

https://dspace.org/

Problem solving in mathematical and everyday contexts : teachers' practices and knowledge.

Fonseca, Kathleen

2012-02-29
http://hdl.handle.net/10539/11379

Downloaded from a DSpace repository. https://dspace.org/

UNIVERSITY OF THE WITWATERSRAND

Title: Problem solving in mathematical and everyday contexts: Teachers' practices and knowledge.

By: Kathleen Fonseca

0517102F

Supervisor: Professor Karin Brodie

A research report submitted to the Faculty of Humanities, University of the Witwatersrand, in partial fulfillment for the degree of Masters of Education by coursework and research report.

Johannesburg, June 2011

DECLARATION

I, Kathleen Fonseca, declare that this research report is my own unaided work. It is being submitted for the degree of Masters of Education in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any other degree or examination at any other university.

Kathleen Fonseca

Sign …….

June, 2011

ABSTRACT

The Revised National Curriculum requires that mathematics teachers make shifts in their pedagogical content knowledge about teaching non-routine problems. One of the shifts is to move from a traditional approach of teaching routine tasks to an approach which includes problem solving. Teaching non-routine problems develops flexible forms of knowledge, which allow learners to construct their own strategies and not merely follow steps shown to them by the teacher. In this study I explore what pedagogical content knowledge grade 6 teachers have about problem solving in everyday and mathematical contexts. A qualitative study was used to explore Grade 6 teachers' pedagogical content knowledge about problem solving in these contexts. Data was collected by means of interviews and lesson observations. The study found variations in the teachers' content knowledge but very similar pedagogical content knowledge among the teachers. Teachers have different understandings of teaching non-routine problems and they are faced with the challenge of how much to make knowledge accessible to their learners in order to lay a conceptual foundation to solve non-routine problems while not lowering the demands of the task. Teachers taught everyday context tasks differently to mathematical context tasks by focusing on the linguistic aspects of the tasks.

ACKNOWLEDGEMENTS

I firstly want to thank God for giving me the strength to complete this research report.

My sincere appreciation and heartfelt gratitude to my supervisor, Professor Karin Brodie for her support, patience and professional advice throughout the process of this research. Without her assistance this project would not have been possible.

Thank you to the three teachers and learners for allowing me into their classrooms and for their anonymous contribution to this study.

I thank my husband, Charles and daughter, Schaedenet for believing in me and encouraging me.

To my friends and colleagues, thank you for listening to my ideas and sharing your opinions with me.

Thank you.

TABLE OF CONTENTS

DECLARATION **1**
ABSTRACT **2**
ACKNOWLEDGEMENTS **3**
TABLE OF CONTENTS **4**

CHAPTER 1: INTRODUCTION
6.
1 INTRODUCTION 6
1.2 CRITICAL QUESTIONS 8
1.3 RATIONALE 9
1.4 STRUCTURE OF THE REPORT 12

CHAPTER 2: LITERATURE AND THEORETICAL FRAMEWORK
13
2.1 INTRODUCTION 13
2.2 PERSPECTIVES ON MATHEMATICAL PROBLEM SOLVING 13
2.3 MATHEMATICAL THINKING 15
2.4 PEDAGOGICAL CONTENT KNOLWEDGE IN EVERYDAY AND MATHEMATICAL CONTEXTS 17
2.5 CONSTRUCTIVIST AND SOCIOCULTURAL PERSPECTIVES 22
2.6 DEALING WITH MISCONCEPTIONS 24
2.7 CONCLUSION 25

CHAPTER 3: METHODOLOGY AND RESEARCH DESIGN **26**
3.1 METHODOLOGY OF THE STUDY 26
3.2 RESEARCH DESIGN AND METHODOLOGY 28
3.2.1 The context and sample 28
3.2.2 Ethics 29
3.2.3 The tasks and tasks analysis 30
3.3 INTERVIEWS 33
3.4 PILOTING 33
3.5 CLASSROOM OBSERVATION 35
3.6 THE TRANSCRIPTS 35
3.7 ANALYSING THE TRANSCRIPTS 36
3.8 RIGOUR 36
3.9 THE REPORT 38

CHAPTER 4: DATA ANALYSIS **39**
4.1 INTRODUCTION 39
4.2 CONTENT KNOWLEDGE 39
4.2.1 Teacher A 40

4.2.2 Teacher B 43
4.2.3 Teacher C 46
4.2.4 Similarities in teachers 49
4.3. PEDAGOGICAL CONTENT KNOWLEGDE 50
4.3.1 Teacher A 50
4.3.2 Teacher B 56
4.3.3 Teacher C 64
4.4 MATHEMATICAL AND EVERYDAY CONTEXT 68

4.5 CONCLUSION 71

CHAPTER 5 **72**
5.1 DISCUSSION OF FINDINGS 72
5.2 IMPLICATIONS 72

REFERENCES **77**

APPENDICES **83**
APPENDIX 1.1 Teachers' consent form 84
APPENDIX 1.2 Parents' consent form 88
APPENDIX 1.3 Interview schedule 93
APPENDIX 1.4 Interview Transcripts 97
APPENDIX 1.5 Teacher C's worksheets 123

CHAPTER 1

1.1 INTRODUCTION

Since the inception of Curriculum 2005, the rhetoric has been for mathematics teachers to adapt a learner-centred and activity based approach to mathematics education (Revised National Curriculum, 2002). This learner-centred and activity based approach requires of mathematics teachers to make shifts in their knowledge of mathematics and their teaching approaches.

Prior to Curriculum 2005 mathematics teaching was informed by a traditional approach and in many cases still is (Modau & Brodie, 2008). In a traditional approach to teaching, learners often learn mathematics as "a set of arbitrary rules rather than as a part of a connected system that makes sense and can be used to understand and solve problems." (Resnick, 1987). In the new curriculum the approach is different. The Revised National Curriculum Statement Policy Document (2002:4) states:

Mathematics is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. Through this process, new mathematical ideas and insights are developed.

The notion of mathematics being a human activity was defined and further elaborated on by Freudenthal (1973 in Vos, 2002), in terms of Realistic Mathematics Education (RME). The core of Realistic Mathematics Education is that learners learn mathematics best from a context that is relevant and challenging to learners' experiences (Vos, 2002). Vos (2002) further argues that the context referred to within RME, is the learners' everyday life. So the RME approach strongly advocates the importance for mathematics teachers to teach mathematics from learners' everyday contexts.

The importance of contexts is also evident in the Revised National Curriculum Statement but in a different way. In every one of the five learning outcomes of Mathematics it is stated that the mathematical content should be placed within a context which is relevant to the learners' everyday life, which will enable learners to utilize the mathematical content they are to learn (Revised National Curriculum Statement-2002). For example an assessment standard for Grade 6

states: “Write number sentences to describe a problem situation, including problems within contexts that may be used to build awareness of human rights, social, economic, cultural and environmental issues” (Revised National Curriculum Statement, 2002:47). From this quote, it is clear that the notion of contexts in the Revised National Curriculum Statement is more closely related to the definition of mathematics as being a human activity, and includes more than everyday contexts, for example social, economic, cultural and environmental issues.

In addition to these contexts we can also think about mathematics as a context in and of itself. This means that pure mathematics problems are not decontextualized but part of a mathematical context. The mathematical context refers to the process of identifying which mathematical concepts are required in order to solve the problem.

Curriculum 2005 not only requires from teachers to shift their views of mathematics but they are also encouraged to make a shift in their teaching practices. One shift is from teaching only algorithmic procedures to that of open-ended problem solving as suggested by Boaler (1997). Boaler (1997:16) asserts that an open-ended problem solving approach develops “more flexible forms of knowledge” in learners, meaning that learners are able to use these forms of knowledge in various situations in both school contexts and real life contexts. A learner who can solve problems and use mathematical knowledge in various situations is the type of learner Curriculum 2005 envisages. The Revised National Curriculum Statement (2002:1) states that one of the aims of the curriculum is to provide learners who are able to “identify and solve problems and make decisions using critical and creative thinking.”

Shifts in teaching practice require shifts in teachers’ knowledge. Burns and Lash (1988:370), allude to the shifts in teaching practice by stating that “many problem solving curricula focus on higher-order skills and the development of cognitive flexibility, with a presumably different and more difficult set of pedagogical concerns facing teachers’ planning and problem solving instruction.” In planning to teach problem solving teachers need to access the knowledge they possess and come to see different ways of doing and teaching mathematics. According to Sherin, Sherin and Madanes (2000:358) teachers teach the way they do because “they do (or do not) possess certain knowledge.”

Shulman (1986) asserts that there are three[1] important kinds of knowledge for teachers. The first is content knowledge which refers to the amount and organization of knowledge a teacher has of a particular learning area or subject, as well as an understanding of the structures of the subject matter. The second type of knowledge, pedagogical content knowledge (PCK) refers to the knowledge of how to make the subject matter accessible to learners. Pedagogical content knowledge enables teachers to understand why learners understand certain topics easily and why they have more difficulty in understanding other topics. The third type of knowledge is curriculum knowledge, which is knowledge of the required material that needs to be taught for the specific learning area or subject, as well as how the subject relates to other subjects contents (Shulman, 1986).

In this study I aim to explore teachers' pedagogical content knowledge about problem solving in two contexts, everyday and mathematical contexts. When using the concept of problem solving I am referring to the notion that mathematics is seen as an activity of problem solving. In the act of problem solving, learners are required to solve problems in a range of contexts, mathematical contexts and the other contexts, including everyday life contexts, social, economic, cultural and environment. Teacher's pedagogical content knowledge about problem solving is key in determining whether a teacher's practice is informed by a problem solving approach.

1.2 CRITICAL QUESTIONS

What pedagogical content knowledge do grade 6 mathematics teachers have about problem solving in everyday contexts and mathematical contexts?

Sub-questions

1. How do grade 6 mathematics teachers understand problem solving in everyday and mathematical contexts?
2. Do grade 6 teachers respond differently to learners' responses in everyday and mathematical contexts?

[1]Shulman (1986) actually mentions seven types of knowledge and of these seven I am only focusing on three types of knowledge.

3. How do grade 6 teachers deal with learners' misconceptions in everyday and mathematical contexts?

1.3 RATIONALE

As mentioned in my introduction, the Revised National Curriculum Statement mathematics policy document advocates the importance of an activity based approach, which includes problem solving. In each of the five learning outcomes, numbers and operations, patterns and functions, shape and space, measurement and data handling, problem solving is mentioned. In each of the five learning outcomes learners are expected to solve problems in a variety of contexts (Revised National Curriculum Statement, 2002).

Learning Outcome one states "learners will be able to recognize, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence in solving problems" (Revised National Curriculum Statement, 2002:40). The mathematical skills relate strongly to Kilpatrick's notion of strategic competence (Kilpatrick, J., Swafford, J., and Findell, B., 2001). Strategic competence is one of the five interwoven strands, (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition) needed in order for a learner to be proficient in mathematics (Kilpatrick et al, 2001). It is therefore important for mathematics teachers to develop strategic competence within their learners to support them to solve problems within everyday and mathematics contexts.

In my own experience as a learner, strategic competence and problem solving were not explicitly developed through my teachers' instructional practices. As a learner I was not taught how to apply different strategies when solving a problem. For example I was never taught first to understand the situation including its key features (Kilpatrick et al, 2001). The strategy I was taught that when solving everyday contexts, which were usually word problems, I had to look for the "magic" words such as "altogether" which meant that I had to add quantities given in the problem. The word "less" meant the given quantities had to be subtracted. In my experience as a learner, the solving of problems in both everyday and mathematical contexts had to be done by applying algorithms. In order to apply algorithms it was required of learners to remember a set of rules or steps. Access to mathematics was by remembering how to follow a set of rules. When errors arose, they were as a result of not following the steps correctly.

The following of steps in my classroom as a learner meant that learners' solution strategies were not made public or accessible. In this type of classroom as learners we had to apply the teacher's solution strategies in order to solve the word problems, meaning that opportunities were not created by the teacher for learners to construct their own solution strategies. One way of creating opportunities for learners to construct their own solution strategies is through the setting or selecting of tasks. The nature of tasks can have an impact on the way learners think, for it can either expand or limit their view (Henningsen & Stein, 1997). Agreeing with Henningsen and Stein (1997), Crespo (2003:243) argues that the posing of tasks by teachers requires thoughtful consideration for tasks can "open or close the learners' opportunities for meaningful mathematics learning."

In a problem solving approach there is a shift from traditional word problems to problem solving in everyday and mathematical contexts. The difference between the two contexts is that with the everyday contexts learners have to first extract the mathematical concepts from the everyday events before they can develop solution strategies. When learners have to solve a problem within a purely mathematical context they work only with mathematical symbols (e.g. numbers, operational signs, etc.). Problem solving in both contexts are synonymous in the sense that both contexts develop mathematical proficiency in learners (Kilpatrick et al, 2001).

Kilpatrick et al (2001) distinguished a mathematical problem as being a routine or a non-routine problem. A routine mathematical problem is a problem learners can solve easily based on their past experiences, meaning that the learners have solved similar tasks previously. A non-routine problem is when a person is required to find a solution for a task for which s/he does not have a prior solution strategy or procedure at hand to solve the problem (Kilpatrick et al, 2001).

The kinds of problems advocated by the Revised National Curriculum Statement are problems for which learners do not have a readily available procedure, i.e.: non-routine, and they require of learners to "expand effort to find a solution" (Charles and Lester, 1984). An example of a non-routine mathematical problem:

Douglas Ming and Omar each has a bag of marbles. Ming has 3 less marbles than Douglas. Omar has 12 less marbles than Douglas. Which statement is true?

(A) Ming has 15 more marbles than Omar.

(B) Omar has 15 more marbles than Ming.

(C) Omar has 9 more marbles than Ming.

(D) Ming has 9 more marbles than Omar.

This everyday context task is taken from the International Competitions and Assessments for Schools (ICAS, 2006) and is an example of an everyday context problem that learners may face when writing systemic tests. In 2006 a sample of grade 6 learners in Gauteng wrote the ICAS tests, which were set by the University of New South Wales, Australia and administered by the Gauteng Department of Education to a sample of schools in Gauteng. The items in these tests are mainly problems in everyday contexts. The test items are spread across all five learning outcomes: numbers, patterns, measurement, shape and space and data handling.

Results from these tests were poor. The overall performance of learners was low across all learning outcomes. For example the overall correct response to the above Grade 6 task was 22%. In thinking about these results, I began to question the extent to which the everyday contexts made problem solving easier or more difficult for learners and decided to investigate teachers' knowledge about both everyday context and mathematical context problem solving and the similarities and differences between them.

This study forms part of the Data Informed Practice Improvement Project (DIPIP). DIPIP is a joint venture between the Gauteng Department of Education (GDE) and Wits School of Education (Curriculum Division). The aim of DIPIP is to influence teaching and learning practices through discussion about the test items. Discussions take place between teachers whose learners participated in ICAS tests and Wits staff and post-graduate students, who serve as facilitators in the project. As a DIPIP facilitator I have found that teachers are acquiring knowledge through conversations about the test items which may challenge them to adapt their teaching practices. I would like to find out the extent to which this is happening. Findings of my study will be of benefit to:

1. The Data Informed Practice Improvement Project (DIPIP) for the findings will indicate whether the project is achieving its aim to influence teaching and learning practices.

2. The Gauteng Department of Education (GDE), which can gain greater understanding of the strengths and weaknesses of the development project.

3. Teachers in general can gain knowledge on how to deal with these types of tasks and what teaching practices are needed to teach them.

4. Curriculum developers will benefit, for the findings of this study can serve as a guide as to what types of tasks, teaching and learning practices are needed in the curriculum. Textbook writers can also use the findings as guide to determine which types of tasks are needed for textbooks.

5. Other researchers could benefit from the findings of this study or could further this study based on its limitations.

1.4 STRUCTURE OF THE REPORT

In Chapter 2 I will discuss the theories which underpin my study by focusing on how these theories relate to my study. These theories will serve as a guide in the process of analyzing my data. Chapter 3 focuses on the design of my research. In this chapter I will explain the methods that I have used to collect data. In Chapter 4 I will analyze the data I have collected in order to answer my research question. I use the categories that I develop in chapter three to analyze in detail the kinds of practices that the three teachers employ to teach mathematics. This chapter provides the analysis that will answer my research questions. Chapter 5 will be my concluding chapter, wherein I will give an overview of what I have found in the study with the aim of answering my research questions. I will also look at how my research study helped me understand teaching practices in mathematics and how these can be used to assist other teachers to better understand what can help them to implement the new curriculum. A discussion of the implications that teachers' practices have for the profession and the research community will also be given. I will also make recommendations and suggest methods by which the practices that I have analyzed in this research study can be disseminated to other teachers.

CHAPTER 2

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 INTRODUCTION

The literature I have reviewed in this chapter serves as a lens to look at the participating teachers' knowledge and practices. The key concepts which frame this study are:

- Perspectives on mathematical problem solving.
- Mathematical thinking and justification of solutions.
- Pedagogical content knowledge in mathematical and everyday contexts.
- Socio-cultural theories of learning.
- Dealing with learners' misconceptions.

2.2 PERSPECTIVES ON MATHEMATICAL PROBLEM SOLVING

A non-routine problem is when a person is required to find a solution for a task which s/he does not have a prior solution strategy or procedure at hand to solve the problem (Kantowski, 1977; Charles and Lester, 1984; Wood, Cobb and Yackel, 1991; Kilpatrick et al, 2001), meaning that learners have to construct their own solution strategies to solve the problem. Doerr and Lesh (2003) describe a model-eliciting perspective, where it is crucial for learners to first establish what the problem is and once they have done this they are able to select and execute a suitable solution strategy.

Schoenfeld (1985) argues that a mathematical problem requires of the solvers to construct their own solution strategies, and therefore that problem solving is "relative." In classifying whether a task is a non-routine mathematical problem one has to look at who is solving the problem as well as at the nature of the task. Schoenfeld (1985:74) argues that a task might be a non-routine problem for one person and the same task might be a "routine exercise" for another person. If a learner has previously solved the same task or similar tasks the purpose of constructing solution strategies is defeated, for the solver will make use of previous strategies to solve the problem.

Reznick (1994) agrees with Schoenfeld when he refers to a particular type of pure mathematical problem namely the "Putnam problem." This "Putnam problem" is a test written by college and university students. The purpose of this test is to assess the students' "abstract problem-solving abilities" (Reznick, 1994:23). In setting this "Putnam problem" the assessors are cautious not to set problems that "sound familiar or might have appeared somewhere" (Reznick, 1994:24).

Schoenfeld's (1985) and Reznick's (1994) studies are based on problem solving in Geometry and Algebra, this type of problem solving is in "purely" mathematical contexts and not connected to the learners' everyday life. With the arrival of the Outcomes Based Education era in South Africa high school teachers were faced with the challenging task of connecting the pure mathematics to their learners' everyday life. Not only was it challenging for teachers to connect the mathematics to their learners' real life (Sethole, 2004) but it was even more challenging to set everyday problems which would require of learners to construct their own solution strategies.

Exercises in textbooks focus mainly on the practicing of procedures and skills (Bauersfeld, 1995). Authors like Charles and Lester, (1984) distinguish word problems in terms of simple translation problems, complex translation problems and process problems. Simple translation problems are the "familiar one-step problems", where learners can solve the problem by applying one of the four basic operations. Complex translation problems are called "multi-step problems', to solve the problem learners have to engage in more than one operation. Process problems require from learners to think about obtaining problem solutions and being able to justify their solutions (Charles and Lester, 1984). According to Kilpatrick et al's (2001) task categories, simple translation problems can also be referred to as routine problems due to their "familiar one-step nature". Complex translation problems can also be referred to as routine problems because they are also "familiar problems" but require more than one operation to be solved. Process problems are similar to non-routine problems for learners need to construct their own solution strategies.

The importance of learners obtaining and justifying their solution strategies is evident in how teachers present word problems to their learners. I agree with Silver and Thomson (1984:537) that there is "no single best way to teach problem solving." There are, however studies which have documented strategies teachers used to teach word problems. Carpenter et al (1988) found that teachers showed their learners specific strategies to solve word problems like finding key

words which suggest a specific operation. Fraivillig, Murphy and Fuson, (1999) describe a range of strategies. On the one hand there are teachers who teach specific strategies and expect their learners to use those same strategies to solve word problems. For these teachers it is easy to accept the learner's strategy as correct for the teacher's strategy is always correct. On the other end there are teachers who allow their learners to use their own strategies. The reason for allowing their learners to make use of different strategies is for learners to see that there are a number of strategies to finding a solution for one problem as well as creating opportunities for them to justify their solutions.

When learners make use of strategies taught by their teachers it is found that teachers are more comfortable to support the learners' mathematical thinking without eliciting their learners' mathematical thinking (Fraivillig, Murphy and Fuson, 1999). When learners make use of their teachers' solutions only, they are denied the opportunity to engage in mathematical thinking where they have to make inferences and conjectures, it thus become meaningless for them to justify their solution strategies. When learners make use of their own solution strategies teachers should first elicit their mathematical thinking by asking them to justify why they selected a particular solution strategy in order to determine both the learners' conceptions and misconceptions.

2.3 MATHEMATICAL THINKING

Mathematical problem solving is a key part of doing mathematics (National Council of Teachers of Mathematics, 1989). Doing mathematics requires of learners to engage in mathematical thinking (Stein, M. K., Grover, B. W., and Henningsen, M.A., 1996).Learners engage in mathematical thinking when they frame and solve problems, look for patterns, make conjectures, examine constraints, make inferences from data, abstract, invent, explain, and justify (Stein et al, 1996). Kilpatrick et al (2001) describe mathematical thinking in terms of being mathematically proficient. Mathematically proficient learners are learners who are competent in the five interwoven strands. These five strands are defined by Kilpatrick et al (2001:116) as the following:

Conceptual understanding*: comprehension of mathematical concepts, operations and relations.*

Procedural fluency*: skill in carrying out procedures flexibly, accurately, efficiently and appropriately.*

Strategic competence*: ability to formulate, represent, and solve mathematical problems.*

Adaptive reasoning*: capacity for logical thoughts, reflection, explanation, and justification.*

Productive disposition*: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.*

Traditional notions of doing mathematics only emphasized the importance of one strand, procedural fluency. Doing mathematics means memorizing presented facts, applying algorithms or procedures with little conceptual understanding (Stein et al, 1996). A problem solving approach to doing mathematics as suggested in the new curriculum emphasizes the importance of learning important mathematical ideas by inventing multiple strategies to solve problems (Stein et al, 1996; Lubienski, 2000). Verschaffel, L., De Corte, E., and Vierstraete, H. (1999) documented that grade 5 and 6 learners solved non-routine addition problems incorrectly by applying the wrong strategies. Learners applied incorrect strategies due to a "lack of conceptual understanding in informal strategies like counting, insufficient proficiency in heuristic methods and an inability to think about their erroneous thinking whilst solving problems" (Verschaffel, De Corte, Vierstraete, 1999:282). In terms of informal strategies like counting and heuristic methods, Kilpatrick et al, (2001) refer to "less sophisticated approach' and "more sophisticated algebraic approach." A less sophisticated approach or strategy is when the learners make use of trial-and- error or "guess and check". A more sophisticated approach would be when learners can construct an equation and to solve the equation (Kilpatrick et al, 2001:126).

Incorrect strategies by learners are not only the result of a lack of conceptual understanding but also consequence of a lack in procedural fluency, strategic competence, adaptive reasoning and productive disposition. Learners can follow procedures in the absence of conceptual understanding due to the fact that they have memorized the steps to follow the procedures. When procedures are memorized without conceptual understanding, learners may apply those procedures incorrectly. Incorrect strategies can be circumvented if learners are able to engage in adaptive reasoning. When learners engage in adaptive reasoning during the process of constructing solution strategies they are able to logically think about the conceptual

understanding and correct procedures needed. They are also able to justify why they selected a particular strategy. Kilpatrick et al (2001) state that adaptive reasoning interweaves with the other strands, conceptual understanding, procedural fluency and strategic competence, specifically during problem solving. Without the development of the already mentioned strands productive disposition will also not develop. Kilpatrick et al (2001) argue that when learners develop in strategic competence when solving non-routine problems, their attitudes about themselves as mathematical problem solvers become more positive. Teachers can encourage a positive attitude towards mathematics through non-routine problem solving (Kilpatrick et al, 2001 and Lubienski, 2000).

2.4 PEDAGOGICAL CONTENT KNOWLEDGE IN EVERYDAY AND MATHEMATICAL CONTEXTS

Pedagogical content knowledge in my view can be referred to as a network consisting of knowledge links essential for the teaching of mathematical problem solving. Pedagogical content knowledge (PCK) links content with pedagogy (Ball, 2000). Ball (2000) further argues that pedagogical content knowledge (PCK) is embedded in content knowledge which consists of more than having an understanding of the mathematical content for personal use. The network of pedagogical content knowledge consists of knowledge of the mathematical content, referred to as content knowledge, knowledge about how learners learn mathematical concepts, how to make mathematical knowledge accessible to learners and how to support them when they make errors. In this network of knowledge there could be links of knowledge that are weaker than other links of knowledge. Ma (1999) found that USA teachers in her study had weaker content knowledge and pedagogical content knowledge than the Chinese teachers in teaching the four basic operations. In their study, Carpenter, Fennema and Frank (1996) found that there were teachers with a weak knowledge of how to elicit learners' solution strategies.

Studies of Ma (1999); Ball and Bass (2000) and Hill, Ball and Schilling (2008) have expanded on Shulman's notion of pedagogical content knowledge. For the purpose of this study I am using a reformulated definition of Shulman's (1986) pedagogical content knowledge:

Pedagogical content knowledge includes knowledge of the conceptual and procedural knowledge that students bring to the learning of a topic, the misconceptions about the topic that they have

developed, and the stages of understanding that they are likely to pass through in moving from a state of having little understanding of the topic to mastery of it. It also includes knowing of techniques of assessing students' understanding and diagnosing their misconceptions, knowledge of instructional strategies that can be used to enable students to connect what they are learning to the knowledge they already possess, and knowledge of instructional strategies to eliminate the misconceptions they may have developed (Carpenter, Fennema, Peterson and Carey, 1988).

This definition of pedagogical content knowledge encapsulates the role of teachers in a problem solving approach. The reason why teachers engage their learners in problem solving is not only for learners to learn about solving a particular problem but for them to develop important mathematical concepts (Lubienski, 2000). The role of teachers is to highlight the intended concepts through the learners' problem solving activities (Lubienski, 2000). The reason for developing important mathematical concepts is for learners to connect these concepts to their existing knowledge and thus construct new knowledge. Knowledge of how to create opportunities for learners to construct their own knowledge is different than knowledge of how to teach algorithms. Ma (1999) argues that teachers need to "package knowledge" and how they package their mathematical knowledge is based on their knowledge of conceptual understanding and procedural fluency. Ma (1999:23) further found that teachers with knowledge of conceptual understanding had a well-structured knowledge package consisting of "procedural topics, conceptual topics and basic principles of the subject." The manner in which teachers package their knowledge will influence their knowledge of their learners' conceptual and procedural understanding, as well as developing or hindering the construction of mathematical concepts. Knowledge of algorithms is an important part of constructing one's own knowledge and cannot be discounted in learning mathematics; it however, is only one part of constructing mathematical knowledge.

In a problem solving approach is it required of mathematics teachers to have both a sound knowledge of the mathematical concepts they teach as well as knowledge of how to make mathematical knowledge and problem solving accessible to their learners. For this study, such knowledge is examined both in mathematical contexts and everyday contexts. How teachers make knowledge accessible to their learners with regards to mathematical contexts and everyday contexts will depend on their notions of the two contexts. A common notion is that learners can

only engage in problem solving by means of word problems. Learners however, can engage in solving problems in everyday context which are not word problems. According to Briars and Larkin (1984), word problems are the primary contexts in which children are asked to apply mathematical knowledge to solve problems, rather than simply to perform algorithms. I question whether this notion of Briars and Larkin (1984) is also held by other mathematics teachers. The implication of this notion could be the result of teachers packaging their knowledge about problem solving in mathematics contexts as purely procedural. It is important for teachers to package their knowledge on both a "conceptual topic" and "procedural topic" (Ma, 1999).Knowledge packages illustrate teacher's level of understanding of the mathematics content or level of content knowledge.

Teachers' level of understanding of the mathematics content is fundamental in planning and implementing instruction as seen in Shulman's (1987) model of pedagogical reasoning and action. Pedagogical reasoning is a cyclic activity of "comprehension, transformation, instruction, evaluation, reflection and new comprehension (Shulman, 1987:112). Understanding or comprehension of the content is the beginning and end of Shulman's (1987) model of pedagogical reasoning because it begins with comprehension and a new cycle starts with new comprehension. Pedagogical reasoning is the process of rigorous reasoning about the teaching practice (Shulman, 1987). Pedagogical reasoning entails both the preparation of a lesson as well as the instruction thereof. According to the model of pedagogical reasoning and action, preparation is defined as a transformation activity. Transformation not only includes preparation which is the analytical understanding of the content and the setting of goals for a particular lesson. Once an analysis of the content is done and clear goals are set for the lesson the teacher needs to decide how the content would be represented. Content can be represented in "analogies, metaphors, examples, demonstrations and explanations" (Shulman, 1987:113). A selection of different teaching approaches need to be chosen, adjusted and designed to meet the conceptions, misconceptions, social background, race and gender of the learners (Shulman, 1987). The action of transformation is followed by instruction; instruction is the implementation of the above mentioned actions of transformation which includes the social factors of whether learners will sit and work in groups or individually and whether learners will be allowed to engage in questioning.

Planning is essential for effective construction to take place but teachers should bear in mind that "no amount of planning can predict what all learners might say and do in the classroom, and so teachers also need to be able to draw on their knowledge to hear and respond to unanticipated learner ideas in the classroom" (Brodie, 2003:144). During as well as after the lesson the teacher has to evaluate whether the learners understood that which was taught to them. The teacher also has to evaluate whether the lesson goals that were set have been reached. The evaluation of whether goals have been reached creates an opportunity for the teacher to reflect and review the lesson by focusing in particular on instances that can be improved. The above mentioned process of comprehension, transformation, instruction, evaluation and in particular reflection generates new comprehension. The teacher thus has an enriched understanding of the content, the learners and instruction practices (Shulman, 1987). I also want to think of the gaining of new comprehension as gaining enriched pedagogical content knowledge

Pedagogical content knowledge is closely linked to practice for a "practice is an emergent phenomenon rather than an already-established way of reasoning and communicating into which students are to be inducted" (Cobb, Stephan, McClain and Gravemeijer, 2001:121). Kilpatrick et al (2001) advocate for classroom practices which promote the development of all five strands of mathematical proficiency, especially strategic competence during the teaching of non-routine problem solving. Developing strategic competence and the four other strands require from the learners and teacher to discuss their mathematical reasoning amongst each other by explaining and justifying their mathematical concepts (Brodie, 2008). These types of discussions where both the teacher and learners express their mathematical thinking are characterized by socio-mathematical norms (Kazemi and Stipek, 2001).

Socio-mathematical norms encourage learners to engage in "conceptual mathematical thinking and conversations" (Kazemi and Stipek, 2001). Socio-mathematical norms are established when the teacher creates opportunities for learners to engage in these norms by "high-press" interaction (Kazemi and Stipek, 2001). According to Kazemi and Stipek (2001) a high-press teacher-learner interaction (i) allows the learners to respond with a mathematical argument and not just describing the following of procedures; (ii) mathematical thinking requires from learners to identify links between different strategies; (iii) errors generate opportunities for learners to search for mistakes and construct new strategies.

A high-press teacher-learner interaction not only allows the learners to engage in mathematical argument with each other and with the teacher but it also requires from the teacher to listen to the learners' arguments. The types of listening required to listen to learners' mathematical arguments are interpretative and hermeneutic listening (Davis, 1997). Davis (1997) describes three types of listening, evaluative, interpretative and hermeneutic listening. Evaluative listening does not allow learners to engage in mathematical arguments because the teacher is only listening for the response s/he expects .Interpretive listening is when the teacher listens to the learners' responses but in view of their mathematical thinking. Hermeneutic listening is the type of listening where the roles of the teacher and learners are not as explicit as with evaluative listening. Both the teacher and learners engage in open-ended discussions meaning that the purpose of the discussion is not directed at gaining specific knowledge. Engaging in interpretive and hermeneutic listening requires thoughtful planning for the role of the teacher has shifted.

The role of the teacher is to institute and steer the development of socio-mathematical norms in order to facilitate conversations among learners while they engage in collaborative problem solving and to support learners' understanding of what acceptable mathematical justifications are (Yackel, 1995). Ways in which teachers can facilitate such discourse in their classrooms is through eliciting their learners' solution strategies, supporting learners' conceptual understanding and extending the solution strategies (Fraivillig, Murphy and Fuson, 1999). In the teacher's role in supporting whole-class collaborative inquiry, Staples (2007:172) refers to the "tripartite model" which comprises of three components "supporting students in making contributions; establishing and monitoring a common ground and guiding the mathematics." Staples (2007:174) further elaborates on supporting students in making contributions in terms of "eliciting student ideas, scaffolding the production of ideas and creating contributions." In discussing the meaning of establishing and monitoring a common ground Staples (2007:180) refers to "creating a shared context and maintaining continuity over time." In terms of guiding the mathematics Staples (2007) refers to guiding that the tasks implemented are of high-cognitive demand and that the level of the task is maintained (Stein et al, 2000).

The above discussion has brought together pedagogical content knowledge and teachers' practices. As will be shown in my analysis these are not easily separable. In the next section I discuss the theoretical framework that frames discussion and justification in my study.

2.5 CONSTRUCTIVIST AND SOCIOCULTURAL PERSPECTIVES

Constructivists are of the notion that learners construct new knowledge by means of a viable fit between their prior knowledge and new knowledge (Piaget, 1977). Prior knowledge is thus important in order for learners to assimilate or accommodate new knowledge. According to constructivists, learners' existing schemas experience perturbations when they learn new knowledge, meaning their schemas are in disequilibrium. In order for their schemas to reach the state of equilibrium the learners have to accommodate or assimilate. Accommodation refers to the way in which people adapt their ways of thinking to new experiences. Assimilation refers to the way in which people transform information so that it fits within their existing way of thinking (Siegler, 1995). When encountering a non-routine mathematical problem learners experience a perturbation in their existing schemas because they cannot make use of a ready algorithm.

My particular interest for this study is on the role of discussions in the classroom according to constructivist perspective. Dialogue within a constructivist classroom has its origins in Piaget's clinical interview. The purpose of these interviews was for the interviewer to build a model of the learners' thinking processes. Discussion in a classroom is similar to Piaget's clinical interview for the teacher asks the learners to justify or explain their solutions or solution strategies in order to gain insight to the learners' mathematical thinking. According to Sherin (2002) class discussions involve the manner in which questions and comments are elicited and how the class reaches consensus. Teachers thus can create "discourse communities" (Sherin, 2002), by instilling socio-mathematical norms within their classrooms. This form of discussion is different to the Initiate-Response-Evaluate pattern of classroom discourse where the teacher initiates a question which is followed by a learner's answer and the teacher evaluates whether the learner's response is correct (Mehan, 1979 in Brodie, 2007). Dialogue in a mathematical problem solving class is a two-way process. The onus for initiating discussions in the classroom is not just on the teacher but the learners can also initiate discussions by posing problems to fellow learners.

Discussions are important within a constructivist perspective but there are many challenges which teachers are faced with. First teachers are not always sure how to begin and to end a discussion (Brodie, 2007). Second, teachers are also faced with the dilemma of whether to promote a classroom environment which encourages conversations or having a classroom which encourages mathematical conversations (Sherin, 2002). Teachers may be knowledgeable of the

importance of learner participation but may not have the knowledge of how to engage their learners in mathematical discussions. By this I mean that teachers may engage in the Initiate-Response-Evaluate (IRE) sequence of classroom discourse but do not know how to use this sequence to engage their learners in mathematical thinking. Brodie (2007) has found that through the IRE- pattern of interaction teachers can create opportunities for their learners to engage in true mathematical discussions by not lowering the task demands and by not directing learners to the correct answer through funneling. Funneling is the process of fragmenting the task by asking simpler questions and when the learners are unable to respond the teacher resorts to giving the learners the answer (Bauersfeld, 1988). A pattern of practice which has been shown to enhance mathematical discussions is: (i) eliciting learners' solution methods and (ii) supporting learners' conceptual understanding and extending learners' mathematical thinking (Yackel, Cobb and Wood, 1991). It is difficult to know how to do this if you have not been trained to or taught in this way yourself.

Confrey and Kazak (2006) are of the opinion that for a better understanding of learning mathematics it is important for constructivism to bridge with socio-cultural perspectives because "mathematics learning entails critical elements of grounded activity and socio-cultural communication and that these components interact in important ways" (Confrey and Kazak, 2006:333). For these components to interact, I see a need for language to serve as a bridging tool. In both perspectives, constructivism and socio-cultural, language is one of the main building blocks. Language however plays different roles in the two perspectives. For constructivists, language "does not create learning but follows as a result of learning" (course notes, Brodie, 2007). A socio-cultural perspective is based on the notion that language is a tool which "mediates and creates learning" (course notes, Brodie, 2007). The role of language being a tool to create learning is especially evident within the Zone of Proximal Development (ZPD). According to Vygotsky the ZPD is the "distance between the actual development level as determined by independent problem solving and the higher level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, cited in Daniels, 2001:57). The ZPD serves as a two-way mechanism for learning because in as much as learners learn from the teacher's guidance, the teacher learns from the learner's ideas and actions (Goos, Galbraith and Rentshaw, 2002). Teachers however should be aware of the fact that the learner's own prior knowledge may come into conflict with

the support given (Daniels, 2001). As the more capable other assists the novice to reach the stage of independence, the capable other can do so by making use of scaffolding. The more capable other scaffolds when s/he gives graduated assistance in order to simplify learners' tasks without lowering the difficulty of the task (Greenfield, cited in Thorpe and Gallimore, 1991). An important aspect of teachers' pedagogical content knowledge is the aspect of how the teacher makes knowledge accessible to learners through the process of mediation and scaffolding in the ZPD. In order for teachers to elicit learners' conceptions and misconceptions they do so via the use of language.

On route to the level of independent problem solving learners may experience difficulty in developing mathematical concepts when solving problems. Teachers thus have to intervene and as argued by Lubienski (2000:460), teachers need to prompt learners' "explorations in fruitful directions." This process of prompting learners' explorations is part of scaffolding in a sociocultural approach. It is important for the difficulty of the task to remain constant, since higher cognitively demanding tasks engage learners in mathematical thinking (Stein et al, 1996). Scaffolding by the teacher not only promotes mathematical thinking in the form of eliciting learners' solution strategies, supporting their conceptual understanding (Fraivillig, Murphy and Fuson, 1999) but also raises an opportunity for the teacher to elicit learners' misconceptions. In order to use learners' ZPD's as a tool in teaching it is important to understand learners' current knowledge, in particular their misconceptions.

2.6 DEALING WITH MISCONCEPTIONS

Part of teachers' pedagogical content knowledge is how to deal with learner errors and misconceptions. In this study, I distinguish errors from slips, which can be easily corrected (Olivier, 1989). Errors made by learners arise from misconceptions, "the notion of misconceptions denotes a line of thinking that causes a series of errors all resulting from an incorrect underlying premise" (Nesher, 1987:35). Misconceptions are not easily detected by teachers because they are not necessarily sensitive to them and on "some occasions the mistaken rule is disguised by a "correct answer" (Nesher, 1987:38). In a problem solving approach teachers can detect learners' misconceptions as mentioned earlier by eliciting their solution strategies (Fraivillig, Murphy and Fuson, 1999). Once these misconceptions are elicited teachers

can probe the thinking underlying the errors in order to create opportunities for all learners to deepen their mathematical understanding (Fraivillig, Murphy and Fuson, 1999).

In eliciting misconceptions teachers will come across a variety of misconceptions, for learners are not all at the same levels within Zones of Proximal Development. Teachers can make use of these misconceptions when they teach. An important factor which teachers need to take into account, when using misconceptions to teach is that misconceptions could be deeply rooted and robust to confront (Smith, diSessa and Rochelle, 1993). A strategy teachers may want to use to confront learners' misconceptions is to make use of correct strategies used by capable peers (Fraivillig, Murphy and Fuson, 1999). However it may take a lot of mediation for learners to understand the correct strategies and be willing to give up their misconceptions.

In Vygotsky's theory the capable other refers to the importance of the social plane of the learner. When defining the Zone of Proximal Development Vygotsky (1976) referred to the actual level as the individual level and the potential level as the social level. The social plays an important role not only in learning but in the construction of misconceptions as well. Misconceptions have their origin from learners' prior knowledge, which could be acquired in the classroom or from interacting with their physical or social world (Smith, diSessa and Rochelle, 1993). Teachers thus can make use of this social aspect of learning to confront learners' misconceptions. Their instructional practices can be of such a nature which will allow for classroom discussions in which students make sense and explain problematic phenomena and give justification for their ideas (Smith, diSessa and Rochell, 1993).

2.7 CONCLUSION

In this chapter I have developed an analytical and theoretical framework in relation to what pedagogical content knowledge is needed to teach problem solving in mathematical and everyday contexts. I have argued that pedagogical content knowledge is embedded in content knowledge and that pedagogical content knowledge is linked to practice. Although my study focuses on pedagogical content knowledge, I will need to look at content knowledge and practice as well. The five interwoven strands of mathematical proficiency (Kilpatrick et al, 2001) forms the basis for my analysis of content knowledge; the model of pedagogical reasoning (Shulman, 1987) and the Zone of Proximal Development (Vygotsky, 1978) form the basis for analysing

pedagogical content knowledge. I have further argued that I see the need for constructivism and socio-cultural perspectives to bridge through language. Language is a tool to create learning and is especially evident in the Zone of Proximal Development (ZPD). I have also looked at how teachers can create opportunities for learners to engage in mathematical thinking through discussions. In the next chapter I will discuss the Research Design of my study.

CHAPTER 3

METHODOLOGY AND RESEARCH DESIGN

In this chapter I discuss the methodology of the study and the different data collecting methods which underpin this study. Included in this chapter is a brief discussion on the nature of the tasks and why the tasks were selected for the study.

3.1 METHODOLOGY OF THE STUDY

The nature of this study is qualitative and to define what a qualitative approach is, I selected one of many definitions from the literature.

"research procedures which produces descriptive data: people's own written or spoken words and observable behavior. [It] directs itself at settings and the individuals within those settings holistically; that is, the subject of the study, be it an organization or an individual, is not reduced to an isolated variable or to a hypothesis, but is viewed instead as part of a whole" (Bogdan and Taylor, 1978:213).

The emphasis of my study is on describing the practices of three grade 6 mathematics teachers as observed through interviews and lesson observations. These three teachers are referred to as the subjects of this study. Coinciding with the above definition of a qualitative approach my study emphasized the following characteristics of a qualitative approach (Hatch, 2002:6).

1. Natural Settings
2. Participant Perspectives
3. Researcher as Data Gathering and Analytic Instrument

In order to find out what content knowledge and pedagogical content knowledge is needed to teach mathematical problem solving in both mathematical and everyday contexts, the object of my study was the teachers' "lived experiences" obtained through a semi-structured interview. To experience the teachers' "real setting" (Hatch, 2002:6), I also observed lessons taught by the teachers. The emphasis of this study is on "real people in real setting" (Hatch, 2002:6). Taking this into account I had to discontinue my study with one of the teachers. Initially my study

focused on four grade 6 mathematics teachers but during the lesson observation I suspected that one of the teachers taught rehearsed lessons. This means that the lessons were not a real setting and would influence the validity of my results so I excluded this teacher from the study. The validity of my research design is discussed later in this chapter.

Interviews were conducted before the lesson observations. A reason for this approach was to gain insights into the teachers' perspectives on mathematical problem solving and the content and pedagogical content knowledge needed to teach it. Another reason was to establish whether there was a link between the teachers' perspectives and their actual teaching practice.

As the researcher I set and structured the interview schedule, based on my research questions, in order to gain the necessary data which will address the research question. Lessons were observed by me taking notes in the class as well as through the means of a video-recorder. The data gathering methods I have used in this study are further discussed in the next section.

3.2 RESEARCH DESIGN AND METHODOLOGY

3.2.1 The context and sample

This study was conducted in November 2008, in three Grade 6 classes at three different schools in Gauteng. Table1 describes the profiles of the teachers in the study.

Table 1: Profile of teachers in study

	Teacher A	**Teacher B**	**Teacher C**
Number of years teaching.	12 Years	15 Years	10 Years
Number of years teaching grade 6 mathematics.	8 Years	10Years	10 Years
Grades currently teaching	Grade 6	Grades 5-7	Grades6-7
Kind of school.	Township school. Moderately resourced	Township school. Moderately resourced	Ex-Model C-School. Well resourced.
Language of Learning and Teaching.	English	English	English
First Language	Xhosa	Sesotho	English
HighestQualification	Advanced Certificate inEducation	B Ed Honours	B Ed.

The selection of my sample was purposive. As I have mentioned in Chapter 1, this study forms part of the broader DIPIP project (Data Informed Practice Improvement Project). The three

teachers formed part of a group of 41 teachers who participated in the DIPIP program. I chose these teachers purposively because they had some experience of the test items and some discussions on misconceptions in the DIPIP sessions. This was important for the purpose of the study which was to look at how teachers distinguished between everyday and mathematical contexts in problem solving as well as how they dealt with learners' misconceptions.

3.2.2 Ethics

In order to undertake my study I needed to abide by the University of the Witwatersrand ethics protocol. I therefore submitted an application to the School of Education ethics committee for ethics clearance, and to the Gauteng Department of Education for permission to do research in the respective schools. I have received permission from both the University of the Witwatersrand Education Ethics Committee (Protocol 2008 ECE 82) and the Gauteng Department of Education to conduct the study.

Letters accompanied by a description of the study were given to the teachers, their principals (who I also asked for permission personally to do the study) and the learners' parents.(See APPENDIX 1.1 and 1.2 for letters). These letters asked permission to undertake the study, including videotaping lessons. Letters issued to the participants ensured that the participants' anonymity will be guaranteed, and that they were free to withdraw from the study at any time.

I acted ethically in the field by respecting and appreciating the teachers and learners participating in this study. During the interviews I respected the teachers' views and responses to the questions by not imposing my views. After the lesson observations I expressed my gratitude towards the participating learners, teachers and their principals. I further respected the fact that the teachers were individuals with different levels of pedagogical content knowledge as well as different levels of problem solving skills (Griffiths, 1998). I have done my best to put all participants at ease so they would be comfortable with the study.

3.2.3 The Tasks and task analysis

Tasks were selected from the grade 6 International Competitions on Assessment in Schools, (ICAS, 2006) question paper which were used in the DIPIP project. Four problem solving tasks were selected, two mathematical context tasks and two everyday context tasks. These tasks were given to the three teachers to solve, to gain an understanding and insight of their knowledge as to how they would teach these tasks to their learners or use them as part of a lesson.

When selecting tasks one has to understand the nature of the tasks because "the nature of tasks can potentially influence and structure the way students think and can serve to limit or broaden their views of the subject matter with which they are engaged" (Henningsen & Stein, 1997:525). I selected four tasks that are of a non-routine problem solving nature. Stein et al (2000) refers to the nature of tasks in terms of cognitive-level demands. They distinguish between low- level demand tasks which requires of learners to make use of previously memorized procedures, which are not connected to the context. Higher-level demand tasks requires of learners to make use of procedures which are connected to the context of the task in order to find a solution. Learners are also required to make use of different strategies. In Stein et al's (2000) terms, non-routine problems are the higher demand tasks. As discussed in Chapter two, authors such as Charles and Lester (1988) also refer to the nature of tasks but in terms of simple translation problems, complex translation problems and process problems. Simple problems are the common story problems which require any one of the four basic operations to solve and are there fore also referred to as routine problems, whereas the complex problems require more than one step in order to solve them but are still routine problems. Process problems require much more thinking for learners have to construct their own solution strategies and therefore are non-routine problems.

To describe the four tasks I used in this study I made use of both the descriptions of Stein et al (1996) and Charles and Lester (1984). Charles and Lester's (1984) description mainly refers to "word problems" or the everyday context. Stein et al (2000) description refers to both mathematical contexts and everyday contexts. Table 2 below, sets out the tasks, while Table 3 gives a description of the tasks.

Table 2: Tasks

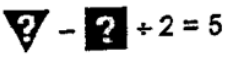
Task 1  Which of these makes the number sentence true? (A)  = 6  = 2 (B)  = 14  = 4 (C)  = 18  = 8 (D)  = 20  = 10	**Task 2:** Look at the pattern of numbers. 7x11x13x1=1001 7x11x13x2=2002 7x11x13x3=3003 Find the pattern of 49x55x26 which equals to 70070
Task 3: **Four children ate some berries.** **Ben and Nina ate half of the berries.** **The other half was eaten by Ali and Sarah.** **Ben ate 4 berries.** **Nina ate twice as many berries as Ben.** **Ali ate half as many berries as Ben.** **Sarah ate 10 berries. How many berries were there?**	**Task 4** **Douglas, Ming and Omar each has a bag of marbles.** **Ming has 3 less marbles than Douglas.** **Omar has 12 less marbles than Douglas.** **Which statement is true?** **A. Ming has 15 more marbles than Omar.** **B. Omar has 15 more marbles than Ming.** **C. Omar has 9 more marbles than Ming.** **D. Ming has 9 more marbles than Omar**

Table 3: Task analysis

Tasks	Task 1	Task 2	Task 3	Task 4
Context	Mathematical context	Mathematical context	Everyday context	Everyday context
Nature of tasks	**Kilpatrick et al (2001):** Non-routine. **Stein et al (2000):** Higher cognitive demand: procedures with connections. **Charles and Lester (1988):** Complex translation: Translation from place holders to numbers.	**Kilpatrick et al (2001):** Non-routine. **Stein et al (2000):** Higher cognitive demand: procedures with connections. **Charles and Lester (1988):** Process problem: Multi-step problem. Translation of multiplication in terms of given pattern in order to find the solution.	**Kilpatrick et al (2001):** Routine. Stein et al (2000): Lower cognitive demand: following procedures **Charles and Lester (1988):** Simple multi-step. The original ICAS-task was a process problem for learners had to construct their own solution strategy. The task thus has changed to a simple but multi-step problem.	**Kilpatrick et al (2001):** Non-routine. **Stein et al (2000):** Higher cognitive demand: procedures with connections. **Charles and Lester (1988):** Process problem: Although learners are given four options they have to find the solution themselves by reasoning who has less or who has more.
Concepts needed	Learners have to substitute the different values by being able to connect the correct place holder with the correct number. They then have to solve the problem by identifying the correct order of operations. Learners need concepts of the BODMAS-rule.	Learners have to use prior knowledge of multiplication. In order to find multiples of 49, 55 and 26. Learners then have to write the multiples following the pattern, 7x7x11x5x13x2. They then have to multiply 7x11x13x5x2x7=7x11x13x70=1001x70, which is 70x1000+70=70070. Learners have to see the pattern and importance of multiplying with 1001.	The ICAS-problem was a higher cognitive demand task. The cognitive demand was lowered. Learners basically need conceptual understanding of the words half as many and twice as many. Since the problem tells the learners that Ben ate 4 berries, they have to calculate what twice as many is of 4 and they have to calculate what half of 4 is.	Learners have to calculate the difference between Ming and Omar in relation to Douglas. Once they have worked out the difference between Ming and Omar they can select the correct statement Not only is it important to focus on the relation between quantities but the fact that we can focus on relations between quantities without knowing any of the quantities as found in this task is very important. This task thus may be useful as an introduction to variable quantities in algebra.

I have analyzed the tasks in terms of their context, mathematical and everyday as well as by looking at the nature of the tasks by categorizing them according to Kilpatrick et al (2001), Stein et al (2000) and Charles and Lester (1984). I have also given a brief description of the concepts needed in solving the tasks which I will bring into the analysis.

3.3. INTERVIEWS

The purpose of the interview questions was to gain insight into the teachers' content and pedagogical content knowledge. The first section of the interview questions is on the teachers' solution strategies and their knowledge about their learners' solution strategies. The second section is to find out how the teachers will make knowledge accessible to their learners by explaining how they would go about in planning to teach the tasks. The third and last section asks how the teachers will identify possible misconceptions. A second purpose of the interview questions is to establish whether the teachers have different pedagogical content knowledge in the mathematical context and everyday context.

Teachers were interviewed about their task solutions using a task based interview schedule. I interviewed the teachers individually for one and a half hours each after school. In order to ensure the accuracy of the transcription of the interview recordings I had to interview the teachers in a quiet surrounding. (See APPENDIX1.3 for interview-schedule).

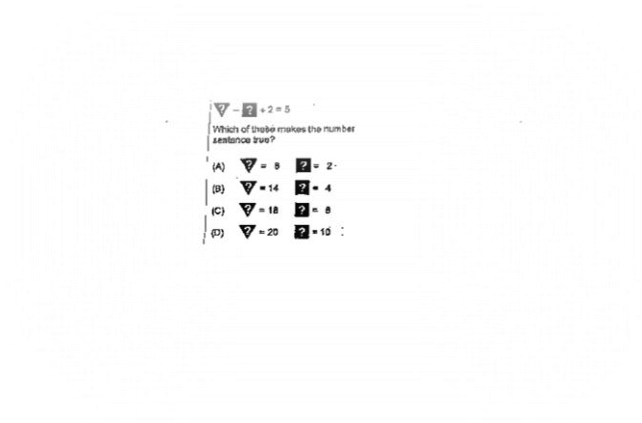
The teachers were asked to solve the tasks and to explain their solution strategies. As the teachers worked through the tasks, questions were asked from the interview schedule. I asked probing questions to give teachers an opportunity to give in-depth explanations. The purpose of the task-based interviews was to gain insight into both content and pedagogical content knowledge. In order to ensure that the questions in the task-based interview yielded answers which would be suitable for analysis, the interview was piloted.

3.4 PILOTING

The task-based interview was piloted with two other grade 6 mathematics teachers. The reason for piloting the task-based interviews was to eliminate questions which might be confusing. It was also important to establish whether the time allocated would be enough, (Opie, 2004). The

main reason for piloting the interview was to establish whether the questions gave the data required. Based on the outcome of the piloted interviews I had to amend two of the tasks. The teachers in the pilot study felt that I needed to amend Tasks 2 and 3 to make them more accessible for the learners. Originally all four tasks were multiple-choice tasks. I have amended these two tasks to problems where learners have to construct their own solutions and not select a solution from the choices given. In task 3, I omitted the four options given and I have asked the learners to find the pattern by giving them the answer. In task 4, I also omitted the four choices and added the sentence, "Ben ate four berries." The amendments to task 2 did not change the cognitive demands of the task but the changes did lower the demands of task 3 (see Table 4).

Table 4:Original and Amended Tasks

Task 1  	**Task 2: Original task** *Look at the pattern of numbers.* *7x11x13x1=1001* *7x11x13x2=2002* *7x11x13x3=3003* *Find the pattern of 49x55x26 which equals to* *(A) 7007 (B) 7070 (C) 70007 (D) 70070* **Task 2: Amended task** Look at the pattern of numbers. 7x11x13x1=1001 7x11x13x2=2002 7x11x13x3=3003 Find the pattern of 49x55x26 which equals to 70070
Task 3: Original task *Four children ate some berries.* *Ben and Nina ate half of the berries.* *The other half was eaten by Ali and Sarah.* *Nina ate twice as many berries as Ben.* *Sarah ate 10 berries. How many berries were there to start?* *(A) 12 (B) 20 (C) 24 (D) 40* **Task 3: Amended task** Four children ate some berries. Ben and Nina ate half of the berries. The other half was eaten by Ali and Sarah. Ben ate 4 berries. Nina ate twice as many berries as Ben. Ali ate half as many berries as Ben. Sarah ate 10 berries. How many berries were there?	**Task 4** Douglas, Ming and Omar each has a bag of marbles. Ming has 3 less marbles than Douglas. Omar has 12 less marbles than Douglas. Which statement is true? A. Ming has 15 more marbles than Omar. B. Omar has 15 more marbles than Ming. C. Omar has 9 more marbles than Ming. D. Ming has 9 more marbles than Omar

3.5 CLASSROOM OBSERVATIONS

Two lessons for each teacher were observed. The three teachers taught the same four tasks used in the task-based interview to their classes. The reason for using the same tasks was to view teachers' pedagogical content knowledge in action. By this I mean that when the teachers taught the same items to their learners, I was in a position to actually view how they responded to their learners' understanding of problem solving in everyday and mathematical contexts as well as how they dealt with the learners' misconceptions. The classroom observations also allowed me to view how constructivism bridged with socio-cultural perspectives. Constructivism served as a lens to view the importance and role of learners' misconceptions for the construction of concepts. Socio-cultural perspectives served as a lens for how teachers dealt with learners' misconceptions as well as how they assisted learners by providing scaffolding to help them move through their different Zones of Proximal Development.

One of my colleagues did the video-taping. She does not have much experience in video-recording lessons for research purposes. As the researcher I explained to her what the aim of my study was and that she had to focus on the teachers' teaching and how they interacted with their learners' responses. While the three teachers were busy explaining, the video-recorder was positioned where she could get a good view of the teacher. As the children were working through the tasks the video-recorder focused on different groups in the class. My role during the lesson observation was that of a non-participant observer. By being a non-participant observer I had no interaction with the teachers and learners while observing their lessons (Opie, 2004). My role in the classrooms was to take field notes of the lessons.

3.6 THE TRANSCRIPTS

The interviews were not difficult to transcribe due to the fact that the questions and responses were guided by the interview schedule.(See APPENDIX 1.4 for interview-transcripts). As I have mentioned earlier the purpose of the interview was to gain insight into the teachers' content and pedagogical content knowledge. To establish whether the teachers' responses in the interviews corresponded with their practices, I had to transcribe the lessons observed. I did not transcribe the full lessons due to the length of the lessons. My focus when transcribing the lessons was on how the teachers made the knowledge accessible to their learners and how they respond to

learners' responses. Transcribing the lessons was challenging because two of the teachers code switched, from explaining in English to SeSotho and IsiZulu. With the help of a colleague those sections of the lessons were translated into English.

3.7 ANALYSING THE TRANSCRIPTS

I used the task based interview schedule as a tool to analyse the interview transcripts. I focused on the teachers' content knowledge and pedagogical content knowledge. To analyse the teachers' level of content knowledge I focused on Kilpatrick et al's (2001) five interwoven strands of mathematical proficiency. To analyse the teachers' pedagogical content knowledge I focused on how teachers would make the mathematical knowledge in the tasks accessible to their learners, their knowledge about their learners' strategic competence and how they dealt with their learners' misconceptions. The usage of the interview schedule as a guiding tool did not stop me from seeing other data arising from the transcripts.

In analyzing the interview transcripts I had to find out if there would be differences or similarities between what the teachers said and what they actually did in their teaching practices. The lessons and partial transcripts were firstly viewed on their own and then in conjunction with the interview transcripts.

3.8 RIGOUR

Validity refers to the appropriateness, meaningfulness, and effectiveness of the conclusions a researcher makes (Fraenkel and Wallen, 1990). An appropriate conclusion of this study will be based on what pedagogical content knowledge the teachers have in this study with regards to problem solving in two contexts, mathematical and everyday. A meaningful conclusion would be based on the meanings I as the researcher derived from how the teachers responded to the tasks and how they taught the tasks in their classrooms. A useful conclusion would be in the form of an answer to the questions of my study. Validity of a study thus depends on the quantity and quality of the evidence there is to support the conclusions I as a researcher wish to make regarding the data I have collected (Fraenkel and Wallen, 1990).

A crucial question I need to ask myself is whether the task-based interviews and lesson observation yield useful information about grade 6 mathematics pedagogical content knowledge with regards to problem solving in two contexts, mathematical and everyday. According to Fraenkel and Wallen (1990) there are three main types of evidence a researcher might collect namely, content-related evidence of validity, criterion-related evidence of validity and construct-related evidence of validity.

Content-related evidence of validity: To ensure the validity of the content of the task-based interview schedule I piloted it with other two grade 6 teachers as I have mentioned earlier. Earlier in this chapter I also stated the reasons for piloting the interview schedule. The reasons that I have mentioned are key in determining the validity of the content of the task-based interview. The content in the form of questions in the task-based interview focus on gaining insight into the teachers' content and pedagogical content knowledge and this study intends to find out what content and pedagogical content knowledge is needed. I have also audio-recorded the interviews and video-taped the lessons as well as transcribed the interviews. Transcripts serve as evidence of the teachers' concepts and also as a means to circumvent that the researcher's ideas do not dominate the views of this study.

Criterion-related evidence of validity: To obtain criterion-related evidence of validity I need to compare the relationship between the teachers' responses of the interviews with their actual classroom practices. A positive relationship would be if what the teacher said during the interview correlates with what they did in the class. A negative relationship would be if what the teachers said during the interviews differs vastly from what they do in the classes and vice versa. *Construct-related evidence of validity:* Construct validity is more appropriate for the purpose of this study because it is the broadest of the three kinds of validity(Fraenkel and Wallen, 1990). In ensuring validity I need to be careful not to analyze the teachers' pedagogical content knowledge about problem solving in mathematical and everyday context based on my predetermined notions of pedagogical content knowledge. To circumvent this from happening I have discussed the theories and literature which underpin this study in chapter 2. In analyzing this data I will do so through the lens of the same theories and literature I have discussed in chapter 2. I will also try to be clear about my own notions when doing the analysis.

Reliability refers to the consistency of results obtained by a data analysis, whether similar results will be found a different time and by a different researcher (Fraenkel and Wallen, 1990). The role of reliability is not useful for this study for this is a qualitative study and reliability is mainly used in a quantitative study.

3.9 THE REPORT

In writing up a research report the aim is to connect the theories underpinning the study with the data analysis. The theoretical framework serves as a guide for the data analysis in the sense that the theories inform the data analysis just as the data analysis informs the theories (Opie, 2004). In writing up a qualitative study the quality of the findings is just as essential as connecting the theories with the data analysis.

In Chapter 2 I have selected literature which relates to the context of my study. It is important to understand that ideas and stances do not happen in isolation but in context (Opie, 2004). In creating the appropriate contexts I had to be reflective about the literature I read and whether I understood it in the appropriate contexts. In order for me to understand whether the literature was suitable for my study I had to be "open-minded" about the literature (Opie, 2004). This means that I had to be open to the possibility of not agreeing with everything I have read and that I was able to disagree with the evidence from another author.

When I analyzed my data in chapter 4 I had to ensure that it linked up with the theoretical underpinnings. I also had to ensure that the literature I reviewed served as evidence for my claims in analyzing the data.

In finalizing the report it is important for the research questions to be answered. My research report is concluded with a discussion on how the research questions are answered based on the data analysis and findings of the study.

CHAPTER 4

DATA ANALYSIS

4.1 INTRODUCTION

I divided my data analysis into three sections; content knowledge, pedagogical content knowledge and mathematical and everyday contexts. For both content knowledge and pedagogical content knowledge I discuss each teacher and each task in turn. In content knowledge I discuss their mathematical proficiency based on how they responded to the tasks. In pedagogical content knowledge I discuss their pedagogical reasoning during the interviews and their actual classroom practice. In everyday and mathematics contexts I discuss whether there is a difference in the teachers' approach to the teaching in the two contexts.

4.2 CONTENT KNOWLEDGE

In categorizing the teachers' content knowledge, I have looked at how they responded to the tasks as well as at the examples they used in presenting their lessons. I have categorized the three teachers' content knowledge based on their level of mathematical proficiency. The categories are represented in the table below.

Table 1: Teachers' level of content knowledge

Teacher	**Content knowledge**
Teacher A	Weak
Teacher B	Partial
Teacher C	Strong

Table 1 represents the teachers' level of content knowledge. The level of the content knowledge is classified based on teachers' conceptual knowledge of the concepts to be taught, their level of procedural fluency, whether they were able to find solution strategies, how they engaged in adaptive reasoning to find solution strategies as well as the disposition they portrayed during the interviews.

4.2.1Teacher A

In solving task 1 (see chapter 3), Teacher A showed that either she did not understand the order of operations, or she did not apply it in the problem. Her first response, having worked the problem out mentally, was that all options were correct. From the task, it can be seen that working from left to right, without using the applicable order of operations, options B, C and D would be seen as correct, but not A.

The interviewer (myself) then directed her that she should think about the order of operations. She then did the task in writing, substituting each number and said that option A is correct. In talking about the task the teacher said that it was "tricky" because it did not state that she had to use BODMAS[2] and because there were no brackets indicating which operation should be done first.

The examples used by Teacher A to teach the concept of the order of operations to her learners, indicated that she understood the order of operations as a rule which can only be applied when a mathematical problem has brackets. She explained what each letter of the acronym BODMAS represents and learners were reminded that brackets always come first.

To demonstrate to her learners that brackets always come first she made use of two examples. The first example was a very simple problem, $(7 + 3) - 4 =$, and the learners could respond with ease to the problem. A second problem $(24 \times 2) \div (6 + 7) =$, was written on the board but the teacher did not request an immediate solution to the problem. She requested from the learners to tell her where she should start, to solve the problem. One learner's response was that she should start with multiplication. Multiplication was not the answer she was looking for since she repeated the question and requested from the learners to look at the word BODMAS. She further reminded them that she had explained to them what each letter in the acronym BODMAS stands for. She then asked a specific learner to tell her where she should start and the learner responded that she should start with division.

[2]BODMAS: Is the acronym representing the order of operations. (**B**rackets, **O**f, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction).

Not satisfied with this response she asked the learners (the whole class) what it is that they should do? She emphasized that she wants her learners to understand that they need to follow the rule.

She concluded that the task was too complicated for her learners and therefore changed the example to $(24 \div 2) \times (6 + 7)$. Teacher A explained to her learners that they need to start with division because it is in brackets. She asked the learners to give the answer to $(24 \div 2)$ and the learners gave the correct response. She then asked for the answer for the second bracket, and the learners gave the answer of 13 and then multiplied 12x13, giving the answer of 156. In passing she mentioned to the learners, when working with BODMAS, they should also be able to subtract.

The above example indicates that the teacher had a weak conceptual understanding of the order of operations for she only understands it in terms of brackets. The first example she gave required of learners to add the numbers in brackets first and then subtract. The usage of brackets in the second example emphasises her difficulties in conceptual understanding, for her reason why the learners should start with division was that it was in brackets. This shows a procedural understanding, for she could have used the same example without the brackets. Using the same example or a different one without brackets could have indicated a deeper conceptual understanding of the order of operations.

For the remaining three tasks I will only give a description and analysis of how Teacher A solved the tasks, for she did not make use of any of these examples in her lessons. In solving task 2, the teacher did not understand how to sequence the factors of 49x55x26 in a similar pattern to the one given in the task. (see chapter 3). I explained to her, by showing her which factors were needed for the pattern and how she could group them to make it easy to find the product. The teacher, however, found the product with the use of a calculator which was acceptable for this task. In discussing the task she explained that she found the task very challenging because her first response to the task was to find the product of 49x55x26. She said she forgot about following patterns.

As I have mentioned in chapter 3, due to the difficulty the teachers experienced in solving task 3 I altered the task, the teachers solved the original task in the interview but taught the altered one

to their learners. In solving task 3, it appeared to me that Teacher A did not understand ratio for she repeatedly read the task. I then mentioned to her that it is a ratio problem she then stated that she and her colleagues do not teach ratio to their learners. She however attempted to solve task 3 but could not work out what half of and twice the amount of berries was. Sarah's 10 berries (see task 3), confused her, she halved the 10 berries. She could not move beyond the 10 berries to solve the task. After working through the task with her, she said that the language, "twice as many and half of" was also very difficult for her.

The concepts of "more than and less than" in task 4, also seemed to be a barrier for Teacher A to determine the correct statement. She observed that the statements did not ask about the amount of marbles of Douglas but that the question was about Ming and Omar's marbles, in relation to Douglas. She understood that if Ming has 3 less marbles than Omar she must have more marbles than Omar. She wanted to work out who had more or less in terms of Douglas. She selected option A, which meant that Ming has 15 more marbles than Omar. I indicated to her that Douglas has 15 marbles. In discussing the task she said the task really requires thinking because the inclusion of Douglas is confusing she also raised concerns about the "English" referring to the concepts of "more than and less than."

From the responses to the tasks and the examples presented by Teacher A, it can be seen that for her the following of procedures are more prevalent than conceptual understanding. In being unable to identify task 1 as an order of operations problem, due to the absence of brackets, shows that she understands the order of operations as a rule which needs to be followed. The rule is that the acronym, BODMAS must be followed in how the letters are sequenced. By not making use of different examples, such as problems with brackets and others without brackets, show that she only understands the order of operations as a rule to be followed. She did not make use of examples which required strategic competence and more adaptive reasoning.

In solving task 2, Teacher A stated that she forgot about the number pattern and just wanted to calculate the answer of 49x55x26, showing that she did not understand the concept of sequencing the numbers in that particular pattern. By finding the product of the three numbers further shows the importance of following of procedures with a lack in strategic competence and adaptive reasoning. She was unable to make use of a strategy, such as trial-and-error or any other strategy. Her difficulties with these strands are further seen in how Teacher A responded to task

3. She was unable to begin to solve the problem because there was a lack of conceptual understanding of ratio. The lack in a strategy was the result in the lack of procedural fluency and adaptive reasoning. In solving task 4, shows that she understood the concept of "more than and less than" but lacked in adaptive reasoning because she could not move beyond the mentioning of Douglas.

In solving all four tasks, Teacher A was positive and was willing to gain a better understanding of the underlying concepts. It seems by opting for the calculator in task 2, she became impatient with herself. It could be that she was concerned about taking up to much time in trying to understand.

4.2.2 Teacher B

In solving task 1, Teacher B showed that she did not apply the order of operations to the problem. Her first response, having worked the problem out mentally, that option D was correct. The reason for her first response was, she saw the numbers 20, 10 and 2 and the answer seemed correct. I then indicated to her that she should think about the order of operations. She then did the task in writing, substituting each number and said that option A is correct. In talking about the task the teacher said that it was "challenging" because it did not state that she had to use BODMAS.

In the examples used by teacher B in class she emphasized the importance of the brackets and the concept of working from left to right when following the order of operations. She started off by asking the learners what each letter of the acronym, BODMAS means. In the example, 3(14+2), she wanted to know what was the meaning of the brackets. Learners gave various answers such as addition, subtraction and division. To all these responses the teacher said no, until a learner said multiplication and she accepted the answer as correct. She further tried to bring across the concept of place holders in the form of squares and question marks. With the example, 3+2 = , she explained that they call this problem an open number sentence and that it is closed when there is an answer. In the third example, 12-4+8, Teacher B wanted to explain the concept of working from left to right. She explained to her learners, they must work from right to left for if they are going to work from left to right they will get negative integers and working with negative integers are above their grade 6 level. She showed them that 12-4 is 8 and

that 8+8 =16. She concluded by saying that when addition and subtraction are together they should work from left to right.

When explaining how to solve a problem with multiple operations like addition and subtraction as seen in the example of 12- 4 +8 it appears that the teacher confused herself. She wanted to explain to her learners that they should work from left to right, meaning that they first need to subtract 4 from 12 and then add the answer to 8. She however explained to her learners that they should work from right to left but she actually meant working from left to right because she concluded that when the two operations, addition and subtraction are together in a problem they should work from left to right.

In solving task 2, Teacher B did not understand what the requirements were for solving the task; she did not know where to start. By explaining to her how to break the numbers in multiples and how she ought to group them, she tried many ways of grouping the numbers before she could get them in the correct sequence. Once she sequenced the numbers, she multiplied the different groups of numbers. In talking about the task she said, she found the task to be very challenging because the task required from you to find the product of 49x55x26 but you have to find the pattern first.

In making use of examples to teach patterns she wanted the learners to come up with their own examples. She however gave a simple example of a set of odd numbers: 1,3,5,7,9,11,13. She wanted to know from the learners, what is happening to the numbers. The learners replied that you add 2 every time. The learners were then requested to take their time-tables out and to recite the 2-time-table. She then asked each group to use their multiplication-tables and to make their own number patterns. The first group's pattern was counting in 5's; Teacher B asked her learners what are they noticing? The learners said that you skip 5 every time. So the rest of the four groups gave their patterns of counting in 6's, 10's, 2's and 5 again. She did not get to the more difficult pattern required for task 2.

In task 3, Teacher B showed that she had a partial understanding of ratio. In solving task 3, she could represent the concepts of half and twice in terms of fractions but could not express them in terms of ratio. She only went as far as representing the quantities as 1:2 and 2:4, the other quantity of 10 confused her. This partial understanding of ratio is further seen in the examples

she used to teach. The first example was an example of "3 boys fighting with 2 girls." She wrote 3:2 and then she mixed 2 glasses of lime juice with 3 glasses of orange juice. With this example, she wanted to know what the whole is, of the concentrated juice. Learners responded, the whole is 5, she explained the whole of the concentrated juice is different to the whole when working with fractions. She gave the example of a pizza, divided into 8 equal parts, emphasizing that the whole is one pizza.

The examples on ratio indicated that Teacher B knew how to represent the ratio of two quantities. The examples also show that she had a partial conceptual understanding of ratio for she could explain to her learners the difference of what the whole would be for quantities in proportion and the whole that is divided into fractions. She has partial knowledge of ratio for she did not link fractions to ratio. She did not show her learners that you can write the ratio as a fraction and how to find the equivalent ratio values. In the two ratio examples the teacher made use of the same quantities but in two different everyday contexts. This shows that she wanted to emphasis what the whole is even in two different scenarios for she wanted the learners to see the difference between the whole of quantities in proportion to the whole of a fraction. Showing the learners the difference between the two concepts of whole was important for laying a conceptual foundation for the solving of task 3 for the task required from learners to find the whole of the different ratio's given.

In solving task 4, Teacher B showed that she understood the concepts of "more than and less than". She could identify that Ming had more marbles but she was deliberating whether Ming had 15 more marbles than Omar or 9 more marbles than Omar. She read the question again and decided that Ming has 9 more marbles than Omar. In being able to select the correct statement showed Teacher B conceptually understood the task but not enough to create examples when she taught the task in class. She did not make use of examples when her learners had to solve task 4.

Teacher B mentioned earlier that she could not think about the order of operations, for the task did not state apply BODMAS to solve the problem. By indicating that the task had to specify that BODMAS should be applied suggests that Teacher B can only apply the procedures when the task specifically indicates that the rule should be followed. Her examples however showed that she h ad a partial conceptual understanding. With her examples she not only wanted to demonstrate the importance of the brackets but the other operations as well. The example where

addition and subtraction are together in a problem indicates that she had a misconception of when to work from left to right. In the example she used, with addition and subtraction working from left to right does not apply for you get the same answer. The two examples did not require strategic competence and adaptive reasoning.

The solving of task 2 shows that Teacher B was not strategically competent to solve the task; because she did not understand which concepts were required to solve the task. Only after she understood how to solve the task, was she able to follow procedures. Her examples in class were simple and basic for grade 6 learners and did not enable the understanding of the concepts needed. The example on the pattern that is formed with consecutive odd numbers is easy for grade 6's for learners had to add 2 to each number. This example was relevant in the sense that she wanted to show them how a number pattern is formed by adding 2 every time but not relevant for the concepts needed to solve task 2. Instead of using the multiplication tables to show learners how the product can be broken up into its multiples the teacher asked the learners to create patterns by counting in multiples of 2, 5, 6 etc.

In task 3, Teacher B showed that she had a partial understanding of ratio because she could represent half as a ratio of 1:2 as well as find the equivalent ratio. She could also explain the different concepts of what a whole is. This partial understanding was the result of her not being able to construct a solution strategy and engage in adaptive reasoning. The teacher was unable to construct a solution strategy for she could not make use of the knowledge she already had to find the next ratio of 4:8.In solving task 4, she demonstrated procedural fluency for she understood that if Ming has 3 less marbles than Douglas than she must have more marbles than Omar.

In solving all four tasks, Teacher B was positive towards the tasks even when she experienced difficulty with some of them.

4.2.3 Teacher C

In solving task 1, Teacher C showed that she did not apply the order of operations to the problem. Her first response, having worked the problem out mentally, was that there were three possibilities, B, C, and D. As indicated earlier options B, C, and D seem correct if the appropriate order of operations were not used. I therefore indicated to her that she should think about the order of operations. She then did the task in writing, substituting each number and said

that option A is correct. In talking about the task the teacher said that at first she did not think in terms of the order of operations only after I told her to think about the order of operations she understood by saying, “now I see”.

The examples selected by her indicated that she understood the order of operations, for she made use of six examples presenting the concepts of order of operations and that of place holders. Her selection of examples emphasized that division and multiplication precede addition and subtraction. I have selected three examples; the first example, 15 x +6= 36 presents the concept of working with the order of operations. One learner responded that 15 x 2 +6 =36. Teacher B wanted to know who said that he must do times before addition, he replied BODMAS. The second example, 20 + 9 ÷  = 23, presents the concept of division preceding addition and the third example, 17- 3 x  =11 presents the concept of multiplication preceding subtraction.

In solving task 2, Teacher C showed that she understood what was required from her to solve the task. She was also able to solve the task by writing the sequence until she reached 7x7x11x5x13x2 and then multiplied the numbers with my assistance. In discussing the task she said, the task is very difficult because it requires the understanding of two concepts; the concept of finding the pattern and then finding the product of the three numbers. She also said that the task would be difficult for her learners and therefore she would never teach the task. She later agreed that she would teach the task if I needed her to do so. The day of the lesson observation she did not teach a lesson on patterns.

To solve task 3, Teacher C said that she will solve the task in the same way she would teach it to her learners. In solving the task she showed that she understood ratio but her understanding was not adequate for the solving of task 3. She started by drawing the four characters in the problem and distributed the berries among them by drawing the berries. This diagrammatical distribution of berries did not work. She experienced difficulty with finding the other three quantities and said that she needed another unknown. By trial-and error, she found there were 24 berries to start with.

Being able to find a solution through trial-and error shows that her level of conceptual understanding of ratio allowed, Teacher C to engage in adaptive reasoning. Her engaging in

different strategies showed that she did not memorize the following of a certain procedure for solving ratio problems but she was able to build on the procedures in order to find the solution.

The examples selected by her in class shows that she understands the concepts needed to answer task 3. Together with the learners they worked through 6 examples of which I am only discussing two. One of the examples asked learners to find the quantity of two different fractions:

John eats 10 sweets. Sarah eats ½ of what John eats. Mary eats $\frac{1}{5}$ *of what John eats. How many sweets are there altogether?*

The following example presents the concepts of twice as many and half of:

Nancy shares sweets between George, Nelly and Peter. She gives Nelly half of her sweets and she gives Peter 10. She gives George twice more than what she gave Peter. How many sweets are there to start with?

The 2 examples above show that Teacher C understood which concepts would be needed to solve task 2. The examples also show that she understood the relation of quantities not just in a ratio form but as fractions too. She did not only focus on one aspect such as using the numerical fractions but making use of words indicating fractions.

Teacher C could easily solve task 4 by selecting D as the correct answer. By selecting the correct option she shows that she understands the comparison and difference in the amount of marbles of Ming and Omar. She started by trying to calculate how many marbles Douglas would have by calculating the difference between Ming and Omar. She explained that Douglas would have 15 marbles which meant that Omar would have 3 and Ming would have 12. Her understanding of the task is evident in her selection of examples. Again 6 examples were selected which presented the concepts of "more than and less than", I will only focus on one for they are all similar (see appendices).

Candice has 9 toys. Peter has 6 more and David has 13 more than Candice. How many toys does David have?

The above task indicated a strong understanding of the concepts needed in order for her learners to solve task 4. The problems she used were based on the same concepts as that found in task 4. The concepts needed such as “more than” and comparing quantities were used to lay a conceptual foundation for the solving of task 4.

In solving task 1 Teacher C was more procedurally fluent than having conceptual understanding. In her selection of examples for the order of operations she demonstrates not only conceptual understanding but procedural fluency as well. The nature of the tasks however does not allow for different strategies.

Teacher C understood how to find the pattern in task 2, even if it was by trial- and error. She showed strategic competence by being able to group the numbers in a way that will make it easy to multiply. She showed strategic competence even if her strategy was not a formal one.

In solving tasks 1, 3, 4, Teacher C was positive in solving the tasks. In solving task 2, she believed that the task would be too challenging for her learners. It seems as though the task was too daunting for her to present to her learners. In not wanting to teach the task and actually not teaching the task portrays a negative disposition to the content needed to be taught.

4.2.4 Similarities across the teachers

The first unsuccessful attempt to solve task 1 indicated that all three teachers were more procedurally fluent in the order of operations. All three believed that it is important to state that the problem is a BODMAS problem. Likewise all three initially experienced difficulty in solving task 3; this was due to a lack in conceptual understanding of ratio and how to use this understanding in finding a solution strategy. Both Teachers A and C could not teach a pattern lesson. Teachers A and B did not select examples to present the concepts needed to solve task 4 but the tasks were given to their learners to solve.

The similarities show that although I have categorized the teachers’ level of understanding as weak, partial and strong there are some gaps in the conceptual understanding of Teachers B and C as well.

Teachers should not only be able to solve mathematical problems but must have knowledge of how to make the mathematical knowledge accessible to their learners. The knowledge I am

referring to is pedagogical content knowledge, this knowledge is not static but is under an ongoing development process (Shulman, 1987).

4.3 PEDAGOGICAL CONTENT KNOWLEGDE

Planning in the form of pedagogical reasoning (comprehension, transformation and instruction) is fundamental for pedagogical content knowledge. "Teachers with more explicit and better organized knowledge tend to provide instruction that features conceptual connections, appropriate and varied representations, and active and meaningful student discourse" (Stein, Baxter and Leinhard, 1990:641). I thus start by discussing the teachers' responses to how they would plan to teach as indicated during the interviews and then their pedagogical content knowledge. I only discuss the teachers' responses about what they would plan for I did not investigate their actual planning. In discussing their pedagogical content knowledge I draw from their pedagogical reasoning as explained during the interviews, and their actual classroom practice. I also draw links between their pedagogical reasoning and classroom practice. I have used categories: making knowledge accessible, knowledge about learners and how teachers responded to their learners' misconceptions to analyze the data. I start with discussing the teachers' knowledge about their learners, then how they made knowledge accessible and then dealing with misconceptions. I start with tasks selected to make knowledge accessible and whether the demands of the four tasks were maintained during implementation. I then discuss the patterns of interaction in the respective classrooms.

4.3.1 Teacher A

Task 1

Pedagogical reasoning

In her descriptions of how she would go about teaching lessons of which the four tasks would form part Teacher A based her planning on how she understood the concepts. Teacher A indicated that she would teach the task to her Grade 6 learners and that she and her colleagues do teach order of operations to their learners but not in-depth like in task 1. They teach it at a much "easier level." Her goal for teaching the order of operations, or as she understood it "the BODMAS-rule," would be for her learners to use the rule when working with word problems.

She therefore would teach the concept of order of operations just after she has taught the four basic operations which will be followed by word problems which would require calculating multiple operations. According to her the "rule" makes it easier for her learners to solve word problems. Teacher A explained that she will start her lesson by explaining to her learners, what BODMAS stands for. She further explained that as a teacher you need to explain to your learners, when they calculate they must follow BODMAS as well explaining to them where to start.

In describing her knowledge about her learners Teacher A explained that task 1 would be very difficult for her learners, for she too struggled to solve it. She explained that her learners will struggle because the task does not indicate to the learners that they should use the BODMAS-rule. In identifying what the possible causes would be why her learners might experience difficulty in solving the task the teacher explained that it could be that her learners did not understand the rule properly. To assist learners with incorrect responses and dealing with misconceptions she would ask her learners to explain their solution strategies. Based on their solution strategies she would explain to them that they have started with the incorrect operation and would then give the learners easier problems. Learners who were successful in solving the simple problems would explain to the group.

Classroom practice

I observed two lessons of Teacher A, during the first she taught the concepts of order of operations and number patterns, and during the second she taught tasks 3 and 4. The learners were seated in groups; each group consisted of 6 to 7 learners but they did not work in groups. Learners were asked questions and were allowed to ask the teacher clarifying questions.

Knowledge of learners

It is important for teachers to have knowledge about their learners' mathematical thinking as to decide how to guide and assist learners in constructing of knowledge (Even and Tirosh, 1995). Teacher A expressed that all four tasks would be very difficult for her learners to solve for they are mathematically weak but she did not really know at which levels of understanding her learners were. Teacher A knew that her learners had limited conceptions about the order of operations but these limitations were based on her limited content knowledge. She explained that

learners who might experience difficulty in solving task 1 would be as a result of them not following the rule. As mentioned earlier the teacher's understanding of the order of operations was limited to the following of the rule and was evident in how she represented the concept in her examples.

Making knowledge accessible through:

(a) Tasks

The tasks used in the form of examples to make the concept of order of operations accessible to the learners were simple familiar one step, routine and low cognitive demanding tasks (Charles and Lester, 1988, Kilpatrick et al, 2001and Stein et al, 2000).Teacher A represented the concepts of order of operations in three similar examples that I discussed previously. The tasks were suitable for learners with limited conceptions about the order of operations but the teacher could have helped build more "sophisticated ones" by giving more cognitive demanding tasks (Even and Tirosh, 1995:3).

The tasks did not challenge or enhance the learners' prior knowledge about the order of operations to serve as a foundation for solving task 1 (Stein, Baxter and Leinhard, 1990). The cognitive demands of task 1 were maintained as being a non-routine problem. In finding a solution strategy the learners experienced difficulty not because they did not follow the rule of brackets first, for the task had no brackets. They experienced difficulty because the teacher was unable to identify their actual level within their Zones of Proximal Development. Teacher A did assist her learners by explaining to them in IsiZulu what the symbols meant and that they should substitute, for the learners did not know what the task required. Scaffolding was absent for she did not elicit learners' mathematical thinking.

(b) Patterns of Interaction

The questions asked by the teacher did not promote mathematical thinking. The teacher was only listening for the answer she was looking for (Davis, 1997). When learners responded with a response other than the one she expected she did not ask them to justify their answers. By asking her learners to justify their responses she could have gained insight to their mathematical thinking (Kazemi and Stipek, 2001). The type of questions asked by Teacher A created an IRE

structure. The teacher initiated a question, the learners gave a response, the teacher was not satisfied with the response, initiated another question directed more to the response she wanted. When the learners could not respond with the answer she was looking for she changed the problem as mentioned earlier. The extract below indicates an IRE structure based on the example: (24×2) ÷ (6+7) =.

> Teacher: Who can tell me when you look at the sum 24 multiply by 2, divide by 6 plus 7.Let's look at BODMAS; the BODMAS rule tell us which one we must start with. Where we? The way to go. You cannot just start, so where must I start when I look at that sum?
>
> Learner 1: Multiply.
>
> Teacher: Where's multiplication? It doesn't matter, where Multiplication starts first or what. Where must I start I want you to look at this words. As I have explained to you the meaning. (Asks another learner for his response).
>
> Learner 2: Starting with divide sign.
>
> Teacher: The divide sign he said we must start with the divide sign. Look at our number here. What must we do or maybe I should have. [Changes the example on the board to $(24 \div 2) \times (6 + 7)$]. I want you to understand this that we need to follow this rule. Ok maybe I've made an example that is complicated so. Ok let's start here because here is divide and then there is brackets. Let's start there 24 divide by 2 is 12, multiply right and the other bracket there is addition 6 plus 7 is 13. 12 times 13 is?

The above extract shows that Teacher A's instruction on the concept of order of operations was informed by her comprehension of the concept. It might be that she did not realize that if you work from left to right, you sometimes get a different result than if you had used BODMAS, as seen with her interaction with the examples she used. The operations she chose in her examples were close to BODMAS which revealed a possible misconception, that she can only apply the rule to operations in the correct BODMAS-sequence. Her question was not with which operation should we start but where should we start indicating a particular position. The position she wanted from her learners was brackets for the rule as she mentioned, "wants you start with the

brackets first". During the interview she explained that she would allow her learners to justify why they have given those particular reasons. In the above extract we see that she did not ask the two learners to explain why they gave those responses. The first learner was correct with the response of multiplication but the teacher did not press to gain the learner's mathematical thinking because she was set on finding the answer of brackets. It is not clear as to whether the first learner was guessing or understood that if division and multiplication are together in a problem you should work from left to right. Likewise with the second learner who responded with the answer divide.

(c) Mathematical proficiency

The teacher could have used the responses given by the two learners as opportunities to promote conceptual understanding by allowing them to justify their responses. Teacher A promoted the following of procedures without the necessary conceptual understanding. The task did not require from the learners to follow the procedures the teacher taught them and the learners were unable to construct solution strategies and reason adaptively. These strands are interwoven with conceptual understanding as the foundation for becoming mathematical proficient. My study focuses on how the teachers promoted strategic competence within their learners but without a solid knowledge foundation and an understanding of the task, learners will not be able to construct their own solution strategies and engage in adaptive reasoning (Kilpatrick et al, 2001).

Dealing with learners' misconceptions

Teacher A was not able to elicit learners' solution strategies and therefore could not elicit learners' misconceptions (Fraivillig, Murphy and Fuson, 1999). As I have explained earlier she indicated that she would elicit learners' misconceptions by allowing them to explain their solution strategies. This shows that Teacher A has knowledge of how to elicit learners' misconceptions but she does not know how to implement this knowledge in class.

Tasks 2, 3 and 4

Pedagogical reasoning

In planning to teach task 2, Teacher A explained that the task would be suitable for her grade 6 learners but that the task would be very tricky for her learners. She further explained that her

goal for teaching the task would be for her learners to think logically and know their multiplication. She would start the lesson by showing her learners different number and geometric patterns. She further explained that she would start with easy patterns where the learners have to give the missing number by following the example given.

In gaining insight as to how the teacher would know whether the problem solving was successful, Teacher A explained that she would ask the learners to explain their solution strategy. She would ask both learners who got correct responses and incorrect responses. She then explained that the underlying causes for learners' errors would be as a result of not knowing their multiplication tables. In order to assist her learners she would show them how to break the numbers up (49, 55 and 26) and how to use all the numbers to multiply in the given pattern. She highlighted the fact that the task is very challenging and that the learners would take very long to solve the problem and that she would have to guide them all the way.

Classroom practice

Knowledge of learners

Teacher A did not put her knowledge about her learners' mathematical understanding into practice by not laying a conceptual foundation in order for them to solve tasks 2, 3 and 4. This shows that she based her knowledge of her learners' mathematical understanding on her level of understanding as seen in task 1 for she did not have an in-depth understanding of her learners. Teachers who are knowledgeable about their learners' mathematical understanding are able to challenge their learners' thinking by constructing appropriate tasks for learners (Even and Tirosh, 1995).

Making knowledge accessible

Teacher A did not make the needed concepts to solve tasks 2, 3, and 4 accessible by not making use of tasks to build a conceptual foundation. Not only were the tasks absent but interaction as well. The only interaction from this point on was the learners engaging in the task to find a solution strategy. No interaction between the learners and the teacher or amongst learners themselves meant no eliciting of solution strategies and misconceptions.

Summary

Teacher A's classroom practice shows how her level of content knowledge limited her attempts to make knowledge accessible to her learners. She could only make the knowledge she had about the concepts accessible to the learners. This was the result of her over emphasizing the limited concept of the order of operations in the form of following the rule (Stein, Baxter and Leinhard, 1990). The tasks remained non-routine tasks throughout the two lessons but this was at the expense of laying a conceptual foundation for the learners. She did not understand the knowledge in various ways and therefore could not represent it in various ways (Shulman, 1987).

4.3.2 Teacher B

Tasks 1

Pedagogical reasoning

According to Teacher B task 1, would be very challenging for her learners because they struggle in general with BODMAS, for learners forget that they have to solve the task procedurally meaning to solve the operations in the brackets first and then, Of, Division, Multiplication, Addition and Subtraction. She further explained that some learners would struggle with the task due to the nature of the task for they are used to solving straightforward questions. In focusing on the strategies her learners would use to solve the task the teacher explained that her learners would make use of counting strokes by grouping them.

The goal of her lesson would be for her learners to gain understanding of how to apply the BODMAS-rule and for her learners to use "strategic competence approaches". The order of operations lesson will be taught after the teacher has taught the four basic operations. She would teach it by incorporating other topics such as place value.

Teacher B further explained the reason why some learners would have difficulty to solve the problem would be as a result of a lack in understanding the BODMAS- rule. In order to assist these learners she would request from them to explain their working out to see where they have gone wrong. She then would explain the BODMAS procedures again and then ask the learners to give their own examples. Teacher B explained that when learners are able to give their own example then the problem solving process was successful.

Classroom practice

I observed two of Teacher B's lessons. In the first she taught the order of operations and patterns, and during the second she taught the concepts of ratio and asked the learners to solve task 4 without teaching the needed concepts. The learners did find solution strategies as a group, they only had to discuss and come up with their own examples as a form of group work. Teacher B attempted to gain an understanding of her learners' thinking but at a very low level. It "sounded" as if her class operated by socio-mathematical norms but she struggled to engage in high-press. She engaged in low-press (Kazemi and Stipek, 2001). Teacher B repeatedly mentioned that she is interested in her learners' strategic competence and reasoning but wanted to elicit them through the examples they gave and not when solving the tasks. Her partial understanding of the concepts was seen in how she interacted with her learners as well as how she promoted mathematical proficiency.

Knowledge of learners

Teacher B expressed that her learners would find task 1 challenging due to the nature of the task for they are used to solving simple tasks, demonstrating her knowledge of her learners' level of conceptual understanding. She also knew what strategies her learners would engage in, in finding a solution that being the grouping of counting strokes. Learners did make use of counting strokes in solving task 1. The counting of strokes is an informal strategy which indicates a lack in procedural fluency.

Making knowledge accessible through:

(a) Tasks

Teacher B did not make use of tasks for learners to solve in order to enrich her learners' conceptual understanding but she made use of examples. With the examples Teacher B presented she tried to enrich her learners' conceptual understanding of the order of operations. She wanted to move beyond the following of the rule of brackets first by taking a different approach to the brackets. In her example brackets represented the operation of multiplication; this example however did not enrich the learners' understanding for the solving of task 1. She further tried to enrich the understanding of what place holders are, for substituting numbers with place holders

was one of the concepts needed in solving the task. In order for her learners to understand that the order of operations is more than following of the rule she tried to show them that certain operations precede other operations. This example of working from left-to-right did not enrich their understanding fully for it was not applicable for the two operations in question. The demands of task 1 were maintained even with the teacher's attempts to enrich the learners' conceptual understanding.

(b) *Patterns of interaction*

In her attempt to enrich her learners understanding of the order of operations, with the example of: 3(14+2), an IRE-pattern was evident. Teacher B initiated when she asked what operation the brackets represented. The learners responded with responses different to the response the teacher expects. The teacher was listening evaluatively (Davis, 1997). The learners were guessing what the correct response was and when a learner responded with the answer multiplication the teacher was satisfied. When the learners were unable to respond with the correct answer, the teacher told them the answer.

(c) *Mathematical proficiency*

In the examples she wanted to promote conceptual understanding and procedural fluency. She however did not create opportunities which would engage her learners in the process of constructing their own solution strategies and reasoning. In solving task 1 Teacher B did not elicit the learners' solution strategies by giving learners an opportunity to explain their different solution strategies for the learners solved the task and the teacher reminded them that she is interested in their working out. She asked them to show their working out on the side of the worksheet. This shows that she believes that the learners' written working out would give her insight to their mathematical thinking.

Dealing with learners' misconceptions

The examples used did not elicit learners' misconceptions or create an opportunity for these misconceptions to surface. In solving task 1 the teacher did not deal with the learners' misconceptions because she gave the task as a test to the learners. She repeatedly reminded them that she is interested in their thinking but did not engage with their thinking.

The above examples show the partial understanding of the concept of order of operations. Teacher B attempted to lay a conceptual foundation for the learners to draw from by making use of more than one representation. She did not over emphasize one concept such as following of the rule. Her knowledge however was not explicit and organized to engage her learners in mathematical talk (Stein, Baxter, Leinhardt, 1990).

Task 2

Pedagogical reasoning

In pedagogically reasoning about a lesson which would include task 2, Teacher B indicated that group work and learner participation is very important for the promotion of mathematical thinking. This is seen in how she would assist learners with difficulties in solving the task. Teacher B indicated that task 2 would be too difficult for grade 6's and that the task would be suitable for grade 7's. Her goal would be to teach her learners finding a number pattern and to see number patterns in other forms. She would start with simple examples of odd numbers and skip-counting and then move to more complex tasks.

In determining whether the problem solving process was successful she would ask her learners to explain how they got their solution and to give their own examples of patterns. In guiding learners who were unsuccessful in solving the task she would ask the other learners in the group who understand to explain to those learners. She believes in "each one teaches one". She further indicated that only when both of the learners do not understand then she will intervene by explaining easier examples and then moving on to more complex examples.

Classroom practice

Knowledge of learners

Teacher B knew that her learners would experience difficulty in solving task 2 for learners do not know their time-tables. She further explained that it is routine for her learners to recite their time-tables before she starts with her mathematics lessons. By mentioning that learners do not know their time-tables shows that she had an understanding of the learners' Zones of Proximal

Development for solving task 2. Teacher B understood that the concept of multiples was needed to solve task 2 and wanted to assist her learners without lowering the cognitive demands of the task.

Making knowledge accessible through:

(a) Tasks

The concept of creating number patterns was made accessible to learners through the examples she used and by giving the learners an opportunity to construct their own examples. The examples given by learners indicated the different Zones of Proximal Development. The learners needed the assistance of their teacher without lowering the demands to guide them to the level where they could solve the tasks independently. By asking the learners to create their own examples and by listening evaluatively did not assist the learners to create their own solution strategies. By asking the learners to explain why they gave incorrect examples Teacher B could assist her learners in gaining conceptual understanding of ratio.

(b) Patterns of interaction

The examples used by Teacher B were simple but she could have high-press in order for the learners to gain a deeper conceptual understanding of the concept of breaking numbers into multiples to form a pattern as required by task 2. The extract below shows how the teacher missed on an opportunity to promote mathematical thinking.

> Teacher: Read me your 2 times table.
>
> Learner 1: Learners read 1 times 2 is 2, 2 times 2 is four, 3 times 2 is 6, 4 times 2 is 8 up until 12 times 2 is 12.
>
> Teacher: Thank you. What I want you to do, your group do 4, you do 5, 6,7 and 10. I want you to tell me what did you discover? What have you observed, look at your multiplication. Mam what we have observed this number is doing this and that. When you work with even numbers there is something constant that you must see. Who can try?
>
> Learner: Multiples of 5, jump with 5.

Teacher: [Writes on board 5+5+5] 5 plus 5 is 10 plus 5 is15. Skip Count in 5. Make a pattern of what Lucas?

Learner: 5.

Teacher: [Asks another group]. You will make a pattern of?

Learner: 1, 3, 5, 7, 9. Thank you.

Teacher: What is that you observe? Which numbers are those, What patterns do they make? I want your thinking, what is that you can say in relation to odd numbers. The rule says that for that pattern we always add 2. Give me your own number pattern in 3 minutes. Share ideas, you can always work as pairs, work with your neighbor. Collaborate with your neighbor. Right Lucas tell us your pattern.

In the above extract Teacher B indicated that she is interested in her learners' mathematical thinking by stating that she wants their thinking. She however did not know how to high-press in order to gain a deeper understanding of their thinking. She started by asking her learners to give simple examples of counting in different multiples. She could have used those simple examples as a springboard to engage the learners in deeper thinking by asking them to break numbers in different multiples. Teacher B did not move her learners beyond those simple examples.

(c) Mathematical thinking

Teacher B stated that she wanted to access her learners' thinking but was unable to create opportunities for them to engage in mathematical thinking. The example she used of odd numbers or adding 2 every time did not require any mathematical thinking from the learners. The same with the examples she requested the learners to give, those examples were very simple and did not indicate an understanding of breaking numbers into multiples and then grouping them in a certain pattern. The examples only indicated that the learners could count in multiples of 4, 5, 6, 7 and 10.The teacher was unable to move the learners to a higher level of thinking by raising her level of questioning.

Dealing with misconceptions

Not only did the examples given by both the teacher and learners not create opportunities for learners to engage in mathematical thinking but they also did not create opportunities for the

learners' misconceptions to surface. The type of examples did not leave enough space for error for learners were given small basic numbers and were asked to make use of the time table sheet.

Task 3

Pedagogical reasoning

Teacher B described that she would teach task 3 to her grade 6 learners but that it would be very difficult for them. The purpose of this lesson would be for her learners to make use of different strategies when working with word problems. The teacher would start her lesson with solving of fractions and more specifically division of fractions.

Features like the language and repetition of other characters' names in the task would make the task very tricky for learners. Teacher B further explained that learners normally struggle with word problems for they have to take the mathematics out of the language. By mathematics she meant the mathematical concepts. Learners might also find the task difficult for they are not familiar with ratio in word problems. To assess whether the actual problem solving process was successful the teacher would ask the learners to give their own examples. In assisting learners who struggle to solve the task she would ask a peer to explain to the learners.

Classroom practice

Making knowledge accessible through:

(a) Tasks

Teacher B had the opportunity to make use of the learners' simple examples as scaffolding to help them move through their Zones of Proximal Development. Some of the learners' examples indicated that they did not understand the concept of ratio. Teacher B did not make use of the opportunity to assist her learners in gaining a better understanding of the concept of ratio through scaffolding whether in the form of giving more examples or questioning.

(b) Patterns of interaction

The pattern of interaction during this session was mainly IRE, the teacher initiated questions the learners responded and the teacher either accepted the response or rejected it. The teacher did not encourage learners to justify their responses. The type of examples she used did not allow for much interaction from the learners. There are three occurrences of initiating that took place and that was when the teacher gave the example on the ratio of 2 boys: 3 girls and 2 glasses of lime juice: 3 glasses of orange juice. The teacher wanted to know what the whole would be for these two quantities and she asked the learners to come up with their own ratio examples.

Learners gave different examples but their examples showed an understanding of fractions and not ratio. One group gave an example of having 12 sweets and dividing in the group. The teacher responded to the learners that they are not talking about fractions but about "ratio and we talk about 2 quantities". She moved to each group to listen to their responses but was not satisfied with the responses. All the groups except for one group gave an example on ratio. Their example was they are 6 learners in the group, 1 has uniform on and 5 do not have uniform on. The teacher responded by affirming their answer with a, "good".

(c)Mathematical thinking

Teacher B reminded her learners often that she wanted their reasoning and that she'll never say their reasoning is wrong. Teacher B did not tell her learners that their examples were incorrect and neither did she create opportunities for her learners to engage in adaptive reasoning. She also did not create an environment where learners were expected to construct their own solution strategies.

Dealing with misconceptions

The examples given by the learners of fractions instead of ratio show that they did not understand the concept of ratio but the teacher did not deal with this misconception. The reason for this could be because of her statement that she would never say their reasoning is wrong or she did not have adequate understanding to know why her learners gave those responses and how to move them to a higher level of understanding. Teacher B mentioned during the interview that she would assist her learners by asking those who understand to explain to those who do not

understand. It seems as though one group understood what she wanted but they did not explain to learners in other groups

Task 4

Pedagogical reasoning

Teacher B indicated that she would teach task 4 to her grade 6 learners. She would incorporate the task in a basic operations lesson where learners have to compare different quantities. She will start the lesson by comparing different numerical values and then give learners word problems.

She explained that her learners would experience difficulty in solving the problem because it is a word problem and the learners, first need to understand the language before they can understand the mathematics required. To assist the learners regarding the language difficulty she will break the task up into sections and explain each term such as "more than and less than. She would also ask those learners who can solve the task to explain to those who are unable.

Classroom practice

Teacher B did not give any examples to lay a conceptual foundation to solve tasks neither did she require from her learners to give their own examples as seen in the previous three tasks. Learners were asked to solve the task on their own and were reminded that they had to show their working out on the side of the page.

4.3.3 Teacher C

Task 1

Pedagogical reasoning

Teacher C indicated that she would teach task 1 to her grade 6 class but she has never taught a task of that nature. She further explained that previously when she had to teach order of operations she never made use of problems where they first had to substitute. She however would teach her learners how to substitute values and then apply the order of operations. When teaching the order of operations she gave them examples where they have to solve the operation in the bracket first as well as problems without brackets. When teaching the concept of order of

operations with problems without brackets she asked her learners to underline the operation they need to do first. She taught the underlining of operations in grade 5 and in grade 6 she taught her learners to work from left to right. Teacher C explained that a lesson on the order of operations will start with showing her learners the different ways in solving problems as well as how to substitute values and only at the end of the lesson she would give them a task like task 1.

In order to evaluate the success of the problem solving process the teacher will give the learners a spot test. In asking her how she would assess whether her learners had gained any understanding while they are busy solving problems she explained that even if the teacher asks the learners whether they understand the content they would say yes even if they don't. Teacher C further explained that the reason why some learners may struggle to solve problems is because they do not know how to transfer knowledge. In assisting learners who are experiencing difficulty to solve the problems she would lead them through the task step by step.

Classroom practice

(a) Patterns of interaction.

Throughout the two periods there was not much verbal interaction which took place in Teacher C's classroom. There was one occurrence where she engaged in low-press interaction. An IRE-sequence was evident when she assisted learners who experienced difficulty in solving the tasks. She identified specific learners whom she knew would struggle with the task (at the end of the lesson she told me that she knew the learners, for she taught them in grade 5 as well). In order to assist them she would ask them a question on which they responded. She evaluated the response and asked another question. The rest of the time the learners interacted with the worksheets. (See APPENDIX 1.5 for worksheets).

Teacher: [Wrote $15 \times \Delta + 6 = 36.$] 15 times what, plus 6 equals 36?

Learner 1: Two.

Teacher: Why did you do, who said you must do times before addition?

Learner: BODMAS

Teacher: BODMAS, says you do multiplication first 15 x 2. You underline what you must do first.

Teacher C elicited her learners understanding of the order of operations by making use of various examples but her questioning in the above extract was more procedural. By asking the learner, "who said you must times first" Teacher C's questioning required from the learner to give a procedural response but could be seen as an opportunity for justification as well.

Mathematical thinking

By giving the learners examples as an introduction to the concept of order of operations, Teacher C did not start by telling the learners that they are going to work with the order of operations. Neither did she explain to them what each letter in the acronym BODMAS stands for instead she made use of different examples which presented the different operations. During the teaching of the concept of order of operations the teacher did not deal with learners misconceptions.

Tasks 2

Pedagogical reasoning

Teacher C explained that she would not know where to begin to teach task 2 for it would be too difficult for her learners. She normally teaches patterns by adding on numbers finding the geometric patterns. She said that she would teach the task but did not.

Tasks 3 and 4

In teaching tasks 3 and 4, Teacher C would teach it the way she solves it and that would be to identify the characters in the problem and draw pictures. The purpose of her lesson would be for her learners to gain conceptual understanding of ratio and what more and less means. She further explained that when she teaches ratio she focus on writing quantities as ratios. She does not teach the concepts as specified by the assessment standards for the particular grade.

In order to evaluate the success of the problem solving process the teacher will give the learners a spot test. In asking her how she would assess whether her learners have gained any understanding while they are busy solving problems she explained that even if the teacher asks the learners whether they understand the content they would say yes even if they don't. In

assisting learners who are experiencing difficulty to solve the problems she would lead them through the task step by step.

Classroom practice

Making knowledge accessible through:

(a) Tasks

As I have mentioned earlier that learners were given worksheets which were similar to task 3 and task 4. These worksheets were used to lay a conceptual foundation of fractions. After they have completed the worksheets the tasks were corrected.

(b) Patterns of interaction

Throughout this period the teacher was the only one talking by explaining the procedures of solving the tasks. Learners listened and checked whether their solutions were the same as the teacher's. By using similar tasks as the non-routine tasks she lowered the demands and not by asking questions.

(c) Dealing with learners misconceptions

In dealing with learners misconceptions Teacher C did not elicit learners' misconceptions she asked one specific learner if she understood. She then explained the task to the learner by working through the different steps.

The three teachers' pedagogical content knowledge was influenced by their level of content knowledge. All three teachers made knowledge accessible to their learners based on how they understood the concepts. Teacher A's weak content knowledge enabled her to make the needed mathematical concepts accessible to her learners by promoting procedural fluency. Promoting procedural fluency limited the interaction in the classroom. Interaction in her classroom was limited to her initiating a question and listening evaluative to the learners' responses. Listening evaluative did not allow Teacher A to elicit learners' mathematical thinking and misconceptions.

Teacher B's partial understanding enabled her to make partial representations accessible to her learners in order to understand the mathematical concepts. She has knowledge of the five interwoven strands but do not know how to promote these strands in her classroom. Being unable to promote the five interwoven strands limited the interaction in her classroom to "talk" but not mathematical discussions. The "talk" in her classroom did create opportunities for her to elicit learners' misconceptions.

Teacher C's strong content knowledge enabled her to lay a strong conceptual foundation in order for her learners to solve tasks 1, 3 and 4. Teacher C made the knowledge accessible to her learners by giving them worksheets to work through. The usage of worksheets limited mathematical discussions in the classroom

Pedagogical content knowledge is always emerging and begins with understanding (Shulman, 1987). It is always emerging for at the end of a lesson teachers could gain new ways of making knowledge accessible and dealing with learners' misconceptions. This can only happen if teachers have adequate content knowledge for if teachers do not understand the concepts themselves they will not be able to identify learners' errors.

4.4 MATHEMATICAL AND EVERYDAY CONTEXTS

In this section I argue that the three teachers understood the solving of non-routine tasks differently in the mathematics contexts and everyday contexts. In approaching the mathematical contexts tasks the teachers' foci was on the manipulation of symbols. The teachers approached tasks 3 and 4 differently to tasks 1 and 2 by focusing on the linguistic aspects. According to De Corte and Verschaffel (1985) it is important for problem solvers to understand and interpret the words correctly in everyday tasks in order to build correct mathematical representations of the problems. Teachers can enable this understanding by describing the task in a way "consistent with its desired outcome" (Shuell, 1990:105).The three teachers in this study made use of coping strategies to teach the everyday task (Verschaffel and De Corte, 1997, Chapman, 1999). Coping strategies are strategies teachers show their learners to assist them in order to solve the problems. The key-word strategy is when the teacher draws the learner's attention to identify the key words and to represent them into mathematical manipulatives, such as addition or subtraction (Verschaffel and De Corte, 1997, Chapman, 1999). In identifying the known and unknown in a

problem the teacher shows the learners to identify all the known quantities in order to make use of the known to find the unknown quantity. I will show how each teacher matches these strategies. In enabling learners to understand the meaning of words, Teacher A focused on the key-word strategy (Verschaffel and De Corte, 1997, Chapman, 1999). Teacher B focused on determining the known and then determining the unknown (Chapman, 1999). Teacher C focused on a combination approach of identifying the known and unknown as well as the key-word strategy.

In teaching task 3 Teacher A taught her learners one of the key-word strategy (Verschaffel and De Corte, 1997, Chapman, 1999). She started by reading the task to the learners. She drew their attention to the following phrases, "half and twice as many". She read the task again but explained each phrase in Isizulu. Whilst conversing in Isizulu she asked the learners what twice as many meant. One learner said "times" the teacher repeated the word times and continued to explain the task in Isizulu. In solving task 4 the teacher read the task to the learners and explained in Isizulu that they had to select or circle the correct statement.

Teacher A showed an understanding of the importance of phrases such as "more than, less than, half and twice as many" and identified that those particular phrases needed to be represented mathematically in order for her learners to solve the tasks. By asking the learners the meaning of "twice as many" she further showed that she understood the teaching of everyday tasks by identifying the key-words and enabling the understanding of these words. Teacher A however only focused on the one phrase "twice as many" and ignored the other phrases. Learners were required to solve the tasks without the teacher's assistance in gaining understanding of the rest of the phrases.

In teaching task 1 Teacher A's focus was on the manipulation of the symbols, the triangle and square. She wanted her learners to understand what these two shapes represented by giving them an example. The example was:  = 4. She explained to the learners that the square represents 2 and the triangle, 6. She showed them to substitute the 2 and the 6 and showed them that 2+6= 4 is incorrect. She then asked her learners in to use the numbers in task1 and substitute them in the given square and triangle. In teaching task 2 she did not show them any strategies to use learners; were expected to construct their own strategies.

In teaching tasks 3 Teacher B did not focus on the language aspects as she explained during the interview for she did not focus on the meanings of the problematic phrases. In solving task 3, Teacher B focused her learners' attention to identify what they need to find out and what they know. She explained to her learners that they needed to find out how many berries there were to start out with. She also explained to the learners that the amount of berries is unknown and that the unknown amount of berries will be represented by an "x". She then explained to them that they know that Ben ate 4 and Sarah ate 10 and that they have to start with what they know to solve the problem. In solving task 4, the learners were encouraged to focus on how many Ming and Omar had and were asked to solve the task.

Based on her knowledge of her learners' language proficiency Teacher B identified that her learners will experience difficulty with the key phrases. She however did not focus on explaining the meanings of these words in order for her learners to construct mathematical representations, but she focused on what is known in order to find the unknown. She introduced her learners to a mathematical representation of the unknown in the form of "x" but did not use this representation to extend the learners' understanding to solve the task.

To assist her learners in learning to manipulate the symbols in task 1, Teacher B gave her learners the following examples: ? + 3 - ? = 7. She explained to her learners that when the answer is given in a number sentence the number sentence is closed. She further explained that the question marks represent numbers that would make the number sentence true. The second example was: 6 ? 4 ? =7, with this example she explained to the learners that the question marks must be substituted with the correct basic operations which will make the number sentence true.

In teaching task 1 Teacher B encouraged her learners to make use of the trial-and error strategy but in task 2 learners were required to construct their own solution strategies. To solve task 3 Teacher B showed them how to identify the known and then the unknown.

In teaching task 3, Teacher C requested from her learners to represent the different characters in the tasks. She represented them on the board by writing the first letter of each name and by allocating the different quantities of items to each character. She emphasized that the tasks required logical thinking and that the learners had to take one sentence at a time.

Teacher C combined identifying the known and unknown-approach and the key-word approach in teaching task 4. In one example (see Appendix1.5), a phrase read, Danny has 6 more than Joe. Learners had to identify the known quantities and draw the characters. The teacher then asked one specific learner for the meaning of "more than". The learner's response was "add." The teacher repeated that "more than" means adding and asked to who's quantity should the 6 be added to.

In teaching task 1, Teacher C explained to her learners that the squares and triangles are place holders of numbers. She started with, $13-\square = 7$ and asked the learners what the answer was. The learners gave the answer of 6. She responded by saying that the square is a place holder for the number 6.She gave two more similar examples based on the same concept of place holders.

As mentioned during the interviews Teachers A and B distinguished the everyday tasks and mathematical tasks in terms of the language difficulty. Both teachers explained that they would have to focus on the meanings of key concepts in the tasks. Teacher C however mentioned that the procedure to solving of everyday tasks would be different to that of mathematical tasks. By mentioning that the procedure would be different Teacher C was referring to her understanding of how problems in everyday tasks should be taught. Her understanding of when problem solving of everyday tasks should be taught was evident in her teaching. In teaching tasks 1, 3 and 4 she created an opportunity for her learners to be competent in the operations by giving them worksheets to work through before she gave them the tasks. The learners had to solve tasks 1, 3 and 4 as tests and then after working through the worksheets, she would ask the learners if they are ready for a test.

4.5 CONCLUSION

In this chapter I have tried to categorize the teachers' content knowledge and to gain an understanding of their pedagogical content knowledge. This analysis implies that the teachers' content knowledge varied from weak, partial and strong. This analysis also suggests that the teachers had different levels of pedagogical content knowledge. In teaching mathematical context tasks the teachers' foci were on the manipulation of symbols and teaching everyday context tasks they focused on the linguistic aspects. In the next chapter I will present a discussion on the findings and limitations of this study.

CHAPTER 5: CONCLUSION

5.1 DISCUSSION OF FINDINGS

This study has explored the pedagogical content knowledge that grade 6 teachers have about problem solving in two contexts, mathematical and everyday contexts. I have responded to the following questions in my analysis.

1. How do grade 6 mathematics teachers understand problem solving in everyday and mathematical contexts?

2. Do grade 6 teachers respond differently to learners' responses in everyday and mathematical contexts?

3. How do grade 6 teachers deal with learners' misconceptions in everyday and mathematical contexts?

As the study progressed it became clear that one cannot explore pedagogical content knowledge without focusing on content knowledge because teachers' pedagogical content knowledge is strongly informed by their content knowledge (Ball, 2000; Fraivillig et al, 1999; Stein, Baxter and Leinhard, 1990). I have found that the teachers had different levels of conceptual understanding of the concepts they had to teach in these tasks. The teachers' level of content knowledge is only based on the four tasks that the teachers interacted with in this specific study. It is not a general conclusion on their overall mathematical content knowledge because these four tasks do not provide me with sufficient knowledge about their total mathematical content knowledge.

The study indicates that Teacher A had a weak conceptual understanding of the concepts to be taught as she over emphasized limited knowledge fragments. She was unable to lay a conceptual foundation which would support learners' problem solving (Stein, Baxter and Leinhardt, 1990). Teacher A's weak conceptual understanding was further seen in the instructional representations she used to make the knowledge accessible to her learners. By making use of suitable representations teachers "communicate their own knowledge to learners" (Sowder, Philipp, Flores and Schappelle, 1995:255). Teacher B had a partial understanding of the concepts needed to be taught. This partial knowledge was the result of her confusing some concepts and making

use of mathematical representations which were not really suitable for grade 6 learners. Teacher C had strong content knowledge but not strong pedagogical content knowledge for she understood the concepts needed to solve the tasks but did not always understand it in a way to make it accessible to the learners.

The study further shows that teachers understand non-routine problem solving differently. This difference was evident in their teaching approaches of non-routine problems. The teaching approaches found in this study to teach non-routine problems include both a traditional approach and an "idea" of what a reform approach requires. Teacher C's approach to teaching non-routine tasks in both mathematics and everyday contexts was a traditional approach. This traditional approach was based on allowing learners to solve similar problems as a means to develop the concepts needed for the non-routine tasks. Learners were no longer finding strategies for non-routine tasks but were applying already known strategies; the tasks thus became routine tasks. Teacher C may know how to select and construct tasks that would extend learners' mathematical knowledge but she did not know how to use these tasks to elicit learners' mathematical thinking, especially strategic competence. Interaction in Teacher C's classroom was limited to the worksheet, meaning learners only interacted cognitively by solving the tasks. Opportunities were not created for them to share their mathematical thinking with the teacher and fellow learners. Teacher C saw her role as being to guide the learners through the task in order to find the correct solution. The purpose of solving non-routine problems was to test whether learners could apply the same procedures.

Teacher A's teaching approach to non-routine tasks was also a traditional approach in the sense that she did not elicit learners' mathematical thinking and she listened evaluatively. The interaction in the class was a one-way interaction, the teacher asked questions and the learners were expected to respond with an answer the teacher required. Learners did not ask any questions.

Teacher B had an "idea" of a reform approach to teaching non-routine problems in two contexts; everyday and mathematical. She seemed to have a different understanding of a problem solving approach. Her vocabulary indicated that she had some knowledge of promoting mathematical proficiency through her teaching. She referred to concepts such as reasoning and strategic competence. She however did not successfully promote this through her teaching. She also

articulated the "idea" of learners working together or "collaborate" as she stated it but this collaboration did not promote learners' mathematical thinking. Responses from learners in Teacher B's classroom were in the form of constructing their own examples in both everyday contexts and mathematics contexts. Teacher B did not ask for learners to explain why they gave those particular examples.

In this study I have found that Teachers A and B were faced with the challenge of not knowing how much conceptual foundation was needed in order to solve the non-routine tasks. Both Teachers A and B, maintained the cognitive demands of the tasks but did not develop conceptual understanding neither did they know how to elicit learners' mathematical thinking to engage in actual problem solving. Even though Teacher B could articulate some ideas of different ways to teach, she did not display these in her practice. Teacher C developed conceptual understanding but lowered the tasks to routine tasks. The tasks in the worksheet were similar in nature to the given tasks and as such were non-routine. However because the learners have worked through six similar tasks applying the same strategies, the ICAS- tasks thus became routine tasks.

This study also found that the three teachers promoted strategic competence differently, Teacher A expected her learners to construct their own solution strategies in order to solve the mathematical contexts tasks without laying a conceptual foundation. In order to solve the everyday task 3 she taught her learners to make use of one of the coping strategies, finding the key-words. In solving task 1, Teacher B explained to her learners to make use of the trial-and-error method. In solving task 3 she explained to her learners to identify the known and unknowns. In both contexts Teacher C made use of the worksheet-strategy where learners have to work through problems applying the same strategy.

In summary, the two teachers with weak and partial content knowledge also had weak pedagogical content knowledge and were not successful in teaching problem solving. The teacher with stronger content knowledge also showed weak pedagogical content knowledge as she consistently reduced the task demands. As I have found that the teacher constantly lowered the tasks demands I need to take into account that the teacher's short term goal was to teach the tasks in two lessons, as well as for learners to answer the tasks. I do not know if she would have taught the same way in the longer term.

5.2 IMPLICATIONS

The main findings of this study is that although content knowledge cannot be separated from pedagogical content knowledge in the analysis of teachers' work, content knowledge is not enough to support strong pedagogical content knowledge. Although the teachers had differing levels of content knowledge, their pedagogical content knowledge and their practice was weak. Teacher C's stronger content knowledge was not sufficient to support a stronger form of pedagogical content knowledge.

This finding has important implications for teacher development, and suggests that both content knowledge and pedagogical content knowledge must be attended to in teacher education. Current calls for development of content knowledge only are therefore misguided.

Curriculum planners will also be informed by the findings of this study. The Revised National Curriculum has been reviewed and it was recommended that "content had to be brought into the curriculum, and specified" (Curriculum and Assessment Policy Statement, 2009:12). This study shows that not only is it recommendable to include and specify the content to be taught but teachers should undergo training to gain deeper content knowledge on specific mathematical concepts. The Curriculum and Assessment Policy Statement (2009) further recommends for teachers to set a variety of tasks, tasks should be spread across at all four cognitive levels, approximately 25% knowledge, 45% routine, 20% complex procedures and 10% problem solving. Teachers should not only be able to set the different cognitive demanding tasks but also need to know how to teach these types of tasks to their learners, especially complex procedural tasks, without reducing the task demands. Teachers need new roles in the classroom by reviewing their instructional goals and interaction with their learners (Manoucheri and Goodman, 2000). To successfully implement complex problem solving instruction teachers also need to have an in-depth mathematical understanding, a thorough knowledge of how their learners learn and think about mathematics (Manoucheri and Goodman, 2000).

According to Curriculum and Assessment Policy Statement (2009:66), a national textbook catalogue needs to be developed and these textbooks should offer "appropriate content and methodology." The findings of this study allude to this recommendation and further recommend

that teachers should not be bound to the methodologies in a textbook but should create opportunities for learners to construct their own solution strategies.

This study is limited in that it only explored the content knowledge and pedagogical content knowledge of three teachers. Other researchers can further the findings of this study. Further studies can explore teachers' level of content knowledge and pedagogical content knowledge in light of the implementation of The Curriculum and Assessment Policy Statement.

REFERENCES

Ball, D. L. (2000). Bridging Practices: Intertwining Content and Pedagogy in Teaching and Learning to Teach. *Journal of Teacher Education*, 51 (3), 241-247.

Ball, D. L. & Bass, H. (2000). Interweaving context and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, C7: Ablex

Boaler, J. (1997). Creating flexible knowledge in mathematics classrooms. *Pythagoras*, 48, 10-16.

Bauersfeld, H. (1995). The Structuring of structures: Development and Function of mathematizing as a social practice. In L. Steffe & J. Gale (Eds), *Constructivism in education*

137-158.

Bogdan, R., & Taylor, S. J. (1978). Introduction to Qualitative Research methods: A Phenomological Approach to the Social Sciences. *Teaching Sociology.* 5 (2), 213-216.

Briars, D. J., & Larkin, J. G. (1984). An integrated model of skills in solving elementary word problems. *Cognition and Instruction*, 1, 245-296.

Brodie, K. (2003). Being a facilitator and a mediator in mathematics classrooms: a multi-dimentional task. In S. Jaffer & L. Byrgess (Eds.), *Proceedings of the 9th national congress of the Association for Mathematics Education of South Africa* (pp 135-146). Cape Town: University of Cape Town.

Brodie, K. (2007a). Teaching with conversations: beginnings and endings. *For the learning of mathematics*, 27 (1), 19-25

Brodie, K. (2007b). Dialogue in mathematics classrooms: beyond question and answer methods. *Pythagoras*, 66, 3-13.

Brodie, K. (2007c). Masters in Education: Course Material. Johannesburg. University of the Witwatersrand.

Brodie, K. (2008). Describing teacher change: interactions between teacher moves and learner contributions. In Matos, J.P., Valero, P. and Yasukawa, K. (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. University of Lisbon and Aalborg.

Burns, B. & Lash, A. (1988). Nine Seventh-Grade Researchers' Knowledge and Planning of Problem-Solving Instruction. *The Elementary Journal*, 88 (4), 369-386.

Calvert, G. A. (2001). *Mathematical conversations within the practice of Mathematics.* 37-56. New York :Peter Lang.

Carpenter, T.P., Fennema, E., Peterson, P.L., & Carey, D.A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education,* 19, 385-401.

Carpenter, T. P., Fennema, E. and Franke, M.L. (1996). Cognitively Guided Instruction: A Knowledge Base for Reform in Primary Mathematics Instruction. *The Elementary School Journal* 97 (1), 3-20.

Charles, R. I., & Lester, F. K. Jr.(1984). An Evaluation of a Process-Orientated Instructional Program in Mathematical Problem Solving in Grades 5 and 7. *Journal for Research in Mathematics Education,* 15 (1), 15-34.

Chapman, O. (2006). Classroom Practices for Context of Mathematics Word Problems. *Educational Studies in Mathematics,* 62 (2), 211-230.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in Classroom Mathematical Practices. Rethinking Methodology in the Learning Sciences. *The Journal of the Learning Sciences,* 10(1/2) 113-163.

Confrey, J. & Kazak, S. (2006). A thirty-year reflection on constructivism in mathematics education in PME. In Guiterrez, A. & Boero, P. (Eds). *Handbook of research on the psychology of mathematics education: Past, Present and Future*. Rotterdam: Sense Publishers.

Crespo, S. (2003). Learning to Pose Mathematical Problems: Exploring Changes in Pre-service Teachers' Practices. *Educational Studies in Mathematics,* 52 (3), 243-270.

Daniels, H. (2001). Vygotsky and Pedagogy. Routledge Falmer.

Davis, B. (1997). Listening for differences: an evolving conception of mathematics teaching. *Journal for research in mathematics education,* 28(3), 355-376.

De Corte, E., & Verschaffel, L. (1985). Beginning first graders' initial representation of arithmetic word problems. *Journal of Mathematical Behavior,* 4, 3-21.

Department of Education. (2009). *Curriculum Assessment Policy Statement National. Final Draft-Intermediate Phase.* Department of Education: Pretoria.

Doerr, H., & Lesh, R. (2003). A modeling perspective on teacher development. In Lesh, R. & Doerr, H. M. (Eds.), *Beyond constructivism. A models and modeling perspective on mathematics teaching, learning and problem solving,* 125-139. Mahwah, NJ: Lawrence Erlbaum Associates.

Even, R. (1993). Subject-Matter Knowledge and Pedagogical Content Knolwegde: Prospective Secondary Teachers and the Function Concept. *Journal for Research in Mathematics Education.* 24 (2), 94-116.

Even, R. & Tirosh, D. (1995). Subject-Matter Knowledge and Knowledge about Students as Sources of Teacher Presentations of the Subject-Matter. *Educational Studies in Mathematics*, 29 (1), 1-20.

Fraenkel, J. V., & Wallen, N. (1990). How to design and evaluate research in education. New York: Mc Graw-Hill Publishing Company.

Fraivillig, J. L., Murphy, L. A., & Fuson, K. C. (1999). Advancing Childrens' Mathematical Thinking in Everyday Mathematics Classrooms. *Journal for Research in Mathematics Education,* 30 (2), 148-170.

Franke, M. L., & Kazemi, E. (2001). Learning to Teach Mathematics: Focus on Student Thinking. Realising Reform in School Mathematics. *Theory into Practice,* 40 (2), 102-109.

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics,* 49, 193-223.

Griffiths, M. (1998). *Educational Research for Social Justice: getting of the fence.* (pp. 29-140).Open University Press: Buckingham. Philadelphia.

Henningsen, M., & Stein, M (1997). Mathematical Tasks as Student Cognition: Classroom Based Factors that Support and Inhibit High –Level Mathematical Thinking and Reasoning. *Journal for Research in Mathematics Education,* 28 (5), 524-549.

Hatch, J. A. (2002). *Doing qualitative research in educational settings.* 147-210. New York. SUNY.

Hill, H. C., Ball, D.L., Schilling, S. G. (2008). Unpacking Pedagogical Content Knowledge. Conceptualising and Measuring Teaching Topic Specific Knowledge of students. *Journal for Research in Mathematics Education,* 39 (4), 372-400.

Kantowski, M. G. (1977). Processes involved in Mathematical Problem solving. *Journal for Research in Mathematics Education,* 30, 164-180.

Kazemi, E. & Stipek, D. (2001). Promoting Conceptual Thinking in Four Upper- Elementary Mathematics Classrooms. *The Elementary School Journal,* 102 (1). The University of Chicago.

Kilpatrick, J., Swafford, J., &Findell, B. (2001). *Adding it up: Helping children learn mathematics,* 115-155. Washington DC: National Academy Press.

Lubienski, S. A. (2000). Problem solving as a Means toward Mathematics for All: An Exploratory Look through a Class Lens. *Journal for Research in Mathematics Education,* 29 (2), 143-163.

Ma, L. (1999). Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Mahwah, NJ: Lawrence Erlbaum Associates.

Modau, S. & Brodie, K. (2008). Teachers selection and use of tasks in the old and new curriculum. *Proceedings from the South African Association for Research in Mathematics and Technology Education.*

Manoucheri, A., & Goodman, T. (2000). Implementing Mathematics Reform: The Challenge within. *Educational Studies in Mathematics,* 42 (1), 1-34.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.*Reston, VA: National Council of Teachers of Mathematics.

Nesher, P. (1987). Towards an instructional theory: The role of students' misconceptions. *For the learning of mathematics,* 7 (3), 33-39.

Opie, C. (2004). *Doing Educational Research.* 111-128. SAGE. London.

Olivier, A. (1989). *Handling pupils misconceptions.* Department of didactics. University of Stellenbosch.

Piaget, J. (1977). *The Development of Thought: Equilibration of Cognitive Structures*. Viking, New York.

Resnick, L. (1987). Constructing knowledge in school. In L. Liben (Ed.), *Development and learning: Conflict or congruence?* (pp.19-50).Hillsdale, NJ: Erlbaum.

Revised National Curriculum Policy Document. (2002). Department of Education.

Reznick, B. (1994). Some thoughts on writing for the Putman. *In Schoenfeld, A. (1994). Studies in Mathematical Thinking and Learning*.

Schoenfeld, A. H. (1985). "Teaching mathematical thinking and problem solving in L. Resnick and L. Klopfer (eds). *Toward the Thinking curriculum. Current Cognitive Research.* ASCD, Alexandra. V.A. 83-103.

Sethole, G. (2004). Meaningful contexts or dead mock reality: Which form will the everyday take? *Pythagoras,* 59, 18-22.

Sherin, M. (2002). When Teaching Becomes Learning. *Cognition and Instruction,* 20 (2), 119-150.

Sherin, M. G., Sherin, B. L., & Madanes, R. (2000).Exploring Diverse Accounts of Teacher Knowledge. Journal of Mathematical Behavior, 18 (3), 357-375.

Shuell, T. J. (1990). Teaching and Learning as Problem Solving Mataphors we learn. *Theory into Practice,* 29 (2), 102-108.

Showder, J. T., Phillip, R.A., Flores, A., & Schapelle, B. P. (1995). Instructional Effects of Knowledge of and About Mathematics: A Case Study. *Providing a Foundation for Teaching Mathematics in the Middle Grades,* 251-274.

Shulman, L. S. (1986). Those Who Understand Knowledge Growth in Teaching. *Educational Researcher,* 15 (2), 4-14.

Shulman, L.S. (1987). Teaching a Community Properly. Essays on Higher Education. *"Knowledge and Teaching Foundations of the New Reform,"*57 (1), 1-22

Siegler, R.S. (1995). 'Reading and developmental processes' in Lee, V and das Gapta, P (1995). *Children's cognitive and language development*. Oxford: Blackwell.

Silver, E.A., & Thompson, A.G. (1984). Research Perspectives on Problem Solving in Elementary School Mathematics. *The Elementary School Journal,* 84 (5), 529-545.

Smith, J. P., Di Sessa, A. A., & Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *The Journal of the learning science,* 3 (2), 115-163.

Staples, M. (2007). Supporting Whole-Class Collaborative Inquiry in a Secondary Mathematics Classroom. *Mathematics Education Research Group.* MERG. University of Connecticut.

Stein, M. K., Baxter, J.A., &Leinhardt, G. (1990). Subject-Matter Knowledge and Elementary Instruction: A case from Functions and Graphs. American Educational Journal, 27 (4) 639-663.

Stein, M. K., Grover, B. W., & Henningsen, M. A. (1996). Building student capacity for mathematical thinking in reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal,* 33 (20), 455-488.

Thorpe, R. & Gallimore, R. (1991). 'A theory of teaching as assisted performance' in Light, P. Sheldon, S., & Woodhead, M. (Eds.). *Learning to Think.* London Routledge: The Open University.

Verschaffel, L., De Corte, E. & Vierstraete, H. (1999). Upper Elementary School Pupils' Difficulties in Modeling and Solving Nonstandard Additive Word Problems Involving Ordinal Numbers. *Journal in Mathematics Education,* 30 (3), 256-285.

Verschaffel, L., & De Corte, E. (1997). Promoting Authentic Mathematical Understanding and Problem Solving in the Primary School? In T. Nunes and P. Bryant (Eds.), *Learning and teaching mathematics. An International Perspective,* (pp. 69-97).Phycology Press.

Vos, P. (2002). Like an ocean-linear changing course: The Grade 8 mathematics curriculum In the Netherlands. In *Realistic Mathematics Education,* 30-33.

Vygotsky, L. S. (1976). *Mind in society:* The development in higher psychological processes. Cambridge, MA:Harvard University Press.

Wood, T. Cobb, P., & Yackel, E. (1991). Change in Teaching Mathematics: A Case Study:*American Education Research Journal,* 28 (3), 587-616.

Yackel, E. (1995). Children's talk in inquiry mathematics classrooms. In P. Cobb and H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures,* (pp. 131-162). Hillside, NJ.Erlbaum.

Yackel, E., Cobb, P., & Wood, T. (1991). Small-group interactions as a source of learning opportunities in second grade mathematics. *Journal for Research in Mathematics Education,* 22, 390-408.

Appendices

APPENDIX 1.1: Teachers' consent form

School of Education

Private Bag 3, Wits 2050, South Africa · Fax: +27-11-717-3005 · Telephone: + 27-11-717-3003/3006

e-mail: debrainef@educ.wits.ac.za Physical address (not for correspondence): 27 St Andrews Road, Parktown 2193

Kathleen Fonseca

Ph: 011-551-5800

20 October 2008

Dear

Information and Consent to participate in a research project

DESCRIPTION

You are invited to participate in a research study on mathematics teaching and learning. The study aims to understand teaching practices in primary school mathematics classrooms focusing on how teachers teach problem solving and interact with their learners. This study is being conducted by me, Kathleen Fonseca. I am a student at the University of the Witwatersrand and am studying towards obtaining a Masters Degree in Mathematics Education. This research project is part of my studies. As part of the research project I would like to observe and videotape two to four of your lessons.

Please feel free to contact me at any time if you have questions or concerns about the research

CONFIDENTIALITY

Unless you request otherwise, your name will be kept completely confidential at all times and in all academic writing about the study. If you give permission, video-clips with you in them may be shown at conferences or in teacher-education programmes.

RISKS AND BENEFITS/PAYMENT

There are no foreseeable risks in participating in this study. You will not be paid for participating in the study. Benefits of the project will be a contribution to understandings of mathematics teaching and learning. If you have any concerns about participation, or any questions that you would like to ask, please contact me at any time.

TIME INVOLVEMENT

Classroom observations will take place during class time. Your class will be videotaped for 2-4 lessons.

SUBJECT'S RIGHTS

If you have read this form and have decided to participate in this project, please understand your participation is voluntary and you have the right to withdraw your consent or discontinue participation at any time without penalty. You have the right to refuse to answer particular questions. Your individual privacy will be maintained in all published and written data resulting from the study.

VIDEO TAPES

If you agree to participate in the study but choose not to allow video clips with you in them to be shown at conferences or in teacher-education programmes, these tapes will be kept strictly confidential. Once the tapes are no longer needed for research or teaching purposes, they will be destroyed.

CONSENT

Please complete, sign and return the form below.

CONSENT

I am willing to participate in the study:

_____ Yes

_____ No

I am willing to be videotaped as I teach my classes:

_____ Yes

_____ No

I am willing to be audiotaped during interviews:

_____ Yes

_____ No

I give consent for video tapes with me in them resulting from this study to be shown at academic conferences

_____ Yes

_____ No

I give consent for video tapes with me in them resulting from this study to be used for teaching purposes, in teacher education programmes

_____ Yes

_____ No

Signature _____ Date: _____

Note: The extra copy of this consent form is for you to keep.

APPENDIX 1.2: Parents' consent form

School of Education

Private Bag 3, Wits 2050, South Africa · Fax: +27-11-717-3005 · Telephone: + 27-11-717-3003/3006

e-mail: debrainef@educ.wits.ac.za Physical address (not for correspondence): 27 St Andrews Road, Parktown 2193

Kathleen Fonseca

Ph: 011-551-5800

20 October 2008

Dear Learner and Parent/Guardian

Information and Consent to participate in a research project

DESCRIPTION

You are invited to participate in a research study on mathematics teaching and learning. The study aims to understand teaching practices in primary school mathematics classrooms focusing on how teachers teach problem solving and interact with their learners. This study is being conducted by, me Kathleen Fonseca. I am a student at the University of the Witwatersrand and am studying towards obtaining a Masters Degree in Mathematics Education. This research project is part of my studies. As part of the research project I would like to observe and videotape a number of lessons in your/your child's mathematics class.

Please feel free to contact me at any time if you have questions or concerns about the research

CONFIDENTIALITY

Unless you request otherwise, your/your child's name will be kept completely confidential at all times and in all academic writing about the study. If you give permission, video-clips with you/your child in them may be shown at conferences or in teacher-education programmes.

RISKS AND BENEFITS/PAYMENT

There are no foreseeable risks in participating in this study. The study will have no effects on your/your child's mathematics marks. You/your child will not be paid for participating in the study. Benefits of the project will be a contribution to understandings of mathematics teaching and learning. If you have any concerns about participation, or any questions that you would like to ask, please contact me at any time.

TIME INVOLVEMENT

Classroom observations will take place during class time. Your class will be videotaped for 2-4 lessons.

SUBJECT'S RIGHTS

If you/your child have read this form and have decided to participate in this project, please understand your participation is voluntary and you/your child has the right to withdraw your consent or discontinue participation at any time without penalty. You/your child has the right to refuse to answer particular questions. Your individual privacy will be maintained in all published and written data resulting from the study.

VIDEO TAPES

If you/your child agree to participate in the study but choose not to allow video clips with you/your child in them to be shown at conferences or in teacher-education programmes, these tapes will be kept strictly confidential. Once the tapes are no longer needed for research or teaching purposes, they will be destroyed.

CONSENT

Please complete, sign and return the form below, indicating whether you agree or do not agree to participate.

CONSENT (Learner)

I am willing to participate in the study:

_____ Yes

_____ No

I am willing to be videotaped in my mathematics class:

_____ Yes

_____ No

I give consent for video tapes with me in them resulting from this study to be shown at academic conferences

_____ Yes

_____ No

I give consent for video tapes with me in them resulting from this study to be used for teaching purposes, in teacher education programmes

_____ Yes

_____ No

Learner's name: _____

Signature: _____

Date: _____

CONSENT (Parent/Guardian)

I give consent for my child to participate in the study:

_____ Yes

_____ No

I give consent for my child to be videotaped in his/her mathematics classes:

_____ Yes

_____ No

I give consent for video tapes with my child in them resulting from this study to be shown at academic conferences

_____ Yes

_____ No

I give consent for video tapes with my child in them resulting from this study to be used for teaching purposes, in teacher education programmes

_____ Yes

_____ No

Learner's name: _____

Parent/Guardian's name: _____

Signature: _____

Date: _____

APPENDIX 1.3: Interview schedule

INTERVIEW SCHEDULE

Purpose of the interview: The purpose of this interview is to understand how you think about your learners' problem solving. I will first ask you to solve the problem and then ask you how you found it. When you have finished the problem I will ask you whether and how you would use it in the classroom and ask you to imagine what difficulties your learners might have with it. The important thing here is that I am not interested particularly in your problem solving; this is not a test of your knowledge in any way. The idea is for you to share your insights about your learners with me, so that I can learn more about those. The idea is also for you to discuss your thinking with each other.

Questions

I see you found this question easy/ challenging to answer.

(a) What makes this type of question easy /challenging to answer for you? Explain.
(b) Can you reflect on your own mathematical problem-solving. What mathematical strategies did you use to solve the problem?
(c) Do you think your learners will be able to answer this question? Why do you think so?
(d) Think about how your learners might solve this problem. What strategies might they use and what might they find difficult?
(e) Would you use this problem to teach with? , If not, why not? , If yes, at what level?
(f) What would your goals be in using this problem?
(g) How would you plan to teach it? As part of which topic?
(h) Describe a lesson of which this might be part of.
(i) How would you evaluate whether the problem solving had been successful, and how would you know what learners had gained from solving the problem?
(j) If some learners come up with incorrect responses, what would you say are the real underlying difficulties of your learners' errors?
(k) How would you go about in assisting your learner with incorrect responses?
(l) These were the results from learners who wrote these tests and the answers they selected as the correct answers.

Achieved: 19%

A= 19%

B = 15%

C =19%

D= 42%

(m) Do you think the results of your learners will be similar or different? Explain.

TASK 2

Questions

(a) What challenges did you experience with this task?
(b) Can you reflect on your own mathematical problem-solving. What mathematical strategies did you use to solve the problem?
(c) Think about how your learners might solve this problem. What strategies might they use to solve the problem?
(d) What aspects of this task will make it difficult for your learners to answer it?
(e) What would you say will be possible incorrect answers from your learners? Which answer will they choose?
(f) What in your view are the actual difficulties your learners are experiencing in working with number patterns?
(g) Would you use this problem to teach with? , If not, why not? , If yes, at what level?
(h) What would your goals be in using this problem?
(i) How would you plan to teach it? As part of which topic?
(j) Describe a lesson of which this might be a part?
(k) How would you evaluate whether the problem solving had been successful, and how would you know what learners had gained from solving the problem?
(l) How would you go about in assisting learners who were unsuccessful in solving the problem?

(m) These were the results from learners who wrote these tests and the answers they selected as the correct answers.
Achieved: 22%

A= 27%

B= 25%

C= 22%

D= 22%

(n) Do you think the results of your learners will be similar or different? Explain.

TASK 3

Questions

(a) What makes this type of question easy/challenging to answer for you? Explain.
(b) Can you reflect on your own mathematical problem-solving. What mathematical strategies did you use to solve the problem?
(c) Think about how your learners might solve this problem. What strategies might they use and what might they find difficult?
(d) Do you think they will find this task different from the previous two tasks? What would be the difference?
(e) If there are learners who are unable to solve this problem, what would you say are the underlying causes?
(f) How would you go about in assisting your learners regarding this underlying causes or difficulties?
(g) Would you use this problem to teach with? , If not, why not? , If yes, at what level?
(h) What would your goals be in using this problem?
(i) How would you plan to teach it? As part of which topic?
(j) Describe a lesson of which this might be a part.
(k) These were the results from learners who wrote these tests and the answers they selected as the correct answers.

Achieved: 20%

A= 31%

B= 20%

C= 20%

D= 20%

(l) Do you think the results of your learners will be similar or different? Explain.

TASK 4

Questions

(a) What challenges did you experience with this task?
(b) Can you reflect on your own mathematical problem-solving. What mathematical strategies did you use to solve the problem?
(c) Think about how your learners might solve this problem. What strategies might they use and what might they find difficult?
(d) Do you think your learners will find this task to be different to the previous tasks? What would be the difference?
(e) If your learners experience difficulty in answering this task. What do you think would be the underlying difficulties?
(f) Would you use this problem to teach with? , If not, why not? , If yes, at what level?
(g) What would your goals be in using this problem?
(h) How would you plan to teach it? As part of which topic?
(i) Describe a lesson of which this might be part.
(j) How would you evaluate whether the problem solving had been successful, and how would you know what learners gained from solving the problem.
(k) These were the results from learners who wrote these tests and the answers they selected as the correct answers.

Achieved: 20%

A= 31%

B= 20%

C= 20%

D= 20%

(l) Do you think the results of your learners will be similar or different? Explain.

APPENDIX 1.4: Interview Transcripts

Teacher A.

Researcher: Did you find it easy or challenging.
Teacher: For my learners?
Researcher: No for you.
Teacher: It's challenging. It was challenging because we start with. I didn't think of BODMAS-rule.
Researcher: Yah.
Teacher: And then I had to work all of them out before I could come to the correct one.
Researcher: So what do you think makes it tricky or difficult?
Teacher: I think what makes it tricky. I think if it could have said use BODMAS-rule maybe it was going to be quicker to get the answer but now I was working with all of them just to check which one is correct and immediately saw 20 and 10 and worked it out mentally and saw D was correct and C and B and A.
Researcher: So what mathematical strategies did you use to solve the problem?
Teacher: What are you talking about strategies? What are you referring to?
Researcher: Like what methods did you use?
Teacher: Oh.
Researcher: Ok, so you substituted.
Teacher: I divided. I substituted the blocks according to the way they are put in here. Then divide again, substitute the other one to get the correct one.
Researcher: Ok, so to get the correct answer what did you apply first?
Teacher: I had to, to use BODMAS-rule and not just do them as they look I had to use the BODMAS-rule which is mathematically correct. The other one I was just to get the answer and unfortunately all the answers are correct but they are not right.
Researcher: Mathematically not right.
Teacher: Yes mathematically it's not right.
Researcher: Do you think your learners will be able to answer this question?
Teacher: They will struggle; they could be able, maybe after struggling. They will struggle the problem is tricky because all the answers gives 5. So he or she might not use the correct mathematical law or rule that is supposed to.
Researcher: Oh, ok, why do you think they would struggle?

Teacher: I think maybe if it could have been written here that calculate use the BODMAS-rule it was going to be easier for them to get the correct. But now it is just open for them to think whether how must they, will work it out.

Researcher: Yah, maybe do you think maybe if you have others in here that will not give you 5. That also will help them quicker get the answer.

Teacher: Yes but they not going to use the BODMAS-rule. If there were different answers for this one they not going to get it quicker but they not going to use the BODMAS-rule. So now getting the same answers they pushed to use the correct rule to calculate but they going to take time to think.

Researcher: Think about how your learners will solve this problem. What strategies might they use and what might they find difficult. How would they solve this problem?

Teacher: I think they going to maybe take use all 4, A, B, C, D start with the first one calculate and check whether answer is correct until the end. But the tricky part is that all of them are going to give them 5 and then they will think they can make a mistake that its they are correct according to the way they calculate.

Researcher: But if they just substitute this one by the way it is it won't give 5.

Teacher: Yah, ok.

Researcher: Yah, can you see if you put 6 minus 2, divided by 2 it won't.

Teacher: It won't and then 14 minus 4 it give 10 divided by 2 give 5 and then 18 minus 4, so 14 divided by 2.

Researcher: Yah all those, this is 18 minus 8. So you say they substitute first.

Teacher: According to the sign they will write 6 there and 4 there but then you say 8 but this is the correct one.

Researcher: Yes, unless they have to find out that they have to use BODMAS.

Teacher: Yes, so it's quite tricky.

Researcher: Would you use this problem to teach with.

Teacher: To teach with.

Researcher: Yah.

Teacher: Yes when they advanced and maybe understand the BODMAS-rule and tell them to use the BODMAS and if you don't tell them they won't get it right unless. Unless you know they remembers or you use exercise immediately after you have done BODMAS-rule. Because maybe you decide you going to do the test at the end of the term then they can get a problem cause they can use the easiest

according to them they can take the quicker one and and yet it is the wrong one.

Researcher: Ok, at what level will you teach it?

Teacher: BODMAS, from grade 5 but we do it in grade 6.

Researcher: You will teach it in grade 6?

Teacher: Yah we do it even if we don't go in-depth, we do it for them to calculate easier.

Researcher: What would your goals be in using this problem?

Teacher: My goals would be for them to be able to calculate easier for BODMAS-rule make the word sum easier rather than if you did not use multiply or divide first. You answer in that word sum becomes wrong you need to use the BODMAS-rule for it to be correct.

Researcher: How would you plan to teach it as part of which topic?

Teacher: It falls under numbers, maybe after numbers maybe after like I've did after we've done multiplication, addition and we've started with word numbers. Because you find that there is no format like this so we read maybe we went to town and we bought five. What, how much I had and multiply by 4 so she must find the word sum. So when you do the word sums that is when you use it more so that they can calculate properly.

Researcher: So if you just give them like this without word sums with brackets and division in one without word sums?

Teacher: You can also but you have to teach them BODMAS-rule because if I can say maybe 7 plus 2 and then minus 3 times 12, they won't go for 3 times 12 first. They will start with this one first and get their answer wrong. So the first have to teach them BODMAS-rule before they get those exercises.

Researcher: Describe a lesson of which this might be part of. Just like how you will start and like you said previously you'll explain BODMAS and stuff like that.

Teacher: You actually need to explain what BODMAS stand for and why its brackets, multiply, of. You must explain if they calculate they must follow BODMAS. You must explain what they start with.

Researcher: How would you evaluate, say now the children are busy in the class and how would you evaluate whether problem solving have been successful and how would you know whether the problem solving have been successful.

Teacher: I think after they have written down they have to explain how did they come up with this answer. So maybe they'll get the same answer and you'll find out that they don't understand. When they got to, how did you reach 10, when they get 5, how did you reach 5 and both of them. I won't

say that they are wrong then I'd say if you have started with this one go back to your BODMAS-rule and see that you cannot start with addition when there is multiplication. You need to start with division, multiplication before you can start with subtraction or whatever because you'll find that maybe the one that is wrong and the one that is right is also did not use the BODMAS-rule. So to emphasis that they have used they must explain how did they reach their answer because sometimes you find that they are mistakes or they don't understand but as long as they reach the answer. And if I don't show explain it means the other answers won't get it.

Researcher: So you would ask them to explain how they got the answer. So that is also how you would find out what they have gained, what they have learned from solving the problem. If some learners come up with incorrect responses what would you say are the real underlying difficulties of their errors.

Teacher: Maybe they did not understand properly the rule then I need to explain and find out why. She added like this and maybe say I did not add, I multiplied then I must use and say next time you start with this one but if you start with addition in this particular sum of BODMAS. So if they know that even in other sums they will be able to work them out.

Researcher: So how would you go about to assist your learners. You said you will?

Teacher: Explain make groups because in their groups, explain further give more simpler ones for them to understand. If they understand they can do difficult ones so we go back to the simple one and also the one that they understand in the group to explain in front of me. How did they reach the answer and if she has followed the correct rule.

Researcher: These are the results of the learners who wrote the test only 19% got the correct answer.

Teacher: It could be similar or maybe depending on like I explain before. Like if you have given the word sums using plus, minus, brackets immediately I can give them exercise maybe it can be more. But if I can give others like maybe notation I can make one big test then it's going to be tricky. So if they have done this its' the BODMAS-rule.

Researcher: So if you give them this and just go back to your class now and give it to them will they also get 19%.

Teacher: Same or lesser.

Researcher: So if you give a lesson on BODMAS and then give it to them.

Teacher: then they can get higher more but after some time they forget you need always to remind them or before they write I need to explain to them that this is the BODMAS-rule. But if I can just give it like this they won't.
Researcher: We moving on to question 2. You have to ok, you can carry on I'll explain afterwards.
Teacher: For learners it will be difficult because I see a pattern.
Researcher: Yah, you have to break these numbers into that pattern then it's easier to multiply.
Teacher: Ok, break these numbers.
Researcher: You see this first number is 7, so that 49 must be 7 times what?
Teacher: 7 so I do 7 times 7 like this?
Researcher: Yah, times and times, now 55 you have to break up. See it's 11 and the 26 must have a 13 in.
Teacher: So why the answer is like this, oh it will give.
Researcher: You have to work it out, it's one of these 2. So now you can group it like this 7 times 11.
Teacher: Oh, ok, ok, like this.
Researcher: 13 times 5 times 40.
Teacher: 35 times 2.
Researcher: I think you must times.
Teacher: But see they have 4, 1, 2, 3, 4.
Researcher: So it's 7x11 then you have 13x5 and then you have 7x5x2.
Teacher: So it's 13x5 again, is that what you saying?
Researcher: You have 7x11.
Teacher: ok.
Researcher: Then it must be 13x5, you could have 13x7 then it's just the 5 and the 2.
Teacher: What, ok why do I have 2, 7's here and 1 there.
Researcher: You must have that one there.
Teacher: I am going to use my calculator.
Researcher: What is 49x36x55?
Teacher: I don't know I've made a mistake because it's also jumps and come to subjection.
Researcher: Oh, the calculator.
Teacher: 70070.
Researcher: So the trick is that you have to get it in that format. You see you had to multiply by 10 then it makes it easier to get it. This is right here if you times by 10, you just add the zero.
Teacher: Yah.
Researcher: That was right just had to add the zero so your format is right there for the pattern.

Teacher: But for learners it's tricky, it was also tricky for me, for the pattern to think that I have to you know.
Researcher: So what challenges did you experience with this task?
Teacher: It was really, really challenging because immediately saw the question I forgot about the instruction that I must make a pattern. I wanted to count and also that the pattern must follow so if I didn't know what is 49, 7x7 it was going to be tricky. Maybe I was only going to take one 7, yah, it was really, really challenging.
Researcher: What mathematical strategies?
Teacher: Number 1, the time-table for this numbers if you don't know your time tables then there will be a problem because you won't make 7x7, 11x5 and also that you have to follow the pattern it has to be 11x5 and this one have to be 13x2.
Researcher: Or you could say the 5 and 2 and then it's 13x2. So then you have 7x11 and then 13x10 then you still have that other 7.
Teacher: But I think I wouldn't have think about 13x10. I want to to do the pattern so I was looking at a smaller number not a bigger and yet if you do it times 10 it is bigger.
Researcher: So I see you first tried to multiply that out and then you went to the pattern because you said earlier it was important for you to find the answer.
Teacher: Yes more than the pattern.
Researcher: The pattern.
Teacher: So the question to find the pattern more than the answer, it means I was distracted really because I was worried of doing the answer. Rather than finding the pattern and find the answer from it.
Researcher: Think about your learners' strategies.
Teacher: They have to know time tables maybe they will try with 7 and 11 like I did here before and it is also easier. When you do it this way they will use those 7 and 11 and 13 as long as they are able to use their time tables.
Researcher: Yes, to find the multiples.
Teacher: So would, some would be distracted just like me or forget about the pattern and think about the answer multiply. I wonder I just want to check if you don't do the pattern do you come up with the same answer.
Researcher: Yes.
Teacher: 49 but now the pattern is gone.
Researcher: Yes, so they won't be able, they won't be allowed to use a calculator.
Teacher: Yes.
Researcher: What aspects of the tasks would make it difficult for your learners to solve the task?

Teacher: I think they have to if they have had some sums like this in class for them to do maybe they can tackle this one but if you have done patterns without such challenging questions and this distraction.
Researcher: It would be very difficult for them.
Teacher: They would get the answer but they won't use the correct thing that they supposed to do.
Researcher: So the pattern.
Teacher: If maybe there was no answer here like multiply and there was no answer and say give the pattern maybe they could.
Researcher: So because the answer is there?
Teacher: Because the answer is there, you completely forget about the pattern. I wanted to get the answer and yet you need to get the pattern correct.
Researcher: What would be possible incorrect answers?
Teacher: You know when I calculated I added this one A,but I noticed that I subtracted I made a mistake.
Researcher: Ok.
Teacher: If they have to calculate without a calculator they could come up with different answers which are wrong.
Researcher: And if they just look at the pattern like if they forgot about that and follow the pattern which one would you say?
Teacher: They would take.
Researcher: If they had to follow the pattern?
Teacher: Maybe they would take A the first one because there is a pattern here.
Researcher: So if they look at the answer which one will they take?
Teacher: They'll take the 4, they won't look at the answer, and they'll just take the one that starts with 4.
Researcher: What would the actual difficulties be?
Teacher: There is sometimes for instance maybe you've done like, you've done 19 I'm just trying to think 20, 25, they forgot to maybe they must add.
Researcher: The common difference.
Teacher: The common difference especially when there are gaps in between. It becomes difficult until you give them more and more exercise like that still they won't know. They want you to show a sign and they want you to put, you must add 5 and if you didn't they can make a mistake, it's tricky.
Researcher: Would you use this task to teach with?
Teacher: Yes but after I have made other patterns simpler ones, maybe with small numbers before I come to 49 and 55 and big ones. I need to start with the simplest one and make a pattern. Yes then I would say make a pattern. I would not say what is 49 because how the pattern is gone. I'll just

work with they must give an answer working with the answer only. I'll say the answer is 430, make a pattern with it.

Researcher: At which grade will you teach it?

Teacher: Grade 6.

Researcher: Grade 6 will be appropriate?

Teacher: Still in grade 6 they going to help I will start in grade 6.

Researcher: What would your goals be for teaching the problem?

Teacher: It would be to make sure that they think logically but you must really, really focus. You must know your time tables; you must know your numbers and calculations otherwise if you don't know your time tables she could take a wrong number. She could take a wrong number or adding or make 55, two numbers to add to 49 not using multiples.

Researcher: What in your view would be the actual difficulties your learners will experience working with number patterns?

Teacher: They could because they don't think the same others they have challenging minds. They could come up or they could make a pattern or find out they make a mistake but if they 7, 11, 13 they could miss up that 5.

Researcher: yes, yes.

Teacher: In our pattern there is two 7's, they could miss up 1 and if they add it, it would give the answer.

Researcher: How would you plan to teach this topic?

Teacher: It's patterns I can use it in patterns in numbers not when we what do we call the other patterns that we use.

Researcher: Geometric patterns.

Teacher: Yah, no just after all addition, subtraction and what we were talking about before they must know but this one I wouldn't ask it like this one it's quite tricky.

Researcher: That would take the focus of the pattern if you give it like that.

Teacher: Talk about patterns give me the pattern, follow the example not actually make them to multiply.

Researcher: How would you go about in evaluating whether the problem solving has been successful?

Teacher: I'll weigh how much they get, how many got, maybe 20% or so and then from that I can see that they didn't understand it properly and then try again. Those who got it right must explain how did they get it right. What did you do and then those who got it wrong what did you do and those maybe did not use the multiples maybe they didn't use the multiples got it wrong.

Researcher: So those who didn't use the multiples, how would you go about to assist them?

Teacher: Like this is 49 and you can see that you have to follow the pattern, you have been given. Here you have 49 which is 7x7 and then after you have done that you have to use both 7's don't throw the other one away. To also make a pattern, and also come to 55, multiples of 55 and 5 and also for 26, 13 and 2. Then they will see there is a pattern, maybe give them another one, then I can see that they understand when I can give them this one. Because it's going to be very challenging it is maybe going to take them an hour or, or 30 minutes, they won't get it just like that.

Researcher: You'll have to guide them.

Teacher: Yes.

Researcher: These were the results 22% so after ok, before you're given the lesson and you just go to your class and you give them that, what would the results be?

Teacher: I don't know in our school these learners take time. We'll have to repeat it and repeat it and give them exercise until they understand. If I give them after I've explained maybe they can get 20.

Researcher: After you've explained?

Teacher: I mean I need to explain and explain it if I had to give them exercise like this one. If I have given them and I have just explained patterns this one is going to be difficult for them. I need to also give them exercises of similar questions otherwise if I give them this.

Researcher: They won't be able to do it?

Teacher: No unless it's guess work especially if you going to give no answers.

Researcher: Let's move on to task 3. This one of the 4 children, a ratio one.

Teacher: I have to be honest you know we don't realy do ratio with our learners we rob, we do it quick, quick.

Researcher: Yah.

Teacher: I don't know if we only do maybe graphs. Like I remember when we did with people going to a park but I did it only in passing just to say I have done it but we don't do it. Where there is 12 the correct answer?

Researcher:: It's a ratio, there's 4 children.

Because I see Nina ate twice as Ben, Ben ate half as Sarah ate 10. Ok Ali ate half as many berries as Ben, so this Ben and Nina ate twice as many berries as Ben so this

Teacher: If Sarah ate 10 and Ali ate half of Sarah.

Researcher: No, no, Ali's amount and Sarah's amount is half of the total amount. This two is right if you have 10 there, so her's is half but he had, if you look at Ali' has half as many berries as Ben.

Teacher: So they ate 5, 5?
Researcher: So two of them, he ate 2 so then you work from there. So if he ate 2 Ben ate, Ali ate half as many berries as Ben, 2 is half of what?
Teacher: One.
Researcher: No, 2 is half of which number
Teacher: 4.
Researcher: So Ben ate 4.
Teacher: I think the language, the language.
Researcher: And then Nina.
Teacher: Nina twice as many berries as Ben. Twice as many berries as Ben ate 8
Researcher: 8?
Teacher: 8, he ate 8 and Nina ate twice as many berries as Ben and Ali ate half as many berries as Ben is 2. This one ate 10, so 12, 24. The language is tricky, the language, the second language to understand the language.
Researcher: You'll have to break it up step-by-step.
Teacher: You have to read and re-read for them to understand. Cause if there is only two they all maybe understand but there is 4 and the other one is specific, they had 10. So they must understand this language. I must understand exactly, properly.
Researcher: So you would say what makes this challenging is the language.
Teacher: The language.
Researcher: Not so much the Maths?
Teacher: Also the Maths I mean the ratio but the language you'll get confused this Nina and this Ali ate half of someone or twice, yah, language. For learners in our schools have to have exercise. I don't know how many but similar exercises under normal conditions they won't be able to get it no ways because you must understand this double, half and twice. We use them but not in this format but now if you have to halve the number and you got 10 you will never think of you know.
Researcher: Which mathematical strategies did you make use of?
Teacher: I used the 10, the one they gave the.
Researcher: The known one.
Teacher: Yes the known and also the mathematical language those double, half of something from the number that has been given because if there was no answer I was going to struggle a lot.
Researcher: Yah, they have to at least give you one number so that you can start from there you can't start from nowhere. So you didn't make use of fractions?

Teacher: I did not at the moment think of any fractions my main problem was the language also to understand exactly what is wanted. Yes fractions is also included there but.

Researcher: You didn't work it out just worked with the names and the quantities as it is given there trying to work from there?

Teacher: Yes.

Researcher: What strategies would your learners use.

Teacher: The language.

Researcher: Like a really bright learner who does not have a language problem. How would they solve it?

Teacher; Even if he is bright he will have difficulty.

Researcher: Difficulty?

Teacher: If I teach them I find it difficult myself. What about them because we speak English at work only when we do school work we speak English. So they'll have to think and think for 20 minutes or 30 minutes or 1 hour to work out the language. The language we need to prioritize, don't do ratio on its own.

Researcher: But if you look at the type of question this is just numbers so they can find the maths quickly but here do you think they need to take the maths out or?

Teacher: They have to use maths and the English they must understand. So you as the teacher if ever you have to explain it's more language then mathematically. You tell them this is how you must do it but you have to emphasise twice.

Researcher: The difference between the three, the main difference apart from the difficulty and language.

Teacher: Maybe this one is simpler.

Researcher: If it was just numbers?

Teacher: They could, they could get it quicker than this one, they could.

Researcher: What would be the underlying causes for why learners will struggle with this task?

Teacher: The language is the main barrier cause if they don't understand. If I don't understand what you are asking I can never get it. They will try, they will struggle, they will try and miss-up twice, half but if they will maybe be a number than they can do ratio.

Researcher: So if you give them?

Teacher: This is the 10 you use to get the answer but those other they could miss-up and don't know twice as many, half, comparing.

Researcher: So how would you go about in assisting your learners regarding underlying cause do you think they would have a problem with fractions or just with comparing.

Teacher: I think just with comparing fractions are numbers but when you put them in such a situation. We don't teach them. Go back to fractions and language, mathematical language need to understand that he knows a quarter but quarter of.
Researcher: Will you teach this task?
Teacher: Challenging but will teach it and see what they do. Will show them how to work it out, the language need to learn it.
Researcher: What would your goals be for teaching this?
Teacher: Improve mathematical language and how to work with ratio.
Researcher: How would you plan to teach it;
Teacher: Ratio doing after money. I had an exercise where they had to compare with money. I did not use it for quantities.
Researcher: Describe a lesson of which this task would be part of.
Teacher: I'll start with fractions because they have to know fractions.
Researcher: The result for this problem was 20%. Will the results for your learners be similar or more?
Teacher: Similar after a lesson it would be a bit more even if you know your, learners like now we busy with angles. They are struggling to use a protractor. I need to give them more and more exercise, similar exercise then they can improve.
Researcher: Ok, the last one.
Teacher: Why did they put that one there?
Researcher: Douglas is just there to confuse you.
Teacher: So Ming has got more than Omar. I think Ming has got more that Omar because Omar has got 12 less because if Omar has 12 less then it means Ming has got more. But we don't know how much Douglas have. Which makes it trickier, so I need to think?
Researcher: Was the task easy or challenging and what made it challenging?
Teacher: I was confused by Douglas did not know how much Douglas had but when Ming has 3 less, Omar has 12 less.
Researcher: What mathematical strategies did you use to solve the task?
Teacher: I was comparing the quantities. This Douglas, Ming and Omar.
Researcher: What strategies might your learners use and what might they find difficult?
Teacher: They will try if I have already taught less than, more than. It is confusing but they could try. We have spoiled them, we always teach straight numbers.

Researcher: Do you think your learners will find this task different to the previous ones. What would be the difference?
Teacher: They are all challenging, they must compare therefore they are similar.
Researcher: If you look at the other two questions the first two and this one. Is there a difference?
Teacher: The other two were just numbers where you have to work it out. This one you must think deeper to compare and find the answer.
Researcher: What would be the underlying difficulty if learners experience difficulty in solving the problem?
Teacher: Understand, reading with understanding, compare subtracts or add 3 plus 12 get 15. The other one get more.
Researcher: Will there only be a language problem or mathematical problems too?
Teacher: Only the language problem.
Researcher: Will you use this task to teach with and at which level?
Teacher: Yes in grade 6.
Researcher: What would your goals be in using this problem?
Teacher: To make them think and to also challenge them to think mathematically. The terms actually encourage me to teach the correct terms.
Researcher: How would you go about in planning to teach this task and describe a lesson of which this would be part of.
Teacher: Start with comparing, start with difficult words. Give examples of less than, more than and use practical examples which they understand.
Researcher: How would you evaluate whether the problem solving had been successful? How would you assist your learners?
Teacher: I need to start again find another way> I must do it practically. It is with us teachers we don't use the language. I change to my language when they don't understand. I change to my language.
Researcher: The results of the learners who wrote these tests were 20 %. Will the results of your learners be similar or different?
Teacher: If I don't teach them the terms of more than and less than it would be similar or even lesser. If I give them after I have taught it might be more.

Teacher B

Researcher: You have to do the first task for me look at it from here.
Teacher: What is this is this subtraction.
Researcher: Yes, that's minus and this is divide.
Teacher: 10 divide by 2 is 5, 18 minus 8 divide by 2 is 5 it is D, C and B.
Researcher: If you think of the order of operations?
Teacher: Brackets, BODMAS, if I maybe think of BODMAS to get the answer.
Researcher: See if you take this one and put it there and do this division first. Because BODMAS you do division first. How did you find this question easy or challenging? What made it challenging? What do you think will make it challenging for your learners. The fact that it is BODMAS, or not so clearly?
Teacher: To me it's not so clearly learners struggle with BODMAS. When they see the task like this they don't think what is divisible they go straight to find the solution, without thinking they have to do it procedurally.
Researcher: What mathematical strategies did you use?
Teacher: To me what I normally do I look at the question; think critically, trial-and error to get the solution.
Researcher: Will your learners be able to answer the question?
Teacher: Some would but some wouldn't struggle with it. The way the set-up of the question.
Researcher: What strategies would your learners use and how will they approach it? Would they see BODMAS, would they know its division then subtraction?
Teacher: To me they won't see BODMAS they would approach it as division and from there they would just group, group like they do as if they are from grade 5 but they in grade 6. Group, group, write strokes until they find a solution, they won't get it through BODMAS. They will even use trial-and error, group until they accidentally come up with a solution.
Researcher: Would you use this task to teach with and at what level?
Teacher: As a teacher we do teach them BODMAS.
Researcher: But like this?
Teacher: No not like this.
Researcher: What level?
Teacher: Grade 7.
Researcher: Not grade 6?
Teacher: No.

Researcher: Why not grade 6, don't they do BODMAS?
Teacher: They do but that language.
Researcher: But that one doesn't have language.
Teacher: BODMAS to them is another language, Chinese on its own.
Researcher: What would your goals be for using this task to teach?
Teacher: As a teacher I would say division, subtraction, multiplication using strategic competence when approach a test. They shouldn't jump into a solution they should take into consideration the BODMAS-rule.
Researcher: How would you plan to teach this problem?
Teacher: LO 1, basic operations.
Researcher: Will you incorporate it anywhere else?
Teacher: I will prefer it with something like BODMAS.
Researcher: On its own?
Teacher: No with other topics.
Researcher: Describe a lesson of which this task would be part of. What would you introduce? How would you start out with before you eventually give them the task?
Teacher: I would introduce addition then ask them if they have other strategies that they can come up with to find the same solution. There I will be testing their strategic competence then from there I'll tell them when you do BODMAS you start with what is in the brackets then from there you can show multiplication then gradually you can put in other numbers.
Researcher: How would you evaluate whether problem solving had been successful?
Teacher: When the learners can give their own examples and explain maybe to their peers and others. Then I'd know now they've grasped.
Researcher: What would be the underlying difficulties for learners who may struggle to solve the problem?
Teacher: I'd like to find out how they do it first. They should explain to me why they did it that way. Then I'll pick it up from there and see where their problems are. Where they have gone wrong and see what maybe they did not do the correct procedure using the BODMAS rule. My understanding is that they can come up with their own strategies but I'll go back and emphasise the BODMAS rule they are not wrong they are correct but the right way is how to do it with the BODMAS-rule.
Researcher: You say it's a lack in the BODMAS-rule that they would have?
Teacher: Yes.

Researcher: The results of the learners who wrote these tests were 19%. Do you think your learners' results will be similar for grade 6?
Teacher: What's this grade 6?
Researcher: Do you think yours would be a bit higher?
Teacher: I think after explaining mine would be a bit higher.
Researcher: After you have explained?
Teacher: Yes after I have explained and given them examples, pair them in groups and as division.
Researcher: We're going to task 2. So that's the pattern you have to see what is 49x55x26.
Teacher: Without working it out?
Researcher: You have to work it out. You have to get it in that pattern see it must be the same format, its starting by 7. You have to break up these numbers to get the same pattern.
Teacher: Break up, work out what 49, 55 and 26 is? Can I use a calculator?
Researcher: You can use, yah but then you must find the pattern. The learners won't be able to use one. The trick lies in here that you have to break that up. What gives you 49, 7 times what gives you 49. You must also break up 7 times what is 49. Its times hey? Its all times, 49x55x26.
Teacher: And after breaking it up?
Researcher: Then you can multiply that and that. You can multiply all of them. If you look at the pattern so it's going to be 7x7x11x5x13x2.
Teacher: Must give me one of this?
Researcher: One of that yah.
Teacher: So it will be 70070.
Researcher: But you'll have to multiply that out. What challenges did you experience with this problem? Was it challenging for you?
Teacher: Yes it was a challenge for me because multiplication was difficult what about the learners?
Researcher: In relation to patterns was it easy to pick up the pattern?
Teacher: It wasn't easy to pick up a pattern for you to pick up a pattern. You really had to work.
Researcher: Work, break up the numbers/
Teacher: Yes break up the numbers like in BODMAS we have to multiply this and this multiply this and this.
Researcher: What mathematical strategies did you use to solve the problem?
Teacher: Simple multiplication before I could even see the pattern there.

Researcher: Just by multiplying did you at first wanted to go to 49x55x26/ or did you want to break them up?

Teacher: I think the best thing was to break them up first before I multiplied. Remember they are not allowed to use a calculator because I know it's easy just to grab a calculator. Do you think they'll notice the patterns or would they just multiply 49x55x26, or would they break it up differently from the pattern/

Teacher: I think they'll just jump to multiplication because they will see the multiplication sign. They won't even see the pattern. They'll just multiply but not all of them, some will break it up in place value notation first Tens and Hundreds but just a few of them.

Researcher: What aspects of the task would be difficult for your learners?

Teacher: The pattern to find the pattern.

Researcher: Why do you think that would be difficult to do? You teach patterns?

Teacher: Because when we teach multiplication we don't do it in patterns we just do multiplication in BODMAS and ordinary multiplication. Because what I know once they see the basic operation they just jump to multiplication.

Researcher: What would be the underlying difficulties for learners who will struggle to solve the problem? Actual difficulties with number patterns?

Teacher: Consistency.

Researcher: The constant difference?

Teacher: The consistency in pacing.

Researcher: Would you use this task to teach with and at what level?

Teacher: Grade 7 because there is progression of patterns in grade 5, 6, 7.

Researcher: If you explain patterns to the grade 6's this way?

Teacher: I think they will be able if we try it and we break it up.

Researcher: What would be your goals for teaching this problem?

Teacher: Number patterns.

Researcher: Would you also show them that there isn't just this one way of finding patterns. Like we always do with the constant difference or this type of way where you have to break up numbers?

Teacher: Yes, they can break 26 into different patterns.

Researcher: How would you plan to teach this task as part of which topic?

Teacher: As part of basic operations under number patterns. I don't want to teach it in isolation. They should actually see it.

Researcher: How would you start your lesson?

Teacher: First find out if they realize with the odd numbers, the even numbers what is it that they observe. And then they'll tell me we skip two numbers before we can go to the next.
Researcher: Then you'll move on to more difficult ones.
Teacher: From simplex to complex.
Researcher: How would you go about in evaluating whether the problem solving had been successful?
Teacher: I'll give them tasks like this one where they add and multiply. Use other strategies not just jump to conclusions.
Researcher: How would you know if your lesson was successful?
Teacher: If I could have an activity and when they can give examples.
Researcher: How would you go about in assisting your learners who are experiencing difficulty?
Teacher: Normally what I do I explain then each one teach one and those who understand better explain to those who don't. If they still don't understand then I'll intervene.
Researcher: How do you intervene, what do you do? Do you ask questions?
Teacher: I'll ask questions and give simple examples and more examples.
Researcher: The results for this task were low. Would the results of your learners be similar or different?
Teacher: Before I teach the task it would be very low but after I have taught it, it could be higher.
Researcher: This is task 3.
Teacher: What is it that what you want here?
Researcher: You have to find the answer.
Teacher: If I say 20 divided by 2 and divide it I can come up with a different strategy.
Researcher: Ok Ben and Nina ate half. Sarah and Ali ate the other half. So 1:2; 2:4; 4:8 and the 10.Let's add them all together. What makes this type of question easy or challenging to answer for you?
Teacher: The language and the repetition of other people's names. Learners have problems with word sums.
Researcher: What mathematical strategies did you use to solve this task?
Teacher: I started by looking at the number of people then broke it up into half. Then it becomes complicated.
Researcher: What strategies might your learners use to solve the problem and what might they find difficult?
Teacher: Learners are not familiar with ratio in word sums. Repetition of people's names. English is a second language.
Researcher: Do you think your learners will find this task different to the previous two tasks?

Teacher: Yes they have to take the mathematics out of the language.
Researcher: What would be the underlying cause why learners will struggle to solve the problem?
Teacher: Language and ratio.
Researcher: Do you work with fractions?
Teacher: We have to work with fractions to ratio, move from fractions to ratio.
Researcher: Will you use this task to teach with and at which level?
Teacher: Grade 7.
Researcher: Why not grade 6?
Teacher: They do not understand the context and they are not up to standard.
Researcher: What would be the goals for teaching this problem?
Teacher: Other strategies, using other strategies that you can get a solution, word sums and mathematics.
Researcher: Can they use the same strategies in pure mathematics and word sums?
Teacher: They can pick out mathematics using the same strategies.
Researcher: How would you plan to teach a lesson of which this task would be part of?
Teacher: Fractions start with fractions, division of fractions.
Researcher: Do they teach division of fractions in grade 6?
Teacher: Yes.
Researcher: The last task you need to solve for me.
Teacher: Who's got more than Douglas?
Researcher: You see Ming has 3 less than Douglas. You have to work it out. Omar has 12 less than Douglas. Which statement is true? You see for the answer they're not asking you about Douglas.
Teacher: So 9 more than Omar.
Researcher: Yes. What challenges did you experience with this task? What made the task challenging?
Teacher: The story about that which is not revealing about Douglas.
Researcher: What else do you find difficult about this task?
Teacher: Some questions repeating of other people's names become confusing.
Researcher: Which strategies did you use to solve the problem?
Teacher: I first add and then I saw I had to subtract.
Researcher: But did you first reason out how much they have?
Teacher: Yes, mathematical reasoning.
Researcher: What strategies would your learners use to solve the task?

Teacher: I think they would just add how many times does the people's names appear and by trial- and-error.
Researcher: What would be the underlying causes for why learners would struggle to solve the problem?
Teacher: They'll struggle with basic operations whether they should add or subtract.
Researcher: Would your learners find this task to be different to the other tasks?
Teacher: The word problems that word problems. They first have to understand the language then take the mathematics out of the problem. They are not use to them.
Researcher: Would you use this task to teach with and at which level?
Teacher: At grade 6.
Researcher: What would your goals be for teaching this task?
Teacher: Is to let them know that you can take mathematics out of word sums and you can do basic operations in word sums.
Researcher: Is this more logical reasoning?
Teacher: They have to reason it out.
Researcher: How would you plan to teach this task and as part of which topic?
Teacher: Basic operations first.
Researcher: How would you assess whether the problem solving had been successful?
Teacher: When they can relate the mathematics into word sums and when they can take the mathematics from the word sums.
Researcher: The results of the learners who wrote these tests were 20%. Would you're learners' results be similar or different. After you have taught it?
Teacher: After I have taught it, it would be more.

Teacher C

Researcher: I want you to work out the task because I need your working out.
Teacher: Ok, for 19?
Researcher: Yes work it out and then I'll ask you questions. I want to see your strategies.
Teacher: Do you to see how I'm working it out sorry. I'm doing it the way I'm teaching it.
Researcher: No problem.
Teacher: This is why I'm doing it this way alright. So there is actually 3 possibilities here.
Researcher: There's only one right answer ok, so you got to figure out which is the right answer.

Teacher: It's fine let me have a look at it which of these make the number sentence true. Ok, something divide that question mark 14-4 is 10 divided 2 is, this got to equal that.

Researcher: No the 2 you put in there and the 6 you put in there.

Teacher: Ok, 6 minus 2, is 6 it's not 16, 4 minus 2 is 12, 14 minus 4 is 10 divided by 2 is 5. 18 minus 8 which is 10 divided by 2 is 5, that also give you 10, 20 minus 10 also gives you 10, ok.

Researcher: If you think of the order of operations.

Teacher: Oh, if you use BODMAS, so I got to do that first. Ok now I see if ok, 2 divided by 2 is 1 minus that will give 5. The first one is true.

Researcher: Yes.

Teacher: But the kids if you don't show them the brackets they won't see it. You know what have had been fine if they've asked them to do 2 different things. They actually asked them to work out this and once you've done that you forget about the BODMAS .Ok I see it now.

Researcher: Yes.

Teacher: Alright.

Researcher: What makes this type of question easy or challenging to answer for you? Was it easy

Teacher: It was challenging. You know the first part was easy to substitute the question marks, triangles and the squares that was the easy part. But working out what your final answer would be once you have all three you see something's wrong.

Researcher: Yes.

Teacher: And if somebody hadn't said remember your BODMAS-rule or reminded me about the BODMAS-rule. You have to put in the brackets for the kids or else they won't see it.

Researcher: Do you think I have to put the brackets in?

Teacher: You will get your bright kids that will remember, they will say BODMAS but I promise you in a class of 30 it will be 2 kids. Its only when you said order of operations. So I'll teach it like this, I'd say before you do your calculations look at your signs. You see the division come before subtraction then they would have got it I think.

Researcher: Ok, so the next question is what mathematical strategies did you use? If you knew about the BODMAS, you would look at the signs.

Teacher: Before they even got to this part I would have said to them now that we have substituted the question marks for it's correct number and before they work out the correct answer. I would tell them that they musn't forget

BODMAS because if they work it and they get these answers some kids get stuck here and they can't go back.

Researcher: Do you think your learners will be able to answer this question?

Teacher: Well they will get it right if I tell them to remember that division comes before subtraction.

Researcher: Will you use this task to teach with and in which grade?

Teacher: Actually I've never taught this one. I've taught patterns which are similar ok, substituting. Trying to find a pattern but no I've never taught this one.

Researcher: But would you use this one to teach with? If you would say explain the BODMAS-rule and give this one to see?

Teacher: You know when I teach BODMAS I never start by asking them to substitute things. I don't ask them to start there. We'll start with say for instance 8 plus 3 divided by 6 times 3 but then we'll move on and I'll show them the brackets. When I teach it I actually underline the way BODMAS is done. So if I do it like this without the brackets I'll actually tell them to underline it. That's in grade 5 in grade 6 we say to them work from the left to right. For some kids they ask them to do multiplication and division on the same line too soon especially in grade 5. They in the stage where they have to do the multiplication first and then the division.

Researcher: What would your goals be for teaching the lesson?

Teacher: This one would be definitely to make sure that the sequence of BODMAS is used no matter what the problem is.

Researcher: How would you plan to teach a lesson?

Teacher: This one would fall into operations. LO 1, number 11 but I'd also teach it in patterns as well cause some kids don't know how to substitute. Some will see this and say what do they want me to do. I don't know what they want me to do.

Researcher: Describe a lesson of which this task would be part of.

Teacher: This would definitely be part of BODMAS now that I see it. So I'll teach it like this first only at the end I'll give something like this. I won't do this right in the beginning because then I'll be teaching too many concepts, skills rather. Because then I will be teaching a skill of substituting and a skill of BODMAS.

Researcher: How will you evaluate whether problem solving has been successful.

Teacher: I'll give them a test.

Researcher: And if they busy working?

Teacher: I'll give them a spot test.

Researcher: And if they busy working and you move around. How would you evaluate whether they've gain something from it?

Teacher: Well there's three types of children, the children that rushes through their work and asks can I have more. There are kids who actually understand what's going on, there are kids that just sit and look around and continually ask those are the ones who don't know. You know it can't be done by walking around because most of the time you ask kids do they understand and they say yes.

Researcher: Yah.

Teacher: The best way is to give them class work and ask them to swop which is the same thing just in a different form.

Researcher: What would be the underlying causes for your learners' incorrect responses?

Teacher: They can't transfer information that's the number one, they can't substitute, they won't identify.

Researcher: How would you assist learners with incorrect responses?

Teacher: Verbal responses I'll just correct them. I'll just say this is how you can do it. It's easier to correct them when they've written because now you can actually say now start again. But verbally you have to do it step-by-step.

Researcher: These were the results. Will your learners' result be similar or lesser?

Teacher: Similar, definitely similar.

Researcher: And after you've given a BODMAS lesson?

Teacher: If I include BODMAS and give something similar like this it would be higher but if I just taught BODMAS without this it would be similar even though you have taught it. If you don't give it in the similar format there are only a few children who can transfer knowledge. It's not the whole class about 20 % of your class can transfer knowledge. They can remember that you did it that way, the rest you actually have to show.

Researcher: Let's move to task 2

Teacher: Do you want me to solve this?

Researcher: Yes.

Teacher: A lot of kids will not look at the pattern they will just multiply. Some will start 7x11x4 and after the fifth one they will say they not getting a pattern. Kids won't be able to do that, they won't see the relationship.

Researcher: Will you use this task to teach with?

Teacher: No, I will never teach it. If we want to ask them to find a pattern. Find the missing one, I wouldn't even know where to start with this one. How do you explain that to a child? If

you want a child to multiply just do this. It's just too difficult for the learners. It's just hard because we don't teach these types of patterns. I will not teach this patterns we don't ask them to jump from something so simple to so difficult without the ladder in between.

Researcher: How would you plan to teach this task and with which topic will you teach it?

Teacher: I will look at a variety of patterns by starting off very basic. Use those three-legged pots, being very graphic. Previously they had to find the patterns on their own. We did it in worms and fishes and some found it very difficult.

Researcher: What would be the underlying cause for learners with incorrect responses?

Teacher: Learners will not know where to start. They would want to find a short cut to get the answer quicker. They will have to get at least 5 more steps before they can find a short-cut.

Researcher: How would you go about in assisting learners with incorrect responses?

Teacher: The whole class will be battling. Take it one step at a time. There's just too much information for them.

Researcher: Let's move to task 3.

Teacher: Problem sums, I'll do it the way I teach it. I ask kids to draw pictures show them step- by-step because others leave information out.

Researcher: Do you teach ratio?

Teacher: Yes but they won't see it they are not going to move from there to there. I need another number, x and 2x, Sarah floating somewhere. 2:1 I need this number; I want to think like they do. It's too wordy for them. I teach it separately it's a word problem. Word problems, a lot of our learners don't understand the technicalities, how much more second language learners.

Researcher: I think I'll have to add another number so that the learners will be able to solve it. Was the task challenging or easy and what were the challenges?

Teacher: I think you have to add another number or else the kids will never get it. Number one I had to do it step-by-step for children tend to leave out steps.

Researcher: What mathematical strategies did you use to solve the task?

Teacher: Draw pictures of different people try and do what they have asked them to do. If you can't work forward work backwards. Some kids don't know how to work forwards will have to show them then learners will only know.

Researcher: What mathematical strategies will your learners use to solve the problem?

Teacher: Some would drawings and some would actually work it out.

Researcher: Will they find it different to the other two tasks?

They will find it easier because they will start to understand what you're asking. They know that this is an operation, the whole procedure is different. I tell them to look at a word problem and do the actual working out. They know the answer must be in a sentence. They have to write it out in words because the question is in words. It's a three step procedure.

Researcher: If there are learners who are unable to solve the task. What would you say are the underlying causes? How would you assist them?

Teacher: Learners will not be able to solve this task. I'll take them step by step. I teach word problems by asking them to read through it, underline what is important. Step-by-step and don't forget and don't lose track of what they doing.

Researcher: Would you use this task to teach with and if you will at which level?

Teacher: The NCS document is awful to teach. We have to teach all 5 LO'S in a term and normally problem solving comes at the end after they have taught operations. 10 minutes of the lesson must be problem solving. Have you seen the Foundations for Learning?

Researcher: What would your goals be in using this problem and how would you plan to teach a lesson?

Teacher: I will tell them a box 12 -  . I will never teach this to the kids. It would be too hard to explain to the kids. What do you think this box would be? Let's just say and then we can substitute the box by trial-and error. I'll ask them to draw it because some kids don't understand ratio. I'll start with the basic, start with things they understand. Every one liter of cool drink takes 2 liters of water. I've just done fractions and simplified that. With grade 7's we do 75 divided into the ratio of 2:3 and express 7:12 as a sum of 2 numbers.

Researcher: The results of this task was 20%, will your learners' results be similar or different?

Teacher: Definitely similar.

Researcher: Lets' move to task 4. What challenges did you experience with this task?

Teacher: The task starts where there are 3 people involved and then it moves to only 2 people it is a word problem and you are not given an exact answer.

Researcher: What mathematical strategies did you use to solve the problem?

Teacher: I started with the two people what they have given me. Draw the characters and distribute the marbles.

Researcher: What strategies might your learners use and what might they find difficult?

Teacher: They would make use of their teachers' strategies, identify the three people involved.

Researcher: Do you think your learners will find this task to be different to the previous ones?

Teacher: They'll know it is a word problem, more than, less than are not given as an exact question.

Researcher: What would be the underlying difficulties for learners who struggle to solve the problem?

Teacher: They will battle because they struggle with the whole concept of working with something like that.

Researcher: Would you use this problem to teach with and how would you plan to teach a lesson.

Teacher: I will start by giving them problems with only two people like Peter has so many sweets, then put three people in Tell them to work with the answers given in the problem, guide them. Give them another one and another one. Look at where the people are in the sentence.

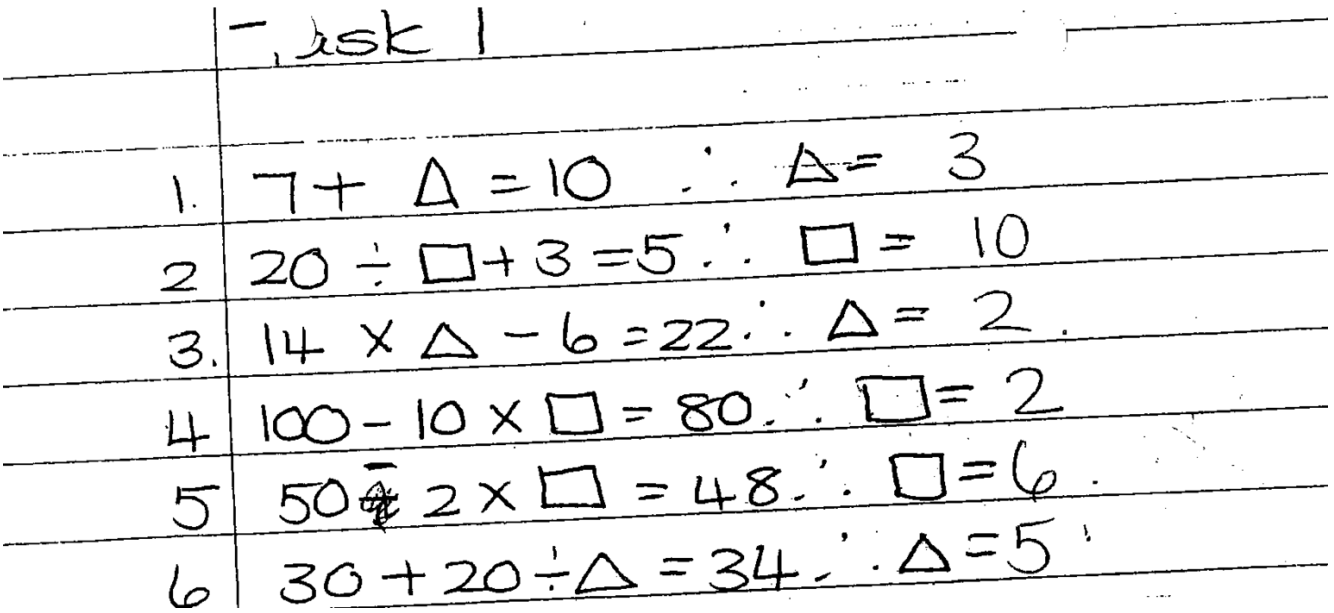
Researcher: How would you know whether the problem solving has been successful?

Teacher: By giving them a spot-test.

Researcher: These were the results of learners who wrote these tests, 20%. Do you think the results of your learners will be similar or different?

Teacher: I think it would be higher if I give them the task after I have given them similar tasks to solve.

APPENDIX 1.5: Teacher C's worksheets



1. Manny [illegible]
Danny has 6 more than Joan. How many sweets altogether?

2. John eats 10 sweets. Sarah eats 1/2 of what John eats, Mary eats 1/5 of what John eats. How many sweets altogethe

3. Nancy shares sweets between George, Mary ar Peter. She gives Mary 1/2 of her sweets, she gives Peter 10 sweets and George twice more of what she gave Peter. How ma sweets were there to start with.

4. There are cars in a car park. 1/2 belong to women, 1/4 belong to men over 40 and the rest which is 20 belong to men und

Task 4.

1. David has 3 more sweets than Sahar. Sahar has 16. How many does David have

2. Candice has 9 toys, Peter has six more than Candice and David has 13 more than Candice. How toys does David have:

3. Mark has 7 sweets. Jan has 3 less than Mary. Mary has 2 more than Mark. How many sweets does Mary have

4. Frances has 30 balls. Joan has 6 less than Frances but Sally has 5 more than him. Doug has 3 less than Sally. How many balls does doug have.