34,346,740	1,821,099
2	36,167,839	2,080,032
3	38,247,871	2,692,977
4	40,940,848	3,495,507
5	44,436,355	1,997,341
6	46,433,696	469,981
7	46,903,677	5,065,898
8	51,969,575	1,628,668
9	53,598,243	5,061,201
10	58,659,444	3,714,986
11	62,374,430	3,059,252
12	65,433,682	5,403,875
13	70,837,557	1,741,656
14	72,579,213	10,001,555
15	82,580,768	24,046,972
16	106,627,740	970,288
17	107,598,028	6,440,356
18	114,038,384	4,925,651
19	118,964,035	1,259,029
20	120,223,064	2,755,196
21	122,978,260	5,245,373
22	128,223,633	3,974,385
23	132,198,018	3,249,424
24	135,447,442	1,510,480
25	136,957,922	3,394,092
26	140,352,014	3,837,658
27	144,189,672	994,010
28	145,183,682	29,749,635
29	174,933,317	4,208,157
30	179,141,474	5,203,514
31	184,344,988	4,012,494
32	188,357,482	1,057,556
33	189,415,038	1,983,766
34	191,398,804	1,745,344
35	193,144,148	2,025,923
36	195,170,071	2,135,450

37	197,305,521	7,848,979
38	205,154,500	13,166,779
39	218,321,279	634,927
40	218,956,206	230,383
41	219,186,589	-1,926,463
42	217,260,126	-8,032,417
43	209,227,709	-3,245,235
44	205,982,474	-3,064,254
45	202,918,220	-4,833,891
46	198,084,329	-2,017,383
47	196,066,946	-4,160,256
48	191,906,690	-2,190,480
49	189,716,210	-1,640,782
50	188,075,428	-2,714,846

Appendix 2 Ultimate pit result for the different models at different cut-off grades

Cut-off= 0.5g/t					
Block Size	5x5x2.5	7.5x7.5x3.75	10x10x5	12.5x12.5x6.25	15x15x7.5
Average grade above the cut-off	2.61	2.26	2.19	2.04	1.96
Tonnes above the cut-off	6,875	7,383	7,570	7,625	7,776
NPV (\$ mil)	174.97	145.98	140.03	125.14	116.73
Strip ratio	6.79	5.59	5.36	4.80	4.55
Life of mine	5.26	5.14	5.18	5.03	5.08
Cut-off= 1g/t					
Block Size	5x5x2.5	7.5x7.5x3.75	10x10x5	12.5x12.5x6.25	15x15x7.5
Average grade above the cut-off	2.90	2.59	2.53	2.42	2.34
Tonnes above the cut-off	6,003	6,158	6,141	6,046	6,017
NPV (\$ mil)	177.88	151.18	146.04	132.89	126.71
Strip ratio	8.01	7.10	6.90	6.62	6.28
Life of mine	5.14	4.93	4.84	4.61	4.51

Cut-off= 2g/t					
Block Size	5x5x2.5	7.5x7.5x3.75	10x10x5	12.5x12.5x6.25	15x15x7.5
Average grade above the cut-off	3.61	3.45	3.39	3.29	3.24
Tonnes above the cut-off	3,929	3,480	3,374	2,178	2,158
NPV (\$ mil)	156.35	121.33	115.08	123.16	112.91
Strip ratio	11.77	12.30	12.20	7.12	7.71
Life of mine	4.50	3.96	3.82	1.63	1.73
Cut-off= 3g/t					
Block Size	5x5x2.5	7.5x7.5x3.75	10x10x5	12.5x12.5x6.25	15x15x7.5
Average grade above the cut-off	4.56	4.45	4.36	4.35	4.20
Tonnes above the cut-off	1,469	1,199	1,141	1,001	1,015
NPV (\$ mil)	126.28	95.35	88.01	76.31	73.48
Strip ratio	11.44	12.67	12.52	12.84	12.51
Life of mine	1.56	1.40	1.32	1.19	1.17
Cut-off= 4g/t					
Block Size	5x5x2.5	7.5x7.5x3.75	10x10x5	12.5x12.5x6.25	15x15x7.5
Average grade above the cut-off	5.54	5.38	5.32	5.14	5.13
Tonnes above the cut-off	802	570	535	508	438
NPV (\$ mil)	81.72	63.55	56.69	52.44	43.79
Strip ratio	17.56	13.98	14.95	13.91	14.80
Life of mine	1.27	0.73	0.73	0.65	0.59