

**THE STUDY OF A NOVEL FLAT-TOPPING
RESONATOR FOR MORE INTENSE
PROTON BEAMS OF BETTER
QUALITY FROM
CYCLOTRONS**

JOHN GARRETT DE VILLIERS

THE STUDY OF A NOVEL FLAT-TOPPING RESONATOR FOR MORE INTENSE PROTON BEAMS OF BETTER QUALITY FROM CYCLOTRONS

by

JOHN GARRETT DE VILLIERS

**Promoter: Dr. S. H. Connell
Co-Promoter: Dr. J. L. Conradie**

**Dissertation presented in fulfillment of the requirements
for the degree of Doctor of Philosophy in Science
Department of Physics
University of Witwatersrand**

December 2006

Declaration

I declare that this thesis is my own, unaided work. It is being submitted for the Degree of Doctor of Philosophy in the University of the Witwatersrand, Johannesburg. It has not been submitted for any degree or examination in any other university.

.....

John Garrett de Villiers

..... day of 2006

ACKNOWLEDGEMENTS

I thank my promoter Dr. S.H. Connell for his encouragement, continued interest and assistance.

My sincere appreciation goes to Dr. J.L. Conradie for suggesting the dissertation topic. I thank him for his supervision of this work and allowing me the time at iThemba LABS to prepare this thesis. His guidance and assistance is highly appreciated.

A special word of thanks to Dr. A.H. Botha for his guidance to always find a solution when obstacles were encountered, his patient assistance and his continuous interest.

I would like to thank my colleagues for their assistance with the completion of the project:

Mr. J. van Niekerk for the use of the measuring equipment of the rf-division and also for the discussions during the design phase and the measurement of some data.

Mr. D. Muller for providing the drawings of some components.

Mr. P.F. Rohwer for his proofreading of the dissertation.

I hereby gratefully acknowledge the financial support from the National Research Foundation (NRF) and the opportunity to use the facilities at iThemba LABS.

Finally I would like to thank my family for their patience, support and encouragement throughout all the years.

To my wife Jacqueline

THE STUDY OF A NOVEL FLAT-TOPPING RESONATOR FOR MORE INTENSE PROTON BEAMS OF BETTER QUALITY FROM CYCLOTRONS

John Garrett de Villiers

iThemba LABS, P.O. Box 722, Somerset West 7129, Republic of South Africa
March 2006

ABSTRACT

The multi-disciplinary accelerator based facilities at iThemba LABS are used intensively for nuclear physics experiments, radiotherapy and the production of radioisotopes. To increase the beam intensity for radioisotope production and to improve the beam quality of the 66 MeV proton beam, a double-gap horizontal half-wave flat-topping resonator has been developed for the separated-sector cyclotron to operate at the associated fixed frequency. This type of flat-topping resonator has never before been implemented in a cyclotron and this study is the first to show that it can be done, featuring a special characteristic not offered by other types. The resonator is reviewed against the other types of resonators that are already in use at other institutes around the world.

The flat-topping voltage of the selected type has a sinusoidal half-wave distribution along a radial line in each of its acceleration gaps with the nodal points located on the injection and extraction orbits. The flat-topping voltage therefore progressively increases from zero at both of the two most critical orbits in a cyclotron to a maximum at about halfway between them. As a result will this resonator, apart from its basic function to reduce the energy spread in the beam, not decrease the orbit separation at the injection and extraction orbits in the cyclotron, as is the case with other types of flat-topping resonators. This advantageous feature implies that the beam pattern in the cyclotron is not affected in the regions of the delicate injection and extraction components and therefore will these components or the operational control of the beam not require any modification to accommodate the resonator.

In order to design a resonator that will meet our requirements, the theory of the beam dynamics and resonator characteristics were studied to ascertain the expected improvement in beam quality and beam intensity when a flat-topping resonator is implemented with the cyclotron. All resonator types were considered and studied in terms of their power dissipation, voltage distribution, harmonic number, space requirements and influence on the beam. The horizontal half-wave resonator type, with two acceleration gaps, was selected as the most suitable for our application, because of its preferred electromagnetic characteristics and its geometric shape that permits the installation inside an existing vacuum chamber through an existing flange. Initially a half-scale resonator model was build to test the feasibility of such a resonator and also to verify the calculation methods.

Transmission line methods and numerical field analysis in 3D were applied to determine the resonator characteristics. In the former method a computer program, POISSON, was used to calculate curvilinear squares on sections through a triangular-shaped transmission line and in the latter method a commercial computer program, SOPRANO, was used. SOPRANO is part of an internationally acclaimed suite of programs and the acquired knowledge and skill to use this state-of-the-art software for the studying and designing of such and other electromagnetic devices also put the institute amongst the front-runners in the world.

The calculated characteristics of the half- and full-scale resonator models, the study of the different electromagnetic modes that resonate in close proximity to the required frequency, the heat transport modelling and the theory and implementation of the coupling and tuning devices are all in good agreement with their respective measured results and are reported in this document.

This study lead the way to have the first-ever double-gap horizontal half-wave flat-topping resonator in a cyclotron successfully commissioned at iThemba LABS and the first tests with beam report very stable operation. Accelerator physicists now have another option to utilize for the establishing of flat-topped acceleration voltages.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION AND MOTIVATION	1
1.1 Introduction	1
1.2 Motivation	4
1.3 Outline of the dissertation	7

CHAPTER 2

THE CYCLOTRONS AT iTHEMBA LABS	9
2.1 Introduction	9
2.2 Layout of the cyclotrons and their main parameters	9
2.2.1 The separated-sector cyclotron (SSC)	9
2.2.2 The light-ion solid-pole injector cyclotron (SPC1)	12
2.2.3 The second solid-pole injector cyclotron (SPC2)	12

CHAPTER 3

THE THEORY OF BEAM DYNAMICS IN RESONATORS AND A REVIEW OF RESONATOR SYSEMS	19
3.1 Introduction	19
3.2 The energy spread and beam width in a cyclotron with and without flat-topping	20
3.2.1 Energy spread, beam separation and orbit separation	20
3.2.2 Canceling of longitudinal space-charge (LSC) effects	27
3.3 Limitations on the implementation of a flat-topping resonator in the SSC	28
3.3.1 Geometric restrictions	28
3.3.2 The azimuthal angle, radial length and harmonic number	29
3.4 Types of resonators	30
3.4.1 Single-gap resonator	31
3.4.2 Double-gap resonators	33
3.4.2.1 Vertical half-wave resonator	33
3.4.2.2 Horizontal quarter-wave resonator	34
3.4.2.3 Horizontal half-wave resonator	34

3.4.3	Advantage of having two flat-topping resonators in opposite valleys	36
3.5	Selecting a type of flat-topping resonator	36
3.6	Summary	38

CHAPTER 4

COMPUTATIONAL PHYSICS FOR THE STUDYING OF THE RESONATOR PHYSICS **39**

4.1	Introduction	39
4.2	The transmission line method for calculating the resonator characteristics	39
4.2.1	Impedance distribution	41
4.2.2	Calculation of the resistance per meter	43
4.2.3	Calculation of the resistance of the short-circuit plates	48
4.2.4	Calculation of the current and voltage distributions along the resonator as well as the power dissipation and the Q-value	49
4.3	Numerical field analysis for calculating the resonator characteristics in 3d	52
4.3.1	Introduction	52
4.3.2	The eigen-value solver SOPRANO-EV	53
4.3.3	The SOPRANO post-processor	54
4.3.4	The power dissipation in the resonator	54
4.3.5	Calculation of the stored energy and the Q-value	55

CHAPTER 5

RESONATOR DESIGN AND MEASUREMENT **57**

5.1	Introduction	57
5.2	Resonator dimensions and construction	57
5.3	Calculated results for the full-scale resonator, using the transmission line method.	63
5.4	Calculated and measured results for the half-scale model	65
5.4.1	Measuring techniques and equipment	65
5.4.2	Half-scale resonator	67
5.5.	Results of calculations with numerical field analysis in 3d and measurements on the full-scale resonator	70
5.5.1	Electromagnetic field distribution	70
5.5.2	Calculated resonance frequency, Q-value, stored energy, scaling of the calculated power dissipation and the radial voltage distribution	74

5.5.3	Measuring the Q-value	77
5.6	Separating the resonance frequency of the unwanted mode from the wanted mode	79
5.7	Calculated influence of temperature change on the resonance frequency	85
5.7.1	Introduction	85
5.7.2	Change in the length of the resonator	85
5.7.3	Change in the width of the resonator	86
5.7.4	Change in the height of the resonator	86
5.7.5	Summary	86
5.8	The acceleration gap and the transit-time factor	87
5.8.1	Introduction	87
5.8.2	The transit-time factor for the SSC flat-topping resonator	88
5.9	Tolerance on difference in dee voltage on the top and bottom dee halves due to a vertical displacement of the dee plates	90
5.10	Summary	92

CHAPTER 6

THEORY AND IMPLEMENTATION OF THE TUNING AND COUPLING SYSTEM OF THE RESONATOR		93
6.1	Introduction	93
6.2	The inductive loops	94
6.2.1	General description and location of the loops	94
6.2.2	The coupling loop	95
6.2.3	The tuning loop	100
6.2.4	Probes for dee voltage measurement	104
6.3	The capacitors	104
6.3.1	General description and location	104
6.3.2	Electrical characteristics of the capacitor plates	106
6.3.3	Measured and calculated values	109
6.4	The effect of the loops and capacitors on the resonance frequency: results of calculations with SOPRANO	111

CHAPTER 7

HEAT TRANSPORT MODELLING 115

7.1 Introduction 115

7.2 Distribution of the pipes for water cooling 115

7.2.1 Calculation of the distances between adjacent cooling water pipes 115

7.2.1.1 Calculation of the cooling pipe distribution on segments
of the resonator 117

7.2.1.2 SOPRANO calculations of adjacent pipe distances 123

7.2.2 The layout of the cooling pipes according to the calculated spacing 128

CHAPTER 8

CONTEXTUAL REVIEW OF THE DOUBLE-GAP HORIZONTAL HALF-WAVE FLAT-TOPPING RESONATOR 131

CHAPTER 9

SUMMARY AND CONCLUSIONS 135

APPENDIX A: Basic flat-topping principles 139

A.1 The harmonic numbers of a cyclotron 139

A.2 Beam quality with a flat-topping resonator operating at the 3rd harmonic 145

A.2.1 Harmonic number 145

A.2.2 Flat-topping voltage 145

A.2.3 Energy gain per turn 146

A.2.4 Energy spread 146

A.3 Azimuthal angle of a flat-topping resonator 148

APPENDIX B: Hard-edge formulae for the separated-sector cyclotron at iThemba LABS 151

APPENDIX C: A spreadsheet program to calculate the energy spread and radial beam width of the beam in the SSC, using the hard-edge formulation 157

C.1 Introduction 157

C.2 The expressions used for the calculation 158

C.3 The input parameters for the program 160

C.4 The calculation procedure 162

C.5 Example 163

APPENDIX D: Cancelling of the longitudinal space-charge effects in the beam	169
D.1 Introduction	169
D.2 Compensation without flat-topping	170
D.3 Compensation with flat-topping	173
APPENDIX E: Electrical properties of some materials	179
APPENDIX F: The transit-time factor	181
APPENDIX G: Calculation of the spacing between cooling ducts and the amount of water required for the resonator	185
G.1 Fourier's law for heat conduction	185
G.2 Distances between cooling pipes and required water flow	187
G.2.1 Minimum distance from any heat source on a conducting plate to the nearest cooling pipe	187
G.2.2 Dissipation of heat from a plate with homogeneous heat generated between cooling ducts that are kept constant but different temperatures	189
G.2.3 The water flow required to remove the generated heat away from the resonator	192
APPENDIX H: Kilpatrick's criterion	195
APPENDIX I: Scaling of the resonator parameters for a model	197
APPENDIX J: A spreadsheet program to calculate the phase history of the vertical motion in the SSC due to a vertical displacement of the dee plates in relation to the median plane	199
J.1 Introduction	199
J.2 Theory	199
J.3 Input parameters and calculation procedure	203
J.4 Results	207
REFERENCES	209

CHAPTER 1

INTRODUCTION AND MOTIVATION

1.1 INTRODUCTION

iThemba Laboratory for Accelerator Based Sciences (iThemba LABS) is a multi-disciplinary research centre, which provides beams of accelerated charged particles and facilities for particle radiotherapy, basic and applied nuclear physics research and the production of radioisotopes for nuclear medicine and industry.

In the nineteen sixties the future need and demand for improved accelerated particle beams in South Africa were discussed amongst physicists in South Africa. The main accelerators in operation at that stage were the variable-energy cyclotron at the Council for Scientific and Industrial Research (CSIR), the 6 MV Van de Graaff accelerator of the Southern Universities (SUNI) near Faure, the 6 MV Tandem Van de Graaff accelerator at the University of Witwatersrand and a 4 MV Van de Graaff at the then Atomic Energy Board, nowadays called the Nuclear Energy Corporation of South Africa (NECSA). The locally-built CSIR cyclotron could accelerate light ions to the following maximum energies: 15.3 MeV for protons, 17 MeV for deuterons, 34 MeV for alpha particles and 39 MeV for Helium-3 ions. Deuteron and Helium-3 beams were, apart from for nuclear physics research, also used for the production of radioisotopes for South African hospitals and industries. Radioisotopes were also regularly exported to Europe and the USA. Experience gained with the design, construction and use of the CSIR cyclotron gave physicists the confidence to start with the design of a separated-sector cyclotron (SSC) - a new type of cyclotron then under construction at the University of Indiana and the Swiss Institute for Nuclear Research (SIN), which later became the Paul Scherrer Institute (PSI). A feasibility study for a national accelerator facility followed and its recommendation [Rau75, Rep75, Abr75] for a $K = 200$ MeV facility with a separated-sector cyclotron [Wil63] and two solid-pole injector cyclotrons (SPC1 and SPC2) was

approved by Government in 1975 and the National Accelerator Centre (NAC) was established in 1977 as a multi-disciplinary research institute under the auspices of the Council for Science and Industrial Research (CSIR). Construction of these, by the NAC custom-designed, cyclotrons and associated beam lines started in 1978 and became operational during 1986.

The Southern Universities Nuclear Institute (SUNI), which has been incorporated into NAC, commissioned a commercially acquired 6 MV CN Van de Graaff accelerator in 1963 [Kon64]. It is capable of delivering beams of light and heavy ions with a low energy spread. A commercial linear accelerator was also incorporated with the existing facilities in 2001, in conjunction with private medical physicians.

In 2002 National Accelerator Centre was renamed as iThemba Laboratory for Accelerator Based Sciences, iThemba LABS.

Currently the above mentioned Van de Graaff and linear accelerators, the main accelerator, a $K = 200$ separated-sector cyclotron, SSC, and two $K = 8$ solid-pole injector cyclotrons, SPC1 and SPC2, are in operation at iThemba LABS. The injector cyclotron, SPC1, with its internal ion source, is routinely used for the acceleration of protons only, whereas the SPC2 provides beams of heavy ions and polarized protons from two external ion sources.

A separated-sector cyclotron, which requires an injector cyclotron, was chosen as the main accelerator [Rau75] for, amongst others, its good beam transmission for the high beam intensities required at high energies for the production of radioisotopes. Because the dees in a separated-sector cyclotron are located in the spaces between the sector magnets, the magnet pole gaps can be made small, which results in good vertical focusing of the beam, and the dee voltages can be much higher than in a solid-pole cyclotron, thereby leaving relatively large open spaces between adjacent orbits at extraction for the insertion of extraction components. Because of the small pole gaps it is possible to make the magnetic field isochronous up to the last orbit in the cyclotron, after which the field drops fast enough with radius to extract the beam in a single turn. This further increases

the orbit separation and enhances beam extraction. The open spaces, called valleys, between magnet sectors also allow the installation of a flat-topping system [Con92a] with which the energy spread, and consequently the radial width of the beam, is decreased to yield better beam separation between adjacent turns at extraction, thereby limiting activation of extraction components, which makes maintenance and repair work less hazardous for staff.

A solid-pole cyclotron with a maximum energy of 8 MeV for protons was chosen as an injector cyclotron for the SSC for the acceleration of light-ion beams, because of its compactness, its relative simple construction, the high beam intensities that can be obtained with an internal ion source and the fact that it does not require another pre-accelerator. SPC1 and the SSC have since the commissioning at the end of 1986 operated on a 24 hour per day and 7 day per week schedule. A second solid-pole injector cyclotron, SPC2, for acceleration of heavy ions and polarized hydrogen ions was later built.

One of the initial design requirements for the cyclotrons at iThemba LABS was to deliver proton beams at energies of below 80 MeV with a beam intensity of 100 μA for the production of radioisotopes for medical and industrial applications. Eventually it was decided to use a 66 MeV proton beam for the production of radioisotopes because this would be the most suitable beam energy for neutron therapy, and time-consuming energy changes could be eliminated by using the same beam for both applications. The radioisotopes that are produced regularly on a weekly basis at iThemba LABS are: ^{67}Ga , ^{123}I (solution, capsules and diagnostic agent, MIBG), $^{81}\text{Rb}/^{81}\text{mKr}$ generators and ^{18}F -FDG. Long-lived radioisotopes produced on demand are: ^{22}Na , ^{22}Na positron sources, ^{68}Ge , ^{82}Sr , ^{139}Ce and ^{88}Y . These radioisotopes are delivered to all the major hospitals in the Republic of South Africa and most of the radioisotopes with long half-lives are exported to the USA [Van04].

The external beam intensities from SPC1 and the SSC for the 66 MeV proton beam are limited to 300 μA and 150 μA , respectively. In SPC1, one of the main contributing

factors to this limitation is the increase in the energy spread in the beam, and consequent radial broadening of the beam, with increasing beam intensity due to space-charge forces. By accelerating longer beam pulses the influence of space charge on the beam can be reduced, but then the energy spread in the beam and the radial width also increases due to the sinusoidal dee voltage. With wider beams the free space between turns for the extraction components are smaller and the extraction efficiency decreases, resulting in further activation and even overheating of these components. In the SSC space-charge forces play a less significant role. The beam pulse length has to be kept small to limit the energy spread and the beam width. Therefore beam pulses from SPC1 are focused longitudinally in the transfer beam line with a beam buncher before injection into the SSC. The pulse length to which the beam can be focused in the SSC is limited by the energy spread in the beam from SPC1.

1.2 MOTIVATION

The demand for the carrier-free radioisotopes produced at iThemba LABS, from both the local and overseas market, has increased over the past few years because new advances in the medical applications of radioisotopes not only demand higher activities, but also require the development of new procedures and products. Two turn-key, low energy cyclotrons have been commissioned in Gauteng since mid-2005, mainly for the production of ^{18}F -FDG. The present and future demand for quality radio-pharmaceuticals [Nor02] from both the local and international market warrants upgrading of the existing facilities at iThemba LABS to be able to produce the quantities required by the clients, thereby ensuring long-term partnerships and generate income. Since the cyclotrons have from the start been operated 24 hours per day and 7 days per week, beam time for the production of radioisotopes cannot be increased. Therefore, to increase the income from the production of radioisotopes, the beam intensity has to be increased. Acquisition of a dedicated cyclotron has also been considered, but has been discarded, because of the cost involved, especially since a new building with shielded areas are required.

To increase the maximum extracted beam current for the 66 MeV proton beam from the present 150 μA to 500 μA , longer beam pulses will have to be accelerated in both SPC1 and the SSC, without increasing the energy spread in the beam. Since this is not possible with sinusoidal dee voltages, additional resonators will have to be installed to flatten the dee voltages in the time intervals in which particles are accelerated, i.e. at the dee voltage peaks, hence the descriptive name flat-topping. The implementation of such flat-topping rf systems in SPC1 and the SSC will increase the external beam intensities delivered by these machines and also their extraction efficiencies for longer beam pulses than are presently accelerated.

The implementation of a flat-topping system in SPC1 has already been described [Con92b]. It was demonstrated experimentally that a proton beam with an intensity of 600 μA can be extracted from SPC1 with such a flat-topping system, operating at the fifth harmonic of the main rf frequency; the first cyclotron to operate with the flat-top dee voltage superimposed on the main dees.

The longer beam pulses delivered by SPC1 with a flat-topping system, for injection into the SSC, will cause a large energy spread during further acceleration in the SSC and therefore increase beam losses in the extraction system and beam lines to the isotope production facilities unless a flat-topping system is also installed in the SSC. Such a system would permit acceleration and extraction of 66 MeV proton beams with intensities of up to 500 μA with practically no losses. The main components of such a system are a resonator, a power amplifier and a feedback system, with which the amplitude and phase of the dee voltage can be stabilized to within very tight tolerances [Con92c].

The resonator will have to fit in the available space in one of the valley vacuum chambers, that are partly filled with diagnostic equipment and injection and extraction components, and has to be installed through one of the existing ports of the chamber to exclude costly re-machining of the chamber and long down-time of the SSC, which will result in a loss of the present income from the production of radioisotopes, and suspension of the nuclear physics experiments as well as the treatment of patients for six

to nine months. The design has also to allow for access to service the resonator and other equipment in the cyclotron. These severe restrictions have a strong influence on the design of the resonator. To optimize the resonator design the beam characteristics in the SSC have to be calculated.

Flat-topping systems for the improvement of the beam intensity and quality have been implemented at several accelerator facilities. The system for the RIKEN AVF cyclotron [Koh98] is modelled on the SPC1 flat-topping system, in which the main dees and resonators resonate simultaneously at two different frequencies. The first successful flat-top acceleration in a cyclotron was achieved in 1979 [Bis79] with the 590 MeV ring cyclotron at the Swiss Institute for Nuclear Research (SIN) at Villigen, Switzerland (later renamed the Paul Scherrer Institute (PSI)). A single cavity-type flat-topping resonator operating at the third harmonic of the main rf frequency of 50 MHz is used. At the Research Center for Nuclear Physics (RCPN) at the University of Osaka, Japan, a 30 kW single-gap flat-topping resonator was installed [Sai86, Sai89, Sai92] in the ring cyclotron during the late nineteen eighties. It operates in the frequency range 90 MHz to 155 MHz, the third harmonic of the main rf system. At the Japanese Institute of Physical and Chemical Research (RIKEN) a flat-topping resonator is used in the 4-sector intermediate stage ring cyclotron (IRC). A similar flat-topping resonator is planned for the 6-sector super-conducting ring cyclotron (SRC). These flat-topping resonators are modified versions of the flat-topping resonator used in the ring cyclotron of RCPN [Sak01]. During 2000 the Japanese Atomic Energy Research Institute (JAERI) at Tagasaki, tested models of flat-topping resonators for the AVF cyclotron [Kur01]. A theoretical study of a flat-topping resonator with variable frequency for the SSC was done but never implemented [Con92d].

This dissertation deals with a fixed-frequency flat-topping resonator for the high-intensity 66 MeV proton beam at iThemba LABS. The resonator that has been selected for this purpose is of a new type and design and is different from those used in any of the existing cyclotrons. It is therefore important to study its characteristics and influence on the beam.

1.3 OUTLINE OF THE DISSERTATION

The structure of the dissertation is as follows:

- The facilities at iThemba LABS are briefly described in chapter 2.
- Chapter 3 deals with the theory of beam dynamics in resonators, particularly energy spread, a review of the types of resonators and the selection of a resonator type.
- Chapter 4 deals with the computational physics used for studying the resonator characteristics.
- Chapter 5 deals with the calculated and measured resonator characteristics for both the half scale and the final resonator.
- Chapter 6 deals with the theory and implementation of the tuning and coupling systems of the resonator.
- Chapter 7 deals with the heat transport modeling and its application for the cooling of the resonator.
- Chapter 8 gives a contextual review of the selected resonator.
- Finally, the conclusions drawn from this study and recommendations for further work on intense proton beams of high quality at iThemba LABS are given in chapter 9.

-o-O-o-

CHAPTER 2

THE CYCLOTRONS AT iTHEMBA LABS

2.1 INTRODUCTION

The solid-pole injector cyclotron for light ions, SPC1, has an internal ion source, capable of delivering intense beams of light ions and is mainly used for acceleration of proton beams for proton and neutron-therapy as well as for the production of radioisotopes. Light and heavy ion beams produced in an external electron cyclotron resonance (ECR) ion source and polarized protons from a second external ion source are accelerated in the second solid-pole injector cyclotron, SPC2, before injection into the SSC. SPC1 and SPC2 are cyclotrons with maximum proton energies of 8 MeV and the beam energies of all three machines can be varied by a factor greater than 10.

Beams from the SSC can be directed to vaults for radioisotope production, neutron therapy, proton therapy, a 1.5 m scattering chamber, a $K = 600$ QDD magnetic spectrometer and AFRODITE, a gamma spectrometer array, for nuclear physics experiments. The 66 MeV proton beam is routinely switched between neutron therapy and radionuclide production.

The facilities of iThemba LABS are shown in figure 2.1. The facilities are operated according to the typical tight time schedule shown in figure 2.2.

2.2 LAYOUT OF THE CYCLOTRONS AND THEIR MAIN PARAMETERS

2.2.1 The separated-sector cyclotron (SSC)

The variable-energy SSC is the main accelerator at iThemba LABS and was designed to accelerate protons up to 200 MeV, though occasionally it accelerated protons to 227 MeV for special purposes. The layout of the SSC with its four sector magnets, SM1, SM2, SM3 and SM4 and vacuum chambers with its resonators and diagnostic equipment is shown in figure 2.3. The main parameters of the SSC, together with those of SPC1 and SPC2, are listed in a table 2.1 at the end of this chapter [Con92e].

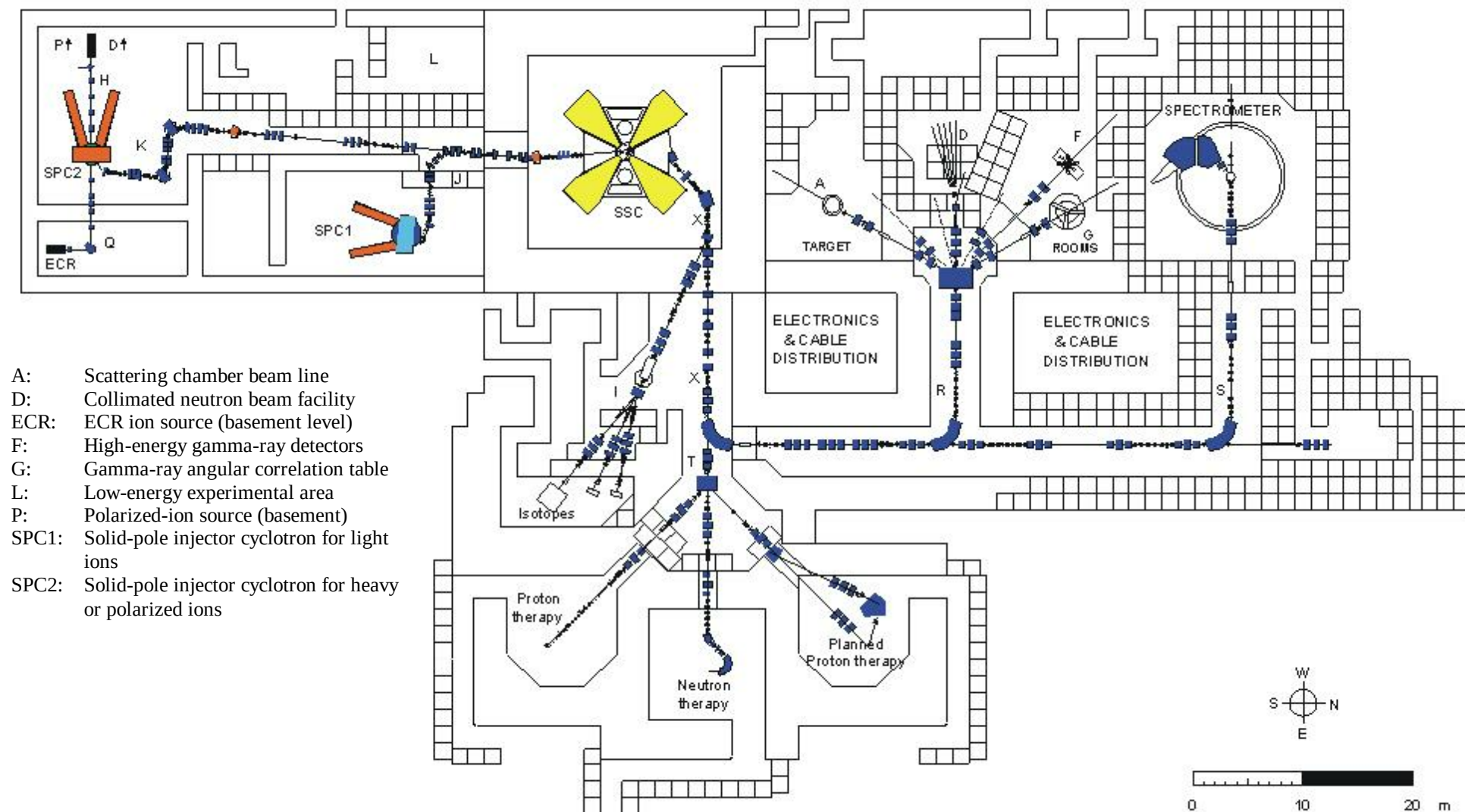


Figure 2.1: *Layout of the cyclotron facilities at iThemba LABS.*

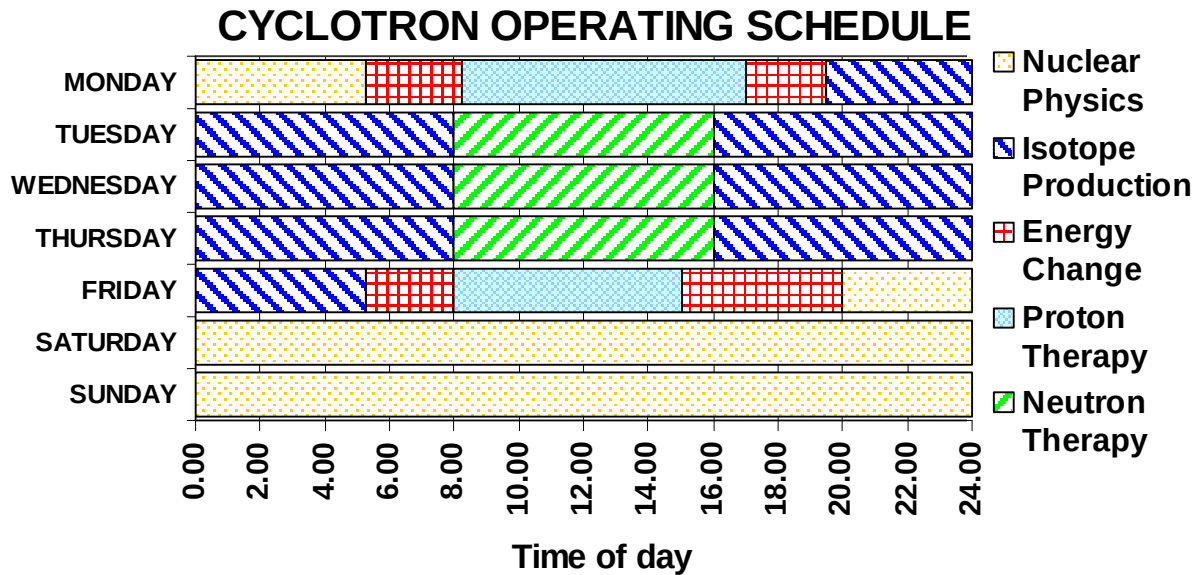


Figure 2.2. The continuous utilization of the beam is shown in the operating schedule of the cyclotron facilities at iThemba LABS.

The SSC has a diameter of 13.2 m and a height of 7 m. The vacuum chambers are mounted in the pole gaps and in two valleys. The four sector magnets, with a sector angle of 34° , weigh about 1400 tons in total and are positioned to an accuracy of 0.1 mm. The maximum magnetic flux density of 1.3 T can be obtained in the 66 mm pole gap by means of the main coils around the poles and additional coils around the yoke pieces. The additional coils are also used to correct differences between the four sector magnets. The currents through 29 trim-coils mounted on each pole surface are adjusted with every energy change to isochronize the magnetic field. The 4 magnet sectors ensure that the beam repeatedly traverse the two $\lambda/2$ -resonators, that are each capacitively coupled to a 150 kW power amplifier with a frequency range of 6 to 26 MHz. The maximum dee voltage is 220 kV, resulting in an energy gain of 0.88 MV per turn. Several feedback systems keep the dee voltage and phase of the high Q-value resonators stabilized and tuned by compensating for beam loading and temperature changes. Similar feedback systems are used for the injector cyclotrons.

The injection system of the SSC consists of two bending magnets and a magnetic inflection channel (MIC). The beam is extracted with two septum magnets (SPM1 & SPM2). An electrostatic extraction channel (EEC) is also available but has seldom been used to avoid the complication of aligning the beam through three extraction components. The large spaces between magnet sectors allow resonators with high dee voltages to obtain good orbit separation at extraction. With the dees located outside

the pole gaps of the magnets it permits the use of small pole gaps and thus resulting in strong vertical beam focusing.

The SSC accelerates beams of light and heavy ions as well as beams of polarized protons. Proton beam intensities of more than 100 μA , at 66 MeV, are extracted from the SSC for the production of radioisotopes. Activation of the components of the SSC is limited, owing to the high extraction and transmission efficiency of the SSC, which is typical 99.8% [Con01].

2.2.2 The light-ion solid-pole injector cyclotron (SPC1)

SPC1 pre-accelerates and provides beams of light ions, mainly protons, that can be injected into the SSC for further acceleration. The layout of SPC1 is shown in figure 2.4. It uses an internal Penning Ionization Gauge (PIG) ion source to produce the intense beams of protons required for neutron therapy and radioisotope production. It has two 90° dees, connected to quarter-wave resonators, and driven by 20 kW power amplifiers, for acceleration on harmonic numbers either 2 or 6, depending on the charge/mass ratio and final energy of the accelerated particle. Dee voltages of up to 60 kV and three constant-orbit geometries are required to accommodate the desired energy ranges and beam currents for the light ions. The beam is extracted from this four-sector cyclotron, with radial magnet sectors, at an extraction radius of 0.476 m, with an electrostatic channel and two active magnetic channels. A third passive channel steers the beam into the beam line.

2.2.3 The second solid-pole injector cyclotron (SPC2)

The layout of SPC2 is shown in figure 2.5. SPC2, which has a K-value of 10 for heavy ions, pre-accelerates and provides beams of both heavy ions and polarized hydrogen ions for the SSC, from its two external ion sources respectively, i.e. an ECR ion source and a polarized ion source, as shown in figure 2.6. The diagnostic equipment includes a harp, which can be positioned near the extraction radius for minimization of the energy spread by adjustment of the magnetic field. SPC2 is in many respects similar to SPC1 but uses axial injection through spiral inflectors from the two external ion sources.

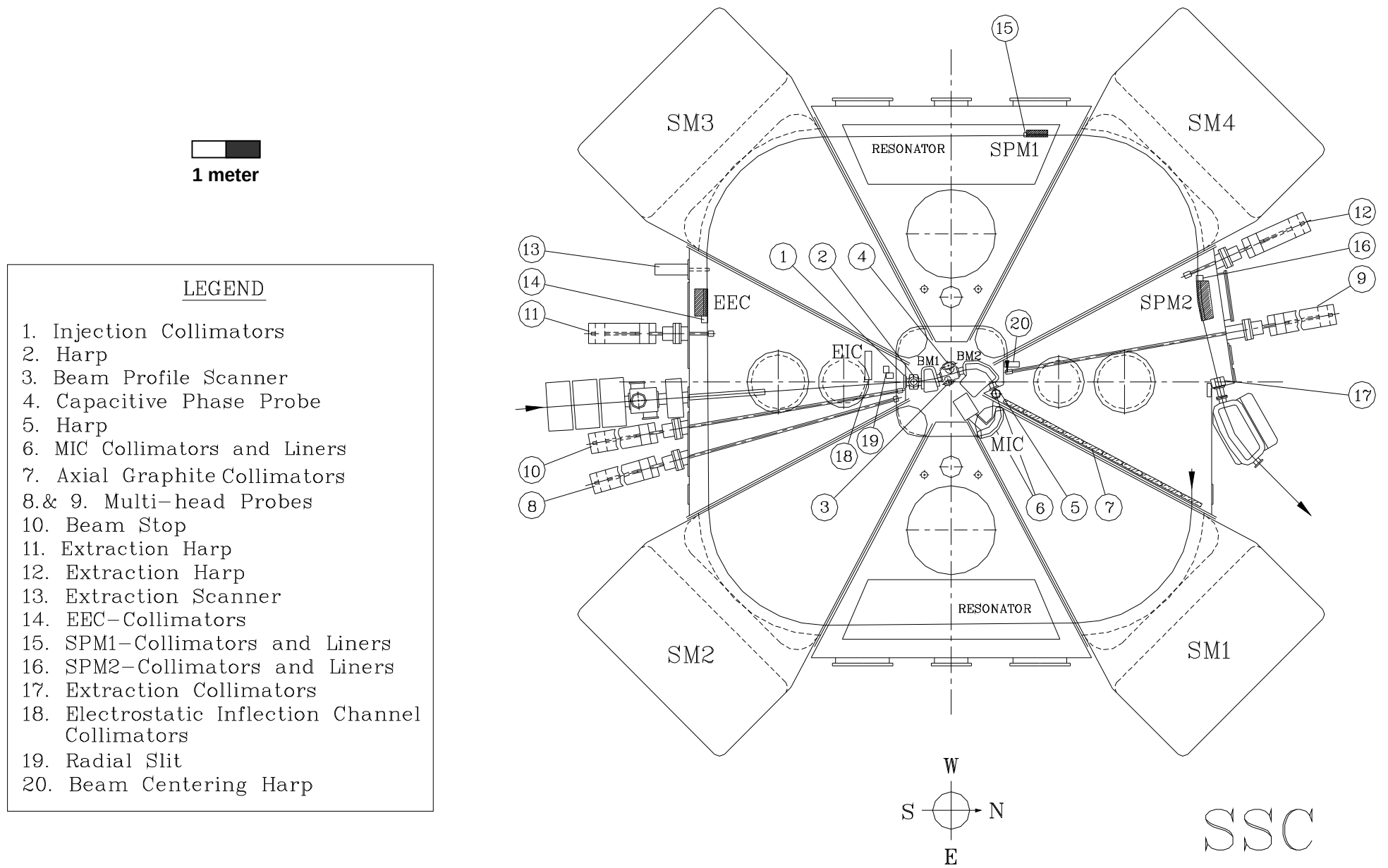
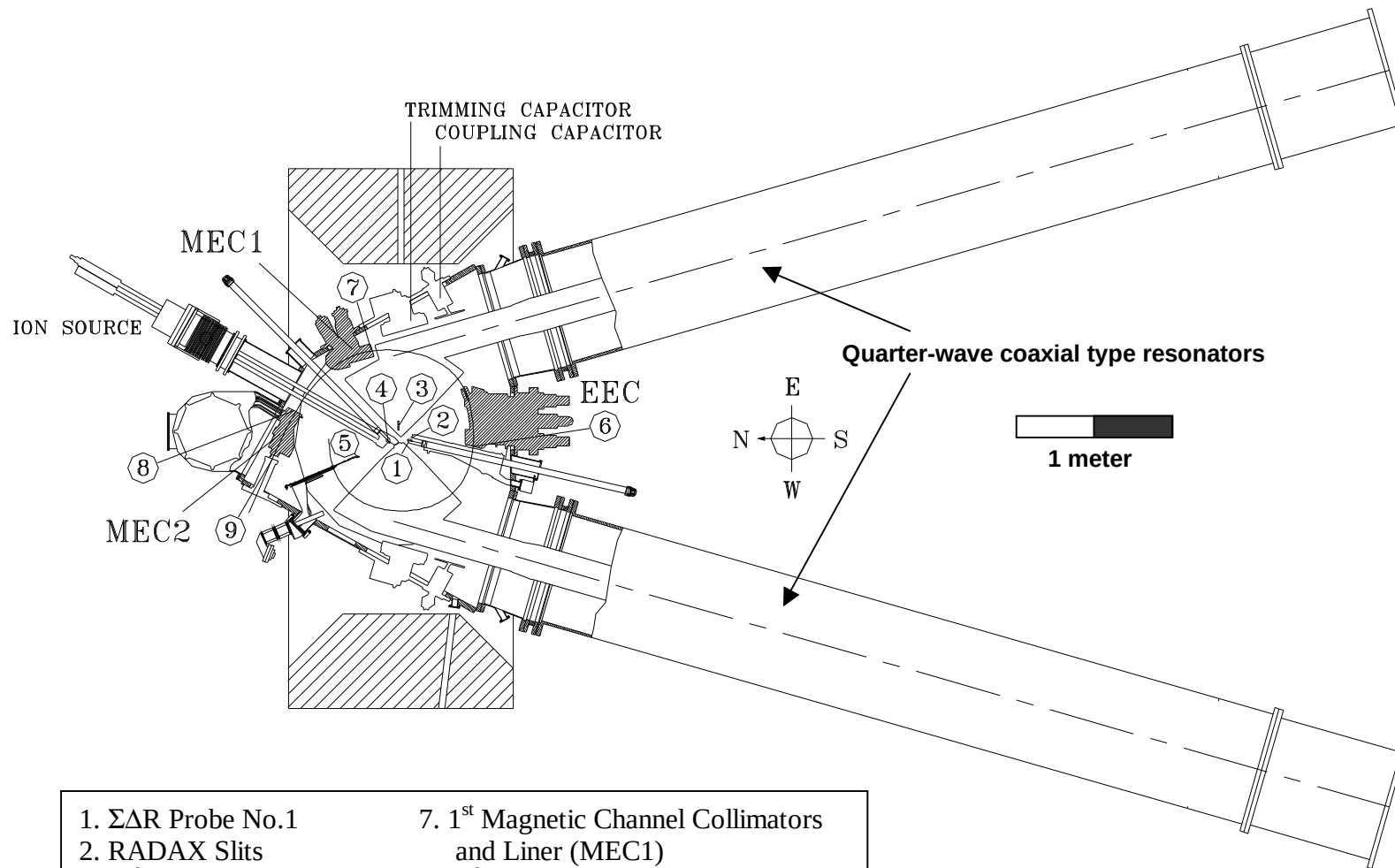


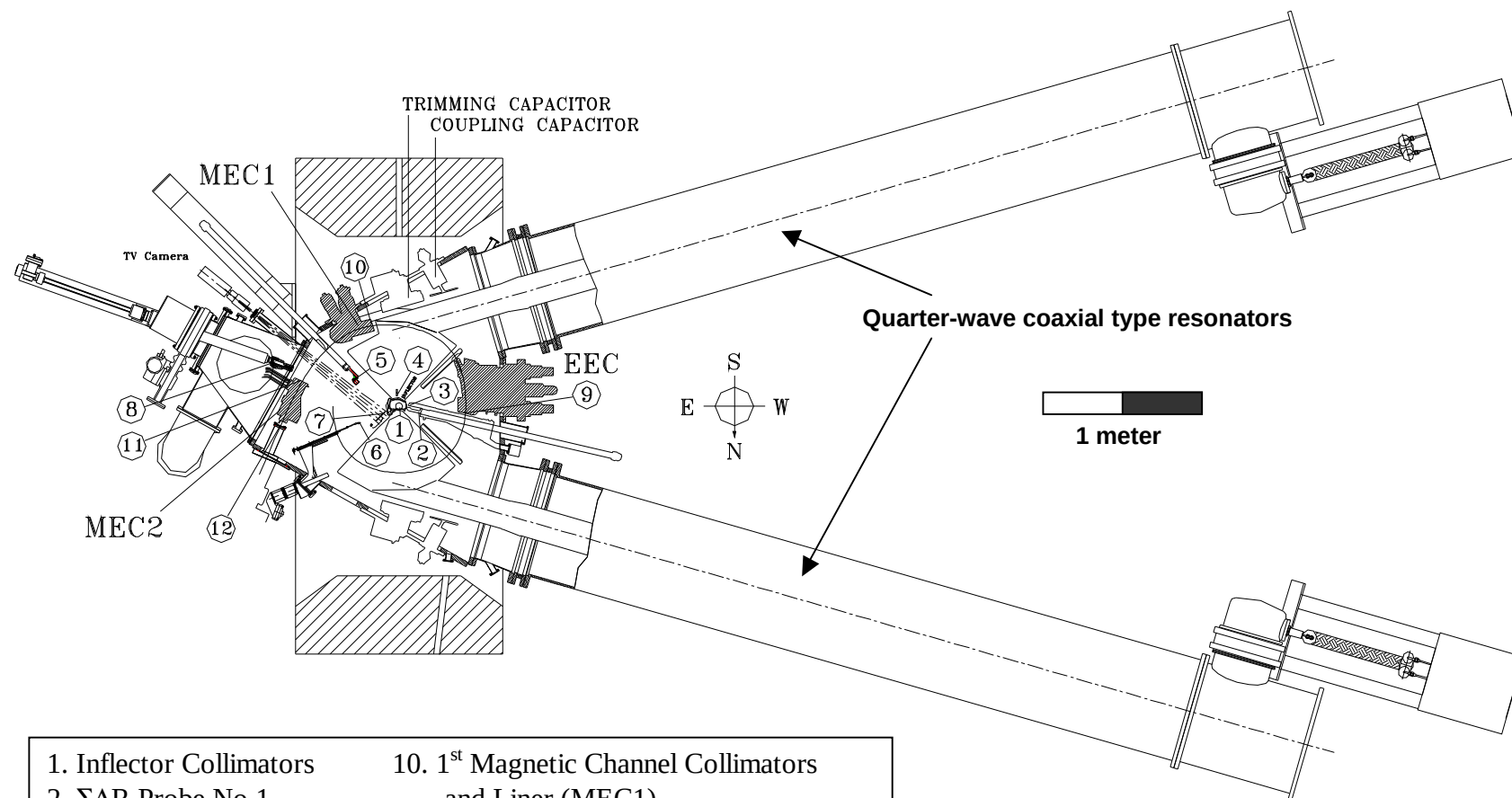
Figure 2.3. *Layout of the separated-sector cyclotron.*



- | | |
|--------------------------------|--|
| 1. $\Sigma\Delta R$ Probe No.1 | 7. 1 st Magnetic Channel Collimators and Liner (MEC1) |
| 2. RADAX Slits | 8. 2 nd Magnetic Channel Collimators and Liner (MEC2) |
| 3. 2 nd Radial Slit | 9. Passive Magnetic Channel Collimators |
| 4. $\Sigma\Delta R$ Probe No.2 | |
| 5. 2 nd Axial Slit | |
| 6. EEC Collimators | |

SPC-1

Figure 2.4. Layout of SPC1.



- | | |
|--------------------------------|---|
| 1. Inflector Collimators | 10. 1 st Magnetic Channel Collimators and Liner (MEC1) |
| 2. $\Sigma\Delta R$ Probe No.1 | 11. 2 nd Magnetic Channel Collimators and Liner (MEC2) |
| 3. RADAX Slits | 12. Passive Magnetic Channel Collimators |
| 4. 2 nd Radial Slit | |
| 5. $\Sigma\Delta R$ Probe No.2 | |
| 6. 2 nd Axial Slit | |
| 7. Visual Display Plates | |
| 8. Harp | |
| 9. EEC Collimators | |

Figure 2.5. *Layout of SPC2.*

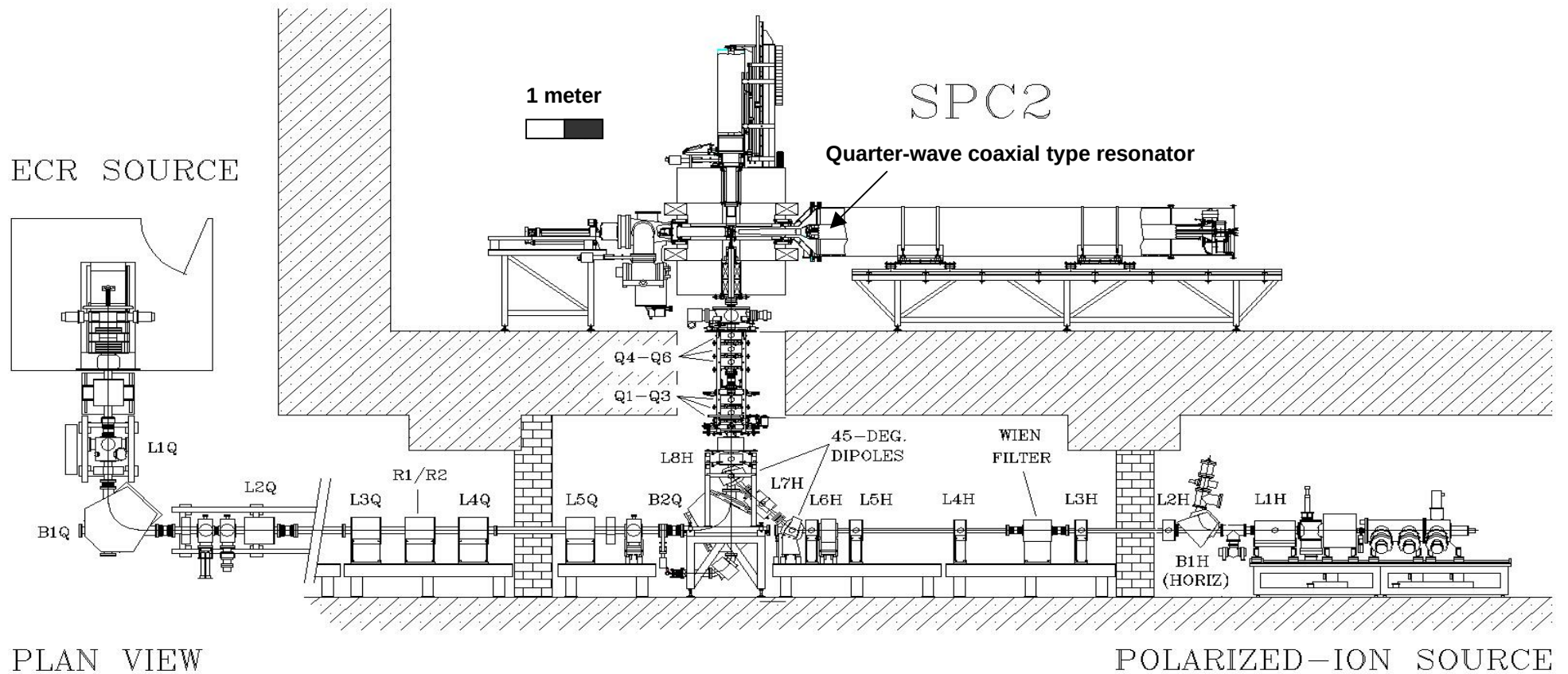


Figure 2.6. Side view of SPC2 and the polarized ion source, combined with a top view of the ECR source, respectively together with the H- and Q-beam lines leading to the axial injection from below the cyclotron. The acronyms of the magnets starting with B and Q respectively refer to Bending Magnets and Quadrupole Magnets and L and R both refer to Solenoid Magnets.

Table 2.1: iThemba LABS cyclotron parameters.

SSC	K-value	200
	Injection radius (hill)	1.01 m
	Extraction radius (hill)	4.422 m
	Magnetic flux density	1.26 T (max.)
	Magnetic sectors	4 x 34° sectors
	Resonators	2 x 49° deltas, $\lambda/2$
	Dee voltage	250 kV (max.)
	Power amplifier	150 kW
	RF power dissipation	80 kW/resonator
	Frequency range	5 – 27.5 MHz
	Harmonic numbers	4 & 12
	Frequency variation	Short-circuit plates & variable capacitors
SPC1	K-value	8
	Extraction radius	0.476 m
	Magnetic flux density	0.86 T (max.)
	Number of sectors	4
	Resonators	2 x 90° dees, $\lambda/4$
	Frequency range	5 – 27.5 MHz
	Harmonic numbers	2 & 6
	Ion source (internal)	Internal PIG source
SPC2	K-value	10
	Extraction radius	0.476 m
	Magnetic flux density	0.86 T (max.)
	Number of sectors	4
	Resonators	2 x 90° dees, $\lambda/4$
	Frequency range	5 – 27.5 MHz
	Harmonic numbers	2 & 6
	Ion sources (external)	ECR source & polarized ion source

CHAPTER 3

THE THEORY OF BEAM DYNAMICS IN RESONATORS AND A REVIEW OF RESONATOR SYSTEMS

3.1 INTRODUCTION

The extraction efficiency of a cyclotron with high beam power must be close to 100% in order to minimize operational problems with cooling and activation of its extraction components. This can only be achieved by ensuring that the energy spread in the beam, and consequently the radial width of the beam, is minimized [Gor67, Sch76]. The extraction efficiency and energy spread of the beam in the SSC depend on the pulse length and energy spread of the beam at injection. It is therefore important to calculate the beam width and orbit separation inside the machine, especially at extraction. The beam intensity from SPC1 is limited to 300 μA without flat-topping, because the beam pulse length in the machine has to be restricted to ensure single-turn extraction in the SSC. Beam pulses from SPC1 are focused longitudinally in the transfer beam line with a buncher, just before injection into the SSC. The available beam intensity is restricted to 150 μA to limit the beam loss to less than 1 μA on the extraction components of the SSC and thereby it is ensured that the activation of these components are kept below 1 mSv.h^{-1} , which is essential for the safety of personnel when maintenance is required. To increase the maximum extracted beam current for the 66 MeV proton beam from the present maximum of 150 μA to 500 μA , longer beam pulses will have to be accelerated in both SPC1 and the SSC, without increasing the energy spread in the beam. This can be done with the implementation of flat-topping rf systems in both SPC1 and the SSC.

The design of any additional equipment for the accelerator facilities must be such that no costly modifications to existing equipment are required. Cost and restrictions posed by the existing valley vacuum chambers and existing equipment mounted inside these chambers, are important determining factors in the selection and design of a flat-topping system for the SSC. The resonator must fit into the existing space available inside a vacuum chamber, other than those that are already occupied by the

main resonators and it must be installed through one of the existing ports of the chamber. The design must also allow for access to service the resonator and other equipment in the cyclotron.

This chapter will deal with the expected improvements in beam quality and intensity, different types of resonators and the limitations on a flat-topping system by the existing facilities in the SSC. For this purpose use has to be made of the results of the calculation of the resonator characteristics that will be described in chapter 4, except in the case of a cavity resonator for which simple analytic expressions are applicable. To select a suitable resonator type and determine its influence on the beams, several iterations between orbit and resonator calculations had to be carried out. For the sake of brevity these iterations are not described in detail here.

3.2 THE ENERGY SPREAD AND BEAM WIDTH IN A CYCLOTRON WITH AND WITHOUT FLAT-TOPPING

3.2.1 Energy spread, beam separation and orbit separation

The energy gained per turn by a particle in a cyclotron that is operated with a sinusoidal dee voltage and without a flat-topping resonator, is given by:

$$\Delta T = QV_d \cos(f + dq) \quad (3.1)$$

with Q the charge of the particle, V_d the dee voltage at the acceleration gap, f the phase of the main rf with respect to the beam and dq the phase deviation of the particle from the central particle in the beam bunch.

The beam quality in a cyclotron with a sinusoidal accelerating voltage is determined to a great extent by the energy spread and consequently the radial beam width at the extraction orbit. The undesirable energy spread causes radial broadening of the beam in the cyclotron and therefore reduces the separation between two successive turns. It is essential to have these turns separated at extraction to ensure a beam of good quality to the user and prevent activation and possible overheating and damage to the extraction components. The beam separation between any two successive turns in a cyclotron is mainly important at the injection and extraction orbits in a cyclotron, to

ensure a clear beam pass through the injection and extraction components [Gor67]. The orbit separation is calculated as the distance between the radii of two successive orbits. The radial position of any orbit in a separated-sector cyclotron is not constant and has its maximum at the azimuthal center of each sector magnet (hill radius) and the minimum radial position (valley radius) at the azimuthal center of the valleys between the sector magnets. Beam and orbit separation and the radial broadening of the beam are schematically illustrated in figure 3.1. The area in grey represents the intrinsic width of the beam without energy spread and the shaded area is the tail of beam, which develops due to the energy spread in a beam bunch. Any change in acceleration voltage will be most noticeable in a change in the orbit separation.

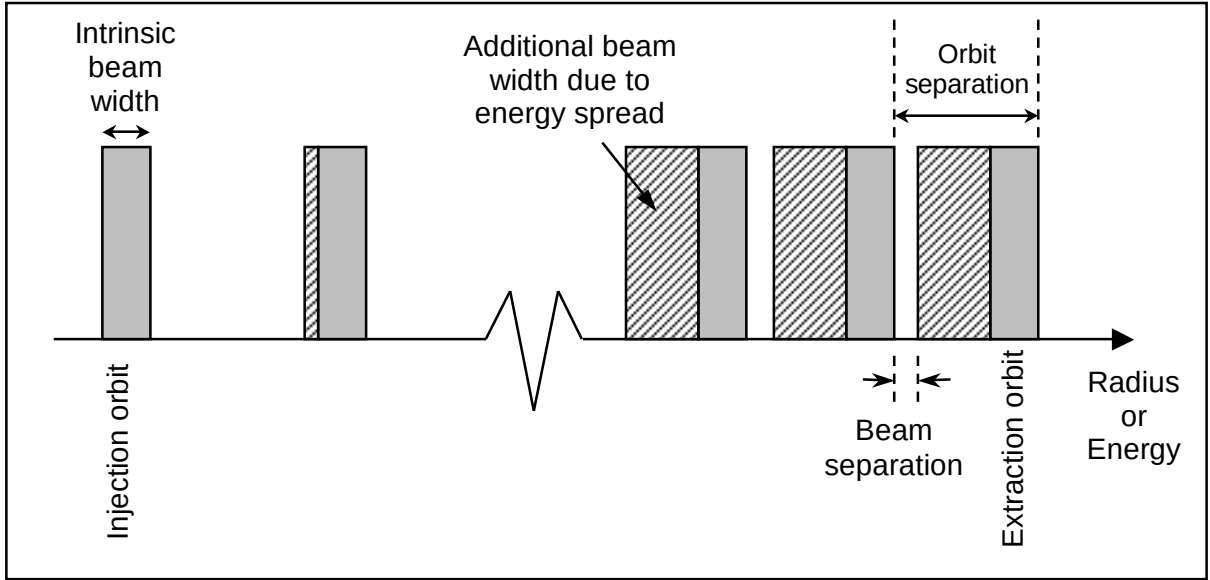


Figure 3.1. A schematic representation of the orbit separation (i.e. is the distance between two successive orbits), the beam separation (i.e. the radial gap between two orbits containing no beam) and the beam width due to the energy spread, which is added to the intrinsic beam width.

When a flat-topping resonator, which operates at the n^{th} harmonic number of the main resonator frequency, is added to the cyclotron, the **gain in energy per turn** becomes:

$$\Delta T = QV_d \cos(f + dq) - QV_f \cos(nj + ndq) \quad (3.2)$$

with V_f the flat-topping voltage at its acceleration gap, and j the phase of the resonator with respect to the beam.

The **energy spread** in a cyclotron without a flat-topping system is [Con92f]:

$$\frac{\Delta E}{E} = 1 - \cos \frac{dq}{2}, \quad (3.3)$$

with ΔE equal to the maximum difference in energy between any two particles in a bunch, E the energy of a particle at the center of the bunch and dq the length of the beam bunch in main dee voltage rf-degrees. The comparative energy spread in a cyclotron with a flat-topping acceleration system operating at the n^{th} harmonic of the main frequency, can be calculated from [Con92g]

$$\frac{\Delta E}{E} = \left[\frac{n^2}{n^2 - 1} \right] \left[1 - \frac{1}{n^2} - \cos \frac{dq}{2} + \frac{1}{n^2} \cos \left(n \frac{dq}{2} \right) \right]. \quad (3.4)$$

The energy spread in a cyclotron, with and without a flat-topping system, is shown in figure 3.2 for flat-topping harmonics $n = 2, 3, 4, 5$ and 6 . This shows that a cyclotron with a fixed maximum allowable energy spread, can accelerate larger bunch lengths when a flat-topping system is implemented at a harmonic frequency of the fundamental accelerating voltage.

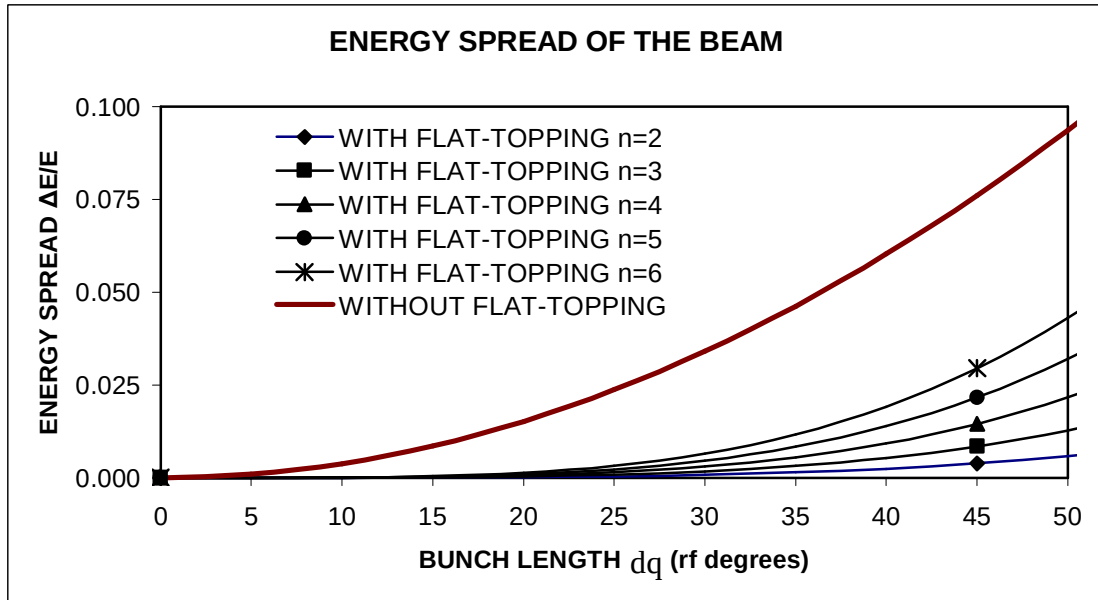


Figure 3.2. A cyclotron with a flat-topping system can accelerate larger bunch lengths than when without a flat-topping system, for any given energy spread. The calculated energy spread without a flat-topping resonator is compared to that with a resonator operating at harmonic numbers $n = 2, 3, 4, 5$ and 6 .

The increase of the **radial beam width**, ΔR , on a specific orbit in a cyclotron is directly proportional to the energy spread and is given by:

$$\frac{\Delta R}{R} = \frac{1}{g(g+1)} \frac{\Delta E}{E}, \quad (3.5)$$

with R the radius of that orbit and γ the ratio of its energy to its rest energy.

In the non-relativistic case the increase in radial beam width becomes [Gor67]:

$$\frac{\Delta R}{R} = \frac{\Delta E}{2E} \quad (3.6)$$

From equations 3.3 and 3.6, a 66 MeV proton beam bunch with an injection phase length of 18 rf-degrees, will have an energy spread of 0.0123 MeV at the SSC valley extraction radius of 3.62 m and an additional radial beam spread of 20.1 mm. The same cyclotron with the same injection conditions and a flat-topping system installed in a valley can accelerate much bigger bunch lengths with the same energy spread and additional radial beam spread at extraction. See Appendix B that describes the hard-edge formulas for the orbit pattern of the SSC. These increased bunch lengths and the factor by which the beam currents can be expected to increase for different harmonic numbers of the flat-topping system, are tabulated in table 3.1.

Table 3.1 : The calculated bunch length that can be accelerated and the factor of increase when implementing a flat-topping system for 66 MeV protons, tabulated for a typical energy spread of 0.0123 MeV and its specific additional radial beam spread of 20.1 mm, due to the energy spread.

Expected gain (with 66 MeV protons)	Without flat-topping	With flat-topping				
		n=2	n=3	n=4	n=5	n=6
Bunch length accepted (rf-degrees)	18	60.4	49.6	43.1	38.7	35.5
Bunch length gain factor	x1	x3.35	x2.76	x2.39	x2.15	x1.97

The increased bunch lengths and therefore higher currents obtained with the flat-topping system from the SSC, necessitates improved beam quality from the injector cyclotron with its own flat-topping system and also from the longitudinal bunching in the transfer beam line. The flat-topping system on the injector cyclotron has already been implemented and tested [Con92h]. The particle density, J , extracted from the ion source in relation to the dee voltage, V_{dee} , of the injector cyclotron, is given by

$J \propto \sqrt{V_{dee}^3}$. The increased particle intensity, due to the longer time that the dee voltage

has a high value, with the addition of a flat-topping system, will improve the calculated beam intensity ratios for the SSC.

These calculated additional beam widths due to the energy spread in a cyclotron with and without a 2nd harmonic flat-topping resonator, are shown in figure 3.3 as a function of the length of a 66 MeV proton beam pulse in the SSC. Later in this chapter it will be shown that with the second harmonic and for crossing of the acceleration gaps at the peak of the dee voltage, the dee width is too large to fit in the vacuum chamber.

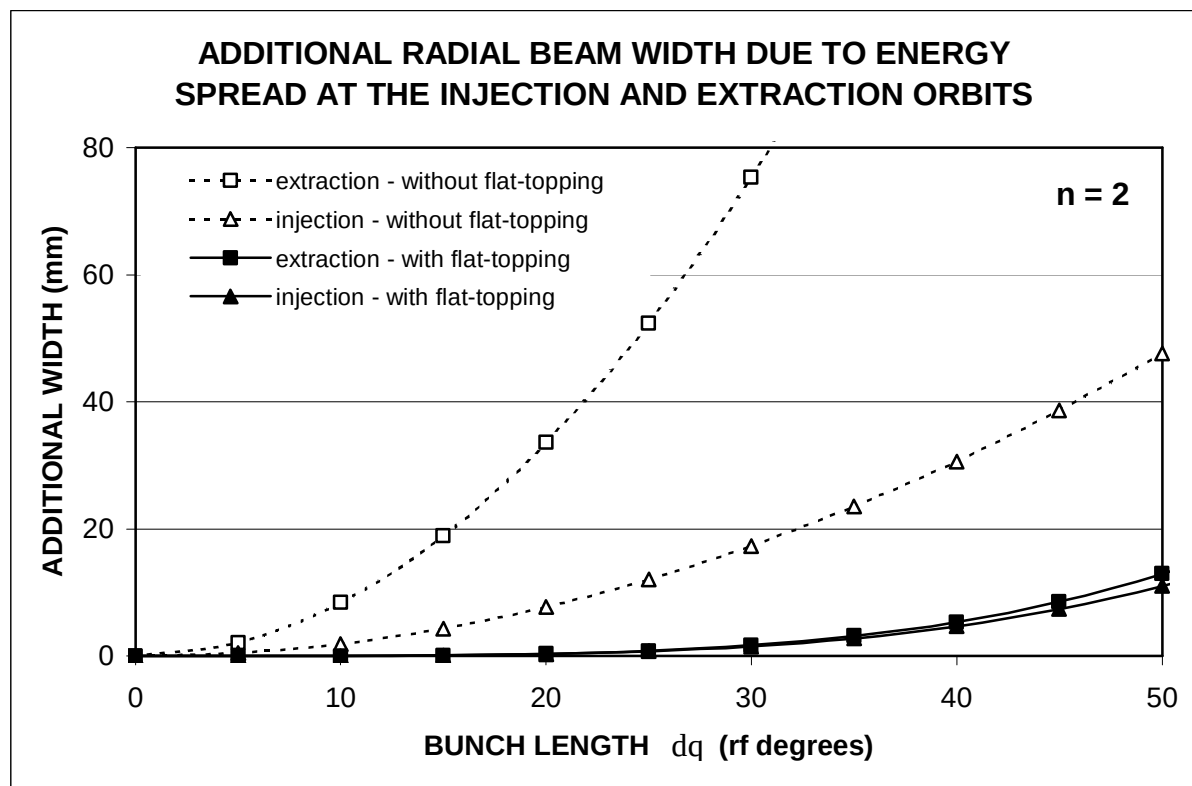


Figure 3.3. The additional radial beam width due to the energy spread in the SSC, with and without a 2nd harmonic flat-topping acceleration system and using 1.016 m and 4.422 m as radii for the injection and extraction orbits, respectively. For any given pulse length the beam width at injection and extraction will be reduced.

The additional beam width due to energy spread, for the same flat-topping resonator, operating at the third harmonic, is shown in figure 3.4.

The time structures of particles crossing the accelerating gaps of different dee shapes are illustrated in Appendix A.

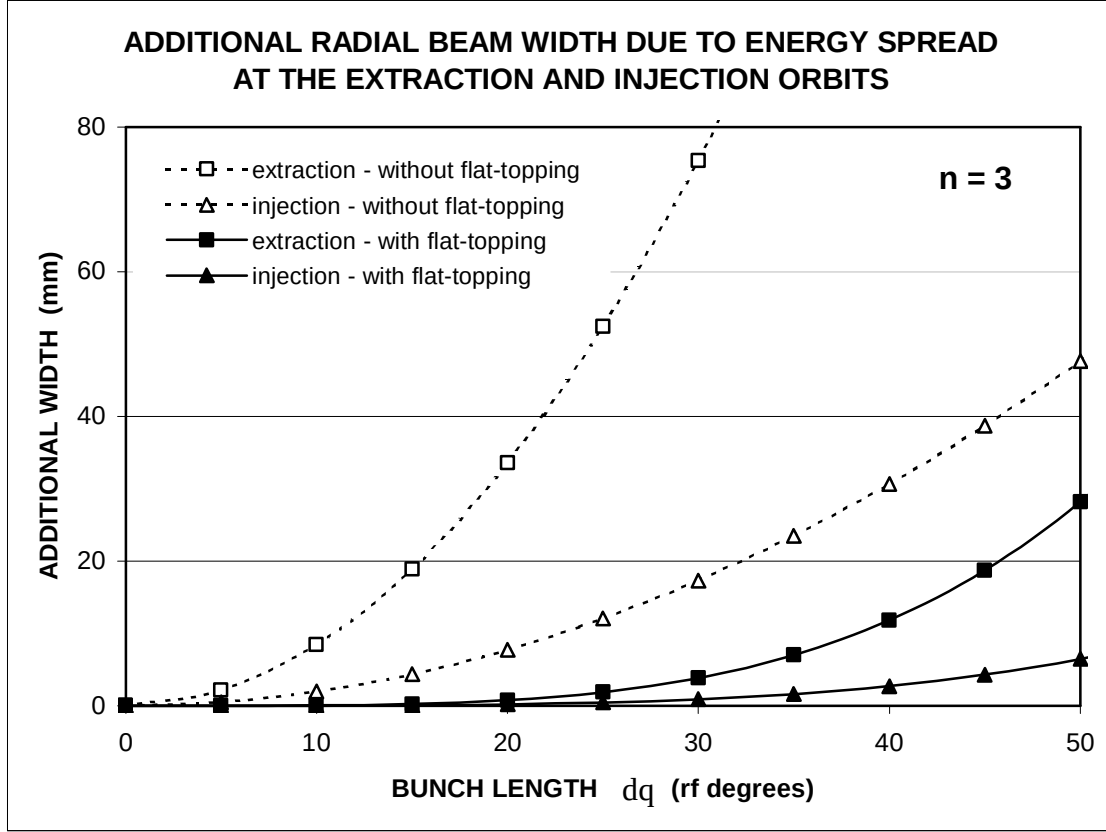


Figure 3.4. The additional radial beam width due to the energy spread in the SSC, with and without a 3rd harmonic flat-topping acceleration system and using 1.016 m and 4.422 m as radii for the injection and extraction orbits, respectively. For any given pulse length the beam width at injection and extraction will be reduced with flat-topping.

The calculations to predict the improvement of the beam in the SSC with a flat-topping system, operating at the third harmonic number of the main rf, was further improved by using the **hard-edge values** of the cyclotron (see appendix B) into a computer program (see appendix C). In the computer model it is assumed that the magnetic field is isochronous, a linear main voltage gradient exists from injection to extraction and a sinusoidal change of flat-topping voltage with radius, with the maximum value about halfway between the injection and extraction orbits, at which the voltage is zero. A bunch of particles is represented by three particles: the central one, which is accelerated at the peak of the main dee voltage, accompanied by a particle on either side of the peak of the rf cycle. The leading and lagging particles are initially separated from the central particle by one half of the bunch length (in main rf-degrees). The different phase angles are q_{lead} , q_{lag} and q_{cent} . A beam of zero inherent

width and typical injection conditions for a final energy of 66 MeV protons are used as the initial parameters to begin the calculations on the injection orbit. Calculating the change in energy spread and accompanying beam width as the beam progresses from injection to extraction requires that the energy and radial position of each of the three particles is calculated for every orbit. The results of each orbit are used as the starting conditions to calculate values for the next orbit and the process is repeated until the extraction orbit position is reached.

The energy spread, the additional beam width due to the energy spread, the orbit separation and beam separation were calculated at the middle of the valley for every orbit from injection to extraction for different flat-topping voltage values, V_f , and particle bunch phase length values, $dq = |q_{cent} - q_{lag}| + |q_{cent} - q_{lead}|$. Combinations of voltages and bunch phase lengths were used to calculate and compare the beam quality at extraction, with and without flat-topping. The results at the extraction orbit in the middle of the valley, at a radius of 3.62 m, are summarized in table 3.2. The sets of variables that were used in the calculation are listed in tables C.1a to C.1f in Appendix C.

Table 3.2: The cyclotron hard-edge formulae were used to calculate the improvement in the quality of the beam at extraction with the flat-topping voltage of 72.3 kV.

Flat-topping voltage (kV)	AT INJECTION			AT EXTRACTION				Figure that shows the calculated values for al the orbits
	PHASE of PARTICLE with respect to the MAIN RF (rf-degrees)			Energy Spread (MeV)	Additional Beam Width (mm)	Orbit Separation (mm)	Beam Separation (mm)	
	θ_{lag}	θ_{cent}	θ_{lead}					
0	-7.5	0	7.5	0.54	13.4	21.9	8.5	3.5a, 3.6a
72.3	-7.5	0	7.5	0.014	0.34	21.8	21.3	3.5b, 3.6b
72.3	-15	0	15	0.029	0.72	21.8	21.1	3.5c, 3.6c

The energy spread at extraction improves significantly from 0.54 MeV to 0.014 MeV for a bunch length of 15 rf-degrees with flat-topping. Even a bunch length of 30 rf-degrees, with flat-topping, corresponds to an energy spread of only 0.029 MeV. The

additional beam width due to the energy spread also significantly reduces from about 13 mm to about 0.5 mm for both beam bunch lengths of 15 and 30 rf-degrees. The orbit separation at extraction is practically unchanged, because the flat-topping voltage is zero at extraction. The combined effect is that the beam separation increases from 8.5 mm to about 21 mm with flat-topping, which is a dramatic improvement in beam quality.

The different initial conditions of flat-topping voltages and particle phases that were used in the computer program for these hard-edge formulation results, are tabulated in Appendix C, together with graphical displays of the calculated energy spread and additional beam width due to the energy spread, for every orbit at the center of a valley. The calculated additional beam width for these conditions is also shown for every orbit at the middle of a hill, though not important, since the flat-topping resonator can physically only be installed in the valley regions.

3.2.2 Canceling of longitudinal space-charge (LSC) effects

Space-charge causes an azimuthal electric field component in a beam bunch and can destroy the turn separation in the cyclotron [Joh68a]. The voltage gain per turn due to this electric field has to be added to the energy gain due to the main rf-voltage. The leading particles of the beam bunch will be accelerated and the lagging particles will be decelerated, thereby increasing the energy spread in the beam. This increase in energy spread of the beam has to be compensated as far as possible or minimized in order to ensure single-turn extraction for high beam intensities. Quantitative analysis of the longitudinal space-charge effect is complex because in the calculation of the resultant force on a particle in the beam bunch, not only the contribution of the remaining particles in the bunch has to be taken into account, but also those of all the other beam bunches along the same radial line, as well as the contributions of the mirror charges in the horizontal walls of the vacuum chambers and in the dee plates. In addition, the fact that the intrinsic length of a beam bunch increases from injection to extraction by a factor of about 4 (depending on particle type and the energy) and the change in the shape of the bunches due to the space-charge force have to be taken into account. According to Joho [Joh68a] the effect becomes evident for beam

intensities higher than about 170 μA . Up to this level the LSC-effect is masked by other more prominent effects, like the accrued energy spread, as described earlier in the chapter.

For bunches with a small phase length, and without a flat-topping resonator, the LSC-effect can partially be compensated for by acceleration of the beam bunches at a leading phase angle with respect to the main rf.

For longer bunch lengths, it is necessary to operate a flat-topping resonator slightly out of phase with respect to the main resonator, in order to partially compensate for the space-charge effect on the beam. This phase difference between the flat-topping and main voltages causes a tilting of the flat-topped voltage, with which the leading and lagging particles receive different energy gains with respect to the central particle.

The LSC-effect can therefore be eliminated to a first order of magnitude by a mere shifting of the phase between the two resonators. See Appendix D for a more detailed analysis, as described by Joho [Joh68a, Joh81]. However a proper study of the LSC-effect has to be done in order to quantify the parameters that have an influence.

3.3 LIMITATIONS ON THE IMPLEMENTATION OF A FLAT-TOPPING RESONATOR IN THE SSC

3.3.1 Geometric restrictions

The choice of geometry and harmonic frequency for the flat-topping resonator are, amongst others, determined by cost, the number of acceleration gaps, the vertical, azimuthal and radial space available in the cyclotron and access to other equipment inside the vacuum chambers. Extending such a resonator to regions outside the vacuum chamber will be too costly to implement in the SSC. These prerequisites severely restrict the design of the resonator and limit the width of the resonator to half the width of a valley vacuum chamber of the SSC. The position of the collimator blocks on the eastern side of the north valley vacuum chamber poses a problem for the positioning of the resonator in this chamber. A flat-topping resonator in this chamber would also prevent access to the magnetic inflection channel. It is therefore not possible to install two additional resonators, one in each of the opposing valley

vacuum chambers. The location of a single flat-topping resonator will therefore have to be in the south valley vacuum chamber of the SSC. A resonator design for the south valley vacuum chamber must be such that it will not block the radial path of the injected beam through the chamber. The resonator has to be installed through the existing port for the electrostatic channel and the pulse selector will remain in its position on the center line of the vacuum chamber.

3.3.2 The azimuthal angle, radial length and harmonic number

The azimuthal angle of a double gap resonator, for a given particle energy, can be selected from a range of possible values to best suit the required harmonic frequency and the geometric restrictions. The largest useful phase range for any given energy spread can be obtained with the harmonic frequency of double the main frequency [Joh68b], but the azimuthal angle and radial length available for the resonator also determine the selection of the harmonic frequency (see Appendix A). The resonator angles for the respective harmonic numbers were calculated from one half period of the dee voltage under consideration, and the speed of the particle. Higher numbers of half periods is not an option, because of the already restricted azimuthal space in the vacuum chamber. The radial space required is determined by the resonator type for which one half wavelength has to fit in the vacuum chamber. Such calculated resonator angles and radial lengths of a flat-topping resonator, dedicated to a 66 MeV proton beam in the SSC, are shown in figure 3.7 for different harmonic numbers.

The restricted radial space in the existing valley vacuum chamber of the SSC is one of the main reasons for excluding the second harmonic as an option for the horizontal half-wave resonator. A quarter-wave resonator operating harmonic number two would fit radially into the vacuum chamber, but the dee angle would be too large for peak crossing of the particles through the acceleration gaps and the orbit separation at injection and extraction would decrease drastically as will be discussed in section 3.4.2 below.

The largest double-gap resonator that can be fitted into the azimuthal space has a dee angle equal to one-third of those of the main resonators and will therefore operate at three times the main rf frequency.

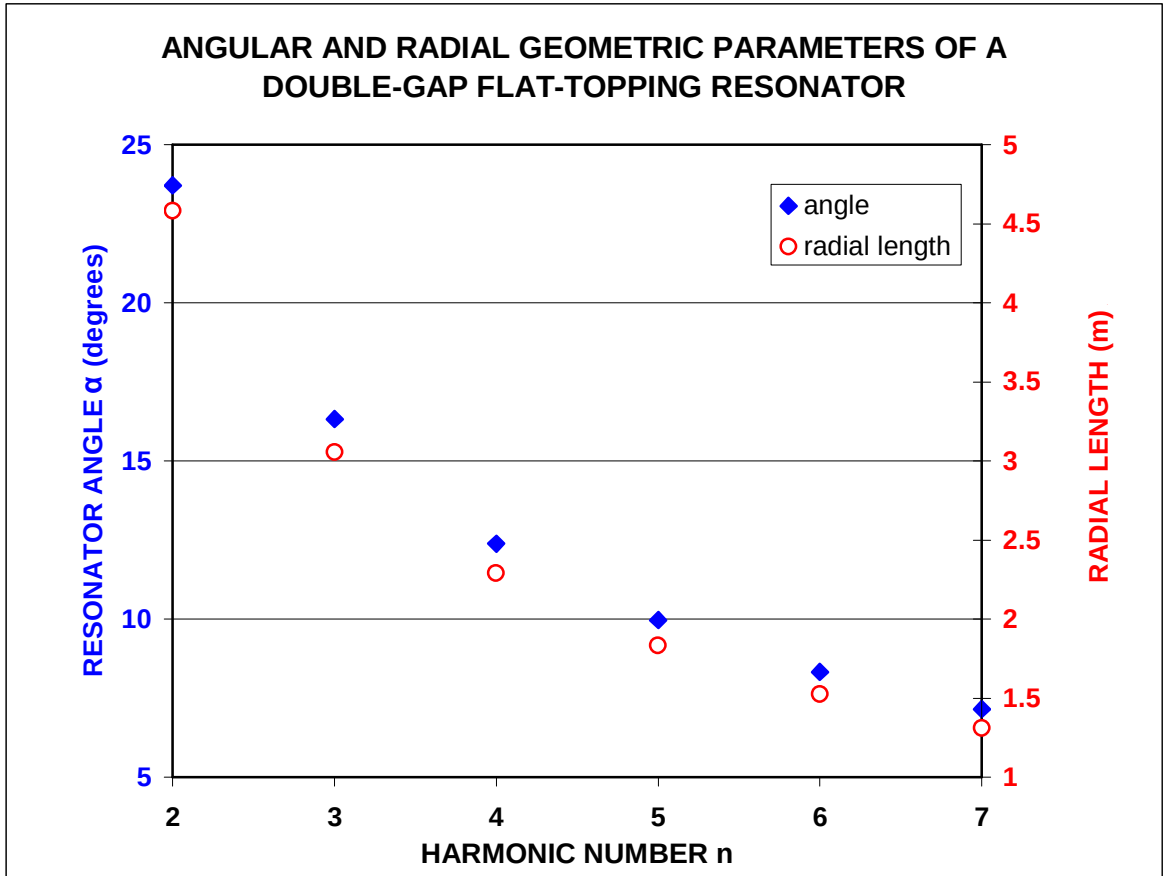


Figure 3.7. Calculated flat-topping resonator angles and the radial space required, at a few harmonic frequencies. The angles were calculated using one half period of the resonance frequency as the transit time for the particle to cross the resonator from the middle of one acceleration gap to the next and the particle speed at injection and extraction.

The third harmonic is a good choice, because the required radial distance of just over 3 m lies within the geometry of the vacuum chamber.

The azimuthal angle of about 16.5° between the centre-lines of the accelerating gaps can be accommodated with minor local modifications to the geometry of the resonator to reserve space for the EEC on the extraction orbit, and the pulse selector.

3.4 TYPES OF RESONATORS

Different types of resonators were investigated [Con98] and compared with the aid of analytical formulae, the computer program COC [Cro87] that uses the measured magnetic field and radial voltage distribution of the main and flat-topping resonators to calculate the orbits in the SSC and the 3d numerical field analysis computer program, SOPRANO Version 1.4 and OPERA-3d Version 2.6 from Vector Fields

[Vec_1]. Wherever possible the results were verified with calculations using transmission-line models for the resonators. To understand the SOPRANO software, calculations were initially done for a resonator for which accurate analytical expressions are available.

3.4.1 Single-gap resonator

The resonance frequency, f , of a rectangular cavity resonator, as shown in figure 3.8, can be calculated from its basic dimensions and is given by [Con92i]:

$$f = \frac{c\sqrt{d^2 + a^2}}{2ad},$$

with c = the velocity of light in vacuum

d = radial length of the cavity

a = height of the cavity

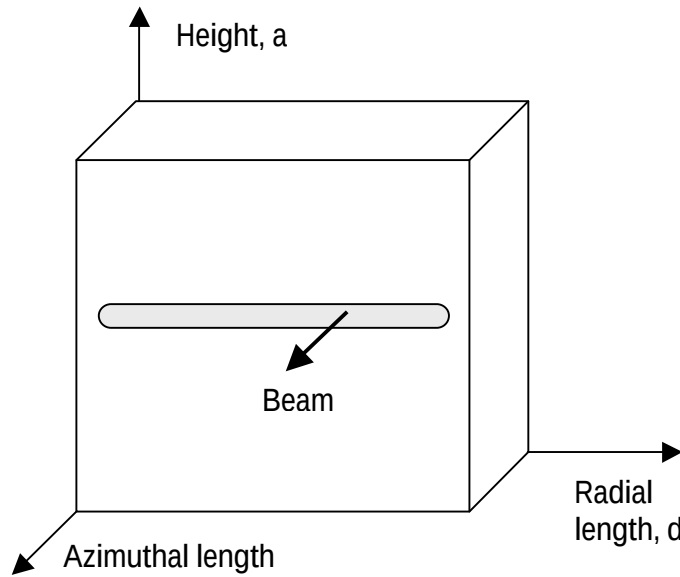


Figure 3.8. A schematic diagram of a cavity resonator of which the frequency is determined by the ratio of its radial length, d , and its height, h .

For a particular resonance frequency there exist many possible combinations of length and height as shown in figure 3.9 for the frequency spectrum from 30 MHz to 100 MHz at intervals of 5 MHz. A cavity, which operates at a frequency of 50 MHz, will require a height of about 5.75 m for a radial length of about 3.5 m, and is therefore not suitable for installation in the vacuum chamber of the SSC, that has a height of only 1 m, which implies that a completely new vacuum chamber would be required.

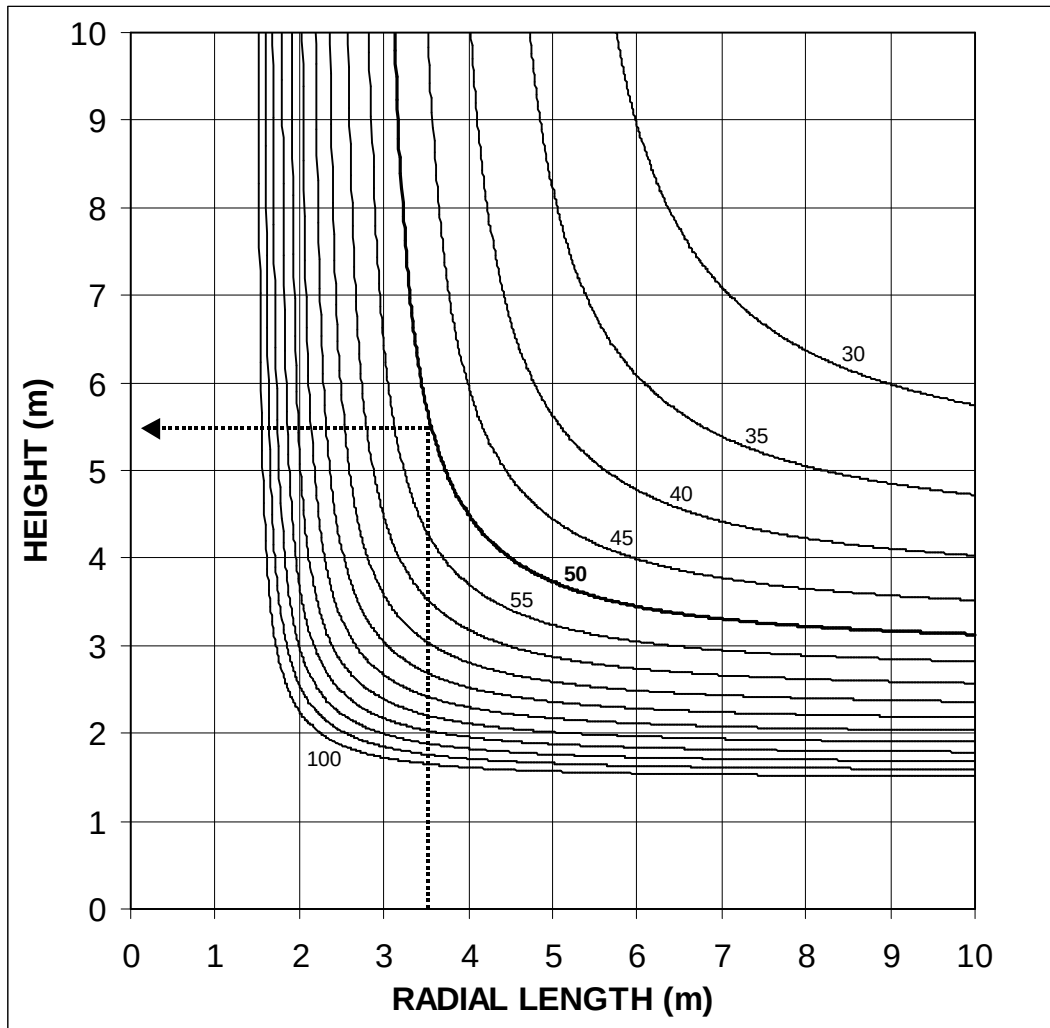


Figure 3.9. Dimensions of a rectangular cavity-type resonator for the frequency spectrum from 30 to 100 MHz at intervals of 5 MHz.

Although its radial voltage distribution across the acceleration gap is advantageous for the control of the beam, the applied voltage would be double the required voltage of a double gap resonator of related design.

A variation of the cavity resonator as described by Saito [Sai86] was also calculated with the program SOPRANO. The unsuitability of a single-gap resonator for our SSC, taking the space requirements into account, was verified. This type of resonator was therefore not considered further.

3.4.2 Double-gap resonators

Vertical half-wave and horizontal quarter-wave resonators are well studied and used in various cyclotrons around the world [Rog84], but no double-gap horizontal half-

wave resonator exists anywhere at accelerator facilities. The resonator characteristics shown here were calculated with the computer programs COC and SOPRANO.

3.4.2.1 Vertical half-wave resonator

For the flat-topping in the SSC the vertical half-wave resonator, shown in figure 3.10, was discarded on the basis that it would be very difficult to install through an existing flange and in the available space of the valley vacuum chamber. The required height of 0.94 meter implies that the upper and lower halves cannot be mounted and installed as a single unit through the existing ports into the vacuum chamber.

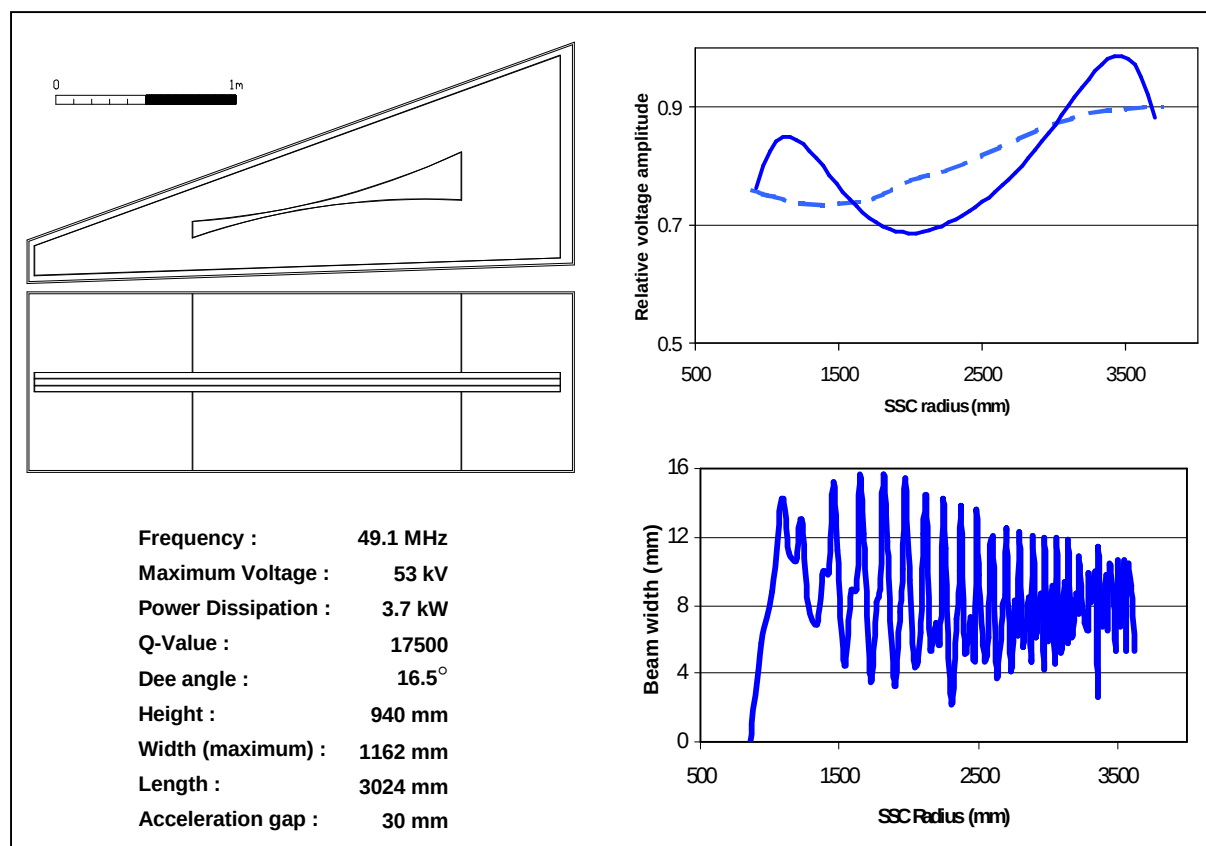


Figure 3.10. *The geometry, electro-mechanical and beam properties of the vertical half-wave resonator.*

The radial distribution of the acceleration voltage is very sensitive to characteristics of the resonator geometry and it will therefore require the building of a model with several geometric adjustments to ensure that the correct distribution is obtained. Any such deviations may cause unacceptable reduction in orbit separation at the injection and extraction orbits. A higher order of accuracy will also be required with the manufacturing of such a resonator, which implies extra costs.

3.4.2.2 Horizontal quarter-wave resonator

A double-gap horizontal quarter-wave resonator, as is frequently used in cyclotrons, was also considered. It would cover only half the radial width up to the extraction orbit of the SSC and would therefore drastically reduce the orbit separation at extraction, since the resonator has to operate out of phase with the main resonators. The basic geometry and main characteristics are shown in figure 3.11.

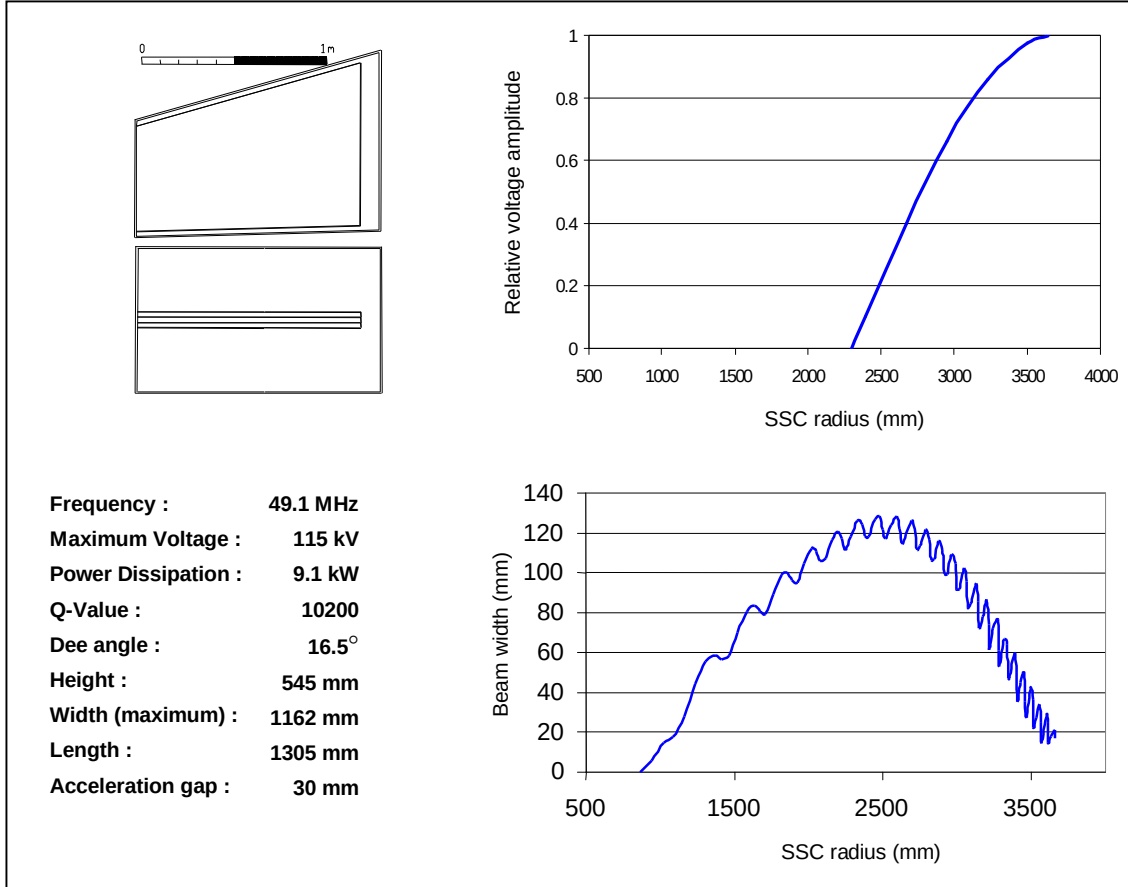


Figure 3.11. *The geometry, electro-mechanical and beam properties of the horizontal quarter-wave resonator.*

3.4.2.3 Horizontal half-wave resonator

The development of this type of resonator for a cyclotron is a novel design and therefore will require a detailed study of its characteristics. It has a radial voltage distribution that is zero at the injection and extraction radii and therefore will have no influence on the beam separation at these orbits. The geometrical shape of such a resonator is also relatively simple and can be accommodated within the geometrical constraints, with good mechanical stability and comfortable access to regions where

water-cooling will be required and also for remote control of adjustable components. The geometry and main characteristics are shown in figure 3.12.

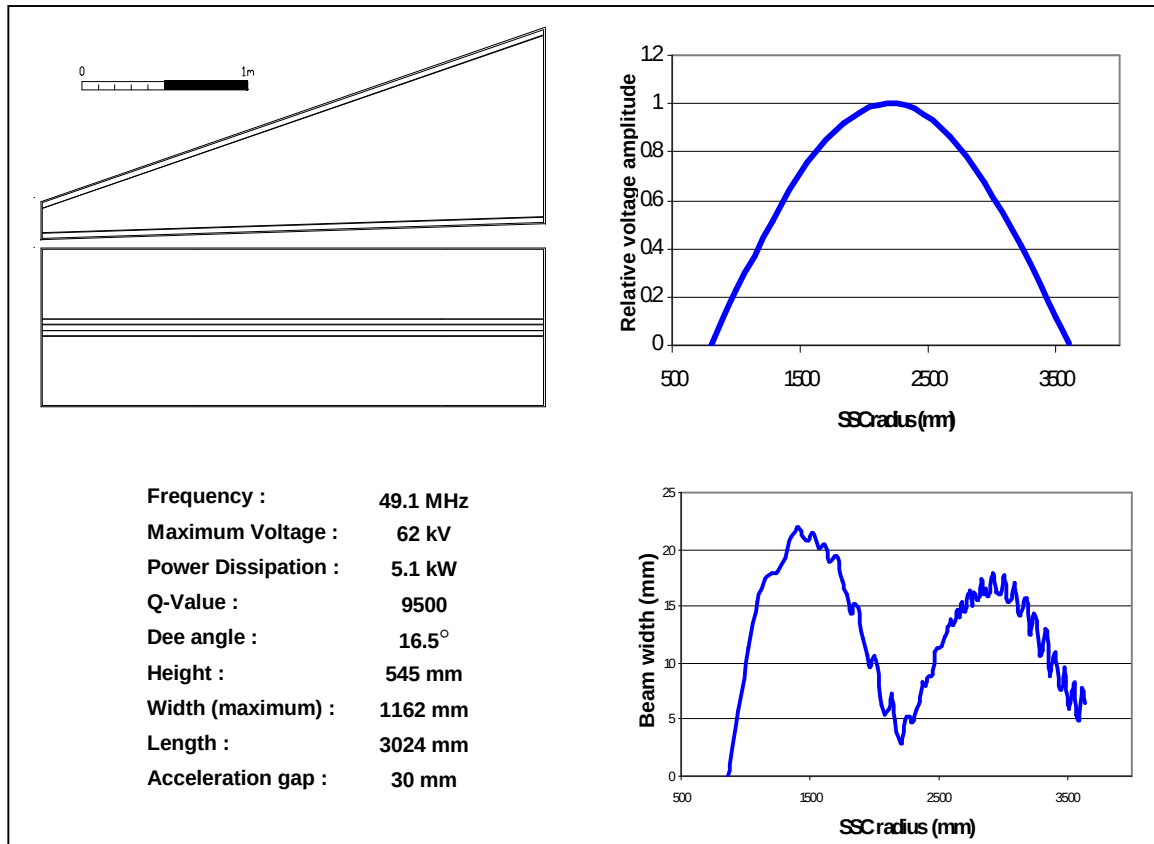


Figure 3.12. *The geometry, electro-mechanical and beam properties of the horizontal half-wave flat-topping resonator.*

The double hump in the beam width can be explained by taking the voltage distribution into account. Initially, along a radial line, the flat-topping voltage is to compensate for the energy spread introduced by the main resonators. At larger radii the flat-topping voltage is higher and begins to decrease the energy spread in the beam until a minimum beam width is reached. Further at larger radii the energy spread is over-compensated and the beam width starts to increase, but the flat-topping voltage decreases to zero at extraction, where the over-compensated energy spread is decreased by the additional energy spread introduced into the beam by the main resonators.

3.4.3 Advantages of having two flat-topping resonators in opposite valleys

Because of the relatively low injection energy of 3.15 MeV and the high flat-topping dee voltage, it would have been better (depending on the radial voltage distribution of the flat-topping resonator) to use two flat-topping dees in opposite valleys in order to keep beam centering errors small and therefore to limit the influence of resonances in the betatron oscillations. The improvement in beam quality that can be obtained with two flat-topping resonators is slightly better than when only one flat-topping resonator is used. Figure 3.13 shows the calculated beam width for beam pulse length of 40° , with two flat-topping resonators that have the same radial voltage distribution as the main resonator, to be about 5 mm in comparison with a 20° pulse length without flat-topping. For the two flat-topping resonators the beam width at extraction is marginally better than in the case of a single flat-topping resonator.

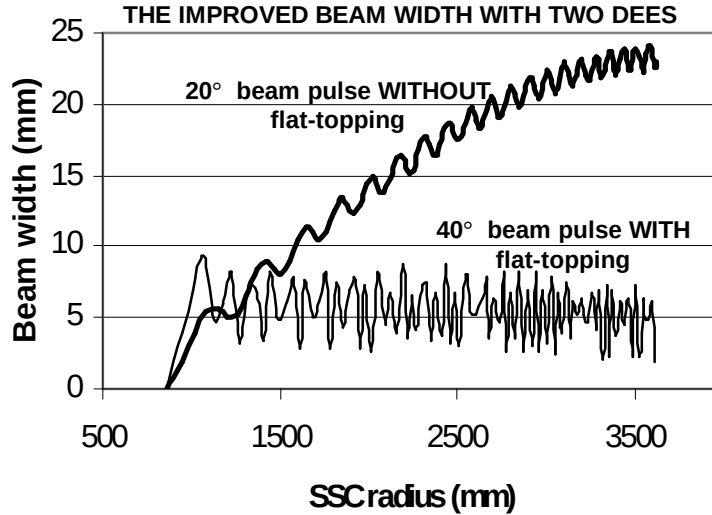
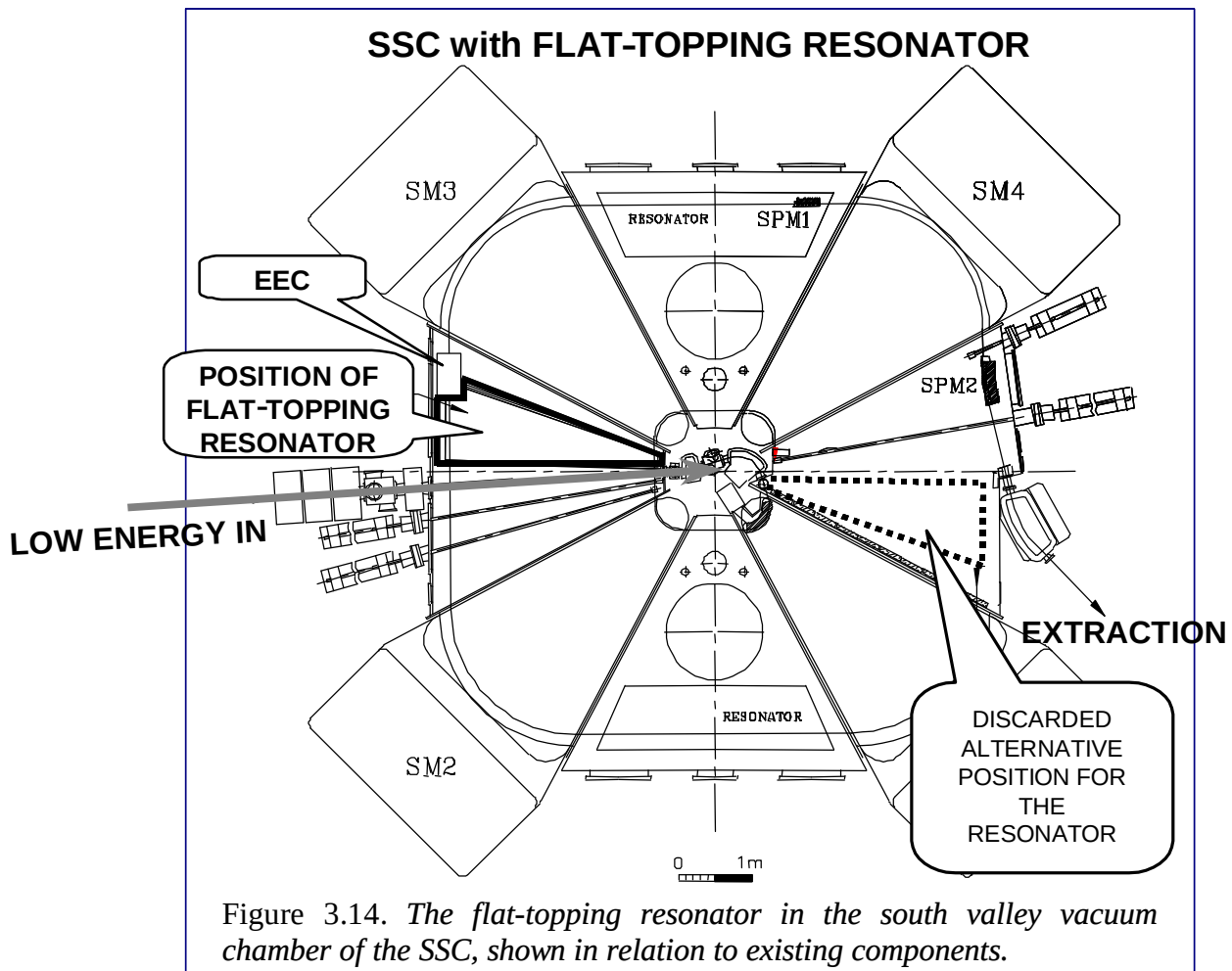


Figure 3.13. Beam width due to energy spread as a function of radius for a 20° beam pulse without flat-topping (thick line) and for a 40° beam pulse with a flat-topping system, consisting of two dees in opposite valleys. The profiles of the radial voltage distributions of the flat-topping resonators were taken to be identical to that of the main resonator. The result for the two flat-topping resonators is slightly better at extraction than for a single resonator, as was shown in figure 3.12.

3.5 SELECTING A TYPE OF FLAT-TOPPING RESONATOR

Considered all these restrictions, it was decided to study a double-gap horizontal half-wave resonator to be implemented in the south valley vacuum chamber of the SSC [DeV04], as shown schematically in figure 3.14.



The resonator will operate at 49.12 MHz, i.e. at the 3rd harmonic of the main rf frequency. Figure 3.15 shows a 3d drawing of the final resonator geometry with the top half partly removed to show its inner structure.

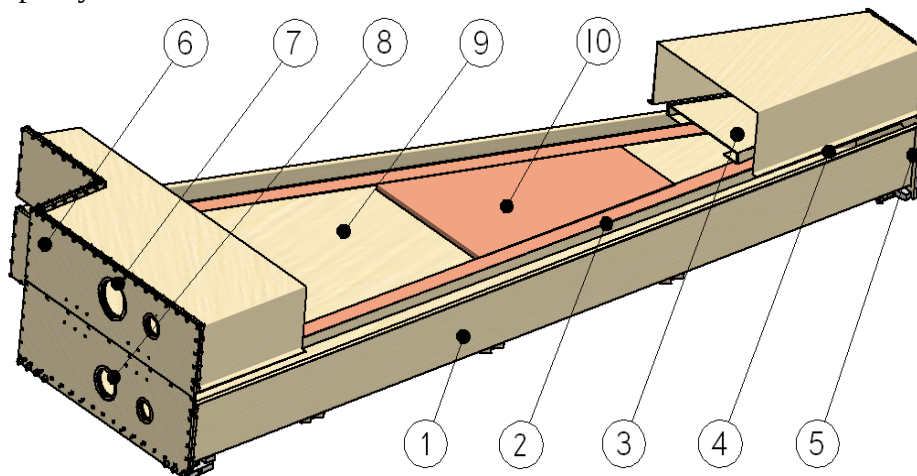


Figure 3.15. Three-dimensional drawing of the flat-topping resonator with the top part cut away, showing the: 1. lower dee housing, 2. acceleration gap, 3. top of the upper dee plate, 4. beam gap, 5. short-circuit plate at injection, 6. short-circuit plate at extraction, 7. and 8. ports for coupling and tuning components, 9. bottom dee plate, 10. plate for detuning of an unwanted resonance mode.

3.6 SUMMARY

- Physical constraints in the north valley vacuum chamber prevent the use of two flat-topping resonators.
- A single-gap resonator does not fit into the available space of the vacuum chamber.
- The installation of a vertical half-wave resonator will be difficult and its voltage distribution is very sensitive to any deformation of the resonator.
- The beam separation at extraction with a horizontal quarter-wave resonator is not as good as can be achieved with other resonators.
- A double-gap horizontal half-wave flat-topping resonator that operates at the 3rd harmonic frequency of the main rf, can be implemented in the existing south valley vacuum chamber, without any major changes to the existing equipment.
- This type of resonator does not exist anywhere at cyclotron facilities and therefore requires a detailed study of its characteristics.
- This type of resonator does not affect the acceleration voltage at the injection and extraction orbits and therefore will it not decrease the orbit separation at these orbits. This advantageous feature is unique to the selected type of resonator only.
- The addition of such a resonator implies that higher intensities (larger beam pulse lengths) can be accelerated with lower energy spread and reduced radial spreading of the beam in the machine. From the calculations it is estimated that the maximum current of 150 μA without flat-topping can be increased to about 350 μA . The flat-topped dee voltage of the injector cyclotron will result in beam bunches to be extracted from the ion source with a higher particle density and result in even higher currents to be finally extracted from the SSC.
- Longitudinal space-charge effects can be partially compensated for by adjusting the phase of the flat-top dee voltage away from its optimum value for low beam intensities.

-o-O-o-

CHAPTER 4

COMPUTATIONAL PHYSICS FOR THE STUDYING OF THE RESONATOR CHARACTERISTICS

4.1 INTRODUCTION

The method of simulating a resonator by a number of homogeneous, but different, transmission line segments connected in series has been used to design the resonators of the SSC [Bot75] and SPC1 as well as the flat-topping system of SPC1 [Con92j]. The same method has also been applied to make the initial design and determine the approximate dimensions of a horizontal half-wave flat-topping resonator for the SSC. The more sophisticated computer program, SOPRANO, was then used to calculate the characteristics of the resonator with a more detailed 3d geometrical description of the electrodes. The different methods used to calculate the SSC flat-topping resonator are discussed in this chapter.

4.2 CALCULATING THE RESONATOR CHARACTERISTICS WITH THE TRANSMISSION LINE METHOD

Figure 4.1 shows the eleven segments used for the calculation of the resonator characteristics. The dashed lines indicate the centerlines of the different segments. The end plates, P and Q, in the figure are short-circuit plates between the inner and outer conductors of the corresponding segments and are simulated as resistors for the purpose of the calculations.

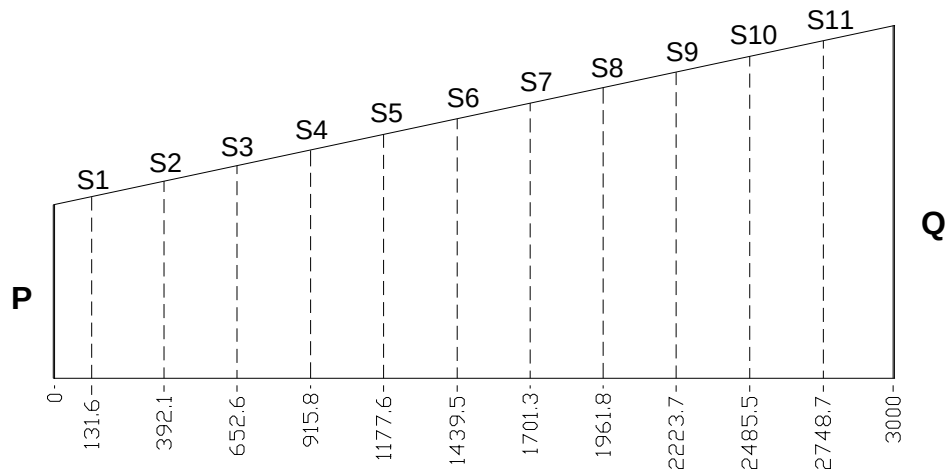


Figure 4.1. *Schematic representation of the outline of the half-wave resonator. Transverse sections were made for calculating the resistance per meter and the characteristic impedance of each segment of the resonator.*

A typical cross-section of the top half of a resonator segment with calculated equipotential and field lines is shown in figure 4.2. The first step is to determine the resonance frequency for given resonator dimensions. The sending-end impedance of the first segment is calculated with the short-circuit plate as termination. This impedance is then used as the receiving-end impedance of the next segment for which the sending-end impedance is again calculated. This procedure is carried out up to the last segment at which the sending-end impedance in parallel with the resistance of the short-circuit plate of the last segment is calculated, in the case of a half-wave resonator as shown in figure 4.1. The frequency at which the calculation is done is then varied until resonance is obtained, i.e. until the reactance is zero. Thereafter the voltage and current distributions as well as the power dissipation in the resonator can be calculated, as described below. The frequency at which the calculation is done can also be kept constant and the length of one of the end resonator segments can be varied to obtain resonance.

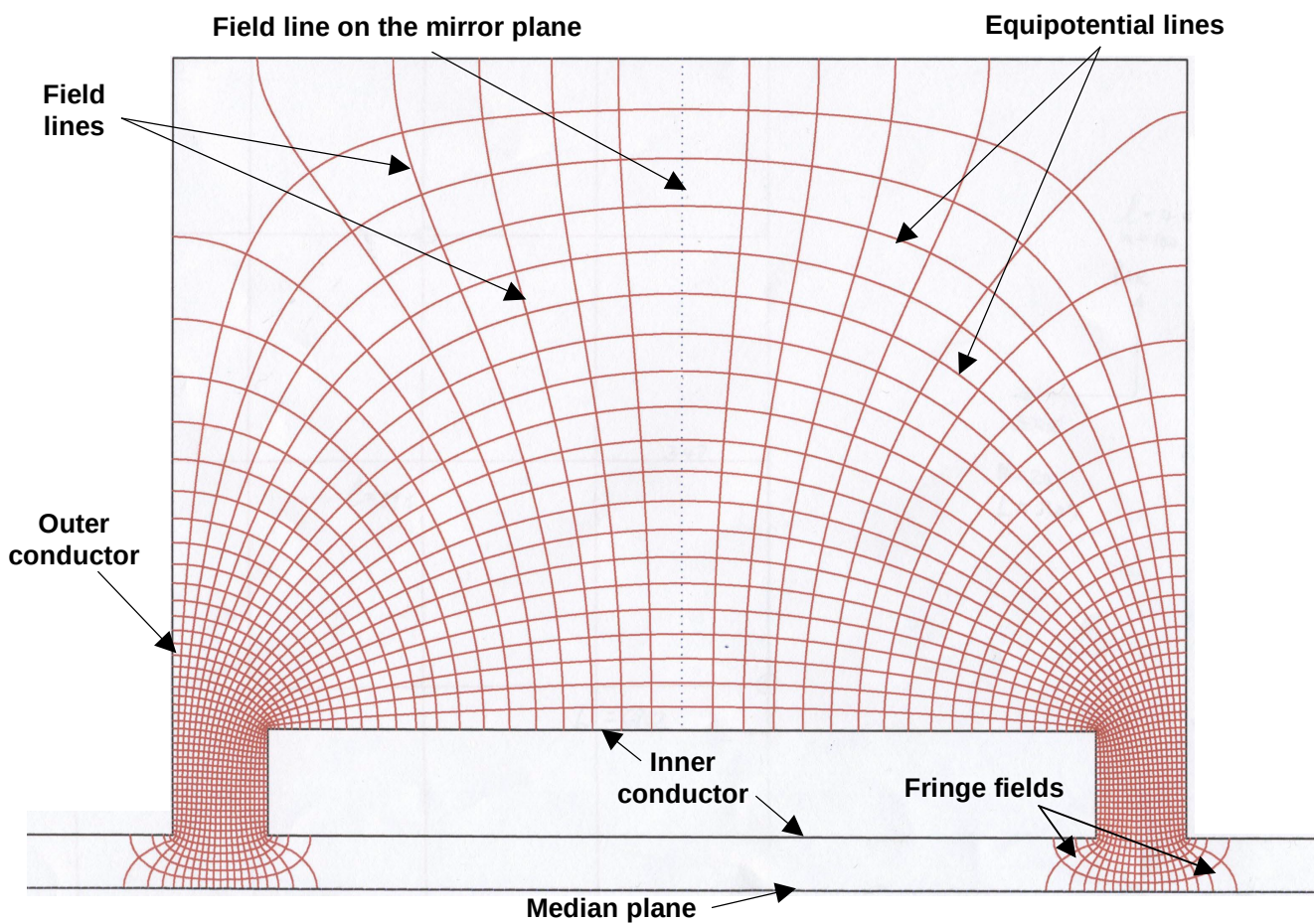


Figure 4.2. The upper halve of a cut through the resonator showing the inner and outer conductors with the calculated curvilinear squares.

4.2.1 Impedance distribution

The characteristic impedance of a transmission line with resistance R per meter, inductance L per meter and capacitance C per meter, at a frequency $f = \frac{w}{2p}$ is given by:

$$\begin{aligned} Z_0 &= \sqrt{\frac{R + Ljw}{Cjw}} \\ &= \sqrt{\frac{L}{C}} \sqrt{1 - j \frac{R}{Lw}} \\ &= Z_{0i} \sqrt{1 - j \frac{R}{Lw}} \end{aligned} \quad (4.1)$$

with $j = \sqrt{-1}$ and Z_{0i} the characteristic impedance of an ideal line without resistance. In the case of cyclotron resonators the conductance between the inner and outer conductors is zero and has therefore been omitted from the equations above.

The sending-end impedance, Z_s , of a transmission line at a distance d from the receiving-end terminated with an impedance, Z_r , is given by [Joh50a]:

$$Z_s = Z_0 \frac{Z_r \cosh g d + Z_0 \sinh g d}{Z_0 \cosh g d + Z_r \sinh g d} \quad (4.2)$$

where $g = a + jb$ is the propagation constant of the transmission line.

The real term of the propagation constant is the attenuation constant and the complex term gives the phase constant of the traveling wave. The following high-frequency approximations can be made [Joh50b]:

$$a = \frac{1}{2} R / Z_0 \text{ and} \quad (4.3)$$

$$b = \frac{w}{c} = w \sqrt{LC} = \frac{2p}{l} , \quad (4.4)$$

$$v = \frac{1}{\sqrt{LC}}$$

where l = the wavelength,

v = is the phase velocity, i.e. the speed of the traveling electromagnetic wave.

It is assumed that the speed v of the transverse electromagnetic, TEM, waves in a cyclotron resonator is the speed of light in vacuum due to the good vacuum conditions inside the resonator.

The constants R , L , C and Z_0 have to be determined for every segment of the resonator by analyzing curvilinear squares [Kra92a], such as in figure 4.2, that were calculated with the computer program POISSON, for 2d numerical field analysis. The space between the outer and inner conductors of each segment is divided into curvilinear squares, which represent transmission-line cells, each with an arbitrary length, d , and each cell carrying the same total electrical current. For the calculation of the characteristic impedance cells between two field lines are in series and those between two equipotential lines are in parallel, as shown in figure 4.2 for a typical sectional geometry of the upper halve of the SSC flat-topping resonator. Due to the time consuming process of creating curvilinear plots with complicated boundary conditions for each section through the resonator, the detail variations in geometry, like the capacitors and the rectangular cut in the outer conductor for the electrostatic extraction channel, were not built into the geometrical description for these calculations. The fact that each cell carries the same total electrical current and has the same voltage across it is used in the calculations below and also in chapter 7 for determining the cooling requirements.

The inductance per meter, L_0 , and capacitance per meter, C_0 , for each curvilinear square, is given by [Kra92b]:

$$L_0 = m_0,$$

$$C_0 = e_0,$$

with $m_0 = 4\pi \times 10^{-7} \text{ H.m}^{-1}$ = permeability of vacuum and

$e_0 = 8.85 \times 10^{-12} \text{ F.m}^{-1}$ = permittivity of vacuum.

The characteristic impedance of each cell is therefore approximated by:

$$Z_{0v} = \sqrt{L_0 / C_0} = \sqrt{m_0 / e_0} = 376.7 \, \Omega \approx 120p \, \Omega,$$

which is the characteristic impedance of free space.

The characteristic impedance and capacitance and inductance per meter for a resonator segment with m cells between the dee and outer conductor and n cells along the surface of the dee, can now be calculated:

$$\begin{aligned} Z_{0i} &= \frac{m}{n} Z_{0v} \\ C &= \frac{n}{m} \epsilon_0 \text{ (F.m}^{-1}\text{)}, \\ L &= \frac{m}{n} \mu_0 \text{ (H.m}^{-1}\text{)}. \end{aligned} \quad (4.5)$$

4.2.2 Calculation of the resistance per meter

Electric currents in the resonator flow in thin layers on the side of the conductors that border the electromagnetic field. The surface resistance on the outer surface of the dee and the inner surface of the outer conductors can be calculated by making use of the surface resistance of the material from which the resonators are made [Ram65a]:

$$R_s = \sqrt{\rho f_{rf} \mu / s} \text{ } (\Omega) \quad (4.6)$$

with s the conductivity of the material in mho.m⁻¹,
 μ the permeability in H.m⁻¹ and
 f_{rf} the frequency in Hz.

In appendix E the electrical properties of some conductor materials are listed. For copper, the surface resistance is given by:

$$R_s (Cu) = 2.61 \times 10^{-7} \sqrt{f_{rf}}$$

The skin depth for copper [Ram65b] is given by:

$$d = 0.066 / \sqrt{f_{rf}} \text{ (m)} \quad (4.7)$$

The electrical resistance of a copper plate of the resonator, assuming a homogeneous current distribution, is given by:

$$R_{Cu-plate} = R_s \times \frac{l}{w} = 2.61 \times 10^{-7} \sqrt{f_{rf}} \times \frac{l}{w} \text{ } (\Omega), \quad (4.8)$$

where l is the length (in m) of the plate in the direction of current flow and w (in m) is the width of the plate, perpendicular to the direction of current flow, as shown in figure 4.3. The resistance per meter can be calculated from equation 4.8 for $l = 1$ m.

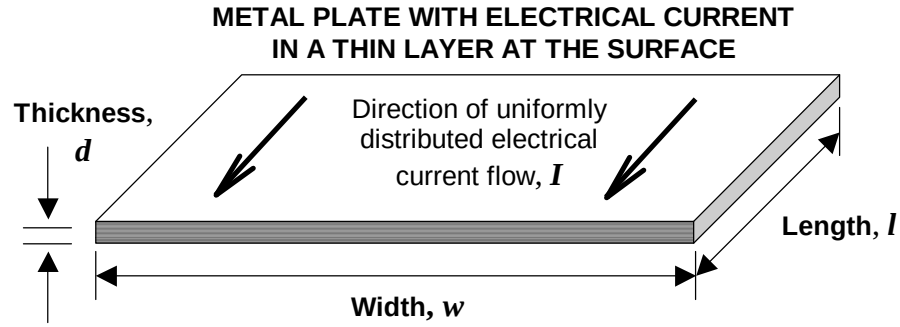


Figure 4.3. Schematic representation of a metal plate, with length, l , width, w , and thickness, d , carrying a current I for the calculation of the resistance.

For calculation of the power dissipation in the resonator it is assumed that current density over the width of a single curvilinear square, at the surface of an electrode, is constant. However, the current density varies over the length of a segment, as defined in figure 4.1. The resistance per meter of a curvilinear square, R_{cs} , can be calculated from the surface resistance and its width, w :

$$R_{cs} \text{ (per meter)} = R_s / w$$

The rate of power dissipation per meter at any point along a segment in a curvilinear square is then:

$$P_{cs} = \frac{I_{cs}^2 R_{cs}}{2} \text{ (W)} \quad (4.9)$$

with I_{cs} the current that flows in the curvilinear square. The aim is to find a resistance per meter that can be used in an equation similar to the one above, but with I the total resonator current at any point along a segment. The width of the different curvilinear squares at the electrode surfaces have to be used for this purpose. To simplify the calculation the curvilinear squares at the electrode surfaces have been divided in two groups, each with its own average width of a square.

The structures of all the segments shown in figure 4.1 are the same, but the width dimensions differ. The parameters used in the calculations are shown in figure 4.4. The **X**-parameters define the width of the 11 segments and are listed in table 5.1. The calculations were done for an acceleration gap, **G**, of 60 mm. The following parameters were kept constant for the calculations: **H** = half height of the resonator = 272.5 mm, **h_d** = height of the dee = 35 mm and **g** = half vertical beam gap = 17.5 mm.

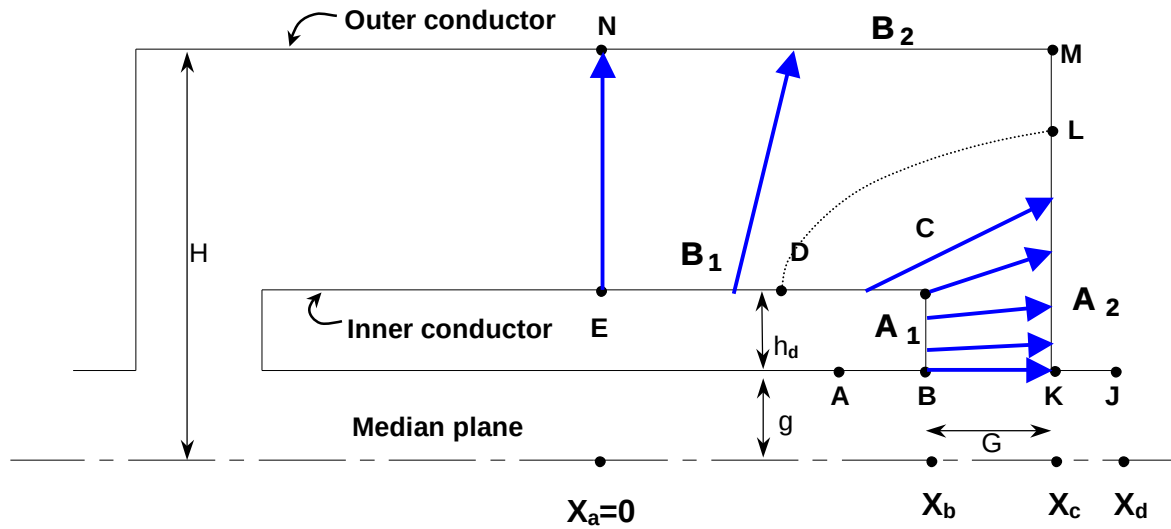


Figure 4.4. A schematic drawing of a typical cross section of the upper half of the flat-topping resonator, that was used in the program POISSON to calculate the field distributions. The positions indicated on the conductor surfaces with letters **A-E** and **J-N** were used for the counting of the number of curvilinear squares. Paths **A₁=ABCD** and **A₂=JKL** are chosen such as to contain the same number of parallel cells along the conductor surface and likewise for paths **B₁=DE** and **B₂=LMN**. The bold arrows schematically indicate the general direction of current flow in the end plates only, as indicated by **P** and **Q** in figure 4.1.

The calculated curvilinear cells of each segment are divided into two groups, which are the high current density (paths **A₁=ABCD** and **A₂=JKL** in figure 4.4) regions and the remaining ones, representing the regions of lower current density (**B₁=DE** and **B₂=LMN**). The former are located mainly on the vertical conductor surfaces and the latter mainly on the horizontal conductor plates. Points **A** and **J** were chosen to be half the vertical beam gap away from the nearest corner, in order to ensure that the fringe fields across the acceleration gap are included in the calculation. Point **D** on the inner conductor is selected one acceleration gap away from point **C**, the top corner of the upper inner conductor. Point **L**, on the outer conductor, is located at the end of the electrical field line that stems from **D**. For each cross-section made through the

resonator, the length of path $\mathbf{A}_1 = \mathbf{AB} + \mathbf{BC} + \mathbf{CD} = \mathbf{g} + \mathbf{h}_d + \mathbf{G}$. The length of path $\mathbf{A}_2 = \mathbf{JK} + \mathbf{KL} = \mathbf{g} + \mathbf{KL}$ and it will have the same number of field cells as on $\mathbf{A}_1$, due to the method of selecting the position of $\mathbf{L}$. The paths $\mathbf{B}_1 = \mathbf{DE}$ and $\mathbf{B}_2 = \mathbf{LMN}$ are the remaining lengths along the respective conductors and represent the regions of lower current density on the section that also have the same number of cells.

The curvilinear cell plot of each section was used to calculate the resistance of each segment. The total resistance of the segment is calculated from the resistance of the two subgroups and then adjusted to account for the four-fold symmetry of conductors. The derivation of the equation for the total resistance in terms of the number of curvilinear cells, is described below:

For a total current I the currents I_{A1} , I_{A2} , I_{B1} and I_{B2} flow, respectively, along the paths $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{B}_1$ and $\mathbf{B}_2$ and are given by:

$$I_{A1} = I_{A2} = \left(\frac{n_A}{n_A + n_B} \right) I \quad \text{and}$$

$$I_{B1} = I_{B2} = \left(\frac{n_B}{n_A + n_B} \right) I$$

where n_A and n_B are the number of curvilinear cells in regions $\mathbf{A}$ and $\mathbf{B}$, respectively

The respective resistances per meter of paths $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{B}_1$ and $\mathbf{B}_2$ are:

$$R_{A1} = R_s \left(\frac{1}{w_{A1}} \right),$$

$$R_{A2} = R_s \left(\frac{1}{w_{A2}} \right),$$

$$R_{B1} = R_s \left(\frac{1}{w_{B1}} \right) \text{ and}$$

$$R_{B2} = R_s \left(\frac{1}{w_{B2}} \right), \quad (4.10)$$

with w_{A1} and w_{A2} the respective total widths of the inner and outer conductors in the selected region of high current density, w_{B1} and w_{B2} the respective total widths of the inner and outer conductors in the region of low current density.

The power dissipation per meter in the segment is the sum of the power dissipations in all the defined sub-regions and can be expressed in terms of the counted number of cells and the resonator geometry:

$$\begin{aligned}
P &= P_{A1} + P_{A2} + P_{B1} + P_{B2} \\
&= \frac{1}{2} I_{A1}^2 R_{A1} + \frac{1}{2} I_{A2}^2 R_{A2} + \frac{1}{2} R_{B1} I_{B1}^2 + \frac{1}{2} R_{B2} I_{B2}^2 \\
&= \frac{1}{2} I^2 \left[R_{A1} \left(\frac{n_A}{n_A + n_B} \right)^2 + R_{A2} \left(\frac{n_A}{n_A + n_B} \right)^2 + R_{B1} \left(\frac{n_B}{n_A + n_B} \right)^2 + R_{B2} \left(\frac{n_B}{n_A + n_B} \right)^2 \right] \quad (4.11)
\end{aligned}$$

Therefore the effective resistance per meter in ohm (of one quarter of the segment) is:

$$\begin{aligned}
R_{eff} &= R_{A1} \left(\frac{n_A}{n_A + n_B} \right)^2 + R_{A2} \left(\frac{n_A}{n_A + n_B} \right)^2 + R_{B1} \left(\frac{n_B}{n_A + n_B} \right)^2 + R_{B2} \left(\frac{n_B}{n_A + n_B} \right)^2 \\
&= R_s \left[\frac{1}{w_{A1}} \left(\frac{n_A}{n_A + n_B} \right)^2 + \frac{1}{w_{A2}} \left(\frac{n_A}{n_A + n_B} \right)^2 + \frac{1}{w_{B1}} \left(\frac{n_B}{n_A + n_B} \right)^2 + \frac{1}{w_{B2}} \left(\frac{n_B}{n_A + n_B} \right)^2 \right] \\
&= \frac{R_s}{(n_A + n_B)^2} \left[n_A^2 \left(\frac{1}{w_{A1}} + \frac{1}{w_{A2}} \right) + n_B^2 \left(\frac{1}{w_{B1}} + \frac{1}{w_{B2}} \right) \right]. \quad (4.12)
\end{aligned}$$

From the four-fold symmetry the total resistance per meter of the segment is given by:

$$R_t = R_{eff} / 4 \quad \Omega.$$

The effective surface resistance per meter can also be expressed in terms of the average cell widths in each of the different defined paths. w_{A1} , w_{A2} , w_{B1} and w_{B2} in equation 4.12 can be replaced by $n_A \cdot \overline{w_{A1}}$, $n_A \cdot \overline{w_{A2}}$, $n_B \cdot \overline{w_{B1}}$ and $n_B \cdot \overline{w_{B2}}$ respectively, where $\overline{w_{A1}}$, $\overline{w_{A2}}$, $\overline{w_{B1}}$ and $\overline{w_{B2}}$ are the respective average widths of a cell. Equation 4.12 then becomes:

$$R_{eff} = \frac{R_s}{(n_A + n_B)^2} \left[n_A \left(\frac{1}{\overline{w_{A1}}} + \frac{1}{\overline{w_{A2}}} \right) + n_B \left(\frac{1}{\overline{w_{B1}}} + \frac{1}{\overline{w_{B2}}} \right) \right] \quad (4.13)$$

4.2.3 Calculation of the resistance of the short-circuit plates.

The vertical end plates of the resonator are short-circuit plates, although with very small but nevertheless finite resistances, in the transmission line model. The direction of the currents that flow from the inner conductor to the outer conductor are schematically shown in figure 4.4 with bold arrows in one half of the figure and they vary in direction from horizontal in the acceleration gap to vertical near the azimuthal center of the resonator. For the calculation of the resistance of these plates, the currents were again divided into the regions of low and high current density, where the former is regarded as flowing along the vertical distance $\mathbf{EN}$ and the latter along the horizontal distance $\mathbf{G}$.

The current in a curvilinear square in a plate, I_{cs} , is given by:

$$I_{cs} = \frac{I}{4(n_A + n_B)}$$

The associated power dissipation of such a short-circuit plate P_{scp} , with the average width of a curvilinear square in the region of high and low current density, $\overline{w_A}$ and $\overline{w_B}$, respectively, is calculated as follows from its four-fold symmetry:

$$\begin{aligned} P_{scp} &= 4 \frac{I_{cs}^2 R_{scp}}{2} \\ &= 2 \left[\frac{I}{4(n_A + n_B)} \right]^2 R_s \left[\frac{G}{n_A \overline{w_A}} + \frac{(H - h_d - g)}{n_B \overline{w_B}} \right] \\ &= \frac{1}{2} I^2 R_s \left[\frac{1}{4(n_A + n_B)^2} \right] \left[\frac{G}{n_A \overline{w_A}} + \frac{(H - h_d - g)}{n_B \overline{w_B}} \right] \end{aligned} \quad (4.14)$$

The resistance of a short-circuit plate R_{scp} is therefore given by:

$$R_{scp} = R_s \left[\frac{1}{4(n_A + n_B)^2} \right] \left[\frac{G}{n_A \overline{w_A}} + \frac{(H - h_d - g)}{n_B \overline{w_B}} \right] \quad (4.15)$$

A shunt, or parallel resistance can also be calculated for a resonator. The shunt resistance at a point along the dee where the dee voltage is V_d , is given by:

$$R_{SH} = \frac{V_d^2}{2P}$$

with P the total power dissipated in the resonator.

4.2.4 Calculation of the current and voltage distributions along the resonator, as well as the power dissipation and the Q-value

A locally developed computer program, SSCFLAT [Bot04a], was used to calculate the impedances and subsequently the voltage, current and power distribution, at a specified frequency, along the resonator, using the division of the resonator in segments as in figure 4.1 and the sending-end impedance calculated as described above. The program is an adapted version of the program FLATOP that was successfully used with the design of the flat-topping resonator of SPC1 [Con92h].

The program SSCFLAT uses the calculated impedance and resistance per meter values obtained from the curvilinear plots and as well as the resistances of the vertical short-circuit plates.

A voltage is applied at one side of the resonator. The voltage and current at the junction of this segment and the next one are then calculated. These voltage and current values are used as the sending-end values for the next segment as illustrated in figure 4.5. That receiving-end becomes the sending-end of the next section, until the complete resonator is covered. The resistance of plate A is used as the receiving-end impedance of segment #11. The resistance of the other vertical plate, B, is added in parallel to the impedance of segment #1.

From these values the power dissipation can be calculated.

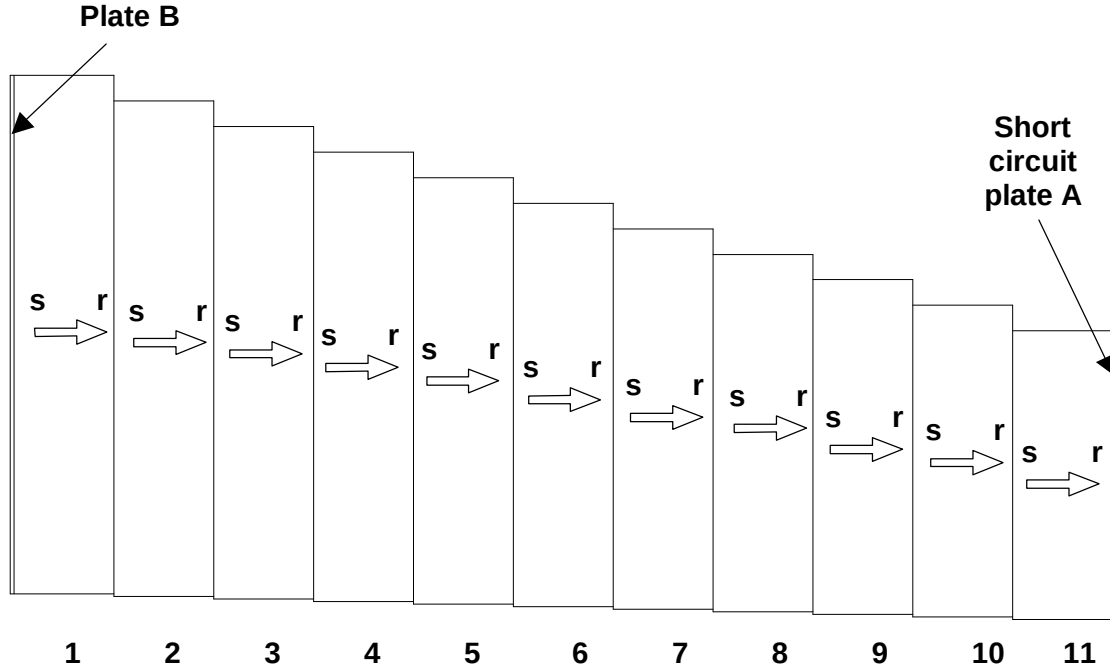


Figure 4.5. A schematic representation of the eleven resonator segments for the calculation of the voltage and current distributions at the resonance frequency. A voltage is applied and adjusted at plate B until the required maximum voltage is obtained somewhere along the length of the resonator, and thereby creating the voltage, current and power distribution along the length of the resonator.

The applied voltage is adjusted until the required maximum acceleration voltage of about 72 kV is reached, which occurs at a position about halfway between the vertical plates.

The voltages and currents at the sending- and receiving-ends are expressed as phasors in terms of their respective peak values. To obtain actual voltages the sinusoidal time variation has to be included. The real and imaginary components, E_r and E_i , of the dee voltage at the receiving-end of a segment are calculated as follows from the sending-end values [Joh50c]:

$$\begin{aligned}
 E_r &= E_s + jE_i \\
 &= E_s \left(\cosh gl - \frac{Z_0}{Z_s} \sinh gl \right)
 \end{aligned} \tag{4.17}$$

and the current at the receiving-end is:

$$\begin{aligned} I_r &= I_r + I_i \\ &= I_s \left(\cosh gl - \frac{Z_s}{Z_0} \sinh gl \right), \end{aligned} \quad (4.18)$$

where

l = the distance from the sending-end to receiving-end, i.e. the length of a segment,

E_s = the voltage at the sending-end,

I_s = the current at the sending-end, and

Z_s = the sending-end impedance calculated above.

The amplitudes of the voltage and current at the receiving-end are respectively:

$$\begin{aligned} E_{Ar} &= \sqrt{E_r^2 + E_i^2} \quad \text{and} \\ I_{Ar} &= \sqrt{I_r^2 + I_i^2}. \end{aligned} \quad (4.19)$$

The phases of the voltage and current at the receiving-end are respectively:

$$\begin{aligned} q_{Er} &= \arctan \frac{E_i}{E_r} \quad \text{and} \\ q_{Ir} &= \arctan \frac{I_i}{I_r}. \end{aligned} \quad (4.20)$$

The power dissipation, P , at the receiving-end of a segment, is the dissipated power in all the remaining segments up to and including the short-circuit plate. It is calculated from:

$$P = \frac{1}{2} (E_r I_r + E_i I_i) \quad (4.21)$$

The Q-value, is calculated as follows:

$$Q = 2p f \frac{\text{Power}}{\text{Dissipated power}} \quad (4.22)$$

4.3 NUMERICAL FIELD ANALYSIS FOR CALCULATING THE RESONATOR CHARACTERISTICS IN 3D

4.3.1 Introduction

The commercial computer program SOPRANO, which uses finite-element methods to perform 3d numerical field analysis of rf systems, was used to calculate the resonance frequencies of the different modes of a finite-element mesh-geometry that accurately describes the resonator in three dimensions. SOPRANO is one of the analysis modules of the software suite OPERA-3d [Vec_1] and is used to compute high-frequency electromagnetic fields in three dimensions. SOPRANO has two modules:

- a) SOPRANO-EV for solving resonator problems containing lossless, isotropic dielectrics. It calculates the different modes of the resonator and returns the characteristic frequencies (eigenvalues) and the related modes of vibration (eigenvectors).
- b) SOPRANO-SS for solving the wave equation to calculate the electric field. It calculates the steady-state fields at a specified frequency.

Only the SOPRANO-EV-module was used in this study to determine the resonator characteristics. The calculated SOPRANO database contains all pre-defined geometrical structures, the electromagnetic field information and the stored energy in the system for each of the calculated eigen-values within the range requested by the user.

The methods and accuracy of obtaining information with the SOPRANO program were tested by performing calculations for a co-axial cable resonator, for which the characteristics can be calculated from exact analytic expressions. A half-scale model was specially built for testing the basic principles of the horizontal half-wave resonator and also to some extent validate the calculation techniques used with the finite-element method. The measured and calculated results show good agreement within the limitations of the model and are presented in chapter 5.

4.3.2 The eigen-value solver SOPRANO-EV

In the calculations of the resonance frequencies and field distributions SOPRANO automatically assumes that all the electrodes of the resonator are perfect electrical conductors (PEC) in vacuum. These are good approximations at high frequencies, since resistance per meter is typically in the order of $m\Omega$ and Z_0 in the order of several tens of ohms. The current and voltage distributions are therefore mainly determined by the Z_0 -values. The potential difference e_{ij} , between any two nodes i and j in the SOPRANO model, along a path, l_{ij} , is given by [Vec_2]:

$$e_{ij} = \int_{l_{ij}} \bar{\mathbf{E}} \cdot d\mathbf{l}$$

where $\bar{\mathbf{E}}$ is the electric field and $d\mathbf{l}$ the line segment along the path of integration.

At high frequencies the electric field in the cavity obeys the vector wave equation:

$$\nabla \times \frac{1}{m} \nabla \times \bar{\mathbf{E}} = -\frac{\partial^2 (e \bar{\mathbf{E}})}{\partial t^2},$$

with the electric field perpendicular to the electrodes at their surfaces, since they are assumed to be PEC. The Neumann boundary condition, $\bar{\mathbf{E}} \times \bar{\mathbf{n}} = 0$, is therefore applied by SOPRANO at all the conductor surfaces of the cavity. When it is assumed that all the materials within the cavity have linear electromagnetic properties, as in the case of the flat-topping resonator for the SSC, the vector wave equation can be reduced to the vector Helmholtz equation for each individual mode m , using the Lorentz gauge to ensure a uniquely defined set of equations :

$$\nabla \times \frac{1}{m} \nabla \times \bar{\mathbf{E}} - e v_m^2 \bar{\mathbf{E}} = 0,$$

with v_m the eigenvalues of the modes. SOPRANO reduces the above equation to an eigenvalue equation from which the eigenvalues and eigenvectors are solved [Vec_1].

To obtain the required resonance mode at 49.1 MHz and optimize other characteristics, like the stored energy, power dissipation, Q-value, voltage and current distributions with SOPRANO and its post-processor, described below, the dimensions of the resonator electrodes were varied.

4.3.3 The SOPRANO post-processor

The post-processing module, OPERA-3d POST permits easy access for the user to create graphical displays of the results from calculations with SOPRANO and to do further in-depth numerical analysis as required by the user. The graphical displays are, apart from portraying results, handy tools for immediate assessment of the legitimacy of the definition of the model and its boundary conditions. In complex 3d-structures it also often leads to insight into characteristics and phenomena not expected and subsequently lead to new avenues of calculations and design. User-defined command programs were used to extract information from the calculated database. These programs and other post-processing features permit calculation of any user-defined integral over any line, surface or volume within the finite element structure. The radial voltage distribution along the acceleration gaps, the electric and magnetic energy stored in the resonator can be obtained by calculating appropriate line and volume integrals over the volume enclosed within the resonator.

4.3.4 The power dissipation in the resonator

In a resonator at high frequencies, at which the skin depth is small in comparison with the thickness of the conductor and the internal inductance can be neglected, the linear current density is equal to the tangential component of the magnetic field:

$$\bar{\mathbf{J}} = \bar{\mathbf{n}} \times \bar{\mathbf{H}} \quad (4.23)$$

with $\bar{\mathbf{n}}$ the normal vector to the surface and $\bar{\mathbf{H}}$ the magnetic field.

The power dissipation P in the resonator can be calculated from the linear current density distribution at the walls of the resonator, obtained for perfect conductors, and the finite conductivity of the resonator material [Ram65c]:

$$P = \frac{1}{2} R_s \int_S \mathbf{H}^2 \cdot d\mathbf{S} \quad (\text{W.m}^{-2}) \quad (4.24)$$

with S the total inner surface of the resonator and R_s the surface resistance of the resonator material at the resonance frequency of f_{rf} .

4.3.5 Calculation of the stored energy and the Q-value

The stored energy in the resonator can be calculated either by integrating the peak magnetic or electric energy density over the volume included by the resonator electrodes:

The maximum stored magnetic energy in the volume V included between the resonator electrodes is:

$$U_m = \frac{1}{2} m_0 m_r \iiint_{\text{volume } V} \mathbf{H}^2 \cdot dV \quad (4.25)$$

with m_0 = the permeability of vacuum and m_r the relative permeability.

The quality factor, Q , of the resonator can be calculated from the stored energy, U , and power loss, P , in the resonator.

$$Q = \frac{\nu_0 (\text{stored energy})}{\text{average power loss}} = \frac{2p f_{rf} U_m}{P} \quad (4.26)$$

with f_{rf} = the resonance frequency of the resonator [Ram65d].

The peak stored energy in the magnetic field is equivalent to the maximum energy stored in the inductance L :

$$U_m = \frac{1}{2} L I_{\max}^2 ,$$

with $I_{\max}$ the maximum amplitude of the current.

The Q-value of the inductor at resonance is given by:

$$Q = \frac{\nu_0 L}{R} .$$

Many conditions may contribute to differences in calculated and measured Q-values, like for example, the quality of the polished surfaces, the direction of the polishing action and the quality of welds and movable contacts. It can be expected that

$$Q_{\text{measured}} < Q_{\text{calculated}} .$$

-o-O-o-

CHAPTER 5

RESONATOR DESIGN AND MEASUREMENT

5.1 INTRODUCTION

Both methods described in the previous chapter were used to make an initial design of the flat-topping resonator before a half-scale model was built to verify some of the calculations. Thereafter calculations were mainly done with the computer program SOPRANO to optimize and finalize the design. To avoid repetition, all the intermediate steps in reaching a final design are not described here. In the next section the dimensions and drawings of the final design are discussed. Thereafter the results of calculations with the transmission line method are presented. The half-scale model and measuring techniques in general are then described. Thereafter follows the calculations of the 3d electromagnetic field distributions in the resonator with SOPRANO. Finally measurements on the full-scale resonator and some refinements in the basic design, such as separation of the unwanted nearby resonance frequency from the required frequency, the effect of dimensional changes due to temperature changes on the resonance frequency, the transit time factor and the effect on the beam due to vertical electrical field components in the acceleration gap are dealt with in this chapter. The adjustable capacitors for coarse tuning, the coupling and tuning loops and the power dissipation are discussed in following chapters.

5.2 RESONATOR DIMENSIONS AND CONSTRUCTION

The schematic layout of the orientation of the double-gap horizontal half-wave flat-topping resonator in relation to the center of the SSC is shown in figure 5.1. The radial length of the beam-exit acceleration gap is shorter than the entrance gap due to the cut-away section for the EEC. The centerlines along the middle of the acceleration gaps do not intersect at the center of the cyclotron, which implies that the resonator is slightly displaced on a radial line. The azimuthal angle of the resonator was selected such that the path distance between the centers of the acceleration gaps will ensure peak voltage crossings of the gap for the selected 3rd harmonic mode. The lengths of

both acceleration gaps vary from 60 mm at injection to 100 mm at extraction. Several geometrical dimensions were varied to achieve the required and optimum characteristics for the resonator.

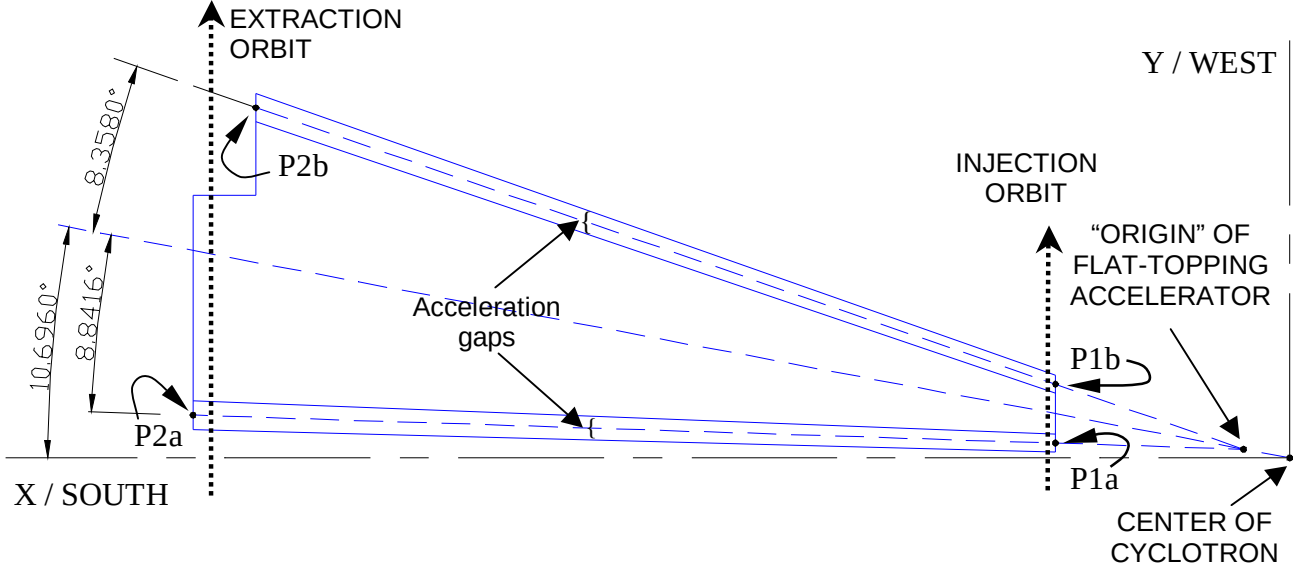


Figure 5.1. *Schematic layout and orientation of the flat-topping resonator in relation to the center of the SSC showing the variation in azimuthal distance between the acceleration gaps. The acceleration gap varies from 60 mm at injection to 100 mm at extraction.*

The coordinates and parameters for defining the geometry of the resonator are given in figure 5.2. The dimensions are shown without considering the thickness of the conducting surfaces, but are accurate indications of the distances between electrodes inside the resonator, i.e. the dimensions that determine the resonant modes of the resonator and its associated electromagnetic field distributions.

Coordinates that are fixed for all calculations are (in mm):

- X- and Y-axis of the coordinate system aligns with the South-North-line and East-West-line, respectively, and cross at the center of the cyclotron
- O: Center of the cyclotron at coordinates $(X;Y) = (0; 0)$
- R: Origin of the flat-topping resonator at $(-158.562; 29.95)$
- A: $(-803.07; 20.816)$
- B: $(-3820; 98.496)$
- C: $(-3820; 900)$
- D: $(-3544.47; 900)$

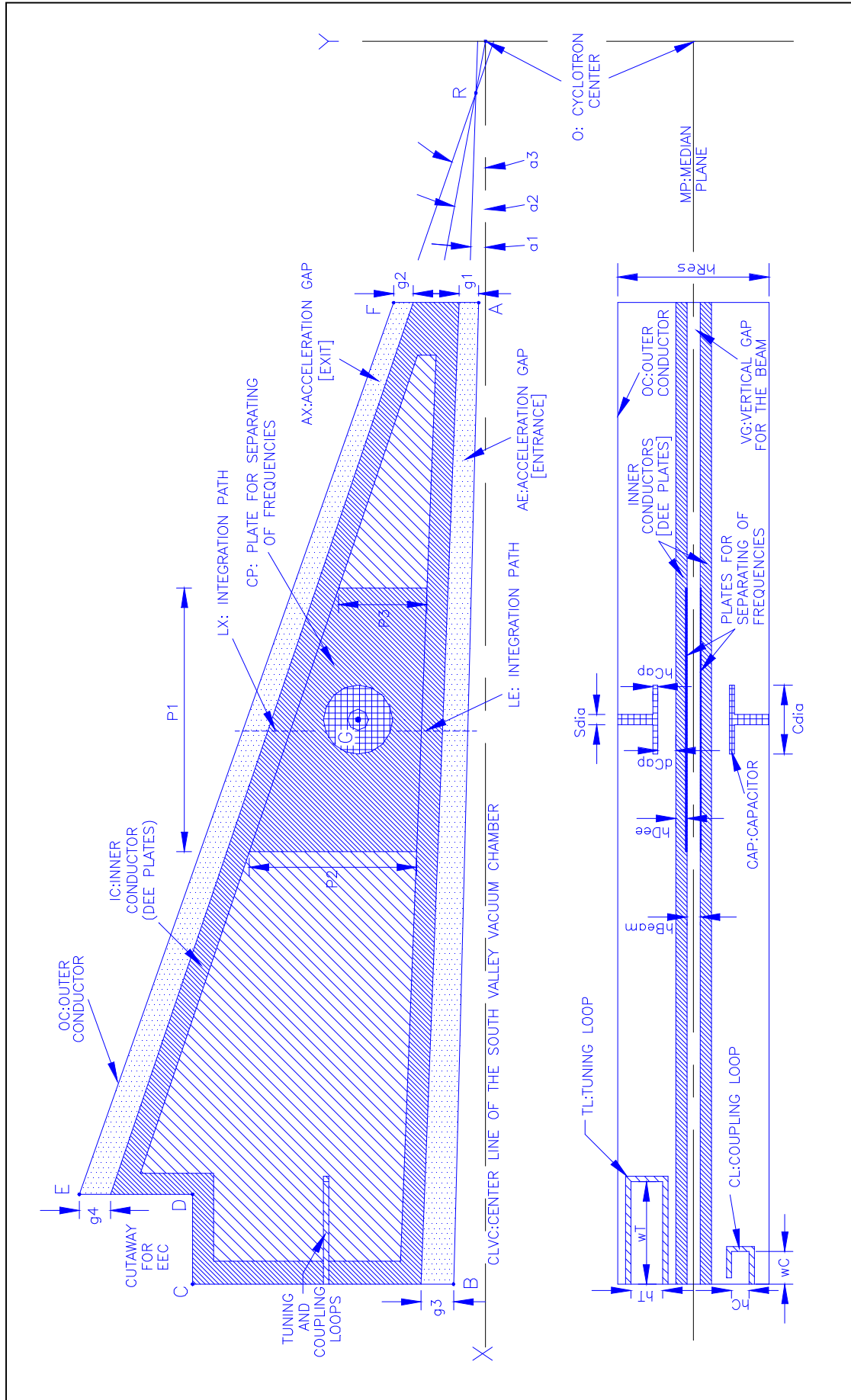


Figure 5.2. Schematic drawing showing the coordinates and parameters that were used in the 3d numerical field analysis of the resonator.

- E: (-3544.47; 1247.555)
- F: (-803.07; 282.551)
- G: Position of the central axis of the capacitor (-2085; 391.47)

Parameters that are fixed for all calculations are:

- Length AB = 3017.93 mm
- Length BC = 801.504 mm
- Length CD = 275.53 mm
- Length DE = 347.924 mm
- Length EF = 2906.288 mm
- Length AF = 261.735 mm
- Length BE = 1182 mm = maximum width of the resonator
- $\alpha_1 = 1.8542^\circ$ = the angle subtending the center line in the first acceleration gap and due south
- $\alpha_2 = 10.6960^\circ$ = the angle GOX
- $\alpha_3 = 19.0540^\circ$ = the angle subtending the center line in the second acceleration gap and due south
- $g_1 = 60$ mm = the length of the acceleration gap at the first beam orbit through the resonator at the entrance to the resonator
- $g_2 = 60$ mm = the length of the acceleration gap at the first beam orbit through the resonator at the exit of the resonator
- $g_3 = 100$ mm = the length of the acceleration gap at the last beam orbit through the resonator at the entrance to the resonator
- $g_4 = 96.3$ mm = the length of the second acceleration gap adjacent to the cutaway section for the EEC.

Parameters that were varied to obtain different results are:

- h_{Res} = height of the resonator (inner dimension; final value = 465 mm)
- h_{Dee} = height of the dee (final value = 35 mm)
- h_{Beam} = vertical gap for the beam between dees (final value = 40 mm)
- C_{dia} = diameter of capacitor plate (final value = 212 mm)
- S_{dia} = diameter of the capacitor stem (final value = 32 mm)
- h_{Cap} = height of the capacitor plate (final value = 15 mm)

- d_{Cap} = height between capacitor plate and the surface of the dee
- w_{T} = inner width dimension of the tuning loop (final value = 315 mm)
- h_{T} = inner height dimension of the tuning loop (final value = 98 mm)
- w_{C} = inner width dimension of the coupling loop (final value = 54.1 mm)
- h_{C} = inner height dimension of the coupling loop (final value = 100 mm)
- P_1 = length of the capacitance plates on the dees (final value = 810 mm)
- P_2 = side length of plate at larger radius (final value = 514 mm)
- P_3 = side length of plate at smaller radius (final value = 271 mm)
- Integration paths LE and LX on median plane.

In all the discussions in this chapter, except where deviations are explicitly mentioned, these parameter values are used. The total inner conducting surface of the resonator is 14.36 m^2 and the resonator volume is equal to 1.0689 m^3 .

Figures 5.3 and 5.4 show the resonator during construction and installation in the SSC, respectively.



Figure 5.3. *The assembled lower half of the resonator.*

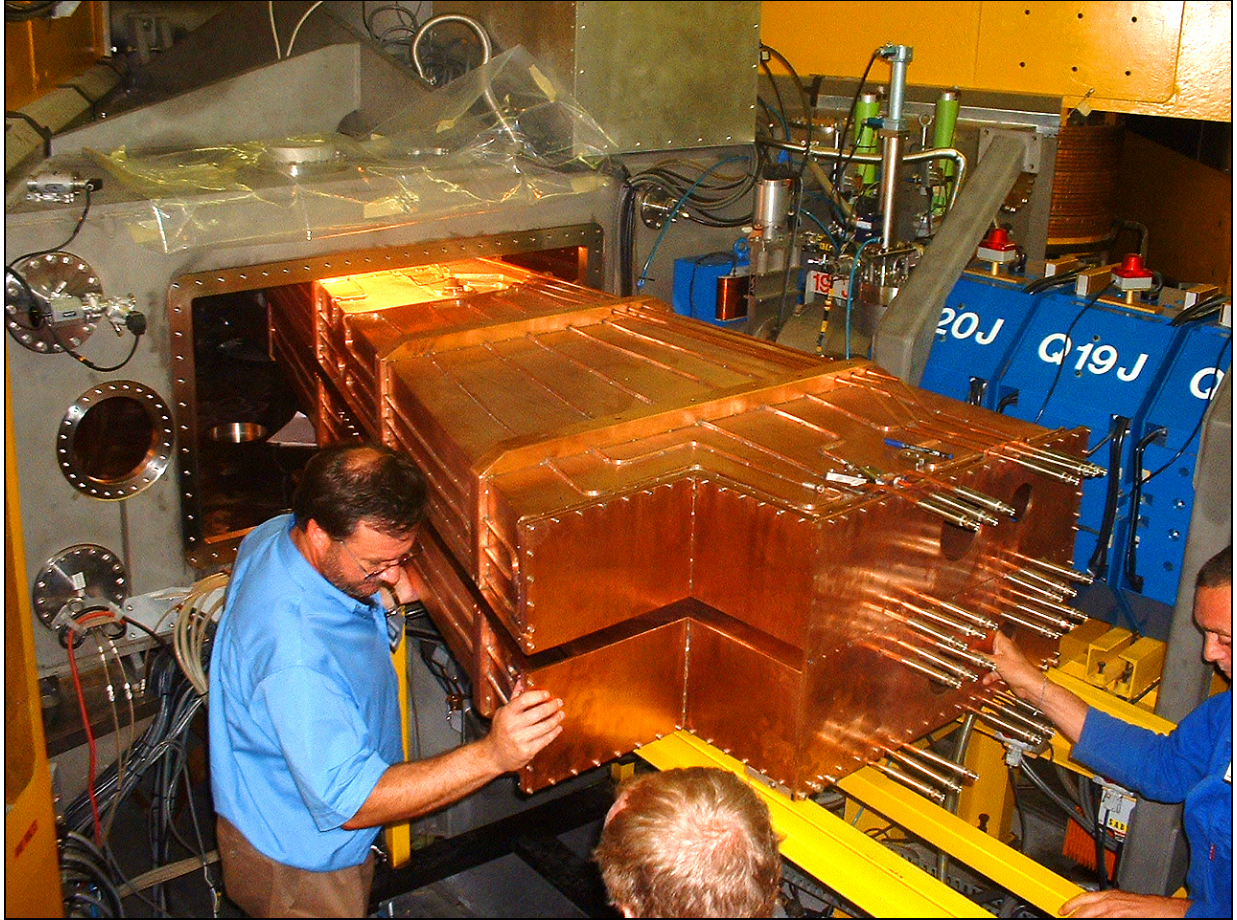


Figure 5.4. *Installing the resonator in the south valley vacuum chamber of the SSC, through an existing port.*

Copper plate of 2 mm thickness, which is more than adequate for the calculated depth at which currents are expected to flow, was used for the construction of the resonator. The vertical end plates (short-circuit plates) and the vertical sides of the dees were, however, made of 10 mm thick copper plates for additional mechanical stability of the structure and also for better heat conduction in the regions of highest current density. The large flat copper surfaces were welded together to obtain the required length. The top and bottom dee plates are welded to the two halves of the short-circuit plate at extraction. Near injection, home-made contact fingers were used to connect the dee plates to the short-circuit plate.

5.3 CALCULATED RESULTS FOR THE FULL-SCALE RESONATOR, USING THE TRANSMISSION-LINE METHOD

The characteristics of the flat-topping resonator were calculated according to the methods described in chapter 4. The surface resistivity, R_s , for copper at 49.1 MHz, is 1.829 m Ω and the penetration depth in the surface is 9.4 μm . The current density at a depth of 9.4 μm below the surface is 36.9 % of the density at the surface. At this depth the current is 45° out of phase with the current at the surface. The ratio m/n , obtained from the counted number of series and parallel field cells at each section, is substituted into equation 4.5 to calculate the characteristic impedance, capacitance and inductance per meter for every section. The calculated results are listed in table 5.1 for the case of a constant acceleration gap distance of 60 mm. The segment length is 0.279 m.

Table 5.1. The calculated surface resistance, R_t , characteristic impedance, capacitance and inductance (per meter) at each of the sections of the resonator for an acceleration gap of 60 mm. The length of a segment is 0.279m.

Section/ segment	X_b (mm)	X_c (mm)	X_d (mm)	n_A	n_B	R_t (Ω/m)	m/n	$Z_{oi}(\Omega)$	C (pF/m)		L (nH/m)	
									$\epsilon_0 * n/m$	$1/c/Z_{oi}$	$m_0 * m/n$	Z_{oi}/c
S1	66.6	126.7	176.7	57.9	11.1	6.1×10^{-3}	0.123	46.3	72.0	72.0	154.5	154.4
S2	108.0	168.1	218.1	57.9	11.1	4.6×10^{-3}	0.122	46.1	72.3	72.4	153.8	153.8
S3	149.4	209.5	259.5	60.0	19.0	4.0×10^{-3}	0.110	41.6	80.1	80.2	138.8	138.8
S4	190.8	250.9	300.9	62.1	26.9	3.5×10^{-3}	0.100	37.5	88.9	89.0	125.1	125.1
S5	232.2	292.3	342.3	57.7	31.3	3.1×10^{-3}	0.098	36.8	90.6	90.6	122.8	122.8
S6	273.6	333.7	383.7	60.0	39.0	2.8×10^{-3}	0.089	33.6	99.2	99.3	112.1	112.1
S7	314.9	375.0	425.0	56.1	42.9	2.6×10^{-3}	0.085	31.9	104.5	104.6	106.4	106.4
S8	356.3	416.4	466.4	52.9	46.1	2.3×10^{-3}	0.077	29.0	115.0	115.0	96.7	96.7
S9	397.7	457.8	507.8	54.4	54.6	2.1×10^{-3}	0.072	27.1	123.0	123.1	90.4	90.4
S10	439.1	499.2	549.2	56.4	62.6	2.0×10^{-3}	0.069	25.9	128.7	128.8	86.4	86.4
S11	480.5	540.6	590.6	54.7	65.3	1.9×10^{-3}	0.066	24.8	134.4	134.5	82.7	82.7

The calculated surface resistances of the front plate (nearest to the injection orbit) and the rear plate (near the extraction orbit) are listed in table 5.2 with the respective dimensions used in the calculation.

Table 5.2. The calculated surface resistance, R_{scp} , of the two short-circuit plates of the flat-topping resonator and the geometrical parameters used for the calculation.

Parameter	Unit	FRONT PLATE	REAR PLATE
R_s	$m\Omega$	1.829	1.829
G	m	0.06	0.06
H	m	0.22	0.22
h_d	m	0.35	0.35
g	m	0.175	0.175
w_{A1}	m	0.1125	0.1125
w_{A2}	m	0.2282	0.2044
w_{B1}	m	0.0066	0.4205
w_{B2}	m	0.1709	0.6087
R_{scp}	mW	5.179	1.475

The applied voltage is adjusted until the required dee voltage, as shown in chapter 3 calculated with the program COC, of about 62 kV is reached, which occurs at a position about halfway between the vertical plates. The sending- and receiving-end currents and the power dissipation of each segment is also calculated simultaneously. The calculated voltages and currents at the junction between segments are shown in figure 5.5. The power consumption is 15.2 kW and the Q-value is 6020.

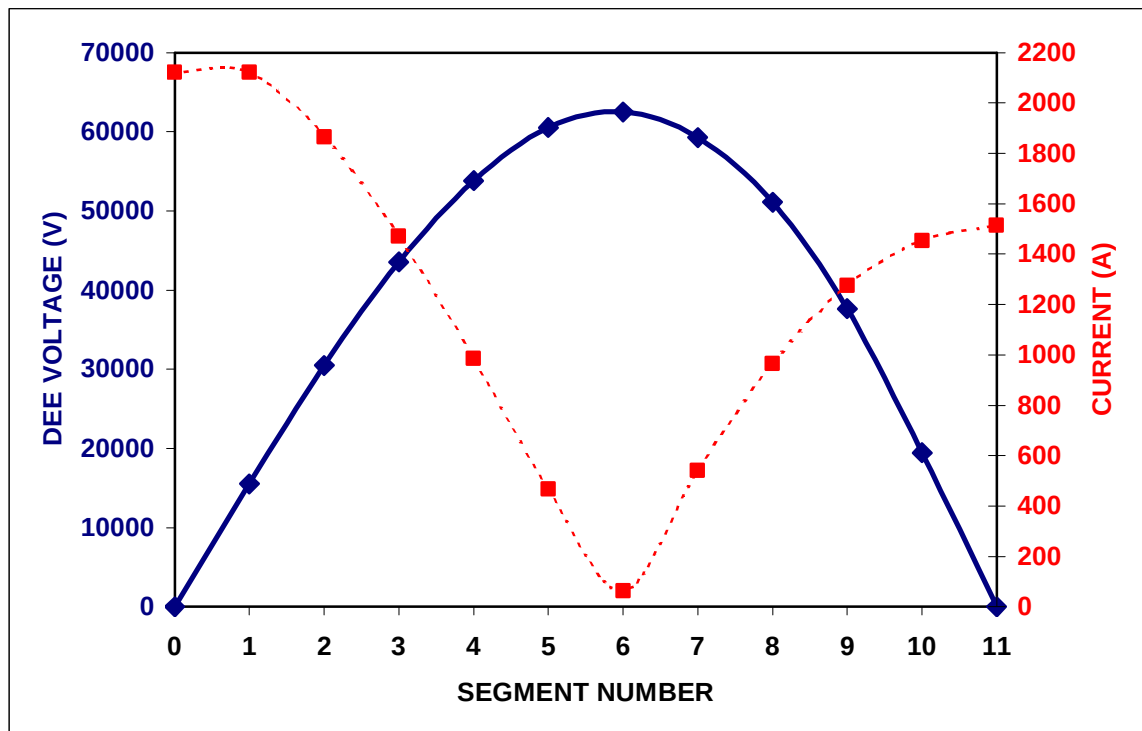


Figure 5.5. The calculated voltage and current distribution in the resonator. Segment #0 represents the values at the sending-end of segment #1.

5.4 CALCULATED AND MEASURED RESULTS FOR THE HALF-SCALE MODEL

5.4.1 Measuring techniques and equipment

The main resonator characteristics to be measured are the resonance frequencies, the Q-value and the power dissipation. If unwanted resonances close to, or at multiples of the required frequency, are present, it is important to modify the resonator dimensions to shift these resonances away from the frequency at which they can influence the operation of the resonator. Since the output of a power amplifier in general contains harmonics, resonances at these frequencies can be excited and may lead to damage of contact fingers or feed-throughs. To change the frequency of an undesired resonance the field pattern at this frequency has to be understood.

To measure the resonance frequencies, power can be coupled to the dee through a loop or a capacitor. Another loop or capacitor can be used to detect the signal, for instance an rf-voltmeter, as shown in figure 5.6. A small transformer can also be inserted in series with the loop or capacitor, coupled to the generator. If a network analyzer is available there is no need for a second loop or capacitor. All these methods, as well as a vector impedance meter connected directly between the dee and the outer conductor, have been used to do measurements on the half-scale model and the full-scale resonator.

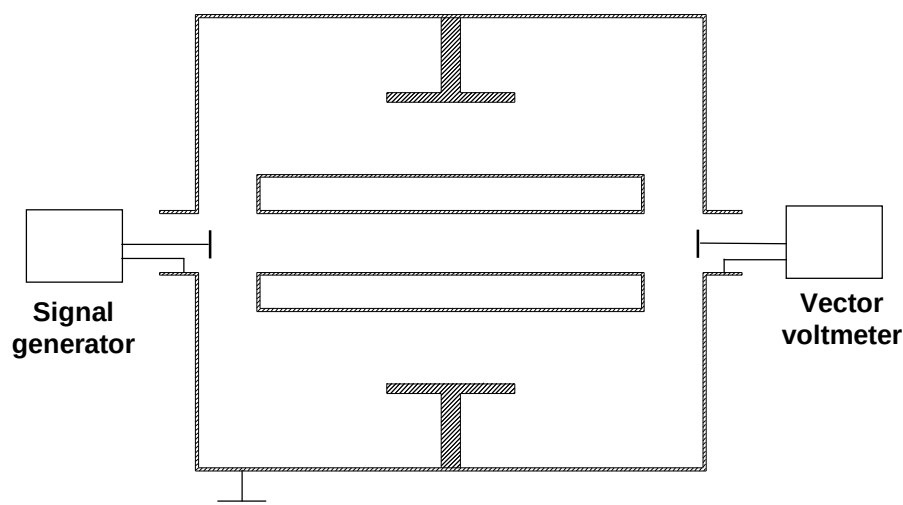


Figure 5.6. Schematic representation of the measuring setup with the signal generator and vector voltmeter connected to probes that are positioned in the acceleration gap of the resonator.

The signal generator and the vector voltmeter, that were used to measure the dee voltage, were coupled capacitively to the dee. The smallest coupling, with sufficient signal strength for measurement, was used for both the signal input and measuring probe, in order to ensure minimal disturbance of the field distribution in the resonator. The cable of the signal input probe was terminated with a commercial 50 Ω -resistor at the probe plate in order to minimize changes in the input voltage on the plate during a change in the frequency of the signal generator. Care was taken with the input signal to have symmetric coupling to the upper and lower dees and thereby reducing the excitation of unwanted resonances at which a voltage difference may exist between the two dee plates. The Q-value was also determined with the signal generator and vector voltmeter by measuring the frequency interval Δf between the two points at which the measured voltage is down by a factor $\frac{1}{\sqrt{2}}$ from the maximum value on the response curve at the resonance frequency f_r . The Q-value is given by:

$$Q = \frac{f_r}{\Delta f}$$

On the full-scale resonator the Q-value was also measured with a network analyzer. The results of the two methods agree.

The following equipment was used for various purposes:

1. Signal generator: Rohde & Schwartz 100 kHz–1000 MHz; SMX 826.457.62
2. Network analyzer: Agilent 4395A; 10 Hz – 500 MHz.
3. RF Vector impedance meter, Hewlett Packard model 4815A.
4. Vector voltmeter, Hewlett Packard model HP 8405A.
5. Current transformer, Tektronix CT-1.

5.4.2 Half-scale resonator

The resonator structure consisting of the outer conductor plates and dee, with the cutaway section for the EEC as shown in figure 5.7, was assembled with nuts and bolts. The power dissipation and the Q-value will be very different from the calculated values because of the poor connections between the different electrodes. The resonance frequency and the voltage distribution should, however, be in good agreement with what was calculated since both depend to a large extent on the Z_0 -values. The scaling laws for resonator models are listed in Appendix I [Bot74]. With a radial length of about 1.5 m, the horizontal half-wave model should resonate at approximately 100 MHz. The power is inductively coupled via a loop and tuned by another loop.

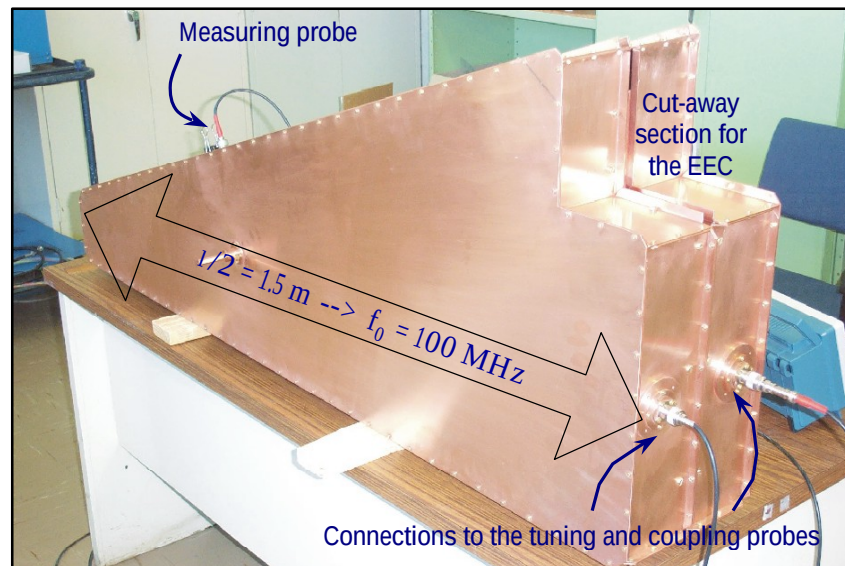


Figure 5.7. The half-scale resonator with the cutaway section where the EEC fits in and the connections to the coupling and tuning loops. Here the tuning loop is not shorted, but used as a loop for measurement.

The inner structure and components of the half scale resonator are shown in figure 5.8 with the top lid (upper outer conductor) removed to the side. One of the manually adjustable capacitors, in the region of maximum dee voltage, is also visible.

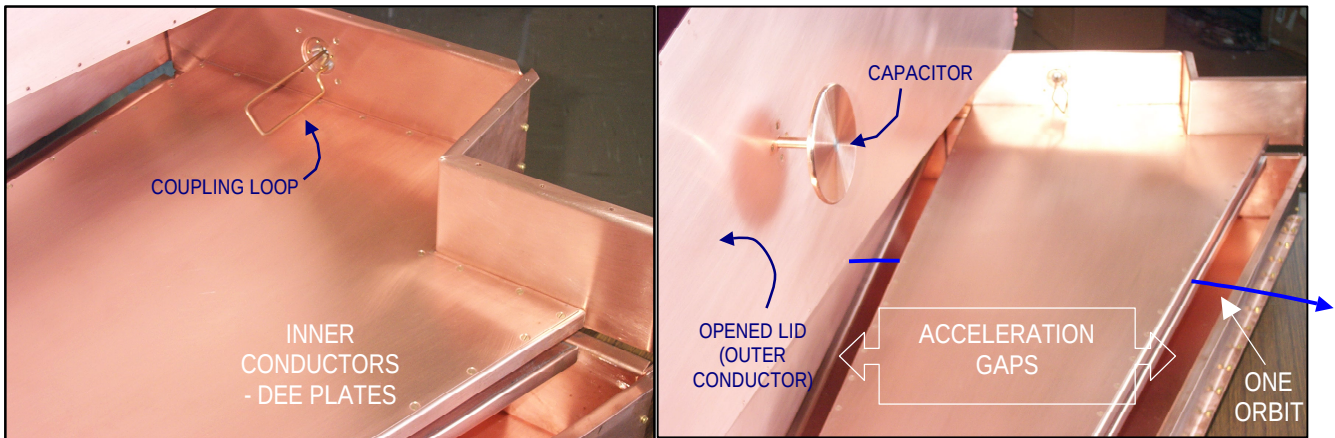


Figure 5.8. *The half-scale resonator model with the top lid (upper outer conductor) opened to show the dee plates, acceleration gaps, coupling loop and adjustable capacitor.*

Good agreement was obtained between the measured and calculated resonance frequencies. The measured frequency was 101.45 MHz and the calculated frequency was 99.6 MHz, as shown in figure 5.9, together with the electric field distribution on the finite-element structure of the outer conductor. Tight mechanical tolerances are required both in the manufacturing of the resonator and positioning of the probe for measurement of the dee voltage distribution. The distance between the probe and the dee has to be kept constant as it is moved along the gap in the radial direction. The amount of detuning of the resonator by the probe depends on the radial position. This further distorts the measured results, depending on at which radial position the probe was when the frequency of the signal generator was adjusted for resonance. The voltage distribution was measured in the respective acceleration gaps with the pick-up probe inserted into a gap and without retuning of the resonance frequency for every radial position of measurement. The measured results are shown in figure 5.10 for one acceleration gap as a normalized electric field, together with the calculated results for both gaps.

Every calculated point on the graph was obtained by integration of the electrical field component across each of the acceleration gaps. The measured distribution shows reasonable agreement with those calculated using the finite-element model and SOPRANO. The next step was to perform the calculations for a full-scale finite-element model and then build and measure the resonator.

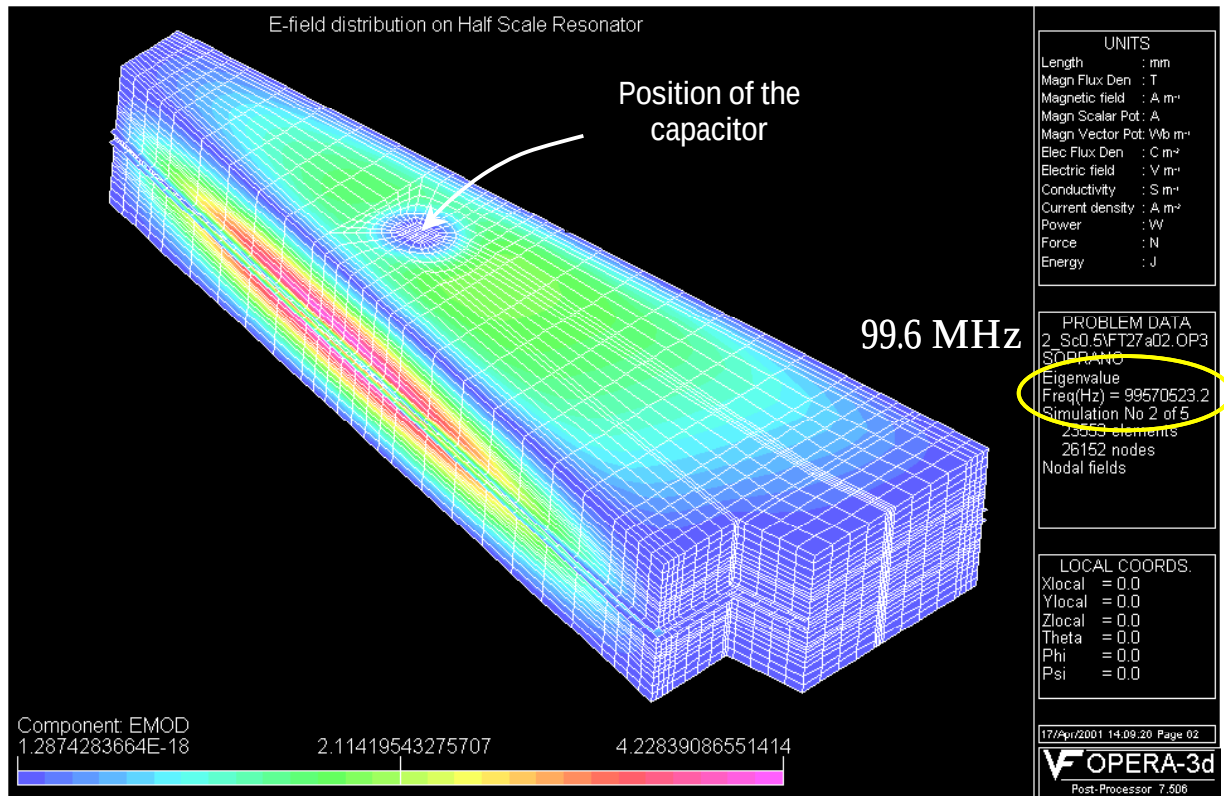


Figure 5.9. The calculated resonance frequency of 99.6 MHz for the horizontal half-wave model shows good agreement with the measured value of 101.45 MHz, without paying attention to the fine tuning of the capacitors or tuning loops. The electrical field distribution that is superimposed on the finite-element structure of the outer conductor, shows the half-wave in the radial direction (longest dimension) with red denoting the highest and blue the lowest value of electrical field. The small electrical field value at the two vertical end plates and at the location of the capacitor stem is clearly visible.

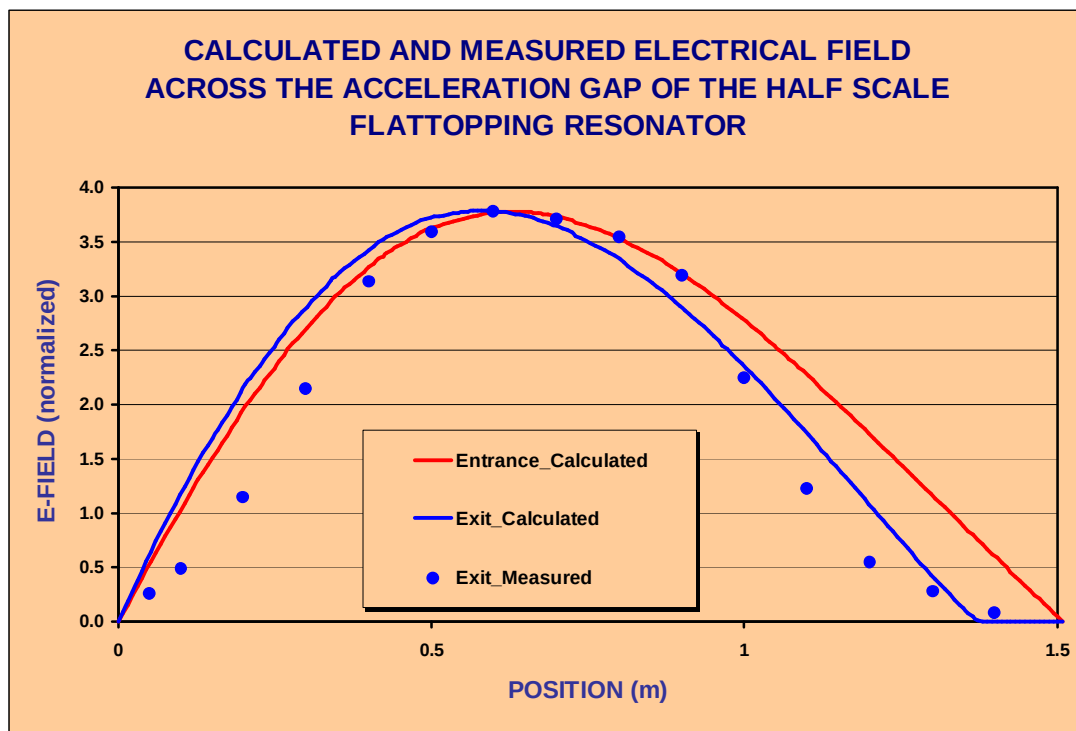


Figure 5.10. The measured normalized electrical field in one of the acceleration gaps of the half-scale resonator, with the calculated SOPRANO results for both gaps.

5.5 RESULTS OF CALCULATIONS WITH NUMERICAL FIELD ANALYSIS IN 3D AND MEASUREMENTS ON THE FULL-SCALE RESONATOR

5.5.1 Electromagnetic field distribution

The resonator electrodes are defined in terms of infinitely thin, perfect conductors in SOPRANO. These conductors represent the inside of the outer conductors and the outside of the inner conductors, to define the space between them where the electromagnetic field exists. The detail of the electromagnetic field distribution in the resonator is best shown with drawings, as those in figures 5.11 and 5.12, of different resonator sections showing the electrical field and magnetic field distributions in pairs at the same instant. The bottom outer conductor, the bottom capacitor and the coupling and tuning loops with the respective field distributions, with the maximum values indicated in red and the minimum values with blue, are shown. The electric field is almost zero at the vertical end plates, with a maximum value about halfway between them, where the capacitor plates are located. The magnetic field is at its strongest at the vertical end plates, which indicate the regions of high current density and therefore heat dissipation. In figures 5.13 and 5.14 the bottom dee plate (shown without the capacitor plate that was later added to shift the unwanted frequency) is included. In figures 5.15 and 5.16 the top dee plate is also included.

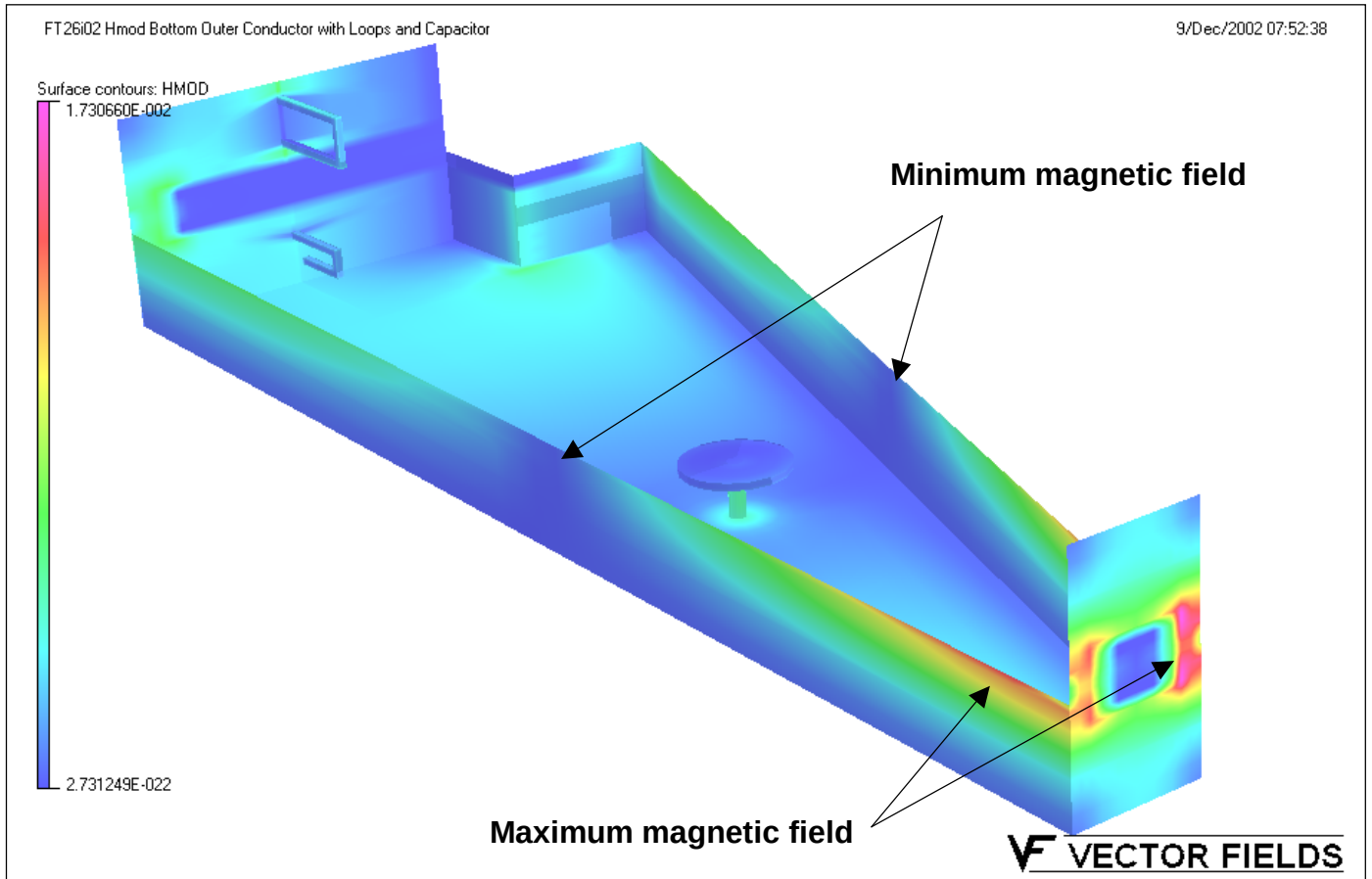
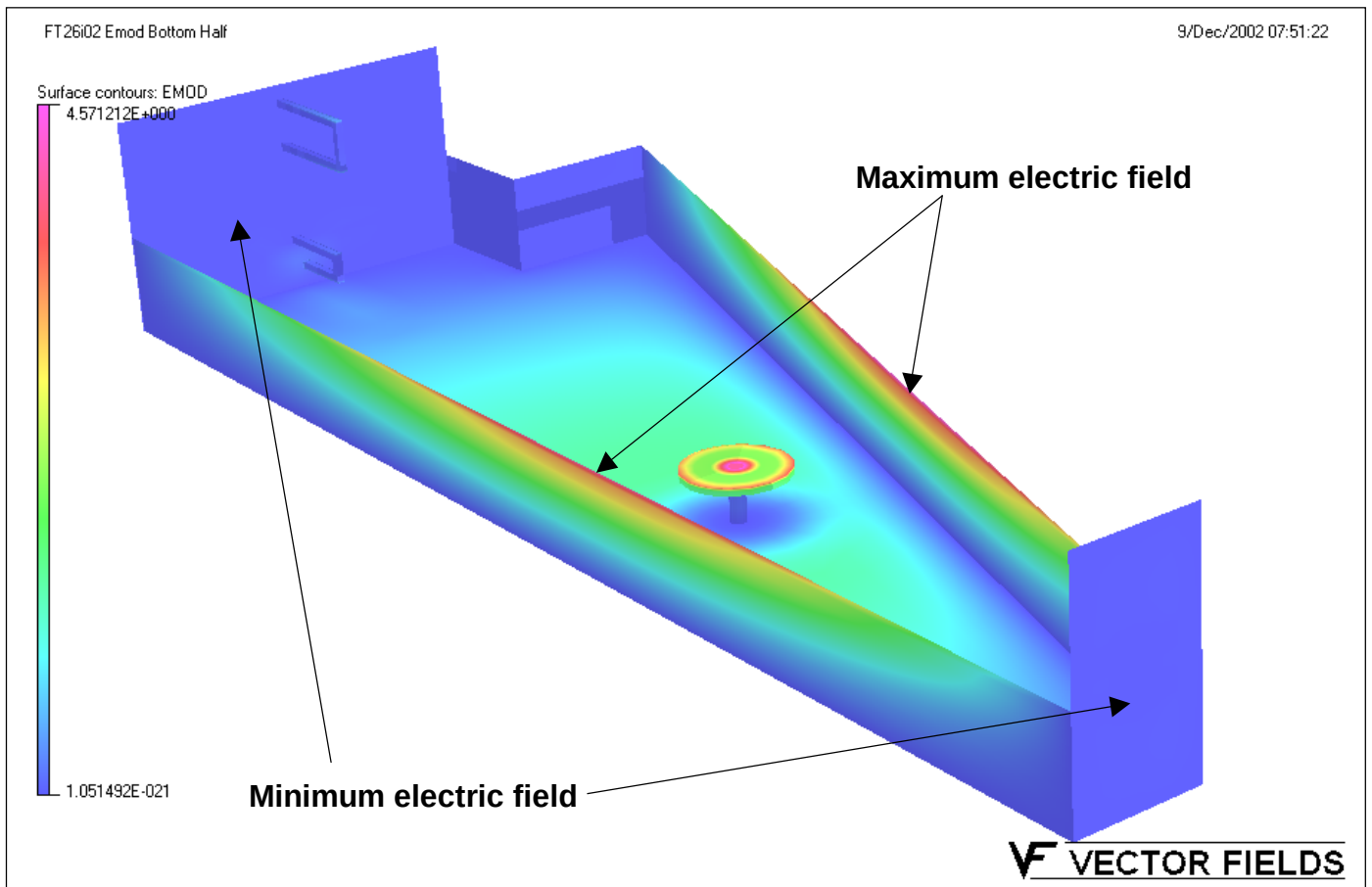


Figure 5.11 and 5.12. Calculated electric (top) and magnetic (bottom) field distributions for the bottom outer conductor, vertical end plates, bottom capacitor and coupling and tuning loops. The maximum electric field occurs about halfway between the end plates and the maximum magnetic field at the end plates. At the regions of maximum magnetic field the current densities are the highest.

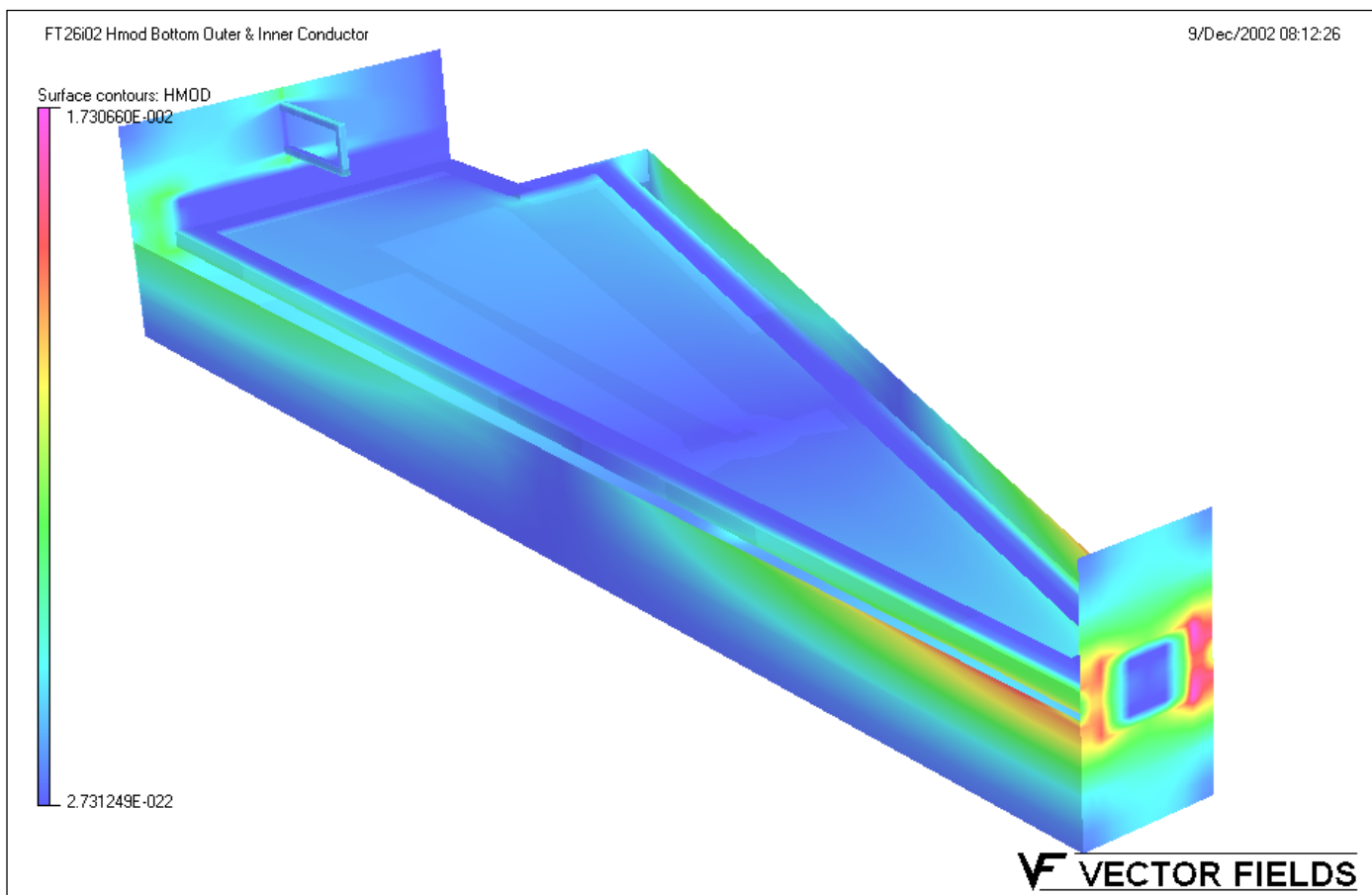
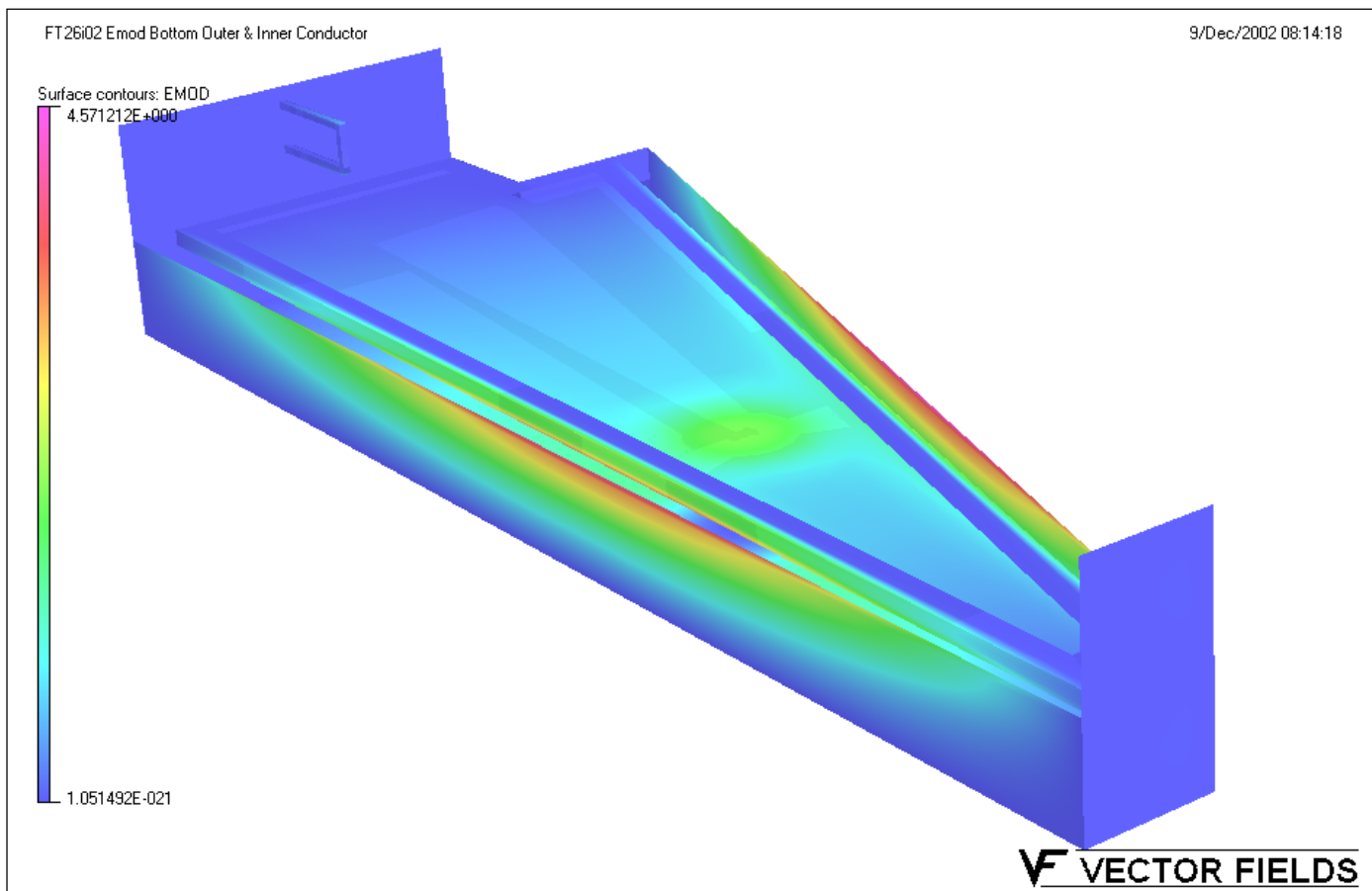


Figure 5.13 and 5.14. Calculated electric (top) and magnetic (bottom) field distributions for the same geometry as in the previous two figures, but with the bottom dee plate included.

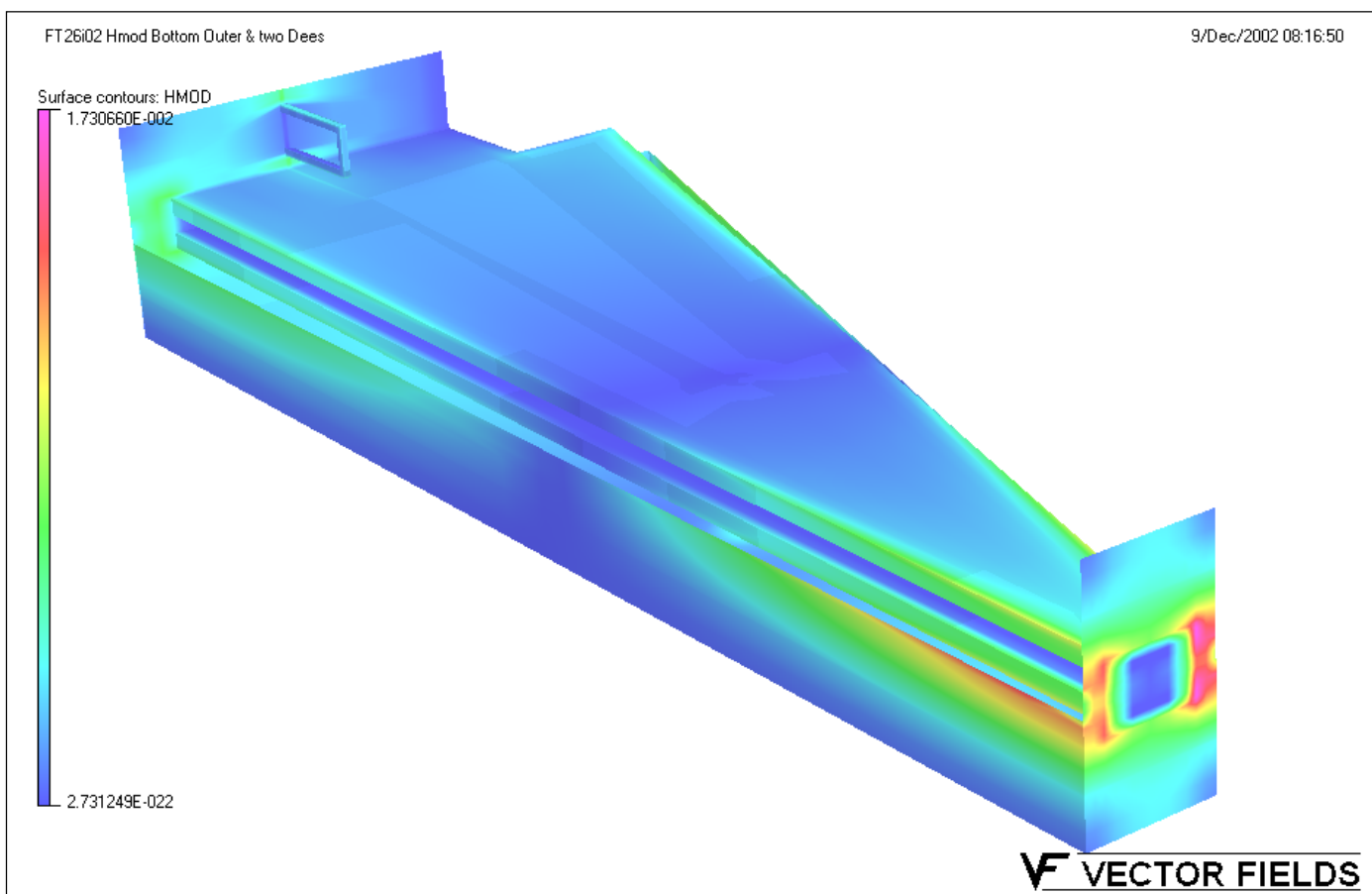
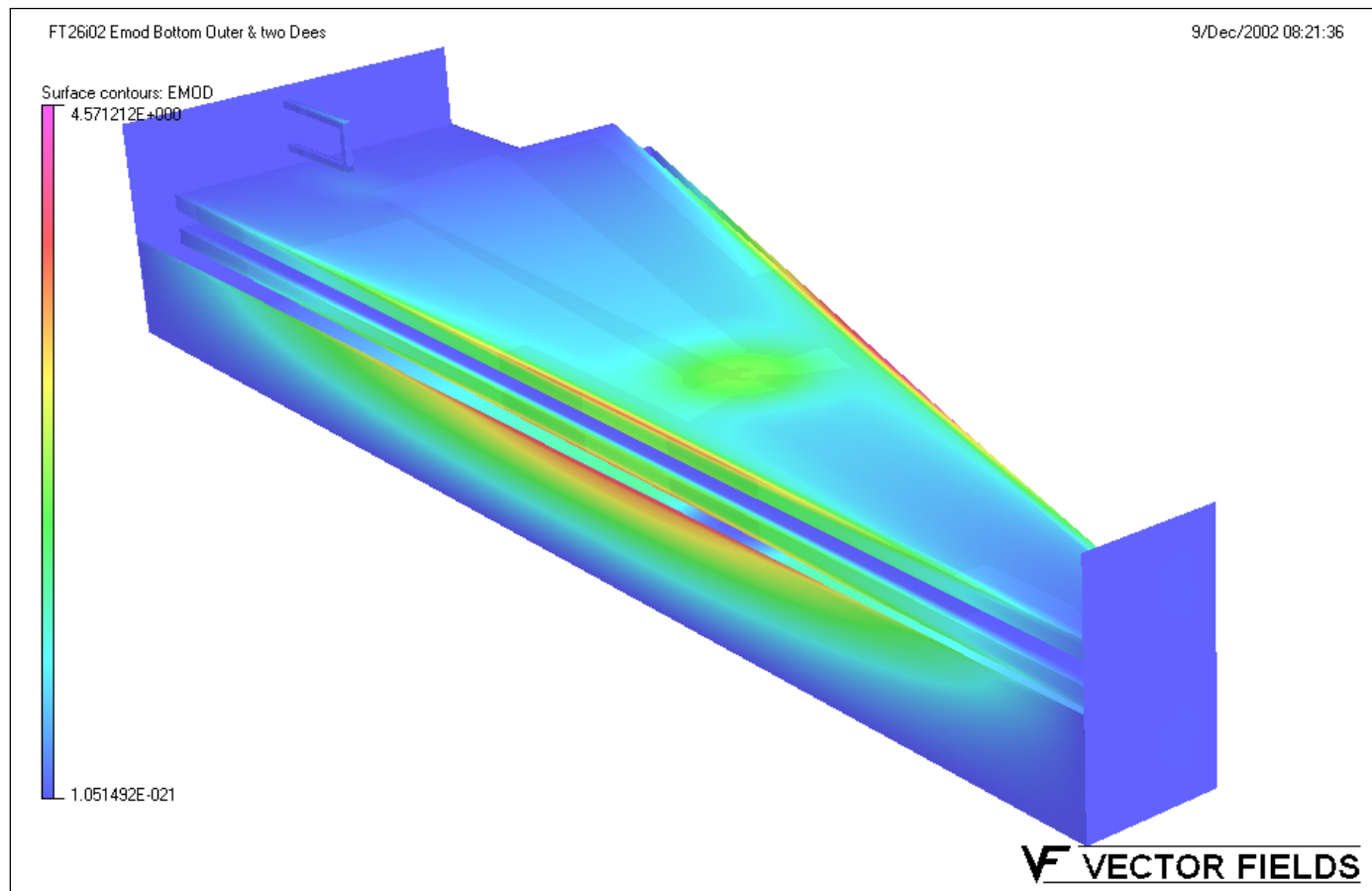


Figure 5.15 and 5.16. Calculated electric (top) and magnetic (bottom) field distributions for the same geometry as in the previous two figures, but with the top dee plate included.

5.5.2 Calculated resonance frequency, Q-value, stored energy, scaling of the calculated power dissipation and the radial voltage distribution

The rms-values were calculated for all the integral-values in the post-processor, but adjusted within the control program to represent the peak-values. Applying equations 4.23 to 4.26 give the calculated Q-value in terms of the stored energy. The average calculated Q-value obtained with SOPRANO is 10573, with a difference of less than 2% between the values obtained with the stored magnetic and electric energy. In private communication with Support Services of Vector Fields Ltd. it was learned that such a low percentage difference is an indicator that the finite-element simulation is adequately subdivided into sufficient finite elements over all regions for whatever field gradient may exist in that region.

The frequency, Q-value and non-scaled results of the SSC flat-topping resonator calculations with SOPRANO are listed in table 5.3, where all the final dimensions were used except for $d_{Cap} = 70$ mm, $w_T = 234$ mm, $h_T = 82$ mm, $w_C = 173$ mm and $h_C = 34$ mm. The conductivity of copper is taken as 5.8×10^{-7} mho.meter⁻¹.

Table 5.3. Calculated parameters of the flat-topping resonator

Frequency	49.126	MHz
Power dissipation due to surface resistance (RMS)	1.62×10^{-7}	W
Stored energy, magnetic (E_{st} -RMS)	5.50×10^{-12}	J
Stored energy, magnetic (E_{st} -peak)	1.10×10^{-11}	J
Power loss, P	6.37×10^{-7}	W
Q-value	10573	

The calculated results are scaled by calculating the potential difference value at a position where the required operational voltage is known, e.g. at the position of maximum acceleration voltage across an acceleration gap. Such calculated potential differences were obtained by calculating the integrals of the electric field along paths, LE and LX, respectively, in figure 5.2. This path forms a straight line along the orbital path across the acceleration gap in the median plane. The length of the integration

path is typically 300 mm, from a point on the defined boundary of the configuration, through the gap and in between the dee plates.

The integrals of the electric field component along these selected paths, at the position $X = -2085$ mm where the maximum acceleration voltage is expected, are:

$$y_{LE} = \int_{LE} \mathbf{E}_y \cdot d\mathbf{L} = 0.266515 \text{ V} \quad \text{and} \quad y_{LX} = \int_{LX} \mathbf{E}_y \cdot d\mathbf{L} = 0.268485 \text{ V}$$

and are shown in figures 5.17 (a) and 5.17 (b), respectively.

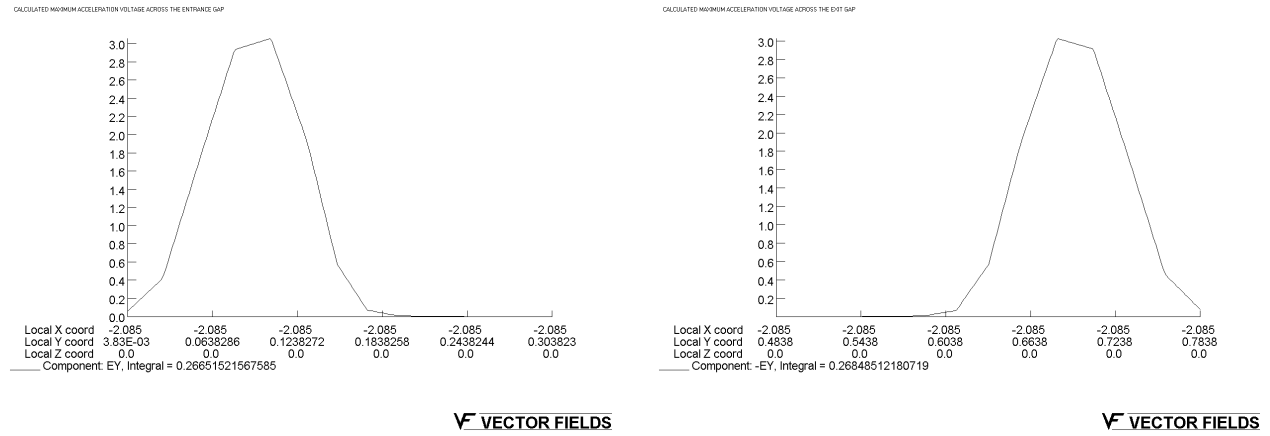


Figure 5.17 (a) and 5.17 (b). The maximum acceleration voltages, in arbitrary units, across the entrance (left) and exit (right) acceleration gaps are calculated by integration of the electric field component ($\mathbf{E}_y$) along paths LE and LX, in figure 5.2, at a projected position of 2085 mm from the center of the cyclotron along the center line of the south valley vacuum chamber.

For a maximum applied voltage of $V_{max} = 62$ kV, the average scaling factor that was used is:

$$Scale = \frac{V_{max}}{((y_{LE} + y_{LX}) / 2)} = 231776$$

The calculated power consumption in the resonator when operated at 62 kV is then:

$$P_{62kV} = P_{calc} \times [Scale]^2 = 6.37 \times 10^{-7} \times \left[\frac{62000}{\sqrt{(0.266515 + 0.268485) / 2}} \right]^2 \approx 9154 \text{ W}.$$

The radial voltage distribution was calculated by integrating the potential difference across the acceleration gaps at radial positions varying from the injection orbit radius to the extraction radius. Wherever these integrals were calculated, the path never extended further than the center-line of the resonator, in order to have maximum inclusion of the effects of the acceleration gap and minimum influence of the other gap. These radial voltage distributions at the entrance and exit gaps are shown as a function of the radius in the cyclotron in figure 5.18. The capacitors of the resonator were at the radial position of maximum acceleration voltage, as was shown at point G in figure 5.2.

The maximum electric field strength is about 8.9 kV/cm for a gap size of 7 cm, which is far below the limit of 88.5 kV/cm, according to Kilpatrick's criterion [Cha99] (see Appendix H). Sparking-over in the acceleration gaps should therefore not be a problem.

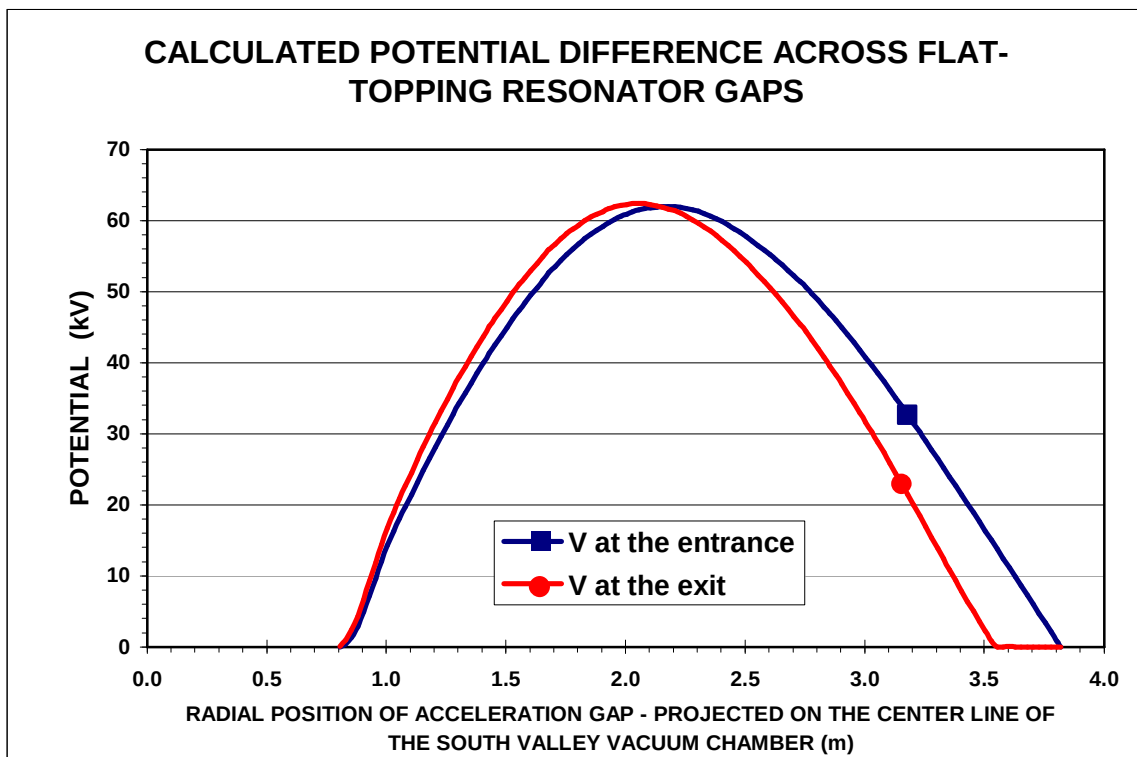


Figure 5.18. The calculated radial voltage distribution across the acceleration gaps of the flat-topping resonator, scaled to a maximum value of 62 kV, as a function of the radial distance from the cyclotron center. The difference in voltages at the higher radii is mainly due to the cut-away section for the EEC.

5.5.3 Measuring the Q-value

The signal generator was coupled capacitively to the dee with small input and measuring signal probes, for reasons as described in paragraph 5.4.1. The cable of the signal input probe was, as described before, again terminated with a commercial 50 Ω -resistor at the probe plate. The input signal was also symmetrically coupled to the upper and lower dees whereby the chance of a voltage difference between the two dee plates was minimized. The plate of the coupling probe was removed as far as possible from the dee plates, though close enough to have sufficient signal strength for the measurement.

A screen print from the network analyzer shows a measured Q-value of 8335 at a resonance frequency of 49.81 MHz in figure 5.19.

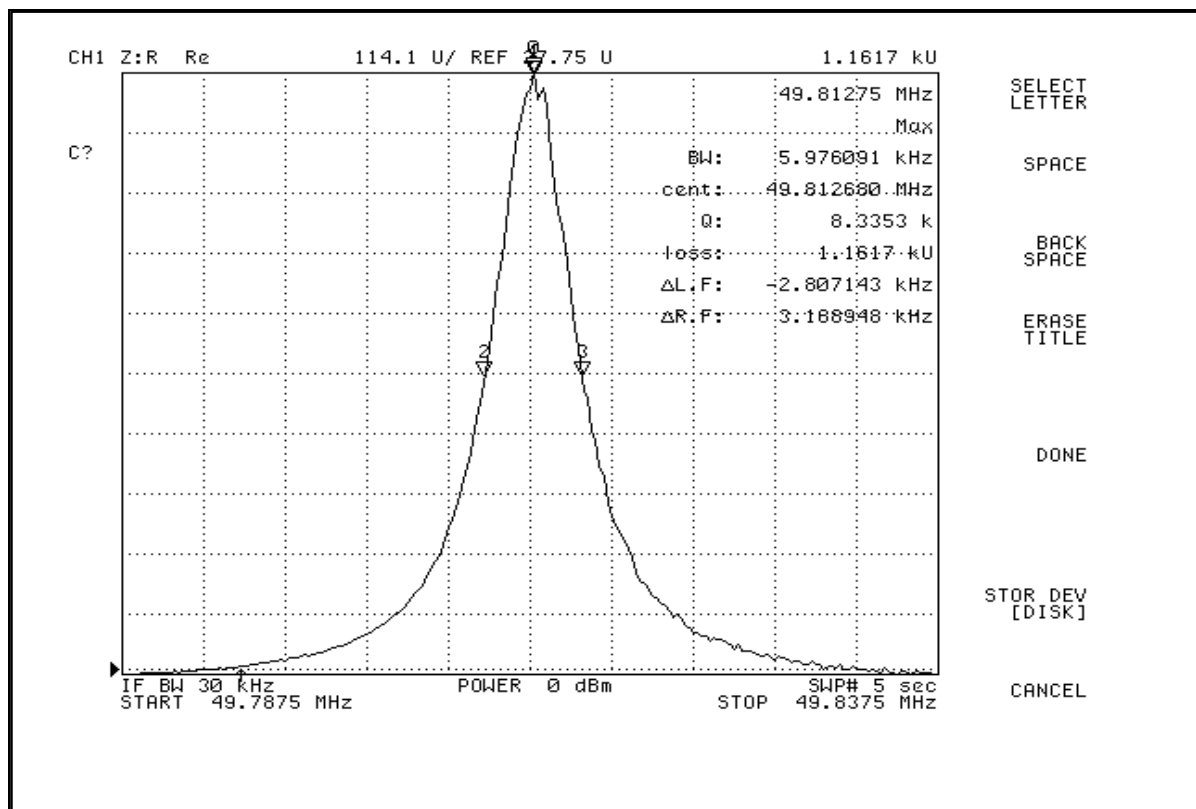


Figure 5.19. A screen print of the network analyzer showing the measured resonance frequency of 49.81 MHz and a Q-value of 8335.

In conjunction with the personnel of the rf-division of iThemba LABS a final set of measurements was obtained with the resonator installed in its position inside the south valley vacuum chamber and with all its tuning devices connected. The Q-value was measured with the network analyzer connected to the coupling loop. The tuning loop was set to about 20° with respect to the median plane. The top and lower capacitor plates were adjusted to obtain the maximum Q-value at the required frequency. The results are listed in table 5.4.

Table 5.4. The final measured Q-values and resonance frequencies for different settings of the capacitor gap distances.

#	GAP TOP (mm)	GAP BOTTOM (mm)	Q-VALUE	FREQUENCY (MHz)
1	21.90	19.15	7200	49.1100
2	18.15	19.15	7600	49.0545
3	15.20	19.15	5700	49.0077
4	18.80	19.15	7700	49.0645
5	19.50	19.15	7800	49.0777
6	20.30	19.15	7700	49.0880
7	20.30	17.95	6900	49.0625
8	20.30	19.70	7700	49.0900
9	20.30	20.20	7780	49.1050

Deductions from the measured values:

- The frequency changes by 17.55 kHz/mm change of plate position (average of changes for measurements #3 and #5 and measurements #7 and #9).
- The Q-value changes by 604 per mm for a large change in the position of a plate (see measurement #3 and #5).
- The Q-value changes by 391 per mm for a small change in the position of a plate (see measurement #7 and #9).
- The Q-value of the resonator is very sensitive to a change of the gap of one of the capacitor plates, i.e. in a deviation from symmetry. A change of 0.5 mm already has a measurable effect on the Q-value (see measurements #8 and #9).
- The power dissipation of the resonator changes by about 100 W for a 0.5 mm displacement of one capacitor plate.

It is to be taken into account that the measurements of the capacitor plate positions are not very accurate and this should be considered when the data is analyzed.

5.6 SEPARATING THE RESONANCE FREQUENCY OF THE UNWANTED MODE FROM THE WANTED MODE

The resonator, of which a cross-section is shown in figures 5.20 (a) and (b), can also be considered as three parallel strip-line resonators. Since the three lines have the same lengths the resonance frequencies are close together. Two of these lines are formed by the top and bottom dee plates and the top and bottom outer conductors, respectively. The third line is formed by the two dee plates only. These lines have two oscillating modes near 50 MHz. In the desired mode, the dee plates have the same voltage and phase and there is no electrical field in the gap between the two plates. In the other mode, the voltages on the dee plates are 180° out of phase and there is a strong field in the dee gap with high currents flowing on the inner surfaces.

The schematic drawings show the instantaneous electromagnetic field distributions for the two modes with the electric field, $\vec{E}$, denoted with solid arrows and the magnetic field, $\vec{H}$, with dashed lines or conventional symbols representing field lines perpendicular to the surface of the paper. The density of the lines is an indication of the field strength. In the unwanted mode electromagnetic waves are strongly radiated from the acceleration gap.

The coupling loop excites the desired mode, but any deviation from symmetry with respect to the median plane will cause excitation of the unwanted mode, if its frequency is close to the operating frequency. The coupling loop itself is also unsymmetrical with respect to the median plane. The same applies to the tuning loop. Space limitations prevented the use of symmetrical loops. The frequency of the unwanted mode had therefore to be changed to a value away from the operating frequency. The effects of the unsymmetrical loops and other deviations from symmetry have been studied with SOPRANO and the computer program MICROCAP [MIC88], in which the three lines and the loops were presented by a large number of lumped-parameter circuit elements [Bot04b]. The effect of symmetrical coupling and tuning as well as the influence of the additional dee plates were also studied. Figure 5.21 shows the circuit diagram.

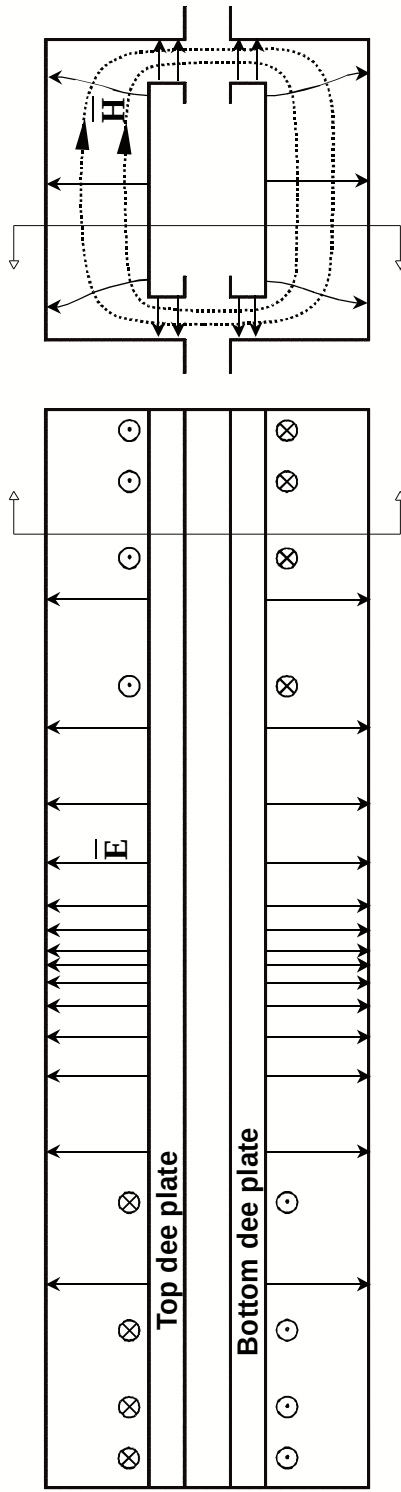


Figure 5.20 (a). Schematic representation of an instantaneous electromagnetic field distribution inside the resonator where the upper and lower halves resonate in phase, resulting in the wanted electric fields across the acceleration gaps of the resonator and practically no electric field between the top and bottom plates, which implies that there is no voltage difference between the dee plates. The strength of the fields is indicated by the density of the lines.

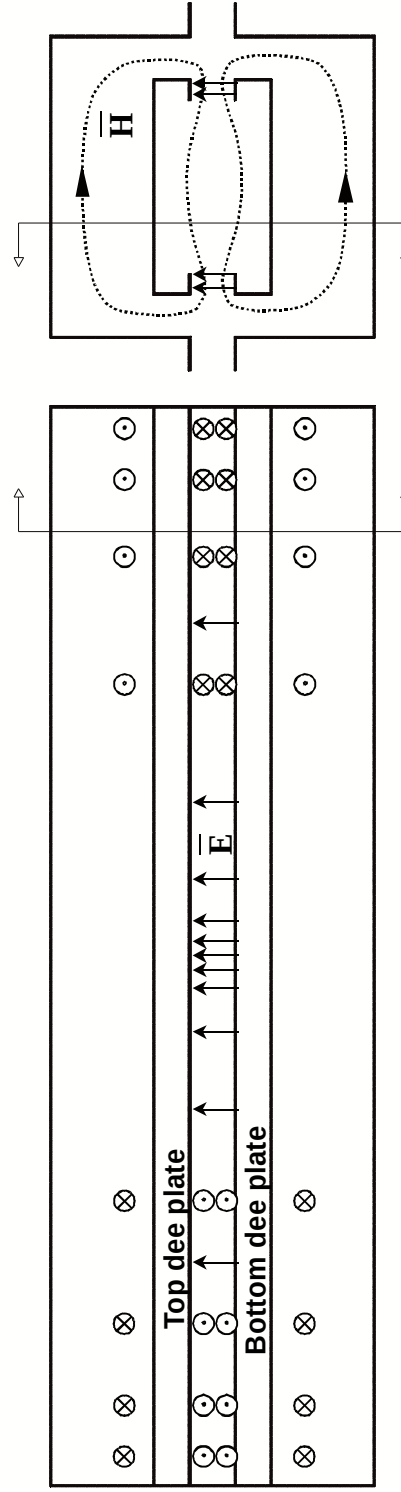


Figure 5.20 (b). Schematic representation of an instantaneous electromagnetic field distribution inside the resonator where the upper and lower halves resonate out of phase, resulting in an electric field between the dees, implying a voltage difference between the top and bottom dee plates.

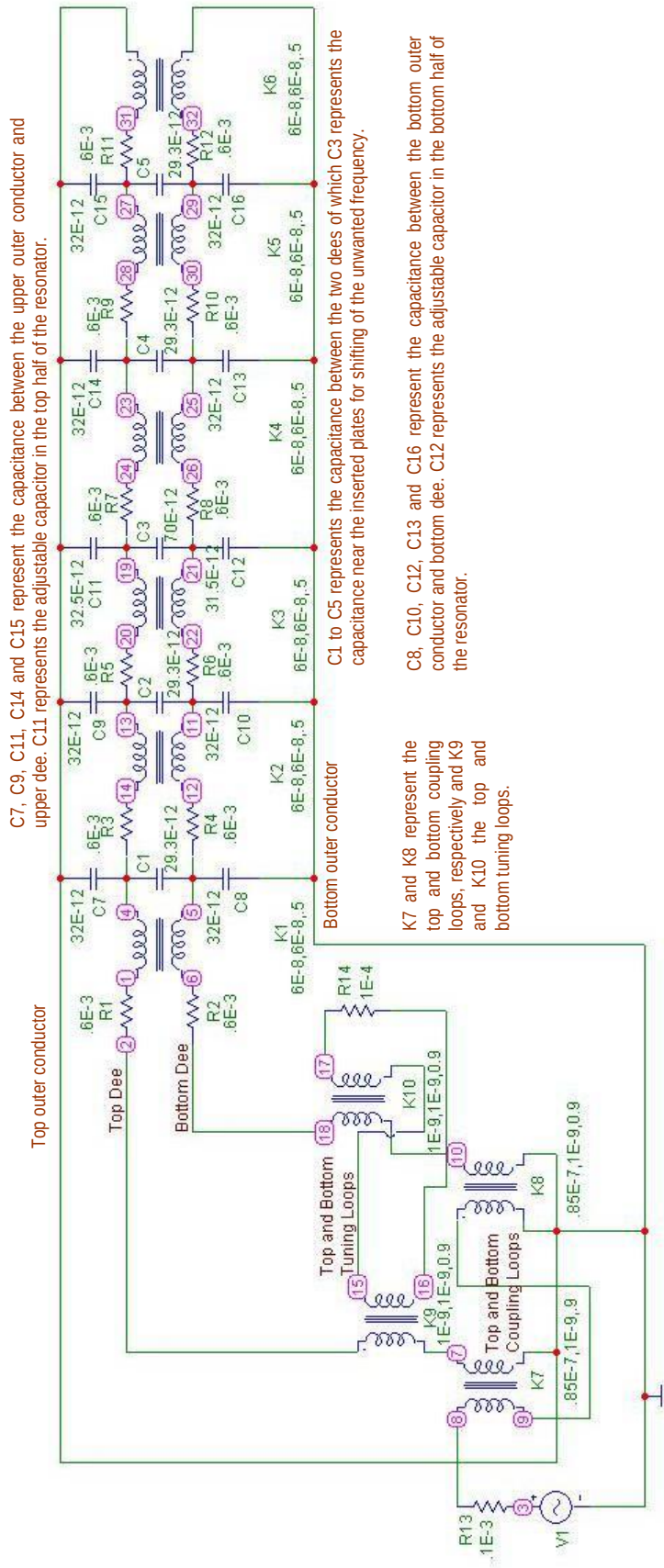


Figure 5.21. The lumped-parameter circuit diagram that was used for analyzing the unsymmetrical configuration of the coupling loops and capacitors of the flat-topping resonator. Two coupling loops and two tuning loops are included in the diagram to determine the effect of symmetrical coupling. The lumped-parameter transmission lines on the top right-hand side are used to simulate the resonator.

Adequate separation of the frequencies was obtained by inserting a capacitor plate, CP, on the inside of each dee plate, parallel and near to the median plane at a radial position where the electric field has a maximum. Figure 5.22 shows one of these plates welded in its position inside the dee plate. Some of the water-cooling pipes on the inside of the dee plate can also be seen. The dimensions of the plate are 810 mm, 514 mm and 271 mm (see figure 5.2).



Figure 5.22. A copper plate welded to the inside of a dee plate (indicated with a dotted line) for separation of the two resonance frequencies. The addition of the plate decreased the resonance frequency of the unwanted mode by about 6 MHz without affecting the frequency of the wanted mode.

To understand the change in frequency of the unwanted mode the two dee plates with their end connections between them can be considered as two quarter-wave resonators joined in the center (opposite the installed plate). The resonance condition for such resonators of length l is [Joh50d]:

$$\frac{1}{C_d w} = Z_0 \tan b l, \quad \left(b = \frac{2p}{l} = \frac{w}{c} \right). \quad (5.1)$$

By installing the plates the capacitance between the dees, C_d , was increased, thereby reducing the frequency.

To measure the resonance frequencies of the two modes simultaneously with a network analyzer, a loop was inserted in an acceleration gap at an angle of 45° with respect to the median plane and also with respect to the gap, and protruding inside the dee, to excite both modes. The measured separation of the frequencies is shown in figure 5.23 with the unwanted mode at 43.3 MHz and the required one at 49.81 MHz.

There is also a distinct difference in the Q-value of the two resonance modes. From the sharpness of the displayed resonance peaks it can be seen that the wanted mode is a much sharper resonance and therefore has a much higher Q-value.

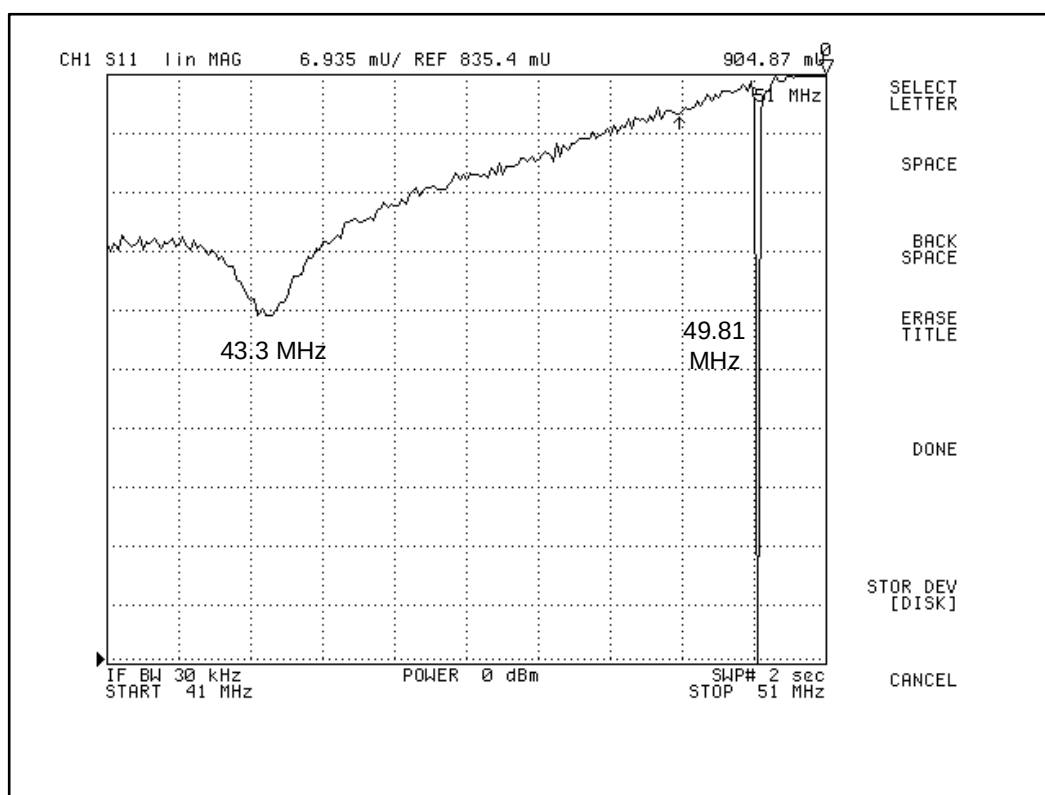


Figure 5.23. A screen-dump of the network analyzer showing the resonance frequencies of the two horizontal half wave modes after the capacitor plates were installed inside the dee plates. The measured frequency of the wanted mode decreased by 0.7 MHz, whereas the frequency of the unwanted resonance decreased by 6.0 MHz. The changes in the calculated values agree.

The calculated shift in frequency, obtained with SOPRANO, agrees with the measured results and is listed in table 5.5. The calculated magnetic field distributions on the lower dee and one vertical plate, after the addition of the capacitor plates, are shown in figures 5.24(a) and (b), for the two modes respectively.

Table 5.5. Calculated and measured resonance frequencies of the flat-topping resonator, before and after the addition of the capacitor plates inside the dee plates.

FREQUENCY (MHz)		WANTED MODE	UNWANTED MODE	DIFFERENCE
		in-phase	out-of-phase	
Before adding the plate	CALCULATED	50.56	49.50	1.06
	MEASURED	50.51	49.34	1.17
After adding the plate	CALCULATED	50.62	43.68	6.94
	MEASURED	49.81	43.30	6.51

3/Dec/2002 14:08:48

Surface contours: HMOD

1.644412E-002

1.057759E-011

Figure 5.24(a). The calculated magnetic field distribution is shown on the lower dee and vertical plate near injection, for the **wanted mode** and its associated resonance frequency of 50.62 MHz, after the insertion of the capacitor plate. The region of highest field density is between the dees and outer conductors, which are not shown.

UNITS

Length	mm
Magn Flux Density	T
Magn Field	A/m
Magn Scalar Pot	A
Magn Vector Pot	Wb/m
Elec Flux Density	C/m ²
Elec Field	V/m
Conductivity	S/mm ²
Current Density	A/mm ²
Power	W
Force	N
Energy	J

PROBLEM DATA

ft26h14.op3
SOPRANO Eigenvalue
Freq(Hz) = 5.0623037813E+07
Simulation No 2 of 4
29184 elements
32129 nodes
Nodally interpolated fields

Local Coordinates

Origin: 0.0, 0.0, 0.0
Local XYZ = Global XYZ

V VECTOR FIELDS

3/Dec/2002 14:09:26

Surface contours: HMOD

2.382378E-002

8.612456E-007

Figure 5.24(b). The calculated magnetic field distribution is shown on the lower dee and vertical plate near injection, for the **unwanted mode** and its associated resonance frequency of 43.68 MHz, after the insertion of the capacitor plate. The region of highest field density is between the two dees, of which the upper dee is not shown here.

UNITS

Length	mm
Magn Flux Density	T
Magn Field	A/m
Magn Scalar Pot	A
Magn Vector Pot	Wb/m
Elec Flux Density	C/m ²
Elec Field	V/m
Conductivity	S/mm ²
Current Density	A/mm ²
Power	W
Force	N
Energy	J

PROBLEM DATA

ft26h14.op3
SOPRANO Eigenvalue
Freq(Hz) = 4.3681777761E+07
Simulation No 1 of 4
29184 elements
32129 nodes
Nodally interpolated fields

Local Coordinates

Origin: 0.0, 0.0, 0.0
Local XYZ = Global XYZ

V VECTOR FIELDS

5.7 CALCULATED INFLUENCE OF TEMPERATURE CHANGE ON THE RESONANCE FREQUENCY

5.7.1 Introduction

The change in resonance frequency due to an increase in the temperature of the electrodes of the resonator was calculated analytically and also with SOPRANO, which gives more accurate results since the full 3d geometry is incorporated in the simulated finite-element model.

The length, L , of an electrode, at a temperature T , is given by [Hal74]:

$$L = L_0 \left[1 + a (T - T_0) \right] \quad (5.2)$$

where L_0 is the length at t_0 ,

a is the linear expansion coefficient of the material.

The change in the length DL is given by:

$$\Delta L = L - L_0 = a L_0 (T - T_0). \quad (5.3)$$

The effect on the resonance frequency was calculated for copper using a linear expansion coefficient of $1.68 \times 10^{-5} \text{ K}^{-1}$ and temperature increase of 15 K.

5.7.2 Change in the length of the resonator

Equation 5.18 is used to calculate the order of magnitude that can be expected before doing calculations with SOPRANO. For a resonator length of 3 m, the increase in length is:

$$\Delta L = 0.756 \text{ mm}.$$

The fractional change in resonance frequency is

$$\frac{\Delta f}{f} = -\frac{\Delta L}{L} = -\frac{0.756}{3000}.$$

This means a decrease of 12.4 kHz in the resonance frequency.

In the finite-element configuration the length of the resonator was increased by 1 mm through a radial displacement of coordinates A and F in figure 5.2. A decrease of 17.3 kHz in the resonance frequency was calculated.

5.7.3 Change in the width of the resonator

The increase in the average width of the resonator for a 15 K temperature increase is calculated to be 0.17 mm, using equation 5.3.

The width is not in a simple way related to the resonance frequency of the resonator and was therefore implemented in two ways on the finite-element model for calculations with SOPRANO. Firstly the increase in the width was applied evenly on either side and in the second case the total increase was introduced at the exit side only. The calculated decrease in resonance frequency was 1 kHz for the symmetric adjustment and 1.3 kHz for the non-symmetric case.

5.7.4 Change in the height of the resonator

Equation 5.3 was again used to a change in the height of the resonator for a 15 K temperature increase. The change in height of the resonator was implemented in two ways in the finite-element model. It was applied firstly by raising only the upper lid and in the second instance dividing the increase evenly between the upper and lower lids. Care was taken that the capacitors that are fixed to the respective lids, were adjusted as well. The calculated increase in resonance frequency was 0.33 kHz for both the symmetric and non-symmetric cases.

5.7.5 Summary

From these calculated results it is clear that the change in frequency due to the increase in the length of the resonator for a temperature increase of 15 K, is significant and has to be compensated by adjusting the tuning loop. The results with SOPRANO are summarized in table 5.6.

Table 5.6: Calculated changes in resonance frequency due to changes in the dimensions of the resonator for a temperature increase of 15 K.

Change in :	Symmetric/ Non-symmetric	Location of adjustment	Geometric adjustment (mm)	Change in frequency (kHz)
Length	Non-symm	Resonator nose	1.00	-17.29
Width	Symmetric	Entrance & exit	0.08	-1.05
	Non-symm	Exit	0.17	-1.38
Height	Symmetric	Top & bottom	0.06	0.33
	Non-symm	Top	0.12	0.33

Changes in the resonator dimensions due to buckling of the electrodes could not be calculated and are therefore not included in the calculations of changes in the resonance frequency above.

5.8 THE ACCELERATION GAP AND THE TRANSIT-TIME FACTOR

5.8.1 Introduction

The ions of the beam bunches move through regions of time-varying electric fields in the acceleration gaps of the resonator. It is therefore necessary to calculate the effects of such transit time in the acceleration gaps of the flat-topping resonator.

The time-dependant voltage across the gap is:

$$V = V_0 \cos(2p f_{rf} t) \quad (5.4)$$

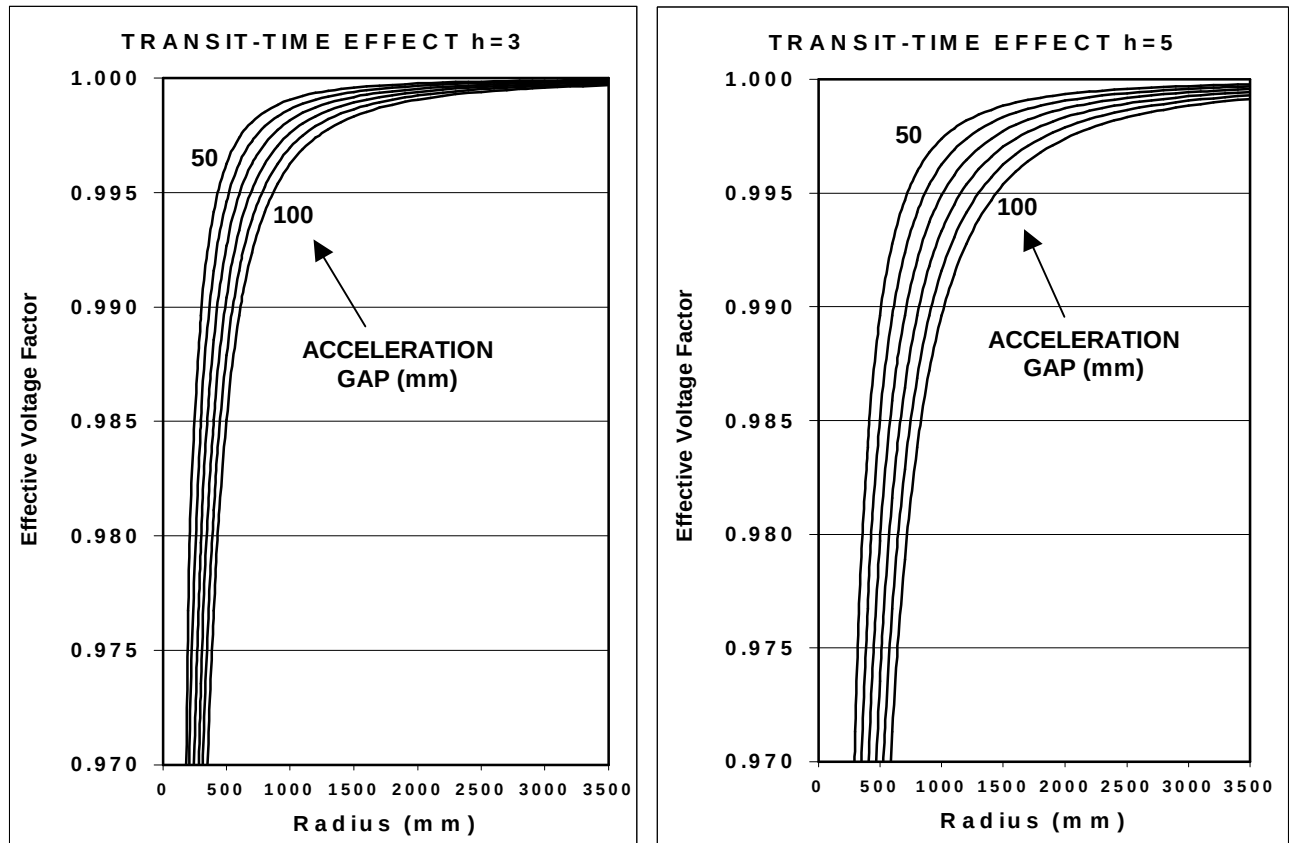
with V_0 the maximum value of the dee voltage, f_{rf} the frequency and t the time.

At a radius R , a gap size g and harmonic number h , the transit-time effect reduces the dee voltage by a factor [Joh68c]:

$$\frac{\sin\left(\frac{gh}{2R}\right)}{\left(\frac{gh}{2R}\right)} \quad (5.5)$$

As shown in Appendix F this factor is independent of the phase of the ion.

Figures 5.25 (a) and (b) show the change in the effective acceleration voltage for harmonic numbers $h=3$ and $h=5$ respectively and for different acceleration gaps as the beam progresses through the cyclotron.



Figures 5.25 (a) and (b). The effective acceleration voltage of the resonator decreases as either the harmonic number or the acceleration gap increases and it increases as the radial position in the cyclotron increases, due to the transit time factor. The transit-time factors are shown for acceleration gap lengths of 50 mm to 100 mm with step size of 10 mm in between.

5.8.2 The transit-time factor for the SSC flat-topping resonator

The effect of the transit-time is particularly pronounced at smaller radii in the cyclotron and for high harmonic numbers. It is therefore advisable to select the lowest permissible harmonic number for the application and to have small acceleration gaps near injection, but since the dee voltage in the flat-topping resonator is zero at injection there can be little effect from the transit-time in this radial region.

Applying equation 5.5 with values derived from the geometrical parameters listed in table 5.7, the transit-time factors were calculated for the SSC flat-topping resonator at the respective acceleration gaps and for harmonic frequency number $h=3$. The results are shown in figure 5.26 and the maximum reduction of 0.2% at injection is negligible.

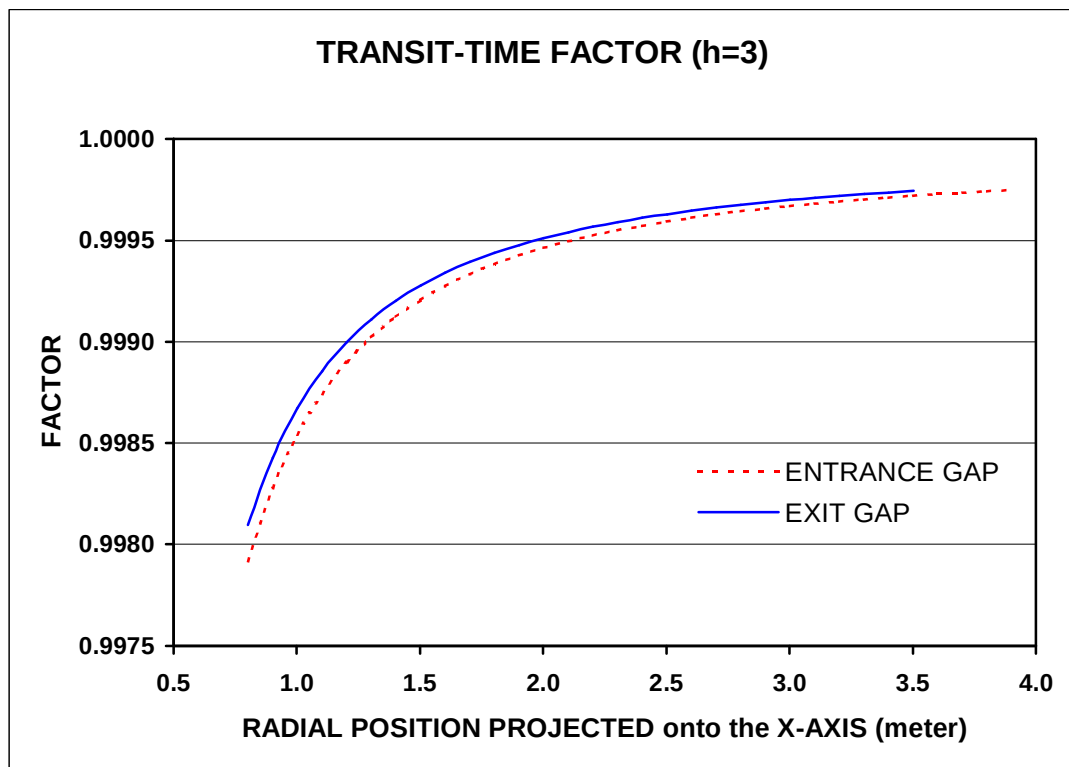


Figure 5.26. The calculated transit-time factor is shown for the acceleration gaps that vary from 60 mm at injection to 100 mm at extraction, and harmonic frequency number 3. The values for the exit gap is shown only up to about 3.5 meter because of the cut-away section to leave space for the EEC.

Table 5.7. Dimensions (in meter) for the calculation of the transit-time factor at harmonic number 3.

distance (meter)	at P1a (fig 5.1)	at P1b (fig 5.1)	at P2a (fig 5.1)	at P2b (fig 5.1)	Origin	
					resonator	cyclotron
Coord X	-0.8031	-0.8031	-3.820	-3.5445	-0.1586	0
Coord Y	0.0508	0.2526	0.1485	1.1994	0.0299	0
Acc. Gap	0.0600	0.1000	0.0600	0.0971		

5.9 TOLERANCE ON DIFFERENCE IN DEE VOLTAGE ON THE TOP AND BOTTOM DEE HALVES DUE TO A VERTICAL DISPLACEMENT OF THE DEE PLATES

The upper and lower halves of the flat-topping resonator must resonate in phase and therefore always be at the same voltage and consequently have no vertical electrical field between the upper and lower plates. However, any vertical offset of the resonator or any other asymmetry with respect to the median plane may cause a potential difference, and therefore an electrical field, between the two dee plates. This electrical field will deflect the passing beam vertically (z-direction), but also have a focusing effect and was studied to determine its influence on the beam dynamics.

The frequencies of the vertical betatron oscillations for every equilibrium orbit of a proton with extraction energy of 66 MeV from the SSC, were calculated with the computer program COC and implemented with a spreadsheet program to calculate the vertical motion created by the combined effects of the inherent betatron oscillations of the SSC and the displacement of the dee plates of the flat-topping resonator.

The calculated vertical betatron frequencies, n_z , are shown as a function of the kinetic energy of the particle in figure 5.27.

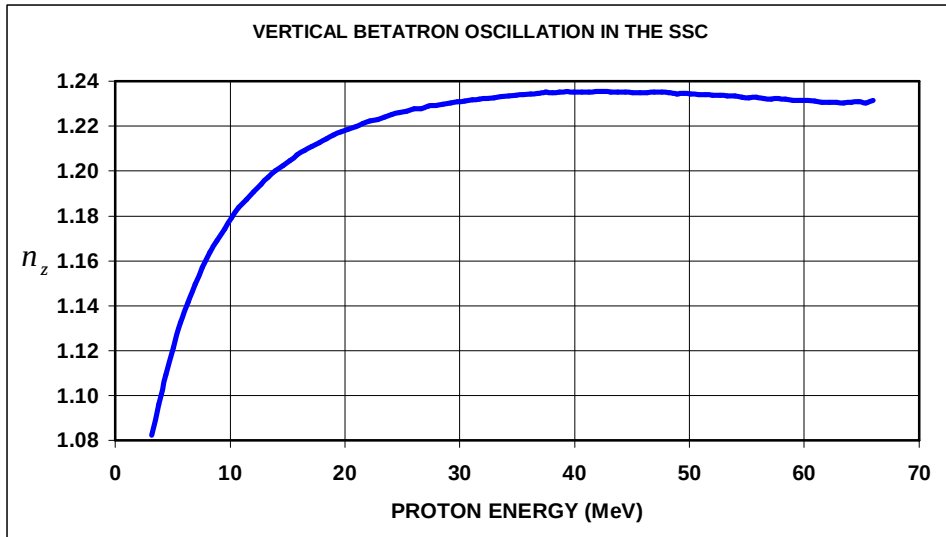


Figure 5.27. The vertical betatron oscillations in the SSC are calculated with the computer program COC for equilibrium orbits of a proton with extraction energy of 66 MeV.

The voltages that appear between the dee plates, V_z , due to vertical displacements of the dee plates relative to the median plane, were calculated with SOPRANO at the exit acceleration gap and expressed as a percentage of the acceleration voltage V_{acc} . The vertical displacements and divergences of the beam at extraction were obtained from the calculated phase-plots of the beam for the different calculated V_z/V_{acc} -ratios and an initial particle displacement and divergence of 0 mm and 0 mrad respectively (see Appendix J). The relation between the vertical displacement of the dee plates with respect to the median plane, the subsequent voltage between the dee plates and the eventual vertical displacement of the beam at extraction, is shown in figure 5.28.

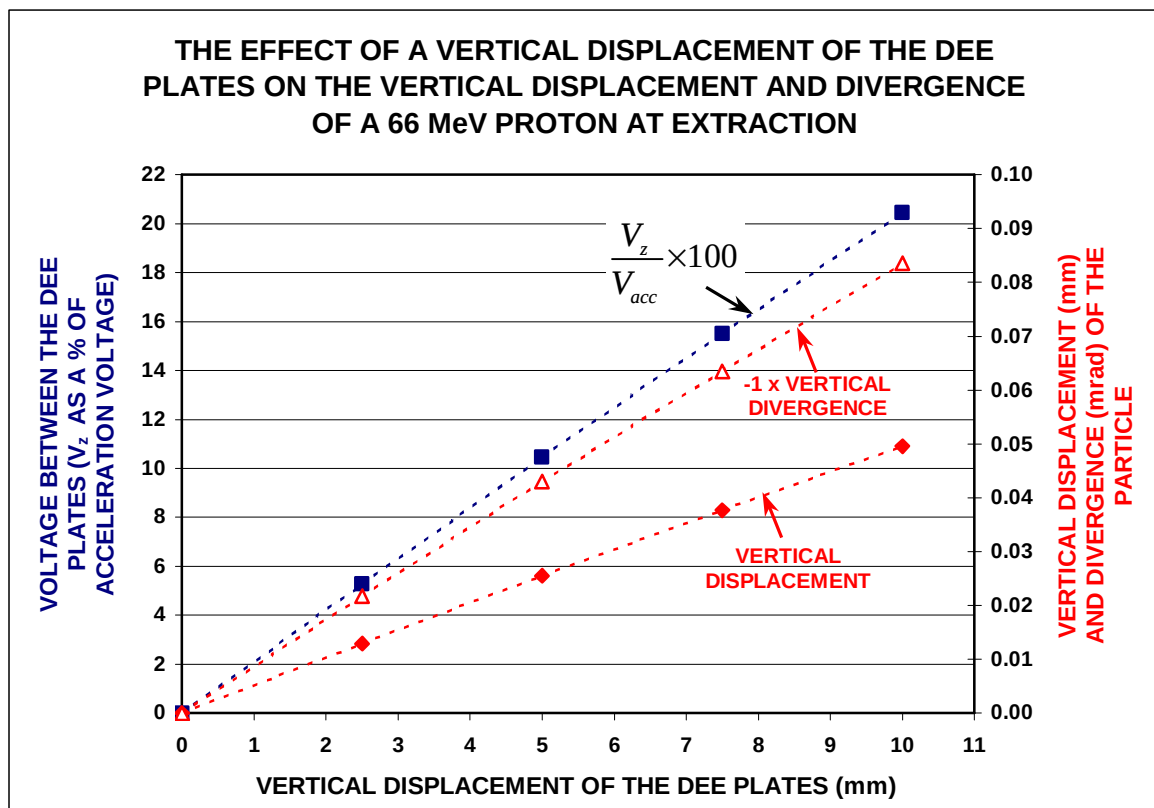


Figure 5.28. The calculated vertical displacement and divergence of a 66 MeV proton at extraction due to a voltage (V_z) between the dees, caused by a vertical displacement of the dees relative to the median plane.

The calculated vertical displacement and divergence of the particle at extraction due to a 10 mm vertical displacement of the dee plates (equivalent to $V_z/V_{acc} \approx 20\%$) are only a fraction of a millimeter and milliradian respectively. Therefore it can be concluded that, if a vertical displacement of the dee plates do exist, its effect on the vertical motion of the beam will be insignificant and calculations of a higher order of accuracy are not required.

5.10 SUMMARY

- The selection of the horizontal half-wave resonator is very well suited for our purpose, despite all the physical restrictions, pre-requisites and the fact that such a type of resonator was never before implemented in a cyclotron.
- The calculated and measured results show that the required electro-magnetic characteristics fulfill the requirements for acceleration of beams of higher intensity and also for the improvement of their quality.
- The construction of the half-scale model also has its worth for confirmation of the calculation methods and for gaining experience with the measurements. The model is still available at the iThemba LABS site and is used for training of students.
- The calculation methods complemented and confirmed each other and were in general satisfactory. The calculations occasionally led to a better insight and the 3-dimensional understanding of the electromagnetic functioning of the resonator, particularly in the regions where analytical formulae or ordinary transmission line techniques are not adequate to predict the fields.
- The higher power dissipation and lower Q-value that was obtained with the transmission line method can be explained in terms of the smaller acceleration gaps that were used in the calculations and the consequently higher currents.
- The temperature effects on the characteristics will be minimal, but not negligible. In retrospect it can be said that it would have been better to use thicker copper for most of the electrodes, mainly to have a more rigid structure that would not require internal support structures and pre-tensioning of the structure. However, the components would then have been more difficult to machine and handle in the workshop.
- The reduction in the effective dee voltage due to the transit-time factor can be ignored.
- Minor vertical displacements of the dee plates relative to the median plane will have little effect on the beam and can therefore also be ignored.

-o-O-o-

CHAPTER 6

THEORY AND IMPLEMENTATION OF THE TUNING AND COUPLING SYSTEM OF THE RESONATOR

6.1 INTRODUCTION

It was shown in chapter 5 that the required horizontal half-wave mode has a field distribution with high magnetic fields and currents on and near the vertical end plates and a low magnetic field about halfway between the plates. The electric field is high near the middle of the resonator and low at the end plates. The coupling of the power amplifier can be done inductively with a loop where the magnetic field is high, or capacitively where the electric field is high.

The geometry of the existing valley vacuum chamber and its access ports will permit either inductive coupling at the rear of the resonator or capacitive coupling from above and below the resonator at a radial position about halfway between the vertical plates of the resonator. The ports in the vacuum chamber at the rear side of the resonator permit easy access for a coupling loop, whereas the offset of the ports in the region for capacitive coupling, will require more complicated driving mechanisms for the capacitor plates. Because of its inaccessibility the vertical plate near injection is not suitable for mounting of the coupling and tuning components. It was therefore, from a mechanical point of view, decided to couple the power inductively at the rear of the resonator. Since all other existing resonators at iThemba LABS are excited capacitively, this resonator will be the first with inductive coupling. An adjustable tuning loop will also be installed for the same reasons as above at the rear of the resonator below the coupling loop. Two smaller fixed loops for measuring the dee voltage are also installed at the rear. Two adjustable capacitors are installed symmetrically with respect to the median plane at a radial position of maximum dee voltage, to be used as a coarse tuning device, but without the capability of remote adjustment. The sizes of these devices and their effect on the resonator have been calculated.

6.2 THE INDUCTIVE LOOPS

6.2.1. General description and location of the loops

The power amplifier will be connected to the resonator via a 50 Ω -cable through a coupling loop, which can be rotated by remote control. The coupling loop is mounted on the rear plate in the bottom half of the resonator and the short-circuited tuning loop in the top half of the resonator. The smaller pick-up probes are mounted symmetrically about the median plane. The locations of the rotatable coupling and tuning loops are shown in figure 6.1 with their orientation adjusted for zero coupling to the dee.

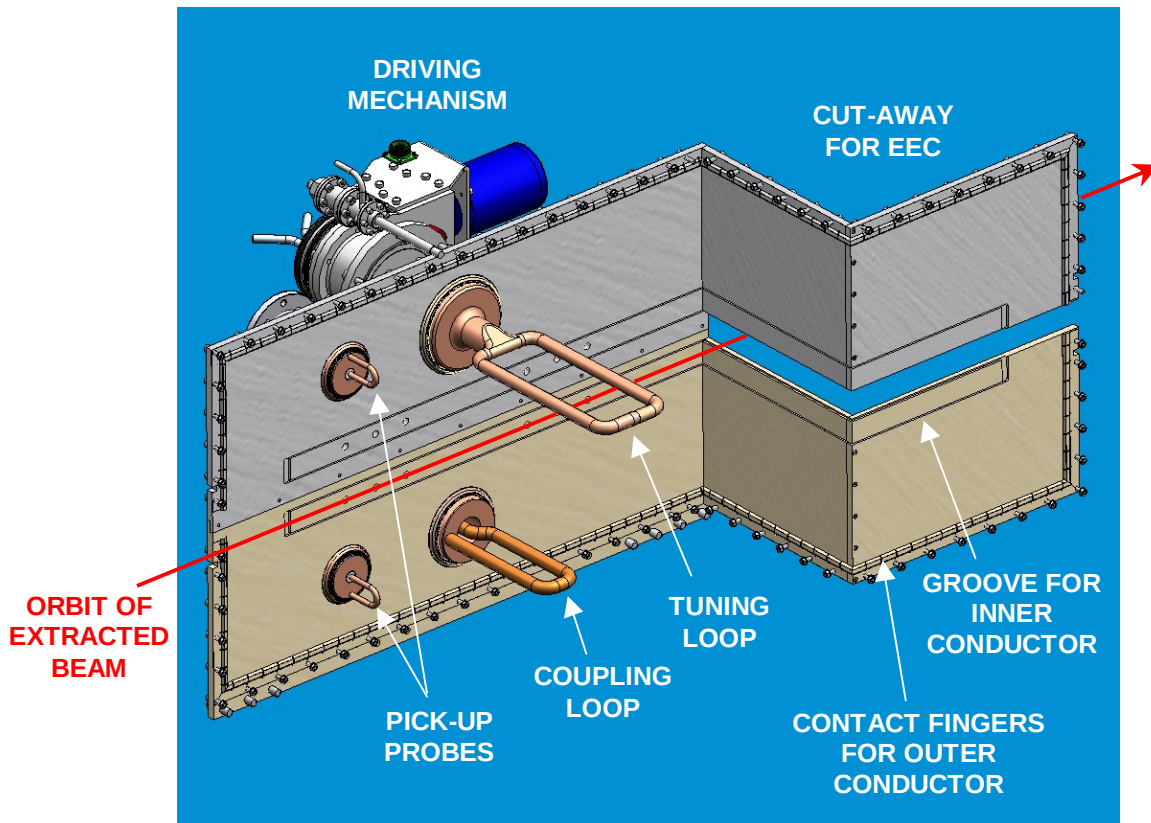


Figure 6.1. A drawing of the inside of the rear vertical plate of the resonator with the coupling and tuning loops and pick-up probes mounted on it. The two major loops are shown rotated to the 0°-position, that is parallel to the median plane and the position of zero flux linkage. The grooves and contact fingers in the plate at which the end of the horizontal inner and outer conductors are embedded in the plate are also visible. A part of the driving mechanism for the tuning loop is visible at the extreme rear, outside the vacuum chamber. The approximate position of the extracted beam orbit is shown schematically.

To determine the dimensions of the coupling and tuning loops the resonator is considered as a series resonant circuit that has, near the resonance frequency, the same characteristics as the resonator [Bot79]. The coupling and tuning loops are each considered as an inductance in series with a resistance. The loops are coupled to the resonator through their mutual inductances.

The tuning loop is a closed loop that reflects a reactance and resistance in series with the dee plates of the resonator opposite the loop. The reflected reactance retunes the resonator and the amount of retuning can be adjusted by turning the loop, thereby varying the mutual inductance between the loop and the resonator dee.

The coupling loop has to present the desired impedance to the power amplifier at the terminals of the loop outside the vacuum chamber. For the amplifier to see a pure resistive load of $50\ \Omega$, the reactance must be zero. These two conditions are met by turning the loop, thereby varying its mutual inductance with the dee, and by detuning the resonator with its resonance frequency away from the operating frequency, i.e. the standard condition for a cyclotron resonator coupled to an amplifier. The voltage and current distributions in the resonator and the positions where the loops are located in the resonators have to be taken into account in determining the parameters of the equivalent series RLC-circuit of the resonator.

6.2.2. The coupling loop

The equivalent circuit of the resonator and coupling loop is shown in figure 6.2.

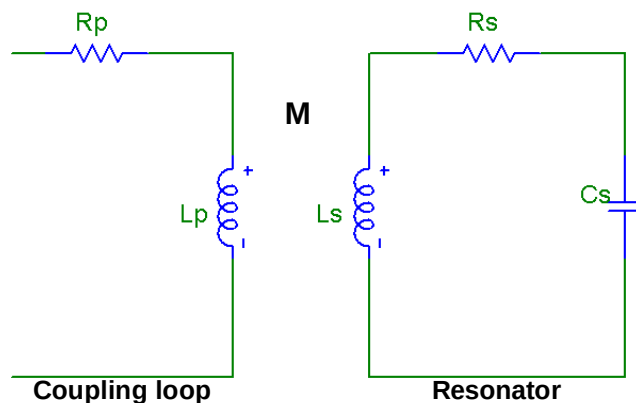


Figure 6.2. The equivalent circuit of the coupling loop and the resonator, linked with the mutual inductance M . L_s , R_s and C_s form the equivalent circuit of the resonator. R_p and L_p describe the electrical characteristics of the coupling loop.

This circuit can further be described by the circuit diagram in figure 6.3, in which the resonator characteristics are incorporated in the primary circuit as a reflected resistance R_r and reactance X_r [Bot05]:

The reflected components are given by:

$$R_r = \frac{M^2 \omega^2 R_s}{X_s^2 + R_s^2} \text{ and} \quad (6.1)$$

$$X_r = \frac{-M^2 \omega^2 X_s}{X_s^2 + R_s^2} \quad (6.2)$$

$$\text{with } X_s = L_s \omega - \frac{1}{C_s \omega}. \quad (6.3)$$

The impedance, Z_i , of this circuit must now be adjusted to obtain the condition of a resistance of 50Ω and zero reactance :

$$Z_i = R_i + jX_i = R_p + R_r + j0 = 50 + j0, \quad (6.4)$$

which means [Ter55]:

$$X_i = X_p + X_r = L_p \omega - \frac{M^2 \omega^2 X_s}{X_s^2 + R_s^2} = 0. \quad (6.5)$$

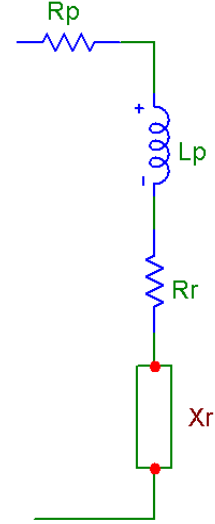


Figure 6.3. *The equivalent circuit diagram where the resonator is incorporated in the primary circuit.*

Since the resistance R_p of the coupling loop can be made much smaller than 50Ω , the equation 6.4 can be simplified to:

$$R_r = \frac{M^2 \omega^2 R_s}{X_s^2 + R_s^2} = 50 \Omega. \quad (6.6)$$

The mutual inductance can now be eliminated from equations 6.5 and 6.6 to obtain an expression for:

$$X_s = \frac{L_p \omega R_s}{50} \quad (6.7)$$

This expression for X_s can now be used to calculate M from (6.6):

$$M = \sqrt{\frac{50(X_s^2 + R_s^2)}{w^2 R_s}}.$$

The value of, L_s or C_s , for a 50Ω impedance can now be calculated from:

$$X_s = L_s w - \frac{1}{C_s w} \quad (6.8)$$

and

$$f_r = \frac{1}{2p \sqrt{L_s C_s}}. \quad (6.9)$$

Instead of the above procedure the resonance frequency can be kept fixed and the operating frequency changed. The amount of detuning in a cyclotron is typically in the order of tens of kilohertz, which is small in comparison to the operating frequency.

The self-inductance of the coupling loop was calculated by applying the law of Biot and Savart piecewise for the different parts of the loop to calculate the flux through the loop for a given current. The mutual inductance was calculated from the magnetic flux in the resonator, considering the lower half of the resonator as a parallel-plate transmission line.

The loop in the resonator is assumed as located at a single point in the resonator, as shown in figure 6.4, where the total impedance of the resonator and the loop is the sum of the impedances to the right and left of the selected point, $Z_T = Z_L + Z_R = R_s + jX_s$.

The coupling loop has been made from round copper tubing with an outer diameter of 15.9 mm. The main dimensions are shown in figure 6.5. The loop with its mounting sleeves is shown in figure 6.6.

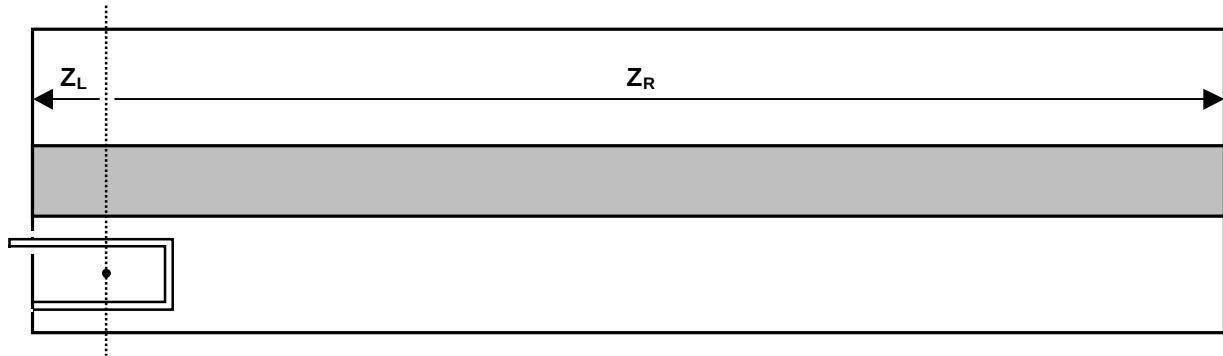


Figure 6.4. Schematic representation of the coupling loop in the resonator. The resonator impedance is the sum of the impedances to the left and the right of the center line of the loop.

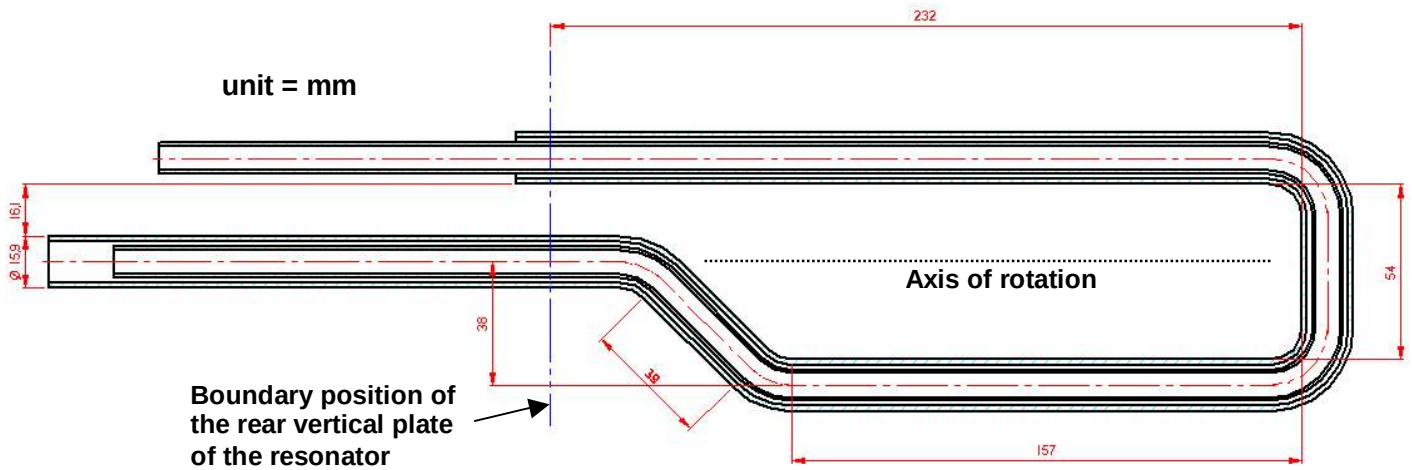


Figure 6.5. A side view section of the coupling loop in relation to the position of the vertical back plate of the resonator and the axis of rotation. The loop is made from round copper tubing, with 15.9 mm outer diameter.

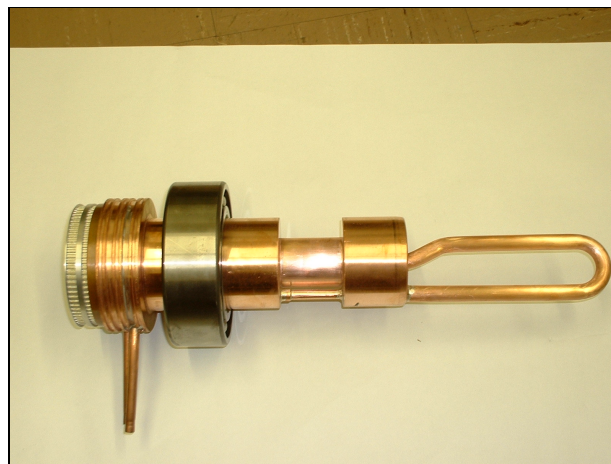


Figure 6.6. The coupling loop with its mounting sleeves.

The currents and voltages mentioned here and below are phasors and do not include the sinusoidal time variation, which should be included to obtain physical currents.

The resistance of the coupling loop itself, excluding the reflected resistance is:

$$R_{CL} = 2.61 \times 10^{-7} \sqrt{f} \times \frac{l}{w} = 2.61 \times 10^{-7} \sqrt{49100000} \times \frac{0.54}{0.05} = 19.8 \text{ m}\Omega, \quad (6.10)$$

with $f = 49.1$ MHz, the resonance frequency, $l = 250$ mm, the length of the loop and w the circumference of the loop. The outer circumference of the copper tube is πd , with $d = 15.9$ mm the outer diameter of the tube used to manufacture the loop. (The surface resistance and other electrical properties of copper are tabled with a few other materials in Appendix E.)

The current required in the coupling loop to supply the calculated power of about 10 kW to the 50Ω resonator load, is given by:

$$I_{CL} = \sqrt{\frac{2P}{R_L}} = \sqrt{\frac{2 \times 10000}{50}} = 20 \text{ A}, \quad (6.11)$$

with P the power dissipation in the resonator and R_L , the load resistance of 50Ω .

The voltage across the loop is:

$$V_{CL} = I_{CL} R_L = 20 \text{ A} \times 50 \Omega = 1000 \text{ V}. \quad (6.12)$$

The power dissipation in the loop due to its resistance is:

$$P_{CL} = \frac{1}{2} I_{CL}^2 R_L = \frac{1}{2} \times (20 \text{ A})^2 \times 50 \Omega = 10000 \text{ W}, \quad (6.13)$$

which is significantly less than the power dissipation in the resonator.

6.2.3. The tuning loop

The resonator and tuning loop are approximated, similar to what has been done for the coupling loop, by the following circuit:

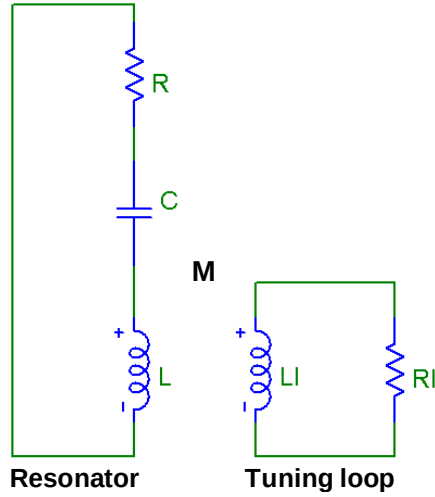


Figure 6.7. The tuning loop is coupled to the resonator through their mutual inductance M . R , C and L form the equivalent circuit of the resonator. L_i and R_i describe the electrical characteristics of the tuning loop.

The parameters of the tuning loop are L_i and R_i and those of the resonator R , C and L . Without the tuning loop, or with the loop parallel to the dee, the resonance frequency of the resonator is given by:

$$f_r = \frac{1}{2p\sqrt{LC}} \quad (6.14)$$

The resonator and tuning loop can, as before, be described by the following circuit (figure 6.8), but now with the reflected resistance R_r and reactance X_r in series with the resonator:

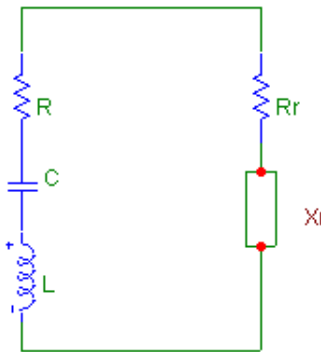


Figure 6.8. The tuning loop described as a reflected resistance and reactance in series with the resonator.

again given by equations 6.1 and 6.2, i.e.:

$$R_r = \frac{M^2 \omega^2 R_s}{X_s^2 + R_s^2} \quad (6.15)$$

and

$$X_r = \frac{-M^2 \omega^2 X_s}{X_s^2 + R_s^2} \quad (6.16)$$

with $R_s = R_l$ and $X_s = L_l \omega$.

The resonance frequency with the tuning loop can be calculated from:

$$L\omega - \frac{1}{C\omega} + X_r = 0, \quad (6.17)$$

which becomes, with X_r substituted:

$$L\omega - \frac{1}{C\omega} - \frac{M^2 \omega^2 L_l \omega}{(L_l \omega)^2 + R_l^2} = 0. \quad (6.18)$$

Since the reactance of the loop $L_l \omega$ is much larger than the resistance, which can be made small by using copper tubing with a large diameter, a further simplification is possible by neglecting R_l in the equation above:

$$L\omega - \frac{1}{C\omega} - \frac{M^2 \omega}{L_l} = 0, \quad (6.19)$$

which yields the following expression for ω :

$$\omega = \sqrt{\frac{L_l}{C(LL_l - M^2)}}. \quad (6.20)$$

The resonance frequency can therefore be adjusted by adjusting M , i.e. by turning the tuning loop. The self and mutual inductances of the tuning loop has been calculated in a similar way as those of the coupling loop.

The voltage induced in the tuning loop is given by:

$$V_l = M j \omega I_d \quad (6.21)$$

with I_d the current in the dee opposite the loop center.

The absolute value of the current, considered as a phasor, in the loop is given by:

$$I_l = \frac{V_l}{\sqrt{R_l^2 + L_l^2 \omega^2}} \approx \frac{V_l}{M \omega}. \quad (6.22)$$

The power dissipation in the loop has been calculated similar to what was done for the coupling loop.

The water-cooled tuning loop is made of round copper tubing with an outer diameter of 15.9 mm. The main dimensions are shown in figure 6.9. The loop with its mounting sleeve, the external motor drive and cooling water couplings is shown in figure 6.10.

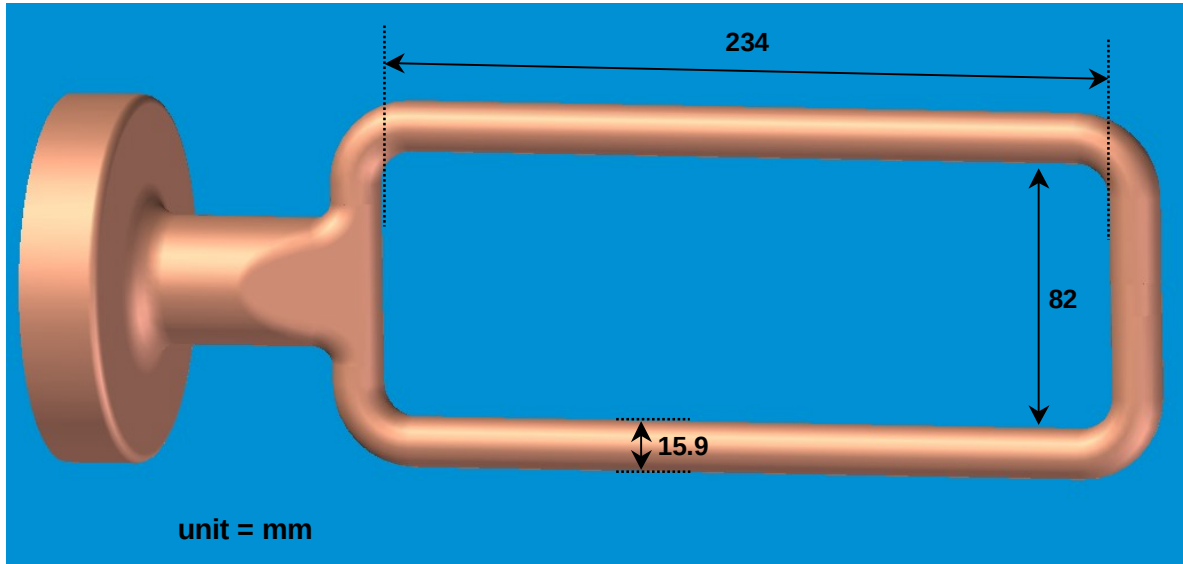


Figure 6.9. A 3d-drawing of the tuning loop with the main dimensions that determine the magnetic flux linkage.

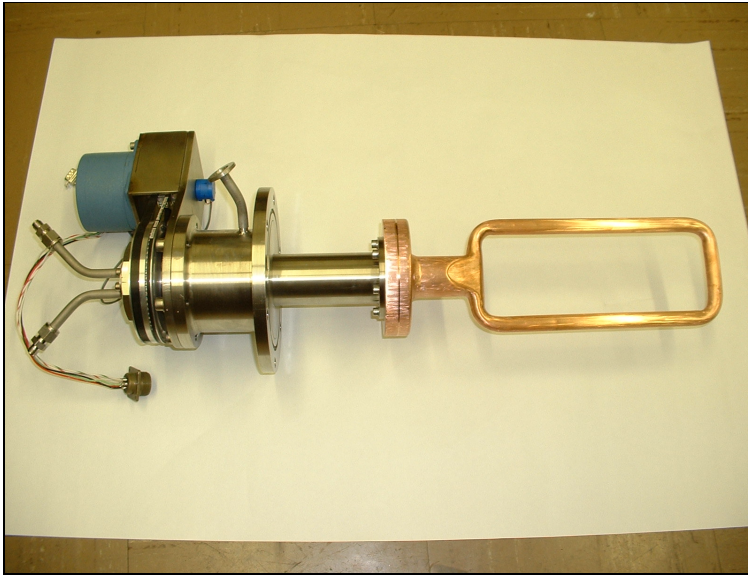


Figure 6.10. A picture of the tuning loop with its mounting sleeve and external motor drive.

The tuning effect of rotating the tuning loop from the angle of minimum coupling to the magnetic field, which is 0° (parallel to the median plane), up to the angle of maximum coupling, 90° , was measured and is shown in figure 6.11. The change in the frequency and the resonance frequency are shown. The tuning range of the loop is about 180 kHz.

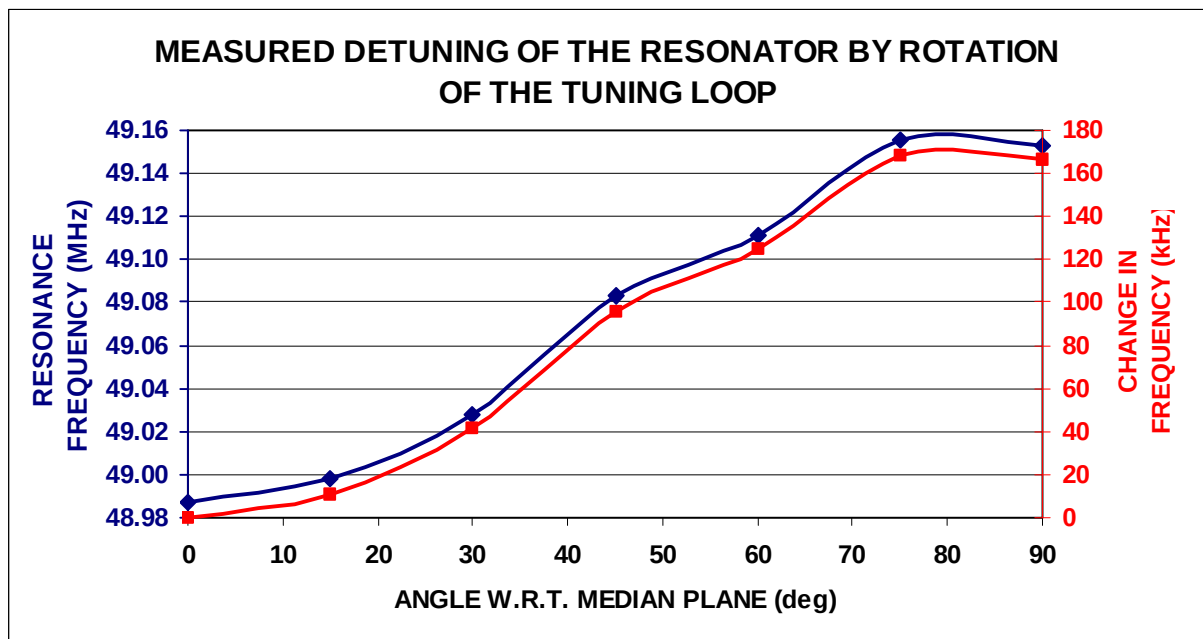


Figure 6.11. The resonance frequency of the resonator as a function of the angle with respect to the plate of the tuning loop.

6.2.4. Probes for dee voltage measurement

Two identical small probes for measurement of the dee voltage are installed on the rear vertical plate, symmetrically about the median plane. The induced voltages in these loops are used for stabilization of the dee voltage and phase. Figure 6.12 shows a picture of one of these probes with its extensions to the regions outside of the resonator. The locations of the probes are shown in figure 6.1.

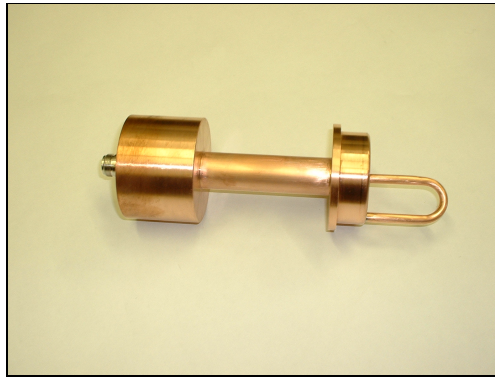


Figure 6.12. A *pick-up probe*. The induced signal is proportional to the current in the dee and consequently the dee voltage.

6.3 THE CAPACITORS

6.3.1 General description and location

A capacitor is installed on each of the horizontal plates of the outer conductors, about halfway between the two vertical end plates (see figure 5.2) for coarse tuning of the resonance frequency and to balance any asymmetries between the upper and lower halves of the resonator. The capacitors are not adjustable by remote control and have therefore to be set to a position that will permit the tuning loop an adequate adjustment range for fine-tuning. The capacitors are positioned in the region where the electric field strength between the inner and outer conductors is a maximum and therefore have its maximum effect on the resonator. Once in operation, the vacuum chamber has to be ventilated to adjust the capacitor plates. This happens seldom, a few times per annum. One of the

capacitors is shown mounted on the inside of an outer conductor in figure 6.13. In figure 6.14 the upper capacitor is shown in relation to the inner conductors and vertical plates, without the outer conductor.



Figure 6.13. *The capacitor connected to the outer conductor of the resonator. It is used for coarse tuning and electro-magnetic balancing of the upper and lower halves of the resonator.*

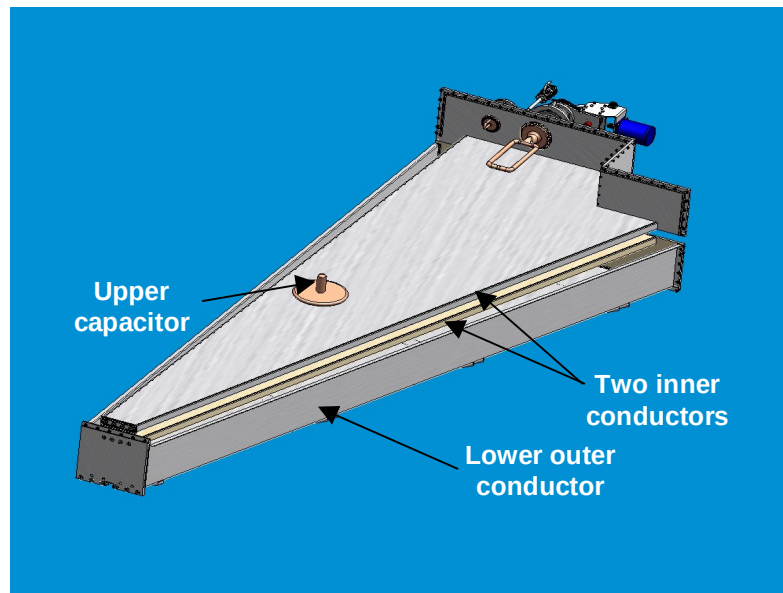


Figure 6.14. *A drawing of the resonator without the upper outer conductor.*

The capacitor plates have a diameter of 212 mm and a thickness of 15 mm and have been made with rounded edges to prevent sparking. The stem has a relatively large diameter of 32 mm to ensure good heat conduction. The capacitors are electrically connected to the outer conductor through contact fingers. Figure 6.15 shows a capacitor with its contact fingers.

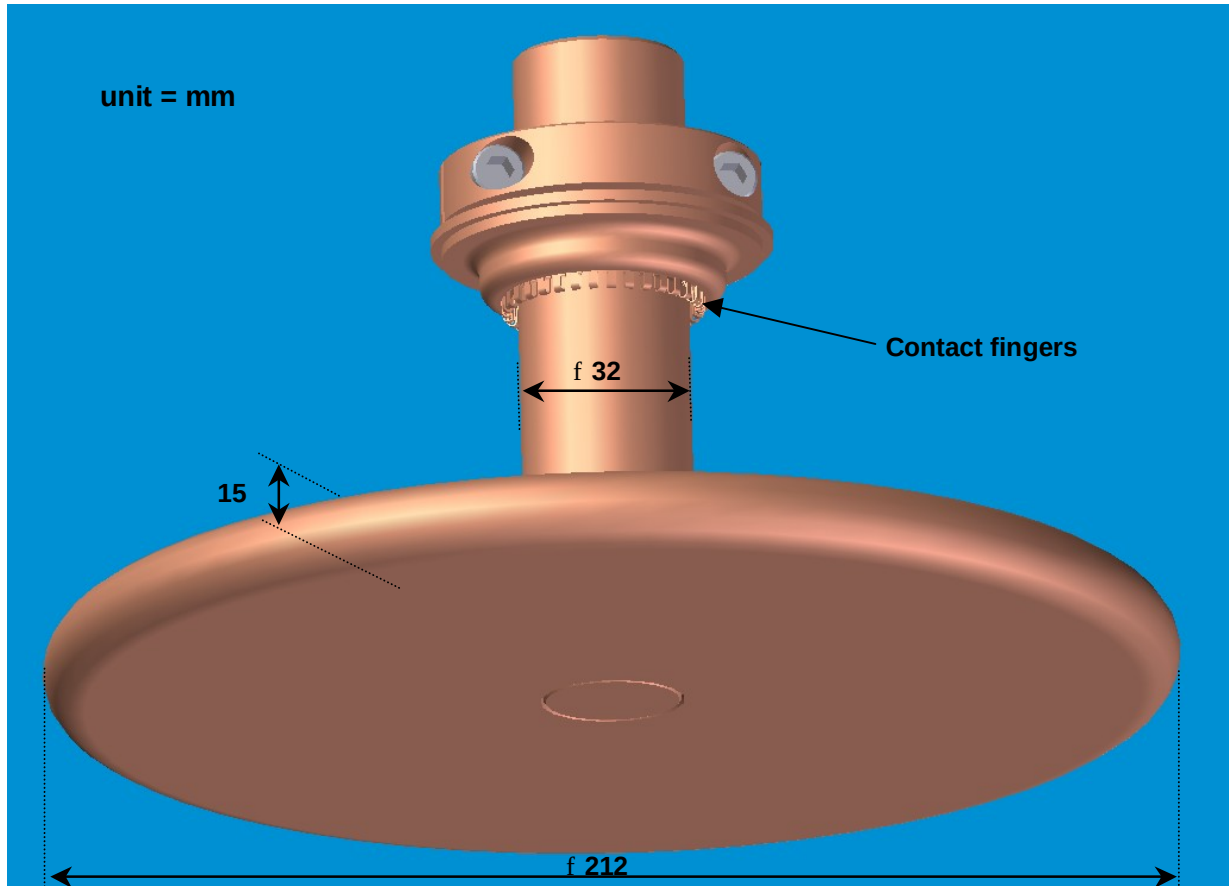


Figure 6.15. A 3d-drawing of a capacitor plate with its contact fingers.

6.3.2 Electrical characteristics of the capacitor plates

The capacitance, C , between a circular capacitor plate, with diameter, D_C , and the flat surface of the inner conductor, which is a distance, d_C , away from the plate, is roughly approximated by (the accurate calculations of the change in the resonance frequency due to the capacitor adjustments have been done with SOPRANO):

$$C = 2e_0 \frac{p \left(\frac{D_C}{2} \right)^2}{d_C}, \quad (6.23)$$

where ϵ_0 is the permittivity of free space. The factor 2 in the equation is included to take edge-effects into account. With the capacitor located at the known maximum dee voltage, V_d , and the resonance frequency, $f = \frac{w}{2p}$, the current in the stem of the capacitor is:

$$I_s = V_d C w . \quad (6.24)$$

The current density on the stem, which has a diameter of D_s , is:

$$J_s = \frac{I_s}{p D_s} . \quad (6.25)$$

The dissipated power in the stem, which has a length of l_s , is:

$$P_s = \frac{1}{2} I_s^2 \left(2.61 \times 10^{-7} \sqrt{f} \frac{l_s}{p D_s} \right) . \quad (6.26)$$

The good approximation of the resistance of the capacitor plate is:

$$R_{PL} = \frac{2.61 \times 10^{-7}}{2p} \sqrt{f} \ln \left(\frac{D_C}{D_s} \right) . \quad (6.27)$$

The dissipated power in a capacitor plate is:

$$P_{PL} = \frac{1}{2} I_s^2 R_{PL} . \quad (6.28)$$

Tuning of the resonator by the adjustment of the gap between a capacitor plate and the inner conductor (dee) was calculated with a spreadsheet program. It was assumed that the resonator is a homogeneous co-axial transmission line with short-circuit plates one half-wavelength apart and an average impedance was used for the full length of the resonator.

The impedance value was adjusted to obtain a calculated length for the resonator, at the operating frequency, that equals the actual length. For the calculation it was also assumed that the capacitor is located exactly halfway between the vertical end plates. The reactance of the parallel combinations of the two capacitors, and the reactances X_L and X_R , of the left and right-hand sections of the resonator as shown in figure 6.16, are calculated. The frequency is then adjusted to obtain zero total reactance. The parameters are shown in the schematic representation of the resonator as a co-axial transmission line.

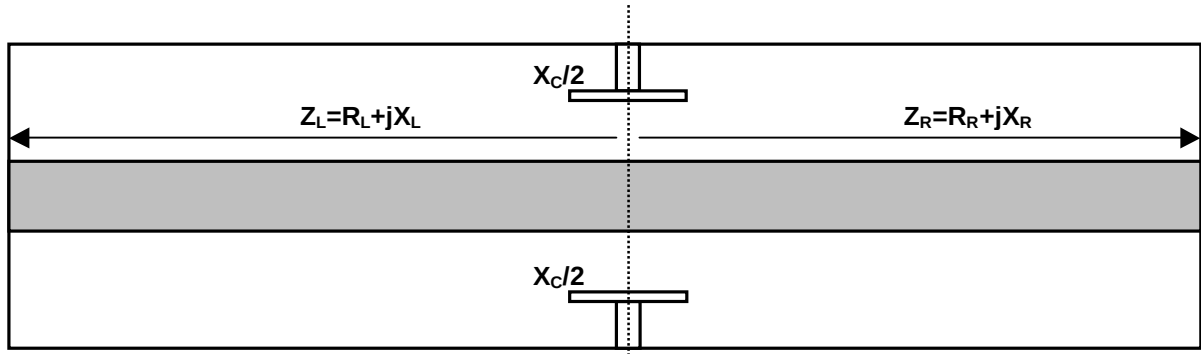


Figure 6.16. A schematic representation of the calculation procedure to determine the tuning effect of the capacitors set at a specific distance from the inner conductor of the flat-topping resonator. The resonance frequency is adjusted until the total reactance of the resonator with the capacitors included is zero. The total reactance is calculated by first adding the reactance of the capacitor in parallel to the reactance to the left and then add the sum in parallel to the reactance to the right.

The reactances to the left and right hand sides of the capacitor are:

$$\begin{aligned} X_L &= Z_0 \tan(b l_L) \text{ and} \\ X_R &= Z_0 \tan(b l_R) \end{aligned} \quad (6.29)$$

with $b = w / c$ and l_L and l_R the resonator lengths to the respective sides of the capacitor.

The total reactance of the two capacitor plates in parallel is:

$$X_C = -\frac{1}{2wC}. \quad (6.30)$$

The total reactance is calculated by adding the reactance of the capacitors connected in parallel to the parallel reactance of the two resonator reactances:

$$X_T = \frac{\left(\frac{X_L X_C}{X_L + X_C} \right) X_R}{\left(\frac{X_L X_C}{X_L + X_C} \right) + X_R}$$

$$= \frac{X_L X_C X_R}{X_L X_C + X_C X_R + X_R X_L}. \quad (6.31)$$

The frequency was iteratively adjusted (using internal functions of the spreadsheet programs) until the total reactance, X_T , is zero, which implies that a new resonance condition is obtained and the difference from the original resonance frequency is the tuning effect of the capacitors, for the specific gap between the capacitor and dee plate.

6.3.3 Measured and calculated values

The calculated results obtained with the analytical formulas and assumptions above, compare well with the calculated results with the SOPRANO program and measured data. The calculated deviation in resonance frequency due to the symmetrical insertion of the capacitor plates are shown in figure 6.17(a) and (b) for different gap distances between the capacitor plates and the dee plates. When the capacitor plates are set at 38 mm from the dee plates the frequency is about 4.5 MHz lower than with the capacitor plates fully retracted. Similarly, the resonance frequency drops by about 1.1 MHz with the plates separated by 162 mm from the dee plates, which is the maximum gap attainable. The associated capacitance of one capacitor plate, which varies from about 16.5 pF to 3.8 pF is also shown in the same figure.

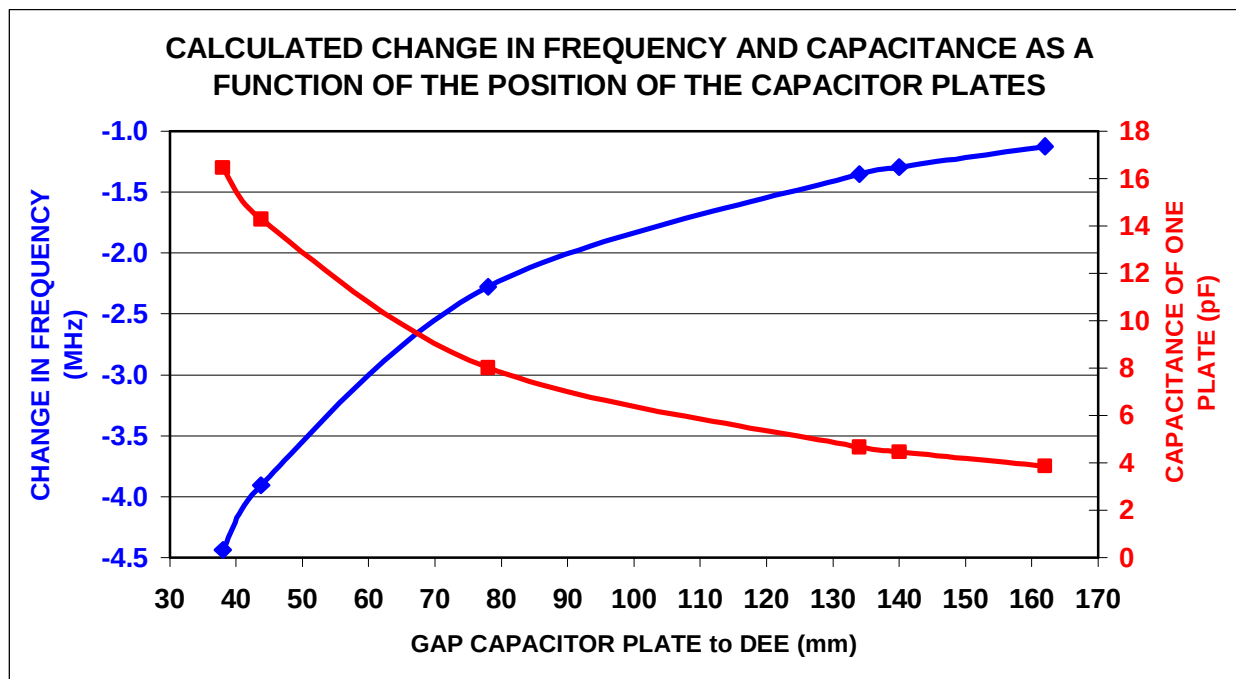


Figure 6.17(a). The calculated variation in the resonance frequency of the flat-topping resonator as a function of the position of the capacitor plates.

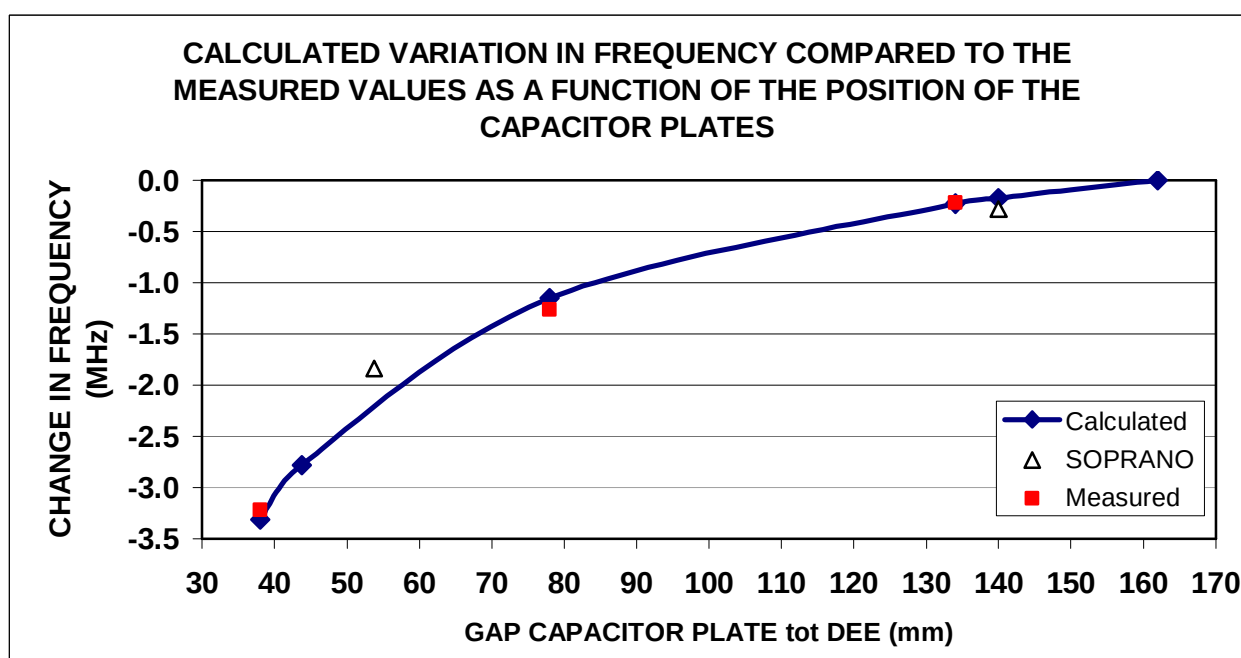


Figure 6.17(b). The calculated variation in resonance frequency of the flat-topping resonator as a function of the position of the capacitor plates compared to the measured values. The changes in frequency are w.r.t. the resonance frequency with the capacitor plate gap at 162 mm. The calculated values from SOPRANO were normalized to the other data.

6.4 THE EFFECT OF THE LOOPS AND CAPACITORS ON THE RESONANCE FREQUENCY : RESULTS OF CALCULATIONS WITH SOPRANO

The resonance frequency of the resonator with, and without the tuning devices and the plate for separation of the frequencies, as calculated with SOPRANO, are listed in table 6.1 (the default geometric parameters of the resonator, as given in Chapter 5, were used for the calculations):

Table 6.1. The calculated effects of the tuning devices and coupling loop on the resonance frequency.

#	PLATE FOR SEPARATION OF FREQUENCIES	DIMENSIONS COUPLING LOOP (mm)	DIMENSIONS TUNING LOOP (mm)	CAPACITOR GAP (TOP) (mm)	CAPACITOR GAP (BOTTOM) (mm)	FREQUENCY (MHz)	COMMENT
1	Absent	0	0	Absent	Absent	50.7950	Gap = 177 mm
2	Absent	0	0	20	Absent	49.7560	
3	Absent	0	0	140	Absent	50.6277	
4	Absent	0	0	140	140	50.4707	
5	Absent	240x70	240x70	140	140	50.5569	
6	Absent	240x70	240x70	53.75	53.75	48.9980	
7	Absent	315x98	0	140	140	50.5598	Compare to #5
8	Absent	0	315x98	140	140	50.4939	Less FE than #9
9	Absent	0	315x98	140	140	50.4922	More FE than #8
10	Absent	100x54.1	315x98	140	140	50.7016	Less FE than #11
11	Absent	100x54.1	315x98	140	140	50.6981	More FE than #10
12	Present	0	0	140	140	50.6230	Compare to #4
13	Present	0	0	70	70	49.0890	
14	Present	234x82	0	70	70	49.1170	
15	Present	234x82	173x34	70	70	49.1260	

The absence of a capacitor implies that the capacitance gap between the outer conductor and the adjacent dee plate is 177 mm. Comparison of the values listed in the table can be used in various ways to illustrate the effect of the tuning devices:

- Introduction of the capacitors with a gap of 140 mm from the dee plates, changes the resonance frequency by -324 kHz. (Compare #1 and #4).

- b. If only one capacitor is introduced at a 140 mm gap, the change in frequency is -167 kHz. (Compare #1 and #3).
- c. Reducing the gap of the capacitor in (b) from 140 to 20 mm changes the frequency by -872 kHz. (Compare #2 and #3).
- d. Changing the capacitor gaps (same for both capacitors) results in the following changes in frequency as listed in table 6.2:

Table 6.2. The effect on the resonance frequency of an adjustment in the capacitor plate position.

CHANGE IN GAP (mm)	CHANGE IN FREQUENCY (kHz)	CHANGE IN FREQUENCY (kHz/mm)	COMPARE CALCULATIONS
-37	-324	~9	#1 and #4
-70	-1534	~22	#12 and #13
-86.25	-1559	~18	#5 and #6

The change in frequency should be considered together with the different geometrical configurations that were used for the calculations.

- e. Introduction of the two loops, each 240x70 mm, changes the frequency by +86 kHz. (Compare #4 and #5).
- f. Turning the tuning loop from a position of minimum flux linkage, i.e. parallel to the median plane, to a position of maximum flux linkage, i.e. 90° with respect to the median plane, will change the frequency by -22 kHz and -9 kHz, respectively for loop sizes of 315x98 mm (compare #4 and #8) and 173x34 mm (compare #14 and #15). Apart from the difference in loop dimensions, the capacitor settings were significantly different for the two sets of calculations, because the plates for the shifting of the frequency were not installed for the first set. These calculated ranges of the tuning loop do not correlate with the measured values shown in figure 6.11. The most probable reason for this discrepancy is the crude representation of the loops, which are relatively small in relation to the complete resonator. However, the measured detuning range of a tuning loop on the resonance frequency of the half-scale resonator was 26.3 kHz, which is equivalent to 13.15 kHz for the full-scale resonator.

- g. In calculation #7 the dimensions of a single loop, that will have the same resonance frequency as the two loops in calculation #5, was iteratively determined. It provided guidance and confirmation as to the length of the coupling loop that will be required and calculation #8 followed.
- h. The introduction of the capacitor plates for shifting the unwanted frequency mode by about -6 MHz also had a minor shift of $+152$ kHz on the frequency of the wanted mode, as one would expect from the fact that in this mode the field inside the dee is limited to the dee edges at the acceleration gaps. Compare #4 and #12. This change could easily be compensated for by other means.
- i. The accuracy of the results from the numerical procedures can depend on the selection of the finite-element distribution and density in the defined structure of the FE model. Comparison of calculations #8 and #9 show that the number of the finite elements used for the lower resolution calculations are adequate, since the configurations with the larger number of elements changed the frequency by only -1 kHz. Similarly, for calculations #10 and #11 the change in the frequency is only -3.5 kHz

-o-O-o-

CHAPTER 7

HEAT TRANSPORT MODELLING

7.1 INTRODUCTION

With a power dissipation of 10 kW in the resonator it has to be water-cooled in order to prevent over-heating of the components and to prevent changes in the dimensions, which would cause detuning of the resonator. A maximum temperature rise at any point in the resonator of 10 K above the water temperature was therefore specified. The cooling pipes also have to be fixed to the resonator components in such a way that it will not cause vibration of the electrodes. The two methods used to calculate the distribution of the cooling pipes, using basic analytic formulae, are discussed in this chapter together with the calculated results.

7.2 DISTRIBUTION OF THE PIPES FOR WATER COOLING

7.2.1 Calculation of the distances between adjacent cooling water pipes

The currents in the resonator electrodes will flow on the outer surfaces of the inner conductors (dee plates) and the inner surfaces of the outer conductors. For a skin depth of 9.4 micron (see chapter 5) and minimum thickness of 2 mm for the copper plates of the electrodes, the plate thickness does not determine the electrical resistance.

For a plate with length, l , width, w , thickness, d , surface resistance, R_s , through which a current flows with a peak value I_{peak} , homogeneously distributed over the width, at frequency f_{rf} the following parameters can be calculated:

1. electrical resistance (Ω)

$$R = R_s \frac{l}{w} \quad , \quad (7.1)$$

2. power dissipation (W)

$$P = \frac{I_{peak}^2}{2} R \quad \text{and} \quad (7.2)$$

3. power dissipation per unit volume (W.m^{-3})

$$\dot{Q} = \frac{P}{lwd} = \frac{I_{peak}^2}{2lwd} R = \frac{I_{peak}^2}{2w^2d} R_s \quad . \quad (7.3)$$

In a resonator, made from copper and operating at 49.1 MHz, the equations above can reduce to (refer to Appendix E for the electrical properties of copper):

$$R = R_s \frac{l}{w} = (1.829 \times 10^{-3}) \frac{l}{w} \quad (7.4)$$

$$P = \frac{I_{peak}^2}{2} R = \frac{I_{peak}^2}{2} (1.829 \times 10^{-3}) \frac{l}{w} \quad (7.5)$$

$$\dot{Q} = \frac{P}{lwd} = \frac{I_{peak}^2}{2lwd} (1.829 \times 10^{-3}) \frac{l}{w} = (1.829 \times 10^{-3}) \frac{(I_{peak})^2}{2w^2d} \quad (7.6)$$

and with the surface current density, $J = I_{peak} / w$, expressed in A.m^{-1} , equation 7.6 becomes:

$$\dot{Q} = (1.829 \times 10^{-3}) \frac{(J_{peak})^2}{2d} \quad (7.7)$$

The distance D , in meter, from any point on the surface of the conductor to the nearest cooling water pipe is given by (see appendix G for the derivation of the formula):

$$D = \sqrt{\frac{2h\Delta T}{\dot{Q}}} \quad , \quad (7.8)$$

where h = the heat conductivity of the conductor ($382 \text{ W.K}^{-1}.\text{m}^{-1}$ for copper) and ΔT the allowed rise in temperature, in Kelvin, above the cooling water temperature [Hol72].

The formulae were applied on the calculation with the resonator divided into segments and with the SOPRANO post-processor model to calculate the required distribution of the cooling pipes for the flat-topping resonator in two different ways.

7.2.1.1 Calculation of cooling pipe distribution on segments of the resonator

The power dissipation and the placing of the cooling pipes were calculated by viewing the resonator as a number of transmission line sections, as described in chapter 4, and using the calculated currents for each segment to calculate the cooling requirements at different points on each segment. The position of the center of each segment and their respective distances from the vertical plate nearest to the injection orbit, **P**, are shown in figure 4.1. Calculations were also done for sections flush against the vertical plates, and also for each of the vertical plates themselves.

For each segment the current density and distance to the nearest cooling pipe were calculated on radial lines at the positions **a** to **I** as shown in figure 7.1. The respective positions of these lines were selected to cover the range of expected current densities. The calculations were done for a temperature rise of $\Delta T = 10$ K above the water temperature.

Except for the vertical plates **P** and **Q** and the plate that forms the vertical part of the inner conductor plates, which has a thickness of 10 mm, all the other conducting surfaces are made from copper plate with a thickness of 2 mm.

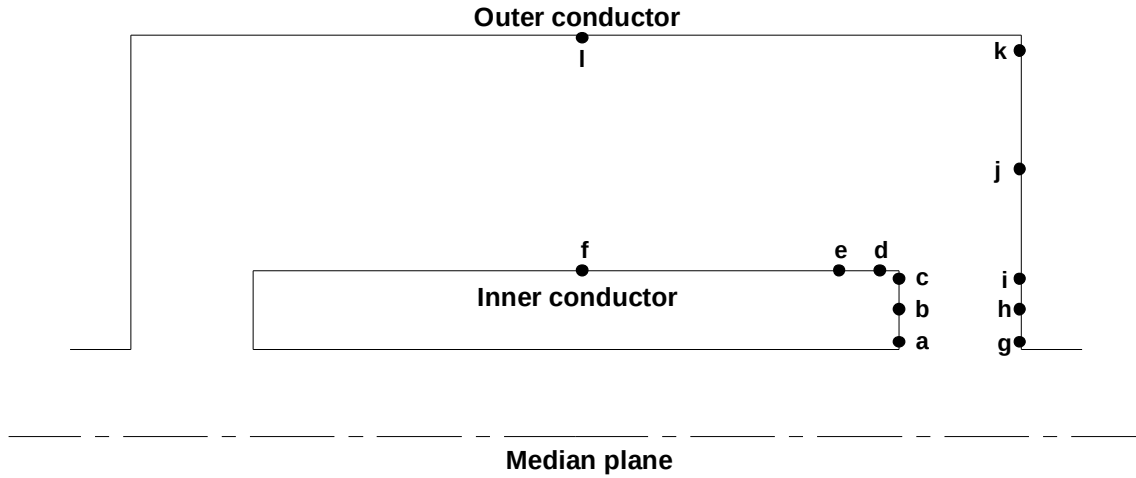


Figure 7.1. A typical cross-section of the upper half of the flat-topping resonator that was used in the POISSON program to calculate the field distribution for given boundary conditions and different dimensions of each section. The positions of radial lines along the length of the resonator surface where the current densities and distances to the nearest cooling pipe were calculated, are indicated by a •. The plate thickness is 2 mm throughout, except for at the dee lip region (a,b,c and d) and the vertical end plates, which have a thickness of 10 mm.

The current density is highest in the region where the lip (a, b, c and d) of the dee and the outer conductor next to the lip (g, h and i) are connected to the vertical plates **P** and **Q**. The current density at the smaller plate **P** near injection is higher than at the larger plate, **Q**, near the extraction orbit. Figures 7.2 and 7.3, respectively, show the high and low current densities on the radial lines along the positions **a** to **l**, from vertical plate **P** to plate **Q**. The graphs were smoothed to emphasize the trend. The actual calculated values are listed in table 7.1, together with the calculated values for $\dot{Q}$ (listed as Q_p).

The distances from points on these lines to the nearest cooling pipe were also calculated at 10 K above the cooling water temperature. The overall picture of how the distance to the nearest cooling pipe varies along the already indicated radial lines, are shown in figure 7.4 and the data is also listed in table 7.1.

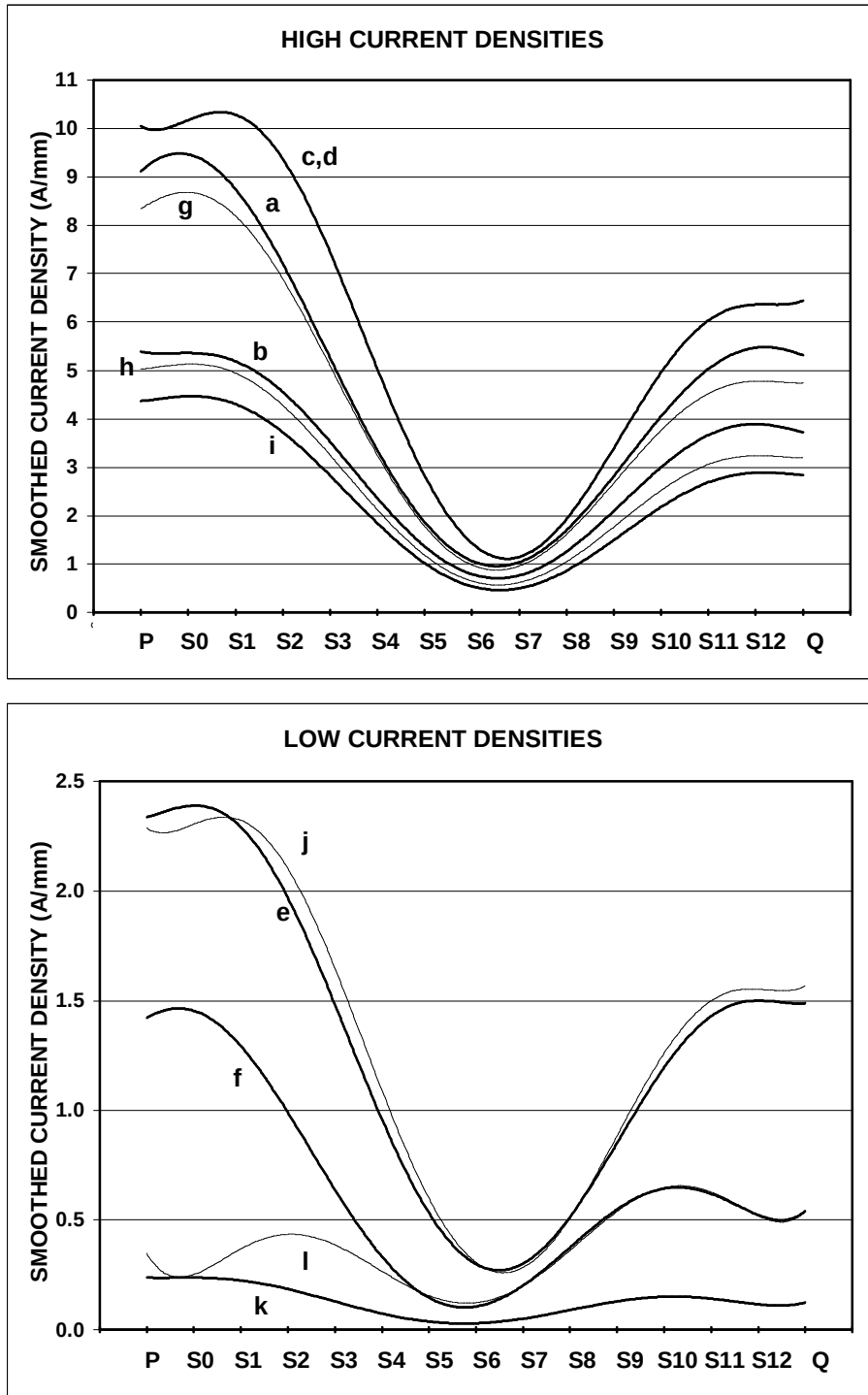


Figure 7.2 (top) and 7.3 (bottom). The smoothed current density in A/mm along different radial lines on the resonator surface (a to l), calculated for the vertical plate P, at each of the 13 sections through the resonator and for plate Q. The actual distances between the sections are given in figure 4.1. The highest current densities are near and on the vertical plates, especially plate P, which is nearest to the injection orbit. The dee lips and the conducting surfaces nearest to them have the highest current density. The current densities are listed in table 7.1.

DISTANCE TO THE NEAREST COOLING PIPE

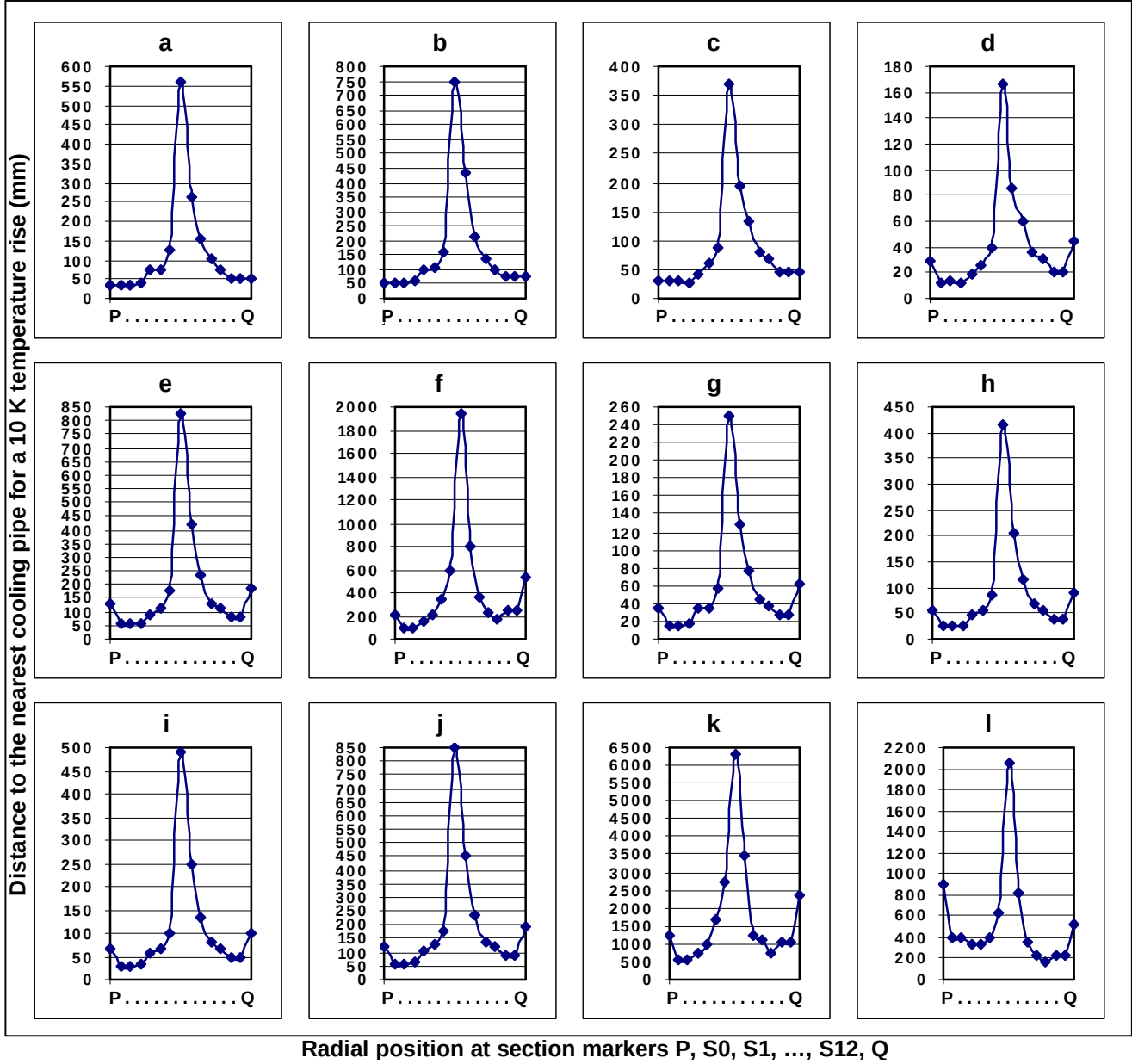


Figure 7.4. The calculated distances in mm from points on radial lines (a to l) along the surface of the resonator to the nearest cooling duct, in order to comply with the permitted temperature rise of 10 K above the water temperature. Each horizontal axis has the longitudinal position starting at vertical plate P, through the 13 cross-sections, to the vertical plate Q. The actual values are listed in table 7.1.

Table 7.1. The calculated current density, J (A.mm⁻¹), the density of the heat dissipation, Q_p (W.m⁻³), and the distance, d (mm), to the nearest cooling pipe along radial lines at each of the cross-sectional positions (a to l). Each radial line runs from plate P through S0 to S12 and ends at plate Q. (The table continues on the next page for positions g to l.)

POS. a	z(mm)	J	Q_p	d
PLATE P	0	9.2	7.701E+06	31.5
S0	0	9.2	7.701E+06	31.5
S1	131.6	9.0	7.399E+06	32.1
S2	392.1	7.8	5.487E+06	37.3
S3	652.6	4.0	1.476E+06	71.9
S4	915.8	4.0	1.462E+06	72.3
S5	1177.6	2.3	4.832E+05	125.7
S6	1439.5	0.5	2.427E+04	561.0
S7	1701.3	1.1	1.087E+05	265.1
S8	1961.8	1.9	3.151E+05	155.7
S9	2223.7	2.9	7.512E+05	100.8
S10	2485.5	3.8	1.285E+06	77.1
S11	2748.7	5.3	2.598E+06	54.2
S12	3000	5.3	2.598E+06	54.2
PLATE Q	3000	5.3	2.598E+06	54.2

POS. b	z(mm)	J	Q_p	d
PLATE P	0	5.4	2.637E+06	53.8
S0	0	5.4	2.637E+06	53.8
S1	131.6	5.3	2.533E+06	54.9
S2	392.1	4.7	1.975E+06	62.2
S3	652.6	3.0	8.020E+05	97.6
S4	915.8	2.8	6.908E+05	105.2
S5	1177.6	1.8	2.859E+05	163.5
S6	1439.5	0.4	1.372E+04	746.1
S7	1701.3	0.7	4.060E+04	433.8
S8	1961.8	1.4	1.711E+05	211.3
S9	2223.7	2.1	4.016E+05	137.9
S10	2485.5	3.0	8.222E+05	96.4
S11	2748.7	3.8	1.295E+06	76.8
S12	3000	3.8	1.295E+06	76.8
PLATE Q	3000	3.8	1.295E+06	76.8

POS. c	z(mm)	J	Q_p	d
PLATE P	0	10.1	9.319E+06	28.6
S0	0	10.1	9.319E+06	28.6
S1	131.6	9.9	8.953E+06	29.2
S2	392.1	10.3	9.754E+06	28.0
S3	652.6	6.8	4.271E+06	42.3
S4	915.8	4.9	2.183E+06	59.2
S5	1177.6	3.3	9.862E+05	88.0
S6	1439.5	0.8	5.576E+04	370.2
S7	1701.3	1.5	2.055E+05	192.8
S8	1961.8	2.2	4.288E+05	133.5
S9	2223.7	3.5	1.148E+06	81.6
S10	2485.5	4.3	1.650E+06	68.0
S11	2748.7	6.4	3.742E+06	45.2
S12	3000	6.4	3.742E+06	45.2
PLATE Q	3000	6.4	3.742E+06	45.2

POS. d	z(mm)	J	Q_p	d
PLATE P	0	10.1	9.319E+06	28.6
S0	0	10.1	4.659E+07	12.8
S1	131.6	9.9	4.477E+07	13.1
S2	392.1	10.3	4.877E+07	12.5
S3	652.6	6.8	2.135E+07	18.9
S4	915.8	4.9	1.092E+07	26.5
S5	1177.6	3.3	4.931E+06	39.4
S6	1439.5	0.8	2.788E+05	165.5
S7	1701.3	1.5	1.028E+06	86.2
S8	1961.8	2.2	2.144E+06	59.7
S9	2223.7	3.5	5.741E+06	36.5
S10	2485.5	4.3	8.250E+06	30.4
S11	2748.7	6.4	1.871E+07	20.2
S12	3000	6.4	1.871E+07	20.2
PLATE Q	3000	6.4	3.742E+06	45.2

POS. e	z(mm)	J	Q_p	d
PLATE P	0	2.3	4.813E+05	126.0
S0	0	2.3	2.407E+06	56.3
S1	131.6	2.3	2.312E+06	57.5
S2	392.1	2.3	2.469E+06	55.6
S3	652.6	1.4	8.997E+05	92.1
S4	915.8	1.2	6.124E+05	111.7
S5	1177.6	0.7	2.360E+05	179.9
S6	1439.5	0.2	1.115E+04	827.7
S7	1701.3	0.3	4.324E+04	420.3
S8	1961.8	0.5	1.340E+05	238.8
S9	2223.7	1.0	4.342E+05	132.6
S10	2485.5	1.1	5.614E+05	116.7
S11	2748.7	1.6	1.113E+06	82.9
S12	3000	1.6	1.113E+06	82.9
PLATE Q	3000	1.6	2.226E+05	185.3

POS. f	z(mm)	J	Q_p	d
PLATE P	0	1.4	1.849E+05	203.3
S0	0	1.4	9.243E+05	90.9
S1	131.6	1.4	8.880E+05	92.8
S2	392.1	0.9	3.652E+05	144.6
S3	652.6	0.6	1.678E+05	213.4
S4	915.8	0.4	6.460E+04	343.9
S5	1177.6	0.2	2.192E+04	590.4
S6	1439.5	0.1	2.030E+03	1940.0
S7	1701.3	0.2	1.169E+04	808.3
S8	1961.8	0.4	6.125E+04	353.2
S9	2223.7	0.5	1.378E+05	235.5
S10	2485.5	0.7	2.425E+05	177.5
S11	2748.7	0.5	1.344E+05	238.5
S12	3000	0.5	1.344E+05	238.5
PLATE Q	3000	0.5	2.687E+04	533.2

Table 7.1. (continued)

POS. g	z(mm)		Q_p	d
PLATE P	0	8.4	6.471E+06	34.4
S0	0	8.4	3.236E+07	15.4
S1	131.6	8.3	3.109E+07	15.7
S2	392.1	7.8	2.743E+07	16.7
S3	652.6	3.8	6.629E+06	33.9
S4	915.8	3.7	6.141E+06	35.3
S5	1177.6	2.3	2.416E+06	56.2
S6	1439.5	0.5	1.214E+05	250.9
S7	1701.3	1.0	4.568E+05	129.3
S8	1961.8	1.7	1.285E+06	77.1
S9	2223.7	2.9	3.756E+06	45.1
S10	2485.5	3.4	5.280E+06	38.0
S11	2748.7	4.7	1.027E+07	27.3
S12	3000	4.7	1.027E+07	27.3
PLATE Q	3000	4.7	2.053E+06	61.0

POS. h	z(mm)	J	Q_p	d
PLATE P	0	5.1	2.330E+06	57.3
S0	0	5.1	1.165E+07	25.6
S1	131.6	5.0	1.119E+07	26.1
S2	392.1	4.7	9.876E+06	27.8
S3	652.6	2.7	3.248E+06	48.5
S4	915.8	2.3	2.449E+06	55.8
S5	1177.6	1.5	1.074E+06	84.3
S6	1439.5	0.3	4.460E+04	413.9
S7	1701.3	0.6	1.822E+05	204.8
S8	1961.8	1.1	5.837E+05	114.4
S9	2223.7	1.9	1.575E+06	69.6
S10	2485.5	2.3	2.455E+06	55.8
S11	2748.7	3.2	4.677E+06	40.4
S12	3000	3.2	4.677E+06	40.4
PLATE Q	3000	3.2	9.354E+05	90.4

POS. i	z(mm)	J	Q_p	d
PLATE P	0	4.4	1.762E+06	65.9
S0	0	4.4	8.808E+06	29.5
S1	131.6	4.3	8.462E+06	30.0
S2	392.1	4.0	7.468E+06	32.0
S3	652.6	2.4	2.529E+06	55.0
S4	915.8	2.0	1.827E+06	64.7
S5	1177.6	1.3	7.457E+05	101.2
S6	1439.5	0.3	3.163E+04	491.5
S7	1701.3	0.5	1.243E+05	247.9
S8	1961.8	1.0	4.235E+05	134.3
S9	2223.7	1.6	1.112E+06	82.9
S10	2485.5	2.0	1.757E+06	65.9
S11	2748.7	2.8	3.696E+06	45.5
S12	3000	2.8	3.696E+06	45.5
PLATE Q	3000	2.8	7.391E+05	101.7

POS. j	z(mm)	J	Q_p	d
PLATE P	0	2.3	5.040E+05	123.1
S0	0	2.3	2.520E+06	55.1
S1	131.6	2.3	2.421E+06	56.2
S2	392.1	2.1	2.041E+06	61.2
S3	652.6	1.3	7.137E+05	103.5
S4	915.8	1.0	4.568E+05	129.3
S5	1177.6	0.7	2.360E+05	179.9
S6	1439.5	0.2	1.049E+04	853.6
S7	1701.3	0.3	3.729E+04	452.7
S8	1961.8	0.5	1.340E+05	238.8
S9	2223.7	0.9	3.938E+05	139.3
S10	2485.5	1.1	5.378E+05	119.2
S11	2748.7	1.5	1.012E+06	86.9
S12	3000	1.5	1.012E+06	86.9
PLATE Q	3000	1.5	2.024E+05	194.3

POS. k	z(mm)	J	Q_p	d
PLATE P	0	0.2	5.111E+03	1222.6
S0	0	0.2	2.555E+04	546.8
S1	131.6	0.2	2.455E+04	557.8
S2	392.1	0.2	1.365E+04	748.2
S3	652.6	0.1	7.679E+03	997.5
S4	915.8	0.1	2.722E+03	1675.4
S5	1177.6	0.0	1.040E+03	2710.4
S6	1439.5	0.0	1.902E+02	6337.3
S7	1701.3	0.0	6.463E+02	3438.1
S8	1961.8	0.1	5.186E+03	1213.7
S9	2223.7	0.1	6.010E+03	1127.5
S10	2485.5	0.2	1.311E+04	763.3
S11	2748.7	0.1	6.919E+03	1050.8
S12	3000	0.1	6.919E+03	1050.8
PLATE Q	3000	0.1	1.384E+03	2349.7

POS. l	z(mm)	J	Q_p	d
PLATE P	0	0.3	9.635E+03	890.5
S0	0	0.3	4.817E+04	398.2
S1	131.6	0.3	4.628E+04	406.3
S2	392.1	0.4	6.974E+04	331.0
S3	652.6	0.4	6.956E+04	331.4
S4	915.8	0.3	4.781E+04	399.8
S5	1177.6	0.2	1.892E+04	635.4
S6	1439.5	0.1	1.830E+03	2043.5
S7	1701.3	0.2	1.169E+04	808.3
S8	1961.8	0.4	6.125E+04	353.2
S9	2223.7	0.5	1.378E+05	235.5
S10	2485.5	0.7	2.425E+05	177.5
S11	2748.7	0.5	1.344E+05	238.5
S12	3000	0.5	1.344E+05	238.5
PLATE Q	3000	0.5	2.687E+04	533.2

7.2.1.2 SOPRANO calculations of adjacent pipe distances

The same formulae as above were applied in the post-processor of the SOPRANO database to calculate the $\dot{Q}$ -values and distances to the nearest cooling pipe. The average current density, $\bar{\mathbf{J}}$, and direction of the current at any point on the surface were calculated from $\bar{\mathbf{J}} = \bar{\mathbf{n}} \times \bar{\mathbf{H}}$, where $\bar{\mathbf{n}}$ is the area vector of the finite-element and $\bar{\mathbf{H}}$ the calculated tangential magnetic field component at that surface element. The results are illustrated in figures 7.5(a) and (b), which respectively show the calculated magnetic field (black arrows) and current density distribution (red arrows) on a perspective view of the rear vertical panel of the resonator. Only a small section of the dee plates and outer conductors are shown connected to the rear plate, but the remaining parts of the resonator are not shown. The coupling and tuning loops were not included in this calculation. This method is convenient for calculating the magnetic field at every finite-element of the SOPRANO database and thereby obtain a distribution of $\dot{Q}$ over the complete resonator surface.

The magnetic field distribution, which is equivalent to the electric current density, is highest on the vertical plate of the resonator near injection, where the dee plates connect to it, as is shown in figure 7.6 with the top outer conductor removed from the figure. The magnetic field distribution on the conducting surfaces with the maximum values on the inside of the vertical end plate near extraction is also shown. The magnetic field direction is indicated by the arrows on the conductor surfaces.

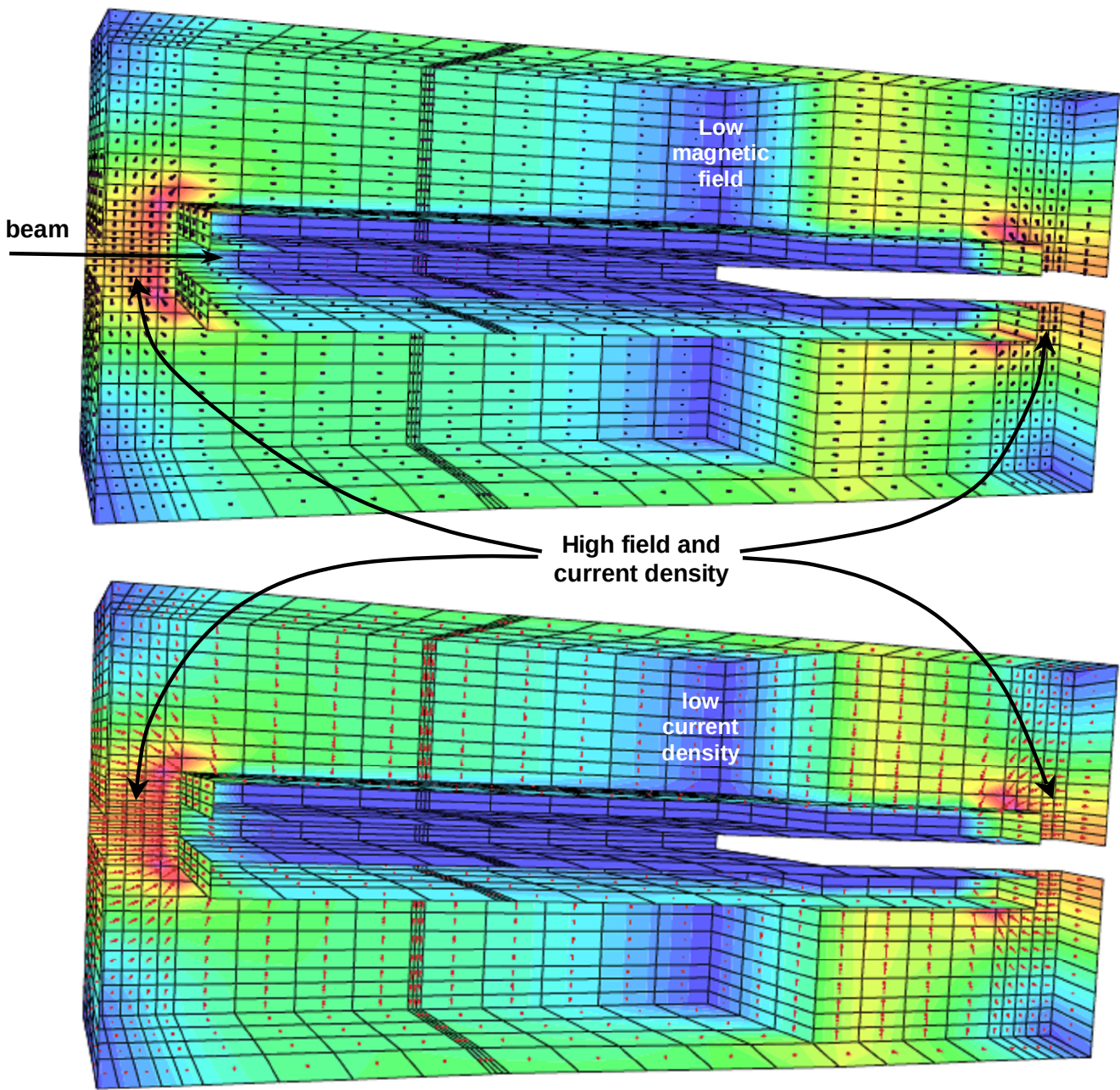


Figure 7.5(a) and (b). A perspective view of the inside of the rear section of the resonator showing that high currents on the surface occur where high magnetic fields are located. The figure at the top shows the calculated magnetic field (black arrows) on each of the finite-elements that form the conducting surfaces and the bottom figure shows the current density distribution (red arrows) on the same selected geometry. The coupling and tuning loops were not included in these calculations. The beam entrance acceleration gap is on the left side of each picture. The size of the arrows and the colour spectrum indicate the magnitude of the calculated field. Red indicates a maximum field strength and blue the least.

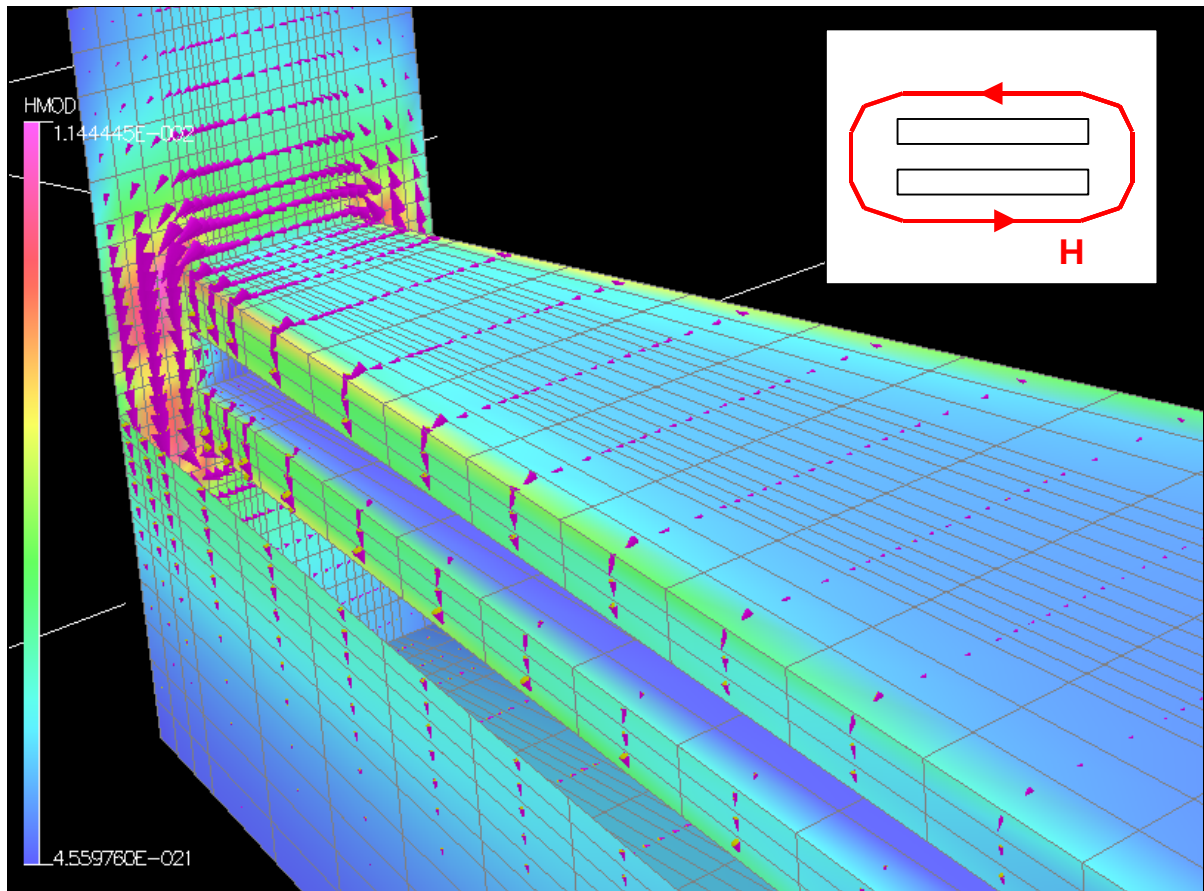


Figure 7.6. *The calculated magnetic field density is highest (red) where the dee plates connect to the front end plate, near injection. The upper outer conductor is not shown and the arrows indicate the direction of the magnetic field that circulates the dee plates.*

The result of one such calculation of distance to the nearest cooling pipe is shown in figure 7.7. As with the previous figure, only the region where the maximum current density is expected is shown. The range for the distances to be displayed on the finite-element geometry are pre-selected and applied with the post-processing features of the SOPRANO database, which then gives a spectrum of distances on the copper surface. The spectrum shown in the figure varies between 40 mm (blue) and 200 mm (red). It implies that any point in the red region should have a cooling duct within 200 mm from it and similarly any point in the blue region should have a cooling duct within 40 mm from it, in order to comply with the selected maximum temperature rise of 10 K. This method of plotting a selected distance region, permits determination of the distance to the nearest cooling duct for any point on the conducting surface of the resonator.

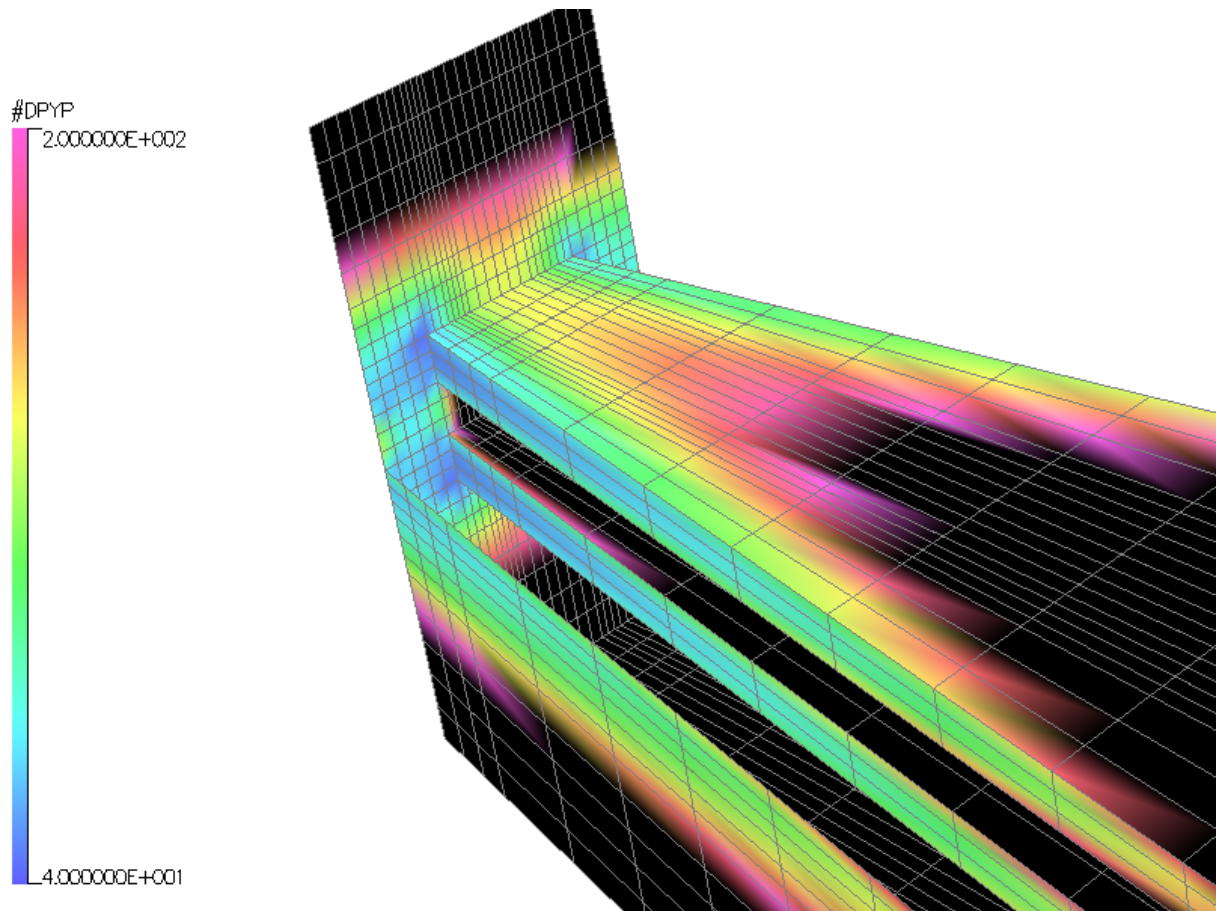


Figure 7.7. The required distance from any point on the surface of the copper to the nearest cooling pipe, in order to allow a maximum temperature rise of 10 K, is shown as a contour spectrum on a part of the FE model for the flat-topping resonator. The pre-selected boundaries are from 40 mm (blue) to 200 mm (red).

The regions outside the limits of the selected colour spectrum (shown in black) will require a distance greater than 200 mm to the nearest cooling pipe. These regions can also be analyzed by selecting other boundaries for display, as is shown in figures 7.8(a) for the full length of the resonator and in 7.8(b) for a view of the vertical plate near extraction. With the smallest calculated distance from a point on the surface to the nearest cooling pipe to be in the order of 40 mm, is easily attainable with the geometry available. These calculated results agree with those obtained with analytic expressions.

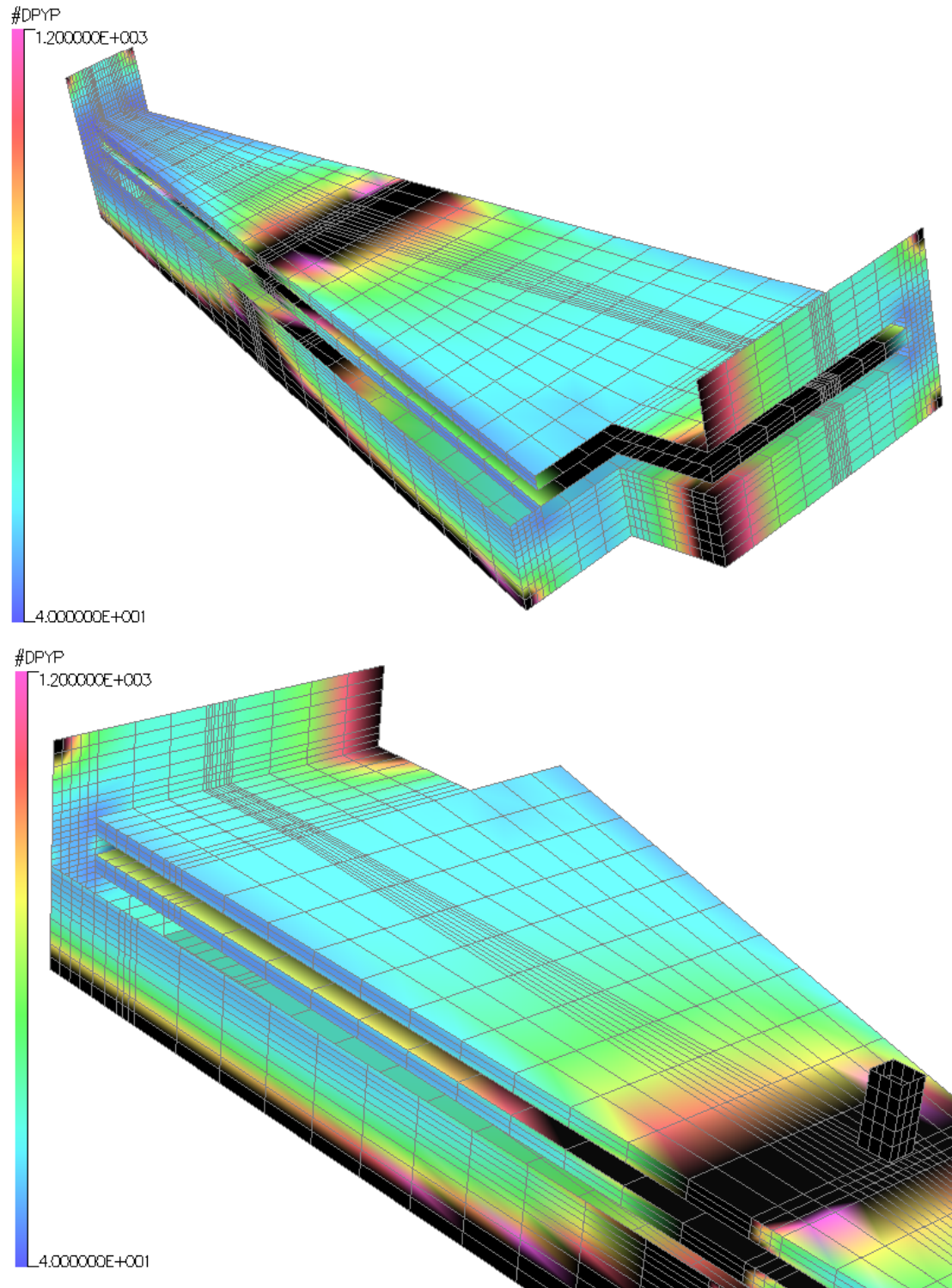


Figure 7.8 (a) and (b). The calculated distance to the nearest cooling pipe, shown over the full length of the finite element model of the resonator (picture at the top), for selected limits of 40 mm (indicated by blue) and 1200 mm (red). The upper outer conductor plates and some tuning components are omitted from the display. In the bottom picture the inside of the vertical plate near the extraction orbit can be seen. The areas indicated by the black colour has the lowest calculated current densities and will require a cooling pipe at a distance beyond the red scale.

7.2.2 The layout of the cooling pipes according to the calculated spacing

The calculated distribution of the cooling ducts is shown in figure 7.9 for the outer conductor and in figure 7.10 for the dee, which provides more coolant to the conducting surfaces than what was calculated for the 10 K limit.

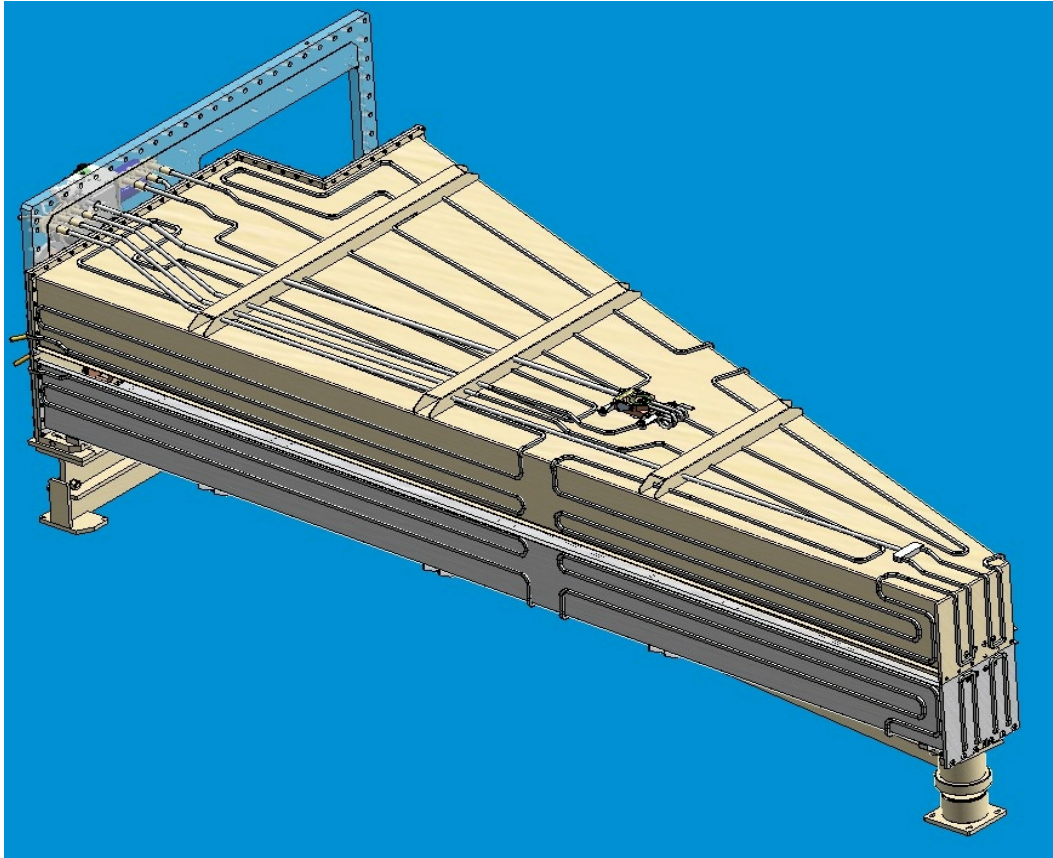


Figure 7.9. A drawing of the flat-topping resonator with its cooling pipe distribution on the outer conductors and end plate near the injection orbit, mounted on its pods and with the vacuum chamber flange at the back.

The flow of the water through the cooling ducts may cause vibration, which can detune a resonator system with a high Q-value and cause amplitude and phase modulation of the dee voltage with detrimental effects on the beam quality. The vibrations caused by water flow can be minimized by keeping the water velocity below 2 m.s^{-1} and by rigidly fixing the pipes to the conducting surfaces. The pipes leading to and from the cooling pipes fixed to the resonator surfaces, are positioned in regions of low magnetic field to prevent the induction of electro-magnetic forces in the loops formed by the pipes and conductor

surfaces. These pipes are also bent away from the copper surface to minimize the transfer of vibration to the resonator.

The cooling pipes were first tack-welded and then soft soldered to the conducting surfaces, except in regions of high electric fields where hard soldering was used to prevent sputtering. Soft solder has a high surface resistance and tends to sputter in regions of high electric fields and will deposit a thin layer on the conducting surfaces, which will increase the power dissipation in the resonator.

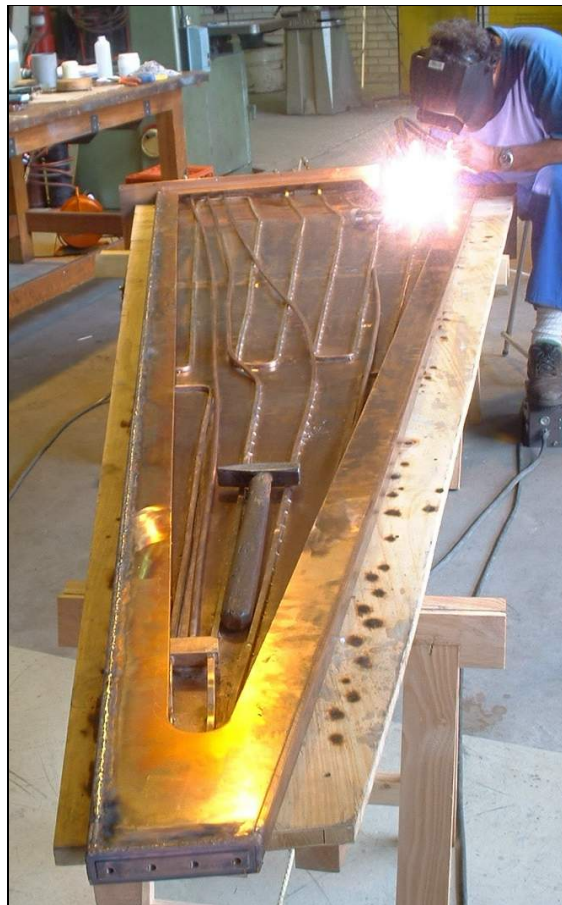


Figure 7.10. *The cooling ducts on the inside of the dee.*

-o-O-o-

CHAPTER 8

CONTEXTUAL REVIEW OF THE DOUBLE-GAP HORIZONTAL HALF-WAVE FLAT-TOPPING RESONATOR

The double-gap horizontal half-wave resonator type that was implemented with the SSC at iThemba LABS, is a novel design and therefore required a thorough study of its characteristics. The flat-topping resonators in operation at cyclotrons around the world are listed in table 8.1.

Table 8.1. The flat-topping resonator types in operation with cyclotrons around the world.

COUNTRY	LOCATION	CYCLOTRON	FLAT-TOPPING RESONATOR TYPE
CANADA	Vancouver	TRIUMF	auxiliary quarter-wave rectangular transmission line
JAPAN	Osaka (RCNP)	AVF Cyclotron	1x quarter-wave coaxial
	Osaka (RCNP)	Ring Cyclotron	1x single-gap cavity
	Saitama (RIKEN)	RRC	1x single-gap cavity
	Saitama (RIKEN)	fRC	1x single-gap cavity
	Saitama (RIKEN)	IRC	1x single-gap cavity
	Saitama (RIKEN)	SRC	1x single-gap cavity
	Saitama (RIKEN)	AVF Cyclotron	1x quarter-wave coaxial
SOUTH AFRICA	iThemba LABS	SPC1	1x quarter-wave coaxial
	iThemba LABS	SSC	1x double-gap horizontal half-wave
SWITZERLAND	Villigen (PSI)	Injector 2	2x single-gap cavities
	Villigen (PSI)	Ring Cyclotron	1x single-gap cavity

Like all the other flat-topping resonator types, the selected type of resonator will also reduce the energy spread that is acquired during the acceleration of the beam. Though, it has the unique advantage above the other types, that it does not reduce the orbit separation at the injection and extraction orbits, which are the most critical orbits in a cyclotron, both for determining the quality of the extracted beam and from an operational point of view. The combined effect of the reduction in energy spread and maintaining the

orbit separation at injection and extraction will ensure a beam of highly improved quality. Consequently it will be possible to accelerate beams of much higher intensity and with a reduced chance of unnecessary activation of accelerator components.

The choice of resonator type was determined, amongst others, by physical factors such as the available space in the existing vacuum chambers and access to components of the resonator for remote control, but the important considerations from an electromagnetic and beam dynamics point of view were the relative simplicity of the system and the fact that the orbit separation in the cyclotron is unaffected at injection and extraction.

The option of instituting an acceleration voltage that has the main and flat-topping voltages both superimposed on the main resonator, as in the case of the SPC1, was not used because this method is applied in cyclotrons only when the available space prevents the implementation of a dedicated separate resonator. The separated-sector magnets of the SSC and the existing vacuum chambers allow enough space to have a separate resonator that can be operated independently from the main resonators.

The single-gap, cavity-type flat-topping resonator [Bis79] is used in cyclotrons where the space between the sector magnets is very restricted, like in high energy machines where many sectors of spiraled contour shape is used. This type was ruled out for the SSC because the dimensions of such a resonator for the required resonance frequency implied the costly manufacturing of a new vacuum chamber for the cyclotron. Apart from the cost it will also require that the present accelerator schedule is halted for a few months, which is unacceptable. Its power consumption will also be relatively high if compared with double-gap resonators of similar design. The design of the selected type permitted the installation of the complete assembled resonator through one of the ports of the vacuum chamber. It was done during a normal service period and therefore did not affect the operational schedule at all.

The double-gap vertical half-wave type of resonator, which is often used in cyclotrons [Rog84], is very useful for its ability to be adjusted over a relative wide frequency range.

Though calculations showed that its established radial voltage distribution is very sensitive for the geometry of the electrodes and will require the building of a model with several iterations of geometric adjustments to ensure that the requirements are met. The manufacturing tolerances will be of a higher order than for other types. Such a resonator will also have to be assembled inside the vacuum chamber, because it will be too high to fit through the entrance port.

The horizontal quarter-wave flat-topping resonator is a type that could be considered, though it will reduce the orbit separation at extraction and have higher power consumption than the selected horizontal half-wave resonator.

The voltage distribution in our design has a simple half-wave sine structure with its nodes fixed at the radial extremities of the resonator. The near zero voltages in the regions of the most important orbits in the SSC, injection and extraction, are advantageous for the orbit separation and from an operational viewpoint. The set procedures of injecting and extracting the beam is not affected and therefore will not require any changes to the existing diagnostic equipment that are located on these orbits. The main disadvantage of the chosen type of resonator is that the beam width is not a minimum everywhere along a radial line in the cyclotron, due to the fact that the voltage distributions of the flat-topping and main resonators do not match. The beam may cross betatron resonances in the cyclotron and subsequently cause severe blow-up of the beam, which will be worse for a broad beam. However, in the case of the SSC at iThemba LABS, the beam width is a minimum when it crosses the $v_x = v_z$ resonance. The resonance frequency of our resonator is fixed for a specific energy only, but in principle it can be adapted to accommodate other resonance frequencies as well.

-o-O-o-

CHAPTER 9

SUMMARY AND CONCLUSIONS

The beam intensity of the 66 MeV proton beam, accelerated at an rf frequency of 16.373 MHz, was limited to 150 μA by beam losses at extraction in the separated-sector cyclotron. The maximum beam intensity that can be obtained from the injector cyclotron SPC1 is 320 μA , as was originally designed. At this intensity, in SPC1, the effect of longitudinal space-charge forces is noticeable and an increase in the internal beam intensity does not lead to an increase in the external beam current. Longer beam pulses and higher intensities, up to 600 μA , are now available from SPC1, since a flat-topping system has been installed in SPC1. To limit the energy spread acquired in the SSC, with the long beam pulses now available from SPC1, a flat-topping system for the SSC is required. Various types of resonators were considered and their respective effects on the beam were studied. This led to the selection of a horizontal half-wave resonator for the SSC. The choice was determined, amongst others, by physical factors such as the available space in the existing vacuum chambers and access to components of the resonator for remote control. Important considerations from an electromagnetic and beam dynamics point of view were the relative simplicity of the system and the fact that the orbit separation in the SSC is unaffected at injection and extraction. Extraction will be much easier, because of the reduced energy spread in the beam at extraction.

The upgrading of the SSC to provide higher beam intensities for the 66 MeV proton beam required the studying of the major factors that influence the quality and intensity of the beam. The flat-topping of dee voltage has already been well established at several accelerators around the world, though the selected type of resonator for the flat-topping of the SSC has never been implemented in a cyclotron before and therefore required a thorough design study.

The resonator operates at the third harmonic of the main rf, 49.12 MHz, to obtain maximum reduction of energy spread in the beam. The second harmonic could not be

used due to the size of the required resonator and the physical constraints of the available space. Beam bunches with a length of about 50 rf-degrees can now be accelerated for the same energy spread as before.

The improved beam characteristics that can be expected from the implementation of such a flat-topping system were also calculated with a spreadsheet program using the hard-edge formulation for the SSC. The energy spread at extraction will reduce from 0.54 MeV to 0.014 MeV for an initial beam bunch of 15 rf-degrees. The orbit separation at extraction will change from 8.5 mm to 21.3 mm. Even increasing the injection phase length to 30 rf degrees will have an energy spread of only 0.029 MeV and a similar beam separation at extraction. The additional beam width due to the energy spread will change from 13.4 mm to 0.34 mm and to 0.72 for the respective cases.

It was shown that with adjustments of flat-topping dee voltage phase, the first order effects of the longitudinal space-charge effect, that are of importance at high beam intensity beams can be compensated for.

The use of the transmission line method with a division of the resonator in segments again proved its worth in determining the approximate resonator characteristics. The computer program POISSON for 2d finite-element numerical field analysis was used to find the characteristic impedances of the segments by careful selection of boundary conditions on each segment to create curvilinear squares, from which the calculations were done. Since the resonator is a complex 3d structure, the characteristics of the resonator were calculated to a higher order of accuracy by use of a commercial computer program, SOPRANO, for numerical field analysis with finite elements. The versatility and accuracy of the latter method proved to be indispensable for detailed insight. Once a finite-element model of the resonator is created, changes to the dimensions and shape are relatively easy to implement, which permits a high turn-over of calculations. Simulating other known geometries and the comparison of the calculated and measured results for the half-scale resonator, confirmed both the accuracy of the mathematical codes used by the software and the discretisation strategy that was followed for the full-scale resonator.

A method was found to shift an inherent unwanted resonance in the resonator away from the desired frequency.

The distribution of the cooling pipes for the resonator was calculated.

The coupling and tuning loops as well as capacitors were studied and their characteristics calculated. In retrospect symmetrical coupling and tuning components with respect to the median plane should have been considered at an early stage in the design of the resonator or remotely adjustable tuning capacitors from outside the vacuum system should have been used to optimize the Q-value and minimize the power dissipation. Both these options would however have complicated the design and construction of the resonator.

The resonator was built, commissioned and successfully tested with beam.

With the completion of the second buncher for the transfer beam line and the high intensity target stations, iThemba LABS will be in a position to increase its production of radioisotopes. The better quality of the flat-topped 66 MeV proton beam can be utilized for the neutron therapy beam as well.

Beam intensities of 200 μA have been extracted from the SSC with the flat-topping system in operation at full power. The resonator operates according to expectations and is stable at full power. The coupling and tuning loops function without any problems. Further tests and optimization are required to obtain the expected beam intensity of about 500 μA .

Finally, it can now be stated that the first-ever horizontal half-wave flat-topping resonator has been successfully implemented in a cyclotron.

-o-O-o-

APPENDIX A

BASIC FLAT-TOPPING PRINCIPLES

A.1 THE HARMONIC NUMBERS OF A CYCLOTRON

In some cyclotrons the orbital frequency, f_p , of the ions is the same as the frequency, f_d , of the dee voltage. This is often, but not always, the case. For continuous beam delivery there must be a fixed relationship between the frequencies and orbital periods, in fact they have to be integer multiples of each other. The ratio between the rf frequency and the particle frequency is called the harmonic number, h :

$$h = f_d / f_p = t_p / t_d \quad h = 1, 2, 3, 4, \dots \quad (\text{A.1})$$

with t_d and t_p the rf and orbital periods, respectively.

The choice of a harmonic number depends not only on practical considerations such as the available frequency range of the RF system but also the shape of the dees, and more particularly the angles which the dees subtend at the center of the machine. All the harmonic numbers are not permissible for a given dee angle. For some harmonic numbers, so called dead harmonics, acceleration is not possible. Closely related to the harmonic number of a cyclotron is the number of dees and the phase difference, if any, between the dee voltages. Many combinations of these parameters are used. Only cyclotrons with two dees are discussed here. In this special case, which applies to the three cyclotrons at Faure, the following rules apply [Bot04b]:

For uneven harmonic numbers the dee voltages have to be 180° out of phase, since after half a turn, from the entrance to the first dee, the voltage on the second dee must have the same polarity as that which the first dee had when the ion entered it. Also with an uneven harmonic number an uneven number of RF half-periods have elapsed during the time interval that the ion moved from the entrance of the first dee to the entrance of the second

dee, with the consequence that the phase of the voltage on the second dee has to be 180° out of phase with the first for acceleration. This is illustrated in figure A1.

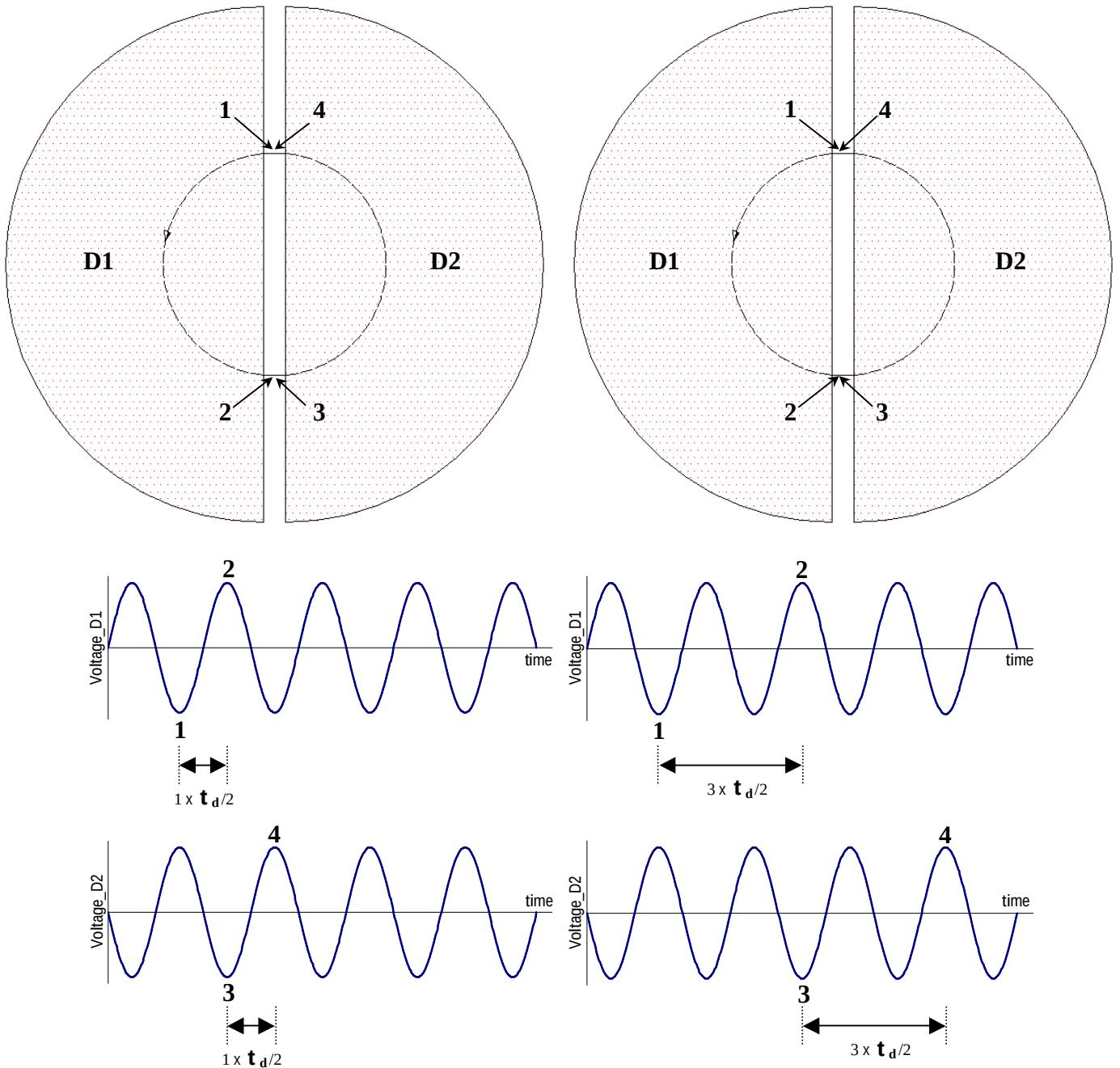
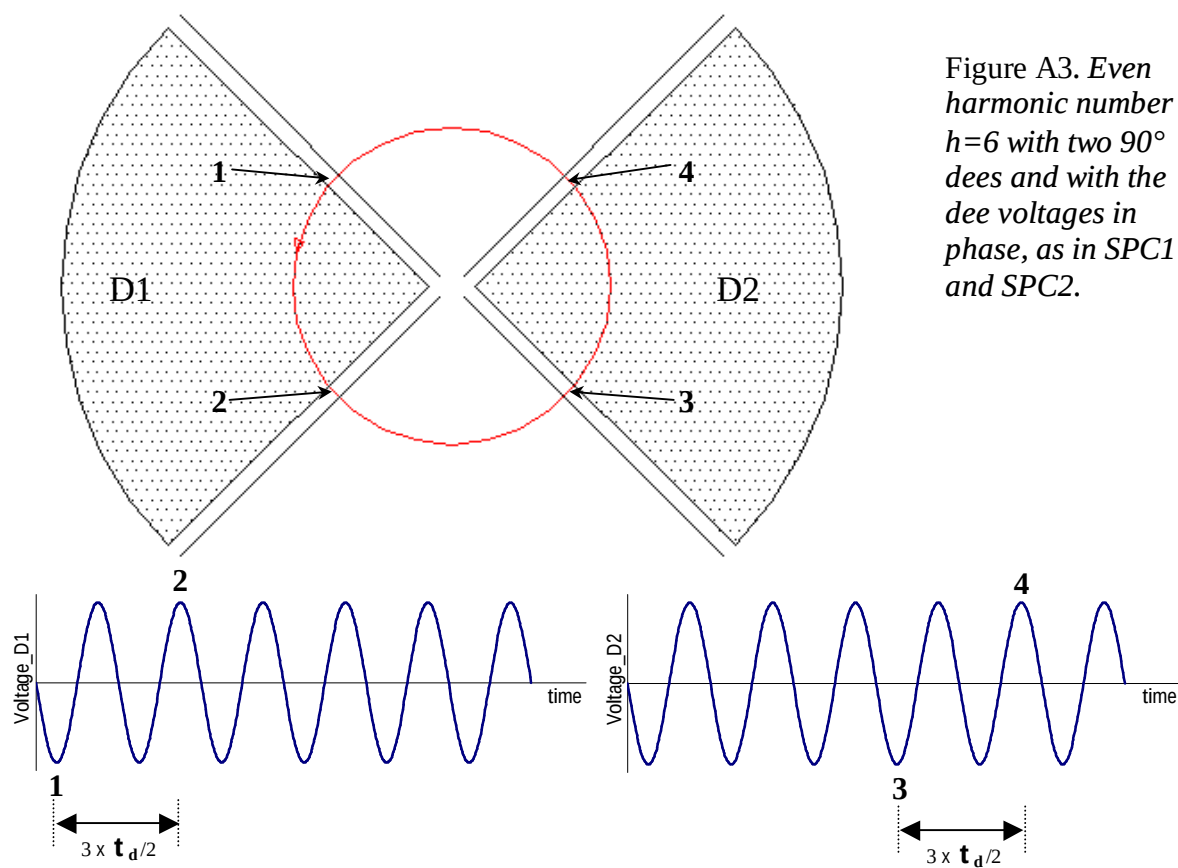
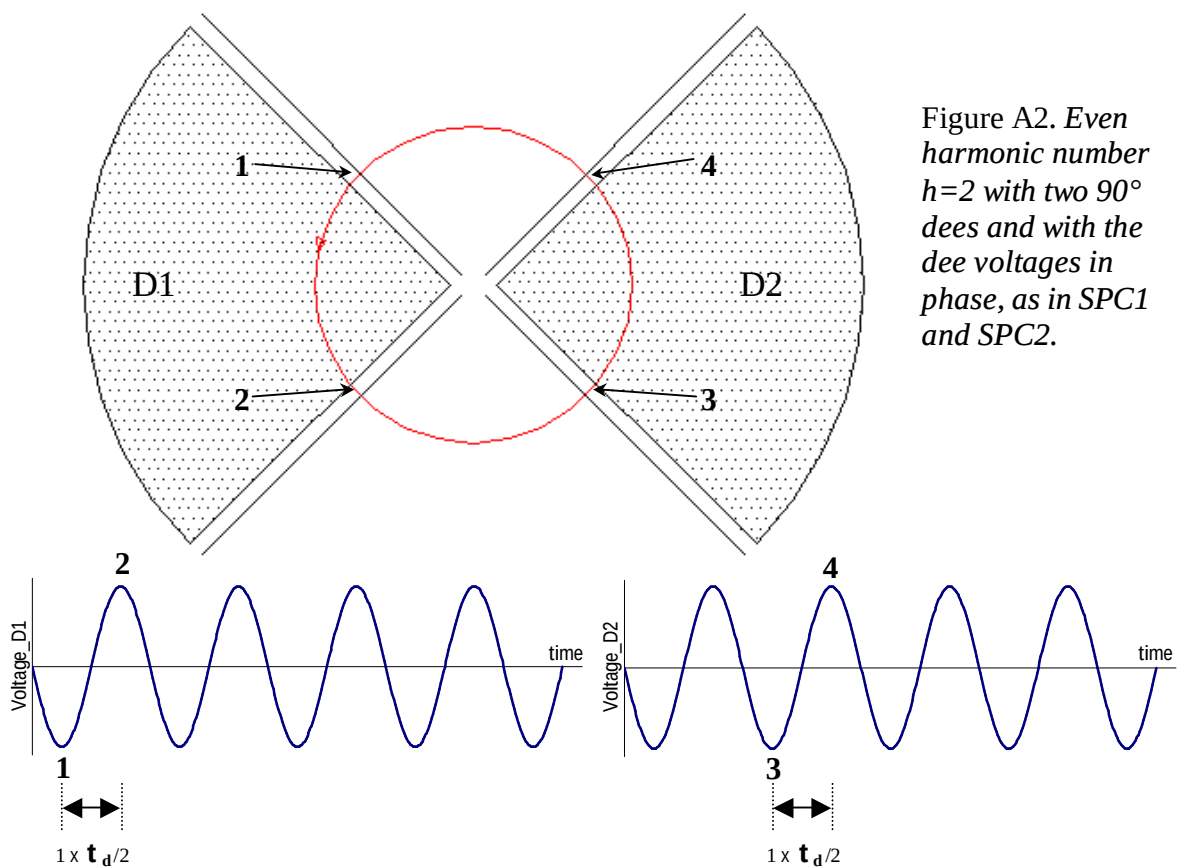


Figure A1. Uneven harmonic numbers $h=1$ (left) and $h=3$ (right) and 180° dees with the dee voltages 180° out of phase.

For **180° dees only uneven harmonic numbers** will lead to acceleration, because with an even harmonic number the dee voltage would have the same polarity when the ion leaves the dee as when it entered it since in this case a full number of RF periods have elapsed during the time that the particle traversed the dee. If an uneven harmonic is used, as it should be, an uneven number of half periods would have elapsed while the ion traversed the dee and the polarity of the dee voltage would be the opposite of what it was when the ion entered the dee. Acceleration therefore takes place as the particle leaves the first dee and enters the second dee. In summary: for acceleration an ion has to traverse a dee in an uneven number of RF half periods and it takes half an orbital period to move through a 180° dee. The harmonic number has therefore to be uneven.

For even harmonic numbers the dee voltages have to be in phase, since after half a turn, from the entrance to the first dee, the voltage on the second dee must have the same polarity as that which the first dee had when the ion entered it, but with an even harmonic number an even number of RF half-periods (a full number of RF periods) have elapsed during the time interval that the ion moved from the entrance of the first dee to the entrance of the second dee, with the consequence that the phase of the voltage on the second dee has to be the same as that of the first dee to be ready for acceleration. This is illustrated in figures A2 and A3.

For **90° dees only even harmonic numbers** will lead to acceleration, because with an uneven harmonic number (such as $h=1$ for instance) the dee voltage would have the same polarity when the ion leaves the dee as when it entered it, since in this case an uneven number of quarter RF periods have elapsed during the time that the ion traversed the dee. Though not all even harmonic numbers, like $h=4$, will result in acceleration.



If an even harmonic is used an uneven number of half periods has to elapse while the ion traverses the dee and the polarity of the dee voltage would now be the opposite of what it was when the ion entered the dee, for $h=2$ and $h=6$, but not for $h=4$. Acceleration therefore takes place as the particle leaves the first dee. In summary: for acceleration an ion has to traverse a dee in an uneven number of rf half periods and it takes a quarter of an orbital period to move through a 90° dee. The harmonic number has therefore to be at least two. A harmonic number of four means four quarter-periods or two full RF periods, for the ion inside the dee, and are therefore not suitable for acceleration. A harmonic number of six implies that the ions will spend six quarter RF periods, i.e. three half periods, inside the dee and that satisfies the condition for acceleration.

The dee shapes of the separated-sector cyclotron at Faure are shown in figure A4.

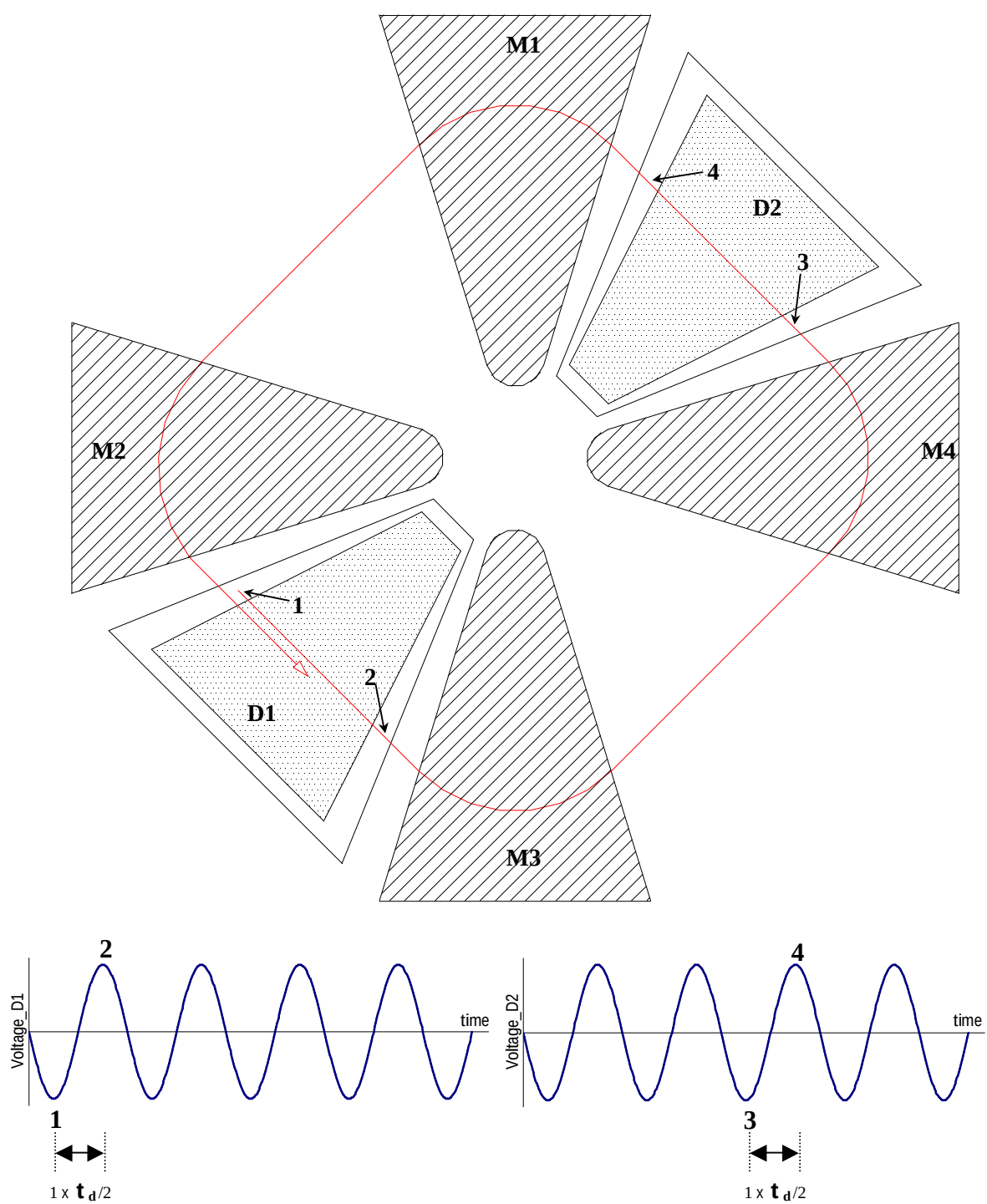


Figure A4. Acceleration in the separated-sector cyclotron with harmonic number 4.

A.2 BEAM QUALITY WITH A FLAT-TOPPING RESONATOR OPERATING AT THE 3rd HARMONIC

A.2.1 Harmonic number

For an orbital frequency f_p and harmonic number, h , with the main and flat-topping resonators operating at frequencies, f_m and f_n , respectively, the relation between these parameters are given by:

$$f_n = n f_m = nh f_p, \quad (\text{A.2})$$

with n the harmonic number of the flat-topping resonator.

A.2.2 Flat-topping voltage

A harmonic voltage of suitable amplitude and phase is added to the main accelerating voltage, which results in a waveform with a flat top. This is the flat-topped voltage that is used to accelerate the beam. The principle is illustrated by adding the 3rd harmonic voltage to the main accelerating voltage for one rf cycle as shown in figure A5.

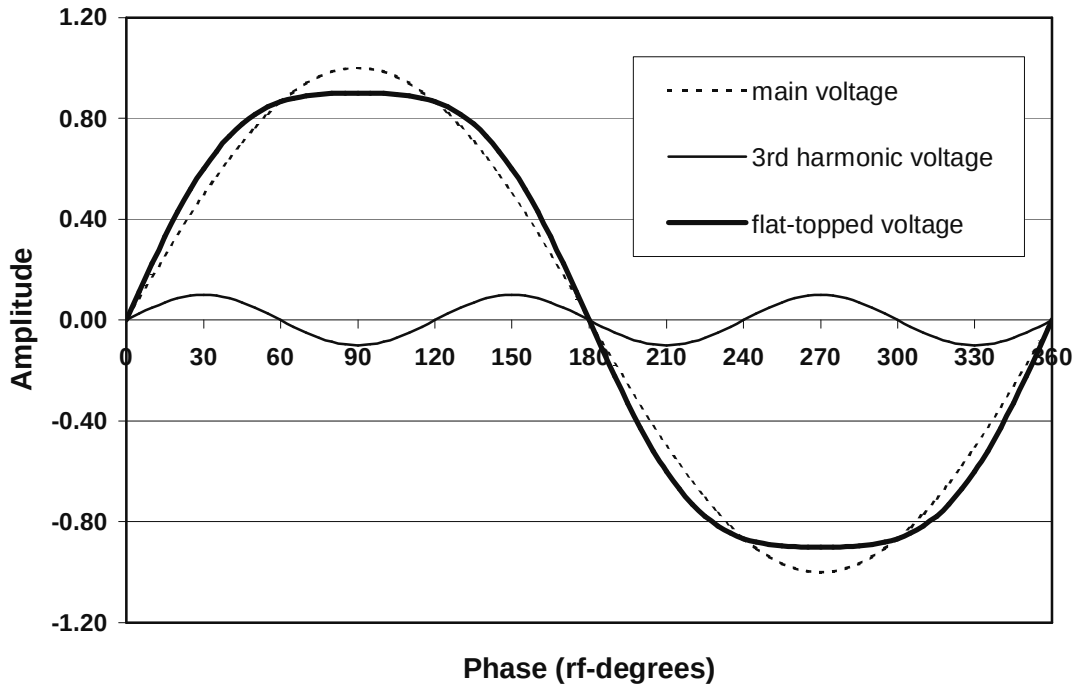


Figure A5. Addition of a 3rd harmonic voltage of suitable amplitude to one cycle of the main dee voltage, to obtain an effective flat-topped voltage for acceleration of the beam.

A.2.3 Energy gain per turn

The energy gained by a particle during one revolution at radius r in a cyclotron, is given by:

$$E_p = h Q V_p[r] = h Q k V_m[r] \cos q + h n Q h V_n[r] \cos(nq) \quad (\text{A.3})$$

with h = the harmonic number at which the cyclotron is operated,

n = the harmonic number of the flat-topping frequency,

Q = the charge state of the ion

$V_p[r]$ = total acceleration voltage per revolution,

$V_m[r]$ = amplitude of the main rf voltage,

$V_n[r]$ = amplitude of the flat-topping rf voltage,

q = phase angle between particle and main rf voltage

k = the number of acceleration gaps in the main resonator

h = the number of acceleration gaps in the flat-topping resonator.

A.2.4 Energy spread

The particles in a beam bunch do not traverse an acceleration gap in a cyclotron at the same instant and are therefore not accelerated at the same phase angle of the dee voltage. This leads to an energy spread in the beam, determined by the maximum difference in applied acceleration voltage to any two particles in the bunch. The flat-topping system will allow the use of longer beam bunches and reduce the energy spread in the beam for a given bunch length, as is illustrated in figures A6(a) and A6(b) for $n = 3$. The optimum

shape of the flat-topped wave is obtained when $\frac{d^2 V_p}{dq^2} = 0$ at $q = 0^\circ$, which implies that

$V_n[r] = -\frac{k}{hn^2} V_m[r]$. This reduced energy gain at the acceleration gap can be

compensated for by an increase in the main dee voltage or by allowing more turns in the cyclotron, i.e. more crossings of acceleration gaps.

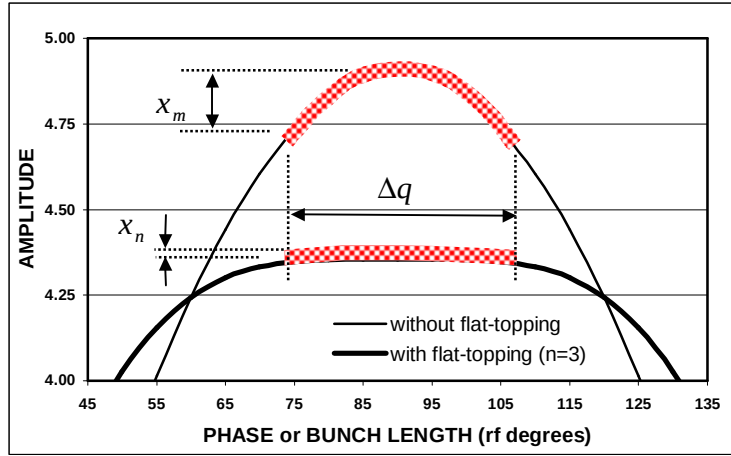


Figure A6(a). A beam bunch with a phase length of Δq rf-degrees is shown superimposed on the dee voltage, with and without flat-topping. The energy spread in the beam is determined by the maximum difference in the dee voltage at the acceleration gap, which is x_n and x_m , respectively, for the two conditions.

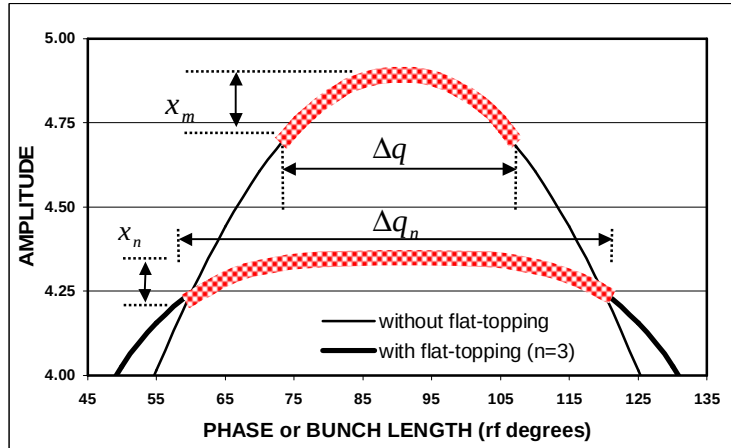


Figure A6(b). Higher beam intensities can be accelerated by using a longer phase length, Δq_n , with the flat-topping system and still have less energy spread than that obtained without flat-topping.

A.3 AZIMUTHAL ANGLE OF FLAT-TOPPING RESONATOR

The azimuthal angle, α , subtended between the center-lines of the two acceleration gaps, for a given particle energy, can be selected from a range of possible values to best suit the required harmonic frequency and the geometric and power specifications. The largest useful phase range for any given energy spread can be obtained with a harmonic frequency of double the main frequency [Joh68a], but the angle and radial length available for the resonator also determine the harmonic number that can be used.

The azimuthal position of a resonator has to be such that the center-lines of the two acceleration gaps are symmetric w.r.t. a radial line from the center of the cyclotron, as is schematically shown in figure A7.

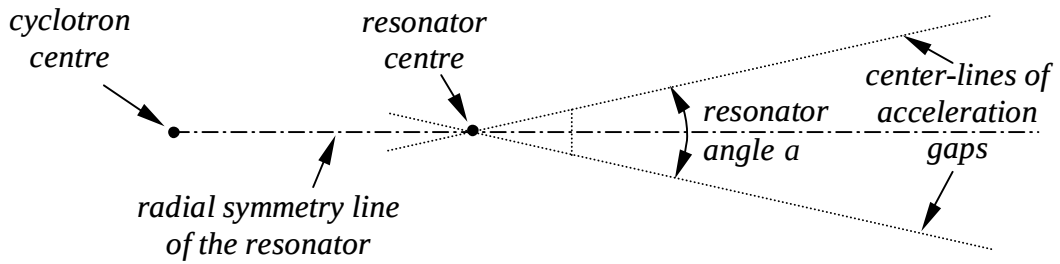


Figure A7. Alignment of the flat-topping resonator.

The flat-topping voltage changes phase during the time in which a particle traverses the double-gap resonator with an angle of α subtended between its two acceleration gaps. The time the particle takes to traverse the flat-topping resonator, from the center of the first to the center of the last acceleration gap, must be equal to an uneven number of half-periods of the harmonic frequency, in order to ensure in-phase crossing of the particles at the peak value of the voltage. If the time is equal to an even number of half-periods between the acceleration gaps will result in no acceleration. The azimuthal path length, L , between the centers of the entrance and exit gaps, over which the uneven number of half-periods of the flat-topping resonator voltage elapses during the crossing of a particle, therefore has to be:

$$L = \frac{(2m-1)v}{2f_n}, \quad m = 1, 2, 3, \dots, \quad (\text{A.4})$$

with v the speed of the particle on the orbit.

The azimuthal angle of the resonator is:

$$\begin{aligned}
 a &= 2 \arctan \left(\frac{(2m-1)(L_e - L_i)}{2(r_e - r_i)} \right), \\
 \text{or } a &= 2 \arctan \left(\frac{(2m-1)L_e}{2(r_e - r_f)} \right), \\
 \text{or } a &= 2 \arctan \left(\frac{(2m-1)L_i}{2(r_i - r_f)} \right), \tag{A.5}
 \end{aligned}$$

with L_e and L_i the path lengths inside the resonator, and r_e and r_i orbital radii on the center line of the resonator, at extraction and injection, respectively, and r_f the radial position of the intersection of the center-lines of the acceleration gaps, as shown in figures A7 and A8.

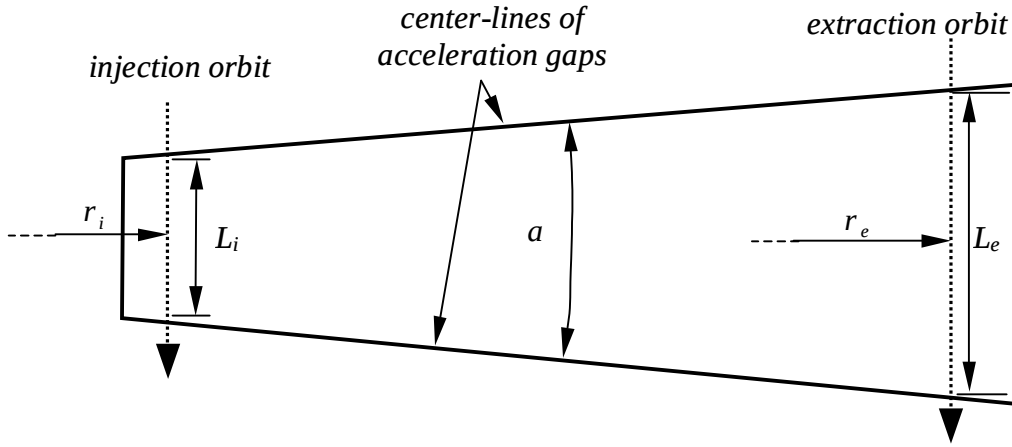


Figure A8. The resonator angle for a given harmonic number can be determined from the orbital parameters and the particle speed on the two indicated orbits.

The calculated azimuthal angles of such a flat-topping resonator for different harmonic numbers and the uneven number of half-periods that are associated with the distance traveled by the particle through the resonator, are shown in figure A9.

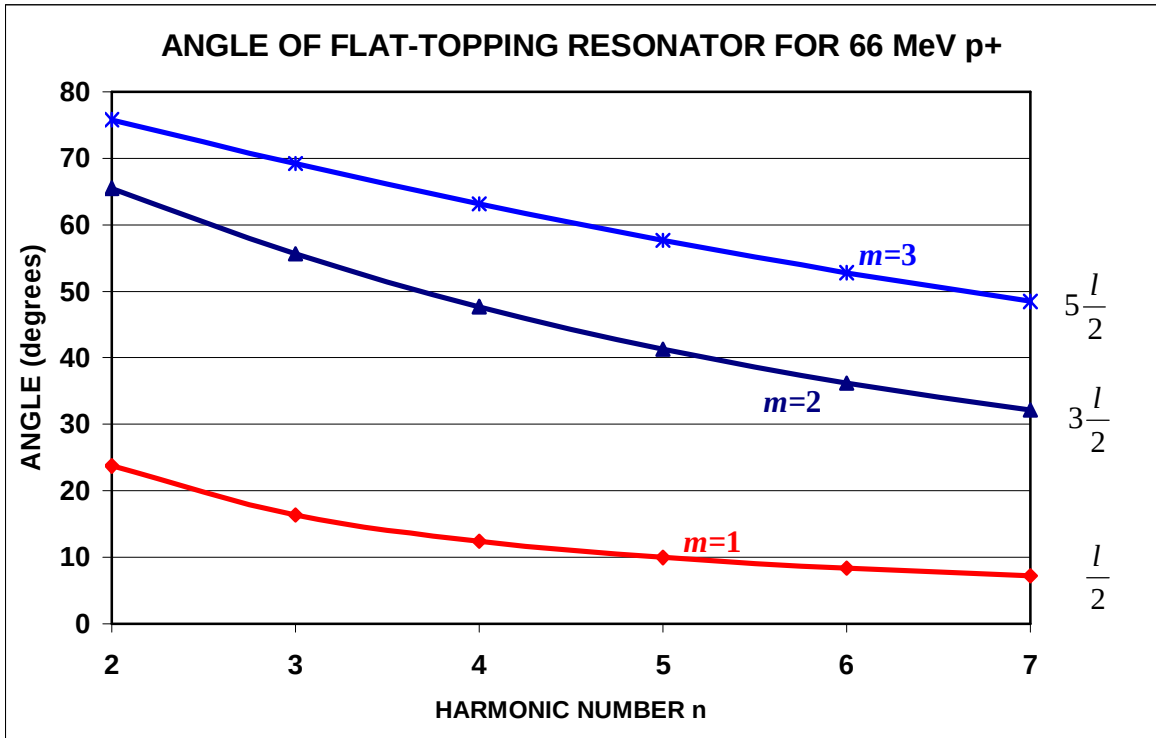


Figure A9. The calculated azimuthal angles of such a flat-topping resonator for different harmonic numbers of the fundamental frequency and different numbers of half-periods that are associated with the distance traveled by the particle through the resonator for acceleration.

APPENDIX B

HARD-EDGE FORMULAE FOR THE SEPARATED-SECTOR CYCLOTRON AT iTHEMBA LABS

The definitions of geometrical parameters used in the hard-edge formulae that apply to a single orbit in a separated-sector cyclotron with a magnet sector-angle of $2a$, are shown figure B1, with the center of the cyclotron at O [Bot88]. The figure represents one half of one sector and one half valley of the cyclotron. Particle M moves along an orbit path VDK , with $\overline{VD}$ the linear path in the valley region and arc DK the circular path in the sector magnet. The distance of the particle from the cyclotron center is R inside the valley region, varying from the shortest distance, L in the middle of the valley to OD at the edge of the magnet sector. The radius of curvature in the magnet is r and the maximum distance from the cyclotron centre, $\overline{OK}$, is at the middle of the sector magnet and is also called the hill radius R_H .

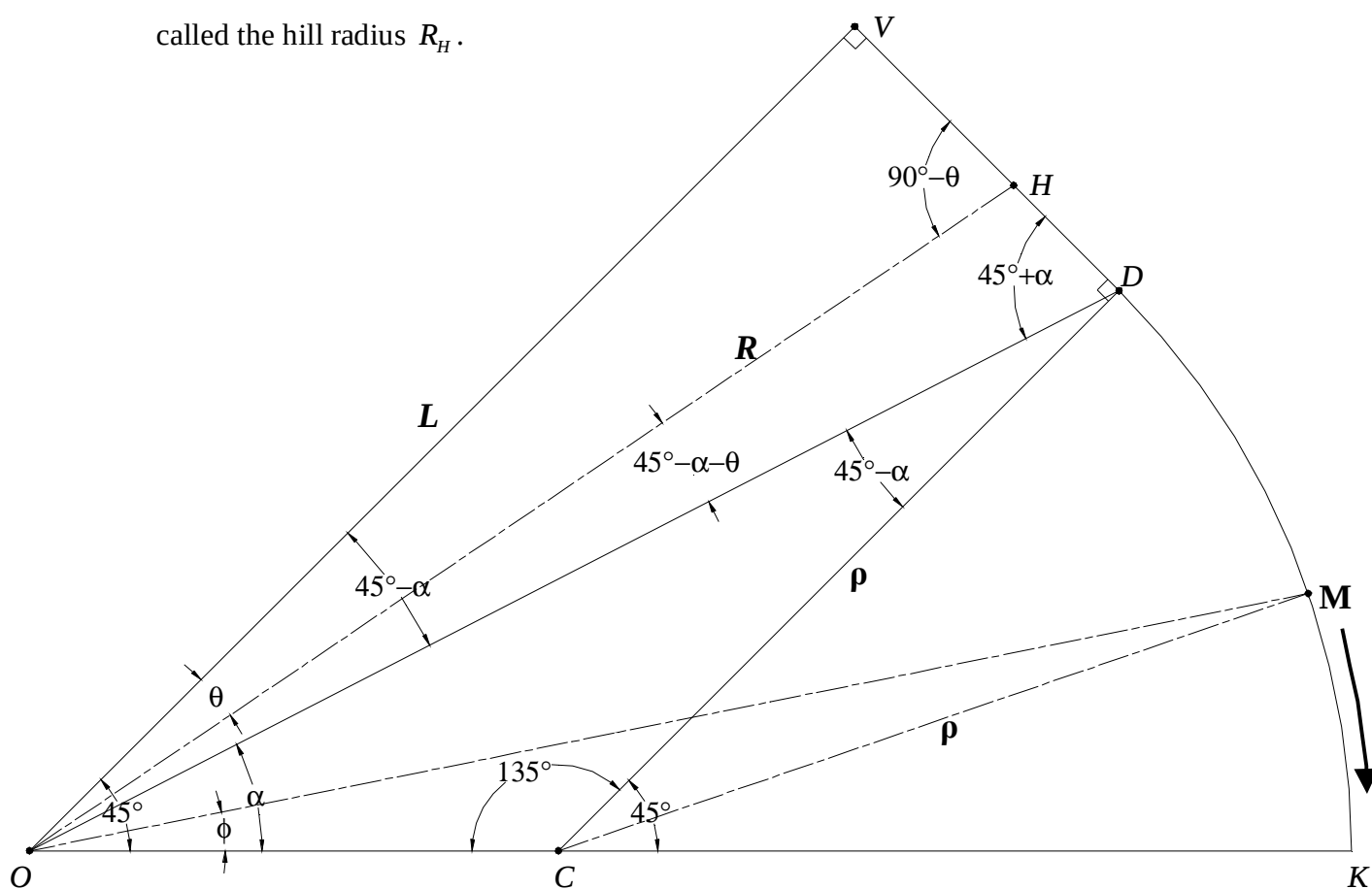


Figure B1. *The geometric parameters in the SSC used for deriving the hard-edge formulae.*

The following relations can now be derived:

$$\frac{VD}{L} = \tan(45^\circ - a), \quad (\text{B.1})$$

$$\frac{L}{OD} = \cos(45^\circ - a), \quad (\text{B.2})$$

$$\frac{\sin a}{r} = \frac{\sin 135^\circ}{OD}, \quad (\text{B.3})$$

$$\frac{\sin(45^\circ - a)}{OC} = \frac{\sin a}{r}, \quad (\text{B.4})$$

$$R = OH = \left(\frac{L}{\cos q} \right). \quad (\text{B.5})$$

The hill radius, $R_H = OC + r$, can also be expressed as:

$$R_H = r \left(\frac{\sin(45^\circ - a)}{\sin a} \right) + r \quad (\text{B.6})$$

$$= r \left(\frac{\sin(45^\circ - a)}{\sin a} + 1 \right) \quad (\text{B.7})$$

$$= OC \left(\frac{\sin a}{\sin(45^\circ - a)} \right) \left(\frac{\sin(45^\circ - a)}{\sin a} + 1 \right) \quad (\text{B.8})$$

$$= OC \left(1 + \frac{\sin a}{\sin(45^\circ - a)} \right). \quad (\text{B.9})$$

$$R_H = OD \left(\frac{\sin a}{\sin 135^\circ} \right) \left(1 + \frac{\sin(45^\circ - a)}{\sin a} \right). \quad (\text{B.10})$$

$$\begin{aligned}
R_H &= L \left(\frac{\sin a}{\cos(45^\circ - a) \cdot \sin 135^\circ} \right) \left(1 + \frac{\sin(45^\circ - a)}{\sin a} \right) \\
&= L \left(\frac{\sin a + \sin(45^\circ - a)}{\cos(45^\circ - a) \cdot \sin 135^\circ} \right). \tag{B.11}
\end{aligned}$$

The radius of curvature in the sector magnet is given by:

$$r = OC \left(\frac{\sin a}{\sin(45^\circ - a)} \right), \tag{B.12}$$

$$r = OD \left(\frac{\sin a}{\sin 135^\circ} \right), \tag{B.13}$$

$$r = L \left(\frac{\sin a}{\cos(45^\circ - a) \cdot \sin 135^\circ} \right), \tag{B.14}$$

$$r = R_H \left(\frac{\sin a}{\cos(45^\circ - a) + \sin a} \right). \tag{B.15}$$

The distance from the center of the cyclotron to the center of radius of curvature is given by:

$$OC = r \left(\frac{\sin(45^\circ - a)}{\sin a} \right), \tag{B.16}$$

$$= OD \left(\frac{\sin(45^\circ - a)}{\sin 135^\circ} \right), \tag{B.17}$$

$$= L \left(\frac{\sin(45^\circ - a)}{\cos(45^\circ - a) \cdot \sin 135^\circ} \right), \tag{B.18}$$

$$= R_H \left(\frac{\sin(45^\circ - a)}{\sin(45^\circ - a) + \sin a} \right). \tag{B.19}$$

The radial distance from the center of the cyclotron to the orbit at the edge of the sector magnet is given by:

$$OD = r \left(\frac{\sin 135^\circ}{\sin a} \right), \quad (\text{B.20})$$

$$= OC \left(\frac{\sin a}{\sin(45^\circ - a)} \right), \quad (\text{B.21})$$

$$= L \left(\frac{1}{\cos(45^\circ - a)} \right), \quad (\text{B.22})$$

$$= R_H \left(\frac{\sin a}{\sin(45^\circ - a) + \sin a} \right) \left(\frac{\sin 135^\circ}{\sin a} \right), \quad (\text{B.23})$$

$$= R_H \left(\frac{\sin 135^\circ}{\sin(45^\circ - a) + \sin a} \right). \quad (\text{B.24})$$

The minimum distance from the cyclotron center, the valley ‘radius’, L , is:

$$L = R_H \left(\frac{\cos(45^\circ - a) \cdot \sin 135^\circ}{\sin a + \sin(45^\circ - a)} \right), \quad (\text{B.25})$$

$$L = r \left(\frac{\cos(45^\circ - a) \cdot \sin 135^\circ}{\sin a} \right). \quad (\text{B.26})$$

The total length of an orbit is given by:

$$\begin{aligned}
L_B &= 8(arcDK + VD) \\
&= 8 \left(r \frac{p}{4} + L \tan(45^\circ - a) \right) \\
&= 2p \left(\frac{\sin a}{\cos(45^\circ - a) \cdot \sin 135^\circ} \right) + 8L \tan(45^\circ - a). \quad (B.27)
\end{aligned}$$

Constant ratios apply to several of these parameters. In the SSC, with a sector angle of $2a = 34^\circ$, the hill-radius ratios are given by:

$$\frac{R_H}{r} = 2.6057, \quad (B.28)$$

$$\frac{R_H}{OC} = 1.6228, \quad (B.29)$$

$$\frac{R_H}{L} = 1.2202, \quad (B.30)$$

the radius-of-curvature ratios are:

$$\frac{r}{OC} = 0.6228, \quad (B.31)$$

$$\frac{r}{OD} = 0.4135, \quad (B.32)$$

$$\frac{r}{L} = 0.4683, \quad (B.33)$$

$$\frac{r}{R_H} = 0.3838 \quad (B.34)$$

and the length of an orbit is given by:

$$\begin{aligned} L_B &= 7.196 L \\ &= 5.8974 R_H . \end{aligned} \quad (\text{B.35})$$

In a cyclotron with harmonic number h and frequency f_{rf} of the main resonator, the kinetic energy T of a particle with rest energy E_0 is obtained from:

$$\text{the velocity of the particle is } v = \frac{7.196 L f_{rf}}{h}, \quad (\text{B.36})$$

$$\text{the relativistic velocity is } b = \frac{v}{c} = \frac{7.196 L f_{rf}}{hc}. \quad (\text{B.37})$$

The kinetic energy of the particle is given by:

$$\begin{aligned} T &= E_0 \left[\left(1 - b^2 \right)^{-\frac{1}{2}} - 1 \right] \\ &= E_0 \left[\left(1 - \left(\frac{7.196 L f_{rf}}{hc} \right)^2 \right)^{-\frac{1}{2}} - 1 \right] \\ &= E_0 \left[\left(1 - \left(\frac{L_B f_{rf}}{hc} \right)^2 \right)^{-\frac{1}{2}} - 1 \right]. \end{aligned} \quad (\text{B.38})$$

The length of any orbit is:

$$L_B = \frac{hc}{f} \left(1 - \left(\frac{E_0}{E_0 + T} \right)^2 \right)^{-\frac{1}{2}}. \quad (\text{B.39})$$

The injection and extraction orbit lengths, L_i and L_e , at respective proton energies of 3.14 and 66 MeV in the SSC, operating at 16.3736 MHz and with harmonic number 4, are:

$$\begin{aligned} L_i &= 5.9765 \text{ m} \\ L_e &= 26.1113 \text{ m}. \end{aligned} \quad (\text{B.40})$$

-o-O-o-

APPENDIX C

A SPREADSHEET PROGRAM TO CALCULATE THE ENERGY SPREAD AND RADIAL BEAM WIDTH OF THE BEAM IN THE SSC, USING THE HARD-EDGE FORMULATION.

C.1 INTRODUCTION

Acceleration with a sinusoidal accelerating voltage in a cyclotron introduces energy spread into the beam and consequent radial broadening of the beam. The orbit separation at extraction is therefore decreased and the beam quality deteriorates. It is essential to have these turns separated at extraction to ensure a beam of good quality to the user and to prevent activation and possible overheating and damage to the extraction components. The pulse selector in the SSC also only works with single-turn extraction. The beam and orbit separation and the radial broadening are schematically illustrated in figure C1. The area in grey represents the intrinsic beam width, due to its finite emittance, and the shaded area the tail of beam, that developed due to the energy spread in a beam bunch.

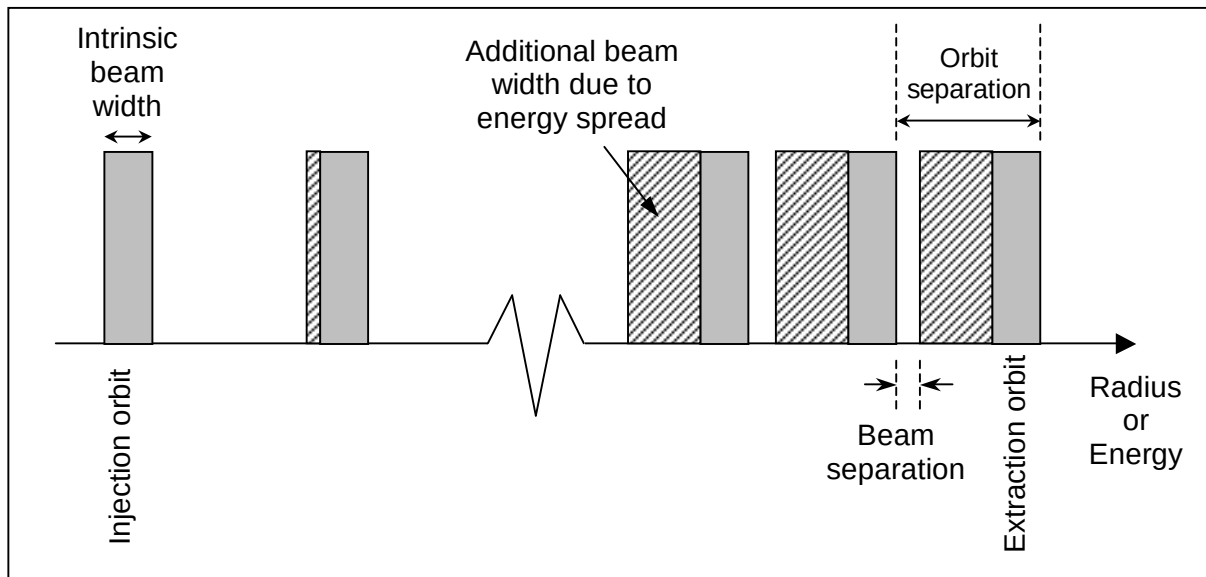


Figure C1. A schematic representation of the orbit separation, which is the distance between two successive orbits; the beam separation, which is the gap between two orbits and the beam width due to the energy spread, that is added to the intrinsic beam width.

The energy spread in the beam and the resulting increase in the beam in the SSC, with and without a 3rd harmonic double gap horizontal half-wave flat-topping resonator, was calculated by implementing the hard-edge values of the cyclotron in a computer program. These calculations were done at the azimuthal position in the middle of a valley and scaled to obtain the corresponding values at the middle of the sector magnet (hill). In the model it is assumed that the magnetic field is isochronous. The bunch of particles is represented by three particles: the central one, which is accelerated at the peak of the main dee voltage, accompanied by one on either side of the peak of the dee voltage.

A beam of zero initial inherent width and typical injection conditions for a final energy of 66 MeV protons are used as the initial parameters to begin the calculations on the injection orbit. The results for each orbit are used as the starting conditions to calculate values for the next orbit and the process is repeated until the extraction orbit is reached. Calculation of the change in energy spread and accompanying beam width as the beam progresses from injection to extraction, requires that the energy of each of the three particles are calculated for every orbit.

The program also provides means to calculate the effect of a positional error in placement of the resonator position on the injection orbit.

C.2 THE EXPRESSIONS USED FOR THE CALCULATION

The gain in kinetic energy per turn with a double-gap flat-topping resonator, operating at the n^{th} harmonic number of the main resonator frequency, with 4 acceleration gaps, is given by:

$$\Delta T = 4QV_d \cos(f + \Delta q) - 2QV_f \cos(nj + n\Delta q) \quad (\text{C.1})$$

with V_d the main dee voltage, V_f the flat-topping dee voltage, Q the charge on the particle, f the phase angle of the beam w.r.t. the main dees, j the phase angle of the beam w.r.t. the phase of the flat-topping dee voltage and Dq the phase length of the

beam bunch. All phase angles are in terms of main dee voltage rf-degrees. Without a flat-topping system the second term becomes zero.

The energy spread in a cyclotron without flat-topping is given by:

$$\frac{\Delta E}{E} = 1 - \cos \frac{\Delta q}{2}, \quad (\text{C.2})$$

with E the energy of a particle at the center of the bunch. The energy spread in a cyclotron with flat-topping using the n^{th} harmonic of the main frequency, can be calculated from:

$$\frac{\Delta E}{E} = \left[\frac{n^2}{n^2 - 1} \right] \left[1 - \frac{1}{n^2} - \cos \frac{\Delta q}{2} + \frac{1}{n^2} \cos \left(n \frac{\Delta q}{2} \right) \right]. \quad (\text{C.3})$$

The increase in the radial beam width due to the energy spread, ΔR , on a specific orbit in the cyclotron is directly proportional to the energy spread and is given by:

$$\Delta R = \frac{R}{g(g+1)} \frac{\Delta E}{E}, \quad (\text{C.4})$$

with R the radius of that orbit and γ the ratio of total energy to the rest energy. In the non-relativistic case the increase in radial beam width becomes:

$$\Delta R = \frac{R \Delta E}{2E}. \quad (\text{C.5})$$

The orbit separation is calculated as the distance between the radii of the central particle on two adjacent orbits. The radial position of a particle on any orbit in a cyclotron is not constant and has its maximum at the azimuthal center of each sector magnet and the minimum radial position at azimuthal center of the valleys between the sector magnets. The radial position and the beam width, due to the energy spread in the beam, do not only change from orbit to orbit, but also along an orbit. With flat-topping the orbit separation will in general decrease, but the decrease in the radial width is much larger.

The beam separation between any two successive turns in a cyclotron is mainly of importance at the injection and extraction orbits in a cyclotron where the septa of the injection and extraction components are inserted between adjacent orbits.

C.3 THE INPUT PARAMETERS FOR THE PROGRAM

The input parameters are given in table C.1(a-f) below for the respective groups of particle, cyclotron geometry, main rf, flat-topping rf, beam bunch size and positional error. Some obvious parameters like the universal constants (e.g. c = speed of light) are not shown.

Table C.1a : PARTICLE proton			
Charge	Q	1.6×10^{-19}	C
Mass	M_p	1.67262×10^{-27}	kg
Rest energy	E_0	938.27234	MeV
Kinetic energy at injection	E_i	3.1484	MeV
Kinetic energy at extraction	E_e	66.0	MeV
Beta at injection	b_i	8.17×10^{-2}	
Beta at extraction	b_e	3.57×10^{-1}	

Table C.1b : SSC			
Harmonic number	h	4	
Magnet sector angle	a	34	deg
Valley radius injection orbit	R_{vi}	0.852	m
Valley radius extraction orbit	R_{ve}	3.620	m
Length of orbit / valley radius	L_{orb}/R_v	7.196	
Radius hill / radius valley	R_h/R_v	1.220	

Table C.1c : Main RF			
Frequency of the rf	f_{rf}	16.3736	MHz
Frequency of the particle	f_d	4.093	MHz
Main dee voltage at injection	V_{di}	180000	V
Main dee voltage at extraction	V_{de}	220000	V
Phase difference to the beam at injection	f_i	0	Rf rad
Phase difference to the beam at extraction	ϕ_e	0	Rf rad

Table C.1d : Flat topping RF			
Frequency of flat topping rf	f_f	49.1	MHz
Peak voltage (use sinusoidal distribution)	V_f	0 or 72.3	kV
Phase difference to beam at extraction	φ_e	0	Rf rad
Phase difference to beam at injection	φ_i	0	Rf rad

Table C.1e : Beam Bunch			
Phase of the lagging particle with respect to the main dee phase	θ_r	-Y	Rf rad
Phase of the central particle with respect to the main dee phase	θ_c	0	Rf rad
Phase of the leading particle with respect to the main dee phase	θ_l	+X	Rf rad

Table C.1f : Positional Error			
Positional error on injection orbit	A_{err}	0	mm

C.4 THE CALCULATION PROCEDURE

The three particles that represent the particle bunch will each gain an amount of energy as it progresses from orbit k to orbit $k+1$. The following equations are applicable to the “central” particle only, but calculations for the other two are done with the appropriate substitution of their phases and positional parameters.

A linear dee gradient in the dee voltage is assumed, using the selected voltage values at the injection and extraction orbit radii. The main dee voltage at the k -th orbit linearly scaled with the radial position is given by:

$$V_{d_k} = V_{d_i} + \left(\frac{V_{d_e} - V_{d_i}}{R_{V_e} - R_{V_i}} \right) (R_{V_k} - R_{V_i}) \quad (C.6)$$

and, similarly, to include a phase history of the particles, the phase of the main rf at the k -th orbit is given by:

$$f_k = f_i + (R_{V_k} - R_{V_i}) \left(\frac{f_e - f_i}{R_{V_e} - R_{V_i}} \right) \quad (C.7)$$

The flat-topping voltage, which is zero at the injection and extraction orbits, varies sinusoidally with radius in between, and is given by:

$$V_{f_k} = V_f \sin \left(p \frac{R_{V_k} - R_{V_i}}{R_{V_e} - R_{V_i}} \right) \quad (C.8)$$

for the k -th orbit and the phase of the flat-topping resonator w.r.t. the phase of the beam is:

$$j_k = j_i + (R_{V_k} - R_{V_i}) \left(\frac{j_e - j_i}{R_{V_e} - R_{V_i}} \right) \quad (C.9)$$

The kinetic energy, E , of the central particle at orbit $k+1$ is then given by:

$$T_{k+1} = T_k + 4QV_{d_k} \cos(f_k + q_c) - 2QV_{f_k} \cos(3j_k + 3q_c). \quad (\text{C.10})$$

The beam width is calculated from the radial positions of the three representative particles in the bunch and the orbit and beam separations are calculated from the particle positions at two adjacent turns. The beam width is the maximum difference in radial position of any two of the three particles. The orbit separation is the radial distance between the central particles of two successive turns, and the beam separation is the radial distance between the particle at the largest radius and the particle on the next turn at the smallest radius, all at the same azimuthal position.

The energy spread, the additional beam width due to the energy spread, as well as the orbit and beam separations can now be calculated at the center of the valley for every orbit from injection to extraction. Combinations of voltages and bunch lengths can be used to calculate and compare the beam characteristics, with and without flat-topping.

C.5 EXAMPLE

The results for an orbital radius of 3.62 m in the middle of the valley for phase lengths of 15 and 30 rf-degrees, are summarized in table C.2. The complete sets of variables that were used in the calculation are listed in tables C.1(a-f).

Table C.2: Calculated energy spread and beam widths with and without flat-topping using the hard-edge formulae. The variables that were used in the calculation are listed in tables C.1(a-f).

Flat-topping voltage (kV)	AT INJECTION			AT EXTRACTION				Figure that shows the calculated values for all orbits
	PHASE of PARTICLE with respect to the MAIN RF (rf-degrees)			Energy Spread (MeV)	Additional Beam Width (mm)	Orbit separation (mm)	Beam Separation (mm)	
	q_{lag}	q_{cent}	q_{lead}					
0	-7.5	0	7.5	0.54	13.4	21.9	8.5	C.2a, C.3a
72.3	-7.5	0	7.5	0.014	0.34	21.8	21.3	C.2b, C.3b
72.3	-15	0	15	0.029	0.72	21.8	21.1	C.2c, C.3c

The energy spread at extraction decreases significantly from 0.54 MeV to 0.014 MeV for a bunch length of 15 rf-degrees with flat-topping. Even for a bunch length of 30 rf-degrees, the energy spread is only 0.029 MeV with flat-topping. The additional beam width due to the energy spread also reduces significantly from about 13 mm to a fraction of a millimeter for both beam bunch lengths of 15 and 30 rf-degrees. The orbit separation remains practically unchanged, but the beam separation increases from 8.5 mm to about 21 mm with flat-topping.

The calculated energy spread and additional beam width due to the energy spread, for the same initial conditions of flat-topping voltages and particle phases as above, are shown in figures C2(a-c) on the center line of a valley. The latter is also shown on the center-line of a hill. Similarly the energy spread is shown together with the beam separation in figures C3(a-c). In all these cases the energy difference between the central and lagging particle is exactly the same as between that of the central and leading particle.

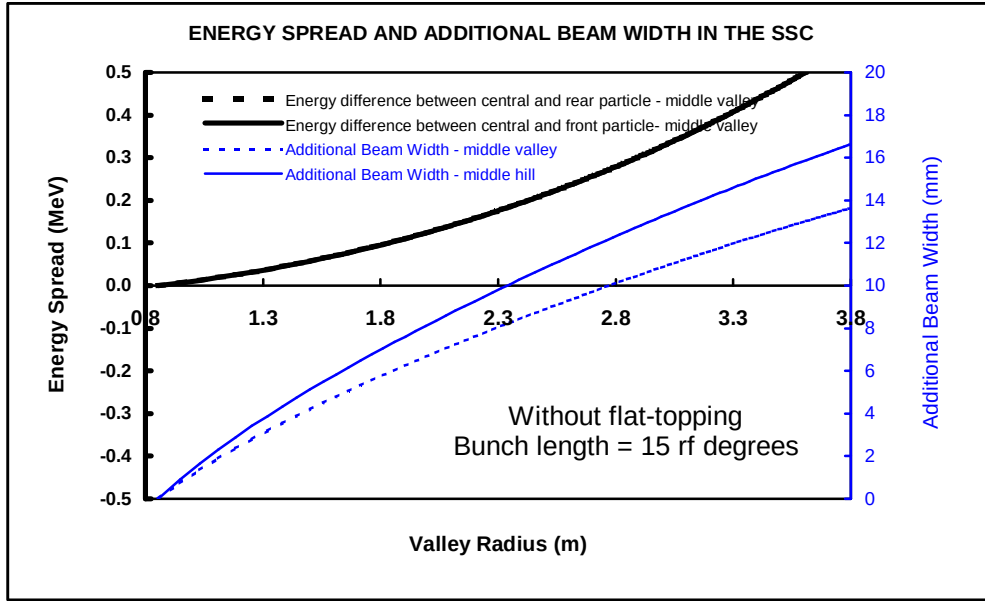


Figure C2(a). Calculated energy spread and additional beam width, due to the energy spread, along a radial line in the SSC for a 66 MeV proton beam with a beam bunch of 15 rf degrees, without flat-topping, at the azimuthal position in the middle of a valley, using the hard-edge formalism and assuming that the magnetic field is isochronous. An extraction radius of 3.62 m was used in the calculations.

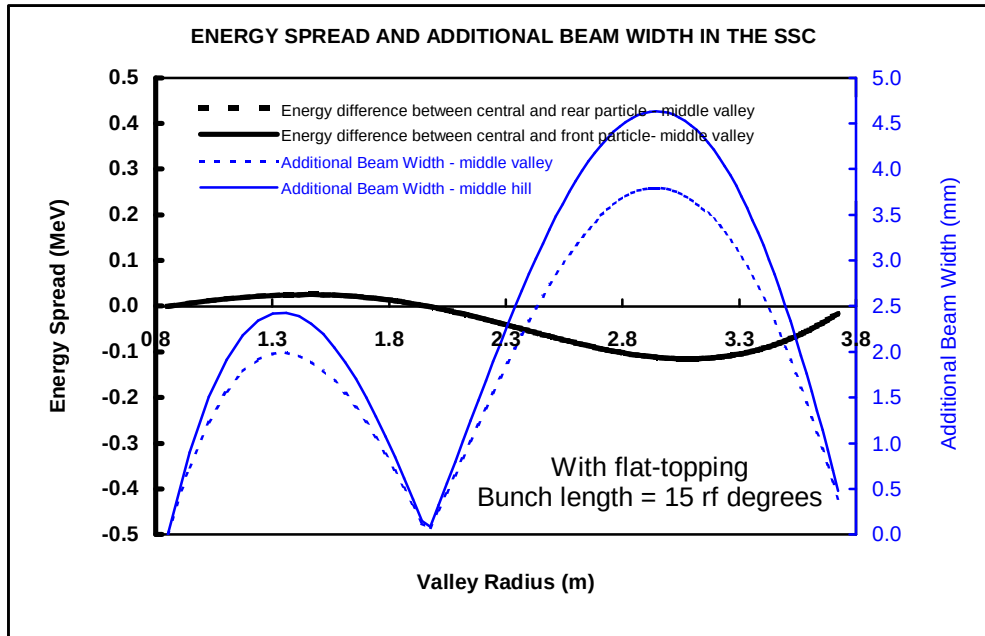


Figure C2(b). The same calculated parameters and conditions as in figure C2(a) but for a flat-topping voltage of 72.3 kV.

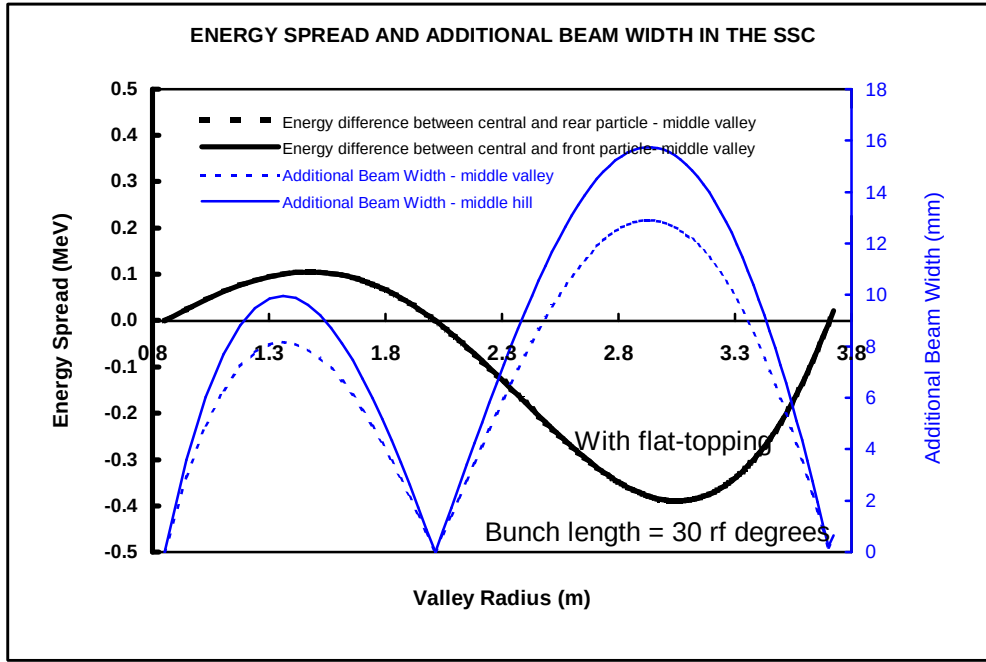


Figure C2(c). The same calculated parameters and conditions as in figure C2(b) but for a much larger bunch length of 30 rf degrees, with still an acceptable energy spread and beam width.

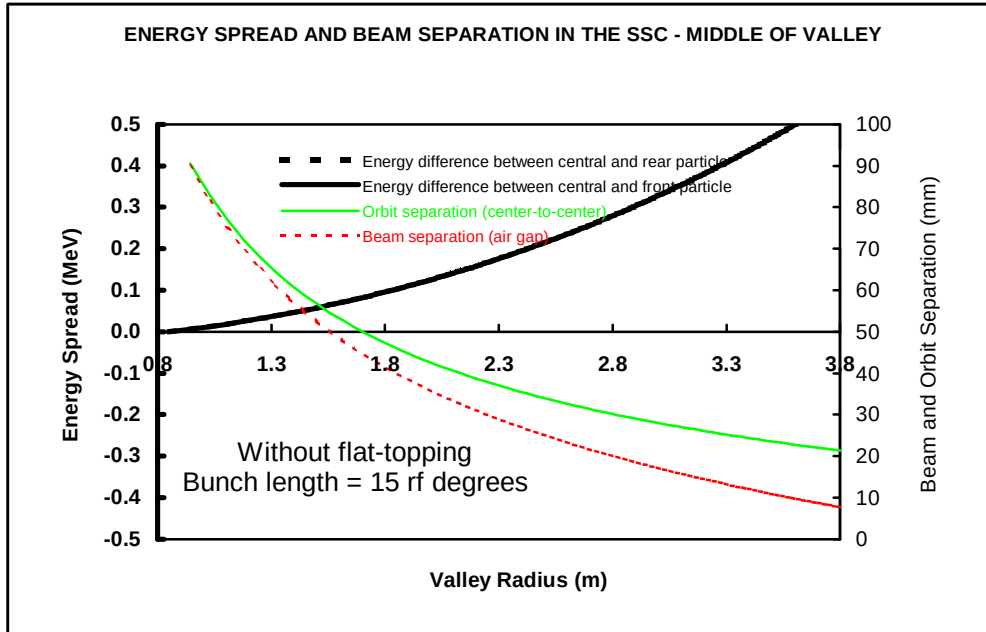


Figure C3(a). Calculated energy spread and beam and orbit separation, due to the energy spread, along a radial line in the SSC for a 66 MeV proton beam with a beam bunch of 15 rf degrees, without flat-topping, at the azimuthal position in the middle of a valley, using the hard-edge formalism and assuming that the magnetic field is isochronous. An extraction radius of 3.62 m was used in the calculations.

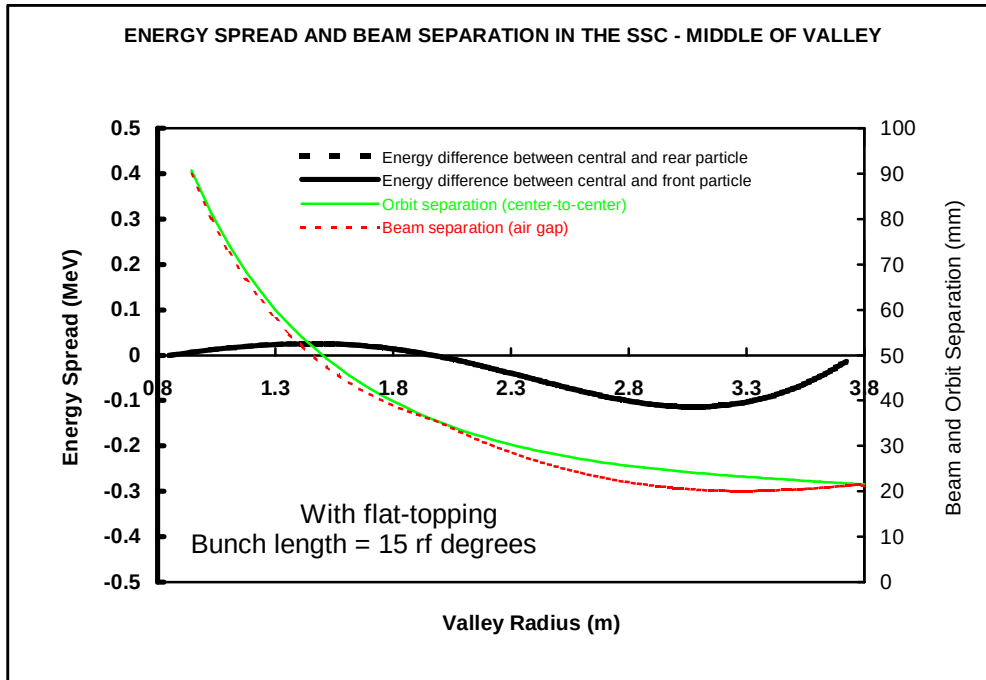


Figure C3(b). The same calculated parameters and conditions as in figure C3(a) but for a flat-topping voltage of 72.3 kV.

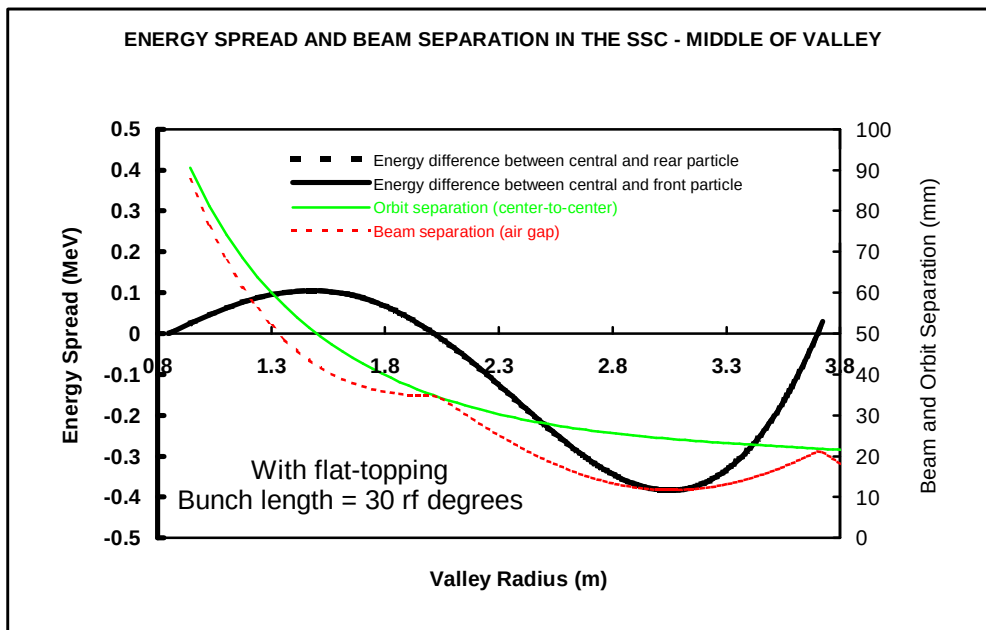


Figure C3(c). The same calculated parameters and conditions as in figure C3(b) but for a much larger bunch length of 30 rf degrees, with still an acceptable energy spread and beam width.

APPENDIX D

CANCELLING OF THE LONGITUDINAL SPACE-CHARGE EFFECTS IN THE BEAM

D.1 INTRODUCTION

Due to longitudinal space-charge (LSC) forces in a high-intensity beam the energy spread in the beam is increased and consequently the turn separation at extraction decrease and may cause overlapping of adjacent turns. To ensure single-turn extraction these forces have to be compensated [Joh68a]. The quantitative analysis of longitudinal space-charge effects is complex since the forces between the particles in the beam bunch, the particles in the adjacent beam bunches on a radial line, as well as the mirror charges in the resonator electrodes and the vacuum chambers have to be taken into account. The shape and length of the beam bunches vary from injection to extraction, even if space-charge forces are not taken into account

The azimuthal electric field component, e_q , across a beam bunch of phase length, dq , due to space-charge forces can be written as [Joh68a]:

$$e_q = k_1(f - f_0) + k_2(f - f_0)^2 + \dots, \quad (D.1)$$

where f_0 = the phase of the central particle in a symmetric bunch,

$$k_1 \approx \frac{8 I n_{tot}}{e_0 w_0 R_{max}^2 dq^2}, \quad (D.2)$$

I = the beam current,

n_{tot} = total number of revolutions,

R_{max} = maximum orbital radius in the cyclotron,

e_0 = permittivity of vacuum and

w_0 = angular velocity of the main dee voltage.

The voltage gain per turn due to the electric field is:

$$\frac{dV_{LSC}}{dn} = 2p R e_q \quad (D.3)$$

and has to be added to the main rf-voltage, with R = the average radius of the orbit. Since e_q is strongly dependant on the azimuthal charge distribution in the beam bunch, it is assumed that the distribution is symmetric about the central particle of the bunch. The leading particles in the beam bunch will be accelerated and the lagging particles will be decelerated. In order to ensure single-turn extraction with high beam intensities, this elongation of the beam bunch has to be minimized by introducing methods to cancel the longitudinal space-charge effect described by equation D.3. This can be done by introducing a phase difference between the beam and the main rf resonator, or, if a flat-topping resonator is used, by operating it at a phase offset w.r.t. the main resonator. If only the linear component of the azimuthal voltage is considered, the additional voltage becomes [Joh68a]:

$$\frac{dV_{LSC}}{dn} = 2p R k_1 dq \quad (D.4)$$

D.2 COMPENSATION WITHOUT FLAT-TOPPING

For a short rf-phase length of dq , the linear component of the force can be eliminated by accelerating the bunch at a leading phase angle, f , of the fundamental accelerating voltage

$$V(f) = V_1 \cos(f) = V_1 x(f) \quad (D.5)$$

The phase shift causes an increase in the total number of turns, $n \propto x^{-1}(f)$, and is described by the dimensionless parameter, K , that is proportional to the slope of the dee voltage at the phase position of the center of the beam bunch, a :

$$K \equiv x(a) \frac{dx(a)}{dn} = x_0 x'_0 = -\cos a \sin a \frac{da}{dn} = -0.5 \sin 2a = -0.5 \sin a \quad (D.6)$$

The maximum value of K is 0.5 for $a = -45^\circ$.

In figure D1 the phase of the center of the beam bunch is shifted towards negative values by a constant value $f_0 = a$ to illustrate the effect. A voltage, ΔV_a , is created across the azimuthal width of the beam bunch, that can be used to cancel the longitudinal space-charge force by decreasing the final energy spread, ΔE , in the beam bunch (figure D2). The requirement to eliminate the linear part of the LSC-effects for all the turns, obtained from equations D.3 and D.6, is:

$$\Delta V_{LSC} - \Delta V_a = \Delta V_{LSC} - n_{tot} x'(a) dq = 0 \quad (D.7)$$

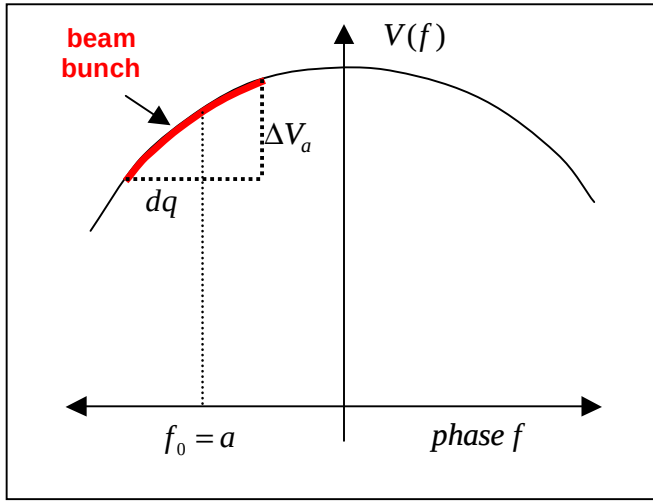


Figure D1. Accelerating the beam bunch, dq , at a phase offset of a rf-degrees ahead of the voltage peak, creates a voltage difference, ΔV_a , across the bunch, that can be used for partial compensation of the longitudinal space-charge effect on short beam bunches by a constant phase shift of the main acceleration voltage.

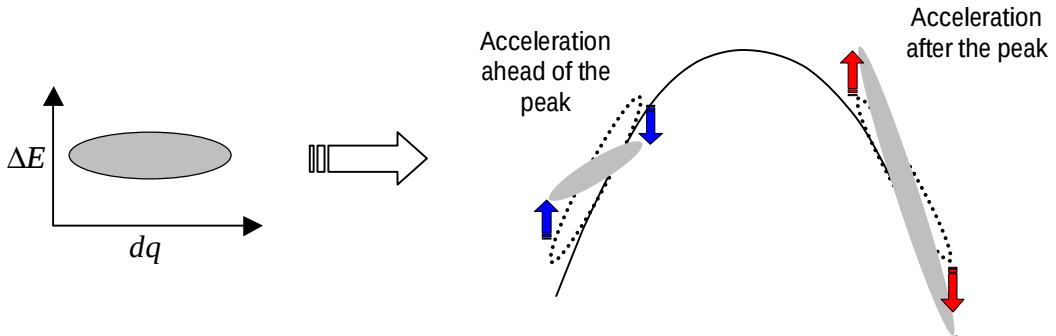


Figure D2. The energy spread in the beam bunch due to space-charge forces, is reduced by accelerating the bunch before the peak of the main dee voltage.

With this constant phase shift, the applied voltage across the azimuthal width of the beam

bunch is :

$$\Delta V_a = V_1 \left[\cos \left(a + \frac{dq}{2} \right) - \cos \left(a - \frac{dq}{2} \right) \right] \approx -V_1 dq \sin a, \quad (D.8)$$

which is used to cancel the voltage gain from the linear part of the space-charge field.

From equations D.3 and D.8 the required constant phase shift a at a radius R , is then given as:

$$a = \arcsin\left(\frac{k_1 2p R}{V_1}\right). \quad (D.9)$$

Overall compensation for all the orbits can be done with a constant phase shift by using of a representative average radius for all the orbits in the cyclotron, e.g. $\langle R \rangle = \frac{2}{3} R_{\max}$. The reduction of the voltage gain per turn due to the phase shift, will increase the number of turns in the machine by:

$$n_{\text{tot}}(a \neq 0) = \frac{n_{\text{tot}}(a = 0)}{\cos a}. \quad (D.10)$$

From equations D.2, D.9 and D.10

$$\sin a = \frac{8 I n_{\text{tot}}(a \neq 0) 2p \langle R \rangle}{e_0 w_0 R_{\max}^2 dq^2 V_1} = \frac{32 I n_{\text{tot}}(a = 0)}{3 e_0 n_0 R_{\max} dq^2 V_1 \cos a} \quad (D.11)$$

From $\sin 2a = 2 \sin a \cos a$ and $n_0 R_{\max} = \frac{cb_{\max}}{2p}$ follows that:

$$\sin 2a = \frac{32 I n_{\text{tot}}(a = 0)}{3 e_0 n_0 R_{\max} V_1 dq^2} \approx 70 \frac{I (E_f - E_i) / e}{e_0 c b_{\max} V_1^2 dq^2} \approx 2.5 \times 10^4 \frac{I (E_f - E_i) / e}{b_{\max} V_1^2 dq^2} \quad (D.12)$$

where c = the velocity of light in vacuum,

E_f = the extraction energy,

E_i = the injection energy and

e = average energy gain per turn.

Compensation is only possible up to $a = 45^\circ$. Beyond this angle the number of turns increases faster than the energy difference between the front and the rear particles in the bunch. The limiting current for single-turn extraction in the cyclotron, without flat-topping is therefore given by:

$$I_{\max} = 4.5 \times 10^{-5} \frac{b_{\max} V_1^2 dq^2}{(E_f - E_i)/e} \quad (\text{D.13})$$

For any given energy the maximum current will only depend on the acceleration voltage and bunch length.

D.3 COMPENSATION WITH FLAT-TOPPING

For larger bunch lengths it is necessary to operate the flat-topping resonator with a phase angle w.r.t. the main dee voltage, in order to partially cancel the space-charge effect on the beam. This phase error between the flat-topping and main voltages will cause a tilting of the flat-topped voltage.

The applied voltage, with a flat-topping resonator operating at harmonic number n , and phase error, df , is [Joh68b]:

$$V(f) = V_1 [\cos(f) - h \cos(nf - df)] = V_1 x(f), \quad (\text{D.14})$$

where V_1 is the peak voltage of the main resonator and $h V_1$ is the peak voltage of the flat-topping resonator. The tilting of the flat-topped voltage is shown in figure D3 for different phase errors $df = 0^\circ, 0.05^\circ, 0.1^\circ$ and 0.2° . The voltage V_1 is normalized to 1 and $h V_1$ is fixed at $\frac{1}{n^2} V_1 = \frac{1}{9} V_1 = 0.111$ for the purpose of showing the tilting effect.

For a quantitative analysis of the phase error, it is assumed that the azimuthal charge distribution in the bunch is symmetric around the phase of the central particle, f_0 , which

implies that the electric field due to space-charge, e_q , has the property that $\frac{d^2 e_q}{df_0^2} = 0$.

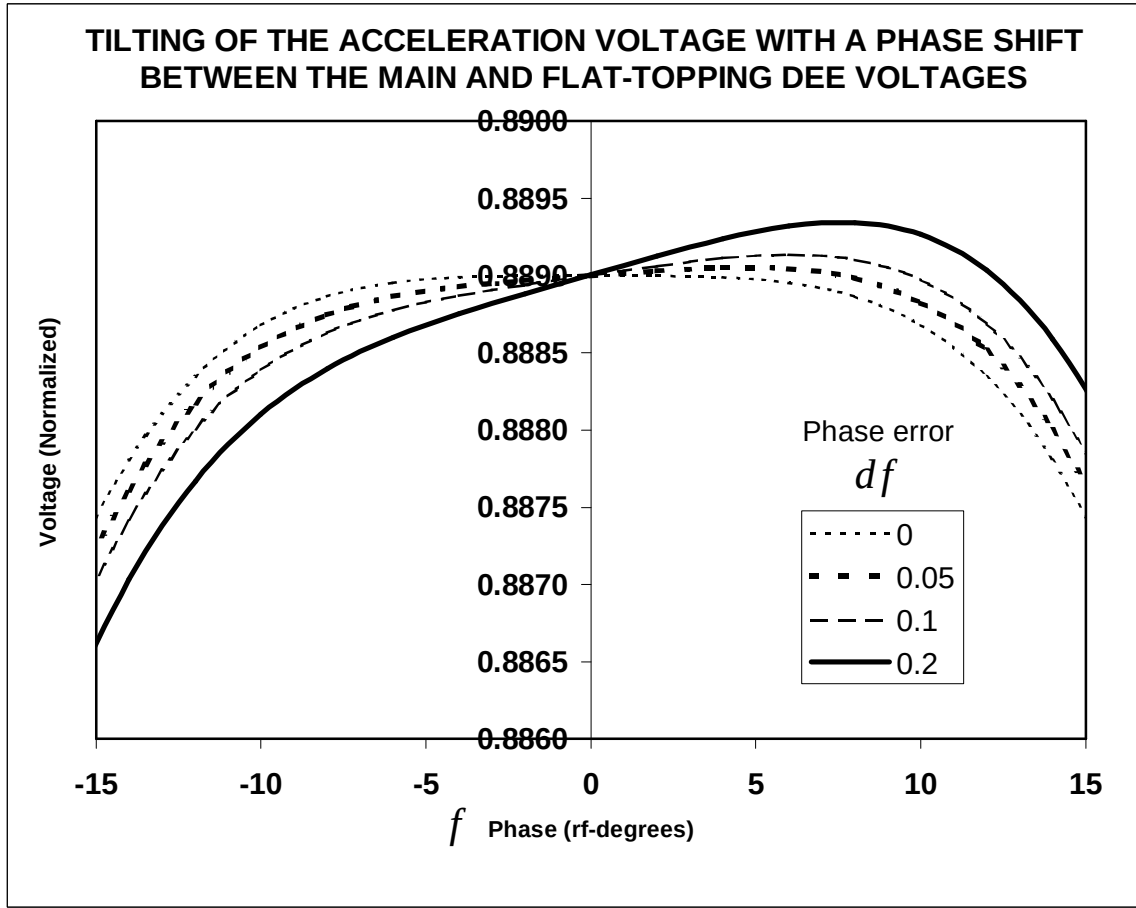


Figure D3. A phase difference between the flat-topping and main dee voltages will cause the tilting of the effective dee voltage. Leading and lagging particles therefore experience different energy gains. The effect is shown for selected phase errors of $df = 0^\circ, 0.05^\circ, 0.1^\circ$ and 0.2° and an harmonic number 3. The main voltage is normalized to 1 and that of the flat-topping resonator is fixed at $\frac{1}{n^2} = 0.111$.

With these assumptions the optimum cancellation of the space-charge effects is achieved when the linear term of the electric field in equation D.1 is cancelled. Therefore $x(f_0)$ is independent of the ratio of the two dee voltages, h .

$$x(f_0) = x_0 = \left(1 - \frac{1}{n^2}\right) \cos f_0 \quad (\text{D.15})$$

The current-dependant parameter for the cancellation of the linear part of the space-charge is proportional to the slope of the dee voltage at f_0 , and is given by equation D.6 as:

$$K \equiv x_0 x'_0.$$

The beam-loading angle, y_n , between the beam and the n -th harmonic introduced and f_0 is selected as the independent parameter to solve for h and df [Joh81]:

$$y_n = n f_0 - df = \tan^{-1} \left(\frac{n^3 K}{(n^2 - 1) \cos^2 f_0} + n \tan f_0 \right) \quad (D.16)$$

The flat-topping dee voltage due to a phase shift, that is $\propto h$, then becomes:

$$h = \frac{\cos f_0}{n^2 \cos y_n} \quad (D.17)$$

In a cyclotron with flat-topping operation at a harmonic number $n = 3$ of the main rf, the relation between a phase offset and the flat-topping voltage, is shown in figure D4 for different K -values.

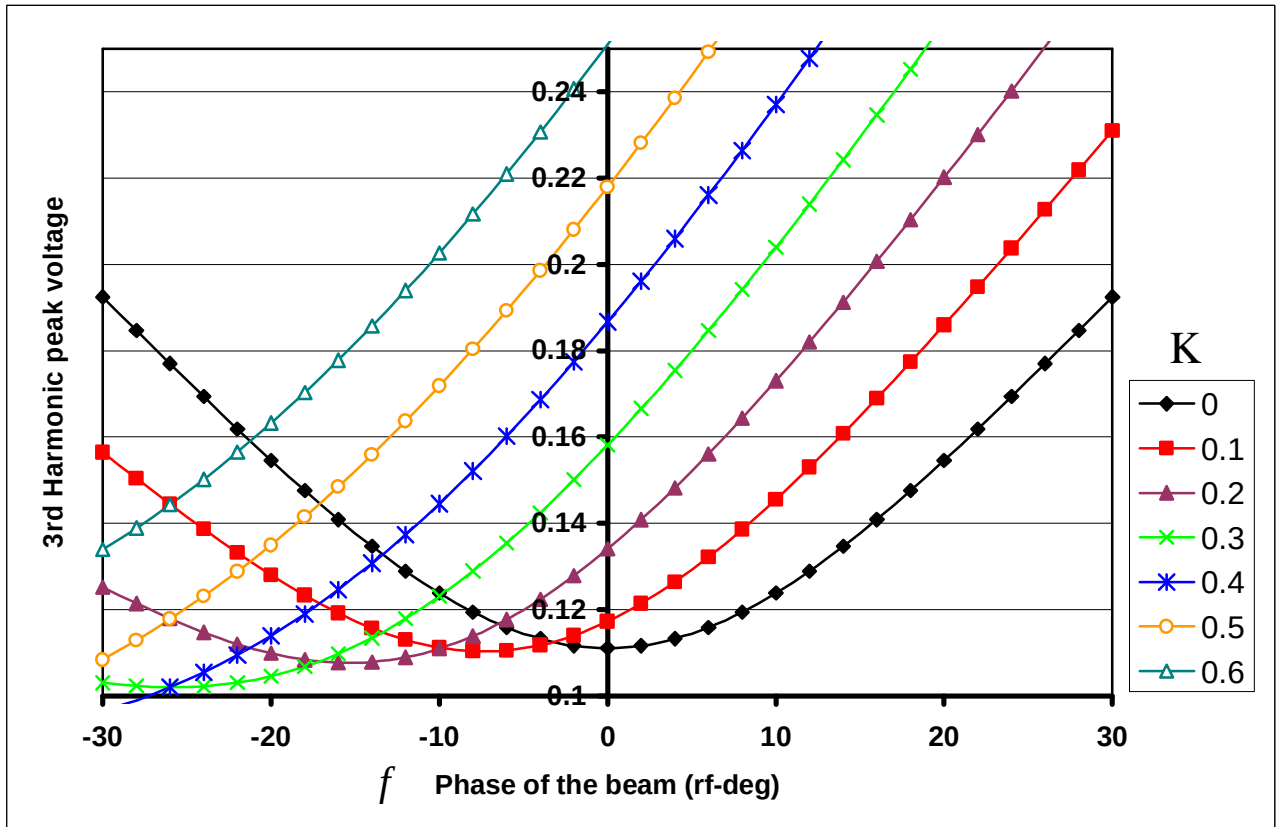


Figure D4. Compensation of the linear part of the longitudinal space-charge forces with a 3rd harmonic rf-component, h , and a shift in the central beam phase, f_0 , w.r.t. the main rf. K is a dimensionless parameter that is proportional to the slope of the accelerating voltage at $f=0$. The optimum value of K depends on the detailed shape of the azimuthal charge distribution and increases proportional to the beam current.

The total accelerating voltage with a 3rd harmonic component h is:

$$V(f) = \cos f - h \cos(3f - df). \quad (D.18)$$

The total accelerating voltage is shown in figure D5 for different values of f_0 , df and h . The position of the current-dependant parameter, K , which is the slope of the total voltage at the center of the beam bunch, is indicated with an “o” on each graph. K is dimensionless and an indication of the degree of compensation of the space-charge field.

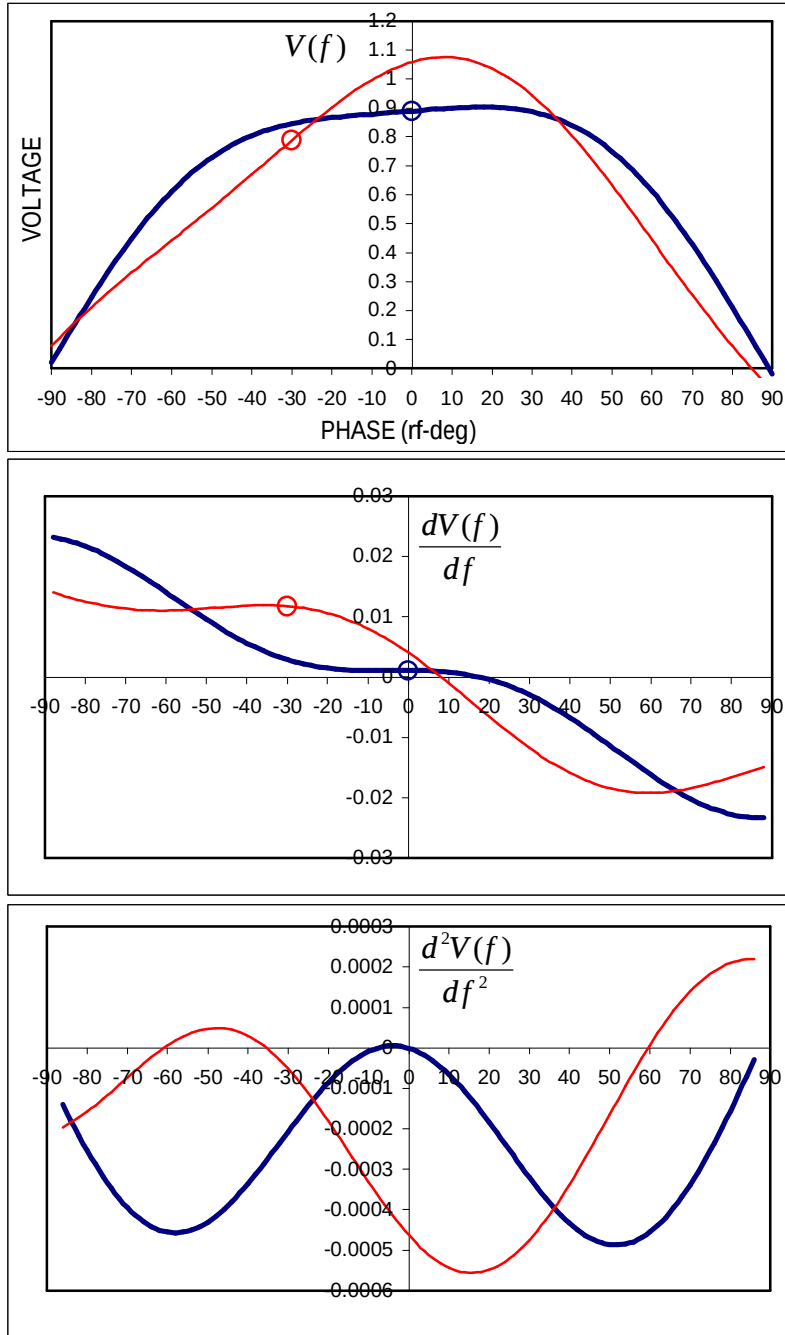


Figure D5. The normalized total acceleration voltage with 3rd harmonic flat-topping at selected values for f_0 , df , K and h . For the thick line the values are:

$f_0 = 0^\circ$,
 $df = -10.5^\circ$,
 $h = 0.113$ and
 $K = 0.055$.

For the thin line the values are:
 $f_0 = -30^\circ$,
 $df = -126.7^\circ$,
 $h = 0.097$ and
 $K = 0.55$.

The first and second derivatives for the different graphs are also shown in separate figures.

In the separated-sector cyclotron at iThemba LABS, with 4 main acceleration gaps and flat-topping with harmonic number 3 and two acceleration gaps, the voltage gain per turn becomes:

$$V(f) = 4V_1 \cos f - 2V_3 \cos 3(f + df). \quad (D.19)$$

The change in acceleration voltage due to the introduced phase error is given by [Joh68d]:

$$\Delta V = V(df \neq 0) - V(df = 0) = 3V_3 \sin 3(f + df) = V_{tot} f df \quad (D.20)$$

Integrating this voltage gain over all the orbits gives

$$\Delta V_{tot}(f) = V f n_{tot} df, \quad (D.21)$$

which has to cancel the effect of the space-charge field:

$$\Delta V_{LSC}(f) = \int_0^{n_{tot}} 2p R e_q dn \approx 4 R_{max} n_{tot} k_1 f \quad (D.22)$$

The phase detuning between the two resonators is determined from equations D.21, D.22 and D.2:

$$df = \frac{4k_1 R_{max}}{V_{rev}} = \frac{5 I n_{tot}}{e_0 V_{rev} n_0 R_{max} dq^2}. \quad (D.23)$$

Since the acceleration voltage per turn is inversely proportional to the number of turns in the cyclotron, it will also be inversely proportional to the square of the phase offset between the flat-topping resonator and the main dee voltage:

$$n_{tot} \propto 1/V_{rev}$$

and

$$df \propto 1/V_{rev}^2 \quad (D.24)$$

-o-O-o-

APPENDIX E

ELECTRICAL PROPERTIES OF SOME MATERIALS

Table E.1: Electrical properties of some materials at frequency f (Hz).

	CONDUCTIVITY s Mho/meter	RESISTIVITY 1/s Ohm.meter	RELATIVE PERMEABILITY m	SURFACE RESISTANCE R_s Ohm	SKIN DEPTH d mm
SILVER	6.17×10^7	1.62×10^{-8}	1	$2.52 \times 10^{-7} \cdot \sqrt{f}$	$64.2 / \sqrt{f}$
COPPER	5.80×10^7	1.72×10^{-8}	1	$2.61 \times 10^{-7} \cdot \sqrt{f}$	$66.0 / \sqrt{f}$
ALUMINIUM	3.72×10^7	2.69×10^{-8}	1	$3.26 \times 10^{-7} \cdot \sqrt{f}$	$82.6 / \sqrt{f}$
BRASS (REPRESENTATIVE)	1.57×10^7	6.37×10^{-8}	1	$5.01 \times 10^{-7} \cdot \sqrt{f}$	$127.0 / \sqrt{f}$
IRON	0.971×10^7	10.3×10^{-8}	~2000	$2.85 \times 10^{-5} \cdot \sqrt{f}$	$3.61 / \sqrt{f}$
SOLDER (REPRESENTATIVE)	0.706×10^7	14.2×10^{-8}	1	$7.73 \times 10^{-7} \cdot \sqrt{f}$	$185.0 / \sqrt{f}$
STAINLESS STEEL	0.137×10^7	73.0×10^{-8}	1	$1.70 \times 10^{-6} \cdot \sqrt{f}$	$430.0 / \sqrt{f}$

PERMEABILITY of VACUUM $\mu_0 = 4\pi \times 10^{-7}$ Henry/meter

APPENDIX F

THE TRANSIT-TIME FACTOR

In a cyclotron, acceleration of ions is achieved by creating a suitable rf-field in one or more gaps between conducting electrodes. In the gaps the ions of the beam bunches move through regions of time-dependant electrical fields and if the time spend in such a field is of the same order as the period of the field, the result can be that the beams are not accelerated at all. It is therefore necessary to calculate the effect of the transit time in the acceleration gaps of the flat-topping rf system. Figure F1 shows a simple schematic representation of an acceleration gap of width g between two electrodes, connected to a rf source, with the beam direction and electric field across the gap assumed uniform and in line with the z-axis.

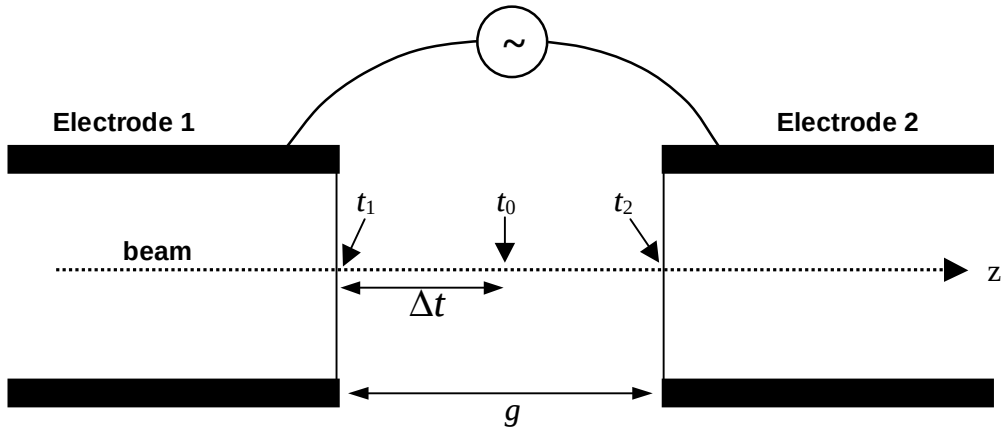


Figure F1. A schematic representation of an acceleration gap in an rf system with the beam direction and acceleration direction in line with the z-axis. The length of the gap is g and the applied sinusoidal alternating electric field has a peak value of E_0 .

An average speed $\bar{v}$ is assumed for the passing of the particles through the gap from time t_1 to t_2 . This implies that the gain in energy is small in comparison to the energy of the ion. Further it is assumed that the ion moves in line with the electrical field. An ion with

charge q travels in a time $\Delta t = \frac{t_2 - t_1}{2}$ from the entrance of the gap to the center of the gap.

The time-dependant electric field along the axis of the gap is:

$$E_z = E_0 \cos(2p f_d t) \quad (\text{F.1})$$

with E_0 the maximum value of the applied field and f_d the frequency of the dee voltage.

The force on an ion with charge q is $\vec{F} = q\vec{E}$ and the gain in energy across the gap is:

$$\begin{aligned} T_k &= \int_{\text{accros gap}} \vec{F} \cdot d\vec{s} \\ &= \int_{\text{accros gap}} q\vec{E} \cdot d\vec{s} \\ &= |\vec{v}| q \int_{t_1}^{t_2} |\vec{E}| \cdot dt \end{aligned}$$

The effective applied acceleration voltage is :

$$\begin{aligned} V_{\text{eff}} &= T_k / q \\ &= |\vec{v}| \int_{t_1}^{t_2} E_0 \cos(2p f_d t) \cdot dt \\ &= \frac{2p R}{t_p} \int_{t_1}^{t_2} (V_0 / g) \cos(2p f_d t) \cdot dt \\ &= \frac{2p R}{h t_d} \int_{t_1}^{t_2} (V_0 / g) \cos(2p f_d t) \cdot dt \end{aligned}$$

where R is the radius of the orbit, t_p is the period of the ion, t_d the period of the dee voltage and V_0 the peak acceleration voltage.

Assuming that the ion moves half the gap distance in a time:

$$\Delta t = (t_2 - t_1) / 2 = \frac{g t_p}{4p R} = \frac{g h}{4p f_d R},$$

the effective voltage becomes:

$$\begin{aligned}
 V_{eff} &= \frac{2pR}{ht_d} \int_{t_1}^{t_2} (V_0 / g) \cos(2p f_d t) . dt \\
 &= \frac{2pR}{ht_d} \frac{V_0}{g} \cos(2p f_d t_0) \frac{\sin(2p f_d \Delta t)}{(p f_d)} \\
 &= V_0 \cos(2p f_d t_0) \frac{\sin\left(\frac{gh}{2R}\right)}{\left(\frac{gh}{2R}\right)} .
 \end{aligned}$$

The effective acceleration voltage is therefore reduced by a factor

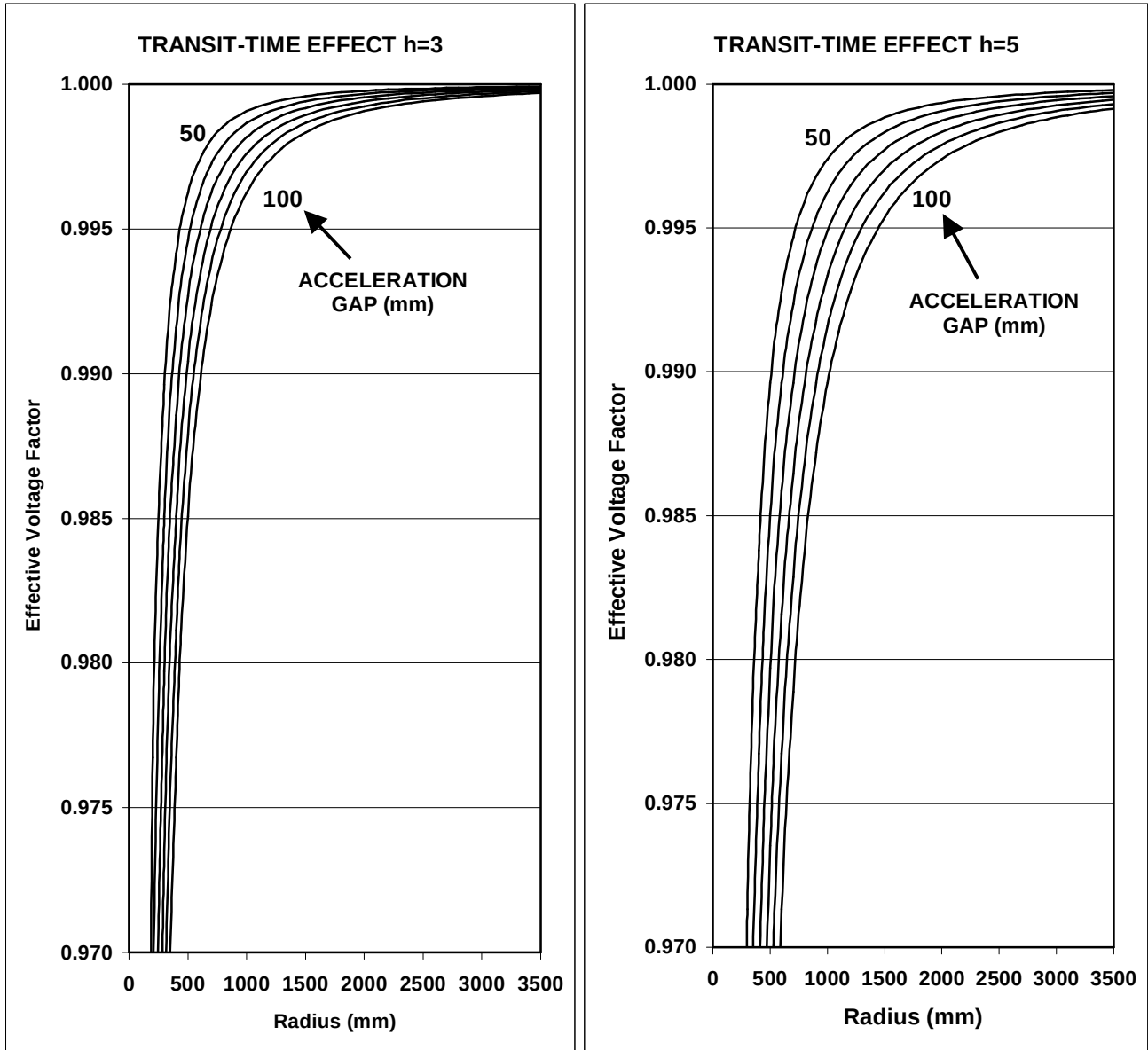
$$\frac{\sin\left(\frac{gh}{2R}\right)}{\left(\frac{gh}{2R}\right)}, \quad (F.2)$$

which is independent of the phase of the ion. This reduction in dee voltage has to be considered in the selection of an harmonic number and acceleration gap. The transit-time factor for different harmonic numbers and acceleration gaps are given at different cyclotron radii in table F.1.

Table F.1. The transit-time factor for different harmonic numbers and acceleration gap sizes at different cyclotron radii.

R (mm)	h=1			h=2			h=3			h=4			h=5		
	acc gap (mm)			acc gap (mm)			acc gap (mm)			acc gap (mm)			acc gap (mm)		
	50	100	150	50	100	150	50	100	150	50	100	150	50	100	150
100	0.99	0.96	0.91	0.96	0.84	0.66	0.91	0.66	0.35	0.84	0.45	0.05	0.76	0.24	-0.15
200	1	0.99	0.98	0.99	0.96	0.91	0.98	0.91	0.80	0.96	0.84	0.66	0.94	0.76	0.51
300	1	1	0.99	1	0.98	0.96	0.99	0.96	0.91	0.98	0.93	0.84	0.97	0.89	0.76
400	1	1	0.99	1	0.99	0.98	0.99	0.98	0.95	0.99	0.96	0.91	0.98	0.94	0.86
500	1	1	1	1	0.99	0.99	1	0.99	0.97	0.99	0.97	0.94	0.99	0.96	0.91
750	1	1	1	1	1	0.99	1	0.99	0.99	1	0.99	0.97	1	0.98	0.96
1000	1	1	1	1	1	1	1	1	0.99	1	0.99	0.99	1	0.99	0.98
1500	1	1	1	1	1	1	1	1	1	1	1	0.99	1	1	0.99
2000	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.99
2500	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3000	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Figure F2(a) and F2(b) show more detail in the change of the effective acceleration voltage for harmonic numbers $h=3$ and $h=5$ respectively and for different acceleration gaps as the beam progresses through a cyclotron.



Figures F2 (a) and (b). The effective acceleration voltage of a resonator, due to the transit-time effect, decreases as either of the harmonic number and acceleration gap increases and increases as the radial position in the cyclotron increases. The effect is shown for acceleration gaps from 50 mm to 100 mm with intervals of 10 mm.

-o-O-o-

APPENDIX G

CALCULATION OF THE SPACING BETWEEN COOLING DUCTS AND THE AMOUNT OF WATER REQUIRED FOR THE RESONATOR

G.1 FOURIER'S LAW FOR HEAT CONDUCTION

Heat generated in a body flows to regions of lower temperature by conduction. In a block of material with no internal heat sources or sinks, and therefore a uniform conduction the rate of heat flow per unit area is proportional to the normal temperature gradient. With the proportionality constant, k , inserted, Fourier's law for heat conduction states:

$$q = -kA \frac{\partial T}{\partial x} \quad (\text{Watt}) \quad (\text{G.1})$$

where q is the heat, A the cross section area of the material, $\frac{\partial T}{\partial x}$ the temperature gradient in the selected x -direction of the heat flow, k is a positive constant specifying the thermal conductivity of the material and the minus sign reflects the fact that heat-flows from regions of higher to the lower temperature boundary.

The rate of heat-flow generated per unit volume is $\dot{q} = \frac{q}{Adx}$ (W/m^3).

The complete conduction equation can be derived from the principle of energy conservation in an infinitesimal element of material, as illustrated in figure G1.

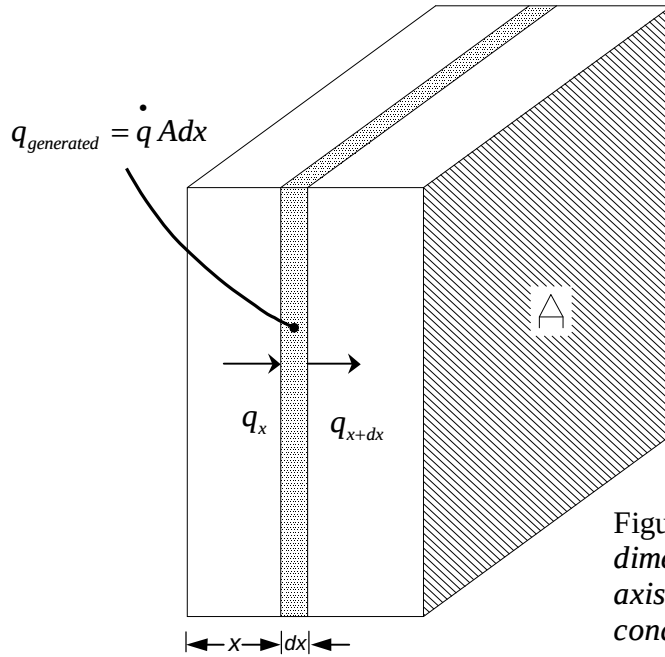


Figure G1. A volume element $A dx$ for one-dimensional heat conduction along the x -axis, from which the equation for heat conduction can be derived.

For a one-dimensional transfer of heat such an element has a volume $A dx$ in the cartesian coordinate system, with A the surface area perpendicular to the direction of conduction. The heat conducted through one face of the element dx , plus the heat generated within the volume element $A dx$ must equal the change in internal energy plus the heat conducted out through the other face of the element dx . This leads to the general three-dimensional heat-conduction equation, through a Taylor series expansion, using only the first two terms for the derivatives at $x+dx$ [Hol72]:

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = r c \frac{\partial T}{\partial t} \quad (\text{G.2})$$

with $\dot{q}$ the rate at which the heat is generated per unit volume,
 c the specific heat of the material,
 r the density of the material and
 t the time.

If a constant thermal conductivity is assumed, the equation becomes:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{a} \frac{\partial T}{\partial t} \quad (\text{G.3})$$

where $a = \frac{k}{rc}$ is the thermal diffusivity of the material.

Equation G.3 can be transformed into either cylindrical or spherical coordinates with standard calculus techniques.

G.2 DISTANCES BETWEEN COOLING PIPES AND REQUIRED WATER FLOW

Constant temperature in a resonator is essential for stable operation. In practice water ducts are thermally soldered to the conducting surfaces in order to remove heat safely away from the resonator. Equations are derived below for the minimum distance from points of heat generation to the nearest cooling duct, which complies with the prescribed allowed temperature increase. Another equation is derived for the temperature on a conducting surface between parallel cooling ducts. Finally a formula for the required water flow is derived.

G.2.1 Minimum distance from any heat source on a conducting plate to the nearest cooling pipe

In order to determine the minimum distance required to the nearest cooling duct it is assumed that the thermal conductivity is constant and that all the heat is conducted one-dimensionally. Selecting the direction of conduction along the Cartesian x-axis, equation G.3 is at equilibrium reduced to:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0 \quad (\text{G.4})$$

In figure G2 a conducting copper plate is kept at a constant temperature, T_0 , by a cooling duct at $x=0$. Heat is generated at a constant and homogeneous rate in the plate with length l .

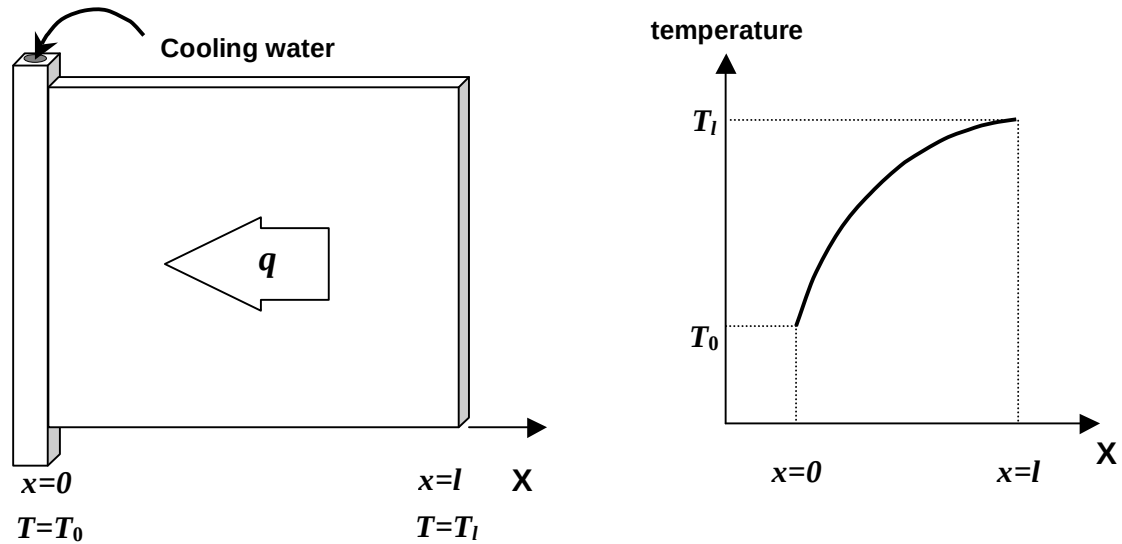


Figure G2. *Schematic presentation of a conducting plate and cooling duct.*

At the border $x=l$ there is no heat flow, which implies that the temperature gradient must be zero, as shown in figure G2. Integrating equation G.4 twice, gives:

$$\frac{\partial T}{\partial x} = -\frac{\dot{q}}{k}x + C_1 \quad (\text{G.5})$$

and

$$T = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 \quad (\text{G.6})$$

with C_1 and C_2 constants that are determined by applying boundary conditions.

Substituting the known temperature of T_0 (at $x=0$) into equation G.6, gives:

$$C_2 = T_0$$

and applying the zero temperature gradient at $x=l$ to equation G.5, gives:

$$C_1 = \frac{\dot{q}l}{k}.$$

The equation for temperature therefore becomes:

$$T = -\frac{\dot{q}}{2k}x^2 + \frac{\dot{q}l}{k}x + T_0 \quad (\text{G.7})$$

The temperature at the maximum distance from the cooling pipe is:

$$T = T_l = -\frac{\dot{q}}{2k}l^2 + \frac{\dot{q}l}{k}l + T_0 = \frac{\dot{q}l^2}{2k} + T_0, \quad (\text{G.8})$$

which agrees with Fourier's law for heat conduction.

The maximum distance, l , between a cooling duct and any point that generate heat, is obtained from equation G.8:

$$l = \sqrt{\frac{2k(T_l - T_0)}{\dot{q}}} \quad (\text{G.9})$$

G.2.2 Dissipation of heat from a plate with homogeneous heat generated between two cooling ducts that are kept at constant but different temperatures

Suppose the maximum temperature is at point, **P**, on a conducting copper plate somewhere between two parallel cooling ducts. The two cooling ducts are separated by a distance l that have temperatures T_1 at $x=x_1$ and T_2 at $x=x_2$. Figure G3 schematically shows the plate with the two conductors, as well as the expected shape of the temperature chart for the set-up with the unknown maximum temperature, T_m , at **P**, which is higher than the water temperature of both adjacent cooling ducts.

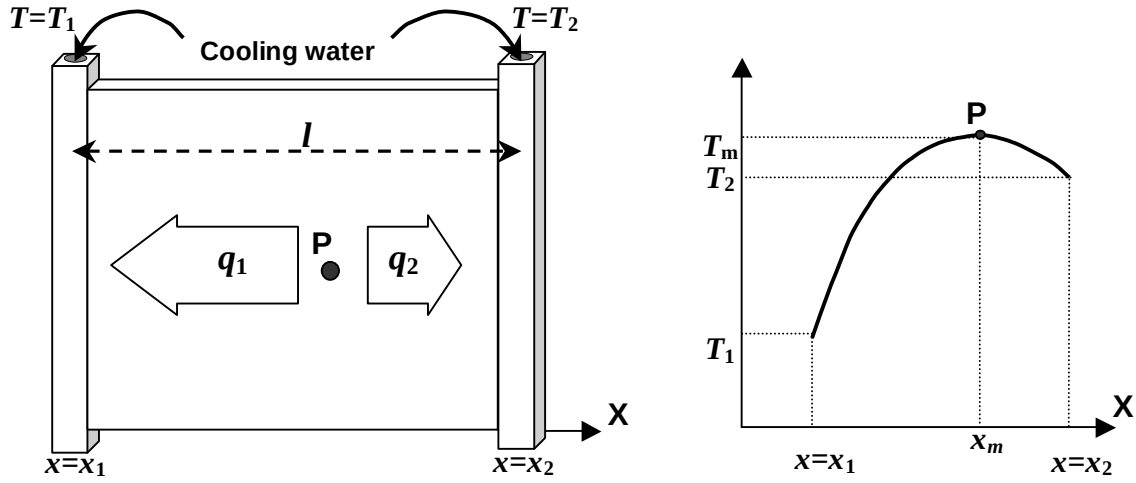


Figure G3. Temperature distribution in a conductor between two cooling ducts that are kept constant but at different temperatures T_1 and T_2 respectively.

At the unknown position of maximum temperature, all the heat generated on the left-hand side of that point will flow to the cooling duct at temperature T_1 . Similarly all the heat that is generated at the right hand side of **P** will flow towards the cooling duct at temperature T_2 . For simplicity of the analysis the boundary position x_1 is selected to be at the origin of the x-axis and therefore x_2 is at coordinate position l . The boundary conditions are now applied to equation G.6 to obtain the solution for the temperature distribution between the two cooling ducts and find the position of and value for the maximum temperature, T_m .

At $x = x_1 = 0$ the temperature is:

$$T_1 = C_2. \quad (\text{G.10})$$

At $x = x_2 = l$ the temperature is:

$$T_2 = -\frac{\dot{q}}{2k} l^2 + C_1 l + C_2. \quad (\text{G.11})$$

Subtracting equation G.10 from G.11 to find a value for C_1 :

$$T_2 - T_1 = -\frac{\dot{q}}{2k}l^2 + C_1l$$

$$\therefore C_1 = \frac{(T_2 - T_1)}{l} + \frac{\dot{q}l}{2k}$$

The equation for the temperature distribution between the two cooling ducts becomes:

$$T = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 = -\frac{\dot{q}}{2k}x^2 + \left(\frac{T_2 - T_1}{l} + \frac{\dot{q}l}{2k}\right)x + T_1 \quad (\text{G.12})$$

The maximum temperature between the two cooling ducts is located where the temperature gradient is zero:

$$\therefore \frac{\partial T}{\partial x} = 0$$

$$\therefore -\frac{\dot{q}}{k}x + C_1 = 0$$

$$\therefore \frac{\dot{q}}{k}x = \frac{T_2 - T_1}{l} + \frac{\dot{q}l}{2k}$$

$$\therefore x = \frac{k(T_2 - T_1)}{\dot{q}l} + \frac{l}{2}$$

Therefore the maximum temperature between the cooling ducts, which are kept at fixed temperatures, is located at:

$$x_m = \frac{k(T_2 - T_1)}{\dot{q}l} + \frac{l}{2} \quad (\text{G.13})$$

Substituting equation G.13 into G.12 gives the maximum temperature between the two cooling ducts as:

$$\begin{aligned}
 T_m &= -\frac{\dot{q}}{2k} x_m^2 + \left(\frac{T_2 - T_1}{l} + \frac{\dot{q}l}{2k} \right) x_m + T_1 \\
 &= -\frac{\dot{q}}{2k} \left(\frac{k(T_2 - T_1)}{\dot{q}l} + \frac{l}{2} \right)^2 + \left(\frac{T_2 - T_1}{l} + \frac{\dot{q}l}{2k} \right) \left(\frac{k(T_2 - T_1)}{\dot{q}l} + \frac{l}{2} \right) + T_1 \\
 &= \frac{k(T_2 - T_1)^2}{2\dot{q}l^2} + \frac{\dot{q}l^2}{8k} + \frac{T_2 + T_1}{2} \\
 &= \frac{(T_2 + T_1)}{2} \left[\frac{k(T_2 - T_1)}{\dot{q}l^2} + 1 \right] + \frac{\dot{q}l^2}{8k} \tag{G.14}
 \end{aligned}$$

G.2.3 The water flow required to remove the generated heat away from the resonator

The mass, m , of water per second required to remove heat produced at a rate P from an object, for an increase of DT in the water temperature, is given by:

$$m = \frac{P\Delta T}{c}, \tag{G.15}$$

with c = the specific heat capacity of water = $4200 \text{ J.kg}^{-1}.\text{K}^{-1}$.

The formula for the power dissipation through cooling water can be written in different forms, depending on the choice of units:

$$P = m \times 4.2 \times \Delta T, \quad \text{with } P \text{ in watt, } m \text{ in gram/second and } \Delta T \text{ in Kelvin.}$$

$$P = m \times 4200 \times \Delta T, \quad \text{with } P \text{ in watt, } m \text{ in kilogram/second and } \Delta T \text{ in Kelvin.}$$

The density of water is about 1 gram.cm⁻³ or 1 kilogram.liter⁻¹. The equations above for the power can be used to calculate the volume of water required per unit time.

$$\begin{aligned} V(\text{liter/second}) &= \frac{P(\text{watt})}{4186 \times \Delta T(K)} \\ V(\text{liter/minute}) &= \frac{P(\text{watt})}{69.77 \times \Delta T(K)} \\ V(\text{m}^3/\text{second}) &= \frac{P(\text{watt})}{4.186 \times 10^6 \times \Delta T(K)} \end{aligned} \quad (\text{G.16})$$

The cooling duct for the water is usually rectangular copper tube with a round aperture for the water to flow through. The volume of water in 1 meter length of such a tube is $V_{1m} = 1 \times \pi r^2 \text{ m}^3$, with r the radius of the cooling duct in meter.

The minimum temperature of the water in the cooling duct must be kept above wet bulb temperature to prevent the formation of water on the outside of the copper duct and the maximum temperature should not exceed 50°C to prevent damage to sensitive equipment or injury to personnel. If a temperature rise of ΔT K is permitted from the inlet to the outlet water temperature, the temperature rise per meter length of the cooling duct is $\Delta T / l \text{ K.m}^{-1}$, with l the total length of the tube between the inlet and outlet. If l is large the permitted temperature rise per meter becomes non-practical and the cooling duct has to be split into parallel water ducts.

The flow speed required for the dissipation of P (W.m^{-1}) in n cooling ducts of length l and radius r and restricting the temperature rise to ΔT K, is:

$$k = \frac{P \times l}{p r^2 \times \Delta T \times n \times c} = k_1 \frac{l}{n} \text{ m.s}^{-1}. \quad (\text{G.17})$$

The flow speed of the water, k , is if practically possible, kept below 2 m.s^{-1} , to prevent corrosion of the copper surfaces in the bends of the cooling duct, due to turbulence. The volume of water (m^3) that moves through the cooling duct at velocity k (m.s^{-1}) in a time Δt (seconds), is:

$$V(t) = V_{1m} \times k \times \Delta t. \quad (\text{G.18})$$

The volume of water per minute is:

$$V_{1m} \times k \times 60 \text{ m}^3 \text{ or}$$

$$V_{1m} \times k \times 60000 \text{ liter.}$$

The water pressure over the length of a water duct that will ensure enough flow for the required power dissipation, can be calculated from [Sch73]:

$$\Delta\Theta / L \text{ (bar.m}^{-1}\text{)} = 1.2 \times 52 \times V^{7/4} \times D^{-19/4} \quad (\text{G.19})$$

with $\Delta\Theta$ the pressure drop (bar) over the length, L (meter), of the cooling duct, the 1.2 is a safety factor for bends in pipes and 52 is a unit conversion factor. The water flow, V , is in liter.min^{-1} and D , the diameter of the cooling duct, in millimeter.

-o-O-o-

APPENDIX H

KILPATRICK'S CRITERION

The criterion for prevention of excessive sparking due to rf electric fields in vacuum, has been described by Kilpatrick as $f = 1.643 (E_k)^2 e^{(-8.5/E_k)}$, with f the frequency in MHz and E_k the maximum allowed electric field strength in MV/m [Edw93].

The calculated maximum electric field strength, using Kilpatrick's criterion, is shown in figure H1. The maximum electric field strength in the flat-topping resonator of the SSC is about 8.9 kV/cm at 49.12 MHz.

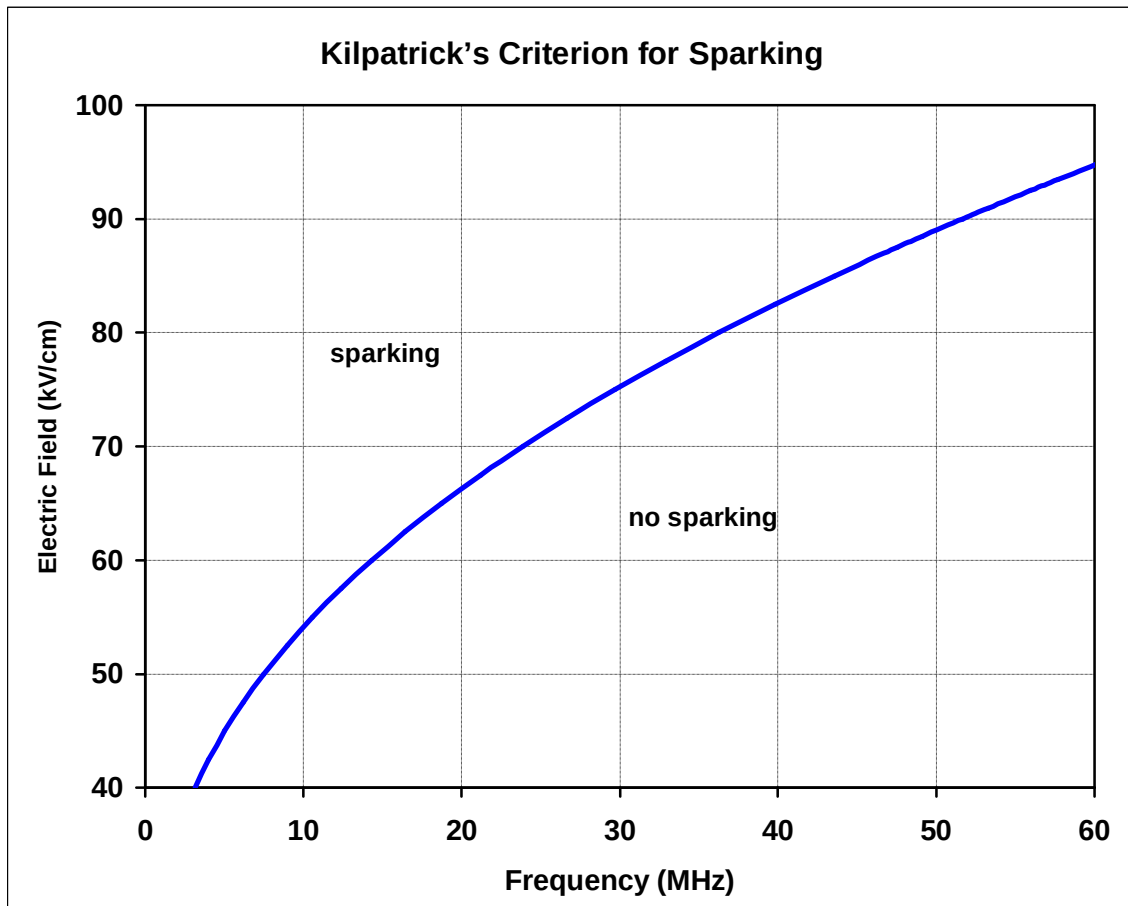


Figure H1. *Kilpatrick's criterion to prevent sparking in an rf-field in vacuum.*

-o-O-o-

APPENDIX I

SCALING OF THE RESONATOR PARAMETERS FOR A MODEL

The parameters of a resonator model that is reduced by a factor F from the full-scale resonator, are [Bot74]:

PARAMETER	FULL-SCALE	MODEL
Length	D	D/F
Surface area	A	A/F^2
Volume	V	V/F^3
Resistance	r	r/F
Capacity	K	K/F
Inductance	I	I/F
Resonance frequency	f	$f.F$
Impedance	Z	Z
Specific surface resistance	ρ	$\rho \cdot \sqrt{F}$
Surface resistance	R_s	$R_s \cdot \sqrt{F}$
Transmission line parameters:		
Resistance/meter	R	$R.F. \sqrt{F}$
Capacitance/meter	C	C
Inductance/meter	L	L
Characteristic impedance	Z_0	Z_0
Power dissipation (for the same voltage)	P	$P \cdot \sqrt{F}$
Stored energy	E	$E.F$
Q-value	Q	$Q/\sqrt{F}$

-o-O-o-

APPENDIX J

A SPREADSHEET PROGRAM TO CALCULATE THE PHASE HISTORY OF THE VERTICAL MOTION IN THE SSC DUE TO A VERTICAL DISPLACEMENT OF THE DEE PLATES IN RELATION TO THE MEDIAN PLANE.

J.1 INTRODUCTION

Many electromagnetic effects in cyclotrons may have field components that cause betatron oscillations (vertical and radial oscillations of the particle about the position of its equilibrium orbit). An offset of the resonator dee plates with respect to the median plane will cause an electrical field, which will influence the vertical sinusoidal movement of the particles inherent to the design of the cyclotron. This electrical field will not only deflect the passing beam vertically (z-direction), but also have a focusing effect and was studied with the aid of a spreadsheet program that was developed to calculate the beam phase plot parameters of the motion caused by such a displacement of the dee plates of the flat-topping resonator in the SSC.

J.2 THEORY

It is assumed that the driving force is a vertical electrical field, E_z , in the gap, g , between the dee plates of the resonator due to the vertical displacement of the dee plates. The associated voltage between the dee plates is assumed to be a constant fraction of the acceleration voltage of the flat-topping resonator and is given by $V_z = F V_p$, where V_p is the flat-topping acceleration voltage and F the fraction. This voltage changes polarity during the transit of the particle at a frequency, f , given by:

$$\omega = 2\pi f = 2\pi n f_h = 2\pi n h f_d = n h \omega_d, \quad (\text{J.1})$$

where f_h is the frequency of the main resonator, f_d the orbit frequency, w_d the orbital velocity of the particle, n the harmonic number of the flat-topping resonator and h the harmonic number at which the cyclotron is operated. The voltage between the dee plates will therefore have a similar sinusoidal profile in the radial direction as for that of the flat-topping resonator voltage.

The force that drives the vertical oscillation due to the vertical displacement of the dee plates is given by:

$$qE_z = \frac{qV_z}{g} \sin wt, \quad (\text{J.2})$$

where q is the charge on the particle and t the time dependence parameter. It is assumed that the particle crosses the acceleration gaps at the peak values of the flat-topping voltage ($wt = \frac{p}{2}$ and $wt = -\frac{p}{2}$). The effect of the voltage on the transit time of the particle will be insignificant.

At a maximum kinetic energy of 66 MeV, the mass of the particle, m , is regarded as non-relativistic. Therefore are the vertical acceleration, velocity and displacement respectively given by:

$$\ddot{z} = \frac{d^2 z}{dt^2} = \frac{qV_z}{mg} \sin wt, \quad (\text{J.3})$$

$$\dot{z} = \frac{dz}{dt} = \frac{-qV_z}{mgw} \cos wt + k_1 \quad (\text{J.4})$$

and

$$z = \frac{-qV_z}{mgw^2} \sin wt + k_1 t + k_2 \quad (\text{J.5})$$

For the initial boundary conditions at the entrance gap of the resonator, with $wt = -\frac{p}{2}$,

is $\dot{z} = \dot{z}_a = -\frac{qV_z}{mgw} \cos\left(-\frac{p}{2}\right) + k_1 = k_1$

and $z = z_a = -\frac{qV_z}{mgw^2} \sin\left(-\frac{p}{2}\right) + \dot{z}_a \left(-\frac{p}{2w}\right) + k_2.$

Therefore $k_1 = \dot{z}_a$ and $k_2 = z_a - \frac{qV_z}{mgw^2} + \dot{z}_a \frac{p}{2w}.$

Equations J.4 and J.5 respectively becomes:

$$\dot{z} = \frac{-qV_z}{mgw} \cos wt + \dot{z}_a \quad (\text{J.6})$$

and $z = -\frac{qV_z}{mgw^2} \sin wt + \dot{z}_a t + z_a - \frac{qV_z}{mgw^2} + \dot{z}_a \frac{p}{2w} \quad (\text{J.7})$

At the exit edge of the resonator is $wt = \frac{p}{2}$ and the vertical velocity is the same as at the entrance edge, as determined from equation J.6:

$$\dot{z} = \frac{-qV_z}{mgw} \cos \frac{p}{2} + \dot{z}_a = \dot{z}_a,$$

which implies that the axial momentum is not changed by the resonator.

The vertical displacement at the exit edge is:

$$\begin{aligned} z &= -\frac{qV_z}{mgw^2} \sin \frac{p}{2} + \frac{\dot{z}_a p}{2w} + z_a - \frac{qV_z}{mgw^2} + \frac{\dot{z}_a p}{2w} \\ \therefore z &= -\frac{2qV_z}{mgw^2} \sin \frac{p}{2} + \frac{\dot{z}_a p}{w} + z_a \\ &= -\frac{2qV_z}{12^2 w_d^2 mg} \sin \frac{p}{2} + \frac{\dot{z}_a p}{12w_d} + z_a \end{aligned} \quad (\text{J.8})$$

If it is assumed that the vertical displacement of the particle due to the resonator happens only at a single azimuthal position in the cyclotron and it oscillates vertically

with frequency n_z , the vertical motion in terms of the angular position, q , is given by:

$$z = A \cos n_z q + B \sin n_z q \quad (\text{J.9})$$

and

$$\dot{z} = -A n_z \sin(n_z q) \frac{dq}{dt} + B n_z \cos(n_z q) \frac{dq}{dt}. \quad (\text{J.10})$$

With $\frac{dq}{dt} = w_d = \frac{v}{R}$, where v is the angular velocity of the particle in the orbit and $\bar{R}$ the average radius of the orbit, equation J.10 becomes:

$$\dot{z} = \frac{v}{R} [-A n_z \sin n_z q + B n_z \cos n_z q]. \quad (\text{J.11})$$

When the vertical momentum of the particle is substituted by its vertical divergence,

$z' = \frac{dz}{ds} = \frac{dz}{v dt} = \frac{\dot{z}}{v}$, equation J.11 becomes:

$$z' = \frac{1}{R} [-A n_z \sin n_z q + B n_z \cos n_z q]. \quad (\text{J.12})$$

Assuming that vertical displacement and divergence at the exit side of the resonator is respectively $z = z_b$ and $z' = z'_b$, equations J.9 and J.12 are used to derive the expressions for A and B .

$$z_b = A \cos n_z q + B \sin n_z q \quad (\text{J.13})$$

$$z'_b = \frac{1}{R} [-A n_z \sin n_z q + B n_z \cos n_z q] \quad (\text{J.14})$$

These two equations are written as:

$$z_b \frac{n_z}{R} \sin n_z q = A \frac{n_z}{R} \sin n_z q \cos n_z q + B \frac{n_z}{R} \sin^2 n_z q$$

and

$$z_b' \cos n_z q = \frac{1}{R} \left[-A n_z \sin n_z q \cos n_z q + B n_z \cos^2 n_z q \right].$$

Adding the last two equations leads to the expressions for B :

$$z_b \frac{n_z}{R} \sin n_z q + z_b' \cos n_z q = B \frac{n_z}{R} \left[\sin^2 n_z q + \cos^2 n_z q \right]$$

$$\therefore B = z_b \sin n_z q + z_b' \frac{\bar{R}}{n_z} \cos n_z q \quad (\text{J.15})$$

Substituting the value of B into equation J.13 gives the value for A :

$$A = \frac{z_b - \left(z_b \sin n_z q + z_b' \frac{\bar{R}}{n_z} \cos n_z q \right) \sin n_z q}{\cos n_z q} \quad (\text{J.16})$$

Substituting the values of A and B into equations J.9 and J.12 give the final expressions for the vertical displacement and divergence of the particle.

J.3 INPUT PARAMETERS AND CALCULATION PROCEDURE

Shown in figure J.1 are the frequencies of the vertical betatron oscillations, n_z , for every equilibrium orbit of a proton with extraction energy of 66 MeV from the SSC that were calculated with the computer program COC and implemented with the spreadsheet program to calculate the additional vertical motion created by the displacement of the dee plates of the flat-topping resonator.

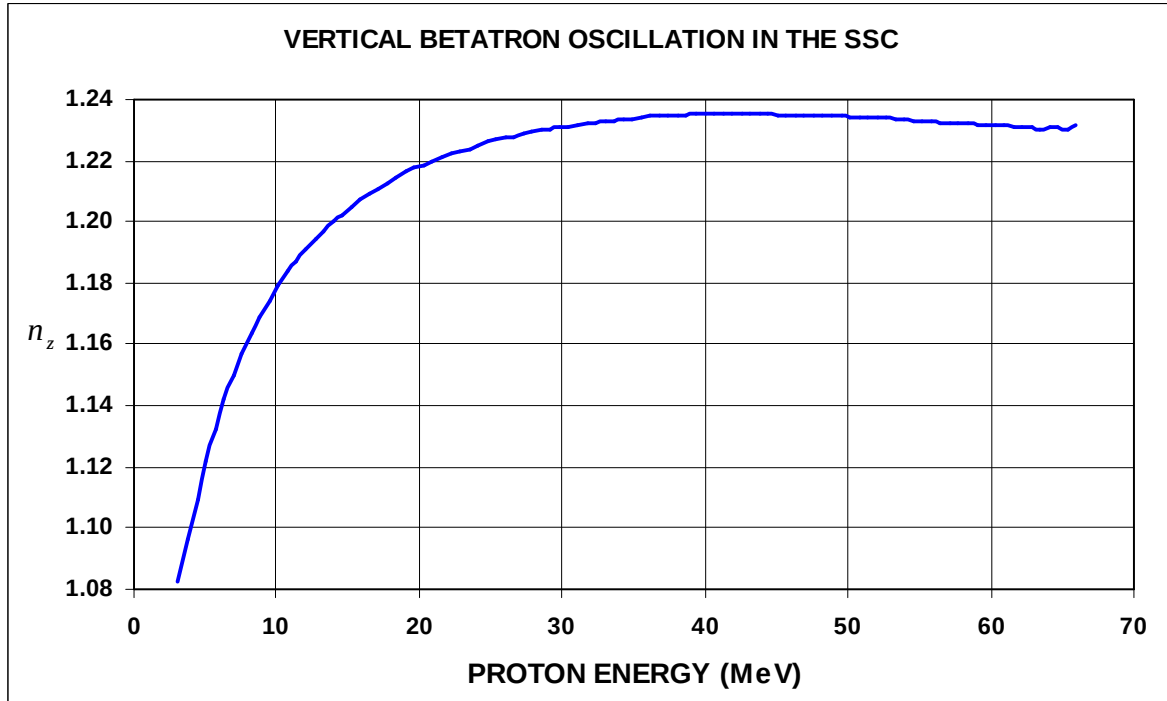


Figure J.1. *The vertical betatron frequency for equilibrium orbits in the SSC, calculated with the computer program COC for a proton with extraction energy of 66 MeV.*

The vertical motion of the particle per turn in the cyclotron is calculated for a section that describes the movement while it is between the dee plates and another section that comprises the rest of the orbit until it reaches the resonator again. These values of vertical displacement and divergence for the second portion of the turn are added when the similar parameters for the next turn in the flat-topping section are calculated. In the program it is assumed that the resonator occupies $\frac{2\pi}{24}$ radians of each turn.

A turn in the cyclotron is assumed to start and end at the entrance gap of the resonator for the calculation purposes. An injection energy of 3.14 MeV is used and an initial vertical displacement of 0 mm and divergence of 0 mrad are assumed before entering the first entrance gap of the flat-topping resonator for the first time.

The change in vertical position and divergence is subsequently calculated at the exit gap of the resonator and then again at the entrance gap to complete the first turn. The calculated values of each turn are used to calculate the vertical motion at these azimuthal positions for the following turn.

The constants and variables that are used for the calculation are listed in table J.1.

Table J.1. constants and variables used for calculating the beam phase plot of the particle due to a vertical displacement of the dee plates of the flat-topping resonator.

DESCRIPTION		VALUE	UNIT
Velocity of light	c	2.99792458×10^8	m/s
Rest mass of proton	m_d	$1.6726231 \times 10^{-27}$	kg
Charge on proton	q	$1.60217733 \times 10^{-19}$	C
Rest energy of proton	E_0	938.27234	MeV
Frequency of the main resonator	f_h	16373000	Hz
Injection energy	T_i	3.14	MeV
Extraction energy	T_e	66	MeV
Voltage on main resonator	V_h	207019	V
Voltage on flat-topping resonator	V_p	62000	V
Fraction of V_p between dee plates	F	0.15	
Harmonic number of the main resonator	h	4	
Harmonic number of the flat-topping resonator	n	3	
Orbit velocity of particle	w_d	25718648	rad/s
Valley radius at injection	L_i	0.83	m
Valley radius at extraction	L_e	3.62	m
Vertical distance between dee plates	d	0.11	m

The equations used for the calculations on 84 turns in the cyclotron, are given below.

The average kinetic energy of the particle (in MeV) on orbit number j is:

$$T_j = T_{j-1} + \left(\frac{4V_h Q - 2V_{p_{j-1}} Q}{1 \times 10^6} \right),$$

where Q is the charge state of the particle.

The radial position on the orbit in the middle of the valley (in m) is:

$$R_j = \frac{hc}{7.196 f_h} \sqrt{\frac{T_j^2 + 2T_j E_0}{T_j^2 + E_0^2 + 2T_j E_0}}.$$

The flat-topping voltage (in Volt) is:

$$V_j = V_p \sin \left(p \frac{R_j - L_i}{L_e - L_i} \right).$$

The velocity of the particle (in m/s) is:

$$u_j = \frac{c}{g_j} \sqrt{g_j^2 - 1}, \text{ where } g_j = \frac{T_j + E_0}{E_0}.$$

The distance traveled between the two acceleration gaps of the flat-topping resonator (in m) in a half period of the resonator is:

$$s_j = \frac{u_j}{2n f_h}.$$

The vertical displacement and vertical divergence of the particle at the exit gap of the resonator are respectively given by:

$$z_{r_j} = z_{j-1} + \frac{2p}{24} z'_{j-1} + \frac{z_m V_j}{V_p} \quad (\text{J.17})$$

and

$$z'_{r_j} = \frac{z'_{j-1}}{n_{z_j}}, \quad (\text{J.18})$$

where n_{z_j} is the vertical betatron frequency of the orbit and z_m is the maximum vertical displacement per turn (in mm), given by:

$$z_m = \frac{-2qFV_p}{h^2 n^2 w_d^2 m_d d} \times 1000.$$

For the first orbit the first two terms on the right hand side of equation J.17 and the value of equation J.18 are zero.

These intermediate values at the exit gap of the resonator are used to complete the calculation for the rest of the turn. The displacement and divergence are respectively given by:

$$z_j = z_{r_j} \cos\left(\frac{23}{24} 2pn_{z_j}\right) + \frac{z'_{r_j}}{n_{z_j}} \sin\left(\frac{23}{24} 2pn_{z_j}\right)$$

and

$$z'_j = -z_{r_j} n_{z_j} \sin\left(\frac{23}{24} 2pn_{z_j}\right) + z'_{r_j} \cos\left(\frac{23}{24} 2pn_{z_j}\right).$$

The calculation process is terminated once the extraction radius is reached.

J.4 RESULTS

The calculated vertical displacement and divergence of the particle for the above set of parameters, where the voltage between the dee plates is 15 % of the flat-topping voltage, are 0.036 mm and -0.061 mrad respectively. The calculated beam phase plot to show the progress per turn is shown in figure J.2, indicated with solid markers. The values for injection are at the origin of the graph. The calculated values are artificially connected with a smoothed curve to visually show the progress of the vertical motion per turn.

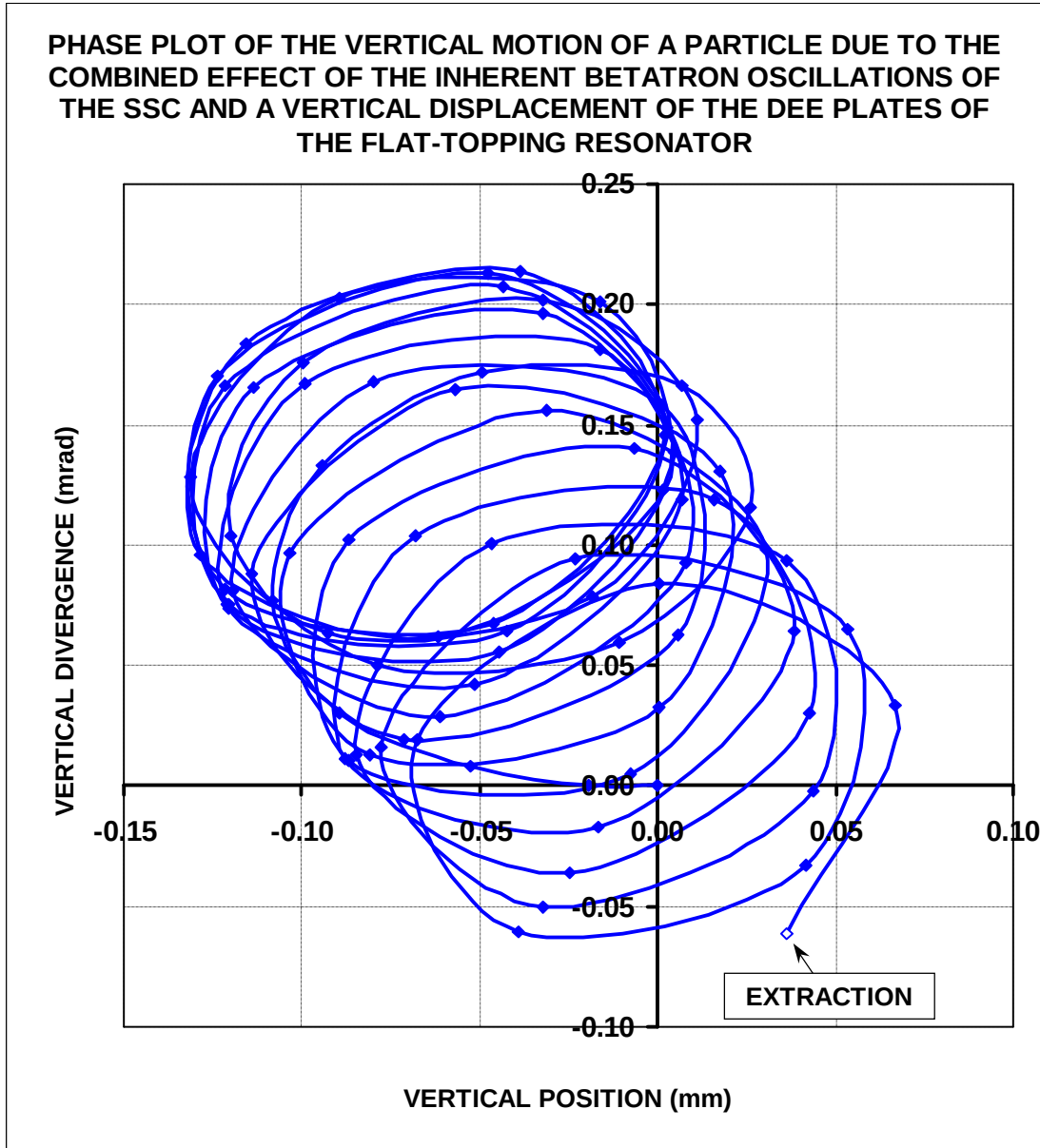


Figure J.2. The calculated beam phase plot of the vertical motion in the SSC due to a voltage between the dee plates of the flat-topping resonator caused by a vertical displacement of the dee plates. The energy of the proton at the extraction orbit is 66 MeV and the calculated vertical displacement and divergence are a fraction of a millimeter and a milliradian respectively.

REFERENCES

- [Abr75] Abridged report on the requirements for and the choice of a National Accelerator Facility for the Republic of South Africa and the placing thereof, iThemba LABS, unpublished (1975).
- [Bis79] Bischof B., *The rf-system of the flat-top-acceleration structure in the SIN 590-MeV-Ring-Cyclotron*, (IEEE Transactions on Nuclear Science, Vol. NS-26, No. 2, 1979).
- [Bot74] Botha A.H., *Skaalfaktore vir resonator-modelle*, NAC Report I20-Rfre-0001, (1974).
- [Bot75] Botha A.H. and Kritzinger J.J., *Design of an Rf System for an Open-sector Cyclotron*, Proceedings of the 7th International Conference on Cyclotrons and their Applications, (Zürich, 1975), Birkhäuser Verlag, Basel, Switzerland (1975), p.156.
- [Bot79] Botha A.H. and Van der Merwe F.S., *Design of circuits for power amplifiers to resonators*, IEEE Transactions on Nuclear Science, VolNS-26, (1979) No. 3, p.3962-3964.
- [Bot88] Botha A.H. and De Villiers J.G., Notes on formulas for a hard-edge separated-sector cyclotron, iThemba LABS, Unpublished.
- [Bot04a] Botha A.H., Conradie J.L. and De Villiers J.G., Locally developed computer program for calculating resonator characteristics using transmission line theory, iThemba LABS, Unpublished.
- [Bot04b] Botha A.H. Private Notes, iThemba LABS, Unpublished.
- [Bot05] Botha A.H., Notes on Inductive Coupling Systems, iThemba LABS, Unpublished.
- [Con92a] Conradie J.L., *Improved proton Beam Quality and Intensity from a 200 MeV Cyclotron System*, Ph.D. Thesis, (University of Stellenbosch, 1992), p.2.

- [Con92b] Ibid, p.122.
- [Con92c] Ibid, p.190.
- [Con92d] Ibid, p.158.
- [Con92e] Ibid, p.11.
- [Con92f] Ibid, p.39.
- [Con92g] Ibid, p.69.
- [Con92h] Ibid, pp.65-124.
- [Con92i] Ibid, p.159.
- [Con92j] Ibid, p.100.
- [Con98] Conradie J.L., De Villiers J.G. and Botha A.H., *A Flat-topping System for the NAC Separated-sector Cyclotron*, Proceedings of the 15th International Conference on Cyclotrons and Their Applications (Caen, 1998), Institute of Physics Publishing, Bristol, UK (1999), p.215.
- [Con01] Conradie J.L., *Cyclotron specifications: entry C-38*, Proceedings of the 16th International Conference on Cyclotrons and their Applications (East Lansing 2001), American Institute of Physics, New York, USA (2001), p.523.
- [Cro87] Cronje P.M., *Orbit studies for the NAC separated-sector cyclotron*, Proceedings of the 11th International Conference on Cyclotrons and their Applications (Tokyo, 1986), Ionics Publishing, Tokyo (1987), pp.244-247.
- [DeV04] De Villiers J.G., et al., *A flat-topping System for the Separated Sector Cyclotron at iThemba LABS*, Proceedings of the 17th International Conference on Cyclotrons and their Applications (Tokyo, 2004), Copyright Particle Accelerator Society of Japan (2005), p.344.
- [Edw93] Edwards D.A. and Syphers M.J., *An Introduction to the Physics of High Energy Accelerators*, Copyright John Wiley & Sons, Inc. (1993), p.54.

- [Gor67] Gordon M.M., *Design Considerations for a Separated Turn Isochronous Cyclotron*, Nuclear Instruments and Methods 58, (1968), Copyright North-Holland Publishing Co., pp.245-252.
- [Hal74] Halliday D. and Resnick R., *Fundamentals of Physics*, Revised Printing, John Wiley & Sons, Inc. (1974), p.351.
- [Hol72] Holman J.P., *Heat transfer*, International Student Edition, Edition 2, McGraw-Hill Kogakusha Ltd. (1972).
- [Joh50a] Johnson W.C., *Transmission lines and networks*, International Student Edition, McGraw-Hill Book Company, Tokyo, Kagakusha Company, Ltd (1950), p.105.
- [Joh50b] Ibid, p.93.
- [Joh50c] Ibid, pp.100-105.
- [Joh50d] Ibid, p.167.
- [Joh68a] Joho W., *Tolerances for the SIN Ring-cyclotron*, SIN Report TM-11-4, (Zürich 1968), p.IX-4.
- [Joh68b] Ibid, p.IV-5.
- [Joh68c] Ibid, p.VIII-7.
- [Joh68d] Ibid, p.IX-5.
- [Joh81] Joho W., *High intensity problems in cyclotrons*, Proceedings of the 9th International Conference on Cyclotrons and their Applications (Caen, 1981), Les Editions de Physique, Paris (1982), p.337.
- [Koh98] Kohara S., et al., *Model Study of a Resonator for Flat-top Acceleration System in the RIKEN AVF Cyclotron*, Proceedings of the 15th International Conference on Cyclotrons and Their Applications (Caen, 1998), Institute of Physics Publishing, Bristol, UK (1999), p.219.
- [Kon64] Koning P. and Kritzinger J.J., *Report 1* (Southern Universities Nuclear Institute, 1964-1968), Annual Reports, p.1.

- [Kra92a] Kraus J.D. and Carver K.R., *Electromagnetics*, Fourth Edition, Copyright by McGraw-Hill, Inc. (1992), p.206.
- [Kra92b] Ibid, p.500.
- [Kur01] Kurashima S., et al., *Design of the Flat-top Accelerator System for the JAERI AVF Cyclotron*, Proceedings of the 16th International Conference on Cyclotrons and their Applications (East Lansing 2001), American Institute of Physics, New York, USA (2001), p.303.
- [Mic88] Micro-cap, Evaluation copy version 7.1.7.1, Copyright 1988-2002, Spectrum Software.
- [Nor02] Nortier F.M. et al., *Production Plans for the new 100 MeV Isotope Production Facility at LANL*, Proceedings of the 9th International Workshop on Targetry and Target Chemistry, Turku, Finland (2002).
- [Ram65a] Ramo S., Whinnery J.R. and Van Duzer T., *Fields and Waves in Communications Electronics*, Wiley, New York (1965), p.289.
- [Ram65b] Ibid, p.251.
- [Ram65c] Ibid, p.290.
- [Ram65d] Ibid, p.8.
- [Rau75] Rautenbach W.L. and Botha A.H., *Proposal for a South African National Accelerator Facility for Physics and Medicine*, Proceedings of the 7th International Conference on Cyclotrons and their Applications, (Zürich, 1975), Birkhäuser Verlag, Basel, Switzerland (1975), pp.117-120.
- [Rep75] Report on the requirements for and the choice of a National Accelerator Facility for the Republic of South Africa and the placing thereof, unpublished (1975).

- [Rog84] Rodgers R.C., *Design of R.F. Systems for Compact Cyclotrons*, Proceedings of the 10th International Conference on Cyclotrons and their Applications (East Lansing 1984), IEEE, New York, USA (1984), p299.
- [Sak01] Sakamoto N. et al., *Construction of the RF-resonator for the Riken Intermediate-Stage Ring Cyclotron, (IRC)*, Proceedings of the 16th International Conference on Cyclotrons and their Applications (East Lansing 2001), American Institute of Physics, New York, USA (2001), p.306.
- [Sai86] Saito T. et al., *The RF System for the RCNP Ring Cyclotron*, Proceedings of the 11th International Conference on Cyclotrons and their Applications (Tokyo, 1986), Ionics Publishing, Tokyo (1987), pp. 341-344.
- [Sai89] Saito T. et al., *The RF System for the RCNP Ring Cyclotron*, Proceedings of the 12th International Conference on Cyclotrons and their Applications (Berlin 1989), World Scientific Publishing Co., Singapore (1991), pp. 201-204.
- [Sai92] Saito T. et al., *Initial Operation of the RF System for the RCNP Ring Cyclotron*, Proceedings of the 13th International Conference on Cyclotrons and their Applications (Vancouver 1992), World Scientific Publishing Co., Singapore (1993), p.538.
- [Sch76] Schneider S., Botha A.H. and Jungwirth H.N., *Beam Injection and Extraction for a 150 MeV Separated-sector Cyclotron*, Internal Report for the national Accelerator Project, I2O–EIAA-0005 (1976), p.90.
- [Sch73] Schnell G., *Magnete*, Thiernig Taschenbucher – Band 49, Copyright 1973, Verlag Karl Thiernig, München, p.107.
- [Ter55] Terman F.E., *Electronic and Radio Engineering*, International Students Edition, Copyright 1955, McGraw-Hill, Kogakusha Ltd., Tokyo, p.59.
- [Van78] Van der Merwe F.S., *RF-Koppelstelsel vir Resonator van 'n Soliede-Pool Siklotron – Ondersoek na moontlikhede*, NAC Reports RF 78-06.

- [Van04] Van der Walt T.N., et al., *Thick targets for the production of some radionuclides and the chemical processing of these targets at iThemba LABS*, Nuclear Instruments and Methods in Physics Research A 521, (2004), pp.171-175.
- [Vec_1] Vector Fields Ltd, 24 Bankside, Kidlington, Oxford OX5 1JE, England.
www.vectorfields.co.uk , info@vectorfields.com
- [Vec_2] *User Guide*, Vector Fields Ltd., 24 Bankside, Kidlington, Oxford OX5 1JE, England.
- [Wil63] Willax H.A., *Proposal for a 500 MeV isochronous cyclotron with ring magnets*, International conference on sector-focused cyclotrons and meson factories (CERN 63-19, Geneva 1963), p.386.

-o-O-o-