GRADE 12 LEARNERS' DIFFICULTIES IN APPLYING THE CONCEPT OF PROPORTIONALITY IN TRIANGLES

MASTER OF EDUCATION (M.Ed)

(By Coursework and Research Report)

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DECLARATION

I **Khayalethu Myoli** declare that this research report is my own work and all contributions to this body of work are acknowledged through references. This report is submitted in partial fulfillment to meet the requirements of the degree of Master's in Education at the University of the Witwatersrand, Johannesburg. This research report has not been submitted to any university or for any other degree or examination.

Khayalethu Myoli

Signature: Date: 15 March 2025

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ABSTRACT

This research examines Grade 12 learners' difficulties when dealing with geometric problems involving proportionality in triangles. The study also explores the kinds of difficulties learners' experience when responding to similarity and proportional division theorem problems, and how their difficulties in similarity and proportional division affect their difficulties in complex geometric problems involving proportionality in triangles. Grade 12 learners' difficulties in these concepts are investigated through the theoretical lens of Kilpatrick et al. (2001) four strands of mathematical proficiency: conceptual understanding, procedural fluency, adaptive reasoning and strategic competence. A mixed method design is utilized to present, analyze and interpret learners' difficulties from their Preliminary 2023 and June 2024 Paper 2 examination scripts.

Grade 12 learners (two different cohorts) from two participant schools at Gauteng East district were purposefully selected to be part of this study. A total of 139 learners' scripts were selected for this study from both participant schools representing both the 2023 and 2024 cohorts. A total of 89 learners' scripts for the 2023 cohort from both participant schools were selected for quantitative analysis. A total of 50 learners' scripts for the 2024 cohort from both participant schools were also selected for both quantitative and qualitative analysis, but only those learners' scripts which showed relevant information were qualitatively presented and analyzed. Learners' difficulties in the concept of similarity due to their struggles with circle geometry concepts and similarity procedures negatively affected their chances of success in proportionality problems. Although learners' performances in proportional division theorem were better, this did not translate to the correct application of the concepts in complex proportionality problems. The study found that Grade 12 learners experience difficulties associated with all four strands, but these difficulties are worse in situations where learners are confronted with complex proportionality geometric problems involving strategic competence. The study also explored how different teaching approaches including the use of ICT's can be useful in addressing learners' difficulties.

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LIST OF ACRONYMS

AAA – Angle-Angle-Angle

AAS – Angle-Angle-Side

BWP - Box and Whisker Plot

CAPS – Curriculum and Assessment Policy Statement

CK – Content Knowledge

DBE - Department of Basic Education.

FET – Further Education and Training

GDE – Gauteng Department of Education

HOD – Head of Department (Mathematics)

HOTS – Higher Order Thinking Skills

ICT – Information and Communications Technology

KZN – KwaZulu Natal

NR – No Responses

NSC – National Senior Certificate

PISA – Programme for International Student Assessment

PST – Pre-service Teachers

STEM – Science Technology Engineering and Mathematics

TIMSS – Trends in International Mathematics and Science Study

USA – United States of America

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CHAPTER 1: CONCEPTUALISATION OF THE STUDY

1.1. Chapter 1 introduction

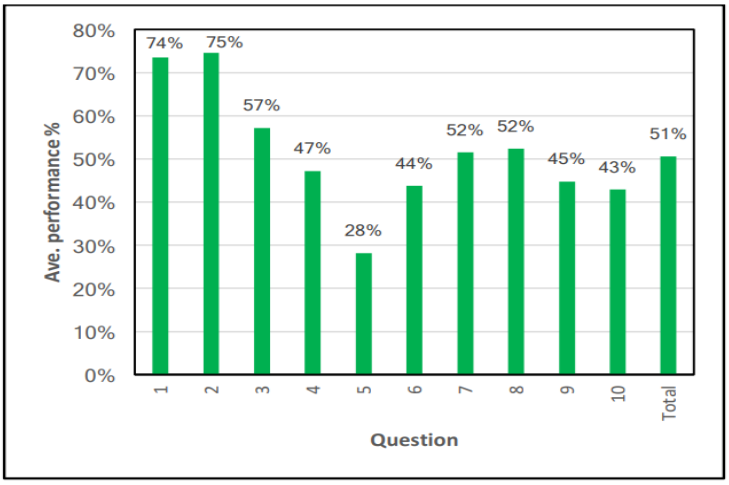
This chapter provides the background of the study by drawing from DBE annual diagnostic reports to reflect on the status of Mathematics education in South Africa. The problem statement, research purpose and significance of the study are outlined in detail. Brief summaries of all the seven chapters which form part of this study are also provided.

1.2. Background of the study and motivation

Mathematics Education in South Africa is at a crisis point, with far-reaching consequences for the countries' growth prospects since our country is experiencing a shortage of Science, Technology, Engineering and Mathematics (STEM) professionals (Sutherland, 2020; Langeni, 2020). Further, many learners in township schools are opting for Mathematics Literacy, leading to a further reduction of learners enrolled for Mathematics at Further Education and Training (FET) phase and one of the reasons is that they experience difficulties in geometry (Gweshe, 2014). These learners are somehow convinced that Mathematics presents challenges that are way beyond their capabilities.

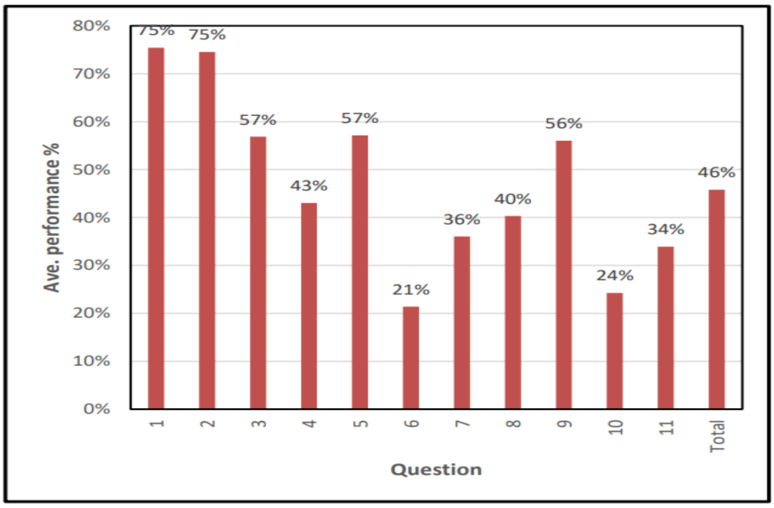
Mathematics as a school subject is not necessarily a global problem since there are a number of countries in Asia which are doing considerable well in this subject for example, Singapore, Japan and China (Leung, 2006). These countries have consistently performed well in international studies such as Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) (Leung, 2006), while South Africa continued to produce poor performances (Mji & Makgato, 2006; Ndlovu & Mji, 2012; Makhubele, 2014). In order to propose lasting solutions to this problem, it is crucial that learners' difficulties be thoroughly investigated and understood.

At the conclusion of the matric examination every year, the Department of Basic Education (DBE) conducts a question analysis of Grade 12 results and provides a diagnostic report based on sampled scripts. The analysis of learner performance in geometry (see Figure 1(b), 2(b) and 3(b)) for previous years does not show any significant improvement on problems involving proportionality. Quite frequently, questions involving proportionality are largely left unanswered or partially answered by learners (DBE, 2020; 2021). In instances where learners attempted solving these problems, several common errors and misconceptions leading to incorrect solutions are identified, such as, the inability to notice the connection between proportion and multiplication as observed in $3KN = 4SM$ which can be represented as $\frac{KN}{SM} = \frac{4}{3}$. This underscores learners' difficulties with the concept of proportionality and how it relates to both multiplication and division. Another notable finding by DBE (2021) is on learners' difficulties when dealing with problems of finding the missing side of a triangle by using similarity and proportionality. The ability to see the relationships between concepts and procedures plays such a crucial part in these types of problems since learners are required to make connections between how the sides of equiangular triangles relate to each other, giving rise to the concept of their proportionality.



Q	Topic/s
1	Data Handling
2	Data Handling
3	Analytical Geometry
4	Analytical Geometry
5	Trigonometry
6	Trigonometry
7	Trigonometry
8	Euclidean Geometry
9	Euclidean Geometry
10	Euclidean Geometry

Figure 1(a): Average performance per question in P2 (DBE, 2021, p.195).



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11	Euclidean Geometry

Figure 2(a): Average performance per question in P2 (DBE, 2022, p.198).

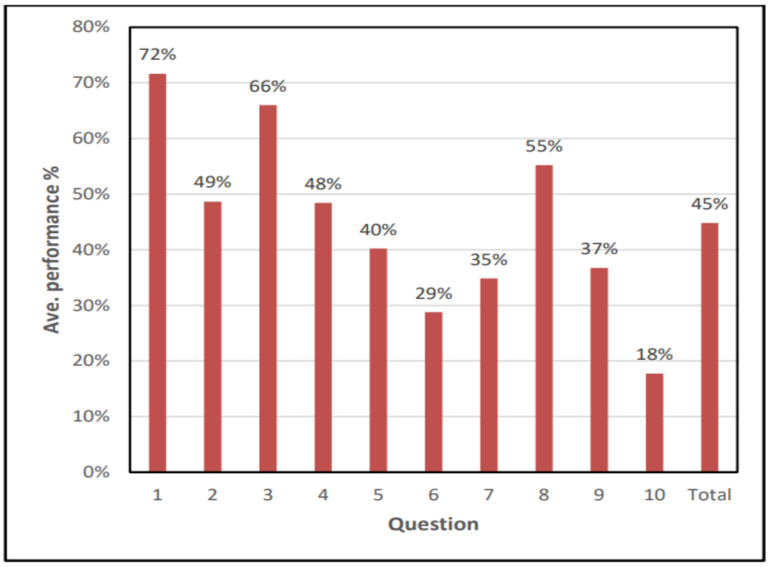

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Figure 3(a): Average performance per question in P2 (DBE, 2023, p.208).

A large number of learners seem to be taking advantage of the geometric concepts which require lower order thinking skills as can be seen through the higher average performances in Question 8 (DBE, 2021; 2022; 2023). These questions generally have slightly more marks compared to those that require *Higher Order Thinking Skills* (HOTS), consequently this serves as a saving grace for the geometry topic overall. This explains why, despite extremely poor performances in problems involving proportionality, geometry is not the worst in terms of average performance per sub-question (see Figure 1(b), 2(b) and 3(b)).

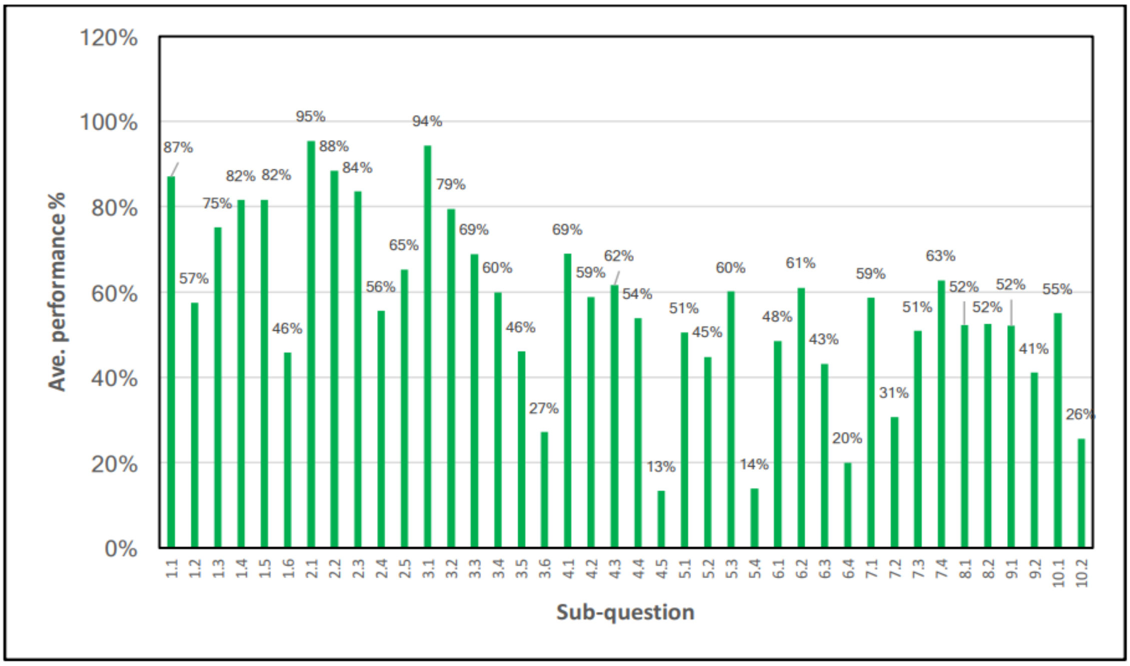


Figure 1(b): Average performance per sub-question in P2 (DBE, 2021, p.196).

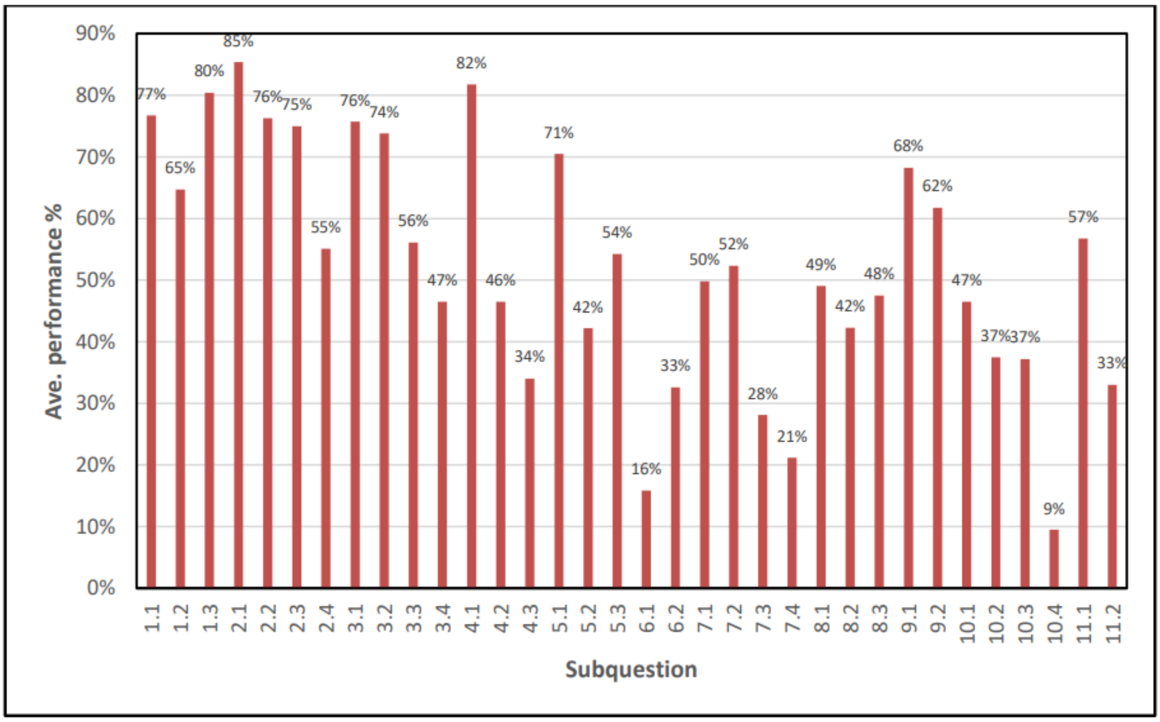


Figure 2(b): Average performance per sub-question in P2 (DBE, 2022, p.198).

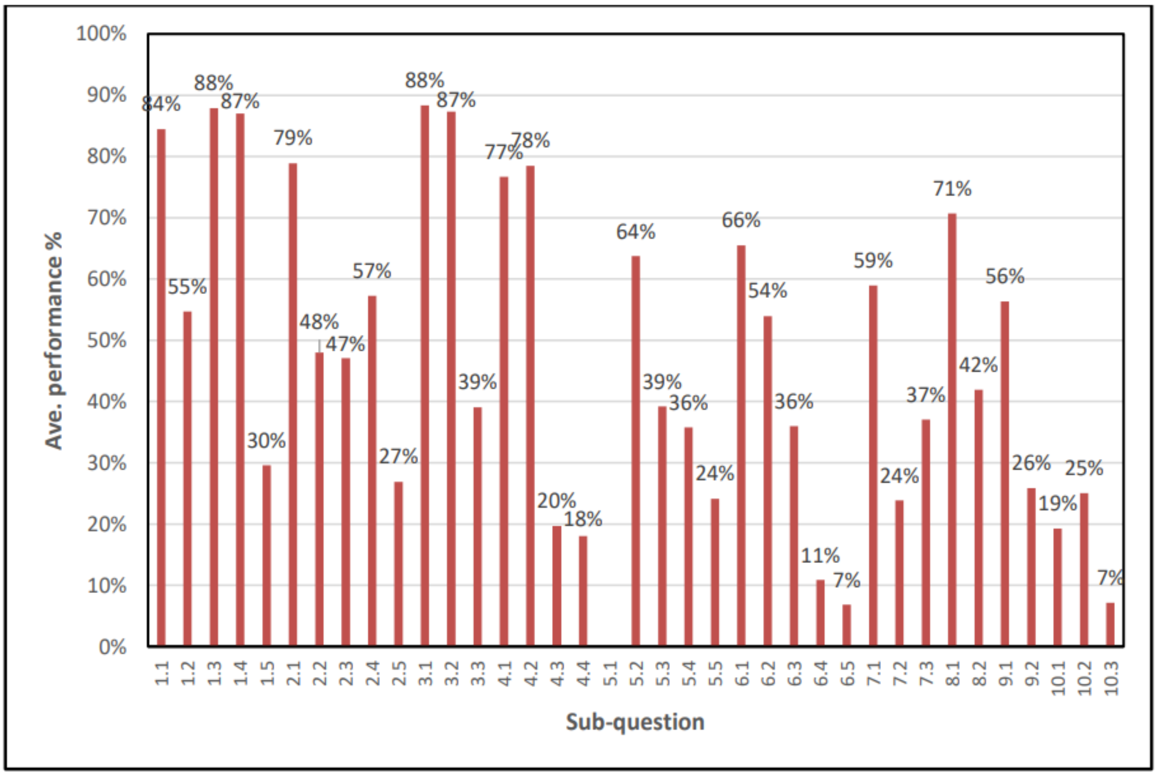


Figure 3(b): Average performance per sub-question in P2 (DBE, 2023, p.208).

Learners experience difficulties when applying the concept of proportionality in triangles as can be seen in previous NSC Grade 12 Mathematics Paper 2 Diagnostic reports (see attached graphs on **Figure 1(a) – 3(b)**). A common trend can also be observed when learners must deal with these types of questions, wherein many of them either do not attempt the questions or when they do, their solutions are riddled with errors due to various difficulties on ratio and proportionality. Moreover, it is noteworthy how the average performance for Question 10 is the lowest at 18% as seen in **Figure 3(a)**. The average performance for Question 10.2 is reported to be at 25% and 10.3 at 7% as seen in **Figure 3(b)** (DBE, 2023), and these are questions involving proportionality of sides of a triangle which require HOTS. In Question 10.2, learners seemed to experience difficulties in proving equal ratios since they incorrectly used proportionality without first proving the similarity of triangles involved (DBE, 2023). This points to learners' lack of conceptual understanding on the concepts of similarity and proportionality. In Question 10.3, learners failed to determine either of the two proportions (one from similarity and the other from proportional division theorem) and as a result they could not successfully solve the problem (DBE, 2023). This further shows how dire the situation is regarding learners' performance on the concept of geometric problems involving similarity and proportionality.

Gauteng East District continues to experience difficulties regarding Grade 12 learners' performance in geometry. Challenges in township schools are complex since some learners' difficulties may emanate from a range of factors such as socio-economic conditions, social issues and access to learning resources (Bayat et al., 2024). Further, a majority of learners in South African township schools are not English first-language speakers while they are expected to learn geometric concepts in English. However, this study's focus is on learners' difficulties as these relate to their experiences in Mathematics classrooms. Our preoccupation as Mathematics educationists is in that which is within our control, that is, the Mathematics classroom and our commitment in ensuring that every learner can succeed in their pursuit to learn Mathematics despite these challenges. Gerdes (2003) argue that all human beings can do geometry, hence Mathematics educationists have the responsibility to investigate learners' difficulties in geometry to help improve on learning and teaching outcomes. This study, therefore, adds to existing knowledge on learners' difficulties in geometry.

1.3. Problem Statement

The inclusion of the topic of geometry in the Mathematics curriculum is crucial since it connects to other topics, including problem solving in Algebra. Zita et al. (2021) provides three reasons why geometry is significant in the Mathematics curriculum: firstly it promotes reasoning capabilities; secondly it enhances spatial awareness; and thirdly it stimulates cognitive faculties in handling diverse information. Geometry provides students with tools for reflection and interpretation of their physical environment and serves as a gateway for studying other mathematical concepts (Clements, 2003). Clearly, learners' difficulties with geometry have implications for Mathematics as a whole. There's however consensus that secondary school learners across all learning phases tend to struggle in learning various subtopics of geometry due to a myriad of different reasons (Haj-Yahya, 2022; Makhubele, 2014). Grade 12 learners find it difficult to apply the concept of proportionality in triangles (DBE, 2025), and this may be rooted in their lack of proficiency in the geometric concepts of proportional division and similarity.

1.4. Research Purpose

The purpose of this research was to investigate Grade 12 learners' difficulties in the application of the concept of proportionality in similar triangles. This study also aimed to examine the extent to which these difficulties link to learners' proficiency in the concept of similarity and proportional division in triangles.

1.5. Research Questions

This research aims to contribute to knowledge already known about Grade 12 learners' struggles with geometry from DBE annual diagnostic reports by providing more insight on the kinds of difficulties learners' experience. The following three research questions were formulated to pursue this goal:

1.5.1. What are Grade 12 learners' difficulties when dealing with problems involving proportionality in triangles?

1.5.2. To what extent are these difficulties linked to learners' proficiency in the concept of similarity in triangles?

1.5.3. To what extent are these difficulties linked to learners' proficiency in the concept of proportional division in triangles?

1.6. Significance of the study

This study is significant because it illuminates learners' difficulties in geometric problems involving similarity and proportional division through Kilpatrick et al. (2001) four strands of mathematical proficiency. One hopes that once these learners' difficulties are fully exposed and understood, educators will be in a better position to address them. Examiners may also consider developing geometric problems which assess learners' proficiency through Kilpatrick et al. (2001) strands of mathematical proficiency, and to also include a wider pool of possible solutions in their marking guidelines. Further, resource materials developers can benefit from this study by including approaches which promote a deeper understanding of geometric concepts such as proportionality and ratio. Furthermore, curriculum developers may be encouraged to reconsider the exclusion of the concept of ratio and proportionality in Grade 11 CAPS curriculum.

1.7. Chapters' outlines

Chapter 1 draws from DBE (2021, 2022, 2023 and 2025) annual diagnostic reports for Mathematics to provide the background and motivation for the study. This chapter also provides the research questions, problem statement, purpose and significance of the study.

In Chapter 2, some of the literature relevant to the concept of proportionality and similarity is discussed. Further, the possible causes of learners' difficulties in the concepts of similarity and proportionality are discussed. Educators' struggles with these concepts which have the potential to affect learners' concept development are also discussed. Lastly, different possible remedies to these difficulties, including resource materials and ICT's are explored.

Chapter 3 provides the theoretical framework for this study. Learners' difficulties are investigated through Kilpatrick et al. (2001) four strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence and adaptive reasoning.

In Chapter 4, research design and methods are discussed. Sampling, data collection and analysis, validity and reliability issues are also discussed.

Chapter 5 provides an analysis of the September 2023 Preparatory or Preliminary (Prelim) and June 2024 question papers and marking guidelines of geometric problems involving similarity and proportionality. The Preliminary or Preparatory examination is administered in September in order to prepare matriculants for their final examination. This analysis is done through Kilpatrick et al. (2001) four strands.

Chapter 6 provides both quantitative and qualitative data analysis and presentation. Both Prelim 2023 and June 2024 learners' scripts are analysed quantitatively while only June 2024 scripts are analysed qualitatively.

Chapter 7 presents research findings, recommendations and conclusions. These findings are drawn from the data analysed and presented through a mixed-method design in chapter 6. This Chapter also links research findings to theory and highlights limitations.

1.8. Summary and conclusion

This chapter provided a background of the study drawing from DBE annual diagnostic reports for Mathematics. These reports have indicated that learners continue performing poorly in geometry and that there's no improvement in problems involving proportionality. This chapter also outlined the research problem statement, purpose and questions. The problem area is learners' poor performances in geometric problems involving similarity and proportionality because of the difficulties they experience when responding to these problems. The purpose is to investigate these difficulties to answer the research questions which were provided in this chapter.

CHAPTER 2: LITERATURE REVIEW

2.1. Chapter 2 Introduction

This chapter presents some challenges regarding pedagogy of geometry in the South African context. The historical background of the concept of proportionality is outlined. The notions of proportionality and difficulties are defined and discussed. This chapter also delves into how difficulties in the concept of proportionality can become obstacles to learning geometry, and how learners’ definitions of similarity can lead to difficulties with proportionality. Educators’ proportional reasoning in geometric contexts is also discussed. Lastly, the following remedies in addressing learners’ difficulties with these concepts are discussed: Task sequencing and development of different approaches to problem-solving; Integrating an instructional module and Information Communications Technologies (ICT’s) in teaching the concept of proportionality; and support of the development of the concept of proportionality in Mathematics school textbooks globally.

2.2. Pedagogy of geometry in South Africa

Although there’s several studies on similarity and proportionality in triangles in the topic of Euclidean geometry in earlier Grades as evidenced by the literature discussed in the following paragraphs, there’s paucity of studies on learners’ difficulties with these concepts in Grade 12, especially in the South African context. It is important to further highlight that the South African context presents unique challenges with regards to the pedagogy of geometry since this topic was removed from the core curriculum in 2008 and later returned with the introduction of the CAPS curriculum (Ubah, 2022; Ndlovu & Mji, 2012). This had far-reaching implications not only for learners but for pre-service teachers (PST’s) as well, especially those who did not learn geometry at FET level (Ubah, 2022) and those who’s undergraduate studies did not include this topic at university level.

2.3. Historical background of the concept of proportionality

Malet (1990) provides a historical background of the notion of ratio and proportionality in pre-modern era right up to the beginning of modernity. In medieval times, ratio and proportionality were used for representing relations between numerical measurements mainly for practical purposes (Malet, 1990). With the advent of symbolic Algebra, the notion of proportionality and ratio evolved to algebraic representations involving variables as seen through the work of

Galileo and Newton in the 17th century (Malet, 1990). During the 17th century, proportionality played a prominent role in expressing functional relationships and dependency similar to how it was used in pre-modern times (Malet, 1990). It was also during the 17th century that the concept of fractions replaced proportions since it is only feasible to perform mathematical operations when proportion is expressed in the form of a fraction (Błaszczyk & Petiurenko, 2019), for example, one cannot simple apply the multiplicative inverse on a proportion to solve for the unknown while this can easily be done once the proportion has been expressed as a fraction. Malet (1990) further argues that it was only in the 18th century that proportionality and ratio gained their modern status when the notion of a function fully emerged. In post-modern times, the development of the concept of proportionality through the ages may have significance on the difficulties Mathematics (geometry) educators and learners experience, because how these mathematical concepts are defined has an impact on how they are understood.

2.4. Proportionality and difficulties

Wijayanti (2019) defines proportionality as a relationship between two quantities such as length and area in such a way that correspondence between them exists. In geometry this may be a relationship between two similar figures or a relationship between line segments divided by a line parallel to one side of a triangle as can be seen in Diagram 1 and 2.

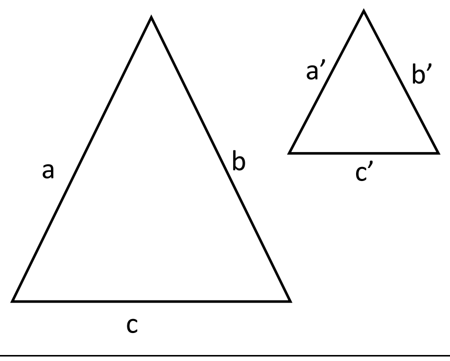
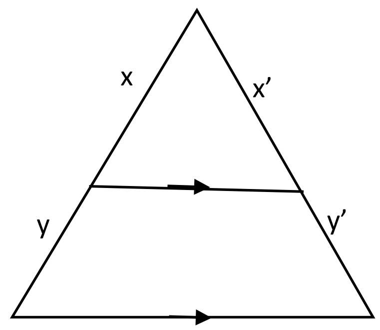
	

Diagram 1: Similar triangles with corresponding sides	**Diagram 2: Line parallel to one side of a triangle**

Hornby (2015) defines difficulty as a challenging situation presenting problems. Consequently, learners’ difficulties in Mathematics education can be defined as challenges learners experience as they attempt to solve mathematical problems, resulting in incorrect or inadequate solutions. These difficulties may originate from lack of comprehension of the foundational concepts of mathematical problems, or challenges with relating mathematical concepts to each other and to relevant procedures (Kilpatrick, 2001). This is an important skill in geometry since learners frequently must prove proportionality of sides of triangles from given geometric riders. Lack of this skill may lead to learners experiencing difficulties.

Casanova et al. (2021) investigated learners' geometric thinking on triangles and concluded that there is still more that needs to be done to help learners develop proficiency. This study found that learners had difficulties connecting concepts to each other and mostly relied on memorization and could not use relevant reasons in supporting their solutions. The authors bemoan the global crisis of learners' experiencing difficulties in mastering geometric concepts, especially those involving formal proof. Learners' difficulties with formal proof are relevant to this study since both examinations under investigation assess learners' proficiency in the formal proof of similarity and proportional division theorems.

Abakah and Brijlall (2023) conducted a qualitative study on grade 11 learners' proficiency in geometry through Kilpatrick et al. (2001) five strands of mathematical proficiency. Their study analyzed learners' marked solutions, oral responses, investigation activities and classroom observations. Although this study's focus was on learners who attained proficiency, it found that a majority of the Grade 11 learners had difficulties associated with all the five strands in geometry.

In this study, learners' difficulties were investigated in terms of Kilpatrick et al. (2001) four strands of mathematical proficiency through a mixed method design.

2.4.1. Difficulties encountered by learners in proportional reasoning and challenges in learning geometry

Proportionality has been described as both the pinnacle of elementary education and the foundation of advanced mathematical studies (Dole, 2010). Given its strong association with numerous concepts and skills introduced in the middle grades, it is seen as a unifying theme to integrate the middle-grade school curriculum (Dole, 2010). The DBE (2025) annual diagnostic reports reveal that learners continue to struggle with geometric problems involving proportionality. This is evident in Figure 4, where Question 11.4 – focused on similarity and proportionality – had the lowest performance, with only 10% of learners answering it correctly. This shows that Grade 12 learners nationally continue to have difficulties with similarity and proportionality, and that more still needs to be done in making sense of their difficulties.

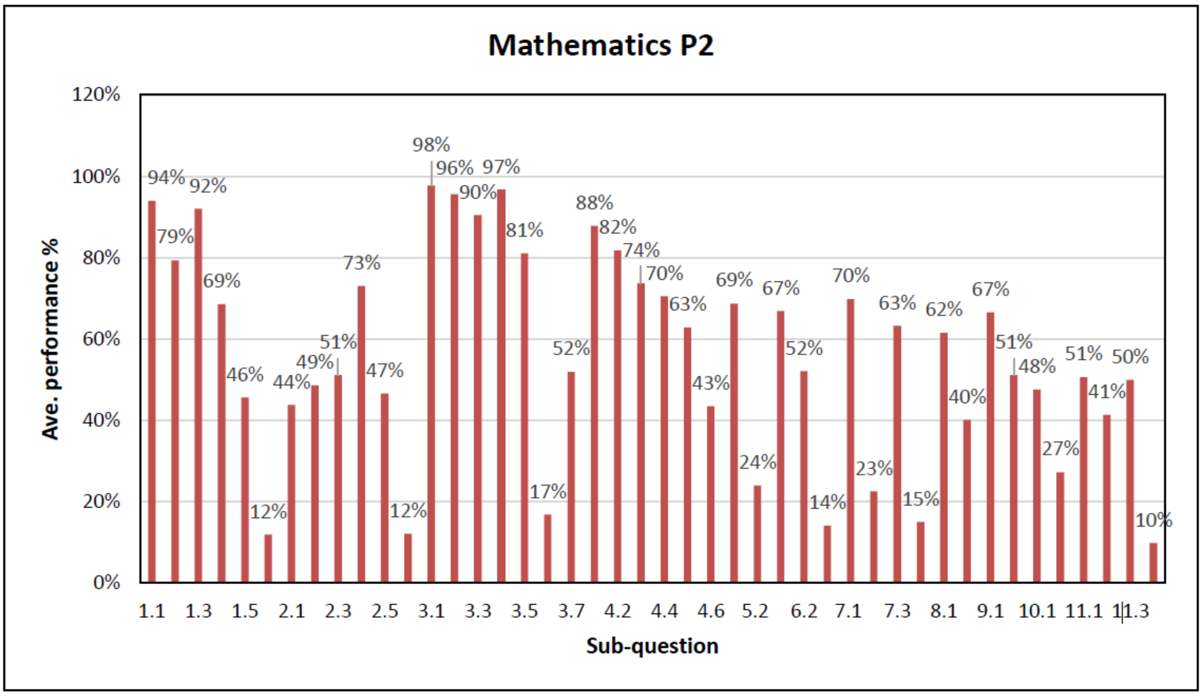


Figure 4: Average performance per sub-question in P2 (DBE, 2025, p.232)

Fathiyah et al. (2024) explored Grade 8 learners' proportional reasoning abilities in solving proportional and non-proportional problems at a Bandung city school in Indonesia. Their study focused on learners' abilities in solving problems involving the missing value, comparison of numbers and qualitative comparison. Fathiyah et al. (2024) study found that learners continue to experience difficulties when dealing with problems involving proportionality, and that they struggle to differentiate between proportionality and non-proportionality contexts. Further, they found that learners incorrectly use cross-multiplication procedures and do not seem to have developed meaningful understanding of the correct use of this procedure in proportionality problems. There were instances where some learners who managed to get correct solutions by using cross-multiplication, still could not explain why this procedure was effective, and this points to weaknesses in learners' proportional reasoning competencies (Fathiyah et al., 2024).

Mahlabela's (2012) study investigated learners' errors and misconceptions on the concepts of proportion and ratio for rural KZN Grade 9 learners. The study used unmarked learners' scripts as part of a qualitative approach in the investigation of learners' errors and misconceptions. Mahlabela's (2012) study found that learners' struggles with these concepts emanate from the lack of deeper understanding and incorrect use of procedures in solving problems involving proportion and ratio. Mahlabela's (2012) main concern from the results of the study is that a majority of learners had major challenges in formulating proportions correctly due to their struggles with cross multiplication procedures. This is an indication that learners as far back as Grade 9, have difficulties in reasoning proportionally.

Govender (2016) compared the 2014 and 2015 geometry sections of Mathematics paper 2 and argued for innovative ways of assessing geometry to promote its accessibility to learners. These are the first two consecutive Paper 2 external examinations in which geometry reappeared as a compulsory component of the Mathematics assessment, following its inclusion as an optional Paper 3. When comparing 2014 and 2015 eternal examinations, Govender (2016) points out that the 2015 geometry component of the examination was difficult for learners as compared to the 2014 examination. For instance, some learners were unable to determine ratios emanating from similarity and proportional division, and also failed to find the necessary links between questions. Another finding by Govender (2016, p.241) was on learners' struggles with non-routine problems which require "higher order-reasoning and processes" such as dismantling the problem to reveal its underlying structure. Govender relied on DBE diagnostic reports and the deliberations and submissions by Association for Mathematics Education of South Africa (AMESA) to identify trends in learner performance. However, in this study, learners' difficulties in similarity and proportional reasoning were investigated in terms of the analysis of learners' marked scripts and the results will be compared with DBE's national trends.

2.4.2. The impact of learners' incorrect definition of similarity on their understanding of proportionality

Haj-Yahya (2022) examined learners' conceptions regarding the definitions of congruent and similar triangles. Haj-Yahya (2022) study is an investigation of the acceptance or non-acceptance of similarity or congruency theorems as sources of definitions and how this can bring insight on how learners conceive mathematical definitions. One finding by Haj-Yahya (2022) is that knowledge of similarity and congruency theorems did not translate into knowledge about their definitions. This means that some learners may have faulty definitions of similarity and congruency although they may have knowledge of the theorems. For instance, some learners who are proficient in proving both similarity and congruency lack the knowledge that all congruent triangles are similar and not all similar triangles are congruent. Another finding is that the acceptance of one theorem as constituting a definition for the concept did not translate to the acceptance of the other theorem (Haj-Yahya, 2022). For example, a learner may have accepted that the similarity theorem constitute the definition of the concept of similarity but fail to accept that the same is true for congruency. This points to learners' struggles resulting from their inability to effectively analyze theorems. Consequently, this contributes to learners' misconceptions with regards to the concept of proportionality since proportionality and similarity are interlinked, because similarity is one of the necessary conditions for the proportionality of sides of triangles. Further, similarity is linked to congruence because two congruent triangles are also similar, but similar triangles are not necessarily congruent (Haj-Yahya, 2022; Güneş, 2022; Shahbarri & Daher, 2020; Mapedzamombe, 2020). This highlights the importance of correct mathematical definitions since difficulties in defining similarity and congruency can hinder learners' ability to define and apply proportionality.

Güneş (2022) also argues that definitions are very important in teaching and learning Mathematics. Moreover, the lack of adequate definitions of concepts among educators often results in senior high school learners struggling to define geometric terms and describe the characteristics of geometric shapes (Güneş, 2022). Güneş (2022) study showed that only 15% of participant learners were at the third level of van Hiele levels for definition of congruent triangles, and only 2% for similarity since they opted to provide both minimal and non-minimal definitions of similar and congruent triangles. Most participant learners were at van Hiele level 2, highlighting difficulties in defining these important geometric concepts.

Casanova et al. (2021) study also found that learners had difficulties differentiating between congruence and similarity of triangles. Learners had difficulties understanding triangles since they used incorrect terminologies and representations due to their poor comprehension of definitions (Casanova et al., 2021). The role of notation and language in conceptual understanding development is crucial since resource materials also use language and notation for development of concepts. Shahbarri and Daher (2020) also found that learners confused definitions of congruent triangles with those of similarity before ethnomathematics-based learning was introduced as an intervention strategy. Although this study is not about congruence, the researcher anticipated that some learners may experience difficulties in problems involving similarity and proportionality because of confusing congruence with similarity.

2.4.3. Teachers’ proportional reasoning in geometric contexts

Arican and Özçakir (2021) investigated and reported on over-application of proportionality by 17 Pre-service Teachers (PST’s) when solving similarity problems with non-proportional relationships in geometric contexts. Initially, PST’s experienced difficulties due to their inability to work out areas of irregular figures, but there was improvement after the implementation of the augmented reality tasks (Arican & Özçakir, 2021). These augmented reality tasks enabled PST’s to discover different representations of similar figures, thereby significantly decreasing their overuse of proportionality in non-proportional contexts. The overuse of proportionality in non-proportional contexts is also an indicator of conceptual understanding difficulties.

Orrill and Burke (2013) investigated four in-service middle grades educators’ proportional reasoning in geometric contexts. These in-service educators were given three proportionality tasks and the focus was on observing how they would draw on their knowledge base to solve each task, with emphasis on similarities and differences in their unique approaches (Orrill & Burke, 2013).

The findings of their study are that there were differences in how these participant educators invoked multiplicative reasoning, although this was deemed not necessarily sufficient for the three proportionality tasks since there were other important resources such as understanding equality of proportions and a constant in proportions. Interestingly, all four participant educators did not use the constant of proportions in their problem-solving (Orrill & Burke, 2013), and this underscores the need for teacher development in this regard.

Another prominent finding by Orrill and Burke (2013) is that educators may not necessarily invoke all the knowledge they possess in specific contexts. For example in cross-multiplication problems involving a missing value, these educators heavily relied on their procedural fluency to address the problem. This may deprive learners of the opportunity to acquire deeper understanding of the concept of proportionality even though the educators may have this conceptual understanding in their knowledge repertoire (Orrill & Burke, 2013). Similarly in this study, although the focus is on learners rather than educators, their solutions may reveal difficulties with proportionality when they rely on procedures instead of deeper conceptual understanding.

2.5. How can these difficulties be addressed in order for learners to develop proficiency in solving problems involving similarity and proportionality?

The following is a list of teaching strategies and resources discussed in this section: Task sequencing and development of different approaches to problem-solving; Integrating an instructional module and Information Communications Technologies (ICT's) in teaching the concept of proportionality; and how school mathematics textbooks support the understanding of proportionality. These strategies and resources may be crucial in developing recommendations for helping learners overcome their challenges and achieve proficiency in the concept of proportionality.

2.5.1. Task sequencing and development of different approaches to problem-solving

Lee and Yim (2014) investigated how problem-solving (coffee problem) supported PST's in recognizing proportionality as an integrated concept through the task of sequencing the works of others and their own from a developmental standpoint. Sequencing the works of others involves providing a step-by-step sequence of how someone solved a problem in order to make sense of their different forms of representations. Lee & Yim (2014) study found that the sequencing of others' work and their own by PST's seemed to carry some benefits since it provided them with opportunities to reflect on the adequacy of the depth of their knowledge in helping others to improve. In this way, as PST's improved their understanding through sequencing of a problem involving proportional reasoning, they also developed different approaches for solving the problem and identified potential challenges that their learners may experience when solving these types of problems (Lee & Yim, 2014). This created a double developmental trajectory for PST's because as they developed their Content Knowledge (CK), their didactical perspectives also developed (Lee & Yim, 2014).

Maweya and Pule (2024) provided a study to help Grade 10 learners make sense of the concept of similarity in triangles by investigating the effectiveness of classroom design intervention strategies. Maweya and Pule (2024) used learners' solutions in their investigation through a mixed-method design. They found that learners continue to struggle with some parts of Euclidean geometry, especially similarity concepts and its vocabulary, and that learners' understanding and engagement can be improved through code switching. Further, they advocate for the consistent and gradual introduction of the concept of similarity in triangles throughout FET. The researcher agrees with this view because the current curriculum excludes similarity in Grade 11 until it reappears in Grade 12.

Malcı (2021) used eye tracking technology to investigate how learners' difficulties and geometric questions affected them as they were attempting to find solutions. According to Malcı (2021) the eye tracking technology is a device which tracks learners' eye movements as they solve problems to determine their eye-fixation techniques. The study found that some learners struggled to relate geometric concepts to each other and to remember rules, hence they could not complete tasks (Malcı, 2021). Most importantly, the study found that when learners had to deal with problems involving similarity, those who experienced difficulties and were unsuccessful tended to be fixated on the known and unknown variables (Malcı, 2021). This calls for educators to emphasize on teaching learners effective problem-solving approaches to improve their ability to develop proficiency in solving non-routine problems.

Although this study is on learners' difficulties, the researcher holds the view that these difficulties can be addressed by educators through effective learner-centered classroom practices.

2.5.2. Integrating an instructional module and ICT's in teaching the concept of proportionality

Information Communications Technology (ICT) tools have rendered many traditional Mathematics teaching approaches irrelevant in modern classrooms because they put focus on concepts and help educators save time (Jojo, 2016). More than ever before, it is crucial to help learners attain a higher level of proficiency in Mathematics, and to achieve this the teacher must also be empowered with the skills to leverage technology (Coronado et al., 2017). Jojo (2016) conducted a case study on an FET educator's reflections in using the GeoGebra app. The study found that by using the GeoGebra app, this inexperienced Mathematics educator was able to cover content timeously while addressing knowledge gaps on learners' understanding of concepts and while enhancing conceptual understanding through discovery in geometry. The participant educator also acknowledged that the use of GeoGebra in geometry and other Mathematics topics promotes meaningful teaching and learning (Jojo, 2016).

Restrepo-Ochoa et al. (2023) conducted a study aimed at the creation of an instructional module with the primary objective of fostering the advancement of learning processes. They achieved this through the examination of the learning trajectories undertaken by learners in the context of studying geometric proportionality within topics involving similarity, homothecy and

Thale's theorem. This instructional module has been carefully created in line with the fundamental principles derived from the levels in the van Hiele model, mathematical visualization and the utilization of GeoGebra (Restrepo-Ochoa et al., 2023). Their findings showed a progression in learners' cognitive development, specifically learners evolved from lacking coherent comprehension of the concept of similarity to successfully formulating a comprehensive definition of similarity, proportionality and homothecy (Restrepo-Ochoa et al., 2023). Moreover, learners developed the ability to effectively determine criteria for similarity of differently positioned geometric figures in space and improved their visualization skills and mathematical language (Restrepo-Ochoa et al., 2023). The utilization of GeoGebra software as a mediating resource in teaching the concept of similarity and proportionality clearly has benefits in Mathematics classrooms, and these benefits include the effective treatment of learners' difficulties.

Coronado et al. (2017) investigated the influence of the Geometer Sketchpad (GSP) on van Hiele levels and geometry proof writing of second and third year high school science learners in the Philippines. The study relied on analysis of learners' pre-test and post-test marked scripts. The Geometer Sketchpad is a computer software which allows learners to construct diagrams, adjust measurements and drag items in order to observe relations (Coronado et al., 2017). It also promotes learning through discovery by allowing learners to explore patterns, properties and to approach concepts from various perspectives (Coronado et al., 2017). The pre-test results show that the majority of learners obtained a score of between 0 and 1 on proportionality division proof before learning through GSP (Coronado et al., 2017). After the use of GSP, there was an improvement in post-test performance showing the effectiveness of the GSP on teaching and learning proof writing (Coronado et al., 2017).

It is clear that ICT's play a significant role in addressing learners' difficulties in geometry. So, although this study is not about ICT's, one hopes to not just identify learners' difficulties in geometric problems involving proportionality but also make recommendations which can help address these difficulties.

2.5.3. How school mathematics textbooks support the understanding of proportionality

Mathematical resources used by both educators and learners have an impact on the difficulties learners encounter. da Ponte and Marques (2011) selected Mathematics textbooks between Grade 6 and 7 from four different countries (Portugal, Brazil, Spain and USA) in order to show

how the concept of proportion is introduced and developed. They found some similarities and differences on how the different textbooks from these countries supported development of the concept of proportionality in learners. Wijayanti's (2015) study also focused on investigating how Indonesian textbooks introduce and deal with the concept of proportion in geometry and arithmetic, and how textbooks connect these concepts to each other. Wijayanti's (2015) study found that the introduction of the concept of proportion in arithmetic topics is more informal whereas in geometric topics the definition is through similarity and hence formal. A further finding by Wijayanti (2015) is that textbooks distinguished between two mathematical related kinds of tasks, between the arithmetic and geometric topics but the connection between these tasks is weak. Consequently, this may lead to learners experiencing difficulties with this concept of proportionality as they may perceive Mathematics as a disjointed body of knowledge.

Textbooks play an important role in learning Mathematics, and how geometric concepts such as similarity are dealt with in these textbooks has an impact on learners' ability to attain proficiency (Mapedzamombe, 2020). The researcher anticipates that some of the difficulties learners' experience in geometric problems involving proportionality and similarity may be caused by how these concepts are presented in textbooks, and that recommendations in this regard may be helpful.

2.6. Summary and conclusion

This chapter provided the historical background of how the concept of proportionality developed over the years and the implications as a result of how it is now defined by learners and educators. A literature-based discussion on learners' incorrect definitions of similarity and how these may contribute to difficulties in the concept of proportionality has been presented. Educators' struggles with the concept of proportionality were discussed. The review of literature revealed that learners and educators globally have difficulties with the concepts of similarity and proportionality. Possible remedies in addressing these difficulties were explored, including those involving development of resource materials (textbooks), integration of ICT's and development of better teaching strategies and teacher content knowledge.

CHAPTER 3: THEORETICAL FRAMEWORK

3.1. Chapter 3 introduction

In this chapter, the study's theoretical framework is discussed, followed by linking it to the research questions. The theoretical framework of this study is based on Kilpatrick et al. (2001) five strands of mathematical proficiency and these strands are discussed in line with the requirements of the study. The basis for selecting this theoretical framework and how it connects to the research questions is also outlined.

3.2. Theoretical foundations of the study

A theoretical framework is the research plan and it creates a structure for the definition of the study drawing from formal theory (Grant & Osanloo, 2014; Ngulube et al., 2015). In this way, a theoretical framework is composed of the theories which underpin research thinking and plans about the topic of study (Grant & Osanloo, 2014). According to Vos et al. (2011), research begins with the organising picture of that which is to be investigated. This image is essentially the framework of the study which includes concepts and applicable theory, and how these relate to each other. Ngulube et al. (2015) posits that the theoretical framework is important in focussing the study by shaping research questions, and is also useful in the interpretation of research findings. This study, therefore, focusses on learners' difficulties with the concept of proportionality in geometry of triangles, and how these difficulties may be connected to their experiences in the concept of similarity and proportional division theorem.

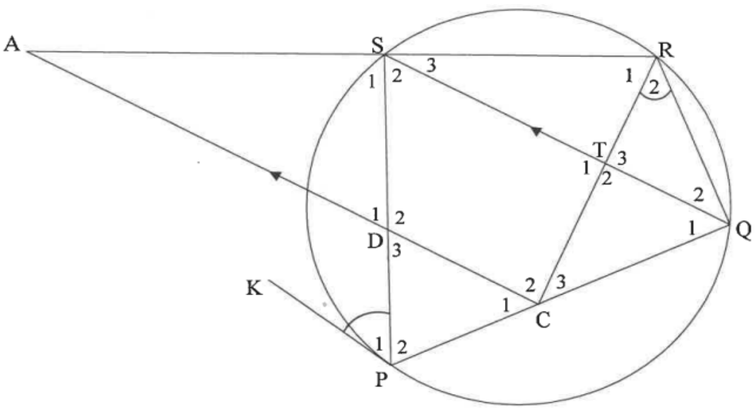
It becomes evident that learners are experiencing difficulties when they lack the proficiency to present correct and error-free solutions. Hence, in this study the researcher investigated these difficulties through Kilpatrick et al. (2001) 'Five strands of mathematical proficiency' as the main Theoretical Framework.

This framework is selected on the basis that it provides a useful theory for the analysis of learners' scripts in the investigation of the difficulties they experience as they struggle with geometric problems involving proportion. An in-depth investigation of the kinds of difficulties learners experience is bound to illuminate the difficulties learners also experience with regards to the concept of proportional division and similarity since these three are interconnected.

3.3. The 'Five strands of mathematical proficiency'

Grade 12 learners experience difficulties with the concept of proportionality (DBE, 2025) due to their lack of proficiency in this mathematical concept (Kilpatrick, 2001). The difficulties learners experience in this concept prevent them from succeeding in Mathematics. Kilpatrick et al. (2001) defines mathematical proficiency in terms of the five strands as that which is necessary for mathematical learning to be successful. Kilpatrick et al. (2001) further argues that mathematical proficiency is not a zero-sum game since learners may have proficiency in certain aspects of the topic and experience difficulties in others. A typical example of this is how some learners may be proficient in proving similarity of triangles (one of the necessary conditions for proportionality) yet experience difficulties in representing proportionality of corresponding sides. These learners will have difficulties in solving a problem such as Question 10.2 (Diagram 3), but could easily prove similarity in other problems where the triangles involved are identified beforehand.

In the diagram, PQRS is a cyclic quadrilateral. KP is a tangent to the circle at P. C and D are points on chords PQ and PS respectively and CD produced meets RS produced at A. CA || QS. RC is drawn. $\hat{P}_1 = \hat{R}_2$.



Prove, giving reasons, that:

10.1	$\hat{S}_1 = \hat{T}_2$	(4)
10.2	$\frac{AD}{AR} = \frac{AS}{AC}$	(5)
10.3	$AC \times SD = AR \times TC$	(4)

Diagram 3: National Senior Certificate Mathematics P2 November 2022 (DBE, 2022, p.13)

According to Kilpatrick et al. (2001), mathematical proficiency is composed of five intertwined strands. In the following sub-sections the following five strands for mathematical proficiency as a theoretical framework for this study are presented: conceptual understanding, procedural fluency, strategic competency, adaptive reasoning and productive disposition.

3.3.1. Conceptual understanding

This strand refers to a deeper attainment of mathematical concepts in an interconnected way (Kilpatrick et al., 2001). Kilpatrick et al. (2001) posits that lack of conceptual understanding may lead to the overextension of knowledge beyond its context, however, learners with conceptual understanding are aware of the applicability limitations of their mathematical ideas. Learners' prior-knowledge plays such a crucial role in the acquisition of conceptual understanding, since learners build on previously acquired knowledge to develop new mathematical ideas (Ay, 2017). When facts and concepts are integrated it becomes easier to remember and apply them correctly in problem situations (Kilpatrick et al., 2001).

Further, Kilpatrick et al. (2001) argue that conceptual understanding does not have to be proved through verbal utterances since learners often attain conceptual understanding before they can even be able to verbalize it. Therefore, as educators we are able to observe their conceptual understanding or lack thereof in Mathematics through their solution attempts. One way of knowing that learners have attained conceptual understanding in proportionality is through their ability to transition from graphical representations (for example given triangles) to equations depicting proportionality of corresponding sides. A notable finding by the DBE (2023) is that on Question 10.3 (Diagram 3) learners had difficulties in determining either of the two proportions and subsequently failed to develop the required equation depicting proportionality of sides, which had to be transformed through cross-multiplication to obtain the solution. These kinds of difficulties are also prevalent in Provincial Preliminary examinations where the level of difficulty is usually slightly higher than the final NSC examinations.

It is evident that the ability to relate concepts and procedures plays such a crucial role in conceptual understanding since learners can attain mathematical proficiency when they are able to make meaningful connections between different types of mathematical representations, procedures and concepts (Kilpatrick et al, 2001). A typical example of this is how proficiency in Question 10 (Diagram 3) is dependent on learners' ability to make connections between several different concepts such as the Tan-Chord theorem, Exterior angle of a triangle, Exterior angle of a cyclic quadrilateral, similarity, proportional division theorem and cross-multiplication. Further, learners' definitions of these concepts have an impact on whether or not they understand them (Haj-Yahya, 2022). Meaningful connections cannot be possible when learners' definitions of concepts are faulty. Furthermore, learners are struggling to identify these concepts from the given diagram and question statement, and to represent them in the form of mathematically acceptable geometric statements and reasons.

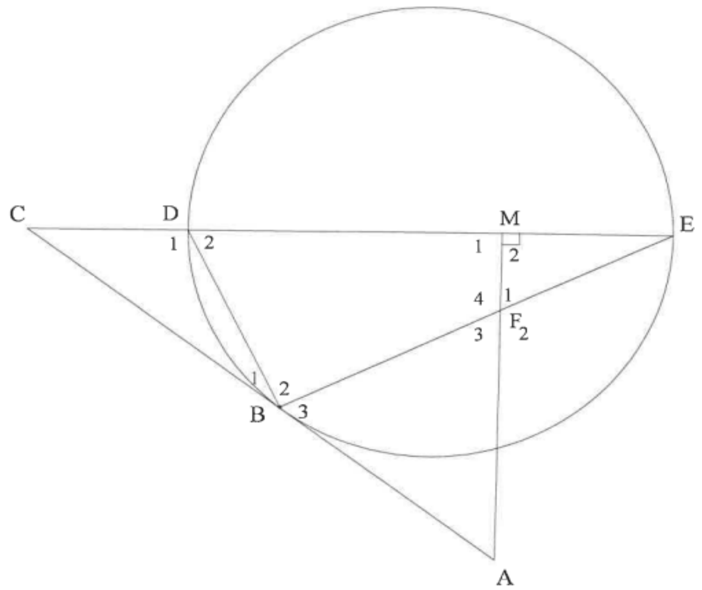
Kilpatrick et al. (2001) points out that conceptual understanding is important in minimizing errors. Nokes et al. (2009) argued that experts differ from novices in that experts are fluent in correcting their errors. A learner (novice) who lack conceptual understanding on proportion may not be able to correct the following error: 3KN = 4SM translates to $\frac{KN}{SM} = \frac{3}{4}$ whereas an expert can quickly realize that there's a mistake since the solution is not reversible to the first equation through cross-multiplication. Although the focus of this study is not on errors, but learner difficulties may reveal themselves in the form of errors.

3.3.2. Procedural fluency

This strand refers to one's knowledge of mathematical procedures and their ability to utilize them correctly, efficiently, flexibly and accurately within relevant contexts (Kilpatrick et al., 2001). In geometry, procedural fluency is particularly important especially when it comes to deductive geometric proving (Ndlovu & Mji, 2012). Some of the difficulties learners experience relate to their lack of fluency in writing logical and relevant geometric statements and reasons. A typical example of this can be observed in Question 10.2 (see Diagram 3) where a number of learners tend to begin their proof by making the assertion that $\frac{AD}{AR} = \frac{AS}{AC}$ which is the very same thing which still needs to be proved, and the common erroneous reason provided by learners in this case is "Given". This is clearly a difficulty originating from learners' lack of fluency in proving procedures.

QUESTION 10

In the diagram, a circle passes through D, B and E. Diameter ED of the circle is produced to C and AC is a tangent to the circle at B. M is a point on DE such that AM ⊥ DE. AM and chord BE intersect at F.



10.1	Prove, giving reasons, that:		
	10.1.1	FBDM is a cyclic quadrilateral	(3)
	10.1.2	$\hat{B}_3 = \hat{F}_1$	(4)
	10.1.3	ΔCDB ||| ΔCBE	(3)
10.2	If it is further given that CD = 2 units and DE = 6 units, calculate the length of:		
	10.2.1	BC	(3)
	10.2.2	DB	(4)
			[17]

Diagram 4: National Senior Certificate Mathematics P2 November 2020 (DBE, 2020, p.14)

Another crucial aspect in procedural fluency is one's ability to anticipate results from a specific procedure (Kilpatrick et al., 2001), for example in Question 10.2 (see Diagram 4) a large number of learners could not see the link between Pythagoras Theorem, similarity and proportionality which should lead to the determination of the length of the missing sides BC and DB.

It is also important for learners to use mathematical procedures in a flexible way since not all problem situations require the same procedure (Kilpatrick et al., 2001). A good example of this, is how some geometric problems may require learners to prove proportional division leading to proportionality of sides while others may require similarity. Lack of procedural fluency in this regard has led many learners to choose an inappropriate procedure in their struggle to solve problems involving proportionality.

Moreover, it is worth noting that procedural fluency require understanding of procedures and concepts. This agrees with Kilpatrick et al. (2001) view that these strands are intertwined. It might prove to be a difficult task for learners to be fluent in procedures when they still have conceptual misunderstandings. Also, Kilpatrick et al. (2001, p.134) assertion that "it is not always necessary, useful or even possible to distinguish concepts from procedures because understanding and doing are interconnected in such complex ways" gives more credence to the argument for the interconnectedness of these strands.

3.3.3. Strategic Competence

This strand refers to one's ability to create mathematical problems, represent them mathematically and determine their solutions (Kilpatrick et al., 2001). So, in secondary school geometry, learners are frequently given complex geometric problems which are not clearly defined such as Question 10.2 (see Diagram 3), where they are required to prove that $\frac{AD}{AR} = \frac{AS}{AC}$ and there's nothing said about similarity and proportional division. In this scenario, learners must first formulate a problem of either proving similarity or proportional division on some unidentified pair of triangles. Once a decision has been taken on the applicable procedure, then the procedure must be presented following a logical reasoning structure which will eventually lead to the proportionality of sides as the required solution. Now, a large number of learners seem to lack the strategic competency to formulate this problem within Question 10.2, hence their struggles in adequately answering the question. Further, some learners exhibit difficulties in correctly representing their problem solutions as demonstrated by no responses or by a series of meaningless and incoherent geometric steps.

In geometry, the effective approach in dealing with these kinds of problems involving proportionality is through a process of deconstruction, i.e. taking apart of proportions to create a pair of triangles which might have led to the proportionality of sides. In this study we refer to this process as **Problem Interpretation**. In instances where no triangles are possible, one must consider the proportional division theorem from the given geometric rider. Depending on one's findings, the problem to be formulated could be that of proving similarity or proportional division or both. In this way, learners develop the required flexibility to solve non-routine problems, i.e. problems that are not clearly defined (Kilpatrick et al., 2001).

Strategic competence may also involve multiple strategies, representations and the learners' ability to select the appropriate strategy (Kilpatrick et al., 2001). Problems involving multiple strategies require learners to make connections with different representations for example the use of the given triangle diagrams or the reliance on the order of appearance of the letters of triangles to determine corresponding sides to formulate proportionality. Maweya and Pule (2024) refer to the use of the order of appearance of letters of triangles in determining corresponding sides as the 'Happy face' approach.

There's clearly a connection between conceptual understanding, procedural fluency and strategic competency since learners cannot achieve strategic competency while they still lack procedural fluency and while possessing conceptual misunderstandings.

3.3.4. Adaptive Reasoning

This strand refers to one's ability to relate concepts and contexts logically by carefully considering all possibilities and the justification of their conclusions by making use of sound reasoning (Kilpatrick et al., 2001). Kilpatrick et al. (2001) further point out that adaptive reasoning is the cornerstone of all Mathematics. The deductive reasoning structure in geometry plays a similar prominent role (Ndlovu & Mji, 2012), and without it, difficulties are bound to be compounded.

A majority of learners who continue to provide long strands of incoherent geometric statements and reasons may be an indication of the adaptive reasoning difficulties learners experience when dealing with problems involving proportionality. Another common difficulty learner's experience which resonates with adaptive reasoning relates to their inability to understand the procedure for geometric reasons, i.e. when do certain conclusions require reasons? For example, the conclusion in Question 10.1 (Diagram 3) does not require a reason since the preceding statements adequately justify the conclusion, whereas the conclusion in 10.2 does. Self-evident conclusions that do not draw from geometric theorems but from logical and algebraic deductions do not necessarily require reasons, e.g proceeding from $\hat{S}_1 = \hat{Q}_1 + \hat{Q}_2$ and $\hat{T}_2 = \hat{Q}_1 + \hat{Q}_2$ to concluding that $\hat{S}_1 = \hat{T}_2$.

Again, adaptive reasoning is intertwined with the other strands since one can only be able to provide valid reasons when concepts are understood, when they possess strategic competency and procedural fluency. A learner can only be able to successfully make the connection between alternating angles, sum of the angles of a triangle, similarity and proportionality when they have understood all these concepts, and when they are fluent in their associated procedures which are inclusive of the ability to represent problem solutions mathematically with valid justifications for solutions.

3.3.5. Productive disposition

This strand refers to one's capacity to view Mathematics as a productive and necessary endeavor which can be pursued successfully (Kilpatrick et al., 2001). In the case of geometric problems involving proportionality, learners' motivation to attempt solutions is directly proportional to their conceptual understanding, procedural fluency, strategic competency and adaptive reasoning. The more they succeed in the other four strands, the more their attitude towards the concept of proportion improves, reaffirming Kilpatrick et al. (2001) position on the interconnectedness of these strands.

3.4. Linking theoretical framework with research questions

The discussion on Kilpatrick et al. (2001) strands in the previous section connects the framework to learners' difficulties in the concept of proportionality. This framework is suitable for the analysis of learners' difficulties when dealing with problems involving similarity and proportionality. This is because this framework provides the theoretical basis for the researcher to make sense of the root causes and the classification of difficulties learners experience in their struggle to solve problems involving proportionality, similarity and proportional division theorem. In this way, the framework plays an important role in responding to the three research questions.

In light of the unavailability of Grade 12 learners for interviews in Gauteng districts, this study is limited to the four strands of mathematical proficiency, thereby excluding productive disposition as a tool for analysis. The researcher also holds the view that the first four strands are sufficient in the investigation of learners' difficulties since one cannot imagine that a learner who lacks in all of the other four strands can somehow be highly motivated to learn the concept of proportionality.

3.5. Summary and conclusion

This chapter discussed the rationale for selecting Kilpatrick et al. (2001) strands of mathematical proficiency as a theoretical framework for the study. A discussion of the five strands was presented in line with potential learner difficulties in the concept of proportionality, drawing from DBE diagnostic reports and past NSC examination papers. A link between the theoretical framework and research questions has also been established.

CHAPTER 4: RESEARCH DESIGN AND METHODS

4.1. Chapter 4 introduction

This chapter provides a discussion of the research design and methods used in the investigation of Grade 12 learners' difficulties in applying the concept of proportionality in triangles. It begins by outlining the research strategy which includes methods and sampling. The process of collecting data and methods of analysis is also outlined. Moreover, issues pertaining to validity, reliability and ethics are also dealt with in detail.

4.2. Research strategy

4.2.1. Methods

This study uses a mixed-method design to investigate Grade 12 learners' difficulties in the concept of proportionality. A mixed-method research design is defined as the type of research that weaves together qualitative and quantitative methods, approaches or language into a single research project in order to enhance understanding of research problems (De Vos et al, 2011). The use of both quantitative and qualitative methods is very useful in making sense and contextualizing data trends. A qualitative research approach is dominant in this study and it involves the analysis of Grade 12 learners' June Paper 2 examination on the topic of similarity of triangles and proportionality. Qualitative research is rooted in the notion that social phenomenon can be understood from the views of participants (McMillan & Schumacher, 2014).

In circumstances where phenomenon cannot be studied through observation and/or interviews, a documentary analysis is more useful (Richie & Lewis, 2003). Due to a very tight academic schedule for Grade 12 learners, the researcher was unable to obtain permission to conduct interviews and observations with learners in addition to the analysis of their scripts. However, learners' voices can also be found in their solution scripts, as Sfard (2020) and Lavie et al. (2019) argued that learners are essentially involved in mathematical discourse within themselves and with the teacher when they are engaged in mathematical problem solving. The difficulties they experience as they struggle to solve geometric problems involving proportions and similarity are echoed in their solutions as can be seen in Solution 1 to 27 of this study.

The goal of this study was to use a mixed-method design to answer three research questions. Table 1 shows how these research questions relate to the methodology and the theoretical framework used in this study.

Research Questions	Theoretical Framework	Methodology
What are Grade 12 learners' difficulties when dealing with problems involving proportionality in triangles?	Four strands of mathematical proficiency as they relate to all problems involving proportionality.	Prelim 2023 and June 2024 exam learners' scripts were analyzed for difficulties on problems involving proportionality through a mixed method design.
To what extent are these difficulties linked to learners' proficiency in the concept of similarity in triangles?	Four strands of mathematical proficiency as they relate to the concept of similarity.	Prelim 2023 and June 2024 exam learners' scripts were analyzed, through a mixed method design for difficulties on problems involving similarity. Learners' difficulties will be compared across Question 9 and 10 to categorize difficulties per question according to a specific strand.
To what extent are these difficulties linked to learners' proficiency in the concept of proportional division in triangles?	Four strands of mathematical proficiency as they relate to proportional division theorem.	Prelim 2023 and June 2024 exam learners' scripts were analyzed through a mixed method design for difficulties on problems involving proportional division theorem. Learners' difficulties were compared across question 9 and 10 to categorize difficulties per question according to a specific strand.

Table 1: Relating research questions, theoretical framework and methodology.

Both Grade 12 Prelim 2023 and June 2024 Paper 2 learners' scripts in geometric problems involving similarity and proportionality were quantitatively analyzed. Descriptive (univariate) statistics were used in the analysis of these learners' scripts in line with Kilpatrick et al. (2001) four strands of mathematical proficiency. Descriptive statistics present an effective way of summarizing data and are crucial in the interpretation of quantitative research results (McMillan & Schumacher, 2014). These descriptive statistics involve determining the mean, the standard deviation, minimum, maximum, median, mode and range of learners' marks in the entire geometry section and specifically in problems involving similarity and proportionality. This enabled the researcher to investigate learners' difficulties in concepts such as similarity and proportionality in comparison to other geometric problems of less complex nature. Box and Whisker Plots (BWP's) in Figure 5 are used to provide a detailed graphical representation of measures of central tendency, measures of dispersion and outliers in learners marks.

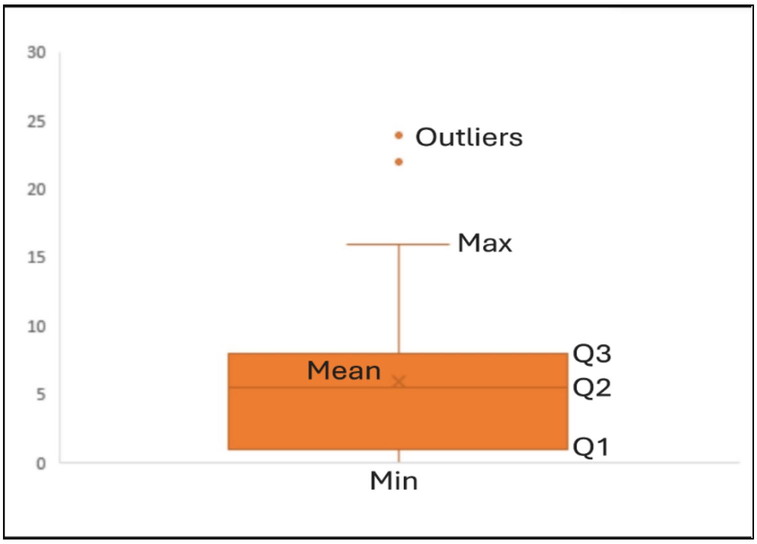


Figure 5: Labelling descriptions on Box and Whisker Plots (BWP's)

The labels, Min; Q1; Q2; Q3; Max and Outliers are provided on the Box and Whisker Plots (BWP's) as seen in Figure 5. These are variables used in describing the distribution of learners' marks. Min represents the lowest mark obtained by learners in a particular question or sets of questions, while Max represents the highest mark obtained. The lower quartile i.e. the 25th percentile is represented by Q1 on the BWP's, whereas the upper quartile i.e. the 75th percentile is represented by Q3. Q2 is the median and represents the 50th percentile of learners' marks. The Mean or average of learners' marks is represented by **X** on all BWP's. Outliers are represented by the dots above the Maximum.

4.2.2. Sampling

The population of this study is the 2023 and 2024 Grade 12 Mathematics learners from two Gauteng East township secondary schools. These schools were purposefully selected on the basis of poor performance in Mathematics, and 'sampling by case' type was used to focus the analysis of learners' scripts only to those which present interesting information, but initially all scripts were analysed. Then all scripts which exhibit difficulties on the concept of proportionality and similarity were further investigated for the kinds of difficulties they showed. McMillan and Schumacher (2014) define purposeful sampling as non-probability sampling which involves a selection of certain elements of the population which present some informative aspects of the study or topic under investigation.

A total of 139 learners’ scripts were selected for this study from both participant schools representing both the 2023 and 2024 cohorts. A total of 89 (56 from School 1 and 33 from School 2) learners’ scripts for the 2023 cohort from both participant schools were selected for quantitative analysis only, since these learners were not available to provide assent. A total of 50 (School 1, n = **30** and School 2, n = **20**) learners’ scripts for the 2024 cohort from both participant schools were selected for both quantitative and qualitative analysis, but only those learners’ scripts which showed information relevant to the research questions of this study were qualitatively presented and analysed.

4.3. Data collection

In this study, data was collected from Grade 12 learners’ solutions to geometric problems for the 2023 Preliminary and 2024 Mid-year examinations. These examinations are standardized provincial tests with marking guidelines. This data was used to investigate learners’ difficulties in problems involving similarity and proportionality in triangles.

4.3.1. Preliminary Examination 2023

The Mathematics Grade 12 preliminary examinations are provincial preparatory formal examinations which are compiled and moderated at provincial level, for example, in the case of Gauteng schools the question papers and marking guidelines are supplied by GDE. All Gauteng schools write the same examination at the same time and internal invigilation is strictly monitored by district or provincial officials. Every cluster within a district has a common location where principals collect sealed question papers and specialized answer sheets, in the case of Mathematics Paper 2, shortly before the commencement of the examination. After the examination, learners’ scripts are internally marked by educators and moderated by relevant HOD’s. Moderation of sampled scripts is also done at district level. These learners’ scripts must then be kept in the school for record keeping since the marks allocated form part of the year mark.

Once permission was granted from the schools, the researcher collected learners’ raw marks from learners’ scripts per question on the geometry section of Paper 2 from the respective educators per school. The researcher had direct access to the marking guideline since he also teaches Mathematics in Grade 12.

4.3.2. Mid-term examination 2024

In Gauteng province, the Mathematics Grade 12 mid-term examinations are also common provincial examinations set and moderated by GDE. This examination is formal and closely monitored by districts to ensure that it is administered at the same time to all Gauteng Grade 12 learners. This is done mainly to prevent leaks and to ensure the credibility of results. Sealed question papers and specialized answer sheets for Paper 2 are collected from a common location for each cluster shortly before the start of the examination. After the examination, learners' scripts are marked internally by educators and moderated by the HOD at school level. Further moderations of sampled learners' scripts are done at district level.

Upon receiving permission from the school principal and all participants, the researcher collected de-identified copies of learners' scripts on geometry section of Paper 2. As a Grade 12 Mathematics educator in the same district cluster as the participating schools, the researcher had direct access to the same marking guideline used by participant educators. The researcher also re-marked the scripts in instances where there were discrepancies in marking or mark allocation, and this re-marking is shown in green colour on learners' scripts.

4.3.3. Research protocols on data collection

The researcher only started approaching the two participant schools once permission was granted by Wits University Ethics committee (Protocol: 2024ECE0013M) and GDE (2024/89 - 8/4/4/1/2). Principals and Grade 12 Mathematics educators from both participant schools also granted permission in writing. The 2024 matric cohort and their parents were available to provide assent and consent after the purpose of the study was explained and they were provided with information participation sheets (see Annexure B). The unavailability of the 2023 matric cohort meant that the researcher could only collect 2024 (June) learners' scripts for both quantitative and qualitative analysis while data from 2023 matric cohort could only be collected for quantitative analysis purposes.

All this data collected from schools is kept safe in a locked container (storage box) at the researcher's house and will remain there for 5 years, thereafter it will be destroyed to prevent any unauthorized access. Insight on the nature of learners' difficulties through document analysis, is then used to gain better understanding of their struggles and to make useful recommendations on how these difficulties may be addressed in line with constructivist classroom practices.

4.4. Data analysis

Analysis of data in this study followed a sequential process. Firstly, the Prelim 2023 and Mid-term 2024 question papers and their respective marking guidelines were analysed according to Kilpatrick et al. (2001) four strands of mathematical proficiency: Conceptual understanding, procedural fluency, adaptive reasoning and strategic competency. The fifth strand, productive disposition, was left out since interviews were not possible with Grade 12 learners. Secondly, a quantitative analysis of the Prelim 2023 learners' marks was done according to the four strands. Thirdly, a quantitative analysis of the Mid-term 2024 learners' marks was also done according to the four strands. Lastly, a presentation and qualitative analysis of the Mid-term 2024 learners' scripts was done according to the four strands.

Learners' difficulties as observed in their solutions were analysed according to Kilpatrick et al. (2001) four strands. The Mathematics Teaching and Learning Framework for South Africa DBE (2018) also provides a good example of how Kilpatrick et al. (2001) five strands can be operationalized, and this study adds to this body of knowledge by focusing mainly on learners' difficulties with the concepts of proportionality in similarity and proportional division problems in Grade 12. The analysis of learners' scripts is per specific learner solution per line, for example a single geometric statement from the learner's response may highlight procedural fluency difficulties while a reason from the same learner solution may show adaptive reasoning difficulties. Further, an analysis of the Prelim 2023 and June 2024 question paper problems involving proportionality, proportional division and similarity theorems and memorandum was done to determine Kilpatrick et al. (2001) four strands per question. Learners who provide long lines of incoherent geometric statements and reasons are experiencing conceptual understanding and adaptive reasoning difficulties, since they lack the ability to relate concepts and contexts logically and the ability to support their conclusions with valid reasons.

Learners with procedural fluency difficulties exhibit procedural misunderstandings such as incorrectly structuring a proof as shown in the example provided on Page 23, where some in Question 10.2 (Diagram 3) began with the very same statement which must essentially be their conclusion. Errors which emanate from the incorrect transition from ratio to fractions in representing proportionality also points to procedural fluency difficulties (see example on Page 23).

According to DBE (2018), strategic competence involves thinking unconventionally. Learners' difficulties associated with strategic competence can be seen through their approach to solving problems that are not clearly defined, for example Question 10.2.3 (see Appendix A) where learners are expected to think out of the box and prove that $AD^2 = CD.DE$ by first formulating a problem of proving similarity which will lead to the proportionality of sides. The complexity in these types of problems is that learners are not asked to prove similarity or even proportionality of sides, yet these may be essential in finding the solution. It is important to note that what makes this problem non-routine is the fact that all concepts involved are hidden (they are not made explicit to the learner) making it difficult for those learners who lack strategic competence to be able to solve the problem. Being able to see the need to prove similarity between $\Delta AED\ and\ \Delta CDB$ is a very good sign that a learner has the strategic competency to see the link between these two triangles, the relationship between AD and DB, and the statement to be proved. It may well be possible that a learner fails to successfully solve this problem due to procedural fluency difficulties along the way, and these will be highlighted and analyzed accordingly, for example a finding may be that some learners who possessed strategic competence to solve Question 10.2.3 (Appendix A) lacked the procedural fluency to successfully prove the statement.

4.5. Ethical considerations

In order to ensure ethical issues, this study had to first obtain ethical clearance from Wits University and thereafter the participating schools were asked to provide permission for the collection of learners' scripts. Further, the schools were asked to de-identify learners' scripts by making copies of learners' geometry solutions only, thereby excluding the cover pages with learners' names. Furthermore, the names of all individual participants and the participant schools were not used in the research report, but pseudonyms were used instead, for example learner 1 (L1), Learner 2 (L2), School 1, etc.

Participant learners (2024 cohort) from both participant schools were required to provide written assent and their parents to provide consent before the scripts were collected. Participant Mathematics educators from both participant schools were also required to provide written consent for their 2023 and 2024 marked learners' scripts. All participants were informed in writing and verbally that participation is voluntarily and that the findings of this study will not negatively affect them in any way.

4.6. Validity

McMillan and Schumacher (2014) define validity as the measure of how close research explanations are to reality and how truthful are the research findings and conclusions. In this study, the researcher relies on national trends from DBE NSC reports and these are compared with findings. In this way, results are cross-validated to increase the credibility of findings (McMillan & Schumacher, 2014).

According to McMillan and Schumacher (2014), a mixed method design produces credible results when both the quantitative and qualitative methods merge and produce non-contradicting results. In this way, the findings are said to be triangulated, leading to "credible conclusions" (McMillan & Schumacher, 2014, p.433). In this study both the quantitative and qualitative methods converge in non-contradictory ways to highlight the kinds of difficulties Grade 12 learners experience when solving geometric problems involving proportionality.

4.7. Reliability

The central method used in this study is the analysis of learners' scripts originating from the Provincial (GDE) June 2024 and GDE Preparatory 2023 examination Paper 2, which are standardised and moderated common question papers, benchmarked from the NSC examination papers. The purpose of these examination papers is to prepare Grade 12 learners for their final examination by ensuring that all concepts applicable to the CAPS curriculum are covered and pitched at the correct level of difficulty. These question papers and their marking guidelines serve as instruments used to investigate learners' difficulties in geometric problems involving proportionality in this study. It can be said that these research instruments are reliable, since all mathematics learners in Gauteng East districts are subjected to the same examination.

4.8. Generalisation

McMillan and Schumacher (2014) posits that the quantitative approach in mixed method research designs helps improve generalizability of results. Quantitative results are used in this study to improve on generalizability by quantifying the performances of learners in geometric problems involving similarity and proportionality. Quantifying learners' performances tells us whether or not they generally experience difficulties in a particular question or topic. This is also important in highlighting the kinds of difficulties learners generally experience in these geometric topics.

Further, findings of this research are linked to the National Senior Certificate (NSC) diagnostic report findings since the main aim of this research is to gain better insight on learners' difficulties when dealing with geometric problems involving proportionality. So, although this study is limited to a single Cluster in Gauteng East district, the findings can be generalized in so far as they relate to the national findings since the benchmark for preliminary and mid-term exams (Provincial) is the National Senior Certificate examination.

4.9. Summary and conclusion

This chapter outlined research methods and design for the study. This study uses a mixed-method design to investigate Grade 12 learners' difficulties in solving problems involving proportionality in triangles. Certain quantitative and qualitative aspects were used in a mixed-method design to enhance feasibility and credibility of the study. This study involves two matric cohorts (2023 and 2024), and this chapter outlined how data was collected and analysed from both groups. Research questions, methodology and theoretical framework were linked together to summarise the study. Lastly, issues of ethics, reliability, validity and generalisation of results were outlined and dealt with.

CHAPTER 5: ANALYSIS OF QUESTION PAPERS AND MARKING GUIDELINES

5.1. Chapter 5 introduction

This chapter provides an analysis of both the question papers and associated marking guidelines of Preliminary 2023 and June 2024 geometry sections of the examinations. These question papers and marking guidelines are analyzed according to Kilpatrick et al. (2001) four strands of mathematical proficiency: Conceptual understanding, Procedural fluency, Strategic competency and Adaptive reasoning. The fifth strand: Productive disposition has been excluded since it was not possible to interview Grade 12 learners due to a congested academic schedule and GDE research requirements.

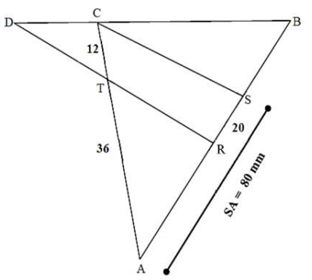
5.2. Analysis of 2023 Prelim Question Paper 2 and Marking Guideline according to Kilpatrick et al. (2001) four strands of mathematical proficiency

This analysis focusses on Question 9 and 10 of the 2023 Prelim examination (Gauteng Province) since these are the questions that are relevant to this study. A total of 28 marks are allocated to problems involving similarity and proportionality for this examination.

5.2.1. Question 9.1.

QUESTION 9

In the diagram below, ΔABC is constructed such that BC is produced to D. DR is drawn, with point T on AC and R on BA. CS is drawn. CT = 12 mm, TA = 36 mm, SR = 20 mm and SA = 80 mm.



9.1 Prove that CS $\parallel$ TR. (3)

9.2 It is further given that $AR = \frac{2}{3}RB$, $BC = 2x$ and $CD = \frac{1}{2}x + 1$.

Calculate the value of x. (6)
[9]

9.1	$\frac{CT}{TA} = \frac{12}{36} = \frac{1}{3}$		✓S	
	$\frac{SR}{RA} = \frac{20}{60} = \frac{1}{3}$		✓S	
	$\therefore \frac{CT}{TA} = \frac{SR}{RA}$			
	$\therefore CS \parallel TR$	line divides the sides of Δ in proportion/*lyn verdeel die sye van die* Δ *in verhouding*	✓R	(3)
	OR/*OF*			
	$\frac{SR}{SA} = \frac{20}{80} = \frac{1}{4}$		✓S	
	$\frac{CT}{CA} = \frac{12}{48} = \frac{1}{4}$		✓S	
	$\therefore \frac{CT}{CA} = \frac{SR}{SA}$			
	$\therefore CS \parallel TR$	line divides the sides of Δ in proportion/ *lyn verdeel die sye van die* Δ in verhouding	✓R	(3)

Diagram 5: Preliminary Exam 2023 Question 9.1 and Solution (GDE, 2023)

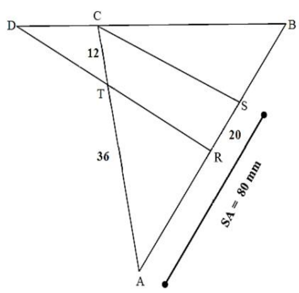
In Question 9.1, learners are required to prove that CS//TR based on the given statement and the diagram as shown in Diagram 5. This question require learners to have **conceptual understanding** of the converse of proportional division theorem i.e. if a line divides two sides of a triangle proportionally, then that line must be parallel to the third side of the triangle. According to the marking guideline for this question, learners needed to use the given length of AS = 80 and RS = 20 to determine AR. Determining the length of AR requires both the understanding of the relationship between the divided line segments and the **procedural fluency** to work out the missing part of a line segment given the overall length and one part of the same line segment. Once AR is known, learners are then expected to possess the **procedural fluency** to work out the ratio of the divided line segments to investigate whether they will become equal once written in fraction form and simplified. This requires **conceptual understanding** of the converse proportional division theorem, so that once learners have achieved equality of the two ratios (proportional division) then they can be able to conclude, informed by the converse proportional division theorem, that the line (TR) dividing two sides of the triangles must be parallel to the third side (CS).

The application of the converse proportional division theorem in this question amounts to the assessment of learners’ **adaptive reasoning** skills. Learners are expected to reason deductively as they first have to see the theorem involved in this problem and then work logically from a specific context in triangle ACS by making use of the given lengths and by establishing how these given lengths relate to each other with regards to the concept of proportionality, and then use these relationships to conclude on the relationship between TR and CS by making reference to the applicable theorem (converse proportional division). This is more of a routine geometric problem which does not really require any **strategic competence**.

5.2.2. Question 9.2.

QUESTION 9

In the diagram below, ΔABC is constructed such that BC is produced to D. DR is drawn, with point T on AC and R on BA. CS is drawn. CT = 12 mm, TA = 36 mm, SR = 20 mm and SA = 80 mm.



9.1 Prove that CS ∥ TR. (3)

9.2 It is further given that $AR = \frac{2}{3}RB$, $BC = 2x$ and $CD = \frac{1}{2}x+1$.

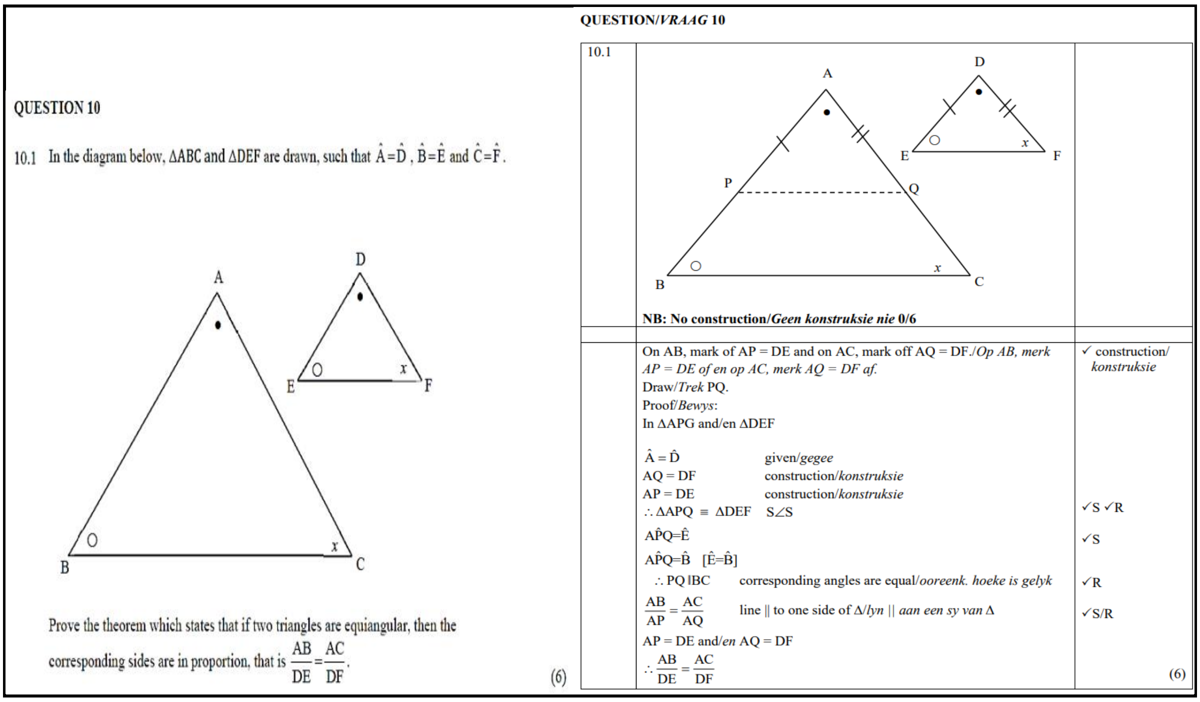
Calculate the value of x. (6)
[9]

9.2	$\frac{AR}{RB}=\frac{2}{3}$	
	$\frac{60}{RB}=\frac{2}{3}$	✓S
	$RB = 90$ mm $\therefore SB = 70$ mm	✓value of/*waarde van* RB
	$\frac{BS}{SR}=\frac{BC}{CD}$ proportion theorem/*eweredigheidstelling* CS‖TR	✓S ✓R
	$\frac{70}{20}=\frac{2x}{\frac{1}{2}x+1}$	✓substitution/ *substitusie*
	$40x = 35x + 70$ $x = 14$ mm	✓answer/*antwoord* (6)
	OR/*OF*	
	$2k = 60$ $\therefore k = 30$	✓value of/*waarde van k*
	$\therefore 3k = 90$	✓value of/*waarde van 3k*
	$\frac{SR}{RB}=\frac{DC}{DB}$ prop. theorem/*eweredigheidstelling* CS‖TR	✓S ✓R
	$\frac{20}{90}=\frac{\frac{1}{2}x+1}{\frac{5}{2}x+1}$	✓substitution/ *substitusie*
	$5x+2=\frac{9}{2}x+9$ $\frac{1}{2}x=7$ $\therefore x = 14$ mm	✓answer/*antwoord* (6)
		[9]

Diagram 6: Preliminary Exam 2023 Question 9.2 and Solution (GDE, 2023)

In Question 9.2 the size of AR is given as two-thirds of RB while the length of BC is given as $2x$ and that of CD as $\frac{1}{2}x+1$, and learners are required to determine the value of x. This question requires **strategic competency** from learners to be able to deconstruct $AR = \frac{2}{3}RB$ in such a way that the given ratio between AR and RB is made explicit. Without the ratio AR:RB = 2:3 it becomes impossible to determine the length for SB which is crucial in finding the value for x through the application of the proportional division theorem. Further, learners' ability to see the connection between the ratio of AR:RB and the value of x is dependent on their **conceptual understanding** of ratio and length of line segments. In this case x represents some constant which when substituted to the expression produce the actual length of the line segment for example $CD = 2x = 2(14) = 28\,mm$. Furthermore, learners are required to possess **procedural fluency** to manipulate the ratio $\frac{AR}{RB}=\frac{2}{3}$ to determine the length of RB by drawing from the solution in Question 9.1 where it should have been determined that AR = 60. Through substitution learners are expected to find the length of RB to be 90 mm, which then leads to the determination of the length of SB as 70 mm. The next stage to solving this problem involves **adaptive reasoning** abilities where learners are expected to apply the proportional division theorem in triangle BDR since CS has been proved to be parallel to DR (TR) in Question 9.1.

5.2.3. Question 10.1.



QUESTION 10

10.1 In the diagram below, ΔABC and ΔDEF are drawn, such that $\hat{A}=\hat{D}$, $\hat{B}=\hat{E}$ and $\hat{C}=\hat{F}$.

Prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE}=\frac{AC}{DF}$. (6)

QUESTION/*VRAAG* 10

10.1

NB: No construction/*Geen konstruksie nie* 0/6

On AB, mark of AP = DE and on AC, mark off AQ = DF./*Op AB, merk AP = DE of en op AC, merk AQ = DF af.* ✓ construction/*konstruksie*
Draw/*Trek* PQ.
Proof/*Bewys*:
In ΔAPG and/en ΔDEF

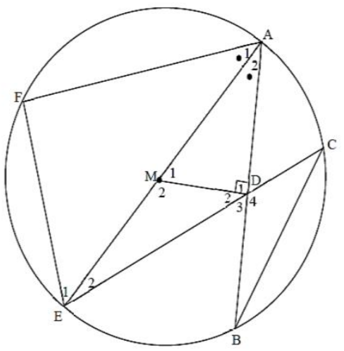
$\hat{A}=\hat{D}$ given/*gegee*
$AQ = DF$ construction/*konstruksie*
$AP = DE$ construction/*konstruksie*
$\therefore \Delta APQ \equiv \Delta DEF$ S∠S ✓S ✓R
$A\hat{P}Q=\hat{E}$ ✓S
$A\hat{P}Q=\hat{B}$ $[\hat{E}=\hat{B}]$
$\therefore PQ \parallel BC$ corresponding angles are equal/*ooreenk. hoeke is gelyk* ✓R
$\frac{AB}{AP}=\frac{AC}{AQ}$ line ∥ to one side of Δ/*lyn* ∥ *aan een sy van* Δ ✓S/R
$AP = DE$ and/*en* $AQ = DF$
$\therefore \frac{AB}{DE}=\frac{AC}{DF}$ (6)

Diagram 7: Preliminary Exam 2023 Question 10.1 and Solution (GDE, 2023)

Theorems essentially involve all of Kilpatrick et al. (2001) five strands of mathematical proficiency if they are to be understood meaningfully. The similarity theorem in Question 10.1 requires one to think ahead in terms of what is to be proved, since that is what determines the type of construction involved. Getting this construction correct is a crucial procedure **(procedural fluency)** to not just proving the entire theorem but also for its application as can be seen in subsequent questions. Learners who have developed the ability to genuinely and proficiently prove theorems understand (**conceptual understanding**) that a theorem is essentially a mathematical explanation, and as such it should adhere to Aristotle's criteria of adequate explanations (Luxomo, 2019). This theorem builds on the concept of proportional division and proving of similarity by equivalence of corresponding interior angles. Deductive reasoning plays such a crucial role in proving the proportionality of corresponding sides, hence learners' **strategic competency** and **adaptive reasoning** skills are put to the test.

5.2.4. Question 10.2.1.

10.2 In the diagram below, diameter EMA of a circle with centre M bisects $F\hat{A}B$. MD is perpendicular to the chord AB. ED produced meets the circle at C. Chords CB and FE are drawn.



10.2.1 Prove that $\Delta AEF\ |||\ \Delta AMD$. (4)

10.2.2 Determine the numerical value of $\frac{AF}{AD}$. (3)

10.2.3 Prove that $AD^2 = CD \times DE$. (6)

[19]

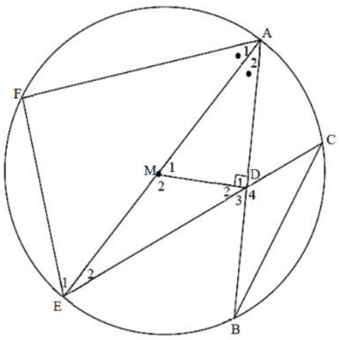
10.2.1	$\hat{F} = 90°$	$\angle$ in semi-circle/*halwe sirkel*	✓S/R
	In ΔAEF and/*en* ΔAMD		
	$\hat{A}_1 = \hat{A}_2$	given/*gegee*	✓S
	$\hat{F} = \hat{D}_1 = 90°$	proved/*bewys*	✓S
	$\therefore\ \Delta AEF\ |||\ \Delta AMD$	$\angle\angle\angle$	✓R (4)
	OR/OF		**OR/OF**
	$\hat{F} = 90°$	$\angle$ in semi-circle/*halwe sirkel*	✓S/R
	In ΔAEF and/*en* ΔAMD		
	$\hat{A}_1 = \hat{A}_2$	given/*gegee*	✓S
	$\hat{F} = \hat{D}_1 = 90°$	proved/*bewys*	✓S
	$\hat{E}_1 = \hat{M}_1$	sum of $\angle$'s in Δ/*binne* $\angle$*e van* Δ	✓R
	$\therefore\ \Delta AEF\ |||\ \Delta AMD$		(4)

Diagram 8: Preliminary Exam 2023 Question 10.2.1 and Solution (GDE, 2023)

In Question 10.2.1, learners' **conceptual understanding** of similarity and the resultant proportionality of sides is assessed. Proving that $\Delta AEF///\Delta AMD$ also requires **procedural fluency** in proof techniques such as the ordering and organization of corresponding angles to be equated through valid geometric reasons. Although the first two angles ($\hat{A}_1$ and $\hat{A}_2$) are given, the second pair requires **conceptual understanding** since learners have to draw from previously acquired knowledge (Grade 11 Circle geometry) in order to be able to find that AF is perpendicular to FE, since AE is given as a diameter to the circle. This question also require **adaptive reasoning** skills due to the deductive reasoning nature of geometric statements and reasons involved – for example, in order to be able to equate $\hat{E}_1$ with $\widehat{M}_1$ learners are required to link the two angles through one of the important characteristics of triangles which states that the sum of the interior triangles is equal to 180°.

5.2.5. Question 10.2.2.

10.2 In the diagram below, diameter EMA of a circle with centre M bisects $F\hat{A}B$. MD is perpendicular to the chord AB. ED produced meets the circle at C. Chords CB and FE are drawn.



10.2.1 Prove that $\Delta AEF \mathbin{|||} \Delta AMD$. (4)

10.2.2 Determine the numerical value of $\frac{AF}{AD}$. (3)

10.2.3 Prove that $AD^2 = CD \times DE$. (6)

[19]

10.2.2	$\frac{AF}{AD} = \frac{AE}{AM}$	$\Delta AEF \mathbin{\|\|\|} \Delta AMD$	✓S/R
	but/*maar* AE = 2AM		✓S
	$\frac{AF}{AD} = \frac{2AM}{AM} = 2$		✓answer/*antwoord*
			(3)

Diagram 9: Preliminary Exam Question 10.2.2 and Solution (GDE, 2023)

In Question 10.2.2, learners are required to work out the value of $\frac{AF}{AD}$ in numerical terms. This numerical value is obtainable when learners have **conceptual understanding** that this question is aimed at assessing their understanding of the relationship between AF and AD in terms of ratio. It is also linked to the results obtained in Question 10.2.1 in terms of the relationship between corresponding sides of equiangular triangles. A key concept in this problem lies in learners' understanding of the concept of radii of a circle and their relationship to the diameter. That is why learners must be able to identify that AM:AE = 1:2, otherwise they have no way of finding the solution. It is important to note that there are other ways of representing this one to one ratio between radius AM and ME, such as through the depiction of unknown lengths by a combination of ratios and variables as shown in Diagram 10.

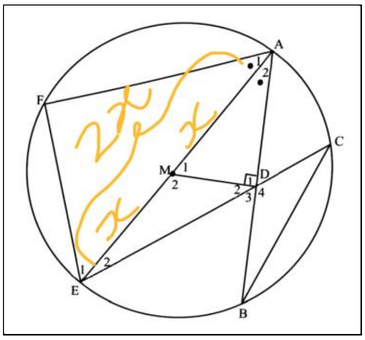
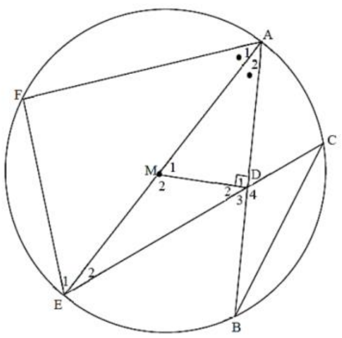


Diagram 10: Alternative approach to solving Preliminary Exam Question 10.2.2 (GDE, 2023)

It is unfortunate that marking guidelines never show this approach as an alternative solution. The **procedural fluency**, **conceptual understanding** and **adaptive reasoning** to connect $\frac{AF}{AD}$ to $\frac{AM}{AE}$ is fundamental in learners ability to successfully solve this problem and demonstrate proficiency.

5.2.6. Question 10.2.3.

10.2 In the diagram below, diameter EMA of a circle with centre M bisects $F\hat{A}B$. MD is perpendicular to the chord AB. ED produced meets the circle at C. Chords CB and FE are drawn.



10.2.1 Prove that $\Delta AEF \,|||\, \Delta AMD$. (4)

10.2.2 Determine the numerical value of $\frac{AF}{AD}$. (3)

10.2.3 Prove that $AD^2 = CD \times DE$. (6)

[19]

10.2.3	In ΔCDB and/*en* ΔADE		✓identifying Δs/ *identifiseer* Δe
	$\hat{C} = \hat{A}_2$	$\angle$s in same segment/$\angle$e *in dieselfde segment*	✓S/R
	$\hat{D}_4 = E\hat{D}A$	vertically opposite $\angle$'s/*regoorstaande* $\angle$e	✓S
	$\Delta CDB \,|||\, \Delta ADE$	$\angle\angle\angle$	✓R
	$\therefore \frac{CD}{AD} = \frac{DB}{DE}$		✓S
	but/*maar* $AD = DB$	line from centre $\perp$ to chord/*lyn vanaf midpt.* $\perp$ *op koord*	✓S/R
	$\frac{CD}{AD} = \frac{AD}{DE}$		
	$\therefore AD^2 = CD \times DE$		(6)
	OR/OF		**OR/OF**
	In ΔCDB and/*en* ΔADE		✓identifying Δs/ *identifiseer* Δe
	$\hat{C} = \hat{A}_2$	$\angle$s in same segment/$\angle$e *in dieselfde segment*	✓S/R
	$\hat{D}_4 = E\hat{D}A$	vertically opposite $\angle$'s/*regoorstaande* $\angle$e	✓S
	$\hat{B} = \hat{E}_2$	$\angle$s in same segment/$\angle$e *in dieselfde segment*	✓R
	$\Delta CDB \,|||\, \Delta ADE$		✓S
	$\therefore \frac{CD}{AD} = \frac{DB}{DE}$		
	but/*maar* $AD = DB$	line from centre $\perp$ to chord/*lyn vanaf midpt.* $\perp$ *op koord*	✓S/R
	$\frac{CD}{AD} = \frac{AD}{DE}$		
	$\therefore AD^2 = CD \times DE$		

Diagram 11: Preliminary Exam 2023 Question 10.2.3 and Solution. (GDE, 2023)

Question 10.2.3 is a classical example of a geometric problem which requires **strategic competence** over-and-above **conceptual understanding**, **procedural fluency** and **adaptive reasoning**. Learners are required to be able to identify triangles that are involved in this equation by taking apart its constitutive elements. This deconstruction requires procedural fluency in reversing the process of cross-multiplication and the identification of equal sides since the second triangle is not explicit. The location of hidden triangles is dependent on learners' prior-knowledge of Grade 11 circle geometry which explains how a line drawn from the centre of a circle and perpendicular to the chord also bisects the chord. Still, the key to solving this problem lies in planning and a well-coordinated strategy as shown in **Problem Interpretation 1**. This does not imply that other geometric problems do not require planning and interpretation but that these types of problems require extensive planning and a clearly defined and well-coordinated strategy. Such a strategy must be written down in order to be effective.

Required To Prove (R.T.P): $AD^2 = CD \times DE$ $AD.AD = CD.DE$ $\frac{AD}{CD} = \frac{DE}{AD}$ But $AD = DB$ (Line from centre perpendicular to chord) $\therefore \frac{AD}{CD} = \frac{DE}{DB}$ Hence the only possible triangles is ΔADE and ΔDBC (horizontal tracing)

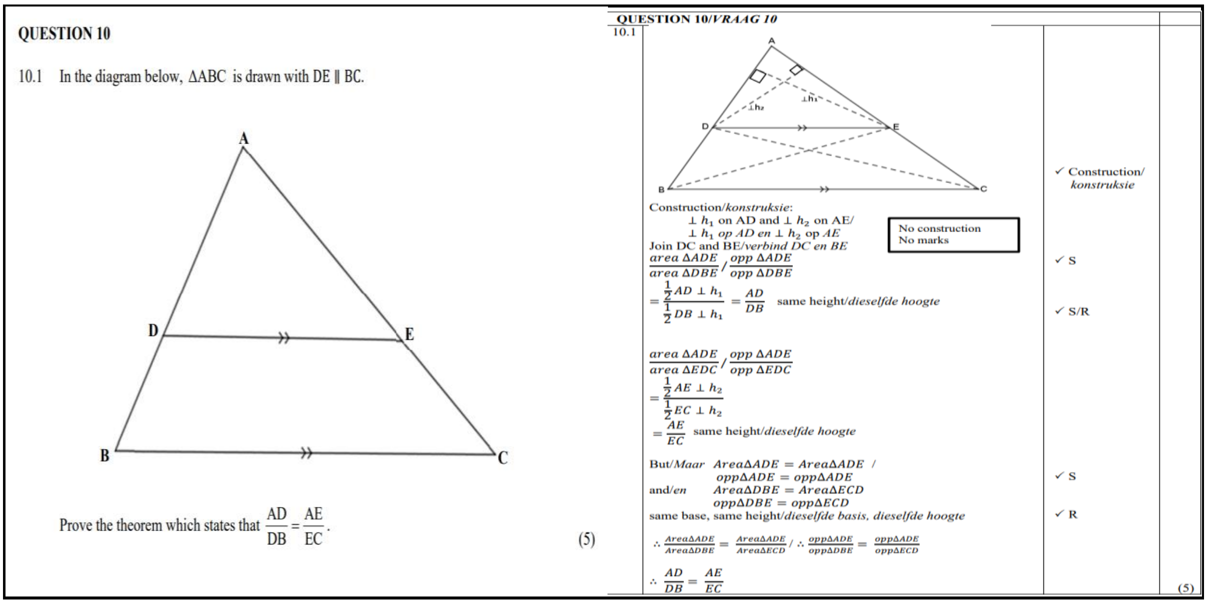
Problem Interpretation 1:

This strategy does not appear in marking guidelines since it is not part of the solution but rather a crucial and organized interpretation of the problem at hand. The fascinating thing about this strategy is that it then simplifies the required deductive reasoning or formulation of geometric statements and reasons which must be provided since learners can simply work from the bottom up i.e. the interpretation of the problem becomes the solution when read from the bottom up, further simplifying the logical flow of learners’ solutions. The solution for Question 10.2.3 is essentially the reversal (bottom up) of this problem interpretation.

5.3. Analysis of 2024 June Question Paper 2 and Marking Guideline according to Kilpatrick et al. (2001) four strands of mathematical proficiency.

This analysis only focusses on Question 10 of the 2024 June P2 examination question paper and marking guideline since this is the only question which involves similarity, ratio and proportionality. A total of 19 marks are allocated to problems involving these concepts.

5.3.1. Question 10.1.


QUESTION 10

10.1 In the diagram below, ΔABC is drawn with DE ∥ BC.

Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$. (5)

QUESTION 10/*VRAAG 10*

10.1

Construction/*konstruksie*:
$\perp h_1$ on AD and $\perp h_2$ on AE/
$\perp h_1$ *op AD en* $\perp h_2$ op *AE*
Join DC and BE/*verbind DC en BE* ✓ Construction/*konstruksie*

No construction
No marks

$\frac{area\ \Delta ADE}{area\ \Delta DBE} / \frac{opp\ \Delta ADE}{opp\ \Delta DBE}$ ✓ S

$= \frac{\frac{1}{2} AD \perp h_1}{\frac{1}{2} DB \perp h_1} = \frac{AD}{DB}$ same height/*dieselfde hoogte* ✓ S/R

$\frac{area\ \Delta ADE}{area\ \Delta EDC} / \frac{opp\ \Delta ADE}{opp\ \Delta EDC}$

$= \frac{\frac{1}{2} AE \perp h_2}{\frac{1}{2} EC \perp h_2}$

$= \frac{AE}{EC}$ same height/*dieselfde hoogte*

But/*Maar* $Area\Delta ADE = Area\Delta ADE$ /
$opp\Delta ADE = opp\Delta ADE$ ✓ S
and/*en* $Area\Delta DBE = Area\Delta ECD$
$opp\Delta DBE = opp\Delta ECD$
same base, same height/*dieselfde basis, dieselfde hoogte* ✓ R

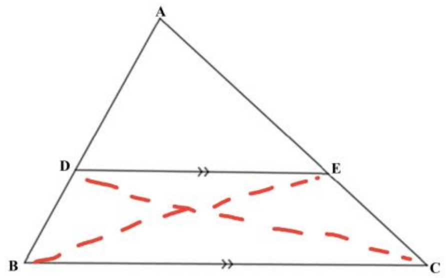
$\therefore \frac{Area\Delta ADE}{Area\Delta DBE} = \frac{Area\Delta ADE}{Area\Delta ECD} / \therefore \frac{opp\Delta ADE}{opp\Delta DBE} = \frac{opp\Delta ADE}{opp\Delta ECD}$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (5)

Diagram 12: June 2024 Exam Question 10.1 and Solution (GDE, 2024)

In Question 10.1 learners are required to prove the theorem which states that 'a line drawn to one side of a triangle cuts the other two sides so as to divide them in the same proportion'. The proportional division theorem is centered around the relationship between areas of triangles which lie between two parallel lines when the base is common and triangles which have different bases and a common height. Learners who lack this **conceptual understanding** are bound to struggle with this theorem even when they resort to memorization since the required constructions are dependent on deeper understanding of the concepts involved. The procedure for selecting triangles produces the required proportional division and as a result it must be executed with care so that the common heights should cancel each other out when the areas of triangles are divided. Equating areas of triangles between parallel lines when the base is common requires learners to possess **adaptive reasoning** abilities, since linking up $\frac{AD}{DB}$ with $\frac{AE}{EC}$ is dependent on valid and logical geometric justifications.

Question 10.1 requires **conceptual understanding** of area of triangles and its component parts in different arrangements, i.e. how (**procedural fluency**) these components can be manipulated to determine the relationship between bases of triangles which are created by a line parallel to one side of triangle, and **adaptive reasoning** abilities to provide valid justifications for conclusions. Proofs of theorems in CAPS examination do not require **strategic competence** since they are clearly defined and because learners know exactly how to proceed based on what would have been done in the classroom setting i.e. proofs do not present any form of complexity associated with the element of surprise. The following proofs in Diagram 13 which involve the Area rule (Trigonometric approach) and Sine rule are also valid although not shown on the marking guideline.



Proof: Construction: Join DC and BE

$$\frac{Area\ \Delta ADE}{Area\ \Delta BEC} = \frac{\frac{1}{2}.AE.DE.\sin A\hat{E}D}{\frac{1}{2}.EC.BC.\sin \hat{C}}$$

But $A\hat{E}D = \hat{C}$ $(Corresp. \angle's; DC//$BC)

$$\therefore \frac{Area\ \Delta ADE}{Area\ \Delta BEC} = \frac{AE.DE}{EC.BC}$$

$$\frac{Area\ \Delta ADE}{Area\ \Delta BDC} = \frac{\frac{1}{2}AD.DE.\sin A\hat{D}E}{\frac{1}{2}.BD.BC.\sin \hat{B}}$$

But $A\hat{D}E = \hat{B}$ $(Corresp. \angle's;\ DC//$BC)

$$\therefore \frac{Area\ \Delta ADE}{Area\ \Delta BDC} = \frac{AD.DE}{BD.BC}$$

$Area\ \Delta ADE = Area\ \Delta ADE\quad (Same\ \Delta)$

$Area\ \Delta BEC =$

$Area\ \Delta BDC\ \ (Same\ base, same\ height)$

$$\therefore \frac{AE.DE}{EC.BC} = \frac{AD.DE}{BD.BC}$$

$$\therefore \frac{AE}{EC} = \frac{AD}{BD}$$

Proof by Sine Rule Application: No construction necessary

In ΔADE: $\dfrac{sin\ A\hat{D}E}{AE} = \dfrac{\sin A\hat{E}D}{AD}$

$\therefore \dfrac{AD}{AE} = \dfrac{\sin A\hat{E}D}{sin\ A\hat{D}E}$ (Cross multiplication).

In ΔABC: $\dfrac{\sin \hat{B}}{AC} = \dfrac{\sin \hat{C}}{AB}$

$\therefore \dfrac{AB}{AC} = \dfrac{\sin \hat{C}}{\sin \hat{B}}$ (Cross multiplication).

But $A\hat{E}D = \hat{C}\ (Corresp\ \angle's; DE//BC)$

And $A\hat{D}E = \hat{B}\ (Corresp. \angle's; DE//BC)$

$$\therefore \frac{AD}{AE} = \frac{AB}{AC}$$

$\therefore \dfrac{AD}{AB} = \dfrac{AE}{AC}$ (Cross multiplication).

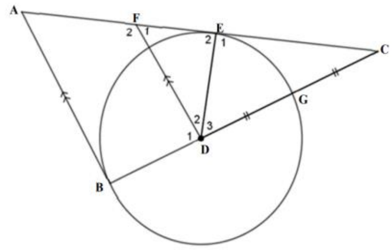
$$\therefore \frac{AE}{EC} = \frac{AD}{BD}$$

Diagram 13: Alternative solutions for Question 10.1 (GDE, 2024)

Lastly, the way this question is examined tends to encourage memorization, what Bishop (1986) refers to as the instrumentalization of the proof, since in this case learners can simply memorize the procedure to generate the required constructions and the Area formula to reproduce the proof. Examiners can mitigate against this memorization by stipulating the specific approach to be used and change how triangles are arranged and labeled to adequately assess learners' understanding of the theorem.

5.3.2. Question 10.2.1.

10.2 In the diagram below, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C. DG = CG. F is a point on AC such that DF ∥ AB.



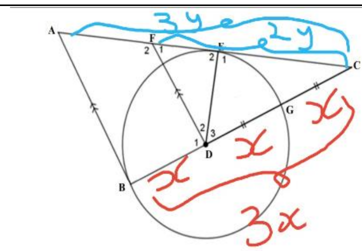
10.2.1	Find, with reasons, the ratio of $\frac{AC}{FC}$.	(3)
10.2.2	Prove, giving reasons, that ΔABC /// ΔDEC.	(3)
10.2.3	Prove that $DE^2 = \frac{AE.EC}{3}$	(5)
10.2.4	Find the ratio of: $\frac{\text{Area}\Delta FDC}{\text{Area}\Delta ABC}$	(3)
		[19]

10.2.1	$DB = DG$ $In\ \Delta ABC$: $DF \parallel AB$ $\frac{AC}{FC} = \frac{BC}{DC} = \frac{3}{2}$ line ∥ to one side of Δ/(Prop theorem,DF∥AB) *aan een sy van Δ/Eweredigheid stelling* (DF∥AB)	✓ radii/*radiusse* ✓ $\frac{AC}{FC} = \frac{BC}{DC} = \frac{3}{2}$ ✓ R	(3)

Diagram 14: June 2024 Exam Question 10.2.1 and Solution (GDE, 2024)

In Question 10.2.1 learners are required to determine the ratio of $\frac{AC}{FC}$ given the diagram and given statement. This problem requires **conceptual understanding** of circle geometry and ratio since learners are required to identify the one-to-one ratio between BD, DG and GC since these three line segments are equivalent in length. **Procedural fluency** in representing this relationship both on the diagram by using variables as shown in Diagram 15 and by following acceptable geometric procedures in writing valid and logical statements and reasons is crucial. For example, how the reason 'prop theorem' provided on the marking guideline (as shown in Diagram 14) for $\frac{AC}{FC} = \frac{BC}{DC}$ must strictly reference the relevant parallel lines (DF and AB) to be considered adequate as valid justification.

The use of different variables on different sides of the triangles plays a crucial role in emphasizing that although the ratios are the same the actual length of line segments are not necessarily the same. Learners' ability to reason sufficiently in justifying their statements or solutions is in-line with Kilpatrick et al. (2001) **adaptive reasoning** strand.



$BD = DG\ (radii)$

$But\ DG = GC\ (Given)$

$\therefore BD = DG = GC = x$

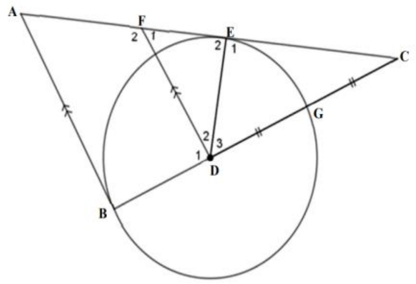
$\frac{AC}{FC} = \frac{BC}{DC}\ (Prop\ theorem, DF//AB)$

$\therefore \frac{AC}{FC} = \frac{3x}{2x} = \frac{3}{2}$

Diagram 15: Alternative approach to solving Question 10.2.1 (GDE, 2024)

5.3.3. Question 10.2.2.

10.2 In the diagram below, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C. DG = CG. F is a point on AC such that DF ∥ AB.



10.2.1 Find, with reasons, the ratio of $\frac{AC}{FC}$. (3)

10.2.2 Prove, giving reasons, that $\Delta ABC \mathbin{///} \Delta DEC$. (3)

10.2.3 Prove that $DE^2 = \frac{AE.EC}{3}$ (5)

10.2.4 Find the ratio of: $\frac{Area\Delta FDC}{Area\Delta ABC}$ (3)

[19]

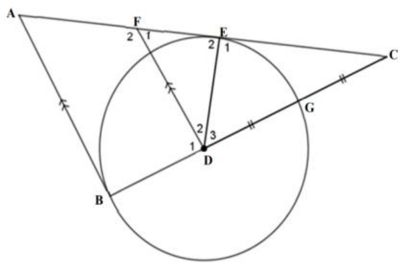
10.2.2	In ΔABC and/*en* ΔDEC: 1) $\hat{B} = 90°$ tan ⊥ rad/*rlyn* ⊥ *radius* $D\hat{E}C = 90°$ tan ⊥ rad/*rlyn* ⊥ *radius* $\therefore \hat{B} = D\hat{E}C$ 2) $\hat{C} = \hat{C}$ common ∠/*gemeenskaplike* ∠ 3) $\hat{A} = E\hat{D}C$ ∠'s of a Δ/∠'e *van* Δ $\therefore \Delta ABC /// \Delta DEC$ ∠∠∠	✓ $\hat{B} = D\hat{E}C$ S/R ✓ $\hat{C} = \hat{C}$ ✓ $\hat{A} = E\hat{D}C$ **OR** ✓ $\hat{B} = D\hat{E}C$ S/R ✓ $\hat{C} = \hat{C}$ ✓R ∠∠∠

Diagram 16: June 2024 Exam Question 10.2.2 and Solution (GDE, 2024)

In Question 10.2.2 learners are required to prove that ΔABC and ΔDEC are similar. This is a fairly routine task since the triangles involved are clearly identified, but learners still have to dig deep from their prior-knowledge of circle geometry and their **conceptual understanding** of similarity to be able to prove that these triangles are equiangular. The **procedural fluency** in the effective application for proving techniques and presentation of geometric statements and reasons is equally important in learners' ability to solve this problem successfully. **Adaptive reasoning** also plays a crucial part in the reasons provided for conclusions as it can be seen on how finding the numerical value for $\hat{B}$ and $\hat{E}_1$ led to a conclusion that $\hat{B} = \hat{E}_1$ since they are both equal to 90°.

5.3.4. Question 10.2.3.

10.2 In the diagram below, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C. DG = CG. F is a point on AC such that DF ∥ AB.



10.2.1 Find, with reasons, the ratio of $\frac{AC}{FC}$. (3)

10.2.2 Prove, giving reasons, that ΔABC /// ΔDEC. (3)

10.2.3 Prove that $DE^2 = \frac{AE.EC}{3}$ (5)

10.2.4 Find the ratio of: $\frac{Area\Delta FDC}{Area\Delta ABC}$ (3)

[19]

10.2.3	$\frac{AB}{DE} = \frac{BC}{EC}$ $\Delta ABC///\Delta DEC$	✓ S/R
	but/*maar* $AB = AE$ tans from same point/*rlyne vanaf dies. punt* and/*en* $BC = 3 \times$ radius = 3DE	✓ S/R
	$\therefore \frac{AE}{DE} = \frac{3DE}{EC}$	✓ substitute AB with AE/*vervang AB met AE*
	$AE.EC = 3DE.DE$	✓ substitute BC with 3DE/*vervang BC met 3DE*
	$AE.EC = 3\,DE^2$	
	$DE^2 = \frac{AE.EC}{3}$	✓ $AE.EC = 3\,DE^2$

Diagram 17: June 2024 Exam Question 10.2.3 and Solution (GDE, 2024)

Question 10.2.3 is another classical example of a problem which require **strategic competence** since by simple glancing at the problem statement to be proved it is neither easy to see any proportionality nor any triangles involved. This is a complex problem which requires the dismantling of the equation into its constitutive parts. This process of deconstruction requires **procedural fluency** in reversing cross-multiplication, identification of triangles and replacement through substitution of equivalent line segments until valid triangles are made explicit. What further complicates this problem is the two layers of substitutions which are required before the two triangles can be illuminated. Further, finding equal line segments is dependent on learners' **adaptive reasoning** abilities, for example the ability to deduce with valid justifications that BC is equal to three times the length of DE through circle geometry theorems. An in-depth **conceptual understanding** of circle geometry theorems such as the equivalence of tangents drawn from the same point, also proves to be crucial in replacing AE with AB in order to be able to arrive at the proportionality $\frac{DE}{AB} = \frac{EC}{BC}$ which is an outcome of the similarity of ΔABC and ΔDEC. Once the dismantling of the given equation is done to a point where the triangles involved are exposed as can be seen in **Problem Interpretation 2**. Thereafter, all that remains for the learner to do as part of the solution, is to work from the bottom up by putting the constitutive parts together until they arrive at the very same thing they were required to prove in the first place. It is in this process of dismantling and reconstruction that **strategic competence** plays a significant part.

Required To Prove: $DE^2 = \frac{AE.EC}{3}$

$DE.DE = \frac{AE.EC}{3}$

$3DE.DE = AE.EC$

But $BC = 3DE$

$BC.DE = AE.EC$

$\frac{DE}{AE} = \frac{EC}{BC}$

But $AE = AB$ (Tans. From same pt.)

$\frac{DE}{AB} = \frac{EC}{BC}$

Hence the only possible triangles is $\Delta DEC\ and\ \Delta ABC$ (horizontal tracing).

Problem Interpretation 2

5.3.5. Question 10.2.4.

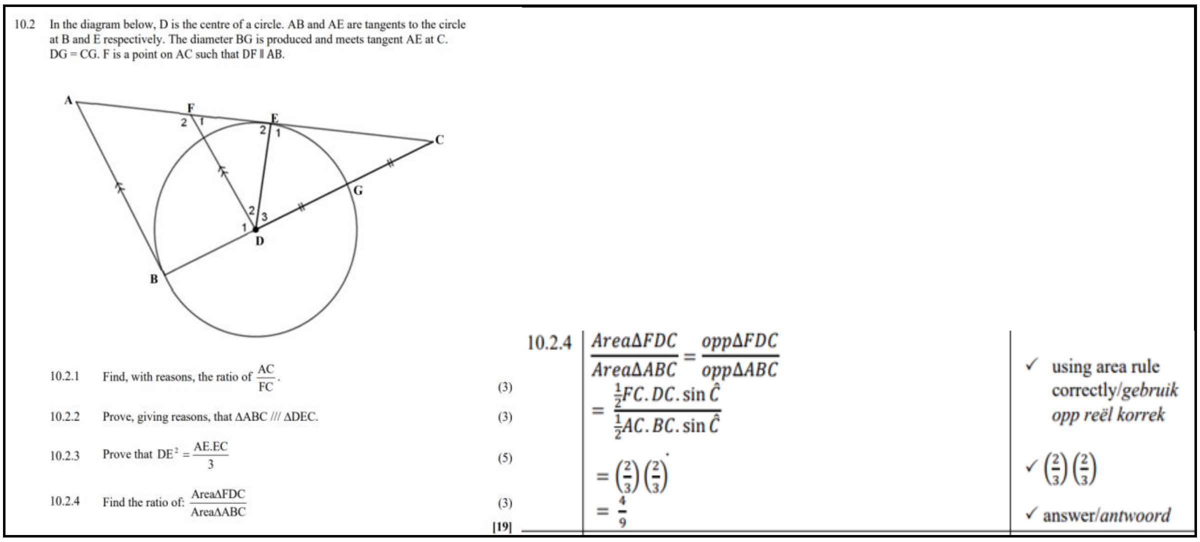


Diagram 18: June 2024 Exam Question 10.2.4 and Solution (GDE, 2024)

In Question 10.2.4, learners are required to find the ratio of $\frac{Area\ \Delta FDC}{Area\ \Delta ABC}$. This problem can be solved in two different ways since there are two formulas for area of triangles applicable to the CAPS curriculum. According to the marking guideline, it is clear that the examiner expected learners to use the Area rule (Trigonometric approach) instead of using the standard Area formula which involves a height perpendicular to the base (see Diagram 19). Notice how this approach draws from the work done in Question 10.2.1 hence the diagram is the same.

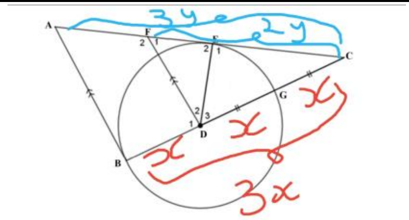
 $\hat{B} = 90°\ \ (Proved)$ $\hat{B} = \widehat{D}_2 + \widehat{D}_3 = 90°\ \ (Corresp.\angle's;\ AB//\text{DE})$ $\frac{Area\ \Delta FDC}{Area\ \Delta ABC} = \frac{\frac{1}{2}.FD.DC}{\frac{1}{2}.AB.BC}$ $= \frac{FD.2x}{AB.3x}$	$= \frac{2FD}{3AB}$ $In\ \Delta ABC\ and\ \Delta FDC$ $\hat{C}\ is\ common$ $\hat{B} = \widehat{D}_2 + \widehat{D}_3 = 90°\ \ (Proved)$ $\hat{A} = \hat{E}_1\ \ (Corresp.\angle's;\ AB//\text{DE})$ $\therefore \Delta ABC///\Delta FDC\ \ (A, A, A)$ $\therefore \frac{FD}{AB} = \frac{2}{3}\ \ (///\ \Delta's)$ $\therefore \frac{Area\ \Delta FDC}{Area\ \Delta ABC} = \frac{2.2}{3.3} = \frac{4}{9}$

Diagram 19: Alternative solution approach to solving Question 10.2.4.

Despite the approach learners may use in tackling this problem, **conceptual understanding** of Area of triangles, ratio, circle geometry, proportional and similarity theorems are paramount. Equally important is the **procedural fluency** and **adaptive reasoning** abilities to combine all these concepts together to form a ratio which is representative of this expression with valid justifications. The solution in 10.2.1 play a major role in finding the correct ratio.

5.4. Summary and Conclusion

It is clear in this section that both the Prelim 2023 and June 2024 Paper 2 question papers include problems encompassing the four strands of mathematical proficiency. It is not surprising that **strategic competence** and **adaptive reasoning** are not as dominant as **conceptual understanding** and **procedural fluency** since there is a long history of poor performances by Grade 12 learners in geometric problems involving proportionality. It is easy for struggling learners who possess some conceptual understanding to memorize procedures and obtain some marks in less complicated problems. Marking guidelines for both examination papers, have minimal alternative solutions and lack important details on **strategic competence.**

CHAPTER 6: DATA PRESENTATION AND ANALYSIS

6.1. Chapter 6 introduction

This chapter provides a quantitative analysis and presentation of Preliminary 2023, a quantitative analysis and presentation of June 2024, and qualitative analysis and presentation of June 2024 examination geometry sections. Statistical tools such as Box and Whisker Plots (BWP's), graphs and tables are used to present quantitative data, while qualitative data is presented through learners' marked solutions. The focus is on geometric problems involving similarity and proportionality in line with the research questions. The qualitative analysis of the Prelim 2023 learners' scripts was not possible due to unavailability of the 2023 matric cohort to provide assent and parental consent.

6.2. Quantitative analysis of Grade 12 2023 Preliminary examination Paper 2 scripts according to Kilpatrick et al. (2001) four strands of mathematical proficiency

In the following passages descriptive statistics are used to provide quantitative analysis of the geometric section for the Prelim 2023 learners' scripts. The aim is to make sense of learners' difficulties in problems involving similarity and proportionality as compared to the entire geometry section.

The total number of marks for the geometry section of the Preliminary examination 2023 (Gauteng Province) is 40, while 28 marks are allocated for problems involving similarity and proportionality. Geometry accounts for 27% while geometric questions involving similarity and proportionality account for 19% of P2, which is essentially a significant component of the examination.

In analyzing learners' scripts from both participant schools (herein referred to as School 1 and School 2) we found that no learner was able to obtain 50% or above in geometry. The highest mark in geometry is 18 from School 1, further highlighting the difficulties learners experience in this topic. From both participant schools, only one learner demonstrated **conceptual understanding** of the converse proportional division theorem in question 9.1 and the **procedural fluency** to formulate the ratios $\frac{AT}{TC}$ and $\frac{AR}{RS}$ leading to the same ratio of 3, but the very same learner had **adaptive reasoning** difficulties since the learner could not provide a reason why CS is parallel to TR.

The following graphs in Figure 6 and Figure 7 provide an analysis of learners' performance per question for both participant schools. The actual number of learners who obtained a pass (≥50%) is shown per geometry sub-question.

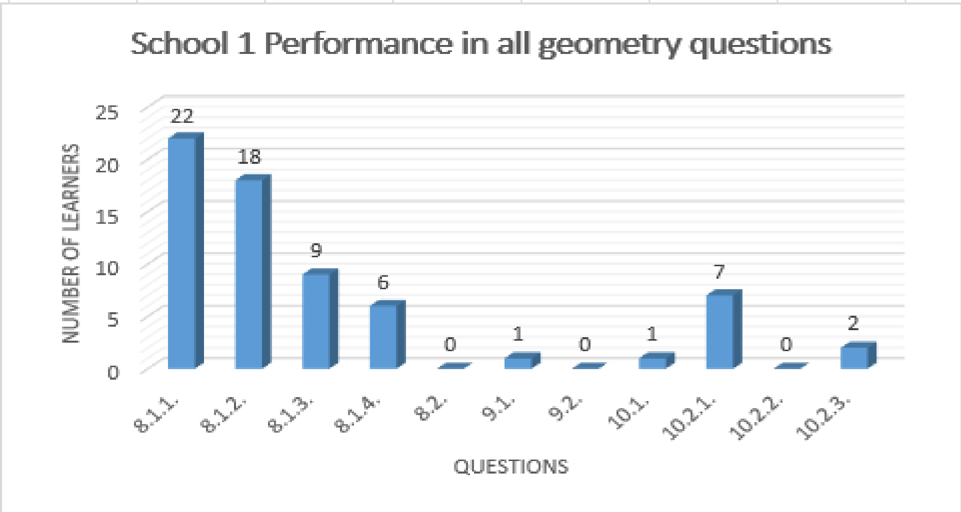


Figure 6: School 1 Prelim 2023 Question by Question analysis (n=56)

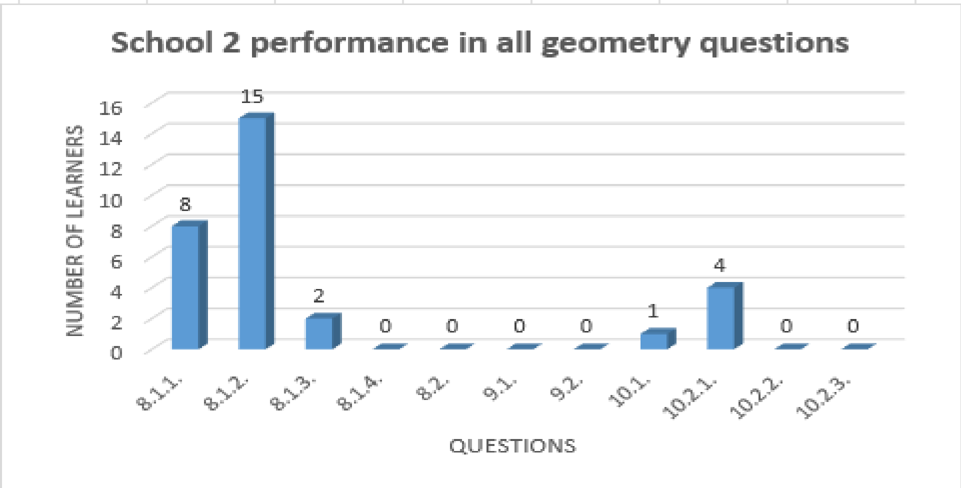


Figure 7: School 2 Prelim 2023 Question by Question analysis (n=33)

The overall average mark (mean) for the entire geometry section in School 1 is 2.8 and for School 2 is 2.1. The average mark (mean) for problems involving similarity and proportionality in School 1 is 1 and for School 2 is 1.4. This underscores the difficulties learners experience in geometry, and how their difficulties are further worsened when confronted by problems involving proportionality. A total of 32 learners obtained 0 marks in problems involving similarity and proportionality from School 1, while School 2 had 14.

Another concern is the number of learners who did not provide any responses for the entire geometry section of the examination. School 1 has 3 learners who did not attempt any of the geometric questions. Table 2 provides a summary of questions involving proportionality and similarity which were not attempted by learners. A total of 30 learners from School 1 and 15 learners from School 2 had No Responses (NR's) for 10.2.3, which is a problem requiring **strategic competence**.

	Similarity	Proportionality				
Questions	10.2.1.	9.1	9.2	10.1	10.2.2	10.2.3
SCHOOL 1	11	18	22	13	30	30
SCHOOL 2	2	8	7	7	12	15

Table 2: No Responses

The analysis of the geometry section data uses learners' raw marks as can be seen in the tables and BWP's in Table 3, providing descriptive statistics for School 1. Learners' marks from School 1 for the whole geometry section and questions involving proportionality are positively skewed since in both categories the mean is larger than the median. Positively skewed data is an indication of unbalanced large number of low marks. The higher standard deviation for the entire geometry section indicates that learners' marks are more spread out from the mean, showing greater variability in performance. The relatively lower standard deviation for geometric questions involving proportionality shows how learners' marks are tightly clustered around the mean.

	Overall geometry	Similarity	Proportionality
Number of learners (n)	56	56	56
Standard Deviation	4	1,03	1,3
Mean	2,8	0,6	0,4
Median	1	0	0
Mode	0	0	0
Minimum	0	0	0
Maximum	18	4	7
Range	18	4	7
Distribution	**Positively skewed**	**Positively skewed**	**Positively skewed**

Table 3: School 1 Distribution and descriptive statistics (Grade 12 Prelim Exam 2023)

The outliers as can be seen in both Figures 8 and 9 have an impact on the standard deviation since the whole data set is used in calculating the standard deviation. It is also worrying that these outliers which represent learners' raw marks are all below 50%. The BWP's (Figure 8, 9 and 10) for School 1 show that the first quartile is at zero due to the large number of learners who obtained a mark of zero. Figure 9 shows that the median mark is also zero. There's no box in Figure 10 since all the quartiles are zero's.

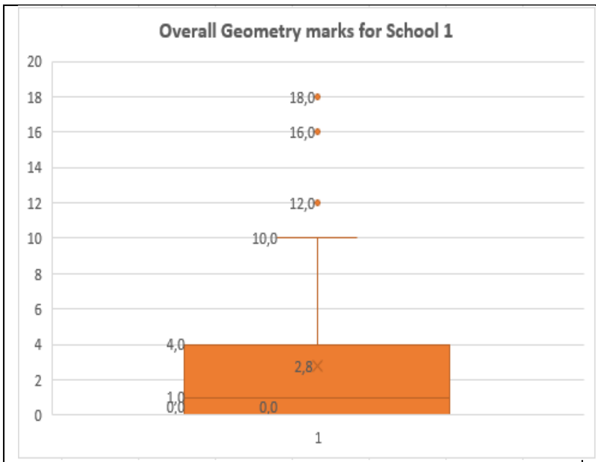


Figure 8: Prelim geometry BWP for School 1

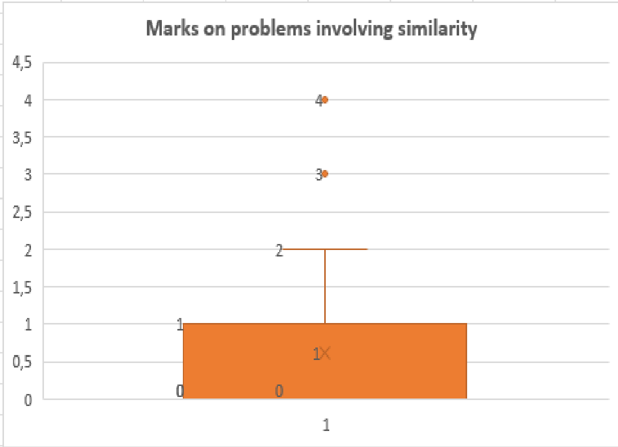


Figure 9: Prelim similarity BWP for School 1

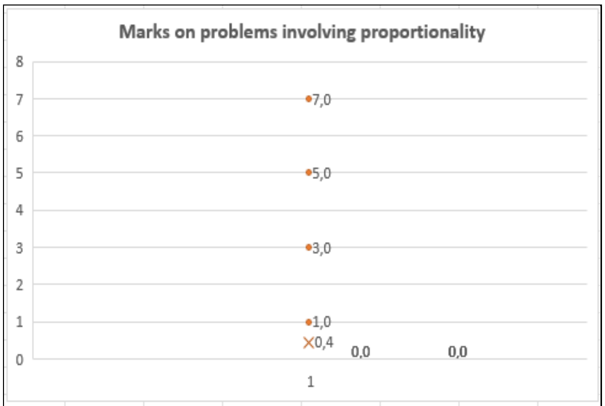


Figure 10: Prelim prop BWP for School 1

Table 4 provides descriptive statistics for School 2. Learners' marks from School 2 for the whole geometry section and questions involving proportionality are positively skewed since in both categories the mean is larger than the median. Positively skewed data is an indication of how disproportionately high is the number of low marks. The mean values in all three categories indicate very poor performances. The lower standard deviation for the entire geometry section shows how learners' marks are closely clustered around the mean, meaning the data is less spread out. The relatively lower standard deviation for geometric questions involving proportionality also shows how learners' marks are tightly clustered around the mean.

	Overall geometry	Similarity	Proportionality
Number of learners (n)	33	33	33
Standard Deviation	2,9	1,2	0,9
Mean	2,1	0,5	0,7
Median	1	0	0
Mode	0	0	0
Minimum	0	0	0
Maximum	13	4	3
Range	13	4	3
Distribution	**Positively skewed**	**Positively skewed**	**Positively skewed**

Table 4: School 2 Distribution and descriptive statistics (Grade 12 Prelim Exam 2023)

The outliers as can be seen in Figures 11, 12 and 13 have an impact on the standard deviation since the whole data set is used in calculating the standard deviation. It is also worrying that these outliers which represent learners' raw marks are all below 50%. The Box and Whiskers (Figure 11, 12 and 13) for School 2 show that the first quartile is at zero due to the large number of learners who obtained a mark of zero. Figures 12 and 13 show that the median mark is also zero. In Figure 12, the maximum and the upper quartile are in the same position of 1 mark.

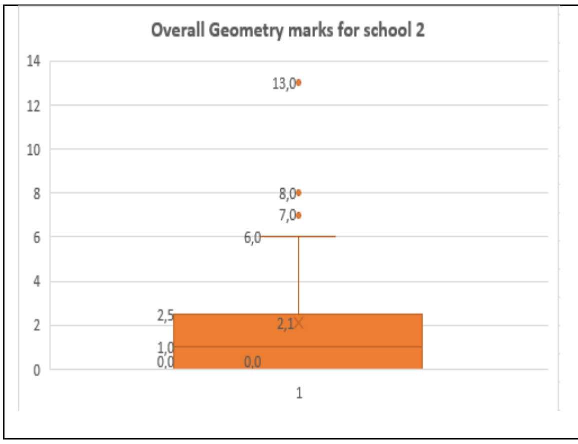


Figure 11: Prelim geometry BWP for School 2

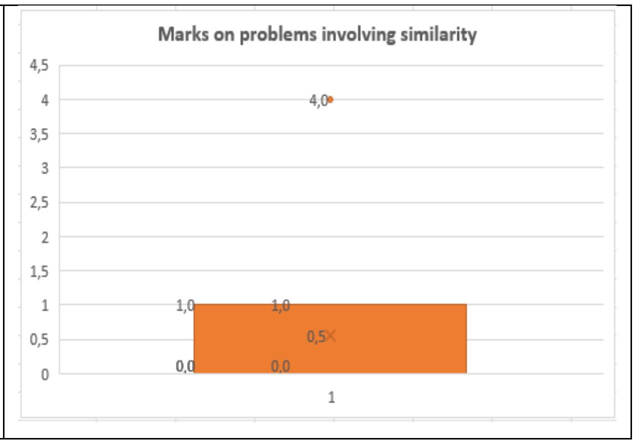


Figure 12: Prelim similarity BWP for School 2

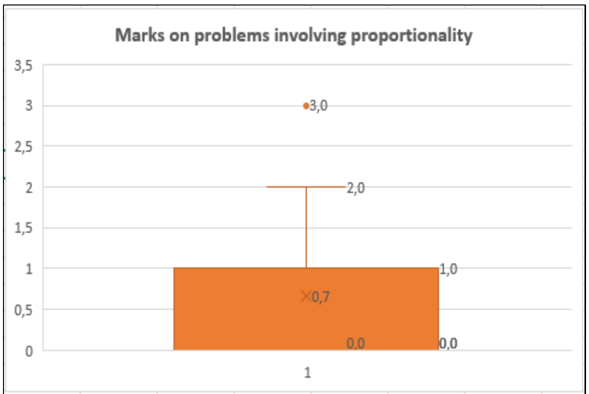


Figure 13: Prelim prop BWP for School 2

The descriptive statistics for both School 1 and School 2 (Figure 13) clearly show poor performances for learners in the concept of similarity and proportionality as highlighted by large number of learners who obtained a zero mark in geometric problems involving these concepts. A zero mark is a clear indication of lack of mathematical proficiency due to difficulties with the strands. Learners' difficulties on problems requiring **strategic competence** such as 10.2.3 are worse, as exhibited by a higher number of NR's.

6.3. Quantitative analysis of Grade 12 2024 Mid-term examination Paper 2 scripts according to Kilpatrick et al. (2001) four strands of mathematical proficiency

In the following passages descriptive statistics are used to provide quantitative analysis of the geometric section for the June 2024 learners' scripts. The aim is to make sense of learners' difficulties in problems involving similarity and proportionality as compared to the entire geometry section. The total number of marks for the geometry section of the Mid-term (June, Gauteng Province) examination 2024 is 43, while only 19 marks are allocated for problems involving similarity and proportionality. The overall geometry topic accounts for 29%, while geometric problems involving similarity and proportionality account for 13% of this Paper 2.

In analyzing learners' scripts from both participant schools, we found that only two learners from School 1 were able to obtain marks which are 50% and above in geometry. The highest mark in geometry is 24 out of 43 marks, and this further underscores learners' difficulties in this topic. Figures 14 and 15 provide a question-by-question analysis for both participant schools in geometry questions. Notice how proportionality problems involving **strategic competence** such as Question 10.2.3 have no learners who managed to pass from both participant schools.

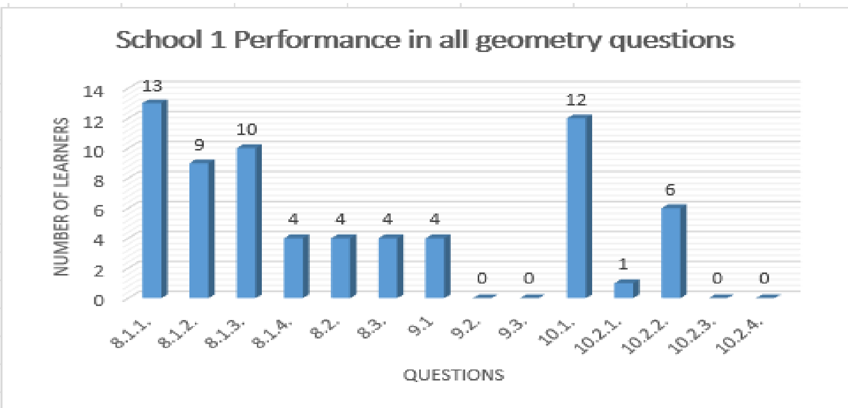


Figure 14: School 1 June 2024 Exam Question by Question Analysis (n=30)

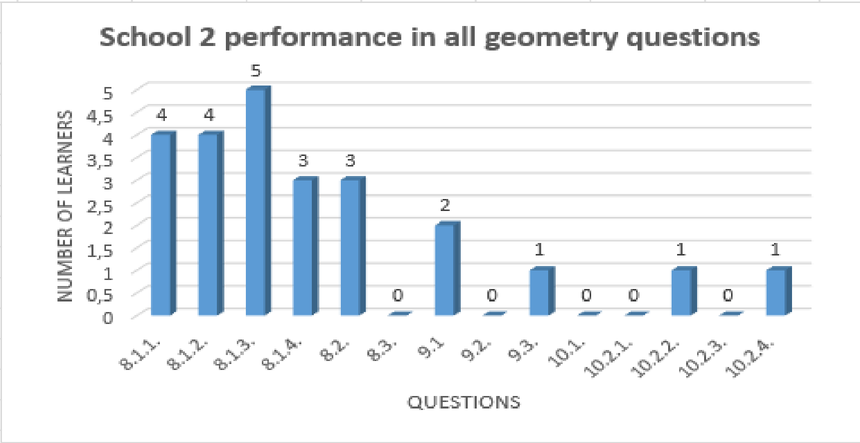


Figure 15: School 2 June 2024 Question by Question analysis (n=20)

The overall average mark (mean) for the entire June exam geometry component in School 1 is 5.96 (approximately 6) and for School 2 is 2.7. The average mark for problems involving similarity in School 1 is 0.8, while in School 2 is 0.1. The average mark for problems involving proportionality in School 1 is 2.3, while in School 2 it is 0.1. This highlights the difficulties learners experience in geometry, and how these are further exacerbated when they have to deal with problems involving proportionality. A total of 16 participant learners from School 1 and 18 learners from School 2 obtained a zero mark on problems involving similarity. A total of 10 participant learners from School 1 and 19 from School 2 obtained a zero mark for problems involving proportionality.

Another disconcerting matter is that of a higher number of learners who do not provide any responses for problems involving proportionality in Paper 2. All participant learners attempted the geometry section of the June 2024 examination, although several of them did not provide responses for problems involving similarity and proportionality. Table 5 provides a summary of questions involving similarity and proportionality which were not attempted by learners.

	Similarity	Proportionality			
Questions	10.2.2.	10.1.	10.2.1.	10.2.3.	10.2.4.
SCHOOL 1	4	0	7	15	8
SCHOOL 2	4	3	7	11	12

Table 5: No Responses

Table 6 provides descriptive statistics for School 1's entire geometry section, problems involving similarity and proportionality for the Mid-term examination. This data is positively skewed since in all three categories the mean is greater than the median. The larger value for standard deviation of the geometry section indicates how learners' marks are loosely clustered around the mean, i.e. the data is more spread out. The lower standard deviation for problems involving similarity and proportionality is indicative of how learners' marks are closer to the mean.

	Overall geometry	Similarity	Proportionality
Number of learners (n)	30	30	30
Standard Deviation	6,1	1,1	2,4
Mean	5,96	0,8	2,3
Median	5,5	0	1
Mode	0	0	0
Minimum	0	0	0
Maximum	24	3	8
Range	24	3	8
Distribution	**Positively skewed**	**Positively skewed**	**Positively skewed**

Table 6: School 1 Distribution and descriptive statistics (Grade 12 June Exam 2024)

The outliers as can be seen in Figures 16, 17 and 18 have an impact on the standard deviation since the entire data set is used to determine the standard deviation. In the Box and Whiskers (Figure 17 and 18) the 1st Quartile is zero due to a higher number of learners who continue to obtain a zero mark in geometric problems. In Figure 17, the median mark is also zero.

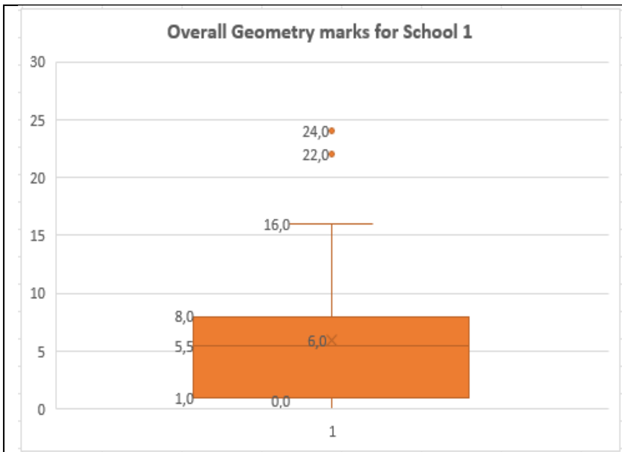


Figure 16: June geometry BWP for School 1

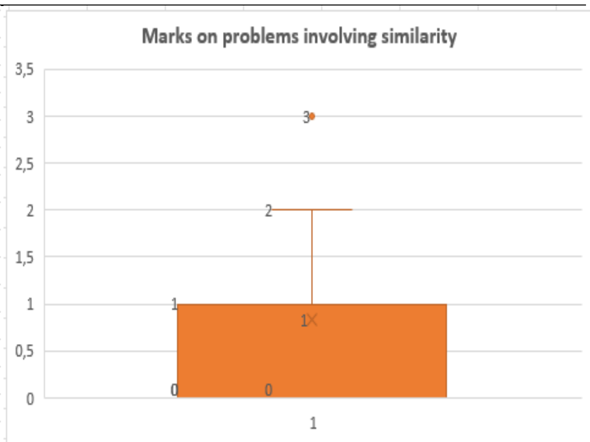


Figure 17: June similarity BWP for School 1

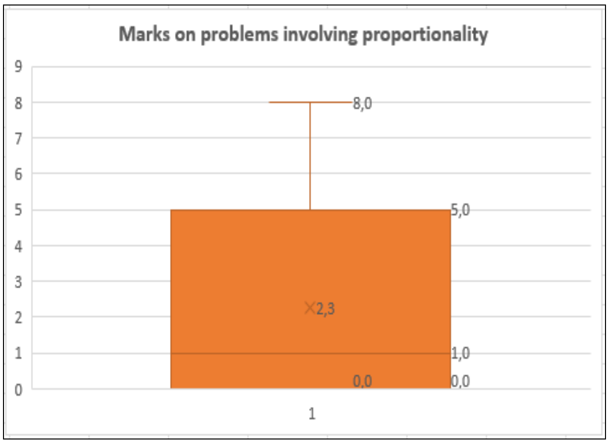


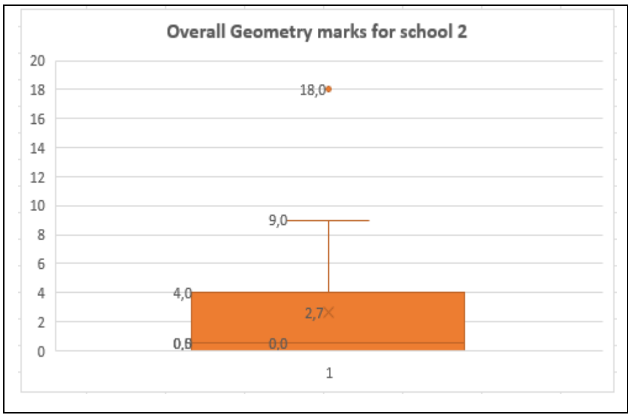
Figure 18: June prop BWP for School 1

Table 7 provides descriptive statistics for School 2. Again, learners' marks from School 2 for the three categories: Entire geometry section, problems involving similarity and proportionality are positively skewed, since in all categories the mean is greater than the median. The lower mean values in all three categories indicate very poor performances. Learners' marks for the entire geometry section are closely clustered around the mean, hence we have a relatively lower standard deviation. The standard deviation for problems involving similarity and proportionality is very low, showing how learners' marks are essentially clustered around the mean.

	Overall geometry	Similarity	Proportionality
Number of learners (n)	20	20	20
Standard Deviation	4,3	0,3	0,4
Mean	2,7	0,1	0,1
Median	0,5	0	0
Mode	0	0	0
Minimum	0	0	0
Maximum	18	1	2
Range	18	1	2
Distribution	**Positively skewed**	**Positively skewed**	**Positively skewed**

Table 7: School 2 Distribution and descriptive statistics (Grade 12 June Exam 2024)

The outliers in Figures 19, 20 and 21 affect the standard deviation since the entire data set is used in determining the standard deviation. It is also concerning that these outliers for School 2 are below 50% of the total mark for geometry, and how all the three quartiles are at zero. There's no box in Figures 20 and 21 since all the quartiles are zero's.



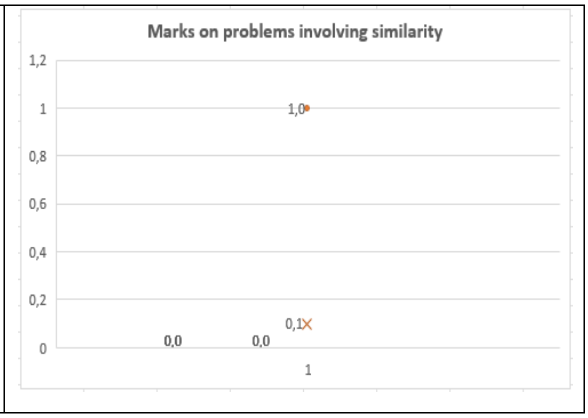


Figure 19: June geometry BWP for School 2

Figure 20: June similarity BWP for School 2

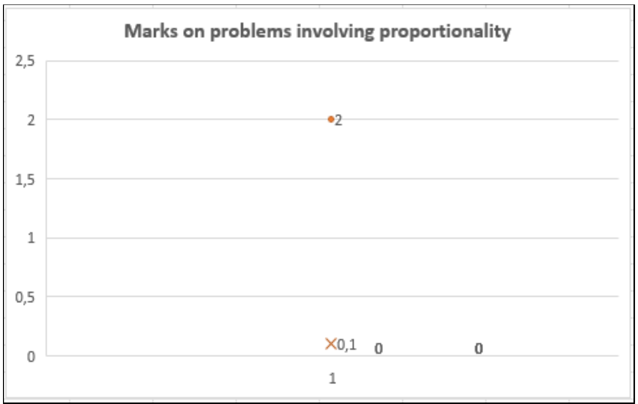


Figure 21: June prop BWP for School 2

The descriptive statistics presented for both School 1 and 2 indicate poor performances for learners in the concept of similarity and proportionality as can be seen through a large number of learners who obtained zero in geometric problems involving these concepts. These zero's expose **conceptual understanding**, **procedural fluency**, and **adaptive reasoning** difficulties learners experienced. In questions involving **strategic competence** such 10.2.3, learners' performances are dismal and the number of 'no responses' is high.

6.4. Qualitative analysis of 2024 Grade 12 June Paper 2 geometry section of learners' scripts: Proportionality and Similarity

In this section, learners' solutions in geometric problems involving proportionality and similarity for June 2024 examination is qualitatively analyzed through Kilpatrick et al. (2001) four strands of mathematical proficiency. Further, the researcher provides a question-by-question qualitative analysis of learners' solutions, clearly outlining difficulties for different learners (Learner 1: L1; Learner 2: L2; …. and L30 respectively) in geometric questions involving similarity and proportionality. Learners' responses are presented as italics in the presentation. The entire Question 10 of this examination is composed of geometric problems involving similarity and proportionality, which is in line with the focus of this study.

6.4.1. Difficulties in Question 10.1

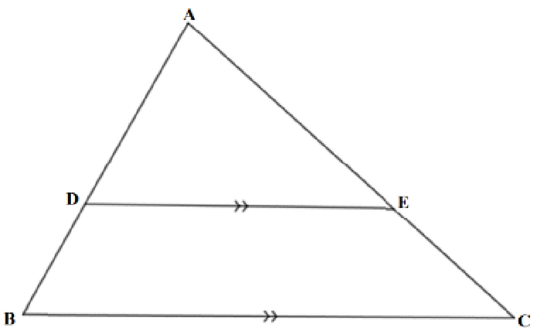
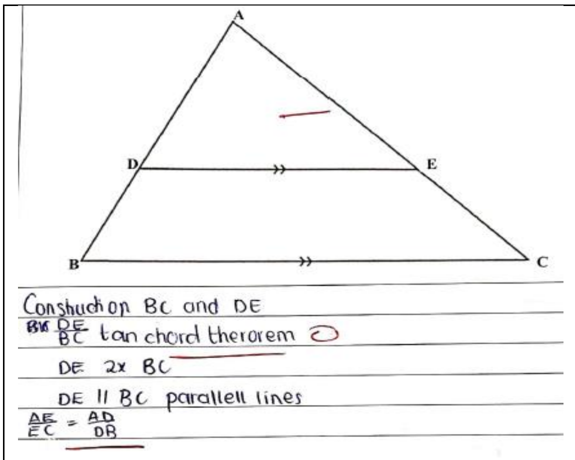
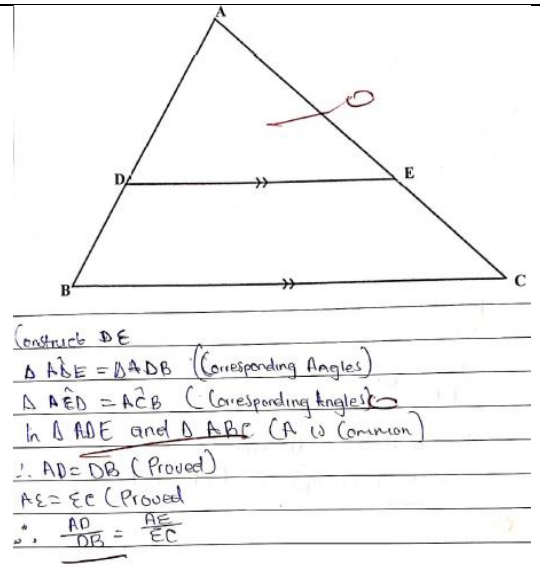
10.1	In the diagram below, ΔABC is drawn with DE ∥ BC.  Prove the theorem which states that $\frac{AD}{DB}=\frac{AE}{EC}$.	5 Marks

Table 8: June 2024 Question 10.1. (GDE, 2024, p.12)

6.4.1.1. School 1

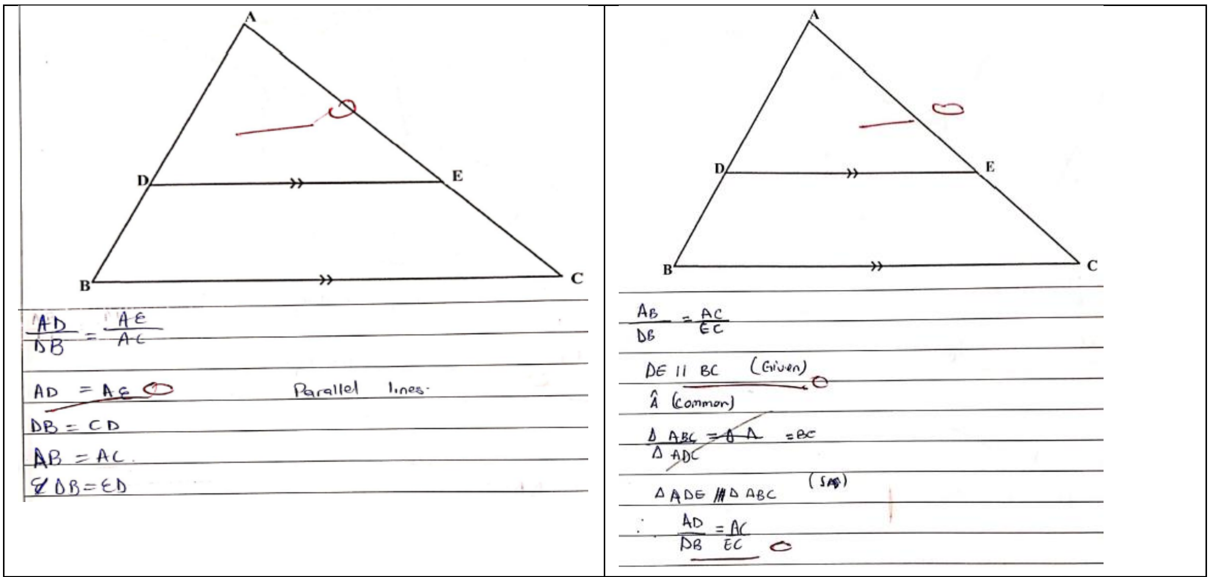




Solution 1: L1 and L7 Constructing existing line segments

In Solution 1, L1 and L7 started by constructing line segments which already existed in the given diagram, and this can be observed on how L1 referred to the construction of BC and DE, while L7 referred to the construction of DE. Both these learners' constructions are only stated and not executed on the actual diagrams. L1 wrote: $\frac{DE}{BC}$ which is an incomplete and irrelevant geometric statement followed by a reason: *tan chord theorem* that does not make any sense since there are no circles or tangents on the given diagram. The same pattern can be observed with regards to L7's solution where the learner confused triangles and angles and proceeded to provide incoherent steps. In line 4, L7 attempted to prove congruency by making comparisons of sides AD to DB and AE to EC from triangles ADE and ABC, and this is in no way helpful in proving the proportional division theorem.

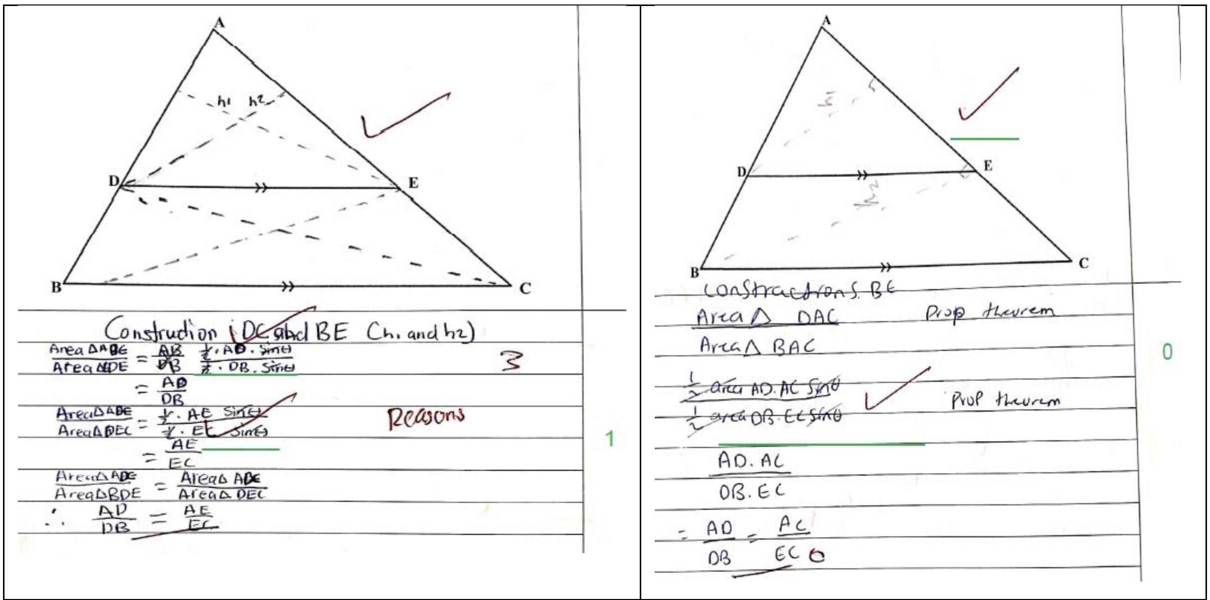
L1 experienced difficulties associated with **conceptual understanding** of the Area of a triangle and the important role it plays in proving proportional division theorem. This learner's response also exhibits difficulties associated with lack of **procedural fluency** with regards to geometric constructions as can be seen in Solution 1, where the learner referred to construction of already existing line segments. L7 exhibits difficulties similar to L1 as this learner also constructed already existing line segments and went on to provide illogical geometric statements and reasons. The provision of invalid and irrelevant reasons by both participant learners highlights **adaptive reasoning** difficulties.



Solution 2: L5 and L8's failure to construct including an incorrect proof

In Solution 2, L5 and L8 did not see the need for any construction and instead went straight to proving the theorem. L5 may have thought that $\frac{AD}{DB} = \frac{AE}{EC}$ only when AD = AE which is a common misunderstanding of the concept of proportionality. L5 justified the incorrect statement: *AD = AE* by using the incorrect reason: *Parallel lines*. The assumption by L5 that *AD = AE* and *AB = AC* shows that the learner viewed triangle ABC as isosceles whereas there's no indication on the given information that this is the case. L8 started by stating the very same statement that needed to be proved: $\frac{AB}{DB} = \frac{AC}{EC}$ instead of providing a series of logical statements and reasons drawing from the concept of the area of triangles to eventually formulate the required proportions. L8 also attempted to prove similarity between triangle ADE and ABC but could not find all three corresponding angles except for angle A.

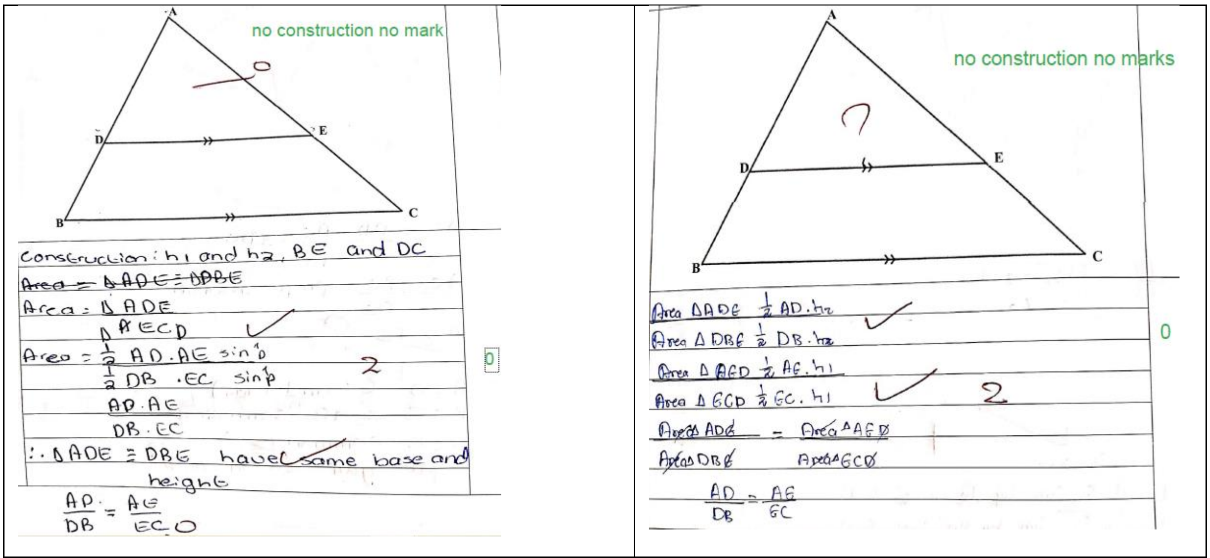
L5's decision to equate AD to AE and AB to AC shows **conceptual misunderstanding** of proportionality because the learner assumed that proportional division can only occur when line segments are equivalent. L8 lacks the **conceptual understanding** of the role of area in proving proportionality. Both L5 and L8 lacked the **procedural fluency** to provide the required constructions and to know that one cannot begin a proof by using the very same statement which needs to be proved. $\frac{AB}{DB} = \frac{AC}{EC}$ should appear as a concluding statement, and not as an opening statement. There's also no logical flow between the statements, pointing to difficulties associated with **adaptive reasoning** abilities. The statements $\hat{A}$ $(Common)$ and $\Delta ADE/// \Delta ABC$ demonstrate that this learner also has difficulties emanating from **conceptual misunderstandings** of similarity and proportional division.



Solution 3: L18 and L11's incorrect use of the Area rule

L18 opted to use the Area rule but also included h_1 and h_2 which are unnecessary constructions when one uses the Area rule (Trigonometric approach). All that is needed when one proves the proportional division theorem through the Area rule is the construction of BE and DC. Another notable observation on L18's solution is the incorrect formulation of the Area rule since the learner only refers to one side for example AD in Δ ADE and DB in Δ BDE. The Area rule requires that two sides and an included angle be used for it to be effective. L11 incorrectly constructed h_1 and h_2, and proceeded to refer to AD and AC from a non-existent Δ DAC. Another incorrect formulation of the Area rule by L11 can be seen through the use of the word *area* in the Area rule: $\frac{\frac{1}{2} area\ AD.AC\ \sin\theta}{\frac{1}{2} area\ DB.EC\ \sin\theta}$ as can be seen in Solution 3. The reason *prop theorem* provided by L11 is incorrect and irrelevant since one cannot reason using the same theorem they are proving.

The Area rule was applied incorrectly by many learners including L18 and L11. This highlights difficulties geometry learners experience with regards to **conceptual understanding** of the proportional division theorem. These learners had difficulties associated with **conceptual understanding** and **procedural fluency** of the Area rule as it relates to proving the proportional division theorem. Irrelevant reasons provided by L11 such as *prop theorem* show difficulties associated with **adaptive reasoning**.

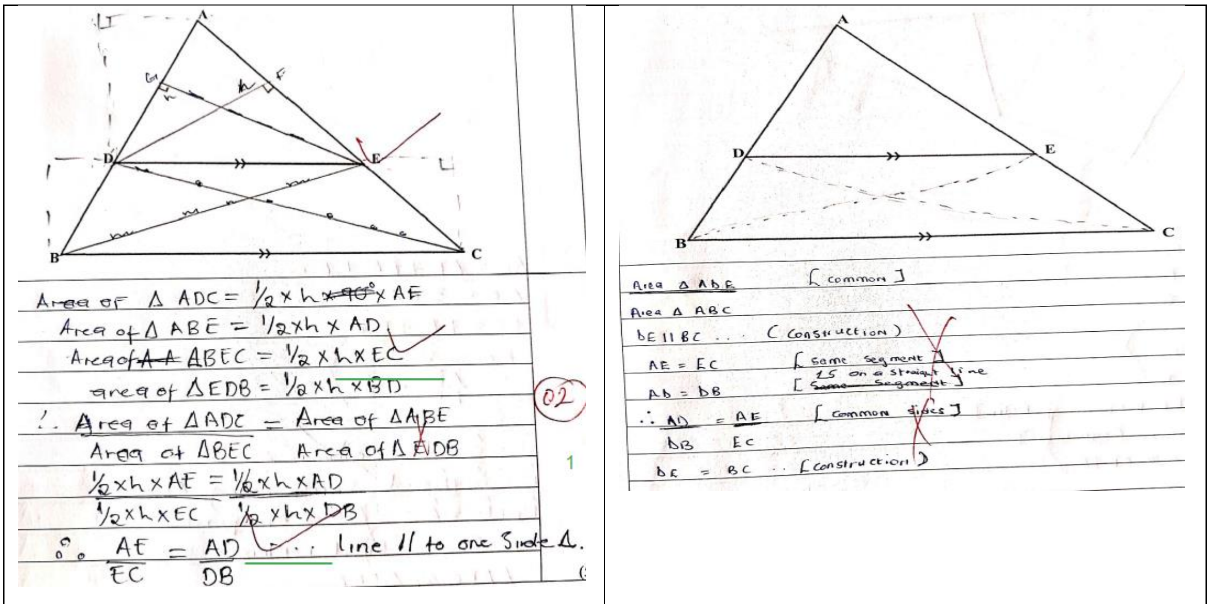


Solution 4: L26 and L27’s reproduction of proof without construction

L26 failed to execute the required constructions for the proof but attempted to reproduce the proof through the Area rule. This attempt is not successful because without constructing DC and BE it becomes difficult for the learner to select the correct line segments and the included angles which will be common between the two triangles involved. L27 provided a proof without any construction, and although this attempt is unsuccessful there’s some understanding of the Area of a triangle through the standard formula for area. L27 also failed to correctly formulate the required areas of triangles ADE, BDE and CED to prove the proportionality of sides. The statement by L27: $\frac{Area\,\Delta ADE}{Area\,\Delta DBE} = \frac{Area\,\Delta AED}{Area\,\Delta ECD}$ require a valid geometric reason and this learner did not provide any.

L27 is an example of a learner who lacked proficiency in the concept of proportional division but was able to reproduce some parts of the theorem through memorization since it is not possible to formulate the ratios from the area of triangles without any constructions, unless if this was proof through the Sine Rule. This learner also could not proceed beyond the second step of the theorem due to lack of **conceptual understanding**. It is as if the learner was not aware of what needed to be done but recalls partly what was either in the textbook or done in class. L26 also lacked the **conceptual understanding** and **procedural** fluency to formulate the required areas of triangles involved that produce the required proportions. Learners may only be awarded marks without construction if they opt to use a Sine Rule Application approach as already discussed in the analysis of the question paper and marking guideline section. The equating of triangles and their areas without any justification highlights learners' difficulties with **adaptive reasoning.**

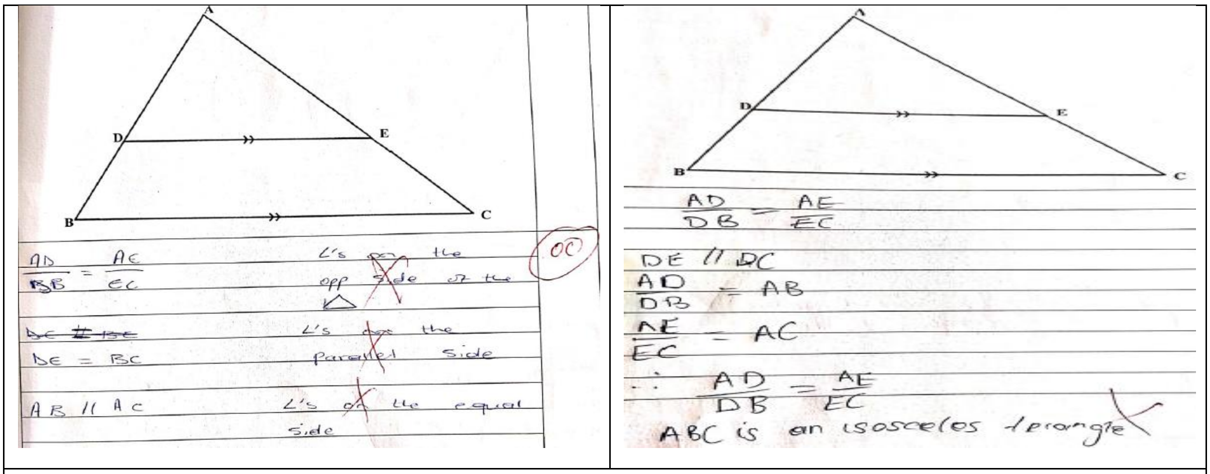
6.4.1.2. School 2



Solution 5: L11 and L15's incorrect labelling and constructions

L11's labelling of the constructed perpendicular heights is problematic since there's no distinction between the height relative to base AD and height relative to base AE. The two heights are not equivalent and labelling them with the same variable implies that they are equal in magnitude, which is not the case. The extended line segments outside L11's diagram are also unnecessary since they do not play any part in proving the theorem. Further, L11 opted to label the constructed heights as EG and DF, and one would have expected that these would have been substituted in the standard area formula instead of just substituting h for height. L11 also failed to correctly formulate the standard area formula because the base used in the Area for Δ ABE: $Area\ \Delta\ ABE = \frac{1}{2}h \times AD$ should have been AB instead of AD. AD is a base for the Area of Δ ADE. The same can be said about base AE in Δ ADC. This is what led to the following incorrect and unjustifiable equating of the Areas of triangles: $\frac{Area\ of\ \Delta ADC}{Area\ of\ \Delta BEC} = \frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta EDB}$. L15's construction gives the impression that the learner wanted to prove the theorem through the Area rule since perpendicular heights are not constructed. What then follows is a disjointed series of statements and reasons which do not indicate any concept involved for example $\frac{Area\ \Delta ADE}{Area\ \Delta ABC}$ (*common*) does not show any justification why the Areas of triangles are divided and what is it that is common between the two triangles. Reasoning that *DE//BC* because of construction is equally problematic since this was a given statement, and not a construction. L15 also reasoned by stating that [$\angle's$ *on a straight line*] in support of the statement *AD = DB*, thereby mixing up line segments with angles.

L11 and L15's faulty constructions show a lack of the **procedural fluency** to make the constructions which are important in formulating the required proportional division. Also, difficulties associated with **conceptual understanding** of the theorem are evident in how L11 selects triangle BEC instead of DEC and triangle ABE instead of ADE. No relevant reasons are provided by this learner, further highlighting **adaptive reasoning** difficulties. The reasons provided for the conclusion by L11 are not correct, and this underscores the difficulties learners experience with regards to **adaptive reasoning** in problems involving proportional division. L15 also possesses **conceptual understanding** difficulties regarding the role of perpendicular heights in determining the Areas of triangles. The reason: $\angle's$ *on a straight line* provided in line 5 for the statement *AD = DB* points to **adaptive reasoning** difficulties since the learner reasons with angles in equating line segments.



Solution 6: L6 and L9 no construction and starting with conclusion

L6 and L9 provided no construction and started with the very same statement which requires proving as can be seen in Solution 6. L6 equated DE to BC although the two lines are visually unequal, and then justified this incorrect equality by stating that: $\angle$ '*s on the parallel side*. L6 also stated that AB//AC although these are clearly two intersecting line segments, and this claim is then supported by the incorrect reason: $\angle$ '*s on the equal side*. L9 stated that $\frac{AD}{DB} = AB$ and $\frac{AE}{EC} = AC$ which essentially imply that the learner incorrectly understands the length of AB to be the same magnitude as the length of AD divided by DB, and that of AC as the length of AE divided by EC. This kind of interpretation is worrisome since one would imagine that learners at this level should at least understand the relationship between constitutive parts of straight lines. The closing statement by L9, that *ABC is an isosceles* triangle is both incorrect and irrelevant since the question asked for the proportionality of sides and not the description of the triangle.

The practice of starting the proof by stating the very same thing which needs to be proved points to both lack of **conceptual understanding** and **procedural fluency** with regards to proportional division theorem. Learners who reason with angles to justify equating sides as it is the case with L6 possess difficulties associated with **adaptive reasoning**.

Two recurring themes identified in learners' difficulties for Question 10.1 from both participant schools are poor or incorrect constructions and illogical and irrelevant geometric statements and reasons. Learners' whose constructions were correct were let down by their **conceptual misunderstandings** of the area of a triangle.

6.4.2. Difficulties in Question 10.2.1.

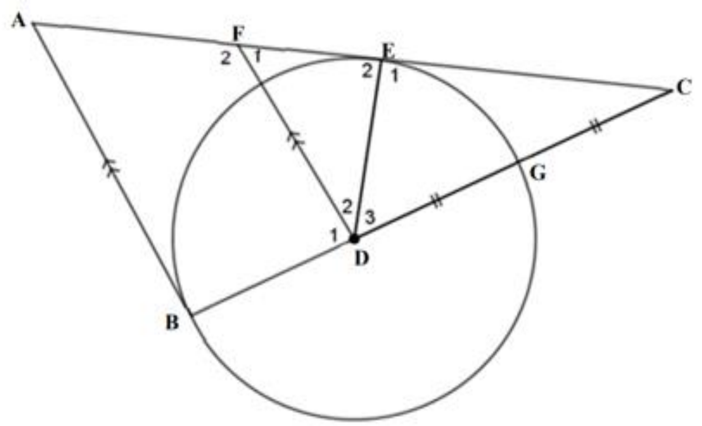
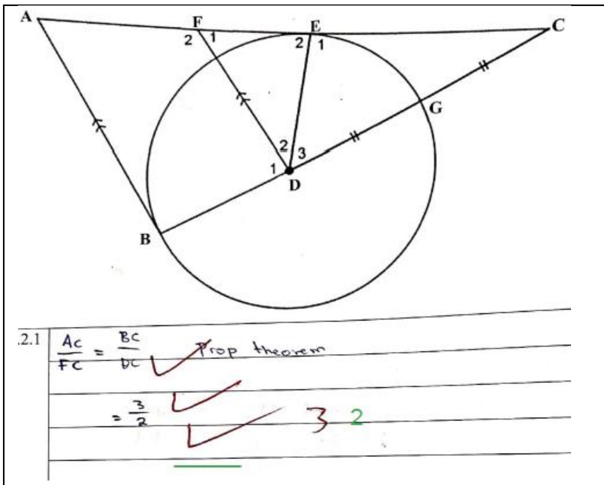
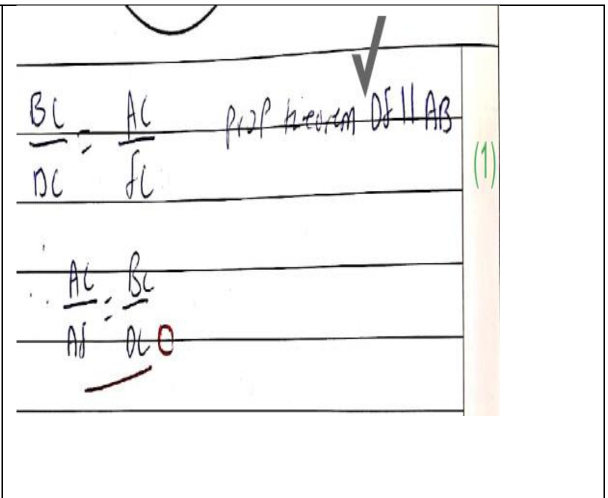
10.2.1	10.2 In the diagram below, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C. DG = CG. F is a point on AC such that DF ∥ AB.  10.2.1 Find, with reasons, the ratio of $\frac{AC}{FC}$.	3 Marks

Table 9: June 2024 Question 10.2.1. (GDE, 2024, p.13)

6.4.2.1. School 1

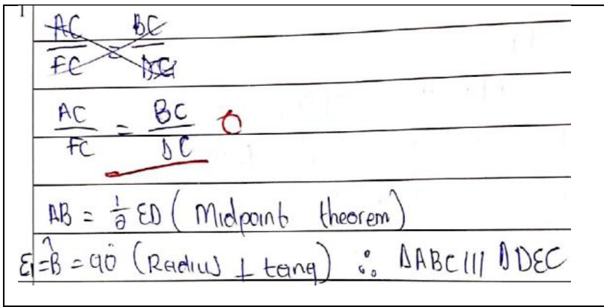


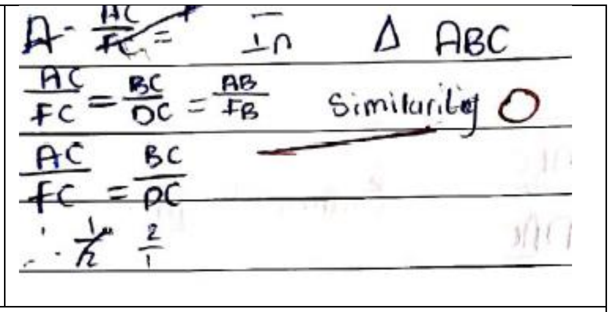


Solution 7: L9 and L11's correct proportion and reason

L9 was able to find the correct ratio for $\frac{AC}{FC}$, although this was done without showing the process and no labelling of the diagram was made. A critical process of showing the one-to-one relationship between BD, DG and GC emanating from the equality of radius BD and DG was not done by this learner. This approach to solving problems involving ratio and proportionality without labelling the ratios on the diagram is notorious for causing difficulties for learners in subsequent questions as can be observed in L9's inability to use the solution from this question to solve the problem in 10.2.3. L11 was able to correctly formulate the proportionality and provided a reason but could not work out the ratio. This learner had difficulties identifying the relationship between BD, DG and GC. The learner showed **adaptive reasoning** skills by correctly providing the proportionality and the correct reason which also makes reference to the applicable parallel lines.

Clearly, L9 has **conceptual understanding** of ratio, but lacks **procedural fluency** to adequately and logically present this understanding by showing all statements and reasons, evident in failing to show that BD = DG, and the reason provided for proportionality without making reference to the parallel lines (shown in Solution 7). Failure to reason adequately is a symptom of **adaptive reasoning** difficulties. L11's inability to identify the relationship between BD, DG and GC underscores difficulties associated with **conceptual understanding** and **procedural fluency.**

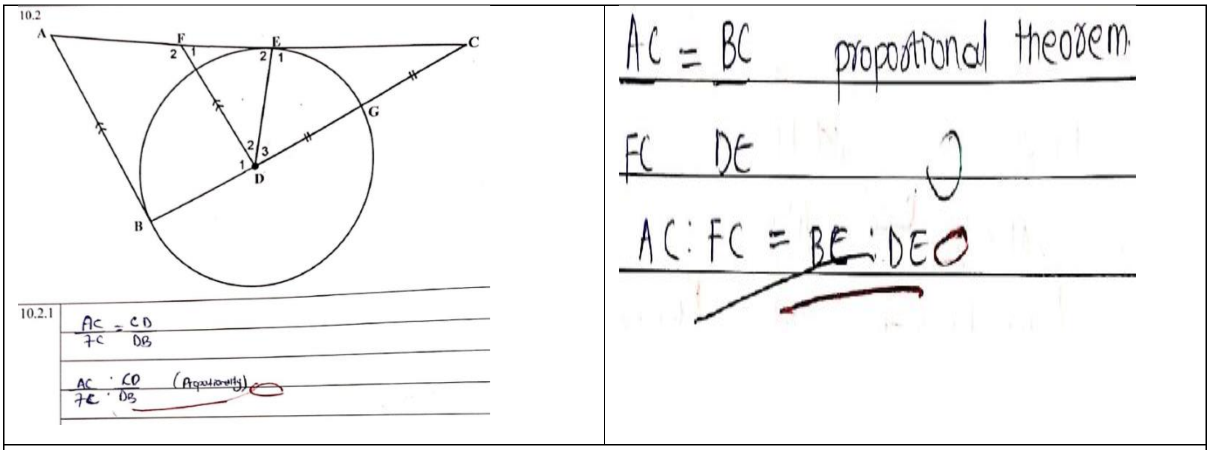




Solution 8: L7 and 29's failure to distinguish between proportional division and midpoint theorem

L7 could not differentiate between the application of the proportional division and the mid-point theorem. This learner was able to set up the proportionality but had difficulties in justifying this statement with a valid geometric reason. The statement E_1 = B = 90 is not relevant to the problem at hand. The learner finally concluded through similarity: $\Delta ABC\ ///\Delta DEC$ when the question was about ratio. This, tragically, is a norm in geometry where some learners resort to writing whatever that comes to mind when they encounter difficulties. L29 has difficulties differentiating between proportionality emanating from similarity and proportionality from proportional division as can be seen in the reason: *similarity* in Solution 8. The fraction $\frac{2}{1}$ provided by L29 after the statement: $\frac{AC}{FC} = \frac{BC}{DC}$ shows that the learner interpreted this question as a midpoint problem, because the ratio 2:1 for both AC:FC and BC:DC is only applicable when FC = AF and BD = DC respectively. So, L29 also confused proportional division with the midpoint theorem.

L7 experienced **conceptual misunderstandings** in differentiating between the application of the proportional division and the mid-point theorem. Irrelevant statements provided by learners also highlight **procedural fluency** difficulties. This learner was also able to set up the proportionality $\frac{AC}{FC} = \frac{BC}{DC}$ but had difficulties in justifying this statement with a valid geometric reason, and this highlights the learner’s difficulties associated with **adaptive reasoning**. L29’s solution shows **conceptual misunderstandings** of similarity, midpoint theorem and proportional division. The substitution of the ratio 2:1 by L29 highlights **procedural fluency** and **conceptual understanding** deficiencies, since determining the required ratio is dependent on the correct interpretation of the relationship between line segments BC and DC, and how these can be formulated as a proportion involving AC and FC.



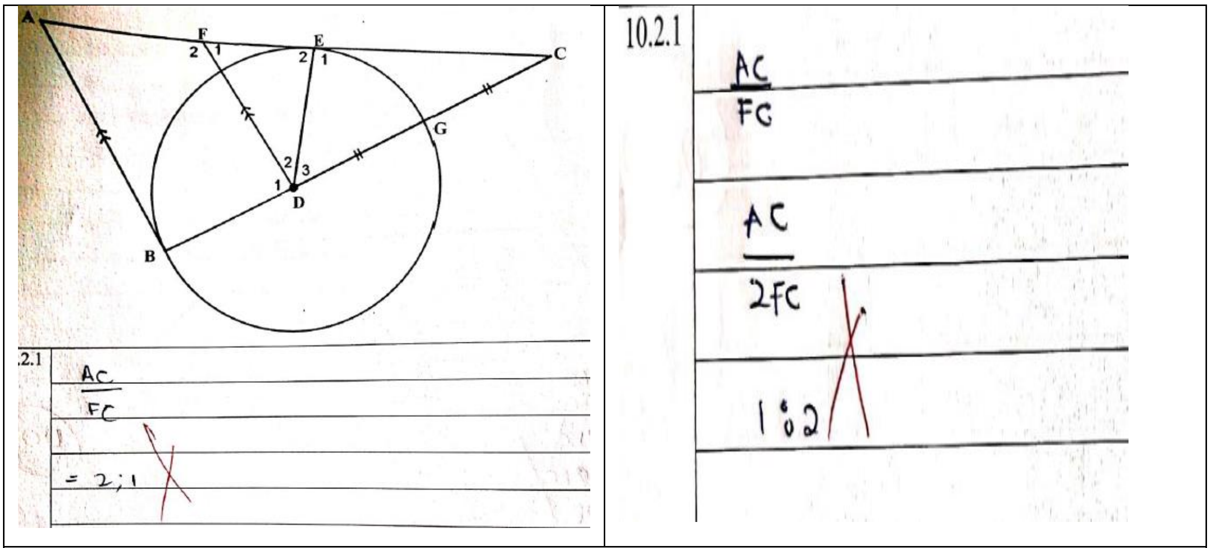
Solution 9: L8 and L13’s failure to understand that ratio can be expressed as a fraction

L8 does not seem to understand that ratio can be expressed as a fraction as can be seen in the statement $\frac{AC}{FC} : \frac{CD}{BD}$ in Solution 9. The reason: *(proportionality)* is incorrect. L13 represented proportionality both as a fraction and as a ratio. The reason: *proportional theorem* provided by this learner is also inadequate since it does not make reference to the parallel lines.

L8 had difficulties associated with **conceptual understanding** of proportionality and the **procedural fluency** to arrange corresponding line segments proportionally as can be seen from the first line of the solution. The incorrect reason provided by this learner highlights **adaptive reasoning** difficulties. L13 provided two different representations of proportionality representing the same thing due to **conceptual misunderstandings** of ratio and also how (**procedural fluency)** it can be expressed as a fraction. The reason provided by this learner is also insufficient, pointing to difficulties associated with **procedural fluency** in using geometric reasons.

A concerning pattern is on how none of the participant learners were able to demonstrate their **conceptual understanding** of ratios of lines which are equal in length. Another notable recurring theme is that no labelling of the diagram was done to simplify the problem.

6.4.2.2. School 2



Solution 10: L2 and L14 mixing up proportional division with midpoint theorem

L2 conjured up the ratio without relying on any logical geometric reasons with no labelling of the diagram, and further used a semicolon instead of a colon to represent ratio as can be seen in Solution 10. The ratio *2;1* indicates that the learner also interpreted this problem as that which involves the midpoint theorem. L14 proceeded from writing $\frac{AC}{2FC}$ and concluded that the required ratio is *1:2* which implies that FC's length is twice the length of AC. This misinterpretation shows that L14 also confused the proportional division with the midpoint theorem.

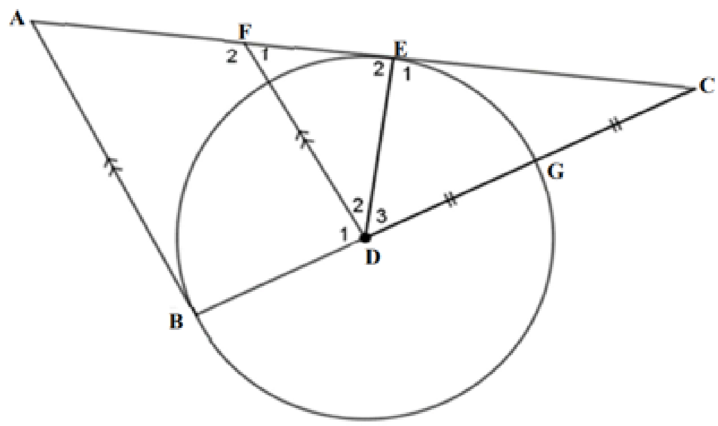
L2 and L14's solutions highlight both **conceptual misunderstandings** and **procedural fluency** difficulties on the concept of ratio and proportional division. The absence of any geometric reasons in both L2 and L14's solutions underscores difficulties associated with **adaptive reasoning**.

$AC = AF + FE + EC$ $AC = 3\ \cancel{EA}$ $FC = FE + EC$ $FC = 2.$ $\therefore \frac{AC}{FC} = \frac{3}{2}$
Solution 11: L11's incorrect procedure leading to correct ratio

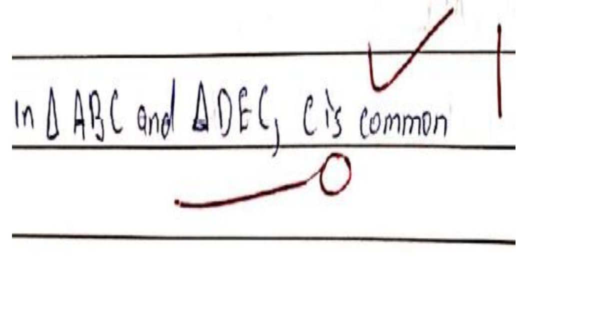
L11 presented an interesting case of how an incorrect or insufficient procedure may lead to a correct solution in geometry. Procedurally, this learner did everything wrong but was able to produce the correct ratio. The educator correctly did not award any marks. The statements AC = 3 and FC = 2 emanate from the assumption that AF = FE = EC.

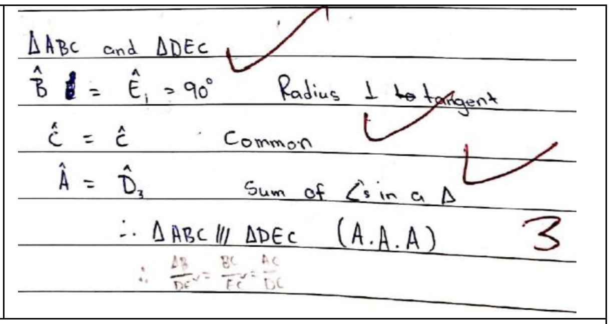
The first four lines from L11's solution clearly illuminate **conceptual understanding** and **procedural fluency** difficulties since this learner assumed that AF = FE = EC without providing any justifications. This lack of justifications highlights **adaptive reasoning** difficulties.

6.4.3. Difficulties in Question 10.2.2.

10.2.2	10.2 In the diagram below, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C. DG = CG. F is a point on AC such that DF ‖ AB.  10.2.1 Find, with reasons, the ratio of $\frac{AC}{FC}$. 10.2.2 Prove, giving reasons, that $\Delta ABC\ ///\ \Delta DEC$.	3 Marks
Table 10: June 2024 Question 10.2.2. (GDE, 2024, p.13)		

6.4.3.1. School 1

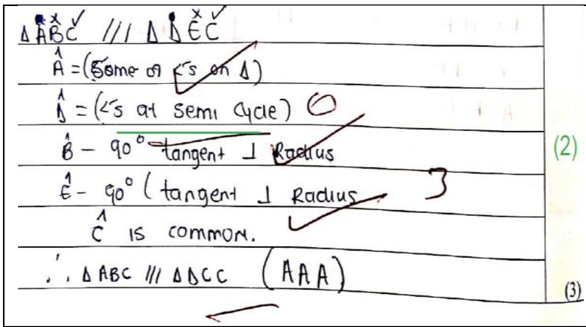


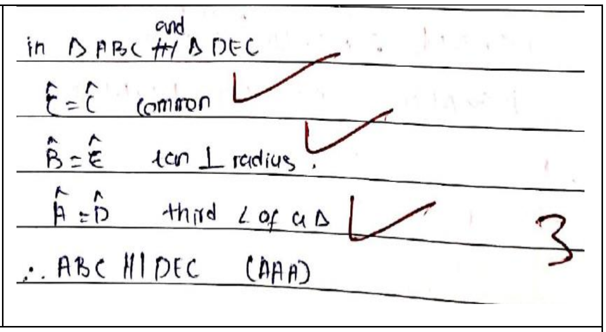


Solution 12: L23 and L9's listing of triangles involved in similarity

L23 began very well by listing the two triangles involved and the common angle C, but was unable to continue due to difficulties in circle geometry theorems which are crucial in finding the other pairs of equal angles ($\hat{A} = \hat{D}_1$ and $\hat{B} = \hat{E}_1$) necessary for proving similarity. L9 also listed the two triangles ABC and DEC, and went on to successfully prove their similarity.

L23 has **conceptual understanding** difficulties regarding circle geometry theorems, hence the learners' inability to determine the remaining two pairs of equal corresponding interior angles. L9 on the other hand showed **conceptual understanding**, **procedural fluency** and **adaptive reasoning** abilities by proficiently proving the required similarity and providing valid justifications as seen in Solution 12.

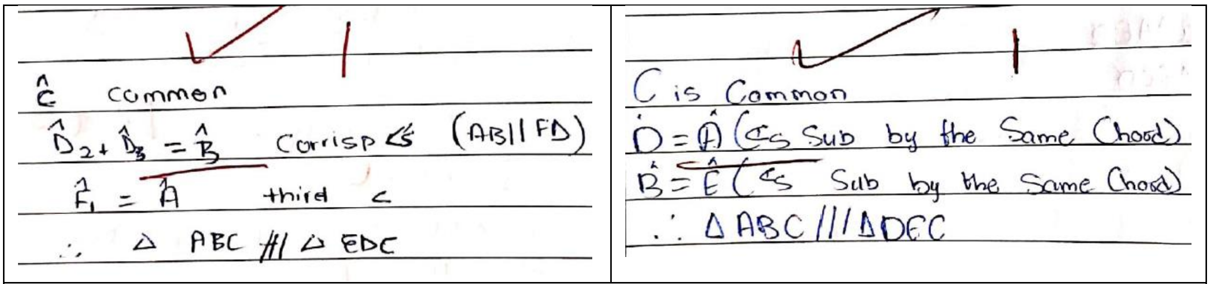




Solution 13: L2 and L4's starting with conclusion instead of listing triangles

L2 had difficulties in coming up with two pairs of angles which are equal through circle geometry. This learner does not understand how geometric statements and reasons should be compiled and arranged, since angles are incorrectly equated to reasons in line 2: *A = (Some of ∠ 's on Δ)* and line 3: *D = (∠ 's at semi circle)* as can be seen in Solution 13. The first line is also problematic because the learner began by declaring the two triangles as similar: Δ ABC///DEC, which is the very same thing which still needed to be proved. Interestingly, L4 also made the mistake of beginning with the concluding statement: ΔABC///DEC, but eventually corrected this error and used "*and*" in listing the two triangles involved.

L2's failure to list the two triangles involved highlights difficulties associated with **procedural fluency** on how proof cannot simply begin with the very same thing which must be proved. The equating of angles to reasons is a symptom of **procedural fluency** difficulties since the learner does not seem to understand how geometric statements and reasons should be compiled and arranged. This learner also had **conceptual understanding** difficulties on circle geometry theorems which became an obstacle in determining the outstanding two pairs of equal interior corresponding angles. L4, despite the initial failure to list triangles which was eventually corrected, showed proficiency in proving similarity in triangles. One's ability to recover from mistakes is also a sign of expertise (Nokes et al., 2009).

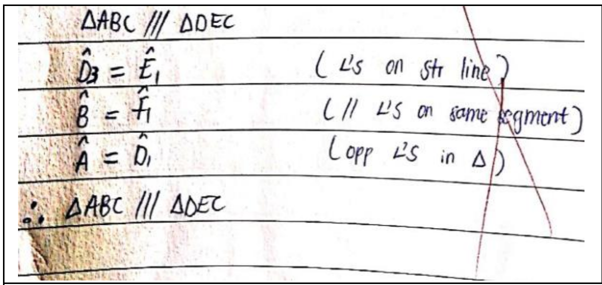


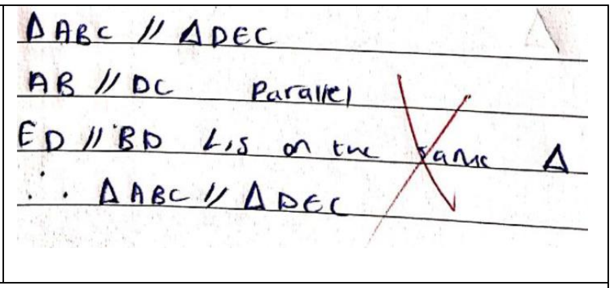
Solution 14: L21 and 27's failure to list triangles involved in similarity to be proved

L21 failed to begin by stating the two triangles involved and this subsequently led to the incorrect comparison involving an angle $\widehat{D}_2 + \widehat{D}_3$ that does not belong to any of the two triangles under consideration. Also, angle F_1 does not belong to any of the two triangles being compared for similarity. L27 also did not list the two triangles involved but was able to demonstrate understanding of the process of pairing angles to prove similarity. The two statements: $D = A$ and $B = E$ are problematic since there's no clarity as to which D or E was the learner referring to, and the learner could have easily used the numbers provided on the diagram to provide clarity but opted not to. The reasons: *(∠'s sub by the same chord)* are incorrect and irrelevant since it is not possible to have a common chord subtending angles which are outside the circumference of a circle, for example angle A and B.

Although L21 possessed the **conceptual understanding** that similarity requires three corresponding pairs of angles to be compared and equated with valid justifications, the learners' ability to solve this problem was hindered by **procedural fluency** deficiencies in listing the triangles involved and **conceptual understanding** difficulties of circle geometry. L27 is another example of a learner who was let down by **conceptual understanding** difficulties in circle geometry theorems and the **procedural fluency** to list triangles. L27's incorrect reasons illuminate **adaptive reasons** difficulties.

6.4.3.2. School 2

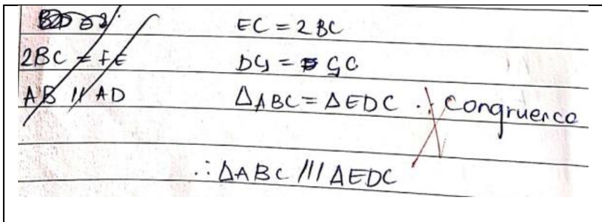




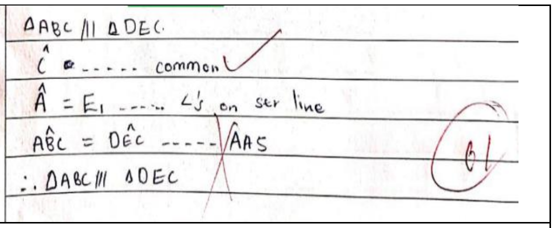
Solution 15: L1 and L4's failure to list triangles involved in similarity

L1 started with the concluding statement and proceeded to compare angles which are not relevant to the triangles involved in the required similarity. The statement: $D_3 = E_1$ is incorrect and it implies that Δ DEC is isosceles. The reason: *(∠ 's on a str line)* is irrelevant since angle D_3 and E_1 are not on the same straight line, besides angles on a straight are not necessarily equal. L1's statement: B = F_1 is also incorrect and is not helpful in proving similarity since angle F_1 does not belong to any of the triangles being compared. L4 presents an interesting case of a learner who confused parallel and similarity symbols. It is clear that L4 was also trying to start with the concluding statement instead of listing the two triangles first, hence the first and last statements are the same: $\Delta ABC\ //\Delta DEC$. The statement and reason: *ED//BD ∠ 's on the same* Δ highlights lack of coherence and understanding of the process of proving similarity.

L1 had **procedural fluency** difficulties regarding the identification of corresponding interior angles which can be equated in order to prove similarity. Further, the reasons used by this learner underscores **conceptual understanding** difficulties with circle geometry theorems and **adaptive reasoning** difficulties since there's no deductive reasoning and logical flow in this solution attempt. L4's mixing up of parallel and similarity symbols points to both **procedural** and **conceptual understanding** difficulties. This learner also possessed **adaptive reasoning** difficulties since the whole proof lacks coherence.



10.2.2



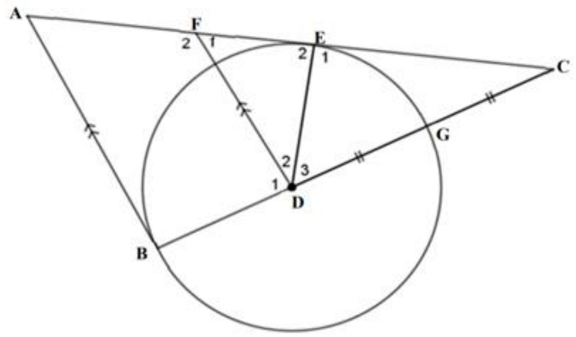
Solution 16: L3 and L10’s confusion of similarity with congruency

L3 in Solution 16 did not list the two triangles involved in this question, and this exposes difficulties regarding proving similarity techniques. The learner then focussed on equating sides and ends up by concluding congruency by linking it up with similarity. L10 began with the very same thing which needed to be proved. This learner then proceeded to equate angle A and E_1 by relying on invalid justification: ∠ *‘s on str line.* The reason: *AAS* provided by L10 in line 4 points to confusion between congruency and similarity.

L3’s inability to list the two triangles involved in similarity exposes **procedural fluency** difficulties, and the confusion between similarity and congruency highlights **conceptual understanding** difficulties on what these concepts are (definitions). How L10 began proving with the concluding statement is symptomatic of **procedural fluency** difficulties regarding proving techniques. The careless use of geometric reasons such as: ∠ *‘s on a str line* in situations where the angles being equated are not on the same point and on the same straight line underscores **adaptive reasoning** difficulties.

6.4.4. Difficulties in Question 10.2.3.

This question requires learners to possess **conceptual understanding**, **procedural fluency**, **strategic competence** and **adaptive reasoning** to be able to solve it successfully. In fact, without **strategic competence**, learners may not be able to provide any meaningful attempt.

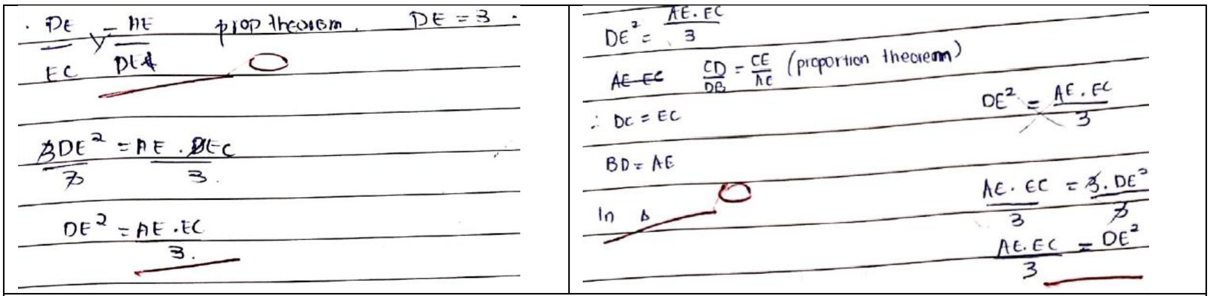
10.2.3	In the diagram below, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C. DG = CG. F is a point on AC such that DF ∥ AB.  10.2.1 Find, with reasons, the ratio of $\frac{AC}{FC}$. 10.2.2 Prove, giving reasons, that $\Delta ABC \mathbin{///} \Delta DEC$. 10.2.3 Prove that $DE^2 = \frac{AE.EC}{3}$	3 Marks
Table 11: June 2024 Question 10.2.3. (GDE, 2024, p.13)		

6.4.4.1. School 1

$\frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC}$ 0
Solution 17: L21 identified the two triangles involved but failed to formulate the required proportionality

L21 was able to identify the two similar triangles involved as shown by the proportionality provided, but since this learner failed to sufficiently prove the similarity of triangle ABC and DEC in question 10.2.2, and also failed to determine the applicable ratios in question 10.2.1, there was no way of correctly formulating the required proportions and substitutions necessary to prove the statement.

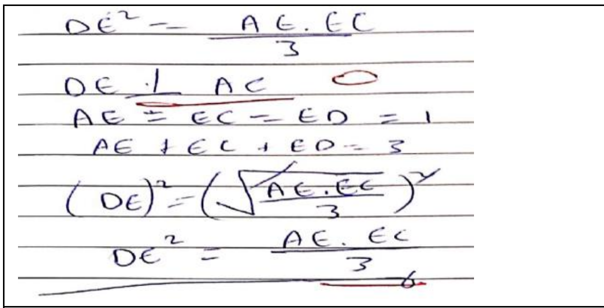
L21 lacked the **conceptual understanding** of circle geometry theorems which is essential in replacing AB with AE, and BC with 3DE. Further, lack of **strategic competency** by L21 led to the unnecessary inclusion of the two irrelevant sides AC and DC, and this prevented the learner from proceeding although the proportionality was partly formulated correctly. The lack of justification for the proportionality provided by the learner, points to difficulties associated with **adaptive reasoning**.



Solution 18: L4 and L20's confusion of proportionality from similarity with proportionality from proportional division theorem

L4 could not identify the two triangles involved which were already proved to be similar in Question 10.2.2, and as a result the learner mistakenly interpreted this problem as that which involves proportional division theorem instead of similarity. The proportionality: $\frac{DE}{EC} = \frac{AE}{DE}$ by L4 is incorrect, and its justification: *prop theorem* is misplaced since the required proportionality emanates from the similarity proved in 10.2.2. L4 then got preoccupied with the manipulation of the statement to be proved as can be seen in the 2nd line in Solution 18: $\frac{3DE^2}{3} = \frac{AE.EC}{3}$ which is an unjustified statement that does not logically flow from the previous statement. L20 also belongs to a category of learners who got fixated on the statement or claim which needed to be proved, and focussed on manipulating it to conveniently produce the required solution without providing any justification. This learner also incorrectly thought that this problem involves proportional division due to the inability to identify the similar triangles involved.

L4's transition from the incorrect proportionality in line 1 to the convenient formulation of $DE^2 = \frac{AE.EC}{3}$ by dividing with 3 on both sides and without any reasons, indicates **adaptive reasoning** difficulties. L4 and L20's confusion between proportionality from proportional division and similarity is as a result **strategic competency** difficulties, since the learners should have been able to identify the two triangles involved through a process of deconstruction. There are issues of **conceptual misunderstanding** and lack of **procedural fluency** for both L4 and L20 hence they struggled with the required procedure to dismantle the statement to be proved in order to expose its underlying structure.


$DE^2 = \frac{AE.EC}{3}$

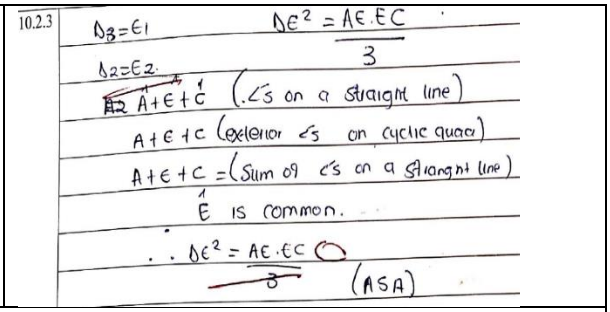
$DE \perp AC$

$AE = EC = ED = 1$

$AE + EC + ED = 3$

$(DE)^2 = \left(\sqrt{\frac{AE.EC}{3}}\right)^2$

$DE^2 = \frac{AE.EC}{3}$


10.2.3

$D_3 = E_1$ $DE^2 = \frac{AE.EC}{3}$

$D_2 = E_2$

A + E + C (∠'s on a straight line)

A + E + C (exterior ∠s on cyclic quad)

A + E + C = (sum of ∠'s on a straight line)

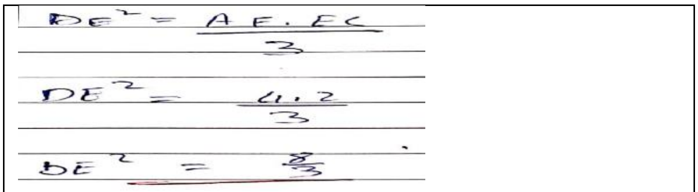
$\hat{E}$ is common.

∴ $DE^2 = \frac{AE.EC}{3}$ (ASA)

Solution 19: L11 and L2 starting and concluding with the statement to be proved

L11 began with the very same statement which needed to be proved and proceeded to formulate an equation involving a square root on the right-hand side so that it can conveniently lead to the required solution when squared on both sides. Learner 2 also started with the very same statement: $DE^2 = \frac{AE.EC}{3}$to be proved. The statements $D_3 = E_1$ and $D_2 = E_2$ imply the existence of isosceles triangles when non exist or were given. The learner then incorrectly wrote A + E + C twice supported by different reasons, firstly: *(∠ 's on a straight line)* and secondly: *(Exterior ∠ 's on cyclic quad)* which are both incorrect and irrelevant. The equating of statements and reasons is incorrect and has been a recurring theme from Learner 2 (see also Solution 13): *A + E + C = (sum of ∠ 's on a straight line).*

No justification is provided for the setup provided by L11, and since it does not flow logically from any acceptable geometric statement(s), it illuminates **adaptive reasoning** deficiencies. Both L11 and L2's inability to identify similar triangles which is the only starting point in solving this problem, points to **strategic competence** difficulties. L2's equating of statements and reasons highlights **procedural fluency** difficulties regarding correct procedures for formulating geometric statements and reasons.


$DE^2 = \frac{AE.EC}{3}$

$DE^2 = \frac{4.2}{3}$

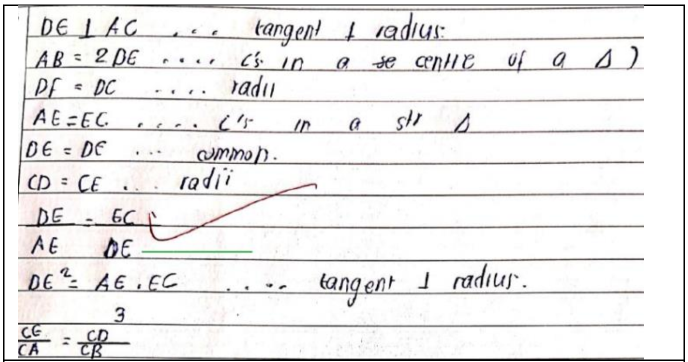
$DE^2 = \frac{8}{3}$

Solution 20: L16's substituting random values for ratios

L16 also started with the very statement which needed to be proved and then randomly created numbers which were then substituted on the right-hand side to produce a fraction. The learner then proceeded to substitute random incorrect values: 4 for AE and 2 for EC. It appears as if the learner misinterpreted this problem as that which require the determination of some ratio.

L16 shows **conceptual understanding** difficulties in the concept of proportionality, since there's no indication that the learner fully understood the question. **Procedural fluency** difficulties are also evident because this learner does not seem to understand the procedures associated with geometric proving. The substitution of random numbers without any justification is symptomatic of **adaptive reasoning difficulties**. Solution 20 shows no sign of strategy, thereby highlighting L16's **strategic competence** deficiencies.

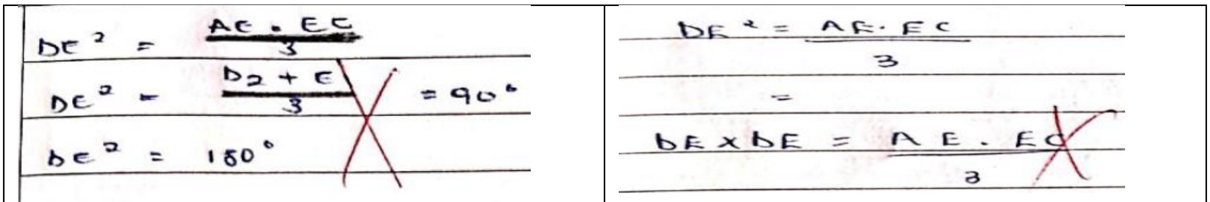
6.4.4.2. School 2


DE ⊥ AC ... tangent ⊥ radius
AB = 2DE .... ∠'s in a se centre of a Δ)
DF = DC .... radii
AE = EC .... ∠'s in a str Δ
DE = DE ... common.
CD = CE ... radii
$\frac{DE}{AE} = \frac{EC}{DE}$
$DE^2 = \frac{AE.EC}{3}$ .... tangent ⊥ radius.
$\frac{CE}{CA} = \frac{CD}{CB}$

Solution 21: L20's irrelevant statements and reasons

L20 wrote $\frac{DE}{AE} = \frac{EC}{DE}$ without any valid geometric reason, notwithstanding that the proportion provided is also incorrect and is preceded by a series of irrelevant statements and reasons. The statement AB = 2DE is incorrect together with the reason provided: ∠ '*s in a centre of a* Δ*)* and this shows that the learner confused the midpoint theorem with the required similarity.

The incorrect statements and reasons provided by L20 in Solution 21 highlight **conceptual understanding**, **procedural fluency** and **adaptive reasoning** difficulties. This learner's failure to identify similar triangles involved points to difficulties associated with **strategic competence**.


$DE^2 = \frac{AE.EC}{3}$
$DE^2 = \frac{D2+E}{3} = 90°$
$DE^2 = 180°$

$DE^2 = \frac{AE \cdot EC}{3}$
=
$DE \times DE = \frac{AE.EC}{3}$

Solution 22: L14 and L15's failure to correctly start proof

L14 also belongs to a category of learners who began the required proof by stating the very same statement which must be proved. This learner then proceeded to equate sides to angles without providing any reasons. L15 started with the same claim which needed to be proved and dismantled DE^2 into DE.DE, but this should have been strategic planning (Problem Interpretation) and not part of the solution set.

L14's inability to formulate a well-structured geometric proof which starts from locating triangles involved, highlights **strategic competence** difficulties. L15's mixing up of solution formulation and strategic planning highlights **procedural fluency** and **strategic competency** difficulties since this learner did not seem to have differentiated between formulating a solution and Problem Interpretation or strategy.

6.4.5. Difficulties in Question 10.2.4.

This question requires learners to determine the ratio of $\frac{Area\ \Delta FDC}{Area\ \Delta ABC}$, and this may involve the Area rule (trigonometric approach) or the standard formula for Area with a perpendicular height, since it can be proved that both triangles involved are right-angled. The 2nd approach requires **adaptive reasoning** skills because similarity between triangle FDC and ABC must be proved first. Both participant schools used both the trigonometric and standard formula for Area approach, although no learner was able to demonstrate proficiency in solving the problem.

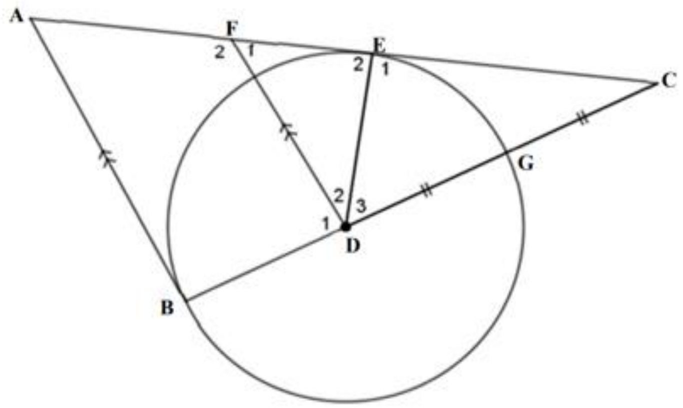
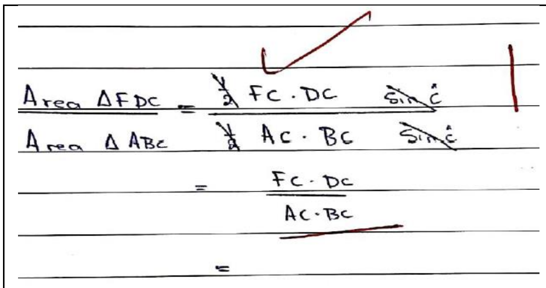
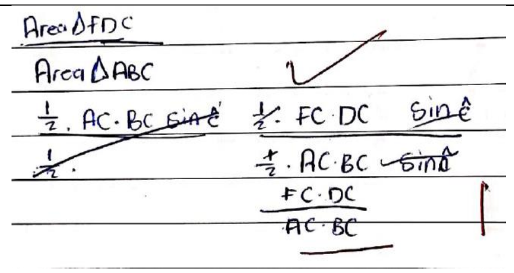
10.2.4	10.2 In the diagram below, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C. DG = CG. F is a point on AC such that DF ∥ AB.  10.2.1 Find, with reasons, the ratio of $\frac{AC}{FC}$. 10.2.2 Prove, giving reasons, that ΔABC /// ΔDEC. 10.2.3 Prove that $DE^2 = \frac{AE.EC}{3}$ 10.2.4 Find the ratio of: $\frac{\text{Area}\Delta FDC}{\text{Area}\Delta ABC}$	3 Marks

Table 12: June 2024 Question 10.2.4. (GDE, 2024, p.13)

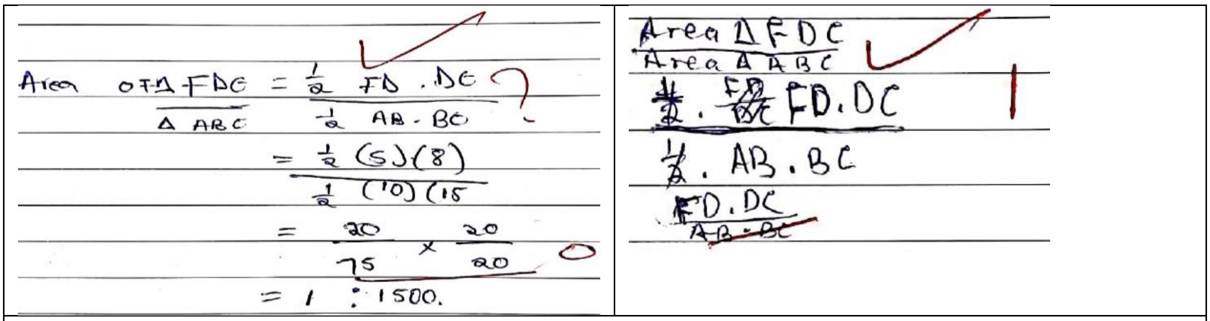
6.4.5.1. School 1

Solution 23: L9 and L29's failure to use the Area rule to find the required ratio

Both L9 and L29 could not substitute any ratios for $\frac{FC.DC}{AC.BC}$ because they were not successfully determined in Question 10.2.1. These learners were able to correctly substitute the correct sides on the correct Area rule formula as can be seen in Solution 23.

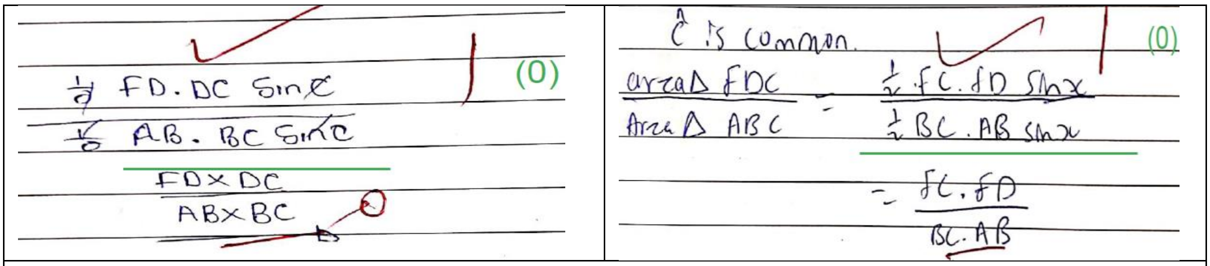
L9 and L29 correctly formulated the proportions by using the area rule (trigonometric approach), but could not proceed past the second step due to difficulties associated with **conceptual understanding** of ratios.



Solution 24: L10 and L23's failure to use the standard Area formula to determine the required ratio

Both L10 and L23 started well by substituting the correct sides on the standard Area formula but they could not proceed further due to the difficulties experienced in finding the correct ratio in Question 10.2.1. L10 resorted to substituting non-existent values as can be seen in solution 24: $\frac{\frac{1}{2}(5)(8)}{\frac{1}{2}(10)(15)}$. L23 was only able to cancel out the halves and could not proceed any further.

L10 and L23 were unable to effectively determine the required ratio due to **conceptual understanding** difficulties linked to circle geometry in Question 10.2.1. Also, the effectiveness of this approach is dependent on **procedural fluency** and **adaptive reasoning** necessary for proving similarity. So, the absence of similarity highlights learners' difficulties with these strands.



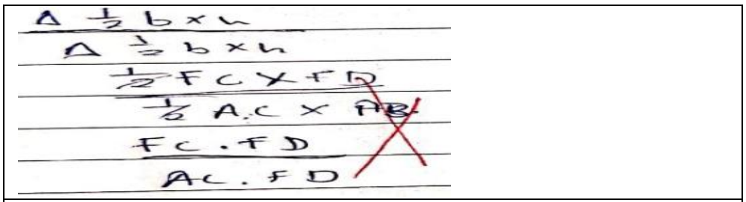
Solution 25: L7 and L11's failure to correctly formulate the Area rule

L7 selected FD and DC with angle C, whereas angle C is only included between side FC and DC. L7 also selected side AB and BC linking them with angle C on the denominator, and this further exposes the learners' confusion of the Area rule with standard formula for Area. L11 did something similar in selecting FC and FD linking them with angle *x* after having stated that angle C is common between the two triangles ABC and FDC.

L7 and L11's formulation of the Area rule is procedurally flawed, highlighting **procedural fluency** difficulties. The confusion between Area formulas by L7 underscores **conceptual**

understanding difficulties on what the two formulas represent in triangles. L11 referred to angle *x* when there was no such labelling on the diagram and this highlights **procedural fluency** difficulties on the introduction and use of variables in geometry.

6.4.5.2. School 2

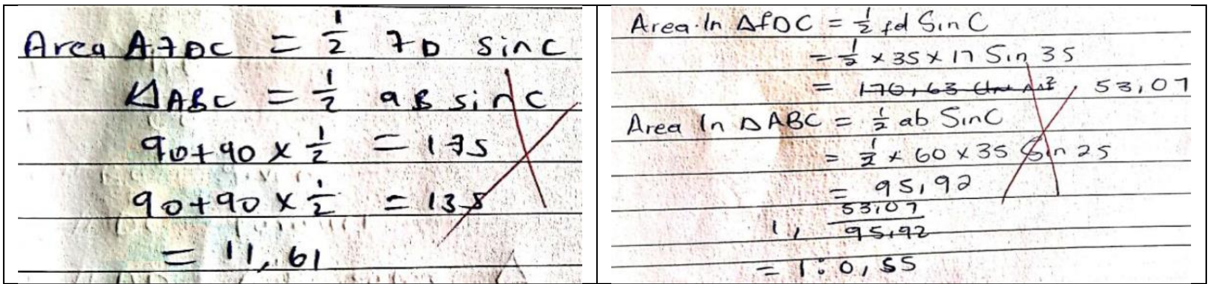


$\Delta\ \frac{1}{2}\, b \times h$
$\Delta\ \frac{1}{2}\, b \times h$
$\frac{1}{2}\, FC \times FD$
$\frac{1}{2}\, A.C \times AB$
$\frac{FC \cdot FD}{AC \cdot FD}$

Solution 26: L10's incorrect use of standard formula for Area

L10 chose the incorrect bases on both the denominator and numerator since FD must be paired with base DC, and AB with BC. The selection of any pairs of line segments which are not perpendicular to each other as base and height is bound to create problems when using this approach.

L10's incorrect use of the standard formula for Area approach highlights **procedural fluency** and **conceptual understanding** difficulties of the area of a triangle with perpendicular height. As with many learners in both participant schools, this learner's difficulties regarding the ratios to be substituted are linked to the difficulties experienced in Question 10.2.1.



Area $\Delta FDC = \frac{1}{2}\ FD \sin C$
$\Delta ABC = \frac{1}{2}\ aB \sin C$
$\frac{90+90 \times \frac{1}{2}}{90+90 \times \frac{1}{2}} = \frac{135}{135}$
$= 11,61$

Area in $\Delta fDC = \frac{1}{2} fd \sin C$
$= \frac{1}{2} \times 35 \times 17 \sin 35$
$= \cancel{170,63}$, $53,07$
Area in $\Delta ABC = \frac{1}{2} ab \sin C$
$= \frac{1}{2} \times 60 \times 35 \sin 25$
$= 95,92$
$\frac{53,07}{95,92}$
$= 1 : 0,55$

Solution 27: L4 and L5's faulty Area rule

Both L4 and L5's substitution of sides into the Area rule were incorrect. They both substituted as follows: Area Δ FDC = $\frac{1}{2} fD \sin C$ and Area Δ ABC = $\frac{1}{2} ab \sin C$. These faulty substitutions were then followed by some random large values substituted in place of sides for example 90, 60 or 35 for *ab, a* and *b* respectively.

L4's procedurally flawed Area rule underscores **procedural fluency** and **conceptual understanding** difficulties. This learner also substituted some random angles on both sides of the equation without providing any reasons and this highlights **adaptive reasoning** difficulties. L5's incorrect use of the Area rule and substitution of random values to determine the ratio highlights difficulties which emanate from a lack of **conceptual understanding** of ratio and **procedural fluency** deficiencies of the Area rule in geometry.

6.5. Summary and conclusion

This chapter presented quantitative data analysis for the Prelim 2023 and June 2024 Paper 2 examinations. Descriptive statistics were used to present and analyse data from both participant schools. These descriptive statistics showed that learners continue to perform poorly on geometric problems involving similarity and proportionality. This is evident through the BWP's which showed very low quartiles where in some instances all the quartiles and the maximum are at zero for problems involving proportionality. Learners' performances on problems involving other geometric concepts and similarity were slightly better compared to those involving proportionality. Qualitative analysis for only the June 2024 examination was done to provide more context in highlighting the kinds of difficulties learners experienced when dealing with geometric problems involving proportionality. Learners' performances on problems requiring strategic competence were dismal, and a large number of learners could not attempt these problems. Solutions from learners' scripts were presented as evidence to the kinds of difficulties learners experienced and these were also categorised according to Kilpatrick et al. (2001) four strands. Lastly, the qualitative presentation and analysis of June 2024 learners' scripts provided context on what exactly learners did which demonstrates the kinds of difficulties they experienced.

CHAPTER 7: FINDINGS, RECOMMENDATIONS AND CONCLUSIONS

7.1. Chapter 7 introduction

This chapter is a discussion on the research results and how these findings answer the research questions. It also provides research recommendations and conclusions. The main aim is to interpret and answer the three research questions based on the data collected, organised and presented in the previous chapter. Learners' scripts in geometry Prelim 2023 and June 2024 were quantitatively analysed and their June 2024 scripts were qualitatively analysed in order to answer the research questions. Recommendations on how these difficulties may be addressed are also made, drawing from available literature.

7.2. Research overview

This study aimed to investigate the kinds of difficulties Grade 12 learners experience when dealing with geometric problems involving proportionality, and how these difficulties may be connected to their difficulties in similarity and proportional division. Chapter 1 provided the background and motivation of the study by drawing from DBE annual diagnostic reports. In Chapter 2, relevant literature on learners' struggles in proportional division and similarity and how these may be addressed was discussed. Kilpatrick et al. (2001) four strands for mathematical proficiency were presented and discussed as the theoretical framework for the study. Research methodology and design involving a mixed-method design was discussed in Chapter 4. The two research instruments, 2023 Preliminary and 2024 June examination Paper 2 question papers and marking guidelines, were presented and analysed in Chapter 5. Chapter 6 presented quantitative analysis of learners' scripts for both 2023 Prelim and 2024 June examination, and qualitative analysis for 2024 June examination.

7.3. Research question 1 answered:

"What are Grade 12 learners' difficulties when dealing with geometric problems involving proportionality in triangles?"

7.3.1. Preparatory Examination 2023

Learners' performance in Question 8.1.1 to 8.1.4 was better, since these were geometric problems largely requiring procedural fluency. Question 10.2.1 (similarity) also recorded better performances from both participant schools as can be seen in Figures 6 and 7. Only 2% of learners from both participant schools were able to obtain a mark of 50% and above (pass) in Question 10.2.3, which involves proportionality requiring **strategic competence** and comprehensive **adaptive reasoning** abilities. Learners' responses for Question 9.1 to 10.1 are also very poor, and these are proportionality problems requiring **conceptual understanding**, **procedural fluency** and **adaptive reasoning**. The average mark (mean) for both Schools 1 and 2 at 0,4 and 0,7 respectively indicate very poor performances in problems involving proportionality.

7.3.2. June Examination 2024

Learners' performances in less complicated geometric problems between Questions 8.1.1 to 9.3 are better. Question 10.2.3, a geometric problem involving proportionality, recorded amongst the worst performances at 0% for both participant schools. No learner was able to obtain a pass, and this Question 10.2.3 is a problem requiring **strategic competence** and **adaptive reasoning**. Learners' performances in Question 10.2.1 and 10.2.4 were also very poor, and these were problems involving proportionality requiring **conceptual understanding**, **procedural fluency** and **adaptive reasoning.** The average mark (mean) for both School 1 and 2 at 2,3 and 0,1 respectively still indicate very poor performances in problems involving proportionality.

Learners experienced difficulties in Question 10.2.1 associated with **conceptual understanding** of the concept of ratio, **procedural fluency** difficulties in representing proportionality of sides including representing and labelling lengths of sides on diagrams according to given ratios. All participant learners could not demonstrate **conceptual understanding** of the ratio of equal line segments. **Adaptive reasoning** difficulties were prominent across all problems involving proportionality where learners either provided statements without reasons, incorrect or irrelevant reasons as can be seen in Solution 7 to 14 and 17 to 27. Only one learner (L9) was able to obtain a pass in this question from both participant schools. Learners' difficulties in Question 10.2.1 meant that they stood no chance of success in Question 10.2.3 and 10.2.4 since the ratios are a prerequisite for these subsequent questions. There were instances where learners confused the application of proportionality theorem with the midpoint theorem as can be seen in Solution 8 and 10, underscoring learners' difficulties with **conceptual understanding** of both the midpoint and proportional division theorems at application level.

A clear pattern which emerged from both qualitative and quantitative analysis of learners' scripts is that learners' performances are very poor on problems requiring **strategic competence**. The fact that no learner obtained a pass in Question 10.2.3 from both participant schools and in both examinations under investigation, highlights this issue. The box and whisker plots presented in Figure 8 to 21 paints a gloomy picture of a higher number of zero's learners obtained in geometric questions, especially those involving proportionality. This leads to a situation where the minimum mark and the first-quartile are in the same position, which then affects the distribution of learners' raw marks.

In both participant schools, more than 50% of learners had no responses for problems involving proportionality. Failure to even begin to solve these problems highlights **strategic competence** difficulties since the first step requires proper strategy and planning. The issue is related to their limited proficiency in **conceptual understanding**, **procedural fluency**, **adaptive reasoning** and **strategic competence**. Learners will have to overcome these difficulties if they are to achieve proficiency in geometric problems involving proportionality.

7.4. Research question 2 answered:
"To what extent are these difficulties linked to learners' proficiency in the concept of similarity in triangles?"

7.4.1. Preparatory Examination 2023

Learners' performances in Question 10.2.1, which is a problem involving similarity, are better as can be seen in Figure 6 and 7. A total of 11 learners (12%) were able to obtain a pass in this question and only 13 (15%) had no responses from both participant schools as can be seen in Table 2. The average mark (mean) for both School 1 and 2 at 0,6 and 0,5 respectively indicate poor performances in problems involving similarity. A majority of learners were also able to obtain a mark by taking advantage of the two given equal angles $\hat{A}_1$ and $\hat{A}_2$. Although these learners possessed the conceptual understanding for similarity, they were hindered by **conceptual understanding** difficulties on circle geometry theorems, and consequently failed to successfully prove the similarity of the two triangles.

7.4.2. June Examination 2024

The average mark (mean) for both School 1 and 2 at 0,8 and 0,1 respectively indicate poor performances in problems involving similarity. Although there were some notable **conceptual understanding** difficulties in Question 10.2.2, such as confusing similarity and congruency, and some **procedural fluency** difficulties such as the inability to list the two triangles involved, learners' performances were better compared to those in proportionality problems. This is perhaps due to the common $\hat{C}$ which allowed a majority of learners to at least obtain a mark in this question. Further, similarity problems are fairly routine when the concepts are understood and procedures known. The same cannot be said about proportionality problems since similarity usually precedes the proportionality of sides and that in many instances learners are further required to possess **strategic competence** to identify such similar triangles before they can establish the required proportionality of sides. Since similarity is one of the conditions for proportionality of sides, then learners with difficulties (**procedural and conceptual)** in similarity are bound to also have difficulties in proportionality problems. Indeed, not all proportionality emanates from similarity as can be seen in the following discussion.

7.5. Research question 3 answered:

"To what extent are these difficulties linked to learners' proficiency in the concept of proportional division in triangles?"

7.5.1. Preparatory Examination 2023

Learners performed very poorly in Question 10.1 since only one learner managed to obtain a pass in this question out of a total of 89 learners from both participant schools. This question involves proving proportionality of sides through congruency by drawing from the proportional division theorem. Learners' **adaptive reasoning** difficulties in the concept of proportional division meant that they could not reason correctly and sufficiently in proving the proportionality of corresponding sides. Some learners failed to start the proof correctly due to **strategic competence** difficulties which prevented them from planning and executing the required constructions.

7.5.2. June Examination 2024

Learners' performances in proving the proportional division theorem are better, especially for School 1, since 12 learners out of 30 were able to obtain a pass. However, School 2 had dismal performances in proving this theorem since no learner was able to obtain a pass. As already discussed in Chapter 5, theorems essentially involve all the strands but are also memorisable since they are often examined in the exact same way they are presented in textbooks and in previous question papers. So, learners' performances in proof may not be a good measure of proficiency as can be observed in how the very same learners in School 1 who performed better in proving the proportional division theorem suddenly experienced difficulties in Question 10.2.1 (June 2024), which is the application of the very same proportional division theorem.

In analysing learners' marked scripts qualitatively, it became clearer that a significant number of learners were able to produce the proof by either using the Area rule or standard area formula without providing any constructions. This reproduction is only possible through memorisation, hence examiners do not permit any allocation of marks when constructions are not provided by the learner. Theorem proofs, when examined in non-routine ways and executed with understanding are quite complex and involve thorough planning and that requires **strategic competency**. A significant number of learners also presented faulty or insufficient constructions, leading to incorrect proof as can be seen in Solution 5.

Learners' **conceptual understanding** and **procedural fluency** difficulties on concepts such as Area of triangles and perpendicular heights played a significant role in learners' overall poor performances in this question. **Adaptive reasoning** difficulties were also quite prominent, since learners had difficulties in reasoning sufficiently, including those learners who performed better in this question.

Unsurprisingly, these difficulties in proving the proportional division theorem were then carried over to the application of the theorem in Question 10.2.1 and 10.2.4. Clearly, learners' proficiency in proving the proportional division theorem has far-reaching implications for their proficiency in proportionality problems rooted in proportional division such as Question 10.2.4.

7.6. Connecting research findings to literature

Review of literature in Chapter 2 showed that learners and educators in South Africa and other parts of the world have difficulties in geometric problems involving similarity and proportionality (DBE, 2025; Fathiyah et al., 2024; Govender, 2016; Mahlabela, 2012; Orrill & Burke, 2013). The mean mark of 0,6 and 0,5 for School 1 and 2 respectively in Prelim 2023 examination for Question 10.2.1 indicates that learners performed poorly in this similarity question. A similar theme of poor performances is also evident in the June 2024 examination where learners obtained a mean mark of 0,8 and 0,1 for School 1 and 2 respectively. This study found that some learners confused similarity with congruency, and this resonates with findings by Haj-Yahya (2022), Güneş (2022), Shahbarri and Daher (2020), and Casanova (2021) on learners' definitions of the concepts of similarity and congruency. This finding answers the question: "To what extent are these difficulties linked to learners' proficiency in the concept of similarity in triangles?" since results show that learners experienced some procedural fluency and conceptual understanding difficulties in listing triangles and on the meaning or definition of similarity. This study also found that even in cases where learners were able to successfully prove similarity of triangles, this did not necessarily translate into successful formulation of proportional sides. This is evident on how learners who demonstrated proficiency in proving similarity in Question 10.2.2. had difficulties in responding 10.2.3 and 10.2.4, for example L4 in Solution 13 and 18.

The findings of this study on learners' difficulties with formal proof of the proportional division theorem agree with Casanova et al. (2021) that learners have difficulties in formal proof situations across the globe. The performances of learners in Question 10.1 were dismal for both participant schools since only two learners out of 89 managed to obtain a pass in the Prelim 2023 examination. The June 2024 examination saw only 12 passing Question 10.1 out of 30 learners from School 1 and no learner obtained a pass out of 20 from School 2. This study's results showed that some learners may have resorted to memorizing the theorem since there were instances where learners reproduced correct statements and reasons without any constructions when such geometric statements should come from the required constructions, for example L26 and L27 in Solution 4 in response to Question 10.1. In response to the research question: "To what extent are these difficulties linked to learners' proficiency in the concept of proportional division in triangles?", the results show that learners had difficulties with this proof and those difficulties were carried over to any question which involved proportionality including those involving areas of triangles.

In complex problems involving proportionality, learners obtained a mean mark of 0,4 and 0,7 in the Prelim 2023 examination for both School 1 and 2 respectively and this underscores the difficulties learners experience in this concept. Similar poor performances were also evident in the June 2024 examinations where learners managed a mean mark of 2,3 and 0,1 for both

School 1 and School 2 respectively. Grade 12 learners' difficulties with problems involving proportionality in triangles are further exacerbated by their inability to dismantle complicated geometric figures in order to make sense of their constitutive parts such as similar triangles involved which might have led to the proportionality of sides. This resonates with Govender (2016) and DBE (2025) that learners' inability to dismantle and put together numerous geometric parts involving complicated geometric shapes with angles, circles, line segments etc, in order to observe and understand how they relate to each other, contributes to the difficulties they experience in geometry. In addition to this, learners' difficulties with formal proof of the proportional division theorem and their comprehension and application of the similarity concept meant that they stood no chance in solving complex geometric problems involving proportionality. These complex proportionality problems, as can be seen in Question 10.2.3 and 10.2.4 of the June 2024 examination, are embedded with similarity and proportional division. The findings of this study answered the research question: "What are Grade 12 learners' difficulties when dealing with problems involving proportionality in triangles?" by identifying and categorising learners' difficulties through Kilpatrick et al. (2001) strands of mathematical proficiency. Moreover, a link has been established between learners' difficulties with similarity and proportional division proof and how these concepts contribute to learners' difficulties in solving complex geometric problems involving proportionality.

This study also confirms national trends from DBE's diagnostic reports on learners' poor performances in geometric problems involving proportionality. The DBE (2022) report as evident in Figure 2(a) shows how learners' performances in question 10, which is a problem involving proportionality, was the worst performing question nationally at 18%. This further highlights learners' difficulties with strategic competency, since both Question 10.2 and 10.3 required learners to think out of the box and plan thoroughly as can be seen in Diagram 3.

Kilpatrick et al. (2001) strands for mathematical proficiency provided a useful and relevant theoretical tool for teaching Grade 12 geometry, because DBE uses the same framework in their drive to enhance the teaching of Mathematics with understanding. As a result, South African learners in the FET phase can benefit from being taught geometry through Kilpatrick et al. (2001) strands of mathematical proficiency.

There is general consensus, as already demonstrated in Chapter 2 (literature review) that learners experience difficulties in geometric problems involving similarity and proportionality, both in the local context and abroad. The South African government also holds the position that these difficulties may be addressed through teaching Mathematics for understanding in pursuit of mathematical proficiency for learners of all age groups (DBE, 2018). This study provides more clarity on the kinds of difficulties learners experience in proportionality and similarity problems for Grade 12’s, with the aim to empower educators and other stakeholders to be in a better position to address these difficulties.

7.7. Emergent findings

It was evident from the marked scripts that some learners confused the mid-point theorem with similarity, for example Solution 21 for L20. This was an unexpected finding. The midpoint theorem is a special case of the proportional division theorem and is taught in Grade 10, so one would expect learners to be proficient at least in its application. Learners’ lack of proficiency in the application of the midpoint theorem raises concerns about how it is taught in Grade 10. The mixing up of the midpoint theorem with similarity by learners also highlights deep seated difficulties with both concepts.

Another unexpected finding is on learners’ difficulties with the concept of Area of triangles. Some learners could not correctly formulate the Area rule or the standard formula for the area of triangles as can be seen in Solution 25 to 27. The standard formula for Area of triangles (perpendicular height) is taught in the Senior phase while the Area rule (trigonometric approach) is taught in Grade 11 (FET phase), and one would expect that both of these concepts should have been mastered by the time a learner gets to Grade 12.

7.8. Research recommendations

The analysis of learners’ scripts and their performances revealed that learners’ difficulties in geometric problems involving similarity, proportional division and proportionality are due to lack of conceptual understanding, procedural fluency, adaptive reasoning and strategic competence abilities.

According to Lee and Yim (2014), task sequencing is crucial in understanding a problem involving proportional reasoning. One of the findings in this study involves learners’ strategic competence difficulties when figuring out the proportionality of sides where similar triangles are not made explicit, and this require learners to be able to generate a sequence of statements and reasons which are helpful in locating triangles involved. Educators should arrange their classroom tasks in such a way that learners should be able to develop the conceptual understanding and procedural fluency that similarity of triangles or proportional division precedes the formulation of proportional sides and ratios. This approach promotes coherence and prevents conjured up ratios and proportions as can be seen in Solution 10 and 11.

The use of Information Communications and Technologies (ICT's) such as the GeoGebra App in helping learners make sense of geometric concepts has been proved to be effective (Restrepo-Ochea et al., 2023; Jojo, 2016; Coronado et al., 2017). Educators may use these ICT's to promote learners' understanding of the criteria for similarity and proportionality, and this may help learners overcome some of the conceptual misunderstandings found through this study. A typical example of this, is how GeoGebra can be effectively used to demonstrate the differences between similar and congruent objects in space through dragging and enlarging.

Examiners should consider setting up geometric questions, especially theorem proofs, in such a way that learners should not be able to reproduce these proofs through memorization. This can be achieved by rearranging diagrams, letters or by providing specific instructions on which approach should be utilised to prove the theorem. Further, marking guidelines should cater for a wider range of alternative solutions to geometric problems and be more elaborate on processes involved in solving problems involving strategic competence. Furthermore, these examination question papers should be structured in such a way that they are accessible to learners with different abilities (Govender, 2016).

Resource materials developers such as CAPS Mathematics textbook creators may consider including additional approaches to solving problems involving proportionality and designing textbooks with mathematical proficiency in mind, i.e. textbooks which promote and maintain a delicate balance between conceptual understanding, procedural fluency, adaptive reasoning and strategic competence. A clear and strong connection of the concept of ratio, proportion and similarity through learner activities between the grades and phases is also crucial (Wijayanti, 2015).

Curriculum developers should seriously consider including the concept of similarity and proportionality in Grade 11, since its absence may also be a significant factor in learners' poor performances.

Lastly, further studies on learners' difficulties in proving and in the application of the midpoint theorem in Grade 10 may clarify learners struggles with this concept. The researcher suspects that proficiency in the midpoint theorem may provide learners with a solid foundation to be proficient in proportional division theorem and its application.

7.9. Limitations of the study

This study was conducted in a single cluster in Gauteng East district and as such generalizability of results is a valid concern since the sampling used is not representative, for example across socio-economic sectors or even race in South Africa. Further, all participants are relatively from similar socio-economic and cultural backgrounds. Furthermore, matriculants have a very congested academic schedule which presents challenges in terms of any research methodology which necessitates interaction with learners, and as such the researcher believes that future studies with learner interviews and observations may yield more insight on the difficulties learners experience when responding to geometric problems involving proportionality.

7.10. Final reflections

I hope this study contributes to making educators teach in line with Kilpatrick et al. (2001), which may lead to improving our learners' understanding of the geometric concepts of similarity and proportionality. I hope that it sheds light on learners' struggles and help us find better ways of teaching them to become proficient in geometry, especially in the concept of proportionality and similarity.

The review of literature on topics relevant to this study has opened my eyes widely to mathematical ideas I did not even know existed. I have learned more than I have anticipated and one can only hope that these profound lessons will continue to permeate to my learners since this research is born out of their struggles in geometry.

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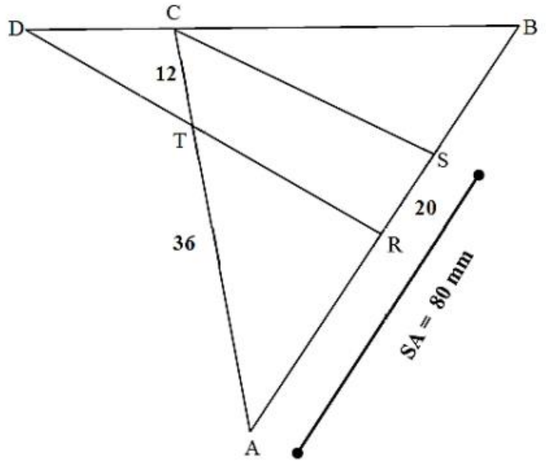
Appendix A: GDE Preparatory Exam Question Paper 2 2023

QUESTION 9

In the diagram below, ΔABC is constructed such that BC is produced to D. DR is drawn, with point T on AC and R on BA. CS is drawn. CT = 12 mm, TA = 36 mm, SR = 20 mm and SA = 80 mm.



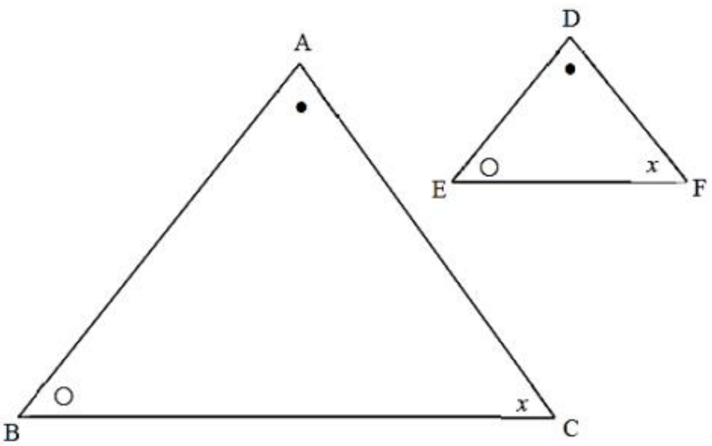
9.1 Prove that $CS \parallel TR$. (3)

9.2 It is further given that $AR = \frac{2}{3}RB$, $BC = 2x$ and $CD = \frac{1}{2}x + 1$.

Calculate the value of x. (6)
[9]

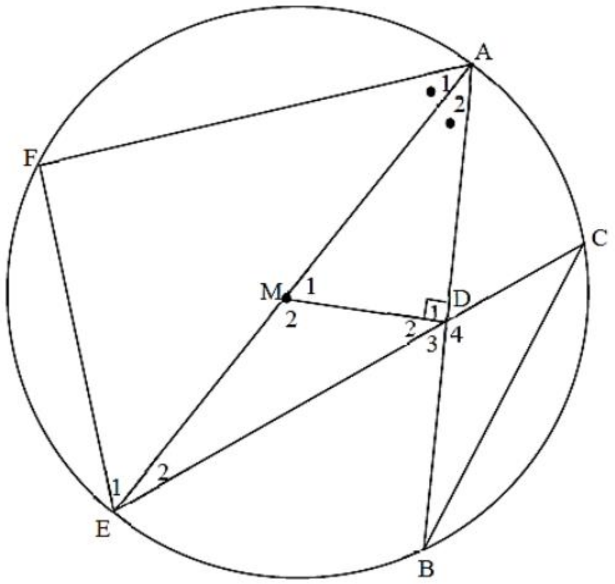
QUESTION 10

10.1 In the diagram below, ΔABC and ΔDEF are drawn, such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE} = \frac{AC}{DF}$. (6)

10.2 In the diagram below, diameter EMA of a circle with centre M bisects $F\hat{A}B$. MD is perpendicular to the chord AB. ED produced meets the circle at C. Chords CB and FE are drawn.



10.2.1 Prove that $\Delta AEF \,|||\, \Delta AMD$. (4)

10.2.2 Determine the numerical value of $\frac{AF}{AD}$. (3)

10.2.3 Prove that $AD^2 = CD \times DE$. (6)

[19]

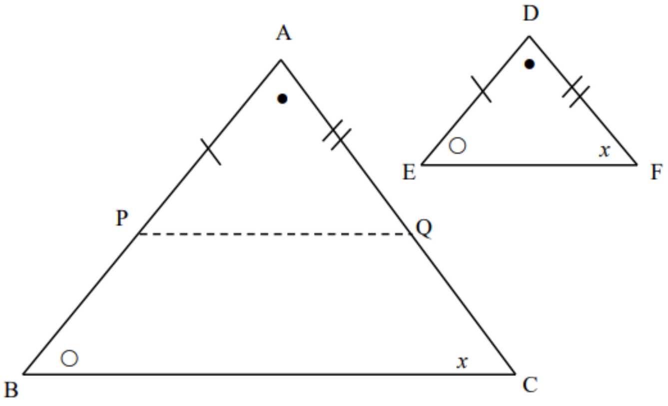
Appendix B

Gauteng Preparatory Examination Question 9 and 10 (2023) Memorandum (marking guidelines) – Gauteng Department of Education.

9.1	$\frac{CT}{TA}=\frac{12}{36}=\frac{1}{3}$		✓S
	$\frac{SR}{RA}=\frac{20}{60}=\frac{1}{3}$		✓S
	$\therefore \frac{CT}{TA}=\frac{SR}{RA}$		
	$\therefore CS \parallel TR$	line divides the sides of Δ in proportion/*lyn verdeel die sye van die* Δ *in verhouding*	✓R (3)
	OR/*OF*		
	$\frac{SR}{SA}=\frac{20}{80}=\frac{1}{4}$		✓S
	$\frac{CT}{CA}=\frac{12}{48}=\frac{1}{4}$		✓S
	$\therefore \frac{CT}{CA}=\frac{SR}{SA}$		
	$\therefore CS \parallel TR$	line divides the sides of Δ in proportion/ *lyn verdeel die sye van die* Δ in verhouding	✓R (3)

9.2	$\frac{AR}{RB}=\frac{2}{3}$		
	$\frac{60}{RB}=\frac{2}{3}$		✓S
	$RB = 90$ mm		✓value of/*waarde van* RB
	$\therefore SB = 70$ mm		
	$\frac{BS}{SR}=\frac{BC}{CD}$	proportion theorem/*eweredigheidstelling* CS‖TR	✓S ✓R
	$\frac{70}{20}=\frac{2x}{\frac{1}{2}x+1}$		✓substitution/ *substitusie*
	$40x = 35x + 70$		✓answer/*antwoord* (6)
	$x = 14$ mm		
	OR/*OF*		
	$2k = 60$		✓value of/*waarde van k*
	$\therefore k = 30$		
	$\therefore 3k = 90$		✓value of/*waarde van 3k*
	$\frac{SR}{RB}=\frac{DC}{DB}$	prop. theorem/*eweredigheidstelling* CS‖TR	✓S ✓R
	$\frac{20}{90}=\frac{\frac{1}{2}x+1}{\frac{5}{2}x+1}$		✓substitution/ *substitusie*
	$5x+2=\frac{9}{2}x+9$		
	$\frac{1}{2}x=7$		
	$\therefore x = 14$ mm		✓answer/*antwoord* (6)
			[9]

QUESTION/*VRAAG* 10

10.1	 **NB: No construction/*Geen konstruksie nie* 0/6**	
	On AB, mark of AP = DE and on AC, mark off AQ = DF./*Op AB, merk AP = DE of en op AC, merk AQ = DF af.* Draw/*Trek* PQ. Proof/*Bewys*: In ΔAPG and/en ΔDEF $\hat{A} = \hat{D}$ given/*gegee* AQ = DF construction/*konstruksie* AP = DE construction/*konstruksie* $\therefore \Delta APQ \equiv \Delta DEF$ S∠S $A\hat{P}Q = \hat{E}$ $A\hat{P}Q = \hat{B}$ $[\hat{E} = \hat{B}]$ $\therefore$ PQ ǀǀ BC corresponding angles are equal/*ooreenk. hoeke is gelyk* $\frac{AB}{AP} = \frac{AC}{AQ}$ line ǀǀ to one side of Δ/*lyn* ǀǀ *aan een sy van* Δ AP = DE and/*en* AQ = DF $\therefore \frac{AB}{DE} = \frac{AC}{DF}$	✓ construction/ *konstruksie* ✓S ✓R ✓S ✓R ✓S/R (6)

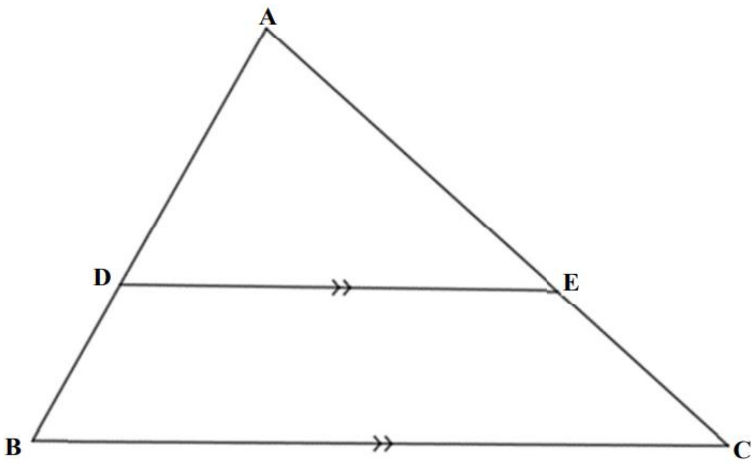
10.2.1	$\hat{F} = 90°$ ∠ in semi-circle/*halwe sirkel* In ΔAEF and/*en* ΔAMD $\hat{A}_1 = \hat{A}_2$ given/*gegee* $\hat{F} = \hat{D}_1 = 90°$ proved/*bewys* $\therefore$ ΔAEF ǀǀǀ ΔAMD ∠∠∠ **OR/OF** $\hat{F} = 90°$ ∠ in semi-circle/*halwe sirkel* In ΔAEF and/*en* ΔAMD $\hat{A}_1 = \hat{A}_2$ given/*gegee* $\hat{F} = \hat{D}_1 = 90°$ proved/*bewys* $\hat{E}_1 = \hat{M}_1$ sum of ∠'s in Δ/*binne* ∠*e van* Δ $\therefore$ ΔAEF ǀǀǀ ΔAMD	✓S/R ✓S ✓S ✓R (4) **OR/OF** ✓S/R ✓S ✓S ✓R (4)

10.2.2	$\frac{AF}{AD}=\frac{AE}{AM}$ but/*maar* $AE = 2AM$ $\frac{AF}{AD}=\frac{2AM}{AM}=2$	$\Delta AEF \mid\mid\mid \Delta AMD$	✓S/R ✓S ✓answer/*antwoord* (3)
10.2.3	In ΔCDB and/*en* ΔADE $\hat{C}=\hat{A}_2$ $\hat{D}_4=E\hat{D}A$ $\Delta CDB \mid\mid\mid \Delta ADE$ $\therefore \frac{CD}{AD}=\frac{DB}{DE}$ but/*maar* $AD = DB$ $\frac{CD}{AD}=\frac{AD}{DE}$ $\therefore AD^2 = CD \times DE$	 $\angle$s in same segment/$\angle$e *in dieselfde segment* vertically opposite $\angle$'s/*regoorstaande* $\angle$e $\angle\angle\angle$ line from centre $\perp$ to chord/*lyn vanaf midpt.* $\perp$ *op koord*	✓identifying Δs/ *identifiseer* Δ*e* ✓S/R ✓S ✓R ✓S ✓S/R (6)
	OR/OF In ΔCDB and/*en* ΔADE $\hat{C}=\hat{A}_2$ $\hat{D}_4=E\hat{D}A$ $\hat{B}=\hat{E}_2$ $\Delta CDB \mid\mid\mid \Delta ADE$ $\therefore \frac{CD}{AD}=\frac{DB}{DE}$ but/*maar* $AD = DB$ $\frac{CD}{AD}=\frac{AD}{DE}$ $\therefore AD^2 = CD \times DE$	 $\angle$s in same segment/$\angle$e *in dieselfde segment* vertically opposite $\angle$'s/*regoorstaande* $\angle$e $\angle$s in same segment/$\angle$e *in dieselfde segment* line from centre $\perp$ to chord/*lyn vanaf midpt.* $\perp$ *op koord*	**OR/OF** ✓identifying Δs/ *identifiseer* Δ*e* ✓S/R ✓S ✓R ✓S ✓S/R

Appendix C: GDE June Exam Question Paper 2 2024

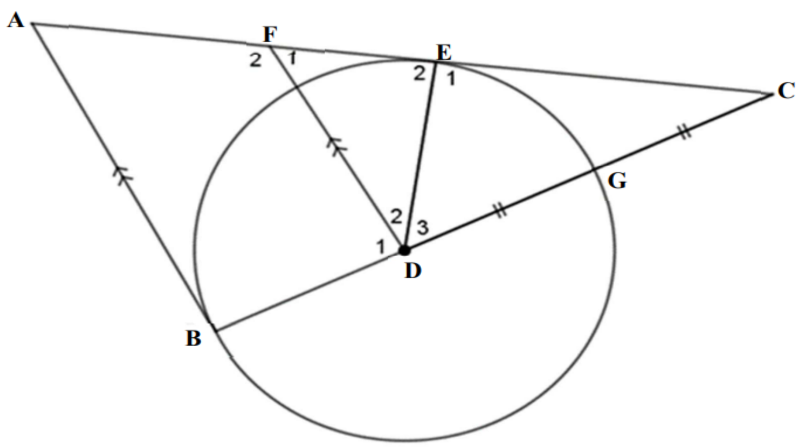
QUESTION 10

10.1 In the diagram below, ΔABC is drawn with DE ∥ BC.



Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$. (5)

10.2 In the diagram below, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C. DG = CG. F is a point on AC such that DF ∥ AB.



10.2.1 Find, with reasons, the ratio of $\frac{AC}{FC}$. (3)

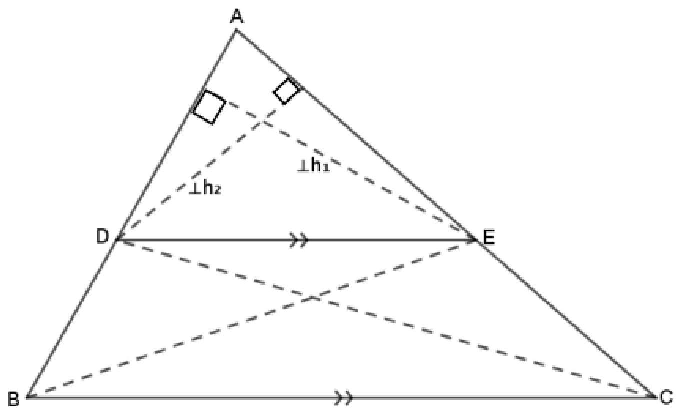
10.2.2 Prove, giving reasons, that ΔABC /// ΔDEC. (3)

10.2.3 Prove that $DE^2 = \frac{AE.EC}{3}$ (5)

10.2.4 Find the ratio of: $\frac{Area\Delta FDC}{Area\Delta ABC}$ (3)

[19]

Appendix D: GDE June Exam Paper 2 2024 Marking Guideline

QUESTION 10/*VRAAG 10*			
10.1	 	✓ Construction/ *konstruksie*	
	Construction/*konstruksie*: $\perp h_1$ on AD and $\perp h_2$ on AE/ $\perp h_1$ *op AD en* $\perp h_2$ op AE Join DC and BE/*verbind DC en BE* No construction No marks		
	$\frac{area\ \Delta ADE}{area\ \Delta DBE} / \frac{opp\ \Delta ADE}{opp\ \Delta DBE}$	✓ S	
	$= \frac{\frac{1}{2} AD \perp h_1}{\frac{1}{2} DB \perp h_1} = \frac{AD}{DB}$ same height/*dieselfde hoogte*	✓ S/R	
	$\frac{area\ \Delta ADE}{area\ \Delta EDC} / \frac{opp\ \Delta ADE}{opp\ \Delta EDC}$ $= \frac{\frac{1}{2} AE \perp h_2}{\frac{1}{2} EC \perp h_2}$ $= \frac{AE}{EC}$ same height/*dieselfde hoogte*		
	But/*Maar* $Area\Delta ADE = Area\Delta ADE$ / $opp\Delta ADE = opp\Delta ADE$ and/*en* $Area\Delta DBE = Area\Delta ECD$ $opp\Delta DBE = opp\Delta ECD$	✓ S	
	same base, same height/*dieselfde basis, dieselfde hoogte*	✓ R	
	$\therefore \frac{Area\Delta ADE}{Area\Delta DBE} = \frac{Area\Delta ADE}{Area\Delta ECD} / \therefore \frac{opp\Delta ADE}{opp\Delta DBE} = \frac{opp\Delta ADE}{opp\Delta ECD}$		
	$\therefore \frac{AD}{DB} = \frac{AE}{EC}$		(5)

10.2	10.2.1	$DB = DG$ $In\ \Delta ABC$: $DF \parallel AB$ $\frac{AC}{FC} = \frac{BC}{DC} = \frac{3}{2}$ line ∥ to one side of Δ/(Prop theorem,DF∥AB) *aan een sy van Δ/Eweredigheid stelling* (DF∥AB)	✓ radii/*radiusse* ✓ $\frac{AC}{FC} = \frac{BC}{DC} = \frac{3}{2}$ ✓ R	(3)
	10.2.2	In ΔABC and/*en* ΔDEC: 1) $\hat{B} = 90°$ tan ⊥ rad/*rlyn* ⊥ *radius* $D\hat{E}C = 90°$ tan ⊥ rad/*rlyn* ⊥ *radius* $\therefore \hat{B} = D\hat{E}C$ 2) $\hat{C} = \hat{C}$ common ∠/*gemeenskaplike* ∠ 3) $\hat{A} = E\hat{D}C$ $\angle's$ of a Δ/$\angle'e$ *van* Δ $\therefore \Delta ABC /// \Delta DEC$ ∠∠∠	✓ $\hat{B} = D\hat{E}C$ S/R ✓ $\hat{C} = \hat{C}$ ✓ $\hat{A} = E\hat{D}C$ **OR** ✓ $\hat{B} = D\hat{E}C$ S/R ✓ $\hat{C} = \hat{C}$ ✓ R ∠∠∠	(3)
	10.2.3	$\frac{AB}{DE} = \frac{BC}{EC}$ $\Delta ABC /// \Delta DEC$ but/*maar* $AB = AE$ tans from same point/*rlyne vanaf dies. punt* and/*en* $BC = 3 \times$ radius = 3DE $\therefore \frac{AE}{DE} = \frac{3DE}{EC}$ $AE.EC = 3DE.DE$ $AE.EC = 3\,DE^2$ $DE^2 = \frac{AE.EC}{3}$	✓ S/R ✓ S/R ✓ substitute AB with AE/*vervang AB met AE* ✓ substitute BC with 3DE/*vervang BC met 3DE* ✓ $AE.EC = 3\,DE^2$	(5)
	10.2.4	$\frac{Area\Delta FDC}{Area\Delta ABC} = \frac{opp\Delta FDC}{opp\Delta ABC}$ $= \frac{\frac{1}{2}FC.DC.\sin\hat{C}}{\frac{1}{2}AC.BC.\sin\hat{C}}$ $= \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$ $= \frac{4}{9}$	✓ using area rule correctly/*gebruik opp reël korrek* ✓ $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$ ✓ answer/*antwoord*	(3)

Annexure A

Request for permission from the school principal to conduct research at a school:

University of the Witwatersrand,
Faculty of Education, Department of Mathematics Education.

Wits Parktown Education Campus
27 St Andrews Rd,
Parktown,
Johannesburg,
2193

Tel: 011 717 3007

Date:…………………….

Dear Sir/Madam,

Re: Permission to conduct research at your school.

My name is Khayalethu Myoli.

I have been teaching Mathematics at Vezukhono Secondary School for the past three years. I am deeply passionate about Geometry but I am concerned about learners' continued poor performances in this topic.

I am studying for a Masters' Degree (Mathematics Education) in the Faculty of Education at the University of the Witwatersrand. I am seeking permission to do research at your school. I am conducting research on Grade 12 learners' difficulties in applying the concept of proportionality in triangles, and to what extent do these difficulties connected to learners' proficiency in the concept of similarity and proportional division.

The research will entail collecting data from learners' June Examination scripts, the Question Paper and Memorandum. Grade 12 learners will be passive participants, i.e. they will not be disrupted in their academic program.

I will invite individuals from your organization to participate in this study. I am requesting all Grade 12 Mathematics learners to provide assent for their June Examination Paper 2 scripts to be used for the study, and their parents to provide consent. Grade 12 Mathematics educators are requested to provide consent for the marked scripts and also make de-identified copies to be used for the study. If they agree, they will be asked to return completed assent and consent forms. Teachers will be requested to provide de-identified learners' scripts. The researcher will require about 15 minutes to explain the purpose of the study to learners and educators. After the return of assent and consent forms and subsequent collection of copies of learners' scripts, no further engagements will be required from participants.

Participants' responses will be treated confidentially, and identities (their names and the name of the organization) will be anonymous. Individual privacy will be maintained in all published and written data resulting from the study.

The results will be communicated in the form a research report for Masters' degree purposes.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

All research data will be de-identified and preserved in a safe place for record purposes.

I therefore request permission in writing to conduct my research at your school. The permission letter should be on your organization's headed paper, signed and dated, and specifically referring to myself by name and the title of my study.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

Khayalethu Myoli.

Cell:
Email: kmyoli@gmail.com

Supervisor: Dr Batseba Mofolo-Mbokane
Wits contact number: 0117173411
Wits email address: batseba.mbokane@wits.ac.za

Annexure B:

Participant Information Sheet School Principal

Dear Sir / Madam

My name is Khayalethu Myoli. I am a Masters student in Mathematics Education at the University of the Witwatersrand, Johannesburg. My supervisor is Dr Batseba Mofolo-Mbokane. I am conducting a research study about Grade 12 learners' difficulties in applying the concept of proportionality in triangles, and to what extent do these difficulties connected to learners' proficiency in the concept of similarity and proportional division. This investigation involves the analysis, interpretation and categorization of learners June Examination solutions in Geometry with the aim of providing more insight on the kinds of difficulties they experience when dealing with geometric problems involving proportionality. The study title is: Grade 12 learners' difficulties in applying the concept of proportionality in triangles.

I am inviting you to take part in this study by giving permission for the use of learners Preliminary Examination scripts for research purposes. If you decide to take part, your participation in this research study will last about 15 minutes. The research activity will take place at …………………………(place) at……. ..……(time).

With your permission, I would like to take de-identified copies of the geometry section of learners June Examination scripts. This data will be stored in my house for 5 years and deleted after 5 years. Only the researcher will have access to the data. When I share the results of the research study, I will not include your name or anything else that could identify you. With your permission, other researchers may use the data collected from this research study, but your name and any personal information will not be used or passed on.

If you decide to take part in the research study, it should be because you want to volunteer. You do not have to take part. You can stop being in the study at any time. You do not have to answer any questions if you do not want to. You will not get any direct benefits if you choose to join the research study. You will not lose any services, benefits or rights you would normally have if you decide not to join. Taking part in the research study will not cost you anything. You will not be paid for being in this research study. The risks for this research study are no more than what happens in everyday life. This research study will be written up as a research report. The report will be available on the university library website. If you would like to receive a summary of this report, I will be happy to send it to you.

If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely,

Researcher: Khayalethu Myoli. Wits email: kmyoli@gmail.com Cell:
Supervisor: Dr Batseba Mofolo-Mbokane Wits email: batseba.mbokane@wits.ac.za
Tel: 0117173411

Annexure C:

Consent form for Teachers:

Grade 12 learners' difficulties in applying the concept of proportionality in triangles.

Name of Researcher: Khayalethu Myoli

I, …………………………………….., agree to participate in this research project.

I agree to the following:

(Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
I understand that I can volunteer to take part in the study	YES	NO
I agree that my learners' June Examination geometry marked solutions will be photocopied and used for this study.	YES	NO
I agree that direct quotations from my work may be used by the researcher in their research report.	YES	NO
I agree that my participation will remain anonymous (my name or other identifying data will not be used by the researcher in their research report)	YES	NO
I agree that other researchers may use the information I provide through my work and my learners' work (depending on their own ethics clearance being obtained) but my name and any personal information will not be used or passed on	YES	NO

…………………………………… (signature)
…………………………………… (name of participant)
…………………………………… (date)

…………………………………… (signature)
…………………………………… (name of researcher/person seeking consent)
…………………………………… (date)

Annexure D:

Consent Form for a child's parent/legal guardian

Grade 12 learners' difficulties in applying the concept of proportionality in triangles.

Name of researcher: Khayalethu Myoli

I, …………………………………….., agree that my child ……………………..… can participate in this research project.

I agree to the following:

(Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
I understand that my child can volunteer to take part in the study	YES	NO
I agree that my child's work can be copied and used in the study.	YES	NO
I agree that copies of my child's work may be used by the researcher in their research report.	YES	NO
I agree that my child's participation will remain anonymous (my name or other identifying data will not be used by the researcher in their research report.	YES	NO
I agree that other researchers may use the information provide by my child in their interview/focus group/other activity (depending on their own ethics clearance being obtained) but that their name and any personal information will not be used or passed on	YES	NO

…………………………………… (signature)
…………………………………… (name of parent/guardian)
…………………………………… (date)

…………………………………… (signature)
…………………………………… (name of researcher/person seeking consent)
…………………………………… (date)

Annexure E:

Learners' Assent Form:

Title of the Study: Grade 12 learners' difficulties in applying the concept of proportionality in triangles.

Name of Researcher: Khayalethu Myoli

I, ……………………………………….., agree to participate in this research project.

I agree to the following:

(Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
I understand that I can volunteer to take part in the study	YES	NO
I agree that my geometry solutions for June 2024 Examination will be photocopied and used in this study.	YES	NO
I agree that direct quotations from my work may be used by the researcher in their research report.	YES	NO
I agree that my participation will remain anonymous (my name or other identifying data will not be used by the researcher in their research report)	YES	NO
I agree that other researchers may use the information I provide through my work (depending on their own ethics clearance being obtained) but my name and any personal information will not be used or passed on	YES	NO

…………………………………… (signature)
…………………………………… (name of participant)
…………………………………… (date)

…………………………………… (signature)
…………………………………… (name of researcher/person seeking consent)
…………………………………… (date)

Annexure F: Ethics clearance certificate

UNIVERSITY OF THE WITWATERSRAND JOHANNESBURG | WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	PROTOCOL NUMBER: 2024ECE0013M
PROJECT TITLE	Grade 12 learners' difficulties in applying the concept of proportionality in triangles.
INVESTIGATOR	Khayalethu Myoli
SCHOOL/DEPARTMENT OF INVESTIGATOR	WSoE
DATE CONSIDERED	08 April 2024
DECISION OF THE COMMITTEE	Approved unconditionally
RISK LEVEL	Minimal Risk
EXPIRY DATE	Date of submission of the Research Report
ISSUE DATE OF CERTIFICATE	24 April 2024
CHAIRPERSON	Dr. Lawan Abdulhamid

cc: Dr. Batseba Mbokane

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am are authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

Signature

Date 29 , 04 , 2024

27 St Andrews Road, Parktown, Johannesburg, 2193 | Pvt Bag 3, WITS, 2050 | T +27 11 717 3007 | www.wits.ac.za/education

Annexure G: Turn-it-in report

Myoli Masters RR Final Turn it in report-2.docx

ORIGINALITY REPORT

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