

CYCLIC STRAIN INDUCED CREEP IN THIN WALLED ALUMINIUM ALLOY TUBES

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CYCLIC STRAIN INDUCED CREEP IN THIN WALLED ALUMINIUM
TUBES SUBJECTED TO CYCLIC TORSION AND STEADY TENSION

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Science in Engineering.

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(ii)

DECLARATION

I hereby declare that all the work contained herein is my own and has not been submitted to any other university for a higher degree.

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(iii)

SUMMARY

An investigation was carried out into the progressive axial deformation, or cyclic strain induced creep, occurring in thin walled aluminium alloy tubes subjected to strain controlled plastic torsion cycles in the presence of a steady tensile load. The investigation aimed to examine whether it was possible to obtain cyclic creep predictions using basic mechanical test data.

An existing testing machine was modified and some control and transducer equipment built to achieve the desired loading conditions. Cyclic creep tests were carried out over a range of plastic shear strain amplitudes and axial loads and the results were compared with theory.

Two existing theoretical approaches were modified and used for creep predictions. An expanding yield surface theory gave reasonable estimates of the steady state cyclic creep rates provided that the plastic shear strain range was greater than a tensile stress dependent value. A viscoplastic theory was used to obtain total creep curve predictions, but did not give satisfactory results.

(iv)

An Mutti und Papi. Vielen Dank für alles.

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LIST OF SYMBOLS USED

σ_{ij}	= General stress tensor
ϵ_{ij}	= General strain tensor
$\sigma_1, \sigma_2, \sigma_3$	= Principal stresses
s_{ij}	= Deviatoric stress $= \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$
$\bar{\sigma}$	= Effective stress $= \sqrt{\frac{3}{2}s_{ij}s_{ij}}$
e_{ij}	= Deviatoric strain $= \epsilon_{ij} - \frac{1}{3}\delta_{ij}\epsilon_{kk}$
$\bar{\epsilon}$	= Effective strain $= \sqrt{\frac{2}{3}e_{ij}e_{ij}}$
E	= Elastic modulus in tension
G	= Elastic modulus in torsion
σ	= Tensile stress
τ	= Shear stress
ϵ	= Axial strain
γ	= Shear strain (Engineering definition)
N	= Number of loading cycles

Prefixes

Δ = Range

Subscripts

y	= Yield quantity
c	= Critical quantity
m	= Maximum quantity
max	= Maximum quantity

Superscripts

p = Plastic quantity

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CHAPTER 1

INTRODUCTION

There are many situations in which plastic flow may occur in metal structures or components subjected to mechanical and thermal loading conditions. Plastic deformation may take place unintentionally due to accidental damage during manufacture or may occur at stress concentrations arising due to machining errors or incorporated in the design. In other situations plastic flow is unavoidable such as creep at high temperatures. Under conditions of variable loads and temperatures, plastic flow may also occur. It has been found that in applications such as steam and gas power plant, pressure vessels and heat exchangers, there are large temperature gradients and hence high transient stresses during start up and shut down of the machinery. These stresses can produce plastic strains not accounted for in the steady state analysis of continuous running conditions.

In some instances a component may be deliberately designed to operate in the plastic region so as to utilize the added strength of the strain hardened metal at the expense of a reduced life. This is common in the aerospace industry where minimum weight is a very important design criterion. The designer thus needs information about the plastic behaviour of the material he is using to enable life predictions to be made. Empirical knowledge is useful, but often difficult to obtain and only of limited applicability. Extrapolation from data obtained for other, seemingly similar, situations may often be unwarranted and erroneous. It should rather be attempted to develop a sound framework of plasticity theories which enable the prediction of plastic behaviour from a minimum of readily obtainable basic data.

One particular aspect of plasticity is the behaviour of metals under conditions of cyclic loading where plastic strains occur during each cycle. Failure is then caused by either high strain fatigue fracture or cyclic creep. Cyclic creep, the subject of the present investigation, is caused by a steady or follow up load superimposed on plastic loading cycles. Although the follow up load alone is insufficient to

cause yield, it is responsible for a steadily accumulating creep deformation in the follow up load direction. A component thus loaded may then fail by gross deformation long before high strain fatigue fracture occurs.

Two types of cyclic creep can be distinguished:

- (a) When the cyclic and follow up loads are in the same direction, the cyclic stress-strain curve is asymmetric, causing deformation to accumulate with each cycle.
- (b) In the case of the follow up and cyclic loads not having the same direction, the cyclic stress-strain curve is symmetric, but creep occurs in the direction of the follow up load. This type of creep is sometimes called cyclic strain induced creep, ratchetting or incremental collapse.

The purpose of the present investigation was to study cyclic strain induced creep in an aluminium alloy for a range of cyclic and follow up loads. To do this, an existing testing machine was modified to enable testing of thin walled tubular specimens in alternating torsion and steady tension. Some theoretical approaches were investigated with the aim of predicting cyclic creep behaviour from a minimum of material data.

CHAPTER 2.

REVIEW OF PREVIOUS WORK

2.1 Introduction

When a metal is subjected to plastic loading cycles in the presence of a steady follow up load, a steadily accumulating deformation, termed cyclic creep, will occur in the direction of the follow up load, even though the latter alone is insufficient to cause yield. Cyclic creep has been observed for a variety of metals and loading conditions. Of particular interest is the case where the cyclic and follow up loads are not in the same direction. The term "cyclic strain induced creep" has been suggested by Coffin (1) for this situation.

Edmunds and Beer (2) found that cyclic strain induced creep occurred in cantilevers subjected to alternating bending in one plane with follow up bending or tensile stresses in another direction, as well as in pipes subjected to internal pressure and cyclic bending. Tests conducted by Ronay (3) in aluminium specimens subjected to reversed torsion cycles showed that axial elongation occurred, even in the absence of a tensile follow up load. The magnitude of the accumulated creep, however, was far smaller than could be obtained with a nonzero follow up load. Udoguchi and Asada (4) observed cyclic strain induced creep for several commercial steels subjected to axial strain cycling and a steady torque. They also obtained some empirical relations between axial strain range, accumulated shear strain, number of cycles and applied shear stress.

Two different types of cyclic strain induced creep have been observed, which are analogous to high temperature and room temperature thermally induced creep (fig. 2.1). For creep analogous to high temperature creep, Wood and Cousland (5) recognized four stages of creep in cylindrical specimens subjected to cyclic torsion and steady tension. The first torque application resulted in a relatively large axial strain. Upon further cycling, the creep increment per cycle fell rapidly, similar to the primary creep region in

high temperature thermal creep, until a region of constant cyclic creep rate was observed. This corresponded to the steady state or secondary creep region. Finally, just before fracture, the cyclic creep rate increased again as in tertiary creep.

The logarithmic type of creep was investigated by Reimann and Wood (6) who tested armco iron in alternating torsion and steady tension. It was found that this type of creep occurred for small shear strain ranges (0,003 - 0,012).

2.2 Plastic Behaviour of Metals

2.2.1 The crystalline structure of metals

All commercially available metals consist of a large number of crystals or grains in which the atoms are stacked in a regular order. The stacking orientation, however, varies from grain to grain. In addition, each grain contains numerous stacking faults or dislocations. Permanent plastic deformation is caused by dislocations moving through the crystal under the influence of a stress greater than the yield stress. Strain hardening may be understood in terms of dislocations interacting with each other and piling up against grain boundaries and other obstacles. A comprehensive description of crystal defects and their role in plastic deformation may be found in (7).

Any investigation into plasticity should thus take account of the polycrystalline structure of metals. The slip theory of Batdorf and Budiansky (8, 9) has, in fact, been developed from considerations of dislocation slip but at present it is still of limited applicability. The same is true for Mura's continuum theory of dislocations (10) which attempts to formulate a plasticity theory in terms of a continuous distribution of dislocations. Similar work by other authors on the Bauschinger effect (11) and cycling loading (12) is essentially qualitatively useful, but difficult to apply quantitatively. Alternative approaches to plasticity thus need to be considered.

2.2.2 Yield surfaces

A yield surface is a surface in 9-dimensional stress space σ_{ij} which gives the locus of all possible stress combinations which will cause a given material to yield. Plastic flow will occur when the stress vector touches the yield surface while any stress changes inside the surface will not cause yield. During plastic flow the yield surface moves with the stress vector. It is common to call the initial surface for the unstrained metal a yield surface while subsequent surfaces are termed loading surfaces. Drucker (13) has derived from quasi-thermodynamic considerations the following results.

- (a) The increment of plastic strain $d\epsilon_{ij}^p$ must be normal to the yield surface. If we define the yield surface as

$$f(\sigma_{ij}) - \text{CONST} = 0 \quad 2.1$$

then
$$d\epsilon_{ij}^p = d\lambda \cdot \partial f / \partial \sigma_{ij} \quad 2.2$$

where $d\lambda$ is a scalar value depending on the stress, strain, stress increment and loading history.

- (b) $d\lambda$ is related to the stress increment

$$d\lambda = G (\partial f / \partial \sigma_{kl}) d\sigma_{kl} \quad 2.3$$

where G , also a scalar, now depends only on the stress, strain and loading history.

It is generally assumed that the hydrostatic stress component $\frac{1}{3}\sigma_{ii}$ has no effect on plastic deformation. In that case the yield surface may be represented graphically as a cylinder in principal stress space $(\sigma_1, \sigma_2, \sigma_3)$ along the hydrostatic stress axis $\sigma_1 = \sigma_2 = \sigma_3$, or simply as a closed curve on the π -plane, this being a plane through the origin of the stress space, normal to the hydrostatic stress axis.

To use the equations above, some information is required about the behaviour of the yield and loading surfaces. Paul (14) reviews some experimentally obtained yield and loading surfaces and it is shown that the shape and size of the surfaces are affected to a very large degree by the definition of yield used. A permanent set definition, for example, would indicate that the yield strength in one direction is affected by prestraining in another direction, while for a proportional

limit definition of yield this cross effect is absent, as also shown by Phillips and Tang (15).

Michno and Findley (16, 17) determined yield and loading surfaces for thin-walled SAE 1017 steel tubes under combinations of axial and torsional loads. It was found that the normality and convexity rules were obeyed. Sharp corners in the loading surface, as predicted by the slip theory, were not encountered, but occasionally well rounded corners were formed under combined tension and torsion. The behaviour of the loading surfaces, however, was quite complex.

It seems thus that an exact description of the loading surface behaviour during plastic straining will be very complicated. An alternative approach has been to assume a form of loading surface behaviour and then check how this predicted actual stress-strain behaviour. Some of the more commonly used loading surface theories are discussed below.

(a) Piecewise linear theory

Any yield surface can be described in terms of a series of linear segments on the π -plane. To decrease the complexity, the number of planes should be small. An example of this is Tresca's yield criterion which has six planes (fig. 2.2). During yield the stress vector carries with it that particular plane it is in contact with. The theory is completely general if the planes move independently. For the theory to have any predictive value, the behaviour of the planes must be specified as has been done in the following theories.

(b) Isotropic hardening theory

The yield surface expands uniformly while the centre remains fixed (fig. 2.3). Von Mises yield criterion is an example of this. The theory works fairly well for single direction loading, but during reversed loading the actual yield stress is lower than the original yield stress. This is called the Bauschinger effect which the isotropic theory makes no provision for. For plastic load cycling with a superimposed follow up load, deformation in the direction of the follow up load will, according to this theory, occur only

during the first quarter loading cycle.

(c) Kinematic hardening theory

This theory assumes a yield surface of constant size which translates in stress space (fig. 2.4). The centre moves in the direction of the plastic strain increment by an amount which depends on the plastic work parameter $b_{ij} \epsilon_{ij}^p$. The Bauschinger effect is accounted for by this theory. However, the theory is only of limited applicability to the cyclic creep problem as, for a tubular specimen subjected to alternating torsion - steady tension conditions, it predicts a limit to the creep strain far below that observed in practice (18).

Combinations of the isotropic and kinematic theories are also possible but again are only of limited applicability to the creep problem as they tend to predict small limiting creep values. Such theories may be useful, however, for describing logarithmic cyclic creep.

2.2.3 Viscoplasticity and Thermodynamics

Viscoplasticity theories attempt to describe more general non linear time-, temperature - and strain rate dependent behaviour of inelastic solids. In linear viscoelasticity the stress δ in a solid subjected to a strain history $\epsilon(t)$ is given by the hereditary integral

$$\delta(t) = \int_0^t E(t-\tau) \cdot d\epsilon(\tau) / d\tau \cdot d\tau \quad 2.4$$

Nonlinear behaviour can also be represented in a similar fashion (19) although the actual equations are more complex.

One approach is to mathematically model the solid as a combination of springs (elastic elements), dashpots (linear viscoelastic elements) and friction blocks (perfectly plastic elements). Hilton (20) has shown how non linear creep for time independent temperatures and stresses may be obtained from such a model.

A comparatively recent development is the application of the laws and concepts of classical thermodynamics to inelastic strained solids. The subject is reviewed by Martin

(21), and it is shown that the general framework thus developed includes the description of viscoelastic materials, for example. This approach is promising, especially as regards the development of constitutive relations.

2.3 Yield surface theories for cyclic plasticity

The yield surface theory of Mroz (22) consists of a series of nested loading surfaces surrounding the yield surface (fig. 2.5). Each surface grows and translates once it has come into contact with the stress vector or another surface. Once a particular surface has started to move, the work hardening modulus stays constant until the next surface is contacted. This amounts to approximating the uni-axial stress-strain curve by a series of linear segments. During nonproportional loading, such as preloading in one direction and loading in another, it is assumed that the surfaces cannot intersect each other and in fact behave like a set of nested frictionless rings lying on a flat surface. This theory is then applied by Mroz (23) to the problem of alternating torsion and steady tension. For simplicity, only two loading surfaces are assumed, the inner and outer ones behaving in a purely kinematic and isotropic fashion respectively. Despite this simple model the strain equations become quite complex. No experimental results for comparison are presented.

A similar theory by Krieg (24) is described as a two surface Mroz theory with a continuum of intermediate loading surfaces whose distribution is assumed beforehand. The model consists of an isotropic - kinematic yield surface and an isotropic - kinematic limit surface. These two do not touch but are both affected by plastic straining. The work hardening modulus is determined by the distance between the surfaces at the tip of the stress vector. The theory is then applied to tension compression cycling but the resulting equations become quite complex. Agreement with experiment is good, but a large number of empirical relations had to be used.

Chandler (18) has suggested, from considerations of the cyclic behaviour of mild steel under steady tension and alternating torsion, a steady-state yield surface which can be described as a semi-ellipse in σ - τ stress space expanding within a limiting isotropic ellipse (fig. 2.6). It is also shown that the plastic shear strain during each loading half cycle must exceed a critical value γ_c^p before any significant axial creep can occur. The steady state axial creep rate $d\epsilon^p/dN$ is obtained from the following equation.

$$\frac{d\epsilon}{dN} = \frac{2}{3\tau_m} \oint \tau d\gamma^p \quad \text{FOR } \gamma^p > \gamma_c^p \quad 2.5$$

where σ = applied tensile stress

τ_m = maximum shear stress reached during cycle

The theory works well for larger plastic shear strain ranges (>0.025 for En 3 and En 19 steels) while it overestimated creep rates for smaller shear strain ranges.

Freudenthal and Ronay (25) showed that cyclic strain induced creep can also be interpreted as an irreversible second order effect amplified by an applied follow up load. The second order effect arises from a quadratic term in the constitutive equation which is usually ignored for monotonic loading. In a cyclic loading case its effect accumulates, however, and cyclic strain induced creep is predicted even in the absence of a follow up load, as was actually observed.

2.4 Viscoplastic theories for cyclic plasticity

Some work has already been done in the application of conventional creep equations to the cyclic loading case (26, 27), but only for coincident cyclic and follow up loads.

Rashid (28) briefly reviews some methods of creep analysis of structures subjected to variable mechanical and thermal loading histories at high temperatures. These methods include expressing the creep rate $\dot{\epsilon}_{ij}^c$ as the gradient of a potential function f .

$$\dot{\epsilon}_{ij}^c = \partial f / \partial b_{ij}$$

as well as using non linear viscoelastic equations. Some methods of numerical analysis are described as is the

obtaining of material data. Three numerical examples are worked out to compare different equations but no experimental comparison is made.

It has been pointed out by Valanis (29) that, for time independent plasticity, the stress in a strained solid "depends on the set of all previous states of deformation of that neighbourhood, but it does not depend on the rapidity at which such deformation states have succeeded one another". For this reason, it is suggested that the history of deformation be related to a "time" scale, which is not measured by a clock, but intrinsically related to the deformation history of the material. This concept may be extended to time dependent plasticity by having the intrinsic "time" depend on both the deformation history and the external, real, time. It is suggested that an intrinsic time increment $d\xi$ may be written as

$$d\xi^2 = g^2 dt^2 + h^2 d\epsilon^2 \quad 2.6$$

where dt is a real time increment

$d\epsilon$ is a deformation increment

g and h are scalar material parameters

Valanis then developed a theory of viscoplasticity from thermodynamical considerations and showed that stress and strain can be related by a hereditary integral similar to eq. 2.4 except that the integral is evaluated over an interval of intrinsic, rather than real, time. For the particular case of tension and torsion respectively.

$$\sigma = \int_0^{\xi} E(z-z') \cdot \delta \epsilon / \delta z' \cdot dz' \quad 2.7$$

$$\tau = \int_0^{\xi} G(z-z') \cdot \delta \gamma / \delta z' \cdot dz' \quad 2.8$$

where z is an internal time scale, which is a function of the time measure ξ

$$dz = d\xi / f(\xi) \quad 2.9$$

$d\epsilon$, the deformation increment, may be written as

$$d\epsilon^2 = k_1 d\epsilon_{ij} d\epsilon_{ij} + k_2 d\epsilon_{ij} d\epsilon_{ij} \quad 2.10$$

where K_1 and K_2 in general depend on ϵ_{ij}

$E(z)$, $G(z)$ are expressed as a sum of decaying exponential terms

$$E(z) = \sum_{r=0}^m E_r e^{-\alpha_r z} \quad 2.11$$

$$G(z) = \sum_{r=0}^m G_r e^{-\alpha'_r z} \quad 2.12$$

where E_r , G_r , α'_r and α_r were positive material constants. In further work on this endochronic theory (30, 31, 32) Valanis found that:

(a) The form of $f(\xi)$ in eq. 2.9 that gave the best results was

$$f(\xi) = 1 + \beta \xi$$

$$z = 1/\beta \cdot \log(1 + \beta \xi) \quad 2.13$$

(b) For monotonic loading a one term approximation of $E(z)$ and $G(z)$ could be used while for loading - unloading a two term approximation sufficed.

$$E(z) = E_0 e^{-\alpha_1 z}$$

$$G(z) = G_0 e^{-\alpha_1 z} \quad \text{for loading} \quad 2.14$$

where E_0 and G_0 are the initial slopes of the respective stress-strain curves.

$$E(z) = E_1 + E_2 e^{-\alpha_2 z}$$

$$G(z) = G_1 + G_2 e^{-\alpha_2 z} \quad \text{for loading - unloading} \quad 2.15$$

where

$$E_1 + E_2 = E_0$$

$$G_1 + G_2 = G_0$$

E_1 and G_1 are limiting slopes for large strains. The theory was then applied to various aspects of plasticity including cyclic strain induced creep (32) and good agreement with experimental results was obtained.

2.5 Concluding remarks

Of the foregoing theories, it was decided to select two for further investigation as these seemed the most promising. These were the expanding ellipse yield surface theory (18)

and the endochronic theory of viscoplasticity by Valanis
(32). They will be developed and discussed in more detail
below.

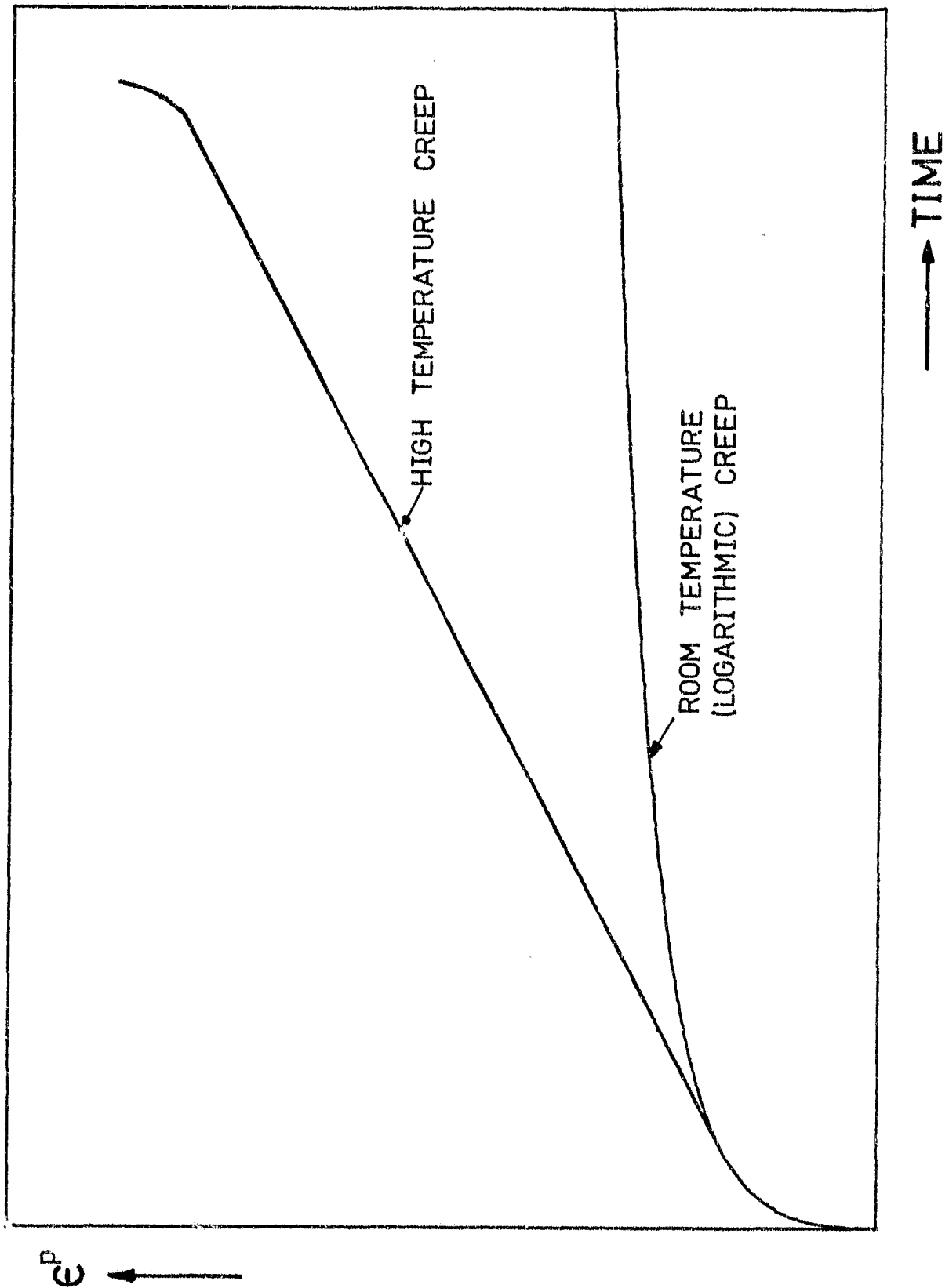


Fig. 2.1 High Temperature and Room Temperature
Thermally Induced Creep

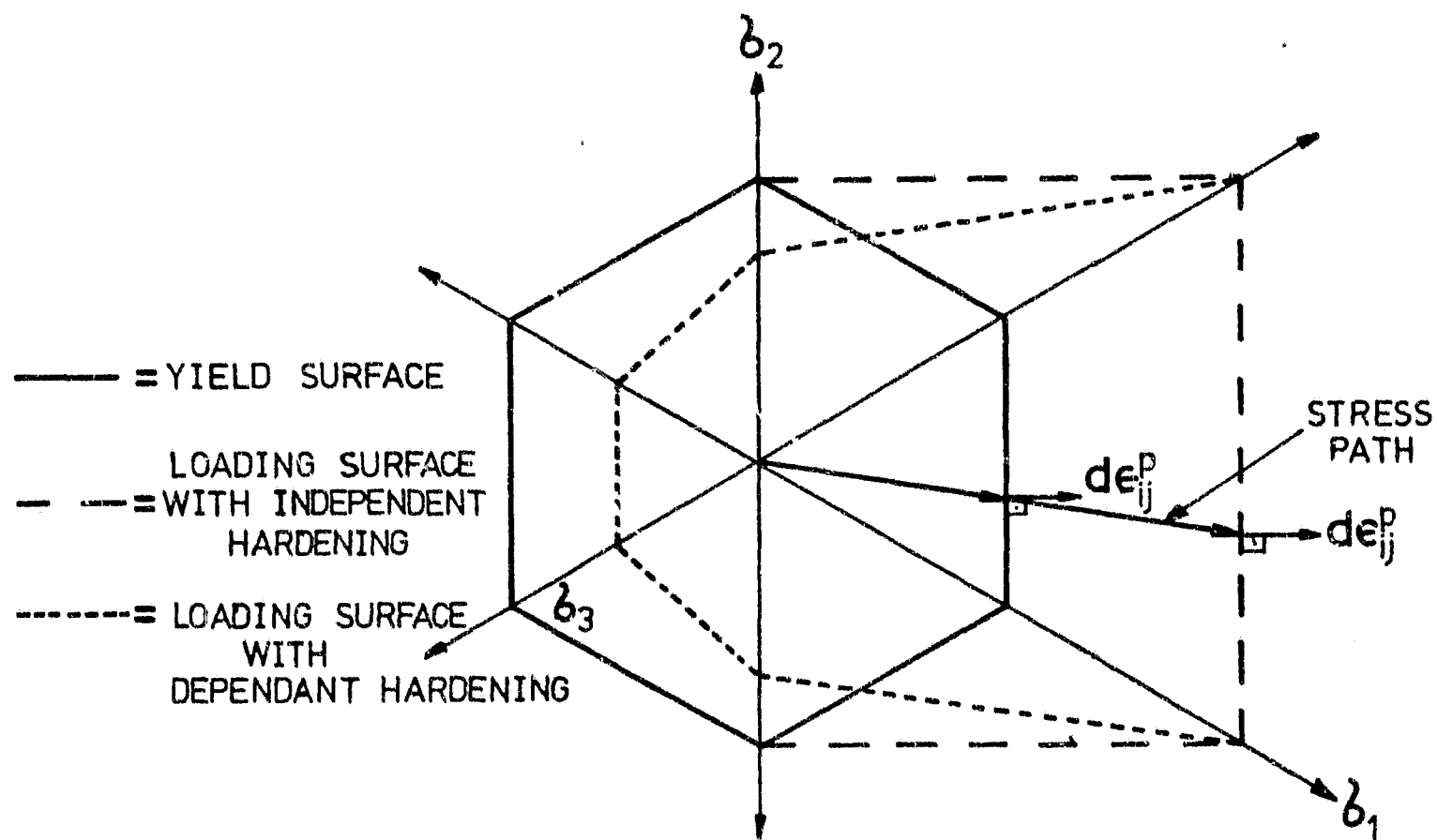


FIG. 2.2 PIECEWISE LINEAR YIELD SURFACE THEORY

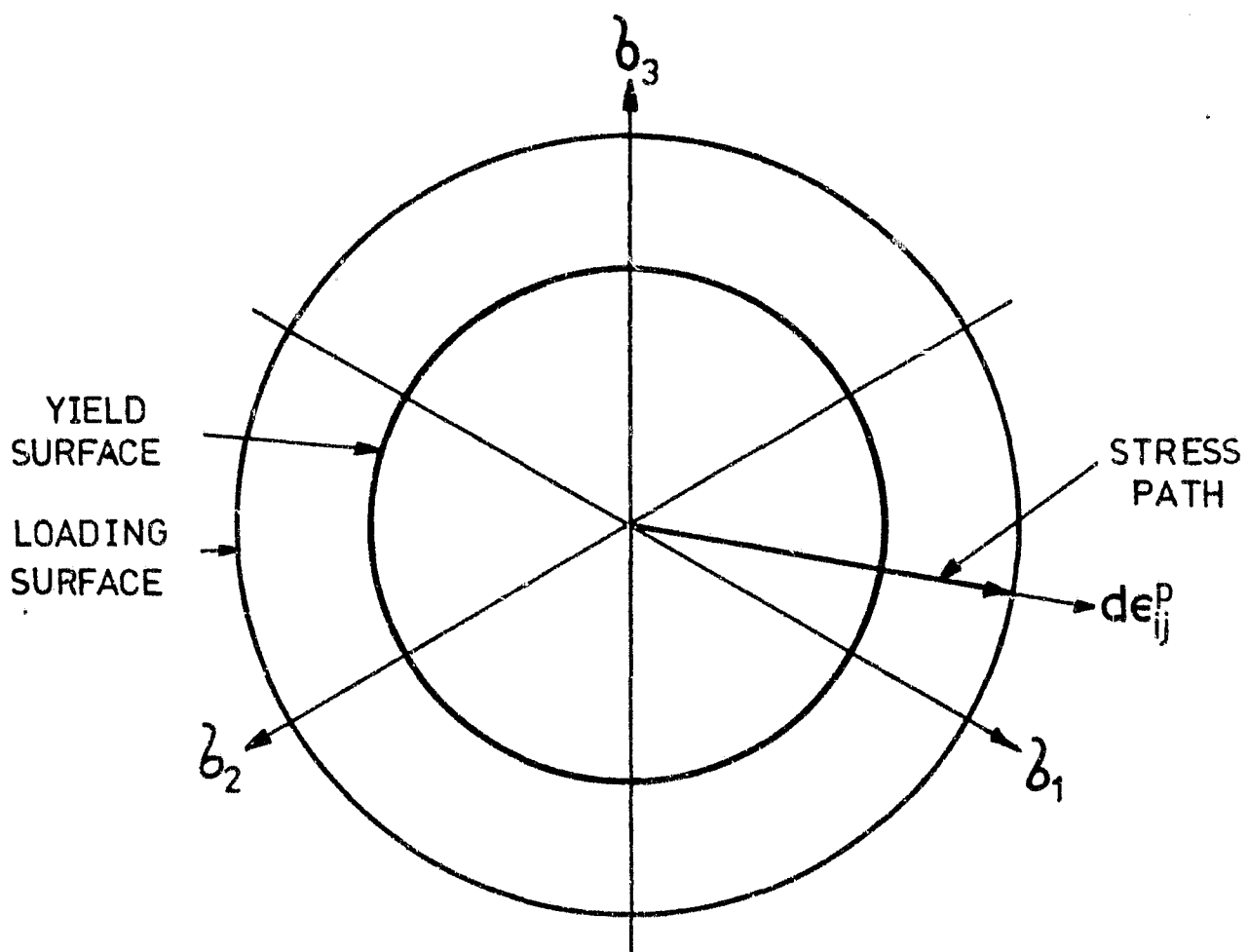


FIG. 2.3 ISOTROPIC HARDENING THEORY

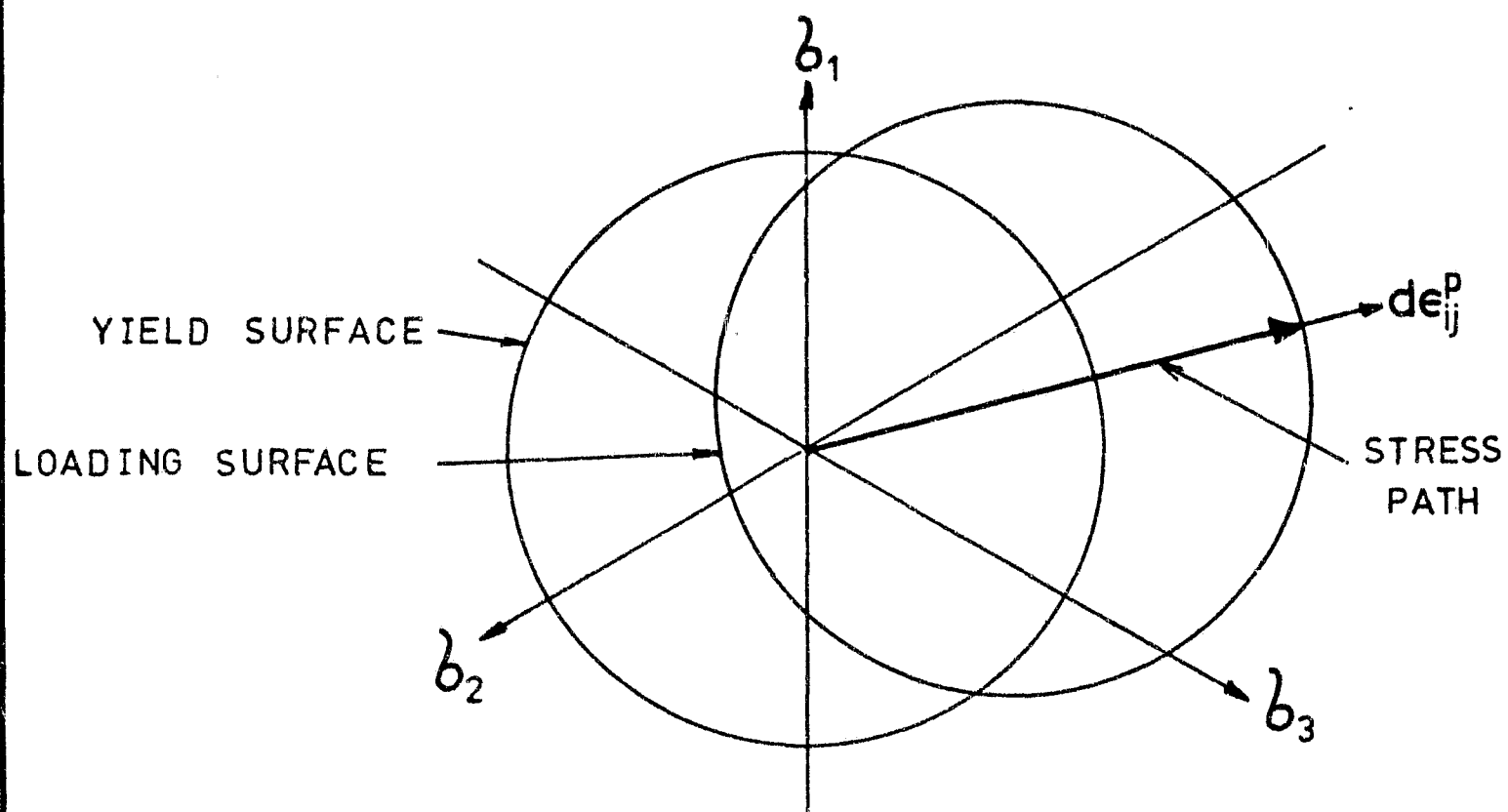
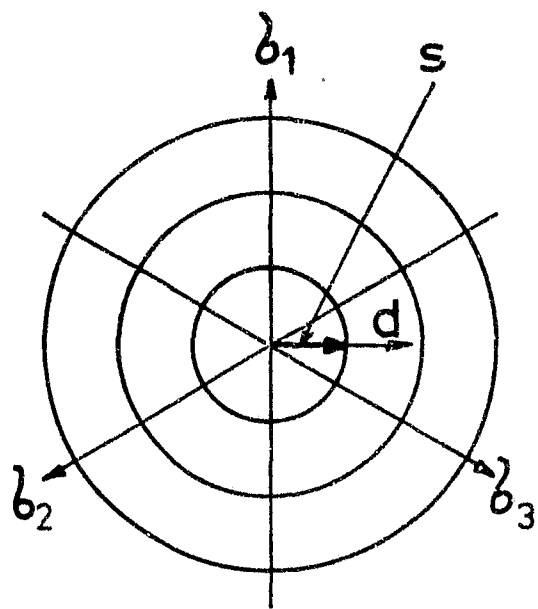
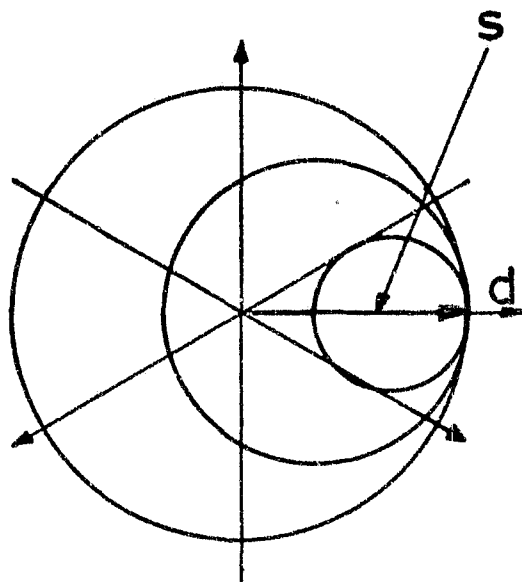


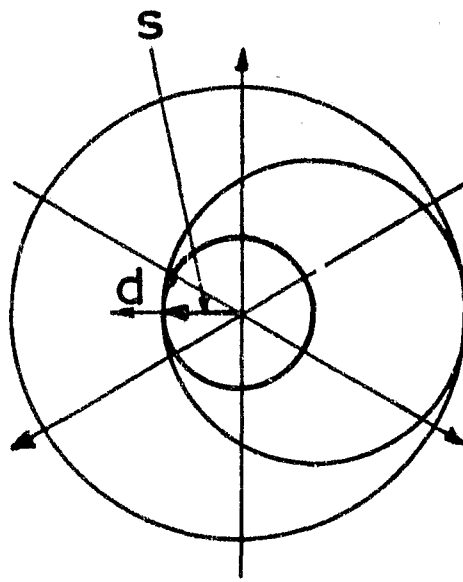
FIG. 24 KINEMATIC HARDENING THEORY



①



②



③

$S = \text{STRESS VECTOR}$
 $d = d\epsilon_{ij}^p$

FIG.2.5 MULTIPLE LOADING SURFACE THEORY OF MROZ(22)

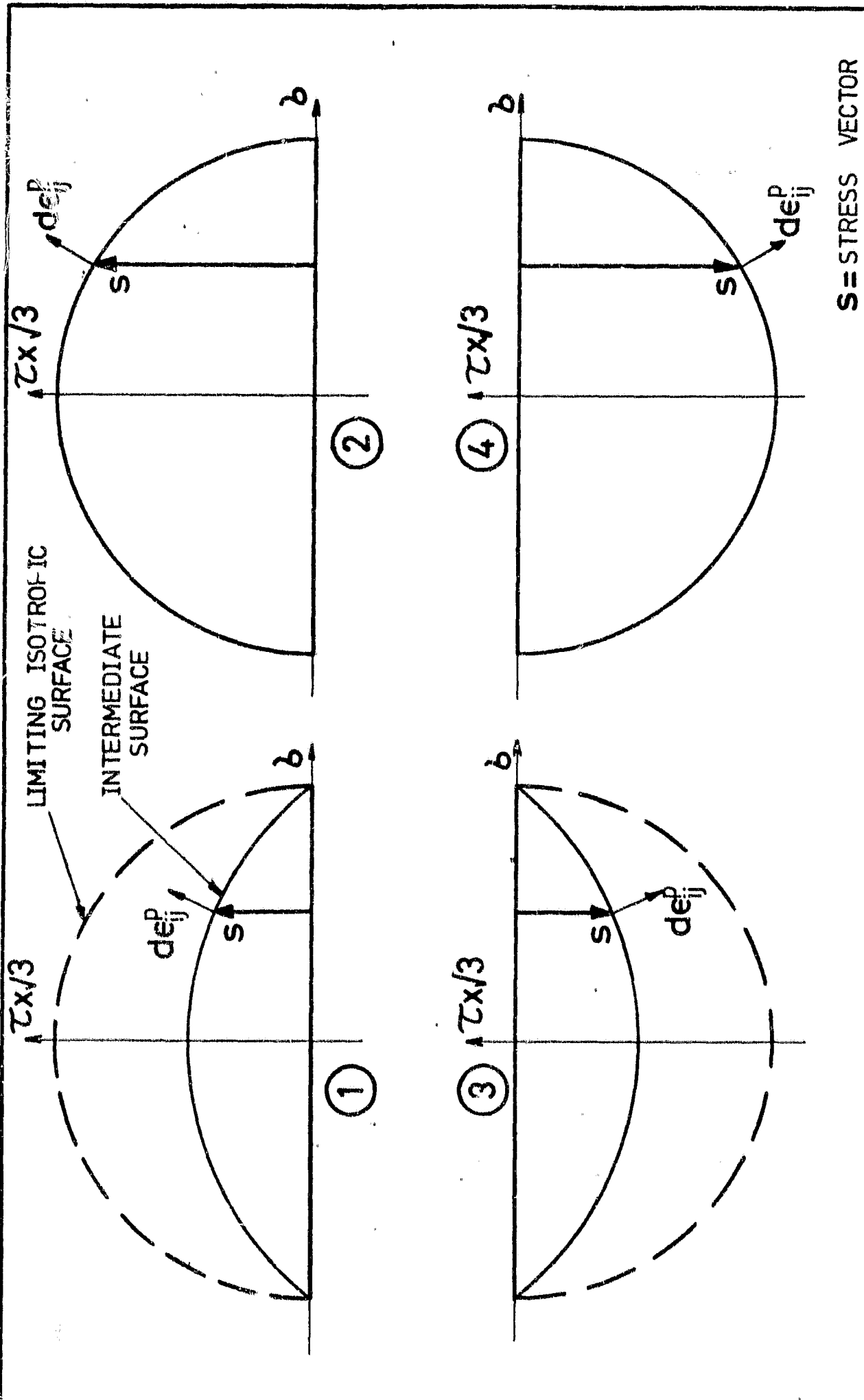


FIG. 2.6 EXPANDING YIELD SURFACES (18) IN $b - \sqrt{3} \tau$ STRESS SPACE

CHAPTER 3

EXPERIMENTAL EQUIPMENT

3.1 General

The equipment consisted of an Instron TT-CM testing machine of 50 kN capacity which was modified so that cyclic torsional loads could be superimposed on the normal tensile operating mode.

The modifications consisted of a tension-torsion drive train fitted between the Instron cross heads. The torque was applied to the drive train by an electrical motor via a V-belt, a 3600 : 1 reduction gear box and a chain drive. Tensile and torsional loads were measured by load cells incorporated in the drive train. The tensile load was applied by the Instron machine and the standard Instron equipment was used for load control and measurement. Shear and axial strains were obtained from a strain gauged clip on extensometer. Torque reversal was achieved by a purposely built electronic circuit capable of monitoring either the shear stress or shear strain signal voltage. Shear stress τ and shear strain δ were plotted on an X - Y pen recorder while axial strain ϵ as well as δ were recorded on an X-t recorder.

Fig. 3.1 is an schematic representation of the experimental equipment. Relevant specifications of equipment may be found in Appendix A.

3.2 Material and Specimens

The material used was a Huletts B51S-TF aluminium silicon alloy, which was received and tested in the solution heat treated and aged condition. Fig. 3.2 gives its chemical composition and other properties stated in the test certificate of the batch. The material was received as 5m lengths of 27mm diameter extruded round bar and all material was from the same cast.

The specimens were thin walled tubes with ends designed to transmit both a torque and a tensile load to the gauge

length (fig. 3.3). Similar specimens had been used in other investigations (18) and it had been found that the wall thickness was small enough to justify the assumption of constant stresses, yet large enough to prevent severe torsional buckling. In order to further inhibit torsional buckling, a lubricated mandrel was inserted into the specimen. To prevent binding and erroneous torque readings, especially when large negative diametral strains occurred, the mandrel was machined down to give a clearance of 1 mm between it and the tube wall over the gauge length (fig. 3.3). Even after considerable axial straining of the specimen, the mandrel could still be rotated freely.

Because of the dimensional accuracy required in the specimens, especially as regards the uniformity and constancy of the wall thickness, the machining procedure was as follows.

- (a) The required lengths of round bar were cut, faced to the correct overall length and centres drilled in both faces.
- (b) The bar was inserted between a centre and a driving centre and turned down to the maximum outer diameter (26 mm).
- (c) It was then placed in a collet (high precision chuck) and the central hole drilled and reamed to the correct depth.
- (d) A mandrel was placed in the hole and the specimen fixed between centres on a profile lathe where the profile was cut in three roughing cuts of 1 mm each and two finishing cuts of approximately 0,25 mm each.
- (e) Finally the flats on the thickened specimen ends were milled.

3.3 Torsion test modification

3.3.1 Motor, gear box and chain drive

A three phase electric motor was chosen to apply the torque because it could be reversed almost instantaneously by changing over two of the phases. Since the torque required from the motor was very low (0,014 Nm) compared to its rated capacity of 4,8 Nm, the motor speed was practically

synchronous (750 rev/min) and produced a saw tooth twist-time curve.

The speed reduction required between motor and specimen was achieved by a Renold Crofts 3600 : 1 double reduction gear box mounted on the Instron frame and driven by the motor via two 80 mm PCD pulleys and an A-section V-Belt (fig. 3.4). The belt tension could be adjusted by lowering or raising the motor on its support.

Sprockets were fitted on the vertical output shaft of the gear box as well as on the top of the drive train and connected by a single $3/8$ in. pitch chain. The chain tension could be adjusted by shifting the gear box on its supporting rails. Several sprocket sizes, all interchangeable, were available to obtain different twisting speeds. Throughout the present investigation, however, the twisting rate was maintained at $0.78^\circ/\text{s}$.

3.3.2 Drive train and load cells

The upper part of the drive train consisted of a bearing housing bolted onto the upper Instron cross head, a torsion load cell and a thrust bearing (fig. 3.5). The bearings in the bearing housing were designed to accommodate the bending moment caused by the one-sided pull of the drive chain, while the thrust bearing prevented tensile loading of the torsional load cell.

The torsional cell was hollow for greater sensitivity and a bolted on cap provided for the driven sprocket. Four strain gauges were accurately mounted on the cell to form a complete bridge. The cell was calibrated by removing the sprocket cap and fitting the two adaptors (fig. 3.6) for insertion into a previously calibrated Avery torsion testing machine. The resulting calibration graph was found to be linear (fig. 3.7).

The tensile load cell (see fig. 3.8) was pinned to the fixture on the lower Instron cross head. Four strain gauges were accurately mounted on this cell, two at right angles and two parallel to its centre line, and spaced in 90°

intervals around its circumference. This was found to give good results. The calibration of this cell which was used with the Instron chart recorder was carried out by dead weight loading, using the standard weights provided. Pinned connections were used throughout the drive train to facilitate assembly.

3.3.3 Specimen holders

The jaws holding the specimen were of the self aligning type which had been used in other investigations (18). The jaw blocks (see fig. 3.9), made of LTAH hardened to $R_c 62$, were able to slide in the jaw carrier, which in turn was free to move in the jaw head. The jaw blocks were tightened against the milled flats on the specimen by means of the bolt shown and a recess was provided for the unmilled specimen end (see fig. 3.10).

3.4 Tensile load application

The steady tensile load was applied to the specimen by the Instron machine which was set to operate in the load cycling mode. Thus the tensile load was varied and was kept to $\pm 5\%$ about the selected mean value. The variation was sufficiently small for the load to be regarded as static.

3.5 Instrumentation

3.5.1 Axial and shear strain transducers

The axial and shear strains were measured by a specially constructed strain gauge extensometer. This consisted of two spring loaded jaw pieces joined by two curved spring steel strips, one of which could deform when the specimen elongated, the other when it twisted. Each strip was fitted with a four strain gauge bridge for maximum sensitivity. Three pointed screws on each jaw piece fixed the extensometer firmly to the specimen. Fig. 3.11 shows the extensometer clamped in position on the specimen. During preliminary investigations with other types of extensometers, it was found that this arrangement was the best and that the cyclic creep rate was unaffected by the chosen clamping method.

The extensometer was calibrated for twist by fixing it to a standard torsion specimen of the same diameter as the tension-torsion specimen and twisting it in the Avery torsion testing machine. The linear relation between the shear strain in the torsion specimen and the number of drive wheel revolutions had been determined previously by means of a twist meter. The resulting calibration curve was linear (fig. 3.12). The slight deviation from linearity for $\delta > 0.025$ did not affect the results since torsional cycling was never carried out in that region. It was found that the effect of twisting on the axial strain signal was negligible.

Axial strain calibration was carried out by fitting the extensometer to a long mild steel tensile specimen, together with a dial gauge extensometer, and straining the specimen in an Amsler Universal testing machine. Care was taken to prestrain the specimen past the yield region, where strains were susceptible to local variations over the specimen length, before the extensometers were connected. The calibration was approximately linear (fig. 3.13). An axial strain also produced a reading on the extensometer's shear strain bridge. The effect of this during steady tension - cyclic torsion testing would be, due to the manner in which torque reversal was achieved, to cause a torsional cyclic creep accumulating with axial deformation. However visual observation of the extensometer after prolonged cycling showed no such accumulated twist, indicating that the cross effect was absent during torsional cycling.

3.5.2 Recording and amplification equipment

The strain gauge bridges on the torsional load cell as well as on the axial - and shear strain transducers were supplied with a constant voltage (6.00 V) by a stabilized single DC power supply operating in the constant voltage mode. The tensile load cell bridge was supplied directly from the Instron machine.

Because of the small signal voltage generated by the torsional load cell and the strain transducers, a high gain DC amplifier was used to pre-amplify the signal voltages.

Gains of the order of 1000 were employed. The amplifier was also used to zero any initial imbalance in the bridge circuits.

The pre-amplified signal voltages were then fed into the respective recorders (fig. 3.1). The reason for duplicating the shear strain on the X-t recorder together with the axial strain ϵ was to provide a time and cycle reference for ϵ .

3.5.3 Torsional load reversal

To enable torque reversals at either preset shear stress or shear strain limits, an electronic comparator capable of reversing the electric motor was designed and built. This comparator was triggered when the input signal reached a preset maximum or minimum value. The comparator circuit was supplied with -12V - 0V - +12V by a stabilized dual DC power source. Fig. 3.14 is a schematic view of the control system. Throughout the present investigation the pre-amplified shear strain signal voltage was used to trigger the torque reversals.

3.6 Setting up the equipment

The gear box and motor were installed as shown in fig. 3.4 and the belt tension adjusted. The upper part of the drive train (fig 3.5) was lowered through the hole in the upper Instron cross head after the torsional load cell wires had been threaded through the hollow cross head. The assembly was bolted down, the sprockets and chain placed in position and the chain tension adjusted. The tensile load cell and the jaws were inserted (fig. 3.8) and the load cell connected to the Instron machine.

The recording, amplification and control equipment was arranged as shown in figs. 3.15 and 3.16 and connected (fig. 3.1). The polarities of the load cell and extensometer outputs, the comparator input and the motor phases were checked for proper recording direction and torque reversal.

3.7 Testing procedure

A warm up period of at least half an hour was allowed before testing commenced so that the electronic components, and especially the strain gauge bridges, could attain thermal equilibrium.

3.7.1 Pure tension testing

Pure tensile tests were conducted on the Instron machine using an extension rate of 0,2 cm/min. No mandrel was required in the specimen. Stress and strain values were obtained from Load-deflection curves from the Instron's chart recorder.

3.7.2 Combined tension-torsion testing

The tensile load cell was calibrated, the Instron switched to the load cycling mode and the tensile load limits set. The axial extension rate was retained as 0,2 cm/min. The required amplifier gains and recorder sensitivities for , and were calculated and set so as to give a simple relation between the signal voltage and the recorder pen deflection. Thus 1 cm on the Y-channel of the X-Y recorder was set to correspond to a shear stress of 20 MPa in the specimen, 20 cm on the X-channel of the X-Y recorder corresponded to the total shear strain range, while 1 in. on the X-t recorder represented 0,005 axial strain.

The specimen was then inserted, tightened and the extensometer fixed to it. A spacer between the extensometer jaw pieces was required to ensure conditions identical to those during calibration. After withdrawal of the spacer the axial strain signal was zeroed and the specimen preloaded in tension.

The zero screw on the amplifier was used to apply a simulated shear strain signal to the comparator in order to set the upper and lower shear strain limits. The shear stress and shear strain signals were then zeroed and the chart drive and motor switched on.

It was found to be advantageous to select a higher chart speed (~ 60 mm/min) on the X-t recorder during the first ten torsional loading cycles so as to note the transient axial strain behaviour. The calibration of the extensometer and load cells was checked after every few tests but very little variation was found.

3.8 Testing Programme

The main part of the testing consisted of steady tension - cyclic torsion tests to find the cyclic creep behaviour of

the aluminium . Shear strain ranges ranged from 0,01 to 0,04, while the applied tensile stresses varied from 0 MPa to 200 MPa. A maximum of four hours was allotted to each test.

In addition a number of pure tensile tests up to fracture as well as fully reversed torsion tests were conducted to obtain the elastic constants and the monotonic leading behaviour.

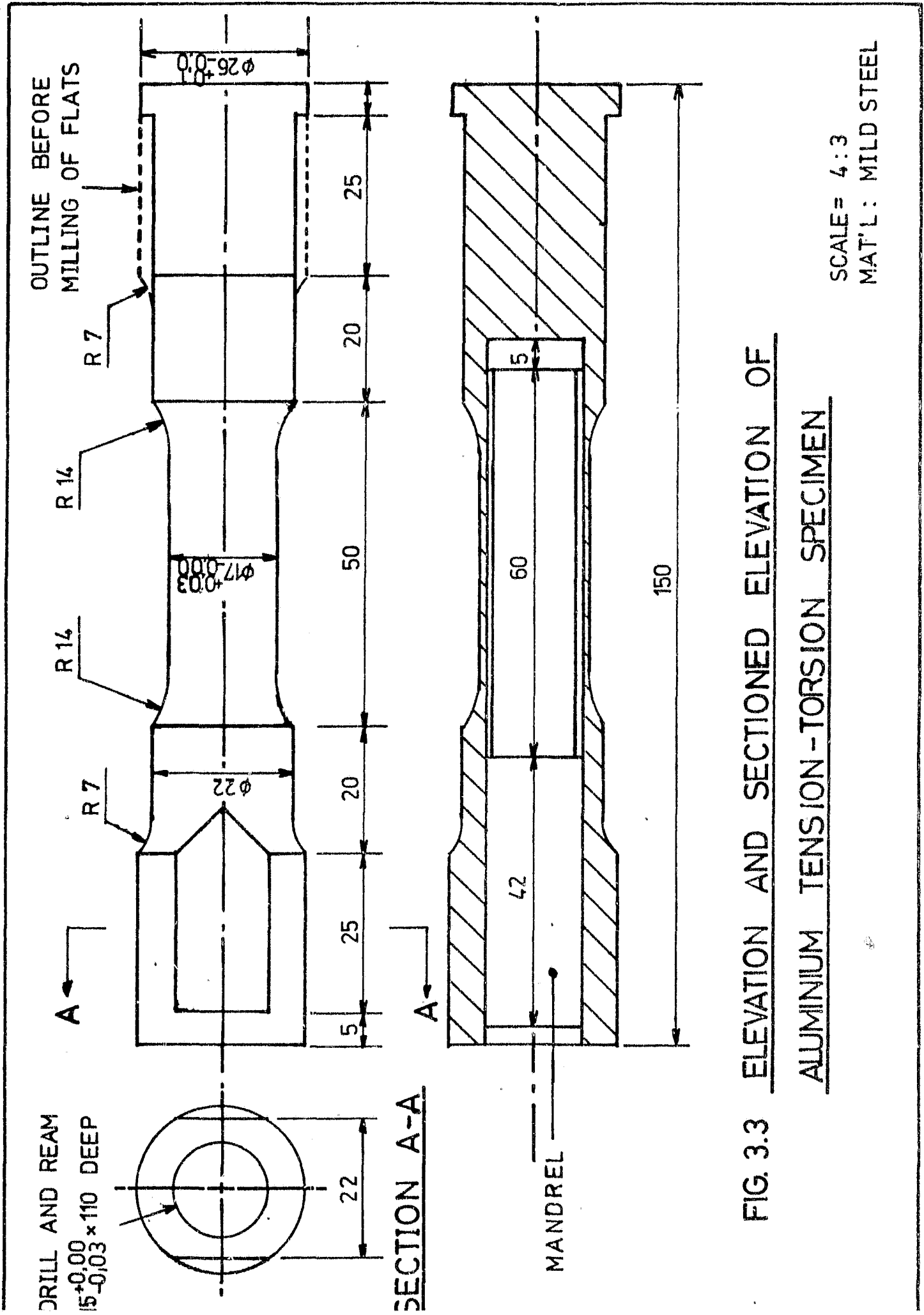
Additional tests consisted of subjecting specimens to a steady tension and a cyclic torque until the torsional hysteresis loop had settled down to a steady state. The specimens were then tested in pure tension up to fracture and the yield behaviour noted. This was done for a high and a low shear strain range (0,032 and 0,012) as well as for tensile stresses ranging from 0 MPa and 200 MPa.

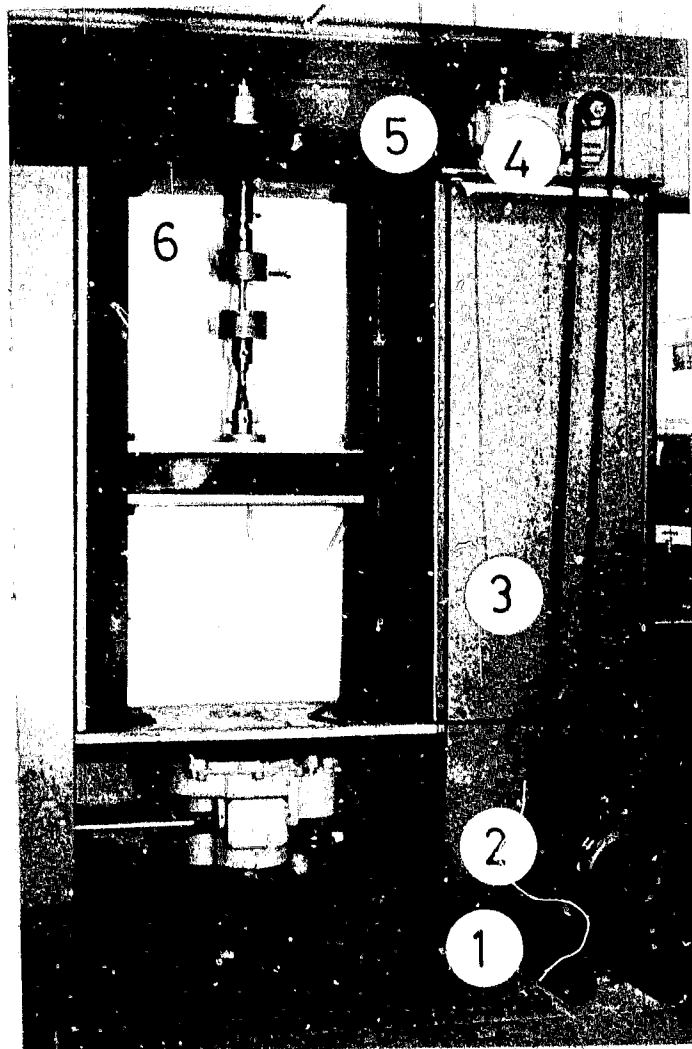
fig. 3.2 Properties of Aluminium Alloy Tested

Chemical analysis (%)

Mg	Fe	Si	Mn	Ti	Cu	Zn	Cr	Al
0,58	0,28	1,03	0,53	0,018	0,079	0,015	0,010	Remainder

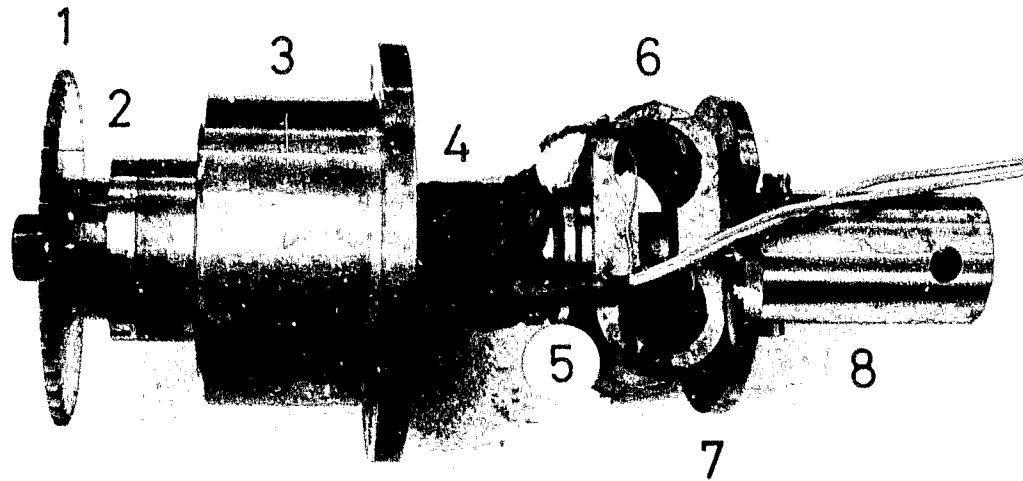
Alloy + temper	: B51S-TF
B.S. designation	: 1474/HE 30 TF
I.S.O designation	: AlSi1MgMn
0,2% Proof stress	: 265 MPa
UTS	: 313 MPa
Elongation	: 14%
Gauge length	: 50 mm





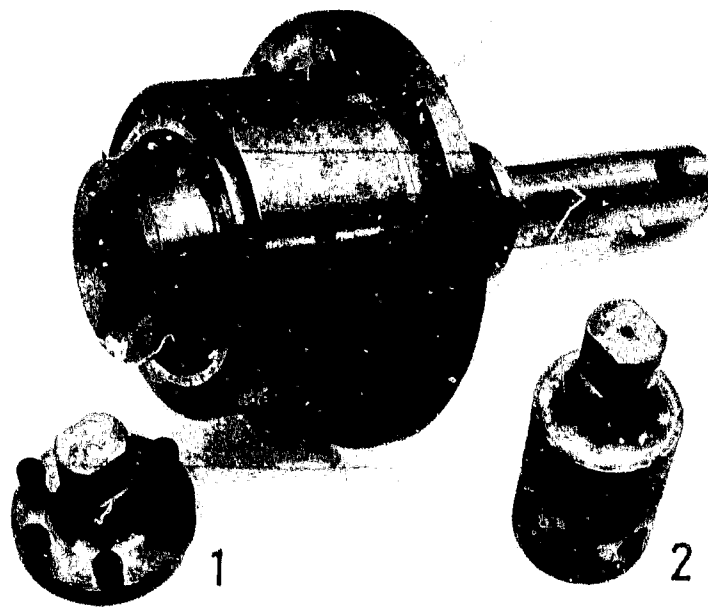
1. Motor supports
2. Motor
3. V-belt
4. Gear box on supporting rails
5. Chain
6. Drive train

Fig. 3.4 Rear View of Instron Machine



1. Sprocket
2. Sprocket cap
3. Bearing housing
4. Torsion load cell
5. Thrust bearing
6. Old tensile load cell used as spacer
7. Supporting plate
8. Adaptor piece

Fig. 3.5 Torsion Load Cell Assembly



- 1. Adaptor no. 2
- 2. Adaptor no. 1

Fig. 3.6 Torsion Load Cell for Calibration

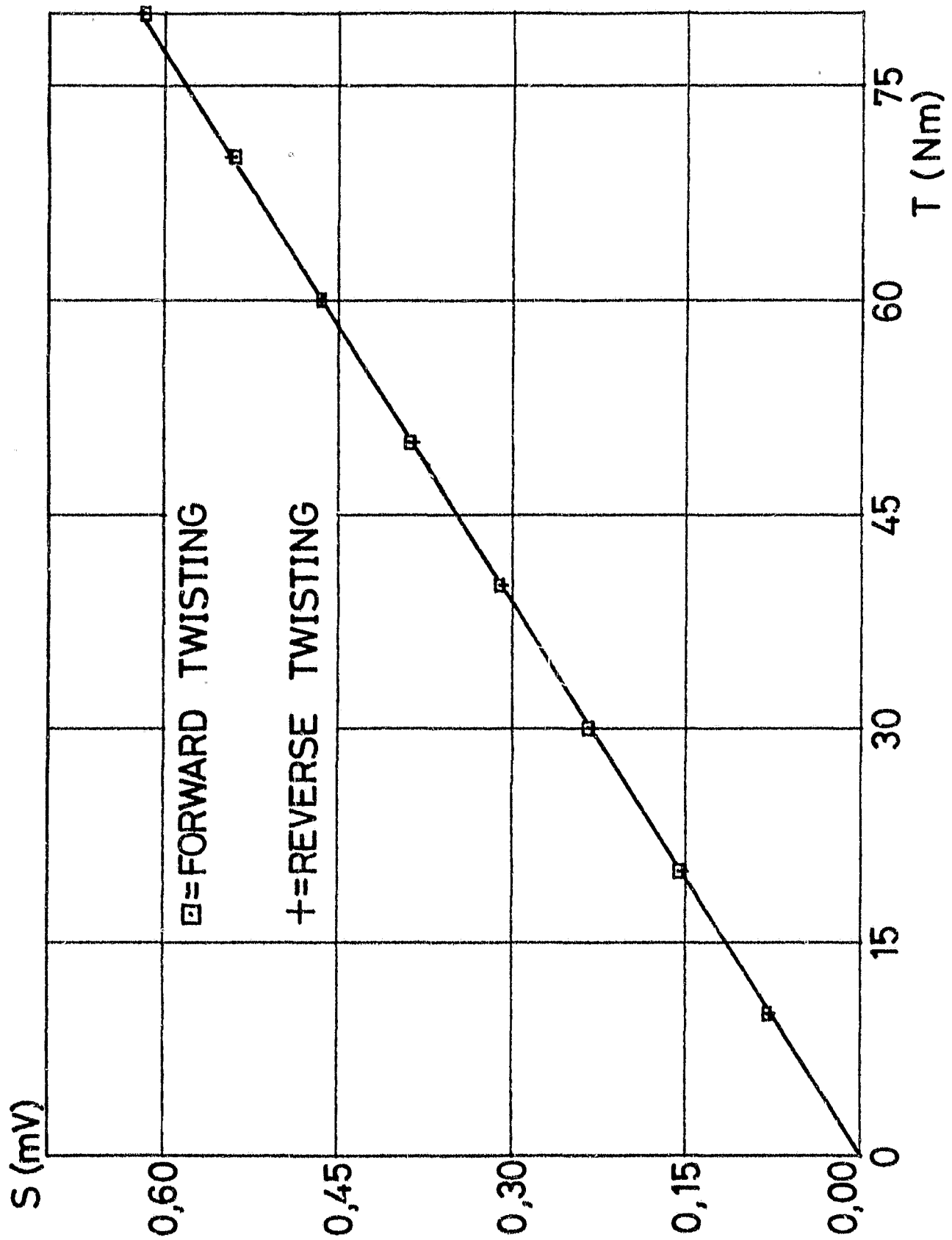
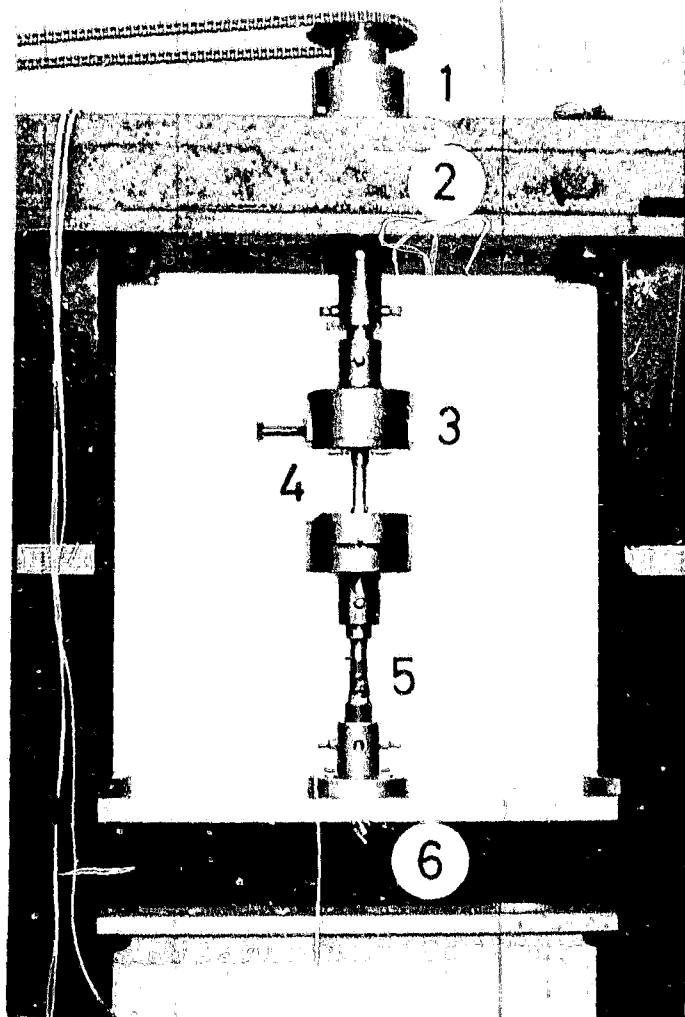
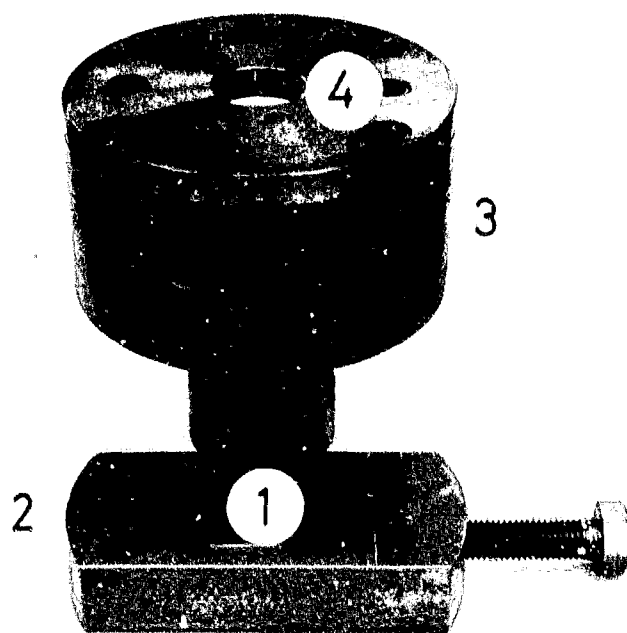


Fig. 3.7 Calibration Curve of Signal Voltage (S)
against Torque (T) for Torsion Load Cell.



1. Torsion load cell assembly
2. Upper Instron cross head
3. Jaw
4. Specimen
5. Tensile load cell
6. Lower Instron cross head

Fig. 3.8 Drive Train



- 1. Jaw block
- 2. Jaw carrier
- 3. Jaw head
- 4. Hole for admitting specimen

Fig. 3.9 Specimen Holder

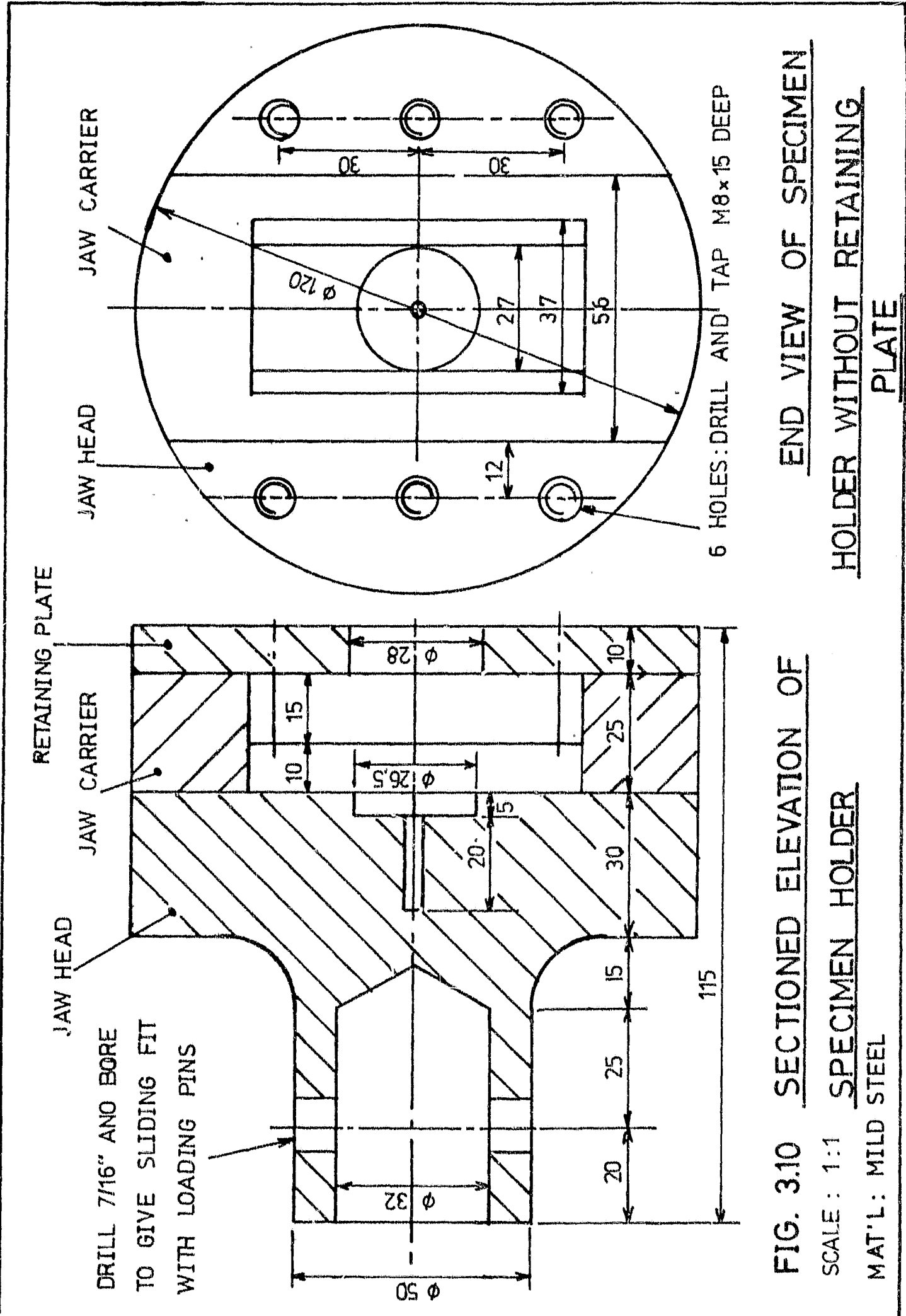
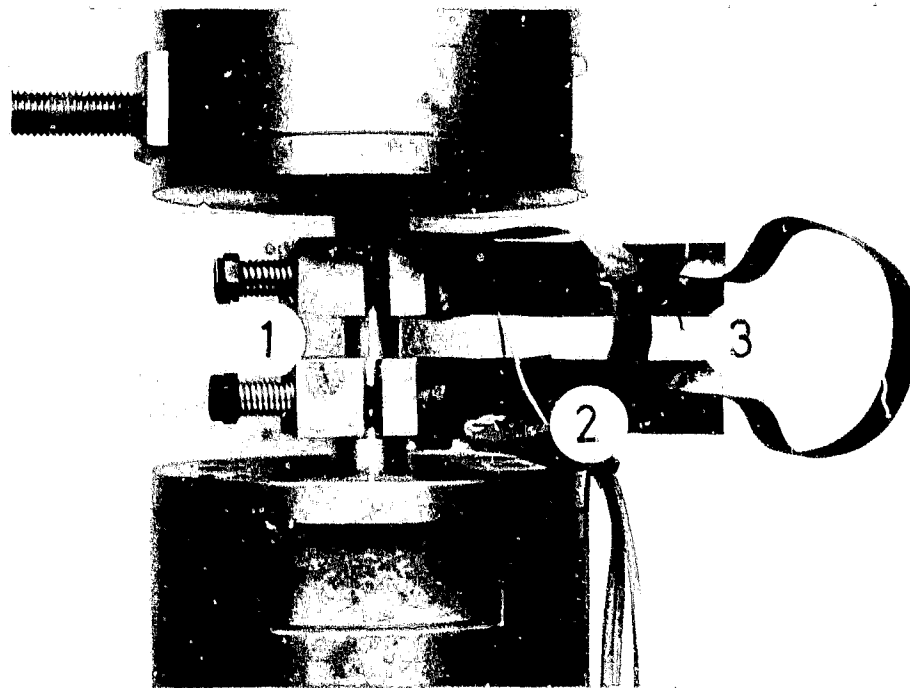


FIG. 3.10 SECTIONED ELEVATION OF
 SCALE : 1:1 SPECIMEN HOLDER
 MAT'L: MILD STEEL
 END VIEW OF SPECIMEN
 HOLDER WITHOUT RETAINING
 PLATE



- 1. Spring loaded jaw pieces
- 2. Axial strain transducer strip
- 3. Shear strain transducer strips

Fig. 3. 11 Extensometer Fitted to Specimen

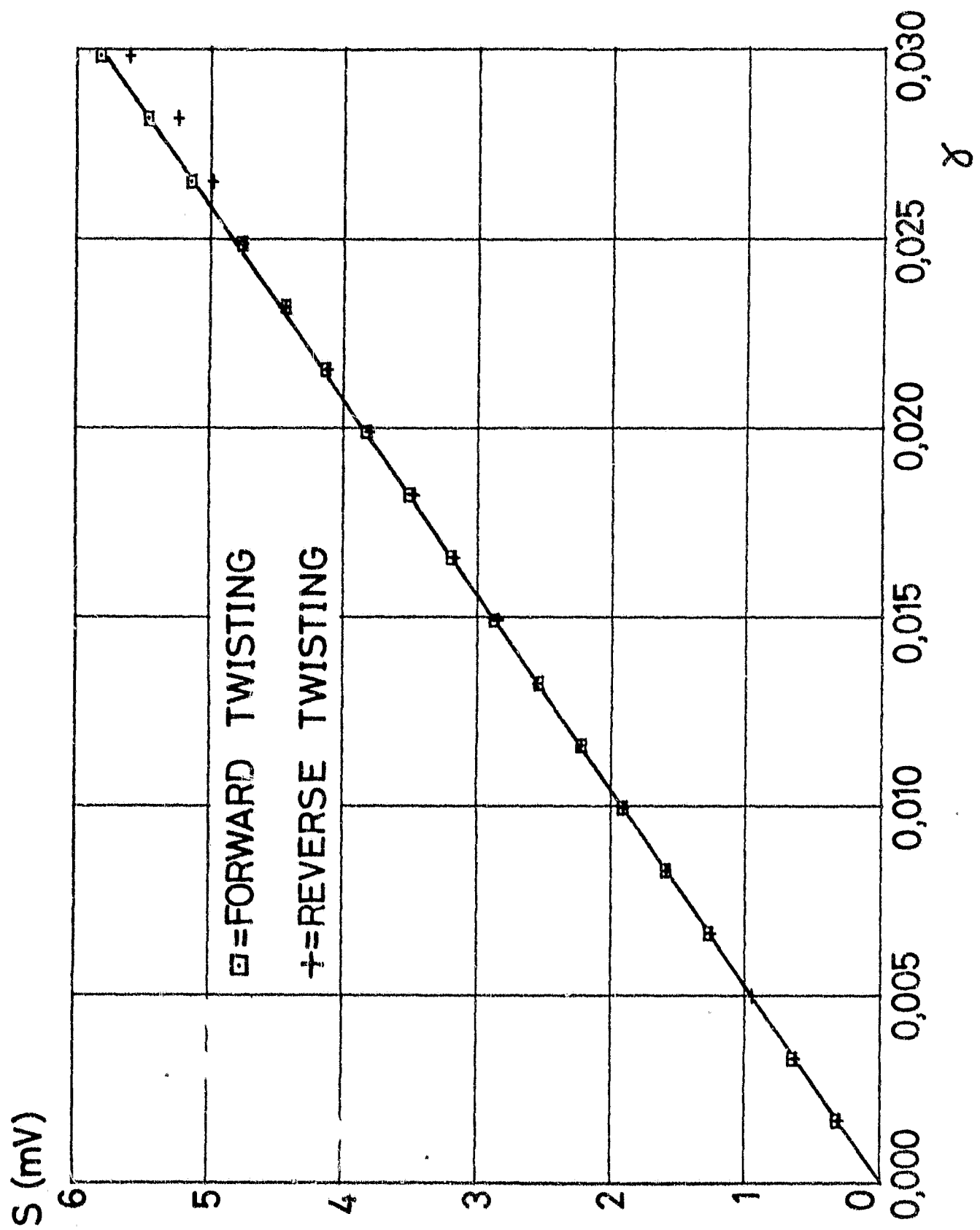


Fig. 3.12 Calibration Curve of Signal Voltage (S) against Shear Strain (δ) for Shear Strain Transducer.

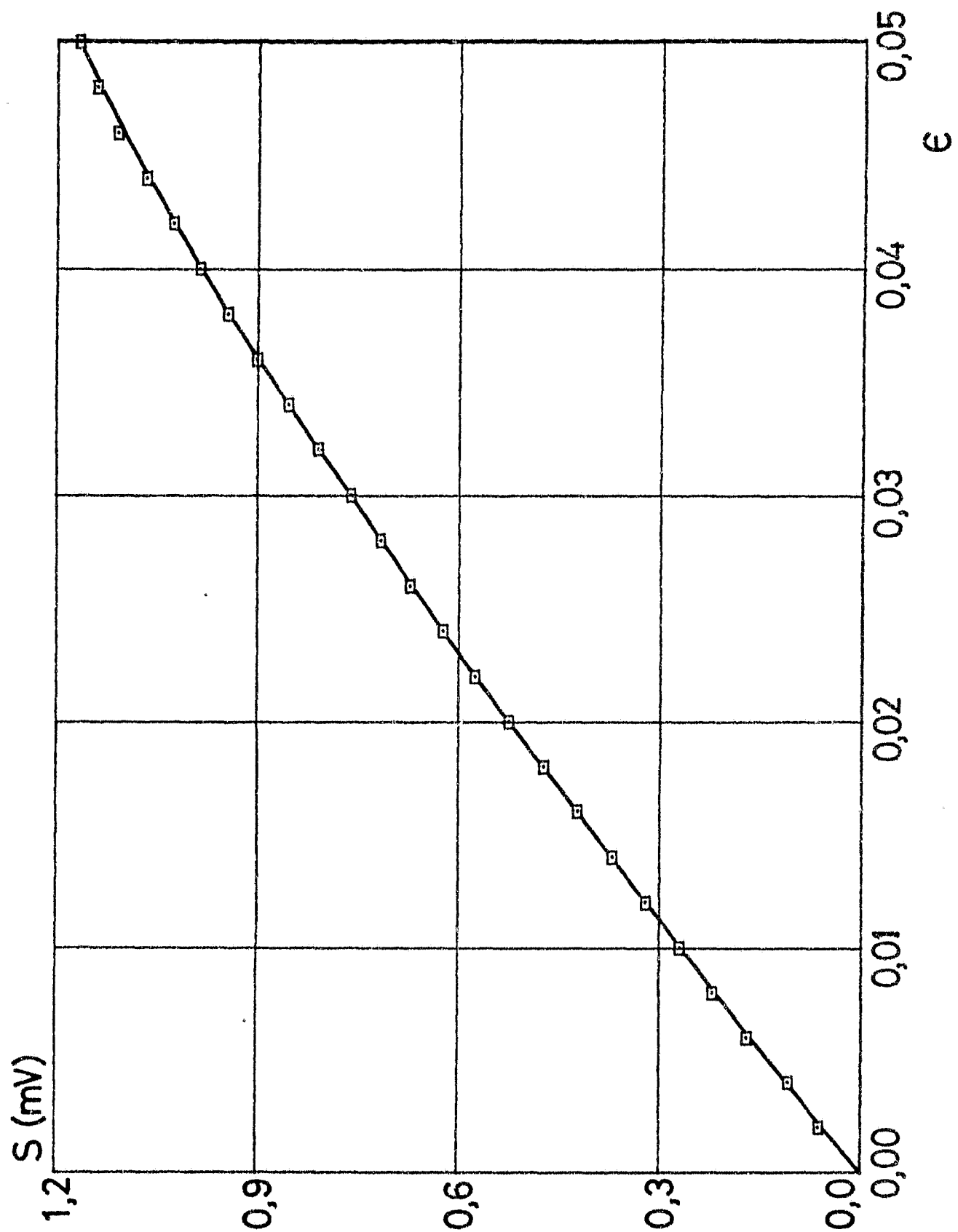


Fig. 3.13 Calibration Curve of Signal Voltage (S)
against Axial Strain (ϵ) for Axial Strain
Transducer.

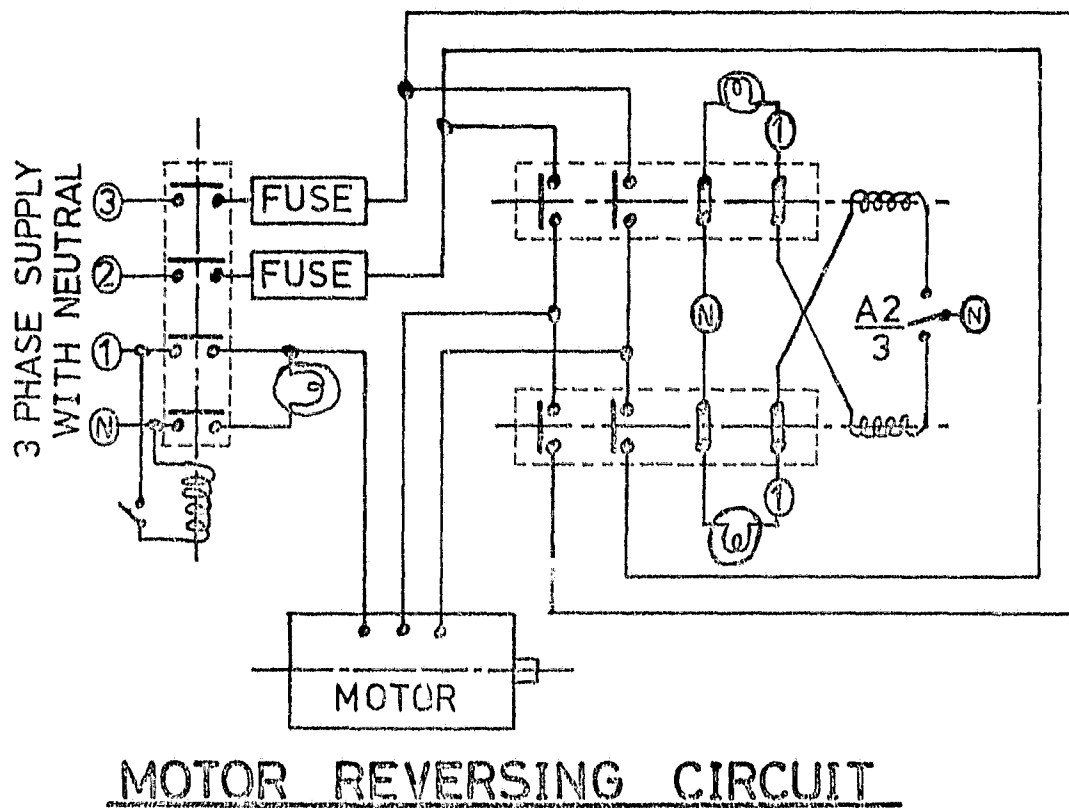
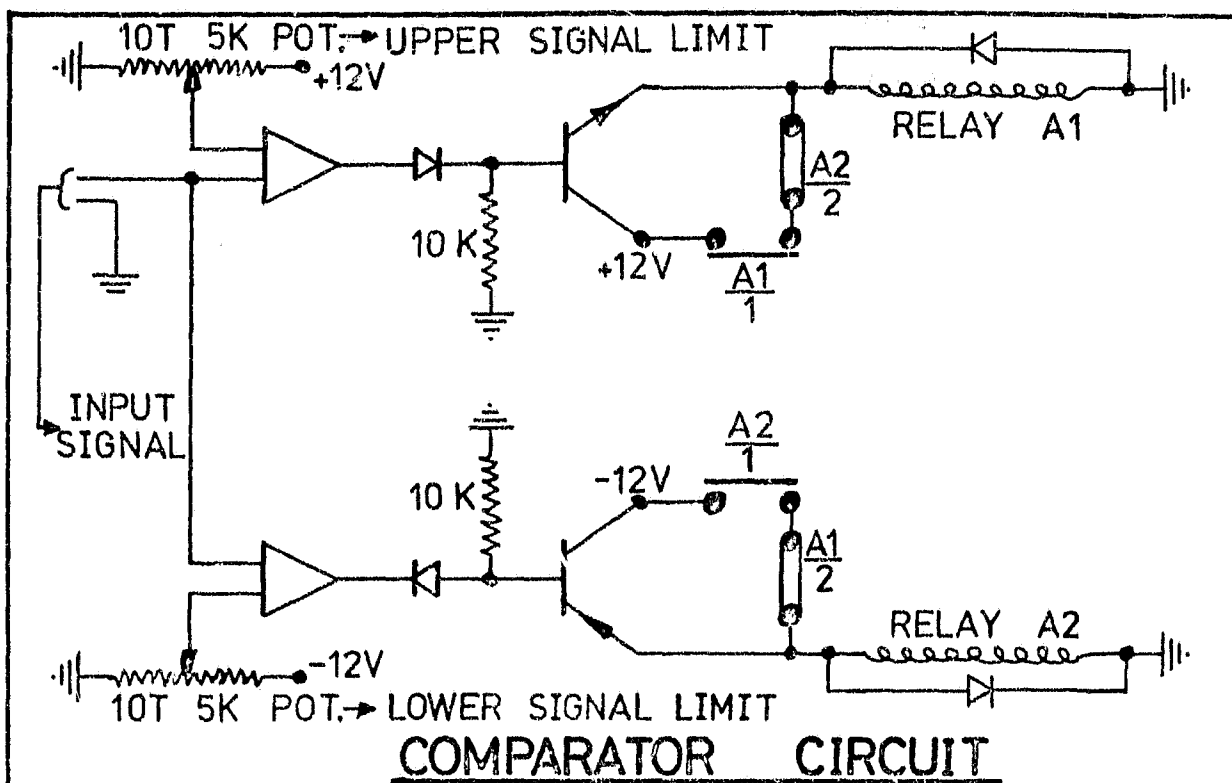
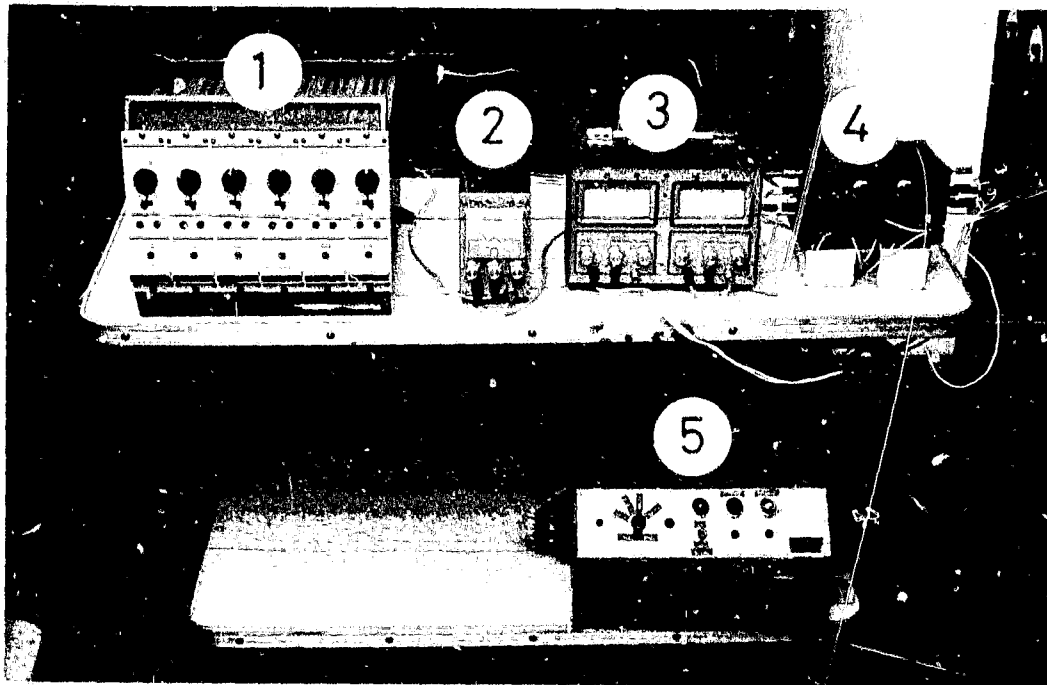
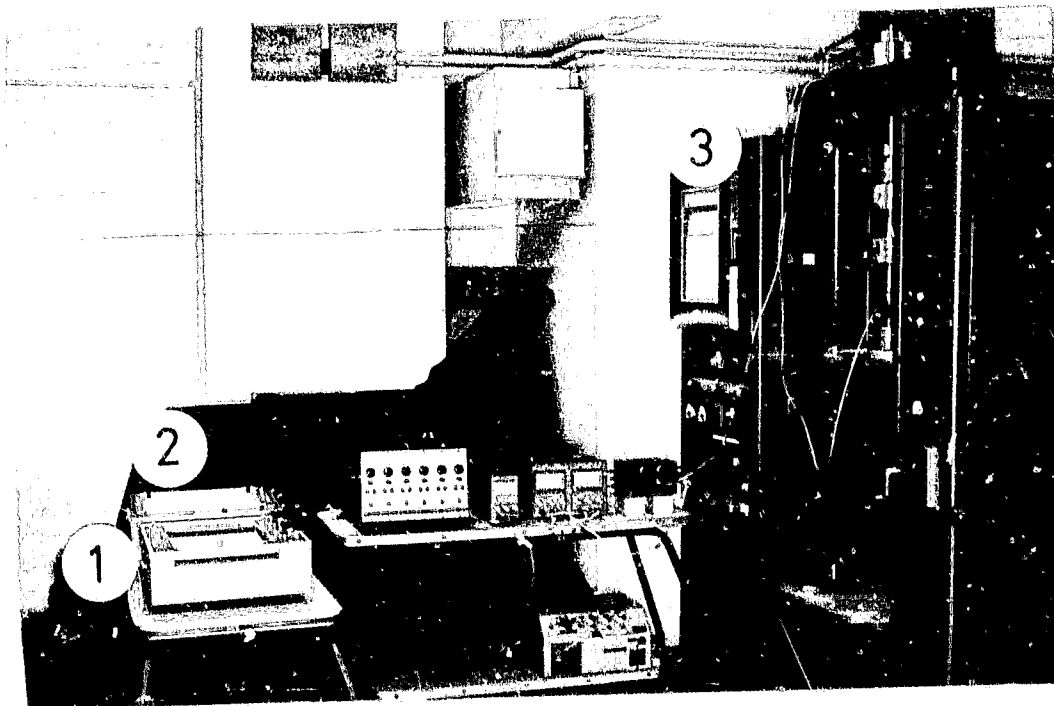


FIG. 3.14 SCHEMATIC VIEW OF CIRCUIT
MONITORING SHEAR STRAIN AND
REVERSING TORQUE



1. DC amplifier
2. Single DC power supply for strain gauges
3. Dual DC power supply for comparator circuit
4. Comparator
5. Switching box

Fig. 3.15 Layout of Control and Amplification Equipment.



1. K - Y recorder
2. K - t recorder
3. Instron recorder

Fig. 3.16 Overall layout

CHAPTER 4.

EXPERIMENTAL RESULTS

4.1 Pure reversed torsion and pure tension

Two examples of the reversed torsion stress-strain curves are shown in fig. 4.1. It can be seen that, immediately after the drive motor reversal, slack in the drive train had to be taken up, allowing stress relaxation rather than elastic unloading to occur. This indicates that the material was strain rate sensitive and for that reason the twisting rate was kept constant throughout this investigation. The stress-strain curves were not quite identical which possibly indicates small variations in specimen dimensions.

Fig. 4.2 shows one tensile stress-strain curve. For comparison the effective stress-strain curve in torsion has been included in this figure. It can be seen that the aluminium possessed a considerable degree of plastic anisotropy although the work hardening slopes were very similar. The ratio of tensile to torsional yield strengths was obtained as 2,369. This anisotropy arose most probably from the extrusion process used in manufacture. Unfortunately further heat treatment failed to remove this and so subsequent tests were carried out on the as received material.

The elastic shear modulus G_e was obtained from several torsional hysteresis loops and the mean value calculated to be $G_e = 26\ 750$ MPa, which is similar to published figures. G_l , the limiting tangent shear modulus, was obtained from fig. 5.1 and found to be $G_l = 260$ MPa. Young's modulus E_e was found as 67 000 MPa.

4.2 Steady tension and alternating torsion.

Although the total shear strain range $\Delta\gamma$ was one of the controlled variables, a more meaningful measure of deformation was the plastic shear strain range $\Delta\gamma^p$, which varied from 0,00119 to 0,02817. The creep strain values obtained from these tests were all referred to the strain at the end of the first quarter loading cycle.

4.2.1 Cyclic strain induced creep

Contrary to expectations, not all the creep curves were monotonically increasing. Figs. 4.3 to 4.6 are representative examples of the four types of creep curves observed. The only common feature of all curves was a region of constant cyclic creep rate. In some duplicated tests with the same loading parameters, the steady state cyclic creep rate was the same even though the transient behaviour differed (fig. 4.7).

It was noticed that the primary creep rate was usually negative for small values of $\Delta\gamma^p$ and b . For larger $\Delta\gamma^p$ or b it became positive while for very large b no primary creep occurred at all. The steady state secondary creep rates were negative for small values of $\Delta\gamma^p$ or b but increased rapidly with increasing $\Delta\gamma^p$ and b . The tertiary creep behaviour was actually a tensile instability phenomenon and not of interest in the present investigation. The steady state creep rates have been plotted against b for each $\Delta\gamma^p$ in figs. 4.8 - 4.10.

It was interesting to note that Freudenthal and Ronay (25) had found a positive axial strain accumulation for aluminium subjected to pure torsional cycling, while the opposite occurred here, even for a nonzero tensile stress. The tube shortening was probably due to anisotropy and the results were as to be expected from Hill's analysis (33). The aim of this investigation, however, primarily was to study the steady state cyclic creep behaviour, which was unaffected by any initial anisotropy.

4.2.2 Torsional hysteresis loops

Several of these were obtained during each cyclic strain induced creep test. It was found that the steady state loop was reached very soon (within 5 cycles) and changed little with progressing axial straining. Some cyclic hardening occurred which was more noticeable for the low plastic shear strain ranges (0,00119 - 0,00252) as evidenced by a shrinkage of the loop (fig. 4.11).

The steady state hysteresis loop was little affected by the presence of a tensile stress. This can be seen in fig.4.12 , where the areas of the loops have been plotted against b for different values of $\Delta\gamma^p$. The large deviations for a few of the

tests were most probably caused by variations in the specimen dimensions.

4.2.3 Fracture behaviour

Two types of fracture were observed in those specimens that failed within the allotted time. Specimens subjected to large tensile stresses failed by local necking and tensile fracture, the fracture plane being inclined at 45° to the specimen centre line. In some cases the crack did not completely propagate around the circumference, indicating a nonuniform wall thickness. High strain fatigue fracture, evidenced by a longitudinal crack was observed in specimens subjected to smaller tensile stresses. If cycling continued after formation of the fatigue crack, further cracks started from the crack tip and spread in circumferential direction.

4.3 Tensile response after previous torque cycling

To obtain information about the material anisotropy after some torsional cycling, specimens were subjected to a cyclic torque and a steady tensile load until the hysteresis loop had reached a steady state. The specimens were then tested in tension up to fracture and the yield stresses determined by a Lode extrapolation technique, taking into account the reduction in cross sectional area caused by the tensile load during previous cycling. The ratios of the tensile yield stress to the maximum shear stress attained during cycling have been plotted against the tensile stress during cycling for each $\Delta\sigma^P$ (fig. 4.13). For comparison the ratio of tensile to torsional yield stress for the previously unstrained material has also been included. It can be seen that the initial anisotropy was decreased by cycling at large plastic shear strain ranges and small applied tensile stresses. However some hardening was caused by large applied tensile stresses.

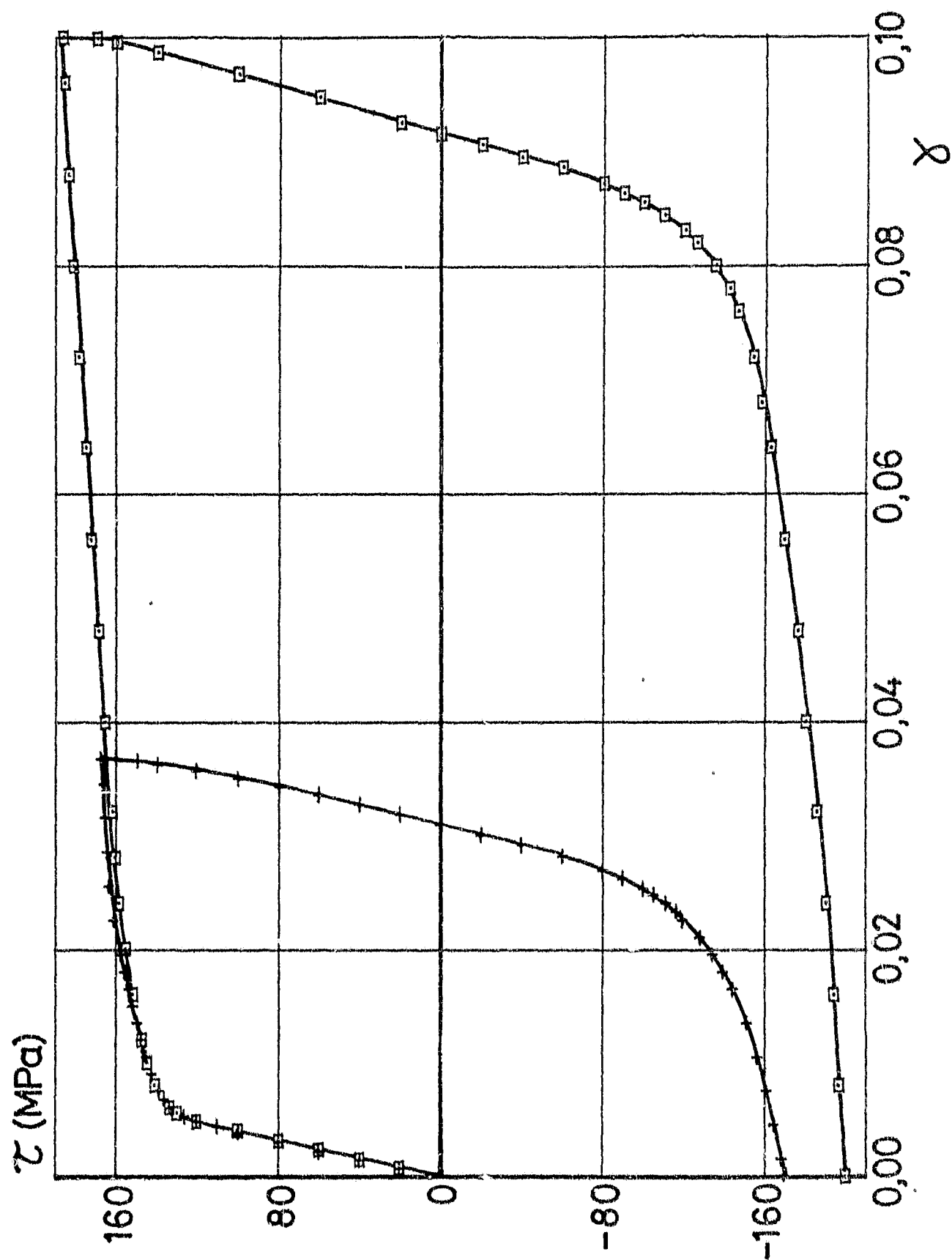


Fig. 4.1 Fully Reversed Stress - Strain Curves
in Torsion.

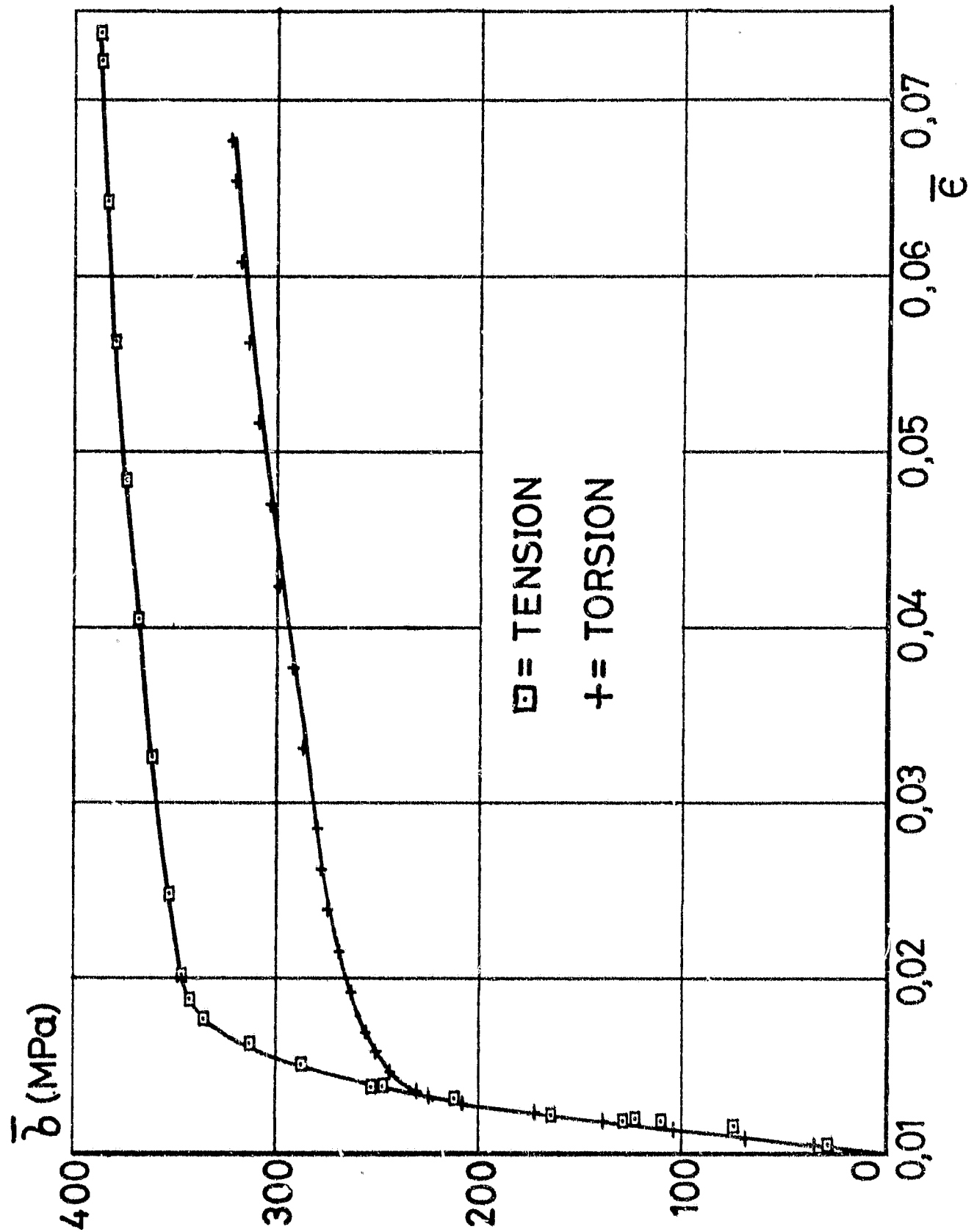


Fig. 4.2 Effective Stress ($\bar{\delta}$) against Effective Strain ($\bar{\epsilon}$) for Tension and Torsion.

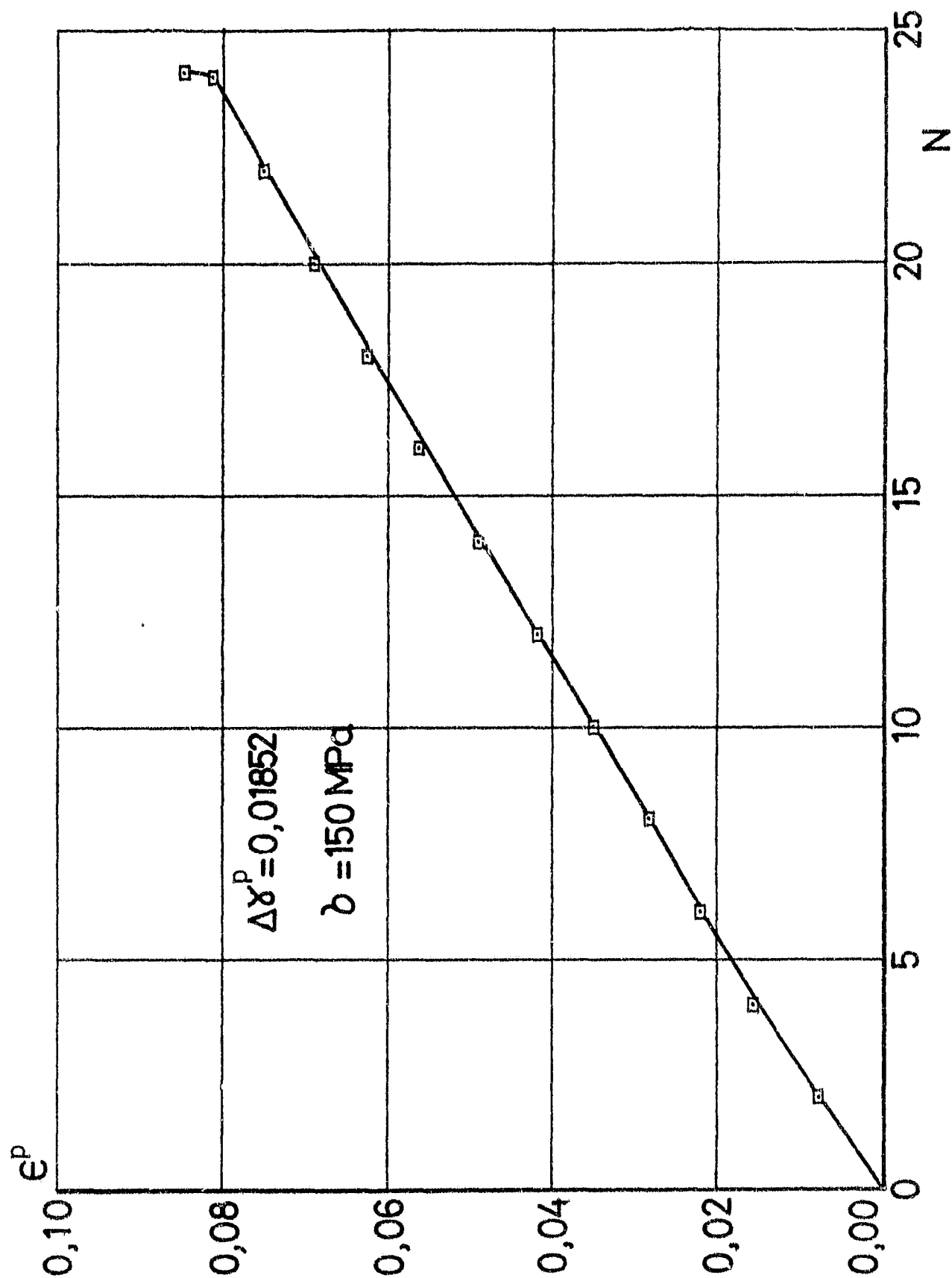


Fig. 4.3 Experimental Cyclic Creep Strain (ϵ^P)
against Number of Cycles (N).

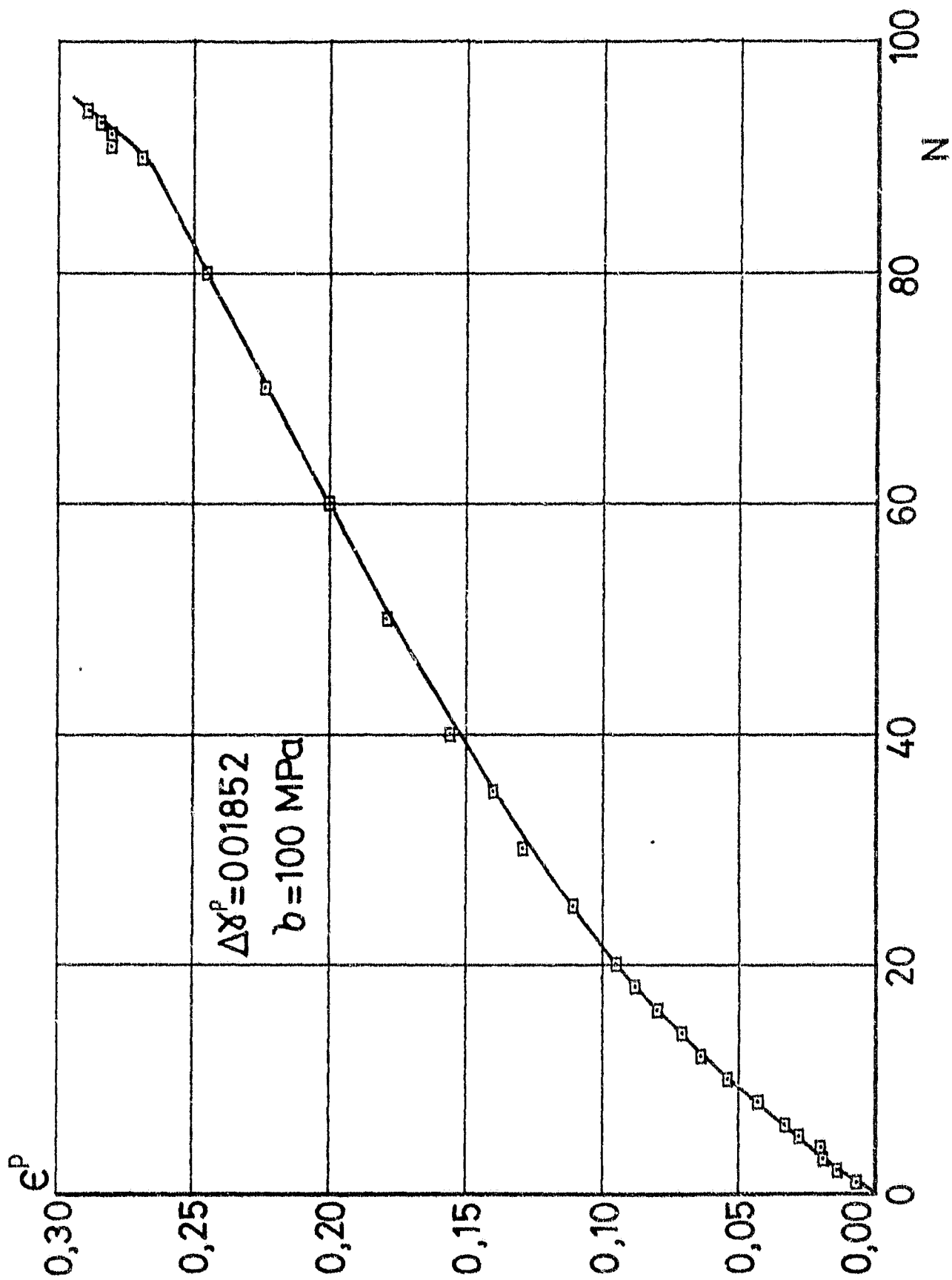


Fig. 4.4 Experimental Cyclic Creep Strain (ϵ^p)
against Number of Cycles (N).

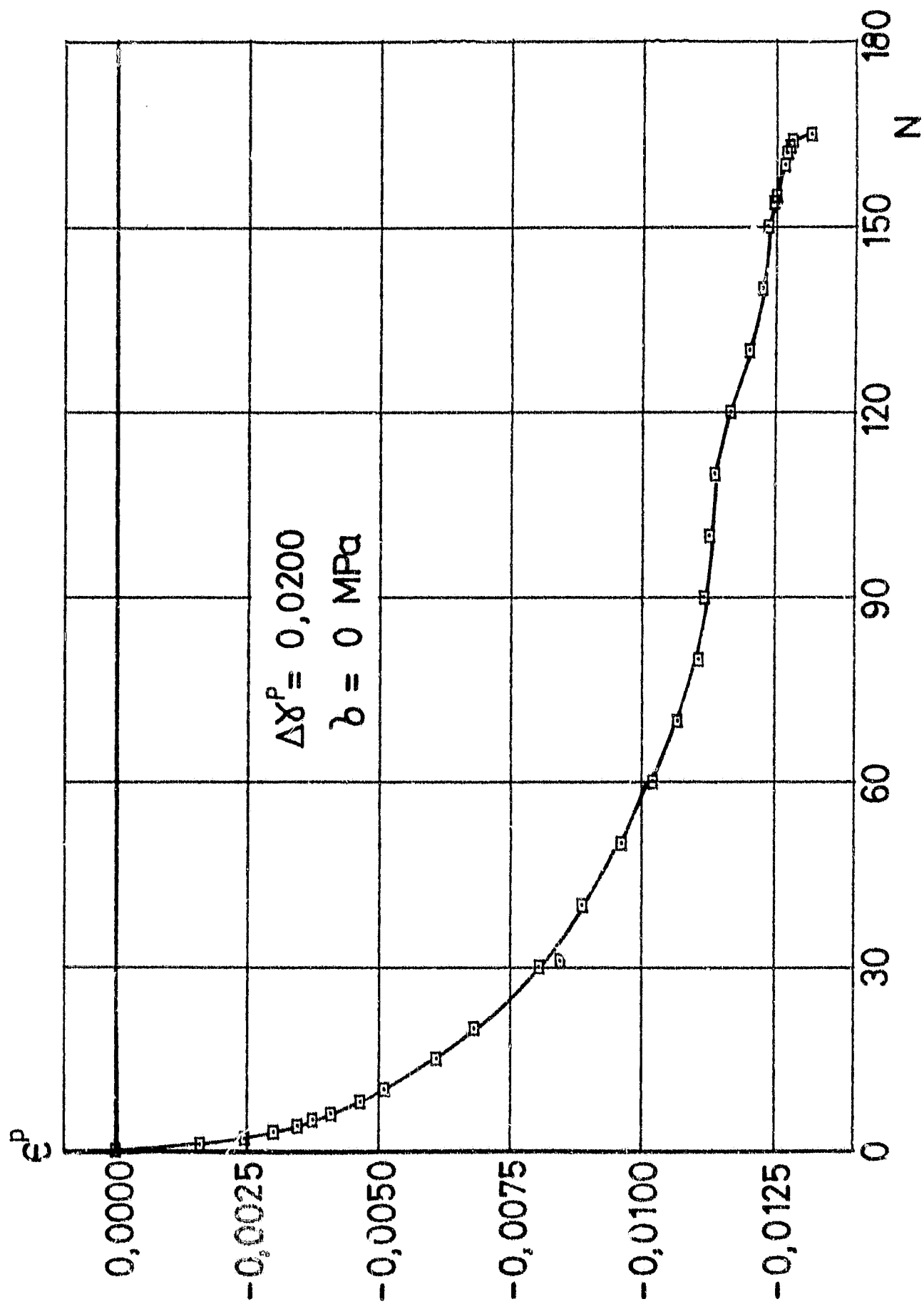


Fig. 4.5 Experimental Cyclic Creep Strain (ϵ^p)
against Number of Cycles (N).

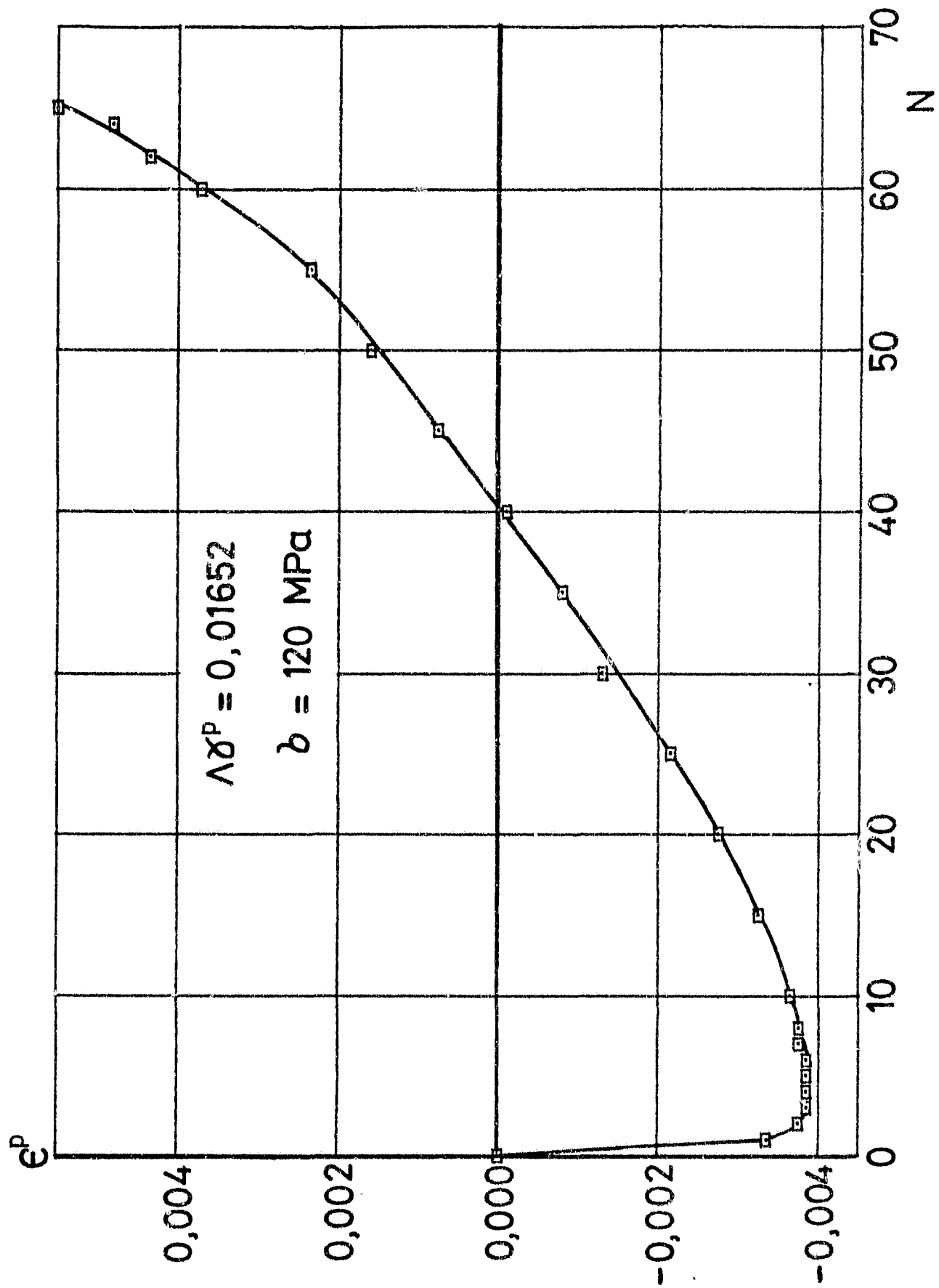


Fig. 4.6 Experimental Cyclic Creep Strain (ϵ^P)
against Number of Cycles (N).

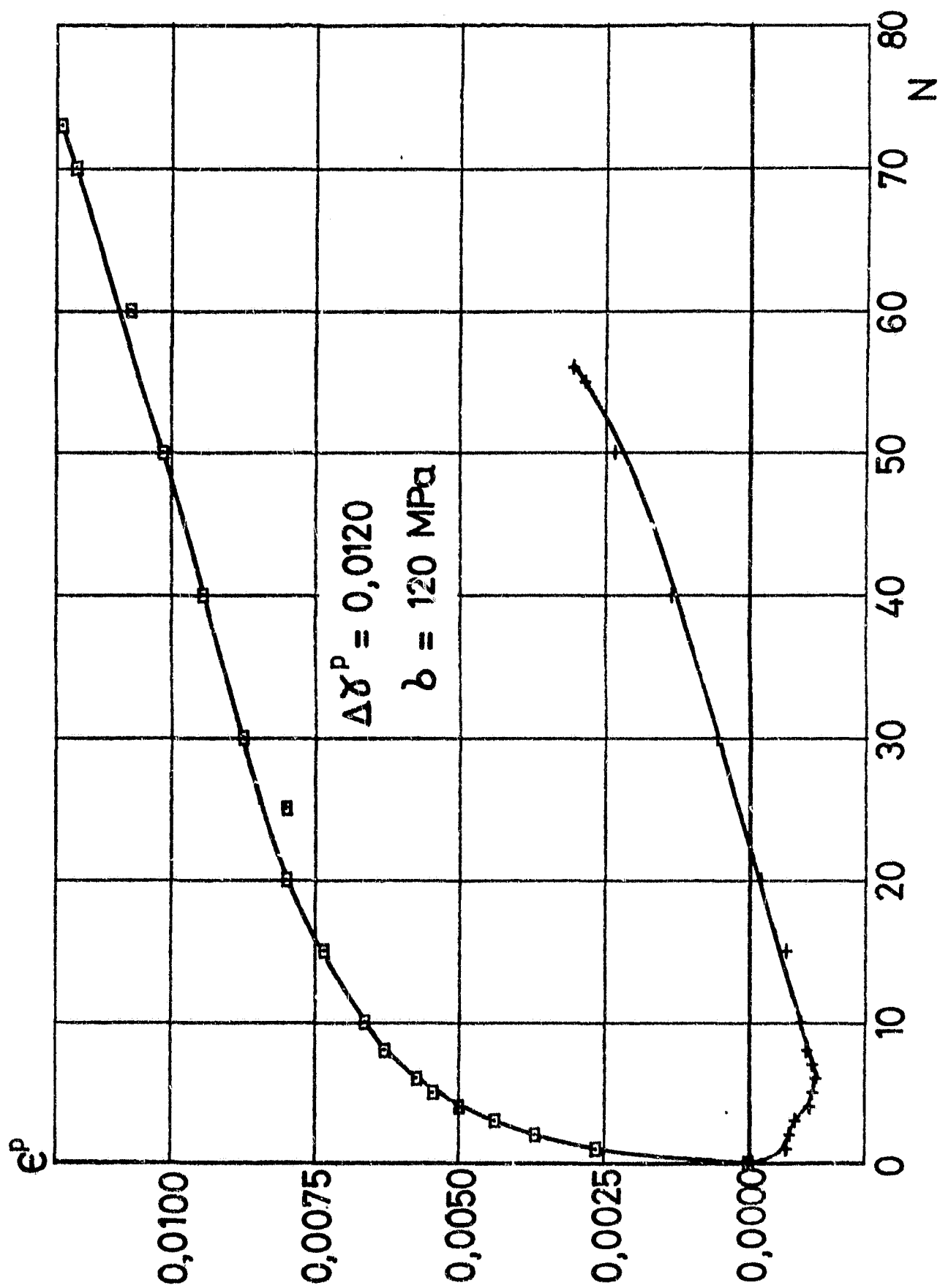


Fig. 4.7 Experimental Cyclic Creep Strain (ϵ^p)
against Number of Cycles (N).

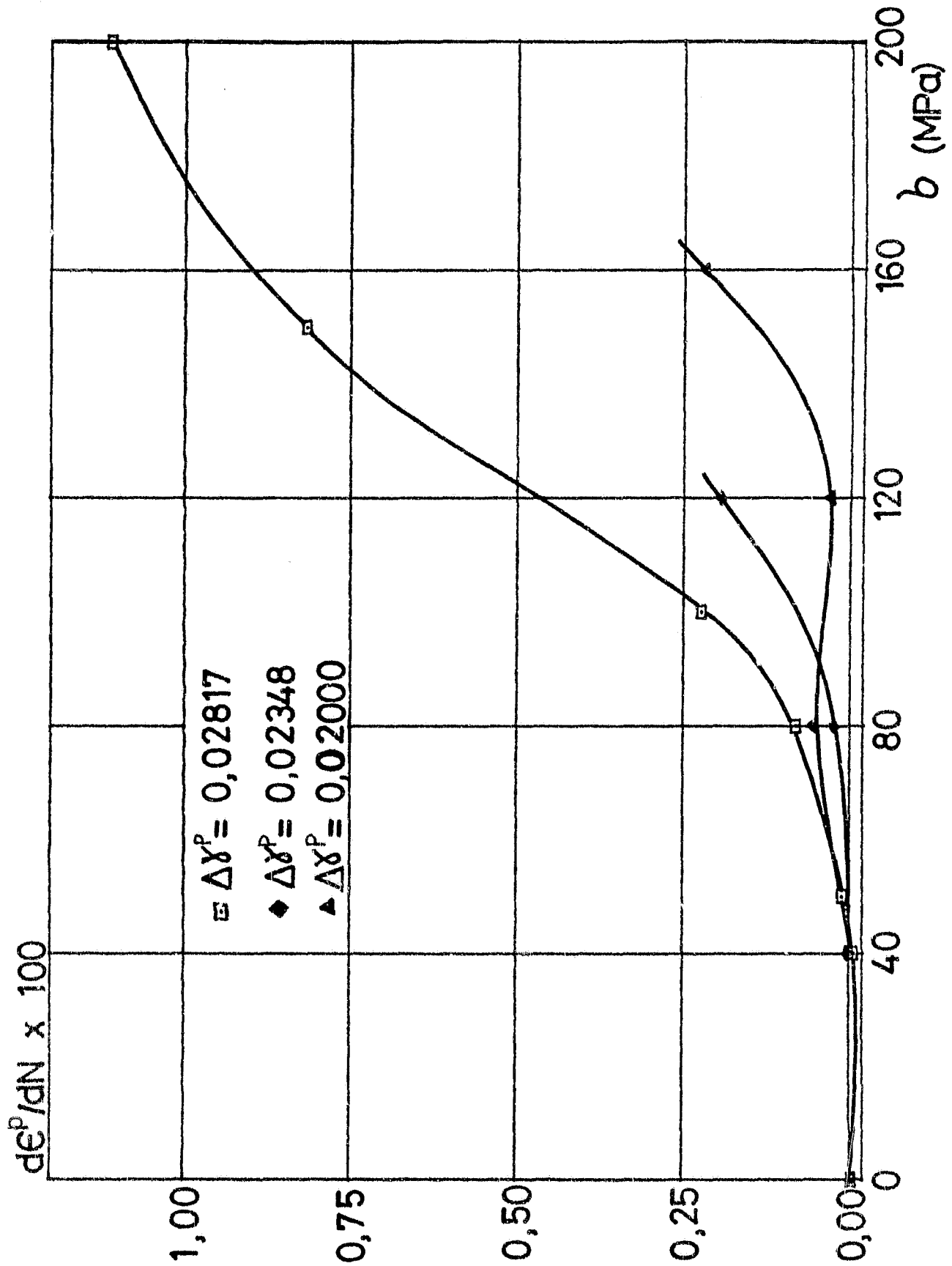


Fig. 4.8 Experimental Steady State Cyclic Creep Strain Rates ($d\epsilon^p/dN$) against Tensile Stress During Cycling (σ) for Different Plastic Shear Strain Ranges ($\Delta\gamma^p$).

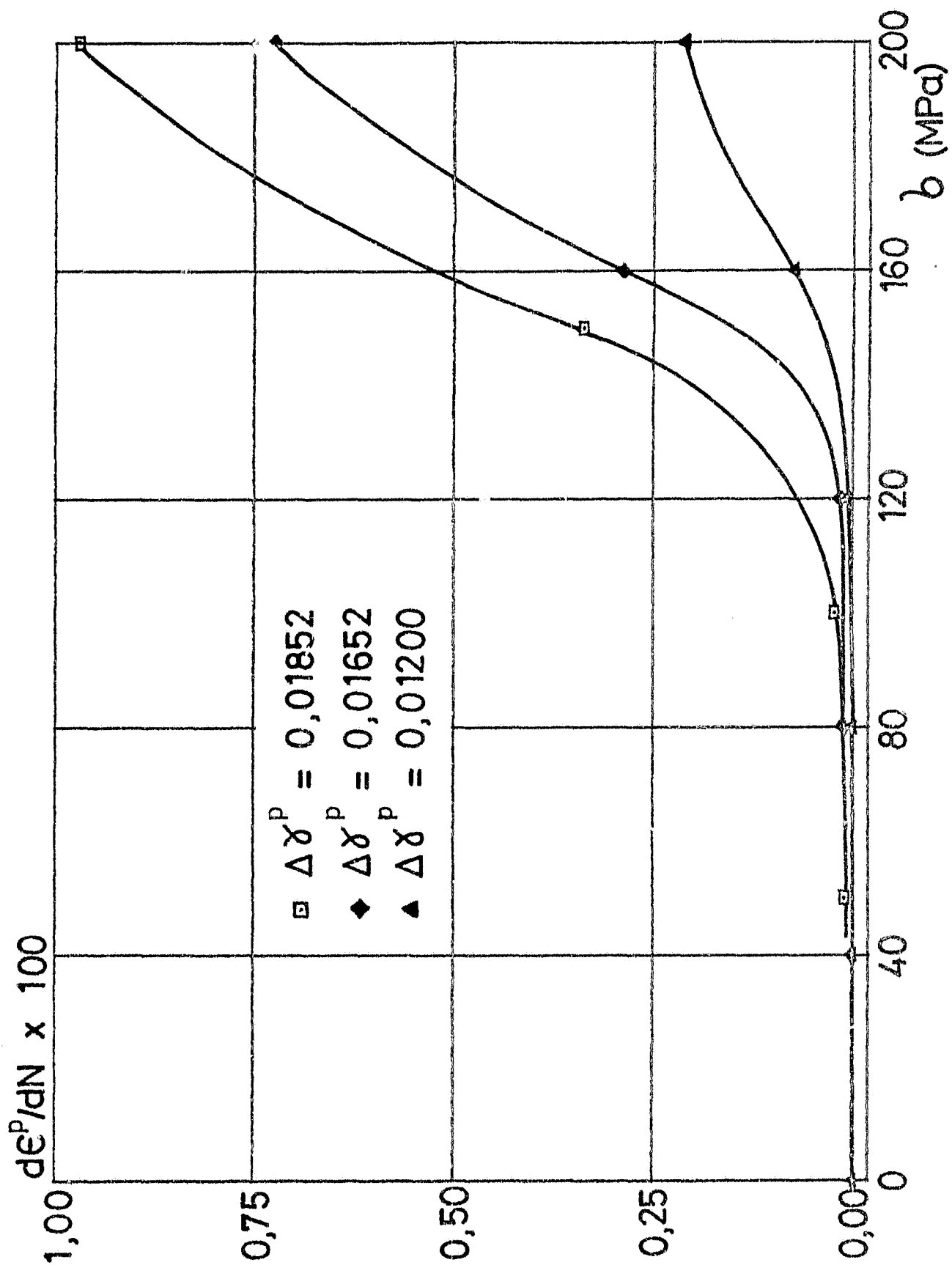


Fig. 4.9 Experimental Steady State Cyclic Creep Strain Rates ($d\epsilon^p/dN$) against Tensile Stress During Cycling (σ) for Different Plastic Shear Strain Ranges ($\Delta\gamma^p$).

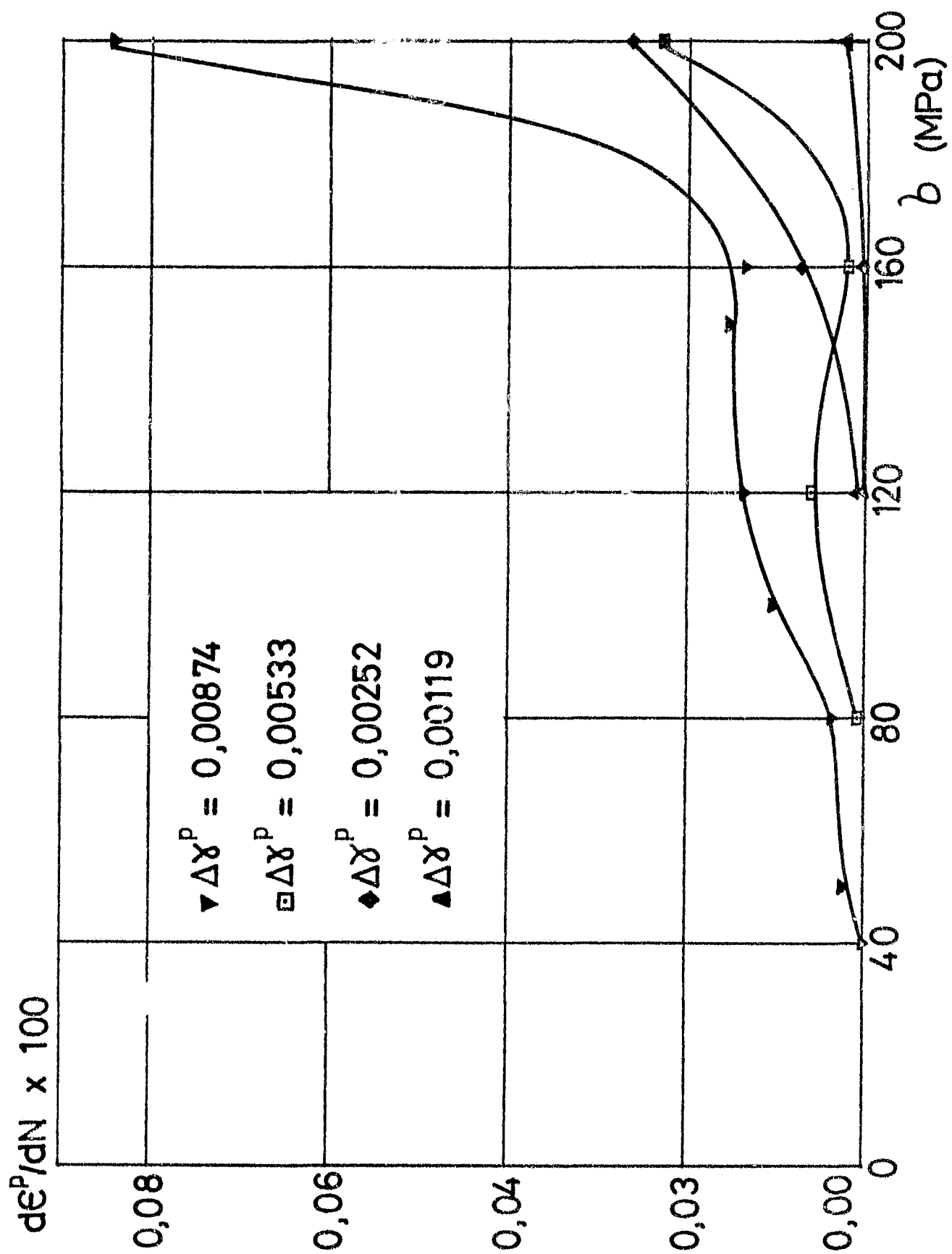


Fig. 4.10 Experimental Steady State Cyclic Creep Strain Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ) for Different Plastic Shear Strain Ranges ($\Delta\gamma^P$).

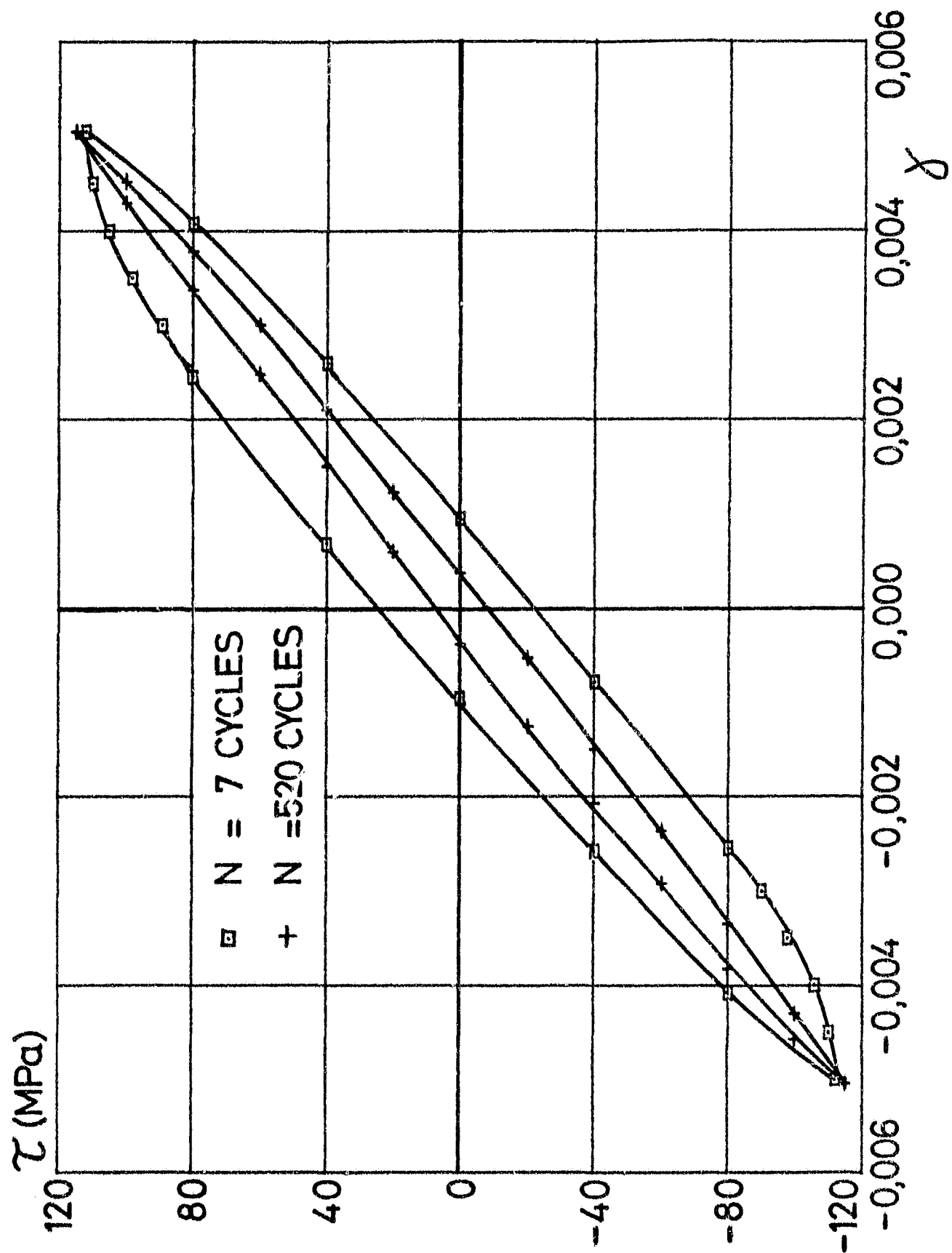


Fig. 4.11 Torsional Hysteresis Loops for $\Delta\gamma^P$
 = 0,00119 and $\delta = 200$ MPa at $N = 7$ Cycles
 and at $N = 520$ Cycles.

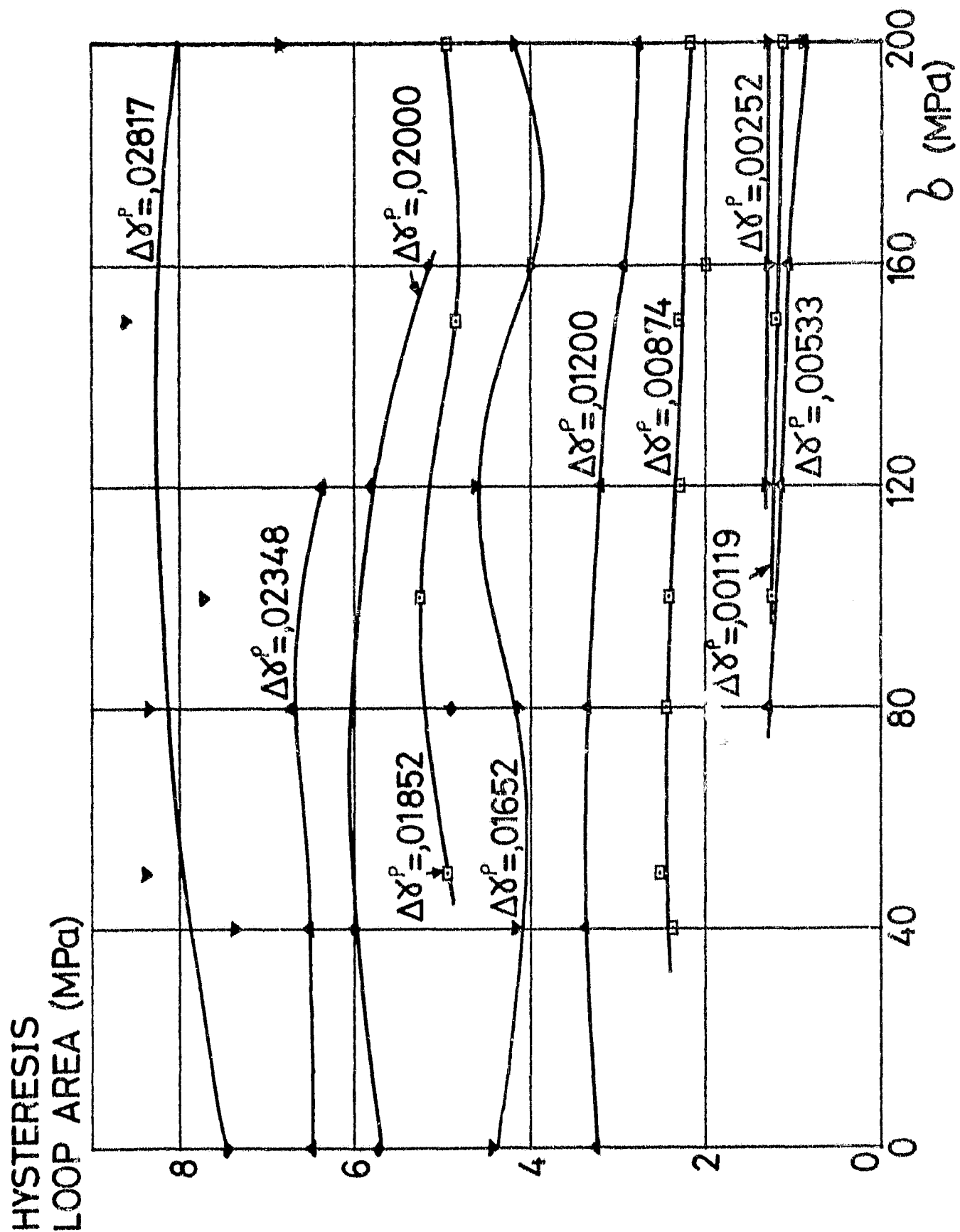


Fig. 4.12 Areas of Torsional Hysteresis Loops
against Tensile Stress during Cycling
(b) for Different Plastic Shear Strain
Ranges ($\Delta\gamma^p$)

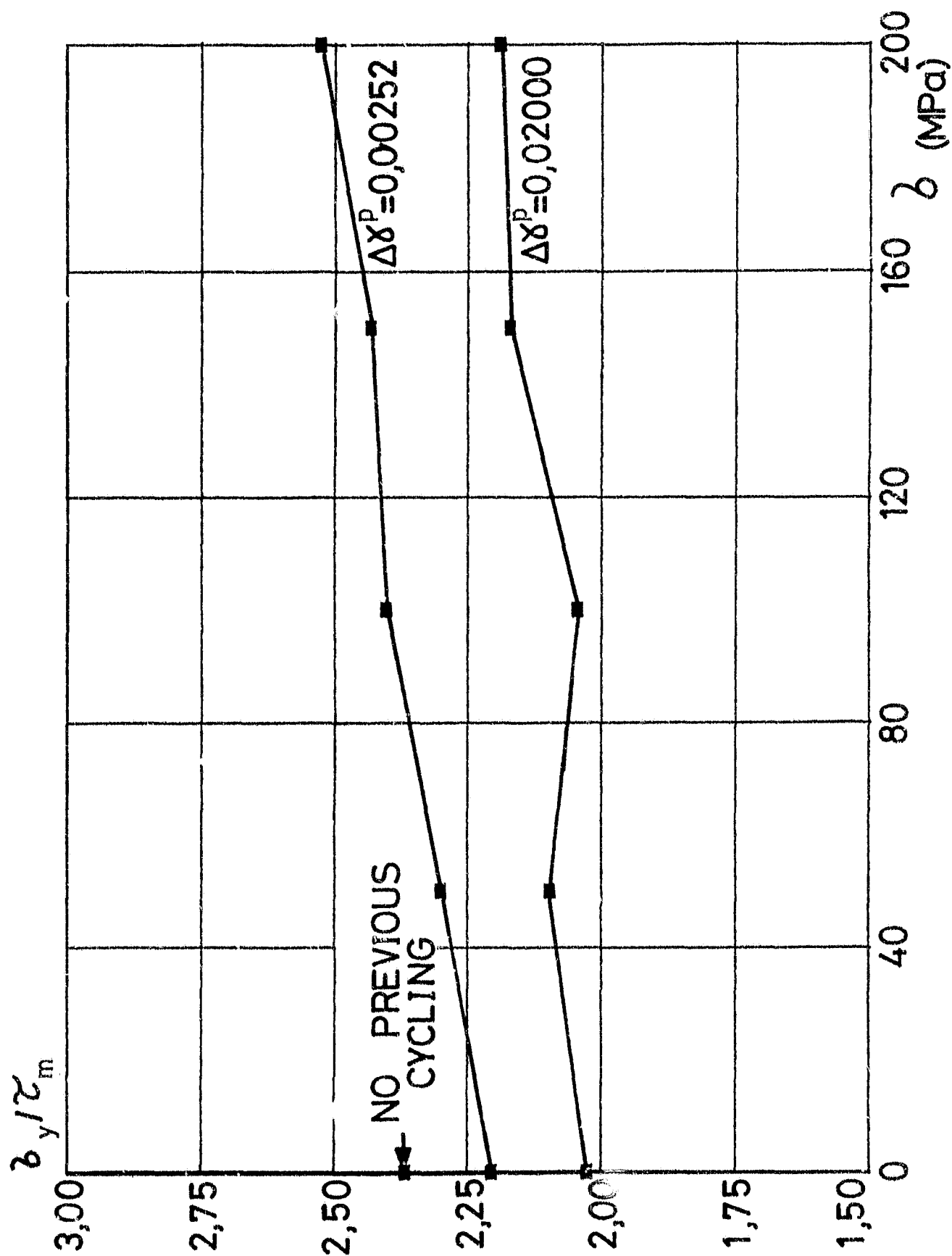


Fig. 4.13 Ratio of Tensile Yield Stress (σ_y) to
Maximum Shear Stress Attained during Previous
Cy ling (τ_m) against Tensile Stress during
Cycling (σ) for Different Plastic Shear Strain
Ranges ($\Delta \gamma^P$)

CHAPTER 5

YIELD SURFACE THEORY

5.1 General

It has been suggested (18) that, for a material subjected to cyclic torsion and steady tension, the loading surface in $b-\tau$ stress space may be represented by an expanding semi-ellipse within a limiting isotropic ellipse (fig. 2.6). This makes it possible to calculate the steady state cyclic strain induced creep rates.

For the material co-ordinate axes as in fig. 5.1, the equation of an intermediate loading surface may be written as

$$f = \frac{b_{11}^2}{a^2} + \frac{1}{2} \frac{\tau_{12}^2}{b^2} + \frac{1}{2} \frac{\tau_{21}^2}{b^2} - 1 = 0 \quad 5.1$$

where a is a constant $= \sqrt{3} \tau_{\max}$

b varies from 0 to $\tau_{\max}$

Since the plastic strain increment is normal to the yield surface

$$d\epsilon_{ij}^p = d\lambda \frac{\partial f}{\partial b_{ij}} \quad 5.2$$

Note that in eq. 5.1 the shear stresses τ_{12} and τ_{21} are numerically equal, but, as far as the differentiation in eq. 5.2 is concerned, they are not identical. Thus

$$d\epsilon^p = d\epsilon_{11}^p = d\lambda \frac{2b_{11}}{a^2} = d\lambda \frac{2b}{a^2} \quad 5.3$$

$$d\gamma^p = 2 d\epsilon_{12}^p = d\lambda \frac{2\tau_{12}}{b^2} = d\lambda \frac{2\tau}{b^2} \quad 5.4$$

From eqs. 5.3 and 5.4

$$d\epsilon^p = \frac{b^2}{a^2} \frac{b}{\tau} d\gamma^p \quad 5.5$$

From eq. 5.1

$$b^2 = \frac{\tau^2}{1 - b^2/a^2} \quad a^2 = 3\tau_{\max}^2$$

Substituting into eq. 5.5

$$d\epsilon^p = \frac{b\tau}{3\tau_{\max}^2 - b^2} d\gamma^p \quad 5.6$$

From eq. 5.1, for $b = \tau_{\max}$ and $\tau = \tau_m$

$$3\tau_{max}^2 - b^2 = \tau_m^2$$

Substituting into eq. 5.6

$$d\epsilon^p = \frac{b\tau}{3\tau_m^2} d\gamma^p$$

Integrating this over one complete torsional loading cycle, the steady state cyclic creep rate is obtained

$$\frac{d\epsilon^p}{dN} = \frac{b}{3\tau_m^2} \oint \tau d\gamma^p \quad 5.7$$

Note that $\oint \tau d\gamma^p$ is the specific plastic work done over one cycle. Since no net elastic work is done over that period.

$\oint \tau d\gamma^p = \oint \tau d\gamma =$ area inside torsional hysteresis loop

Because the hysteresis loop is hardly affected by either the tensile stress or by progressing axial deformation, cyclic creep calculations in a design situation may be based on the results of cyclic torsion tests. In the present work, however, each cyclic creep estimate has been calculated from the properties of the corresponding hysteresis loop. The experimental steady state creep rates together with those obtained from eq. 5.7 have been plotted against b in figs. 5.2a - 5.2j. It can be seen that the theory overestimates the creep rates considerably, especially for the smaller shear strain ranges and stresses.

5.2 Determination of the Approximate Yield Surface

The tests described in section 4.3 were used to determine whether the assumption of an isotropic limiting loading surface used in eq. 5.7 was justified. This assumption was based on the notion that cycling in the plastic region removes the effects of any previous deformation history (34). It would thus be expected that the initial material anisotropy as shown in fig 4.2 would have been removed once steady state cycling had been achieved. For a material, which has been cycled in torsion with a superimposed steady tension, the limiting yield surface, assumed to be elliptical, is given by

$$\frac{b^2}{b_y^2} + \frac{\tau^2}{\tau_{max}^2} = 1 \quad 5.8$$

where δ_y is the tensile yield stress. For the particular value of δ during cycling, when $\tau = \tau_m$

$$\tau_{max} = \tau_m / \sqrt{1 - \delta^2 / \delta_y^2} \quad 5.9$$

From the tests mentioned above, the values of τ_{max} and δ_y / τ_{max} have been calculated the latter plotted in fig 5.3. It seems that the limiting loading surface, for this material at least, is not isotropic. There is a small dependence of δ_y / τ_{max} on the shear strain range and the tensile stress, but as it is small, a mean value of 2,07 has been used in rederiving eq. 5.7. The latter is now changed by replacing the factor 3 by $2,07^2$. Hence

$$\frac{d\epsilon^p}{dN} = \frac{\delta}{4,28 \tau_m^2} \oint \tau d\delta^p \quad 5.10$$

This decreases the previous estimates of the cyclic creep rate by a factor of 1,43 which is still not sufficient to give close agreement with actual behaviour. These results have been plotted in figs. 5.2a - 5.2j from which it can be seen that, although the theory considerably overestimates the creep rates, theoretical and experimental curves tend towards having similar slopes at higher tensile stress values.

5.3 Assumption of a Critical Plastic Shear Strain δ_c^p for Creep

The results shown in figs. 5.2a - 5.2j indicate that the expanding ellipse loading surface theory could be used provided modifications are made to the loading surface shape. It seems that the loading surface does not expand from the zero shear stress axis as the curvature, and hence the plastic strain increment, is too high. An alternative form has been assumed in which the loading surface translates along the τ -axis until a critical value of τ has been reached after which it expands elliptically as before (See fig. 5.4). To determine the loading surface behaviour, it is more convenient to introduce a critical shear strain δ_c^p at which the ellipse starts to expand, rather than at a critical τ_c . δ_c^p is clearly a function of the tensile stress δ , increasing with decreasing δ . Approximate values of δ_c^p could be obtained from experimental results by considering the value of δ at which the cyclic creep rates, for a particular plastic shear strain

range $\Delta\delta^p$, began to increase rapidly. Values of $\Delta\delta^p$ against b have been plotted in fig. 5.5 which may now be used to find δ_c^p from b . The modified theory is only approximate as it now only predicts creep that occurs for $\Delta\delta^p > \delta_c^p$.

Referring to fig. 5.5, it was assumed that the intermediate and limiting surfaces were semi-ellipses with the major axis situated at $\tau = \tau_c$. The equation of the ellipse is thus:

$$\frac{b^2}{a^2} + \frac{(\tau - \tau_c)^2}{b^2} = 1 \quad 5.11$$

where a is a constant

b varies from 0 to $\tau_{max} - \tau_c$

τ varies from τ_c to τ_m

When $\tau = \tau_m$, $b = \tau_{max} - \tau_c$ and

$$\tau_m = \tau_c + (\tau_{max} - \tau_c) \sqrt{1 - b^2/a^2} \quad 5.12$$

The value of τ_c for each cyclic creep test was obtained in the following way. Using fig. 5.5, δ_c^p was found from b . A line parallel to the unloading line, but displaced from it by δ_c^p , was drawn on the actual torsional hysteresis loop. This line intersected the loop at $\tau = \tau_c$ (fig. 5.6).

Using this procedure and eq. 5.12, the values of τ_{max} were obtained for the specimens tested in tension after previous torsional cycling. It was found that the ratio of b/τ_{max} was virtually the same as before. The constant a in eq. 5.11 could thus be set equal to $2.07 \tau_{max}$.

The cyclic creep rate was then derived as before.

$$\frac{d\epsilon^p}{dN} = \frac{b}{2.07^2 \tau_{max}^2 - b^2} \oint (\tau - \tau_c) d\delta^p \quad 5.13$$

where the cyclic integral is evaluated only for $|\tau| > \tau_c$ and is in fact equal to the shaded area in fig. 5.6.

The creep rates predicted by eq. 5.13 have been compared to the actual creep rates in figs. 5.7a - 5.7j. The agreement is fairly good, especially at higher shear strain ranges.

5.4 Discussion

For design purposes, the original theory (eq. 5.7) is obviously preferable as it can be used to predict cyclic strain induced creep from a knowledge of the steady state torsion hysteresis loop only, the latter being theoretically

obtainable from tests on a single specimen. Unfortunately its agreement with experiment is rather poor as it considerably overestimates the axial creep rates. The modified loading surface (eq. 5.13) gives quite good results but predicts creep only for $\Delta\gamma' > \gamma'_c$. Unfortunately the creep rates can no longer be predicted from simple mechanical data. For the modified theory to be of use in a design situation, some simple method of determining γ'_c would have to be devised. Further investigation of the theory would have to be carried out to enable creep below the critical plastic shear strain range to be calculated.

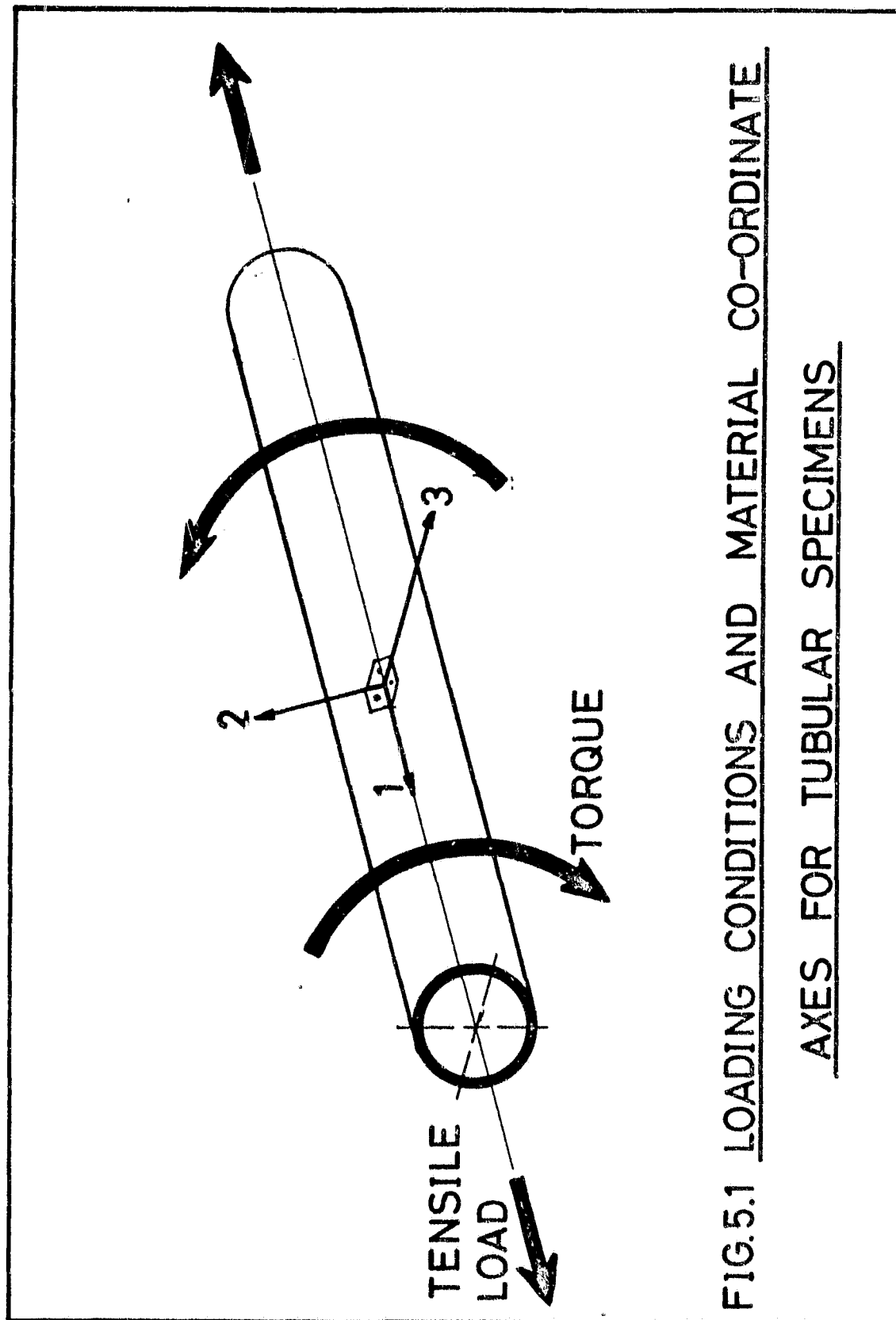


FIG.5.1 LOADING CONDITIONS AND MATERIAL CO-ORDINATE
AXES FOR TUBULAR SPECIMENS

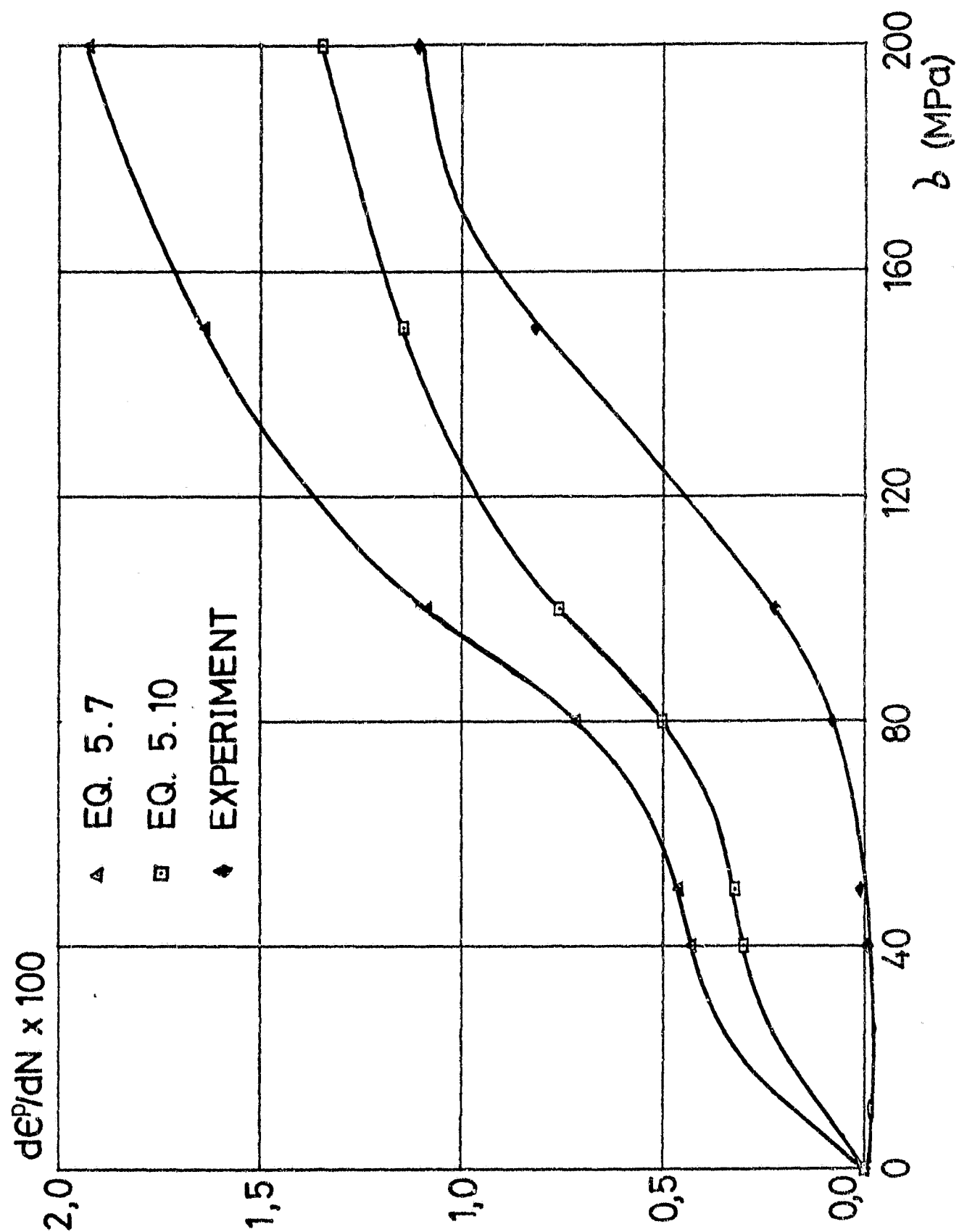


Fig. 5.2a Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^p/dN$) against Tensile Stress During Cycling (σ) for $\Delta\sigma^p = 0,02817$

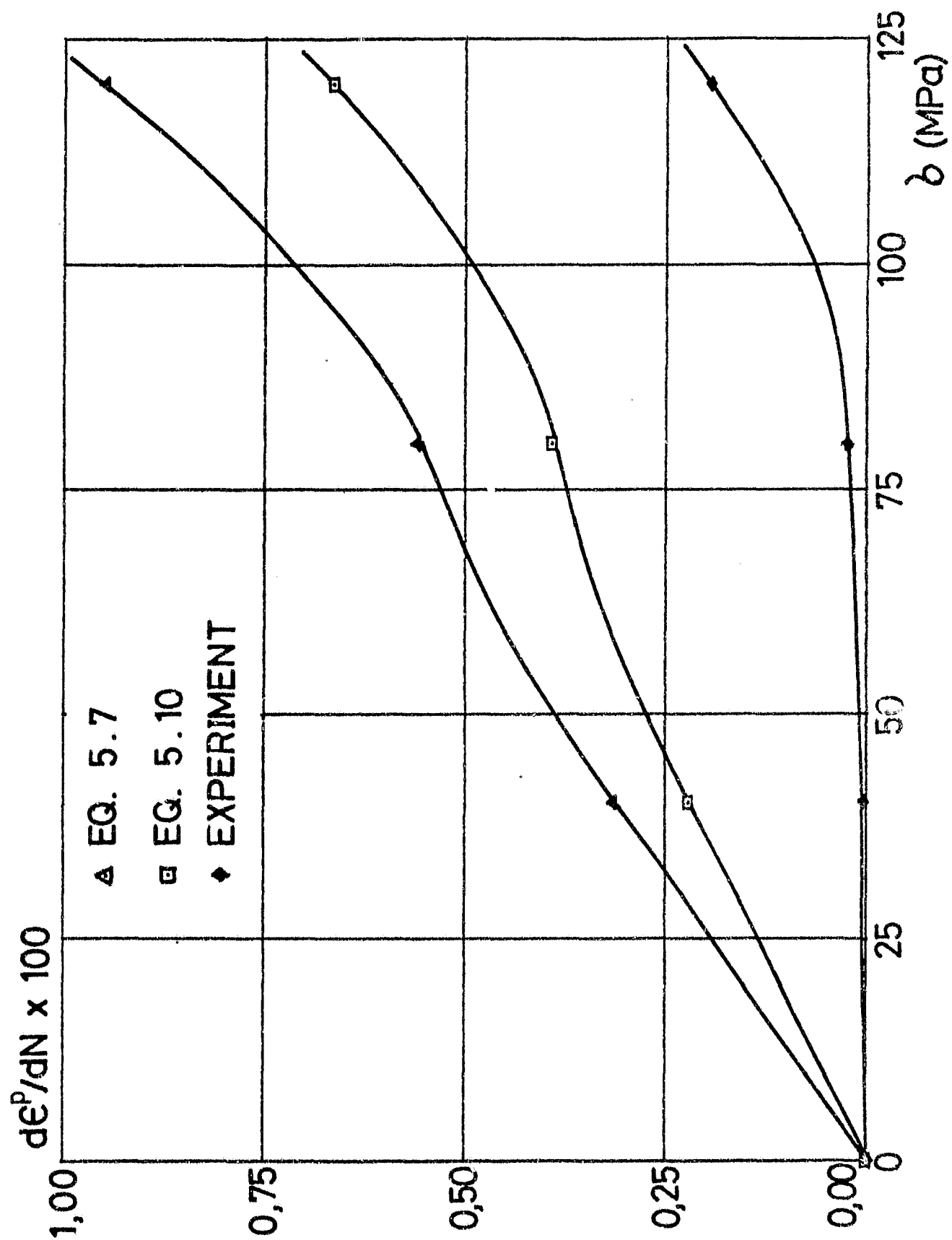


Fig. 5.2b Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ), $\Delta\sigma^P = 0,02348$

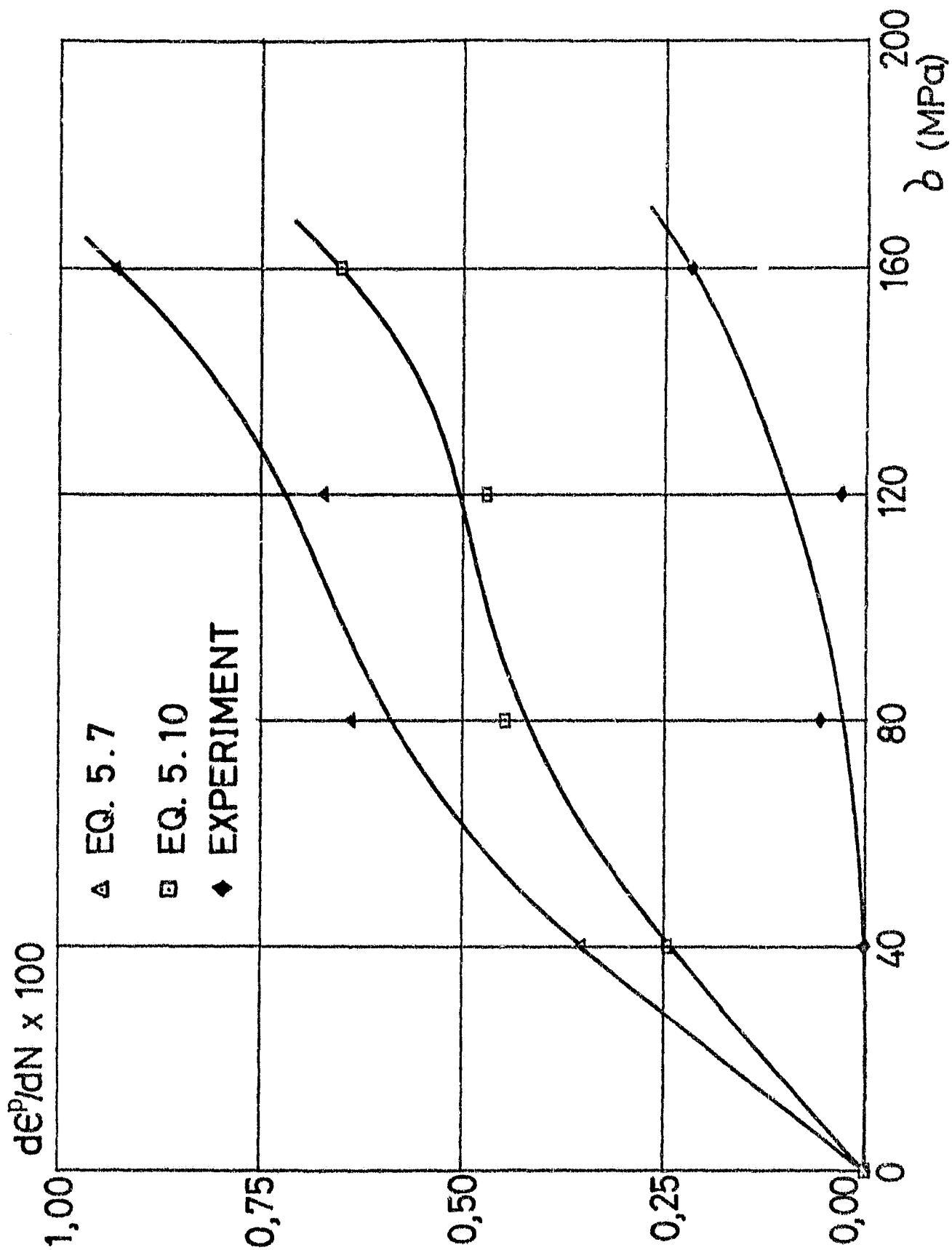


Fig. 5.2c Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^p/dN$) against Tensile Stress During Cycling (σ), $\Delta\sigma^p = 0.02000$

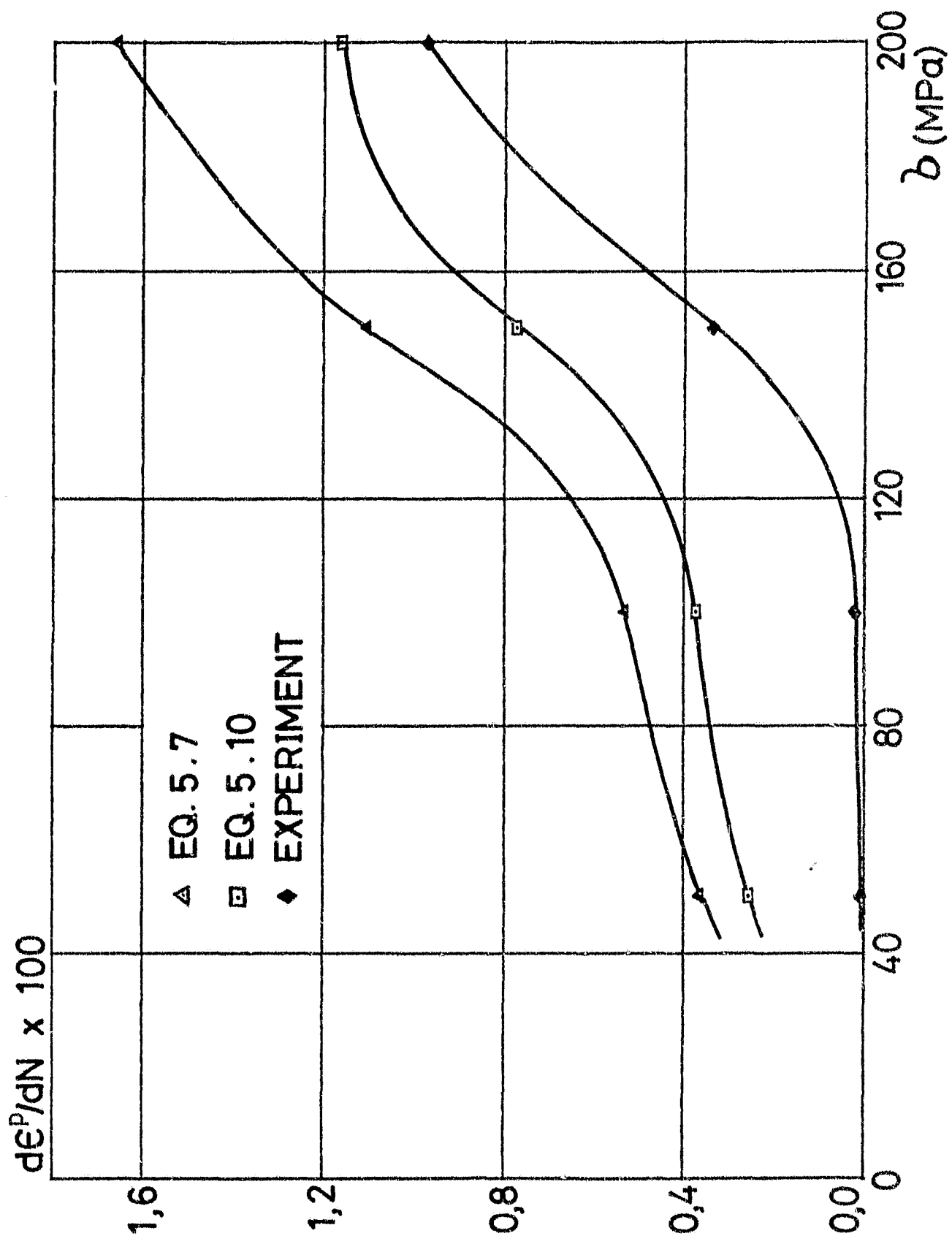


Fig. 5.2d Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ) for $\Delta\sigma^P = 0.01852$

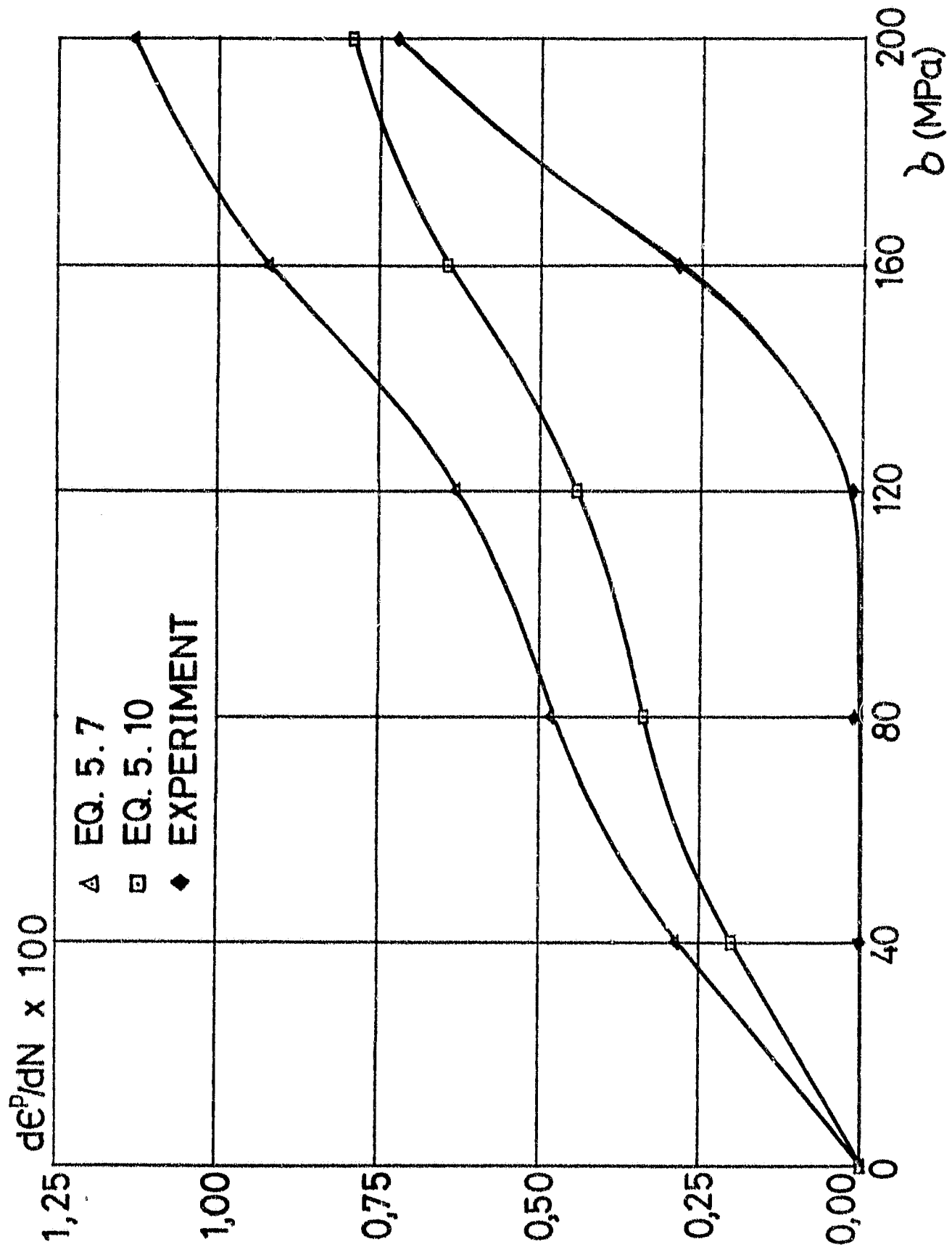


Fig. 5.2e Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^p/dN$) against Tensile Stress During Cycling (σ) for $\Delta\epsilon^p = 0,01652$

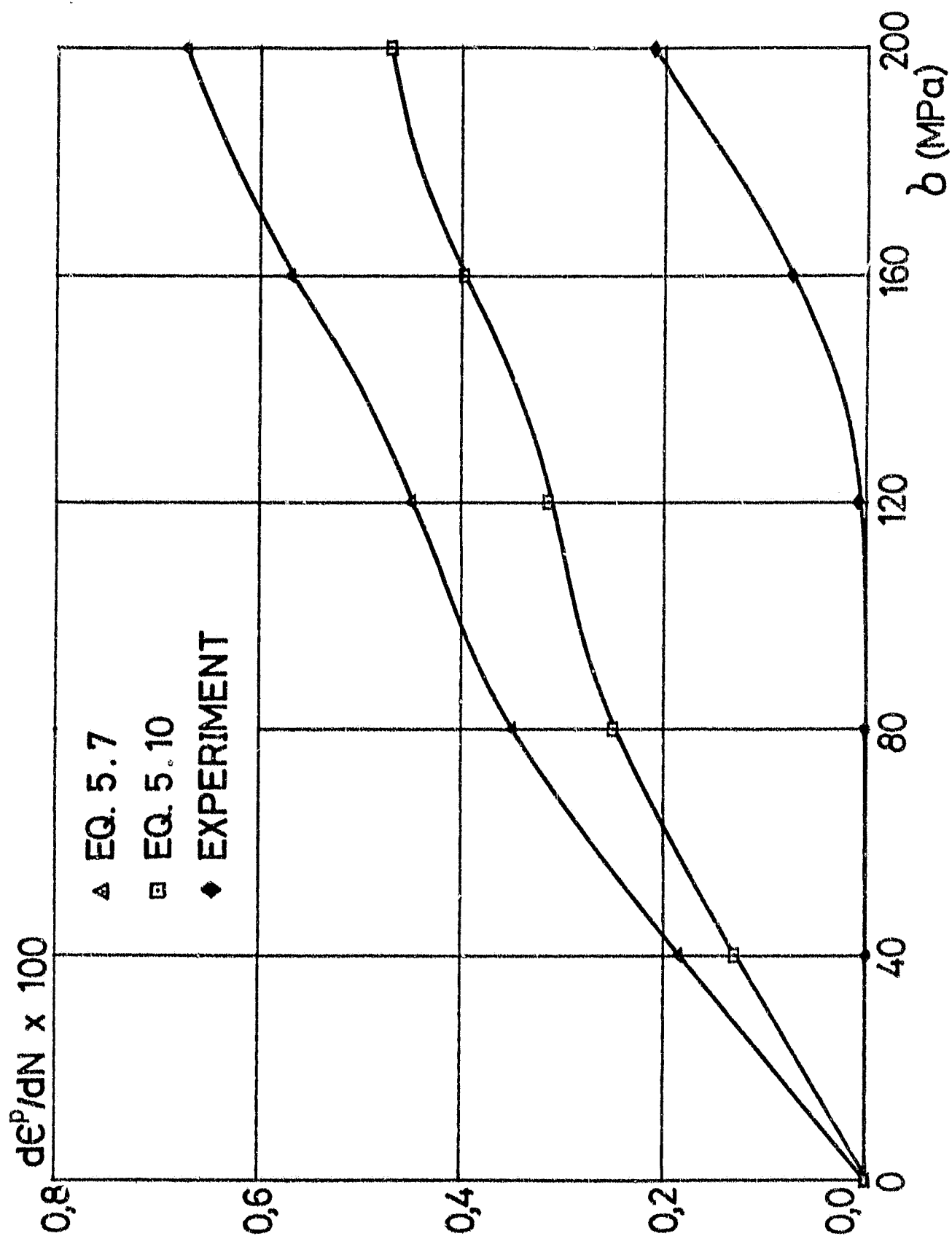


Fig. 5.2f Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^p/dN$) against Tensile Stress During Cycling (σ) for $\Delta\sigma^p = 0,01200$

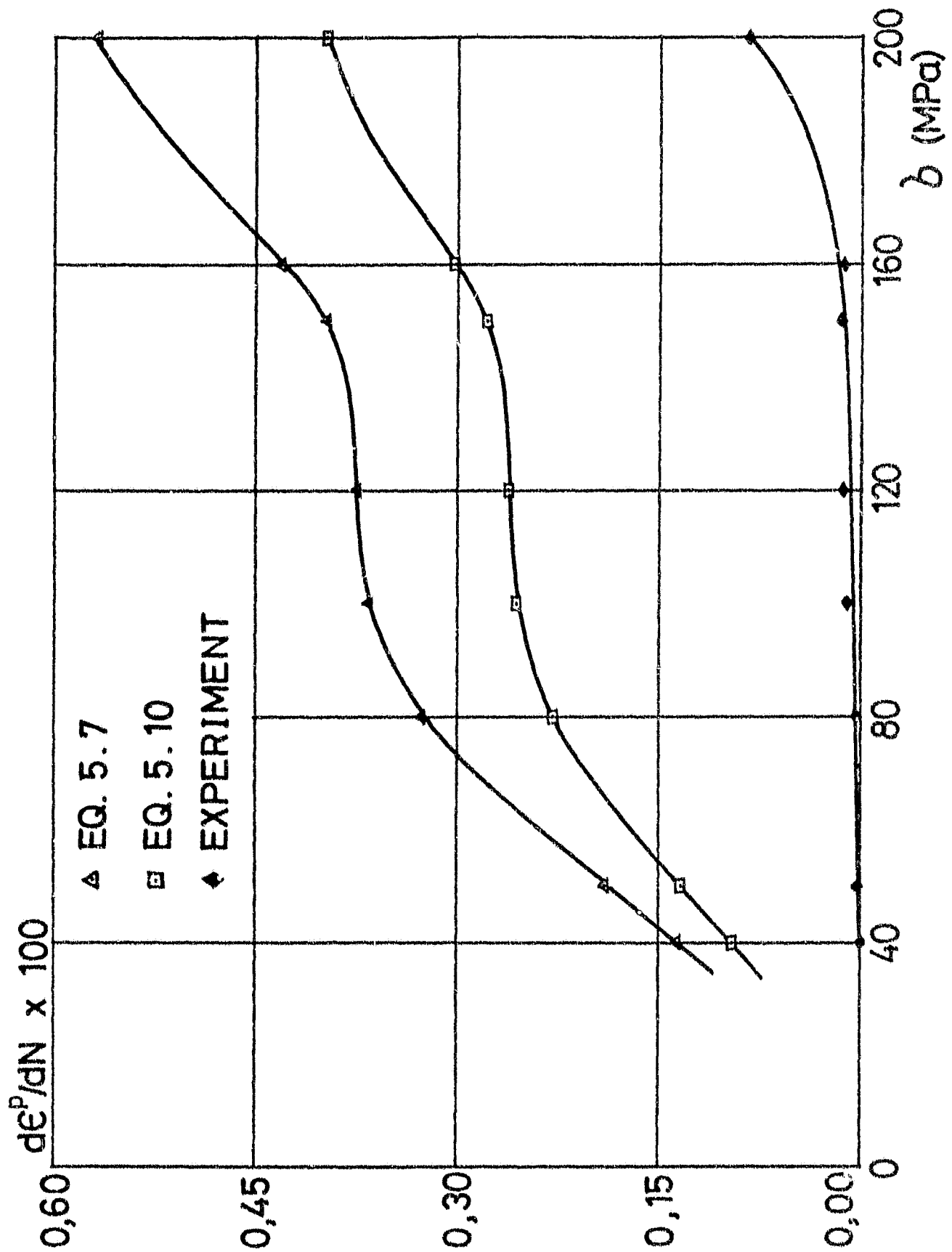


Fig. 5.2g Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling ($\bar{\sigma}$) for $\Delta\sigma^P=0,00374$

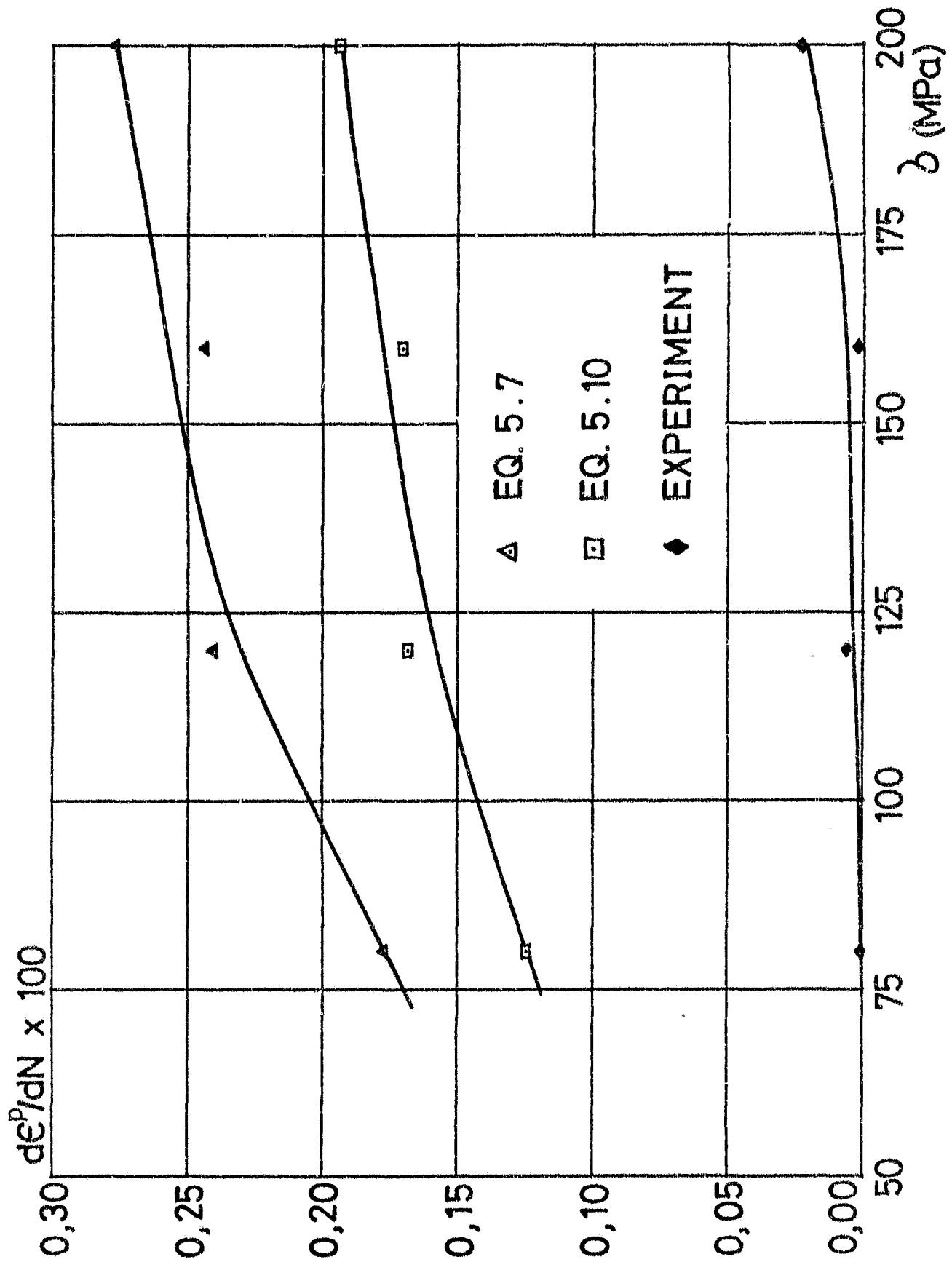


Fig. 5.2h Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^p/dN$) against Tensile Stress During Cycling (σ) for $\Delta\sigma^p = 0.00532$

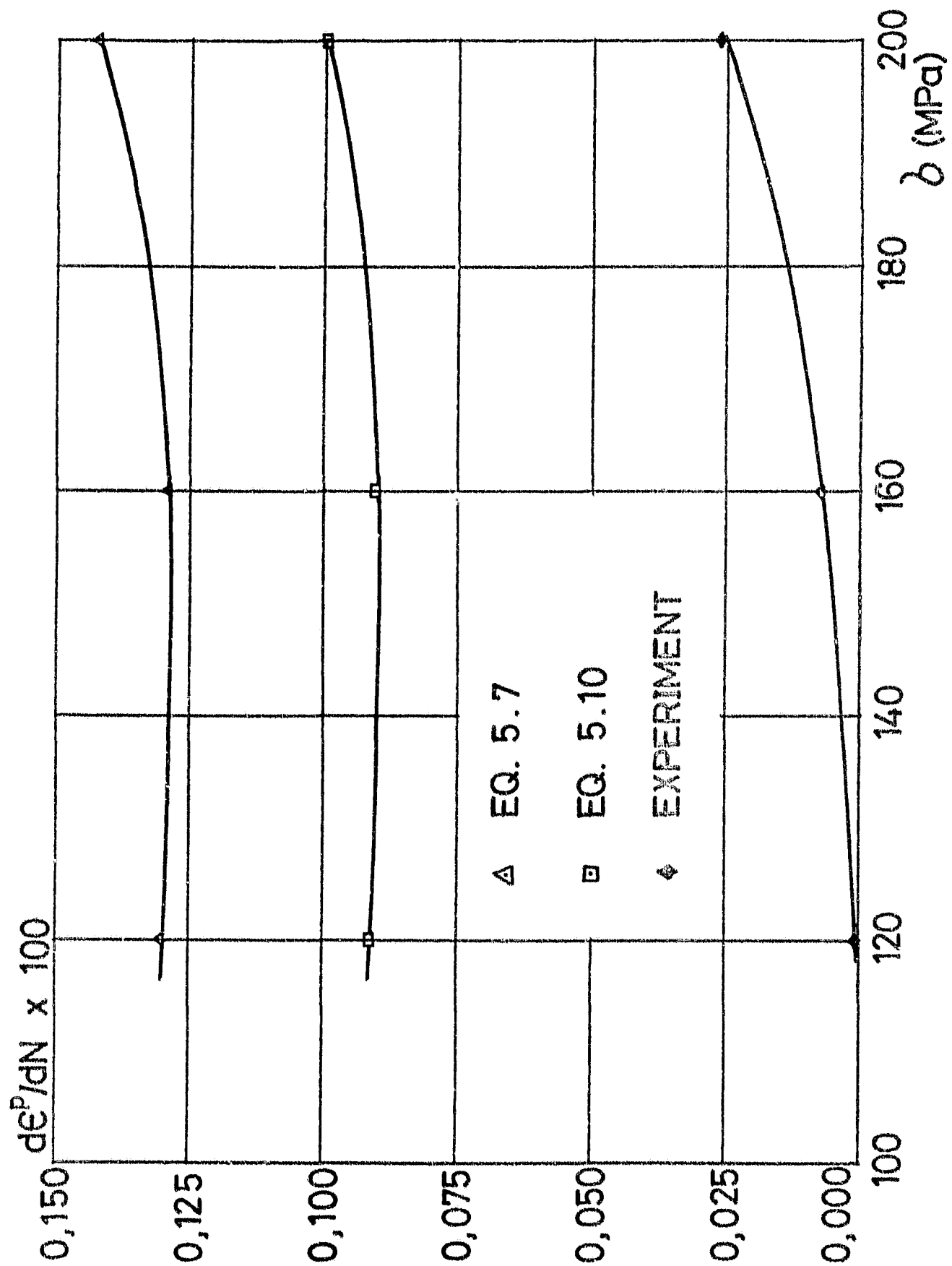


Fig. 5.2i Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (δ) for $\Delta\sigma^P = 0.00252$

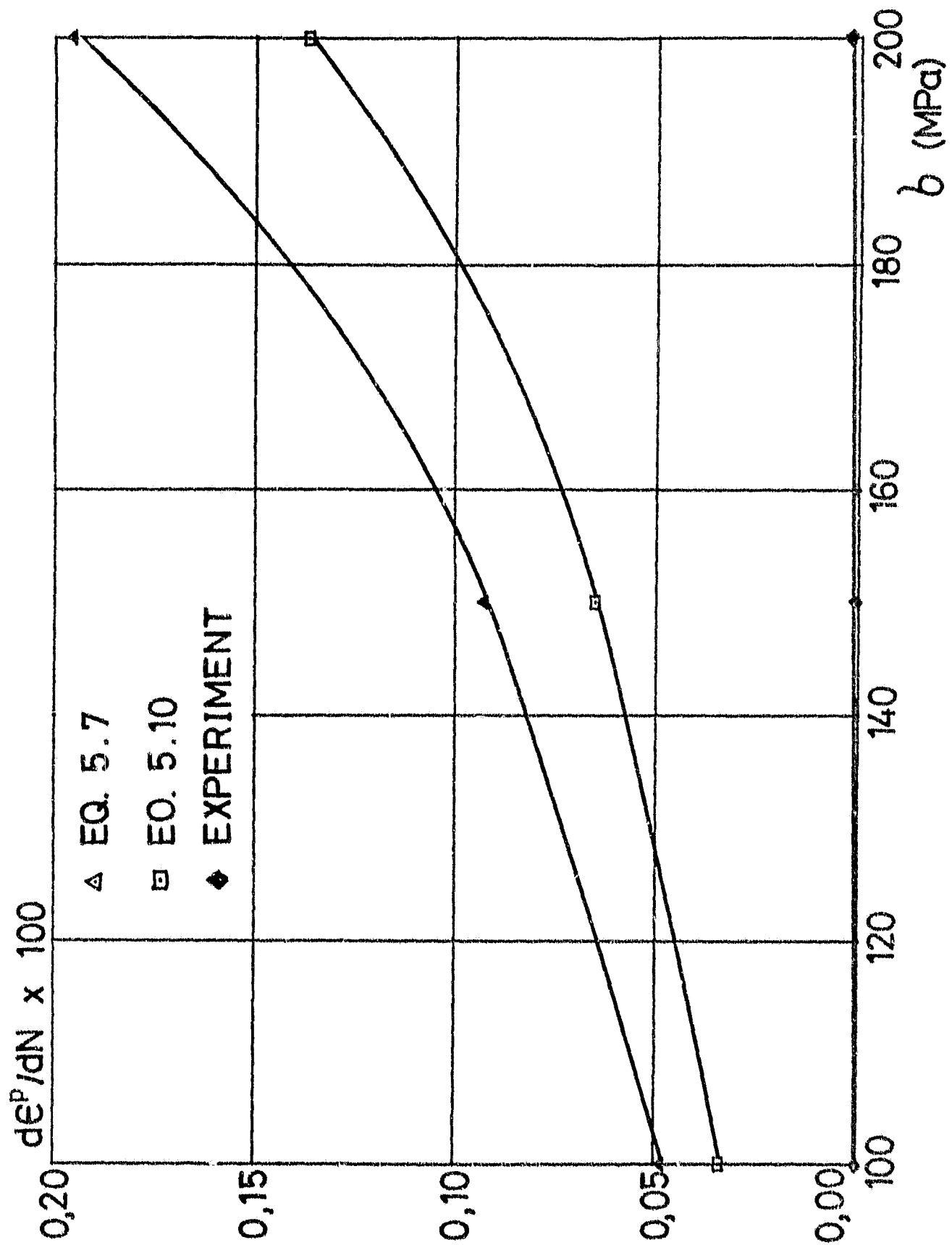


Fig. 5.2j Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ) for $\Delta\sigma^P = 0.00119$

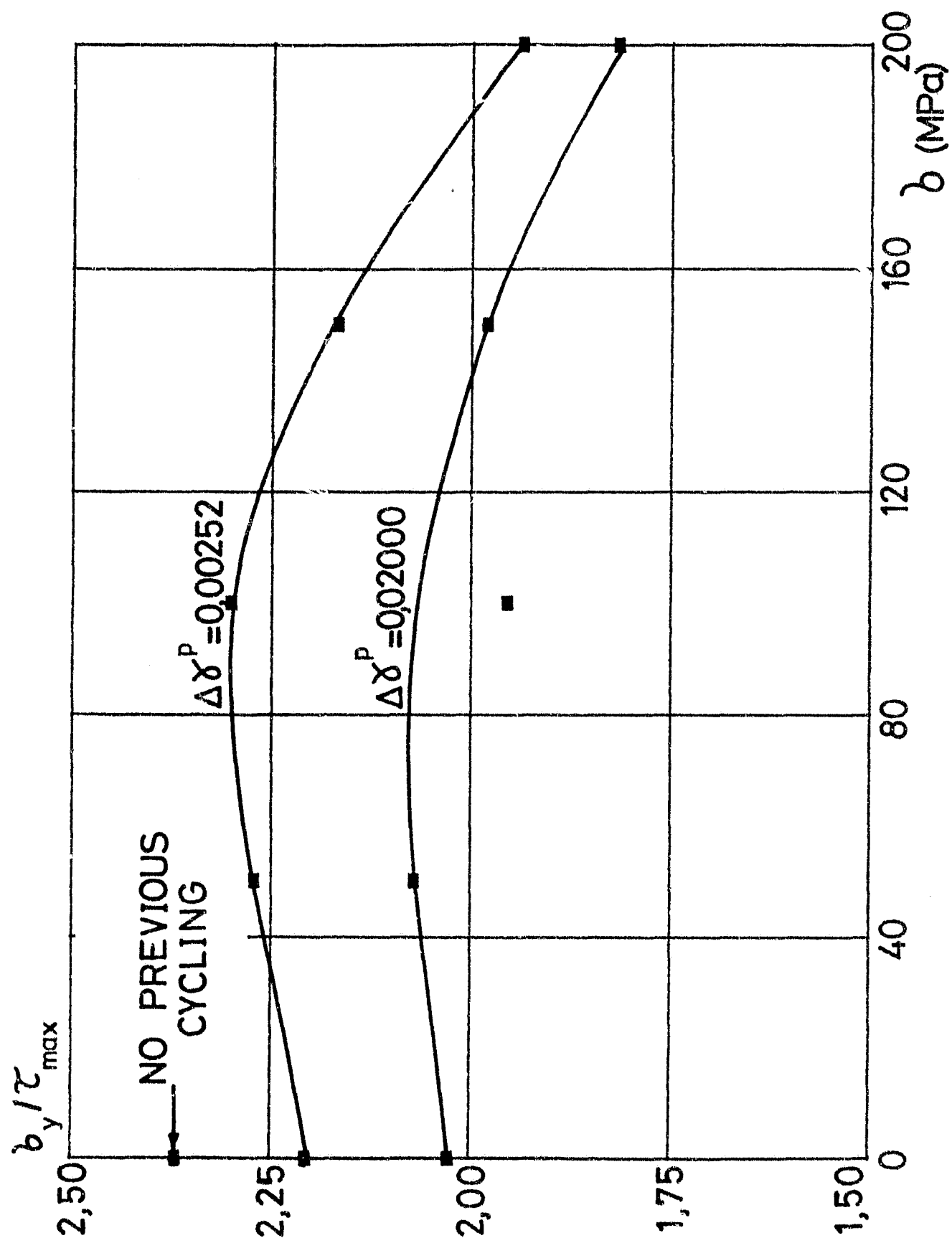
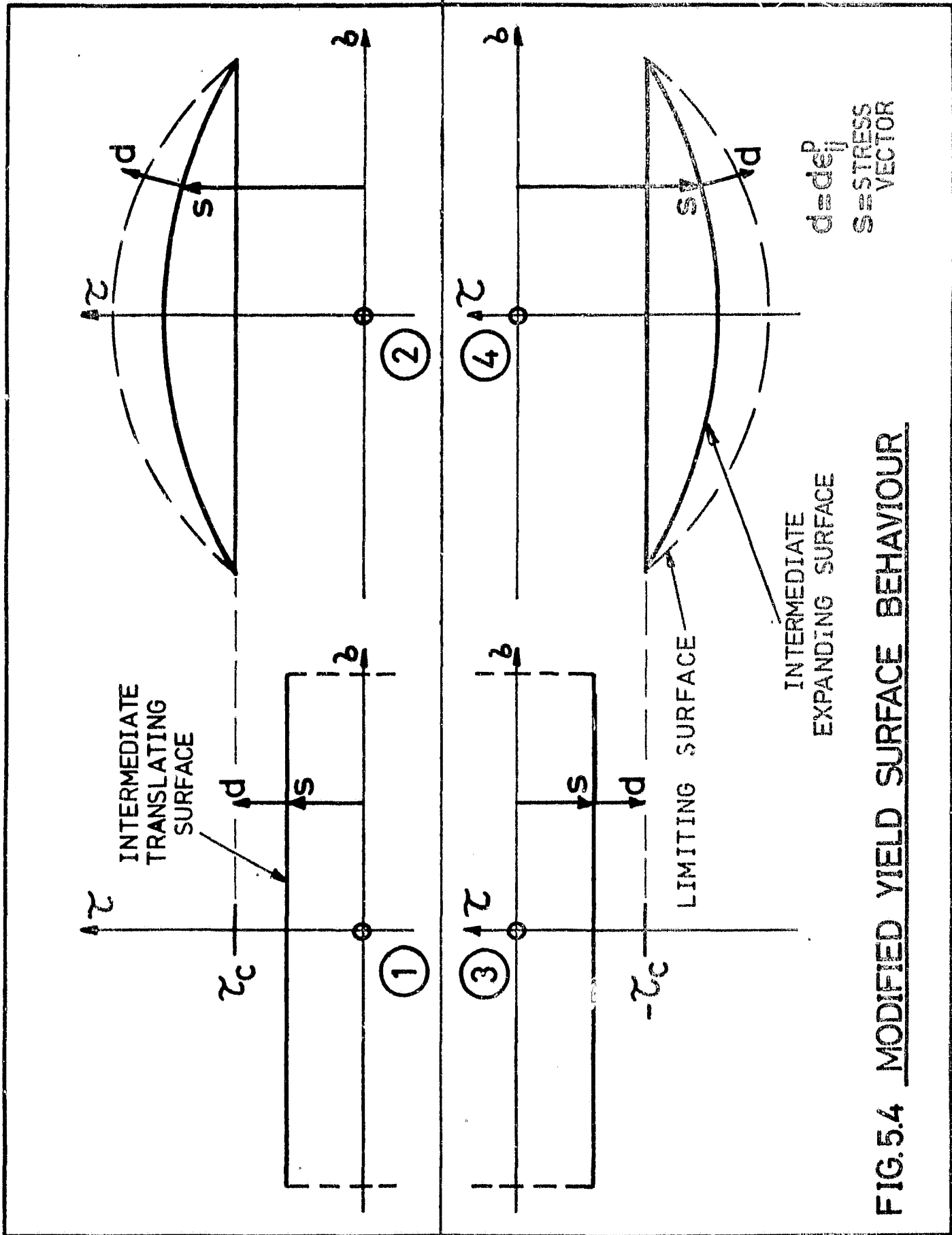


Fig. 5.3 Ratio of Tensile Yield Stress (b_y) to Maximum Shear Stress (τ_{max} from Eq. 5-9) against Tensile Stress During Cycling (b) for Different Plastic Shear Strain Ranges ($\Delta \gamma^P$)



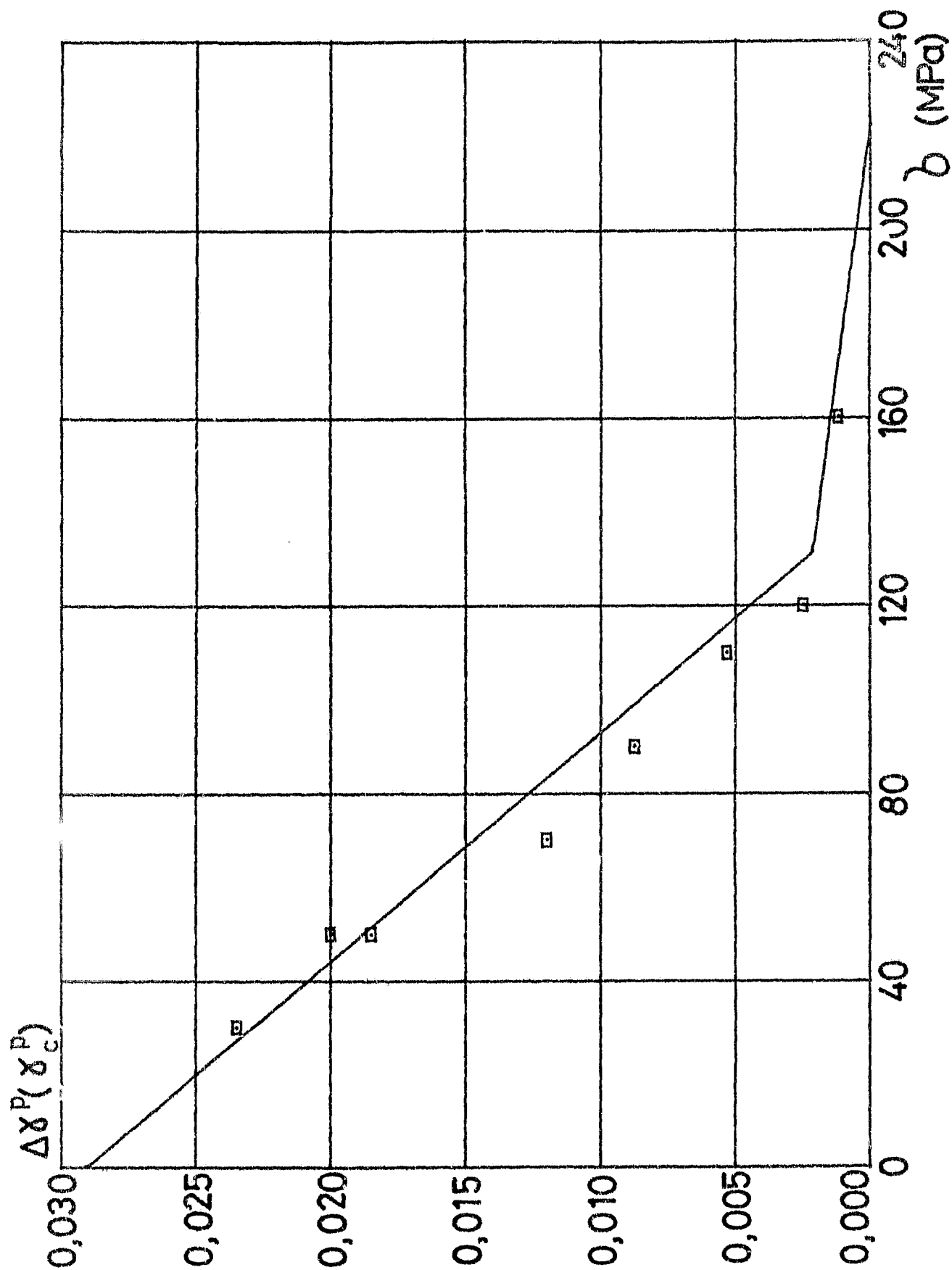


Fig. 5.5 Critical Plastic Shear Strain Range ($\Delta\gamma^P$ or γ_c^P) against Tensile Stress During Cycling (**b**)

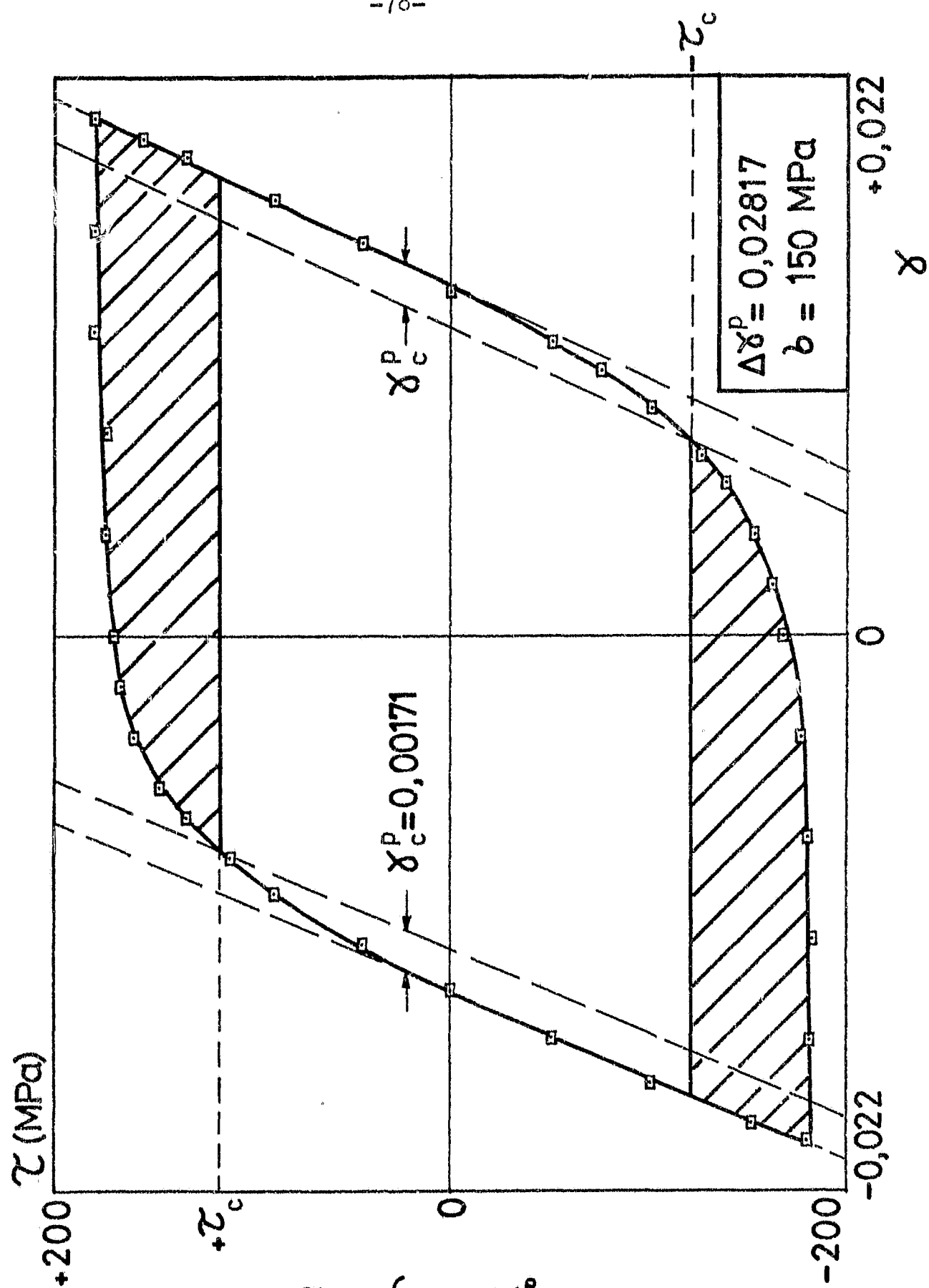


Fig. 5.6 Obtaining τ_c and $\oint (\tau - \tau_c) d\delta^p$ from Actual Torsional Hysteresis Loop

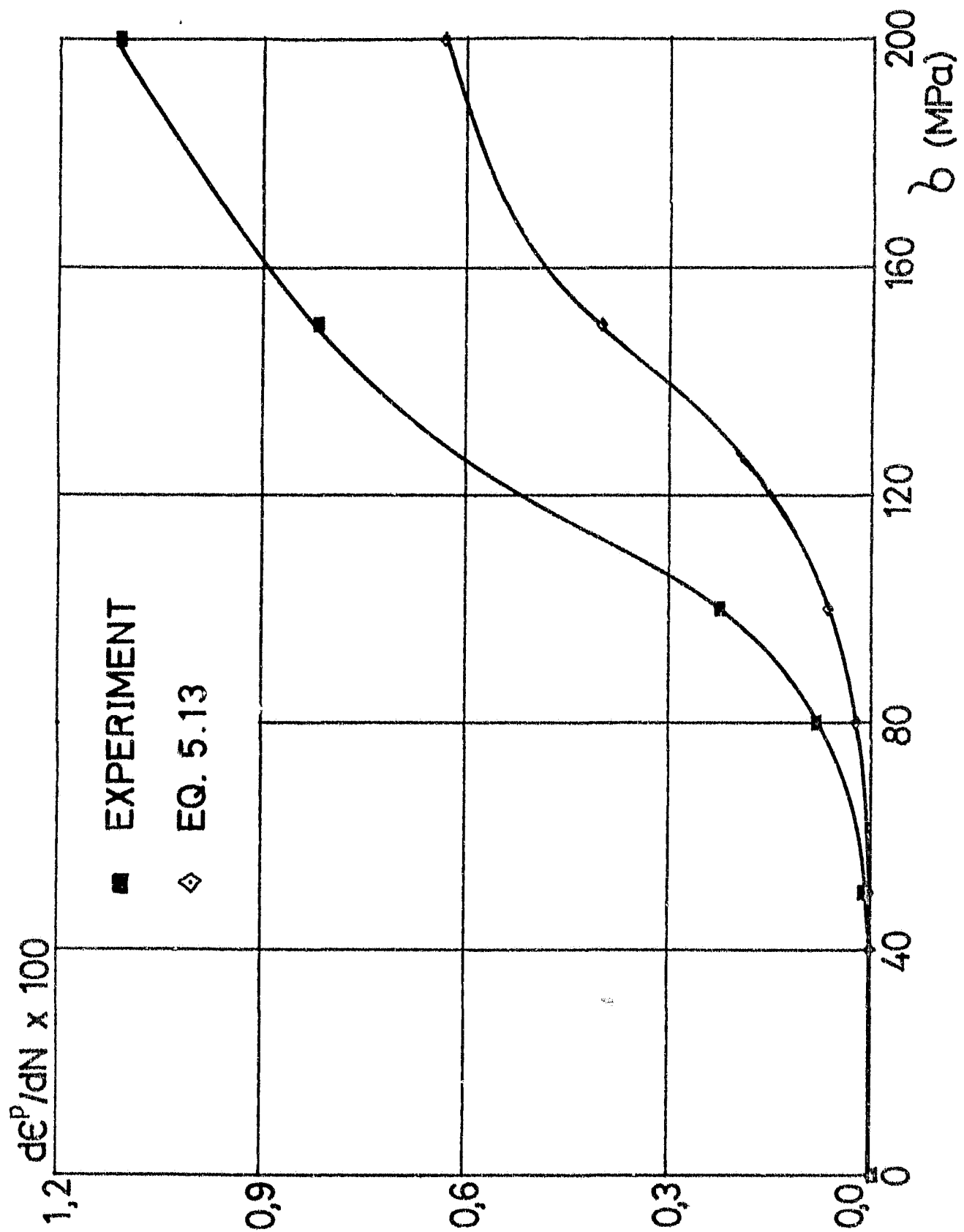


Fig. 5.7a Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ) for $\Delta\gamma^P = 0,02817$

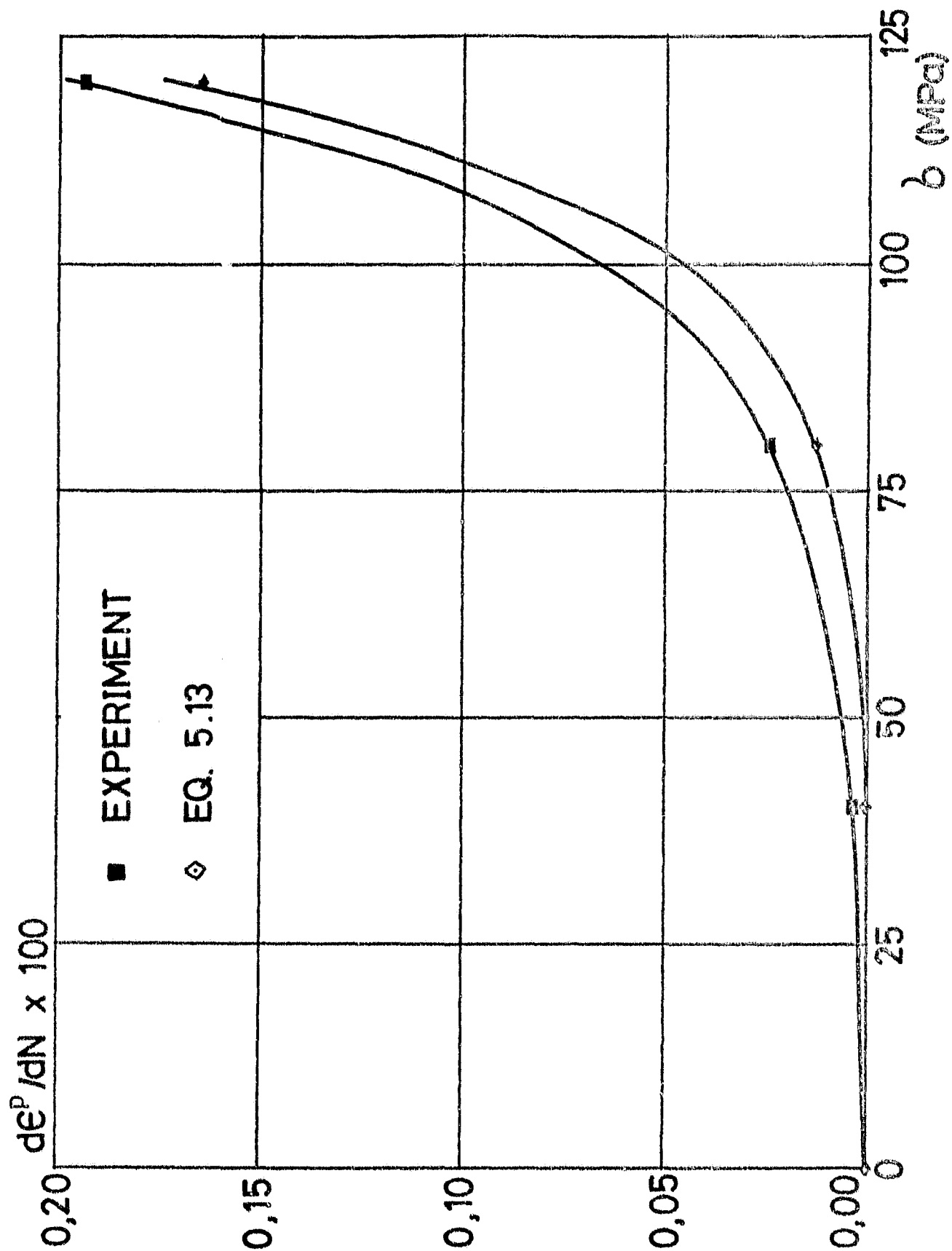


Fig. 5.7b Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ) for $\Delta\epsilon^P=0,02348$

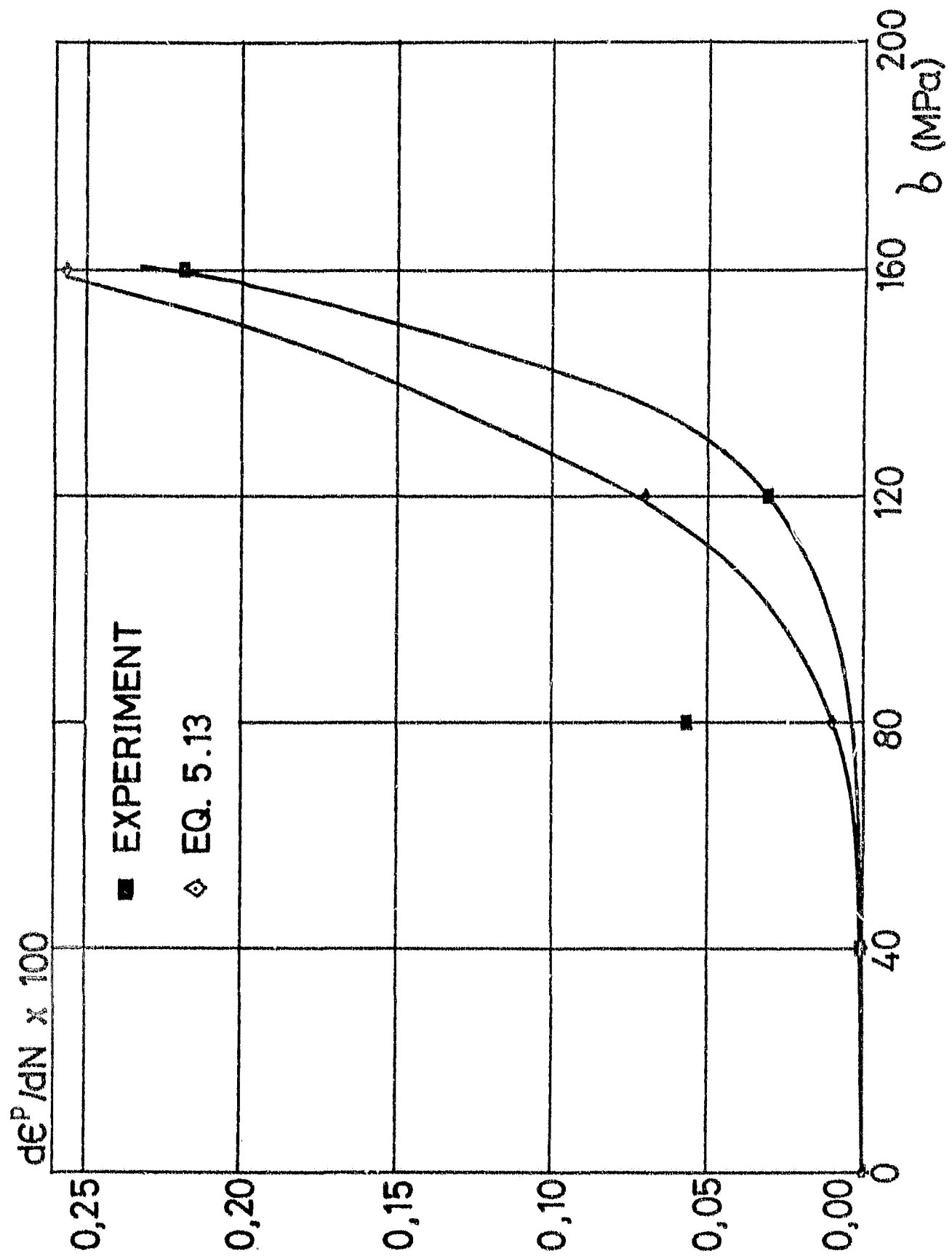


Fig. 5.7c Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^p/dN$) against Tensile Stress During Cycling (σ) for $\Delta\epsilon^p=0,02000$

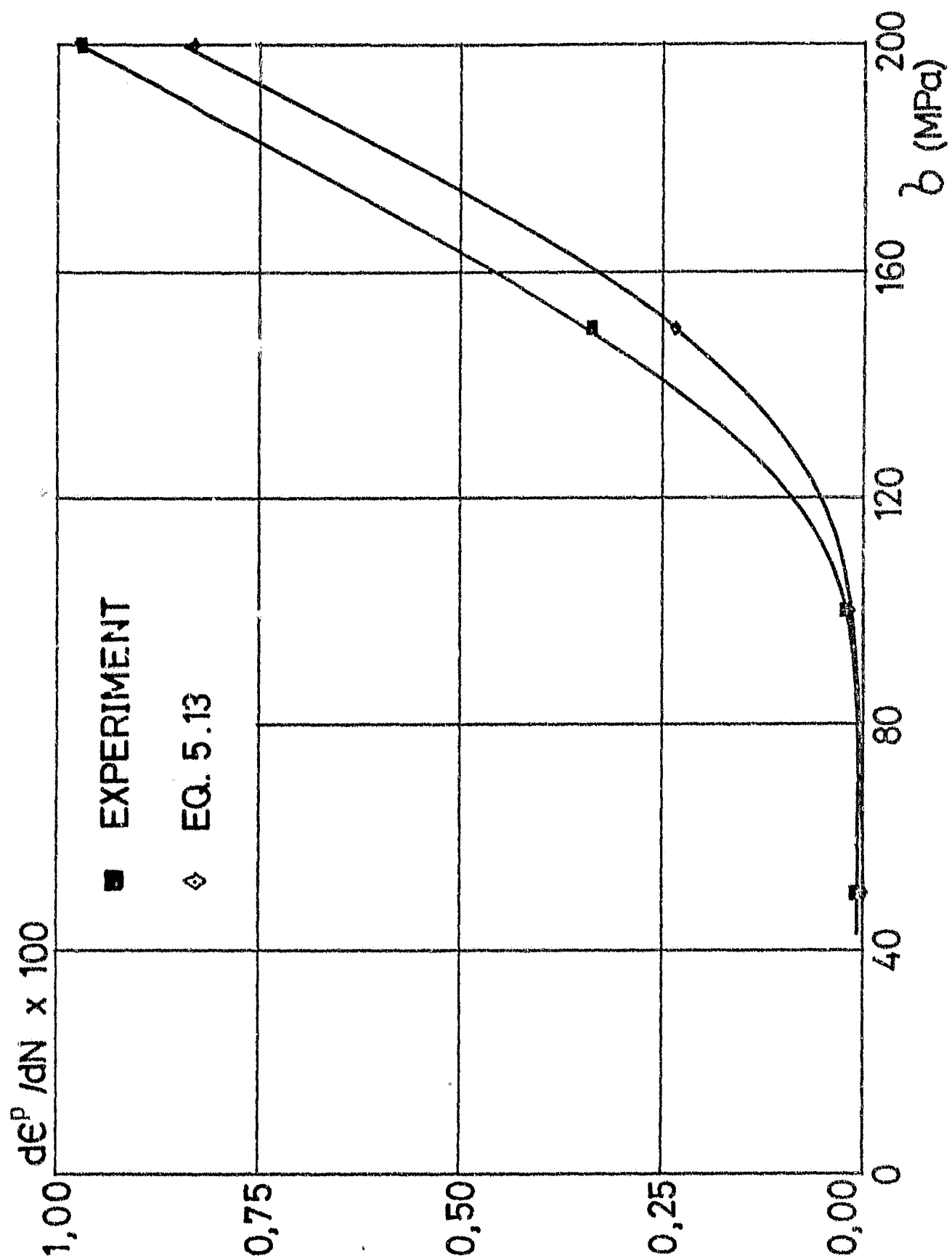


Fig. 5.7d Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ) for $\Delta\epsilon^P = 0,01852$

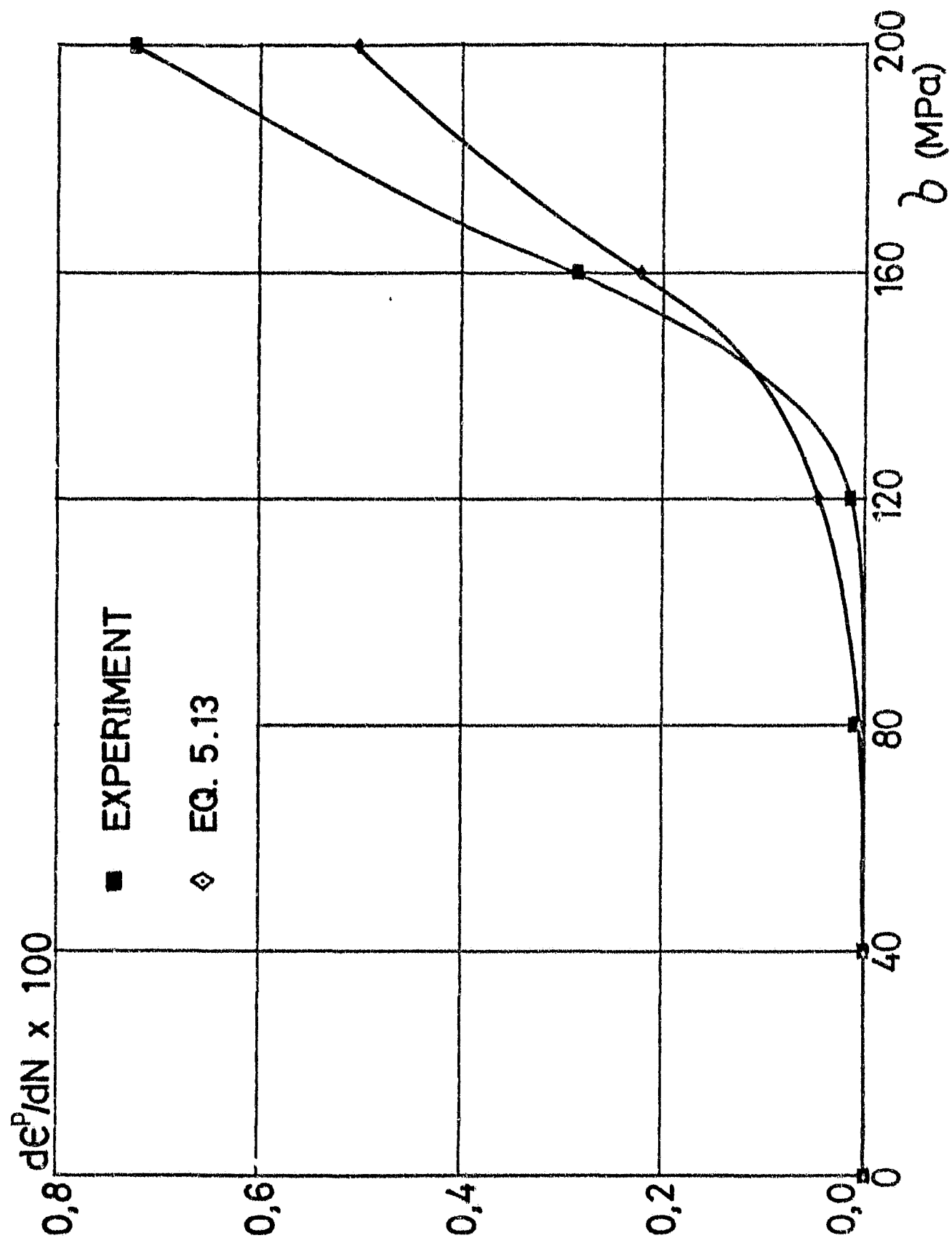


Fig. 5.7e Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ) for $\Delta\sigma^P = 0,01652$

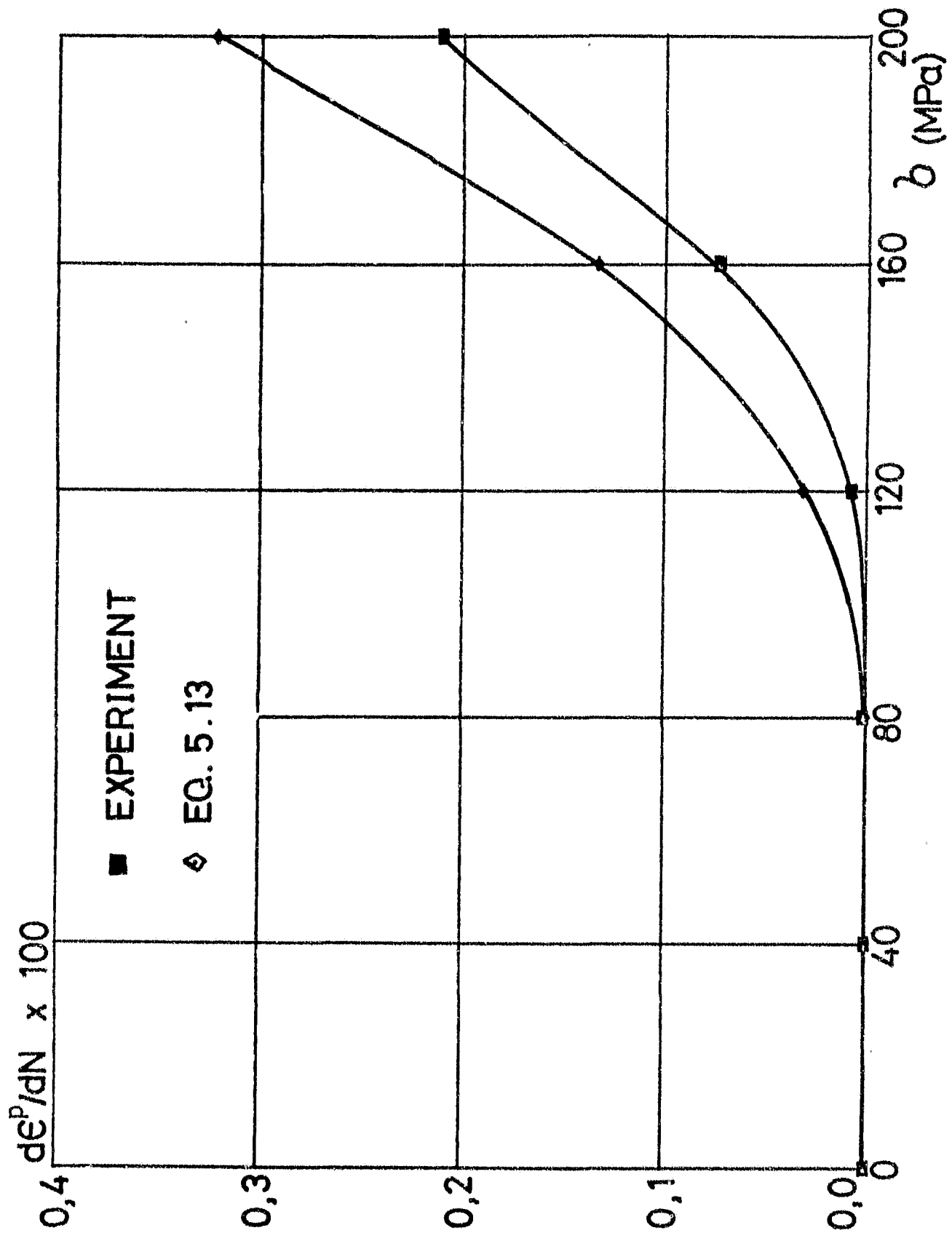


Fig. 5.7f Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^p/dN$) against Tensile Stress During Cycling (σ) for $\Delta\sigma^p = 0,01200$

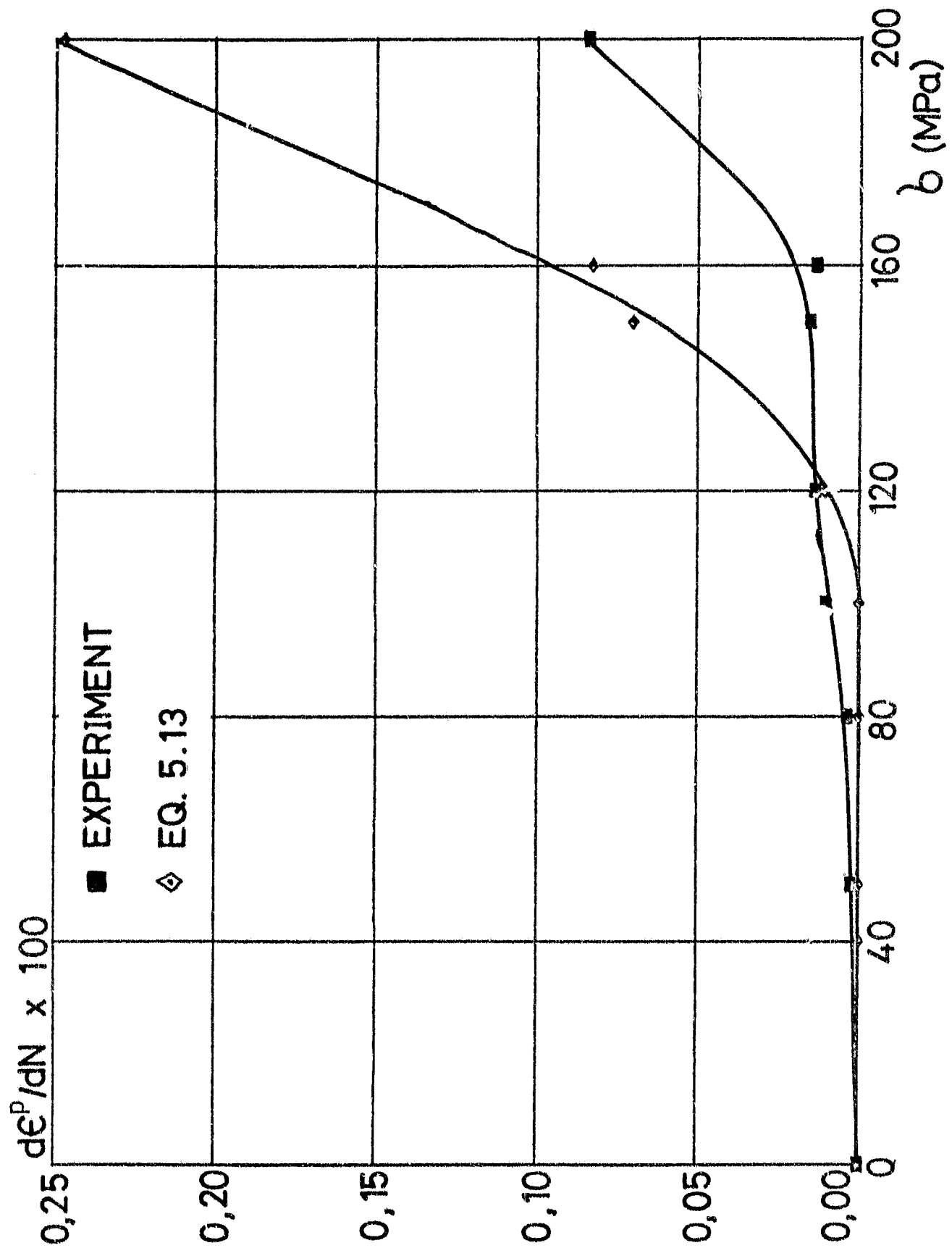


Fig. 5.7g Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ) for $\Delta\epsilon^P = 0.00874$

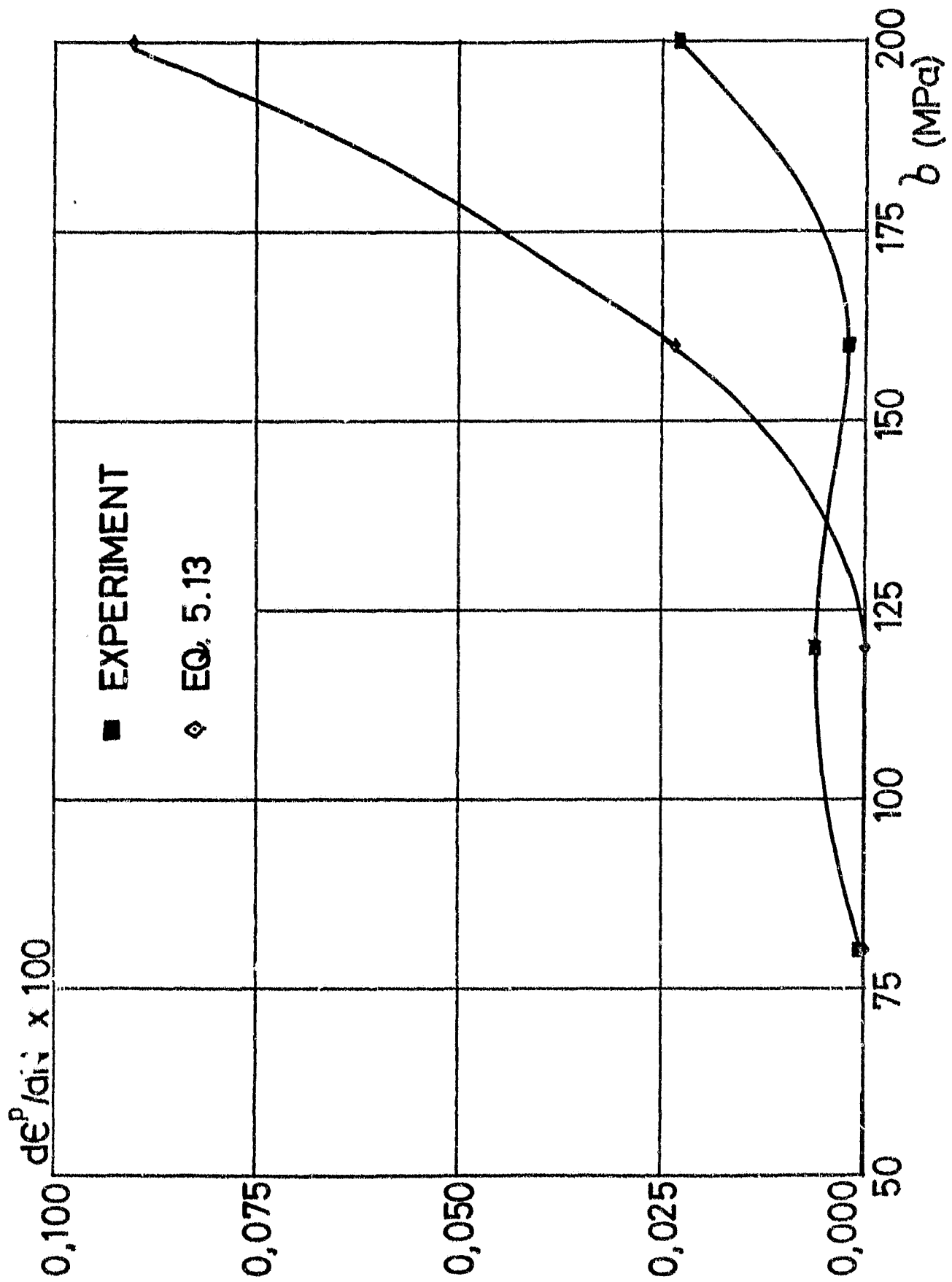


Fig. 5.7h Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ) for $\Delta\epsilon^P = 0.00532$

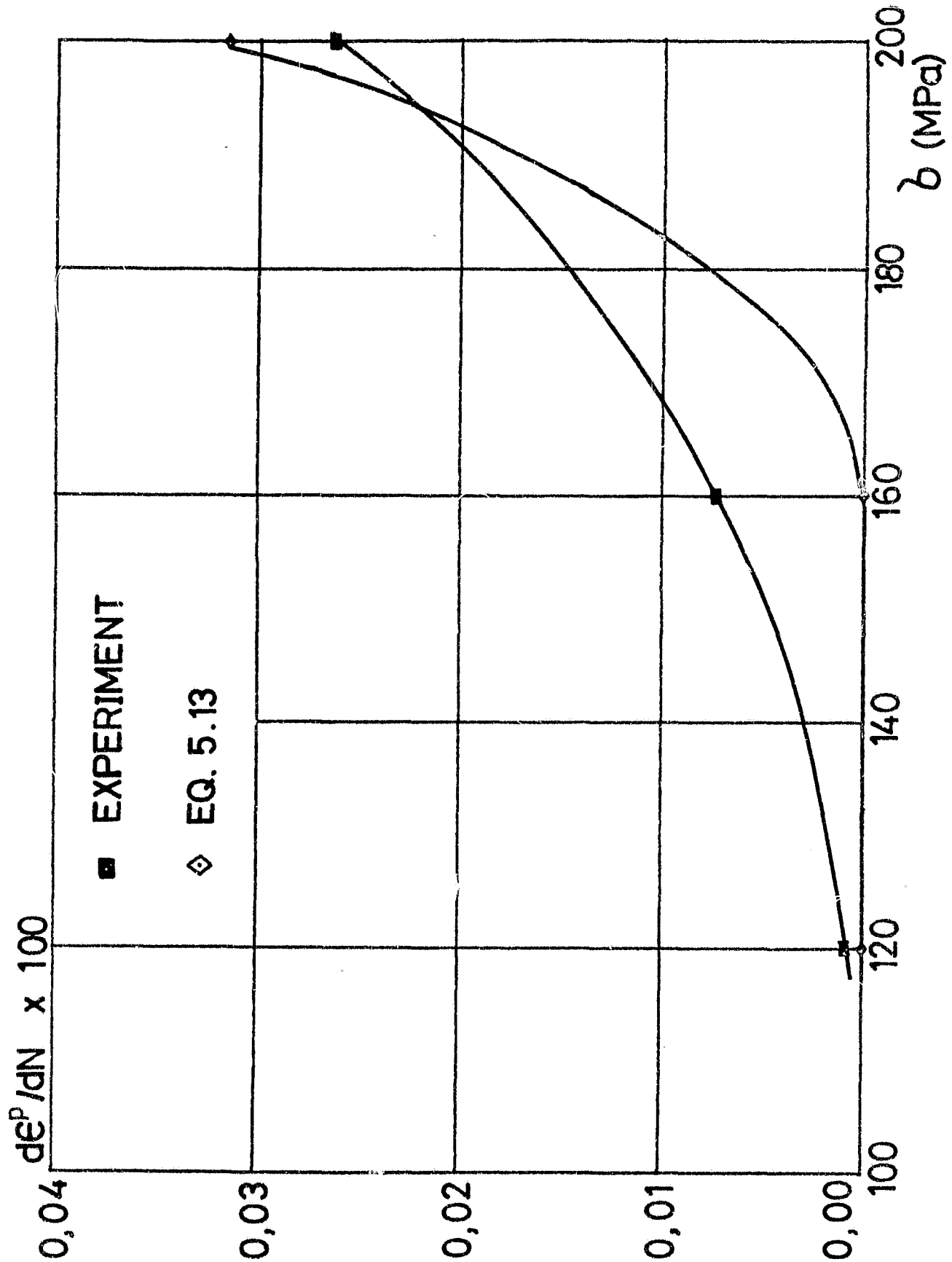


Fig. 5.7i Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ) for $\Delta\gamma^P = 0,00252$

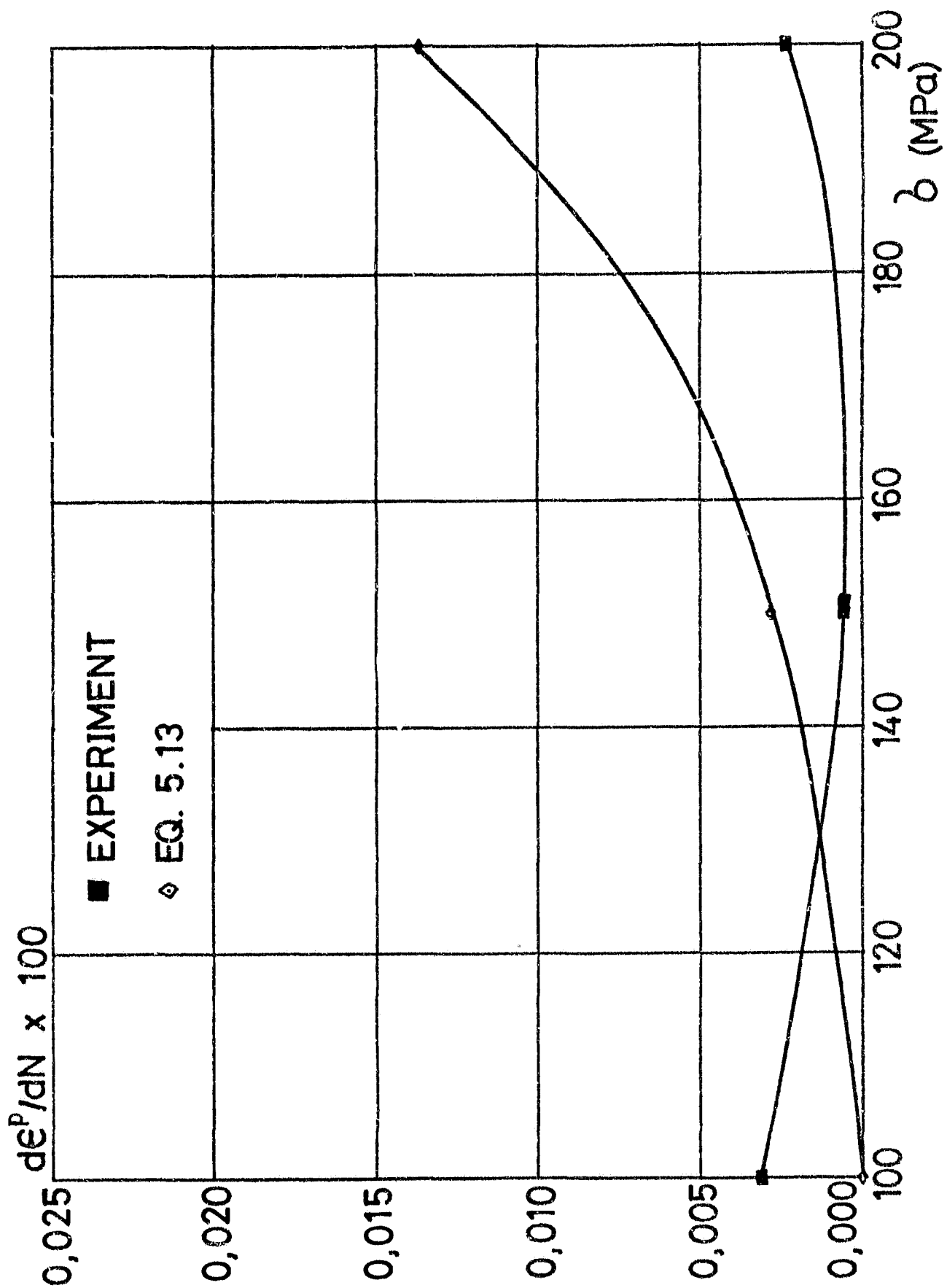


Fig. 5.7j Experimental and Theoretical Steady State Cyclic Creep Rates ($d\epsilon^P/dN$) against Tensile Stress During Cycling (σ) for $\Delta\sigma^P = 0,00119$

CHAPTER 6

ENDOCHRONIC THEORY OF VISCOPLASTICITY

6.1 Basic Theory

The endochronic theory of viscoplasticity, introduced in section 2.4, was applied by Valanis and Wu (32) to cyclic strain induced creep in metals subjected to alternating torsion and steady tension. The approach used will be briefly reviewed here.

Since the material behaviour is assumed to be time independent, the intrinsic time measure $d\xi$ is equivalent to the deformation increment $d\zeta$. Thus

$$\zeta = \frac{1}{p} \log (1 + \beta \zeta) \quad 6.1$$

$$d\zeta^2 = k_1^2 d\epsilon_{11} d\epsilon_{11} + k_2^2 d\epsilon_{ij} d\epsilon_{ij} \quad 6.2$$

For the material co-ordinate axes and loading conditions as in fig. 5.1

$$\begin{aligned} d\epsilon_{11} &= d\epsilon & d\epsilon_{22} &= d\epsilon_{33} \approx -\frac{1}{2} d\epsilon \\ d\epsilon_{12} &= d\epsilon_{21} = \frac{1}{2} d\gamma & d\epsilon_{13} &= d\epsilon_{31} = d\epsilon_{23} = d\epsilon_{32} = 0 \\ b_{11} &= b & b_{22} &= b_{33} = 0 \\ b_{12} &= b_{21} = \tau & b_{13} &= b_{23} = b_{31} = b_{32} = 0 \end{aligned}$$

Sq. 6.2 thus becomes

$$d\zeta = K(\epsilon, \gamma) \left[\frac{3}{2} d\epsilon^2 + \frac{1}{2} d\gamma^2 \right]^{\frac{1}{2}} \quad 6.3$$

During cyclic strain induced creep, the axial strain increment is far smaller than a corresponding shear strain increment. In addition, it was assumed that $K(\epsilon, \gamma) = K(\gamma)$ only. Hence

$$d\zeta = K'(\gamma) |d\gamma|$$

where $K(\gamma) = K(-\gamma)$ and $|d\gamma|$ ensure a monotonically increasing "time" ζ . For a one term approximation of $E(\zeta)$ and with ζ as in eq. 6.1, the tensile response is

$$b = E_0 (1 + \beta \zeta)^{\frac{1}{p}} \int_0^{\zeta} (1 + \beta \zeta')^{\frac{1}{p}} \frac{\partial E}{\partial \zeta'} d\zeta' \quad 6.4$$

Differentiating w.r.t. $\dot{\gamma}$ and noting that $b = b_0$
= constant:

$$\begin{aligned} d\epsilon &= \frac{a \cdot b_0}{E_0} \frac{1}{1 + \beta \dot{\gamma}} d\dot{\gamma} \\ \epsilon &= \frac{a \cdot b_0}{\beta E_0} \log (1 + \beta \dot{\gamma}) \end{aligned} \quad 6.5$$

The creep strain is thus explicitly expressed in terms of the total deformation $\dot{\gamma}$. After N torsional loading cycles

$$\dot{\gamma} = N \int K'(\gamma) |d\gamma|$$

which by the mean value theorem, becomes

$$\dot{\gamma} = 4N K'(\bar{\gamma}) \gamma_{max}$$

where $0 < \bar{\gamma} < \gamma_{max}$

$\gamma_{max} = \frac{1}{2} \times$ shear strain range $\Delta\gamma$

$\bar{\gamma}$ is some function of γ_{max} , such that

$$K'(\bar{\gamma}) \gamma_{max} = \bar{K}(\gamma_{max})$$

Eq. 6.5 now becomes

$$\epsilon = \frac{a \cdot b_0}{\beta E_0} \log (1 + 4N \beta \bar{K}(\gamma_{max})) \quad 6.6$$

The value of $a \cdot b_0 / \beta E_0$ was obtained from one experimental cyclic creep curve and used to find the variation of $4\beta \bar{K}(\gamma_{max})$ with γ_{max} from a few other creep curves. Eq. 6.6 was then used to predict creep results for tests with the same applied stress b_0 but intermediate values of γ_{max} . Although the agreement with experiment was good, the theory in this form is of limited use since the material constants need to be obtained from a selection of creep curves before creep behaviour for intermediate loading conditions can be predicted. It is thus essentially an empirical approach.

This approach was used in the present investigation to predict cyclic creep in specimens subjected to a tensile stress of $b_0 = 200$ MPa and a shear strain half range of $\gamma_{max} = 0,005$ to $0,02$. The material constants were determined for $\gamma_{max} = 0,015$, $0,01$, $0,006$ and $0,005$ by choosing a value of $4\beta \bar{K}(\gamma_{max})$ and recasting eq. 6.6 into the following form

$$\epsilon = A x$$

where

$$A = \frac{d_1 b_0}{\beta E_0}$$

$$x = \log (1 + 4N\beta \bar{K}(\gamma_{max}))$$

The value of A was then determined by a least squares technique and the sum of the squared errors calculated. It was found that the value of A giving the least sum of squared errors varied from 0,007 (for $\gamma_{max} = 0,005$) to 0,2 (for $\gamma_{max} = 0,015$). The best compromise was to take $A = 0,02$. The corresponding values of $4\beta \bar{K}(\gamma_{max})$ for a least squares fit were then calculated and plotted against γ_{max} in fig. 6.1. Note the similarity between fig. 6.1 and the modified yield surface theory in the previous chapter where it had been shown that very little creep occurred below a critical shear strain range. This critical value would correspond to the point where the value of $4\beta \bar{K}(\gamma_{max})$ becomes zero.

From fig. 6.1 the value of $4\beta \bar{K}(\gamma_{max})$ were read off for $\gamma_{max} = 0,008$, 0,012 , 0,014 and 0,02. With $d_1 b_0 / \beta E_0$ set to 0,02, the theoretical creep curves were calculated from eq. 6.6 and plotted in figs. 6.2 - 6.5 together with the experimental curves. It can be seen that approximate agreement between theory and experiment is only attained for intermediate values of γ_{max} . Although this approach is empirical, it may be satisfactory in many design situations as it is capable of giving a reasonable estimate of creep behaviour over a smaller range of loading conditions.

The purpose of the following sections is to attempt a modification of the theory to see whether it can be used to predict creep behaviour from simple material data, such as monotonic tensile and cyclic torsional stress-strain curves. The starting point used was to determine the function $K(\epsilon, \delta)$ in eq. 6.3 from tension and torsion tests.

6.2 Pure Tensile behaviour

Since shear strains are absent, $\gamma = \gamma(\epsilon)$ only and eq. 6.4 becomes

$$b = E_0 (1 + \beta \gamma(\epsilon))^{\frac{d_1}{\beta}} \int_0^\epsilon (1 + \beta \gamma(\epsilon'))^{\frac{d_1}{\beta}} d\epsilon'$$

Differentiating w.r.t. ϵ and rearranging:

$$\frac{\alpha_1 \frac{df}{d\epsilon}}{1 + \beta f(\epsilon)} = \frac{E_0}{b} - \frac{1}{b} \frac{db}{d\epsilon} = \chi(\epsilon) \quad 6.7$$

This is an ordinary differential equation in ϵ with the following solution

$$f = e^{\int \frac{\beta}{\alpha_1} \chi(\epsilon') d\epsilon'} \int_0^\epsilon \frac{1}{\alpha_1} \chi(\epsilon') e^{-\int \frac{\beta}{\alpha_1} \chi(\epsilon'') d\epsilon''} d\epsilon' \quad 6.8$$

$\chi(\epsilon)$ and $\int \chi(\epsilon) d\epsilon$ were found numerically from the tensile stress-strain curve in fig. 4.2 and plotted in fig. 6.6. The erratic behaviour of χ at small values of ϵ was due to numerical errors. Three possible analytical approximations for χ suggested themselves, as shown in fig. 6.6. The corresponding solutions for 6.8 are given below

$$\begin{aligned} (a) \chi &= k_3 \text{ for all } \epsilon \\ 1 + \beta f &= e^{\frac{k_3 \beta \epsilon}{\alpha_1}} \\ df &= \frac{k_3}{\alpha_1} e^{\frac{k_3 \beta \epsilon}{\alpha_1}} d\epsilon \end{aligned} \quad 6.9$$

$$\begin{aligned} (b) \chi &= \frac{k_3}{\epsilon_c'} \epsilon \text{ for } \epsilon \leq \epsilon_c' \\ 1 + \beta f &= e^{\frac{k_3 \beta \epsilon^2}{2 \alpha_1 \epsilon_c'}} \\ df &= \frac{k_3 \epsilon}{\alpha_1 \epsilon_c'} e^{\frac{k_3 \beta \epsilon^2}{2 \alpha_1 \epsilon_c'}} d\epsilon \end{aligned} \quad 6.10a$$

$$\begin{aligned} \chi &= k_3 \text{ for } \epsilon > \epsilon_c' \\ 1 + \beta f &= e^{\frac{k_3 \beta (\epsilon - \epsilon_c')}{\alpha_1}} + e^{\frac{k_3 \beta \epsilon_c'^2}{2 \alpha_1 \epsilon_c'}} - 1 \\ df &= \frac{k_3}{\alpha_1} e^{\frac{k_3 \beta (\epsilon - \epsilon_c')}{\alpha_1}} d\epsilon \end{aligned} \quad 6.10b$$

$$\begin{aligned} (c) \chi &= 0 \text{ for } \epsilon \leq \epsilon_c \\ 1 + \beta f &= 1 \\ df &= 0 \end{aligned} \quad 6.11a$$

$$\begin{aligned} \chi &= k_3 \text{ for } \epsilon > \epsilon_c \\ 1 + \beta f &= e^{\frac{k_3 \beta (\epsilon - \epsilon_c)}{\alpha_1}} \end{aligned} \quad 6.11b$$

The tensile stress-strain equation resulting from the three forms of are briefly presented for comparison

$$(a) \quad \sigma = \frac{E_0}{k_3} (1 - e^{-k_3 \epsilon}) \quad 6.12$$

(b) No simple solution exists except maybe an error function with complex variables

$$(c) \quad \begin{aligned} \sigma &= E_0 \epsilon & \epsilon \leq \epsilon_c \\ \sigma &= E_0 \epsilon_c + \frac{E_0}{k_3} (1 - e^{-k_3 (\epsilon - \epsilon_c)}) & \epsilon > \epsilon_c \end{aligned} \quad 6.13$$

6.3 Pure Torsional Behaviour

In this case $\gamma = \gamma(\delta)$ only and a two term approximation will be used for $G(\delta)$. Thus

$$\begin{aligned} \tau &= \int_0^\delta (G_1 + G_2 e^{-\alpha_2 \delta} e^{\alpha_1 \delta'}) \frac{\delta \delta}{\delta \delta'} d\delta' \\ \tau &= G_1 \delta + G_2 (1 + \beta_\gamma(\delta))^{-\frac{\alpha_2}{\beta}} \int_0^\delta (1 + \beta_\gamma(\delta'))^{+\frac{\alpha_2}{\beta}} d\delta' \end{aligned}$$

Differentiating w.r.t. and rearranging, an ordinary differential equation identical to eq. 6.7 is obtained

$$\frac{\alpha_2 \frac{d\gamma}{d\delta}}{1 + \beta_\gamma(\delta)} = \frac{G_0 - \frac{d\tau}{d\delta}}{\tau - G_1 \delta} = \phi(\delta) \quad 6.13a$$

$\phi(\delta)$ and $\int \phi(\delta) d\delta$ were again found numerically and proved to be similar in appearance to $\chi(\epsilon)$ and $\int \chi(\epsilon) d\epsilon$ respectively (fig. 6.7). The equations for $d\gamma$ and $1 + \beta_\gamma$ (eqs. 6.9 - 6.11) are thus unchanged for torsion except that $k_3, \alpha_1, \epsilon, \epsilon_c$ and ϵ_c now become $k_4, \alpha_2, \delta, \delta_c$ and δ_c respectively. It will now be attempted to develop a unified relation for γ for the case of simultaneous tension and torsion.

6.4 Combined Tension-Torsion Behaviour

During torsional cycling with a steady tensile load, plastic deformation, and hence an increase in γ , will occur when δ , not ϵ , exceeds its critical value. The critical axial strains ϵ_c' and ϵ_c will thus be assumed to be zero. Considering the

form $k(\epsilon, \gamma)$ in eq. 6.3 must take in order to predict the pure tensile and pure torsional material responses, the simplest is to let

$$\begin{aligned} k(\epsilon, \gamma) &= k_1(\epsilon) \cdot k_2(\gamma) \\ \text{where } k_1(0) &= C_1 \\ k_2(0) &= C_2 \end{aligned} \quad 6.14$$

For pure tension, $\gamma = d\gamma = 0$, and from eq. 6.9

$$k_1(\epsilon) = \sqrt{\frac{2}{3}} \frac{1}{C_1} \cdot \frac{K_3}{\alpha_1} e^{\frac{K_3 \beta \epsilon}{\alpha_1}} \quad 6.15$$

For pure torsion, $\epsilon = d\epsilon = 0$, and from the torsional equivalents of eqs. 6.9 - 6.11, three possibilities exist for $k_2(\gamma)$

$$(a) \quad k_2(\gamma) = \sqrt{2} \frac{1}{C_1} \cdot \frac{K_4}{\alpha_2} e^{\frac{K_4 \beta \gamma}{\alpha_2}} \quad 6.16$$

$$\begin{aligned} (b) \quad k_2(\gamma) &= \sqrt{2} \frac{1}{C_1} \cdot \frac{K_4 \gamma}{\alpha_2 \gamma_c'} e^{\frac{K_4 \beta \gamma^2}{2 \alpha_2 \gamma_c'}} & \gamma \leq \gamma_c' \\ k_2(\gamma) &= \sqrt{2} \frac{1}{C_1} \cdot \frac{K_4}{\alpha_2} e^{\frac{K_4 \beta (\gamma - \gamma_c')}{\alpha_2}} & \gamma > \gamma_c' \end{aligned} \quad 6.17$$

$$\begin{aligned} (c) \quad k_2(\gamma) &= 0 & \gamma \leq \gamma_c \\ k_2(\gamma) &= \sqrt{2} \frac{1}{C_1} \cdot \frac{K_4}{\alpha_2} e^{\frac{K_4 \beta (\gamma - \gamma_c)}{\alpha_2}} & \gamma > \gamma_c \end{aligned} \quad 6.18$$

From eq. 6.14, the additional result is obtained

$$\begin{aligned} \sqrt{2} \frac{K_4}{\alpha_2} &= \sqrt{\frac{2}{3}} \cdot \frac{K_3}{\alpha_1} \\ \frac{\alpha_1}{\beta} &= \frac{1}{\sqrt{3}} \cdot \frac{K_3}{K_4} \cdot \frac{\alpha_2}{\beta} \end{aligned} \quad 6.19$$

From eq. 6.15 - 6.19 three possible relations for $d\gamma$ may be derived.

$$(a) \quad d\gamma = \sqrt{2} \frac{K_4}{\alpha_2} e^{\frac{K_3 \beta \epsilon}{\alpha_1} + \frac{K_4 \beta \gamma}{\alpha_2}} \left[\frac{3}{2} d\epsilon^2 + \frac{1}{2} d\gamma^2 \right]^{\frac{1}{2}} \quad 6.20$$

$$\begin{aligned} (b) \quad d\gamma &= \sqrt{2} \frac{K_4 \gamma}{\alpha_2 \gamma_c'} e^{\frac{K_3 \beta \epsilon}{\alpha_1} + \frac{K_4 \beta \gamma^2}{2 \alpha_2 \gamma_c'}} \left[\frac{3}{2} d\epsilon^2 + \frac{1}{2} d\gamma^2 \right]^{\frac{1}{2}} & \gamma \leq \gamma_c' \\ d\gamma &= \sqrt{2} \frac{K_4}{\alpha_2} e^{\frac{K_3 \beta \epsilon}{\alpha_1} + \frac{K_4 \beta (\gamma - \gamma_c')}{\alpha_2}} \left[\frac{3}{2} d\epsilon^2 + \frac{1}{2} d\gamma^2 \right]^{\frac{1}{2}} & \gamma > \gamma_c' \end{aligned} \quad 6.21$$

$$\begin{aligned} (c) \quad d\gamma &= \sqrt{2} \frac{K_4}{\alpha_2} e^{\frac{K_3 \beta \epsilon}{\alpha_1}} \left[\frac{3}{2} d\epsilon^2 + \frac{1}{2} d\gamma^2 \right]^{\frac{1}{2}} & \gamma \leq \gamma_c \\ d\gamma &= \sqrt{2} \frac{K_4}{\alpha_2} e^{\frac{K_3 \beta \epsilon}{\alpha_1} + \frac{K_4 \beta (\gamma - \gamma_c)}{\alpha_2}} \left[\frac{3}{2} d\epsilon^2 + \frac{1}{2} d\gamma^2 \right]^{\frac{1}{2}} & \gamma > \gamma_c \end{aligned} \quad 6.22$$

These will now be used in a rederivation of the cyclic strain induced creep equation.

6.5 Cyclic Strain Induced Creep Behaviour

Eq. 6.5, as originally derived by Valanis, will be used as a starting point. In addition, since $d\epsilon^2 \ll d\gamma^2$

$$d\dot{\gamma} = k(\epsilon, \gamma) \frac{1}{\sqrt{2}} |d\gamma|$$

From eq. 6.5

$$e^{\frac{k_2 \beta \epsilon}{\alpha_1}} = (1 + \beta \dot{\gamma})^{\frac{k_2 \beta \epsilon_0}{E_0}} \quad 6.23$$

Substituting in 6.20 - 6.22, we get

$$\frac{d\dot{\gamma}}{(1 + \beta \dot{\gamma})^{\frac{k_2 \beta \epsilon_0}{E_0}}} = f(\gamma) |d\gamma| \quad 6.24$$

where $f(\gamma)$ depends on the form of $k_2(\gamma)$. During one torsional cycle, γ varies as follows

$$\gamma = 0 \rightarrow \gamma_{max} \rightarrow 0 \rightarrow -\gamma_{max} \rightarrow 0$$

This implies, however, that the material "remembers" the original unstrained condition. Since this memory effect is absent in real metals, it would be better to let γ vary from 0 to $2\gamma_{max}$ during any half loading cycle. Integrating eq. 6.24 over N complete torsional cycles, and back substituting for eq. 6.23, the following result is obtained.

$$\epsilon = \frac{\alpha_1 \beta \epsilon_0}{\beta E_0} \frac{E_0}{E_0 - k_2 \beta \epsilon_0} \log \left[1 + 2N \frac{E_0 - k_2 \beta \epsilon_0}{E_0} F(\gamma_{max}) \right] \quad 6.25$$

$$\text{where } F(\gamma_{max}) = \int_0^{2\gamma_{max}} \beta f(\gamma) d\gamma$$

Consider the qualitative implications of eq. 6.25. For large tensile stresses, the creep strains are large but the factor $(E_0 - k_2 \beta \epsilon_0)/E_0$ is small. This will cause the logarithmic term to increase linearly with N, as was actually observed. The same behaviour is predicted for small shear strain ranges but this did not occur in practice. It seems thus as if eq. 6.25 can only predict the logarithmic type of cyclic creep and hence is applicable only to tests with small shear strain ranges.

$f(\gamma)$ and $F(\gamma_{max})$ are, for the three cases of $k_2(\gamma)$

$$\begin{aligned} (a) \quad f(\gamma) &= \frac{k_4}{\alpha_2} e^{\frac{k_4 \beta \gamma}{\alpha_2}} \\ F(\gamma_{max}) &= e^{\frac{2k_4 \beta \gamma_{max}}{\alpha_2}} - 1 \end{aligned} \quad 6.26$$

$$\begin{aligned} (b) \quad f(\gamma) &= \frac{k_4 \gamma}{\alpha_2 \gamma_c'} e^{\frac{k_4 \beta \gamma^2}{2 \alpha_2 \gamma_c'}} \\ F(\gamma_{max}) &= e^{\frac{2k_4 \beta \gamma_{max}^2}{\alpha_2 \gamma_c'}} - 1 \quad 2\gamma_{max} \leq \gamma_c' \\ f(\gamma) &= \frac{k_4}{\alpha_2} e^{\frac{k_4 \beta (\gamma - \gamma_c')}{\alpha_2}} \\ F(\gamma_{max}) &= e^{\frac{k_4 \beta (2\gamma_{max} - \gamma_c')}{\alpha_2}} + e^{\frac{k_4 \beta \gamma_c'}{2 \alpha_2}} - 2 \quad 2\gamma_{max} > \gamma_c' \end{aligned} \quad 6.27$$

$$\begin{aligned} (c) \quad f(\gamma) &= 0 \quad F(\gamma_{max}) = 0 \quad 2\gamma_{max} \leq \gamma_c \\ f(\gamma) &= \frac{k_4}{\alpha_2} e^{\frac{k_4 \beta (\gamma - \gamma_c)}{\alpha_2}} \\ F(\gamma_{max}) &= e^{\frac{k_4 \beta (2\gamma_{max} - \gamma_c)}{\alpha_2}} - 1 \quad 2\gamma_{max} > \gamma_c \end{aligned} \quad 6.28$$

6.6 Cyclic Strain Induced Creep in Aluminium

Eq. 6.25 - 6.28 above can now be used to predict cyclic strain induced creep in aluminium. The required material constants were:

$E_0, G_0, G_1, \alpha_1/\beta, \alpha_2/\beta, k_3, k_4, \gamma_c'$ and γ_c

Although G_0 and G_1 do not appear in the creep equations, they were required in the calculation of $\phi(\gamma)$ and hence in determining k_4 . E_0, G_0 and G_1 were assumed to be constant regardless of the state of deformation and the values obtained in chapter 4 were used.

k_3 was found from the slope of the numerically obtained plots of $\int \chi(\epsilon) d\epsilon$ vs ϵ for specimens tested in pure tension after initial torsional cycling. Little dependance of k_3 on either the shear strain range or on the tensile stress during cycling was observed (fig. 6.8).

A similar procedure was followed in determining k_4, γ_c' and γ_c from torsional hysteresis loops with shear strain ranges varying from 0.016 - 0.04. It was found that k_3, γ_c' and γ_c

remained relatively constant, regardless of the shear strain range (fig. 6.9).

The values of α_1/β and α_2/β could not be determined from the stress-strain curves eqs. 6.12 and 6.13. In section 6.1, however, it was found that for a similar type of creep equation and for $b_0 = 200$ MPa

$$\frac{\alpha_1 b_0}{\beta E_0} = 0,02$$

From this approximate values for α_1/β and α_2/β could be determined.

The values of the material constants used in predicting cyclic strain induced creep were:

$$E_0 = 67\ 000\ \text{MPa}$$

$$G_0 = 26\ 750\ \text{MPa}$$

$$G_1 = 260\ \text{MPa}$$

$$k_3 = 192$$

$$k_4 = 73,8$$

$$\delta_c' = 0,0176$$

$$\delta_c = 0,0097$$

$$\frac{\alpha_1}{\beta} = 0,02 \times \frac{67000}{200} = 6,7$$

$$\frac{\alpha_2}{\beta} = \sqrt{3} \frac{k_4}{k_3} \frac{\alpha_1}{\beta} = 4,46 \text{ (from eq. 6.19)}$$

Figs. 6.10a - 6.10c compare the predicted and actual cyclic creep curves for $b_0 = 50$ MPa - 200 MPa and $\gamma_{max} = 0,005 - 0,02$. Fair agreement was achieved for large tensile stresses and shear strain ranges while the creep strains were overestimated considerably for small shear strain ranges. It is thought that this is due to the choice of the value of α_1/β . A smaller value would give better agreement, but only for smaller values of b_0 and γ_{max} . Note also that the three forms of $F(\gamma_{max})$ in equations 6.26 - 6.28 only affect the theoretical creep curve to a small extent.

6.7 Discussion

The endochronic theory as applied in this investigation does not offer a unified approach to cyclic strain induced creep in metals. The form of the creep equations derived suggests, however, that the theory could be used to predict logarithmic cyclic creep for small tensile stresses and shear strain

ranges. This type of creep, although not observed in the present investigation, has been found to occur in steel, for example (6).

Alternatively, the theory could be used in a semi empirical manner in design situations where the range of loading conditions is small. The material constants would then have to be obtained from actual creep tests with similar loading parameters.

It is nevertheless felt that the endochronic theory does offer a possible theoretical approach to cyclic creep. More work in the field will be necessary, however, especially a more thorough investigation into the background of the theory.

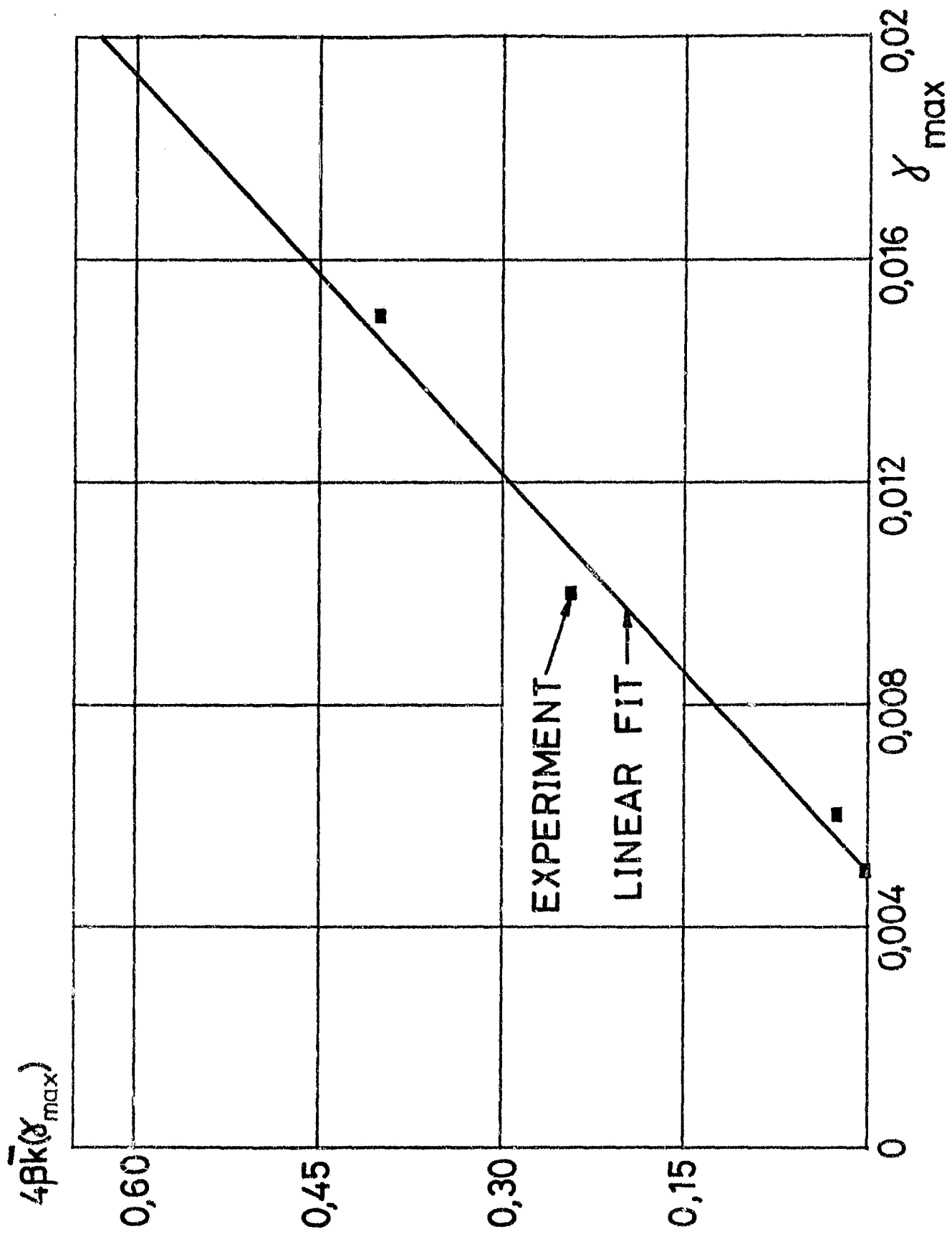


Fig. 6.1 Variation of $4\beta\bar{k}(\gamma_{\max})$ with $\gamma_{\max}$ (Eq. 6.6)

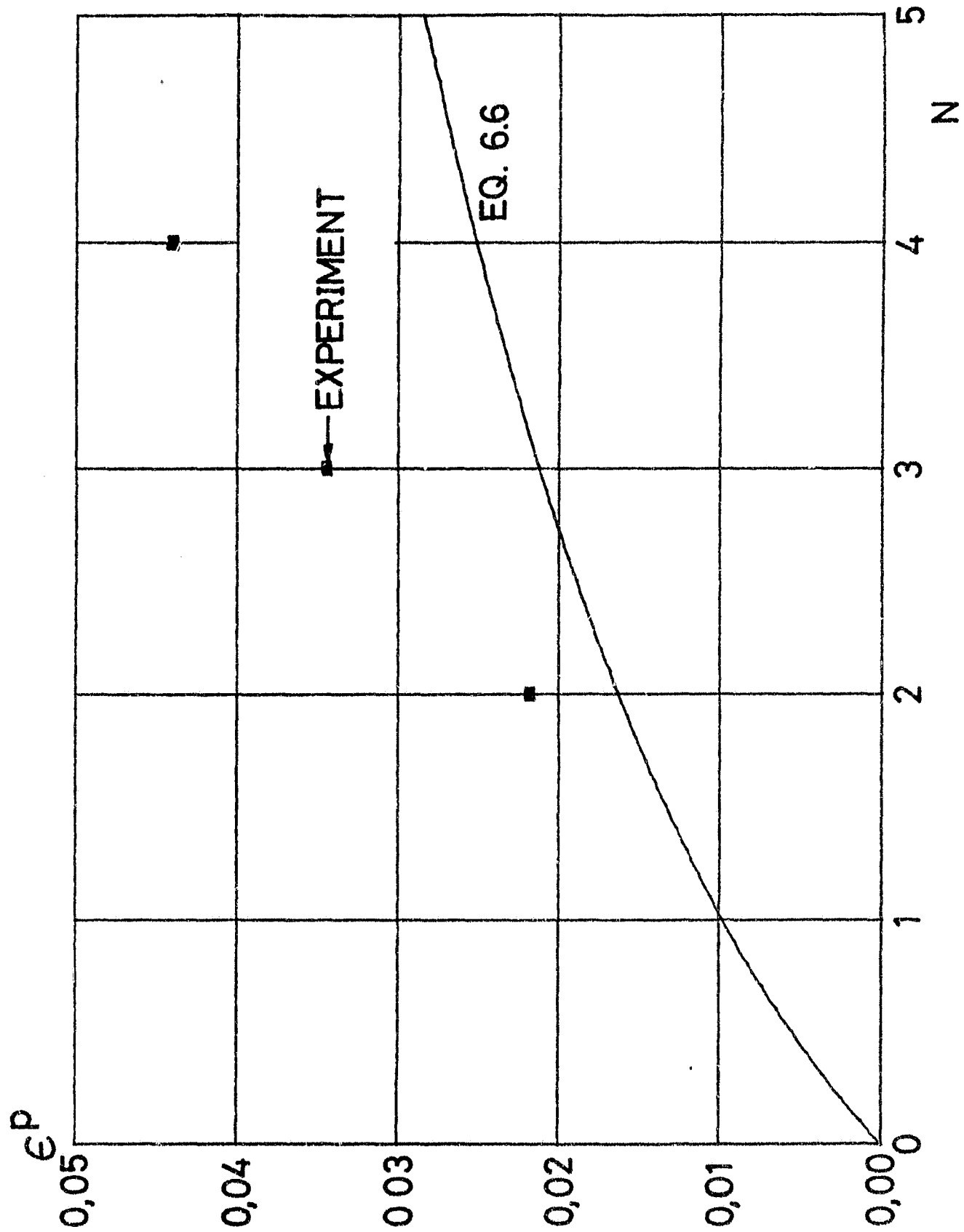


Fig. 6.2 Experimental and Theoretical Cyclic Creep Strain (ϵ^p)
against Number of Cycles (N) for $\gamma_{max} = 0,02$ and
 $b_0 = 200$ MPa

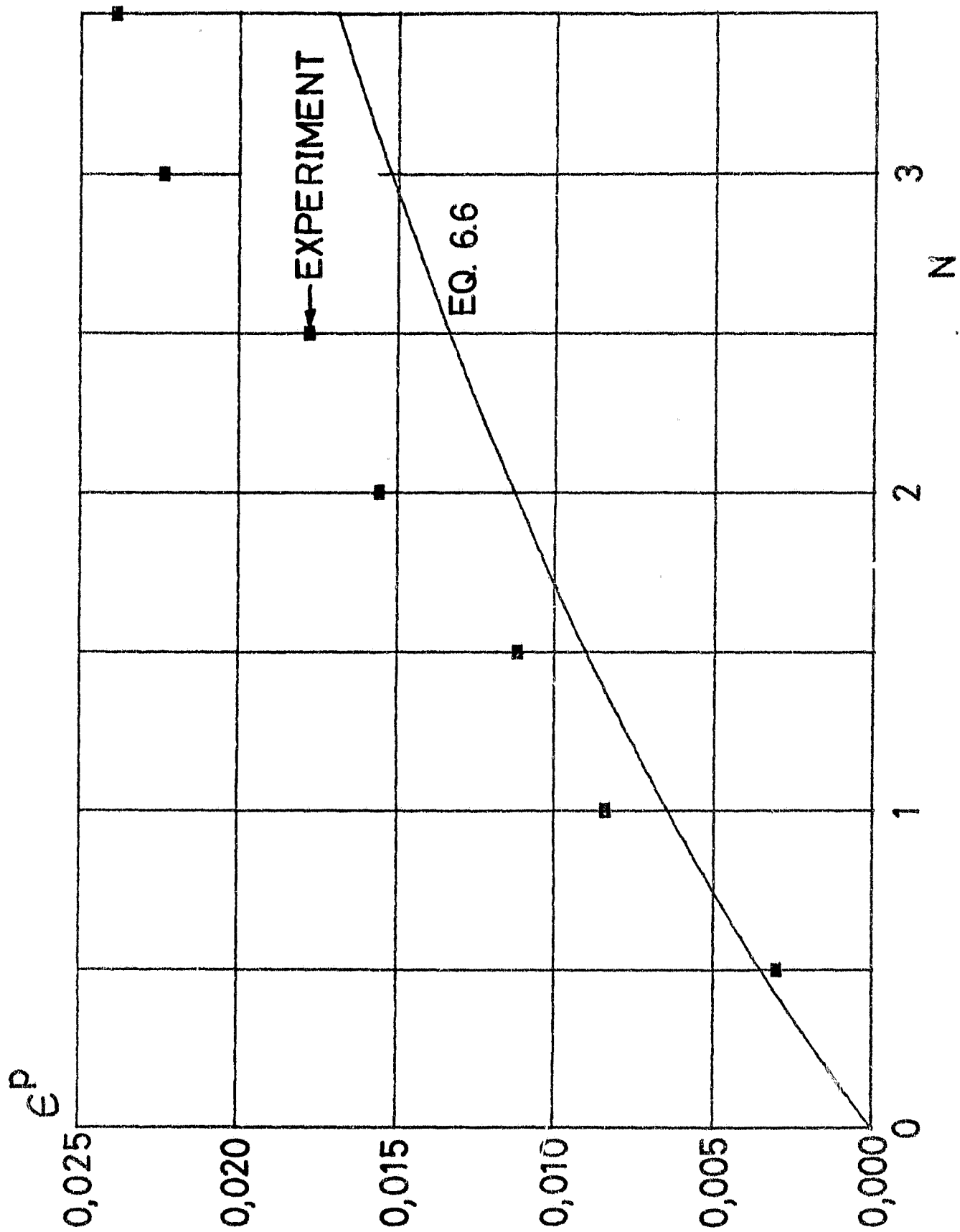


Fig. 6.3 Experimental and Theoretical Cyclic Creep Strain (ϵ^p) against Number of Cycles (N) for $\delta_{max} = 0.014$ and $\sigma_o = 200$ MPa

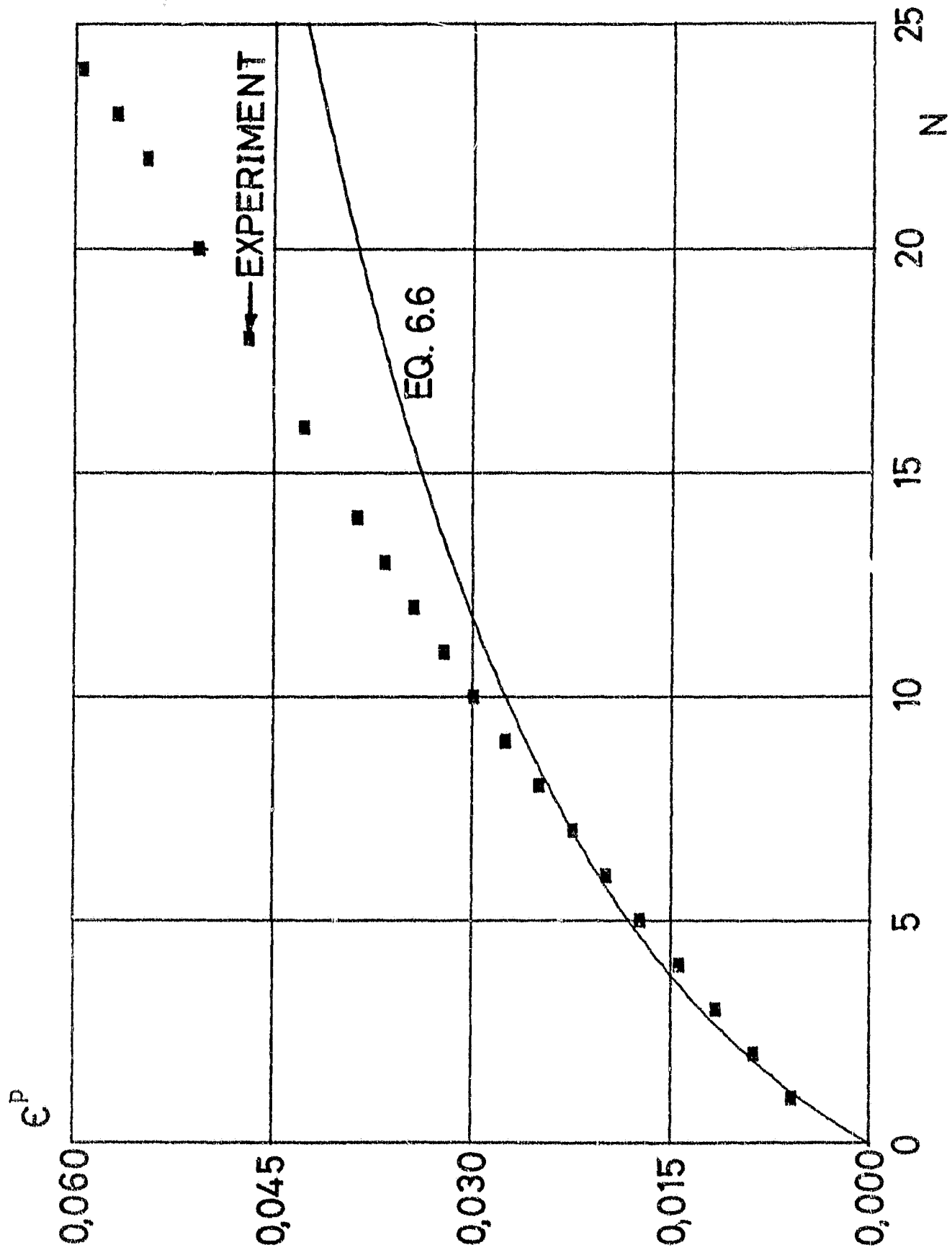


Fig. 6.4 Experimental and Theoretical Cyclic Creep Strain (ϵ^P)
 against Number of Cycles (N) for $\delta_{max} = 0.012$ and
 $\sigma_0 = 200$ MPa

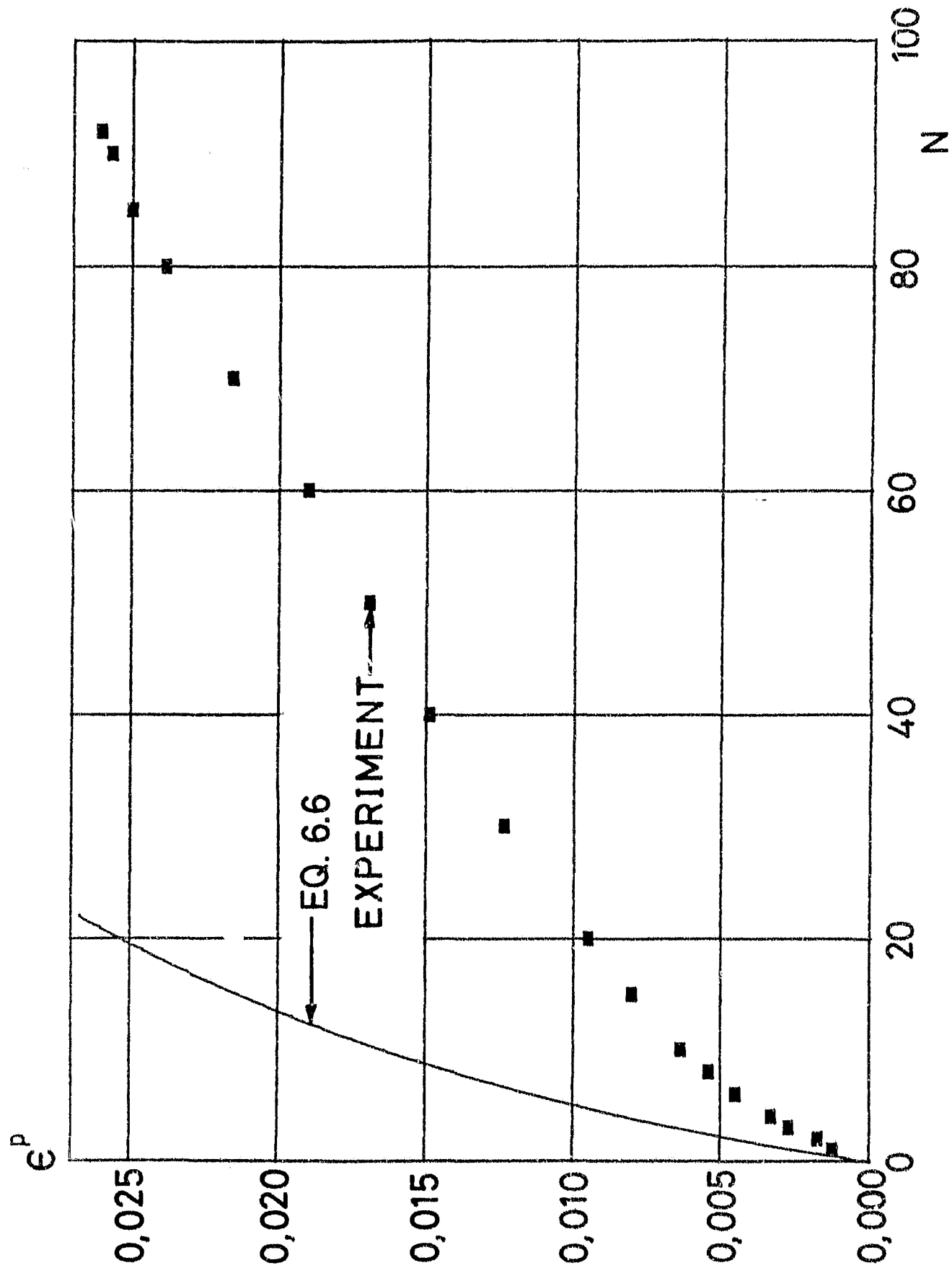


Fig. 6.5 Experimental and Theoretical Cyclic Creep Strain (ϵ^p) against Number of Cycles (N) for $\gamma_{max} = 0.008$ and $b_0 = 200$ MPa

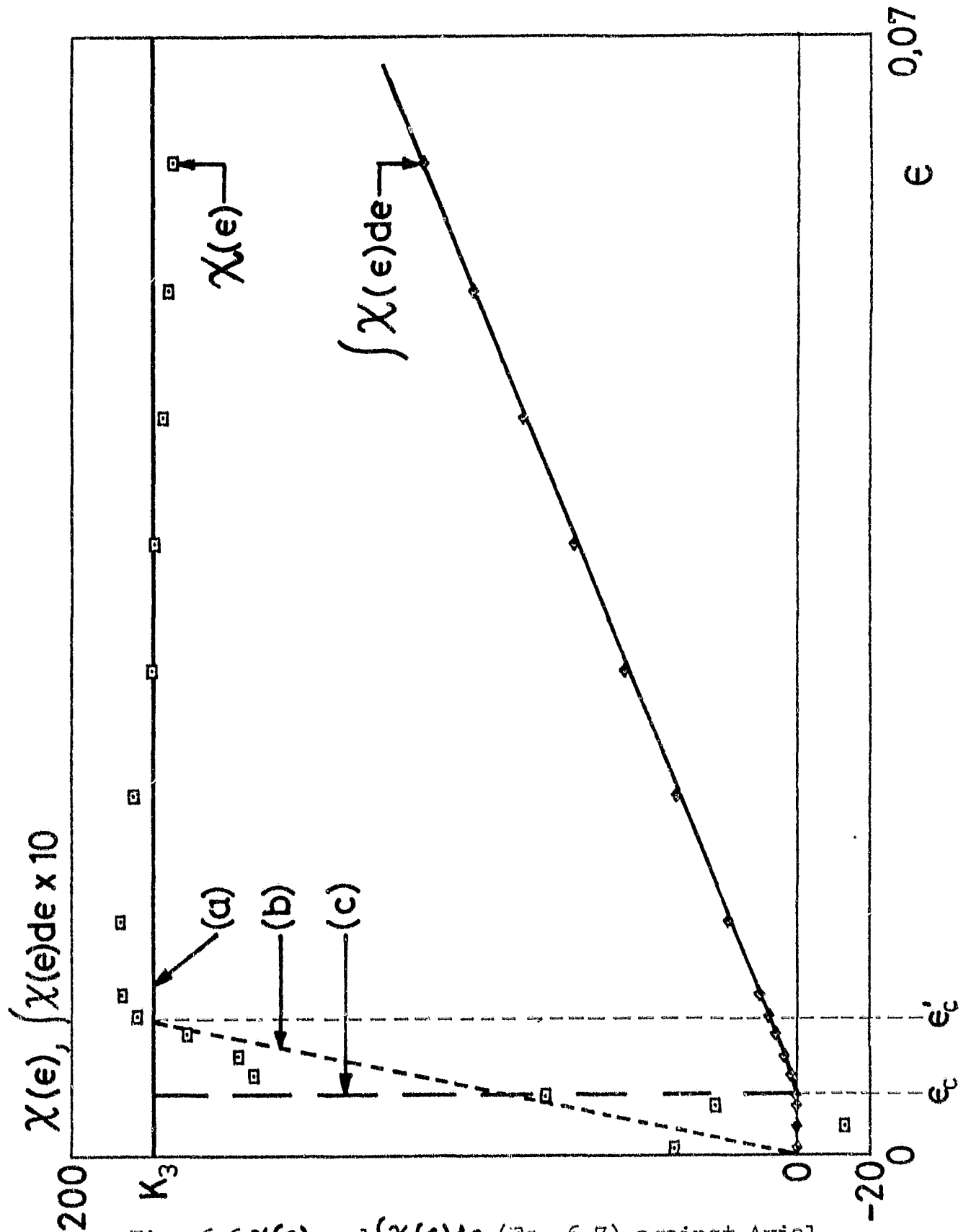


Fig. 6.6 $\chi(e)$ and $\int \chi(e)de$ (Eq. 6.7) against Axial Strain (ϵ) from Pure Tension Test

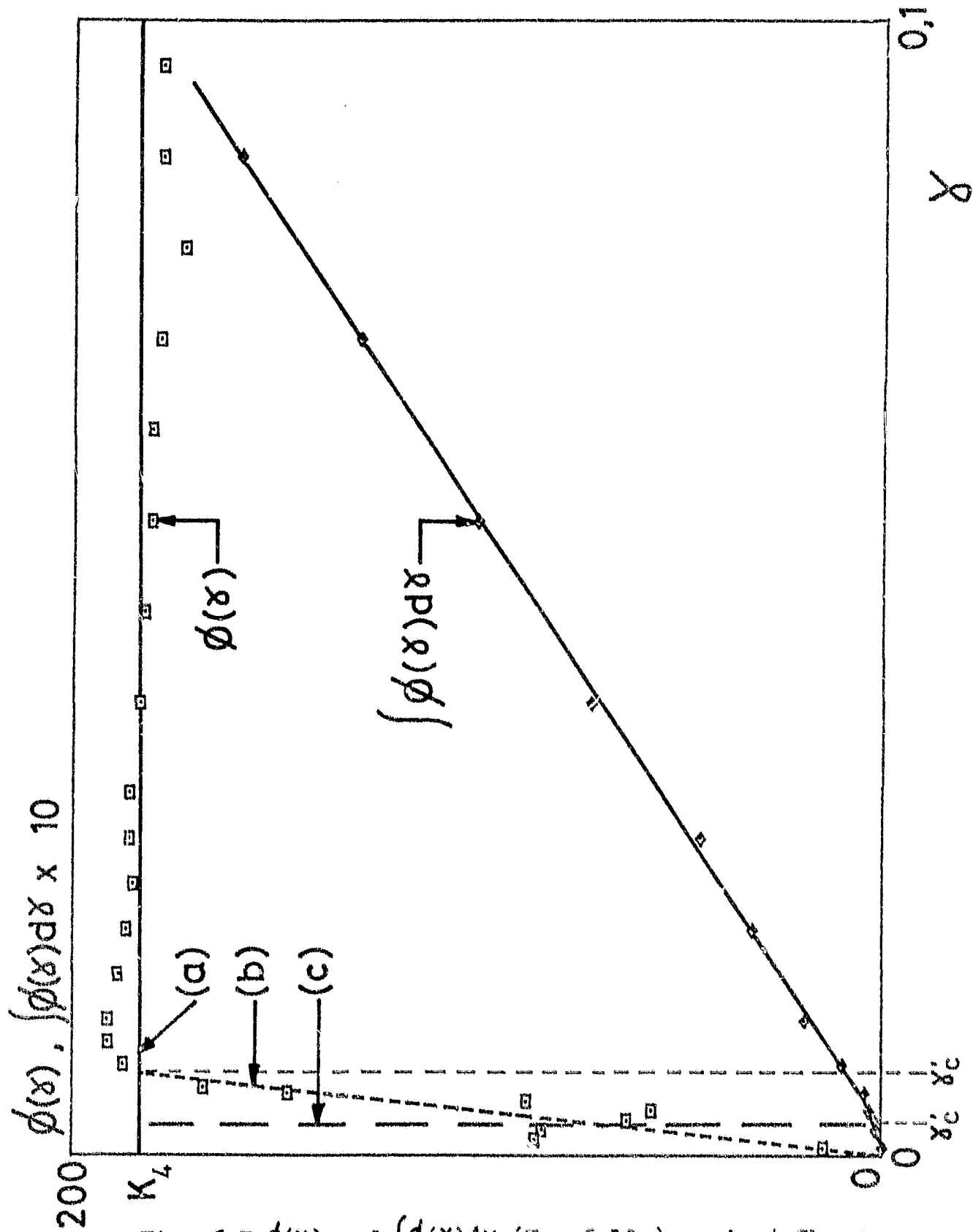


Fig. 6.7 $\phi(\gamma)$ and $\int \phi(\gamma) d\gamma$ (Eq. 6.13a) against Shear Strain (γ) from Pure Torsion Test

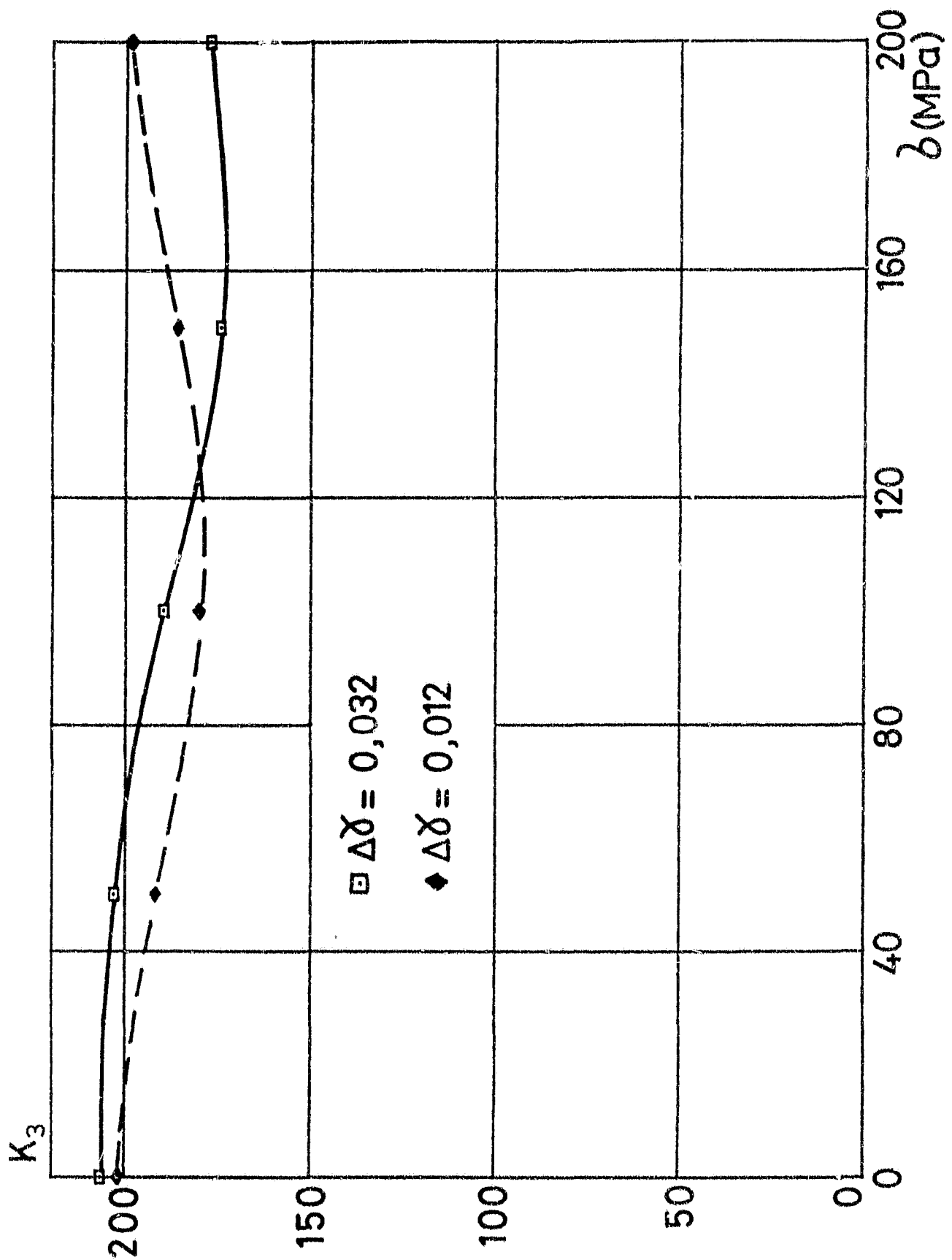


Fig. 6.8 K_3 Obtained from Tension Tests after Initial Torque Cycling against Tensile Stress During Cycling (δ) for Different Cyclic Shear Strain Ranges ($\Delta\gamma$)

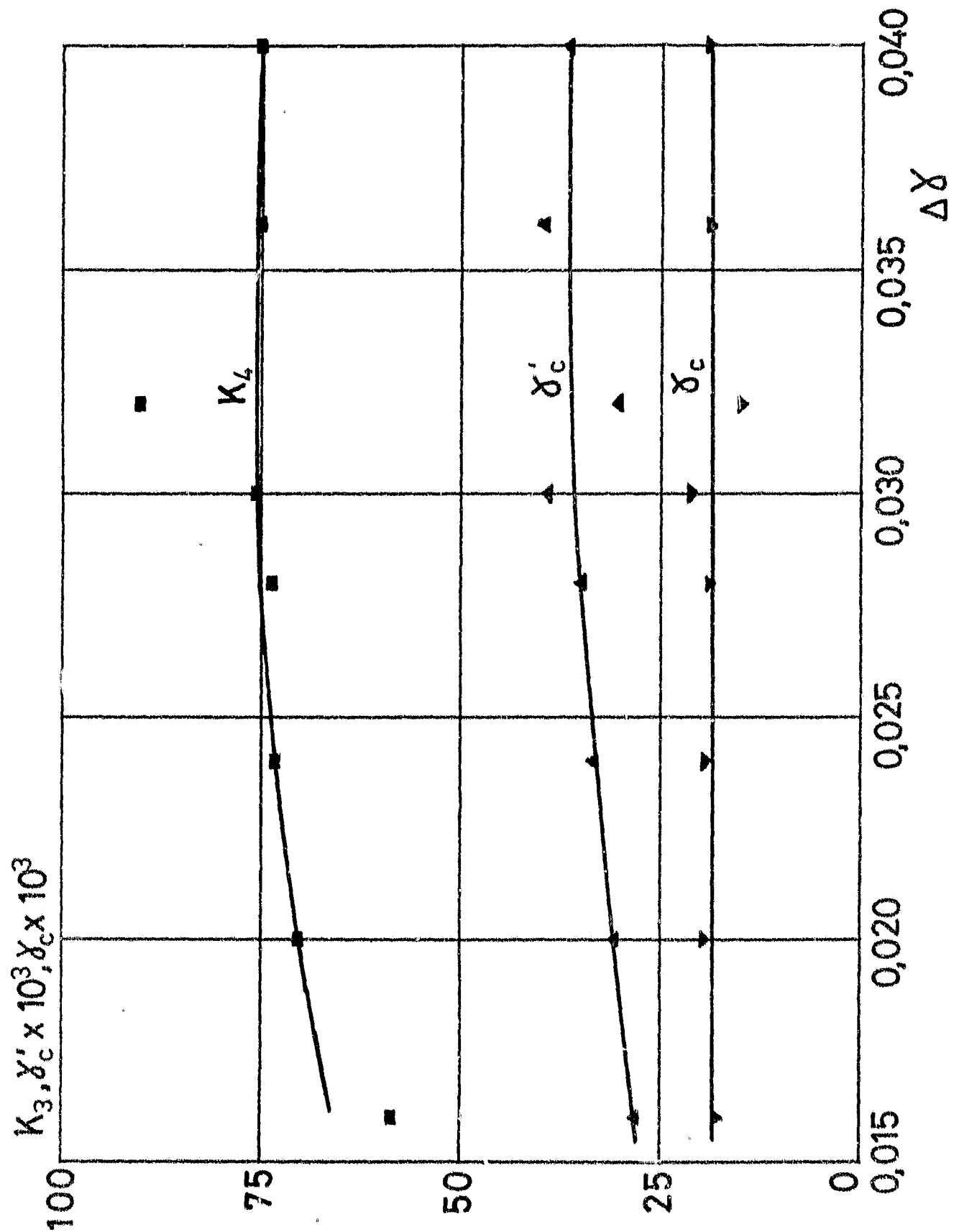


Fig. 6.9 k_4 , γ'_c and γ_c Obtained from Steady State Torsional Hysteresis Loops against Cyclic Shear Strain Range ($\Delta\gamma$)

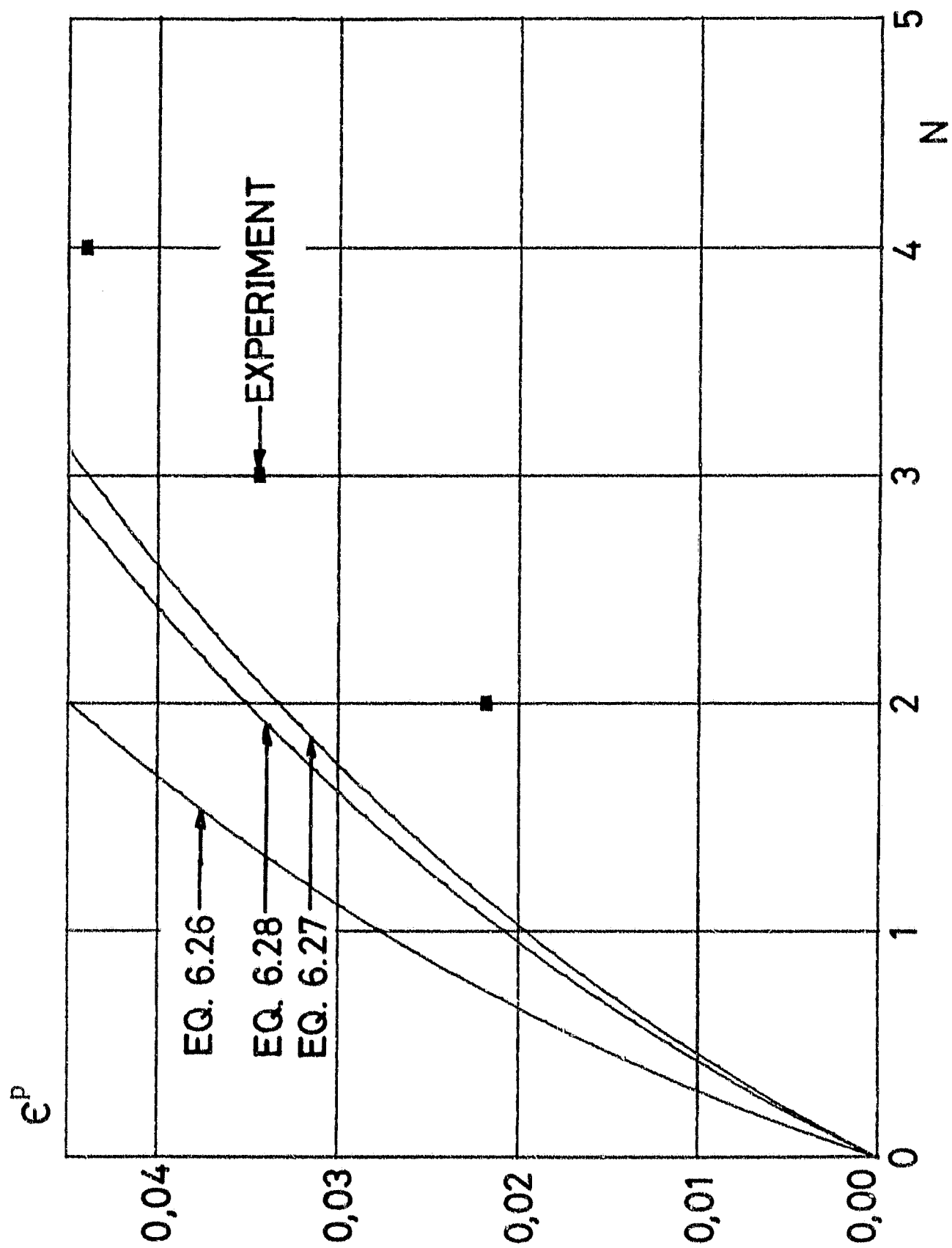


Fig. 6.10a Experimental and Theoretical Cyclic Creep
Strain (ϵ^p) against Number of Cycles (N)
for $\delta_{max} = 0.04$ and $\delta_0 = 200$ MPa

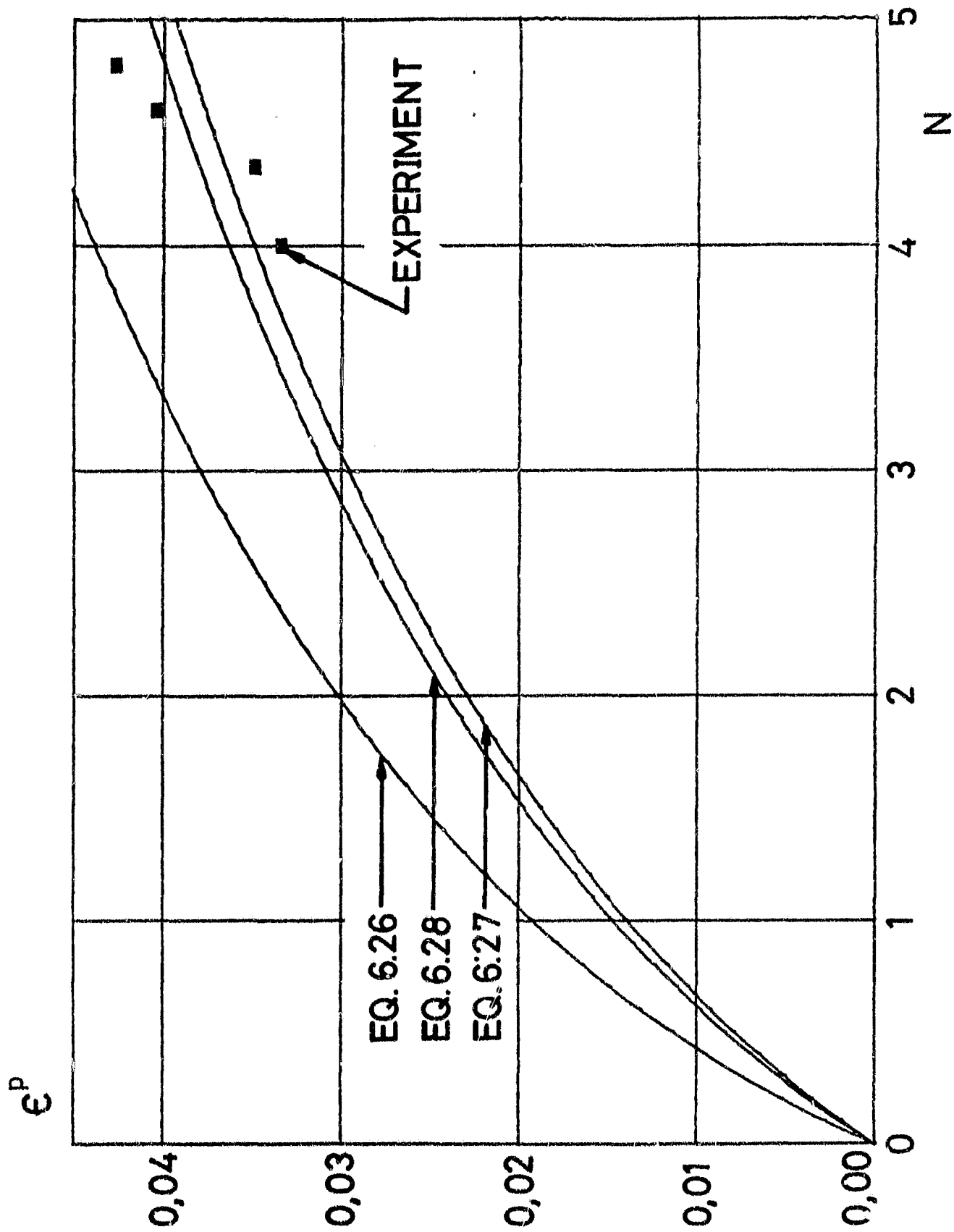


Fig. 6.10b Experimental and Theoretical Cyclic Creep Strain (ϵ^p) against Number of Cycles (N)
for $\gamma_{max} = 0.04$ and $\sigma_0 = 150$ MPa

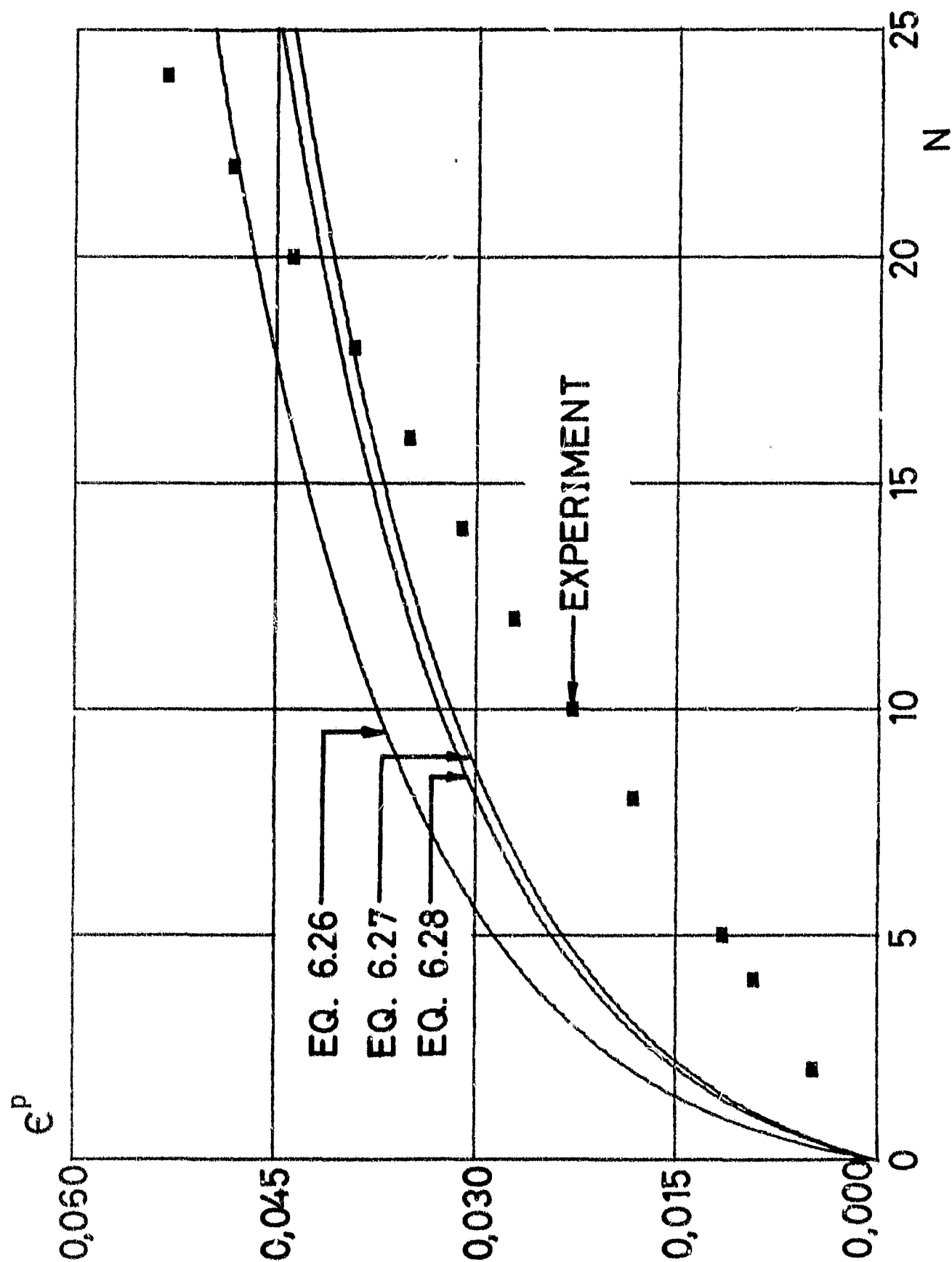


Fig. 6.10c Experimental and Theoretical Cyclic Creep Strain (ϵ^p) against Number of Cycles (N) for $\delta_{max} = 0.04$ and $\delta_0 = 1.00$ MPa

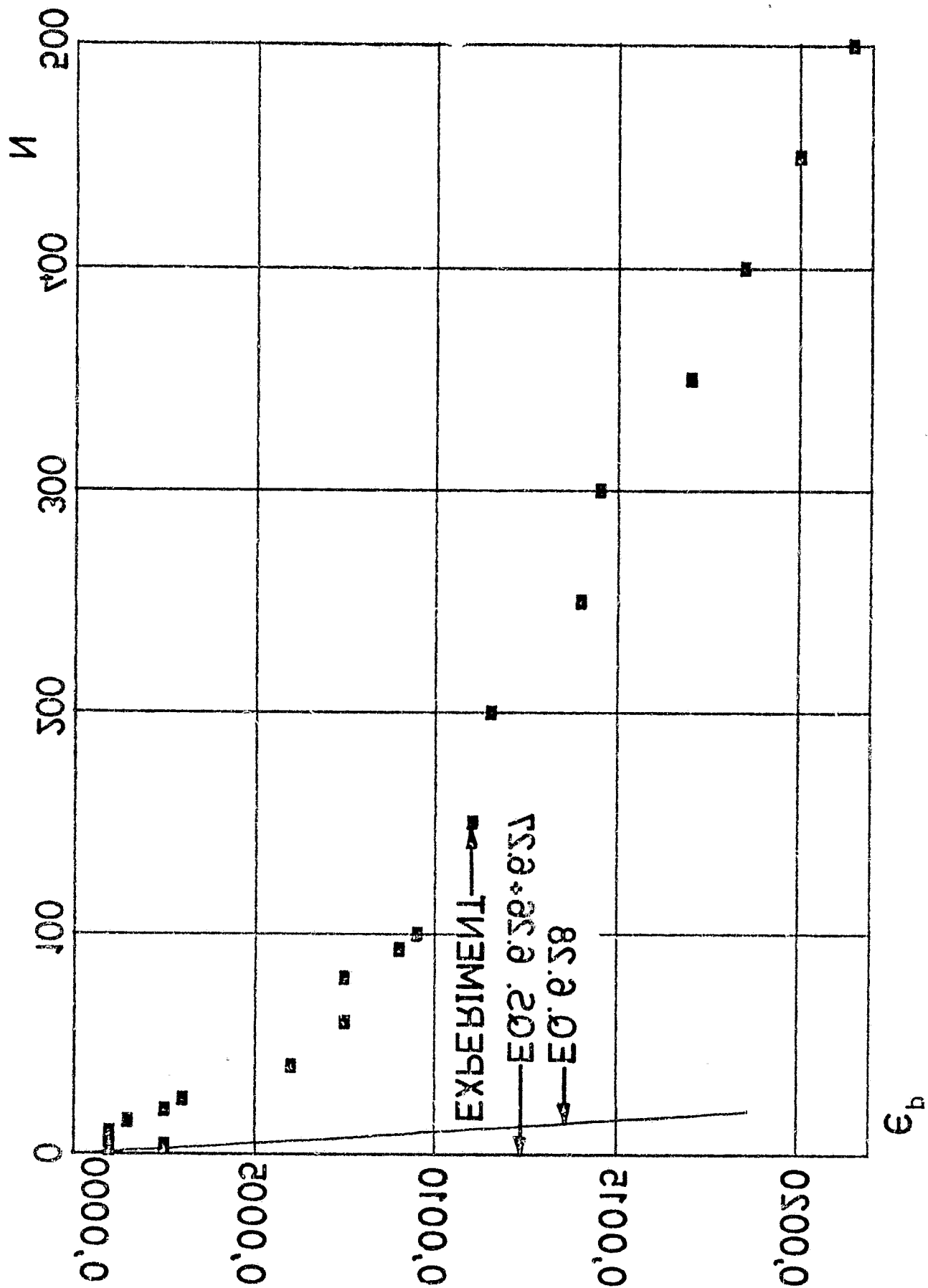


Fig. 6.100 Experimental and Theoretical Cyclic Creep
Strain (ϵ_p) against Number of Cycles (N)
 $\sigma_{max} = 0.01$ and $\sigma_0 = 100$ MPa

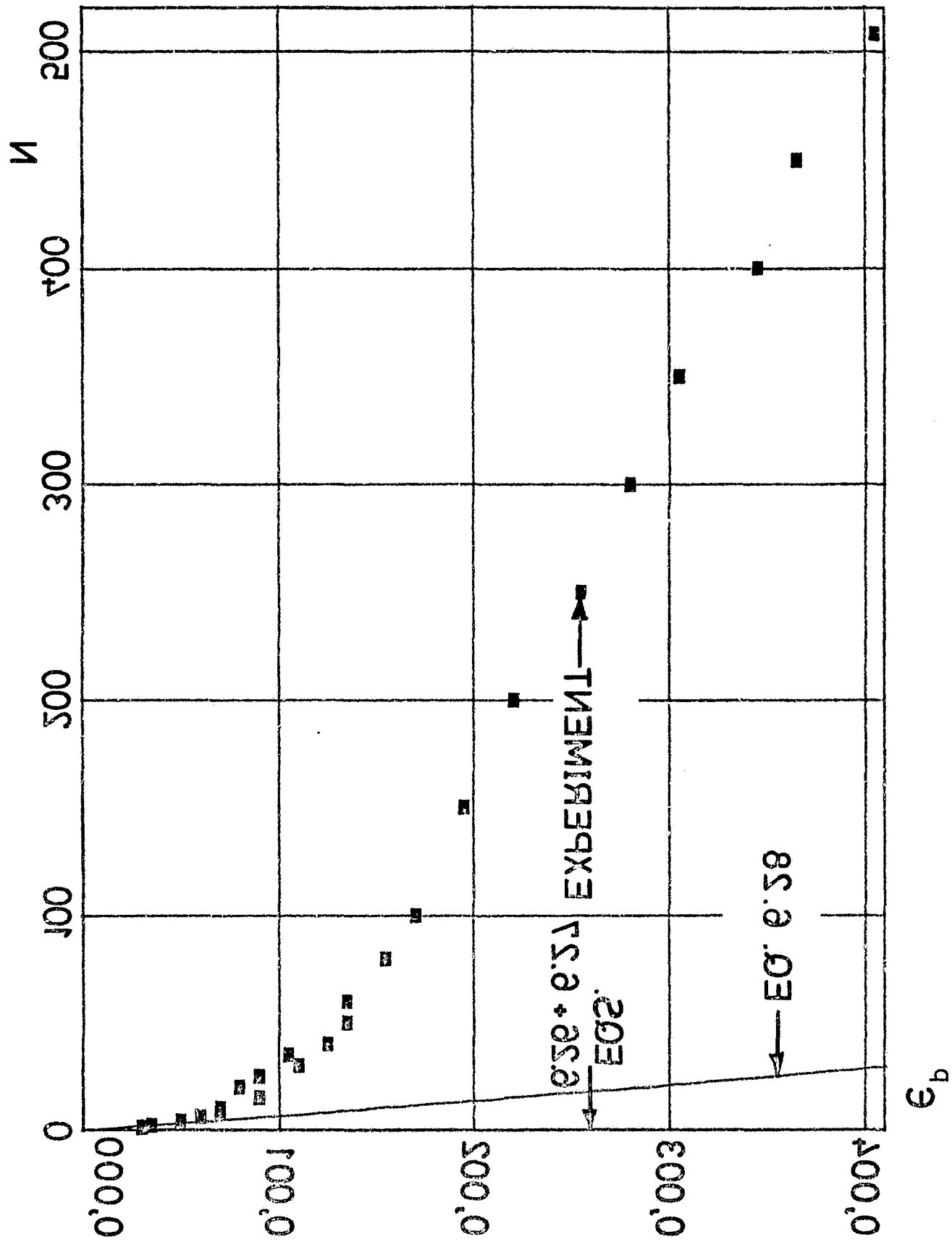
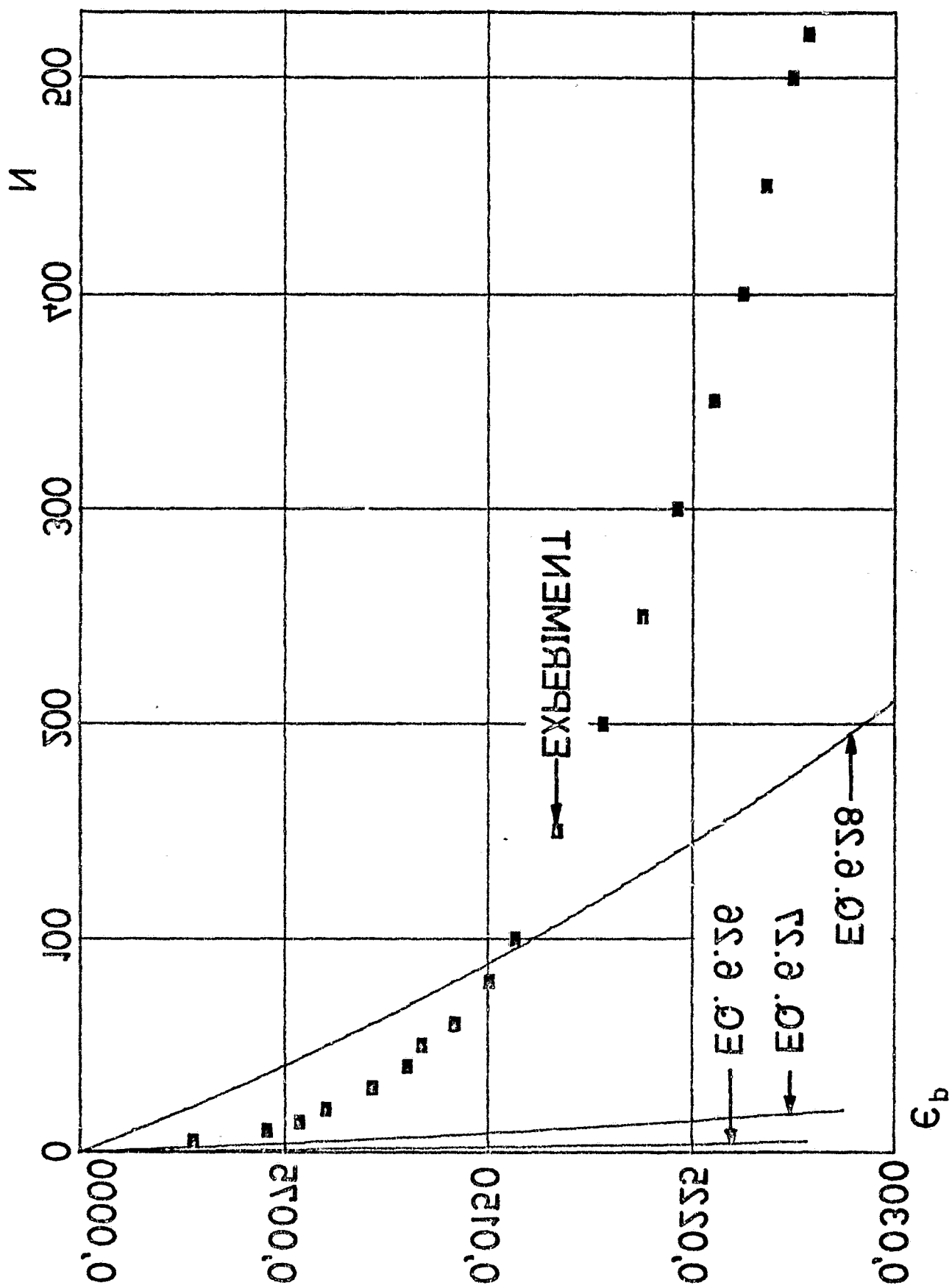


Fig. 6.10m Experimental and Theoretical Cyclic Creep
Strain (ϵ) against Number of Cycles (N)
For $\sigma_{max} = 0.01$ and $\sigma_0 = 150$ MPa



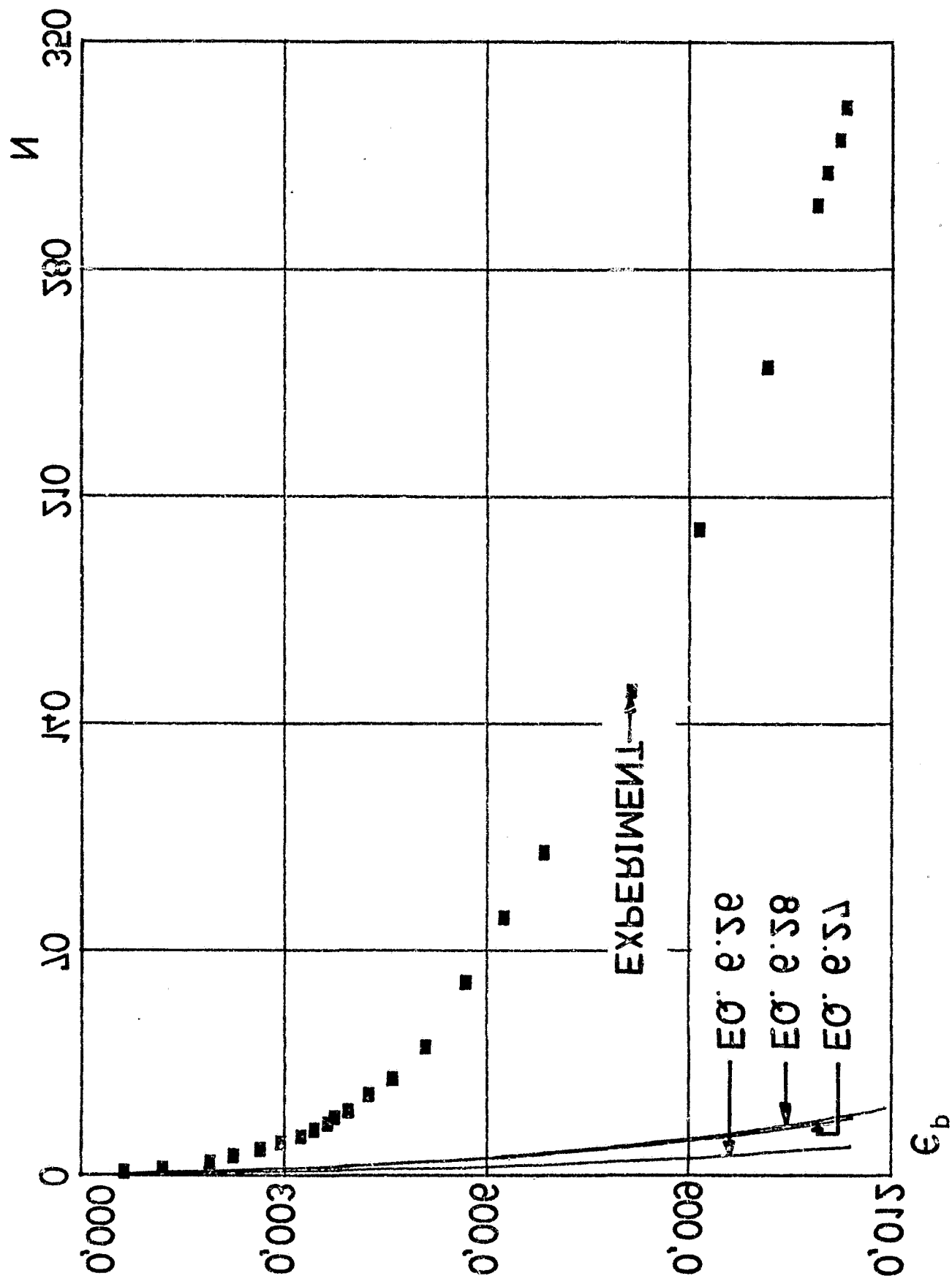


Fig. 6.10K Experimental and Theoretical Cyclic Creep
Strain (ϵ_p) against Number of Cycles (N)
for $\delta_{max} = 0.02$ and $\delta_0 = 100$ MPa

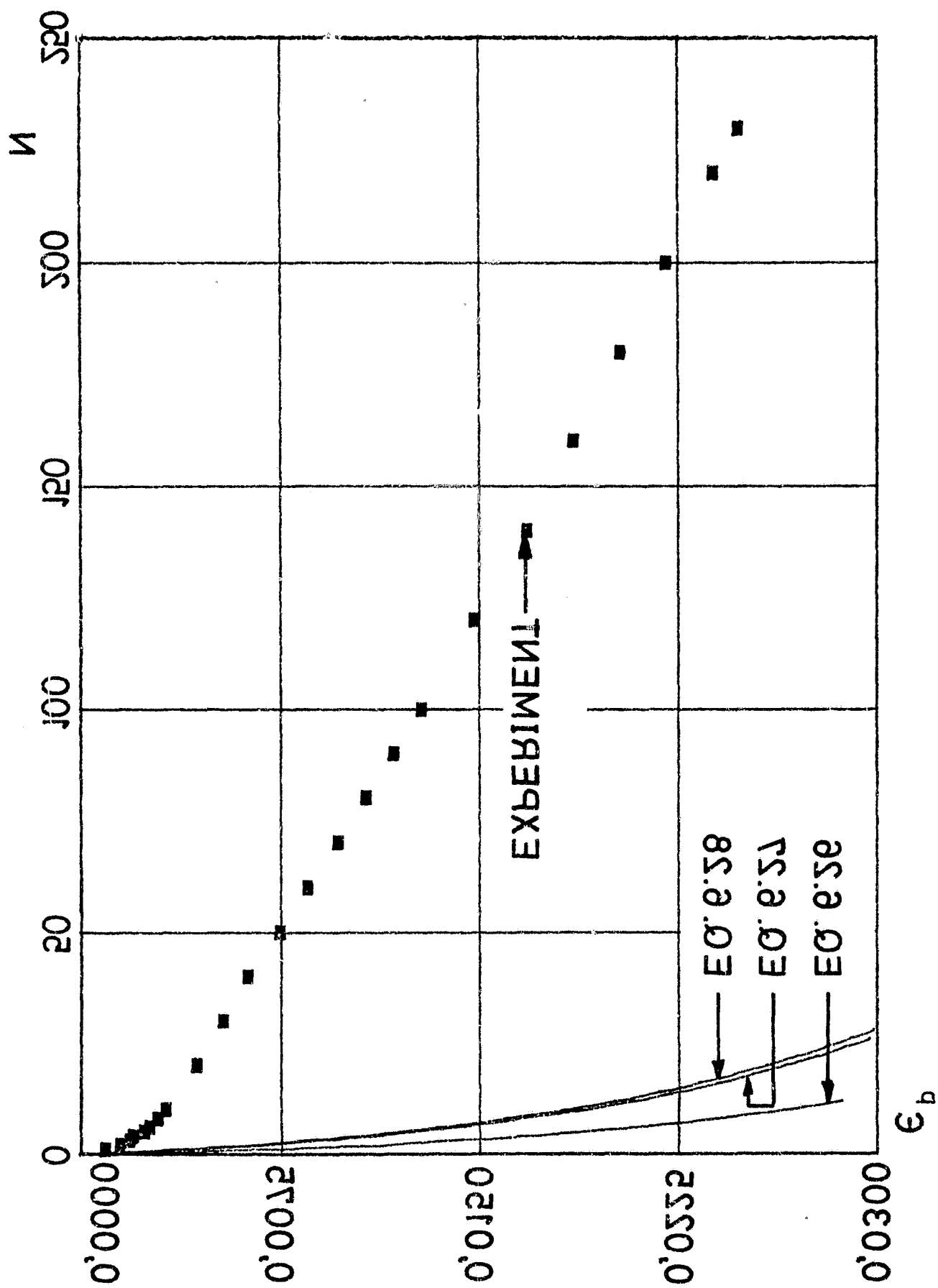
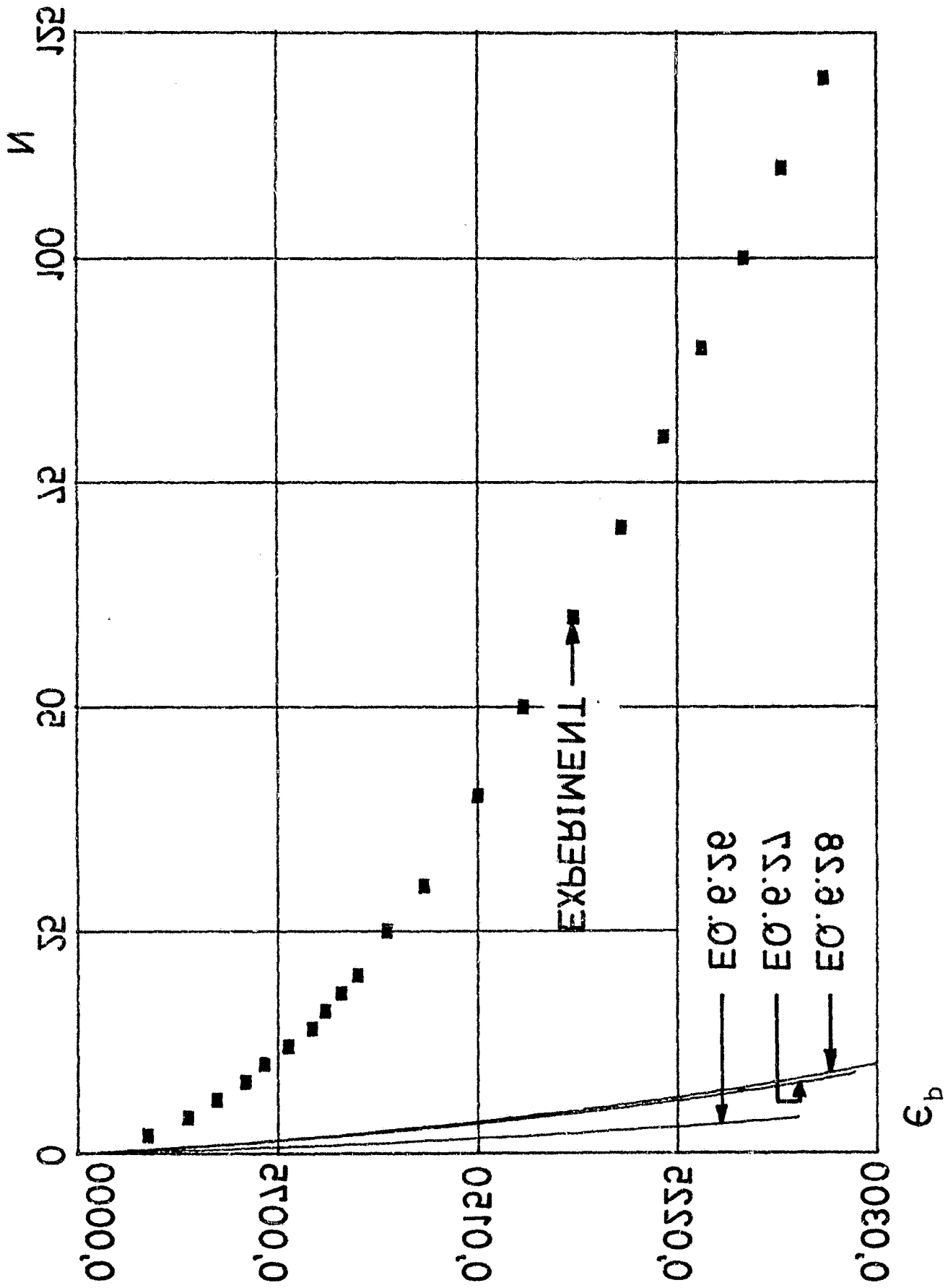
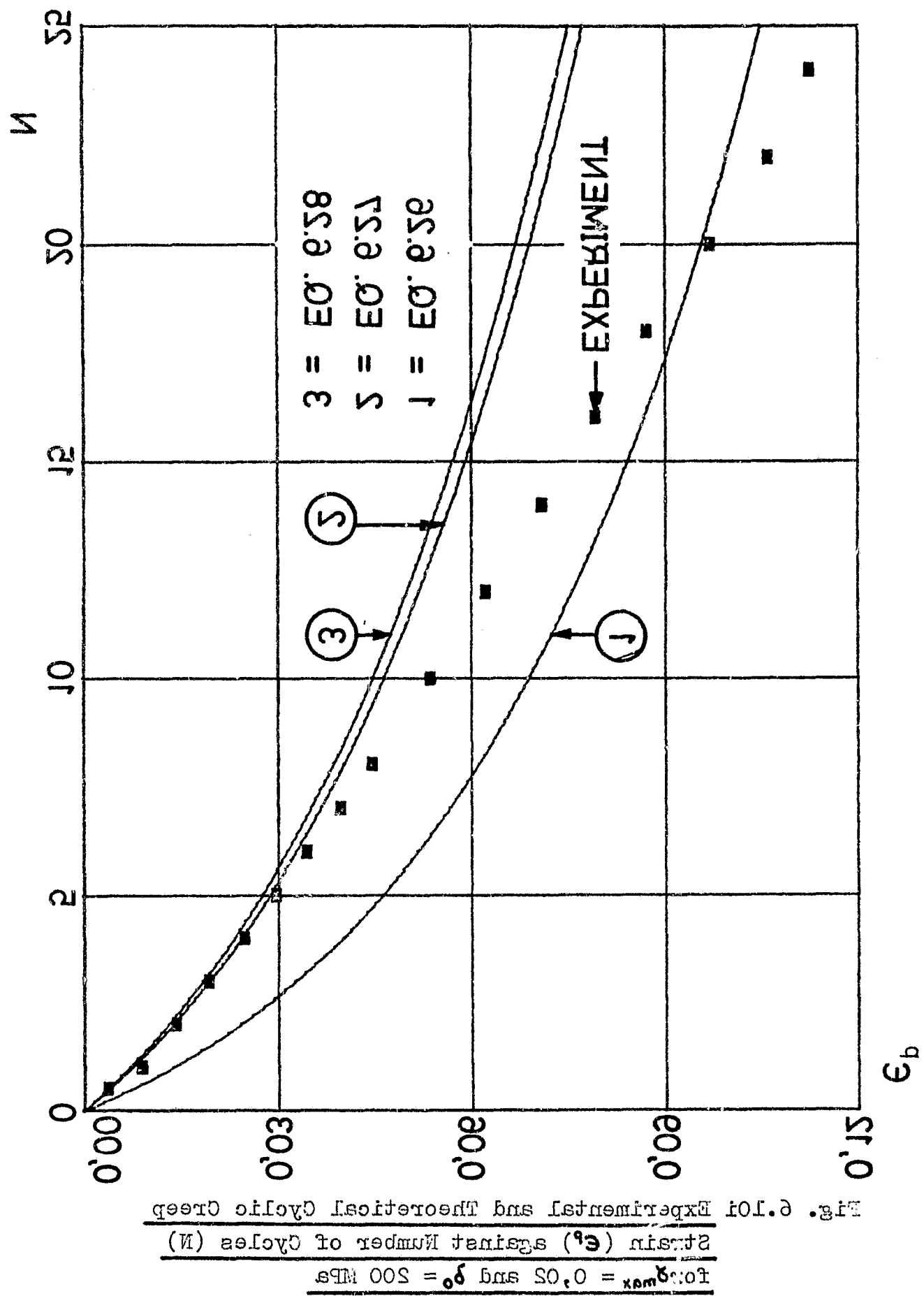
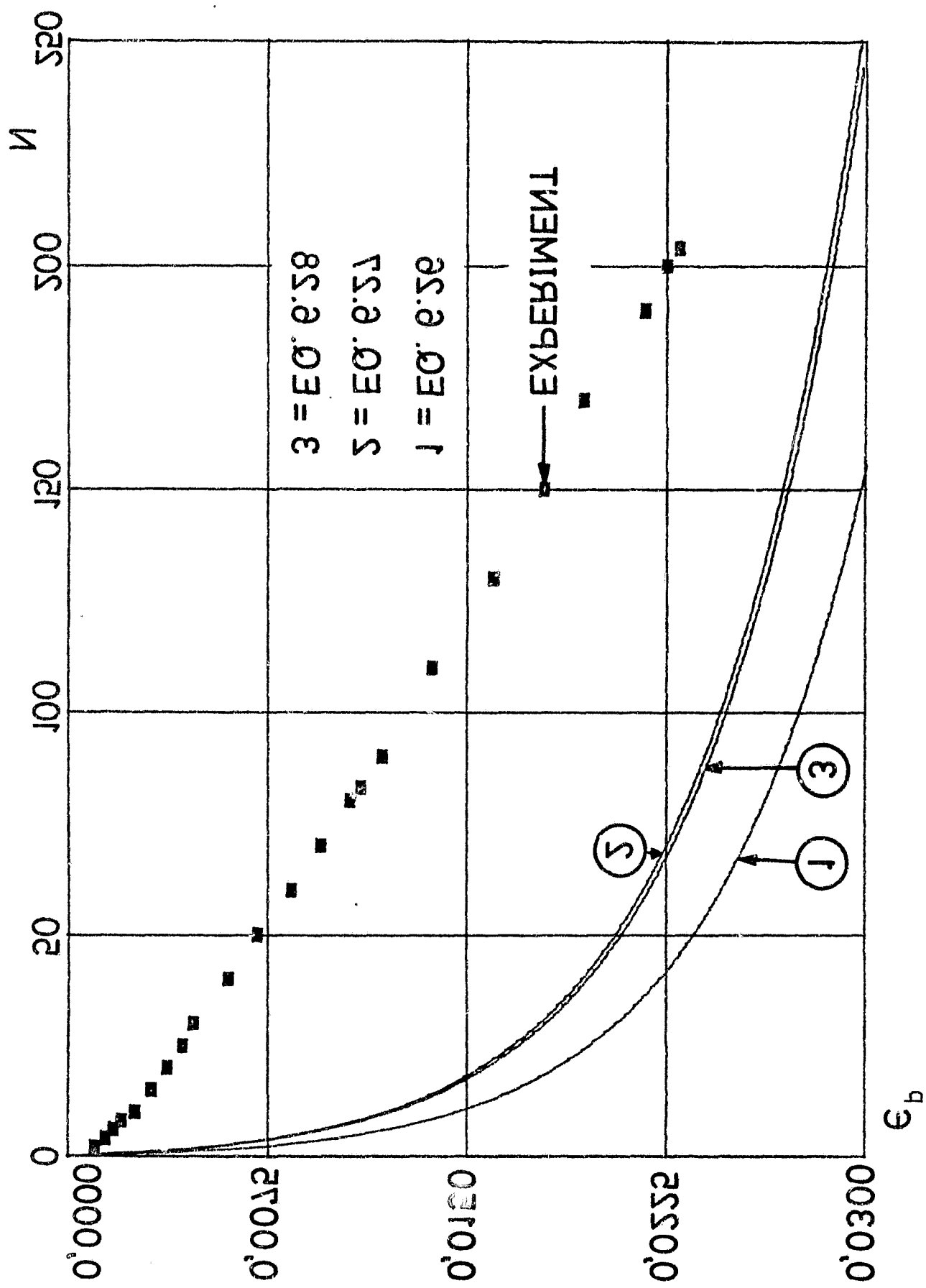


Fig. 6.10 Experimental and Theoretical Cyclic Creep
Strain (ϵ_p) against Number of Cycles (N)
for $\sigma_{max} = 0.02$ and $\rho = 150$ MPa







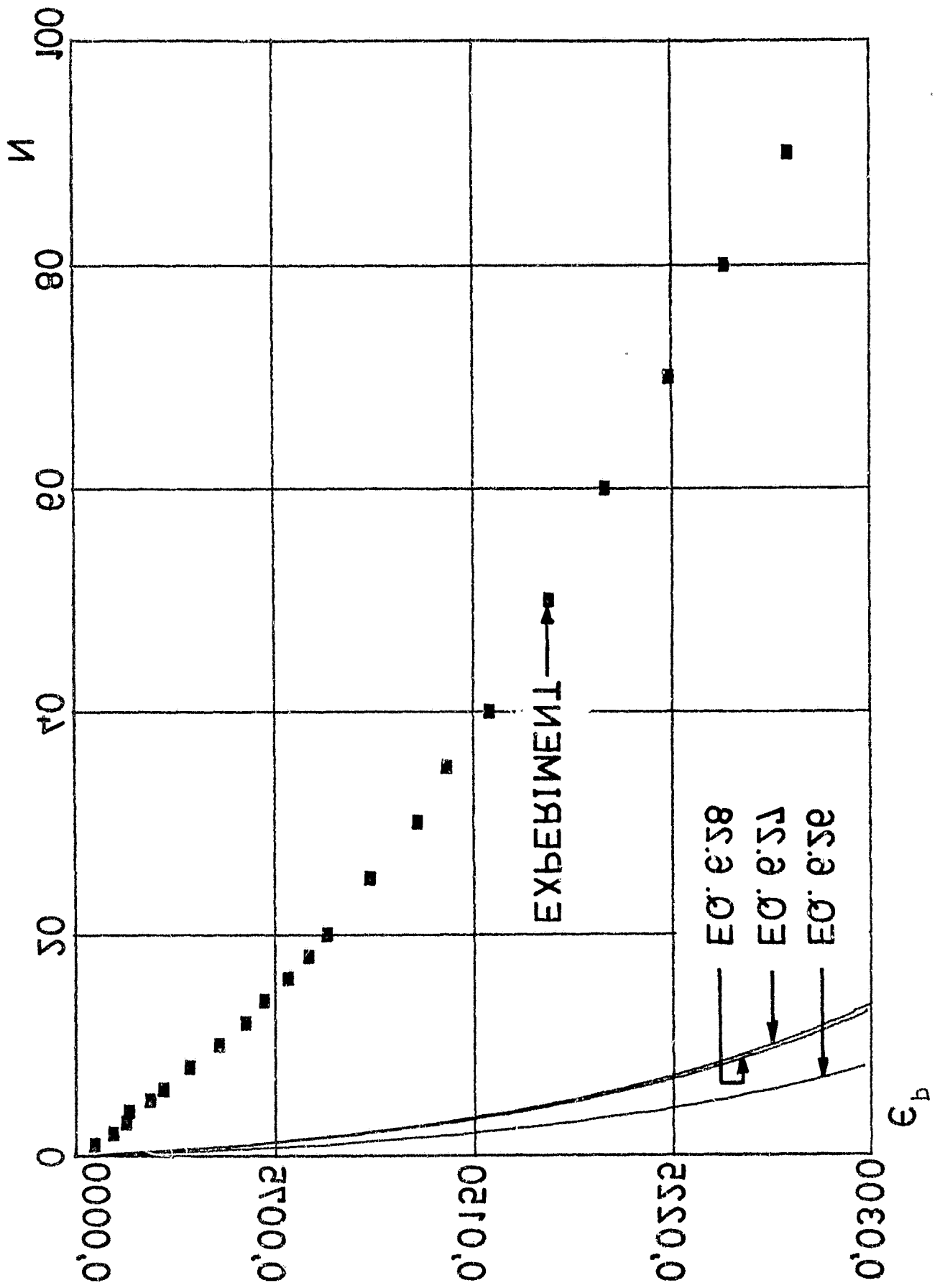


Fig. 6.10g Experimental and Theoretical Cyclic Creep

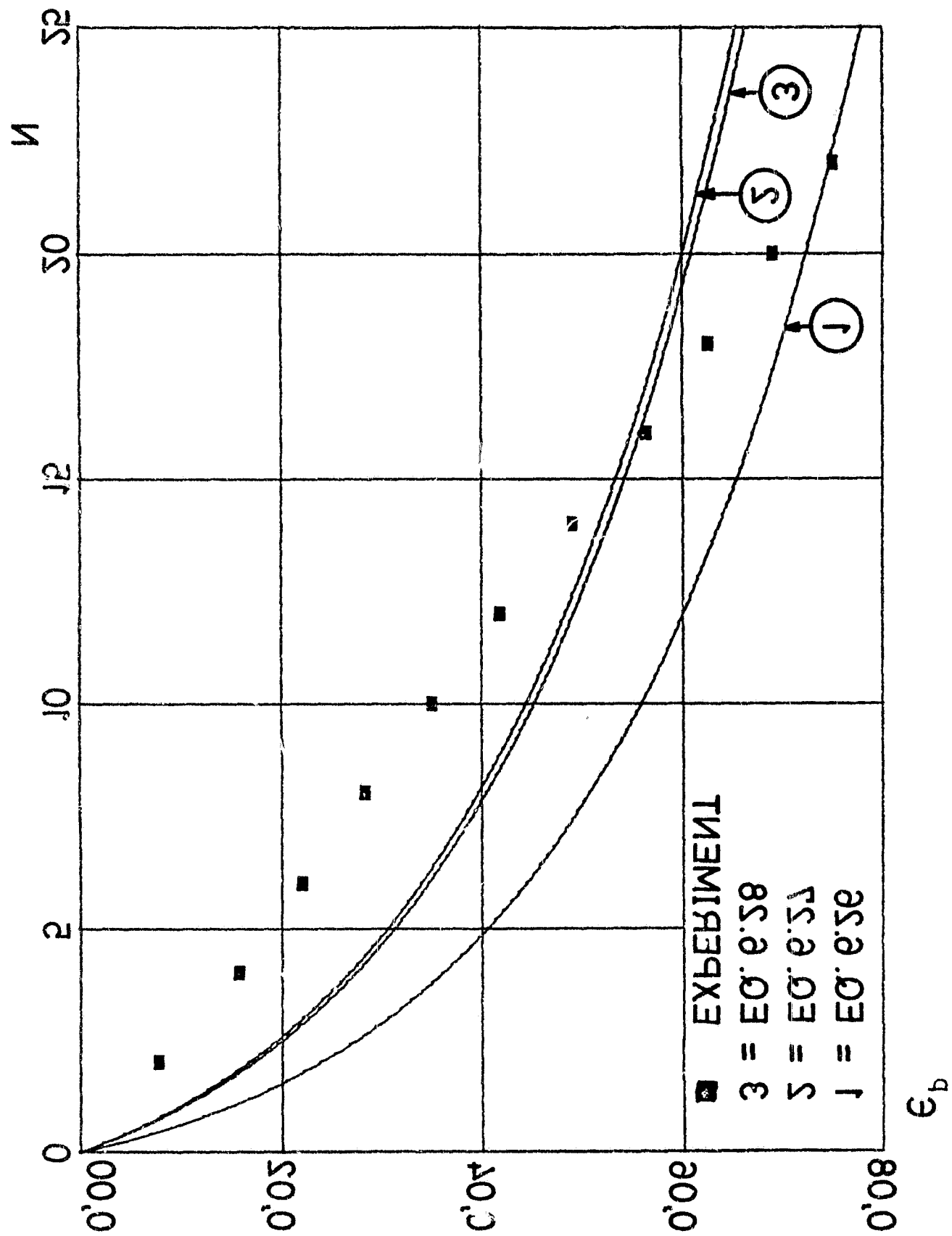
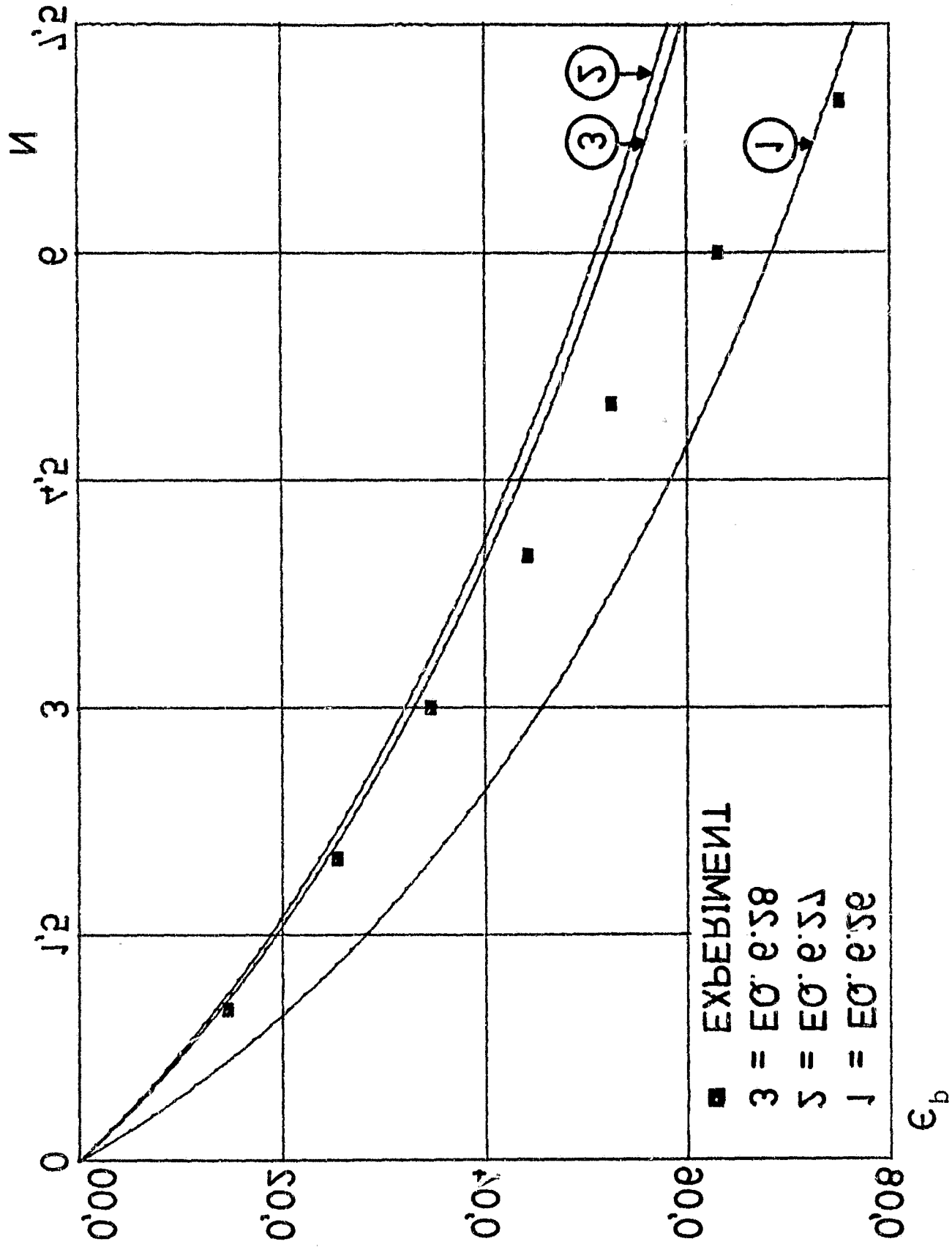


Fig. 6.101 Experimental and Theoretical Cyclic Creep
Strain (ϵ_p) against Number of Cycles (N)
for $\sigma_{max} = 0.03$ and $\sigma_0 = 150$ MPa

Fig. 6.10e Experimental and Theoretical Cyclic Creep
Strain (ϵ_p) against Number of Cycles (N)
for $\delta_{max} = 0.03$ and $p_0 = 500$ lbs



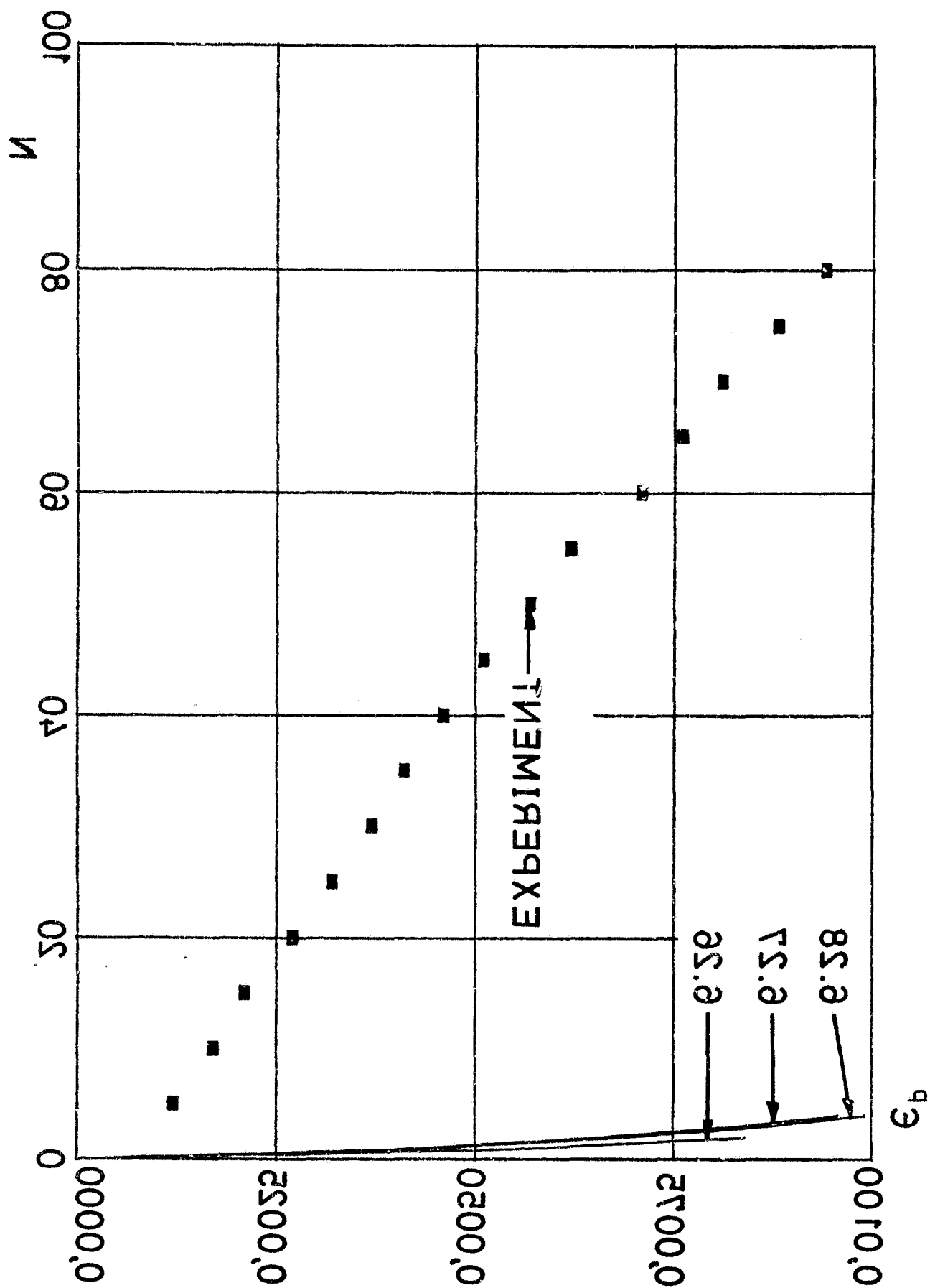


Fig. 6.104 Experimental and Theoretical Cyclic Creep
 Strain (ϵ_p) against Number of Cycles (N)
 for $\delta_{max} = 0.04$ and $\delta = 50$ MPa

APPENDIX B

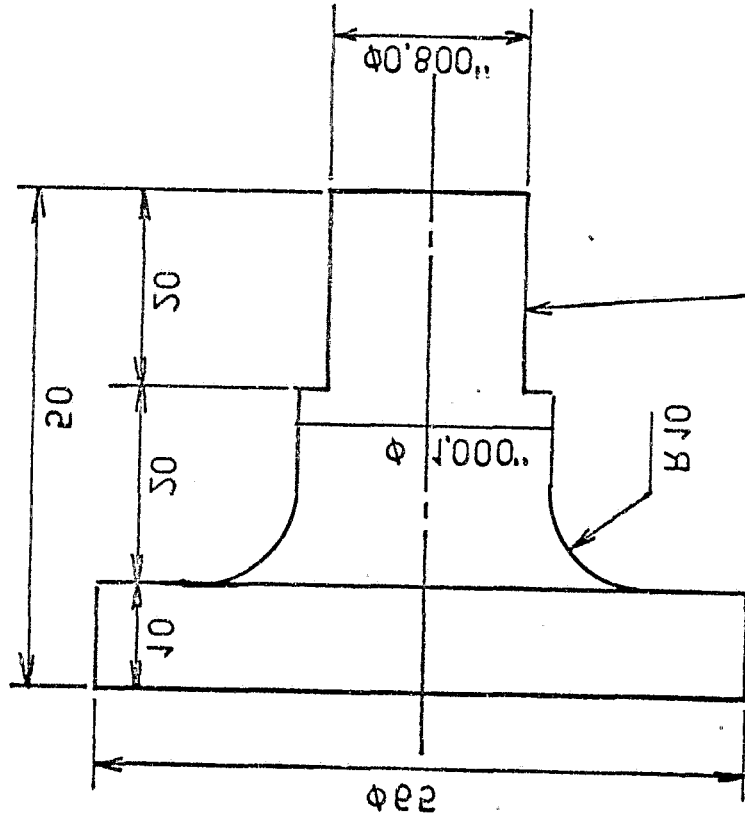
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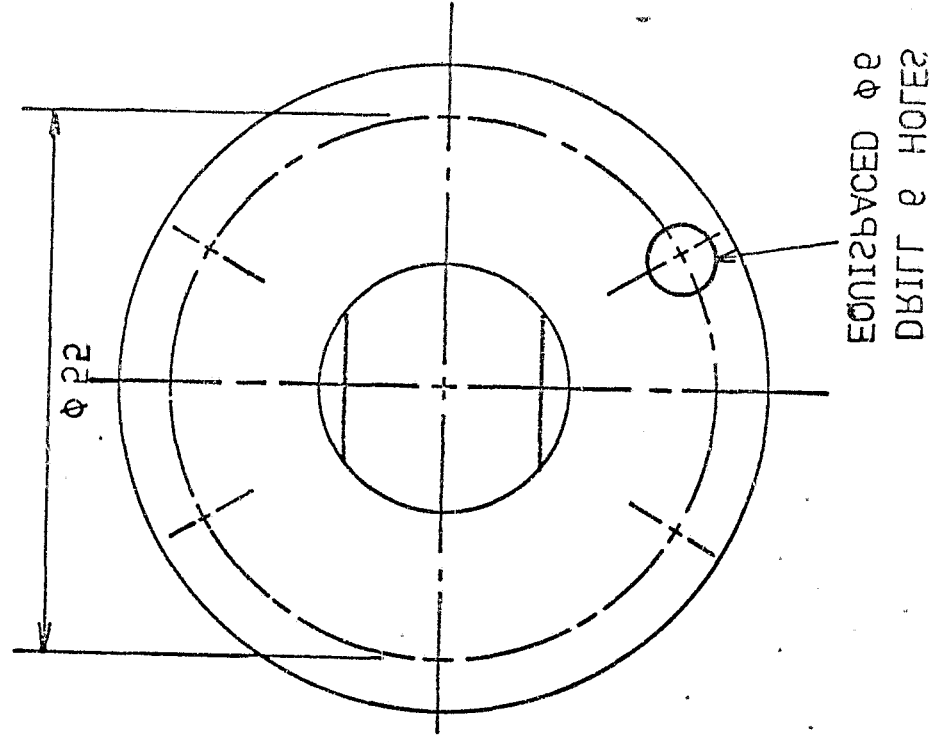
MATERIAL : MILD STEEL
 SCALE : 1:3

FIG. A.2 ADAPTOR FOR TORSION LOAD CELL CALIBRATION

SIDE VIEW



END VIEW



1:1:3JAS2
J3E7S QJIM:J7AM

ИСТИТАРБИЛАС ЦЕЛ ДАДЛ ИОИСРОТ РОФ I.OИ РОТРАДА ЗА ДИР

SIDE VIEW

END VIEW

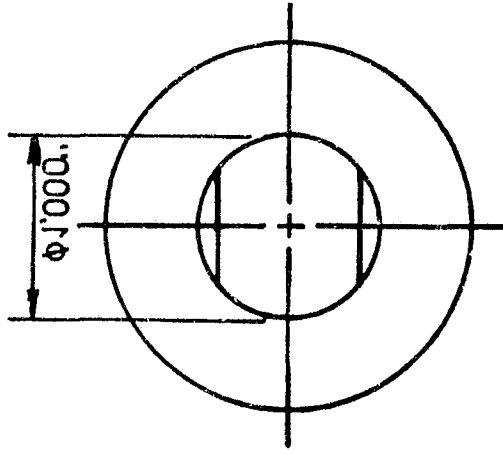
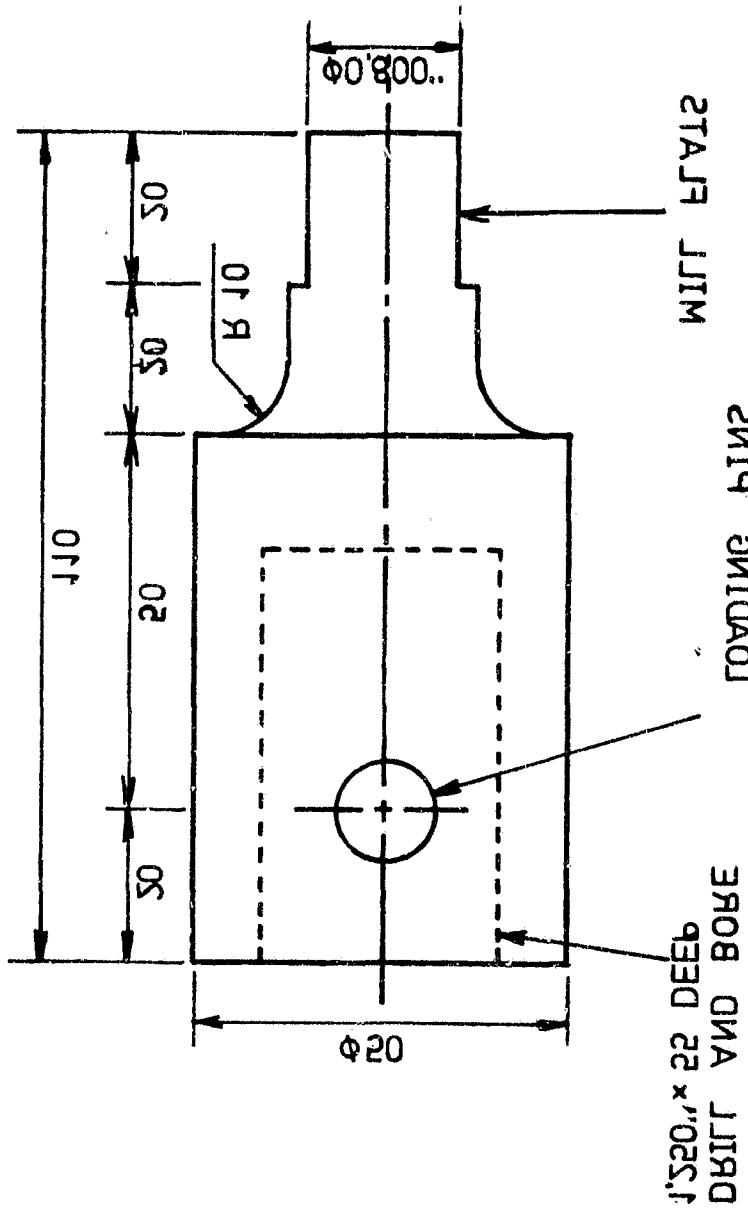
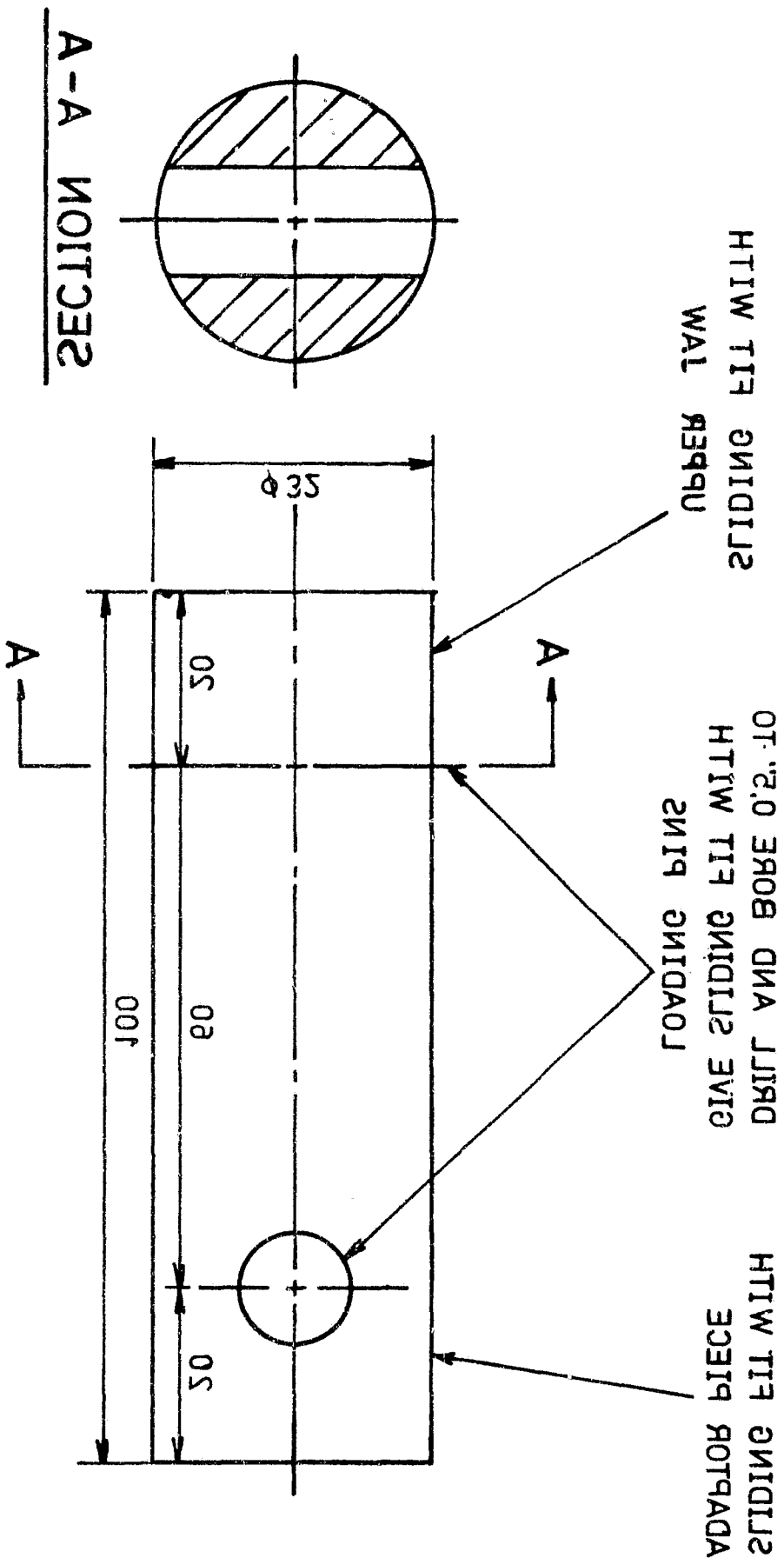
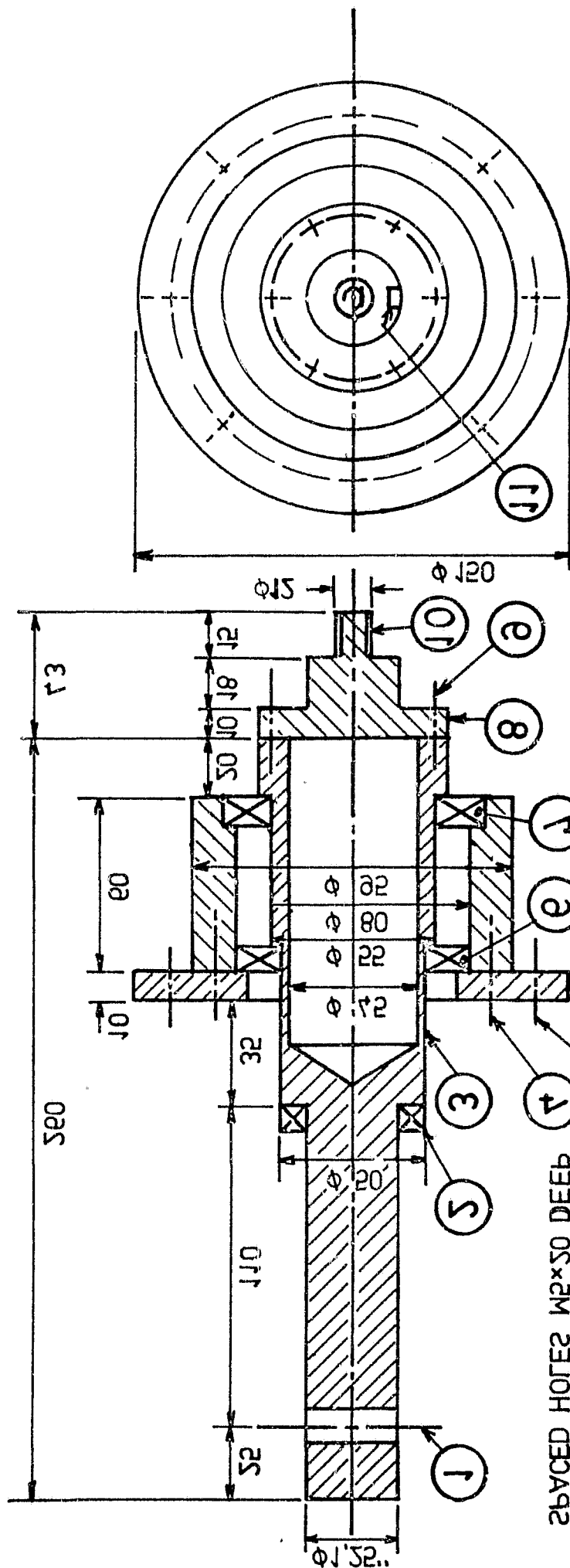


FIG. 2. A. OF
SHORT CONNECTOR BETWEEN
PIECE AND UPPER WALL
SCALE: 1:3
MATERIAL: MILD STEEL



SECTIONED SIDE VIEW

END VIEW



"BNT JLRQ : 1
 TW QIRAB TUBHT DAF : 5
 IARTS ROF NOTTIOF : 3
 ZSDUAD
 - IUO3 8 PAT DND JLRQ : 4
 PEPD QSOX2M ZLOH ZCEDAQ9

РАС ТЕКТОР2: 8

MIADATS 207 NOTTISOP : 3

2017 FW CIRCA 2017 : 2

1 : DRIFF VLE."

01 001 011A3B 11A 0A7 : d
11 001 011A3B 11A 0A7 : T

18 LONG
KEYWAY 4 : DEEP* 8 WIDE*
STIM OT DEEDED HT : 07
SPACED HOLES NO* 50 DEEP
-TUB* 3 PAT AND LIFT : 2

A.9 Constant Dual DC Power Supply Type 101100

Line regulation	: $\pm 0.01\% + 1mV$
Load regulation	: $0.03\% + 3mV$
Ripple voltage	: $0.005\% + 0.5mV$ peak to peak
Temperature coefficient	: $0.02\% / ^\circ C$

A.10 Constant Single DC Power Supply Type 10100/30

Line regulation	: $\pm 0.01\% + 1mV$
Load regulation	: $0.03\% + 3mV$
Ripple voltage	: $0.005\% + 0.5mV$ peak to peak
Temperature coefficient	: $0.02\% / ^\circ C$

APPENDIX A

SPECIFICATIONS OF EQUIPMENT

- A.1 Instron TT-CM Tensile Testing Machine
Gross head speed : 0.05 - 50 cm/min
Full scale load range : 0.1 - 50 KN
- A.2 Amaler 50 SBD4 Universal Testing Machine
Gross head speed : 0 - 170 mm/min
Full scale load range : 50 - 500 KN
- A.3 Avery 660999 Torsion Testing Machine
Full scale torque : 1500 (5), 750 (2)
(and Resolution) : 300 (1) and 150 (0.5) Nm
Twisting speeds : 3 1/2, 10, 30, 90 /min
- A.4 3 Phase Induction Motor
Synchronous speed : 750 rev/min
Power : 0.5 HP continuous
- A.5 Remold Grotha Double Reduction Gear Box
Reduction ratio : 3600 : 1
- A.6 Riken Denchi X-Y Recorder Type W-3D
Accuracy : within $\pm 0.3\%$
Sensitivity : 0.1mV/cm - 10V/cm
- A.7 Riken Denchi X-t Recorder Type SP-H3B
Accuracy : within $\pm 0.3\%$
Sensitivity : 1mV - 100V full scale
- A.8 San-Ei DC Amplifier Type GLL-P
Input and output : Fully floating
Gain : 1 - 10 000
Accuracy : Better than 0.2%

CHAPTER 8

SUGGESTIONS FOR FURTHER WORK

1. The modified form of the yield surface theory (ed. 5.13) requires a knowledge of the critical plastic shear strain and the pure torsional hysteresis loop. The latter is easily obtainable but some simple method should be developed to find it.
2. The yield surface theory should also be further investigated to see whether cyclic creep below the critical plastic shear strain range can be predicted.
3. The creep equations arising from the endochronic theory of viscoplasticity in its modified form (eds. 6.25 - 6.28) are of the logarithmic type. This suggests that the theory should be further investigated as to its applicability to logarithmic cyclic creep.
4. It is also felt that the endochronic theory offers a useful avenue of approach to plasticity in general and that further work should be carried out.

4. The endochronic theory of viscoplasticity (32) was found to be of limited applicability since the material constants had to be determined from several actual creep curves before creep behavior could be predicted for intermediate loading conditions. A modification of the theory was attempted in order to obtain the material constants from tensile and cyclic torsional stress-strain curves. This attempt was not successful and one of the constants had to be obtained from an actual creep curve. Even then a single value of this constant could correctly predict creep strains only for a limited range of loading conditions.

CHAPTER 7

CONCLUSIONS

1. A testing machine has been modified to enable the testing of tubular specimens in alternating torsion and steady tension. Although the equipment behaved satisfactorily, improvements could be made in future to the extensometer, the specimen machining procedure and the electronic comparator.

2. Tests on an aluminum alloy showed that it possessed a large initial anisotropy which was only partially removed by torsional cycling. When cycled in torsion with a superimposed tensile load, most specimens exhibited a continuing lengthening, but some shortened initially. The latter effect was thought to be due to the material anisotropy. The torsional hysteresis loop was found to change little with both the tensile load and progressive axial deformation. Variations in the area of the loop for different specimens were attributed to machining inaccuracies.

3. The expanding yield surface theory (18) was used to predict the steady state cyclic creep rates in specimens subjected to cyclic plastic shear strains ranging from 0.001 to 0.03 and superimposed tensile stresses from 0 MPa to 200 MPa. The creep rates were found to be overestimated considerably, especially for small plastic shear strains and tensile stresses. A modification was made to the theory, taking into account the anisotropy of the aluminum as well as the tensile stress dependent critical plastic shear strain range, below which very little creep occurred. This modification gave fair agreement with experiment but could not predict any creep strains below the critical plastic shear strain range. In its modified form, the theory could no longer predict creep strains from pure torsional hysteresis loops only.

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