M Ed RESEARCH REPORT

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TOPIC:

Teachers' choice of examples and talk in mediating the chosen examples in Grade 8 multilingual classrooms.

NAME

Sehowa Mmatsela Kate

STUDENT NUMBER

940176

PROTOCOL NUMBER

2024ECE004M

SUPERVISOR

Prof. Anthony A. Essien

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Abstract

Examples that teachers of mathematics choose to relay the mathematical narrative to their learners plays a crucial role in the kind of mathematics that learners then have access to (or not). Furthermore, how the teachers bring the chosen examples to life through classroom talk, which aims to mediate the chosen examples is what determines what learners truly learn. Data in the study was collected through consecutive video recordings of lessons during which linear and exponential equations were covered from start to finish and post-lesson interviews with the two teachers. In this study, I have used Essien's (2021b) Dyadic framework to analyse the findings. One of the interesting findings in the study is that although the two teachers involved teach in the same school, they teach differently, for example, one teacher used talk to mediate the examples through code-switching (to Sesotho and IsiXhosa), while the other teacher strictly used the LoLT (English). Additionally, both teachers allowed their learners to write corrections on the board, which then allowed teachers to conceptualise their learners' lived object of learning (and whether or not it aligns with the intended object of learning). Through the work learners wrote on the board, teachers were then able to invite mathematical discussions (with the whole class). One other interesting finding is that the teachers dominantly accepted learners' choral responses which limited prospects for meaningful discussions, which led to missed opportunities for further learning.

Keywords: examples, example space, variation theory, choice of examples, meaning making, talk, mediate, multilingual, algebra, equations, linear and exponential equations, code-switching, resources

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Declaration

I, Sehowa Mmatsela Kate, declare that this research report is my own work with appropriate APA referencing style. This work is submitted for the Master of Education degree, at the University of the Witwatersrand. I have not submitted this work before, for any other degree.

Sehowa MK

Signature

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Acronyms

ATP	Annual Teaching Plan
CAPS	Curriculum and Assessment Policy Statements
LoLT	Language of Learning and Teaching
NSC	National Senior Certificate
PST	Pre-Service Teacher

Chapter 1: Introduction to the study

1.1. Background of the study

This research is based on investigating the teachers' choice of examples and the talk teachers use to mediate the examples they choose in Grade 8 multilingual mathematics classrooms. The teachers' choice of examples and teacher talk to mediate examples are analysed through the use of the dyadic framework by Essien (2021a) which merges two theories in mathematics. These theories are the variation theory by Marton and associates, as well as Scott and Mortimer's meaning making as a dialogic process. The data for this research project is collected in one of the public schools in South Africa, with the focus on algebra, specifically on linear and exponential equations.

The teachers' whose lessons were observed are central to this research project as the findings of this project are based on the observations of these teachers' choice of examples and the teachers' talk to mediate the examples in teaching the concept of linear equations and exponential equations. In this section, I aim to provide a rationale and significance of the study, provide a detailed problem statement and outline the research questions that inform the study.

Research has shown that the choice of examples and way teachers use classroom talk to mediate the examples they choose in mathematics classrooms is an important process which aids the kind of mathematics learners get exposed to and learn (Essien, 2021a; Kullberg, Runesson & Marton, 2017). In addition, examples that are chosen and used during instruction communicate meaning in terms of the mathematics the learners then have access and/or are limited to. The examples that learners are exposed to by the teacher, in or outside the class, provide learning opportunities for learners. It is through the use of examples that mathematical meaning can be drawn (Rowland, Thwaites & Huckstep, 2008). Thus, the choice of examples, and how these examples are then used, remain central to the meaning making process, and make clear what is made possible to learn in each mathematics lesson (Kullberg et al., 2017; Rowland et al., 2008).

Anecdotal evidence on the ground shows that teachers do not necessarily choose examples based on or by considering the context in which they teach. Additionally, the talk that the teachers use to mediate the chosen examples does not necessarily maximise all the learning resources (such as the different languages present) in order to enable meaning making.

In order for teachers to choose and use meaningful examples and communicate these effectively through classroom talk, teachers must do this in light of the context in which they teach. In South Africa, for example, most classes are multilingual, and Setati and Adler (2000) contend that many more classes in South Africa are more linguistically diverse. As Essien (2021a) contends, the choice of examples and how they are used in a multilingual classroom should be a carefully made decision which considers which language is used and how such language is used to accommodate the dynamics of the multilingual classroom in enhancing mathematical understanding.

From my experience as an educator, in teaching in multilingual classrooms, I have realised that the examples that I choose and enact, during instruction is imperative to the mathematics that the learners are exposed to. The type of examples I choose to use is not usually based on the fact that I teach in multilingual classrooms. The use of learners' home languages (HLs) is what I use to get my learners interested in the lesson and in most instances, learners start engaging more. But this does not attend to what makes for a good choice and a good use of examples in a multilingual context such as the one I teach in. What then does it take to choose and use examples in the context of multiple languages to enable mathematical sense making? For the purpose of investigating this question, I have chosen to focus on the topic of algebraic equations (specifically, linear and exponential equations). Katz (2007) argues that algebra is one of the most important topics to learn in mathematics as it encompasses many of the sub-topics in mathematics. Such sub-topics include plotting graphs of functions, calculus, trigonometric equations, linear programming etc. (Tsamir & Almog, 2001).

My motivation to embark on the journey of investigating linear and exponential equations is guided for one, by the concerns emanating from the diagnostic analysis for the attainment of the National Senior Certificate (NSC), in South Africa. In the past recent years (2019-2023), they report that majority of learners generally find it hard to solve algebraic equations. Furthermore, and most importantly, the focus on algebraic equations is influenced by research findings (Capraro & Joffrion, 2006; Katz, 2007), some of which include that learners find it more challenging to solve linear equations, which then poses a lot more issues for learners if they have to deal with higher order equations such as exponential equations. In other instances, it has been further established that most learners are unable to relate the solutions to exponential equations to any other concepts learned in mathematics (Kieran, 2007).

Furthermore, research indicates that learners of mathematics in multilingual classrooms perform poorly and are not motivated to learn mathematics thus, leading to many of such learners not pursuing mathematics (Setati-Phakeng, 2016). Additionally, by resolving linear and exponential equations, with adequate conceptual and procedural understanding, Hewson

(2007) suggests that this is considered an important feature to enabling the ability to plot graphs of functions, for example.

Due to these concerns and challenges, it is then that I decided to carry out this research project so as to ascertain the importance of the teachers' choice of examples and the teacher talk used to mediate the examples in mathematics classrooms, especially in multilingual contexts. Although there is a plethora of examples to choose from learning materials such as textbooks, online resources and those that can be instantly made-up during instruction, it is my argument that not all the examples available for use, that are chosen and used in each mathematics classrooms are either considerate of the language dynamics (multilingual) and how these examples are used is affected by such dynamics.

1.2. Research questions

The purpose of this study is, therefore, to investigate teachers' choice of examples and the teachers' talk used to mediate the examples in Grade 8 multilingual classrooms in the topics of linear and exponential equations. Thus, the following research questions inform the study.

1. What examples do teachers in Grade 8 multilingual classrooms choose in the topics of linear and exponential equations? Where do these examples come from?
2. How do teachers use talk to mediate these examples in their multilingual classrooms?
3. What is made possible to learn through the chosen examples? How does the teachers' talk, in mediating the examples enable meaning making in multilingual classrooms?

With the above research questions, I hope to advance my understanding about the type of examples that are appropriate for multilingual classrooms and how it is that these examples enhance meaning making in learning mathematics (in the case of this study, linear and exponential equations). Additionally, by gaining insight, I should also be able to use the findings from the study to improve my own teaching.

1.3. Significance of study

I consider the study to be focused on a significant area in mathematics because as a teacher, I am hoping to improve my teaching from the findings in this study for my own benefit as well as for my learners. I am hoping that my teaching will improve in terms of my ability to choose appropriate examples and use talk (effectively) to mediate examples in the multilingual context I teach in. Additionally, I am hoping that the findings from the study will benefit other teachers of mathematics and help them improve their teaching. The study should also benefit the research fraternity in mathematics education in terms of realising and emphasising the importance of the choice of examples and teacher talk used to mediate the examples in mathematics multilingual classrooms, not only applicable to Grade 8, but other grades as well.

1.4. Conclusion

In this chapter, I have provided the background of the study regarding the teachers' choice of examples and the way in which teachers use talk to mediate the examples they choose in multilingual classrooms. As highlighted, the types of examples chosen and later mediated through talk is a crucial process to the mathematical knowledge that becomes accessible to learners.

In what follows, I provide a detailed discussion regarding the subsequent chapters in the study.

- In Chapter 2, I discuss the literature which relates to the topic I intend to study. The literature is carefully chosen by relevance to my study, which provides insight into research that has been done in the field, already.
- In Chapter 3, I discuss the research design and methodology. Through this, I provide details of the data collection methods used in the study and the way in which analysis is done.
- In Chapter 4, I discuss findings that emerge from the data collection process.
- In Chapter 5, the concluding chapter, I provide a summary of the findings in the study through answers to the research questions, the limitations in the study and implications for future research.

Chapter 2: Literature review

2.1. Introduction

In this chapter, I aim to review literature in the field of mathematics with regards to the teaching and learning of mathematics through variation theory, exemplifying as a crucial mathematical practice, how mathematics (specifically, linear and exponential equations) is taught in multilingual classrooms.

2.2. Variation theory and the teaching and learning of Mathematics

Research conducted in mathematics education (Marton, 2015; Essien, 2021b; Watson & Mason, 2006) reveal that the use of variation theory enabled understanding of how it is that learners learn mathematics. It has also been reported that teachers who receive training on how to teach with the use and careful consideration of variation theory, teach for understanding rather than processing and mimicking procedures carried out by the teacher (Essien, 2021b). The implication of the type of training that the teacher(s) receives plays an important role in the way in which the teacher thinks about the examples they should choose and use during instruction, and also, the way in which these should be presented. This is especially the case for a teacher who understands the elements of variation theory. The aim of this is to maximise and exhaust all avenues and possibilities to learn any concept.

Additionally, research (e.g. Kullberg, Martensson & Runesson, 2016; Kullberg et al., 2017) shows that teachers who receive training on teaching using variation theory are able to present examples with a form of structure.

2.3. Exemplifying as a crucial mathematics practice

In order to teach mathematics, teachers are expected to use examples in order to communicate meaningfully (Essien, 2021a) and order for learners to learn mathematics, they ought to be exposed, in order to develop familiarity, with mathematical examples (Watson & Mason, 2005). Watson and Mason (2005) argue that this is the one way of enabling access of mathematical knowledge for and to the learners. The examples that are chosen and used in class (by the teacher) may at times emerge on the spot during classroom discussions, while some are planned prior to the lesson which Zodik and Zaslavsky (2008) distinguish between the two as pre-planned examples and spontaneous examples.

The type of examples chosen and used during classroom discussions determine the mathematical knowledge that learners are afforded to learn. Thus, the discernment of each concept in mathematics is dependent on the examples that the teacher choses and uses during instruction, which in turn determines the learning opportunities that emerge from the classroom discussions. Furthermore, the examples that are chosen and used by the teacher can be as limiting (or limited) as they do enable and/or enhance mathematical understanding (Kullberg et al., 2013). Thus, exemplifying is a fundamental mathematical practice (Essien, 2021a). Essien (2021b) refers to examples as tasks that are used to communicate mathematical meaning, in and out of the classroom.

Arzarello, Ascari and Sabena (2011) support the notion that through the use of examples, learners gain access to the mathematical knowledge and further state that examples help support the learners' way of thinking of and about mathematics.

A group of examples together aimed at achieving the objective of what it is that is made possible to learn is referred to as the example space (Watson & Mason, 2005; Arzarello et al., 2011). When one develops an example space it is important that the example space reveals connections and is varied and this is what should constitute the structure of the example space (Arzarello et al, 2011). Olteanu (2018) contends that a set of examples used during instruction should supposedly aim at revealing patterns of variation, which should assist learners to be able to reach a stage of generalisation and in order for this to be, the examples to be used must be well thought of.

Given that exemplifying is central to learning mathematics, Essien (2021b) asserts that it is not only the type of examples chosen and used during instruction that is important. The wording also holds a significant role. This implies that the wording (the way the language of learning and teaching (LoLT), and other languages available for use, which constitute a multilingual classroom) is just as important. Thus, examples chosen and used in multilingual contexts should be expected to differ from the way these are used in a monolingual classroom. Goldenberg and Mason (2008) support the notion that the aim of using examples is to exemplify. Thus, it is significant that if examples are to be used for the purpose of exemplifying, then it is crucial that the context (such as a multilingual context) in which exemplifying as a necessary mathematical practice is to be internalised is prioritised.

2.4. Teaching and learning in multilingual classrooms

Research has shown that most PSTs, in South Africa, are most likely to teach in multilingual settings at the end of their qualification (Essien, 2021b). Thus, this is the reality for majority of teachers in South Africa. Perhaps it is time that the type of examples chosen and used in mathematics classrooms be chosen and used based on the context they are to be used in, such as a multilingual context. Smit and van Eerde (2011) suggest that scaffolding of the language is a necessary condition for learning in multilingual classrooms. Scaffolding is the "temporary, intentional, responsive support that assists second language learners to move towards new skills, concepts or levels of understanding." (Gibbons, 2002, as cited in Smit & van Eerde, 2011, p. 890).

Smit and van Eerde (2011) agree that there is a need for research to be done in the area of teaching and learning in multilingual classrooms in terms of scaffolding the language(s) that learners are to learn mathematics in and how appropriate and effective the scaffolding becomes for a multilingual context.

Research by Wessel (2020) has revealed the importance of learning vocabulary in language as a necessary condition to learning mathematics. Thus, in order for learners to learn mathematics effectively and meaningfully, learners must first attain the vocabulary contained in the mathematics itself. Additionally, Schleppegrell (2007) asserts that in order for learners to learn mathematical concepts, they must first learn the correct mathematical language, which is or has the correct use of the mathematics register. Thus, as Barwell (2020) argues, learners whose home language is not the same as the LoLT in learning mathematics are faced with the challenge of first learning the LoLT, and then the mathematics language.

Given the significant and groundbreaking amount of work done in mathematics education, in classrooms regarding how teachers teach, the type of examples they use and how they communicate meaning and interact with their learners, I have realised that there is not a lot of research done on/in multilingual contexts wherein the examples that the teacher chooses and uses during instruction carefully consider that learners who are multilingual may learn differently from those who are monolingual, thus examples chosen and the talk used to mediate such examples in multilingual classrooms must be chosen with this context in mind. This is supported by research done in multilingual (and monolingual) context by Smit and van Eerde (2011) ; Wessel (2020), for example.

2.4.1. Code-switching for meaning making

In many developing countries such as South Africa, mathematics (and other subjects), in high school, is usually not offered officially in the learners' home language(s) (Jantjies & Joy, 2015). Thus, it is important that teachers are aware of the learners' home languages and that they can be used as resources for attaining mathematical knowledge (Setati, 2016).

Mathematics is taught and communicated through the use of language as a medium (Wessel, 2020). Given that in South Africa there are 12 official languages, the use of language(s) as a medium to access mathematical “goods”, differs dynamically from one class to the next due to the multilingual nature that exists (Setati & Adler, 2000). There is not a clear-cut path that is/can be set out to be accommodative for all classes due to the languages that exist and differ from class to class. Thus, for the existing languages in each class to be used effectively to attain mathematical knowledge and understanding, teachers ought to be aware and deliberately conscious of their language use in terms of which language to use, at what point it would be appropriate to use, how the use of any other language other than the LoLT support what the learners learn and so forth (Smit & van Eerde, 2011).

Essien (2021b) argues that it is through the “quality” of training that PSTs (in the context of this study, in-service teachers) receive that one can be able to determine whether or not such teachers are trained in a manner that prepares them to teach in multilingual contexts, for example, which then allows them to pedagogically “know” when and when not to use other languages available in the classroom as resources.

One of the ways that teachers make use of other languages to enable meaning making is through code switching. Code switching refers to the act of using more than one language in the same conversation in order to communicate meaning (Rose & van Dulm, 2006) and is one of the practices that can be used in multilingual contexts to support learning (Rose & van Dulm, 2006). In most cases, the teacher would code switch to the learners’ home language(s) and if code switching is used to enable meaning making, it would mean that the teacher makes use of the learners’ home language(s) to assist learners to understand a certain mathematical concept.

2.5. Teaching and learning algebra in a multilingual classroom

Algebra is one of the fundamental concept and topic to be learn in mathematics. Algebra is defined as the mathematical ‘science’ which one may use arithmetic ways to determine the unknown value through the use of the known numeric (or otherwise) through the use of signs and symbols, which are referred to as variables (Katz, 2007). This definition has evolved overtime, however, the evolution has not changed the true meaning of algebra, instead, this has shed light into ways of interpreting and carrying out algebraic procedures to solving mathematical problems.

Learners in multilingual classrooms of mathematics are faced with more than just the challenge of learning mathematics; these learners are also faced with the challenge of attaining mathematical knowledge through a language other than their home language (Setati-Phakeng, 2016). Kasmer and Billings (2017) concede that for a multilingual mathematics classroom to be considered effective, shared and common understanding of what they term 'intercultural competence' is a necessary condition. By intercultural competence, they refer to ways in which different people's languages may express ideas differently or express the same idea in a different way, as it may be, with some aspects of certain languages being under develop or inability to be directly translated dependent on the language infrastructure development.

Research in mathematics reveals the importance of negotiating meaning in a social setting, such as a classroom, through language (Gorgorio & Planas, 2001). Thus, a multilingual classroom in which algebra is taught, definitely has to be accommodative of the language diversity. By so doing, in learning algebra, learners are expected to draw meaning of mathematical symbols used and the way in which these mathematical symbols may be embedded in the 'words' (Caprapro & Joffrion, 2006). Algebra is considerably a big discipline in mathematics, hence for the scope of this research report I specifically focus on exponential equations which I discuss next.

2.6. The concept of algebraic equations

Hewson (2007) suggests that many learners have difficulties understanding linear and exponential equations due to the way in which they are structured and how it is that learners tend to memorise only the procedures and the true meaning of the conceptual and procedural understanding they should be focusing on. Simply put, exponential equations are algebraic equations in which exponential laws are applies to solve or find the unknown. Hewson (2007) argues that it is important that learners learn exponential equations with understanding so as to allow them, for example to plot graphs of functions such as the one for the logarithm function.

2.7. Conclusion

Given the literature review presented above, and how it links to my own study, beyond the theoretical framework, I aim to look into the details of how the research was carried out which provide deeper insight regarding the study.

Chapter 3: Theoretical framework

3.1. Introduction

In this study, I make use of Essien's (2021a) dyadic framework in order to understand the choice and use of examples in multilingual mathematics classrooms. In this dyadic framework, Essien (2021a) merges variation theory by Marton and associates and Mortimer and Scott's framework on meaning making as a dialogic process. Although the framework was initially developed for pre-service teachers (PSTs), it is considered relevant to this study because it contains elements of how examples may be engaged with, in mathematics classrooms, especially those that are multilingual. The model is seen in the figure below.

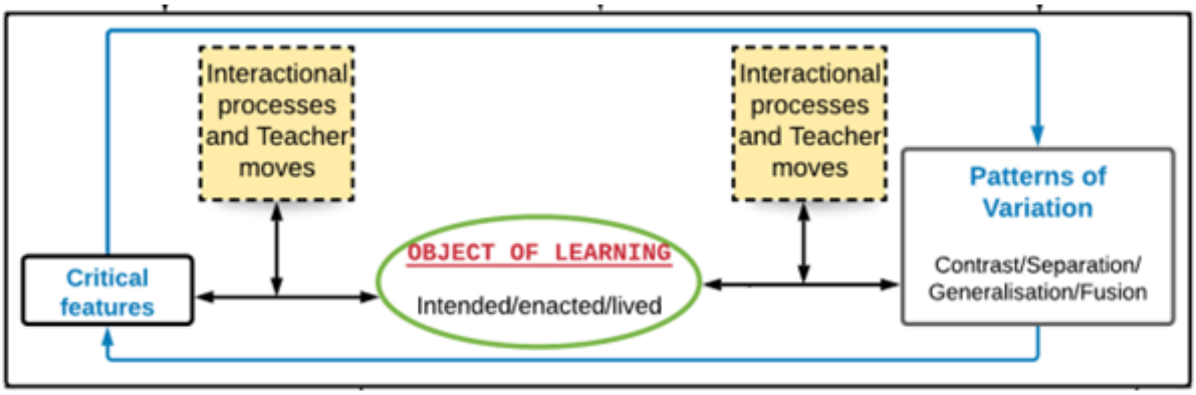


Figure 2. 1: Essien's (2021a) dyadic approach to understanding examples in multilingual classrooms(pg.480)

Figure 1 shows the dyadic framework as a diagrammatic representation of how variation theory and meaning making as a dialogic process come together. The dyadic framework contains elements of both theories and reveals continuous learning in the form of a cycle. The object of learning, patterns of variation and critical features are elements of variation theory, and the interactional process and teacher moves are elements of meaning making as a dialogic process. In what follows, I expound further on the dyadic framework for understanding examples in multilingual classrooms, taking care to first engage with each of the constituent framework.

3.2. Variation theory of learning

Variation theory is a learning theory whose roots lie in phenomenography. Phenomenography refers to the qualitative research methods that contend that learning occurs through seeing and experiencing the parts of the object of leaning in novel situations, so as to attain novel meanings (Marton & Pang, 2013). Further, phenomenography helps us to understand how the various ways of seeing something differ. The attainment of novel meanings is referred to as discernment. Novel meanings refer to the ability to recognise unique features of a concept (object of learning) whose goal is to be one's own (the learners' own) in a lesson (Marton &

Pang, 2013). The tenets of variation theory are the object of learning, patterns of variation and critical features. These are discussed in detail, below.

3.2.1. Object of learning

In order for learners to learn, it is imperative that the object of learning is identified and engaged with. The object of learning in a mathematics classroom refers to a concept to be learned (Marton & Pang, 2013) The object of learning is in fact, the focus of attention in each lesson. It is what the teacher or class is focusing on at a particular point in time. In mathematics, this is the mathematical concept at hand, and this may be different for each learner (Kullberg et al., 2017). The object of learning can be intended, enacted and lived.

The dyadic framework considers the object of learning as a central feature of attaining novel meanings for mathematical understanding and knowledge from an example space in order to engage with examples that are initially chosen, and then used during instruction. The object of learning is what both the teacher and learners engage with in order to acquire novel meanings about a particular concept in mathematics (Mårtensson & Ekdahl, 2021). Novel meanings are curated in novel situations.

The way in which the teacher may understand, view and aim to teach, using the chosen examples does not always directly correlate with what the learners think about what the teacher intends as the novel meaning the learners are to discern (Marton & Pang, 2013). In other words, although the teacher may plan to carry out instruction in a certain way, what decides what learners really learn is what learners discern through the novel situations and meanings that the teacher presents the learners with. This reveals what it is that is made possible for learners to learn (Kullberg et al., 2017). According to Pang and Marton (2003), this refers to how it is that the object of learning is meant to be contextualised and intentional (Lam, 2013). Thus, they differentiate between the intended, enacted and lived objects of learning in a mathematics classroom. The intended object of learning refers to what it is that the teacher aims to make available for learners to learn (Bills & Bills, 2005), and the enacted object of learning refers to the interactions and discussions that take place during instruction. These can be amongst learners on their own, or learners with the teacher (Runesson, 2005). The enacted object of learning reveals what is made possible to learn (Kullberg et al., 2017). The lived object of learning refers to what it is that learners truly learn-the novel meanings learners come to own (Essien, 2021a; Marton & Pang, 2013). Furthermore, the lived object of learning may differ for each learner in the same classroom, taught by the same teacher and engaging with the same example space. This is because although the learners may arrive at the same answer, their learning experiences in engaging with the object of learning may differ (Watson & Mason, 2006). Each aspect of the object of learning has to always align with the other, for each lesson.

Once the object of learning is identified and evident in the example set/space, it is possible to spot elements of the example space that make the example space coherent. In order to do this, an example space can be analysed in terms of identifying what is similar, different, brings everything together and what helps to generalise. This leads to the discussion of patterns of variation in an example space which is below.

3.2.2. Critical features

Critical features refer to the aspects of a phenomenon outlined based on the teacher's knowledge of the mathematical concept at hand (Kullberg et al., 2017). Critical features are dependent on the aspect of the phenomenon each person chooses to focus on, at a time, and all these ways are parts of a whole depending on the lens (or perspective) used. In order for one to discern one or more critical features, they need to have knowledge about that aspect in order to be able to discern it (Essien, 2021b; Marton & Pang, 2013). There are many critical features for each example space, and these may include, in an example space based on discerning the concept of exponential equations, that in order to compare (equate) the exponents, the bases must be the same, thus, emphasising and highlighting the importance of ensuring a common base on both sides of the equation.

Learning through variation theory implies that learners discern the critical aspects of an object of learning by experiencing differentiation (Kullberg et al., 2017). Thus, in order for a learner to discern what an exponential equation is, they must first know/discern what it is not, for example, identifying exponential equations as those that contain a variable as an exponent ($2^x = 4$), and a quadratic equation as one that has the exponent as a base ($x^2 = 4$), for example.

Like any other theory, variation theory has its own limitations, some of which include, but are not limited to, the fact that variation theory alone, does not spell out how it is that a structured and well organised example space may be used effectively in the classroom (put into action). According to Lo (2012), no learning theory is perfect that it has no limitations. Thus, it is important to "allow" the measure of the theory to be extended in a multilingual classroom, to the language used and the "talk" involved in engaging with the example space. It is due to this that this study uses a theoretical framework that merges variation theory with meaning making as a dialogic process in order to mitigate the limitations.

3.2.3. Patterns of variation

Another important feature of variation theory is patterns of variation. The constructs of patterns of variation include contrast, separation, generalisation and fusion (Essien, 2021b). Contrast refers to how examples in the same example space are presented differently. This implies that the focused meaning is varied while the other features are kept the same (Marton & Pang, 2013). The aspects that remain invariant is what is referred to as similarity (Essien, 2021b). Separation, on the other hand, refers to how it is that in one example space, one feature is varied (or not varied) at a time (Marton & Pang, 2013). Fusion refers to "…simultaneous variation of several critical aspects of the object of learning in an example space" (Essien, 2021b, p.477). With fusion, it is expected that the critical aspects of the object of learning that were made variant, or invariant are realised and merged. Lastly, generalisation refers to the ability to spot patterns in the variations presented in the same example set.

In the above, I have provided the descriptions of each tenet of variation theory. It is important to note that each of these does not exist in isolation, and that each one informs the other. One way of seeing the links amongst the tenets can be through suggesting that once the teacher decides on the topic to teach for a particular lesson, taught the topic and evaluated what the learners have learned, they would have covered the object of learning. In addressing the object of learning, it is inevitable that the critical features themselves are embedded in this, thus, leading to an understanding of the patterns of variation in terms of what is similar or different within the example space(s), for example.

3.3. Meaning making as a dialogic process

According to Mortimer and Scott (2003), meaning making refers to the way in which the teacher handles the conversations and interactions (talk) that take place in a mathematics classroom which grant learners epistemic access into the mathematics. The teacher is then seen as the one who guides the classroom interactions because the teacher is aware of the learning goal. As indicated in the dyadic model (Essien, 2021a), the focus is on the three aspects of analysis, namely: communicative approach, patterns of discourse and teacher interventions. These aspects of analysis are discussed below.

Mortimer and Scott (2003) indicate that talk can either be dialogic or authoritative, at the same time, talk can either be interactive or non-interactive. Next, I discuss elements of meaning making as a dialogic process, namely, communicative approach, patterns of discourse and teacher interventions.

3.3.1. Communicative approach

Communicative approach is divided into four aspects. These are the dialogic/authoritative as well as interactive/non-interactive. The dialogic communicative approach refers to the point at which a lesson becomes permissible of more than one idea to be explored which then intends that more than one point of view is considered (Mortimer & Scott, 2003). The authoritative communicative approach refers to a position wherein only one point of view is considered, in most cases being the teacher's view, however it is imperative to note that mathematics, as communicated by the teacher, also acts as the authority.

The talk in the classroom may be dialogic or authoritative as already mentioned, and another important aspect to consider is that which involves participation of learners in the classroom talk. Whenever learners participate in the classroom talk, this aspect of the lesson would be considered to be interactive whereas when learners' active participation in the lesson is not visible then this part of the lesson is considered to be non-interactive (Mortimer & Scott, 2003). The four abovementioned communicative approaches may be merged as can be seen in *Figure 2.2* below.

	INTERACTIVE	NON-INTERACTIVE
DIALOGIC	**A** Interactive/ dialogic	**B** Non-interactive/ dialogic
AUTHORITATIVE	**C** Interactive/ authoritative	**D** Non-interactive/ authoritative

Figure 2. 2: Forms that talk can take on in a classroom (Mortimer & Scott, 2003, pg.50)

In most (if not all) mathematics classrooms, "talk" is central to communicating meaning. Thus, in most instances, in order to check for understanding, teachers would pose questions to learners to either respond verbally, or in writing. I now turn my attention to a discussion around patterns of discourse.

3.3.2. Patterns of discourse

The patterns of discourse refer to how the teacher interacts with the learners, how the learners interact with one another and how it is that both the teacher and learners interact with a mathematical concept in the form of 'talk'. 'Talk' refers to the way in which both the teacher, and learners engage in a conversation that allows each party (especially the teacher), to understand what it is that the learners think, what it is that they understand and further initiate/ask questions that would deepen the learners' understanding of a concept (Mortimer & Scott, 2003). The first step is usually the initiation(I), followed by a response(R), thereafter follows a sequence of either evaluation(E), response or prompt(P). Participation is key to the patterns of discourse (Essien, 2021a).

3.3.3. Teacher interventions

The teacher interventions aspect refers to how the teacher is able to amalgamate, all the contributions by the learners and give directive in terms of what the contributions mean mathematically. When the teacher intervenes, at any point of the lesson, they do so by allowing themselves and most importantly the mathematics to be the authority.

The features above of the model of the dyadic approach by Essien (2021a) reveal how it is that the features, together, contain the dialectic relationship between variation theory and meaning making as a dialogic process.

3.4. Dyadic framework

The dyadic framework used in the study merges variation theory with meaning making as a dialogic process. The two theories support one another in enhancing the choice and use of examples in multilingual mathematics classrooms. Thus, a good example space on its own is not sufficient to ensure the attainment of novel meanings and ensuring minimal to no missed opportunities during engagement with the example space. A model of the dyadic framework can be seen at the beginning of the discussion of the theoretical framework above. Merging the two theories aims to afford opportunities for variation theory to be put into practice through realising its effectiveness through 'talk' which is mainly focussed on by Mortimer and Scott's (2003) meaning making as a dialogic process. Variation theory focuses more on the chosen examples, whereas meaning making as a dialogic process focuses more on the talk that is used to mediate the example spaces.

3.5. Conclusion

In conclusion, the theoretical framework I have described above is the one I use to inform the analytical framework which I present and describe in the next chapter. The analytical framework will be used to analyse the findings and provide answers to research questions posed earlier, later in the study.

Chapter 4: Research design and Methodology

4.1. Introduction

In this chapter, I intend to provide a discussion of the methodology and the data collection methods I made use of, in the study. I also intend to present the details of the analytical framework which I later make use of, in Chapter 4 to analyse the findings.

4.2.The setting

Data in this study was collected in a school situated in a coloured community (township), in the Free State, South Africa. The area where the school is situated is dominated by spoken Sesotho, Setswana and Afrikaans. The school offers instruction in two languages, English and Afrikaans. The language infrastructure in the school is such that English, Sesotho and Afrikaans are offered as Home Languages (HL). For learners who take English as their HL, they learn mathematics in English, together with those who do Sesotho as their HL. Whereas, for those who do Afrikaans as HL, they learn mathematics (and all the other subjects) in Afrikaans and take English as their First Additional Language (FAL). The school is considered multilingual because there is more than one language that is used as the LoLT, alongside other languages (including their dialects) that are potentially available for use. Table 3.1 below shows the breakdown of the language infrastructure in the school.

	HL	**FAL**	**MATH (and other subjects)**
Group 1	English	Afrikaans	English
Group 2	Afrikaans	English	Afrikaans
Group 3	Sesotho	English	English

Table 3. 1: Language infrastructure in the school

Both teachers are multilingual, and their HL is Sesotho and both teachers share a HL with a vast majority of their learners. Other languages that both teacher Thembi and teacher Mercy are fluent in include Afrikaans, Isizulu, IsiXhosa and English. Further, both teachers teach mathematics in English.

4.3. Sampling

There are two educators who were observed and whose lessons were video recorded in the study. In order to protect the teachers' identity, I make use of pseudonyms, teacher Thembi and teacher Mercy for the teachers. In total, I observed three classes. Teacher Thembi taught two classes (class T1 and class T2), and teacher Mercy taught only one class (class M1).

4.4. Research design and Methodology

This research project made use of the qualitative research approach (qualitative study) of phenomenography. When conducting such research, a particular phenomenon's meaning is studied. Qualitative research within phenomenography is concerned with observing how people interpret the world through their lived experiences and how they create their own reality of the world based on their understanding(s) of the experiences they encounter (Merriam & Tisdell, 2015). Phenomenography refers to how it is that a single phenomenon can be encountered by different people in different ways, depending on how each person thinks (perception) about such phenomenon and the life experiences that shape their thinking (Merriam & Tisdell, 2015; Marton & Pang, 2013).

Since the study makes use of a qualitative approach, I have specifically used classroom observations and interviews as methods of data collection. I observed two teachers in Grade 8 classrooms teaching linear and exponential equations to multilingual learners in three classrooms. The video recording was used to capture the concepts of linear and exponential equations. The two teachers involved in this research project teach in the same school and the same grade. The choice of the school in this linguistic setting (multilingual) was motivated by the desire for a multilingual context where there is a dominance of one South African indigenous language. Subsequent to lesson observations, the teachers were then interviewed with regards to each teachers' choice of examples and how the teachers used talk to mediate the examples they chose, in their classrooms.

4.5. Validity and Reliability

Cohen, Manion and Morrison (2011) highlight the importance of conducting research with an open mind and not allowing preconceived ideas to cloud the judgment and analysis of findings. Validity is one of the crucial features of a research study, however, what makes a good research study is: in addition to the study being valid, it must also be reliable (Guest, Kathleen, MacQueen & Namey, 2014). According to Guest et al. (2014), validity can be defined in many ways, one of which is that validity is more concerned with how it is that an explored phenomenon contains elements of accuracy which can assist in understanding how it is that meaning is drawn from assessing and focusing on the aspects of the phenomenon one intends to study, throughout. Reliability refers to the ability to measure with consistency "yielding comparable results each time…" (Guest et al., 2014, p. 4). Thus, reliability can be thought of a good measure of how the same result may be achieved over and over again, so long, all the conditions remain unchanged. In order to ensure that this research study is valid and reliable, below, I provide a rationale of the research methods made use of, looking into the importance of each, and what each of them focuses on.

4.6. Data collection methods

4.6.1. Classroom observations

Video recording and field notes were used as part of the classroom observations. It is through observations that one may be able to provide a description of the behavioural patterns, events and the context in which such events occur, (Maxwell, 2018) which may be moments not captured by video recordings of the lesson, for example. Lesson observations are vital in this study as the observations have enabled me to infer what truly happens in the classrooms. As indicated in the dyadic framework, the object of learning can either be intended, enacted or lived. Classroom observation is aimed at enabling me to capture the enacted and lived objects of learning. During the time that I observed lessons, I also took field notes which I then use to support the inferences I make in analysing the data provided in the video recordings.

4.6.2. Audio-recorded Interviews

In addition to observations, "interviews can also provide additional information that was missed in observation and can be used to check the accuracy of the observations" (Maxwell, 2018, p. 15). Thus, through interviews, one can be able to gain insight into what had happened before, that might have led to the occurrence(s) of the current event(s) (Maxwell, 2018). Furthermore, interviews are useful, when the questions posed show a direct link between the research questions and those posed to the interviewee (Maxwell, 2018) which, through the interviewer-interviewee interaction, shared common understanding can be reached as the ultimate goal (Alshenqeeti, 2014). The interviews are semi-structured and focus specifically on the teacher's choice and use of examples.

4.7. Analytical framework and Data Analysis

The dyadic framework (Essien, 2021a) from which this study draws, provides an analytical framework for analysing the teachers' choice of examples and how the teachers use talk to mediate the examples in multilingual classrooms. This analytical framework is the one I make use of, for analysis in my study. This framework (adapted from Essien, 2021b) draws from both variation theory and Mortimer and Scott's meaning making as a dialogic process framework and is represented below:

Analytical Framework

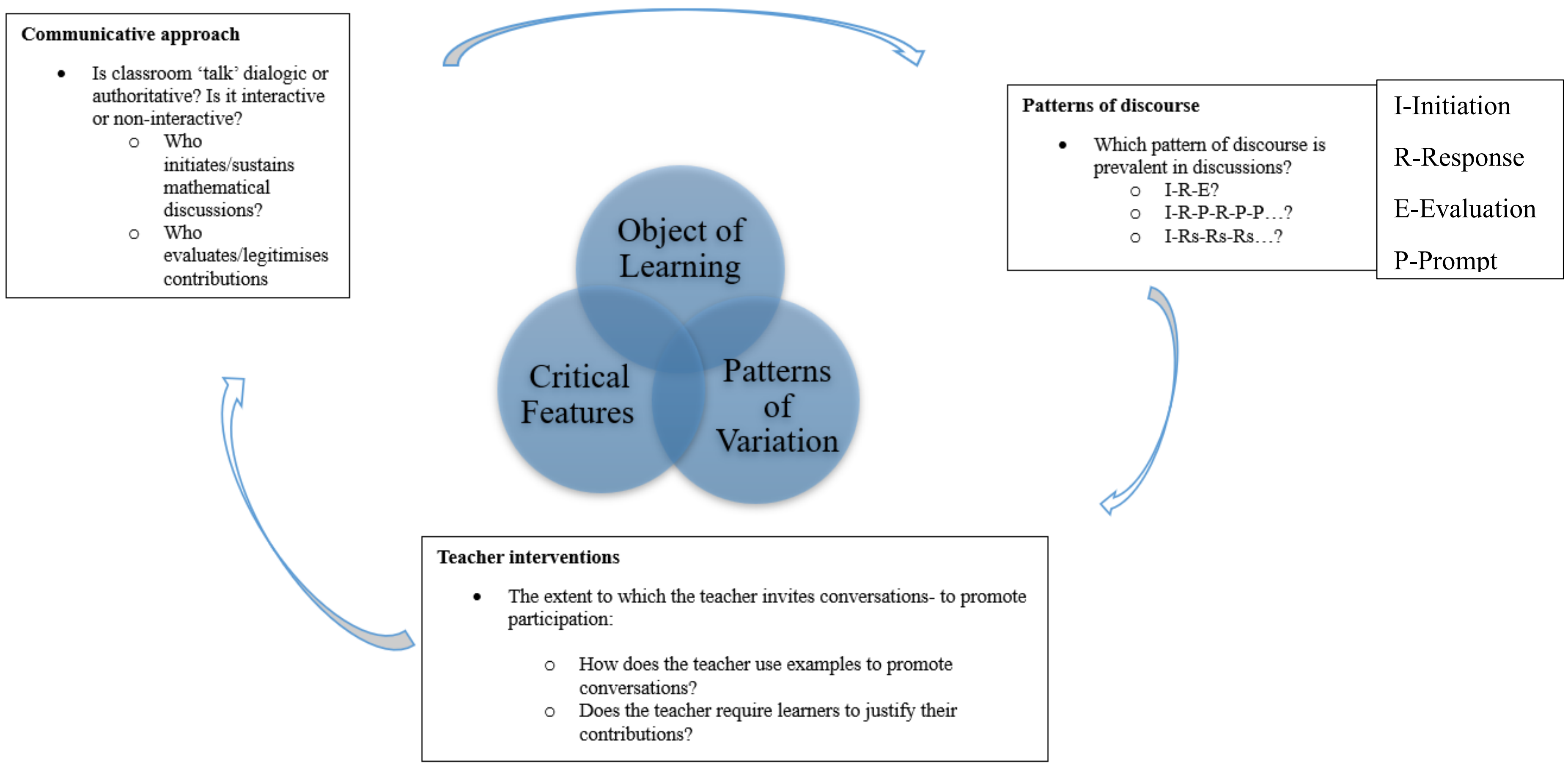


Figure 3. 1: Guiding analysis questions/descriptors (Analytical framework)

At the core of the above analytical framework, are the elements of variation theory, which have allowed me to analyse the example space in terms of the teachers' choice of examples and on the peripheral is Mortimer and Scott's (2003) meaning making as a dialogic process, which should allow me to analyse how the teachers used talk to mediate the examples in their respective classes. Furthermore, the analytical framework has helped me to answer the research questions I posed earlier, in Chapter 1.

Analysis in this study is done in two parts. The first part investigates the teachers' choice of examples through the use of variation theory within the Dyadic framework. The second part investigates how it is that the teachers use talk to mediate the examples they chose, through the use of the dyadic framework's elaboration of Mortimer and Scott's (2003) framework on meaning making as a dialogic process.

4.8. Ethical considerations

Applications for ethics clearance were submitted and approved by the provincial education department in which the data for the research project was collected, as well as to the ethics committee at the Wits School of Education.

This research study does not aim to negatively affect the participants or advantage or disadvantage the employment standing of any of the participants and participating in the study is solely voluntary. The identity of all participants is be kept confidential at all times. The real names of participants are not used in the study and all participants remain anonymous. In the case where analysis of data requires the use of a name, pseudonyms have been used. The school's identity is also kept confidential in all the writing related to the study. Since participation is voluntary, participants were made aware that they are allowed to withdraw their permission to participate in the study at any point. Following raw data collection, the data was kept secure, digitally, with encrypted and authorised access in a memory stick and upon completion of the study, will be destroyed in the period between three and five years.

4.9. Conclusion

In this chapter, I have discussed the data collection methods, together with the way in which analysis in the study is done. In what follows, I turn to a discussion and presentation of findings, which I then analyse.

Chapter 5: Analysis and Discussion of Results

5.1.Introduction

The purpose of this study is to investigate the choice of examples and talk in mediating the chosen examples by two teachers in Grade 8 multilingual classrooms when they teach linear and exponential equations. The present chapter aims to present the findings for the study through the use of the earlier presented analytical framework- outlined in Chapter 3. Analysis in this study is done in two main parts and the analyses are provided per class that each teacher teaches in three Grade 8 multilingual classrooms. The first part seeks to analyse the choice of examples *per se*. In doing this, I use variation theory (and its constructs) to analyse the examples that the teachers choose for inclusion, to be able to enact the object of learning. The second part of the analysis investigates the teachers' talk in mediating the examples they initially chose, and this part of the analysis is done through the use of the dyadic framework's elaboration of Mortimer and Scott's (2003) framework on meaning making as a dialogic process.

The two teachers involved in the study were interviewed (one at a time) subsequent to the video recording of lessons in the classes they teach in Grade 8. The findings in the study seek to provide answers to the research questions below, which guide the study:

1. What examples do teachers in Grade 8 multilingual classrooms choose in the topics of linear and exponential equations?
2. How do teachers use talk to mediate these examples in their multilingual classrooms?
3. What is made possible to learn through the chosen examples? How does the teachers' talk, in mediating the examples enable meaning making in multilingual classrooms?

5.1. Part 1: Teachers' choice of examples

The object of learning informs and is informed by the examples that the teachers choose and use during instruction (Kassa & Ding, 2019). In part 1, I present the analysis of the choice of examples, separately for each teacher. As previously stated, I analyse the teachers' choice of examples through the use variation theory as a lens. The choice of examples by each teacher suggests that the teachers teach differently. Teacher Thembi teaches linear equations followed by exponential equations, whereas Teacher Mercy teaches both linear and exponential equations combined. I first present a discussion of the resources that both teachers choose their examples from and thereafter, I present the analysis of Teacher Thembi's choice of examples and follow with Teacher Mercy's choice of examples.

5.1.1. Resources that teacher Thembi and teacher Mercy choose examples from.

Both teacher Thembi and teacher Mercy choose examples dominantly from the Mind Action Series Grade 8 textbook. First, I present a discussion of the resources that teacher Thembi chooses examples from and follow with those for teacher Mercy.

5.1.1.1. Resources that teacher Thembi chooses examples from.

Teacher Thembi makes use of more than one resource that she chooses examples from, and many of these examples are taken from the Mind Action Series Grade 8 textbook. Additionally, the teacher makes use of past examination papers to choose examples from. This is one way of allowing and developing familiarity for the learners with the style of questioning which their formal assessment shall be based on. Apart from resources already mentioned, the teacher also chooses examples from resources that prepare learners for the next level (grade). For example, the teacher chooses some of the examples from Grade 9 textbooks, even though the learners are only in Grade 8.

For example, exercise example 9 ($2.2^x = 16$) (see Figure 4.2) is not from the textbook that the teacher generally chooses examples from. It is not clear where this example is chosen from. Despite this not being clear, it is important to note that this example fits into example space (through its features) and was surely carefully chosen from a source that can easily be the teacher herself. As this may be, the teacher may have chosen these examples from other sources such as Siyavula, as can be seen in Excerpt 4.1 below.

33	**Researcher**	Okay…and where do you get your examples?
34	**Thembi**	Ehhh examples, normally I use…uhmm Mind Action…
35	**Researcher**	…or just questions in general?
36	**Thembi**	The Mind Action textbook
37	**Researcher**	Oh, the blue one?
38	**Thembi**	Yes, the blue one and ehh, what is this? Siyavula, the online one
39	**Researcher**	Okay…
40	**Thembi**	Siyavula, and then sometimes I use past question papers
41	**Researcher**	Okay…
42	**Thembi**	Even if I find something on the Grade 9 textbook, I use it
43	**Researcher**	Oh okay, in Grade 8?

44	**Thembi**	Yes, just to expose them uhmm so that we don't struggle much when we get to Grade 9. Yes.

Excerpt 4.1: Interview Transcript, Resources teacher Thembi chooses examples from

Excerpt 4.1 reveals the resources that teacher Thembi chooses her examples from. I now turn to a discussion of the resources that teacher Mercy chooses examples from.

5.1.1.2. Resources that teacher Mercy chooses examples from

All the examples that teacher Mercy chooses are taken from different exercises in the Mind Action Series textbook for Grade 8. At times, teachers tend to have an (over) reliance on the resources they find effective. This may also be because teachers do not think that they have much 'freedom' in deciding on what examples to choose because at times, subject advisors (from the education department) expect teachers to use the materials and resources they prescribe. Thus, by so doing, the department expects teachers to choose examples that the subject advisors know to be in the materials.

It can be challenging at times for teachers, I imagine, knowing when and where to draw the line in terms of how far they can go in terms of choosing examples from other materials either than from those that are prescribed and making up their own examples. From my own experience as a teacher, I know and understand that the subject advisors from the education department expect things to be done in a certain way, which is to be reflected in the teacher's preparation and learners' books. Moreover, subject advisors from the education department would be more pleased to see that as a teacher, I implement what they prescribe, especially if it is done to the latter. Thus, it is understandable that teacher Mercy would say that she does not have any choice, but to choose examples from what the education department prescribes. When teacher Mercy was asked how she decides on the examples to choose, she responded in the following way, which can be seen in Excerpt 4.2 below.

25	**Researcher**	Interesting. And then how do you decide which examples to use and where do you get those examples?
26	**Mercy**	Okay, actually, we have work schedule. There are examples made for us, coming from the department.
27	**Researcher**	Okay…

28	**Mercy**	…because even the lesson plans, they do it for us. So, what we only have to do, the resources, you know *mos* you cannot rely on only one thing. You have to use as many resources as possible to make sure that you cater all, every learner in the classroom and then as much DBE books concerned, every term they provide us with DBE books. First term and second term, third term and fourth term, yes, we do have it.
29	**Researcher**	Okay.
30	**Mercy**	Yes, we also used to have 'hey math'. Even though because of technical problems we no longer have, we used to have, we used to give 'hey math'
31	**Researcher**	Okay, was it a hard copy book or was it…
32	**Mercy**	It was laptop, like yes. Learners would just watch and look at all the examples that are just there and then you see that the teacher at some point, is using the protractor, at some point you can even play some videos, pause…(*inaudible)*
33	**Researcher**	Oh, so it plays videos?
34	**Mercy**	Yes, it was very nice that one, 'hey math' one, very nice. Even slow learners *neh* (right)? There are those learners who learn through visual. It also helped for them, but then technicalities happened

Excerpt 4.2: Interview Transcript, How Teacher Mercy decides on her choice of examples and resources

Although the department provides teacher with lesson plans and prescribe materials to use, it is important to note that the decision to choose appropriate examples for the context and type of learners each teacher has to teach, lies with the teacher's pedagogical knowledge. Thus, even the implementation of the chosen examples depends on the teacher. Furthermore, the education department ensures, in most cases, that there are different resources freely available (in print and online) to both teacher and learners that teachers may choose examples from. Other materials that teachers may choose examples from are in the form of online learning management systems (LMS) such as the 'hey math' that the teacher mentions.

5.1.2. Teacher Thembi's choice of examples

In teaching exponential equations, teacher Thembi chooses examples specifically based on the content she aims to cover. There are several divisions of and under algebraic equations, these include linear, exponential, word problems, quadratic equations etc., at Grade 8 level. For this teacher, each type of equation was taught separately. Prior to teaching exponential equations, teacher Thembi taught linear equations and in the previous term, she taught exponential laws.

More often than not, the timeframes and content (thus informing the examples the teachers choose) that the teachers have to cover is stipulated by the department in the documents (such as the Curriculum and Assessment Policy Statements, CAPS and Annual Teaching Plan, ATP) that they make available to the teachers. Thus, in a sense, in 'suggesting' what to teach and what materials to choose examples from, the department does 'dictate' the types of examples to choose in teaching a particular concept. Of course, by so saying, I do not imply or suggest that the teachers are compelled, must abide and be limited to and by the examples that are listed in the departmental documents, it simply means that the teachers' choice of examples is sort of channelled in a certain way. Therefore, the teachers may still well choose their own examples, because after all, as a teacher, one is expected to be indispensable in ensuring that they can rely on their own knowledge as a teacher to make informed decisions (Ball, Thames, & Phelps, 2008) that allow them to make up their own examples too.

When asked what informs her choice of examples, teacher Thembi responded in a manner I engage with shortly in Excerpt 4.3 taken from the interview. The examples that teacher Thembi chose are presented directly after Excerpt 4.3.

Maxwell (2018) maintains that interviews are useful in that they provide information beyond what observations may not be able to, when conducted well. As previously stated, teacher Thembi was interviewed subsequent to the lesson observations. Below is an excerpt (Excerpt 4.3) of the interview audio transcription with Teacher Thembi regarding her choice of examples. This excerpt alludes to the teacher's response when asked what it is that informs the choice of examples. Above and beyond presenting the excerpt below, the argument I intend to reemphasise is that teachers do not truly feel that they have much liberty in deciding what examples to choose in teaching a concept. This is also because much as the teacher teaches the types of equations separately, this is a decision informed and guided by what the department expects of her.

29	**Researcher**	And then, for each topic that you teach, how do you decide on which examples to use in class? And where do you get those examples?
30	**Thembi**	Uhmm…How do I decide on the examples…?
31	**Researcher**	Mhmm
32	**Thembi**	Ehhh…like, for each and every topic they have the subtopics *akere (*right)? So, I choose the examples based on the concepts. Like for example in the topic that we are busy with now, functions, mxm, what is this? Solving equations neh…like we have different kinds of equations, like we have the linear equations, we have the exponential (*stresses term*) equations. Okay, we have the linear equations, then we have those equations that we have, ehhh…unknowns on both sides of the equations, then we have the exponential equations. So I have to separate the different types of equations and choose…for example, let's say I'm starting with the linear equations, the easy ones, I will choose…the heading will be: Solving equations, and the examples will be based on that particular concept, and then when we are done we move on to the next type of equation, those ones that have the unknowns on both sides of the equation. And then I look for problems that have unknowns on both sides of the equation and the exponential equations.

Excerpt 4.3: Interview Transcript, How Teacher Thembi decides on her choice of examples

In Excerpt 4.3 above, it can be argued that what informs teacher Thembi's choice of examples is simply reliant on the topic she teaches which is a decision that heavily depends on the material the teacher decides to use, rather than it being about choosing examples that are specifically suited for the context they teach in. Perhaps, the material(s)/resource(s) that the teacher uses provide examples that the teacher thinks of as being suitable for the multilingual context they teach in.

Additionally, because the teacher chooses examples based on the concept they are to teach, it is pertinent to highlight that the teacher carefully chooses these examples in a manner that allows her to focus on one object of learning for each type of equation, rather than teaching different kinds of equations all in one lesson, especially the introductory lesson.

Teacher Thembi is particular about the way in which the whole topic of algebraic equations should be broken down in order for her learners to understand better, as is evident in her response in the Excerpt 4.3 above. Below are the examples that Teacher Thembi uses in both her classes (first class T1, then class T2), in Figure 4.1 and Figure 2 below. I will provide an analysis of each class's example space, and thereafter talk about the two classes together.

Class	Examples chosen
T1	1. $5^{2x} = 25$ 2. $3^x = 1$ 3. $3.3^x = 27$ **Worked examples**
	1. $2^x = 4$ 2. $3^{x-1} = 9$ 3. $2.2^x = 8$ 4. $2.3^x = 18$ 5. $2^{3x-1} = 32$ 6. $3^{2(x-1)} = 81$ 7. $8^x.2 = 128$ 8. $3^{2x}.3^x = 9$ 9. $\frac{1}{3}.3^{2x} = 1$ 10. $2^{2x} + 8 = 9$ 11. $6^{x-2} = 1$ **Exercise examples**

Figure 4. 1: Example space for Teacher Thembi in class T1

5.1.2.1.Examples teacher Thembi chose in Class T1

In class T1, the teacher chose a total of fourteen examples to teach the concept of exponential equations. These examples were not all presented in one lesson. The first three examples were chosen as worked examples which the teacher used to introduce and explain the concept of exponential equations, for the first time. The exercise examples were chosen to be used over four lessons (including doing corrections). The teacher chose the worked examples as they appear in the textbook but did not make use of the ordinality that the textbook uses when choosing exercise examples. What this implies is that the order in which the worked examples are listed in the textbook is not the same order or sequence that the teacher follows.

5.1.2.1.1. The example space's object of learning

The initial object of learning was introduced to the learners by the teacher, as solving exponential equations, wherein the teacher continued to give a description of what exponential equations are. With this, then, it can be argued that the intended object of learning is that the teacher intends to use the chosen examples to teach learners how to solve exponential equations with the understanding of first, how to apply exponential laws, and secondly how to solve linear equations because exponential equations themselves require the understanding of solving linear equations in order to find the value of the unknown which is the variable, once the point of comparing/equating exponents is reached. I shall delve into the details of the other constructs of the object of learning (enacted and lived) later in the chapter when I provide a discussion of how the teacher used talk to mediate the examples. Prior to learning exponential equations, the learners, in the previous term had learned exponential laws, and the topic that was taught right before exponential equations was linear equations. The understanding of both concepts is necessary to serve as prior knowledge and gateway into the attainment of the newly learned concepts, especially if these were understood well by the learners (Rittle-Johnson et al, 2009).

5.1.2.1.2. Patterns of variation in the example space

The first two worked examples ($5^{2x} = 25$ and $3^x = 1$, respectively) are similar in structure in that they both have a power on the left-hand side and a whole number (to be converted to a power) on the right-hand side, while the second and third ($3.3^x = 27$) examples both use the base three. The third worked example differs from the first two in that prior to converting the whole number on the right-hand side to a power, it is expected that the coefficient of the power on the left-hand side must be used as a divisor on both sides. None of the worked examples have the same solution when solved.

The teacher used a total of eleven exercise examples in four lessons. Examples 1, 2, 5, 6, 9, 10 and 11 ($2^x = 4$, $3^{x-1} = 9$, $2^{3x-1} = 32$, $3^{2(x-1)} = 81$, $\frac{1}{3}.3^{2x} = 1$, $2^{2x} + 8 = 9$, $6^{x-2} = 1$, respectively)are all similar in structure in that they all contain a power on the left-hand side and a whole number on the right-hand side. Examples 3, 4 and 7 ($2.2^x = 8, 2.3^x = 18, 2.8^x = 128,$ respectively) are also similar in structure in that they all contain a coefficient attached to the power placed on the left-hand side. Example 8 ($3^{2x}.3^x = 9$) is the only one that differs from the rest of the examples in structure and requires high order thinking in terms of solving it due to the way in which it is structured. This example requires learners to first identifying base 3 as a common factor on the left-hand side, and thereafter follow the same solving procedure as in examples 1, 2, 5, 6, 9, 10 and 11.

However, example 8 in the first step resolves into a structure similar to examples 2, 5, 6, and 11. Examples 1, 2, 5, 6, 9, 10 and 11 are similar in structure, to examples 1 and 2 of the worked examples and example 3 of the worked examples is similar to examples 3, 4 and 7. All these examples chosen in class T1 make use of the variable x which is placed on the left-hand side of the equation, throughout.

In totality (both worked and exercise examples), seven out of the fourteen examples make use of the base three (worked: 2 and 3; exercise: 2, 4, 6, 8 and 9) and seven out of the fourteen examples have a solution: $x = 2$ (worked: 3; exercise: 1, 3, 5, 6, 8 and 11). All the solutions to the equations in this example space have a whole number as a solution, except for examples 8 ($x = \frac{2}{3}$) and 9 ($x = \frac{1}{2}$). Examples 2 (worked) and 10(exercise) have the solution $x = 0$ while examples 2 and 6 have the solution $x = 3$. Example 8 is distinguished from the rest in that it contains two powers that are the same on the left side. The issue of how learners receive and understand this type of example is engaged in when I explore the use of examples. Although the teacher does not follow the sequence in which the examples are presented in, in the textbook, the teacher, however, chose these examples from the same exercise

The conversion of the whole numbers on the right-hand side of the equation, to a power is a form of generality that is contained in this example space. It is the kind of generality that can be further developed in solving each equation into ensuring that in order to compare (equate) the exponents, the bases on either side of the equation must be the same.

5.1.2.1.3. Critical features of the example space

In order to solve any of these examples, all the whole numbers (on the right-hand side) must first be converted to a power, and this is one of the critical features in this example space. Given that the critical features of any example space lie in the mathematics itself (Marton & Pang, 2013), some of these critical features are listed in Table 4.1 below, above and all relate to the object of learning which is solving exponential equations. Another critical feature that is crucial to solving these examples is that learners need to be equipped with the understanding of the laws of exponents. Learners must be able to use/apply the exponential laws in relation to solving equations (especially linear) in order to develop their confidence in talking about the steps they undertake to solving any of these equations.

Critical features in class T1
• Converting whole number to a power. • When bases are the same on both sides of the equation, compare (equate) the exponents. • Use of the additive inverse. • Dividing both sides by the coefficient of the power containing the variable. • Use of laws of exponents • Any base raised to the exponent zero is equal to one. • Use of laws of exponents

Table 4. 1: Critical features in teacher Thembi's class T1

Given that there is a number of critical features from the chosen examples, it is important to note that at any given point, should it be that, for example, learners experience more than one critical feature, it would mean that the learners experience fusion (Baskoro, 2021).

5.1.2.2. Examples teacher Thembi chose in Class T2

Figure 4.2 below, shows the examples that were chosen by teacher Thembi in class T2.

Class	Examples chosen
T2	1. $3^{2x} = 9$ 2. $3^x = 1$ 3. $3.3^x = 27$ 4. $2^{x-1} = 8$ **Worked examples**
	1. $2^x = 4$ 2. $3^{x-1} = 9$ 3. $2.3^x = 18$ 4. $2^x = 1$ 5. $2^{3x-1} = 32$ 6. $8.2^x = 128$ 7. $3^{2(x-1)} = 81$ 8. $6^{x-2} = 1$ 9. $2.2^x = 16$ 10. $9.3^{x-1} = 81$ 11. $5.5^{x-5} = 5$ **Exercise examples**

Figure 4. 2: Example space for Teacher Thembi in class T2

5.1.2.2.1. The example space's object of learning

The intended object of learning was identified as solving exponential equations by the teacher. In this class, Teacher Thembi chooses four worked examples, one of which ($2^{x-1} = 8$) is not taken directly from the textbook, and a total of eleven exercise examples, of which, only one (example 9: $2.2^x = 16$) is not chosen from the textbook (chosen from another source, not clear which). Both the worked and exercise examples were chosen for three lessons.

5.1.2.2.2. Patterns of variation in the example space

Worked examples 1, 2 and 4 ($3^{2x} = 9,\ 3^x = 1, 3.3^x = 27,\ 2^{x-1} = 8$, respectively) are similar to exercise examples 1, 2, 4, 5, 7 and 8 ($2^x = 4,\ 3^{x-1} = 9, 2^x = 1,\ 2^{3x-1} = 32,\ 3^{2(x-1)} = 81,\ 6^{x-2} = 1$, respectively) in structure and the steps to follow in solving these in order to get the solution. All these examples require that, in solving, the first step is to convert the whole number on the right-hand side to a power such that the power this whole number is converted to contains the same base as in on the left-hand side. The application of this understanding is what is identified as one of the critical features that enhance and support the object of learning. Similarly, worked example 3 ($3.3^x = 27$) is similar to the exercise examples 3, 6, 9, 10 and 11 ($2.3^x = 18,\ 8.2^x = 128,\ 2.2^x = 16,\ 5.5^{x-5} = 5$, respectively) in their structure. The structure in these examples is such that the power on left side of the equation has a coefficient attached to it and there is a whole number (to be converted to a power) on the right side of the equation.

In total, five of the examples (worked: 3 ($3.3^x = 27$); exercise: 1, 3. 5, 6 & 8, $2^x = 4, 2.3^x = 18,\ 2^{3x-1} = 32, 8.2^x = 128,\ 6^{x-2} = 1$) give $x = 2$ as the solution, while four of them (exercise: 2, 7, 9 & 10) give $x = 3$ as the solution. In all the examples, the variable x is used as the unknown, placed on the left side of the equation, the left side comprises at least one power, while the right side has a whole number to be converted into a power prior to solving. Base 3 is a commonly used base in this class.

The kind of generality the teacher sort of aims is that learners ought to be able to convert whole numbers to powers depending on the variable-containing-base. Furthermore, one other generality that the teacher may have achieved, judging from the chosen examples (not necessarily on the use/effectiveness) is that the exponent that the base on the right side of the equation should be raised to is the number of times that the base has to be multiplied by itself.

5.1.2.2.3. Critical features of the example space

The critical features that are evident in the examples chosen by teacher Thembi in this class are listed in Table 4.2 below. One of the most important critical features that is necessary to understanding exponential equations is that which learners learned of previously (linear equations). This is with regards to the use of the additive inverse. Another important critical feature that is evident in the examples in the example space for this class is that in order to reach a point where a value of the variable is calculated correctly, bases on both sides of the equation must be the same. Other critical features in this example space include the discernment of comparing exponents once the bases are the same on both sides.

Critical features in class T2
• Use of the additive inverse. • Converting whole number to a power. • When bases are the same on both sides of the equation, compare (equate) the exponents. • Any base raised to the exponent zero is equal to one. • Dividing both sides by the coefficient of the power containing the variable. • Use of laws of exponents

Table 4. 2: Critical features in teacher Thembi's class T2

5.1.2.3. How teacher Thembi chooses examples differently in classes T1 and T2

As previously stated, both class T1 and class T2 are taught by Teacher Thembi. It is my contention that the two example spaces from these two classes are not exactly the same but have some similar features. Not all learners learn in the same way. One of the things that remain central to ensuring that learners' learning needs are catered for is through simplifying concepts and adapting the concepts in a manner that the teacher knows the learners would receive it better (Ball et al., 2008). Thus, it may not always be best to choose the same examples for different groups of learners, even so, perhaps, the same examples may be chosen and not necessarily sequenced in the same way for different groups. Below, in Excerpt 4.3, is teacher Thembi's response when asked why it is that she does not choose (and/or use) the same examples for both classes. The teacher's response supports the notion that catering for learners' needs is crucial and this can be easily done once the teacher 'knows' their learners.

60	**Researcher**	Okay, I see. And then, I noticed that in some of the questions you would use in some of your classes, they would differ, so is there any motive behind that?
61	**Thembi**	*Yah* (yes), there is. So, the 8F (Class T1), in the 8G class (Class T2), not G, in 8F (Class T1), we have more of the strong learners than in 8G (class B). So, in 8G (class B) I use examples that are at least easy*nyana (*a bit easy).
62	**Researcher**	Okay, okay. So that's 8F (Class T1)?
63	**Thembi**	8F (Class T1), I give them examples that are a little bit challenging,

Excerpt 4.3: Interview Transcript, Choosing different examples for the two classes

In classes T1 and T2, although both classes are taught by teacher Thembi, she chooses different examples and/or chooses the same examples, but sequences them differently in these two classes. Teacher Thembi chose three worked examples in class T1 and chose four worked examples for class T2, and for the exercise examples, the teacher chose eleven examples in both classes. For the worked examples, examples 2 and 3 ($3^x = 1,\ 3.3^x = 27$, respectively) are exactly the same and appear in the same order (2 then 3) in both classes.

The eleven exercise examples in each class have seven examples that are similar, however, these examples are not all sequenced in the same way in each class. Below is Table4.3 which shows the way in which examples are listed in class T1 in comparison to class T2. As indicated by teacher Thembi, she does this because she thinks that learners in class T1 are capable of dealing with examples that are a bit challenging as compared to those in class T2, hence the order. Exercise examples 1 and 2 ($2^x = 4, 3^{x-1} = 9$) are listed in the same way in both classes.

Examples chosen in class T1	Examples chosen in class T2
1. $5^{2x} = 25$ 2. $3^{x} = 1$ 3. $3.3^{x} = 27$ **Worked examples**	1. $3^{2x} = 9$ 2. $3^{x} = 1$ 3. $3.3^{x} = 27$ 4. $2^{x-1} = 8$ **Worked examples**
1. $2^{x} = 4$ 2. $3^{x-1} = 9$ 3. $2.2^{x} = 8$ 4. $2.3^{x} = 18$ 5. $2^{3x-1} = 32$ 6. $3^{2(x-1)} = 81$ 7. $2.8^{x} = 128$ 8. $3^{2x}.3^{x} = 9$ 9. $\frac{1}{3}.3^{2x} = 1$ 10. $2^{2x} + 8 = 9$ 11. $6^{x-2} = 1$ **Exercise examples**	1. $2^{x} = 4$ 2. $3^{x-1} = 9$ 3. $2.3^{x} = 18$ 4. $2^{x} = 1$ 5. $2^{3x-1} = 32$ 6. $8.2^{x} = 128$ 7. $3^{2(x-1)} = 81$ 8. $6^{x-2} = 1$ 9. $2.2^{x} = 16$ 10. $9.3^{x-1} = 81$ 11. $5.5^{x-5} = 5$ **Exercise examples**

Table 4. 3: Comparison of the sequence of examples in classes T1 and T2

5.1.2.3.1. Reflections on teacher Thembi's choice of examples

Although the teacher stated that learners in class T1 are more capable than those in class T2, it is not clear why and how it is that the teacher came to choose example eleven ($6^{x-2} = 1$). as the last example in class T1. One would expect that for such capable learners, given that example eleven is similar (in structure) to examples that may be considered easier to solve as compared to examples such as example 8 ($3^{2x}.3^{x} = 9$) (which one would expect that learners would find it challenging to solve) and that most of the examples structured similarly to example eleven are generally at the top of the list, example eleven should have been listed earlier. Thus, in Table 4.4 and Table 4.5 below, I present example spaces for both classes should it have been that the examples in the example spaces were variation theory (VT) informed.

Ideally, the lessons would start off with examples considered to be 'easier' to solve, followed by examples considered to be more cognitively demanding; thus, likely to introduce an increase in the level of difficulty as one goes down the list. The VT informed example space does not introduce any new examples, it only re-orders the examples that teacher Thembi chose for both her classes. I make use of the abbreviations W (for worked example) and E (for exercise example) to indicate what kind of example each one was chosen as, by teacher Thembi, in the VT informed column.

Teacher Thembi's chosen examples	**VT informed example space**
1. $5^{2x} = 25$ 2. $3^x = 1$ 3. $3.3^x = 27$ **Worked examples**	1. $2^x = 4$ (e) 2. $3^x = 1$ (w) 3. $3^{x-1} = 9$ (e) 4. $3.3^x = 27$ (w) **Worked examples**
1. $2^x = 4$ 2. $3^{x-1} = 9$ 3. $2.2^x = 8$ 4. $2.3^x = 18$ 5. $2^{3x-1} = 32$ 6. $3^{2(x-1)} = 81$ 7. $2.8^x = 128$ 8. $3^{2x}.3^x = 9$ 9. $\frac{1}{3}.3^{2x} = 1$ 10. $2^{2x} + 8 = 9$ 11. $6^{x-2} = 1$ **Exercise examples**	1. $5^{2x} = 25$ (w) 2. $2^{2x} + 8 = 9$ (e) 3. $6^{x-2} = 1$ (e) 4. $2^{3x-1} = 32$ (e) 5. $3^{2(x-1)} = 81$(e) 6. $2.2^x = 8$ (e) 7. $2.3^x = 18$ (e) 8. $2.8^x = 128$ (e) 9. $\frac{1}{3}.3^{2x} = 1$ (e) 10. $3^{2x}.3^x = 9$ (e) **Exercise examples**

Table 4. 4: VT informed example space for class T1

Teacher Thembi's chosen examples	**VT informed example space**
1. $3^{2x} = 9$ 2. $3^x = 1$ 3. $3.3^x = 27$ 4. $2^{x-1} = 8$ **Worked examples**	1. $2^x = 4$ (e) 2. $3^x = 1$ (w) 3. $3^{x-1} = 9$ (e) 4. $3.3^x = 27$ (w) **Worked examples**
12. $2^x = 4$ 13. $3^{x-1} = 9$	1. $3^{2x} = 9$ (w) 2. $2^x = 1$ (e)

14. $2.3^x = 18$ 15. $2^x = 1$ 16. $2^{3x-1} = 32$ 17. $8.2^x = 128$ 18. $3^{2(x-1)} = 81$ 19. $6^{x-2} = 1$ 20. $2.2^x = 16$ 21. $9.3^{x-1} = 81$ 22. $5.5^{x-5} = 5$	3. $2^{x-1} = 8$ (w) 4. $6^{x-2} = 1$ (e) 5. $2^{3x-1} = 32$ (e) 6. $3^{2(x-1)} = 81$ (e) 7. $2.2^x = 16$ (e) 8. $2.3^x = 18$ (e) 9. $9.3^{x-1} = 81$ (e) 10. $5.5^{x-5} = 5$ (e) 11. $8.2^x = 128$ (e)
Exercise examples	**Exercise examples**

Table 4. 5: VT informed example space for class T2

Unlike the traditional methods of teaching, variation theory aims to enhance understanding of mathematical concepts and advance learners' learning experiences through ensuring that the object of learning aligns with the critical aspects that arise from the use of the examples chosen by the teacher. Further, there is no linearity to be assumed in ensure such alignment, the two are meant to inform one another (Holmqvist & Mattisson, 2008).

The main objective of using variation theory to teach is to allow teachers to be able to identify the critical aspects of what they teach and by so doing, improving the way in which the teachers teach effectively through prioritising what it is that learners truly learn (Holmqvist & Mattisson, 2008). Thus, leading to expanded learning outcomes and improved understanding of concepts for the learners.

In my opinion, listing the examples in both classes (as though they are VT informed) is necessary because from teacher Thembi's example spaces, it is not easy to see structure in terms of how it is that the example that is chosen before the next leads to a 'build-up' or accumulation of critical features that lead to different forms of generality, as one continues down each example space.

Moreover, at times, in order to decide on the appropriate examples to choose in order to enhance learning, depends on the teacher's reflections upon teaching one or more lessons covering a concept such as exponential equations, to become aware and understand how best they should choose the examples in a manner that learners would be able to understand better and order the examples in a way that optimises the discernment of the object of learning and its critical features.

More and more example spaces lack structure, and this is considered to be popular in most classrooms and many teachers do not find it to be an easy task and do not undermine the power that the structure and order of the examples in an example space may have in terms of the mathematics that their learners attain (Arzarello et al., 2011).

In this study, teacher Thembi seems to have her examples as, what Arzarello et al. (2011) refer to as a ‘collection of examples’, since the set of examples used lack structure, rather than ‘example spaces’. The teacher simply has examples chosen based on the topic that they taught and not necessarily about how it is that these examples should be structured and/or sequenced.

Given the presented example spaces used by teacher Thembi, there is no evidence of the use of counterexamples (or non-examples). Counterexamples allow learners to think about generalisations and how it is that these may allow the learners to understand what is true, and what it not in attaining their mathematical understanding of a certain concept. Further, counter-examples may be a way of enhancing the learners’ critical thinking. Thus, it becomes a difficult task in this instance, to assume many other generalisations that the lessons led to, or those that the teacher may have intended for her teaching to lead to (Arzarello et al., 2011). An example of a counterexample that the teacher could have use is one such as $3^x = 6$. With this kind of example, one can expect that learners may want to change the whole number, 6 to a power with base 3 and through this then they would hopefully realise that there is no way that when using base 3 can anyone figure out an exponent that base 3 should be raised to in order to get 6.

Additionally, one way that the teacher could have developed a generality about equating/comparing exponents once the bases are the same, is through the use of an example that has the exponents equal on both sides and the bases not necessarily the same in order to help learners when and when not to apply the ‘equating’ of exponents. Such an example can be $2^x = 3^x$. This will most likely lead learners to wonder with questions around 'if the exponents are the same, can I equate the bases?'. In what follows, I provide a discussion of teacher Mercy’s choice of examples.

5.1.3. Teacher Mercy’s choice of examples

Teacher Mercy taught algebraic equations as a broad concept that includes both linear and exponential equations. The examples she chooses are a mixture of linear and exponential equations and this can be seen in her example space which I shall present shortly.

As previously mentioned, teacher Mercy teaches both linear and exponential equations in the same lesson(s) and teaches only one Grade 8 class. Teacher Mercy chose examples that are in Figure 4.3 below which the teacher chose for three consecutive lessons.

Class	Examples chosen
M1	1. $2^x = 4$ 2. $3^x = 27$ 3. $4x + 10 = 38$ 4. $3(x + 4) = 9$ 5. $5x + 5 = 3x + 15$ 6. $\frac{x}{2} = 5$ 7. $5^x = 25$ 8. $9^x = 27$ 9. $3(1 + 2x) = 8 + x$ **Exercise examples**

Figure 4. 3: Example space for Teacher Mercy in class M1

In the three consecutive lessons, teacher Mercy chose a total of nine examples. Five of these examples ($4x + 10 = 38,\ 3(x + 4) = 9,\ 5x + 5 = 3x + 15,\ \frac{x}{2} = 5$ and $3(1 + 2x) = 8 + x$) are linear, and four are exponential ($2^x = 4,\ 3^x = 27,\ 5^x = 25,\ 9^x = 27$).

5.1.3.1. The example space's object of learning: Linear and exponential equations

As previously stated in Chapter 3, the teacher had already started with linear equations prior to data collection for this study, thus, I was unable to capture the introduction of linear equations. The intended object of learning for this example space, as stated by the teacher, is solving algebraic equations. I shall delve into the details of the other constructs of the object of learning (enacted and lived) later in the chapter when I provide a discussion of the use of the examples. The analysis I provide below is in a manner that I analyse linear equations separately from exponential equations and I start with the analysis of linear equations first.

5.1.3.2.Analysis of the Linear equations

5.1.3.2.1. Patterns of variation in the example space

Three ($4x + 10 = 38,\ 3(x + 4) = 9,\ 5x + 5 = 3x + 15$) of the four linear equations chosen by teacher Mercy are generally similar in that they all require the understanding of transposing terms (either to the left or right side or both), besides the fact that they are all linear and make use of the variable x. For examples 3, 4 and 6 ($4x + 10 = 38,\ 3(x + 4) = 9,\ \frac{x}{2} = 5$) the variable is placed on the left side of the equation, whereas for example 5 ($5x + 5 = 3x + 15$), the variable appears on both sides of the equation.

One thing that is not similar amongst all four of the linear equations is that, when solved, none of them give the same value of x. Additionally, example 4 ($3(x + 4) = 9$) is the only example that requires the use of the distributive law in order to get it to the form that examples 3 and 5 ($4x + 10 = 38\ and\ 5x + 5 = 3x + 15$) are already in. example 5 differs from the rest of the linear equations in that it is the only one that contains the variable (x) on both sides of the equation. Example 6 is distinguished from the rest in that it is the only example that requires and develops the understanding of the critical feature of cross multiplication.

5.1.3.2.2. Critical features of the example space

One of the critical features of the linear equations is generally the use of the additive inverse (as already indicated) and one other critical feature from these linear equations is one that which may be expected to remind and enhance learners' understanding of grouping like term. This is one critical feature that if discerned with the use of the additive inverse may be expected to lead to fusion. The critical features expected to be discerned from the chosen linear equations can be seen in Table 4.6 below. With this example space, although it sorts of lacks structure, the kind of generality that the teacher might have aimed to achieve is that which is also a critical feature: the use of the additive inverse. Next, I provide an analysis of the exponential equations from the same example space as the linear equations.

Linear equations: Critical features for class M1
• Use of the additive inverse. • Grouping of like terms. • Cross multiplication. • Use of distributive law.

Table 4. 6: Linear equations, Critical features in teacher Mercy's class M1

5.1.3.3. Analysis of the Exponential equations

Below, I provide an analysis of the exponential equations in the example space.

5.1.3.3.1. Patterns of variation in the example space

What is interesting with teacher Mercy is that she does not solve (work out) any examples with and/or with the learners; instead, when the teacher chooses the exponential equations, she reminds learners that they have covered exponential laws and that learners must apply their understanding of the exponential laws, to now solve exponential equations. With regards to the exponential equations' teacher's choice of examples, they are all the same in structure; in that they all have the variable x which is placed on the left side of the equation. The four exponential equations are also similar because they all contain a power on the left side of the equation, and a whole number (to be converted to a power in order to solve) on the right side of the equation. One other similarity amongst the exponential equations is that two of them, examples 1 and 7 ($2^x = 4\ and\ 5^x = 25$), when solved, give $x = 2$ as the value of x.

One of the things that is dissimilar in this example space of exponential equations is that all of them make use of a different base for each variable-containing base. The bases include bases 2, 3, 5 and 9.

5.1.3.3.2. Critical features of the example space

In identifying the similarities and contrasts of an example space, one can then be able to use these to identify the critical features. One crucial critical feature to finding the value of the variable is converting the whole number on the right side of the equation to a power. Another critical feature is ensuring that bases on either side of the equation is the same in order to compare (equate) the exponents, and this can also be regarded a form of generality. Thus, in solving each equation, learners are expected to engage, experience and discern both critical features, hence, fusion. The critical features of the exponential equations 'example space' can be seen in Table 4.8, below.

Exponential equations: Critical features for class M1
• Converting a whole number to a power. • When bases are the same on both sides of the equation, compare (equate) the exponents.

Table 4. 7: Exponential equations, Critical features in teacher Mercy's class M1

5.1.3.4. Reflections on teacher Mercy's choice of examples

Teacher Mercy did not make use of any counterexamples (non-examples) and this in a way leaves much to desire when it comes to the many more generalities that this entire example space could have led to. Furthermore, the teacher's choice of examples is acceptable, however, in order to ensure that the learners are allowed to discern the critical features fully, I would suggest a variation theory (VT) informed example space which focuses on one object of learning at a time. This can be done through the separation of the two types of equations into linear equations, and then follow with exponential equations: instead of a broad object of learning such as "algebraic equations". By separating the two types of equations, the implication is also then that these are to be the chosen examples for at least two separate lessons. I present the VT informed type of example space below in Tables 4.8 and 4.9. In the tables, there are no new examples added, I only sequence the examples from teacher Mercy's example space, for the examples she chose for linear equations. I do not sequence the examples chosen for exponential equations because these, in my opinion are already sequenced in a way that agrees with a VT informed example space. I would, however, choose example 3 ($4x + 10 = 38$) as a worked example (for linear equations) and choose example 1 ($2^x = 4$) as a worked example.

The sequencing is done based on starting with less difficult to more difficult questions as suggested by Kassa and Ding (2019) that it is better to have an example space that is sequenced in terms of the cognitive demands of each example, without abandoning the aim the object of learning aims to achieve.

<table>
<tr><th>Teacher Mercy's chosen examples: Linear equations</th><th>VT informed example space</th></tr>
<tr><td rowspan="2">1. $4x + 10 = 38$
2. $3(x + 4) = 9$
3. $5x + 5 = 3x + 15$
4. $\frac{x}{2} = 5$

Exercise examples</td><td>1. $4x + 10 = 38$

Worked examples</td></tr>
<tr><td>1. $\frac{x}{2} = 5$
2. $5x + 5 = 3x + 15$
3. $3(x + 4) = 9$
Exercise examples</td></tr>
</table>

Table 4. 8: Linear equations, VT informed example space for class M1

<table>
<tr><th>Teacher Mercy’s chosen examples:
Exponential equations</th><th>VT informed example space</th></tr>
<tr><td rowspan="2">1. $2^x = 4$
2. $3^x = 27$
3. $5^x = 25$
4. $9^x = 27$

Exercise examples</td><td>1. $2^x = 4$
Worked examples</td></tr>
<tr><td>1. $3^x = 27$
2. $5^x = 25$
3. $9^x = 27$
Exercise examples</td></tr>
</table>

Table 4. 9: Linear equations, VT informed example space for class M1

5.2. Part 2: Teachers’ use of examples

So far, the analysis I have provided focused on the teachers’ choice of examples by using variation theory as a lens (for understanding teachers’ choice and use of examples). In this part of the analysis, I turn to using the second dimension of the Dyadic framework that deals with meaning making as a dialogic process to explore the ways in which the teachers use the examples they initially chose. The analysis of the use of examples by each teacher also draws on the two aspects of the object of learning, namely, the enacted and lived object of learning.

As previously indicated, in Part 1, teacher Thembi and teacher Mercy choose examples differently. Each teacher’s talk in mediating the examples further suggests that, despite teaching in the same school, teacher Thembi code switches (to Sesotho and IsiXhosa) during the enactment of the example spaces, whereas teacher Mercy, on the other hand, teaches strictly in the LoLT (English). In order to understand how meaning making is made possible in both teachers’ classes, through the enactment of the examples, I draw on interview excerpts and lesson transcript excerpts to bear on the findings and arguments I make. The excerpts I present are strategically chosen based on what generally (not only one case) happens in all the lessons, and not chosen based on only what is suitable to respond to the arguments I make. First, I present the analysis of the way that teacher Thembi uses talk to mediate the chosen examples, and then follow with the analysis of the teacher talk to mediate the examples in teacher Mercy’s class.

5.2.1. How teacher Thembi used talk to mediate the chosen examples in classes T1 and T2

In dealing with how teacher Thembi mediated the examples she chose, I will first talk about her use of code-switching practice, then follow with the communicative approaches and teacher interventions that are evident in both classes. Code-switching was a prominent practice teacher Thembi used and it is one of the practices that sets her apart from the second teacher.

5.2.1.1.Code switching practice

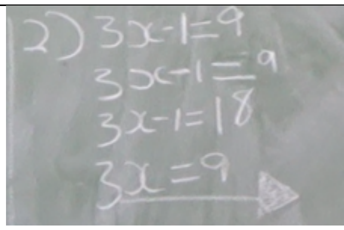
Given that mathematics is communicated and taught through the use of language (Wessel, 2020), it is important that teachers consider and be cautious of how they use the language(s) that allows them to enhance learners' understanding of mathematical concepts. This is especially imperative in multilingual contexts (Smit & van Eerde, 2011) such as the one teacher Thembi teaches in. Although many of the teachers in South Africa are more likely to teach in a multilingual context (Setati & Adler, 2000), teacher training does not make clear how pre-service teachers (PSTs) are better prepared to teach in such a context (Essien, 2021b), beyond their own experience(s) as a learner. Thus, some teachers, such as teacher Thembi would make use of the other languages (usually the learners' HL) available for use, in the classroom, to 'teach' mathematics.

Although teacher Thembi code-switches in both her classes, at times it was for meaning making, and other times, for control. Below, I provide two excerpts (Excerpts 4.4. and 4.5) wherein (as examples) teacher Thembi used code-switching for meaning making.

Thembi	Now, *re raiser (*we raise) five to exponent what, to get twenty-five? Five...? It's five multiplied by five, so it's five exponent what?

Excerpt 4.4: Teacher Thembi's use of code-switching for meaning making

In Excerpt 4.4 above, the teacher used the practice of code-switching in order to ask learners about the exponent that five should be raised to in order to get twenty-five, and Excerpt 4.5 below, is another example of how the teacher has used code-switching for meaning making through inviting other learners' perspectives by asking the learners "what is happening here?" and later asking for other learners to make their contribution by asking if there is anyone that has a different answer.

		
46	**Thembi**	Number two, ehhhhh (*shocked). Ho etsahalang mona* (What is happening here)? (*Teacher refers to the learner's solution, above)*
50	**Thembi**	…Okay, *ke mang anang le* (Who has a) different answer?

Excerpt 4.5: Teacher Thembi's use of code-switching for meaning making

In excerpt 4.6 below, I show how it is that the teacher uses code-switching for control as the learners were not responding to what the teacher asked and some were chatting to one another.

Thembi	We divide?! (*Raises voice). Ye* (what)? *Ay, lona ketlo le pota nou (*I am going to punish you now).

Excerpt 4.6: Teacher Thembi's use of code-switching for class control

The practice of code switching becomes more effective when and depends on the intended and lived object of learning aligning with both the teacher's and learners' proficiency in the LoLT, and any of the languages used to attain mathematical concepts (Marton & Pang, 2013**)**. Below is an excerpt of the interview audio transcript with teacher Thembi which alludes to how she considers her learners' proficiency in the LoLT when she teaches.

56	**Thembi**	…I don't know if you've noticed that when I teach, I always switch between the languages, which is English and Sesotho. I do give them the instruction in English, then try to translate it in their mother tongue

Excerpt 4.7: Interview Transcript, How Teacher Thembi considers the learners' proficiency in the LoLT.

In response to the question around code switching and the use of learners' HLs as a resource, this is what the teacher said (see excerpt 4.7 above). The teacher indicated that she code-switches in order to help learners understand instructions.

5.2.1.2. Communicative approaches

In both her classes, teacher Thembi started off with an introduction to exponential equations by explaining what exponential equations are, thereafter she used worked exponential equations' examples to elaborate further on her explanations. At a later stage, learners engaged in exercise examples, which learners volunteered (through the teacher) to do corrections for. Teacher Thembi taught exponential equations individually, in both classes. The teacher mediated the example spaces in both classes through engaging learners in different forms of communicative approaches. Communicative approaches refer to how it is that the teacher assists learners to extend their understanding of concepts (Mortimer & Scott, 2003).

At different points of the lessons, teacher Thembi taught without inviting conversations from (and by) the learners (authoritative and non-interactive), invited conversations with the learners, acknowledged and explored more than one viewpoint (dialogic), engaged in classroom talk/discussions (or not) (interactive). None of the communicative approaches outweighs or is better than the other. The decision for each category of the communicative approaches as being appropriate, lies with the teacher's view (at times, even the learners') of what they consider to be efficient and necessary for each part of the lesson and the direction that the lesson assumes during instruction.

Teacher Thembi's first lessons are generally non-interactive, at different points such as the introductory part of each lesson. The teacher starts off by writing down the definition of exponential equations and proceeds to write down the examples (on the board). At this point, as the teacher explains, she poses questions that she, herself answers- as can be seen in excerpt 4.6 below.

	Utterances
Thembi	Okay, please put your pens down for a moment, let me explain this. Alright, today's story, solving equations involving exponents. Alright, we are still solving for x, we are still solving for the unknown. But today, we have a story se (which is) a little bit different. As you can see here, the examples we have, number 1, five exponent two x is equal to twenty-five. The unknown is not in the base, *akere* (right)? It's on the what? (*learners remain silent)* It's on the exponent. So, now, how do we solve exponential equations? (*learners remain silent)* Step number 1 (*teacher moves towards the board),* you check your left-hand side, on the left-hand side

	we have a power, *akere (*right)*? (learners remain silent)* On the right-hand side we have a whole number, it's not a power. O*n* the left-hand side we have five exponent two x your left-hand side you have a what? So, what you are going to do is, you convert the right-hand side into a power, with a base what? With a base five, because on the left-hand side, the base is what? The base is base five we have a base five. So, when you convert this *(points to twenty-five)*. You have a power, *akere* (right) (*no response from learners)*? So, what we are going to do is, convert the right-hand side into a what? (*learners remain silent)* Into a power, because on the left-hand side, the base is a what? Is five. So, when you convert this, it must do what? It must be the same as the left -hand side. *Iyavakala (*Is it clear)*?* (*learners remain silent)* Okay, first step, we have five exponent two x is equal to, now we convert this (*points to twenty-five)* to what? (*learners remain silent, teacher responds)* A power, *akere (right)?* With the base what? (*learners remain silent)* With the base five. The base must be the same as the one in the left-hand side. *Iyavakala* (is it clear)?

Excerpt 4.8: Lesson Transcript, Evidence of non-interactive/authoritative nature of teacher Thembi's lesson

In this part of the lesson, since there is no evidence of the learners' participation, it is considered to be non-interactive/authoritative. Only one point of view was explored by the teacher. The teacher, during this time, could have used the examples to remind learners of the components of a power as part of the introductory narrative like I have done in Figure 4.3. below.

Given: $\boldsymbol{a^x}$

$$a: base$$

$$x: exponent$$

$$a^x: power$$

Figure 4. 4.: Identification of the components of an exponential expression

Learners had learned this already, through the laws of exponents. This would have allowed for the lesson to be interactive/dialogic and allowed the teacher to engage the learners in a way that tests their prior knowledge instead of the teacher posing questions, that she, herself answers without giving learners an opportunity to think about the questions she asked and how to answer them. As argued by Tsamir and Almog (2001) most of the concepts in mathematics are related, thus an earlier learned concept usually serves as a gateway into learning a new concept (Rittle-Johnson et al., 2009). In this case, in the previous term, learners learned exponential laws. At another point in the lesson, teacher Thembi attempts to engage learners by testing their prior knowledge about exponential laws. This is one way that the teacher used to check learners' prior knowledge.

Generally, the expectation is that learners would be familiar with exponential laws and that they would be able to respond correctly to questions related to exponential laws. When the teacher tests the learners' prior knowledge, the learners respond with an incorrect answer (see excerpt 4.7; lines 27-29). At this point, one can think of the ways in which the learners' prior knowledge could not be fully used effectively as a resource. In excerpt 4.7 below, the teacher attempts to remind learners of the ways they are expected to use in applying exponential laws and how it is that they are meant to determine the exponent, a base is to be raised to, by multiplying a base by itself and count the number of times they had to use same base. As a starting point, the teacher reminds learners that in order to identify the exponent to which the base must be raised to, the base should be multiplied by itself 'n' number of times. Learners seem to not understand this well, as they continue to give an incorrect answer, until the teacher tells them what they are supposed to do and what the correct answer is. The teacher shares a 'method' with the learners as they solve the question $5^{2x} = 25$ together.

17	**Thembi**	Now, *re raiser (*we raise) five to exponent what, to get twenty-five? Five...? It's five multiplied by five, so it's five exponent what?
18	**L**	Exponent five
19	**Thembi**	Five? (*astonished)* Five exponent five? *(chuckles)*. Five exponent five?
20	**Ls**	*(inaudible)*
21	**Thembi**	Okay, now, I'm *gonna* give you the easy way to determine the exponent, *neh (*right)? So, how do we determine the exponent? (*learners remain silent)* We start with what? (*learners remain silent, teacher responds)* We start with the smallest number. Let us take zero. Let us raise five to exponent zero and see if we are going to get twenty-five. What is five exponent zero?
22	**L**	It's zero
23	**Thembi**	It's zero *akere* (right)? Not twenty-five. Therefore, let us go to the next number. Let us raise five to the exponent one to see if we get twenty-five. What's five exponent one?
24	**Ls**	Five
25	**Thembi**	Five, but we are looking for twenty-five. We go to the next number, *akere (*right)?
26	**Ls**	Yes ma'am
27	**Thembi**	Let us raise five to exponent two. Five exponent two is…?
28	**Ls**	Ten
29	**Thembi**	Ten?! (*raises voice, astonished)*
30	**Ls**	Twenty-five
31	**Thembi**	*(laughs)* What does five exponent two mean?
32	**L**	Five multiplied by two
33	**Thembi**	It's five multiplied by five, not five multiplied by two, *haw! Huh-uh (*No). It's five multiplied by five, gives us what?
34	**Thembi**	Twenty…
35	**Ls & Thembi**	…five
37	**Thembi**	Therefore, the exponent for five is what?
38	**Ls & Thembi**	It's two

Excerpt 4.9: Lesson Transcript, Evidence of interactive/dialogic nature of teacher Thembi's lesson

One of the interesting engagements that the teacher had with the learners in this class was when the teacher realised that the learners were struggling with determining the value of the exponent (when converting the whole number). Upon this realisation, teacher Thembi indicated to the learners that she was going to share a method of determining the exponent. With the method that the teacher shared, one would expect that learners would be able to determine exponents and multiples. Interestingly, despite the teacher's efforts, some learners were still responding with incorrect answers (see lines 27-28, for example). Initially, the learners were unable to determine the exponent that five should be raised to in order to get twenty-five. The learners responded with an incorrect answer, five (see line 18). This shows that learners did not understand what an exponent is and how to determine it. It is evident in this interaction that teacher Thembi's method, was only effective to a point because learners continued to answer incorrectly. This only shows the understanding of those that chose to respond when the teacher posed the question and not necessarily true that the teacher's method did not work at all. Eventually, some learners were able to answer correctly, with the teacher's help.

Perhaps, one thing that could have helped in order to ensure that learners attain the mathematics around/about the object of learning that the teacher aimed to convey, the teachers could have revisited the learners' prior knowledge regard to exponential laws before they expect learners to immediately translate their understanding of exponential laws into using exponential laws in solving exponential equations. This aspect refers to the importance of awareness of what learners know and understand already, in relation to the concept to be taught and/or covered next (Holmqvist & Mattisson, 2008).

The dialogic/interactive approach in the excerpt above is, however, superficial since in most instances, the teacher's tone is suggestive of the answer that the learners are expected to give. If not suggestive, then the teacher answers the beginning part, and expects learners to finish off. As an example, see lines 34-35. The teacher says 'twenty…' and learners say '…five' once they have 'read' the teacher's lips. These types of contributions cannot be considered mathematically meaningful and do not enhance understanding of mathematical concepts (Essien, 2017). Besides the 'yes ma'am' responses that the teacher 'expects' from learners, the suggestive tone is more prevalent in all the lessons by teacher Thembi. At times, learners' contributions in teacher Thembi's class would simply be a response to 'what is two plus one' type of questions that the teacher asks, and these types of questions do not give a sense of what learners truly understand.

Meaningful participation (and contribution) in a mathematics class is not just an ‘at least I said something’ exercise, it is rather a contribution that reveals understanding of a mathematical concept, in which ‘talk’ means the ability to use correct mathematical language to say why it is that a certain procedure works, for example.

With the interaction that the teacher had with the learners, it is inevitable that learners are giving choral answers, and it would be difficult for anyone, including the teacher to ‘measure’ the effectiveness of the method she used during this interaction because it is not clear who is responding and who is not responding. Moreover, choral answers can be misleading. Thus, it is necessary that for any interaction to be fruitful, one of the things that the teacher may want to consider is holding learners accountable for their responses by creating an environment wherein questions are directed at individual learners rather than them being directed to the whole class (Bauersfeld, 1988). Furthermore, the benefits of probing individual learners’ understanding are far reaching in terms of developing learners’ ability to ‘talk’ about mathematics meaningfully (Mortimer & Scott, 2003) and such meaningful discussions and interactions ensure that more and more learners reap the benefits of the discussions.

Additionally, classroom talk is central to attaining mathematical concepts, especially in modern classrooms wherein even curriculum specialists do not support teaching that aims to only feed and/or transmit knowledge in a manner that does not develop learners to think inquisitively about the concepts they learn (Essien, 2017).

Teacher Thembi acknowledges the mathematics to be the authority (Essien, 2017), through herself legitimising and evaluating learners’ contributions, especially when she marks the work that learners write on the board as corrections. The teacher indicated mark allocations as she corrected the work the learners wrote on the board and this is one way that the teacher uses to indicate to learners that it is not only the final answer that is important when solving equations, but the steps that lead to the final answer are just as important.

As the learners engage with the exercise examples, teacher Thembi encourages them to work together, with the person they are seated next to, if there is anything they do not understand (see excerpt 4.9 below). This is one way that the teacher used to promote conversations amongst learners, with the hope that the learners are able to help one another understand mathematical concepts without entirely relying on the teacher.

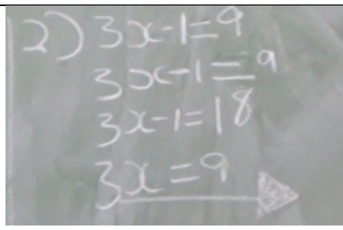
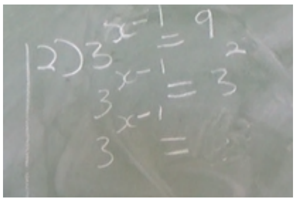
5.2.1.3. Patterns of discourse

As the teacher interacted with the learners, teacher Thembi created a one-to-many dialogue. This is due to the nature of the choral responses she received and accepted, as discussed above. Many of teacher Thembi's lessons, in both classes are dialogic in that she, at times, posed questions, allowed learners to think about the question and how they should respond and acknowledged the learners' responses and explored the learners' ideas.

In most cases, the teacher is the one that initiates the mathematical discussions in the classroom. The discussions are generally sustained by both parties (teacher and learners). Thus, the I(initiate)-R(response)-E(evaluation) pattern of discourse is more prevalent in teacher Thembi's class(es) because learners would not be asked to justify their responses and the teacher too, does not provide justifications for her contributions (Essien, 2017), such as when she simply corrects or ignores the learners' incorrect answers. Furthermore, the teacher is the one who evaluated the learners' responses without stating why it is that 'her' method works. The teacher's evaluation, in most instances comes across as a form of an unopposed authoritative guidance (Essien, 2017). If anything, the teacher is dismissive of incorrect answers, as opposed to inviting meaningful conversations about what it is that learners do not understand through the incorrect answers that the learners gave.

5.2.1.4. Alignment of the intended object of learning with the lived object of learning

In one instance, the teacher gave learners an exponential equation, $3^{x-1} = 9$ and one learner, when he was writing corrections, wrote this equation as a linear equation (see Excerpt 4.9 below). First, this shows that this particular learner has not yet developed the understanding of the difference between linear and exponential equations. Additionally, the learner's lived object of learning is not aligned with the teacher's initial intended object of learning. What is interesting is that for the intended and lived object of learning to align, the enactment is expected to bridge the gap between the two. Should it be that the bridge does not connect the two, then the learner suffers in terms of not understanding concepts, especially those related to the concept they were unable to discern. Of course, this is just one instance of the intended object of learning not being in alignment with the lived object of learning, but it is one challenge that many learners may be facing, and as suggested by Engle and Conant (2002); Kullberg et al. (2017), many learners are faced with the same challenge. Thus, it is important that teachers become aware, intentional and cautious of the way in which they deliver their teaching to learners in order to ensure that the intended object of learning ‘matches’/aligns with the lived (Kullberg et al., 2016).

		
	Thembi	Number two, ehhhhh (*shocked). Ho etsahalang mona* (What is happening here)? (*Teacher refers to the learner’s solution, above)*
	Ls	(*inaudible)*
	Thembi	Okay, number two, do we all have three x equal to nine?
	Ls	No
	Thembi	Okay, *ke mang anang le a different answer?* (Who has a different answer)?
	L	x is equal to two
	Thembi	*Eya, tloyo* (Yes, come) *Tloo, o refe (*Come give us) $x = 2$. (*Learner walks up to the board)*
	L	*(second learner writes on the board, attempting the same question)* 
	Thembi	Okay, from here, we have? x?
	Ls & Thembi	x minus one equals to two (*Teacher writes this down, continuing from the second learner’s solution)*

Excerpt 4.9: Lesson Transcript, Evidence of no alignment between intended and lived OL of teacher Thembi’s lesson.

For the second learner, he was able to rewrite the exponential equation correctly and converted the whole number, 9, to a power but could not proceed thereafter. The teacher then intervened and wrote the correct answer. Again, this is what Gorgorio and Planas (2001) would regard a missed opportunity to learn because the teacher could have, instead, probed each of the two learners’ thinking.

The disadvantage is that the same learner who converted the exponential equation to a linear equation might not know why or even notice the difference on his/her own; of which the learner most probably copied down the teacher's solution as expected by the teacher and as fulfilment of the part of the lesson when and where learners write corrections. It would have been beneficial for the teacher to lead the discussion by asking the learner (or even the whole class, at one point) questions that would probe their understanding in a way that gets the learners to talk about their thinking process. The whole point of engaging in conversation with the learner would have been to get to understand why the learner solved the equation in the way that they did. Some of the questions that the teacher could have used in leading the discussion include, but are not limited to: refer to the given equation, is it the same as the one that was initially given? What type of equation were you given and what type of equation have you written on the board? The way in which the 'interaction' unfolded in class, has in this instance, posed limits in the way learners talk to and about mathematics.

Additionally, the teacher could have used this as an opportunity to contrast exponential equations with other types of equations such as quadratic equations, in order to help learners identify the differences between types of equations learned already, or still to be learned, whether in the same grade or in the same grade. This is one way that the teacher could have used to enrich classroom discussions. For example, how is $3^x = 9$ different from $x^3 = 9$?

As already indicated, it is important that the teacher ensures that the learners' lived object of learning aligns with the intended object of learning. This can be do through encouraging talk through the provision of rich learning situations that require/allow learners to use language (Setati & Adler, 2000). Furthermore, the teacher may want to ask questions that invite meaningful mathematical conversations; rather than posing questions that require one-word answers or does not aim to test or contribute to the lessons' objectives. I now turn to a discussion of how teacher Mercy enacted and mediated the examples in her class.

5.2.2. How teacher Mercy used talk to mediate the examples in class M1

Teacher Mercy as previously indicated, taught both linear and exponential equations in the same lesson(s). The teacher enacted the examples through the use of the LoLT (English) throughout the lessons. At no point does the teacher code switch during instruction, just as she indicated during the interview. The teacher does not code-switch because according to her, because English (LoLT) is not the learners' HL, it is important that she gets them used to the LoLT so that they are better prepared for assessments as well, for example. Teacher Mercy further indicated that the only time that the learners are really exposed to English is when they are in class. See Excerpt 4.10 below.

37	**Researcher**	Okay, and then is your learners' proficiency in the language of learning and teaching, which would be English, I would assume that you teach in English?
38	**Mercy**	Actually, I teach only in English, it's only that with those learners because they are the Grade 8s from different schools whereby some of the teachers they teach the content in Sesotho. That is a challenge to me, I don't want to lie. It becomes a challenge for the learner to read the instructions because if the learner gets the wrong instruction, it tells you something, that learner is not going to pass…*(inaudible).* So that is the barrier of language that I experience in the class.

Excerpt 4.10.: Interview Transcript, Language that teacher Mercy teaches in

What the teacher indicated in her response is what Barwell (2020) alludes to as the notion that learners in multilingual classroom are faced with a number of linguistic challenges which include the learners having to learn the LoLT, and 'translate' their understanding of the LoLT into learning mathematics as a subject.

Thus, language becomes a barrier that learners in multilingual classrooms stumble upon Schleppegrell (2007) and teachers find this to be their reality in their classroom, which they may not know how to overcome. This is one of the issues that teacher Mercy raised, as can be seen in excerpt 4.10 below.

39	**Researcher**	So would you say that the language of learning and teaching is a barrier?
40	**Mercy**	Is a barrier, of course yes
41	**Researcher**	So would you consider that when you choose the examples?
42	**Mercy**	…there is nothing that I can do because basically you know *mos,* when the questions come, they come in English. What I must do, is to explain and then, get them used to it because the moment they can give it to me in the other language, I will be killing them, definitely I will be killing them. So, they will come slowly, but

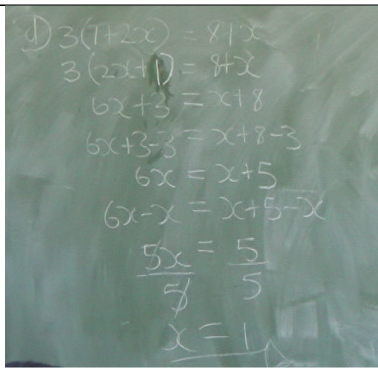
		then slowly, you have even seen, even confidence wise language is not there but then I try to do it a certain way

Excerpt 4.11.: Interview Transcript, LoLT as a barrier

5.2.2.1. Communicative Approaches

One of the most commonly used communicative approaches in teacher Mercy's class is the interactive/dialogic approach. This is because the teacher gives learners exercise examples to work on (on their own or with their peers) and later ask learners to volunteer to do corrections on the board. The learners seemed to already know that volunteering to write corrections on the board in this class meant that they would have to explain what they did to get to the answer. Surprisingly, the learners were not too reluctant to take part in this. By so doing, the learners become open to scrutiny of their work by others (Essien, 2017) and more than one point of view is explored. In excerpt 4.12 below, I present one case where a learner had volunteered to write corrections on the board and explain their thinking process. Although this is just one case that I use to support my argument, it is important to note that this excerpt covers what and how it is that the interactive/dialogic approach is brought out. In this instance, learners were expected to solve $3(1 + 2x) = 8 + x$.

	Mercy	…let us have someone to do for us number (letter) *d.*
	L	*(a learner stands up to write the solution on the board)*
	Mercy	Remember that either you get it wrong, or right, it is okay. It is okay to get it wrong, right?
	Ls	Yes ma'am
	Mercy	…let us focus so that we can check if she is correct or not correct, or whether she is on the right track or not, right?
	Ls	Yes ma'am
	Mercy	Let's see
	L	*(learner begins to write on the board)* Ma'am, can I write first and explain after?
	Mercy	Okay
		(learner continues to write on the board)

		 *(learner's solution)*
	L	Okay then, here, (*pointing and underlining terms in the first step)* we had to first rearrange the terms, even here we need to rearrange the terms. So, this three, is a positive three, so, I take it as if this three is a negative one and…
	Mercy	Please multiply ma'am, please. Don't skip, write it there for us please.
	L	So, ma'am, this is how I rearranged the terms  Then I said, three multiplied by two is six x, then I said three multiplied by one is three. This part, (*referring to the right-hand side)* I had to rearrange again. Then I took x to this side, plus eight, the way it is, but then took this one to this one, and this one to this one (*learner indicates how she swapped the position of x and 8).* And then, here I said, six x plus three, this positive one (*referring to the positive three on the LHS),* and I said here, it's x plus eight because x I take it as one (*referring to the coefficient of x)*. x plus eight minus this three…then I said six x minus x is equal to five x, x plus five minus x. Five, I took it the way it is, then I said five divided by five, five x divided by five. Five when it goes to five it's x (*referring to the RHS),* it does go, but once, it leaves x and five divided by five is one.
		(Learners clap hands)

Excerpt 4.12: Lesson transcript; Evidence of interactive/dialogic approach

Perhaps learners in this class are not discouraged to participate because the teacher would emphasise that there is nothing wrong with getting an incorrect answer indicating to the learners that they are there to learn.

Although teacher Mercy allows her learners to take part in doing corrections, and explain what they did to the final answer, it is important to note that there was one instance where the teacher was dismissive of other methods that learners use to get their answers, thus making the teacher authoritative and considered a "sole custodian of mathematical knowledge" (Essien, 2017, p.93). Two learners had correctly attempted the same question, the first learner made use of the additive inverses to solve while the second learner simply transposed. The teacher's method was similar to both learners' methods, except that she kept the variable on the right side of the equation. An example of such an instance can be seen in the excerpt below, where the teacher suggests to the learners that her method is the simplest, in a way implying that her method is the one that learners should stick with. This type of approach is referred to as the interactive/authoritative since the teacher became the authority by using/exploring her method comparing to the learners' methods.

Mercy	I will reserve my comment on this one as well. You need to simplify maths, right?
L	Yes ma'am
Mercy	Simplicity is the order of the day, are we together people?
L	Yes ma'am
Mercy	I am going to show you a simple, simple method of solving this…. (*teacher solves the problem on the board)* …This one (*pointing to the first learner's method)* I can see what she did, but I can see that you will get lost in some way. This one *(pointing to the second learner's method),* I can see, but this one is simple (*pointing to her own method).* Simplicity is key; therefore, you must never get confused in the test.

Excerpt 4.13: Lesson transcript; Evidence of interactive/authoritative approach

Excerpt 4.13 is used to elucidate more details about the common communicative approach (interactive/authoritative) in teacher Mercy's class as it captures patterns of occurrences generally, in every lesson I observed.

5.2.2.2. Patterns of Discourse

It is due to the language barrier and challenge that at times, the teacher would find it difficult to invite conversations in mathematics classrooms because these are conversations learners may be reluctant to take part in, especially if the learners are to be expected to hold conversations with one another in English (LoLT). This implies that most lessons, in multilingual contexts, such as the one teacher Mercy teaches in are more likely to be authoritative and non-interactive. It is usually the teacher's voice and ideas that are explored, especially when it comes to classroom talk. Furthermore, learners in teacher Mercy's do not participate much, and would remain silent, at times when she asks questions, and the teacher would then answer the questions that she poses. As suggested by the teacher, the learners lack confidence in terms of speaking mathematically, especially, in English.

At times, a mathematical conversation becomes difficult, even if the learners are to carry it out in their HL because the understanding of what a 'word'/ phrase means in English does not imply guaranteed automated translation to correct mathematical understanding. Thus, even though teacher Mercy stated that she allows her learners to use their HLs to assist one another, because these conversations are not monitored for one (even by the teacher), there is no guarantee that the discussions are meaningful and lead to enhanced understanding of mathematical concepts.

It is due to the challenges and dynamics of a multilingual mathematics classroom, such as teacher Mercy's one that the patterns of discourse are strangely, without responses, except for "yes ma'am" responses, that the pattern goes from initiation straight to a response that is already an evaluation (I-R&E), instead of the usual I-R-E. The 'yes ma'am' type of responses may not be considered a point wherein a lesson is to be considered interactive, as there is no evidence of mathematical sense making and that the learners respond in a choral manner to questions that they already know the teacher expects them to say yes already. Thus, though responses may superficially be regarded as interaction, it is rather the teacher acting as the authority as opposed to enhancing meaningful interactive discussions. Additionally, one-word responses leave much to be desired in terms of what it is that structures the response and in its true sense, what it implies.

5.2.2.3. Alignment of the intended object of learning with the lived object of learning

During video recording, some of the learners' work (in their books) was captured and learners whose work were captured were seated in different rows in the class. In this part of my analysis, I make use of the snapshots of the learners' written work. Judging from the captured written work, the learners' lived object of learning does not align with the intended object of learning. As can be seen in Figures 4.5 and 4.6 below, the learners seem to not understand what it is that they are expected to do in order to solve, in this case exponential equations. The learner's work in Figure 4.5 shows that the learner attempted the two questions $2^x = 4$ and $3^x = 27$ but the learner seems to barely understand what he must do. Although the learner, for some reason gives the correct answer to $3^x = 27$, as $x = 3$, it is not clear where this answer came from.

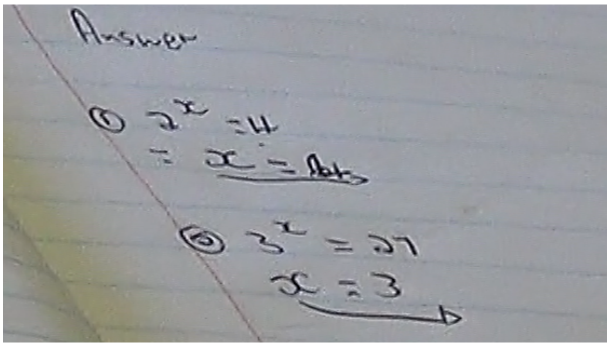


Figure 4. 5: Learner 1, the intended OL not aligned with the lived OL

With the second learner (Figure 4.6), firstly, the learner copied the question incorrectly as $2^x = 8$ instead of $2^x = 4$. This learner, just like the first learner, does not seem to understand the concept of converting the whole number on the right-hand side of the equation, such that the bases on either side of the equation are the same and then compare (equate) the exponents.

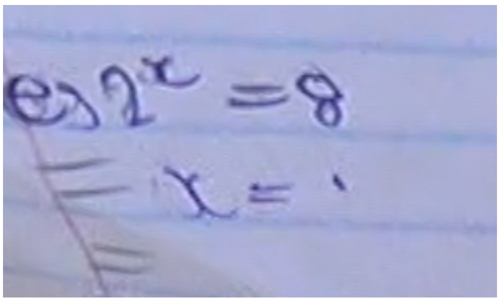


Figure 4. 6: Learner 2, the intended OL not aligned with the lived OL

Thus, the way in which the teacher enacted the examples has not enabled the learners to understand what the teacher intended them to.

5.3. Conclusion

To conclude, in this chapter, I have provided the analysis of the data that I collected from two teachers who taught in three classes. Teacher Thembi, who taught two of the three classes, taught exponential equations, whereas teacher Mercy taught both linear and exponential equations in her class. I have made use of the analytical framework (Dyadic framework) to present the analysis in two parts. The first part investigated the teachers’ choice of examples, and the second part focused on the teachers’ talk to mediate the examples they chose. One of the key findings that the Dyadic framework has helped me see is that the two teachers teach differently. In elaborating on this, I used a selective choice of the teachers’ (post lessons’) interviews, as well as lesson transcripts to bear on the arguments that emerged through the findings. In the next chapter, I provide a summary of the findings, limitations implications for future research.

Chapter 6: Summary of Findings and Conclusion

6.1. Introduction

In this concluding chapter, I present discussions that aim to summarise the findings that emerged from the study. The summary of the findings is presented in a way that responds to the research questions posed earlier, in Chapter 1. Additionally, I provide the limitations and the implications the study has for future research.

6.2. Summary of Findings based on Research Questions

To provide a summary of the findings from this study, I provide answers to the research questions I posed in Chapter 1.

6.2.1. What examples do teachers in Grade 8 multilingual classrooms choose in the topic of linear and exponential equations? Where do these examples come from?

The examples that teachers chose are presented in Chapter 4. Both teachers chose examples according to what it is that they introduce to the learners as the object of learning. For teacher Thembi, when she started with exponential equations, she stated the object of learning as "solving equations with exponents". Thus, the examples the teacher chose, were examples of exponential equations only. Most of the examples that the teacher chose were taken from the Mind Action Series Grade 8 textbook. Other examples that the teacher chose came from the online platform Siyavula. Although the teacher had indicated, during the interview, that she also makes use of Grade 9 textbooks at times, to choose examples from, this was not evident in the example spaces, in both her classes, that she chose for the period that I observed and collected data. Additionally, teacher Thembi chose a few examples that she worked out with the learners, and the rest of the examples were chosen as exercise examples.

Teacher Mercy, on the other hand, introduced the concept differently. She taught both linear and exponential equations together, in the same lesson(s). The teacher introduced the concept as "solving algebraic equations". Since algebraic equations is an umbrella term which incorporates many kinds of equations (such as quadratic, surd etc.), this then implies that with the examples the teacher chose meant that there was more than one object of learning to explore beyond the main one. All the examples that teacher Mercy chose are taken from the Mind Action Series Grade 8 textbooks. The teacher had indicated in the interview that she uses more than one resource to choose examples from, however, from her example space, there is no evidence of the use of other resources besides the one I have already mentioned. There were no worked examples that the teacher chose to work out with the learners, she chose examples only, which learners had to work through.

6.2.2. How do teachers use talk to mediate these examples in their multilingual classrooms?

Both teachers use classroom talk in order to mediate their example spaces, as would be expected in any classroom. The teachers consider and accommodate the language dynamics in their classrooms differently through the use of different practices. Teacher Thembi code-switches from the LoLT (English) to Sesotho and IsiXhosa in both her classes, while teacher Mercy does not code-switch. Teacher Thembi does this because of her awareness of teaching in a multilingual context and uses the learners' HL as a resource to attain mathematical knowledge. As Barwell (2020) contends, teachers who teach in multilingual classrooms are faced with linguistic challenges that require that they adapt their teaching to make use of learners HLs as resources. In support of this notion, Essien (2021a) further suggests that even the training that PSTs receive does not stipulate how teachers are prepared to teach in multilingual contexts such that they optimise and enhance the attainment of mathematical concepts through the use of the available HLs in their classes.

Teacher Mercy believes that in order to ensure that her learners are better prepared to respond to questions during assessment, she would rather teach using the LoLT (English) only. She does this in order to build the learners' confidence to be able to speak of and about mathematics in the language the learners are to be assessed in. This suggests that teacher Mercy sees her role as that of developing learners who would be proficient users of English for the purpose of doing mathematics (Essien, 2017).

Other practices that both teachers make use of in order to mediate the examples they choose include allowing learners to be involved in the learning process by requiring that the learners display their understanding of the concepts by writing corrections on the board. In this way, the teachers created environments wherein they were able to depict the learners' lived object of learning, in terms of what it is that the learners understood and anything else that they needed to address. Furthermore, teachers were able to make their lessons interactive, rather than the teachers having to do all the work and only waiting until they administer formal assessments to reflect on their teaching to know what learners understood. Both teachers were able to initiate discussions in their classes, and in general, learners would respond to the teachers' questions and the teachers would then evaluate the learners' responses. In both classes, when learners responded to the teachers' questions, they would do so dominantly in a chorus form because in most instances, the teachers would not direct the questions to specific learners, questions were dominantly directed and posed to the whole class.

6.2.3. What is made possible to learn through the chosen examples? How does the teachers' talk, in mediating the examples enable meaning making in multilingual classrooms?

The answer to this question is not clear cut, it is not easy to answer this question because for one, there was no formal assessment that learners had to write in order to determine whether or not they understood well. Secondly, it was not every single learner that was involved in doing corrections and sharing their understanding with others. Thirdly, both teachers barely invited conversations with the learners because, in most instances, learners would give choral answers, making it difficult to say who understands, and who does not. Additionally, in instances where teachers directed questions to individual learners, if the learner got the answer correct, the teacher would move on to the next item of the lesson or in instances where the learners gave an incorrect answer, the teachers would go straight to the correct answer. Both teachers would do this without requiring learners to justify their answers, which would have enabled access into the learner's thinking. Thus, by not promoting meaningful conversations, the teachers were limiting themselves in terms of gathering knowledge about what their learners truly understand. Furthermore, this type of missed opportunity for learning meant that learners might not see the importance of learning mathematics meaningfully such that they would be able to see how mathematical concepts are linked and how it is that justifying procedures, for example would help learners understand why certain procedures are appropriate for some instances and not for others.

Nonetheless, the examples that the teachers chose enabled the learners access to mathematical knowledge in terms of making learners aware of the concept of linear equations and exponential equations. One way that the teachers enabled meaning making in their multilingual classrooms was through highlighting the critical features of each example when it was solved.

According to the textbook that both teachers chose examples from (Mind Action Series), the sub-topic that concludes algebraic equations is solving word problems, which both teachers did not cover. It would have been interesting though, to observe how the teachers would have taught the word problems, especially given the fact that they both teach in a multilingual context. Perhaps, teaching word problems would have allowed teachers to invite meaningful discussions with their learners since word problems are ‘language’ problems and require the translation of ‘words’ into mathematics, through the use of mathematical objects and symbols.

6.3. Limitations

Given that in the school where I collected data for my study there is more than one medium of instruction (offered in both English and Afrikaans), perhaps it would have been interesting to observe classrooms wherein mathematics is taught in the learners’ HL (in this case, Afrikaans). I am hoping this would have provided me more insight to understand the kind of examples and how they are used in a monolingual classroom wherein learners learn in their HL and also ‘see’ how the intended object of learners differs or coincides with the learners’ lived object of learning. Furthermore, this would have allowed me as a researcher to draw more insights about whether or not it is easier for the teachers to invite meaningful discussions with their learners if the learners are taught mathematics in their HL.

6.4. Implications for Future Research

Given that the teachers involved in the study did not receive any training on variation theory, prior to data collection, it means that teachers taught linear equations and exponential equations the best way they know how. I believe that better results could have been obtained if the teachers had received training on variation theory. Perhaps, the example spaces would have been more structured in a manner that would allow teachers to sequence the examples they choose in terms of their level difficulty (from easy to difficult) and use talk to effectively elucidate meaningful discussions, by requiring learners to justify their contributions, for example. Thus, I would suggest that in future research should focus more on ensuring that teachers receive training on variation theory in order to measure the effectiveness of variation theory and how it is that when teachers enact the examples they choose, they invite meaningful discussions in order to enhance learners’ understanding of mathematical concepts.

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Annexures

Annexure A: Wits Ethics Clearance Certificate

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	PROTOCOL NUMBER: 2024ECE004M
PROJECT TITLE	Teachers' choice and use of examples in algebra in Grade 8 multilingual mathematics classrooms. The choice and use of examples in mathematics has proven to be an important
INVESTIGATOR:	**Mmatsela Sehowa**
SCHOOL/DEPARTMENT OF INVESTIGATOR	WSoE
DATE CONSIDERED	05th February 2024
DECISION OF THE COMMITTEE	Approved conditionally.
RISK LEVEL	Low Risk
EXPIRY DATE	Date of submission of the Research Report
ISSUE DATE OF CERTIFICATE	16th February 2024
CHAIRPERSON	Dr. Lawan Abdulhamid

cc: Prof. Anthony Essien

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am are authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

Signature **Date** 20 / 02 / 2024

27 St Andrews Road, Parktown, Johannesburg, 2193 | Pvt Bag 3, WITS, 2050 | T +27 11 717 3007 | www.wits.ac.za/education

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Appendix A: Interview Schedule

Interview schedule (Semi-structured)

- What are the learners' home language(s)? Am I correct in assuming that Sesotho is your home language? What languages do your learners speak generally? Which languages are officially offered in the school?
- How do you decide, for each topic to be taught, the examples to use in class? Where do you take your examples from?
- Do you consider the order of the questions to be an important feature to learning mathematics? Do you arrange questions according to their level of difficulty or not?
- How are learners grouped into classes in the school? Is there any streaming according to merit?
- Do you consider your learners' proficiency in the language of learning and teaching (LoLT) when choosing examples?
- When you notice that there is a learner(s) in your class that does not understand a concept, what do you generally do?
- Any other comments you would like to make?

Line	Thembi/Researcher	Utterance
1	Researcher	Okay ma'am, thank you for availing yourself. I am going to ask you a few questions about the way in which you conduct teaching and learning in your class. So, the first question that I've got is: What are the learners' home languages in the school?
2	Thembi	Most learners they speak Sesotho
3	Researcher	Okay, most of them?
4	Thembi	Yes
5	Researcher	Are there any other languages?
6	Thembi	Yes, uhm others they speak Afrikaans and…I think others, I suppose others they are speaking…their home language is English because we are having English home language…
7	Researcher	But what language do they speak at home? Their mother tongues?
8	Thembi	Mother tongue…most of them it's Sesotho.
9	Researcher	Okay, and there are those whose home language is Afrikaans?
10	Thembi	Yes.

11	Researcher	Oh okay, interesting. And then would I be correct in assuming that your home language is Sesotho?
12	Thembi	Yes(*laughs).* Why?
13	Researcher	Nah, I'm just asking because I heard you speak Sesotho in class and here and there Zulu, and then there is Xhosa, so I wasn't really sure.
14	Thembi	Okay
15	Researcher	But dominating language would be Sesotho?
16	Thembi	yah
17	Researcher	Okay, then in general, what do the learners speak? Which language do the learners speak in class? Or during lunch…?
18	Thembi	It's mostly Sesotho because we are having more learners who… who are speaking Sesotho at home *neh*? So *yah,* mostly Sesotho.
19	Researcher	Okay, okay. And which languages are offered in the school, as official languages? Which official languages are offered in the school?
20	Thembi	We have Sesotho, English and Afrikaans

21	Researcher	Okay, which ones are home languages in the school?
22	Thembi	All of them. There those who are doing Sesotho home language, Afrikaans home language, and English home language
23	Researcher	Okay…and then, if they do…
24	Thembi	Those who are doing Sesotho home language, they are doing English first additional. Afrikaans they are doing English first additional and those who are doing English as home language they are doing Afrikaans as first additional
25	Researcher	That's a lot
26	Thembi	It's a lot *neh*? (*laughs)*
27	Researcher	Only you would understand it hey? I will have to listen to this over and over again to get that
28	Thembi	Alright

29	Researcher	And then, for each topic that you teach, how do you decide on which examples to use in class? And where do you get those examples?
30	Thembi	Uhmm…How do I decide on the examples…?
31	Researcher	Mhmm
32	Thembi	Ehhh…like, for each and every topic they have the subtopics *akere (*right)? So, I choose the examples based on the concepts. Like for example in the topic that we are busy with now, functions, mxm, what is this? Solving equations neh…like we have different kinds of equations, like we have the linear equations, we have the expo-ne-n-ti-al equations. Okay, we have the linear equations, then we have those equations that we have, ehhh…unknowns on both sides of the equations, then we have the exponential equations. So I have to separate the different types of equations and choose…for example, let's say I'm starting with the linear equations, the easy ones, I will choose…the heading will be: Solving equations, and the examples will be based on that particular concept, and then when we are done we move on to the next type of equation, those ones that have the unknowns on both sides of the equation. And then I look for problems that have unknowns on both sides of the equation and the exponential equations.
33	**Researcher**	Okay…and where do you get your examples?
34	**Thembi**	Ehhh examples, normally I use…uhmm Mind Action…
35	**Researcher**	…or just questions in general?
36	**Thembi**	The Mind Action textbook
37	**Researcher**	Oh, the blue one?
38	**Thembi**	Yes, the blue one and ehh, what is this? Siyavula, the online one
39	**Researcher**	Okay…
40	**Thembi**	Siyavula, and then sometimes I use past question papers
41	**Researcher**	Okay…

42	**Thembi**	Even if I find something on the Grade 9 textbook, I use it
43	**Researcher**	Oh okay, in Grade 8?
44	**Thembi**	Yes, just to expose them uhmm so that we don't struggle much when we get to Grade 9. Yes.
45	Researcher	Ohhh okay…and then when you… this is one of the questions that was not there when I sent you this. But when you, when you, how do I put this? When you give the learners the questions, in class, whether it be an example or an activity question, do you consider the order of the questions to be important for the learners to learn maths? So in saying that, what I am trying to find out from you is do you consider the level of difficulty when you give them the questions?
46	Thembi	Yes! Yes, we have to consider the level of difficulty. (*responds with lots of enthusiasm).* So, we start with the easy, the easy one and then we move on to the ones where you have to solve something first before you, you solve the actual equation. So, *yah* the level of difficulty is very important
47	Researcher	Okay, so you start with the easy ones…?
48	Thembi	Yes, we start with the easy ones so that they understand the concept, then we move on to the ones that are a little bit challenging
49	Researcher	Okay. And then in your school, how are learners grouped? Are they put into class according to their merit or is it just done…
50	Thembi	No, they are just grouped
51	Researcher	Okay, so it's not on merit?

52	Thembi	No, it’s not on merit
53	Researcher	At all? Across the board?
54	Thembi	Across all Grades
55	Researcher	Huhhh, okay. And then when you choose questions to use in class, as examples or in an activity, do you consider the learners’ proficiency in the language of learning and teaching?
56	Thembi	Well, I cannot say that I do consider the, the proficiency. But uhm, what I do is, when I give them the instructions *neh*, I make sure that I give them the instructions in such a way that they will understand the instructions on the question paper. So, I use, like I said, sometimes I use examples from the past question papers just to expose them on how questions are asked in the test or examinations. I don’t know if you’ve noticed that when I teach, I always switch between the languages, which English and Sesotho. I do give them the instruction in English, then try to translate it in their mother tongue
57	Researcher	*Yah,* I’ve noticed that. If you teach a certain concept to learners, and you realise that they do not understand the concept or the grapple with that concept, what do you generally do?
58	Thembi	Uhm, normally I would refer a struggling learner to a learner that understands what we are doing. Why? The reason why I do that is that I want to give the learner who understands the concept to understand or grasp the content, *ya* (of), what is this?...the concept better. So, uhm like you know that when you explain something, the more you explain something, the more you understand it better. So, by doing that, I am giving the one who understands the concept a chance to understand the concept better and uhm, I’ve noticed that learners understand something if their peers explain to them. So that’s why most of the time I refer them to the ones that understand the concept. But if, if both of them are struggling, that’s when I come in. So, I don’t address, I don’t explain to the individual learner, I just explain to whole class.

60	**Researcher**	Okay, I see. And then, I noticed that in some of the questions you would use in some of your classes, they would differ, so is there any motive behind that?
61	**Thembi**	*Yah* (yes), there is. So, the (Class T2-pseudonym), in the B class, not B, in Class T1, we have more of the strong learners than in class B. So, in class B I use examples that are at least easy*nyana.*
62	**Researcher**	Okay, okay. So that's class B? Class T1?
63	**Thembi**	Class T1, I give them examples that are a little bit challenging,
64	**Researcher**	Okay, alright. Are there any other comments that you would like to make?
65	**Thembi**	Uhm, no, I don't think so (*laughs)*
66	**Researcher**	Okay, thank you so much for your time, and that will be the end of the interview. Thank you so much.
67	**Thembi**	Thank you

Appendix B: Lesson Excerpts

Line	Mercy/Researcher	Utterance
1	**Researcher**	So, what are the learners' home languages in the school?
2	**Mercy**	The school is Afrikaans school, actually, it was Afrikaans school. Then they realised that the intake of Afrikaans learners was too little, so they agreed with the department to make it a bilingual, meaning that English and Afrikaans, now it is not an Afrikaans school. So, there are two streams, stream for, *ntho* (this), English and Sesotho as a home language and then stream for Afrikaans, just Afrikaans, all subjects. Maths in Afrikaans, everything in Afrikaans.
3	**Researcher**	And then, what about your home language? Would I be correct in assuming that it's Sotho?
4	**Mercy**	Sesotho
5	**Researcher**	okay, any other languages that you are proficient in?
6	**Mercy**	Mmm, Zulu yes
7	**Researcher**	Okay
8	**Mercy**	Tswana yes, it dominates Tswana around (*teacher mentions name of the city the school is located in)*
9	**Researcher**	Really? Not Sesotho?
10	**Mercy**	No
11	**Researcher**	Oh, I thought its Sesotho
12	**Mercy**	Yeah, even the learners are speaking Setswana in school here. Some of the learners are speaking Setswana, yes.
13	**Researcher**	Then which languages are offered in the school?

14	**Mercy**	it's only that when they come here, on the side of the language, they are forced to do Sesotho and English, but then they speak Setswana anyway
15	**Researcher**	So, there are those that do English, curriculum in English, and then Sesotho as their home language?
16	**Mercy**	Yes, as their home language.
17	**Researcher**	And then the Afrikaans ones is everything Afrikaans?
18	**Mercy**	Everything Afrikaans
19	**Researcher**	So, they do not do any additional language?
20	**Mercy**	Not at all. Okay, it's FAL…
21	**Researcher**	English
22	**Mercy**	Yes, as additional only.
23	**Researcher**	Okay
24	**Mercy**	Everything, all the contents in Afrikaans.
25	**Researcher**	Interesting. And then how do you decide which examples to use and where do you get those examples?
26	**Mercy**	Okay, actually, we have work schedule. There are examples made for us, coming from the department.
27	**Researcher**	Okay…
28	**Mercy**	…because even the lesson plans, they do it for us. So, what we only have to do, the resources, you know *mos* you cannot rely on only one thing. You have to use as many resources as possible to make sure that you cater all, every learner in the classroom and then as much DBE books concerned, every term they provide us with DBE books. First term and second term, third term and fourth term, yes, we do have it.
29	**Researcher**	Okay

30	**Mercy**	Yes, we also used to have 'hey math'. Even though because of technical problems we no longer have, we used to have, we used to give 'hey math'
31	**Researcher**	Okay, was it a hard copy book or was it…
32	**Mercy**	It was laptop, like yes. Learners would just watch and look at all the examples that are just there and then you see that the teacher at some point, is using the protractor, at some point you can even play some videos, pause…(*inaudible)*
33	**Researcher**	Oh, so it plays videos?
34	**Mercy**	Yes, it was very nice that one, 'hey math' one, very nice. Even slow learners *neh* (right)? There are those learners who learn through visual. It also helped for them, but then technicalities happened
35	**Researcher**	Okay. How long ago was it stopped?
36	**Mercy**	I think it's… during Covid-19, most of the things were being affected by Covid-19
37	**Researcher**	Okay, and then is your learners' proficiency in the language of learning and teaching, which would be English, I would assume that you teach in English?
38	**Mercy**	Actually, I teach only in English, it's only that with those learners because they are the Grade 8s from different schools whereby some of the teachers they teach the content in Sesotho. That is a challenge to me, I don't want to lie. It becomes a challenge for the learner to read the instructions because if the learner gets the wrong instruction, it tells you something, that learner is not going to pass…*(inaudible).* So that is the barrier of language that I experience in the class.
39	**Researcher**	So would you say that the language of learning and teaching is a barrier?
40	**Mercy**	Is a barrier, of course yes
41	**Researcher**	So would you consider that when you choose the examples?

42	**Mercy**	No, when I choose examples, there is nothing that I can do because basically you know *mos,* when the questions come, they come in English. What I must do, is to explain and then, get them used to it because the moment they can give it to me in the other language, I will be killing them, definitely I will be killing them. So, they will come slowly, but then slowly, you have even seen, even confidence wise language is not there but then I try to do it a certain way
43	**Researcher**	Yeah, I saw even during class when one learner was trying to explain
44	**Mercy**	Ma'am, can I speak Sesotho? At some point what I say is okay my baby, just write it down *neh*? That's where the problem is.
45	**Researcher**	But do you not think if you allow them to speak their home language then maybe they would….
46	**Mercy**	Yah, in their groups, I always say that *(inaudible)* because really if you can just get away from the language like English, I will be killing them really, I don't want to lie. They need to get used to it. Definitely, because they are not going to be able to read the instructions, they are not going to be able to understand what they are busy with because the question paper comes with English. There is nothing that I can do, with that one, I always tell them at the beginning of the year that "my kids, ma'am is going to teach in English". Then you have to do is to get used to it. I know that can really be a problem, but really, there is nothing that I can do. I would rather slow the pace, so that I go as slow as possible for them to get used to it. Maybe responding they might not be good, but I want them to understand that this must be in English. And then the other thing is that there are those learners, few, maybe 5 percent of the class or 10 percent of the class that are from Afrikaans class that are also doing in English stream. Therefore now, they also need to be included. When you teach you have to ensure that you cater for every child because sometimes those learners are sitting in the class, their home language is only

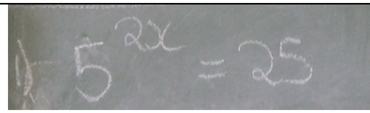
		Afrikaans. They opted to do the content in English, therefore the teacher has to ensure that let's stick to the English in the class, to the learners, that's what you have to do.
47	**Researcher**	Okay, so as a school, are you the ones who decide if the learner should go for the English stream or the Afrikaans stream? Or is it the parents that decide?
48	**Mercy**	You know this area here, the area, apparently is Afrikaans speaking area. This community here is Afrikaans, it's Afrikaans speaking. Even if I go to the shop there, it's Afrikaans people, it is Afrikaans area. Therefore now, all learners, African learners, they just come enroll in this school because the parents are interested in the language. That is why you see them in numbers. When they play together, they are going to get used to the language; when they do this, they tend to get used to it. The primary schools around here, they are all Afrikaans primary schools. So, some of our black learners, the Sesotho speaking at home, they also started Afrikaans, from Afrikaans schools. It becomes a challenge when they grow to that levels because it is supposed to be home language Afrikaans, so it challenges them. Yes, but then they are doing Afrikaans as home language.
49	**Researcher**	Shuu! There is a lot about the languages here. And then, in class, when you notice that there is a learner that's finding it difficult to understand a concept, in this case, the algebraic equations, what do you generally do?

50	**Mercy**	You need to attend to the learner. You need to go and assist. You need to give yourself time that this five-minute time or ten minutes time, at the end of the lesson, I need to ensure that I move around. Like you've seen what I was doing *neh?* I give them a presentation and then I go to them because some learners are not writing the corrections. So should you sit down or turn a blind eye, you are going to find yourself with only 20 learners doing, writing correction, the rest will be doing nothing. So, you have to ensure that you keep the eye to every child, and then you have to ensure that even when they are marking themselves, okay, let us mark and do the corrections. Still, you have to ensure that you are doing 70% of the work, you need to be smart mathematician.
51	**Researcher**	Okay, so you do mark the learners' work as well?
52	**Mercy**	Of course, yes
53	**Researcher**	Shuu! With such big numbers? I must applaud you for that.
54	**Mercy**	I'm sitting with 53, those 53, it decrease to 45, now they are 45 in a class, even though in those 45, it's not everyone that comes to school, no. Like 41 sometimes you find that for two weeks, they are not at school, and it is your baby as a teacher that when they come back those learners, they understand the content because still they are going to write the examination, what are you going to do? Because there is a target that we need to get to. You have to reach the target, the target says 50, I mean 90 50? So how are you going to be able to do that? So, at some point we are asked to say that set your own target, so what are you going to do? You cannot say that they are absent forever, the learners are…the department will say that they are the learners, you are the teacher, so what do you do as a teacher to ensure that all the learners are here? You have to do extra classes for these learners.
55	**Researcher**	*Yah,* it's a difficult task hey.

56	**Mercy**	More especially with the content subject, because if it is the numbers, you can say it's fine, they speak it at home, they can follow even if they have been absent for a certain time, but that does not work in the case of the content. They have to sit and then *yah* that's how it is.
57	**Researcher**	Hmm…so how are learners streamed in the school, how are they grouped into classes? Are they streamed according to their academic abilities or are they just put into one class?
58	**Mercy**	No, you know what? That is what one can do, you decide to do in your class. Remember, at the beginning of the year, they write a test. A, B, C are normally Afrikaans classes, at some point you can find that 8D they are English and Afrikaans class. The only thing, the difference is that when they go to their home languages, some will go to Afrikaans, and some will go to Sesotho but then they are in English stream. To say that these learners, specifically we stream them according to cognitive areas or whatever intellectual ability, no we do not do that, except for a case when we have realised that there was a high failure rate, so as a school we say that let us form one class for these learners specifically, so that we now see how we can deal with them, how we can help them and how we can assist them. But when they are coming from primary school, we just *argh man,* we just make the classes the same. So, the only thing that you can do as a teacher is, after March, now you know your learners better, is then that you can decide to restructure and say okay, these are my level 4s, these are my level 3s, level 2s and level whatever, so that you can know where to put your attention to.
59	**Researcher**	Okay, in one of the examples that the learners were doing, you realised that the learners were struggling, especially with the distributive law, "removing the brackets" as you would call it, what strategies do you use, and what informs which strategies you use to help them?

60	**Mercy**	You know what I do? Okay, yesterday actually, I just gave only one and I realised, I picked up to see, they have mastered all, but they are struggling with this one, so today I came with only, only those ones. I have to focus onto them, then they've got no option. Then they've got no option, because you know learners, they can say, let's not do this one, let's focus on this one. So, I said, today, let us focus on these ones. So the minute you realise that these learners are lacking in the class, don't only be one teacher in class, that's when I decided, okay, let me stand at the back, let them do the work themselves so that they can end up enjoying, when I realise that this one is getting it wrong, I call the other one and then, you reserve your comments, don't say anything because you are going to demoralize them. Just let one to go, just play around with the numbers, they end up feeling like going. If you can be only one teacher in the class, you are just leaving them behind, you are totally leaving them behind. When you realise that this one is challenging them, give them chance, give them chance, let them do it on the board, let them do all that work. Just say okay, okay, not correct, the other one, okay. They are going to enjoy maths. Because finding ourselves doing the work alone, *aww shame,* that's when you are just losing them, just to let it go, but when they realise they are stuck now, okay, okay, just stand at the back. I realised yesterday that these ones, they are just ignoring, they just left that one and I decided okay, they are just enjoying and doing that one, no! Let me only put these four. When you are in the class, do not give them more, they are going to get bored. Just 4 or 5.
61	**Researcher**	Then they will be fine?
62	**Mercy**	Of course, yes.
63	**Researcher**	Alright, do you have any other comments that you would like to add?
64	**Mercy**	*Argh man,* I don't have, it's only that one can say, learners do not always give what you want. I love mathematics, I enjoy teaching mathematics, I feel like I could see everyone do well, everyone passing. Those are the things

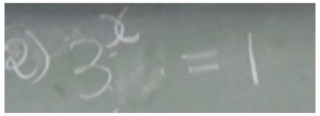
		that I am looking forward for. The other thing is I never demoralise them, I want to, just to simplify mathematics, make it simple so that every child can be able to see that she can take mathematics to the next level. I really like encouraging them. I don't like for learners to do maths lit, for me I feel like all learners can have a chance to study mathematics to the second level. That's why I'm always trying to build their confidence, even if it is lacking, even if I can see that really, here, it will take so long. 'yes, my baby, yes, go, go, go! What would you like to become? 'ma'am I *wanna* become…' yes, yes! Just push, push until they realise that they can do this.
65	**Researcher**	Ohhh, thank you so much ma'am. That will be the end of the interview. I really learned so much that I can really apply to my own teaching

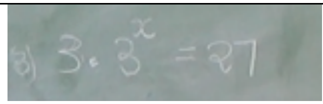
Line	Thembi / Learner(s)	Utterances
11	**Thembi**	 Okay, please put your pens down for a moment, let me explain this. Alright, today's story, solving equations involving exponents. Alright, we are still solving for x, we are still solving for the unknown. But today, we have a story se (which is) a little bit different. As you can see here, the examples we have, number 1, five exponent two x is equal to twenty-five. The unknown is not in the base, *akere* (right)? It's on the what? It's on the exponent. So, now, how do we solve exponential equations? Step number 1 (*teacher moves towards the board), you* check your left-hand side, on the left-hand side we have a power, *akere (*right)*?* On the right-hand side we have a whole number, it's not a power. *on* the left-hand side we have five exponent two x your left-hand side you have a what? So, what you are going to do it, you convert the right-hand side into a power, with a base what? With a base five, because on the left-hand side, the base is what? The base is base five we have a base five. So, when you convert this *(points to twenty-five).* You have a power, *akere* (right)? So, what we are going to do is, convert the right-hand side into a what? Into a power, because on the left-hand side, the base is a what? Is five. So, when you convert this, it must do what? It must be the same as the left -hand side. *Iyavakala (*Is it clear)*?* Okay, first step, we have five exponent two x is equal to, now we convert this (*points to twenty-five)* to what? A power, *akere?* With the base what? With the base five. The base must be the same as the one in the left-hand side. *Iyavakala* (is it clear)?
12	**Ls**	Yes ma'am

17	**Thembi**	Now, *re raiser (*we raise) five to exponent what, to get twenty-five? Five...? It's five multiplied by five, so it's five exponent what?
18	**L**	Exponent five
19	**Thembi**	Five? (*astonished)* Five exponent five? *(chuckles)*. Five exponent five?
20	**Ls**	*(inaudible)*
21	**Thembi**	Okay, now, I'm gonna give you the easy way to determine the exponent, *neh (*right)? So, how do we determine the exponent? We start with the what? We start with the smallest number. Let us take zero. Let us raise five to exponent zero and see if we are going to get twenty-five. What is five exponent zero?
22	**L**	It's zero
23	**Thembi**	It's zero *akere* (right)? Not twenty-five. Therefore, let us go to the next number. Let us raise five to the exponent one to see if we get twenty-five. What's five exponent one?
24	**Ls**	Five
25	**Thembi**	Five, but we are looking for twenty-five. We go to the next number, *akere(*right)?
26	**Ls**	Yes ma'am
27	**Thembi**	Let us raise five to exponent two. Five exponent two is…?
28	**Ls**	Ten
29	**Thembi**	Ten?! (*raises voice, astonished)*
30	**Ls**	Twenty-five
31	**Thembi**	*(laughs)* What does five exponent two mean?
32	**L**	Five multiplied by two

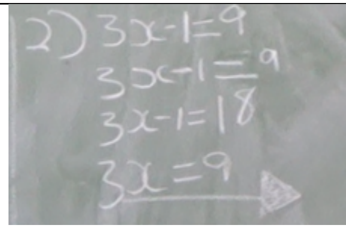
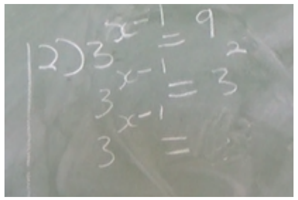
33	**Thembi**	It's five multiplied by five, not five multiplied by two, *haw! Huh-uh (*No). It's five multiplied by five, gives us what?
34	**Ls & Thembi**	Twenty-five
35	**Thembi**	Therefore, the exponent for five is what?
36	**Ls & Thembi**	It's two
37	**Thembi**	It's two, *akere (*right)? Now, once you have the same bases on both sides of the equation, we ignore the bases, and work out the what? And work out the exponents. But before you work out the exponents, the bases must be what? The bases must be the…
38	**Ls & Thembi**	…The same
39	**Thembi**	The bases must be the same. Alright, now we have five exponent two x is equal to five exponent two. Bases are the same, we have five on the left-hand side and five on the right-hand side. When we have the same bases, we ignore the bases, and we work out the exponents. What is the exponent on the left-hand side?
40	**Ls**	Two x
41	**Thembi**	It's two x. Two x is equal to?
42	**Ls**	Two
43	**Thembi**	Okay, now, what is the value of x? What is the value of x?
44	**Ls**	One, two, one

45	**Thembi**	How did you get one? *(pauses, waiting for learners to respond and they do not)* We have two x equals to two, *akere* (right)? Remember, we are looking for the value of what? Of x, not two x. So, we have to get rid of coefficient
46	**Ls & Thembi**	Two
47	**Thembi**	How do we get rid of the coefficient? By…?
48	**Ls**	Dividing
49	**Thembi**	Dividing what by what?
50	**Ls**	Two x over two, two over two
51	**Thembi**	Why two divide by two?
52	**Ls**	Because… (*inaudible)*
53	**L**	What is always…you also do…
54	**Thembi**	Whatever change you make, on the left-hand side, must also be made on the right-hand side. So that the equation balances. This cancels this (*refers to* $\frac{2x}{2}$). Therefore x is equal to one. Now, let us take one, and substitute it into the equation and see if the left-hand side is equal to the right-hand side. *Akere (*Isn't it) I said to you, after finding the value of x you verify the value of x? By what? By substituting in the original equation. So, when we substitute x is equal to one, it should give us what? It should give us x is equal to twenty-five. Five exponent two multiplied by one gives us… two multiplied by one is 2, it gives us five exponent two. And this gives us five multiplied by five. And this gives us what? Twenty- five. So indeed, the value of x is equal to one gives us what? It gives us twenty-five. Alright, example number two. Is there anyone *o batlang ho e tryer* (who wants to try it)?

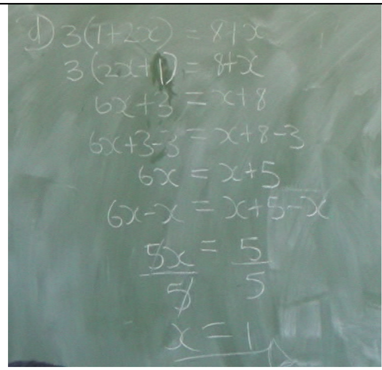
55		
56	**L**	Three x is equal to…
57	**Thembi**	*(interjects)* Three x?
58	**L**	Three exponent x
59	**Thembi**	Three exponent x, uh-huh…?
60	**L**	Is equals to…
61	**Thembi**	Equal to?
62	**L**	Three
63	**Thembi**	Equal to three. Three exponent what? Does three exponent one give us one?
64	**Ls**	No
65	**Thembi**	So, we raise three to exponent what to get one? Exponent?
66	**Ls**	Zero
67	**Thembi**	*Akere (*Isn't it) we know that anything raised to the exponent zero is one? *Bathong, ke itseng ho wena* (Hey, what did I say to you)? Left-hand said is a power and right-hand side is not a power, *akere* (right)? Now on the left-hand side we have three exponent x equal to three exponent zero. Now the bases are the same, what do we do?
68	**Ls**	We divide
	Thembi	We divide?! (*Raises voice). Ye* (what)? *Ay, lona ketlo le pota nou (*I am going to punish you now).
	L	We multiply

	Thembi	*Ke itse ho wena* (I said to you) once we have the same bases, we ignore the bases, now we work out the exponents. *Ha ke tlo e pheta* h*ape* n*eh* (I am not going to say it again, okay)?
	Ls	*Eya mme* (Yes ma'am)
	Thembi	We have the same bases, so we work out the exponents. Exponent on the left-hand side is what?
	Ls	x
	Thembi	x, equal to…?
	Ls	Zero
	Thembi	So, if we take zero and substitute it in place of x, we are going to get what?
	Ls	Zero
		
	Thembi	Alright, number three. *Letle le ba bone bao multiplier* (You'll see those that multiply). (*Teacher calls learner by name)*? Hi!
	L	Three comma, three exponent x…
	Thembi	(*interjects)* Comma???
	L	Full stop *(learners laugh)*
	Thembi	Three comma? *Ke* (it's) three comma or *ke* (is it) three multiply?
	Ls	Ke three multiply
	L	Three multiplied by three exponent is equal to…

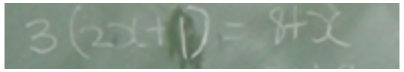
	Thembi	Is equals to what? Okay, uhmm, number three, we have three multiplied by three exponent x is equal to twenty-seven. If you look at the left-hand side, we have how many powers on the left? How many powers are we having here (*teacher points to the left side of the equation)?*
	Ls	Two. One.
	Thembi	Two or one?
	Ls	three
	Thembi	Make up your mind
	Ls	One. Two
	Thembi	*Ke bo mang bareng two* (who says two)? *(Some learners raise their hands)*. Okay, *hare sheba* (when we look on the) left-hand side, we have a power multiplied by another power. Now you have to work the…okay, three is the same as three exponent one. Okay, no (*teacher erases what she had written).* We have a number that is not raised to an exponent, multiplied by a power. So, if we have a power that is multiplied by a number that is not raised to an exponent, the number that is not raised to an exponent is called the coefficient of the power. So, three is the coefficient of three exponent x. So, before we convert this (*points to 27)* to a power, we have to get rid of this three. How do we get rid of the coefficient?
	L	We divide
	Thembi	We divide. Left-hand side, this cancels with this (*teacher referring to* $\frac{3.3^x}{3}$)

		
	Thembi	Number two, ehhhhh (*shocked). Ho etsahalang mona* (What is happening here)?
	Ls	(*inaudible)*
	Thembi	Okay, number two, do we all have three x equal to nine?
	Ls	No
	Thembi	Okay, *ke mang anang le a different answer?* (Who has a different answer)?
	L	x is equal to two
	Thembi	*Eya, tloyo* (Yes, come) *Tloo, o refe (*Come give us) $x = 2$. (*Learner walks up to the board)*
	L	*(learner writes on the boards)* 
	Thembi	Okay, from here, we have? x?
	Ls & Thembi	x minus one equals to two (*Teacher writes this down, continuing from the second learner's solution)*

	Mercy	…let us have someone to do for us number (letter) *d.*
	L	*(a learner stands up to write the solution on the board)*
	Mercy	Remember that either you get it wrong, or right, it is okay. It is okay to get it wrong, right?
	Ls	Yes ma’am
	Mercy	…let us focus so that we can check if she is correct or not correct, or whether she is on the right track or not, right?
	Ls	Yes ma’am
	Mercy	Let’s see
	L	*(learner begins to write on the board)* Ma’am, can I write first and explain after?
	Mercy	Okay
		(learner continues to write on the board)



(learner’s solution)

	L	Okay then, here, (*pointing and underlining terms in the first step)* we had to first rearrange the terms, even here we need to rearrange the terms. So, this three, is a positive three, so, I take it as if this three is a negative one and…
	Mercy	Please multiply ma'am, please. Don't skip, write it there for us please.
	L	So, ma'am, this is how I rearranged the terms  Then I said, three multiplied by two is six x, then I said three multiplied by one is three. This part, (*referring to the right-hand side)* I had to rearrange again. Then I took x to this side, plus eight, the way it is, but then took this one to this one, and this one to this one (*learner indicates how she swapped the position of x and 8).* And then, here I said, six x plus three, this positive one (*referring to the positive three on the LHS),* and I said here, it's x plus eight because x I take it as one (*referring to the coefficient of x)*. x plus eight minus this three…then I said six x minus x is equal to five x, x plus five minus x. Five, I took it the way it is, then I said five divided by five, five x divided by five. Five when it goes to five it's x (*referring to the RHS),* it does go, but once, it leaves x and five divided by five is one.
		(Learners clap hands)
	Mercy	I will reserve my comment on this one as well. You need to simplify maths, right?
	L	Yes ma'am
	Mercy	Simplicity is the order of the day, are we together people?
	L	Yes ma'am

	Mercy	I am going to show you a simple, simple method of solving this…. (*teacher solves the problem on the board)* ...This one (*pointing to the first learner's method)* I can see what she did, but I can see that you will get lost in some way. This one *(pointing to the second learner's method),* I can see, but this one is simple (*pointing her own method).* Simplicity is key; therefore, you must never get confused in the test.