CREATING OPPORTUNITIES TO LEARN THROUGH RESOURCING LEARNER ERRORS ON SIMPLIFYING ALGEBRAIC EXPRESSIONS IN GRADE 8

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Date: 5 June 2017

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A Research Report submitted to the Wits School of Education, Faculty of Science, University of Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Education.

Declaration

I declare that a Research Report hereby handed in for the qualification for the degree of Master of Science in Mathematics Education at the University of Witwatersrand, South Africa is my own unaided work and that all the sources I have used and quoted are indicated and acknowledged by means of complete references.

Olinah Matuku

24 March 2017

Abstract

This research problematised the teaching and learning of the grade 8 topic of simplifying algebraic expressions via the errors and misconceptions learners' show on that topic. The study conducted at a secondary school in Johannesburg identified the nature of grade 8 learners' errors and misconceptions on simplifying algebraic expressions. A teaching intervention through using those errors as resource to help learners reduce them was undertaken. There was an implementation of discovery learning as an intervention strategy to help learners to explore algebraic concepts with the minimum involvement of the researcher. The researcher used constructivism, sociocultural learning and variation theories since these theories affect the learners learning of algebra. The researcher used an interpretive paradigm which is concerned about the individuals' interpretation of the world around them. Purposive and convenience sampling were used in the study. Data was collected using a sample of thirty grade 8 learners. The learners wrote a pre-test as one of the assessment task in the study. The purpose of the pre-test was to identify learners' errors on simplifying algebraic expressions. After the learners' errors were identified and analysed, the researcher conducted a semi-structured, focus group interview with six learners in the study. The selection of the interviewees depended on the type and frequency of errors they have displayed in their pre-test scripts. The purpose of the interview was to investigate the reasons behind the learners' errors as identified in the pre-test. An intervention strategy which implemented guided discovery learning was employed to learners with the use of the identified errors as a resource to help learners reduce them. After the intervention, the learners wrote a post-test to check if there was an improvement in learners' performance after the intervention. Pre- and post-tests results were analysed for errors revealed by learners. The teaching intervention periods were introduced to create learning opportunities for learners. The findings of the study revealed that before intervention learners encountered a lot of difficulties when simplifying algebraic expressions but the learners' performance improved after the intervention. The recommendations of the study are, teachers should welcome learners' errors in teaching and learning of mathematics and use them as a resource to help learners reduce them in solving mathematical problems

Key words: Learners' errors, misconceptions, simplification of algebraic expressions.

Acknowledgements

- My God, the Almighty, I thank you for your protection, guidance, blessings and strength which you bestowed upon me to finish my research study.

My deepest gratitude goes to the following people:

- My supervisor Dr Makonye for his patience, irreplaceable advices and suggestions. Dr Makonye you never gave up on me during those difficult times when there seemed to be no progress in my research journey. Your help, support, motivation and inspiration for successful compilation of this research study are greatly appreciated.
- The Gauteng Department of Basic Education for permitting me to conduct my research in one of its urban schools in Johannesburg.
- My Family; husband Victor and children Anotida, Anesu and Anashe for their endless love and untiring moral support and encouragement for me to do the best throughout the research study. My dear husband Victor your unconditional love will always be remembered, you stood by me in everything, most of the family responsibilities were yours and you afforded me an opportunity to concentrate on my research. My beloved family all your efforts are valued.
- My parents the late Pheneus and Tereiza Rusinga for their unconditional love, advices, perseverance and sacrifices towards my education.
- My friend Tsitsi Mugwagwa who stood by me through thick and thin and offered me unwavering support and motivation throughout the research study.
- Professor Marrisa, for her financial assistance for me to complete my studies.
- Finally, I would like to thank my principal and my grade 8 mathematics class. These people were amazing, understanding, cooperating and offered unwavering support throughout the study.

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LIST OF ABBREVIATIONS

EFRM	Examination Feedback Resource Material
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
NRC	National Research Council
NSC	National Senior Certificate
ZPD	Zone of Proximal Development

CHAPTER 1: BACKGROUND OF THE STUDY

1.1 Introduction

In South Africa, the Curriculum and Assessment Policy Statement (CAPS) is a National Curriculum Strategy document which has replaced the Subject and Learning Area statement implemented in schools in 2012. CAPS under the Department of Basic Education (DBE) defined algebra as the language for exploring and collaborating Mathematics which can be extended to the study of functions and other associations between variables (DBE, 2011). Algebra is used as a tool for expressing some associations between variables; it is generalised arithmetic which can be referred to in a given situation to solve mathematical problems. Learners start doing pre-algebraic skills in primary school. The formal algebra is introduced in grade 7, this includes algebraic expressions. Learners need to have a strong basic foundation in arithmetic for them to learn algebra with less difficulties because these two topics connect with each other. In preparing for algebraic expression in grade 8, learners are expected to solve problems in context, this involves the addition, subtraction, multiplication and division of directed numbers.

In Senior Phase, CAPS aims at developing learners in grade 7, 8 and 9 to attain mathematical skills in algebra where learners' algebraic skills are improved. According to the DBE (2011) in the mathematics guidelines for grade 8 and 9, learners are expected to recognise and interpret rules, recognise and distinguish symbols for writing algebraic terms, exponents and coefficients to represent and describe situations, interpret and manipulate algebraic expressions (DBE, 2011). Learners are also expected to use commutative, associative and distributive laws for rational numbers to add, subtract, multiply and divide terms in simplifying algebraic expressions (DBE, 2011).

Amerom (2002) distinguishes algebra in four basic perspectives as follows; algebra as comprehensive arithmetic, a problem-solving instrument, a study of relations and constructions. Algebra is a branch of mathematics which deals with symbols and letters to represent relationships between numbers and quantities through the use of some mathematical rules. Kieran (1992) states that algebra is a shift from a reliance on knowledge of numbers and arithmetic operations to a reliance on algebraic operations, rules, and numerical structure. Algebraic terms are general terms/formula representing some mathematical connections between variables, while arithmetic terms are specific numeric values. In algebra learners must be able to differentiate between specific and general situations when solving problems in mathematics.

Algebra is one of the most critical topics in mathematics that develop learners' problem solving and reasoning skills (Schoenfeld, 2007). This is so because algebraic concepts offer learners a very strong foundation in problem solving skills. Learners who are good in algebra are liable to perform well in other areas of mathematics or in other subjects. Learners who understand algebra well possess strong reasoning and analysing skills which they can use in any other mathematics topics including other problem solving situations which are non-mathematical. Algebra takes a great portion of the schools' curriculum since it is an important subject in the learning of mathematics.

Algebra is often described as a doorway to advanced mathematics, it offers the language in which mathematics is taught (Stacey & Chick, 2004). It is vital that learners have a strong elementary understanding of algebraic concepts for them to prosper in any topics/courses in mathematics through the application of strong analysing, justifying and reasoning abilities. Algebra lays a concrete foundation for learners to understand some high order mathematics courses like actuarial science, engineering and medicine in tertiary education. Proficiency in algebra remains vital for understanding any mathematics. Algebra is the cornerstone for mathematical proficiency. By so doing teachers need to do their utmost best to help learners understand algebraic concepts for the learners to do well in other mathematics topics.

Algebra is a significant section of mathematics and yet, "often the first mathematics subject that requires wide-ranging intellectual thinking, a provocative new skill for many students, (Star, Pollack, Durkin, Rittle-Johnson, Lynch, Newton & Gogolen, 2015:1). From my two decades teaching experience and from other researchers done, I have realised that most of the grade 8 and 9 learners in South Africa struggle with algebra, when simplifying algebraic expression. This is the topic the researcher is concerned about in the study. Herscovics and Linchevski (1994) state that learning to operate algebraically is a difficult process that relies on a broad understanding of arithmetic and pre algebraic principles. Understanding of algebra depends on the learners' understanding of arithmetic since these concepts are interrelated. Learners must be able to note some similarities and differences between the new approach of simplifying algebraic expressions and their old approach of simplifying arithmetic expressions. This is important for the new algebraic information to be accommodated into the pre-existing conceptual framework of arithmetic concepts.

Many students at the fundamental level are challenged when moving from arithmetic world to an algebraic world since they cannot understand the construction and outlines of arithmetic (Warren & Cooper, 2003). Some learners in secondary schools struggle with algebra due to lack of basic pre-algebraic (arithmetic) skills. These learners have a very weak knowledge base in algebra; their conceptual and procedural understanding of algebraic concepts is very low and insufficient for them to be able to simplify algebraic expressions. This was supported by Witzel, Mercer and Miller (2003) who state that many students find algebra to be abstract and difficult to comprehend despite its importance in school mathematics curricula. Algebra is a double-edged topic since teachers find it hard to impart to learners and learners find it hard to apprehend. Algebra is a difficult topic to understand because of its complex nature which is sometimes difficult to apply in real life situations.

The difficulty of students' learning of algebra is accredited to the intellectual gap between arithmetic and algebra; students hold misconceptions as they changeover from arithmetic to algebra thinking, (Herscovics &Linchevski, 1994). Arithmetic deals with the manipulation of numbers and algebra deals with symbols and variables and focuses on relationships between terms. The difficult in transition from arithmetic to algebra is supported by Filloy and Rojano (1989) who state that changing from arithmetic thinking to algebraic thinking requires students to understand the concepts of variables. The complexity of algebra is due to the use of symbols which are not specified but generalised and representing unknown relations. For instance learners struggle to simplify expressions like $5x-4y-3y-3z+2x$ because all variables x, y and z are representations of unknown quantities. . Mathematics literacy learners have a better understanding of the topics involving some representations and relationships because their variables are specified unlike in algebra.

Amerom (2002) states that students struggle to obtain an operational idea of algebra which is basically different from an arithmetic viewpoint. In the transition from arithmetic to algebra learners need to change their mind set since the two concepts are not exactly the same. If learners do not change their way of thinking and handle algebraic concepts in the same way they did in arithmetic this can be misleading when it comes to the manipulation of algebraic expressions. For instance in arithmetic $6+5 = 11$ but $6a+5 \neq 11a$, this is so because 6a and 5 are unlike terms which cannot be combined to a single term. Learners who move with incomplete knowledge or misconceptions from arithmetic can definitely pass on these misconceptions in simplification of algebraic terms. These misconceptions hinder the learners' performance in simplifying algebraic expressions. This is supported by Linchevski and Livneh (1999) who state that students who make errors in arithmetic expressions repeat some of these errors when dealing with algebraic expressions. Learners who lack basic arithmetic skills find it difficult to simplify algebraic expressions.

The main reason for this struggle according Sfard (1991) is due to learners failing to understand the double existence of algebraic expressions as a process which put in place both working instructions as well as represent a number which is a result of these operations. According to Kieran (1992), students' learning difficulties are centralised on the meaning of letters, the change from arithmetic to algebraic symbols and the acknowledgment and use of structures. The researcher as an educator has realised that learners from grade 8 to 12 perceive algebra as a topic of the classroom, not applicable anywhere else because of the existence of the variables x, y and z. They fail to understand the meaning of expressions like 2x-3y +3z because the variables are generalised. In simplifying algebraic expressions learners will end up with errors because what they are simplifying is unspecified unlike in arithmetic.

Working with negative numbers is naturally difficult for students who are moving from arithmetic to algebraic thinking due to the abstract nature of algebra (Linchevski & Williams, 1999). These difficulties are related to addition, subtraction, multiplication and division of directed numbers and this includes the expansion of brackets. Learners encounter some problems in algebra when they enter high school because they cannot interpret and comprehend the meaning of letters in algebraic statements. This occurs because in primary school learners manipulate specific numerical values in arithmetic unlike in high school where learners are exposed to symbolised terms in algebra. For learners to have a sound knowledge in simplifying algebraic expressions these learners must have strong arithmetic skills which are the basic skills needed for them to understand algebra.

Teachers need to handle the transition from arithmetic to algebra with great care so as to reinforce correct mathematical concepts to minimise some occurrence of learners' errors and misconceptions. Demana and Leitzel (1988) argue that students have insufficient ideas of some algebraic values. There is a need for primary school teachers to reinforce some arithmetic principles within learners to reduce learners' cognitive difficulties in algebra. The learners need to be thoroughly and adequately taught in addition, subtraction, multiplication and division of directed numbers since these concepts are necessary in simplifying of algebraic expressions. The manipulation of integers is very crucial since it equip learners with pre-algebraic skills and prepare learners with some representations and operations of algebra.

1.2 Problem statement

I am a mathematics educator who has been teaching grade 8-12 for the past 19 years in 5 different government schools with different school settings. From all these teaching years, I have realised that the majority of mathematics learners struggle with algebra no matter how resourceful the school is. In South Africa there are challenges related to the teaching and learning of mathematics causing learners to perform badly in mathematics (Reddy, 2006; Luneta & Makonye, 2010). Algebra is one of the topics which is a stumbling-block that hinders learners to perform well in mathematics. Most of the learners find it very hard to manipulate algebraic problems using accepted rules, procedures, or algorithms because algebra is abstract (Mamba, 2012). Learners encounter some problems when simplifying algebraic expressions because the concepts are nonconcrete, the variables used are unspecified and most of the times learners find it difficult to apply them in real life situations.

Greens and Rubenstein (2008) state that in grade 8, 9 and 10 learners find it difficult to understand some elementary algebraic concepts and skills. Most learners at my school and other schools in South Africa are not performing well in algebra in general and algebraic expressions to be specific. Overall this contributes to poor learners' performance in mathematics in South Africa and this is a national problem. The Umalusi (2015) under the Department of Basic Education and Gabriel et al. (2013) indicate that the widely held errors and misconceptions shown by learners in their National Senior Certificate mathematics examination (NSC) scripts are due to deficiency in basic algebra. This is so because algebra covers more than half of the concepts examined in the NSC examinations. One reason why learners struggle with algebra in grade 12 is lack of basic algebraic skills in earlier grades. This is supported by the DBE (2016) which reports that candidates' showed poor algebraic skills which is an indication of lack of basic pre-algebraic skills which could not allow learners to effectively understand mathematical concepts in the examinations.

The government through the Department of Education need to find ways of helping learners to minimise their errors one way could be resourcing of the learners' errors. The low learners' attainment in mathematics is linked to errors and misconceptions (Makonye, 2010). One of the reasons why grade 8 learners fail to simplify algebraic expressions is due to misapplication of arithmetic rules to algebra or its incomplete knowledge gaps between arithmetic and algebra. Most learners struggle with algebra, and end up failing mathematics examinations since algebra contributes to a lot of marks in these examinations. The researcher as an educator is very worried about this low achievement in mathematics in algebra. That is why it was of great importance for the researcher to investigate learners’ misunderstandings on simplifying algebraic terms and investigate the probable cause for errors in order to help learners reduce them.

1.3 Purpose of the study

One way of examining why algebra is problematic is to find the kinds of errors learners usually make in the topic and explore the likely reasons for the errors (Booth, 1988).The study aimed at investigating the nature of errors and misconceptions grade 8 learners have in simplifying algebraic expressions and get a deep understanding of the possible reasons for these errors. The study sought to investigate if a learning opportunity can be created through resourcing learners’ errors to help learners reduce them on simplifying algebraic expressions. The researcher allowed learners to explore the concepts surrounding the simplification of algebraic expressions before the teacher demonstrated how to simplify the expressions in the pre-test. Also the researcher introduced variation theory during the intervention where the researcher contrasted a correct answer with a wrong answer for learners to note the differences between the two. If learners know their misconceptions in any mathematical topics, they can easily avoid them when solving mathematical problems. During intervention the researcher incorporated the concepts of integers as revision to fill up the gap between arithmetic and algebra.

1.4 Research questions

1. What are the types of errors and misconceptions grade 8 learners reflect on simplifying algebraic expressions?
2. What are the possible reasons for these errors?
3. To what extent do resourcing learners’ errors and misconceptions through the discovery method in teaching help learners to diminish them?

1.5 Significance of the study

Graeber (1999) highly motivated me when he states that an appreciative approach to common student misconceptions, and active plans to help students avoid them, is an important aspect of educational content information. I felt like its high time now for me and other teachers to understand learners’ errors and misconceptions in algebra in order to implement meaningful intervention strategies which can help learners diminish the errors. I enthusiastically wanted to know how best I could create opportunities for learners to learn through using their errors as correctional devices in the teaching and learning of mathematics. My intentions were greatly supported by Nesher (1987) who stated that misconceptions can be used as a response mechanism for real learning on the basis of the actual presentation. Khazanov (2008) gave me hope of succeeding in my study when he highlighted that teachers must know learners’ errors and misconceptions so as to cautiously and systematically prepare lessons to sufficiently address them. Teachers who know and use learners’ errors and misconceptions successfully escalate in their teaching methods and achieve long-lasting results in their goals (Riccomini, 2005). I needed to understand learners’ thinking which ended up as errors and misconceptions when simplifying algebraic expressions in the pre-test. These errors were used as a resource to remediate learners in simplifying algebraic expressions.

Nesher (1987) argues that the students’ errors should not be looked down upon, since they disclose the learners’ imperfect and inadequate knowledge (misconceptions) and encourage teachers to find ways to improve on their intervention strategies. Teachers need to understand and appreciate learners’ errors and use them as correctional devices in the teaching and learning of algebra. In my entire teaching career I have never used errors as a resource to improve my teaching. I eagerly want to use learners’ errors as a resource and I hope this intervention strategy is going to help learners overcome their problems in simplifying algebraic expressions and will improve their performance in this regard. The knowledge about learners’ errors and misconceptions can be a useful asset to the teachers, subject advisors, curriculum planners, policy makers and the entire field of mathematics education, since it proposes improvement for teaching. I hope that all the stake holders will gain some useful information from the research findings.

Teachers must be mindful of learners’ errors and misconceptions in algebra for them to come up with some significant intervention approaches to help the learners to minimise their productions of errors when solving algebraic tasks. These errors and misconceptions can be used as a resource tool for correcting erroneous learning in order to improve performance. Khazanov (2008) states that teachers who are attentive to their learners’ errors and misconceptions in a specific topic cautiously and systematically prepare their lessons in order to sufficiently address the errors and misconceptions. Knowing learners' errors in algebra is an important aspect of teaching which can encourages teachers to attentively prepare their lesson with an aim of helping learners to avoid the errors if it's possible or reduce them. It's difficult for doctors to treat patients if they do not know what each patient is suffering from. This also applies to learners in the classroom; teachers can only work for an improvement in assisting learners if they know the learners' shortfalls in simplifying algebraic expressions. The researcher hopes that the findings from this study might lead to the improvement of teaching strategies in this topic for the benefit of the learners.

Teachers can exploit learners' errors as a resource that helps them to understand learners' thinking to the advantage of the learner (Makonye &Luneta, 2013; Nesher, 1987). An understanding of learners' reasoning which can result in errors and misconceptions can be used as a resource to improve one's teaching methods. This research can be useful and effective to teachers and all other education professionals since it spearhead for quality and improved teaching and learning through remediation in the classroom. Nesher (1987) has argued that the students' errors should be considered as useful in teaching and learning since they reveal the incompleteness of learners' knowledge. This enables the teacher to reteach the concept or contribute additional knowledge to learners for them to correct their errors. The knowledge about learners' errors and misconceptions in algebraic expressions can motivate the researcher and other teachers to strategically improve their teaching styles. This helps learners to minimise their errors and misconceptions in the topic thereby improving learners' performance in mathematics. If teachers become competent in their teaching, learners will benefit from this improved teaching which can result in better learning and improved learner performance in mathematics. The study can help the researchers to learn more about learners' errors and misconceptions in algebra and to inform policy and curriculum makers to emphasise what needs to be done in order to improve in the learning and teaching of algebra.

1.6 The research methods

The research study used a mixed methods research approach. This method was employed mainly because of its strengths since the merits of the qualitative and quantitative approaches complement each other resulting in a strong research study that yields more valid and reliable findings (Opie, 2004). The combined evidence from the two different research methods will provide strong arguments about the research. The data was collected from 30 grade 8 learners through the use of a pre-test, interview and post-test. Data from the pre and post-test was quantitatively analysed and the data from the interview was qualitatively analysed. The researcher assumed that collecting diverse types of data could provide better understanding of the research problem which is; creating opportunities to learn through resourcing learner errors on simplifying algebraic expressions.

1.7 Outline of the chapters in the study

In chapter 1 the researcher gave an overview of the expectations of the study through discussing the problem that gave rise to the study, the research questions and significance of the study. In chapter 2 the researcher focused on the theoretical framework that guided the study and the literature-based on a conceptual framework that helped the researcher to understand the data in the study. The literature brought different pieces of research into relationships with each other and included some discussions on learner errors on simplifying algebraic expressions. Chapter 3 represented the methodology that was used in the study. This included some discussions on research design, research instruments, target population, sampling, data collection methods/procedures, data analyses and ethical considerations in the study. Chapter 4 focused on data analysis, data representations, data interpretations from the pre and post-test and the interview. In chapter 5 a summary of the research findings was presented. This includes the insights obtained and the problems encountered during the study and changes to be done if given another opportunity to do the study. Recommendations were made from the findings which included some suggestions for further investigations in ways of helping learners to reduce errors on simplifying algebraic expressions.

1.8 Conclusion

In this chapter, the background of the study was established. The problem that led to the study was discussed and the purpose and rationale of the study were briefly stated. The three research questions which the study intends to answer were asked. The outline of the chapters in the study was presented. The investigation was about identifying learners' errors and misconceptions on simplifying algebraic expressions and to find possible reasons for these errors. The other focus of the study was to find out if an intervention strategy of resourcing the learners identified could help learners to reduce them.

CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 Introduction

A theoretical framework is the researcher's choice of theories to be used as lens that guide and explain the study in order to answer the research questions. The study was informed by cognitive constructivism Piaget (1968) and von Glasersfeld (1990) and social constructivism Vygotsky (1978). Variation theory by Marton and Booth (1997) and the concept image and concept definitions by Tall and Vinner (1981) were used as bridging theories to support the major theories stated above. These theories directed the study and helped in interpreting the data on learners' errors and misconceptions.

2.2 THEORETICAL FRAMEWORK

2.2.1 Constructivism

Constructivism is a cognitive learning theory which focuses on the human mind which constructs knowledge (Piaget, 1968; von Glasersfeld, 1990). This theory is useful in the study, as it helps the researcher to understand learners' thinking which leads to learners' errors and misconceptions on simplifying algebraic expressions. Learning is a thinking process. In this process the mind can conceive misconceptions which hinders learners to successfully solve problems. In problem solving situations, the mind reasons within some conceptual structures called schemas. According to Marshall (1995), schemas are mechanism in human memory that allow for the storage, synthesis, generalization and recovery of similarities of experiences. The schema is the most crucial area of learning, a conceptual arrangement stored in the learner's memory that combines previous experience with current information and retrieve some resemblances in both information and use it to solve problem at hand. In algebra, learners need to identify some similarities in concepts between algebra and integers (directed numbers) and use them to simplify algebraic expressions.

Piaget (1968) states that learning occurs in three stages namely; assimilation, the placing of new information into schemas, accommodation, the transforming of existing schemas to new information and equilibration, attaining a balance between oneself and the environment. During learning, learners usual simplify and rearrange new information to fit into their existing schemas (assimilation and accommodation). If this fails, misconceptions can occur due to overgeneralization (Piaget, 1968). In this study overgeneralisation can occur in a situation where learners fail to link algebraic concepts to already existing arithmetic concepts. For instance learners can incorrectly simplify the expression 6a+5 to 11a using knowledge of arithmetic where 6+5=11 (misapplication of arithmetic rules to algebra). This erroneous answer 6a+5=11a is as a result of learners failing to rearrange their thinking mechanism in arithmetic in order to accommodate the algebraic concepts to fit in their schema. Generally learners' errors in mathematics are as a result of learners failing to retrieve the correct methods from their long term memory and consequently apply incorrect schemas in problem solving situations. To avoid misconceptions, the learners' schemas need to be fitting and suitable to assimilate and accommodate new information into the schema which is already pre-occupied with previous experiences.

Another source of learners' errors and misconceptions is due to misunderstanding of mathematical information as the teacher teaches or when discussing with other learners or when reading from mathematics textbooks. This can force learners to learn concepts through memorisation, a key cause of many mistakes in mathematics as pupils try to recall partially remembered and distorted rules (Olivier, 1989). Memorisation is short lived, since it hinders learners to recap previous learnt concepts in order for them to connect the concepts to the new information at hand in mathematical problem situations. Learners who learn by memorisation have high chances of producing inaccurate information when solving mathematical problems. Learners can produce errors when solving mathematical problems if they fail to remember concepts involved in the problem solving situation. Teachers need to understand the learners' mental processes which acquire knowledge to understand how learners conceive concepts including misconceptions. This can be done by regular check-ups of the learners' learning progress in class, homework, tests activities and the like. If learners' errors and misconceptions are noted teachers must find ways to help learners to reduce them.

Learning depends on the prior knowledge of the learner which serves as a schema in which a new schema is fitted (Smith, DiSessa & Roschelle, 1993). Learners with rich previous learnt concepts for instance in arithmetic will easily understand concepts in algebra since the two topics relate to each other. The learner's new understandings are dependent on the prior knowledge a learner retrieves from the earlier learnt concepts (Piaget, 1968) and students without this prior knowledge are disadvantaged as far as learning is concerned (Booth, 2008). Whatever information a learner brings to the class influences what that learner will benefit from the experience. This is supported by Piaget (1972) who argues that concepts are not taken directly from experience but the person's ability to learn from an experience depends on the quality of ideas that he/she brings to that experience. Learning is cumulative; whatever knowledge learners have from the previous lessons influences what he/she will learn in future. Learners with misconceptions from previous learnt concepts will apply them when they encounter new information related to the previous information. For instance learners can transit from arithmetic to algebra with misconceptions like failing to manipulate directed numbers which can negatively affect the learners' performance in simplification of algebraic expressions. The researcher revisited the concepts of arithmetic as part of the intervention plan in order to help learners clear their misconceptions in this topic. This was done through the teachers' clarification of concepts and differentiation of rules applicable and inapplicable in arithmetic and algebra.

Equilibration as stated by Piaget (1968) can only occur if and only if the learners' current knowledge correlates with the learners' previous knowledge. If this happens the mind will be satisfied with this experience and will maintain a state of balance. Disequilibrium usual comes in when a child experiences a new event until a time when the learner is able to assimilate and accommodate new information (Piaget, 1968). A state of disequilibrium which is a challenge to the learner occurs when the former knowledge fails to connect with the current knowledge. This result in a state of perturbation referred as mental disequilibrium where the mind anxious and seriously wants to solve a mathematical problem (Piaget, 1968) but the concepts presented are unfamiliar to the mind. This happens when learners are exposed to new information too isolated to fit into their current understanding but at the same time the learners eagerly want to solve the problem at hand. For instance learners can simplify an algebraic expression $2x + 3x$ to $5x$ using the concepts of like terms. This same learner can struggle with simplifying $2x^2 + 3x$ because of confusion of not knowing whether $2x^2$ and $3x$ are like or unlike terms since both terms have the same variable x. To clear up this state of disequilibrium, the learner can forcibly conjoin the two unlike terms to a single term $5x^2$. This distortion of rules results in errors and misconceptions.

Robinson, Eve & Tirosh (1994) state that students often make sense of the subject matter in their own ways which do not correlate with the structure of the subject matter. In constructivist classrooms learners create their own understanding and interpret it according to their own experience. During teaching and learning each individual learner uses his/her mental framework to constructs his/her own conceptions sensible to him/her (von Glasersfeld, 1990). Some learners strongly believe in their wrong knowledge constructions because they are sensible to them even though they do not relate with the subject matter. The constructed knowledge can be a correct conception of concepts or it can be a misconception when interpreted mathematically. Piaget (1970) and Bruner (1996) state that every person has a different interpretation and construction of knowledge since knowledge acquisition and schemata are personal depending on one's past experiences and cultural factors. The construction of knowledge is personal because mental structures of individual learners are different and assimilate and accommodate information differently. Learners arrange their experiences in their own personal ways which can be meaningful or meaningless to the mathematics community. Constructivism acknowledges that each learner has a unique background and unique needs and the knowledge he/she brings to the classroom is distinctive depending on what type of prior knowledge he/she had been exposed to. In the study there was need for the researcher to understand each and every learner's understandings and knowledge gaps in algebra in order to help them. The researcher appreciated every learner's contributions in the tests and interview since these unique answers led to an improved strategy of teaching and learning (intervention) in simplifying algebraic expressions

The best way to overcome a misconception is by engineering a cognitive conflict which allows learners to realise that their misconceptions are not mathematical as compared to the concepts of experts or mathematical textbooks (Makonye, 2013; Luneta and Makonye, 2013). Learners need to accept that their solutions are incorrect (cognitive conflict to be felt) for them to be helped by the teacher or mathematics textbook to clear their misconceptions. However in teaching and learning of mathematics there are some learners who are so attached to their misconceptions and believe that their constructions are correct. These learners hardly reject their misconceptions; they fail to experience the state of cognitive conflict and hence cannot learn from the experience. According to Nesher (1987) errors are difficult to eradicate from learners because these errors are valuable and meaningful to them since it's their own creation. This is one of the reasons why some learners keep on repeating the same errors in mathematical topics like algebra irrespective of how many times these errors and misconceptions are rectified.

The curriculum should allow the learners to continually build upon what they have already learned (Bruner, 1996). On top of the curriculum efforts to relate topics, the teachers need to do a regular recap of concepts when new concepts are introduced to fill up the gap between previous information with current information. In the study there was a need for a researcher to revisit the concepts of directed numbers (integers) before algebraic expressions were tackled. Teachers must teach learners according to the learners' educational level so that the information fits in the learners current cognitive schemas. Tasks which do not meet the learners' cognitive level can discourage or demotivate the learners to work. Any mathematical concept which is too isolated from the learners' prior knowledge is meaningless to the learners because they cannot understand it.

Learners' errors and misconceptions can be minimised if learners are provided with appropriate intervention strategies which enable them to attain a high level of development in learning of algebra. The researcher's main aim in the study was to find the root cause of learners' errors and misconceptions in simplifying algebraic expressions. This was done through a thorough scrutiny of learners' written work in the pre- and post-test and through learners' responses in the interview. Thereafter, the teacher uses the errors as a resource to help learners minimize the errors. Any errors or misconceptions which arise from learners' work shows learners' incomplete knowledge and it's a witness of learners' misunderstandings which results in inefficiency to solve the mathematical problems (Nesher, 1987). It is very important for teachers to check learners' understanding during learning. This can be done through giving learners some class activities and tests and check their progress in these tasks. By so doing, no learners are left behind since their misconceptions would have been corrected.

This constructivist's theory of learning is useful in explaining how learners conceive mathematical ideas including their misconceptions (Piaget, 1968; Von Glasersfeld, 1990, Hatano, 1996; Smith, Disessa, & Roschelle, 1993). Teachers need to understand learners' thinking which results in misconceptions and errors when solving mathematical problems. Pupils' erroneous thinking is an important part of teaching (Olivier, 1992). Errors contribute to the learning process because they allow one to think deeply about his/her answers in order to reduce errors which can arise from solving a mathematical problem. If learners' errors and misconceptions are identified they must be rectified immediately to avoid recurring misconceptions in the next coming concepts. Piaget (1970) states that constructivism guides teachers to intervene with learners through questioning and discussions and skilfully responding to the learners' ideas and allowing learners to discuss relationships between concepts and predict future events. Teachers need to create an environment in which they and their learners are encouraged to think and explore. Errors are an indication of learners' thinking which caused misconceptions and teachers need to help learners to rectify this problem

2.2.2 Socio-cultural learning

Vygotsky (1986), states that social interaction and interdependence of social and individual processes in the construction of knowledge give rise to learning opportunities. The theory believes that learners can learn through assisted discovery from knowledgeable persons. Learning involves learner-learner interaction, learner- teacher interaction and sharing of information to improve the learners' understanding of mathematics. In social constructivism, learners construct their own knowledge through social and cultural interaction, sometimes receiving misconceptions from their teachers and other learners through misinterpretations of concepts. The theory emphasise that children will always learn when motivated by an adult who is always there to assist the children. Teachers must always expose learners to some mathematical concepts like algebraic expressions before giving learners such tasks to do. The teacher's lesson plans need to be up to date so that whatever the teacher assess the learners on must go hand in hand with what was covered. A thoroughly prepared lesson plan can help the teacher to impact correct information to learners and avoid giving learners wrong information which can results in errors and misconceptions.

The Zone of Proximal Development (ZPD) is one of the most crucial principles in Vygotsky's theory of social cultural learning. The ZPD is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more knowledgeable peers (Vygotsky, 1978, p.86). In this zone learners cannot solve problems on their own but require guidance and support from the teacher for them to be capable of solving mathematical problems. Mediation is the assistance given to learners if they fail to solve the mathematical problems by themselves referred to as the Zone of Proximal Development (Vygotsky, 1978). Here the teacher will assist a learner to solve mathematical problems during a time of incompetence. The ZPD helps the teachers to identify the gap in understanding of concepts by learners and this will help the researcher to understand the learners' struggles in simplifying algebraic expressions. The ZPD enlightens the teacher about the extent of assistance learners need for them to solve mathematical problems by themselves. The zone of proximal development must be approached with careful thought instructions to avoid instilling misconceptions in learners' mind. This higher level of development which results from the assistance given to the child enables the child to independently solve problems at a new level (Osei, 2000). Teachers need to assist learners to a certain point and leave them to proceed with the solution. By so doing the learners will learn something from that and will reach a higher level of understanding.

Learning is a social activity where learners and the teacher cooperatively solve mathematical problems. Teachers need to encourage learners to share ideas and the teacher needs to be fully prepared to help any learner at any time. The development of conceptual thinking does not come naturally through experience but is dependent on specific types of social interactions (Vygotsky and Luria, 1994).

The interactions in the classrooms must be well organised, manageable, and beneficial to all participants in the learning process. Learners must be aware of some concepts and procedures involved in the learning of algebra before being afforded some opportunities to construct their own knowledge. Teachers need to do the recapping of previous learnt concepts so that learners connect them to the new concepts. In social interactions in and outside the classrooms misconceptions can be passed from learners to other learners or teachers to learners. Teachers need to be alert of the class discussions, to clarify concepts to minimise learners' errors and misconceptions.

Vygotsky (1962) argues that language is a very powerful tool for the development of thought and helps learners to understand concepts cognitively. In cognitive development, language enables people to master and understand concepts established in their culture and be able to realise what's going on in their environment. Learners who have acquired the language of mathematics like symbols, variables and constants in algebra can master and understand algebraic concepts fairly well. They will be in a position to correctly apply rules applicable to algebra and effectively simplify algebraic expressions; their schemas can easily assimilate this mathematical information. Children learn language in a social-interaction, then think in terms of that language which the most important tool or symbol in life of an individual to move to a higher level of cognitive awareness (Vygotsky and Luria, 1994). Learners derive their mathematical language from the classroom or society they are exposed to. If learners understand a language which relates to a mathematical problem, it will be easier for them to solve the problem since learners will have a clear understanding of what they are expected of.

The grade 8 learners in an algebraic lesson ought to understand the language of that particular topic in that culture for them to successfully simplify algebraic expressions. According to Mamba (2012) learner errors and misconceptions can arise if learners do not understand the medium of communication such as language and symbols used in the teaching and learning of mathematics. In teaching and learning, sometimes learners fail to solve some given mathematical tasks due to lack of understanding of mathematical language involved. Teachers and learners need to use correct mathematical language in the classrooms to reduce learner errors and misconceptions.

2.2.3 Concept image and concept definition

The concept image and concept definition by Tall and Vinner (1981) is another suitable theory that gave more meaning to learners' errors and misconceptions in algebra in the study. Tall and Vinner (1981) referred to concept image as a cognitive amount of ideas that a learner has formed in his/her mind regarding all aspects of specific concept. The concept images are all mental representations together with a set of properties associated with a certain concept (Vinner, 1983). Learners thoughtfully create their imaginary pictures which represent certain concepts and retrieve these representations in solving mathematical problems. The concept image is associated to Piaget's schema; it is a model of the learners' inner thoughts of reality constructed by learners as a result of experience with a particular concept. The concept images can be correct, partially correct or erroneous and they are a function of maturity and experience with the concept, (Luneta & Makonye, 2014). If a learner's concept images are faulty and incorrectly formed this results in learner's errors and misconceptions on solving mathematical problems.

Vinner (1983) highlights that a concept definition is a verbal definition that accurately explains the concept in a more understandable way. A concept definition is a precise description in words of a concept as is agreed by mathematicians. If learners' concept images do not correlate with the learners' concept definition misconceptions do arise in the learners' minds which results in learners' errors. For instance, a teacher can define like terms in algebra as algebraic expressions with the same variables. So learners can take 2x and $3x^2$ as like terms since they have the same variable x without noticing that the two terms have different exponents so they are unlike terms. Here learners are trying to integrate the new concepts of like terms in a corrupted concept image which views x and x^2 as the same quantities with the same variables x. Learners always refer back to their incorrect concept images instead of focusing on the correct concepts presented to them by the knowledgeable teachers and the resourceful mathematics textbooks. That's the reason why learners always have misconceptions in their minds and produce errors in mathematical solutions.

Learners' errors are therefore as a result of naive concept images that do not measure up to the concept definition (Tall & Vinner, 1981). The learners can construct concept images differently from what they are taught because of confusion due to lack of understanding or misinterpreting of concepts. The learners' constructed images can be mathematically incorrect (misconceptions) as a result of learners failing to link the concept image to the correct concept definitions. In teaching and learning, learners are not taught misconceptions but they sensibly construct them using the structure of the current knowledge they possess. The misconceptions reveal that learners did not understand the concepts taught or were unable to connect the current knowledge to their previous learnt concepts. Teachers need to understand that concept images and concepts definitions work hand in hand. It's not very easy for learners to understand concepts through definitions but understand concepts better if some representations or pictures or diagrams are involved. Also, presenting learners with concept images without explaining what they mean does not make sense. For instance the learners can understand better the concept of a right angled triangle if a diagram is presented and the properties of the triangle are defined or explained. Teachers need to give learners some examples that build up the desired concept image regularly when defining concepts or vice versa.

2.2.4 Variation Theory

Variation is about what changes, what stays constant and the underlying rule that is discerned by learners in the process. From a variation theory, learning is defined as a change in the way something is seen or understood. Teachers must provide learners with the means to experience variation through strategically designed activities. In the study the researcher employed variation theory through using errors as a resource to help learners reduce them. Learners need to know the difference between correct and wrong answers to avoid or reduce errors. If $2x + 4x = 6x$ then $2x + 4y \neq 6xy$ since these are unlike terms. In this study, the researcher made the learners to write a pre-test. The purpose of the pre-test was to identify learners' errors in simplifying algebraic expressions. Thereafter, an intervention strategy was done, where the concept of variation was applied. By exposing learners' errors, teachers will enable learners to see what is wrong and what is right (variation) in the simplification of algebraic expressions.

2.2.4.1 Critical features

During teaching and learning, learners must be able to detect the very important concepts to remember for them to solve a mathematical problem. These crucial concepts are called critical features that help learners to have a better understanding of any mathematical problem given in the classroom. Learners can only recognize a particular critical aspect in a mathematical concept, if they experience variation in the dimension of that aspect. Learners must experience variation in the critical aspects of a concept, within limited space and time, for the concept to be learnable (Marton & Tsui, 2004; Marton & Pang 2006). In the study learners recognised critical features in the pre-test, post-test and interview they attended to. In these tasks errors and misconceptions are also revealed. Marton and Tsui (2004) state that teachers need to effectively engage their students to grasp variations in understandings and knowledge so they can take account of this diversity in structuring the learning activities in a lesson. In the study the researcher used errors as an intervention tool. Learners need to know the difference between right solution and wrong solution through discernment of critical features. This can help learners to effectively solve mathematical problems.

2.2.4.2 Contrast

Marton (2009) describes the awareness brought about by experiencing the variation between two values as contrast. Concepts are learned only when they have been grasped through variation/contrast. In the classroom most teachers demonstrate concepts to learners by giving them a lot of similar examples without letting learners know dissimilarity between concepts. Errors are prominent in the learners' work because the learners are not aware of them. If a phenomenon is combined with contrast, this allows us to separate the essential features of the phenomenon from irrelevant features", (Akerlind, 2008, p.638). Learners can easily understand the correctness of a concept if they also know its misconception. For instance learners need to be clarified that the expression $y \text{ x } y = y^2$ but the expressions $y \text{ x } y \neq 2y$, this is a common misconception in learners' minds. Learners need to be exposed to variety of concepts which seem to be the same but different for the learners to understand and appreciate the difference. For example, 2+3+4=9 is an arithmetic expression but 2x+3x+ 4=5x+4 not 9x is an algebraic expression. Errors and misconceptions arise if learners fail to discern the differences between the two concepts. In algebraic expressions there is a need for collecting like terms and simplify whereas in integers, learners only deal with numbers which can be simplified to one term.

2.2.4.3 Separation

Leung (2012) states that separation is an awareness awakened by a systematic refined contrast obtained by purposely varying or not varying certain aspects in an attempt to differentiate the invariant parts from the whole. Separation is based on the view that everything is multifaceted and this gives rise to different understanding of that thing. Separation is a refined contrast which focuses on the uniqueness of individual concepts but have similarities with other concepts. For instance $3x$, $3x^2$ and $3x^3$ have something common, they are all algebraic expressions with 3 as their coefficient. But we can vary them by looking at the exponent of x, 3x is a linear, $3x^2$ is quadratic and $3x^3$ is cubic.

2.2.4.4 Generalisations

Generalisation is a variation and conjecture-making activity enabling learners to check the general validity of a separated pattern which is often a goal of mathematics exploration, (Chick & Marton, 2012). Generalisation of concepts comes about when there are similarities between the concepts. For instance we can treat the algebraic expressions3x, $3x^2$ and $3x^3$ as a sequence; as shown on the table below;

Table 1: Showing a sequence of algebraic terms

Term 1 = (T_1)	Term 2 = (T_2)	Term 3 = T_3	Term n = T_n
3x	$3x^2$	$3x^3$	$3x^n$

The sequence will generalise to formula which is $T_n = 3x^n$. This formula is applicable to all, for instance we can calculate T_{10}, T_{50}, and T_{1000} by just substituting in the formula. Learners must be provided with some generalisations for them to be able to tackle mathematical problems in different ways. Generalisations help grade 8 learners to simplify exponents by applying exponential rules (general terms) in adding, subtracting, multiplying and dividing exponents.

2.2.4.5 Fusion

Fusion takes place when the learner’s attention is focused on several aspects of a concept that vary at the same time. Fusion incorporates critical features or dimensions of variation into a whole under simultaneous co-variation. Ling (2012) states that if the learners can only discern individual critical features but fail to achieve the simultaneous discernment of all of the critical features and the relationships amongst them (fusion) possibly they will not thoroughly understand the object of learning and unable to apply such knowledge to solve new problems (Ling, 2012). In any given scenario learners need to identify all the critical features in solving mathematical problems and combines them and solve the problem. For instance in simplifying algebraic expressions learners need to note what is critical in each question for them to be able to simplify them.

$2(x + 3) = 2x + 6$ $\qquad$ $2(x – 3) = 2x – 6$

$-2 (x + 3) = -2x – 6$ $\qquad$ $2(x – 3) = -2x + 6$

Here learners need to know the critical concepts of multiplication of integers and combine it with the simplification of algebraic expression (fusion) to come up with different answers representing the linear graph or equation differently. All the above solutions represent one linear graph which has been transformed in different ways like reflection, translation. In fusion there is synchronic simultaneity when learners focus on different aspects of an object of learning at the same time and diachronic simultaneity which connects variation experiences gained in previous and present interactions (Leung, 2012). This follows the view that learners always bring what they have met before (previous knowledge) to bear on what they are learning now; hence, simultaneous co-variation can be explained in terms of both synchronic and diachronic simultaneity.

2.2.5 Conceptual framework

The conceptual framework is a visual or written product that explains the concepts or variables and the relationships among them to be studied (Miles, & Huberman, 1994). This involves the bringing together of a number of related concepts to have a better understanding of the research problem. The conceptual framework includes the actual ideas and beliefs including some concepts in the theoretical framework, the researcher will follow to understand learners' errors and misconceptions on simplifying algebraic expressions.

2.2.5.1 Conceptual knowledge and procedural knowledge

Conceptual knowledge is the understanding of mathematical concepts or ideas, operations and relations. Learners with conceptual understanding know the usefulness and applicability of a mathematical idea depending on its context. Conceptual understanding involves building relationships between existing bits of knowledge; there is significant cognitive recognition when previously independent networks are related, (Skemp, 1976 and Davis, 1984). Teachers should help pupils to connect new knowledge to previous knowledge. Learners must be able to make connection between incoming information and existing knowledge. According to Schifter and Fosnot1 (993), the creation of conceptual algebraic understanding calls for the teacher to give learners constructive and interpretive activities. Teachers need to carefully prepare their lessons with meaningful tasks which motivate learners to construct their own knowledge. Conceptual understanding supports retention and helps learners to avoid many critical errors in solving algebraic problems.

Hiebet and Leferve (1986) argue that procedural understanding is mastery of computational skills where procedures (rules) and algorithms are carried flexibly, appropriately, accurately and efficiently in solving mathematical problems. Learners who use mathematical procedures without understanding cannot solve more complex problems independently (Nesher, 1987).

Learners, who lack procedural knowledge, learn concepts without understanding ending up failing to solve mathematical problems. Errors and misconceptions come about if learners memorise facts and formulas from previous learnt concepts which they cannot apply properly when challenged by a mathematical problem. Conceptual and procedural knowledge are inseparable since they depend on each other. A learner who is able to execute the procedures correctly displays a good grasp of conceptual knowledge (Siegler, 2003). Possession of both conceptual and procedural knowledge allows a learner to solve mathematics of advanced or difficult form.

2.2.5.2 Relational learning

Skemp (1976) defines relational understanding as knowing how to apply mathematical rules and be able to justify why the rule applies in that particular mathematical problem. This involves knowing both what to do and how to do it, this involves mathematical reasoning and meaning. In relational learning, learners know methods to use and why using them and relate the methods to new problems and there is much less memory work involved. The learners would also find their learning naturally pleasurable and many would continue it willingly. Learners who get satisfaction from relational understanding always try to understand relationally new material which is put before them, but also seek out new material and explore new areas.

2.2.5.3 Instrumental learning

Kilpatrick, Swafford and Findell (2001) state that instrumental understanding is using rules without reasons and there is no conceptual understanding. It provides learners with more quick and easy solutions; learners immediately come up with solutions and this boost the learners' confidence in the subjects. Instrumental learning does not take learners far as long as simplifying algebraic expressions is concerned because this type of learning is short lived and results in memorisation of concepts. Premature teaching serves only to inculcate empty procedures-instrumental understanding (Skemp, 1976). Instrumental learning promotes learners to memorise concepts without a clear idea of steps involved in solving mathematical problems like in algebraic tasks. Here learners will have problems with accommodating new concepts in the learners' existing schemas which will result in errors and misconception. Both relational and instrumental understanding should be used during learning. Teachers can teach for instrumental understanding if relational understanding would take too long to achieve or too difficult to be understood for examination purposes.

2.2.5.4 Scaffolding

Scaffolding is the help given to learners by the teacher at the beginning of a learning activity and suddenly withdrawn, (Makonye and Khanyile, 2015). Sometimes learners can be assisted especially if they fail to connect the current knowledge to the previous learnt concepts. Sometimes learners need to be assisted if they fail to understand concepts. The teacher needs to do the recapping of concepts so that learners can start to work in a mathematics classroom. Scaffolding learners is like you are removing a stumbling block in a learners learning path for him/her to be able to move and solve mathematical problems. Reflection done individually or with others assisted or unaided is important in the teaching and learning of mathematics (Ruhl, Balatti and Belward, 2011). Teachers need to help learners if they are faced with some difficulties which cannot allow them to proceed with the solution of mathematical problems.

2.2.5.5 Discovery learning

Guided discovery learning is a constructivist approach of learning where learners construct knowledge through self-discovery with minimum assistance from the knowledgeable person (Bruner, 1961; Vygotsky, 1986). Bruner argues that “Practice in discovering for oneself teaches one to acquire information more readily viable in problem solving” (Bruner, 1961, p.26). Guided discovery learning model is useful in the teaching and learning of mathematics since it creates an opportunity for learners to solve problems through exploration. This type of learning encourages learners to independently and critically think about ways of solving any problem situation at hand. For guided discovery learning model to be effective learners need to be intrinsically motivated, responsible and focused for them to be able to discover concepts by themselves to solve mathematical problems. Hernandez et al. (2011) emphasised that guided discovery learning could help learners learn various problem learning strategies, transfer cognitive data to be more useful and know how to commence learning. Discovery learning exposes learners to a variety of learning activities. These activities promote critical thinking in learners and exploration of different problem solving methods.

Constructivism is a process whereby learners actively construct their understanding of the world through the interaction of previous and current knowledge with the environment (Piaget, 1968; von Glasersfeld, 1990; Bruner, 1996). Learners construct their knowledge through connection of concepts by active participation and interaction with others in the classroom. Knowledge cannot be transferred intact from one person to the other (Piaget, 1967, Hatano, 1996). The teacher cannot transfer his/her knowledge directly to the learner but the learner needs to create his/her own knowledge. Instead, the teacher must encourage the learners to discover some of the mathematical concepts by themselves. To encourage discovery learning, teachers must allow learners to work independently and give them some tasks that provoke their thinking and incites them to think and discover mathematical principles by themselves.

Bruner (1961) suggests that students are more likely to remember concepts if they discover them on their own as opposed to those that are taught directly. In discovery learning learners mainly discover concepts with little assistance from the teacher. These learners can understand and remember these concepts well because they originate from the learners' own constructions. The important role of the teacher in guided discovery learning is to monitor the learners' progress and check if learners do not have misconceptions which need to be rectified. Guided discovery model encourage students to think for themselves, analyse themselves so that they can find the general principles based on material or data provided by the teacher. Here learners are acquainted with the necessary skills to independently solve mathematical problems through some profound investigations. Discovery inquiry is expected to reduce the occurrence of mathematics misconceptions among students.

Discovery-inquiry learning model allows students to gain learning experience by doing activities that enable them to find the concepts and principles for themselves. The teacher needs to have a well-planned lesson for him/her to address all the objectives set in order to meet the goals of learning. Guided discovery learning is ineffective if the lessons and activities are not carefully planned for. This type of learning can be time wasting if the teacher does not manage it properly through giving clear instructions of what tasks to do, when to do them and for how long. Learners are active participants in this learning process where they take responsibility of constructing their own knowledge. In discovery learning model, learners are not told how to solve mathematical problems but they investigate/explore various methods until they come up with a solution.

The teacher must provide all necessary background knowledge to lead the student to discovery learning. The teacher must know his/her learners' level of understanding before any guided discovery model is implemented. This is important because in the classroom learners are unique and have unique learning needs. Some learners need a lot of attention from the teacher and some need minimum assistance so the teacher's questioning/probing technique can differ from one learner to another. The teacher must give learners some mathematical activities which provoke their thinking. For discovery learning to work, the teacher must give learners task which suit their level of education. The task must be appropriate and relevant to the learners depending on the learners' experience in the assessment topic.

In problem solving situations, learners connect their previous learnt concepts to the current information. However the teacher needs to guide if there is a realisation that learners are not heading anywhere as far as solution of mathematics problems are concerned. Learners find solutions to problems in context by themselves through perseverance and hard work. They practically investigate the concepts and procedures involved in any problem solving situation. The teacher is a facilitator who guides and motivates learners to discover concepts involved in any problem solving scenario. The teacher controls the learning process through managing time, checking learners' progress and listening to the learners' conversations and intervenes when necessary. This is done in order to reduce learners' misconceptions during class discussions and in their written work.

Discovery learning learners learn about time management, they actively engage themselves with the content for they gain a sense of independency as learners construct their own knowledge. The researcher employed discovery learning during intervention as a way of encouraging learns to find concepts by themselves. In teaching and learning, learners learn less if all knowledge is given to them. Learners who are always provided with mathematical information are not self-motivated to discover or investigate concepts. These learners encounter some challenges in situations where critically thinking or analysing or justifying of mathematical facts is required. Teachers must give learners a leeway to work on their own to improve their mathematical reasoning. Learners' errors and misconceptions are essential in teaching and learning. They can be used as a resource to help learners reduce them.

For guided discovery learning model to be effective, the learning environment must be conducive allowing learners to think, discuss and share ideas. The ideas from each and every learner are important and must be considered despite the fact that the ideas are right or wrong. The teacher must give learners freedom to solve problems in their own ways and to ask questions if they need clarification in any problem in context. Teachers must expose learners to various ways of solving same problems so that if they encounter problems involving these, the learners would have a wide variety of methods to choose from.

2.3 LITERATURE REVIEW

Literature review is a data gathering technique which was used in this study to understand errors and misconceptions in algebraic expressions. The literature review represents the first phase of empirical study that entails reviewing other authors' work in a specific field. The purpose of the literature review is to understand the concept of errors and misconceptions, the types of errors, possible reasons why learners have errors and misconceptions in the subject and investigate the usefulness of errors and misconceptions in teaching and learning.

2.3.1 Errors

An error is a mistake, slip, blunder or inaccuracy and deviation from accuracy which can result in wrongly answered problems, and it's an incorrect application and conclusion of mathematical expressions and ideas (Erbas, 2004). An incorrect application of mathematical rules, algorithms, and formulae results in errors which lead to wrong answers in learners' tasks. Errors are systematic, persistent and pervasive patterns of mistakes performed by learners across a range of contexts due to a faulty line of thinking (Green, Piel and Flowers, 2008). If learners continue to live with misconceptions they repeatedly produce systematic errors which are carefully constructed or reconstructed within a given period of time. Unsystematic errors are unintentionally, non-recurring wrong answers, which learners can correct independently or learners may not repeat them (Lukhele, Murray and Olivier, 1989); Riccomini, 2005). Learners accidentally produce errors which cannot be repeated and learners can correct them without the teacher's assistance. In data analysis I will use Hodes and Nolting (1998)'s four types of errors defined as follows;

1. *Careless errors: mistakes made which can be caught automatically upon revising one's own work.*
2. *Conceptual errors: mistakes made when the learner does not understand the concepts covered in the textbook or due teaching and learning.*
3. *Application errors: mistakes that learners make when they know the concept but cannot apply it to a specific situation or question.*
4. *Procedural errors: these are errors which occur when learners omit some steps or misunderstand some instructions but still answer the question.*

2.3.1.1 **Careless errors**

Slips are careless errors which appear in learners tasks even if they know the right answers of a mathematical problem due to learners' carelessness in their calculations. These are wrong answers due to processing (Olivier, 1989). Teachers can easily take note of slips in learners' workings and correct them immediately. Learners commit careless mistakes due to lack of concentration and miss the correct solutions even though the learners know the right answers of a mathematical problem.

2.3.1.2 Application errors

An incorrect application of mathematical rules, algorithms, and formulae results in errors which lead to wrong answers in learners' written tasks (Erbas, 2004; Luneta & Makonye, 2010). If learners continue to live with misconceptions they keep on producing errors in their knowledge construction over a given period of time. A misconception is a display of an already acquired system of concepts and algorithms that have been wrongly applied (Nesher, 1987). Sometimes learners will be fully aware of some correct concepts acquired earlier on but inappropriately apply them. For instance learners know that 6+5=11 but this concept becomes wrong in algebra if learners think that 6a+5=11a because 6a and 5 are unlike terms which cannot be conjoined to a single term.

2.3.1.3 Conceptual errors

Conceptual errors occur when learners fail to understand the concepts involved in a given task or when learners fail to connect the relationships between concepts (Hiebet & Lefevre 1986). Conceptual errors arise when learners do not understand a mathematical concept and apply it in a wrong context. For instance in the study some learners incorrectly wrote $x(x) = 2x$ instead of writing $x(x) = x^2$. Learners who lack conceptual knowledge fail to connect new information to previous existing knowledge.

2.3.1.4 Procedural errors

Procedural errors are displayed by learners when they fail to carry out calculations or algorithms although they understand concepts in a given problem (Hiebet & Lefevre 1986). Learners who lack procedural knowledge cannot accurately and appropriately carry out mathematical procedures when solving mathematical problems. These learners learn concepts without understanding but through memorisation of rules or algorithms without a deep understanding of them. Learners who lack procedural knowledge cannot apply mathematical knowledge to new context accurately and flexibly.

2.3.2 Misconceptions

A misconception is a display of an already acquired system of concepts and algorithms that have been wrongly applied (Nesher, 1987). This wrong conceiving of concepts (misconceptions) results in learners errors. Learner misconceptions are often persistent and resistant to instruction designed to correct them and distract knowledge acquisition especially in mathematics (Smith, DiSessa and Roschelle, 1993). These misconceptions are important to learners as they result from the learners' attempts to construct their own knowledge. In making errors students are attempting to make sense in their experiential world, they deviate from the teacher's expected path (Jaworski, 1994). Errors results from learners' attempt to construct knowledge. These errors are meaningfully to the learners and they conceive them as correct answers for a mathematical problem. These misconceptions are intelligent constructions based on correct or incomplete previous knowledge. According to Smith, et al. (1993), misconceptions are sensible to learners and can be resilient to instructions designed to correct them. Misconceptions are unavoidable no matter how effectively teachers teach; even brilliant learners can show some misconceptions in their work.

Teachers need to understand and appreciate learners' errors and use them efficiently for the benefit of the learners. If learners' errors are exposed to the learners in a detrimental way, this may reduce learners' confidence to solve problems and increase confusion in the learners' minds. Instead, making errors is best regarded as part of learning. If misconceptions are not attended to they affect the students' success in problem solving (Knuth et al., 2005) and hinder their learning of new material (Booth & Koedinger, 2008). Errors are important in learning since they show the learners' incomplete knowledge and misunderstanding in algebra. Learners' misconceptions can be solved through class discussions, practice and writing down of corrected answers.

2.3.3 Reasons for learners' errors

Students often have difficulty with algebra because of misconceptions in various areas. When these are corrected, the students seem to grasp the concepts more clearly.

2.3.3.1 Lack of procedural and conceptual knowledge

Learners fail to use the correct procedures to solve a mathematical problem because they incorrectly perceive concepts involved. The use of incorrect procedures is common when learning algebra (Lerch, 2004), and by nature, this behaviour inhibits accurate solution of problems. Learners try to use short cuts in solving mathematical problems. Improvement in students' procedural knowledge requires both reduction in use of incorrect procedures and construction and strengthening of correct procedures (Siegler, 1996). Teachers need to be aware of learners' procedural and conceptual errors in order to rectify them. These errors and misconceptions might be as a result of learners' memorisation of mathematical rules and using procedures without connection and understanding (Stein; Grover and Henningsen; 1996;

Skemp, 1976). Learners often apply algorithms from other procedures in new concepts, Mamba (2012) pointed out that the abstract nature of algebraic expressions such as understanding or manipulating them according to accepted rules, procedures, or algorithms posed problems to learners. During learning, learners' misconceptions usually are revealed when learners use formulas or procedures inadequately.

Conceptual knowledge is the understanding of mathematical concepts or ideas, operations and relations. Learners with conceptual understanding know the usefulness and applicability of a mathematical idea depending with its context. Conceptual understanding involves building relationships between existing bits of knowledge; there is significant cognitive recognition when previously independent networks are related, (Skemp, 1976 and Davis, 1984). Teachers should help pupils to connect new knowledge to previous knowledge. Learners must be able to make connection between incoming information and existing knowledge. According to Schifter and Fosnot1 (993), the creation of conceptual algebraic understanding calls for the teacher to give learners constructive and interpretive activities. Teachers need to carefully prepare their lessons with meaningful tasks which motivate learners to construct their own knowledge. Conceptual understanding supports retention and helps learners to avoid many critical errors in solving algebraic problems.

2.3.3.2 Distributive property

Demana and Leitzel (1988) cited in Leibowitz (2016) maintain that a sound understanding of the distributive property is essential for algebraic functioning. Terms must be distributed appropriately in algebraic expressions but this is a challenge to most learners because they mix up signs when simplifying algebraic expressions. For instance the possible answers for -3(x-4) simplified are; -3x – 12, -3x + 12, -3x – 4 etc. Learners should be aware of the brackets usage since brackets convey a strong message in algebra. If learners fail to expand brackets their answers will have errors. So to avoid this, learners need to be able to expand brackets correctly. Students make all kinds of computational errors when simplifying algebraic expressions because they read their expressions from left to right and see no need for brackets.

2.3.3.3 Deletion error

The difficulty of algebra is attributed to deletion errors. Researchers have attributed the deletion error to the over-generalisation (or false generalisation) of certain mathematically valid operations. The source of the deletion error can be traced back to arithmetic, where simplification gives a single numerical value. For instance who lack an understanding of algebraic language can simplify an expression like 4a+5+2a to 6 where like and unlike terms are conjoined and variables are deleted. Learners cancel algebraic expression if a common factor exists; they do this in the same way that they simplify common fractions.

2.3.3.4 Computational error

Computational errors arise from misapplication of rules and not from carelessness or not knowing how to proceed but can be caused by learners through application of failing strategies (Ashlock, 1994). Learners can be requested to simplify m^7 x m^2 and write the answer as $m^{7-2} = m^5$ instead of adding exponents and come up with m^5. Here the learners are aware of exponential laws but incorrectly apply the law of division when there is a multiplication concept. Students should be aware of the conventions of algebraic syntax since this gives meaning to algebraic expressions. Teaching should therefore focus on the conventions of algebraic syntax.

2.3.3.5 Conjoining

Bosse and Faulconer (2008) affirmed that conjoiners constitute an important component of learners' source of errors in algebra. Conjoining is combining unlike terms in the multiplication, addition or subtraction of algebraic expressions. For example learners simplify; 9x + 4 = 13x which is a misconception because 9x and 4 cannot be added since they are unlike terms.

2.3.3.6 Language of algebra

Pyke (2003) and Boulet (2007) postulate that language plays an important role in the teaching and learning of mathematics, fluency in it provides learners with greater understanding of mathematical concepts. Learners need to understand the language of algebra for them to understand the concepts of algebra. This special language refers to symbolic notation which fills a dual role as an instrument of communication and thought, (Roux, 2003). In algebra symbols, algebraic expressions and numbers are used which learners need to master. Knowledge cannot be transferred to the student by linguistic communication but language can be used as a tool in the process of guiding the students' construction (Jaworski, 1994). Teachers use mathematical language to help or guide learners in the construction of knowledge not as a way of impacting knowledge. Arzarello (1998) reports that many learners do not understand algebraic language correctly and as a result, their thinking and performance are badly affected. In simplification of algebraic expressions learners struggle with the language of like and unlike terms, order of operation, distributive property and so on. Burton (1988, p.2) state that students are unable to make sense with the language of algebraic system but this need to be rectified by considering algebra in a linguistic sense. Teachers need to let learners familiarise themselves with algebraic language. The words like coefficients, variables, constants, like and unlike terms must be well defined in the introduction of algebraic expressions. The improper use of algebraic language results in structural errors. An improper use of algebraic language contributes to misconceptions which can result in errors in simplifying algebraic expressions.

2.3.4 Resourcing learners' errors

The study investigated if resourcing learners' errors could help learners to reduce them. Errors and misconceptions are useful resources in the teaching and learning of mathematics as they inform instruction. Teachers can exploit learners' errors as a resource that helps them to understand learners thinking to the advantage of the learner, (Makonye and Luneta, 2013, Nesher, 1987; Smith DiSessa and Roschelle, 1993). Teachers have to accept and appreciate learners' errors as making errors is a student's contribution to learning. These errors are sensible to the learners since they rise from learners' constructions. By resourcing learners' errors this can create an opportunity for learners to learn and reduce the errors. Students become more aware of their errors and take more responsibility for their own thinking and increase their understanding and appreciation of other people's thinking, (von Glasersfeld, 1990). The use of errors as a resource during intervention can help learners to increase their understanding of algebraic expressions. The student's errors reveal the incompleteness of their knowledge which enables the teacher to contribute additional knowledge or lead them to realise where they went wrong, (Nesher, 1987). Errors indicate learners' misunderstanding of concepts; this alone can motivate teachers to find improved ways of helping learners reduce their errors.

CHAPTER 3: METHODOLOGY

3.1 Introduction

The purpose of the study was to explore grade 8 learners' errors and misconceptions on simplifying algebraic expression. The study sought to find possible reasons why learners have errors. Thereafter, the researcher created opportunities to learn through resourcing errors to help learners reduce them. A research, methodology is the plan of the study which gives some highlights of the purpose of the study. It shows how data is to be collected, analysed and the sample to be used. Mixed methods paradigm which involved the use of both quantitative and qualitative research methods was employed. A paradigm is a broad viewpoint of something. It is an imaginary interpretation of ideas, beliefs and feelings by individuals who agree on a definite kind of reality.

The paradigm mainly concentrates on how each and every person deliberately solves real life problems. The paradigm appreciates the distinctiveness of individuals and their numerous ways of viewing the world. The paradigm is appropriate in the classroom setting up, where learners uniquely understand mathematical concepts. As educators we need to understand and appreciate every learner's contributions to learning despite the fact that the contribution is right or wrong. The learners' contributions in the classroom will benefit all the participants in education. Qualitative research is fixed in the interpretive paradigm which is connected more with plans that understand the participants' views and worries in a research study (Weaver and Olson, 2006). In pre-tests, interviews and post-test learners revealed their thoughts, in terms of their understandings/misconceptions on simplifying algebraic expressions.

Quantitative research is placed in a positivist paradigm which studies human behaviour witnessed under organised environments to come up with scientific facts or theories. The quantitative research paradigm search for the truth through studying some connections between variables (Creswell, 2012). Quantitative research believes in the existence of a complete or fixed truth without considering individual persons' viewpoints. In this study learners' errors and misconceptions were viewed on the basis of the agreed logical facts.

3.2 Research Design

Yin (1994) defines a research design as the reasoning that contacts the data to be collected to the initial research question of the study. The research design is the implementation of the overall plan which demonstrates how the research is going to be directed. The research design was applied in order to gain full knowledge of learners' errors on simplifying algebraic expressions. The research design shows how the research is to be conducted.

3.3 Case Study

A case study method was employed in this study where one of the grade 8 classes at a certain school was studied as the case. A case study is an experimental investigation that strongly explores the actions of individuals within their real life context (Yin, 2009). In the study a class of grade 8 learners was used as the case in a classroom setting where teaching, learning and assessment were taking place. This research approach provided a true picture of grade 8 learners' performance; errors and misconceptions were revealed in the simplification of algebraic expressions.

3.4 Quantitative and qualitative research

Quantitative research method is a data collection method which involves representation and statistical analysis of numbers (O' Leary, 2004). This method involves the use of arithmetical inquiry of data. The quantitative research method was used in the study as Mamba (2012) points out that quantitative research explores the occurrence of errors in the learners' test scripts. The quantitative inquiry was done on the learners' pre and post-test scripts to find the frequency and popularity of each and every type of error revealed by learners when simplifying algebraic expressions. The different error types were noted in order to trace the misconception underlying them. The possible reasons for the errors were investigated through the use of a focus group semi structured interview.

Qualitative research method was appropriate for the study, as it provided a full explanation of learners' thinking which results in learners' errors in algebra (Merriam, 1992). The research method allowed the researcher to come up with a complete report about learners' errors and misconceptions in algebraic expressions. Qualitative research is concerned with what people say or write as a way of expressing their thinking in certain situations (Olds, Moskal and Miller, 2005). Qualitative research gave learners an opportunity to give a justification of why they think their answers are correct when they are incorrect. The researcher probed the learners in their explanations in order to find out why they were making errors. Qualitative research aimed at understanding the possible reasons for learners' mistakes in algebraic expressions. Strauss and Corbin (1997) and Makonye (2011) state that the qualitative method focuses on understanding how people view the world, giving detailed information from carefully selected cases revealing people's feelings, emotions and thoughts that cannot be found using quantitative methods. The research method was very necessary in this study since it revealed learners' understanding/misunderstandings of algebraic concepts.

Interviewing of six grade 8 learners in the study was an illustrative example of the use of qualitative research. The learners' interview responses enabled the researcher to understand the learners' perceptions which led them into errors when simplifying algebraic expressions. The interview responses enabled the researcher to understand learners' struggles/ misunderstanding and misconceptions in algebra. The learners' responses in the interview together with the direction identified in the literature review were greatly valued and used in data analysis in chapter 4.

3.5 Mixed methods research

The following diagram shows how a mixed method research works

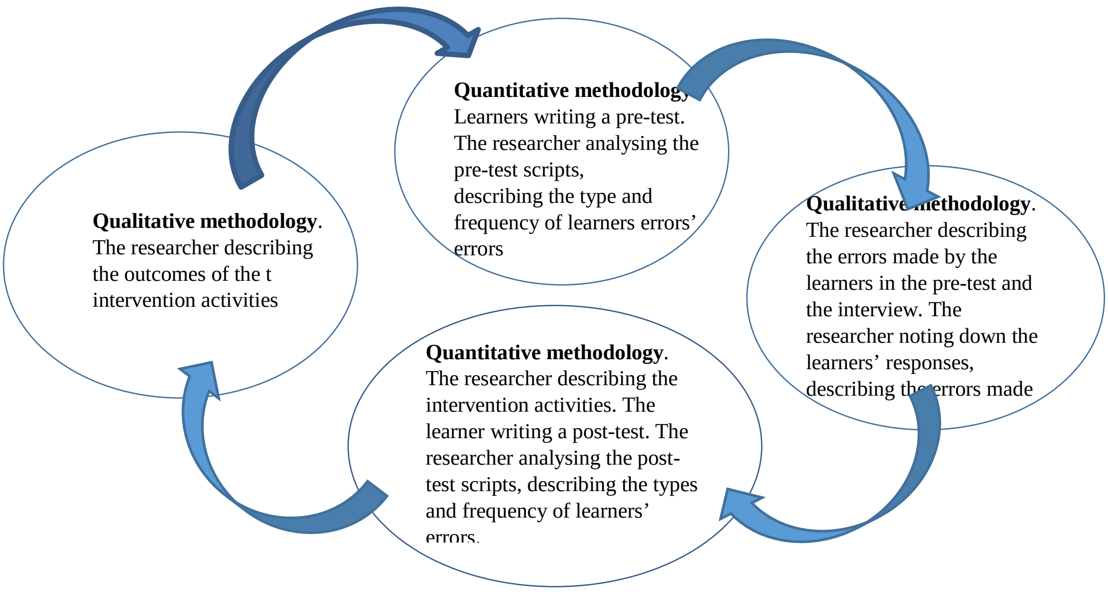


Figure 1: Summary of how the mixed method works

The study used a mixed research design where both quantitative and qualitative methods were employed in a single study in order to obtain a full detailed understanding of the research problem. Mixed method research provides a powerful mix in research with complementary strengths which allows for the collection of various sets of data (Miles & Huberman, 1994). The mixed method research methods supplemented each other and provided a broader picture of learners' errors and misconceptions in algebraic expressions. The researcher was able to investigate different types of data through this mixed method research and obtained clear and meaningful results which were used in data analysis in chapter 4. The researcher obtained quantitative data from both the pre and post-test where the different types of errors and their frequency were shown. Most of the qualitative data was obtained from learners' interviews. Here the learners' misconceptions, weaknesses and reasons for their errors on simplifying algebraic expressions were noted.

3.6 Sample and the sampling methods

Maree (2007, p.79) defines a sample as "the selected portion of the population for study". A reliable sample represents the entire population well. The population of the study was represented by hundred and twenty five learners of the grade 8 learners at the school where the study was done. The participant learners consisted of ten girls and twenty boys from one class of the entire grade 8 learners at the school. The average age of the participant learners was fourteen years. Sampling refers to the process of selecting a sample from a population to obtain information about an area of interest in a way that ensures that the population is well represented, (Brink, 1991). In this research the participants were purposively and conveniently selected. Convenience sampling involves choosing the nearest individuals to serve as respondents and purposive sampling involves intentionally selecting of participants that have the desired characteristics suitable for the research study (Cohen & Manion, 1995). The sample was conveniently selected in the sense that it was available and accessible to the researcher, since the researcher was teaching them mathematics. The sample was purposively chosen; they suited the study since they were mathematics learners with some knowledge of algebraic expressions.

3.7 Data collection methods

The data collection methods consisted of a pre and post-test and a semi-structured interview.

3.7.1 Pre and post-test

The researcher highlighted what was expected of learners in the study. The data was collected using a pre-test, an interview, observation and a post-test over a period of 6 weeks. Both the two tests were 1 hour long and consisted of questions 1.1 to 1.7 with 3 marks per question. These tests were conducted on different days after school to avoid disturbing the learners' normal contact time. These pre and post- tests were not for assessment purpose but were used only for the purpose of the research. Before the learners wrote the pre-test, the researcher revisited the concepts of integers as a way of recapping concepts before introducing the concepts of algebraic expressions. Some learners failed to add or subtract integers involving positive and negative numbers or both negative numbers. For instance some learners hardly simply problems like -9 + 4, the possible answers obtained were -5, +5, -13, and +13. Learners with such misconceptions can hardly simplify expressions like 5x – 4y + 12y -14x + 4.

The thirty learners wrote the pre and post-tests. Through the use of these tests, the researcher came up with some errors as they were shown on the learners' test scripts. The researcher invigilated the learners when writing the tests so that each and each and every learner show their understanding or misconceptions on simplifying algebraic expression. The researcher carried out a self-observation and observation of the learners during data collection and intervention processes. The purpose of the observation was for the researcher to have an idea of how learners view the tests in terms of how easy or hard the tests were. The researcher carried an interview with six learners who performed the least in the pre-test.

3.7.2 Interviews

Maree (2007, p.87) defines an interview "as a two-way conversation in which the interviewer asks the participant questions to collect data and to learn about the opinions of the participant". The researcher interviewed learners to understand why learners have produced some errors in the pre-test. An interview can be used to notice learners' behaviour and allows the researcher to collect data in greater depth (Creswell, 2012). The interview enabled the learners to say their thoughts of the pre-test. Qualitative data was gathered via this interview and this data was found from the learners' written scripts and explanations. The researcher audiotaped the learners' responses for accuracy purposes. After the interview the learners' responses were transcribed and this information was used during intervention and in data analysis. The researcher used a semi-structured interview guideline with open ended questions. Semi-structured interviews are flexible both to the researcher and the participant. This type of interview allows the participants to respond to any questions the way they feel like without deviating from relevant context of research study in order to accomplish what the study wanted to achieve.

A focus group interview enabled the research to obtain information from a group of people (Creswell, 2012). The focus group interview was used as a way of making learners feel at ease with the researcher as a one to one interview can be difficult to grade 8 learners. Learners were freely and confidently explaining their answers in the company of other interviewees. During the interview the individual learners were allowed to respond to the questions when his/her turn came. The learners were encouraged to listen to each other so that they learn from what other learners were saying. The researcher paid special attention to the learners' interview responses to avoid missing out any important information which was given. The researcher was very vigilant in controlling the interview procession in order to listen and note down any conversation said during the interview.

3.7.3 Procedure of the study

A pre-test was administered to a sample of thirty grade 8 learners at a certain school in Johannesburg in South Africa to check on their knowledge of simplifying algebraic expressions. The aim of the pre-test was to identify the nature of errors and misconceptions grade 8 learners have in algebra. The researcher marked the learners' scripts and analysed the identified errors from the learners' pre-test scripts. Thereafter a sample of six grade 8 learners was selected for the interview. These six learners were selected for the interviews based on the types of errors they had displayed in the pre and post-tests. These learners' performance in the pre-test was bad. The learners showed a lot of errors on the pre-test scripts that's why the researcher selected them for an interview. The interviews were conducted during break time for privacy purposes and to avoid disturbing the learners during their normal contact time. Numbers were used to represent learners' names so that the participant learners remain anonymous throughout the research study. The researcher sorted and grouped learners' written scripts depending on the types of errors identified on the scripts. A follow-up focus group interview was administered to six participant learners to have an idea of learners' misunderstandings which resulted in errors. The interview was audio-taped after the six learners agreed to that for clarity and accuracy purposes. The audio-tapes and verbatim transcripts were analysed in order for the researcher to be able to answer research questions 2.

3.8 Data analysis

3.8.1 Methods of data analysis

The researcher used a mixed method design comprising of both qualitative and quantitative data analysis methods. The quantitative analysis was done on the learners' pre and post-tests results and a detailed qualitative analysis was discussed with an emphasis on learners' interviews. The researcher wanted to see if learners would confirm their errors in the pre-test through their interview responses. In this data analysis some numerical values were used to represent the learners' names for anonymity purposes as from Miles and Huberman (1994).

The participants remained anonymous to protect their privacy as it was decided upon at the beginning of the research study.

3.9 Intervention

Hansen (2006) states that mathematics teachers need to understand how learners' errors arise and how to correct them. After the errors were identified from the learners' pre-test-scripts and the learners' interview responses, the researcher remediated the participant learners in the study. The lesson plans prepared by the researcher indicated the activities to be done in each day of the intervention. Guided discovery learning was the main priority of the intervention where learners' errors identified in the pre-test were used as a resource to help learners reduce errors on simplifying algebraic expressions. The researcher contrasted the correct answers with an error using variation theory by Tall and Vinner (1991). This theory lay its basis on the fact that for people to know what is right they must also be exposed to what is wrong for them to avoid it. Bruner (1966) argues that allowing oneself to engage into independent discovery of concepts enables one to acquire information fast and easily. The discovery learning used in the intervention afforded learners some opportunities to discover concepts by themselves. Specifically the researcher selected the learners in groups of five to scrutinise each and every learner's pre-test scripts, checking each and every learner's workings for correct conception of concepts, errors and misconceptions. The groups consisted of five learners of low, medium and high level of understanding of algebraic concepts.

All the learners were expected to participate in the group discussions and their contributions were written down since they were all important in the research study. Bruner (1966) suggests that students are more likely to remember concepts if they discover them on their own. The main reason why the researcher implemented discovery learning was for the learners to construct their on knowledge. The learners were given 10 minutes to do an individual and group inspection of each and every learners' pre-test script to try and check the errors on their scripts. The researcher strictly monitored learners' progress to avoid learners wasting of time, making noise and disturbing others during the intervention process.

In the group discussions the researcher appointed a group leader to manage the group for progress sake. The learners volunteered to be group leaders. Everything which was discussed in learners' groups was written down by one of the learners in the group for later use during the interventions. As the researcher was supervising the group discussion it was noted that arguments occurred among learners in defending the accuracy of their answers even though some were wrong. The learners were supposed to listen to each other and give each other time to speak since every learner's contributions were crucial in the study. According to Discovery-inquiry learning model allows the teachers to encourage students to gain experience by doing activities that enable them to find the concepts and principles for themselves. The researcher did not contribute anything in the discussions excerpt in situations where learners were asking for clarification in certain questions.

The researcher only guided the learners for them to discover concepts by themselves. There was a need for learners to discover why some of their solutions were considered right and some of the solutions were reflecting errors and misconceptions. The researcher while, supervising the learners noted down the essential parts of the learners' statements. On the second day of the intervention, the learners in their groups revealed their previous discussions to the entire class for class discussions to begin. The researcher wrote the pre-test questions on the board and allowed each group to present their agreed answers. Some groups presented more than one answer since they did not agree on one. For instance in item 1.1 of the pre-test the learners were required to simplify the expression, 4a+5+2a. The following answers were given by learners confirming them from their pre-test solutions; 6a+5, 11a, $6a^2$+5, 11 and $11a^2$. The researcher demonstrated to the learner why the answer 6a+5 was considered the correct and why the other answers were wrong. The researcher used variation theory by Tall and Vinner (1991) as stated above where a correct answer of 6a+5 was contrasted with the wrong answers 11a, $6a^2$+5, 11 and $11a^2$ for learners to differentiate the critical features involved in each. If learners are exposed to their errors in algebraic expressions, they can learn from their mistakes in future algebraic tasks and minimise their errors. The learners were discouraged to conjoin unlike terms, to remove variables from algebraic terms and to inappropriately apply exponential law of multiplications to addition of algebraic terms where 4a+2a were treated as $6a^2$ instead of 6a.

During the class discussions, learners were allowed say what they think about their solutions. There was room for learners to agree or disagree on the correct answers give. Those who disprove the answers given as correct were allowed to give a justification of why they think their incorrect answers were correct. After the teacher's and other learners' explanations to remediate learners with wrong answers some learners ended up realising their misconceptions but others continue to believe that their wrong answers were correct. The best way to get rid of a misconception is by causing a cognitive conflict (Makonye, 2013). In the teaching and learning of mathematics learners need to come to a point where they accept that their answers are wrong for them to be assisted by the teacher, other learner or mathematics textbooks for them to clear up their misconceptions. On this day of the intervention, the researcher observed that some learners were still attached to their misconceptions (failed to experience the cognitive conflict). The researcher took an extra mile during break time to help learners who seemed not to understand how to simplify algebraic simplifications. The intervention lasted for 1.5 weeks where each and every correct answer of each pre-test item was contrasted with some erroneous answers (variation).

As part of the intervention, the researcher revisited concepts of integers as according to the Curriculum and Assessment Policy Statements (CAPS, 2011) learners are expected to carry some manipulations on integers to solve problems in context. The research used a number line learner as a teaching aid to demonstrate how integers were added or subtracted. The concepts of adding and subtracting of integers were emphasised using the following examples, +4+(+)3 =+7(positive number + positive number = positive number), -4+(-3)=-7 (negative number plus + negative number=negative number) and -4+(+3)= -1 (the answer takes the sign of the bigger number if we consider the numbers undirected)). The researcher also used practical examples of the use of integers in real life situations in order for learners to understand that maths is real in their day to day lives. Examples given on directed numbers include the concepts of temperature, where high and low temperatures were represented by positive and negative numbers respectively. Other examples include the bird on top of the tree, the sea level and the fish under the sea. On multiplication and division of integers the following concepts were also emphasised; + x + = +, + x - = -, - x - = +. The learners were given a class activity consisting of 10 short questions to check on the learners' adding, subtracting, multiplying and dividing skills. The activity was marked and feedback was given and some errors were rectified as a way for preparing learners for the post-test.

3.10 Rigour

Rigour refer to the issue of reliability and validity of the research. Validity focuses on trustworthiness of the research as far as people's experiences are represented in the study. Contrary to this, reliability concerns with uniformity of the research results if the research is conducted elsewhere using the same data collection methods. Validity and reliability are talked about in the next paragraphs.

3.10.1 Validity

A valid research reveals the honesty truth of what transpired in the research (it's not biased). There were three types of validity which applied in the study as follows; interpretive-validity of the test, descriptive-factual accuracy of the tests and interview results and theoretical-ability to generalize findings to the target population (Maxwell, 1992). The researcher used all the three types of validity stated above. To improve the internal validity, the researcher gave the pre-test, post-test and interview guides to the other mathematics teachers to check if the tests were up to scratch and meaningful. After that the assessment tasks were sent to the researcher's supervisor, the Department of Education for a thorough check for errors, suitability and appropriateness before they were administered to the learners. To improve the external validity the researcher avoided using false and irrelevant results to suit the study. The key criteria of validity in qualitative research are trustworthiness, transferability, constancy and conformability, (Lincoln and Guba, 1985). All what is needed for the study to be valid is honesty, accountability and responsibility of all the research participants.

3.10.2 Reliability

Reliability is the extent to which a process produces comparable results under constant conditions at all times, (Creswell, 2012). A measure is reliable if results which are alike are produced somewhere else using similar research participants. Reliable tests of the study will produce similar results if the sample remains constant even if the researcher changed. The results of the tests and interviews were honestly presented in chapter 4. The researcher wanted the teachers, curriculum planners, subject advisors and the entire mathematics community to benefit from this study which is worthy trusting.

3.10.3 Ethical considerations

The researcher sought permission to do the study from Human Research Ethics Council (HREC), Gauteng Department of Education (GDE), and School Governing Body (SGB), the principal, the learners and their parents for authorisation to carry out the study. The study commenced after permissions were granted by all the stakeholders stated above. The research site and the research participants were conducted with respect. The researcher explained to the participant learners the purpose of the study before data was collected. The learners were clarified of what they were expected to do before they agreed to participate. No learner was persuaded or forced to take part in the study and no incentives were used to lure learners to agree to be part of the study. The participation of every learner was open and free and every participant had a right to withdrawal from the study at any time he/she felt like.

The researcher protected the anonymity of the participants by assigning numbers to the learners instead of using their real names. All the data collected from the pre-test, post-test and interview were analysed without revealing the identities of the learners who contributed to this data. The researcher did not inconvenience or harm or discomforts to any of the participating learners. The researcher had increased the learners' self-confidence by avoiding naming and shaming of learners who did not perform well in the two tests. The results of the study were honestly reported. Studies which were completed by others were credited and referenced to avoid plagiarism. The researcher had tried to make all the information in this study to be understandable. The practical significance of the research study was well communicated to my supervisor, the faculty of science, the Gauteng Department of Education to check for clarity and a mistake before a go ahead was given. The learners' test scripts are going to be stored and locked in a safe for about 3-5 years and destroyed thereafter.

3.10.5 Informed Consent

The purpose of the study was communicated to the parents and learners before they signed some consent forms. Informed consent gave each learner the freedom to choose what they wanted as far as taking part in the study was concerned. The informed consent offered the participants the right to withdraw from the study at any time if necessary.

3.10.6 Voluntary participation

A good researcher does not coerce the learners into the study but show empathy to the participants (Creswell, 2012). The grade 8 learners were notified of what they were expected to do before they agreed/disagreed to participate in the study. The parents signed letters of consent to allow their learners who were minors to participate in the study. There after the learners voluntarily signed the consent forms for agreeing to participate in the study. The learners freely made their choices of participating in the study without being persuaded to do so.

3.10.7 Violation of privacy and anonymity

The researcher protected the identity of all the participants by keeping their names anonymous through assigning fake names to the learners. No data was gathered using the actual names of the participants. The researcher assigned numbers to learners instead of using their real names.

3.10.8 Publication of findings

The researcher reported the findings of the study in an exact way as they came out to be. Honesty took the pivotal role in this study. The researcher referenced all the quotations used in this study to avoid plagiarism and accredited all the work used by other researchers. The information used in the research study was clear and understandable to those who took part in the study participants and the readers.

3.11 Conclusion

This chapter gave a detailed description of the research design and methodology that was employed when carrying out the study. In the discussion, sampling methods, data collection methods, procedure used in the collection of data, validity and reliability of the research methods and ethical issues were discussed.

CHAPTER 4: DATA ANALYSIS

4.1Introduction

In this chapter, the research findings which emerged from the collected data were presented and the results from the research findings are analysed. The objective of the study was to identify learners' misconceptions underlying their errors and describe and discuss the errors in the learners' micro worlds (Davis, 1984).The purpose of the analysis was to identify grade 8 learners' misconceptions which resulted in errors on simplifying algebraic expression and to explore the possible reasons for the errors. The identification of learners' errors was necessary for the researcher to use them as a resource during intervention to help learners reduce them when simplifying algebraic expressions. Data was analysed in order to answer the following research questions;

1. What are the types of errors and misconceptions grade 8 learners reflect on simplifying algebraic expressions?
2. What are the possible reasons for these errors?
3. To what extent do resourcing learners' errors and misconceptions through the discovery method in teaching help learners to diminish them?

4.2 Data analysis.

The data analysis was based on the grade 8 assessment tasks done. The tasks consisted of a pre-test, an interview, an intervention and a post-test. Specific errors were identified by assessing and evaluating the students" independent work and this error analysis informs priorities for teaching (Riccomini, 2005). The researcher strictly supervised the learners when writing the pre and post-test because the researcher was interested to know the individual learners' thinking in simplification of algebraic expressions. The researcher analysed all the thirty pre-test scripts of the learners so as to identify the learners' misconceptions which resulted in errors in simplifying algebraic expressions. The researcher critically analysed and evaluated each and every learner's scripts and noted down all the learners' misconceptions in anticipation of understanding learners' thinking which caused pre-test errors. The analysis of the pre-test was done for the researcher to be able to answer Research Question 1.

After the analysis of the pre-test, the researcher conducted a focus group interview to six learners. The six learners were chosen on the basis of the types of errors they displayed in their pre-test scripts. The purpose of the interview was to gain an insight of the learners' misunderstandings or knowledge gaps on simplifying algebraic expressions. The learners were given an opportunity to explain how they came up with the pre-test solutions and the reasons for the solutions. The possible reasons for the errors were inferred from the learners' interview responses together with information from literature. The researcher analysed the data from the interviews to understand the learners' mathematical thinking which resulted in errors and misconceptions when simplifying algebraic expressions as revealed in the pre-test in order to answer Research Question 2.

Teachers can use learners' errors as a resource that helps them to understand learners' thinking for the benefit of the learner, (Makonye and Luneta; 2013, Nesher, 1987). After the analysis of the pre-test and analysis of the interview responses the researcher employed an intervention strategy to the same learners who wrote a pre-test. This was done through discovery learning and use of learners' pre-test errors as a resource to help learners reduce them on simplifying algebraic expressions. The learners' performance in the post-test was observed to check if the performance would have improved after the above stated intervention strategy was administered in order to answer Research Question 3. The study investigated the extent to which resourcing of errors helped learners to minimise them.

4.3 Type of errors and error analysis

Luneta (2008) states that error analysis is a process of discovering what errors learners are making, why those errors are made and the most suitable methods of dealing with the identified errors. Error analysis was done to identify the types of errors learners made when simplifying algebraic expressions and possible reasons for the errors. The learners' errors identified were diagnosed and named depending on how the error was committed. These errors were classified using Hodes and Nolting (1989)'s four error categories which are; procedural, conceptual, carelessness and misapplication errors. Error analysis means that teachers need to use students' mathematical errors to improve instructions and correct misunderstandings, (Riccomini, 2005). The error analysis was done so that the researcher understand the learners' challenges when simplifying algebraic expressions and to find ways to improve teaching of algebraic expressions for the benefit of the learners. The tests provided evidence of learners' misconceptions underlying their errors, this showed learners' thinking and reasoning when simplifying algebraic expressions. The learners' solutions and explanations of their answers in both the tests and the interview helped to identify reasoning behind them (Osei, 2000). Some learners' errors and misconceptions were shown on scanned vignettes illustrated below. The researcher identified the types of learners' errors by critically looking at the all the learners' workings in the tests together with using Hodes and Nolting (1998)'s four types of errors which were defined in chapter 2 are as follows;

- Careless errors
- Conceptual errors
- Application errors
- Procedural errors

For more information about the above stated errors refer to page refer to page 30 in chapter 2.

4.5 Quantitative analysis of the learners' performance in the pre-test

This section gives a description of the quantitative analysis of learner performance in the pre-test. In the pre-test the learners were supposed to answer algebraic questions from item 1.1 to item 1.7. Each item consisted of 3 marks each and total marks in the pre-test were 21. The pre-test items were testing learners on adding, subtracting, multiplying and expanding of brackets of algebraic terms. The marks obtained by learners in each test item were categorised as follows;

- Level 0-referred to a mark of $\frac{0}{3}$
- Level 1-referred to a mark of $\frac{1}{3}$.
- Level 2–referred to a mark of $\frac{2}{3}$.
- Level 3-referred to a mark of $\frac{3}{3}$.

The marks obtained by each learner in each pre-test from item 1.1 to item 1.7 were noted and tabulated as shown in table 2 below;

Table 2: Summary of learners' performance levels in each of the pre-test items

Item	**Level 0**	**%**	**Level 1**	**%**	**Level 2**	**%**	**Level 3**	**%**	**Unattempt**	**%**
1.1	3	10	2	6	6	20	19	64	0	0
1.2	6	20	7	23	8	27	9	30	0	0
1.3	23	77	4	13	3	10	0	0	0	0
1.4	6	20	8	27	5	17	9	30	2	6
1.5	9	30	8	27	4	13	5	17	4	13
1.6	21	70	3	10	1	3	0	0	5	17
1.7	16	54	9	30	0	0	1	3	4	13

The following graph shows the learners' performance in each of the pre-test item

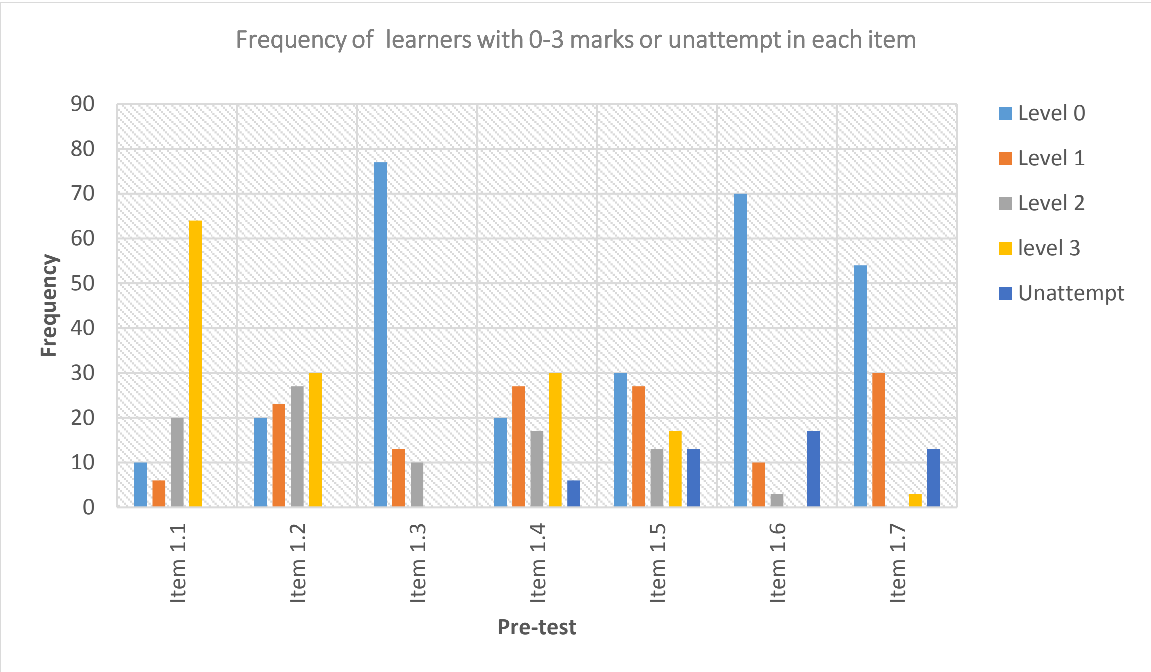


Figure 2: Frequency of learners with 0 to 3 marks in the pre-test items 1.1 to 1.7

Table 2 and figure 2 show the learners' performance in each and every item of the pre-test. The learners' best performed item was in item 1.1 with 84% of the learners obtaining marks above 50% and 64% of these learners getting full marks in that item and 10% of learners obtained a mark of zero. The second best performed item was item 1.2 with 57% of the learners obtaining a mark above 50% and 37% of these learners getting full marks. This indicated that most learners were able to add and subtract algebraic terms using the pre-conceived concepts of directed numbers. The learners' performance in item 1.1 and 1.2 was fair; some learners showed a good understanding of addition and subtraction of algebraic terms. In item 1.1, 1.2 and 1.4, 10%, 20% and 20% of the learners got zero marks respectively.

In items 1.3, 1.6 and 1.7, learners did not perform well with 77%, 70% and 54% of the learners getting zero marks. In items 1.3, 1.6 and 1.7 the learners failed to simplify the algebraic expressions due to an incomplete distribution of terms, incorrect expansion of brackets and reversing of signs of algebraic terms. An example of where learners changed the operational signs of terms was observed in the expansion of the second algebraic terms in item 1.7. Here the learners simplified the expression – $(-2-9x+3x^3)$ to $2-9x+3x^3$or $2-9x-3x^3$ and so on. The least performed item was item 1.3 where learners incorrectly simplified the expression 3-5(2x-1) to -2(2x-1) which was equated to -4x+2 or -4x+1 or 4x+2 etc. The learners subtracted 5 from 3 before expanding brackets. Learners perform badly in item 1.3 because the order of operation (BODMAS rule) was not adhered to. Learners subtracted 5 from 3 before the expanded the bracket -5(2x-1).

Table 3: Summary of learners' marks as a percentage in the pre-test

Level	Marks	Number of learners	Percentage of learners
1	0-29	12	40
2	30-39	7	23
3	40-49	4	13
4	50-59	2	8
5	60-69	3	10
6	70-79	2	6
7	80-100	0	0

The number of learners who achieved level 1 to 6 as expected in the senior phase performance standards are shown in table 3 above. In grade 8 and 9, 40% is the minimum promotional mark. Overall the learners' performance in the pre-test was poor. Out of the 30 learners who wrote the pre-test, 63% of these learners failed the test, 40% of these learners had marks ranging from 14-29% and 23% of the learners having marks ranging from 30-39. Most the learners obtained marks ranging from 0-29%, followed by marks ranging from 30-39% then followed by the marks ranging from 40-49%. There is an indication that 79% of the learners (majority) had marks below 50%. Only 37% of the learners passed the test with most of the marks ranging from 40-49% and no learners obtained distinctions.

Table 4: Summary of grade 8 learners' results of the pre-test

Grade	Number of learners wrote	Number of learners passed	Number of learners failed	Grand total	Pre-test average	Percentage pass	Percentage fail
8	30	11	19	1129	38	37	63

From table 4 above, 30 grade 8 learners who wrote the pre-test. The percentages of learners who passed and failed the test were 37% and 63% respectively and a pre-test average of 38%, a mark below the minimum requirement. The marks the learners got on each question item and the overall mark in the pre-test was an indication of learners' understanding of the algebraic tasks done. The results showed that most learners failed the test implying that these learners encountered some difficulties when simplifying algebraic expressions.

4.6 Qualitative analysis of the learners' errors in the pre-test

This section gives a description of the qualitative analysis of learners' errors identified in the pre-test. It gives some highlights of types and frequency of learners' errors as identified from their pre-test scripts. The researcher critically analysed all the 30 pre-test scripts for each and every learner in the sample. This was done so that the errors' types and frequency in the pre-test was determined. These types of errors and their frequencies in the learners' pre-test scripts were noted and recorded. Table 5 was prepared to show the categorised the errors as shown in the learners' pre-test scripts from item 1.1 to item 1.7. This table gives detailed information about learners' errors and the possible reasons for the errors as analysed from the learners' pre-test scripts.

Table 5: Shows the type of learners' errors on the pre-test scripts from item 1.1 to item 1.7

Item	Application	Careless	Conceptual	Procedural
1.1	**Inappropriate application of rules** $4a+2a+5=6a^2+5$.	**Changing addition sign to subtraction sign** $4a+5+2a=4a-5-2a=2a-5$, **Omitting of terms** $4a+5+2a=4a+5$.	**Incorrect addition of terms** $4a+2a+5=5a+5$ or$10a$ **Conjoining of terms**, $4a+2a+5=11a$ and removing of variables, $4a+2a+5=11$.	No procedural errors
1.2	**Changing an algebraic expression into an equation** $5x+2x-3y-4y-3a=7x-7x = 3A$	**Omission some algebra terms** $5x+2x-3y-4y-3a=7x$, $5x+2x-3y-4y-3a=7x-7y+2a$, $5x+2x-3y-4y-3a=7x-7y-3$, $5x+2x-3y-4y-3a=7x-7y-2a$.	**Incorrect addition and subtraction of terms** $5x+2x-3y-4y-3a=7x-3a$, $2x-3y-4y-3a=6x-7y-3a$, $5x+2x-3y-4y-3a = 6x-6y-3a$**, conjoining of unlike terms** $5x+2x-3y-4y-3a=6xy-3a$	**Changing the signs of terms of the whole algebraic expression** $5x+2x-3y-4y-3a = 7x+7y+3a$ or $7x+7y+4y$
1.3	**Creation of learners' own rules**, $3-5(2x-1) =3-5+1-2x=-2-2x$, **misinterpretation of information** $3-5(2x-1) =3-5x2-1=-8$, **application of own rules**, $3-5(2x-1) =3-5-1-2x=2-1-2x$ or$3-5+2x-1$.	**Obtaining right answer from wrong working** $3-5(2x-1) =3-10x-5=8-10x$, **omission of algebraic terms**, $3-5(2x-1)=3-10-5=12$	**Incomplete distribution of terms and incorrect multiplication of terms** $3-5(2x-1)=3-10x-1$or $3-10x-5$, **changing of algebraic terms and conjoining of terms**, $3-5(2x-1)=3-5+1-2x=-2+2x=4x$	**Incorrect order of operation, and incorrect multiplication** of $3-5(2x-1)=-2(2x-1)=4x-1$, **creation of own rules and conjoining**, $3-5(2x-1)=-2(-10x-5)= -20x+10=-10x$, $3-5(2x-1)=3(-10x+5)=-30x+8$, $3-5(2x-1)=15-(6x-3)$.
1.4	**Creation of learners' own rules**, $5(p^2+2p-3) =5-4-3$, $5(p^2+2p-3) =5(p+2p-3) (p+2p-3)$, $5(p^2+2p-3) =5+p^4-3$, **application of own rules and conjoining** $5(p^2+2p-3) =2p^2+2p-3=7p^2-3=14p-3$.	**Omission of a variable in an expression** $5(p^2+2p-3)=5p^2+10p-3$ or $5p^2+10-15$	**Incomplete distribution of terms**, $5(p^2+2p-3) =5p^2+10p-3$, **incorrect multiplication of terms** $5(p^2+2p-3) =5p^2-10p+15$, **conjoining of terms** $5(p^2+2p-3) =5p^2+10p-15=15p^2-15$.	**Incomplete distribution of terms and conjoining**, $5(p^2+2p-3)=5p^2+7p+2$, $5(p^2+2p-3)=5p^2+2p-3=27p-3$
1.5	**Creation of learners own rules**, $x(x-1)-5x=(x-1)(1-x)=5x-1$, $x(x-1)-5x=x(-x)-5x=x(-x-5x)=-6x$	**Omission of terms** $x(x-1)-5x=x^2-x-5x=x^2-5x$, **incorrect distribution of terms and omission of variables**, $x(x-1)-5x=x-x-5=-5x$	**Incorrect and incomplete distribution of terms** $x(x-1)-5x=2x-1-5x$, **conjoining of terms**, $x(x-1)-5x=x^2-x-5x=-5x$, **incorrect addition of terms**, $x(x-1)-5x=x^2-x-5x=x^2-4x$	**Changing of operation signs**, $x(x-1)-5x=x^2-x-5x=1-5x-x^2=-4-x$.
1.6	**Creation of learners' own rules and conjoining**, $a^2-5a+6-(a^2+4a+4) = a^2-5a+6-(6a^2+24a+24) =a^2-5a+6-54a$, **application of own rules**, $a^2-5a+6-(a^2+4a+4) =6a^2+5a-4a+4a^2=5a+12a-4a+8a=17a-12a=5a$.	**Incomplete distribution and addition of term**, $a^2-5a+6-(a^2+4a+4) =a^2-a^2-5a-4a-6+4=a^2-a-10$.	**Conjoining of terms**, $a^2-5a+6-(a^2+4a+4) =11a^2-(10a^2) =a^2$, **incomplete distribution of terms**, $a^2-5a+6-(a^2+4a+4) =a^2-5a+6-a^2+4a+4=9a+10$.	**Incorrect procedures used**, $a^2-5a+6-(a^2+4a+4)=5a+4a-a^2-a^2-6+4=9a-a^2-10=8a^2-10$, **no distribution of terms**, $a^2-5a+6-(a^2+4a+4)=a^2-5a+6-a^2-5a+4a+6-4=-a+2$
1.7	No application errors.	No careless error	**Incorrect and incomplete distribution of terms and incorrect addition**, $3(x^2+3x-2)-(-2-9x+3x^3) = 3x^3+9x-6-2-9x+3x^3=6x^3+x-4$.	**Incorrect distribution of terms**, $3(x^2+3x-2)-(-2-9x+3x^3)= 3x^3+9x-6+6-27x+9x^3=12x^6+18x$, **incomplete expansion of terms and conjoining** $3(x^2+3x-2)-(-2-9x+3x^3)= 3x^3+3x-2+2-9x+3x^3=9+9-2=16x$ or $3x^2+3x-2-11x+12x^3=12x^6+8x-0$.

The reasons for the errors as shown by table 5 were, inappropriate application of rules, creation of learners' own rules, incorrect addition/subtraction of algebraic terms, inappropriate order of operation, omission of algebraic terms, changing of the sign of the algebraic terms, incorrect/incomplete distribution of terms and conjoining.

Table 6: Shows the frequency of each type of error per item

Item	**Application**	**%**	**Careless**	**%**	**Conceptual**	**%**	**Procedural**	**%**	**None**	**%**	**Unattempt**	**%**
1.1	1	3	2	7	7	24	0	0	20	66	0	0
1.2	1	3	5	17	12	40	3	10	10	30	0	0
1.3	6	20	1	3	7	24	13	43	0	0	3	10
1.4	4	13	2	7	7	23	5	17	11	37	1	3
1.5	2	7	3	10	12	40	4	13	4	13	5	17
1.6	5	17	1	3	9	30	5	17	6	20	4	13
1.7	0	0	0	0	8	27	15	50	4	13	3	10
Total	19		14		62		45		55		16	

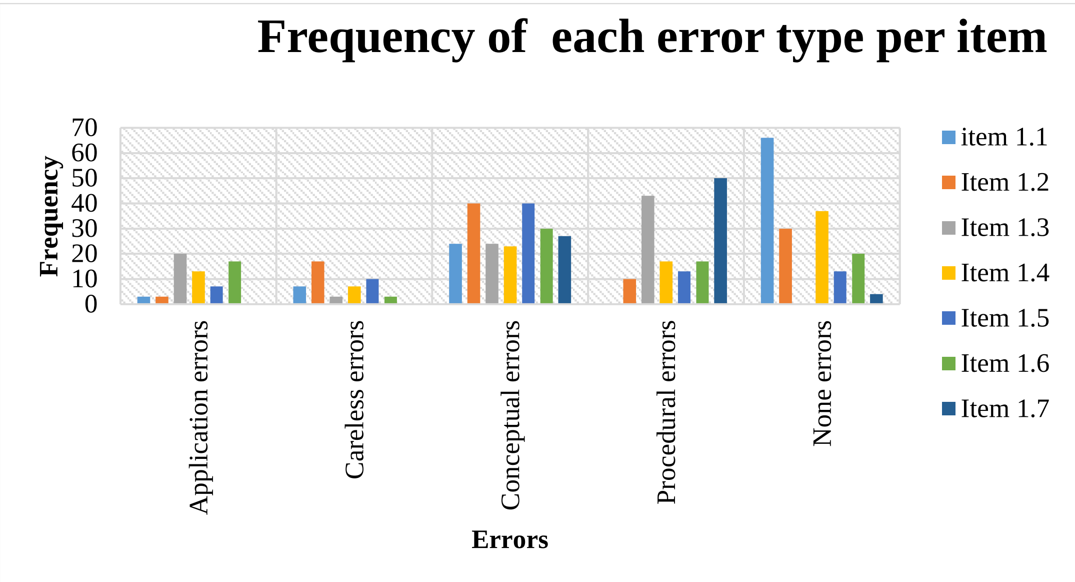


Figure 3: Showing the types and frequency of learners' errors in the pre-test

In item 1.1 the learners' performance relating to their errors was fairly done with only 34% of the learners having errors in that item The errors with the highest frequency were conceptual errors, followed by careless then followed by application errors and no procedural errors with frequencies of 24%, 7%, 3% and 0% respectively. The reason for the errors were due to incorrect addition of terms, conjoining of unlike terms, omission of terms or variables, reversing of the operational signs of the algebraic terms and inappropriate application of algebraic concepts for. These were revealed in learners' solutions when they simplified the expression 4a+2a+5 to 5a+5 or 6a or 6a-5 or $6a^2$+5.

In item 1.2 the learners' performance relating to their errors was not good with 70% of learners having errors in this item. The errors with the highest frequency were conceptual errors, followed by careless then followed by procedural errors then followed by application errors with frequencies of 40%, 17%, 10% and 3% respectively. The reason for the errors were due to incorrect addition of terms, conjoining of unlike terms, incorrect procedures where the operational signs of terms were reversed and learners' creation of own rules. These were revealed in learners' solutions when they simplified the expression 5x+2x-3y-4y-3a to 7x-y-3a or 6x-7y-3a or 6xy-3a or 7x+7y+3a or 7x+7y=3A.

In item 1.3 the learners' performance relating to their errors was very bad with 100% of learners displaying errors in their pre-test scripts. The errors with the highest frequency were the procedural errors, followed by conceptual errors then followed by application errors then followed by careless errors with frequencies of 43%, 24 %, 20% and 3% respectively. The reason for the errors were due to learners failing to follow the BODMAS, the learners subtracted the algebraic terms before the bracket was expanded, incorrect subtraction and multiplication of terms, omission of terms and creation of learners' own rules. These were revealed in learners' solutions when they simplified the expression 3-5(2x-1) to -2(2x-1) =4x-1 or 3-10-5=12 or 3-5+2x-1.

In item 1.4 the learners' performance relating to their errors was bad with 60% of learners having errors in this item. The errors with the highest frequency were conceptual errors followed by procedural errors then followed by application errors then followed by careless errors with frequencies of 23%, 17%, 13% and 7% respectively. The reasons for the errors were due to incorrect or incomplete expansion of brackets, conjoining of unlike terms and creation of learners' own rules. These were revealed in learners' solutions when they simplified the expression $5(p^2+2p-3)$ to $5p^2+7p+2$ or $5p^2+2p-3$ or $15p^2-15$ or $5+p^4-3$

In item 1.5 the learners' performance relating to their errors was bad with 70% of learners having errors in this item. The errors with the highest frequency were conceptual errors, followed by procedural errors, then followed by careless errors, then followed by application errors with frequencies of 40%, 13%, 10% and 7% respectively. The reason for the errors were due to conjoining of unlike terms, incorrect/incomplete distribution of terms, incorrect addition or subtraction of terms, omission of algebraic terms and creation of learner's own rules. These were revealed in learners' solutions when they simplified the expression, $x(x-1)-5x$ to $x^2-x-5x=-5x$ or $2x-1-5x$ or x^2-4x or $x(-x-5x)=-6x$.

In item 1.6 the learners' performance relating to their errors was bad with 67% of learners having errors in this item. The errors with the highest frequency were conceptual errors, followed by both procedural and application errors then followed by careless errors with frequencies of 30%, 17% and 3% respectively. The reason for the errors were due to incorrect addition of terms, incomplete or incorrect expansion of brackets and learners' creation of own rules. These were revealed in learners' solutions when they simplified the expression $a^2-5a+6-(a^2+4a+4$ to $a^2-a^2-5a-4a-6+4=a^2-a-10$ or $a^2-5a+6-(6a^2+24a+24)=a^2-5a+6-54a$,

In item 1.7 the learners' performance relating to their errors was extremely bad with 77% of learners displaying errors on their pre-test scripts. The errors with the highest frequency were procedural errors followed by conceptual errors; there were no careless and application errors then with frequencies of 50%, 27%, 10% and 0% respectively. The reasons for the errors were due to incorrect addition, incomplete distribution of and conjoining of unlike terms. These were revealed in learners' solutions when they simplified the expression $(x^2+3x-2)-(-2-9x+3x^3)$ to $3x^3+9x-6+6-27x+9x^3=12x^6+18x$ or $3x^3+9x-6-2-9x+3x^3=6x^3+x-4$.

The researcher put overall total frequency for the conceptual, procedural, application and careless errors. Overall, conceptual errors were the most prominent errors followed by procedural errors, followed by application errors and the errors with the least frequency were careless errors with total frequencies of 62, 55, 19 and 14 respectively. This showed that most learners encountered some problems when simplifying algebraic expression due to lack of conceptual and procedural understanding of algebraic concepts. The contribution of errors due learners' carelessness and misapplication of concepts were not very significant as compared to the errors due to misunderstandings of concepts and use of wrong procedures on simplifying algebraic terms.

Table 7: Showing the percentage of learners with errors in each pre-test item

Pre-test items	1.1	1.2	1.3	1.4	1.5	1.6	1.7
% of learners with errors	34	70	100	60	70	67	77

The percentages of learners with errors in each and every item from item 1.1 to item 1.7 are shown on table 7 above. In descending order the most difficulty items were item 1.3, followed by item 1.7, followed by both item 1.2 and 1.5 followed by item 1.6 and the item 1.1 where, 100%, 77%, 70% , 67%, 60% and 34% of learners with errors in each item.. Overall the results indicated that most learners failed to follow the BODMAS, failed to expand brackets, applied learners' own rules, omit some algebraic terms and reversed the signs of algebraic terms. For more information about learners' errors and their possible reasons refer to table 5 on page 52.

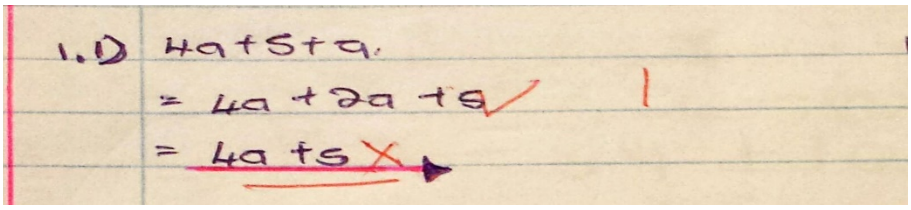
4.7 Qualitative analysis of the pre-test using scanned vignettes

In this section the researcher selected some learners' scripts and scanned them to put them in this section. These scripts were scanned so as to practically show the learners' thinking which resulted in misconceptions and errors. The researcher scanned vignettes to show the types of errors made by learners in item 1.1, 1.2, 1.3, 1.4, 1.5, 1.6 and 1.7 in the pre-test. The number of learners used in each and every item were not consistent. This was so because some the learners displayed on the learners' scripts were not possible to leave them. The learners were varied.

Item 1.1 *Simplify the following algebraic expression 4a+5+2a*

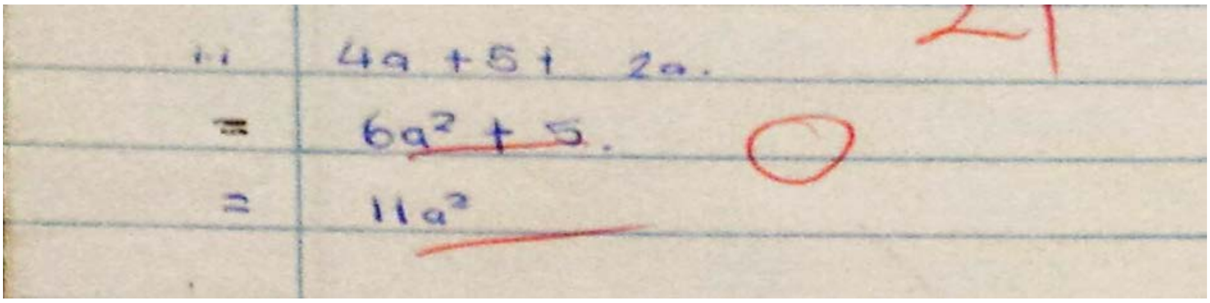
In item 1.1, learners were required to simplify the algebraic expression 4a + 5 + 2a+5 to 6a+5. The objective of this question was to find out if learners can add algebraic expressions. The learners were tested on their understanding of the concepts of like and unlike terms in adding algebraic terms. Also the learners' pre-knowledge of addition of integers was put on the spot light.

Learner 1 of item 1.1



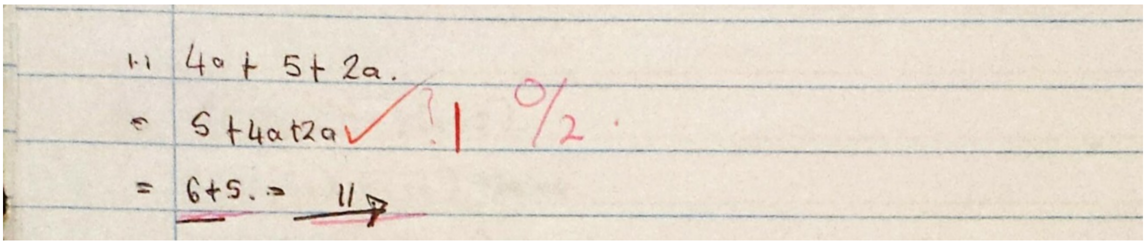
The learner incorrectly copied the question and wrote 4a+5+a instead of 4a+5+2a, this is a slip because the mistake was immediately rectified in step 1. The learner did not pay much attention to the solution process. In the last step the learner wrote an incorrect answer forgot to add 4a to 2a and got a wrong answer 4a+5 instead of 6a+5. One of the possible reasons for learners' two slips was due to math anxiety which caused the learner to forget and lost one's self-confidence (Tobias, 1993). The type errors indicated here were careless errors.

Learner 2 of item 1.1



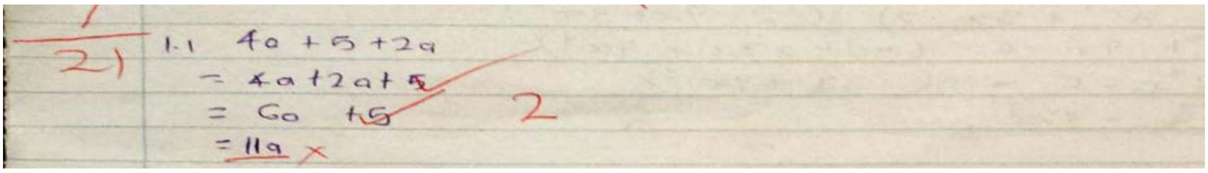
The learner thought that $4a+2a=6a^2$ since the learner viewed the combination of the two a's as a^2. The addition of exponents is an exponential law applicable when multiplying terms with the same base. The learner applied knowledge that is correct in one domain to another different domain which it is not applicable (Smith, DiSessa & Roschelle, 1993). An error the learner made was failing to accept 6a+5 as answer and was contented with answer $11a^2$). The learner viewed the plus sign as procedure to calculate something. There is an indication of an application error together with a conceptual error.

Learner 3 of item1.1



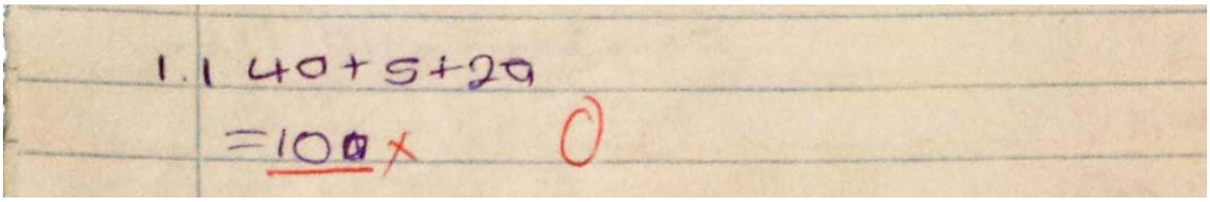
In the last step of the solution, the learner removed the variable a in the algebraic expression and ended up with a numerical values 6+5=11. This is supported by Collins & Romberg (1975) and Kuchemann (1981) who state that children are reluctant to record an algebraic statement as an answer they tend to give numerical values to letters or variables. The learner tended to ‘overgeneralise’ new knowledge of algebra into pre-existing correct knowledge of arithmetic (Olivier, 1989).This is an indication of a procedural and application errors.

Learner 4 of item 1.1



The learner successfully simplified 4a+2+2a to 6a+5 but finally wrote an incorrect answer of 11a. The learner combined unlike terms to a single term as Robinson and Tirosh (1994) point out that procedural learner think that open expression is incomplete so they tend to find a final answer. The learner viewed 6a+5 as an incomplete solution and further simplified it to 11a. The learner has some difficulties in computing algebraic terms he/she conjoined unlike terms to a single term. This resulted in conceptual and procedural errors.

Learner 5 of item 1.1



The learner did not add the terms correctly 4+5+2 is equal to 11 not 10, the skill of addition of terms is lacking. According to Booth (1999), learners have a tendency of simplifying algebraic answer to a single term. The learner had combined the unlike terms to a single term 10a (conjoining). The learner lacks some understanding of concepts of like and unlike terms which cannot conjoin to one term. This resulted in a conceptual and procedural error

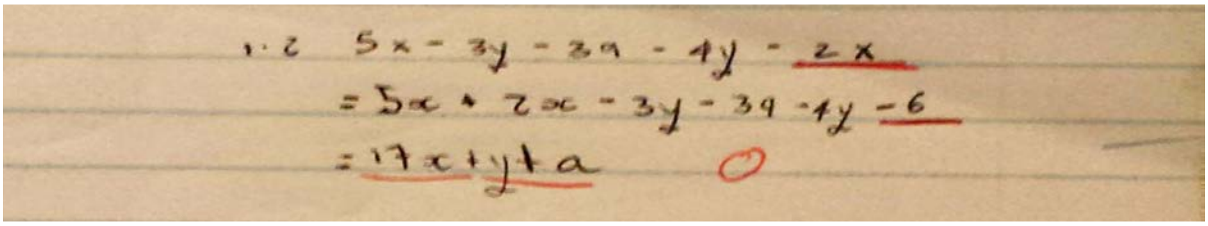
4.7.1 A summary of learners’ errors in item 1.1 of the pre-test

In item 1.1, all the learners attempted this question despite the fact that some learners failed to come up with the correct solution. The most frequent error was the conceptual error where learners were conjoining unlike terms to a single term and were incorrectly adding terms. Some learners used wrong procedures to simplify algebraic expressions for instance learners changed the algebraic expression to arithmetic. They removed the variable in the algebraic expression and remain with one numerical value for instance 4a+2a+5=4+2+6=11. Careless errors were committed when learners mistakenly omitted some terms from the algebraic expression. Some learners failed to add the terms due to carelessness.

Item 1.2 *Simplify the following algebraic expression 5x-3y-3a-4y+2x*

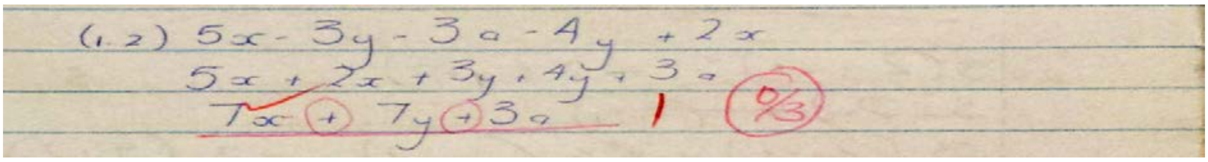
The learners were supposed to simplify 5x+2x-3y+4y-3a to 7x-7y-3a. The learners were tested on the concepts of addition and subtraction of algebraic expressions involving three different variables. The learners were supposed to use the concepts of like and unlike terms for them to be able to simplify the algebraic expression. There was need for learners to use their previous acquired knowledge of addition and subtraction of integers to help them to simplify the algebraic expressions which was at hand.

Learner 1 of item 1.2



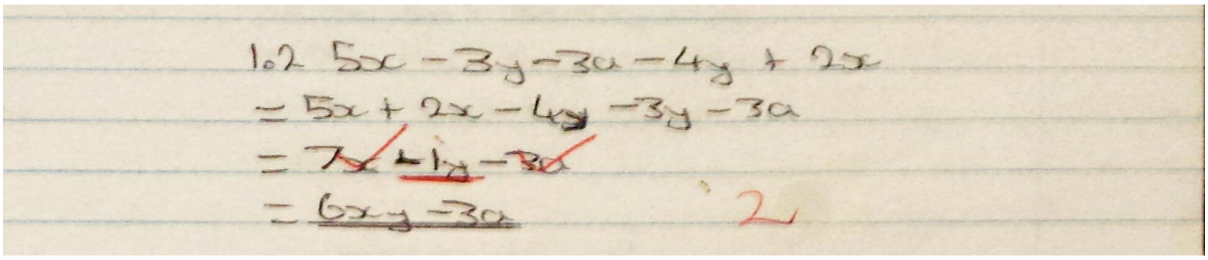
The learner made a mistake by changing +2x to -2x and in step 1. According to Riccomini (2005) the mistake made was a slip because the learner corrected it by him/herself without outside help. There is an indication of learners’ lack of seriousness, -6 was introduced in step 2 from nowhere and changed the subtraction operations into addition operations. In the last step of the solution the learner added all the numerical values to get 17x .Many rules of algebra seem arbitrary and meaningless to children, that’s why the learner invented his/her own rules the procedures were never taught in class. There is an indication of procedural, conceptual and careless errors.

Learner 2 of item 1.2



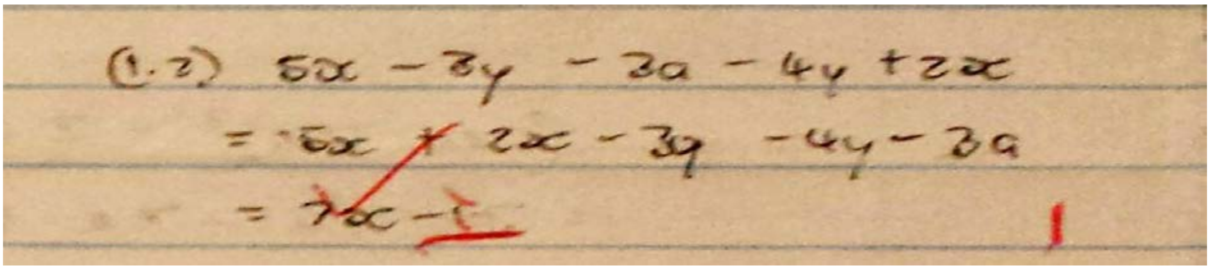
The learner unnecessarily changed the subtraction signs in the initial algebraic expressions to addition signs where -3y-4y and -3a were changed to +3y+4y and +3a respectively. The reversing of operational signs is possible when solving equations not in adding or subtracting of algebraic expressions (Smith, DiSessa & Roschelle, 1993). This is an indication of application and procedural errors.

Learner 3 of item 1.2



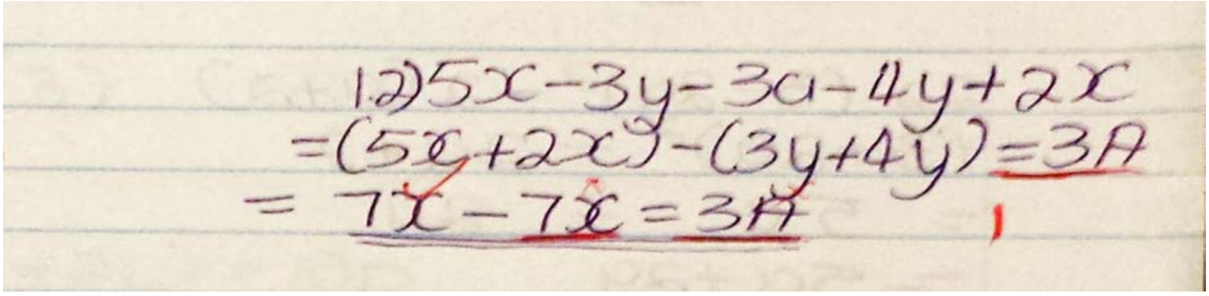
The learner simplified -4y-3y as –y, which shows that the learner struggles with addition of integer cannot add and subtract algebraic expressions. The learner simplified 7x-y to 6xy combining unlike terms 7x-y to single term of 6xy which is a computational error. According to Ashlock (1994), computational errors arise from misapplication of rules caused by learners through application of failing strategies. The learner conjoined 7x-y to 6xy, incorrectly combining two unlike terms. The learner lacked an understanding of like and unlike terms and hence failed to simplify the algebraic expression. There is indication of a conceptual error.

Learner 4 of item 1.2



In the last step of the solution, the learner calculated -3y-4y as -1 instead of being equal to -7y, the variable y and the term -3a were all eliminated from the algebraic expression. The learner failed to add and subtract terms in the algebraic expression. This result indicated learners' lack of seriousness. The learner hurriedly simplified the expression thereby omitting some of the algebraic terms. There is an indication of conceptual and careless errors.

Learner 5 of item 1.2



The learner partly simplified -4y-3y to -7 xs instead of -7y this is due to carelessness. This shows lack of concentration of the learner. This might have been caused by memory overload or tiredness (Higgins et al., 2002) There was an inclusion of an equal which changed the algebraic expression to an equation with an intention of solving it. There was creation of learners' own rules since the algebraic did not make sense to the. There is an indication of a procedural and careless error. According to Riccomini (2005), unsystematic

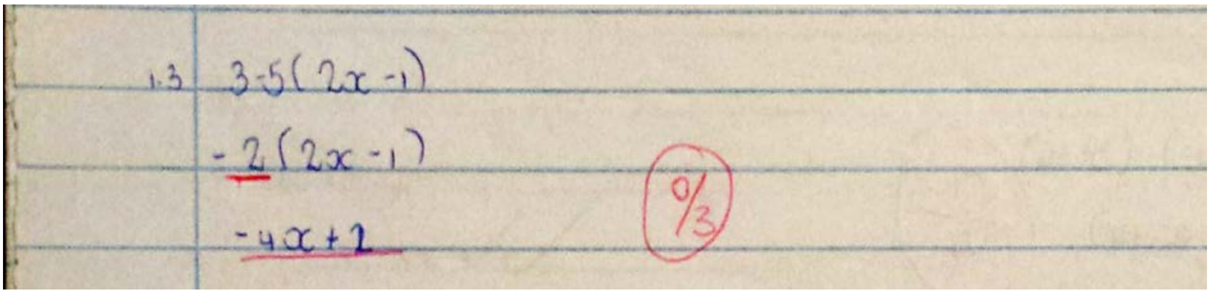
4.7.2 A summary of learners' errors in item 1.2 of the pre-test

Item 1.2 was attempted by all the learners but it was noted that some learners faced some challenges when simplifying algebraic expressions as revealed by errors in their pre-test scripts. Learners failed to add and subtract algebraic terms because of carelessness. The reasons of failing to add or subtract terms were omission of terms, inclusion of learners' own terms and changing of operation in the algebraic expression. It might be learners thought of changing signs of terms of the algebraic expression from positive to negative or negative to positive as is done to transposed terms of an equation. The results from 1.2 indicated learners did not understand the concepts of integers (directed numbers) from previous learnt concepts. This was reflected by the learners' answers in simplifying -3y -4y where the following answers were obtained; -y, +7y, +y, - (-y) instead of the correct answer -7y. Some learners conjoined unlike terms for instance 7x-y=7xy, there was a lack of understanding of adding and subtracting of like and unlike terms. Performance of learners could have improved if learners concentrated fully in this item or revisited their answers before submission because some errors incurred from this item could have been avoided.

Item 1.3 *Simplify the following algebraic expression 3-5(2x-1)*

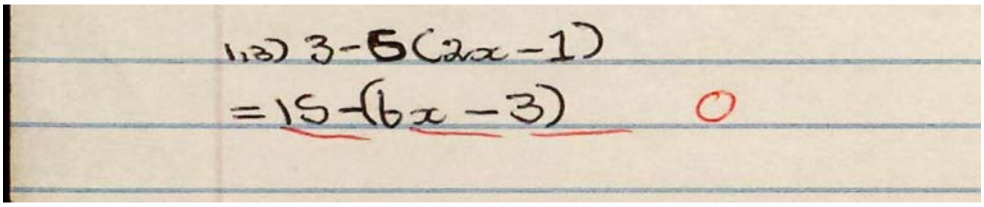
The learners were supposed to simplify the algebraic expression 3-5(2x-1) to 8-10x. Learners were being tested on the concepts of expanding brackets or distributing of terms and following the correct order of operation using the rule of BODMAS.

Learner 1 of item 1.3



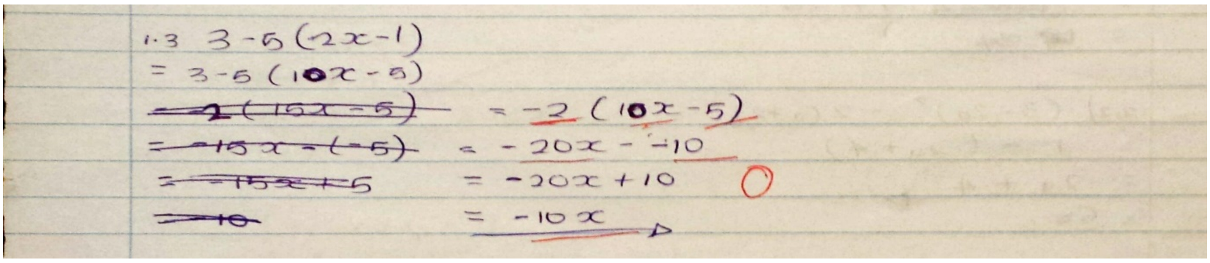
Firstly the learner subtracted 5 from 3 to get -2. This misconception, the learner did not follow the BODMAS rule where brackets were supposed to be expanded before subtraction. The solution is incorrect; the learner did not follow the BODMAS rule. Use of incorrect procedures is common when learning algebra (Lerch, 2004), this behaviour inhibits accurate solutions.

Learner 2 of item 1.3



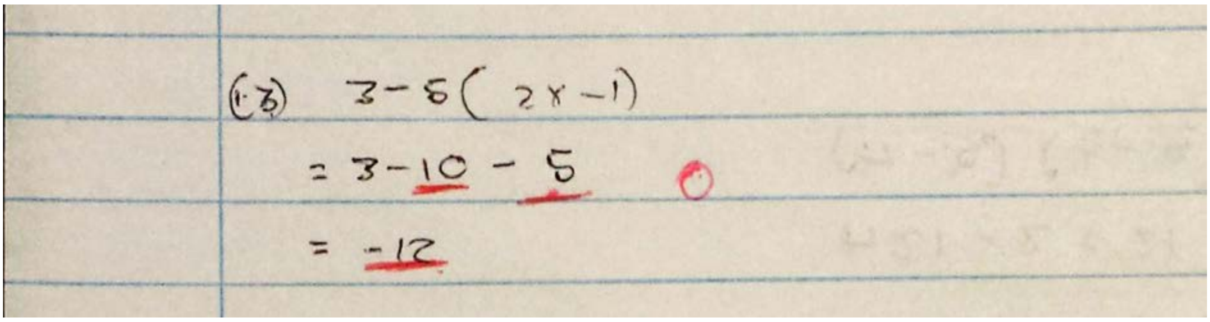
The learner misinterpreted the question by multiplying 3 by 5 and 3 by (2x-1). This is a misconception because the learner introduced a multiplication operation thus creating his/her rules which were never taught in class. These incorrect procedures inhibit accurate solutions (Lerch; 2004). A procedural error resulted in the learners' solution because the learner created his/her own rules which were never taught in class.

Learner 3 of item 1.3



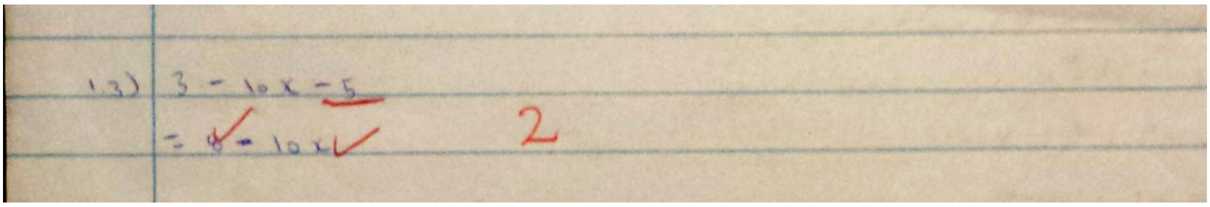
The learner simultaneously subtracted 5 from 3 and incorrectly expanded the brackets to 10x-5 instead of -10x+5. The learner did not follow the BODMAS rule where multiplication comes first before subtraction. In step 2 the learner expanded the bracket -2(10x-5) to -20x+10. The learner viewed the expression -20x+10 as a set of instructions to do something. Consequently the learner simplified the algebraic expression to a single term -10x. Here the learner conjoined to unlike terms to a single term. There is an indication of procedural and conceptual errors.

Learner 4 of item 1.3



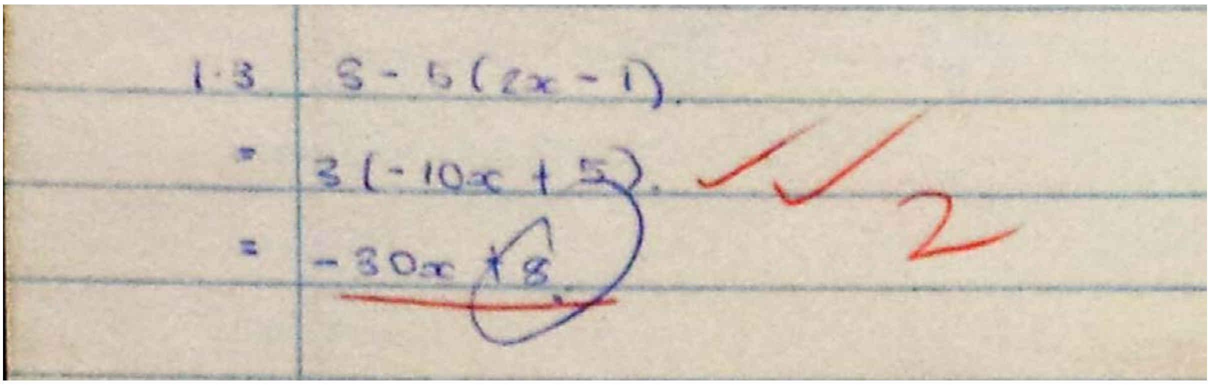
Here the learner incorrectly expanded the brackets and changed the algebraic expressions into arithmetic. According to Amerom (2002) students struggle to acquire a structural conception of algebra which is basically different from an arithmetic perspective. The learner switched from algebra to arithmetic because algebra is complex unlike arithmetic which is a simple manipulation of numerals. The above solution indicates that the learner struggled with simplifying algebraic expression and distributing of terms.

Learner 5 of item 1.3



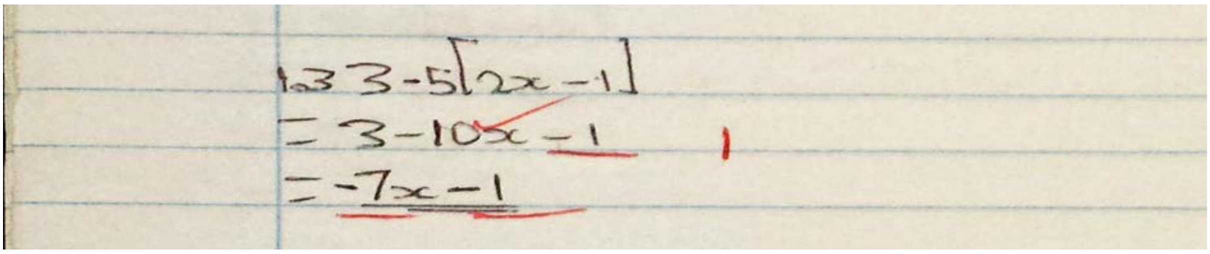
The learner incorrectly distributed the terms in step 1 but managed to come up with the correct answer. Another viewpoint is that it might be 8-10x which happened to be the right answer was an incorrect answer for the learner. According to the working above the learners' correct answer was supposed to be -2x-10x. The error in step 1 was a slip since the learner managed to correct the mistake without any assistance (Olivier, 1989). This resulted in conceptual and procedural errors.

Learner 6 of item 1.3



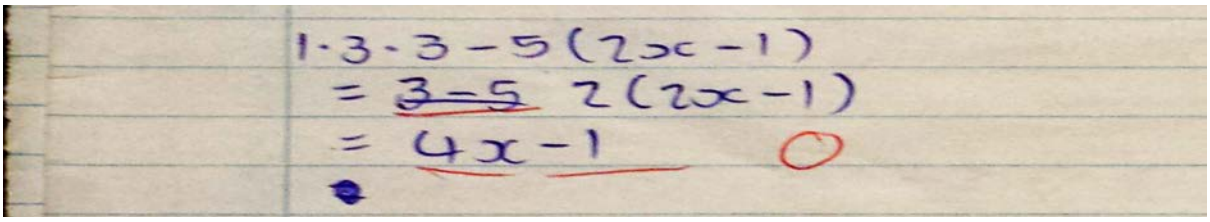
In step 1, the learner distributed the terms correctly but brackets were not removed. This is a misconception because expanding of brackets involves multiplication of terms and removing of brackets. The learner then multiplied the bracket by 3, this procedure was wrong because 3 was supposed to be added to the distributed terms -10x+5. In the last step, the terms were incorrectly distributed. The learner multiplied 3 by -10x and added 3 to 5 to get 8. This was a misconception which was created by mixing of mathematical concepts or by mistakenly adding the 3 and 5 or by learner creating his/her own rules. There is an indication of procedural, conceptual and careless errors.

Learner 7 of item 1.3



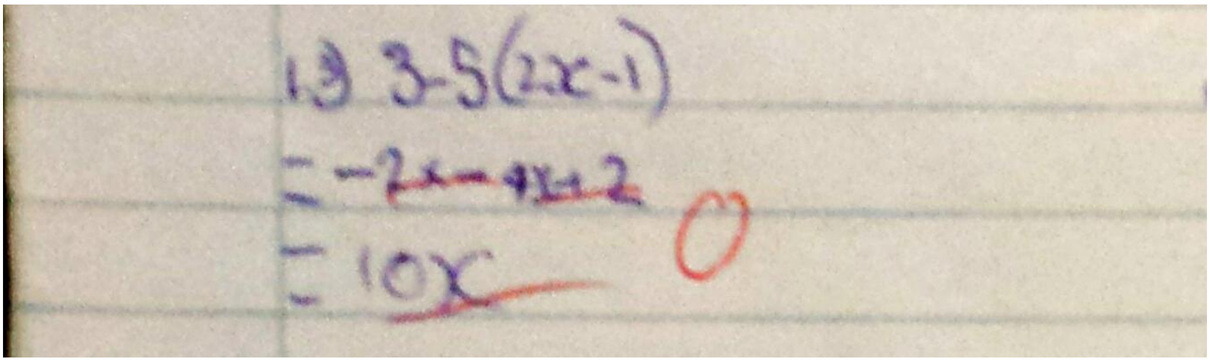
The learner did not expand the brackets fully. Inside the bracket only 2x was multiplied by -5. The learner wrote 3-10x =-7x, two unlike terms were conjoined which is a misconception because the learner wanted to leave the algebraic expression in a simplified form or conjoined to 'finish' the algebraic expressions. (Robinson & Tirosh, 1994)) This resulted in conceptual and procedural error

Learner 8 of item 1.3



Here the learner created his own question, 3-5 was cancelled completely and the learner remained with 2(2x-1). The learner failed to rearrange his/her schema because the previous knowledge failed to connect with the new concepts so the learner created a new question in order to balance his/her state of disequilibrium. In step 2 the learner failed to distribute the terms.

Learner 9 of item 1.3



Here the learner did not follow the BODMAS rule, 5 was subtracted from 3 to get -2 before brackets were distributed (distribution of terms). The learner wrote -2×-4x+2 which is mathematically wrong, -2(2x-1) = -4x+2 so this expression was not supposed to be multiplied by -2 again. Learners tend to be unwilling to stop before getting to an answer they are content with. The learner combined the two unlike terms, 8x and 2 to 10x. Learners usually conjoin terms because they feel that their answers are incomplete and not correct that's why they simplify them to a single term. They conjoined the terms.

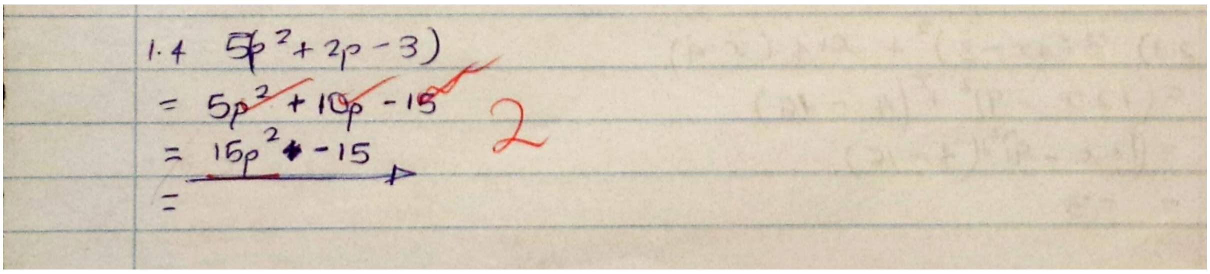
4.7.3 Summary of learners' errors in item 1.3 of the pre-test

Generally learners did not perform well in this item. There was no learner who successfully simplified the algebraic expression 3-5(2x-1). Errors were identified in every learners who wrote the pre-test. Most learners did not follow the BODMAS rule; they did subtraction of terms before multiplication (expansion of brackets). Some learners followed the BODMAS rule but failed to follow the distributive property. The errors emanated from incomplete distribution of terms, reversing of signs of algebraic terms, careless and applying of learners' own rules or creating learners' own questions. Some errors were as a result of conjoining of numerals and variables to a single term. Most learners viewed of the algebraic solutions as incomplete and combined them to one term despite the fact that they were unlike terms. Some errors arose from learners' misinterpretation of concepts for instance one 3-5(2x-1) was simplified to 3(-10x+5) =-30x+15 which is an incorrect answer. The highest numbers of errors were found in this item 1.3.

Item 1.4 *Simplify the following algebraic expression $5(p^2+2p-3)$*

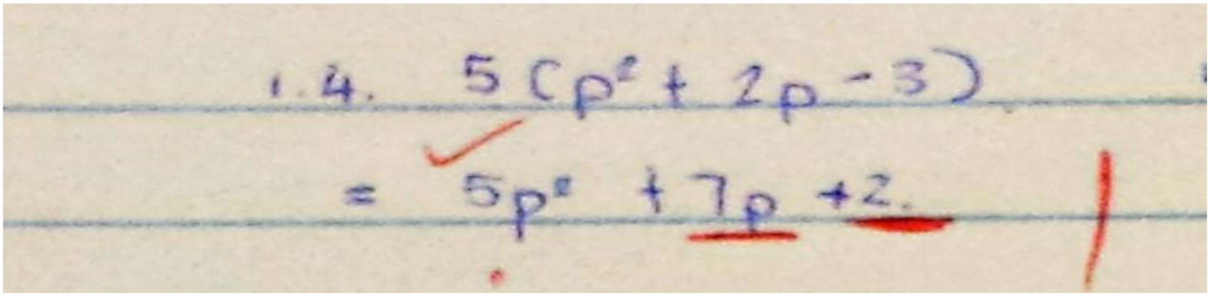
In Question 1.4 learners were supposed to simplify the expression $5(p^2+2p-3)$ to $5p^2+10p-15$. Learners were being tested on expanding of brackets where they were expected to recall the following concepts; positive x positive=positive, positive x negative= negative, negative x negative=positive and negative x negative=positive.

Learner 1 of item 1.4



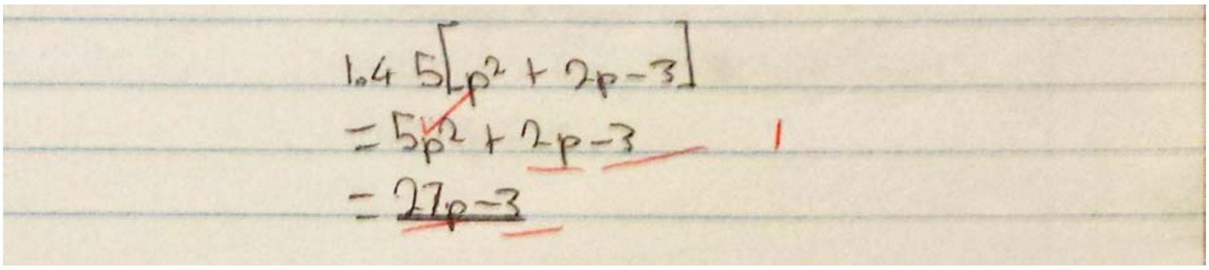
In the first step of the solution the learner correctly distributed the terms. In the last step of the working the learner conjoined $5p^2$ and 10p because p is common term so the terms were considered as like terms. In the conjoining process, the learner tended to take the highest exponents of the variables (Makonye & Matuku, 2016). This resulted in conceptual error.

Item 2 of item 1.4



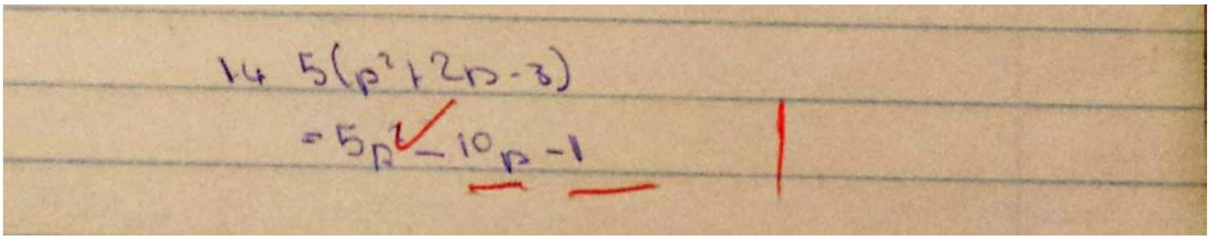
There is an incomplete distribution of terms where the learner only multiplied p^2 by 5. The learner added 5 to the other terms, 2p and -3 to get the answers 7p and +2 respectively. The learner applied his/her own rules which were never taught in class. The learner has a misconception of conjoining terms, for instance the learner simplified 5+2p to 7p. The two terms are unlike terms so they were not supposed to be conjoined. According to Brodie and Berger (2010) this misconception is referred to as conceptual structure which sensible to the learner in relation to his/her current knowledge but not related to the mathematical knowledge required. There is an indication of both procedural and conceptual errors.

Learner 3 of item 1.4



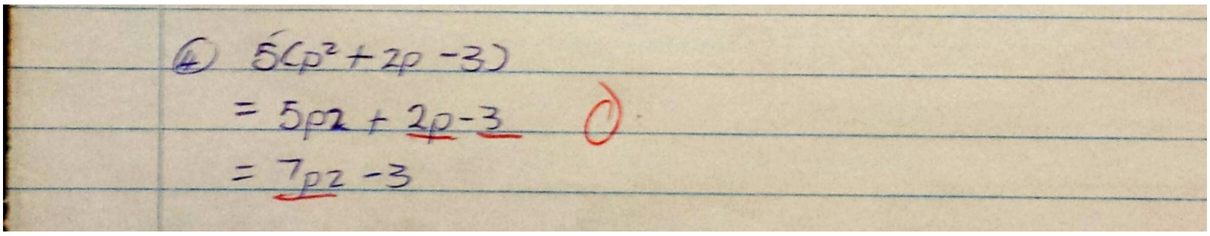
The learner partly distributed the terms. In the last step the learner misinterpreted the exponential laws by assuming that $5p^2=5^2p=25p$. Thereafter the learner added 25p to 2p to come up with 27p. There is creation of learners own rules. This resulted in procedural and conceptual errors.

Learner 4 of item 1.4



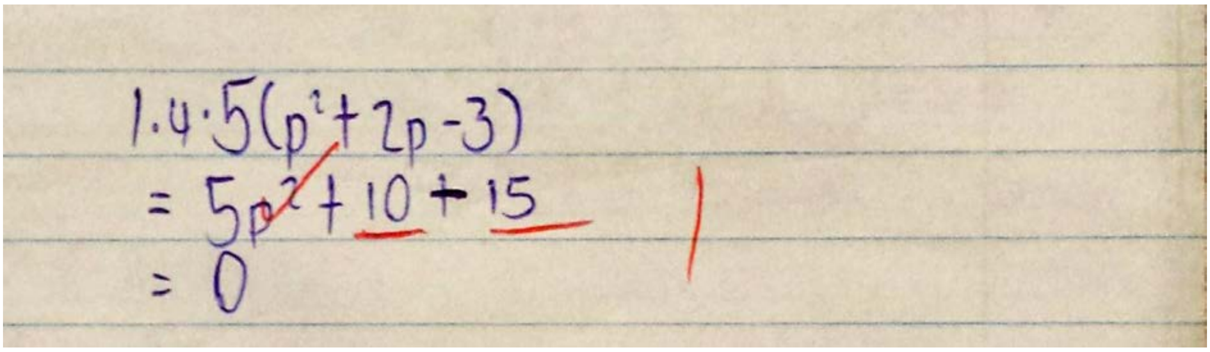
The expansion of the brackets was incomplete. The learner incorrectly simplified 5(2p) to -10p instead of the correct answer 10p. Also the learner did not multiply 5 by -3. This shows that the learner lacked a skill of how to multiply algebraic expressions and what sign is involved for instance if a positive number x a negative number. This is supported by Makonye and Matuku (2016) who state that learners have problems in removing parenthesis. This resulted in a conceptual error.

Learner 5 of item 1.4



The learner did not distribute the terms instead the learner introduced a wrong notation which is not mathematical. The learner incorrectly wrote $5(p^2)$ =5p2 and added this answer to 2p to get 7p2. The learner introduced his/her own rules which never taught in class. To the researcher the solution shown above is senseless but to the learner, his /her relation to her or his current knowledge, but which is not aligned with the conventional mathematical knowledge.

Learner 6 of item 1.3



The learner made a slip by writing 10 instead of 10p (careless error). A slip can be corrected without getting assistance from anyone (Riccomini, 2005). Also the learner combined all terms to a single term by adding $5p^2$ to 10 and subtracts 15 to come up with the answer zero. Children are unable to interpret algebraic letters as generalised numbers or even as specific unknowns, instead, they ignore the letters, replace them with numerical values, or regard them as shorthand names. The learner conjoined unlike terms to a single numerical value. This resulted in procedural and conceptual errors.

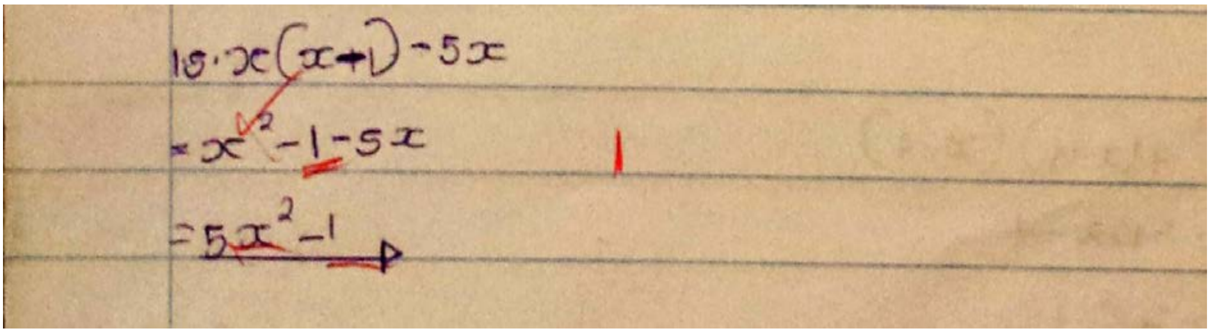
4.7.4 Summary of learners' errors in item 1.4 of the pre-test

Learners performed well in item 1.4. The least number of errors were found in item 1.4. However there were some few errors displayed by learners in this item. The most common was the conceptual error where, learners conjoined unlike terms to a single term. Some learners were applying their own rules which were never taught in class. For instance $5(p^2+2p-3)$ was simplified to $5p^2+7p+2=5^2p+7p+2=25p+2p+2=27p+2$. Here exponential rules were misinterpreted. There was an incomplete distribution of terms. Learners were putting wrong operation signs after brackets were expanded. Some learners failed to distribute the terms due to carelessness, where some terms were omitted.

Item ***1.5: Simplify the following algebraic expressions x(x-1)-5x***

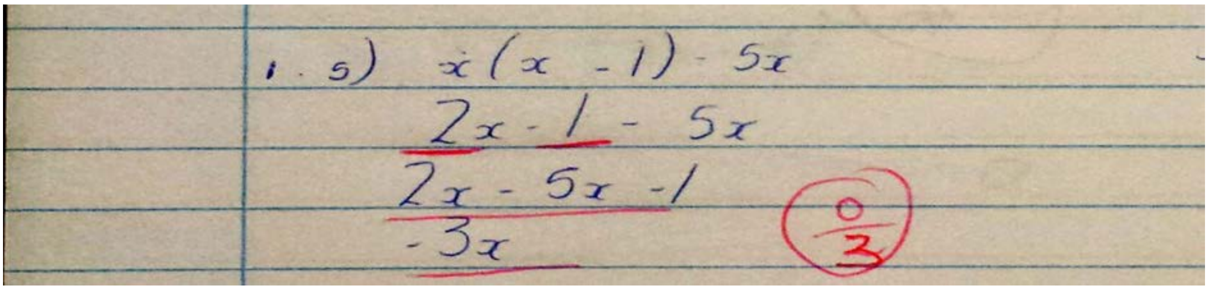
The learners were supposed to simplify x(x-1)-5x to x^2-x-5x = x^2-6x. Learners were being tested on BODMAS rule, distributing of terms and adding and subtracting of term

Learner 1 of item 1.5



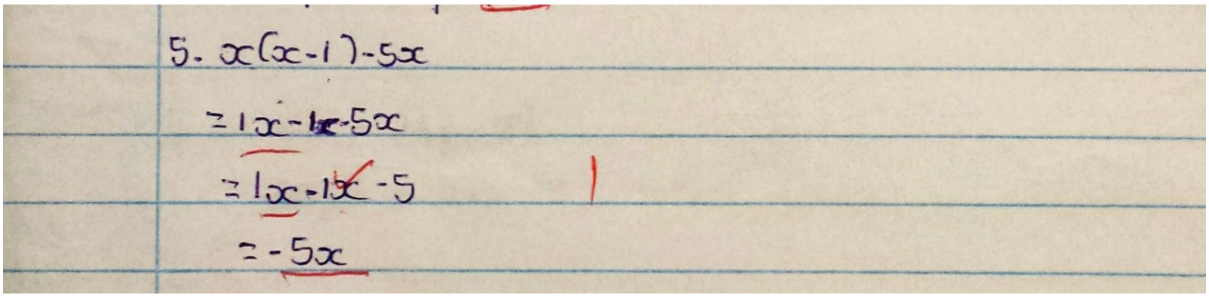
The solution shows an incomplete distribution of terms and conjoining of unlike terms. $x^2$and -5x were considered to be like terms since both terms have x as the variable. In the last step of solution the learner incorrectly added terms where x^2-1-5x was simplified to 5x^2-1. This resulted in conceptual error.

Learner 2 of item 1.5



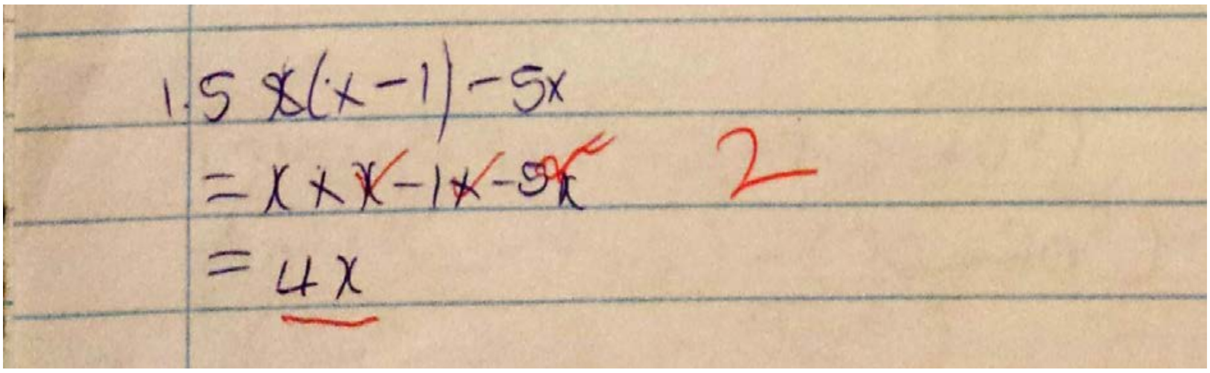
Learner wrote x(x-1)-5x = 2x-1-5x = -3x. The learner did not the use the laws of exponents correctly by writing x(x) = 2x instead of writing the correct answer x^2, also the learner did not multiply x by -1. The learner simplified incorrectly 2x-5x-1 to -3x. The term -1 was omitted. This resulted in conceptual and procedural error

Learner 3 of item 1.5



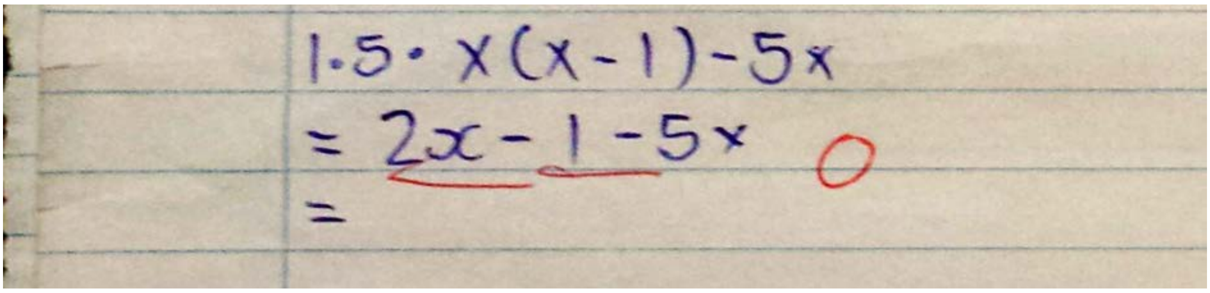
The learner failed to distribute terms, x(x) was simplified to x instead of the correct answer x^2. In step 2 the learner made a slip by omitting x on the term 5x. This could be a slip because the learner came out with a correct answer of -5x on the last step. There is an indication of conceptual and careless errors

Learner 4 of item 1.5



The learner failed to expand the brackets and to add terms. The learner incorrectly wrote x(x) =x and x-5x =4x instead of correct solutions $x^2$and -4x respectively. The addition and subtraction of algebraic terms is lacking. There is an indication of a conceptual error.

Learner 5 of item 1.5



The learner failed to distribute terms, x(x-1) was incorrectly simplified to 2x-1instead of x^2-x. This resulted in conceptual.

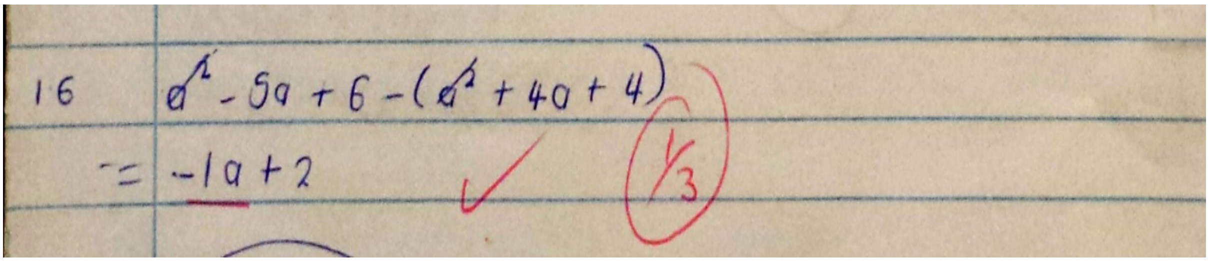
4.7.5 Summary of learners' performance in item 1.5 of the pre-test

This item was poorly done and, conceptual error was the most common error followed by procedural. Some errors were caused by conjoining of unlike terms, incomplete distribution of terms, and incorrect addition of terms. There were some few careless errors noted in the learners' pre-test script.

Item 1.6 ***Simplify the algebraic expressions a^2-5a+6-(a^2+4a+4).***

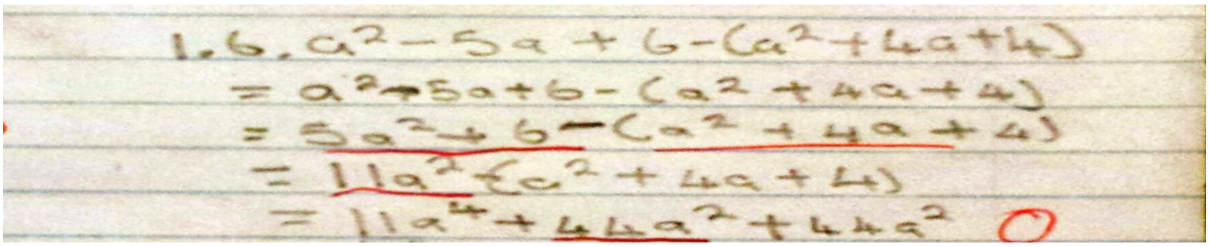
The learners were supposed to simplify $a^2-5a+6-(a^2+4a+4)$ to $a^2-5a+6-a^2-4a-4 = -9a+2$. The learners were tested on distribution and addition of terms. The recalling of previous learnt concepts of addition and subtraction of integers was of great importance.

Learner 1 of item 1.6



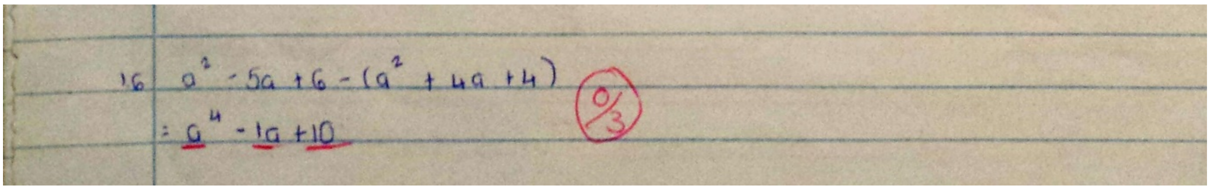
The learner subtracted the terms in the second algebraic expressions from the terms in the first algebraic expressions without expanding the brackets where the signs of terms in the second algebraic terms were going to change. The learner did not distribute the terms but created his/her own rules.

Learner 2 of item 1.6



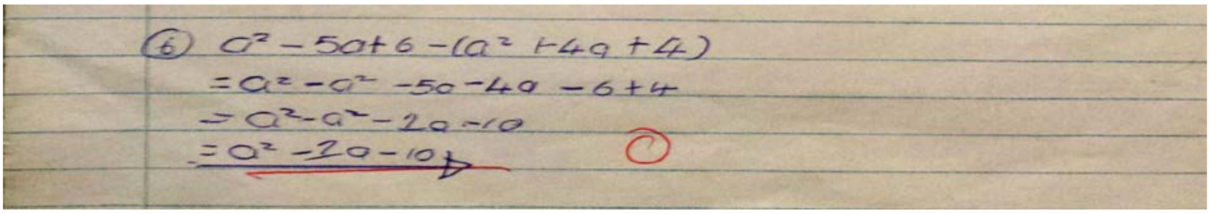
Here the learner combined all the terms in the first algebraic expression to a single term of $11a^2$ (combining unlike terms) thereafter the learner multiplied all the terms in the second algebraic expression by $11a^2$. The created his/her own mathematical rules which were never taught in class. The learner misinterpreted the question. The learner conjoined unlike terms and incorrectly distributed terms. The learner lacked an understanding of exponential laws. This is an indication of conceptual and misapplication errors.

Learner 3 of item 1.6



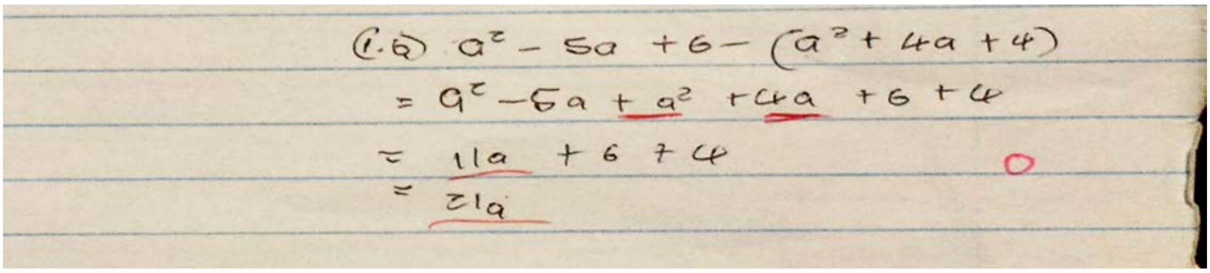
The learner wrote a^2-5a+6-(a^2+4a+4) = a^4-a+10. The learner ignored the negative sign after incorrectly simplifying a^2-a^2. The learner was just adding the terms from the first algebraic expression to the terms in the second algebraic expression. The learner applied his/her own mathematical rules which were never taught in class. The learner was supposed to distribute the terms in the second bracket before any addition or subtraction of terms was done. The learner incorrectly simplified a^2-(a^2) to a^4, the lacked an understanding of exponential laws which were inappropriately applied. This resulted in conceptual, procedural and misapplication error

Learner 4 of item 1.6



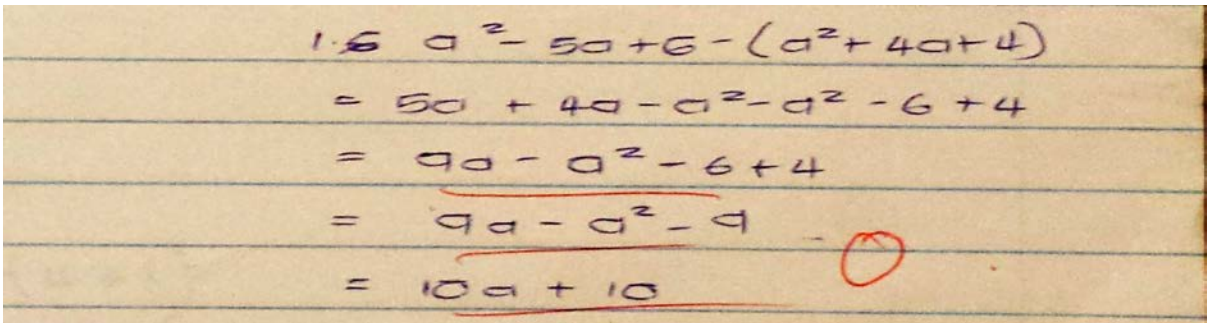
The learner distributed the terms fairly well, the error was on changing 6 to -6 and simplifying – (4) to 4, the last term. In step 2 the learner failed to add and subtract terms where a^2-a^2, 5a-4a, and -6+4 were incorrectly simplified to a^2, '-a' and -10 respectively. The learner lacked some basic skills of addition and subtraction of algebraic terms.

Learner 5 of item 1.6



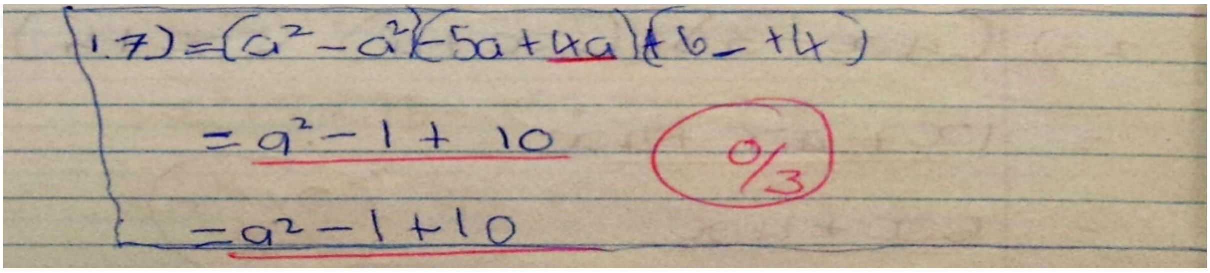
The learner did not expand the bracket as the sign of terms of the second algebraic terms were supposed to be negative after being multiplied by -1. The learner incorrectly added algebraic terms. The unlike terms were conjoined to a single term. The expression a^2-5a+a^2+4a was incorrectly written as 11a where the exponent 2 of base ‘a’ was discarded. Students conjoin terms to a single term because students face cognitive difficulty in ‘accepting lack of closure’. They conceive open expressions as incomplete and tend to finish them (Booth, 1998; Collis, 1975 & Davis, 1975). This resulted in conceptual and procedural errors.

Learner 6 of item 1.6



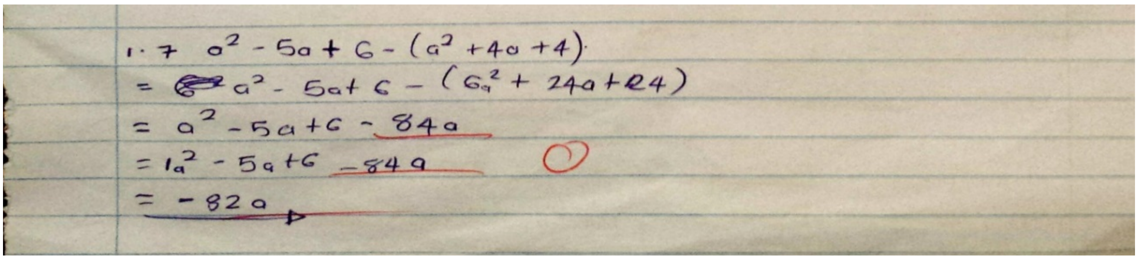
The learner failed to distribute the terms in the second algebraic expression. Also the learner changed the sign of terms unnecessarily for instance, -5a, 6 and a^2were changed to 5a, -6 and $–a^2$ respectively. The learner failed to add and subtract terms, he/she conjoined terms. This resulted in conceptual and procedural errors.

Learner 7 of item 1.6



The learner wrote a^2-5a+6-(a^2+4a+4) = (a^2-a^2) (-5a+4a) (+6-+4) = a^2-1+10. The learner was not supposed to introduce some brackets because this will change the concept of the question all together. The simplification was not correct, the learner wrote (a^2-a^2) = a^2which is incorrect, (a^2-a^2) = 0. The learner wrote (-5a+4a) = -1 instead of ‘–a’ and (+6-+4) = 2.

Learner 8 of item 1.6



The learner took the notation 6-(a^2+4a+4) as indication of multiplication so the learner incorrectly simplified that expression to 6a^2+24a+24. The learner made a mistake and simplified this answer to 84a. Finally the miscaculated the whole algebraic expression in step three and came with an incorrect answer of -82. This learner has a lot of confusion which me as a researcher I can not even follow them. The learner created his/her own rules which were incorrectly manipulated. This resulted in conceptual , procedural and misapplication errors.

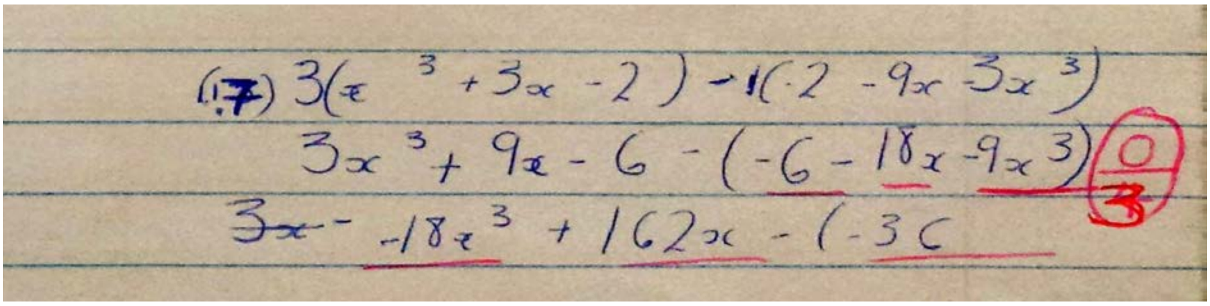
4.7.6 Summary of learners' errors in item 1.6 of the pre-test

Item 1.6 was poorly done. Most of the learners struggled to simplify the algebraic expressions. A number of learners did not attempt this question. The errors with the highest number of errors were conceptual and procedural errors. Learners directly subtracted terms of the second algebraic expression from term of the first algebraic expression without distributing of terms. The reasons for learners' errors in this item were, conjoining of unlike terms, creation of learners' own rules, unnecessary changing of operational signs, lack of basic skills of addition, subtraction and exponents.

Item 1.7 ***Simplify the following algebraic expression 3(x^3+3x-2)-(2-9x+3x^3)***

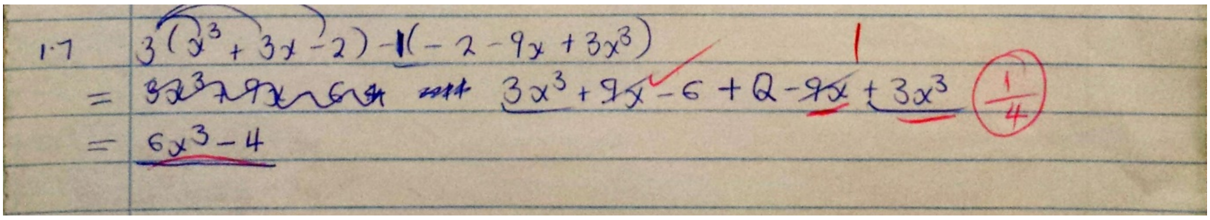
The learners were supposed to simplify 3(x^3+3x-2)-(2-9x+3x^3) to 3x^3+9x-6-2+9x-3x^3 = 18x-8. The learners were tested on the concepts of addition, subtraction, distribution, order of operation of on simplifying algebraic expression.

Learner 1 of item 1.7



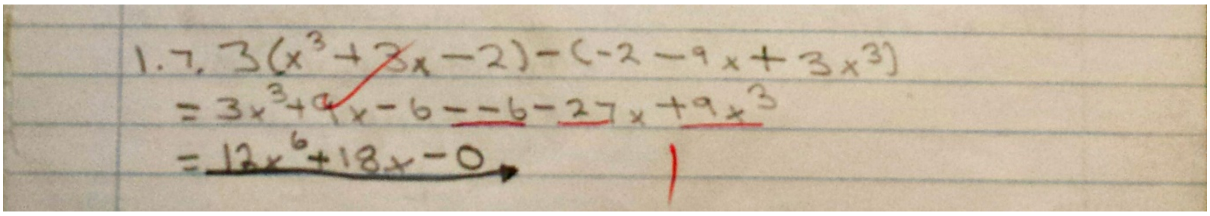
Here the learner multiplied every term in the two algebraic expressions by 3. This was an incorrect way of simplifying the expression because 3 is an only multiplier of the first algebraic expression. The second expression is multiplied by -1.

Learner 2 of item 1.7



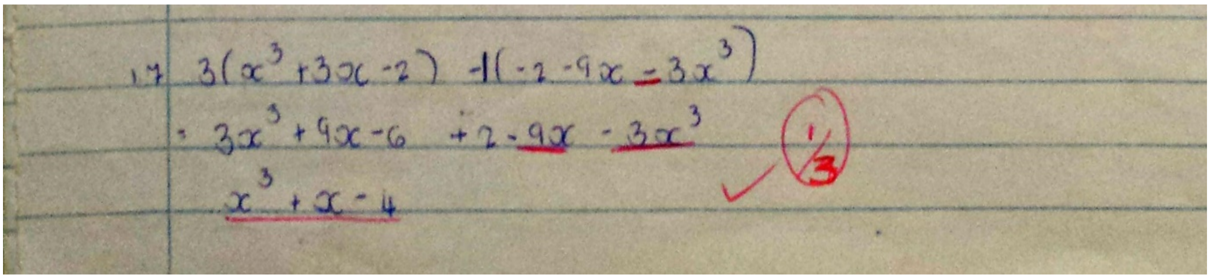
Here the learner incorrect distributed the terms of the second algebraic expression where – (-2-9x+3x^3) was simplified to 2-9x+3x^3 instead of 2+9x-3x^3. This shows that the learner struggles with multiplication of integers or struggle with negative sign in front of brackets.

Learner 3 of item 1.7



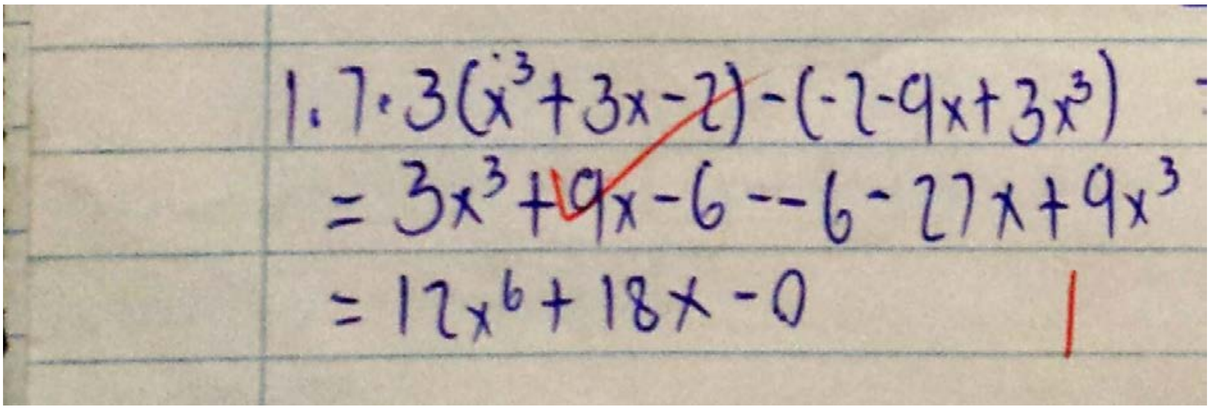
Here the learner incorrectly multiplied all the terms in the entire algebraic expression by 3. The 3 and the -1 were multipliers of the first and second algebraic expressions respectively. This is a conceptual error. Siegler (1996) also asserted that a learner without a good understanding of a concept result in using procedures of solving problems inappropriately.

Learner 4 of item 1.6



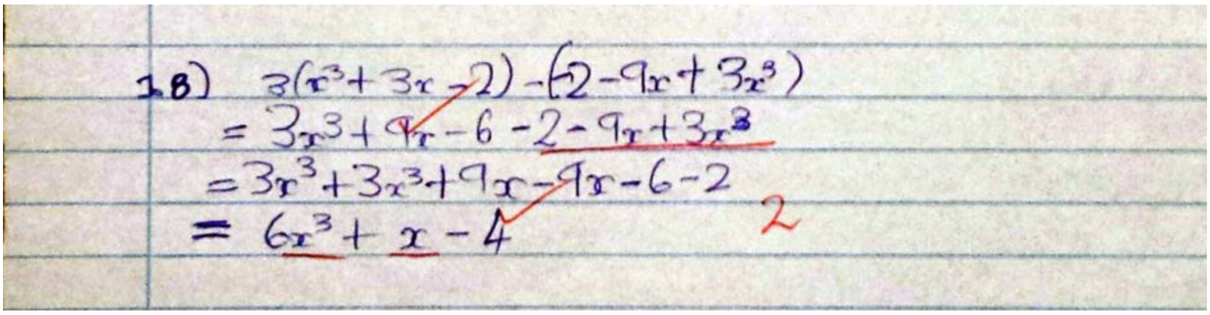
The learner wrote 3(x^3+3x-2)-(2-9x+3x^3) = 3x^3+9x-6+2-9x-3x^3). Here the learner had failed to calculate correctly – (-9) to get 9 and – (-3x^3) to get 3x^3 but the learner wrote -9x and -3x^3 instead. From the incorrect simplification of terms in step 1, the learner incorrectly added and subtracted terms for instance $3x^3$-$3x^3$ = 0 and 9x-9x = 0 but the learner wrote the answers x^3 and +x-4 respectively.

Learner 5 of item 1.7



Learner 5 wrote $3(x^3+3x-2)-(2-9x+3x^3) = 3x^3+9x-6--6-27x+9x^3)$. Here the learner multiplied the first and second bracket all by 3 which is mathematical wrong because the learner was only supposed to multiply the first algebraic expression by 3 and the second algebraic expression by -1. The learner changed the -27 in step 1 to +27 also the learner incorrectly wrote 27x+9x = 9x instead of 36x and $3x^3+9x^3$ as 39^3x. Brodie and Berger (2010) refer to a misconception as “a conceptual structure, constructed by the learner, which makes sense to the learner but not mathematical.

Learner 6 of item 1.7



Learner 6 wrote $3(x^3+3x-2)-(2-9x+3x^3) = 3x^3+9x-6-2-9x+3x^3)$. Here the learner only distributed the terms in the first algebraic expression, the second algebraic expression was not multiplied by -1, and the learner ignored the negative sign before the brackets.

4.7.7 Summary of learners' errors in item 1.7 of the pre-test

The learner's performance in item 1.7 was very poor. Learners failed to distribute terms, they conjoined unlike terms, they misinterpreted concepts, they unnecessarily change the signs of terms, and they applied their own mathematical rules.

4.8 Learners' interview

A focus group interview was conducted with six of the thirty learners in the study depending on the amount of errors these six learners displayed in their pre-test scripts. The learners who showed the most number of errors on their pre-test scripts were selected for the interview. The purpose of the interview was for the learners to explain the procedures they used when simplifying algebraic expressions in the pre-test. Through this interview the researcher wanted to gain an understanding of the learners' thinking process which was not revealed in their working. The names used in the transcriptions are fake names for all the six learners who were involved in the interview. The section gives an over view of findings from the interview. The purpose of the interview was to investigate the possible reason for the errors displayed in the leaners' pre-test scripts. In this section the researcher identified some reasons from the learners' interview responses for learner errors on simplifying algebraic expressions.

Question 1.1

Teacher: Rendy can you look at question 1.1 on your script there is something I need to ask you about. Right Rendy on 1.1 if I go on with your working there was a time when you were writing 4+5+2+a+what was going on?

Rendy: I removed 1a from 4 and 1a from 2 and then I put them and I say plus a+a then I put them together.

Teacher: Ok you separated the 'a' from 4 and 'a' from 2. Why were you separating these variables from 4 and 2?

Rendy: So that there they stand on their own.

Teacher: Rendy you wrote4+5+2+a+a=11+a=11a. Explain to me what is going on here.

Rendy: I took all the whole numbers add them together then I took the 2a's and put them together with 11.

Teacher: Ok that's why you are getting an 11a?

Rendy: Yes

Teacher: In step number 2 I am seeing you wrote 4+2+5+a+a and in step 3 you wrote 11a, there is one a where is the other one?

Rendy: Oh …Shh… I have done a mistake.

Teacher: Right the last one on this one. Rendy I am seeing you say 11+a is 11a. Why are you writing 11a?

Rendy: I combined the terms together

Teacher: Why did you combine the terms?

Rendy: So that there they stand on their own

Teacher: Khaya in your solution for question 1.1 you wrote 5+4a+2a = 6+5 = 11 so why were you cancelling out the 'a'?

Khaya: I cancelled out a's because I want to stay with whole numbers.

Teacher: Tracey I can see that you wrote 6a+5 as part of your working, but you wrote 6a-5 as your final answer why?

Tracey: I know ma'am that positive always change to a negative.

Teacher: What do you mean when you say a positive always change to a negative?

Tracey; But ma'am you said that sometime…. When finding x.

Teacher: Ok I said that when we were solving equations, this time we are not solving equations but are simplifying algebraic expressions.

Tracey: Ok. So it must be 6a+5 then.

Teacher: If we look at the answer in the first step it was 4a+2a+5 then you wrote 6a+5 but finally you wrote 11a why?

Tshwanelo: I thought that we have to combine them in this way 6+5 is 11 then I saw the 'a' then I have to combine.

Teacher: Ok you say 6+5 is 11 then you combine 11 with the 'a' which you were seeing?

Tshwanelo: Yes.

4.8.1 Summary of learners' interview responses in item 1.1

Some learners cancelled out the 'a' when simplifying the expression 4a+5+2a to 6+5 because they wanted to remain with numerals only. Learners have a good understanding of adding arithmetical expression but struggles with addition of algebraic expressions, to clear up their misconceptions the learners just cancelled a and simplified 6+5 to 11. The learners said they cancelled the 'a' so that the variables stand on their own. Some learners justified why they conjoined unlike terms. The learners simplified 6a+5 to 11a because they thought that the operation of addition meant that the terms were supposed to be combined. The learners' interview indicated that grade 8 learners struggle with simplifying algebraic expressions. The learners wrongly simplified the expression 4a+2a+5 to $6a^2+5$ because that's what they were taught in exponents that if the bases are the same and the exponents are the same they add the exponent. The misconception here is due to misapplication of rules, $4a+2a+5 \neq 6a^2+5$ but 4a+2a+5= 6a+5. Some learners admitted that they made some errors because integers confused them a lot. Some learners indicated that they could not remember properly what they did to come up with their solutions and others

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Question 1.2

Teacher: Levison look at your final step of 1.2, you wrote 5x+2x=6x. Are you seeing that?

Levison: Yaah…

Teacher: Levison what is 5x+2x?

Levison: 7x.

Teacher: But I am seeing you wrote 6x what happened?

Levison: Oh… Eish... I wrote 6x instead of 7x. I miscalculated.

Teacher: Tshwanelo in 1.2 you were supposed to simplify the expression 5x-3y-3a-4y+2x but I am seeing you were changing the sign of terms look at your step 2, you wrote +3y instead of -3y, why were you changing the sign of that term?

Tshwanelo: I thought we were supposed to change it.

Teacher: You also change the -4y in the question to +4y and -3a to +3a why were you changing the operational signs

Tshwanelo: That's the way we do it ma'am.

Teacher: We only change signs of transposed term when solving equations. In algebraic expressions we do not change.

Tshwanelo: Ok ma'am.

Teacher: Hazel I want your to look at you step 2. I am seeing in your step 2 you wrote 7x+7y where are you getting +7y because it was -7y?

Hazel: Oh it was a mistake

Teacher: Right in your final answer you wrote 17x+y, where were you getting the 17x? Is it 14+3?

Hazel: This one?

Teacher: Yes your last answer to get 17, were you adding 7+7+3 to give you 17?

Hazel: Yes.

Teacher: Right I am seeing in your last answer there is no 'a' you wrote x and y only Why did you leave the 'a' out?

Hazel: I cancelled out the 'a'.

Teacher: Why?

Hazel: I was gonna add it up in my final answer.

Teacher: Why didn't you add it?

Hazel: I forgot.

4.8.2 Summary of learners' interview responses in item 1.2

Some learners were able to realise their mistakes after the researcher probed them of their errors. For instance one learner wrote 7x+2x=6x but the researcher asked the learner what is 7x+2x until the learner realised that the correct answer was 7x. The learners' errors in their pre-test were as a result of learners forgetting some algebraic concepts to use in simplifying algebraic expressions. Some learners were having errors in the solutions because that was the way they were taught. According to literature these misconceptions could be an indication of learners' misinterpretation of concepts from the teacher or teacher passing some misconceptions to learners. Some learners inappropriately reversed the operational signs of some algebraic terms because they thought that the sign of algebraic terms always change when simplifying algebraic expressions as they do in equations.

Question 1.3

Teacher: We are going to look at question 1.3
Leaners: All of us?
Teacher: Yes. Hazel look at your step 2 in 1.3, you wrote -2 where did you get that-2?
Hazel: I said 3-5?
Teacher: You started by subtracting 5 from 3. What does the BODMAS rule say do you still remember?
Hazel: I cannot remember properly

Teacher: Which one comes first subtraction or multiplication?
Hazel: its subtraction because it comes first before anything else

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Teacher: You were supposed to multiply everything in the bracket by 5 before you subtract. We always follow the rule of BODMAS where multiplication comes first before subtraction.
Hazel: Ok.

Teacher: Levison from your first step you wrote 3 which you changed to -3. Why were you changing 3 to -3?
Levison: My second step?
Teacher: Yes there is a -3-5 are you seeing that?
Levison: Oh yes.
Teacher: Why did you put that -3?
Levison: I thought thereby in brackets there is 2x-1 you have to do the: which you have to add
Teacher: Oh you thought you supposed to say 2x+1 so that's where you are getting that -3?
Levison: Yaah.
Teacher: Let us go to your third step where did you get that -8 from?
Levison: I said -3-5 which I got -8.
Teacher: You said 3-5 what about the numbers in the brackets?
Levison: I just thought of subtracting before multiplying.
Teacher: Why did you do that?
Levison: I just thought that any of the two signs will do.

Teacher: Which signs?
Levison: Addition or subtraction
Teacher: Rendy in step 2 you wrote $4x^2+2$ *are you seeing that?*
Rendy: Yes.
Teacher: Right go to step 2 where did you get that $4x^2$*?*
Rendy: I multiplied the -2 with the 2 inside the bracket and I got 4.
Teacher: You got 4?
Rendy: $4x^2$.
Teacher: Where is the squared coming from?
Rendy: I multiplied the x with -2.

Teacher: the x inside.
Rendy: Yes.
Teacher: My question still stands where did you get that x^2 from? Why were you writing x^2 instead of x?
Rendy: Silent. I don't know I think it's a mistake.

Teacher: Tracey I am seeing a -2 in your step 2. Where did you get that -2?
Tracey: I just say 3-5 and I got -2.
Teacher: Khaya can you look at your question 1.3. You introduced another bracket on -10x+5, you wrote 3(-10x+5). Were you not multiplying -5 with (2x-1) to get -10x+5 so why are you introducing some other brackets?
Khaya: Yes I was I was.
Teacher: Why were you putting the brackets for the second time?
Khaya: Yaah I still wanted to distribute the 3 into the bracket.
Teacher: That's why you got -30x+15?
Khaya: Yes.

Teacher: John can you look at your last step where did you get that 8 from?
John: I made a mistake I added instead of multiplying
Teacher: You added what to what?
John: I added 3 to 5 to get 8.
Teacher: Ok.

4.8.3 Summary of learners' interview responses in item 1.3

The learners thought in an algebraic expression signs need to be reversed that's why some learners changed 2x-1 to 2x+1 in item 1.3. The learners incorrectly applied valid concepts from equations to algebraic expressions. Some learners' errors emanated from miscalculation. Learners did not follow the correct order of operation due to the assumption that any of the two operation (subtraction and multiplication) can start. Learners thought the order of operation does not matter, some argued that they subtracted before multiplication because subtraction was coming first before multiplication. Some stated that they knew the BODMAS rule but failed to follow it because they were confused and they were in hurry and some said they did not pay much attention to the question.

Question 1.4

Teacher: Tshwanelo look at your answer on 1.4, you wrote $5p^2+2p-3$. Where did you get that from?

Tshwanelo: Hesitantly I just… thought that it was like terms.

Teacher: Are you seeing the 5 there, were you not supposed to multiply everything in the bracket by 5?

Tshwanelo: I just thought I was supposed to multiply only 5 by p^2.

Teacher: You did not multiply the +2p-3 all by 5 that's why your answer was incorrect. You were supposed to multiply everything inside the bracket by 5.

Teacher: Rendy Look at your last answer. I am seeing there is a 5 inside a bracket there is a $2p^2$-3. Where did you get that $2p^2$?

Rendy: I just put the 2ps together…. because I also thought that they were like terms

Teacher: You thought p^2 and 2p will go together? Then you combined into $2p^2$ why didn't you put it to $2p^3$? How many p's are you seeing?

Rendy: There is two.

Teacher: Are they two or they three?

Rendy: They is two.

Teacher: You are seeing two.

Rendy: Yes.

Teacher: Ok so Rendy you said you combined the$2p^2$ and 2 p to come up with$2p^2$.

Rendy: Yes.

Teacher: Levison you did not write 1.4 why?

Levison: 1.4 confused me so I left it out.

Teacher: Right the next one is John, John look at your 1.4. Where is the 7p coming from and the +2?

John: I did the same thing

Teacher: What is that same thing?

John: I did multiplication before addition I mean I did addition instead of multiplication.

Teacher: You did addition instead of multiplication.

John: Yes.

4.8.4 Summary of learners' interview responses in item 1.4

In item 1.4, the learners were supposed to simplify the expression $5(p^2+2p-3)$ to $5p^2+10p-15$ but some learners simplified $5(p^2+2p-3)$ to $5p^2+2p-3$ because they thought p^2 was the only term to be multiplied by 5. Some learners conjoined the $5p^2$ and 10p because they assumed that the terms were like since they all contain variable p. Overall, the reasons for learners' errors were as follows; learners' carelessness, miscalculations, forgetfulness, they were taught to do so, were confused and the like. Some learners misinterpreted the questions and introduced their own rules which were never taught in class for instance learner 3 in item 1.4 on page 67 simplified the expression $5(p^2+2p-3)$ to $5p^2+2p+2= 5^2p+2p+2=27p+2$. Some learners' responses were not very useful as far as the study was concerned. The researcher probed the learners to explain their answers but others could not explain but gave answers like I don't know. Some learners made some errors in item 1.4 because of carelessness or forgetfulness or failed to remember the concepts involved on simplifying the expressions.

Question 1.5

Teacher: Levison look at your answer on 1.5 where did you get that –1x in step 2. Look at your step 2.

Levison: I ….. Oh.

Teacher: I you seeing that –x inside a bracket where did you get that?

Levison: I swapped it around because the first one in bracket says x-1, I swapped it around and I then put it -1x.

Teacher: You thought of swapping and starting with a minus and then you wrote your x, why?

Levison: Just….

Teacher: Can you look at your last answer where did you get that -6x?

Levison: -6x?

Teacher: Yes on your last answer.

Levison: I have said -1x and I then minus it from -5x and I got -6x.

Teacher: Where did that x go?

Levison: I used the same as x as I got from -5x.

Teacher: Are you seeing there is an x (demonstrating). Because you say -x-5x gives you -6x where is the x which was outside the bracket.

Levison: I must have left it out.

Teacher: Rendy at your first step of you wrote -1-5 where did you get this from?

Rendy: I done the same thing as in question 1.1

Teacher: What have you done?

Rendy: I took the whole numbers and put them together and the variables and put them together.

Teacher: You put that -1 inside the bracket together with the -5 with the x?

Rendy: Yes.

Teacher: You thought of putting the whole numbers together?

Rendy: Yes.

Teacher: Where did you get the three x which you add as x+x+x?

Rendy: I took the outside x and the inside x and the x from 5.

Teacher: Where is the addition sign coming from because initially there was a subtraction sign?

Rendy: By putting the plus sign I was putting the three x together.

Teacher: Ok that's why you ended up with the plus sign as a way of putting the x's together.

Rendy: Yes

Teacher: In the final answer there is -3x where did you get that from?

Rendy: I got it from -6+3x gives you -3x.

Teacher: Is it -6 and +3x unlike terms do the combine.

Rendy: They are unlike terms but I did not think of that when I was writing the test.

Teacher: Tshwanelo look at your second step are you there?

Tshwanelo: Yes.

Teacher: Where did you get that 2x?

Tshwanelo: I said….pausing.

Teacher: You said what?

Tshwanelo: x outside multiply the x inside and I say 2x

Teacher: Other learners may you please assist. What is x multiply x?

Other learners: The answer is x^2*.*

Teacher: It was supposed to be x^2 *not 2x, 2x we get it from x+x.*

Tshwanelo: Ok.

Teacher: Where is that -1 next to 2x coming from?

Tshwanelo: I left it

Teacher: Where is the -1 next to 2x coming from?

Tshwanelo: From the -1 inside the bracket.

Teacher: John where did you get that -4x in your last answer?

John: I said –x-5x.

Teacher: Why did you write -4x?

John: Because integers confuse me a lot I thought… I said 5x-x which gives me 4x.

4.8.5 Summary of learners' interview responses in item 1.5

The findings from the interview indicated that learners struggled with subtraction of algebraic expressions involving negative numbers. John indicated that 5x-x=4x then –x-5x=-4x this was an incorrect answer, the correct answer was -6x. Levison wrote x(x-1)-5x=x-1-5x. The learner left out the x outside the bracket because he did not want to repeat it since it was the same x inside the bracket. This misconception indicated that the learner lacked an understanding of expanding of brackets. The expansion of brackets was supposed to be done before subtraction. Learners conjoined terms, Tshwanelo simplified the expression -6x+3x to -3x, the learner conjoined unlike terms. Levison stated that he/she swapped the terms around, he wrote x-1=1-x because that's the way to do. Some indicated that they produced errors because they could not understand the algebraic language used. Some learners did not attempt some questions in the pre-test because the questions were confusing.

Question 1.6

Teacher: Hazel you wrote a^4 where is a^4 coming from?

Hazel: Because when doing exponents we were told that if the bases are the same and the exponents are the same we add the exponents.

Teacher: The law only applies when multiplying numbers with the same base not in addition.

The negative sign outside the bracket affects all the terms inside the bracket after multiplication.

Hazel: Ok.

Teacher: What about the '–a' after 4a?

Hazel: I said since before the bracket there is minus.

Teacher: there is -5a?

Hazel: No 6 minus

Teacher: Yes so what did you do?

Teacher: Which one?

Hazel: I said a negative multiply positive is negative so I put a minus sign.

Teacher: On which one?

Hazel: For this one. I said minus times because this is positive.

Teacher: Are you seeing the first -5a, is that the one you added to 4a.

Hazel: Yes. Teacher: Tshwanelo where did you get the $2a^2$*?*

Tshwanelo: I forget what I have done.

Teacher: You added the first a^2 *to the second* a^2*?*

Tshwanelo: Yes.

Teacher: you wrote $a^2+a^2 = 2a^2$.

Tshwanelo: Yes.

Teacher: you were supposed to multiply the second bracket by -1 which is invisible before you add the terms to the first terms of the algebraic expression, thereafter you collect like terms.

Teacher: Rendy where did you get $5a^2+6$ *in your first step?*

Rendy: I took the a^2 *and added it to 5a to get* $5a^2$.

Teacher: Rendy just remember that unlike terms cannot combine

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Teacher: Levison in your final answer there is a^2-1+10.

Levison: In question 1.7?

Teacher: No in question 1.6 how did you come up with that answer.

Levison: I thought here by the first step, the one in the bracket; I thought it was one thing it will give.

Teacher: You thought a^2-a^2 *gives one term which is* a^2*?*

Levison: Yaah it was one thing.

Teacher: Where did you get -1 from? Its -5+4 which is -1 but where is the 'a'?

Levison: Oh I must have left it by mistake.

Teacher: Does it mean that you combined 6 with 4 to get 10?

Levison: Yaah are say 6+4.

Teacher: are you seeing there is a minus sign?

Levison: No... I say +6 - +4 gives you a positive 10.

4.8.6 Summary of learners' interview responses in item 1.6

In item 1.6 learners failed to add and subtract algebraic expressions for instance Levison simplified +6-+4 as 10 instead of 2. They failed to subtract algebraic expressions for instance a^2-a^2 was simplified as a^2 instead of 0. The learners stated that they produced errors in their solutions because the question was too complicated. Some confirmed that they do not know the signs to use if multiplying positive number with negative number or negative number and negative number. Some learners simplified – (a^2+4a-4) as a^2+4a-4 because they assumed that the multiplier -1 was insignificant.

Question 1.7

Teacher: Hazel if you look at this one how did you come up 9x-3x?

Hazel: I said +3x multiply by +x.

Teacher: Where did you get -9x+3x?

Tracey: I took it exactly as it is from the question.

Teacher: You were supposed to expand the brackets not to take the question as it is.

Teacher: Tshwanelo can you look at your final answer where did you get this part in bracket

Tshwanelo: I…….

Teacher: Did you multiply all the terms by the first 3 outside the first bracket.

Tshwanelo: Yes

Teacher: You were not supposed to that. The 3 only multiply the first bracket and the -1 only multiply the second bracket

Teacher: We have come to the end of our interview. I would like to thank all for your full participation in this interview. All your responses are useful in this study. Enjoy the rest of your day.

Learners: Thank you ma'am.

4.8.7 Summary of learners' interview responses in item 1.7

Learners said they failed to simplify the expressions in item 1.7 because it was confusing. Some learners said they were careless and others did not understand what the question wanted them to do. Some struggled with multiplication of positive and negative numbers.

4.9 Analysis of the intervention

This section gives a description of the qualitative analysis of what transpired during the intervention. The purpose of the intervention was to use learners' pre-test errors as resource to help learners reduce them in simplifying algebraic expressions. In the intervention strategy there was an implementation of discovery learning by (Bruner, 1961) and concept image and concept definitions by (Tall and Vinner, 1991) and variation theory. For more information of what transpires during the intervention refer to page 40 in chapter 3 under procedure of the study.

The participating learners worked in groups of 5 during intervention. The learners were mixed according to their performances and the selection was done by the researcher. In those groups the learners were doing corrections for the pre-test. Here learners were supposed to help each other with addition, subtraction of algebraic terms and expansion of brackets, concepts of directed numbers were of paramount importance. The learners were able to help each other because those groups there were learners who did well in pre-test, so they helped others. The aim of the intervention was for the learners to minimise errors when simplifying algebraic concepts. The researcher defined the following words; like and unlike terms, expansion of brackets, BODMAS rule, multiplier, exponents so that learners' concept images connects well with learners concept definitions. Also the learners assisted the learners by introducing variation theory Lung (2012) where correct answers were contrasted with wrong answers. By so doing learners were able to know why some of the answers were wrong and accept the corrections. An opportunity to learn was created during the intervention because most of the learners were able to simplify the expressions on their own when they were given the pre-test to rewrite.

Generally the intervention process proceeded well after some algebraic concepts were clarified. Most of the learners were able to add, subtract, expand brackets and follow the BODMAS rule on simplification of algebraic expressions. The researcher emphasised the concepts of unlike terms through using of a variety of teaching aids like adding bananas to apples or subtracting chalks from A-4 exercise books and the like. The learners helped each other in some instances and misconceptions were cleared. The teacher emphasised to the learners that there is a need to identify the critical features in each and every algebraic task. For instance in simplifying the expression 3-5(2x-1), the researcher demonstrated that the critical features of this item were the BODMAS rule multiplication by a negative multiplier -5, reminding the learners that sign inside the brackets were supposed to be reversed. The resourcing of learners' pre-test error had improved the learners' performance on simplifying algebraic expressions. This was reflected in class activity learners were given to do almost at the final stage of the intervention. In the class activity learners were supposed to simplify the following algebraic expressions;

1. 2x+6y-3x+4z-10z+6y
2. 5(2x-5)-6(3x^2+4x-2)
3. 5+3(2x-4)-7x
4. 10x(x+1)-7x+2(3x-1)
5. 9-9(5x+5)+2(-x+7x^2-4)

According to Smith et al. (1993) students errors are the symptoms of misunderstanding. The findings from the pre-test and interview clearly showed that grade 8 learners lack basic algebraic skills hence they struggle to simplify algebraic expressions. The learners' errors emanated from learners failing to add and subtract algebraic terms, conjoining unlike terms, failing to follow BODMAS rule, incorrect distribution of terms, carelessness, misapplication/misinterpretation of concepts, and incorrect application of exponential laws. The identified errors were classified as conceptual, procedural, careless and misapplication/misinterpretation errors according to Hodes and Nolting (1998)'s four types of errors.

4.10 Quantitative analysis of the learners' performance in the post-test

This section gives a description of the quantitative analysis of learner performance in the post-test. In the post-test learners were supposed to simplify algebraic expressions from item 1.1 to item 1.7. Each item consisted with 3marks and the total marks of 21 in the entire post-test as it was with the pre-test. The post-test items tested the learners on adding, subtracting, multiplying and expanding of brackets of algebraic terms. The marks obtained by learners in each test item were categorised as follows;

- Level 0-number of learners who got a mark of $\frac{0}{3}$.

- Level 1-number of learners who got a mark of $\frac{1}{3}$.
- Level 2-number of learners who got a mark of $\frac{2}{3}$.
- Level 3-number of learners who got a mark of $\frac{3}{3}$.

The unattempt was not considered as zero. The researcher could not assign a zero to all learners because the results of the study were not going to be a true reflection of the learners' performance on the pre- and post-tests.

The marks obtained by each learner in each post-test from item 1.1 to item 1.7 were noted and tabulated as shown in table 2 below;

Table 8: Summary of learners' performance level in each of the post-test items

Item	**Level 0**	**%**	**Level 1**	**%**	**Level 2**	**%**	**Level 3**	**%**	**Unattempt**	**%**	
1.1	0	0	0	0	3	10	26	87	1	3	
1.2	2	6	3	10	8	27	17	57	0	0	
1.3	6	20	6	20	2	6	16	54	0	0	
1.4	3	10	5	17	9	30	13	43	0	0	
1.5	3	10	7	23	9	30	11	37	0	0	
1.6	3	10	10	34	4	13	9	30	4	13	
1.7	4	13	11	37	7	23	5	17	3	10	

The following graph shows the learners' performance in each of the post-test item

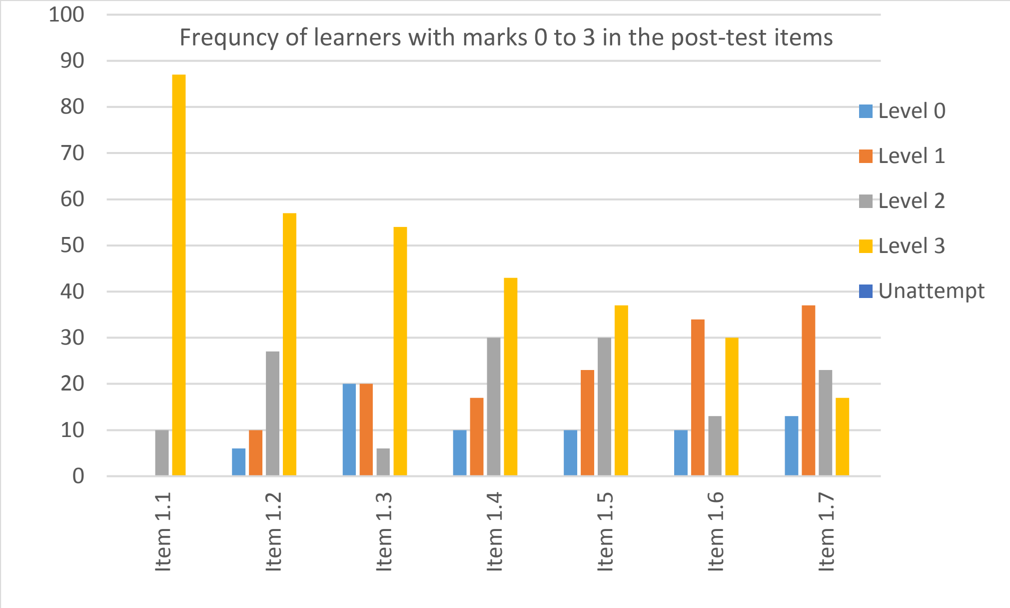


Figure 4: Frequency of learners with 0 to 3 marks in the post-test items from 1.1 to 1.7

From table 8 and figure 4, items 1.1, 1.2, 1.3, 1.4, 1.5 were attempted by all the learners with item 1.6 with the highest number (13%) followed by item 1.7 (10%) of no attempts. Item 1.1 was at the top in terms of learner performance with 87% of the learners obtaining marks above and 87% of these learners getting full marks. This was followed by item 1.2 with 84% of the learners obtaining a mark above $\frac{1}{3}$ and 57% of these learners got full marks. The item with highest number of zeros was item 1.3 followed by item 1.7, followed by both items 1.5 and 1.6 with 20%, 13%, and 10% of learners who obtained zeros in these items. Item 1.6 was the least in terms of learner performance with only 3% of the learners obtaining a mark above $\frac{1}{3}$ and no learner got full marks. The second least item was item 1.7 with 3% of the learners obtaining a mark above $\frac{1}{3}$ and 17% obtaining full marks. Items 1.6 and 1.7 were the least performed items with 43% and 40% of the learners getting a mark above $\frac{1}{3}$ in these two items respectively. Generally learners were able to add and subtract algebraic terms. The conjoining of unlike terms was minimal, learners did not remove variables to remain with numerals, and carelessness was minimal. The learners were able to follow the BODMAS rule and to expand the brackets. Only a few learners were still confused about changing of signs if terms are multiplied by negative multiplier. The misinterpretation of questions was minimal, no learners changed the algebraic expressions to equations. The performance of the learners was better as compared to the performance of the learners in the pre-test.

Table 9: Summary of learners' marks as a percentage in the post-test

Level	Marks	Number of learners	Percentage of learners
1	0-29	5	17
2	30-39	4	13
3	40-49	3	10
4	50-59	3	10
5	60-69	5	17
6	70-79	3	10
7	80-100	7	23

The marks obtained by learners in the post-test are shown in table 9. The table show the number of learners who achieved level 1 to 7 as expected in the senior phase performance standards. The highest and lowest marks obtained in the post-test were 100% and 19% respectively with a range of 81% and post-test average of 54%. Most of the learners (23%) were having marks ranging from 80-100 (7 distinctions) followed by 17% of the learners having marks ranging from 60-99 and 0-29%.

Table 10: Summary of grade 8 learners' results in the post-test

Grade	Number of learners wrote	Number of learners passed	Number of learners failed	Grand total	Subject average	Percentage pass	Percentage fail
8	30	21	9	1627	54	70	30

4.10.1 Summary of learners' performance in the post-test

Overall the learners' performance in the post-test was good with only 30% of the learners failing to meet the minimum requirement of 40%.. The 70% of the learners passed the post-test. The number of learners who passed or failed the test by considering a minimum requirement of 40%, the subject percentage and percentage pass and fail were noted as shown in table 10.

4.11 Quantitative analysis of the learners' errors in the post-test

This section gives a description of the quantitative analysis of learners' errors identified in the post-test. It gives some highlights of types and frequency of learners' errors as identified from their post-test scripts. Some descriptions of the learners' misconceptions which resulted in errors in simplifying algebraic expressions were carefully noted from some learners' scripts and recorded as shown on table 5. The researcher critically analysed all the thirty post-test scripts for each and every learner in the sample. This was done to determine the errors' types and frequencies in the post-test. The errors and their frequencies in the learners' post-test scripts were noted and recorded as shown in table 11. The researcher classified the errors which were identified during the analysis of learners 'pre-test scripts, using Hodes and Nolting (1998)'s four types of errors which were conceptual, procedural, careless and application errors. Table 11 was prepared to show the categorised the errors as shown in the learners' post-test scripts from item 1.1 to item 1.7.

The main focus of the study was to identify grade 8 learners' errors and misconceptions when simplifying algebraic expressions after an intervention strategy of resourcing the pre-test learners' errors. There was need to investigate the effects of resourcing the errors as far as learners' performance was concerned. All this was done in order to answer research question 3 which says to what extent does resourcing of learners' errors through discovery learning to help learners reduce them? The reasons for the errors were inappropriate application of rules, creation of learners' own rules, incorrect addition/subtraction of algebraic terms, inappropriate order of operation, omission of algebraic terms, changing of the sign of the algebraic terms, incorrect/incomplete distribution of terms and conjoining. The learners' errors were reduced after the intervention.

The table below show the errors identified in the analysis.

Table 11: Summary of learners' errors in each of the post-test items

Item	Application	%	Careless	%	Conceptual	%	Procedural	%	None errors	%	Unattempt	%
1.1	0	0	1	3	1	3	0	0	28	94	0	0
1.2	2	7	3	10	7	23	3	10	14	47	0	0

1.3	1	3	0	0	11	37	1	3	14	47	3	10
1.4	3	10	2	7	8	27	2	7	11	36	4	13
1.5	1	3	1	3	13	44	2	7	11	37	2	6
1.6	3	10	2	7	10	33	1	3	12	40	2	7
1.7	3	10	1	3	16	53	3	10	4	14	3	10
Total	13		10		67		12		94		14	

Table 12: Showing the percentage of learners with errors in each post-test item

	Post-test items						
	1.1	1.2	1.3	1.4	1.5	1.6	1.7
Percentage of learners with errors in each item	13	10	67	12	70	67	77

The graph below showing the type and frequency of learners' errors in the post-test

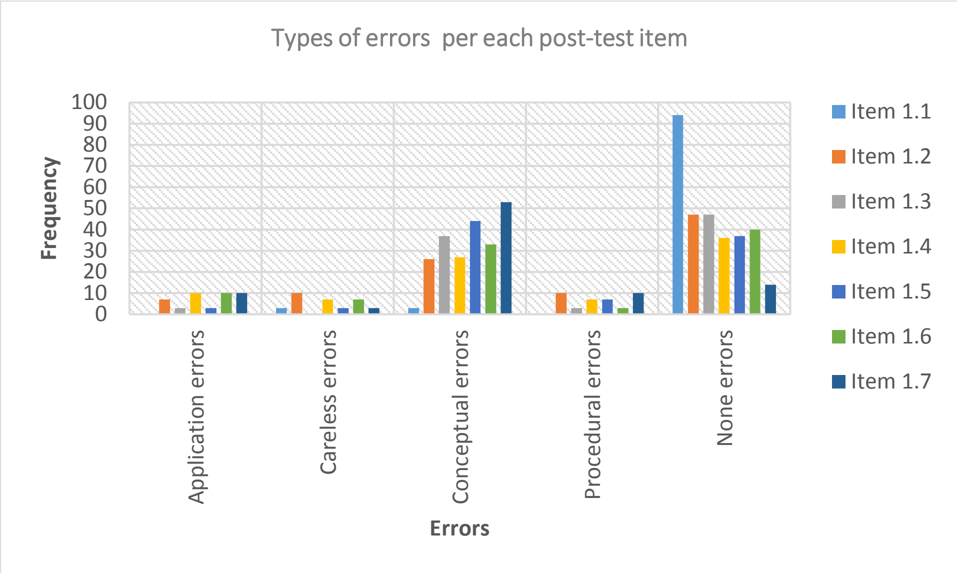


Figure 5: Frequency of each type of error per item in the post-test

In item 1.1 the learners' performance relating to their errors was fairly done with only 94% of the learners having errors in that item. The learners revealing no errors in this item were 28%. There were no application and procedural errors indicated on the learners' scripts except careless and conceptual errors with an occurrence of only 3%. There was a reduction of errors in the post-test as compared to the pre-test. The reason for the few errors identified on the learners' scripts was due to some learners were due to omission of terms or variables, reversing of the operational signs of the algebraic terms and inappropriate application of algebraic concepts for.

In item 1.2 the learners' performance relating to their errors was good with only 10% of learners having errors in this item. The learners revealing no errors in this item were 47%. There were no careless errors revealed in this item, indicating that learners were serious and concentrated well on simplifying algebraic expressions. The errors with the highest frequency were conceptual errors followed errors, followed by procedural and careless errors and then followed by application errors with percentage occurrence of 23%, 10%, 10% and 7% respectively. The reason for the errors were due to incorrect addition of terms, conjoining of unlike terms, incorrect procedures where the operational signs of terms were reversed and learners' creation of own rules.

In item 1.3 the learners' performance relating to their errors was fair 47% of learners displaying no errors in their post-test scripts. There were no careless errors, the errors with highest frequency were conceptual errors followed by application and procedural with a percentage occurrence of 37%, 3%, 3% respectively. The reason for the errors were due to some few learners who failed to expand brackets by incomplete distribution of terms and omission of terms and creation of learners' own rules.

In item 1.4 the learners' performance relating to their errors was fair with only 12% of learners having errors in this item. There 36% of the learners revealing no errors in the post-test. The errors with the highest frequency were conceptual errors followed by application errors then followed by procedural and careless errors with frequencies of 27%, 10%, 7% and 7% respectively. The reasons for the errors were due to incorrect or incomplete expansion of

In item 1.5 the learners' performance relating to their errors was not good with 70% of learners having errors in this item. There were37% of the learners revealing no errors in the post-test. The errors with the highest frequency were conceptual errors, followed by procedural errors, then followed by careless and application errors with frequencies of 44%, 7%, 3% and 3% respectively. The reason for the errors were due to conjoining of unlike terms, incorrect/incomplete distribution of terms, incorrect addition or subtraction of terms, omission of algebraic terms and creation of learner's own rules.

In item 1.6 the learners' performance relating to their errors was not good with 67% of learners having errors in this item. There were 40% of the learners revealing no errors in the post-test. The errors with the highest frequency were conceptual errors, followed by both application and procedural errors then followed by careless errors with frequencies of 33%, 10% and 7% respectively. The reason for t the errors were due to incorrect addition of terms, incomplete or incorrect expansion of brackets and learners' creation of own rules.

In item 1.7 the learners' performance relating to their errors was extremely bad with 77% of learners displaying errors on their pre-test scripts. The errors with the highest frequency were conceptual errors followed by both application and procedural errors and then followed by careless errors with frequencies of 53%, 10% and 3% respectively. The reasons for the errors were due to incorrect addition, incomplete distribution of and conjoining of unlike terms. These were revealed in learners' solutions when they simplified the expression $(x^2+3x-2)-(-2-9x+3x^3)$ to $3x^3+9x-6+6-27x+9x^3=12x^6+18x$ or $3x^3+9x-6-2-9x+3x^3=6x^3+x-4$.

The researcher put overall total frequency for the conceptual, procedural, application and careless errors. Overall, conceptual errors were the most prominent errors followed by procedural errors, followed by application errors and the errors with the least frequency were careless errors with total frequencies of 62, 55, 19 and 14 respectively. This showed that most learners encountered some problems when simplifying algebraic expression due to lack of conceptual and procedural understanding of algebraic concepts. The contribution of errors due learners' carelessness and misapplication of concepts were not very significant as compared to the errors due to misunderstandings of concepts and use of wrong procedures on simplifying algebraic terms.

4.12 Analysis of both the pre and post-test items

The analysis of the combined results of the pre and post-test were for the researcher to investigate if the intervention strategy of resourcing the learners' errors through the use of discovery learning would have helped the learners to reduce errors in the post-test.

Table 13: Showing the combined learners' performance in both the pre and post-test items

	Marks	Pre-test		Post-test		Post and pre-test
Level	Marks	Number of learners	Percentage of learners	Number of learners	Percentage of learners	Percentage points increase or decrease
1	0-29	12	41	5	17	24
2	30-39	7	23	4	13	10
3	40-49	4	13	3	10	3
4	50-59	2	6	3	10	4
5	60-69	3	10	5	17	7
6	70-79	2	6	3	10	4
7	80-100	0	0	7	23	23

The results shown in table 13 indicated that learners' performance in the post-test had improved. The number of learners with a mark ranging from 0-29% and 30-39% has reduced by 24% and 10% respectively. The reduction of percentage of learners in these two intervals is a positive move, this means that 34% of the learners who were in the interval 0-39% have moved to intervals between 40-100%. These learners have passed the post-test unlike what they did in the pre-test. The number of learners with a mark ranging from 40-49%, 50-59%, 60-69%, 70-79% and 80-100% had increased which is an indication an improvement in learners' performance as follows, 3%, 4%, 7%, 4% and 23% respectively. In the post-test most of the learners have strong conceptual and procedural understanding of algebraic terms unlike the knowledge they revealed in the pre-test.

Table 14: Showing combined statistics for the pre and post-test

	Pre-test	Post test	Percentage points increase or decrease	Comment
Percentage pass	37	70	33	Improvement
Percentage fail	63	30	33	Reduction in failure rate
Test average	38	54	16	Improvement

The improvement in the learners' performance is also shown in table 14 where the percentage pass rate of the learners increased from 37% to 70% which is a 33% improvement. On the other hand the percentage fail rate of the leaners decreased from 63% to 30% which is a decrease of 33%. The decrease in the number of learners failing is an indication of an increase of the number of learners passing. The learners' performance in the post-test was good and of high quality as compared to the learners' performance in the pre-test.

Table 15: Showing combined errors from item 1.1 to item 1.7 from the pre and post test

	Application	Careless	Conceptual	Procedural
Pre-test	19	14	62	45
Post-test	13	10	67	12
% decrease/increase	6	4	5	33

The combined analysis of the errors in both the pre and post-test indicated that in each and every error category there was reduction in the number of errors displayed in the learners' post-test scripts as compared to the number of errors displayed by learners' in the pre-test. There were a drop of 19% to 13% of application errors and a drop of 14% to 10% of careless errors and an increase of 62% to 67% of conceptual errors and a drop of 45% to 12% of procedural errors. In application , careless, procedural errors there was a reduction of 6%, 4%, 33%

respectively except the conceptual errors which increased by 5%. The frequency of the conceptual did not decrease in the post-test but increased. The learners were more careful in their simplifying of algebraic expressions, most of the learners avoided applying their own rules, their procedural understanding of algebra improved greatly. The percentages of the conceptual errors have increased instead of decreasing. This indicated that some grade 8 learners were still struggling on simplify algebraic expressions due to a lack of conceptual knowledge in algebra.

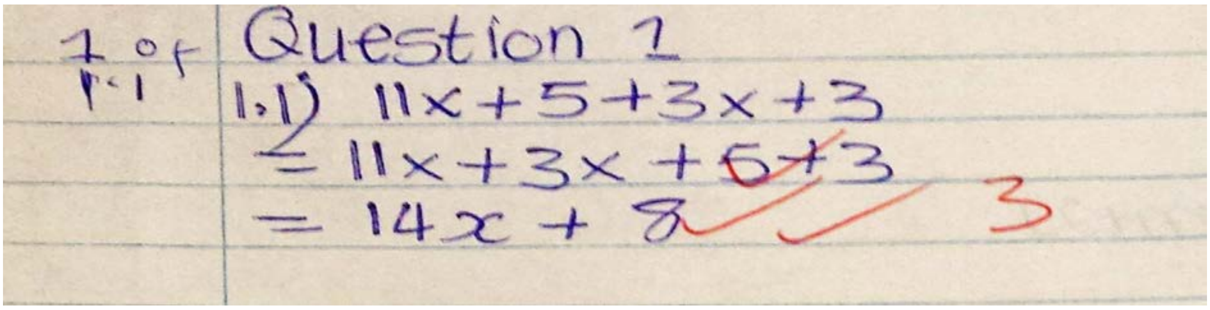
4.13 Qualitative analysis of the post-test using scanned vignettes

In this section the researcher analysed the same learners' post-test scripts as in the pre-test. The purpose of the post-test was to check if the intervention strategy employed through using learners' errors from the pre-test as a resource would have helped the learners to reduce the errors in the post-test. The researcher scanned vignettes to show learners' performance in the post-test from item 1.1 to item 1.7

Item 1.1 *Simplify the following algebraic expression 11x+5+3x+3*

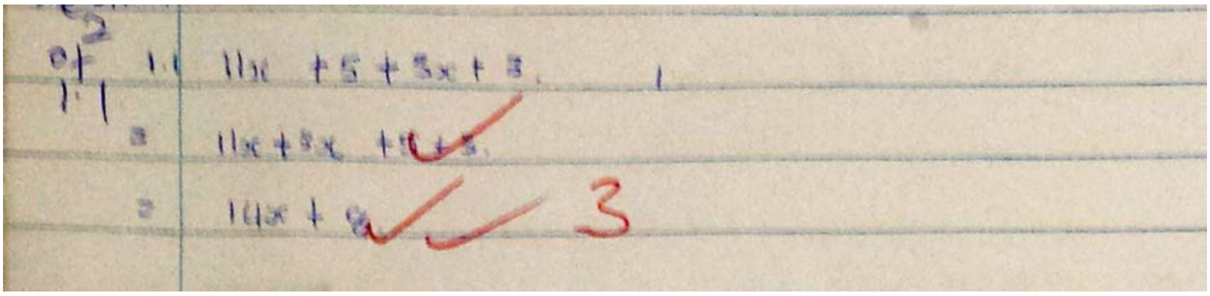
In item 1.1 of the post-test, learners were required to simplify the algebraic expression 11x + 5 + 3x+3to 14x+8. The objective of this question was to find out if learners could have improved in the addition of algebraic terms in an algebraic expression after a discovery learning was employed during intervention. Also the learners' pre-knowledge of addition of integers was put on the spot light.

Learner 1 of item 1.1



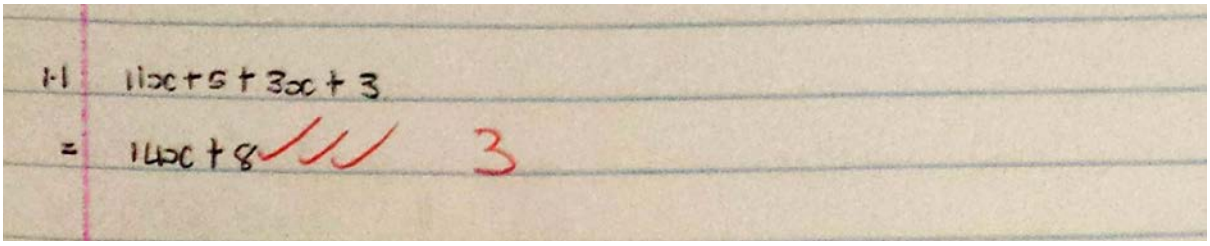
The learner managed to add the terms in this item and managed to come up with the correct answer. The careless error which was shown in the pre-test was avoided in the post-test indicating carefulness of working by the learner was reduced.

Learner 2 of item 1.1



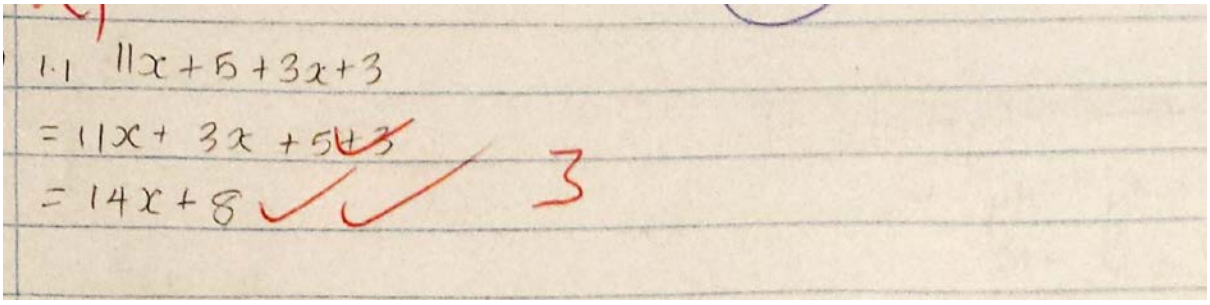
The learner correctly simplified the expression as shown above. Conjoining of terms as was done in the pre-test was avoided. Also the learner did not misapply rules as in the pre-test, he/she managed to simplify 11x+3x to 14x not $14x^2$. The application error was avoided.

Learner 3 of item 1.1



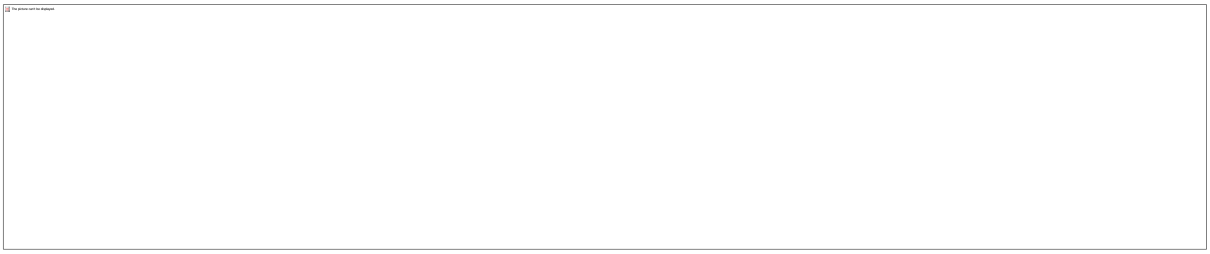
The learner simplified the algebraic expression correctly. This time the learner did not remove the variable in the expression to remain with numerical values. The learner showed an understanding of how to add terms in an algebraic expression without combining terms to one numerical value. The learner's procedures were perfect and no application errors were indicated.

Learner 4 of item 1.1



The learner correctly simplified the algebraic expression. The learner showed a good understanding of concepts and procedures in this regard. Conjoining of terms which the learner did in the pre-test was avoided as shown above. The learner took the answer 14x+8 as a complete solution, since it is an algebraic solution, not an arithmetic expression.

Learner 5 of item 1.1



The learner conjoined the terms as he/she did in pre-test the learner showed a lack of understanding of like and unlike terms. The learner viewed 14x+8 as an incomplete solution thus the learner combined the two terms to single term 22x. In that wrong answer of 22x, the learner managed to add 14 to 8 to get 22 which was an improvement from the pre-test where the learner wrote 4a+5+2a=10a.

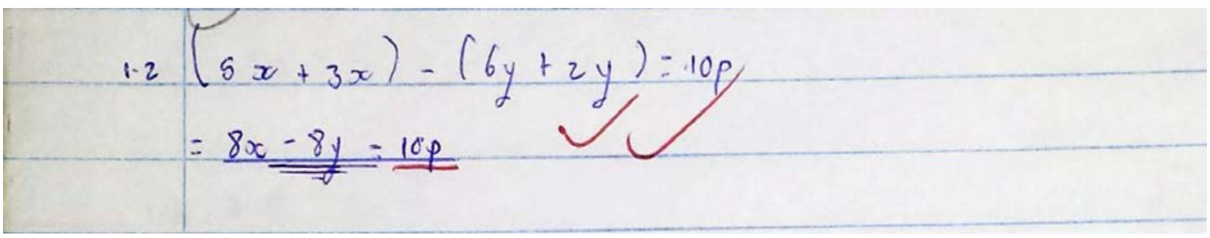
4.13.1 A summary of learners' errors in item 1.1 of the post-test

Generally learners performed well in item 1.1 of the post-test with 94% of the 30 learners getting all the answers correct and only 6% of the learners getting a mark of zero. The learners' performance in the post-test had greatly improved as compared to the learners' performance in the pre-test. The frequency of conceptual, procedural, careless and application errors had reduced in most of the learners' post-test scripts. Most of the learners managed to add the terms in the algebraic expression and conjoining of terms was avoided in most situations.

Item 1.2 *Simplify the following algebraic expression 5x-6y+3x-6y-2y+10p*

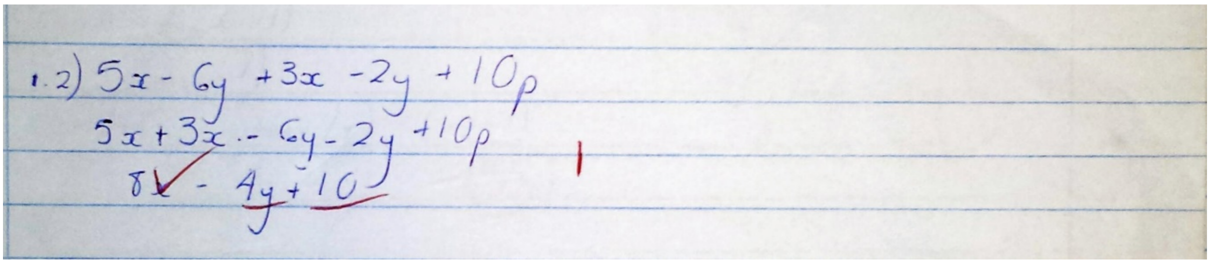
The learners were supposed to simplify 5x-6y+3x-6y-2y+10p to 8x-8y+10p. In this item the researcher wanted to find out if learners would have improved in their addition and subtraction of terms in an algebraic expression as compared to their performance in the pre-test. The learners were supposed to use their previous acquired knowledge including the knowledge acquired from the intervention to manipulate the above stated algebraic expressions.

Learner 1of item 1.2



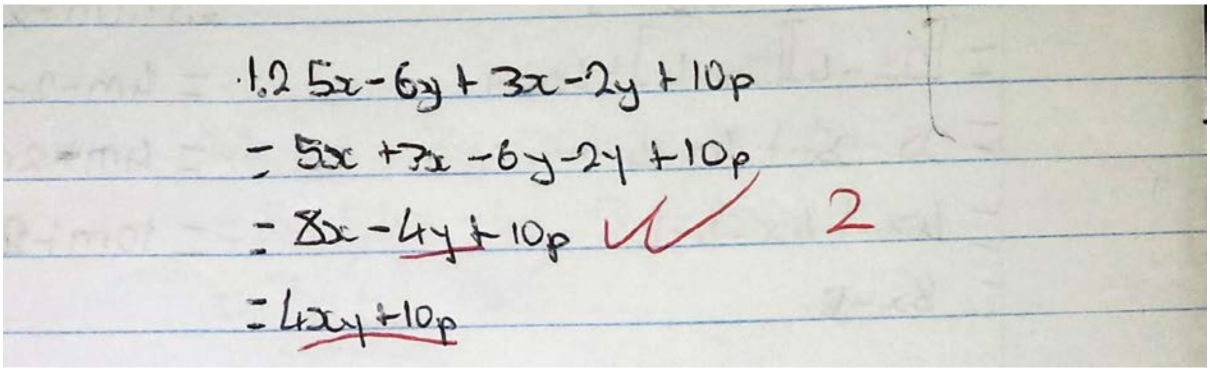
The learner managed to get the correct answer for the first two terms of the expression. However the learner mistakenly changed +10p to -10p, this is an indication of careless error. The learner incorrectly copied the question, an indication of learners' lack of concentration in this item. Conjoining of terms was avoided, where the answer 8x-8y-10p could have been written as 16x+y-10p as was done in the pre-test.

Learner 2 of item 1.2



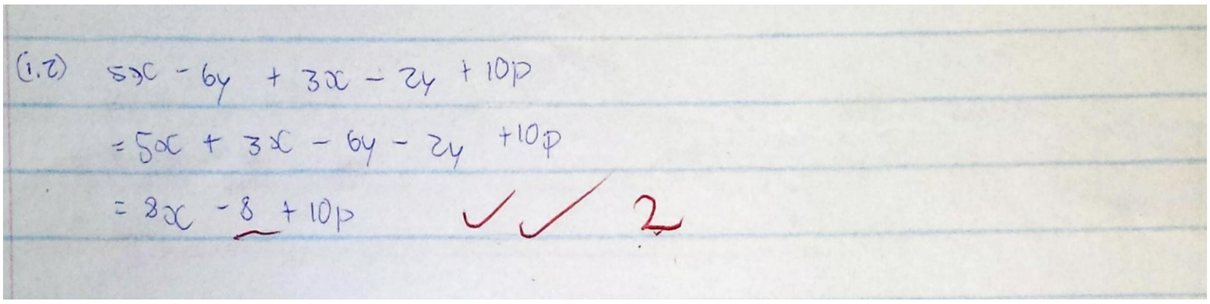
Only the first term of the algebraic expression was done correctly. The learner failed to simplify the expression -6y—2y to -8y instead the learner wrote -4y. Also in the final step the learner wrote the third term as 10 instead of 10p. The learner's wrong answer is an indication of conceptual, procedural and careless errors.

Learner 3 of item 1.2



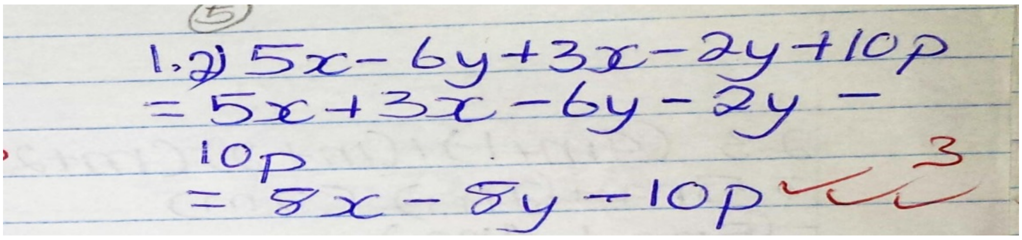
In his/her collection of like terms, the learner managed to get the first and second term of the expression in step 2 correct. The learner incorrectly simplified -6y-2y to -4y instead of -8y and conjoined 8x and 4y to 4xy. This indicated that the learner still have some misconceptions in subtraction of negative terms and still thought that two unlike terms can be combined to a single term to make the expression complete.

Learner 4 of item 1.2



The learner showed a good understanding of addition and subtraction of terms of the expression, no conceptual and procedural errors were shown. However the learner wrote -8 instead of -8x, an x was omitted, an indication of careless error. Learner 4 made what Olivier (1992b) refers to as a slip.

Learner 5 of item 1.2



The learner collected like terms but he/she made an error of changing +10p to -10p. This negatively affected the learner's answer. However the learner managed to simplify5x+3x and -6y-2y to 8x and -8y. The learner realised that unlike terms cannot be conjoined to a single term. The researcher was supposed to give the learner 2 marks instead of 3 because of the change of sign indicated above.

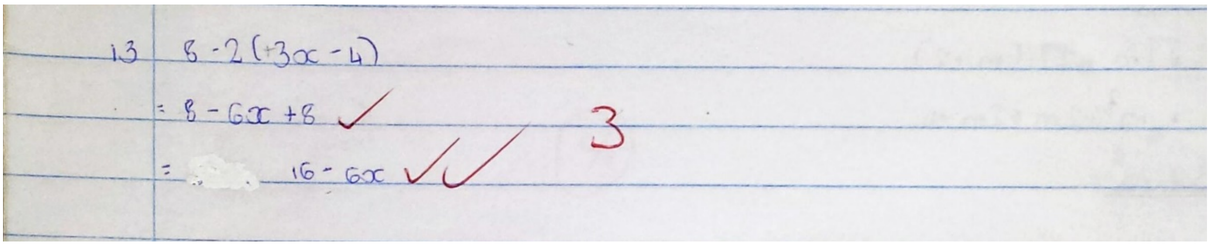
4.13.2 Summary of learner's errors in item 1.2 of the post-test

Item 1.2 of the post test was fairly done. Most of the learners managed to successful simplify the expression. However there were some few learners who were still struggling with addition and subtraction of terms. The conjoining of terms was minimal with a lot of learners' procedural, conceptual and careless errors reduced in the post-test as compared to the pre-test. Some learners were still omitting terms for instance some learners wrote -8 or 10 instead of -8y or 10p respectively. This is an indication of careless errors where learners hurriedly simplify algebraic expression without paying much attention to what they were doing. The reasons for failing to add or subtract terms were omission of terms, inclusion of learners' own terms and changing of operation in the algebraic expression. The results from 1.2 indicated learners were now having a better understanding of the addition and subtraction of directed number as emphasised in the intervention process through the use of a number line.

Item 1.3 Simplify the following algebraic expression 8-2(3x-4)

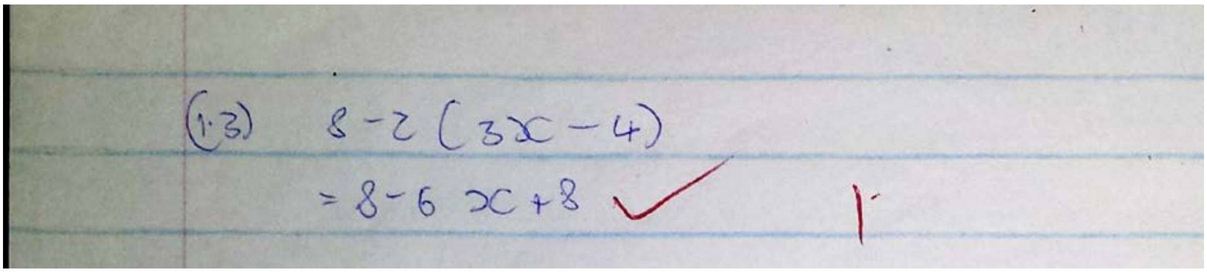
In item 1.3 of the post-test learners were supposed to simplify the algebraic expression 8-2(3x-4) to 16-6x. The purpose of the item was to follow up on learners' progress on appropriate order of operation, expansion of brackets, addition and subtraction of algebraic terms after an intervention was done before the writing of the post-test

Learner 1 of item 1.3



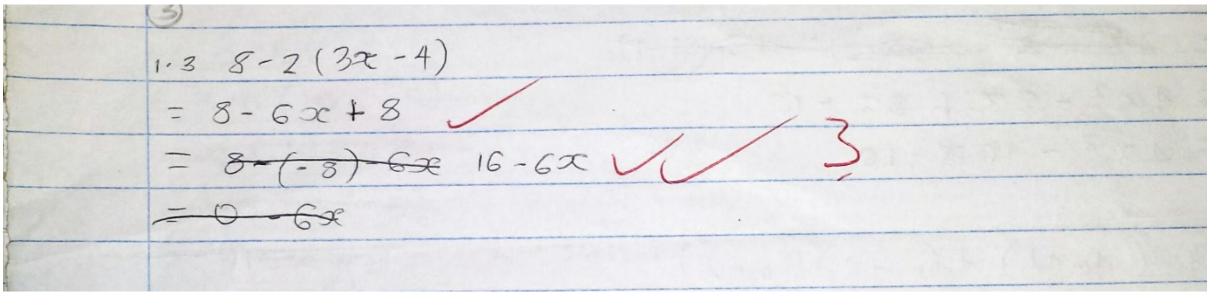
The learner successfully simplified the expression as shown above. This was a great improvement as compared to what was done in the pre-test. The BODMAS rule was adhered to where the bracket was expanded before the subtraction operation was done. The learner used correct concepts and procedures.

Learner 2 of item 1.3



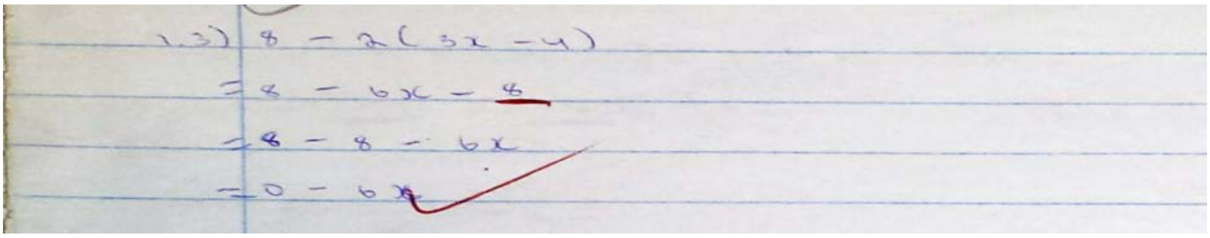
The learner's interpretation of the question was better than what was done in the pre-test where the 8 would have been multiplied by all the other terms in the expression. The learner managed to distribute the terms. The learner was supposed to fully simplify the expression to 16-6x. There is an improvement of learners performance in this item as compared to what was done in the pre-test.

Learner 3 of item 1.3



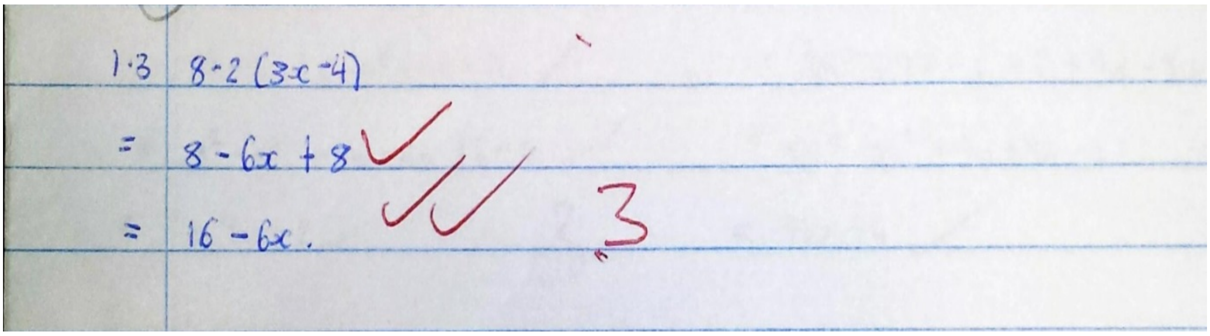
The learner followed the BODMAS rule where brackets were expanded before subtraction of terms. The learner just expanded the brackets once unlike expanding the brackets twice as in the pre-test. Conjoining of terms was avoided indicating that the learner knew that unlike terms cannot be combined to a single term. No error was indicated here.

Learner 4 of item 1.3



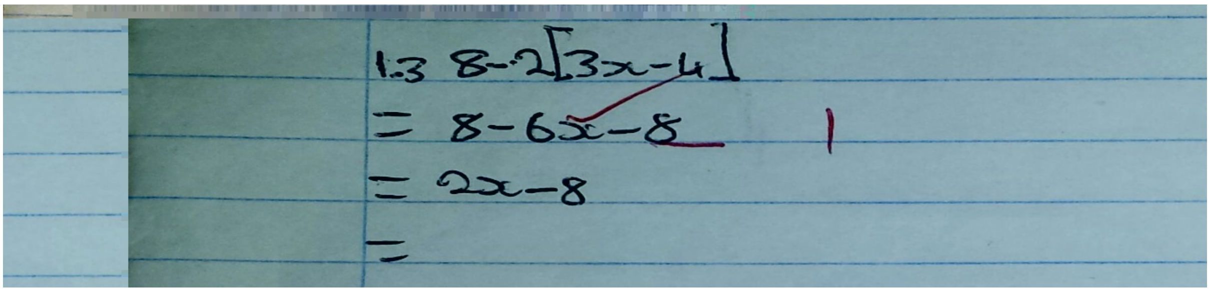
The learner committed the same error of failing to distribute the terms where -2(-4) was simplified to -8 instead of +8. This resulted in a conceptual error.

Learner 5 of 1.3



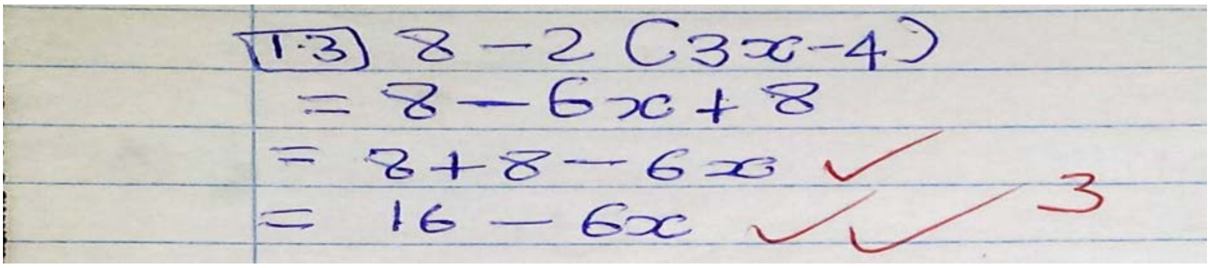
The learner managed to distribute the terms correctly and no other brackets were introduced after the distribution as was done in the pre-test. No learner's own rules were applied. The learner appropriately followed correct procedures involved in simplifying the algebraic expression. No procedural, conceptual and careless errors indicated in this item.

Learner 6 of item 1.3



The partial distributed the terms, -2 was multiplied by 3x but -2 was not multiplied by -4. Here the learner showed that he/she did not master the concepts of simplifying algebraic expressions well. Also the learner's misconception of conjoining terms was not cleared during intervention. This is so because some errors are resistant to instructions (Nesher, 1987).

Learner 7 of 1.3



The learner successfully simplified the expression. The learners' conceptual and procedural knowledge had improved if we compare the learner's post-test results and the pre-test result. There was no creation of the learners' own question as was done in the pre-test.

Learner 8 of item 1.3

The learner successful managed to simplify the expression. A great improvement in mastery of algebraic concepts as compared to what the learner had done in the pre-test where an expression 3-5(2x-1) was incorrectly simplified to -2(10x-5) = -20x+10 = -10x. In the post-test the learner managed to follow the BODMAS rule and did not conjoin unlike terms.

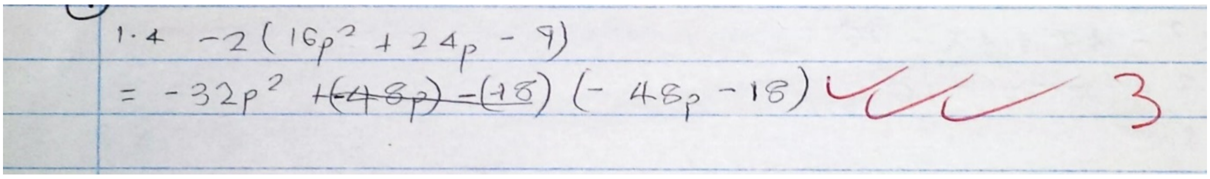
4.13.3 Summary of learners' errors in item 1.3 of the post-test

Generally most learners performed well in this post-test item with 70% of the learners getting all correct answers, successfully simplified the algebraic expression 8-2(3x-4). Most learners were able to follow the BODMAS rule where expansion of brackets was done before subtracting the resulting terms from 8. Most learners showed that their procedural and conceptual understanding had improved as compared to their performance in the pre-test. The learners were able to add and subtract algebraic terms, careless errors, misapplication of rules and conjoining of terms were minimal.

Item 1.4 *Simplify the following algebraic expression* $-2(16p^2+24p-9)$

In item 1.4 of the post-test learners were supposed to simplify the expression $-2(16p^2+24p-9)$ $= -32p^2 - 48+18$. The aim of this item was to check learners' understanding in distribution of terms where the concepts of multiplication of negative and positive algebraic terms were tested. Learners were expected to recall the following concepts; positive x positive=positive, positive x negative= negative, negative x negative=positive and negative x negative=positive in the multiplication of integers.

Learner 1 of item 1.4

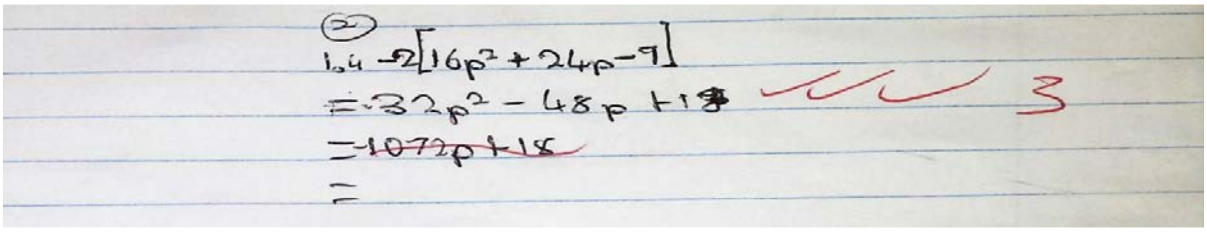


The learner distributed the terms fairly well, the first and second terms of the solution are correct except the third term which was presented -18 instead of +18. The learner struggled with the multiplication of negative numbers. Conjoining of terms was avoided. The researcher was supposed to give the learner two marks instead of 3 because -2(-9) = 18 not -18 as written by the learner.

Learner 2 of item 1.4

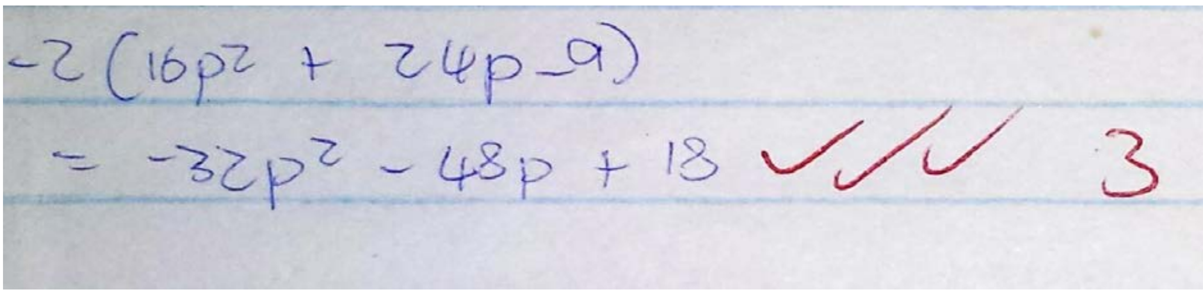
The learner simplified the expression correctly. The terms were well distributed unlike in the pre-test where the distribution of terms was incomplete. The learner expanded the brackets $-2(16p^2+24p-9)$ correctly to $-32p^2$ $-48p+18$ not $-32p^2+-2+24-2-9=-32p^2+22p-11$ where the learner applied his/her own rules, rules which were never taught in class. Conjoining of terms was avoided. No indication of any procedural, conceptual, application and careless errors. Correct concepts and procedures were used.

Learner 3 of item 1.4



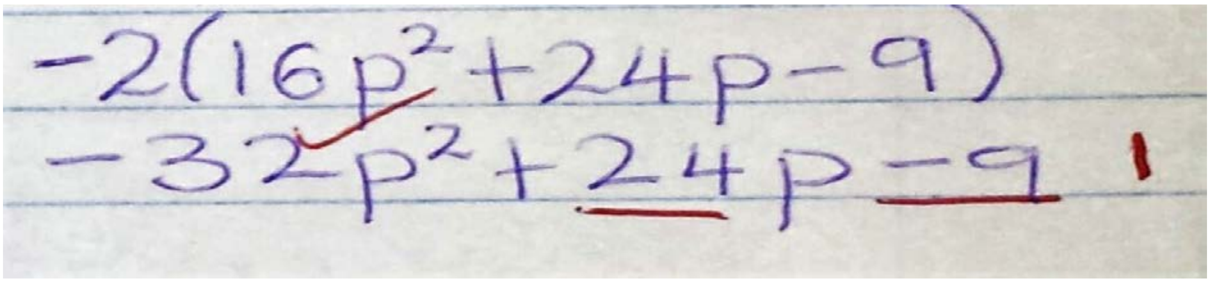
The terms were distributed correctly in step 2. The learner again misinterpreted the meaning of $-32p^2-48p$ as $-32^2p-48p=-1024p-48p=-1072p$ as was done in the pre-test. This learner's misconception was not eradicated in spite of the discovery learning intervention which was employed. The learner conjoined unlike terms to a single term. There is an indication of conceptual, procedural and careless errors.

Learner 5 of item 1.4



The learner simplified the expression correctly, a great improvement as compared to what the learner did in the pre-test where the expression $5(p^2+2p-3)$ was simplified to 5p2+2p-3, wrong notation and an incomplete distribution of terms. In this post-test item there is no indication of an error, procedural, conceptual, application or careless.

Learner 6 of item 1.4



The learner did partially distributed the terms where $16p^2$was the only term multiplied by -2. However the learner did not multiply -2 by -9. This error could be due to learner's carelessness where the multiplication of terms was omitted. This careless error could be due to lack concentration where learners hurriedly simplify the expression with no rechecking of the answer. Conjoining of terms was avoided. There is an indication of conceptual and procedural errors.

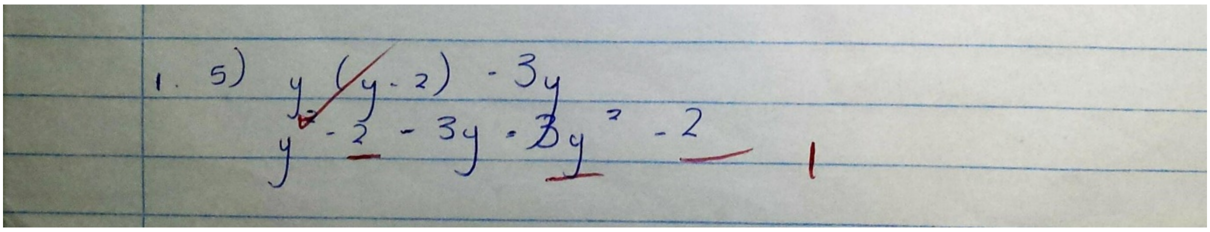
4.13.4 Summary of learners' errors in item 1.4 of the post-test

Generally most of the learners performed well in this item. About 55% of the learners in the post-test compared to 36% of the learners in the pre-test managed to come up with correct answers. The results from this item indicated that learners have improved greatly in their conceptual and procedural understanding of algebraic expressions. There was no application of learners' own rules and careless errors and conjoining of terms was minimal. However some errors were noted when learners incompletely distributed terms and incorrect signs for the algebraic expressions after the terms were multiplied.

Item 1.5 ***Simplify the following algebraic expressions y(y-2)-3y***

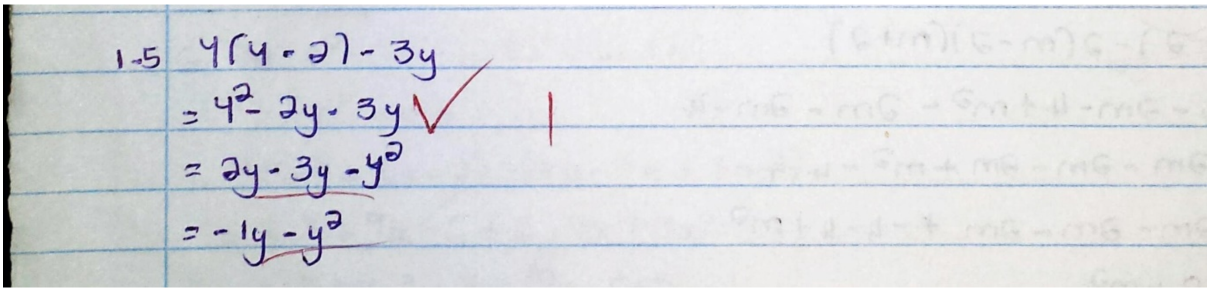
The learners were supposed to simplify ***y(y-2)-3y to*** $\boldsymbol{y^2}$***- 5y.*** The purpose of the item was to check learners' understanding of multiplication (distribution of terms), addition and subtraction of algebraic expressions after the learners' pre-test errors were used as a resource to help learners reduce errors in simplifying algebraic expressions.

Learner 2 of item 1.5



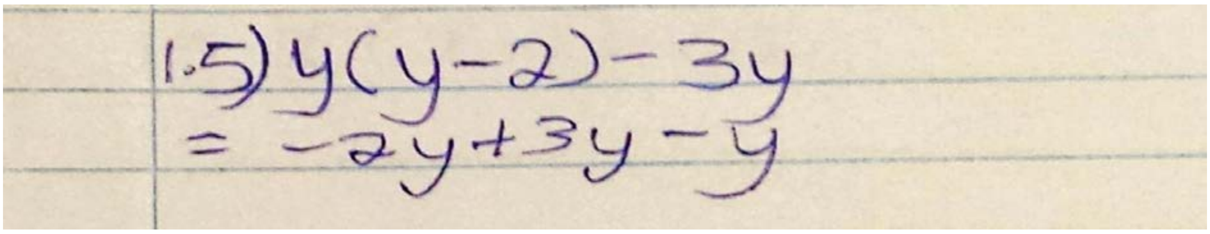
The learner was able to multiply y by y and got y^2, an improvement from the pre-test where the learner wrote x(x) as 2x where laws of exponents were not adhered to. The learner did no multiply y by -2 (an incomplete distribution of terms). Also the learner multiplied y by -3y which was outside the brackets. The learner introduced rules which were never taught in class. This showed that this leaner was stilling struggling with simplifying algebraic expressions; misconceptions were still present in the learner's mind. There is an indication of conceptual, procedural and application errors.

Learner 3 of 1.5



The learner distributed the terms correctly in step 1. This was a great improvement from what the learner has done in the pre-test where x(x-1) -5x was expressed as x-x-5x=-5x. In step 2 the learner made a mistake by rearranging the expression where-2y and y^2 were written as 2y- and - y^2 respectively. The learner made a slip resulting in careless error.

Learner 5 of item 1.5



The learner incorrectly simplified the expression and no improvememt was made from the pre-test. This showed that the learner was still struggling with simplifying algebraic expression. This is an indication of conceptual and procedural errors.

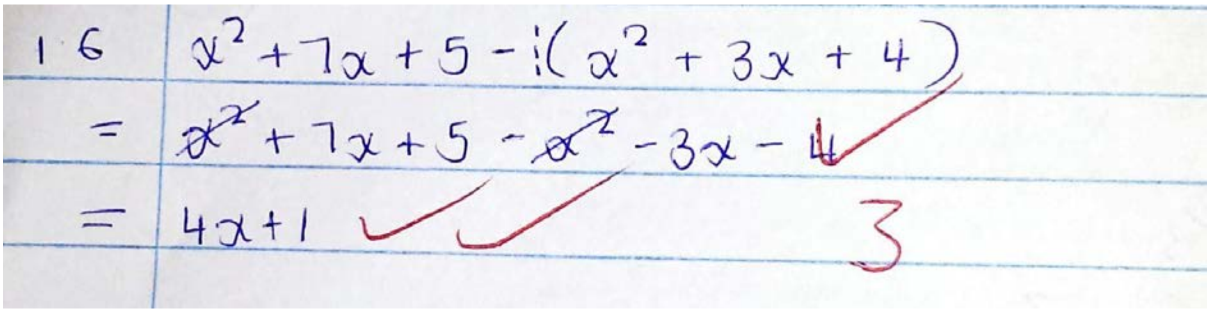
4.13.5 Summary of learners' errors in item 1.5

This item was fairly done with 44% of the learners in this item getting all answers correct as compared to only 13% of the learners in the pre-test getting all answers correct. There is an indication of learners' improvement in conceptual and procedural understanding of algebraic expressions. Not any learner conjoined the algebraic terms. The common error in this item was as a result of learners failing to assign correct signs to the terms distributed and an incomplete distribution of terms was another problem. Minimal careless errors were noted in this item.

Item 1.6 Simplify *the algebraic expressions x^2+ 7x+5-(x^2+4x+4).*

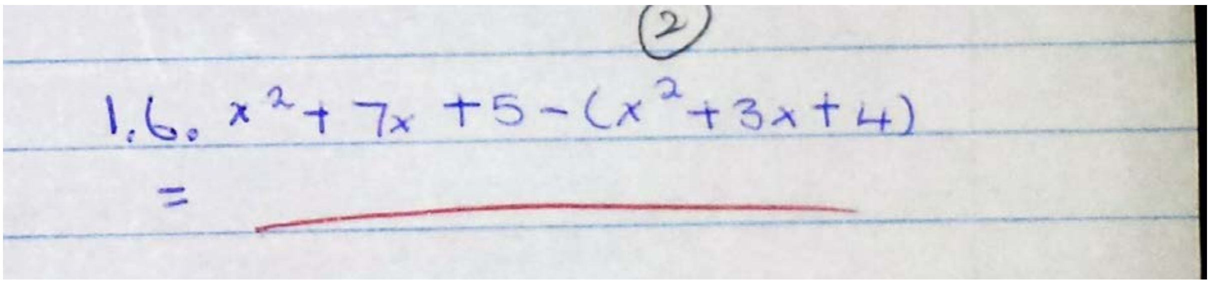
The learners were supposed to simplify x^2+ 7x+5-(x^2+4x+4) to 4x+1. The purpose of the post-test was to check if learners were able to recall some previous learnt concepts including those concepts taught during intervention. The recalling of previously learnt concepts of addition and subtraction of integers was of great importance.

Learner 1 of item 1.6



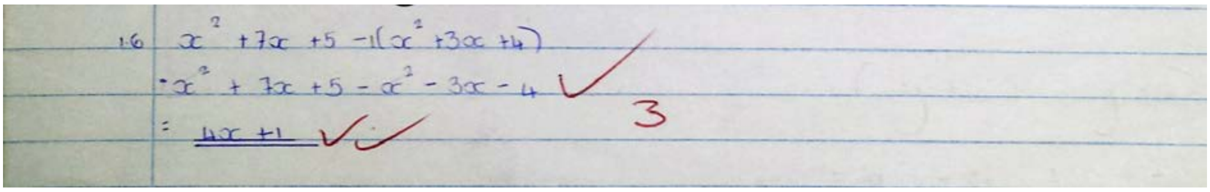
The learner simplified the expression correctly, there was a correct distribution of terms in the second algebraic expression before it was added to the first algebraic expression. The learner's performance in this post-test item greatly improved as compared to the learner's performance in the pre-test where the learner hardly got anything correct.

Learner 2 of item 1.6



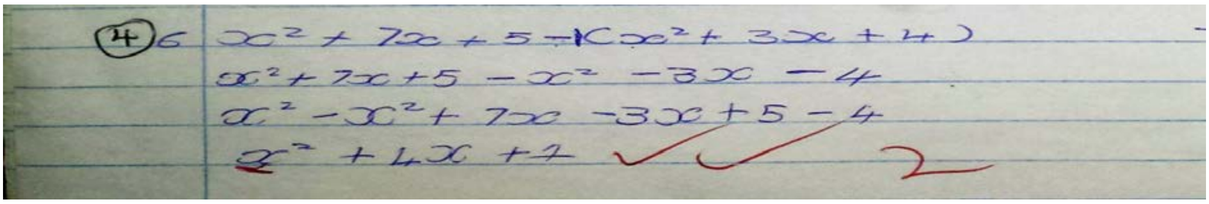
The learner just wrote the question down and no working was done thereafter. Hence no conclusion can be made about this item in this post-test as far as performance is concerned.

Learner 3 of item 1.6



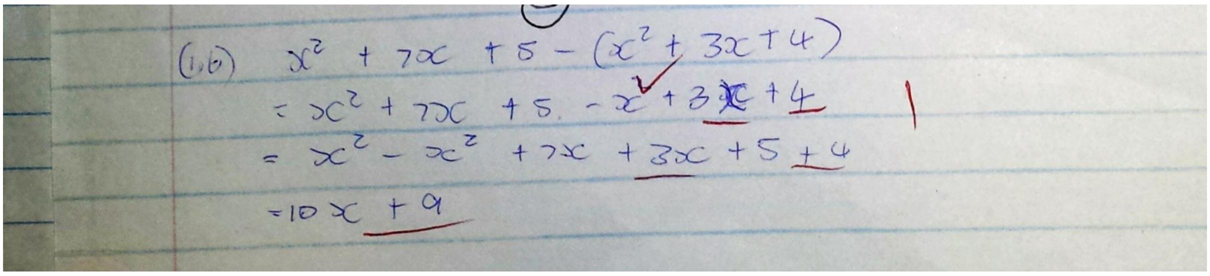
The learner simplified the expression correctly. The learner got everything correct an indication of learner understands of simplifying algebraic expressions. The learner managed to rectify his errors of the pre-test by expanding the brackets of the expression before subtracting it from the first expression. The learner knew that x^2-x^2 = 0 not x^4 as was done in the pre-test. No errors were indicated in this learner's post-test item.

Learner 4 of item 1.6



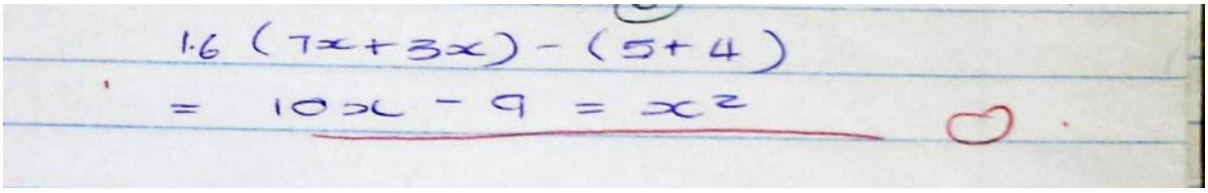
The learner distributed the terms well, he/she showed conceptual and procedural understanding in simplifying algebraic expressions. Step 1 and 2 of the solution are correct. The only problem was the final step where the learner mistakenly wrote $x^2 – x^2 = x^2$ instead of 0 (incorrect addition of terms). This error could also been there because of a slip. There is an indication of either a conceptual or careless error.

Learner 5 of item 1.6



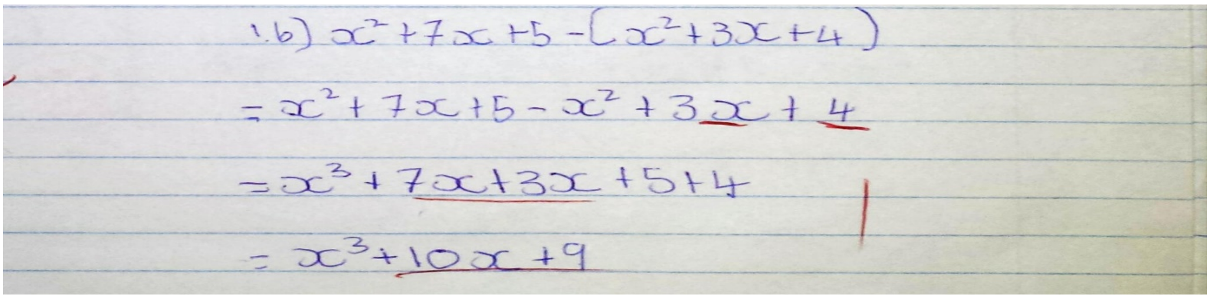
There was an incorrect simplifying of the expression due to an incomplete expansion of the brackets in the second expression where x^2 was the only term multiplied by -1. The learner simplified 7x+3x+5+4 to 10x+9 instead of 7x+3x+5-4 = 10x+1.The performance of this learner in this post-test item indicated that the learner was still struggling with simplifying algebraic expressions despite the fact that an intervention strategy of resourcing errors from the pre-test was done.

Learner 6 of item 1.6



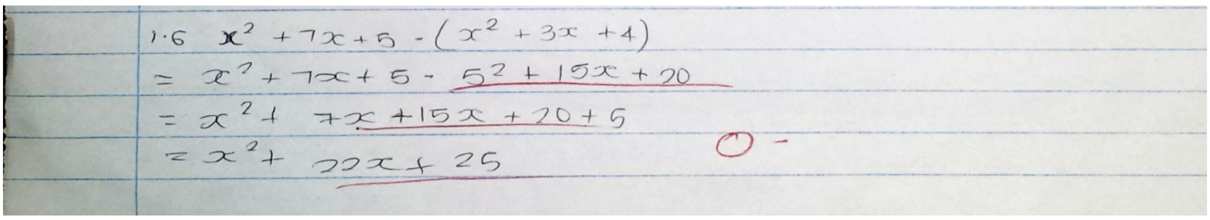
The learner did not simplify the terms correctly as was done in the pre-test. The leaner did not improve in his/her performance after an intervention strategy was administered before the learner's writing of the post-test. The terms were not distributed correctly, there was conjoining of terms and applying of leaner's own rules where 10x-9 was simplified to x^2 (application of learner' own rules). This resulted in conceptual and procedural errors.

Learner 7 of item 1.6



The learner did not simplify the expression correctly. There was an incorrect distribution of terms in the second algebraic expression, x^2 was the only term which was multiplied by -1. Also the learner failed to add the terms in step 3 where $x^2 - x^2$ was simplified as x^3 instead of 0. The learner's performance did not improve. This learner showed that he/she struggles with the simplification of algebraic expression. There is an indication of conceptual and procedural errors in this learner's post-test item.

Learner 8 of item 1.6



The learner failed completely to simplify the algebraic expressions. There was an incorrect distribution of terms in the second algebraic expression where the terms were multiplied by 5 (the third term of the first expression). The learner created his/her own rules which were never taught in class. The learner's misconception in this regard resulted in conceptual, procedural and application errors.

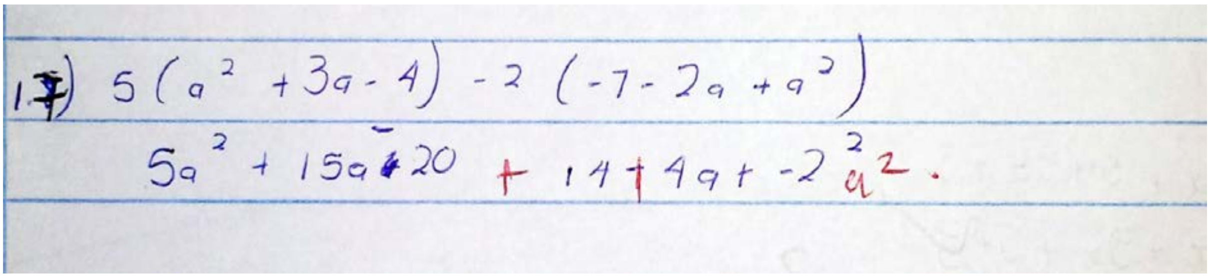
4.13.6 Summary of learners' errors in item 1.6 of the post-test

Item 1.6 was not done well even though there were some improvements in learners' performance in the post-test as compared to the learners performance in the pre-test where 6% compared to 0% of the learners got all the answers correct. Most learners showed that they struggle with simplifying this algebraic expressions. A number of learners did not attempt this question. Conceptual, procedural , application and careless errors were still on the rise in this item The reasons for learners' errors in this item were conjoining of unlike terms, creation of learners' own rules, unnecessary changing of operational signs, lack of basic skills of addition, subtraction and exponents.

Item 1.7 Simplify *the following algebraic expression $5(a^2+3a-4)-2(-7-2a+a^2)$*

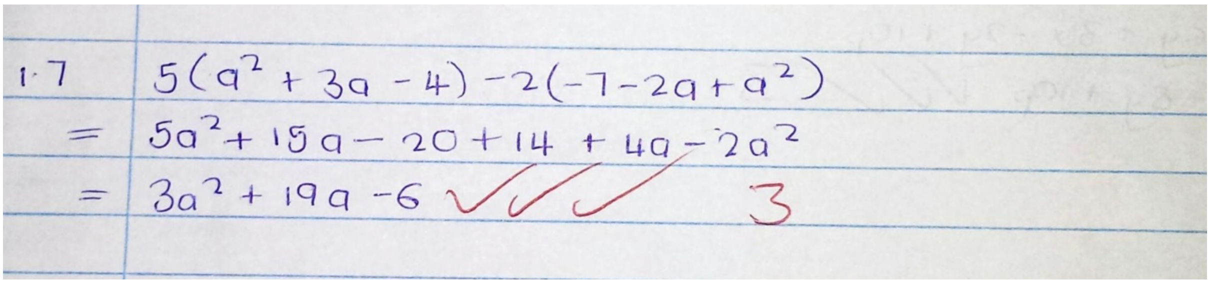
The learners were supposed to simplify $5(a^2+3a-4)-2(-7-2a+a^2)$ to $3a^2+19a-16$. The aim of this item was to check learners' understanding of algebraic expressions after an intervention strategy was employed to learners before they write the post-test. The learners were tested on the concepts of addition, subtraction, distribution, order of operation of on simplifying algebraic expression.

Learner 1 of item 1.7



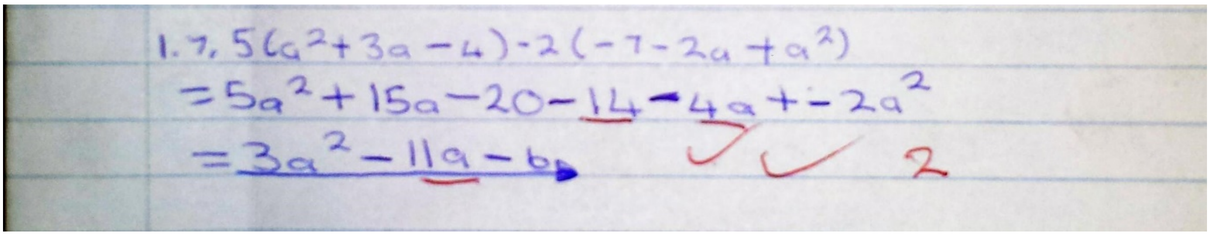
The learner distributed the terms in the first expression correctly but failed to distribute the terms in the second bracket where $-2(-7-2a+a^2)$ was simplified to $-14-4a-2a^2$ instead of $14+4a-2a^2$. This result showed that that this learner still have some problems with multiplication of terms by a negative number where a negative number x negative number = positive number.

Learner 2 of item 1.7



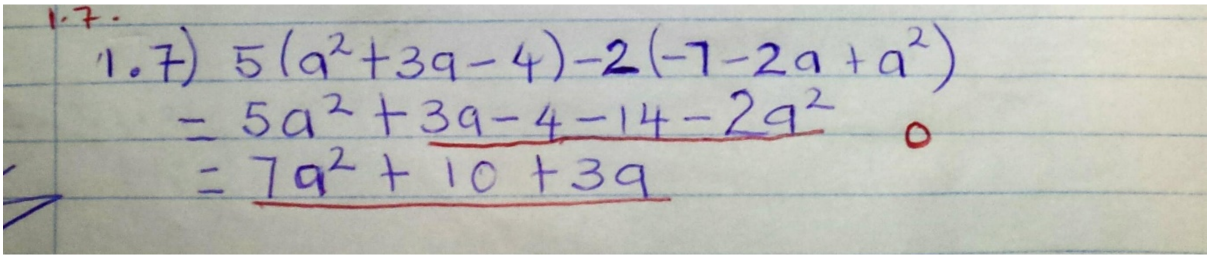
The learner successfully simplified the expression not like what the learner did in the pre-test where nothing was correct in the learner's solution. This result is an indication of an improved conceptual and procedural understanding of the learner in simplifying algebraic expressions. No error was indicated in this learner's solution.

Learner 3 of item 1.7



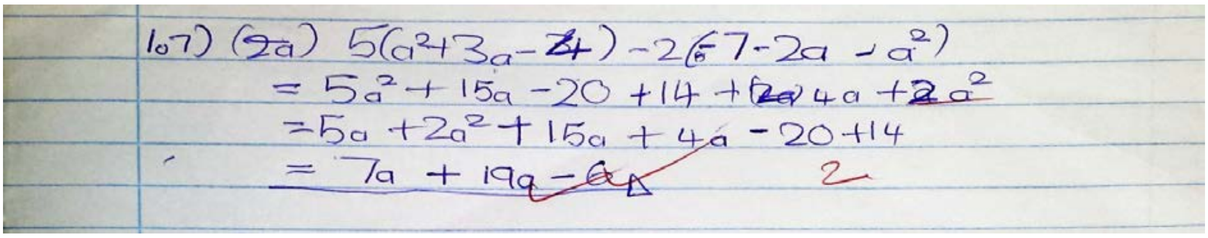
The learner simplified the first algebraic expression correctly but failed to distribute terms in the second expression where $-2(-7-2a+a^2)$ was simplified as $-14-4a-2a^2$ instead of $14+2a -2a^2$. This showed the learners misunderstanding in the multiplication of negative algebraic terms with a negative number. This resulted in conceptual and procedural errors.

Learner 4 of item 1.7



The learner failed to distribute the terms in both the expressions. The learner struggled with the simplification of algebraic expressions. This resulted in conceptual and procedural errors.

Learner 5 of item 1.7



The learner managed to distribute terms in the first expression and in the second expression. Step 2 was correctly done where the learner managed to group and collect like terms together. However the learner failed to come up with correct answers in the last step because of conjoining of terms. The learner simplified $5a+2a^2$ to $7a$. This is an indication of conceptual and procedural errors.

4.13.7 Summary of learners' errors in the post-test item

The learners' performance in item 1.7 was very poor. Learners failed to distribute terms, they conjoined unlike terms, they misinterpreted concepts, they unnecessarily change the signs of terms, and they applied their own mathematical rules.

4.14 Discussions

The constructivist's theory of learning was useful in the study in explaining how learners conceived mathematical ideas including their misconceptions (Piaget, 1968; Von Glasersfeld, 1990, Hatano, 1996; Smith, Disessa, & Roschelle, 1993). Learners were afforded some opportunities to construct knowledge through answering questions in both the pre and post-test. The study revealed that learners encountered some difficulties when simplifying algebraic expressions through errors displayed on learners' pre and post-test scripts. The underlying reasons for the errors were misconceptions which learners conceived in their minds during teaching and learning of algebra. The learners' misconceptions held up learners not to successful simplify algebraic expressions in both the pre and post-test. Learning is a thinking process which occurs in three stages which are assimilation, accommodation and equilibration (Piaget, 1968). If one of the processes fails then learning becomes ineffective and erroneous. This was the reason why learners failed to simplify algebraic expressions because their conceptual framework (schema) failed to reorganise their current algebraic concepts to fit in their existing arithmetic concepts. The learners' minds were supposed to link the previous learnt arithmetic skills to the current algebraic skills and retrieve some information which is alike and use it to simplify algebraic expressions. For instance learners were supposed to use the concepts of addition and subtraction of integers for them to be able to simplify the algebraic expression 5x-3y-4y-3a+2x. Complement to this the concepts of like and unlike terms was a priority.

In the study, some learners' misconceptions which resulted in errors as revealed in the learners' pre and post-test scripts were due to overgeneralisation (Piaget, 1968). This overgeneralisation was as a result of the learners' minds failing to assimilate/ accommodate new algebraic concepts in their existing schemas. In the study overgeneralisation was reflected in some instances where learners failed to connect the algebraic expressions to the previous learnt concepts of arithmetic which were earlier learnt about in primary school. In the pre-test learners overgeneralised concepts and incorrectly simplified 6a+5 to 11 or 11a. They conjoined unlike terms or removed the variables 'a' for them to remain with a single algebraic term or numerical value. The learners' errors in simplifying algebraic expressions resulted in learners failing to retrieve the correct methods from their long term memory and applied incorrect schemas in this regard. Some errors were not due to the mind failing to retrieve concepts but were caused by learners failing to rearrange their thinking mechanism in order to accommodate the algebraic concepts.

In the study another source of errors was misinterpretation of concepts either from the teacher during teaching or from mathematics textbooks, work sheets and the like. Misinterpretation of concepts usual occur when the mind fail to accommodate new information in their current schemas. Thereby forcing learners to learn concepts by memorisation, a key cause of many mistakes where learners try to recall incomplete or distorted rules (Olivier, 1989). This was reflected in the scanned script of learner 6 of item 1.3 of the pre-test where the expressions 3-5(2x-1) was simplified to 3(-10x+5) =-30x+8. The learner expanded the brackets -5(2x-1) to -10x+5 but misinterpreted the meaning of 3-(-10x+5). The learner wrote 3(-10x+5) where 3 was multiplied by -10x and 3 was added to 5 to get -30x+8 which was an error. Another example of learners' misinterpretation of concepts was shown by the scanned script of learner 2 of item 1.4 of the pre-test where the expression $5(p^2+2p-3)$ was simplified to $5p^2+7p+2$. The 7p+2 was obtained by simplifying adding 5 to 2p and adding 5 to -3. The learner partially remembered to multiply 5 to p^2but did not know what to do thereafter. The root cause of the misinterpretation was due to memorisation of concepts so learners failed to recap previous learnt concepts and created their own rules in an attempt to simplify the algebraic expressions in the tests. This was supported by Robinson, Eve and Tirosh (1994) who state that students often make sense of the subject matter in their own ways which do not correlate with the structure of the subject matter.

The learner's new knowledge understandings are dependent on the prior knowledge a learner retrieves from the earlier learnt concepts and learners without this prior knowledge are disadvantaged as far as learning is concerned (Piaget, 1968; Booth, 2008). In the study some learners errors were due to the learners' misconceptions resulted from incomplete knowledge from previous learnt concepts like arithmetic concepts. This was reflected in the pre-test when learners incorrectly simplify the expression 5x-3y-4y-3a+2x to 7x+7y-3a or 7x-y-3a or 7x+y-3a. This reflected that learners did not grasp the concepts of addition and subtraction of integers. They had misconceptions from the manipulation of directed numbers which they passed on to algebraic concepts and failed to simplify the expression -3y-4y to -7y. After the analysis of both the pre and post-test the researcher noted that teachers need to identify errors learners have in each and every topic in order to help learners to reduce them. If learners are not helped to rectify their errors when solving mathematical problems, the learners will pass misconceptions inherited from previous learnt concepts to the next topic and continue marking errors. This is so because learning is cumulative, meaning to say whatever information a learner brings to class influences what the learner will benefit from the next experience.

Piaget (1968) states that the mind sometimes reaches a state of disequilibrium where the mind eagerly wants to solve problems but the concepts presented are unfamiliar.

Constructivism acknowledges that each learner has a unique background and unique needs and brings to the classroom distinctive knowledge depending on the learners previous acquired knowledge (Piaget, 1968; Wittrock, 1986) and Bruner (1996) states that every person has a different interpretation and construction of knowledge. This view were confirmed in the study when learners revealed some wide range of errors as shown in their test scripts and interview responses but all pinned under conceptual or procedural or application or careless errors. The results from the analysed indicated the uniqueness of learners in relation to their knowledge acquisition. Some learners' unique characters in the construction of knowledge were witnessed in items like item 1.2 of the pre-test. Learners were supposed to simplify the expression 5x-3y-3a-4y-3a+2x to 7x-7y-3a. The table show how learners differently interpret and construct knowledge depending on one's past experiences.

Table 16: Showing learners' different constructions of knowledge

7x-y-3a	7x-1	7x+y-3a	7x-7x=3A	7x+7y+3a	17x+y+a	6xy-3a

The research noticed different types of learners' knowledge constructions; some correct some incorrect. The researcher gained a better understanding of different ways in which learners conceive mathematical concepts despite the fact that the learners were exposed to the same teacher with the same resources and given the same attention. This was supported by von Glasersfeld (1990) who states that each individual learner uses his/her schema to construct his/her conceptions sensible to him. The researcher appreciated every learner's contribution in the pre and post-test in order to come up with strategies for improved teaching. Teachers need to understand that in the classroom learners have a unique ways of constructing knowledge so as to help learners to solve problems depending on the misconceptions they have.

The uniqueness of learners in constructing knowledge was also shown through the way learners performed in both the pre and post-test. Some learners showed some good conceptual and procedural understanding of algebraic expressions as early as in the pre-test while others struggled with simplifying algebraic expressions in both the pre and post-test in spite of all the intervention done. Some learners continued with their misconceptions from the pre-test, intervention up to the post-test. This was so because some errors were resistant in spite of how many, how much, how strong the intervention was (Nesher, 1987). These learners completely failed to assimilate and accommodate algebraic concepts.

One reason why learners failed to assimilate and accommodate algebraic concepts in schemas was due to the learners failing to feel a state of cognitive conflict. Makonye (2013) states that engineering a cognitive conflict is the best way to overcome a misconception Learners with wrong answers were supposed to accept that their answers were wrong after being corrected by the teacher or textbook during intervention for them to clear misconception. The problem was that some learners were so attached to their errors and have a belief in them because to them the answers were sensible even though mathematical they were wrong. This was supported by Nesher (1987) states that errors are difficult to eradicate from learners because these errors are valuable and meaningful to them since it is their own creation. That's why learners kept repeating the same errors irrespective of how many times the errors are rectified. These learners continued with conjoining of unlike terms, application, and subtraction errors in the post-test even though the errors were discouraged during intervention.

Vygotsky's (1986), social learning theory helped in the study through teacher-learner and learner-learner interactions during the intervention. The research employed this theoretical framework after being motivated by Vygotsky (1978) who states that children will always learn when motivated by an adult who is always there to assist the children. Through this theory, the researcher was able to identify learners' misconceptions which resulted in learners' errors (conceptual, procedural) in solving algebraic task. In minimising the errors a discovery learning strategy was employed where learners were given an opportunity to construct their knowledge through exploring of concepts by themselves. The researcher managed to help the learners after realising that they were in a zone of proximal development where learners could not solve problems alone but through adult guidance (Vygotsky, 1978). The importance of helping learners in the zone of proximal development is to help the learner to have a good feel of what they are required to solve for them to improve in their performance. This was supported by Osei (2000) who states that the higher level of development which results from assistance given to the child enables the child to independently solve problems at a new level. This improvement of learners after guided discovery was indicated by learners' great improvement in simplifying algebraic expressions in the post-test as compared to their performance in the pre-test. The development of conceptual thinking does not come naturally through experience but is dependent on specific types of social interactions (Vygotsky, 1994). Learners were able to improve in their conceptual and procedural understanding through the use of the class interactions during intervention. Learners were given an opportunity to share algebraic knowledge with minimal supervision from the researcher.

Vygotsky (1962) argues that language is a vital tool for the development of thought and helps learners to understand concepts cognitively. Learners who are proficient in algebraic language have a good mastery of symbols, variables, coefficients, constants, and algebraic terms and have strong conceptual and procedural knowledge and can easily simplify algebraic expressions. One reason why learners have errors when simplifying algebraic expressions is due to learners failing to understand the language of algebra, learners were not able to follow the prescribed instructions algebraically. Here the learners did not understand the language of algebra used. The first learner incorrectly multiplied everything by 3. The reason of doing that was because the brackets indicated multiplication. The second learner only multiplied 5 by p^2 and added the 5 to 2p and -3 to get $5p^2+7p+2$, a very big misconception. These learners failed to understand the algebraic language involved in this regard. Learners failed to simplify the expression 3-5(2x-1) because of misinterpretation of knowledge or learners misunderstood the language of algebra used; thereby order of operation was not adhered to. The learners subtracted the algebraic terms before expanding of brackets.

The concept image and concept definition by Tall and Vinner (1981) was used in the study for the researcher to understand learners' errors and misconceptions in algebra. Some learners' errors in the study were due to learners' concept images not correlating with learners concept definitions. For instance in item 1.4 learners were supposed to simplify the expression $5(p^2+2p-3)$ to $5p^2+10p-15$ but learner 1 of item 1.4 of the pre-test and others simplified the expression to $15p^2-15$. Learners understood that in algebraic expressions like terms are terms with the same variable. This explanation created a picture in the learners' mind that if they see terms with same variables then they know the terms are like. That's why some learners further simplified the above expression to $15p^2-15$ because they thought that $5p^2$ and $10p$ are like terms because they have the same variable p. Another example of errors due to concept images not correlating with concept definitions was illustrated in item 1.7 of the pre-test where learners were supposed to simplify the expression $3(x^3+3x-2)-(-2-9x+3x^3)$ to $3x^3+9x-6+2+9x-3x^3=18x-4$ but simplified the expressions to $3x^2+9x-6+6+18x+9x^3=9x^3+3x^2+27x$ which is an error.

However, some learners understood from teachers' definitions or explanations that when distributing terms in algebraic expressions the multiplier outside the bracket always multiplies everything inside the bracket. So these learners had a picture in their minds that in expansion of brackets, the multiplier must multiply everything inside the bracket. That's why learner 5 of item 1.7 of the pre-test and other learners simplified the expression $3(x^3+3x-2)-(-2-9x+3x^3)$ to $3x^3+9x-6+6+27x-9x^3$because they thought that the multiplier was multiplying everything in those two algebraic expressions. These learners' errors in this situation were as a result of naïve concept images which did not measure up to the concept definition (Tall & Vinner, 1981). These misconceptions might have resulted from teacher's definition of algebraic terms or from the learners misinterpreting definitions. Teachers need to find ways to explain the meanings of some mathematical concepts through the use of some representations or pictures because giving some learners some definitions can be misleading.

Variation theory was used in the study to assist the researcher with rectifying learners' errors and misconceptions through the use of contrasting correct answers with their errors. Leung (2010) states that teachers must provide learners with the means to experience variation through strategically designed activities. The researcher exposed the learners to correct solutions and errors of the algebraic expressions for learners to understand why their solutions were considered to be right or wrong. Akerlind (2008) states that critical features of any mathematical problem can be identified if the phenomenon with a contrast. This concept was emphasised in all question items of the pre-test. Some errors were noticed in item 1.4 of the pre-test when learners simplified the expression x(x-1) -5x to 2x-x-5x=-3x. The researcher rectify the common error among learners that x(x) =2 instead of x^2. This misconception was common to most learners in the pre-test.

After contrasting correct answers with their errors, learners were encouraged to identify some critical features in each question item from item 1.1 to item 1.7. The identification of critical features on simplifying algebraic expressions enabled the learners to improve in their performance in the post-test. For instance in item 1.1 of the pre and post-test there was need for the learners to group like terms together as their first step of the solution .So the critical feature was to group like terms together, where like refer to terms with the same variable or the same exponent. Most of the learners remembered that and successfully simplified the expression11x+5+3x+3 to 14x+8 in the post-test. In item 1.2 of the pre and post-test the critical features was ordering and of grouping like terms.

The learners put their critical features down successfully. In additional to the use of the concepts of contrast and critically features the learners were introducing to the concept of fusion another concept of variation. Learners cannot solve problems if they just identify critical features but fail to understand the relationship between each of them. This concept was well illustrated in items 1.6 and 1.7 of the post-test. For instance in item 1.7 learners were supposed to identify the critical features of the two algebraic expression when simplify the expression $3(x^3+3x-2)-(-2-9x+3x^3)$. Here the learner were supposed to expand and collect like terms in the first expression and likewise in the second expression.

On top of that the learners were supposed to realise that the two expressions were related through a subtraction operation so the terms of the second expression were supposed to be subtract from the terms of the first expression. Discovery learning together with variation theory were used during intervention where learners were requested to discover concepts by themselves before assistance was given. This learning model is useful in the teaching and learning of mathematics since it gives learners some opportunities to solve problems by exploration. Learners were supposed to discover what was wrong or what right in their simplifications of algebraic expressions by themselves with minimum assistance. Bruner (1961) believes that practice in discovering for oneself teaches one to acquire information more readily viable in problem solving. In this study, constructivism, social cultural learning together with the other bridging theories like concept image and concept definitions and variation theory helped to understand how learners conceive mathematical concepts together with misconceptions. This assisted the researcher to understand how learners' misconceptions by using the theories together with discovery learning to help learners simplify algebraic expressions.

4.15 Conclusion

The improved learners' performance in the post-test is an indication that learners' errors and misconceptions were minimised after a resourcing of the pre-test errors was done. The researcher have discovered that to be able to assist learners to improve in their performance teachers need know to the learners misconceptions, find ways of rectifying them, and remediate the learners. There is a need to implement some varying teaching methods as these can assist learners to reduce their errors as indicated in this study.

CHAPTER 5: FINDINGS, RECOMMENDATIONS AND CONCLUSIONS

5.1 Introduction

In this chapter, errors identified in chapter 4 through the analysis of learners' written scripts, interview responses and the outcome of the intervention process will be discussed. The purpose of the study was to identify grade 8 learners' errors in simplifying algebraic expressions and find the possible reasons for the errors. The overall aim of the study was to investigate if a learning opportunity was created after resourcing of errors to help learners reduce them on simplifying algebraic expressions. Thereafter some implications of the findings and recommendations arising from the study will be presented. All this was done in order to answer the following research questions;

1. *What are the types of errors and misconceptions grade 8 learners display in simplifying algebraic expressions.*
2. *What are the possible reasons for these errors?*
3. *To what extent do resourcing learners' errors and misconceptions through the discovery method in teaching help learners to diminish them?*

In order to answer the research question the researcher collected data from a sample of thirty grade 8 learners at a secondary school in Johannesburg. The data collection methods included a pre and post-test and interview. A one hour pre-test was administered to the learners through strict supervision since individual input from each learner was required. The purpose of the pre-test was to identify learners' errors on simplifying algebraic expressions. After the errors were identified a semi-structured focus group interview was conducted with six learners. The selection of the learners depended on the type and frequency of errors the learners displayed in their pre-test scripts. After the analysis of the learners' pre-test scripts and the learners' interview responses, the researcher employed guided discovery learning in the intervention. After the intervention the learners wrote a post-test. The purpose of the post-test was to check if a learning opportunity was created by resourcing identified errors

A mixed method research approach was used in the study where both the pre and post-tests provided quantitative and qualitative data showing different types of errors, their frequency and prominence in the learners' scripts. The interview provided qualitative data mainly consisting of the individual learners' opinions about their difficulties in simplifying algebraic expressions and possible reasons for their errors. A mixed method research was important in the study since it offered a large variety of important data, quantitative and qualitative methods complement each other resulting in a very strong research. The study was informed by cognitive constructivism and social constructivism as the major theories of the study together with variation theory and the concept image and concept definitions as bridging theories to support the major theories. The theories directed the study and to helped the research in the interpretation of research data on learners' errors and misconceptions.

5.2 Research findings

In this section the findings of the study are presented through the use of the analysed data from the learners' pre and post-test and learners' interview responses. The research data was collected in three stages. This was done as follows; the learners' errors where identified through a pre-test to answer the research question "*What are the types of errors and misconceptions grade 8 learners display in simplifying algebraic expressions*" then the possible reason for the errors were investigated through a focus group interview to answer research question "*What are the possible reasons for these errors?*" and then after an intervention, the effects of the intervention were investigated through a post-test to answer research question "*To what extent do resourcing learners' errors and misconceptions through the discovery method in teaching help learners to diminish them?*"

5.3 What are the types of errors and misconceptions grade 8 learners display in simplifying algebraic expressions?

The analysis of learners' errors showed that a lot of misconceptions exits in learners' minds and this were shown in the learners' pre and post-test scripts. In this research study, errors identified were classified using the four types of errors by Hodes and Nolting (1998). The four types of errors are conceptual, procedural, application and careless. The learners showed a lot of different types of errors which were classified into the four types of errors stated due to lack of conceptual or procedural understanding. Some errors were due to carelessness and misapplication/ misinterpretation of concepts. The errors which were identified in the pre-test were the same errors identified in the post-test but they reduced in quantity. For more information about the error refer to the scanned vignettes and table 5 on page 52.

5.3.1 Incorrect addition and subtraction of algebraic terms

In the study learners failed to add and subtract terms. For instance in item1.1 and item 1.2 of the pre-test learners simplified the expression 4a+2a+5=5a+5 or10a and 5x+2x-3y-4y-3a to 6x-7y-3a or 7x-y-3a or 7x+7y-3a.

5.3.2 Conjoining of algebraic terms

The learners failed to simplify some algebraic terms because of conjoining of unlike terms to a single term or more than one term. For example in item 1.1, 1.2 and 1.4 of the pre-test learners simplified the expression 4a+2a+5 to 11a, simplified the expression 5x+2x-3y-4y-3a to 6xy and simplified the expression $5(p^2+2p-3)$ to $5p^2+10p-15=15p^2-15$.

5.3.3 Removing of variables from algebraic expressions

Learners failed to simplify some algebraic expressions because they removed variable/variables of algebraic expressions to remain with numerals only. For instance in item 1.1 and 1.4 learners simplified the expression 4a+2a+5 to 11 and simplified the expression 3-5(2x-1)=3-10-5=-12.

5.3.4 Incomplete/incorrect distribution of terms

In the pre-test learners failed to to expand the brackets due to incomplete or incorrect distribution of terms. For instance in items 1.3, 1.4, 1.5 and 1.6 learners simplified the expression 3-5(2x-1) to 3-10x and simplified $5(p^2+2p-3)$ to $5p^2+2p$ and simplified x(x-1)-5x to 2x-1-5x = -3x-1

5.3.5 Changing of operational signs

Learners failed to simplify algebraic expressions because they reversed the signs of terms. For instance in items 1.1 and 1.2 learners simplified the expression 4a+5+2a to 6a-5 and simplified the expression 5x+2x-3y-4y-3a to 7x+7y+3a.

5.3.6 Changing algebraic expression to equations

Learners failed to simplify algebraic expressions because they changed algebraic expressions into equations. For instance in item 1.2 a learner simplified the expression 5x+2x-3y-4y-3a to 7x-7x = 3A.

5.3.7 Omission of algebraic terms

Learners failed to simplify algebraic expressions due to omission of algebraic terms. For instance in items 1.1, 1.2, 1.4 and 1.5 of the pre-test learners simplified the expression 4a+5+2a to 4a+5 and simplified the expression 5x+2x-3y-4y-3a to 7x-7y-3 and simplified the expression $5(p^2+2p-3)$ to $5p^2+10p-3$ (-3 not multiplied by 5) or $5p^2+10-15$ (the variable p was omitted) and simplified the expression x(x-1)-5x to $x^2-x-5x=x^2-5x$.

5.3.8 Creation of learners' own rules

Some learners encountered some difficulties in simplifying algebraic expressions instead they tried to clear their misconceptions by creating their own rules. For instance in items 1.3, 1.4

and 1.6 learners simplified the expression 3-5(2x-1) to 3(-10x+5)=-30x+8 and simplified the expression 3-5(2x-1) to 15-(6x-3) and simplified the expression $5(p^2+2p-3) = 5p^2+2p-3$ and simplified the expression $a^2-5a+6-(a^2+4a+4)$ to $a^2-5a+6-(6a^2+24a+24)$.

5.3.9 Inappropriate application of rules

Some learners failed to simplify algebraic expressions by applying valid concepts taught to invalid situations. A leaner inappropriately applied exponential rules to addition of algebraic terms. For instance in item 1.1 a learner simplified the expression $4a+2a+5=6a^2+5$.

5.3.10 Slips

A learner managed to produce a correct answer, the error made was a slip, and the error was easily corrected the error in last step of the solution. In item 1.3 a learner simplified the expression 5(2x-1) to 3-10x-5=8-10x.

5.4 What are the possible reasons for these errors?

The purpose of the interview was to investigate from the learners' themselves the possible reasons for their errors. The researcher referred to the learners' interview responses to find out why learners displayed errors on their pre-test scripts. The possible reasons for the errors given by learners were:

5.4.1 Separation of coefficients from variables

On simplifying algebraic expressions learners separated coefficients from variables so that they stand on their own. They took numerals and add them and took the variables and add them. Cancelled out the variables for them to add the numbers but forgot to put the variables back.

5.4.2 Lack of concentration/ forgetfulness

Learners stated that they made some errors on simplifying algebraic expression because of lack of concentration or they were confused. Learners confirmed that they made errors in the pre-test because they forgot how algebraic expressions were simplified.

5.4.3 Production of a single answer

Learners agreed that the conjoined terms because they wanted their answers to be expressed in simplified forms or single terms. Some learners conjoined terms like $5p^2+2p$ because they thought that the terms were like terms.

5.4.4 Answers to be whole numbers

Some learners said they cancelled out variables because they wanted to stay with whole numbers only.

5.4.5 Misconceptions from previous learnt concepts

Some learners confirmed that the reversed the operational signs of algebraic terms because that's how they were taught. This is an indication of misinterpretation of concepts. The reversing of signs usually occurs on solving of equations.

5.4.6 Miscalculation

Learners agreed that they made errors when simplifying algebraic expressions due to miscalculation.

5.4.7 Manipulation of first operational sign

Some learners argued that the subtracted 3 from 5 in item 1.3 because subtraction comes first before multiplication.

5.4.8 Partially remembering of concepts

Learners revealed that they created their own rules because they were confused. Some leaners said they created their own rules because they could not remember some algebraic rules.

5.4.9 Incomplete multiplication of terms

Learners did not distribute terms fully because they thought that the multiplier outside the brackets was supposed to multiply only the first algebraic expression.

5.4.10 Avoiding repetition of terms

Some learners simplified the expression x(x-1)-5x to x-1-5x left out the x outside the bracket because it's the same with the x inside they left it out to avoid repetition of terms.

5.4.11 Integers difficult to work with

The learners confirmed that they made some errors when simplifying algebraic expressions because integers are confusing.

5.5 To what extent do resourcing learners' errors and misconceptions through the discovery method in teaching help learners to diminish them?

Data analysis has revealed that learners' performance in the post-test improved as compared to the learner's performance in the pre-test. The number of errors displayed on the learners' post scripts decreased as compared to the number of errors on the pre-test. There were 19 application, 14 careless, 62 conceptual and 45 procedural errors in total from item 1.1 to item 1.2 on pre-test scripts which decreased to 13 application, 10 careless, 67 procedural (increase) and 12 procedural errors which gives some percentage decrease except conceptual of 32%, 29%, 8% and 73% respectively. The only errors which did not reduce but increase were conceptual errors. This was so because some errors resist instructions no matter how strong the intervention might be. Beside the reduction of errors learners had improved after the intervention. The post average was 54% which was higher than the pre-test average of 38% (16% improvement). The percentage pass rate of the post test was 70% and was higher than the percentage pass rate of the pre-test which was 37%. Overall there was an improvement in learners' performance of 33%. These results clearly showed that a learning opportunity was created when errors were resourced to help learners reduce them.

5.6 Limitations of the study

The researcher collected a lot of data from both the pre and post-test for instance the scanned vignettes from the learner' scripts. The analysis was so hectic and very time consuming. However the researcher managed to do a thorough analysis of each and every learners' scripts in spite of the process being vigorous and strenuous. The researcher spent some sleepless night for the set goals to be accomplished. The researcher was supposed to have used 33 learners in the sample but 30 learners participated, the three did not participate because of fear of failing the tests even though the researcher explained to them that the tests were not for assessment. However those 30 participant learners were the best in terms of their contributions, participation, cooperation, willingness and patience despite their busy schedules at school. The researcher managed to come up with some findings which can help teachers, policy makers and all other educational professionals in helping South African learners to improve in mathematics through resourcing of errors to help learners reduce them.

5.7 Recommendations

5.7.1 Practice

Teachers need to accept that learners are unique individuals in the classroom who grasp concepts differently where some are fast and some are slow. So teachers need to help each and every learner accordingly. The learners who are not performing need more attention in each and every topic. These learners need to be attended when necessary to help them minimise their errors. Extra lessons must be put in place not to cover the syllabus but to help slow learners to improve in their problem solving skills. For teachers to effectively help the learners, they need to understand learners' thinking which contribute into errors. After knowing learners' misconceptions and errors teachers must accept the errors as they are an indication of learners' misunderstandings or incomplete knowledge and use them as a resource to help learners simplify algebraic expressions. Teachers need to do some regular checks to see if learners have some misunderstandings on some particular topics. This can be done through assessing learners using class activities, tests, assignments and controlled group works. Also recapping of previous learnt concepts is important for learners to clear up some misconceptions if they exist. The learners' performance on these tasks will enable teachers to gain an insight about learners' misconceptions in order to help them. If misconceptions and errors are identified, teachers need to rectify them as early as possible for the learners not to pass them on in the oncoming topics. Teachers need accept learners' errors as they show learners misunderstandings so they can be used as correctional devices. Teachers must allow learners to independently construct their knowledge. This gives learners some opportunities to explore concepts by themselves and learn from the experiences since people learn more by doing than observing.

5.7.2 Policy

The policy makers must encourage the use of learners' errors in teaching and learning of mathematics. On top of providing resources to schools, the policy makers need to provide schools with some activities which involve the use of errors where learners are able to identify some errors in some given solutions. Teachers must be given some workshops on accepting learners' errors and using them as a resource to help learners to reduce them. In teaching and learning of mathematics most teachers emphasise on getting correct answers and discourages learners who obtain wrong answers to continue with the wrong workings. Most of the times these learners are not talked about but the policy makers must start to enlighten the teachers about the importance of learners' errors where they can be used as a resource to help learners reduce them. In each and every national examinations like NSC, the policy makers must compile all learners' errors in each topic and every subject and send them to school for teachers to use them in helping learners to solve problems in different subjects.

5.7.3 Theory

Further research is needed to help learners accomplish the goal of defeating the resistant errors in algebraic expression which are so persistent and difficult to eradicate in algebra. Some more other research need to be done to come up with ways to help learners minimise or avoid errors in problem solving situations in mathematics and other subjects.

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Appendix 1:Pre-test

SUBJECT		**MATHEMATICS**
GRADE		**8 PRE-TEST**
TOPIC		**ALGEBRAIC EXPRESSIONS**
TIME		**1HOUR**
MARKS		**21**

INSTRUCTIONS

1. Answer all questions Write neatly and eligibly
2. Show all your workings

QUESTION 1

Simplify the following expressions

1.1	$4a + 5 + 2a$	(2)
1.2	$5x – 3y - 3a – 4y + 2x$	(3)
1.3	$3 - 5(2x – 1)$	(3)
1.4	$5(p^2 + 2p – 3)$	(3)
1.5	$x(x - 1) - 5x$	(2)
1.6	$a^2 – 5a + 6 – (a^2 + 4a +4)$	(3)
1.7	$3(x^3 + 3x -2) - (- 2 -9x + 3x^3)$	(4)
		Total [21]

Appendix 2: Memorandum for the pre-test

1.1 4a+5+2a

= 4a+2a+5

= 6a+5 (3)

1.2 5x+-3y-3a-4y+2x

= 5x+2x-3y-4y-3a

= 7x-7y-3a (3)

1.3 3-5(2x-1) (3)

= 3-10x+5

= 3+5-10x

= 8=10x

1.4 $5(p^2+2p-3)$ (3)

$= 5p^2+10p-15$

1.5 x(x-1)-5x

$= x^2-x-5x$

$= x^2-6x$ (3)

1.6 $a^2-5a+6-(a^2+4a+4)$

$= a^2-5a+6-a^2-4a-4$

$= a^2-a^2-5a-4a+6-4$

= -9a+2 (3)

1.7 $3(x^3+3x-2)-(-2-9x+3x^3)$

$= 3x^3+9x-6+2+9x-3x^3$

$= 3x^3-3x^3+9x+9x-6+2$

$= 18x-4$ (3)

Appendix 3: Interview guide

After marking the learners' scripts, the teacher is going to select six learners for an interview in review of errors displayed on the scripts. The learners will explain their errors and try to give possible reasons for the errors. The teacher is going to audio-tape the learners with the learners and their parents' permission. The interview is going to be semi structured. This interview is going to be a focus group interview where the research will interview the 6 learners in group but each individual learner is going to be given his/her time to speak. The questions are not fixed and specific but the researcher can just ask any questions to any learner depending on the learners' responses.

Questions like these will be asked…

Teacher: I need your help to find out more about your challenges when simplifying algebraic expression. The information I will get from you will help me understand your thinking which results in errors.

Learners… yes maam we will help you.

Teacher: In question 1.1, why did you write that 4a + 5 + 2a = 11a

Learner…We just say 4 + 5 + 2 = 11 but the answer is 11a.

Teacher: That's interesting, tell me why at first you work with numbers then you finally put a.

Learner: …That's how we do the calculations numbers first then variables will be added.

Teacher: Now I can see in your calculations for question 1.2 in step 2 you wrote 7x -3ay-4y what was the reason?

Learner: I added 5x to 2x to get 7x then I combined -3y and -3a to make -3ay because -3 is a common factor.

Teacher: I need to know more about what you are saying. So why didn't you also combine -3ay with -4y since y is common?

Learner: -3 and -4 do not have a common factor.

Teacher: In question 1.3 I can see that you first simplify 3 – 5(2x – 1) to -2(2x – 1) why?

Learner: ….Maam can't you see that 3 – 5 = -2, I will then multiply this -2 with 2x-1

Teacher: In your second step you wrote -2(2x – 1) = -4x -2 can clarify where you get -2 from?

Learner: I know maam that a negative x a negative = negative

Teacher: Go on, I'm listening

Appendix 4: Observation guide

I am going to be a participant observer as I wish to observe while I work with my group of participants. I will refer to myself as the teacher and to my participants as the learners.

- Learners will work individually to answer all algebraic questions in the pre-test and post-test to be given by the researcher.
- All the 30 grade 8 learners will write the two algebraic tests since the researcher has already covered the topic in term 2.
- In both tasks the learners will all simplify algebraic expressions.
- Teacher will invigilate the learners so that an individual effort is noted since this will be useful in data analysis.

The researcher will observe the learners in terms of how they handle the task and the interview. There is a need to check if learners are finding the task easy or challenging through their reactions when writing the test. No any learner will be assisted during the tests.

- The researcher need to encourage learners to follow instructions. Learners are not to share ideas or answers the task is strictly to be done individual.
- Slow learners can be given some additional minutes to finish their tasks.
- At the end of the session, the teacher will collect all the scripts from all the 33 learners.

Appendix 5: Post-test

SUBJECT		**MATHEMATICS**
GRADE		**8 POST-TEST**
TOPIC		**ALGEBRAIC EXPRESSIONS**
TIME		**1HOUR**
MARKS		**21**

Instructions

1. Answer all questions
2. Write neatly and eligibly
3. Show all your workings

Question 1

Simplify the following algebraic expressions

1.1 $11x + 5 + 3x + 3$ (3)

1.2 $5x - 6y+3x-2y+10p$ (3)

1.3 $8-2(3x-4)$ (3)

1.4 $-2(16p^2 + 24p - 9)$ (3)

1.5 $y(y-2)-3y$ (3)

1.6 $x^2+7x+5-(x^2+4x+4)$. (3)

1.7 $5(a^2+3a-4)-2(-7-2a+a^2)$ (3)

Total [21]

Appendix 6: Memorandum for the pre-test

1.1 $11x+5+3x+3$

$= 11x+3x+5+3$

$= 14x+8$ (3)

1.2 $5x-6y+3x-2y+10p$

$= 5x+3x-6y-2y+10p$

$= 8x-8y+10p$ (3)

1.3 $8-2(3x-4)$

$= 8-6x+8$ (3)

$= 8+8-6x$

$= 16-6x$ (3)

1.4 $-2(16p^2+24p-9)$

$= -32x-48p+18$ (3)

1.5 $y(y-2)-3y$

$= y^2-2y-3y$ (3)

$= y^2-5y$

1.6 $x^2+7x+5-(x^2+3x+4)$

$= x^2+7x+5-x^2-3x-4$

$= x^2-x^2+7x-3x+5-4$

$= 4x+1$ (3)

1.7 $5(a^2+3a-4)-2(-7-2a+a^2)$

$= 5a^2+15a-20+14+4a-2a^2$

$= 5a^2-2a^2+15a+4a-20+14$

$$= 3a^2+19a-6 \tag{3}$$

Appendix 7: GDE Approval letter

For administrative use only:
Reference no: D2017 / 318
enquiries: 011 843 6503

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

UNITY IN DIVERSITY

GDE RESEARCH APPROVAL LETTER

Date:	4 November 2016
Validity of Research Approval:	6 February 2017 to 29 September 2017
Name of Researcher:	Matuku O.
Address of Researcher:	295 Sparrowgate, 1 Lark Street; Meredale; 2091
Telephone / Fax Number/s:	011 837 8577; 073 001 2233
Email address:	olinahmatuku@yahoo.com
Research Topic:	Creating opportunities to learn thorough resourcing learners' errors and misconceptions on simplifying algebraic expressions in Grade 8.
Number and type of schools:	ONE Secondary School
District/s/HO	Johannesburg North

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved. A separate copy of this letter must be presented to the Principal, SGB and the relevant District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted. However participation is VOLUNTARY.

The following conditions apply to GDE research. The researcher has agreed to and may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

CONDITIONS FOR CONDUCTING RESEARCH IN GDE

1. *The District/Head Office Senior Manager/s concerned, the Principal/s and the chairperson/s of the School Governing Body (SGB.) must be presented with a copy of this letter.*
2. *The Researcher will make every effort to obtain the goodwill and co-operation of the GDE District officials, principals, SGBs, teachers, parents and learners involved. Participation is voluntary and additional remuneration will not be paid;*

2016/11/07

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management ER&KM)
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix 8: Human Research Ethics Council clearance letter

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

27 January 2017

Student Number: Dr Judah Makonye

Protocol Number: 2016ECE061M

Dear Olinah Matuku

Application for ethics clearance: Master of Science

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate, has considered your application for ethics clearance for your proposal entitled:

Title of Research: Creating opportunities to learn through resourcing learners' errors and misconceptions on simplifying algebraic expressions in grade 8.

The committee recently met and I am pleased to inform you that **clearance was granted**.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

MMabeta

Wits School of Education

011 717-3416

cc Supervisor - Dr Judah Makonye

Appendix 9: Letter for the Principal

THS Langlaagte

Corner Dutoit & Kruger Street

Private Bag x1

Langlaagte 2012

30 July 2016

Dear Mr T.B Molefe

My name is Olinah Matuku. I am a teacher at your school and a second year MSc mathematics Education student in the School of Education at the University of the Witwatersrand who is conducting a research on creating opportunities to learn through resourcing learners' errors and misconceptions on simplifying algebraic expressions in grade 8**.**

I am going to work with a grade 8 mathematics class at this school and the reason I choose this school is for convenience and accessibility since I teach this class. The purpose of the study is to identify the nature of errors and misconceptions grade 8 learners have in simplifying algebraic expressions and to get a deep understanding of the possible reasons for the errors and to investigate if resourcing of these errors can help learners reduce them. The research questions of the study are as follows;

- What are the types of errors and misconceptions grade 8 learners display in simplifying algebraic expressions.
- To what extent do resourcing learners' errors and misconceptions in teaching helps learners to diminish them?

My research involves a mixed method research where both qualitative and quantitative research methods are employed. The data collection methods include a pre and post-test, observation and interview which will be administered for 1 hour for three different days. The research is to be done after school hours in order not to disturb the proposed daily schedules. Letters informing the parents seeking their permission for their child to participate in this research are to be sent.

I am asking for permission to do this research at this school. The research results are not for marks and will not be used for class assessment in any way. The research participants will not be advantaged or disadvantaged in any way and will be informed that they are free to withdraw from participating from the research at any time during this project without any penalty. There are no foreseeable risks in participating in this study; participation is voluntary as no one is forced to participate. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. If quotes are to be used in the final research report, pseudo-names will be used to ensure confidentiality and anonymity. Individual privacy will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Find below is the information of my supervisor.

NAME OF SUPERVISOR: Judah Makonye

TELEPHONE/CELL NUMBER: 0117173206 / 0786894572

EMAIL ADDRESS: **Judah.Makonye@wits.ac.za**

Yours Sincerely

Olinah Matuku PROTOCOL NUMBER: 2016ECE061M

SIGNATURE:

EMAIL: olinahmatuku@yahoo.com

TELEPHONE/ CELL NUMBERS: 0118378577 / 0848649216

Appendix 10: Letter to the learner

295 Sparrow gate

1 Lark Street Meredale

Johannesburg South Africa

The learner

THS Langlaagte

Corner Dutoit &Kruger Street

Private Bag x1

Langlaagte 2102

30 July 2016

Dear Learner…………………………………………..

My name is Mrs O. Matuku your mathematics teacher at the above mentioned School and a second year MSc mathematics Education student in the School of Education at the University of the Witwatersrand. I am doing research on creating opportunities to learn through using learners' errors and misconceptions as a resource in simplifying algebraic expressions in grade 8**.** The purpose of the study is to identify the nature of errors and misconceptions grade 8 learners have in simplifying algebraic expressions and to get a deep understanding of the possible reasons for the errors and to investigate if resourcing of these errors can help learners reduce them. The research questions of the study are as follows;

- What are the types of errors and misconceptions grade 8 learners display in simplifying algebraic expressions.
- To what extent do resourcing learners' errors and misconceptions in teaching helps learners to diminish them?

My investigation involves two algebraic expressions tasks which you are going to write her at school for 1 hour after school. After marking the tasks, 6 are going to be selected for an interview with me. The interview will involve very simple questions on errors which may have been identified. There is going to be an intervention using your errors to be found in task 1 and thereafter a second task is to be written after to check if your performance will improve after the intervention.

I kindly invite you to participate in this research. If you are going to be chosen for the interview you are going to be audiotaped for me to get accurate information from you for my data analysis.

Remember, this is not a test, the tasks are not for marks, and the results are not going to be used for assessment. You are not forced to participate and you are free to withdraw from the research at any time you feel like and you not going to be penalysed for that. I will use fake names for you in the research and no one will identify you. All information about you will be kept confidential in all my writing about the study. All collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join us in the study.

I look forward to working with you. Please feel free to contact me or my supervisor if you have any questions.

Thank you.

Find below is the information of my supervisor.

NAME OF SUPERVISOR: Judah Makonye

TELEPHONE/CELL NUMBER: 0117173206 / 0786894572

EMAIL ADDRESS: **Judah.Makonye@wits.ac.za**

Yours Sincerely

Olinah Matuku PROTOCOL NUMBER: 2016ECE061M

SIGNATURE:

EMAIL: olinahmatuku@yahoo.com

TELEPHONE/ CELL NUMBERS: 0118378577 / 0848649216

Learner Consent Form [**Topic: Creating opportunities to learn through resourcing learners' errors and misconceptions on simplifying algebraic expressions in grade 8**].

Please fill in the reply slip below if you agree to participate in my study called:

My name is: ________________________

Permission to review/collect documents/artifacts	**Circle one**
I agree that (Pre-test, post-test, interview) can be used for this study only.	YES/NO
Permission to observe you in class	
I agree to be observed in class.	YES/NO
Permission to be audiotaped	
I agree to be audiotaped during the interview or observation lesson	YES/NO
I know that the audiotapes will be used for this project only	YES/NO
Permission to be interviewed	
I would like to be interviewed for this study.	YES/NO
I know that I can stop the interview at any time and don’t have to answer all the questions asked.	YES/NO
Permission to be photographed	
I agree to be photographed during the study.	YES/NO
I know that I can stop this permission at any time.	YES/NO

I know that the photos will be used for this project only. YES/NO

Permission for questionnaire/test

I agree to fill in a question and answer sheet or write a test for this study. YES/NO

Permission to be videotaped

I agree to be videotaped in class. YES/NO

I know that the videotapes will be used for this project only. YES/NO

Informed Consent I understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time. · I can ask not to be audiotaped, photographed and/or videotape
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign______________________________ Date____________________________

Appendix 11: Letter to the parent/guardian

295 Sparrow gate

1 Lark Street Meredale

Johannesburg South Africa.

THS Langlaagte

Corner Dutoit &Kruger Street

Private Bag x1

Langlaagte 2102

30 July 2016

Dear Parent/Guardian of…………………………………

My name is Mrs O. Matuku your child's mathematics teacher at the above mentioned school and I am a second year MSc mathematics Education student in the School of Education at the University of the Witwatersrand I am doing a research on creating opportunities to learn through using learners' errors and misconceptions as a resource in simplifying algebraic expressions in grade

8**.**

I am conducting a mixed method research which involves working with a grade 8 class at the school. The purpose of the study is to identify the nature of errors and misconceptions grade 8 learners have in simplifying algebraic expressions and to get a deep understanding of the possible reasons for the errors and to investigate if resourcing of these errors can help learners reduce them. The research questions of the study are as follows;

- What are the types of errors and misconceptions grade 8 learners display in simplifying algebraic expressions.
- To what extent do resourcing learners' errors and misconceptions in teaching helps learners to diminish them?

The research is to be done after school when your learner will write a pre and post-test and attend an interview for 1 hour after school to avoid disturbing the stipulated daily programs. Letters informing the principal, The Gauteng Department of Education about this research are to be sent. The research results are not for marks and will not be used for class assessment in any way. There are no foreseeable risks in participating in this study; participation is voluntary as no one is forced to participate. The participants will not be paid for this study.

The reason why I have chosen your child's class is because I teach them and it is easy for me to give them the tasks. Your child will not be disadvantaged in any way and will be reassured that he/she can withdraw his/her permission at any time during this project without any penalty. Participation is voluntary. There are no foreseeable risks in participating in this research and your child will not be paid for this study hence, participation is

I am asking for permission to collect data for my research from your child. I kindly request that your child participates in the research through writing the tests and responding to an interview and audiotaped if he/she is chosen for the interview. The purpose of the interview is to identify reasons why learners make errors when simplifying algebraic expressions.

The results are to be kept private and confidential only to be used in the research and are not for assessment. All research data will be destroyed between 3-5 years after completion of the project.

Your child's name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

Please let me know if you require any further information. Thank you very much for your help.

Find below is the information of my supervisor.

NAME OF SUPERVISOR: Judah Makonye

TELEPHONE/CELL NUMBER: 0117173206 / 0786894572

EMAIL ADDRESS: **Judah.Makonye@wits.ac.za**

Yours Sincerely

Olinah Matuku PROTOCOL NUMBER: 2016ECE061M

SIGNATURE:

EMAIL: olinahmatuku@yahoo.com

TELEPHONE/ CELL NUMBERS: 0118378577 / 0848649216

Parent's Consent Form [**Topic: Creating opportunities to learn through resourcing learners' errors and misconceptions on simplifying algebraic expressions in grade 8**].

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called:

I, __________________________ the parent of ________________________

Permission to review/collect documents/artifacts **Circle one**

I agree that my child's (SPECIFY DOCUMENT) can be used for this study only. **YES**/NO

Permission to observe my child in class

I agree that my child may be observed in class. YES/NO

Permission to be audiotaped

I agree that my child may be audiotaped during interview or observations. YES/NO

I know that the audiotapes will be used for this project only YES/NO

Permission to be interviewed

I agree that my child may be interviewed for this study. YES/NO

I know that he/she can stop the interview at any time and doesn't have to answer all the questions asked. YES/NO

Permission to be photographed

I agree that my child may be photographed during the study. YES/NO

I know that I can stop this permission at any time. YES/NO

I know that the photos will be used for this project only. YES/NO

Permission for questionnaire/test

I agree that my child may be fill in a question and answer sheet or write a test for this study. YES/NO

Permission to be videotaped

I agree my child may be videotaped in class. YES/NO

I know that the videotapes will be used for this project only. YES/NO

Informed Consent I understand that:

- My child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- He/she does not have to answer every question and can withdraw from the study at any time. · he/she can ask not to be audiotaped, photographed and/or videotape
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign________________________________ Date______________________________