High school learners' mathematical identities: The case of an after-school mathematics club

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Declaration

I, Lovejoy Comfort Gweshe declare that this thesis is my own, unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any other degree or examination at any other university.

Lovejoy Comfort Gweshe

05th day of March 2025

Dedication

To my loving parents:

Herbert and the late Ivy Gweshe

May the Almighty forever be with you.

To my wife:

Netsai Melody Nyamangodo

May the Almighty be pleased with you and elevate your status among his children.

And to my two children:

Nigel Nqobile and Nobuhle Hannah Gweshe

May the Almighty guide you and shower you with his blessings.

Amen

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Abstract

Some researchers have tried to understand why some learners engage in and others disengage from mathematics by exploring learners' mathematical identities. Other researchers have viewed mathematics clubs as contributing to learners' engagement in mathematics, but learners' disengagement from mathematics continues to be a concern despite these efforts. Bringing these two important ideas together, I investigated the relationship between learners' mathematical identities and an after-school mathematics club.

A part of this study focuses on the collective role of significant others, who are teachers, peers and family members, in supporting and constraining learners' mathematical identities. Some researchers view productive disposition as important for learners to successfully learn mathematics. So, another part of the study focuses on learners' productive dispositions as linked to their mathematical identities.

Building on previous definitions, I define mathematical identity, be it participative or narrative, as constituted by personal and social mathematical identities, each of which can be robust-leaning, mixed or fragile-leaning. I define a mathematics club as an informal supportive space where learners engage in problem-solving activities.

The study is guided by Wenger's theory of learning as social practice. Learning is considered as involving four interconnected components: mutual engagement, meaning-making, a sense of belonging and identity construction, and occurring in different communities of practice that people move across. The theory guided learning in the club and it helped me to analyse and explain the club and learners' identities.

I used a largely qualitative design to explore the identities of Grade 10 learners who volunteered to participate in the study. The first group of 133 learners responded to a mathematical identity questionnaire and their responses were used to purposively select four learners for a semi-structured interview about their mathematical identities. The first group of learners did not participate in the mathematics club activities. A second group of fifty learners, at a different school from the first group, participated in the mathematics club activities. The learners responded to a similar mathematical identity questionnaire to learners in the first group, before they started in the club and after the club finished, and to five short questions about their expectations of and experiences in the club. Six out of the fifty learners were purposively selected to participate in two semi-structured interviews, before and after participating in the club activities. A teacher teaching four of the six interviewed learners participated in a semi-structured interview about the learners' mathematical identities in the class after the learners participated in the club activities. Questionnaire responses were analysed using descriptive and inferential statistical techniques. Interview and written responses were analysed using deductive and inductive coding.

The analyses showed that many of the learners' mathematical identities were fragile-leaning, and the learners had fragile-enhancing relationships with at least two groups of significant others. A few of the learners' mathematical identities were robust-leaning, and these learners had robust-enhancing relationships with at least two groups of significant others. The learners' relationships with their significant others collectively contributed to their mathematical identities. The club supported the learners' productive dispositions and identities. The learners preferred club discussions and the club's sociability over being given information in their classes. I recommend that significant others work together to support learners' mathematical identities. Teachers can discourage learners from labelling each other based on their mathematical abilities by, for example, fostering the belief that any learner can learn mathematics, and they can advise parents on how to support their children in mathematics. Clubs and classrooms operate under different constraints, but teachers can try to promote the sharing and explanation of ideas in their classes. The teachers can volunteer to start clubs at their schools as complementary spaces for supporting learners' identities.

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Chapter 1: Introduction

1.1 Background to the study

The context in which mathematics is presented influences how learners position themselves concerning mathematics. In South African high schools, mathematics lessons are characterised by a timetable, a prescribed Annual Teaching Plan, formal tasks and completion dates. As a result, the pressures of syllabus and formal task completion allow for little or no time for the teacher to illustrate the fun, beautiful, engaging and useful side of mathematics. A negative mathematics learning environment that is boring and rule-dominated is created, leading to negative mathematical experiences (Graven, 2015; Tachie & Chireshe, 2013).

The challenges in mathematics education are not unique to South Africa. Bishop (2012) provided evidence that 50% of Grade 12 learners involved in the United States of America's 1996 National Assessment of Educational Progress (NAEP) did not like mathematics and would quit if given a choice, reflecting a lack of confidence in mathematics. Darragh (2015) showed that after repeated negative mathematical experiences, learners at a school in New Zealand felt as if they were not good at mathematics. Darragh (2015) concluded that repeated negative mathematical experiences go towards explaining why learners lose their confidence in mathematics and choose not to identify with mathematics. For Darragh (2015) and several other researchers, learners' mathematical experiences are related to their relationships with mathematics.

Some researchers have argued that limited time for teacher support and negative beliefs about mathematics acquired through interaction with peers and family members negatively influence learners' mathematical experiences and relationships with mathematics (e.g., Graven, 2015; Ramohapi et al., 2015). After-school mathematics clubs are an example of communities where learners' relationships with mathematics can be enriched through experiencing mathematics differently from regular mathematics classrooms (Graven, 2015;

Lampen & Brodie, 2020). The clubs can improve learners' confidence (Frenzel et al., 2019; Graven, 2015), persistence (Park et al., 2012; Schlosser & Balzano, 2014), beliefs about mathematics (Can et al., 2017; Valentine et al., 2002) and sense of belonging to mathematics communities (Papanastasiou & Bottiger, 2004; Seow & Pan, 2014), which for some researchers are indicators of mathematical identities (e.g., Bishop, 2012). I now define mathematical identity and then mathematics clubs.

1.2 Mathematical identity

Identity has become increasingly useful in exploring why many learners disengage from mathematics (Graven & Heyd-Metzuyanim, 2019). Various researchers have defined and operationalised identity differently but there is a general agreement that past and present mathematical experiences contribute to learners' mathematical identities (e.g., Aydeniz & Hodge, 2011; Black et al., 2010; Gee, 2001; Grootenboer & Zevenbergen, 2008; Sfard & Prusak, 2005). For Darragh (2016), identity is constructed in social contexts and can be participative, i.e., constructed through participation, and narrative, i.e., constructed through stories. Some researchers focus on the narrative view of identity (e.g., Sfard & Prusak, 2005) and others focus on the participative view of identity (e.g., Darragh, 2013). I focused on the narrative view of identity since people usually narrate their participation.

Marks and O'Mahoney (2014) argue that an identity is constituted by a personal identity, which is a personal perspective, and a social identity, which is a community construction. For Marks and O'Mahoney (2014), a person's personal and social identities are mutually constitutive and unified in reality, although separable analytically. In other words, what a learner says about themself in relation to mathematics, i.e., the learner's personal mathematical identity, influences how the learner's mathematics community members view and choose to interact with the learner, i.e., the learner's social mathematical identity. What a learner says about their interactions with their mathematics community members and how the community members choose to interact with the learner, i.e., the learner's social mathematical identity, influences how the learner views themself in relation to mathematics, i.e., the learner's personal mathematical identity.

McGee (2015) argues that the strength of a learner's mathematical identity depends on how the learner deals with persistent negative mathematical experiences. McGee (2015) categorises learners' mathematical identities as robust or fragile. A learner who deals well with persistent negative mathematical experiences has a robust mathematical identity while a learner who struggles with persistent negative mathematical experiences has a fragile mathematical identity (McGee, 2015). Linking McGee's (2015) idea of robust and fragile identities to Marks and O'Mahoney's (2014) idea of personal and social identities, a learner who has positive mathematical experiences may demonstrate a positive perspective of themself in relation to mathematics and therefore a robust personal mathematical identity. The learner may also demonstrate a positive perspective of their interactions with their mathematics community members during mathematical activities and consequently a robust social mathematical identity. Similarly, a learner with negative mathematical experiences may demonstrate a fragile personal mathematical identity and a fragile social mathematical identity.

Drawing on Hall et al.'s (2018) idea of identity as fluid and linking to Marks and O'Mahoney's (2014) idea of personal and social identities and McGee's (2015) idea of robust and fragile identities, a personal mathematical identity can be robust-leaning, have a mix of some robustness and fragility, or fragile-leaning, and so can a social mathematical identity. Different researchers have used different indicators to operationalise mathematical identity. Cunningham (2021) used confidence and persistence, Cribbs et al. (2015) used competence, interest and recognition, Bishop (2012) used persistence and beliefs, and Darragh (2013) used confidence and a sense of belonging to mathematics communities.

In this study, I defined mathematical identity as how we describe ourselves, how our mathematics community members describe us, and how we think the community members describe and treat us in relation to mathematics. I drew on Marks and O'Mahoney (2014) and considered a mathematical identity constituted by personal and social mathematical identities. Linking with McGee's (2015) idea of robust and fragile mathematical identities and the idea that identity is fluid, I considered personal and social mathematical identities to be robust-leaning, mixed or fragile-leaning. Confidence and persistence in and beliefs about mathematics operationalised a personal mathematical identity while a sense of belonging to mathematics communities operationalised a social mathematical identity.

1.3 Mathematics clubs

Researchers have developed interventions for supporting learners' mathematical identities. The intervention that I investigated was a mathematics club – a voluntary supportive learning community in which learners engaged in mathematical tasks and reasoning and their learning experiences complemented classroom experiences (Lampen & Brodie, 2020). Mathematics clubs are usually conducted during break-time, after school or during weekends or school holidays and have been given different names by different researchers, for example, out-of-school programmes and co-curricular activities. Key in many clubs is that the way of learning mathematics should be different from that in regular mathematics classrooms. Making sense of mathematics and problem-solving are encouraged in clubs and learners work together in small groups under the guidance of skilled mentors who regularly offer scaffolding (Graven, 2015; Jacob & Jacob, 2018; Lampen & Brodie, 2020), rather than the usual procedural learning and individual work encouraged in regular mathematics classrooms. Many of the clubs encourage the enjoyment of mathematics (e.g., Lampen & Brodie, 2020; Papanastasiou & Bottiger, 2004).

Different researchers have shown that mathematics clubs can develop learners' confidence, persistence, beliefs about mathematics and sense of belonging to mathematics communities (e.g., Amit & Naaman, 2014; Graven, 2015; Papanastasiou & Bottiger, 2004; Prescott & Pressick-Kilborn, 2015). For Graven (2015) and Sherman and Catapano (2011), clubs offer learners extended opportunities to engage with mathematics resulting in increased learning. Turner et al. (2011) showed that a club that encourages collective problem-solving can support learners' mathematical identities.

1.4 Purpose of the study

The purpose of this study was to contribute to the conversation on identity in mathematics education by investigating if and how an after-school mathematics club supports high school learners' mathematical identities through the lens of a four-component social theory of learning by Wenger (1998) and learners' and a class teacher's narratives of the learners' experiences of mathematics. I wanted to offer a framework to conceptualise identity in mathematics education and insights into influences on high school learners' mathematical identities. I explored characteristics of an after-school mathematics club that made the club a suitable intervention strategy for supporting learners' mathematical identities based on the club's mathematical activities and guiding principles, which differed from those in regular mathematics classrooms. I explored learners' interactions with the club activities, their teachers, peers and family members.

1.5 Research questions

The following are the main research questions that guided the study:

1. What kinds of mathematical identities do high school learners narrate?
2. What are some of the influences on high school learners' mathematical identities and how do they influence mathematical identity?

3. Can an after-school mathematics club support high school learners' mathematical identities? If so, what are the characteristics of such a club?

1.6 Rationale for the study

Several researchers have distinguished two categories of learners' mathematical identities depending on the learners' relationships with mathematics. Oppland-Cordell & Martin (2015) distinguished between a mathematical identity of participation and non-participation depending on whether a learner participated or not in mathematical activities. McGee (2015) distinguished between a robust and fragile mathematical identity depending on whether a learner could deal well or not with negative mathematical experiences. Some researchers have argued that identity is a multi-component concept (e.g., Marks & O'Mahoney, 2014) and other researchers have argued that identities are fluid (e.g., Hall et al., 2018). The idea that identities are multi-component and fluid justified investigating the possibility of distinguishing more than two categories of learners' mathematical identities.

It is well known that many learners do not like mathematics. Several researchers have explored teacher pedagogy as linked to learners' dislike of mathematics (e.g., Ramohapi et al., 2015). Bishop (2012) and Sfard and Prusak (2005) argue that the construct of identity, which has had a growing interest in the research community may help explain why many learners dislike mathematics. In response to Bishop (2012) and Sfard and Prusak (2005), several researchers have explored teacher pedagogy as linked to learners' mathematical identities (e.g., Gardee, 2019). The scarcity of studies exploring other influences on learners' mathematical identities besides teacher pedagogy justified exploring how teachers, peers and family members support or constrain learners' mathematical identities.

Many of the studies on learners' mathematical identities have been conducted in regular mathematics classrooms and have shown among other findings that learning in the classrooms is largely procedural (e.g., Boaler & Selling, 2017) and that learners do not enjoy such lessons. Many of the studies on mathematics clubs have shown that learners enjoy making sense of mathematics (e.g., Brodie, 2022; Papanastasiou & Bottiger, 2004). Some studies on mathematics clubs, though not investigating learners' mathematical identities directly, have shown improvements in indicators of learners' mathematical identities. A rationale for this study was whether an after-school mathematics club supports multiple indicators of learners' mathematical identities.

Studies exploring learners' mathematical identities have focused on understanding why many learners do not like mathematics. Studies exploring mathematics clubs have focused on developing an intervention for persuading learners to like and choose mathematics. The studies exploring learners' mathematical identities and those exploring mathematics clubs have largely been conducted separately. The rationale to develop an appropriate intervention for supporting learners' mathematical identities formed the focus of this study. The study investigated the relationship between an after-school mathematics club and learners' mathematical identities.

1.7 The papers

The study is made up of four papers that collectively focus on high school learners' mathematical identities as the learners participated in an after-school mathematics club. In this section, I offer a brief overview of each of the four papers with a focus on the research questions of each paper, which are the sub-questions of the main research questions. I relate the sub-questions to the main research questions.

I conducted a pilot study to test the data collection instruments and analysis procedures and used the findings to write the first paper (chapter 2). The pilot study was conducted at a different school from the school in the main study. Even though the learners who participated in the pilot study were different from those in the main study, they shared similar characteristics: they lived in a township area where people of low socio-economic status live, and many of the learners did not like mathematics. According to Yin (2009), carrying out a study at more than one site may provide more compelling and robust findings. So I decided to consider the findings of the pilot study as part of the findings of the main study.

Table 1.1 shows the paper number, the paper's research questions and findings, and the main research questions linked to the paper's questions.

Table 1.1 Findings of the four papers

Paper number	**Research question of paper**	**Findings**	**Main research question answered**
1	1.1 What are some of the influences on learners' mathematical identities?	Three kinds of mathematical identities were discussed: largely robust, modest and largely fragile identities.	1
		Peers and family members influenced high school learners' mathematical identities.	2
2	2.1 What kinds of mathematical identities do high school learners narrate?	Nine kinds of mathematical identities were discussed. The identities ranged from robust-leaning personal and social identities through mixed personal and social identities to fragile-leaning personal and social identities.	1
	2.2 What kinds of relationships, with different groups of significant others, do high school learners narrate and how do these relationships collectively afford learners' mathematical identities?	Two kinds of relationships between learners and their significant others were discussed: robust-enhancing relationships, which supported learners' mathematical identities; and fragile-enhancing relationships, which constrained learners' mathematical identities. Teachers, peers and family members collectively supported or constrained learners' mathematical identities.	2
3	3.1 Does a mathematics club offer learners conditions for developing their productive dispositions?	The club supported learners' productive dispositions and therefore their mathematical identities by encouraging mutual engagement and meaning-making.	3
4	4.1 Can an after-school mathematics club support learners' mathematical identities?	The club supported learners' mathematical identities. Robust-leaning mathematical identities shifted toward more robust identities and so did fragile-leaning mathematical identities.	3

In summary, the first and second papers collectively combined to respond to the first and second main research questions. The third and fourth papers collectively combined to respond to the third main research question. I now describe the theoretical orientation of the study, followed by a discussion of the research design and methodology.

1.8 Theoretical framing

In this section, I present the overarching theoretical framework for the study. The framework brings together Wenger's (1998) social theory of learning, Marks & O'Mahoney's (2014) theory of identity as constituted by personal and social identities, McGee's (2015) theory of identity as robust or fragile and Hall et al.'s (2018) theory of identity as fluid. Sections of the framework framed each of the four papers and are further discussed in each of the papers.

Wenger's (1998) social theory of learning guided learning in the club and helped to explain the meaning and nature of learners' mathematical identities. For Wenger (1998), a community is characterised by pursuing enterprises deemed meaningful to the development of the community's practices and we learn and construct our identities through participating in the practices of our community. Mathematics may be viewed as a practice with which learners identify or do not identify and it can be practiced in regular mathematics classrooms, after school and home communities. Wenger (1998) argues that learning involves four interconnected and inseparable components: mutual engagement, which is about sharing ideas and resources; meaning-making, which is about seeing meaning in a community's practices; a sense of belonging, which is about being a part of a community through participating in its practices; and identity construction, which is about developing a more robust relationship with a community's practices.

Drawing on Marks and O'Mahoney (2014), learners' mathematical identities are constituted by their personal mathematical identities, which are their understandings of their relationships with mathematics, and their social mathematical identities, which are their feelings of belonging to mathematics communities. The learners' mathematical identities are constructed through interaction with their significant others who are their teachers, peers and family members respectively in regular mathematics classrooms, after school and at home (Darragh, 2015). The significant others offer learners different opportunities for mathematical identity support which the learners accept or reject leading to the construction of different mathematical identities (Aydeniz & Hodge, 2011; Epstein et al., 2010; Gardee, 2019; McGee, 2015).

Linking McGee's (2015) and Hall et al.'s (2018) theories to Marks and O'Mahoney's (2014) theory, learners' personal and social mathematical identities can be robust-leaning, have a mix of some robustness and fragility, or fragile-leaning. The three kinds of personal mathematical identities and three kinds of social mathematical identities suggest that learners can construct nine different possible combinations of personal and social mathematical identities. Figure 1.1 presents a framework showing the potential relationships between mathematics communities and learners' mathematical identities.

MATHEMATICS COMMUNITIES

Classroom:
Procedure repetition
Teacher; Peers

After-school club:
Mutual engagement
Meaning-making
Peers

Home:
Family

MATHEMATICAL IDENTITIES				
		PERSONAL IDENTITY		
		Robust-leaning	**Mixed**	**Fragile-leaning**
SOCIAL IDENTITY	**Robust-leaning**	Robust-leaning personal and social identity	Mixed personal identity and robust-leaning social identity	Fragile-leaning personal identity and robust-leaning social identity
	Mixed	Robust-leaning personal identity and mixed social identity	Mixed personal and social identity	Fragile-leaning personal identity and mixed social identity
	Fragile-leaning	Robust-leaning personal identity and fragile-leaning social identity	Mixed personal identity and fragile-leaning social identity	Fragile-leaning personal and social identity

Figure 1.1 A framework relating mathematics communities and mathematical identities.

Wenger's (1998) four components of learning: mutual engagement, meaning-making, a sense of belonging and identity construction, assisted me in analysing and explaining learners' mathematical identities and the practice of mathematics in regular mathematics classrooms and the club. The activities of the after-school mathematics club were guided by Wenger's (1998) four components of learning.

1.9 Research design and methodology

The study consisted of a pilot and main study that focused on understanding a social construct: mathematical identity. The purpose of the pilot study was to test some of the data collection instruments and analysis procedures that I wanted to use in the main study. For both the pilot and main study, I adopted a qualitative approach that incorporated a quantitative aspect. This means that the emphasis was on values and context, and interpreting participants' socially constructed realities from the participants' perspectives (McMillan & Schumacher, 2010). Using qualitative and quantitative data collection methods ensured that each set of evidence would strengthen the weaknesses of the other set (Johnson & Christensen, 2012; Yin, 2009). Lodico et al. (2010) argue that researchers may use more than one method of data collection and compare the results to ensure thoroughness and depth in understanding a phenomenon.

The quantitative aspect of the study offered a general picture of learners' mathematical identities before and after participating in the after-school mathematics club activities. Through the qualitative design, I captured and uncovered some of the learners' and a class teacher's perspectives on the meanings of their socially constructed realities about the learners' mathematical identities, thus comprehensively explaining the learners' mathematical identities in the context under investigation.

The pilot study was conducted for four weeks and learners attended their regular mathematics lessons during normal school hours where they were taught by their class teachers. No pilot club was started but I gained the experience of running a club at another school, as will be discussed later. The main study was conducted for eight months and learners attended their regular mathematics lessons during normal school hours where they were taught by their class teachers. Learners in the main study also attended club sessions once a week after school where they engaged in interactive problem-solving activities under the guidance of club mentors.

1.9.1 Samples

Schools and grade

The pilot study was conducted at a school located in a township area near Johannesburg. This was a convenience sample because the school is close to the school where I teach, which made travelling to the site easy. I wanted to conduct the main study at the school where I teach so I chose to work with Grade 10 learners in both the pilot and main study because it was a grade that I did not teach that year, which minimised my attachment to the learners.

Another reason for choosing to work with Grade 10 learners is that, except for Lampen and Brodie (2020) who worked with Grade 10 learners, none of the studies that have focused on mathematics clubs in South Africa has worked with Grade 10 learners or learners in the Further Education and Training phase (FET), i.e., Grades 10 to 12.

The main study was conducted at the school where I teach, also a convenience sample. Drawing on Merriam (2009), selecting the school where I teach was expected to ensure that participants were comfortable with someone whom they already knew allowing them to act more naturally and minimise the time it took them to adjust to the mathematics club environment. Drawing on McMillan and Schumacher (2010), since the main aim of the study was to contribute insights into how a mathematics club supports high school learners' mathematical identities, convenience sampling was suitable. The pilot and main study were thus carried out in learners' natural learning settings (Creswell, 2012), that is to say, at the participants' usual schools so following the philosophy of a qualitative study.

Club mentors

In the main study, I took the position of the main club mentor who sourced club activities and created an environment that encouraged mutual engagement, meaning-making and a sense of belonging to mathematics communities. One teacher at the school consented to be a co-club mentor. The co-club mentor did not teach Grade 10, ensuring that club activities would only be done in the club. The co-club mentor assisted me in running the club by facilitating some of the club sessions.

Learners

One hundred and thirty-three learners participated in the pilot study by responding to a questionnaire about their mathematical identities. The questionnaire results were used to understand learners' mathematical identities and to select four learners with different mathematical identities who had performed differently in their end-of-term mathematics test to participate in a semi-structured interview about their mathematical identities. Table 1.2 shows the four selected learners (all names are pseudonyms), the kinds of mathematical identities they narrated and their end-of-term mathematics test scores.

Table 1.2 Pilot study selected learners

Learner	**Identity**	**Test score**
Thabo	Largely robust	86%
Mpho	Modest	73%
Themba	Largely fragile	41%
Fana	Largely fragile	34%

Fifty learners participated in the main study by joining the after-school mathematics club and responding to a questionnaire about their mathematical identities before and after participating in the club. The learners' responses to the pre-club questionnaire were analysed and used to understand their mathematical identities and to select six learners with different mathematical identities for pre- and post-club semi-structured interviews about their mathematical identities. The six learners were selected based on the kinds of mathematical identities observed in the pre-club questionnaire data, the number of learners who had narrated each kind of mathematical identity, and their willingness to participate in the interviews. Table 1.3 shows the six selected learners and the kinds of mathematical identities they narrated.

Table 1.3 Main study selected learners

Learner	**Mathematical identity**
Rose	Robust-leaning
Ben	Robust-leaning
Kelly	Fragile-leaning
Neo	Fragile-leaning
Tom	Fragile-leaning
Cindy	Fragile-leaning

Teacher

The teacher teaching Rose, Ben, Kelly and Cindy responded to a teacher post-club semi-structured interview about his views of these four learners' mathematical identities in their regular mathematics classrooms. The teacher did not take part in club activities.

1.9.2 The club

The club that I started was the third club in a project carried out by a team of researchers from the University of the Witwatersrand and Stellenbosch University that I was a part of. The project had three clubs around South Africa, in high schools (The other two clubs are reported in Brodie, 2022; Frenzel et al., 2019; Lampen & Brodie, 2020). The three clubs shared a curriculum that was premised on developing learners' mathematical reasoning based on their everyday reasoning and experiences and supporting their mathematical identities (For more detail see Lampen & Brodie, 2020). The team of researchers met during school holidays to discuss theories framing the clubs and to develop club activities and data collection instruments. We discussed preliminary findings from each of the clubs and shared challenges and ways of dealing with the challenges, for example, the pressure from learners to help them prepare for a class test or to reteach concepts not fully understood in the class was managed by modifying club activities to include concepts expected to be in the test or not fully understood in the class.

I gained the experience of club mentoring by taking part in one of the first two clubs, which was run by a team of researchers from the University of the Witwatersrand and led by Professor Karin Brodie, before establishing the third club. As a team, we took turns to lead the club sessions. During group work, we moved around providing scaffolding to learners. We carried out after-club reflection sessions where we discussed whether the goals of the club session had been achieved, challenges and possible improvements, and prepared for the next session.

Fifty learners volunteered to participate in the after-school mathematics club which was conducted once a week for one and a half hours. The actual total attendance per session ranged between 25 and 50 learners. The weather and clashes with other school and community activities affected attendance on some days. Eighteen club sessions were conducted and each learner attended an average of 15 sessions. The club venue had posters with messages that encouraged learners to persist in mathematics and believe that they could learn mathematics, i.e., positive mathematical identity messages. The school principal gave the club space on a bulletin board outside the club venue where we posted additional messages. To dispel some of the myths around mathematics, for example, that mathematics is only about numbers, we posted problem-solving activities that could be solved without involving numbers.

1.9.3 Instruments and data collection

Learners in the pilot study responded to a 22-item mathematical identity questionnaire. The questionnaire asked the learners about their confidence and persistence in and beliefs about mathematics, and their sense of belonging to mathematics communities and was on a Likert scale. Questions were adopted from previous studies that had investigated identities and related constructs (e.g., Bishop, 2012; Darragh, 2013). The learners' responses to the questionnaire gave a general picture of their mathematical identities. Six redundant items were removed from the questionnaire because they were repetitive or unclear and 16 items remained.

I also used the questionnaire responses to purposively select four learners with different mathematical identities and having performed differently in their end-of-term mathematics test for an audio-recorded semi-structured interview about their mathematical identities. The four learners were asked about their confidence and persistence in and beliefs about mathematics, and sense of belonging to mathematics communities. Some items in the interview schedule were adopted from previous studies and others were constructed with the help of colleagues who have investigated identities and related constructs. Some questions, phrases and words not fully understood by learners in the learner semi-structured interview were changed after the pilot study.

The pilot study offered little information about the learners' academic achievement besides their end-of-term mathematics marks so I changed the focus of my study from mathematical identity and academic achievement to mathematical identity only. For the same reason, the procedure for purposively selecting interesting learners was changed from focusing on mathematics marks and questionnaire mathematical identity scores to mathematical identity scores only. Learners' mathematical mindsets had been a focus in the pilot study but after the pilot, I decided to consider mathematical mindsets as part of beliefs about mathematics since beliefs are a more encompassing concept.

Learners in the main study responded to the same 16-item mathematical identity questionnaire, developed through the pilot, before the club started and after it ended (Appendix A). The questionnaire asked learners about their personal mathematical identities (i.e., their confidence and persistence in and beliefs about mathematics) and their social mathematical identities (i.e., their sense of belonging to mathematics communities) and was on a Likert scale. The learners' responses to the pre-club questionnaire gave a general picture of their mathematical identities before participating in the club. I also used the pre-club questionnaire responses to purposively select six learners for audio-recorded pre- and post-club semi-structured interviews. Responses to the post-club questionnaire gave a general picture of the learners' mathematical identities after participating in the club.

To have a deeper understanding of learners' mathematical identities and the mathematics club from the learners' perspectives, six purposively selected learners participated in a pre-club semi-structured interview (Appendix B) and a post-club semi-structured interview (Appendix C). The learners were asked about their personal and social mathematical identities and views of the club. The purpose of the learner pre-club semi-structured interview was to understand the learners' mathematical identities and views of the after-school mathematics club before participating in the club. The purpose of the learner post-club semi-structured interview was to determine whether there was a change in the learners' mathematical identities after participating in the club and how the club had contributed to the change, if any. I compared the learners' mathematical experiences in the club with their experiences in their regular mathematics classrooms to identify aspects of the club that potentially made the club a suitable intervention strategy for supporting learners' mathematical identities.

The learners responded in writing to five short questions about their views of the club, with one question being asked at the end of each of five club sessions (Appendix D). The questions were constructed with the help of colleagues in the mathematics club project. The purpose of the five questions was to add depth to understanding the club and learners' mathematical identities through the learners' productive dispositions.

To contribute to rich descriptive data about learners' mathematical identities, a teacher teaching four of the six selected learners was interviewed using a semi-structured interview schedule (Appendix E). The teacher interview findings largely confirmed the learners' mathematical identities, as narrated by the learners.

Tables 1.4, 1.5, 1.6 and 1.7 summarise the data collection methods discussed in each of the four papers, the main research questions answered by the data, and from whom, how and when the data were collected.

Table 1.4 Data collection methods in paper 1

Data collection method	Date	Data source	Nature of data	Main research questions answered
Learner mathematical identity questionnaire	March 2017	Learners’ written responses	Quantitative	1
Learner semi-structured interview	March 2017	Interview notes; audio recordings	Qualitative	1; 2

Table 1.5 Data collection methods in paper 2

Data collection method	Date	Data source	Nature of data	Main research questions answered
Learner pre-club mathematical identity questionnaire	February 2018	Learners’ written responses	quantitative	1
Learner pre-club semi-structured interview	February 2018	Interview notes; audio recordings	qualitative	1; 2

Table 1.6 Data collection methods in paper 3

Data collection method	Date	Data source	Nature of data	Main research questions answered
Learner post-club semi-structured interview	September 2018	Interview notes; audio recordings	Qualitative	3
Five short written questions, one question per club session	February 2018; April 2018; June 2018; August 2018; September 2018	Learners’ written responses	Qualitative	3

Table 1.7 Data collection methods in paper 4

Data collection method	Date	Data source	Nature of data	Main research questions answered
Learner pre- and post-club mathematical identity questionnaire	February 2018; September 2018	Learners' written responses	Quantitative	3
Learner pre- and post-club semi-structured interviews	February 2018; September 2018	Interview notes; audio recordings	Qualitative	3
Teacher post-club semi-structured interview	September 2018	Interview notes; audio recordings	Qualitative	3

1.9.4 Trustworthiness

A study is trustworthy when there is evidence of rigour and quality in its findings. To enhance the study's trustworthiness, I have described the study setting, participants and the data collection procedures above and in each of the papers, to convince the reader that the findings make sense in relation to the collected data. Data analyses procedures are discussed in the papers.

I was the main club mentor, interviewer and researcher, which drawing on Yin (2009), minimised chances of disrupting the natural flow of events while increasing chances of gaining more insights into learners' mathematical identities and an after-school mathematics club. Performing these roles provided opportunities for me to understand the participants' viewpoints (Yin, 2009), resulting in a more accurate portrayal of the learners' mathematical identities and the club. Drawing on Gay et al. (2011) and Savin-Baden and Major (2013), confidence and confidentiality among participants were maintained and the acquisition of nuanced contextual information that contributed to a deeper understanding of learners' mathematical identities and the club was promoted. My supervisor and other colleagues acted as peer debriefers by providing critical comments and "searching questions helpful in understanding my position and its role in the study" (McMillan & Schumacher, 2010, p.334). Peer debriefing sessions were carried out at various stages of the study while being audio-recorded for further use. Changes to the study were made, where appropriate, for example, focusing on the collective role of significant others rather than their separate roles in mathematical identity development.

I kept a field journal in which events in the club were written. Furthermore, "interim data analyses, including preliminary comparisons and corroborations of findings were carried out during data collection to refine ideas and ensure a match between the study findings and learners' realities" (McMillan & Schumacher, 2010, p.331).

I searched for, analysed and acknowledged participants' views and situations that contradicted common themes. Plausible explanations for such disconfirming evidence were investigated and recorded, where necessary, for example, despite many of the learners saying that they enjoyed learning in the club, a few learners said that they enjoyed learning in their regular mathematics classrooms because for them, classroom lessons where shorter than club sessions. I acknowledge that I was a non-neutral researcher seeking to transform learners' mathematical identities through a mathematics club.

1.9.5 Ethical considerations

Educational research focuses primarily on human beings and as such, I was ethically responsible for protecting the rights and welfare of the participants in the study (McMillan & Schumacher, 2010). I applied for ethical clearance from the University's Ethics Committee, per the University's policy on ethics. Clearance was provided and the protocol number is 2017ECE005D. After ethics approval, I applied to the Gauteng Department of Education (GDE) to access the participating school, and the application was successful.

I disclosed the purpose of the study in letters that I wrote to the parents and guardians of participating learners, the school principal, a representative of the School Governing Body (SGB) and the participating teachers and learners. The parents and guardians of participating learners, the school principal and representative of the SGB, and participating teachers and learners were assured of confidentiality, anonymity, protection, voluntary participation and exit before taking part in the study.

1.10 An overview of the thesis

The thesis is divided into six chapters. In this chapter (chapter One), I provided the background to the study, the study purpose, the research questions and the rationale for the study, theoretical framing and the research design and methodology for the study. The next four chapters consist of interrelated manuscripts that form the central body of the thesis. Chapter 6 is the conclusion, where I summarise the findings of the study and discuss its contribution, implications and limitations.

For the completion of the thesis, it is required that I publish a minimum of four papers in reputable, peer-reviewed journals. Of the four papers required by the University, one should be published, one in press and two accepted for publication. I have three published papers (chapters 2, 3 and 4) and one accepted paper (chapter 5). Table 1.8 shows a list of the papers, their statuses and references.

Table 1.8 Details of manuscripts that make up the Chapters of the thesis.

Chapter	Paper	Name of paper	Status	Reference
2	1	High school learners' mathematical identities	Published	Gweshe, L. C., & Brodie, K. (2019). High school learners' mathematical identities. *African Journal of Research in Mathematics, Science and Technology Education*, *23*(2), 254-262. https://doi.org/10.1080/18117295.2019.1662642
3	2	Learners' mathematical identities: exploring relationships between high school learners and significant others	Published	Gweshe, L.C., & Brodie, K. (2024). Learners' mathematical identities: exploring relationships between high school learners and significant others. *Mathematics Education Research Journal*, *36*, 917–944. https://doi.org/10.1007/s13394-023-00479-5
4	3	Learners' productive dispositions: The case of an after-school mathematics club	Published	Gweshe, L. C. (2023). Learners' productive dispositions: The case of an after-school mathematics club. *Didactica Mathematica, 45*, 5-31. https://doi.org/10.14708/dm.v45i0.7285
5	4	Mathematics club learners' mathematical identities: Narratives of learners and their class teacher	Accepted	Gweshe, L. C. (2024). Mathematics club learners' mathematical identities: Narratives of learners and their class teacher. Journal of Education, *97*, 196 – 220, http://dx.doi.org/10.17159/2520-9868/i97a10

1.11 References

Amit, M., & Naaman, K. (2014). Kidumatica – the mathematics club for creativity and excellence among multicultural pupils: Practice and research. *Procedia - Social and Behavioral Sciences*, *141*,1403-1411. https://doi.org/10.1016/j.sbspro.2014.05.242

Aydeniz, M., & Hodge, L. L. (2011). Identity: A complex structure for researching students' academic behavior in science and mathematics. *Cultural Studies of Education, 6*, 509-523. https://doi.org/10.1007/s11422-011-9331-9

Bishop, J. P. (2012). "She's Always Been the Smart One. I've Always Been the Dumb One": Identities in the Mathematics Classroom. *Journal for Research in Mathematics Education*, *43*(1), 34-74. http://www.jstor.org/stable/10.5951/jresematheduc.43.1.0034

Black, L., Williams, J., Hernandez-Martinez, P., Davis, P., Pampaka, M., & Wake, G. (2010). Developing a 'leading identity': the relationship between students' mathematical identities and their career and higher education aspirations. *Educational Studies in Mathematics*, *73*, 55–72. https://doi.org/10.1007/s10649-009-9217-x

Boaler, J., Selling, S. K. (2017). Psychological imprisonment or intellectual freedom? A longitudinal study of contrasting school mathematics approaches and their impact on adults' lives. *Journal for Research in Mathematics Education*, *48*(1), 78–105. https://doi.org/10.5951/jresematheduc.48.1.0078

Brodie, K. (2022). Learning mathematics in an after-school mathematics club. *African Journal of Research in Mathematics, Science and Technology Education*, *26*(3), 237–247. https://doi.org/10.1080/18117295.2022.2131267

Can, I., Koydemir, S., Durhan, S., Ogan, S., Gozukara, C., & Cokluk, G. (2017). Changing high school students' attitudes towards mathematics in a summer camp: Happiness matters. *Educational Sciences-theory & Practice*, 1625-1648. https://doi.org/10.12738/estp.2017.5.0373

Creswell, J. W. (2012). *Educational Research. Planning, conducting, and evaluating quantitative and qualitative research.* (4th edn.). Pearson Education, Inc.

Cribbs, J. D., Hazari, Z., Sonnert, S., & Sadler, P. M. (2015). Establishing an explanatory model for mathematics identity. *Child Development*, *00*(0), 1–15. https://doi.org/10.1111/cdev.12363

Cunningham, J. A. (2021). "We made math!": Black parents as a guide for supporting black children's mathematical identities. *Journal of Urban Mathematics Education*, *14*(1), 24–44. https://doi.org/10. 21423/jume-v14i1a414

Darragh, L. (2013) Constructing confidence and identities of belonging in mathematics at the transition to secondary school. *Research in Mathematics Education*, *15*(3), 215-229. https://doi.org/10.1080/14794802.2013.803775

Darragh, L. (2015). Recognising 'good at mathematics': using a performative lens for identity. *Research in Mathematics Education*, *27*, 83–102. https://doi.org/10.1007/s13394-014-0120-0

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, *93*, 19-33. https://doi.org/ 10.1007/s10649-016-9696-5

Epstein, D., Mendick, H., & Moreau, M. (2010). Imagining the mathematician: Young people talking about popular representations of maths. *Discourse Studies in the Cultural Politics of Education*, *31*(1), 45–60. https://doi.org/10.1080/01596300903465419

Frenzel, J., Lampen, C. E., & Brodie, K. (2019). Learners' awareness of their emotions and their engagement with mathematics tasks in a mathematics club. *Journal of Education 77*, 45-59. http://dx.doi.org/10.17159/2520-9868/i77a03

Gardee, A. (2019). Social relationships between teachers and learners, learners' mathematical identities, and equity. *African Journal of Research in Mathematics, Science and Technology Education*, *23*(2), 233-243. https://doi.org/10.1080/18117295.2019.1662641

Gay, L. R., Mills, E., & Airasian, P. (2011). *Educational Research: Competencies for analysis and application.* (10th edn.). Pearson Education, Inc.

Gee, J. P. (2001). Identity as an Analytic Lens for Research in Education. *Review of Research in Education*, *25*, 99-125. https://doi.org/10.2307/1167322

Graven, M. (2015). Strengthening maths learning dispositions through 'math clubs', *South African Journal of Childhood Education*, *5*(3), Art. #342, 7 pages. http://dx.doi. org/10.4102/sajce.v5i3.342

Graven, M., & Heyd-Metzuyanim, E. (2019). Mathematics identity research: The state of the art and future directions. *ZDM – The International Journal on Mathematics*

Education. The special issue on identity in mathematics education, *51*, 361-377. https://doi.org/10.1007/s11858-019-01050-y

Grootenboer, P.J., & Zevenbergen, R. (2008). Identity as a lens to understand learning mathematics: Developing a model. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions. Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, Brisbane,* Vol. 1 (pp. 243-250). Brisbane: MERGA.

Hall, J., Towers, J., & Martin, L. C. (2018). Using I poems to illuminate the complexity of students' mathematical identities. *Educational Studies in Mathematics*, *99*, 181–196. https://www.jstor.org/ stable/45184656

Jacob, R., & Jacob, B. (2018). New evidence on the benefits of small group math instruction for young children. *Evidence Speaks Reports, 2*(55).

Johnson, B., & Christensen, L. (2012). *Educational Research. Quantitative, Qualitative, and Mixed Approaches.* (4th edn.). SAGE Publications, Inc.

Lampen, C. E., & Brodie, K. (2020). Becoming mathematical: Designing a curriculum for a mathematics club. *Pythagoras, 41*(1), a572. https://doi.org/10.4102/pythagoras.v41i1.572

Lodico, M. G., Spaulding, D. T., & Voegtle, K. H. (2010). *Methods in educational research: from theory to practice.* (2nd edn.). John Wiley & Sons, Inc.

Marks, A., & O'Mahoney, J. (2014). Researching identity: a critical realist approach. In: Edwards, P., O'Mahoney, J., & Vincent, S. (Eds.). *Putting Critical Realism into Practice: A Guide to Research Methods in Organization Studies*, Oxford University Press, pp. 66-85.

McGee, E. O. (2015). Robust and fragile mathematical identities: A framework for exploring racialised experiences and high achievement among black college students. *Journal for Research in Mathematics Education*, *46*(5), 599-625. https://doi.org/10.5951/jresematheduc.46.5.0599

McMillan, J. H., & Schumacher, S. (2010). *Research in education: evidence-based inquiry*. (7th edn.). International edition. Pearson Education Inc.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation. Revised and expanded from Qualitative research and case study application in education*. John Wiley & Sons, Inc.

Oppland-Cordell, S., & Martin, D. B. (2015). Identity, power, and shifting participation in a mathematics workshop: Latin@ students' negotiation of self and success.

Mathematics Education Research Journal, *27*, 21–49. https://doi.org/10.1007/s13394-014-0127-6

Papanastasiou, E. C., & Bottiger, L. (2004). Maths clubs and their potential: Making mathematics fun and exciting. A case study of a maths club. *International Journal of Mathematical education in Science and Technology*, *35* (2), 159–171. https://doi.org/10.1080/00207390310001638395

Park, H., Byun, J., & Jo, S. (2012). Do after-school programs matter? A longitudinal study on the effectiveness of participating in after-school programs in Korea. *Journal of Educational Policy*, *9*(1), 3–27.

Prescott, A., & Pressick-Kilborn, K. (2015). It's great to be doing maths! Engaging primary students in a lunchtime club. *Australian Primary Mathematics Classroom*, *20*(3).

Ramohapi, S., Maimane, J. R., & Rankhumise, M. P. (2015). Investigating factors contributing to learner performance in mathematics: A case study of some selected schools in Motheo District. *International Journal of Science Education*, *8*(3), 445-451. https://doi.org/10.1080/09751122.2015.11890266

Savin-Baden, M., & Major, C. H. (2013). *Qualitative research. The essential guide to theory and practice*. Routledge.

Schlosser, L.K., & Balzano, B. (2014). Playing to learn: How after school clubs influence teachers' beliefs about instruction. Sage Open, https://doi.org/10.1177/2158244014558031

Seow, P., & Pan, G. (2014). A literature review of the impact of extracurricular activities participation on students' academic performance. *Journal of Education for Business*, *89*, 361–366. https://doi.org/10.1080/08832323.2014.912195

Sfard, A., & Prusak, A. (2005). Telling Identities: In Search of an Analytic Tool for Investigating Learning as a Culturally Shaped Activity. *Educational Researcher*, *34*(4), 14-22. https://doi.org/10.3102/0013189X034004014

Sherman, H.J., & Catapano, S. (2011). After-school elementary school mathematics club: Enhancing achievement and encouraging future teachers. *Education Research Quarterly*, *35*(1), 3–16.

Tachie, S. A., & Chireshe, R. (2013). High failure rate in mathematics examinations in rural senior secondary school in Mthatha district, Eastern Cape: Learners' attributions. *Stud Tribes Tribals*, *11*(1), 67-73. https://doi.org/10.1080/0972639X.2013.11886667

Turner, E. E., Gutiérrez, R. J., & Sutton, T. (2011). Student participation in collective problem solving in an after-school mathematics club: Connections to learning and identity. *Canadian Journal of Science, Mathematics and Technology Education*, *11*(3), 226-246. https://doi.org/10.1080/14926156.2011.595884

Valentine, J. C., Cooper, H., Bettencourt, B. A., & DuBois, D. L. (2002). Out-of-School Activities and Academic Achievement: The Mediating Role of Self-Beliefs. *Educational Psychologist*, *37*(4), 245–256. https://doi.org/10.1207/S15326985EP3704_4

Wenger, E. (1998). *Communities of practice. Learning, meaning, and identity.* Cambridge University Press.

Yin, R. K. (2009). *Case study research: design and methods*. (4th edn.). SAGE Publications, Inc.

Chapter 2: High school learners' mathematical identities

2.1 Introduction

In this chapter, I drew on Sfard and Prusak (2005) and defined mathematical identity as stories about learners' relationships with mathematics told by the learners and indicated in their confidence and persistence in and beliefs about mathematics. From McGee (2015), I borrowed the concept of identity as robust or fragile and suggested three kinds of mathematical identities: largely robust, modest and largely fragile, addressing the first research question. I drew on Wenger's (1998) theory of learning as occurring in communities of practice and involving identity construction and showed how learners' interactions with their peers and family members influenced their mathematical identities, helping me to address the second research question. The chapter helped me to have a sense of the kinds of mathematical identities that learners narrate before participating in an intervention aimed at supporting their mathematical identities, and to understand why they construct such mathematical identities.

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High School Learners' Mathematical Identities

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We investigate Grade 10 learners' mathematical identities, with a focus on influences on the learners' identities. A mathematical identity questionnaire was given to learners and their test scores accessed. Four learners were purposively selected for interviews on the basis of interesting relationships between their questionnaire responses and test scores. The interview data were analysed for stories of the learners' confidence, beliefs and persistence with mathematics, as indicators of mathematical identity. The findings showed that learners can hold largely robust, modest or largely fragile mathematical identities, which are related to these three indicators. Past and present mathematical experiences, what family members and peers said about the learners and about mathematics, including peer labelling, were related to the learners' identities. Career opportunities associated with mathematics, pleasing others and gaining prestige contributed to how learners formed their mathematical identities.

Keywords: *Mathematical identities; confidence; beliefs; mindsets; persistence*

Introduction

There are concerns over South African high school learners' success in mathematics. Spaull (2013) argues that 'more pupils are opting for easier subjects like mathematical literacy, compared to more challenging subjects like mathematics' (p. 8). Negative mathematical experiences and beliefs, including low performance, influence learners to feel unconfident about their mathematical abilities and to give up on achieving mathematical success (Graven, 2015). Similar learner mathematical identities have been found by researchers in other countries (e.g. Darragh, 2015), making negative learner mathematical identity development a global concern. Many researchers concede that learners' disengagement and underachievement in mathematics have become persistent problems, frustratingly difficult to address (Yeager & Dweck, 2012). A disconnect between learners' fun with interactive after-school leisure activities and school mathematics contributes to: beliefs that school work and the formal mathematics curriculum are irrelevant and boring (Aydeniz & Hodge, 2011); the formation of fragile mathematical identities (McGee, 2015); and doing mathematics for career opportunities rather than for enjoyment (Andersson, Valero, & Meaney, 2015; Boaler, William, & Zevenbergen, 2000).

We suggest that a possibility to address the problems of learner disengagement and underachievement in mathematics may be in understanding learners' mathematical identities and developing strategies likely to positively transform their identities. However, identity is still not well conceptualised in the mathematics education literature (Radovic, Black, William, & Salas, 2018). Also, few studies focusing on learners' identities have been conducted in Africa (Darragh, 2016), despite the situated nature of identities (Bishop, 2012) and the importance of socio-cultural experiences in identity formation (McGee, 2015; Sabbah & Heyd-Metzuyanim, 2019). As an initial exploration into South African learners' mathematical identities, this paper focuses on the question: what are some of the influences on learners' mathematical identities?

Literature Review: Mathematical Identities

Identities may provide better insights into the persistent problems of mathematical disengagement and underachievement as they encompass our life histories, affective qualities and cognitive dimensions (Grootenboer & Zevenbergen, 2008). Confidence (Darragh, 2015), beliefs (Bishop, 2012) and persistence (Bishop, 2012) are closely related to and reflect learners' mathematical identities. Confidence is the level of trust in one's mathematical responses (Sander & Sanders, 2009). Beliefs are personal views and judgements formed by past experiences (Eleftherios & Theodosios, 2007), for example that mathematics is valuable. Mathematical mindsets, which include growth and fixed mindsets, are beliefs about mathematical intelligence (Boaler, 2013; Dweck, 2016). Growth mindsets are beliefs that mathematical intelligence can improve through effort and engaging in challenging mathematical activities while fixed mindsets are beliefs that mathematical intelligence cannot be changed even through hard work (Boaler, 2013). Persistence is about not giving up on mathematics, as a subject and when faced with challenging mathematical problems, seen by for example purposefully struggling with a challenging problem (Boaler & Selling, 2017). We postulate that if learners have confidence in their mathematical abilities, they are likely to persist, learn and form positive mathematical beliefs. Likewise, if learners are persistent, they are likely to learn, gain confidence and develop positive mathematical beliefs.

Identity can be viewed as reifying, significant and endorsable stories about a person, told by her/himself or by others (Sfard & Prusak, 2005). In this paper, we adopt the view of mathematical identity as stories about a person's confidence, beliefs and persistence with mathematics. Two forms of mathematical identities, depending on the strength of a person's relationship with mathematics, have been distinguished in the literature: robust and fragile identities (McGee, 2015). Robust identities are related to positive mathematical experiences and are largely intrinsically motivated while fragile identities are related to negative mathematical experiences and are extrinsically motivated (McGee, 2015). Learners with robust identities tend to be confident and persistent and they hold positive beliefs about mathematics; those with fragile identities are less confident and less persistent and hold negative beliefs about mathematics.

The Kinds of Identities Constructed by Learners

Research shows that many learners hold fragile mathematical identities, finding mathematics boring (Andersson et al., 2015), abstract and not useful in their everyday lives (Boaler et al., 2000). Many learners give up mathematics as soon as they are given the chance to do so, despite knowing the career opportunities associated with the subject (Boaler et al., 2000). Fear and dislike of mathematics are rife among high school learners such that even those performing well are likely to choose to discontinue with the subject after high school (Hall, Towers, & Martin, 2018). Also, many learners hold a fixed mindset (Yeager & Dweck, 2012). Some researchers have shown that learners' identities are fluid, contested, negotiated and closely related to significant others in their lives, who are their teachers, peers and family members, making them complex (Hall et al., 2018). The contradictory and multifaceted nature of self-narratives, and therefore identities, adds to their complexity (Hall et al., 2018).

Influences on Learners' Identities

Identities are linked to other people and communities, and therefore are a social construction occurring within communities of practice (Hall et al., 2018; Wenger, 1998). The kinds of social relations among community members contribute to how members form their identities (Wenger, 1998), and social context is central to how mathematical identities are defined (Darragh, 2016). Within different social contexts, most researchers have focused on the influence of teachers on learners' mathematical identities (e.g. Varelas, Martin, & Kane, 2012), arguing that the ways teachers convey mathematics in class influence identity formation (Boaler et al., 2000; Gardee & Brodie, 2019). Some studies have focused on the influence of family members and peers on learners' identities but more can still be done, especially on the extent of their influence. Past mathematical experiences have been found to influence future mathematical identities (Andersson et al., 2015).

Despite some researchers showing that mathematical identities, learning and mathematical competence are related (Bishop, 2012; Boaler & Selling, 2017), other researchers argue that academic achievement does not necessarily imply holding robust identities (e.g. Sfard & Prusak, 2005). Darragh (2015), for example, notes that achievement does not always imply that all of the components of mathematical identity are positive, meaning that a learner may perform well and even go on to a career that needs mathematics despite some component(s) of her/his identity being negative. Learners' sense of belonging within communities of practice (Wenger, 1998), has been shown to influence their mathematical identities, rather than their ability (Boaler et al., 2000). However, other literature shows a strong correlation between academic achievement and mathematical identities of black learners (Varelas et al., 2012). Learners taught using a project-based approach showed robust identities and performed better than those taught to follow rules and to memorise, suggesting that teaching methods have the potential to link academic achievement and mathematical identities (Boaler & Selling, 2017). There appear to be no clear results on the relationship between mathematical identities and academic achievement, and evidence of the relationship between identities and achievement—as related to learners' test scores—is scarce.

Conceptual Framework

The conceptual framework for this study shows the relationship between mathematical identities and communities that practice mathematics (Figure 1). It mainly draws from Sfard and Prusak's (2005) construct of identity as stories that people tell and Wenger's (1998) notion of learning as identity construction, occurring within communities of practice. Influences on learners' identities, for example what other people say, are found within mathematics communities. Three interrelated components of identity—confidence, beliefs and persistence—are explored. Learners can hold varying levels of confidence, beliefs and persistence, constructing different forms of identities.

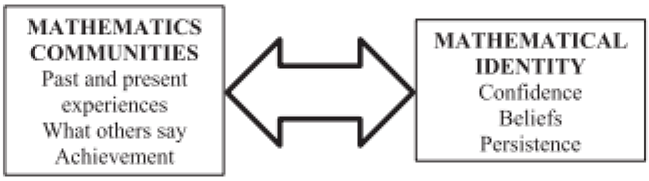

Figure 1. A conceptual framework for mathematical identity.

We anticipate past and present mathematical experiences, achievement and what others say about a learner and mathematics to influence the learner's mathematical identity. Reifying, significant and endorsable stories about a learner, told by the learner and other people, show the learner's levels of confidence, beliefs and persistence, and therefore constitute mathematical identity. The development of a robust mathematical identity is expected to lead to better achievement, a change in what others say about the learner and the learner pursuing the subject in the future.

Methodology

Given the interacting nature of identities, learning and communities, and our stance to understand identity construction from participants' perspectives (McMillan & Schumacher, 2010), an interpretivist paradigm was adopted. As learners responded to a questionnaire and an interview, they told stories showing their confidence, beliefs and persistence with mathematics. Influences on learners' mathematical identities were revealed during story telling.

Sample and Data Collection

The school that participated in this study was selected by convenience sampling. The closeness of the school to the first author's place of work made it easy to travel to the site. The school is located in a township area and serves mainly black South Africans of low socio-economic status. Ethics clearance was granted for the study by the University's Ethics Committee.

Stratified purposive sampling of learners was conducted. A group of Grade 10 learners who volunteered to participate in the study responded to a 22-item mathematical identity questionnaire and allowed us to access their first term's mathematics test scores. A term test is a school-based test written at the end of a school calendar term. Teachers are guided by previous national question papers to set the test, which they send to district subject specialist facilitators to check for quality, for example cognitive levels, language and mark allocation, before learners write it. A single term's test score, covering topics covered over about two months, was collected for each learner. The validity of the questionnaire was enhanced by adapting items from previous studies that had measured learners' confidence, beliefs and persistence (e.g. Boaler, 2013). Examples of the questionnaire items are: I am confident with mathematics; if it was up to me I would continue with mathematics next year; and working hard can make you to be good at maths. Learners chose responses ranging from strongly agree, coded as 5, to strongly disagree, coded as 1. Each learner's total mathematical identity questionnaire score was calculated by adding individual item scores. The learner's total identity score was divided by the total possible identity score of 110 and converted to a percentage, which was used to describe the identity of the learner.

The sample of learners was stratified into three groups based on learners' test and identity scores. The learners' identities were classified based on the questionnaire responses as largely robust (>70%), modest (69–50%) or largely fragile (<49%). A largely robust identity suggested that the learner was largely confident, largely persistent and held more positive than negative beliefs about mathematics, or the learner showed more elements of robust than fragile identities. A largely fragile identity suggested that the learner was largely less confident and less persistent and held more negative than positive beliefs about mathematics, or s/he showed more elements of fragile than robust identities. A learner holding relatively equal mixes of robust and fragile identities was considered as having a modest identity. Within each identity class, learners were separated into high, average and low achievers based on their mathematics test results.

Four interesting learners—with different mathematical identities and having performed differently—were selected for interviews, which allowed us to investigate and re-analyse mathematical identities more deeply. The test and identity scores of the four selected learners, whose pseudonyms are Thabo, Mpho, Themba and Fana, are shown in Table 1. Responses to a question probing the learners on their belief about whether *everyone can do mathematics* (Boaler, 2013) were found to be interesting because three of the four selected learners did not believe that everyone can do mathematics, irrespective of their identities.

Individual interviews were conducted before or after school, depending on what suited each learner. Before conducting the interviews, learners were asked to be truthful and were told that their responses would be confidential and anonymous and they could choose not to answer any question. Follow-up questions were sometimes asked to gather detailed information. Examples of the interview questions probing learners about their confidence, beliefs and persistence are:

- Do you think that you are good at maths?
- Are people born good at maths?
- How much effort do you put into maths?

Table 1. The four learners' identities based on questionnaire responses.

Learner	Thabo	Mpho	Themba	Fana
Test score	86%	73%	41%	34%
Identity score	94%	55%	32%	41%
Everyone can do mathematics	No	No	No	Yes
Description of identity	Largely robust	Modest	Largely fragile	Largely fragile

Learners could show more than one of the components of identity in a single response. For example, considering that mindsets are beliefs that effort (i.e. persistence) can improve intelligence, single responses showing beliefs and persistence were expected. Questions probing learners about their significant others were also asked, for example: what do other learners say about you as a math learner?

Transcribed interviews were coded for responses showing confidence, beliefs and persistence, after which learners were classified as holding a largely robust, modest or largely fragile identity. Other significant codes were also identified, for example peer influence. A story that was reifying and significant was considered to be endorsed (Heyd-Metzuyanim & Sfard, 2012), therefore focus was on establishing whether stories were reifying and significant, for them to be identities. Reification was operationalised as utterances showing who a person is (Andersson et al., 2015; Sfard & Prusak, 2005), for example, 'I am good at mathematics'. A story was considered significant if it showed a learner's feelings, including feelings of belonging or exclusion, or indicators of what the learner 'presumably considered important, appropriate or desirable' (Heyd-Metzuyanim & Sfard, 2012, p. 133), for example 'you regret', which also shows how learners may seem to be talking about others yet they will be referring to themselves (Heyd-Metzuyanim & Sfard, 2012). Quotes that best illustrate the learners' identities are provided.

Findings

Questionnaire

The questionnaire identity scores suggest that the four learners held different mixes of robust and fragile identities, except possibly for Thabo, thus revealing a broader range of identities than those emphasised in the literature. The two high performers, Thabo and Mpho, had different mathematical identities. Thabo held a largely robust identity while Mpho had a modest identity (Table 1). Both learners did not believe that everyone can do mathematics. Themba and Fana were the two low performers, who both showed largely fragile identities. Themba's questionnaire responses showed little confidence, little persistence and largely negative beliefs about mathematics, but he claimed to be good at mathematics, a reason why we selected him for deeper understanding. Like Thabo and Mpho, Themba did not believe that everyone can do mathematics. Some of Fana's questionnaire responses contradicted each other, for example, believing that everyone can do mathematics and that he could get smarter in mathematics by trying hard, while at the same time not believing that ability in mathematics increases when one studies hard.

Interviews

For the most part, the interviews confirmed the questionnaire findings on identity. They also showed that past and present mathematical experiences gained through interactions with peers and family members were related to learners' identities and that learners' identities were related to what others said about the learners. The learners showed different mixes of robust and fragile identities.

Thabo: largely robust identity

Thabo's questionnaire had a very high score suggesting a largely robust identity, backed by his interview responses which suggested some elements of a fragile identity. Thabo showed confidence in the way that he spoke. When asked whether he thought that he was good at mathematics he said, 'I don't think that I am good, actually I know that I am good', showing that confidence was a component of his identity. He saw himself as the best mathematics learner to whom others went when they needed help. Thabo praised himself saying that 'I assist others if they come to me', illustrating a relationship between how others saw Thabo and how he saw himself with respect to mathematics. Thabo added that, 'I am only assisted by learners who are as good as me, on just a few questions and not always because most of the times I am good'. The influence of past experience in constructing future identities, as well as a relationship between achievement and identities, was highlighted when Thabo talked about his achievements in mathematics from primary school up to high school saying 'I have done well in tests at primary and even now I still do them well, I will never drop'. Thabo's response suggested that his past performances were related to how he constructed his present identity. Thabo showed confidence and persistence in both his questionnaire and interview responses. A change was observed in his response about giving up on challenging problems. In the questionnaire he agreed to giving up when stuck, but in the interview he said that 'when stuck I don't give up because the same problem can come in the test and you regret why you gave up', suggesting that identity formation was a process of constant negotiation, related to context. Thabo understood that mathematics improves creativity when he said that 'mathematics teaches you to solve your own things', illustrating a growth mindset and therefore a positive belief within his identity.

Elements of a fragile identity were shown when Thabo spoke about career opportunities and mindset. When asked about the uses of mathematics, Thabo did not mention uses of mathematics besides career opportunities, saying 'mathematics is useful because a lot of jobs need mathematics'. When asked about how long he would continue doing mathematics he said, 'I will do mathematics forever because not everyone can do mathematics, therefore I will be unique and special', suggesting that the performance of others influenced him to construct an identity constituted by negative mathematical beliefs, a component of fragile identities. Thabo believed that people are born good at mathematics, and his reason was that, 'because others try but still fail', suggesting that the academic performance of his peers was related to his view of himself as strong in mathematics, suggesting some potential fragility in his identity. Thabo's mathematical identity was related to what others said about mathematics and about him. He did mathematics because he wanted to be different from other learners, who he said believed that mathematics was difficult. Thabo said, 'what other learners say means that I have a gift others don't have which makes me unique and is cool. It encourages me to do what is difficult to others'. While we do not know what Thabo's career trajectory will be, it is possible that if he confronts mathematics that he finds extremely difficult in the future, a more fragile identity will come to the fore, given his fixed mindset.

Mpho: modest identity

Mpho's interview responses confirmed the modest identity score in the questionnaire and identified some robustness and some fragility in her identity. The questionnaire responses suggested that Mpho did not believe that everyone can do mathematics. In the interview, Mpho said she believed that everyone can do mathematics, suggesting a change in mindset. Mpho believed that people are not born good at mathematics 'but can practise'. She believed in learning as mutual engagement saying that 'we can challenge each other as learners, practise together and understand each other', demonstrating an identity constituted by some elements of a growth mindset. Mpho's belief in working with others was further highlighted when she said, 'we learners sometimes understand each other more than we understand the teacher'.

Mpho showed little confidence during the interview, suggested by saying that she was a 'little bit good at mathematics' and 'sometimes' assisted others. She said that when she got stuck she sometimes left the question and waited for the teacher to assist her, suggesting little persistence and a fixed mindset. Mpho said that she lacked practice. Doing mathematics was motivated by career opportunities associated with mathematics and gaining prestige, as suggested by her saying, 'I will carry on doing mathematics because it is useful for jobs, may be you can get enough salary and it's like you are doing something big'. What peers and family members said appeared related to Mpho's fixed mindset, highlighted by saying, 'what others say sometimes makes me think that mathematics is difficult'. Mpho also said that 'if stuck other learners laugh at me', suggesting that what her peers said about her negatively influenced her mathematical identity.

Themba: largely fragile identity

During the interview, Themba did not show any elements of a robust identity, underscoring that he had a largely fragile identity. Themba felt that he was not good enough to be doing mathematics. He said that he was not committed to mathematics and would not do mathematics after Grade 12. Themba seemed to change most of his questionnaire responses, for instance when he said that he did not like mathematics during the interview. Themba said, 'sometimes I get bored and do other homework in a mathematics period', suggesting a fragile identity, related to a particular context. When asked about doing a job that needed mathematics he said he would not, because 'I want to be a sound engineer and that does not need mathematics', suggesting a fragile mathematical identity constituted by a negative belief about mathematics.

Themba believed that people are born good at mathematics and, as in the questionnaire, that not everyone can do mathematics, arguing that 'people have different gifts'. According to Themba, 'making a mistake means you don't know or you are dull', showing a fixed mindset. Themba believed that constant practice was important to learning mathematics but he sometimes gave up on challenging problems, illustrating contradicting identities.

Like with Thabo and Mpho, Themba's interview responses showed that what peers and family members said was related to his mathematical identity. Themba said that 'other learners say mathematics is difficult and it's for clever and special people, they say mathematics is not cool because they believe that even if they try they won't get it right', comments likely to have influenced Themba to construct a fragile identity. Other learners sometimes told Themba that he was 'dull', did not 'know mathematics and should change the subject'. Themba's academic achievement appeared related to what his peers said about him, which was in turn related to his fragile identity.

Fana: largely fragile identity

Fana showed some elements of a robust identity, believing in mutual engagement and that people are not born good at mathematics. In agreement with his questionnaire response, he believed that 'everyone can do mathematics if you practice', suggesting a growth mindset.

Similarly to Themba, Fana felt that he was not good enough to be doing mathematics. He said that he was not committed to mathematics and would not do mathematics after Grade 12. He believed that constant practice was important to learning mathematics, but like Themba, he sometimes gave up on challenging problems. In one of his responses he said that 'making a mistake means I didn't work hard', suggesting a fixed mindset. Fana just wanted to pass mathematics because 'jobs need mathematics', even though he would have preferred not to do mathematics because 'mathematics is very complicated and it's difficult, I don't like doing mathematics all the time', illustrating that he was doing mathematics because of career opportunities associated with mathematics, and a negative belief.

Like the other three learners, Fana's interview responses suggested that what peers and family members said was related to his mathematical identity, for example when he also said that other learners believed that mathematics was difficult, an utterance likely to have influenced him to form a fragile identity. Fana's family influenced him into believing that mathematics was difficult as suggested by saying 'my family, they don't like mathematics because they say it's difficult'.

Discussion and Conclusion

In this section, we respond to the research question and make recommendations for areas of future study. The four learners presented in this study, two high and two low achievers, showed different mathematical identities, and highlighted the contestations, contradictions and therefore complexities associated with identities (Aydeniz & Hodge, 2011; Hall et al., 2018), and the challenge of describing them. Interconnectedness between confidence, beliefs and persistence was evident. The two low achievers showed largely fragile identities and chose not to follow careers that need mathematics while one of the high achievers showed a modest identity. Both high achievers incorporated some negative components of mathematical identities (Sfard & Prusak, 2005), showing that strategies for developing the identities of high achievers are as necessary as they are for low achievers.

Some of the influences on the learners' mathematical identities, revealed in the interviews were pleasing others, gaining prestige and accessing career opportunities associated with mathematics, adding nuance to our understanding of learners' identities. The challenge of defining learners' mathematical identities, was significant. Peers, especially how they labelled each other and family members, played an important role in how learners formed their mathematical identities.

The learners' mathematical identities were related to past and present experiences gained within mathematics communities. We found that mathematical identity formation is a constant social negotiation (McGee, 2015; Varelas et al., 2012), in that learners chose to allow or refuse their past or present mathematical experiences to influence their identities. What peers and family members said about mathematics and about the learners, including the academic performance of peers, contributed to learners' mathematical identity formation. Peers had a largely negative influence on the learners' mathematical identities. Learners' views about the usefulness of mathematics were limited to good careers, gaining prestige and pleasing others. The interviews suggested that academic achievement might be related to how a learner was seen by others, which was related to how the learner saw her/himself with respect to mathematics.

Drawing on the above findings, we argue that the issue of learners labelling each other deserves more attention from researchers. Teaching strategies that show mathematics as creative (Boaler, 2013), useful for more than career opportunities and fun might contribute to improved engagement and formation of more robust mathematical identities. Given that all learners showed some elements of robust and fragile identities, strategies aimed at improving and maintaining the mathematical identities towards and at a robust level are essential for high, average and low achievers. More studies investigating the role of peers and family members on learners' mathematical identities are necessary. It is now time to focus on studies that attempt to conceptualise mathematical identities more clearly, rather than alluding to the problem, if we are to get any closer to solving the challenges of mathematical disengagement and underachievement.

Disclosure Statement

No potential conflict of interest was reported by the authors.

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References

Andersson, A., Valero, P., & Meaney, T. (2015). 'I am [not always] a maths hater': Shifting students' identity narratives in context. *Educational Studies in Mathematics*, *90*, 143–161.

Aydeniz, M., & Hodge, L. L. (2011). Identity: A complex structure for researching students' academic behavior in science and mathematics. *Cultural Studies of Education*, *6*, 509–523.

Bishop, J. P. (2012). 'She's always been the smart one. I've always been the dumb one': Identities in the mathematics classroom. *Journal for Research in Mathematics Education*, *43*(1), 34–74.

Boaler, J. (2013). Ability and mathematics: The mindset revolution that is reshaping education. *Forum (Chicago, Ill.)*, *55*(1), 143–153.

Boaler, J., & Selling, S. K. (2017). Psychological imprisonment or intellectual freedom? A longitudinal study of contrasting school mathematics approaches and their impact on adults' lives. *Journal for Research in Mathematics Education*, *48*(1), 78–105.

Boaler, J., William, D., & Zevenbergen, R. (2000). *The construction of identity in secondary mathematics education.* Paper presented at the International Mathematics Education and Society Conference, Montechoro, 26–31 March.

Darragh, L. (2015). Recognising 'good at mathematics': Using a performative lens for identity. *Research in Mathematics Education*, *27*, 83–102.

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, *93*, 19–33.

Dweck, C. S. (2016). *Mindset: The new psychology of success.* (Updated edn.). New York, NY: Ballantine Books.

Eleftherios, K., & Theodosios, Z. (2007). Students' beliefs and attitudes about studying and learning mathematics. In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.). *Proceedings of the 31st conference of the international group for the Psychology of mathematics education* (Vol. 3, pp. 97–104). Seoul, South Korea: PME.

Gardee, A., & Brodie, K. (2019). The influence of teachers on learners' mathematical identities. In M. Graven, H. Venkat, A. Essien, & P. Vale (Eds.), *Proceedings of the 43rd conference of the international group for the psychology of mathematics education* (Vol. 2, pp. 248–255). Pretoria, South Africa: PME.

Graven, M. (2015). Strengthening maths learning dispositions through 'math clubs'. *South African Journal of Childhood Education*, *5*(3), Art. #342, 7 pages.

Grootenboer, P. J., & Zevenbergen, R. (2008). Identity as a lens to understand learning mathematics: Developing a model. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions. Proceedings of the 31st annual conference of the mathematics education research group of Australasia, Brisbane* (Vol. 1, pp. 243–250). Brisbane: MERGA.

Hall, J., Towers, J., & Martin, L. C. (2018). Using I poems to illuminate the complexity of students' mathematical identities. *Educational Studies in Mathematics*, *99*, 181–196.

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggle in the mathematics classroom: On learning mathematics as an interplay of mathematising and identifying. *International Journal of Educational Research*, *51-52*, 128–145.

McGee, E. O. (2015). Robust and fragile mathematical identities: A framework for exploring racialised experiences and high achievement among black college students. *Journal for Research in Mathematics Education*, *46*(5), 599–625.

McMillan, J. H., & Schumacher, S. (2010). *Research in education: Evidence-based inquiry* (7th edn.). International edition. Boston, MA: Pearson Education Inc.

Radovic, D., Black, L., William, J., & Salas, C. E. (2018). Towards conceptual coherence in the research on mathematics learner identity: A systematic review of the literature. *Educational Studies in Mathematics*, *99*, 21–42.

Sabbah, S., & Heyd-Metzuyanim, E. (2019). *Agency and identity of female arab students entering a technological university*. Paper presented at the 11th Congress of the European Society for Research in Mathematics Education, Utrecht, 06-10 February.

Sander, P., & Sanders, L. (2009). Measuring academic behavioural confidence: The ABC scale revisited. *Studies in Higher Education*, *34*(1), 19–35.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14–22.

Spaull, N. (2013). *South Africa's education crisis: The quality of education in South Africa 1994–2011*. Johannesburg: Centre for development and enterprise (CDE).

Varelas, M., Martin, D., & Kane, J. (2012). Content learning and identity construction (CLIC): A framework to strengthen African American students' mathematics and science learning in urban elementary schools. *Human Development*, *55*, 319–339.

Wenger, E. (1998). *Communities of practice. Learning, meaning, and identity*. Cambridge: Cambridge University Press.

Yeager, D. S., & Dweck, C. S. (2012). Mindsets that promote resilience: When students believe that personal characteristics can be developed. *Educational Psychologist*, *47*(4), 302–314.

Chapter 3: Learners' mathematical identities: Exploring relationships between high school learners and significant others

3.1 Introduction

In this chapter, I brought together Marks and O'Mahoney's (2014) idea of identity as constituted by personal and social identities, McGee's (2015) idea of mathematical identity as robust or fragile, and Bishop's (2012) idea of identity as fluid, addressing the first research question. Drawing on Wenger's theory of communities of practice, I developed a framework for understanding mathematical identity that links three communities of mathematics practice: the classroom, after-school and home communities, with nine different mathematical identity combinations. Two kinds of relationships between learners and three different groups of significant others (i.e., teachers, peers and family members): robust-enhancing and fragile-enhancing relationships were defined and explored. I showed how the three different groups of significant others collectively supported and constrained learners' mathematical identities thus addressing the second research question. The chapter showed how the developed framework can be used to analyse and describe learners' mathematical identities.

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ORIGINAL ARTICLE

Learners' mathematical identities: exploring relationships between high school learners and significant others

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Abstract

Many researchers have tried to understand why some learners engage in and others disengage from mathematics by exploring learners' mathematical identities. Significant others (i.e., teachers, peers and family members) offer learners' different opportunities for mathematical identity support, but no study has explored their collective role in supporting or constraining learners' identities. In this paper, we explore how relationships between high school learners and different groups of significant others shaped learners' narrated mathematical identities. Fifty Grade 10 learners responded to a mathematical identity questionnaire. After analysing the results of the questionnaire, six learners were purposively selected for a semi-structured interview, and the interview data were analysed thematically. Identities are described as robust-leaning, mixed or fragile-leaning, depending on learners' narrated confidence, persistence, beliefs and sense of belonging to or exclusion from mathematics communities. Relationships with teachers, peers and family members are described as robust-enhancing or fragile-enhancing, depending on the kind of identity encouraged by the relationships. The findings show that many learners narrated fragile-leaning identities, and all the learners showed some robustness and fragility in their identities. The more groups of significant others a learner developed robust-enhancing relationships with, the more likely the learner narrated a robust-leaning identity. We argue that understanding learners' identities requires understanding their relationships with the three different groups of significant others as a collective.

Keywords Mathematical identity · Significant others

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Introduction

Repeated negative mathematical experiences have been linked to a decline in participation and interest in mathematics, with many learners choosing not to pursue mathematics as they move to higher grades (Darragh, 2015; Grootenboer & Jorgensen, 2009; Spaull, 2013). Some researchers have argued that the abstract nature of mathematics supports feelings among learners that mathematics is boring and is only useful for career opportunities (Andersson et al., 2015; Boaler et al., 2000). Learners' disengagement from mathematics has led researchers to investigate learners' mathematical identities and their relationships with mathematics, as a way of trying to understand why some learners enjoy mathematics and others hate it (e.g., Darragh, 2015; Hall et al., 2018; Sfard & Prusak, 2005). Understanding how learners construct their mathematical identities may suggest strategies likely to encourage more learners to enjoy and choose mathematics.

Mathematical identities are constructed in relation to others (Hall et al., 2018). Significant others in learners' lives (i.e., their teachers, peers and family members) offer learners' identity support which the learners can accept or reject (Gardee, 2019). Learners' mathematical identities are related to how they think their significant others position them and how they are positioned by their significant others (Hall et al., 2018).

A number of researchers have investigated the role of teachers in supporting learners' identities and found that how teachers communicate with learners and how they present mathematics in class are linked to learners' identities (e.g., Aydeniz & Hodge, 2011; Gardee, 2019). Some researchers have explored the role of peers and family members in how learners enact their identities and found that learners' views of the expectations of their peers and family members support or constrain their identities (e.g., Darragh, 2015; Hall et al., 2018; Zhang & Wang, 2021). Previous studies have tended to focus on the role of each group of significant others separately, and the extent of the collective role played by significant others in affording learners' identities has not been investigated. We therefore explored relationships between learners and three different groups of significant others: teachers, peers and family members, and how these relationships afforded learners' narrated identities. The following research questions guided the study:

1. What kinds of mathematical identities do high school learners narrate?
2. What kinds of relationships, with different groups of significant others, do high school learners narrate and how do these relationships collectively afford learners' mathematical identities?

In the next section, we outline our perspective on mathematical identities, followed by a discussion of research focusing on how significant others support learners' mathematical identities.

Learners' mathematical identities and relationships between learners and their significant others

Identity is a social construct strongly linked to a person's interactional experiences with others. Specifically, learners' mathematical identities are shaped by their interactions with their teachers, peers and families in mathematics classrooms and after-school and home communities.

Learners' mathematical identities

Many researchers do not precisely state what identity is and how it can be empirically studied. Identity has been difficult to define and operationalise (Fellus, 2019; Graven & Heyd-Metzuyanim, 2019; Ntow & Adler, 2019). Wenger's (1998) social practice theory places identity as central to practice and learning in communities of practice. Identity is therefore enacted in community with others (see also Darragh, 2016). Sfard and Prusak (2005) argue that identity is discursive and thus is produced as we narrate ourselves to ourselves and others (see also Bishop, 2012). These two seemingly contrasting positions, that identity can be enacted or narrated, are intimately interrelated because our actions are never separate from our discourse. Marks and O'Mahoney (2014) argue that identity is composed of personal identity, which is a view of the self, and social identity, which is how a person relates to the collective. Investigating both personal and social identities contributes to understanding identity since personal identity is supported or constrained by the social identities made available by society (Marks & O'Mahoney, 2014), and social identities are supported or constrained by the personal identity that a person enacts or narrates.

McGee (2015) argues that a learner's identity can be robust or fragile, depending on how the learner deals with persistent negative mathematical experiences. A learner with a robust identity is self-motivated to persist with mathematics despite persistent negative mathematical experiences, which means that their persistence with mathematics is intrinsically motivated, showing a strong relationship with mathematics. A learner with a fragile identity may not like mathematics but may persist with the subject to please others, which means that their persistence with mathematics is extrinsically motivated, showing a delicate relationship with mathematics (McGee, 2015). Intrinsically motivated persistence is motivated by a personal desire to conquer challenging activities, while extrinsically motivated persistence is motivated by external factors such as career opportunities, rewards or punishment or trying to please other people (McGee, 2015; Woolfolk, 2010).

We define mathematical identities, be they enacted or narrated, as composed of personal and social identities, which can be robust or fragile. Four different possible combinations of personal and social identities are suggested: robust personal and social identities, fragile personal and social identities, robust personal identities and fragile social identities and fragile personal identities and robust social identities.

Different researchers focus on different indicators of identities. Sabbah and Heyd-Metzuyanim (2019) focused on ethnicity and religion, Bishop (2012) focused on beliefs and persistence, Darragh (2013) focused on confidence and a sense of belonging to mathematics communities and McGee (2015) focused on race and persistence. Researchers who want to understand the identities of populations focus on indicators specific to groups of people (e.g., Sabbah & Heyd-Metzuyanim, 2019), while those who want to understand identities at a personal level focus on indicators specific to individuals (e.g., Bishop, 2012). Some researchers focus on both sets of indicators (e.g., McGee, 2015).

Similarly to McGee (2015), we wanted to understand identities at both a social and personal level so we focused on confidence, persistence, beliefs about mathematics and a sense of belonging to or exclusion from mathematics communities. Confidence is a person's trust in their mathematical responses (Sander & Sanders, 2009). Persistence is not giving up when solving mathematical problems and pursuing mathematics despite facing challenges (Sutherland et al., 2013). Beliefs are personal views about mathematics formed by past experiences (Eleftherios & Theodosios, 2007). A sense of belonging is a feeling of being a member of a community of mathematics practice (Wenger, 1998), while a sense of exclusion is a feeling of not being a member of a community of mathematics practice. Confidence, persistence and beliefs are personal characteristics and are therefore indicators of personal identities, while a sense of belonging or exclusion shows how a person relates to the collective and is an indicator of social identities. Since personal and social identities are mutually constitutive (Gardee, 2019; Marks & O'Mahoney, 2014), and no indicator of identity operates in isolation from the other indicators (Fellus, 2019), all of the indicators influence each other.

McGee (2015) focused on a single indicator of personal identity, persistence, and a single indicator of social identity, race, and distinguished learners' overall identities as robust or fragile. For a deeper understanding of identity, we focus on three indicators of personal identity and a single but more encompassing indicator of social identity, a sense of belonging. We also make further distinctions in relation to robust and fragile identities, which will be discussed below.

Learners' mathematical identities and significant others

Goos and Bennison (2019) argue that a person's identity is constructed across different mathematics communities, suggesting that understanding how these communities help produce identities will help us understand learners' identities. While some studies have focused on how relationships between learners and teachers afford learners' identities (e.g., Aydeniz & Hodge, 2011; Boaler et al., 2000; Gardee, 2019; Gardee & Brodie, 2022; Grootenboer, 2013), others have focused on how relationships between learners and peers or learners and family members afford learners' identities (e.g., Heyd-Metzuyanim & Sfard, 2012; Sfard & Prusak, 2005; Turner et al., 2013; Zhang & Wang, 2021). We could not find any study that focused on how relationships between learners and the three different groups of significant others: teachers, peers and family members, support learners' identities.

Many learners find mathematics unappealing because they view the subject as dealing only with numbers rather than with ways of thinking (Epstein et al., 2010), suggesting fragilities in identities related to how mathematics is presented by teachers. Some learners pursue mathematics for career opportunities and not for enjoyment, attributing this fragile position to the abstract nature of mathematics (Andersson et al., 2015; Aydeniz & Hodge, 2011; Boaler et al., 2000). If teachers teach mathematics for deep understanding, for example, by encouraging classroom discussion and showing learners that mathematics is about making connections between ideas, then learners' mathematical experiences may shift positively (Anderson et al., 2018; Andersson et al., 2015; Boaler et al., 2018; Cribbs et al., 2020), leading to the creation of more robust identities. Grootenboer (2013) found that teachers who exposed learners to tasks with multiple solutions or solution pathways supported robust identities. He also found that some teachers were quick to assist learners, which he argued robs learners of the opportunity to develop robust identities. He argued that teachers need to be skilled in finding a balance between protecting learners' identities and allowing learners to struggle with challenging problems if learners are to develop more robust identities.

Teachers' social relationships with learners can also support learners' mathematical identities (Forster, 2000; Gardee, 2019, 2021; Gardee & Brodie, 2022; Noren, 2011). Teachers who relate to their learners with care, concern and thoughtfulness, attend to learners' emotions and treat learners with respect and courtesy support learners' identities (Grootenboer, 2013). A teacher's explanation to a learner that it is normal and necessary to struggle to understand mathematics can reduce the learner's fears of mathematics (Braathe & Solomon, 2015), supporting the learner's identity. Some teachers label learners as good or weak at mathematics and believe that these are characteristics inherent in the learners (Gardee, 2019), contributing to the enactment of robust or fragile identities. In a follow-up paper, Gardee (2021) argued further that if teachers offer learners similar opportunities to participate in class irrespective of their abilities then the learners are more likely to develop more robust identities. Some teachers still publicly compare learners with each other, and this constrains many learners' identities, because of feeling embarrassed and inadequate (Grootenboer, 2013).

Many learners are pressured to pursue mathematics by their parents, and some are discouraged from pursuing the subject by their peers, which constrains the learners' identities (Gweshe & Brodie, 2019; Hall et al., 2018; McGee, 2015). Gardee and Brodie (2023) showed that a learner can be offered an identity of high, equal or low status by a peer and can construct their identity by affiliating with or resisting the offered identity. Gardee and Brodie (2023) argued that the kind of identity that a learner offers and constructs when working with others depends on the learner's personal and social identities.

Heyd-Metzuyanim and Sfard (2012) argued that if learners are not guided by their teacher, they may negatively influence each other's identities showing the importance of the collective role of teachers and peers in affording learners' identities. In another study, teachers facilitated class discussions in which peers attended to the explanations of problem-solving strategies of other learners. The peers were allowed to disagree with a learner's explanation but not in ways that positioned the learner as not capable of contributing to a group's discussion (Turner et al., 2013), a response usually offered to other learners by peers that constrain identities. Instead, the teachers encouraged peers to request clarifications and justifications in ways that helped each other feel capable of contributing to the discussion (Turner et al., 2013). Langer-Osuna et al. (2020) found that learners who work on a task without guidance from the teacher can dominate the discussion by giving commands and ideas, which affects participation patterns and identity development. They argued that teachers need to teach learners how to give each other opportunities to participate fully in a group. The studies by Heyd-Metzuyanim and Sfard (2012), Langer-Osuna et al. (2020) and Turner et al. (2013) showed that peers usually constrain each other's identities, but if they receive proper guidance from their teachers, they can support each other's identities, strengthening the idea that teachers and peers can collectively support learners' identities.

Several researchers have found that family members can support or constrain learners' identities. Sfard and Prusak (2005) found that if parents' educational expectations for their children are accompanied by family support rather than pressure, then learners can develop robust identities. If parents speak positively about mathematics, then their children will believe them, aim to meet the parents' educational expectations and develop more robust identities (McGee & Martin, 2011). Learners with parents who have high educational expectations are likely to develop more positive beliefs about mathematics than learners with parents who have low educational expectations (Zhang & Wang, 2021). Parents who praise their children when successful and encourage them when struggling, support their children's identities (Cunningham, 2021). Showing children the usefulness of mathematics outside of school and telling children stories of people similar to them who pursued mathematics and became successful in life support the children's identities (Cunningham, 2021). The collective role played by parents and teachers in supporting learners' identities was suggested when a learner frequently referenced her parents' and teachers' educational expectations, which she embodied and tried to meet, leading to the development of a robust identity (Hall et al., 2018).

The research reviewed here has shown that teachers, peers and family members support or constrain learners' mathematical identities. Despite the importance of these three groups of significant others for learners' identities, and some investigations into the collective role of two groups, no study has attempted to explore the collective role of all three groups in affording learners' identities.

Conceptual framework

Learners' personal and social identities are constructed through interaction with their teachers, peers and family members in mathematics classes, and after-school and home communities. Significant others in these communities offer learners different opportunities for identity support which the learners accept or reject, leading to the construction of different identities (Aydeniz & Hodge, 2011; Epstein et al., 2010; Gardee, 2019; McGee, 2015).

Identities can be robust or fragile (McGee, 2015) and are fluid (Bishop, 2012), so they can be constituted by different mixes of robustness and fragility. In this study, we consider a range of identities—from robust-leaning identities, those constituted by more robustness than fragility, through mixed identities, those constituted by

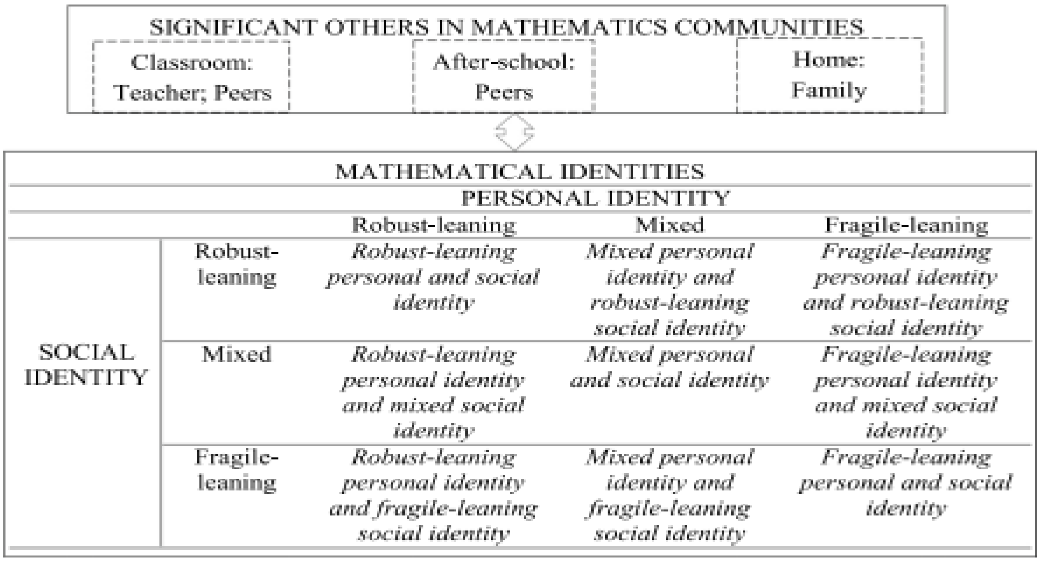

Fig. 1 A framework for learners' identities

relatively equal mixes of robustness and fragility and to fragile-leaning identities, those constituted by more fragility than robustness. Since a person's identity is constituted by personal and social identities (Marks & O'Mahoney, 2014), their personal identity can be robust-leaning, mixed or fragile-leaning, and so can their social identity. The three kinds of personal identities and three kinds of social identities suggest that learners can construct nine different possible combinations of personal and social identities.

Figure 1 presents a framework showing relationships between learners' significant others and learners' personal and social identities.

Method

As we argued above, identities can be enacted or narrated. In this study, we focus only on narrated identities. The focus of this study was to understand how relationships between learners and their significant others as narrated by learners afforded learners' narrated personal and social identities. A qualitative research design within an interpretive paradigm was adopted (Lodico et al., 2010) because the focus was on learners' socially constructed realities, and data were analysed to show how the learners interpreted their and others' thoughts, actions and feelings (McMillan & Schumacher, 2010).

Permission to conduct the study was granted by the university's ethics committee. The participating high school was selected because of its convenience. The school is a township school similar to many other South African schools that learners from low socio-economic households attend. Parents do not pay school fees and learners

Table 1 Questions assigned to each indicator and total possible scores

Indicator	No. of questions	Total possible score	Example of a question
Confidence	4	20 (4×5)	I am confident in maths
Persistence	4	20 (4×5)	In maths, when faced with a hard problem I continue trying until I get it
Beliefs	5	25 (5×5)	Ability in maths increases when one studies hard
Sense of belonging	3	15 (3×5)	In maths, my contributions are valued

are given free lunch, textbooks and stationery. The provincial department of education and the school principal permitted us to conduct the research at the school. All learners and their parents gave consent for learners' participation in the study.

Sample, instruments and data collection

Fifty out of 328 Grade 10 learners volunteered to take part in the study and were assured of confidentiality and anonymity. For ethical reasons, we could only work with the 50 volunteers, which we did not consider to be a setback since the purpose of the study was to understand relationships rather than to generalise findings. The 50 learners responded to a 16-item mathematical identity questionnaire using a Likert scale, and after analysing the questionnaire responses, we selected some of the learners to participate in a semi-structured interview about their identities and their relationships with significant others. The questionnaire and interview schedules were piloted at a similar school during the previous year, and some changes were made to the instruments, for example, simplifying the language in the questionnaire. Items in the questionnaire and semi-structured interview were adapted from previous studies investigating learners' identities (e.g., Bishop, 2012; Darragh, 2015).

The purpose of the questionnaire was twofold: to gain an overview of the fifty learners' narrated identities and to help us purposively select learners for the semi-structured interview. The questionnaire consisted of 13 questions asking learners about their personal identities (i.e., confidence, persistence and beliefs) and 3 questions about their social identities (i.e., sense of belonging). Questionnaire responses ranged from strongly agree, scored as 5, to strongly disagree, scored as 1. The questionnaire was answered after school, and learners ticked their responses on paper using pencils or pens.

A total personal identity score was calculated by adding together the scores for confidence, persistence and beliefs. A learner's sense of belonging score was their social identity score. Although identities are far more complex than such quantification allows, this was the best way to get an overview from a number of learners, and we found that the questionnaire data were confirmed in the interviews.

Table 1 shows the indicators of learners' narrated identities, the number of questions asked, the total possible score and an example of a question for each indicator.

Table 2 shows the nine possible identity groups (see also Fig. 1). Four out of the 9 identity groups were observed in the data (as will be discussed later in the paper). These are cells 1, 5, 8, and 9 (as in italics).

We intended to select two learners from each of the four identity groups observed in the data to participate in a semi-structured interview. The two learners narrating robust-leaning personal and social identities, Rose and Ben (all names are pseudonyms), agreed to participate in the interview. None of the learners narrating mixed personal and social identities or mixed personal identities and fragile-leaning social identities (cells 5 and 8 in Table 2) agreed to participate in the interview. Since many of the learners had narrated fragile-leaning personal and social identities (cell 9 in Table 2), we felt that it was reasonable to select four learners from cell 9: Kelly, Neo, Tom and Cindy. The purpose of the interview was twofold: to understand the learners' narrated identities more deeply and to understand the relationships between the learners' significant others and the learners' narrated identities, from the learners' perspectives.

Individual interviews were conducted after school in the learners' classrooms. The interviews lasted for about 40 min each, and they were audio-recorded and transcribed. Before conducting the interview, learners were put at ease regarding the purpose of the interview and informed that no one except the researcher would see their responses, and their responses would not affect their school results. They were asked to answer honestly and told that they could choose not to answer any question. Some examples of the interview questions were:

- How do you see yourself as a maths learner?
- Please tell me about how you relate with your teacher in maths.
- How do you relate with other learners in maths in your class?
- What does your family say about you as a maths learner?

Data analysis

The questionnaire

Since we had 13 questions asking learners about their personal identities and each question's score ranged from 5 = strongly agree to 1 = strongly disagree, we considered the minimum total score indicating robustness in personal identities to be $13 \times 4 = 52$ and the maximum to be $13 \times 5 = 65$. So, learners in the personal identity score range of 52–65 were considered having more robustness than fragility in their personal identities and were therefore narrating robust-leaning personal identities. The score range indicating fragile-leaning personal identities was from $13 \times 1 = 13$ to $13 \times 2 = 26$, suggesting more fragility than robustness in personal identities. Total personal identity scores of 27–51 indicated relatively equal mixes of robustness and fragility in personal identities, and learners in this score range were considered narrating mixed personal identities. Learners in the social identity score range of 12–15 were considered narrating robust-leaning social identities. The score range indicating fragile-leaning social identities was 3–6, while social identity scores of 7–11

Table 2 The nine possible identity groups

	Personal identity			
		Robust-leaning	**Mixed**	**Fragile-leaning**
Social identity	**Robust-leaning**	*1. Robust-leaning personal and social identity*	2. Mixed personal identity and robust-leaning social identity	3. Fragile-leaning personal identity and robust-leaning social identity
	Mixed	4. Robust-leaning personal identity and mixed social identity	*5. Mixed personal and social identity*	6. Fragile-leaning personal identity and mixed social identity
	Fragile-leaning	7. Robust-leaning personal identity and fragile-leaning social identity	*8. Mixed personal identity and fragile-leaning social identity*	*9. Fragile-leaning personal and social identity*

Table 3 Score ranges for narrated identities

Identity	Score range
Personal identity	
Robust-leaning	52–65
Mixed	27–51
Fragile-leaning	13–26
Social identity	
Robust-leaning	12–15
Mixed	7–11
Fragile-leaning	3–6

indicated mixed social identities. Table 3 shows the score ranges indicating the different learners' personal and social identities.

The interview

The learners' interview responses were thematically analysed, and the coding process took place in two stages. First, to understand learners' narrated identities, we coded for responses indicating confidence, persistence, beliefs and a sense of belonging to mathematics communities. Second, we coded for responses indicating relationships between learners and significant others. Two kinds of relationships were considered: robust-enhancing relationships, which encouraged the narration of robust-leaning personal and social identities; and fragile-enhancing relationships, which encouraged the narration of fragile-leaning personal and social identities. Table 4 shows the codes for relationships between learners and significant others, code descriptors and examples of learner quotes. The codes in Table 4 are predetermined codes drawn from the literature. The reported codes are not exhaustive since codes considered insignificant in relation to the study were left out.

Findings

All of the learners' narrated identities were constituted by some robustness and fragility. Relationships between learners and their significant others collectively supported or constrained learners' narrated identities.

Learners' narrated identities

The questionnaire

Table 5 shows the nine different possible combinations of learners' personal and social identities and the learners who narrated each identity combination. Four out of the nine possible combinations were seen in our data and five were not.

Table 4 Codes for relationships between learners and significant others

Code	Code descriptor and example of a quote	
	Robust-enhancing actions	**Fragile-enhancing actions**
Teacher pedagogy	Teacher teaches for understanding: *He encourages us to try harder and not leave a problem incomplete*	Teacher does not teach for understanding: *When the teacher teaches it's boring, it's just equations and sums*
Peer beliefs	Positive reaction to negative peer beliefs: *They say maths is difficult. I see that they don't understand what they are saying and what maths is*	Negative reaction to negative peer beliefs: *They say maths is boring and difficult. It makes me have a mind that says maths is difficult*
Peer responses	Positive peer responses: *When we are given homework we stay after school and help each other*	Negative peer responses: *They laugh at me*
Family pressure	Family does not pressure learner to pursue mathematics: *My family says they are very proud of me and will still love me whether I pass or fail*	Family pressures learner to pursue mathematics: *I'm the only one who has Grade 10 in my family and I feel that I have to carry on and make my parents proud*
Family support	Family supports learner: *She encourages me not to stop but to keep trying*	Family does not support learner: *They are busy most of the time. They don't even have time to sign my books*

Table 5 Identity combinations and narrated identities

Personal identities				
		Robust-leaning	**Mixed**	**Fragile-leaning**
Social identities	**Robust-leaning**	1. L26, L50 (2 learners)	2	3
	Mixed	4	5. L1, L19, L24, L30, L44 (5 learners)	6
	Fragile-leaning	7	8. L2, L5, L12, L27, L29, L32, L37, L42 (8 learners)	9. L3, L4, L6, L7, L8, L9, L10, L11, L13, L14, L15, L16, L17, L18, L20, L21, L22, L24, L23, L28, L31, L33, L34, L35, L36, L38, L39, L40, L41, L43, L45, L46, L47, L48, L49 (35 learners)

Table 6 Selected learners' identity scores

Identity	Learner code	Confidence (20)	Persistence (20)	Beliefs (25)	Personal identity (65)	Social identity (15)
Robust-leaning personal and social identities	Rose (L26)	15	17	24	56	14
	Ben (L50)	18	18	23	59	15
Fragile-leaning personal and social identities	Kelly (L7)	5	8	9	22	5
	Neo (L18)	6	7	9	22	5
	Tom (L28)	5	5	6	16	4
	Cindy (L2)	9	7	8	24	4

The majority of learners in the study were in cell 9, with some in cells 5 and 8 and two in cell 1. Each learner's personal and social identity score combination fits into one of the nine cells allowing for the differentiation of the learners' identities (see also in the Appendix). All the learners showed different mixes of robustness and fragility in their narrated identities suggesting that identities are narrated along a continuum. For example, L42 in cell 8 and L22 in cell 9, respectively, had personal identity scores of 27 and 26 and social identity scores of 6 each.

Six learners were selected for the semi-structured interview based on the questionnaire data. They were Rose (L26) and Ben (L50), narrating robust-leaning personal and social identities, and Kelly (L7), Neo (L18), Tom (L28) and Cindy (L2), narrating fragile-leaning personal and social identities (Table 6). Findings from the interview, which we discuss in the next section, confirmed these six learners' narrated identities from the questionnaire.

The interview

Robust-leaning personal and social identities The interviews showed that Ben and Rose narrated robust-leaning personal and social identities. The two learners described their mathematical experiences as follows:

> Ben: I'm very good at maths because there is nothing that I think I'm unable to do. I'm willing to do more work and I put much effort into maths that I never believed I could because it seems like a fun subject. My teacher has been teaching me since last year so he understands how I do things. Maths is useful except for doing classwork.
> Rose: Maths increases my confidence, when I get a sum right I want to do another one. I find it [maths] interesting because sometimes we just say maths is difficult and we find something right and we laugh in class. You can even do it [maths] just for fun. We [peers] help each other because we know each other from primary.

These two learners trusted their mathematical capabilities, and the trust was linked to past experiences with mathematics. Both learners suggested a personal desire to pursue mathematics. Positive descriptions of previous experiences with mathematics by the two learners suggested positive beliefs about mathematics. The learners' positive comments about mathematics, their teacher and peers indicated a sense of belonging to their classroom mathematics community and the narration of robust-leaning social identities. Ben's negative views of classwork suggested that learners narrating robust-leaning personal and social identities can show some fragility in their narrated identities.

Fragile-leaning personal and social identities The interviews showed that Kelly, Tom, Neo and Cindy narrated fragile-leaning personal and social identities. Kelly, Tom and Neo described their mathematical experiences as follows:

> Kelly: Sometimes I feel that I can't do maths but I work hard because I want to make my teacher proud. I feel like my teacher doesn't know me very well, he doesn't talk to me that much. My mind says maths is boring and difficult.
> Tom: I'm not so good at maths. I don't think I will pass maths this term, even if I try it's just hard. My family says that I can't do anything without maths so I have to put in the effort, but maths is a hard and horrible subject that can't be conquered. I don't talk with other learners that much, I believe I'm an introvert.
> Neo: I don't think I'm good at maths because I failed last year. Some [peers] say I won't make it, and it discourages me. I'm the only one with Grade 10 in my family and I feel that I have to carry on doing maths and make my parents proud.

These learners' comments showed that they had low confidence in mathematics. External factors, such as teachers and family members, influenced the learners to pursue mathematics, suggesting the narration of extrinsically motivated persistence. The learners' past experiences with mathematics were unpleasant for them, possibly leading to the development of negative beliefs about mathematics. The learners suggested that they did not feel a connection with other members of their classroom mathematics communities, indicating a sense of exclusion from their classroom mathematics communities. Tom constantly described mathematics as a subject that could not be "conquered", which meant that for him, mathematics could not be understood.

Cindy's narrative did not indicate low confidence but it indicated extrinsically motivated participation:

> Cindy: I participate because I look forward to succeeding and making a fortune. If maths is not needed for jobs I won't focus on it, I will focus on the job. My family says I can't go anywhere without maths so I try by all means to practice and love maths. I can say that maths is difficult, that's why I don't compromise on my time for maths. Other learners are selfish, they don't share what they know with me but I share everything with them.

Table 7 Robust-enhancing relationships related to teacher pedagogy

Learner	Teacher pedagogy
Rose	He encourages us to try harder and not leave a problem incomplete. Sometimes I get it right and that makes me feel that I can be an achiever. When we don't get it he helps us. I would say I'm improving and gaining confidence because of my teacher
Ben	He gives clues and explains things a little bit further and that makes me put in effort. He praises me and that strengthens me to do more challenging problems
Neo	If I don't know a sum, my teacher gives me examples so that I can understand. It helps me to try harder. He says maths is easy if you practice. It makes me think that I have to try harder because when I try I find new ways of solving problems. If he can do it then I can also do it

Cindy's participation was motivated by a desire to get a well-paying job, and her persistence was motivated by family pressure. Since extrinsically motivated persistence contributes to fragility in identity (McGee, 2015), we argue that extrinsically motivated participation contributes to fragility in identity. Believing that mathematics was difficult influenced Cindy to put effort into mathematics, indicating persistence motivated by negative beliefs about mathematics and fragility in her narrated identity. Cindy felt a sense of exclusion from her classroom mathematics community linked to not sharing ideas with her peers, indicating a narrated fragile-leaning social identity.

Relationships between learners and their significant others

We wanted to understand the role of teachers, peers and family members in supporting or constraining learners' narrated identities. In this section, we present learners' narratives about their relationships with their different groups of significant others. We do not have the significant others' narratives about the learners.

Learner-teacher relationships

Three learners, Rose, Ben and Neo, had robust-enhancing relationships with their teachers. Rose and Ben narrated robust-leaning personal and social identities while Neo narrated fragile-leaning personal and social identities. The other three learners, Kelly, Tom and Cindy, had fragile-enhancing relationships with their teachers, and they narrated fragile-leaning personal and social identities. Rose, Ben, Kelly and Cindy were taught by the same teacher, with Rose and Ben in the same class while Kelly and Cindy were in another class.[1] Neo and Tom were in the same class and were taught by a different teacher from the other four learners.

Robust-enhancing relationships with teachers Rose, Ben and Neo felt that their teachers taught mathematics for understanding. The learners appreciated their

[1] Ability grouping was not practiced at the participating school.

Table 8 Fragile-enhancing relationships related to teacher pedagogy

Learner	Teacher pedagogy
Kelly	He tells us the method of solving a problem and we work on the problem. I don't connect with him most of the time…he doesn't say much about me. He says maths is fun and challenging but that makes me scared to face challenging problems
Cindy	I feel like most learners don't understand the teacher. I don't share my methods with him because he says it's not the right method. He tells us that if you practice you will understand maths and if you love maths you will pass it. I try to love it but it's difficult
Tom	When the teacher teaches us it's boring, it's just equations and sums. He says maths is easy if you work on it every day. It makes me think I can do it but most of the time I just think I can't do it

teachers' pedagogies, and their comments indicated that teachers' pedagogies can support or constrain learners' narrated confidence and persistence (Table 7).

The teachers allowed learners to struggle with problems, which encouraged the learners to "do more challenging problems" and to feel successful. The teachers explained and the learners felt supported by their teachers' pedagogies. Because depth in teaching mathematics is related to positive beliefs about mathematics (Boaler et al., 2018), we argue that these learners' experiences of their teachers' pedagogies strengthened their beliefs about mathematics. Neo's comment indicated that teachers can act as learners' role models.

Fragile-enhancing relationships with teachers Kelly and Cindy felt that their teacher did not teach mathematics for understanding (Table 8). Tom felt that his teacher did not assist him to like mathematics.

Kelly and Cindy felt that their teacher insisted on particular methods and did not explain why and did not give learners a chance to ask questions or give their methods, which negatively affected their participation and sense of belonging to their classroom mathematics community. Tom suggested that his teacher did not change his teaching style, leading to boredom. Not teaching mathematics for deep understanding leads to learners not understanding and developing negative beliefs about mathematics (Boaler et al., 2018). The learners' comments suggested that telling learners that mathematics is fun or easy may not be sufficient to support learners' narrated identities.

Learner-peer relationships

Two learners, Rose and Ben, had robust-enhancing relationships with their peers, and they narrated robust-leaning personal and social identities. Four learners, Neo, Kelly, Tom and Cindy had fragile-enhancing relationships with their peers and they narrated fragile-leaning personal and social identities.

Table 9 Peer beliefs and responses

Learner	Peer beliefs	Peer responses
Rose	Other learners say that maths is hard. I find it interesting because sometimes we just say maths is difficult and we find something right and laugh in class	When we are given homework we stay after school and help each other, we don't laugh at each other. Some learners say I'm good at maths and it increases my confidence, I try to work harder and help other kids
Ben	They say maths is difficult. I see that they don't understand what they are saying and what maths is	We work together, sit in groups and share everything related to maths. They praise and encourage me to do more and that encourages me to help more learners

Robust-enhancing relationships with peers Rose and Ben had good relationships with their peers and resisted their peers' negative beliefs about mathematics (Table 9).

Rose and Ben's beliefs about mathematics were different from their peers' beliefs, indicating different experiences with mathematics from their peers. For Rose, struggling with a problem and eventually solving it was an interesting experience. Ben's comment suggests that not understanding what mathematics is can contribute to developing a negative view of mathematics.

Despite Rose and Ben's peers having negative beliefs about mathematics, they worked together on homework. Rose and Ben were encouraged and praised by their peers, which supported their narrated confidence, persistence and sense of belonging to the classroom and after-school mathematics communities.

Fragile-enhancing relationships with peers Peers' negative beliefs and attitudes constrained Neo, Kelly, Cindy and Tom's narrated identities (Table 10). These learners' comments suggested that they did not believe in their mathematical capabilities and they felt that their peers' attitudes discouraged them from participating in mathematical activities.

Peers' negative beliefs about mathematics supported these learners' negative beliefs. The learners felt excluded from their classroom mathematics communities, linked to being laughed at and made fun of when stuck on problems. The learners' narrated confidence in and beliefs about mathematics were constrained.

Learner-family relationships

Two learners, Rose and Kelly, had robust-enhancing relationships with their families. Rose narrated robust-leaning personal and social identities while Kelly narrated fragile-leaning personal and social identities. Four learners, Neo, Tom, Cindy and

Table 10 Peer beliefs and responses

Learner	Peer beliefs	Peer responses
Neo	They say maths is boring, I sometimes feel like yes it's boring	They laugh at me [when stuck] and it makes me think that maths is difficult. Some say I won't make it
Kelly	They say it's boring and difficult. It makes me have a mind that says maths is difficult and I can't solve all the problems all the time	I participate but not all the time because they sometimes embarrass me
Cindy	They say many negative things about maths. It affects me to listen to them. I begin to have that same belief about maths	They don't assist me [when stuck]. All they do is say hahaha! It's wrong here and stuff. It makes me think that I'm not good enough to learn maths
Tom	They say it's difficult. It affects me negatively because if the entire class doesn't think that they can do maths then I too feel as if I can't because no one is stepping up and trying to conquer maths	[If stuck], very few help me… It makes me perceive maths as a hard subject that can't be conquered

Table 11 Supportive families

Learner	Family support
Rose	Sometimes when I get something wrong I cry but my mom tells me that I'm good at maths. She encourages me not to stop but to keep trying. Sometimes I teach her some sums and that makes me feel that I'm growing up in maths and should not quit
Kelly	My family says they are proud of me and will still love me whether I pass or fail. When I get stuck on a problem my brother helps me

Ben had fragile-enhancing relationships with their families. Neo, Tom and Cindy narrated fragile-leaning personal and social identities while Ben narrated robust-leaning personal and social identities.

Robust-enhancing relationships with family members Rose and Kelly felt that their families supported them, which was robust-enhancing (Table 11). There were no indications of family pressure from Rose or Kelly's responses.

Rose and Kelly's comments indicated that they were encouraged and helped when struggling by their family members. Rose's narrated persistence and confidence were supported and related to a robust-enhancing learner-family relationship.

Fragile-enhancing relationships with family members Family members pressured Neo, Tom and Cindy to pursue mathematics but offered little support (Table 12). The sources of pressure were family members' past experiences with mathematics and the assumption that success in mathematics was a gateway to a better socio-economic status. Family members' past experiences with mathematics and their busy schedules limited the amount of support given to the learners. Ben's family pressured him not to pursue mathematics.

Neo, Tom and Cindy's comments indicated that coming from a family that historically did not study mathematics in the higher grades and the assumption that mathematics is a gateway to a better socio-economic status by family members can contribute to a narrative of extrinsically motivated persistence in mathematics. The learners felt that they were not being supported by their families, which was fragile-enhancing.

Ben felt that he was not being supported in dealing with the challenges that come with studying mathematics, which possibly constrained his narrated identity. Ben's family did not support him and they pressured him not to pursue mathematics. Ben resisted this pressure, illustrating robustness in his narrated identity which was supported by his teacher and peers, and indicated the collective role played by

Table 12 Non-supportive families

Learner	Family pressure	Family support
Neo	I'm the only one with Grade 10 in my family, I feel I have to carry on doing maths and make my parents proud	They also find maths difficult because they didn't do Grade 10, so they can't help me
Tom	My family says that I can't do anything without maths so I have to put in the effort. I feel it's compulsory to do maths to succeed in life	I live with my granny. She doesn't know maths so she can't help me. All she says is that maths is difficult but I must do it to succeed in life. It makes me feel as if I can't do maths but it's something that I need to do to achieve my goals
Cindy	My family says I can't go anywhere and I can't make a living without maths. I feel it's a must to love maths and to learn it	They are busy most of the time. They don't even have time to sign my books
Ben	Aah! [Hesitantly]. Honestly, my family doesn't say anything [about him as a maths learner] because they are not used to encouraging me	Last year they said I must do mathematical literacy[a] because maths was difficult for me. They don't give me the support that I expect

[a]Mathematical literacy is a subject that learners who find mathematics challenging can opt to take in South Africa (Spaull, 2013)

Table 13 Learners' relationships with significant others and their narrated identities

Learner	Rose	Ben	Neo	Kelly	Cindy	Tom
Teacher	RE	RE	RE	FE	FE	FE
Peers	RE	RE	FE	FE	FE	FE
Family	RE	FE	FE	RE	FE	FE
Identity	RL	RL	FL	FL	FL	FL

RE robust-enhancing relationship, *FE* fragile-enhancing relationship, *RL* robust-leaning, *FL* fragile-leaning

significant others in supporting or constraining learners' narrated identities. Ben had a fragile-enhancing relationship with his family but narrated robust-leaning personal and social identities, suggesting that having a fragile-enhancing relationship with only one group of significant others does not necessarily support the narration of fragile-leaning personal and social identities.

Summary

Table 13 shows that Rose had robust-enhancing relationships with all three groups of significant others while Ben had robust-enhancing relationships with two groups of significant others, his teacher and peers. Rose and Ben narrated robust-leaning personal and social identities. Neo and Kelly each had a robust-enhancing relationship with one group of significant others, Neo with his teacher and Kelly with her family. Tom and Cindy did not have robust-enhancing relationships with any of the three groups of significant others. Neo, Kelly, Tom and Cindy narrated fragile-leaning personal and social identities.

Discussion and conclusion

In discussing our findings, we address our two research questions. The first question is as follows: What kinds of mathematical identities do high school learners narrate? We found that while all the learners' narratives indicated some robustness and fragility in identities, more than half of the learners narrated fragile-leaning personal and social identities. This study resonates with other studies that have found that many learners enact fragile identities (Aydeniz & Hodge, 2011; Bishop, 2012; Epstein et al., 2010; Hall et al., 2018). The learners' narrated identities arose from their experiences with mathematics and social interactions with their teachers, peers and family members in different communities of mathematics practice.

The second research question is as follows: What kinds of relationships, with different groups of significant others, do high school learners narrate and how do these relationships collectively afford learners' mathematical identities? Many researchers have investigated the relationships between teachers and learners, peers and learners and family members and learners separately, despite learners being in constant interaction with all three groups of significant others (e.g., Boaler et al., 2000; Gardee, 2019; McGee, 2015; McGee & Martin, 2011). We moved forward by trying to understand how these three groups of significant others afford learners' narrated mathematical identities collectively. The findings showed that the relationships between learners and their significant others, as narrated by the learners, were either robust- or fragile-enhancing. Robust-enhancing relationships with at least two groups of significant others supported the narration of robust-leaning personal and social identities while fragile-enhancing relationships with at least two groups of significant others supported the narration of fragile-leaning personal and social identities.

Two learners, Rose and Ben, reported that they had robust-enhancing relationships with their teacher and peers, and they narrated robust-leaning personal and social identities. Three learners, Kelly, Cindy and Tom, reported that they had fragile-enhancing relationships with their teacher and peers, and they narrated fragile-leaning personal and social identities. This finding supports studies that have shown that teachers can work with peers to support more robust identities (Heyd-Metzuyanim & Sfard, 2012; Turner et al., 2013).

Peers' and family members' relationships with learners were usually fragile-enhancing for the learners. This finding, coupled with the finding that more than half of the learners narrated fragile-leaning personal and social identities, shows the importance of peers and family members in supporting learners' narrated identities. Some peers supported each other by working together on homework and praising each other. Ben was not supported and pressured to pursue mathematics by his family, and he developed a fragile-enhancing relationship with the family.

Ben's case suggests that a learner can resist a fragile-enhancing relationship with one group of significant others because of the support that they get from the other two groups of significant others. Neo and Kelly's cases suggest that having a robust-enhancing relationship with only one group of significant others may not be enough to overcome the other fragile-enhancing relationships.

This study examines learners' narrated identities by analysing learners' narratives about their relationships with their teachers, peers and family members. Confidence, persistence, beliefs and a sense of belonging to mathematics communities are part of learners' narrated identities and are supported or constrained through the kinds of relationships that learners develop with their teachers, peers and family members. Teachers' pedagogies and peers' and family members' experiences with mathematics are key to the kinds of relationships developed between learners and these three groups of significant others and therefore key to the kinds of identities that learners narrate.

We recognise some limitations in our study. Our findings depend on our choice and definitions of the indicators of identities and are based on a limited number of learners at one school and from one grade. The fifty learners who volunteered to participate in the study may have been more mathematically confident than those who did not volunteer. Future research may investigate other indicators or work with a larger sample and several grades, providing a broader range of experiences and allowing for more general results. Separating questionnaire scores indicating robustness and fragility in identity led to learners with score differences of only one or two points located in different identity cells. However, this approach, of quantifying a qualitative construct and creating groups based on this quantification, did help to gain an overview of learners' identities based on their narrated personal and social identities.

We relied on the learners' narrations of their interactions with their significant others, and we did not observe the actual relationships. We did not interview significant others about the learners, which could have added depth to our findings. Interviewing the significant others about their personal and social mathematical identities might have helped us to understand why they offered learners the social identities they offered.

Our study is the first to try to understand the relationship between learners' identities and learners' relationships with their teachers, peers and family members as a collective. The study shows that a deeper understanding of identity requires analysing learners' collective relationships with the three groups of significant others, indicating the importance of all three groups of significant others to learners' identities. We have broadened McGee's (2015) definition of identity, which considers two identity indicators and two identity groups, by considering four identity indicators and nine identity groups. The collective role of significant others in identity and the broader definition of identity can be used by educators and researchers to better understand the development of learners' mathematical identities.

Appendix

Learners' questionnaire scores and narrated identities

Cell number	Identity	Learner code	Confidence (20)	Persistence (20)	Beliefs (25)	Personal identity (65)	Social identity (15)
1	Robust-leaning personal and social identities	L26	15	17	24	56	14
		L50	18	18	23	59	15
5	Mixed personal and social identities	L1	8	12	18	38	8
		L19	8	10	19	37	8
		L24	12	12	14	38	7
		L30	12	12	15	39	7
		L44	8	12	18	38	9
8	Mixed personal identities and fragile-leaning social identities	L3	13	6	9	28	4
		L5	7	12	9	28	6
		L12	12	10	7	29	5
		L27	9	7	11	27	5
		L29	10	9	8	27	5
		L32	8	7	12	27	5
		L37	9	8	10	27	5
		L42	9	8	10	27	6

Cell number	Identity	Learner code	Confidence (20)	Persistence (20)	Beliefs (25)	Personal identity (65)	Social identity (15)
9	Fragile-leaning personal and social identities	L2	9	7	8	24	4
		L4	6	6	9	21	5
		L6	8	7	7	22	6
		L7	5	8	9	22	5
		L8	6	7	9	22	4
		L9	8	8	7	23	6
		L10	8	6	7	21	6
		L11	7	7	6	20	6
		L13	6	7	5	18	3
		L14	8	6	9	26	3
		L15	6	8	7	21	6
		L16	6	7	5	18	5
		L17	5	9	8	22	5
		L18	6	7	9	22	5
		L20	8	7	6	21	5
		L21	4	9	7	20	6
		L22	9	8	9	26	6
		L24	10	4	11	25	6
		L23	8	11	6	25	6
		L28	5	5	6	16	4
		L31	8	9	9	26	6
		L33	8	9	9	26	6
		L34	8	5	11	24	3
		L35	11	8	7	26	6
		L36	11	8	7	26	5
		L38	9	8	9	26	6
		L39	8	6	11	25	4
		L40	8	6	8	22	4
		L41	9	9	8	26	3
		L43	7	8	10	25	6
		L45	7	7	7	21	5
		L46	9	8	9	26	6
		L47	9	8	8	25	6
		L48	8	6	7	21	3
		L49	8	9	9	26	5

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Declarations

Consent to participate All participants and their parents consented that the participants would participate in the study after having been assured of confidentiality and anonymity. Ethical clearance was granted by the University of the Witwatersrand Ethics Committee. Protocol Number: 2017ECE005D.

Conflict of interest The authors declare no competing interests.

References

Anderson, R. K., Boaler, J., & Dieckmann, J. A. (2018). Achieving elusive teacher change through challenging myths about learning: A blended approach. *Education Sciences*, *8*(98). https://doi.org/10.3390/educsci8030098

Andersson, A., Valero, P., & Meaney, T. (2015). 'I am [not always] a maths hater': Shifting students' identity narratives in context. *Educational Studies in Mathematics, 90*, 143–161. https://doi.org/10.1007/s10649-015-9617-z

Aydeniz, M., & Hodge, L. L. (2011). Identity: A complex structure for researching students' academic behaviour in science and mathematics. *Cultural Studies of Education, 6*, 509–523. https://doi.org/10.1007/s11422-011-9331-9

Bishop, J. P. (2012). 'She's always been the smart one. I've always been the dumb one': Identities in the mathematics classroom. *Journal for Research in Mathematics Education*, *43*(1), 34–74. http://www.jstor.org/stable/https://doi.org/10.5951/jresematheduc.43.1.0034

Boaler, J., Dieckmaan, J. A., Pérez Núñez, G., Sun, L. K., & Williams, C. (2018). Changing students' minds and achievement in mathematics: The impact of a free online student course. *Frontiers in Education, 3*(26). https://doi.org/10.3389/feduc.2018.00026

Boaler, J., William, D., & Zevenbergen, R. (2000). *The construction of identity in secondary mathematics*. Paper presented at the International Mathematics Education and Society Conference, Montechoro, 26–31 March.

Braathe, H. J., & Solomon, Y. (2015). Choosing mathematics: The narrative of the self as a site of agency. *Educational Studies in Mathematics, 89*, 151–166. https://doi.org/10.1007/s10649-014-9585-8

Cribbs, J., Hazari, Z., Sonnert, G., Sadler, P., & M. (2020). College students' mathematics-related career intentions and high school mathematics pedagogy through the lens of identity. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-020-00319-w

Cunningham, J. A. (2021). "We made math!": Black parents as a guide for supporting black children's mathematical identities. *Journal of Urban Mathematics Education*, *14*(1), 24–44. https://doi.org/10.21423/jume-v14i1a414

Darragh, L. (2013). Constructing confidence and identities of belonging in mathematics at the transition to secondary school. *Research in Mathematics Education, 15*(3), 215–229. https://doi.org/10.1080/14794802.2013.803775

Darragh, L. (2015). Recognising 'good at mathematics': Using a performative lens for identity. *Mathematics Education Research Journal, 27*, 83–102. https://doi.org/10.1007/s13394-014-0120-0

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics, 93*, 19–33. https://doi.org/10.1007/s10649-016-9696-5

Eleftherios, K., & Theodosios, Z. (2007). Students' beliefs and attitudes about studying and learning mathematics. In Woo, J. H., Lew, H. C., Park, K. S. & Seo, D. Y. (Eds.). *Proceedings of the 31st Conference of the International Group for the psychology of mathematics education, 3*, 97–104. Seoul, South Korea: PME.

Epstein, D., Mendick, H., & Moreau, M. (2010). Imagining the mathematician: Young people talking about popular representations of maths. *Discourse Studies in the Cultural Politics of Education, 31*(1), 45–60. https://doi.org/10.1080/01596300903465419

Fellus, O. O. (2019). Connecting the dots: Toward a networked framework to conceptualizing identity in mathematics education. *ZDM – The International Journal on Mathematics Education. The Special Issue on Identity in Mathematics Education, 51*, 445–455. https://doi.org/10.1007/s11858-019-01053-9

Forster, P. A. (2000). Katie thought she couldn't do it but now she knows she can. *Educational Studies in Mathematics, 43*(3), 225–242. https://doi.org/10.1023/A:1011903213430

Gardee, A. (2019). Social relationships between teachers and learners, learners' mathematical identities, and equity. *African Journal of Research in Mathematics, Science and Technology Education, 23*(2), 233–243. https://doi.org/10.1080/18117295.2019.1662641

Gardee, A. (2021). Relationships between teachers' practices and learners' mathematical identities. *International Journal of Mathematical Education in Science and Technology, 52*(3), 377–403. https://doi.org/10.1080/0020739X.2019.1688403

Gardee, A., & Brodie, K. (2022). Relationships between teachers' interactions with learner errors and learners' mathematical identities. *International Journal of Science and Mathematics Education, 20*, 193–214. https://doi.org/10.1007/s10763-020-10142-1

Gardee, A., & Brodie, K. (2023). A framework for analysing the relationships between peer interactions and learners' mathematical identities: Accounting for personal and social identities. *Educational Studies in Mathematics, 113*, 443–473. https://doi.org/10.1007/s10649-022-10201-0

Goos, M., & Bennison, A. (2019). A zone theory approach to analysing identity formation in mathematics education. *ZDM – The International Journal on Mathematics Education. The Special Issue on Identity in Mathematics Education, 51*, 405–418. https://doi.org/10.1007/s11858-018-1011-8

Graven, M., & Heyd-Metzuyanim, E. (2019). Mathematics identity research: The state of the art and future directions. *ZDM – The International Journal on Mathematics Education. The Special Issue on Identity in Mathematics Education, 51*, 361–377. https://doi.org/10.1007/s11858-019-01050-y

Grootenboer, P. (2013). The praxis of mathematics teaching: Developing mathematical identities. *Pedagogy, Culture & Society, 21*(2), 321–342. https://doi.org/10.1080/14681366.2012.759131

Grootenboer, P., & Jorgensen, R. (2009). Towards a theory of identity and agency in coming to learn mathematics. *Eurasia Journal of Mathematics, Science & Technology Education, 5*(3), 255–266. https://doi.org/10.12973/ejmste/75277

Gweshe, L. C., & Brodie, K. (2019). High school learners' mathematical identities. *African Journal of Research in Mathematics, Science and Technology Education, 23*(2), 254–262. https://doi.org/10.1080/18117295.2019.1662642

Hall, J., Towers, J., & Martin, L. C. (2018). Using I poems to illuminate the complexity of students' mathematical identities. *Educational Studies in Mathematics, 99*, 181–196. https://www.jstor.org/stable/45184656

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggle in the mathematics classroom: On learning mathematics as an interplay of mathematising and identifying. *International Journal of Educational Research, 51–52*, 128–145. https://doi.org/10.1016/j.ijer.2011.12.015

Langer-Osuna, J., Munson, J., Gargroetzi, E., Williams, I., & Chavez, R. (2020). "So what are we working on?": How student authority relations shift during collaborative mathematics activity. *Educational Studies in Mathematics, 104*, 333–349. https://doi.org/10.1007/s10649-020-09962-3

Lodico, M. G., Spaulding, D. T., & Voegtle, K. H. (2010). *Methods in educational research: From theory to practice.* (2nd ed.). John Wiley & Sons, Inc.

Marks, A., & O'Mahoney, J. (2014). Researching identity: A critical realist approach. In P. K. Edwards, J. O'Mahoney, & S. Vincent (Eds.), *Studying organizations using critical realism: A practical guide* (pp. 66–85). Oxford University Press.

McGee, E. O. (2015). Robust and fragile mathematical identities: A framework for exploring racialised experiences and high achievement among black college students. *Journal for Research in Mathematics Education, 46*(5), 599–625. https://doi.org/10.5951/jresematheduc.46.5.0599

McGee, E. O., & Martin, D. B. (2011). ""You would not believe what I have to go through to prove my intellectual value!"' Stereotype management among academically successful black mathematics and engineering students. *American Educational Research Journal, 48*(6), 1347–1389. https://doi.org/10.3102/0002831211423972

McMillan, J. H., & Schumacher, S. (2010). *Research in education: Evidence-based inquiry* (7th ed.). Pearson Education Inc.

Noren, E. (2011). Students' mathematical identity formation in a Swedish multilingual mathematics classroom. *Nordic Studies in Mathematics Education, 16*(1–2), 95–113.

Ntow, F. D., & Adler, J. (2019). Identity resources and mathematics teaching identity: An exploratory study. *ZDM – The International Journal on Mathematics Education. The Special Issue on Identity in Mathematics Education, 51*, 419–432. https://doi.org/10.1007/s11858-019-01025-z

Sabbah, S., & Heyd-Metzuyanim, E. (2019). *Agency and identity of female Arab students entering a technological university*. Paper presented at the 11th Congress of the European Society for Research in Mathematics Education, Utrecht, 06–10 February.

Sander, P., & Sanders, L. (2009). Measuring academic behavioural confidence: The ABC scale revisited. *Studies in Higher Education., 34*(1), 19–35. https://doi.org/10.1080/03075070802457058

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14–22. https://doi.org/10.3102/0013189X034004014

Spaull, N. (2013). *South Africa's education crisis: The quality of education in South Africa 1994–2011*. Johannesburg: Centre for development and enterprise (CDE).

Sutherland, O., Dienhart, A., & Turner, J. (2013). Responsive persistence part II. Practices of postmodern therapists. *Journal of Marital and Family Therapy*. https://doi.org/10.1111/j.1752-0606.2012.00334.x

Turner, E., Dominguez, H., Maldonado, L., & Empson, S. (2013). English learners' participation in mathematical discussion: Shifting positionings and dynamic identities. *Journal for Research in Mathematics Education, 44*(1), 199–234. Equity Special Issue. https://doi.org/10.5951/jresematheduc.44.1.0199

Wenger, E. (1998). *Communities of practice. Learning, meaning, and identity*. Cambridge University Press.

Woolfolk, A. (2010). *Educational psychology*. (11th ed.). Pearson Education International.

Zhang, S., & Wang, G. (2021). Different contributions of parental expectations and teacher's behaviors to students' mathematics-related beliefs. In Vollstedt, M. (Ed.). *Proceedings of the ICME-14 Topic Study Group 18. Students' identity, motivation, and attitude towards mathematics and its study*. Shanghai, China.

Chapter 4: Learners' productive dispositions: The case of an after-school mathematics club

4.1 Introduction

In this chapter, I aimed to understand mathematical identity as linked to productive disposition. I considered confidence and persistence in and beliefs about mathematics as indicators of productive disposition and explored an after-school mathematics club as an intervention strategy for positively influencing the learners' mathematical identities described in chapters 2 and 3, which are usually largely fragile. Wenger's (1998) theory of learning as a social practice that involves mutual engagement, meaning-making, a sense of belonging and identity construction guided the design and running of the club.

Chapter 4 was the beginning of finding answers to the third research question. I asked learners to compare how they learned mathematics in their regular mathematics classrooms and in the club. Comparisons included the kinds of problems that learners solved in the class and club and how they solved the problems, teacher-to-learner interactions and peer-to-peer interactions. Many of the learners liked that the club helped them to understand mathematics. The learners' descriptions of learning in the club, which they enjoyed, was in line with Wenger's (1998) four interconnected components of learning while learning in the classroom, which they did not enjoy, was teacher-dominated. The findings discussed in chapter 4 suggest that clubs can support learners' productive dispositions and therefore their mathematical identities.

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Learners' productive dispositions: The case of an after-school mathematics club

Abstract: Productive dispositions are an important yet neglected part of mathematical proficiency developed through learners' experiences with mathematics. Mathematics clubs are usually intended to develop learners' mathematical proficiency through problem-solving and sense-making activities in mathematics. In this paper, I explore learners' productive dispositions in an after-school mathematics club that encourages learning in the forms of mutual engagement, meaning-making, belonging and identity construction. The study employed a qualitative research design with two data sources. First, club participants responded to five questions about their expectations of and experiences in the club. Second, six learners volunteered to engage in a semi-structured interview so that I could understand their experiences in the club more deeply. Results show that many of the learners preferred challenging club activities solved through discussions and in a friendly club environment rather than being given notes and corrections in their regular mathematics classes. The learners felt that the club had helped them to understand mathematics and that learning mathematics in the club was fun compared to learning mathematics in their regular classes. These results go some way towards explaining why learners tend to struggle with and not like mathematics.

1 Introduction

It is well known that many learners struggle with and do not like mathematics, and researchers are trying to understand why this is the case. Strongly related

Key words: mathematics club; productive disposition.

to learners' struggles with and dislike of mathematics are their mathematical experiences, particularly in their regular mathematics classrooms. Some researchers have explored mathematics clubs as informal spaces for supporting regular class learning and for developing learners' interest in and engagement with mathematics (e.g., Graven, 2012; Lampen & Brodie, 2020). The clubs have been found to offer learners experiences with mathematics that are different from regular classroom experiences by encouraging learners to relate their everyday experiences and reasoning with their mathematical reasoning (Lampen & Brodie, 2020), rather than the drill and practice often encouraged in regular classrooms. Learners have said that clubs are fun and relaxing compared to regular mathematics classes (Amit & Naaman, 2014; Brodie, 2022; Graven, 2015; Papanastasiou & Bottiger, 2004).

Mathematical proficiency describes what is needed for learners to learn mathematics successfully (Kilpatrick et al., 2001). Mathematical proficiency involves five interconnected strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (Kilpatrick et al., 2001). The five strands are interconnected and are equally important for mathematical proficiency (Allsopp et al., 2017; Kilpatrick et al., 2001), but researchers have focused on trying to understand the first four strands such that little is known about the fifth strand - productive disposition (Gilbert, 2014; Graven, 2012). An emphasis on passing examinations that measure learners' procedural fluency and to some degree, conceptual understanding arguably compels teachers to focus on procedural fluency (Brodie, 2017). Conceptual understanding, strategic competence and adaptive reasoning are rarely taught while productive disposition is completely neglected (Brodie, 2017). So, researchers, examination bodies and teachers have not usually attended to learners' productive dispositions. Since the five strands are intertwined, it means that ignoring any one of the strands compromises the development of the other strands (Brodie, 2017). A growing number of researchers are now trying to close this gap with the hope of understanding why many learners are struggling with and do not like mathematics. Some researchers have tried to understand how productive dispositions may be developed in mathematics clubs (e.g., Brodie, 2022; Graven, 2012).

2 Productive Disposition

Productive disposition is the "tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (Kilpatrick et al., 2001, p. 131). Learners with developing productive dispositions believe that with perseverance they can make sense of mathematics (Kilpatrick et al., 2001). Learners' confidence in and beliefs about mathematics can contribute to productive dispositions, particularly if they feel comfortable exploring challenging problems and sharing ideas (Kilpatrick et al., 2001). So, productive disposition is about the affective aspects of learning mathematics, which is key for the development of the other strands of mathematical proficiency.

Learners' emotions contribute to their productive dispositions (Gilbert, 2014). Negative emotions such as anxiety, irritation and exhaustion influence learners to lose focus when learning, which limits the development of their productive dispositions (Gilbert, 2014). Since experiencing negative emotions may be difficult to prevent, learners need to develop ways of managing negative emotions. Frenzel et al. (2019) show that when learners are encouraged to talk about and process their negative emotions, their levels of engagement with mathematics are likely to increase. If learners experience positive emotions, for example, feeling comfortable and happy, then they are likely to develop productive dispositions.

Learners with developing productive dispositions focus on learning and understanding rather than on performance (Gilbert, 2014) or task completion (Jansen, 2012). Such learners believe that confidence is not fixed but can change and they have a feeling of autonomy (Jansen, 2012). The learners see mathematics as interesting and they attribute their success in mathematics to perseverance and understanding (Gilbert, 2014).

Drawing on the reviewed research, I chose the concept of productive disposition because when learners believe that effort leads to success, they are more likely to persevere with challenges, make efforts leading to sense-making, and see mathematics as worthwhile and interesting.

3 Mathematics clubs

After-school mathematics clubs, also called out-of-school programmes, extracurricular activities or pull-out programmes, are voluntary and informal supportive mathematics learning communities where learners engage in problem-solving activities. Club sessions are mainly characterised by pair and small group interactions facilitated by mentors who encourage all five strands of mathematical proficiency, particularly strategic competence and adaptive reasoning (Lampen & Brodie, 2020). Not having formal tests creates less pressure on learners to perform than in regular mathematics classes (Papanastasiou &

Bottiger, 2004; Schlosser & Balzano, 2014), creating the opportunity for developing learners' productive dispositions. Mathematics clubs convey messages to learners that support perseverance and productive struggle, making mistakes, doing challenging problems, making connections between ideas, justifying answers and methods, and assisting others and sharing ideas (Boaler et al., 2018; Brodie, 2022; Lampen & Brodie, 2020).

Some researchers argue that club tasks should be different from regular class tasks but linked to the formal curriculum (e.g., Lampen & Brodie, 2020). Tasks that ensure a feeling of success for all the learners while encouraging everyone to reach new limits are encouraged (Brodie, 2022; Lampen & Brodie, 2020). The tasks are interesting and challenging and they encourage active engagement and mathematical thinking (Brodie, 2022; Scholsser & Balzano, 2014). Learners can sometimes contribute to selecting the club tasks since this has the potential to develop the learners' sense of belonging to and ownership of the club (Stott & Graven, 2013).

The majority of studies that I found focused on after-school mathematics clubs for Grade 2 to 9 learners and most of these studies have reported that learners expressed positive views of learning in the clubs (e.g., Papanastasiou & Bottiger, 2004; Prescott & Pressick-Kilborn, 2015; Schlosser & Balzano, 2014; Sherman & Catapano, 2011). Grade 10, 11 and 12 learners' views of their experiences of learning in after-school mathematics clubs are under-researched. Darragh (2013) asks why learners who previously had positive relationships with mathematics choose to give up on mathematics post-secondary schooling. Brodie (2022), who followed club learners from the time they were in Grade 8 until they reached Grade 10, found that clubs have the potential to develop high school learners' enjoyment of mathematics.

Clubs encourage learners to share, explain and justify their ideas as they solve interesting and challenging problems, which is necessary for developing their productive dispositions. Exploring club learners' productive dispositions may therefore suggest ways of dealing with learners' struggles with and dislike of mathematics. This paper focuses on a mathematics club developed for Grade 10 learners in a school near Johannesburg. I investigate: Does a mathematics club offer learners conditions for developing their productive dispositions?

4 Conceptual framework

Wenger's (1998) theory of learning as social practice argues that learning occurs as we participate in communities of practice, and involves four interconnected components, which I discuss here in relation to learning mathematics:

- Mutual engagement: Involves learners interacting with others and in the process sharing mathematical ideas and negotiating meanings. The learners give and request help and they explain to and request explanations from others. Agreeing and disagreeing is a part of mutual engagement.
- Meaning-making: Making sense of mathematical ideas by connecting them into a structured network of ideas. Guided by past and present mathematical experiences gained through participating in communities of mathematics practice, learners continually renegotiate the meanings of old ideas while adding new ideas and they use these ideas to solve problems.
- Belonging: A feeling of being a part of a mathematics community. The three modes of belonging are engagement, imagination and alignment (Wenger, 1998). Engagement, which should be mutual, is described above. Imagination is about having images of what learning mathematics can be. These images could be positive or negative. Alignment is about lining up with the norms and expectations of a community of mathematics practice, making visible and broader contributions to the practice, for example, when a learner indicates to the class how a particular problem-solving strategy may be easier to use than another strategy.
- Identity construction: Changing how a person defines themself or is defined by others in relation to mathematics. Learners' mathematical identities are a negotiation of their past, present and anticipated experiences of participating or not participating in communities of mathematics practice, their talk and actions, and their competence in mathematics.

Wenger's four components guided the running of the club and the research. The club involved a group of learners who participated in challenging mathematics activities to develop engagement, meaning-making and mathematical identities. Since learners with developing productive dispositions focus on sense-making and need to feel comfortable enough to explore challenging problems and share ideas (Kilpatrick et al., 2001), Wenger's (1998) theory of learning as social practice was a suitable lens for understanding learners' productive dispositions as they participated in the club.

Drawing on Kilpatrick et al. (2001), I considered confidence and perseverance in and beliefs about mathematics as indicators of productive dispositions. Confidence is trust in personal mathematical capabilities, for example, being good at mathematics. Perseverance is not giving up on mathematics or mathematical problems despite facing challenges. Beliefs are views of mathematics, for example, that mathematics makes sense or that it is useful. Figure 1 shows the possible relationships between a mathematics club and learners' productive dispositions.

MATHEMATICS CLUB		PRODUCTIVE DISPOSITION
Mutual engagement Meaning-making Belonging Identity construction	$\Leftrightarrow$	Confidence Perseverance Beliefs

Figure 1. A Framework relating a mathematics club and productive disposition.

Learners are often unconfident in mathematics as they feel that they are not good enough, they believe that mathematics is boring and if given the chance, they disengage from mathematics (Aydeniz & Hodge, 2011; Bishop, 2012; Darragh, 2015). If a club offers learners opportunities to mutually engage, they can make sense of mathematics and possibly develop confidence in and positive beliefs about mathematics. The learners' sense of belonging to mathematics communities, which entails engaging with mathematics, imagining themselves as effective learners and doers of mathematics and aligning with the club's principles and expectations, may develop. Belonging and identity construction are closely linked to learning (Darragh, 2013; Wenger, 1998). Identity construction is linked to perseverance, which is linked to confidence and beliefs (Bishop, 2012). So, if learners' sense of belonging to a club develops then their mathematical identities will develop, and so will their productive dispositions.

In summary, as learners learn mathematics they also learn what it is like to be effective learners and doers of mathematics and may begin to see themselves as such. Learners' mathematical experiences and participation patterns in the club can contribute to their productive dispositions. A change in learners' productive dispositions points to a change in how they mutually engage, their sense-making and views of mathematics, their sense of belonging to mathematics communities and their mathematical identities.

5 Previous research on mathematics clubs and learners' productive dispositions

Research on mathematics clubs has gained insights into how learners act and interact when in a community of mathematics practice different from their regular mathematics classes. The characteristics of mathematics clubs and the definition of productive disposition suggest that clubs can offer conditions for developing learners' productive dispositions. In this section, I discuss themes that came out of previous research on mathematics clubs and learners' productive dispositions. Some studies focusing on mathematics clubs have shown that clubs develop learners' productive dispositions, while other studies have focused on learners' productive dispositions but not on clubs.

I found two studies focusing on mathematics clubs and learners' productive dispositions. Brodie (2022) found that club learners' productive dispositions improved and so did their school results, suggesting a possible relationship between after-school mathematics clubs, mathematics achievement and productive dispositions. Since school tests usually measure learners' procedural fluency, Brodie's (2022) finding supports Kilpatrick et al.'s (2001) argument that the strands of mathematical proficiency are interconnected. The learners were willing to learn from their mistakes rather than trying to avoid them (Brodie, 2022), indicating a possible link between beliefs about making mistakes and productive dispositions. The club encouraged discussion and talking through ideas, to deepen understanding, and learners attended the club from the time they were in Grade 8 until they reached Grade 10 (Brodie, 2022). Graven (2012) established a club for Grade 3 learners, intending to interrupt teacher-dominated mathematics learning in regular mathematics classes. Learner-led interactions in which learners allowed each other to contribute ideas and explanations, and active engagement, were encouraged in the club. Graven (2012) found that when learners believe that they should only do what is required and do not feel free to explore and draw on their sense-making they are likely to develop restricted productive dispositions.

Other studies showed that clubs can develop indicators of productive dispositions, such as confidence, perseverance and beliefs. Reporting about the same club as in Graven (2012), Graven (2015) found that the club developed learners' confidence in mathematics. Learners in Sherman and Catapano's (2011) club were in Grades 2 to 5 and they were more confident to take mathematics tests than non-club learners. Papanastasiou and Bottiger's (2004) study which involved Grade 5 to 8 learners found that learners who participated in club games and contests experienced mathematical success and despite doing well, the learners continued to want to do even better, indicating developing perseverance in mathematics. In four studies attended by learners aged between 8 and 17 years, learners said that learning mathematics in clubs was fun, particularly the club activities and informal environment, compared to learning mathematics in their regular classes (Amit & Naaman, 2014; Brodie, 2022; Graven, 2015; Papanastasiou & Bottiger, 2004), indicating that clubs offer learners opportunities to strengthen positive beliefs about mathematics.

Some studies not focusing on clubs but on learners working in small groups have explored conditions for developing learners' productive dispositions. Gilbert (2014), focusing on Grade 6-8 learners, found that learners who can: manage their emotions, are encouraged to reason and critique each other's reasoning, and believe that understanding mathematics is more important than performance are likely to develop confidence in their mathematical competence and positive beliefs about mathematics, for example, that mathematics is fun. Such learners are not worried about being seen as incompetent in mathematics by others (Gilbert, 2014), suggesting that they believe in the importance of making mistakes in learning mathematics. Jansen (2012) interviewed Grade 6 learners working in small groups and the learners' comments indicated that their productive dispositions were developed in an environment that encouraged independent thinking and taking ownership of learning. Encouraging conceptual understanding and multiple strategies for solving problems and offering strategic scaffolding that does not lower cognitive demand were found to contribute to developing learners' productive dispositions (Jansen, 2012).

In summary, mathematics clubs that offer learners opportunities to discuss their ideas and explore alternative methods for solving problems can support learners' productive dispositions. Researchers have largely focused on clubs attended by learners in Grades 2 to 9 and shown that clubs can improve learners' confidence, perseverance and beliefs about mathematics. Small group work that involves fun activities and encouraging mistakes, reasoning, independent thinking and multiple strategies for solving problems have been shown to support learners' productive dispositions.

6 Method

I wanted to understand learners' productive dispositions from learners' perspectives, so I adopted a qualitative research design within an interpretive paradigm. Several studies have explored learners' productive dispositions from the learners' perspectives, for example, Jansen (2012) interviewed learners and Graven (2012) asked learners to respond to written questions, in writing. My interest was to describe characteristics of learners' productive dispositions and conditions for developing learners' productive dispositions.

6.1 Sample, participant selection

The study is part of a larger research project focusing on a mathematics club at a high school whose catchment area is low socio-economic status households such that parents do not pay school fees and the government offers a nutrition programme where learners receive free lunch every school day. The school had 328 Grade 10 learners enrolled and fifty of them volunteered to take part in the study and joined the club. I chose this school because it was where I was teaching and it was convenient for me to carry out my study there. Similar to other schools, many of the learners at the school struggle and choose to disengage from mathematics the moment they are given the chance to do so. I chose to work with Grade 10 learners because they had just chosen to do mathematics instead of the alternative subject, mathematical literacy[1], but this did not mean that they had strong productive dispositions since learners often choose to pursue mathematics to please their teachers or families.

Ethical clearance was provided by the university's ethics committee. The provincial education department allowed me to carry out the study and the school principal allowed me to talk to the Grade 10 learners about the study. I told the learners that I intended to set up a mathematics club at the school, where they would learn mathematics differently from their regular classes, and was asking them to join. I would also conduct a study on their participation in the club and would ask them to respond to short written questions and an interview about their experiences in the club. The learners were assured of confidentiality and anonymity through learner consent letters, that all participation was voluntary and that they could withdraw at any time. All the learners who agreed to participate, and their parents, returned signed consent forms.

6.2 The club

The club was established at a township[2] school located near Johannesburg that enrols about 2500 learners from Grades 8 to 12 each year. The school is similar to other township schools in that it is poorly resourced and many learners struggle with and do not like mathematics. The club started with 50 Grade 10 learners, who gave consent to participate. For this reason, no learners joined the club after it started. The club took place after school to avoid disrupting the formal school activities.

I conducted a total of 18 club sessions, for one and a half hours each. The total learner attendance per session varied from 25 to 50 learners (see Table 1, below). Some reasons for non-attendance included initial beliefs that the club taught mathematical literacy[3] instead of mathematics, the weather, and violent protests in the community around the school, such as the demand for proper housing.

[1]In South Africa, mathematical literacy is a subject that Grade 10, 11 and 12 learners opt to take when they do not want to take mathematics.

[2]In South Africa, townships are occupied by people of low socio-economic status.

[3]Mathematical literacy draws heavily on real-life situations and is less abstract than mathematics.

Drawing on Lampen and Brodie (2020), I wanted the learners to experience mathematical ideas learned or to be learned in regular classrooms differently so the club activities supported the formal curriculum, while not following it. The activities encouraged using different strategies to support meaning-making and to develop learners' productive dispositions. Learners worked in groups of three or four on problem-solving activities and they chose who to work with. After solving a given question, groups created similar questions for each other or they came to the front and presented their ideas to the club. Explaining ideas did not necessarily have to be done using numbers, and critiquing others' ideas was encouraged. Persevering in challenging problems, sharing ideas and representing thinking in different ways, for example, using words, diagrams and tables, were encouraged in the club. When requested and after seeing indications of productive struggle, club mentors offered assistance but not solutions.

6.3 Instruments and data collection

Two data sources were drawn upon for this study: responses to written questions after club sessions, and a semi-structured interview with learners. Both instruments aimed to understand changes, if any, in the learners' productive dispositions as they participated in the club and the conditions for developing learners' productive dispositions from the learners' perspectives.

Written questions

The learners were asked to respond to five short questions in writing at the end of five club sessions, one question at the end of each session. The questions asked the learners about their expectations of and experiences in the club since these indicate learners' productive dispositions, which were the focus of the study. The questions were answered five minutes before the end of five of the club sessions so that the learners would not have to rely on recalling their club experiences when responding to the questions. The five sessions were selected to have an even spread across 18 sessions:

(1) Why do you come to the club?

(2) If you were a grade 10 maths teacher what would you do?

(3) What are the differences between the club and your maths class?

(4) Which do you like, club or class?

(5) Do you want the club to continue next year? Why?

Table 1 shows the session when each question was asked, the responses received and the total number of attendees. Responding to the questions was voluntary so not every learner who attended a session responded to the question.

	Question number				
	One	Two	Three[4]	Four	Five
Session number	3	6	12	15	18
Total responses received	22	14	6	15	26
Total attendance	29	32	47	25	27

Table 1. Session attendance and responses.

Semi-structured interview

Six learners representing the bigger sample, Tom, Ben, Neo, Rose, Cindy and Kelly (all names are pseudonyms) volunteered to respond to a semi-structured interview after participating in the club activities (For a detailed selection process, see Gweshe & Brodie, 2023). Drawing on McMillan and Schumacher (2010), as a qualitative researcher, I considered every learner's word important and contributing to deeply understanding productive disposition, so I expected the responses of the six learners to strengthen rather than nullify the findings of the study. The purpose of the interview was to understand the learners' productive dispositions (i.e., their confidence, perseverance and beliefs) and conditions for developing learners' productive dispositions more deeply.

The semi-structured interviews, which lasted for about 40 minutes and were audio-recorded, were conducted after school in the learners' classrooms. Some of the interview questions were similar because I wanted to determine whether there would be consistency in the learners' responses to the questions. Examples of semi-structured interview questions are:

(1) Can you compare your days before and after joining the club?

(2) Would you consider continuing to attend the maths club?

(3) What have you learned in the club?

(4) What did you enjoy the most about the club?

[4] There were few responses to question 3 compared to the number of learners who attended the session. During that session we spent more time on the activity than expected, due to exciting discussions among learners, and there was little time left for the learners to respond to the question. Most of the learners needed to leave to get home.

6.4 Data analysis

Written responses

The written responses were inductively coded. The procedure involved cycles of carefully reading and highlighting words and phrases considered meaningful in relation to learners' productive dispositions. Similar words and phrases were highlighted with the same colour, forming codes that were then grouped into themes. Some of the words and phrases were repeatedly said by many of the learners while others were not repeated but were considered significant. Some codes were inferred from what the learners had written, for example, confidence. Other codes were exact words written by the learners, for example, understand. Drawing on the definition of productive disposition (Gilbert, 2014; Jansen, 2012; Kilpatrick et al., 2001), I considered these codes significant for understanding productive disposition but acknowledge that learners' understanding of the words may differ from mine. The learners' responses were usually multidimensional such that more than one code could be assigned to a response. Table 2 shows the codes that came out of the data and were significant to the study, the meanings of the codes, examples of responses containing the codes and the associated themes.

Themes	Codes & meanings	Examples of responses
Learners' productive dispositions	Confidence: self-trust	I'm now able to solve, interpret and analyse problems.
	Perseverance: not giving up	It [maths] just needs practice and determination.
	Beliefs: personal views	The maths club has helped me know that maths isn't difficult.
Learners' views of learning in the club	Understand: know how to solve problems	It's [club] very useful and you get to understand, ask questions and get an explanation more and more.
	Solve challenging problems: find solutions to challenging problems	I'm now able to tackle problems without fear and doubt. Now I know that I can solve word problems.
	Share ideas: mutually engage	In the club, we work together with other children from other classes, so we learn how to communicate and socialise with others.
	Think: come up with ideas	The club helps educate us more on what we don't know... and ways of thinking in maths. In the club, I can make up my own method and it is correct.

	Additional learning time: more time to learn	Yes, the club has helped me to understand more than my teacher does because sometimes the time is so little that he can't explain everything.
	Fun: enjoyable	The club is more fun because we work in pairs or groups and work with each other.
	Marks: test scores	Yes, the maths club has helped me improve my maths marks.
	Belong: be a part of	I like that the club is friendly.
	Helps: assists	The club has helped me and other learners to communicate and understand maths. Some things I now understand because of the club.
Learners' views of learning in the class	Learning in the class: regular classroom learning style	In the class, there is less communication... only one method comes out.

Table 2. Themes, assigned codes and examples of responses.

Semi-structured interview

The interview data were transcribed and thematically analysed. Since data from the semi-structured interview were more substantive and could add depth to data from the written responses, I conducted deductive and inductive coding. Deductive coding involved assigning codes created from the analysis of the written responses to the interview data. For inductive coding, I created new codes as I carefully read the interview data. Both deductive and inductive coding were repeated multiple times, in the process, I adjusted the codes until I was satisfied that they fit the data. Table 3 shows the assigned codes and examples of learners' quotes exemplifying the codes.

Codes	Examples of quotes
Confidence	*I think I will pass maths because I'm attending the club and starting to see the importance of maths.*
Persistence	*In the club, some posters encourage us to carry on with maths. I have learned that maths is all about making connections and not giving up but trying to solve problems.*
Beliefs	*I did not like to do mistakes but the club has taught me that in maths you have to do mistakes, that's the important part, you don't have to fake perfection for you to do maths.*
Share ideas	*We work together...*
Helps	*It [club] helps me get a better understanding.*

Fun	*Maths can be done just for fun.*
Solve challenging problems	*In the club, we do challenging problems.*
Think	*In the club, we do challenging problems and it helps you develop a mindset of a problem solver. If something is difficult it pushes you to think more and be creative.*
Additional learning time	*In class, you only have one hour for maths and I don't think that's enough for every learner to grasp what he needs to understand maths so the after-school club will help that individual learner to understand what he couldn't understand in class.*
Useful	*The club has taught me that maths is about learning new things and it gives you the ability to create stuff. You can make bridges and skyscrapers using maths.*
Sensible	*The club has taught me that maths is about solving and connecting things. [When stuck] I don't just stay and say I got stuck. I look around at how I can connect that thing with something else.*

Table 3. Assigned codes and examples of quotes.

Two new codes coming out of the interview data were: useful, which I defined as the belief that mathematics is beneficial; and sensible, which I defined as the belief that mathematics is about connecting and using ideas to solve problems. For transparency and to enhance the confirmability of the analytical procedures used, extracts of the interview text have been included.

7 Findings

Many of the learners' responses, both written and spoken, centered on solving challenging problems, sharing ideas and understanding mathematics. Some of the learners felt that the club offered them additional time to learn mathematics and that learning mathematics in the club was fun. Many of the learners showed developing confidence and perseverance in and positive beliefs about mathematics. The learners felt that learning in the club was different from learning in the class, with many of them preferring to learn in the club. The learners' responses suggested that they took on a passive role in their classes and an active role in the club.

I now discuss the findings in detail according to the three themes: learners' productive dispositions, learners' views of learning in the club, and learners' views of learning in the class. For each theme, I begin with a discussion of the findings from the written responses. I document the learners'[5] expectations

[5] Learners who responded to the written responses were not given pseudonyms and were described as learners, while learners who responded to the interview were given pseudonyms.

of the club, captured in their responses to questions 1 and 2, and then their experiences in the club, captured in their responses to questions 3, 4 and 5. I then add depth by discussing the findings from the semi-structured interview.

7.1 Learners' productive dispositions

Table 4 shows the codes assigned to the written responses in relation to learners' productive dispositions, the number of each question where the codes were evident and the number of learners whose responses were assigned the codes. Not every learner who attended a session responded to the question and some codes could not be created from some responses.

Codes	Question					Total
	1	2	3	4	5	
Confidence, perseverance, beliefs	0	10	0	1	14	25

Table 4. Codes and number of responses.

Ten responses to question 2 indicated that the learners hoped that the club would develop their perseverance in or beliefs about mathematics, reflected by talking about, for example, hoping to be encouraged to "never give up" and hoping "to realise how fun maths is". There is evidence that the club offered conditions for developing learners' confidence and perseverance in or beliefs about mathematics, reflected by the responses to questions 4 and 5. Some learners talked about being "able", one learner talked about no longer wanting to "leave maths" and another said that the club had helped them "know that maths isn't that difficult".

Similar to the written responses, the learners' responses in the semi-structured interview indicated that the club had created opportunities for developing learners' productive dispositions. Tom and Ben had this to say in relation to their confidence in mathematics:

> Tom: I think I will pass maths because I'm attending the club and starting to see the importance of maths. I'm not sure about being good at maths because I haven't given it 100% focus but I will now continue doing maths for the rest of my life because you always learn something new in maths.
>
> Ben: When I'm in the club, in one problem I get many ideas and getting all these ideas makes me feel like I'm the man.

These learners' comments indicated that the club had offered them conditions for developing their confidence in mathematics, a component of productive disposition. Tom's confidence in mathematics changed related to seeing mathematics as worthwhile, suggested by talk of seeing "the importance of maths" and "always" learning "something new". Tom's response indicated a relationship between confidence in mathematics and beliefs about mathematics. Ben felt confident in the club, related to sharing ideas and experiencing multiple strategies for solving problems.

The club encouraged perseverance in mathematics and there were indications from some of the learners that they had experienced the benefits of persevering in mathematics. Neo and Rose said the following:

> Neo: In the club, some posters encourage us to carry on with maths. I have learned that maths is all about making connections and not giving up but trying to solve problems.
>
> Rose: In the club, we do challenging problems and it helps because it trains your mind to think further. Sometimes I struggle a lot with some questions but then I try very hard to understand and try working with others so I can get some opinions from them and it helps.

Neo's perseverance in mathematics was related to experiencing messages that encouraged perseverance and productive struggle, indicated by his talk of "posters[6]". Rose spoke about perseverance in mathematics and thinking in solving challenging problems and "working with others". Rose felt helped in the club, after trying and then seeking and getting assistance from others.

Cindy and Kelly's comments suggested that the club had offered them conditions suitable for changing their beliefs about mathematics:

> Cindy: I did not like to do mistakes but the club has taught me that in maths you have to do mistakes, that's the important part, you don't have to fake perfection for you to do maths. I can say it's [math is] not difficult, it just needs you to work hard. Because of the club, now I see that maths is a very interesting subject and it's something that we apply every day in our lives.
>
> Kelly: Before joining the club I didn't know that maths can be done in so many different ways but after joining the club I saw that maths has a lot of things and you can solve problems differently. The club has taught me that everyone can learn maths if they try. I no longer think that people are born good at maths anymore because some people not good at maths right now knew it better in primary school and some get to high school and start doing well.

Cindy reported that her belief about making mistakes in mathematics changed, possibly related to the messages conveyed in the club and she believed that challenging problems could be solved through perseverance. Cindy

[6]The club displayed posters with messages that encouraged learners to persevere with challenging problems, share ideas and not be afraid of making mistakes. Other posters were of challenging problems not usually seen in regular classrooms.

felt that mathematics was "interesting" and relevant in life, related to participating in the club activities. Kelly's belief that mathematical problems could be solved by only one strategy changed because she experienced multiple strategies for solving problems in the club. Kelly justified why she no longer believed that people are born good at mathematics, indicating a change in her beliefs about mathematics.

Ben, Tom and Kelly indicated that the club had assisted them in seeing mathematics as useful, indicating a positive change in their beliefs about mathematics:

> Ben: I used to do maths in the class but I could not see the maths outside, because of attending the club I'm starting to see maths outside and I'm using the maths that is in class on the outside.
>
> Tom: The club has taught me that maths is about learning new things and it gives you the ability to create stuff. You can make bridges and skyscrapers using maths.
>
> Kelly: The club has taught me that everything we do is about maths. When you do maths you think of more great things. Maths can be done just for fun, to know maths and help others with maths.

These three learners began to see mathematics as useful, linked to participating in the club activities. Ben started to see and use mathematics outside of his class. Tom began to see mathematics as creative and in objects. Kelly's response indicated that she believed that mathematics could be used "just for fun", and similar to Tom, for developing people's creativity.

Ben and Neo learned that mathematics is about making connections, in the club, indicating that the club offered conditions for seeing mathematics as sensible and therefore developing learners' beliefs about mathematics:

> Ben: The club has taught me that maths is about solving and connecting things. [When stuck] I don't just stay and say I got stuck. I look around at how I can connect that thing with something else. When I'm in the club, the same thing we did in class seems like the simplest thing that I've ever seen.
>
> Neo: I have learned that maths is all about making connections and not giving up but trying to solve problems.

These learners' statements indicated a relationship between seeing mathematics as sensible and perseverance in mathematics, and therefore a relationship between perseverance in and beliefs about mathematics. Both learners developed a better understanding of what mathematics is and is about.

7.2 Learners' views of learning in the club

Table 5 shows the codes assigned to the written responses in relation to learners' views of learning in the club, the number of each question where the codes were evident and the number of learners whose responses were assigned those codes. Many of the learners felt that the club had "helped" them by offering them the opportunity to "share ideas", "think" and "understand maths" in a "fun" way.

Codes	Question					Total
	1	2	3	4	5	
Understand	18	9	2	1	14	44
Helps	0	0	0	0	21	21
Solve challenging problems	2	4	2	0	3	11
Share ideas	2	0	4	4	9	19
Fun	2	1	3	1	5	12
Additional learning time	0	6	1	1	2	10
Think	1	1	0	0	3	5
Marks	1	0	0	0	3	4
Belong: happy, important, friendly	0	0	0	0	3	3

Table 5. Codes and number of response.

Eighteen responses to question 1 and nine responses to question 2 indicated that many of the learners joined the club with the hope of understanding mathematics. Fourteen responses to question 5 indicated that the club had offered learners conditions for understanding mathematics. Nineteen out of the twenty-one learners whose responses to question 5 contained the code "helps" indicated that they wanted the club to continue because they felt that the club had helped them. Fourteen of these learners felt that they had been "helped to understand maths" in the club, three said that they had been "helped" to "improve" their "maths marks", and two said they had been helped but did not offer further explanations. Two out of the twenty-one learners did not want the club to continue the following year. These learners felt that they were not learning mathematics in the club, but "maths lit[7]", possibly because they were not used to the club's way of learning mathematics. Other negative views of learning in the club, provided by the learners, are discussed in the next section.

Some learners hoped to solve "challenging problems" and share ideas and that the club would "make maths fun", which was somewhat realised. Some of the learners who felt that learning mathematics in the club was fun enjoyed

[7]Mathematical literacy is called maths lit by the learners.

"communicating" and "working in groups", and others enjoyed the "cakes[8]". Some learners hoped that the club would offer them additional time to learn mathematics, suggested by saying that if they were teachers they would offer "extra lessons" to their learners. The club did offer learners additional time to learn mathematics, reflected in the talk about, for example, being offered "time to discuss" and to "ask questions and get an explanation". Some learners hoped that the club would develop their "thinking", one learner felt that the club was "friendly" and two learners felt "important" and "happy", indicating a sense of belonging to the club.

Semi-structured interview responses supported and added depth to the findings from the written responses. Cindy, Tom, Neo and Rose's responses indicated that they shared ideas and got help in the club:

> Cindy: [In the club] You get many ideas from others and maybe you can understand the learners you communicate with. I think if we work together as club members we can pass with distinctions.
>
> Neo: In class, I'm not active but in the club, I am because other learners help me. I'm now confident to raise my hand in class even though it's not that much but I'm starting to be confident in maths.
>
> Tom: In class, I avoid being asked questions because I can't tell him [teacher] that I didn't understand a thing because others may laugh at me. In the club, I'm not scared to answer questions because we don't laugh at each other. I participate more because there are a lot of interactions with other learners. It [club] helps me get a better understanding because I give my opinions and they [peers] also give theirs.
>
> Rose: In the class it's boring but when I get to the maths club I meet other people that I don't learn with and we get to share ideas. When we don't understand something in class we hope that we gonna understand it more in the club.

These learners' statements indicated that they felt that they were helped and they "better understood" mathematics, related to sharing ideas encouraged in the club. Cindy believed that she could "pass with distinctions", indicating developing confidence in mathematics related to sharing and "getting many ideas from others" in the club. Neo showed developing confidence in mathematics related to getting help from peers. Tom's fears of answering questions vanished in the club, because he could share his ideas without fear of shame. Rose enjoyed sharing ideas with different learners in the club. Talk about communicating, participating more, being active and sharing ideas in the club by

[8]The learners would say they were hungry after attending club sessions so I gave them snacks at the end of each session.

these learners indicated engagement, one of Wenger's (1998) three modes of belonging.

Kelly, Rose and Neo talked about solving challenging problems in the club, and Neo believed that the way they learned mathematics in the club was fun:

> Kelly: In the club, we do challenging problems and it helps you develop a mindset of a problem solver. If something is difficult it pushes you to think more and be creative.
>
> Rose: In the club, we do challenging problems and it helps because it trains your mind to think further.
>
> Neo: In the club, we do hard problems. It helps because I get to get stuck and get to see from a different perspective that I should have done this thing like this. When I do hard problems it is hard for me, but I try, and when I get to a point where I know the sum and I do it on my own, I find it fun.

These learners' statements indicated that solving challenging problems encouraged productive struggle. Kelly and Rose's responses indicated that productive struggle was related to thinking creatively and therefore sense-making, while Neo's response indicated that productive struggle was related to seeing mathematics as fun.

Just like in the written responses, the idea of the club offering learners additional time to learn mathematics differently from in their regular classes came up in the interview responses. Kelly, Tom and Neo said the following:

> Kelly: When I'm in class I can't express myself but when I'm in the maths club I can express myself as to how I did certain things but in class, I don't get a chance because there is not enough time and the period ends. In the maths club, I'm afraid to go in front because I'm around people I don't know.
>
> Tom: In class, you only have one hour for maths and I don't think that's enough for every learner to grasp what he needs to understand maths so the after-school club will help that individual learner to understand what he couldn't understand in class.
>
> Neo: If in class we learn something but we don't understand, when we come to the club we can still do the same thing but in a different way.

These learners' comments indicated a possible relationship between additional learning time and sense-making. For Kelly, additional time to learn mathematics offered her the opportunity to justify her ideas, however, her participation in the club was limited by being around people that she was not used to. Kelly's response about being around people that she did not know suggested that some learners preferred to interact with different learners from their regular classes and others did not. For Tom, additional time to learn mathematics offered learners and possibly him the opportunity to "understand" concepts that he previously did not understand. Neo's comment suggested that additional learning time offered learners the opportunity to experience other ways of solving problems, different from those used in regular classes.

7.3 Learners' views of learning in the class

Table 6 shows the code assigned to the written responses, in relation to learners' views of learning in the class, the number of each question where the code was evident and the number of learners whose responses were assigned that code.

Codes	**Question**					**Total**
	1	2	3	4	5	
Learning in the class	0	0	6	15	2	23

Table 6. Themes, codes and number of responses.

Six responses to question 3 were of learners who expressed negative views of learning in the class and positive views of learning in the club. Ten out of the fifteen responses to question 4 were of learners who enjoyed learning in the club and not in the class, one response was of a learner who enjoyed learning in the class, and four responses were of learners who enjoyed learning in both the class and club. The two responses to question 5 indicated negative views of learning in the class.

One learner's written response, with a negative view of learning in the class, was that they were "not challenged" suggesting that they were not offered the opportunity to solve challenging problems. Tom enjoyed class problems over club problems because for him club problems were "too challenging".

Another learner with a negative view of learning in the class, in the written responses, said that they were "given a lot of remedial" and another said that they "just did corrections", suggesting that in the class, learners were considered passive recipients of the information. Rose expressed a view similar to this learner, in the interview, when she said: "In the class, we just write, calculate and do corrections".

In relation to mutual engagement and in the written responses, one learner with a negative view of learning in the class said that they were "quiet and still in class" and another said that there was "less communication", suggesting that learners were not allowed to mutually engage in the class. The learners usually experienced "only one method" of solving a problem, in the class, a view of learning in the class that came from the written responses and was echoed by Ben in the interview, who said that the class offered "limited formulas and methods".

Despite many of the learners' written responses suggesting negative views of learning in the class, one learner said that they enjoyed learning in the class because for them, the lessons were "short", indicating a positive view of learning in the class and a negative view of learning in the club. The four learners, from the written responses, who enjoyed learning both in the class and in the club felt that they were gaining from both learning spaces, as exemplified by one of the learners who said that they "got to understand more" by participating in both learning spaces, "and revise".

A different finding coming from the interview responses and not revealed in the written responses was that the class did not offer learners the opportunity to see the use of mathematics, as suggested by Ben, indicating that the class limited learners' development of their beliefs about mathematics and therefore their productive dispositions:

> Ben: I used to do maths in the class but I could not see the maths outside, because of attending the club I'm starting to see maths outside and I'm using the maths that is in class on the outside.

8 Discussion, limitations and conclusion

Productive dispositions are an important yet neglected part of mathematical proficiency. I explored learners' productive dispositions using Kilpatrick et al.'s (2001) constructs of confidence, perseverance and beliefs, and Wenger's (1998) constructs of mutual engagement, meaning-making, belonging and identity construction. In this section, I discuss the research question: Does a mathematics club offer learners conditions for developing their productive dispositions? I explain my findings in relation to other research.

According to the learners, the club offered them conditions for developing their productive dispositions (i.e., their confidence and perseverance in and beliefs about mathematics). This finding supports studies that have shown that clubs can improve learners' productive dispositions (Brodie, 2022; Graven, 2012). Many of the learners felt that the club had helped them to understand mathematics. Some of the learners began to see mathematics as sensible, useful and fun, related to solving challenging problems and perseverance encouraged in the club. Learners who said they took on passive roles in their regular classes said that they took on more active roles as they participated in the club activities.

A comparison of the learners' views of learning in the club and in the class suggested that developments in the learners' productive dispositions could be attributed to the learning conditions in the club, which were contrary to those in the class. Drawing from the learners' statements, learning in the club involved solving challenging problems, engaging in productive struggle, sharing and explaining ideas, thinking mathematically, helping each other, experiencing multiple strategies for solving problems and experiencing and not just being told what mathematics is and is about. The class did not embrace any of these ideas encouraged in the club, according to the learners' statements. This finding supports Brodie (2022) who found that learning in the club was a social activity while learning in the class was an individual activity. Learning in the class was characterised by one strategy for solving a problem and learners receiving information from their teachers. Some studies have shown that classes are teacher-dominated, which limits the development of learners' productive dispositions (Graven, 2012; Jansen, 2012).

The study had its weaknesses. I being the club mentor and interviewer could have influenced the learners' written and spoken responses. I tried to minimise my effect as a participant researcher by encouraging the learners to be themselves and be truthful in their responses, and assuring them that they were free to withdraw from the study at any time. Talk and speech suggesting negative views of learning in the club indicate some degree of truthfulness in the learners' responses. A similar study in which the club mentor is not the interviewer can be conducted.

Learners said that the club had offered them the opportunity to understand mathematics but I did not explore what the learners meant when they said that they had understood mathematics. My definition of understanding mathematics and the learners' definition may differ, so research on what learners consider as understanding mathematics is needed if we are to fully understand learners' productive dispositions. I worked with a small group of learners so my findings cannot be generalised but they can be extended to similar learners. My findings may not apply to a situation different from the one described in this paper, so there is a need to explore learners' productive dispositions in other situations. I did not see the learners' productive dispositions developing. A similar study that draws upon observations as a data source can be carried out.

In this study, I have shown that learners who feel helped and believe that they have understood mathematics reported on developments in their productive dispositions. Such learners prefer to actively participate in fun challenges over being passive recipients of the information. So, the development of learners' productive dispositions can take place in mathematics clubs that encourage mutual engagement and meaning-making. Finally, mathematics clubs can be an intervention strategy for developing learners' productive dispositions in poorly resourced communities, the main requirement for establishing the clubs is teachers' extra time.

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References

Allsopp, D., Lovin, L. H., & van Ingen, S. (2017). Supporting mathematical proficiency: Strategies for new special education teachers. *Teaching Exceptional Children, 49*(4), 273–283.

Amit, M., & Naaman, K. (2014). Kidumatica – the mathematics club for creativity and excellence among multicultural pupils: Practice and research. *Procedia – Social and Behavioral Sciences, 141*, 1403–1411.

Aydeniz, M., & Hodge, L. L. (2011). Identity: A complex structure for researching students' academic behaviour in science and mathematics. *Cultural Studies of Education, 6,* 509–523.

Bishop, J. P. (2012). 'She's always been the smart one. I've always been the dumb one': Identities in the mathematics classroom. *Journal for Research in Mathematics Education, 43*(1), 34–74.

Boaler, J., Dieckmann, J. A., Pérez-Núñez, G., Sun, K. L., & Williams, C. (2018). Changing students' minds and achievement in mathematics: The impact of a free online student course. *Frontiers in Education, 3*(26).

Brodie, K. (2017). Mathematics pedagogy in South Africa: A human and academic project. In Webb, P., & Roberts, N. (Eds.), *The pedagogy of mathematics in South Africa: Is there a unifying Logic?* (pp. 175–183). Real African Publishers.

Brodie, K. (2022). Learning mathematics in an after-school mathematics club. *African Journal of Research in Mathematics, Science and Technology Education, 26*(3), 237–247.

Darragh, L. (2013). Constructing confidence and identities of belonging in mathematics at the transition to secondary school. *Research in Mathematics Education*, *15*(3), 215–229.

Darragh, L. (2015). Recognising 'good at mathematics': Using a performative lens for identity. *Mathematics Education Research Journal*, *27*(1), 83–102.

Frenzel, J., Lampen, C. E., & Brodie, K. (2019). Learners' awareness of their emotions and their engagement with mathematics tasks in a mathematics club. *Journal of Education, 77*, 45–59.

Gilbert, M. C. (2014). Using productive disposition to differentiate between students' levels of precision when critiquing a peer's work. *School Science and Mathematics*, *114*(7), 339–348.

Graven, M. (2012). Accessing and assessing young learners' mathematical dispositions. *South African Journal of Childhood Education*, *2*(1), 49–62.

Graven, M. (2015). Strengthening maths learning dispositions through 'math clubs'. *South African Journal of Childhood Education, 5*(3), Art. #342, 7 pages.

Gweshe, L. C., & Brodie, K. (2023). Learners' mathematical identities: exploring relationships between high school learners and significant others. *Mathematics Education Research Journal*.

Jansen, A. (2012). Developing productive dispositions during small-group work in two sixth-grade mathematics classrooms: Teachers' facilitation efforts and students' self-reported benefits. *Middle Grades Research Journal*, *7*(1), 37–56.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics.* National Academy Press.

Lampen, C. E., & Brodie, K. (2020). Becoming mathematical: Designing a curriculum for a mathematics club. *Pythagoras, 41*(1), a572.

McMillan, J. H., & Schumacher, S. (2010). *Research in education: Evidence-based inquiry* (7th ed.). International edition. Pearson Education Inc.

Papanastasiou, E. C., & Bottiger, L. (2004). Maths clubs and their potential: Making mathematics fun and exciting. A case study of a maths club. *International Journal of Mathematical Education in Science and Technology*, *35*(2), 159–171.

Prescott, A., & Pressick-Kilborn, K. (2015). It's great to be doing maths! Engaging primary students in a lunchtime club. *Australian Primary Mathematics Classroom*, *20*(3), 34–39.

Schlosser, L. K., & Balzano, B. (2014). Playing to learn: How after-school clubs influence teachers' beliefs about instruction. Sage Open.

Sherman, J., & Catapano, S. (2011). After-school elementary school mathematics club: Enhancing achievement and encouraging future teachers. *Educational Research Quarterly, 31*(1), 3–16.

Stott, D., & Graven, M. (2013). The dialectical relationship between theory and practice in the design of an after-school mathematics club. *Pythagoras, 34*(1), Art. #174, 10 pages.

Wenger, E. (1998). *Communities of practice. Learning, meaning, and identity.* Cambridge University Press.

Produktywne postawy uczniów: przypadek pozalekcyjnego koła matematycznego

Streszczenie

Produktywne postawy są ważną, lecz często zaniedbywaną częścią biegłości w matematyce, rozwijaną poprzez doświadczenia uczniów z matematyką. Koła matematyczne zazwyczaj mają na celu rozwijanie biegłości matematycznej uczniów poprzez rozwiązywanie problemów oraz angażowaniu w aktywności, których celem jest zrozumienie. W niniejszym artykule badam produktywne postawy uczniów uczęszczających na koło matematyczne pozalekcyjne, którego celem było zachęcenie do nauki poprzez wzajemne zaangażowanie, tworzenie znaczenia, poczucie przynależności oraz budowanie tożsamości. Badanie oparte było na jakościowym projekcie badawczym z dwoma źródłami danych. Po pierwsze, uczestnicy koła odpowiadali na pięć pytań dotyczących ich oczekiwań i doświadczeń z zajęć. Po drugie, sześciu uczniów zgłosiło się do udziału w wywiadzie półstrukturalnym, dzięki czemu mogłem lepiej zrozumieć ich doświadczenia z zajęć w kole. Wyniki pokazują, że wielu uczniów preferowało wymagające zadania z kółka, rozwiązywane poprzez dyskusje w przyjaznej atmosferze klubowej, niż uzyskiwanie notatek i wskazówek podczas ich regularnych lekcji z matematyki. Uczniowie czuli, że zajęcia w kole pomogły im zrozumieć matematykę i że nauka matematyki w kole była przyjemniejsza w porównaniu z nauką matematyki na regularnych lekcjach. Wyniki te w pewnym stopniu wyjaśniają, dlaczego uczniowie napotykają trudności w nauce matematyki oraz mają negatywne nastawienie do matematyki.

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Chapter 5: Mathematics club learners' mathematical identities: Narratives of learners and their class teacher

5.1 Introduction

Chapter 5 consolidated the finding discussed in chapter 4, that a mathematics club can support learners' mathematical identities. A change in learners' mathematical identities as they participated in the club was shown in the learners' responses to a mathematical identity questionnaire answered before and after participating in the club, learners' pre-and post-club semi-structured interview responses and a class teacher's post-club semi-structured interview responses about the learners' mathematical identities. Through chapter 5, I showed that an after-school mathematics club that encourages mutual engagement, meaning-making and a sense of belonging to mathematics communities can support learners' mathematical identities.

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Mathematics club learners' mathematical identities: Narratives of learners and their class teacher

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Abstract

Research into mathematics clubs has shown that learners enjoy mathematics in the clubs more than in their regular classrooms, while research on mathematical identities has shown a relationship between learners' mathematical identities and classroom practices. Bringing these two ideas together, I investigated learners' mathematical identities in a mathematics club and their regular mathematics classrooms. I used a mixed-methods design to explore the mathematical identities of learners who participated in an after-school mathematics club. The learners responded to a mathematical identity questionnaire before they started in an after-school mathematics club and after the club ended. Some learners were purposively selected to respond to two semi-structured interviews, one before and one after participating in the club activities. A teacher who was teaching the interviewed learners responded to a semi-structured interview about their mathematical identities in class after they had participated in the club activities. Two paired samples t-test analyses of the questionnaire responses show a statistically significant difference between the learners' mathematical identity scores before and after participating in the club activities. Interview findings show that the learners' mathematical identities were supported through participating in the mathematics club, which encouraged mutual engagement, meaning-making, and a sense of belonging. The teacher observed changes in the learners' participation in the class, suggesting that participation in a mathematics club that supports strong mathematical identities can influence participation in the mathematics classroom.

Keywords: mathematical identities, mathematics club, confidence, persistence, beliefs, sense of belonging

Introduction

Many learners have negative mathematical experiences and do not enjoy mathematics, and this is related largely to how the subject is presented in class (Epstein et al., 2010; Graven, 2015). For How (2022), mathematics has been perceived persistently as difficult. The thought of mathematics, especially in relation to tests, induces stress, anxiety, and fear in some learners (Shone et al., 2023; Utha et al., 2021). Other learners have described mathematics as boring (Aydeniz & Hodge, 2011) and would choose to drop mathematics if given the chance

(Bishop, 2012). Utha et al. (2021) found that many learners feel that because they do not understand mathematics they rely on memorising large volumes of information and this becomes overwhelming. For them, mathematical content is irrelevant. Other learners feel that they are not good at mathematics (Darragh, 2015) and that mathematics is reserved for bright learners (Shone et al., 2023), or that they are marginalised from mathematics (Gardee, 2019). Several researchers have turned to the idea of mathematical identity to understand these worrying experiences of learners. Their mathematical identities are based on their relationship with mathematics and can be seen in how they describe themselves and are described by others in relation to mathematics (Cribbs et al., 2015; Hall et al., 2018).

Researchers have used different approaches to try to understand why some learners enjoy mathematics and others do not. Some researchers, like Gardee (2019), have investigated learners' mathematical identities and have shown a relationship between classroom practices and these identities. Turner et al. (2011) investigated the relationship between learners' mathematical identities and mathematics clubs and found that a club that encourages collective problem-solving and that values learners' contributions can support their mathematical identities. Lampen and Brodie (2020) suggested that a club curriculum responsive to learners' strengths and weaknesses can do this. This study and that of Turner et al. (2011) pointed to the idea that learners' mathematical identities can be supported in communities in which the learners feel a sense of belonging. Studies have shown a difference between learners' experiences of mathematics in mathematics clubs and in regular mathematics classrooms (Brodie, 2022; Jacob & Jacob, 2018; Jensen & Sjaastad, 2013).

To offer learners opportunities for positive mathematical experiences and to study their developing mathematical identities and relationships with mathematics, I established an after-school mathematics club. The research questions guiding this study were:

- Can an after-school mathematics club support learners' mathematical identities?
- Can participating in an after-school mathematics club support learners' mathematical identities in the classroom?

The study contributes to understanding learners' mathematical identities by shedding light on the relationship between after-school mathematics clubs and these identities. In addition, I explored the relationship between participating in an after-school mathematics club on the one hand, and in regular mathematics classes on the other.

Previous research on mathematics clubs and learners' relationships with mathematics

Mathematics clubs encourage active participation and knowledge acquisition to support learners' interest in mathematics (Neher-Asylbekov & Wagner, 2023); learners have expressed positive experiences of mathematics in these clubs. Learners have reported increases in their confidence in, and enjoyment of, mathematics, and these were related largely to their receiving opportunities for interaction and affirmation of their mathematical competence (Kennedy & Smolinsky, 2016). Jensen and Sjaastad (2013) showed that learners' confidence and persistence in, and beliefs about, mathematics improved when they participated in a mathematics club, and this was related to being helped to believe that with effort they could learn mathematics and with feeling comfortable in asking for help. Since club mentors were friendly, they encouraged learners to ask questions (Jensen & Sjaastad, 2013). Different ways of solving problems were encouraged in these clubs and the problems were solved collaboratively. Zhou et al. (2021) found that enjoying mathematics in a club was accompanied by an increase in confidence and a sense of belonging to the club. Learners in clubs were encouraged to be involved, by, for example, asking questions, the answers to which, along with explanations, were provided by club mentors (Zhou et al., 2021). In Brodie's (2022) club, the improvement of learners' productive dispositions was related to social interaction, collaboratively solving problems, and being helped to feel comfortable.

In some studies, class teachers observed positive shifts in learners' relationships with mathematics following their experience of such clubs. Some class teachers observed a positive shift in club learners' participation and confidence in mathematics in the class (Graven, 2015). Other class teachers said that mathematics clubs offered learners extra support and time to think and reflect on their tasks, thus increasing the chances for low achievers to succeed (Prescott & Pressick-Kilborn, 2015). Graven's (2015) and Prescott and Pressick-Kilborn's (2015) findings suggest that clubs can support learners' mathematical identities in the classroom.

Many learners enjoy mathematics in mathematics clubs but not in their regular mathematics classrooms (Brodie, 2022; Jacob & Jacob, 2018; Jensen & Sjaastad, 2013). For Jacob and Jacob (2018), learners' negative views of mathematics in their regular classrooms did not change after attending a club. These learners said that in their regular classrooms, they were often told that they were wrong but were not given any explanation and noted that there was no interaction. In the club, they had worked together with their club mentors until they solved problems so that no-one was found to be wrong, according to these learners. The club encouraged fun-filled and interactive activities aligned with the formal curriculum (Jacob & Jacob, 2018). Learners in Jensen and Sjaastad's (2013) club said that the club offered them new ways of seeing mathematics which increased their interest. According to the learners, club mentors explained in understandable ways, and the focus was on problem-solving, but in class, the focus was on calculations (Jensen & Sjaastad, 2013). Learners in Brodie's (2022) club felt that mathematics in class was an individual activity that had to be made easy by the teacher while in the club mathematics was an enjoyable, challenging, social activity to be shared.

Participating in clubs has been shown to improve learners' mathematics achievement (Brodie, 2022; Lynch et al., 2023; Schuepbach, 2015). Several studies have shown significantly better mathematics achievement outcomes among learners who participated in clubs than in control groups because the clubs allowed learners to catch up and enrich their learning (Lynch et al., 2023). Regularly attending a club in which interaction was encouraged improved learners' mathematics achievement compared to learners in a control group (Schuepbach, 2015).

Brodie (2022) showed that the mathematics results of learners who regularly attended a club increased from Grade 8 to Grade 10 while the mathematics results of non-club learners at the same school decreased. For Brodie (2022), additional time to practice doing mathematics in the club, and not having to worry about assessments may have influenced the club learners' results and attending the club prevented the decrease in the club learners' Grade 10 mathematics results that is common in South Africa.

In summary, clubs offered learners time for interaction, explanation, and for experiencing alternative strategies for solving problems. This cannot happen when class teachers are under pressure to finish the syllabus and prepare for examinations. The clubs increased learners' confidence and persistence in and beliefs about mathematics, suggesting that they can support learners' mathematical identities. Class teachers observed a positive shift in club learners' participation and confidence in mathematics in regular classrooms and noted that they enjoyed mathematics in clubs more than in their regular mathematics classrooms. Findings from the studies above suggest that clubs can support learners' mathematical identities, a suggestion that I hope to have validated in this study. Generally, studies are inconclusive about the relationship between clubs and learners' mathematical identities in class, so in this study I aimed to contribute to understanding whether clubs can support learners' mathematical identities in class.

Theorising mathematical identity

Identity is constructed in social contexts and can be participative or narrative (Darragh, 2016). The view of identity as participative holds that identity is constructed through participation and a sense of belonging to communities. Identity as narrative considers identity as constructed through stories and words, be they written or spoken (Darragh, 2016; Fellus, 2019). A person's participation and narratives are closely linked. The strength of a mathematical identity can (and often does) depend on how a person deals with persistent negative mathematical experiences. The ability to deal with these suggests a robust mathematical identity, while the inability to deal with persistent negative mathematical experiences suggests a fragile mathematical identity (McGee, 2015).

Identities are constituted by personal and social identities (Marks & O'Mahoney, 2014). Personal mathematical identities are based on personal ideas and understandings of oneself in relation to mathematics (Gardee, 2019). A personal mathematical identity takes on a stronger consideration of the self and a weaker consideration of the social. Social mathematical identities are related to how a person feels they are perceived by other mathematics community members and how others perceive them in relation to mathematics (Gardee, 2019). A social mathematical identity takes on a stronger consideration of the social.

Personal and social mathematical identities are mutually constitutive (Gardee, 2019) since how we act and describe ourselves to people in relation to mathematics can influence how people act toward us and describe us. Social mathematical identities are constitutive of personal mathematical identities since personal mathematical identities are shaped by social relationships, as seen when a person tries to be different from or similar to others (Hall et al., 2018). Even though personal and social mathematical identities are unified in reality, they are analytically distinct (Gardee, 2019). A separate analysis of personal and social mathematical identities can show how the two components of mathematical identity relate to building mathematical identity and thus offer a deeper understanding of mathematical identities. A personal or social mathematical identity can be: robust-leaning, which means it has more robustness than fragility; mixed, which means it has relatively equal mixes of robustness and fragility; or fragile-leaning, which means it has more fragility than robustness (Gweshe & Brodie, 2024). In this study, mathematical identity, both participative and narrative, is defined as constituted by personal and social mathematical identities, each of which could be robust-leaning, mixed, or fragile-leaning.

Sfard and Prusak (2005) viewed identity as narratives that are reifying, endorsable, and significant. Cribbs et al. (2015) argued that mathematical identities are influenced by multiple internal constructs, and they offer a theoretical framework for mathematical identity that consists of three constructs: competence/performance—self-belief about understanding and performing in mathematics; interest—the desire to learn mathematics; and recognition—a person's view of how others view them. Cribbs et al. (2015) combined competence and performance since they are strongly correlated. In this study, I drew on Cribbs and colleagues' (2015) view of mathematical identity because mathematical identity was considered as indicated by multiple constructs seen in narratives, and not as narratives. These scholars focused on three constructs related to people's personal characteristics: competence/performance; interest; and recognition.

For this study, I developed four indicators of mathematical identity. Confidence and persistence in mathematics and beliefs about this subject are indicators of learners' personal mathematical identities, while a sense of belonging to mathematics communities indicates learners' social mathematical identities. Confidence in mathematics, which encompasses Cribbs and colleagues' (2015) notion of competence/performance, is trust in personal mathematical competence and has been used by Darragh (2013) to indicate mathematical identity. Persistence in mathematics is about not giving up despite facing setbacks such as failure or experiencing difficulties and has been used by Bishop (2012) to indicate mathematical identity. Persistent learners reported less of a decline or an increase in interest as they solved more problems than non-persistent learners (Tulis & Fulmer, 2013). Beliefs about mathematics are personal views formed by past experiences. Beliefs about mathematics as linked to mathematical identity were included, since several researchers such as Bishop (2012) have explored these. A sense of belonging to a community, which encompasses the notion of recognition listed by Cribbs et al. (2015), is a feeling of being a member of a community of practice (Wenger, 1998). A person who feels recognised in relation to mathematics is likely to feel that they belong to mathematics communities while, of course, the opposite is true. A sense of belonging has been used by Darragh (2013) to indicate mathematical identity.

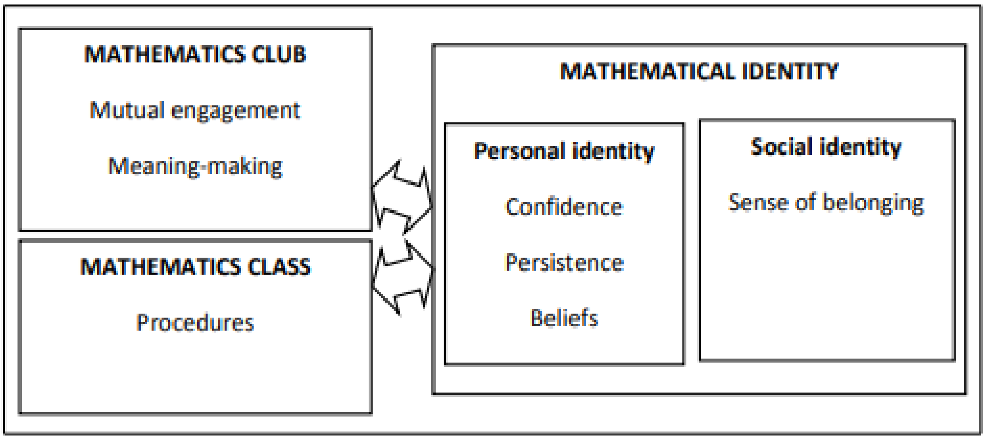
A framework for mathematics clubs, classrooms, and mathematical identities

Wenger (1998) argued that learning involves four interconnected components—mutual engagement, meaning-making, a sense of belonging, and identity construction. Mutual engagement can be seen to operate when learners share repertoires such as strategies for solving problems, definitions, theorems, and laws and symbols in pursuit of a community's joint enterprise, in this case, learning mathematics. Social relationships, for example, being friendly and caring, develop through mutual engagement as learners and their club mentors or teachers work towards developing a common understanding of what mathematics is, and is about, and what it means to learn mathematics. Mutual engagement offers learners opportunities to experience many different strategies aimed at solving problems and to connect related ideas into a structured network useful for solving problems. This leads to their experiencing the world of mathematics as meaningful. In other words, mutual engagement supports meaning-making since an idea developed in a certain context used to solve problems in other contexts points to this. A learner's mutual engagement and their meaning-making signifies their past participation experiences, indicates what they have learned and are ready to learn, and is an indicator of their sense of belonging to a mathematics community. Changes in a learner's confidence and in their beliefs about mathematics as well as persistence in wanting to belong to a mathematics community indicate *identity construction*; the learner changes from seeing mathematics as boring to enjoying engaging with this subject.

A learner's mathematical experiences in the different mathematics communities like, for example, a mathematics club and a regular mathematics classroom through which the learner moves, contribute to their mathematical identity. The learner's mathematical experience in one of the communities may be more influential on their mathematical identity than their experience in the other community. This depends on the learner's choices and decisions that are often made in consideration of past and present experiences of mathematics.

Wenger's (1998) theory of interconnected components of learning lends itself to the exploration of a mathematics club, mathematics classrooms, and learners' mathematical identities since it relates practices useful for pursuing mathematics, which include but are not limited to mutual engagement and meaning making, to identity. The components of learning assist in analysing and describing the practice of mathematics in a mathematics club and a regular mathematics classroom community. Figure 1 presents a framework that shows the potential relationships among mathematics clubs, regular mathematics classrooms, and learners' mathematical identities. Clubs usually encourage mutual engagement and meaning-making while regular mathematics classrooms encourage the quick and accurate performance of mathematical procedures.

Figure 1
The relationships between and among clubs, classrooms, and identities



The mathematics club

Site, participants, and sessions

The club under discussion was an informal learning space that engaged learners in problem-solving activities to encourage mathematical reasoning and positive shifts in mathematical identities (see Lampen & Brodie, 2020). The club was established at a school located in a township area in which people of low socio-economic status reside. I was the volunteer main club mentor while a volunteer teacher joined me and assisted as a co-mentor. He attended the club sessions frequently, observed my facilitation, and also facilitated some of the club sessions. The club took place after school to offer learners time and space to work collaboratively on challenging problem-solving activities. Eighteen club sessions were conducted over eight months. Learners attended an average of 15 sessions, each of which lasted for 90 minutes.

Principles

The club adopted key principles that drew on the work of Boaler et al. (2018) and Lampen and Brodie (2020).

- Productive struggle and making mistakes are valuable opportunities to learn.
- Mathematics is about making connections between ideas.
- Assisting others and sharing ideas provide valuable opportunities to learn.
- Trying to explain your thinking to others before asking for assistance is a valuable learning opportunity.
- Mathematics is interesting.

These principles were explicitly stated regularly and explained to learners in the club.

Curriculum and activities

The club was part of a larger project involving three clubs in South Africa and I gained the experience of running a club by participating in one of the first two clubs before establishing the third one.[1] The three clubs shared a curriculum responsive to learners' ideas, strengths, and weaknesses; for more detail, see Lampen and Brodie (2020). The curriculum was premised on developing learners' mathematical reasoning and supporting their mathematical identities rather than on repetition of mathematical procedures as is often the case in many regular mathematics classrooms and extra-lesson sessions. Mathematical reasoning was seen as a mode of thinking involving taking ideas apart and putting them together and using the ideas to solve problems (Lampen & Brodie, 2020).

Learners were expected to represent their reasoning in different ways, for example, graphically, diagrammatically, in tables, equations, or expressions, or with physical objects and they had to critique their own and others' reasoning (Lampen & Brodie, 2020). Opportunities to engage in problem-solving activities not normally seen in the classroom were offered in the club. The activities had to be solved using more than one strategy and through mutual engagement and meaning-making to align them with Wenger's (1998) components of learning.

Method

I adopted a mixed-methods design in which a qualitative analysis helped explain and contextualise the results from a quantitative analysis. Using this design allowed me to incorporate the strengths of two designs while minimising their weaknesses (see McMillan & Schumacher, 2010) and this enhanced the chances of having a comprehensive understanding of learners' mathematical identities. The quantitative design was a quasi-experiment because I did not randomly select participants and did not involve a control group. The qualitative design was a narrative inquiry because I wanted to understand learners' mathematical identities through their and their class teachers' narratives.

Sample

Of 328 Grade 10 mathematics learners at a selected school, 50 volunteered to participate in the study along with one class teacher. I chose this school because I was teaching there at the time of conducting the study, so it was convenient for sampling. I decided to involve learners in Grade 10 because few studies focus on mathematics clubs attended by learners at this level in South Africa; most studies involve Grade 2 to 9 learners (e.g., Graven, 2015; Stott et al., 2019).

The university's ethics committee provided ethical clearance for the study. The provincial education department and the school principal allowed me to conduct the study at the school. The parents consented to their children taking part in this study as did the participating

[1] The other two clubs are reported on in Brodie (2022), Frenzel et al. (2019), and Lampen and Brodie (2020).

teachers. All the participants were assured of confidentiality and anonymity before the study commenced.

Instruments and data collection

The learners responded to the same mathematical identity questionnaire before they joined the club and after its activities ended. The purpose of the questionnaire was to compare the learners' mathematical identities before and after participating in the club activities. It consisted of 16 items and used a Likert scale with responses ranging from strongly agree which was scored as 5 to strongly disagree which was scored as 1. Questionnaire items were adapted from previous studies that had investigated the indicators of learners' mathematical identities under study (e.g., Boaler, 2013; Darragh, 2013). Table 1 shows the four indicators of learners' mathematical identities about which the learners were asked, the corresponding number of questions for each indicator, and an example of a question for the indicator.

Table 1
Identity indicators and the number of questions asked

Identity indicator	No. of questions	Example of a question
Confidence	4	I am good at maths
Persistence	4	Even if I fail maths this term, I will try again next term
Beliefs	5	Practising can improve my ability in maths
A sense of belonging	3	In maths, I feel important

As can be seen, thirteen questions asked learners about their personal mathematical identities (i.e., confidence, persistence, and beliefs) and three questions asked about their social mathematical identity (i.e., a sense of belonging). A learner's personal mathematical identity score was calculated by adding the scores from the four confidence questions, four persistence questions, and five beliefs questions, while their social mathematical identity score was calculated by adding the scores from the three questions based on their sense of belonging. The total possible score for each question was five, so, the total possible personal mathematical identity score was 65 and the total possible social mathematical identity score was 15. Personal mathematical identity scores were put into three groups: robust-leaning (52–65), mixed (27–51), and fragile-leaning (13–26). Social mathematical identity scores were put into three groups: robust-leaning (12–15), mixed (7–11), and fragile-leaning (3–6).

The learners' personal and social mathematical identity scores before they started in the club were also used to purposively select a smaller group of learners for two semi-structured interviews, the first before the learners participated in the club activities and the second after this. Interviewing a small group of learners meant that a small quantity of data would be collected that could lead to a deeper understanding of mathematical identity, based on the learners' narratives of how this came about, that could offer insights into the relationship between mathematics clubs and mathematical identities.

Since the study had different personal and social mathematical identity groups, this resulted in learners narrating different combinations of these. Learners with different combinations of personal and social mathematical identities were asked to participate in the interviews. In this paper, I discuss the interview responses of four learners who agreed to be interviewed: Kelly and Cindy[2] who narrated fragile-leaning personal and social mathematical identities before participating in the club; and Ben and Rose, who narrated robust-leaning personal and social mathematical identities before participating in the club. These four learners were more articulate than the other learners who volunteered to be interviewed and their class teacher, Mr. Kent, who was not involved in the club activities, also agreed to be interviewed. I wanted to determine what he had observed in relation to the club learners' mathematical identities in the classroom after their participation in club activities.

I conducted both learner and teacher interviews and these were audio-recorded and transcribed.[3] Each interview lasted for about 40 minutes. Some of the learner interview questions were adapted from previous studies that had investigated learners' mathematical identities or constructs influencing mathematical identities (Bishop, 2012; Boaler, 2013; Darragh, 2015; Vanayan et al., 1997). Teacher interview questions and other learner interview questions were constructed with the help of researchers who have conducted studies on mathematics clubs, mathematical identities, and related constructs. Table 2 shows the three instruments and examples of questions in each of the instruments.

Table 2
Instruments and questions

Instrument	Examples of questions
Learner pre-interview	What do you think about maths? How do you see yourself as a maths learner?
Learner post-interview	What do you think about the after-school maths club? Can you compare your days before joining the club and after joining the club?
Teacher interview	Please tell me about what type of maths learner s/he was before and after joining the maths club. Do you see the learner pursuing maths in the future?

Questionnaire analysis

The null hypothesis for the quantitative analysis was formulated and tested at 0.05 level of significance, a level commonly used in education research to indicate that the observed differences are not attributed to chance.

- There is no statistically significant difference between the mean pre- and post-personal and social mathematical identity scores of Grade 10 learners who participated in the activities of an after-school mathematics club.

2 All names are pseudonyms.
3 The participants' responses have not been edited.

The questionnaire was on a Likert scale, so each learner's personal and social mathematical identity scores before and after participating in the club activities were calculated, as discussed above. Therefore, there were two sets of data from one group of learners, so two paired-sample t-tests were computed, one for personal mathematical identities and the other for social mathematical identities. Rejecting the null hypothesis meant that the differences between the club learners' mean personal and social mathematical identity scores before and after participating in the club activities were statistically significant. Not rejecting the null hypothesis meant that the differences were not statistically significant.

Interview analysis

The learner interview transcripts were deductively coded to draw on theory and inductively coded to allow new themes to emerge. Deductive coding involved carefully assigning predetermined codes drawn from the literature and considered relevant to the data. Inductive coding involved creating codes, while reading the data carefully many times. Some responses were multi-coded and cycles of checking that the assigned codes fit the data were conducted.

Three categories representing learners' descriptions of their experiences of learning mathematics in a club and their classrooms were constructed from the codes: Personal identity, Social identity, and Learning. The first two categories were constructed in line with Marks and O'Mahoney's (2014) components of identity, and they supported an analysis of the learners' mathematical identities in the club and classroom. The last category was constructed in line with Wenger's (1998) theory of learning as meaning-making and it supported an analysis of how learners experienced mathematics in the club and classroom. Table 3 shows the categories, codes, code descriptors, and examples of learner statements represented by the codes.

Table 3
Learner categories and codes

Category	Code	Descriptor (bold) and example of a learner's statement
Personal identity	Confidence	**The club supports learners' confidence.** When I'm in the club, in one problem I get many ideas and getting all these ideas makes me feel like I'm the man.
	Persistence	**The club encourages learners to persist in mathematics.** When stuck on a problem I no longer just stay and say I got stuck, I now look around at how I can connect that thing with something else.
	Beliefs	**The club changes learners' beliefs about mathematics.** It [is] very useful because it changes people's attitudes towards maths, what makes maths hard is wiped off.
Social identity	Belonging	**Participating in mathematics activities.** I try to work with others so I can get some opinions from them, and I also give them mine.
	Sharing ideas	**The club encourages discussions.** In the club, I enjoy working in groups because different people come up with different ideas.

Category	Code	Descriptor (bold) and example of a learner's statement
Learning	Multiple strategies	**The club encourages multiple ways of solving problems.** Someone comes up with another method that you don't know and it's new to you.
	Mistakes	**The club encourages making, noticing and correcting mistakes.** I enjoy noticing our mistakes in the club; that helps me not to repeat them.
	Enjoyment	**The learner enjoys learning mathematics in the club.** We do different stuff that is more interesting than textbook stuff.
	Identity messages	**The learner adopted ideas from identity messages.** I enjoy noticing our mistakes in the club.
	Supplement	**The club provides additional practice activities.** I'm helped with those things that maybe I didn't understand in class.

The teacher interview transcript was coded using the same approach as the learner interview transcripts. Some codes describing learner interview data did not fit the teacher interview data and this suggested that not everything said by the learners was noticed by the teacher, and that the teacher's perception of some of the learners' relationships with mathematics was different from the learners' perceptions. Similarly to the analysis of the learner interviews, the three categories of Personal identity, Social identity, and Learning were constructed. Table 4 shows the categories, codes, code descriptors, and examples of teacher statements represented by the codes.

Table 4
Teacher categories and codes

Category	Code	Descriptor (bold) and example of a teacher's statement
Personal identity	Confidence	**Learner shows supported confidence.** After the maths club, I noticed that she now wants to go to the board and do something.
	Persistence	**Learner shows supported persistence.** These days he is coming up with extra problems and he asks me if his answers are correct.
	Beliefs	**Learner shows supported beliefs.** She didn't want to be wrong. . . Now she can ask if she solved it correctly.
Social identity	Belonging	**Learner participates in mathematics activities.** She would make noise and wouldn't cooperate. After the maths club. . . her attitude has changed.
Learning	Assists others	**Learner helps others.** After the club, if the person who was writing on the board asked how they did something then she'd assist them.
	Alternative strategies	**Learner suggests alternative strategies for solving problems.** She can even say, "Sir, let's try this question this way."

Category	Code	Descriptor (bold) and example of a teacher's statement
	Mistakes	**Learner is not afraid of making mistakes.** Now she is one of the learners who jumps first to say, "I want to give you the corrections on the board."

Findings

The results from the questionnaire analysis showed that the club learners' personal and social mathematical identities were supported. Analysis of the learners' interview responses also suggested that their personal and social mathematical identities were supported through experiencing meaning-making in the club. The teacher's interview response suggested a positive shift in the learners' personal and social mathematical identities in the class.

Learners' mathematical identities

Questionnaire

The paired samples t-test for personal mathematical identity showed that there was a statistically significant difference between the club learners' personal mathematical identity scores, calculated as discussed above, before participating in the club activities (M=26.62, SD=8.18) and after participating in them (M=29.58, SD=9.54); $t(49)=4.108$, $p=0.0002$. The paired samples t-test for social mathematical identity showed that there was a statistically significant difference between the club learners' social mathematical identity scores, calculated as discussed above, before participating in the club activities (M=5.70, SD=2.22) and after doing so (M=8.86, SD=2.14); $t(49)=7.99$, $p<0.0001$. The null hypothesis on mathematical identity was thus rejected. The club learners' mean personal mathematical identity score increased by 2.96 points while their mean social mathematical identity score increased by 3.16 points. This result showed a positive shift in the learners' mathematical identity scores, suggesting that their mathematical identities had shifted toward more robust-leaning identities, and, therefore, had been supported.

Learner interviews

As discussed above, three categories: Personal identity, Social identity, and Learning were constructed. I provide excerpts illustrating how the learners constructed their Personal mathematical identities, indicated by changes in their personal characteristics: confidence; persistence; and beliefs. The learners' Social mathematical identities were indicated by the social characteristic of belonging. The third category included opportunities for meaning-making: sharing ideas; multiple strategies; productive struggle; identity messages; mistakes; and supplementation.

I start by discussing Kelly and Cindy, learners who narrated fragile-leaning personal and social mathematical identities before participating in the club activities and robust-leaning personal and social mathematical identities after having done so. I then discuss Ben and Rose, learners who narrated robust-leaning personal and social mathematical identities before and after participating in the club activities. For each learner, I draw comparisons between their comment before participating in the club activities and their comment after doing so.

For Kelly, mathematics was learned differently in the club, which she enjoyed, and this led to her personal and social mathematical identities shifting towards the more robust.

Table 5
Kelly's comments before and after participating in the club

	Learner Comment	
	Before	**After**
Kelly	I don't participate because I'm afraid that my answers will be wrong, and I get embarrassed. Some [peers] say I should go to maths lit [when stuck on a problem]. I feel I should leave maths.	In the club, I enjoy working in groups because different people come up with different ideas. I'm helped with those things that maybe I didn't understand in class.
	I will not be doing maths for long because it's difficult, I feel like I just can't do it.	The club is different. It's not what I'm used to doing. I didn't know that maths could be done in so many ways. I feel that maths is getting easier.

Kelly's beliefs about mathematics were supported by the club, shown by changing from believing that mathematics is difficult, before participating in the club, to viewing mathematics as getting easier, after doing so. The change in Kelly's beliefs about mathematics was related to learning through sharing ideas, getting help, and experiencing many strategies for solving problems in the club (i.e., practices that support meaning-making) and suggests a shift in her personal mathematical identity from fragile-leaning toward robust-leaning.

Kelly's comment before participating in the club suggested that she experienced fear and discouragement in her regular mathematics classroom while her comment after doing so suggested that she felt helped in the club, thus indicating a sense of belonging to the club and a sense of not belonging in her regular classroom. The support to Kelly's sense of belonging was related to a difference in the way learners interacted in the club and classroom, and it suggests a supported social mathematical identity by the club.

Cindy's comments offered insights into changes in her narrated personal and social mathematical identities, related to experiencing meaning-making in the club.

Table 6
Cindy's comments before and after participating in the club

	Learner Comment	
	Before	**After**
Cindy	Maths is difficult, that's why I don't compromise on my time for maths.	The club is messing with our minds because when the teacher corrects our mistakes, we realise it wasn't that difficult. Now I see certain things, it's funny that I didn't know these things, and I've become more capable of doing maths. In class, other learners are selfish, they don't share what they know with me but in the club, we share everything. Since I'm a club member, people consider me as very good at maths.
	I participate because I'm looking forward to succeeding and making a fortune.	

Cindy's beliefs about mathematics were supported by the club, shown by her changing from believing that mathematics is difficult before participating in the club, to believing that mathematics is not that difficult after having done so, thus suggesting that her personal mathematical identity shifted from fragile-leaning toward robust-leaning. Cindy's comment before participating in the club indicated confidence in mathematics related to getting a well-paying job, suggested by wanting to make a fortune. Her comment after participating in the club indicated supported confidence and meaning-making related to experiencing mathematical identity messages, suggested by talking about "mistakes" as helping her to "see certain things" that she previously could not see and resulting in her becoming "more capable." The change in Cindy's confidence suggests a shift in her personal mathematical identity from fragile-leaning toward robust-leaning.

Cindy's talk about sharing ideas suggested a sense of belonging to the club and not belonging to her regular mathematics class and therefore a social mathematical identity that was supported by the club. Her comment after participating in the club suggested that she learned mathematics differently in the club through experiencing meaning-making, suggested when she talked about the club as "messing" with her mind and experiencing mathematical identity messages through learning from "mistakes." Despite Cindy's response after participating in the club being largely positive, her talk about being considered "very good at maths" suggests that she attended the club with the belief that by attending, she would be viewed as good at mathematics by others, indicating some fragility in her supported mathematical identity.

Ben's robust-leaning personal and social mathematical identities, before participating in the club, were supported in relation to experiencing meaning-making in the club.

Table 7
Ben's comments in relation to meaning-making

	Learner Comment	
	Before	**After**
Ben	I'm very good at maths because there is nothing that I think I'm unable to do. He [teacher] was teaching me from last year, so he understands how I do things. Everyone can learn maths, they just need to work hard and avoid laziness. They [peers] praise and encourage me and that encourages me to help more learners.	When I'm in the club, in one problem I get many ideas and getting all these ideas makes me feel like I'm the man. Someone comes up with another method that you don't know and it's new to you. I enjoy noticing our mistakes in the club; that helps me not to repeat them because as you know, we learn from our mistakes. It [is] useful because it changes people's attitudes towards maths, what makes maths hard is wiped off. When stuck on a problem I no longer just stay and say I got stuck, I now look around at how I can connect that thing with something else.

Ben's confidence in mathematics, before participating in the club, was supported by the club through his being exposed to "many ideas", thus related to sharing ideas, a learning practice that supports meaning-making. Ben's positive beliefs about mathematics before participating in the club, were supported by the club through his experiencing mathematical identity messages, suggested by talking about mistakes and their importance. Ben's talk about "attitudes towards maths" changing and mathematics as not "hard" after participating in the club suggests supported beliefs about mathematics in other club learners and possibly in himself. His persistence in mathematics and meaning-making were supported by experiencing mathematical identity messages, suggested when he talked about connecting ideas. The changes in Ben's confidence and persistence in and beliefs about mathematics suggest a strengthened robust-leaning personal mathematical identity by the club related to experiencing meaning-making.

Before participating in the club, Ben felt that he was understood by his regular class teacher and was encouraged by his peers, thus suggesting a sense of belonging to his classroom community. His feeling "like a man" in the club, suggested a supported sense of belonging to the club community and therefore a strengthened robust-leaning social mathematical identity, related to experiencing meaning-making.

Similarly to Ben, Rose's robust-leaning personal and social mathematical identities before participating in the club were supported in relation to the meaning-making encouraged by the club.

Table 8
Rose's comments before and after participating in the club

	Learner Comment	
	Before	**After**
Rose	As I said maths is fun. I started liking maths in Grade 8, it was difficult, but I found it interesting because sometimes when you think that you know something then something comes up. They [peers] try to help me when I'm wrong, we don't laugh at each other.	Maths hasn't been that difficult ever since I started working together with others in the club. It's interesting and you are free to ask questions. I didn't think that my mind could think this further. Sometimes I struggle a lot with some equations but then I try very hard to understand and work with others so I can get some opinions from them, and I also give them mine. We do different stuff that is more interesting than textbook stuff [in the club]. In the class, we just write, calculate and do corrections, so the club is cool because we don't just calculate but we do shapes with crayons and it's very interesting.

Before participating in the club, Rose believed that mathematics was fun, indicating a robust-leaning personal mathematical identity. Her comment after participating in the club suggested that her beliefs about mathematics were supported through sharing ideas in the club, being offered the opportunity to "think further" than she could imagine, and learning mathematics differently, all practices that support meaning-making. Rose's persistence in mathematics was supported by the club, suggested by talking about struggling leading to mutual engagement. The changes in Rose's persistence in and beliefs about mathematics suggest a robust-leaning personal mathematical identity that was strengthened by belonging to the club.

Rose felt a sense of belonging to her classroom and the club communities as indicated by feeling helped in the classroom and free in the club, suggesting that the club helped to strengthen her robust-leaning social mathematical identity.

The four learners reported that their personal and social mathematical identities shifted toward more robust identities. Talk about "working in groups", getting "many ideas", connecting "that thing with something else", and getting and giving "opinions" suggest that the club encouraged meaning-making. Opportunities to experience mathematical identity messages were offered by the club.

Class teacher's views of club learners' mathematical identities

Mr. Kent's interview response corroborated Kelly's, Cindy's, and Ben's pre- and post-club interview responses. His interview response corroborated Rose's post-club interview response but not her pre-club interview response. Provided excerpts show observed improvements in the club learners' participation and relationship with mathematics in the class.

See the table below for Mr Kent's comments on Rose and Kelly.

Table 9
Teacher's comments on Rose and Kelly

Teacher	Learner	Teacher's comment
Mr. Kent	Rose	She was a little bit reserved, shy to participate and not that enthusiastic about maths but now she is one of the learners who jumps first to say "I want to give you the corrections on the board." I've noticed that if she knows what the next topic will be she tries to go and prepare herself before I start the topic.
	Kelly	Before the maths club, I just saw a quiet child. After the club, she doesn't like to stand up but what I've noticed is that if she had a group that she was working with and the person who was writing on the board asked how they did something then she'd assist them.

Mr. Kent observed a positive shift in both Rose's and Kelly's participation in the class that suggested a shift in confidence in mathematics and a sense of belonging to the classroom mathematics community and, therefore, a shift in these two learners' personal and social mathematical identities towards more robust ones. Rose's interview response before participating in the club suggested that she had confidence and persistence in, and positive beliefs about, mathematics and felt a sense of belonging to her classroom mathematics community, which was not suggested by her teacher. Rose's response after participating in the club was corroborated by her teacher's response, thus suggesting a supported mathematical identity.

According to Mr. Kent's comment, Cindy's personal and social mathematical identities shifted positively after participating in the club activities.

Table 10
Teacher's comments on Cindy

Teacher	Learner	Teacher's comment
Mr. Kent	Cindy	Before the club, sometimes when she got stuck, she got frustrated and gave up quickly. She would make noise and wouldn't cooperate. She would say, "Sir, I don't know" before she even tried the problem. She didn't want to be wrong and complained a lot. The other thing that worried me was that she thought too little of herself and used to tell me that she was practicing to pass and not to know these things. After the maths club, I noticed that she now wants to go to the board and do something. Now she can come up with extra work from somewhere and ask if she solved it correctly. Her attitude has changed. She can even say, "Sir, let's try this question this way."

According to the teacher's comment, Cindy tended to ask for help without attempting questions and she thought "too little of herself" before participating in the club, thus indicating low confidence and low persistence in mathematics in the class and therefore a fragile-leaning personal mathematical identity. Cindy's confidence and persistence in mathematics in the class shifted positively after she participated in the club, indicated by "going to the board" and doing "extra work." Cindy's beliefs about mathematics were supported by the club, shown by her changing from being afraid of making mistakes, before participating in the club, to not being afraid of doing so after such participation, and this was related, possibly, to experiencing positive mathematical identity messages and therefore meaning-making. The change in Cindy's confidence and persistence in and beliefs about mathematics suggest a shift in her personal mathematical identity from fragile-leaning, before participating in the club, toward robust-leaning, after doing so.

There was no mention of Cindy's disruptive behaviour after participating in the club, but she suggested alternative ways of solving problems and offered to participate productively in mathematical activities, thus indicating a positive shift in her sense of belonging to the class and, therefore, her social mathematical identity.

According to Mr. Kent, Ben had a positive relationship with mathematics before and after participating in the club activities.

Table 11
Teacher's comments on Ben

Teacher	Learner	Teacher's comment
Mr. Kent	Ben	He was and is still a good learner. He helps others, yes that one I know. These days he is coming up with extra problems and he asks me if his answers are correct. One problem that he asked me was whereby you make one of the variables the subject of the formula, yeah! He came up with something interesting and I think it took me two days to figure it out.

Ben's robust-leaning mathematical identity, before participating in the club, was supported by the club and indicated by his doing challenging "extra problems" and suggesting an improved understanding of the importance of engaging in productive struggle and a positive shift in persistence in mathematics and therefore personal mathematical identity. His helping others suggested a sense of belonging to the classroom community and therefore a robust-leaning social mathematical identity before and after participating in the club.

In summary, Mr. Kent observed a positive shift in how club learners participated in the class, suggesting a shift in the learners' mathematical identities toward more robust identities. Learners like Rose, who were "reserved" before participating in the club became active after doing so.

Discussion, limitations, and conclusion

In this section, I show how the findings add to previous research about mathematics clubs and learners' mathematical identities. In response to the first research question, "Can an after-school mathematics club support learners' mathematical identities?" I suggest that the club supported the learners' personal and social mathematical identities. Their mean personal and social mathematical identity scores before they started in the club and afterwards were significantly different. The mean scores showed that the learners' personal and social mathematical identities shifted toward more robust-leaning ones. The four learners' interview responses showed that their personal and social mathematical identities were supported by the club. These findings support the study by Turner et al. (2011) that showed that clubs can support learners' mathematical identities.

Club activities encouraged sharing ideas and helping others, contributed to seeing mathematics as "not that difficult" and gave learners a sense of belonging to a group and of feeling "capable." Problems were solved using many different methods, thus supporting the learners' meaning-making, and they were encouraged not to give up on solving problems and this supported their persistence in mathematics.

In relation to the second research question, "Can participating in an after-school mathematics club support learners' mathematical identities in the class?" Mr. Kent observed a positive shift in the club learners' participation, confidence, and persistence in and beliefs about mathematics, and their sense of belonging to their mathematics classrooms, thus indicating a shift in the learners' personal and social mathematical identities in the class. Other class teachers have observed a positive shift in club learners' participation and confidence in mathematics in the class (see Graven, 2015). This finding suggests that clubs can support other indicators of learners' mathematical identities in the class, in addition to the previously known ones. The study contributes to understanding how club experiences can be replicated in the classroom, which, according to Brodie (2022), who argued that clubs can run side-by-side with regular mathematics classrooms, might be difficult to achieve given the different constraints in clubs and classrooms.

Despite suggested mathematical identity changes in regular classrooms by their class teacher, the learners said that they did not enjoy learning mathematics in their regular classrooms. Several studies have found no change in these negative views of learning mathematics in regular classrooms after the learners participated in clubs (Jacob & Jacob, 2018; Zhou et al., 2021). This finding supports McGee's (2015) argument that mathematical identities can strengthen despite persistent negative mathematical experiences. In this study, the strengthening was achieved through participating in an after-school mathematics club, but there was no transfer to the class.

The learners' interview responses showed a comparison between the club and class ways of learning mathematics providing insights into how the club supported the learners' mathematical identities. Kelly's and Rose's responses suggested that they worked collaboratively on club activities, which was usually not the case in their regular mathematics classrooms. This finding supports studies that have shown that mathematics classes encourage individual activities while mathematics clubs encourage social activities (Brodie, 2022; Jacob & Jacob, 2018). Kelly did not feel comfortable participating in the class because of her fear of being laughed at by peers and being embarrassed but she felt comfortable participating in the club because she got help from peers, and this indicated a difference in peer-to-peer relationships in the club and classroom. Club activities were engaged in using strategies not normally used in the classroom, thus influencing Cindy to "see certain things" that she could not see previously and Rose to believe that mathematics is interesting. Several studies have explored ways of learning mathematics that are different from those normally used in regular mathematics classrooms and have found that learners enjoy these ways, and the learners' relationships with mathematics are thus supported (Jacob & Jacob, 2018; Kennedy & Smolinsky, 2016). Cindy's interview response indicated that she attended the club with the belief that by attending she would be viewed as good at mathematics by others. Papanastasiou and Bottiger (2004) argued that such learners might eventually develop positive perceptions of mathematics become of the mathematical experiences gained in the club, which are different from regular class experiences.

This study has limitations that offer opportunities for further studies. It has a bias toward the narrative view of identity as constructed through stories and words and does not pay attention to the participative view of identity as constructed through participation, which, if investigated, may add depth to the findings. The participating school was selected because of its convenience. Conducting a similar study but with learners of a different socio-economic status and different experiences with mathematics to those in this study may offer additional insights into the relationship between mathematics clubs and learners' mathematical identities. Participating in this study was voluntary so the participants may have had some interest in mathematics. Working with a control group in the quantitative design will allow for comparisons, thus adding depth to the findings.

Based on the findings, it can be concluded that learners enjoyed experiencing mathematics differently in employing different strategies to solve problems, engaging in productive struggle, and sharing ideas in the club. So, teachers need to find ways of presenting mathematics in a way that is motivating for learners, which requires creativity on their part, and an understanding of what it is that learners describe as motivating. This is not to say that classrooms should or can be turned into clubs since they operate with different constraints (Brodie, 2022). Clubs that encourage mutual engagement, meaning-making, and a sense of belonging to mathematics communities, however, can be considered as complementary spaces to classrooms for supporting learners' mathematical identities. The club supported learners' narrations of different mathematical identities, indicating the flexible nature of the club. A relationship between learners' mathematical identities in the club and their mathematical identities in the class was suggested.

References

Aydeniz, M., & Hodge, L. L. (2011). Identity: A complex structure for researching students' academic behaviour in science and mathematics. *Cultural Studies of Education*, *6*, 509–523. https://doi.org/10.1007/s11422-011-9331-9

Bishop, J. P. (2012). 'She's always been the smart one. I've always been the dumb one': Identities in the mathematics classroom. *Journal for Research in Mathematics Education*, *43*(1), 34–74. https://www.jstor.org/stable/10.5951/jresematheduc.43.1.0034

Boaler, J. (2013). Ability and mathematics: The mindset revolution that is reshaping education. *Forum*, *55*(1), 143–153. https://doi.org/10.2304/forum.2013.55.1.143

Boaler, J., Dieckmann, J. A., Pérez-Núñez, G., Sun, K. L., & Williams, C. (2018). Changing students' minds and achievement in mathematics: The impact of a free online student course. *Frontiers in Education*, *3*(26). https://doi.org/10.3389/feduc.2018.00026

Brodie, K. (2022). Learning mathematics in an after-school mathematics club. *African Journal of Research in Mathematics, Science and Technology Education*, *26*(3), 237–247. https://doi.org/10.1080/18117295.2022.2131267

Cribbs, J. D., Hazari, Z., Sonnert, S., & Sadler, P. M. (2015). Establishing an explanatory model for mathematics identity. *Child Development*, *86*(4), 1048–1062. https://doi.org/10.1111/cdev.12363

Darragh, L. (2013). Constructing confidence and identities of belonging in mathematics at the transition to secondary school. *Research in Mathematics Education*, *15*(3), 215–229. https://doi.org/10.1080/ 14794802.2013.803775

Darragh, L. (2015). Recognising 'good at mathematics': Using a performative lens for identity. *Mathematics Education Research Journal*, *27*, 83–102. https://doi.org/10.1007/s13394-014-0120-0

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, *93*, 19–33. https://doi.org/ 10.1007/s10649-016-9696-5

Epstein, D., Mendick, H., & Moreau, M. (2010). Imagining the mathematician: Young people talking about popular representations of maths. *Discourse Studies in the Cultural Politics of Education*, *31*(1), 45–60. https://doi.org/10.1080/01596300903465419

Fellus, O. O. (2019). Connecting the dots: Toward a networked framework to conceptualizing identity in mathematics education. *ZDM – The International Journal on Mathematics Education: The special issue on identity in mathematics education*, *51*, 445–455. https://doi.org/10.1007/s11858-019-01053-9

Frenzel, J., Lampen, C. E., & Brodie, K. (2019). Learners' awareness of their emotions and their engagement with mathematics tasks in a mathematics club. *Journal of Education*, *77*, 45–59. http://dx.doi.org/10.17159/2520-9868/i77a03

Gardee, A. (2019). Social relationships between teachers and learners, learners' mathematical identities, and equity. *African Journal of Research in Mathematics, Science and Technology Education*, *23*(2), 233–243. https://doi.org/10.1080/18117295.2019.1662641

Graven, M. (2015). Strengthening maths learning dispositions through 'math clubs.' *South African Journal of Childhood Education*, *5*(3). http://dx.doi.org/10.4102/sajce.v5i3.342

Gweshe, L. C., & Brodie, K. (2024). Learners' mathematical identities: Exploring relationships between high school learners and significant others. *Mathematics Education Research Journal*, *36*, 917–944. https://doi.org/10.1007/s13394-023-00479-5

Hall, J., Towers, J., & Martin, L. C. (2018). Using I poems to illuminate the complexity of students' mathematical identities. *Educational Studies in Mathematics, 99*, 181–196. https://www.jstor.org/stable/45184656

How, M. L. (2022). Advancing multidisciplinary STEM education with mathematics for future-ready quantum algorithmic literacy. *Mathematics*, *10*(7), 1146. https://doi.org/10.3390/math10071146

Jacob, R., & Jacob, B. (2018). New evidence on the benefits of small group math instruction for young children. *Evidence Speaks Reports*, *2*(55). https://www.brookings.edu/articles/new-evidence-on-the-benefits-of-small-group-math-instruction-for-young-children/

Jensen, F., & Sjaastad, J. (2013). A Norwegian out-of-school mathematics project's influence on secondary students' stem motivation. *International Journal of Science and Mathematics Education*, *11*, 1437–1461. https://doi.org/10.1007/s10763-013-9401-4

Kennedy, E., & Smolinsky, L. (2016). Math circles: A tool for promoting engagement among middle school minority males. *Eurasia Journal of Mathematics, Science & Technology Education*, *12*(4), 717–732. https://doi.org/10.12973/eurasia.2016.1223a

Lampen, C. E., & Brodie, K. (2020). Becoming mathematical: Designing a curriculum for a mathematics club. *Pythagoras*, *41*(1). https://doi.org/10.4102/pythagoras.v41i1.572

Lynch, K., An, L., & Mancenido, Z. (2023). The impact of summer programs on student mathematics achievement: A meta-analysis. *Review of Educational Research*, *93*(2), 275–315. https://doi.org/10.3102/00346543221105543

Marks, A., & O'Mahoney, J. (2014). Researching identity: A critical realist approach. In P. K. Edwards, J. O'Mahoney & S. Vincent. (Eds.), *Studying organizations using critical realism: A practical guide* (pp. 66–85). Oxford University Press.

McGee, E. O. (2015). Robust and fragile mathematical identities: A framework for exploring racialised experiences and high achievement among black college students. *Journal for Research in Mathematics Education*, *46*(5), 599–625. https://doi.org/10.5951/jresematheduc.46.5.0599

McMillan, J. H., & Schumacher, S. (2010). *Research in education: Evidence-based inquiry.* (7th ed.). Pearson Education Inc.

Neher-Asylbekov, S., & Wagner, I. (2023). Effects of out-of-school STEM learning environments on student interest: A critical systematic literature review. *Journal for STEM Education Research*, *6*, 1–44. https://doi.org/10.1007/s41979-022-00080-8

Papanastasiou, E. C., & Bottiger, L. (2004). Maths clubs and their potential: Making mathematics fun and exciting. A case study of a maths club. *International Journal of Mathematical Education in Science and Technology, 35*(2), 159–171. https://doi.org/10.1080/00207390310001638395

Prescott, A., & Pressick-Kilborn, K. (2015). It's great to be doing maths! Engaging primary students in a lunchtime club. *Australian Primary Mathematics Classroom*, *20*(3), 34–39.

Schuepbach, M. (2015). Effects of extracurricular activities and their quality on primary school-age students' achievement in mathematics in Switzerland. *School Effectiveness and School Improvement*, *26*(2), 279–295. https://doi.org/10.1080/09243453.2014.929153

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14–22. https://doi.org/10.3102/ 0013189X034004014

Shone, E. T., Weldemeskel, F. M., & Worku, B. N. (2023). Strategies of enhancing students' mathematics perception and self-efficacy to improve their mathematics achievement. *Cogent Education*, *10*(2). https://doi.org/10.1080/2331186X.2023.2285642

Stott, D., Baart, N., & Graven, M. (2019). Partnering with districts to expand an after-school maths club programme. *Africa Education Review*, *16*(6), 183–200. https://doi.org/10.1080/18146627.2018.1464690

Tulis, M., & Fulmer, S. M. (2013). Students' motivational and emotional experiences and their relationship to persistence during academic challenge in mathematics and reading. *Learning and Individual Differences*, *27*, 35–46. http://dx.doi.org/10.1016/j.lindif.2013.06.003

Turner, E. E., Gutiérrez, R. J., & Sutton, T. (2011). Student participation in collective problem solving in an after-school mathematics club: Connections to learning and identity. *Canadian Journal of Science, Mathematics and Technology Education*, *11*(3), 226–246. https://doi.org/10.1080/14926156.2011.595884

Utha, K., Subba, B. H., Mongar, B. B., Hopwood, N., & Pressick-Kilborn, K. (2021). Secondary school students' perceptions and experiences of learning science and mathematics: The case of Bhutan. *Asia Pacific Journal of Education*, *43*(2), 350–367. https://doi.org/10.1080/02188791.2021.1901652

Vanayan, M., White, N., Yuen, P., & Teper, M. (1997). Beliefs and attitudes toward mathematics among third- and fifth-grade students: A descriptive study. *School Science and Mathematics*, *97*(7), 345–351. https://0-doi-org.innopac.wits.ac.za/10.1111/j.1949-8594.1997.tb17375.x

Wenger, É. (1998). *Communities of practice. Learning, meaning, and identity.* Cambridge University Press.

Zhou, L., Flores, E. S., Sapkota, B., Mbewe, R., & Newton, J. (2021). Image of mathematics in- and out-of-school: A case study of two original participants in an afterschool stem club – girls excelling in math and science (gems). In D. Olanoff, K. Johnson & S. Spitzer (Eds.), *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. (pp. 1210–1219). Philadelphia, PA.

Chapter 6: Conclusion

6.1 Introduction

In this chapter, I discuss the empirical and theoretical contributions that the study makes in the field. I explore the implications of the study and note the study's limitations.

The research questions addressed are:

4. What kinds of mathematical identities do high school learners narrate?
5. What are some of the influences on high school learners' mathematical identities and how do they influence mathematical identity?
6. Can an after-school mathematics club support high school learners' mathematical identities? If so, what are the characteristics of such a club?

I wanted to understand the relationship between high school learners' mathematical identities and an after-school mathematics club. First, I needed to understand learners' mathematical identities more generally so with research question 1, I addressed the kinds of mathematical identities constructed by learners and with research question 2, I addressed how teachers, peers and family members support or constrain learners' mathematical identities. I then explored an after-school mathematics club as an intervention strategy for supporting the learners' mathematical identities.

In chapter 2 (paper 1), I used figure 6.1, which reveals the relationship between mathematical identities and mathematics communities of practice to help me answer the first and second research questions. The figure draws on Wenger's (1998) social theory of learning. The focus of chapter 2 was on learners' mathematical identities as indicated by their confidence, persistence and beliefs about mathematics. Three kinds of mathematical identities were discussed in chapter 2: largely robust, modest and largely fragile.

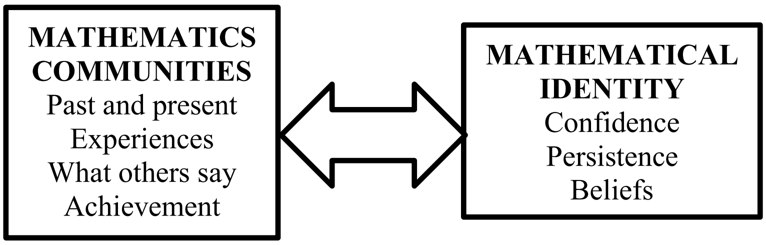
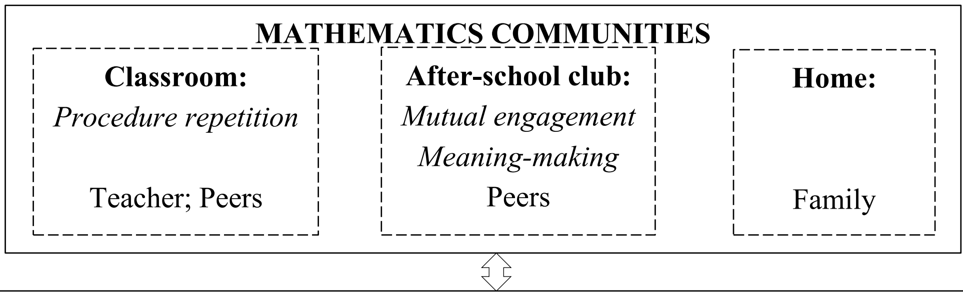


Figure 6.1 A Conceptual framework for mathematical identity

In chapter 3 (paper 2), I used figure 6.2, which reveals the different kinds of mathematical identities that learners constructed as they interacted with their teachers, peers and family members to help me answer the first and second research questions in greater detail than in chapter 2. Through continuously reading the literature as the study progressed, I developed figure 6.2 from figure 6.1 by relating Marks & O'Mahoney's (2014) theory of identity as composed of personal and social identities, McGee's (2015) theory of mathematical identity as robust or fragile and the idea of identity as fluid to Wenger's (1998) social theory of learning, allowing for a more detailed analysis and description of learners' mathematical identities. I focused on learners' personal mathematical identities (i.e., confidence, persistence, beliefs) and their social mathematical identities (i.e., a sense of belonging), and nine kinds of mathematical identities, which was deeper than focusing on learners' personal mathematical identities and three kinds of mathematical identities only, as I did in chapter 2. Modest identity, in chapter 2, was renamed mixed identity in chapter 3 but the meaning remained the same.



MATHEMATICAL IDENTITIES				
		PERSONAL IDENTITY		
		Robust-leaning	**Mixed**	**Fragile-leaning**
SOCIAL IDENTITY	**Robust-leaning**	Robust-leaning personal and social identity	Mixed personal identity and robust-leaning social identity	Fragile-leaning personal identity and robust-leaning social identity
	Mixed	Robust-leaning personal identity and mixed social identity	Mixed personal and social identity	Fragile-leaning personal identity and mixed social identity
	Fragile-leaning	Robust-leaning personal identity and fragile-leaning social identity	Mixed personal identity and fragile-leaning social identity	Fragile-leaning personal and social identity

Figure 6.2 A framework relating mathematics communities and mathematical identities.

In chapter 4 (paper 3), I used figure 6.3 to investigate the relationship between mathematics clubs and learners' productive dispositions. The chapter helped me to answer the third research question.

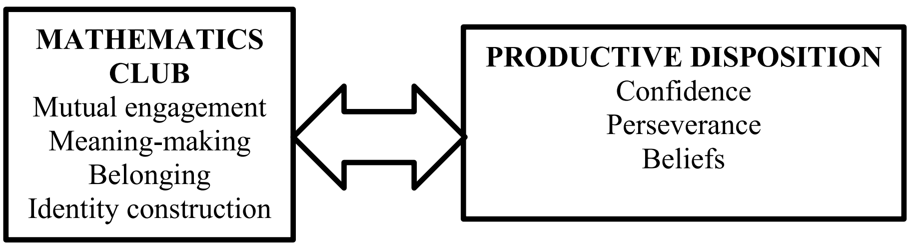


Figure 6.3 A framework relating a mathematics club and productive disposition.

The idea of investigating learners' productive dispositions came up when I wanted to write a paper to present at the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE) conference held in 2023. While reading the literature, I noticed a link between productive disposition and mathematical identity resulting in writing a conference paper that was later developed into the journal paper in chapter 4.

Kilpatrick et al. (2001) consider learners' confidence, persistence and beliefs about mathematics as indicators of their productive dispositions. Researchers, for example, Bishop (2012), consider learners' confidence, persistence and beliefs about mathematics as among indicators of their mathematical identities, suggesting that productive disposition can be considered a part of mathematical identity. Drawing on Marks & O'Mahoney's (2014) idea of personal identity as a self-view and social identity as about social relations, I considered learners' confidence, persistence and beliefs about mathematics as indicators of their personal mathematical identities. So, productive disposition was a part of personal mathematical identity.

In chapter 5 (paper 4), I used figure 6.4, which reveals the relationship between mathematics clubs, regular mathematics classrooms and learners' mathematical identities to help me answer the third research question. I compared the learners' mathematical identities before and after participation in the club. I also compared the learners' mathematical identities in the class with their mathematical identities in the club.

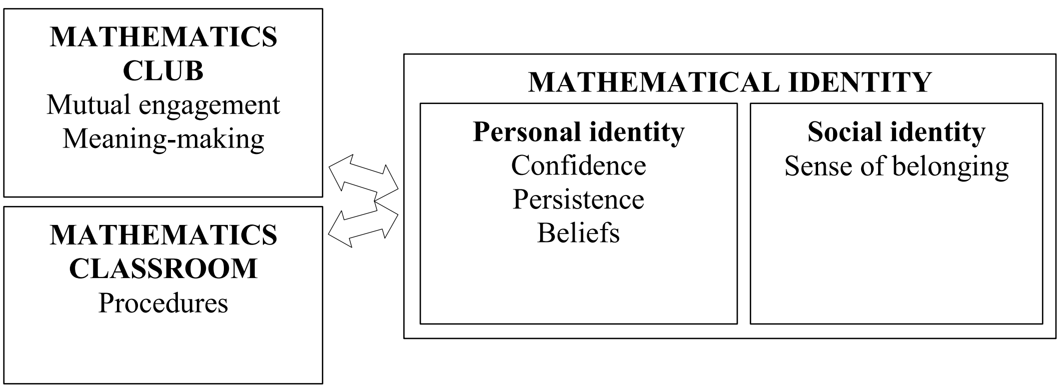


Figure 6.4 A framework relating mathematics clubs, classrooms and mathematical identities.

6.2 The study's contribution and answers to the research questions

In this section, I describe how chapters 2 to 5 holistically contribute to answering the research questions and I present some key contributions to the field.

Research question 1

In chapter 2 (paper 1), Wenger's (1998) theory of learning as involving mutual engagement, meaning-making, a sense of belonging and identity construction was presented as suitable for analysing and describing learners' narrated mathematical identities. The chapter considered mathematical identity as robust or fragile (McGee, 2015) and fluid (Hall et al., 2018). The learners reported on in chapter 2 narrated largely robust, modest or largely fragile mathematical identities.

Chapter 3 (paper 2) built on chapter 2 by considering mathematical identity as composed of personal and social identities (Marks & O'Mahoney, 2014), as well as robust or fragile (McGee, 2015) and fluid (Hall et al., 2018). Through chapter 3, I showed that learners can narrate different combinations of personal and social mathematical identities, which has not been shown before. Many learners, reported on in chapter 3, narrated fragile-leaning personal and social mathematical identities. A few learners narrated robust-leaning personal and social mathematical identities, mixed personal and social mathematical identities, or mixed personal mathematical identities and fragile-leaning social mathematical identities. Only four of the nine possible cells in figure 6.2 were narrated by the learners in this study. The study supported other studies showing that many learners construct fragile mathematical identities (e.g., Hall et al., 2018).

Research question 2

Many researchers have explored the role of teachers in learner mathematical identity development. Not many researchers have explored the role of peers or family members in learner mathematical identity development. The studies focusing on teachers, peers or family members as contributing to learners' mathematical identities have largely been conducted separately despite Wenger's (1998) argument that people move across different communities in which their identities are constructed. Given this background, chapter 2 (paper 1) explored how peers and family members constrain learners' mathematical identities while chapter 3 explored how teachers, peers and family members, i.e., three groups of significant others, support or constrain learners' mathematical identities together.

Chapter 3 showed that teachers, peers and family members supported learners' mathematical identities in the following ways:

- **Teachers**: explaining and giving examples when learners got stuck on problems.
- **Peers**: encouraging and praising each other.
- **Family members**: encouraging and helping learners when struggling with problems.

The teachers, peers and family members constrained learners' mathematical identities in the following ways:

- **Teachers**: encouraging one strategy of solving problems, not allowing interaction and giving learners information.
- **Peers**: expressing their negative views of mathematics to others and labelling others.
- **Family members**: constantly telling learners about their past negative mathematical experiences but expecting the learners to carry on doing mathematics, pressuring learners to carry on doing mathematics because of the belief that mathematics is a gateway to a better life but offering little support and having little time to check on learners' schoolwork because of busy schedules.

Most importantly, chapter 3 showed that teachers, peers and family members collectively supported or constrained learners' mathematical identities, a finding that has not been shown before. Having robust-enhancing relationships with two groups of significant others helped to resist a fragile-enhancing relationship with the third group. A learner having robust-enhancing relationships with at least two groups of significant others narrated a robust-leaning mathematical identity. A learner having fragile-enhancing relationships with at least two groups of significant others narrated a fragile-leaning mathematical identity.

Together, chapters 2 and 3 showed that teachers sometimes supported learners' mathematical identities while peers and family members rarely did. Teachers sometimes constrained learners' mathematical identities while peers and family members often did. I, therefore, argue that understanding learners' mathematical identities requires understanding their relationships with their three different groups of significant others as a collective.

Research question 3

Chapters 4 and 5 (papers 3 and 4) focused on an intervention, an after-school mathematics club, intended to support learners' mathematical identities. The functioning of the club drew on Wenger's (1998) social theory of learning as involving mutual engagement, meaning-making and a sense of belonging to mathematics communities, Lampen and Brodie's (2020) responsive club curriculum and explicitly stating and explaining positive mathematical identity messages (i.e., messages for supporting learners' mathematical identities). In chapter 4, I explored the relationship between learners' productive dispositions and the after-school mathematics club. As discussed above, I considered productive disposition a part of learners' personal mathematical identities. The learners reported on in chapter 4 said that they:

- felt helped to understand mathematics in the after-school mathematics club, suggesting a sense of belonging and meaning-making;
- were not discouraged from participating by their club peers, which was often the case in their regular mathematics classrooms, suggesting a sense of belonging in the club;
- worked together on problems, suggesting mutual engagement;
- shared and explained ideas to each other, suggesting mutual engagement.

Kilpatrick et al. (2001) have suggested that mutual engagement and a sense of belonging are linked to confidence and beliefs about mathematics, and that meaning-making is linked to persistence. So, these learners' responses suggested that the after-school mathematics club had supported their confidence, persistence and beliefs about mathematics and therefore their productive dispositions and personal mathematical identities.

In chapter 5, I explored the relationship between learners' mathematical identities and the after-school mathematics club and showed that learners' mathematical identities, both robust-leaning and fragile-leaning, can shift towards more robust mathematical identities after participating in an after-school mathematics club. A statistically significant difference between the learners' mean personal and social mathematical identity scores before and after participating in the after-school mathematics club was shown by two paired samples t-test analyses. The learners' mean personal and social mathematical identity scores improved after the learners participated in the after-school mathematics club. The learners said that they:

- enjoyed the way of learning mathematics in the club because it was different from that in their regular mathematics classrooms, suggesting that the club supported their mathematical identities;
- got ideas on how to solve challenging problems from the club, suggesting meaning-making.

A class teacher teaching some of the interviewed learners reported a positive shift in the learners' participation, confidence and persistence in and beliefs about mathematics, and sense of belonging to their classroom mathematics communities indicating a shift in the learners' personal and social mathematical identities towards more robust identities.

Chapters 4 and 5 showed that learners' productive dispositions and mathematical identities can strengthen despite persistent negative mathematical experiences. Positive mathematical experiences in the after-school mathematics club helped resist the negative mathematical experiences in regular mathematics classrooms suggesting that clubs can be complementary spaces to classrooms, for supporting learners' productive dispositions and mathematical identities (see also Brodie, 2022). Characteristics of the club, linked to supported learner productive dispositions and mathematical identities were mutual engagement, meaning-making, a sense of belonging to mathematics communities, and regularly stating and explaining positive mathematical identity messages.

6.3 Implications of the study

Empirical

Learners in the study narrated four out of the nine possible combinations of personal and social mathematical identities (see figure 6.2). I am interested in finding out the combinations of personal and social mathematical identities that might be narrated by learners in other situations, including in other grades. It would also be interesting to explore the combinations of personal and social identities narrated by learners in other subjects.

The study showed that teachers, peers and family members collectively supported or constrained learners' mathematical identities. Understanding the kinds of mathematical identities constructed by teachers and family members through, for example, exploring the influences on teachers' and family members' own mathematical identities in the different communities of mathematics practice that these two groups of significant others participate in may help to understand further learners' mathematical identities.

Theoretical

I developed a framework consisting of two inseparable components (Figure 6.1):

- **Component 1**: different groups of significant others in different communities of mathematics practice collectively contribute to how learners construct their mathematical identities.

- **Component 2**: nine combinations of robust-leaning, mixed and fragile-leaning personal and social mathematical identities that learners can construct, and therefore nine kinds of mathematical identities.

The framework drew on Wenger's (1998) social theory of learning, Marks & O'Mahoney's (2014) theory of identity as composed of personal and social identities, McGee's (2015) theory of mathematical identity as robust or fragile and the idea of identity as fluid. The framework implies that understanding learners' mathematical identities requires understanding the relationships between learners and their different groups of significant others and that mathematical identities are constituted by personal and social mathematical identities and are fluid. Drawing on Gardee (2019), learners can narrate different combinations of personal and social mathematical identities depending on offered opportunities for mathematical identity support and the learners' choices to accept or reject the opportunities, which are dependent on the learners' past and present mathematical experiences. The framework may be used to understand learners', teachers' and family members' mathematical identities and their identities in other subjects.

Practical

The study was part of a larger project with three after-school mathematics clubs in high schools, which showed that after-school mathematics clubs can support learners' mathematical identities (see Brodie, 2022; Lampen & Brodie, 2020). Lampen and Brodie's (2020) club focused on designing a club curriculum responsive to learners' strengths and weaknesses. The club that I established followed Lampen and Brodie's (2020) club curriculum. Brodie's (2022) club, which also followed Lampen and Brodie's (2020) club curriculum, compared learners' relationships with mathematics in the club and their regular mathematics classrooms. Brodie (2022) found that learners expressed positive relationships with mathematics in the after-school mathematics club and negative relationships with mathematics in their regular mathematics classrooms, a finding supported by my study, as discussed in chapters 4 and 5.

Given the scale of the larger project and its findings, after-school mathematics clubs can be considered as extra-curricular activities similar to sports. Wenger's (1998) four components of learning, Lampen and Brodie's (2020) responsive club curriculum, and regularly and explicitly stating and explaining positive mathematical identity messages can be used as a guide for developing club activities and running the clubs. Teachers can volunteer to run the clubs. Training on how to run the clubs is required since the way of learning in the clubs should be different from the way of learning in regular mathematics classrooms. As a way of giving back to the community, community businesses may be asked to assist the clubs by providing club learners with snacks and possibly transportation.

The different constraints in clubs and classrooms make it difficult to fully implement club activities in classrooms but clubs can run side-by-side with classrooms (see Brodie, 2022). The idea of regularly and explicitly stating and explaining positive mathematical identity messages to learners can however be adopted in regular mathematics classrooms. There is a need to find ways of discouraging peers from labelling and embarrassing each other in mathematics, and these ways can be regularly stated and explained by teachers to learners in the class. Parents who believe that they need to know mathematics to support their children in mathematics and busy parents need advice on how to support children in mathematics and this advice can be communicated to the parents by teachers.

6.4 Limitations

Limitations of a study may be viewed as weaknesses in the methodology that may compromise the utilisation of the study's findings (Rosa, 2010). The limitations do not invalidate a study's findings but they offer opportunities for further studies. In this study, participating learners were from a township area where people of low socio-economic status reside. Working with learners of a different socio-economic status to those reported in the study and from a different location may lead to a deeper understanding of learners' mathematical identities and after-school mathematics clubs since the learners may have different experiences of mathematics.

My findings depend on the questionnaire responses of fifty learners, between six and twenty-six written responses per five short questions, and narratives of six learners and one class teacher. Working with a bigger sample size in the quantitative design of the study and more than one class teacher in the qualitative design may add depth to the findings. My findings are based on written responses and narratives, so they depend on participants' recollections of events. I regularly observed the six interviewed learners in their regular mathematics classrooms and in the club so that I could see their mathematical identities develop. The observation data was video recorded, but I later lost the data when my laptop crashed. The observation data might have added depth to the findings. I do not have written responses or narratives of peers and family members, which may add depth to the findings. I did not exhaust all the possible influences on learners' mathematical identities. Other studies may explore the other influences on learners' mathematical identities, for example social media. Additional limitations of the study are provided in each of the four papers.

6.5 Final conclusion

Learners constantly move across different communities of mathematics practice where they interact with different groups of significant others. The different groups of significant others offer learners different opportunities for mathematical identity support, some of which are accepted and others rejected by the learners, leading to learners showing different mathematical identities.

Since many learners have fragile-leaning mathematical identities, I explored an after-school mathematics club as an intervention strategy for shifting learners' mathematical identities towards more robust identities. Findings showed that the club supported the learners' mathematical identities.

6.6 References

Aydeniz, M., & Hodge, L. L. (2011). Identity: A complex structure for researching students' academic behavior in science and mathematics. *Cultural Studies of Education, 6*, 509-523. https://doi.org/10.1007/s11422-011-9331-9

Brodie, K. (2022). Learning mathematics in an after-school mathematics club. *African Journal of Research in Mathematics, Science and Technology Education*, *26*(3), 237–247. https://doi.org/10.1080/18117295.2022.2131267

Gardee, A. (2019). Social relationships between teachers and learners, learners' mathematical identities, and equity. *African Journal of Research in Mathematics, Science and Technology Education*, *23*(2), 233-243. https://doi.org/10.1080/18117295.2019.1662641

Graven, M., & Heyd-Metzuyanim, E. (2019). Mathematics identity research: The state of the art and future directions. *ZDM – The International Journal on Mathematics Education. The special issue on identity in mathematics education*, *51*, 361-377. https://doi.org/10.1007/s11858-019-01050-y

Hall, J., Towers, J., & Martin, L. C. (2018). Using I poems to illuminate the complexity of students' mathematical identities. *Educational Studies in Mathematics*, *99*, 181–196. https://www.jstor.org/ stable/45184656

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. National Academy Press.

Lampen, C. E., & Brodie, K. (2020). Becoming mathematical: Designing a curriculum for a mathematics club. *Pythagoras, 41*(1), a572. https://doi.org/10.4102/pythagoras.v41i1.572

Marks, A., & O'Mahoney, J. (2014). Researching identity: a critical realist approach. In: Edwards, P., O'Mahoney, J., & Vincent, S. (Eds.). *Putting Critical Realism into Practice: A Guide to Research Methods in Organization Studies*, Oxford University Press, pp. 66-85.

McGee, E. O. (2015). Robust and fragile mathematical identities: A framework for exploring racialised experiences and high achievement among black college students. *Journal* *for Research in Mathematics Education*, *46*(5), 599-625. https://doi.org/10.5951/jresematheduc.46.5.0599

Rosa, M. (2010). *A mixed method study to understand the perceptions of high school leaders about ELL students: The case of mathematics*. (Corpus ID: 154400538). [Doctoral dissertation, California State University]. Semantic Scholar.

Wenger, E. (1998). *Communities of practice. Learning, meaning, and identity.* Cambridge University Press.

Appendix A Learner pre- and post-club mathematical identity questionnaire

School name: **Learner Name:** **Date:**
Gender: **Age:** **Grade:**

This questionnaire is **voluntary AND confidential**.
You may skip any question that you do not wish to answer.
Please write your School name, your name, date, gender, age and grade in the spaces above.
Carefully read each question and then **put a tick** in the appropriate box.
Please be as **TRUTHFUL** as you can.

		Strongly agree	Agree	Uncertain	Disagree	Strongly disagree
1	I am confident with maths					
2	I can do hard maths problems					
3	I raise my hands during maths lessons					
4	I am good at maths					
5	In maths, when faced with a hard problem I continue trying until I get it					
6	Even if I fail maths this term, I will try again next term					
7	In class, if I make a mistake I work until I have corrected it					
8	If it was up to me, I would continue with maths next year					
9	Maths is a worthwhile and necessary subject					
10	I study maths because I know how useful it is					

11	Practicing can improve my ability in maths					
12	I can get smarter in class maths if I try hard					
13	Ability in math increases when one studies hard					
14	In maths, I feel important					
15	I feel there are people who can help me with maths					
16	In maths, my contributions are valued					

Appendix B Learner pre-club semi-structured interview schedule

This interview is voluntary and confidential.

You can refuse to answer any question that you do not want to answer.

You may ask me to clarify on a question that you do not understand.

Please be as truthful as you can.

Can you please tell me the following information about yourself?

<u>**Your school name; Your name; Your age; Gender; Your grade.**</u>

Interview questions

1. What can you tell me about your confidence in mathematics? Please explain while giving a few examples.
    a. Do you think that you are good at mathematics? Why?
    b. Do you assist other learners in mathematics? Please explain?
    c. Do you think that you will pass mathematics this term? Why?
2. Please tell me about how you relate with your teacher in mathematics?
    a. Do you think you can convince your teacher to use your methods in math? Why?
    b. Does your teacher assist you when you need help? Please explain?
    c. What does your teacher say about you as a math learner?
    d. How has that made you to think about yourself?
3. How do you relate with other learners in mathematics?
    a. Do you think you can convince them to use your methods? Why?
    b. Do they assist you when you need help? Please explain?
    c. What do they say about you as a math learner?
    d. How has that made you to think about yourself?
4. What does your family say about you as a math learner?

a. How has that made you to think about yourself?

5. When you are stuck on a mathematics problem or when you get something wrong, what do other learners say?
    a. How has that made you to think about mathematics?
    b. How has it made you to think about yourself?
6. What does your teacher say when you are stuck?
    a. How has that made you to think about mathematics?
    b. How has it made you to think about yourself?
7. What does your family say when you are stuck?
    a. How has that made you to think about mathematics?
    b. How has it made you to think about yourself?
8. What do you think about mathematics? Please explain.
    a. Is it difficult or challenging? Why?
    b. Can it be understood? Why?
    c. Is it fun or boring? Why?
    d. Is it useful or it is a waste of your time? Why?
9. Besides being needed at good jobs, are there any other uses of mathematics?
10. What do other learners say about mathematics?
    a. Do they think of mathematics as something that is "cool" to do or not? Why?
    b. How has that made you think about mathematics?
11. What does your teacher say about mathematics?
    a. Does your teacher think of mathematics as something that is "cool" to do or not? Why?
    b. How has that made you think about mathematics?
12. What does your family say about mathematics?
    a. Do they think of mathematics as something that is "cool" to do or not? Why?

b. How has that made you think about mathematics?

13. Can everyone do mathematics? Please explain.
    a. What do you think is required to be good at mathematics?
    b. Is being good at math something that a person is born with? Why?
    c. Can practice make someone to be good at math? Why?
    d. If a person is good at memorising math rules can you say that person is good at math? Why?
    e. Can an after-school mathematics club make someone to be good at math? Why?
14. What does making a mistake mean to you in mathematics? Please explain?
    a. Is making mistakes a good thing? Why?
    b. What do you do about the mistakes in your classwork book?
15. Are you committed to mathematics? Please explain how.
    a. How much effort do you put in mathematics? Please explain.
16. How do you deal with hard mathematical problems?
    a. What do you do when you get stuck?
    b. Do you give up or you continue trying? Why?
17. If you get low marks in mathematics, what do you do?
18. If it was your choice, for how long would you want to carry on doing mathematics? Why?
    a. So, if you fail this term what would you do? Why?
    b. After grade 12, would you continue doing math?
    c. Do you see yourself doing a job that needs mathematics? Why?
    d. If math was not needed in a job that you wanted to do, would you still do math? Please explain.
    e. What are some of the reasons why you chose to do mathematics?

19. Please tell me about your primary school mathematics
    a. Did you enjoy it or not? Why?
20. Please tell me about Grade 8 and 9 mathematics
    a. Did you enjoy it or not? Why?
21. What do you think about joining a mathematics club? Please explain why?
22. What is the "coolest" thing about your school?
23. What is the "coolest" thing about your class?
24. Is being good at mathematics a "cool" thing or not? Why?
25. Do you have anything else you would like to tell me about you and mathematics?

Thank you very much.

Appendix C Learner post-club semi-structured interview schedule

This interview is voluntary and confidential.

You can refuse to answer any question that you do not want to answer.

Please understand that some questions relate to the club and others to the classroom.

You may ask me to clarify on a question that you do not understand.

Please be as truthful as you can.

Can you tell me the following information about yourself?

<u>School name; Your name; Your age; Gender; Your grade.</u>

Interview questions

1. What do you think about the after-school mathematics club? Please explain.
    a. What did you enjoy the most? Why?
    b. What did you enjoy the least? Why?
    c. Did you gain anything by joining the club?
    d. Has anything about you changed?
    e. How has the club made you to think about mathematics?
    f. How has the club made you to think about yourself?
2. What do other learners say about the club?
    a. How has that made you to think about mathematics?
3. What does your teacher say about the club?
    a. How has that made you to think about mathematics?
4. What does your family say about the club?
    a. How has that made you to think about mathematics?
5. What can you say about the kind of mathematics that you do at the club?
    a. Is the math that you do at the club a waste of time or useful? Please explain.
    b. Is the math that you do at the club fun or boring? Why?

6. Is there anything that you think may be changed to make mathematics at the club fun and educational at the same time?
7. What do you think about doing mathematics in your usual mathematics classroom? Please explain.
   a. What do you enjoy the most in your usual math class? Why?
   b. What do you enjoy the least in your usual math class? Why?
   c. Is doing math in your class fun or boring?
   d. Is there anything that you think may be changed to make mathematics in the classroom fun and educational at the same time?
8. What do other learners think about the mathematics they do in class?
   a. Does that affect how you think about math? Please explain.
9. Can you compare your days before joining the club and the days after joining the club?
10. If people want to do well in mathematics what can they do?
    a. Does making mistakes help? Why? Please explain.
    b. Does doing hard problems help? Why? Please explain.
    c. Does memorising rules help? Why? Please explain.
    d. Does joining an after-school mathematics club help? Why? Please explain.
11. If given a choice, would you continue doing mathematics? Why?
    a. Until when?
    b. Would you do a job that needs mathematics? Why?
    c. Would you consider continuing to attend the math club? Why?
12. What do you think about what you do in a mathematics class and what you do at a mathematics club?
    a. Are there any differences? Please explain.

b. Are there any similarities? Please explain.

c. Which one would you prefer and why?

13. Can you tell me about your confidence in mathematics? Please explain.

    a. Why is your confidence like that?

    b. Do you think that you will pass mathematics this term? Why?

14. Can you tell me about your beliefs about mathematics?

    a. Why are your beliefs like that?

    b. Can everyone do mathematics? Please explain.

    c. Can math be understood or not? Why?

    d. Is mathematics challenging or difficult? Why?

15. Are you committed to mathematics? Please explain why.

16. If you are asked to solve a hard mathematical problem, would you give up or continue trying? Please explain why.

17. Would you give up or continue with mathematics? Please explain why.

18. How is your performance in mathematics? Please explain why.

19. Please tell me about how you solve mathematical problems.

    a. What are some of the methods that you use? Please explain.

    b. Do you use formulas, tables, graphs, diagrams, form equations or expressions?

    c. Where do you get the methods from? From your class teacher, textbooks, your class friends or from the club?

    d. Do you sometimes make or think of your own methods? Please explain how.

20. Can one problem be solved by many methods?

    a. If so, how do you decide on the best method to use?

21. Can one problem have more than one answer?

    a. If so, how do you decide on which answer makes sense?

22. How do you feel about the mathematics club? Please explain.
23. How has the club affected you?
24. Did you gain anything at the club which changed you in the class? Please explain.
    a. What about confidence in mathematics?
    b. Did the way you participate at the club change how you participate in class? Please explain.
    c. Has your view about giving up or continuing when solving hard problems changed by participating in the club? Why?
    d. Has your view about giving up or continuing with mathematics in the future been changed by participating in the club? Why?
    e. Did the mathematics done at the club change your beliefs about mathematics? Please explain.
25. Depending on whether you are at the club or in class:
    a. Does your confidence in mathematics change? Please explain.
    b. Do your feelings about helping others and getting help from others change? Please explain.
    c. Do your beliefs about mathematics change? Please explain.
    d. Does your desire to continue trying even after failure or after facing difficulty change?
26. Which mathematics is “cool”, the one done in class or the one done at the club? Why?
27. Is it “cool” to be good at mathematics? Why?
28. In your class, does what other learners say about mathematics affect you? How?
29. Does what your teacher say about mathematics affect you? How?
30. Does what your family say about mathematics affect you? How?
31. In your class, does what other learners say about you in math affect you? How?

32. Does what your math teacher say about you affect you? How?

33. Does what your family say about you affect you? How?

34. In relation to mathematics, is there anything else you would like to tell me?

Thank you very much.

Appendix D Learner short written questions

School name: Name: Date:

Gender: Age: Grade:

This reflection prompt sheet is **voluntary AND confidential**. Please be as **TRUTHFUL** as you can.

1) Why do you come to the club?
2) If you were a grade 10 maths teacher what would you do?
3) What are the differences between the club and your maths class?
4) Which do you like, club or class? Why?
5) Do you want the club to continue next year? Why?

Appendix E Teacher post-club semi-structured interview schedule

This interview is voluntary and confidential.

You can refuse to answer any question that you do not want to answer.

You may ask me to clarify on a question that you do not understand.

Please be as truthful as you can.

Can you please tell me the following information?

School name: Teacher name: Learner name:

Interview questions

1. Please describe the learner before and after joining the maths club.
   a. What about the learner's confidence? Please explain.
   b. What about participation in class? Please explain.
   c. What about how the learner relates with others, including yourself? Please explain.
   d. Does s/he assist others when they need help?
   e. What does the learner now say about mathematics?
      i. In terms of it being useful or a waste of time?
      ii. What about mathematics being fun or boring?
      iii. Being difficult or challenging?
      iv. Being understandable or not?
   f. What does s/he now say about making mistakes?
   g. Please tell me about the learner's persistence in mathematics.
   h. Does the learner complete given activities? Please explain.
   i. How does s/he deal with hard problems? Please explain.
   j. Does s/he sometimes do extra work besides the one that you give them?
   k. When stuck what does s/he do?

2. Do you see the learner pursuing mathematics in the future? Please explain.
    a. Up to what level?
    b. What about doing a job that needs mathematics?
3. Please tell me about the learner's performance in mathematics. Please explain.
    a. Has it changed in any way since joining the club?
    b. Has anything changed about the learner using different methods to solve the same problem? Please explain.
    c. Does the learner show the understanding that some problems can be solved using more than one method? Please explain.
    d. Does the learner accept or suggest other methods in class? Please explain.
4. Do you have anything else you would like to tell me about the learner pertaining to mathematics?
5. What are your views about the mathematics club? Why? Please explain?

Thank you very much.