

Kelvin-Helmholtz Instability on the Mach Reflection Shear Layer

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Declaration

I declare that this dissertation is my own, unaided work, except where otherwise acknowledged. It is being submitted for the degree of Master of Science in Engineering in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other university.

Signed this ____ day of _____ 20____

Sidney Richard Rubidge.

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Abstract

The Kelvin-Helmholtz Instability (KHI) is an instability that takes the form of repeating wave like structures. It forms on a shear layer where two fluids are moving at a relative velocity to one another. Such a shear layer forms as part of the Mach reflection. This dissertation focuses on experimentally visualising the presence of the KHI on the Mach reflection shear layer, as well as its evolution.

Experimentation was performed at Mach numbers of 1.34, 1.46 and 1.61. Plain and parabolic entrance test pieces with varying wedge angles of 30° to 38° were tested. Flow field visualisation was performed with a schlieren optical system. The camera-side knife edge orientation was altered to vary the axis of sensitivity of the optical system.

It was found that the KHI did form on the Mach reflection shear layer; more readily with increasing Mach number and wedge angle. The evolution of the instability was found to be almost scalable, with $M_0 = 1.46$ and $M_0 = 1.61$ achieving the best scaling of the length and average width of the KHI respectively. The KHI was best visualised with the camera-side knife edge perpendicular to the shear layer (i.e. the axis of sensitivity along the length of the shear layer), the visualisation of the KHI was completely obscured when camera-side knife edge was parallel to the shear layer.

Second order Euler, Navier-Stokes and k- ϵ numerical simulations of the flow field were also conducted using FLUENT 13. It was found that the Euler and Navier-Stokes solvers achieved very similar results, both producing the KHI but at a much less developed state than the experimental cases. The k- ϵ solver, however, did not produce the KHI.

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1 INTRODUCTION

1.1 Shear Layer

A shear layer is the interface between two flows of differing properties, most notably velocity. Theoretically a shear layer is a clean discontinuity, but in practice the shear layer has a thickness as boundary layers form on both sides of the layer and mixing between the layers occur, thus a shear layer is often referred to as a mixing layer [10]. The mixing across a shear layer has been thoroughly researched as it plays a large role in combustion [11]. It has been found that the shear layer thickness, and thus the magnitude of mixing, decreases with increasing velocity difference, this pattern is exaggerated as the shear layer flow becomes compressible [11].

Shear layers are often experimentally created with two wind tunnels whose flows of differing velocities are combined over a splitter plate, this creating a shear layer [12].

1.2 Shock Waves

A shock wave is discontinuity in a flow field, after which abrupt rises in pressure, temperature and density occur. A shock wave is a means by which supersonic flow adjusts itself to changes in flow geometry or is formed by sudden compression of a fluid, an example being an explosion.[13]

1.2.1 Shock Reflection

When a supersonic flow field encounters a change in geometry it will have to adapt its direction of flow. As the flow is supersonic it has no knowledge of the geometry change before it is encountered, thus the change in flow direction is brought about by means of a shock wave, this is known as an oblique shock wave (see Figure 1.1,

adapted from Ben-Dor 2007 [1]). Now, for the same flow field assume there is also an upper wall, thus the flow would contain the original oblique shock caused by the wedge, but due to the presence of the upper wall, the direction of the flow again has to be bent so that it is parallel to this wall thus creating a second oblique shock. If the angle of the wedge is greater than the critical angle, Regular Reflection (RR) occurs, else Mach Reflection (MR) occurs (see Figures 1.2 (a) and (b) respectively).[1]

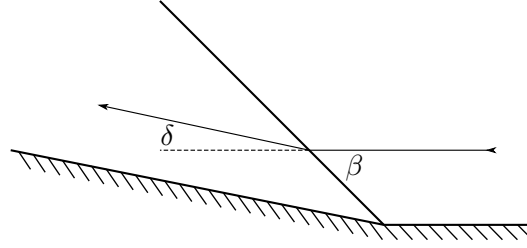


Figure 1.1: Schematic of formation of an oblique shock wave [1].

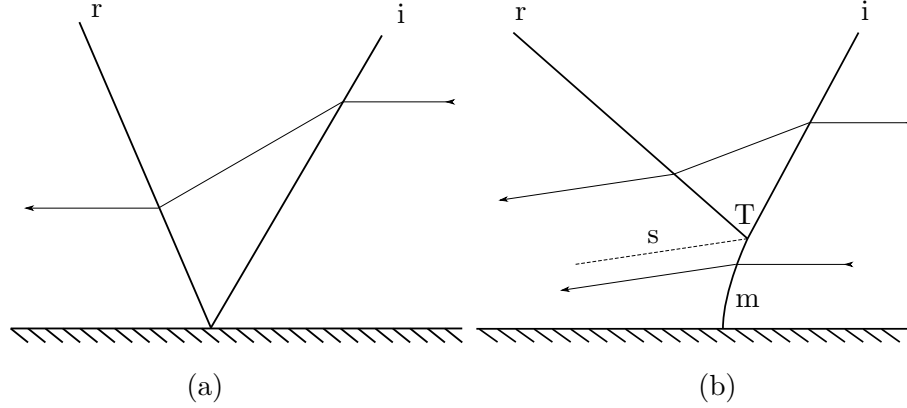


Figure 1.2: (a) Schematic of regular reflection, (b) Schematic of Mach reflection.

The pressure and density ratios across an oblique shock can be calculated using Equations 1.1 and 1.2 respectively [14],

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 \sin^2 \beta - (\gamma - 1)}{\gamma + 1} \quad (1.1)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) M_1^2 \sin^2 \beta}{2 + (\gamma - 1) M_1^2 \sin^2 \beta} \quad (1.2)$$

The Mach Number of the flow resulting from it crossing an oblique shock wave can be calculated using Equation 1.3 [14] below,

$$M_2^2 \sin^2 (\beta - \delta) = \frac{M_1^2 \sin^2 \beta + \frac{2}{\gamma - 1}}{\frac{2\gamma M_1^2 \sin^2 \beta}{\gamma - 1} - 1} \quad (1.3)$$

The deflection angle of the flow as it crosses an oblique shock wave can be calculated using Equation 1.4 [14] below,

$$\tan\delta = \frac{2\cot\beta (M_1^2 \sin^2\beta - 1)}{2 + M_1^2 (\gamma + \cos 2\beta)} \quad (1.4)$$

The ratio of sound speeds can be calculated using Equation 1.5 altered from [4].

$$\frac{a_2}{a_1} = \frac{[(\gamma - 1) M_1^2 \sin^2\beta + 2]^{1/2} [2\gamma M_1^2 \sin^2\beta - (\gamma - 1)]^{1/2}}{(\gamma + 1) M_1 \sin\beta} \quad (1.5)$$

In the above equations, β is the angle of the shock wave to its upstream flow direction (see Figure 1.1), M_1 and M_2 are the Mach Numbers before and after the shock wave respectively, γ is the ratio of specific heats of the fluid being analysed.

1.2.2 Mach Reflection

Mach reflection was discovered and first described by Ernst Mach in 1878, it was later formally researched by von Neumann in 1943 [15] who named it after its discoverer. The Mach reflection consists of three shock waves; the incident shock wave (i), reflected shock wave (r) and the Mach stem (m) all of which meet at a triple point (T), see Figure 1.2 (b). The flow above the triple point has to pass through two shocks (i and r) whereas the flow below it only has to pass through one (m). The pressures of the two flows have to be equal thus their conditions have to differ, resulting in different velocities, densities and temperature, thus yielding a shear layer [15]. The foot of the Mach stem is the reflection point and is always perpendicular to the reflection surface [1]. The Mach reflection is grouped into three categories based on the motion of their triple point:

1. Direct Mach reflection (DiMR); Occurs if the triple point is moving away from the reflecting surface and is possible in both pseudosteady and unsteady flows [1].
2. Stationary Mach reflection (StMR); Occurs if the triple point stays at a constant distance from the reflecting surface and is only possible in unsteady flows [1].
3. Inverse Mach reflection (InMR); Occurs if the triple point moves towards the reflecting surface and is only possible in unsteady flows. When the triple point

comes into contact with the reflecting surface transitional regular reflection (TRR) occurs [1].

1.2.3 Shock Tube

Shock waves are most often experimentally created and studied in a shock tube. The basic construction of a shock tube has two main regions; a high pressure region known as the driver and a low pressure expansion section. The driver and expansion section are separated by a diaphragm. A normal shock is produced by bursting the diaphragm and stabilises as it propagates down the expansion section. After the expansion section the shock wave usually travels through to a test section where its interaction, most commonly with geometry changes, is studied [16]. A schematic layout of the basic shock tube is shown in Figure 1.3.

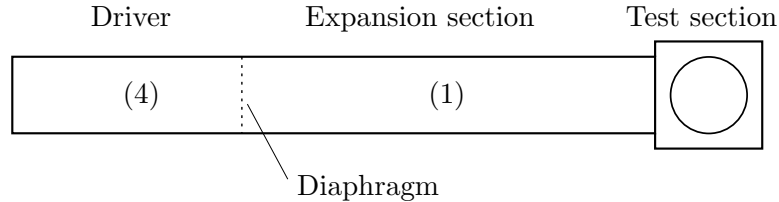


Figure 1.3: Shock tube initial conditions.

The diaphragm can either be burst by pricking it, or by the pressure difference across it creating a force greater than its strength (natural burst). The strength of the resulting shock wave is a function of the pressure ratio across the diaphragm, which can be made out of any material that is impervious to the testing fluids; for gasses thin plastic sheets are most commonly used and liquid shock tubes often use a scored metal plate.[16]

1.3 Kelvin-Helmholtz Instability

The Kelvin-Helmholtz Instability is often brought about by a relative velocity between parallel flowing fluids, thus creating a shear layer (see Section 1.1). If the relative velocity across the shear layer is greater than a critical velocity (dependent on various factors including the surface tension between the layers and the densities of the fluids) small perturbations on this layer evolve into wave-like structures, which develop into vortices, often referred to as “cat’s eye” type structures [17]. An example is shown in Figure 1.4. Ultimately the whole system breaks down into turbulence, the understanding of which makes the study of the KHI important [18].



Figure 1.4: Kelvin-Helmholtz Instability [2].

The most recognised form of the KHI is the formation of water waves caused by the wind blowing over surface of the ocean. A visually appealing form of the KHI is formed within cloud structures by the relative motion between atmospheric strata, as shown in Figure 1.5.



Figure 1.5: Kelvin-Helmholtz Instability visible within a cloud structure [3].

Another extensively researched form of the instability is that formed on a planet's (particularly the Earth's) magnetopauseⁱ. This was first hypothesised in 1955 by Dungey [20] and after which many studies were undertaken using incompressible simulations, but satellite observations suggested that the assumption of an incompressible flow is not accurate, thus much research into the compressible case has now been performed, whose results are confirmed by data collected by spacecraft such as the ISEE [21]. Later work by González et al. [19] shows that an incompressible model can be used near the front of the magnetopause as the relative velocities across it are lower, but further down the magnetopause compressibility needs to be taken into account. The effects on the formation of the KHI in a compressible flow, as opposed to in an incompressible flow, are: a lower critical relative velocity across the shear layer before the onset of the instability, and reduced growth rate of the instability [19, 21].

1.4 Flow Visualisation

Density sensitive optical visualisation techniques allow for flow visualisation, both qualitatively and quantitatively, whilst being non-intrusive. These techniques rely on

ⁱA planetary magnetopause is the abrupt layer caused by the solar wind being repelled by the planet's magnetic field. This results in the solar wind being deflected around the the planet causing a shear layer on the magnetopause [19]

the refractive index of gasses being a function of their density; this enabling density gradients within the flow to be observed due the variation of the angle through which light, passed through the flow, is diffracted [16].

The most common forms of flow visualisation are shadowgraphy and schlieren, shadowgraphy being the simplest. It is not direction sensitive and visualises the second derivative of density. On the other hand schlieren (a simple z-configuration layout of which is shown in Figure 1.6) can either be simple or coloured (where light deflection can be determined by the colour that it is visualised), it visualises the first derivative of density (density gradient), and is direction sensitive to one axis. The direction of sensitivity is perpendicular to the camera-side knife edge, thus can be altered by changing the orientation of the knife edge.[16]

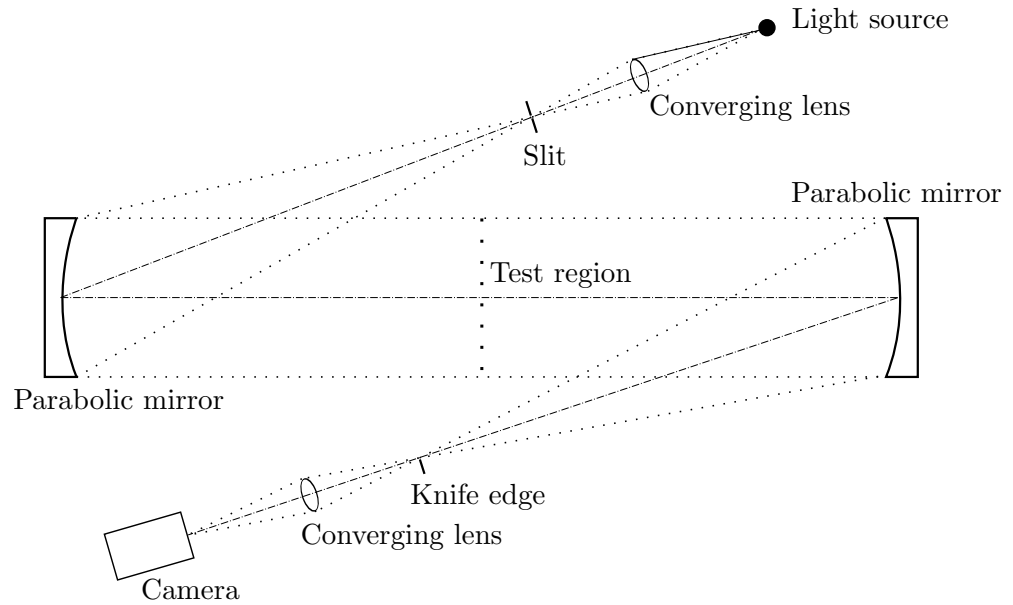


Figure 1.6: Typical layout schematic of a z-configuration schlieren system

2 OBJECTIVES

The objectives to be achieved is as follows:

1. Describe the evolution of the Kelvin-Helmholtz Instability, in a compressible fluid, from its origin to its breakdown into turbulence. The instability is to develop on the shear layer created by a Mach reflection.
2. Perform numerical simulations of the experimental flow field and compare the results to those achieved experimentally.

3 LITERATURE SURVEY

3.1 Mach Reflection

A study by Li and Ben-Dor [4] addresses the shortcomings in von Neumann's three shock theory [15], in that it is void of a length scale, thus the size of the Mach reflection (and more importantly the length of the Mach stem) is not calculated. The solution they proposed can be used to calculate the important conditions surrounding the triple point, namely the Mach numbers, flow direction, temperatures, pressures and sound speeds in all the regions of a Mach reflection (see Figure 3.1). The solution is for the steady state case and assumes an ideal gas with a constant heat capacities ratio and assumes that three oblique shock waves meet at a point (the triple point) and form the Mach reflection geometry. Standard oblique shock equations 1.1, 1.2, 1.3, 1.4 and 1.5 (see Section 1.2.1) are applied to each of the three shocks, together with the three boundary conditions below (Equations 3.1, 3.2 and 3.3) results in 18 equations and 18 unknowns, thus the system can be solved.

Note: The equations in Section 1.2.1 use β for the variables represented by ϕ in Figure 3.1.

$$\theta_1 = \theta_w \tag{3.1}$$

$$\theta_1 - \theta_2 = \theta_3 \tag{3.2}$$

$$p_2 = p_3 \tag{3.3}$$

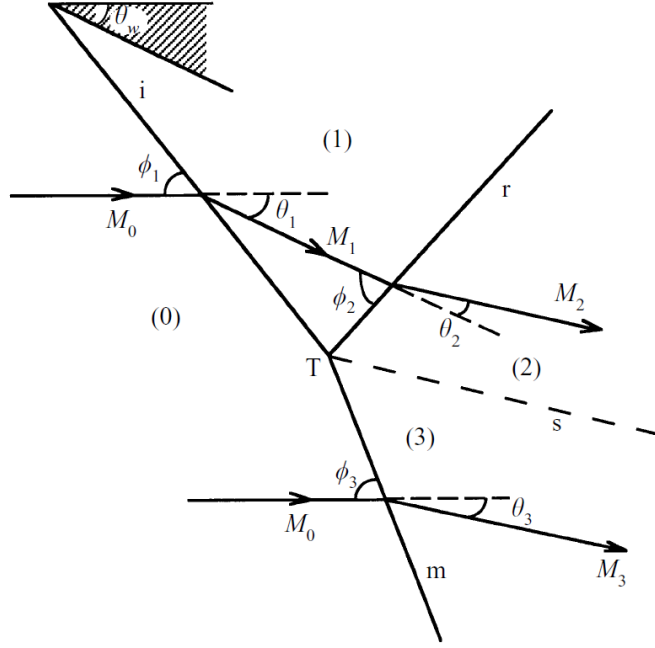


Figure 3.1: Geometry and parameters of a steady state Mach reflection from Li and Ben-Dor (1997) [4].

3.2 Instability on the Mach Reflection Shear Layer

A study by Rikanati et al. [5] compares the experimental growth rate of the Mach reflection shear layer to that of the theoretical large-scale Kelvin-Helmholtz Instability turbulent mixing zone, by comparing the spread angles. The experimental spread angles were measured off holographic interferometry images of experimentally created Mach reflections; an example is shown in Figure 3.2. The theoretical spread angle was obtained by using Equation 3.4 from Brown et al. [22], the theoretical width of the KHI turbulent mixing zone against time, and converting it so that it represented the width of the KHI turbulent mixing zone against distance, as can be seen in Equation 3.5. Adding in corrections for the differing densities of the fluids and for flows where the velocity difference is greater than Mach 1 results in Equation 3.6. Finally, using Equation 3.6, it was shown that the spread angle could be calculated using Equation 3.7. The values required for Equation 3.7 were found using a similar procedures to that used by Li and Ben-Dor [4], but due to this case being transient the first boundary condition (Equation 3.1) did not apply thus the system had to be iterated. It is additionally noted that if the Mach Stem is assumed normal to the surface of the wedge the angle of the triple point trajectory can be calculated.

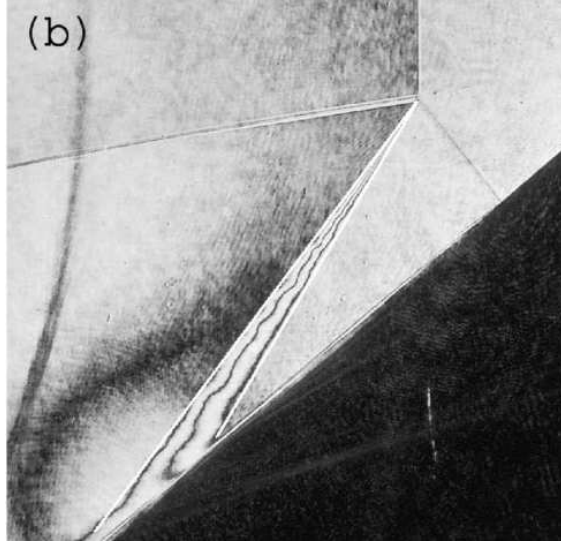


Figure 3.2: Holographic interferometry image of a Mach reflection on a wedge angle $\theta_w = 40^\circ$ and incident shock strength $M_0 = 1.9$ from Rikanati et al. (2006) [5].

$$\delta h(t) = c\Delta vt \quad (3.4)$$

Where δh is the width of the turbulent mixing zone and Δv is the difference in velocity across the shear layer; t represents the evolution of time and $c = 0.16 \pm 0.01$ derived experimentally in [22].

$$\delta h(x) = 2c[(v_1 - v_2) / (v_1 + v_2)]x \quad (3.5)$$

Where v_1 and v_2 represent the fluid velocity above and below the shear layer, x the distance from the mixing starting point and $c = 0.16 \pm 0.01$ as above.

$$\delta h(x) = (0.38 \pm 0.02) \frac{S(v_1, v_2)}{1 - 2f_d(\rho_1, \rho_2)S(v_1, v_2)} \times x f_{HiMach} \left(\frac{\Delta v}{a} \right) \quad (3.6)$$

Where:

$$f_d(\rho_1, \rho_2) = \frac{1}{2} \left| \frac{1 - \sqrt{\rho_2/\rho_1}}{1 + \sqrt{\rho_2/\rho_1}} \right|$$

$$S(v_1, v_2) = \frac{v_1 - v_2}{v_1 + v_2}$$

$$f_{HiMach} \left(\frac{\Delta v}{a} \right) = 0.5 \left(1 - \tanh \left(2 \left(\frac{\Delta v}{a} - 1.2 \right) \right) \right)$$

ρ_1 and ρ_2 are the densities above and below the shear layer, and a is the sound speed. The other variables are as explained above.

$$\theta_{spread} = \arctan\left(\frac{\delta h(x)}{2x}\right) \quad (3.7)$$

It was found that the theoretical and experimental spread angles were comparable for Reynolds numbers greater than 2×10^4 . It was inferred that the KHI is the cause for Mach Reflection shear layer growth, though the instability was not visualised. The plot of the experimental results and theoretical predictions for the spread angle of the Mach reflection shear layer as a function of incident shock wave Mach number is shown in Figure 3.3.

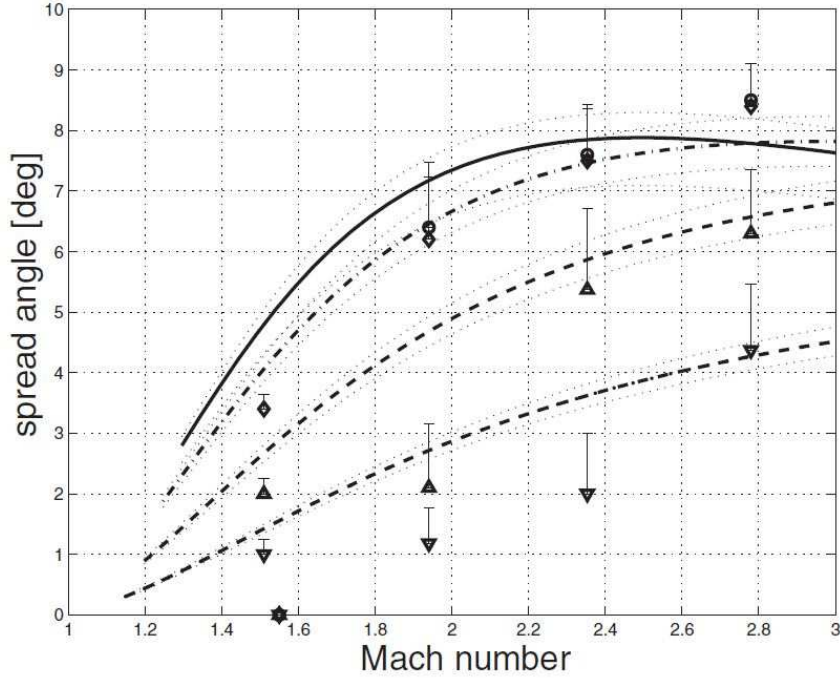


Figure 3.3: Mach reflection shear layer spread angle as a function of incident shock wave Mach number. The experimental results for wedge angles of 45° , 40° , 30° , and 20° are represented by $\circ$, $\diamond$, Δ , and ∇ respectively. The theoretical predictions of the above mentioned wedge angles are represented by the solid line, the dash-dotted line, the upper dashed line, and the lower dashed line respectively. [5]

The instability was however unintentionally visualised (Figure 3.4) in a study performed by Skews et al. [6] where the transition point from initial Mach Reflection to a Transitioned Mach Reflection was being investigated. The KHI was visible on the shear layer of the Mach reflection with an initial Mach number of 1.5. There were pieces of tape stuck to the surface of the curved surface to create perturbation

waves. In Figure 3.4 it can be seen that these perturbation waves are in contact with the larger loops of the KHI; thus it was stated that they were triggered by the perturbation waves.

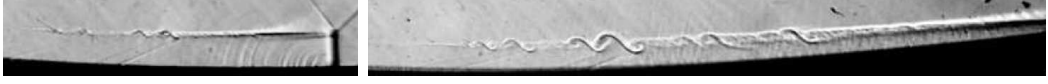


Figure 3.4: Experimental images of the Kelvin-Helmholtz Instability on a Mach reflection (off a curved surface) shear layer, $M = 1.5$ [6].

Previous work done by Ashlin Ramdas [9] at the Flow Research Unit, University of the Witwatersrand also yielded an unintentional visualisation of the KHI whilst investigating the Mach reflection development on various wedges with parabolic entrances, shown in Figure 3.5.

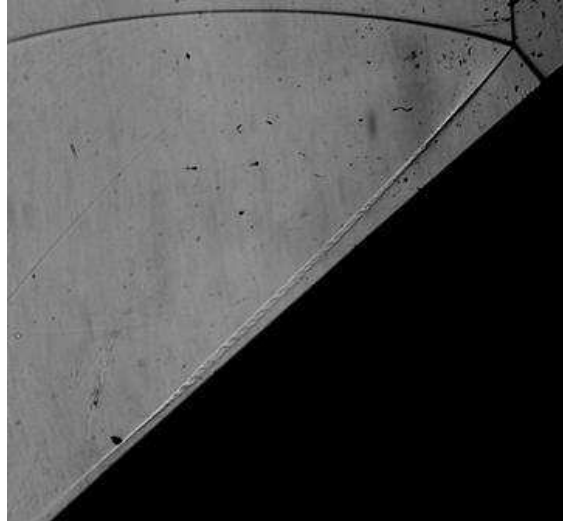


Figure 3.5: Experimental Images of the Kelvin-Helmholtz Instability on a Mach reflection (off a 38° wedge with a parabolic ($y = x^2$) entrance) shear layer, $M = 1.42$ [6].

A study by Gvozdeva [7] investigates the magnitude of mixing across shear layers (caused by both Mach reflection and shock wave diffraction) relative to the ratio of specific heats (γ) of the working fluid. The specific heat ratios tested ranged from $\gamma = 1.18$ to $\gamma = 1.66$ and were achieved with the use of different gasses namely: argon, air, nitrogen, carbon dioxide and freon. It was found that the magnitude of the mixing across the shear increased with decreasing specific heat ratios, Figure 3.6 shows an instability on the shear layer (resulting in a high intensity of mixing) of a Mach reflection in nitrogen, caused by a initial Mach number $M_0 = 2.12$ and a wedge angle $\theta_w = 24^\circ$. Gvozdeva states that these shock interactions are self similar.

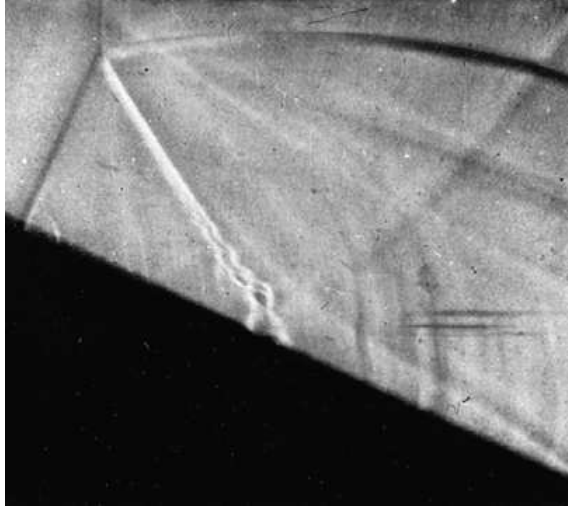


Figure 3.6: Mach reflection, in nitrogen, with instability on the shear layer, $M_0 = 2.12$ [7].

3.3 Numerical Simulations of Shear Layers

Sun and Takayama noted that small vortices (that had never been observed experimentally) often form on the shear layer behind a diffracted shock or a Mach reflection when solved with a very fine mesh [23]. They study the effects of mesh size and different viscous solvers on the formation of this vorticity. It was found that very fine meshes cause the formation of these small vortices when solved with Euler (inviscid) and Navier-Stokes (laminar) solvers, but these vortices are suppressed when solved with a $k-\epsilon$ turbulence solver. A benchmark study into Mach reflections co-ordinated by Takayama and Ziang [8] illustrates this scenario where vorticity forms on the shear layer of a Mach reflection when a the numerical simulation is solved with a very fine mesh and solved with a non-turbulence solver. The cases studied in this benchmark test were the Mach reflections caused by a normal shock, with a Mach number of 2, propagating along a 46° and a 49° wedge. An example of the vorticity forming on a Mach reflection shear layer can be seen in Figure 3.7.

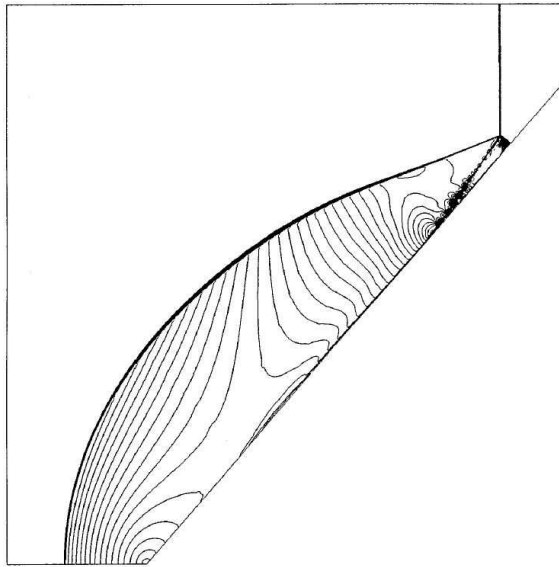


Figure 3.7: Mach reflection numerical simulation (off a 49° wedge) with vorticity on the shear layer, $M = 2$ [8].

4 PRELIMINARY WORK

4.1 Three Shock Calculator

Three shock theory was used to understand what the effect of altering the Mach number or wedge angle has on the conditions surrounding the shear layer. This was a case of using Equations 1.3 and 1.4 to calculate the Mach Numbers and deflection angles over each of the shocks.

Assume a geometry configuration of a wedge with an angle of θ_w and an incident shock wave traveling from left to right with a Mach Number of M_S . This would produce a Mach Reflection configuration which is shown with a triple point frame of reference in Figure 4.1. The angle of the triple point trajectory from the wedge surface is χ . The regions of concern are 0 (upstream of the incident shock wave), 1 (between the incident and reflected shock waves), 2 (downstream of the reflected shock wave and above the shear layer) and 3 (downstream of the Mach Stem and below the shear layer). β_{01} , β_{12} and β_{03} are the angles between the flow direction and the incident shock wave (i), reflected shock wave (r), and Mach Stem (m) respectively. All of the above are represented in Figure 4.1.

From the geometry of the Mach Reflection configuration the following can be calculated:

$$\beta_{01} = 90^\circ - \chi - \theta_w \quad (4.1)$$

$$\beta_{03} = 90^\circ - \chi \quad (4.2)$$

This, and M_S , is used to calculate the direction and Mach Number (M_0) of the flow into the system with respect to the triple point. From Figure 4.2 it can be shown that:

$$M_0 = \frac{M_S}{\sin \beta_{01}} \quad (4.3)$$

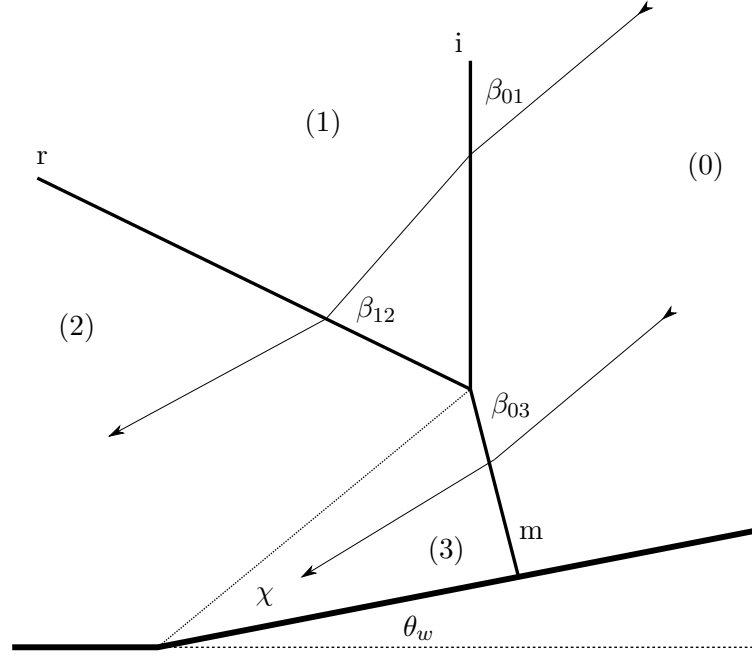


Figure 4.1: Geometry of a Mach Reflection

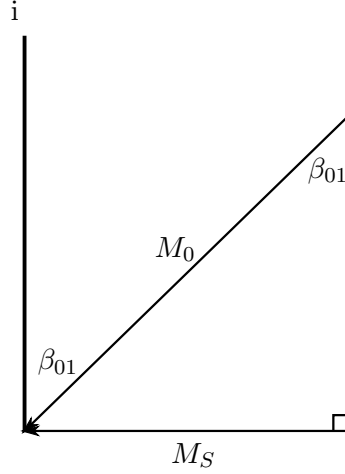


Figure 4.2: Transformation of M_S into the triple point frame of reference M_0

Assuming a value for χ and using Equations 4.1, 4.2 and 4.3 the flow can be transformed to the triple point frame of reference, and the angles between the flow and the incident shock wave and Mach Stem can be calculated. Substituting these values into Equations 1.4, 1.3 and 1.1, the deflection of the flow as it passes through, the Mach Number, and the pressure after the incident shock wave, then can be calculated. This analysis can also be performed across the Mach Stem, thus the direction, velocity and pressure of the flow in regions 0, 1 and 3 are now known. It can be seen that the conditions for region 2 are still unknown, this is due to the

angle β_{12} being unknown so, as with χ , this must be assumed. The pressures and flow directions above and below a shear layer are equal, thus the values of χ and β_{12} should be iterated until the difference between these properties in regions 2 and 3 are minimised.

The independent variables in the KHI are the velocity and density differences across the shear layer (See Section 1.3). The theoretical values for velocity and density differences, across a Mach reflection shear layer, vs Mach number can be seen in Figures 4.3 and 4.4 respectively. From this it can be seen that both the velocity and density differences increase with increasing wedge angles and incident shock wave Mach number.

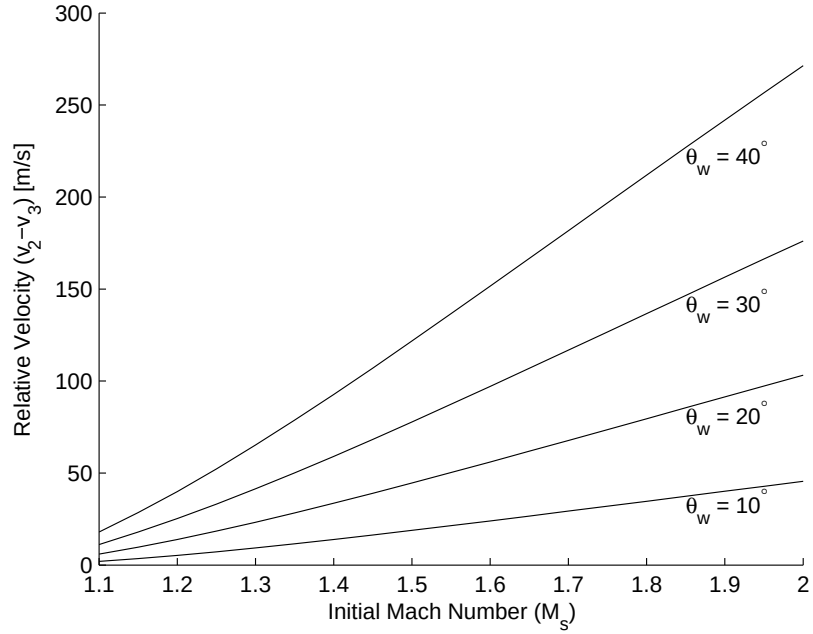


Figure 4.3: Theoretical velocity difference across the shear layer vs initial Mach number for various wedge angles

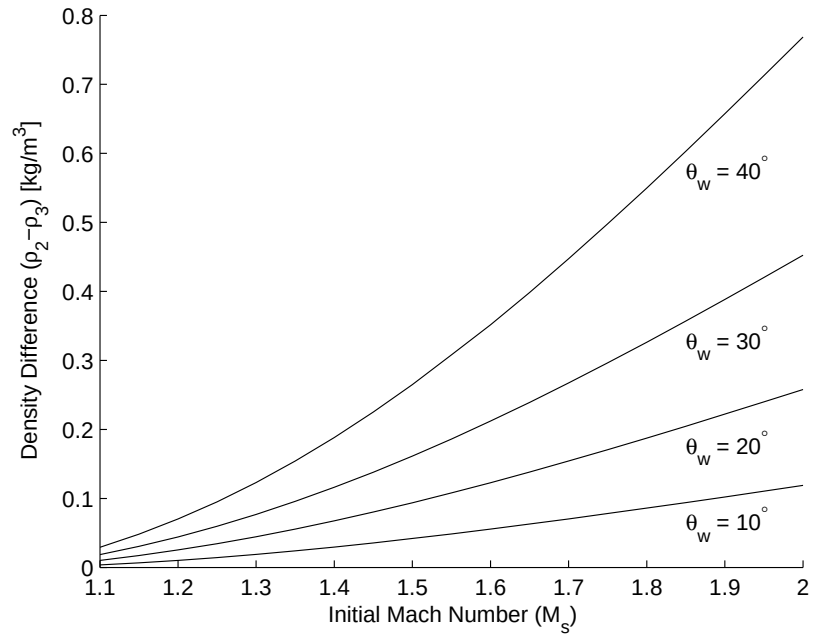


Figure 4.4: Theoretical density difference across the shear layer vs initial Mach number for various wedge angles

5 EXPERIMENTAL FACILITIES

5.1 Michael Seitz Automated Shock Tube (MSAST)

The Michael Seitz Automated Shock Tube (MSAST) was designed by Michael Seitz as part of his PhD dissertation [24] and is located in the Flow Research Unit at the University of the Witwatersrand in Johannesburg, South Africa. Its layout is shown in Figure 5.1 and critical internal dimensions are outlined below:

Test section:

Length: 450mm

Height: 180mm

Width: 76mm

Window Diameter: 300mm

Expansion Chamber:

Length: 6000mm

Height: 180mm

Width: 76mm

The MSAST creates a shock wave by using a double diaphragm bursting technique (as opposed to the more common technique of mechanically bursting the diaphragm) that, once calibrated, allows very repeatable shock wave Mach numbers. A maximum Mach number of approximately 1.84 can be attained; this can be increased by evacuating the expansion chamber. The diaphragm bursting technique is as

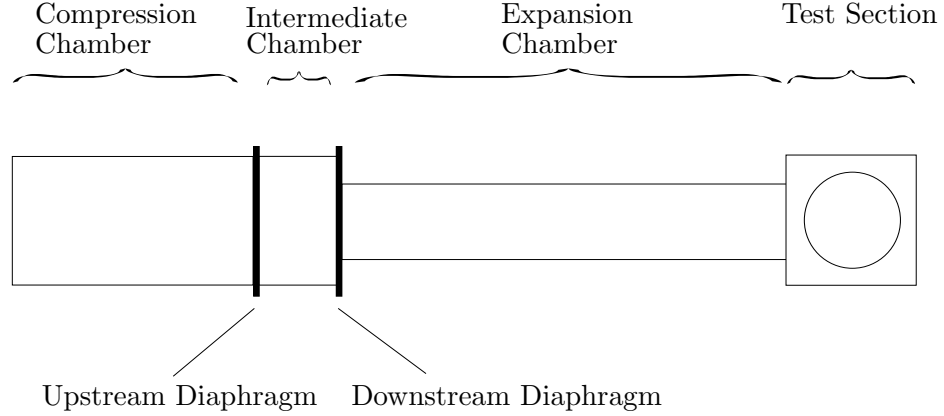


Figure 5.1: Schematic of the Seitz Automated Shock Tube layout

follows: both the intermediate and compression chambers are initially pressurised simultaneously, the compression chamber is then pressurised higher than that of the intermediate chamber. Once pressurisation is complete the intermediate chamber is vented, increasing the pressure ratio across the upstream diaphragm until it bursts causing a compression wave that bursts the downstream diaphragm.

A 25mm spaced grid is present on the outside of one of the test section windows, resulting in the grid being overlayed over the experimental images. This is used to determine image scaling and distortion.

5.2 Test Pieces

Initially test pieces (Models A though F), designed and used by Ashlin Ramdas for his final year research project [9], were used. These were PVC wedges with parabolic entrances ($y = x^2$) and ramp angles ranging from 30° to 38° . These test pieces were later remanufactured out of aluminium, with mirror finished surfaces to minimise perturbation effects. In addition to the parabolic entrance test pieces, a 30° plain walled test piece (Model E) was also manufactured out of aluminium. The critical dimensions of the wedges are detailed in Table 5.1 and drawings are available in APPENDIX A.

The parabolic entries of models A and B are similar with the exception of the doubled scale on model B. During testing with the initial PVC test pieces, the tip of Model A broke. The remainder of the parabolic entrance was then machined off, leaving a 38° plain PVC wedge which is referred to as Model F.

Note: the “parabola length” in Table 5.1 refers to the horizontal length that is

Table 5.1: Critical test piece dimensions

Model Name	Parabola length [mm]	Ramp angle [°]	Effective length [mm]	Effective height [mm]
A	75	38	250	165
B	150	38	262.5	150
C	70	30	262.5	130
D	60	34	210	120
E	N/A	30	260	150
F	N/A	38	171	135

covered by the parabolic portion of the test piece.

5.3 Optical Visualisation

The schlieren optical visualisation technique with a z-configuration (as outlined in Section 1.4) was used. The components of the system were as detailed in Figure 1.6 with the addition of a cylindrical lens on the light source side, after the slit. This lessens the image distortion, resulting in a circular focal point from the camera-side parabolic mirror, as opposed to the band focal point that is usually the case with a z-configuration.

The orientation of the camera-side knife edge was altered to adjust the axis of the system's sensitivity; this will be discussed in more detail in Section 6.

A Xenon light source (see APPENDIX B for details) was set up so that its arc was perpendicular to the slit, creating a close approximation of a point source. The light source, lenses, slit, knife edge and camera (see APPENDIX B for details) were mounted on rails and the parabolic mirrors, of diameter 254mm and focal length $\frac{f}{6}$, were positioned on adjustable stands for easy alignment of the system.

6 EXPERIMENTAL PROCEDURE

6.1 Overview

A photograph of the flow field for each test was captured with a high resolution single shot camera, thus only capturing one point of the flow progression. To capture the entire progression, a series of tests were run for each condition whilst altering the delay of the light source, and thus the position of the photographed flow field shock wave progression. Delay increments of $25\ \mu s$ were used for $M_0 = 1.34$ and $M_0 = 1.46$, and increments of $20\ \mu s$ were used for $M_0 = 1.61$.

6.2 Testing procedure

It is established that the high pressure supply is charged to an adequate pressure for testing, and that the required test piece is installed in the test section and is securely bolted in place.

6.2.1 Start up procedure

The following procedure should be observed at the beginning of each testing day:

1. Switch on all instrumentation.
2. Open the pressure gauge and set the pressure limiter to the pressure required for the tests to follow.
3. Uncover and switch on all optical visualisation equipment.

6.2.2 Testing procedure

The following procedure should be observed for each test:

1. Ensure the test section door is open.
2. Remove the burst diaphragm remains from the compression and intermediate chambers.
3. Clean out the venting tube, expansion chamber, and compression chamber with the high pressure supply.
4. Add the diaphragm, required for the test, to the compression and intermediate chambers.
5. Close and secure the test section door.
6. Close the room door.
7. Initialise the testing wizard and follow the prompts, entering the required Mach number and light source delay.
8. Switch off the room lights.
9. Open camera shutter.
10. The shock tube will automatically fire at the correct pressure ratio.
11. Once the shock has passed through the test section, close the camera shutter.
12. Switch on the room lights.
13. Ensure the tube has been vented before opening the compression and intermediate chambers.
14. Open the test section.

6.2.3 Shut down procedure

The following procedure should be observed at the end of each testing day:

1. Open test section door.
2. Switch off all instrumentation.

3. Close the pressure valve.
4. Cover and switch off all optical visualisation equipment.

6.3 Precautions

1. Hearing protection should be worn whilst cleaning the venting tube, compression and expansion chambers.
2. Hearing protection should be worn whilst testing.
3. Ensure the test piece is securely fastened in the test section.
4. Ensure the test section is securely closed.
5. Check that the camera has not automatically switched off at the start of each test.
6. Ensure the optics are clean.
7. Do not bump the optics.
8. Ensure the test section door is not left closed overnight.

7 NUMERICAL TECHNIQUES

Fluent 13, a commercially available CFD package, was used to obtain numerical results for the experimentally tested $M_0 = 1.34$, $M_0 = 1.46$ and $M_0 = 1.61$. Tecplot 360 (2009 and 2010) was used to process these results. This section details the setup of these numerical simulations.

7.1 Physical geometry

The numerical geometry was setup to mimic that of the experimental cases as closely as possible, but only modeling the flow field above the test piece; as that below it has no effect on the flow region of interest. The entire expansion section of the shock tube was not modeled either, but rather just a section of length 1.5 times that of the flow field height to allow the formation of a steady normal shock wave. An example of the geometry of Model A is shown in Figure 7.1.

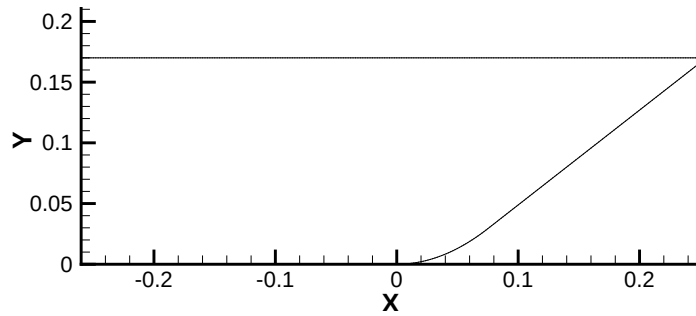


Figure 7.1: Physical geometry of Model A

7.2 Mesh

A 2mm mesh was used across the entire flow field. Mesh refinement was used to refine the mesh at the Mach reflection shear layer (see Figure 7.2), as this is the

region of greatest interest. The refinement settings used are detailed in Section 7.3.3.

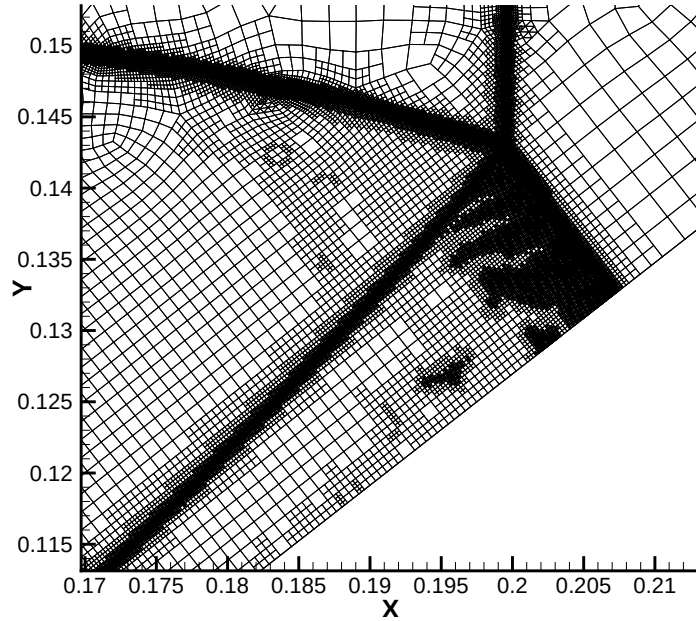


Figure 7.2: Mesh refinement on the Mach reflection shear layer

7.3 Settings

The working fluid was air treated as an ideal gas.

A journal file with the standard settings was read into Fluent to keep the settings the same across all simulation cases.

7.3.1 Solver

Time - Unsteady first order

Type - Density based: As this is more suitable to highly compressible flows with shock interactions, than a pressure based solver.

Energy - On: It is necessary to include the energy equation when compressible flows are being simulated.

Viscous solver - Inviscid, laminar and k- ϵ : Three simulations of each geometry and boundary condition configuration were run each with a different viscous solver because, as mentioned in Section 3.3, Inviscid and Laminar solvers

allows the formation of flow vorticity, whereas the k- ϵ solver dampens it.

7.3.2 Solution method

Formulation - Explicit: An explicit solution is suited for high Mach number cases, where the transient flow features are of interest.

Flux type - Roe-FDS

Energy - On

Spacial Discretisation :

Gradient - Least squares cell based

Flow - Third order MUSCL: MUSCL was used as it has third order accuracy, which is the highest available in Fluent 13, it is also most suitable for anticipating vortices.

Transient formulation - Second order implicit As this is the highest available in Fluent 13.

7.3.3 Mesh refinement

Mesh refinement was used to enhance resolution without causing too much of an increase in computational expense. The coarsen and refine thresholds were increased for the $M_0 = 1.61$ simulations as the density gradients between the shear layer and test piece were very high.

Refinement method - Gradient

Gradients of - Density and Mach Number

Refine threshold :

$$M_0 = 1.34, 1.46 - \mathbf{0.01}$$

$$M_0 = 1.61 - \mathbf{0.03}$$

Coarsen threshold :

$$M_0 = 1.34, 1.46 - \mathbf{0.005}$$

$$M_0 = 1.61 - \mathbf{0.015}$$

Maximum level of refine - 5

Adaption interval - Every 4 time steps

7.3.4 Solution

Ambient pressure - 83300 Pa

Ambient temperature - 293 K

Courant Number - 2

Mach number specific settings for the inlet are shown in Table 7.1.

Table 7.1: Mach number specific settings for the inlet.

Mach number	Gauge total	Supersonic	Inlet	Time step
	pressure	initial pressure	temperature	
	[Pa]	[Pa]	[K]	
1.34	184411.32	160619.06	370.70	6.8e-8
1.46	240528.17	193272.66	403.52	6.3e-8
1.61	329501.56	238025.59	448.50	5.71e-8

8 RESULTS AND DISCUSSION

The experimental and numerical results are presented and discussed below. The project was focused on the experimental aspects since there are no previous detailed studies of the KHI on Mach reflection shear layers.

8.1 Camera-side knife edge position

Section 1.4 mentions that the axis of sensitivity of a schlieren system is perpendicular to the direction of the camera-side knife edge. The effect of changing the sensitivity direction on the visualisation of the KHI is made clear in Figures 8.1, 8.2, 8.3 and 8.4 which were taken with the camera-side knife edge orientated perpendicular to the shear layer, parallel to the shear layer, vertical, and completely removed, respectively. The Model D test piece was used with an incident shock Mach number of $M_0 = 1.46$ and a light source trigger delay of $t = 1750\mu s$. Notice the 25mm grid overlay over the images as is explained in Section 5.1.

In the case of the camera-side knife edge being perpendicular to the shear layer (Figure 8.1) the KHI can be easily discerned on the shear layer, as the sensitivity axis of the optical system runs along the shear layer, thus highlighting the repeating pattern of the instability.

The adverse case to this is the knife edge parallel to the shear layer (Figure 8.2), resulting in the shear layer appearing as a thick black line, with no indication of the KHI that has developed on it. This is due to the axis of sensitivity running across the shear layer, a direction not utilised by the chief components of the instability. A thick black line behind the shear layer along the test piece surface, is also visualised. This is most probably the shear gradient formed from the curved incident shock wave (before the formation of the Mach reflection) as it traversed up the parabolic section of the test piece. As the shear gradient is formed along the parabolic section of the test piece, it is not present on plain wedges. This is shown in the comparison



Figure 8.1: Schlieren image, with camera-side knife edge perpendicular to the shear layer, of flow field progression for Model D ($M_0 = 1.46$).

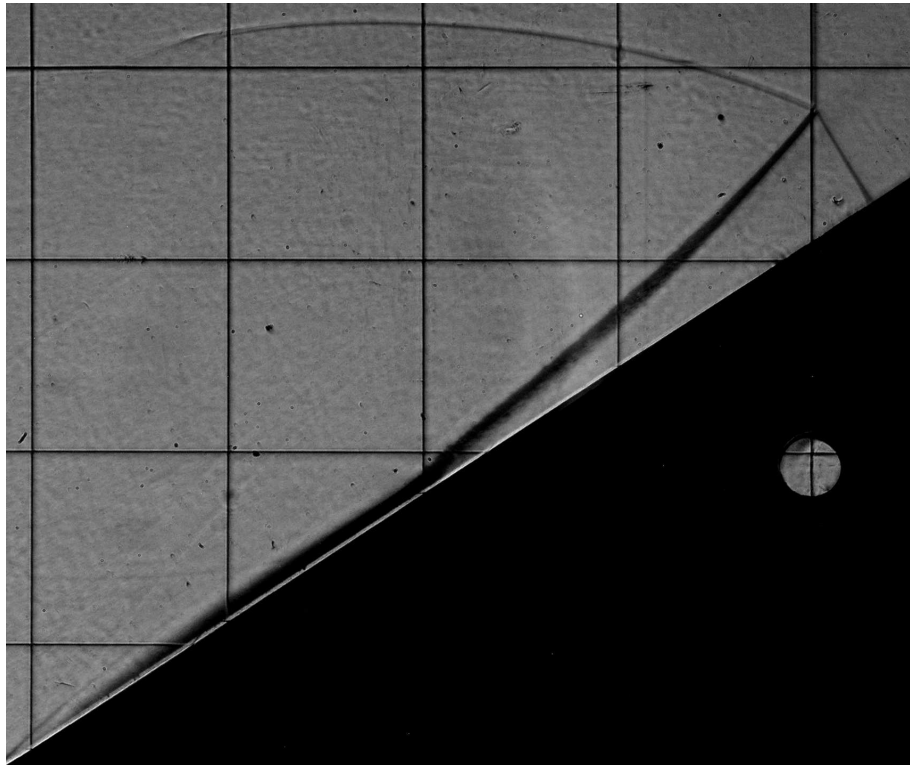


Figure 8.2: Schlieren image, with camera-side knife edge parallel to the shear layer, of flow field progression for Model D ($M_0 = 1.46$).

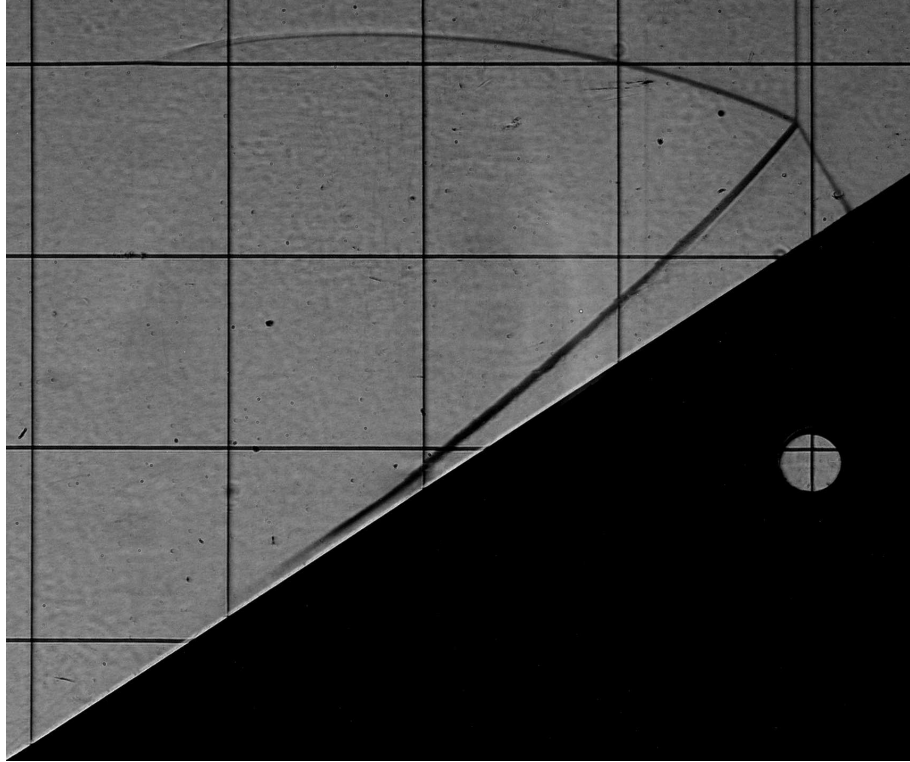


Figure 8.3: Schlieren image, with vertical camera-side knife edge, of flow field progression for Model D ($M_0 = 1.46$).

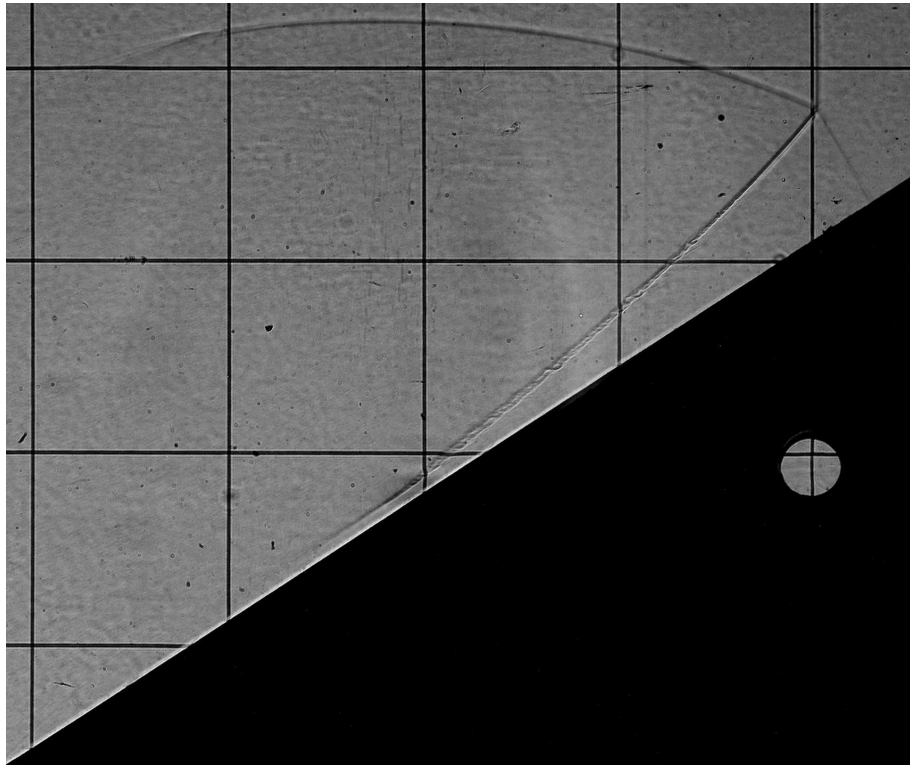


Figure 8.4: Shadowgraph (no camera-side knife edge) image of flow field progression for Model D ($M_0 = 1.46$).

between the numerical results: Model C shows the presence of the shear gradient (Figure 8.5 (a)), whereas Model E does not (Figure 8.5 (b)). This shear gradient was not visualised in the perpendicular knife edge case. It can be seen that the shear gradient does not touch the test piece surface as there is a white line below it indicating the boundary layer.

For the case of a vertical knife edge (Figure 8.3) the blurring of the shear layer is less pronounced and if viewed closely perturbations can be seen along it, which could be attributed to the presence of the KHI. The slightly greater clarity of shear layer features in this case, when compared to that of the knife edge parallel to the shear layer, is due to the knife edge being vertical thus the sensitivity is not on the opposite axis to the shear layer features. The presence of the shear gradient behind the shear layer is also less apparent.

Figure 8.4 shows a shadowgraph image (no camera-side knife edge) of the aforementioned case. The less pronounced definition of the shock waves reveal the lack of sensitivity of this optical system compared to its schlieren counterparts, but this would be expected of a shadowgraph system. The KHI on the shear layer, however, is clearly visible. The contrast between the individual KHI loops is less than that of the perpendicular knife edge case, but internal features of the instability can be better discerned; due to features of the instability that run along the length of the shear layer not being along the schlieren system's axis of sensitivity.

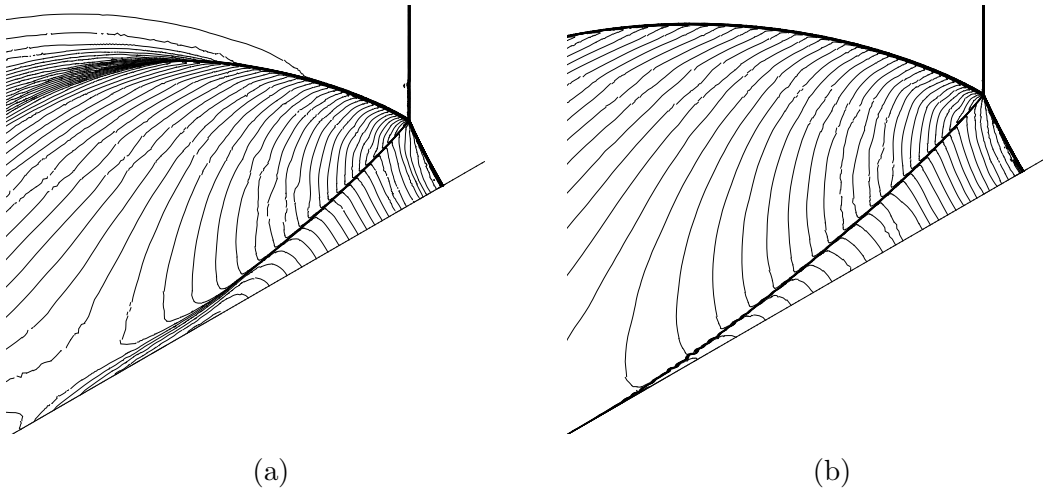


Figure 8.5: Numerical contours of density showing (a) the presence of the shear gradient on Model C and (b) no presence of the shear gradient on Model E ($M_0 = 1.46$).

8.2 KHI initiation

It is difficult to determine the point where the KHI initiates, but close inspection of images early in the flow field development often yields the presence of a repeating pattern along the shear layer. As the KHI is a repeating feature and is clearly present on the shear layer later in the flow field development, it can be reasonably assumed that this repeating pattern is the KHI. Thus the point of KHI initiation is taken as the earliest frame that this repeating pattern is visualised. The horizontal distance that the incident shock has traveled along the test piece at the point of the KHI initiation was plotted against the incident shock wave strength for each test piece, shown in Figure 8.6.

Note: the scatter in Figure 8.6 is high as the time step between each image was not small enough to determine the exact point of KHI initiation.

Figure 8.6 shows that the overall position of the initiation of the KHI follows an exponential decay type function with increasing Mach number. Models E and F causing the earliest initiation of the KHI this due to them being plain wedges, allowing an earlier formation of the Mach reflection, in turn allowing an earlier formation of a suitable shear layer, thus leading to the early formation of the KHI. As with the plain wedge test pieces, the parabolic entrance test pieces the promptness of the KHI formation is related to the test piece wedge angle; the greater the wedge angle the sooner the KHI initiation. This is most probably due to the greater relative velocity across the shear layer, caused by a steeper wedge angle (see Figure 4.3). Exceptions to this are Models B and D: Model B has the latest KHI initiation due to its parabolic entrance being twice the scale of Model A, and Model D initiates just before Model A due to its parabolic section being relatively shorter.

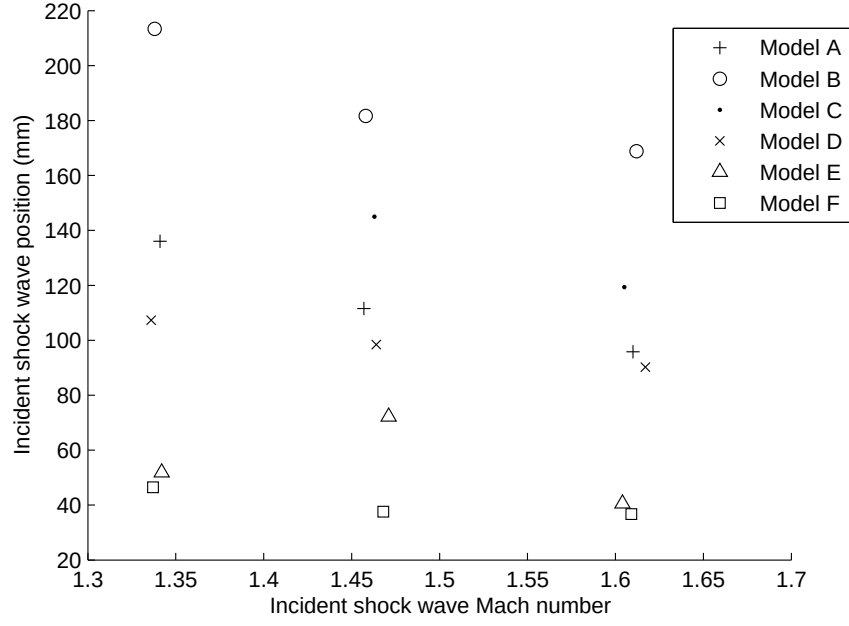


Figure 8.6: Incident shock wave position at KHI initiation vs Mach number

8.3 KHI evolution

This section details the evolution of the KHI on the Mach reflection shear layer. A schlieren optical setup (as detailed in Section 5.3), with the camera-side knife edge perpendicular to the shear layer, was used to obtain the images. The original images were rotated so that the wedge surface was horizontal. They were then cropped to show just the shear layer progression and the test piece surface, which is the region of interest. Each Figure shows the flow field progression of a particular incident shock strength on a particular model. The progressions start just before the formation of the KHI, and end when the Mach reflection has traversed the entire length of the visible test section. The KHI was magnified for a selection of images from (and are displayed after) each progression.

The flow field progressions for Models A, C and E are shown below. The full set of progressions are available in APPENDIX C.

8.3.1 Model A

Figures 8.7, 8.10 and 8.12 detail the KHI evolution on Model A with incident shock wave strengths of $M_0 = 1.34$, $M_0 = 1.46$ and $M_0 = 1.61$ respectively. They show that as the Mach reflection develops the length of the shear layer increases, the

KHI initiates along the shear layer and grows in both length and width ultimately forming well defined KHI loops. There is the presence of the KHI at all the tested Mach numbers and it starts earlier in the progression the greater the Mach number, this concurs with the results presented in Figure 8.6. It is also apparent that the greater the Mach number the longer the respective Mach stem and steeper the angle of the shear layer to the wedge surface, resulting in it being shorter. This is standard Mach reflection behavior [1].

There is a clear perturbation along the shear layer of only the $M_0 = 1.34$ case, starting from Figure 8.7 (g). As the flow field progresses this perturbation grows and develops into a well defined “S” loop of the KHI.

There are perturbation shocks coming off the wedge surface at the final stages of the $M_0 = 1.46$ (Figure 8.10) case, but are clearly visible throughout the $M_0 = 1.61$ (Figure 8.12) case. As the test piece had a polished surface, which is regarded as hydraulically smooth by Ben-Dor [1], the most probable cause for these perturbation shocks is the thin tape that was stuck onto the side of the test piece to protect the window. The tape was trimmed to be flush with the test piece surface, but the finish of this surface can never match that of polished aluminium. These shocklets would only be present along the test section window/wall and their effect on the rest of the flow field would be negligible.

The boundary layer is clearly visible, throughout Model A progressions, as a white line along the test piece surface behind the Mach stem. A magnified portion (approximately the first $\frac{1}{4}$) of the the boundary layer of Figure 8.7 (i) ($M_0 = 1.34$), is shown in Figure 8.9. This shows that there is a clear repeating pattern to the boundary layer and on close inspection a wave-like structure to the boundary layer can be seen, indicating the possible presence of the KHI.

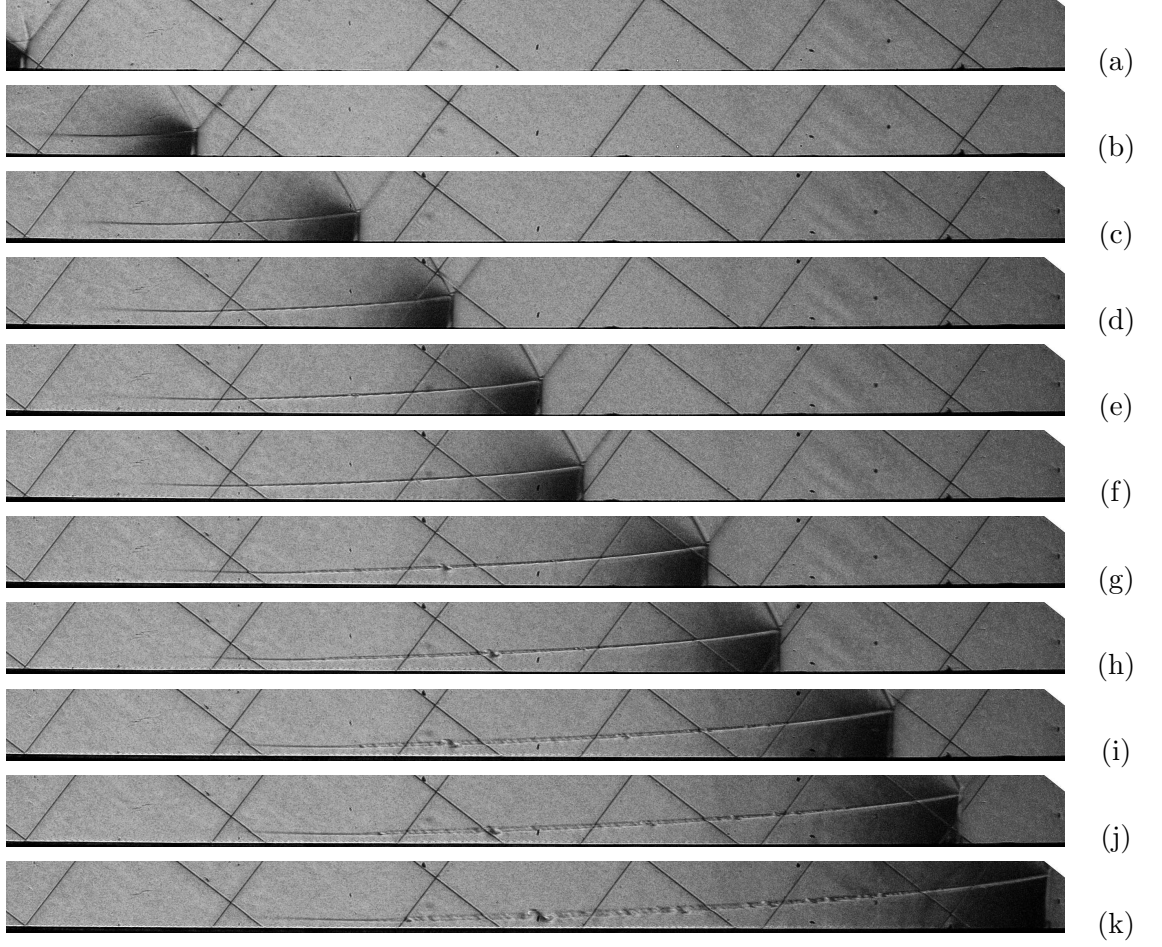


Figure 8.7: KHI evolution for Model A, $M_0 = 1.34$. At times of (a) $1650\mu s$, (b) $1700\mu s$, (c) $1750\mu s$, (d) $1775\mu s$, (e) $1800\mu s$, (f) $1825\mu s$, (g) $1850\mu s$, (h) $1875\mu s$, (i) $1900\mu s$, (j) $1925\mu s$, and (k) $1950\mu s$.

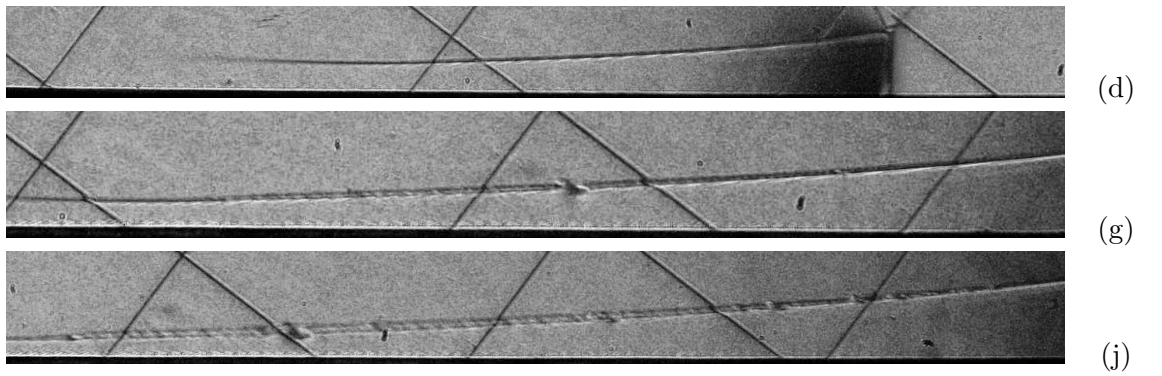


Figure 8.8: Magnified KHI of Figure 8.7 (d), (g) and (j).



Figure 8.9: Magnification of the boundary layer of Figure 8.7 (i) ($M_0 = 1.34$)

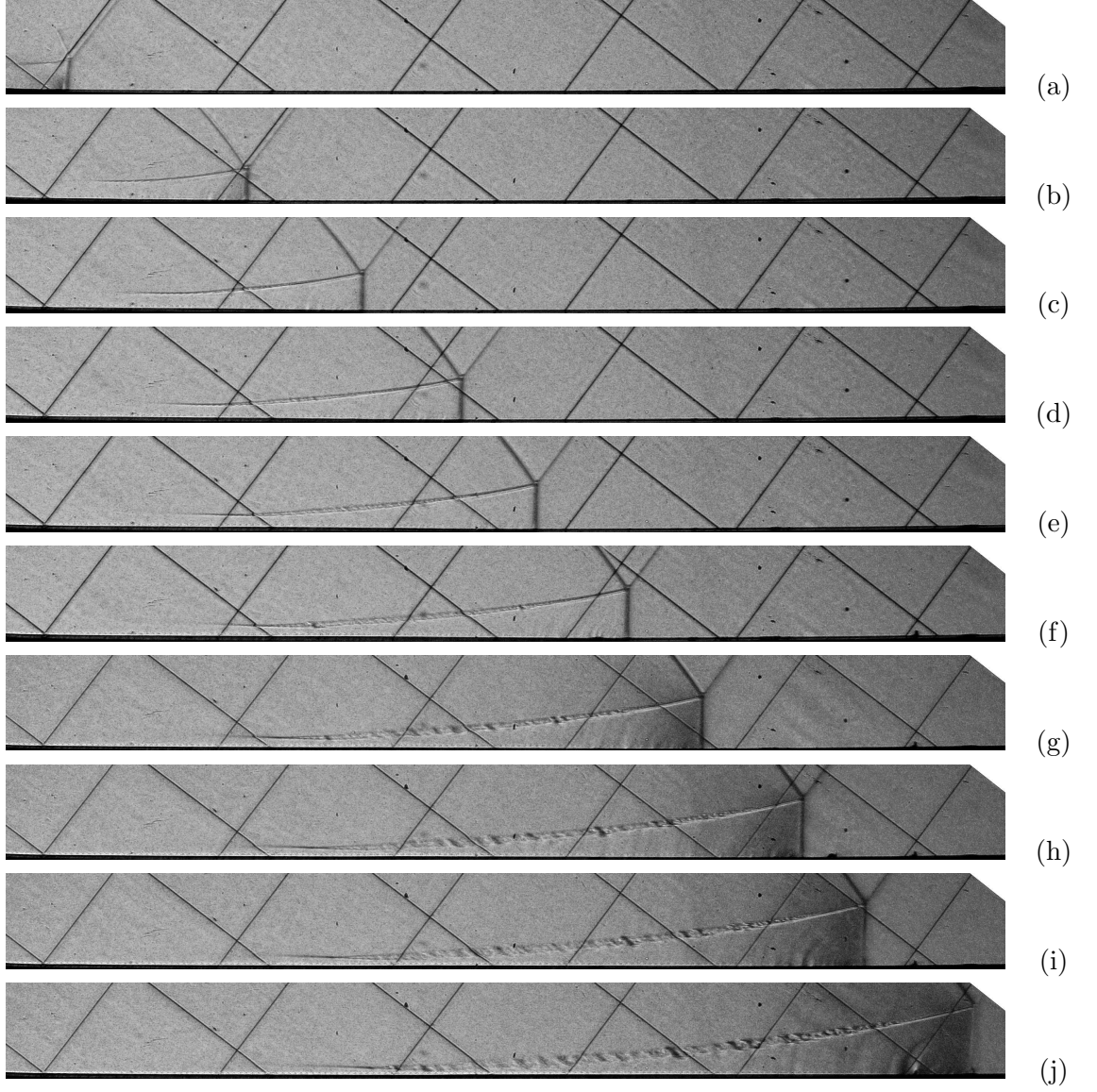


Figure 8.10: KHI evolution for Model A, $M_0 = 1.46$. At times of (a) $1525\mu s$, (b) $1575\mu s$, (c) $1600\mu s$, (d) $1625\mu s$, (e) $1650\mu s$, (f) $1675\mu s$, (g) $1700\mu s$, (h) $1725\mu s$, (i) $1750\mu s$, and (j) $1775\mu s$.

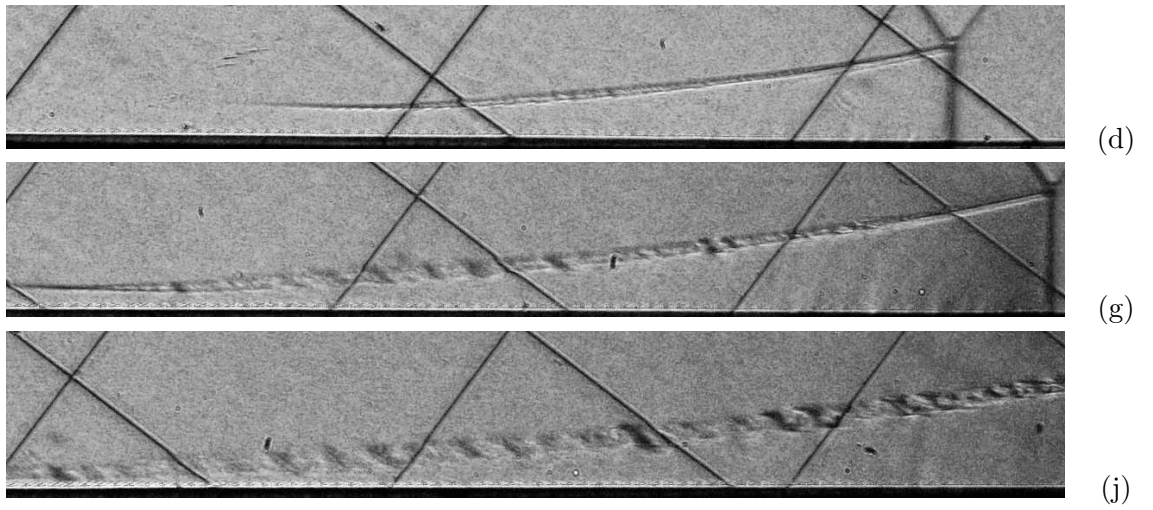


Figure 8.11: Magnified KHI of Figure 8.10 (d), (g) and (j).

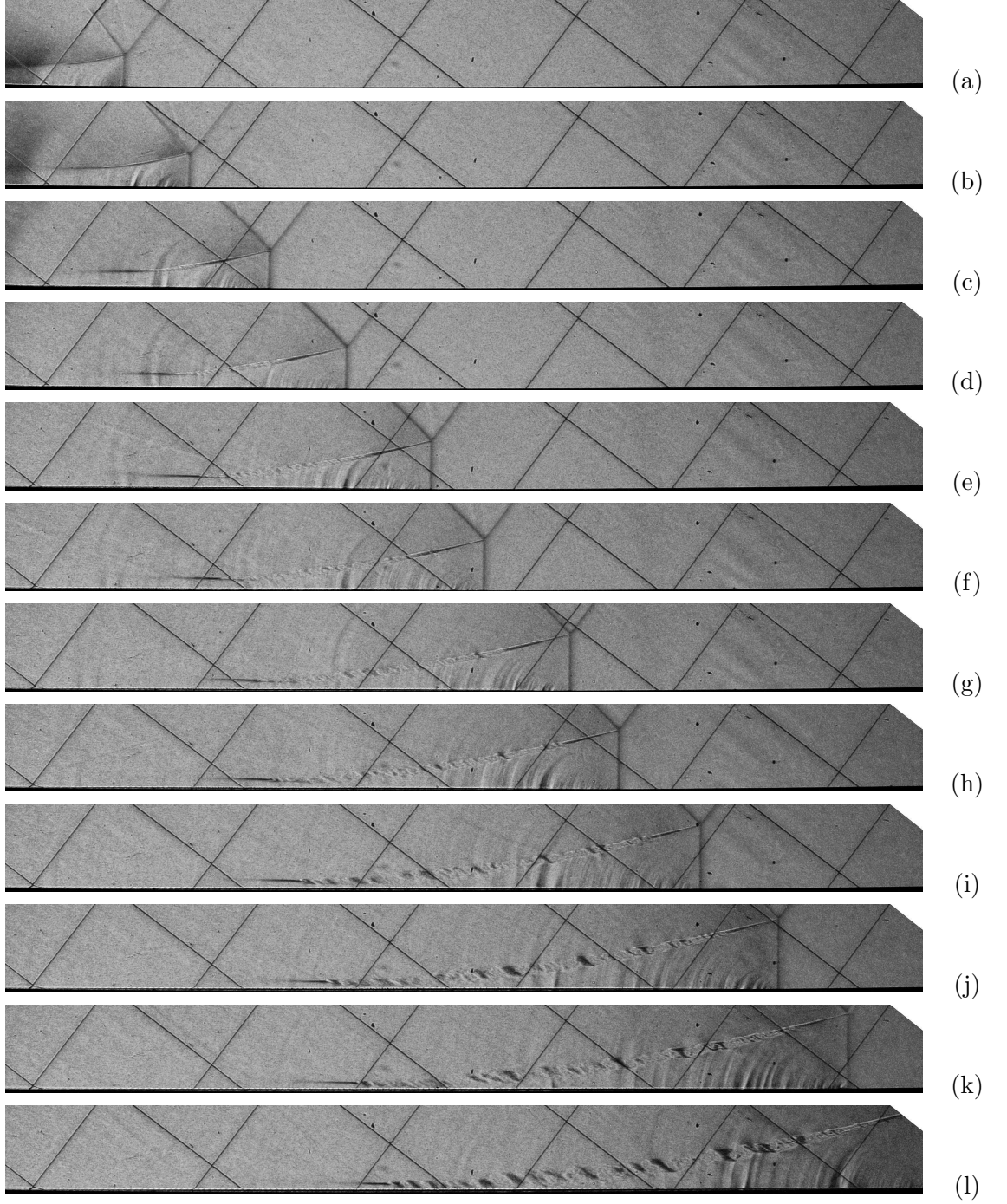


Figure 8.12: KHI evolution for Model A, $M_0 = 1.61$. At times of (a) $1400\mu s$, (b) $1420\mu s$, (c) $1440\mu s$, (d) $1460\mu s$, (e) $1480\mu s$, (f) $1500\mu s$, (g) $1520\mu s$, (h) $1540\mu s$, (i) $1560\mu s$, (j) $1580\mu s$, (k) $1600\mu s$, and (l) $1620\mu s$.

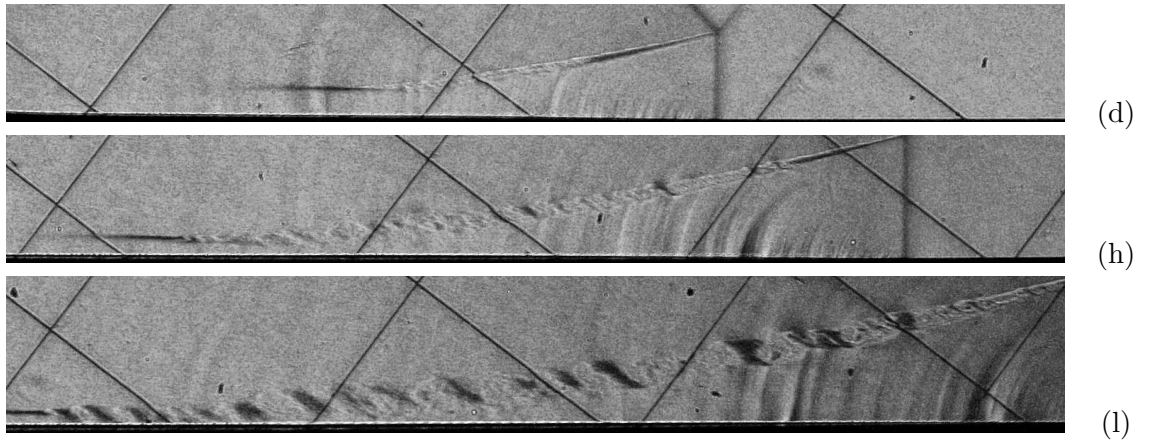


Figure 8.13: Magnified KHI of Figure 8.12 (d), (h) and (l).

8.3.2 Model C

Figures 8.14, 8.16 and 8.18 detail the KHI evolution on Model C with incident shock wave strengths of $M_0 = 1.34$, $M_0 = 1.46$ and $M_0 = 1.61$ respectively.

The progression of Model C is very similar to that of Model A with some exceptions:

1. The length of the Mach stem is greater and the angle of shear layer less steep, relative to that formed on Model A, thus allowing for the formation of a longer shear layer.
2. There is no formation of the KHI along the shear layer of the $M_0 = 1.34$ (Figure 8.14) case, this would be due to the lower relative velocity across the shear layer due to the gentler wedge angle (see Figure 4.3).
3. There is also the presence of a perturbation on the shear layer of the $M_0 = 1.34$ case (starting at Figure 8.14 (d)) as well as the $M_0 = 1.46$ and $M_0 = 1.61$ cases (starting at Figures 8.16 (f) and 8.18 (f) respectively) although the relative size of the perturbation to the shear layer is greater for the $M_0 = 1.34$ case. Later on in the flow field, after the formation of the first perturbation, multiple perturbations form on the shear layers of the $M_0 = 1.46$ and $M_0 = 1.61$ cases. The position of these perturbations are consistent throughout the progression, thus they are repeatable as each image was from a different test.
4. In the later images of Figure 8.18, and to a lesser extent the later images of Figure 8.16, the KHI abruptly constricts and ceases near the end of the shear layer. This constriction is also visible in all the Mach number cases on Model A, although it is not as abrupt. This is most probably due to the junction between shear gradient (before the formation of the Mach reflection) and the shear layer (after the formation of the Mach reflection) inducing the relative velocity at this point.
5. The presence of the boundary layer along the test piece surface is apparent, but there is no clear indication of the KHI as there was on Model A.

The blur approximately $\frac{3}{4}$ across each image, just visible above the test piece surface, was due to a bubble within one of the test section windows.

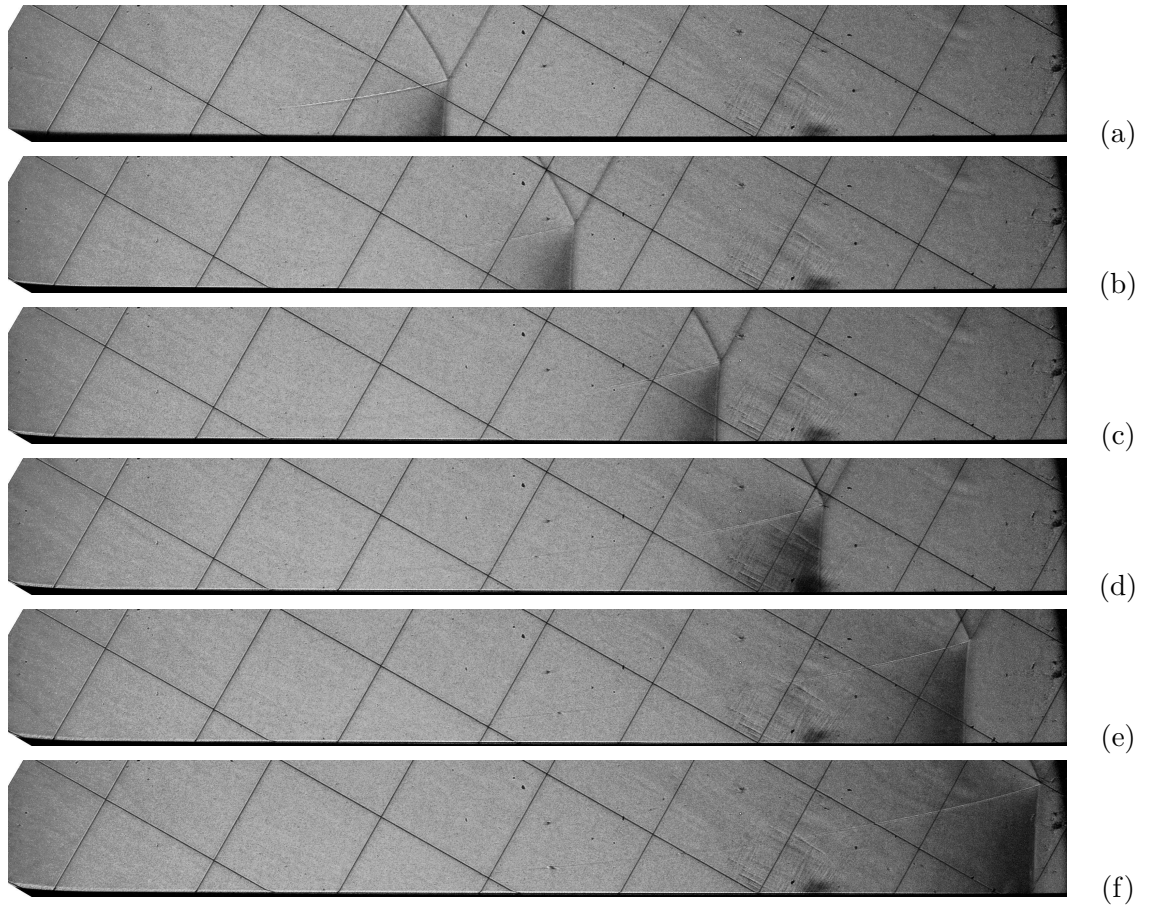


Figure 8.14: KHI evolution for Model C, $M_0 = 1.34$. At times of (a) $1750\mu s$, (b) $1800\mu s$, (c) $1850\mu s$, (d) $1900\mu s$, (e) $1950\mu s$, and (f) $1975\mu s$.

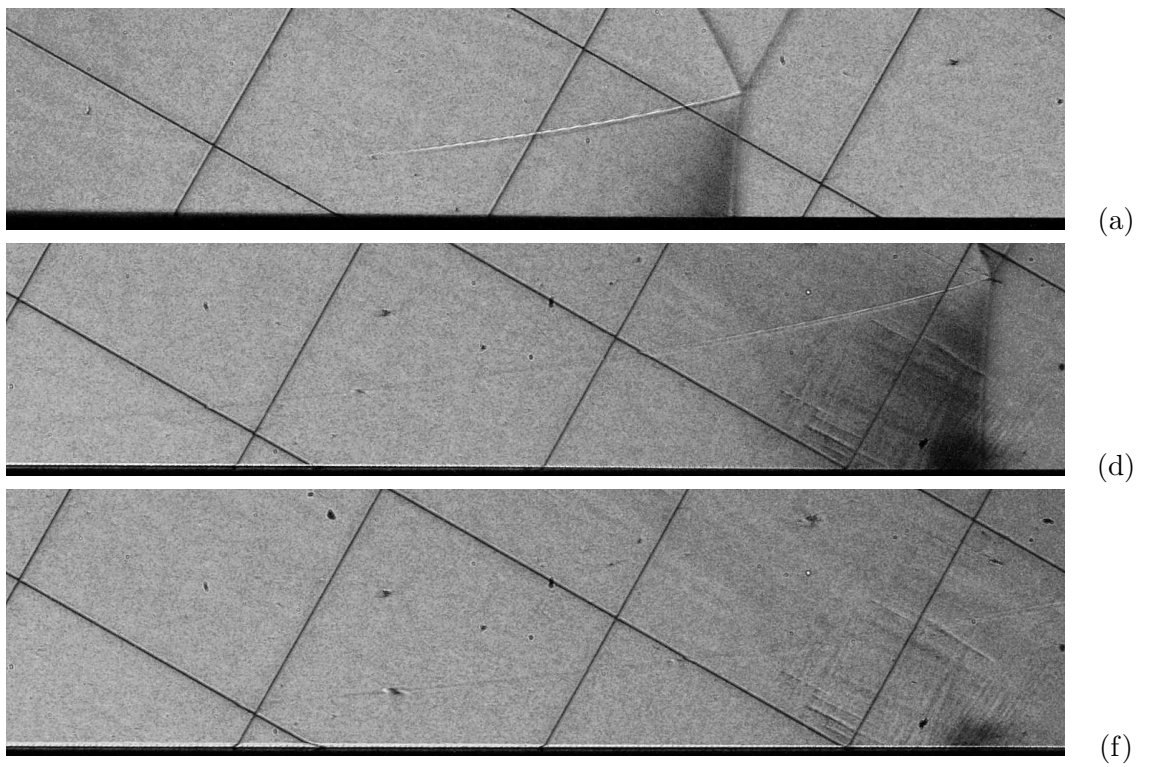


Figure 8.15: Magnified KHI of Figure 8.14 (a), (d) and (f).

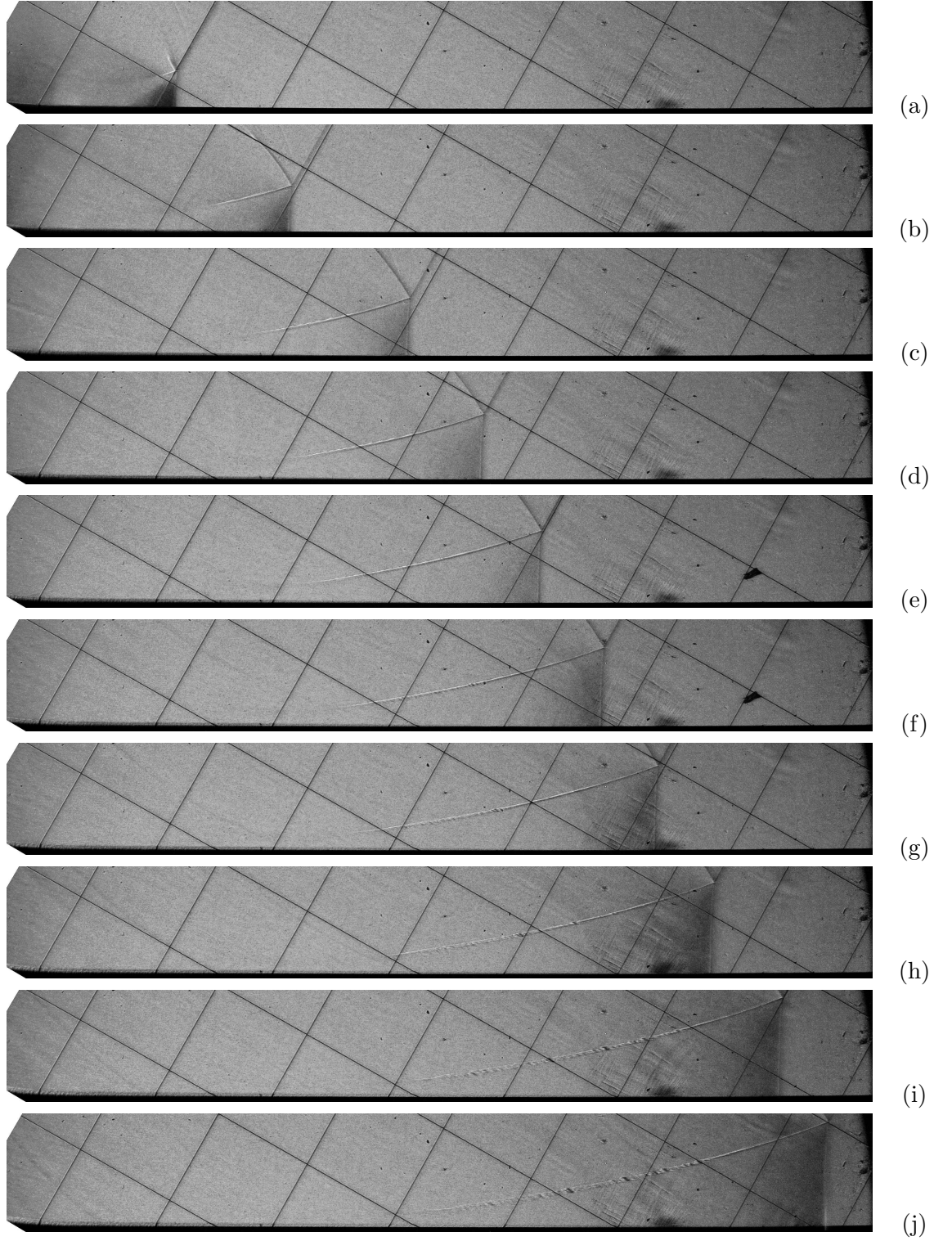
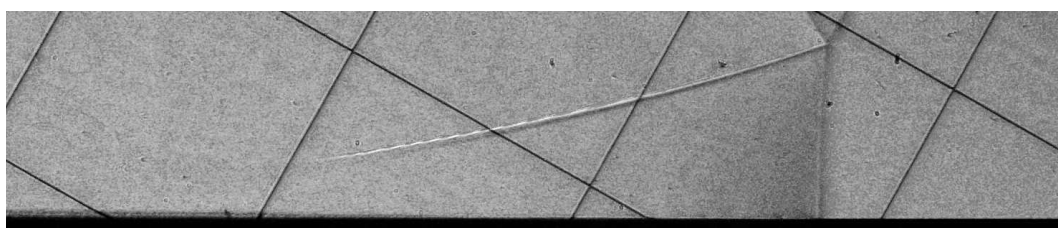


Figure 8.16: KHI evolution for Model C, $M_0 = 1.46$. At times of (a) $1425\mu s$, (b) $1525\mu s$, (c) $1625\mu s$, (d) $1650\mu s$, (e) $1675\mu s$, (f) $1700\mu s$, (g) $1725\mu s$, (h) $1750\mu s$, (i) $1775\mu s$, and (j) $1800\mu s$.



(d)



(g)



(j)

Figure 8.17: Magnified KHI of Figure 8.16 (d), (g) and (j).

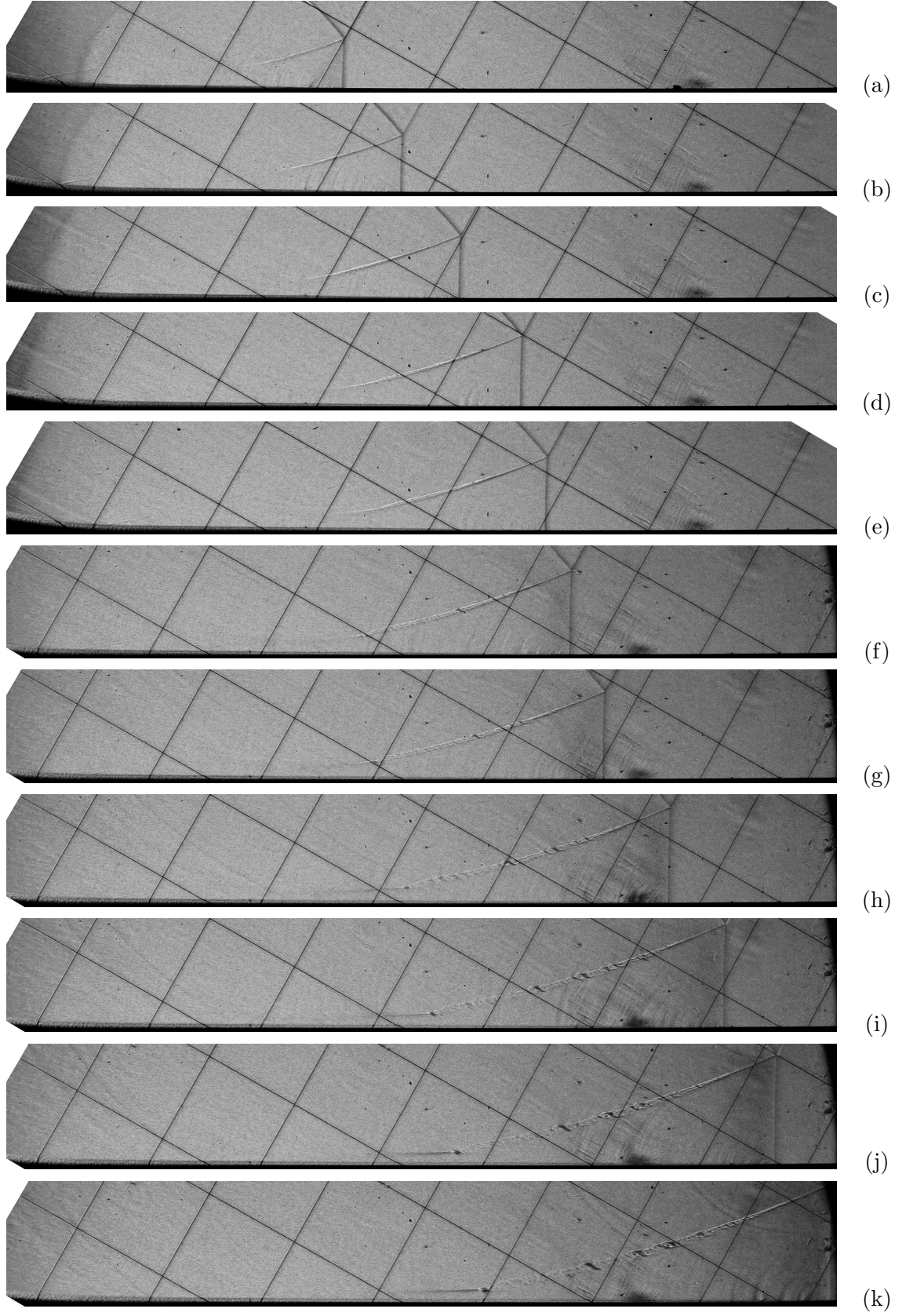
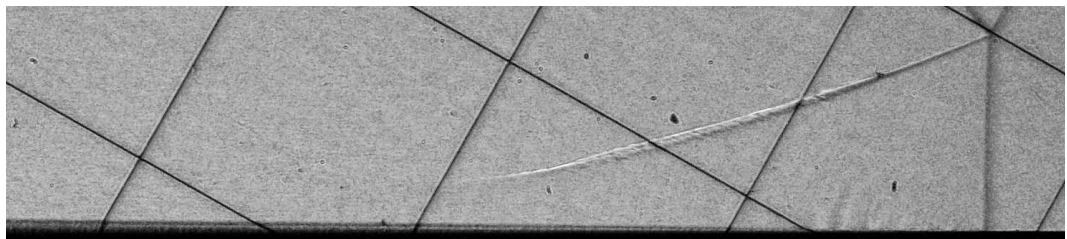


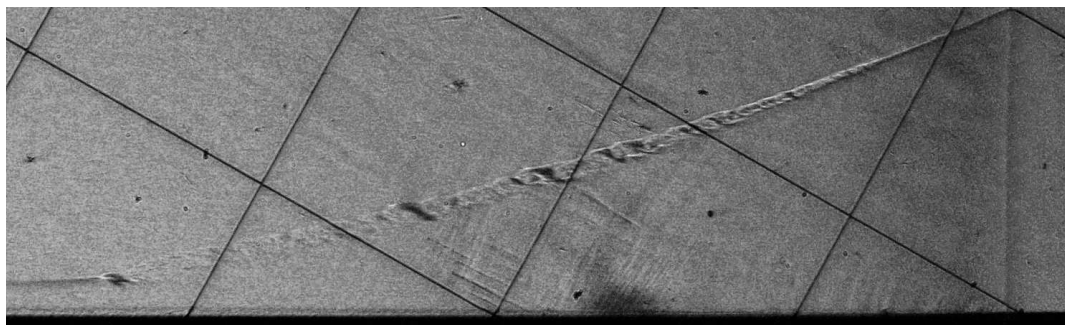
Figure 8.18: KHI evolution for Model C, $M_0 = 1.61$. At times of (a) $1440\mu s$, (b) $1460\mu s$, (c) $1480\mu s$, (d) $1500\mu s$, (e) $1520\mu s$, (f) $1540\mu s$, (g) $1560\mu s$, (h) $1580\mu s$, (i) $1600\mu s$, (j) $1620\mu s$, and (k) $1640\mu s$.



(d)



(g)



(j)

Figure 8.19: Magnified KHI of Figure 8.18 (d), (g) and (j).

8.3.3 Model E

Figures 8.20, 8.22 and 8.24 detail the KHI evolution on Model E with incident shock wave strengths of $M_0 = 1.34$, $M_0 = 1.46$ and $M_0 = 1.61$ respectively.

Note: Figures 8.20 (a); 8.22 (a) and (b); 8.24 (a) and (b) are taken with slightly repositioned optics so as to capture an earlier part of the flow field, not visible with the default setup.

Unlike Models A and C, Model E does not have a parabolic entrance, but is a plain wedge. The progression of Model E is very similar to that of Models A and C (it should be noted that Models C and E both have a 30° wedge angle) with some exceptions:

1. The shear layer is longer and extends to the test piece surface rather than terminating above it. This is due to Model E being a plain wedge allowing for prompt formation of the Mach reflection when the incident shock wave interacts with the test piece allowing the triple point to form close to the surface, leaving behind a shear layer from that point. In the case of a parabolic test piece entry, the Mach reflection forms later thus the shear layer starts away from the surface.
2. The shear layer appears to have a more noticeable curve when compared to any of the parabolic entrance test pieces. This would be because the flow field behind the Mach stem is different to that of a test piece with a parabolic entrance, resulting in a different shear layer shape.
3. The KHI seems to develop from the wedge surface (rather than from within the shear layer as in Models A and C), with very defined and large KHI loops close to the surface, this is most visible in the $M_0 = 1.46$ and $M_0 = 1.61$ cases. These large loops, unlike those that developed from small perturbations along the shear layer as mentioned for the Model A and C cases, are not consistent between images, thus indicating that their formation is random. These loops are in contact with the surface, and thus the boundary layer, the latter most probably the trigger for their formation. A boundary layer is turbulent, which is a random phenomena, and could explain the randomness of the large KHI loops.

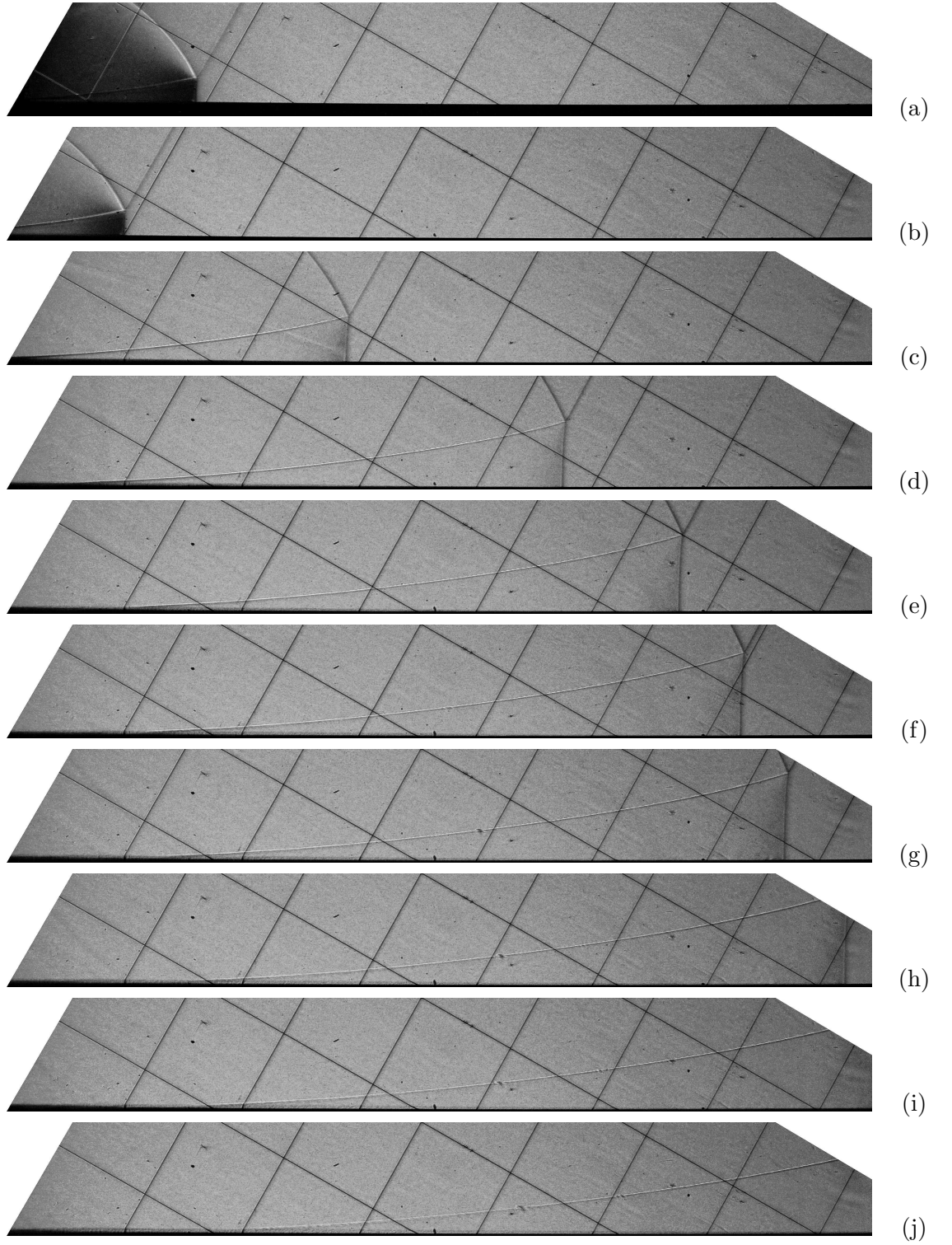


Figure 8.20: KHI evolution for Model E, $M_0 = 1.34$. At times of (a) $1600\mu s$, (b) $1600\mu s$, (c) $1700\mu s$, (d) $1800\mu s$, (e) $1850\mu s$, (f) $1875\mu s$, (g) $1900\mu s$, (h) $1925\mu s$, (i) $1950\mu s$, and (j) $1975\mu s$.

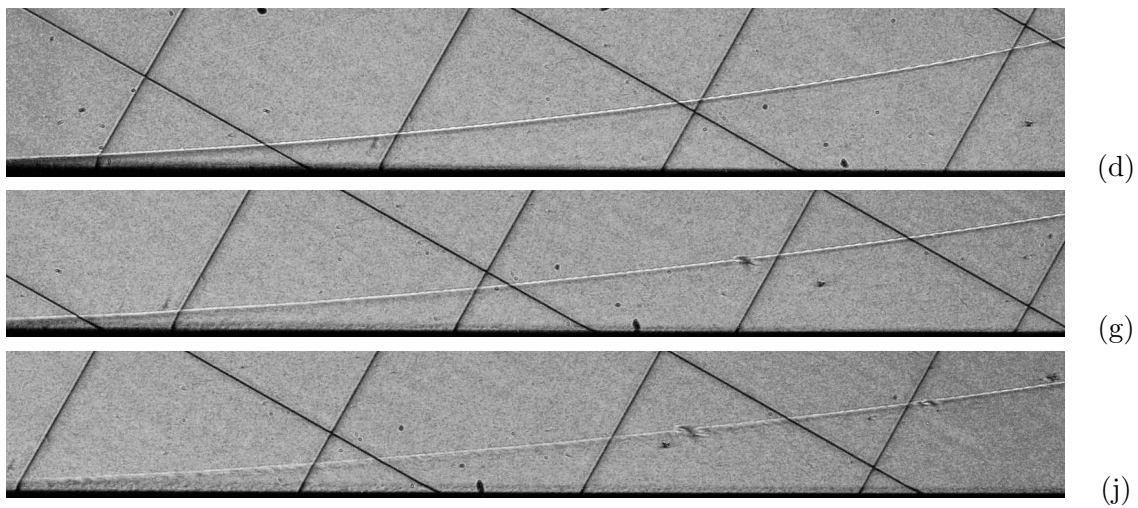


Figure 8.21: Magnified KHI of Figure 8.20 (d), (g) and (j).

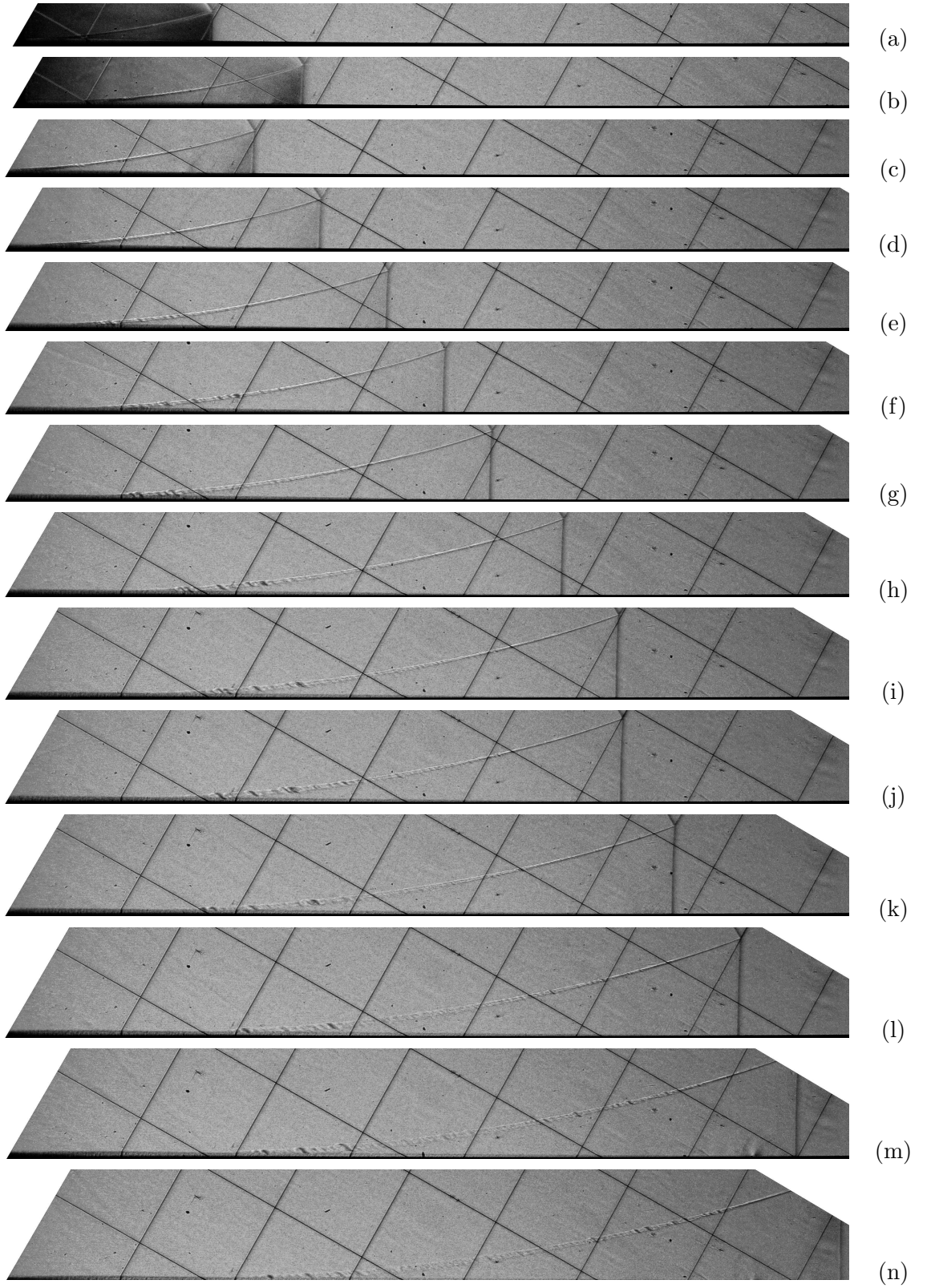


Figure 8.22: KHI evolution for Model E, $M_0 = 1.46$. At times of (a) $1475\mu s$, (b) $1500\mu s$, (c) $1525\mu s$, (d) $1550\mu s$, (e) $1575\mu s$, (f) $1600\mu s$, (g) $1625\mu s$, (h) $1650\mu s$, (j) $1675\mu s$, (k) $1700\mu s$, (l) $1725\mu s$, (m) $1750\mu s$, and (n) $1775\mu s$.

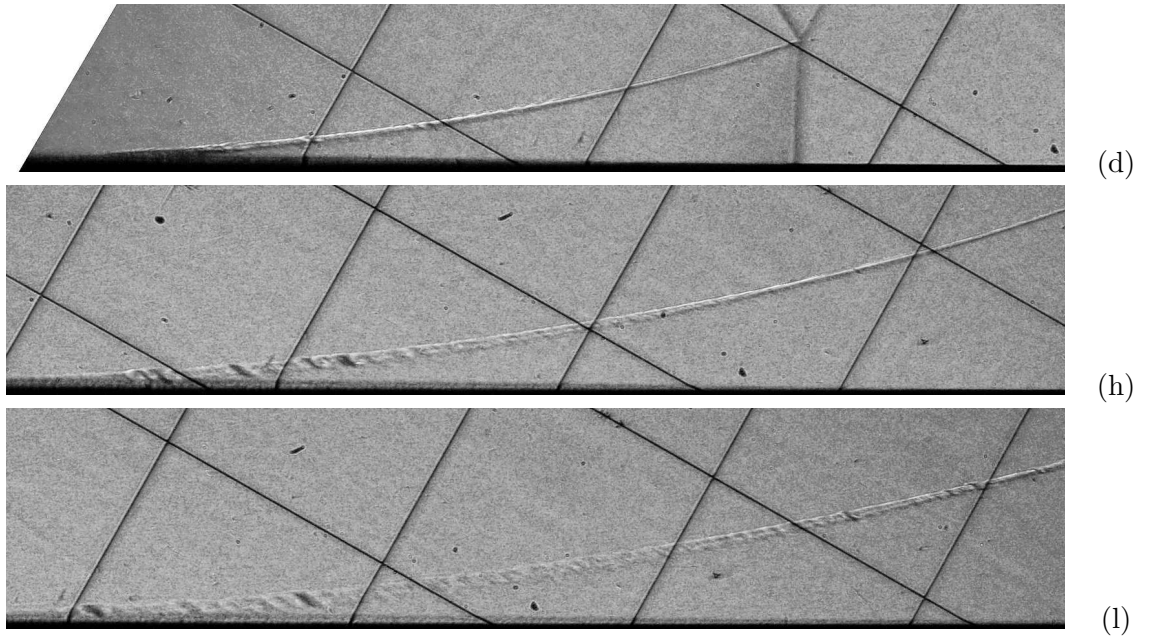


Figure 8.23: Magnified KHI of Figure 8.22 (d), (h) and (l).

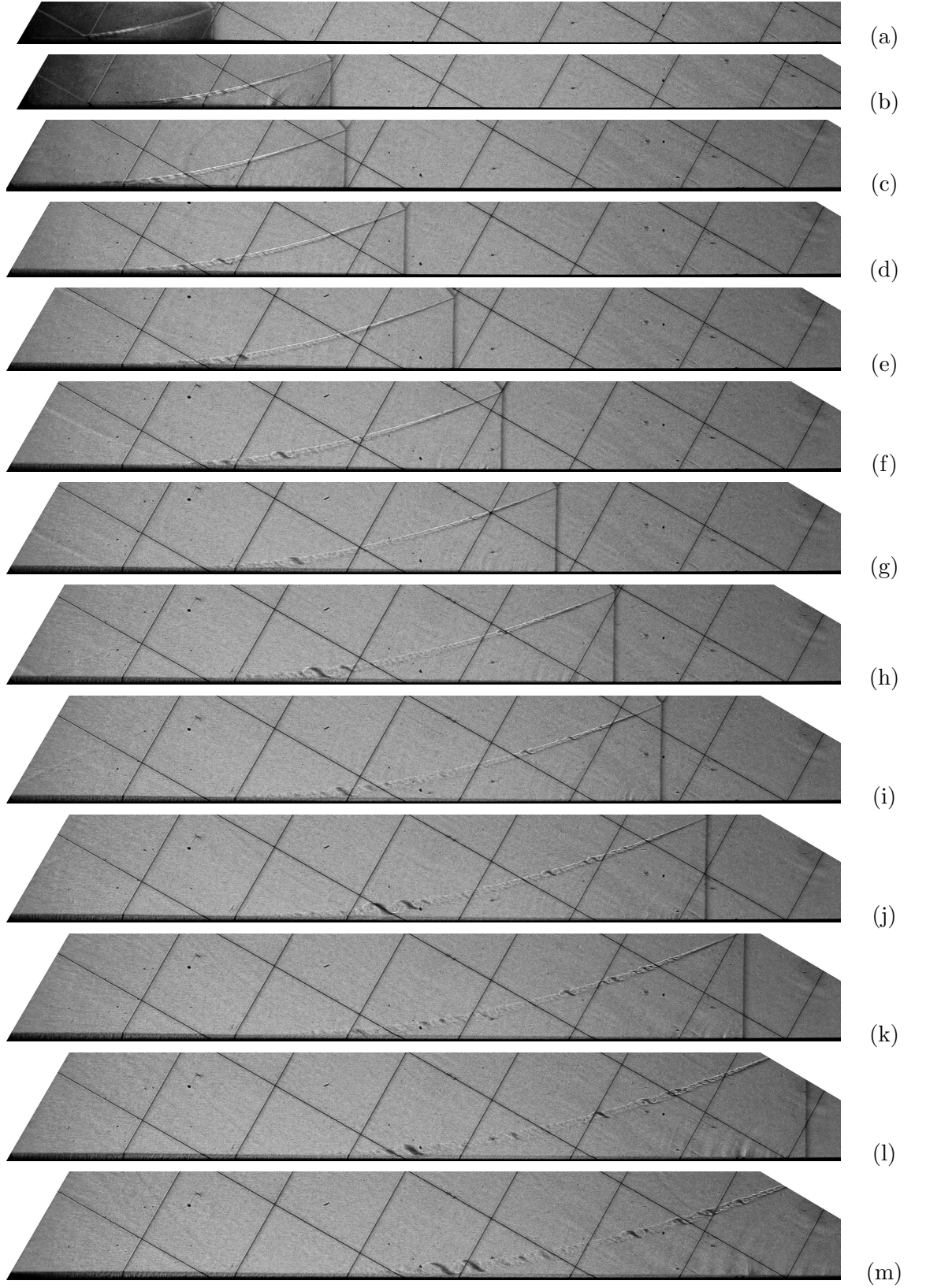


Figure 8.24: KHI evolution for Model E, $M_0 = 1.61$. At times of (a) $1340\mu s$, (b) $1380\mu s$, (c) $1420\mu s$, (d) $1440\mu s$, (e) $1460\mu s$, (f) $1480\mu s$, (g) $1500\mu s$, (h) $1520\mu s$, (i) $1540\mu s$, (j) $1560\mu s$, (k) $1580\mu s$, (l) $1600\mu s$, and (m) $1620\mu s$.

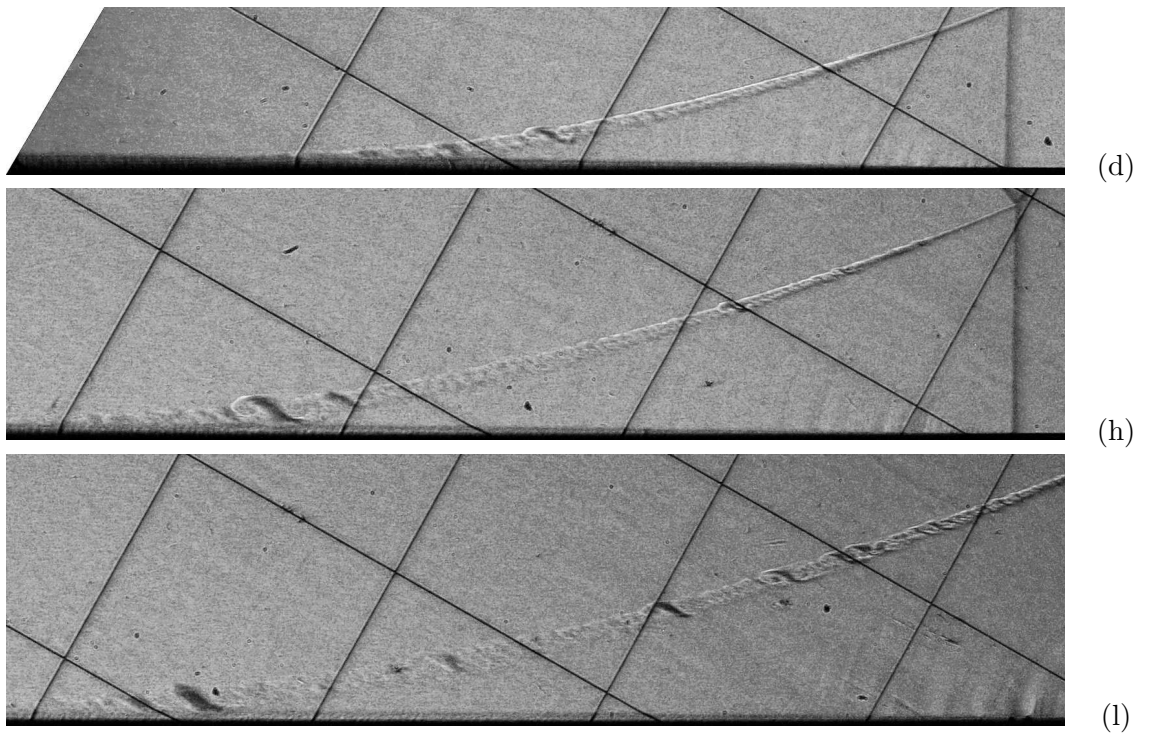


Figure 8.25: Magnified KHI of Figure 8.24 (d), (h) and (l).

8.3.4 Boundary layer

Figure 8.9 shows the presence of KHI like structures on the boundary layer of Model A with an incident shock Mach number of $M_0 = 1.34$. These structures are also present on the boundary layer of the $M_0 = 1.46$ and $M_0 = 1.61$ cases but the position of the structures move closer to the Mach stem the greater the Mach number. This is most probably because the reaction forces within the boundary layer are greater with greater Mach numbers, thus causing quicker initiation of the structures. These structures also break down into turbulence further down the flow field. The aforementioned features suggest that the KHI like structures are actually the KHI.

The presence of the KHI on the boundary layer is not noticeable on Models other than that of A, for the schlieren optical setup. The testing of Models C, D, E and F was done in a different session to that of Models A and B, thus a slightly different optical setup could be the cause for this.

8.4 Processed images

To extract dimensional data from the experimental images, DigXY (a computer program details of which can be seen in APPENDIX B) was used to extract the co-ordinates of 18 points from each image. A MATLAB code was written to read in these co-ordinates and output the required graphs. Many of the points were taken on very small regions of the image, thus it was difficult to get a completely accurate result.

The points of each extracted co-ordinate is listed below:

Position point This is a specific intersection of the wire grid who's position within the test section is known.

Scaling point This is a specific intersection of the wire grid, used for scale as the dimensions of the grid were known.

Incident shock The point where the incident shock wave intersects a specific horizontal wire, used to calculate its position within the test section.

Triple point The point where the Mach stem, incident and reflected shock waves meet.

Mach stem and wedge surface contact point This was used to calculate the length of the Mach stem

End of shear layer The point after which the shear layer can no longer be identified. It was used to calculate the length of the shear layer.

KHI start The point where the KHI starts (close to the triple point).

KHI end The point where the KHI ends (close to the end of the shear layer).

KHI edge points Five pairs of points (above and below the shear layer) were taken at equal intervals along the entire length of the KHI. These were used to calculate the average width of the KHI.

Basic geometry was used to calculate the distances. In the case of calculating the length of the curved shear layer: the centre point of the “KHI edge points” pairs were calculated. A cubic spline was then created through these points with the “Triple point” and “End of shear layer” points as the start and end points. The length of the spline was then regarded as the length of the shear layer.

The images were magnified for a more accurate extraction of the points, but due to the lack of definition of the flow features at this magnification the accuracy to which a point could be extracted was approximately 0.3mm. The oscillating nature of the instability on the shear layer results in an accuracy of the “KHI edge points” of 0.6mm, thus the width of a single section of the shear could only be calculated to a accuracy of 1.2mm. The resulting accuracy of the average width of the KHI, taken over 5 points, is 0.54mm. The error bounds of the following graphs sometimes overlap another set of results, but clear progressions can be indentified nevertheless.

Note: The graphs below are for illustrative purposes, a full set of the graphs are shown in APPENDIX D.

8.4.1 Average KHI width

Figure 8.26 indicates that the average width of the KHI increases almost linearly as the flow field progresses, this is due to the instability having more time to develop and become wider. The KHI is also wider the greater the Mach number, this being due to the greater relative velocity across the shear layer (afforded by a stronger incident shock wave as is shown in Figure 4.3) causing greater mixing.

The increase of the KHI thickness from $M_0 = 1.34$ to $M_0 = 1.46$ is noticeably greater than the increase from $M_0 = 1.46$ to $M_0 = 1.61$ even though the relative increase in Mach number is smaller. This would be expected as results usually become less sensitive to Mach number as it increases.

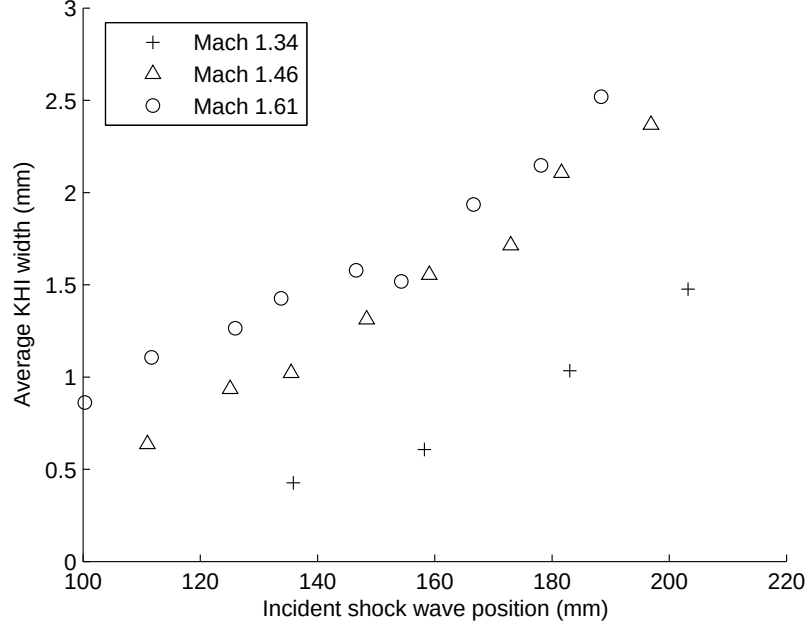


Figure 8.26: Average KHI width vs incident shock wave position on Model A.

8.4.2 KHI length

Figure 8.27 shows that the length of the KHI increases linearly as the flow field progresses, again due to the instability having more time to develop. As with the average width, the length of the KHI also increases with the increase in incident shock wave strength.

Again the increase of the KHI thickness from $M_0 = 1.34$ to $M_0 = 1.46$ is noticeably greater than the increase from $M_0 = 1.46$ to $M_0 = 1.61$ even though the relative increase in Mach number is smaller.

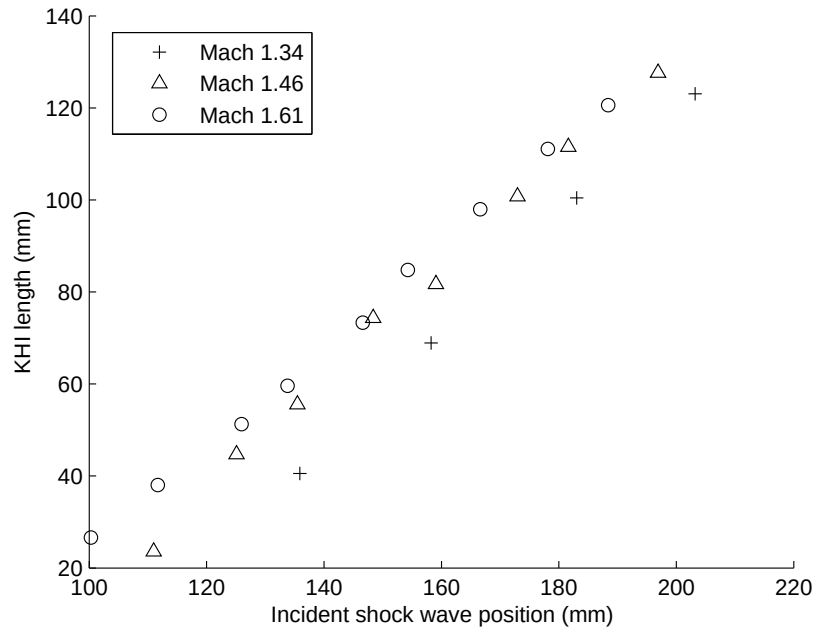


Figure 8.27: KHI length vs incident shock wave position on Model A.

8.4.3 Average KHI width relative to length

The width of the KHI relative to its length initially decreases then flattens out and becomes relatively constant as is shown in Figure 8.28. This indicates that the initial growth of the instability is not self similar but becomes more so as the flow progresses.

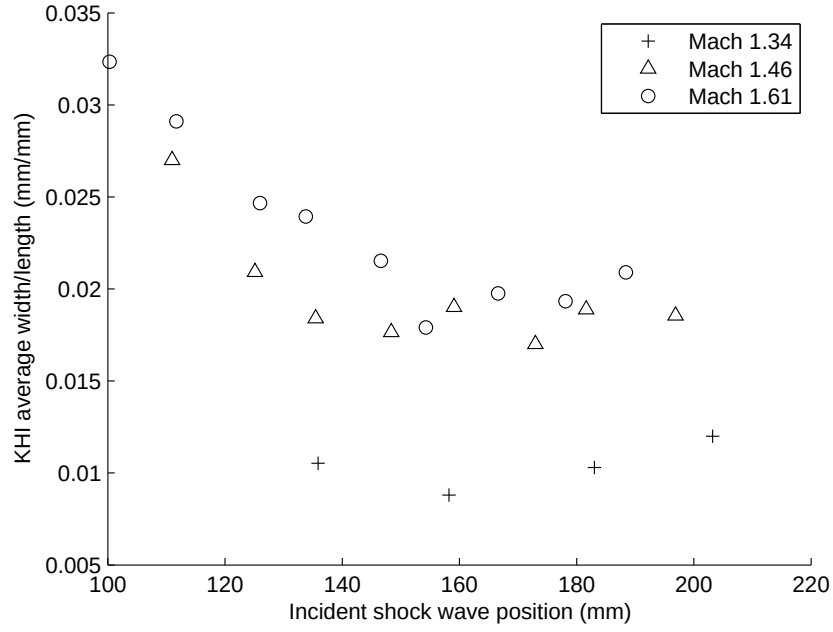


Figure 8.28: KHI average width/length vs incident shock wave position on Model A.

8.4.4 Average KHI width relative to Mach stem length

The average width of the KHI relative to the length of the Mach stem shows the growth of the KHI with respect to the growth of the Mach reflection as a whole. A positive gradient of the plot represents the average KHI width developing faster than the Mach stem and visa versa, with a 0 gradient representing equal development. Figures 8.29, 8.30, 8.31 and 8.32 show how test piece geometry affects this:

Figure 8.29 shows results for the steepest angle tested (38°) and it can be seen that, for all incident shock Mach numbers, the average KHI width over Mach stem length ratio increases, indicating that the average width of the KHI grows faster than the length of the Mach stem. The gradient grows at a similar ratio for the $M_0 = 1.34$ and $M_0 = 1.46$ cases, but the ratio grows at a lesser rate for the $M_0 = 1.61$ case.

Figure 8.30 shows the ratio development for the gentlest angle tested (30°). It can be seen that both the $M_0 = 1.46$ and $M_0 = 1.61$ cases are relatively flat indicating no relative growth of the average KHI width to Mach stem ratio.

The results for median test piece angle of 34° is shown in Figure 8.31. This shows an interesting result as the $M_0 = 1.61$ is relatively flat whereas the $M_0 = 1.46$ is increasing. It has been previously mentioned in this dissertation (Sections 8.3.1

and 8.3.2) and by Ben-Dor [1] that an increase in Mach number and decrease in wedge angle results in an increase in the length of the Mach stem. The flattening on the above plots with increased Mach number and decreased wedge angle is due to this phenomena as the dependent variable contains the Mach stem length in its denominator.

Figure 8.32 also indicates ratio of the average KHI width to the Mach stem length, but this being for Model E which is a plain wedge. The Mach stem grows faster than the relative KHI development thus the plot has a negative gradient for the $M_0 = 1.46$ and $M_0 = 1.61$ cases.

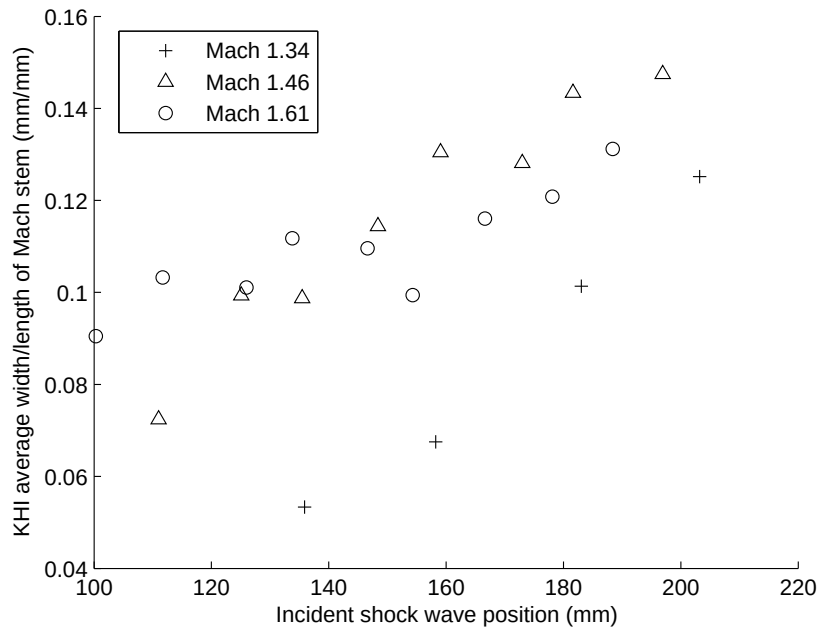


Figure 8.29: KHI average width/Mach stem length vs incident shock wave position on Model A.

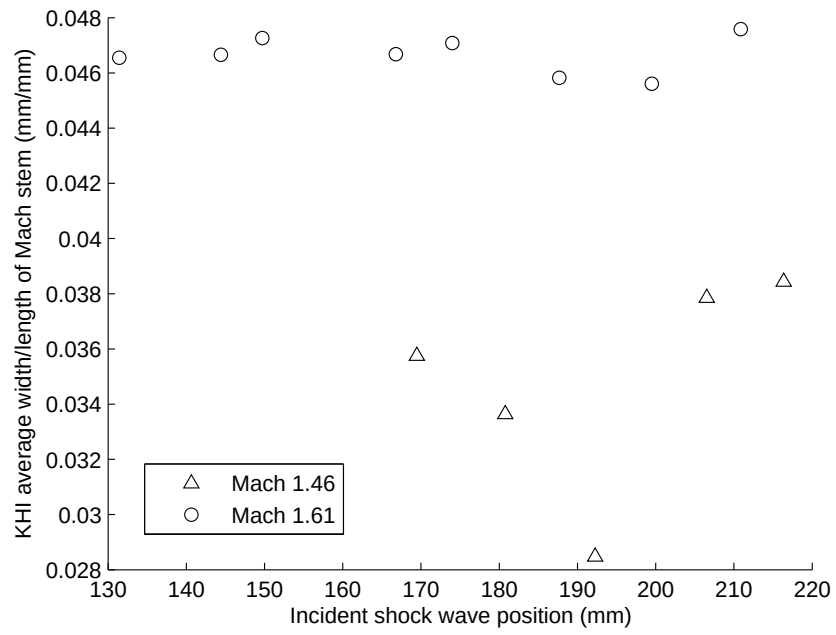


Figure 8.30: KHI average width/Mach stem length vs incident shock wave position on Model C.

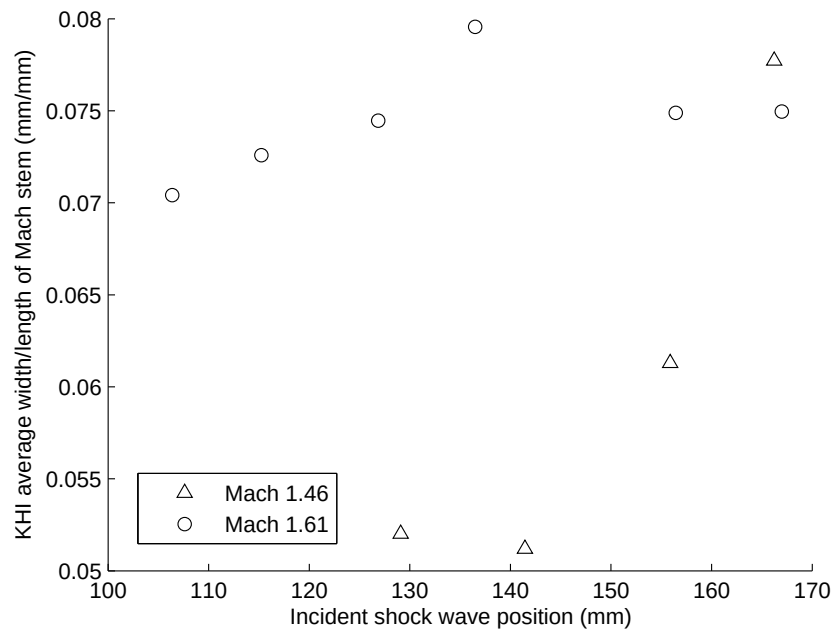


Figure 8.31: KHI average width/Mach stem length vs incident shock wave position on Model D.

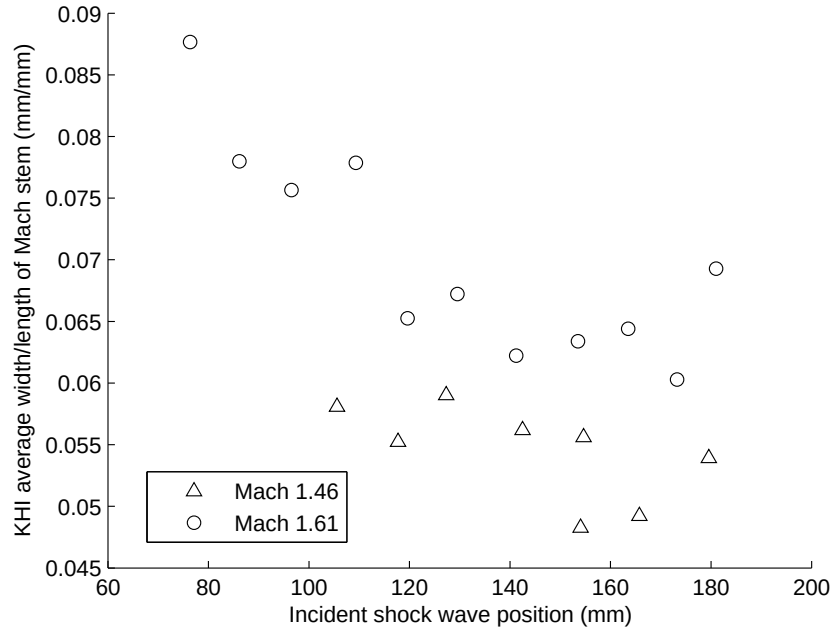


Figure 8.32: KHI average width/Mach stem length vs incident shock wave position on Model E.

8.4.5 KHI length relative to Mach stem length

Figure 8.33 indicates that the ratio of the length of the KHI to the Mach stem length grows steeply and then flattens out. As the growth of the KHI length is linear (Figure 8.27) this shows that the growth of the Mach reflection is not self similar, but this would be expected because of the parabolic entrance of Model A introduces a length scale. The ratio is greater, on average, the weaker the incident shock wave; as a weaker incident shock wave results in a shorter Mach stem and a longer shear layer as previously mentioned in Section 8.3.1.

It is interesting to note that the first point of each incident shock strength case is below the point of corresponding distance of the higher shock strength case. This is because of the steep initial growth of the KHI length, and the fact that it develops earlier the higher the incident shock strength as is shown in Figure 8.6.

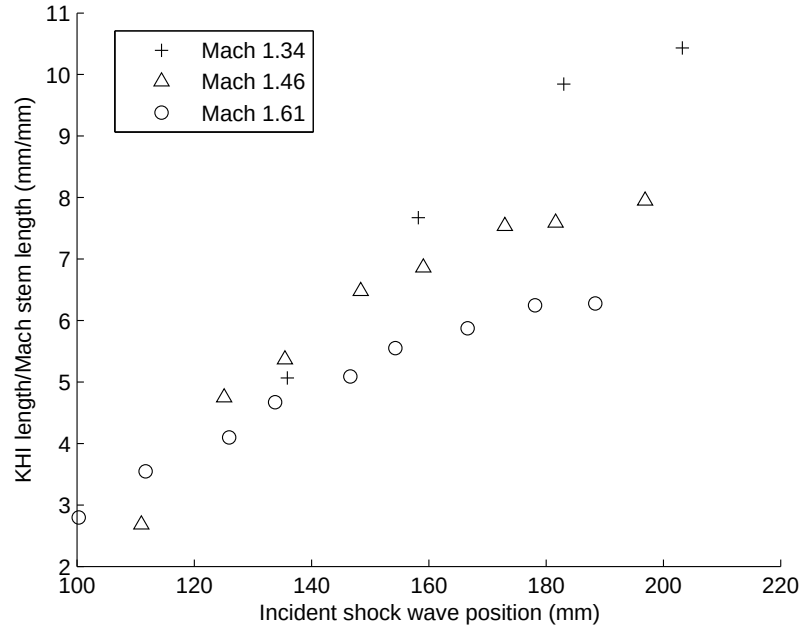


Figure 8.33: KHI length/Mach stem length vs incident shock wave position on Model A.

8.4.6 Shear layer length before KHI

Figure 8.34 shows the length of the shear layer before the formation of the KHI (i.e. the distance from the triple point to the start of the KHI). It is shown that the length of this portion of the shear layer remains reasonably constant, and in the case of $M_0 = 1.46$ and $M_0 = 1.61$ there is even a slight decrease as the flow field progresses. This indicates that the length of the KHI increases more rapidly than the size of the Mach reflection, this is supported by Figure 8.33.

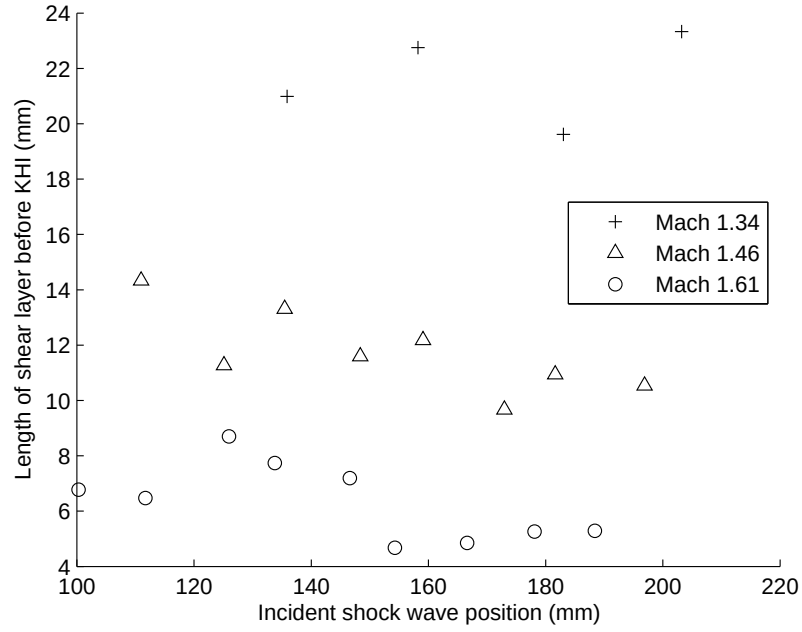


Figure 8.34: Shear layer length before KHI initiation vs incident shock wave position on Model A.

8.5 Test piece scaling

The results from Models A and B were compared to each other as Model B is twice the scale of A. The length and average width of the KHI vs the incident shock wave position for both test pieces were plotted, the values for Model B were halved so that they could be easily compared with those from Model A. These results are shown in Figures 8.35 and 8.36 respectively.

Studying Figure 8.35 reveals that the loci of points of Model A closely match that of Model B, more so the higher the Mach number. Figure 8.36 also reveals similar loci comparing the average KHI widths on Models A and B, although the average width is notably less on Model B for the highest Mach number of $M_0 = 1.61$ with the closest match being that of the $M_0 = 1.61$ case. Thus these figures indicate a scalable growth of the length of the instability and an almost scalable growth of the width, that deteriorates with higher Mach number. To be considered completely scalable, the number of KHI loops for each case would have to be the same, but this is not the case as the number of loops increase with increasing instability size. Thus the scalability of the KHI mentioned above is in overall dimension only.

Figures 8.37 (a) and (b) are a scaled comparison between Models A and B respectively, at the same scaled position along the test piece. It can be seen that the

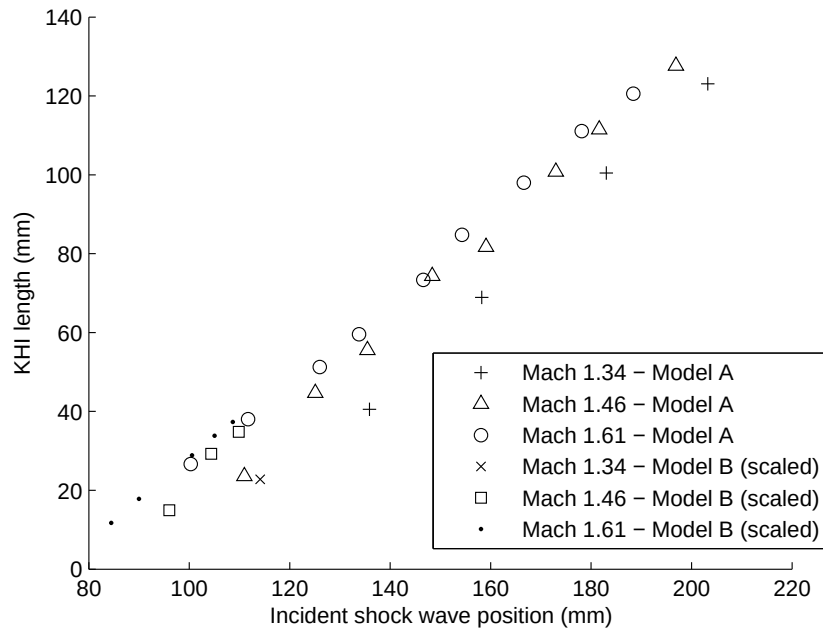


Figure 8.35: Comparison of KHI lengths vs incident shock wave position for Models A and B. Note: The results for Model B are at half scale.

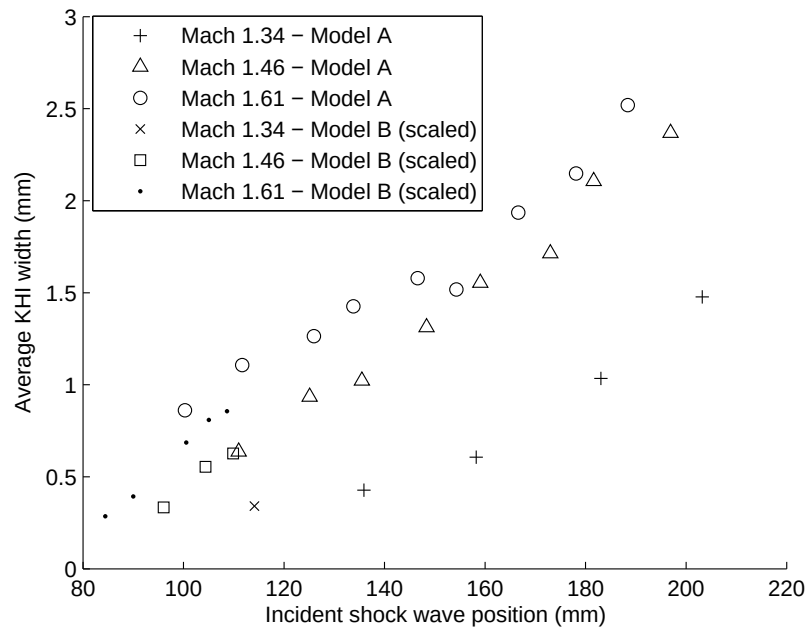


Figure 8.36: Comparison of average KHI widths vs incident shock wave position for Models A and B. Note: The results for Model B are at half scale.

clarity of the KHI on Model A is lower than that on Model B. It must be noted that the size of the entire Mach reflection on Model A is half that of B, thus the loss of clarity is most probably due to three dimensional effects, across the width of the test section, being relatively larger compared to the size of the KHI, thus blurring it out.

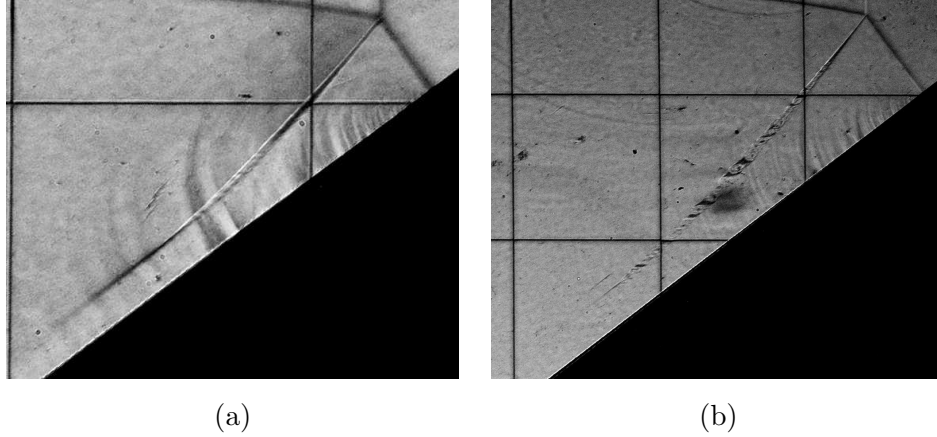


Figure 8.37: Comparison between (a) Model A and (b) Model B, with a parabolic entrance of twice the scale, with the incident shock wave at the same scaled position, with half the magnification, as (a). ($M_0 = 1.61$)

8.6 Plain vs parabolic entrance

The effect of having a parabolic entrance as opposed to a plain one can be established by comparing the results produced by Models C and E as they both are of similar length and have a ramp angle of 30° , the major difference being that Model C has a parabolic entrance and Model E is a plain wedge (see Table 5.1).

There are two cases of comparison:

1. Direct comparison between like incident shock wave strengths and horizontal distance were performed as this allowed for comparison of the flow fields being in contact with the test piece for the same horizontal distance. Model C is $2.5mm$ longer than Model E, but because of the slight variability of the incident shock wave Mach numbers this distance has little effect on the result.
2. Comparison between like incident shock strengths and the horizontal distance that the incident shock wave/Mach stem has been in contact with the plain section of each wedge (i.e. not including the length of the parabolic section of Model C).

Graphs were plotted to compare the average width and length of the KHI verses the horizontal incident shock wave position, these are shown in Figures 8.36 and 8.35 respectively. Both the direct (including parabolic portion) and plain wall (excluding parabolic portion) cases, as mentioned above, were plotted for Model C so they could be compared to the single set of results of Model E.

From Figure 8.38 a direct comparison (including the parabolic section of Model C) of the average KHI width reveals that it is notably lower on Model C than it is on Model E. This is because Model E allows for the prompt formation of a Mach reflection as the incident shock wave interacts with the test piece, thus promptly forming a shear layer and thus the KHI. This early formation of the KHI allows it more time to develop. The average KHI width on Model C excluding the parabolic section is much closer to that of Model E but is still slightly less. This is most probably due to the KHI being most developed near its centre but constricting on either end, whereas the KHI on a plain wedge (Model E) is most developed closest to the wedge surface and constricting at its opposite end. As the average width of the KHI is calculated from points along the length of the instability its double constriction on Model C reduces the average width when compared to the single constriction of Model E.

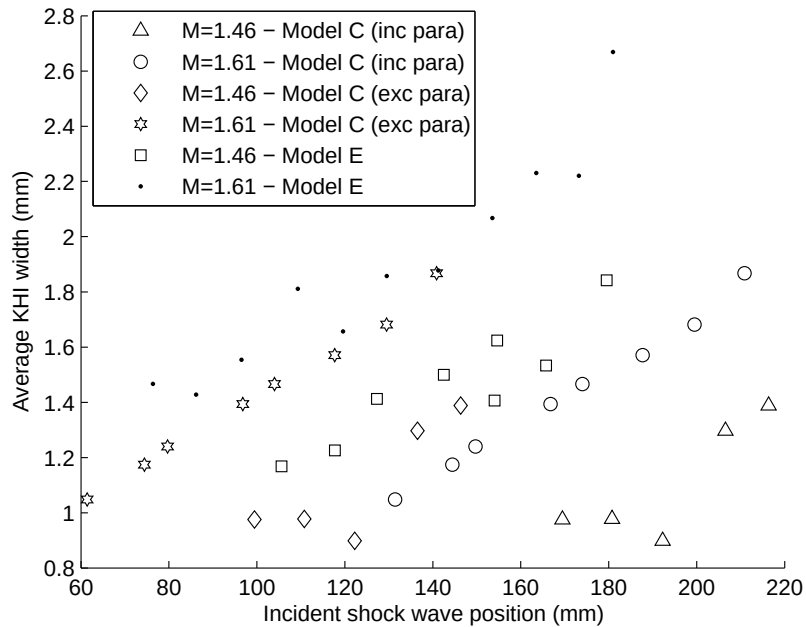


Figure 8.38: Comparison of average KHI width vs incident shock wave position for Models C and E.

Figure 8.39 reveals that the comparison of the KHI lengths is very similar to the widths, as mentioned above. The progressions of the plain wedge sections (excluding

the parabolic section on Model C) are relatively closer than those of the, above mentioned, average KHI width case; the locus of the $M_0 = 1.61$ being particularly close. These plots are parallel, the difference between them is due to the difference in the early progression of the flow field over the two test pieces.

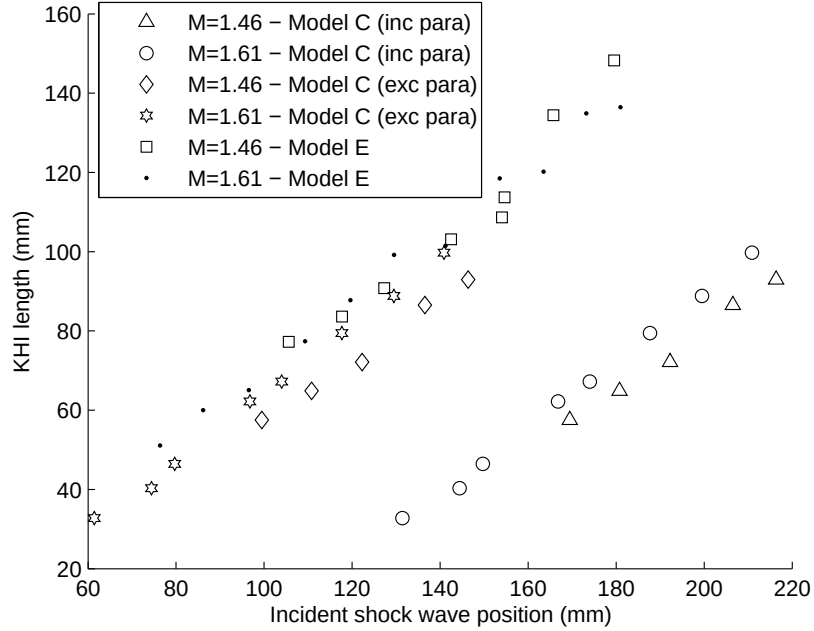


Figure 8.39: Comparison of KHI length vs incident shock wave position for Models C and E.

8.7 Comparison with Rikanati et al.

Rikanati et al. [5] plotted the effect of Mach number and wedge angle on the spread angle of the Mach reflection shear layer, their plot can be seen in Figure 3.3.

A direct comparison to Rikanati's plot (Figure 3.3) was done with experimental data from Models E as this was the only data that could be compared directly. Figure 8.40 shows this comparison. Rikanati et al. did most of their experimentation with $M_0 \geq 1.55$ thus only results in the $M_0 = 1.55-1.61$ range could be compared. Interpolation reveals that the results achieved in this dissertation are approximately half of those achieved by Rikanati et al. This could be due to the holographic interferometry system used by Rikanati not accurately identifying the shear layer boundary as it might not have landed exactly on one of the system's fringes of density. It is also notable that the interferogram shows little evidence of the characteristic KHI vortices, this is probably due to the fringe spacing being too wide.

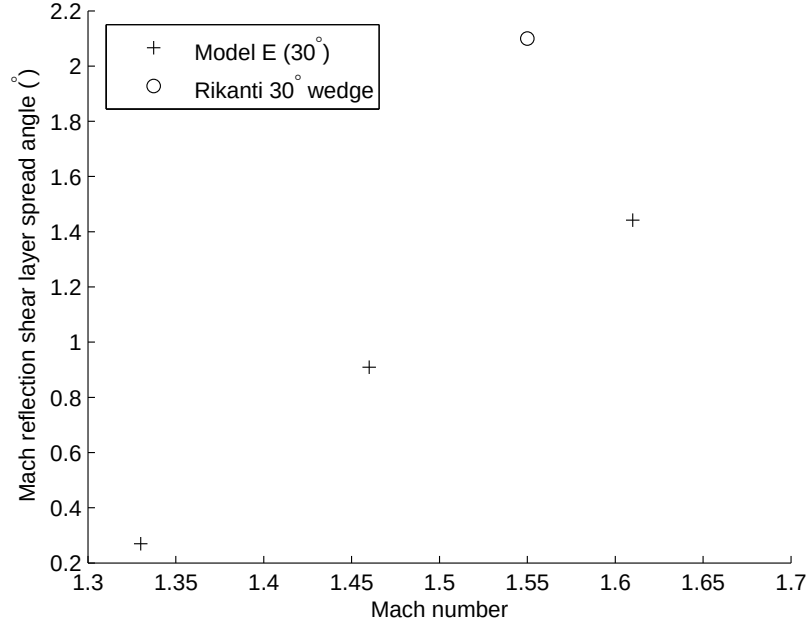


Figure 8.40: Mach reflection shear layer spread angle vs incident shock wave Mach number for a wedge angle of 30° .

8.8 CFD

This section compares the effects of three solvers (inviscid, laminar and $k-\epsilon$) on the formation of the KHI, and the numerical results are compared to their experimental counterparts. Contours of constant density are plotted as this mimics the visualisation of the schlieren optical system most closely.

The setup details of the following numerical simulations are available in Section 7.

8.8.1 Solver comparison

Figures 8.41, 8.42 and 8.43 are plots of Euler, Navier-Stokes and $k-\epsilon$ solvers respectively, of Model A at the same time step with the same boundary conditions and meshes. It can be seen that the results produced by the Euler (Figure 8.41) and Navier-Stokes (Figure 8.42) solvers are almost identical. The general Mach reflection geometry is clearly visible. The shear gradient, caused by the incident shock wave interacting with the parabolic section of the wedge (mentioned previously in Section 8.1), can be seen at the bottom end of the shear layer. The presence of the KHI can clearly be seen along the shear layer. It is interesting to note that the KHI has perturbations along it even though this is a numerical solution. This could

be due to numerical noise, or could indicate that these perturbations are flow field specific, which is more likely because these perturbations were noted in the experimental cases and discussed in Section 8.6. The $k-\epsilon$ solution, however, dampens out vorticity, thus the KHI does not form. The shear layer of the $k-\epsilon$ solution is much wider than in the Euler and Navier-Stokes solutions.

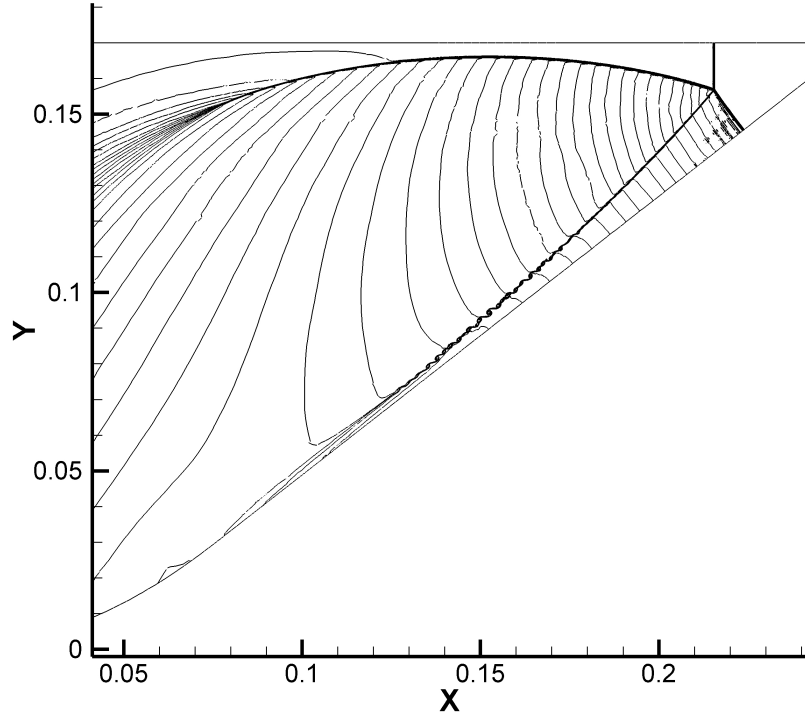


Figure 8.41: Euler numerical simulation of Model A at $M_0 = 1.46$.

8.8.2 Experimental comparison

Figures 8.44, 8.45 and 8.46 compare the numerical and experimental cases for all the tested Mach numbers on Models A, C and F respectively. As in the experimental cases, the images were rotated so that the test section was horizontal and then cropped to just the region of the shear layer.

8.8.2.1 Model A

The $M_0 = 1.34$ (Figure 8.44 (a)) numerical simulation does not show the presence of the KHI, even though it is present in the experimental case. There is, however, a very slight “wobble” of the shear layer at its bottom end which would develop into

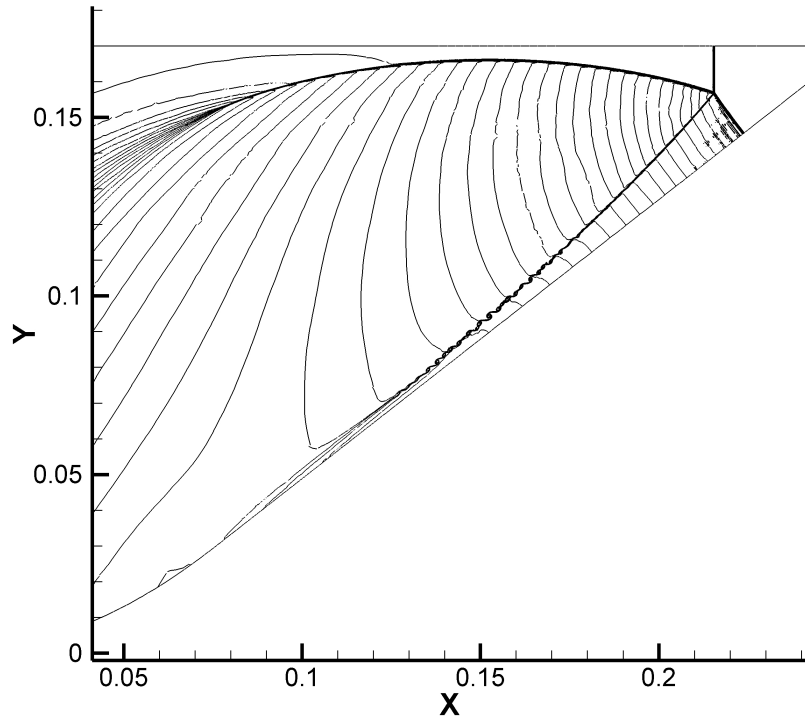


Figure 8.42: Navier-Stokes numerical simulation of Model A at $M_0 = 1.46$.

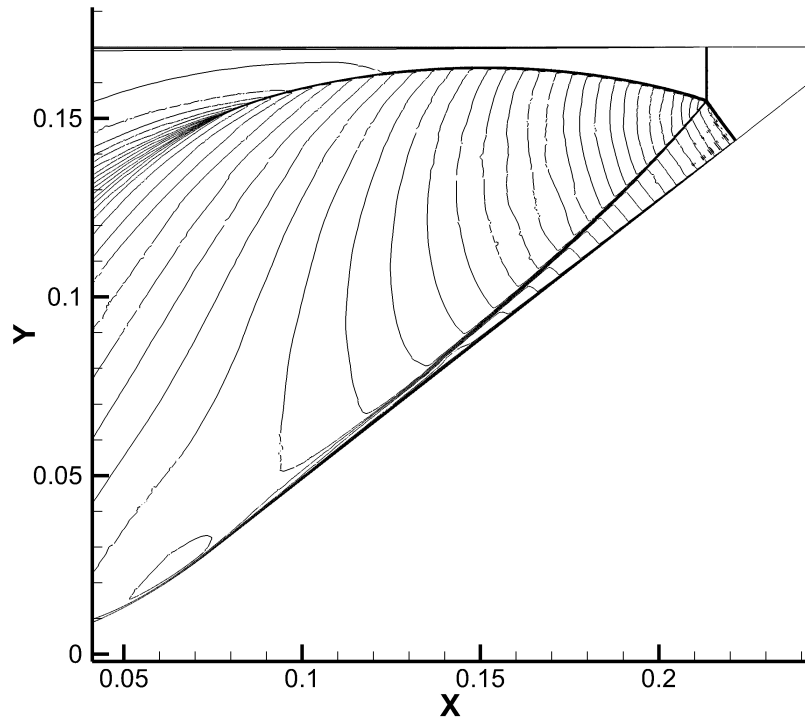


Figure 8.43: K- ϵ numerical simulation of Model A at $M_0 = 1.46$.

the KHI if the flow was allowed to develop.

The $M_0 = 1.46$ (Figure 8.44 (b)) case shows a very visible series of disturbances along the shear layer which could almost be called the KHI. Again, the instability is much less developed than in the experimental case.

The KHI is fully developed in the $M_0 = 1.61$ (Figure 8.44 (c)) numerical simulation, although most of it is obscured by many contours below the shear layer. This is caused by the mesh being very refined in this region due to a large density gradient, thus causing numerical instability. Again, the instability is less developed than in the experimental case.

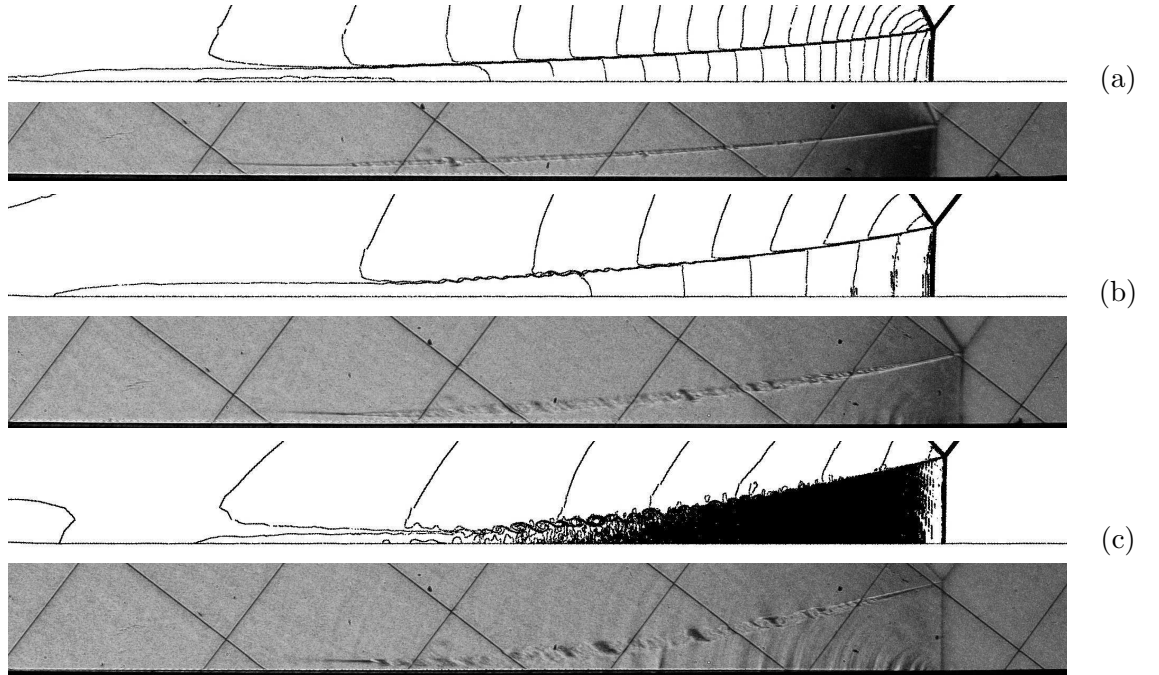


Figure 8.44: Euler numerical simulations of Model A compared to their experimental counterparts: (a) $M_0 = 1.34$ at $1900\mu s$, (b) $M_0 = 1.46$ at $1750\mu s$, and (c) $M_0 = 1.61$ at $1580\mu s$.

8.8.2.2 Model C

There is no presence of the KHI in the numerical simulations of Model C (Figure 8.45), but there is a “wobble” along the bottom end of shear layer of the $M_0 = 1.61$ case. Following the guide of the previous case (Figure 8.44) (where the simulated instability is less developed than its experimental counterpart) this would be expected because the KHI in these experimental cases is not very developed.

Comparing the numerical and experimental images of the $M_0 = 1.61$ case (Figure

8.45 (c)) it can be clearly seen that the shear gradient, behind the shear layer, is the cause for the abrupt termination of the KHI.

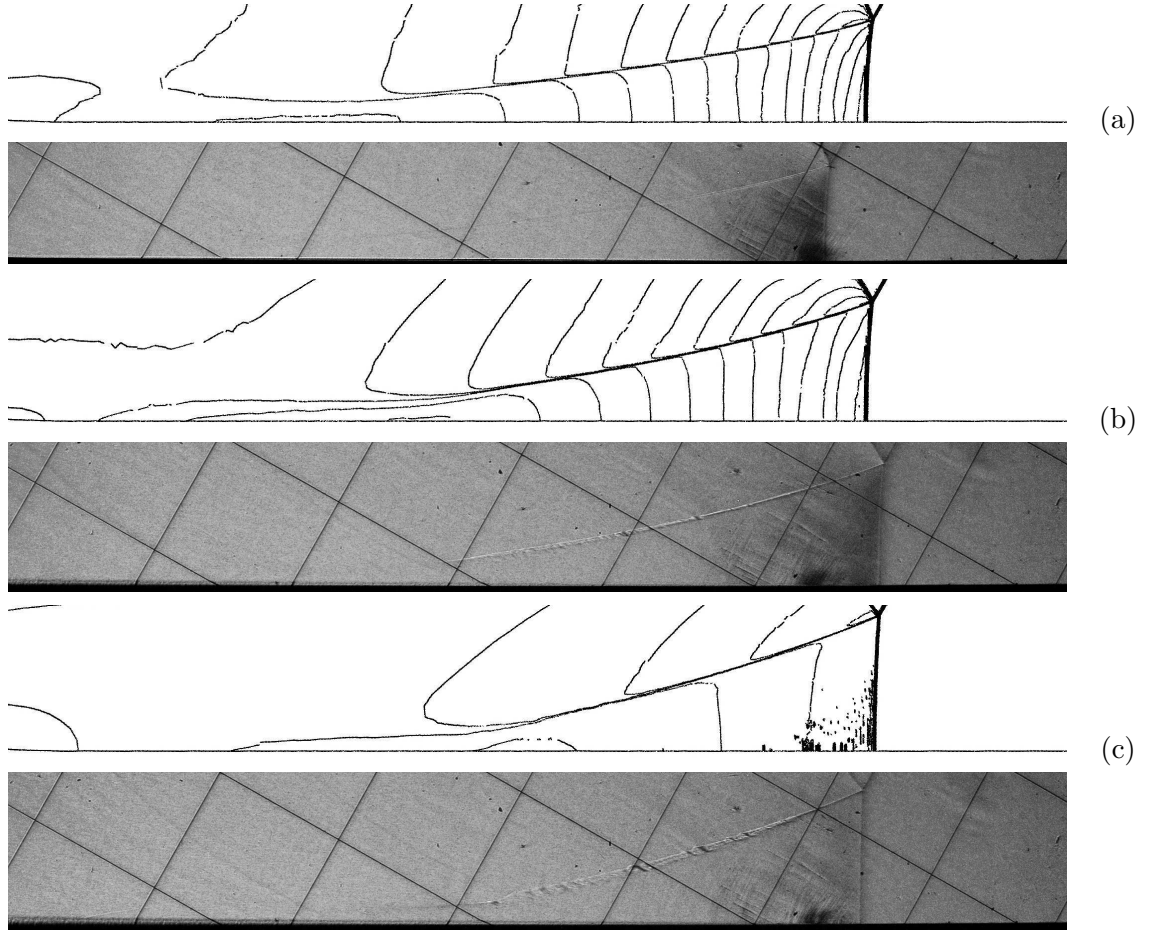


Figure 8.45: Euler numerical simulations of Model C compared to their experimental counterparts: (a) $M_0 = 1.34$ at $1900\mu s$, (b) $M_0 = 1.46$ at $1750\mu s$, and (c) $M_0 = 1.61$ at $1580\mu s$.

8.8.2.3 Model E

Figure 8.46 shows that the numerical simulations of the plain wedge (Model E) case allow for a more accurate development of the KHI but it is still notably less developed than the experimental cases. However, the most developed section of the KHI in the numerical simulation of the $M_0 = 1.61$ case is of similar width to the experimental case, but the length of the numerical instability is shorter than its experimental counterpart, thus the length of stable shear layer between the KHI and the triple point is greater.

The less developed KHI (when compared to the experimental cases) is a common

thread throughout the numerical part of this study. This could be due to the simulations being run on a commercial package where a maximum of a second order solver can be used. If a solver with a greater order were used the KHI would probably match that of the experimental cases more closely.

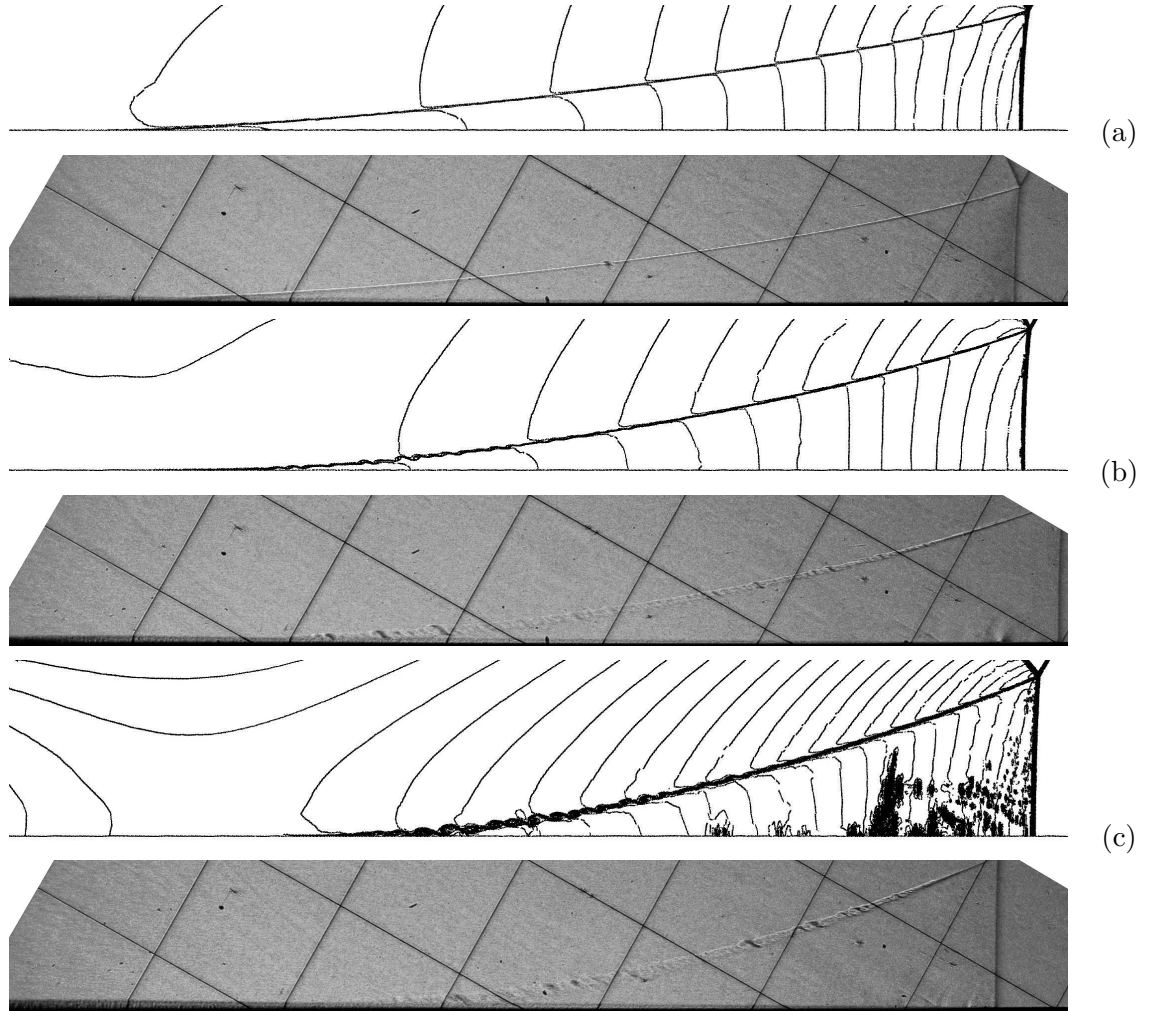


Figure 8.46: Euler numerical simulations of Model E compared to their experimental counterparts: (a) $M_0 = 1.34$ at $1900\mu s$, (b) $M_0 = 1.46$ at $1750\mu s$, and (c) $M_0 = 1.61$ at $1580\mu s$.

9 CONCLUSIONS

The presence of the KHI is apparent on the shear layer of the Mach reflection. It initiates and grows more readily the greater the incident shock wave strength and the greater the test piece wedge angle.

The orientation of the schlieren camera-side knife edge plays a large role in visualising the KHI. The KHI is most visible when the camera-side knife edge is perpendicular to the Mach reflection shear layer and becomes less visible the further the camera-side knife edge is from this orientation, so much so that if the knife edge is orientated parallel to the shear layer the instability is completely obscured.

The evolution of the KHI is almost scalable. Out of the tested incident shock strengths, $M_0 = 1.46$ allowed for the best scaling of the KHI length, and $M_0 = 1.61$ allowed for the best scaling of average width.

The initiation of the Mach reflection plays a role in the formation and evolution of the KHI. A plain wedge allows prompt formation of the KHI causing the instability to grow from the test piece surface and be most developed at this point. A parabolic entrance to the test piece causes the Mach reflection to form later on, with a shear gradient forming before the shear layer, this causing an abrupt termination of the KHI above the test piece surface.

A comparison of the Mach reflection spread angle results to those of Rikanati et al. [5] was done. It revealed that the results obtained in this dissertation were approximately half of those obtained by Rikanati et al, which could be due to inaccuracies in their holographic interferometry system not picking up the exact shear layer boundary.

The Euler and Navier-Stokes solutions are almost identical and show the presence of the KHI. The k - ϵ , however, does not show the presence of the KHI. The numerical simulations are comparable to their experimental counterparts, with the exception of the KHI being less developed. This being due to the limitation of a second order

solver; higher order solvers would most probably create a more accurate representation of the KHI development. Another possibility is that experimental imperfections and perturbations could be triggering the KHI, thus enabling it to become more developed than the numerical solutions.

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APPENDIX A DETAILED TEST PIECE DRAWINGS

Figures A.1, A.2, A.3 and A.4 were taken from Ramdas (2008) [9].

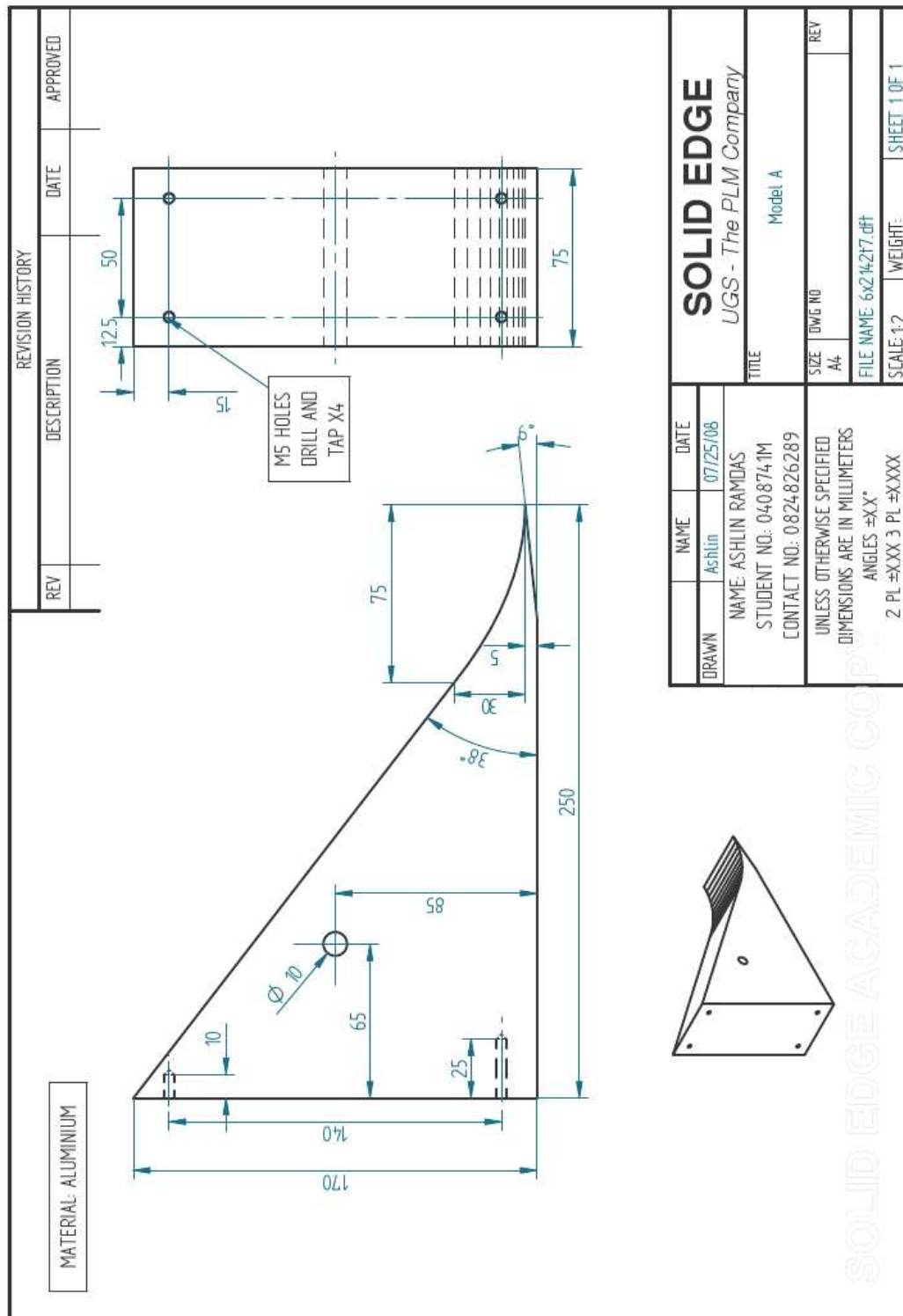


Figure A.1: Model A [9].

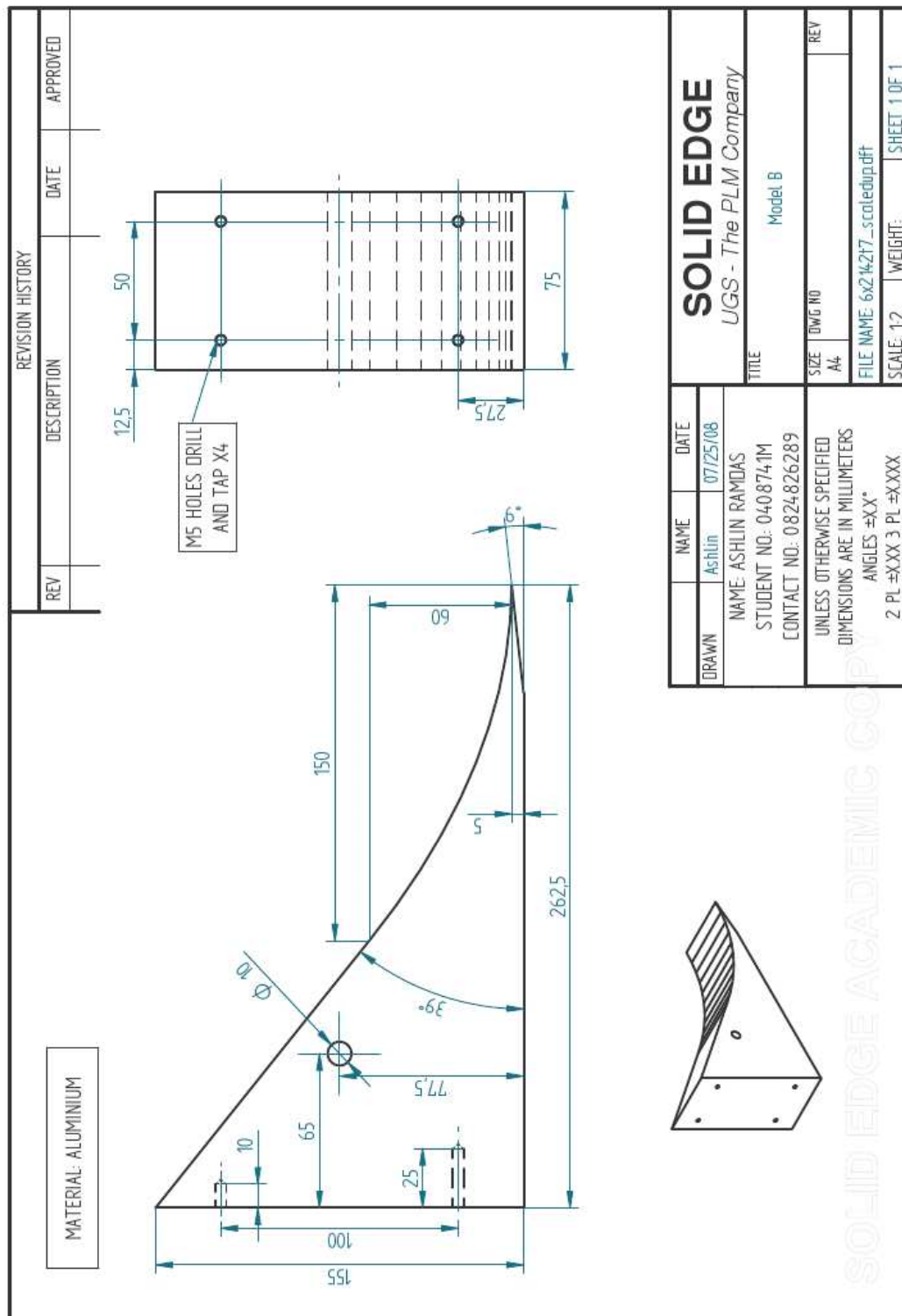


Figure A.2: Model B [9].

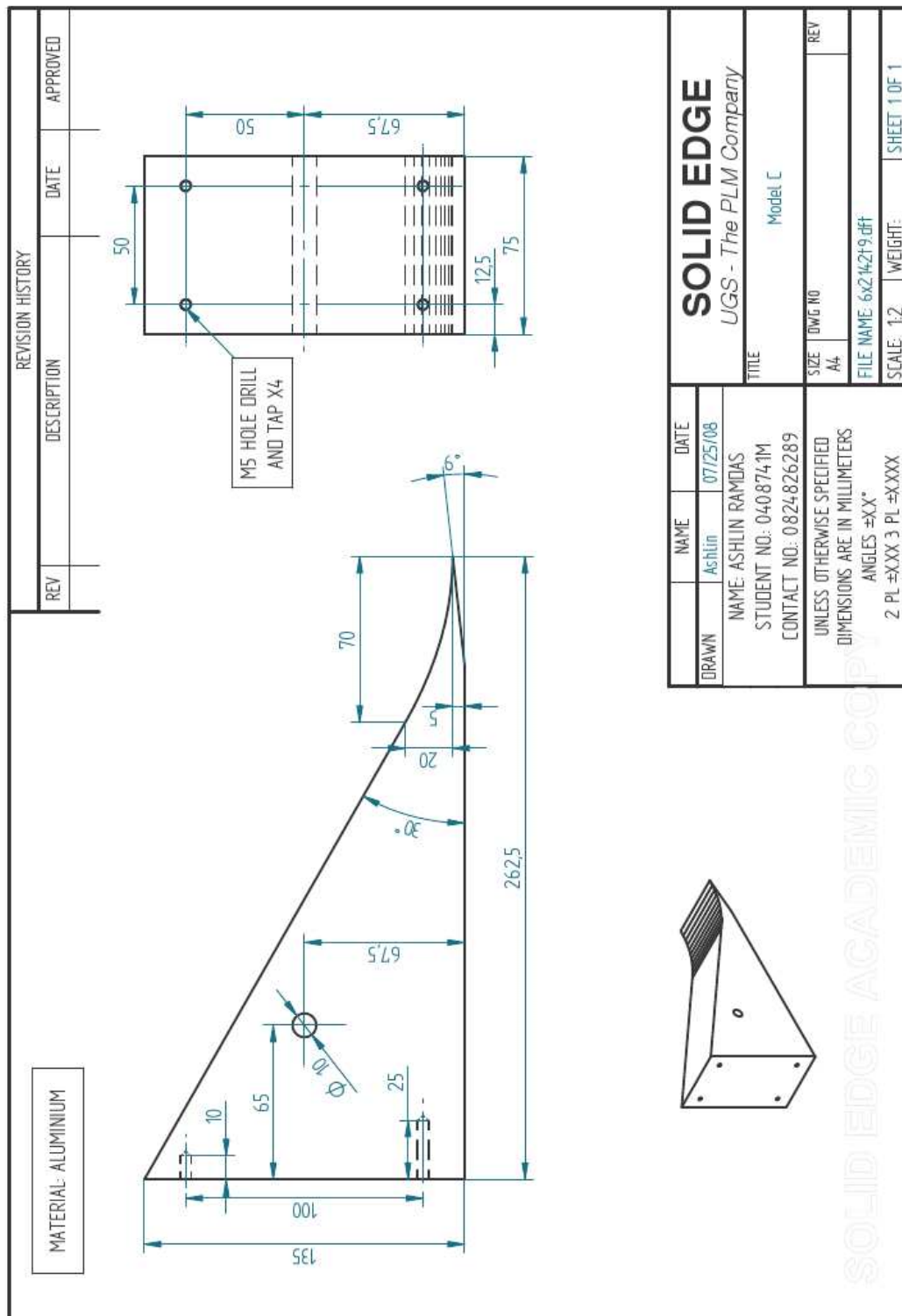


Figure A.3: Model C [9].

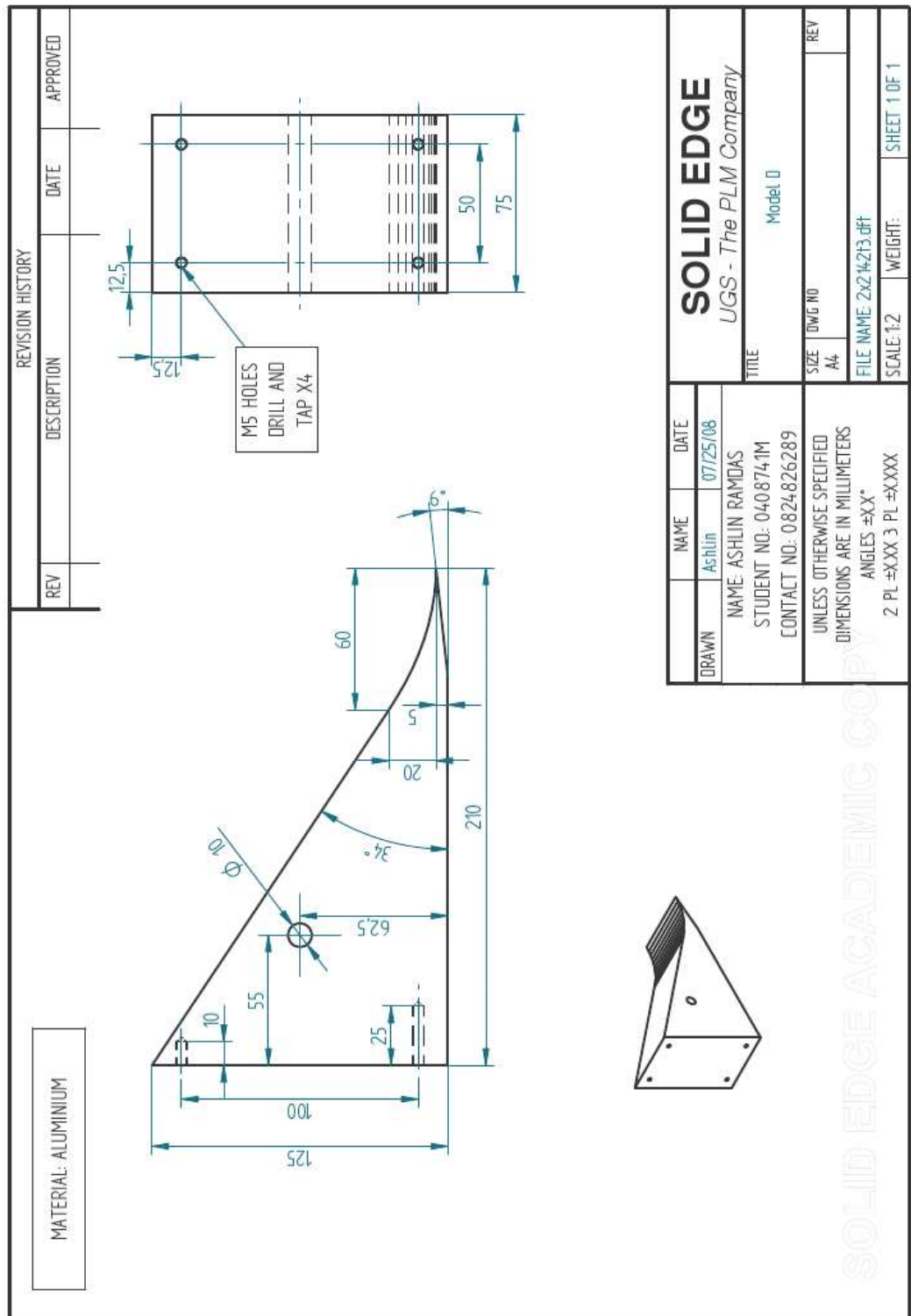


Figure A.4: Model D [9].

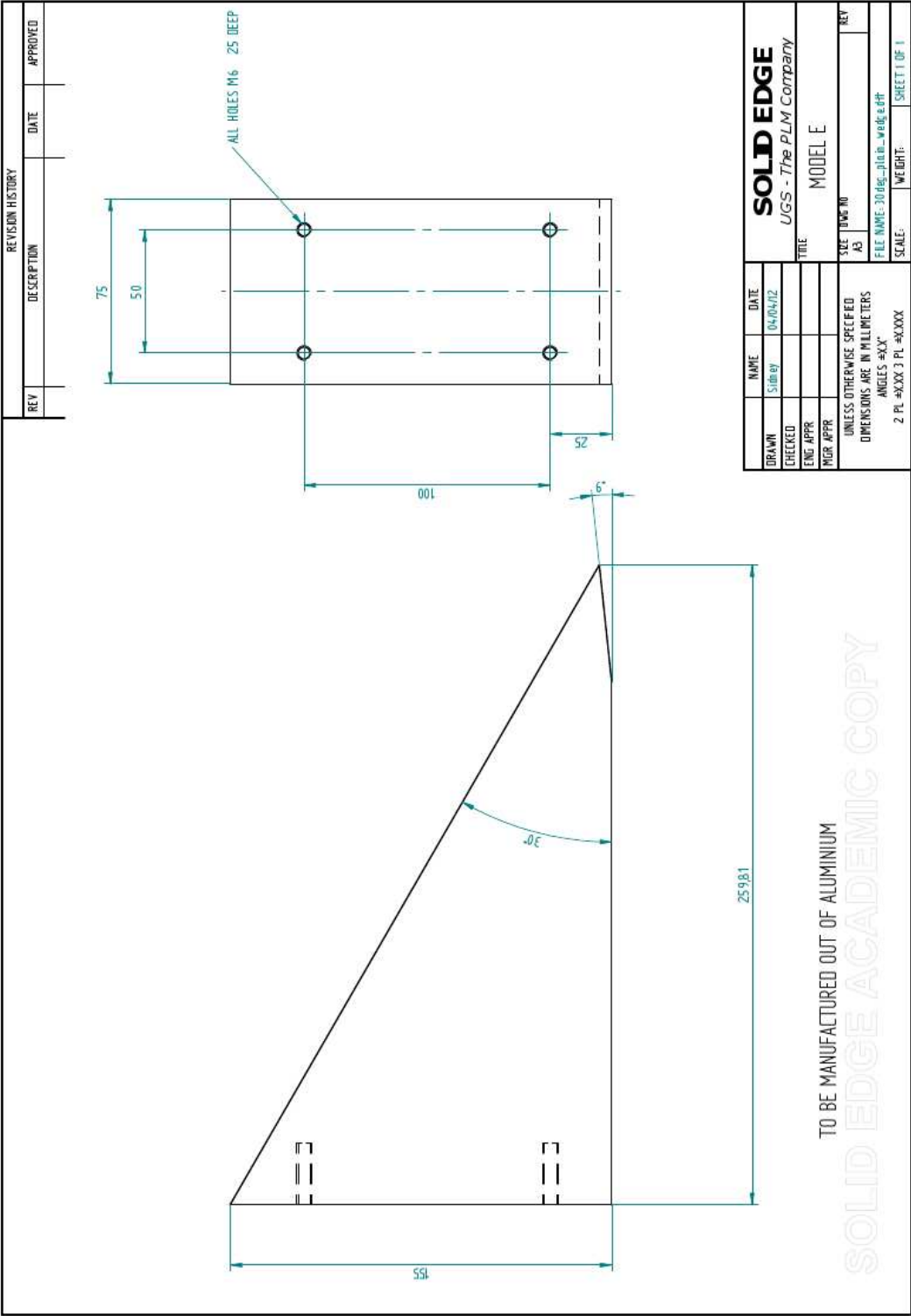


Figure A.5: Model E.

APPENDIX B APPARATUS DETAILS

B.1 Hardware

Xenon light source:

Manufacturer: Hamamatsu

Model: SQ type L2437

Digital camera:

Manufacturer: Fujifilm

Model: FinePix S3 Pro

Serial number: 62Q03532

Resolution: 12 Megapixels

Format: RAW

ISO: 1600

Shutter setting: BulbF

B.2 Software

FLUENT:

Version: 13.0.0

MATLAB:

Version: 7.7.0.471 (R2008b)

DigXY:

Version: 1.2

APPENDIX C EVOLUTION IMAGES

C.1 Model A

Figures C.1, C.2 and C.3 detail the KHI evolution on Model A with incident shock wave strengths of $M_0 = 1.34$, $M_0 = 1.46$ and $M_0 = 1.61$ respectively.

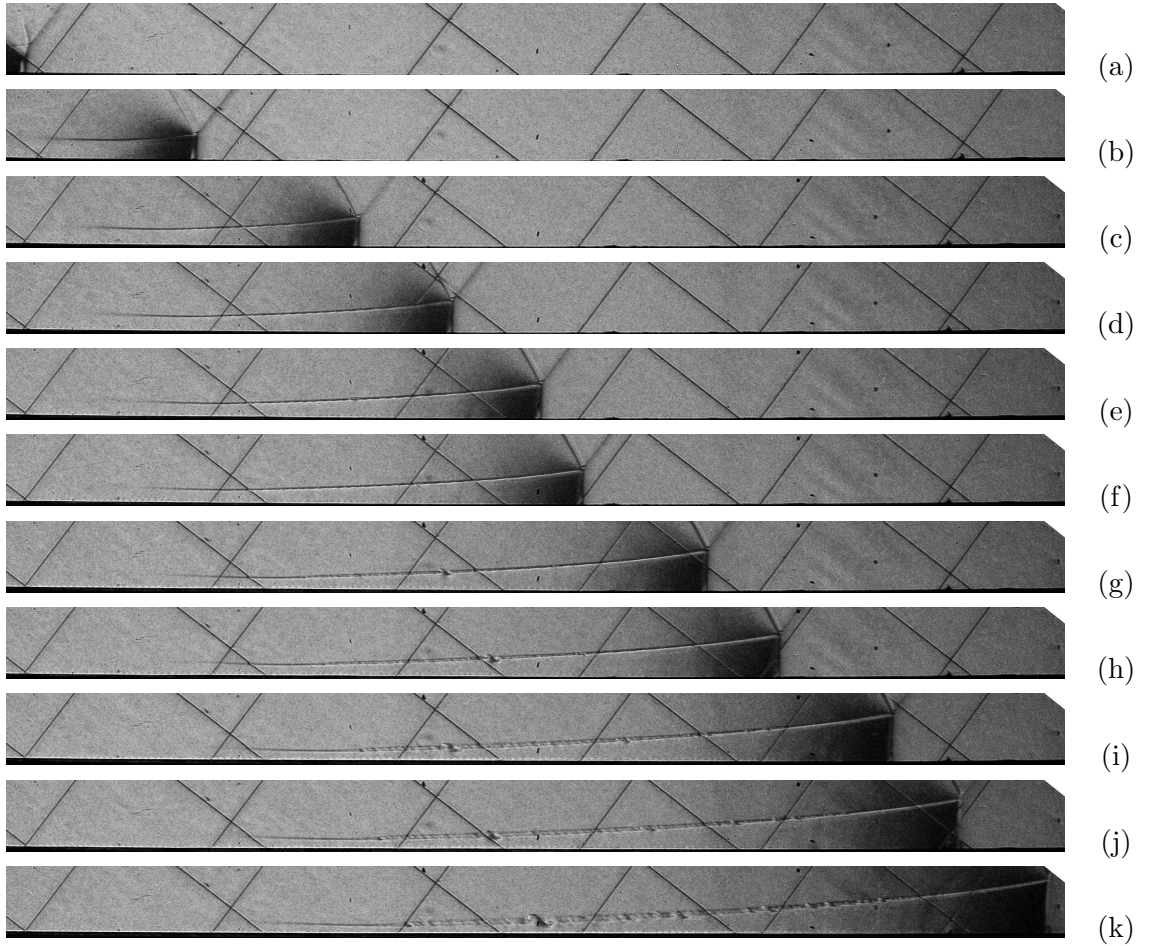


Figure C.1: KHI evolution for Model A, $M_0 = 1.34$. At times of (a) $1650\mu s$, (b) $1700\mu s$, (c) $1750\mu s$, (d) $1775\mu s$, (e) $1800\mu s$, (f) $1825\mu s$, (g) $1850\mu s$, (h) $1875\mu s$, (i) $1900\mu s$, (j) $1925\mu s$, and (k) $1950\mu s$.

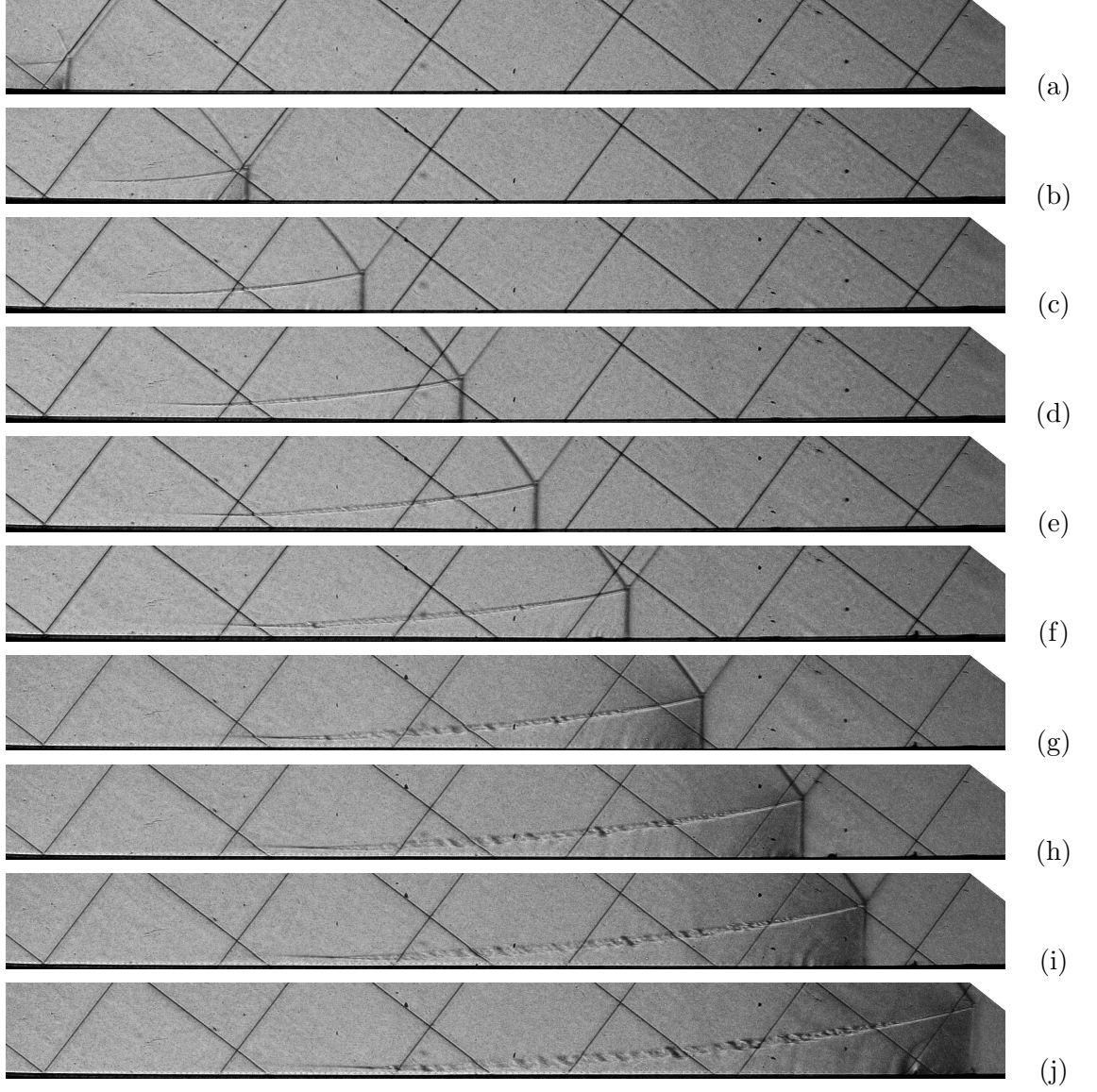


Figure C.2: KHI evolution for Model A, $M_0 = 1.46$. At times of (a) $1525\mu s$, (b) $1575\mu s$, (c) $1600\mu s$, (d) $1625\mu s$, (e) $1650\mu s$, (f) $1675\mu s$, (g) $1700\mu s$, (h) $1725\mu s$, (i) $1750\mu s$, and (j) $1775\mu s$.

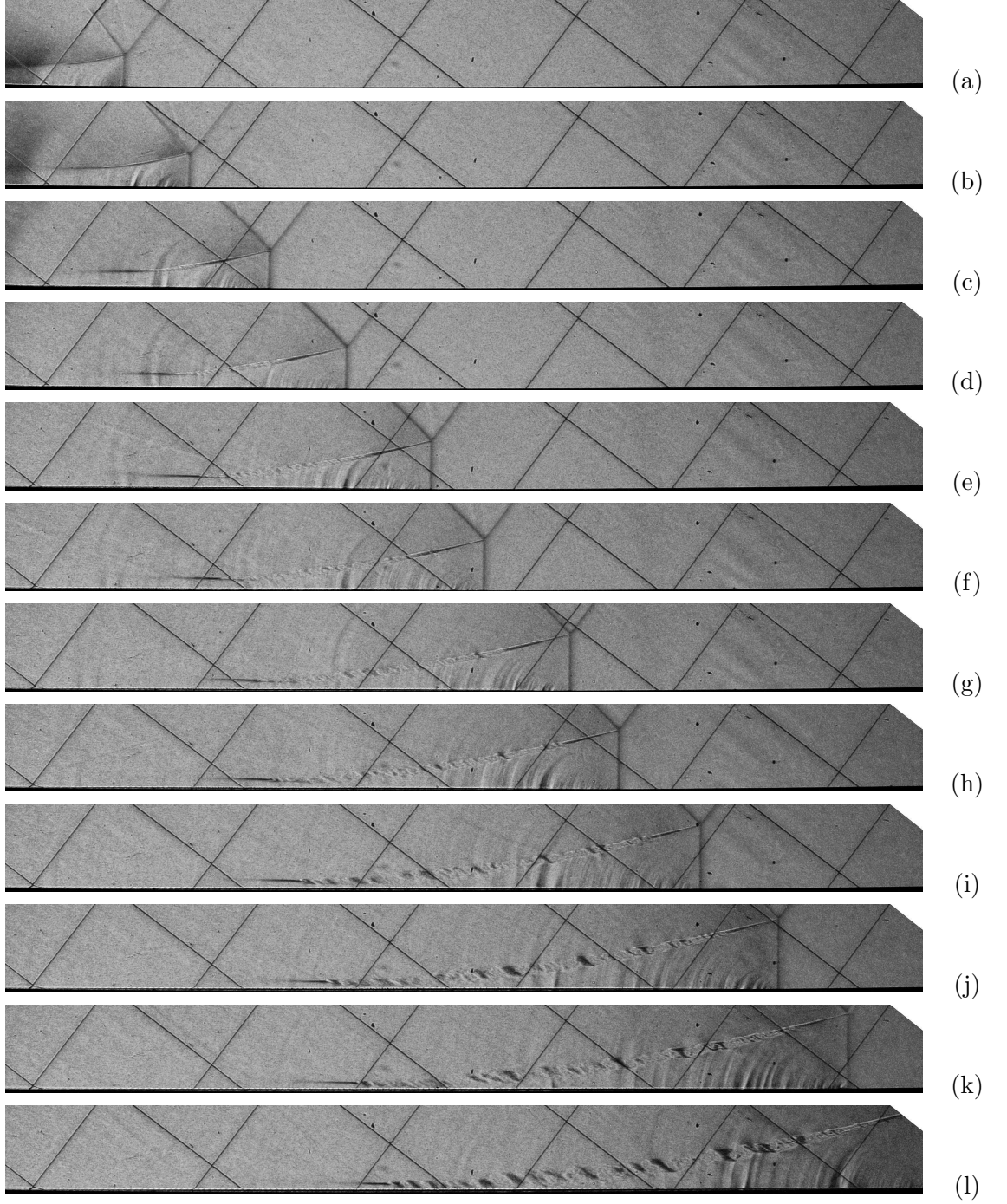


Figure C.3: KHI evolution for Model A, $M_0 = 1.61$. At times of (a) $1400\mu s$, (b) $1420\mu s$, (c) $1440\mu s$, (d) $1460\mu s$, (e) $1480\mu s$, (f) $1500\mu s$, (g) $1520\mu s$, (h) $1540\mu s$, (i) $1560\mu s$, (j) $1580\mu s$, (k) $1600\mu s$, and (l) $1620\mu s$.

C.2 Model B

Figures C.4, C.5 and C.6 detail the KHI evolution on Model B with incident shock wave strengths of $M_0 = 1.34$, $M_0 = 1.46$ and $M_0 = 1.61$ respectively.

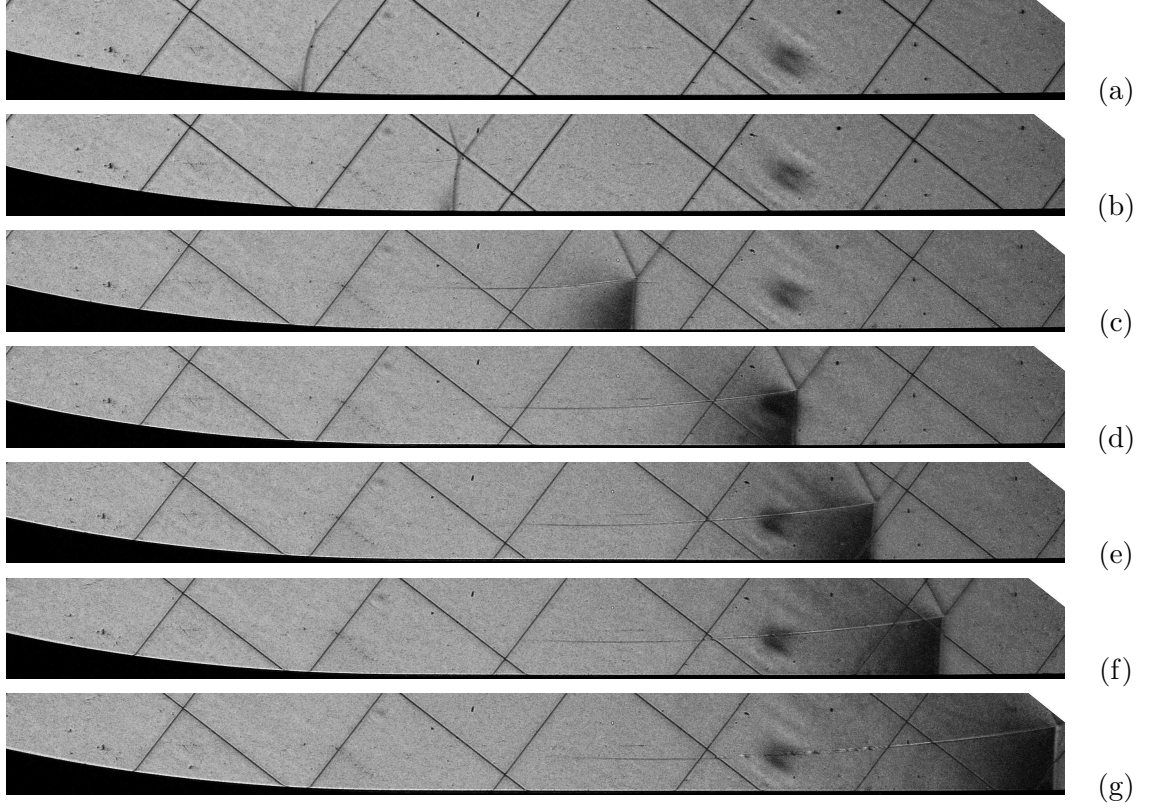


Figure C.4: KHI evolution for Model B, $M_0 = 1.34$. At times of (a) $1750\mu s$, (b) $1800\mu s$, (c) $1850\mu s$, (d) $1900\mu s$, (e) $1925\mu s$, (f) $1950\mu s$, and (g) $1975\mu s$.

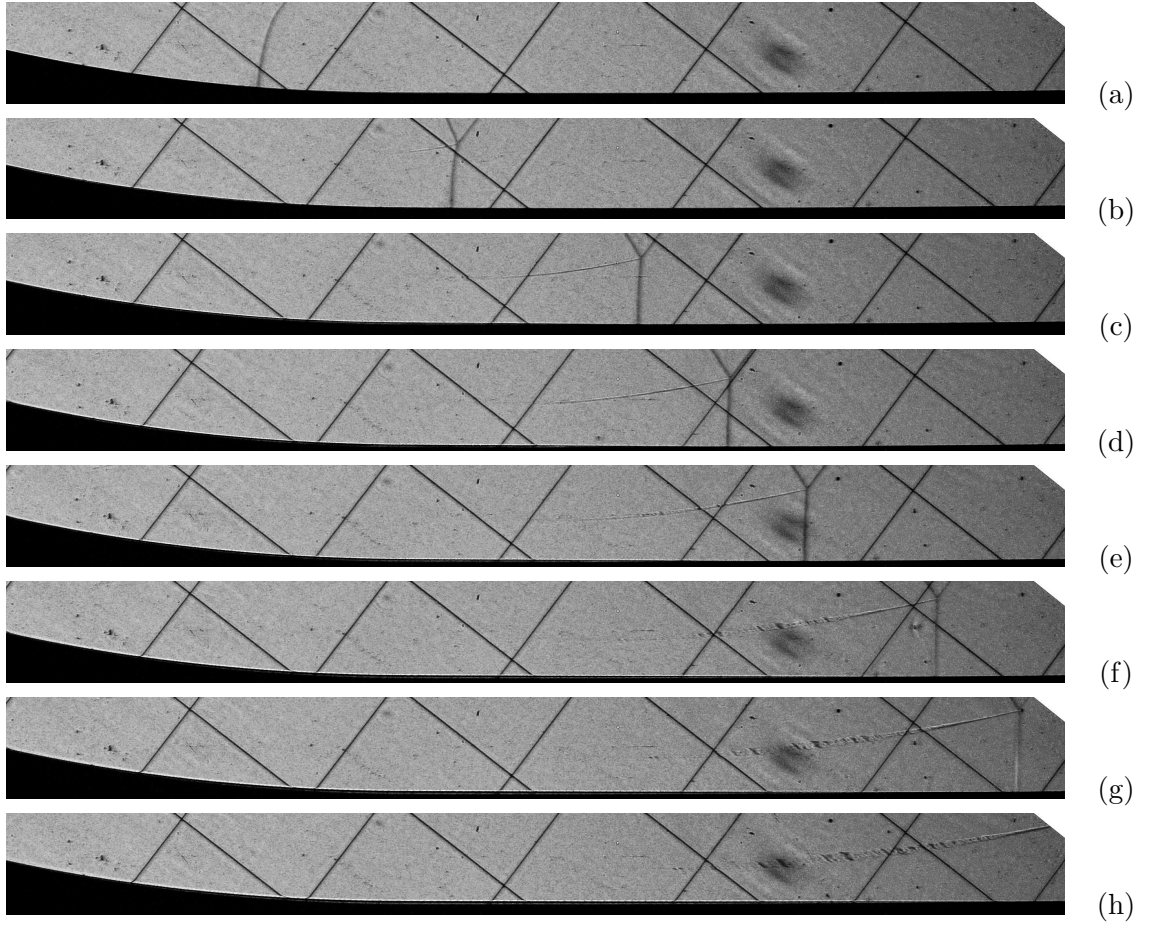


Figure C.5: KHI evolution for Model B, $M_0 = 1.46$. At times of (a) $1600\mu s$, (b) $1650\mu s$, (c) $1700\mu s$, (d) $1725\mu s$, (e) $1750\mu s$, (f) $1775\mu s$, (g) $1800\mu s$, and (h) $1825\mu s$.

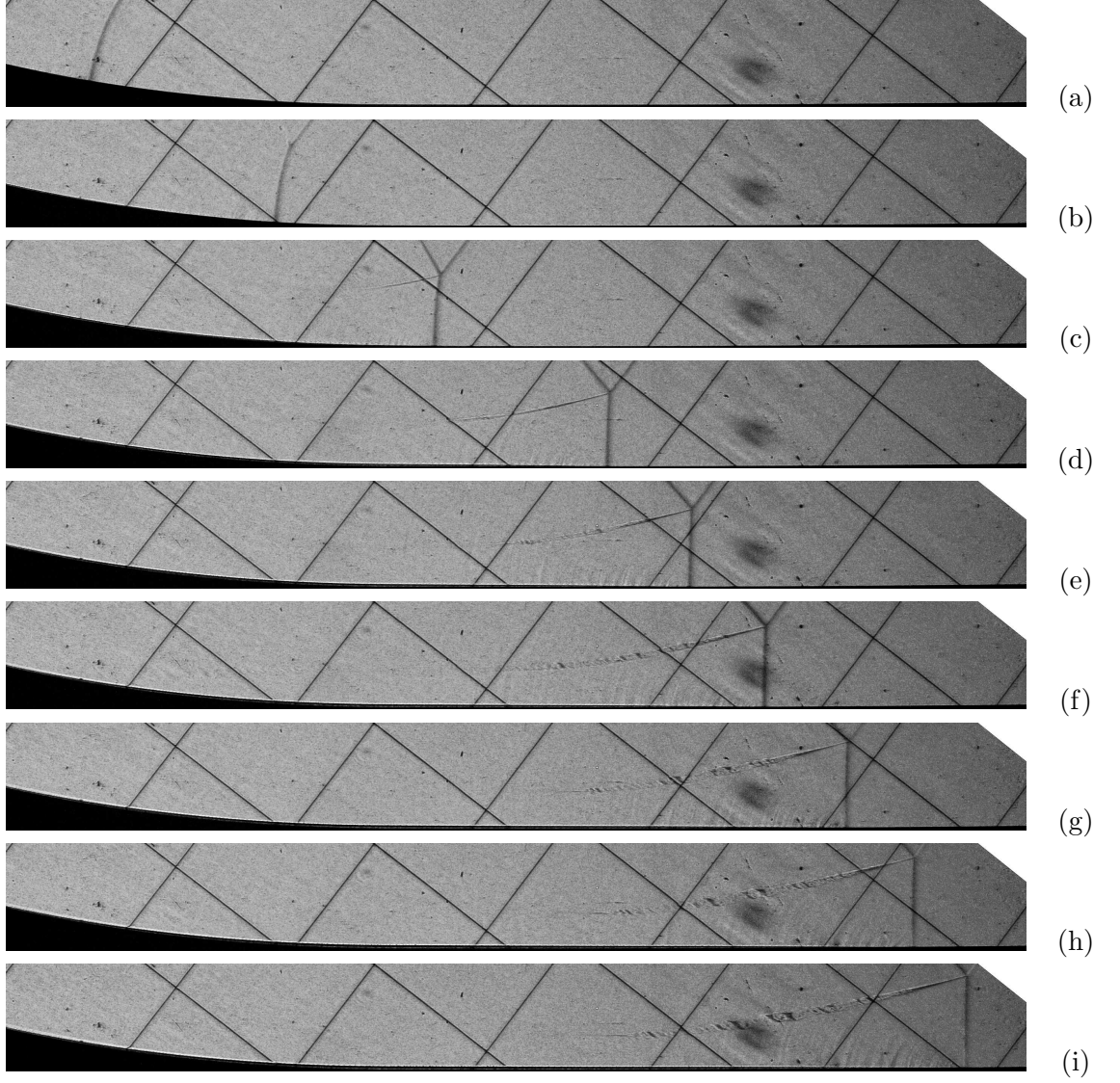


Figure C.6: KHI evolution for Model B, $M_0 = 1.61$. At times of (a) $1420\mu s$, (b) $1460\mu s$, (c) $1500\mu s$, (d) $1540\mu s$, (e) $1560\mu s$, (f) $1580\mu s$, (g) $1600\mu s$, (h) $1620\mu s$, and (i) $1640\mu s$.

C.3 Model C

Figures C.7, C.8 and C.9 detail the KHI evolution on Model C with incident shock wave strengths of $M_0 = 1.34$, $M_0 = 1.46$ and $M_0 = 1.61$ respectively.

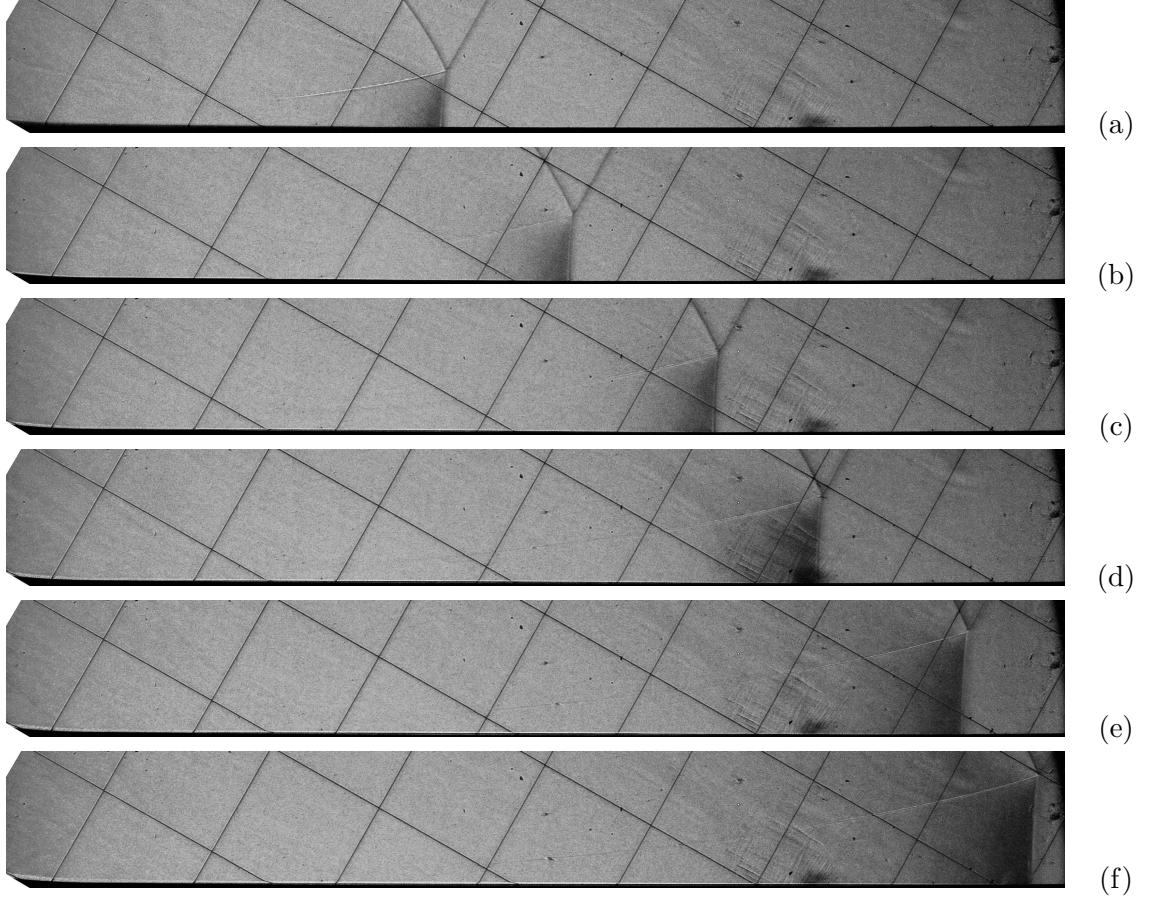


Figure C.7: KHI evolution for Model C, $M_0 = 1.34$. At times of (a) $1750\mu s$, (b) $1800\mu s$, (c) $1850\mu s$, (d) $1900\mu s$, (e) $1950\mu s$, and (f) $1975\mu s$.

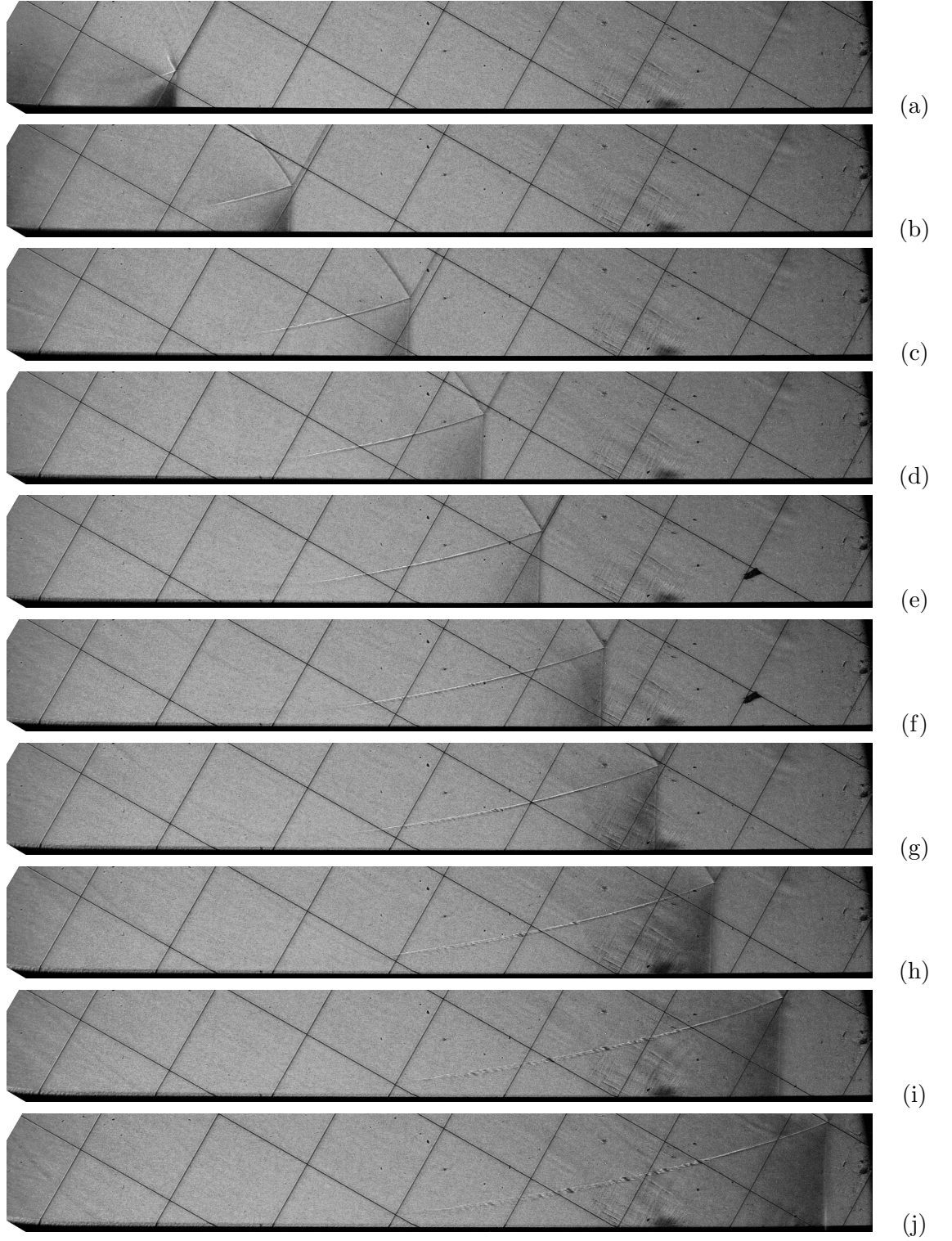


Figure C.8: KHI evolution for Model C, $M_0 = 1.46$. At times of (a) $1425\mu s$, (b) $1525\mu s$, (c) $1625\mu s$, (d) $1650\mu s$, (e) $1675\mu s$, (f) $1700\mu s$, (g) $1725\mu s$, (h) $1750\mu s$, (i) $1775\mu s$, and (j) $1800\mu s$.

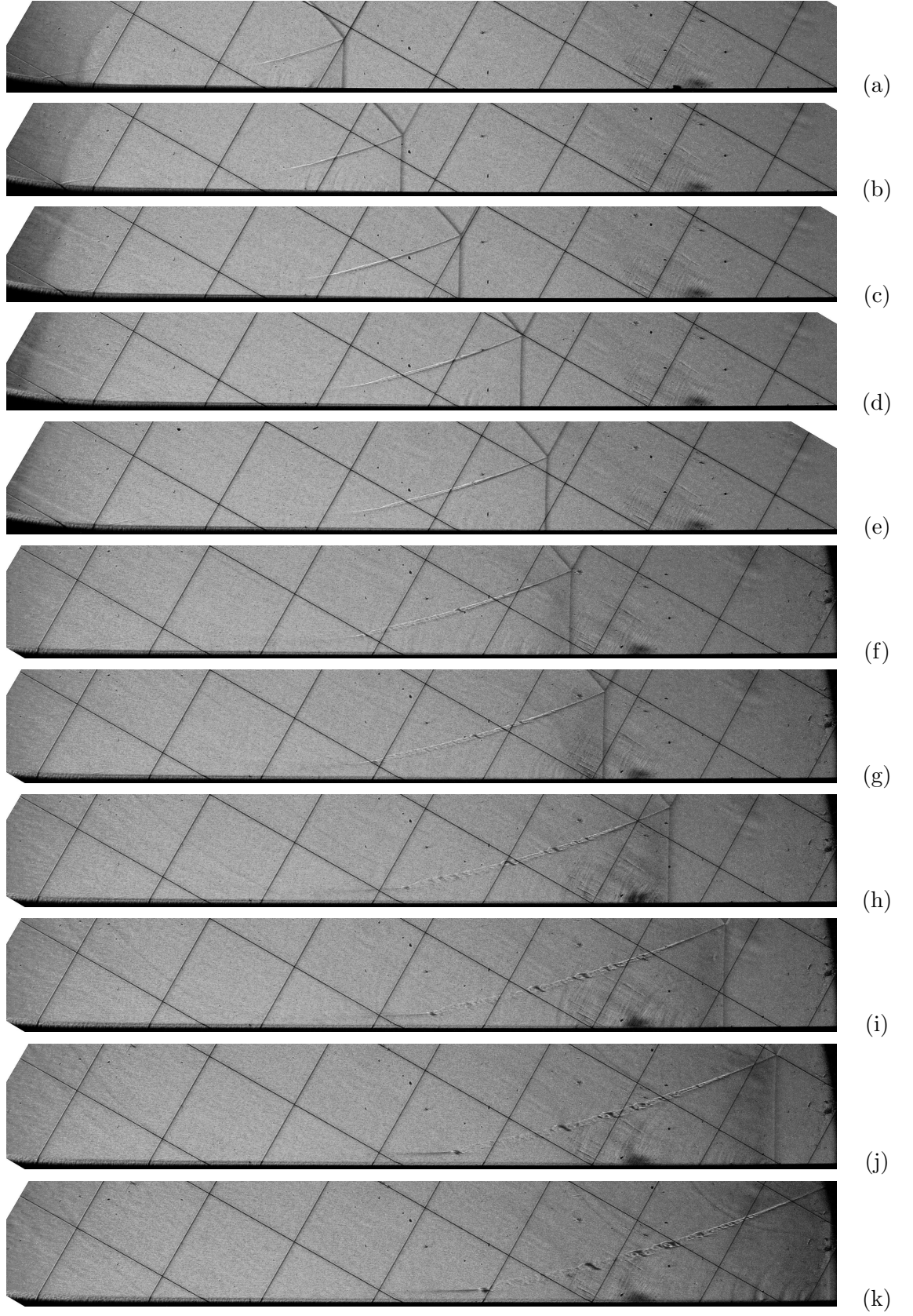


Figure C.9: KHI evolution for Model C, $M_0 = 1.61$. At times of (a) $1440\mu s$, (b) $1460\mu s$, (c) $1480\mu s$, (d) $1500\mu s$, (e) $1520\mu s$, (f) $1540\mu s$, (g) $1560\mu s$, (h) $1580\mu s$, (i) $1600\mu s$, (j) $1620\mu s$, and (k) $1640\mu s$.

C.4 Model D

Figures C.10, C.11 and C.12 detail the KHI evolution on Model D with incident shock wave strengths of $M_0 = 1.34$, $M_0 = 1.46$ and $M_0 = 1.61$ respectively.

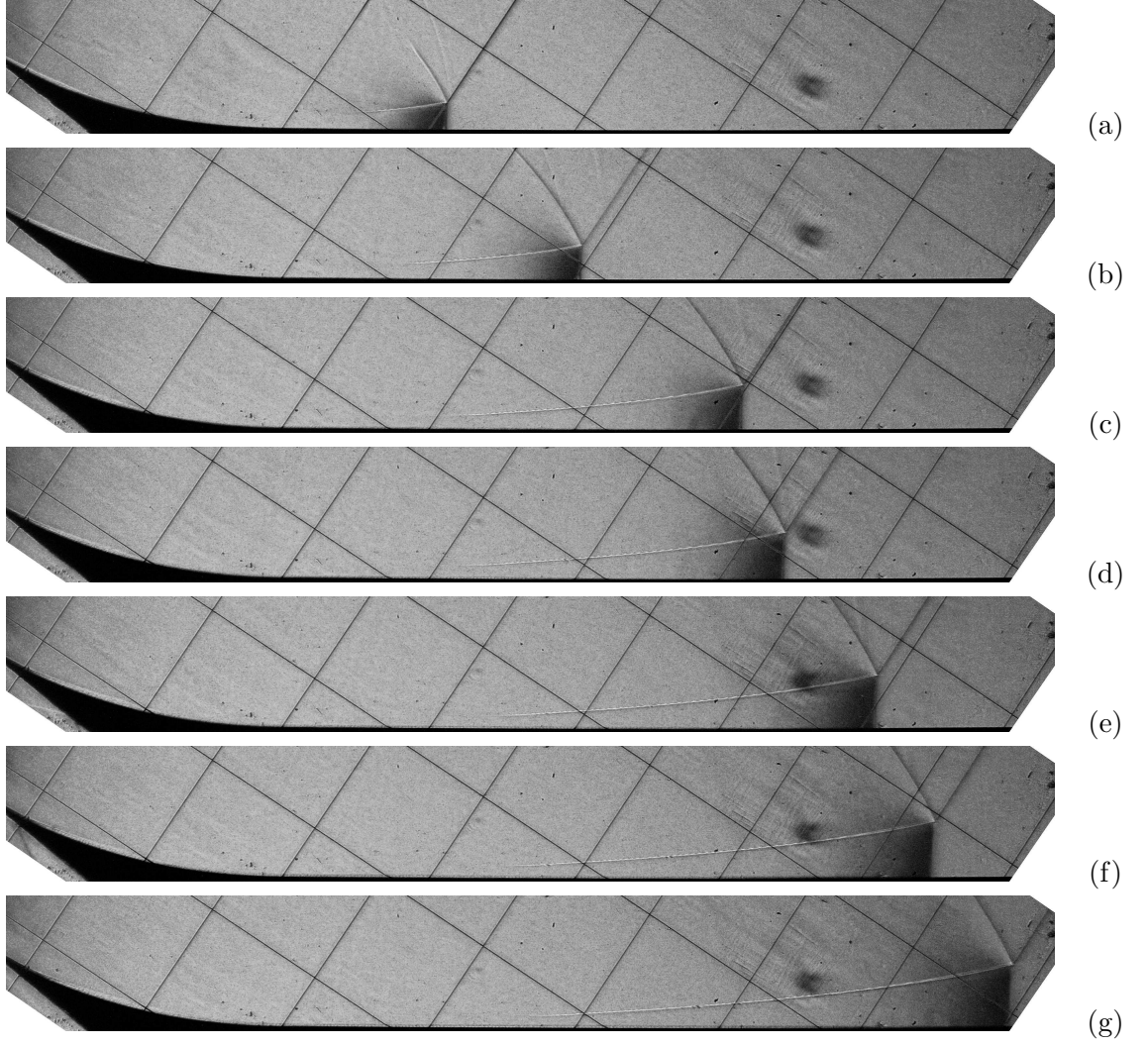


Figure C.10: KHI evolution for Model D, $M_0 = 1.34$. At times of (a) $1750\mu s$, (b) $1775\mu s$, (c) $1825\mu s$, (d) $1875\mu s$, (e) $1900\mu s$, (f) $1925\mu s$, and (g) $1950\mu s$.

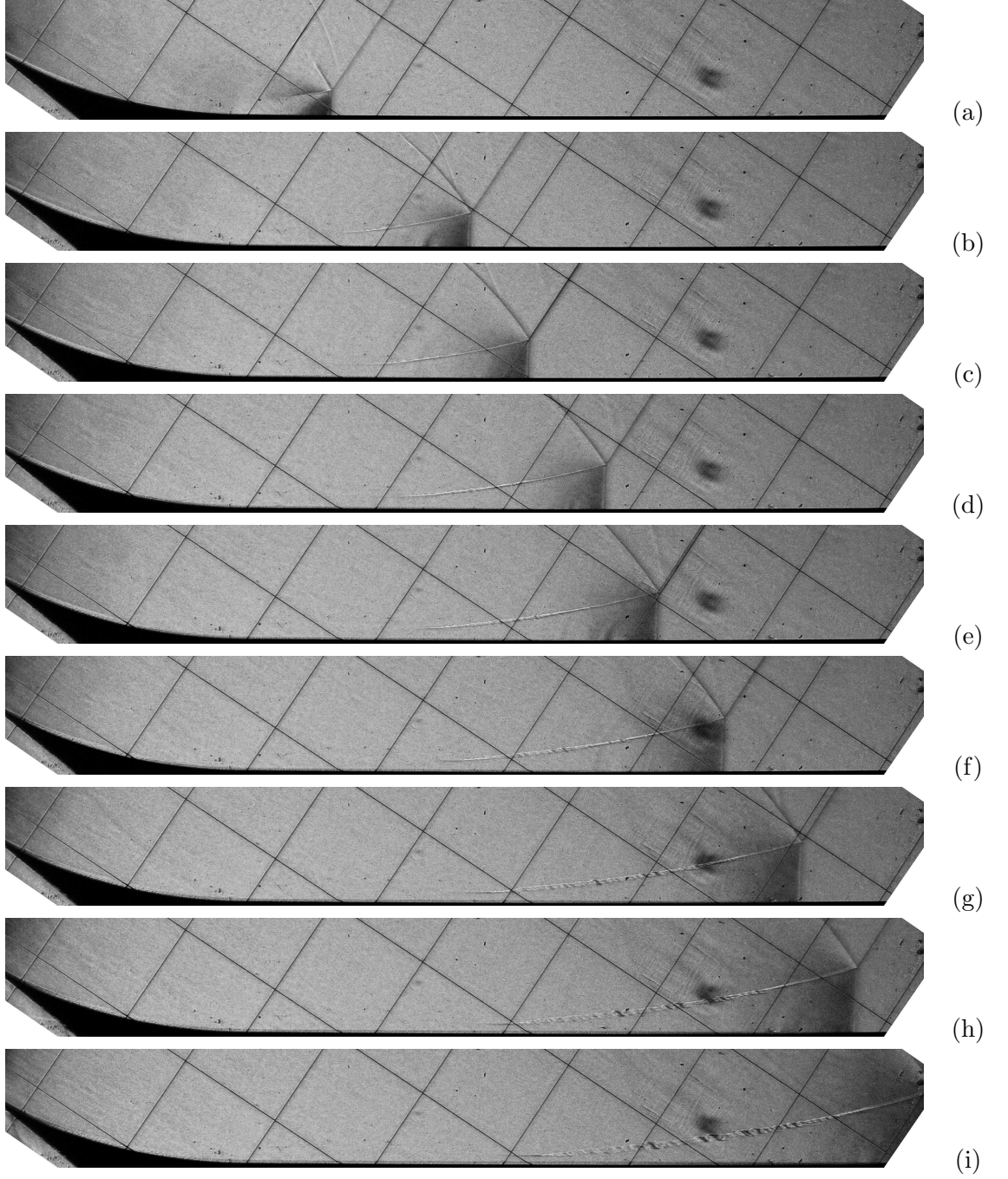


Figure C.11: KHI evolution for Model D, $M_0 = 1.46$. At times of (a) $1750\mu s$, (b) $1600\mu s$, (c) $1650\mu s$, (d) $1675\mu s$, (e) $1700\mu s$, (f) $1725\mu s$, (g) $1750\mu s$, (h) $1775\mu s$, and (i) $1800\mu s$.

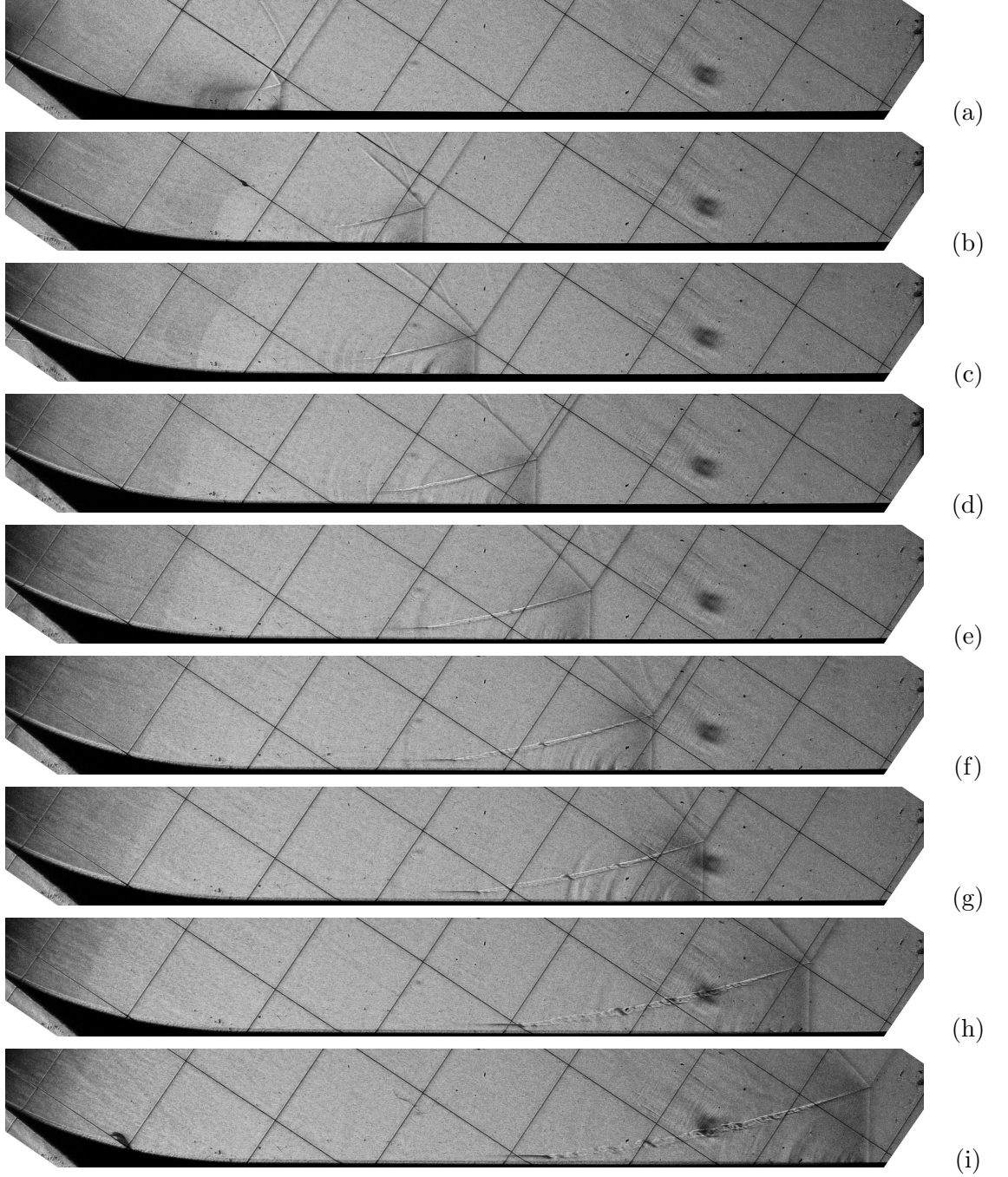


Figure C.12: KHI evolution for Model D, $M_0 = 1.61$. At times of (a) $1340\mu s$, (b) $1440\mu s$, (c) $1480\mu s$, (d) $1500\mu s$, (e) $1520\mu s$, (f) $1540\mu s$, (g) $1560\mu s$, (h) $1580\mu s$, and (i) $16000\mu s$.

C.5 Model E

Figures C.13, C.14 and C.15 detail the KHI evolution on Model E with incident shock wave strengths of $M_0 = 1.34$, $M_0 = 1.46$ and $M_0 = 1.61$ respectively.

Note: Figures 8.20 (a); 8.22 (a) and (b); 8.24 (a), (b), (c) and (d) are taken with slightly repositioned optics so as to capture an earlier part of the flow field, not visible with the default setup.

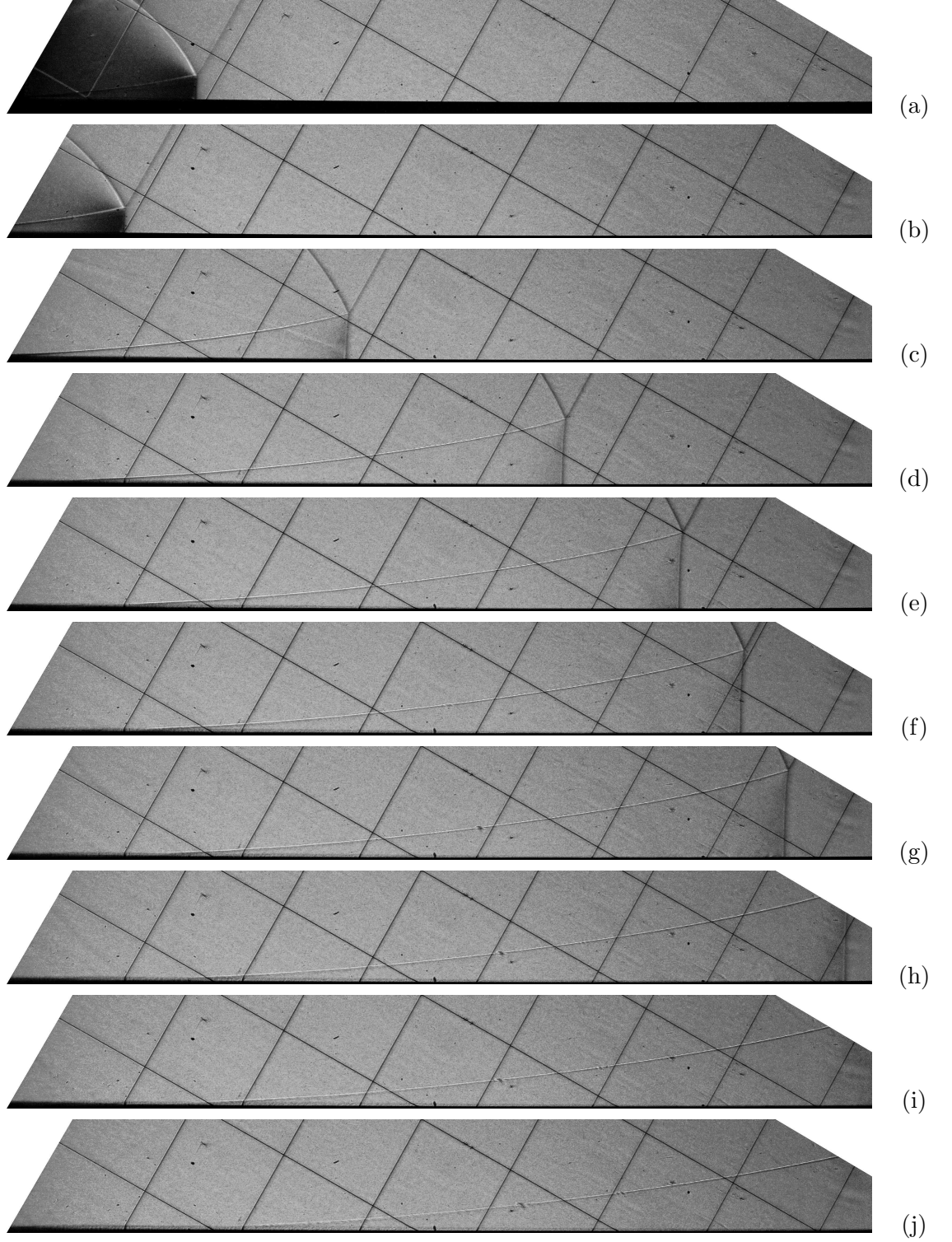


Figure C.13: KHI evolution for Model E, $M_0 = 1.34$. At times of (a) $1600\mu s$, (b) $1600\mu s$, (c) $1700\mu s$, (d) $1800\mu s$, (e) $1850\mu s$, (f) $1875\mu s$, (g) $1900\mu s$, (h) $1925\mu s$, (i) $1950\mu s$, and (j) $1975\mu s$.

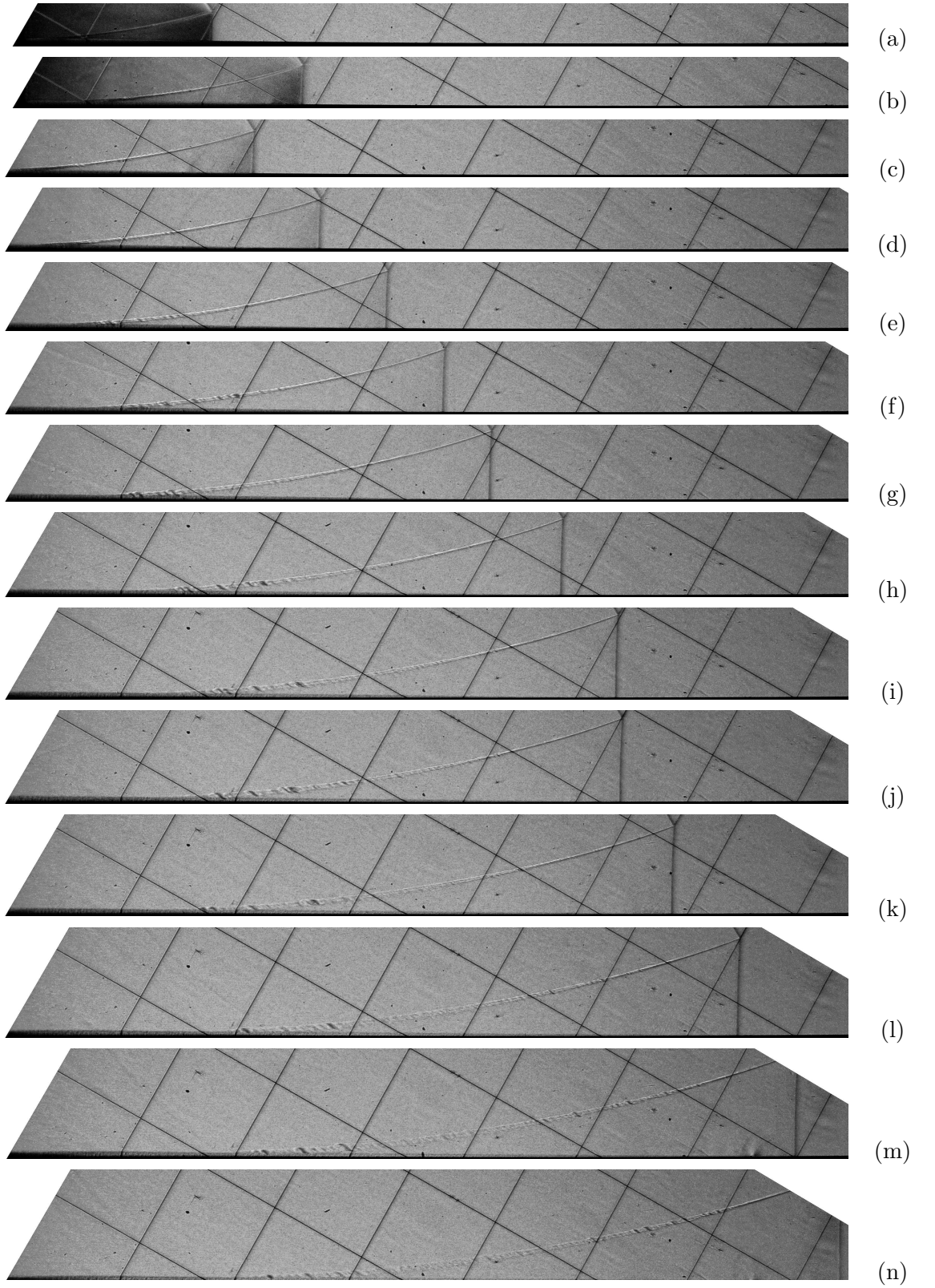


Figure C.14: KHI evolution for Model E, $M_0 = 1.46$. At times of (a) $1475\mu s$, (b) $1500\mu s$, (c) $1525\mu s$, (d) $1550\mu s$, (e) $1575\mu s$, (f) $1600\mu s$, (g) $1625\mu s$, (h) $1650\mu s$, (j) $1675\mu s$, (k) $1700\mu s$, (l) $1725\mu s$, (m) $1750\mu s$, and (n) $1775\mu s$.

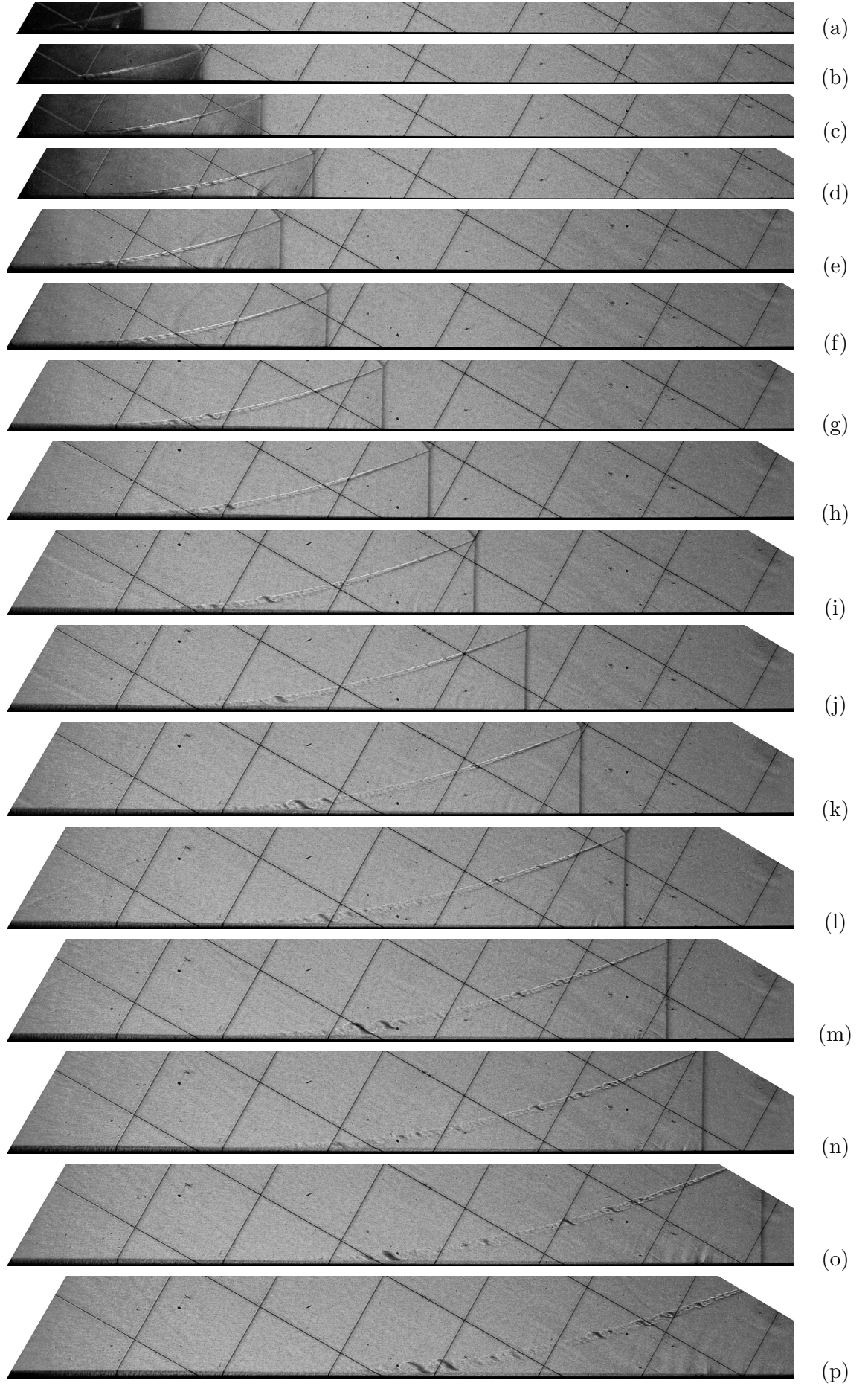


Figure C.15: KHI evolution for Model $\mathbb{E}03M_0 = 1.61$. At times of (a) $1320\mu s$, (b) $1340\mu s$, (c) $1360\mu s$, (d) $1380\mu s$, (e) $1400\mu s$, (f) $1420\mu s$, (g) $1440\mu s$, (h) $1460\mu s$, (i) $1480\mu s$, (j) $1500\mu s$, (k) $1520\mu s$, (l) $1540\mu s$, (m) $1560\mu s$, (n) $1580\mu s$, (o) $1600\mu s$, and

C.6 Model F

Figures C.16, C.17 and C.18 detail the KHI evolution on Model F with incident shock wave strengths of $M_0 = 1.34$, $M_0 = 1.46$ and $M_0 = 1.61$ respectively.

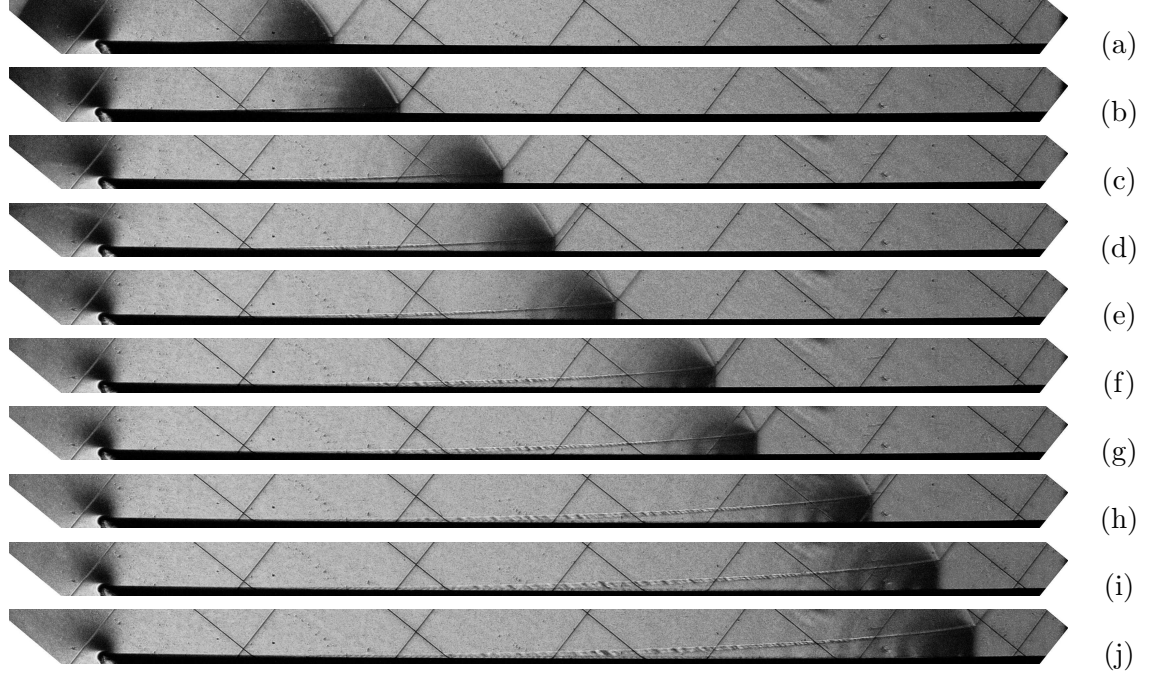


Figure C.16: KHI evolution for Model F, $M_0 = 1.34$. At times of (a) $1750\mu s$, (b) $1775\mu s$, (c) $1800\mu s$, (d) $1825\mu s$, (e) $1850\mu s$, (f) $1875\mu s$, (g) $1900\mu s$, (h) $1925\mu s$, (i) $1950\mu s$, and (j) $1975\mu s$.



Figure C.17: KHI evolution for Model F, $M_0 = 1.46$. At times of (a) $1600\mu s$, (b) $1625\mu s$, (c) $1650\mu s$, (d) $1675\mu s$, (e) $1700\mu s$, (f) $1725\mu s$, (g) $1750\mu s$, (h) $1775\mu s$, and (i) $1800\mu s$.



Figure C.18: KHI evolution for Model F, $M_0 = 1.61$. At times of (a) $1440\mu s$, (b) $1460\mu s$, (c) $1480\mu s$, (d) $1500\mu s$, (e) $1520\mu s$, (f) $1540\mu s$, (g) $1560\mu s$, (h) $1580\mu s$, (i) $1600\mu s$, (j) $1620\mu s$ and (k) $1640\mu s$.

APPENDIX D EVOLUTION GRAPHS

D.1 Average KHI width vs incident shock position and Mach number

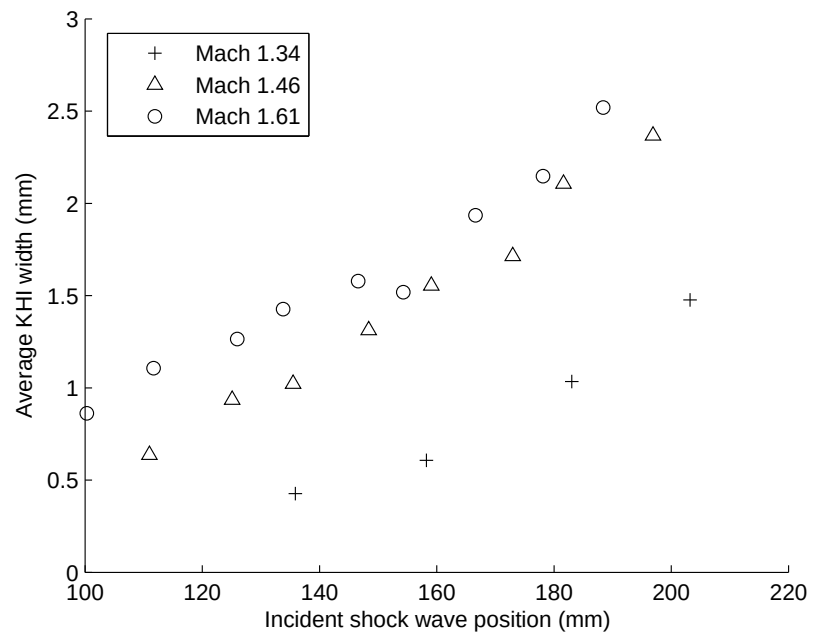


Figure D.1: Average KHI width vs incident shock wave position on Model A.

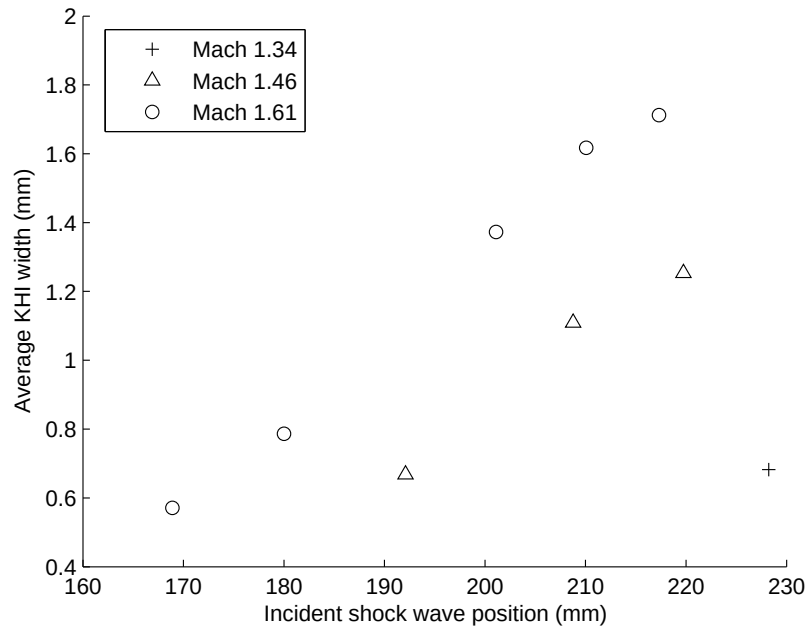


Figure D.2: Average KHI width vs incident shock wave position on Model B.

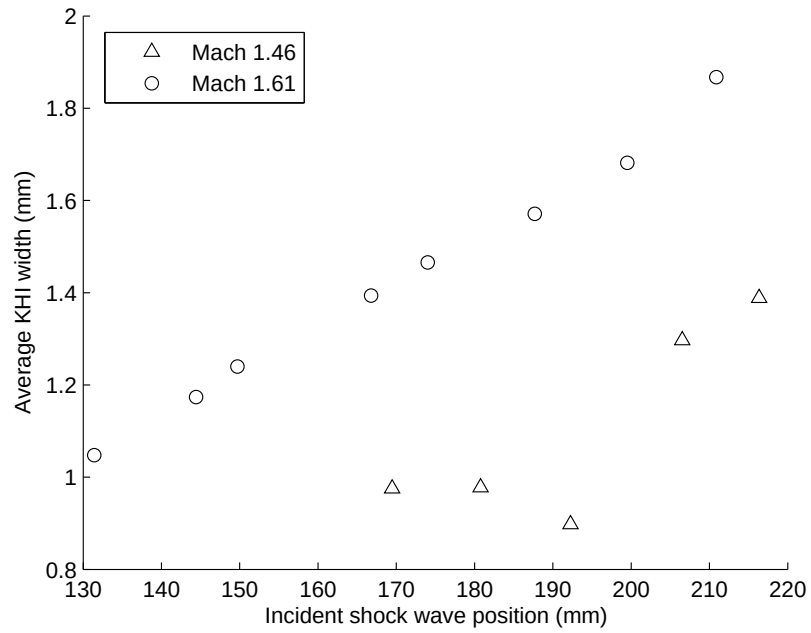


Figure D.3: Average KHI width vs incident shock wave position on Model C.

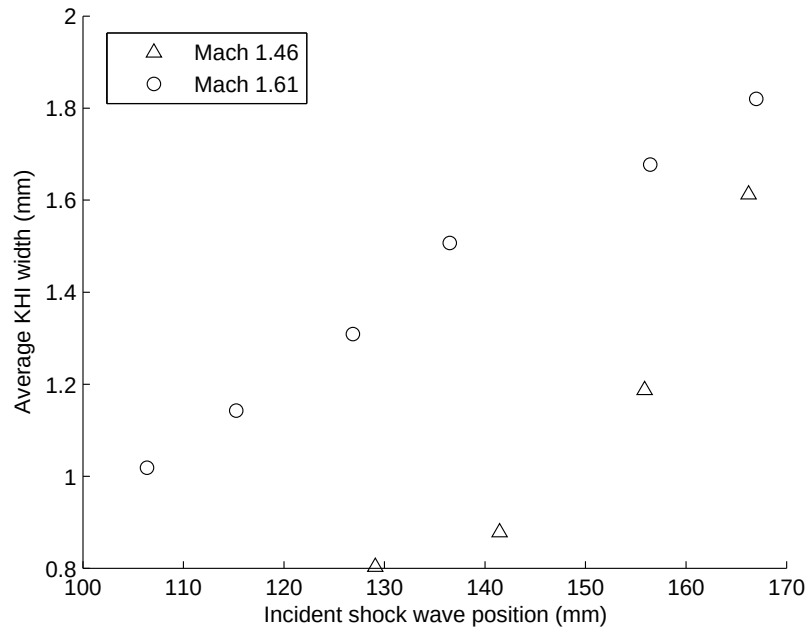


Figure D.4: Average KHI width vs incident shock wave position on Model D.

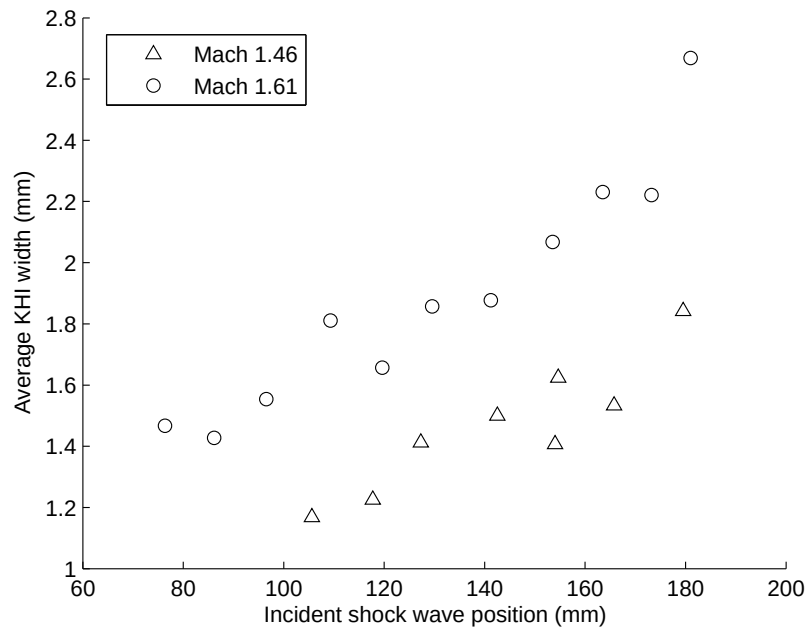


Figure D.5: Average KHI width vs incident shock wave position on Model E.

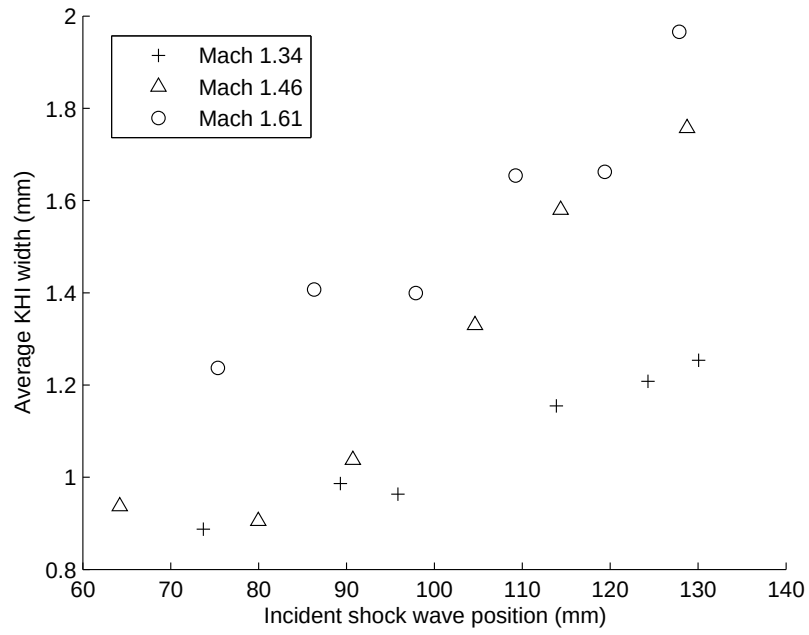


Figure D.6: Average KHI width vs incident shock wave position on Model F.

D.2 KHI length vs incident shock wave position and Mach number

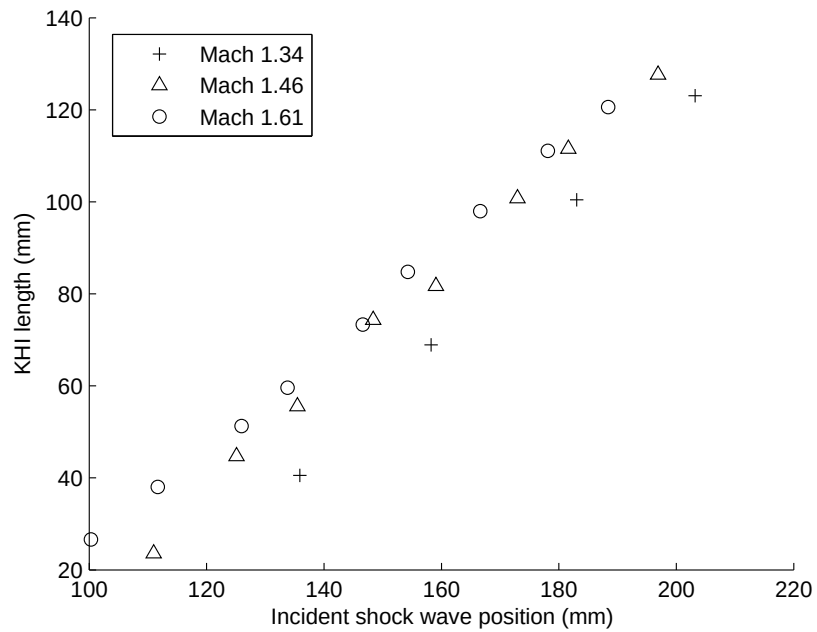


Figure D.7: KHI length vs incident shock wave position on Model A.

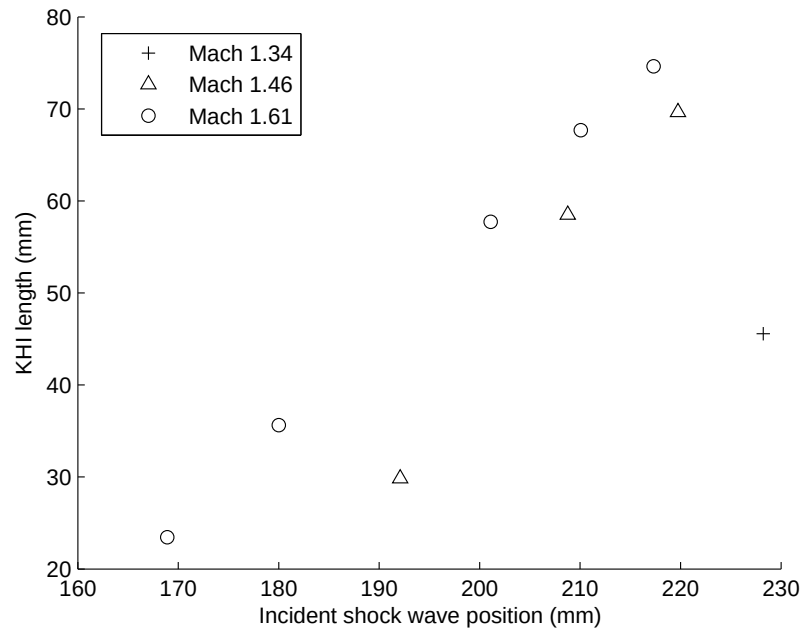


Figure D.8: KHI length vs incident shock wave position on Model B.

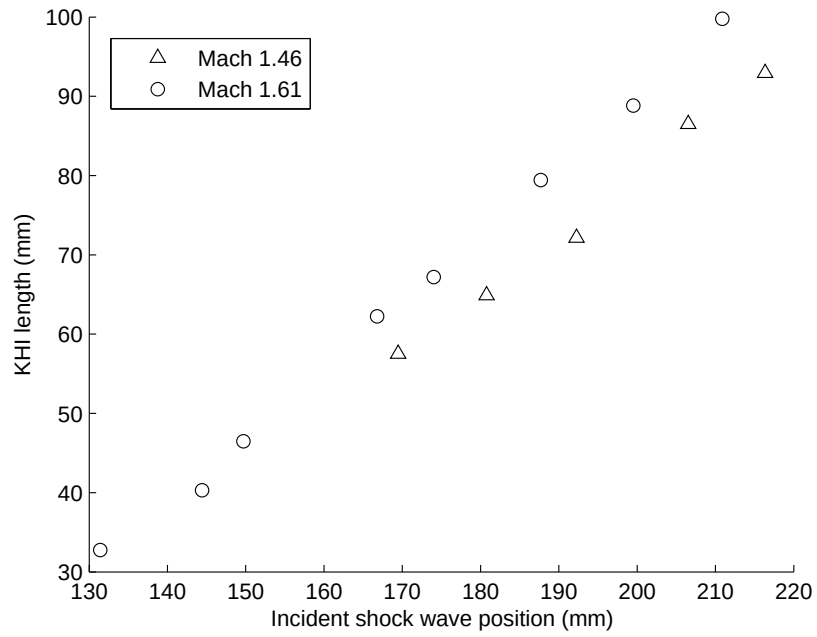


Figure D.9: KHI length vs incident shock wave position on Model C.

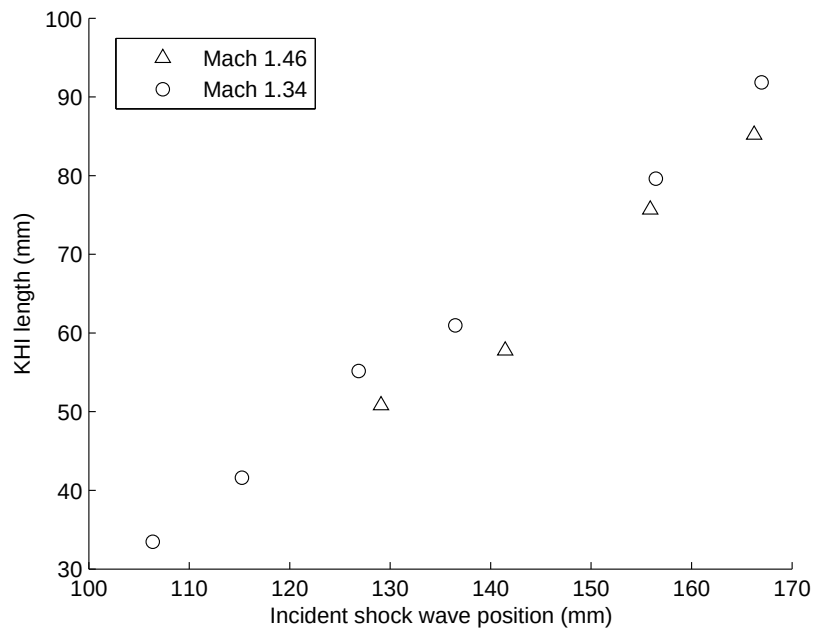


Figure D.10: KHI length vs incident shock wave position on Model D.

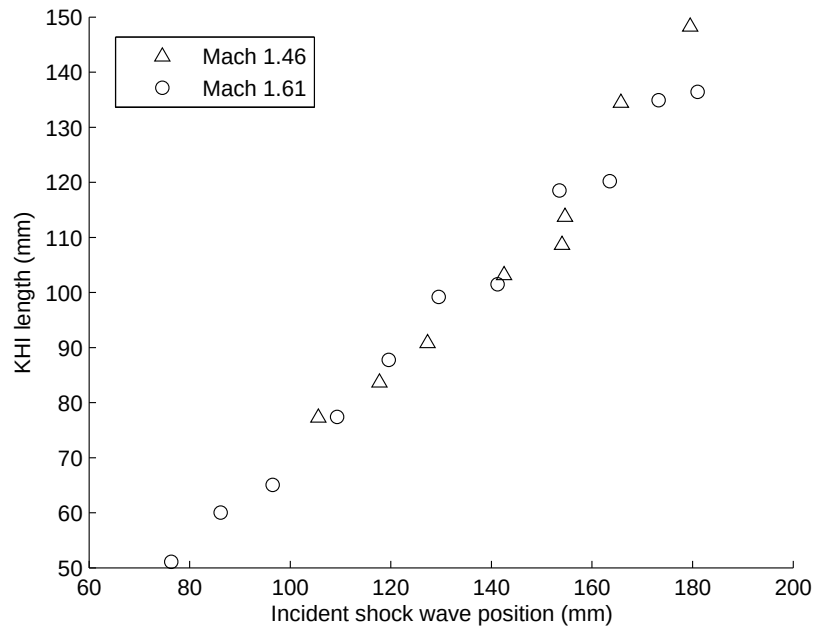


Figure D.11: KHI length vs incident shock wave position on Model E.

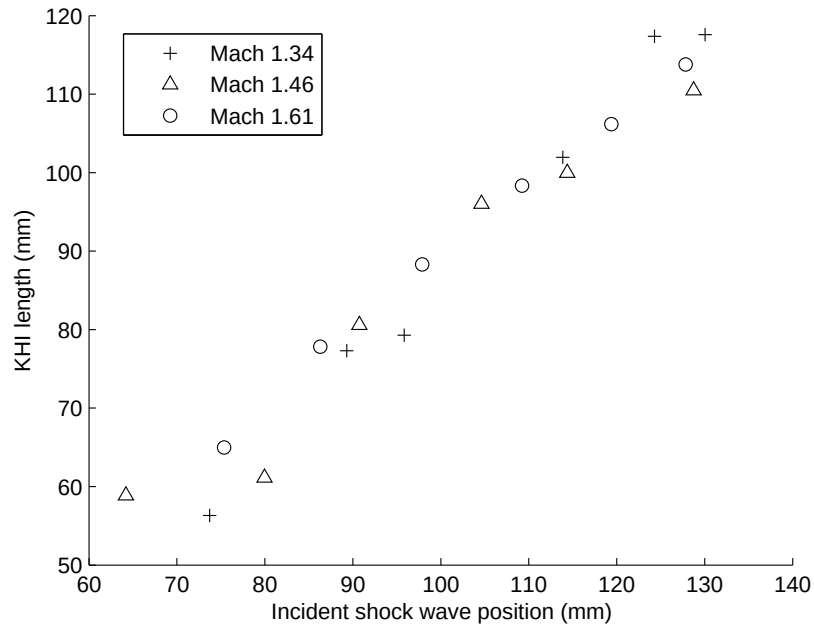


Figure D.12: KHI length vs incident shock wave position on Model F.

D.3 KHI average width/length vs incident shock position and Mach number

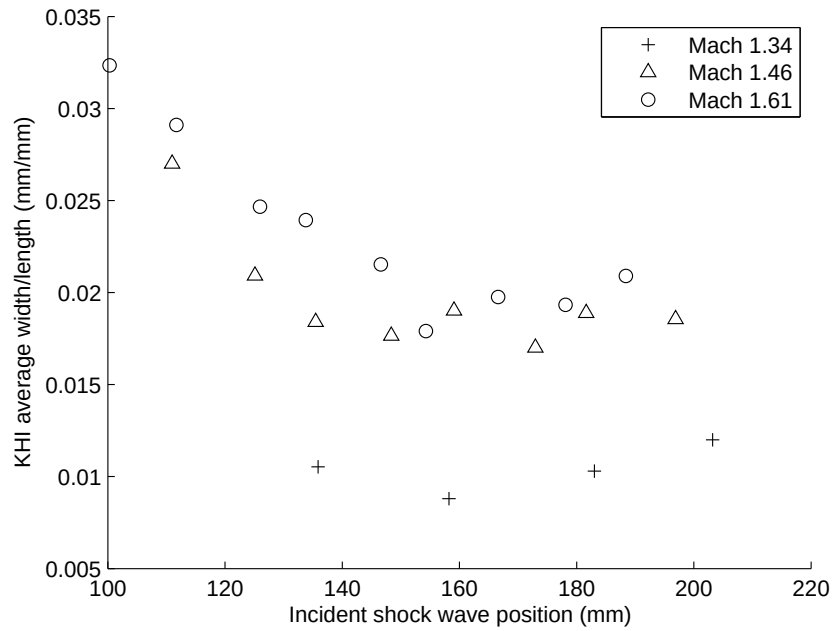


Figure D.13: KHI average width/length vs incident shock wave position on Model A.

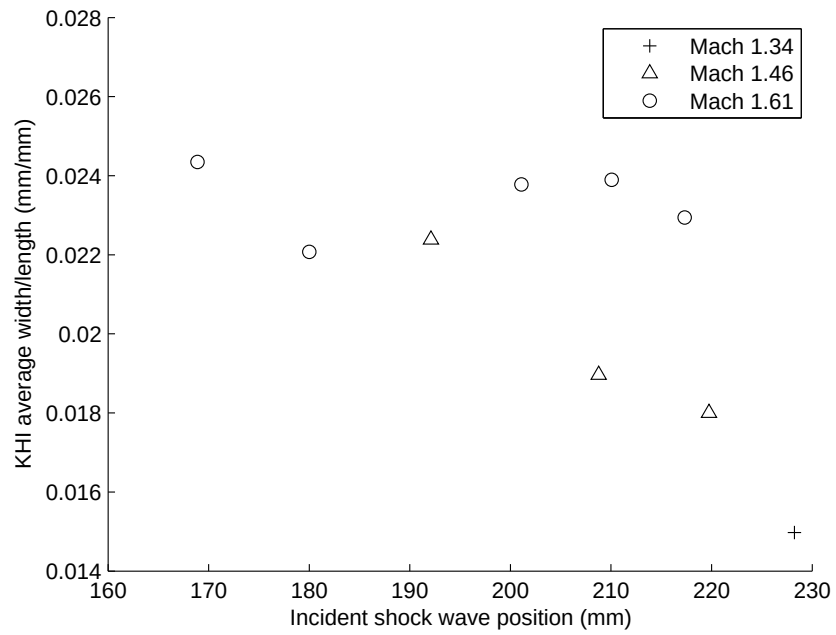


Figure D.14: KHI average width/length vs incident shock wave position on Model B.

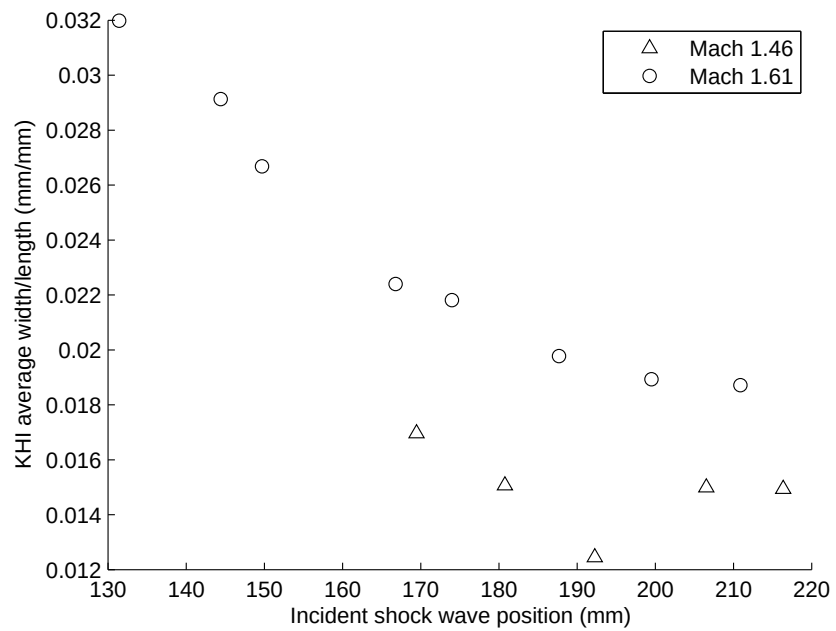


Figure D.15: KHI average width/length vs incident shock wave position on Model C.

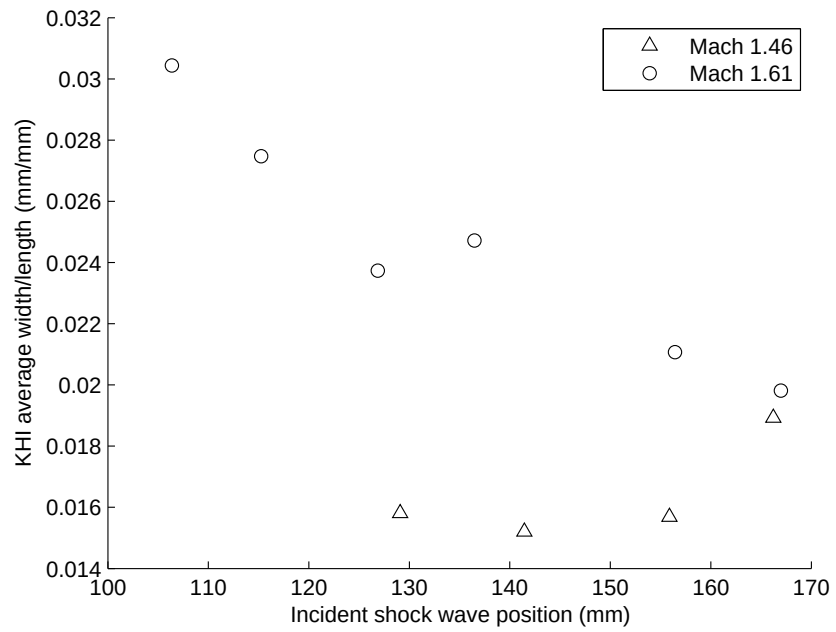


Figure D.16: KHI average width/length vs incident shock wave position on Model D.

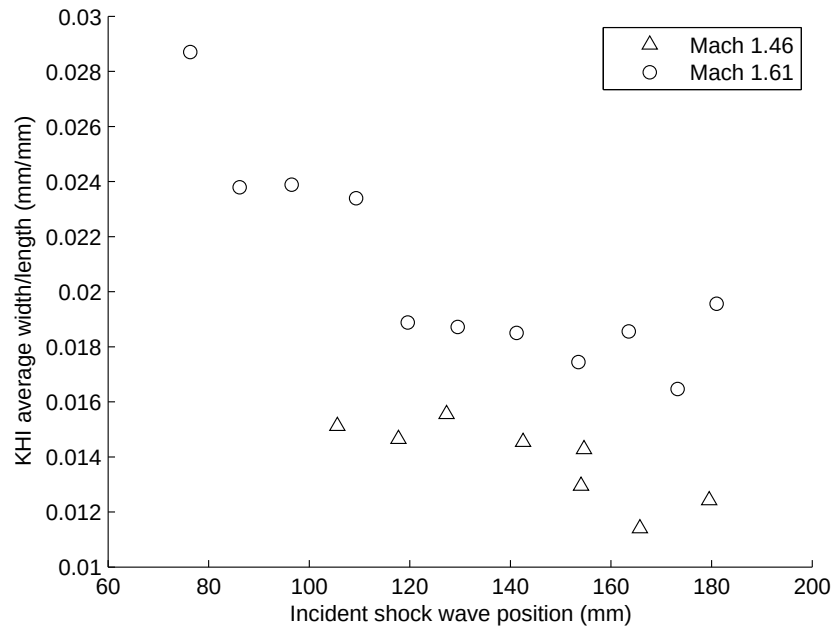


Figure D.17: KHI average width/length vs incident shock wave position on Model E.

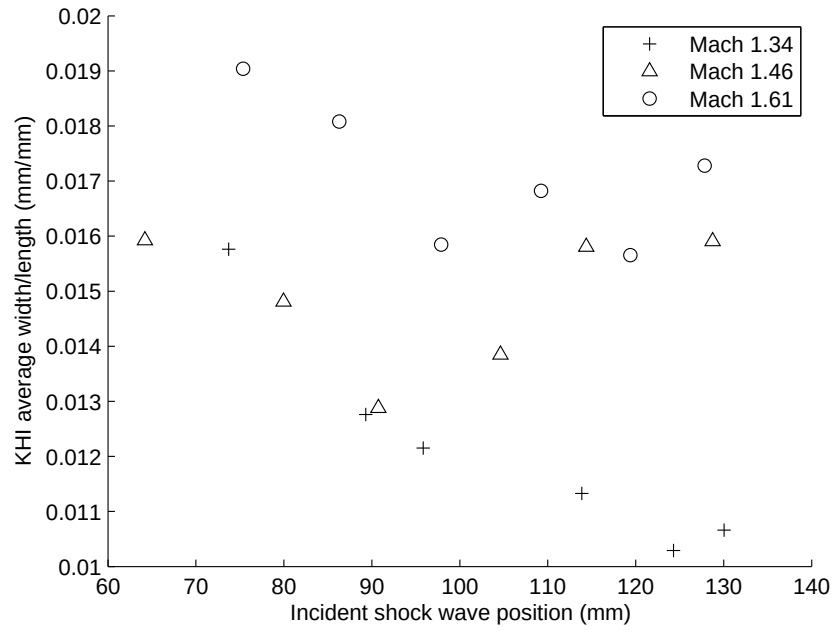


Figure D.18: KHI average width/length vs incident shock wave position on Model F.

D.4 KHI average width/Mach stem length vs incident shock position and Mach number

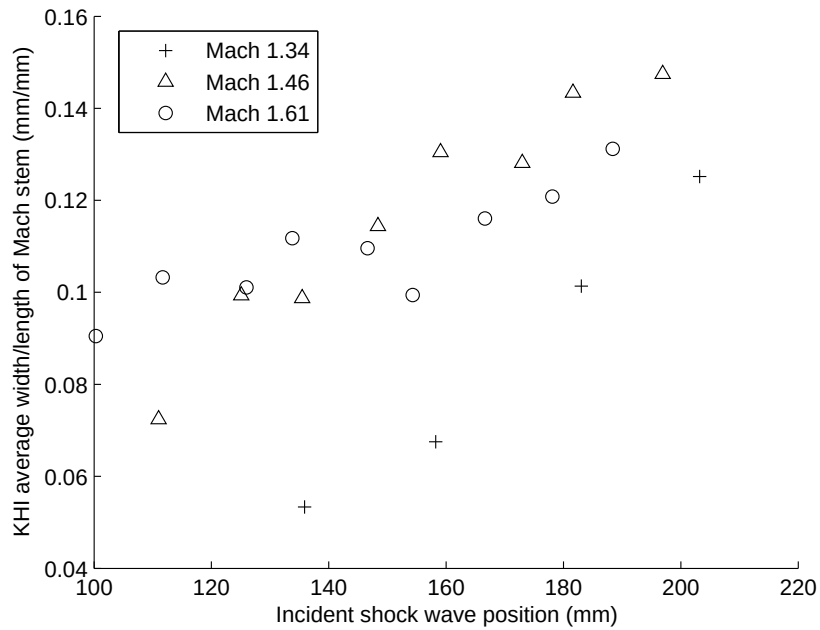


Figure D.19: KHI average width/Mach stem length vs incident shock wave position on Model A.

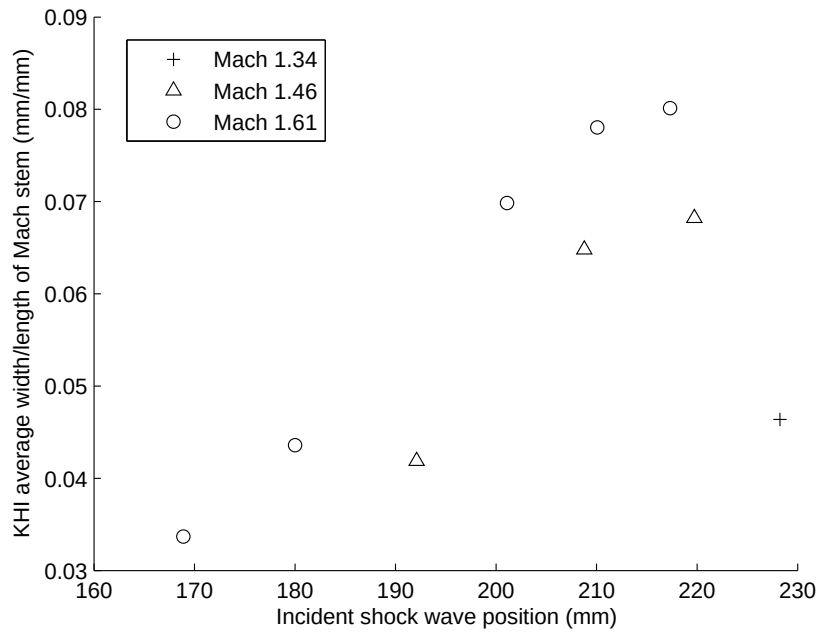


Figure D.20: KHI average width/Mach stem length vs incident shock wave position on Model B.

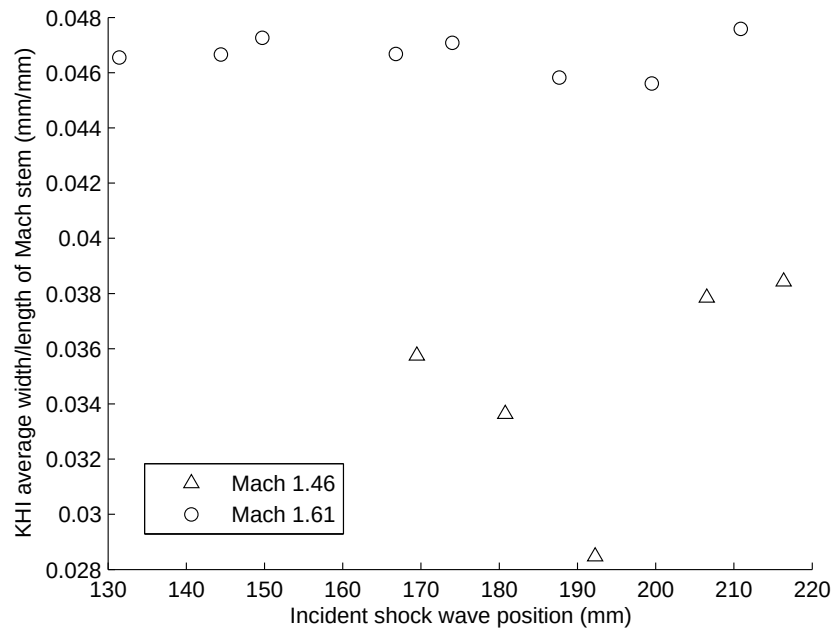


Figure D.21: KHI average width/Mach stem length vs incident shock wave position on Model C.

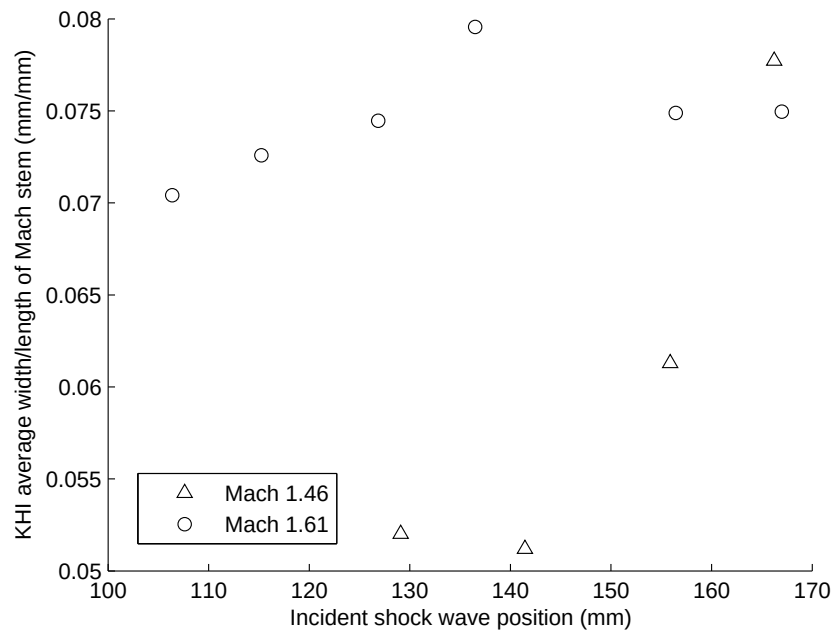


Figure D.22: KHI average width/Mach stem length vs incident shock wave position on Model D.

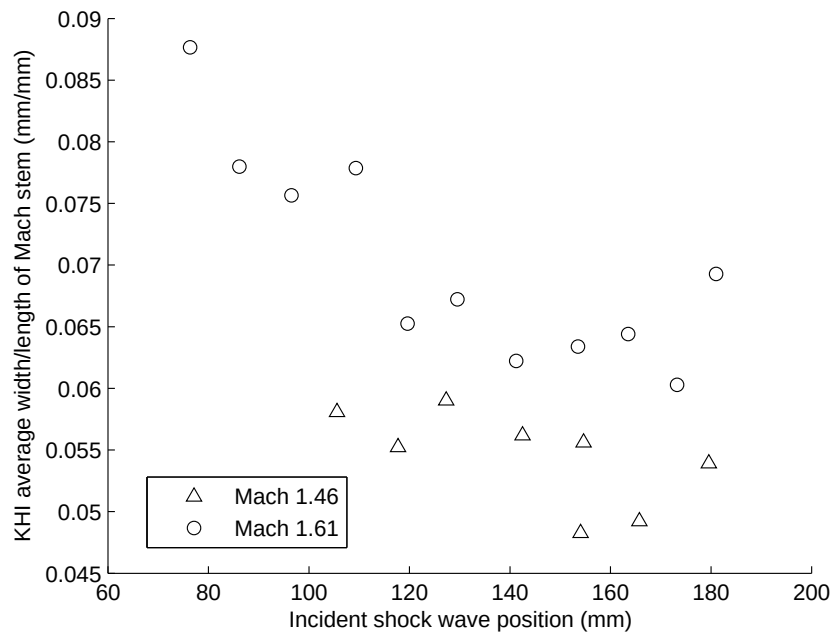


Figure D.23: KHI average width/Mach stem length vs incident shock wave position on Model E.

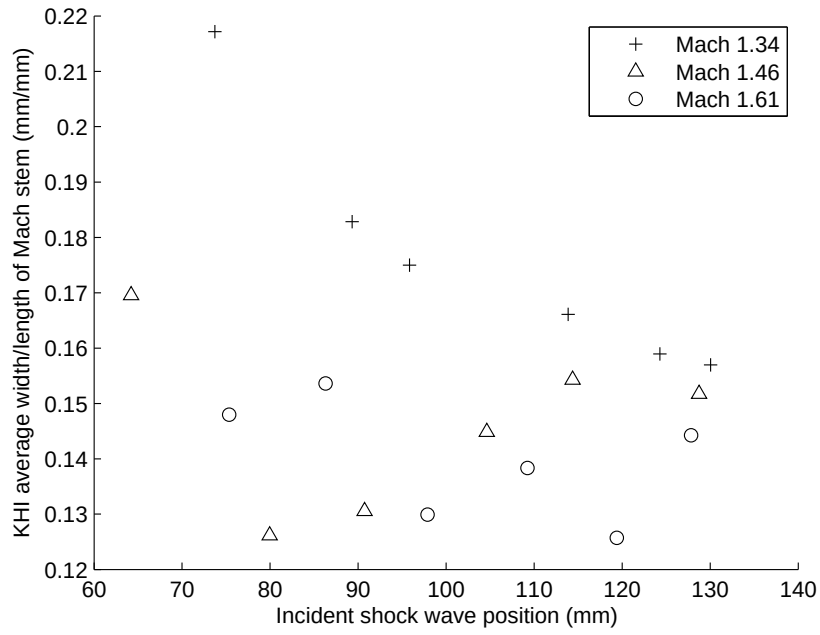


Figure D.24: KHI average width/Mach stem length vs incident shock wave position on Model F.

D.5 KHI length/Mach stem length vs incident shock wave position and Mach number

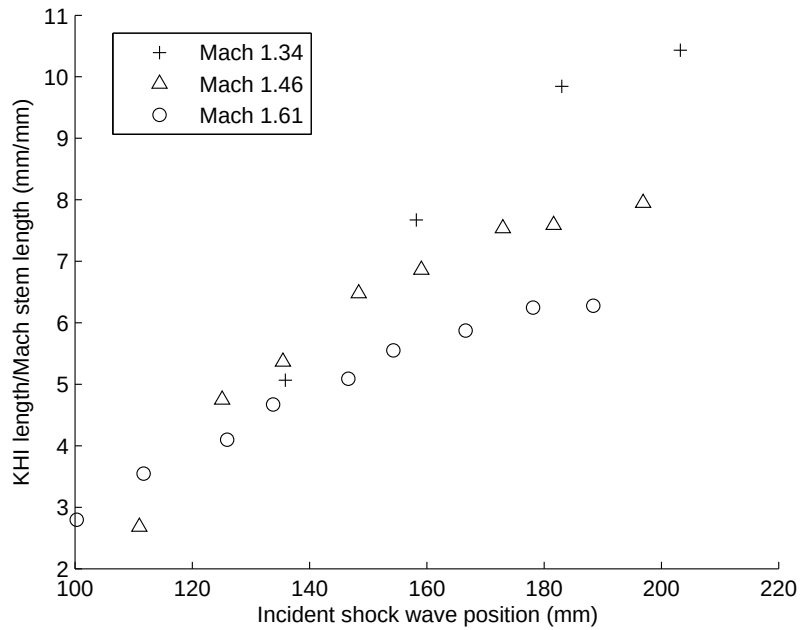


Figure D.25: KHI length/Mach stem length vs incident shock wave position on Model A.

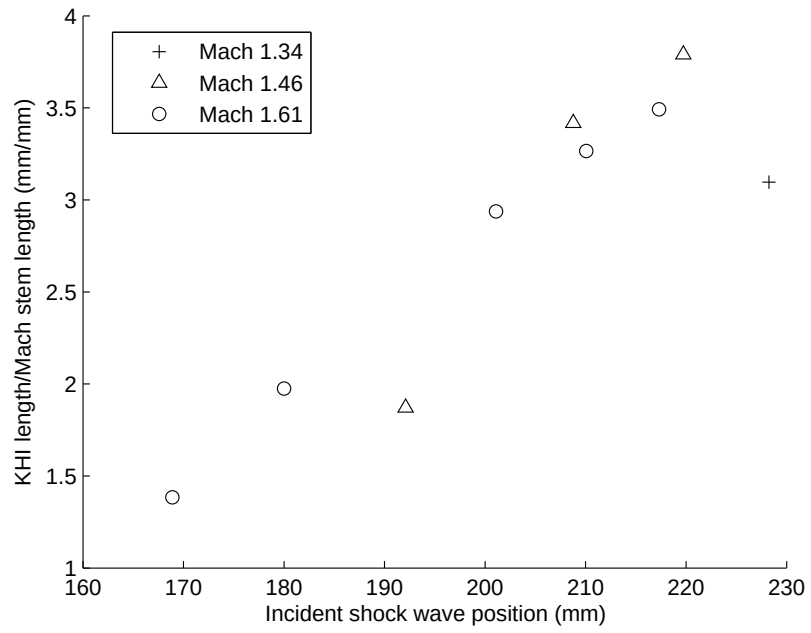


Figure D.26: KHI length/Mach stem length vs incident shock wave position on Model B.

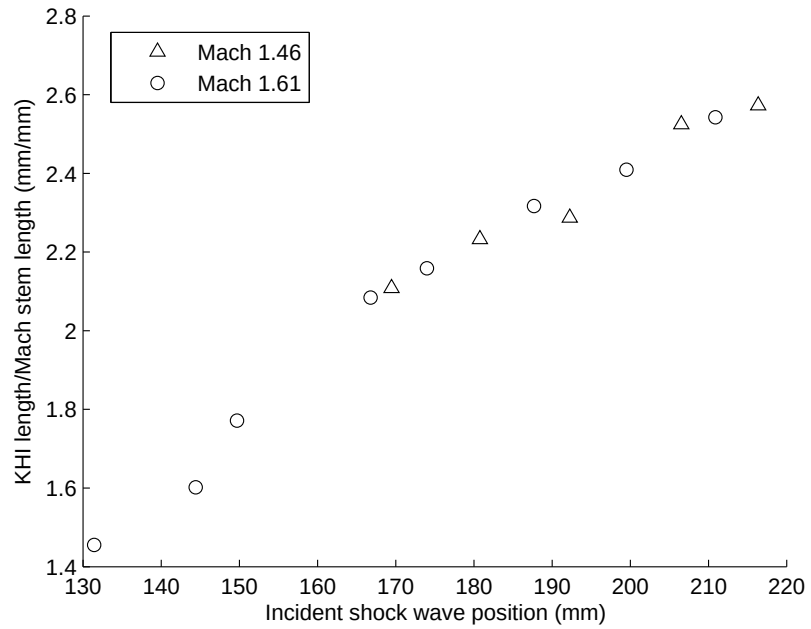


Figure D.27: KHI length/Mach stem length vs incident shock wave position on Model C.

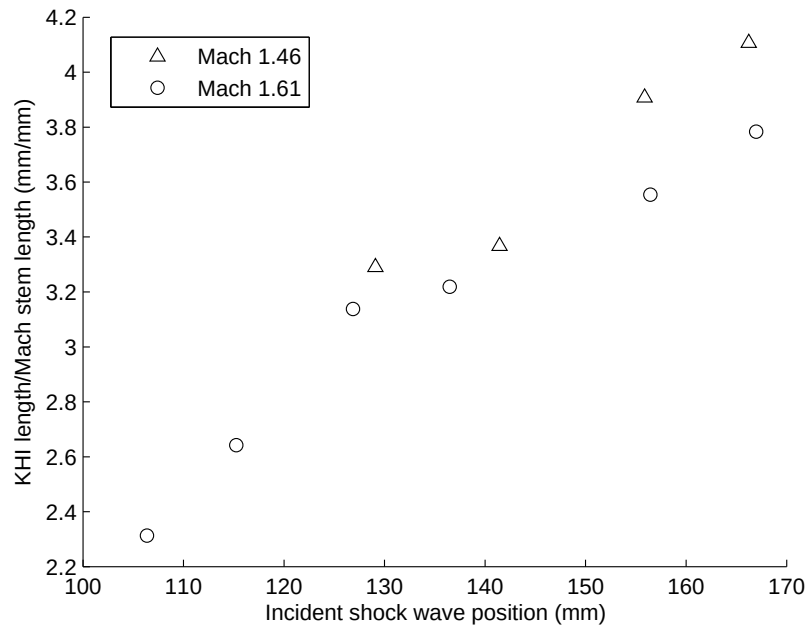


Figure D.28: KHI length/Mach stem length vs incident shock wave position on Model D.

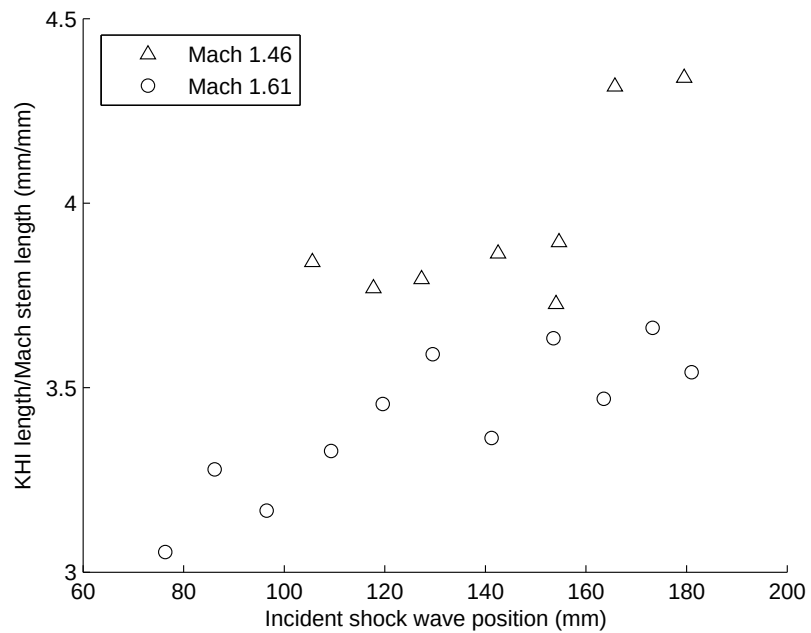


Figure D.29: KHI length/Mach stem length vs incident shock wave position on Model E.

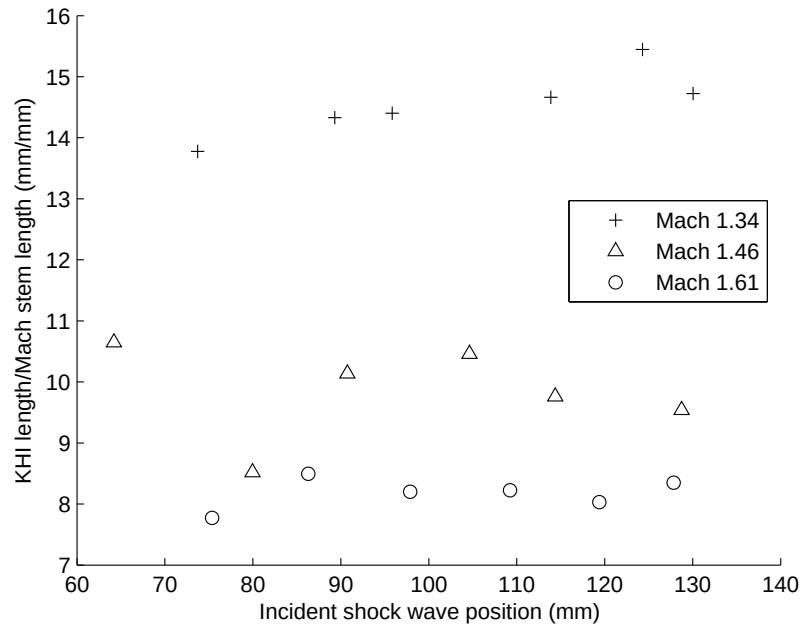


Figure D.30: KHI length/Mach stem length vs incident shock wave position on Model F.

D.6 Shear layer length before KHI initiation vs incident shock wave position and Mach number

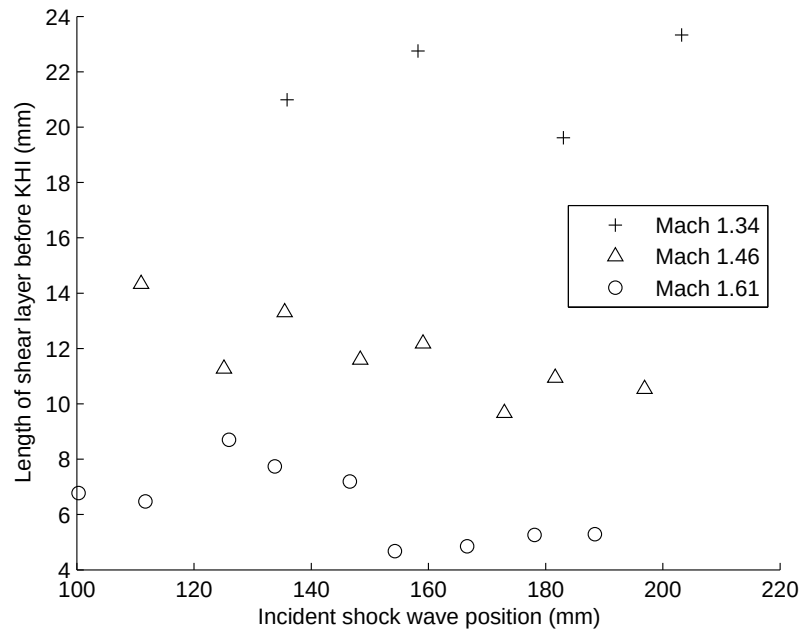


Figure D.31: Shear layer length before KHI initiation vs incident shock wave position on Model A.

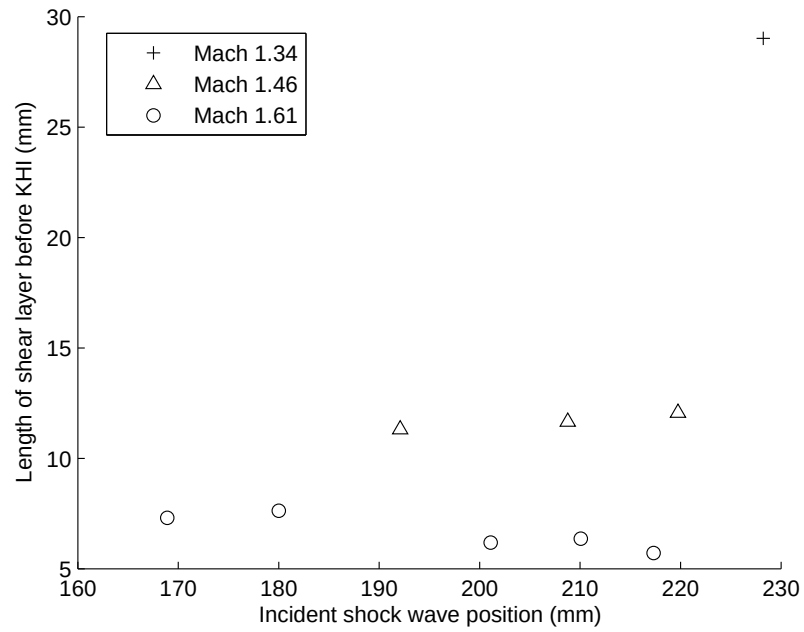


Figure D.32: Shear layer length before KHI initiation vs incident shock wave position on Model B.

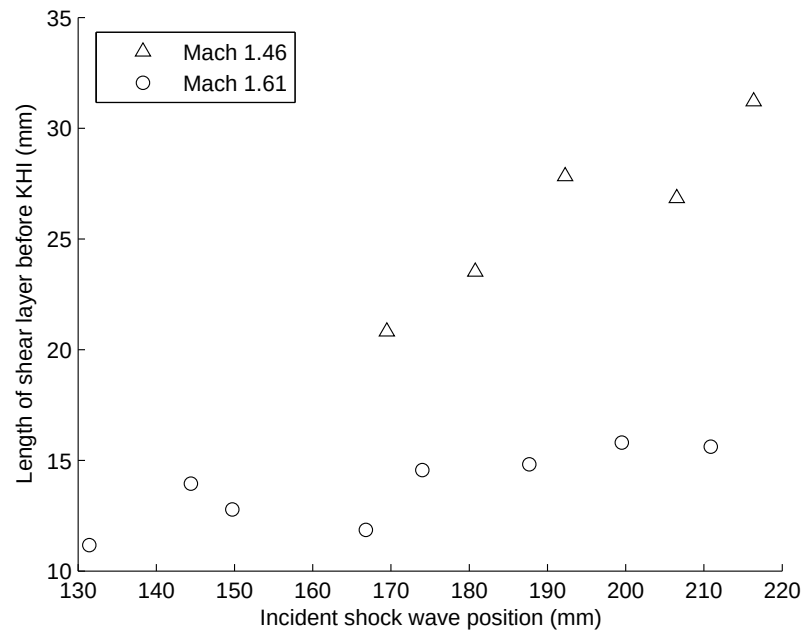


Figure D.33: Shear layer length before KHI initiation vs incident shock wave position on Model C.

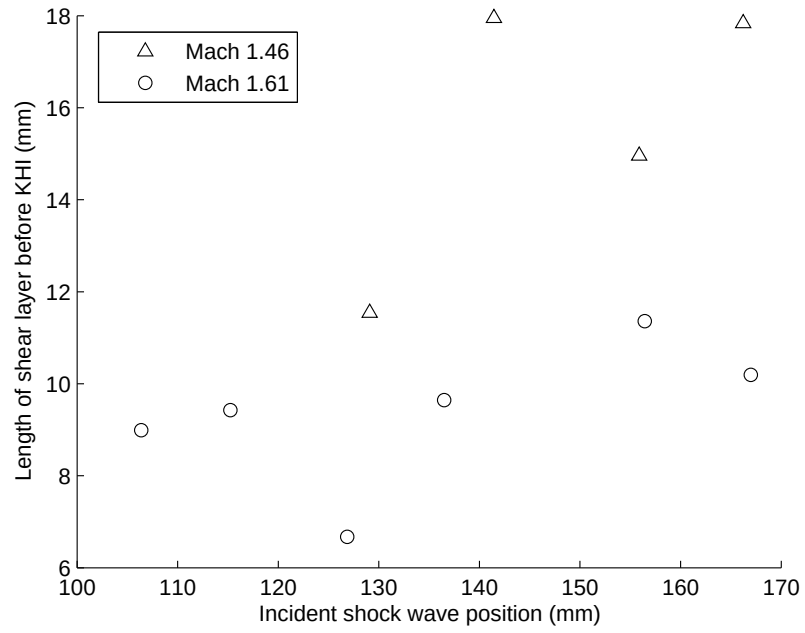


Figure D.34: Shear layer length before KHI initiation vs incident shock wave position on Model D.

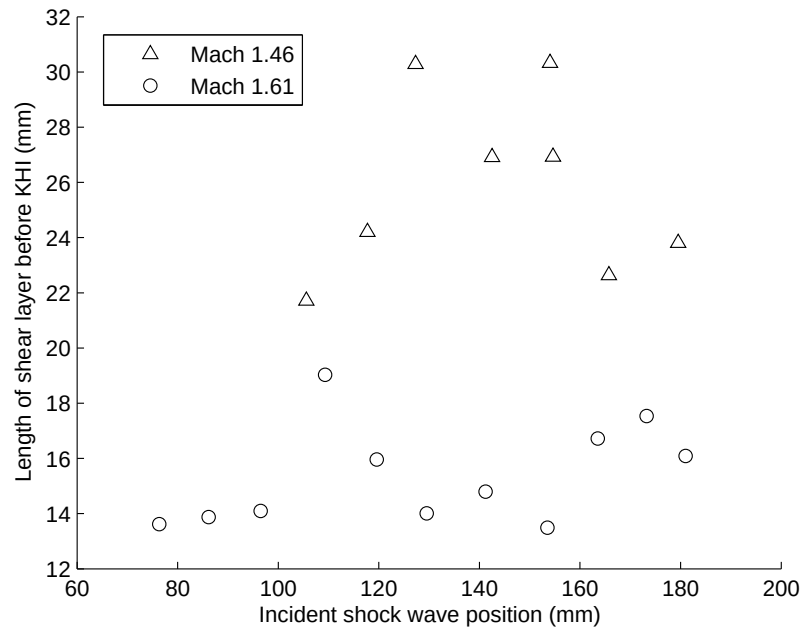


Figure D.35: Shear layer length before KHI initiation vs incident shock wave position on Model E.

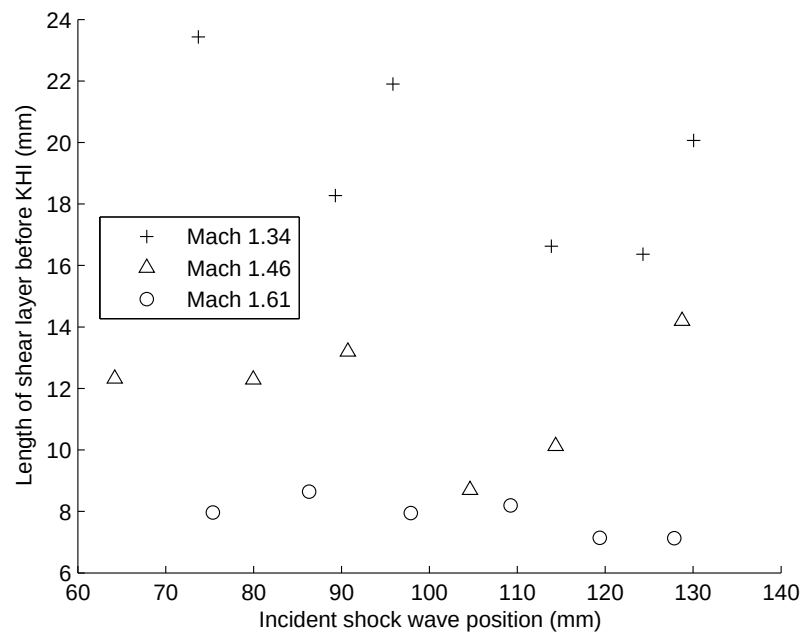


Figure D.36: Shear layer length before KHI initiation vs incident shock wave position on Model F.